CASUALTY ACTUARIAL SOCIETY FORUM

Winter 1996
Including the
Ratemaking Call Papers

CASUALTY ACTUARIAL SOCIETY
ORGANIZED 1914

Printed for the Society by
Colortone Press
Landover, Maryland

1996 CAS *Forum*
Winter 1996 Edition
Including the Ratemaking Call Papers

The Winter 1996 edition of the CAS *Forum* is a cooperative effort of the CAS Continuing Education Committee on the CAS *Forum* and the Research and Development Committee on Ratemaking.

The CAS Committee on Ratemaking is pleased to present for discussion eight papers prepared in response to its 1996 Ratemaking Call Paper Program. Some topics addressed are territorial ratemaking including protection class determination, the use of catastrophe models in ratemaking, the risk-return relationship, loss development in excess ratemaking, and claims-made tail pricing. These papers will be discussed by the authors at the 1996 CAS Seminar on Ratemaking, March 14-15 in Las Vegas, Nevada.

Please note the Brubaker paper, "Geographic Rating of Individual Risk Transfer Costs Without Territorial Boundaries," has a patent application pending with regard to the procedure described. The rights for use of the procedure may be subject to restriction.

In addition to the ratemaking call papers, this issue of the *Forum* also contains one discussion of a *Proceedings* paper, the *CAS Dynamic Financial Analysis (DFA) Handbook (Release 1.0)*, and the Society of Actuaries' *Report to the Membership* by the Board Task Force on Education. The *Report on the Findings of the Limited Attendance Workshop on Financial Risk Theory*, held on October 1, 1995, in Boston, Massachusetts, is not published in this edition of the *Forum*, contrary to the announcement published in the workshop's preliminary brochure. Copies of this report are available from the CAS Office, (703) 276-3100.

The members of the Committee on the CAS *Forum* encourage authors to submit papers and articles for future editions. Submissions may be sent to any of the Committee on the *Forum* members.

Both committees hope you find value in this edition of the *Forum*.

Table of Contents

CAS Dynamic Financial Analysis Handbook
CAS Valuation and Financial Analysis Committee
Subcommittee on the DFA Handbook

Casualty Actuarial Society

Dynamic Financial Analysis

Property/Casualty Insurance Companies

Handbook

Release 1.0 (Final)
September 1995

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Introduction

Introduction

Dynamic Financial Analysis (DFA) is the process by which an actuary analyzes the financial condition of an insurance enterprise. Financial condition refers to the ability of the company's capital and surplus to adequately support the company's future operations through an unknown future environment.

The purpose of this *Dynamic Financial Analysis Handbook* is to provide suggestions and guidance to actuaries in performing DFA studies. As such, the *Handbook* is not a Standard of Practice and is not binding upon any actuary. Nor is the *Handbook* intended to define an acceptable standard of care which-if not followed-would indicate the actuary has acted negligently. Rather, the *Handbook* provides a list of considerations for actuaries to refer to when performing DFA. The *Handbook* is not exhaustive, but is intended to be revised and edited regularly as knowledge of DFA evolves. The release date of the *Handbook* appears on the cover page of the document, as well as at the top of each page.

The *Handbook* does not prescribe reporting requirements regarding DFA. The actuary performing DFA should decide on the format of any required report and comply with regulatory or professional requirements regarding such reports. The report allows the reader to clearly determine the key material threats to the company's solvency. The report assists in quantifying the company's surplus over the projection period and allows the reader to better understand the impact of alternative business scenarios on surplus. The report is not an absolute statement regarding the financial condition of a company, but rather a tool to identify material risks to solvency faced by the company.

In addition to assisting management and regulators with understanding solvency risks, the DFA process generally permits management to gain a better understanding of both the risks and opportunities inherent in the company under various business conditions and stress factors. This understanding allows management to better control the company's risk profile and to allocate surplus more effectively and efficiently. It also allows management to test the impact of various proposed business strategies under a variety of possible future conditions.

The *Handbook* does not prescribe a specific projection period for the entire process of analyzing the company's financial condition. The length of the projection period is determined by either the actuary performing the testing, or the regulators. However, if a long projection period is used, the actuary must use greater care in choosing assumptions and generally test a broader array of assumptions.

The process of DFA involves testing a number of adverse and favorable scenarios regarding an insurance company's operations. DFA assesses the reaction of the company's surplus to the various selected scenarios. This assessment of the test results is contained in the DFA report. The *Handbook* does not present the scenarios to be used in the testing process. However, normally, the company's business plan will serve as the base scenario for this process. The choice of additional scenarios is determined by actuarial judgment, and/or regulatory guidance. Scenarios may vary greatly depending on an individual company's circumstances.

The actuary is expected to select a set of plausible scenarios sufficient to test all material threats to the company's solvency. It is expected that an actuary performing solvency testing will focus most heavily on those scenarios for which a material adverse impact on surplus is plausible. The reporting actuary, therefore, should define plausible scenarios and a materiality standard. By definition, large balance sheet items like claims reserves, unearned premium reserves, invested assets, and other material receivables and payables, as well as future profitability, should be tested under various scenarios. Influences such as pricing strategy, reserving methodology, reinsurance arrangements, growth targets, and investment policy should be analyzed. Items the actuary reasonably believes to be relatively immaterial, such as a slightly higher than average broker commission level, need not be addressed. It may be interesting to management, but if the situation is not likely to impair solvency, or materially impact profitability, then it need not be rigorously tested.

In performing DFA, as in any actuarial analysis, the actuary should assess the credibility of the data used to perform the analysis. If the data is not credible, the actuary should augment it with external data sources. Indeed, many of the potential threats to the solvency of a company are external, and the actuary should gather information from many external data sources, such as information on the economy, reinsurers, and emerging environmental risks. Each actuary performing DFA should assess the reliability and quality of each company's management information systems, and policy information systems. This can become complicated if a company owns many subsidiaries, particularly in foreign or non-U.S. locations. To properly analyze the financial condition of a company with subsidiaries, each subsidiary should be analyzed separately.

The actuary preparing the financial condition report may choose to rely on the work of another professional. Such professionals include auditors, both external and internal, investment professionals, insurance company senior management, and other actuaries who have expertise in areas that may be useful to the actuary preparing the report. Any actuary who relies on another professional should establish a basis for doing so. In addition, the actuary should formally communicate the significance of the process to those professionals whose advice is to be included in the report, so the professional is aware of and understands the significance of their contribution.

To properly assess the financial condition of a company, the actuary should have access to all relevant documents, systems, and employees. This *Handbook*, does not grant authority for that access. The actuary should look to the regulatory body of the jurisdiction requiring the DFA for access to those areas, or to the company's senior management if the analysis is being performed for internal purposes.

When an actuary identifies one or more plausible scenarios as a material threat to solvency, the actuary should suggest possible corrective actions or control strategies. Further action steps that may be required, such as possible notification of regulators, external auditors, or audit committees of boards of directors, are beyond the scope of this *Handbook.*

This *Handbook* is divided into six sections that provide guidance in particular facets of DFA. These sections are pricing/business planning, reserve considerations, mass tort exposure, reinsurance considerations, invested assets, and other assets and liabilities.

These major sections focus on the most common exposure risks to the *typical* property/casualty insurance company. The major categories of *risk* identified are:

> *Inappropriate pricing*—generally underpricing and often coupled with excessive growth.
>
> *Inappropriate business plan*—generally (excessive) growth in areas with significant underpricing, or areas for which there is little data or limited company expertise.
>
> *Inappropriate reserving*—under-reserving due to lack of data, inadequate techniques, and/or management pressure, often coupled with underpricing.
>
> *Inappropriate reinsurance program*—a company retains too much risk relative to surplus, or over-relies on one or a few reinsurers who subsequently experience financial difficulty.
>
> *Inappropriate investment portfolio*—the company invests too much of its portfolio in asset classes that are overly volatile, poorly understood, overly concentrated with a few issuers who subsequently experience financial difficulty, or the portfolio is severely mismatched relative to the cash flow demands of the liabilities during a time when the portfolio is weak.

Each section contains a commentary focusing the topic, and an outline that can serve as a checklist for the actuary conducting DFA.

Other risks, beyond those enumerated in this *Handbook,* may at times overwhelm the enumerated risks. Examples of such risks include management fraud or incompetence, successful unanticipated shareholder lawsuits, significant off-balance sheet guarantees, or unusually adverse circumstances that go beyond what the actuary believes constitute reasonably plausible adverse scenarios. Therefore, it is incumbent upon the actuary to clearly express that these tests in no way constitute an implicit or explicit guarantee of future solvency.

Section I

Pricing/Business Planning

Pricing/Business Planning
Preface

Adequate pricing and sound business planning are paramount to the sound financial condition of property/casualty insurance companies. Two of the most serious risks associated with the pricing and business planning process are:

- inadequate rates (or overly aggressive pricing), and
- excessive growth in areas where rates are inadequate, or where the company has limited expertise.

Inadequate rates can impair financial results of the company for several years if, for example, regulatory constraints prevent approval of more adequate rates, or the rate inadequacy is not identified for several years, which might be the case for new products. Inadequate rates can also result if management is persistently optimistic in its projections of ultimate losses, selection of trend factors, or ability to take effective remedial actions. Exit barriers in certain lines or geographic areas can exacerbate these problems. In short, an adequate rate structure and a sound realistic business plan are the cornerstones of the company's future financial health.

Pricing

The ratemaking and pricing process involves numerous components each of which may play a key role in overall profitability. A company may initiate the pricing process using adequate manual rates but may end with inadequate rates via the injudicious use of schedule credits, preferred rate programs, inappropriate use of dividend plans or retrospective rating plans, etc. Therefore, the actuary should be aware of rate modifiers as well as the technical details of initial manual rate adequacy.

The actuary should be knowledgeable about significant expense items such as commission schedules and changes thereto, significant changes in staffing levels, and significant reinsurance purchase decisions. For example, the cost of catastrophe reinsurance may overwhelm virtually all other expense items for certain lines and markets.

The actuary needs to consider a host of both external and internal issues relevant to pricing decisions. Examples of external issues include anticipated inflation rates, interest rates, general economic strength/growth in the lines being priced, market cycles, nature of and growth in involuntary market mechanisms, and various regulatory issues. Examples of internal considerations include changes in underwriting programs, subline or classification mix changes, changes in claim department settlement practices and use of attorneys, marketing initiatives, etc.

The actuary should be knowledgeable about likely investment returns and needed profit loads. Consideration should be given to achieving a return on equity sufficient to provide adequate capital growth to support the company's business plan objectives.

Business Planning

The business plan should be consistent with the results of the pricing review as well as overall economic and market conditions. The plan should be realistic in that it is within the financial and managerial capacity of the company.

During the business planning process, the actuary should keep in mind the length of the planning horizon. As the horizon increases, additional uncertainty is added to the process. In addition to reviewing the company's internal activities, it is important to make assessments of the perceived market rate adequacy, activities of competitors, and regulatory environments (including exit barriers).

The business plan should show a sufficient level of detail and identify any significant items that impact cash flow. The written and earned premium components, planned growth, rate/price levels and exposure growth assumptions should be consistent with the pricing cycle, regulatory environment, and anticipated changes in these environments. Projections of loss ratios should also be consistent with the pricing cycle, while allocated loss adjustment expense ratios should be consistent with trends in legal environments and claim department practices.

The actuary should also include ceded and assumed reinsurance in the business planning process, and consider the type of coverage, attachment points, limits, risk tolerance, cost and financial strength of the reinsurer.

Changes in the mix of business can impact on expense, profits and geographic concentration. Several scenarios should be considered to determine the sensitivity of the plan to various changes in operating, economic, and regulatory environments. Items to consider include the impact of catastrophes, changes in internal operations impacting payout patterns, significant changes in interest rates or investment strategies, and rate approvals that are less than originally anticipated.

The plan should also consider possible changes in anticipated reserve needs, emanating from prior accident years and possible mass tort activity during the plan horizon.

The plan should reflect the differences between the various accounting methods to which insurance companies are subject (statutory, GAAP, tax), in addition to likely changes in these accounting methods. Lastly, the actuary should consider the impact of the above scenarios on surplus, regulatory monitors such as RBC, rating agency perceptions, and the ability to raise capital.

The nature of any changes in business direction and the company's ability to monitor the shift are two very important considerations to include in the DFA analysis. A company that has adequate management information system capabilities, and procedures in place to monitor their plan progress, will be in a position to react to potentially adverse outcomes and take prompt corrective action. On the other hand, a company with inadequate management information systems may not recognize when their plans are not being followed, or when conditions and underlying assumptions have changed enough to warrant changes in the basic plan.

Pricing

I. Source of rates
   A. Bureau
      1. Is individual company experience consistently better or worse than bureau average? If so, what is the cause?
         a. Different underwriting guidelines
         b. Level of underwriting expertise
         c. Different claims handling practices (for example, more or less aggressive in defending/litigating claims)
      2. Are individual company determined expense loads (or pure premium multipliers or loss cost multipliers) appropriate?
   B. Deviation from bureau—What is the motivation for the deviation?
      1. Is the deviation justified by company experience?
      2. Is the company trying to grow significantly by cutting rates?
   C. Company filed—Are rates justified by company experience or based heavily on competitors' rate structure?
   D. Account specific

II. How frequently are rates reviewed and filed? How frequently is pricing adequacy reviewed?

III. What ratemaking data is available?
   A. Industry
      1. Is it applicable to individual company book?
      2. What is the level of integrity?
      3. What level of detail is available?
      4. How many years of history are available?
   B. Company
      1. Is the data sufficiently credible?
      2. What is the level of integrity?
      3. What level of detail is available?
         a. Policy year, accident year, calendar year
         b. Line, class, subline, limit, deductible, account, etc.
         c. Direct, assumed, ceded, net
      4. How many years of history are available? Is this adequate for the line being reviewed?
      5. Has mix of business by class, deductible, policy limit, attachment point, etc., been consistent? That is, is past experience representative of future experience? If not, can data be adjusted to make it representative?

IV. Ratemaking considerations—for each ratemaking component, what should be considered/analyzed?

A. What type of data is used? (How responsive is the method to changes?)
   1. Accident year
   2. Policy year
   3. Exposure year
   4. Contract year
   5. Pool year

B. Current and historical loss and ALAE Development—Have appropriate development factors been selected given consideration to the following? (See Section II, "Reserve Considerations for DFA.")
   1. Incurred loss
      a. Have past development patterns been distorted due to any of the following?
         (1) Changes in claim department practices that would affect
            (A) Case reserve levels (new case reserve philosophy)
            (B) Reporting patterns (For example, telephone reporting may reduce time lag between accident and notice date.)
            (C) Settlement patterns (incentives or disincentives to close claims more quickly, workload per adjuster, change in management)
         (2) Changes in mix of business by class, limit, state, etc.
         (3) Changes in underwriting standards/guidelines
         (4) Changes in type of policy (guaranteed cost, retro, large deductible, excess of SIR, service only)
         (5) Changes in type of coverage (occurrence, claims-made)
         (6) Changes in policy language or exclusions and legal interpretation of such
         (7) Changes in policy limits or deductibles
         (8) Changes in reinsurance purchased (net basis)
         (9) Changes in laws (For example, Superfund, workers compensation state benefits, administration rules, etc.)
         (10) Changes in judicial or administrative decisions that establish precedents (new dispute resolution procedures)
         (11) Changes in discounting or escalation procedures (inflation assumptions)
         (12) Catastrophes
         (13) Indirect changes in reporting patterns (For example, for workers compensation, fewer medical only claims may be reported under large deductible policies to reduce the experience mod)
         (14) Changes in medical management

- (A) Impact of health maintenance organizations (HMO's) and preferred provider organizations (PPO's)
- (B) Use of capped rates versus fee-for-service
- (C) Incentives for HMO's

b. If so, can the data be adjusted to be consistent with planned future business?

c. If not, what is the likely impact on ultimate loss projections?

2. Paid loss (same as incurred loss)
3. Incurred ALAE

a. Have development patterns been distorted due to any of the following?

- (1) Changes in the definition of ALAE
- (2) Changes in treatment of ALAE (within the limit versus in addition to the limit)
- (3) Changes in claim department practices or new expense items such as medical cost containment that would affect
  - (A) Case reserve levels (if case reserves are established)
  - (B) Reporting patterns
  - (C) Payment patterns (For example, partial payments of legal fees versus pay at the end, new type of expenses that occur early on such as medical cost containment and management)
- (4) Changes in defense philosophy that would impact
  - (A) Amount of litigation
  - (B) Cost of litigation (use of in-house versus independent attorneys)
  - (C) Success of litigation
- (5) Changes in mix of business by class, limit, state, etc.
- (6) Changes in underwriting guidelines
- (7) Changes in type of policy (guaranteed cost, retro, large deductible, excess of SIR)
- (8) Changes in type of coverage (occurrence, claims-made)
- (9) Changes in policy language/interpretation that may impact duty to defend
- (10) Changes in policy or deductible limits
- (11) Changes in reinsurance purchased (net basis)
- (12) Changes in laws (For example, Superfund, workers' compensation state benefits, administrative rules, etc.)
- (13) Changes in judicial or administrative decisions that establish precedents

(14) Indirect changes in reporting patterns
b. If so, can the data be adjusted?
c. If so, what is the likely impact on ultimate ALAE projections?
4. Paid ALAE (same as incurred ALAE)

C. Loss trend—Have appropriate trend factors been applied given consideration to the following?
1. Trend period
a. What is the length of the trend period?
b. Is it consistent with the effective policy period?
2. Magnitude of trend
a. Is the trend being applied consistently with industry trends or are differences explainable?
b. Is the trend being applied consistently with internal and/or external indices?
c. What external indices are considered?
3. Type of trend—what type of trend is justified?
a. Linear
b. Exponential
c. Other (For example, econometric)
4. Consistency of trend indications based on the various considerations above

D. Treatment of large losses
1. Basic limits versus total limits
a. At what level are losses capped if at all? Is this level appropriate?
b. Are the losses above the cap spread back and if so, on what basis are they spread?
2. How are increased limits rates made?

E. Catastrophe provision
1. Historical
a. How many years of history are considered in determining the load?
b. Have changes in geographical exposure been considered when applying past experience to current exposure?
c. Are event frequency and PML estimates reasonable?
2. Simulation based?
3. What perils have been considered?
a. Hurricane
b. Tornado
c. Earthquake
d. Hail storm
e. Freeze

F. Premium development

1. Is audit premium included?
2. Are loss sensitive premium adjustments included?

G. Premium trend
1. Trend period
   a. What is the length of the trend period?
   b. Is it consistent with the effective policy period?
2. Magnitude of trend
   a. Is the trend being applied consistently with industry trends or are differences explainable?
   b. Is the trend being applied consistently with internal and/or external indices?
   c. What external indices are considered?
3. Type of trend—what type of trend is justified?
   a. Linear
   b. Exponential
   c. Other (For example, econometric)
4. Consistency of trend indications based on the various considerations above
5. Exposure base (Is appropriate recognition given to inflation sensitive versus non-inflation sensitive exposure bases?)

H. Earned premium at current rates (adjusted for both rate and benefit level changes)

I. Weighted trended on-level loss ratio
1. How many years of experience are averaged? Is this appropriate for the line of business?
2. How are the loss ratios weighted together? That is, how responsive is the method to change?
   a. All receive equal weight
   b. Weight increases for more recent accident (or policy) years
   c. Exclude outliers
   d. Is there a trend or pattern to the loss ratios?

J. Credibility
1. What form of credibility is applied?
   a. Square-root rule
   b. P/(P+K)
   c. Other
2. To what is the complement of credibility applied? (class, state, countrywide, industry data, peer group data, etc.)

K. Unallocated loss adjustment expense
1. Have appropriate adjustments been made for planned changes in volume and staffing?
2. Have charges been appropriate in the past (especially for large accounts demanding high quality and quantity of service)?

L. Commissions
   1. Are any changes in commission structure (including contingent) appropriately reflected?
   2. Are agents incented only for volume or volume and profit?

M. Taxes, licenses and fees
   1. Second injury funds (Are past costs representative of future costs?)
   2. Other assessments (Are past costs representative of future costs?)

N. Profit and contingency
   1. Considerations for the profit load
      a. What method is used to calculate the profit loading?
         (1) Discounted cash flow
         (2) Internal rate of return
            (A) Capital asset pricing model
            (B) Arbitrage pricing theory
            (C) Option pricing theory
         (3) Other models
      b. Are the assumptions used appropriate?
         (1) Discount rate
         (2) Risk charge
         (3) Premium to surplus
         (4) Investment yield
         (5) Other
      c. Is the load appropriate for the risk being taken?
   2. Is the appropriate credit risk reflected for national accounts?
   3. How is the contingency factor, if any, determined?
      a. Historical need
      b. Future potential

O. General expense
   1. Does it accurately reflect expected expenses during the period the rate will be in effect?
   2. Are special expenses for a particular line or state adequately reflected?

P. Policyholder dividends (Do rates reflect the appropriate dividend rate for the selected loss ratio?)
   1. Sliding scale
   2. Fixed

Q. Assigned risk overburden—Are estimates of assigned risk pool deficiencies accurate and appropriately reflected in pricing where possible?

R. Reinsurance (See Section IV, "Reinsurance Considerations for DFA.")
   1. Should more or less reinsurance be purchased based on cost, past results and management's level of tolerance for variability in operating results?
   2. Is the type of reinsurance purchased appropriate for the lines of business being covered?

3. Is the cost appropriately spread back to business unit and line?
4. Is the cost appropriately reflected in rates or are rates calculated on losses gross of reinsurance?

S. Investment income (See Section V, "Invested Asset Issues for the Appointed Actuary.")
1. How was the investment yield determined?
   a. New money rate
   b. Imbedded yields
2. What type of investments underlie the selected yield?
   a. "Safe" yields—Treasury bills
   b. Risky yields—higher risk bonds or stocks or others such as derivatives
3. Is there appropriate recognition of asset/liability mismatch risk?
4. Are various interest rate, loss ratio and payout scenarios considered?

T. Implied ROE
1. Are some lines, states, etc., being subsidized by others? Is this acceptable?
2. Does overall ROE meet shareholders' expectations for stock companies and allow for adequate capital growth for mutual companies?
3. Does by line and overall ROE meet management's expectations?

U. Guaranty fund assessments

V. Pricing Considerations

A. What pricing practices may lead to inadequate prices in spite of adequate rates?
1. Are degree of use and amount of schedule rating credits justified?
2. Are preferred rate programs overused?
3. Are retrospective rating and dividend plans used appropriately?
4. Are loss limits and maximums used appropriately for business being underwritten?
5. Are premium audits accurate and adequate?
6. Are agents/brokers reviewed for profitability?
   a. Are there appropriate incentives?
   b. Are contingent commissions based on growth only or growth and profitability?
7. Are MGA's used?
   a. Level of authority
   b. Use of sub-agents
8. Misapplication of rates
9. Change in underwriting standards
10. Shifts in distribution among rating classes to inadequately priced classes

B. Issues impacting level of accuracy of rates

1. Are individual risk premiums rated on a loss sensitive basis? How wide a swing?
2. Is the class plan highly segmented or broad?

C. Other considerations
1. In general, what is the company's degree of retained risk versus risk sharing with policyholders, reinsurers, etc.?
2. Is coverage on a claims made or occurrence basis?
3. Have aggregate limits been taken into account?
4. Elasticity of demand

VI. New product

A. Level of expertise of actuaries (hired externally or developed internally), underwriters, management, reinsurers

B. Source and adequacy of initial rates or underlying data used to construct rates? Determined relative to competitors? Competitors profitable?

C. Surplus requirement

D. Start-up versus on-going expense costs

E. Profitability
1. Is a higher loss ratio expected to begin with?
2. How long before profitable?

F. What has the experience of other carriers with a similar product been?

G. Is there sufficient demand for the product relative to supply, both presently and as anticipated in the future?

Business Planning

I. What is the planning horizon? 1 year, 3 years, 5 years?

II. Are the planning assumptions consistent with the actuarial pricing reviews or indications?

III. Ability to achieve the plan goals. This risk may be the major risk (particularly for a new line of business or for a management with an unsatisfactory historical track record relative to achieving plan).

IV. Assessment of environment/market conditions
   - A. Underwriting cycle/perceived rate adequacy
   - B. Competition
     1. Who is the competition?
     2. Is competition growing or shrinking?
     3. How do you compare?
        - a. Rate level
        - b. Profitability
        - c. Coverage provided
        - d. Service
        - e. Strengths and weaknesses
     4. Are you a major or minor player?
     5. Are you new to the market?
        - a. Burn your way in?
        - b. Other strategy?
     6. What do you bring to the table that makes you unique?
     7. Why will insureds do business with you?
   - C. Regulatory environment
     1. Product
     2. State
     3. Territory

V. Level of detail? (line of business, market, product) (guaranteed cost, retro, service only, large deductible)

VI. Components
   - A. Written and earned premium
     1. Are growth assumptions realistic given regulatory environments and the underwriting cycle?
     2. Are premium equivalents for servicing type business appropriately reflected?
     3. Are assumed exposure level changes reasonable given economic

trends?

4. Are assumed rate level and pricing changes reasonable given the regulatory and competitive environment?

B. Paid and incurred loss ratios—Are loss ratio projections reasonable given past experience, underwriting cycle (projected rate adequacy) and underwriting guidelines?

C. Paid and incurred ALAE ratios—Are ALAE ratio projections reasonable given claim department practices, legal environments and recent trends in ALAE costs?

D. Unallocated loss adjustment expense

E. Other insurance expense

F. Commissions (including contingent commissions) (See "Pricing" in this section.)

G. Taxes, licenses and fees (including assessments) (See "Pricing" in this section.)

H. Policyholder dividends (See "Pricing" in this section.)

I. Underwriting income

J. Net investment income (See "Pricing" in this section.)

K. Other income

L. Federal income tax (Are any net operating losses properly reflected?)

M. Reinsurance (same components as A-C)

1. Ceded
   - a. What are the attachment points, deductibles, limits and aggregates?
   - b. Risk tolerance
   - c. Degree of risk transferred
   - d. What is the degree of swing if sliding scale commission?
   - e. Adequate coverage (estimate PML under various scenarios)
   - f. Proportional/Nonproportional
   - g. Facultative/Treaty
   - h. Acceptable cost/Market conditions
   - i. CAT assumption
     - (1) Historical—For example, a 1 in 20 year event
     - (2) Simulation based
     - (3) Perils considered (hurricane, tornado, earthquake, hail storm)
   - j. Reinstatement premium
     - (1) Is there a reinstatement provision?
     - (2) What is the cost?
     - (3) How many?
   - k. Sunset clauses? Deductibles? Aggregates?
   - l. Financial strength of reinsurers
   - m. Write-off for uncollectability (model various plausible

scenarios)
n. Traditional/Finite risk
2. Assumed
a. Lines of business, exclusions, layers and limits
b. What has been the general loss experience by line and type?
c. What is the degree of swing if sliding scale commission?
d. Maximum and minimum if retrospective plan
e. Profit plan sharing parameters
f. Credit risk of cedent
g. Risk charge
h. Degree of risk assumed
i. Proportional/Nonproportional
j. Facultative/Treaty
k. PML—risk to surplus
l. International—foreign exchange risk
m. Level of expertise
n. CAT assumption
(1) Historical—For example, a 1 in 20 year event
(2) Simulation based
(3) Perils considered (hurricane, tornado, earthquake, hail storm)
N. Retention ratios—new versus renewal mix impacts loss ratio
O. New business
1. Written premium (growth strategy)
2. Loss ratio
3. Expense ratio
P. Retro reserve
Q. Other loss sensitive reserves (dividend reserves, contingent commission, sliding scale commission on reinsurance etc.)
R. Credit risk
1. Credit-worthiness of creditor? Credit rating (if available)?
2. Type of collateral?
a. Letters of credit
b. Trust accounts
c. Cash
d. Surety bond
e. Other
3. Write-off for uncollectability

VII. Other considerations
A. Mix of business
1. Changes to current mix or volume
a. Over-concentration in any line, subline, state, or territory

   - b. Impact on expense ratios and profits
2. Changes in policy limits sold
3. New lines
4. Lines in runoff

B. Variability of cash flow assumptions
1. Payout patterns
2. Interest rates
3. Other

C. Accounting method
1. Statutory
2. GAAP
3. Tax

D. Miscellaneous adjustments
1. Salvage and subrogation recoveries (if not already considered)
2. Discount (workers' compensation tabular) or other statutorily permitted discounts
   - a. Accretion of discount—impact on calendar year results
   - b. Special amortization requirements (if any)
3. Asbestos and environmental reserve increases
4. Reserve increases or decreases for principal or runoff lines
5. FASB/NAIC accounting initiatives (changes to rules and regulations)

E. Level of underwriting input into the planning process

F. State strategy
1. Growth or lack thereof
2. Withdrawal
3. Undue CAT concentration (decrease writings)

G. Involuntary market
1. Type
   - a. Assigned risk with assignments
   - b. Reinsurance pool
   - c. JUA
2. Size
3. Rate adequacy or size of burden (impact on voluntary prices and results)
4. Level of exposure by state
5. State programs—For example, take out credits
6. Servicing carrier income offsets

H. Potential exit barriers in certain lines/geographic areas

Pricing/Business Planning

I. Trends
   A. Economic
      1. Inflation
         a. Medical
         b. Legal fees
         c. Wages
         d. Specific to line of business (For example, car repair costs, home construction costs, etc.)
         e. Overall (CPI, etc.)
      2. Interest rates
      3. Unemployment
      4. General economic growth by industry group and state
      5. Business failures and formations
   B. Pure premium
   C. Frequency (For example, the number of accidents per exposure unit changes because number of miles driven decreases during recessions, highway improvements, number of hours worked, age of workers, level of experience, unemployment, etc.)
   D. Severity
   E. Litigation
      1. Outcomes of key cases (for relevant states, lines, etc.)
      2. Extent of general litigation and general outcomes (pro defendant versus pro litigant)
   F. Exposure bases (sales, payroll, etc.)
   G. Policy interpretations (extensions of coverage that are unintended by insurer)
   H. Social—non-economic
      1. Judicial
      2. Claim consciousness
      3. Court practices
      4. Morality
   I. Demographics (general aging of population may impact medical costs or accident frequency may increase or decrease)
   J. Public health (mortality and morbidity trends)
   K. New technology (may change how various services are delivered, claim estimates are made or may impact frequency of accidents and severity of accidents) (For example, airbags, improved braking systems, etc.)

II. Environmental changes
   A. Regulatory
   B. Judicial
   C. Legislative

D. Government intervention/involvement

III. Operational changes
   A. Underwriting
   B. Claim handling
   C. Case reserves
   D. Marketing

IV. Field input
   A. Experience of field and key managers
   B. Ability to execute successfully (historical track record)
   C. Adequate staff levels
   D. Appropriate field compensation plans and incentive plans

V. Adequacy of MIS
   A. Monitor results (results = expected results)
   B. Feedback loop (ability to diagnose and fix problems)
   C. Adequacy of data items captured
   D. Real time or significant lag of information

VI. Capital issues
   A. RBC and impact of various business strategies on RBC results
   B. Rating agency formula/perceptions (possible upgrades or downgrades and impact on ability to achieve business plan)
   C. Ability to raise capital (access to borrow, equity markets, private investors, etc.)
   D. Dividend requirements to parent or receivable from subsidiaries
   E. Regulatory perceptions (premium and reserve leverage, IRIS tests, etc.)

Section II
Reserve Considerations For Dynamic Financial Analysis

Reserve Considerations For Dynamic Financial Analysis

The largest liability on an insurer's balance sheet is usually the reserve for losses and loss adjustment expenses. A significant portion of the remainder is often in the unearned premium reserves. These reserves represent provisions an insurer makes to carry out the promise it has made to its insureds to pay for covered losses. As such, the reserves are subject to substantial potential variability due to many causes: random fluctuation, imprecise forecasts, or changes in law or interpretation. Such variability can have a significant impact on the insurer's solidity. In addition, other risk-bearing mechanisms (for example, self-insurance or state pools) will also be affected by variability in reserve estimates.

By its nature, DFA is concerned with a range or distribution of potential outcomes and not merely a point estimate. The notion of range or distribution is particularly significant in evaluating reserves within a DFA framework. The final payout for a book of business is uncertain until all claims are closed and all payments are made. Thus, quantification of a range of potential reserve outcomes arising from a set of specific scenarios or an estimate of the distribution of possible reserve outcomes, with corresponding probability estimates, is critical to any DFA model. For this reason, much of the attached outline is directed toward identifying sources of uncertainty for reserve estimates.

An actuary performing DFA for a risk-bearing enterprise should be aware of the various types of variability and sources of uncertainty in reserve estimates. The types of variability include:

- process (inherent in any random process, even if that process is perfectly known),
- parameter (inherent in the fact that even if models are perfectly known, parameters usually should be estimated), and
- specification (reality may not follow the model selected).

In addition to uncertainty in the overall reserve estimates, the actuary faces additional uncertainty in estimating the timing of the payment of those liabilities.

Most statistical models for estimating loss reserves will recognize process variability. However, for most insurance applications, the "law of large numbers" significantly reduces the influence of process variability on reserves. These statistical models may also provide estimates of the variability inherent in the model parameters. For most insurance applications, parameter uncertainty contributes far more to the variability than process variability and may not be reduced by the "law of large numbers."

The actuary should be familiar with the various methods that can be used for the analysis of reserves. Each of these methods has specific assumptions, strengths and weaknesses. Selection of a method usually results in a compromise between stability and responsiveness. Thus, the actuary should have knowledge of these various assumptions, given the particular situation under analysis, and exercise appropriate actuarial judgment in the selection of models and in the final estimates used for reserves.

The actuary should be aware, however, that because of the choice of specific models, substantial variability still exists. This last source may be unquantifiable but can be substantial and may explain why ranges implied by various statistical methods may not overlap for a specific situation. Furthermore, the actuary should also be aware of the distribution estimates provided by the model. A significant difference exists between the distribution of the expected reserves and the distribution of reserves. An example may make this distinction clearer. The distribution of the expected outcomes of one throw of a fair die is 3.5 with probability 1 and probability 0 for any other value. However, the distribution for one throw has 1/6 probability assigned to each integer from 1 through 6, and 0 to all other values. The first is concerned with the expected value, while the second is concerned with possible values. Many forecasting models provide an estimate of the former while the latter is of concern for DFA.

Any statistical model used to estimate reserves is based on a specific set of assumptions. The actuary using any such model should be familiar with its inherent assumptions, as well as the extent that actual conditions can influence the forecasts and the resulting estimates of the reserve distribution.

Some events influence specific coverages: Specific judicial decisions, legislative benefit changes, or shifts in marketing emphasis for a particular line of business. Other events can influence different lines of insurance: economic recession or growth, the insurance underwriting cycle, and internal processing changes. Still, others may effect both sides of the balance sheet; for example, an unexpected change in inflation can affect both claims costs and the value of the company's assets. The actuary should be aware of the effects of these influences on reserves and the distribution of potential reserves used in the DFA model.

Accounting considerations can affect the structure of a DFA model. For example, the presence of discounting in the statutory reserves may affect balance sheet entries, but not necessarily the cash flow models used in modeling of reserves in a DFA model (assuming, of course, that the reserves are treated appropriately).

Reinsurance is another significant issue. Although the results should be independent of accounting conventions, the approaches in constructing a DFA model may differ if reinsurance is considered a contra-liability rather than an asset. In the former case, one could concentrate on net reserves, leaving collectability as a separate, asset-related issue; whereas, in the latter situation, the actuary may construct separate but interconnected models for direct and ceded losses.

Although often treated when considering the income side, payments on future claims will also affect the DFA model. The actuary should then consider the effect of the various factors that influence losses, as well as those that influence rates, to address uncertainty in payments on future claims. Variation in such future payments will affect the adequacy of the unearned premium reserve.

The attached outline presents additional details and is intended to be used as a guideline for the actuary addressing the reserve component of a DFA model.

Reserving Considerations for the Dynamic Financial Analysis

I. Variability in what?
   A. Expected ultimate loss (and LAE)?
   B. Actual ultimate loss (and LAE)?
   C. Example: If X is the outcome of the roll of a fair die then $E(X)=3.5$ *with certainty;* that is, the expected ultimate loss is known but the actual value can be any integer between 1 and 6 with equal probability and is thus uncertain.

II. Sources of uncertainty for loss and loss adjustment expense (LAE) reserves
   A. For loss and LAE reserves for a single line and single exposure year
      1. Process—uncertainty due to the randomness of the process, even if the process is perfectly known. For example, a single throw of a fair die will come up with an integer between 1 and 6, but which one is unknown. The "Law of Large Numbers" may help to mitigate this source of uncertainty in insurance situations if there is a sufficiently large number of independent events. Some refer to this as "diversifiable" risk.
      2. Parameter—uncertainty that the parameters of the selected model are correct. For example, in the die analogy, what is the certainty that the die itself is a fair die. This risk may not be able to be diversified by use of the "Law of Large Numbers," though it is possible that, in some situations more data may lead to better estimates of parameters.
      3. Specification—uncertainty that the models used to approximate reality are correct. For example, in the die analogy, are the underlying numbers really generated from another distribution, Poisson for example, rather than from the throw of a die? More significantly, if the actuary is using some overall model fitting to the development patterns, then there is uncertainty that the model selected (regression, Horel curve, etc.) actually reflects the underlying loss emergence process.
      4. Other—uncertainty that the future will not be like the past with legal and possibly other changes. To the extent that reserve estimates are based on past patterns, such changes can affect the applicability of using past patterns to forecast future losses.
      5. Coverage specific issues
         a. Data quality
         b. Credibility of the data
         c. Frequency and severity characteristics of the coverage
         d. Limits written
         e. Salvage, subrogation or collateral sources
         f. Reinsurance
         g. Catastrophes
         h. Unique characteristics of the coverage (For example, surety,

D&O, E&O, financial guarantee, etc.)

i. Occurrence versus claims made
   (1) Length in claims made (first year claims made may be different than mature claims made development)
   (2) Tail coverage
   (3) Prior acts ("nose") coverage

j. Unique internal influences
   (1) Changes in contract or coverage
   (2) Insurer experience in coverage/market segment, a "neophyte" may fare worse than a seasoned veteran in some markets
   (3) Unusual growth (or shrinkage) in particular coverage—is new (or lost) business significantly different from remainder?
   (4) Changes in rate of claims settlement (may impact forecasting methods)
   (5) Changes in reserving practices (may impact forecasting methods)
   (6) Changes in claims staffing, significant additions or subtraction to staff can affect both reserving and payment practices
   (7) Accounting changes
   (8) Implementation of loss control methods and procedures, of potential significance (though not the only ones) use of utilization review and audits for medical bills or case management can affect costs
   (9) Changes in the defense philosophy of claim management
   (10) Claims procedure changes
      (A) Opening practices
      (B) Adjuster authorization level
      (C) Field practices
   (11) Claim department organization
   (12) Insurer organization (in the process of centralizing/decentralizing, etc.)
   (13) Presence of discount in the reserve

k. Appropriateness of the selection of projection methods used to estimate reserves given the credibility and volatility of the data

l. "Track Record" of projection methods, if methods have been historically "noisy" or particularly accurate, this should be reflected in the actuary's assessment

m. External influences unique to coverage
   (1) Claims inflation

(2) Local economic conditions
   (A) Local recession
   (B) Local expansion
   (C) Unusually high (or low) demand for services purchased by insurers. For example, scarcity of contractors and building materials after a large property catastrophe.
   (D) Employment levels
(3) Underwriting cycle
(4) Unique market characteristics
   (A) Residual market
   (B) Behavior of major players in market
   (C) Management market objectives (growth, profit, etc.)
   (D) Market position and changes that may affect losses (and LAE)
(5) Weather
(6) Profitability of coverage
(7) Reliability of exposure base in measuring loss potential
(8) Legislative changes
   (A) "Retroactive" liability
   (B) Fee schedules
   (C) Changes in statutory benefits
(9) Judicial changes
   (A) Covered but unanticipated damages
   (B) Reinterpretation of policy language
(10) Administrative changes in resolving disputes
(11) International considerations (exchange, etc.)

n. Large or unusual losses can have significant impact on reserves and are likely to have significantly different expected emergence patterns than more "usual" claims. The following are some examples but should not be considered to be exhaustive
(1) Catastrophes
   (A) Cost effects of supply and demand shifts after a major catastrophe
   (B) Moral hazard
   (C) Additional burden on staff or use of additional outside adjusters with different reserving practices
   (D) Interpretation of coverage that differs from insurer's interpretation
   (E) Cost to construct to new, more stringent,

requirements
- (2) Structured settlement agreements
- (3) Continuing trauma/industrial disease
- (4) Hazardous waste
- (5) Asbestos
  - (A) Products bodily injury
  - (B) Products property damage
  - (C) Other coverages (?)
- (6) DES
- (7) Bendectin
- (8) Silicon implants (?)
- (9) Electromagnetic Fields (?)

o. Other (Son of asbestos?)

p. Do the projections of the various methods make sense? That is, are various diagnostic statistics such as frequency, severity, pure premiums, loss ratios, etc., explainable?

q. Actuarial judgment should be exercised throughout the entire process. How does this affect the results?

6. Effects on various reserve categories (if separate analysis is performed and some may be combined in the analysis)
   - a. Case reserves
   - b. Provision for development on known claims
   - c. Reopened claims reserve
   - d. Provision for claims incurred but not reported
   - e. Provision for claims incurred and reported but not recorded

7. Loss adjustment expenses (LAE)
   - a. Presumably allocated treated in conjunction with losses. Same considerations apply along with the possibility that ALAE may be correlated to losses.
   - b. Changes in internal organization that may shift LAE costs between allocated and unallocated
   - c. Changes in reporting requirements may shift costs between allocated and unallocated LAE
   - d. Catastrophes and the need to bring in additional claims processing resources.
   - e. Unallocated
     - (1) Appropriateness of forecasting method
     - (2) Any change in costs due to financial condition of insurer? Will it cost more (or less) to run off a book than to service an on-going book.
   - f. Are case reserves separately set for allocated expenses?
   - g. Relationship of loss expenses to losses

B. For an insurer's book (all coverages and all years)

1. Data quality
2. Process
3. Parameter
4. Specification
5. Correlation among lines for a single exposure year
6. Correlation among various exposure years
7. Correlation of reserve amounts with environmental factors:
   a. Interest rates
      (1) Risk free rate
      (2) Risk premium
      (3) Yield curve
   b. Economy-wide inflation
   c. Economy-wide business cycle (depression, recession, economic growth?)
   d. Employment levels
   e. Local economies of influence to insurer. For example, if an insurer has significant concentration in one jurisdiction unique characteristics of that jurisdiction's economy may impact results and hence appropriate reserve levels
   f. Movements in financial markets
   g. Underwriting cycle
   h. Tax law changes
   i. Exchange rate variations (international business)
   j. Weather
8. Correlation of reserve amounts to the insurer's operational factors
   a. Changes in rate of claims settlement (may impact forecasting methods)
   b. Changes in reserving practices (may impact forecasting methods)
   c. The rate of growth (positive or negative) in business
   d. Changes in mix of business
   e. Changes in claims staffing
   f. Implementation of loss control methods and procedures
   g. Changes in the defense philosophy of claim management
   h. Claim department organization
   i. Claims department staffing
   j. Insurer organization (in the process of centralizing/decentralizing, etc.)
   k. Weather
   l. Current insurer profitability (or lack thereof)
   m. Insurer's financial strength
9. Pools, associations and residual market
   a. Adequacy of current reserve share

b. Variability in reserve share
c. Reliance on the work of others?
d. Assessability
e. Changes in residual market size that may reduce appropriateness of historic data for projecting future results

III. Impact of reinsurance (retrocessional) coverage on carrier's retained book of loss and LAE reserves
A. Accounting treatment may dictate where reinsurance is considered in the balance sheet and hence how addressed in the modeling
1. Asset?
2. Contra-liability?
3. Impact of various accounting requirements, for example FAS 113
B. Approach to analyzing ceded and retained losses
1. Net/Ceded
2. Direct and assumed/Ceded
3. Direct and assumed/Net
4. Other?
C. Characteristics of the coverages
1. Pro rata
a. Aggregate maxima/minima
b. Ceding commissions
c. Loss sensitive rating
d. Cash flow impact
e. Other
2. Excess (including catastrophe)
a. Per claim coverage
b. Per risk coverage
c. Per occurrence coverage
d. Aggregate limits
e. Loss sensitive rating
f. Ceding commissions
g. Reinstatement premiums
h. Cash flow impact
i. Other
3. Financial
a. Impact on ultimate losses
b. Cash flow impact
c. Degree of risk transfer (accounting treatment)
d. Other
4. State reinsurance pools, associations or funds
5. Commutations
6. Other

D. Solidity of reinsurers/retrocessionaires (if reinsurance is an asset, this belongs in asset considerations, otherwise, in reserve considerations)
  1. Exposure years and amounts at risk
  2. Calendar years effected (reinsurer may go broke three years from now)
  3. Security available from reinsurer
  4. Cash flow influences
  5. Will offsets against cash outflows to troubled reinsurer provide additional protection?

IV. Reinsurance assumed
  A. Most of above considerations also relate to assumed reinsurance
  B. Nature and effect of retrospective or reinstatement premiums on cash flows
  C. Catastrophe potential
  D. Solidity of reinsureds
    1. Drop-down potential?
    2. Cut-through potential?
    3. Offset potential between premiums receivable and losses payable in case of insolvency?

V. Unearned premium reserves (UEPR)
  A. Underlying pricing assumptions
  B. Uncertainties in outcome (see reserving topics above)
    1. Process uncertainty
    2. Parameter uncertainty
    3. Specification uncertainty
    4. Other
  C. Market influences on price adequacy
    1. Underwriting cycle
    2. Insurer market position
    3. Effects of competition
    4. Regulatory effects
  D. Mismatch between UEPR and future obligations
    1. Equity in UEPR
      a. Prepaid acquisition expenses
      b. Taxes
      c. Profit (positive or negative) built into rates
      d. Other
    2. Timing differences between loss emergence and premium earning
      a. Long term coverages (warranties)
      b. Seasonality in losses
    3. Recovery of prepaid expenses

Section III
Mass Tort Exposure

Mass Tort Exposure

Estimation of ultimate liabilities for any significant mass tort exposure can be an actuary's most difficult challenge.

The outline that follows is intended to guide the actuary through various significant considerations that will impact this analysis. Although this outline is applicable to generalized mass tort situations, it mainly focuses on two well-known mass tort exposures: asbestos and environmental liability.

Principally, the actuary will encounter one of three situations when evaluating a company:

1. The company provided coverage that can reasonably be expected to produce material levels of asbestos and/or environmental impairment liability claims activity *and* has experienced material levels of asbestos and/or environmental claims activity to date.

2. The company provided coverage that can reasonably be expected to produce material levels of asbestos and/or environmental impairment liability claims activity *and* has experienced non-material levels of asbestos and/or environmental claims activity to date.

3. The company has *not* provided coverage that could reasonably be expected to produce material levels of asbestos and/or environmental impairment liability claims activity *and* has experienced little or no asbestos and environmental claims activity to date.

For the first two situations above, the actuary may choose to review the relevant language used in the company's 10K (Securities and Exchange Commission [SEC] document for publicly held companies) and possibly the 10K's of other similar companies as a first step to determine the company's reserve practice and philosophy relative to its peers. The actuary of a non-public company may also find it useful to review 10K language filed by public companies. The actuary's review should consider the following items.

First, the actuary should determine whether or not there appears to be a "material" exposure. The following outline enumerates various statistical items to use as a guide when making that determination. This may assist the actuary in determining the appropriate general magnitude of a reasonable range from which to draw scenarios (that is, millions versus billions).

Second, the actuary should gain an understanding of current reserving practices. This item includes the following:

- Identify the aggregate dollar amount of reserves (direct and net) held for this exposure (if possible).

- Identify current payment levels for the most recent three to five years. The ratio of reserves to average annual payments can be used as one indicator of reserve strength relative to peer companies. (The actuary should perform this comparison on both a direct and a net of reinsurance basis. The actuary should also be aware of the nature of the coverage provided [primary versus excess] and shifts in payment activity between coverages.)

- Identify whether the recorded reserves are intended to cover the unpaid portion of the "ultimate" losses and loss adjustment expenses for both reported and unreported claims (or only some subset thereof).

- Identify whether these liabilities are being handled by a dedicated experienced claim/legal unit (an indication of the reliability of the case reserves).

- Identify whether the carried IBNR reserve has been produced by management judgment or by an actuarial estimate.

- Identify management's philosophy concerning these reserves (for example, management asserts no coverage and therefore establishes no reserves). (Even in this situation the actuary may wish to test plausible alternative scenarios.)

Third, the actuary should determine if a reasonable actuarial estimate of IBNR can be made. The outline lists various considerations for the actuary to review to make this determination. Even if the actuary believes that a reasonable estimate of IBNR cannot be made, some modeling of "what if" situations may be appropriate.

If the actuary is using a type of Monte Carlo simulation model, the actuary may randomly draw numbers from a reasonably pre-determined range of possible outcomes, then evaluate the associated strain on the company under each scenario. However, this approach may prove to be somewhat unsatisfactory since the probability of each such outcome may be unknown. Use of a statistical distribution (if known) may be preferable.

Alternatively, the actuary may examine historical average payment streams for these types of claims and run various scenarios where payments for one or more subsequent model years are "shocked" to be $2X$, $3X$, $4X$, etc., of the average historical amount. This approach may increase the "plausibility" of the test and therefore its acceptance by management.

The actuary may also approach testing from a "maximum possible withstandable strain" from this item under various broad business plan scenarios. This approach may best be described as "How much can I afford before I trigger some unpleasant circumstance?"

(For example, the RBC falls below acceptable levels, the rating agency downgrades, there is an inability to pay dividends, or outright insolvency.)

Clearly, a considerable degree of actuarial judgment will be applied in this area. The actuary should emphasize reasonably plausible adverse scenarios and not "doomsday" scenarios.

The fourth critical item to be considered together when modeling the previous item is to gain an understanding of case law and judicial trends in key states relevant to the company, or significant new federal legislation. For example, historical trends may be quite benign, but a relevant new judicial decision can reasonably cause future trends to be considerably more pessimistic (or vice versa). Similarly, passage of significant federal Superfund legislation can materially alter historical payment pattern trends and/or estimates of ultimate liabilities.

The final critical item for the actuary to consider is the availability of reinsurance recoveries on these claims. Reconstruction (or retrieval) of the various ceded reinsurance program information is an integral step in the process. The actuary should also consider modeling time lags in reinsurance collection, the presence of any disputed claims and outright uncollectible reinsurance. Similarly, the actuary should consider inward assumed reinsurance exposures likely to produce these types of claims.

In modeling the business plan, the implications for new business being written should be considered in addition to the potential for adverse reserve development arising from older years. Although a virtual pollution exclusion has been in effect since 1985, voluntary pollution coverage may be offered and similarly may require model consideration.

Furthermore, the actuary should consider generic mass torts the company may have (such as pharmaceuticals, exposure to toxic chemicals or other types of cumulative exposure) in the model. The actuary should review whether policy or underwriting exclusions have been put in place to reduce or eliminate such exposure on newly-written (or future) business and whether rate levels and reserves adequately reflect the cost of these exposures.

In addition to modeling the implications on the business plan, the balance sheet, and the company's cash flows due to liability payouts, the actuary may examine the asset side of the balance sheet to determine whether these liabilities are backed by appropriate assets (quality, duration, liquidity and yield). In particular, the actuary should review liquidity if there is reason to believe that significant cash payments will need to be made in the near future to (for example) effect settlement of a major case. The actuary should also consider the presence of structured settlements in these cases.

Mass Tort Exposure Outline

NOTE: An excellent reference on this topic is the Property Casualty Practice Note 1994-1, Statements of Actuarial Opinion on P+C Loss Reserves as of December 31, 1993, prepared by the Committee on Property and Liability Financial Reporting, American Academy of Actuaries.

I. Scope of the Exposure
   A. Determination of materiality
      1. Historical claim data
         a. Claim counts reported to date (Obtain counts by site when possible. Methodology used to count claims should be identified as it can vary materially between companies and adjusters.)
         b. Dollars paid to date (Loss + ALAE)
         c. Dollars future potential exposure (Loss + ALAE) - case reserves + IBNR
      2. Premium exposure
         a. Premium derived from lines/sublines/classes which potentially gives rise to exposure
         b. Market share of lines/subline/class which potentially gives rise to exposure

II. Current reserving practices
   A. Who sets reserves?
      1. Case Reserves? Level of expertise of adjusters/lawyers in this area?
      2. IBNR? Actuary? Management?
   B. What is intended to be included in reserves?
      1. Management asserts no coverage, therefore, sets no reserves?
      2. Reported claims only? Loss? ALAE? ULAE?
      3. Provision for adverse development? Loss? ALAE? ULAE?
      4. Provision for unreported claims? Loss? ALAE? ULAE?
   C. Historical development (runoff) of reserves
      1. Generally adequate?
      2. Generally inadequate?
      3. Review report year analysis of case reserves?

III. Can a reasonably reliable actuarial estimate of IBNR be made for the company?
   A. Adequacy of data base?
   B. Adequacy of actuarial methodology?
   C. Degree of variability of possible outcomes? Shape of outcome distribution?
   D. Dependency on (consideration of) exogenous variables? (Federal legislation, judicial outcomes, general economics, technology, "how clean is clean," etc.?

E. Range of outcomes? Reasonableness of range?
F. Select low end, midpoint or other point(s) within range?
G. Modeling feasibility? (Actuarial estimation model? Monte Carlo simulation? multiple of current cash flows, etc.?)

IV. Understanding case law, judicial and legislative trends
A. Key historical judicial decisions (often state by state, owned versus non-owned sites, definition of occurrence, joint and several liability, etc.)
B. Current judicial decisions
C. Trends in such decisions
D. Differences in jurisdictions
E. Implications of such trends
F. Federal legislation (Superfund reform, impact of such reform on non-NPL sites, etc.)

V. Reinsurance
A. Ability to cede to reinsurers? Schematic of cession program? Disputed claims? Uncollectible reinsurance? Delays in collection? Commutations?
B. Potential in assumed reinsurance book? Schematic of assumed program? Attachment points? Limits? Layers of coverage?

VI. Implications for new business being written
A. Policy exclusions?
B. Underwriting exclusions?
C. Rate levels reflect exposure?
D. Reserves on new business reflect exposure?

VII. Implications on business plan if material adverse future reserve development is reasonably possible (Ability to meet profit goals, pay dividends, maintain ratings, etc.?)

VIII. Implications for cash flow testing under various selected scenarios within range (Ability to meet organization cash flow needs, define need to borrow or otherwise raise cash, etc.)

IX. Implications for investment portfolio—Selection of appropriate assets, durations, liquidity to back mass tort liability portfolio

X. Implications for reserve opinion (Can the actuary give clean opinion? If not, how does this impact ratings, business plan, ongoing operational ability of company, etc.?)

Section IV
Reinsurance Considerations for Dynamic Financial Analysis

Reinsurance Considerations for Dynamic Financial Analysis

Insurance companies purchase reinsurance for many reasons, such as to

1. stabilize calendar year results,
2. provide large line capacity,
3. finance growth, and
4. provide catastrophe protection.

A properly structured reinsurance program placed with a set of financially strong, stable reinsurers should successfully meet all of the above needs, thus enhancing the financial position of the reinsured. Alternatively, many risks are associated with reinsurance that can impair the financial results of even the strongest reinsured, such as

1. insolvency of a significant reinsurer ("significant" relative to the reinsured's ceded book),
2. inadequate catastrophe protection,
3. inadequate casualty clash protection, and
4. over-reliance on proportional reinsurance for financing.

In the context of DFA, the actuary should construct scenarios that not only test the adequacy of the current and future (as contemplated in the company's business plan) reinsurance programs, but also scenarios that test the adequacy of the reinsurance programs purchased historically.

In reviewing the financial condition of a property/casualty insurance company, the actuary should note the historical benefit that has been derived from reinsurance, while at the same time review the efficiency and effectiveness of the prospective reinsurance strategies. With respect to the historical reinsurance programs, the actuary should review the ceded loss and loss adjustment expense reserve calculations using standard actuarial techniques. This review should, if possible, be conducted on a contract-by-contract basis. Beyond simply reviewing the accuracy of, and potential volatility associated with, the ceded loss reserve calculation, three other questions should be answered for each treaty:

1. Does the treaty provide adequate reinsurance protection for the underlying risks written by the reinsured, or is there a possibility that the reinsured will be forced to retain losses net following the exhaustion of its reinsurance treaty

limits?

2. What is the uncollectible reinsurance exposure on each treaty? Given that reinsurance recoverables can be generated from some very old accident years, and involve treaties on which scores of reinsurers participated, the probability of having some amount of uncollectible reinsurance is high. The actuary should confirm that the reinsurance recoverable assumption has been confined to the collectible portion only and does not include any unrealistically optimistic assumptions regarding recoveries from impaired, or insolvent, reinsurers. In the scenarios constructed for DFA, these two items represent an exposure to the financial strength of a company.

3. Does the treaty contain any loss-sensitive provisions, such that a change in ceded losses may be at least partially offset by a change in ceded premiums and/or ceding commissions? Examples of loss-sensitive contracts include retrospectively-rated (swing-rated) non-proportional covers and proportional covers with sliding scale ceding commissions. For each contract containing such provisions, premium and/or ceding commission accruals should be established at a level consistent with ceded losses to accurately estimate the net benefit derived from the reinsurance.

The questions posed above also apply to testing the company's future operations under the various selected DFA scenarios as well as reviewing its current position. The actuary should test the reinsurance program to confirm that it provides a proper level of protection for the company, assuming everything is fully collectible. Furthermore, various assumptions regarding the percent of reinsurance that will ultimately become uncollectible should be included in the actuary's tests.

These three questions are not confined to the casualty lines of business, but are considerations for property catastrophe treaties as well. To be certain that an adequate amount of catastrophe protection has been purchased, a company should collect detailed risk information by zip code (or its foreign equivalents) for each of its catastrophe-exposed areas, and model the full range of possible results to estimate the loss potential contained within the book of business. Many such models are commercially available, if the "in-house" development of such models is not feasible. Once a company's loss potential has been established, the actuary should confirm the availability and affordability of a sufficient amount of catastrophe reinsurance protection. Furthermore, even if a sufficient amount of reinsurance is purchased, an uncollectible reinsurance exposure remains, emanating from any single reinsurer that may have assumed too large an aggregate level of exposure across all of its catastrophe treaties, thereby creating an insolvency situation once the catastrophe occurs.

Reinsurance is an area that is not easily subjected to standard actuarial techniques, but the following outline serves as a guideline for the actuary concerning the many reinsurance considerations incorporated into a DFA model. While one company's reinsurance purchasing strategy may be very straightforward and easily testable, another company's reinsurance program may include exotic and complicated treaties. It is important that the actuary fully understand not only the protections provided by these coverages, but also the factors that might "stress" these protections, thereby jeopardizing the financial position of the company. In constructing the various scenarios for DFA, the actuary should incorporate a portion of the potential reinsurance risk into the model.

Reinsurance Considerations for Dynamic Financial Analysis

I. Types of reinsurance
   A. Facultative
      1. How often is it used?
      2. What is the split between property cessions and casualty cessions?
   B. Treaty
      1. Proportional
         a. Quota share
            (1) What is the impact on surplus due to the existence of the Q/S?
            (2) Does the Q/S treaty contain :
               (A) occurrence caps
               (B) Loss corridor deductibles, or
               (C) Sliding scale commissions that serve to increase the ceding company's retained loss/risk?
         b. Surplus share
      2. Non-proportional
         a. Per risk excess
            (1) Does the risk excess program cover the maximum policy limits?
         b. Per occurrence excess
            (1) Are clash layers purchased?
            (2) What are the retentions/limits/lines of business covered by the treaties?
            (3) What is excluded from coverage?
            (4) What is the treatment of extra contractual obligations and/or excess of policy limit exposures?
            (5) Has the company ever had a large loss that it had to retain due to treaty wording?
         c. Aggregate excess
            (1) Have results ever been worse than the limit of the aggregate excess treaty?
            (2) What is the net loss ratio impact due to a
               (A) Single large risk loss
               (B) Property catastrophe loss
               (C) Casualty clash loss
            (3) How volatile is the company's net loss ratio?

   C. Non-traditional/Finite risk/Financial
      1. Loss portfolio transfers
      2. Financial quota shares
      3. Funded catastrophe covers

4. General considerations:
    a. Do these treaties pass the risk transfer tests of FAS 113?
    b. Has the company properly accrued for any additional premiums payable or profit commissions receivable?
    c. Have the historical net loss results been impacted at all by the presence of a loss portfolio transfer?

D. Non-reinsurance alternatives
1. Chicago Board of Trade Catastrophe Insurance Futures/Options
2. Other derivative products
3. Lines of credit (for example, surplus notes)
4. Other pure financing alternatives
5. General considerations:
    a. Have any of these alternatives ever been utilized?
    b. If so, how is it accounted for?
    c. To what degree have the actual price movements in these products offset the company's actual property catastrophe loss?

II. Functions of Reinsurance

A. Financing
1. How much would surplus decrease by if all quota share treaties were cancelled?

B. Capacity
1. Are maximum policy limits covered by either facultative or treaty excess protections?
2. Is the clash protection sufficient to guard against a large casualty clash claim?

C. Stabilization
1. Does the distribution of net underwriting results display less volatility than the distribution of gross underwriting results?

D. Catastrophe protection
1. Are the limits of the property catastrophe treaty sufficient to cover the company's worst-case catastrophe loss? If not, how many areas of the country expose the company to a catastrophe loss in excess of treaty limits?

III. Considerations for ceded claims liabilities

In reviewing the potential variability associated with the reinsurance recoverables posted as either an asset or a reduction to liabilities on the company's books, the impact from all of the following items need to be considered.

A. Homogeneity
1. Type of reinsurance
    a. Facultative versus treaty
    b. Proportional versus non-proportional

2. Statutory line of business
3. Layer
   a. Primary
   b. Working
   c. High excess
   d. Clash
4. Type of cedent
5. Contract terms
   a. Flat-rated versus retro-rated
      For loss-sensitive contracts, a change in ceded losses may be at least partially offset by a change in ceded premiums and/or ceding commissions. If premium and/or commission accruals are not established on a basis that is consistent with the ceded loss reserves under these contracts, a mismatch of income and outgo will result.
   b. Claims-made versus occurrence
   c. Method of handling ALAE
   d. Risks-attaching versus losses-occurring
6. Type of reinsurer
   a. Broker market
   b. Direct writer

B. Credibility of historical results

C. Emergence patterns

D. Settlement patterns

A commonly used source for reinsurance industry loss development information is the biannual study produced by the Reinsurance Association of America.

E. Frequency/Severity of claims

F. Reopened claims potential

G. Sunset clause provisions

A sunset clause provides that the reinsurance treaty only covers claims reported to the company during a fixed time period (either from the inception date of the treaty, or from the policy expiration date). Thus, the treaty with a sunset clause is providing less coverage than a treaty without a sunset clause, and the company's net results will be subject to more volatility as the reinsurance coverage "sunsets."

H. Aggregate limits

I. Reserving techniques/Methods and assumptions
1. Appropriateness of techniques for long/medium/short tailed business
2. Sensitivity of results due to changes in assumptions
3. Provision for adverse deviations

J. Salvage/Subrogation/Other recoveries

K. Uncollectible reinsurance exposure

1. Coverage disputes
   a. Non-uniform contract wording might be susceptible to differing interpretations by the various parties to the transaction
2. Actual insolvencies
3. Potential insolvencies
   a. The financial strength and commitment of each current reinsurer should be assessed, with the extent (level of detail and frequency) of the analysis depending on the amount of reinsurance recoverable from the reinsurer. For unauthorized reinsurers, the amount of collateral held by the company should be sufficient to meet all future obligations.
4. Right of offset
   Allows the reinsured (or the reinsurer) to offset balances due from one party to the other

L. Impact of commutations
Be aware of treaties with automatic commutation provisions. The cedant should establish reserves for any liabilities re-assumed as part of the commutation.

M. External influences (For example, changes in tort law)

N. Operational changes (For example, changes in the reinsurance program)

O. Historical exposure to "Mass Tort" losses
In order to assess whether sufficient reinsurance coverage has been purchased historically to cover these types of claims, it is necessary to evaluate the reasonableness of current gross reserve estimates for each class of claims. Also, there may be disputes between the company and its reinsurers over how the treaties were meant to respond to certain classes of claims. These disputes may lead to much less historical reinsurance protection being available to protect the company against these "mass tort" claims than had previously been assumed.
1. Asbestos
2. Pollution
3. Others

P. Impact of partial placements and/or co-insurance clauses
Some treaties provide only a portion of the intended protection, due to less than 100 percent participation by reinsurers. Other treaties mandate that the ceding company should maintain a partial participation within the reinsured layers. In either instance, the ceding company's retained liabilities should be accounted for in their net reserves.

IV. Pricing/Coverage considerations

A. Method of handling ALAE
1. Shared in the same proportion as loss, and not limited by the reinsurance treaty limit

2. ALAE added to loss, with this sum being subject to the reinsurance treaty limit
3. Within or outside reinsurance treaty limits?

B. Occurrence versus claims-made coverage

C. Reinstatement provisions
1. Number of reinstatements
    a. Is there a possibility that coverage may be exhausted due to limited reinstatements?
2. Cost of reinstatements

D. Additional coverages
1. Sunrise cover on prior years
    a. Sunrise cover reinstates coverage that was eliminated from prior treaties via the sunset clause. If current treaties contain sunrise covers for prior treaty years, then one potential source of volatility has been removed.
2. Excess of policy limits coverage
3. Extra-contractual obligations coverage

E. Coverage restrictions
1. Sunset clause provisions
2. Treaty exclusions
3. Limited reinstatements

F. Other provisions
1. Per occurrence loss limits
2. Corridor deductibles
3. Overall ceded loss ratio cap
4. Sensitivity of treaty cost to ceded losses
    a. Sliding scale ceding commission
    b. Profit/Contingent commission
    c. Swing-rated treaties
    d. Reinstatement premium provisions

V. Solvency considerations

A. Adequacy of current reinsurance program
1. Property per risk treaty
    a. Comparison of attachment point/limit of treaty to the ceding company's distribution of risks by policy limits
    b. Presence of facultative reinsurance on risks that are larger than the treaty limit
    c. Number of reinstatements provided
    d. Presence of a per occurrence limitation or an aggregate loss ratio cap
2. Property catastrophe treaty
    a. Adequacy of the reinsurance limit provided relative to the

company's catastrophe probable maximum loss (PML)

(1) Detailed exposure information by zip code (or its foreign equivalents) should be collected by the ceding company

(2) In order to accurately estimate a catastrophe PML and assess the adequacy of the reinsurance limit purchased, the ceding company should either create or purchase a catastrophe loss modeling system, which uses the exposure information by zip code (or its foreign equivalents) as an input

b. Number of reinstatements provided, and their cost

c. Other means of financing, to be used in the event of a catastrophe loss that exceeds the limits of the treaty, or the insolvency of a major catastrophe treaty reinsurer.

(1) Chicago Board of Trade Catastrophe Insurance Futures/Options

(2) Lines of credit

3. Casualty excess of loss treaty

a. Number of reinstatements provided, and their cost

b. Sunset clause impact/sunrise cover exposures

c. Adequacy of clash cover protection

B. Adequacy of historical reinsurance program

1. Responsiveness to mass tort claims

2. Responsiveness to changing tort law

3. Uncollectible reinsurance exposure

C. Ceding company insolvency—Issues from the reinsurer's perspective

1. Insolvency clause

Required to be present in all reinsurance treaties, the insolvency clause obligates the reinsurer to reimburse an insolvent reinsured company in full, even though the reinsured may not be able to pay its claimants in full.

2. Offset clause

Allows the reinsured (or the reinsurer) to offset balances due from one party to the other. The handling of multiple (across) treaty offsets may differ from the handling of single (within) treaty offsets.

3. Consistency of claims handling

The liquidator will be handling claims settlements for the insolvent company, and historical claims settlement practices of the ceding company may not be followed.

VI. Accounting issues

A. Presence of risk transfer—FAS 113

A transaction cannot be accounted for as reinsurance unless an adequate amount of risk transfer can be demonstrated.

B. Accrual of future benefits/Obligations—EITF 93-6
FASB's Emerging Issues Task Force issued EITF 93-6 in the third quarter of 1993. The purpose of EITF 93-6 is to ensure that multiple-year retrospectively rated reinsurance contracts containing provisions which create future rights and/or obligations as a result of past events are appropriately accounted for.

C. New or upcoming issues—FAS 115
The actuary needs to stay abreast of any emerging accounting issues. For example, the "mark-to-market" aspect of the newly-adopted FAS 115 may result in GAAP surplus decreases for some companies. If this ultimately leads to a company holding a different asset mix than what it held historically, the potential impact on future solvency should be assessed.

Section V
Invested Asset Issues for the Appointed Actuary

Invested Asset Issues for the Appointed Actuary

Historically, actuaries have been responsible predominantly for the liability side of the balance sheet. The actuary's focus has included reserves for losses, loss adjustment expenses, retrospective premiums, dividends, and other loss sensitive reserves. There has been minimal actuarial involvement on the asset side of the balance sheet.

Recently, however, actuarial responsibilities have been expanding to include asset-related issues. These expanded responsibilities include duration studies (such as asset/liability matching studies) and investment decision-making. The DFA concept ultimately requires actuaries to examine assets as well as liabilities, thereby requiring that actuaries have detailed knowledge on the asset side. Since the actuarial syllabus did not generally include investment or finance topics until about 1990, the asset side is most likely an underdeveloped area for many practicing casualty actuaries.

The risk associated with investment activities has been emphasized recently by the bankruptcies of Orange County, California, and Barings Bank in the United Kingdom (due to investment losses) and by the large losses on derivatives incurred by several major corporations such as Proctor & Gamble and Dell Computer. These problems appear to have been at least partially driven by either over-reliance on the expertise of outside advisors, inadequate internal audit controls, or both. Given the size of the bankrupt entities and the estimated costs of the bankruptcies, it is clear that investment activities can financially impair almost any entity if sufficient care and diligence is not exercised in performing necessary investment activities.

To participate in the evaluation of assets and/or investment policy, the actuary should understand the objectives of an insurer's investment policy. The primary goals are generally to preserve the insurer's claims-paying ability and to earn favorable risk-adjusted returns on an after-tax basis. In other words, the preservation of asset values while earning an attractive rate of return is the ultimate goal. With current knowledge of the insurer's liabilities, the actuary should add value to the investment results of an insurer.

The following outline provides a basic listing of issues related to investments on the asset side of an insurer's balance sheet. To adequately understand the issues associated with assets, the actuary should be familiar with numerous other issues that are discussed in the many volumes of published research. Additionally, the actuary should review appropriate tax publications or consult with appropriate tax experts to understand the company's tax obligations and potential associated strategies.

In the following outline, the first section focuses on the general risk factors of assets. These items are not necessarily specific to any particular type of asset, but deal with either the overall financial structure of the insurer or with systematic risk in general. The financial structure of the insurer includes leverage of the insurer, the distribution of assets across both type and quality of the asset, and other items. Systematic risk refers to risks that are inherent to the process of investing, such as the spread of issuers of the assets, the economic environment, and other items.

The second section, which deals with types of assets, lists various categories of each specific asset, descriptive features of certain assets issued by a particular type of entity, and risk factors associated with each type of asset. Each type of asset is described as to various issuers (such as various issuers of bonds) and, if applicable, the various investment objectives that the particular investment may satisfy (such as the growth or income-producing aspects of common stocks). The descriptions include characteristics that may differentiate one asset from another within an asset type (such as a callability provision of a preferred stock or the risk measurement of a common stock). Finally, each type of asset may have risk factors especially pertinent to the particular asset (such as interest rate movement for bonds, or economic growth for real estate). The outline also lists risk factors specific to each type of asset.

The investment background information section reviews factors specific to each insurer that impact investment decisions and, therefore, the analysis of assets. These factors include historical investment performance, the propensity of the insurer to incur large or catastrophic losses, and the impact on the risk based capital calculation.

The management controls section describes management involvement and responsibilities in the investment function. Issues included in this section are management information systems, management oversight, and audit controls, which all impact management's ability to ensure that adequate controls exist to mitigate the risks associated with the investment function.

The actuary should be aware of the numerous interactions between cash inflows due to new premium inflows and cash streams produced from various investments (bond coupons, stock dividends, sales, and redemptions), and the various payment outflows due to claims, expenses, or dividends. DFA models involving numerous cash flow scenarios under diverse sets of interest rate environments should generally be reviewed to understand the portfolio risks of significant unanticipated cash requirements that may arise under various "stressed" scenarios. Additionally, changes in investment strategies that involve changes in asset allocation mix, tax minimization strategies, etc., should also be reviewed.

The portfolio should be reviewed for over-concentration of assets (lack of diversification) that may render the portfolio unusually susceptible to downturns in particular economic or geographic sectors, or unusually susceptible to the economic conditions of a particular issuer. Over-concentration should also include consideration of large receivables, such as large retrospective rating premium balances, as well as equities and bonds emanating from a single issuer.

The actuary should be alert to the presence of assets that may be poorly understood, or highly complex (such as derivatives), that may react with unusual volatility under certain conditions (usually linked to changes in interest rates).

Finally, the actuary should review management's investment policy, information systems, and degree of control over significant investment decisions to ascertain that reasonable controls have been established. The actuary should consider review of these issues with the company's independent auditors.

Invested Asset Issues for the Appointed Actuary

I. General risk factors of assets

A. Financial leverage—Ratios of "assets to surplus" and "premiums to surplus" reflect the financial leverage of insurers. Higher ratios indicate higher levels of leverage and generally warrant a more conservative investment portfolio.

B. Investment quality—The quality of investments can range from investment grade to "junk". The quality of bonds and equities are evaluated by various organizations including the Security Valuation Office of the NAIC, Standard and Poor's, and Moody's. Real estate and private placements are generally not evaluated on a qualitative basis by rating organizations.

C. Distribution of assets

1. Asset allocation—Different assets (short-term investments, bonds, stocks, etc.) have different historic average returns and a wide variance from the average return. As a result, different portfolio distributions of bonds, stocks, and other assets will experience different levels of volatility. Further, the variables that affect the value of the investment portfolio will have different impacts on different companies, depending on the asset allocation.

2. Asset/Liability/Surplus proportions—It would be prudent to invest a portion of total assets relating to recorded liabilities so that the likelihood of loss of principal or investment income is minimized.

3. Amount of risky investments—Risky investments include "junk" bonds, certain real estate, and volatile common stocks. The amount of any risky investments should not be excessive given the insurer's obligations and other assets.

D. Duration of assets, liabilities and surplus—Duration measures the weighted average of the present value of a particular cash flow. It is used to measure the sensitivity of an asset or liability to changes in interest rates. The cash flow can be either incoming (such as an investment portfolio or a particular investment) or outgoing (such as for a liability or a particular claim). A gap can result if the incoming asset duration differs substantially from the outgoing liability duration. This gap can be measured by the duration of surplus, which is an indicator of the sensitivity of surplus to changes in interest rates.

E. Liquidity—Several types of assets do not have liquid markets for acquiring and disposing of assets. These include real estate, certain foreign stocks and debt, private placements and certain "junk" bonds. If the need arises to liquidate assets, the lack of liquidity of these assets may translate into either a longer time period to divest at the desired price or selling at a less desirable price.

F. Public versus private placements—Private placements have limited or no markets for buying and selling equity interests or debt of the issuer. Large amounts of assets acquired through private placement might create liquidity concerns.

G. Volatility of asset values—The volatility of asset values may be measured by the beta of an individual stock, the variance of returns on a portfolio of stocks, both the term and coupon rate of bonds, and both the economic conditions and vacancy rate for real estate.

H. Spread of assets—The spread is a reflection of the concentration or diversification of the investment portfolio across either industries, issuers or geographic regions. A more concentrated portfolio increases the reliance on the conditions within the segmentation that is owned (namely the particular industry, the issuer of the debt or equity, or the geographic region). A more diversified portfolio reduces reliance on the individual segmentation.

I. Economic environment—The direction of the economy has a direct impact on the value of assets, although the impact varies for different assets. Economic factors that affect asset values include interest rates, direction of interest rates, inflation level, growth in GNP (which is a measure of recession and health of the economy), corporate profits, and many other factors.

J. Potential inaccuracies in cash flow assumptions—Cash flow models (which are also referred to as stress tests) are used to assess the differences between cash inflows derived from investments and cash outflows to satisfy liabilities. Either cash flow stream can be inaccurately modeled, especially the cash outflow stream. Greater risk of a material inaccuracy should translate into a more conservative investment philosophy.

II. Types of assets

A. Cash—United States cash is the safest investment. Foreign currency may not be as safe since two additional risks (currency exchange risk and, to a lesser degree, political risk) are present. However, foreign currencies that are used to fund liabilities in the same foreign currency can be considered as reducing currency exchange risk.

B. Short-term investments—Defined as non-cash assets with a maturity of one year or less

1. U.S. Government Treasury Bills are debt of the U.S. government.
2. Certificates of Deposit (CD's) are interest-bearing short-term debt issued by banks, either domestic or foreign.
3. Commercial paper represents the unsecured short-term promissory notes of corporations that can be either interest bearing or sold at a discount.
4. Banker's acceptances are issued by banks to support demands for money of the bank's customers. The demands should first be accepted by the bank.
5. Repurchase agreements (repos) are the transfer of a security, generally a U.S. Treasury security, where the seller agrees to repurchase the security on a certain date at a specified price. Repos are similar to secured borrowing and lending of funds generally at lower-than-market

interest rates. They can also be sold as reverse repos, whereby the investor assumes the credit risk of the other party.

6. Money market funds are funds that invest in short-term instruments and are operated by mutual funds, banks or insurance companies.
7. Eurodollars are dollar-denominated deposits at foreign banks or foreign branches of U.S. banks, both of which are not regulated by the Federal Reserve Board.

C. Bonds—The principal investment of insurers providing higher yields and higher risk than short-term investments and lower risks and lower long-term returns than equities.

1. Types of bonds
   a. U.S. Government issued bonds include treasury notes (which have maturities of between two and ten years) and treasury bonds (which have maturities of between ten and thirty years). These bonds include both coupon-bearing bonds and non-interest bearing (zero coupon) bonds (referred to as STRIPS or CATS).
   b. U.S. Government agencies issue debt to allow them to carry out their function. The debt is not guaranteed by the U.S. government, but by the agency. This debt is considered to be very low risk.
      (1) Government sponsored enterprises—Six in total including Tennessee Valley Authority and the Export/Import Bank.
      (2) Government agencies—Twenty-four including Government National Mortgage Association (GNMA) and Federal Home Loan Bank (FHLB).
   c. Municipal bonds receive favorable tax treatment and include the following two types of bonds issued by states and political subdivisions of states.
      (1) General obligation bonds are backed by the full faith and credit of the issuer.
      (2) Revenue bonds are backed by revenues from a specific project, such as a toll road.
   d. Corporate bonds are debt obligations of the issuer.
   e. Foreign bonds include those issued by governments and corporations. They are valued in the currency of the issuer and are, therefore, subject to currency exchange risk.
2. Major risk factors of bonds
   a. Interest rate risk refers to the price movement in the value of the bond due to changes in interest rates. The price movement will vary based on the term to maturity, the coupon rate and the quality of the bond.

b. Liquidity risk refers to the cost of having to liquidate assets. The asset liquidation may be completed due to the cash flow needs of the insurer or due to the financial condition of the bond issuer.
c. Inflation risk involves the erosion of the value of future coupon receipts and the principal repayment by inflation.
d. Credit risk is reflected by the potential non-payment of principal and interest by the bond issuer due to financial impairment. Diversification reduces credit risk by reducing reliance on the financial health of one issuer. The quality of the bond holdings further impacts the credit risk, since it relates to the likelihood of financial impairment.
e. Call risk represents the risk that bonds may be called by the issuer before they mature. Bond calls usually occur when interest rates are low since the issuer can place debt at lower interest rates. Conversely, the bondholder should reinvest at lower interest rates.
f. Event risk refers to the impact that an event can have on bond values. Events that can impact bond values include mergers, nuclear power plant accidents, product tampering, and class action litigation brought against the bond issuer.

D. Preferred stocks—Represent equity interests in a corporation, similar to common stock, that pays a dividend that is generally fixed, similar to the interest payment on a bond. In a corporate liquidation, preferred equity interests are subordinated to debt issues (bonds) but receive preference over common stock.

1. Preferred stock features
   a. Callability refers to the company's option to repurchase the preferred stock at a certain price that may decrease over time. Callability is more important for preferred stock issuers as compared to bond issuers since bonds have a natural maturity date that retires the debt. The only way, other than calling the preferred issue, to retire preferred stock is through open market repurchases.
   b. Dividend yield represents the dividend payment as a percentage of the stock price.
   c. Cumulative dividends indicate that any preferred dividend payments that the company has missed should be paid prior to paying common stock dividends.
   d. Convertibility indicates that the preferred stock is convertible into common stock at the option of the stockholder for a certain price during a specified time period.
2. Risk factors of preferred stocks

a. Interest rate risk refers to the price movement in the value of the preferred stock due to changes in interest rates. Preferred stock prices tend to reflect interest rate movement more than common stocks with the price reflecting dividend yield, convertibility provisions, and the credit-worthiness of the company.
b. Liquidity risk refers to the cost of having to liquidate assets, similar to bond liquidity issues.
c. Credit risk is reflected by potential non-payment of preferred dividends and sinking fund obligations.
d. Call risk represents the risk that the preferred stock may unexpectedly be called by the issuer. Preferred stock calls are not as frequent as bond calls and usually occur to retire a source of financing.
e. Event risk refers to the impact that an event can have on stock values, similar to the potential impact on bond values.

E. Common stocks—Represent equity interests in corporations. These common equity interests are subordinated to both debt issues (bonds) and preferred stocks in a corporate liquidation.

1. Common stock features
    a. Types of stock
        (1) Growth stocks
        (2) Cyclical stocks tend to grow and contract depending on the phase of the economic cycle.
        (3) Income-producing stocks are frequently purchased based on dividend yield and, to a lesser degree, growth prospects.
    b. Sector/Industry
    c. Risk level
        (1) Price-to-earnings (PE) ratio of individual stocks equals the ratio of the stock price to the earnings per share of the company. Weighted average PE ratios can be calculated for a portfolio of stocks. The PE ratio is generally an indication of the public perception of the growth prospects and riskiness of both the company and the industry of the company. The size of the PE ratio also tends to be negatively correlated (the higher the PE, the larger the price decrease) with the price impact of unanticipated unfavorable news. The impact of unanticipated favorable news tends to be positively correlated (the higher the PE, the larger the price increase) but not as strongly as

the unfavorable news correlation.

(2) Beta of a stock is a measure of the sensitivity of the stock price to price movements of the overall stock market. It is calculated as:

$$\frac{\text{Cov (return of the asset, return of the market)}}{\text{Var (return of the market)}}$$

(3) Variance of the returns of a stock portfolio measures the consistency of the investment returns. Riskier portfolios with heavier weights to growth stocks, high PE stocks or high beta stocks tend to have higher variances.

d. Dividend yields tend to indicate the level of risk of a stock. Non-dividend paying stocks tend to be companies in a growth mode that frequently have higher risk. High dividend paying companies are frequently more stable companies (including utilities).

e. Country of origin has become more prevalent as foreign stocks have gained in popularity. The strength and political stability of the country and currency fluctuation become additional concerns when investing in foreign stocks.

2. Major risk factors of common stock

a. Market risk is considered a systematic risk (risks that are dependent on macro factors, such as the national economy). It refers to the impact of the performance of the stock market and its impact on any individual stock or a portfolio of stocks.

b. Interest rate risk is also a systematic risk that relates asset values to the movement of interest rates. The correlation to interest rates of common stock prices is lower than the correlation to bonds and preferred stocks.

c. Credit or company risk refers to the financial strength and future prospects of the company. In addition, the business risk (or the risk inherent in the business) impacts the risk of the company.

d. Sector/industry risk refers to the risks of a particular industry or sector of the economy. These risks tend to affect all companies in the particular industry or sector.

F. Mortgages

1. Insured mortgages (those insured by the Federal Housing

Authority [FHA] or the Veterans Administration [VA]) reduce the credit risk of investing in mortgages.

a. Payment status indicates if the mortgage payments are current, late or in default.

b. The location and type of property is an indicator of the quality of the collateral.

c. The level of equity of the property owner is significant to ensure that the owner has a vested financial interest in the condition and financial health of the property.

2. Collateralized mortgage obligations (CMO's) are securities backed by payments of mortgagees of principal and interest. They are created as trusts and have numerous major variations, two of which are discussed below:

a. Payment stream variations of CMO's

(1) Interest only (IO) CMO's are the purchase of only the interest payments on the mortgages. They are priced based on the anticipated (and uncertain) interest payment stream, which reflect likely pre-payments of mortgages due to either refinancing or home sales. If interest rates decline, pre-payments typically increase, thereby eliminating expected interest receipts and creating a loss for IO owners. Conversely, increasing interest rates tend to decrease pre-payments, thereby providing additional, unanticipated interest receipts and added value for IO owners. Interest rate movements provide the greatest risk to IO values. Reinvestment risk and credit risk provide additional risk.

(2) Principal only (PO) CMO's are the purchase of only the principal repayment on mortgages. The price is based on the anticipated (and uncertain) repayment of the principal due to refinancing or sale of the property. If interest rates decline, the balance of the principal is typically paid sooner than expected, thus producing an unanticipated gain (since the pricing of the PO CMO was based on a lower interest rate than was actually realized). Reinvestment risk is also evident, however, in that the repaid principal should be reinvested at the lower interest rate. If interest rates increase, principal repayment typically slows down, thereby delaying receipt by the

investor. The initial pricing of the PO CMO would, therefore, have been based on too high an interest rate.

b. PAC's refer to planned amortization classes. They can be viewed as a scheduled sinking fund mechanism that provides nearly certain payments over a predetermined time frame, thereby reducing the risk of other CMO products. The near certainty is achieved since PAC holders have priority over other CMO holders in the receipt of payment.

G. Real estate

1. Owner-occupied reduces the risk of vacancy of the building and deterioration in the condition of the building.
2. The geographic location and type of property is an indicator of the likely price appreciation or price depreciation of the property. Given the wide variation in return by geographic location, the degree of concentration/diversification becomes a major factor in real estate.

H. Other invested assets

1. Derivatives are financial instruments whose price relies on the price movement of a different security index, interest rate, commodity, or other financial instrument. The risk level of derivatives ranges from risk-reducing to extremely high.

a. Types of derivatives

(1) Forwards are obligations to complete a transaction at a future date for a specified price. Forwards are generally used to reduce risk by hedging currency risk or commodity risk, although certain speculative forwards can be high risk.

(2) Futures are similar to forwards except that they are regulated and trade on exchanges. They are frequently used to hedge risks such as interest rates and commodity prices.

(3) Options give the investor the right, not the obligation, to buy (call) or sell (put) a specified asset at a given price (called the strike price) before a certain date. They are frequently used to protect against adverse changes in either a stock price, a commodity price, an interest rate, or a foreign currency exchange rate. An owner of an asset can:

(A) Sell a call that gives another investor the right to purchase the asset at a stipulated price on or before a certain date.

(B) Purchase a put that gives the owner the right to sell the asset at a stipulated price on or before a certain date.

An investor who does not own the particular asset can:

(A) Purchase a call, which gives the investor the right to purchase the asset at the stipulated price on or before a certain date.

(B) Sell a put that gives another investor the right to sell the asset at the stipulated price on or before a certain date.

Options are also used speculatively and are very high risk when written as "naked options" (selling call options to buy an asset that is not owned by the option writer). The risk of loss in selling "naked options" is unlimited for the option writer.

(4) Swaps are used to exchange certain financial instruments, such as interest rates, principal denominated in different currencies, or any other payment stream. The major risks are of movement in the financial instrument in the unanticipated direction (such as interest rates increasing when the purchased swap anticipates a decrease) and of default by the other party. The risk can be mitigated through the purchase of caps (an upper limit on an interest rate), collars (an upper and a lower limit on an interest rate) and floors (a lower limit on an interest rate).

2. Asset-backed securities represent the repackaging of certain pooled assets of an issuer into collateralized securities. These pooled assets may be mortgages, bank loans or other debt. The investor is exposed to pre-payment risk and credit risk.

3. Stripped securities are securities created by investment firms by separating the bond principal and the coupons, similar to CMO's discussed above. The new securities are also called "interest-only" (IO) or "principal-only" (PO) securities and are sold to produce a reasonable yield-to-maturity. The major risks

are pre-payment risk and interest rate risk (especially for PO's, which generally have both long durations and long weighted average maturities).

III. Investment background information

A. Relative return on investments—The historical return of the investment portfolio and the variability of these returns, relative to appropriate indices, can be an indicator of the acumen of the investment management team.

B. Experience with current types of investments—Management should have a clear understanding of the characteristics, risks and features of each type of asset that is being included in the investment portfolio. The greater the risk of the asset, the greater the knowledge that management should have. Management should be aware of any high risk investments (such as derivatives and "junk" bonds) that have been made and the risks associated with each investment.

C. Recent actual large losses and exposure to large losses—Large losses can necessitate the unanticipated liquidation of assets in certain situations. Variables that affect the likelihood of needed liquidation include degree of concentration of insured values, the reinsurance program (including catastrophe reinsurance), amount of upcoming cash receipts, and other anticipated cash outflows.

D. Impact on the risk based capital (RBC) calculation—The impact of the investment portfolio on the RBC calculation should be assessed. If the insurer's RBC result is clearly above the minimum acceptable level, the impact may be immaterial. If the RBC result is marginal or unacceptable, the insurer might consider modifying the investment portfolio to improve the result.

IV. Management controls

A. Adequacy of management information systems—Inadequate or inaccurate management information systems may impair management's investment decision-making and may impede management's ability to uncover problems in the investment area.

B. Management oversight of investment activities—Management is ultimately responsible for the investment function. This responsibility includes establishing an investment philosophy (regarding types of investments, acceptable risk level, and other factors), reviewing the investment performance, monitoring the level of risk in the portfolio, and, in general, ensuring that the investment philosophy is properly implemented. The amount of oversight depends on how the investment function is handled. There are three common ways to handle the function:

1. An in-house investment department performs the investment activities and maintains responsibility for the investment philosophy and performance. Management oversight is naturally maintained.

2. An in-house investment department establishes an investment philosophy and retains an investment advisor to provide investment selection services. Management oversight is necessary to ensure that the advisor is providing the needed services at an acceptable level of risk.
3. An in-house department (perhaps comptrollers or financial) retains an external investment advisor to provide more complete investment services. Substantial oversight is required to ensure that management is aware of the performance and risks of the investment portfolio.

C. Adequacy of the auditing function—The auditing of the investment function, as performed by both internal auditors and the independent auditors, can determine if the investment portfolio is excessively risky, uncover "hidden" or "problem" transactions, protect against the physical disappearance of assets by either theft or destruction, and assess the overall integrity of the investment function.

Section VI
Other Assets and Other Liabilities

Other Assets and Other Liabilities

The previous sections of this *Handbook* deal with risks associated with pricing/business planning, reserving, mass torts, reinsurance and invested assets. While the so-called "Other Assets" and "Other Liabilities" typically present relatively minor issues, they can be quite significant for certain companies.

A key attribute of any model is to accurately portray the *interrelationship between the various balance sheet accounts* for a given issue. For example, an unexpected increase in the loss ratio experience of a company above planned levels may also trigger an increase in accruable retrospective premium, a decrease in accrued policyholder dividends, an increase in reinsurance recoverables, a decrease in the liability for contingent commissions to agents, and an increase in reinsurance payables under sliding scale commission contracts. The actuary should be aware of such interactions between the various balance sheet accounts since they can act to either minimize or magnify the impact of the tested scenario.

Similarly, the actuary should be aware of capital and debt related items and transactions between affiliates. For example, a company that raises capital through a preferred stock offering should adequately recognize the related dividend payouts in the model. A company that issues debt should accurately reflect the future stream of debt service charges incurred. A subsidiary that is expected to contribute capital to a parent via dividends should also be modeled appropriately.

Other expenses, taxes, licenses and fees, and federal and foreign income taxes should be appropriately considered in the model.

In addition to considering the amounts of such items and interactions between the various balance sheet accounts, the model should consider cash flow issues related to the timing of receipts for items recorded in the various receivable accounts, and the timing of payouts for items recorded in the various payable accounts.

The actuary should be aware of any significant off-balance sheet liabilities; for example, guaranty fund assessments (that can be significant if a major company became insolvent or an unusual amount of mass tort liabilities were put into the guaranty fund), special assessments (such as second injury funds), guarantees relating to the sale of a subsidiary, shareholder suits, or bad faith claims. Finally, the actuary should be aware of likely short-term future events such as material changes in accounting pronouncements that may have a material impact on the company.

Other Assets and Other Liabilities

I. Other assets
   A. Premium receivables
      1. Agent's balances or premiums in course of collection (agent's credit-worthiness, disputes, aging of accounts, reasonable default provisions)
      2. Booked but deferred and not yet due (typically installment payments—same considerations as item 1 above)
      3. Accrued retrospective premium (accuracy of reserve estimate relative to plan parameters, credit-worthiness of insured, security held such as LOC's, etc.)
      4. Any other loss sensitive premium or dividend receivable (same considerations as item 3 above)
   B. Reinsurance related
      1. Funds held by or deposited with reinsured companies (credit-worthiness of reinsured, funds escrowed, ability to offset)
      2. Reinsurance recoverables on paid losses and LAE (credit-worthiness of reinsurers, security held [if any], presence of disputes, aging of accounts, ability to offset, etc.)
   C. Other receivables
      1. Bills receivable taken for premium (at times done with credit impaired risks)
      2. Federal income tax recoverable (accuracy of estimate, strength of position advocated, etc.)
      3. Interest, dividends and real estate income due and accrued (look through to quality of underlying invested asset, credit-worthiness, reasonable default provision, collateral)
      4. Receivable from parent, sub or affiliate (credit-worthiness of affiliate, etc.)
      5. Other receivables (usually write-ins)

II. Other liabilities
   1. Contingent commissions (agents' commissions, sliding scale reinsurance contracts, etc., accuracy of reserve estimate relative to plan parameters)
   2. Other expenses (accuracy and completeness of estimate)
   3. Taxes, licenses and fees (same as item 2 above)
   4. Federal and foreign income tax (same as item 2 above)
   5. Borrowed money (understand debt obligations of company and parent company, look through on debt structure to determine if subsidiaries, etc., have sufficient cash flows to meet parent's obligations)
   6. Interest on borrowed money (same as item 5 above)
   7. Unearned premium (accuracy of estimate)
   8. Dividends declared but unpaid (accuracy of estimate)

(a) Stockholders (also look through to long-term capital needs including dividend capacity and debt service obligations, consider preferred and common stock, market reactions to dividend reduction or cessation, etc.)

(b) Policyholders (also consider undeclared but due under filed dividend plans and accuracy of estimate relative to plan parameters)

9. Funds held by company under reinsurance treaties (adequacy of estimate)
10. Amounts withheld or retained by company for account of others (generally payroll tax and other withholdings)
11. Foreign exchange adjustments (exposure to currency fluctuations, materiality, hedging used to mitigate swings, etc.)
12. Drafts outstanding
13. Payable to parent, subs and affiliates
14 Payable for securities

III. Contingent Liabilities—Off-balance sheet

1. Review contingency footnotes in GAAP and Stat statements for particular items noted.
2. General considerations would include items such as: guaranty fund assessments, bad faith claims, special assessments such as for second injury funds, rate rollback potential, guaranty of loss reserves (or other) for a previously sold subsidiary, general litigation, shareholder suits, etc.

IV. Other

1. New accounting pronouncements that may materially impact the company (within the next calendar year, for example).

Report to the Membership
SOA Board Task Force on Education

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REPORT TO THE MEMBERSHIP

by the
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REPORT TO THE MEMBERSHIP

Introduction

In the past few years, SoA members and leaders have raised questions about future needs for actuarial skills, globalization of markets, and the appropriate focus of the education system in preparing actuaries for their professional futures. The SoA Board of Governors set aside a major portion of its January 1994 meeting to discuss the education of actuaries, with primary consideration being given to a longer horizon than the immediate future. As part of the discussion, the Board invited Chris Daykin, representing the Institute of Actuaries (U.K.), who reported that the Institute was incorporating fundamental changes to its basic education and examination system. These changes were aimed at focusing education more on fundamental principles, making it easier for actuaries to move into nontraditional areas of business.

Following this discussion, the Board named a Board Task Force on Education (a presidential task force) and asked it to recommend the best way to educate actuaries in the future. In particular, the Task Force was asked to address how to enhance the core competencies that distinguish actuaries from professionals in related business areas. After several months of study, the Task Force concluded that fundamental change in the SoA education system was necessary for actuaries to operate with maximum effectiveness in the future, and the Task Force recommended specific educational principles which the Board of Governors has accepted.

What ultimately convinced the Task Force that fundamental change in basic education is needed? Let's consider the case for change.

The Case For Change

The actuary is a highly skilled and well-trained professional with a strong analytical and practical approach to the financial problems posed by the uncertainty of the future. As a goal, the actuary should be—and be perceived to be—*the* professional who assesses and manages the financial aspects of risk (or uncertainty). The Task Force concluded that this goal calls for an education process that successfully meets the challenges posed by a rapidly changing environment. The process should be designed to focus on the current, recognized strengths of the actuary and then to expand and enhance those strengths. In that way, the value of the actuary is enhanced and the actuary is increasingly distinguishable, in a positive sense, from potential competitors.

The effective actuary brings a distinctive set of competencies to his/her business environment. Those competencies must be in sync with the needs of the business environment, not only today but also in the future, for the actuary's potential value to be recognized and appreciated.

Because the business environment changes at an accelerating rate, flexibility and tolerance for unstructured environments are becoming increasingly essential. Clearly, the following skills and attributes are of increasing importance to actuaries and should be emphasized in the education system:

- Unstructured problem solving
- Flexibility
- Adaptability to change
- Expertise in modeling techniques
- Global thinking
- Stochastic/dynamic approaches
- Expanded application of contingencies
- Imaginative responses (e.g., to regulation)
- Business value added.

Actuaries have been in an enviable position. Stable employment markets in major industries have valued their skills. Career paths within the company or the consulting firm were clear and well-defined, and led to desirable places. The CEO and the primary financial executive (such as the CFO) of an insurance company were often actuaries. At present, other professionals compete for many of the same business opportunities, and computers and technical software provide less expensive ways to obtain some of what the actuary has offered. The future is no longer virtually guaranteed; fewer actuaries may be needed to provide the same or even an enhanced level of traditional services to the traditional client/employer.

In the future, the environment will place demands on all actuaries to develop the full range of skills and knowledge needed to assume strong, challenging, and rewarding roles, and to assume those roles both within and outside traditional markets. The SoA is committed to maintain and further enhance the value of the FSA as we move toward the future. To that end and to counter competition from other business professionals, it must be firmly established that the actuary adds value to the business enterprise that extends well beyond technical proficiency.

Currently actuaries possess knowledge and skills that are recognized, appreciated, and valued in fulfilling traditional (technical and practice or product-line) roles within traditional markets. In addition, individual actuaries have effectively demonstrated their value outside that range in new roles and new markets (e.g., banking, investment firms). Nonetheless, for actuaries in general to expand into new roles (e.g., investment analysis) in traditional markets and to move into new markets (e.g., manufacturing), there is a real need to demonstrate the value that the actuary adds in those areas. The demanding and concentrated practice-specific basic education attained with the present system does not translate directly into the broad, adaptable skills and knowledge required for these new roles.

To demonstrate therefore to potential clients and employers not familiar with what the actuary can do (and to traditional employers who may not recognize the full range of the actuary's skills) it must be apparent from the outside that the education system provides for the development of essential mathematical and business-related knowledge and capabilities, with clear applicability beyond traditional markets. Further, the education system must recognize that a professional education builds on the general education and experiences that individuals acquire elsewhere.

What Distinguishes Actuaries from Other Professional and Business People?

Actuaries deal with the intersection of risk and finance. They solve problems, primarily in the insurance and pension industries, involving contingencies and financial risk. It is the mathematical rigor applied to this type of practical problem-solving that makes actuaries unique. This uniqueness, or competency, need not, however, be limited to the insurance and pension industries. In fact, with proper training, actuaries should be able to use these same elements to solve problems for all kinds of businesses, thus suggesting new and different roles in the future. It is conceivable that the actuary, as the expert in modeling techniques applied to any discipline, will often be part of a team, working with other experts who provide the comprehensive knowledge of a specific industry.

Effectively accomplishing this, however, will call for actuaries to focus on enhanced development of mathematical education with broader business application. The education system, therefore, must specifically develop and further enhance the core competencies of the profession, including construction of models, setting of assumptions, testing of data, sensitivity testing, and the interpretation, communication, and management of results. The specific knowledge of the actuary should also include mathematics and logic, economic security programs, investment and finance vehicles, and asset/liability management. The Society of Actuaries must provide these essentials to ensure the future viability of the profession.

There are other basic subjects that contribute not only to the competency of the actuarial profession but also to the competency of many other technical and scientific disciplines. Mastery of these subjects is essential to becoming an actuary, but specifically providing the education for that mastery in the SoA system is redundant when it is widely available elsewhere.

What then becomes the focus of the education process and, specifically, the syllabus for the actuary? The Task Force believes the SoA education system should encompass the following four principles. The system should:

(1) Examine only those subjects that cover essential elements of an actuary's education

(2) Provide a business context with rigor consistent with that of the current mathematical education

(3) Include all kinds of contingencies, not just life contingencies

(4) Include models from outside the insurance and pension fields.

In addition, the Task Force believes that applying these principles suggests a restructuring of actuarial education into four categories, with a defined role for the SoA in each category (as shown in the table):

1. Preliminary Education: subjects that are probably necessary but are not actuarial and are generally taught in universities and colleges. Examinations given in the other categories will explicitly assume knowledge of these preliminary subjects.

2. Basic Education: subjects that are actuarial and encompass significant mathematical rigor along with business knowledge that all actuaries need to master.

3. Advanced Education: subjects that actuaries in a particular field need to master, but that are relatively stable over time and are not primarily country-specific.

4. Professional Development: subjects that are highly specialized, are primarily country-specific, and/or can change quite rapidly. The SoA would require some minimum initial Professional Development content before granting the FSA designation. The Task Force also recognizes that fulfilling the responsibility for professional development will require the individual to take ownership for determining and meeting his/her own needs.

Table 1

Categories of Education	SoA Role
Preliminary Education	Recommend sources and advise
Basic Education	Provide, with explicit testing
Advanced Education	Provide, with explicit testing
Professional Development	Enable (one of many providers)

E&E System Redesign: "Tweaking Won't Do It"

The present E&E system is remarkably *comprehensive*, covering topics from set theory to the Black-Scholes option valuation method, from the normal distribution curve to managed competition. The present system is remarkably *effective*, consistently producing a high quality actuarial professional. And the present system is also extraordinarily *complex*.

Comprising 60 distinct courses and 5 different fellowship specialty tracks, the E&E system is the source of numerous inefficiencies. Students spend hours planning the best path to take from calculus to fellowship. Employers of actuaries spend hours deciphering course catalogs to develop effective study programs. And finally, E&E committees spend hours developing courses and exams, coordinating study material, and maintaining full topic coverage while minimizing overlap.

The desired changes could be implemented by again tinkering with the present E&E system, although it is hard to envision fitting in any more courses. However, the Task Force believes that tinkering is a strategy that would lead to increased *complexity*, *confusion*, and *frustration* for everyone involved—students, employers, and E&E volunteers. A new E&E system could be streamlined and enhanced, but still rooted in fundamental actuarial skills and based on the essential principles of actuarial science.

The Task Force has therefore concluded that the E&E system should focus its educational investment on the core competencies of actuaries. The E&E system has been a proven, effective method of educational testing and training at the preliminary levels of actuarial education. However, the significant effort expended on these courses could be more effectively invested in increasing the coverage of business and modeling topics and in expanding the coverage of contingencies to all kinds of contingencies. In addition, each category of education (preliminary, basic, advanced, and professional development) should be obtained from the best available source. For example, preliminary education should be obtained through recommended undergraduate university and college courses and tested only indirectly through later examinations.

A new entry-level, "attractor" examination would act as a recruiting examination in a manner superior to that of Courses *100* and *110*. The examination would comprise work problems consistent with the challenges actuaries face every day, and might also include case studies of actuarial problems and touch on topics found in standard business school course work. It would have the kind of rigorous mathematical content needed to demonstrate the level of mathematical skills required of an actuary and would also provide some insight into the work of an actuary.

What Will the New Syllabus Look Like?

While the detailed new syllabus is not yet in the design stage, some general comments can be made about how the syllabus would likely evolve.

Calculus, linear algebra, introductory probability and statistics, numerical analysis, and operations research might be labeled Preliminary and, if so, would not be tested by the SoA.

Essential topics such as contingencies, interest theory, survival models, credibility, and loss distributions would be labeled Basic. They would be required courses and therefore would be tested by the SoA. Life contingencies would be expanded to include problems showing applications in various situations involving contingencies, such as survivorship rate of light bulbs, municipal bond default rates, and the like. A business context would be built into all this syllabus material.

The nonmathematical Basic courses would cover the fundamentals of such topics as: valuation of liabilities; financial reporting; pension funding; design, administration, marketing, underwriting, and pricing of financial security programs; employee benefit programs; risk management programs; and banking and securities programs. The Basic courses in investment would cover an introduction to asset management, corporate finance, and the principles of asset/liability management.

Advanced material would build on the Basic material and educate the actuary in a chosen practice area. However, these courses would not require the actuary to master detailed, nation and time-specific legislation or regulation. While some legislative and tax material might be introduced, it would only provide background on the genesis and framework of the systems being studied. Nation and time-specific material would move to Professional Development. It is anticipated that some pre-Fellowship Professional Development requirement would exist. However, fulfilling this requirement would include the flexibility of several alternatives, such as conferences, seminars, and colloquia.

Where Do We Go from Here?

The Task Force believes that this new structure for the E&E system will provide the effectiveness, efficiency, and clarity of purpose needed to meet the educational needs of actuaries for the 21st century and has recommended to the Board of Governors that this new structure be adopted.

The Board strongly endorsed this direction at its January 1995 meeting and encouraged the Task Force and a Design Team to proceed with the full redesign of the SoA's basic education system.

As we proceed with a redesign of the SoA's basic education system, considerable time and effort will be required to ensure that the right steps are taken, the right courses are developed and the fairest provisions are made to protect the legitimate interest of clients and employers as well as current and future candidates.

A Design Team has been formed to develop the syllabus for the new system in accordance with the four principles approved by the Board of Governors. The Design Team will start by determining the subjects that must be covered within the Basic and Advanced categories of the new education system. Each course will be defined in terms of specific objectives, topics, testing methods, credit value, and so forth. The work of the Design Team will be presented to a Review Group of diverse composition for critique/comment. A Board-level task force is charged with overseeing the design and implementation of the new system.

Input from the SoA membership and related constituencies is being actively solicited by the Task Force and the Board of Governors. The ideas generated and obtained from the membership will be given to the Design Team for its use.

The Design Team will report its progress on the proposed new system at each Board meeting. We anticipate that final approval of the proposed syllabus and system will occur in 1996, with full implementation starting in 1998.

Presentations on the redesign of the education system and the work of the Task Force have already been made to several audiences of actuaries: sessions at each of the SoA spring meetings, the Chief Actuaries Forum, and the Nebraska and the San Francisco Actuaries Clubs. Presentations have also been made to the Canadian Institute of Actuaries Council and the Casualty Actuarial Society Executive Council. In total, several hundred interested actuaries have listened to these ideas, and many of them have provided ideas and input.

Support and approval for the general principles and framework guiding the redesign have been strong. Those who have attended and participated in the presentations have endorsed the goals and direction and made positive suggestions.

Naturally, concerns have been raised. The need for early and effective screening to benefit both prospective actuaries and potential employers has been expressed. The need to maintain sufficiently high standards for attaining membership in the SoA has been voiced. Study time and expense are also concerns. These concerns are all valid and will be among the factors carefully considered in designing the new education system and the transition to it.

Q & A

(1) This is a radical change in the education process. How are you going to involve the membership?

We have, are and will continue to solicit input via a series and variety of communication opportunities. We need the ideas of the membership to develop an education system that will provide current and future actuaries with the knowledge and tools needed to practice effectively in the future.

The educational and examination requirements are the responsibility of the SoA Board of Governors, but the support of the membership is essential to the success of such a major effort. The SoA membership and other constituents will have a real voice in this process as it moves forward.

(2) Is the change intended to attract more people to the profession?

The intention is not particularly to attract more people, but rather to do a better job of attracting and educating the people who want to become actuaries. Business skills are needed as well as mathematical skills, critical thinking, communication skills and facility with all aspects of modeling. Enhancing these skills will enable entrants (and seasoned practitioners) to put their skills and knowledge to the most effective use in the future professional environment.

(3) If the SoA no longer tests the rigorous mathematics, how will the high standards of the profession be maintained?

The mathematics will be tested, but in an applied business setting. The new courses, such as those covering all aspects of modeling and all types of contingencies, will be mathematically rigorous. The "attractor" exam as it is envisioned will be rigorous. The shift in focus is not away from mathematics but towards placing the mathematics in a context of real-world actuarial applications. Standards on the mathematical examinations will not be compromised. What is being eliminated is the extensive testing of general mathematical subjects.

(4) Screening has always been a major function of the examinations. How is that changed by the new education system as currently envisioned?

Effective screening will still be a major function of the examinations in the new system. High standards will be established for the Basic and Advanced courses; the result may be fewer but higher hurdles.

An area of concern for employers has been the need for a front-end screen. The "attractor" examination is intended to fill that need, testing candidates on general/fundamental mathematical skills such as calculus, probability and statistics within a real-world context. The candidate passing this attractor exam will thereby have a more realistic sense of the environment in which the actuary operates and will bring to the employer more than a facility with pure calculus and statistics.

(5) What happens to candidates who are currently in the system or ready to start taking actuarial examinations?

The changes envisioned for the education system are not going to take place immediately. Candidates who are ready to start taking examinations should go ahead and start. They may have made significant progress by the time these changes take effect.

One of the principles established to guide the transition process is that candidates should be minimally dislocated by the changes. Transition rules will be equitable in crediting achievement in the current system, and the focus will be on encouraging candidates to take the most essential courses within the new system.

(6) Within the Professional Development category, it appears that there may be an examination component for some candidates. Does the Task Force envision a need to make the examination requirements the same for everyone?

Candidates who need to satisfy the requirements of organizations or agencies outside the SoA may have to write examinations to satisfy those requirements. For example, the enrolled actuaries (EA) examinations are required for pension practitioners in the US. The CIA could have some additional requirements for actuaries practicing in Canada. Such specialty and regulatory topics would be expected to fit within the Professional Development category to meet the professional needs of the individual, not the SoA per se. A strict equivalence of examination requirements within the category is not viewed as necessary. We see this category as a way for the individual to fill in the gaps in his/her own professional education.

(7) What about the potential for overlap or conflict with the CAS? Is that a concern?

One of the guiding principles for the design of the new system is to obtain education from the best available source. For subjects that cover traditional property/casualty topics we regard the CAS as that source and will rely on the CAS for the appropriate courses and examinations. Discussion is under way on developing a joint CAS/SoA course that encompasses risk theory, credibility theory, and loss distributions.

We have had discussions with the leadership of the CAS and have been pleased by their openness/receptivity to the proposed changes. Every effort will be made for the two organizations to work cooperatively as we progress.

(8) How will the new ASA be defined?

There are two likely possibilities for the level of the ASA.

First, the level of ASA requirements could be set at the point most comparable to the July 31, 1995 standard: the required basic mathematical courses (155 credits), the four core courses covering basic practice and investments (100 credits), and 45 candidate-selected elective credits. Under this proposed structure, the comparable level would be at completion of the Basic category and would represent demonstrating attainment of the core knowledge and competencies needed by all actuaries.

Second, the level of ASA requirements could be set at the point that signifies full exposure to fundamental principles within a selected practice area as well as the attainment of the universal core knowledge and competencies. Under this proposed structure, the comparable level for ASA would come at completion of the Basic and Advanced categories of education.

Review of "Risk Loads for Insurers"
PCAS LXXVII, (1990 by Sholom Feldblum)
by Glenn G. Meyers, FCAS

"Risk Loads for Insurers" by Sholom Feldblum

Discussion by Glenn Meyers

1. Introduction

For many years now, a theoretical war has been raging on the subject of risk loads. Some favor the classical premium calculation principles, such as the standard deviation principle, the variance principle or the expected utility principle. Others favor the modern portfolio theories, represented most often by the Capital Asset Pricing Model, also known as the CAPM. Mr. Feldblum presents arguments against the classical premium calculation principles, calling them theoretically unsound, and presents arguments for the Capital Asset Pricing Model.

In this discussion, I will address the same issues as Mr. Feldblum from a different viewpoint. Historically, actuaries have not always derived premium calculation principles from economic and/or statistical assumptions. More often their approach would be to simply state a principle, then check to see if it has desirable properties[1]. While a mathematical derivation from explicitly stated economic principles is certainly desirable, I see no reason why it should be required. I find it difficult to attach much meaning to Mr. Feldblum's use of the term "theoretically unsound" in this context.

However, the list of "desirable properties" can be, and often is, at issue. My personal view is that the list of "desirable properties" should be consistent with competitive market economic principles. Moreover, we should be able to observe behavior in the insurance marketplace which is consistent with these desirable properties.

From this viewpoint I will make the following arguments.

1. The standard deviation principle is not acceptable. It predicts behavior that is opposite of what is observed in the insurance marketplace.
2. The variance principle and the expected utility principle predict some behavior which can be observed in the insurance marketplace, but much is left unexplained.

[1] A recent analysis of this type can be found in "Why Standard Deviation should be replaced by Absolute Deviation" by D. Dennenberg, *Astin Bulletin*, November, 1990, p. 181.

8/4/91

3. *The CAPM was designed as a tool for pricing securities (including those of insurance companies).* While some may argue that it is oversimplified, it provides a tremendous amount of economic insight and predicts behavior which is consistent with activity observed in the securities market. Many, however, try to make the CAPM into a premium calculation principle by treating a line of insurance or even an individual insurance policy as if it were a security in which one could invest. I will argue that such an treatment is inappropriate. I will further argue that many statements made by those who attempt this treatment are inconsistent with behavior observed in the insurance marketplace.

4. Instead of trying to mold premium calculation principles into the framework of the CAPM result, one should apply the principles underlying CAPM to the problem that exists -- calculating premiums. This work has been done. The result is a premium calculation principle called the Competitive Market Equilibrium risk load formula[2].

2. The Classical Premium Calculation Principles

Let X be a random loss faced by a prospective insured. Let μ_X, σ_X and σ_X^2 denote the mean, standard deviation and variance of X respectively. The classical premium calculation principles provide different formulas for calculating the premium, P, to be charged for insurance against this loss[3]. The standard deviation principle can.be stated as:

$$P = \mu_X + \lambda \cdot \sigma_X$$

The risk load, R, for the standard deviation principle is given by $\lambda \cdot \sigma_X$.

The variance principle can be stated as:

$$P = \mu_X + \lambda \cdot \sigma_X^2$$

The risk load, R, for the variance principle is given by the expression $\lambda \cdot \sigma_X^2$.

[2]This formula is described in detail in "The Competitive Market Equilibrium Risk Load for Increased Limits Ratemaking", by Glenn Meyers *PCAS LXXVII*, 1992.

[3]Here, *and elsewhere, parameters which are extraneous to the argument, such as insurer* expenses or initial wealth of the insured are suppressed.

8/4/91

It turns out that the standard deviation principle and the variance principle imply contradictory behavior with respect to excess of loss reinsurance. For a random loss, X, let:

$$X_1 = \begin{cases} X \text{ if } X \leq L \\ L \text{ if } X > L \end{cases} \qquad \text{and} \qquad X_2 = \begin{cases} 0 \quad \text{if } X \leq L \\ X - L \text{ if } X > L \end{cases}$$

We have:

$$X = X_1 + X_2$$

Let ρ be the coefficient of correlation between X_1 and X_2. If $\mu_{X_2} \neq 0$, we have that $0 < \rho < 1$[4].

We have that:

$$\sigma_X^2 = \sigma_{X_1}^2 + 2 \cdot \rho \cdot \sigma_{X_1} \cdot \sigma_{X_2} + \sigma_{X_2}^2 > \sigma_{X_1}^2 + \sigma_{X_2}^2$$

This implies that total risk load is reduced by excess of loss reinsurance for the variance principle.

We also have that:

$$\sigma_X = \sqrt{\sigma_{X_1}^2 + 2 \cdot \rho \cdot \sigma_{X_1} \cdot \sigma_{X_2} + \sigma_{X_2}^2} < \sqrt{\sigma_{X_1}^2 + 2 \cdot \sigma_{X_1} \cdot \sigma_{X_2} + \sigma_{X_2}^2} = \sigma_{X_1} + \sigma_{X_2}$$

This implies that the risk load is *increased* by excess of loss reinsurance for the standard deviation principle.

The fact that excess of loss reinsurance arrangements are common in the insurance business provides evidence that the variance principle predicts results which are consistent with observed marketplace behavior, while the standard deviation principle predicts results which are contradictory to observed marketplace behavior. The insurance industry does not take on the extra expense of reinsurance for the purpose of increasing its total risk.

[4]Shown as part of a demonstration of risk reduction by layering in "Increased Limits and Excess of Loss Pricing" by Robert S. Miccolis *PCAS LXIV*, 1977.

8/4/91

The expected utility principle is generally regarded as the most complete of the classical premium calculation principles. It usually addresses the problem from the point of view of the insured. If the insured, with utility function, u, is faced with a random loss, X, it calculates the risk load, R, as the solution of the equation:

$$E[u(X)] = u(\mu_X + R) \equiv u(P)$$

i.e. the insured is indifferent between the variable loss, X, and the certain premium, $P = \mu_X + R$.

It should be noted that this premium represents the maximum premium the insured will pay for insurance against the random loss. If an insurer offers a lower price, the insured will surely accept it.

The variance principle and the expected utility principle are closely related. Consider the approximation[5]:

$$P \approx \mu_X + \tilde{\lambda} \cdot \sigma_X^2$$

where[6]:

$$\tilde{\lambda} = -\frac{1}{2} \cdot \frac{u''(-\mu_X)}{u'(-\mu_X)}$$

This approximation is based on a Taylor series expansion in which the approximation becomes increasingly accurate as σ_X^2 gets smaller.

The formula is exact in some cases. One example is when the utility function is exponential and the losses have a normal distribution[7].

[5]This expression is derived on page 21 of *Actuarial Mathematics*, by Bowers, Gerber, Hickman, Jones and Nesbitt, Society of Actuaries, 1986.

[6]$\tilde{\lambda}$ will be positive under the usual assumptions that $u' > 0$ (more is better) and $u'' < 0$ (risk averse).

[7]Bowers, *et al, op. cit.*, p. 11.

8/4/91

Thus far, we have addressed utility from the point of view of the insured. We now consider utility theory from the point of view of the insurer. It should be noted that many proponents of CAPM say that it is improper to use utility theory in this context. This will be addressed below.

The minimum premium, G, necessary for an insurance company to voluntarily write an insured is given by[8]:

$$u(0) = E[u(G - X)]$$

i.e. the insurer is indifferent between doing nothing and accepting the uncertain liability, X, in exchange for the premium, G.

If the maximum premium, P, an insured is willing to pay is greater than the minimum premium, G, an insurer must receive, a deal can be made to benefit both parties. Utility theory says nothing about where the final price of the insurance policy will lie between P and G. This is determined by the economic laws of supply and demand. For this reason, it could be said that utility theory provides an incomplete description of insurance pricing.

Insurance Services Office (ISO) originally used a risk load based on the variance principle, but in the mid 1980's it was changed to the standard deviation principle. It is true, as Mr. Feldblum states, that "ISO simply chooses an overall risk load by line of business, and then spreads this risk load by size of policy limit using the standard deviation or variance method." This is done by adjusting the λ parameter so that the average risk load, in ISO's judgment, is reasonable. In describing this practice he uses terms such as "theoretically unsound". It is certainly true that the λ parameter is not derived with the consideration of any kind of utility function, or risk aversion. I tend to think this reflects a narrow view of what is "theoretically sound". ISO has always viewed this as a good practical solution which has been deemed acceptable by many actuaries. It should be mentioned that the Competitive Market Equilibrium risk load formula, referenced below, does provide an explicit justification for its version of this practice.

[8] Bowers, *et al*, *op. cit.*, p. 10.

3. The Capital Asset Pricing Model

The recognition of the ability of individual investors to diversify their investment risk has been the main contribution of the modern portfolio theories. The CAPM models the effect of the ability to diversify on the price of securities. The significance of these models has been recently recognized by the awarding of the Nobel Prize in Economics to three of the originators of the theory.

We begin with an examination of a derivation of the Capital Asset Pricing Model. I have found the following derivation based on a constrained optimization to be particularly illuminating. What follows is a direct quote of the statement of the problem by Thomas E. Copeland and J. Fred Weston[9].

"We assume that portfolio cash flows for the *i*th individual are generated at the end of the period and that they are normally distributed with mean, e_i, and variance, σ_i^2. The *i*th individual's utility is a function of the mean and variance of his end-of-year cash flows. His utility function is written

$$U_i(e_i, \sigma_i^2)$$

"We further assume that the marginal utility of expected cash flows is positive, and the marginal utility of the variance of cash flows is negative.

$$\delta U_i/\delta e_i > 0, \qquad \delta U_i/\delta \sigma_i^2 < 0$$

"Finally, all assets are marketable and infinitely divisible, transactions costs and taxes are zero, and there are no constraints on short sales. The expected end-of-period cash flows to an individual are the payments from risky assets less any interest on debt:

$$e_i = \sum_j X_{ij} \cdot E[\tilde{D}_j] - r \cdot d_i$$

where

X_{ij} = fraction of *j*th firm held by the *i*th individual.

r = $(1 + R_f)$, where R_f is the one period risk-free borrowing/lending rate.

d_i = the net personal debt issued by the *i*th individual.

$\tilde{D}_j$ = net end of period cash flow paid by the *j*th firm.

[9] Copeland, T. E., and Weston, J. F., *Financial Theory and Corporate Policy*, Addison-Wesley, 1979, Appendix to Chapter 7: An Alternative Derivation to the CAPM.

"The variance of the end of period cash flows for the *i*th individual is

$$\sigma_i^2 = \sum_j \sum_k X_{ij} \cdot X_{ik} \cdot \mathrm{Cov}[\widetilde{D}_j, \widetilde{D}_k]$$

"The individual investor's problem is to find the set of weights, X_{ij}, and borrowing, d_i, which maximize his expected end-of-period utility subject to his budget constraint.

$$\underset{X_{ij},\, d_i}{\mathrm{MAX}}\ EU_i(e_i, \sigma_i^2)$$

subject to

$$\sum_j X_{ij} \cdot V_j - d_i = W_i$$

where

V_j = the total market value of the *j*th firm at the beginning of the period.

W_i = the total wealth of the individual at the beginning of the period."

The derivation of the CAPM assumes that all investors behave in the manner described above, and that the market is in equilibrium. The equilibrium value of the *j*th asset is then demonstrated to be:

$$V_j = \frac{1}{r} \cdot \left(E[\widetilde{D}_j] - \theta \cdot \mathrm{Cov}[\widetilde{D}_j, \widetilde{D}_m] \right) \tag{1}$$

where

θ = $(E[\widetilde{D}_m] - r \cdot V_m)/\mathrm{Var}[\widetilde{D}_m]$

$\widetilde{D}_m$ = the cash payouts for all firms in the market.

V_m = the value of the market portfolio at the beginning of the period.

The above equation can be converted into rates of return if we define the rate of return on the jth asset as

$$\widetilde{R}_j = (\widetilde{D}_j - V_j)/V_j \tag{2}$$

Using Equation 2 in Equation 1, we obtain

$$E[\widetilde{R}_j] = R_f + \lambda \cdot \mathrm{Cov}[\widetilde{R}_j, \widetilde{R}_m] \tag{3}$$

where

$\widetilde{R}_m = \sum_j V_j \cdot \widetilde{R}_j / V_m.$

$\lambda \quad = (E[\widetilde{R}_m] - R_f)/\mathrm{Var}[\widetilde{R}_m].$

Equation 3 is the familiar CAPM.

As noted above, the CAPM was put forth as a model to explain the price of securities. Mr. Feldblum, along with many others, has tried to use the CAPM to calculate risk loads for insurers. I believe this attempt has failed. I offer two complaints.

My first complaint has to do with the treatment of risk as it applies to insurers. The most direct statement of the prevailing sentiment by proponents of CAPM is given by Cummins who states[10]: "Firms should not be risk averse." The reasoning behind such a statement is that individual investors can "eliminate this type of risk by holding diversified portfolios." The implication of such a statement is that an insurance firm should be indifferent between insuring low and high limit policies.

By my own observations, and by the observations of others, managers of insurance firms are risk averse. The existence of reinsurance provides an objective verification of these observations.

Another way to view this complaint is to note that much of the risk that insurers face is deemed "diversifiable" and CAPM proponents claim that the market should not reward such risks. Examples given of such diversifiable risks include the risk faced by insurers who accept high limit policies.

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[10]Cummins, J.D., "Asset Pricing Models and Insurance Ratemaking", *ASTIN Bulletin*, November, 1990, p.125.

8/4/91

The flaw in these statements can be addressed by the CAPM itself. Nowhere in the above development of the CAPM is one required to label a particular risk as being diversifiable or non-diversifiable. Diversification of the investor's risk is a result of the investor finding the optimal X_{ij}'s. Consider the case when there are two otherwise identical insurance firms, Firm #1 which writes only low limit policies and Firm #2 which writes only high limit policies. Each firm uses the same percentage risk load. Since the variability of the results will be less for Firm #1, each risk averse investor will compete more for Firm #1's securities. Firm #1 will have a higher value. The only counter to this problem is for the managers of Firm #2 to charge a higher risk load.

My second complaint is about the allocation of surplus. In their attempt to use CAPM to calculate risk loads, proponents treat a line of insurance as if it were a free-standing insurance company. It is then maintained that multiline insurers can allocate their surplus by line of business, and all will be well. What is missing from the argument is an acceptable method of allocating surplus by line of insurance. Most methods used by insurance regulators for allocating surplus simply choose an arbitrary premium to surplus ratio. Others, including myself, believe that it is not appropriate to even attempt to allocate surplus. The case against allocating surplus was made most eloquently at the March 1990 CAS Ratemaking Seminar by Charles F. McClenahan who, after noting that the purpose of surplus was protection against insolvency, stated "The protection against insolvency afforded by a \$100 million dollar surplus for a free-standing automobile insurance company is not comparable to the protection afforded by a multiline insurance company with \$100 million dollars of surplus allocated to automobile insurance."

4. The Competitive Market Equilibrium Risk Load Formula

The case has been made that it is inappropriate to adapt the CAPM to calculate risk loads for insurers. In this section, I will discuss an alternative model, called the Competitive Market Equilibrium (CME) risk load formula. Curiously enough, its development parallels the above development of the CAPM. Since it is being described in a separate paper, I will only show the parallels between its development and the CAPM.

Both the CAPM and the CME risk load formula assume that each market participant (investor or insurer) behaves in a "rational" manner. By matching supply with demand, an equilibrium price for each product (security or insurance policy) is calculated. What follows are the definitions of "rational".

8/4/91

CAPM Statement of Individual Investor's Problem

The individual investor's problem is to select investments in such a way as to maximize his utility subject to a constraint on his total wealth.

CME Statement of Insurer Management's Problem

The insurer management's problem is to select amounts of exposure in lines of insurance and policy limits in such a way as to maximize the total risk load subject to a constraint on the variance of the insurer's book of business.

The CME risk load formula addresses most of the shortcomings of the premium calculation principles described above. It provides a more complete description of the premium than that provided by utility theory. Since each insurer has a constraint on the variance of its book of business, the insurer is assumed to be risk averse. Since each insurer chooses the amount of exposure for each line of insurance, it is not necessary to allocate surplus by line of insurance.

Many of these issues are discussed more fully in the CME paper.

8/4/91

Geographic Rating of Individual Risk Transfer Costs Without Territorial Boundaries

by Randall E. Brubaker, FCAS

Notice: A patent application is pending with regard to the procedure described in this paper. The rights to use this procedure may be subject to restriction.

"Geographic Rating of Individual Risk Transfer Costs Without Territorial Boundaries"

Abstract

This paper describes a geographic ratemaking procedure that does not require territories or territory boundaries. The procedure develops a unique rate for every point on the map. The result can be visualized as a smooth surface over a map, with the height of the surface at any point representing the rate for that point. Abrupt changes in rates such as those which occur at territory boundaries are eliminated, though "natural boundaries" can be provided on an exception basis.

The procedure described uses massive computing power (as available on personal computers), and geo-coded loss data. Policy rating requires use of a personal computer, and geographic software to determine latitude/longitude for a risk.

The paper discusses credibility and data-weighting concepts, and determination of the effect of a rate change. It also compares the traditional territorial ratemaking model to the proposed method in the context of a generalized model for determining rates for individual locations.

An example of the described method is provided based on an adaptation of zip code data.

Introduction

The third principle of ratemaking of the Casualty Actuarial Society is "a rate provides for the costs associated with an individual risk transfer." Risk location is known to be an important determinant of the cost of an individual risk transfer for certain types of insurance. This paper discusses the estimation of cost of an individual risk transfer, as it would be done one risk at a time, taking into consideration the unique geographic location of each risk. A ratemaking procedure is proposed that effectively repeats this process for all locations. The proposed ratemaking procedure does not use rating territories.

A ratemaking procedure that evaluates costs for every location will likely result in rates that respond gradually to changes in geographic location. Two risks not far apart will in most cases largely share similar influences of geographic location on expected losses. Thus expected losses evaluated separately for two close-by risks should not be much different and rates should change gradually in response to location. This will eliminate the discontinuities that occur at territorial boundaries. There may be situations in which "natural boundaries" such as a river separate risks that are close-by in distance, and the rate development procedure may need to recognize these particular situations. As will be described below, it is possible in the calculation of rates for individual geographic points to select the data that is most appropriate for the calculation of a rate at each point.

The proposed procedure for geographic pricing implies massive data processing requirements. It is necessary to perform separate calculations of indicated rates for many different geographic locations. It isn't necessary (and would be impossible) to calculate rates for all points, however. Rates may be calculated for a finite number of evenly-spaced points close enough together that an interpolation procedure is reasonable to determine rates for points in between. By this approach only a finite number of experience-based rate calculations are necessary.

The development of massive computing power (even in a desk-top PC) is one of the developments of the last decade or so that makes this proposed approach to geographic pricing practical.

An additional capability necessary to determine premiums based on individual risk location is the means to rate policies without the need for the policy-rater to manually determine latitude/longitude for each risk. The recent development of geographic software offers a practical solution to this requirement. Software is available that will determine latitude/longitude based on street address, and/or based upon indication by the policy rater of risk location on a map shown on a computer screen. These capabilities make it possible to determine an individual risk location's premium based on knowledge of street address, and/or indication of map location.

Ratemaking for Geographic Points

The orientation towards insurance rate-making offered in this paper is most different from current methods in that the objective is to develop rates for specific geographic points, as opposed to territories. It also is necessary to develop a ratemaking algorithm that the computer will use to calculate rates for a large number of points. As described above, the end product of the ratemaking process will be rates for a large number of evenly spaced pre-determined points, and an interpolation procedure that allows determination of a rate for any point that lies among the predetermined points.

Before proceeding further, the definition of a term will be useful. The predetermined points for which rates are calculated by the ratemaking algorithm will be called "grid" points.

The first task for an actuary in use of this procedure will be to establish the grid points. Since rates for points in between grid points will be based on interpolation, the consideration that must be made is how far apart the grid points may be for interpolation among the grid points to still be reasonable. This will depend upon the degree of variation in expected loss based upon location. This judgment may vary among different regions of a state. In metropolitan areas where expected loss varies over relatively short distances, grid points may need to be only a mile or less apart. In rural areas five or ten miles may be acceptable.

The number of grid points will affect the computing resources necessary. For a state 200 miles-square, grid points one mile apart will mean that the ratemaking algorithm must be repeated 201 x 201 times, or 40,401 iterations.

The next step in the ratemaking process is to provide the computer instructions on what loss and exposure data to use for calculation of the rate for each grid point. The procedure described here assumes that available data consists of historic geo-coded individual risk loss and exposure data. (The term "geo-coded" means that latitude and longitude for each risk location are part of the statistical loss and exposure records). Selection of data for calculation of a rate for each grid point should be based on criteria that indicate expected similarity to the grid point in terms of expected loss per unit of exposure. The most obvious criteria for similarity is geographic proximity. This can be translated into a rule for the ratemaking algorithm such as "for each grid point, calculate a rate using all loss and exposure data within a ten mile radius of the grid point." More sophisticated criteria are possible. If a radius criteria such as the above example is used, the radius could be different for different grid points. The geographic shape of the data set used for each grid point also does not need to be a circle. The actuary is free to use judgment in deciding what data to use for each grid point. There may be data from regions of a state relatively far away that may be considered useful for ratemaking for a particular grid point, or set of grid points. There also may be data close-by that is considered not useful.

Data for calculation of a rate for a grid point may also be selected from other locations based on criteria other than geographic. For example, it may be decided that population density is a useful criteria for similar expected loss. Data for calculation for a grid point could be selected on the combined criteria of geographic proximity and similar population density.

"Natural boundaries" may be dealt with in selection of data for a grid point. The computer could be instructed that for grid points in a certain region, data from an inappropriate region shall not be used.

Another way in which the ratemaking algorithm can be refined is by varying the weight assigned to data records selected for each grid point based upon a relative similarity criteria. For example, if data within a certain radius of a grid point is used it may be desired that data closer to the grid point within that radius receive a greater weight than data from further away. Carrying the example further, the actuary might choose a ten-mile radius, and also decide that data nearest the grid point should have three times the weight of data ten miles away. Data at points in between could have weights with appropriately varying proportions. A formula weighting scheme of this type will be described in the next section of this paper. Varying weights can also be based on criteria other than distance, e.g. similarity of population density.

A Data Weighting Example

One form of weight for individual loss and exposure records used in the calculation of the rate for a grid point would be a fraction with distance from the grid point in the denominator. Such a weight would decrease as distance from the point being rated increases. A general formula for such a weight is

$$W=\left(\frac{1}{D+1}\right)^{P}$$

W in the expression above is the weight assigned to a particular loss and exposure record. D is the distance from the location of the loss and exposure record to the grid point being rated. P is an exponent that varies the sensitivity of W to D. If P is near zero, all W's will be close to 1. As P increases, the sensitivity of W to D increases. The actuary will want to choose a P value that varies weight based on distance in a reasonable manner. Further discussion of this is provided in Appendix A.

The quantity +1 in the denominator above is present to prevent division by zero in case of a data point having the same location as a grid point. This quantity also prevents an inordinately large weight being given to a data point that is only a very small distance from the grid point.

The quantity D, or distance from the data point to the grid point can be calculated in the following manner based on latitude/longitude.

$$D = 3958 \text{ miles} \times \text{Arc Cos}\,[(\sin a \sin b) + (\cos a \cos b \cos g)]$$

where a = latitude of the grid point
b = latitude of the data point
g = degrees of longitude between the grid point and the data point.

The distance 3,958 miles is approximately one radian on the earth's surface. The formula above is adapted from a formula on page 35 of Elements of Cartography, a cartography textbook by Arthur Howard Robinson (Wiley, New York, 1969).

After W's are determined for all individual data records, each W would be divided by the sum of all W's.

The example above assumes that there is exactly one exposure unit associated with each data record. If the number of exposure units varies then the formula above would be used to assign a weight to each exposure unit.

Credibility

Credibility considerations and procedures should apply to ratemaking for points in the same manner as for ratemaking for territories. "Full credibility" criteria generally are based on the volume of historic data necessary for an estimated rate to have a specified probability of falling within a desired percentage of the true expected loss per unit of exposure. Credibility criteria for rates for points should be the same as those rates for territories, unless an uneven weighting of data based on relative similarity is used. An uneven weighting will increase the expected variance of an estimated rate, for a given amount of data. It follows that uneven weighting will require an increase in data required for full credibility. An example of the sensitivity of a credibility standard to weighting is discussed in Appendix A. In this example, a credibility standard is shown to increase by a factor of 1.1 to 2.05 depending on the degree of unevenness of weighting.

A credibility procedure requires an alternate rate indicator against which to apply the complement of credibility. In ratemaking for points, there are a number of candidates for an alternate indicator, including the following:

- The prior rate for the grid point, trended to current cost level (this is possible only if this ratemaking procedure has been used before).

- A prior rate for a grid point based upon a territorial rate structure, trended to current cost level (see the section below - "Use of Summarized Data").

- An indicated rate based on a relationship of expected losses to a variable other than location, e.g. population density.

Other alternate indicators may exist.

Supplemental Smoothing

After development of rates for grid points, and application of credibility procedures, it may be desirable to view a three-dimensional representation and/or contour charts of a "surface" made of the rates for the grid points. This will illustrate if there is "bumpiness" in the surface due to apparent randomness of underlying claim experience. If this is the case then supplemental smoothing of the grid point rates may be desired. This could be done by capping the influence of large claims in the underlying data and recalculating the rates. It would also be possible to smooth a surface of the grid points by running the grid point rates themselves through the rate calculation algorithm. It is anticipated that additional smoothing procedures will be developed as experience is gained using the general approach proposed in this paper.

Use of Summarized Data

If a company does not have geo-coded individual risk data, the procedure described in this paper can be applied to loss and exposure data that is summarized by geographic areas such as zip codes or rating territories. Generally, this can be done by establishing evenly - spaced simulated data locations over the area to be rated, and apportionment of the summarized data to the simulated locations. The simulated data could be based upon territorial data such as losses or territorial rates, or it could be zip code data. An example of this procedure is included in this paper.

Loss Development, Trend, Class Distribution

Loss development and trend must also be part of the ratemaking process, and may be taken care of either by adjusting the individual risk data appropriately, or by uniformly adjusting the rates for all grid points at the end of the process to balance to an aggregate indication that incorporates development and trend.

As in territorial ratemaking, there may be an inter-relationship between location of risks and non-geographic rating factors such as driver class for automobile or construction class for property. If this is true then adjustment of loss experience to offset these factors before calculation of the rates for the grid points would be appropriate. The procedures that are used to address this issue for territorial ratemaking should also apply to the calculation of rates for grid points.

Calculation of the Effect of Rate Changes

With the proposed rating procedure it will still be desired to know the overall effect of a rate change. Also, it may be desired to determine premium at present rates for the purpose of determining a rate indication. Even though each risk has its own unique rate based on this procedure, there is a way to calculate the effect of rate changes or premium at present rates other than re-rating all historic data. The procedure to do this relies on the recording of exposure units allocated to each grid point. While every risk in the rating process gets its own rate based on interpolation among the rates for grid-points, the rating process still generates exposure data assignable to grid points. This is because by the interpolation process every risk's rate is a weighted combination of rates from grid points. The identity of the grid points used for each risk, the exposures of the risk, and the weights assigned to each grid point can be retained as statistical data. The sum of weighted exposures so recorded for each grid point for all risks rated will be the exposures allocated to each grid point to be used in developing the effect of a rate change, or in calculating premium at present rates. With exposures allocated to grid points, the effect of a rate change can be calculated as the exposure-weighted average of the changes at the grid points. Similarly premium at present rates can be calculated by extending historic exposures recorded for the grid points by the current rates for the grid points.

Policy Rating - An Interpolation Example

Suppose a rate is desired for a location with latitude/longitude (x,y), and (x,y) lies among four grid points with latitude/longitude as shown below:

.a,b		.a,c
	* (x,y)	
.d,b		.d,c

Let $R_{(x,y)}$ represent the rate for the point (x,y), and assume that rates for the four grid points have been determined based on loss and exposure data.

A formula that can be used to calculate a rate for the point (x,y) is

$$R_{(x,y)} = R_{(a,b)}\left(\frac{d-x}{d-a}\right)\left(\frac{c-y}{c-b}\right) + R_{(a,c)}\left(\frac{d-x}{d-a}\right)\left(\frac{b-y}{b-c}\right) + R_{(d,b)}\left(\frac{a-x}{a-d}\right)\left(\frac{c-y}{c-b}\right) + R_{(d,c)}\left(\frac{a-x}{a-d}\right)\left(\frac{b-y}{b-c}\right)$$

The above formula can be thought of as a simultaneous two-way linear interpolation. It gives reasonable answers but there may also be other ways of interpolating. It seems likely that any two reasonable interpolation methods will give similar answers, considering that the rates at the four grid points should not be significantly different.

A Generalized Model and Territorial Rates as a Specific Case

A general model for determination of rates for specific points based on historic claim, exposure, and location data will now be described. It will also be shown that territorial rating as a means to develop rates for specific points is a special case of this model.

Let the coordinates (x,y) specify the latitude and longitude for the point to be rated.

Let it be assumed that loss and exposure data are available from N previously recorded insurance contracts.

For each contract the following data is available:

L_i denoting losses incurred for contract i

$(x,y)_i$ denoting the coordinates for the geographic location of the earned exposure for contract i.

It is assumed that there is one unit of exposure for each historic contract.

The general model for determination of a rate $R_{(x,y)}$ for a point (x,y) is

$$R_{(x,y)} = \sum_{i=1}^{N} W_{(x,y)_i} L_i$$

with

$$\sum_{i=1}^{N} W_{(x,y)_i} = 1$$

where $W_{(x,y)i}$ is the weight assigned to data from contract i in determination of the rate for the point (x,y). Note that the $W_{(x,y)i}$ may be different among the various contracts. Also, a different set of weights may be assignable among the recorded contracts for every different (x,y) location being rated.

The Traditional Rating Territory Model

In the traditional model, rates are developed based on geographic location by grouping data into mutually exclusive territories. Assuming full credibility, the rate for all points in a territory is based on the total historic losses divided by total historic exposures for all contracts provided in the territory.

By this procedure

$$R_{(x,y)} = \frac{\sum_{i=1}^{N_j} L_i}{N_j}$$

where (x,y) is in a territory which will be designated as territory J, and the first N_j historic contracts are those located in territory J.

The expression above can also be written as

$$R_{(x,y)} = \sum_{i=1}^{N_j} \left(\frac{1}{N_j} \right) L_i$$

which is equivalent in the general model to defining

$$W_{(x,y)_i} = \frac{1}{N_j}$$

for contracts that were in Territory J, and

$$W_{(x,y)_i} = 0$$

for contracts in other territories.

For contracts located in the territory that (x,y) happens to be in the weights are equal, and for contracts in other territories the weights are zero. There is no use of data outside of territory J, and no distinction among data points within territory J.

If a territorial rate is the credibility weighted combination of a territory indication and a statewide indication, the calculation for a rate at a point in territory J is

$$R_{(x,y)} = Z\frac{\sum_{i=1}^{N_j} L_i}{N_j} + (1-Z)\frac{\sum_{i=1}^{N} L_i}{N}$$

which can be written

$$R_{(x,y)} = Z\sum_{i=1}^{N_j}\frac{1}{N_j}L_i + (1-Z)\sum_{i=1}^{N}\frac{1}{N}L_i$$

By rearrangement and combination of terms of the summations the above expression is equivalent to the general model with $W_{(x,y)i}$ equal to a uniform value for contracts within territory J, and another uniform value for contracts outside territory J.

The above discussion provides a framework for comparison of the territorial model to other types of models in considering which might be best for developing rates that accurately estimate expected losses for specific locations. It is apparent that the territorial model is appropriate in situations where there are two uniform degrees of relevancy of data to the point being rated. Also, these two "degrees of relevancy" are independent of the location of a point being rated within a defined territory. This type of rating could be appropriate in a situation where whether or not a point (x,y) belongs within a particular defined territory is the only significant influence on expected losses. This might be the case if a location in a particular political subdivision is the only relevant geographic consideration in determination of expected losses. If the influence of geography on expected losses is more complex than a "two-weight" model allows for, then the generalized model opens the door to other alternatives.

Expense/Profit and Catastrophe Loads

Rates for grid points may be first calculated as expected loss rates. Then expenses and profit may be added on a flat dollar basis and/or a percentage load. Coverages that include catastrophe perils will need an element for expected catastrophe losses. This could be developed from a catastrophe simulation model that develops expected loss rates at individual geographic points, or by using the catastrophe element that is built into an existing territorial rate structure.

Rate Manual Format

It is anticipated that rating of policies using this procedure will be done using a Personal Computer. However a rate manual specifying the rate structure of the company will probably be necessary at least for rate filing purposes. The geographic base rates in a rate manual would consist of a listing of the grid points and their rates. An interpolation rule for rating of policies located in between the grid points would also be part of the rate manual. The rest of the rate manual need not be different from the current format. Only the determination of geographic base rates would change.

Communication Tools

A company using this pricing procedure will need to develop new ways of communicating with regard to rate levels.

Here are some tools that should be useful.

Contour maps of rate levels.
Contour maps of rate change percentages.
Contour maps of premium comparisons.

Average rates and average rate changes over geographic areas of interest.

Maps of grid points and rates at grid points.

Rates and rate changes at individual grid points.

Any of the above maps should be producible as overlays over street maps. This will facilitate use of the maps for underwriting and marketing management.

An Example Using Zip Code Data

If geo-coded individual risk data is not available to a company, an alternative for use of the procedure described in this paper is to use zip code data of exposures and losses. Zip Code areas generally are a finer geographic breakdown than are most territorial rating structures, and much of the refinement in geographic pricing developed by this procedure can be achieved with zip code data.

An example of use of zip code data is provided here, using Private Passenger Automobile Bodily Injury zip code data for 1991 for the industry obtained from the California Insurance Department. The geographic area for which the example is developed is the San Francisco Bay Area. The example to be developed includes use of the data to determine pure premiums for a company's rating territories, so that the results of territory ratemaking can be compared to the procedure described in this paper.

Exhibit 1 attached to this paper is a zip code map of the Bay Area, with the industry pure premium (losses divided by exposure) by zip code shown on each of the zip code areas. Exhibit 2 is a map of average pure premiums for the rating territories of a well known auto insurer, based on this industry data. This company defines their territories using zip codes. (The territory definitions were obtained from documents available to the public at the California Department of Insurance). The pure premium shown on Exhibit 2 for each of the territories is an exposure-weighted average of the zip code pure premiums shown on Exhibit 1, for the zip codes within each territory.

Exhibit 3 attached illustrates the first step in the procedure described in this paper, which is to determine the "grid points" for which rates will be determined. For this example, the intervals between grid points are four-tenths of a mile each.

Exhibit 3 shows the grid points over zip code boundaries. For this example, each grid point is assumed to be a location of risk data as well as a point for which we will determine a rate. For each assumed data location, we will assign an observed pure premium equal to the pure premium (loss/exposure) of the zip code area that the grid point is within.. To each assumed data location we also assign a number of exposures equal to the total exposures for the zip code that the data point is within, divided by the number of data points in the zip code.

The next step in this example is to determine a radius around each grid point from within which data will be used to calculate a rate (pure premium) for each grid point. For this example we used a radius of one and one-half miles. We also used the weighting formula based on distance described earlier in this paper, with a P value (exponent) of 1.0.

Exhibit 4 attached is an illustration of one grid point, (at the longitúde, latitude shown on the exhibit) and the simulated data points around it that are used to determine a rate (i.e. fitted pure premium) for that grid point. A similar picture would apply for all other grid points in this example.

Exhibit 5 details the calculations of the rate for the grid point illustrated on Exhibit 4. All assumed data locations that are used to calculate the rate for the grid point are listed, with their distances from the grid point and weights based on distance and exposure. The total in Column (8) is the rate for the grid point. Rates for all other grid points are similarly calculated.

Exhibit 6 is a contour chart of the rate surface that results from rates calculated for every grid point as illustrated on Exhibit 5. The rates for points in between the grid points are assumed to be based on interpolation. Exhibit 6 also shows the territorial rates shown on Exhibit 2. The territorial rates may be compared to the surface contours, with regard to which is higher or lower, and with regard to how the rates vary by location.

It may be noted that the one and one-half mile radius used for this example was chosen in recognition that the example area is densely populated, and that substantial variation of cost by location may occur. As to credibility, use of more than one year of data would be appropriate for an actual application. Also, credibility procedures can be used as discussed above. To illustrate the volume of data used for a grid point in this example, it is estimated that 946 claims were within the circle shown around the grid point in Exhibit 4. For the zip codes that are only partially within this circle, this estimate includes a proportion of the total claims in each zip code equal to the proportion of data points of the zip code within the circle.

This example includes use of a "natural boundary" in creation of the pure premium surface. Alameda Island, which lies near Oakland in the East Bay, is isolated from the nearby mainland in a manner that could be expected to develop a distinct difference in expected losses. This island has its own zip code, and the data used for the grid points on this island was restricted to be from that zip code only. Also, data from this zip code was excluded from use for any grid points on the mainland. An area known as "Bay Farm Island" lies just below Alameda Island. For the purpose of this example we used the same pure premium as for Alameda Island, instead of the zero shown on Exhibit I. We eliminated rate contours over the area of the Oakland airport.

Summary and Conclusion

The geographic insurance pricing procedure described in this paper offers a new approach to insurance pricing based on geographic location. It eliminates the need to determine territorial boundaries, and also eliminates the discontinuities in territorial rates that occur at boundaries. It should be a more accurate procedure of evaluating insurance costs in relation to location, if such costs vary in a gradual manner. The procedure is made possible by the development in the last few years of massive computing power, and geographic software.

Weighting of Data
Based on Relative Similarity

This appendix discusses the choice of P in the weighting formula

$$W=\left(\frac{1}{D+1}\right)^{P}$$

Generally, the larger P is the more weight will be assigned to nearby data points in calculation of the rate for a grid point. This will increase the sensitivity of rates for the grid points to nearby experience, but it will also decrease credibility and increase random fluctuations in estimated rates.

To illustrate how P effects W, the following table shows how W is affected for data 0 to 9 miles from a grid point by varying P. The W values below have not been adjusted so that their sum is 1. This would be done in an actual application.

			W Values		
Miles from Grid Point	P=0.01	P=0.2	P=0.6	P=1.0	P=2.0
0	1.000	1.000	1.000	1.000	1.000
1	1.000	.871	0.660	0.500	0.250
2	.999	.803	0.517	0.333	0.111
3	.999	.758	0.435	0.250	0.063
4	.999	.725	0.381	0.200	0.040
5	.998	.699	0.341	0.167	0.028
6	.998	.678	0.311	0.143	0.020
7	.998	.660	0.287	0.125	0.016
8	.998	.644	0.268	0.111	0.012
9	.998	.631	0.251	0.100	0.010

A table such as above should aid an actuary in determining a reasonable P value. For example, more sensitivity of weighting to distance is probably desirable than results from P=0.01. Less sensitivity of weighting is probably desirable than results from P=2.0. For P=2.0, data from only two miles away gets only 11.1% as much weight as data very close by. Based on examination of the table above, an actuary might choose a P value in the area of 0.5 to 0.7. The choice of a P value must ultimately be based on a judgment of the relative value of data at varying distances from a point being rated. Such a judgment could also vary between rural and urban points.

The higher the P value, the more uneven is the resulting weighting. This will decrease credibility because it will increase the variance of an estimated rate. Under traditional territorial ratemaking, a rate indication is developed from the sum of recorded losses divided by the number of exposure units. If N is the number of exposure units and σ is the standard deviation of loss for a unit of exposure, the standard deviation of the sum of recorded losses divided by N is $(1/\sqrt{N})\,\sigma$. This assumes even weighting of the observed loss and exposure records, and independence of the loss and exposure records.

A more general formula for the standard deviation of the weighted sum of observed losses is

$$\left(\sum_{i=1}^{n} (W_i)^2\,\sigma^2\right)^{1/2}$$

$$with \qquad \sum_{i=1}^{n} w_i = 1$$

The above expression is minimized by having all Wi's equal 1/N. If the Wi's are uneven, the standard deviation is larger than the minimum.

If the Wi's are uneven and the distribution of exposures by distance from a point being rated is known, it is possible to determine the increase in data required for full credibility. As an example, if exposure units are approximately uniformly distributed over a circle of radius nine miles, and a P value of 0.6 is used in the weighting formula above, it takes approximately 10% more exposure units for the standard deviation of the unevenly weighted indication to be as low as for an evenly weighted indication.

The increase in exposure units needed goes up sharply as P goes up, as shown in the table below:

P Value	Increase in Exposure Units
0.6	+10%
0.8	+25%
1.0	+45%
1.2	+85%
1.4	+105%

The spreadsheet on the following page shows how the increase in exposure units above was determined for P=0.6. The same spreadsheet was used for the other P values. While uneven weighting can be expected to always increase the amount of data needed to meet credibility standards, the percentage increase needed can be expected to vary depending upon the weighting formula used, and the geographic distribution of data.

EFFECT OF UNEVEN WEIGHTING ON CREDIBILITY STANDARD

Miles from Point Grid	# of Risks	Weights Based on Distance (1/((1)+1))^P	Weighted # of Risks (2) x (3)	Normalized Weights (3) / (4 Tot)	Sum of the Variances (2)x(5)^2	Increased # of Risks (2)x(13)	Weighted # of Risks (3) x (7)	Normalized Weights (3) / (8 Tot)	Sum of the Variances (7)x(9)^2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0	10	1.000	10.0	0.002988	0.000089	11	11.000000	0.002716	0.000081
1	30	0.660	19.8	0.001971	0.000117	33	21.771881	0.001792	0.000106
2	50	0.517	25.9	0.001546	0.000119	55	28.450502	0.001405	0.000109
3	70	0.435	30.5	0.001301	0.000118	77	33.516197	0.001182	0.000108
4	90	0.381	34.3	0.001138	0.000116	99	37.692348	0.001034	0.000106
5	110	0.341	37.5	0.001020	0.000114	121	41.294729	0.000927	0.000104
6	130	0.311	40.4	0.000930	0.000112	143	44.491517	0.000845	0.000102
7	150	0.287	43.1	0.000858	0.000110	165	47.383807	0.000780	0.000100
8	170	0.268	45.5	0.000800	0.000109	187	50.037557	0.000727	0.000099
9	190	0.251	47.7	0.000751	0.000107	209	52.498426	0.000682	0.000097
TOTAL	1,000		334.7		0.001113	1100	368.136964		0.001012

Std. Dev. of Est. Rate	Evenly Weighted (11a)	Unevenly Weighted (11b)	Adjusted Weighted (11c)
	0.031623	0.03336325	0.031811

INPUTS:

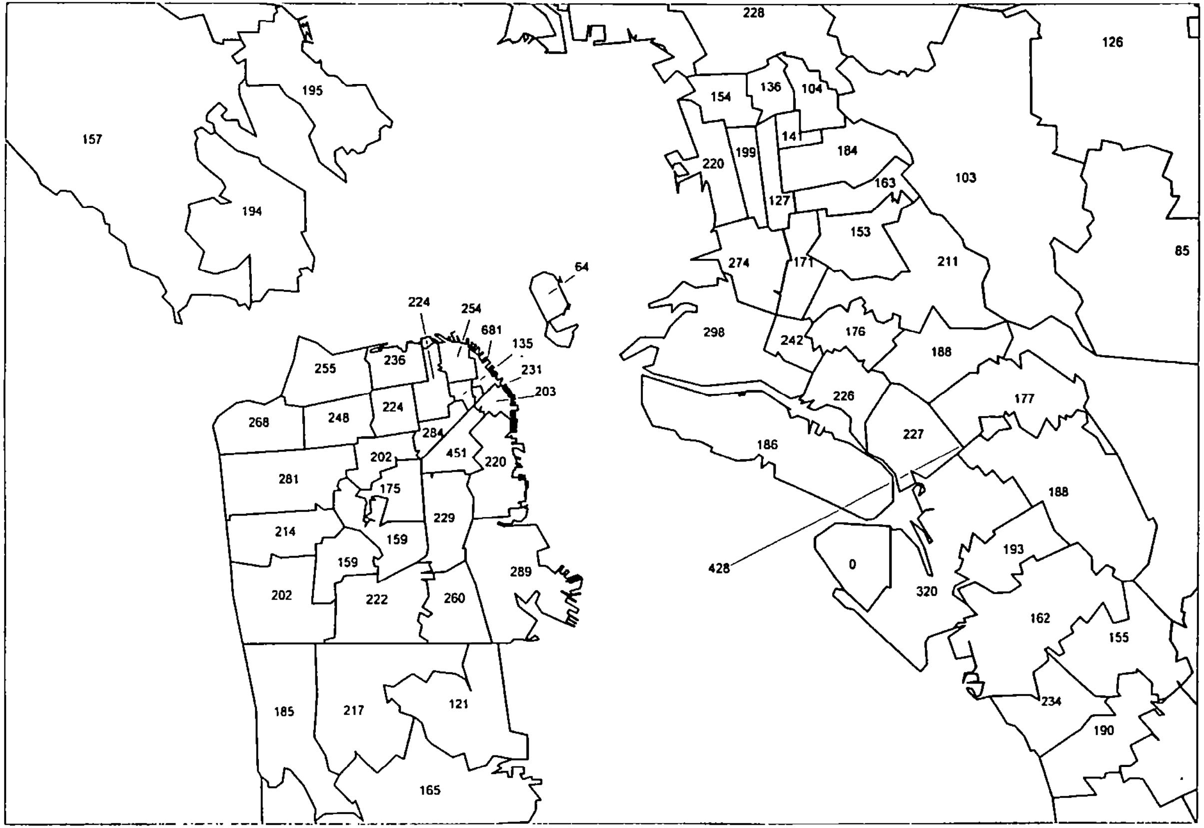
P-Factor (12)	# Risks Increase (13)
0.6	1.1

Notes:

(a) This exhibit assumes that the variance of loss experience for an individual risk equals 1.
(b) Item (11a) = 1 / 1000^.5
(c) Item (11b) = ((6) Total)^.5
(d) Item (11c) = ((10) Total)^.5
(e) Item (13) (# Risks Increase) is determined by trial and error such that Item (11c) is approximately equal to Item 11(a).

SAN FRANCISCO AREA INDUSTRY PURE PREMIUMS BY ZIP CODE BODILY INJURY 1991

Exhibit 1



—— 1994 ZIP CODE BOUNDARIES

Exhibit 2

SAN FRANCISCO AREA INDUSTRY PURE PREMIUMS BY RATING TERRITORY BODILY INJURY

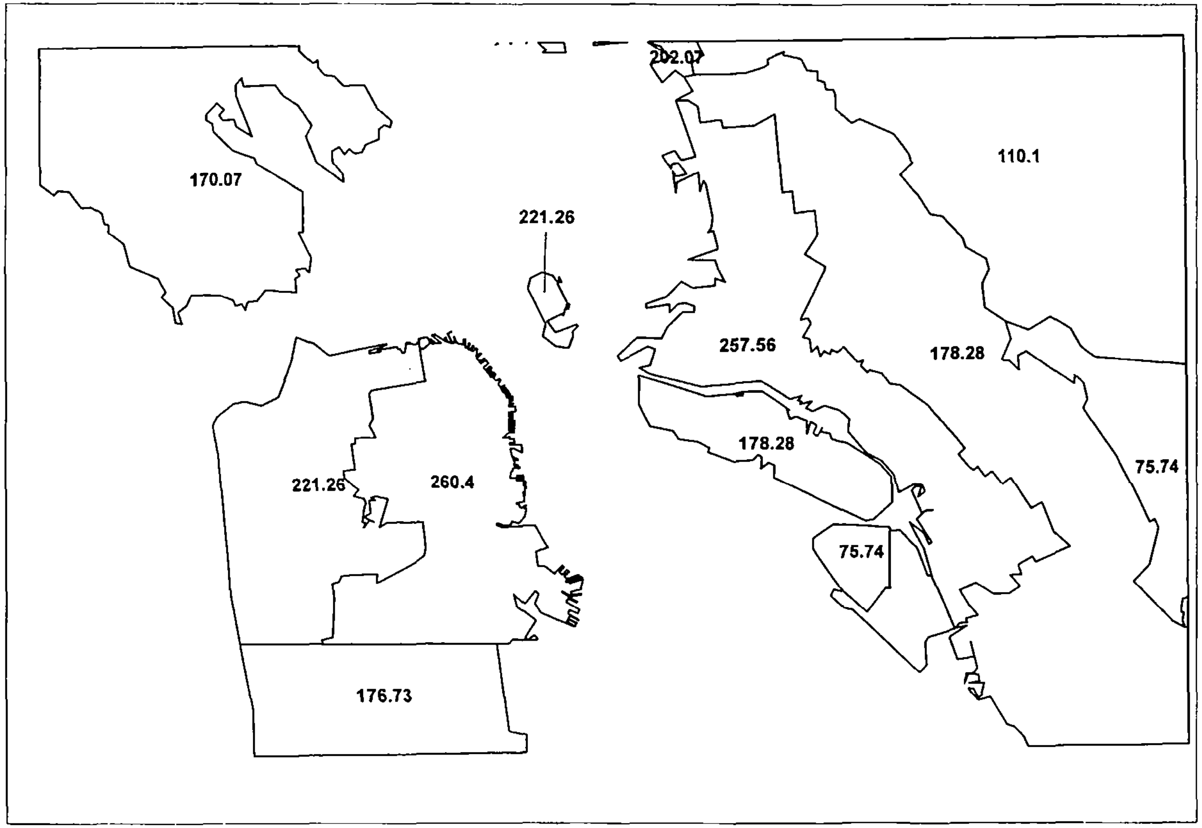
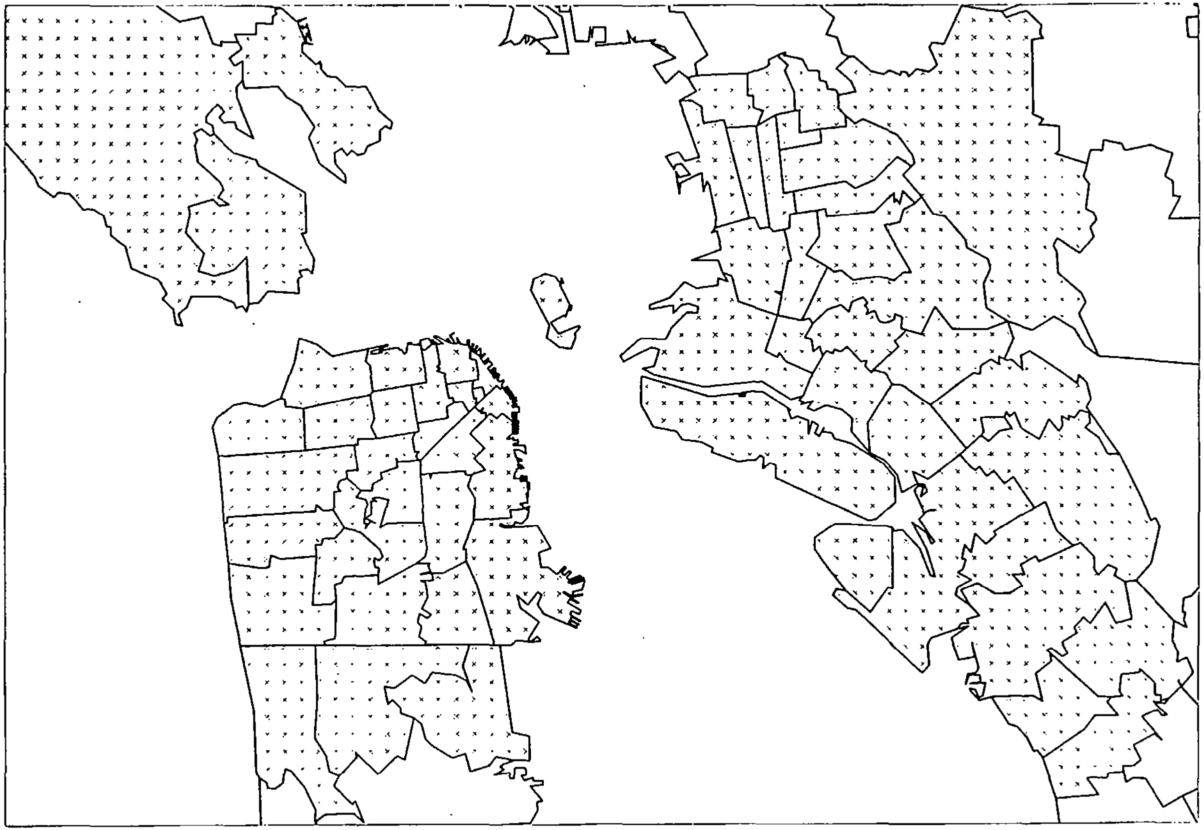


Exhibit 3

SAN FRANCISCO AREA GRID POINTS
0.4 MILES APART

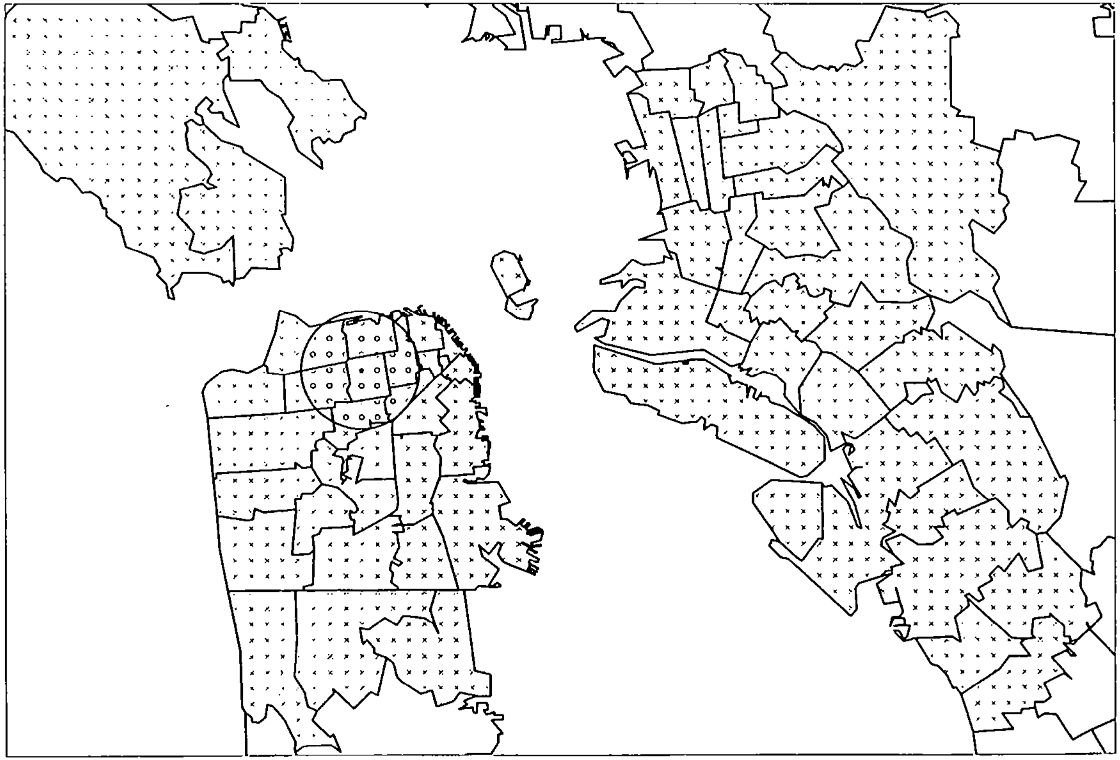


——— 1994 ZIP CODE BOUNDARIES

Exhibit 4

CALCULATION OF BODILY INJURY PURE PREMIUM SURFACE FOR A GRID POINT

LONGITUDE = -122.439362, LATITUDE = 37.788797, RADIUS = 1.5 MILES



• GRID POINT   ∘ DATA POINT (INCLUDES GRID POINT)   ——— 1994 ZIP CODE BOUNDARIES

Exhibit 5
Page 1

CALCULATION OF BODILY INJURY PURE PREMIUM SURFACE
FOR A GRID POINT
LONGITUDE = -122.439362, LATITUDE = 37.788797, RADIUS = 1.5 MILES, P=1.0

	Longitude	Latitude	Distance from Grid Point	Distance Weight	Exposures at Grid Point	Total Weight	Total Weight Normalized	Unweighted Zip Code Pure Premium	Weighted Smoothed Pure Premium
	(1)		(2)	(3)	(4)	(5)	(6)	(7)	(8)
GRID POINT	**-122.439362**	**37.788797**	**0.000**	**1.0000**	**1958.00**	**1958.000**	**6.803%**	**223.66**	**231.48**
	-122.461387	37.800391	1.443	0.4093	46.73	19.127	0.066%	254.86	
	-122.461378	37.794594	1.265	0.4415	46.73	20.631	0.072%	254.86	
	-122.461369	37.788797	1.200	0.4545	1353.09	614.979	2.137%	248.31	
	-122.461360	37.783000	1.264	0.4417	1353.09	597.660	2.077%	248.31	
	-122.461350	37.777203	1.441	0.4097	1353.09	554.361	1.926%	248.31	
	-122.454059	37.806188	1.443	0.4093	46.73	19.127	0.066%	254.86	
	-122.454051	37.800391	1.132	0.4690	46.73	21.916	0.076%	254.86	
	-122.454042	37.794594	0.895	0.5277	46.73	24.659	0.086%	254.86	
	-122.454033	37.788797	0.800	0.5556	1353.09	751.777	2.612%	248.31	
	-122.454025	37.783000	0.894	0.5280	1353.09	714.432	2.482%	248.31	
	-122.454016	37.777203	1.131	0.4693	1353.09	635.005	2.206%	248.31	
	-122.454007	37.771406	1.441	0.4097	799.81	327.682	1.138%	280.85	
	-122.446722	37.806188	1.265	0.4415	1505.71	664.771	2.310%	236.44	
	-122.446714	37.800391	0.895	0.5277	46.73	24.659	0.086%	254.86	
	-122.446706	37.794594	0.566	0.6386	46.73	29.842	0.104%	254.86	
	-122.446698	37.788797	0.400	0.7143	1353.09	966.512	3.358%	248.31	
	-122.446690	37.783000	0.565	0.6390	1353.09	864.625	3.004%	248.31	
	-122.446682	37.777203	0.894	0.5280	1353.09	714.432	2.482%	248.31	
	-122.446674	37.771406	1.265	0.4415	1319.13	582.396	2.023%	202.04	
	-122.439385	37.806188	1.200	0.4545	1505.71	684.345	2.378%	236.44	
	-122.439377	37.800391	0.800	0.5556	1505.71	836.572	2.907%	236.44	
	-122.439370	37.794594	0.400	0.7143	1505.71	1075.529	3.737%	236.44	
	-122.439355	37.783000	0.400	0.7143	1958.00	1398.599	4.859%	223.66	
	-122.439347	37.777203	0.800	0.5556	1319.13	732.909	2.546%	202.04	
	-122.439340	37.771406	1.200	0.4545	1319.13	599.545	2.083%	202.04	
	-122.432040	37.800391	0.894	0.5280	1505.71	795.015	2.762%	236.44	
	-122.432034	37.794594	0.565	0.6390	1958.00	1251.162	4.347%	223.66	

CALCULATION OF BODILY INJURY PURE PREMIUM SURFACE
FOR A GRID POINT
LONGITUDE = -122.439362, LATITUDE = 37.788797, RADIUS = 1.5 MILES, P=1.0

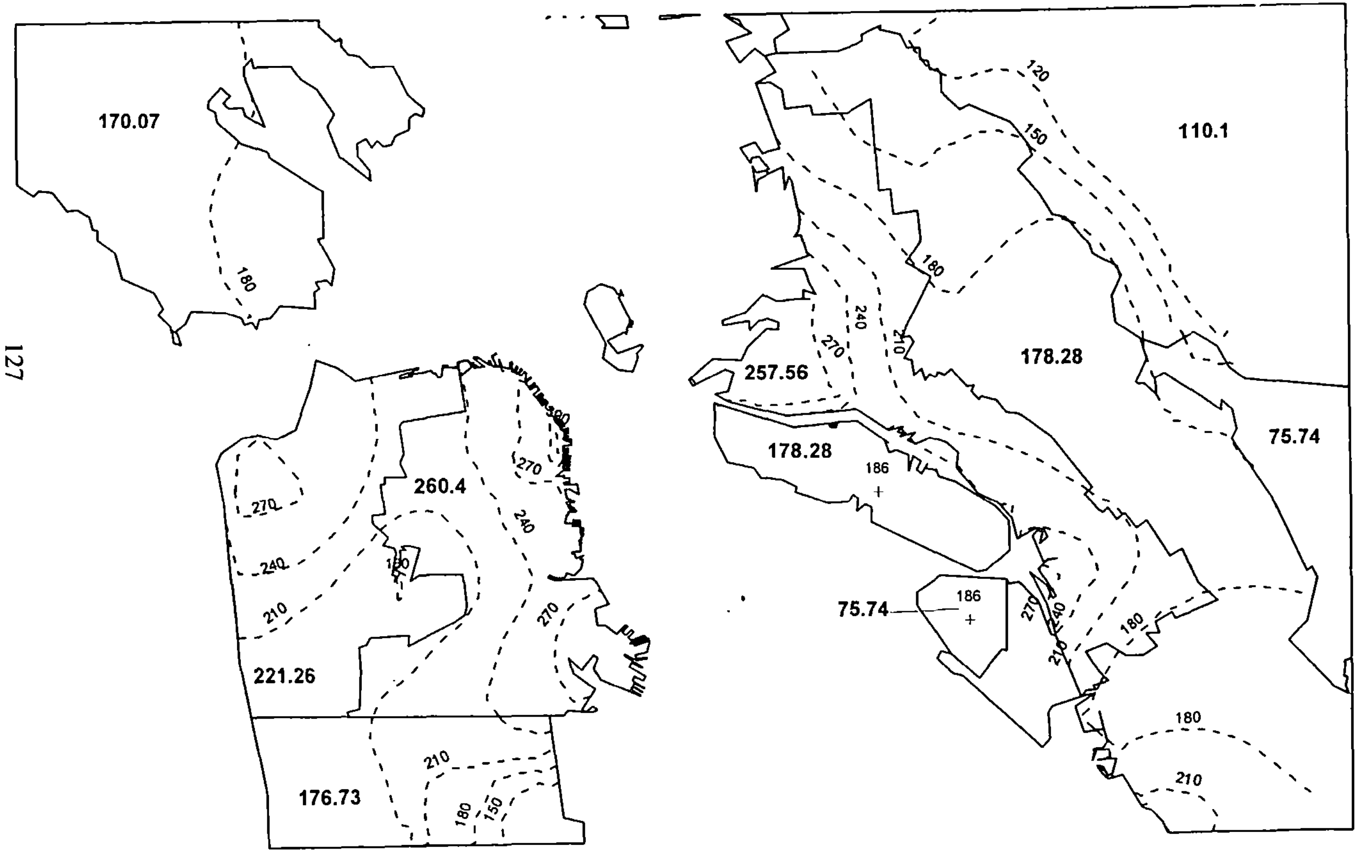
Longitude	Latitude	Distance from Grid Point	Distance Weight	Exposures at Grid Point	Total Weight	Total Weight Normalized	Unweighted Zip Code Pure Premium	Weighted Smoothed Pure Premium
(1)		(2)	(3)	(4)	(5)	(6)	(7)	(8)
-122.432027	37.788797	0.400	0.7143	1958.00	1398.599	4.859%	223.66	
-122.432020	37.783000	0.566	0.6386	1958.00	1250.379	4.344%	223.66	
-122.432013	37.777203	0.895	0.5277	1319.13	696.105	2.419%	202.04	
-122.432006	37.771406	1.265	0.4415	1319.13	582.396	2.023%	202.04	
-122.424710	37.806188	1.442	0.4095	1505.71	616.588	2.142%	236.44	
-122.424704	37.800391	1.131	0.4693	1505.71	706.630	2.455%	236.44	
-122.424697	37.794594	0.894	0.5280	1825.67	963.954	3.349%	223.65	
-122.424691	37.788797	0.800	0.5556	1825.67	1014.342	3.524%	223.65	
-122.424685	37.783000	0.895	0.5277	1825.67	963.406	3.347%	223.65	
-122.424678	37.777203	1.132	0.4690	419.83	196.900	0.684%	284.05	
-122.424672	37.771406	1.443	0.4093	419.83	171.836	0.597%	284.05	
-122.417367	37.800391	1.442	0.4095	1825.67	747.612	2.597%	223.65	
-122.417361	37.794594	1.265	0.4415	1825.67	806.033	2.800%	223.65	
-122.417356	37.788797	1.200	0.4545	1825.67	829.767	2.883%	223.65	
-122.417350	37.783000	1.265	0.4415	419.83	185.355	0.644%	284.05	
-122.417344	37.777203	1.443	0.4093	419.83	171.836	0.597%	284.05	

Notes (by column)

(1) Location of each data point
(2) See formula in text of article
(3) $[1/[(2)+1]]^P$, where P = 1.0
(5) (3) x (4)
(6) (5)/[Sum of (5)]
(7) 1991 California Department of Insurance BI Industry Pure Premiums
(8) Sum of [(6) x (7)]

SAN FRANCISCO AREA
BODILY INJURY PURE PREMIUM SURFACE
Radius = 1.5 miles, P = 1.0

Exhibit 6



Incorporating a Hurricane Model into Property Ratemaking

by George Burger, FCAS
Beth E. Fitzgerald, FCAS
Jonathan White, FCAS, and
Patrick B. Woods, FCAS

Abstract

This paper explains the procedures used to incorporate a hurricane model into the development of state loss costs by territory for personal property and state loss costs by territory and construction class for commercial property. It explains why a modeling approach was used to estimate losses for hurricane perils. Issues discussed in the procedures include the combination of modeled loss estimates with insurance data, the adjustments for deductibles/coinsurance clauses and the application of trend and credibility. The paper also discusses the continuing activities of model use and comments on other applications for hurricane models, such as its use in the redefinition of territories.

Table of Contents

SECTION I - DESCRIPTION OF THE WIND HAZARD

The standard personal and commercial property insurance forms provide coverage for a host of perils, several of which have the potential to generate catastrophic losses -- fire, explosion, riot or civil commotion and windstorm and hail. Of these perils, windstorm has clearly been the leading cause of catastrophic losses. Seventy - four percent of the total $112 billion insured catastrophic losses from 1950 through 1994 were due to windstorms.[1] One type of windstorm in particular stands out - hurricanes. Hurricanes are the number one generators of insured catastrophe losses in the United States. Of the 15 largest catastrophes (as measured by insured losses) in the United States, seven have been hurricanes. Hurricanes have generated 36% of the $71 billion of insured catastrophe losses from 1985 through 1994.

Windstorms

Windstorm is defined as wind, with or without rain, of sufficient velocity to cause damage. Catastrophic wind losses are generated by storms of several types:

1) Tornadoes - strong, violently rotating columns of air extending from the base of a cumulonimbus cloud to the ground

2) Hail-Storms - the falling of hailstones (balls of ice ranging from ½ to 3 inches in diameter), which are generated by the updraft of a thunderstorm

3) Nor'easters (or winter storms) - cyclonic storms of the east coast of North America

[1] Based on Property Claim Services (PCS) estimates.

4) Tropical Cyclones - low pressure weather systems in which the central core is warmer than the surrounding atmospheres; e.g., tropical storms and hurricanes.

Hurricanes

Hurricanes are technically defined as non-frontal, low pressure synoptic scale systems or more commonly tropical cyclones, with sustained winds of 75 mph or more. Hurricanes and their cousins -- Pacific Ocean typhoons and Indian Ocean cyclones -- are the world's most violent storms.

Hurricanes are born in the most placid of climates -- the tropics. The tropics supply the essential ingredients for a hurricane -- wide expanses of warm ocean water; warm, humid air; and normally weak upper air winds blowing from the same direction as winds near the surface. Hurricanes consist of high-speed winds blowing circularly (counter-clockwise in the northern hemisphere) around a low-pressure center, known as the eye of the storm. The low-pressure center develops when the warm, humid air prevalent in the tropics is underrun and forced upward by denser cooler air. The winds attain maximum force close to the point of lowest pressure, just beyond the eye, at a distance called the radius of maximum winds. This distance, the radius of maximum winds, typically ranges from 5 to 15 miles. The central pressure in the eye of the storm is a key parameter of the storm's strength and the resulting windspeeds. The lower the pressure (or in other words the higher the differential with normal pressure) the stronger the storm. Sustained winds[2] can range from 75 mph for the mildest hurricanes (Saffir/Simpson category 1) to greater than 155 mph for the strongest hurricanes (Saffir/Simpson category 5). Hurricanes can be thought of as heat engines that convert the warmth of the tropical oceans and atmosphere

[2] Highest average windspeed over a one-minute period.

into wind and waves. They are made up of bands of thunderstorms, spiraling in toward the center - the eye. The width of a typical hurricane is approximately 300 miles.

Hurricanes inflict property damage from high wind speeds, intense rain, projected missiles, and high water. The resulting storm surge and flooding are responsible for a considerable portion of the damage and loss of life, especially within the first few hundred yards of the shoreline. While damage caused by rain, high winds, or wind-blown debris are covered by standard property insurance policy forms, damage caused by storm surge or flooding is not.

Insurance Coverages

For personal property, hurricane coverage is most frequently provided under a Homeowners policy form. A small portion of the market is serviced under Dwelling forms. The Homeowners policy form provides a package of coverages. Coverage A provides coverage for the building. Coverage B provides coverage for other appurtenant structures, such as garages, pools, barns. Coverage C provides coverage for the personal property (i.e. contents of the residence). Coverage D provides coverage for any additional living expense and/or loss of rents incurred by the policyholder and caused by a covered peril. For the Owners policy forms, the amount of insurance provided for Coverages B, C and D are usually expressed as a percentage of the amounts of insurance provided for Coverage A, the building. The typical policy provides the following:

Coverage B = 10% of the Coverage A

C = 50% - 80% of the Coverage A (selected by insured)

D = 20% of the Coverage A

Under the current ISO statistical plan only the Coverage A amount of insurance is reported by insurers electing to report statistics to Insurance Services Office (ISO) for their Homeowners policies.

For personal property written under Dwelling Forms, hurricane coverage is provided under the Extended Coverage endorsement[3]. For commercial property hurricane coverage is typically provided under the Commercial Basic Group II[4] forms as well as indivisible premium package policy forms (e.g. Businessowners). However, for Dwelling Extended Coverage (EC) and Commercial Basic Group II (BGII), separate records and amounts of insurance are reported to ISO for the building and contents coverage.

[3] Dwelling Extended Coverage is an endorsement that extends the standard fire coverage to a list of perils including windstorm and hail, riot and civil commotion, smoke aircraft, vehicles, and explosion.

[4] Commercial Basic Group II is the coverage form for commercial risks and provides coverage for windstorm and hail, riot and civil commotion, smoke aircraft, vehicle action, and sink hole collapse.

SECTION II - TRADITIONAL METHODS OF CATASTROPHE LOSS ESTIMATION

The traditional approach used by ISO and most of the industry to reflect catastrophic losses and catastrophic loss potential in the calculation of loss costs/rates has been to use various long-term smoothing techniques. This was done by establishing a cut-off for aggregate reported insurance losses above which losses were deemed excess. Losses below the cut-off were termed normal. For Homeowners, individual state cut-offs were based on the long-term average ratio of wind losses to non-wind losses. For Dwelling Extended Coverage and Commercial Basic Group II, those cut-offs were based on loss ratios, and were judgementally established. Reported loss activity that exceeded these cut-offs were deemed excess, were excluded from the ratemaking database, and were replaced with expected excess losses that were loaded in using an excess loss factor. This excess loss factor was calculated as a long-term average ratio of actual excess losses to normal losses. In some situations, the excess loss factor was calculated using both a state and regional component. The regional component provided a broader base for the loss smoothing for the higher layers of loss. States were grouped into regions based on geographical and meteorological considerations. No distinction was made in either the personal or commercial property procedures for the specific type of catastrophic event (hurricane, tornado, winter freeze, et. al.) that gave rise to the excess losses for the coverage.

Unfortunately, traditional loss smoothing approaches have five major limitations in determining loss costs in states that have significant hurricane loss potential.

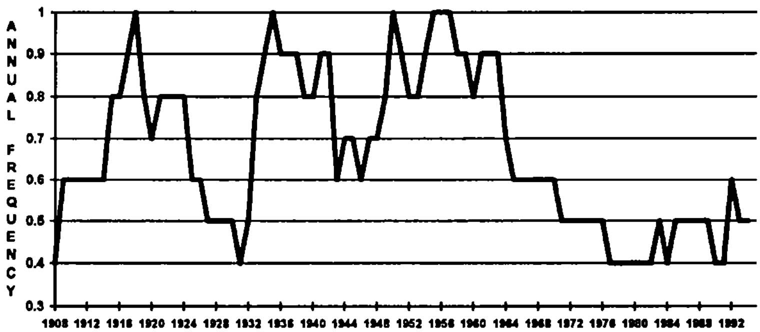
1) <u>Not enough historical insurance data</u>

The available historical insurance statistical data base (approximately 1960 to present for Homeowners, 1950 to present for Extended Coverage/Basic Group II) provides too short an experience period to measure hurricane activity on a state specific or even countrywide basis. Between 1899 and 1994, only 157 hurricanes (as defined by the sustained wind speed) made landfall in the continental United States. With only 1.6 hurricanes per year striking the entire U.S. coast, obviously in any given state many years may pass without hurricane activity.

In addition, the most recent period, 1960-1994, the only period for which we have statistical data for Homeowners, has had unusually low hurricane frequency - particularly for intense hurricanes. Chart 1 below shows that the frequency of intense hurricanes for that period is extremely low when compared with the long-term history. Consequently, any technique that makes exclusive use of meteorological or insurance experience for this period of low hurricane frequency risks understating the hurricane potential.

CHART 1

FREQUENCY@ OF INTENSE* HURRICANES



@ Ten year moving average

* Intense hurricanes are those storms achieving Saffir-Simpson 3, 4 or 5 level as defined by the sustained wind speed at landfall.

The sparsity of hurricanes, only 155 total hurricanes from 1900 to 1994 and 61 intense ones[5], makes the job of estimating prospective hurricane losses quite difficult. This difficulty is compounded by the recent low frequency, only 17 intense hurricanes from 1960 to 1994. While in theory it might be possible to adjust historical insured hurricane losses for the recent low frequency on some broad multi-state basis, this adjusted aggregation would be of little value for state or territory calculations.

[5] Tropical Cyclones of the North Atlantic Ocean - National Climatic Data Center

2) <u>Over reliance on long-term premium and loss information</u>

The traditional approach relies exclusively on the long-term premium and loss information contained in the ISO statistical data base. The long-term statistical data has limited applicability to future catastrophic losses because in the last thirty-five years, land use, population densities, construction techniques and materials, engineering techniques, building codes and their enforcement and the damageability of structures, have changed extensively. For example, the population density in the coastal areas has increased significantly. From 1960 to 1990, the population density of the South East Atlantic coast has increased more than 120%, while the density countrywide has increased less than 40%. Storms that might have generated only moderate losses in 1960 would now generate catastrophic losses. Thus, excess factors derived from insurance experience of the 1960's and 1970's have limited validity when applied to today's or tomorrow's insured portfolios. It would be very difficult to properly adjust the historical insurance exposure, premium, and loss data bases for all the changes that have taken place which have a significant impact on hurricane losses.

3) <u>Grouping states into regions can mask hurricane potential</u>

The traditional approach for Commercial Basic Group II, for example, entailed grouping states into regions in order to calculate a regional excess component in addition to the state component. While all due care was taken to optimize this grouping, in reality each state has its own hurricane potential, due to geographical and other factors. The use of a regional component distorts that potential. The Southeast Region (Alabama, Florida, Georgia, Louisiana, Mississippi, North Carolina, South Carolina, Hawaii) is a good example. Clearly all these states have significant hurricane potential; however, the hurricane potential for Florida is quite different than Alabama's or Mississippi's. A regional factor might be appropriate for the average, but it will not be for all the states in the group.

4) <u>Individual storms can have disproportionate impact</u>

The traditional approach was overly sensitive to recent individual hurricanes. When loss experience for Hurricane Hugo was reflected in the traditional analysis, the indicated loss costs needed for South Carolina increased significantly. When loss experience for Hurricane Andrew was reflected, the indicated loss costs for Florida also increased significantly. The fact that Hurricane Andrew struck Dade County, Florida and Hurricane Hugo struck Charleston, South Carolina did not change the underlying probabilities of hurricanes striking Florida or South Carolina at some future period. Clearly the traditional analysis was flawed when the occurrence or absence of individual storms had such a dramatic impact on the results.

5) <u>Not all portions of a state are equally exposed to hurricanes</u>

The traditional approach generated an excess factor to be used for the entire state. But not all portions of a state are equally exposed to hurricanes. Clearly the coastal areas have a much greater potential for hurricane losses than the inland areas. The traditional excess factor approach provided limited assistance in allocating excess losses to the individual rating territories.

For some lines, a separate territory wind analysis was performed using the available 10 to 15 years of data. Unfortunately, subdividing experience into territory detail and limited years of data available in territory detail precluded having an adequate data base to measure hurricane loss potential.

SECTION III - DESCRIPTION OF THE HURRICANE MODEL

After evaluating the limitations of the traditional loss smoothing approaches, ISO decided to use a computer simulation modeling approach for measuring the hurricane catastrophe peril. There are several models available. The one being used in ISO catastrophe procedures was developed by Risk Management Solutions, Inc. (RMS).

Establishing Probability Distributions

The RMS hurricane model uses the available meteorological data base of 107 years of hurricanes to establish the overall probability of a storm, separately for each of the 31 coastal segments that make up the United States coast from Brownsville, Texas to Maine. Each segment is 100 nautical miles long. The key characteristics of hurricanes are fit to probability distributions, separately for each coastal segment, based on the observed characteristics of historical storms that have made landfall in that segment and adjacent ones. The observed central pressure differentials (the difference between ambient central pressure and the central pressure in the eye of the storm) are fit to Weibull distributions. The observed forward velocities are fit to lognormal distributions. The track angles are fit to normal distributions.

Simulating Hurricanes

For each segment, a few discrete parameter values are selected from the probability distributions for each of the essential hurricane characteristics:

- central pressure differential - 6 values
- forward velocities - 3 values
- track angles - 3 values

These parameter values are concatenated to generate 54 (6 x 3 x 3) simulated storms per segment. The probability of each simulated storm is determined from the probability distributions and the overall probability of a storm in that segment..

Each 100 nautical mile segment is further divided into four equal subsegments. Then, for each 25 nautical mile subsegment, a landfall location is selected randomly. Each of the 54 storms are simulated to landfall in each of the four different selected locations within a 100 nautical mile segment, with one quarter of the previously established probability. Thus, there are 216 (54 x 4) simulated storms per segment. This approach is referred to as a "logic tree" (as opposed to Monte Carlo simulation) approach and results in 6,696 (216 x 31) simulated hurricanes in total. Each of the simulated storms has an associated probability of occurrence derived from the overall probability of a storm and the probability of the central pressure differential, forward velocity and track angle combination for that segment.

For each of the 6,696 simulated hurricanes, a storm track is assigned to each hurricane. The track of each simulated storm is determined by selecting the track of the historical storm in the segment or adjacent segments with meteorological characteristics at landfall closest to the simulated storm. The decay characteristics[6] (rate of energy loss) of the selected historical storm are also used for the simulated storm.

__Wind Field Model__

The maximum wind speed at a particular site due to a simulated hurricane is determined using a wind field model that is based on the meteorological characteristics of the storm near the site (e.g. central pressure difference, forward velocity), the distance/direction from the site to the storm path, distance to coast, and any natural or man-made roughness at the site.

[6] Hurricanes dissipate as they pass over land. That dissipation is termed the decay characteristics and is measured by the increase in central pressure in the eye of the storm.

Damageability

The model's estimate of damages at a particular site is based on the peak gust wind speed as calculated by the wind field model. The RMS model does not estimate any damages when the peak gust wind at a site is less than 75 miles per hour. The damage relationships were derived from a combination of engineering studies and actual insurance loss data. These damage relationships vary by construction, occupancy, number of stories, and other associated variables. Estimated hurricanes damages are measured in terms of a damage ratio, which is defined as the ratio of repair costs (i.e., losses) to the replacement cost. Separate mean damage ratios are calculated and expressed as a percent of total insurable property value for building, contents, and additional living expenses.

Each of the 6,696 simulated hurricanes is run through its assigned path with its assigned decay functions. At any point on its path, the hurricane's central pressure differential is determined by the original value at landfall as modified by the hurricane's assigned decay characteristics. Based on the key characteristics (central pressure differential and forward velocity) the wind field model calculates the peak wind gusts in all zip codes (as defined by the population-weighted centroid) around the storm reflecting distance/direction from storm, distance from coast and local area roughness. Using the damageability relationships, the peak gusts generated from each storm by location are translated into damage ratios. The sum of the products of the damage ratio and the probabilities of the simulated storms is the mean damage ratio (MDR) which is generated by zip code.

Outputs

The standard outputs to the model are a set of mean damage ratios by zip code and construction, occupancy and number of stories separately for buildings, contents and additional living expense coverages.

SECTION IV - USING THE MODEL OUTPUT IN A LOSS COST REVIEW

General Considerations

The calculation of an indicated loss cost change prior to the introduction of a hurricane model was based on using available insurance loss data to calculate prospective loss costs. Incorporating a hurricane model into property ratemaking revises that procedure by developing a prospective hurricane loss cost separately from a prospective non-hurricane loss cost and then combining the two pieces.

The hurricane loss cost is developed using MDRs that are the output of the hurricane model and converting them to an ISO ratemaking and coverage basis for the specific line of insurance. This consists of consolidating the MDRs for each zip code into broader rating territory detail for the particular coverage or policy form; adjusting the MDRs to a common deductible basis and/or coinsurance basis; and reflecting any necessary ratemaking adjustments, such as application of loss adjustment expense factors and/or trend.

The procedure for the development of the non-hurricane loss cost is similar to the prior procedure with two exceptions. First, any hurricane losses in the experience period are removed. Secondly, the traditional catastrophe smoothing procedure is adjusted to a non-hurricane basis by the removal of hurricane experience and the elimination of regional components.

Once a statewide loss cost indication reflecting the hurricane model is calculated, the next step is to calculate the territory relativities. The procedures assume that the hurricane loss costs are fully credible, since the MDRs are based on all available meteorological data on hurricanes over the past century and there is no credibility standard for the volume of data needed to determine reliable estimates from the model. There is also an absence of a source to use for the complement of credibility for the hurricane model. Credibility is thus taken into account only for the non-hurricane loss costs. A detailed description of the specific methods used for homeowners and commercial property basic group II follows.

Use of the Model in Homeowners Ratemaking

A. Development of a Prospective Hurricane Loss Cost

Since the hurricane model provides MDRs by zip code, the first step in using the model is to aggregate the MDRs to conform to broader rating territory boundaries. In the absence of insurance data by zip code, the number of residential units within each zip code available from the U.S. Census Bureau can be used for weighting the zip code MDRs to territory MDRs. (This will work well unless the distribution of risks is believed to be locally concentrated in particular zip codes.)

The MDRs that the model produces are expressed as a percent of the total insurable property value. For homeowners owners policy forms (1-3, 3w/15), the amount of insurance collected in the ISO Statistical Plan is just the Coverage A building amount of insurance. The homeowners owners forms provide coverage for the building, other appurtenant structures, contents, and additional living expenses (and/or loss of rents). For example, a policy insured for $100,000 of building coverage would typically have $10,000 of other appurtenant structures coverage, $70,000 of contents coverage, and $20,000 of additional living expense coverage--for a total amount of insurance at risk of $200,000. In order to calculate the expected hurricane losses for all coverages on the homeowners policy, either the reported amount of insurance for Coverage A needs to be increased to reflect all coverages or a weighted MDR reflecting all coverages needs to be calculated to apply to the Coverage A amount of insurance. The latter method is used in this paper. For homeowners owners policy forms, this requires weighting each building, contents and additional living expense MDR by its percent of the Coverage A amount of insurance. Table 1 shows a sample output of the hurricane model with MDRs aggregated by rating territory.

TABLE 1
SAMPLE OF HURRICANE MODEL OUTPUT FOR PERSONAL LINES

		---------------Single Family MDRs---------------		
Rating Territory	Construction	Building	Contents	Additional Living Expenses
A	Frame	1.0%	0.8%	0.9%
	Masonry	0.5%	0.3%	0.4%
	Superior	0.1%	0.0%	0.1%
B	Frame	2.0%	1.5%	1.8%
	Masonry	1.8%	1.2%	1.5%
	Superior	0.5%	0.3%	0.4%

Table 2 shows a typical calculation of a weighted MDR reflecting the relationship of each individual coverage's amount of insurance to the Coverage A amount of insurance. This sample calculation uses the MDRs from Table 1 for territory A, frame construction.

TABLE 2
SAMPLE CALCULATION OF WEIGHTED MDR

Coverage	(1) Relationship to Coverage A Amount of Insurance	(2) MDR	(1) x (2)
A - Buildings	1.00	.010	.010
B - Appurtenant Structures	0.10	.010	.001
C - Contents	0.70	.008	.0056
D - Additional Living Expense	0.20	.009	.0018
Weighted MDR			.0184

Referring back to our example of a policy with $100,000 of building coverage above, the expected hurricane losses for this policy in territory A (frame) are:

.0184 x $100,000 = $1,840

This is equivalent to applying the individual coverage MDRs to the amount of insurance for each coverage separately as follows:

(.01 * $100,000) + (.01 * $10,000) + (.008 * $70,000) + (.009 * $20,000) = $1,840

Similarly, this weighting of each set of MDRs is done for other homeowners policy forms 4(tenants) and 6(condominiums). For these policy forms, the amount of insurance collected in the ISO Statistical Plan is just the Coverage C contents amount of insurance. See Appendix A for more details.

Deductible Adjustment

The MDRs of the hurricane model are the mean of a probability distribution of all possible damage ratios, on a first dollar basis. The MDRs have not been adjusted to account for any deductible that the insurance policy may include. But, supplementary output from the model can be used to calculate an MDR reflecting a percent deductible.

The standard ISO ratemaking deductible for homeowners is $250 deductible. Thus, the $250 deductible is converted into a percent deductible relative to the average amount of insurance for each territory and policy form. Then, net MDRs are calculated based on the probability distribution of the damage ratios. This calculation is accomplished by computing the net loss for each simulated hurricane event and probabilistically aggregating the net results based upon the annual rate of occurrence of each storm. The steps in this calculation of net MDRs are as follows:

Step (1): Expression of damage ratios on a first dollar basis

For each simulated hurricane event, h, there is a mean damage ratio for zip code j, for each coverage k and construction class l that can be expressed as:

$$MDR(h,j,k,l)$$

Step (2): Derivation of the beta cumulative distribution function

For each MDR(h,j,k,l), there is an associated coefficient of variation based on the probability distribution of the damage ratios. Using the mean and coefficient of variation of the damage ratios, the parameters of a beta cumulative distribution function can be derived and expressed as:

$$F(x|h,j,k,l)$$

where F(x) represents the probability that the damage ratio will be less than or equal to x.

Step (3): Calculation of net MDR for each event

Given a deductible, 100 x d %, expressed relative to the amount of insurance, and the beta cumulative distribution, integration can be performed to calculate the mean damage ratio after the deductible for each event. This can be expressed as:

$$\text{net } MDR(h,j,k,l,d) = MDR \int_d^1 (1-F(x|h,j,k,l))dx$$

Step (4): Calculation of net MDR over all events

The net mean damage ratio over all events is given by:

$$\text{net MDR}(j,k,l,d) = \sum_{h} \text{net } (\text{MDR}(h,j,k,l,d) \times P(h))$$

where P(h) is the annual probability of hurricane event h.

Calculation of Prospective Hurricane Loss Costs

Once the MDRs are adjusted for the $250 deductible, the net weighted MDRs are applied to the reported amounts of insurance for each construction type within each territory to determine expected hurricane losses. The sum of the expected hurricane losses by construction type within a territory are the territory hurricane losses. The statewide expected hurricane losses are then the sum of the hurricane losses across all territories. The results of this calculation for our sample state are shown in Table 3. The hurricane losses are calculated using the latest year earned amount of insurance (Coverage A for the owners policy forms). The hurricane loss cost can then be calculated by dividing by the latest year earned house years. This table also shows the average MDRs by territory and by state, which are calculated by dividing the hurricane losses by the earned amount of insurance.

TABLE 3
CALCULATION OF HURRICANE LOSS COST

Territory	Latest Year Coverage A Amount of Insurance (1)	Expected Hurricane Losses (2)	Latest Year House Years (3)	Average Weighted MDRs (4) = (2)/(1)	Average Hurricane Loss Cost (5) = (2)/(3)
A	10,000,000	2,000	200	0.02%	10.00
B	20,000,000	40,000	300	0.20%	133.33
C	100,000,000	100,000	1,000	0.10%	100.00
Statewide	130,000,000	142,000	1,500	0.11%	94.67

To calculate the prospective hurricane loss cost, the same trend factors (current cost/amount factor and composite projection factor) used for the latest year in the calculation of the non-hurricane loss cost are applied to the statewide hurricane loss cost. This loss cost already is adjusted to a $250 deductible basis, but excludes loss adjustment expenses. Thus, a loss adjustment expense factor must be applied. The same loss adjustment expense factor as used with the non-hurricane loss cost is used here since there is no data to derive a factor appropriate for an average hurricane provision.

Since the hurricane loss cost is an average loss cost for all classes, it must be transformed to a base class basis by dividing by the latest year classification and coverage factor[7]. Table 4 illustrates this calculation and results in a prospective hurricane base class loss cost of $88.37.

[7] The classification and coverage factor is an average rating factor based on the distribution of data by policy form, construction and protection class, and amount of insurance. The base class level for the owners policy forms is Form 3, frame protection class 5, $60,000 Coverage A amount of insurance.

TABLE 4
CALCULATION OF HURRICANE LOSS COST SAMPLE STATE

(1)	Average Modeled Hurricane Loss Cost	94.67
(2)	Loss Adjustment Expense Factor	1.150
(3)	Latest Year Current Cost/Amount Factor	1.005
(4)	Composite Projection Factor	1.050
(5)	Latest Year Class and Coverage Factor	1.300
(6)	Modeled Hurricane Base Class Loss Cost (1) x (2) x (3) x (4) / (5)	$88.37

B. *Development of a Prospective Non-Hurricane Loss Cost*

The calculation of a non-hurricane prospective loss cost begins with the reported incurred losses for the latest five accident years with hurricane losses removed. The standard ratemaking adjustments are then made to the non-hurricane losses, including a modified excess procedure based on non-hurricane experience.

Removal of Hurricane Losses

The first step in calculating the non-hurricane loss cost is to remove any actual hurricane losses from the experience period. The losses removed must be consistent with the types of losses generated by the modeling process. The model does not generate damages if the peak gust is less than 75 mph. For the calculation of state and territory loss cost level changes, the latest five accident years of experience are used. Experience from 1960 to present is used to calculate the long-term excess wind factor. Although there is no need to use the traditional ISO catastrophe procedure for the hurricane peril, there still is a need to use this procedure for other catastrophic perils, such as tornadoes, hail storms, nor'easters and other tropical cyclones below hurricane status. Thus, the hurricane losses must be removed for the period of 1960 to present.

Hurricane losses are not specifically identified in the ISO data base. The meteorological history of all hurricanes that occurred from 1960 to present including storm tracks and wind speeds at 6 hour intervals is used to assist in the removal of hurricane losses. This information identified the states and territories affected by each hurricane (i.e., peak gusts of at least 75 mph).

The details of the process for removal of the hurricane losses vary by the information available in the ISO data base. For the more recent years, monthly wind losses by territory are available. Since it is impossible to isolate the hurricane losses for these years, all the wind losses in any month effected by a hurricane are removed and replaced with average monthly wind losses for the same month from non-hurricane years. For the 1970s and early 1980s, only annual wind losses by territory were available. Here, the annual wind losses by territory are replaced with the average wind losses for that territory from the non-hurricane years. Only statewide annual wind losses were available for the 1960s. For any year in the 1960s in which a hurricane occurred, that year was excluded from the excess wind calculation.

Calculation of Non-Hurricane Excess Wind Factor

The calculation of a non-hurricane excess wind factor is similar to the traditional calculation method in effect before the use of a model--with two exceptions. First, any hurricane losses are removed from the wind losses, or the year in which a hurricane occurred is excluded from the calculation as described above. Second, the calculation no longer includes a regional component for the Southeast region[8]. The Southeast regional component smoothed large excess wind losses mainly accounted for by hurricanes. Since non-hurricane wind experience is generally more stable than experience including hurricanes, the need for a regional component is eliminated. See Exhibit 1 for a sample calculation of a non-hurricane excess wind factor.

Calculation of Prospective Non-Hurricane Loss Costs

Once the hurricane losses accounted for by the model are removed from the experience period, the standard ratemaking adjustments need to be made to calculate the prospective non-hurricane loss cost. These adjustments include the following:

- adjustment of property losses to a common $250 deductible basis using loss elimination ratios,

- application of loss development factors to bring the losses to an ultimate settlement basis,

[8] The prior procedure included a regional component for the Southeast region only.

- removal of non-hurricane excess wind losses and the application of the non-hurricane excess wind factor,

- application of a loss adjustment expense factor,

- adjustment for changes in cost levels and increases in amount of insurance by a two step application of a current cost/amount factor and a composite projection factor,

- adjustment to a base class level by dividing by the classification and coverage factor.

Table 5 displays a sample calculation of a prospective non-hurricane loss cost. Once the projected non-hurricane base class loss costs are calculated for each of the five accident years, a weighted average is determined with the weights shown giving more weight to the latest year. Thus, the weighted prospective non-hurricane base class loss cost is $239.50.

TABLE 5
CALCULATION OF A PROSPECTIVE NON-HURRICANE LOSS COST
SAMPLE STATE

Accident Year	(1) Developed Non-Hurricane Losses on a $250 Deductible Level	(2) Non-Hurricane Excess Losses on a $250 Ded. Level	(3) Non-Hurricane Losses Less Non-Hurricane Excess Losses (1)-(2)
1	325,895	5500	320,395
2	460,686	80200	380,486
3	319,819	6000	313,819
4	300,565	7000	293,565
5	381,499	0	381,499

Accident Year	(4) Non-Hurricane Losses in col. (3) x Loss Adjustment Expense Factor of 1.15	(5) Non-Hurricane Losses in col. (4) x Non-Hurricane Excess Factor of 1.053	(6) Current Cost/ Amount Factor	(7) Earned House Years
1	368,454	387,982	1.050	1,475
2	437,558	460,749	1.030	1,510
3	360,892	380,019	1.020	1,480
4	337,600	355,493	1.010	1,450
5	438,724	461,976	1.005	1,500

Accident Year	(8) Projected Average Non-Hurricane Loss Cost ((5)x(6)/(7))x Projection Factor of 1.05	(9) Classification and Coverage Factor	(10) Projected Non-Hurricane Base Class Loss Cost (8)/(9)	(11) Accident Year Weights
1	290	1.160	250	0.10
2	330	1.179	280	0.15
3	275	1.222	225	0.20
4	260	1.238	210	0.25
5	325	1.300	250	0.30

(12) Weighted Prospective Non-Hurricane Base Class Loss Cost = $239.50

C. *Calculation of Statewide Indicated Loss Cost Change*

To determine the statewide indicated loss cost level change, the prospective non-hurricane base class loss cost is added to a prospective hurricane base class loss cost and is divided by the current statewide average base class loss cost.

Table 6 shows the calculation of the statewide indicated loss cost level change for our sample state. The weighted prospective non-hurricane base class loss cost is added to the prospective hurricane base class loss cost to get a total prospective base class loss cost of $327.87 which when compared to the current base class loss cost of $300, results in a +9.3% indicated loss cost change.

TABLE 6
CALCULATION OF STATE WIDE INDICATED
LOSS COST LEVEL CHANGE SAMPLE STATE

(1)	Weighted Prospective Non-Hurricane Base Class Loss Cost	239.50
(2)	Prospective Hurricane Base Class Loss Cost	88.37
(3)	Total Prospective Base Class Loss Cost (1) + (2)	327.87
(4)	Current Base Class Loss Cost	300
(5)	Indicated Loss Cost Change (3)/(4)	1.093 or + 9.3%

D. ***Calculation of Indicated Loss Cost Changes By Territory***

The calculation of indicated loss cost changes by territory compares individual territory combined non-hurricane and modeled hurricane experience to statewide combined non-hurricane and modeled hurricane experience.

First, the five-year non-hurricane loss cost is calculated for each territory and statewide. These loss costs are then projected to the latest year cost level to be consistent with the modeled hurricane loss costs. For each territory that is not fully credible, the non-hurricane loss cost is credibility-weighted with the statewide non-hurricane loss cost (multiplied by the current territory relativity) to produce a credibility-weighted non-hurricane loss cost for each territory. This adjustment to the statewide pure premium for use as a complement of credibility is needed in order to bring the statewide experience to a cost and frequency level consistent with the territory's long term levels. This credibility-weighted non-hurricane loss cost is then added to the modeled hurricane loss cost. The total loss cost for each territory divided by the statewide loss cost produces a territory experience relativity. The experience relativity for each territory is then compared to the current relativity to produce indicated relative changes by territory.

Exhibit 2 shows a calculation for our sample state with three territories. In territory C, for instance, we calculate a credibility-weighted non-hurricane loss cost of $203.33 and add this to a modeled hurricane loss cost of $76.92 to get a total loss cost of $280.25. This results in an indicated territory relativity of 1.055 for territory C. Thus, the indicated loss cost change for territory C is the change in the territory relativity

(1.055/1.050) multiplied by the statewide indicated loss cost change of +9.3%, which is a +9.8% increase.

This procedure assumes that the modeled hurricane loss costs are fully credible. For the non-hurricane loss cost, the complement of credibility ideally should use the current territory relativity underlying the non-hurricane portion of the current loss cost, since we are trying to calculate only the non-hurricane portion of the loss cost for each territory. The first time that a loss cost review incorporates the hurricane model, though, it is not known what the underlying territory relativity is for just the non-hurricane portion of the current loss cost, since the existing territory relativity reflects both portions. Thus, for the first review incorporating the hurricane model, the current territory relativity for the current loss cost is used.

In subsequent loss cost reviews, the complement of credibility will use the territory non-hurricane relativity calculated in the prior loss cost review. Exhibit 3 shows the sample state's territory review in the second year. In territory C, the complement of credibility is the statewide loss cost of $221.47 multiplied by the non-hurricane relativity from the first loss cost review (see Exhibit 2) of 1.064 (203.33/191.09), which results in a credibility-weighted loss cost of $235.45.

INSURANCE SERVICES OFFICE, INC. EXHIBIT 1

SAMPLE STATE
TABLE 23A
HOMEOWNERS INSURANCE - FORMS 2, 3, 3W/15
DERIVATION OF NON-MODELLED EXCESS WIND FACTOR

Year	(1) Non-Modelled Reported Wind Losses	(2) Non-Modelled Reported Total Losses	(3) Non-Modelled Reported Total-Wind Losses(2) - (1)	(4) Total Wind To Non-Wind Ratio (1)/(3)	(5) Capped Wind Ratio < (5 X MED)	(6) Capped Excess Wind Ratio (5) - AVG(5)	(7) Capped Excess Wind Losses (3) X (6)	(8) Non-Modelled Excess Wind Ratio Above The Cap (4) - (5)	(9) Excess Wind Losses Above The Cap (8) X (3)	(10) Total Non-Modelled Excess Wind Losses (7) + (9)
12/60	108,781	1,799,873	1,691,092	0.064	0.064	0.000	0	0.000	0	0
12/61	338,985	3,465,992	3,127,007	0.108	0.108	0.000	1,049,213	0.000	0	0
12/62	2,123,842	7,449,796	5,325,954	0.399	0.399	0.197	0	0.000	0	1,049,213
12/63	526,094	7,417,475	6,891,381	0.076	0.076	0.000	0	0.000	0	0
12/64	880,812	7,572,784	6,691,972	0.132	0.132	0.000	0	0.000	0	0
12/65	1,023,957	8,234,603	7,210,646	0.142	0.142	0.000		0.000	0	
12/85	5,249,089	35,420,706	30,171,617	0.174	0.174	0.000	0	0.000	0	0
12/86	2,871,522	27,885,394	25,013,872	0.115	0.115	0.000	0	0.000	0	0
12/87	2,174,221	27,464,409	25,290,188	0.086	0.086	0.000	0	0.000	0	0
12/88	14,301,387	39,398,365	25,096,978	0.570	0.570	0.368	9,235,688	0.000	0	9,235,688
12/89	18,962,472	50,844,072	31,881,600	0.595	0.595	0.393	12,529,469	0.000	0	12,529,469
12/90	13,036,475	40,556,412	27,519,937	0.474	0.474	0.272	7,485,423	0.000	0	7,485,423
12/91	14,988,711	40,765,082	25,776,371	0.581	0.581	0.379	9,769,245	0.000	0	9,769,245
12/92	4,067,790	26,930,737	22,862,947	0.178	0.178	0.000	0	0.000	0	0
Total	$128,135,31	$675,234,41	547,099,108	6.449	6.449	1.967	$44,671,51	0.000	0	$44,671,512
Average				0.202	0.202	0.061		0.000		

(11) Normal Wind to Non-Wind Ratio = Average of Column (5) = 0.202

(12) Median Wind to Non-Wind Ratio = 0.148 5 X Median Wind to Non-Wind Ratio = 0.740

(13) Excess Factor = 1.0 + {(Avg. (6) + (Avg. (8)) / (1.0 + Avg. (5) - Avg. (6))}
Excess Factor = 1.0 + {(0.061 + 0.000) / (1.0 + 0.202 - 0.061)} = 1.053

EXHIBIT 2

DETERMINATION OF INDICATED BASE CLASS LOSS COSTS BY TERRITORY
First Review with Hurricane Model
Sample State

(1) Territory	(2) Aggregate Loss Cost Volume At Current Level	(3) Rel To SW of Current Base Class Loss Cost	(4) Projected Experience Non-Hurricane Base Class Loss Cost	(5) Credibility	(6) Credibility Weighted Non-Hurricane Base Class Loss Cost	(7) Modeled Hurricane Base Class Loss Cost	(8) Total Base Class Loss Cost	(9) Relativity of Territory (8) to Statewide (8)	(10) Indicated Relative Change (9)/(3)	(11) Indicated Base Class Change
A	62,500	0.750	165	0.10	148.13	7.69	155.82	0.587	0.783	-14.4%
B	105,000	0.800	180	0.10	158.40	102.56	260.96	0.983	1.229	34.3%
C	500,000	1.050	200	0.30	203.33	76.92	280.25	1.055	1.005	9.8%
Statewide	667,500		195		191.09		265.56			

DETERMINATION OF INDICATED BASE CLASS LOSS COSTS BY TERRITORY
Second Review with Hurricane Model
Sample State

(1) Territory	(2) Aggregate Loss Cost Volume At Current Level	(3) Non-Hurricane Rel To SW in Current Base Class Loss Cost	(4) Projected Experience Non-Hurricane Base Class Loss Cost	(5) Credibility	(6) Credibility Weighted Non-Hurricane Base Class Loss Cost	(7) Modeled Hurricane Base Class Loss Cost	(8) Total Base Class Loss Cost	(9) Relativity of Territory (8) to Statewide (8)	(10) Relativity To SW Of Current Base Class Loss Cost	(11) Indicated Relative Change (9)/(10)
A	65,000	0.775	175.00	0.10	171.98	7.75	179.73	0.605	0.587	1.031
B	115,000	0.829	183.00	0.10	183.54	103.00	286.54	0.964	0.983	0.981
C	550,000	1.064	235.00	0.30	235.45	78.00	313.45	1.054	1.055	0.999
Statewide	730,000		221.47				297.30			

Use of the Model in Commercial Property (Basic Group II)[9] Ratemaking

A. ***Hurricane Loss Costs: Adjusting Modeled Output to be Compatible with ISO'S Commercial Property Program***

The modeled output is in the form of MDRs, which represent a generic, non-insurance measure of dollars of damage. Therefore, as a first step in the ISO process, it is necessary to convert these MDRs to an ISO basis. Specifically, this means recognizing the various nuances of ISO's Commercial Property Basic Group II Program (i.e. both coverage and rating), which were not reflected in the MDRs provided by the model.

The necessary adjustments are as follows:

- Consolidating the MDRs, which the model provided in refined (i.e. zip code) detail, into broader rating territory detail. This was accomplished through ISO exposure distributions available in county detail, through the Commercial Statistical Plan.

- Mapping the construction scheme (i.e. six constructions) underlying the model into ISO's scheme, which utilizes a symbol format. The ISO structure publishes loss costs for three types of construction: Ordinary (Symbol B), Semi-Wind Resistive (Symbol AB), and Wind Resistive (Symbol A).

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[9] Basic Group II (BGII) provides "extended coverage" for windstorm, hail, riot, smoke, aircraft, vehicles, volcanic action, and sinkhole collapse.

- Adjusting the MDRs, which are on a full coverage basis, to a $250 deductible basis. The adjustment was made using available ISO countrywide full-coverage data to determine the $0-$250 discount. This discount is 3% (i.e. .97 factor).

- Accounting for the coinsurance requirement in Commercial Property, which typically requires insureds to insure their properties to at least 80% of value. Since the model reflects full value when calculating MDRs, it becomes necessary for us to multiply these MDRs by 1.25, since ISO loss costs are quoted based on amounts of insurance assumed to be reported at 80% of these full values.

- Loading in loss adjustment expenses, since the model considers indemnity only within their MDRs. These expenses are loaded in via a multiplicative factor.

B. Supplementing RMS Model with Non-Hurricane Experience

Since the MDRs are intended to price the hurricane hazard only, it becomes necessary to supplement these MDRs with non-hurricane (e.g. tornadoes, tropical storms, riots) loss costs based on ISO experience. Essentially, the development of an ISO non-hurricane data base requires the following four steps:

Step (1): For latest ten years, remove hurricane losses:

The experience review of non-hurricane experience will follow standard ISO methodology for BG II reviews of using ten years of experience. The removal of losses for hurricane months (i.e. any month with hurricane experience) is necessary to avoid double-counting with the hurricane-based model's loss cost. The process for accomplishing this is essentially the same as that previously outlined for Personal Lines.

Step (2): Replace (1) with average monthly non-hurricane losses:

The motivation behind this step is to retain the non-hurricane losses removed as part of the more sweeping removal in step (1). To account for this, a proxy is added for these non-hurricane losses. This proxy is an average of the territory's ten-year average of July-to-December losses that remained after the hurricane months are removed. The average is based on six months rather than one month to minimize the volatility that could result from a one-month average. The use of six months also allows for maximum data within a state, thus avoiding the need to group states with perhaps somewhat dissimilar weather patterns. The period from July to December was chosen to avoid the possible impact of tornadoes, which typically strike during the first half of the year.

Step (3): Apply an excess smoothing procedure for the non-hurricane losses:

The traditional excess procedure has been revised to smooth catastrophic BG II losses due to perils other than hurricane. The revised procedure is based on long-term (1950 to present) statewide BG II non-hurricane experience. For those years prior to 1982 (pre-CSP), any year in which a hurricane occurred has been excluded from the excess procedure, since monthly detail is not available for these years. For 1982 and later, total losses for years with a hurricane have been replaced by average non-hurricane losses as described above. The normal loss ratio cutoff for each year included in the excess procedure is 0.50. From this flow the following definitions:

The Normal incurred losses for each year are those losses which do not exceed 0.500 times the earned premium for the year. The Excess incurred losses for each year are equal to the Incurred losses minus the Normal losses for the year. Thus, we have:

$$\text{Normal Loss Ratio (NLR)} = \frac{\text{Normal Losses}}{\text{Earned Premium}}, \text{ for each year}$$

$$\text{Excess Loss Ratio (ELR)} = \frac{\text{Excess Losses}}{\text{Earned Premium}}, \text{ for each year}$$

$$\text{Excess Component} = \frac{\text{Sum of ELR's}}{\text{Sum of NLR's}}, \text{ over the long-term non-hurricane experience period.}$$

The Excess Multiplier is equal to the excess component plus 1.000 and is applied to the normal non-hurricane losses used in the statewide experience review. There is no longer a regional excess smoothing component used in the hurricane-prone states.

(Attached is Exhibit 4, illustrating the calculation of a sample state's excess multiplier.)

This procedure is essentially similar to the traditional long-term excess procedure used for BG II losses (i.e. hurricane and non-hurricane), with the exception of two points. The first point of divergence involves the use of the .50 cutoff. The second point involves the elimination of regional smoothing.

The .50 cutoff is largely judgmental and attempts to strike a balance between two considerations:

- The cutoff should be low enough to recognize that this ratio represents non-hurricane losses compared to total premiums (i.e. including the hurricane peril);

- The cutoff should be high enough to reflect the fact that the non-hurricane peril is not nearly as volatile as the hurricane peril.

The decision to <u>not</u> incorporate a regional component together with the statewide component in the smoothing procedure was based on the following:

- The hurricane model accounts for the majority of the excess loss dollars, reducing the need to smooth across region.
- Non-hurricane experience is more stable than experience including hurricanes.

C. *Calculating the Revised BG II Loss Costs*

The statewide experience review (Exhibit 5) is based on the latest ten years of non-hurricane loss experience. The losses are normal non-hurricane losses (i.e., hurricane losses reflected by the model have been eliminated and the remaining non-hurricane losses have been capped at 0.50 times the earned premium for each year), multiplied by the excess multiplier, loss adjustment expense factor, and trend factors. The aggregate loss costs[10] are at current manual level and have been trended to the average date of writing in the assumed effective period. Note that these current aggregate loss costs which form the denominator of the annual experience ratios[11] reflect both the hurricane and non-hurricane perils. The result of this calculation is an indicated statewide non-hurricane loss cost level change, where the change is from the total loss cost (i.e. hurricane and non-hurricane) to the non-hurricane loss cost.

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[10] Aggregate loss costs are defined as the product of exposures and ISO published loss costs summed over all risks.

[11] Experience ratio is defined as adjusted incurred losses ÷ aggregate loss costs.

In those states with BG II rating territories, territorial relativities are being revised to reflect both hurricane "differentials" based on modeled output, as well as non-hurricane differences based exclusively on loss experience for these other perils. The territorial review is based on the latest ten years of non-hurricane loss experience (Exhibit 6), and the resulting indicated relativities are credibility-weighted with the statewide average relativity (1.000) to determine the revised non-hurricane territorial relativities.

The non-hurricane portion of the revised BG II loss costs for each territory (where applicable), coverage, and symbol is calculated as:

$$\text{Current BG II Loss Cost} \times \left[\frac{\text{Revised Terr. Rel.}}{\text{Current Terr. Rel.}}\right] \times \text{Statewide Monoline Non-Hurr. Change}$$

where the statewide monoline non-hurricane change is the product of the statewide non-hurricane coverage change and the indicated monoline relativity, as outlined on Exhibit 5. This calculation can be found on Exhibit 7, Column (7) for the Beach territory in the sample state. The remainder of Exhibit 7 shows how the revised territorial BG II total (hurricane and non-hurricane) loss cost is derived by simply adding the modeled hurricane loss cost and ISO-experience based revised non-hurricane loss cost. Indicated loss cost changes are simply weighted across coverage/constructions to determine an overall loss cost level change for each territory. Similarly, Exhibit 8 shows the calculation of the statewide change as an average of the previously calculated territorial changes.

TABLE 31A - DEVELOPMENT OF BASIC GROUP II NON-HURRICANE EXCESS MULTIPLIER*

(1) YEAR	(2) EARNED PREMIUMS	(3) NON-HURRICANE INCURRED LOSSES	(4) NORMAL INCURRED LOSSES	(5) NORMAL LOSS RATIO	(6) EXCESS LOSS RATIO
1951	1,217,965	205,643	205,643	0.169	
1952	1,366,016	250,463	250,463	0.183	
1953	1,313,064	257,083	257,083	0.196	
1954	1,380,201	171,129	171,129	0.124	
1955	1,404,337	355,555	355,555	0.253	
1956	1,472,475	454,615	454,615	0.309	
1957	1,579,563	523,177	523,177	0.331	
1958	1,685,836	241,239	241,239	0.143	
1959	1,672,435	433,655	433,655	0.259	
1960	1,744,386	407,607	407,607	0.234	
1961	1,777,632	389,535	389,535	0.219	
1962	1,731,463	782,480	782,480	0.452	
1963	1,685,767	1,107,190	842,884	0.500	0.157
1965	1,524,306	678,493	678,493	0.445	
1966	1,523,018	430,762	430,762	0.283	
1967	1,545,246	884,886	772,623	0.500	0.073
1968	1,460,382	807,921	730,191	0.500	0.053
1970	2,194,332	717,508	717,508	0.327	
1971	2,457,195	1,018,760	1,018,760	0.415	
1972	2,905,485	1,394,539	1,394,539	0.480	
1973	3,266,668	6,195,532	1,633,334	0.500	1.397
1974	3,820,837	8,844,165	1,910,419	0.500	1.815
1976	5,796,692	2,045,130	2,045,130	0.353	
1977	8,079,010	2,786,457	2,786,457	0.345	
1978	9,835,100	3,385,756	3,385,756	0.344	
1980	10,030,050	5,113,011	5,015,025	0.500	0.010
1981	9,854,456	3,798,736	3,798,736	0.385	
1982	10,409,556	3,705,567	3,705,567	0.356	
1983	9,911,647	5,838,705	4,955,824	0.500	0.089
1984	9,523,948	3,633,728	3,633,728	0.382	
1985	10,890,755	6,662,248	5,445,378	0.500	0.112
1986	13,367,099	2,163,341	2,163,341	0.162	
1987	12,696,500	1,750,276	1,750,276	0.138	
1988	12,523,229	4,647,489	4,647,489	0.371	
1989	11,912,271	7,998,260	5,956,136	0.500	0.171
1990	11,798,355	6,110,356	5,899,178	0.500	0.018
1991	12,028,205	5,032,698	5,032,698	0.418	
1992	11,858,947	2,228,857	2,228,857	0.188	
Totals				13.264	3.895

(7) State Excess Component = (EXLR /NLR) = 0.294

(8) State Excess Multiplier = (1 + SEC) = 1.294

* Hurricane Years Have Been Excluded

EXHIBIT 5

STATEWIDE BASIC GROUP II NON-HURRICANE COVERAGE LOSS COST LEVEL EVALUATION

YEAR	(2) AGGREGATE* LOSS COSTS	(3) ADJUSTED** NON-HURRICANE INCURRED LOSSES	(4) EXPERIENCE RATIO (3) / (2)
1983	19,623,050	11,499,094	0.586
1984	17,091,854	8,122,799	0.475
1985	16,113,850	11,906,927	0.739
1986	16,732,892	4,631,881	0.277
1987	15,674,733	3,624,708	0.231
1988	16,614,603	9,377,596	0.564
1989	16,420,308	11,119,852	0.677
1990	16,046,314	10,796,382	0.673
1991	15,637,938	8,740,128	0.559
1992	14,290,651	3,673,133	0.257

(5) Weighted Experience Ratio (Equal Weights) = 0.505

(6) Indicated Non-Hurricane Coverage Change = 0.505 or -49.5%

(7) Indicated Non-Hurricane Monoline Relativity = 1.1293

(8) Indicated Non-Hurricane Monoline Change ϕ 0.505 X 1.1293 = 0.570 or -43.0%

* Aggregate loss costs are adjusted to current ISO loss cost level and 9/01/95 amount of insurance levels.

** Incurred losses are adjusted to current deductible and 3/01/96 cost levels and include all loss adjustment expenses.

Losses incurred during the month of a hurricane have been excluded and replaced with average non-hurricane losses

ϕ This change is from the total loss costs (i.e. hurricane and non-hurricane) to the non-hurricane loss costs.

EXHIBIT 6

CALCULATION OF NON-HURRICANE BGII TERRITORY DIFFERENTIALS

Territory	(1) Current Territory Differential	(2) 1992 Earned Premium at Current Manual Level	(3) Current Average Loss Costs	(4) Weights (2) / (1)	(5) 10-Year Non-Hurricane Loss Ratio	(6) Indicated Change in Differential (5) / SW (5)
Beach	2.646	563,240	0.471	1,195,839	0.079	0.210
Seacoast	1.573	1,318,885	0.280	4,710,304	0.239	0.634
Inland	0.927	10,665,523	0.165	64,639,533	0.412	1.093
Statewide/Wtd. Avg.	1.000	12,547,648		70,545,676	0.377	

Territory	(7) Indicated Territory Differential (1) * (6)	(8) Balanced Indicated Differential (7) / SW (7) (a)	(9) 10 Year Earned Risks	(10) Credibility (b)	(11) Credibility Weighted Differential (c)	(12) Revised Territory Differential (11) / SW(11) (a)
Beach	0.556	0.554	9,770	5.3%	0.976	0.970
Seacoast	0.997	0.993	55,675	24.4%	0.988	0.992
Inland	1.013	1.009	540,055	75.7%	1.007	1.001
Statewide/Wtd. Avg.	1.004	1.000	605,500		1.006	1.000

(a) Balanced to 1.000 Statewide

(b) Credibility = R / (R + 172,931), where R = 10-Year Earned Risks

(c) Credibility Weighted Differential = (10) x (8) + [1 - (10)] x (1.000), where 1.000 = Statewide Average Differential

EXHIBIT 7

CALCULATION OF TERRITORY BG II LOSS COST CHANGES (a)

Territory - Beach

Coverage	Symbol	(1) 1992 Written Premiums	(2) Current Loss Cost (b)	(3) Weights (1) / (2)	(4) Current Territory Differential	(5) Revised Territory Differential
Building	A	207,025	0.334	619,835	2.646	0.970
Building	AB	24,759	0.414	59,804	2.646	0.970
Building	B	202,141	0.790	255,875	2.646	0.970
Contents	A	10,878	0.267	40,742	2.646	0.970
Contents	AB	7,352	0.334	22,012	2.646	0.970
Contents	B	73,170	0.629	116,328	2.646	0.970
Total/Wtd. Avg.		525,325	0.471	1,114,596		

172

Coverage	Symbol	(6) Statewide Monoline Non-Hurricane LC Change	(7) Revised Non-Hurricane Loss Cost (2) * (5) * (6) /(4)	(8) Hurricane Modeled Loss Costs	(9) Indicated Total Loss Cost (7) + (8)	(10) Indicated Percent Change (9) / (2) - 1
Building	A	0.570	0.070	0.118	0.188	-43.7%
Building	AB	0.570	0.087	0.375	0.462	11.6%
Building	B	0.570	0.165	0.427	0.592	-25.1%
Contents	A	0.570	0.056	0.066	0.122	-54.3%
Contents	AB	0.570	0.070	0.211	0.281	-15.9%
Contents	B	0.570	0.131	0.414	0.545	-13.4%
Total/Wtd. Avg.					0.332	-29.6%

(a) All Loss Costs shown are on a per $100 Amount of Insurance basis, $250 Deductible level, 80% coinsurance.

(b) Current loss costs shown are for Non-habitational properties, Occupancy Class A.

EXHIBIT 8

CALCULATION OF STATEWIDE BG II LOSS COST CHANGE

Territory	1992 Exposure Weights (000)	Indicated Monoline Change
Beach	$ 1,114,596	-29.6%
Seacoast	4,376,754	+44.3%
Inland	60,949,343	-30.9%
Statewide		-23.1%

SECTION V - OTHER USES OF THE MODEL

A. *Pricing Optional Coverages*

Homeowners insurance may not always be written with the basic flat uniform property deductible on all perils, particularly in hurricane-prone areas. Various optional endorsements such as an endorsement excluding coverages for wind and hail, credit for the installation of wind resistant shutters, or higher optional wind deductibles may be offered to lower the insurer's risk of catastrophic loss. Since the output of the hurricane model better measures the long term catastrophic loss potential than the shorter historical statistical data base, it provides a tool for more accurate pricing of each of these options.

Wind Exclusion Credits

A coverage option that has been available in several southeast states in their coastal territories has been the windstorm and hail exclusion. This endorsement does just as it states - it excludes windstorm and hail coverage from the standard property policy forms. The insured is able to buy back the coverage for the excluded peril through the state's Wind Pool or Beach Plan. The excluded coverage may or may not include the additional living expense (Coverage D in an ISO homeowners policy) losses due to the wind losses.

Recognizing the nature of the coverage is a key item in developing a pricing algorithm. A second key factor in developing the pricing is to recognize the nature of the wind coverage provided and the risk of loss to each exposure in the territory under consideration. An important question to consider is whether two risks located in the same territory, the first of which is in a town that has a fire protection code of 4 and the second in an unprotected area (Code 10), have different wind peril exposure based solely on the fire protection difference. Clearly, if all other aspects of these risks in terms of exposure to the wind peril are the same, one would expect that the wind risk is the same. For this reason, the credit developed for the wind exclusion coverage will be a flat dollar credit and not a percentage credit.

To develop the pricing for the wind exclusion, it must be recalled that the loss cost was composed of two components, a modeled hurricane loss cost (H), and a non-modeled loss cost (N). The combination of these two components is the base class loss cost (BCLC). Expressed as a formula,

$$BCLC = N + H.$$

The key to determining the credit for the wind exclusion endorsement is to determine the long term wind percentage included in the non-modeled loss costs. The non-wind portion of the non-modeled loss costs (N) is estimated using the ratio of total non-wind losses to the total non-modeled losses (reflecting the long term non-hurricane wind losses). This ratio can be identified as R. The credit (C) for all protection classes is then given by the formula,

$$C = [BCLC - (N)(R)]\ PC,$$

where PC is the average protection-construction relativity.

The resulting credit C is a flat dollar credit for all protection classes and base amount of insurance. In rating each risk, it would be subtracted from the base class loss cost after application of the protection-construction and policy form relativities but before application of any of the appropriate relativities - policy amount, deductible, etc. A sample calculation of the Wind Exclusion Credit is shown on Table 7.

TABLE 7
CALCULATION OF WIND EXCLUSION CREDIT

Terr.	Base Class Loss Cost	Non-Modeled Loss Cost	Non-Wind Portion of Non-Modeled Loss Cost	Average Protection Construction Relativity	Wind Exclusive Credit
A	155	148	.95	1.06	15.26
B	261	158	.80	1.02	137.29
C	280	203	.85	.97	104.23

B. Development of Territory Definitions

When developing territory boundary definitions, a necessary piece of information is the long-term wind loss potential for small geographic areas. It is quite likely that there may not be adequate historical insurance experience to accurately measure the long-term wind loss potential. Since the output of the model provides the long-term average hurricane loss cost by zip code, these estimates can be combined with more current information from all other causes of loss to produce relative indices by zip code. These indices could be grouped, using banding or clustering techniques, to produce revised territory definitions.

C. Building Code Effectiveness Grading

Hurricane Andrew focused attention on the importance of building codes and the enforcement of these codes in potentially mitigating property damage during hurricanes and other windstorm events.

ISO, working with the Insurance Institute for Property Loss Reduction (formerly the National Committee on Property Insurance), building code officials, and academics, developed a program to grade communities on their building code enforcement activities. Risks in communities receiving a better grade will be given a reduction in their property insurance premium.

Since buildings, even when built to code, are more susceptible to damage at higher wind speeds, a key to pricing the appropriate credits is the long-term frequency and severity of hurricanes (classified by Saffir-Simpson scale). The use of a hurricane model provides the necessary measure of loss potential that, when combined with engineering estimates of the effectiveness of the building code, produces an estimate of the appropriate credit.

D. Risk Load

While the focus of this paper has been on the development of expected costs, it should be noted that the distribution of losses around this average (i.e. variance) often has as much impact on the insurer's pricing and underwriting decisions as the average. This is true because the distribution around the average determines the degree of risk underlying the coverage. For Property catastrophe coverage, this risk is magnified because of the high concentration of properties in areas prone to catastrophic events (e.g. South Florida). Risk load is the charge in excess of the expected losses required to cover the cost of the capital needed to support the risk of providing the coverage. Since risk load is ultimately a variance-based concept, a model can be indispensable in providing mathematical-based distributions of losses for calculating such variances.

SECTION VI - LIMITATIONS OF THE MODEL/PROCEDURE

While we are confident that the ISO new procedure employing the RMS computer hurricane model is a dramatic improvement over the prior loss smoothing procedures, we recognize that there are limitations to the model.

1) Limitations of Meteorological History

While the model uses the broadest history for which hurricanes characteristics are available, this is still a very limited history, with a total of only 157 hurricanes and approximately 650 tropical storms making landfall in the continental United States in the period from 1899 to 1994. Not one of the 31 coastal segments have experienced all five Saffir-Simpson categories of hurricanes in that period. Some segments have not experienced a severe storm in the 96 years. Expanding from the available insurance database to the available meteorological data base has not totally solved the problem of sparse data. In the absence of a more complete meteorological history, significant assumptions and extrapolations have to be made, particularly with respect to the central pressure differential distribution.

In addition, it is quite plausible that hurricane frequency is impacted or correlated with large scale climatic and geological cycles that are currently not fully understood The model does not attempt to incorporate any cyclical or other time dependent interpretation of the meteorological data.

2) Limitation of the Understanding of Hurricanes

The model estimates the damages generated from an average hurricane with a given central pressure differential and forward velocity. In reality each hurricane is unique. Due to limited understanding of hurricanes, the complicated physics at the core of the storm (which are difficult to parameterize), and the absence of data for more sophisticated modeling, the model is not able to capture unique features.

3) Limitations of the Exposure Inventory

While the model can produce MDRs by specific location, construction, and other variables, the exposure inventory (amount of insurance data), is rarely available in as fine detail. Of particular importance is the location of the risk. While zip code detail is now being reported for personal lines, only statistical territory detail was available for experience prior to 1994. For commercial lines, county detail is reported under the current ISO statistical plans. For coastal areas the variation of MDRs within an individual zip code can be quite significant. Only three wind-based categories of construction are reported to ISO. Information such as number of stories, roofing type, and other details that can impact hurricane vulnerability are not reported. To the extent that the exposure inventory is not available in fine location detail (as well as other variables), averaging will be required. Thus, the output of the model will always be constrained by the limitations of the input.

4) Demand Surge

The model assumes that the cost of repair -- materials and services -- will be relatively normal. One of the lessons of Hurricane Andrew is that a severe catastrophe can dramatically affect those costs -- a phenomenon described as demand surge. While the current model does not reflect demand surge, it is being considered.

5) Limitations of Damageability Information

Unlike earthquake for which public, government-sponsored studies of damageability (such as the Applied Technology Council publication 13) are available, there is no broadly-accepted and publicly-released analysis of damageability from hurricanes to use as reference or starting point. Thus, the modelers must rely on more limited proprietary information from individual clients.

6) Limitations of the model in specific pricing situations

The output of the hurricane model may not be appropriate for use in all pricing or ratemaking situations. It is necessary to check the assumptions underlying the model before using the output in the pricing.

An example of where this may be true is in developing policy amount relativity factors. The Mean Damage Ratio is generally defined as the ratio of the structure's repair cost divided by its replacement cost. If the hurricane model's Mean Damage Ratio is calculated by averaging the damage ratios which are available for each combination of construction materials (frame, masonry, etc.), building usage (residential, commercial) or unit type (single family, multi family) but does not vary by amount of insurance or value of the property, then it is unclear if the Mean Damage Ratio would be appropriate for use in determining amount of insurance relativities. This may be particularly true if the data underlying the table that was used in calculating the damage ratios can be shown to be some function of the amount of insurance.

The key fact in this situation is to know what data, assumptions and calculations underlie the results of the model.

SECTION VII - RESULTS OF THE MODEL

Observations for Personal Property

In examining the indicated loss cost changes within a state, after introduction of the model, a relationship emerged between the statewide indications and the frequency of hurricanes within the experience period. In states where the 33 years of experience had infrequent hurricanes (vs. the nearly 100 years of meteorological history), the indications were for loss cost level increases and in some cases significant ones. In other states, the indications were negligible or negative if the recent 33 years of hurricane frequency was more frequent than the 100 year meteorological history.

One important advantage of the model is the more accurate estimation of the hurricane peril by territory within a state. Although up to 33 years of wind experience was traditionally used in ISO's catastrophe procedure for the statewide loss cost changes, the distribution of the catastrophe wind losses to territory has been based on a shorter time period of 10-20 years of wind losses.

In reviewing the modeled hurricane loss costs by territory, there is a very strong relationship in the severity of the modeled hurricane loss cost and the territory's distance to coast. The territories on or near the coast typically have the highest modeled hurricane loss cost, with this loss cost decreasing as the territories get further from the coast. This is due to two factors. One, the hurricane will most likely be strongest when passing through the coastal territory. Two, independent of the storm's path, winds are higher by the coast due the absence of local roughness which would have a tempering effect on the wind speeds.

SECTION VII - RESULTS OF THE MODEL

Observations for Commercial Property

In examining the Basic Group II statewide indications, upon introduction of the model, two basic patterns emerged by region. In the Northeast, most states showed clearly positive indications, with some indicating very substantial increases. The Southeastern states generally indicated moderate decreases, with the major exception to this pattern being Florida, which had a large positive indication. The explanations for these patterns fall into two categories, both related to the experience-based methodology used prior to the introduction of the model:

1) Experience Period: As alluded to previously in this paper, a problem with the experience-based procedure is the limited period available for hurricane insurance statistics. While for BG II, this period covers as much as 43 years (i.e. 1950-1992), this still leaves a gap when contrasted with the nearly 100 years of meteorological history underlying hurricane models. The gap between these two periods has opposite impacts on the indications, by region. In the Northeast, the experience period is too recent to reflect major hurricane activity that struck the Northeast throughout the 1930's and 1940's. Hence, the model's inclusion of this period has, in effect, corrected for this via upward indications. In the Southeast, on the other hand, major hurricanes in the 1950's and early 1960's, and in particular the two prominent recent events (Hurricane Hugo and Andrew) have been captured by the experience period, hence resulting in no particular need to "true up" overall loss cost level within the region as a whole.

2) Regional Smoothing: An integral part of the previous methodology was the inclusion of a significant regional component (supplementing the statewide component) within the excess factor meant to account for catastrophic losses. First of all, this had the impact of keeping the southeastern and northeastern regions totally separate, thus preventing at least some spreading of the more recent hurricane activity in the former to the latter. Secondly, the emphasis of the regional component, particularly for highly severe occurrences such as Andrew, may have contributed to an over-spreading of these losses throughout the southeastern region, and away from Florida. The model is likely correcting for this by producing a high increase in Florida, at the expense of loss cost level in the other states within the region.

SECTION VIII - CONTINUING ACTIVITIES

As a user of a hurricane model, it is important to maintain an ongoing relationship with the developers of the model. It is expected that the models will undergo improvements over time, as a result of additional meteorological data becoming available as new hurricanes occur or new meteorological research is done. Any change in the relationship between the meteorological characteristics and the damageability of property could occur either based on new engineering studies or on additional insurance statistics that become available. Thus, it is important to keep up-to-date with any new information that could be reflected in future versions of a model.

APPENDIX A - WEIGHTING MEAN DAMAGE RATIOS FOR HOMEOWNERS TENANTS AND CONDOMINIUM POLICY FORMS

For HO-4 and HO-6, the amount of insurance collected in the Statistical Plan is for Coverage C(contents).

The homeowners tenants policy form (Form 4) provides coverage for contents and additional living expenses and the additional living expenses is usually 20% of the Coverage C amount. Thus, the building MDR is given a weight of 0 for Coverage A and B; the contents MDR is given a weight of 1.0 for Coverage C; and the additional living expenses MDR is given a weight of .2 for Coverage D. The tenants form is written on single-family and multi-family units. Thus, the single-family and multi-family MDRs are weighted together using the distribution of single-family and multi-family houses obtained from the Census Bureau for the state.

The homeowners condominium policy form (Form 6) provides coverage for applicable building structures, contents and additional living expenses and the additional living expenses are usually 40% of Coverage C. Thus, the building MDR is given a weight based on the reported Coverage A amount of insurance limit collected in the Statistical Plan as a percent of the reported Coverage C amount of insurance for each territory and construction class. The content MDR is given a weight of 1.00 for Coverage C; and the additional living expense MDR is given a weight of .40 for Coverage D. Since the condominium policy form is written primarily on multi-family units, only the multi-family MDRs are used.

Glossary:

Additional Living Expenses - a form of coverage which may be included in a Homeowners or Dwelling policy, providing funds to pay for increased living costs which result from damage covered by the policy.

Appurtenant Structures - a structure pertaining or belonging to the insured structure, such as a tool shed.

Base Class Loss Cost - for the homeowners owners forms, the territory loss cost for Policy Form 3, Protection Class 5, Frame Construction and Policy Amount of $60,000. The base class loss cost does not reflect the application of Policy Amount relativities (or Key Factors), Protection/Construction relativities, Policy Form relativities or other applicable discounts or surcharges for a particular policy.

Basic Group II (BGII) - the extension of commercial property insurance to the perils of windstorm, hail, riot, smoke, aircraft, volcanic action and sinkhole collapse.

Classification and Coverage Factor - an average rating factor in homeowners representing the distribution of earned house years by policy form, protection/construction, policy amount and other applicable policy provisions relative to the base class loss cost.

Composite Projection Factor - a trending factor that reflects external loss projection, total loss trend adjustment (if applicable), adjustment for trend from first dollar and amount of insurance projection. The composite projection factor is applied to the loss costs on a current cost/amount level to project losses to the average date of loss (12 months past the effective date) and amount of insurance to the average date of writing for policies written during the period the new loss costs are assumed to be in effect (6 months past the effective date).

Current Cost/Amount Factor - a trending factor which reflects the combined loss trend as measured by the external index and amount of insurance trend on the loss cost from a given accident year to the point in time corresponding to the mid-point of the latest available quarter of the Current Cost Index.

Central Pressure Differential - the difference between the ambient sea-level pressure at the outer limits of hurricane and the lowest sea-level pressure at the center of a hurricane. As this differential increases, the strength of the storm and velocity of the winds generated by the storm increases.

Decay - the reduction in wind speeds of a hurricane due to removal of the oceanic heat/energy source as the hurricane moves from sea to land or over cooler water.

Damage Ratio - the ratio of losses due to a hurricane to the replacement cost.

Dwelling Extended Coverage - a common extension of dwelling property insurance beyond fire and lightning. Extended coverage adds insurance against loss by the perils of windstorm, hail, explosion, riot and riot attending a strike (civil commotion), aircraft damage, vehicle damage, smoke damage and volcanic eruption.

Expected Hurricane Losses - the expected losses due to the hurricane peril as estimated from the hurricane computer model using the latest year amount of insurance years. The losses are on a $250 deductible level and are calculated by multiplying the homeowners Coverage A amount of insurance years by each territory's weighted mean damage ratios for each construction class.

Eye ("of a storm") - the roughly circular area of comparatively light winds and fair weather found at the center of a tropical cyclone (hurricane or tropical storm). The diameter of the eye typically ranges from 10 to 30 miles.

Forward Velocity - the rate of movement of the hurricane center.

Hurricane - a tropical cyclone with sustained winds of 74 mph or more.

Hurricane Loss Cost - the portion of the loss cost attributable to the hurricane peril. The loss cost is determined by dividing the expected hurricane losses by the latest number of house years.

Indicated Loss Cost Change - the percent change that must be made to the current loss costs to achieve adequacy to pay for losses and loss adjustment expenses in the prospective period.

Loss Adjustment Expense Factor - a factor applied to the indemnity losses to load for allocated and unallocated loss adjustment expenses. The factor represents the ratio of the sum of the incurred indemnity losses plus all loss adjustment expenses to the sum of the incurred indemnity losses.

Mean Damage Ratio (MDR) - the expected damage ratio across all simulated storms, calculated as the sum of the products of the individual storm probabilities and damage ratios.

Net Mean Damage Ratio - an MDR adjusted to reflect a deductible. For homeowners, the common ratemaking deductible is $250.

Net Weighted Mean Damage Ratio - an MDR used for Homeowners reflecting the appropriate building, other appurtenant structures, contents and additional living expenses MDRs and weighing them together based on the relationship of their amount of insurance weight to the Coverage A (building) amount of insurance. The net weighted MDR is applied to the Coverage A amount of insurance to develop expected hurricane losses for all coverages on a homeowners policy.

Non-hurricane Loss Cost - the portion of the loss cost that is attributable to all covered perils other than the hurricane peril.

Population-Weighted Centroid - the central location (latitude/longitude) of a zip code based on census tract population weights.

Prospective Loss Cost - the portion of a rate that does not include provisions for expenses (other than loss adjustment expenses) or profit, and is based on historical aggregate losses and loss adjustment expenses adjusted through development to their ultimate value as well as a model-generated hurricane loss provision, both projected through trending to a future point in time.

Radius of Maximum Winds - the radial distance from the hurricane center to the band of strongest winds, the area immediately past the eye.

Roughness - characteristics of a local area (e.g. uneven elevation) which modify the hurricane windspeeds near the surface.

Saffir-Simpson - a scale (from 1-5) used to measure hurricane intensity, with 1 being the least severe and 5 being the most severe.

Territory Relativity - the factor which relates the territory loss cost for a particular territory to the statewide loss cost.

Track Angle - the angle that the forward path of the hurricane makes at landfall as measured clockwise from due North.

Using a Geographic Information System to Identify Territory Boundaries

by Steven Christopherson, and
Debra L. Werland, FCAS

USING A GEOGRAPHIC INFORMATION SYSTEM TO IDENTIFY TERRITORY BOUNDARIES

Steven Christopherson
Debra Werland

ABSTRACT

The location of a risk is an important rating variable in most lines of insurance. The aggregate loss experience of similarly located risks is needed in order to determine an appropriate rate for a particular area. A geographic information system (GIS) can be used to estimate the geographic component of insurance risk at any location. Exposures and losses at nearby locations can be aggregated by a GIS without being constrained by predetermined boundaries. After geographic risk has been estimated for each location, GIS can draw a topographic risk map for an entire state. Risk terraces, created by rounding off the risk estimates to several discrete values, can be shaded according to relative risk, like elevation on a standard topographic map. New territory boundaries could be drawn along the boundaries of the risk terraces. When contrasted with the results of traditional territory rating, our new methodology creates a more detailed and representative picture of geographic risk.

Biography:

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USING A GEOGRAPHIC INFORMATION SYSTEM TO IDENTIFY TERRITORY BOUNDARIES

OVERVIEW

Location of residential property is a key determinant in the rating of Homeowners insurance. Territory boundaries within a state define areas which are demonstrably different from other areas within the state. For most insurance companies, territory boundaries have not changed significantly over the years, although territory relativities have changed because of loss experience or competitive market forces. This paper will demonstrate the power of a geographic information system (GIS) in determining a company's geographic risk relativities within a state.

Relative geographic risk was represented here as a topographic risk surface, which was rounded to discrete values (or risk terraces) for rating purposes. Loss experience was analyzed using a pure premium approach, with exposures defined as amount of insurance years. Although the example was based on the Homeowners insurance experience of a hypothetical company, the basic technique and principles could apply to other lines of business. Where the company's experience was not considered credible, we employed credibility formulas.

The results of our new methodology were contrasted with the results of a traditional methodology. The overall results from the two approaches were similar, but the new approach had the advantage of revealing more of the underlying geographic variability.

USE OF A GEOGRAPHIC INFORMATION SYSTEM (GIS)

Geographic mapping software is now available to enable actuaries and underwriters to see the location and variation of risk levels on computer-drawn maps. Typical geographic information system software comes with the coordinates needed to draw familiar geographic and political features: rivers, streets, county lines, and zipcode boundaries. Any data from external files can be mapped if latitude and longitude, zipcode, or other geographic reference is included. Maps which have heretofore been painstakingly done by hand, such as territory maps or catastrophe exposure maps, can now be generated by computers and multi-color printers.

We used GIS software to geocode (i.e., mark latitude and longitude coordinates) Homeowners insurance exposures and losses in order to identify which data outside of a zipcode was near enough in distance to be used in estimating the local geographic risk of the zipcode. We also used GIS software to draw the boundaries of each zipcode and shade each zip according to its rounded risk estimate. Zipcodes with equivalent risk estimates appeared as same-shaded risk territories, or, in our new terminology, risk terraces.

PROPOSED METHODOLOGY

Homeowners risks are typically rated according to the following primary variables: geographic location, amount of insurance, protection class, and type of construction. For rating geographic risk, we are concerned about the physical and social conditions at and around a location and about significant differences among locations within a state. The most relevant data for estimating geographic risk are those that center on the neighborhood being evaluated. This principle is often violated in current territory ratemaking, because the territory boundaries always cut off nearby data that is relevant to the neighborhoods near the boundaries. The data directly across a territory boundary (often across the street) are more relevant than the most distant data within the territory.

The proposed method is based on the principle that the physical and social conditions ***around*** a location impact the risks associated with homes ***at*** that location. Certain weather-related perils are found throughout a state, such as freezing temperatures, but extreme weather-related perils, such as hailstorms or hurricanes, tend to be geographically constrained. Social conditions, such as crime patterns, are generally the result of actions pertinent to specific areas of a city or region and do not occur with equal frequency within a state. Whether we look at weather conditions or social conditions, the risk level will vary gradually from one location to another location.

Essentially, all nearby relevant data should be used in estimating a neighborhood's geographic risk. This approach would be impractical, however, without automation and the ability to identify the location of each risk geographically. Geographic mapping software makes it possible to assign a latitude and longitude to every customer address, census block, or zipcode and to determine the geographic distance between every data point. From any geographic starting point, we can programmatically collect all the nearby data in order to estimate the relative risk in each geographic neighborhood.

Data Requirements And Adjustments

Five years of policy data and non-catastrophe loss data were used in calculating pure premiums. Losses were developed and trended to current cost levels. In order to diminish the effect of liability losses (Section II of a typical Homeowners policy), the individual incurred liability loss dollars were capped at $100,000. A unit of exposure was defined to be $10,000 worth of coverage for one year (based on Coverage A Dwelling of a typical Homeowners policy). A $100,000 home insured for one year represented 10 exposures; if insured for half a year, 5 exposures were represented.

Statistically, geographic risk is the residual risk after the effects of other ratable variables have been controlled. In other words, geographic risk is the remaining variation in loss experience after subtracting the effects of deductible, amount of insurance, protection class, and construction. We have, therefore, adjusted losses to a common deductible and adjusted exposures for the other major variables in order to remove the bias that would result from any non-random geographic distribution of these ratable variables. We are then left with only the geographic component of risk to measure. The adjustment procedure is similar to adjustments in Personal Automobile whereby the exposures in each zipcode are multiplied by the zipcode's average class factor in order to remove class bias due to different class distributions by zipcode.

Distance And Credibility Formulas

Geographic risk at location L is similar to the geographic risk at locations near L. Loss data in zipcodes contiguous to location L are therefore expected to be similar to the loss experience in L's zipcode. For rating geographic risk, we generally do not have enough data in a local neighborhood L to develop a credible rate, so we aggregated data surrounding L to identify and differentiate groups of neighborhoods, called territories or risk terraces.

In our method, the data from each 5-digit zipcode were supplemented with data from nearby zipcodes. For computing convenience, each zipcode was defined to be a neighborhood. Mapping software provided geographic coordinates representing the center of each zipcode, and all the records for a zipcode were assigned to the zipcode's coordinates. The coordinates made it possible to calculate the distance between every pair of zipcodes. The following formula was used to weight nearby data according to distance from the local zipcode center. The weighting function is graphed in Exhibit 1.

Distance	Weight
$0 <= d <= 5$ km,	1
5 km $< d <$ 35 km,	(35-x)/30
35 km $<= d$,	0 .

This distance function was arbitrary but constrained by the logic that nearer data are more relevant than farther data. The radius could have been longer, shorter, or even variable. The decreasing weight function could have been linear or nonlinear, segmented (as above: 0-5 km and 5-35 km) or not, and it could have accelerated early or late.

Based on traditional Homeowners ratemaking, as discussed in Walters [1], a body of experience could be deemed "fully credible" if there are at least 40,000 earned house years in the experience period. Partial credibility has been represented by the square root rule, as introduced in Longley-Cook [2], i.e., local credibility = square root (local exposures/credible exposures). We converted all our exposures to a $100,000 base coverage and redefined full credibility to be 400,000 $10K exposures. In traditional Homeowners ratemaking, if the result for a group of neighborhoods or territory was less than fully credible, the mean pure premium for the territory was credibility-weighted with the statewide mean pure premium.

In our new method, before the local pure premium was adjusted with the statewide pure premium, an intermediate group adjustment was made according to the local zipcode's MSA (metropolitan statistical area) grouping: rural versus non-rural. The following formulas were used in the credibility adjustments in the new method:

credible exposures=400,000 $10k exposures
local exposures=# $10K exposures in local and nearby zips weighted by distance
group exposures=# $10K exposures in MSA grouping: rural, non-rural
local credibility=sq.rt.(local exposures/400,000)
group credibility=sq.rt.(group exposures/400,000), max = 1-local credibility
state credibility=1-local credibility-group credibility
full credibility=local credibility+group credibility+state credibility=1

The adjusted pure premium (pp) for a local zipcode center was: $pp_{adj.\ local}$ = ($credibility_{local}$ * pp_{local}) + ($credibility_{group}$ *pp_{group}) + ($credibility_{state}$ * pp_{state}).

The resulting credibility-adjusted pure premium for the zipcode center was divided by the unadjusted statewide pure premium to obtain the geographic risk relativity.

RESULTS

To illustrate the results, we selected a traditional ISO territory in an unidentified state. Territory results were calculated using a traditional method, which aggregated all data within the territory boundary without regard to distance, adjusted the results by credibility-weighting with the statewide results, and then applied the results uniformly across the territory. The distance from the center of this ISO territory to the border averaged about 35 km, which was comparable to the 35 km radius circles which we used to aggregate data for each zip in our new method. Exhibit 2 displays this traditional territory with zipcodes inside and outside the boundary.

While the traditional territory method generated one relativity for the entire territory, our new method generated several different relativities. Exhibit 3 shows how the new credibility-adjusted relativities in and around the traditional territory varied from the policyholder-weighted average of the new relativities in the territory. These zipcode-based relativities ranged from below average in the eastern and southern zipcodes of the territory to above average in the western zipcodes. The traditional credibility-adjusted territory relativity deviated only +.01 from this weighted average. Although the two methods derived similar average relativities for this territory, only the new method revealed the underlying variability.

Exhibit 4 gives the credibility-adjusted claim frequencies in and around the traditional territory. In this case the policyholder-weighted average was the same as the traditional credibility-adjusted territory result. The eastern zipcodes of the territory had the lowest claim frequency and the northwestern zipcoes had the highest. For this territory, the two methods gave identical average frequencies, but only the new method showed how the frequencies varied from zipcode to zipcode.

In Exhibit 5 we see that the lowest credibility-adjusted claim severities were in the eastern and southern sections of the traditional territory. The zipcodes with the highest claim severities were in the western section. The traditional credibility-adjusted territory result for claim severity was only $34 below the policyholder-weighted average for the territory's zipcodes. Again, the two methods have a similar overall result, and only the new method isolated the underlying differences.

The last map, Exhibit 6, shows the credibility distribution across the zipcodes. The highest credibility (or, alternatively, the highest number of exposures) was in two zipcodes near the center of the traditional territory. Credibility (or exposures) decreases as we move the focus away from this peak. The traditional territory credibility was .07 higher than the policyholder-weighted average, because the traditional territory method gave all exposures in this 70 km wide territory full weight, whereas the new method gave only partial weight to most exposures in the 70 km diameter circle around each zipcode. Although the two methods weight exposures differently, the relativity, frequency, and severity values that were derived by the new method were comparable to the traditional method's results.

We could say that each zipcode is its own territory, but that is not true in the traditional sense of territory because the local zipcode risk estimate incorporates nearby data from outside the zipcode. In effect, our attempt to identify new territory boundaries has resulted in the elimination of traditional boundaries. For any zipcode, we could draw a boundary around all the nearby zipcodes that form the data pool for the local zipcode estimate. If we do the same thing for an adjacent zipcode, then the second boundary will cut through the first boundary because the two data pools will be overlapping. Continuing this procedure, it is clear that each zipcode is part of multiple data pools. The estimate derived from any data pool is assigned to the zipcode at the center of the data pool.

A key advantage of the new method is that it reveals much of the underlying variability that is obscured by the traditional territory method. This textural detail is evident in Exhibits 3 to 6, where several values appear in what would otherwise be a single-valued traditional territory. Another key advantage is that the natural clustering of zipcodes is revealed without the distortion caused by territory boundaries that split geographically contiguous data. For example, Exhibits 3 to 6 show that the easternmost zipcodes in the territory have more in common with nearby zipcodes ***outside*** the territory than with the other zipcodes ***inside*** the territory.

ADDITIONAL CONSIDERATIONS

Although our new method provides one basic approach for developing geographic relativities, there are several areas which deserve further consideration. While we are not proposing any definitive stance on these issues, we do raise them as deserving more attention and research. These areas include: (1) catastrophe adjustments, (2) impact of large losses, (3) years of experience, (4) credibility issues, (5) optimal number of zipcode groups or territories, (6) geographically-based versus population-weighted centroids, and (7) industry versus company analysis.

Although catastrophes are fortuitous events, there are areas within a state which are more prone to certain natural hazards, e.g., hurricanes in southern Florida, hailstorms in the Dallas-Fort Worth area, brush fires in southern California. Catastrophe loss experience for territory ratemaking should include as many years as possible, not just the standard five years. Ideally, with the use of computer simulation and modeling for certain natural hazards, catastrophe pure premiums can be developed and added to the non-catastrophe pure premiums before zipcode-based territories are determined. This procedure could work for hurricanes, tornadoes, and hailstorms, perils for which models now exist.

Extremely large losses in the experience period may cause unusual results from year to year when the analysis is repeated. Instead of using mean pure premiums in the development of territories, perhaps median pure premiums could be used, or outliers could be eliminated when individual claims are considered for the input file. Otherwise, one could put a cap on individual losses, say twice the statewide average amount of insurance, or some other judgmental but reasonable figure.

While five years of exposure and loss data are typically used in Homeowners ratemaking in the development of an indicated rate change, using more years of loss experience would increase the stability of the risk estimates. Using only five years, as in the method outlined in this paper, many states would not reach full credibility on a statewide basis.

We have presented only one method for addressing full and partial credibility. This area of the paper deserves further attention. Many other formulas could be applied, while not detracting from the essence of the proposed procedure. We used a three-way credibility formula based on exposures. Perhaps claims could have been used instead of exposures. Perhaps a simpler two-way credibility formula could have been applied.

The number of zipcode groups or territories developed is more of a judgment call than the result of a statistical constraint. However, one could argue that the number of territories is optimized if the number selected results in the smallest within variance of the zipcode groups and the largest between variance among the groups, as those terms are normally understood. It is left to actuaries and underwriters to determine the appropriate number of discrete territories. Competitive considerations may also play an important role in this determination. We have not taken market forces into account in our example, but we do realize their importance. Regulators are likewise concerned about the range of premiums by zipcode, county, or city among major competitors.

Through the use of a geographic information system, each zipcode's risk was estimated using data within a specified radius of the zip's centroid (defined by specific coordinates of latitude and longitude). These coordinates were used in a distance formula which gave less weight to the more distant data. Alternatively, if we had more computing and storage capacity, we could calculate population-based centroids. Population-based centroids might give even more detailed and representative estimates of the underlying loss distribution.

The final issue involves industry data versus individual company data analysis. Perhaps territory boundaries should be developed based on a much larger volume of data, such as in Texas Homeowners, while territory relativities should be determined by individual companies, representing their own relative risk within a state. Obviously, large insurance companies can rely heavily on their own experience, while smaller companies need to rely on the direction taken by others in the market to adequately assess their risk.

It is left to the reader to develop the overall statewide indicated rate change and to apply that change, or a selected change, to each individual territory or zipcode group. We have concentrated here on relative geographic risk within a territory.

SUMMARY

The geographic component of risk is a major factor in Homeowners insurance. Although the traditional method generates one pure premium relativity, one frequency, one severity, and one credibility for an entire territory, the true geographic risk varies from point to point inside a traditional territory. An improved method would recognize geographic areas that are higher or lower than the traditional territory average. An improved method would also divide risk estimates into small steps or terraces, instead of the large steps or cliffs that we often see between traditional territories.

It may be tempting to use the boundaries of the risk terraces to define the boundaries of new territories, but such terraces are not the equivalent of traditional territories, because each constituent part, i.e., each zipcode, already has its own credibility-weighted geographic risk relativity. A terrace would be a *pseudo*-territory in the sense that the zipcodes would not be locked into predetermined alignments; zipcodes would be free to shift to higher or lower terraces whenever there is a sufficient change in Homeowners experience.

Traditional methods of isolating and estimating geographic risk have been widely criticized 1) for being slow to respond to realignments of underlying risk drivers and 2) for creating disparate risk estimates for exposures that are separated only by a territory boundary. Our method of estimating risk puts each zipcode at the center of its own pool of distance-weighted data, with data at smaller distances receiving larger weights. Our method has at least two advantages: 1) zipcodes are automatically regrouped into relativity terraces whenever there is a significant change in the data, and 2) adjacent zipcodes will have overlapping data pools, and consequently, similar risk estimates.

We used a geographic information system (GIS) to assign latitude and longitude coordinates to the center of each zipcode so that nearby data could be pooled according to a distance-weighted formula. The resulting small-step terraces, built from the estimated risk relativities for the zipcodes, are consistent with the construct that true risk varies gradually from point to point, but the boundaries of these terraces are not the boundaries of traditional territories. A consequence of our method is that traditional territory boundaries disappear.

REFERENCES

[1] Walters, Michael A., "Homeowners Insurance Ratemaking," PCAS LXI, 1974, p.15.

[2] Longley-Cook, L.H., "An Introduction to Credibility Theory," PCAS XLIX, 1962, p.194.

Exhibit 1:
Distance function to aggregate and weight nearby data

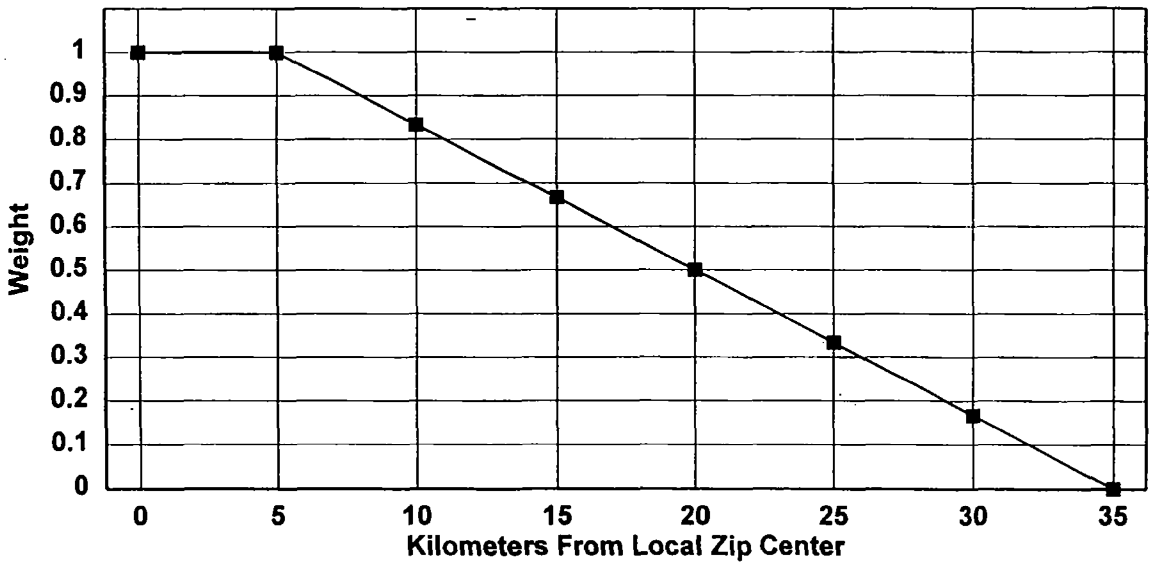
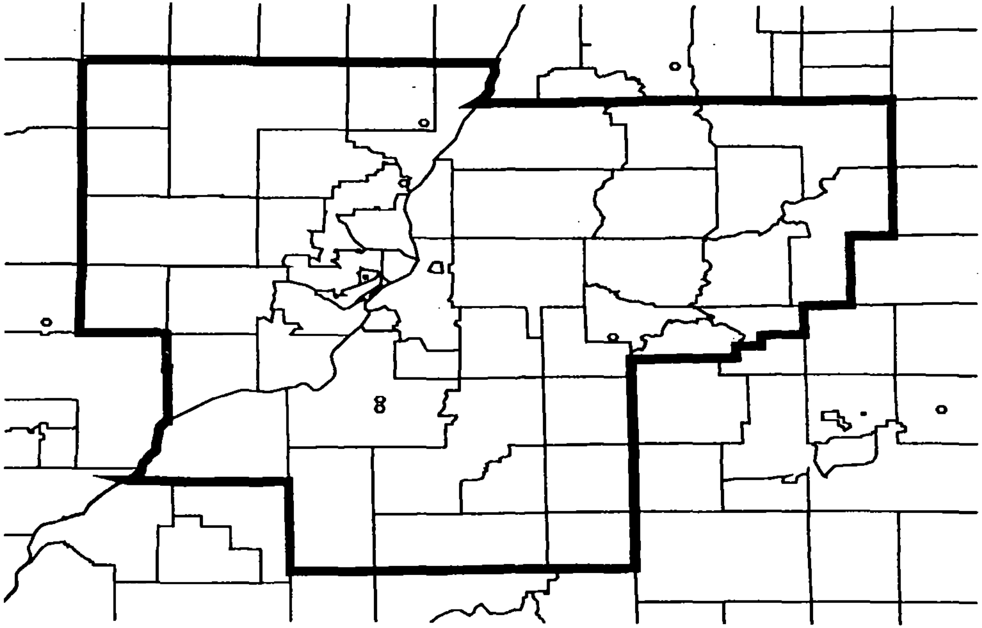


Exhibit 2:

Traditional ISO Territory



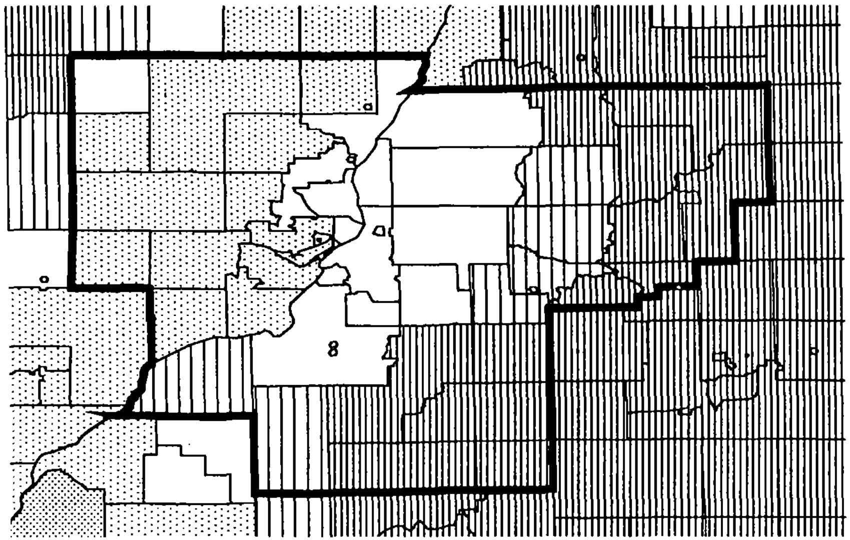
Old Territory

This traditional territory boundary follows county lines. Traditional territory methods aggregate only the data within the boundary and apply the results uniformly across the territory.

Note: The new zip-based method aggregates all nearby data, even data across county lines.

Exhibit 3:

Pure Premium Relativity



Old Territory

Zip Deviation*

- +.06
- +.03
- 0
- -.03
- -.06

Note: The traditional territory result deviates +.01 from average*.

***Deviation relative to weighted average across zips inside old territory: average = sum (p x value) / sum (p), where p = number of policyholders in zip.**

Prepared by:
Geographic Data Analysis
498-5227
9/26/95

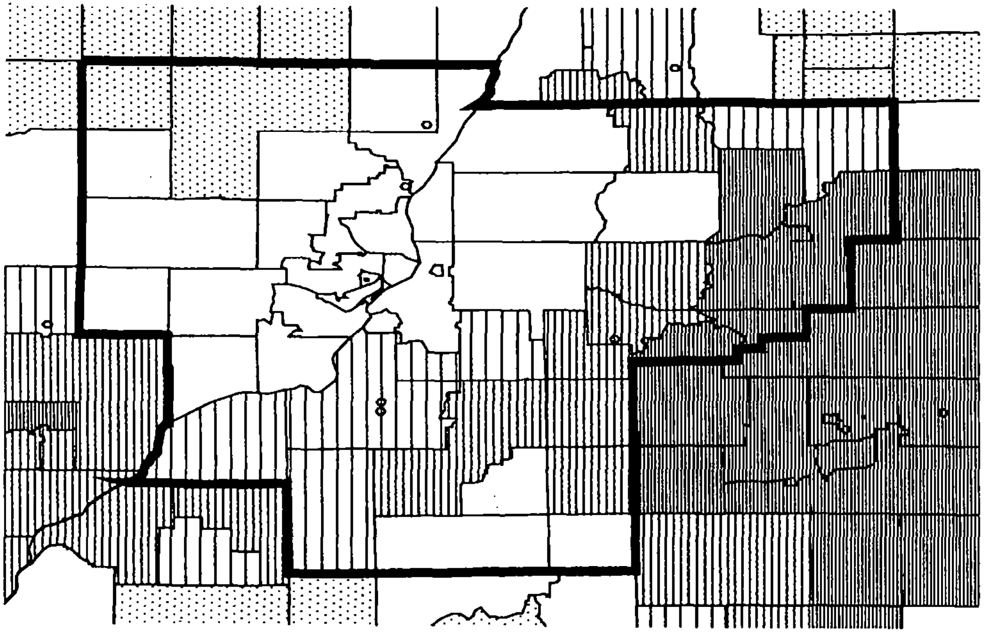


Exhibit 4:

Claim Frequency

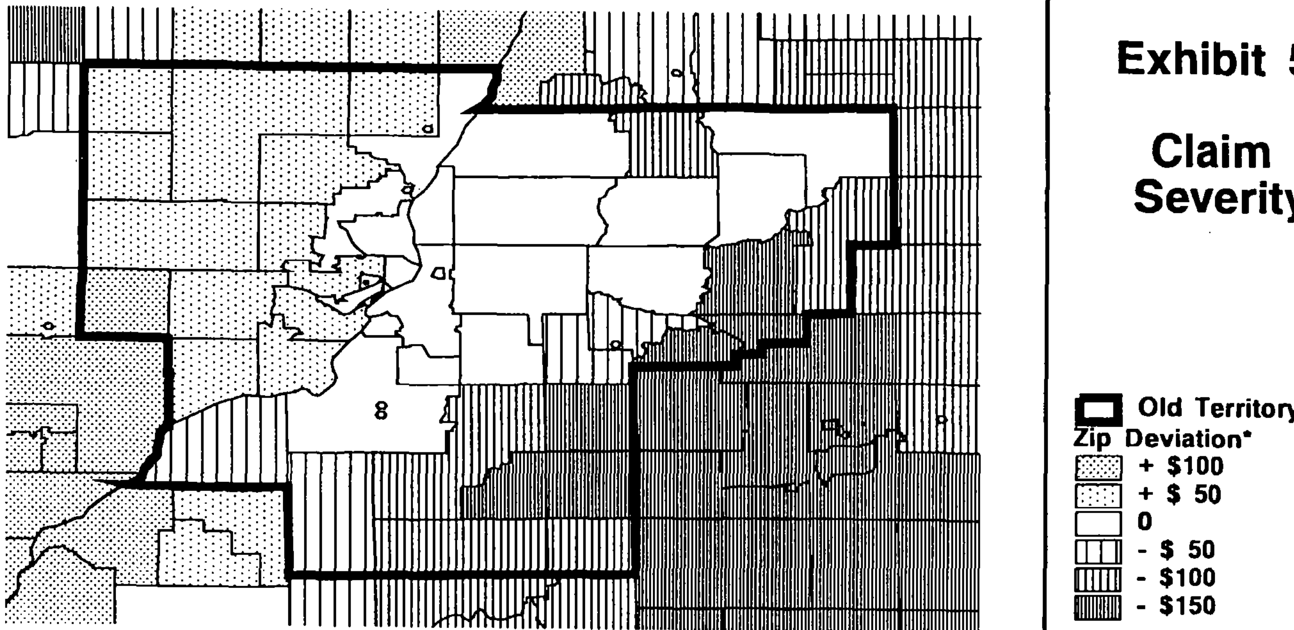
Old Territory
Zip Deviation*
+.001
0
-.001
-.002
-.003

Note: The traditional territory result deviates 0 from average*.

*Deviation relative to weighted average across zips inside old territory:
average = sum (p x value) / sum (p), where p = number of policyholders in zip.

Exhibit 5:

Claim Severity

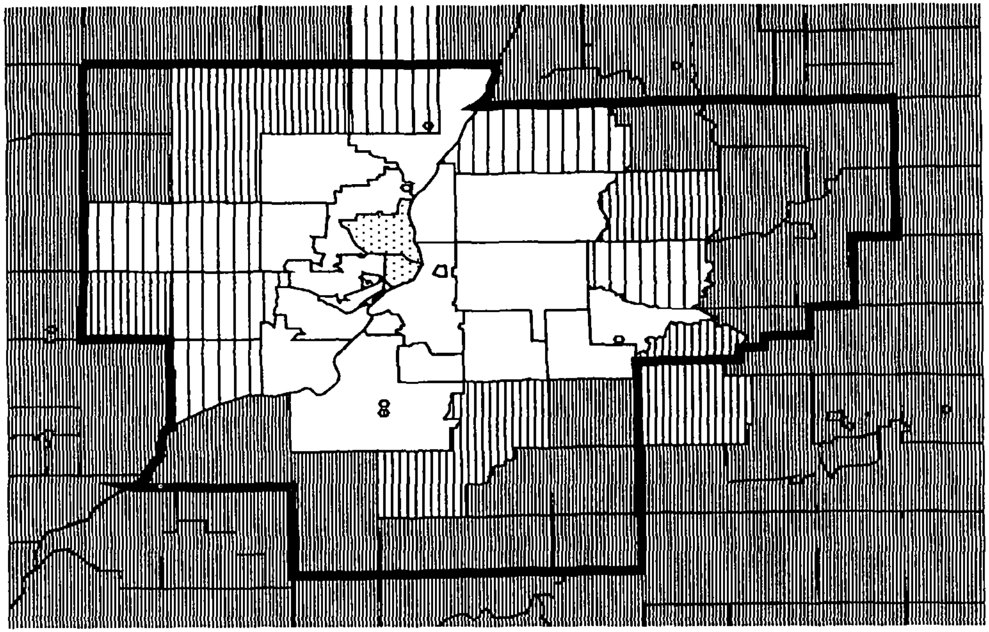


Note: The traditional territory result deviates -$34 from average*.

***Deviation relative to weighted average across zips inside old territory: average = sum (p x value) / sum (p), where p = number of policyholders in zip.**

Exhibit 6:

Credibility



Note: The traditional territory result deviates +.07 from average*.

*Deviation relative to weighted average across zips inside old territory:
average = sum (p x value) / sum (p), where p = number of policyholders in zip.

Pricing to Optimize an Insurer's Risk-Return Relation

by Daniel F. Gogol, FCAS

PRICING TO OPTIMIZE AN INSURER'S RISK-RETURN RELATION

Abstract

The idea of estimating loss discount rates and risk loads for categories of an insurer's premium by using the categories' contributions to surplus variation is an appealing one. However, there has been a theoretical obstacle to this approach, as will be explained in this paper.

A method which overcomes the obstacle will be presented. It produces a surprisingly simple result. The risk load (in dollars) of a category is proportional to the covariance of the category's profit with surplus.

The use of the above result to optimize an insurer's risk-return relation is analyzed in the paper. Some examples of applications of the result to compute risk loads and risk-based discount rates for losses are presented.

The relationship between the method of this paper, the Capital Asset Pricing Model, and several other models is discussed.

1. INTRODUCTION

A few years ago, a Nobel Prize was awarded to Harry Markowitz [10] for developing a method of producing a diversified portfolio of stocks with the optimal relationship between expected rate of return and expected variability. In other words, Markowitz showed how to maximize the expected rate of return for a fixed amount of expected variability and, alternatively, how to minimize the variability at a fixed rate of return. Markowitz's method has been widely used by large investors because of their desire to lower the variability of their results.

Insurance company managers are also interested in reducing variability. Taking steps to reduce risk helps a company with its Best's rating and also increases the security of its employees and its policyholders. These things help in attracting good business and retaining good employees, and produce increased profitability in the long run. Therefore, insurers require a greater profit margin on a risk with greater volatility.

Suppose that an insurer expects to write a certain volume and mix of business in the next year, and that the insurer has a certain target profit. The method of this paper produces a risk load for each risk such that the total expected profit equals the target and each risk is equally advantageous to the insurer in the following sense. If the insurer charges more than the indicated risk load for any type of risk, then by increasing the proportion of that type of risk in the total book of business the insurer can increase the expected return without increasing the surplus variability. Conversely, if the insurer charges less than the indicated price, then increasing the proportion of that type of risk will decrease the expected return if variability is left constant.

The term "risk load" is sometimes used with a different meaning than it is given above. Other meanings of the term include:

1. The risk load that a customer is willing to pay. This may be based on the market, or on the risk aversion of the customer.

2. The risk load that an underwriter desires, based on the possible effect that a contract may have on the total results of the contracts he or she has underwritten, or on the effect on a profit center within the company.

The method presented here produces an indicated price for each risk by discounting losses and loss adjustment expenses at a risk-based rate and then adding a risk load as well as other expenses. As will be explained later, the risk loads and discount rates are produced by allocating surplus to categories of underwriting and loss reserves. This allocation is based on the contribution of these categories to surplus variability. The measure of surplus variability used in this paper is defined as follows.

The "standard deviation of surplus" is the standard deviation of the probability distribution of surplus one year in the future.

A problem with allocating surplus based on each category's contribution to surplus variability is that the effect on the standard deviation of surplus of a category can not be estimated by simply estimating the standard deviation of surplus with and without the category, and then taking the difference. The explanation of this is as follows. (See Gogol [7].)

The standard deviation of surplus equals the standard deviation of the sum of the effects on surplus of all the categories of underwriting, loss reserves, other liabilities, assets, and other sources of income and expense. Suppose those categories are arranged in a list. Suppose the effect of each category on the total standard deviation is defined as the difference between the standard deviation of the sum of the categories up to and including that category on the list, and the standard deviation of the sum of the categories prior to it on the list. The sum of all these "effects" equals the total standard deviation, but the effect of a particular category depends on the order of the list. (Suppose, for example, that there is a list of two independent categories each with standard deviation σ. The standard deviation of the sum is $2^{.5}\sigma$. The effect of the first category in the list is σ, and the effect of the second is $2^{.5}\sigma-\sigma$.)

This dependence on the order in which the categories are listed has been considered a barrier to using contribution to surplus variability to estimate required risk loads. This study will propose a solution. The following quotations from Venter [12] give an interesting description of the problem.

> "In 1953, Harry Markowitz developed a way of selecting optimal holdings for each available security if you were clear about your preferred mean-variance trade-off. This has been applied to optimal line mix strategies for insurers as well."
>
> It's tempting for actuaries to invent (or re-invent) the Mean-Variance Pricing Model (MVPM).
>
> "Presumably the change in variance of your whole portfolio of risks or securities is more important than that of the new entrant by itself."
>
> "MVPM could be applied to the portfolio with and without the new entrant, whose price then becomes the difference. But then the order of entry will influence the price, which it should not. Or you could estimate in advance the make-up of the portfolio and then pro-rate to each unit a credit based on the reduction in variance achieved by the combination. The mind boggles. Besides needing a fair way to allocate credits, which this theory does not provide, any difference from the predicted result will give the wrong price overall. Because of covariance, MVPM does not seem usable for pricing individual risks in a portfolio."

2. ESTIMATING RISK-BASED PREMIUM

A. Return on Allocated Surplus

The surplus considered in this paper is a type of adjusted surplus, using the market value of assets and a risk-based discounted value for loss reserves.[1] Statutory liabilities such as equity in the unearned premium reserve are included in the surplus. The value of the assets necessary to offset the discounted loss reserve liability is considered here to be greater than the discounted value of loss reserves at the "risk-free" interest rate (see Butsic [3]). This is because it would be necessary to pay an insurer more than this amount, as a reward for risk, in order for them to be willing to assume this liability. By using a lower discount rate to determine the loss reserve liability, the following is expected to occur. In the course of a year, the value of the offsetting assets is expected to grow at a greater rate of interest than was used to discount the liability, providing a profit for the risk of having the liability.

Suppose that each category of loss reserves is considered to be offset by an amount of assets which is equal to the risk-based discounted value of the reserves. The expected effect on surplus one year in the future of a category of discounted loss reserves and offseting assets equals the accumulated value of the assets after one year of reserve payouts, minus the discounted value of the remaining reserves and the tax effects of the assets and liabilities.

The expected effect of a category of underwriting on the surplus one year in the future equals the effect of the premium minus the effect of the corresponding paid losses, discounted loss reserves, expenses and taxes.

Suppose an amount of surplus is allocated to a category of underwriting, or to a category of loss reserves and offsetting assets. Then the expected return on the allocated amount during the year is the

[1] In this paper "loss reserves" will mean loss and loss adjustment reserves, net of ceded losses. "Earned premium" will refer to premium net of cessions.

after-tax investment gain on it plus the expected effect of the category on surplus. The rate of return is the return divided by the amount of surplus.

B. Method of Allocation

Just as there is a probability distribution of what surplus may be one year in the future, there are probability distributions of the effects on surplus of each category of underwriting and each category of discounted loss reserves and offsetting assets. A basic part of the method of this paper is the idea that the appropriate amount of surplus to allocate to a category of underwriting, or of discounted loss reserves and matching assets, is equal to

$$\frac{\text{(surplus)(cov(surplus, effect of category on surplus))}}{\text{variance of surplus}}$$

It will be shown below, by Theorem 1, that in a certain sense the above covariance of a category with surplus is proportional to the category's effect on surplus variability. It is shown by Theorem 2 that if surplus is allocated to each category of underwriting according to the above formula, and the appropriate risk-based loss discounting rate is used, the following is true. Each category will improve the risk-return relation of the insurer if, and only if, its rate of return on allocated surplus is greater than the rate of return on the total amount of surplus allocated to underwriting.

It is a property of covariance that the covariance with surplus of a sum of categories equals the sum of the covariances. Therefore, the surplus allocated to a sum of categories is the same whether the surplus is allocated based on the covariance of the sum, or allocated to each individual category based on its covariance. This would not be true if surplus were allocated in proportion to the standard deviation or variance of a category's effect on surplus.

Thus, the amount of surplus allocated to a category is independent of how finely the categories are subdivided. For example, the amount of surplus allocated to private passenger auto does not depend on whether it is considered to be one category or whether it is split into private passenger auto liability and private passenger physical damage.

Surplus variability is caused not only by underwriting and by loss reserves and offsetting assets, but also by other things. For example, the value a year in the future of the surplus assets themselves is not precisely known. If surplus is allocated to all sources of surplus variability, and these sources are referred to as "categories" 1 through n, then

$$\sum_{i=1}^{n} (\text{cov}(\text{surplus, effect of category } i \text{ on surplus})) =$$

$$\text{cov}(\text{surplus, effect of } (\sum_{i=1}^{n} \text{category } i) \text{ on surplus}) =$$

$$\text{cov}(\text{surplus, surplus}) = \text{variance of surplus}$$

Therefore, the proportions of surplus allocated to the categories sum to unity.

C. Risk-Based Underwriting Margin and Discount Rate

In order to explain how to apply the method of this paper, it is helpful to consider the following questions:

1. What risk-based discount rate should be used for loss reserves?

2. How much surplus should be allocated to loss reserves, and how much to underwriting?

Suppose the insurer's loss reserves are discounted, both at the beginning and end of the year, at a discount rate d. Suppose that, with this rate d, surplus is allocated by the above covariance formula to discounted loss reserves and offsetting assets, and to underwriting.

Lastly, suppose that the rates of return on allocated surplus from discounted loss reserves and offsetting assets, and from underwriting, are equal. Call this rate R.

Call the amounts of surplus allocated to discounted loss reserves and offsetting assets, and to underwriting, S_r and S_u, respectively. It was mentioned above that the surplus allocated to a sum of categories by the covariance method is equal to the sum of the amounts allocated to the individual categories. Suppose for the moment that, for each category of loss reserves and offsetting assets, the discount rate d produces the same rate of return on allocated surplus. Since the sum of the amounts of surplus allocated to each category equals S_r, this rate of return equals R.

Suppose that, for some underwriting category C, the rate of return on the surplus allocated to the category, using the discount rate d, is R. Thus, the premium not only provides a rate of return on allocated surplus equal to the rate of return on S_r and S_u, but also provides for the offsetting assets for its loss reserves at the end of the year. Assuming that the required discount rate remains the same, these reserves and offsetting assets are expected to produce a rate of return R on allocated surplus in each following year. This is a key point, since it means that the expected effect on surplus of the loss reserve runoff from category C neither helps nor hurts the insurer's risk-return relation.

It will be shown by Theorem 2 that in a certain sense the covariance method allocates surplus in proportion to a category's effect on surplus, and it follows that the category C neither helps nor hurts the insurer's risk-return relation. This explains what conditions a category or contract must satisfy in order to help optimize that relation.

A discount rate d with the above properties may be found by iteration, as outlined below. (See Example A in section 4 for additional explanation.) Suppose the insurer expects to earn a given amount of premium in the coming year, with a given expected loss ratio and expense ratio. Certain estimates are made relating to loss payout rates, loss reserve variability, asset variability, underwriting variability, and various correlations, and an initial value of the discount rate is selected.

The value of the discount rate affects the estimated amount of surplus as well as:

1. the covariance with surplus of the total effect on surplus of discounted loss reserves and offsetting assets
2. the covariance with surplus of the total effect on surplus of all underwriting categories

3. the total amounts of surplus allocated by the above two covariances
4. the rates of return on the above two amounts of surplus

Iteration is used to find a discount rate d which makes the above two rates of return equal. Call this rate of return R.

It isn't actually necessary to assume that a single discount rate d produces the same rate of return on the amounts of surplus allocated to each category of loss reserves and offsetting assets. The indicated discount rate may vary for different categories, and thus it may be appropriate to use different discount rates in estimating the required risk-based premiums for different underwriting categories. This would require a more complicated iteration than the one described above. This may not be preferable from a practical point of view. The need for a great deal of judgment in estimating covariances with surplus will be discussed further in part E of the next section.

The theoretical significance of the allocation method is indicated by the following two theorems. The proofs[2] are in the Appendix.

Theorem 1

Using any discount rates for each category of loss reserves and for each category of underwriting, suppose a pro-rata share of 1/n of each category of one year underwriting results, loss reserves and offsetting assets, and other assets, liabilities, expenses, and sources of income affecting surplus is added to a list, and this is done n times. The limit as n approaches infinity of the total of the n effects of a category on the standard deviation of surplus[3], divided by the total standard deviation of surplus, equals

(cov(surplus, effect of category on surplus))/(variance of surplus)

[2] It will be assumed in the proofs that the covariance of a category with surplus is not zero. The case in which the covariance equals zero will be left to the reader.

[3] The effect of each category on the standard deviation was defined in the introduction as the difference between the standard deviation of the sum of the categories up to and including that category on the list, and the standard deviation of the sum of the categories prior to it.

Theorem 2

Suppose that an insurer can charge more premium for a category of underwriting than the required risk-based premium described above. Then, by increasing the proportion of that category in the total book of business, the insurer can increase the expected return without increasing surplus variability. Specifically, there is some ϵ such that the expected return on surplus will increase if the following is assumed.

a. The premium for the category is increased by less than ϵ.
b. The expected underwriting return and the standard deviation of underwriting return for the category increase by the same proportion as the premium, and the correlation of its return with surplus is unchanged.
c. The rest of the insurer's premium is reduced by an amount such that total surplus variance remains the same.
d. The expected underwriting return and standard deviation of underwriting return for the rest of the premium decrease by the same proportion as the rest of the premium, and the correlation of its return with surplus is unchanged.

Conversely, a contract written at less than the required risk-based premium will decrease the expected return.

3. DISCUSSION OF THE METHOD

A. Overall Premium Targets

The method presented above indicates what the required risk-based premium is for a contract or category, given certain overall expectations or targets of the insurer. These expected values or targets include the overall loss ratio, expense ratio, payout rate, and mix of business for the coming year. Covariances of categories with surplus are estimated based on these expected values. The method applies to individual underwriting decisions concerning contracts or categories of business, but it does not answer the question of what the overall mix or amount of premium should be. It is assumed that there are practical constraints against making drastic shifts in the current mix of business. An insurer is not free to simply choose any portfolio of business in the way that a stockholder can choose a portfolio of stocks.

If an insurer increases or decreases its premium, or changes the mix of business, this has an immediate effect, as well as an additional long term effect, on the insurer's combined ratio, total return on surplus, and variability of surplus. In the long run, increased variability can make an insurer less attractive to its employees and its clients, and can adversely affect its combined ratio and return on surplus.

If certain estimates are made, it is possible to use the Capital Asset Pricing Model (CAPM) to help in selecting the volume of premium which maximizes the market value of the insurer. This model (Lintner [9] and Sharpe [11]) will be discussed further in the last section of the paper. In actual practice, insurer managements are more likely to use informed judgment than CAPM.

B. One Year Variability

The one year time frame used for optimizing the risk-return relation is also intended to optimize this relation over the long term. Long-term variability may be thought of as a sum of one year random variables.

Sometimes it may be more natural to estimate the long term variability for a category than to estimate the one year variability. Loss reserves for environmental and mass tort (E/MT) claims is an example of such a category. The estimate of the one year variability for E/MT reserves should be selected in a way that is consistent with estimated long-term variability.

The effect of this category on surplus in the coming year i can be represented by a random variable X_i. The effect in the following year will represented by X_{i+1}. If follows from the definition of these random variables that X_{i+1} is independent of X_i. Since the probability distribution for X_{i+1} is not determined until the end of year i, the fact that X_i was greater or less than the mean of its distribution has no bearing on how X_{i+1} will differ from the mean of its distribution.

C. Loss Reserve Variability and Discounting

The estimates of loss reserves referred to in this paper are assumed to be unbiased, although annual statement estimates may be biased.

Thus, the estimates do not necessarily equal the risk-based discounted values of annual statement estimates.

The reader may have noticed that the variability of loss reserves has been addressed in the paper, and not the variability of the unearned premium reserve. This is because the variability associated with this reserve is included in the underwriting variability for the comir.g year.

The definition of surplus in this paper uses a risk-based discounted value for the loss reserves. The corresponding value of surplus is not necessarily the market value of the insurer. For one thing, it excludes franchise value. However, it appears that optimizing the risk-return relation for this surplus, as discussed in this paper, should be a good approximation to optimizing the risk-return relation for market value.

D. Asset Variability

An attempt can be made to minimize the effects of interest rate variability on surplus. A relatively simple method is to choose a mix of assets with a "duration" (see Ferguson [5]) such that interest rate changes have the same effect on the value of assets as on the value of liabilities. To apply this duration method, using the definition of surplus in this paper, it is necessary to estimate the effect of interest rate changes on the risk-based loss discounting rate. The correlation between interest rates and inflation, and the effect of inflation on estimated loss reserves, must also be estimated.

An insurer may find that duration matching of assets and liabilities requires an asset portfolio with a shorter duration than is desired. Shorter duration bonds have a lower interest rate.

Changing the mix of assets, including stocks, can be used as a tool in attempting to optimize an insurer's risk-return relation. The correlation of the insurer's return with "market return" (i.e. the average return for the market of all capital assets) should be taken into account in such an attempt. This is discussed briefly in the final section, which contains a comparison of the method of this paper with the Capital Asset Pricing Model. However, the subject of optimizing an insurer's mix of assets is beyond the scope of this paper.

E. Estimation Problems

The covariance between the effects on surplus of any two categories a and b will be denoted by cov(a,b). The covariance of category c with all other sources of surplus variability will be denoted by cov(c,s-c).

Let the variance of the effect on surplus of a category c be denoted by $(\sigma_c)^2$. Denote the correlation between the category and surplus by $\rho_{c,s}$. Note that

$$cov(c,s)=cov(c,c)+cov(c,s-c)= (\sigma_c)^2+\sigma_c\sigma_{s-c}\rho_{c,s-c}$$

Therefore, for a category c which is small, the estimate of cov(c,s) is very sensitive to the estimate of $\rho_{c,s-c}$. This is a problem, due to the low credibility of the related data. From a practical point of view, it is best to implement the method of this paper by starting with estimates relating to the largest categories.

For example, a practical first step would be to allocate surplus to the category of all loss reserves and offsetting assets and to the category of all underwriting. This determines the risk-based discount rate for the category of all loss reserves, and the risk-based profit margin on discounted underwriting results.

A reasonable second step would be to allocate surplus to the sum of all property underwriting categories and to the sum of all casualty underwriting categories. (Note that the sum of these two amounts of surplus equals the amount of surplus allocated in the first step to the category of all underwriting.) These allocations determine risk-based profit margins for property and casualty as a whole.

The problem of implementing the method is a vast one, and the examples in the next section are only intended as illustrations. In practice, it is necessary to use a considerable amount of judgemental estimation, in addition to making a study of relevant historical data.

4. EXAMPLES OF APPLICATIONS

A. Overall Underwriting Risk Load and Overall Discount Rate

Suppose that for some insurer:

1. Risk-free interest rate on assets = 6%.
2. Loss reserves at start of year discounted at 3% = \$500,000,000.
3. Discounted value of amount of loss reserves expected to be paid during year = \$100,000,000.
4. Present discounted value of loss reserves not expected to be paid during year = \$400,000,000.
5. Expected earned premium for coming year = \$150,000,000.
6. Expected underwriting expenses to be incurred during year = \$40,000,000.
7. Expected current accident year losses to be paid during year = \$45,000,000.
8. Expected value of loss reserves at end of year for current accident year discounted at 3% = \$50,000,000.
9. The pre-tax contributions to surplus of loss reserves and offsetting assets, and of underwriting, are in the same proportion as the corresponding after-tax effects.

Assume that the expected expense and loss ratios equal the targets which were discussed in section 3A. "Risk load" will be taken to mean "risk-based underwriting margin," which was discussed in section 2C. The after-tax effect on surplus of loss reserves and offsetting assets will be called the return from loss reserves. The after-tax effect on surplus of underwriting will be called underwriting return. These returns do not include investment income on allocated surplus.

Using the above 3% discount rate, the expected one year pre-tax return from loss reserves and offsetting assets, assuming loss reserves paid during the year are paid on average in the middle of the year, is (as explained below):

$$(\$500{,}000{,}000)(1.06)-(\$100{,}000{,}000)(1.03)^{.5}(1.06)^{.5}-(\$400{,}000{,}000)(1.03)=\$13{,}511{,}000$$

By the end of the year, the \$400 million in loss reserves which are not expected to be paid during the year grows to \$400 million (1.03) due to one year's unwinding of discounting. The \$400 million in offsetting assets grows, from investment income, to \$424 million, producing a pre-tax return of \$400 million (1.06 - 1.03). A loss reserve payment of \$100 million $(1.03)^{.5}$ is made in the middle of the year (on average), reducing the assets which were offsetting those reserves to \$100 million $((1.06)^{.5}-(1.03)^{.5})$. By the end of the year, these assets grow by a factor of $(1.06)^{.5}$ to \$100 million $((1.06)-(1.03)^{.5}(1.06)^{.5})$.

If it is assumed, for the sake of simplicity, that the earned premium is received in the middle of the year, and that the underwriting expenses and accident year losses are paid in the middle of the year, then the expected pre-tax return on underwriting is

$$(1.06)^{.5}(\$150,000,000-\$40,000,000-\$45,000,000)-\$50,000,000=\$16,922,000$$

Approaches to estimating the covariances of loss reserve return with surplus, and of underwriting return with surplus, will be discussed after the following brief description of the iterative process.

Suppose that, using the above 3% discount rate, the above two covariances, respectively, are in the proportion A:1. The corresponding rates of return on allocated surplus are then in the proportion 13,511/A: 16,922. Call this proportion B:1. Suppose that using a 4% discount rate changes the proportion of rates of return from B:1 to C:1. Since the goal is to make the rates of return equal, a reasonable next step in the iteration would be

$$4\% + (3\%-4\%)((1-C)/(B-C))$$

Suppose for the sake of illustration that the above 3% rate is the solution to the iteration. It then follows from the above formula for pre-tax return on underwriting that

$$\$150,000,000=\$40,000,000+\$45,000,000+(1.06)^{-.5}(\$50,000,000)+(1.06)^{-.5}(\$16,922,000)$$

In other words, the premium equals expected expenses (i.e. \$40,000,000) + expected discounted losses (i.e. \$45,000,000 + $(1.06)^{-.5}$(\$50,000,000))+ risk load (i.e.$(1.06)^{-.5}$(\$16,922,000)).

The covariance of the loss reserve return, and of the underwriting return, with surplus can be estimated based on the insurer's historical data. The insurer's loss reserve runoff variability, its loss ratio and expense ratio variability, the duration of its loss reserves, the duration of its assets, and the historical variability of interest rates are all relevant.

Variability in the loss reserve return is caused by differences between the estimated loss reserve and the one-year runoff, changes in market values of offsetting assets, changes in estimated risk-based discount rates, and changes in estimated payout rates for loss reserves. To some extent, changes in asset values caused by interest rate changes are offset by corresponding changes in discount rates. Variability in the underwriting return results from variability in asset values, loss ratios, expense ratios, payout rates and discount rates.

One way of estimating the covariances is as follows. For some period of years, estimates are made of what the expected increases in surplus, and the expected returns from loss reserves and underwriting, would have been at the beginning of each year. (Note that surplus is increased by the return on other assets as well as those offsetting reserves.) These estimates are then compared with what would have been estimated for each of those returns at the end of the same year.

For each year, all the above estimates can be brought to the level of the current year. The estimated loss reserves return for each year can be multiplied by a factor equal to the reserves at the beginning of the current year divided by the beginning reserves for the year. A similar on-level adjustment can be made for estimated underwriting return, based on the premium for the years. For the on-level factor for return on assets other than those offsetting reserves, the amount of those assets can be used. As mentioned above, the estimated increase in surplus is the sum of the above three estimated returns, so the on-level estimate is the sum of the three on-level estimates.

The covariances of the loss reserves and underwriting returns with surplus can then be estimated as shown in the example below. The example is intended to illustrate a method of computation, but in actual practice many more years of data would be used.

Table 1

	Estimated Loss Reserve Return (000's)		**Estimated Underwriting Return (000's)**		**Estimated Increase in Surplus (000's)**	
Year	1/1	12/31	1/1	12/31	1/1	12/31
1990	$13,600	$12,800	$33,000	$28,600	$81,600	$75,600
1991	$13,200	$14,200	$31,400	$25,600	$80,800	$86,000
1992	$19,400	$18,600	$28,400	$39,600	$77,400	$81,900
1993	$17,000	$15,000	$21,400	$18,200	$62,200	$57,200
1994	$18,900	$14,400	$22,700	$24,200	$63,100	$59,500

The estimated covariances with surplus are as follows (000,000's):

Loss Reserve Return: (1/5)((12,800-13,600)(75,600-81,600) + (14,200-13,200)(86,000-80,800) + (18,600-19,400)(81,900-77,400) + (15,000-17,000)(57,200-62,200) + (14,400-18,900)(59,500-63,100)) = 3,240.000

Underwriting Return: (1/5)((28,600-33,000)(75,600-81,600) + (25,600-31,400)(86,000-80,800) + (39,600-28,400)(81,900-77,400) + (18,200-21,400)(57,200-62,200) + (24,200-22,700)(59,500-63,100)) = 11,448,000

Another method of estimating the covariances of loss reserve return and underwriting return with surplus is to analyze the covariance structure and estimate the component parts.

Let σ_r, σ_u and σ_a denote the standard deviations of the following random variables:

R: return from loss reserves
U: return from underwriting
A: return on assets other than those offsetting loss reserves

Let the correlations between the above returns be denoted by $\rho_{r,u}$, $\rho_{r,a}$, and $\rho_{u,a}$. Let cov(R,S) and cov(U,S) denote the covariances of the indicated returns with surplus. Then,

$$
\begin{aligned}
\text{cov}(R,S) &= \text{cov}(R,R+U+A) \\
&= \text{cov}(R,R)+\text{cov}(R,U)+\text{cov}(R,A) \\
&= (\sigma_r)^2 + \sigma_r \sigma_u \rho_{r,u} + \sigma_r \sigma_a \rho_{r,a} \\
\text{cov}(U,S) &= \text{cov}(U,R+U+A) \\
&= \text{cov}(U,R)+\text{cov}(U,U)+\text{cov}(U,A) \\
&= \sigma_u \sigma_r \rho_{r,u} + (\sigma_u)^2 + \sigma_u \sigma_a \rho_{u,a}
\end{aligned}
$$

B. Risk Loads for Property and Casualty

Since 1980, the variation in industry casualty loss ratios has been much greater than the variation in property loss ratios. Also, casualty loss ratio variation has been significantly correlated with variation in loss reserve estimates. Both loss ratios and reserve estimates were affected by trends in loss severity.

Suppose that, for some insurer:

1. All premiums are either casualty or property.
2. The overall underwriting risk load (discussed in the previous example) is 8% of premium.
3. The covariances with casualty return and with property return of the return on assets other than those offsetting loss reserves are zero.
4. Expected property and casualty earned premiums are $100,000,000 and $150,000,000, respectively, and total risk-based discounted loss reserves are $400,000,000.
5. The expected pre-tax returns from property and casualty premiums are in the same proportion as the corresponding after-tax returns.
6. The estimated covariances of property return, casualty return, and loss reserves return with each other are based on Table 2 below.

Table 2

Year	Change from 1/1/ to 12/31/ in Estimated Property Return (000's)	Change from 1/1 to 12/31 in Estimated Casualty Return (000's)	Change from 1/1 to 12/31 in Estimated Loss Reserves Return (000's)
1983	-$2,500	-$20,800	$14,600
1984	-$6,100	-$29,700	-$16,400
1985	-$400	$6,100	$1,300
1986	$8,700	$16,500	$4.600
1987	$4,100	$28,800	$8,900
1988	-$600	$6,200	$1,400
1989	-$500	$1,500	$4,800
1990	-$6,000	-$1,700	$2,100
1991	-$3,600	-$1,400	$5,700
1992	$2,100	-$2,500	$5,900
1993	$4,800	-$3,800	$1,200
1994	-$1,500	$900	-$1,100

The covariance between any two of the returns in Table 2 is estimated by taking the average of the products of the numbers in each row of the two columns of returns. Let P, C, R and A denote random variables which equal the returns from property, casualty, reserves, and other assets, and let S denote a random variable which equals the change in surplus. Then,

$$\begin{aligned} cov(P,S) &= cov(P,P)+cov(P,C)+cov(P,R)+cov(P,A) \\ &= var(P)+cov(P,C)+cov(P,R)=74.14 \text{ million} \end{aligned}$$

$$cov(C,S)= cov(C,P)+cov(C,C)+cov(C,R)+cov(C,A)$$

$= cov(C,P)+var(C)+cov(C,R)=342.83$ million

The ratio of the risk load, in dollars, for property to that of casualty is 74.14:342.83,i.e. .216:1. It was assumed above that overall underwriting risk load is 8% of premium, so if x represents the casualty risk load in dollars,

$x+.216\,x = .08(\$250 \text{ million})$
$x = \$16.447$ million

Therefore, the risk loads for casualty and property, as percentages of premium, are 16.447/150, i.e. 11.0%, and (.216(16.447))/100, i.e. 3.6%.

Suppose that expenses are 30% of premium for both casualty and property, and that the respective risk-based present value factors for the losses are .800 and .970. It then follows that the target combined ratio for casualty is given by

$$30+(100-30-11)/.800=103.8$$

and the target for property is given by

$$30+(100-30-3.6)/.970=98.5$$

C. Catastrophe Cover Risk Load

In this example, in order to estimate the value of a catastrophe cover to a ceding company, we will suppose that the ceding company re-assumes the cover, and we will estimate the required risk load.

Assume that:

1. The probability of zero losses to the catastrophe cover is .96, and the probability that the losses will be \$25 million is .04. Therefore, the variance $(\sigma_C)^2$ of the losses is 24 trillion, and the expected losses are \$1 million.
2. Property premium earned for the year is \$100 million, and there is no casualty premium.
3. The standard deviation of pre-tax underwriting return is 15 million.
4. The expected pre-tax return from underwriting is \$8 million.

5. Taxes have the same proportional effect on the expected pre-tax returns on total premium and on the catastrophe cover, and on the standard deviations of the returns.
6. The covariance between the catastrophe cover's losses and losses net of the cover is equal to .50 times the variance of the cover's losses.
7. The discount rate for losses is zero.
8. Total underwriting return, and the return on the catastrophe cover, are statistically independent of non-underwriting sources of surplus variability.

It follows from 1 and 5 above that the covariance with surplus of the pre-tax return on the catastrophe cover is 24 trillion + .50 (24 trillion), i.e. 36 trillion. It follows from 3 that the corresponding covariance for total underwriting is (15 million)^2, i.e. .225 trillion. Therefore, it follow from 4 that the risk load for the catastrophe cover should be such that the pre-tax return from the catastrophe cover is given by (36/225)(S8 million)= $1.28 million. This is greater than the cover's expected losses.

The insurer may be able to cede the catastrophe cover for a price which is mutually beneficial to it and a reinsurer. For example, if a reinsurer is much larger and more diversified than the ceding company, and it pools its assumed catastrophe covers with other reinsurers, it may not require as great a risk load for the cover as the ceding company.

D. Risk Load by Layer

Suppose that for some insurer:
1. All premium is property premium.
2. The accident year expected property losses for the $500,000 excess of $500,000 layer, and the 0-$500,000 layer, respectively, are $10 million and $90 million. Expected losses excess of $1 million are zero.
3. The accident year property losses for each of the above layers are independent of all non-underwriting sources of surplus variation.
4. The discount rate is zero.
5. The coefficients of variation (ratios of standard deviations to means) of the higher and lower layers are .30 and .15, respectively.
6. The correlation between the two layers is .5.
7. Taxes have the same proportional effect on the returns of both layers.

Let σ_1 and σ_2 denote the standard deviations of the losses to the higher and lower layers, respectively. Let ρ denote the correlation. With the above assumptions, the pre-tax covariances with surplus for the higher and lower layers, respectively, are given by:

$\sigma_1^2 + \rho\,\sigma_1\sigma_2 = ((10 \text{ million})(.30))^2 +$
$(.5)(10 \text{ million})(.30)(90 \text{ million})(.15) = 29.25$ trillion, and
$\sigma_2^2 + \rho\sigma_1\sigma_2 = ((90 \text{ million})(.15))^2 +$
$(.5)(10 \text{ million})(.30)(90 \text{ million})(.15) = 202.5$ trillion

The allocated surplus for the 0-$500,000 layer is (202.5/29.25) times as great as the allocated surplus for the $500,000 excess of $500,000 layer. The expected losses are nine times as great for the lower layer. Therefore, the required risk load, as a percentage of expected losses, is 1.3 (i.e. ((9)(29.25))/202.5) times as great for the higher layer as it is for the lower layer. This is expected due to the higher layer's larger coefficient of variation.

Note the contrast of the use of covariances to the use of variances or standard deviations. The covariances for the lower and higher layers are 202.5 trillion and 29.25 trillion, respectively. The corresponding variances are 182.25 trillion and 9 trillion, and the corresponding standard deviations are 13.5 million and 3 million. Thus the ratio of total risk loads, in dollars, for the lower and higher layers is about 7 for the covariance method, about 20 for the variance method, and exactly 4.5 for the standard deviation method.

5. SOME RELATED METHODS

It will be shown that the Capital Asset Pricing Model (CAPM) can be useful in selecting the overall premium and combined ratio targets which are used in this paper to set targets for individual categories. Also, the significance of the method of this paper from a CAPM perspective will be discussed.

According to CAPM, the price of a capital asset depends on its expected rate of return and the covariance of this rate with the overall rate of return on the market of all capital assets. (See Brealey and Myers [1], Lintner [9], and Sharpe [11]). There is some similarity between CAPM and the method presented here since CAPM estimates prices based on the

covariance of an asset with the market, and the method presented here estimates prices based on the covariance of a contract with surplus.
The similarity is limited, however. The derivation of the CAPM formula for a capital asset uses the fact that holders of capital assets are able to use Markowitz diversification. The method presented here requires that the mix of business of an insurer is approximated in advance. CAPM applies to the problem of optimizing the relation between variability and return in diversifying a portfolio of assets. The method presented here applies to a risk-return optimization problem, but for an insurer with a stable, or almost stable, book of business.

According to CAPM, each asset j in the market of all capital assets will have a market price such that

$$E_j=R_f+(E_m-R_f)((cov(R_j,R_m))/\sigma^2 m)$$

where

E_j = the expected rate of return on asset j.
E_m = the expected rate of return on the market portfolio.
σ_m = the standard deviation of the rate of return on the market portfolio.
R_r = the risk-free rate of return.
R_m = the market rate of return.
R_j = the rate of return on asset j.

The market value of an insurer's assets, not including franchise value, minus its liabilities will be called the market value of its surplus. Suppose for the sake of illustration that for some insurer, called asset j, the market value of surplus equals the market value of the insurer. In other words, the franchise value is zero. Suppose also that the expected market value of surplus one year in the future equals the expected market value of the insurer one year in the future. It then follows that the expected change in this value of surplus in the coming year, divided by the present surplus, is equal to E_j in the above formula if R_j represents the rate of return on the market value of the insurer.

This expected rate of return, which makes the market value of the insurer equal the runoff value (market value) of the assets and liabilities, could be considered to be the minimum acceptable expected return on surplus for the insurer.

Suppose that due to a change in management, the expected change in surplus in the coming year increases, and there is no change in the expression R_f or

$$(E_m-R_f)((cov(R_j,R_m))/\sigma^2_m)$$

Since E_j does not change, the market value of the insurer (asset j) theoretically increases and becomes greater than the market value of surplus. This creates what is known as franchise value.

The amount of premium which is required for a category in order to neither improve nor worsen the insurer's risk-return relation is not necessarily the same as the amount which neither increases nor decreases the market value of the insurer according to CAPM.

Suppose that surplus is allocated according to the method of this paper, and the estimated rate of return on the surplus allocated to a category A is less than the rate of return of the insurer. Suppose also that, according to the application of CAPM to category A and its allocated surplus, this rate of return is above the acceptable minimum for the insurer discussed above. Also, suppose that according to CAPM the rate of return of the insurer is only equal to the acceptable minimum.

In the above example, category A would be estimated by CAPM to increase the market value of the insurer if certain intangible effects of worsening the risk-return relation are ignored.

Advantages that the insurer gains by improving the risk-return relation were described in the second paragraph of the introduction to this paper. (The risk-return relation has an influence on policyholders, employees, and rating organizations.) In the long run, these advantages can translate into lower expected combined ratios. In the case of the above example, the long-term effects of worsening the risk-return relation should be weighed against a CAPM estimate which ignores them.

An insurer can also use CAPM to evaluate the effects on its market value of changes in its amount of written premium or the composition of its asset portfolio. Here again, the effects on the risk-return relation are important, as well as the effects on the CAPM estimate of market value. Also, the intangible effects of variability on rating organizations, customers and employees should be considered.

Kreps [8] presented a method of determining risk load by marginal surplus requirements. A problem with Kreps' method was discussed in the introduction. The sum of the effects of all categories on the standard deviation of surplus, as measured by Kreps, does not equal the total standard deviation. Kreps does not address the variability of loss reserves or the discounting of losses.

Feldblum [4] suggested a modified version of CAPM for determining risk loads for insurers:

> The market return R_m in the CAPM model should be replaced by the return on a fully diversified insurance portfolio.

Feldblum's method could be used to estimate required return on allocated surplus for an insurance contract. The subscript m for market is replaced in three places in the CAPM formula by i for insurance industry. Feldblum's method does not address the problem of discounting, but it could be expanded to do so.

Feldblum's method is somewhat similar to the method in this paper in that it addresses the problem, for an insurer, of optimizing the risk-return relation. The key difference between Feldblum's method and the method in this paper is the following. Feldblum's method evaluates insurance contracts for an insurer which is free to use an insurance analogue of Markowitz diversification to produce a portfolio of insurance contracts. (In actual practice, there are constraints on an insurer.) The method in this paper estimates the effect of a contract on surplus variance given an approximated mix of earned premium for the coming year.

Brubaker [2] and Ferrari [6] discuss methods of maximizing an insurer's profit, given a constraint on variance, by selecting an insurance portfolio. They don't address the problems of variability of loss reserves or discounting of losses. Underwriting profit margins by category are estimated prior to selecting the portfolio.

6. CONCLUSION

The method in this paper is an attempt to address the problem of risk-based pricing for an insurer in a way which is useful and also meaningful in the context of financial theory. Although there is considerable judgment and effort involved in applying the method, it provides a new theoretical framework for dealing with the challenge of improving an insurer's risk-return relation.

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APPENDIX

Proof of Theorem 1

Let the random variable X equal the effect of a category x on surplus in a one year period. Let the random variabie Y equal the combined effect of all other sources of surplus variation in the one year period.

Suppose a 1/n pro-rata share of each category, including x, which contributes to surplus variation is added in any order. Suppose the process is repeated until category x is about to be added for the $(k+1)^{th}$ time, where $k+1 \leq n$. Let V_1 denote the variance of the effect on surplus of the set of pro-rata shares before x is added, and let V_2 denote the variance afterwards.

In the following argument, the expression $\approx$ will be used to indicate that the ratio of the expression on the left to the one on the right approaches 1 as k and n approach infinity. It can be seen that

$$V_1 \approx 2\,(k/n)^2\,\rho_{x,y}\sigma_x\sigma_y + (k/n)^2\,\sigma_x^2 + (k/n)^2\sigma_y^2$$

$$V_2 \approx 2((k+1)/n)(k/n)\,\rho_{x,y}\sigma_x\sigma_y + ((k+1)/n)^2\sigma_x^2 + (k/n)^2\sigma_y^2$$

The change in standard deviation, Δ std. dev., is $(V_2)^{.5} - V_1^{.5}$.

It can be seen by algebra that

$$\Delta \text{ std dev.} \approx .5((V_2 - V_1)/V_1^{.5})$$

$$\approx .5((2k/n^2)\rho_{x,y}\sigma_x\sigma_y+((2k + 1)/n^2)\sigma_x^2)/(2(k/n)^2\rho_{x,y}\sigma_x\sigma_y+(k/n)^2\sigma_x^2+(k/n)^2\sigma_y^2)^{.5}$$

$$\approx ((1/n)(\rho_{x,y}\sigma_x\sigma_y + \sigma_x^2))/2\,\rho_{x,y}\,\sigma_x\sigma_y + \sigma_x^2 + \sigma_y^2)^{.5}$$

Therefore, it can be seen that

$$\lim_{n\to\infty} \sum_{1}^{n} \Delta \text{ std dev.} = (\rho_{x,y}\sigma_x\sigma_y + \sigma_x^2)/(2\,\rho_{x,y}\,\sigma_x\sigma_y + \sigma_x^2 + \sigma_y^2)^{.5}$$

$$= \text{cov}(X + Y, X)/\text{std.dev. } (X + Y)$$

$$= \text{cov (surplus, X)/std. dev. (surplus)}$$

$$(\lim_{n\to\infty} \sum_{1}^{n} \Delta \text{ std. dev.})/\text{std. dev. (surplus)} = \text{cov(surplus, X)/var(surplus)}$$

Proof of Theorem 2

Let the random variable X equal the effect of the category x on surplus in a one year period. Let the random variable S equal the change in surplus in the one year period.

It was assumed that the insurer gets more than the required risk-based premium for category x. Therefore,

$$E(X)>E(S)(cov(X,S)/var(S)) \tag{1}$$

It follows that,

$$E(S-X)=(E(S)-E(X))<E(S)(1-(cov(X,S)/var(S)))$$

$$=E(S)(cov(S-X,S)/var(S))$$

Therefore, $E(S-X)<E(S)(cov(S-X,S)/var(S))$ (2)

Suppose the premium for category x is multiplied by some number 1+a, where a>o, and that the total premium for the rest of the book is multiplied by some number 1-b, where b>o. Suppose also that the insurer's total surplus variance is unchanged. Therefore,

$$var(S) =\sigma_x^2(1+a)^2 + (\sigma_{s-x})^2\text{-y}(1-b)^2 + 2(1+a)(1-b)\rho_{x,s-x}\sigma_x\sigma_{s-x}$$
$$=\sigma_x^2 + (\sigma_{s-x})^2 + 2\rho_{x,s-x}\sigma_x\sigma_{s-x}$$

Let Δ var(S) represent the first of the above two expressions minus the second. There is an expression f(a,b) such that

$$= \Delta\, var(S)$$

$$= \sigma_x^2(2a) + (\sigma_{s-x})^2(-2b) + (2a-2b)\rho_{x,s-x}\, \sigma_x\sigma_{s-x} + f(a,b)$$

$$= 2a\, \sigma_x(\sigma_x + \rho_{x,s-x}\sigma_{s-x})-2b\sigma_{s-x}(\sigma_{s-x} +\rho_{x,s-x}\, \sigma_x)+ f(a,b)$$

$$= 2a(cov(X,S) + 2b(cov(S-X,S) + f(a,b)$$

and the limit as a and b approach zero of f(a,b)/a, and of f(a,b)/b, is zero.

It follows from the above that

$$a\, E(S)(cov(X,S)/var(S))= bE(S)(cov(S-X,S)/var(S))+g(a,b) \tag{3}$$

where $g(a,b)/a$ and $g(a,b)/b$ approach zero as a and b approach zero.

Now,

$$E((1+a)X+(1-b)(S-X))=E(X+(S-X)+aX-b(S-X))=E(S)+aE(X)-bE(S-X) \qquad (4)$$

It follows from equations 1 and 2 that the formula above equals

$$E(S)+a(E(S)(cov(X,S)/var(S))-b(E(S)(cov(S-X,S)/var(S))+ad+be \qquad (5)$$
where $d>o$ and $e>o$.

It was mentioned above that $a>o$ and $b>o$. As a and b approach zero, d and e above remain constant and, by equations 3, 4 and 5,

$$E((1+a)X+(1-b)(S-X))=E(S)+g(a,b)+ad+be>E(S)$$

This completes the proof of Theorem 2 for the case in which category x is written at more than the required risk-based premium. The proof of the converse is similar.

A Survey of Methods Used to Reflect Development in Excess Ratemaking

by Stephen W. Philbrick, FCAS, and
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Abstract

This paper discusses the strengths, weaknesses, and application of several methods used to obtain an estimated ultimate loss distribution from data whose valuation is less than final. The central issues are introduced by examining several basic methods via a simple example. This foundation is followed by a description of three additional methods which rely on industry loss distributions as a basis for obtaining the ultimate loss distribution using limited data. Finally, a more robust method is introduced which accommodates slightly more refined, but not atypical, data.

Introduction

There is a substantial amount of published material on fitting statistical distributions to sample data[1]. In actual practice the sample data usually consist of individual claims and the distributions fit to this claim data are referred to as loss distributions. Several authors[2] have illustrated the use of loss distributions in estimating various insurance pricing factors, such as deductible credits, increased limits factors, and excess loss factors. However, most of these materials tiptoe around the issue of loss development. For instance, the current staple reference of the profession regarding loss distributions, "Loss Distributions" by Messrs. Hogg and Klugman[3], directly fits loss distributions to property and liability claims and then immediately uses these curves in further computations. This process essentially makes the development assumption that the individual case reserves are correct and that unreported claims will basically be no different in nature than the claims which have been reported. While this assumption may be appropriate for direct use in estimating deductible factors, increased limits factors, etc., for property claims, the errors arising are often too large to be ignored without adjusting the distribution for the effects of loss development for non-property claims. Appendix A discusses some of the literature references to this problem.

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This paper discusses some of the issues regarding the recognition of loss development when estimating liability loss distributions. It is separated into three sections. These sections are organized as follows:

Section I This section will illustrate the nature and potential magnitude of the problem of using an artificially simplified data set containing a handful of claims. This section also demonstrates that several standard adjustments for development do not sufficiently address the problem.

Section II This section illustrates three intermediate techniques which provide a more complete solution to the problem:

- Use shape of industry curves without adjustment
- Use shape of industry curves with adjustment for mean values
- Use industry curves incorporating the latest evaluation date of individual claim data.

Section III This section will discuss a more rigorous approach to account for loss development in the case that more complete historical data on individual claims is available.

Section I - Examining The Basic Problem

In order to examine the basic dynamics surrounding the development issue, this section will discuss a simple empirical sample which at the time of evaluation, consists of 10 claims. Continuous loss distributions, other than the empirical distribution, will not be discussed in this section as the basic concepts are unaffected by the transition from the actual data to a continuous loss distribution that seeks to model the process underlying the actual data. While we realize that this simple example may not totally reflect reality, it is provided to familiarize the reader with some of the fundamental issues surrounding the problem and thereby more fully prepare the reader for the latter sections of the paper.

Given the following ten claims from a specific accident period, the goal will be to estimate a $1,250 deductible credit, a $12,500 increased limits factor (ILF), and a $7,500 excess loss factor (ELF). Assume that the base limit is $5,000.

Claim Number	Status	Paid Value	Incurred Value
1	Closed	$600	$600
2	Closed	800	800
3	Closed	1,100	1,100
4	Closed	1,300	1,300
5	Closed	1,600	1,600
6	Closed	1,800	1,800
7	Closed	2,500	2,500
8	Open	0	3,000
9	Closed	11,000	11,000
10	Open	0	12,000
Total	10	$20,700	$35,700

The goal is to estimate the following quantities for all claims occurring during the specific accident period after they have each been reported, settled, and closed.

Deductible Credit[4] = $\frac{\text{Sum losses limited to the deductible}}{\text{Sum of unlimited losses}}$

Increased Limits Factor[5] = $\frac{\text{Sum of losses limited to \$12,500}}{\text{Sum of losses limited to the basic limit (\$5,000)}}$

Excess Loss Factor = $\frac{\text{(Sum of unlimited losses - Sum of losses limited to \$7,500)}}{\text{Sum of unlimited losses}}$

The process of loss development consists of the reporting of claims to the insurer and the adjustment of those claims until each claim is closed. Typically, as a body of claims develops, the total value and the average value of the claims increase on both a paid and an incurred basis[6]. Assume, for the present, that no IBNR claims are reported. We will revisit this assumption later. Assume that the final settlement values for each claim are as follows:

Claim Number	Status	Final Value
1	Closed	$600
2	Closed	800
3	Closed	1,100
4	Closed	1,300
5	Closed	1,600
6	Closed	1,800
7	Closed	2,500
8	Closed	15,000
9	Closed	11,000
10	Closed	20,000
Total	10	$55,700

Based on the ultimate distribution of the 10 claims from this accident period, the actual factors should be:

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[4] Calculation of deductible credits in the context of workers compensation coverage normally uses the sum of unlimited losses as the denominator. In liability coverages, it is more usual to use losses at some limit as a denominator. The reader is invited to restate the data if a different convention is preferred.
[5] The concept of increased limits factors rarely occurs directly in workers compensation. However, the pricing of excess layers often uses techniques that are mathematically equivalent to an ILF approach, so we believe the issue associated with appropriate adjustments to ILF's to account for development also apply to workers compensation.
[6] This is apparent if one divides the paid or incurred losses by the reported counts for the industry in total using data from Best's Aggregates and Averages. It is also apparent upon examining the average loss by settlement lag implied by ISO's selected loss distributions for general liability.

Item	Calculation	Value
Deductible Credit (DED)	11,250/55,700	0.202
Increased Limits Factor (ILF)	45,700/24,700	1.850
Excess Loss Factor (ELF)	(55,700 - 32,200)/55,700	0.422

These are the actual factors for this accident period that we are trying to estimate. The key is that these factors are usually estimated using a body of claims that are not fully developed. In this example, we need to estimate the actual factors using the original ten claims.

The following are several basic approaches that might be used:

- Estimation using the eight closed claims;
- Estimation using the incurred value of the ten claims;
- Estimation using the incurred values after adjusting by a single loss development factor; and
- Estimation using the incurred values of the ten claims after adjusting the open claims by a single total case reserve development factor, which includes a provision for the unreported claims.

Each of these approaches will mis-estimate the actual factors.

The purported justification for using closed claims is that they are settled and their values will not be subject to change. The problem with using closed claims is that the eight closed claims do not represent the same loss distribution as the ultimate body of ten claims. The estimated factors using the closed claims only are:

Item	Calculation	Value
DED	8,750/20,700	0.423
ILF	20,700/14,700	1.408
ELF	(20,700-17,200)/20,700	0.169

The deductible credit is overstated because the larger claims, which develop and are closed later, add more to the denominator of the calculation of the deductible credit than they do to the numerator. The ILF is understated because the future development of the larger claims tends to push a greater percentage of the losses into the higher layers. The excess loss factor is understated for the same reasons.

One could use the incurred values of the ten claims. The use of incurred values attempts to include more information about the claims than is contained in the actual paid values alone. In essence, the incurred values recognize more of the development in the claims than the paid values. The use of the incurred claim values results in the following estimates:

Item	Calculation	Value
DED	11,250/35,700	0.315
ILF	35,700/22,700	1.573
ELF	(35,700-27,700)/35,700	0.224

The use of incurred claim values produces errors in the same direction as the use of closed claims. However, the magnitude is smaller.

Once again, this is due to the fact that the average claim value and the spread of the claims tend to increase with time. The increase in the average claim value over time is supported by statistics from Best's aggregates and averages[7], while the increase in the spread of claims is supported by actual data and common sense. There are two common sense arguments supporting the latter phenomenon. The first is that if the average value increases, then if all claims have been reported, the total increase must originate from the open claims. If one

[7] Average incurred claim size can be calculated from incurred dollars and reported counts shown in the industry aggregate Schedule P exhibits. A comparison of subsequent years' values will demonstrate the increase over time.

assumes that the average open claim is larger than the average closed claim, then increasing the values of open claims should increase the variance of the total body of claims. The second common sense argument is the "big bang" theory. The big bang theory suggests that adjusters do a pretty good job on most of the claims, but usually get surprised by one or two claims. Thus, given a body of claims with ten open claims, eight might settle within a relatively small percentage of the case reserve, but the remaining two claims "explode" and settle at much more than the case reserve. The net effect is that the two problem claims spread the distribution of all the claims and account for a substantial portion of the total dollar development.

In order to more properly recognize claim development, one might suggest that we simply multiply each of the incurred claims by a development factor and then compute the statistics. This approach has the continuous loss distribution analog of multiplying the individual losses by a development factor before fitting the loss distribution. In the example, assume that the incurred development factor is known with certainty to be 1.560 (55,700/35,700). The resulting factors are:

Item	Calculation	Value
DED	12,184/55,700	0.219
ILF	44,815/29,815	1.503
ELF	(55,700-34,815)/55,700	0.375

If all the claims are reported, the deductible factor is still overstated because the use of a uniform development factor increases the value of the small closed claims too much. The ILF and the ELF for the larger limits are understated because the majority of the true upper layer development is distributed by the use of uniform factor to the more frequent smaller and moderate sized claims. Once again, the actual change in the spread of claims is not completely captured. For more moderate limits, the ILF and the ELF would be overstated because the values of many moderate-valued claims are increased beyond what they will actually settle for. This is due to the dollars that must be accounted for in the loss development factors arising from increases in large value claims.

Finally, one might decide to apply a development factor to the open reserves[8]. This factor can be calculated directly from historical data, or solved for using paid and incurred development factors. In this case, assume that the development factor applicable to open reserves is 2.333 ((20,000 + 15,000)/(12,000 + 3,000)). Applying this factor to the open claim reserves, and calculating the resulting factors yields:

Item	Calculation	Value
DED	11,250/55,700	0.202
ILF	40,200/24,700	1.628
ELF	(55,700-31,700)/55,700	0.431

The deductible factor is right on (although if we construct an example with open reserves well below the deductible amount, we would still get an error). The ELF is reasonably close while the ILF is still not close.

The calculations to this point have been based upon the assumption that no new claims are reported and the only development arises from known, open claims. In the usual situation in which IBNR claims do emerge, the picture becomes more complicated.

[8] The technique of using a factor applicable to open reserves only is not as widespread as other traditional methods. Part of the reason is the correct perception that the factor can be leveraged - at close to maturity, only a small portion of total incurred is still outstanding and the factors may swing widely based upon the actual prior settlements of just a few claims. However, at less mature ages, the perception of instability may be false. While the factors appear larger and more volatile, it should be noted that an incurred development factor can be derived from an outstanding development factor by adding a constant (paid dollars) to both numerator and denominator. While it should be clear that adding such a constant does force the resulting value closer to one, it can be argued that it is an artificial dampening of results. In any event, we think that this method should not be rejected simply because the typical factors are larger and appear to be more volatile.

Let us assume that, at ultimate, our total incurred is supplemented by two IBNR claims, one at $5,000 and one at $25,000. Then our ultimate distribution is as follows:

Claim Number	Status	Paid Value	Incurred Value
1	Closed	$ 600	$ 600
2	Closed	800	800
3	Closed	1,100	1,100
4	Closed	1,300	1,300
5	Closed	1,600	1,600
6	Closed	1,800	1,800
7	Closed	2,500	2,500
8	Closed	15,000	15,000
9	Closed	11,000	11,000
10	Closed	20,000	20,000
11	Closed	5,000	5,000
12	Closed	25,500	25,000
Total		$85,700	$85,700

With these ultimate claims, the correct factors are as follows:

Item	Calculation	Value
DED	13,750/85,700	0.160
ILF	63,200/34,700	1.821
ELF	(85,700-44,700)/85,700	0.478

The calculation of estimated factors using closed only claims does not change. For convenience, the results are repeated here:

Item	Calculation	Value
DED	8,750/20,700	0.423
ILF	20,700/14,700	1.408
ELF	(20,700-17,200)/20,700	0.169

Note that the deductible and ELF factors are further away from the correct values. The ILF is only marginally closer.

Similarly, the calculation using incurred amounts is unchanged and reproduced here:

Item	Calculation	Value
DED	11,250/35,700	0.315
ILF	35,700/22,700	1.573
ELF	(35,700-27,700)/35,700	0.224

Assuming that we have accurate incurred loss development factors reflecting IBNR emergence, we can update the incurred development method with the revised factor of 2.401 as follows:

Item	Calculation	Value
DED	12,500/85,700	0.146
ILF	55,487/37,284	1.488
ELF	(85,700-45,487)/85,700	0.469

Note that the derived deductible factor is now too low (as contrasted to the situation where we assumed no IBNR). This understatement results because the IBNR claims would include a full deductible, but the development factor applied to known claims with incurred values above the deductible produces no new deductible losses. The ELF factor is reasonably close, but the ILF factor is still substantially off.

When we update our open reserve development method, we can consider the possibility that the entire development factor should be applied to the open reserves, but a few moments reflection should indicate that this does not make much sense. To increase open reserves for anticipated development of open claims is plausible, but to increase individual claim amounts to account for newly reported counts seems unreasonable.

Given that our goal is to estimate the ultimate distribution as opposed to the total incurred, it would be entirely appropriate to ignore the IBNR claims if these claims had the same expected size distribution as known claims. However, experience tells us that IBNR claims tend to have higher loss amounts than previously reported claims. Loading pure IBNR dollars into known claims is clearly wrong, but ignoring these counts also introduces errors.

We have reproduced below the open case development approach using only expected development on known claims:

Item	Calculation	Value
DED	11,250/55,700	0.202
ILF	40,200/24,700	1.628
ELF	(55,700-31,700)/55,700	0.431

While this method produces better results than incorporating the entire development into the open reserves, our correct factors have now changed and we see that the resulting factors no longer match the correct factors.

The relative error of these procedures depends upon the context in which the results will be used. In the rating factors being estimated in the example, one must keep in mind how the factors will be applied and the nature of the overall objective. For example, the ILF will be applied to an adequate base rate to estimate losses/premiums above the base limit but below the increased limit. Similarly, the deductible factor and ELF might be used to layer the unlimited losses for workers compensation, which are assumed to be reflected in the rate. We will use the following error functions for each:

Deductible Credit Error:

(Estimated Dollar Deductible Credit -
Actual Dollar Deductible Credit)/
Actual Dollar Deductible Credit

This Equals:

$$\frac{\text{Estimated Deductible Factor}}{\text{Actual Deductible Factor}} - 1$$

Increased Limits Factor Error:

(Estimated dollar increased limits cost -
Actual dollar increased limits cost)/
Actual dollar increase limits cost

This Equals:

$$\frac{(\text{Estimated ILF - Actual ILF})}{\text{Actual ILF - 1}}$$

Excess Loss Factor Error:

$$\frac{\text{Estimated Dollar Excess Losses - Actual Dollar Excess Losses}}{\text{Actual Dollar Excess Losses}}$$

This Equals:

$$\frac{\text{Estimated ELF}}{\text{Actual ELF}} - 1$$

The table below displays the errors associated with the methods discussed under the assumption that no IBNR claims would be reported:

ESTIMATION ERROR - without true IBNR			
Method	**Deductible**	**ILF**	**ELF**
Closed Only	109%	-52%	-60%
Incurred Only	56%	-33%	-47%
Incurred Developed	8%	-41%	-11%
Open Developed	0%	-26%	2%

The next table displays the errors associated with the methods under the assumption of two IBNR claims:

Method	Deductible	ILF	ELF
Closed Only	163%	-50%	-65%
Incurred Only	96%	-30%	-53%
Incurred Developed	-9%	-41%	-2%
Open Developed	26%	-24%	-10%

Although this is a simple example, the magnitude of the errors should be unappealing for most. In subsequent sections of this paper we will discuss several techniques being used to reflect loss development in distributions.

The final introductory topic regards trend. The deductible credits, ILF's and ELF's are probably being estimated for a prospective period. Even if the ultimate loss distribution is estimable based on prior claims, it still must be adjusted to reflect the economic cost levels of the prospective period being considered.

For the sake of simplicity, we will assume in this section that trend is uniform. Loss distributions are very malleable under this assumption[9]. Unfortunately, little research that we are aware of has been performed in the area of non-uniform trend, although a recent article by Philbrick[10] did discuss the issues surrounding the problem.

If the accident period being projected is four years later than the accident period of the sample claims, then the estimated claim values at ultimate for the projected period, assuming a

[9] Hogg and Klugman, p. 179
[10] Stephen W. Philbrick, "Brainstorms," Actuarial Review, August 1994

uniform annual trend of 5%, would be (working only with the original reported claims and ignoring the IBNR claims):

Claim Number	Original Ultimate Claims	Trended Ultimate Claims
1	600	729
2	800	972
3	1,100	1,337
4	1,300	1,580
5	1,600	1,945
6	1,800	2,188
7	2,500	3,039
8	15,000	18,233
9	11,000	13,371
10	20,000	24,310
Total	55,700	67,704

The trended claim values equal the ultimate value multiplied by 1.05^4. The resulting deductible credit, ILF, and ELF are .173, 1.84, and .49. As one might expect, the deductible credit decreases with trend and the ILF and ELF increase. Fortunately, assuming uniform trend, the estimation error can be eliminated by multiplying the original claim values by the trend index of 1.05^4 before fitting the loss distribution. Alternatively, the original claims may be indexed to an overall severity of 1 before fitting. The selected ultimate indexed distribution may then be scaled to the ultimate severity. This approach is discussed in further detail in Section II of this paper.

Section II - Some Practical Methods to Reflect Development

Quite often the data available for use in a loss distribution fitting process consists of individual claim values for a particular valuation and other aggregate projections from more traditional triangular methods. In less optimal cases, only the aggregate data may be available. This section discusses three methods for developing loss distributions, which reflect loss development, using data provided in one of these formats. These methods will rely substantially on "industry" analysis performed by the Insurance Services Office (ISO) and the National Council on Compensation Insurance (NCCI).

The first method uses the projected ultimate unlimited severity (the average claim) for an individual risk or line of business and an appropriate coefficient of variation (CV) based on industry data to obtain parameters for three two-parameter loss distributions. The second method adjusts actual industry distributions to produce the projected average claim of the individual risk being considered. This process is referred to as "scaling" the industry curves and is described by Venter[11]. The third method, unlike the first two methods, requires individual ground up claim information for a single valuation. This claim detail is used to estimate an immature CV. Industry loss distribution development patterns are employed to develop the immature CV to ultimate. The ultimate CV is combined with the projected average claim, as in the first method, to obtain the parameters of several two-parameter loss distributions.

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[11] Gary G. Venter, "Scale Adjustments to Excess Expected Losses," Proceedings, May 23, 24, 25, 26, 1982, Vol. LXIX, Part 1, No. 131.

Using Industry Coefficients of Variation

For each of the three methods discussed, we will assume that an estimated ultimate average severity for the accident year in question is available. This estimate could be developed by projecting ultimate losses and non-zero claim counts to ultimate using traditional actuarial techniques. An exponential regression could then be fit to the indicated historical ultimate average values and used to estimate the ultimate severity for the prospective period[12]..

Some of the more familiar loss distributions such as the Pareto, lognormal, and the gamma distribution have two parameters which define them. Estimates of these parameters can be obtained if two quantities about the distribution are known. These quantities might be the mean, mode, median, second moment, variance, CV, 99th percentile, etc.[13] As discussed, it is assumed that a projected average cost per claim or mean is available. The second quantity used to parameterize the loss distributions in this first method will be CV's based on ISO and NCCI (industry) published information.

In order to obtain an industry CV for workers compensation, the loss distributions used by the NCCI in developing excess loss premium factors for use in retrospective rating may be used. However, we are interested in a total CV, whereas the NCCI distributions are by injury type. Further, the NCCI distributions have been "indexed" so that the expected value for each distribution is one. For a more complete description of this process see Gillam[14].

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[12] It should not be inferred that this is the only or even the best method to determine these values. This approach is suggested as one specific method. Other methods do exist and may be appropriate.
[13] It should be understood that the calculation of parameters from the various quantities cannot be performed arbitrarily. Issues of bias and efficiency are important. Certain pairs of quantities could be very poor choices for the determination of parameters. The selection of parameters is discussed in many good statistics references and is beyond the scope of this paper.
[14] William R. Gillam, Parameterizing the Workers Compensation Experience Rating Plan, PCAS LXXIX, 1992

In order to develop a CV for the general distribution underlying any workers compensation claim, we need to describe the mixture process somewhat further. Appendix B contains a discussion regarding the mixture of models. This process is often confused with the addition or convolution of two random variables. The NCCI developed three loss distributions:

1. Fatal claims (D)
2. Permanent total (PT) and major permanent partial (major)
3. Minor permanent partial (minor) and temporary total (TT)

The cumulative distribution function (cdf) for the general workers compensation claim is:

$$F_T(x) = P_D F_D(x) + P_{PT/major} F_{PT/major}(x) + P_{TT/minor} F_{TT/minor}(x) + P_{mo} F_{mo}(x)$$

Where P_i is the probability that a claim is from injury type *i*, F_i is the cdf for injury type *i*, and MO claims are claims with medical losses only.

The expected value for the general workers compensation claim is:

$$E[X] = P_D E[D] + P_{PT/major} E[PT/major] + P_{TT/minor} E[TT/minor] + P_{MO} E[MO]$$

Where E[*i*] is the expected value of injury type *i*.

The second moment for the general workers compensation claim is:

$$E[X^2] = P_D E[D^2] + P_{PT/major} E[PT/major^2] + P_{TT/minor} E[TT/minor^2] + P_{mo} E[MO^2]$$

Based on NCCI published data and other internal data, we developed the following table.

	INJURY TYPE			
	Fatal	PT/Major	Minor/TT	Med Only
Average Claim	$210,000	$180,000	$8,500	$400
Probability	.0006	.0270	.2264	.7460
Second Moment	1.34E11	2.08E11	2.80E8	213,333

The second moment by injury type is the major item which was estimated using the NCCI loss distributions. It should be noted that the second moment for PT/Major claims is undefined in the NCCI information. The value shown here is calculated by capping the PT/Major loss distribution at $50 million. This capping allows us to calculate a second moment. We believe this is an acceptable adjustment, however, this means that the value shown should not be characterized as an NCCI value. Additionally, we assumed that the distribution for medical only claims is uniform from zero to twice the mean. Finally, the NCCI produces two different loss distributions for fatal claims, based on state benefit types. We averaged the CVs for the two distributions to produce our single second moment. Similarly, the NCCI has two separate distributions for PT/Major claims based on state benefits, which we averaged.

The CV of the total distribution is the standard deviation of the total distribution divided by the mean. The standard deviation can be calculated using total mean and second moment[15]. The resulting total CV is 10.474. If the total claim process being considered excludes medical only claims the CV is 5.441.

Some extensive client data was available for testing the three methods presented in this section, as well as the more robust method presented in Section III. The data consisted of the incurred value, paid value, accident date, report date, and closure date (if closed) for all claims occurring subsequent to 1/1/75 and prior to 3/31/95. The claims were due to products liability self-insured exposures for a diverse manufacturer. We transformed the actual claim amounts to preserve the confidentiality of the client's data. Therefore, all of the loss distributions fit to this sample data are for illustrative purposes only and are not suitable for use in any other circumstances.

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[15] $\text{Variance}[T] = \text{Second Moment}[T] - (\text{Mean}[T])^2$

Given the estimated industry CV of 10.474 and an estimated ultimate average claim size of $133,892, a lognormal and Pareto distribution were parameterized[16]. The table below displays the limited expected values (LEVs) and the cdf for the two loss distributions and the undeveloped trended empirical data.

Loss Limit	Lognormal		Pareto		Empirical	
	F(x)	LEV	F(x)	LEV	F(x)	LEV
1,000	0.121	935	0.015	993	0.215	872
5,000	0.333	3,957	0.070	4,821	0.404	3,519
10,000	0.456	6,953	0.134	9,308	0.481	6,296
25,000	0.622	13,671	0.289	21,081	0.602	13,090
100,000	0.829	32,011	0.671	57,336	0.808	33,321
500,000	0.955	65,037	0.955	105,807	0.965	64,342
1,000,000	0.978	80,676	0.986	118,261	0.984	76,434

This method assumes that no individual claim data is available. This makes it difficult to select the most appropriate loss distribution. Because of development, we would expect that the empirical cdf and LEV would be less than that of the ultimate loss distributions. It appears that both of the distributions had trouble handling the combination of the large CV and the large unlimited severity. Therefore, it would probably be better to use a CV from ISO's products liability distributions, discussed next, as the data consists of products liability losses and the average ISO claim sizes are much more consistent with the client data.

ISO has estimated distributions for the premises and operations (PremOPs) and products and completed operations (products) lines of business[17]. ISO has generated three compound distributions for each, tables 1-3 for PremOps and tables A-C for products. The tables

[16] Given the unlimited mean, M, and the CV, the parameters of the three distributions are: lognormal sigma = sqrt(ln(1+CV^2)), mu = ln(M) - .5 sigma^2; gamma alpha = 1/CV^2, beta = alpha/M; and Pareto alpha = 1/CV^2+2, lamda=Mx(alpha-1).

[17] Insurance Services Office, Inc., Revision of Premises/Operations and Products/Completed operations Increased Limits and Deductible Discount Factors. Filing GL95-ICDD1-Louisiana March 17, 1995.

represent increasing degrees of hazard within the line. The parameters for these tables are included as Section II, Exhibit 1a-1b.

The total loss distribution for a given line for a given hazard group is a compound process. All of the accidents for a given accident year are separated by settlement lag. Settlement lag is the number of years after the accident year in which an individual claim settles. Two Pareto distributions represent the group of claims in each settlement lag. The total cdf for the process is:

$$F(x) = \sum_{i=1}^{7} q_i \left[P_i F_{i2}(x) + (1 - P_i) F_{i2}(x) \right]$$

Where i is the settlement lag, F_{ij} is the jth Pareto distribution for the ith settlement lag, P_i is the weight for the first Pareto distribution in settlement lag i, and q_i is the relative percentage of claims which settle in settlement lag i.

The mean and variance of the compound process are calculated as noted previously for the total workers' compensation process. The resulting CV's for each distribution are displayed in the table below.

Coefficients of Variation					
PremOps			**Products**		
Table	**Mean**	**CV**	**Table**	**Mean**	**CV**
1	10,920	10.298	A	11,372	19.393
2	24,996	13.142	B	66,356	9.392
3	95,772	7.389	C	276,832	4.423

As with the NCCI distributions, we calculated a limited mean and variance assuming the losses were capped at $50 million, where the unlimited variance did not exist. Of interest is the observation that the CVs for the Products tables decrease as the hazard increases.

The estimated ultimate average claim size of the sample data combined with the ISO products table B CV produce the following parameters:

Distribution	Distribution Parameters First Parameter	Second Parameter
Lognormal	9.559	2.119
Pareto	2.011	135,410
Gamma	0.011	8.467E-08

The table below displays the cdf and ILF's, assuming a $25,000 base limit, for two of the loss distributions and the ISO products table B distribution.

Loss Limit	Lognormal F(x)	Lognormal ILF	Pareto F(x)	Pareto ILF	ISO Table B	
10,000	0.435	0.505	0.134	0.441	0.767	0.602
25,000	0.606	1.000	0.289	1.000	0.861	1.000
100,000	0.822	2.353	0.671	2.720	0.947	1.921
250,000	0.912	3.653	0.878	4.146	0.976	2.701
1,000,000	0.978	5.889	0.986	5.611	0.994	3.932

It appears that the two parameter distributions do not capture the skew or diversity present in the ISO multimodal process.

In preparing this paper, we noted several random observations about CV's. First, it is possible for a line which would normally be considered highly skewed to have a CV which appears small. This is partly due to the fact that these lines generally have larger average claim sizes. Because the CV is a ratio to the mean, a smaller CV with a larger mean can still produce a skewed distribution.

Second, some people fall into a normal distribution thought process when considering CV's. A distribution consistent with the normal distribution would have a 97.5 percentile at 1.96 standard deviations. Unfortunately, insurance distributions, such as the Pareto distribution, tend to be skewed. A Pareto distribution with parameters 3 and 10,000 has a 97.5 percentile which is 2.8 standard deviations from the mean.

Third, we have found in other studies of empirical data, support for the statement that inflation does not affect claims uniformly. In these studies, lognormal distributions were fit to claims for individual accident years of common maturities. The distributions were not rejected by various goodness of fit statistics and produced reasonable and increasing overall severities. The CV's for these distributions were clearly decreasing. If inflation impacted claims uniformly, the CV's would have been constant, assuming no substantial changes in loss adjustment procedures or the mix of business. Finally, care must be taken when trying to examine CV's behavior through simulation. CV's are ratios and many simulation techniques produce biased results for ratios.

Scaling the Industry Curves

Given a projected unlimited severity for a risk or book of business, the industry loss distributions can be modified to produce a mean equal to the risk's average claim. This process essentially accepts the shape of the industry loss distribution and shrinks or expands the industry distribution to match the risks average claim. This adjustment is made to the scale parameter of the distribution and basically assumes a uniform inflationary effect to "scale" the original distribution. Hence, the name of the method. This procedure can also be applied with some modification to a limited average claim size, if a credible unlimited severity is unavailable.

The following table displays the approximate parameters for the NCCI distributions, and our estimates of representative probabilities for each injury type.

Workers' Compensation Distributions by Injury Type						
Benefit Type	Fatal NonEscalating or Limited	Fatal Escalating	PT/Major NonEscalating	PT/Major Escalating or Limited	Minor/TT All	Med Only All
Distribution Type	Gamma	Gamma	Shifted T-Beta[18]	Shifted T-Beta	Trans-Beta	Uniform
Alpha	.5500	.4450	1.4900	1.4000	63.4960	N.A.
Lamda	381,818	471,910	66,616	64,772	4,024,363	N.A.
Gamma	N.A.	N.A.	1.2200	1.2500	.6410	N.A.
Tau	N.A.	N.A.	1.5000	1.4000	.9670	N.A.
Mean	210,000	210,000	180,000	180,000	8,500	400
Probability	.06%	.06%	2.70%	2.70%	22.64%	74.60%

The medical only distribution is not based on the NCCI distributions. The Lamda parameter is the scale parameter. The actual Lamda parameters used by the NCCI produce a mean of one for each distribution. However, in order for scaling to be feasible, a representative industry severity for each injury type is required. We have adjusted the Lamda parameters to produce these representative severities.

The overall mean for the industry distribution is $7,209, the weighted averages of the industry severities. In the following calculations, we will assume limited fatal benefits and non-escalating PT/Major benefits. The projected severity for the sample data is $133,892. Given the large difference between the average claim sizes, it is probably inappropriate to scale the NCCI distributions in this example. However, we will continue with the process for purposes of illustration. Each of the Lamda parameters and distribution means need to be increased by 18.573 ($133,892/$7,209). The new and original PT/Major shift points are 25% of the new and original means, respectively. The medical only distribution is adjusted by multiplying the upper bound by 18.573. The adjusted average severities by injury type are displayed in the table below:

[18] The PT/Major distributions are truncated and shifted by 25% of the average claim. Therefore, the actual mean of the distribution is $135,000 or 75% of $180,000.

Adjusted Severities					
Injury Type	Fatal	PT/Major	TT/Minor	Med Only	Total
Adjusted Severity	$3,900,423	$3,343,220	$157,874	$7,429	$133,892

The resulting LEV's, ILF's, and ELF's are displayed in the table below.

NCCI Scaled Distributions			
Limit	LEV[19]	ILF	ELF
$ 10,000	7,793	0.729	0.942
25,000	10,685	1.000	0.920
50,000	14,780	1.383	0.890
100,000	21,340	1.997	0.841
250,000	34,656	3.243	0.741
1,000,000	66,595	6.232	0.503

Establishing a worksheet in Excel or Lotus which readily handles the distributions is not an insurmountable task. Both software packages contain the functions necessary to construct cdf, LEV, and moment functions for each of the distributions used. Even the truncated shifted PT/Major distribution can be handled.

In order to scale the ISO compound Pareto distribution, the Pareto scale parameters must be multiplied by the overall severity adjustment factor. The overall industry mean for the products table B is $66,356. Given the projected severity for the sample data of $133,892, the B

[19] When estimating the PT/Major component of the LEV, if the limit is less than 25% of the average claim (the shift point) then the LEV is the limit. If the limit is greater than the shift point then the LEV is computed using the transformed Beta distribution with the scaled lamda parameter. The actual limit used in computing the LEV with the transformed Beta distribution is the original limit less the shift point. After the LEV is calculated via the transformed Beta distribution, the shift point must be added back in to obtain the final LEV.

parameters need to be multiplied by 2.018 ($133,892/$66,356). The adjusted B parameters are displayed in the following table.

Adjusted B Parameters		
Lag	B1	B2
1	13,504	7,367
2	20,104	10,968
3	58,482	31,905
4	104,184	56,838
5	130,974	71,453
6	153,230	83,595
7	150,894	82,321

The table below displays the adjusted cdf, LEV's, and ILF's which result from scaling the ISO medium hazard products loss distribution.

Scaled Products Table B			
Loss Limit	F(x)	LEV	ILF
$ 10,000	0.665	5,268	0.577
25,000	0.792	9,136	1.000
50,000	0.860	13,347	1.461
100,000	0.911	18,861	2.065
250,000	0.956	28,087	3.074
1,000,000	0.988	44,448	4.865

The major assumption made when one scales industry loss distributions is that the shape of the industry distribution is appropriate for the individual risk or book of business. If the data has an inordinate number of small losses, perhaps due to an incident reporting procedure, the overall projected severity will be reduced. This will distort the accuracy of the adjusted distribution. For example, suppose we are given the following risk and industry losses.

Industry Losses		Risk Losses	
Loss Amount	Number	Loss Amount	Number
$ 100	500	$ 110	500
1,000	50	1,100	50
10,000	10	11,000	10
50,000	5	55,000	5
250,000	1	275,000	1

Obviously, the risk's average severity of $1,360 is 10% higher than the industry average severity of $1,237. Scaling the industry distribution up 10% would be appropriate. However, assume the risk had an additional 500 claims valued at $25 each. The risk's total severity is now $734. The industry distribution scaled down by 41% (1-734/1,237) would be as follows:

Scaled Industry Losses

Loss Amount	Number
$59	500
594	50
5,935	10
29,677	5
148,383	1

Now the ELF for a $25,000 limit based on the scaled industry distribution is .35, whereas the actual ELF is .51. Problems pertaining to a larger than normal or less than expected number of smaller claims can often be discovered by examining the empirical and theoretical cdf and LEV's at smaller loss limits.

Developing Empirical CV's

The general approach for developing an ultimate CV based on an entity's individual ground up claim detail or a book of business and industry data consists of the following steps:

1. Using the individual claim detail available, estimate the risk's CV. This is done by first trending the individual claims to the prospective period. The sample CV can then be computed by dividing the sample standard deviation by the sample mean.

2. Estimate an industry CV which is appropriate for the overall maturity of the sample data and the inflationary level of the prospective period.

3. Estimate an ultimate industry CV for the prospective period based on the industry distributions.

4. Develop the ultimate sample CV by multiplying the sample CV from step 1 by the ratio of the ultimate industry CV and the undeveloped industry CV.

5. Use the projected unlimited severity and the estimated ultimate CV to parameterize a two parameter loss distribution as in method 1.

Unfortunately, attempting this process with NCCI data is problematic. The NCCI curves were developed by fitting a distribution for each injury type for a single policy year at 3rd, 4th, and 5th report. After examining the progression of these parameters, the NCCI selected ultimate parameters. In order to estimate an industry CV for a given maturity mix of data, estimates of the NCCI distributions would be required at additional maturities. While these distributions might be estimable, it is not currently possible to obtain all of the immature total workers compensation distributions. In order to obtain the total claim distribution, weights by injury type for each valuation are needed. This would require a claim count distribution for each maturity by type of injury, which is unavailable other than on a state by state basis for the first few reports. ISO's current methodology lends itself to this procedure much more readily than the NCCI data.

Section II, Exhibit 2a displays the estimated developed CV based on a portion of the settled claims from the sample data. Section II, Exhibit 2b displays the notes to the calculations. Estimated "premiums" for each year are combined with a rate change and trend index to develop a relative volume index by year in column (5). This volume index is combined with the estimated percentage of claims settled to obtain the cumulative lag weights by lag in column (8). The relative weights are used as the probabilities in a compound process to obtain the overall immature industry standard deviation and mean, columns (9)-(10). The ultimate industry CV is divided by the immature industry CV yielding the CV development factor. This is applied to the sample data CV to obtain the estimated ultimate CV for the sample data, row (14).

The industry distributions should be trended to the prospective period level before calculating the standard deviation and mean for each year. This was accomplished by trending the ISO B-parameters. The relative volume weights could be estimated via an ultimate claim count projection for each accident year. The settlement pattern might be adjusted based on the individual risks data. However, the settlement pattern represents the percentage of claims closed for the industry by lag and should be treated accordingly.

Prior to the most recent ISO ILF filings, ISO used the incurred claim data in its ILF estimation process. As a result, loss distributions for each valuation of a current diagonal of claims were developed. One might think that these could be used in a fashion similar to the process described for Exhibit 2 and the settled data. However, the size of loss distributions by valuation age are not independent. Therefore, the CV cannot be estimated without an estimate of the covariances for each distribution pair. As incurred data often contains more information regarding individual large claims than settlements only, it would be helpful if someone developed a process to account for the covariance in the NCCI and ISO incurred distributions by valuation age.

It has been suggested that one might construct a triangle of CV's from incurred claims and develop these CV's to ultimate. This procedure would be similar, in its basic nature, to the prior approach used by ISO. Before such a procedure could be relied upon, a more complete understanding of the underlying statistical assumptions, particularly regarding independence, would need to be obtained. Any additional research in this area would certainly be welcome.

The following table displays the cdf and LEV using the developed CV and the projected average claim to parameterize these distributions. Due to the large CV, the gamma distribution was not tractable.

Loss Distributions Via Developed CV's				
	Lognormal		Pareto	
Limit	F(x)	LEV	F(x)	LEV
10,000	0.338	7,997	0.133	9,314
25,000	0.526	16,326	0.287	21,109
100,000	0.787	39,413	0.670	57,519
250,000	0.899	60,998	0.878	87,710
1,000,000	0.978	95,141	0.986	118,624

This section presented three simplified methods of developing loss distributions using minimal sample data. Method 1 and method 3 result in a two-parameter loss distribution. Such a simple distribution will probably not capture all of the variation in the underlying loss process. This is one reason why ISO and NCCI have developed such robust compound processes.

Scaling the industry distributions as in method 2 retains the same diversity as the industry distributions but may not correctly address the shape issue. However, all of these methods attempt to address development and require only basic summary data for the most part. Section III introduces a more refined method which requires a minimal amount of extra data and a few additional loss distribution parameters.

Section III - A More Refined, Practical Approach

When considering the problem of development and loss distributions, one is tempted to jump to the utopic extreme and begin thinking about what the best procedure would be if there were no limitations on the data available. After this perfect method was created, the minimal amount of alterations could be made to the assumptions of the method to account for the actual data available in a given situation. We decided to approach the problem from another direction. The first question we asked was what is the format of the data most likely to be available for this type of project. For many practical business applications, this data consists of listings of individual claims in excess of a fixed retention and summary loss and claim count information. The procedure described in this section is designed to use data in this format.

A two part loss distribution is developed in order to estimate ILF's and ELF's. Because the data format does not include individual claim information regarding the smaller claims, the distribution developed will not be applicable to smaller deductibles. The first part pertains to the smaller claims for which no individual claim detail is available. This part of the distribution is estimated using aggregate loss data and more traditional triangular approaches. The second part involves fitting a loss distribution based on the individual large claim data. The technique employed is somewhat different from more traditional approaches.

The complete ultimate loss distribution is similar to the old five parameter ISO distributions. The distribution consists of two parts: one for the smaller claims below the loss limit and another for the larger claims about which we have more detail. The distributional formula is:

$$F(x) = \begin{cases} \dfrac{px}{L} & x < L \\ p + (1-p)G(x) & x \geq L \end{cases}$$

Where L is the loss limit, p is the ultimate proportion of claims below the loss limit, and G(x) is the cdf of a truncated lognormal distribution.

The function form below the loss limit is essentially immaterial because (1) we are estimating ILF's and ELF's for limits that are greater than the loss limit and (2) there is no individual loss data below the loss limit. The product of the proportion of all claims less than the loss limit (p) and the average severity of these small claims (S) is important. An ILF for a given limit K is estimated by the following formula:

$$\frac{pS + (1-p)E[X;K]}{pS + (1-p)E[X;B]}$$

Where E[X;Y] is the limited expected value of X limited to Y, B is the basic limit, and S is the average severity of all claims less than the loss limit. One property of this estimator is that it is not distorted by the addition of a large number of very small claims.

Handling the Small Claims

The quantities that must be estimated for the smaller claims are p and S. By subtracting the incurred losses and claim counts for the large claims from the total aggregate information, we constructed a small claim loss and count triangle. These are then developed to ultimate to produce the estimated historical severity by accident year. A regression was fit to these to both smooth the indications and project the severity S for the prospective period.

A total claim count triangle was developed to ultimate. The projected ultimate small claim counts divided by the total count projection yields historic p ratios by accident year. A regression was fit to these to smooth and project the p value for the prospective period.

There are a few points to note here. First, the inclusion or exclusion of a large amount of small claims will effect both p and S. However, the product of pS will not be materially effected. Therefore, if the historic projections for either p or S are not very smooth, one may wish to regress on pS.

Second, the possibility of error from misestimation is greater if the value of p is very large or very small. The final unlimited severity should be multiplied by the projection of ultimate claim counts and checked for reasonableness against an independent projection of the total ultimate losses.

Addressing the Large Claims

The process used to estimate the parameters of the lognormal distribution consisted of the following steps:

1. For each accident year and valuation, count the cumulative number of claims in each of 8 fixed layers;

2. Convert the count distributions to percentage distributions;

3. Develop a function for each layer which most accurately reflects the changes in the percentage distributions as each accident year matures;
4. For each accident year, estimate an ultimate percentage distribution using the development function;

5. Select an ultimate percentage distribution for the prospective period; and

6. Fit a lognormal distribution to the selected ultimate percentage distribution for the prospective period.

In order to determine the shape of the ultimate size of loss distribution, we computed "development factors" for selected size of loss ranges. We were provided with individual claim amounts in the excess of $25,000 evaluated at 1990, in addition to values evaluated at 1995. This information gives us two "snapshots" of the development process separated by five years.

Each of the individual claims greater than or equal to $25,000 were grouped into categories. The following categories were established:

$50,000—claim amount greater than $25,000 but less than $50,000

$100,000—claim amount greater than $50,000 but less than $100,000

$250,000—claim amount greater than $100,000 but less than $250,000

$500,000—claim amount greater than $250,000 but less than $500,000

$750,000—claim amount greater than $500,000 but less than $750,000

$1,000,000—claim amount greater than $750,000 but less than $1,000,000

$2,500,000—claim amount greater than $1,000,000 but less than $2,500,000

$6,000,000—claim amount greater than $2,500,000 but less than $6,000,000

The counts in each category were compared at the 1990 and 1995 evaluations by accident year. For example, the number of claims in the $50,000-$100,000 category for the 1985 accident year as of 1990 was 41, and five years later, the number in that category as of 1995 was 30. From this information, we wish to determine a set of development factors which can be used to estimate the movement of claims between categories.

In Section III, Exhibit 1, we have calculated the relative proportions of the large claims at the two evaluation dates. For example, the 1985 Accident Year had 31.8% of the large claims with an incurred value between $50,000 and $100,000 as of 1990. Five years later, the proportion of large claims in this size category had dropped to 25.2%. Conversely, the proportion of large claims in excess of $1,000,000 had increased from 3.1% to 4.2% over the same time period.

When there is sufficient claim count experience by layer, the actual claim count development factors by layer may be used in the fitting process discussed later to obtain the selected layer claim count development factors. However, the sample data included in this analysis was sparse in some of the upper layers. In particular, there were problems associated with individual cells which had no claims. There were several multimillion dollar claims which needed to be reflected in the procedure. Therefore, each of the accident year claim count distributions were smoothed by fitting a lognormal loss distribution for each of the two valuations. These smoothed distributions are displayed in Section III, Exhibit 2.

The results of the smoothed distributions were employed to obtain the fitted distribution layer development factors in Section III, Exhibit 3. Continuing to focus on Accident Year 1985, the proportion of $100,000 claims as of 1995 is divided by the proportion as of 1990. The resulting ratio is 0.904. Similarly , the factor for the $2,500,000 range is 1.934, reflecting the fact that a higher proportion of the claims are in this size category at the later evaluation. (It is important to keep in mind that the data reflects relative proportions of claims, not the absolute number of claims. The absolute number of claims will be discussed later).

A review of these five-year development factors shows a clear trend. The proportion of claims under $250,000 drops steadily over time, faster at early evaluations and slower at later evaluations. The proportion of claims in the largest categories grows steadily over time, fast at first and then slower. The larger the size category, the larger the growth. In other words, over time we have a migration of claims. At early intervals we have a certain proportion of claims in each size category. Over time, there is a tendency to depopulate the smaller size categories. Some of these claims become larger, and some become smaller. The larger size categories tend to show an overall net increase in the proportion of claims.

At the same time, a small proportion of claims "drop out," that is, are settled with a zero indemnity amount. However, the remaining claims show a pronounced trend toward higher size categories.

The table in Section III, Exhibit 3 essentially has overlapping development factors evaluated at a five year interval. We need to convert these to one-year development factors. One approach is to approximate the annual development factor by the fifth root of the five year age-to-age factor, then calculate the average of the factors with the same "maturities." However, most development factors have the property that the age-to-age factor is not a constant factor over a period of time, but rather a decreasing factor. In order to apportion the five year factors into annual amounts, we fit the development factors for a given layer to a curve of the form EXP[exp(a(x+5)+b)-exp (ax+b)], where x is the development year of each individual accident year. This curve provided a good fit to the factors.

The curve is used to apportion each five-year development factor into an annual amount. The resulting annual factors are accumulated in the normal manner to produce age-to-ultimate factors. The resulting factors are then applied to the current proportion of claims in each size category, which yields an estimate of the ultimate proportion of claims by size for each accident year. Based on the projected distributions for the most recent years, an ultimate distribution is selected for the prospective period in Section III, Exhibit 4. It is to this ultimate distribution that the lognormal loss distribution is fit.

The table below displays the resulting ILF's and ELF's from this method, as well as some of the methods from Section II.

	Section III		ISO CV Pareto		Scaled ISO		Developed CV Lognormal	
Limit	ILF	ELF	ILF	ELF	ILF	ELF	ILF	ELF
$ 25,000	1.000	0.922	1.000	0.843	1.000	0.932	1.000	0.878
50,000	1.759	0.863	1.729	0.728	1.461	0.900	1.605	0.804
100,000	2.973	0.768	2.720	0.572	2.065	0.859	2.414	0.706
250,000	5.315	0.586	4.146	0.347	3.074	0.790	3.736	0.544
1,000,000	9.418	0.266	5.611	0.116	4.865	0.668	5.828	0.289

There are three aspects of this method which are appealing. First, the data required is frequently available. Second, the final loss distribution is fit to data at ultimate. It is possible that a particular family of loss distributions may be rejected if fit to immature data, where the distribution would have been appropriate for the ultimate distribution. This procedure avoids this possibility when the empirical data is not smoothed.

Finally, one may have noticed that after giving the sermon in Section I on the evils of not trending the data, there is no explicit trend adjustment in the method provided in Section III. The method in Section III recognizes trend implicitly in the actual percentage claim count distribution by layer and its migration. The fact that trend is addressed without making the usual uniform assumption is appealing. The method could probably be improved in this respect if the layer boundaries were actually indexed to a smoothed average severity by year for each age. However, because the individual loss data is provided above a fixed retention, indexing would be problematic.

There are some problems with this method, which arise primarily from variability and small sample sizes. For example, the layer boundaries must be sufficiently refined and still contain a sufficient number of claims by grouping for each accident year. In order to accomplish this one might consider grouping accident years together. In addition, the availability of intermediate valuations may not provide sufficiently stable information for use.

Loss distributions are an invaluable tool. However, the actuary should be aware of the possible effects of development on loss distributions used for many casualty exposures. We attempted to illustrate the potential problems which may result if development is not considered via a simple example in Section I. In Section II, some practical methods for reflecting development were discussed. In particular, two methods were provided which did not require any individual claim information. Finally, Section III presented a practical method for use with aggregate data and individual loss data for losses in excess of a fixed amount. There still remains several unanswered questions, such as "what is the utopic procedure for recognizing loss development?" and "what is the actual impact of trend on claims?" However, the concepts presented in the paper do not hinge upon the answers to these questions, so we will leave them for another day.

Appendix A

The observation that loss development has a material impact on the size of loss distributions is made in Harwayne's article "Accident Limitations for Retrospective Rating."[20] He notes the significance of the impact of loss development, yet suggests only that the NCCI use fourth reports instead of third reports. While this increases the development age of claims from 42 months to 54 months, workers compensation claims show a stubborn tendency to continue development beyond 20 years.

A paper presented by Dr. Shaw Mong at the 1980 Discussion Paper Program, "Estimating Aggregate Loss Probability and Increased Limit Factor"[21] recognized the importance of loss development on the size-of-loss distribution but did not feel the need to provide techniques. He states, "In our model, we assume that all the losses have already been adjusted to the present or ultimate level. That is: losses have been developed to the ultimate; IBNR has been adjusted and inflation has been trended to the forecasting year, etc. The reason that we did not discuss those in here is because they are rather standard actuarial techniques practiced in most areas of rate-making and have been covered extensively elsewhere in the literature." He supplies two references in the literature. However, the Hewitt and Lefkowitz paper referenced only deals with inflation adjustments. The Miccolis paper[22] referenced discusses the need to adjust for development. The author notes, "Loss development also poses certain problems in working with severity distributions...It is very likely that this distribution of immature claim values

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[20] Frank Harwayne, "Accident Limitations for Retrospective Rating," Proceedings, May, 1976, Vol. LXIII, Part 1, No. 119.
[21] Dr. Shaw Mong, "Estimating Aggregate Loss Probability and Increased Limit Factor," Pricing Property and Casualty Insurance Products, May 11-14, 1980.
[22] Robert S. Miccolis, "On the Theory of Increased Limits and Excess of Loss Pricing," 1977 Proceedings, Vol. LXIV, p. 49.

will change considerably as these claims develop..." but when it comes to specific techniques, the author notes "Hachemeister describes a technique of estimating such loss development distributions conditioned on the age of the claim and its estimated value...The actual procedure for adjusting a severity distribution for loss development using the Hachemeister technique will be left to the interested reader..." (The methodology used in Section III of this paper is intended to follow the spirit of the Hachemeister proposed technique.)

Gary Patrik notes in his excellent paper on fitting loss distributions, "I decided to use undeveloped and incomplete data for this example so as not to get involved in the question of how to develop and complete it..." [23]

As recently as 1987, Pinto and Gogel noted that, "There is very little information available regarding excess loss development, despite its importance in excess of loss pricing and reserving..." and "There is a paucity of published information regarding both reported and paid excess loss development..." [24] They went on to explore the impact of loss development on various sizes of claims in one of the few papers to address the subject.

One other paper directly discusses specific techniques. Venter's paper, "Scale Adjustments to Excess Expected Losses" [25] illustrates one of the techniques discussed in Section II of this paper.

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[23] Gary Patrik, "Estimating Casualty Insurance Loss Amount Distributions," PCAS, Vol. LXVII, 1980.
[24] Emanuel Pinto and Daniel F. Gogol, "An Analysis of Excess Loss Development," Proceedings, November 4, 5, 6, 1987, Vol. LXXIV, Part 2, No. 142.
[25] Gary G. Venter, "Scale Adjustments to Excess Expected Losses," Proceedings, May 23, 24, 25, 26, 1982, Vol. LXIX, Part 1, No. 131.

Appendix B

Combining Distributions

In the normal course of actuarial work, it is often necessary to combine two or more distributions, and calculate relevant statistics (such as the mean and the variance) of the composite distribution. There is more than one way to create such a composite distribution, and the formulas differ. While the correct calculation of the mean is usually straightforward, the calculation of higher moments is trickier. This appendix will clarify the distinction between a convolution of two variables, and a mixture of two variables, as well as the appropriate formulas for mean and variance of each. The NCCI and ISO distributions discussed in Section II of this paper are mixtures of models.

Severity distributions for workers compensation provide a good working example, because an overall workers compensation severity distribution can be viewed as both a mixture and a convolution. First consider one injury type, such as PT (Permanent Total). A typical PT claim has an indemnity component, and a medical component. Assume that we analyze the severity distribution of indemnity amounts and call this random variable X and its associated distribution F_x. Similarly, analyze the medical distribution, call the random variable Y and the distribution F_y.

Now assume that we are interested in the distribution of a PT claim, including both the indemnity and medical amounts. We can define a new random variable,

$$Z = X + Y$$

which has distribution F_z. F_z is the **convolution** of X and Y.

(One important caution. It is likely that there is some correlation between medical and indemnity. Unfortunately, the calculation of the convolution requires independent distributions. For purposes of this discussion, we will make the simplifying assumption that X and Y are independent.)

Under convolution, the mean of the resulting distribution is the sum of the means of the two distributions being combined. The variance of the resulting distribution is the sum of the variances of the two distributions.

After we calculate a severity distribution for PT, we might also calculate severity distributions for other injury types. Now, we may be interested in an overall workers compensation severity distribution. The process of combining the severity distributions of the various injury types into an overall distribution is a **mixture**. The resulting distribution is not formed by adding a death amount to a PT amount, but by combining the distributions such that the resulting distribution has the appropriate proportion of each injury type.

For simplicity, assume we have only two injury types, death and PT. Assume the proportion of death claims is p and the severity distribution is F_x. Assume that the proportion of PT claims is q, *that is, 1-p , and its distribution is F_y. We form the composite distribution by mixing the two* distributions. Using Z to represent the resulting random variable, we can describe Z as:

$$Z = X \text{ with probability } p$$

$$Y \text{ with probability } q$$

The formula for the resulting mean is straightforward, but the formula for the variance is slightly less intuitive because the exponents on p and q do not allow the usual simplification. The formulas for convolution and mixtures are summarized below:

	Convolution (assuming X, Y independent)	Mixture (assuming X, Y independent)
	$Z = X + Y$	$Z = X \text{ with probability } p$ $Y \text{ with probability } q$
Mean	$E(Z) = E(X) + E(Y)$	$E(Z) = pE(X) + qE(Y)$
Variance	$Var(Z) = Var(X) + Var(Y)$	$Var(Z) = pE\left[X^2\right] + qE\left[Y^2\right] - \left(pE[X] + qE[Y]\right)^2$

It may also be helpful to think of these concepts in terms of an urn model. Assume we can represent severity distributions by values in urns X and Y. We form Z by selecting one value from urn X, one value from urn Y, and adding the values together. Each draw selects two values which are added together. The resulting random variable Z, has distribution F_z which is the convolution of F_x and F_y.

Alternatively, we could form Z by selecting a value from urn X with probability p, and selecting a value from urn Y with probability q. Each draw selects precisely one value. The resulting random variable Z has distribution F_z, which represents the mixture of F_x and F_y.

INCREASED LIMITS FACTORS
GENERAL LIABILITY
SUPPORTING MATERIAL - INDEMNITY

Section II
Exhibit 1a

Table of Mixed Pareto Parameters
Average Accident Date of July 1, 1996
Products/Completed Operations Liability

Table A

Lag	B1	Q1	P	B2	Q2	Lag Weight	$1 million Limited Expected Value *
1	2,515.12	1.20	0.915991663	2,200.53	3.20	0.531773315	1,653.70
2	3,744.31	1.20	0.746928275	3,275.96	3.20	0.286921419	4,301.54
3	10,892.26	1.20	0.624052128	9,529.83	3.20	0.079310877	14,904.08
4	19,404.31	1.20	0.534745107	16,977.18	3.20	0.044616820	28,826.35
5	24,393.84	1.20	0.469836301	21,342.62	3.20	0.025099466	38,599.45
6	28,538.96	1.20	0.422660243	24,969.26	3.20	0.014119858	46,955.28
7	28,103.97	1.20	0.388372440	24,588.68	3.20	0.018158245	48,447.35
Total/Average						1.000000000	7,093.34

Table B

Lag	B1	Q1	P	B2	Q2	Lag Weight	$1 million Limited Expected Value *
1	6,692.52	1.15	0.921341244	3,651.13	3.15	0.418536937	3,419.68
2	9,963.26	1.15	0.752277856	5,435.49	3.15	0.277258043	10,126.17
3	28,983.32	1.15	0.629401709	15,811.95	3.15	0.094095287	34,316.06
4	51,633.12	1.15	0.540094688	28,168.64	3.15	0.064990169	64,650.57
5	64,909.82	1.15	0.475185881	35,411.79	3.15	0.044887711	85,654.41
6	75,939.63	1.15	0.428009824	41,429.14	3.15	0.031003252	103,251.17
7	74,782.16	1.15	0.393722021	40,797.68	3.15	0.069228601	107,070.92
Total/Average						1.000000000	26,127.78

Table C

Lag	B1	Q1	P	B2	Q2	Lag Weight	$1 million Limited Expected Value *
1	9,359.41	1.10	0.816306461	3,701.05	3.10	0.304440622	7,865.08
2	13,933.50	1.10	0.647243074	5,509.81	3.10	0.233816271	18,835.47
3	40,532.82	1.10	0.524366926	16,028.14	3.10	0.091998387	57,430.38
4	72,208.31	1.10	0.435059905	28,553.77	3.10	0.073668491	102,373.05
5	90,775.62	1.10	0.370151099	35,895.95	3.10	0.058990671	132,179.64
6	106,200.66	1.10	0.322975042	41,995.57	3.10	0.047237281	156,654.04
7	104,581.96	1.10	0.288687239	41,355.47	3.10	0.189848278	161,900.30
Total/Average						1.000000001	65,557.43

* Limited Average Severity at the one million dollar policy limit.

SECTION2.XLS ISO
10/2/95

INCREASED LIMITS FACTORS
GENERAL LIABILITY
SUPPORTING MATERIAL - INDEMNITY

Section II
Exhibit 1b

Table of Mixed Pareto Parameters
Average Accident Date of July 1, 1996
Premises/Operations Liability

Table 1

Lag	B1	Q1	P	B2	Q2	Lag Weight	$1 million Limited Expected Value *
1	6,723.15	1.70	0.829092621	1,990.24	3.70	0.555830405	2,203.36
2	8,047.56	1.70	0.436495006	2,382.30	3.70	0.294627117	6,643.19
3	29,432.05	1.70	0.273002096	8,712.70	3.70	0.070332575	28,909.45
4	45,198.29	1.70	0.204917292	13,379.94	3.70	0.037253873	46,656.98
5	59,490.73	1.70	0.176564133	17,610.89	3.70	0.019732692	61,810.38
6	72,580.29	1.70	0.164756775	21,485.76	3.70	0.010452045	74,767.81
7	80,979.51	1.70	0.159839732	23,972.17	3.70	0.011771294	82,770.33
Total/Average						1.000000001	9,928.87

Table 2

Lag	B1	Q1	P	B2	Q2	Lag Weight	$1 million Limited Expected Value *
1	5,033.26	1.30	0.818071267	2,416.55	3.30	0.502090657	3,288.78
2	6,024.78	1.30	0.425473652	2,892.59	3.30	0.299264855	9,588.06
3	22,034.19	1.30	0.261980742	10,578.97	3.30	0.080330909	38,265.66
4	33,837.54	1.30	0.193895938	16,245.95	3.30	0.047845462	59,696.28
5	44,537.52	1.30	0.165542779	21,383.18	3.30	0.028496979	77,342.69
6	54,336.96	1.30	0.153735421	26,088.05	3.30	0.016972933	92,054.25
7	60,625.01	1.30	0.148818378	29,107.04	3.30	0.024998204	101,000.47
Total/Average						0.999999999	16,742.04

Table 3

Lag	B1	Q1	P	B2	Q2	Lag Weight	$1 million Limited Expected Value *
1	6,928.10	1.10	0.914673388	3,319.51	3.10	0.481709595	3,764.32
2	8,292.88	1.10	0.522075773	3,973.43	3.10	0.298779652	16,098.35
3	30,329.25	1.10	0.358582863	14,531.87	3.10	0.083458413	60,279.55
4	46,576.11	1.10	0.290498059	22,316.34	3.10	0.051727363	91,466.07
5	61,304.23	1.10	0.262144900	29,373.13	3.10	0.032060519	115,885.35
6	74,792.81	1.10	0.250337542	35,836.01	3.10	0.019871048	135,446.14
7	83,448.07	1.10	0.245420499	39,983.07	3.10	0.032393410	147,065.78
Total/Average						1.000000000	27,556.07

* Limited Average Severity at the one million dollar policy limit.

SECTION2.XLS ISO
10/2/95

Developing Empirical CV's

Section II
Exhibit 2a

Accident Year	Premium	On-Level Rate Factor	Exposure Trend	On-Level Premium	Relative Volume Weights
	(1)	(2)	(3)	(4)	(5)
88	27,600	1.210	1.160	38,729	0.1530
89	29,400	1.210	1.131	40,249	0.1590
90	28,200	1.100	1.104	34,240	0.1352
91	29,900	1.100	1.077	35,419	0.1399
92	31,700	1.100	1.051	36,635	0.1447
93	32,100	1.000	1.025	32,903	0.1300
94	35,000	1.000	1.000	35,000	0.1382
Total	213,900			253,175	1.0000

Accident Year	Settlement Lag	ISO Lag Weight	Cumulative Lag Weight	Prospective Level Second Moment	Prospective Level Mean
	(6)	(7)	(8)	(9)	(10)
88	7	0.069229	0.010590	2.105E+12	327,291
89	6	0.031003	0.009671	2.021E+12	314,712
90	5	0.044888	0.020073	1.551E+12	248,251
91	4	0.064990	0.038155	1.048E+12	174,762
92	3	0.094095	0.068858	4.366E+11	80,559
93	2	0.277258	0.238929	8.589E+10	19,397
94	1	0.418537	0.418537	1.728E+10	5,362
Total		1.000000	0.804814	2.122E+11	38,005

(11) Adjusted Industry Immature CV	12.0796
(12) Adjusted Industry Ultimate CV	9.1692
(13) Sample Data Immature CV	7.8672
(14) Sample Data Ultimate CV	5.9717

SECTION2.XLS DevCV's
10/18/95

Notes to Section II, Exhibit 2a

(1)-(3) These would be based on the individual client or book of business. The purpose is to essentially develop an estimate of the ultimate number of claims by year, which is column (5).

(4) (1) x (2) x (3)

(5) (4) / (4) Total

(7) From Section II, Exhibit 1a, Lag Weight Table B

(8) [Sum (5)] x (7). For example, for lag 2 column (5) is summed for accident years 88-93. This sum is then multiplied by the lag 2 weight in column (7).

(9) The second moment from ISO Table B for the specific lag. The B parameters have been trended forward from 7/1/96, the midpoint of the filing, to the midpoint of the prospective period. The total second moment is sum [(8) x (9)] divided by (8) Total.

(10) The mean from ISO Table B for the specific lag. The B parameters have been trended forward from 7/1/96, the midpoint of the filing, to the midpoint of the prospective period. (10) Total is [sum ((8) x (10))]/(8) Total.

(11) (9) Total/(10) Total

(12) Based on ISO Products Table B using trended B parameters.

(13) Based on client data for settled claims.

(14) (13) x (12)/(11)

Empirical Claim Count Distribution
Valued as of 3/90 and 3/95

Section III
Exhibit 1

	AY_77		AY_78		AY_79		AY_80		AY_81		AY_82		AY_83		AY_84	
Layer	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*
50,000	25	25	30	30	27	27	45	45	29	30	50	49	41	41	48	51
100,000	11	11	11	11	20	20	29	28	24	22	36	31	45	46	49	42
250,000	11	11	10	10	15	13	16	16	28	28	27	30	26	25	33	26
500,000	5	5	5	5	3	4	4	4	8	8	18	16	12	11	16	13
750,000	1	1	4	4	3	3	2	2	5	5	4	5	4	5	3	4
1,000,000	2	2	1	1	1	1	0	0	2	2	2	1	1	1	1	1
2,500,000	1	1	2	2	0	0	0	0	2	3	0	1	4	3	5	5
6,000,000	0	0	0	0	0	0	0	0	1	1	0	0	2	2	2	2
Total	56	56	63	63	69	68	96	95	99	99	137	133	135	134	157	144

	AY_77		AY_78		AY_79		AY_80		AY_81		AY_82		AY_83		AY_84	
Layer	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*
50,000	0.446	0.446	0.476	0.476	0.391	0.397	0.469	0.474	0.293	0.303	0.365	0.368	0.304	0.306	0.306	0.354
100,000	0.196	0.196	0.175	0.175	0.290	0.294	0.302	0.295	0.242	0.222	0.263	0.233	0.333	0.343	0.312	0.292
250,000	0.196	0.196	0.159	0.159	0.217	0.191	0.167	0.168	0.283	0.283	0.197	0.226	0.193	0.187	0.210	0.181
500,000	0.089	0.089	0.079	0.079	0.043	0.059	0.042	0.042	0.081	0.081	0.131	0.120	0.089	0.082	0.102	0.090
750,000	0.018	0.018	0.063	0.063	0.043	0.044	0.021	0.021	0.051	0.051	0.029	0.038	0.030	0.037	0.019	0.028
1,000,000	0.036	0.036	0.016	0.016	0.014	0.015	0.000	0.000	0.020	0.020	0.015	0.008	0.007	0.007	0.006	0.007
2,500,000	0.018	0.018	0.032	0.032	0.000	0.000	0.000	0.000	0.020	0.030	0.000	0.008	0.030	0.022	0.032	0.035
6,000,000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.010	0.010	0.000	0.000	0.015	0.015	0.013	0.014

	AY_85		AY_86		AY_87		AY_88		AY_89		AY_90		AY_91	AY_92	AY_93	AY_94
Layer	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*95*	*95*	*95*	*95*
50,000	29	40	40	31	31	35	18	34	9	32	0	64	29	22	10	10
100,000	41	30	41	23	44	32	35	44	4	36	2	63	36	45	23	6
250,000	40	31	48	38	44	35	31	42	7	40	4	50	46	41	18	6
500,000	8	6	6	16	12	15	3	8	2	6	0	8	17	13	10	2
750,000	5	5	4	3	3	7	1	3	0	3	0	7	3	1	1	0
1,000,000	2	2	2	0	3	4	0	3	0	4	0	4	0	2	0	0
2,500,000	4	2	2	2	0	2	0	5	1	3	0	2	0	0	1	0
6,000,000	0	3	0	1	0	2	0	0	0	2	0	1	0	1	0	0
Total	129	119	143	114	137	132	88	139	23	126	6	199	131	125	63	24

	AY_85		AY_86		AY_87		AY_88		AY_89		AY_90		AY_91	AY_92	AY_93	AY_94
Layer	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*95*	*95*	*95*	*95*
50,000	0.225	0.336	0.280	0.272	0.226	0.265	0.205	0.245	0.391	0.254	0.000	0.322	0.221	0.176	0.159	0.417
100,000	0.318	0.252	0.287	0.202	0.321	0.242	0.398	0.317	0.174	0.286	0.333	0.317	0.275	0.360	0.365	0.250
250,000	0.310	0.261	0.336	0.333	0.321	0.265	0.352	0.302	0.304	0.317	0.667	0.251	0.351	0.328	0.286	0.250
500,000	0.062	0.050	0.042	0.140	0.088	0.114	0.034	0.058	0.087	0.048	0.000	0.040	0.130	0.104	0.159	0.083
750,000	0.039	0.042	0.028	0.026	0.022	0.053	0.011	0.022	0.000	0.024	0.000	0.035	0.023	0.008	0.016	0.000
1,000,000	0.016	0.017	0.014	0.000	0.022	0.030	0.000	0.022	0.000	0.032	0.000	0.020	0.000	0.016	0.000	0.000
2,500,000	0.031	0.017	0.014	0.018	0.000	0.015	0.000	0.036	0.043	0.024	0.000	0.010	0.000	0.000	0.016	0.000
6,000,000	0.000	0.025	0.000	0.009	0.000	0.015	0.000	0.000	0.000	0.016	0.000	0.005	0.000	0.008	0.000	0.000

REPORT.XLS
10/2/95

Fitted Claim Count Distribution
Valued as of 3/90 and 3/95

Section III
Exhbit 2

Layer	AY_77		AY_78		AY_79		AY_80		AY_81	
	90	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*
50,000	0.398	0.398	0.392	0.392	0.381	0.389	0.467	0.470	0.285	0.289
100,000	0.251	0.251	0.250	0.250	0.279	0.274	0.289	0.287	0.260	0.255
250,000	0.188	0.188	0.190	0.190	0.214	0.207	0.176	0.175	0.250	0.244
500,000	0.077	0.077	0.078	0.078	0.075	0.075	0.047	0.047	0.110	0.110
750,000	0.029	0.029	0.030	0.030	0.023	0.024	0.011	0.012	0.039	0.040
1,000,000	0.015	0.015	0.016	0.016	0.010	0.011	0.004	0.005	0.019	0.020
2,500,000	0.029	0.029	0.031	0.031	0.015	0.016	0.005	0.005	0.029	0.033
6,000,000	0.012	0.012	0.013	0.013	0.003	0.004	0.001	0.001	0.007	0.009

Layer	AY_82		AY_83		AY_84		AY_85		AY_86	
	90	*95*	*90*	*95*	*90*	*95*	*90*	*95*	*90*	*95*
50,000	0.336	0.335	0.341	0.345	0.330	0.370	0.246	0.340	0.274	0.244
100,000	0.276	0.276	0.259	0.261	0.261	0.253	0.274	0.248	0.298	0.265
250,000	0.234	0.235	0.220	0.219	0.226	0.202	0.287	0.210	0.284	0.281
500,000	0.089	0.090	0.092	0.091	0.095	0.084	0.119	0.093	0.099	0.123
750,000	0.028	0.029	0.033	0.032	0.034	0.031	0.036	0.036	0.026	0.040
1,000,000	0.013	0.013	0.017	0.016	0.017	0.016	0.016	0.019	0.010	0.018
2,500,000	0.018	0.019	0.029	0.027	0.028	0.031	0.019	0.037	0.010	0.025
6,000,000	0.004	0.004	0.009	0.008	0.008	0.012	0.003	0.015	0.001	0.004

Layer	AY_87		AY_88		AY_89		AY_90	AY_91	AY_92	AY_93	AY_94
	90	*95*	*90*	*95*	*90*	*95*	*95*	*95*	*95*	*95*	*95*
50,000	0.233	0.260	0.209	0.266	0.340	0.271	0.334	0.203	0.223	0.185	0.381
100,000	0.298	0.248	0.383	0.271	0.261	0.256	0.281	0.306	0.277	0.288	0.307
250,000	0.312	0.254	0.345	0.271	0.222	0.256	0.238	0.342	0.306	0.347	0.224
500,000	0.110	0.121	0.056	0.113	0.092	0.116	0.089	0.112	0.125	0.129	0.063
750,000	0.027	0.045	0.005	0.036	0.033	0.041	0.027	0.024	0.036	0.031	0.015
1,000,000	0.010	0.023	0.001	0.016	0.016	0.020	0.012	0.007	0.015	0.011	0.005
2,500,000	0.009	0.038	0.000	0.022	0.027	0.032	0.016	0.006	0.016	0.009	0.005
6,000,000	0.001	0.011	0.000	0.004	0.008	0.008	0.003	0.000	0.002	0.000	0.000

REPORT.XLS
10/2/95

Fitted Distribution Layer Development Factors

Section III
Exhibit 3

Layer	AY_77	AY_78	AY_79	AY_80	AY_81	AY_82	AY_83
50,000	1.000	1.000	1.022	1.006	1.012	0.995	1.013
100,000	1.000	1.000	0.983	0.993	0.981	0.998	1.007
250,000	1.000	1.000	0.970	0.991	0.974	1.002	0.997
500,000	1.000	1.000	0.988	1.004	0.997	1.007	0.982
750,000	1.000	1.000	1.021	1.023	1.032	1.012	0.969
1,000,000	1.000	1.000	1.052	1.039	1.063	1.015	0.960
2,500,000	1.000	1.000	1.111	1.067	1.126	1.019	0.946
6,000,000	1.000	1.000	1.260	1.136	1.274	1.029	0.918

Layer	AY_84	AY_85	AY_86	AY_87	AY_88	AY_89
50,000	1.122	1.381	0.890	1.113	1.269	0.797
100,000	0.971	0.904	0.890	0.833	0.708	0.981
250,000	0.891	0.734	0.991	0.813	0.786	1.150
500,000	0.884	0.787	1.242	1.095	2.017	1.254
750,000	0.926	0.992	1.571	1.669	7.211	1.265
1,000,000	0.974	1.239	1.877	2.373	20.238	1.245
2,500,000	1.090	1.934	2.494	4.367	85.628	1.174
6,000,000	1.422	5.423	4.642	17.603	3808.624	0.986

REPORT.XLS
10/2/95

Developed Claim Count Distribution

Section III
Exhibit 4

Layer	AY_87	AY_88	AY_89	AY_90	AY_91
50,000	27.31%	27.39%	27.18%	33.14%	20.36%
100,000	26.39%	26.07%	23.62%	25.03%	26.96%
250,000	26.34%	26.06%	23.59%	21.36%	30.50%
500,000	11.80%	11.88%	11.86%	9.03%	12.85%
750,000	3.77%	3.86%	4.66%	3.74%	4.47%
1,000,000	1.70%	1.80%	2.66%	2.38%	2.58%
2,500,000	2.27%	2.46%	4.70%	3.80%	2.03%
6,000,000	0.40%	0.48%	1.72%	1.53%	0.26%

				Selected	Fitted
Layer	AY_92	AY_93	AY_94	AY_96	AY_96
50,000	19.23%	16.83%	37.93%	15.16%	15.27%
100,000	20.20%	21.18%	22.94%	22.46%	20.70%
250,000	22.92%	26.75%	18.39%	27.24%	28.82%
500,000	13.62%	15.17%	8.28%	14.32%	16.98%
750,000	7.60%	8.00%	4.41%	9.71%	6.94%
1,000,000	6.92%	6.86%	4.13%	5.12%	3.62%
2,500,000	6.63%	4.21%	2.79%	4.92%	6.09%
6,000,000	2.88%	0.99%	1.13%	1.07%	1.58%

REPORT.XLS
10/2/95

Fire Protection Classifications of Homeowners Insurance

by William J. VonSeggern, FCAS
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FIRE PROTECTION CLASSIFICATIONS FOR HOMEOWNERS INSURANCE

Acknowledgments

The authors would like to acknowledge the assistance of the Actuarial Administration staff at AAA Michigan. In addition, we would like to thank Ed Baum of the Auto Club of Southern California for his editorial assistance and advice and Michael Toledano for his help in the beginning stages of development. Finally, we would be remiss if we failed to thank Lisa Ivaniszyn and Taunya Burton for keeping us organized and providing clerical and editorial assistance.

FIRE PROTECTION CLASSIFICATIONS FOR HOMEOWNERS INSURANCE

Abstract

For many years the Insurance Services Office (ISO) has classified the fire protection offered by communities for all but the largest cities based upon a complex engineering study of communities' fire departments, water pressure and availability, and communications facilities. These protection classes are used in making rates for homeowners insurance and commercial property insurance. With regard to homeowners insurance, this classification system is effective in distinguishing protected from unprotected communities, and the loss experience is consistent with those results. However, among protected communities the ISO protection classes appear to be less effective at grouping communities in appropriate classes consistent with loss experience.

This paper introduces a methodology which performs the assignment of protection classes and the determination of protection class relativities in one step. This methodology uses actual homeowners experience in conjunction with engineering studies to determine protection class assignments. In using this method, a concept called "partial loss ratio" will be introduced. The partial loss ratio utilizes fire losses with the total adjusted homeowners insurance premium to derive a measure of fire loss experience. It is this experience that is used to develop protection classes and protection class relativities.

FIRE PROTECTION CLASSIFICATIONS
FOR HOMEOWNERS INSURANCE

Introduction

When calculating rates for homeowners insurance, most insurers use ISO fire protection classes to partition their experience into homogeneous groupings for analysis. Some insurers modify the classes to include areas in different protection classes based on slightly different criteria than those used by ISO. Generally these differences involve classifying parts of unprotected communities into protection classes 6 or lower based on some company-specific guidelines. Insurers then apply standard ratemaking methodology to their homeowners experience in order to determine protection class rate relativities.

This paper develops a methodology utilizing loss experience by cause of loss in the assignment of communities to protection classes and in the development of the resulting relativities. This will be done by introducing two concepts, the "partial loss ratio" and the "fire adjusted total loss ratio."

Using these concepts, the paper will then develop an enhanced methodology for better assigning medium and large communities to protection classes utilizing their own fire experience as well as ISO engineering studies. Properly categorized, this experience will be used to assign classes and determine rate relativities for those classes simultaneously.

Finally, we will discuss some potential public policy benefits of this methodology and further uses that may be possible.

Fire Protection Classes

ISO has developed a complex system to evaluate the ability of communities to protect their residents and businesses from damage caused by fire. This system is known as the Fire Suppression Rating Schedule. The system measures three factors:

1. The equipment available to and the training of the fire department in the community,
2. The availability of water of sufficient pressure to extinguish a fire, and
3. The quality and sufficiency of communications equipment.

ISO staff visit a community and its fire department to assess these three factors and assign points to the community based upon specific aspects of each. For example, a certain number of points is assigned to the number of phone lines entering into the dispatch system depending on a community's population. Points are also assigned to reflect the number and quality of fire equipment such as pumper and ladder trucks available to the community. Additional points are determined by measuring sustained water pressure and flow through the hydrant system.

Finally, ISO calculates a point total which is utilized to assign a protection class code to the community. This protection class is assigned based on a ten point scale, with 1 being the most protected community and 10 being a community with virtually no fire protection.

For a number of years there has been one major exception to this classification system for homeowners insurance. Communities with a population of more than 250,000 are known as statistically-rated communities. Statistically-rated communities are not assigned a formula protection class for homeowners insurance. Rather, it is assumed that a statistically-rated community will automatically reflect in its rates the fire protection that is available to its residents. Statistically-rated communities are often assigned unique protection class and territorial codes.

These cities have rates that reflect their own loss experience and are not necessarily subject to the fire protection class system.

It is not always apparent which communities are statistically-rated. For example, in Michigan the only city large enough to be statistically-rated is Detroit. The rate manual for virtually any homeowners insurer in Michigan would assign a protection class code of 2 to Detroit. Further review of the manual would reveal that no other city in Michigan has a protection class code of 2. If one were not aware that Detroit is a statistically-rated community, one might assume that Detroit has the best fire protection available in the state of Michigan. While it is true that Detroit has a fine fire department, it does not automatically follow that Detroiters enjoy the best fire protection in Michigan as measured by the ISO classification system, since the 2 is not derived from that system.

Some Traditional Concepts

Homeowners insurance was originally offered as a combination of several different coverages including fire, allied lines and personal liability. These coverages were priced separately or as optional endorsements. In the 1950's, insurers began marketing homeowners insurance with an indivisible premium, combining the coverages of all three of the above products and providing even broader protection. The homeowners combined product was offered at a price lower than the sum of the predecessor coverages.

This lower price was made possible by a reduction in adverse selection. Because insureds no longer had the option of rejecting allied lines, for example, insurers were not providing coverage only to insureds with a perceived need for the coverage and at greater risk of a loss. Prospective insureds with low risk of a loss now also purchased the coverage in its combined form. Further, homeowners insurance was offered with an indivisible premium making it impossible for insurers or insureds to know exactly what premium was responsible for which losses.

The Partial Loss Ratio and Fire Adjusted Total Loss Ratio

We need a mechanism for dealing with the reality that homeowners insurance is written for an indivisible premium and a methodology to determine which part of the premium is supposed to pay for fire losses, for theft losses, etc.

In order to address that problem, some new ratios will be defined. The first of these is the partial loss ratio. Let us define the fire partial loss ratio as

(1) Fire partial loss ratio (FPLR) = (fire losses)/(total premium).

Similarly, we can define partial loss ratios for theft, liability and other causes of loss as:

(2) Theft partial loss ratio (TPLR) = (theft losses)/(total premium),

(3) Liability partial loss ratio (LPLR) = (liability losses)/(total premium), and

(4) Other partial loss ratio (OPLR) = (other losses)/(total premium).

Since all of these partial loss ratios have the same denominator they can be added together, resulting in the total loss ratio as we traditionally understand it

(5) Total loss ratio (TLR) = FPLR + TPLR + LPLR + OPLR.

In this paper, we are only concerned with fire and non-fire losses. We will define the non-fire partial loss ratio (NPLR) as follows:

(6) NPLR = TPLR + LPLR + OPLR.

Equation (5) then becomes:

(7) TLR = FPLR + NPLR.

In addition, we can define these loss ratios for protection classes:

$FPLR_{PC_i}$ is the fire partial loss ratio for protection class i, for i = 1, 2,...., 10.

The premium weighted average of the total loss ratios for each of the protection classes is the statewide total loss ratio:

(8) $TLR_{S/W} = \text{Weighted Average}(TLR_{PC_1}, TLR_{PC_2}, ..., TLR_{PC_{10}})$.

Similarly, the statewide fire partial loss ratio is a weighted average of the fire partial loss ratios for each protection class:

(9) $FPLR_{S/W} = \text{Weighted Average}(FPLR_{PC_1}, FPLR_{PC_2}, ..., FPLR_{PC_{10}})$.

and the statewide partial loss ratios for non-fire perils can be shown to be the weighted average of the partial loss ratios for each protection class:

(10) $NPLR_{S/W} = \text{Weighted Average}(NPLR_{PC_1}, NPLR_{PC_2}, ..., NPLR_{PC_{10}})$.

Now we can define the Fire Adjusted Total Loss Ratio (FATLR). The FATLR is the loss ratio to be utilized in determining the rate relativities for a protection class or community by including only its own fire experience in the calculation. This is done by adjusting the loss ratio to exclude the effect of other causes of loss on the calculation.

When rate relativities are calculated they are increased or decreased until the resulting loss ratio for each classification is the same. Using this principle we need to adjust the loss ratios for each protection class so that losses from causes other than fire do not affect the calculation of the rate relativity. We do this by assuming that

(11) $NPLR_{PC_i} = NPLR_{PC_j} = NPLR_{S/W}$ for $i, j = 1,2,....,10$.

However, for fire losses we include the actual partial loss ratio for the protection class. When these are added together we have the $FATLR_{PC_i}$ for each protection class.

(12) $FATLR_{PC_i} = FPLR_{PC_i} + NPLR_{S/W}$ for $i = 1,2,....,10$.

For the statewide total loss ratio, the following equation holds:

(13) $TLR_{S/W}$ = Weighted average($FATLR_{PC_1}$,$FATLR_{PC_2}$,...,$FATLR_{PC_{10}}$).

Pure Premium Approach

We have created loss ratios with different premiums in their denominators. These premiums reflect rating factors which may not be uniform across protection class. In fact, they are not uniform, so adjustments will be required. To introduce these adjustments, we will begin with a simple pure premium example (Table 1). Consider a state with only one territory, two protection classes and no other rating factors. Pure premiums and exposures are available.

Table 1

Protection Class	Exposures	Fire Pure Premium	Non-fire Pure Premium	Total Pure Premium
1	1	F_1=50	N_1=100	T_1=150
2	1	F_2=100	N_2=100	T_2=200
Statewide	2	$F_{S/W}$ =75	$N_{S/W}$ =100	$T_{S/W}$ =175
Proportion of Statewide		43%	57%	100%

In this case, the only difference between the protection classes is the fire losses, and they are clearly doubled in protection class 2 as compared to protection class 1. We can compare the total-protection class 2 pure premium to the total statewide average and calculate a relativity of 1.143 (200/175). Algebraically, this is expressed

(14) $T_2/T_{S/W} = (F_2+N_2)/(F_{S/W}+N_{S/W})$.

In this example

(15) $N_{S/W} = N_1 = N_2$.

Substituting into Equation (14) we have

(16) $T_2/T_{S/W} = (F_2+N_{S/W})/(F_{S/W} +N_{S/W}) = F_2/(F_{S/W}+N_{S/W}) + N_{S/W}/(F_{S/W} +N_{S/W})$.

Next, let us illustrate what happens when we assume Equation (15) to be true no matter what differences in other causes of loss actually do exist. Returning to our numerical example, let us change the non-fire pure premium in each protection class (Table 2). For example, the non-fire pure premium differences could result from each protection class being located in a unique territory. We will continue to hold the proportion of a statewide pure premium due to each cause of loss constant, since a change would affect the result.

Table 2

Protection Class	Exposures	Fire Pure Premium	Non-fire Pure Premium	Total Pure Premium
1	1	F_1=50	N_1=50	T_1=100
2	1	F_2=100	N_2=150	T_2=250
Statewide	2	$F_{S/W}$ =75	$N_{S/W}$ =100	$T_{S/W}$ =175
Proportion of Statewide		43%	57%	100%

Using Equation (16) which compares protection class 2 to statewide, we again calculate a relativity of (100+100)/175 = 1.143. Clearly, this calculation does not represent the complete difference in total pure premium for protection class 2. That relativity would be 250/175=1.429. Rather, it reflects only the difference in fire results between protection classes.

We now expand the example to a two territory, two protection class situation (Table 3). The formulae still hold, but the results differ by territory.

Table 3

Protection Class	Exposures	Fire Pure Premium	Non-fire Pure Premium	Total Pure Premium
Territory A				
1	1	F_{A1}=50	N_{A1}=50	T_{A1}=100
2	1	F_{A2}=100	N_{A2}=50	T_{A2}=150
Total	2	$F_{A\bullet}$=75	$N_{A\bullet}$=50	$T_{A\bullet}$=125
Territory B				
1	1	F_{B1}=50	N_{B1}=150	T_{B1}=200
2	1	F_{B2}=100	N_{B2}=150	T_{B2}=250
Total	2	$F_{B\bullet}$=75	$N_{B\bullet}$=150	$T_{B\bullet}$=225
All territories				
1	2	$F_{\bullet 1}$=50	$N_{\bullet 1}$=100	$T_{\bullet 1}$=150
2	2	$F_{\bullet 2}$=100	$N_{\bullet 2}$=100	$T_{\bullet 2}$=200
Statewide	4	$F_{\bullet\bullet}$=75	$N_{\bullet\bullet}$=100	$T_{\bullet\bullet}$=175
Proportion of Statewide		43%	57%	100%

For example, in territory A the relativity between protection class 2 and the statewide total is (100+50)/175 = 0.857. In territory B the relativity is (100+150)/175 = 1.429. Statewide, the figure is (100+100)/175 = 1.143.

We have demonstrated that this methodology produces consistent statewide protection class relativities using pure premiums and that other rating factors (territory, in our example) affect the results. By using the statewide non-fire pure premiums to determine protection class relativities, we were able to adjust for only fire differences in protection class. The statewide protection class 2 relativity of 1.143 was constant across examples:

$[(F_{A2} + N_{\bullet\bullet}) / T_{\bullet\bullet} = (F_{B2} + N_{\bullet\bullet}) / T_{\bullet\bullet} = (100+100) / 175 = 1.143]$.

Transition to Loss Ratio

We wish to make the transition from pure premium to loss ratio for Equation (16). Again, we will restrict ourselves to two rating variables, territory and protection class. We need to review the relationship between exposures and premiums:

(17) Premiums in protection class 2 = Exposure in protection class 2∗base rate∗protection class 2 relativity∗average territorial relativity in protection class 2.

Therefore,

(18) Exposures in protection class 2 = premiums in protection class 2/(base rate∗protection class 2 relativity∗average territorial relativity in protection class 2).

Each pure premium term in Equation (16) can now be rewritten. First,

(19) F_2 = fire losses in protection class 2/exposures in protection class 2 = fire losses in protection class 2/[premiums in protection class 2/(base rate∗protection class 2 relativity∗average territorial relativity in protection class 2)].

Using $FPLR_{PC_2}$ as defined in Equation (1),

(20) $F_2 = FPLR_{PC_2}$∗base rate∗protection class 2 relativity∗average territorial relativity in protection class 2.

Now returning to Equation (16), we have

(21) $T_2/T_{S/W}$ = [$FPLR_{PC_2}$∗(base rate∗protection class 2 relativity∗average territorial relativity in protection class 2) + $NPLR_{S/W}$∗(base rate∗statewide average protection class relativity∗statewide average territorial relativity)]/[TLR∗(base rate∗statewide average protection class relativity∗statewide average territorial relativity)].

The *base rate* term cancels and we are left with

(22) $T_2/T_{S/W}$ = [$FPLR_{PC_2}$∗(protection class 2 relativity∗average territorial relativity in protection class 2) + $NPLR_{S/W}$∗(statewide average protection class relativity∗statewide average territorial relativity)]/[TLR∗(statewide average protection class relativity∗statewide average territorial relativity)].

We have shown that the use of partial loss ratios as defined above is equivalent to using adjusted pure premiums for calculating protection class relativities in our two variable example.

However, homeowners ratemaking encompasses more than our two rating variables, territory and protection class. It also considers amount of insurance, security devices, age of dwelling, age of insured, construction and a myriad of other possible classifications depending on jurisdiction. Utilizing pure premiums without adjusting for differences in each of these factors will lead to double counting and inaccurate relativities. For example, if the protection classes in Table 3 had differing underlying amounts of insurance as well as different loss experience, one would need to adjust for the differences in amount of insurance before calculating the appropriate protection class relativities. The adjustment would include dividing each protection class pure premium and the statewide pure premium by the average amount of insurance relativities in effect for those classes. Conversely, using loss ratios adjusted to a base territory and protection class will achieve the same result as using adjusted pure premium. This, of course, could be done for any and all rating factors.

Testing Protection Classes in Michigan

One of the premises of this paper is that the ISO Fire Suppression Rating Schedule is effective at separating protected from unprotected communities, but less effective at predicting the loss results in protected communities. Using both the traditional methodologies and the one just developed we can test that presumption, utilizing Michigan homeowners data.

We can divide the ISO protection classes into four categories:

1. Statistically-rated community - This is coded as protection class 2 and represents the city of Detroit. Since Detroit has a population of more than 250,000 it is not evaluated by ISO and is statistically-rated.
2. "Protected" communities - Protection classes 3-6 are included in this group which consists of all of the larger communities and most of the metropolitan areas in the state.
3. "Less protected" communities - These communities are assigned protection classes 7, 8 and 9 and include a number of developing suburban communities.
4. "Unprotected" communities - This is protection class 10 and includes many rural areas.

Exhibit 1 shows the TLR's, FPLR's, and FATLR's by protection class. These have been adjusted to a common territorial and protection class level as described above. The loss ratios reveal differences among the four categories cited above. They also distinguish among the less protected and unprotected classes. However, these loss ratios are not as effective in differentiating among the protected classes (3-6), where higher protection classes do not translate to higher loss ratios.

Assignment of Fire Protection Classes and Relativities by Community

We have argued that the ISO methodology is less effective at distinguishing among protected communities. Using the previously defined FATLR's, we will develop a credibility enhanced procedure to assign rate relativities for protected communities. This methodology will highlight distinctions among protected communities, thus measuring the efficacy of a community's fire protection using proprietary loss data.

Let us take the partial loss ratios defined earlier for protection classes and redefine them for individual communities:

$FPLR_{C_i}$ is the fire partial loss ratio for community i

and similarly for $NPLR_{C_i}$.

Equation (23) follows from Equation (12):

(23) $FATLR_{C_i} = FPLR_{C_i} + NPLR_{S/W}$ for i = community 1, 2,....

We can use this definition for all of the communities in the state. As with the protection classes, these loss ratios will average to the respective statewide loss ratios. In addition, all of the loss ratios for communities within a protection class will also average to the protection class loss ratios. However, for credibility reasons not all communities will be analyzed individually.

For each selected community, we need to calculate the FATLR. Premiums and losses by cause of loss are gathered for each community. Adjustments are completed as in Equation (22). The indicated relativity for each community is calculated as

(24) $Rel_{C_i} = [FATLR_{C_i} * Z + FATLR_{S/W} * (1-Z)] / FATLR_{S/W}$, where Z is the credibility of community i's experience.

Establishing a community's credibility is a significant issue in this methodology. We have found through practical experience that three rules have created relatively stable and sound results; however, other alternatives may be appropriate. The criteria we have used are as follows:

1. Only communities of 30,000 residents or more are included in the procedure;
2. As many years of data as are available (up to 10 years) are used;
3. The square root rule is used for partial credibility with a full credibility standard of 683 fire claims.

While these standards have been developed without theoretical study, especially with regard to the variation in claim severity, it does appear that these standards provide for reasonable results.

In addition, different complements of credibility might be appropriate. For example, Equation (24) might be amended to use a fire adjusted loss ratio for a whole protection class rather than $FATLR_{S/W}$. Or, a three-way credibility technique could be developed using the community, protection class and statewide FATLR's.

Returning to our analysis, we can rank each community and select protection class relativities by grouping the communities based on the indicated relativities from Equation (24). Exhibit 2 presents a sample analysis using fifteen communities in Michigan. On page 1, $FATLR_{C_i}$ was calculated and the indicated relativity was determined. On page 2, the communities were ranked and protection classes were assigned. In this example, protection classes 3-6 were divided into seven groups with relativities ranging from 0.90 to 1.10.

<u>Other Relevant Issues</u>

There is public policy value to this analysis, as well. With the ISO Fire Suppression Rating Schedule, communities are encouraged to engage in certain specific fire protection activities in order to achieve a lower fire protection class. However, this lower fire protection class may not yield better loss experience or lower rates. By performing this internal data analysis, insurance companies can report to communities and insureds on the benefit of the fire protection offered. Communities are then free to respond as they deem appropriate. For example, one community may invest in additional fire trucks. Another community may decide that better fire protection could be achieved by rehabilitating communities and developing stable neighborhoods rather than by hiring more firefighters and buying more trucks. Still another community may realize that brush fires or other unusual hazards are affecting them and provide unique or different solutions for their residents.

In addition, insurers may be able to adapt this methodology to other ratemaking classification analyses to produce more accurate rates and more understandable rating plans. For example, insurers could develop theft protection classes or water seepage districts by community. This would provide insurers with the data to support the differences in rates that many insureds and consumer groups regularly challenge. Public officials would also receive valuable information to guide improvements that would benefit the residents of their communities.

Conclusion

ISO fire protection classes which are utilized, at least as a starting point by most insurers, effectively distinguish between protected and unprotected communities. However, they are not sufficiently refined to provide accurate and appropriate distinctions among protected communities. This paper has presented a method for distinguishing among these protected communities using their fire experience. The new methodology also provides some public policy benefits. Finally, we have offered an additional tool for classification ratemaking.

Protection Class Data

Protection Class	Category	Total Loss Ratio	Fire Partial Loss Ratio	Fire Adjusted Total Loss Ratio
2	Statistically-rated	132.9%	54.5%	99.5%
3	Protected	88.1%	38.0%	83.1%
4	Protected	67.1%	24.4%	69.4%
5	Protected	55.0%	21.3%	66.3%
6	Protected	67.7%	27.9%	73.0%
7	Less Protected	68.2%	31.5%	76.5%
8	Less Protected	84.0%	41.8%	86.9%
9	Less Protected	106.1%	57.7%	102.7%
10	Unprotected	110.4%	65.4%	110.5%
Statewide		79.1%	34.1%	79.1%

All premiums are adjusted to a common territory and protection class. Loss ratios are adjusted to protect the proprietary nature of the data.

Protection Class by Community

City	ISO Protection Class	Fire Adjusted Total Loss Ratio	Credibility	Indicated Relativity
Community 1	3	65.6%	68.7%	1.043
Community 2	3	60.4%	58.9%	0.986
Community 3	4	48.5%	26.0%	0.944
Community 4	4	55.4%	47.9%	0.950
Community 5	4	66.2%	33.8%	1.024
Community 6	4	54.4%	64.0%	0.924
Community 7	4	51.2%	72.8%	0.875
Community 8	4	57.0%	100.0%	0.922
Community 9	5	64.6%	49.6%	1.023
Community 10	5	79.2%	61.7%	1.173
Community 11	5	60.1%	59.0%	0.983
Community 12	5	57.2%	82.2%	0.939
Community 13	5	65.3%	63.7%	1.037
Community 14	6	81.6%	43.3%	1.139
Community 15	6	49.9%	58.3%	0.887
Statewide		61.8%	100.0%	

All premiums are adjusted to a common territory and protection class. Loss ratios are adjusted to protect the proprietary nature of the data.

Protection Class by Community

City	**ISO Protection Class**	**Indicated Relativity**	**Possible Selection**
Community 7	4	0.875	0.900
Community 15	6	0.887	0.900
Community 8	4	0.922	0.925
Community 6	4	0.924	0.925
Community 12	5	0.939	0.925
Community 3	4	0.944	0.950
Community 4	4	0.950	0.950
Community 11	5	0.983	0.975
Community 2	3	0.986	0.975
Community 9	5	1.023	1.025
Community 5	4	1.024	1.025
Community 13	5	1.037	1.025
Community 1	3	1.043	1.050
Community 14	6	1.139	1.100
Community 10	5	1.173	1.100

All premiums are adjusted to a common territory and protection class. Loss ratios are adjusted to protect the proprietary nature of the data.

Death, Disability, and Retirement Coverage: Pricing the "Free" Claims-Made Tail

by Christopher P. Walker, FCAS, and
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Abstract

Death, Disability, and Retirement Coverage: Pricing the "Free" Claims-Made Tail

Introduced primarily as a marketing tool, free tail coverage is becoming a standard feature of claims-made insurance policies and is increasingly being used for medical malpractice and other forms of professional liability exposures. In addition, as of December 31, 1993 the NAIC is requiring that reserves be established to recognize this exposure, further elevating the need for proper pricing and reserving.

When free tail coverage is extended due to the death, disability, or retirement of the insured, it is commonly referred to as "DD&R" coverage.

This paper presents three models for determining the cost of DD&R as a function of mature claims-made coverage. Level funding, or constant premium loads, are calculated that take into account mortality, disability, retirement, and lapse profiles of the insured population.

We also examine model assumptions and the implications on unearned premium reserves attributable to the presented pricing models.

Death, Disability, and Retirement Coverage: Pricing the "Free" Claims-Made Tail

By Chris Walker and Don Skrodenis

I. Introduction and Background

Claims-made is currently the most widely used policy form for professional liability insurance, particularly in the area of medical malpractice. A feature of claims-made coverage is that extended reporting endorsements are generally available to insureds who wish to cancel coverage. An extended reporting endorsement is a separate insurance policy that will indemnify the insured for claims with injury dates subsequent to the insured's retroactive date but reported after the insured's final claims-made policy has lapsed. Extended reporting endorsements are also referred to as "tail coverage."

Free tail coverage, or issuing an extended reporting endorsement at no charge, has become a standard feature of many claims-made insurance programs. It is most often extended to insureds meeting certain conditions. Some common conditions include the following:

- Death of the insured;

- Disability of the insured making further participation in professional activities impossible;

- Retirement from practice after attaining a certain age and/or years insured with the same insurance company.

These conditions and the free tail coverage associated with them compose what is commonly referred to as Death, Disability, and Retirement coverage, or "DD&R."

As of December 31, 1993, the NAIC has required that companies establish a reserve for unissued extended reporting endorsements associated with DD&R coverage. The NAIC recommends an incremental premium charge be applied to insureds in order to fund the DD&R reserve. A copy of the applicable section of the NAIC's 1994 *Accounting Practices and Procedures Manual for Property and Casualty Insurance Companies* (provided through the Casualty Actuarial Task Force) is attached as Exhibit 4.

This paper will examine three separate models for the calculation of a DD&R premium load and the corresponding unearned premium amounts that result from the methods. We will also discuss how well each method conforms with the general guidance given by the NAIC. Finally, some ideas for handling changing assumptions and model inputs will be explored.

II. Perspectives on DD&R

As stated above, there are various coverage triggers for the issuance of DD&R. Each of these coverage triggers have implications on the cost of DD&R to the insurer. For example, if all insureds non-renew prior to the event of a DD&R trigger, the cost of DD&R to an insurer is zero. On the other hand, if all policyholders are able to take advantage of the DD&R coverage, then its cost is essentially the difference between claims-made coverage and occurrence coverage.

As such, there are elements that enter the DD&R ratemaking process that are not generally used for other property/casualty insurance coverages. These elements include:

-Policyholder lapse ratios

-Disability rates

-Mortality rates

-Retirement rates

For this discussion, we have made certain assumptions concerning each of these for our model inputs.

We have also made assumptions on other inputs such as loss trend, discount rates, and most importantly, the relationship between the cost of tail coverage and mature claims-made. Each of these must be examined prior to the determination of the DD&R premium load.

In discussions by the NAIC Casualty Actuarial Task Force that investigated the DD&R issue and its accounting treatment, two main viewpoints were apparent:

(1) Insurance companies do not have liability for DD&R until the policy is issued. We refer to this approach as "pay as you go," and relate it to the loss reserving principles of the property/casualty lines of insurance.

(2) Though not contractually, insurers are making a promise to policyholders to provide DD&R. As such, the expected liability of the promise must be recognized, preferably as an unearned premium reserve. This concept implicitly considers a much longer timing horizon than the first approach.

The NAIC currently requires that the second viewpoint must be used for Annual Statement reporting purposes. In light of this NAIC requirement and in recognition of generally accepted accounting principles, it is clear that the second viewpoint should be adopted by insurers. However, the position of the NAIC is interesting due to the fact that, in the case of liquidation, the first view may dictate which insureds are entitled to coverage

In conjunction with (2) above, the NAIC also recommends a level-funding premium load; that is, a load that will not vary due to age shifts in the insured population or to changes in the insurer's business plan.

In this paper, in order to provide concrete examples of the implications of each, we have included one pricing model for viewpoint (1), and two models for viewpoint (2).

For simplicity, we have assumed in each model that

- The cost of tail coverage, as a multiple of mature claims-made, does not vary by age or risk classification of individual insureds;
- All current insureds purchase mature claims-made coverage;
- The insurer has been offering DD&R coverage for many years; and,
- Policyholders are eligible for the coverage with no tenure ("years insured") restriction.

III. Model 1 - "Pay as You Go" Funding (Exhibits 1A and 1B)

This model is based on the concept that an insurer needs only to charge its insureds for DD&R coverage issued during the policy year.

The basics of the model are as follows:

(1) Given a group of insureds at the inception of a policy year, estimate the number of those insureds that will be issued DD&R policies for free during the course of the policy year. This exercise is performed by age using disability, mortality, and retirement assumptions. Given assumptions on the cost of extended reporting endorsements (as a percentage of mature claims-made), a total cost of the DD&R can be estimated for that policy year.

(2) Estimate the total premium to be collected net of DD&R.

(3) The ratio of (1) to (2) tells the insurer the average load to apply to its non-DD&R premium.

In this model, we have assumed that the number of policies that are non-renewed or canceled are offset by new policies written.

The following observations may be made concerning this model:

A. Comparison of Exhibits 1A and 1B shows premium loadings that are clearly dependent on the age of the population. As such, if the population age increases, the costs for DD&R could increase dramatically. In forming their guidelines, the NAIC clearly wished to avoid such a situation.

B. Using this model, the theoretical unearned premium reserve at the end of the policy year would be zero. This is due to the fact that all DD&R that had been charged for would be issued by the end of the policy period and all costs would be reflected as incurred losses or loss reserves.

C. Column (11) in each exhibit shows the costs and loadings attributable to each insured age group. Assuming that the actual charge to each insured is the group overall average, it is apparent that the younger insureds pay significantly more than indicated, with no assurance that they will receive DD&R coverage from the insurer in future policy periods.

D. This model is computationally simple. As assumptions change from year to year, it is a straight-forward exercise to update the DD&R premium loading.

IV. Model 2 - Level Premium Funding by Entry Age of Insured (Exhibits 2A and 2B)

Rather than funding each policy year's DD&R costs only, this model estimates total expected ultimate DD&R costs, by each insured's "entry age." Here, "entry age" is defined as the age of the insured when he first purchased claims-made coverage from the insurer. A flat DD&R loading, as a percentage of non-DD&R premium, is then computed for each insured. This charge is designed to stay constant during the insured's tenure with the insurer.

The entry age of each insured is important for this model. For example, for two insureds of the same absolute age but different entry ages, the insured with the earlier entry age should receive a lower DD&R charge because he has had more years to contribute for the same DD&R benefit.

The basics steps of the model are as follows:

(1) Estimate the expected costs associated with DD&R coverage over the expected lifetime of insurance coverage, by entry age. Exhibit 2A shows a calculation for entry age 42, and Exhibit 2B shows entry age 55.

For example, in Exhibit 2A the "pool" of insureds at entry age 42 starts out at 100,000 people. By the end of the first year of tenure, it is expected that 621 insureds will utilize DD&R coverage while 9,938 insureds will non-renew (further shrinking the eligible population) and will not be eligible for the coverage after their first policy year. Over the course of the next year, 606 insureds utilize DD&R, and 8,883 people non-renew for the next year.

Thus, over time your population shrinks and the actual number that may utilize DD&R is much smaller than your initial population would imply.

(2) Discount the expected future costs of DD&R for each entry age. This is shown as the total of Column (16) in Exhibits 2A and 2B.

(3) Estimate the discounted value of expected non-DD&R premium for each entry age. This is shown as the total of Column (14) in Exhibits 2A and 2B.

(4) The ratio of (2) to (3) above is the flat premium load that should be applied to the entry age in question for the insured's lifetime in order to pay all expected DD&R costs by the time the last policyholder has lapsed, died, become disabled, or retired.

Some observations on this process are warranted:

A. For each insured, the loading calculated is invariant to changes in the age makeup of the population. However, the resulting aggregate population load will change as the population changes. As such it does not follow the exact recommendations of the NAIC, though the problems exhibited in Model 1 are not a concern because there will be no "surprises" for insureds that are members of an aging population. The point here is that the charge to insureds will not change and the entire insurance program (rates and unearned premium reserves) will be in balance.

B. As long as model assumptions do not change, the loadings applied to insureds of the same entry age should always be the same, regardless of the absolute ages of the insureds. For example, if two insureds both begin coverage with the insurer at age 30 but one began in 1996 and the other began in 1999, the DD&R loadings for each should be identical.

C. The estimated unearned premium reserve at the end of each policy year is presented in column (20). As one can see, the reserve increases considerably as the years progress and becomes depleted by the time each insured has lapsed, died, become disabled, or has retired.

At any point in time, the unearned premium reserve is equal to the difference between the discounted expected future DD&R losses and the discounted expected future DD&R premium. As long as model assumptions are followed, this reserve will be identical to accumulating the unused DD&R premium along with its associated interest income.

D. The unearned premium per insured is presented in column (21). These factors could be used as inputs for an insurance carrier to estimate its unearned premium reserve based on its own rates and distribution of insureds.

E. This method is very sensitive to more assumptions than Model 1. In particular, the long term affects of trend, discount rate, and lapse ratio add considerable variation to the loading generated.

V. Model 3 - Level Premium Charge the Same for all Insureds (Exhibit 3)

The final model is a by-product of Model 2 and represents the true spirit of the recommendations made by the NAIC. This model estimates the per-insured charge by entry date, calculates the overall DD&R charge for the pool of insureds, and applies that rate to all insureds regardless of current age or entry age. We have assumed that new insureds over the course of a year will not be older than the current insureds.

Exhibit 3 shows the differences between the charges calculated using the average charge and those calculated using Model 2; we have constructed an example using only two different insured ages. All insureds in each age are assumed to have the same entry date.

Some observations on Model 3:

A. It is clear that those insureds who become insured early will pay more than those who become insured later in life. Thus, there is essentially a subsidy from the former group to the latter. This subsidy becomes less as the insured population's entry ages converge.

B. There is no difference in the aggregate DD&R premium collected in Model 3 than that implied in Model 2. Thus, the unearned premium reserve will not change either.

VI. Remarks on Assumptions

Throughout this paper we have made several assumptions that were applied to the models that simplify the analysis. We would like to comment on those assumptions.

- The cost of tail coverage, as a multiple of mature claims-made, does not vary by age or risk classification of individual insureds. This assumption follows common professional liability rating practices. Until such practices change in the marketplace, we feel this is a reasonable assumption for this analysis.

- All insureds purchase mature claims-made coverage. This premise is clearly not applicable to the actual insurance markets, with new-in-practice insureds often purchasing first-year claims-made coverage at significant discounts from the mature rate.

  Incorporating non-mature claims-made insureds into the analysis would be a relatively straight-forward process. For each entry age, the premium collected and the DD&R losses incurred during the insured's first several years would be different than those presented here. Of course, some insured's will have purchased prior acts coverage, eliminating the need for a non-mature adjustment.

- The insurer has been offering DD&R coverage for many years. Use of this assumption serves to eliminate the need for "catch-up" reserves when an insurer offers DD&R for the first time. Clearly, if an insurer "grandfathers" existing insureds into the coverage, an immediate liability exists.

In this situation, we see the insurer as basically having two choices. First, all insureds can be treated as new policyholders and an appropriate flat premium loading would be charged; i.e., entry age is equal to current age. This would serve to drive up the premiums of older insureds significantly based upon our Model 2 assumptions; using the Model 3 technique would put a smaller burden on older insureds.

Second, the insurer may chose a smaller flat loading recognizing the entry age of each insured. It would then be necessary to establish a beginning unearned premium reserve that would recognize the funding shortfall inherent in the premium load.

Policyholders are eligible for DD&R coverage with no tenure restrictions. Most programs do have "years insured" restrictions, or have discounts applied to the tail coverage cost for insureds with limited tenure. Clearly, the insurer's liability is decreased as eligibility restrictions are applied.

As a model adjustment, costs associated with DD&R could be eliminated from the model for those insured years where restrictions are applied. The computed DD&R loading would be thus be reduced.

VII. Sources of Data

We have used data sources in this analysis that need some identification. In particular, we will discuss possible sources for the mortality, disability, lapse and retirement assumptions.

Mortality. This paper used as its mortality source the 1979-81 U.S. Mortality Table, males only. This table is based on census information made available through the U.S. Department of Health and Human Services. Many other tables are available from the Society of Actuaries and insurance industry groups that may pool individual company data. These tables may reflect more closely the particular make-up of an insurer's insured population.

Disability. We have used the Commissioners' 1985 Individual Disability Table A. The rates used reflect male-only accident and sickness disabilities with a 90-day elimination period. This period was chosen assuming that an insured who recovered within this period would not be eligible for DD&R coverage. The table above is available through the 1985 *Transactions* of the Society of Actuaries.

Lapse Ratios. Insurers should review their book of business to review policyholder lapse ratios. The lapse ratios assumed in this model were hypothetical. Insurers should take a prospective view on lapse ratios: for example, future rate activity may effect persistency levels.

Retirement Rates. Insurers should review their book of business to review policyholder retirement rates. As stated above concerning lapse ratios, such inputs to the DD&R model should be prospective in nature. The retirement rates assumed in our model were hypothetical.

VIII. Additional Observations

In closing, we would like to make a few additional observations.

A. It may be very tedious to estimate flat loadings for each entry age grouping, particularly if distinctions are made between sex, the existence/non-existence of prior acts coverage, etc. Therefore, an insurer may wish to use entry age groupings. For example, insureds with entry ages between 45 and 50 could be "banded."

However, as long as assumptions in the model do not change, each entry age loading would have to be calculated only once.

B. Insurers may believe that insureds that retire may be superior risks or may have scaled back their activities and risk in the last few years before retirement. As such, some experience rating credit may be warranted for insureds that survive to retirement. In theory, such a credit would not be applicable to insureds that utilize DD&R due to death or disability.

C. When model assumptions change, updating of the models is required. Suppose that the cost of tail coverage should have been 3.00 instead of the 2.00 as used in the models. In this case, continuing to fund at the lower tail coverage ratio will prove to be inadequate. One solution to this problem is the following. First, premium loadings should be increased on a go-forward basis to indicated levels. Second, the unearned premium reserve should be in increased in order to make up for the shortfall in the current unearned premium reserve.

D. The unearned premium reserve estimated in Model 2 is not typical in the property/casualty sense due to the fact that the accrued investment income necessary to fund the DD&R is included in the reserve. This fact may pose some interesting tax situations for insurers.

DD&R Model 1
"Pay as You Go" Funding
Assumptions and Column Keys for Exhibits 1A and 1B

Assumptions:

- Premium is collected in the middle of the policy year.
- The average loss date for DD&R is the middle of the policy year.
- The number of lapses = the number of new policies.
- Mature claims-made rate is normalized to equal $1.00.
- All insureds lapse or use DD&R by age 75.

Column Key:

(1) is the insured's age.

(2) is the number of insureds of that age.

(3) is the percentage of the insureds that are that age.
(3) = (2) / total (2)

(4) is the assumed disability rate.

(5) is the assumed mortality rate.

(6) is the assumed retirement rate.

(7) is the resulting expected number of DD&R utilized during the year.
(7) = (2) x [1 - (4)] x [1 - (5)] x [1 - (6)]

(8) is the assumed cost of tail coverage in relation to the cost of mature claims-made.

(9) is the non-DD&R premium collected for the policy year.
(9) = (2) x $1.00

(10) is the expected DD&R dollars utilized during the year.
(10) = (7) x (8)

(11) is the ratio of expected DD&R dollars to non-DD&R premium.
(11) = (10)/(9)

The boxed entry at the bottom of (11) is the resulting DD&R charge that would be applied to all insureds regardless of current age. It is the ratio of total column (10) to total column (9).

DD&R Model 1

"Pay as you Go" Funding

Exhibit 1 - A: Average age = 51

Age (1)	Distribution of Insureds (2)	% of Insureds (3)	Disability Rate (4)	Mortality Rate (5)	Retire Rate (6)	Expected # of DD&R Utilized (7)	Tail/Mature Cost (8)	Premium Collected (9)	DD&R Utilized (10)	Ratio DD&R to Prem. (11)
27	1000	0.16%	0.166%	0.193%	0.000%	3.59	2.00	$1,000	$7.17	0.72%
28	2000	0.32%	0.159%	0.191%	0.000%	6.99	2.00	$2,000	$13.99	0.70%
29	3000	0.48%	0.155%	0.191%	0.000%	10.37	2.00	$3,000	$20.74	0.69%
30	4000	0.64%	0.152%	0.191%	0.000%	13.71	2.00	$4,000	$27.42	0.69%
31	5000	0.80%	0.151%	0.191%	0.000%	17.09	2.00	$5,000	$34.17	0.68%
32	6000	0.96%	0.151%	0.193%	0.000%	20.62	2.00	$6,000	$41.25	0.69%
33	7000	1.12%	0.154%	0.198%	0.000%	24.62	2.00	$7,000	$49.24	0.70%
34	8000	1.28%	0.158%	0.205%	0.000%	29.01	2.00	$8,000	$58.03	0.73%
35	9000	1.44%	0.164%	0.216%	0.000%	34.17	2.00	$9,000	$68.34	0.76%
36	10000	1.60%	0.172%	0.229%	0.000%	40.06	2.00	$10,000	$80.12	0.80%
37	11000	1.76%	0.181%	0.244%	0.000%	46.70	2.00	$11,000	$93.40	0.85%
38	12000	1.92%	0.192%	0.261%	0.000%	54.30	2.00	$12,000	$108.60	0.90%
39	13000	2.08%	0.206%	0.280%	0.000%	63.11	2.00	$13,000	$126.21	0.97%
40	14000	2.24%	0.221%	0.303%	0.000%	73.27	2.00	$14,000	$146.53	1.05%
41	15000	2.40%	0.239%	0.332%	0.000%	85.53	2.00	$15,000	$171.06	1.14%
42	16000	2.56%	0.259%	0.363%	0.000%	99.37	2.00	$16,000	$198.74	1.24%
43	17000	2.72%	0.281%	0.398%	0.000%	115.24	2.00	$17,000	$230.48	1.36%
44	18000	2.88%	0.307%	0.435%	0.000%	133.32	2.00	$18,000	$266.64	1.48%
45	19000	3.04%	0.335%	0.476%	0.010%	155.67	2.00	$19,000	$311.34	1.64%
46	20000	3.20%	0.367%	0.522%	0.020%	181.38	2.00	$20,000	$362.76	1.81%
47	21000	3.36%	0.402%	0.576%	0.040%	213.21	2.00	$21,000	$426.42	2.03%
48	22000	3.52%	0.441%	0.638%	0.060%	249.82	2.00	$22,000	$499.64	2.27%
49	23000	3.68%	0.485%	0.705%	0.080%	291.10	2.00	$23,000	$582.19	2.53%
50	24000	3.84%	0.533%	0.775%	1.000%	549.80	2.00	$24,000	$1,099.60	4.58%
51	25000	4.00%	0.586%	0.846%	1.000%	603.19	2.00	$25,000	$1,206.39	4.83%
52	24000	3.84%	0.645%	0.942%	1.000%	615.63	2.00	$24,000	$1,231.26	5.13%
53	23000	3.68%	0.710%	1.010%	1.000%	620.01	2.00	$23,000	$1,240.02	5.39%
54	22000	3.52%	0.780%	1.105%	1.000%	628.68	2.00	$22,000	$1,257.35	5.72%
55	21000	3.36%	0.858%	1.206%	2.500%	945.49	2.00	$21,000	$1,890.97	9.00%
56	20000	3.20%	0.943%	1.310%	2.500%	936.93	2.00	$20,000	$1,873.85	9.37%
57	19000	3.04%	1.036%	1.423%	5.000%	1391.19	2.00	$19,000	$2,782.38	14.64%
58	18000	2.88%	1.137%	1.549%	5.000%	1356.29	2.00	$18,000	$2,712.59	15.07%
59	17000	2.72%	1.247%	1.690%	5.000%	1320.92	2.00	$17,000	$2,641.84	15.54%
60	16000	2.56%	1.367%	1.846%	10.000%	2059.04	2.00	$16,000	$4,118.08	25.74%
61	15000	2.40%	1.497%	2.016%	10.000%	1970.18	2.00	$15,000	$3,940.36	26.27%
62	14000	2.24%	1.638%	2.201%	10.000%	1879.17	2.00	$14,000	$3,758.34	26.85%
63	13000	2.08%	1.779%	2.398%	10.000%	1783.72	2.00	$13,000	$3,567.44	27.44%
64	12000	1.92%	1.920%	2.604%	10.000%	1683.19	2.00	$12,000	$3,366.38	28.05%
65	11000	1.76%	2.061%	2.817%	20.000%	2624.15	2.00	$11,000	$5,248.31	47.71%
66	10000	1.60%	2.202%	3.044%	20.000%	2414.32	2.00	$10,000	$4,828.64	48.29%
67	9000	1.44%	2.343%	3.289%	20.000%	2199.96	2.00	$9,000	$4,399.91	48.89%
68	8000	1.28%	2.484%	3.563%	20.000%	1981.34	2.00	$8,000	$3,962.69	49.53%
69	7000	1.12%	2.624%	3.868%	20.000%	1757.87	2.00	$7,000	$3,515.74	50.22%
70	6000	0.96%	2.765%	4.207%	20.000%	1529.07	2.00	$6,000	$3,058.14	50.97%
71	5000	0.80%	2.906%	4.571%	20.000%	1293.77	2.00	$5,000	$2,587.53	51.75%
72	4000	0.64%	3.047%	4.951%	20.000%	1051.11	2.00	$4,000	$2,102.22	52.56%
73	3000	0.48%	3.188%	5.338%	20.000%	800.54	2.00	$3,000	$1,601.08	53.37%
74	2000	0.32%	3.329%	5.736%	20.000%	541.98	2.00	$2,000	$1,083.97	54.20%
75	1000	0.16%	3.470%	6.167%	100.000%	1000.00	2.00	$1,000	$2,000.00	200.00%
Total	625,000					37,500		$625,000	$75,000	12.00%

DD&R Model 1
"Pay as you Go" Funding
Exhibit 1 - B: Average age = 44

Age (1)	Distribution of Insureds (2)	% of Insureds (3)	Disability Rate (4)	Mortality Rate (5)	Retire Rate (6)	Expected # of DD&R Utilized (7)	Tail/Mature Cost (8)	Premium Collected (9)	DD&R Utilized (10)	Ratio DD&R to Prem. (11)
27	5000	0.72%	0.166%	0.193%	0.000%	17.93	2.00	$5,000	$35.87	0.72%
28	7000	1.01%	0.159%	0.191%	0.000%	24.48	2.00	$7,000	$48.96	0.70%
29	9000	1.30%	0.155%	0.191%	0.000%	31.11	2.00	$9,000	$62.23	0.69%
30	12000	1.74%	0.152%	0.191%	0.000%	41.13	2.00	$12,000	$82.25	0.69%
31	14000	2.03%	0.151%	0.191%	0.000%	47.84	2.00	$14,000	$95.68	0.68%
32	16000	2.32%	0.151%	0.193%	0.000%	54.99	2.00	$16,000	$109.99	0.69%
33	18000	2.60%	0.154%	0.198%	0.000%	63.31	2.00	$18,000	$126.61	0.70%
34	20000	2.89%	0.158%	0.205%	0.000%	72.54	2.00	$20,000	$145.07	0.73%
35	22000	3.18%	0.164%	0.216%	0.000%	83.52	2.00	$22,000	$167.04	0.76%
36	24000	3.47%	0.172%	0.229%	0.000%	96.15	2.00	$24,000	$192.29	0.80%
37	26000	3.76%	0.181%	0.244%	0.000%	110.39	2.00	$26,000	$220.77	0.85%
38	28000	4.05%	0.192%	0.261%	0.000%	126.70	2.00	$28,000	$253.40	0.90%
39	30000	4.34%	0.206%	0.280%	0.000%	145.63	2.00	$30,000	$291.25	0.97%
40	32000	4.63%	0.221%	0.303%	0.000%	167.47	2.00	$32,000	$334.93	1.05%
41	34000	4.92%	0.239%	0.332%	0.000%	193.87	2.00	$34,000	$387.74	1.14%
42	36000	5.21%	0.259%	0.363%	0.000%	223.58	2.00	$36,000	$447.16	1.24%
43	40000	5.79%	0.281%	0.398%	0.000%	271.15	2.00	$40,000	$542.31	1.36%
44	35000	5.07%	0.307%	0.435%	0.000%	259.23	2.00	$35,000	$518.47	1.48%
45	30000	4.34%	0.335%	0.476%	0.010%	245.80	2.00	$30,000	$491.59	1.64%
46	25000	3.62%	0.367%	0.522%	0.020%	226.73	2.00	$25,000	$453.45	1.81%
47	20000	2.89%	0.402%	0.576%	0.040%	203.06	2.00	$20,000	$406.12	2.03%
48	19000	2.75%	0.441%	0.638%	0.060%	215.75	2.00	$19,000	$431.51	2.27%
49	18000	2.60%	0.485%	0.705%	0.080%	227.81	2.00	$18,000	$455.63	2.53%
50	17000	2.46%	0.533%	0.775%	1.000%	389.44	2.00	$17,000	$778.88	4.58%
51	16000	2.32%	0.586%	0.846%	1.000%	386.04	2.00	$16,000	$772.09	4.83%
52	15000	2.17%	0.645%	0.942%	1.000%	384.77	2.00	$15,000	$769.53	5.13%
53	14000	2.03%	0.710%	1.010%	1.000%	377.40	2.00	$14,000	$754.80	5.39%
54	13000	1.88%	0.780%	1.105%	1.000%	371.49	2.00	$13,000	$742.98	5.72%
55	12000	1.74%	0.858%	1.206%	2.500%	540.28	2.00	$12,000	$1,080.55	9.00%
56	8000	1.16%	0.943%	1.310%	2.500%	374.77	2.00	$8,000	$749.54	9.37%
57	8000	1.16%	1.036%	1.423%	5.000%	585.76	2.00	$8,000	$1,171.53	14.64%
58	8000	1.16%	1.137%	1.549%	5.000%	602.80	2.00	$8,000	$1,205.59	15.07%
59	8000	1.16%	1.247%	1.690%	5.000%	621.61	2.00	$8,000	$1,243.22	15.54%
60	6000	0.87%	1.367%	1.846%	10.000%	772.14	2.00	$6,000	$1,544.28	25.74%
61	6000	0.87%	1.497%	2.016%	10.000%	788.07	2.00	$6,000	$1,576.14	26.27%
62	6000	0.87%	1.638%	2.201%	10.000%	805.36	2.00	$6,000	$1,610.72	26.85%
63	6000	0.87%	1.779%	2.398%	10.000%	823.25	2.00	$6,000	$1,646.51	27.44%
64	6000	0.87%	1.920%	2.604%	10.000%	841.60	2.00	$6,000	$1,683.19	28.05%
65	5000	0.72%	2.061%	2.817%	20.000%	1192.80	2.00	$5,000	$2,385.60	47.71%
66	5000	0.72%	2.202%	3.044%	20.000%	1207.16	2.00	$5,000	$2,414.32	48.29%
67	4000	0.58%	2.343%	3.289%	20.000%	977.76	2.00	$4,000	$1,955.52	48.89%
68	1000	0.14%	2.484%	3.563%	20.000%	247.67	2.00	$1,000	$495.34	49.53%
69	1000	0.14%	2.624%	3.868%	20.000%	251.12	2.00	$1,000	$502.25	50.22%
70	1000	0.14%	2.765%	4.207%	20.000%	254.85	2.00	$1,000	$509.69	50.97%
71	1000	0.14%	2.906%	4.571%	20.000%	258.75	2.00	$1,000	$517.51	51.75%
72	1000	0.14%	3.047%	4.951%	20.000%	262.78	2.00	$1,000	$525.55	52.56%
73	1000	0.14%	3.188%	5.338%	20.000%	266.85	2.00	$1,000	$533.69	53.37%
74	1000	0.14%	3.329%	5.736%	20.000%	270.99	2.00	$1,000	$541.98	54.20%
75	1000	0.14%	3.470%	6.167%	100.000%	1000.00	2.00	$1,000	$2,000.00	200.00%
Total	691,000					18,006		$691,000	$36,011	5.21%

DD&R MODEL 2

Level Premium Funding by Entry Age of Insured
Assumptions and Column Keys for Exhibits 2A and 2B, Sheets 1 and 2

Assumptions:

- Premium is collected in the middle of the policy year.
- The average loss date for DD&R is the middle of the policy year.
- Lapses occur at the beginning of the policy period.
- Premium increases every year by the yearly loss trend.
- Mature claims-made rate at year of entry is normalized to equal $1.00.
- All insureds lapse or use DD&R by age 75.

Column Key:

(1) is the insured's age at the entry year and subsequent years.

(2) is the number of years the insured has been insured with company (tenure).

(3) is the "normalized" expected number of insureds at the beginning of the policy year.
(3) = prior (3) - prior (7) - prior (9)

(4) is the assumed disability rate.

(5) is the assumed mortality rate.

(6) is the assumed retirement rate.

(7) is the resulting expected number of DD&R utilized during the year.
(7) = (3) x [1 - (4)] x [1 - (5)] x [1 - (6)]

(8) is the assumed lapse rate.

(9) is the number of lapses (non-renewals) for the next policy year.
(9) = { (3) - (7) } x (8)

(10) is the assumed loss trend multiple and expresses losses in current year dollars.

(11) is the assumed discount factor to the midpoint of the entry age year.

(12) is the assumed cost of tail coverage in relation to the cost of mature claims-made.

DD&R MODEL 2
Level Premium Funding by Entry Age of Insured
Assumptions and Column Keys for Exhibits 2A and 2B, Sheets 1 and 2
(continued)

Column Key (cont.):

(13) is the non-DD&R premium collected for the policy year.
(13) = (3) x (10) x $1.00

(14) is the present value of the premium collected as of the middle of the entry age year.
(14) = (13) / (11)

(15) is the expected DD&R dollars utilized during the year.
(15) = (7) x (10) x (12)

(16) is the present value of the expected DD&R dollars utilized as of the middle of the entry age year.
(16) = (15) / (11)

(17) is the ratio of the expected DD&R utilized to the non-DD&R premium collected (discounted).
(17) = (16) / (14)

(18) is the discounted value of future expected DD&R losses. (discounted to current year)
(18) = sum of remaining (15) discounted to current year

(19) is the discounted value of future expected DD&R premium. (discounted to current year)
(19) = sum of remaining (13) discounted to current year

(20) is the year-end unearned premium reserve.
(20) = (18) - (19)

(21) is the year-end unearned premium reserve per insured.
(21) = (20)/(3)

The boxed entry at the bottom of (17) is the resulting DD&R charge that should be applied to all insureds with the entry age in question, regardless of current age. It is the ratio of total column (16) to total column (14).

DD&R Model 2
Level Premium Funding by Entry Age of Insured
Exhibit 2 A: Entry Age = 42
(Sheet 1)

Yearly Discount Rate = 1.04
Yearly Loss Trend = 1.05

Age at Beginning of Year (1)	Years Insured at Start of Year (2)	# of Insureds (3)	Disability Rate (4)	Mortality Rate (5)	Retire Rate (6)	Expected # of DD&R Utilized (7)	Lapse Rate (8)	Expected # of Lapses (9)	Loss Trend Factor (10)	Discount Rate Factor (11)	Tail Cost (12)
42	0	100,000	0.259%	0.363%	0.000%	621	10.00%	9,938	1.000	1.000	2.00
43	1	89,441	0.281%	0.398%	0.000%	606	10.00%	8,883	1.050	1.040	2.00
44	2	79,951	0.307%	0.435%	0.000%	592	10.00%	7,936	1.103	1.082	2.00
45	3	71,423	0.335%	0.476%	0.010%	585	10.00%	7,084	1.158	1.125	2.00
46	4	63,754	0.367%	0.522%	0.020%	578	10.00%	6,318	1.216	1.170	2.00
47	5	56,858	0.402%	0.576%	0.040%	577	10.00%	5,628	1.276	1.217	2.00
48	6	50,653	0.441%	0.638%	0.060%	575	10.00%	5,008	1.340	1.265	2.00
49	7	45,070	0.485%	0.705%	0.080%	570	10.00%	4,450	1.407	1.316	2.00
50	8	40,050	0.533%	0.775%	1.000%	917	8.00%	3,131	1.477	1.369	2.00
51	9	36,002	0.586%	0.846%	1.000%	869	8.00%	2,811	1.551	1.423	2.00
52	10	32,322	0.645%	0.942%	1.000%	829	8.00%	2,519	1.629	1.480	2.00
53	11	28,974	0.710%	1.010%	1.000%	781	8.00%	2,255	1.710	1.539	2.00
54	12	25,937	0.780%	1.105%	1.000%	741	8.00%	2,016	1.796	1.601	2.00
55	13	23,180	0.858%	1.206%	2.500%	1,044	8.00%	1,771	1.886	1.665	2.00
56	14	20,366	0.943%	1.310%	2.500%	954	8.00%	1,553	1.980	1.732	2.00
57	15	17,859	1.036%	1.423%	5.000%	1,308	8.00%	1,324	2.079	1.801	2.00
58	16	15,227	1.137%	1.549%	5.000%	1,147	8.00%	1,126	2.183	1.873	2.00
59	17	12,953	1.247%	1.690%	5.000%	1,006	8.00%	956	2.292	1.948	2.00
60	18	10,991	1.367%	1.846%	10.000%	1,414	8.00%	766	2.407	2.026	2.00
61	19	8,811	1.497%	2.016%	10.000%	1,157	2.00%	153	2.527	2.107	2.00
62	20	7,500	1.638%	2.201%	10.000%	1,007	2.00%	130	2.653	2.191	2.00
63	21	6,364	1.779%	2.398%	10.000%	873	2.00%	110	2.786	2.279	2.00
64	22	5,381	1.920%	2.604%	10.000%	755	2.00%	93	2.925	2.370	2.00
65	23	4,533	2.061%	2.817%	20.000%	1,081	2.00%	69	3.072	2.465	2.00
66	24	3,383	2.202%	3.044%	20.000%	817	2.00%	51	3.225	2.563	2.00
67	25	2,515	2.343%	3.289%	20.000%	615	2.00%	38	3.386	2.666	2.00
68	26	1,862	2.484%	3.563%	20.000%	461	2.00%	28	3.556	2.772	2.00
69	27	1,373	2.624%	3.868%	20.000%	345	2.00%	21	3.733	2.883	2.00
70	28	1,008	2.765%	4.207%	20.000%	257	2.00%	15	3.920	2.999	2.00
71	29	736	2.906%	4.571%	20.000%	190	2.00%	11	4.116	3.119	2.00
72	30	534	3.047%	4.951%	20.000%	140	2.00%	8	4.322	3.243	2.00
73	31	386	3.188%	5.338%	20.000%	103	2.00%	6	4.538	3.373	2.00
74	32	277	3.329%	5.736%	20.000%	75	2.00%	4	4.765	3.508	2.00
75	33	198	3.470%	6.167%	100.000%	198	2.00%	0	5.003	3.648	2.00

DD&R Model 2

Level Premium Funding by Entry Age of Insured

Exhibit 2 A: Entry Age = 42

(Sheet 2)

Yearly Discount Rate = 1.04

Yearly Loss Trend = 1.05

Age at Beginning of Year (1)	Years Insured at Start of Year (2)	# of Insureds (3)	Premium Collected (13)	PV Prem. Collected (14)	DD&R Utilized (15)	PV DD&R Utilized (16)	Ratio DD&R to Prem. (17)	Discounted Value of Future DD&R Loss (18)	Discounted Value of Future DD&R Prem (19)	Year-End UPR (20)	Year-End UPR per Insured (21)
42	0	100,000	$100,000	$100,000	$1,242	$1,242	1.24%	$54,790	$49,991	$4,800	$0.05
43	1	89,441	$93,913	$90,301	$1,273	$1,224	1.36%	$55,683	$46,293	$9,390	$0.10
44	2	79,951	$88,146	$81,496	$1,306	$1,207	1.48%	$56,579	$42,797	$13,782	$0.17
45	3	71,423	$82,681	$73,503	$1,355	$1,204	1.64%	$57,461	$39,494	$17,967	$0.25
46	4	63,754	$77,494	$66,242	$1,406	$1,202	1.81%	$58,326	$36,372	$21,953	$0.34
47	5	56,858	$72,567	$59,645	$1,474	$1,211	2.03%	$59,156	$33,425	$25,731	$0.45
48	6	50,653	$67,880	$53,646	$1,542	$1,218	2.27%	$59,950	$30,644	$29,306	$0.58
49	7	45,070	$63,418	$48,193	$1,605	$1,220	2.53%	$60,711	$28,023	$32,688	$0.73
50	8	40,050	$59,172	$43,236	$2,711	$1,981	4.58%	$60,375	$25,554	$34,821	$0.87
51	9	36,002	$55,850	$39,240	$2,695	$1,894	4.83%	$60,041	$23,188	$36,853	$1.02
52	10	32,322	$52,650	$35,568	$2,701	$1,825	5.13%	$59,688	$20,922	$38,767	$1.20
53	11	28,974	$49,555	$32,190	$2,672	$1,735	5.39%	$59,351	$18,752	$40,599	$1.40
54	12	25,937	$46,580	$29,094	$2,662	$1,663	5.72%	$59,010	$16,677	$42,334	$1.63
55	13	23,180	$43,710	$26,251	$3,936	$2,364	9.00%	$57,357	$14,692	$42,665	$1.84
56	14	20,366	$40,323	$23,286	$3,778	$2,182	9.37%	$55,798	$12,834	$42,965	$2.11
57	15	17,859	$37,127	$20,615	$5,437	$3,019	14.64%	$52,486	$11,095	$41,391	$2.32
58	16	15,227	$33,239	$17,746	$5,009	$2,674	15.07%	$49,477	$9,522	$39,955	$2.62
59	17	12,953	$29,689	$15,242	$4,614	$2,369	15.54%	$46,751	$8,102	$38,649	$2.98
60	18	10,991	$26,451	$13,057	$6,808	$3,361	25.74%	$41,678	$6,821	$34,857	$3.17
61	19	8,811	$22,264	$10,567	$5,848	$2,776	26.27%	$37,381	$5,744	$31,637	$3.59
62	20	7,500	$19,900	$9,082	$5,342	$2,438	26.85%	$33,428	$4,766	$28,662	$3.82
63	21	6,364	$17,729	$7,780	$4,865	$2,135	27.44%	$29,804	$3,881	$25,922	$4.07
64	22	5,381	$15,740	$6,642	$4,416	$1,863	28.05%	$26,493	$3,082	$23,411	$4.35
65	23	4,533	$13,925	$5,650	$6,644	$2,696	47.71%	$20,777	$2,360	$18,417	$4.06
66	24	3,383	$10,910	$4,256	$5,268	$2,055	48.29%	$16,236	$1,793	$14,443	$4.27
67	25	2,515	$8,516	$3,195	$4,163	$1,562	48.89%	$12,639	$1,348	$11,291	$4.49
68	26	1,862	$6,621	$2,388	$3,280	$1,183	49.53%	$9,800	$1,000	$8,800	$4.73
69	27	1,373	$5,126	$1,778	$2,574	$893	50.22%	$7,567	$729	$6,838	$4.98
70	28	1,008	$3,950	$1,317	$2,013	$671	50.97%	$5,817	$519	$5,298	$5.26
71	29	736	$3,029	$971	$1,567	$503	51.75%	$4,451	$356	$4,095	$5.57
72	30	534	$2,310	$712	$1,214	$374	52.56%	$3,391	$230	$3,161	$5.91
73	31	386	$1,752	$520	$935	$277	53.37%	$2,573	$133	$2,440	$6.32
74	32	277	$1,322	$377	$717	$204	54.20%	$1,945	$58	$1,887	$6.80
75	33	198	$992	$272	$1,983	$544	200.00%	$0	$0	$0	$0.00
Total				$924,058		$54,968	5.95%				

DD&R Model 2
Level Premium Funding by Entry Age of Insured
Exhibit 2 - B: Entry Age = 55
(Sheet 1)

Yearly Discount Rate = 1.04
Yearly Loss Trend = 1.05

Age at Beginning of Year (1)	Years Insured at Start of Year (2)	# of Insureds (3)	Disability Rate (4)	Mortality Rate (5)	Retire Rate (6)	Expected # of DD&R Utilized (7)	Lapse Rate (8)	Expected # of Lapses (9)	Loss Trend Factor (10)	Discount Rate Factor (11)	Tail Cost (12)
55	0	100,000	0.858%	1.206%	2.500%	4,502	8.00%	7,640	1.000	1.000	2.00
56	1	87,858	0.943%	1.310%	2.500%	4,116	8.00%	6,699	1.050	1.040	2.00
57	2	77,043	1.036%	1.423%	5.000%	5,641	8.00%	5,712	1.103	1.082	2.00
58	3	65,689	1.137%	1.549%	5.000%	4,950	8.00%	4,859	1.158	1.125	2.00
59	4	55,881	1.247%	1.690%	5.000%	4,342	8.00%	4,123	1.216	1.170	2.00
60	5	47,416	1.367%	1.846%	10.000%	6,102	8.00%	3,305	1.276	1.217	2.00
61	6	38,009	1.497%	2.016%	10.000%	4,992	2.00%	660	1.340	1.265	2.00
62	7	32,356	1.638%	2.201%	10.000%	4,343	2.00%	560	1.407	1.316	2.00
63	8	27,453	1.779%	2.398%	10.000%	3,767	2.00%	474	1.477	1.369	2.00
64	9	23,212	1.920%	2.604%	10.000%	3,256	2.00%	399	1.551	1.423	2.00
65	10	19,557	2.061%	2.817%	20.000%	4,666	2.00%	298	1.629	1.480	2.00
66	11	14,594	2.202%	3.044%	20.000%	3,523	2.00%	221	1.710	1.539	2.00
67	12	10,849	2.343%	3.289%	20.000%	2,652	2.00%	164	1.796	1.601	2.00
68	13	8,033	2.484%	3.563%	20.000%	1,990	2.00%	121	1.886	1.665	2.00
69	14	5,923	2.624%	3.868%	20.000%	1,487	2.00%	89	1.980	1.732	2.00
70	15	4,347	2.765%	4.207%	20.000%	1,108	2.00%	65	2.079	1.801	2.00
71	16	3,174	2.906%	4.571%	20.000%	821	2.00%	47	2.183	1.873	2.00
72	17	2,306	3.047%	4.951%	20.000%	606	2.00%	34	2.292	1.948	2.00
73	18	1,666	3.188%	5.338%	20.000%	445	2.00%	24	2.407	2.026	2.00
74	19	1,197	3.329%	5.736%	20.000%	324	2.00%	17	2.527	2.107	2.00
75	20	855	3.470%	6.167%	100.000%	855	2.00%	0	2.653	2.191	2.00

DD&R Model 2
Level Premium Funding by Entry Age of Insured
Exhibit 2 - B: Entry Age = 55
(Sheet 2)

Yearly Discount Rate = 1.04
Yearly Loss Trend = 1.05

Age at Beginning of Year (1)	Years Insured at Start of Year (2)	# of Insureds (3)	Premium Collected (13)	PV Prem. Collected (14)	DD&R Utilized (15)	PV DD&R Utilized (16)	Ratio DD&R to Prem. (17)	Discounted Value of Future DD&R Loss (18)	Discounted Value of Future DD&R Prem (19)	Year-End UPR (20)	Year-End UPR per Insured (21)
55	0	100,000	$100,000	$100,000	$9,005	$9,005	9.00%	$131,221	$118,938	$12,283	$0.12
56	1	87,858	$92,251	$88,703	$8,643	$8,311	9.37%	$127,656	$103,893	$23,762	$0.27
57	2	77,043	$84,940	$78,531	$12,439	$11,500	14.64%	$120,077	$89,816	$30,261	$0.39
58	3	65,689	$76,044	$67,603	$11,460	$10,188	15.07%	$113,193	$77,085	$36,108	$0.55
59	4	55,881	$67,923	$58,061	$10,555	$9,023	15.54%	$106,956	$65,588	$41,368	$0.74
60	5	47,416	$60,516	$49,739	$15,575	$12,802	25.74%	$95,351	$55,221	$40,129	$0.85
61	6	38,009	$50,935	$40,255	$13,380	$10,575	26.27%	$85,520	$46,497	$39,023	$1.03
62	7	32,356	$45,528	$34,598	$12,222	$9,288	26.85%	$76,476	$38,584	$37,893	$1.17
63	8	27,453	$40,560	$29,637	$11,130	$8,133	27.44%	$68,184	$31,420	$36,764	$1.34
64	9	23,212	$36,010	$25,300	$10,102	$7,097	28.05%	$60,610	$24,947	$35,663	$1.54
65	10	19,557	$31,857	$21,521	$15,199	$10,268	47.71%	$47,534	$19,107	$28,427	$1.45
66	11	14,594	$24,960	$16,214	$12,052	$7,829	48.29%	$37,144	$14,513	$22,631	$1.55
67	12	10,849	$19,483	$12,169	$9,525	$5,949	48.89%	$28,916	$10,911	$18,005	$1.66
68	13	8,033	$15,148	$9,097	$7,503	$4,506	49.53%	$22,421	$8,096	$14,325	$1.78
69	14	5,923	$11,727	$6,772	$5,890	$3,401	50.22%	$17,312	$5,903	$11,409	$1.93
70	15	4,347	$9,036	$5,018	$4,606	$2,557	50.97%	$13,307	$4,199	$9,108	$2.10
71	16	3,174	$6,929	$3,699	$3,586	$1,914	51.75%	$10,183	$2,880	$7,303	$2.30
72	17	2,306	$5,285	$2,713	$2,777	$1,426	52.56%	$7,758	$1,861	$5,897	$2.56
73	18	1,666	$4,009	$1,979	$2,140	$1,056	53.37%	$5,886	$1,075	$4,811	$2.89
74	19	1,197	$3,025	$1,436	$1,639	$778	54.20%	$4,450	$468	$3,981	$3.33
75	20	855	$2,269	$1,035	$4,538	$2,071	200.00%	$0	$0	$0	$0.00
Total				$654,080		$137,677	21.05%				

DD&R Model 3

Level Premium Charge the Same for all Insureds
Assumptions and Column Keys for Exhibit 3

<u>Assumptions:</u>

- The same as applied in Model 2.
- Only two ages of insured: 47 and 60.
- Entry dates for each age were five years ago.
- Mature claims-made rate is $6,500 (average).

<u>Column Key:</u>

(1) is the insured's age.

(2) is the age at entry (i.e., age when first insured).

(3) is the number of insureds in each age class.

(4) is the DD&R rate for the insureds based on Model 2 analysis.

(5) is the assumed average mature claims-made rate.

(6) is the non-DD&R premium.
(6) = (3) x (5)

(7) is the resulting DD&R premium.
(7) = (4) x (6)

(8) is the assumed cost of tail coverage in relation to the cost of mature claims-made.

The boxed entry at the bottom of (4) is the resulting DD&R charge that would be applied to all insureds regardless of current age or entry age. It is the ratio of total column (7) to total column (6).

DD&R Model 3

Level Premium Charge the Same for all Insureds

Exhibit 3

Age at Beginning of Year	Age at Entry	# of Insureds	DD&R Rate	Average Mature C-M Rate	non-DD&R Premium	DD&R Premium
(1)	(2)	(3)	(4)	(5)	(6)	(7)
47	42	50	5.950%	$6,500	$325,000	$19,338
60	55	35	21.050%	$6,500	$227,500	$47,889
Total		85	12.168%		$552,500	$67,226

EXHIBIT 4
ATTACHMENT ONE

Adopted Accounting Language on Claims-Made Policies and Reserves

The following language to be inserted on page 10-3 as the third paragraph under the caption "Claims-Made Policies":

Some claims-made policies provide extended reporting coverage at no additional charge in the event of death, disability or retirement of a natural person insured. In such instance, a reserve is required to assure that amounts collected by insurers to pay for these benefits are not earned prematurely and that an insurer with an aging book of business will not show adverse operating results simply because an increasing portion of insureds is earning the benefits for which it has paid. This reserve for "unclaimed coverage extension benefits" is most appropriately treated as part of the unearned premium reserve. However, an insurer may consider it to be a claims reserve and included with unpaid losses if authorized by the commissioner of the state of domicile. For a further discussion of this reserve, see Chapter 12 - Unearned Premiums.

****

The following language to be inserted on page 12-2, immediately preceding the Section - "Unearned Premiums - Unauthorized Reinsurance":

Claims-Made Extended Reporting Coverage Options Relating to Death, Disability or Retirement

Some claims-made policies provide extended reporting coverage at no additional charge in the event of death, disability or retirement of a natural person insured. In such instance, a reserve is required to assure that amounts collected by insurers to pay for these benefits are not earned prematurely and that an insurer with an aging book of business will not show adverse operating results simply because an increasing portion of insureds is earning the benefits for which it has paid.

Insurers should fund this future liability by charging a higher price for insurance, rather than relying on financing from future revenues. The concept of level funding, applied to these grants of extended reporting coverage without additional charge, is that the indicated incremental premium should be the same proportion of premium regardless of whether an insurer: is just starting to write this business and does not expect any extended reporting options to be claimed in the near future; or has provided this type of coverage for several years and continues to write new business; or has ceased writing substantial amounts of new business but continues to renew existing accounts, expecting to grant increasing amounts of extended reporting coverage options without additional premium.

The amount of the reserve, when combined with premium appropriate for an on-going book of business, including some charge for extended reporting coverage, should be adequate to pay for all future claims arising from these coverage features. These future claims include those covered by future grants of extended reporting coverage, without diminishing future profitability below normal expectations for on-going business. If the loss rates for providing this coverage to an aging population are low enough to indicate a negative reserve, then the reserve should be set at zero.

Reserve estimates will normally assume that a portion of the existing population of insureds will not continue with the same insurer until qualifying for the benefit and exercising the option. Funding should not anticipate vesting or cash values for individual insureds unless specifically provided by contract.

These additional factors should be considered in estimating the reserve:

1. Loss trends;
2. Time value of money;
3. Nonrenewal rates;
4. Age and tenure eligibility requirements in the contracts;
5. Age and tenure demographics of the insured population;
6. Mortality considerations;
7. Morbidity considerations;
8. Pricing differentials (if any) related to age of insured;
9. Expected claim costs in relation to age of the insured and the number of years until retirement;
10. Waivers (if any) of charges for specialty changes before retirement;
11. Partial benefits (if any) for termination by either the insured or the insurer prior to retirement; and
12. Other factors that impact the value of future benefits.

Insurers should provide for this contingency as a reserve entitled "unclaimed coverage extension benefits." This reserve should be treated as part of the unearned premium reserve and should be considered to run more than one year from the date of the policy. The amount should be identified in a footnote. When the reserve is revalued at the close of each accounting period, a portion will flow into earned premium corresponding to insureds which have terminated claims-made coverage. A corresponding IBNR loss reserve will be established for those insureds which have exercised the extended reporting coverage option.

This reserve may alternatively be considered a claims reserve and included with unpaid losses by an insurer which has obtained authorization to do so from the commissioner of the state of domicile.

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Effective FY ending
12/31/93

Catastrophe Ratemaking Revisited
(Use of Computer Models to Estimate Loss Costs)

by Michael A. Walters, FCAS, and
François Morin, FCAS

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Abstract
Recent developments in computer technology have significantly altered the way the insurance business functions. Easy access to large quantities of data has rendered some traditional ratemaking limitations obsolete. The emergence of catastrophe simulation using computer modeling has helped actuaries develop new methods for measuring catastrophe risk and providing for it in insurance rates. This paper addresses issues associated with these methods and provides actuaries, underwriters and regulators with an understanding of the features and benefits of computer modeling for catastrophe ratemaking.

Biographies
Michael A. Walters is a principal and consulting actuary with Tillinghast – Towers Perrin in Parsippany, New Jersey. A graduate of Fordham University, he has a master's degree in mathematics from the University of Notre Dame. He is a Fellow and past president of the Casualty Actuarial Society and a past vice president of the American Academy of Actuaries and past chairman of its Casualty Practice Council. He has authored two papers previously included in the CAS Syllabus of Examinations: "Homeowners Insurance Ratemaking" and "Risk Classification Standards." The latter was awarded the 1981 CAS Dorweiler prize.

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Acknowledgment
The authors wish to acknowledge the input to the risk load analysis by Stephen Philbrick (FCAS) and the overall review efforts of Ronald Kozlowski (FCAS), Stuart Mathewson (FCAS), David Sommer (FCAS), and Tom Warthen (FCAS).

Catastrophe Ratemaking Revisited

(Use of Computer Models to Estimate Loss Costs)

WHY MODELING?

According to the CAS Principles of Ratemaking, a rate "is an estimate of the expected value of future costs, provides for all costs associated with the transfer of risk, and provides for the costs associated with an individual risk."

Traditionally, ratemaking has been regarded as the art of projecting scientifically measured past experience into valid conclusions about the future. However, for lines of business with catastrophe potential, questions always arise as to how much past insurance experience is necessary to accurately represent possible future outcomes and how much weight should be assigned to each year's experience. For instance, if a 1954 hurricane was the last severe event in a given state, may one assume that the return period for an event of the same severity is 40 years? What if historical records show that more severe storms occurred in the 1930s, before the advent of homeowners coverage? If the same storm happened today, would it affect the same properties? What level of damage would occur, given that the distribution of insureds had shifted to coastal communities and that the insured values at risk have trended at a pace that has exceeded inflation?

For these rare event calamities, reliance on actual insured experience does not allow accurate measurement of future expected loss. Therefore, one must use a much longer experience period, especially for event frequency. Computer simulation of events to obtain current insured losses has replaced traditional methods based exclusively on reported loss experience. These new methods can now be used not only to measure expected losses, but also to develop risk loadings to compensate for the variance in outcomes, compared to lower-risk insurance products.

The need for catastrophe modeling has existed for some time to aid in reinsurance purchase decisions as well as in insurance ratemaking. However, computer limitations on the amount of data that could be manipulated to develop a catastrophe model had usually rendered the concept impractical. But computer capacity has improved dramatically, which now makes catastrophe simulation feasible. It has also enabled scientists to expand their research and produce better simulations through a better understanding of catastrophic events.

WHAT TO MODEL

A state's most recent past may not be indicative of its true catastrophe potential because what happens in a given year is only a sample of what could have happened. The goal is to build a model to simulate what could realistically happen, based on information relevant to that state and to all refined geographic areas within the state.

Doing this with a computer model requires that the estimation process be separated between frequency and severity. For the frequency of hurricanes, there is a long history (more than 100 years) of recorded information to gauge the relative likelihood of landfall in a given state.

For severity of loss, however, the actual insured damage may not have been recorded. Certainly, the extent of loss if that same storm occurred again would depend on *today's* insured values, coverage and level of windstorm resistant structures. This is the first area utilizing computer simulation — taking the characteristics of a storm and replicating the windspeeds at various locations and times over its course after landfall. Next, the damage to buildings and the effect on insured values flow from the windfield created by the storm. Validation of the model examines actual storms over the recent past, so that the full range of possible storms is the basis for expected loss calculations as well as risk analysis on the possibility of adverse outcomes in any given year.

HOW TO MODEL FOR SEVERITY

The severity component of catastrophe modeling generally comprises three distinct modules with three separate skills required:

- event simulation (science)
- damageability of insured properties (engineering)
- loss effect on exposures (insurance).

The event simulation module is designed to reproduce natural phenomena. For a hurricane model, this involves predicting wind speeds at every ZIP code affected over the course of a single storm. The damageability module estimates the damage sustained by a given property exposed to the simulated event. The majority of the damage functions used in a catastrophe model are developed by engineers who better understand the physics of natural phenomena and can test the resistance of various materials to high windspeeds. (The results of the studies are also used to develop new materials and to implement new building codes to limit the damage from catastrophes.) The insured loss effect module incorporates the results of the first two modules and adjusts them for such factors as deductibles, co-insurance, insurance to value, and reinsurance. This is generally the only company-specific module because it includes all the factors that describe the in-force company book of business.

This part of catastrophe modeling is known as deterministic, because it allows the simulation of a predetermined event with known characteristics. The computer could duplicate this event, if it occurred today, with the resulting effects on the insured exposures calculated. Appendix A provides a detailed description of the process involved in developing and validating the severity component of a catastrophe model.

HOW TO MODEL FOR FREQUENCY

Once the deterministic model has been created, calibrated and validated, the modeler must analyze historical meteorological records and develop a probabilistic facet to the catastrophe model. The first step involves generating distributions for each of the parameters required as input to the hurricane model. A hurricane model may be dependent on a variety of factors, such as the radius of maximum speed, forward moving speed and pressure differential at the eye of the storm. Considerable effort must be spent in constructing these distributions so that accurate representations of realizable events can be obtained by combining the variables. For example, an analysis of the radius of maximum winds of historical events yields a conclusion that they are normally distributed ($N(\mu,\sigma)$), (with parameters of 16.840 and 10.567 in South Florida). Similarly, the forward moving speed of these events follows a lognormal distribution ($\Phi(\mu,\sigma)$) (with parameters of 2.304 and 0.283, respectively in South Florida.) Similar distributions must be built for each of the parameters that are hurricane specific in each geographic zone. One can obtain the historical data from National Oceanic and Atmospheric Administration (NOAA) publications.

The modeler then uses sampling techniques to randomly select the parameters from each distribution. Most catastrophe models rely on a Monte Carlo approach, a stratified sampling approach or a combination of both. Although Monte Carlo is easier to use and to explain to a nonstatistical audience, it does not have the sampling power of a stratified approach. Therefore, the modeler should consider both methods before generating the probabilistic database.

In conjunction with storm intensity distributions, conditional probabilities, storm paths, and landfall locations must be developed for each storm modeled. These parameters are based on actual storm paths of historical events over the last hundred years. The storm probabilities depend on the type of sampling utilized in selecting parameters for storm intensities. By nature, Monte Carlo sampling requires that all storms have the same probability, whereas stratified sampling can be done in such a way that probabilities are not all equally likely.

After selecting the storm intensity parameters and deriving the probabilities, one combines the two. The end result is the probabilistic library, which comprises a large enough number of events (in excess of 5,000) to represent all likely scenarios. For example, the database should include Category 5 storms making landfall in Maine (if they are at all possible) so that the damage associated with such an event can be calculated. (Stratified sampling allows a more efficient handling of this issue because it can cover all possibilities with fewer storms.) Because each event has an associated probability that is conditional on a hurricane making landfall, the sum of all probabilities will, by definition, add up to one. The modeler will then use these probabilities to derive annual expected loss costs.

BASIC OUTPUT OF MODEL

Once the probabilistic database is complete, one can proceed to calculate expected loss costs by ZIP code. To accomplish this, the modeler should run the entire event library against a set of exposures that assumes a constant value (e.g., $100,000 of Coverage A amount for homeowners) in each ZIP code. It is important to ensure that all exposed amounts are included in the simulation. For homeowners, it is customary to increase Coverage A amounts by 10% for appurtenant structures, 50% for contents and 20% for additional living expense (i.e., loss of use). Annual expected loss costs for a given ZIP code are then developed by multiplying the sum of the probability weighted simulated results across all storms by an annual hurricane frequency. The average annual frequency of hurricanes making landfall in the U.S. has been approximately 1.3.

For a given line of business, the expected losses by ZIP code are then:

$$EL_{ZIP} = F \times \sum_{storm} (P_{storm} \times E_{ZIP} \times DF_{storm})$$

Where EL_{ZIP} = Expected Losses for ZIP code for base class
F = Annual Hurricane Frequency
P_{storm} = Probability of storm
E_{ZIP} = Total exposure amount (Base class constant for all ZIP codes)
DF_{storm} = Damage factor for base class by ZIP code by storm

These expected losses represent insured losses for a base class amount of insurance, construction type and deductible. These may be selected as frame building with $250 deductible, with $100,000 Coverage A (building), $10,000 Coverage B (appurtenant structures), $50,000 Coverage C (contents) and $20,000 Coverage D (additional living expense). To convert this to a loss cost expressed as a rate per $1,000 of Coverage A amount requires division by the exposure base times 1,000.

$$ELC_{ZIP} = \frac{EL_{ZIP}}{COVA_{ZIP}} \times 1{,}000$$

Where ELC_{ZIP} = Expected Loss Cost for ZIP code
$COVA_{ZIP}$ = Base class Coverage A amount in ZIP code

A major feature of this calculation is its independence of an individual company's actual loss experience and of its exposure distribution. Being independent of individual company data, it is, in fact, appropriate for each insurer.

The next step is to average the loss costs by ZIP code over the insurer's exposure distribution within the territory structure it selects.

$$ELC_{terr} = \frac{\sum_{ZIP} (ELC_{ZIP} \times COVA_{ZIP})}{\sum_{ZIP} COVA_{ZIP}}$$

Where ELC_{terr} = Expected Loss Cost for territory

In Exhibit 1, the ZIP code loss costs per $1,000 of Coverage A amount for homeowners are averaged to a given territory structure to derive the territorial loss costs for hurricane coverage. It is likely that the more representative territory structure for hurricane will differ from regular homeowners territories. Because the latter evolved over time to respond to homogeneity considerations in setting rates for the perils of fire and theft, a company may wish to create new territories to reflect differences in hurricane loss potential.

ATTRIBUTES OF LOSS COSTS VIA COMPUTER MODELING

Credibility

Through computer simulation and stratified sampling, the most remote cells have complete credibility in the traditional sense. That is, the measurements can be taken at full value, without having to ballast them with actual results on a statewide basis, or on last year's results. One substitutes the random variation of low frequency actual storms with the set of all possible storms via the model. Moreover, the probabilities are assigned by the selection of the input parameters. This solves the problem of low credibility of actual results and the attempt to refine actual statewide data to territory.

While full credibility can be assigned in cell detail from computer simulation, this only means that random statistical variation can be resolved to eliminate the process risk from a ratemaking standpoint. However, there is still parameter risk in the selection of the key variables. It is possible that the event frequencies of the past 100 years are not representative of the next 100 years. This is especially true in the case of earthquake simulation, where the physics of shake intensity are not understood well enough by earthquake experts to generate fully reliable parameters of frequency and severity.

With full credibility in ZIP code detail, one can calculate statewide averages by averaging over ZIP code and territory. This is in stark contrast with the usual homeowners indicated loss costs, which first are developed statewide, and then must be distributed to the different class and territory cells with appropriate credibility weightings. This stems from the experience loss ratio method used to derive the result — actual insured experience that is a sample taken from what might have occurred over time. In contrast, hurricane loss costs are derived from the set of all possible events as constructed in the computer model. Using a hurricane model to produce loss costs is truly a pure premium method of ratemaking, versus the loss ratio method usually used in traditional ratemaking with historical insurance data.

Frequency of Review

Hurricane loss costs derived from modeling do not need frequent updates for two reasons. First, with more than 100 years of actual event characteristics shaping the model design, another year or two of actual results are unlikely to change model parameters much. In the early stages of model building, with each new hurricane to landfall, the potential exists to update some of the damage factors and the estimated effect of deductibles or other class factors. Also, when new class variables are developed, one can refine initial estimates with the loss experience of subsequent actual storms. For example, new kinds of shutters will have been tested, and it would be possible to incorporate their effect in the model.

Secondly, once adequate rate levels are achieved, annual updates are also not critical because the exposure base ($1,000 of Coverage A) is inflation sensitive. For the average territory loss costs, in the early years of implementation, it may be well to test for changing ZIP code distributions, as insureds and insurers react to some high loss costs in certain coastal areas.

Risk Variations

Non-hurricane homeowners loss costs vary significantly by fire protection class, reflecting the large portion of the coverage represented by the fire peril. Yet, the hurricane peril is obviously independent of protection class.

Policy form relativities basically increase as additional perils are covered. In Forms 1 and 2, the perils are specified, while Form 3 gives essentially all risk coverage on the building, but not on contents. Form 5 provides all risk coverage on contents. Thus, the wind coverage is identical in all the homeowners policy forms. Hence, if the hurricane loss costs are a material portion of total homeowners costs, the policy form relativities would have to vary substantially by territory or even by ZIP code if applied to an indivisible homeowners premium.

For construction class, a frame house can be almost as hurricane resistant as one made of brick or stone. For large hurricanes, the key is to protect the envelope of the building from penetration — i.e., the windows and the roof. Hence, the relative fire resistance of the construction is irrelevant for the hurricane peril.

Hurricane (and other catastrophes) ultimately may need a separate class plan because of different risk variation from the traditional covers. For example, for hurricanes, new rating factors will likely emerge for shuttering and for roof type (e.g., gable versus hip roof). Local enforcement of building codes is an early rating distinction that is implementable. Redoing all the traditional homeowners class relativities to meld with the new catastrophe classes would be very cumbersome. Perhaps the traditional homeowners territories could be retained, with a separate set of territory definitions for the hurricane rate.

A possible class plan with sample surcharges and discounts follows:

Category	*Criteria*	*Sample Factor*
Hurricane Shutters	None	+ 0.20
	Add-On	- 0.20
	Built-In	- 0.40
Roof Type	Hip	- 0.25
	Gable	+ 0.30
Location	Shielded by buildings	- 0.20
	Subject to projectiles	+ 0.20
	Beach front or subject to surge	+ 0.10
Town Building Code	Not enforced	+ 0.15
	Enforced; not inspected	- 0.10
	House inspected; within code	- 0.25

FORM OF RATING

If the hurricane peril does not vary by class the same way as the non-hurricane perils, should the hurricane rate be split out from the heretofore indivisible premium for homeowners? Should it have its own class plan? The answer to both questions is yes.

Basically, one can have the best of both worlds. The indivisible premium formerly simplified the review of loss experience and the rating of the homeowners policy, as well as lowering the cost of the monoline coverages, knowing that all the major perils were essentially compulsory. Virtually all of the advantages of the indivisible premium can be kept by still keeping hurricane *coverage mandatory. Yet, it is the very difficulty of the experience review that suggests the* segregation of it for ratemaking — using the pure premium method for hurricane ratemaking and allowing a loss ratio approach for the other perils.

Computer modeling could also be used for other catastrophe perils (e.g., earthquake, tornado and winter storm) such that the remaining non-catastrophe perils in homeowners would use the more traditional methods of ratemaking. Computer modeling for catastrophe perils actually makes ratemaking for the other perils much easier, because of less fluctuating results. With loss costs supplied by modeling and with a separate rate for each catastrophe peril, the actual catastrophe losses only need to be removed from the experience period and nothing need be loaded back to the normal homeowners losses. This means that catastrophe serial numbers ought to be retained for loss coding.

The overwhelming advantages of separate catastrophe rates are the simplification of the normal coverage rating and ratemaking as well as the better class and territory rating of the catastrophe coverages.

This does mean an extra rating step for the catastrophe coverages, but there already are so many endorsements in homeowners that this should not be much of a burden. Furthermore, if hurricane loss costs are left in the indivisible premium, the homeowners classes will become much more complicated to rate. The class relativities will have to vary greatly by hurricane zone, and the actuarial calculation of relativity indications will also be much more complex.

Another simplification via separate hurricane rating is not having to calculate a complicated set of statewide indications including hurricane. Instead, the indications can be produced, and actual rates selected, separately. Ostensibly, this creates a problem in rate filings, where tradition has called for a combined statewide average indicated rate change as well as a filed rate level change. However, this is mere custom, and not strictly required by the rating laws — which usually call for *rates* to be filed, not *rate changes*. In other words, statutory requirements are for *rates* to be reasonable, not excessive, inadequate or unfairly discriminatory. Filed measures of *rate changes* have merely been a convenient way for regulators to monitor reasonableness.

This is not to suggest that a rate filing should repress the estimate of statewide rate change. But given the different ways of calculating the appropriate rates (via a pure premium approach for hurricanes and a loss ratio method for other coverages), the statewide indication does not as readily come out of the ratemaking method as, for example, it does for auto insurance. Hence, other reasonable ways of estimating changes will need to be developed, instead of directly from the ratemaking method.

EXPENSE LOAD CONSIDERATIONS

If the hurricane peril is reinsured in a reasonable fashion, then the primary insurer ought to be able to pass those costs through to the policyholder. The reinsurance premium can be expressed as a function of the primary layer and added to the equation.

Then, the total expected hurricane loss costs would be adjusted to exclude the reinsured portion by having the hurricane computer model simulate the reinsurance layer. This is done by running all probabilistic storms against the insurer's exposure base by ZIP code and line of business. Each storm's losses in the reinsurance layer (1) are then allocated to line and ZIP code in proportion to total losses for that storm (2). Then each storm's probability is multiplied by the losses in the layer and accumulated (3). This produces the expected losses in the reinsurance layer.

(1) $$L_{XS} = \text{MIN}\,(\text{MAX}\,((\sum_{ZIP} E_{ZIP} \times DF_{storm}) - RET,\ 0),\ LIM)$$

Where
L_{XS} = Total Losses in Layer for each storm
RET = Reinsurance Retention
LIM = Reinsurance Layer Size

(2) $$L_{XS,\,ZIP} = L_{TOT,ZIP} \times L_{XS} \div L_{TOT}$$

Where
$L_{XS,\,ZIP}$ = Excess Losses by zip code for each storm
L_{TOT} = Total Ground-Up Losses for each storm
$L_{TOT,ZIP}$ = Ground-Up Losses by zip code for each storm

(3) $$EL_{XS,ZIP} = F \times \sum_{storm} P_{storm} \times L_{XS,ZIP}$$

Where $EL_{XS,ZIP}$ = Expected Losses in Layer by Zip Code

The reinsurance premium can then be allocated to line of business and ZIP code in proportion to the expected excess losses in the reinsurance layer. Those premiums are then ratioed to the primary premium by line and ZIP code to get a factor to add to the indicated rate by line and ZIP code.

The remaining expected loss costs outside the reinsurance layer (above and below) would then be loaded for risk margin and expenses. The reinsurance pass-through would already have included the expenses and risk margin of the reinsurer.

RISK LOAD CONSIDERATIONS

Splitting the homeowners premium into a catastrophe and non-catastrophe component also allows for a separate calculation of a risk margin. As a result, the non-catastrophe component becomes easier to price, with less variability and a lower margin needed for profit. This makes it closer to a line of business like automobile physical damage in its target total rate of return and total target operating margin needed, which can be expressed as a percentage of premium.

Once a target margin is selected for the non-catastrophe component, the margin for the catastrophe piece can be calculated as a multiple of the non-catastrophe component, using some basic assumptions. One assumption is that profit should be proportional to the standard deviation of the losses. (Some actuarial theorists argue that risk load should be proportional to variance. It is important to note that these arguments apply to individual risks. The assumption that the required risk load for an entire portfolio is related to the standard deviation is not inconsistent with a variance based risk margin for individual risks. In addition, the high correlation of losses exposed to the risk of a catastrophe as well as the large contribution of parameter risk to the total risk load requirement provides additional arguments in favor of a standard deviation basis for risk load.)

The calculation of the risk load should be performed on a basis net of reinsurance since the reinsurance premium is being built back into the rates separately. However, calculating the risk load both gross and net of reinsurance may be an important exercise for an insurer analyzing retention levels. By doing so, the insurer may be able to evaluate its reinsurance protection by considering the total risk load required.

In the table below, one starts with a homeowners non-catastrophe pretax operating profit margin of 3%. At a 2.5 to 1 premium to surplus ratio, this is equivalent to about a 9.4% after-tax return on surplus (((2.5 x 3 + 7) x .65) = 9.4), assuming surplus can be invested at 7% pre-tax.

Calculation of the Hurricane Risk Margin as a Function of the Non-Catastrophe Risk Margin

(1)	*% of Loss* *(2)*	*Coefficient of Variation* *(3)*	*Standard Deviation* *(4)=(2)x(3)*	*Relativity* *(5)*	*Risk Margin (% of Mean)* *(6)*	*Dollar Return* *(7)*
Non-Catastrophe	80%	0.08	0.064	1.00	3%	0.0240
Hurricane	20%	3.50	0.700	10.94	131%	0.2625

Next, assume that the total pure premium can be split 80% non-catastrophe and 20% catastrophe. (This split is expected to be state-specific as the hurricane loss cost in hurricane-prone states will represent a greater proportion of the total loss cost.) Based on homeowners industry data adjusted to eliminate catastrophes, the coefficient of variation of non-catastrophe loss ratios has been about 8% over the past 40 years. The corresponding coefficient of variation for hurricane losses, based on computer models, might be 350%, for example. This implies that the standard deviation of hurricane catastrophe losses would be 10.94 times the standard deviation of non-catastrophe losses.

If a 3% operating margin for non-catastrophe homeowners produces a $2.40 operating profit on an $80 pure premium, then the operating profit for the hurricane pure premium should be 10.94 times that, or $26.25 (10.94 x 2.40 = 26.25). Expressed as a percentage of the pure premium, this would result in a risk margin of 131% on top of the expected hurricane loss costs. (These operating margins would include investment income from policyholder-supplied funds, and therefore need to have that quantity subtracted to derive an underwriting profit margin to be applied to loss costs.)

One can actually convert the risk margin to be a direct function of the ratio of CV's , as the risk margin incorporates the ratio of the dollar profit to the mean:

$$\text{Risk Margin}_{CAT} = \text{Risk Margin}_{NON\text{-}CAT} \times CV_{CAT} \div CV_{NON\text{-}CAT}$$

RATE FILING ISSUES

The approval of computer models as the source of expected catastrophe loss and risk margin can be a lengthy process because it changes the way regulators can verify the calculations. Under traditional filings, basic data are included with the filing, and the underlying source data are often part of statistical plan information that has been implicitly approved by the regulators in the past.

With catastrophe modeling, the frequency of events is often taken from published information tracking 100 or more years of event history. For the key catastrophe event simulation, (a hurricane or an earthquake, for example), the source is usually a scientific paper describing the ability of various equations to simulate the event. For the probabilistic model generating expected losses, often thousands of events are used, each with a specific probability derived from past distributions of input parameters.

This presents a dimensionally different approach to the regulatory approval process. It lends itself to a separate evaluation of each independent modeler — to pre-clear each model before an actual rate filing is made utilizing that model's calculation of expected loss costs. This preclearing process can take several months' time, depending on the level of due diligence needed and on the amount of rate level increase implied by the use of models to replace the old ratemaking system.

Once the independent modelers have been approved, the resulting set of indicated loss costs can provide a range of reasonable answers within which to evaluate specific company filings if the insurer has built its own model. If that company-specific model has loss costs within the pre-cleared range, that is usually prima facie evidence of the overall reasonableness of the company model. Even if the insurer model has some results outside the range, that should not necessarily disqualify the result. It merely places an additional burden on the insurer to prove the result is reasonable based on its own assumptions and judgments.

The following steps can be considered in that regulatory approval process (the details of which are included in Appendix C):

- review general design of the model
- examine event simulation module
- test ability of module to simulate known past events
- check distributions of key input variables
- perform sensitivity checks on which inputs are most important
- verify damage and insurance relationship functions
- test output for hypothetical new events
- compare different modelers' results for loss costs
- conduct on-site due diligence and review of actual assumptions.

For independent modelers, and even for insurer specific models, it is important to preserve trade secret information during the approval process and afterwards. This will affect the likelihood of future innovations to know that research and development investments can be preserved.

The on-site due diligence of regulators should keep the inner workings of the models confidential, as long as the examining process is documented by the regulator, much in the same way a financial examination of an insurance company keeps key information confidential.

Even after the approval process of a model, the regulator can preserve the confidentiality of indicated loss costs by ZIP code by not publishing the ranges that it plans to use in reviewing other company filings. First, it is better policy not to disclose the high end of the range lest some insurers be tempted to file that answer rather than using a rigorous model. Second, publishing the rate may be tantamount to the regulator setting the rate instead of approving reasonable filed rates. And third, the regulator would not be receiving the direct public attention on why the rates are so high in certain areas.

FINAL PERSPECTIVE

In summary, computer models are now capable of simulating catastrophic events and creating probabilistic models of reality that can be used to generated expected loss costs for catastrophe perils. These same models also provide a means of including the reinsurance premiums in the primary pricing process and can help quantify the needed risk load in relation to profit margins required for the non-catastrophe perils.

The same model can also be used for insurer or corporate risk analysis including reinsurance purchase decisions, and for insurer marketing and underwriting strategies. These analyses are beyond the scope of this paper.

Use of computer models for ratemaking involves a different approach from the customary one in that it is a pure premium method in contrast to the usual loss ratio method involving past insured loss experience. But that carries advantages as well as challenges, as it attempts to deal with the true underlying probabilities of loss; not just with what appears in the last few years of actual insured loss experience - which is merely a sample of what could have occurred. The computer models attempt to measure what could have occurred.

Thus, the models rely heavily on computer simulations and other technical methods newly emerging as feasible because of the vast improvement in personal computer potential. This also requires a heavy investment not only in research and design, but in resources to have the model evaluated and accepted by regulators and others.

But it is worth the process, not only for the practical results in insurer ratemaking and planning, but also for the insights gained on these catastrophic events and the reduction in uncertainty for society in dealing with them.

Furthermore, the techniques developed in producing these computer models might ultimately be applied to other perils as well. After all, the essence of actuarial work is modeling reality to assess the present financial impact of future contingent events.

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[2] Walters, Michael A., "Homeowners Insurance Ratemaking," PCAS LXI, 1974, P. 15.

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Sample Insurance Company
State XYZ
Expected Loss Cost
Per $1,000 of Homeowners Coverage A

Exhibit 1

Base Class. Frame
Base Deductible: $250

Zip Code Loss Costs

Base Territory	Zip Code	Exposure in Coverage A Amount	Expected Loss Cost
(1)	(2)	(3)	(4)
A	2001	3,227,000	0.351
	2002	12,495,000	0.342
	2003	8,113,000	0.421
	2004	9,204,000	0.482
B	2005	1,198,000	1.232
	2006	3,254,000	1.425
	2007	6,681,000	1.647
	2008	11,341,000	1.552
C	2009	7,295,000	2.565
	2010	6,400,000	2.752
	2011	8,508,000	2.832
	2012	9,212,000	3.011
D	2013	17,346,000	3.742
	2014	15,212,000	3.953
	2015	13,900,000	4.032
	2016	6,573,000	4.211
	Total	139,959,000	2.464

Territory Loss Costs

Base Territory	Exposure in Coverage A Amount	Expected Loss Cost
(1)	(2)	(3)
A	33,039,000	0.401
B	22,474,000	1.545
C	31,415,000	2.806
D	53,031,000	3.937
Total	139,959,000	2.464

Notes:
(2): In-force Coverage A amounts as of June 30, 1995.
(3): Expected Loss Costs derived from probabilistic hurricane modeling.

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HOW TO CONSTRUCT A MODEL

The severity component of catastrophe models generally contain three modules which are initially built separately but eventually integrated. These modules are:

- event simulation (science)
- damagability of properties (engineering)
- loss effect on exposures (insurance)

Described below is the level of research and testing that must be performed to develop a catastrophe model before it can be used for ratemaking purposes.

Science Module

As a first step, the modeler must incorporate the physics of the natural phenomena in a module (also called the event generator module) that simulates as closely as possible the actual event. Examples of input for a hurricane model include the radius of maximum winds, pressure differential at the eye of the storm (ambient pressure minus central pressure), forward speed, angle of incidence, landfall location and directional path. For an earthquake model, factors such as magnitude, location of the epicenter, soil conditions, liquefaction potential and distance from the fault rupture are used to estimate the shaking intensity of the ground at a given location.

Complete testing of the event generator module must be performed to ensure that it can be used both for the reproduction of historical events and for the simulation of hypothetical or probabilistic events. As a first step, actual wind speed records for recent events should be compared to modeled results. Organizations such as the National Hurricane Center can provide actual recorded conditions for historical events. Second, the hurricane model should be used, and its accuracy tested, to predict wind speeds for hypothetical events along the Atlantic and Gulf coasts. Since one of the key drivers of a hurricane model is the terrain or roughness parameter, this testing will help evaluate the sensitivity of the model to this factor and will allow the modeler to perform the necessary refinements to the initial assumptions.

The predictive accuracy of the model is limited by the fact that some site-specific factors that affect the way an event behaves on a given property (e.g., topographic peculiarities that affect wind speeds, liquefaction propensity at a given location for earthquakes) cannot be captured and modeled. Therefore, one should not expect a model to exactly reproduce a single past event, but rather verify that it can simulate adequately hypothetical events with a given set of parameters. Thus, actual future events with other site differences do not require major modifications to the model, but rather provide additional information to further refine it. The two maps attached are modeled replications of Hurricane Hugo and of a simulated earthquake (of a 7.5 magnitude on the Richter scale) on the Newport-Inglewood fault in Southern California.

Engineering Module

Once the event generator has been developed, damageability functions are needed to estimate the damage to a property subject to an event of a given intensity. Input from various fields of the engineering profession, such as wind engineering and structural engineering, must be gathered to develop these functions. For damage by hurricane wind speeds, numerous studies have been performed that estimate these relationships. The functions should vary by line of business, region, construction, and coverage (building versus contents).

As was the case for the event generator module, accuracy of the damage functions is improved by analyzing actual past events. Actual loss experience of insurance companies should be compared to modeled losses in the most refined level of detail available. Whereas only aggregate loss amounts by catastrophe used to be collected by companies, it is now generally possible to see loss data at least by line of business and county (or even ZIP code).

Next, on-site visits to the locations of catastrophes can help assess the damageability of exposed structures. While not imperative, these visits provide additional insight to the modeler, especially in identifying future classification distinctions.

The refinement of the damage functions is an ongoing process that is dependent on input generally provided by the engineering community. Engineering studies and loss mitigation reports are constantly being published, and their conclusions should be adapted and incorporated into the damage functions being used in the catastrophe model.

Insurance Module

Once the science and engineering modules have been developed, they must be integrated with the insurance module to determine the resulting insured loss from a given event. Kozlowski and Mathewson [3] stress the importance of developing and maintaining a database of in-force exposures that captures the relevant factors that can be used in assessing the damage to a given risk. This database will not only include such factors as location, construction type, number of stories, age of building and coverage limits, but also replacement cost provisions, deductibles, co-insurance and reinsurance (both proportional and non-proportional).

Integration of Modules

The table below presents a sample calculation of the loss estimate generated by the model for a sample hurricane after integrating the three modules.

Sample Calculation of Hurricane Losses

Zip Code	*Exposure Amount*	*Deductible*	*Windspeed (mph)*	*Corresponding Damage Factor*	*Gross Resulting Loss*	*Net Resulting Loss*
2001	$180,000	$250	100	.15	27,000	26,750
2002	180,000	500	90	.08	14,400	13,900
2003	180,000	2%	80	.05	9,000	5,400

The example assumes that we have one single family dwelling in each zip code, each with a different deductible. Based on the parameters of the storm simulated, the event generator module calculates the average windspeed sustained by all structures within the zip code. In this case, the windspeeds decrease as the zip codes are further away from the coast.

The damageability module then predicts the damage sustained by each structure as a function of the windspeed. The damage factors generally vary based on factors such as construction type (e.g. frame versus wind-resistive), age of building and number of stories. The gross resulting loss is then calculated by multiplying the exposure amount by the damage factor. The estimate is then adjusted for insurance features such as deductibles and reinsurance. In this example, the gross loss is reduced by the deductible to derive the net resulting loss.

HOW TO VALIDATE

The final task in developing a catastrophe model lies in validating the simulated results. While intermediate levels of calibration are performed for each module, the modeler must verify how they interact by completing an overall analysis of the results.

Because the model purports to simulate reality, actual incurred loss experience is the obvious candidate to be used in testing modeled losses. It is important to realize that all comparisons are dependent on the quality of the data captured from the loss records of insurers. As described above, the modeler should gain access to various sets of insured loss data and verify that all relevant factors are reflected in the model. These would include line of business, construction class, coverage (e.g., building versus contents), and loss adjustment expense (LAE) as a percentage of loss.

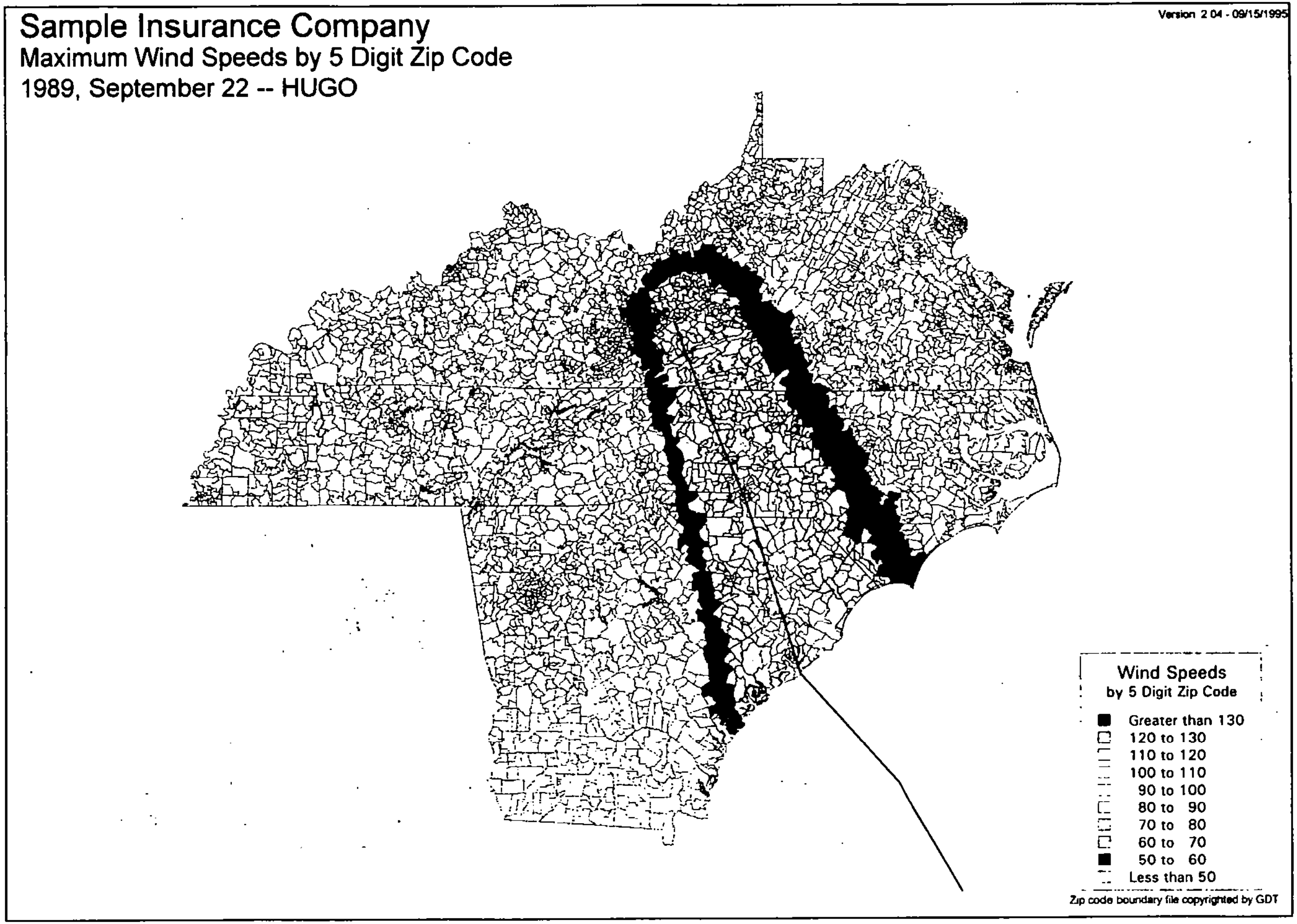
One issue that is often raised when validating a catastrophe model is demand surge (or "price gouging"). Because this phenomenon is dependent on the time, size and location of the event, it should not be incorporated in the damage functions except to the extent it is "expected." For example, most models underestimated the actual losses from Hurricane Andrew. If the models were adjusted to exactly reproduce Andrew's losses, they would effectively include a provision for factors that were specific to Andrew and are not expected in the long run, for example:

- inflation in reconstruction costs due to the excess of demand over supply
- excess claim settlements, as adjuster resources were overwhelmed by the volume of claims.

While these factors can be included separately in the reproduction of a single storm, they should not be part of the base model because they would inappropriately increase the expected level of future losses.

Another issue is storm surge from a hurricane, which as a flood loss is not officially covered by a homeowners policy. However, cynics expect that some adjustment of losses on houses affected will likely construe coverage from wind damage prior to the house being flooded. This can be handled with a small additional factor on those locales in low areas most susceptible to surge. However, from a ratemaking and rate filing standpoint, it is difficult to support much of an increase from a coverage that does not strictly apply to homeowners.

Map 1


Sample Insurance Company
Maximum Wind Speeds by 5 Digit Zip Code
1989, September 22 -- HUGO
Version 2.04 - 09/15/1995
Wind Speeds
by 5 Digit Zip Code
Greater than 130
120 to 130
110 to 120
100 to 110
90 to 100
80 to 90
70 to 80
60 to 70
50 to 60
Less than 50
Zip code boundary file copyrighted by GDT

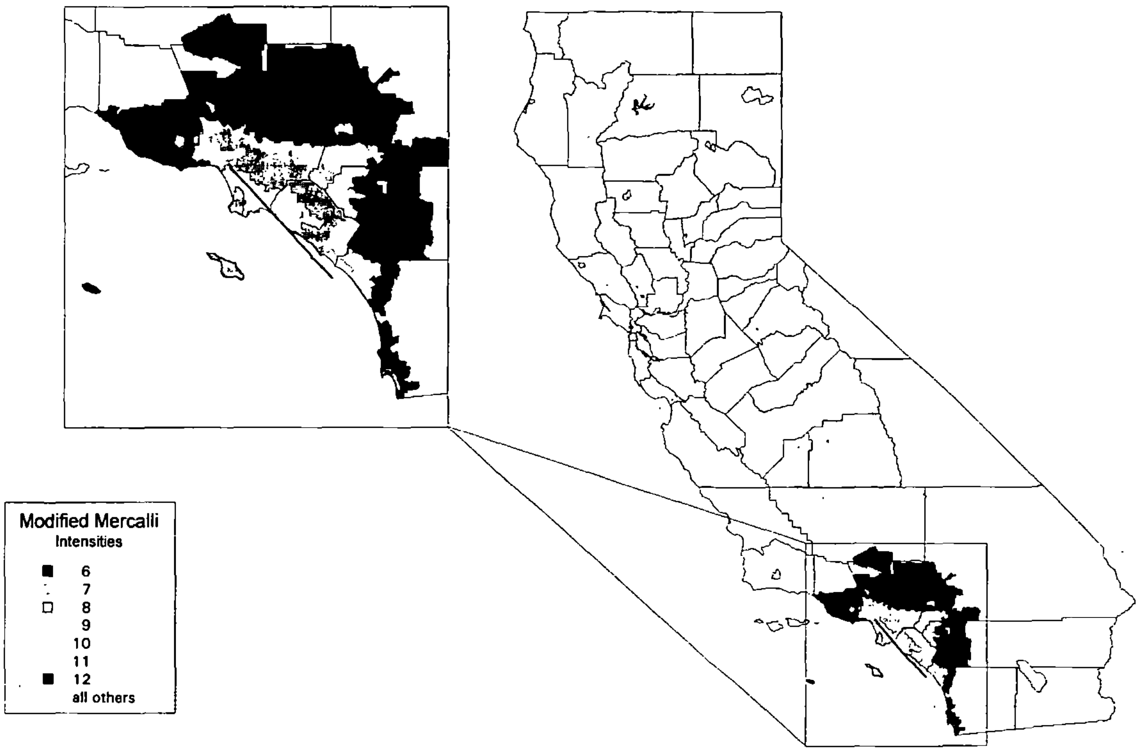
Appendix A

Map 2

Sample Insurance Company

Version: 2.04 - 08/10/1995

Shaking Intensities for California Region - Newport Inglewood Fault - Magnitude 7.50



Zip Code and County Boundaries copyrighted by GDT

Densities based on Zip Code, County Boundaries overlaid on top.

HOW OTHER PERILS ARE MODELED

Earthquake

Given that the library of historical earthquake events producing significant insured losses is scant compared to historical hurricane events, it is generally not expected that the level of precision of a computerized earthquake model will soon reach that of a hurricane model. Nevertheless, numerous models have been developed and a great amount of research has been done to define the various factors and relationships at play.

In the science module, the modeler attempts to reproduce the event by simulating shaking intensities in a ZIP code. As a starting point, the magnitude of an earthquake is generally expressed as a unit on the Richter scale. This implies a rupture length on a fault. Using other factors such as distance to the rupture, soil conditions and the liquefaction potential of the areas affected, the model estimates the shaking intensity for each ZIP code. The resulting shaking intensities are then usually converted to the Modified Mercalli Intensity (MMI) scale. This conversion is made necessary by the fact that most models use the ATC-13 damage functions as a starting point in their models.

The insurance module for an earthquake model is generally similar to a hurricane model. However, the use of percentage deductibles (which is not common on a standard homeowners policy) and separate coverage deductibles present a new twist to the equation. Hence, the model developed must have the capability of handling various deductible combinations. For instance, some earthquake policies apply a building deductible that is distinct from the contents deductible and the additional living expense deductible. A good model will apply the deductible credit separately for each coverage.

The insured loss data available to validate an earthquake model is more limited than for hurricanes. Also limiting is the fact that earthquakes are not all similar. For instance, most major faults in California have been of the strike-slip type. Yet the 1994 Northridge quake was a "blind" thrust-fault earthquake. These two types of earthquakes are by their nature very different and will cause a modeler to adjust the event generator model to reflect different shaking intensities.

Once the deterministic earthquake model has been developed, a probabilistic version must be generated. For earthquake modeling, a set of known faults is generally used as a starting point in building the library of events. Events of various strengths and locations are simulated for each fault. A probability is then assigned to each event in the library. These probabilities are generally expressed in a return time format such as 1 in 400 years. They can be obtained from geological sources such as the United States Geological Survey.

The Northridge event highlighted the fact that serious damage could be caused by earthquakes not located on well-known fault systems. This has implications for earthquake ratemaking because, while the frequency of these events is very much unknown at this time, inclusion of this type of event could increase the expected loss costs substantially. However, the modeler needs to take care that the long-run frequency of earthquakes remains reasonable.

Tornado and Hail

The actual loss experience of tornadoes and hailstorms is more readily available than for any other type of natural catastrophe. Given that there are roughly 1,000 tornadoes in the U.S. each year, the traditional way of developing a tornado/hail catastrophe loading in states with exposure to these perils has been to spread the actual loss experience over a number of years. However, this methodology does not get at the essence of why catastrophe modeling is the preferred approach, which is to estimate the current loss potential of a company given its distribution of exposures. Also implicit in any modeling approach is the simulation of events that have not occurred but are reasonably foreseeable given the historical database of events.

Tornadoes and hailstorms are typically generated by inland storms where moist, warm air masses collide with cooler, drier air masses. Such conditions are often present in the southwestern United States (northern Texas, Oklahoma) and the plains states (Iowa, Kansas, to cite some specific examples) where the Gulf of Mexico provides a continuous source of warm, moist air, and the Rocky Mountains create a source of cooler drier air as weather systems move over them. Tornadoes do, however, occur in all fifty states.

An inland storm capable of generating tornadoes may create one, or tens, of individual funnels over a widely dispersed area. A single funnel will produce damage over the portion of its track making contact with the earth. The length of that ground contact track can range from tens of feet to two hundred miles or more, and the width of the track within which damage can be produced by that funnel can range from tens of feet to a mile or more.

In order to accurately model the loss effects of a single funnel, it is therefore necessary to consider the small scale (nine digit zip code) location of exposures relative to the funnel path.

Because tornadoes and hailstorms are more sudden and unpredictable than hurricanes, most historical information has been the result of human observation. Current tornado databases generally consist of date and time, initial observed location, path width, path length and storm intensity for each event. Tornado intensity is generally measured on the basis of the Fujita scale, which translates an expected degree of damage to a range of windspeeds. For example, a tornado with a Fujita-scale intensity of F2 will be expected to tear roofs from frame houses. Engineering studies indicate that damage of this intensity can be generated by windspeeds of between 113 and 157 miles per hour.

Tornadoes do not behave like hurricanes. The spinning funnel-shaped updraft of a mature tornado is the most damaging windstorm produced by nature. Hence, the damage relationships at a given windspeed for a tornado are quite different from those of a hurricane. This indicates that the results of engineering and damage studies specific to tornadoes must be collected to develop a representative model.

The development of a hail model resembles that of a tornado model. However, difficulties lie in the definition of what is considered a hailstorm and which hailstorms are associated with tornadoes that are already included in a tornado database. The interpretation of the data present in the databases therefore has a significant impact on the overall frequency assumptions used in both models.

The validation of a tornado and/or a hail model against actual loss experience is dependent on the availability of loss data and on how much differentiation between the two perils is possible. (If this cannot be obtained, the modeler may have to calibrate the models on a combined basis. As a result, this would make the development and justification of territorial loss costs for all severe local storm perils easier.)

Winter Storm

Winter storm and freeze activity has been quite severe over the last few years. As a result, the need for better risk measurement and expected loss calculations has increased. Also, some of the same characteristics as hurricane prompt the use of a catastrophe model to simulate winter storm losses - changes in exposure and longer return periods than in an individual insurer's data base.

However, contrary to the other catastrophe perils, winter storms do not have a specific unit of measure that describes the intensity of a given event, and individual temperature is not the only factor that can describe these events. For example, wide temperature swings and absolute highs and lows over consecutive days have been identified as some of the factors that impact the intensity and duration of these events.

The damage functions associated with winter storms are also very different from those of the other perils. Because little of the damage is structural, damage functions are less severe than those of hurricanes, for example.

Similar to a hurricane model, the creation of a probabilistic database requires simulation of multiple events. While the parameters are different, each event is defined by a location (or landfall), size, intensity and duration.

Because individual winter storms have not been as surplus-threatening as hurricanes or earthquakes, the motivation to develop computer models has not been as high for risk analysis and development of PMLs. However, for ratemaking, this peril is equally as compelling as hurricane towards the use of computer modeling. Not only does it yield better expected loss estimates, but it allows the exclusion of past catastrophes from the normal homeowners ratemaking data base for better stability in rate level indications.

METHODS TO REVIEW CATASTROPHE MODELS IN REGULATORY PROCESS

1. **Review general design of model**
   - Examine the credentials of the modeler
   - What is the scientific basis for the key event simulation?
   - What is the engineering support for the damage factors produced by each event severity?
   - Are the insurance limitation features reasonable, e.g., deductibles, coinsurance and reinsurance calculations?

2. **Examine event simulation module**
   - What are the credentials of the scientists who specified it?
   - Has their work been published and/or peer reviewed?
   - What special insights are they offering on the particular event to be simulated?

3. **Test event generator's ability to simulate known past events**
   - Use published information from some critical events, such as Hurricanes Andrew and Hugo, the Loma Prieta earthquake (1989) or even the 1906 San Francisco earthquake
   - Input some key parameters, such as central pressure, landfall, speed and radius of maximum wind, and examine the output windfield at various locations compared to published information on windspeeds. This can be done for any event, even if no current estimates of insured losses are available, as a test of the event simulation accuracy.

4. **Check key input distributions**

- Compare the distributions of key input values among the different modelers, to see if there is any disparity in the key drivers of results. For hurricanes, a possible approach could be to look at the:
  - Distributions of central pressure at ten millibar intervals: 900-909, 910-919 etc.
  - Distributions of radius of maximum winds in five nautical mile ranges, and forward speeds in five knot ranges.
  - Probabilities of landfall for all storms affecting the state (direct hit and nearby landfalls).

5. **Conduct sensitivity checks**

- Use a few sample events
- Promulgate a sample exposure base statewide (e.g., 25 risks)
- Vary the parameters one at a time, or perhaps a few in pairs
- Observe changes is output (insured losses) for incremental changes in input
- The goal is a rough measurement of the effect of changing inputs (e.g., central pressure, radius of maximum winds, forward speed)

6. **Verify damage and insurance relationship functions**

- Examine the credentials of the engineers
- Has the analysis been published and/or peer reviewed?
- Analyze the damage curves (functions of increasing damage for increasing event intensity) separately for types of exposure, class and coverage
- Review the insurance module for effects by deductible and reinsurance or coinsurance
- Review the validation of the two components (damage and insurance effects) via multiple events over the past few years for multiple insurers; each event does not have to be replicated, but that they should average out over all events and all insurers.

7. Test output for hypothetical new events

- Select some new events defined by key parameters
- Use a sample database of exposures by ZIP code
- Compare results for different modelers and ask outside experts for their opinions on the reasonableness of these results.

8. Compare indicated loss costs for different modelers

- Select sample ZIP codes throughout the state
- Have modelers run all events with probabilities for those ZIP codes
- Use several base classes and coverages:
    - homeowners, $100,000 frame house, $250 deductible
    - tenants, $30,000 contents, masonry, $250 deductible
    - businessowners, $200,000, masonry, $1,000 deductible
- Compare modelers' loss costs per $1,000 of coverage by ZIP code
- Ask outliers to explain large differences from average.

9. Conduct on-site due diligence and review of key assumptions

- View a live running of the model, with actual input data
- Review input data sources — published and non-published
    - all key input parameters
    - frequency of events by location
    - key damage factors and sources
- Review output, including color coded maps showing ranges of expected loss costs.