CASUALTY ACTUARIAL SOCIETY
FORUM

Spring 1996
Including the
Prize and Call Papers on
Dynamic Financial Models

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NOTICE

The Casualty Actuarial Society is not responsible for statements or opinions expressed in the papers or reviews in this publication. These reports, papers, and reviews have not been reviewed by the CAS Committee on Review of Papers.

1996 CAS *Forum,* Spring 1996 Edition Including the Prize and Call Papers on Dynamic Financial Models

The CAS Subcommittee on Dynamic Financial Models is pleased to present the results of the Call for Papers on Dynamic Financial Models of Property/Casualty Insurers.

As an adjunct to the Call for Papers, the Casualty Actuarial Society established a prize pool of up to $10,000 for papers describing a dynamic financial model which has actually been applied to a property/casualty insurer. The authors who participated in the prize paper program will present their papers at the CAS Seminar on Dynamic Financial Analysis, July 15-16, Montreal Bonaventure Hilton in Montreal, Quebec.

The prize has been awarded to:

> Douglas M. Hodes, J. David Cummins, Richard D. Phillips, Sholom Feldblum, FCAS, and Tony Neghaiwi, FCAS, for "The Financial Modeling of Property/Casualty Insurance Companies;" and
>
> Stephen P. Lowe, FCAS, and James N. Stanard, FCAS, for "An Integrated Dynamic Financial Analysis and Decision Support System for a Property Catastrophe Reinsurer."

The Subcommittee on Dynamic Financial Models is an adjunct to the CAS Valuation and Financial Analysis Committee (VFAC), chaired by Susan E. Witcraft, FCAS. This project was begun under the leadership of the former VFAC chairperson, Susan T. Szkoda, FCAS. The Subcommittee's first report, "Dynamic Financial Models of Property/Casualty Insurers," was published in the CAS *Forum*, Fall 1995 Edition.

Members of the subcommittee are indebted to Michael J. Miller, FCAS, for the creation of the prize competition, and for securing the funding for the prize. The papers were judged by a panel of six, including four subcommittee members who had no connection to any of the entries, and two volunteers. The judges were not informed of the names of the authors until the judging process was completed. We are indebted to Robert J. Finger, FCAS, and Richard I. Fein, FCAS, for their unswerving assistance to the subcommittee.

This edition of the *Forum* is organized as follows.

- The two prize-winning papers appear first.
- The remainder of the papers competing for the prize appear in alphabetical order based on the last name of the principal author.
- The remainder of the call papers appear in alphabetical order based on the last name of the principal author.

Our thanks go to all of the authors, without whom we would have been unable to present this fine book. And our particular appreciation goes to the authors of the excellent prize-winning papers.

Respectfully Submitted,

Oakley E. Van Slyke, FCAS, ASA

Chairperson, the CAS Subcommittee on Dynamic Financial Models

Apology

It has been brought to the attention of the members of the Committee for the Casualty Actuarial Society *Forum* that the paper, "A Survey of Methods Used to Reflect Development in Excess Ratemaking," by Stephen W. Philbrick, FCAS, and Keith D. Holler, FCAS, that was published in the CAS *Forum*, Winter 1996 Edition, Including the Ratemaking Call Papers, contained data tables on pages 288 and 289 that were the property of the Insurance Services Office, Inc. The authors included the tables in their paper without first obtaining the proper permission from the Insurance Services Office, Inc., to do so.

Members of the Committee for the CAS *Forum* apologize for this and are grateful that officials at the Insurance Services Office, Inc., have subsequently allowed the tables to appear in that paper.

The Casualty Actuarial Society *Forum*

The Casualty Actuarial Society *Forum* is a non-refereed journal printed by the Casualty Actuarial Society. The viewpoints published in the *Forum* do not necessarily reflect the views of the Casualty Actuarial Society.

The Casualty Actuarial Society *Forum* is edited by the members of the Committee for the Casualty Actuarial Society *Forum*. The committee invites all members of the Casualty Actuarial Society to submit papers on topics of interest to the actuarial community. Articles need not be written by a member of the Casualty Actuarial Society, but the content should be relevant to the interest of the CAS membership.

Members of the Committee for the Casualty Actuarial Society *Forum* request that the following procedures be followed when submitting an article to be published:

1. Authors should submit a camera-ready original paper, and two copies.
2. Authors should not number their pages.
3. All exhibits, tables, charts, and graphs should be in original format and camera-ready.
4. Authors should avoid using gray-shaded graphs, tables, or exhibits. All text and exhibits should be in black and white.
5. Authors are responsible for obtaining proper copyright permission for all elements of their contributions.

The Casualty Actuarial Society *Forum* is printed on a periodic basis, based on the number of articles submitted. Its goal is to publish two editions during the calendar year.

All comments or questions may be directed to the Committee for the Casualty Actuarial Society *Forum*.

The CAS Subcommittee on Dynamic Financial Models

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Richard D. Phillips, Ph.D., is an assistant professor of Risk Management and Insurance at Georgia State University. Dr. Phillips's recent publications on topics related to insurance company financial modeling include "Financial Pricing of Insurance in the Multiple-Line Insurance Company" (with J. David Cummins), and "The Economics of Risk and Insurance."

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The Financial Modeling of Property/Casualty Insurance Companies

by Douglas M. Hodes
Tony Neghaiwi, FCAS
J. David Cummins
Richard Phillips
Sholom Feldblum, FCAS, ASA

THE FINANCIAL MODELING OF PROPERTY-CASUALTY INSURANCE COMPANIES

by

Douglas M. Hodes,
Tony Neghaiwi,
J. David Cummins,
Richard Phillips, and
Sholom Feldblum

Abstract

This paper describes a financial model currently being used by a major U.S. multi-line insurer. The model, which was first developed for solvency monitoring purposes, is now being employed for a variety of internal management purposes, including (i) the allocation of equity to corporate units, thereby allowing measurements of profitability by business segment and by policy year, as well as analysis of the progression of "free surplus," (ii) the analysis of major risks, such as inflation risks, interest rate risks, and reserving risks, that have heretofore been difficult to quantify, and (iii) consideration of varying scenarios on the company's financial performance, both of macroeconomic conditions as well as of the insurance environment.

This paper begins with the genesis of the model and with its structure. It moves on to equity considerations and to performance measurement. It then discusses the major risks that have heretofore resisted actuarial analysis, such as interest rate risk (inflation risk), reserving risk, and scenario testing. The paper shows how cash flow financial models can deal with global risks that simultaneously affect various aspects of the insurer's operations, delineating the resulting changes in the company's performance.

The Financial Modeling of Property-Casualty Insurance Companies

(Authors)

Douglas M. Hodes is a Vice President and Corporate Actuary with the Liberty Mutual Insurance Company in Boston, Massachusetts. He oversees the Corporate Actuarial and Corporate Research divisions of the company, and he is responsible for capital allocation, financial modeling, surplus adequacy monitoring, and reserving oversight functions.

Mr. Hodes is a graduate of Yale University (1970), and he completed the Advanced Management Program at Harvard University in 1988. He is a Fellow of the Society of Actuaries, a member of the American Academy of Actuaries, a member of the American Academy of Actuaries Life Insurance Risk-Based Capital Task Force, and a former member of the Actuarial Committee of the New York Guaranty Association. Before joining Liberty Mutual, Mr. Hodes was a Vice President in Corporate Actuarial at the Metropolitan Life Insurance Company, where his responsibilities included the development of a life insurance financial model.

Mr. Hodes is the author of "Interest Rate Risk and Capital Requirements for Property-Casualty Insurance Companies" (with Mr. Sholom Feldblum) and of "Workers' Compensation Reserve Uncertainty" (with Dr. Gary Blumsohn and Mr. Feldblum). These papers apply actuarial and financial techniques to quantify risks associated with interest rate movements and with unexpected reserve developments. In addition, Mr. Hodes is a frequent speaker at actuarial conventions on such topics as dynamic financial analysis and risk-based capital.

* * * * *

Tony Neghaiwi, FCAS, is an Associate Actuary with the Liberty Mutual Insurance Company, where he is responsible for the development and the maintenance of the model described in this paper.

* * * * *

Dr. *J. David Cummins* is the Harry J. Loman Professor of Insurance and Risk Management and Executive Director of the S. S. Huebner Foundation for Insurance Education at the Wharton School of the University of Pennsylvania. Dr. Cummins' primary research interests are the financial management of insurance companies, the economics of insurance markets, and insurance rate of return and solvency regulation. He is the editor of the *Journal of Risk and Insurance* and past president of the American Risk and Insurance Association and the Risk Theory Society.

Dr. Cummins has written or edited fourteen books and published more than forty journal articles in publications such as the *Journal of Finance*, *Management Science*, the *Journal of Banking and Finance*, the *Journal of Economic Perspectives*, the *Journal of Risk and Uncertainty*, and the *Astin Bulletin*. Among his recent publications are "Insolvency Experience, Risk-Based Capital, and Prompt Corrective Action in Property-Liability Insurance," *Journal of Banking and Finance* (1995); "Pricing Insurance Catastrophe Futures and Call Spreads: An Arbitrage Approach," *Journal of Fixed Income* (1995); and "Capital Structure and the Cost of

Equity Capital in Property-Liability Insurance," *Insurance: Mathematics and Economics* (1994). His paper, "An Asian Option Approach to Pricing Insurance Futures Contracts," was awarded the Best Paper prize at the 1995 AFIR Colloquium in Brussels, Belgium.

Dr. Cummins has served as consultant to numerous business and governmental organizations. He has consulted and testified on the cost of capital in insurance for organizations such as the National Council on Compensation Insurance and Liberty Mutual Insurance Group. He has conducted research on insurance cycles and crises for the National Association of Insurance Commissioners, and he has testified for several state departments of insurance and the U.S. Department of Justice. He advised the Alliance of American Insurers on risk-based capital in property-liability insurance.

* * * * *

Dr. *Richard D. Phillips* is an Assistant Professor of Risk Management and Insurance at Georgia State University. He received his Ph.D. in Finance and Insurance in 1994 from the Wharton School, University of Pennsylvania.

Dr. Phillips's recent publications on topics related to insurance company financial modeling include "Financial Pricing of Insurance in the Multiple-Line Insurance Company" (with J. David Cummins) and "The Economics of Risk and Insurance." From March 1992 through January 1994, Dr. Phillips was a principle investigator in the Alliance of American Insurers project to develop a risk-based capital cash flow solvency model, which used an earlier version of the model described in this paper.

Dr. Phillips is a frequent speaker at research seminars on such topics as cash flow modeling, financial pricing, and insurance regulation.

* * * * *

Sholom Feldblum is an Assistant Vice President and Associate Actuary with the Liberty Mutual Insurance Company. He holds the FCAS, CPCU, ASA, and MAAA designations, and he is a member of the CAS Syllabus Committee and of the American Academy of Actuaries Task Force on Risk-Based Capital. He is the author of numerous papers on ratemaking, loss reserving, statutory accounting, insurance economics, competitive strategy, investment theory, solvency monitoring, and finance, which have appeared in *Best's Review*, the *CPCU Journal*, the *Proceedings of the Casualty Actuarial Society*, the *Actuarial Digest*, the *CAS Forum*, the *Journal of Insurance Regulation*, the *Journal of Reinsurance* and the *CAS Discussion Paper Program*. He was the recipient of the CAS Michelbacher Prize in 1993 for his paper on "Professional Ethics and the Actuary."

In addition to the two papers co-authored with Douglas Hodes (see above), Mr. Feldblum is the author of "European Approaches to Insurance Solvency," which describes the British and Finnish foundations upon which the model described in this paper is based, and "Forecasting the Future: Stochastic Simulation and Scenario Testing," which describes the use of scenario testing in financial models.

THE FINANCIAL MODELING OF PROPERTY-CASUALTY INSURANCE COMPANIES

Introduction

The existing literature on the financial modeling of property-casualty insurance companies consists predominantly of theoretical discourses seen through the eyes of the research actuary. The sophistication of complex stochastic simulation is extolled; the practical implementation of the models is rarely considered.

This paper, in contrast, describes a financial model currently being used by a major U.S. multi-line insurer. The first version of the model was developed in 1993 for solvency monitoring purposes. In the three years since then, the model has been greatly expanded and it has been applied to a variety of internal management uses, including (i) the allocation of equity to corporate units, thereby allowing measurements of profitability by business segment and by policy year, as well as analysis of the progression of "free surplus," (ii) the analysis of major risks, such as inflation risks, interest rate risks, and reserving risks, that have heretofore been difficult to quantify, and (iii) consideration of varying scenarios on the company's financial performance, both of macroeconomic conditions as well as of the insurance environment.

Many multiline insurance enterprises are complex organizations, with dozens of distinct yet interrelated parts. This complexity is the major stimulus for financial models that consider the workings of the entire corporation. At times, however, this complexity renders cumbersome the documentation of the models. To facilitate the readability of this paper, the numerical exhibits are contained in the appendices, so that the text flows more easily.

This paper discusses the following topics:

- Genesis: that is, the factors that stimulated the development of the model.
- Structure: that is, the types of underwriting and financial operations and the types of time periods with which it deals. Since this paper is not just a theoretical discourse but also a practical description of a working model, it shows the actual inputs and outputs: what variables must be provided by the user, and several types of tables, charts, and graphs that are produced by the model.
- Equity considerations: how net worth ("economic surplus") is determined by line of business (LOB) and how the progression of "free surplus" is viewed.
- Profitability measures: given the actual (past) or expected (future) cash flows, along with the progression of LOB surplus and of free surplus, how profitability is measured.

The financial model described here is particularly important for evaluating three types of risk that are not easily analyzed by other methods:

- Risks that simultaneously affect several components of an insurance company's operations, such as inflation risks and interest rate risks.
- Risks that results from an overall change in the external economic environment, such as recessions, or from changes in the insurance industry as a whole, such as underwriting cycle movements.

- Risks that depend on complex, random fluctuations, such as reserving risks.

This paper shows how the financial model deals with these types of risk.

Genesis of the Model

The company's modeling efforts were stimulated by several developments:

1. From 1990 through 1993, the NAIC developed new risk-based capital requirements for both property-casualty and life insurance companies. Many observers have criticized the NAIC efforts from three perspectives:

   A. The risk-based capital formulas are based on accounting figures.
   B. Some of the RBC charges seem to be "ad hoc" factors, lacking actuarial or financial justification.
   C. Several important risks are not even considered.

   For example, these critics have said that

   A. The statutory financial statements that underlie the risk-based capital formulas should be replaced by cash flow approaches or by market value accounting, both for solvency monitoring by state regulators and for management evaluation of the company's performance.[1]

   B. The reserving risk charges in the NAIC formula, which are based on the NAIC "worst case year" method coupled with a large dose of "regulatory judgment," should be replaced by rigorous actuarial analyses of reserve variability. Similar analyses should be undertaken for the underwriting risk of new business ("written premium risk" in RBC terminology) and for the risks of reinsurance collectibility.

   C. Interest rate risk, which affects both assets and liabilities, should be incorporated into the formula. Interest rate risk is particularly difficult to model in the NAIC formula, since (i) it is a market value phenomenon, not an accounting phenomenon, and (ii) it is intertwined with other risks, such as inflation risks and reserving risks.

2. Meanwhile, the American Academy of Actuaries has proposed an expanded vision of the Appointed Actuary's role, covering not just opinions on the reasonableness of loss and loss adjustment expense reserves but also statements on the financial strength of the insurance enterprise under varying longer term scenarios and on the resilience of the company to different types of adverse external conditions. The model described in this paper is the practical implementation of the AAA vision: it shows the cash flows of the company under varying future scenarios.

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[1] Compare especially Robert P. Butsic, "Solvency Measurement for Property-Liability Risk-Based Capital Applications," *Journal of Risk and Insurance*, Volume 61, Number 4 (December 1994), pages 656-690, who discusses the "measurement bias" introduced when GAAP or statutory accounting statements are used for solvency monitoring purposes.

❸ Soon after this model was implemented, the authors changed their emphasis from solvency monitoring to profitability measurement. When insurance companies fare poorly, financial models are important for monitoring solvency. In the early 1990's, the multi-line insurer using this model fared extremely well, because of both strong industry profits in its major lines of business and its own favorable performance relative to its peer companies. It elected to expand into new markets, develop new products, and acquire other (related) businesses. It required a sophisticated management model, in order to judge both the immediate risks and the long-term uncertainties associated with the new projects, as well as the capital needed to safely undertake them.

Description of the Model

The financial model described here provides three types of results:

❶ The model itself uses a *cash-flow* approach, following the method developed by the British Solvency Working Party in the 1980's.[2] The cash flow results are particularly important for Appointed Actuary work and for comparing the effects of different scenarios.

❷ For management purposes, the model can generate statutory accounting results, as would be needed for pro-forma financial statements. Statutory accounting is an important constraint on insurance company strategy. These results are useful for analyzing the progression of "free surplus."[3]

❸ By selecting appropriate discount rates for loss outflows and for investment inflows, the analyst can determine market values of the insurance enterprise at various points in time

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[2] For a more complete presentation of the British Solvency Working Party approach, see Chris D. Daykin, G. D. Bernstein, S. M. Coutts, E. R. F. Devitt, G. B. Hey, D. I. W. Reynolds, and P. D. Smith, "Assessing the Solvency and Financial Strength of a General Insurance Company," *Journal of the Institute of Actuaries*, Volume 114, Part 2 (1987), pages 227-310; Chris D. Daykin, G. D. Bernstein, S. M. Coutts, E. R. F. Devitt, G. B. Hey, D. I. W. Reynolds, and P. D. Smith, "The Solvency of a General Insurance Company in Terms of Emerging Costs," in J. David Cummins and Richard Derrig, *Financial Models of Insurance Solvency* (Boston: Kluwer Academic Publishers, 1989), pages 87-149, or in *ASTIN Bulletin*, Volume 117, No. 1 (1987), pages 85-132; Chris D. Daykin and G. B. Hey, "Managing Uncertainty in a General Insurance Company," *Journal of the Institute of Actuaries*, Volume 117, Part 2, No. 467 (September 1990), pages 173-259. The recent text by Chris D. Daykin, Teivo Pentikäinen, and M. Pesonen, *Practical Risk Theory for Actuaries* (Chapman and Hall, 1994), combines the cash flow approach of the British Solvency Working Party and the accounting approach of the Finnish Working Party. In addition, that textbook emphasizes stochastic procedures to develop scenarios, whereas the model described here uses stochastic procedures for risks that are random and "scenario building" for global risks with interdependent elements.

[3] For reasons of space, the translation of net cash flows and market values into statutory values is not shown in the exhibits in this paper. The required work is primarily accounting, not actuarial, and it is not germane to the theoretical framework of the model.

or under various scenarios. These results are important for determining profitability of existing and of new business.

Past and Future Business

For *past business*, the model uses actual company results, along with

- chain ladder paid loss development for the run-off of existing reserves,
- stated coupon rates for fixed income securities, and
- expected dividend yields on common stocks for investment returns.

Two further adjustments are made:

❶ The company has large investments in mortgage-backed securities, with high prepayments as borrowers change homes or simply refinance their mortgages when interest rates are low. The expected cash flows are adjusted in each scenario for these options, and the effects are shown in the exhibits. Similar adjustments are used for other options, such as call provisions in corporate bonds.[4]

❷ About half of the company's workers' compensation business is written on loss sensitive contracts. The premium payment patterns extend for about ten years after the policy expires, as shown in the exhibits.

For *future years' operations*, the cash flows are based on a combination of company business plans and actuarial projections. For instance, written premium by line of business is taken from the business plans. The anticipated loss and LAE ratios and the anticipated underwriting expense ratios are actuarial projections. These figures are combined with the payment and collection *patterns* developed from past business to model the cash flows from new business.

Base Case and Alternative Scenarios

To illustrate the power of the financial model, two scenarios are shown in the exhibits and discussed in the text.

❶ The *base case scenario* assumes an annual inflation rate of 4.0% and growth in real exposures of 2.0%, for a nominal growth in underwriting cash flows of 6.1% per annum. These assumptions affect premiums, losses, and expenses for each line of business. In practice, of course, the assumed growth in real exposures will vary by line, depending on the company's business plans. [The model allows for separate assumptions by line and by policy year, which are used in actual work.]

The average pre-tax yield on the bond portfolio held by the company is 8.3% per annum. The assumed stock dividend yield is 2.75% per annum, and the rate of growth in stock values is 8.0% per annum, providing an annual return on common stocks of 10.75%.

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[4] These expected cash inflows are similar to those required in the new NAIC risk-based capital "supplementary asset schedule" used to measure interest rate risk; see Douglas M. Hodes and Sholom Feldblum, "Interest Rate Risk and Capital Requirements for Property-Casualty Insurance Companies" (CAS Part 10 examination study note).

The federal income tax rate is 35%. Since income taxes are explicitly included in the cash flows, the model uses an after tax discount rate of 5.4% to determine the present values of insurance operations (when present values are used in the analyses). [5.4% is 8.3% * (1−35%).] The expected after-tax yield on the company's investment portfolio is about 5.7%, reflecting the higher returns on the common stocks.

❷ The *alternative scenario* assumes that the inflation rate increases by 200 basis points to 6.0% with a concomitant increase in the pre-tax bond yield on new investments to 10.3%. [The market value of existing fixed-income securities, of course, falls when the interest rate rises, with the magnitude of the effect depending on the duration of the fixed income portfolio.] The growth in real exposures remains 2.0%, for a nominal growth in underwriting cash flows of 8.1%. The immediate effect on underwriting and investment results is twofold:

A. For each line of business, the new rates affect premiums, losses, and expenses. When rates first increase, however, the nominal losses grow more quickly than premiums, leading to an initial increase in the loss ratios.

B. Market values of bonds and of mortgages, as well as of mortgage-backed securities, fall when interest rates rise. Initially, common stock prices also drop when interest rates increase, as noted by many investment economists.[5] This decline, however, is fully recovered in the subsequent two years, since the simultaneous rise in inflation and interest rates causes no change in the real equity value of corporations.

The federal income tax rate remains 35%. The after tax discount rate for insurance cash flows now changes to 6.7%.

The summary assumptions for the base case scenario and for the alternative scenario are shown on Exhibit 1 of Appendix A. Exhibits 2 through 5 of Appendix A show the projected written premium, incurred loss plus loss adjustment expense, and other underwriting expenses, as well as the loss ratios, expense ratios, and combined ratios, for new business, under each of the two scenarios.

The exhibits shows ten years of new business, as would be used in a "going-concern" valuation. For clarity of exposition, the text discusses a single year of new business (policy year 1995), though we show exhibits and graphs for ten years of new writings as well. For the "progression of free surplus," the exhibits also show the anticipated 1996 written premiums.

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[5] See, for example, Eugene F. Fama and G. William Schwert, "Asset Returns and Inflation," *Journal of Financial Economics*, Volume 5 (1977), pages 115-146. Fama and Schwert's paper uses data which is now 20 years "out-of-date." Other analysts have replicated the Fama and Schwert results, though the theoretical explanations vary from author to author; see, for instance, Martin Feldstein, "Inflation and the Stock Market," *American Economic Review*, Volume 70 (December 1980), pages 839-487. To parameterize our model, we replicated the Fama and Schwert study using the most recent 20 years of data from the Ibbotson and Sinquefield indices. The signs of the coefficients in our analysis were generally consistent with the signs found by Fama and Schwert, though the magnitudes of the coefficients were dampened.

Asset Returns

The financial model uses the expected cash flows from each group of securities, not the stated cash flows. The difference is particularly great for mortgages and for mortgage backed securities (which form significant proportions of life insurance and property-casualty insurance investment portfolios, respectively), for two reasons:

1. As borrowers move to different homes, they pre-pay the mortgages. This effect occurs even if interest rates do not change. It is dependent on interest rate changes to the extent that real estate purchases depend on the availability of "affordable mortgages."[6]

2. When interest rates decline, many homeowners refinance their mortgages. Conversely, when interest rates rise, refinancings become less frequent.

The rise in interest rates under the "alternative scenario" has two effects on the market value of mortgage backed securities.

1. Since the payment obligations are in fixed dollar terms, but the appropriate discount rate rises, the market value of these assets decline.

2. When interest rates rise, refinancings become less frequent, causing a further decline in the market value of the assets.

Similar analyses are performed for each class of securities. The model requires cash flows by type of security for each scenario. One begins with the stated cash flows from each category of securities, before consideration of issuer options. For each scenario, the model then incorporates the effects of options, such as calls on corporate bonds and pre-payment options on mortgages.[7]

Exhibits 2 and 3 of Appendix B show graphically the effects of a 2% rise in interest rates on the *cash flows and remaining balances* from mortgage backed securities. Exhibits 4 and 5 of Appendix B shows the overall effects of a 2% rise in interest rates on the entire fixed income portfolio.

Exhibit 1 of Appendix B shows graphically the effect of a 2% rise in interest rate on the *market value* of mortgage backed securities. The effect is divided into two pieces. The dominant portion of the decline in value, from the third bar to the middle bar, stems from the higher discount rate. A second portion of the decline in value, from the middle bar to the top bar, stems from the fewer prepayments.

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6 On the effect of interest rates on real estate values, see Charles A. D'Ambrosio's chapter in J. L. Maginn and D. L. Tuttle (editors), *Managing Investment Portfolio: A Dynamic Process*, Second Edition (Warren, Gorham, and Lamont, 1990).

7 Because of federal income tax provisions, the model is substantially more complex than is described here. Fixed income investments are divided between taxables and tax-exempts (e.g., municipal bonds), and the latter category is further subdivided by date of acquisition (i.e., pre-August 6, 1986 and post August 6, 1986).

The model performs this analysis for each type of security. Since federal income taxes have a great effect on net income, the analysis is performed separately for taxable versus tax-exempt bonds, with the appropriate tax rates applied to each.

Combining Operations

The model described here provides numerous advantages over other analyses of an insurance company's operations:

❶ The different components of the company, such as premium inflows, investment inflows, loss outflows, and expense outflows, are combined. For instance, Exhibits 1 and 2 of Appendix C show the cash flows in future years from the run-off of existing workers' compensation business under each of the two scenarios mentioned above. [Exhibits 3 and 4 show the individual cash flows separately for each component of the company for two years, for the pure run-off case versus the run-off with one additional policy year, and for the base scenario versus the alternative scenario. Exhibits 5 through 10 show the combined cash flows for assets versus liabilities for 35 years, for the pure run-off case versus the run-off with one additional policy year, and for the base scenario versus the alternative scenario. Exhibit 11 shows the loss and loss adjustment expense cash flows for the two scenarios.]

Workers' compensation provides a particularly good example of the model's operations, so we use this example repeatedly in the paper. The statutory benefits in workers' compensation make a chain ladder paid loss development reserving procedure accurate for this line of business, since the cash outflow patterns for loss reserves are relatively stable. The dominance of retrospectively rated contracts in this company's workers' compensation book of business causes retrospective premium payments for about ten years past the expiration date of the policies. Since the company uses primarily paid loss retros for its large accounts, a cash flow approach is most useful.

The expected cash flows from assets are projected by the company's Chief Investment Officer and his staff, and they are incorporated as inputs into the model. Since the company has a large life insurance subsidiary, these cash flow projections under different interest rate scenarios are needed for the "asset adequacy analysis."[8]

Existing vs New Business

❷ Equally important as the combination of the different components of the company is the *differentiation of blocks of business*. In particular, the model separates (i) the cash inflows and outflows from the run-off of the current business from (ii) the cash inflows and

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[8] Life insurance companies must have an "asset adequacy analysis," signed by a Fellow of the Society of Actuaries, showing whether the income of the company from its investments will suffice to meet benefit obligation under sever future interest scenarios. These cash flow projections must take into account both issuer options on the asset side and policyholder options on the liability side. For the property-casualty model, we use the same cash flow projections on the asset side. The casualty liabilities do not have the complication of policyholder options, but they have other complexities, such as inflation sensitivity and dependence on macro-economic conditions. See below in the text for the manner in which these are handled.

outflows resulting from new business. Previous analyses often asked: "Is the company's workers' compensation book of business profitable?" This question is not just simplistic; it misses the point. Rather, the actuary must ask two sets of questions:

1A. What are the net cash flows from the run-off of the existing book of business? We are concerned with business already written, not business already earned. Thus, this "run-off" includes both

- the future (retrospective) premium collections from exposures already earned and payments for losses already incurred, as well as
- the earnings from the unexpired portions of policies already written and the expected loss accruals from these policies.

1B. If external conditions change, how would the net cash flows change, and what is the implied change in profitability for the run-off of existing business?

2A. What are the net cash flows from an additional policy year of business, or from additional policy years of business? As noted above, the future earning of premium and accrual of losses from the most recent policy year is included in the "run-off" section. The present question ties the valuation analysis to the current underwriting procedures, policy provisions, and premium rates. We are asking: "Based on the company's current business plans and our actuarial projections, what are the expected net cash flows from the new business?"

2B. If external conditions change, how would the net cash flows from the new business change, and what is the implied change in profitability of this business?

The difference between questions 2A and 2B is crucial for the valuation analysis, and it demonstrates the power of the financial model. All the exhibits in this paper differentiate between the effects on the existing business and the effects on new business. For instance, Exhibits 5 and 6 of Appendix C shows the investment and the net liability cash flows for the base case scenario for (i) the run-off of existing business and (ii) the run-off of existing business plus one year of new business. In the former case, the net liability cash outflows greatly exceed the investment income cash inflows in the first subsequent calendar year (1995 in the exhibits), since most of the premium has already been collected whereas most of the losses remain to be paid. In the latter case, the net liability cash outflow is small, since the premiums from the new business nearly equal the total loss and expense payments in that year.

External changes have vastly different effects on the run-off of existing business versus the profitability of new business. For example, an important external change that affects property-casualty insurance company profitability is a movement in inflation, with a concomitant movement in interest rates, as we have in the two scenarios. This is interest rate risk, which the financial model quantifies. First, however, let us consider the issue conceptually.[9]

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[9] The more sophisticated analysis described below in this paper quantifies more carefully how loss liabilities are affected by inflationary changes, by calculating "real dollar" link ratios for the chain ladder loss development procedure, projecting future inflation rates that are tied to the assumed future interest rates (which affect the asset values), determining the "inflation

We must consider the effects on the run-off of previously written business versus the effects on new business. We have the following characteristics of workers' compensation business:

Run-Off Valuation and Interest Rate Risk

1. Workers' compensation benefit payments consist of indemnity ("wage-loss") benefits and medical benefits. Incurred losses are about 55-60% indemnity and 40-45% medical. Since medical benefits are paid more quickly, the reserves are about 65-75% indemnity and 25-35% medical.

2. Medical benefits are fully inflation sensitive. If inflation increases by 2%, medical benefits (in nominal terms) will be 2% higher.[10]

3. In about half the U.S. jurisdictions, COLA adjustments make certain indemnity benefits inflation sensitive as well. Generally, the COLA adjustments in these jurisdictions apply only to long-term indemnity benefits, such as benefits two years or more after the accident, and the adjustments are often capped at a relatively low amount, such as 5% per annum.

4. For simplicity, let us assume that overall workers' compensation reserves are 50% inflation sensitive. In other words, a 2% rise in inflation causes *nominal loss costs on previously written business* to rise by about 1%. [As noted earlier, the 1% rise applies to losses paid one year after the valuation date. A loss paid three years after the valuation date would increase by about 3%. The actual model, of course, separately quantifies indemnity and medical workers' compensation benefits and their respective cash flows. The "50%" inflation sensitivity is used in this explanation for heuristic purposes only.]

5. Workers' compensation loss reserves have a long average payment date, generally about 7 to 8 years. [Permanent total claims, or "lifetime pension" cases, form a higher proportion of reserves than they do of incurred losses, thereby greatly lengthening the duration of reserves compared to the duration of incurred losses.] Because of the high retention levels of workers' compensation business and the generally upward sloping yield curves, most companies will choose asset portfolios with longer maturities than the average maturity of the loss reserves. Given the steady benefit outflows in workers' compensation, a common asset liability management strategy would call for high grade corporate bonds or mortgage backed securities with an average maturity of ten years or longer.

6. A 2% rise in inflation, with a corresponding 2% rise in interest rates, would severely

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sensitivity" of each reserve component (such as workers' compensation medical benefits versus workers' compensation indemnity benefits), and then calculating the cash outflows each year.

[10] The textual explanation given here over-simplifies the effects. Since the average payment date of the reserves exceeds one year, the increase in nominal value of reserves exceeds 2%. That is, reserves paid out one year hence will rise in nominal value by 2%; reserves paid out in two years' time will rise by about 4%; and so forth. Since the financial model tracks the cash flows, the true effects are easily seen. The explanation in the text, however, is simplified, to highlight the differences between existing business and new business.

depress the market values of these corporate bonds or mortgage backed securities. The market value of the liabilities would decline by a lower amount, since their nominal value rises (as the liabilities are 50% inflation sensitive) and their duration is shorter.

7. The financial model shows this explicitly, since the cash inflows from the existing bond portfolio do not change in nominal terms, whereas the cash outflows from the reserve portfolio increase in nominal terms. Discounting at the new interest rate shows the loss resulting from interest rate risk.

New Business and Interest Rate Risk

The situation is entirely different for the new business.

1. For simplicity, suppose that inflation increases by 2% just before the new business is written, and interest rates show a corresponding 2% jump. Both medical and indemnity losses will be 2% higher, but since the discount rate is also 2% higher, their economic value does not change. Similarly, the coupon rate on newly issued bonds will be 2% higher, but since the discount rate is also 2% higher, their economic value does not change. There is almost no change to the expected value of new business from interest rate risk.[11]

2. Again, the financial model shows this explicitly. The model shows the cash flows from assets and for benefit payments. There is a 2% increase in the cash inflows from assets and a concomitant 2% increase in cash outflows for benefits, leading to no net change. Alternatively, discounting both sets of flows at the new (higher) discount rate shows no change in the present value of either cash flow.

3. In sum, interest rate risk has a great effect on the run-off of existing workers' compensation business, but little effect (if any) on the expected profitability of new business.

Recessions

There is no reason to assume that changes in the external environment affect only the valuation of run-off business but not the value of new business. Consider the effects of a recession on a workers' compensation carrier.

A recession has two effects on workers' compensation benefit costs.

- During recessions, firms lay off recently hired and inexperienced workers, and overtime work decreases. Conversely, during prosperous years, firms hire young and inexperienced workers, and overtime work increases. Workers' compensation accident frequency is higher for young and inexperienced workers, particularly when they are working long

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[11] Since this is new business, the assets purchased with the newly collected premiums either have higher coupons (for newly issued bonds) or higher yields to maturity (for bonds bought in the secondary market). The change in timing of loss payments and tax payments slightly affect the results for the alternative scenario. In addition, the difference between the increase in workers' compensation benefits and the assume increase in inflation slightly increases the expected profitability.

hours. Thus, accident frequency is higher during prosperous years than during recessions.

Moreover, during recessions, workers are often reluctant to file workers' compensation claims for less severe injuries, for fear that there may not be a job to return to when they have fully recovered from the injury. In addition, some workers are afraid that if they do file a claim during a recession, the employer will look less favorably upon them during promotion and advancement decisions. Thus, even for the same *accident frequency* levels, the *claim filing frequency* is lower during recessions.

❷ During recessions, *durations of disability* lengthen. Group health insurance studies of long-term disability coverage show that as unemployment increases, disabled employees tend to remain on disability for longer periods, apparently because there may be no job to return to. This phenomenon is equally true for workers' compensation: during recessions, unemployment rises and durations of disability lengthen.[12]

For new business, these two effects are offsetting, though the effects of claim frequency are stronger. The exact magnitudes depend on a host of factors, such as the type of industry, unemployment levels, seniority effects on job retention patterns, overtime practices, and the relationships between experience levels and injury rates. A general rule of thumb, though, is that for every 2% decline in claim frequency during recessions, one can expect a 1% increase in loss costs from lengthening durations of disability, for an overall 1% decline in loss costs.

For reserves, there is no effect from a decline in claim frequency. Moreover, workers' compensation reserves are dominated by permanent total cases, permanent partial cases, and medium term temporary total cases. For the latter two types of cases, the increase in durations of disability is particularly noticeable. A recession causing a 2% decline in loss frequency and a 1% increase in loss severity (duration of disability) for new business would cause a 1% or greater increase in reserves.

Recessions are generally accompanied by declines in interest rates. As discussed above, the decline in interest rates would not affect the valuation of new business. For the run-off business, however, the decline in interest rates raises the value of fixed-income assets supporting the reserves more than it raises the value of compensation benefit obligations.

In sum, the effect of a recession on the value of the insurance enterprise holding a block of workers' compensation reserves is unclear. Depending on the input assumptions, the financial model may show either a net increase or a net decrease. For new business, however, the model will generally show an increase in the value of the insurance enterprise.[13]

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[12] For the effects of macroeconomic conditions on workers' compensation claim frequency and durations of disability, see Sholom Feldblum, "Workers' Compensation Ratemaking," (Casualty Actuarial Society Part 6 Study Note, Sept. 1993) and the references cited therein.

[13] Numerous items that we have not discussed in the text have opposing effects. For instance, written premium declines during recessions, (i) first as payrolls decline and the demand for workers' compensation coverage decreases, and (ii) second as carriers compete more strenuously for the remaining business. The decline in written premium raises expense ratios and reduces overall profits. Moreover, the collectibility of premiums receivable

Surplus and Profitability

Many insurance profitability models deal with returns on equity. Some do so directly, such as by setting a target return on equity that the insurance operations must provide. Some do so indirectly, such as by using the target return on equity to determine a risk adjustment to the loss reserve discount rate or to determine a risk margin in the premium.[14] For the internal rate of return models often used in workers' compensation rate setting, the desired return on equity becomes the internal rate of return that the projected premium must achieve.

Most of these models use equity assumptions, such as "assumed premium to surplus" ratios or "reserves to surplus ratios." These models tell us little about the actual profitability of the insurance enterprise. Indeed, the implications are sometimes counter-intuitive. For instance, an internal rate of return model with a fixed reserves to surplus ratio may imply that a company with poor underwriting experience and high reserves is using more surplus. In fact, the company has less surplus, which is precisely the item we are trying to measure.

The financial model described in this paper uses two methods to measure performance.

- For the operating performance of distinct blocks of business, the model uses a return on (economic) surplus. [The calculation of the needed economic surplus is described below.] For instance, in December 1995, the model will simulate the expected return from policy year 1996 workers' compensation business, given assumptions about 1996 compensation underwriting experience, scenarios about interest rates and inflation rates, and analysis of the economic surplus needed to support this business.

- Surplus needed to "support" insurance underwriting is "tied up" in the "day to day" operations of the company. The insurer's management asks: "How much 'free surplus' does the company have?" "Is this 'free surplus' increasing or decreasing?" "What operations of the company are contributing to the increase or decrease?"

To properly measure the returns on surplus and the progression of free surplus, the insurance

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decreases, as employers find it difficult to meet their payment obligations. Both of these effects must be incorporated into the financial model to accurately ascertain the expected results from a recession on the profitability of new business.

14 The former method is used by the Fireman's Fund risk-adjusted discounted cash flow model; see Robert P. Butsic and Stuart Lerwick, "An Illustrated Guide to the Use of the Risk-Compensated Discounted Cash Flow Method," *Casualty Actuarial Society Forum* (Spring 1990), pages 303-347. The latter method is used by Stephen Philbrick to determine a pricing risk margin (or "narrow risk margin," in Philbrick's terms) from the capital requirements (or the "broad risk margin" in Philbrick's terms); see his "Accounting for Risk Margins," *Casualty Actuarial Society Forum* (Spring 1994), Volume I, pages 1-90. The relationship between the narrow risk margin and the broad risk margin, in Philbrick's method, depends on the relationship the risk-free interest rate and the desired return on equity.

company's economic surplus (i.e., the economic net worth) is divided into three components:[15]

❶ *Surplus supporting the run-off business.* This surplus supports the variability in the indicated reserves (i.e., unexpected adverse loss development), as well as credit risk from reinsurance recoverables, and asset risks (such as default risk or market fluctuation risks) on the investments supporting the reserves. The amount of surplus needed is determined by stochastic simulation analyses, using target expected policyholder deficit ratios or target probability of ruin percentiles, and then translated into target *reserves to surplus* leverage ratios, which differ for each line of business.

❷ *Surplus supporting the new business.* This surplus supports the variability in underwriting results, stemming from underwriting cycle movements, from random loss fluctuations, and from natural catastrophes. In addition, this surplus supports the risk from poor reinsurance arrangements, as well as the asset risk as the newly collected premium is held in the investment markets before the losses are paid. Once again, the amount of surplus needed is determined by stochastic simulation analyses, using target expected policyholder deficit ratios or target probability of ruin percentiles, and then translated into target *premium to surplus* leverage ratios, which differ for each line of business.

❸ *Free surplus.* This is the company's economic surplus that is not needed to support its insurance operations. (a) It may be used for other operations, such as surplus supporting overseas expansion, business growth, or an investment company subsidiary, (b) it may be required for regulatory purposes (e.g., it may be needed to achieve a high risk-based capital ratio), or (c) it may be pure "surplus surplus."

Consider first a monoline insurance company, with past workers' compensation reserves and a new policy year of workers' compensation business. The stochastic simulation analyses combined with target expected policyholder deficit ratios are used to set reserves to surplus leverage ratios and premium to surplus leverage ratios for the surplus supporting the run-off

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15 Throughout the surplus allocation process described here, we are concerned with "economic net worth," not "statutory surplus." The distinction is particular important for the surplus supporting workers' compensation reserves. The stability of workers' compensation loss payout patterns, along with the long duration of these patterns, makes the "implicit interest margin" in undiscounted workers' compensation reserves far exceed the capital required to safeguard the company against even highly unlikely adverse scenarios (i.e., low probabilities of ruin or low expected policyholder deficit ratios); see below in the text. In other words, the statutory surplus needed to support workers compensation reserves is negative, since the economic surplus needed is less than the interest cushion in the undiscounted reserve. The exhibits in this paper, however, show positive surplus, since we are looking at economic values of assets and of liabilities, not at statutory figures.

Statutory accounting, however, is a constraint on insurance company operations. For instance, a monoline workers' compensation carrier with steady underwriting results may feel forced to hold significant statutory surplus to support these reserves because of the NAIC's 11% risk-based capital reserving risk charge. For allocating surplus by line of business, we have actually used a combination of surplus determined by the economic allocation described in this paper and surplus as determined by the NAIC's risk-based capital formula.

business and the surplus supporting the new business, respectively.[16]

The leverage ratios determine the amount of surplus needed at the inception of the new policy year. The company's remaining economic surplus at the inception of the new policy year is "free surplus."

Return on Surplus and Surplus Progression

As the year progresses, there are three "returns."

❶ The *expected* return on the surplus supporting reserves is composed of two pieces: The assets corresponding to this surplus earn a return in the investment market. In addition, since the discount rate for loss reserves is generally less than the investment yield on the surplus funds, the difference in the two yield rates times the reserves to surplus ratio is an additional return on these surplus funds.[17] The *actual* return on the surplus supporting reserves includes a third piece: the favorable or adverse development on these reserves.

❷ The *expected* return on the surplus supporting new business is also composed of two pieces: The assets corresponding to this surplus earn a return in the investment market. In addition, the projected underwriting gain or loss on this business is an additional return on these surplus funds. As is true for surplus supporting reserves, the *actual* return on the surplus supporting new business includes a third piece: the favorable or adverse underwriting performance of this business.[18]

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[16] These leverage ratios use market value accounting. For instance, we use an "discounted reserves" to "economic surplus" leverage ratio, not a "statutory reserves" to "statutory surplus" leverage ratio. [Pricing actuaries, in contrast, who must file rate revisions with state insurance departments, use statutory leverage ratios; see Sholom Feldblum, "Pricing Insurance Policies: The Internal Rate of Return Model" (Casualty Actuarial Society Part 10A Examination Study Note, May 1992) for the standard workers' compensastion procedures.] For an illustration of the method in this paper, using a stochastic simulation with 10,000 runs of workers' compensation reserves along with a 1% expected policyholder deficit ratio, see below in the text.

[17] To determine the economic value of the loss reserves, the financial model uses a "risk-free" discount rate, which is the yield rate on Treasury securities of short to medium maturities. The investment yield of the company is somewhat higher than this rate, since the investment portfolio includes also common stocks, corporate bonds, and mortgage-backed securities. Daniel Gogol uses a similar procedure, where the return on surplus allocated to reserves stems from the difference between a risk-free rate used for assets supporting the reserves and a risk-adjusted rate used to value the reserves themselves; see his "Pricing to Optimize an Insurer's Risk-Return Relation," *CAS Forum*, Winter 1996 Edition (Casualty Actuarial Society, 1995), pages 213-242.

[18] The two returns – the return on the run-off of existing business and the return on new business – are not independent. Since the greatest value of the existing consumer base is the retention of insureds and the expected future profits, persistency rates are high in most casualty lines, such as personal automobile and workers' compensation. Meanwhile, insurers

- *Finally, there is a progression of "free surplus."* The total surplus of the company increases by the returns on investable funds plus underwriting gains (or minus losses) minus federal income taxes and minus the unwinding of the interest discount on economic reserves. We assume that the company expects to write a similar volume of business one year from now as it is writing in the new policy year, with appropriate adjustments for inflation and expected real business growth.

  The free surplus at the beginning of the year minus the surplus needed to support both the run-off business and the new business is the initial free surplus. The initial reserves decline over the course of the year, and the new business becomes run-off business, both leading to lower surplus requirements. Conversely, there is a new year of new business, with additional surplus requirements. The third profitability measure shown by the financial model asks: "How will the amount of free surplus progress over the course of the year, given the assumptions for underwriting results, reserve developments, and investment performance?" Similarly, once the year has actually transpired, the financial model asks: "How has the amount of free surplus progressed over the course of the year?"

The three profitability measures overlap. They are used for different purposes; they are not independent. For instance, good expected underwriting results for new business will raise the return on surplus supporting this business and also result in an increase in free surplus. The

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are reluctant to implement (and regulators are equally reluctant to allow) large rate changes. The result is that many of the same policyholders occupy the company's existing book of business as well as its future book of business.

Thus, *unexpected* favorable or adverse results on the run-off of the existing book may portent corresponding favorable or adverse results on the book of new business. For expected results, however, the two pieces are largely independent. The model accrues profit as the premium is written, not as the losses are paid. Expected underwriting profits are included in the new business section. The return on the run-off of existing business (if interest rates do not change) is

   the investment return on the assets supporting the reserves,
\+ the investment return on the surplus supporting the reserves,
− the amortization of discount on the market value of the reserves.

This is different from the approaches used by Robert Butsic and by Stephen Philbrick. Butsic incorporates an "implicit reserve margin" by using a reserve discount rate lower than the risk-free rate. Philbrick incorporates an "explicit reserve margin" (his "narrow risk margin" or NRM) that is embedded in the policy premium, held "above the line," and ultimately paid to equityholders.

The Butsic and Philbrick models seek to align the return with "uncertainty." As long as there is uncertainty in the ultimate loss payments, the insurer, or the insurer's owners, must earn a "risk-compensated" return. The financial model described in this paper seeks to align the return with the insurer's operating decisions. Once the policy has been written and earned, the insurer's action do not much affect the random reserve developments that determine the actual return. [The choice of investments does affect the insurer's returns, so the investment yield on the assets supporting the reserves does influence the return.]

return on surplus supporting the new business shows the profitability of the insurance operations. The progression of free surplus shows how much money is available for other corporate functions, such as expansion into new markets. Exhibits 1 and 2 of Appendix D illustrate graphically the connection between the return on surplus and the progression of "free surplus." Exhibits 3 through 7 detail the results for each line of business.

Scenarios and Returns on Surplus

Exhibits 8 through 12 of Appendix D show the various measures of return described above, for both the base case scenario and for the alternative scenario. Consider first the return on surplus supporting the run-off business.

❶ For the base case, the return is slightly higher than the expected after-tax return on assets, depending on the "discounted reserves to economic surplus" leverage ratio used for each line. For workers' compensation, for instance, the after tax investment yield is 5.7%, and the loss reserve discount rate is 5.4%.[19] With a three to one "reserves to surplus" leverage ratio, the 30 basis point difference in yield contributes 90 basis points to the return on surplus, resulting in a 6.6% return.

❷ In the alternative scenario, the return drops sharply, from 6.6% to –7.3%. The magnitude of the change in the return is driven by several items, particularly the duration of the assets supporting the compensation reserves, the inflation sensitivity of the losses, the sensitivity of workers' compensation retrospective premiums, the federal income tax implications, and the discount rates used.

The return on surplus supporting new business is driven primarily by expected underwriting gains and losses. Industry-wide workers' compensation results have been good in the early 1990's, and the company projects an 80% loss and loss adjustment expense ratio. The company's direct writing distribution system (a salaried sales force) provide for a low underwriting expense ratio of 18%, yielding a combined ratio of 98%. [See Exhibit 3 of Appendix A for these figures.] The long payment lag in workers' compensation produce an anticipated return on surplus of 42.7%.

This return is not much affected by changing interest rates or inflation rates, as discussed earlier. In fact, the alternative scenario, with a 2% rise in inflation and interest rates, causes only an insignificant change in the return, from 42.8% to 42.6%. However, there are differences in the other lines of business, primarily because of the drop in the market value of investments. [See Appendix D, exhibits 8-12, for the total returns, and exhibits 13-19, for an analysis of the sources of return.]

In fact, the high 1994 and 1995 returns in workers' compensation result from strong benefit reforms in many states along with a movement to "managed care" program, with only partially

[19] This difference reflects the lower yielding bonds supporting the reserves, not a "risk adjustment" for reserve variability. The reserve variability is used to determine the target leverage ratio, using an expected policyholder deficit analysis, not the appropriate discount rate. Greater reserve variability means that the insurer must hold more surplus to support the reserves, and it therefore has less surplus for other uses. It does not mean that the insurer "earns" more by holding these reserves.

offsetting rate reductions.[20] The incentive effects of the benefit reforms were greater than anticipated, with large reductions in workers' compensation claim frequencies. Rate level decreases in 1996 (as in Massachusetts) will reduce the anticipated return on new business, though they will have no effect on the anticipated return on surplus supporting existing business.

Exhibits 13 and 14 of Appendix D show the overall company results under both the base scenario and the alternative scenario for (i) surplus supporting the run-off versus (ii) surplus supporting the new business, with additional detail showing types of assets, types of liabilities, and federal income taxes. The return on surplus supporting the run-off business drops sharply, whereas the return on surplus supporting new business does not change significantly.

Surplus Requirements

The measurement of expected profitability requires an assessment of the capital needed to support the business. Past actuarial attempts to assess surplus requirements have several failings, which are avoided in the financial model described here.

❶ Most commonly, analysts use an "assumed surplus requirement," such as a "two-to-one premium to surplus ratio," or a "three-to-one reserves to surplus ratio," to model the needed capital. Such assumptions simply beg the question of how much capital is actually needed.

Another common approach has been to take the company's existing surplus and simply allocate it to lines of business based upon premium volume or reserve volume. This procedure skirts the issue entirely. If the company is profitable, it allocates more capital to each line of business, reducing the expected return on equity. The financial model described here takes a different approach. If the company is profitable, more capital is moved to "free surplus," which the company may use for other purposes.

This approach demands a method of quantifying the capital truly needed, not simply an allocation of existing capital. Similarly, it is insufficient to adopt simple rules-of-thumb, such as the Kenney rule used by some regulators of a "two-to-one premium to surplus ratio" or the ad hoc "reserves to surplus" ratios often used by pricing actuaries for workers' compensation ratemaking.

Actuarial science has developed several methods of quantifying the needed capital. The financial model described here began with leverage ratios determined from "probability of ruin" analyses. [These are the leverage ratios which are reproduced in the exhibits in the Appendix.] The capital requirements used in the model are now being updated, using an "expected policyholder deficit" analysis by line of business from simulation analyses.

❷ Some analysts use the same leverage ratio, such as a premium to surplus ratio or a reserves to surplus ratio, for the entire underwriting risk. This approach has two shortcomings.

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20 Because of the competitive characteristics of the commercial insurance market, one may expect workers' compensation rate levels to decrease in line with costs, as has already occurred in several states in 1995 and 1996.

- First, it fails to recognize that there are two distinct risks, which are important for different insurance personnel. (a) The underwriter seeking to sell new policies or the pricing actuary seeking to make rates for new business must quantify the variability of loss costs over the coming policy period. (b) The corporate accountant seeking to complete the company's financial statements or the reserving actuary seeking to estimate the company's loss obligations must quantify the variability of adverse reserve developments on the company's existing reserve portfolio.

- Second, the proper type of leverage ratio, as well as the needed analysis, differs for these two types of risk. (a) For new business, one must know the volume of exposures, the degree of diversification, and the adequacy of reinsurance arrangements. For instance, to assess the surplus needed to support a new policy year of Homeowners writings, new business volume – as considered in a premium to surplus ratio – is appropriate. A reserves to surplus leverage ratio is irrelevant. (b) To assess the surplus needed to support the run-off of pollution and asbestos claims, a premium to surplus ratio is irrelevant. The needed surplus to support these reserves may be estimated either by a reserves to surplus leverage ratio or by other actuarial techniques.[21]

❸ Past approaches often use leverage ratios of *statutory* figures to statutory surplus. These approaches ignore the implicit interest margins inherent in undiscounted reserves.

For instance, a standard measure of surplus needed to support workers' compensation writings, as used in some internal rate of return pricing models, is an assumed ratio of statutory reserves to statutory surplus. This approach compounds all three errors discussed above:

- Undiscounted workers' compensation loss reserves contain an enormous "implicit interest margin," since workers' compensation loss reserves, like life annuities, have slow but steady payment patterns combined with long durations.[22]

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[21] Insurer liabilities for environmental exposures highlight the difference between returns on surplus and the progression of free surplus. Insurance companies want to know the effects of alternative environmental scenarios on the company's performance. But insurers are not holding pollution and asbestos reserves in order to earn a high return on surplus. And no insurer would say that the great uncertainty in pollution payments necessitate a low discount rate for determining the present value of the reserves, thereby leading to a high expected return on surplus. Rather, the insurer's management asks: "How do different scenarios relating to environmental liabilities affect the company's net worth?" Rephrased in the terms used by the financial model, this question is: "How do these different scenarios affect the progression of 'free surplus'?"

[22] For a rough estimate of the payout pattern of workers' compensation reserves, see Richard G. Woll, "Insurance Profits: Keeping Score," *Financial Analysis of Insurance Companies*, (Casualty Actuarial Society 1987 Discussion Paper Program), pages 446-533. Woll's estimates, which are based on ten years of Schedule P data, are severely understated, since the lifetime pension cases, which form the bulk of workers' compensation reserves for older policy years, have an extremely slow payout pattern. The authors' own analyses, based on

- The major risks in workers' compensation business are not the fluctuations in loss reserves but the uncertainties in new business, whether by the random occurrences of accidents or by macroeconomic conditions, industry underwriting cycles, or the regulatory climate that affect the claim frequency rate, the expected premium levels, or the prospects for rate level increases.

- A "three-to-one" reserves to surplus ratio has no more actuarial support than a "two-to-one" premium to surplus ratio. The latter has a long regulatory tradition, and the former has a short actuarial tradition. Neither has much theoretical foundation.

The Expected Policyholder Deficit Approach

The original leverage ratios used in the financial model were developed from a probability of ruin analysis. These ratios are shown in Exhibit 1 of Appendix F. The supporting exhibits for workers' compensation, based upon 2,000 runs of a stochastic simulation analysis, are shown in Exhibits 2 and 3 of Appendix F.

Most of the deficiencies of past analyses are solved by these leverage ratios:

- The ratios are determined by probability of ruin analyses, not by tradition or by rule of thumb.
- Separate ratios are used for the run-off of existing business (reserving risk) and for the writing of new business (premium risk).
- All measures use "economic surplus" and "economic reserves."

The financial model described here is now being refined by means of the expected policyholder deficit (EPD) concept developed by Robert Butsic. The EPD ratio analysis says that

- The appropriate measure of solvency is the ratio of the expected policyholder deficit to the obligations to policyholders (i.e., the expected losses).[23]

The corollary to this is that the appropriate amount of capital needed to guard against any

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25 years of paid loss experience from 10% of the industry's business, show an average time to payment of about eight years for workers' compensation reserves.

23 *This statement should be qualified. The appropriate measure of solvency for regulators* and for policyholders is the expected policyholder deficit ratio. The appropriate measure of financial strength for investors and for company management is the probability of ruin. Policyholders are indeed concerned about the amount of loss. Regulatory measures of solvency, which serve to protect policyholders, are concerned with the same issue. The "corporate shield," however, insulates investors from the magnitude of the loss after bankruptcy. Similarly, management and employees are concerned with job security, which is not affected by the magnitude of the post-insolvency loss.

Thus, the appropriate measure of financial strength should depend on the use of the model. However, there are other advantages of the EPD approach, particularly when different risks are being considered in combination, so we present the results from this approach.

risk is the amount of capital needed to reduce the EPD ratio to a predetermined figure.

- Capital requirements, expressed in terms of EPD ratios, should be uniform across risks. That is, the capital needed to guard against workers' compensation reserving risk should produce the same EPD ratio as the capital needed to guard against personal auto reserving risk.

The Expected Policyholder Deficit

To properly quantify the surplus needed for each type of risk, we combine a simulation analysis with an expected policyholder deficit approach. We illustrate this for workers' compensation reserving risk, for which we have completed the full analysis.[24] The calculations are as follows:

Were there no uncertainty in the future loss payments, then the insurer need hold funds just equal to the reserve amount to meet its loss obligations. Since future loss payments are not certain, funds equal to the expected loss amount will sometimes suffice to meet future obligations and will sometimes fall short. The insurer holds surplus to ensure that the loss obligations will indeed be met.

When the future loss obligations are less than the funds held by the insurance company to meet these obligations, the "deficit" is zero. When the future loss obligations are greater than the funds held, the "deficit" is the difference between the two. The "expected policyholder deficit" is the average deficit over all scenarios, weighted by the probability of each scenario. In the analysis here, the expected deficit is the average deficit over all simulations, each of which is equally weighted.

The Stochastic Simulation

How should we measure the uncertainty inherent in the loss reserve estimates? We use stochastic simulation of the experience data to ensure statistically meaningful results, with simulation parameters that are based upon the actual experience of the company.

We begin with 25 years of countrywide paid loss workers' compensation experience, separately for indemnity and medical benefits, for accident years 1970 through 1994. From these data we develop 24 columns of paid loss "age-to-age" link ratios, as shown in Exhibit 1 of Appendix H.

We fit each column of "age-to-age" link ratios to lognormal curves, determining "mu" (μ) and "sigma" (σ) parameters for each. We perform 10,000 sets of stochastic simulations. Each simulation produces 24 "age-to-age" link ratios (one for each column). These are the age-to-age factors that drive the actual loss payments.

The 10,000 simulations produce 10,000 reserve amounts. For ease of presentation, we normalize the results to $100 of average undiscounted reserves. We ask: "How tight is this

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[24] A more complete description is contained in Douglas M. Hodes, Sholom Feldblum, and Gary Blumsohn, "Workers' Compensation Reserve Uncertainty" (1996 Casualty Loss Reserve Seminar discussion paper program, forthcoming), which discusses the stochastic simulation, the curve fitting considerations, and the influences on reserve uncertainty.

distribution of reserve amounts?" We answer in two ways.

- We show the standard deviation, the mean, and two other percentiles of the distribution (5% and 95%). For instance, the table below shows that for discounted reserves with no adjustments for inflation, the mean reserve amount is $5.27 million, the standard deviation is $3.4 million, the 95th percentile is $58.7 million, and the 5th percentile is $47.9 million.

- To facilitate the comparison of reserve uncertainty with other types of risk used in the financial model, we use the "expected policyholder deficit (EPD) ratio" as a yardstick. We ask: "How much additional capital must the insurer hold to have a 1% EPD ratio?" The table below shows that for discounted reserves, the required capital for a 1% EPD ratio is $2.4 million.

	Average Reserve Amount	Standard Deviation of Reserve	95th Percentile of Reserve	5th Percentile of Reserve	Capital Needed for 1% EPD Ratio
Undiscounted	100.0	19.5	135.3	74.0	31.0
Discounted: 6.75%	52.7	3.4	58.7	47.9	2.4

Reserve Discounting

We are primarily concerned with the economic values, or discounted values, of the reserves, not with undiscounted amounts. Much of the variation in statutory reserve requirements stems from fluctuations in "tail factors." This fluctuation depends in part on inflation rates. For discounted reserves, the effects of changes in the long-term inflation rate are offset by corresponding changes in the discount rate. Moreover, tail factor uncertainty has a relatively minor effect on the present value of loss reserves, even if the discount rate is held fixed. Thus, the distribution of discounted loss reserve amounts is more compact than the distribution of undiscounted loss reserve amounts.

Because statutory accounting mandates that insurers hold undiscounted reserves, we show analyze results both for discounted and for undiscounted reserves. Moreover, the difference between the discounted and undiscounted reserve amounts is the "implicit interest margin" in the reserves, which is important for assessing the implications of the reserve uncertainty on the financial position of an insurance company.

Length of the Development

The paid loss development for 25 years is based on observed data. Workers' compensation paid loss patterns extend well beyond 25 years. For each simulation, we complete the development pattern as follows:

1. Given the 24 paid loss "age-to-age" link ratios from the set of stochastic simulations on the fitted lognormal curves, we fit an inverse power curve to provide the remaining

"age-to-age" factors.[25] This fit is deterministic.

- The length of the tail is chosen (stochastically) from a linear distribution of 30 to 70 years.

Let us suppose first that the company holds no capital besides the funds supporting the reserves. We ran our analysis For the discounted analysis, the average reserve amount is $52.7 million. About half the simulations give reserve amounts less than $52.7 million. In these cases, the deficit is zero. The remaining simulations give reserve amounts greater than $52.7 million; these give positive deficits. The average deficit over all 10,000 simulations is the expected policyholder deficit, the EPD. The "EPD ratio" is the ratio of the EPD to the expected losses, which are $52.7 million in this case.

Clearly, if the probability distribution of the needed reserve amounts is "compact," or "tight," then the EPD ratio will be relatively low. Conversely, if the probability distribution of the needed reserve amounts is "dispersed" – that is, if there is much uncertainty in the loss reserves – then the EPD ratio will be relatively great.

We now "fix" the EPD ratio at a desired level of financial solidity and determine how much additional capital is needed to achieve this EPD ratio. We use a 1% EPD ratio as our benchmark, since this is the ratio which Butsic uses for risk-based capital applications.

Suppose the desired EPD ratio is 1%. If the reserve distribution is extremely compact, then even if the insurer holds no capital beyond that required to fund the expected loss payments, the EPD ratio may be 1% or less. If the reserve distribution is more dispersed, then the insurer must hold additional capital to achieve an EPD ratio of 1%. The greater the reserve uncertainty, the greater the required capital.

Results

The results for the base case, with discounted reserves, are shown in the table above.[26] The average discounted reserves are $52.7 million, and additional capital of $2.4 million is needed to achieve a 1% EPD ratio.

The corresponding full value reserves are $100.0 million. The company uses tabular discounts on the indemnity portion of life-time pension cases at a 3.5% discount rates, which is the rate used in the NCCI unit statistical plan. The resulting statutory reserves, normalized to a $100

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25 On the use of the inverse power curve, see Richard Sherman, "Extrapolating, Smoothing, and Interpolating Development Factors," *Proceedings of the Casualty Actuarial Society*, Vol. 71 (1984), pages 122-192, as well as the discussion by Stephen Lowe and David Mohrman, vol 72 (1985), page 182, and Sherman's reply to the discussion, page 190.

26 We ran our simulation for several cases: (i) discounted versus undiscounted reserves, (ii) with and without various adjustments for medical inflation, and (iii) with and without consideration of loss-sensitive contracts. The "base case" in our analysis uses discounted reserves with no adjustments for medical inflation or for loss-sensitive contracts. For a full description of the analysis, see Hodes, Feldblum, and Blumsohn, "Workers' Compensation Reserves Uncertainty" (op. cit.).

million undiscounted reserve, are about $92 million.

The difference between the perspective in the financial model described here and the "received actuarial wisdom" warrants further comments. The common view is that workers' compensation reserve estimates are highly uncertain, because of the long duration of the claim payments and because of the unlimited nature of the insurance contract form. This uncertainty creates a great need for capital to hedge against unexpected reserve development. In fact, the opposite is true. There is indeed great underwriting uncertainty in workers' compensation, and regulatory constraints on the pricing and marketing of this line of business have disrupted markets and contributed to the financial distress of several carriers. But once the policy term has expired and the accidents have occurred, little uncertainty remains. The difference between the economic value of the reserves and the reported (statutory) reserves, or the "implicit interest margin," is many times greater than capital that would be needed to hedge against reserve uncertainty.

These results have important implications for our financial model.

- There is no "leverage ratio" between statutory reserves and statutory surplus, since one needs *negative* statutory surplus to support workers' compensation reserves (if undiscounted reserves are indeed held).

- Regulatory requirements, however, such as risk-based capital requirements, force companies to hold more surplus to guard against "reserving risk" than they actually need.[27]

Thus, our financial model assumes that the implicit interest margin in the compensation reserves provides the full economic surplus needed to support the reserves as well as a substantial amount of "free surplus." However, because of the constraints imposed by statutory accounting and by the NAIC's risk-based capital formula, the leverage ratios used in the financial model are still lower than the implicit leverage ratios from the EPD analysis.[28]

Inflation and Interest Rate Risks

For the past twenty years – ever since the dramatic rise in inflation during the late 1970s – casualty actuaries have debated the effects of inflation on the economic worth of insurance companies. Because there are multiple and simultaneous effects, several of which are difficult to quantify without a sophisticated financial model, past analyses of this issue have often been

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27 For instance, the current NAIC risk-based capital formula uses an 11% reserving risk charge for workers' compensation, before company-specific adjustments, such as the company's average reserve development, the percentage of business written on loss-sensitive contracts, and loss concentration factor; see Sholom Feldblum, "Risk-Based Capital Requirements" (Casualty Actuarial Society Part 10 Examination Study Note, Second Edition, July 1995), for a complete description of the NAIC risk-based capital reserving risk charge.

28 The final surplus requirements by line of business and by operational unit used by the company were determined in part by management discretion, with consideration of rating company expectations, peer company practices, and NAIC constraints, not solely by the actuarial analysis reviewed in this paper.

incomplete.

Inflationary changes have several effects on the financial solidity of an insurance enterprise. For the "alternative scenario" discussed earlier, we assume that the inflationary change is matched by a corresponding change in interest rate (the "Fisher effect").

- *They reduce the market value of fixed income securities.* The effects on payment patterns and market values of fixed income securities have been studied by both investment analysts and by actuaries (primarily life actuaries). A full analysis requires consideration also of issuer options, such as calls on corporate bonds and prepayments on mortgages and mortgage backed securities. The effects on the company's portfolio of mortgage backed securities are shown in the exhibits and they are discussed further below.

- *They temporarily reduce the market value of common stock investments*, though not of real estate investments. The effect on all equity investments, however, is generally short-term. Over the long-term, equity investments serve as a "hedge" against inflation.

  Good data on the magnitude of the relationships are lacking. Our model uses in-house studies based on the stock market experience of the past twenty years, as described further below.

- *They reduce the market value of fixed liabilities*, such as some workers' compensation indemnity payments or personal automobile no-fault compensation payments.

- *They increase the nominal value of most casualty loss reserves.* In other words, most casualty loss reserves are "inflation sensitive": if inflation increases, the nominal amounts increase as well, with little change in the market value.

Quantifying the last of these effects is particularly difficult without a financial model. In workers' compensation, for instance, inflation affects *medical* benefits through the payment date. In about half of the U.S. jurisdictions, *indemnity* payments that extend beyond two years have COLA adjustments that depend on inflation.

To properly assess the effects of inflation on the company's loss reserves, we "strip" out past inflation from the historical loss triangles, determine the paid loss "age-to-age" link ratios, then restore expected future inflation to the indicated (future) link ratios. In other words, we make the following adjustments to the loss reserve development analysis used to project future loss payments:

❶ We convert the paid losses to "real dollar" amounts by means of an appropriate inflation index. For workers' compensation medical benefits, we used the medical component of the CPI.

❷ We select a future inflation rate that is consistent with the scenario being analyzed. For instance, if we project future inflation at 6% per annum, we may select 7% per annum as the medical inflation rate.

❸ Finally, we combine the projected link ratios and the projected inflation rate to determine the expected loss outflows.

Exhibits 1 through 5 of Appendix G shows the method of "stripping" inflation out of the loss payment triangle, determining link ratios in real dollars, and then restoring the projected inflation to the assumed future payments.

Results

A comparison of the results from the financial model with the recommendations on the American Academy of Actuaries task force on risk-based capital regarding interest rate risk is most instructive. The *statutory* interest rate risk considered by the AAA task force differs in two respects from the economic effects of a simultaneous shift in interest rates and inflation rates:

❶ The *statutory* effect must consider the valuation rates used for assets and liabilities. For NAIC risk-based capital purposes, this is the rate implicit in amortized values for fixed income assets and a flat 5% rate used for loss liabilities, since this is the rate used to discount losses for the reserving risk charge. The difference between these rates may provide either a cushion or an extra charge for interest rate risk.

The financial model described here deals with cash flows. There are no valuation rates. Accounting conventions serve only as constraints; they do not enter the underlying analysis.

❷ The risk-based capital charge must consider the interplay of reserving risk and interest rate risk. Increases in interest rates that are accompanied by increases in inflation cause adverse development of undiscounted losses. The risk-based capital formula picks up this adverse development in the reserving risk charge. To reflect it also in the interest rate risk charge would be "double-counting" the same risk.

The financial model used here, however, is a scenario based model. The surplus needed to support the reserves is determined by an "expected policyholder deficit" analysis, not by a "worst-case year" approach (see above). The inflation sensitivity of casualty reserves, which causes higher *nominal* cash flows when inflation increases, must be explicitly incorporated into the results.

The resulting differences between the statutory charges developed by the AAA task force on risk-based capital for interest rate risk and the economic effects quantified by the financial model described here are large. For a two point increase in interest rates and in inflation rates, the economic return on the surplus supporting reserves changes from a positive return to a significant negative return. For most companies, the AAA recommendations would show a slight interest rate risk charge, if it would show any at all.

The reason for this difference is the overstatement of the reserving risk charge in the NAIC risk-based capital formula.

- The NAIC formula uses a flat 5% discount rate to determine the implicit interest margin in the reserves. The financial model described here the anticipated cash flows. "Economic values," when needed, are determined by means of market interest rates.

- The NAIC formula uses a "worst-case year" approach to determine the surplus needed to support reserves. The financial model described here uses either an "expected policyholder deficit" analysis or a "probability of ruin" analysis to determine the surplus needed to support reserves.

We have included this comparison of the financial model described here with the NAIC risk-based capital charges to highlight the importance of actuarial analysis. Many observers have pointed out that the NAIC reserving risk charges and written premium risk charges seem to be *ad hoc* numbers only marginally related to the actual risks faced by companies. The AAA interest rate risk recommendations were hampered by the need to fit into an existing formula that did not accurately reflect the actual risks.

The financial model described here shows the expected results under a variety of future scenarios. Some risks are more serious than those implied by the NAIC risk-based capital formula; some are less serious. Valuation actuaries must be careful to consider the risks highlighted by financial models of the type described here.

Scenario Testing

The proper management of an insurance enterprise requires consideration of the overall environment, not of isolated risks. An insurance executive does not ask: "What is the effect of a 200 basis point drop in interest rates, or a 10% rise in personal auto claim frequency, on the company's financial position?" Rather, he or she may ask: "What would be the effect of an economic recession, or of an underwriting cycle downturn, on the company's performance?" The actuary must translate the scenarios into model assumptions and rerun the cash flow projections to answer such questions.

Scenarios may be divided into two categories: economic scenarios and insurance scenarios. Economic scenarios posit changes in the macro-economic environment, such as recessions, high unemployment, or prosperous years. Insurance industry scenarios posit changes in such elements as the underwriting cycle, industry competition, or state regulation.

Scenarios are composed of interdependent elements, each of which may affect multiple elements of an insurance company's operations. For instance, economic recessions are often characterized by falling interest rates and high unemployment, which affect bond prices and stock prices (which are similar for most insurers) as well as claim frequency and claim severity (which vary by line of business). Thus, modeling the effects of scenarios is a two step process:

- The analyst must translate the scenario into a set of model assumptions. For instance, a recession scenario may be characterized by a drop in interest rates, a fall in stock market prices, a drop in inflation, and a rise in unemployment.

- These economic assumptions must be applied to the company's characteristics and run through the financial model. For instance,

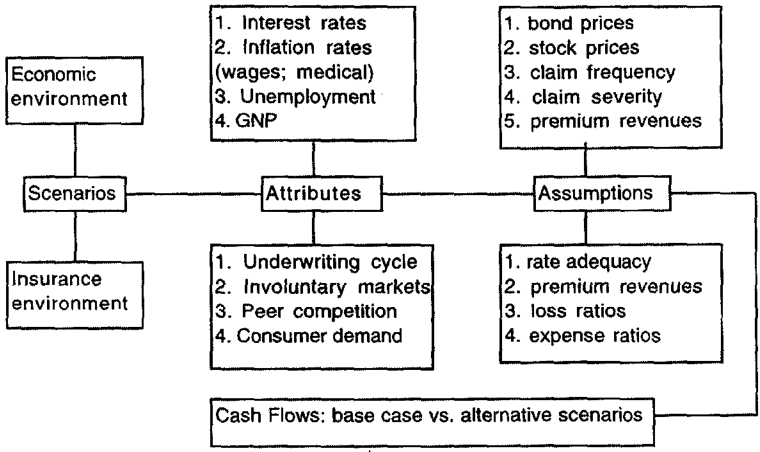
    - a drop in interest rates would increase the market values of fixed income securities,
    - a drop in inflation would decrease the (nominal) required reserve in some lines of business, though the economic value may increase because of the decline in interest rates, and
    - the rise in unemployment would probably decrease workers' compensation claim frequency but increase workers' compensation claim severity (or "durations of disability").

In other words, the effects of the economic scenario depend on the composition of the investment portfolio, the lines of business written, and the nature of the loss reserves.

Insurance scenarios are equally complex. For instance, an underwriting cycle downturn would involve changes in premium revenues (because of changes in rate adequacy), changes in expense ratios (because of changes in the percent of the market turning to self-insurance), changes in involuntary market sizes and "burdens," and changes in the expected loss ratio.[29]

The economic environment and the insurance environment are not independent. For instance, interest rate movements and changes in the cost of capital may affect the course of the underwriting cycle. Specifying the attributes and associated characteristics of a scenario requires a keen understanding both of the macroeconomic environment and of the effects of external factors on company operations.

The types of scenarios, their distinguishing attributes, and the assumptions relevant for the financial model are shown in the graph below.



We have applied this analysis with two scenarios, an economic recession and an underwriting downturn, for the company's workers' compensation book of business. Workers' compensation is particularly appropriate for scenario testing for two reasons:

- The effects of economic conditions, such as high unemployment, and of insurance industry conditions, such as growing involuntary markets, are clearer in this line of business than in most others.

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29 For a more complete discussion of the process of scenario building, see Sholom Feldblum, "Forecasting the Future: Stochastic Simulation and Scenario Testing," in *Incorporating Risk Factors in Dynamic Financial Analysis*, Casualty Actuarial Society 1995 Discussion Paper Program (Landover, Maryland: Colortone Press, 1995), pages 151-177.

Consider several effects of a recession on claim frequency and claim severity in workers' compensation. (a) Workplace accidents are noticeably greater for young and inexperience workers, who are frequently unfamiliar with the hazards of the machines or equipment used. During prosperous years, firms hire new workers, and workers' compensation claim frequency rises. (b) In addition, overtime work increases. Carefulness does not abide well with fatigue, and claim frequency increases even more. (c) Finally, during recession, workers are loath to file claims, lest there be no job to return once they have recuperated.

The effects on claim frequency affect new business results. They have no effect on the run-off of existing business.

Conversely, recessions cause a lengthening of durations of disability. An injured worker is unlikely to declare himself healed if there is no job to return to.[30] [Note the distinction: *working* employees are less likely to file claims during recession. *Disabled* employees are less likely to recover from disabilities during recessions.] This relationships affects primarily the run-off of existing business, not the results of new business.

Some analysts err by assuming that the effects on claim frequency and claim severity (durations of disability) offset each other, so neither need by quantified. In truth, they affect separate components of the insurer's business (new business versus run-off business). Needed rate increases or decreases for the new book of business are should not be offset against unexpected gains or losses on the existing reserve portfolio. The financial model described here separates the effects of the projected scenario on these two components.

- The long duration of workers' compensation business, where a rise in unemployment may cause a lengthening of durations of disability whose financial effects are spread out over thirty years, make cash flow projections essential for proper performance measurement. In other words, claim frequency rates can be quantified by monthly reports. Durations of disability can be quantified only over periods of years.

As noted above, the model allows the analyst to separate individual lines of business or blocks of business. In addition, the analyst must also allocate the assets supporting the reserves of each line of business, and allocate surplus (or economic net worth) backing each line of business.

The use of a financial model for scenario testing has two advantages:

❶ The interdependence of the various scenario components, and the complexity of their effects on insurance operations, makes the problem almost intractable at first glance. Translating the projected scenario into model assumptions, running both the base assumptions and the revised assumptions through the model, and comparing the resulting cash flows, enables the user to see the effects of a changing environment.

❷ Users often have different opinions about the components of scenarios and about their effects on company operations. For instance, users may have different views on the expected stock

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[30] This influence is based on the experience of employer provided group health insurance. During the late 1970s, when unemployment rates rose, durations of disability under these plans rose concomitantly.

market movements during a recession or on the effects of unemployment on workers' compensation claim frequency. With the financial model described in this paper, the analyst can show the cash flows resulting from different model assumptions, as well as the sensitivity of the results to changes in the assumptions.

Conclusion

Corporate financial models are less important for a company writing short-tailed lines of business with an assured consumer base, adequate rates, and little competition. But financial models are essential for companies writing long-tailed lines of business, with fluctuating rate adequacy, severe competition, and volatile consumer bases.

Dynamic financial models take various forms, corresponding to the types of business written by the insurance enterprise and the issues that they address. Unlike traditional ratemaking or reserving techniques, there are no "cookbook" approaches to serve as benchmarks.

Actuarial practice is outracing actuarial literature. Property-casualty insurance companies are adopting models originally designed for life insurance companies and adapting them to their specific risks. The NAIC and the major rating agencies have developed solvency models to help ascertain companies' resilience to adverse future conditions.

This paper documents the cash-flow financial model used by a major commercial line insurance company. It discusses the uses of the model, the types of risks addressed, and the scenarios that it analyses. It should acquaint new actuaries with the many components of dynamic financial analysis, and it would confront experience actuaries with the complexities of reserving risk, interest rate risk (inflation risk), and scenario testing. And it should inform all actuaries that financial modeling is here to stay.

Notes to Underwriting Assumptions

Base Case Scenario

Annual inflation rate	4.00%
Growth in real exposures	2.00%
Resulting growth in underwriting	6.08%

For each line of business, the above rates affect equally losses, expenses, and premium.

Pre-tax Yield on Bonds	8.30%
Stock Dividend Yield	2.75%
Rate of growth in value of stock	8.00%
Resulting annual return on stock	10.75%

Federal income tax rate	35.00%
Discount Rate for Insurance operations cash flow*	5.40%

* This rate is the after-tax yield on bonds. After-tax rate is appropriate because income taxes are explicitly included in the cash flows.

Alternative Scenario:

Additional increase in inflation:	2.00%
Resulting annual inflation:	6.00%
Additional growth in real exposures	0.00%
Resulting growth in real exposures	2.00%
Resulting growth in underwriting	8.12%

For each line of business, the above rates affect equally losses, expenses, and premium. However, when rates first increase, nominal losses grow at a higher rate than premium, resulting in the overall increase in loss ratio.

Additional increase in interest rates	2.00%
Resulting new bond pre-tax bond yield	10.30%
Stock Dividend Yield	2.75%
Rate of growth in value of stock#	10.00%
Resulting annual return on stock	12.75%

The above growth rate for stocks is the long-term rate. In the short term, the market value of stock reacts negatively to increase in interest and inflation rates. In our alternative scenario, we assumed that initially, the market value of stock declines by five percentage points for each percentage point increase in interest rates. This decline was assumed to be eventually recovered after two years.

Federal income tax rate	35.00%
Discount Rate for Insurance operations cash flow	6.70%

Note: Actual input in the model had finer line detail.

Projected Written Premium ($ Millions)

Policy Year	Voluntary, Involuntary, Net of Reinsurance				Total
	Work Comp	GL & CMP	Comm Auto & Other	Pers Auto & HO	
1995	2,415	513	562	2,374	5,864
1996	2,562	544	596	2,518	6,220
1997	2,718	577	633	2,671	6,599
1998	2,883	612	671	2,834	7,000
1999	3,058	650	712	3,006	7,425
2000	3,244	689	755	3,189	7,877
2001	3,441	731	801	3,382	8,356
2002	3,650	776	850	3,588	8,864
2003	3,872	823	902	3,806	9,403
2004	4,108	873	956	4,038	9,974

Projected Loss and Adjustment Expense ($ Millions)

Policy Year	Voluntary, Involuntary, Net of Reinsurance				Total
	Work Comp	GL & CMP	Comm Auto & Other	Pers Auto & HO	
1995	1,966	505	462	1,855	4,787
1996	2,086	535	490	1,968	5,078
1997	2,212	568	520	2,087	5,387
1998	2,347	602	551	2,214	5,715
1999	2,490	639	585	2,349	6,062
2000	2,641	678	620	2,491	6,431
2001	2,801	719	658	2,643	6,822
2002	2,972	763	698	2,804	7,236
2003	3,152	809	741	2,974	7,676
2004	3,344	858	786	3,155	8,143

Projected Other Expense ($ Millions)

Policy Year	Voluntary, Involuntary, Net of Reinsurance				Total
	Work Comp	GL & CMP	Comm Auto & Other	Pers Auto & HO	
1995	482	118	124	478	1,201
1996	511	125	131	507	1,274
1997	542	133	139	538	1,352
1998	575	141	148	570	1,434
1999	610	149	157	605	1,521
2000	647	158	166	642	1,613
2001	687	168	176	681	1,711
2002	728	178	187	722	1,815
2003	773	189	198	766	1,926
2004	820	201	210	813	2,043

Note: Actual input in the model had finer line detail.

Projected Loss and Adjustment Expense Ratio

Policy Year	Voluntary, Involuntary, Net of Reinsurance				Total
	Work Comp	GL & CMP	Comm Auto & Other	Pers Auto & HO	
1995	81%	98%	82%	78%	82%
1996	81%	98%	82%	78%	82%
1997	81%	98%	82%	78%	82%
1998	81%	98%	82%	78%	82%
1999	81%	98%	82%	78%	82%
2000	81%	98%	82%	78%	82%
2001	81%	98%	82%	78%	82%
2002	81%	98%	82%	78%	82%
2003	81%	98%	82%	78%	82%
2004	81%	98%	82%	78%	82%

Policy Year	Voluntary, Involuntary, Net of Reinsurance				Total
	Work Comp Comp	GL & CMP Liab	Comm Auto & Other	Pers Auto & HO	
1995	20%	23%	22%	20%	20%
1996	20%	23%	22%	20%	20%
1997	20%	23%	22%	20%	20%
1998	20%	23%	22%	20%	20%
1999	20%	23%	22%	20%	20%
2000	20%	23%	22%	20%	20%
2001	20%	23%	22%	20%	20%
2002	20%	23%	22%	20%	20%
2003	20%	23%	22%	20%	20%
2004	20%	23%	22%	20%	20%

Projected Underwriting Ratio

Policy Year	Voluntary, Involuntary, Net of Reinsurance				Total
	Work Comp	GL & CMP	Comm Auto & Other	Pers Auto & HO	
1995	101%	121%	104%	98%	102%
1996	101%	121%	104%	98%	102%
1997	101%	121%	104%	98%	102%
1998	101%	121%	104%	98%	102%
1999	101%	121%	104%	98%	102%
2000	101%	121%	104%	98%	102%
2001	101%	121%	104%	98%	102%
2002	101%	121%	104%	98%	102%
2003	101%	121%	104%	98%	102%
2004	101%	121%	104%	98%	102%

Note: Actual input in the model had finer line detail.

Projected Written Premium ($ Millions)					
Policy	Voluntary, Involuntary, Net of Reinsurance				Total
Year	Work Comp	GL & CMP	Comm Auto & Other	Pers Auto & HO	Comp
1995	2,514	534	589	2,421	6,058
1996	2,720	578	637	2,620	6,555
1997	2,944	625	690	2,835	7,093
1998	3,185	676	746	3,067	7,675
1999	3,446	732	807	3,319	8,304
2000	3,729	792	874	3,591	8,985
2001	4,035	857	945	3,885	9,722
2002	4,366	927	1,023	4,204	10,519
2003	4,724	1,003	1,107	4,549	11,382
2004	5,111	1,085	1,198	4,922	12,316

Projected Loss and Adjustment Expense ($ Millions)					
Policy	Voluntary, Involuntary, Net of Reinsurance				Total
Year	Work Comp	GL & CMP	Comm Auto & Other	Pers Auto & HO	Comp
1995	2,175	562	496	1,949	5,182
1996	2,353	608	536	2,109	5,607
1997	2,546	658	580	2,282	6,067
1998	2,755	712	628	2,469	6,564
1999	2,981	770	680	2,672	7,103
2000	3,225	834	735	2,891	7,685
2001	3,490	902	796	3,128	8,316
2002	3,776	976	861	3,385	8,998
2003	4,086	1,056	931	3,662	9,736
2004	4,421	1,143	1,008	3,963	10,534

Projected Other Expense ($ Millions)					
Policy	Voluntary, Involuntary, Net of Reinsurance				Total
Year	Work Comp	GL & CMP	Comm Auto & Other	Pers Auto & HO	Comp
1995	501	122	129	490	1,241
1996	541	132	139	529	1,341
1997	586	143	150	573	1,451
1998	634	154	163	620	1,570
1999	686	167	176	671	1,699
2000	742	181	190	726	1,839
2001	803	195	206	785	1,989
2002	869	211	223	850	2,153
2003	933	229	241	918	2,321
2004	979	243	256	968	2,445

Note: Actual input in the model had finer line detail.

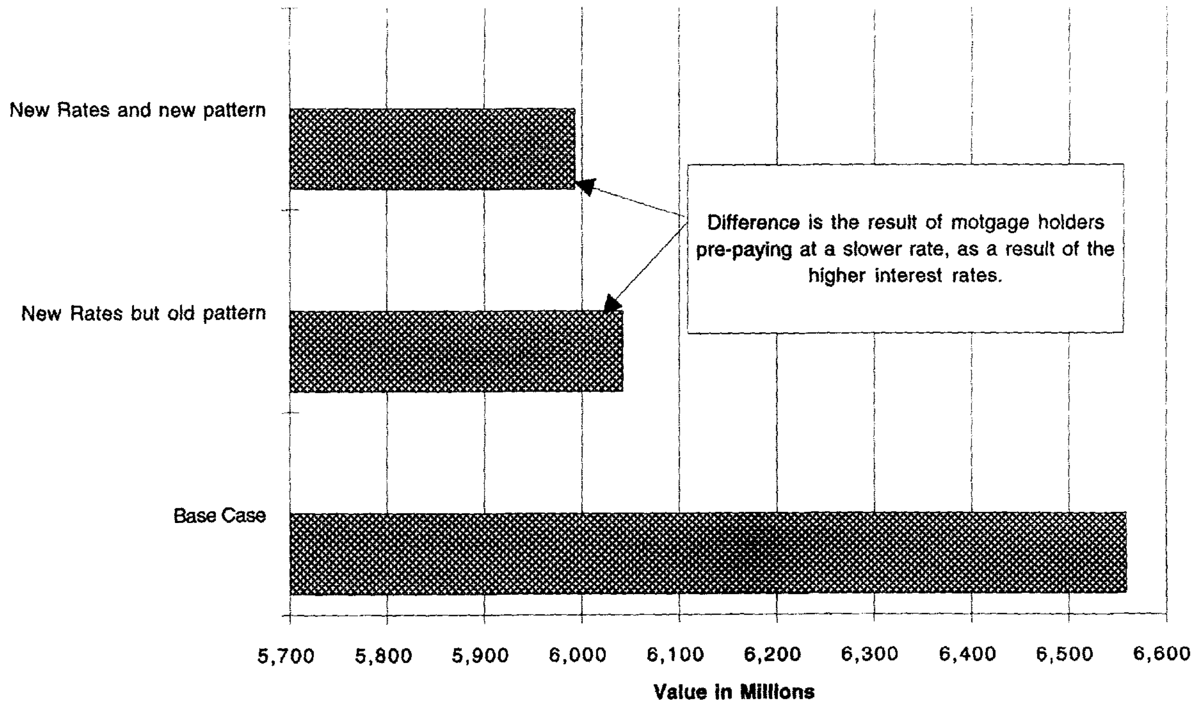
Projected Loss and Adjustment Expense Ratio					
Policy	Voluntary, Involuntary, Net of Reinsurance				Total
Year	Work Comp	GL & CMP	Comm Auto & Other	Pers Auto & HO	
1995	87%	105%	84%	81%	86%
1996	87%	105%	84%	81%	86%
1997	87%	105%	84%	81%	86%
1998	87%	105%	84%	81%	86%
1999	87%	105%	84%	81%	86%
2000	87%	105%	84%	81%	86%
2001	87%	105%	84%	81%	86%
2002	87%	105%	84%	81%	86%
2003	87%	105%	84%	81%	86%
2004	87%	105%	84%	81%	86%

Policy	Voluntary, Involuntary, Net of Reinsurance				Total
Year	Work Comp Comp	GL & CMP Liab	Comm Auto & Other	Pers Auto & HO	Comp
1995	20%	23%	22%	20%	20%
1996	20%	23%	22%	20%	20%
1997	20%	23%	22%	20%	20%
1998	20%	23%	22%	20%	20%
1999	20%	23%	22%	20%	20%
2000	20%	23%	22%	20%	20%
2001	20%	23%	22%	20%	20%
2002	20%	23%	22%	20%	20%
2003	20%	23%	22%	20%	20%
2004	19%	22%	21%	20%	20%

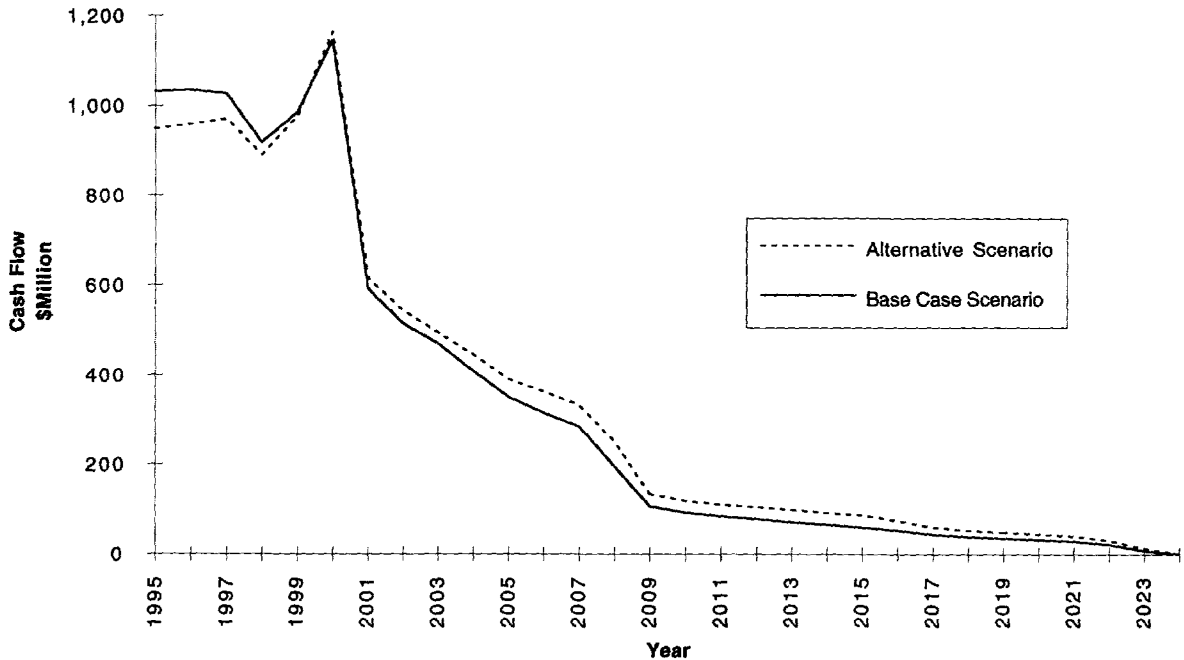
Projected Underwriting Ratio					
Policy	Voluntary, Involuntary, Net of Reinsurance				Total
Year	Work Comp	GL & CMP	Comm Auto & Other	Pers Auto & HO	
1995	106%	128%	106%	101%	106%
1996	106%	128%	106%	101%	106%
1997	106%	128%	106%	101%	106%
1998	106%	128%	106%	101%	106%
1999	106%	128%	106%	101%	106%
2000	106%	128%	106%	101%	106%
2001	106%	128%	106%	101%	106%
2002	106%	128%	106%	101%	106%
2003	106%	128%	106%	101%	106%
2004	106%	128%	105%	100%	105%

Note: Actual input in the model had finer line detail.

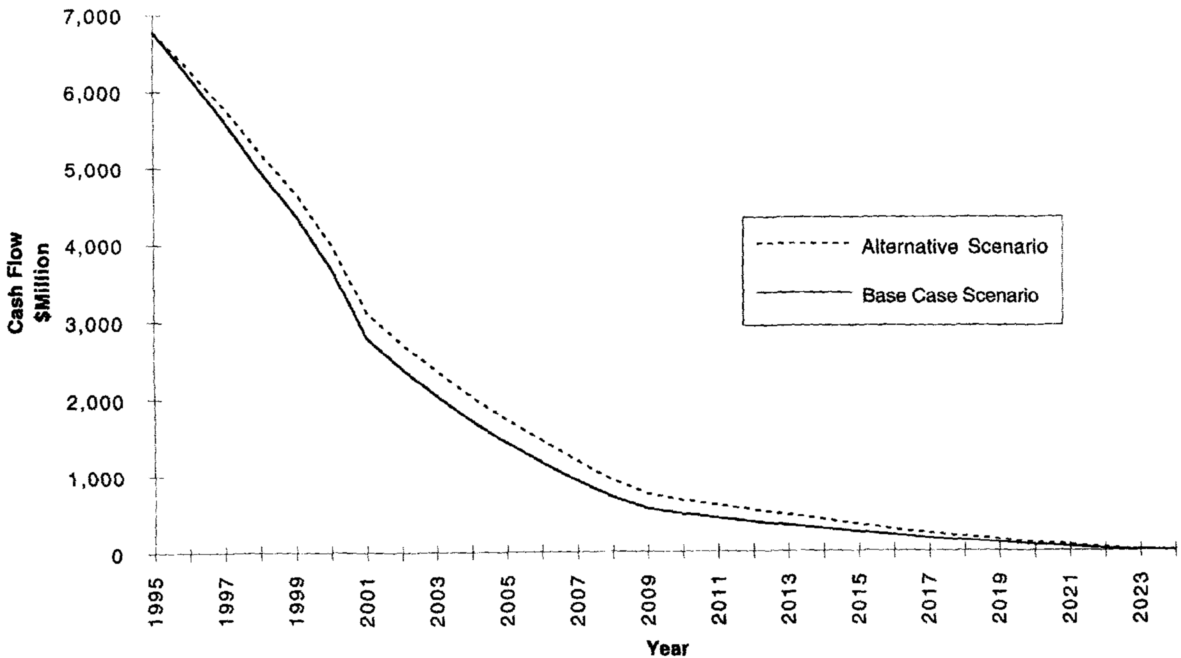
Market Value of Mortgage Backed Securuites



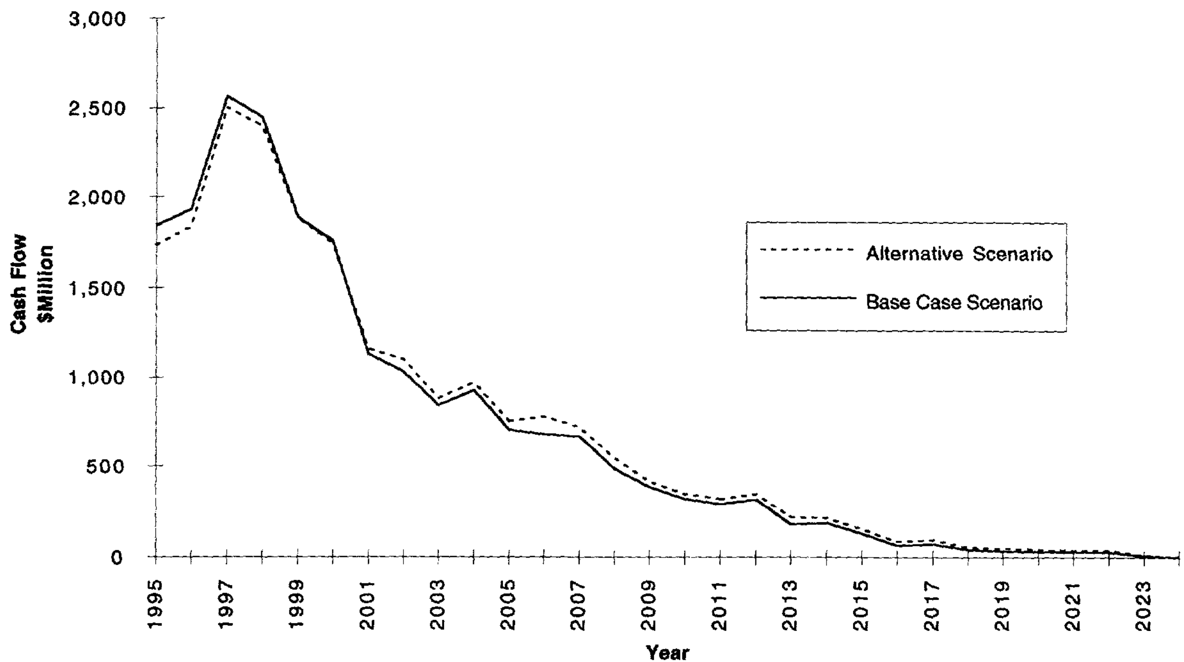
Mortgage Backed Securities Cash Flow
Effect of Prepayment from interest rate increase



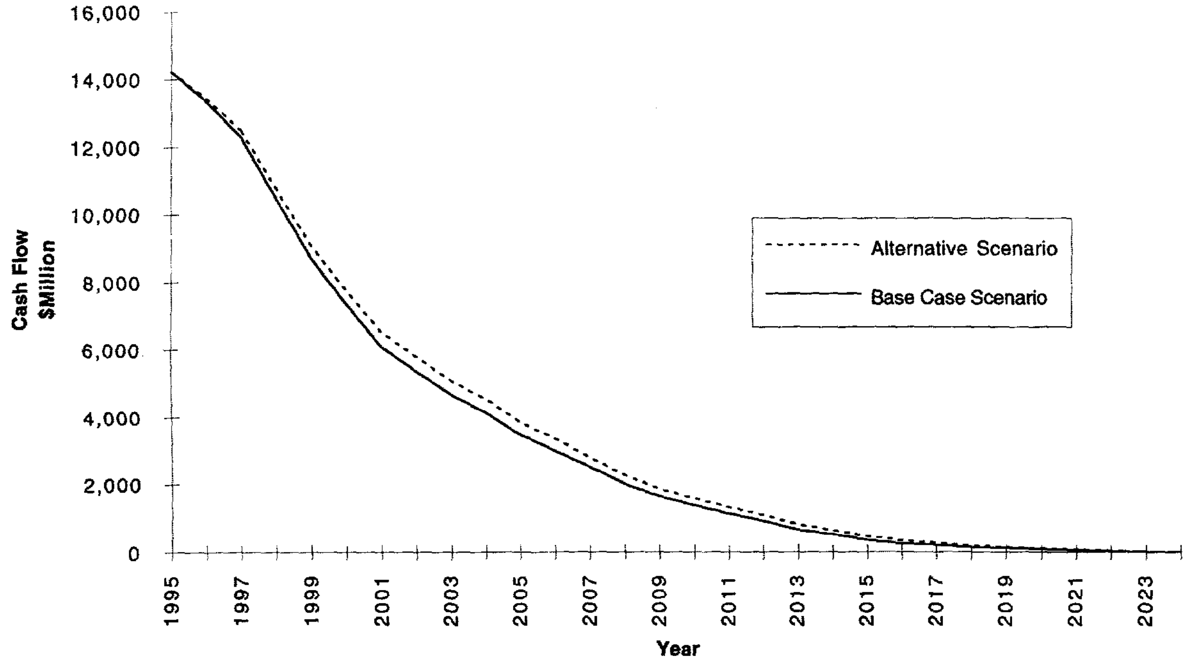
Mortgage Backed Securities Remaining Balance
Effect of Prepayment from Interest rate increase



Total Fixed Income Securities Cash Flow
Effect of Prepayment from interest rate increase



Total Fixed Income Securities Remaining Balance
Effect of Prepayment from interest rate increase



Sample Financial Model
Workers Compensation Insurance Operations Cash Flows (Base Case)
12/94 Runoff Business Only

Appendix C
Exhibit 1

	Premium	Loss & LAE	Other Expense	FIT from Underwriting	Net Insurance Operations
1995	819,224	1,741,739	160,648	(126,161)	(957,003)
1996	360,641	1,216,972	30,747	(124,299)	(762,780)
1997	327,524	874,610	9,744	(102,362)	(454,467)
1998	199,793	679,871	4,050	(89,976)	(394,152)
1999	131,450	542,334	1,481	(77,534)	(334,831)
2000	101,706	452,309	1,293	(73,695)	(278,201)
2001	80,702	390,858	959	(63,945)	(247,170)
2002	65,481	344,704	529	(54,703)	(225,049)
2003	55,504	308,930	310	(44,212)	(209,525)
2004	46,612	279,126	199	(33,199)	(199,514)
2005	38,352	253,854	0	(23,853)	(191,650)
2006	23,969	231,812	0	(16,284)	(191,559)
2007	16,185	213,346	0	(11,254)	(185,907)
2008	9,650	197,115	0	(7,503)	(179,962)
2009	3,517	182,693	0	(3,956)	(175,219)
2010	0	170,360	0	(2,182)	(168,178)
2011	0	158,640	0	(2,033)	(156,607)
2012	0	147,477	0	(1,892)	(145,586)
2013	0	136,635	0	(1,754)	(134,881)
2014	0	126,130	0	(1,621)	(124,509)
2015	0	116,117	0	(1,494)	(114,623)
2016	0	106,497	0	(1,371)	(105,126)
2017	0	97,502	0	(1,256)	(96,247)
2018	0	89,685	0	(1,155)	(88,530)
2019	0	82,569	0	(1,065)	(81,504)
2020	0	75,303	0	(974)	(74,329)
2021	0	67,002	0	(872)	(66,130)
Total	2,280,311	9,509,120	209,961	(873,568)	(6,565,202)

Sample Financial Model
Workers Compensation Insurance Operations Cash Flows (Alternative)
12/94 Runoff Business Only

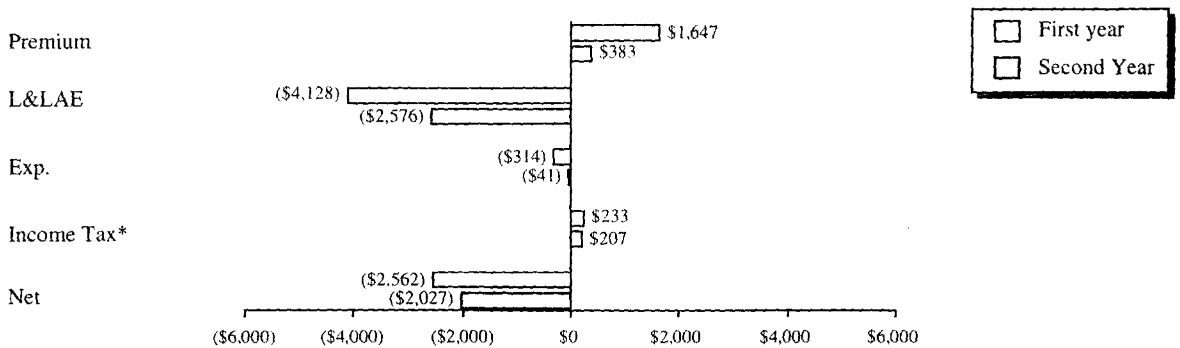
Appendix C
Exhibit 2

	Premium	Loss & LAE	Other Expense	FIT from Underwriting	Net Insurance Operations
1995	917,281	1,767,629	167,816	(132,687)	(885,478)
1996	375,222	1,252,167	31,890	(162,408)	(746,427)
1997	341,376	912,170	10,580	(135,572)	(445,802)
1998	209,446	719,144	4,633	(121,097)	(393,234)
1999	138,075	582,025	1,881	(107,231)	(338,600)
2000	106,499	492,559	1,582	(102,958)	(284,685)
2001	84,030	432,021	1,160	(92,220)	(256,931)
2002	67,708	386,815	664	(81,833)	(237,937)
2003	57,079	352,105	405	(69,608)	(225,823)
2004	47,664	323,218	263	(56,279)	(219,538)
2005	39,040	298,762	42	(44,460)	(215,304)
2006	24,473	277,467	30	(34,404)	(218,621)
2007	16,599	259,747	25	(27,171)	(216,002)
2008	9,984	244,170	20	(21,364)	(212,842)
2009	3,808	230,277	18	(15,624)	(210,862)
2010	43	218,538	3	(12,131)	(206,366)
2011	0	207,191	0	(10,837)	(196,354)
2012	0	196,164	0	(9,624)	(186,540)
2013	0	185,162	0	(8,500)	(176,662)
2014	0	174,220	0	(7,468)	(166,752)
2015	0	163,554	0	(6,520)	(157,034)
2016	0	152,966	0	(5,628)	(147,338)
2017	0	142,804	0	(4,786)	(138,018)
2018	0	133,958	0	(3,988)	(129,970)
2019	0	125,700	0	(3,205)	(122,495)
2020	0	116,804	0	(2,466)	(114,338)
2021	0	105,876	0	(1,809)	(104,067)
Total	2,438,327	10,829,116	221,012	(1,284,129)	(7,327,673)

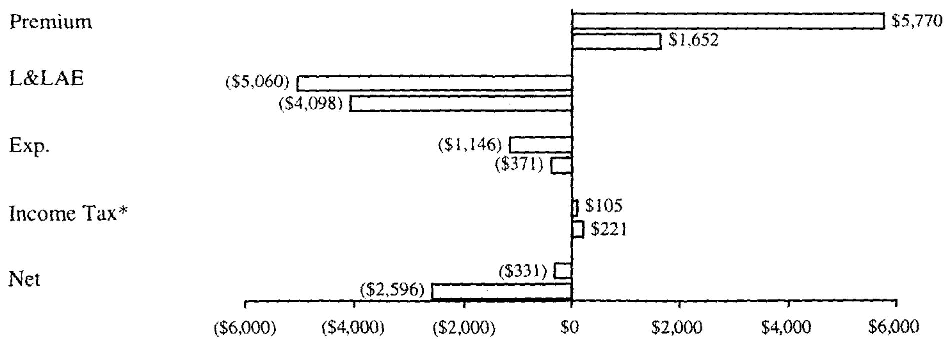
Insurance Operations Base Case Cash Flows
First Year and Second Year Comparison

Appendix C
Exhibit 3

Runoff Only



Runoff and One New Policy Year

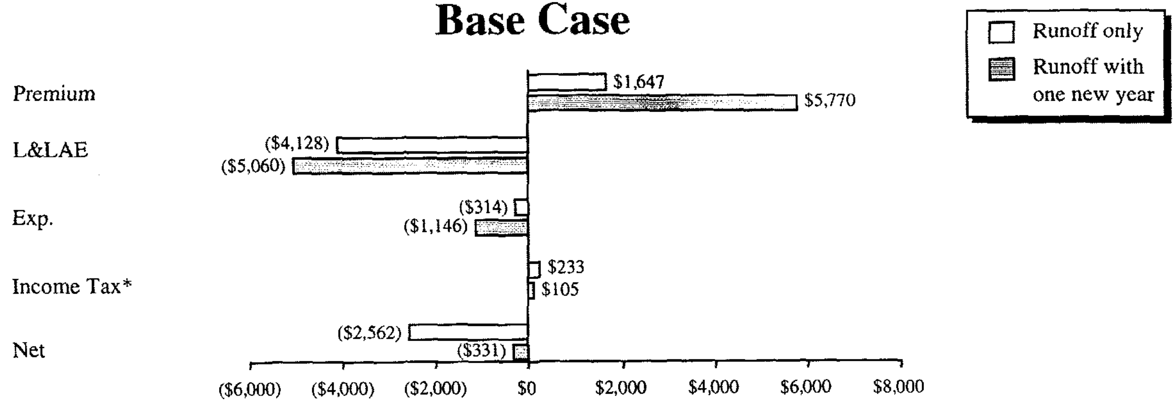


*Income tax excludes tax on investment

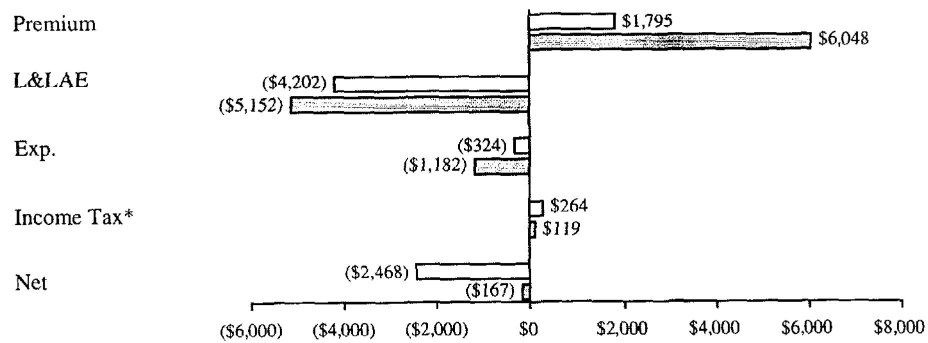
Appendix C
Exhibit 4

Insurance Operations Cash Flow in First Year

Base Case

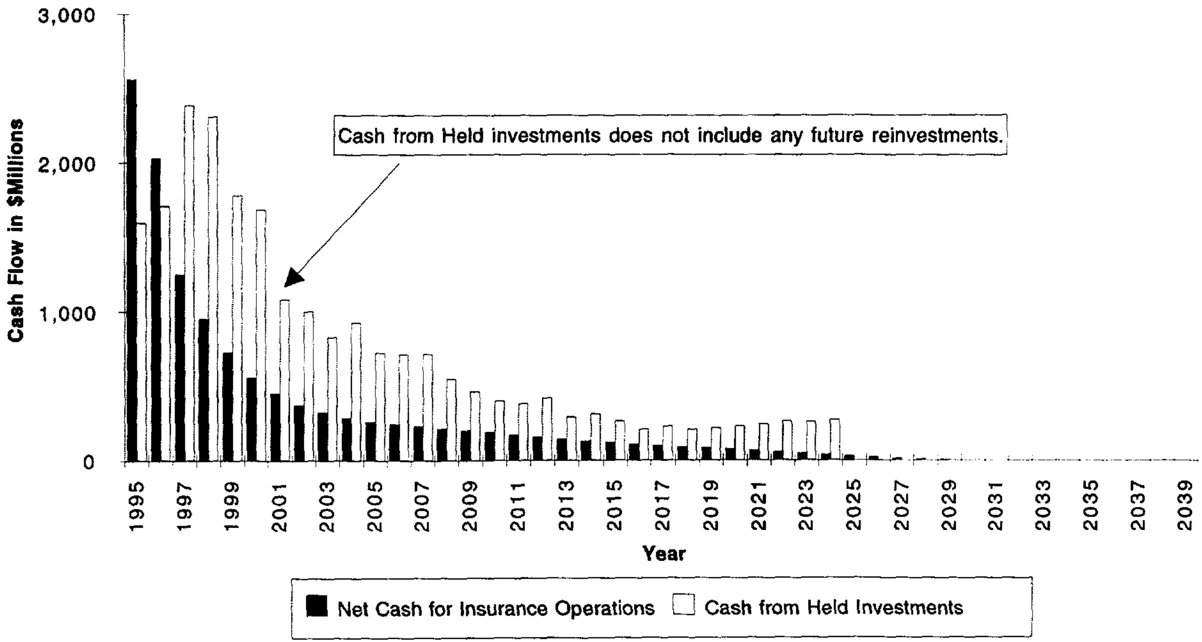


Alternative Scenario

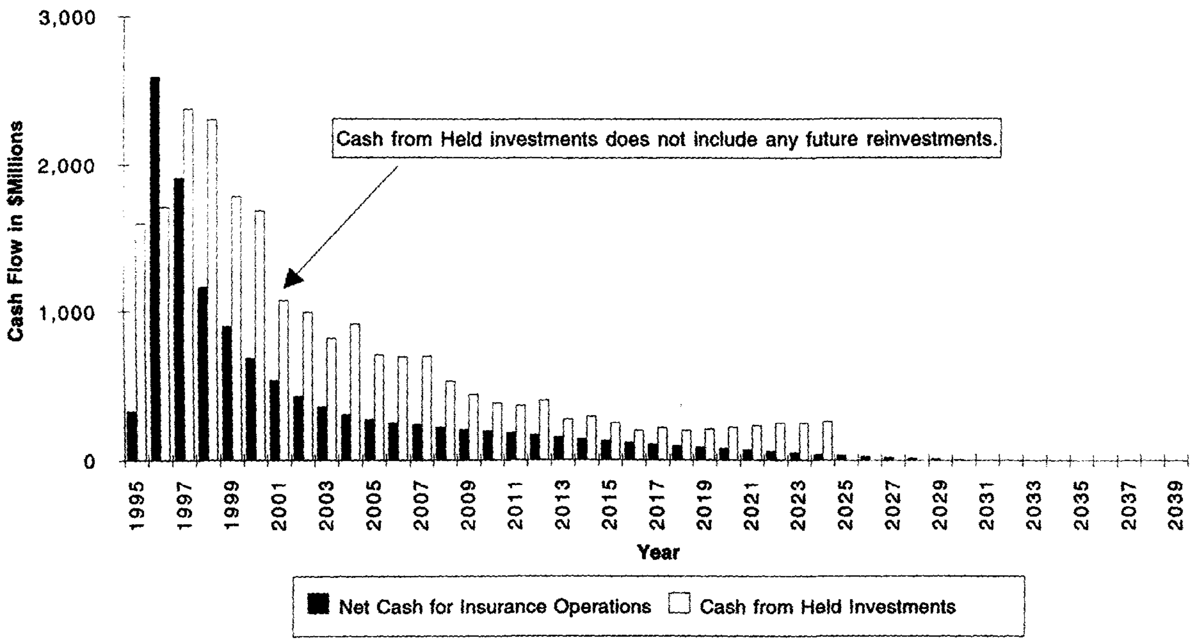


*Income tax excludes tax on investment

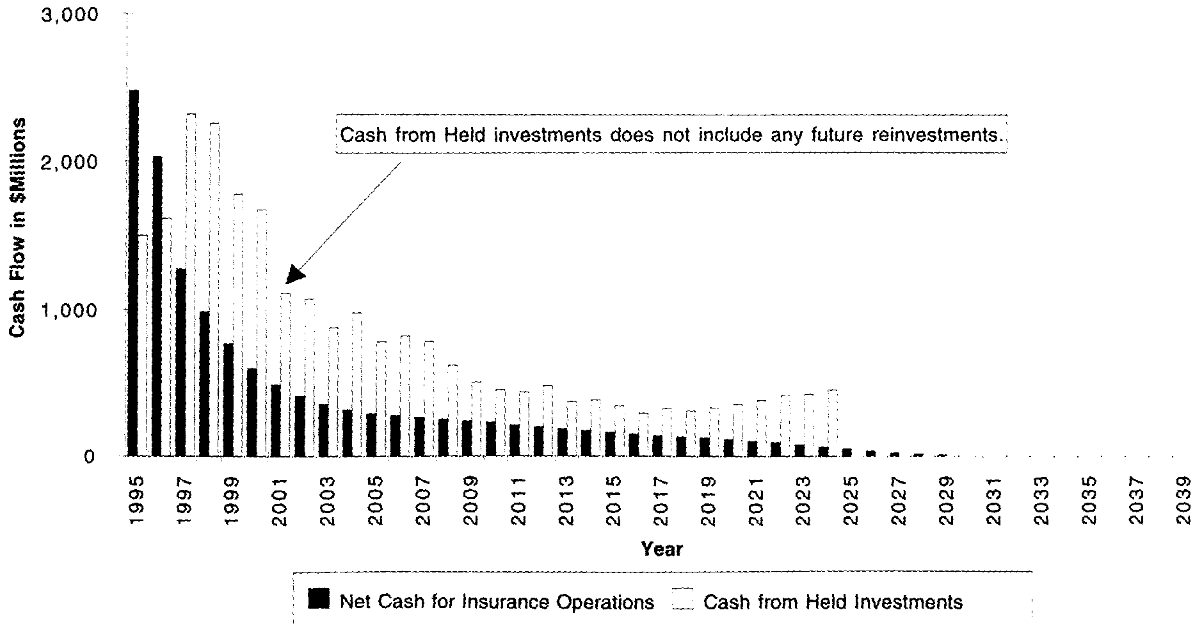
Investments and Net Liability Cash Flows
Cash Flow from Runoff Only
Base Case Scenario



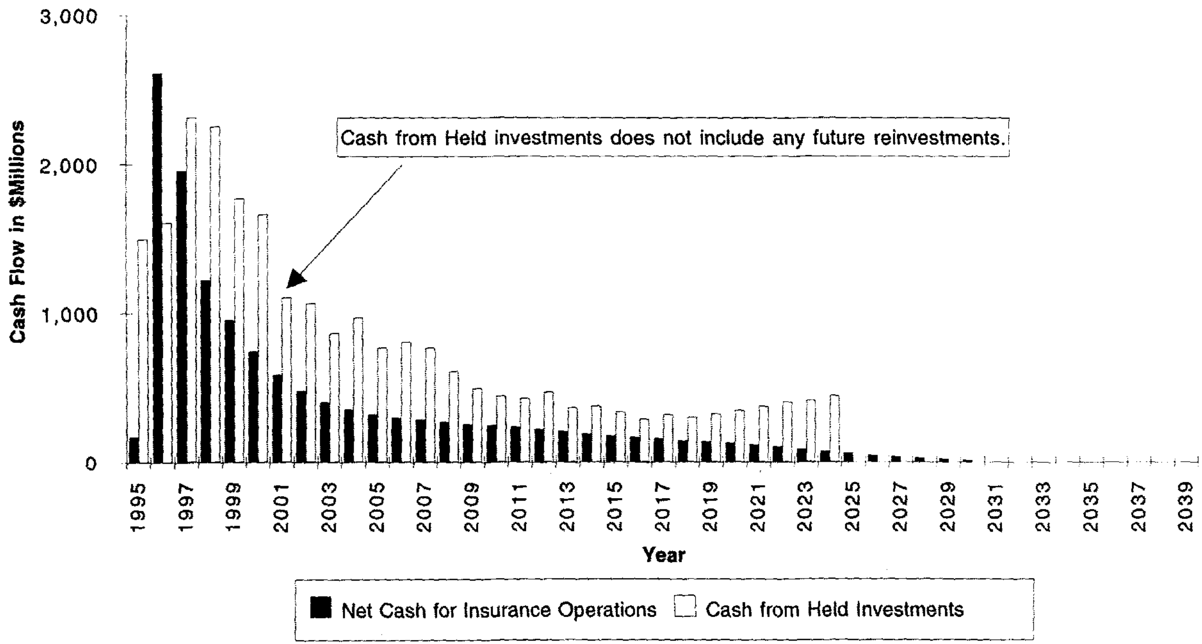
Investments and Net Liability Cash Flows
Cash Flow from Runoff and One New Policy Year
Base Case Scenario



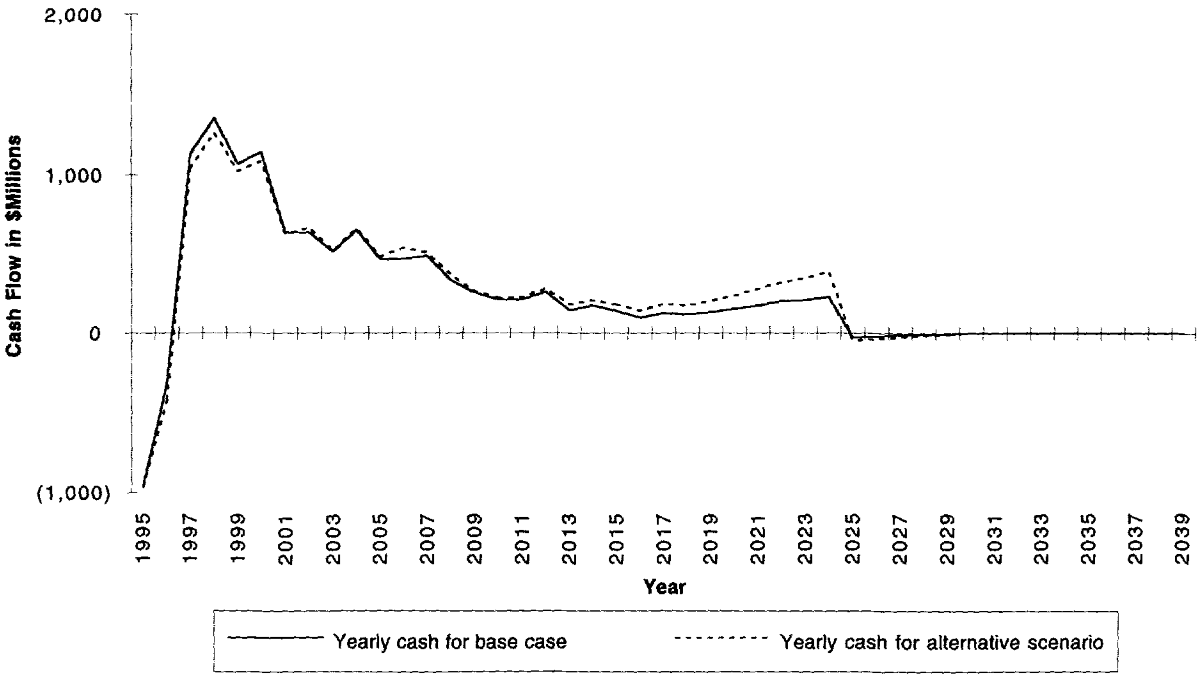
Investments and Net Liability Cash Flows
Cash Flow from Runoff Only
Alternative Scenario



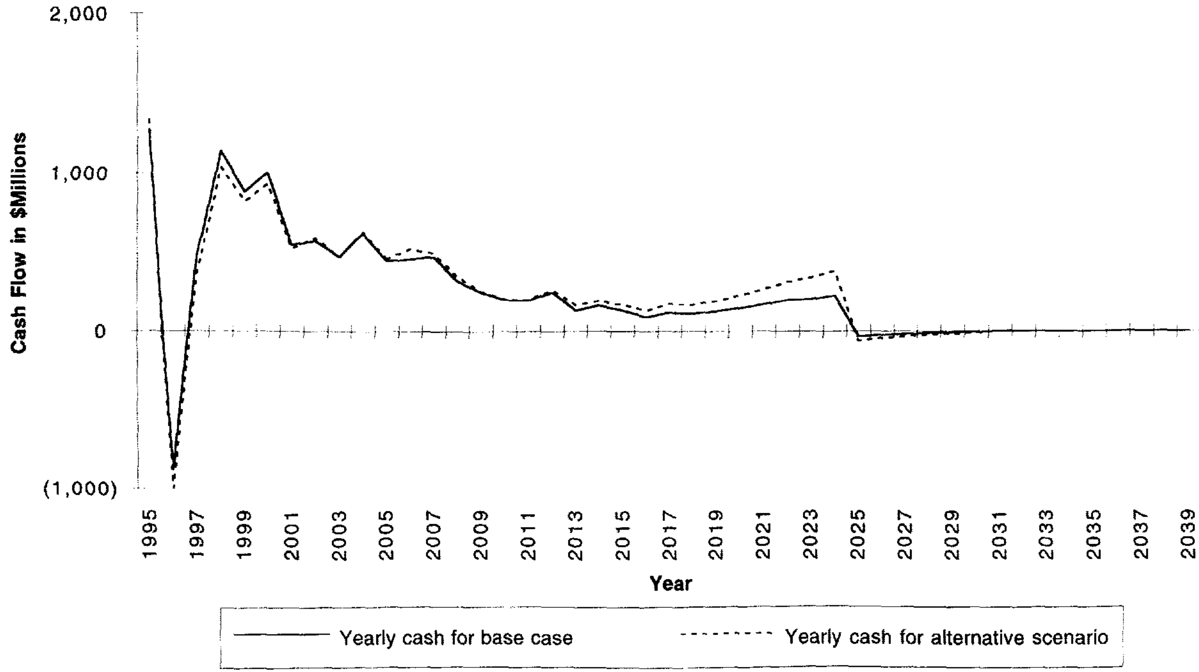
Investments and Net Liability Cash Flows
Cash Flow from Runoff and One New Policy Year
Alternative Scenario



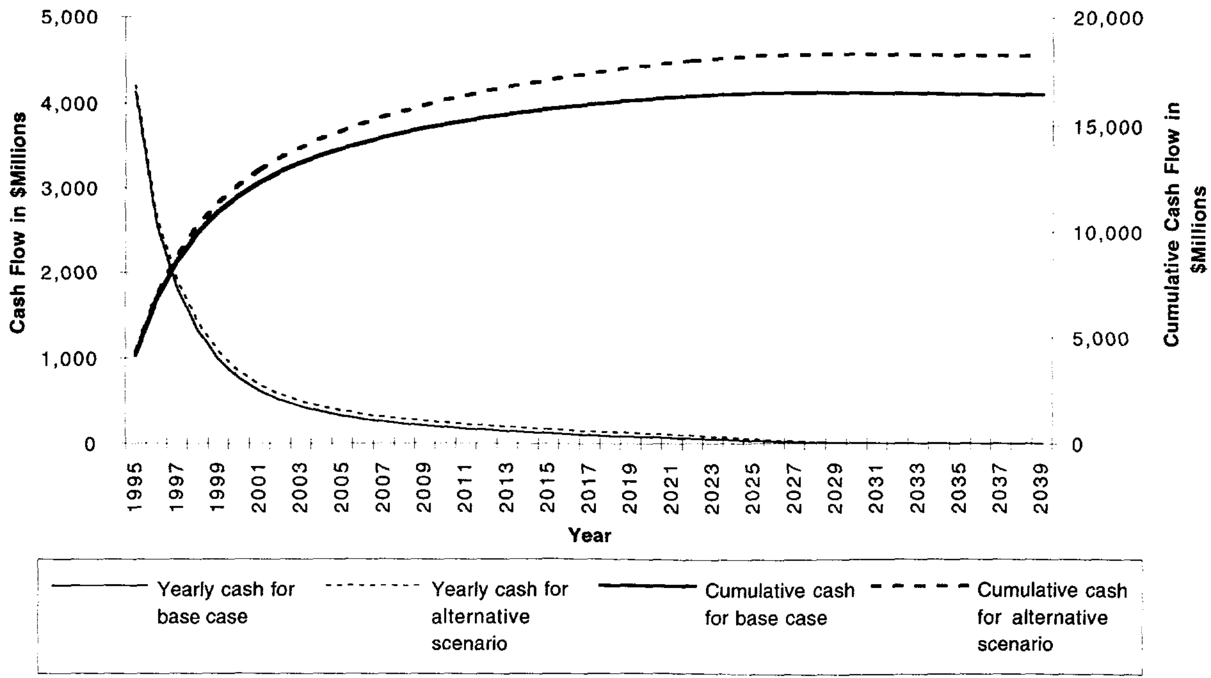
Net cash flows under the two Scenarios
Cash Flow from Runoff Only



Net cash flows under the two Scenarios
Cash Flow from Runoff and One New Policy Year



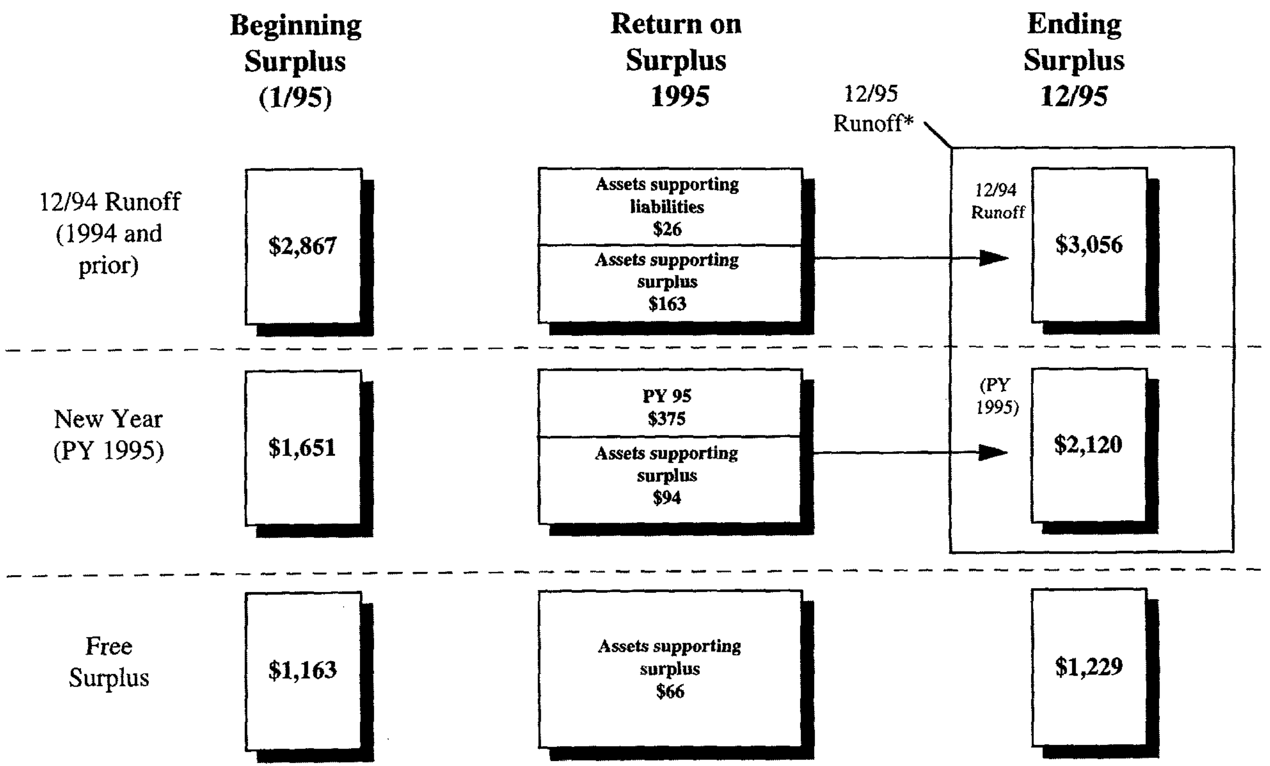
Loss and LAE cash flows under the two Scenarios
Cash Flow from Runoff Only



Appendix D
Exhibit 1

Return on Capital Illustration

Base Case



(Millions of Dollars)

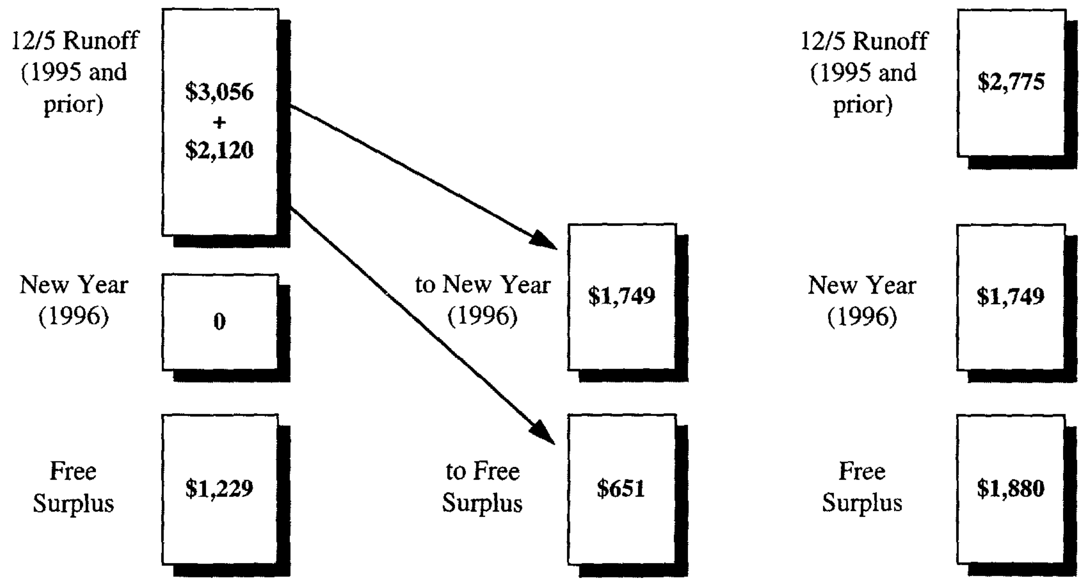
*This combination makes up the 12/95 runoff surplus

Appendix D
Exhibit 2

Release of Capital Illustration

Base Case

1996 Beginning Surplus Before Capital Reallocation	Surplus Release at (1/1/96)	1996 Beginning Surplus After Capital Reallocation



(Millions of Dollars)

Note: Excess runoff surplus is released first to support the new year, and the remainder is released to free surplus.

Summary of return on Capital and release of Capital
Base Case

Total

	beginning surplus	ending surplus before new year U/W	% return	additional return from new year	% return	ending surplus	% return
Runoff	2,867,226	3,055,745	6.6%	–		3,055,745	6.6%
new year	1,650,585	1,744,225	5.7%	375,495	22.7%	2,119,720	28.4%
free surplus	1,162,973	1,228,951	5.7%	–		1,228,951	5.7%
Total	5,680,784	6,028,921	6.1%	375,495	6.6%	6,404,416	12.7%

Surplus allocated at the beginning of the following year

	capital needed	capital available	capital released	reallocated capital needed
Runoff	2,775,303	5,175,465	2,400,162	2,775,303
new year	1,749,224	–	(1,749,224)	1,749,224
free surplus	–	1,228,951	–	1,879,889
Total	4,524,527	6,404,416	650,938	6,404,416

Summary of return on Capital and release of Capital
Base Case

Appendix D
Exhibit 4

WC

	beginning surplus	ending surplus before new year U/W	% return	additional return from new year	% return	ending surplus	% return
Runoff	1,299,488	1,385,532	6.6%	–		1,385,532	6.6%
new year	588,698	622,097	5.7%	218,418	37.1%	840,515	42.8%
Total	1,888,186	2,007,629	6.3%	218,418	11.6%	2,226,047	17.9%

Surplus allocated at the beginning of the following year

	capital needed	capital available	capital released
Runoff	1,292,212	2,226,047	933,835
new year	623,879	–	(623,879)
Total	1,916,091	2,226,047	309,956

GL

	beginning surplus	ending surplus before new year U/W	% return	additional return from new year	% return	ending surplus	% return
Runoff	493,855	524,978	6.3%	–		524,978	6.3%
new year	203,021	214,538	5.7%	(1,050)	-0.5%	213,488	5.2%
Total	696,876	739,516	6.1%	(1,050)	-0.2%	738,466	6.0%

Surplus allocated at the beginning of the following year

	capital needed	capital available	capital released
Runoff	472,917	738,466	265,549
new year	215,153	–	(215,153)
Total	688,070	738,466	50,396

Summary of return on Capital and release of Capital
Base Case

Appendix D
Exhibit 5

CMP

	beginning surplus	ending surplus before new year U/W	% return	additional return from new year	% return	ending surplus	% return
Runoff	150,610	159,616	6.0%	–		159,616	6.0%
new year	82,935	87,640	5.7%	(2,098)	-2.5%	85,542	3.1%
Total	233,545	247,256	5.9%	(2,098)	-0.9%	245,158	5.0%

Surplus allocated at the beginning of the following year

	capital needed	capital available	capital released
Runoff	140,488	245,158	104,670
new year	87,892	–	(87,892)
Total	228,380	245,158	16,778

CA

	beginning surplus	ending surplus before new year U/W	% return	additional return from new year	% return	ending surplus	% return
Runoff	174,893	186,587	6.7%	–		186,587	6.7%
new year	133,752	141,340	5.7%	30,436	22.8%	171,776	28.4%
Total	308,645	327,927	6.2%	30,436	9.9%	358,363	16.1%

Surplus allocated at the beginning of the following year

	capital needed	capital available	capital released
Runoff	190,065	358,363	168,298
new year	141,745	–	(141,745)
Total	331,810	358,363	26,553

Summary of return on Capital and release of Capital
Base Case

Appendix D
Exhibit 6

Other Bus.

	beginning surplus	ending surplus before new year U/W	% return	additional return from new year	% return	ending surplus	% return
Runoff	4,832	5,130	6.2%	–		5,130	6.2%
new year	13,776	14,557	5.7%	(10,182)	-73.9%	4,375	-68.2%
Total	18,608	19,687	5.8%	(10,182)	-54.7%	9,505	-48.9%

Surplus allocated at the beginning of the following year

	capital needed	capital available	capital released
Runoff	10,752	9,505	(1,247)
new year	14,599	–	(14,599)
Total	25,351	9,505	(15,846)

PA

	beginning surplus	ending surplus before new year U/W	% return	additional return from new year	% return	ending surplus	% return
Runoff	317,159	340,008	7.2%	–		340,008	7.2%
new year	461,749	487,945	5.7%	144,259	31.2%	632,204	36.9%
Total	778,908	827,953	6.3%	144,259	18.5%	972,212	24.8%

Surplus allocated at the beginning of the following year

	capital needed	capital available	capital released
Runoff	324,266	972,212	647,946
new year	489,343	–	(489,343)
Total	813,609	972,212	158,603

Summary of Return on Capital and Release of Capital
Base Case

Appendix D
Exhibit 7

HO & Other

	beginning surplus	ending surplus before new year U/W	% return	additional return from new year	% return	ending surplus	% return
Runoff	426,389	453,894	6.5%	-		453,894	6.5%
new year	166,654	176,108	5.7%	(4,288)	-2.6%	171,820	3.1%
Total	593,043	630,002	6.2%	(4,288)	-0.7%	625,714	5.5%

Surplus allocated at the beginning of the following year

	capital needed	capital available	capital released
Runoff	344,603	625,714	281,111
new year	176,613	-	(176,613)
Total	521,216	625,714	104,498

Comparison of return on Capital between base case and alternative case

Appendix D
Exhibit 8

Total

Base Case

	beggining surplus	ending surplus before new year U/W	% return	additional return from new year	% return	ending surplus	% return
Runoff	2,867,226	3,055,745	6.6%	–		3,055,745	6.6%
new year	1,650,585	1,744,225	5.7%	375,495	22.7%	2,119,720	28.4%
free surplus	1,162,973	1,228,951	5.7%	–		1,228,951	5.7%
Total	5,680,784	6,028,921	6.1%	375,495	6.6%	6,404,416	12.7%

Alternative Case

	beggining surplus	ending surplus before new year U/W	% return	additional return from new year	% return	ending surplus	% return
Runoff	2,867,226	2,481,714	-13.4%	–		2,481,714	-13.4%
new year	1,650,585	1,664,141	0.8%	402,158	24.4%	2,066,297	25.2%
free surplus	1,162,973	1,172,522	0.8%			1,172,524	0.8%
Total	5,680,784	5,318,377	-6.4%	402,158	7.1%	5,720,535	0.7%

Comparison of return on Capital between base case and alternative case

Appendix D
Exhibit 9

WC

Base Case

	beggining surplus	ending surplus before new year U/W	% return	additional return from new year	% return	ending surplus	% return
Runoff	1,299,488	1,385,532	6.6%	–		1,385,532	6.6%
new year	588,698	622,097	5.7%	218,418	37.1%	840,515	42.8%
Total	1,888,186	2,007,629	6.3%	218,418	11.6%	2,226,047	17.9%

Alternative Case

	beggining surplus	ending surplus before new year U/W	% return	additional return from new year	% return	ending surplus	% return
Runoff	1,299,488	1,205,016	-7.3%	–		1,205,016	-7.3%
new year	588,698	593,533	0.8%	245,885	41.8%	839,418	42.6%
Total	1,888,186	1,798,549	-4.7%	245,885	13.0%	2,044,434	8.3%

GL

Base Case

	beggining surplus	ending surplus before new year U/W	% return	additional return from new year	% return	ending surplus	% return
Runoff	493,855	524,978	6.3%	–		524,978	6.3%
new year	203,021	214,538	5.7%	(1,050)	-0.5%	213,488	5.2%
Total	696,876	739,516	6.1%	(1,050)	-0.2%	738,466	6.0%

Alternative Case

	beggining surplus	ending surplus before new year U/W	% return	additional return from new year	% return	ending surplus	% return
Runoff	493,855	430,088	-12.9%	–		430,088	-12.9%
new year	203,021	204,688	0.8%	(461)	-0.2%	204,227	0.6%
Total	696,876	634,776	-8.9%	(461)	-0.1%	634,315	-9.0%

Comparison of return on Capital between base case and alternative case

Appendix D
Exhibit 10

CMP

Base Case

	beggining surplus	ending surplus before new year U/W	% return	additional return from new year	% return	ending surplus	% return
Runoff	150,610	159,616	6.0%	–		159,616	6.0%
new year	82,935	87,640	5.7%	(2,098)	-2.5%	85,542	3.1%
Total	233,545	247,256	5.9%	(2,098)	-0.9%	245,158	5.0%

Alternative Case

	beggining surplus	ending surplus before new year U/W	% return	additional return from new year	% return	ending surplus	% return
Runoff	150,610	136,968	-9.1%	–		136,968	-9.1%
new year	82,935	83,617	0.8%	(3,217)	-3.9%	80,400	-3.1%
Total	233,545	220,585	-5.5%	(3,217)	-1.4%	217,368	-6.9%

CA

Base Case

	beggining surplus	ending surplus before new year U/W	% return	additional return from new year	% return	ending surplus	% return
Runoff	174,893	186,587	6.7%	–		186,587	6.7%
new year	133,752	141,340	5.7%	30,436	22.8%	171,776	28.4%
Total	308,645	327,927	6.2%	30,436	9.9%	358,363	16.1%

Alternative Case

	beggining surplus	ending surplus before new year U/W	% return	additional return from new year	% return	ending surplus	% return
Runoff	174,893	152,101	-13.0%	–		152,101	-13.0%
new year	133,752	134,850	0.8%	35,870	26.8%	170,720	27.6%
Total	308,645	286,951	-7.0%	35,870	11.6%	322,821	4.6%

Comparison of return on Capital between base case and alternative case

Other Bus.

Base Case

	beggining surplus	ending surplus before new year U/W	% return	additional return from new year	% return	ending surplus	% return
Runoff	4,832	5,130	6.2%	–		5,130	6.2%
new year	13,776	14,557	5.7%	(10,182)	-73.9%	4,375	-68.2%
Total	18,608	19,687	5.8%	(10,182)	-54.7%	9,505	-48.9%

Alternative Case

	beggining surplus	ending surplus before new year U/W	% return	additional return from new year	% return	ending surplus	% return
Runoff	4,832	4,062	-15.9%	–		4,062	-15.9%
new year	13,776	13,889	0.8%	(10,807)	-78.4%	3,082	-77.6%
Total	18,608	17,951	-3.5%	(10,807)	-58.1%	7,144	-61.6%

PA

Base Case

	beggining surplus	ending surplus before new year U/W	% return	additional return from new year	% return	ending surplus	% return
Runoff	317,159	340,008	7.2%	–		340,008	7.2%
new year	461,749	487,945	5.7%	144,259	31.2%	632,204	36.9%
Total	778,908	827,953	6.3%	144,259	18.5%	972,212	24.8%

Alternative Case

	beggining surplus	ending surplus before new year U/W	% return	additional return from new year	% return	ending surplus	% return
Runoff	317,159	202,110	-36.3%	–		202,110	-36.3%
new year	461,749	465,541	0.8%	140,949	30.5%	606,490	31.3%
Total	778,908	667,651	-14.3%	140,949	18.1%	808,600	3.8%

Comparison of Return on Capital between base case and alternative scenario

Appendix D
Exhibit 12

HO & Other							
Base Case							
	beggining surplus	ending surplus before new year U/W	% return	additional return from new year	% return	ending surplus	% return
Runoff	426,389	453,894	6.5%	–		453,894	6.5%
new year	166,654	176,108	5.7%	(4,288)	-2.6%	171,820	3.1%
Total	593,043	630,002	6.2%	(4,288)	-0.7%	625,714	5.5%
Alternative Case							
	beggining surplus	ending surplus before new year U/W	% return	additional return from new year	% return	ending surplus	% return
Runoff	426,389	351,369	-17.6%	–		351,369	-17.6%
new year	166,654	168,023	0.8%	(6,063)	-3.6%	161,960	-2.8%
Total	593,043	519,392	-12.4%	(6,063)	-1.0%	513,329	-13.4%

Illustration of Return
Overall Company
Base Case Scenario

Return on Runoff Before any New Business				
Item	Market Value @ 12/94	Return	Rate of Return	Market Value @ 12/95
Fixed Income Securities	13,942	786	5.6%	14,728
Stocks	1,200	125	10.4%	1,325
Unrecognized FIT on Investments	(79)	(55)		(134)
Other Assets	(90)	(6)	7.0%	(96)
Collectable Premium	3,009	162	5.4%	3,172
Loss & LAE	(12,960)	(699)	5.4%	(13,659)
Other Expense	(360)	(19)	5.4%	(379)
FIT on Underwriting Income	1,018	55	5.4%	1,073
Net Market Value	5,681	348	6.1%	6,029
Net Value of Investments & Other	14,973	849	5.7%	15,823
Net Value of Insurance Operations	(9,292)	(501)	5.4%	(9,794)

Additional Return from New Policy Year (1995 PY)			
Item	Nominal Value	Dec-95 Market Value	Discount Factor
Premium	5,864	5,839	99.6%
Loss & LAE	(4,787)	(4,257)	88.9%
Other Expense	(1,201)	(1,210)	100.8%
Income Tax	44	4	
Net Return from New Policy Year	(81)	375	

Summary of Results				
	Market Value @ 12/94	Return	Rate of Return	Market Value @ 12/95
All Items	5,681	724	12.7%	6,404

Note: Actual input in the model had finer line detail.

Illustration of Return
Overall Company
Alternative Scenario

Return on Runoff Before any New Business

Item	Market Value @ 12/94	Return	Rate of Return	Market Value @ 12/95
Fixed Income Securities	13,942	(210)	-1.5%	13,732
Stocks	1,200	1	0.1%	1,201
Unrecognized FIT on Investments	(79)	339		259
Other Assets	(90)	(6)	7.0%	(96)
Collectable Premium	3,009	336	11.2%	3,346
Loss & LAE	(12,960)	(1,202)	9.3%	(14,162)
Other Expense	(360)	(34)	9.5%	(394)
FIT on Underwriting Income	1,018	414	40.7%	1,433
Net Market Value	5,681	(362)	-6.4%	5,318
Net Value of Investments & Other	14,973	123	0.8%	15,096
Net Value of Insurance Operations	(9,292)	(485)	5.2%	(9,778)

Additional Return from New Policy Year (1995 PY)

Item	Nominal Value	Dec-95 Market Value	Discount Factor
Premium	6,058	6,016	99.3%
Loss & LAE	(5,182)	(4,418)	85.3%
Other Expense	(1,241)	(1,246)	100.4%
Income Tax	xxxx	50	
Net Return from New Policy Year	(365)	402	

Summary of Results

	Market Value @ 12/94	Return	Rate of Return	Market Value @ 12/95
All Items	5,681	40	0.7%	5,721

Note: Actual input in the model had finer line detail.

Analysis of Returns (Before New Business)
Illustration of Base Case Scenario for the Overall Company

The difference in returns results from the combination of two factors, both of which need to exist simultaneously. The first is because of how assets and liabilities are allocated. The second is because the after-tax rate return on investments is different from the after-tax rate of discount in the reserves.

How are assets and liabilities allocated? Recall the stated Results (Before new year underwriting):

Type of Surplus	Beginning Surplus	Ending Surplus	Dollar Return	Rate of Return
Runoff	2,867	3,056	189	6.6%
New Year	1,651	1,744	94	5.7%
Free Surplus	1,163	1,229	66	5.7%

To get an explanation as to why the return on the runoff surplus is different from the return on the other surplus, one needs to look at the composition of assets and net liabilities in each of these segments. At the beginning of the year, all of the net liabilities, with a starting market value of $9,298 million, is allocated to the runoff surplus. Why? By definition, the runoff surplus is dedicated to support all of the insurance operations that were generated prior to the evaluation date, which is the net liabilities from the insurance operations.

Investments, and other miscellaneous items, can thus be broken into four parts, two of which go to the runoff surplus. First, we allocate enough assets to support the net insurance operations liabilities. Second, we allocate assets to support the runoff surplus. These two portfolios are allocated to the runoff surplus. The rest of the assets are then allocated to support each of the remaining surplus items: new year surplus, and free surplus. This allocation procedure results in the following "summarized balance sheets".

Composition of the Beginning Surplus			
	Investments and Misc.	Ins. Ops Liabilities	Surplus
Runoff	12,160	(9,292)	2,867
New Year	1,651	0	1,651
Free Surplus	1,163	0	1,163

We now turn to the second factor, the difference between the investment rate of return and the rate of discount in the reserves. Reserves are discounted at 5.4%, the after-tax yield rate of a selected bond portfolio. However, the actual investment portfolio was projected to yield 5.7%, mainly because this portfolio includes common stocks with a projected after-tax return of 6.7%. If the rate of return on investments and the discount rate were equal, then the return from the assets supporting the liabilities, would be exactly offset by the unwinding of the discount in the liabilities. However since the rate of return on investments is higher than the discount rate, the unwinding of the discount will not completely offset the return on investments. The table below reconciles the returns.

	Value of Item	Dollar Return	Rate of Return
(1) Assets supporting liabilities	9,292	527	5.7%
(2) liabilities	(9,292)	(501)	5.4%
(3) Net = (1) + (2)	0	26	
(4) pure runoff surplus	2,867	163	5.7%
(5) runoff surplus (3) + (4)	2,867	189	6.6%
(6) new year surplus	1,651	94	5.7%
(7) free surplus	1,163	66	5.7%
(8) total surplus	5,681	348	6.1%

Note that a real quick way to double-check the reasonability of the results:
Return on runoff surplus = 5.7% + ((5.7% - 5.4%) * (9,292 / 2,867)) = 6.6%.
Return on total surplus = 5.7% + ((5.7% - 5.4%) * (9,292 / 5,681)) = 6.1%.

For convenience, we shall call the 5.7% return as the return on assets supporting surplus, and the 0.3% (5.7% - 5.4%) as the return on assets supporting liabilities. The above short cut can be used to confirm the computation for each of the lines. One needs only the ratio of liabilities to surplus (all on a market value basis), in addition to the above returns. The following are examples for some selected lines.

Leverage Ratios			
	Market Values of		Liability-to-
	Insurance Operations	Allocated Surplus	Surplus Ratio
Workers Compensation	4,429	1,299	3.408
General Liability	1,116	494	2.260
Personal Auto	1,745	317	5.503

(1) Return on Investments	5.7%
(2) Insurance Operations Discount Rate	5.4%
(3) Return on Assets Supporting Surplus = (1)	5.7%
(4) Return on Assets Supporting Liabilities = (1) - (2)	0.3%

Return on the Runoff (Base Case)				
	Liability-to-	Return on Assets		% Return
	Surplus Ratio	Supporing Liabilities *	Supporing Surplus **	on Surplus
Workers Compensation	3.408	0.9%	5.7%	6.6%
General Liability	2.260	0.6%	5.7%	6.3%
Personal Auto	5.503	1.5%	5.7%	7.2%

* Return on Assets Supporting Liabilities = product of liabilities-to-surplus ratio and the net yield of assets supporting liabilities

** Return on assets supporting surplus = net yield of assets supporting surplus.

Analysis of Change in Inflation and Interest Rates
Impact on the Return on Runoff Surplus

In the illustrated alternative scenario, we assumed that both inflation and interest rates jump by 200 basis points and remain at the new level. The model assumes that the nominal paid loss and adjustment expenses increase at the 2% annual rate. The only exception is that a portion of workers comp indemnity reserves, in the non-COLA states, are not sensitive to chnages in inflation . All other expenses, except for acquisition expenses, grow at the rate of inflation. There are other consequences that are considered in the model.

First, the retrospective premiums are sensitive to loss experience, and should similarly increase, but not necessarily at the same rate. The increase in premium is dependent on the retrospective contracts. In the illustrated scenario, the sensitivity of retro premium to loss experience was based on the company's historic premium sensitivity to loss and was heavily dependent on the relative age at which the loss development occurs.

Second, one needs to consider the actual reaction of held reserves to the change in inflation outlook. From a market value perspective, held reserves affect only the timing of income taxes. Unfortunately, the reaction of held reserves to changes in inflation are not easily quantifiable. The difficulty stems not from theoretical reasons, since one can easily quantify such impact, rather the difficulty is in attempting to mimic the real world. The standard actuarial techniques do not consider inflation implicitly. True, Inflation is explicitly projected in the standard techniques, but these techniques average out inflation throughout the life of the policy or accident year and, as a result, do not calculate reserves under different inflation scenarios. In our scenario, WC nominal reserves ultimately grow 14% because of the 200 basis points jump in inflation. Company managers do not increase and reduce the reserve levels by such levels just because inflation has gone up. Because of these considerations, the scenario above assumes that the nominal held reserves increase at the rate of inflation.

Third, All the changes above will impact future earned premium and incurred loss and expenses. The tax implications from these earnings and losses are considered in the model. The change of 200 basis points in interest rates causes a change of 130 basis points in the discount rate, since the discount rate is an after-tax rate.

We had discussed previously the impact on investments from a 200 basis points increase in interest rates. Overall, the after-tax return on investments drops from 5.7% in the base case scenario, to 0.8% in the alternative scenario.

The following exhibits will illustrate the change in return on the runoff for some of the lines.

Change in Nominal Values of Insurance Operations				
	Premium	L & LAE	Oth. Exp.	FIT
Workers Compensation	158	(1,320)	(11)	411
General Liability	34	(158)	(1)	44
Personal Auto	N/A	(132)	(1)	47
Total All-Lines	227	(1,800)	(15)	556

Change in Market Values of Insurance Operations				
	Premium	L & LAE	Oth. Exp.	FIT
Workers Compensation	112	(247)	(11)	243
General Liability	27	(74)	(1)	32
Personal Auto	1	(78)	(1)	40
Total All-Lines	174	(503)	(15)	359

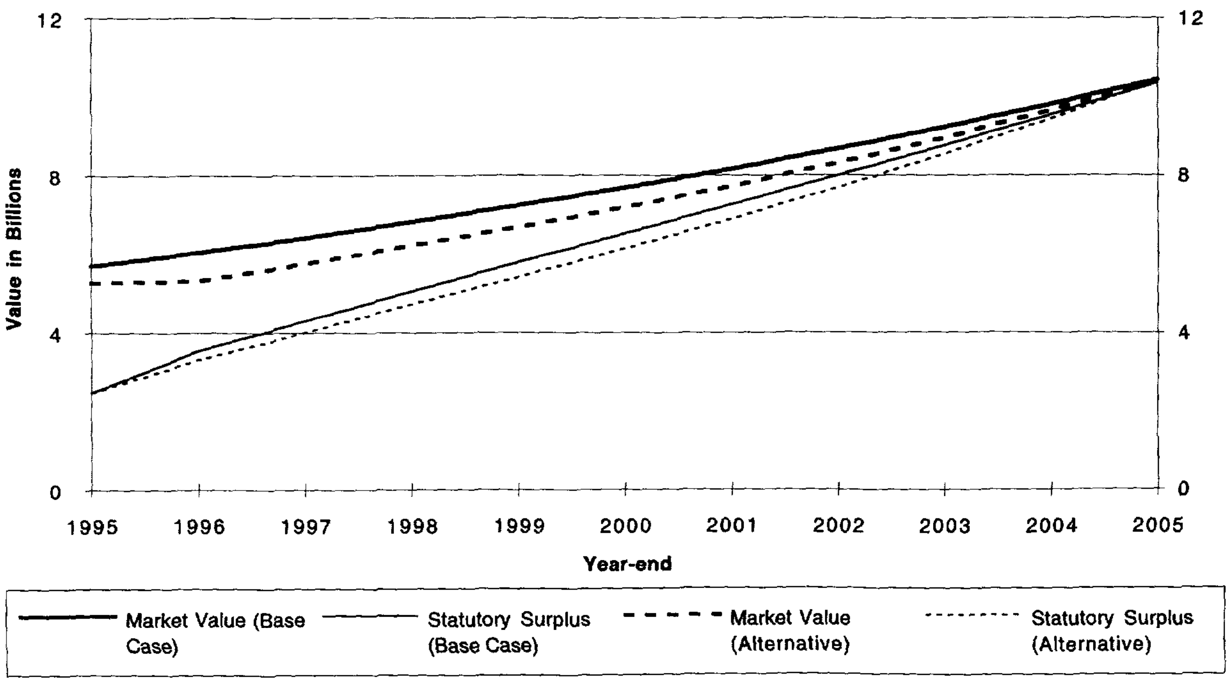
Change in Market Values of Investments			
	Beginning Investments	% Change in Return	Change in Return
Workers Compensation	5,728	-4.9%	(278)
General Liability	1,610	-4.9%	(78)
Personal Auto	2,062	-4.9%	(100)
Total All-Lines	12,160	-4.9%	(590)

Total Change in Return				
	Insurance Operations	Investments	Total Change	Change as % of Surplus
Workers Compensation	97	(278)	(180)	-13.9%
General Liability	(17)	(78)	(95)	-19.2%
Personal Auto	(38)	(100)	(138)	-43.5%
Total All-Lines	16	(590)	(574)	-20.0%

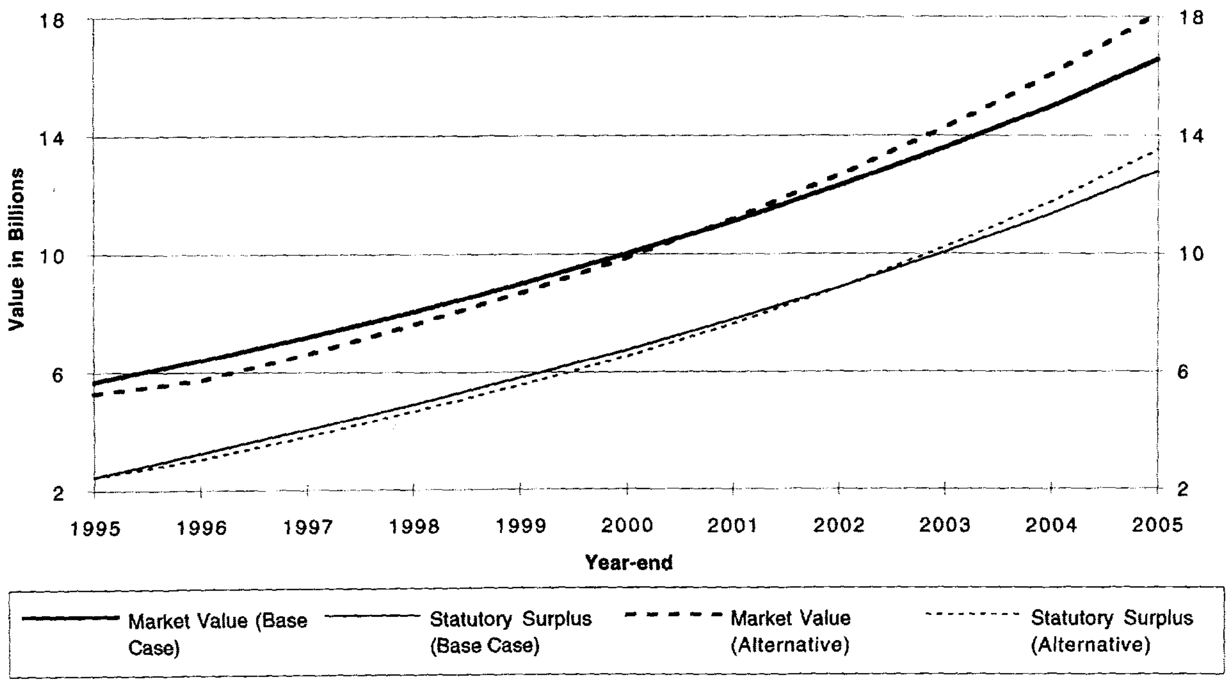
Smaple Financial Model

Appendix E
Exhibit 1

Statutory and Market Value Surplus
Runoff With No New Policy Years



Statistory and Market Value Surplus
Results Reflect Ten Additional Years of Underwriting



Capital Allocation Rules for Base Case Scenario

Capital Allocation Rules	WC	GL	CMP	Other Lines *	Total
Dsic. Reserves-to-surplus ratio	5.3	3.1	2.8	(Mixed)	(Mixed)
Premium-to-surplus ratio	4.1	1.7	2.1	(Mixed)	(Mixed)

1/1/95 Capital Allocation

Information for 1/1/95 Capital Allocation	WC	GL	CMP	Other Lines *	Total
12/94 Discounted Reserves	6,898,630	1,523,676	417,945	4,119,895	12,960,146
1995 Planned premium	2,414,960	342,930	170,109	2,935,919	5,863,918

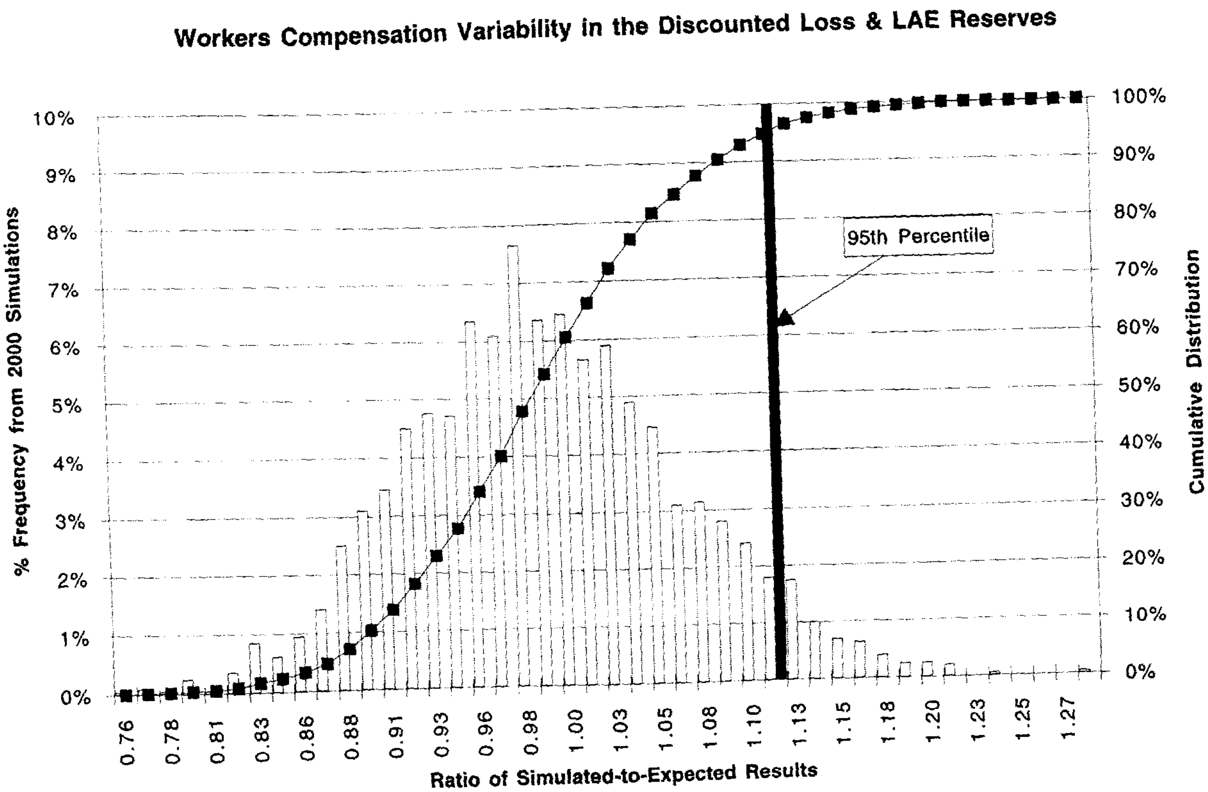
1/1/95 Allocated Capital	WC	GL	CMP	Other Lines *	Total
Runoff Business	1,298,213	493,371	150,462	922,368	2,864,415
New Policy Year	588,121	202,821	82,854	775,170	1,648,966
Total Capital	1,886,335	696,192	233,316	1,697,538	4,513,381

1/1/96 Capital Allocation

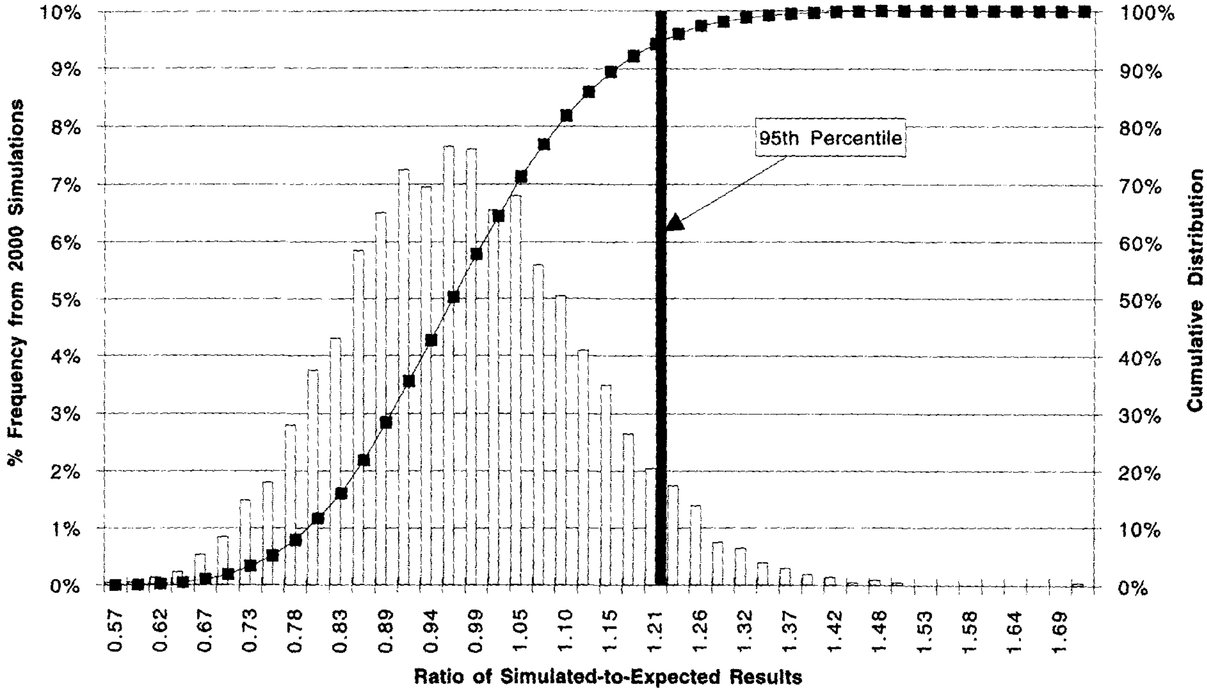
Information for 1/1/96 Capital Allocation	WC	GL	CMP	Other Lines *	Total
12/95 Discounted Reserves	6,866,738	1,460,508	390,241	4,005,847	12,723,333
1996 Planned premium	2,561,790	363,780	180,451	3,114,423	6,220,444

1/1/96 Allocated Capital	WC	GL	CMP	Other Lines *	Total
Runoff Business	1,292,212	472,917	140,488	869,686	2,775,303
New Policy Year	623,879	215,153	87,892	822,300	1,749,224
Total Capital	1,916,091	688,070	228,380	1,691,986	4,524,527

* Other Lines Capital Ratios varied by specific lines.


Workers Compensation Variability in the Discounted Loss & LAE Reserves
10%
9%
8%
7%
6%
5%
4%
3%
2%
1%
0%
% Frequency from 2000 Simulations
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
Cumulative Distribution
95th Percentile
0.76
0.78
0.81
0.83
0.86
0.88
0.91
0.93
0.96
0.98
1.00
1.03
1.05
1.08
1.10
1.13
1.15
1.18
1.20
1.23
1.25
1.27
Ratio of Simulated-to-Expected Results

Workers Compensation Variability in the New Year's Results



Impact of Medical Inflation
Illustration based on a Fifteen-Year Triangle where the Dollar amounts
and the payment patterns are the Fixed Basis for Further Simulations

Appendix G
Exhibit 1

Year	Payments in Nominal Dollars ($Millions)														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1976	600	263	114	50	22	10	4	2	1	0	0	0	0	0	0
1977	658	285	125	55	24	11	5	2	1	0	0	0	0	0	0
1978	713	311	138	61	27	12	5	2	1	0	0	0	0	0	0
1979	778	346	153	68	30	13	5	2	1	0	0	0	0	0	0
1980	864	383	171	74	32	13	6	2	1	0	0	0	0	0	0
1981	956	427	186	79	34	14	6	3	1	0	0	0	0	0	
1982	1,067	465	197	84	36	15	7	3	1	1	0	0	0		
1983	1,161	493	210	90	38	16	7	3	1	1	0	0			
1984	1,233	524	226	96	41	18	8	3	1	1	0				
1985	1,311	564	240	102	44	19	8	4	2	1					
1986	1,409	601	256	110	48	21	9	4	2						
1987	1,502	640	276	120	52	22	10	4							
1988	1,600	689	301	131	56	24	10								
1989	1,723	751	327	140	59	25									
1990	1,878	817	351	149	62										
1991	2,042	877	372	156											
1992	2,193	929	389												
1993	2,323	974													
1994	2,434														

Impact of Medical Inflation
Illustration based on a Fifteen-Year Triangle where the Dollar amounts
and the payment patterns are the Fixed Basis for Further Simulations

Appendix G
Exhibit 2

Year	Loss Development Factors Based on Nominal Payments														
		1 - 2	2 - 3	3 - 4	4 - 5	5 - 6	6 - 7	7 - 8	8 - 9	9 - 10	10 - 11	11 - 12	12 - 13	13 - 14	14 - 15
1976		1.438	1.132	1.051	1.022	1.009	1.004	1.002	1.001	1.000	1.000	1.000	1.000	1.000	1.000
1977		1.434	1.132	1.052	1.022	1.010	1.004	1.002	1.001	·1.000	1.000	1.000	1.000	1.000	1.000
1978		1.437	1.135	1.053	1.022	1.010	1.004	1.002	1.001	1.000	1.000	1.000	1.000	1.000	1.000
1979		1.444	1.136	1.053	1.022	1.009	1.004	1.002	1.001	1.000	1.000	1.000	1.000	1.000	1.000
1980		1.443	1.137	1.052	1.021	1.009	1.004	1.002	1.001	1.000	1.000	1.000	1.000	1.000	1.000
1981		1.446	1.134	1.050	1.020	1.009	1.004	1.002	1.001	1.000	1.000	1.000	1.000	1.000	
1982		1.435	1.129	1.049	1.020	1.008	1.004	1.002	1.001	1.000	1.000	1.000	1.000		
1983		1.425	1.127	1.048	1.020	1.008	1.004	1.002	1.001	1.000	1.000	1.000			
1984		1.425	1.128	1.048	1.020	1.008	1.004	1.002	1.001	1.000	1.000				
1985		1.430	1.128	1.048	1.020	1.009	1.004	1.002	1.001	1.000					
1986		1.426	1.127	1.049	1.020	1.009	1.004	1.002	1.001						
1987		1.426	1.129	1.050	1.021	1.009	1.004	1.002							
1988		1.431	1.131	1.050	1.021	1.009	1.004								
1989		1.436	1.132	1.050	1.020	1.008									
1990		1.435	1.130	1.049	1.020										
1991		1.430	1.127	1.047											
1992		1.424	1.125												
1993		1.419													
1994															
	Five Year Averages														
Link Ratio		1.429	1.129	1.049	1.020	1.009	1.004	1.002	1.001	1.000	1.000	1.000	1.000	1.000	1.000
Factor to Ultimate		1.753	1.227	1.087	1.035	1.015	1.006	1.003	1.001	1.001	1.000	1.000	1.000	1.000	1.000
Cum. Pay Pattern	57.1%	81.5%	92.0%	96.6%	98.5%	99.4%	99.7%	99.9%	99.9%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
Pay Pattern	57.1%	24.5%	10.5%	4.5%	2.0%	0.8%	0.4%	0.2%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

Calculation of Indicated Reserves (Nominal Dollars)															
Year	1994	1993	1992	1991	1990	1989	1988	1987	1986	1985	1984	1983	1982	1981	1980
Paid-to-date	2,434	3,297	3,512	3,447	3,257	3,026	2,811	2,627	2,460	2,295	2,151	2,022	1,876	1,707	1,547
Ultimate	4,266	4,044	3,815	3,569	3,306	3,045	2,818	2,630	2,461	2,296	2,152	2,022	1,876	1,707	1,547
Reserves	1,832	748	304	122	49	19	8	3	1	0	0	0	0	0	0
Total Reserves	3,086														

Appendix G
Exhibit 3

Impact of Medical Inflation
Illustration based on a Fifteen-Year Triangle where the Dollar amounts
and the payment patterns are the Fixed Basis for Further Simulations

	Inflation Factors (1994 $Dollar)														
Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1976	4.057	3.701	3.415	3.127	2.817	2.545	2.280	2.096	1.973	1.856	1.727	1.620	1.521	1.412	1.296
1977	3.701	3.415	3.127	2.817	2.545	2.280	2.096	1.973	1.856	1.727	1.620	1.521	1.412	1.296	1.192
1978	3.415	3.127	2.817	2.545	2.280	2.096	1.973	1.856	1.727	1.620	1.521	1.412	1.296	1.192	1.110
1979	3.127	2.817	2.545	2.280	2.096	1.973	1.856	1.727	1.620	1.521	1.412	1.296	1.192	1.110	1.048
1980	2.817	2.545	2.280	2.096	1.973	1.856	1.727	1.620	1.521	1.412	1.296	1.192	1.110	1.048	1.000
1981	2.545	2.280	2.096	1.973	1.856	1.727	1.620	1.521	1.412	1.296	1.192	1.110	1.048	1.000	
1982	2.280	2.096	1.973	1.856	1.727	1.620	1.521	1.412	1.296	1.192	1.110	1.048	1.000		
1983	2.096	1.973	1.856	1.727	1.620	1.521	1.412	1.296	1.192	1.110	1.048	1.000			
1984	1.973	1.856	1.727	1.620	1.521	1.412	1.296	1.192	1.110	1.048	1.000				
1985	1.856	1.727	1.620	1.521	1.412	1.296	1.192	1.110	1.048	1.000					
1986	1.727	1.620	1.521	1.412	1.296	1.192	1.110	1.048	1.000						
1987	1.620	1.521	1.412	1.296	1.192	1.110	1.048	1.000							
1988	1.521	1.412	1.296	1.192	1.110	1.048	1.000								
1989	1.412	1.296	1.192	1.110	1.048	1.000									
1990	1.296	1.192	1.110	1.048	1.000										
1991	1.192	1.110	1.048	1.000											
1992	1.110	1.048	1.000												
1993	1.048	1.000													
1994	1.000														

Appendix G
Exhibit 4

Impact of Medical Inflation
Illustration based on a Fifteen-Year Triangle where the Dollar amounts and the payment patterns are the Fixed Basis for Further Simulations

Year	Payments in 1994 Dollars ($millions)														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1976	2,434	974	389	156	62	25	10	4	2	1	0	0	0	0	0
1977	2,434	974	389	156	62	25	10	4	2	1	0	0	0	0	0
1978	2,434	974	389	156	62	25	10	4	2	1	0	0	0	0	0
1979	2,434	974	389	156	62	25	10	4	2	1	0	0	0	0	0
1980	2,434	974	389	156	62	25	10	4	2	1	0	0	0	0	0
1981	2,434	974	389	156	62	25	10	4	2	1	0	0	0	0	
1982	2,434	974	389	156	62	25	10	4	2	1	0	0	0		
1983	2,434	974	389	156	62	25	10	4	2	1	0	0			
1984	2,434	974	389	156	62	25	10	4	2	1	0				
1985	2,434	974	389	156	62	25	10	4	2	1					
1986	2,434	974	389	156	62	25	10	4	2						
1987	2,434	974	389	156	62	25	10	4							
1988	2,434	974	389	156	62	25	10								
1989	2,434	974	389	156	62	25									
1990	2,434	974	389	156	62										
1991	2,434	974	389	156											
1992	2,434	974	389												
1993	2,434	974													
1994	2,434														

Impact of Medical Inflation
Illustration based on a Fifteen-Year Triangle where the Dollar amounts and the payment patterns are the Fixed Basis for Further Simulations

Year		Loss Development Factors Based on Payments Converted to 1994 $Dollars													
		1 - 2	2 - 3	3 - 4	4 - 5	5 - 6	6 - 7	7 - 8	8 - 9	9 - 10	10 - 11	11 - 12	12 - 13	13 - 14	14 - 15
1976		1.400	1.114	1.041	1.016	1.006	1.002	1.001	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1977		1.400	1.114	1.041	1.016	1.006	1.002	1.001	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1978		1.400	1.114	1.041	1.016	1.006	1.002	1.001	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1979		1.400	1.114	1.041	1.016	1.006	1.002	1.001	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1980		1.400	1.114	1.041	1.016	1.006	1.002	1.001	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1981		1.400	1.114	1.041	1.016	1.006	1.002	1.001	1.000	1.000	1.000	1.000	1.000	1.000	
1982		1.400	1.114	1.041	1.016	1.006	1.002	1.001	1.000	1.000	1.000	1.000	1.000		
1983		1.400	1.114	1.041	1.016	1.006	1.002	1.001	1.000	1.000	1.000	1.000			
1984		1.400	1.114	1.041	1.016	1.006	1.002	1.001	1.000	1.000	1.000				
1985		1.400	1.114	1.041	1.016	1.006	1.002	1.001	1.000	1.000					
1986		1.400	1.114	1.041	1.016	1.006	1.002	1.001	1.000						
1987		1.400	1.114	1.041	1.016	1.006	1.002	1.001							
1988		1.400	1.114	1.041	1.016	1.006	1.002								
1989		1.400	1.114	1.041	1.016	1.006									
1990		1.400	1.114	1.041	1.016										
1991		1.400	1.114	1.041											
1992		1.400	1.114												
1993		1.400													
1994															
		Five Year Averages													
Link Ratio		1.400	1.114	1.041	1.016	1.006	1.002	1.001	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Factor to Ultimate		1.667	1.190	1.068	1.026	1.010	1.004	1.002	1.001	1.000	1.000	1.000	1.000	1.000	1.000
Cum. Pay Pattern	60.0%	84.0%	93.6%	97.4%	99.0%	99.6%	99.8%	99.9%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
Pay Pattern	60.0%	24.0%	9.6%	3.8%	1.5%	0.6%	0.2%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

Calculation of Indicated Reserves (in 1994 Dollars)															
Year	1994	1993	1992	1991	1990	1989	1988	1987	1986	1985	1984	1983	1982	1981	1980
Paid-to-date	2,434	3,408	3,797	3,953	4,015	4,040	4,050	4,054	4,056	4,056	4,057	4,057	4,057	4,057	4,057
Ultimate	4,057	4,057	4,057	4,057	4,057	4,057	4,057	4,057	4,057	4,057	4,057	4,057	4,057	4,057	4,057
Reserves	1,623	649	260	104	42	17	7	3	1	0	0	0	0	0	0
Total Reserves	2,704														

Impact of Medical Inflation
Illustration based on a Fifteen-Year Triangle where the Dollar amounts
and the payment patterns are the Fixed Basis for Further Simulations

		Expected Payment of Reserves in													
Year	**Total**	**1995**	**1996**	**1997**	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**
1994 Dollars	2,704	1,623	649	260	104	42	17	7	3	1	0	0	0	0	0
		Reinflated Payments													
4% Inflation	2,890	1,688	702	292	121	51	21	9	4	2	1	0	0	0	0
8% Inflation	3,085	1,752	757	327	141	61	26	11	5	2	1	0	0	0	0
10% Inflation	3,187	1,785	785	346	152	67	29	13	6	3	1	0	0	0	0

Appendix H
Exhibit 1
Page 1

Development Factors for Indemnity and ALAE

Period	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108	108 - 120	120 - 132	132 - 144
1970 Dec					1.055	1.040	1.028	1.021	1.018	1.016	1.012
1971 Dec				1.094	1.055	1.041	1.026	1.024	1.016	1.011	1.010
1972 Dec			1.168	1.093	1.065	1.043	1.032	1.025	1.016	1.018	1.018
1973 Dec		1.386	1.169	1.096	1.062	1.049	1.033	1.025	1.020	1.017	1.012
1974 Dec	2.334	1.385	1.164	1.093	1.068	1.044	1.034	1.022	1.019	1.016	1.013
1975 Dec	2.310	1.398	1.190	1.116	1.076	1.051	1.037	1.026	1.021	1.016	1.013
1976 Dec	2.262	1.388	1.195	1.117	1.069	1.048	1.031	1.027	1.020	1.017	1.013
1977 Dec	2.192	1.397	1.191	1.111	1.070	1.048	1.031	1.023	1.019	1.016	1.015
1978 Dec	2.246	1.407	1.193	1.113	1.068	1.048	1.031	1.027	1.022	1.019	1.016
1979 Dec	2.199	1.409	1.192	1.109	1.068	1.045	1.036	1.027	1.023	1.020	1.019
1980 Dec	2.169	1.400	1.209	1.107	1.074	1.050	1.038	1.030	1.023	1.020	1.017
1981 Dec	2.191	1.400	1.185	1.115	1.075	1.055	1.041	1.032	1.025	1.019	1.017
1982 Dec	2.179	1.395	1.207	1.131	1.098	1.059	1.046	1.043	1.026	1.024	1.020
1983 Dec	2.283	1.437	1.227	1.140	1.088	1.064	1.048	1.037	1.025	1.022	1.017
1984 Dec	2.345	1.473	1.228	1.134	1.089	1.064	1.044	1.033	1.027	1.018	
1985 Dec	2.422	1.473	1.245	1.140	1.087	1.057	1.041	1.030	1.020		
1986 Dec	2.377	1.500	1.237	1.133	1.085	1.055	1.036	1.026			
1987 Dec	2.452	1.496	1.234	1.127	1.080	1.053	1.034				
1988 Dec	2.496	1.498	1.228	1.126	1.074	1.047					
1989 Dec	2.502	1.512	1.231	1.121	1.068						
1990 Dec	2.666	1.520	1.232	1.109							
1991 Dec	2.529	1.507	1.217								
1992 Dec	2.454	1.470									
1993 Dec	2.426										
1994 Dec											
	1	2	3	4	5	6	7	8	9	10	11
*Lognormal Parameters:**											
mu	0.30	-0.82	-1.58	-2.16	-2.62	-3.00	-3.33	-3.59	-3.86	-4.03	-4.21
sigma	0.101934	0.113505	0.124266	0.132860	0.154326	0.138741	0.167132	0.182388	0.157487	0.174078	0.203765
Simulated ATA**	2.304	1.354	1.167	1.136	1.063	1.059	1.039	1.019	1.021	1.014	1.016

* Lognormal parameters are based on fitting a lognormal distribution to the column of age-to-age factors. To get a better fit, the distribution is fit to (ATA - 1), rather than to the age-to-age factors themselves.

** The simulated age-to-age factors are used in a Monte-Carlo simulation, and are derived by inverting the Cumulative Density of the lognormal, assuming no correlation among the development at different maturities.

Appendix H
Exhibit 1
Page 2

144 - 156	156 - 168	168 - 180	180 - 192	192 - 204	204 - 216	216 - 228	228 - 240	240 - 252	252 - 264	264 - 276	276 - 288	288 - 300
1.013	1.009	1.008	1.007	1.021	1.001	1.004	1.006	1.005	1.004	1.004	1.004	1.005
1.011	1.007	1.010	1.012	1.009	1.005	1.006	1.005	1.005	1.004	1.006	1.005	
1.010	1.012	1.011	1.008	1.008	1.008	1.007	1.007	1.006	1.006	1.008		
1.013	1.012	1.011	1.008	1.008	1.007	1.007	1.007	1.005	1.007			
1.010	1.013	1.009	1.008	1.009	1.007	1.006	1.008	1.007				
1.014	1.012	1.011	1.010	1.009	1.010	1.010	1.010					
1.013	1.012	1.008	1.011	1.009	1.009	1.009						
1.013	1.011	1.010	1.009	1.009	1.009							
1.014	1.012	1.012	1.011	1.009								
1.013	1.011	1.012	1.012									
1.017	1.011	1.011										
1.016	1.011											
1.015												

12	13	14	15	16	17	18	19	20	21	22	23	24
-4.34	-4.52	-4.61	-4.65	-4.64	-5.21	-5.00	-4.95	-5.19	-5.23	-5.12		
0.163310	0.176673	0.160767	0.170090	0.291440	0.985813	0.279841	0.228982	0.190354	0.226015	0.280225		
1.014	1.013	1.010	1.011	1.007	1.018	1.006	1.005	1.006	1.005	1.007		

An Integrated Dynamic Financial Analysis and Decision Support System for a Property Catastrophe Reinsurer

by Stephen P. Lowe, FCAS
James N. Stanard, FCAS

An Integrated Dynamic Financial Analysis and Decision Support System for a Property Catastrophe Reinsurer

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Abstract

This paper describes the dynamic financial analysis model currently being used by a property catastrophe reinsurer to manage its business. The model is an integral part of the day-to-day operations at the Company; and is used as a decision making tool in the underwriting, investment and capital management processes. The paper begins by describing the framework that the Company uses for risk management. This includes a classification of the risks facing the Company, used to define and prioritize their implementation in the model. Also included is a description of the conceptual approach the Company takes to evaluate the tradeoff between risk and return. The paper then goes on to describe the structure and operation of the dynamic financial analysis model; and provides examples of its use at the Company, along with illustrative examples of the various types of output that is produced by it.

Biographies

Stephen P. Lowe, FCAS, MAAA, is a consulting actuary with Tillinghast – Towers Perrin. He is a Principal of the firm, and currently serves as Chief Actuary for their world-wide property/casualty practice. Prior to joining the firm in 1980, he was employed by Aetna Life & Casualty. He is a past Vice President of the American Academy of Actuaries, and has served on a number of task forces working on key issues facing the insurance industry, including the task force that advised the NAIC on risk-based capital. Steve has a BS in Mathematics from Union College.

James N. Stanard, FCAS, MAAA, is Chairman of the Board, President and Chief Executive Officer of RenaissanceRe Holdings Ltd. He has over twenty-five years of property and casualty experience, most of it in reinsurance. Prior to joining Renaissance at its formation in 1993, Jim was Executive Vice President and member of the Office of the President at USF&G; previously he was Executive Vice President of F&G Re. Jim has a BA in Mathematics from Lehigh University and a PhD in Finance from New York University. He is currently serving as Chairman of the ASTIN Committee.

An Integrated Dynamic Financial Analysis and Decision Support System for a Property Catastrophe Reinsurer

I. Introduction

The Company that is the subject of this paper is a major property catastrophe reinsurer, writing excess of loss coverage on a world-wide basis. It was formed in Bermuda in 1993 to provide additional capacity to the market, capitalizing on the market dislocation following Hurricane Andrew. Since that time the Company has grown to be one of the largest specialist writers in the catastrophe reinsurance market.

Since its formation in 1993, a core strategic premise of the Company has been that an increased level of precision in the measurement and management of risk can be translated into a competitive advantage.

- Improved measurement of underlying exposure and modeling of losses allows underwriters to build a superior insurance portfolio, one that is less risky and/or more profitable than that of peers.

- Improved measurement of financial risk allows management to make more efficient use of capital, leading to superior returns on that capital.

The Company has developed systems and processes to support and implement this premise. Taken as a whole, they are used to facilitate ongoing dynamic financial analysis (DFA) of the enterprise. Perhaps most importantly, dynamic financial analysis activities are not restricted to technical staff operating apart from management. DFA has been integrated directly into the ongoing underwriting and financial management processes of the Company. Every senior manager is trained on the use of the system, so that it is a practical and immediate resource for decision making.

The development of these capabilities has been a collaborative effort between the Company and an actuarial consulting firm (hence this co-authored paper). In addition to the authors, who co-led the development effort, many other people in both organizations contributed to the conceptualization, design, programming, and testing of the system.[1]

Development of the system and its modeling capabilities is an ongoing activity; its design continues to evolve as experience with its use develops. Initially, the model was

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[1] The authors would like to acknowledge specifically the significant contributions made by Jayant Kadilkar, Richard Rafferty, William Riker, and Cary Sparrow towards the development of the Company's DFA system.

relatively simple, and focused only on measuring the principal risks facing the Company. As confidence in the model has grown, new features and additional risk components have been added. While the paper generally describes the model as it exists today, a few features are described that are under active development at the time of this writing, with the full expectation that they will be on line by the time of publication. A major goal of current development activity is to better integrate the various components of the system, strengthening the linkages between the risk elements in the process.

Finally, while the output exhibits presented in the paper are illustrative of those actually produced by the model, they are stylized versions of that output containing figures that have been altered. They are included only to illustrate the varied uses of the model, and represent only a small sample of what has been produced. Many of the output exhibits, as well as the details of the system's implementation are considered proprietary by the Company (key parts of the system are copyrighted). In preparing this paper it has been necessary to balance those interests against the goal of providing readers of the paper with useful insight into the structure, capabilities, and uses of the system.

The paper has three major sections. Section I begins by describing the risk framework that was developed to guide the development of the model. The various types of risks facing an insurer are outlined and defined, and the approach taken to evaluate the tradeoff between risk and return is described. In Section II, the structure of the dynamic financial analysis model is presented. This includes a system schematic, and a description of the various inputs, variables, and calculation steps. Finally, in Section III the uses of the model are described and the output is illustrated.

II. Conceptual Framework

A necessary first step in the development of a dynamic financial model is establishing a conceptual framework to serve as a guide. The structure of the risks to be modeled must be defined in general, and then prioritized based on the business profile of the company. Appropriate measures of risk must also be defined, and threshold values for the risk measures must be chosen.

Classification of Risk

The risks faced by an insurance enterprise have been classified in a variety of ways in the published literature on the subject. There are three basic elements of risk, each of which must be considered in a dynamic financial analysis model:

1. Liability Risk: the risk that the cost of settling the insurance liabilities will be greater than expected (also referred to as obligation risk).
   - Claims on coverage already provided cost more to settle than anticipated.
   - Cost of claims generated on future coverage is greater than anticipated .
2. Asset Risk: the risk that the realizable value of assets will be less than anticipated.
   - The market value of invested assets declines.
   - Invested assets become non-performing.
   - Receivables from outward reinsurers become uncollectable.
   - Receivables from customers become uncollectable.
3. Business Risk: the general business risks faced by all enterprises.
   - Competitors will force market prices below costs to preserve their position/share.
   - Competitors will gain a competitive advantage, taking customers away.
   - Regulators or legislators will interfere in the market in a harmful way.
   - The company will be victimized by a crime.
   - Operations will be adversely affected by a disaster at company premises.

The sub-bullets above are intended to be illustrative of the types of risks included in each element, and are not necessarily exhaustive.

As will be seen, the Company's dynamic financial analysis model is structured around this risk framework, explicitly incorporating each of these three major elements.

Liability Risk

Liability risk (or obligation risk) is viewed as the predominant risk element by most property/casualty insurers. As is indicated, it includes existing claim obligations (whether known or not) on coverage provided in the past as well as new claim obligations arising from future coverage provided on policies currently in force or written in the future. From the perspective of the actuary, liability risk includes what may loosely be referred to as reserving and pricing risk. It is the actuary's responsibility to estimate the cost of claims in each of the two contexts; liability risk stems from the uncertainty of those estimates.

In the definition of liability risk, cost is expressed in terms of present value. Liability risk includes the timing of the claim cash flows, as well as their nominal amounts. It also includes the expenses of settling the claims, as well as the claim payments themselves.

Uncertainty of liabilities includes both process risk stemming from the random nature of claim events and parameter risk stemming from the inability to know the claim frequency and severity distributions from which the events are drawn. These distributions cannot be known in advance, because they are dependent on future social and economic conditions that cannot be predicted with certainty.

For most lines of insurance, a company can write sufficient volumes of business to diversify away process risk. In such a circumstance parameter risk will be the dominant component of liability risk, with process risk considered de minimis. However, in property catastrophe reinsurance process risk is not diversifiable by volume; even on a world-wide market basis the covered events are too few to achieve a stable annual result. (We will have to wait for the market to expand to include a few other worlds beyond earth to achieve diversification by volume.) For this line, both process and parameter risk must be accommodated in a dynamic financial analysis model.

Finally, a complicating factor for an international insurer is the issue of currency. Insurance contracts are typically issued with claims to be settled in a specific currency, typically the local currency of the contract. However, from the perspective of the owner claim costs are ultimately measured by their impact on equity as measured in the owner's currency. Thus the cost of liabilities includes the cost of converting them from the local contract currency to the owner's currency, and liability risk includes movements in exchange rates that affect conversion costs.

Asset Risk

By definition, assets are capable of generating an expected positive cash flow. Asset risk deals with the uncertainty associated with the realization of that cash flow. This uncertainty stems from two fundamental sources. One is the risk of non-performance of the obligor, such as the default of a bond or the insolvency of a reinsurer. The other is a change in conditions that affects the value or performance of the asset. Examples of the latter would include a recession causing a decline in the stock market, or a rise in mortgage interest rates that lowers the rate of refinancing on a CMO.

The inclusion of reinsurance recoverables with asset risk aligns the risk classification structure with contemporary GAAP thinking, and not with traditional US statutory accounting where the financial presentation suggests that obligation risk be measured on a net basis.

As is the case with liabilities, much of the risk associated with individual assets is diversifiable. Thus the movement of individual stock prices, or the default of individual bonds is not usually relevant to asset risk, unless the individual holding is material. Instead the primary focus is on the non-diversifiable components of risk associated with each asset class.

Asset risk also has a currency dimension. To the extent that assets are held in currencies different than that of the owner's, changes in exchange rates contribute to asset risk.

Business Risk

General business risk has been given relatively little attention in the actuarial literature. This is unfortunate, because it is a significant source of risk in insurance. Business risk contributes significantly to underwriting risk, in ways that can not be described by simple random processes. Severe underwriting losses at the bottom of the U.S. property/casualty underwriting cycle are neither random nor unforeseen events. They aren't caused by claim costs being higher than expected (i.e., by liability risk), but rather by prices being set below the level of expected costs. During a down-cycle many companies are aware that their prices are too low and that underwriting results will be bad.

A variety of forces act on price levels in the insurance marketplace, most notably the level of overall capacity in relation to demand. Prices will fall when capacity exceeds demand, and will rebound only when capacity is withdrawn. The operation of these forces depends on the structure of the market and external conditions at the time. External economic conditions play a reinforcing role, particularly such items as the level of interest rates.

Competitive position is also important to the business risk of individual companies operating within the market. One example would be the cost of distribution. Companies with a high cost distribution system should not expect to achieve adequate returns, unless that distribution system offers value to them or their customers sufficient to warrant its excess cost. In a competitive market, the companies with the lower distribution costs will simply set the market price at a level that produces sub-par returns for their high cost competitors.

Competitive advantage is not just about distribution costs. It includes the effectiveness of the company's marketing, underwriting, claim, and capital management functions. While the overall industry results over the last few years have been generally lackluster, many individual companies have produced attractive returns during this period by superior execution in one or more of the above areas. And, the disappearance of several of the national multilines over the same period can be attributed to their inability to successfully execute in these areas. Competitive risks are both significant and real in this industry.

Business risks relating to market competition are not at all unique to insurance. One only has to look as far as the airline industry to witness the same risks playing themselves out in a non-insurance context. There also an excess of capacity in relation to demand has forced a blood-letting as competitors vied to retain market share. While managements have known that fares were inadequate, the market forces have likewise been beyond their control.

From a dynamic financial analysis perspective, the authors believe that business risk should be modeled separately from liability risk. While the two risks are not entirely unrelated, the drivers of each are different, such that modeling them as a single risk (i.e., modeling underwriting risk via loss ratios) is an inherently weak approach.

Measuring Risk and Return

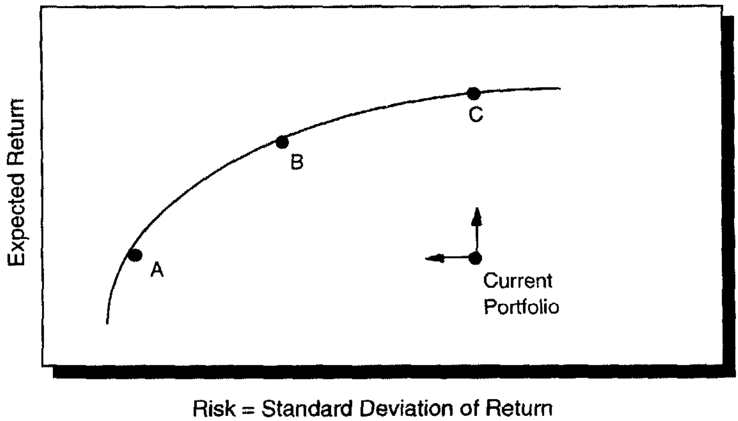
Application of dynamic financial analysis requires that financial constraints be defined. For example, while the results of an analysis might indicate that there is an $x\%$ probability of impairment, defined as the loss of $y\%$ or more of capital, those results alone do not tell management what actions to take. To translate results into action it is necessary for management (or the board) to decide whether or not that level of impairment probability is too high. In a similar vein, while impairment probability might be an appropriate constraint, it is probably not the only constraint relevant to the enterprise. In fact, a variety of constraints are relevant, depending on the question the analysis is designed to answer.

Dynamic financial analysis also requires the definition of financial performance objectives. If a reinsurance program were offered to the company that reduced its probability of impairment from $x\%$ to $x'\%$, management can only judge the benefit of that reduction in relation to the cost of the reinsurance. This issue becomes particularly relevant when there are several alternative reinsurance programs, each with different ruin reductions and different costs. The issue is further complicated when the cost of a particular program is variable, or when its effects are spread into several future accounting periods in a multi-year deal.

In developing its dynamic financial analysis model, the Company has adopted the Asset/Liability Efficient Frontier (ALEF℠) as a basic framework for resolving these issues in a logically consistent manner.[2]

The efficient frontier concept is taken from modern portfolio theory, and is attributed to Markowitz. In its most basic formulation, the investor is presented with several alternative classes of assets in which he can invest. For each class of asset, the investor knows the expected return, the risk associated with that return (as measured by its standard deviation), and the correlation of returns with all other classes of asset. His problem is to choose a portfolio by specifying the mix of assets by class. Markowitz's contribution was to recognize that not all asset mixes are optimal: either a higher return can be achieved for the same level of risk, or the same return can be achieved for a lower risk. There is, however, a frontier to the set of possible asset mixes consisting of those portfolios that are efficient in the sense that one cannot improve upon them.

Classical Efficient Frontier in Modern Portfolio Theory: Mixes A, B, and C Are Efficient - Current Mix Is Not



The investment portfolios on the efficient frontier are all good choices; choosing among them is a matter of the investor's risk/return preferences.

[2] ALEF is described more fully in Buff, Joseph, "Extending the Efficient Frontier", *Emphasis*, 1990/4. See also Doll, Douglas and Sonlin, Stephen, "Optimizing ALEF Studies", *Emphasis*, 1994/2.

ALEF is a generalization of the efficient frontier, along both of the two axes in the chart above. In the ALEF approach the x-axis is labeled generically as 'level of risk' and the y-axis is labeled generically as 'expected performance'. The user must define each of these terms. Similarly, the strategies to be analyzed are generalized from asset mix to any set of decision variables relevant to the enterprise. Once the problem is specified in these terms, the dynamic financial analysis model can be used to find the efficient frontier from the available choices.

The performance objective can be any financial measure that management feels is most important, or any combination of such measures. Generally, the measure should be consistent with the maximization of shareholder value, but can be reflective of any specific component such as profitability or revenue growth. In the case of multiple measures, management must specify the relative weight assigned to each so that they can be combined into a single index. (The function combining the measures need not be linear.) The measures can be based on absolute dollars, returns in relation to capital employed, or relative performance when compared to peers. Finally the measure can reflect any chosen time horizon.

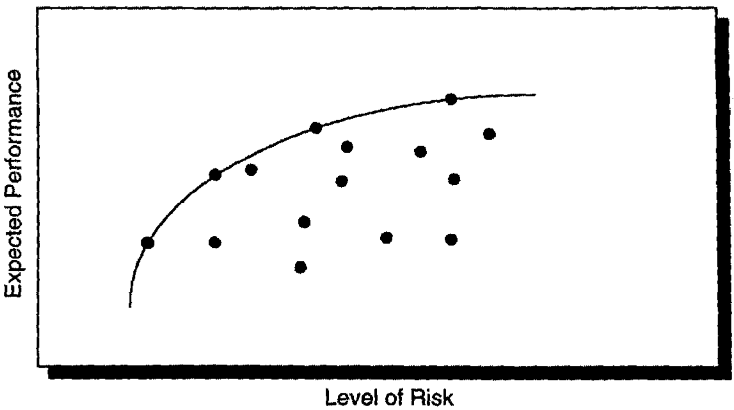
The only overriding requirement of the performance measure used is that it must be consistent: management must always want to choose the strategy that maximizes the measure's expected value, all other things being equal.

As a measure of risk, standard deviation has been the subject of ample criticism. Much of this criticism stems from the fact that standard deviation focuses only on the dispersion of the outcomes, without any special recognition of the greater disutility of the adverse outcomes. While most people equate risk with uncertainty of outcomes, they also equate risk with the likelihood and severity of adverse outcomes. In the ALEF framework, risk can be any measure of adverse outcomes that management feels is most relevant. Examples would include:

- probability of ruin over the next ten years;
- probability of combined ratio above 110% next year;
- expected policyholder deficit on current business;
- probability of suffering a net decline in surplus of 20% or more at the end of three years;
- probability of failing an RBC test at any point in the next five years;
- probability of a ratings downgrade by AM Best;
- probability of a combined ratio two points or more worse than the industry.

As was the case with the measure of performance, several of these measures of risk can be combined to produce an overall index of risk, with weights reflective of their relative importance.

Using the Asset/Liability Efficient Frontier, Strategies Can Be Evaluated in a Generalized Risk/Reward Framework



ALEF is a powerful and flexible tool for managing an insurance company. It can be customized to mirror the business philosophy of the company, both as to the financial objectives to be maximized and the risks to be controlled.

The Company uses the ALEF framework in conjunction with its dynamic financial analysis model to evaluate a variety of strategic issues. The Company has developed a vector of multiple risk constraints that collectively capture its appetite for risk. This vector is used consistently in each analysis. While the types of strategic issues analyzed are discussed in subsequent sections of the paper, the Company considers its risk constraint vector to be confidential.

III. Description of the Model

Overview

A conceptual schematic of the Company's dynamic financial analysis model is presented on the following page. The model consists of the following basic components:

- a liability scenario generator, which produces distributions of aggregate underwriting results for the insurance portfolio;
- an asset scenario generator, which, when combined with the liability generator, produces a distribution of operating results for the combined insurance/investment portfolio;
- a multi-period financial model, which extends the distributions over a longer time horizon.

Each component produces dynamic output that is used to manage different aspects of the business.

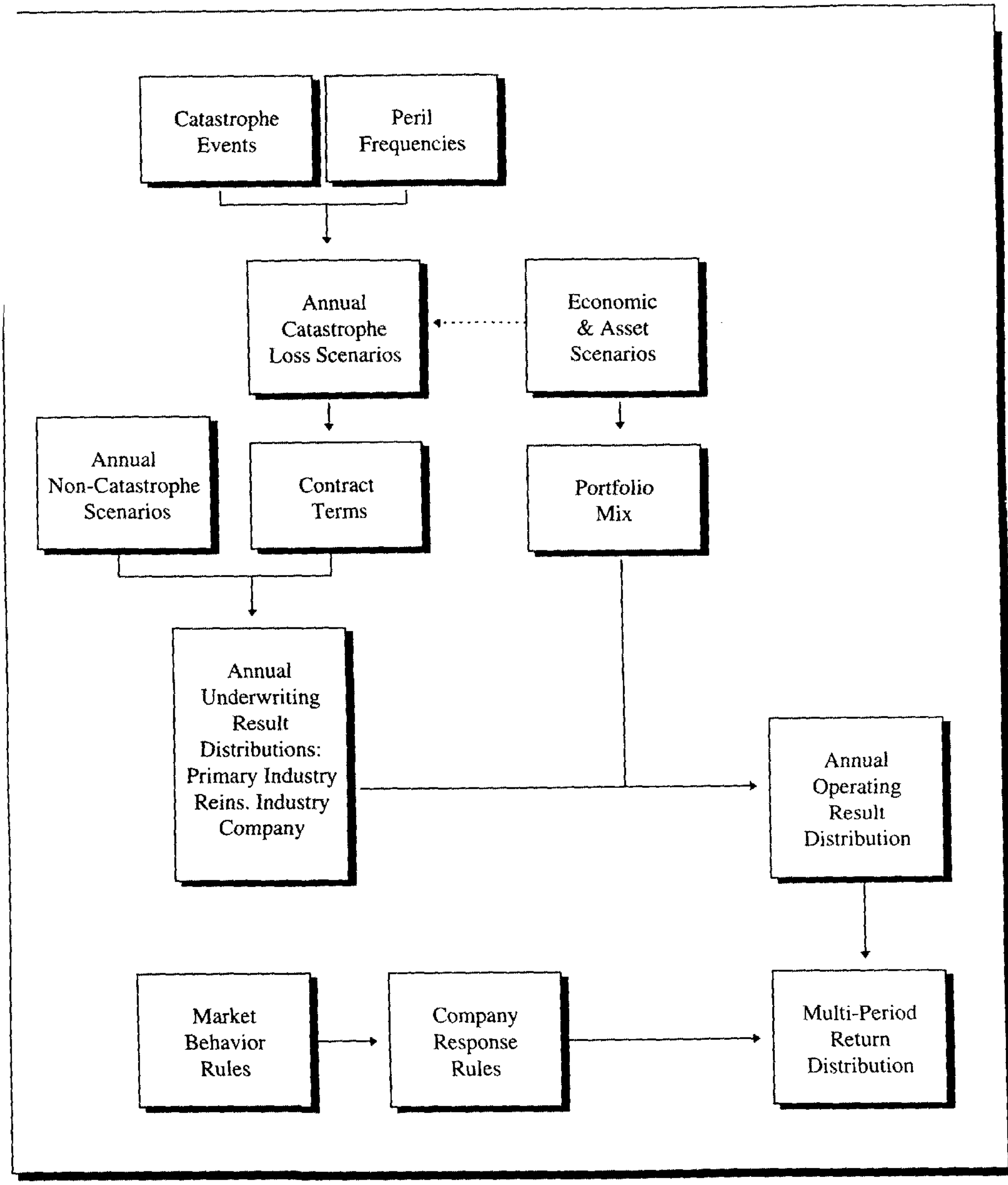
As can be seen from the schematic, the model is not a single system. Rather it is a linked set of programs and databases that can be used in a variety of combinations to facilitate the needs of any given analysis. A key attribute of this structure is flexibility; while the core calculation engines are written in higher order programming languages to achieve efficiency, many of the inputs and outputs of each component are held in spreadsheets to facilitate their manipulation 'on the fly' by the user. This spreadsheet environment also promotes the use of graphical output for analysis of results.

Liability Scenario Generator

Because the Company's core business is property catastrophe reinsurance, a heavy emphasis is placed on detailed modeling of the volatile claim experience inherent to that line. The models are used extensively in the underwriting of individual contracts; however in the context of this paper the focus of presentation is on their use as an input to the enterprise-level DFA model. The advantage of this tightly integrated approach is that the effect of any one underwriting decision on the key DFA objective functions can be easily determined by the underwriter, and therefore taken into account at the point of decision in the underwriting process.

For each peril in each region of the world a set of catastrophic events has been developed. The events vary as to their location, size, and intensity; as well as the ensuing insured damage they would generate. Relative probabilities are also assigned to each event in the set, based on the likelihood of that particular combination of event parameters occurring at once. The probabilities sum to one; in conjunction with a set of insured losses

Conceptual Schematic of the Dynamic Financial Analysis Model



associated with each event they represent a sample severity distribution for the particular peril. Similarly, for each peril in each region a frequency distribution is specified reflecting the likelihood of a given number of events happening within a year. For example, a frequency distribution is specified for the number of landfall hurricanes hitting the U.S. over the course of a season.

Within the system, the frequency and severity distributions for each peril are convoluted to produce annual aggregate catastrophe losses. In the current configuration, 40,000 scenarios of annual losses are created, which is deemed sufficient for analysis purposes. (The sampling process is stratified, not Monte Carlo, such that the tails of the resulting aggregate distribution are considerably more robust.)

At this juncture in the system, the losses in each scenario are those of the primary ceding company. The primary losses are then run through the applicable reinsurance contract terms to obtain the corresponding losses to the reinsurance contract. A database containing the actual terms of all catastrophe reinsurance contracts in the portfolio is maintained, such that the aggregate underwriting results for the entire portfolio, on a world-wide basis, for each scenario can be obtained and analyzed.

The system is on-line, such that portfolio results can be obtained at any time; a complete portfolio run takes about two hours to process through the system on a Silicon Graphics workstation. Analysis of the marginal impact of adding a contract to the portfolio takes less than five minutes. In addition to ongoing ad hoc portfolio analysis, portfolio results are produced and analyzed in detail on a quarterly basis in a formal underwriting meeting, after the latest cycle of contracts have been written.

The Company writes small amounts of other types of reinsurance from time to time, which are incorporated into the system using a less formal modeling approach. A spreadsheet containing the estimated underwriting distributions applicable to this business is maintained, and is incorporated into the overall results as a 'last step' in the overall process. This assures that the complete underwriting portfolio is modeled within the system.

The principal output of this component of the system is a distribution of underwriting results for the Company. The distribution reflects all elements in the underwriting result that vary directly with losses:

- reinstatement premiums;
- losses;
- brokerage;
- federal excise taxes/premium taxes.

These elements are calculated on a contract-by-contract basis, reflecting the actual applicable terms and conditions. Other elements such as operating expenses may be added in as a last step in the process.

In addition to Company underwriting experience, supplemental industry-wide information is produced showing the corresponding losses for the primary industry, and the estimated portion of those losses that would be ceded to the property catastrophe reinsurance industry.

Since the Company's functional currency is the U.S. dollar, contracts written in all other currencies are converted to their U.S. equivalent. Within the system, exchange rates can be varied to test the impact of adverse movements on underwriting results.

Each of the underlying catastrophe events has an associated day of the year, such that each underwriting scenario generated by the model has a pattern of losses throughout the year. At the present time, the models do not consider the variability in the timing from occurrence to payment. Such risk is considered fairly immaterial.

In addition there is no consideration of 'reserving risk', in the sense that actual payments might be greater than estimated in the financial statements.

Parameter risk is not explicitly included within the modeling process itself. Instead, the parameters are sensitivity tested in a variety of ways and the results are used to introduce conservatism into the final parameter assumptions. These sensitivity tests take two forms.

- First, output can be generated using event files created by different vendors. In addition to developing its own event files for various perils and regions, the Company has developed relationships with many of the primary catastrophe modeling consultants including Applied Insurance Research, RMS, Dames & Moore, EQECAT, and Tillinghast. Event files have been constructed and incorporated into the system using the catastrophe models developed by each of these firms. Comparing the results generated by these different event files, reflecting the different approaches and assumptions of each firm, provides a measure of the impact of varying the underlying event parameters, and helps to assure that the results obtained are not dependent on the specific catastrophe model used.

- Second, sensitivity testing is performed by altering the underlying frequency and severity distributions. Results are routinely tested using higher peril frequencies. This is particularly relevant in light of the research being done by global climatologists (such as that published by Dr. Gray and popularized in the media), and the record level of hurricane activity experienced in 1995. The generated peril severity distributions have also been adjusted to consider various factors such as the demand-driven inflation that occurred after hurricane Andrew.

Finally, results can be produced for the entire portfolio of reinsurance contracts or any defined subset. This facilitates analysis of sources of risk, and also can be used to analyze the value of potential retrocessions. Hypothetical portfolios can be run to test alternative underwriting strategies, as well.

Asset Scenario Generator

The Company uses the Global CAP:Link system to obtain scenarios for various economic and investment variables for several different currencies. On request, a CAP:Link output file is provided to the Company containing 1,000 scenarios, with each scenario reflecting a future path of interest rates, inflation rates, currency exchange rates, and rates of return by asset class for each of five major currencies. Each scenario is a plausible path of the annual movement of the variables; taken together the scenarios describe the range of variation in each of the variables.

The CAP:Link system uses a stochastic diffusion model to generate economic and capital market scenarios on a global basis. Scenarios are generated based on a cascading set of stochastic differential equations, structured so that the proper relationship between the modeled variables is maintained over time. These include serial correlation effects, reinvestment risks, and path volatility characteristics. The top of the cascade is a yield curve scenario generator, based on a variant of the two-factor "Brennan-Schwartz" yield model. These yield results are then passed down to generators for other variables such as inflation and stock returns, which are conditionally related in the cascade. The developers of the CAP:Link system believe that it is superior to other popular approaches such as lognormal models, time series models based on ARIMA or Box-Jenkens, or models based on Vector AutoRegression.[3]

The asset scenarios from CAP:Link are convoluted with the liability scenarios, such that each individual annual scenario consists of:

- economic conditions - annual inflation rates by currency, and exchange rate movements for the year;
- capital market conditions - interest rates and annual rates of return by asset class and currency
- catastrophic conditions - a set of catastrophic events, and primary and reinsurance industry losses ensuing from those events.

The Company underwriting result distribution is combined with investment results reflecting the cash flows and investment returns for each scenario, such that an annual operating result distribution for the Company can be obtained. Note that both the liabilities and the assets are dynamically adjusted for changes in exchange rates. The operating result distribution can be produced either for the current mix of investments,

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[3] A more detailed description of the CAP:Link stochastic diffusion model, and comparison of its performance to other models, is presented in Worldwide Asset and Liability Modeling (eds., W.T. Ziembarand and J.M. Mulvey), Cambridge University Press, 1996

or for any hypothetical alternative mix (as well as different insurance portfolios). This facilitates the testing of alternative investment portfolio strategies, including the mix of investments by currency.

At the time of writing, the catastrophe losses at the detailed scenario level are not dynamically linked directly to the economic scenarios (hence the dotted line in the schematic diagram). This is an enhancement that is currently under development; once completed the losses will vary based on the inflation rates in each scenario.

Multi-Period Model

Up to this point, the description of the model has focused on the short-term, annual time horizon. The liability and asset legs of the model focus on annualized results in the context of the current business environment. The multi-period model extends the analysis to a longer-term horizon (currently five years) and introduces business risk into the analysis.

The first step in this process is to encapsulate the behavior of the market in a set of rules. The critical question is how market price levels will move over the five year time horizon, and what factors will affect that movement. In this area the Company has an advantage over the large multiline insurers, for whom this would be a vast and daunting question. Such insurers would need to specify the market behavior and drivers for each product-line they offer in each market, as well as the interrelationships across the different product-lines and markets. In the Company's case only one product-line and market, property catastrophe reinsurance, must be addressed.

The fundamental behavior of prices in the property catastrophe reinsurance market can be stated succinctly.

- If results are good, prices will decline from their current level.
- Prices will continue to decline until results are bad, at which point they rise.
- The rate of decline is related to how good the results are; the rate of increase is related to how bad results are.
- Rises in prices include nominal increases in rates on line, and also implicit increases through higher retentions and other coverage reductions.

The market has exhibited this general behavior over an extended period, such that it is reasonable to assume the behavior will continue. The difficult part of the problem is translating the qualitative behavior rules into quantitative terms. While the historical responsiveness of prices to results can serve as a guide, changes in the market that influence its behavior must also be considered. For example, one could argue that the new capital provided to the reinsurers in Bermuda may be less forgiving, and will be withdrawn more rapidly if and when results are bad. Similarly, the growing use of catastrophe models by the reinsurers in underwriting may inject a greater degree of discipline, reducing the rate of price decline in the face of favorable results.

The approach taken by the Company is to relate catastrophe reinsurance price levels in each subsequent year to the industry-wide catastrophe experience in several preceding years. A market price index has been constructed, the movement of which is dependent on emerging industry experience. The market price index is based on information from several sources: the actual price movements observed by the Company since its formation, historical price movements over a longer time period based on information from several sources, discussions with brokers and other experts in the market, and judgment.

The responsiveness of price levels to experience over several year's time involves significant parameter risk. The Company has performed significant sensitivity testing of this element of the model to gain insight into how alternative assumptions influence results.

The starting point in the multi-period simulation is the current distribution of annual underwriting results. Using a Monte Carlo approach a first year scenario with the associated underwriting result for the Company is chosen from that distribution. Based on the corresponding industry-wide result, the movement in the price level index for year two is determined. The annual underwriting result distribution is then modified to reflect the effect of the change in price level to obtain a distribution for the second year. A second year result is then chosen from the modified underwriting result distribution. This stochastic process continues until five years of results have been generated.

In addition to the market behavior rules, company response rules reflecting the actions of Company management must also be defined. These actions fall in three areas.

- Market share actions must be defined, reflecting the Company's willingness to write business at the prevailing price level. Based on the perceived adequacy of prices, the Company will either seek to grow, hold steady, decrease, or severely reduce its market share. This decision feeds back into Company results as follows: the price level on the Company's portfolio relative to the market price level improves/degrades as the Company's market share declines/grows, due to more/less selective underwriting.

- Capitalization actions must be specified, reflecting the changing needs of the Company over time. For example at some threshold level a portion of excess capital is returned to shareholders. Similarly, if actual capital falls below specified requirements, market share is forced down to the level allowed by the requirements. Both normal and extraordinary dividend policies must be defined.

- Debt/Capital levels over the five year period must be specified, and debt actions in relation to operating losses must be defined.

The multi-period model starts with an opening balance sheet, simulates the underwriting result for the first year, translates that result into a first year operating result, determines the market behavior for the next year, and implements the company responses. This process continues iteratively until the full five years have been generated. Typically, 20,000 trials are run to produce a distribution of five year returns to shareholders, based on the stream of dividends and the final equity at the end of the fifth year. In addition to return measures, appropriate risk measures are also generated. The model can be run using different company response strategies; the risk and return associated with each strategy can be compared by placing it in an ALEF context.

IV. Model Uses and Sample Output

One of the key advantages of a highly integrated system such as the one described is that many different types of decisions can be tested against a consistent risk/return 'yardstick', based on a common set of underlying probability distribution assumptions. These include:

- ongoing evaluation of the adequacy of capital to support the current risks undertaken;
- evaluation of the value of retrocessional coverage offerings;
- analysis of alternative capital structures;
- development of asset mix investment policy;
- analysis of currency risk;
- studies of alternative market and underwriting strategies;
- individual underwriting decisions reflecting the marginal effect of a given contract on risk and return constraints.

Exhibit 1 is an example of output from the liability scenario generator. It shows graphically the right-hand tail of an underwriting result distribution for a portfolio. As has been indicated previously, this information (along with accompanying risk and return statistics) can be generated for any vendor/peril scenario, and any segment of the portfolio of reinsurance contracts.

While Exhibit 1 is a relatively simple graphic, coupled with the risk/return measures it is a very powerful management tool. For example, distributions can be generated with and without a retrocessional cover that is being considered. Comparison of the two allows management to evaluate the marginal impact of the cover on underwriting risk and return, and ultimately to assess the value of the cover. Alternatively, reinsurance accounts that have a particularly detrimental impact on the distribution can be isolated for potential re-underwriting at renewal. Management also tracks changes in the distribution over time, as a measure of underwriting performance.

A variety of diagnostic exhibits are also routinely produced that allow management to gain insight into the sources of adverse underwriting scenarios: perils, regions, reinsurance layers, etc. Comparative information on primary and reinsurance industry losses is also included. Exhibits 2 and 3 are illustrative of these types of exhibits. Exhibit 2 displays industry and portfolio experience on a standard, defined event set. The defined events reflect a range of different likelihoods for various perils and regions. (The 'break' in the exhibit indicates that it is longer than actually shown; only the beginning and end of the exhibit is shown in the illustration.) Exhibit 3 displays percentiles of severity distributions for the portfolio by (illustrative) geographic zone, and the Company's share of the industry loss at that percentile.

In addition to underwriting profit distributions, operating profit distributions reflecting investment as well as underwriting risk are produced by the model, such as those shown on Exhibit 4. These can be used to translate underwriting risk into operating profit terms, or to test the effect of introducing various levels of asset risk via changes to the mix of investments.

Many of the risk measures suggested in Section II can be translated into boundary constraints, reflecting their maximum level of acceptability. For example, one possible risk measure is the probability of suffering a surplus decline of 20% or more. If that were a chosen risk measure, management would presumably seek to minimize that probability for a given level of return, and would only be willing to accept an increase in that probability in exchange for a higher return. Management might also impose a *boundary constraint* that in no event will management allow that probability to exceed 3%.

One can invert the boundary constraint relationship to obtain an implied surplus requirement. For example, if the current annual operating profit distribution for a hypothetical company indicates that there is a 3% chance of suffering an operating loss of $70-million or greater, then the minimum required surplus for the company is $350-million. At that level of surplus, they will be just inside the boundary constraint.

The company has established several such boundary constraints, and uses them to measure surplus employed on an ongoing basis, based on the operating profit distributions generated by the model each quarter. In addition to being directly useful in the capital management of the company, this approach also facilitates the measurement of expected returns on surplus employed. Exhibit 5 illustrates this type of information. In addition to Company results, the model generates the results for an 'index fund' of a cross-section of the entire excess property catastrophe market (for certain regions) labeled as 'XYZ', such that comparative performance can also be measured.

A variety of exhibits can be generated from the multi-period model, as it can be used to test so many different strategy variables: operating leverage, debt/capital ratios, dividend strategies, and responses to changing market conditions. Exhibits 6, 7 and 8 are illustrative of the types of output generated by this analytic tool. Exhibit 6 shows the expected results for the Company generated by the model for four sample strategies. In actual practice, basic exhibits like these have been generated for hundreds of alternative strategies and assumptions sets.

Exhibit 7 is an illustration of an asset/liability efficient frontier for eighteen different strategies, listed on the lower half of the exhibit. In this example, the Company is considering raising or lowering its operating leverage by 15% from current levels, varying its debt/capital ratio from 20% to 40%, and altering its response to changes in market price levels from 'modest' to either 'flat' or 'aggressive'. While the exhibit is a highly stylized version of such an analysis, it is indicative of the approach actually taken.

Finally, Exhibit 8 is a supporting exhibit to Exhibit 7, showing the trade-off between risk and return associated with the operating leverage and debt/capital variables. For a range of values of each variable, risk and return measures from the multi-period planning model have been used to construct a contour map. The contour map shows how risk and return rise and fall in each region of variable values. (The actual contour lines are more involved, with multiple inflection points, than actually shown.) To find an efficient frontier point, one follows a particular return line, looking for the region where the line also achieves minimum risk. Exhibits such as these are used as diagnostics in the efficient frontier analysis.

In addition to varying the decision variables, the model is run with varying assumptions to test how the resulting frontiers and contour maps are affected.

Conclusion

To make the dynamic financial analysis system described in this paper useful to decision making, a significant continuing investment is required in:

- keeping the underlying databases current and error free on a routine basis;
- including all types of business and perils to which the company is exposed;
- training *all* professional staff in the details of the model;
- designing the system so that the DFA results are produced quickly, with easily understandable output reports;
- selecting employees and establishing a culture where decision making in this framework is considered natural and practical.

The substantial investment in building and maintaining the system has clearly been justified -- but only because of its usefulness in many of the practical decisions facing the company.

A final challenge is for employees using this admittedly complex system to develop good judgment as to how much weight to give its results in their actual decision making. This requires a thorough understanding of the theory and the practical details, and an appreciation of the limitations and assumptions underlying the results. A good sense of how to weigh system results with unmodeled factors is the essence of the amorphous term 'underwriting judgment'.

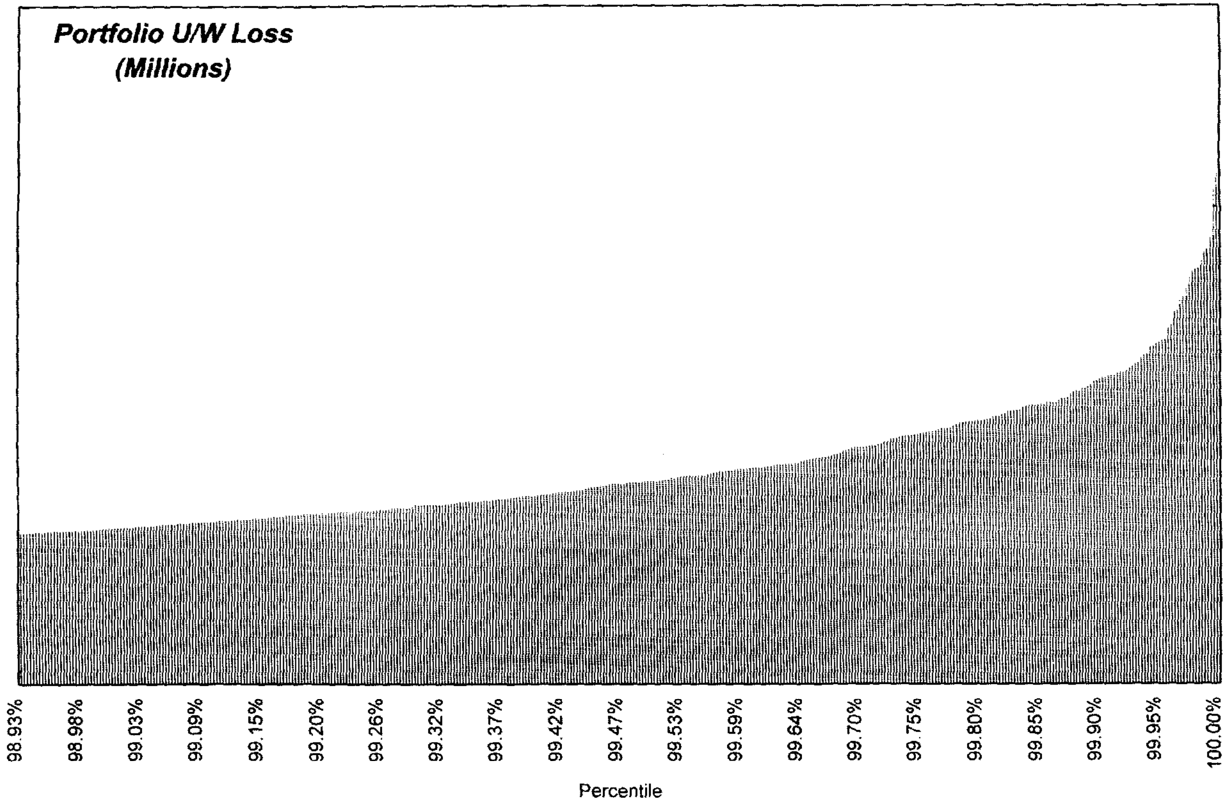

Portfolio U/W Loss
(Millions)
98.93%
98.98%
99.03%
99.09%
99.15%
99.20%
99.26%
99.32%
99.37%
99.42%
99.47%
99.53%
99.59%
99.64%
99.70%
99.75%
99.80%
99.85%
99.90%
99.95%
100.00%
Percentile

Exhibit 1

Exhibit 2

Worldwide Portfolio as of xx.xx.xxxx
Based on Peril/Assumption Set 23

Description	Return Interval	Peril	Industry Loss *(billion)*	Portfolio Gross Loss *(million)*
Northeast/NY	1/xxx	H	xxx	xxx
Northeast/MA	1/xxx	H	xxx	xxx
/NY	1/xxx	H	xxx	xxx
	1/xxx	H	xxx	xxx
	1/xxx	H	xxx	xxx
	xx	H	xxx	xxx
		E	xxx	xxx
			xxx	xxx
			xx	xxx
				xxx
Gulf/				
Gulf/LA				
California				
South CA				
North CA	1/xx			
South CA	1/xxx			
North CA	1/xxx	E		
New Madrid/TN	1/xxx	E	xx	
New Madrid/MO	1/xxx	E	xxx	
Hawaii	1/xxx	H	xxx	xxx
Northwest/WA	1/xxx	E	xxx	xxx
N Europe/UK	1/xxx	W	xxx	xxx
N Europe/UK	1/xxx	W	xxx	xxx
N Europe/GER	1/xxx	W	xxx	xxx
N Europe/GER	1/xxx	W	xxx	xxx
Japan Wind	1/xxx	H	xxx	xxx
Japan Wind	1/xxx	H	xxx	xxx
Japan Quake	1/xxx	E	xxx	xxx
Japan Quake	1/xxx	E	xxx	xxx
Australia	1/xxx	H	xxx	xxx
New Zealand	1/xxx	E	xxx	xxx
Caribbean	1/xxx	H	xxx	xxx
Northridge/CA	1/xxx	E	xxx	xxx
Loma Prieta/CA	1/xxx	E	xxx	xxx
Great NE Hurricane/NY	1/xxx	H	xxx	xxx
Andrew/FL	1/xxx	H	xxx	xxx
Hugo/SC	1/xxx	H	xxx	xxx
90A - Daria/Europe	1/xxx	W	xxx	xxx
90G - Vivian/Europe	1/xxx	W	xxx	xxx
90D - Herta/Europe	1/xxx	W	xxx	xxx
87J - StormV/Europe	1/xxx	W	xxx	xxx
76B - Capella/Europe	1/xxx	W	xxx	xxx
Mirielle/Japan	1/xxx	H	xxx	xxx
New Castle/Australia	1/xxx	E	xxx	xxx

Exhibit 3

US Primary and Retro Business
Inforce Date - xx.xx.xxxx

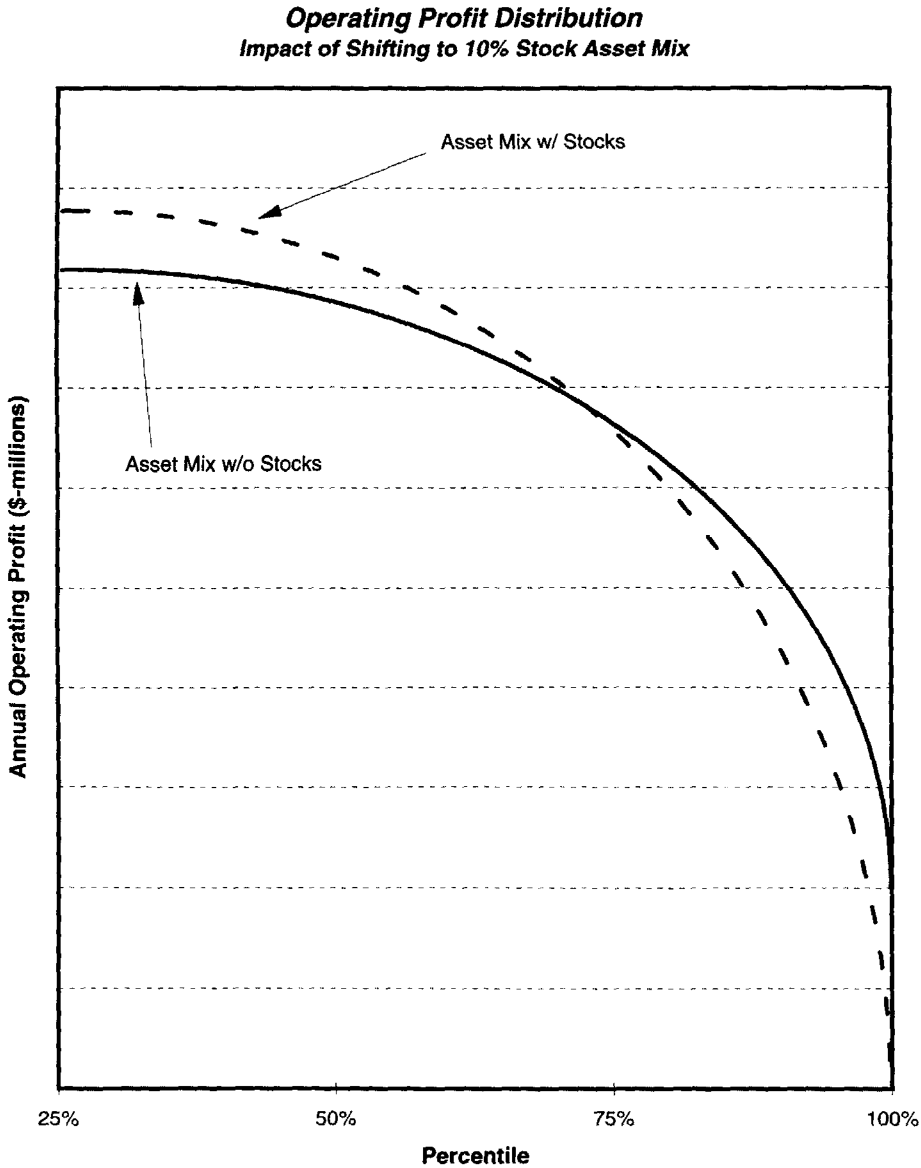
Occurrence Distribution

Zone of Event	Occurrence Probabilities					
	90.0%	*95.0%*	*99.0%*	*99.6%*	*99.8%*	*99.9%*
Eastern New England	xxx	xxx	xxx	xxx	xxx	xxx
Eastern LI & CT	xxx	xxx	xxx	xxx	xxx	xxx
...ern NJ	xxx	xxx	xxx	xxx	xxx	xxx
	xxx	xxx	xxx	xxx	xxx	xxx
	xxx	xxx	xxx	xxx	xxx	xxx
	xxx	xxx	xxx	xxx	xxx	xxx
		xxx	xxx	xxx	xxx	xxx
			xxx	xxx	xxx	xxx
				xxx	xxx	xxx
					xxx	xxx
						xxx
Northern Pla...						
Northwest						
Northern California						
Southern California						
Hawaii						
National Occurrence	xxx	xxx				
Wind Storms	xxx	xxx	xxx	xxx	xxx	
Earthquake	xxx	xxx	xxx	xxx	xxx	xxx
National Aggregate	xxx	xxx	xxx	xxx	xxx	xxx

Share of Market Loss

Zone of Event	Primary Industry Market Share					
	90.0%	*95.0%*	*99.0%*	*99.6%*	*99.8%*	*99.9%*
Eastern New England	x.x%	x.x%	x.x%	x.x%	x.x%	x.x%
...stern LI & CT	x.x%	x.x%	x.x%	x.x%	x.x%	x.x%
... NJ	x.x%	x.x%	x.x%	x.x%	x.x%	x.x%
	x.x%	x.x%	x.x%	x.x%	x.x%	x.x%
	x.x%	x.x%	x.x%	x.x%	x.x%	x.x%
	x.x%	x.x%	x.x%	x.x%	x.x%	x.x%
		x.x%	x.x%	x.x%	x.x%	x.x%
			x.x%	x.x%	x.x%	x.x%
				x.x%	x.x%	x.x%
					x.x%	x.x%
						x.x%
Northern Plains ...						
Northwest						
Northern California						
Southern California						
Hawaii						
National Occurrence	x.x%	x.x%				
Hurricane	x.x%	x.x%	x.x%	x.x%		
Wind Storms	x.x%	x.x%	x.x%	x.x%	x.x%	
Earthquake	x.x%	x.x%	x.x%	x.x%	x.x%	x.x%
National Aggregate	x.x%	x.x%	x.x%	x.x%	x.x%	x.x%

Exhibit 4



Operating Profit Distribution
Impact of Shifting to 10% Stock Asset Mix
Asset Mix w/ Stocks
Asset Mix w/o Stocks
Annual Operating Profit ($-millions)
25%
50%
75%
100%
Percentile

Exhibit 5

Expected Annual Return on Surplus Employed (Worldwide Portfolio As of Date Shown)

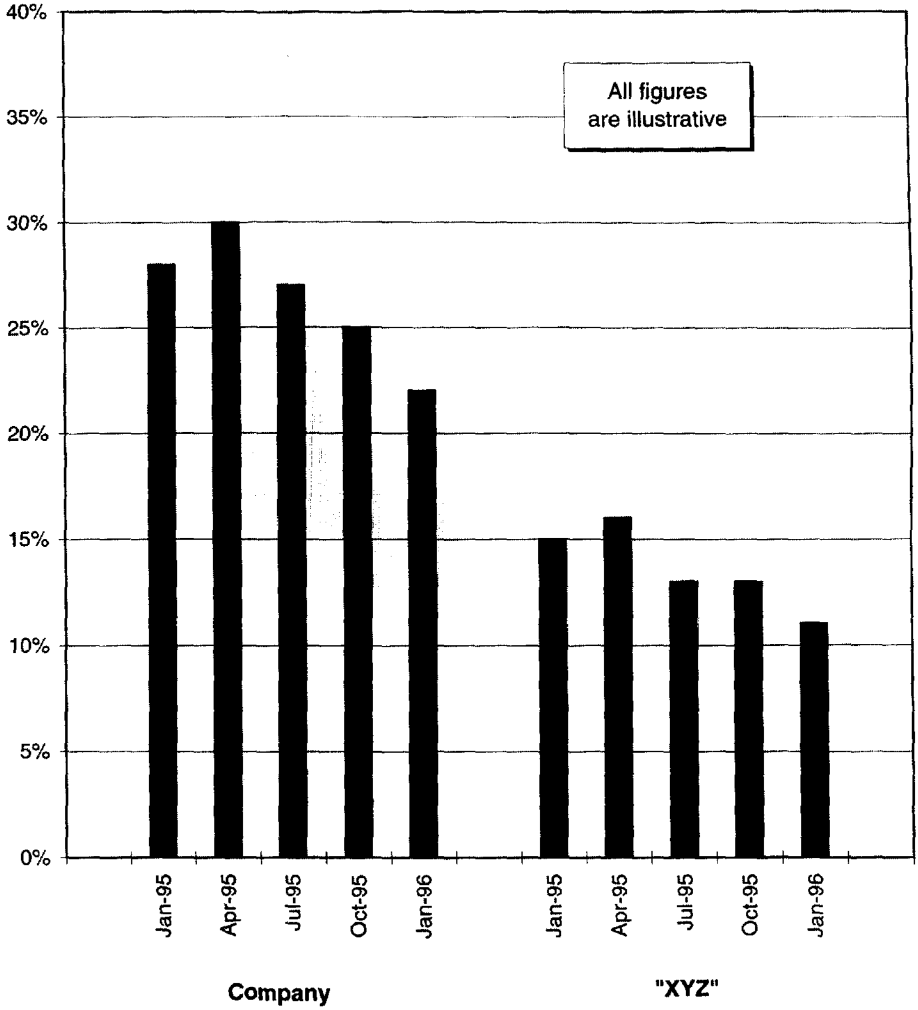


Exhibit 6

Multiperiod Financial Planning Model
Expected Operating Performance by Strategy
Baseline Market Behavior Assumption

Operating Leverage: 50% | Debt/Capital Ratio: 0% | Company Response To Market: Modest

	Year 1	Year 2	Year 3	Year 4	Year 5
Written Premium	191	207	209	210	170
Net Operating Profit	120	129	126	124	98
Dividends	16	60	73	176	
Surplus	381	486	555	608	556

Operating Leverage: 50% | Debt/Capital Ratio: 30% | Company Response To Market: Modest

	Year 1	Year 2	Year 3	Year 4	Year 5
Written Premium	190	207	211	214	180
Net Operating Profit	110	114	110	107	84
Dividends	35	56	64	129	
Surplus	381	489	572	638	608

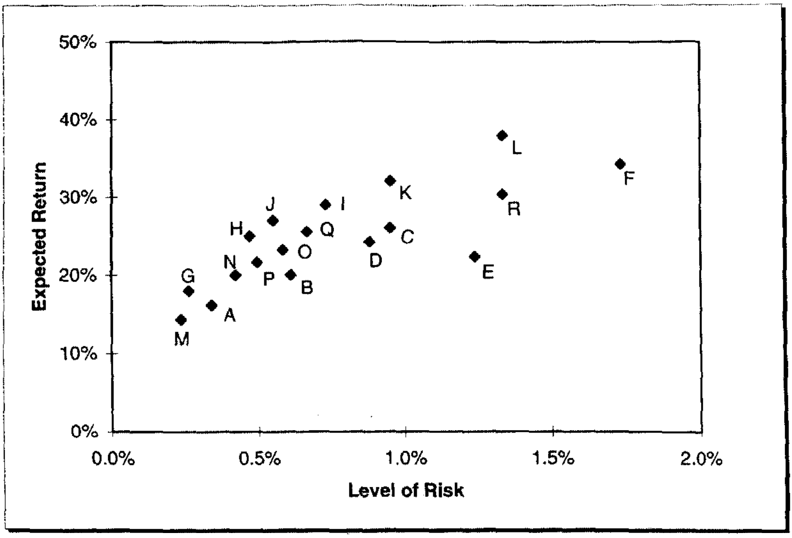
Operating Leverage: 65% | Debt/Capital Ratio: 30% | Company Response To Market: Modest

	Year 1	Year 2	Year 3	Year 4	Year 5
Written Premium	267	290	295	302	270
Net Operating Profit	153	158	152	148	123
Dividends	70	94	87	131	
Surplus	381	501	594	687	714

Operating Leverage: 80% | Debt/Capital Ratio: 30% | Company Response To Market: Modest

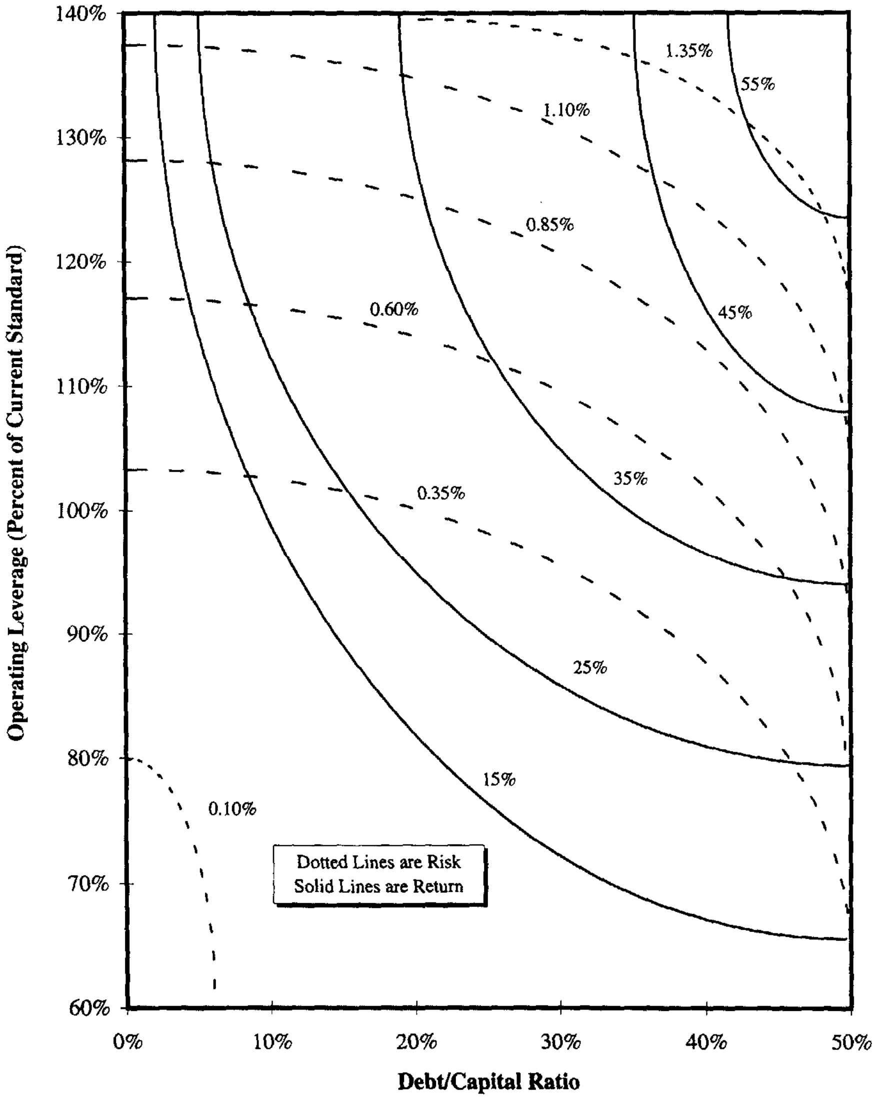
	Year 1	Year 2	Year 3	Year 4	Year 5
Written Premium	305	332	332	333	270
Net Operating Profit	177	181	171	158	138
Dividends	89	113	111	131	
Surplus	381	509	607	708	743

Multiperiod Financial Planning Model
Asset Liability Efficient Frontier
Baseline Market Behavior



Strategy	Operating Leverage	Debt/ Capital	Dividend Policy	Response to Market
A	85%	20%	Standard	Level
B	100%	20%	Standard	Level
C	115%	20%	Standard	Level
D	85%	40%	Standard	Level
E	100%	40%	Standard	Level
F	115%	40%	Standard	Level
G	85%	20%	Standard	Modest
H	100%	20%	Standard	Modest
I	115%	20%	Standard	Modest
J	85%	40%	Standard	Modest
K	100%	40%	Standard	Modest
L	115%	40%	Standard	Modest
M	85%	20%	Standard	Aggressive
N	100%	20%	Standard	Aggressive
O	115%	20%	Standard	Aggressive
P	85%	40%	Standard	Aggressive
Q	100%	40%	Standard	Aggressive
R	115%	40%	Standard	Aggressive

Contour Map Showing Impact of Varying Capital Structure on Risk and Return



MIDAS: A Dynamic Financial Model of a Property and Casualty Insurer

by Steven Thoede
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MIDAS

A Dynamic Financial Model of a Property and Casualty Insurer

ABSTRACT

This paper discusses the development of a dynamic financial planning model of a Property and Casualty insurer. The model provides management with a tool to instantaneously determine the impact of various actions on key financial ratios. It has helped management analyze pricing, reserving, catastrophe and investment risks.

The theoretical basis of the model is the accounting equation (Assets = Liabilities + Surplus). The model incorporates both known and unknown components of the equation. The known components, such as premiums and losses, are developed using various forecasting techniques. The model determines the unknown components of investment income and assets by solving a series of simultaneous equations using linear programming.

In order to integrate the model into the organization's financial planning process, it was necessary for management to agree to the theoretical basis; however, that was only the first step. Throughout the model's five year evolution, it became apparent that the method of delivery was as important as the theoretical basis. Management felt it was important to develop a financial model that was mobile, integrated numerous companies, provided dynamic change capability, analyzed instantaneously the impact on key financial ratios, all in a compact system.

Thus the **M**obile **I**ntegrated **D**ynamic **A**nalysis **S**ystem, **MIDAS**, was born.

ACKNOWLEDGEMENTS

The authors would like to acknowledge the efforts of all former financial forecast analysts who through their hard work and persistence developed the foundation for the MIDAS model. In particular, a special acknowledgement to Hank T. Gartland for the direction he provided which became the theoretical basis for this model and Barbara A. Berger for her helpful comments and suggestions after reading an earlier version of this paper.

1.0 INTRODUCTION

This paper describes the development of a financial planning model that determines investment income and assets given certain assumptions of revenues, expenses, portfolio mix and yield. It begins with an overall view of financial forecasting. Then it introduces the historical events which led to the development of the **M**obile **I**ntegrated **D**ynamic **A**nalysis **S**ystem, referred to as the MIDAS Model. After the historical background of events is discussed, examples of actual applications performed by the model to analyze financial problems are presented. The theoretical basis is reviewed next with a sample problem. Finally, the limitations of the model and suggestions for future refinements are presented.

2.0 FINANCIAL FORECASTING

In the past management had expressed frustration that the financial forecasting process was too cumbersome and time-consuming. By the time a thorough analysis of a problem was performed, the opportunity to make an informed decision had passed and management had reacted by the "seat of the pants".

The following quotation further reflects management's historical frustration with financial forecasting:

> Indeed it (financial forecasting) has been likened to driving a car blindfolded while following directions given by a person looking out the back window. Nevertheless, if this is the best we could do, it is important that it should be done properly, with the appreciation of the potential errors involved. In this way it should at least be possible to negotiate straight stretches of road without a major disaster.
>
> Andrew C. Harvey [1]

With all the uncertainty the business environment has to offer, it is a pity that management would resign themselves to such low expectations. This model has assisted management to negotiate not only through the straight stretches but over, under and around many potential pitfalls by enlightening them immediately of various options and combinations of options.

3.0 BACKGROUND

3.1 Focus on Underwriting

Historically, financial forecasting for a Property and Casualty insurance company consisted of consolidating the various underwriting components to develop a proforma underwriting profit/loss statement. Various departments in the company, called principals, applied the appropriate forecasting technique to project the component for which they were responsible. Some principals, such as Actuary and Loss Reserving, incorporated sophisticated techniques to develop premiums and losses. Others found it more appropriate to use experience and judgement. The underwriting components are listed below in Exhibit 1.

Exhibit 1
Underwriting Components

Premiums
Losses
Underwriting Expenses
Loss Adjustment Expenses
Dividends

The data provided by the principals was very detailed. Each component was broken down by risk and line of business. The proforma underwriting statement allowed management to monitor, by line of business and risk, the operating expense ratio (OER) and the combined ratio to determine the efficiency of the organization and pricing adequacy.

3.2 Focus on Underwriting and Investment Cycles

Due to the external pressures of regulation and competition, the company found it difficult to obtain an underwriting profit. This placed pressure on the yield from the investment portfolio to support surplus growth. Now management required a proforma income statement and balance sheet to monitor both the underwriting and investment cycles of the business and the impact of decisions on the key financial strength ratios of surplus and liquidity.

The additional components of the income statement and balance sheet were developed in a manner similar to the underwriting proforma. Exhibit 2 lists the additional components of the income statement and balance sheet.

Exhibit 2
Income Statement and Balance Sheet Components

Income Statement	Balance Sheet
Investment Income	Investment Assets
Investment Expenses	Cash
Other Income	Other Assets
Federal Income Taxes	Insurance Reserves
	Other Liabilities

With written premiums as a guide, the Investment Company developed a forecast of investment income and assets. This method of projecting investment income and assets before all the other components of the income statement and balance sheet were known worked quite well for some time. However, the company had diversified and made some major changes. A large portion of profits were invested into non-income producing fixed assets. These investments made it difficult to project investment income and assets based solely on written premiums.

Another problem with the method of projecting investment income and assets was the inability to quickly assess the impact of changing variables such as policyholder dividends on the investment portfolio. Under this method, the impact on the investment portfolio was reviewed by the Investment Company then incorporated with the other forecast components to determine the impact on the key financial ratios. Because of the time required for this process, management was unable to use this information to make timely decisions.

As a result, the company developed a new way of projecting investment income and assets based on cash flow. The basic premise of the cash flow forecast was:

> If the forecast could determine how much cash would be generated and used for operating expenses, investment in subsidiaries, and fixed assets, the excess cash could be assumed to be available for investment in stocks and bonds.

Consequently an integrated model that would adjust investment income and assets as a result of a change to any other component in the forecast was built.

3.3 Focus on an Integrated and Dynamic Model

The original proforma underwriting model was a Dynaplan spreadsheet into which all components were keyed. This method was appropriate at the time and worked significantly better than any model previously used. When the income statement and balance sheet were completed, the Dynaplan spreadsheet(s) were expanded to incorporate the new components. The level of detail expanded significantly.

The development of investment income and assets added another dimension to the model. Dynaplan was used to consolidate the income statement and balance sheet components from the principals. The statistical software, SAS, was used to solve for investment income and assets. This was extremely time-consuming and error-prone.

To overcome the timing and validity problems, a data base was set up. The Dynaplan spreadsheets were also converted to Lotus which simplified the retrieval, manipulation and reporting of forecasted information. Then Lotus commands replaced SAS as the method to calculate investment income and assets. This worked very well but was still time-consuming. Macro instructions were written which reduced the execution time by automating many of the manual steps but required extensive maintenance.

3.4 Focus on the MIDAS Model

Management requested a further refinement of the financial forecasting process that would allow for dynamic changes and instantaneous evaluation of the key financial ratios. This led to development of the **M**obile **I**ntegrated **D**ynamic **A**nalysis **S**ystem, the MIDAS Model.

First, to decrease the processing time, the level of data was rolled-up to a company level. The detailed line of business and risk would no longer be available for immediate review. Management believed that this would be appropriate due to their global view of the organization. Then the model was further refined to allow for dynamic changes by modifying user access through a "front-end" written in Visual Basic. The Visual Basic front-end made it easier for management to change assumptions or variables such as premiums, expenses and investment mix/yields and instantaneously determine the impact on the key financial ratios.

Next the model was loaded onto a laptop computer. This made the model portable and further expanded management's ability to incorporate the model into financial discussions at planning conferences or financial presentations. Management now was not dependent upon an analyst to run the model every time a financial option arose. Management could take the laptop and review various financial scenarios on demand. MIDAS also expanded management's ability to easily save each scenario and retrieve previous versions if the situation warranted.

MIDAS currently projects quarterly data for one year and annual information for the next five years. Scenarios can be run to determine the impact on an individual company or on a consolidated basis. A consolidation process including elimination entries is built into the MIDAS model. Outputs of the MIDAS model include summarized income statements and balance sheets along with key financial ratios on either a Statutory or GAAP basis. These reports are provided both by company and on a consolidated basis. Appendix I - III include sample reports. Ad hoc reports are developed as needed when additional information is required. The Appendix IV - VI include sample screens from the Visual Basic front-end.

4.0 ACTUAL APPLICATIONS

The model has helped in the analysis of numerous financial scenarios. Below, in Exhibit 3, is a list of some of the applications of the model. All options are measured at a minimum on the key financial ratios of operating expense, combined, surplus and liquidity.

Exhibit 3

<u>Actual Applications</u>

<u>Investment risk:</u>

Investment yields	Investment yields for stocks and bonds have been adjusted to determine the impact of swings in the market on the portfolio.
Investment mix	The investment mix between common and preferred stocks and taxable and tax exempt bonds have been adjusted to determine the impact on investment income.
Subsidiary dividends	Subsidiary dividends have been adjusted to determine the impact on investment income.
Taxes	The mix of taxable vs. tax exempt bonds have been examined to determine the optimum mix to minimize taxes.

<u>Pricing risk:</u>

Premiums	Premiums have been adjusted for competitive scenarios, volume growth, and new markets to determine the impact on financial ratios.

Expenses	Corresponding expenses are adjusted for each scenario to determine the impact on the operating expense and combined ratios.
Policyholder dividends	Various dividend options have been reviewed to determine the impact on the financial ratios.
New Companies	Diversification into new companies and new markets have been reviewed to determine appropriate pricing and capitalization.

<u>Reserving risk:</u>

Loss reserves	Loss reserves have been adjusted to determine the impact of both favorable and unfavorable development on the financial ratios.

<u>Catastrophe risk:</u>

Probable Maximum Loss (PML)	The PML or exposure of the organization to a large catastrophe loss has been adjusted to allow management to develop methods to reduce risk exposure and prepare contingency plans.
Reinsurance	Various reinsurance options have been reviewed both for cost and benefits and the impact on the financial ratios once the reinsurance is purchased.

Throughout its evolution, MIDAS has proven to be comprehensive, yet flexible enough to analyze an infinite number of scenarios.

5.0 THE ACCOUNTING EQUATION

In this section we discuss the fundamental theoretical accounting assumptions of the model.

5.1 Basic Accounting Equation Assumptions

The basis of the model is the accounting equation:

Assets = Liabilities + Surplus

The general approach of the model is to develop a projected income statement and balance sheet from data provided from various principals. These are the known components of the accounting equation. The unknown components are investment income and assets. This creates a dilemma because investment income cannot be determined until investment assets are known and vice versa. To solve this problem, numerous algebraic equations are developed for each unknown component and linear programming is applied to solve for the unknown values.

Exhibit 4

Components of the Accounting Equation

Assets	= Liabilities +	Surplus
Known Components		
Existing Portfolio	Loss Reserves	Beginning Surplus
Bonds	LAE Reserves	Premiums
Stocks	Unearned Premium	Losses
Affiliated Stocks	Other Liabilities	Underwriting Expenses
Cash		LAE Expenses
Other Invested Assets		Policyholder Dividends
Other Assets		Existing Portfolio
		Bond Interest
		Stock Dividends
		Other Investment Income
Unknown Components		
New Portfolio		New Portfolio Income
Bonds		Bond Interest
Stocks		Stock Dividends

5.2 Development of the Accounting Equation

The model assumes a sources and uses (cash flow) approach to project investment income and assets based on the accounting equation.

Assets = Liabilities + Surplus

If surplus, liabilities and all assets other than stocks and bonds can be projected, the remainder is stocks and bonds.

Stocks + Bonds + All other Assets = Liabilities + Surplus

Or

Stocks + Bonds = Liabilities + Surplus - All other Assets

The projections for liabilities and all other assets are known components supplied by principals. Surplus can be defined as:

Ending Surplus = Beginning Surplus + Net Income + Other Surplus Changes

The projection for other surplus changes is a known component provided by principals and the beginning surplus is the latest actual surplus. All the components that make up net income are also known and provided by principals with the exception of investment income. Net income can be further divided as follows:

Net Income = Investment income + All other income - All expenses

Investment income is referred to as interest and dividends from bonds and stocks. Substituting the above definitions, the resulting equation is defined as follows:

Stocks + Bonds = Liabilities+(Beginning Surplus+Bond Interest + Stock Dividends + All other income-All expenses+Other Surplus Changes) -All other assets

Note that bond interest and stock dividends must be calculated to determine the amount of investment in stocks and bonds. Also the amount of investment in stocks and bonds must be known to determine bond interest and stock dividends. This is a circular argument, so the solution is to resolve the conflicting arguments simultaneously.

5.3 Investment Issues

The following is a discussion of further refinements to the basic theory discussed in section 5.2.

5.3.1 Stock and Bond Yields

The stock portfolio was further subdivided into common stocks and preferred stocks, each with a unique yield. Bonds were subdivided into taxable and tax exempt with corresponding yields.

The portfolio for stocks and bonds is split into existing and new. Existing portfolio is the beginning (Actual) portfolio for the planning period. The yield on the existing portfolio is what the current portfolio is actually returning. The new portfolio yield is provided by the Investment Company and is the best estimate of the current market yield.

Management wanted the model to assume that all stocks and bonds would be purchased throughout the period and not on one day in the period. In order to calculate the appropriate dividend or interest to stocks and bonds, a mid-period convention was applied. The following is the calculation for common stock dividends.

Common Stock dividends = ((Common Stock current period + Common stock previous period)/2) * Yield

The calculation for preferred stock, taxable and tax exempt bonds would be very similar.

5.3.2 Portfolio Mix

The mix of portfolio assets refers to the dollar value of stocks and bonds to total investment assets. There are many ways to split the various components of the portfolio. MIDAS currently calculates portfolio mix as a percentage of total surplus.

The common stock mix is limited to be X% of surplus. The remaining portfolio is further divided into preferred stock, taxable and tax exempt bonds.

Preferred stock	X%
Taxable bonds	X%
Tax exempt bonds	X%
	100% of the remaining portfolio

The investment mix can be easily modified for various options. The mix strategy is currently provided by the Investment Company and the Tax department.

5.3.3 Selling from the Portfolio

Sometimes it is necessary to sell either stocks or bonds from the portfolio to maintain the desired portfolio mix. Selling from the portfolio can occur for two reasons. The first occurs when the actual investment mix is different from the investment mix assumptions included in MIDAS. The second is when management desires to change the portfolio mix assumptions. MIDAS automatically calculates the amount to sell from the portfolio to achieve the desired portfolio balance.

5.4 Examples of Additional Refinements

These issues and many other contingent solutions can be easily incorporated into MIDAS.

5.4.1 Realized Gains

Realized gains have become a significant component in the growth of surplus. Consequently, management has requested that realized gains be incorporated into the model as an unknown component. This would allow MIDAS to solve for this amount with the other unknowns. The following is the calculation for realized gains on common stock.

Realized Gains = (Existing Common Stock + New common Stock)*RealizedGains Yield

Realized gains on the other investment components such as preferred stock and bonds would be calculated in a similar manner.

5.4.2 Taxes

MIDAS also calculates Federal Income Taxes as an unknown component. This enables management to determine the tax effect of various actions immediately instead of waiting for an analysis from the Tax department. Taxes are computed by the following calculation.

Income Taxes =Tax Rate*Taxable Income

Taxable income is adjusted for tax credits and deferrals.

6.0 AN EXAMPLE OF SOLVING THE BASIC ACCOUNTING EQUATION

The following is an example of the theoretical basis of the MIDAS model. For simplicity purposes, this example solves for only eight unknowns of one company for one period. The unknowns are:

Investment Income	Common Stock Dividends
	Preferred Stock Dividends
	Tax Exempt Bonds Interest
	Taxable Bonds Interest
Investment Assets	Common Stock
	Preferred Stock
	Tax Exempt Bonds
	Taxable Bonds

The unknowns may be expanded or contracted as needed for each situation. For example, to calculate taxes, an additional equation would be added to define the calculation. To add multiple companies, separate equations may be developed for each company and then the results consolidated for the insurance group.

6.1 Example Assumptions

Listed below are sample values for the example.

Exhibit 5
Example Assumptions

Variable		Values
A = Dividends on Common Stock	New portfolio	Unknown
B = Dividends on Preferred Stock	New portfolio	Unknown
C = Interest on Tax Exempt Bonds	New portfolio	Unknown
D = Interest on Taxable Bonds	New portfolio	Unknown
E = Common Stock	New portfolio current period	Unknown
F = Preferred Stock	New portfolio current period	Unknown
G = Tax Exempt Bonds	New portfolio current period	Unknown
H = Taxable Bonds	New portfolio current period	Unknown
I = Dividends on Common Stock	Existing portfolio	12
J = Dividends on Preferred Stock	Existing portfolio	7
K = Interest on Tax Exempt Bonds	Existing portfolio	16
L = Interest on Taxable Bonds	Existing portfolio	47
M = Common Stock	Existing portfolio	1,010
N = Preferred Stock	Existing portfolio	247
O = Tax Exempt Bonds	Existing portfolio	752
P = Taxable Bonds	Existing portfolio	1,826
Q = Common Stock	New portfolio previous period	23
R = Preferred Stock	New portfolio previous period	87
S = Tax Exempt Bonds	New portfolio previous period	150
T = Taxable Bonds	New portfolio previous period	278
YCN = Yield on new Common Stock		.035
YPN = Yield on new Preferred Stock		.085
YEN = Yield on new Tax Exempt Bonds		.090
YTN = Yield on new Taxable Bonds		.070
Z = (Net income - Investment income) + Surplus changes		25
AA = Previous surplus		2,951

Example Assumptions

Variable	Values
BB = Total liabilities	4,257
CC = All assets other than stocks and bonds	2,780
MC = Common stocks to surplus mix	.35
MP = Preferred stocks to remaining surplus mix	.10
ME = Tax exempt to remaining surplus mix	.27
MT = Taxable bonds to remaining surplus mix	.63

6.2 Calculation of Unknown Variables

The following algebraic equations incorporate the values for the example.

6.2.1 Investment Income New Portfolio

Dividends on common stock of the new portfolio

Basic Equation	A = .5*((YCN*(Q+E))
Assign Values	A = .5*((.035*(23+E))
Simplify Equation	A = .403+.018E

Dividends on preferred stock of the new portfolio

Basic Equation	B = .5*((YPN*(R+F))
Assign Values	B = .5*((.085*(87+F))
Simplify Equation	B = 3.70+.043F

Interest on tax exempt bonds of the new portfolio

Basic Equation	C = .5*((YEN*(S+G))
Assign Values	C = .5*((.090*(150+G))
Simplify Equation	C = 6.75+.045G

Interest on taxable bonds of the new portfolio

Basic Equation	D = .5*((YTN*(T+H))
Assign Values	D = .5*((.070*(278+H))
Simplify Equation	D = 9.73+.035H

6.2.2 Investment Assets New Portfolio

Common stock new portfolio

Basic Equation	E = MC*((A+B+C+D)+(I+J+K+L+Z+AA))-M
Assign Values	E = .35*((A+B+C+D)+(12+7+16+47+25+2,951))-1,010
Simplify Equation	E = (.35*(A+B+C+D))+60

Preferred Stock new portfolio

Basic Equation	F = MP*((A+B+C+D)+(I+J+K+L+Z+AA+BB-CC-E-M)) - N
Assign Values	F = .10*((A+B+C+D)+12+7+16+47+25+2,951+4,257-2,780-E-1,010)-247
Simplify Equation	F = (.10*(A+B+C+D-E))+106

Tax exempt bond interest on new portfolio

Basic Equation	G = ME*((A+B+C+D-E)+I+J+K+L+Z+AA+BB-CC-E-M) - O
Assign Values	G = .27*((A+B+C+D)+12+7+16+47+25+2,951+4,257-2,780-E-1,010)-752
Simplify Equation	G = (.27*(A+B+C+D-E))+200

Taxable bond interest on new portfolio

Basic Equation	H = MT*((A+B+C+D-E)+I+J+K+L+Z+AA+BB-CC-E-M) - P
Assign Values	H =.63*((A+B+C+D)+12+7+16+47+25+2,951+4,257-2,780-E-1,010)-1,826
Simplify Equation	H = (.63*(A+B+C+D))+395

6.3 Simultaneous Linear Equations

This is the resulting set of simultaneous linear equations which has eight unknowns and will be used to determine one unique solution as follows:

A = .403 + .018E
B = 3.70 + .043F
C = 6.75+.045G
D = 9.73+.035H
E = (.35*(A+B+C+D))+60
F = (.10*(A+B+C+D-E))+106
G = (.27*(A+B+C+D-E))+200
H = (.63*(A+B+C+D-E))+395

Refining and simplifying for the coefficient matrix derives the following equations.

A-.018E = .403
B-.043F = 3.70
C-.045G = 6.75
D-.035H = 9.73
-.35A-.35B-.35C-.35D+E = 60
-.10A-.10B-.10C-.10D+.10E+F = 106
-.27A-.27B-.27C-.27D+.27E+G = 200
-.63A-.63B-.63C-.63D+.63E+H = 395

6.4 Linear Programming

To determine the simultaneous solutions for the unknowns, the equations must be expanded to determine the relationships between each unknown variable. Applying the equations to matrices is the easiest technique to accomplish this relationship.

6.4.1 Coefficient Matrix

Assigning the variables in the equations to a matrix results in the following coefficient matrix.

Exhibit 6
Coefficient Matrix

A	B	C	D	E	F	G	H
1	0	0	0	-.018	0	0	0
0	1	0	0	0	-.043	0	0
0	0	1	0	0	0	-.045	0
0	0	0	1	0	0	0	-.035
-.35	-.35	-.35	-.35	1	0	0	0
-.10	-.10	-.10	-.10	+.10	1	0	0
-.27	-.27	-.27	-.27	+.27	0	1	0
-.63	-.63	-.63	-.63	+.63	0	0	1

6.4.2 Constant Matrix

Assigning the constants from the equations to a matrix produces the following constant matrix.

Exhibit 7
<u>Constant Matrix</u>

A	.403
B	3.70
C	6.75
D	9.73
E	60
F	106
G	200
H	395

6.5 Lotus Commands

Lotus matrix commands were incorporated into MIDAS to perform the matrix calculations of linear programming. The two matrix commands are:

DMI - Data Matrix Invert
DMM - Data Matrix Multiply

The solution matrix must first be inverted using the data matrix invert command (DMI). The basic format is:

DMI"Range of solution matrix"~"Beginning position of resulting inverted matrix"

Then the inverted matrix is multiplied by the coefficient matrix using the data matrix multiply command. The format is as follows:

DMM"Range of inverted matrix"~"Range of coefficient matrix"~"Beginning position of resulting coefficient matrix"

These Lotus commands were required for each period projected. The commands were automated by macro routines which decreased processing time and the exposure to errors.

6.6 Results

The results of the example are as follows:

Variable	Unknown	Value
A	Common stock dividend new portfolio	2
B	Preferred stock dividend new portfolio	8
C	Tax Exempt interest new portfolio	15
D	Taxable interest new portfolio	23
E	Common stock new portfolio	77
F	Preferred stock new portfolio	103
G	Tax Exempt bonds new portfolio	192
H	Taxable bonds new portfolio	377

6.7 Proof of Results

The solutions of the algebraic equations follow incorporating the example results. The results have been rounded to whole numbers.

Dividends on common stock of the new portfolio

Basic Equation	A = .5*((YCN*(Q+E))
Assign Values	A = .5*((.035*(23+E))
Simplify Equation	A = .403+.018E
Solution	2 = .403 + .018(77)

Dividends on preferred stock of the new portfolio

Basic Equation	B = .5*((YPN*(R+F))
Assign Values	B = .5*((.085*(87+F))
Simplify Equation	B = 3.70+.043F
Solution	8 = 3.70 + .043(103)

Interest on tax exempt bonds of the new portfolio

Basic Equation	C = .5*(YEN*(S+G))
Assign Values	C = .5*(.090*(150+G))
Simplify Equation	C = 6.75+.045G
Solution	15 = 6.75+.045(192)

Interest on taxable bonds of the new portfolio

Basic Equation	D = .5*(YTN*(T+H))
Assign Values	D = .5*(.070*(278+H))
Simplify Equation	D = 9.73+.035H
Solution	23 = 9.73 + .035(377)

Common stock new portfolio

Simplify Equation	E = (.35*(A+B+C+D))+60
Assign Values	E = (.35*(A+B+C+D)+(12+7+16+47+25+2,951))-1,010
Simplify Equation	E = (.35*(A+B+C+D))+60
Solution	77 = (.35*(2+8+15+23))+60

Preferred Stock new portfolio

Basic Equation	F = MP*(A+B+C+D)+(I+J+K+L+Z+AA+BB-CC-E-M)-N
Assign Values	F = (.10*(A+B+C+D)+12+7+16+47+25+2,951+4,257-2,780-E-1,010)-247
Simplify Equation	F = (.10*(A+B+C+D-E))+106
Solution	103 = (.10*(2+8+15+23-77))+106

Tax exempt bond interest on new portfolio

Basic Equation	G = ME*((A+B+C+D-E)+I+J+K+L+Z+AA+BB-CC-E-M)-O
Assign Values	G = (.27*(A+B+C+D)+12+7+16+47+25+2,951+4,257-2,780-E-1,010)-752
Simplify Equation	G = (.27*(A+B+C+D-E))+200
Solution	192 = (.27*(2+8+15+23-77))+200

Taxable bond interest on new portfolio

Basic Equation	H = MT*((A+B+C+D-E)+I+J+K+L+Z+AA+BB-CC-E-M) - P
Assign Values	H =.63*((A+B+C+D)+12+7+16+47+25+2,951+4,257-2,780-E-1,010)-1,826
Simplify Equation	H = (.63*(A+B+C+D))+395
Solution	377 = (.63*(2+8+15+23-77))+395

7.0 LIMITATIONS

This section discusses the limitations of the MIDAS model.

7.1 Quality of Data

The quality of data incorporated in the model has a direct impact on the quality of output. For example, when incurred losses are developed, losses paid and losses outstanding must be developed together. A mismatch of paid to outstanding losses would generate an inappropriate buy or sell from the portfolio. To minimize this limitation, data is reviewed by the analyst prior to inputting into MIDAS for reasonableness and integrity. Once a good base case is developed, it is easy for management to manipulate variables and run scenarios.

7.2 Investment Assumptions

Actual investment income and assets will be different from those projected in MIDAS. If part of the portfolio needs to be sold, the Investment Company might sell a different type of security (stocks or bonds) than the security the model is indicating should be sold. The constraints in the model ensure the assumed portfolio mix is correct while the actual Investment Company transactions are based on current market conditions.

7.3 Noncash Transactions

Noncash transactions, such as depreciation, are not taken into consideration in the model. Normally depreciation is an immaterial figure. However, management may discredit the model due to misunderstanding the impact of assumptions. Non-cash transactions can be incorporated as known components if they become material to the financial results or key ratios.

7.4 Double Entry Accounting

MIDAS is a cell-referenced spreadsheet and not a double entry accounting system. When management requests changes to a component in the model, analysis is required to ensure that the impact on the accounting equation impact is reflected. For example, a dividend from a subsidiary would require the following adjustments:

Income statement impact	Increase dividends from subsidiaries
Balance Sheet impact	Reduce investment in subsidiaries

An analyst must modify the model to incorporate both entries or management will misinterpret the impact of this alternative.

7.5 Overall Model Assumptions

The model is built to reflect a basic set of assumptions. If management desires to change a basic assumption, it may take time to review and incorporate the request into the model. Management must communicate the issues to be reviewed so that the analyst can ensure that the assumptions are appropriate for the problem to be studied.

7.6 User Education

MIDAS is a complex tool although the Visual Basic front-end is very user friendly. Users must understand the relationships among components so the correct adjustments are made when running scenarios. For example, growth in premiums could have an impact on both losses and expenses. The analyst may customize the front-end for specific issues so that all effected components are adjusted. An extensive amount of time is spent educating the users of the model to ensure that they understand its theoretical basis and can interpret results correctly.

7.7 Loss of Control

Once the model was made dynamically mobile, its use exploded. Consequently, the analyst had difficulty reconciling the various versions of the model to the original. To minimize this limitation, the analyst must ensure that all users are starting with the same base case and overall model assumptions. This information is coordinated quarterly or as needed to support planning conferences and financial presentations.

8.0 WHERE DO WE GO FROM HERE?

As the regulatory and competitive environment changes, management will be interested in incorporating changes to stay abreast of the times. The analyst will need to respond to these requests by continually refining MIDAS. Some modifications to MIDAS may include incorporating additional companies, more detailed subsidiary analysis, risk-based capital calculations, increased PML analysis, and the latest tax planning strategies.

REFERENCES

[1] Harvey, A.C. (1990). Forecasting, structural time series models and the Kaiman filter. Cambridge University Press, New York.

Appendix I

4q95ff
Actuals through the 3th Qtr. 1995

STATUTORY CONSOLIDATED INCOME STATEMENT SCORECARD
($ in thousands)

11-Nov-95
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ACCOUNT DESCRIPTION	Total 1993	Total 1994	Total 1995	Total 1996	Total 1997	Total 1998	Total 1999	Total 2000	Total 2001
PREMIUMS WRITTEN	1,461,593	1,523,516	1,581,251	1,626,419	4,871,648	1,822,738	1,983,597	2,164,515	2,368,824
PREMIUMS EARNED	1,418,648	1,500,731	1,561,364	1,605,436	1,669,786	1,775,079	1,923,369	2,097,095	2,293,450
CAT LOSSES	58,398	115,301	96,357	105,958	114,238	125,880	139,066	153,306	169,182
NON-CAT LOSSES	915,339	935,013	808,473	930,927	1,067,697	1,133,411	1,232,812	1,345,444	1,474,341
TOTAL LOSSES	973,951	1,050,631	904,830	1,036,884	1,181,935	1,259,291	1,371,878	1,498,750	1,643,523
CAT LAE	3,019	12,823	11,280	4,857	5,264	5,478	5,472	5,465	5,458
NON-CAT LAE	215,074	199,986	186,339	229,285	268,799	274,388	297,044	324,165	355,358
TOTAL LOSS ADJUSTMENT EX	218,093	212,809	197,619	234,141	274,064	279,865	302,516	329,630	360,815
UNDERWRITING EXP	180,978	209,331	202,845	212,590	215,915	231,525	252,030	274,971	299,802
OPERATING EXPENSES	399,071	422,140	400,464	446,731	489,979	511,390	554,545	604,601	660,617
DIVIDENDS	28,084	49,663	218,774	50,229	48,328	48,188	49,098	50,589	52,378
U/W GAIN (LOSS)	17,542	(21,703)	37,297	71,592	(50,456)	(43,790)	(52,152)	(56,844)	(63,068)
INVESTMENT INCOME	153,467	159,033	187,929	201,534	216,347	232,191	250,086	267,174	287,166
REALIZED GAINS (LOSSES)	57,119	14,430	14,278	48,645	52,829	57,362	62,102	66,246	71,276
SUBSIDIARY DIVIDENDS	44,513	91,862	113,554	102,797	97,088	126,319	137,723	145,269	156,120
INVESTMENT EXPENSES	35,394	37,965	41,136	45,776	46,693	48,768	52,885	57,655	63,000
NET INVEST GAIN(LOSS)	175,191	174,525	199,569	229,429	228,958	271,650	295,481	313,110	337,179
OTHER INCOME	1,614	(1,740)	1,355	323	451	79	(271)	(698)	(1,239)
INCOME BEFORE TAXES	149,833	98,246	163,166	223,573	88,340	132,486	141,514	147,644	158,488
FIT (BENEFIT)	5,583	(14,093)	(756)	7,074	10,413	12,345	11,672	4,993	14,492
NET INCOME	144,251	112,340	163,921	216,499	77,927	120,141	129,843	142,651	143,997
NET WORTH PREV PERIOD	1,072,393	1,228,823	1,282,644	1,570,253	1,814,738	1,945,586	2,105,839	2,278,414	2,469,692
NET INCOME	144,251	112,340	163,921	216,499	77,927	120,141	129,843	142,651	143,997
CHANGE IN SUB NET WORTH	28,680	(17,539)	9,571	23,330	52,578	36,738	40,482	48,614	54,691
UNREALIZED GAIN (LOSS)	(22,266)	(28,788)	101,047	0	0	0	0	0	0
OTHER NET WORTH CHANGE	5,737	(12,193)	13,071	4,655	343	3,374	2,251	13	(333)
TOTAL NET WORTH CHANGE	156,401	53,821	287,611	244,484	130,849	160,253	172,575	191,278	198,355
NET WORTH CURR PERIOD	1,228,823	1,282,644	1,570,253	1,814,738	1,945,586	2,105,839	2,278,414	2,469,692	2,668,047
ASSETS	3,015,652	3,361,865	3,502,445	3,795,777	4,148,042	4,487,070	4,825,777	5,199,495	5,596,114
LIABILITIES	1,786,830	2,079,221	1,962,193	1,981,040	2,202,456	2,381,231	2,547,362	2,729,803	2,928,068
OER	28.1%	28.1%	25.6%	27.8%	29.3%	28.8%	28.8%	28.8%	28.8%
COMBINED RATIO	98.4%	101.2%	97.4%	95.4%	94.5%	102.1%	102.3%	102.3%	102.3%
U/W GAIN TO EP	1.2%	-1.4%	2.4%	4.5%	-3.0%	-2.5%	-2.7%	-2.7%	-2.7%
LOSSES TO EP	68.7%	70.0%	58.0%	64.6%	70.8%	70.9%	71.3%	71.5%	71.7%
LAE TO EP	15.4%	14.2%	12.7%	14.6%	16.4%	15.8%	15.7%	15.7%	15.7%
U/W TO EP	12.8%	13.9%	13.0%	13.2%	12.9%	13.0%	13.1%	13.1%	13.1%
U/W TO WP	12.4%	13.7%	12.8%	13.1%	4.4%	12.7%	12.7%	12.7%	12.7%
DIV TO EP	2.0%	3.3%	14.0%	3.1%	2.9%	2.7%	2.6%	2.4%	2.3%
LAE TO LOSSES	22.4%	20.3%	21.8%	22.6%	23.2%	22.2%	22.1%	22.0%	22.0%
LIQUIDITY RATIO	1.24	1.16	1.34	1.46	1.41	1.41	1.41	1.41	1.41
SURPLUS RATIO	84.1%	84.2%	99.3%	111.6%	39.9%	115.5%	114.9%	114.1%	112.6%

Appendix II

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Actuals through the 3rd Qtr. 1995

STATUTORY CONSOLIDATED BALANCE SHEET SCORECARD
($ in thousands)

11-Nov-95
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ACCOUNT DESCRIPTION	Total 1993	Total 1994	Total 1995	Total 1996	Total 1997	Total 1998	Total 1999	Total 2000	Total 2001
ASSETS:									
TAXABLE BONDS	877,775	1,152,431	1,606,443	1,771,870	1,962,253	2,185,312	2,393,400	2,631,338	2,866,871
TAX EXEMPT BONDS	601,531	481,772	161,388	160,648	159,242	158,427	157,176	156,616	156,616
PREFERRED STOCK	117,057	109,288	32,745	42,302	44,651	47,288	50,018	53,008	56,236
COMMON STOCK	387,947	401,306	530,716	643,558	689,081	745,039	803,558	854,725	916,824
CASH & SHORT TERM	48,180	97,853	95,341	47,507	51,707	55,907	60,107	64,307	68,507
TOTAL OTHER INV ASSETS	47,676	49,409	56,448	49,835	50,492	50,977	51,657	52,130	52,617
REAL ESTATE	203,988	206,837	214,308	226,799	227,607	224,194	220,920	216,801	212,808
EXCESS RE (5% LIAB LESS RE	(59,763)	(78,839)	(61,720)	(56,207)	(87,029)	(115,982)	(142,989)	(173,171)	(205,487)
TOTAL QUALIFIED ASSETS	2,224,390	2,420,057	2,635,669	2,886,310	3,098,005	3,351,162	3,593,847	3,855,755	4,124,992
INVESTMENT IN SUBSIDIARIE	501,212	484,479	494,924	520,005	596,957	633,695	674,177	722,791	777,483
PREMIUM BALANCES	223,149	226,750	250,479	241,229	276,817	298,918	326,094	356,678	391,235
FURN, EQUIP & EDP	34,234	29,470	31,050	56,725	53,296	49,961	49,112	48,899	50,078
OTHER ASSETS	92,429	122,270	28,603	35,302	35,938	37,352	39,557	42,201	46,841
EXCESS REAL ESTATE	59,763	78,839	61,720	56,207	87,029	115,982	142,989	173,171	205,487
TOTAL ASSETS	3,015,652	3,361,865	3,502,445	3,795,777	4,148,042	4,487,070	4,825,777	5,199,495	5,596,114
LIABILITIES:									
LOSS RESERVES	893,745	1,105,422	973,713	951,623	1,057,779	1,163,874	1,238,560	1,318,420	1,402,699
LAE RESERVES	232,731	242,785	224,539	219,749	249,201	260,441	272,452	285,144	298,562
UNEARNED PREMIUMS	483,935	506,722	526,609	547,592	582,883	630,542	690,771	758,190	833,565
OUTSTANDING CHECKS	7,140	45,557	0	5,390	5,390	5,390	5,390	5,390	5,390
OUTSTANDING DRAFTS	39,456	45,777	50,036	47,023	53,274	56,577	60,322	64,262	68,616
DIVIDENDS PAYABLE	13,407	14,050	14,262	11,610	49,438	49,525	50,267	51,692	53,508
OTHER LIABILITIES	116,415	118,908	173,034	198,053	204,491	214,882	229,601	246,704	265,727
TOTAL LIABILITIES	1,786,830	2,079,221	1,962,193	1,981,040	2,202,456	2,381,231	2,547,362	2,729,803	2,928,068
SURPLUS:									
TOTAL SURPLUS	1,228,823	1,282,645	1,570,253	1,814,738	1,945,586	2,105,839	2,278,414	2,469,692	2,668,047
LIABILITIES & SURPLUS	3,015,652	3,361,865	3,532,445	3,795,777	4,148,042	4,487,070	4,825,777	5,199,495	5,596,114

Appendix III

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Actuals through 3th Quarter 1995

P&C CONSOLIDATED GAAP SCORECARD
($ IN THOUSANDS)

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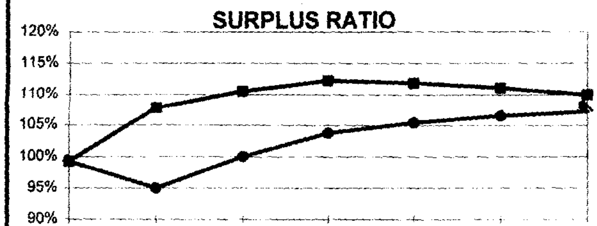
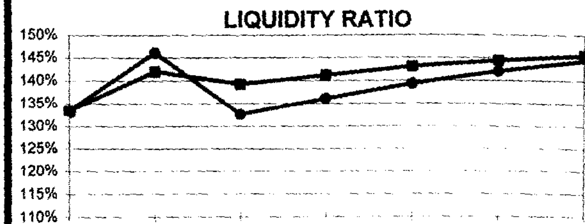
	TOTAL 1993	TOTAL 1994	TOTAL 1995	TOTAL 1996	TOTAL 1997	TOTAL 1998	TOTAL 1999	TOTAL 2000	TOTAL 2001
STATEMENTS									
REVENUES	1,639,457	1,682,509	1,766,605	1,879,067	1,962,816	2,088,628	2,257,526	2,449,881	2,668,615
EXPENSES	1,444,596	1,523,632	1,558,347	1,590,193	1,797,486	1,900,671	2,059,423	2,242,643	2,450,142
P&C OPERATING INCOME	**194,860**	**158,878**	**208,259**	**288,875**	**165,330**	**187,957**	**198,103**	**207,237**	**218,473**
NET INCOME OF SUBS	40,137	38,669	47,359	59,147	71,486	81,829	93,086	104,662	117,026
NET INCOME	234,997	197,546	255,617	348,022	236,816	269,785	291,189	311,899	335,499
ASSETS	**3,418,647**	**3,559,539**	**3,950,593**	**4,121,149**	**4,476,079**	**4,812,139**	**5,149,881**	**5,520,370**	**5,922,661**
LIABILITIES	1,931,711	2,133,729	2,125,339	2,025,050	2,232,562	2,393,029	2,539,791	2,704,916	2,888,908
NET WORTH	1,486,936	1,425,810	1,825,254	2,096,100	2,243,518	2,419,110	2,610,090	2,815,454	3,033,753
TOTAL LIABILITIES & NET WORTH	**3,418,647**	**3,559,539**	**3,950,593**	**4,121,149**	**4,476,079**	**4,812,139**	**5,149,881**	**5,520,370**	**5,922,661**
RECONCILIATIONS									
STATUTORY NET INCOME	**144,251**	**112,340**	**163,921**	**216,499**	**77,927**	**120,141**	**129,843**	**142,651**	**143,997**
DEFERRED POLICY ACQ COSTS	1,096	1,525	(2,946)	5,204	3,554	4,823	6,113	6,849	7,665
DEFERRED TAXES	4,974	41,240	(14,926)	16,323	43	96	(1,078)	(9,450)	(254)
SUBSIDIARY DIVIDEND	44,513	13,809	36,556	52,746	84,138	64,589	65,366	70,577	71,226
OTHER ADJUSTMENTS	144,277	102,303	189,574	214,603	77,594	118,449	127,702	139,261	139,836
P&C OPERATING INCOME	**194,860**	**158,877**	**208,258**	**288,875**	**165,330**	**187,956**	**198,103**	**207,237**	**218,473**
EQUITY IN EARNINGS OF SUBS	40,137	38,669	47,359	59,147	71,486	81,829	93,086	104,662	117,026
GAAP NET INCOME	234,997	197,546	255,617	348,022	236,816	269,785	291,189	311,899	335,499
STATUTORY SURPLUS	**1,763,933**	**1,412,183**	**1,521,176**	**1,763,477**	**1,994,325**	**2,054,578**	**2,988,190**	**3,179,469**	**2,616,756**
VALUATION OF BONDS	340	(113,306)	75,503	78,864	78,864	78,864	78,864	78,864	78,864
NON-ADMITTED ASSETS	56,851	62,164	65,489	60,834	60,491	57,117	54,866	54,854	55,187
DEF POLICY ACQUISITION COSTS	17,731	521,848	17,233	19,054	20,298	21,987	24,127	26,524	29,207
DEFERRED TAXES	58,536	133,988	14,906	31,229	31,271	31,367	30,289	20,839	20,586
OTHER ADJUSTMENTS	(410,453)	(591,067)	130,946	142,643	158,269	175,197	(566,246)	(545,096)	233,123
GAAP SURPLUS	**1,486,936**	**1,425,809**	**1,825,253**	**2,096,101**	**2,343,519**	**2,419,110**	**2,610,090**	**2,815,453**	**3,033,722**

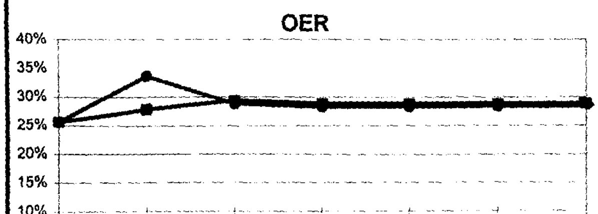
M I D A S

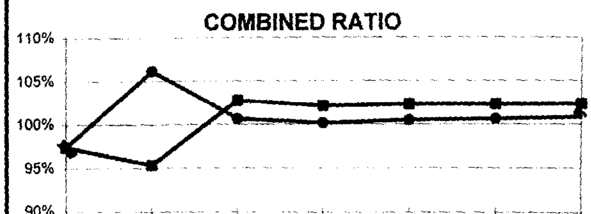




	1995	1996	1997	1998	1999	2000	2001
Base	99.3%	107.8%	110.5%	112.3%	111.9%	111.1%	110.0%
Scenario1	99.3%	95.0%	100.0%	103.8%	105.5%	106.6%	107.3%

	1995	1996	1997	1998	1999	2000	2001
Base	133.5%	142.1%	139.3%	141.2%	143.1%	144.4%	145.5%
Scenario1	133.5%	146.2%	132.6%	136.1%	139.3%	142.0%	144.4%





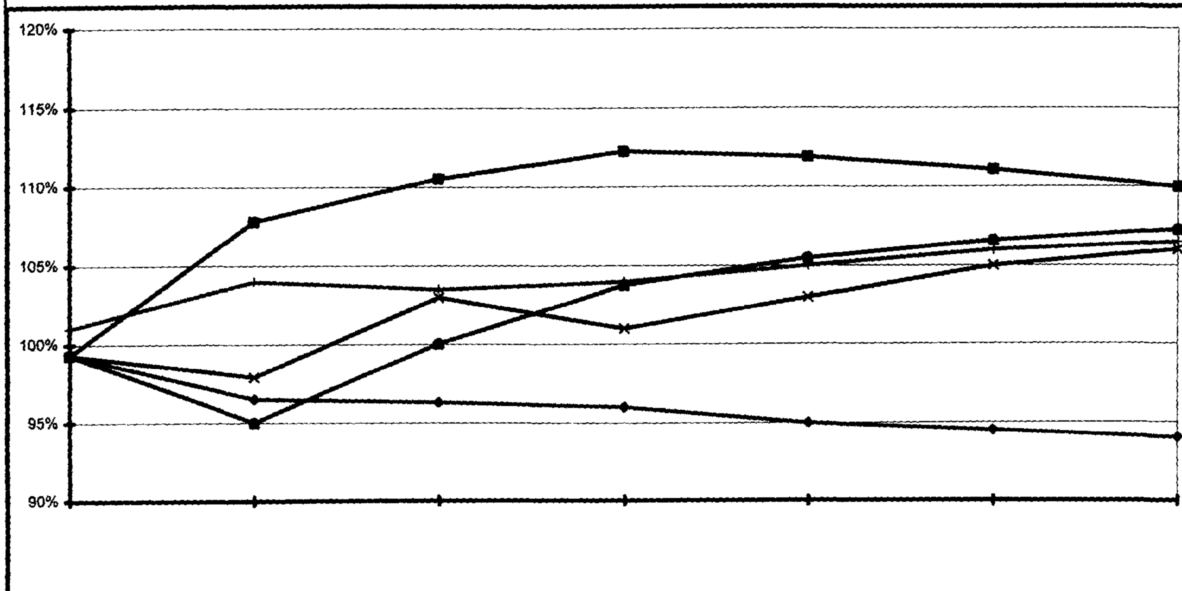
	1995	1996	1997	1998	1999	2000	2001
Base	25.6%	27.8%	29.3%	28.8%	28.8%	28.8%	28.8%
Scenario1	25.6%	33.6%	28.7%	28.3%	28.3%	28.4%	28.4%

	1995	1996	1997	1998	1999	2000	2001
Base	97.4%	95.4%	102.8%	102.1%	102.3%	102.3%	102.3%
Scenario1	97.4%	106.1%	100.6%	100.2%	100.5%	100.6%	100.8%

Clear Scenario | Scenario 1 | Changes | Run | Exit

Appendix V
Detail Screen

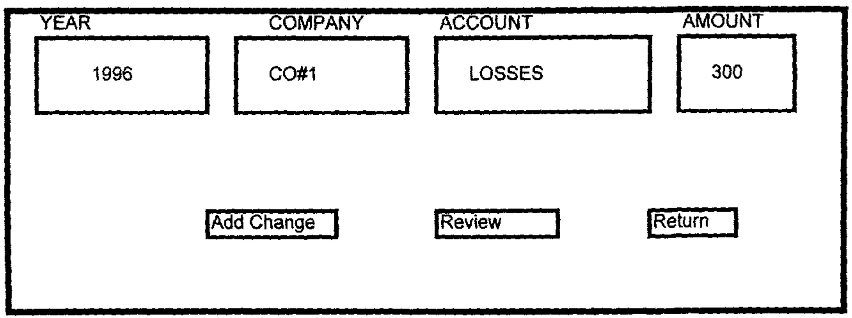
SURPLUS RATIO



	1995	1996	1997	1998	1999	2000	2001
Base	99.3%	107.8%	110.5%	112.3%	111.9%	111.1%	110.0%
Scenario 1	99.3%	107.7%	112.7%	116.8%	118.5%	119.7%	120.3%
Scenario 2	99.3%	95.0%	100.0%	103.8%	105.5%	106.6%	107.3%
Scenario 3	99.3%	97.9%	103.0%	101.0%	103.0%	105.0%	106.0%
Scenario 4	99.3%	96.5%	96.3%	96.0%	95.0%	94.5%	94.0%
Scenario 5	101.0%	104.0%	103.5%	104.0%	105.0%	106.0%	106.5%

Appendix VI
Change Form

CHANGE OPTIONS



A Stochastic Planning Model for the Insurance Corporation of British Columbia

by Rodney E. Kreps, FCAS
Michael M. Steel

A Stochastic Planning Model for the Insurance Corporation of British Columbia

by
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Summary

A stochastic planning model is a representation to an appropriate level of detail of all of the cash flows of an insurance company, where the variables are stochastic (randomly generated). The variables are connected by simple econometric equations whose form and parameters are generated by the relevant underlying data. The main virtue of stochastic planning models is that all the probability levels, and not just the mean, are available for any financial variable. Such a model has been built for a large Canadian automobile carrier, with the primary application to surplus requirements under different management decisions.

Although it is considerably more complex than the spreadsheet approach to risk-based capital being proposed by the NAIC[1] (for reasons of simplicity), a stochastic model gives surplus requirements as a function of both risk appetite and management scenarios. The data and analysis requirements for a detailed model are substantial.

One of the by-products is a model of stochastic loss development involving accident period, development period, and payment period changes. Taken with the stochastic investment treatment and a projected zero future premium income, the run-off position variability can be quantified, i.e. the distribution of the adequacy of loss reserves can be ascertained.

Introduction

Both for proprietary reasons and because the data is specific to a very specialized situation, this paper for the most part discusses methodology rather than numerical results. Our intent is to encourage other actuaries in the creation and use of models such as this one.

What is a Stochastic Planning Model?

A stochastic planning model is a simulation model that represents to an appropriate level of detail all the cash flows of an insurance company. The variables are all stochastic (randomly generated). They are connected by econometric equations generated by the relevant underlying data. The loss model here is Compound Poisson, with the severities being linked to the inflation rate. Claims are incurred on an underwriting period basis. The actual cash flow during each calendar period is a combination of stochastic runoff from all open underwriting periods, with calendar inflation of the severity distributions. Market share is represented by the total number of ultimate claims for each underwriting period. Investment yields and asset growth are correlated both to themselves at prior periods and to inflation. Expenses are also linked appropriately.

There are many places where management policies must be made explicit, e.g., in investment scenarios, in expense projections, and in the size of loads to be added to the pure premium to get the rates. The loads may depend upon management goals for cross-subsidization of lines, for market share, for profitability, for solvency, or for any other goal which is explicitly codifiable.

Any one realization is an explicit random choice of all the stochastic variables, moving forward through time to the desired horizon. The output from one realization is one complete delineation of all of the cash flows of the company under the given management policies. The output from the simulation of many realizations is a probability distribution for any financial quantity at any point in time. **The advantage of stochastic simulation is that the mean values are available, as in most planning models, and the probabilities of being far from the mean are also available.** The primary application is to surplus, for solvency testing, and the resulting distributions are a sophisticated way of doing risk-based capital. In particular, by setting future income and exposures to zero a distribution of reserve adequacy can be obtained which incorporates loss, investment, and expense variability. However, one should remember that not just surplus, but all income statement and balance sheet items are available.

What is the Insurance Corporation of British Columbia?

The Insurance Corporation of British Columbia (ICBC) is a virtually monopolistic carrier of automobile insurance in the Canadian province of British Columbia. Current premium volume is around two billion Canadian dollars annually. It is a Crown corporation that pays no taxes (other than premium taxes) or dividends. It is mandated to provide coverages at cost. It is not subject to regulatory scrutiny in the same manner as private insurance corporations.

At the same time, it has a need for surplus. Although in principle a deficit could be made up by increased taxation, this is politically unpalatable. On the other hand, a large surplus would not only be a tempting target for other uses, but could lead to charges that the rates had been too high, which is also undesirable. Thus, ICBC needed a way of quantifying a defensible surplus.

ICBC represents a particularly simple case for modeling. The market share is essentially constant, so no supply-demand curves or market elasticities need be created. There are no income taxes, so the US. complications of tax status, alternative minimum tax, and balancing between taxable and non-taxable investments need not be considered. There are no dividends or stockholders, so the usual management decisions with respect to investment analysts are irrelevant. The investments are conservative, mostly in British Columbia and Canadian government bonds. Since the writings are confined to one province, the British Columbia consumer price index provides the relevant inflation index.

However, even with all these simplifications, this turned out to be a major project requiring considerable data preparation by and consultation with the actuarial department of ICBC over a period of about a year and a half. Their chief actuary, Dave Lalonde, was instrumental in the creation of the model and in setting many of the key actuarial assumptions.

<u>What are ICBC's uses for the model?</u>

The original and major application for ICBC is to surplus under various management policies. For any given policy, the model gives the distribution of surplus at future times. Management's appetite for risk was stated as "What are acceptable probabilities that the surplus will be negative at the end of one year/five years?" Given those numbers, the first question was what initial surplus was necessary to obtain them with rate increases following projected claim costs. The second question was how the numbers changed if a previously announced set of future rate increases were followed independent of claim costs.

The third question was what impact various forms of reinsurance would have: *a priori,* reinsurance should decrease the negative variability of the results, thus increasing the probability of positive surplus, while at the same time having an average cost, which would reduce the mean surplus growth.

Other possible questions that may be investigated include the effects of expense savings and of re-aligning the investment strategy to accept more risk and more profit.

Annual Model

The actual model used for ICBC was a quarterly model. In the interests of clarity this discussion of an annual model is presented first as it contains all the key concepts. The modifications required for quarterly work along with some of the data are presented later. For any of these models, a salient requirement is parsimony: the individual equations should be simple and intuitive so that the overall model with its complex behavior is believable. There should also be as few equations as possible. A corollary of this is that there is no point in trying to model a detail whose behavior is masked by the random noise created by other terms.

Overview

This model is based[i] primarily on the 1989 work of Pentikainen *et al.*[2] "Insurance Solvency and Financial Strength" and on the 1990 Daykin and Hey[3] paper "Managing Uncertainty in a General Insurance Company". Losses, premiums, expenses, and investments are explicitly modeled. Both of these papers assume that the high end of the loss distribution is well-behaved, if necessary through appropriate reinsurance. One of the major tasks here was to obviate that assumption by treating the reinsurance and large claims explicitly, using parametrized distributions derived from the data.

For each time period (year, in an annual model), the evolution is modeled at three points. The first point is just after the start of the period when all of the last period data is known. At this point, projections of loss characteristics and market size for current underwriting are created. When combined with management-determined loads, the rates are generated. Premium dollars and market share of claims are returned by the market. Reinsurance premium is paid. The premiums and any investment yields are then invested in various assets, again determined by management goals. If asset allocations are to be re-balanced, this is when it happens.

The second point is at the middle of the period, when all losses and expenses are assumed to happen. Explicit realizations of all the stochastic variables are taken: first inflation, then loss payments, expenses, and investment yields and asset growth. If it is appropriate to re-evaluate any fundamental econometric parameters (such as the long-term inflation rate), this is the point. If assets need to be sold to raise additional cash, there is a liquidity penalty. Reinsurance recoveries are taken to be immediate, although a delay could be introduced.

Lastly, just before the end of the period the discounted and undiscounted reserves are calculated. Results of investment yields and asset growth are evaluated at this time. In the more general case, taxes and dividends would be paid. All income statement and balance sheet items are created.

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[i] Subsequent to the writing of this paper, the book "Practical Risk Theory for Actuaries" by Daykin, Pentikainen, and Pesonen was published by Chapman and Hall (1994). This book contains an updating and summarization of all the early papers, and is strongly recommended.

The above is repeated for each period out to the chosen time horizon. It is typically suggested that five years is the most that can reasonably be chosen because of possible divergence of the extrapolation from current data. The whole process comprises one simulation. In order to have confidence in information above the 1% level, it is not unreasonable to use 20,000 to 100,000 simulations.

Inflation

The key external economic variable that connects to all the other variables through econometric equations is the inflation. In the ICBC case, the British Columbia consumer price index was used. The form suggested for the econometric equation for the inflation at time "t" is

infl[t] = avg_infl + regress*(infl[t-1] - avg_infl) + uncertainty

Long-term average inflation is **avg_infl**. This can be allowed to be a function of time in order to allow for updating of this parameter. However, it will be slowly changing since by definition it requires a number of years of data for evaluation. The autoregressive coefficient for inflation is **regress**, and **uncertainty** is a random term reflecting of the variability of the inflation.

If the term **regress** were zero, then inflation would be essentially a random walk about its long-term average. However, it is known that inflation is "sticky": when it is high at one time it tends to remain high and conversely for low values. The autoregressive term reflects this. The value of the coefficient should be below one for stability. The value of **regress** found on annual Canadian data is 0.69. The expected value for the inflation going forward from some fixed time approaches the average geometrically, although the actual values will only exhibit this behavior when the uncertainty is small compared to the other terms. In general, for the inflation and all other variables, the behavior going forward from t=0 will depend upon the parameters, on the immediately preceding (historic) values, and on the realizations of the uncertainties.

The values of the coefficients emerge from regression analysis on the data. In fact, the model structure - the number of lags involved and indeed the form of the equation itself - is given by appropriate analyses. However, it is to be emphasized again that simplicity and ease of interpretation are virtues not lightly to be dismissed in the choice of a model. It happens in the present case that the above form is not only simple, but also works well on the actual data.

Claims

The first problem is to define the claim lines. In the model these are the fundamental pricing units as well as the exposure and reserving bases. For

ICBC eight claim lines were used: Bodily Injury, Property Damage, Comprehensive, Death Benefits, Loss of Use and Collision, Medical Rehabilitation, Special Coverages, and Weekly Benefits. The information needed by the model is the frequencies, the severity distributions, and the runoff patterns for each line. These are needed both to run the histories forward from t=0 and to get the parameters for the distributions and econometric equations.

The underwriting for one period consists of receiving, by line, premium and an exposure (market share) in the form of an ultimate number of claims. This version of the model takes the claims to have one payment and to be closed when paid. One could possibly treat partial payments as separate claims. The number of claims closed from each underwriting period during each calendar period is given by a Poisson draw on the number expected; the latter is determined as the exposure times the appropriate element of the runoff pattern times a stochastic structure function[ii].

For each claim closed in each line, a random draw is made from a severity distribution that is inflating with calendar time. The claim inflation by line is the overall inflation plus a line dependent claim excess and a stochastic term. The claim excess is the average amount by which the claim inflation exceeds the overall inflation. As usual, the parameter values come from the actual data. Thus, during each calendar period, for each line, each open underwriting period contributes a stochastic number of claims which have severities drawn from a distribution whose mean inflates in a stochastic manner with payment time.

It is perhaps worthy of mention why payment date inflation is relevant. When, for example, auto parts are bought, it is the current price *at the time of purchase* which governs, independent of how much earlier the actual accident occurred. This economically very reasonable statement is equivalent to payment quarter inflation. If inflation is constant- which the data on the Exhibit 1 indicates is certainly not true - then payment quarter inflation is equivalent to accident quarter inflation.

This procedure implies that the distribution for claims from a given underwriting period that pay late has the same shape (but different mean) as that for claims that pay early. This is probably not true, but perhaps more true in automobile than in, say, general liability. Given sufficient data and motivation, one could construct distributions with a severity shape dependent upon lag and incorporate them into this type of model.

[ii] The latter term is used to account for some correlation and/or parameter variation, which was otherwise not treated in this early model.

The principal computational difficulty in the model is the creation of an algorithm to evaluate all the claim payments in less than real time[iii]. A workable solution is to separate the Compound Poisson (for a given severity distribution and number of claims) into two groups based on a cutoff value. The severity distribution then becomes two distributions: one limited by the cutoff and one above the cutoff. The compound Poisson process becomes a sum of two compound Poisson processes, with the expected number of claims in each part proportional to the probabilities above and below the cutoff value in the original severity distribution.[4]

The claims above the cutoff are simulated individually. This makes it possible to model excess reinsurance coverage explicitly, with or without aggregate deductibles and/or aggregate limits. In principle, any conceivable reinsurance arrangement can be modeled. The claims below the cutoff are evaluated by calculating the first three moments of the aggregate distribution and constructing the corresponding three-parameter gamma distribution. The stochastic value of the aggregate is then chosen as a random gamma deviate.

The lower the cutoff value is taken the more accurate this procedure becomes but also the more claims have to be simulated individually. In order to check out different possibilities, a test bed was constructed using some five billion simulations of individual claims. This gave us an "exact" cumulative distribution function against which individual approximations were then tested. We found that for our distributions, reducing the cutoff to a point where the skewness of the aggregate was less than 0.3 produced an acceptable compromise between accuracy and speed[iv].

Expenses

Expenses are taken to be those proportional to premium, those proportional to the number of claims, those proportional to size of claims, and those independent of premium and claims. In the ICBC case, ALE was included with loss, and the premium-based expenses were the taxes and agents' commissions. All other expenses were combined and projected from past history including a stochastic term. Management objectives for future expense reduction were included in the projection. Fixed expenses were allocated to claim line by a fixed ratio.

Investments

[iii] That is, in order to be useful a model with a projection horizon of five years should not take five years to run. In fact, if the runs are longer than a few days the model becomes very difficult to use.

[iv] In particular, the error in the approximation was less than 0.5% everywhere in the distribution.

ICBC invests principally in provincial and Canadian government bonds, with some commercial investment-grade bonds and a tiny stock portfolio. The assets were taken as three kinds: cash, bonds, and stocks. Except for the stocks, the investment yield and asset growth indices used track well with the overall inflation. Investment transaction costs were negligible compared to the variability of returns and were ignored. It was also assumed that all instruments were available at any time and that there was no liquidity problem, as outlined below.

Investment performance is split up into two components - cash return (dividends) and asset growth. For both of these, parsimonious equations are defined that link the current value to past investment returns and to the current and past realizations of inflation. The Wilkie[5] approach was used, although further work could involve the use of dynamic investment models.

The investment strategy was taken as keeping a fixed ratio[6] of different classes of assets. No attention was paid to asset-liability matching. If this were desired, then the investments would need to be broken out more finely, including the durations.

If, during a particular realization, the dollar outflow from losses and expenses exceeds the cash available, then other assets are converted to cash to cover the shortfall. No liquidity penalty except for the loss of interest is incurred - that is, there is no loss of value assumed for a forced sale of assets.

Reserves

There are many methods for calculating reserves. This model uses the expected value of the discounted and undiscounted cash flows from the exposures, which is in line with using the cash flows as our primary variables. As mentioned earlier, over time the expected inflation geometrically approaches the long-term average. It is this series of values that is used in estimating the values of the future payments that make up the reserves.

Clearly, the model can be run with no future premium income or exposures, and no assets other than those which make up the reserves. In that case, the distribution of the surplus at the end of the runoff is the distribution of the reserve adequacy. This distribution includes the variability due to losses, investments, and expenses. The model can be run to ascertain how much additional surplus is necessary to achieve any desired probability of non-negative final surplus. Additionally, other reserving methodologies could be explored[7]

The discounted reserves are calculated based on the expected present value with no risk loading. The discount rate is tied to the expected future yield for cash, although any type of method could be used. Perhaps a better alternative would be to tie the yield into the current investment portfolio.

Other Income/Payments

The model also includes income and payments difficult to associate with losses, investments, or expenses. In particular, there are revenues from late fees and penalties. There are also bulk payments relating to the use of ambulance services and hospitals. Both of these cash flows were modeled as linear projections with uncertainty. Both of these were relatively small compared to the large components, but ICBC felt they warranted specific attention.

Quarterly Model

The previous discussion was based on an annual model, as were the European models referenced earlier. ICBC required a more detailed model, one which would identify key seasonal differences throughout the year. It was therefore decided to construct a quarterly model.

In analyzing the data there were clear seasonalities, and therefore even a parsimonious model had to be more complex than described previously. A most interesting result arose in that the severity data exhibited not only accident quarter seasonality, as would be expected, but also strong payment quarter seasonality. That is, regardless of when the claim occurred, there was seasonality according to when it was paid. This apparent mystery was resolved by the ICBC actuary who remarked on the effect of summer vacations taken by people in the legal system.

The description of the quarterly model will parallel the annual. The general commentary is the same and will not be repeated, but only the specifics to the actual equations used.

Inflation

Exhibit 1 shows the British Columbia quarterly CPI change. The average of this data over the period 1968-1992 was used for long-term quarterly mean inflation. For the equation, intuition suggests that there should be a similar form to the annual model: average inflation plus some autoregression. In particular, correlation with the preceding quarter and the preceding year are appealing.

Piecewise linear regression was used to examine correlations between the lags; auspiciously enough, the statistically favored equation was the one suggested by intuition, with lag one and lag four. Exhibit 2 shows the standardized residuals, along with the actual values of the coefficients, and Exhibit 3 shows the check of residuals for Normality. Similar work was prepared for all the appropriate econometric equations.

<u>Claims</u>

The claims data has three pieces: the claim frequencies, the shape of the claim severity distributions, and the runoff and payment quarter severities. The frequencies were obtained by projecting the total number of claim counts to ultimate after extracting the accident quarter seasonality in the data. Fortunately, for this book of business the IBNR counts die out rapidly and we had mostly reliable data for ten years.

The severity distributions by line were fit by using a parametrized distribution- often lognormal- above some demarcation point in the data, and using the empirical moments below. The data we had were binned (numbers of losses in size classes rather than individual losses), so that in order to get the first three moments from the low-end empirical distribution we had to approximate the claims as all at the class midpoint. The error introduced by this becomes more severe as the demarcation point gets larger, so there is a compromise between the goodness of the fit (which typically gets worse as the demarcation point lowers) and the error from the empirical data. It is also necessary to be aware of the necessity of keeping the high-end tail correct, as this is where a substantial fraction of the dollars are. This latter consideration is perhaps the most significant, especially when reinsurance is involved.

As a technical note, although the usual tendency is to use a Chi-squared test on binned data, such tests are notoriously sensitive to the tails and should not be used when the expected frequency is small.[8] In order to create bins with enough points in them to be usable, much of the top end would be collapsed into a single bin, losing a great deal of information. A way around this difficulty is to use a maximum likelihood solution, evaluating the differences of the parametrized CDF at the end points of the bins. This also allows for an infinite end point, such as values listed only as "greater than $5,000,000." The price paid is the technical difficulty of actually doing the calculation and minimizing the negative log-likelihood.

The third piece, for the runoff and payment quarter severities, is one of the more interesting calculations. Each line followed the same analysis: First an accident quarter triangle of incremental quarterly payments was converted into a partial severity triangle by dividing by the appropriate ultimate claim count. Each payment cell represents the product of (1) an accident quarter seasonality, (2) a lag quarter runoff, and (3) a payment quarter mean severity. A least-squares fit to the data was done numerically, which involved many variables. For example, in Property Damage the fit to the data had three accident quarter variables (the four seasonalities are constrained), 28 development quarter variables, and 39 payment quarter variables (there was a data glitch), for a 70-variable minimization! Fortunately, the usual accident quarter runoff gives a good starting point. It was found that whereas the simplex minimization was unusably slow, a less robust but more sophisticated technique based on Powell's conjugate direction set method worked quite well.[9]

The result of the minimization is the accident quarter seasonalities, the runoff factors, and the payment quarter severities. The surprising result alluded to earlier was the presence of seasonality in the payment quarters. When this was removed, the comparison to the historical inflation was made to get the claim excess. Typically, the size of the uncertainty was large compared to the average value of the excess.

Reversing the process for the simulation, the econometric equations and their realizations are on the de-seasonalized data. The consequent realization of the mean severity results from the blending of inflation, accident quarter seasonality, and payment quarter seasonality. Mean frequency is similarly a combination of trend, runoff pattern and accident quarter seasonality.

Premiums

Since the policies were assumed to be annual, as most actually are, there is a significant unearned premium reserve as well as the spread of counts to accident quarter. Although there is a notable fraction of the policies that incept on March 1, the model took uniform premium writings.

Expenses and Investments

The expenses were available only on an annual basis, so they were assumed uniform over the quarters. On the investment side, the quarterly econometric equations were not as clean and satisfying as for inflation. They did give statistically significant coefficients, which tended to be lags 1 and 3 or 4 on the variable, and the current and lag 1 values of the inflation.

Results

A sanitized version of the principal result can be seen on Exhibit 4, which shows the mean level and the 1% and 5% probability levels for surplus at different time horizons beginning at a given fixed initial value. For heuristic reasons, this same exhibit shows in light tones a few of the simulations that underlie the statistical values. The whole calculation was repeated for several different starting points, and comparison of the results provided an answer to the question "What initial value of surplus is necessary in order that the probability of negative surplus at the end of one year is less than 1%?"

Implicit in the result of Exhibit 4 was that the rates were set to exactly cover costs. Thus, as claim costs rose in some realizations, so did the rates. Another question of interest was what would happen if a particular pre-determined set of rates were used. Exhibit 5 shows the values corresponding to Exhibit 4, but generated by this imposed rating structure. Even though the mean values are not too dramatically affected, the risk-significant levels are shifted considerably. For state regulators who want to fix rates or limit rate increases independent of costs, this kind of exhibit should provide food for thought on the increased risk of insolvency.

The third question of interest was the effect of reinsurance on the probability spread. Exhibit 6 shows the effect of reinsurance over a fixed $100,000 excess. The mean line is dropped due to the (on average) cost of reinsurance, but the low percentile levels are significantly raised because the down-side of the loss is considerably restricted. This is equivalent to requiring less initial surplus for a given risk appetite.

Conclusion

The results and further use of the model allow ICBC management to know the risk being assumed under different scenarios and their trade-offs. Prepared with this information and knowledge of external factors, ICBC can take an optimum stance for its risk appetite.

Their optimum surplus value is a dynamic number, depending on risk appetite, rate constraints, investment profiles, developing trends in losses, expenses, and investments, and reinsurance arrangements. In order that it best reflect current reality, the model should be retested in its assumptions and re-parametrized on an annual basis, as well as having the input data updated quarterly. Each quarter, out of all the possible simulation paths implicit in the model, one will be selected by actuality. The management reaction to the situation may require a change in the assumptions of the model, as some of them directly reflect management decisions.

Since the model is a complete but simple version of all the cash flows of ICBC, it can be used to analyze many other questions besides the appropriate value of surplus. For example, it can be used as an "early warning system", in that it highlights potential areas that are likely to have large swings and suggests careful monitoring of these areas. Control engineers often define a band of variability inside of which a process is considered in control, and outside of which action needs to be taken to restore balance. Whereas the claims department is (properly) focused on its payouts, and the investment department on its returns, senior management needs to know each of these and the overall result. It needs to have control bands at all levels.

The model can also be used as a planning tool to explore the consequences of other decisions, such as changing the target investment mix or introducing a new product line. In the latter case, however, one must be extremely careful with the actuarial assumptions involved for the line if they are not supported by comparable data from elsewhere.

For the actuary, the important conclusion is that a model of this type can be built, and it provides a useful tool for management planning and control. For any individual company, the specifics of the claim lines, investments, etc. that are considered will depend upon the company's current environment and plans for the future. One of the areas left untouched here is the whole question of what happens in a competitive environment as rates move. Classically, this means the creation of the supply and demand curves for the company's products. The forthcoming book by Pentikainen[10] has a treatment of this and other approaches to competition.

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[1]As of this writing (4/93), the final version has not emerged. For a preliminary version, see "Property-Casualty Risk-Based Capital Requirement - A Conceptual Framework" in the Spring 1992 edition of the Casualty Actuarial Society Forum.

[2]T. Pentikainen, H. Bonsdorff, M. Pesonen, J. Rantala, M. Ruchonon, Insurance Solvency and Financial Strength, Finnish Insurance Training and Publishing Company Ltd., Helsinki, 1989

[3]C.D. Daykin and G.B. Hey, "Managing Uncertainty in a General Insurance Company", in Journal of the Institute of Actuaries, Vol. 117, Part II, pp. 173ff.

[4] Newton L. Bowers, Hans U. Gerber, James C. Hickman, Donald A. Jones and Cecil J. Nesbitt, Risk Theory, Section 11.4.

[5]A.D. Wilkie, "A stochastic investment model for actuarial use", in Transactions of the Faculty of Actuaries, #39, 1986, pp.341-373.

[6] This is the approach that has been taken by ICBC. A discussion of rebalancing is given in A.J. Wise, "The Investment Return from a Constantly Rebalanced Asset Mix" in the 3rd AFIR Colloquium, 1993 pp. 349-358.

[7] See for example T. Pentikainen and J. Rantala, "A simulation Procedure for Comparing Different Claims Reserving Methods" in the Astin Bulletin, Volume 22, No. 2 (November 1992) p. 191.

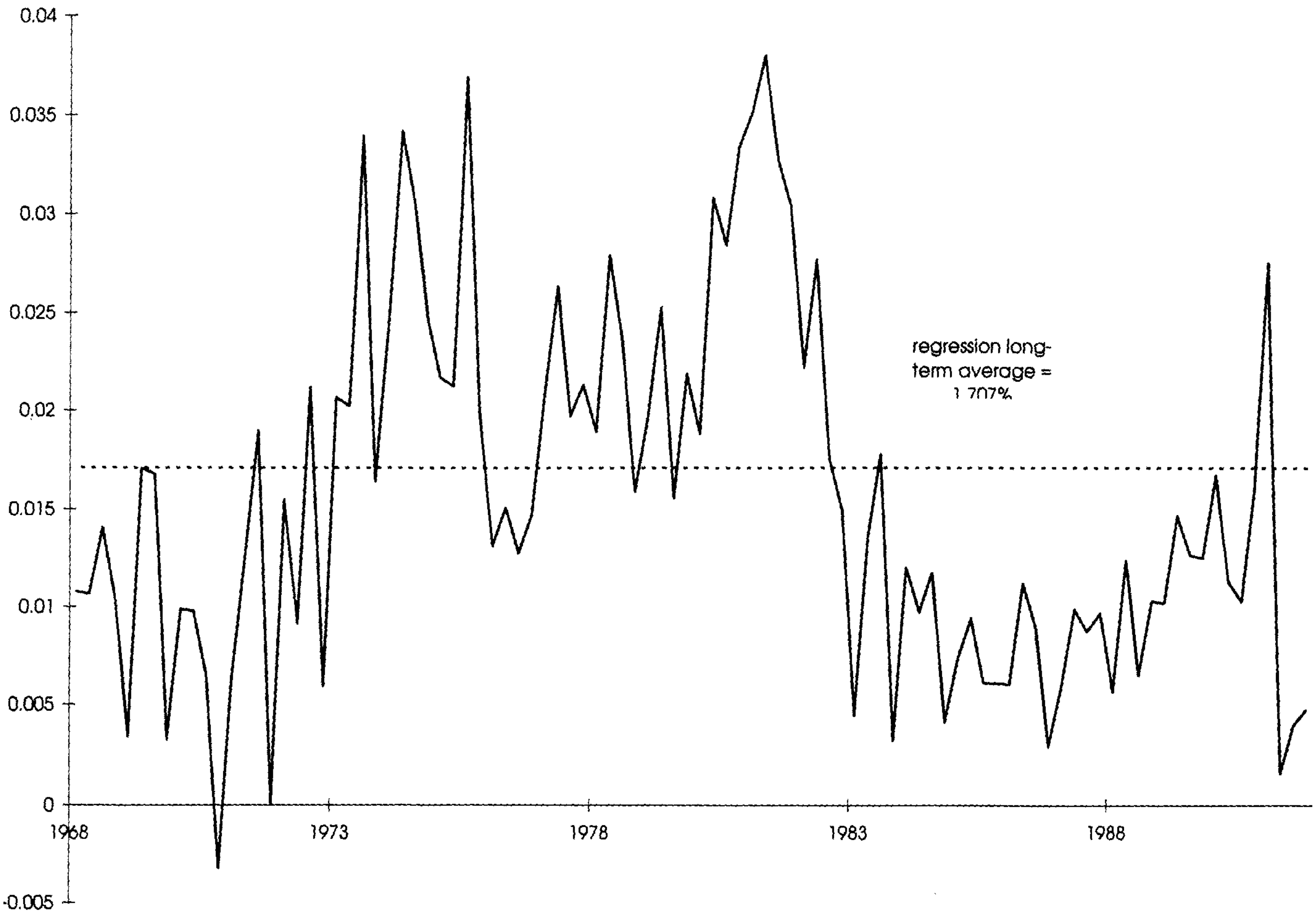
[8] Robert V. Hogg, Allen T. Craig, Introduction to Mathematical Statistics, Macmillan Publishing Co., Inc., 1990, pages 269ff.

[9] See for example Press, Flannery, Teukolsky and Vetterling Numerical recipes in C - The Art of Scientific Computing, Cambridge University Press, 1988 p. 309ff.

[10] T. Pentikainen, private communication at the 3rd AFIR International Colloquium.

Exhibit 1

British Columbia Composite Quarterly CPI Change



B.C. CPI Change Standardized Residuals from regression

long-term mean = 1.707%; autoregressive Lag 1 = 41.0% and Lag 4 = 40.9%; deviation = 0.637%

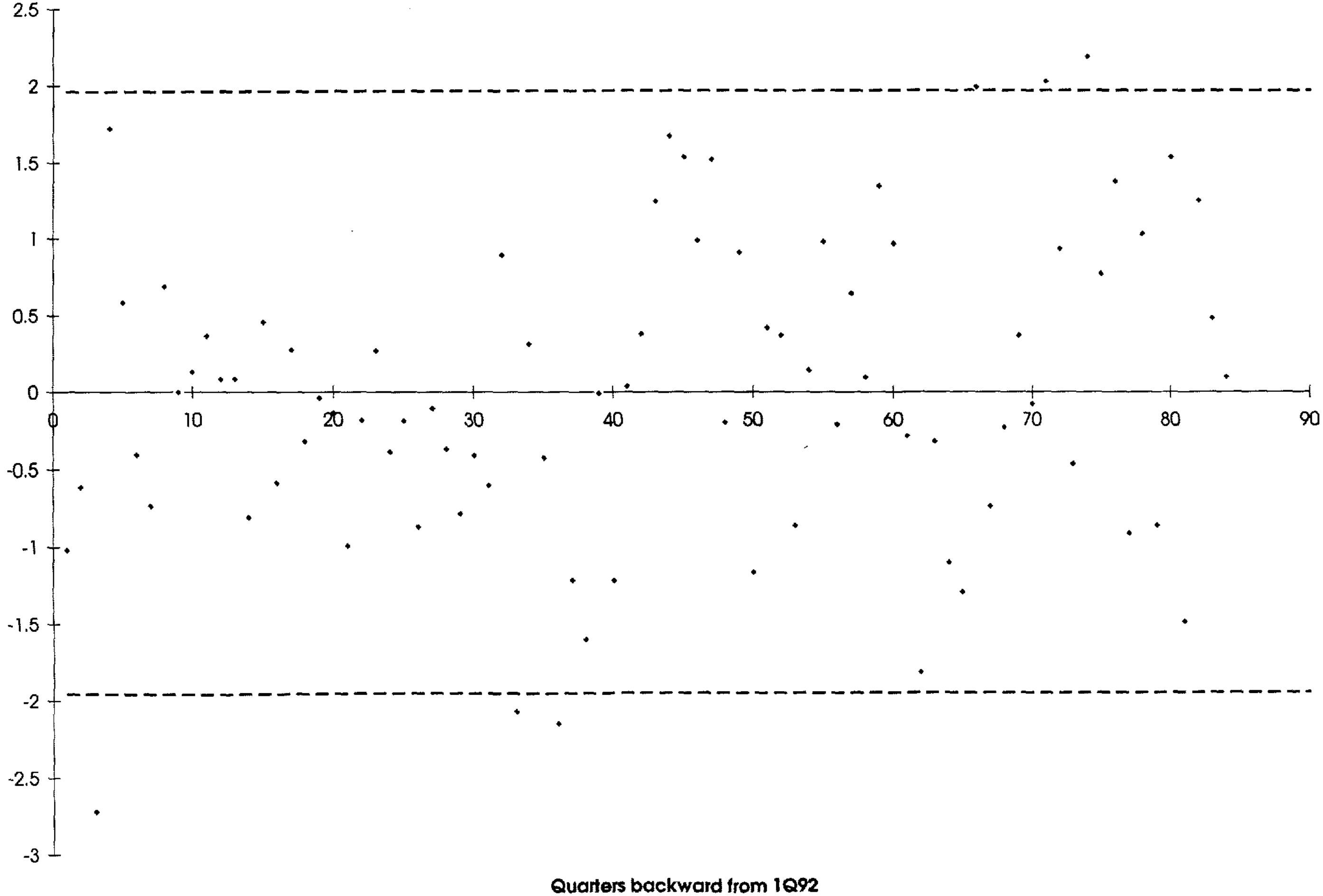


Exhibit 3

Regression on B.C. CPI Change
Check of residuals for normality

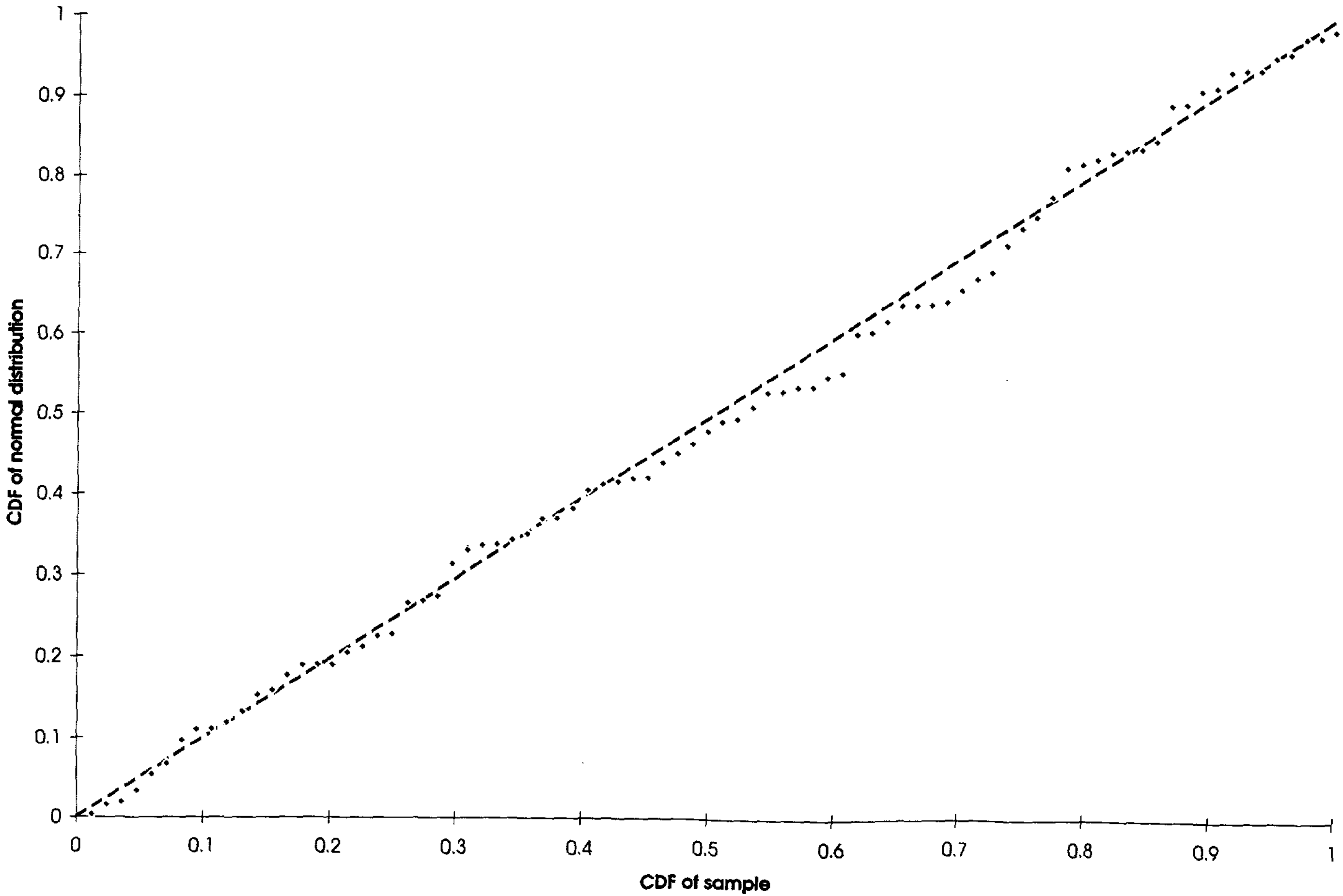


Exhibit 4

Surplus by Quarter
for rates following costs

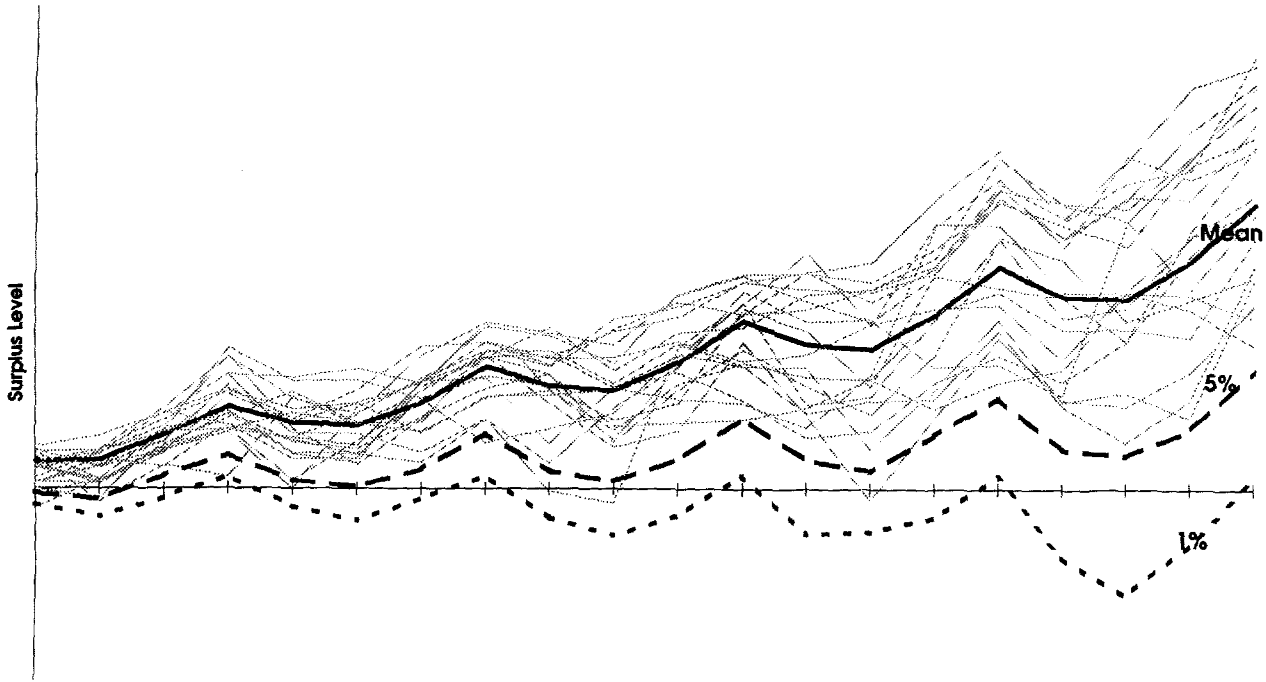


Exhibit 5

Surplus by Quarter
for fixed rates

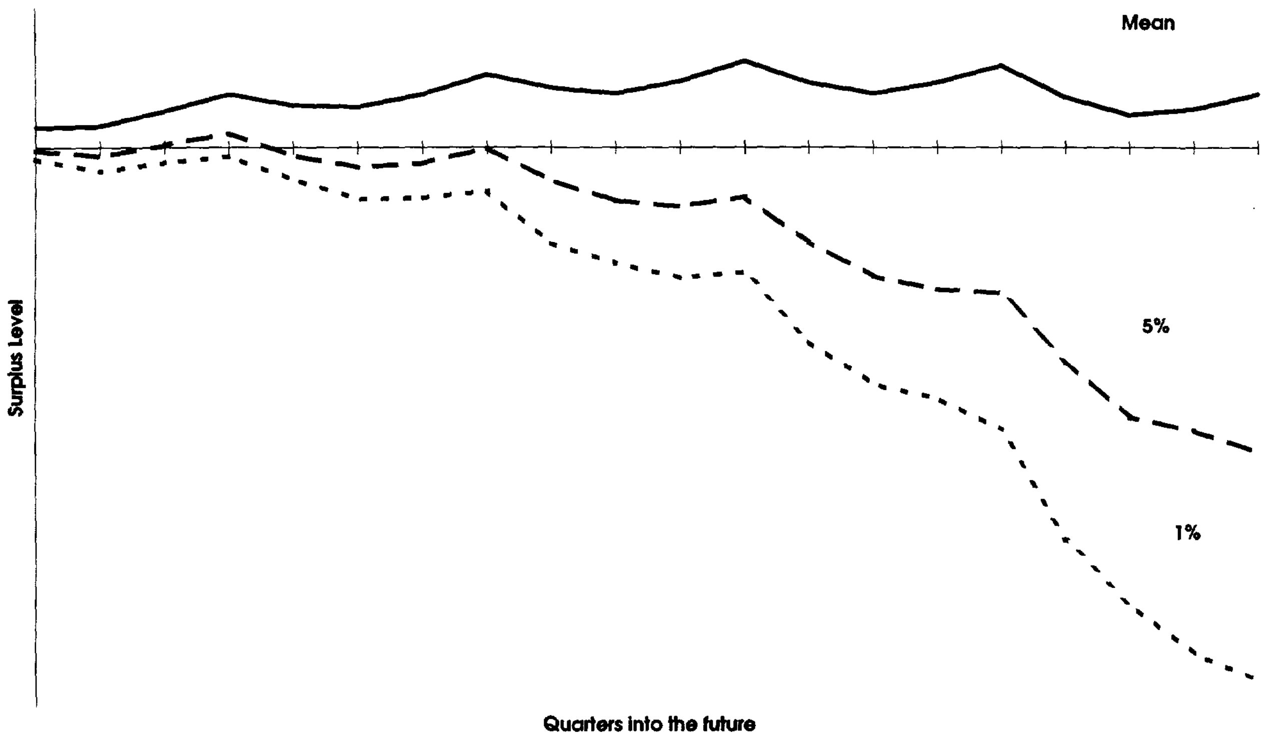
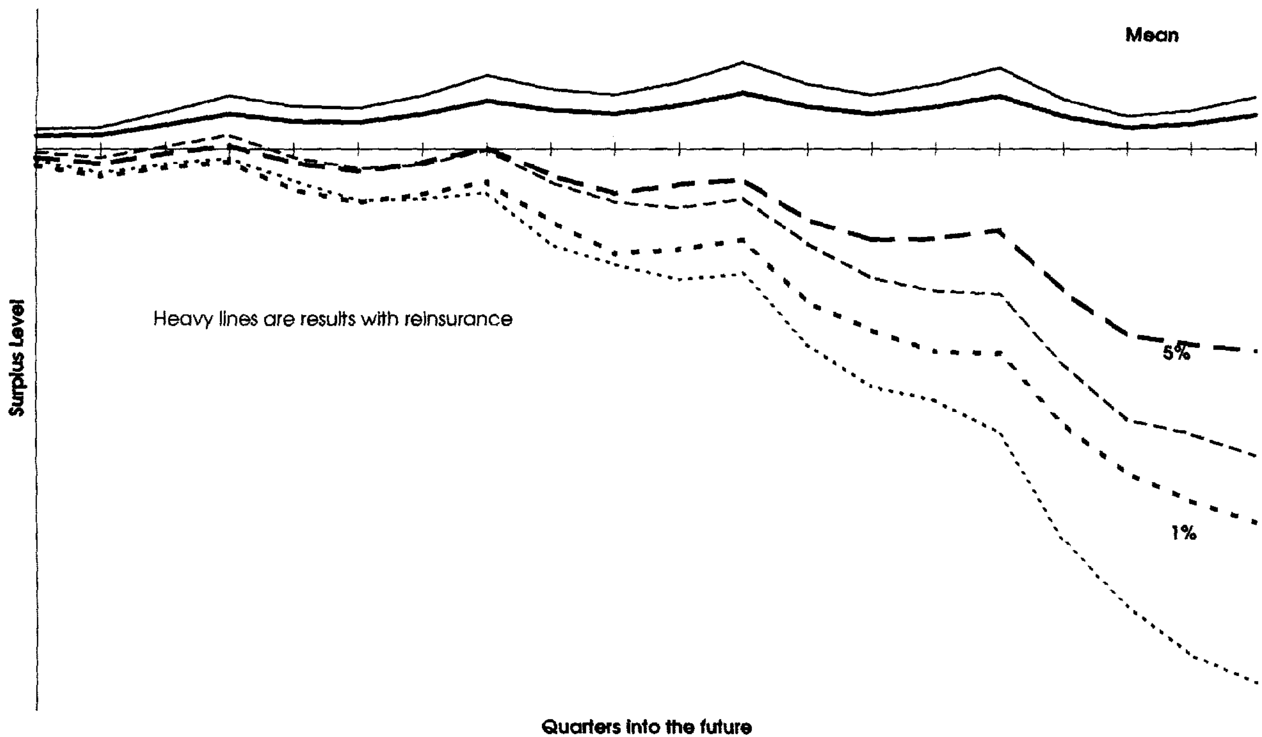


Exhibit 6

Surplus by Quarter
for fixed rates with and without reinsurance



Concepts of the Financial Actuary
by Stephen T. Morgan, ACAS

Concepts of the Financial Actuary

by Steve Morgan

I used to think that I began learning the concepts during 1993 when I began to calculate durations for my employer. A leveraged buyout had just been completed and the new owners wanted us to monitor cash flows more closely. From then till now, much of my time has been spent learning subjects and doing work related to becoming, what I term, a *financial actuary*. While I still consider myself, in many respects, a neophyte, these concepts are starting to come together in terms of their interrelations. It once appeared the various concepts were somewhat disparate. As the realizations occur, there is a better understanding of the concepts.

This is a work in progress. The purpose of this paper is to discuss these concepts and their interrelations. The paper is intentionally not technical for two reasons. One, I do not consider myself to be technical. Second, in my opinion, concepts are best communicated with relatively simple examples. There was a paper written by Charles Hewitt that used to be on Part 9 that had many pages of formulae accompanied by an ongoing example using a die and spinner. I spent a lot of time with the example and a little with the formulae. I passed the exam.

It's Only Cash You Know But I Like It-apologies to the Rolling Stones

I now know I really began learning the concepts in late 1971. When I was a baby actuary, one of my first assignments was analyzing occurrence-based medical malpractice insurance in California. Abandonment. Patients falling off the operating table. Patients not liking the way the doctor treated them. Patients died. How is it best analyzed: policy year, accident year, report year, created year or the ever famous calendar year? At first, these concepts were mind boggling. However, the concepts started to make some sense after I saw a quotation by L. H. Longley-Cook that referred to cutting a cake so many times that you were left with a pile of crumbs. Those concepts were just ways to slice the cake. **Concept 1: Whatever the data (whether it is coupon income on bonds or paid losses or loss reserves, for example), get it organized into groups that make sense for analytical purposes.**

So much work was necessary in analyzing the reserves and paid losses (yes, occurrence malpractice losses eventually were paid) that rarely, if ever, was there a thought about where the cash came from to pay those losses. There was enough to do in estimating what the losses would ultimately cost without having to worry about how they would get paid. Pouring over paid and incurred triangles, I learned how to detect patterns and I learned the Law of Small Numbers: "it is harder to detect patterns in small numbers." I learned how to look at many averages for the factors. It seemed I was tracking the average number of averages I was using to project ultimates. Talk about a nebulous concept!

Even better, I never got a chance to look at the piles of cash being held in reserves. Talk about another nebulous concept! Where were those reserves kept, in the president's office, in the basement, in a bank or in bonds, whatever they are? **Concept 2: Investment income or paid losses are just cash; it is your job to predict how the payments will be made.**

Ultimates. This is the total amount estimated to be paid for all losses that belong to a defined group (accident year, for example). Paids, unlike reserves, are facts, which always appeared to me to act like squeezing a flaccid balloon. Reserves are a continually moving target. Why is it that when reserves are added to the paids (incurreds), a better estimate of ultimates generally results for casualty lines than an estimate by using paids? **Concept 3: Reserves are just estimates of future cash streams.**

IBNR (Incurred But Not Reported), (Incurred But Not Reserved) and my favorite (Interesting But Not Relevant). In making ultimate projections, this is what is left after paid and case reserves, at a particular point in time, are subtracted. IBNR gets paid too, just later. IBNR is for those losses that have already happened but the client was not considerate enough to let you know about. So, they are losses that have happened, but you do not know about them. By the way, IBNR does include amounts for losses you do know about but do not know what they ultimately will cost. Yet another nebulous concept.

Paids. These will be referred to numerous times in this paper. It is not possible to overestimate their importance in learning the concepts of the financial actuary. As indicated above, almost everything in an insurance company (reinsurance company too, but much later) will become cash. You just have to be patient. The first ever Workers' Compensation claim in New York was still open as of a few years ago. The first report of a hurricane loss to a reinsurer may come up to two years after the storm. I have seen losses from a hurricane still open after ten years, okay. **Concept 4: Almost everything on a balance sheet will eventually become cash.**

Sympathy for the Regulator - again apologies

I learned about the annual statement at an early age. I am dating myself but I can remember Schedule P on a policy-year basis. It is so much better now on an accident-year basis (chortle, chortle). The first page I remember seeing is Page 4, Underwriting and Investment Exhibit; Statement of Income; and Gains and (Losses) in Surplus. Earn the premiums, incur the losses and expenses, add in investment income and other income, mix with taxes and net income is the result. It is not quite cash and it is not quite reserves. The net income is added in with other items and you have the change in surplus for the year. Again, we have a mix of paids and reserves. **Concept 5: When performing a financial analysis, segregate the cash items from the non-cash items. Then convert the non-cash items and perform the analysis.**

On page 3, Liabilities, Surplus and Other Funds are shown. Here we see another mix of paids and reserves. Very confusing. Now on to Page 2, Assets, and the equation to end all equations: Assets = Liabilities + Surplus. Very, very confusing. Assets and surplus are "helps" and liabilities are "hurts"; yet they equal each other. Since this equation is apparently true, part of the assets must equal liabilities and the remaining portion must equal surplus. Carried to its logical conclusion, this means the portion of assets that supports surplus can be held in assets that will not necessarily be needed soon. This means they can be held in longer term assets (like bonds). These assets will only need to be used if the assets allocated to the liabilities are not sufficient. That is the textbook definition of surplus. **Concept 6: Align a portion of assets to support the ability to pay liabilities and align the rest to surplus. When properly aligned, better financial management may take place.**

The rest of the assets support the liabilities. That is where the cash is to pay the losses. When liabilities come due, just liquidate the assets to pay them. It is just that simple. I think not. Statutory accounting is really run-off accounting. This is fine if your company is going out of business fairly soon. The way accounting is done, you will sell assets to pay losses. How much sense does it make to sell off 30 year bonds to pay losses when, as an ongoing company, you have a continual new supply of cash coming in the door. Why not use the cash from written premiums instead? It is not earning investment income. Only consider selling the assets if the rate being earned on them is less than the rate being paid on new investments or if cash inflows are insufficient to pay losses (a catastrophe, for example). Consequently, while **the** equation holds, its application in the real world is not as simple as it first appears. **Concept 7: Run-off accounting is generally not a fair**

representation of how a company operates. Realize that cash is continually coming in while it is continually going out.

You may have heard about asset liability management (ALM) or asset liability matching or cash flow matching (discussed below). The purpose is to track how assets and liabilities work together. Remember, the interrelations are not as straightforward as they may first appear. Statutory accounting rarely reflects how an ongoing company conducts its business. Remember, when you match assets with liabilities, different assets than you are matching will probably be used to actually pay the liabilities.

Duration (discussed below) is a run-off concept. It infers that the assets currently held are being related to the liabilities on the books. When the liabilities are finally paid, it is a different ball game. Other assets will most likely be used for cash.

Actuaries' Big Helpers [1]

Now, when looking back on those days when I was an Actuarial Trainee (oxymoronic, maybe), it is fairly easy to say what is important. You know what is said about hindsight, 20-20% of the time. This section's purpose is to talk about the papers and experiences that have contributed to developing the concepts of the financial actuary. In no particular order, they are:

[1] **Author's Note**: Absolutely no slight is intended to any author of any paper I have read that is not included here. There have been too many papers read to list each one. Each had its own particular impact. Necessarily, the list here is somewhat brief.

- **Individual Risk Rating** The Hartford put me in a department doing this work until I passed the first two exams. At the time, I was upset because I was eager to do actuarial work and they were sticking me in a rating department. Little did I realize that experience rating and retrospective rating are the basis for all actuarial work. During this time, experience modifications, insurance charges (Table M) and Plan D basics were calculated by hand. I wish I had a nickel for every one I calculated. The knowledge learned here gave me an appreciation for credibility, patterns and the ability to pass Part 9.

- **Theory of Interest** This was on Part 3 when I took it. Never **discount** the importance of this area for the Concepts. I refined my knowledge of compound interest. I learned about bonds and how they are priced.

  As I look back on it, I do not recall any emphasis about bonds being a stream of cash flows. Also, I did not make a connection that **(Concept 8) the stream of coupons and the return of principal is equivalent to a payout pattern on losses**. Finally, **(Concept 9) a reason that values are discounted is to put them on a comparable economic basis with other values**. Both are important concepts.

- **"Trend and Loss Development Factors"** This is a paper written by Charles F. Cook (*PCAS* LVII, 1970). I read it at a time when I was first leaning about loss development factors. The "overlap" concept has never been forgotten. Knowledge of patterns is an important concept and this paper contributed greatly.

- **<u>Insurance Statutory and GAAP Accounting</u>** For years, I only saw the statutory side of accounting. I saw inconsistencies, as noted above. Why account for almost everything on a run-off basis if you are staying in business? Are assets really liquidated to pay losses? GAAP (Generally Accepted Accounting Principles) endeavors to correct some of these problems. GAAP is comparable to a function of discounting in that, under GAAP, values in insurance are put on an equal economic basis with other industries. **Concept 10: Relating assets to liabilities on an economic basis (GAAP) is an important step towards being able to match them for cash flow purposes.**

- **<u>"Duration"</u>** This is a paper written by Ronald E. Ferguson (*PCAS* LXX, 1984). When I started calculating durations, this was the first paper I read. His explanations of calculating duration, the time value of money and investment related risks (timing, credit and reinvestment) are excellent.

- **<u>"The Measurement and Management of Interest Rate Risk"</u>** This paper was presented at a special interest seminar on *Valuation Issues* in 1989 and was written by Linda A. Dembiec, James D. Pogorzelski and Vincent T. Rowland, Jr. This was the second paper I read. Like "Duration", I refer to it often. Their explanations of ALM, matching and applying ALM to property & casualty insurance are excellent.

- **"Asset/Liability Management: Beyond Interest Rate Risk"** This paper was written by William H. Panning and was published in the *Financial Analysis of Insurance Companies* 1987 *Discussion Paper Program*. His extended discussion of three posed questions related to ALM is excellent.

- **"An Investigation of Methods, Assumptions, and Risk Modeling for the Valuation of Property-Casualty Insurance Companies"** This paper was written by Robert S. Miccolis and was from the same program as Mr. Panning's. I read this when it was first released and it helped make all other related readings since that time easier to understand.

- **"Chapter 8: Investment Issues in Property-Liability Insurance" and "Chapter 9: Special Issues"** Both chapters were written by Stephen P. D'Arcy and are in the *Foundations of Casualty Actuarial Science* book. He is not only an excellent author, but I have enjoyed numerous presentations on related issues by Mr. D'Arcy.

- **"Dynamic Financial Analysis Handbook"** This was written by the DFA Handbook Committee as a subcommittee of the Valuation and Financial Analysis Committee of the CAS. It is a new publication and its purpose is to give guidance to actuaries in the financial analysis of an insurance company. It is intended to evolve over time as actuaries gain experience in DFA modeling.

➢ **Data Quality** I have often joked, "Nobody knows the data I've seen." I have worked for small companies and large companies. The lack of data quality affects them all to a certain extent. Most actuarial papers start with the tacit assumption you have perfect data with which to work. Often, however, this is not the case. It is not just a concept of the financial actuary, but rather any actuary: **Concept 11: Make sure you are familiar with the weaknesses as well as the strengths of your data.**

19th Nervous Breakdown - no apology needed.

There is a concept, which is extremely important, that affects almost all actuarial work, but is most likely not covered to any great extent in the Syllabus of Examinations. This is the oral and written communication of the results of your work The financial actuary will most likely come in contact with both insurance and non-insurance (board members or investment professionals, for example) professionals. Consequently, communication skills are extremely important. Everyone has heard the question: "If a tree falls in the woods and no one hears it, does it make a sound?" **Concept 12: If an actuary cannot relate the results of his/her work, was anything accomplished?**

I, like most actuaries, have dealt with the stereotype that actuaries are introverted, geeky and dull. Often, perception is reality. If enough people perceive you that way, it does not matter how you really are. We deal with a lot of complicated issues. Just think back to the first time you heard some of the things mentioned in this paper or the papers referenced. You were probably confused too, at least for awhile. Translation and communication are the keys to success. A part of being successful includes the following:

- Put your audience at ease. Sometimes the communication deals with topics that may be perceived as "bad news", or even worse, actuarial. Always take the time to have the audience see you as a person, not an ACTUARY. Well-placed humor, especially of the self-deprecating kind, will go a long way towards putting your audience at ease. Also, spend a few moments talking about something completely unrelated to the topic at hand. Once the initial jitters are gone, yours and theirs, start the presentation.

- Always use examples when communicating. Even actuaries enjoy examples. It gives the audience something they can relate to and follow.

- Explain why the topic is important and useful to the audience, even if not at present.

- Relate the subject of the presentation to concepts that the audience is familiar with.

- To be sure the audience is still with you, look for continual feedback during the presentation. Pose unassuming questions to see if they are.
- Take turns addressing each person, by name, if possible. If the audience is large, be sure to scan all portions of the audience.

- Take several mental breaks during the presentation. It is easier to tackle a tough subject if one gets away, if only briefly, from it. Your audience will appreciate it and be more likely to stay with you.

- Try to use words the audience knows the meaning of. If you do not, a vituperative and obstreperous reaction may result. After that, even obsequious and propitiatory behavior will not help.

- Be aware of time constraints, especially around noon and the end of the day.

- It is human nature to enjoy being appreciated or thanked. Do so for your audience.

Now, explanations of duration, matching, and stress testing.

Jumpin' Jack Cash

Insurance companies are in the business of paying losses that insureds, or reinsureds, have incurred. Setting the value of these incurreds in advance is difficult, and determining when they will be paid is not any easier. There are numerous variables that enter the process that add to the uncertainty.

As discussed earlier, assets back up the ability to pay losses. They need to be there at the appropriate time and in the appropriate amount in order to meet this responsibility. One of the main uncertainties on the asset side, even with a large fixed income portfolio, is the movement of interest rates or, maybe better, interest rate risk. In certain circumstances, the ability to pay losses could be affected.

Insolvency is the ultimate risk that you will not be able to pay all of your liabilities. Therefore, protection of solvency is paramount. Answering to your shareholders and understanding the amount of risk they are willing to take is a key ingredient.

Concept 13: The management of interest rate risk through the management of assets and liabilities will help maintain solvency. Strictly matching the assets and liabilities is not the key. It is the prudent management of the mismatch that is important. Duration and convexity are some of the tools that help in this work. More on mismatch later.

When performing a duration analysis for the first time, plan to spend an extensive amount of time with the accountants as well as reserving and pricing actuaries. Attach payout patterns to nearly every asset and liability on the balance sheet. If a holding company is involved, be sure to model any assets or liabilities (debt, for example) at that level too. Anything that is or will become cash should get a duration. Rely on investment professionals to help model investable assets. Other assets, like Agents' Balances, can be modeled by sampling.

The implied duration for economic surplus falls out of these calculations. This calculation makes a statement as to how susceptible surplus is to interest rate changes. In a perfect world, it would be a small number. I would like to go over some examples and definitions to help explain duration and convexity. Then, I will briefly relate some of the key issues. Look at Chart 1.

Column 3 displays a hypothetical overall calendar year future payout pattern. Remember similar calculations are done for bonds too. The stream of coupon income is similar to the loss payout pattern. It is derived from accident-year patterns by line of business. The calendar-year pattern is an accumulation of the projected contributions to each calendar year of the various accident-year payouts. This pattern is then applied to the reserves as of December 31, 1994 ($1,000). The payments are assumed to be made in the middle of each year. That is why the first payment is as of time .5 (six months after December 31, 1994). The payments shown in column 4 are discounted to the present using a selected discount factor (6% in this example). The discount factors are in column 5 and the present values are in column 6. You can think of the present value of the liabilities as their market value or price. Conceptually, that makes the liabilities like the market value of the bonds or their price.

Take a quick look at Chart 2, Page 1, then we will return to Chart 1. It shows the formula for calculating the price of a bond. Remember, the price of a bond is the sum of the present values of the coupon payments and the present value of the redemption of principal. The individual present values are multiplied by the appropriate time period (.5,1.0,1.5) and then summed. The resulting product sum ($3,420.19) is divided by the present value of the payments ($744.51). This value is called the Macaulay duration (4.59). In this context, it can be thought of as the weighted average time to payment. The weights are the individual present values. This is DM on Chart 2, Page 1. Then the convexity (36.12) is the weighted average time squared to payment using the same weights.

The "TOTAL" line in column 6 displays the present value of the payments ($744.51) at 6%. I have also displayed the present value when the interest rate rises 1% (in investment parlance, 100 basis points) to 7%. The present value drops to $713.53, or a decrease of 4.16%.

Instead of rerunning the present values, we can use duration and convexity to give a fairly good approximation of that decrease in present values. First, calculate the modified duration. This is the Macaulay duration of 4.59 divided by 1 plus the interest rate of 6%. The relationship is obtained by taking the first derivative of the present value function with respect to the yield divided by the present value function. Thus, it is the percentage change in present values when the interest rate changes 100 basis points. This change is equal to the modified duration of 4.33 multiplied by -.01 (the change in the interest rate). -.0433 is fairly close to the change in present values of -.0416. Again, the value of -.0433 is the estimate of the change in present value when interest rates change one percentage point. Remember that an increase in rates decreases present values. Modified duration is the first term in a Taylor series expansion.

You may not feel this is close enough. This is where convexity increases the accuracy. If I take one-half times .01 squared times the convexity, I get a value of .0018. If this is added to the estimation of the value change from looking only at duration, -.0433, we get -.0415 which is very close to the real change of -.0416. The convexity expression is derived from the second term of the Taylor series for the present value function. It is the rate of change of duration as interest rates change. It is also termed "duration drift". For background on convexity as well as duration, I refer you to "The Measurement of Interest Rate Risk" by Dembiec, Pogorzelski and Rowland, mentioned earlier.

For those of you that remember calculus and love formulas, I will give you a slightly more rigorous explanation of modified duration. Please look at Chart 2, Page 2.

It is somewhat confusing, but people often use Macaulay and modified duration interchangeably. For example, a duration of 4.5 would mean both a 4.5 year duration and present values will decrease 4.5% if interest rates rise 100 basis points (1%). Since the two values are usually close, it does not present much of a problem.

Satisfaction

You would probably say "great" if I told you the assets and liabilities were matched or that the durations are equal. What does this really mean? Look at Chart 3.

Assume we are going to pay a total of $1,000 in losses over the next 5 years and we have reasonable estimates of the individual payments. This is line one. We can buy 5 bonds that will fund each of the 5 payments. We design the first bond so that the last coupon payment ($8.04) and the return of principal ($66.96) equal the last loss payment ($75.00). The other four coupon payments are used to fund part of the other four loss payments. A second bond with a 4 year term is selected so its last coupon and principal equal $91.96 and so forth. The combined duration of the bonds equals the liability duration. Durations are combined using their respective prices as weights. This calculation makes intuitive sense. Since the individual payments or coupons are the weights and the resulting durations are equal, this implies the weights or payments are equal for each asset and each liability payment.

Why do we care about interest rate changes? I will recap what I said earlier. On the asset side, the market value of bonds decreases as interest rates increase and vice versa. Since the purpose of assets is to back up the ability to pay our liabilities, it is important to keep these values in step with each other or to intelligently know the risk of a mismatched position. The purpose of surplus is to protect the company from extreme changes in the assets or liabilities. These extreme changes might come from investments, for example. If surplus was allowed to move at some multiple of interest rate changes, it would not be in a very good position to protect assets or liabilities. It is being affected by the thing it is supposed to protect against.

As an example, look at Chart 4, Page 1. We all know that assets (A) equal liabilities (L) plus surplus (S). Then the same formula works for their respective market values or present values. Suppose that total liabilities have a duration of 3.5 and are 90% of the market value assets. Assume that the assets have a duration of 5.15. Solving for the surplus duration, we see it is 20.0. That means if interest rates rise 1% then the market value surplus falls 20%. It can also work the other way. Page 2 of this chart shows that asset matching with the liabilities is not the complete answer. The assets backing the liabilities are what is needed to be matched.

For an ongoing concern, I do not believe perfect matching is desirable. As long as the slope of the yield curve is positive, meaning longer term bonds have higher yields, it is possible to pick up yield on assets by purchasing bonds with longer durations. The emphasis is then on the prudent management of mismatch. It does not matter whether your duration is 1.0 or 5.0 or 10.0. It is the management of mismatch that is important. The Dembiec etal paper discusses mismatch and surplus duration.

Now that we have seen the theory and the mechanics, here are some of the issues. When you construct the accident-year payouts, be sure you know what is included. It is probably not necessary to do each and every accident year. When the reserves are small enough, you can add them together to make one combined year. For example, assume you have some losses open for accident years 1974 and prior. They could be paid out with accident year 1975 losses without materially affecting the overall accuracy. As another example, assume that some of your automobile accident years are substantially affected by PIP losses, and a separate payout pattern for these losses is not available. I would suggest using a workers' compensation payout rather than an automobile payout. If you have a stop loss cover, truncate the payout to the extent you expect to utilize the cover. If any reserves are discounted, like workers' compensation, the reserves should be grossed up before applying payout percentages. While this is certainly not an exhaustive list, it is hoped that it gives you a flavor for what to consider.

Doing a duration on only the carried loss reserves is not complete. The losses contemplated in the unearned premium reserve (UPR) need to be considered too. The duration of the UPR is not its earning period. As the UPR is earned, losses will be incurred. I recommend calculating ultimate losses in the UPR by using ultimate loss and loss expense ratios, by line for the most recent accident year, applied to the UPR. These ultimates can then be paid out with accident-year patterns. The accident-year payments can be accumulated into one UPR calendar year payout. The UPR payout is blended in with the carried reserve payout and an overall duration is calculated.

For the carried reserves, it is fairly safe to assume the average payment is in the middle of the year. Assuming quarterly payments would be closer to reality, but I have found that estimating quarterly payments for a reinsurer is subject to considerable variation. This leads to an issue on the UPR payouts: when is the first payment made? We have discussed it internally and agreed to assume that the average UPR earns off in six months and the first payments will average coming six months later. Therefore, the first payments are shown twelve months after the date of the UPR, which, while not perfect, is what we are assuming today while we reconsider the issues.

You Can't Always Get What You Want

Sometimes, you find, you get what you need. **Concept 14: In order to totally protect the company, cash flow or scenario testing is needed.** (I prefer to call it "stress testing".) **Cash flow testing involves developing several scenarios, from best case to a catastrophic case, on timing and amounts of losses.** Please note: comparable comments also apply to the stressing of assets.

- Lengthen and shorten the payouts. It is best to apply the stress on payouts to the next couple of calendar years rather than at some distant point. In this manner, multiple stresses can be applied for maximum impact to roughly the same time frame. The simultaneous application of multiple stresses is in itself, a stress.

- Assume the reserves are deficient or redundant by various percentages. This can be considered as a type of catastrophe. Note, this will have the effect of changing the overall accident-year payout pattern.
- Assume the deficiencies or redundancies emerge at different times. This should still be done in the relative near term.

- If there is a possibility of natural catastrophes occurring in the upcoming time period, multiple occurrences should be included as a part of the stressing.

- For discounting, test different interest rates over the time period of the payout pattern. In real life, interest rates do change. Therefore, in modeling, why not do the same?

- Also, benefit can be derived from testing different interest rates where each is applied over the entire period.

- Consider doing the payouts separately between gross and ceded. Then you could test the effect if some of your reinsurance is not collectible.

- Test annual versus quarterly, or maybe even monthly, payments. This is particularly important for short-tailed lines.

- Prior financials should be stressed for comparison purposes.

- Run scenarios on future premiums and accompanying losses as well as current and future investments. This has particular application for valuations for a possible acquisition.

Several possibilities for stresses have been listed. Obviously, each is not applicable in a given situation. Concentrate on those stresses that will have an impact on the overall projected results. A future large increase in other underwriting expenses may be feasible; but it will likely not have a noticeable impact on future financials. Reasonableness of the occurrence of the stress is integral. Applying the equivalent effect of two Andrew like storms may be interesting, but not applicable in a given situation.

The stresses listed here are being applied in a deterministic manner. By this I mean that the stresses are applied discretely. Stresses applied in this manner are easier to explain than if stochastic methods are used. Stochastic methods involve using probability distributions in which the random variables are the stresses listed above. Stochastic modeling has technical appeal because of the virtually unlimited values that can be given to the variables. Clustering of outcomes can be noted and credence can thus be given to the particular values generating the clustered outcomes.

Deterministic modeling is a subset of stochastic modeling. Its advantages are ease of explanation and the controlling of the values given to the variables. Stochastic modeling has the advantages of giving practically innumerable values to variables and the sense of impartiality when applying stresses.

As I indicated, this is very much a work in progress. The entire area is evolving quickly and very little of what is done has yet to be cast in granite. That is what makes it so fascinating.

RESERVES AT 12/31/94 1,000.00 CHART 1

DISCOUNT RATE 6.0%

(1) YEAR	(2) TIME(T)	(3) CAL. YR. PAYOUT	(4) CAL. YR. PYMTS.	(5) DISCOUNT FACTOR	(6) DISC PYMTS(DP)	(7) (2)*(6) DP* TIME	(8) (6)*(2)^2 DP* TIME^2
1995	0.5	0.100	100.00	0.9713	97.13	48.57	24.28
1996	1.5	0.170	170.00	0.9163	155.77	233.66	350.48
1997	2.5	0.120	120.00	0.8644	103.73	259.33	648.31
1998	3.5	0.090	90.00	0.8155	73.40	256.90	899.15
1999	4.5	0.080	80.00	0.7693	61.55	276.98	1,246.39
2000	5.5	0.050	50.00	0.7258	36.29	199.60	1,097.77
2001	6.5	0.049	49.00	0.6847	33.55	218.08	1,417.49
2002	7.5	0.047	47.00	0.6460	30.36	227.70	1,707.75
2003	8.5	0.046	46.00	0.6094	28.03	238.26	2,025.17
2004	9.5	0.045	45.00	0.5749	25.87	245.77	2,334.77
2005	10.5	0.044	44.00	0.5424	23.86	250.53	2,630.57
2006	11.5	0.042	42.00	0.5117	21.49	247.14	2,842.05
2007	12.5	0.041	41.00	0.4827	19.79	247.38	3,092.19
2008	13.5	0.040	40.00	0.4554	18.22	245.97	3,320.60
2009	14.5	0.036	36.00	0.4296	15.47	224.32	3,252.57
TOTAL		1.000	1,000.00		744.51	3,420.19	26,889.54

DURATION AT 6% =(7)/(6)=3420.19/744.51	4.593880539
DURATION AT 7% =(7)/(6)=3179.08/713.53	4.455425841
CONVEXITY AT 6% =(8)/(6)=26889.54/744.51	36.11709715
CONVEXITY AT 7% =(8)/(6)=24488.87/713.53	34.32072933
PRESENT VALUE OF PYMTS AT 6%	744.51
PRESENT VALUE OF PYMTS AT 7%	713.53
% CHANGE IN VALUE FOR 100BP CHANGE IN INTEREST RATE =713.53/744.51-1	-4.16%
MODIFIED DURATION -4.59/1.06	-4.33%
CHANGE IN VALUE MD+.5*CON*.01^2	-4.15%

BOND PRICE AND MACAULAY DURATION

P = price of a t-year bond

R = redemption value

t = time in years

c = coupon

i = yield

D_M = Macaulay duration

$$V = \frac{1}{(1+i)}$$

$$P = c \bullet v^1 + c \bullet v^2 + \cdots cv^t + R \bullet v^t$$

$$D_M = \frac{1 \bullet c \bullet v^1 + 2 \bullet c \bullet v^2 + \cdots + t \bullet c \bullet v^t + t \bullet R \bullet v^t}{P}$$

DERIVATION OF MODIFIED DURATION

A. Take the first derivative of the price function (P) with respect to i

$$\frac{dP}{di} = -1 \bullet c \bullet v^{2} - 2 \bullet c \bullet v^{3} - \cdots - t \bullet c \bullet v^{t+1} - t \bullet R \bullet v^{t+1}$$

B. Factor out -v or $\frac{-1}{(1+i)}$ from right side of equation

$$\frac{dP}{di} = \frac{-1}{(1+i)} \bullet \left[1 \bullet c \bullet v + 2 \bullet c \bullet v^{2} + \cdots + t \bullet c \bullet v^{t} + t \bullet R \bullet v^{t} \right]$$

C. Divide both sides by P

$$\frac{\frac{(dp)}{(di)}}{P} = \frac{-1}{(1+i)} \bullet \left[\frac{(1 \bullet c \bullet v^{1} + 2 \bullet c \bullet v^{2} + \cdots + t\,c\,v^{t} + t \bullet R \bullet v^{t}}{P} \right]$$

$$\frac{\frac{(dp)}{(di)}}{P} = \frac{-1}{(1+i)} \bullet D_M$$

DISCOUNT RATE 6.0%

CASH FLOW MATCHING

CHART 3

	COUPON	PRINC-IPAL	PRICE	TERM IN YRS.	DURATION	1	2	3	4	5	TOTALS
LIABILITY PAYOUT		$1,000.00	$880.71		2.136	$375.00	$275.00	$ 175.00	$100.00	$ 75.00	$1,000.00
BOND 1	12.0%	$ 66.96	$ 83.90	5	4.146	$ 8.04	$ 8.04	$ 8.04	$ 8.04	$ 75.00	$ 107.16
NET						$366.96	$266.96	$ 166.96	$ 91.96	$ 0.00	
BOND 2	11.0%	$ 82.85	$ 97.19	4	3.489	$ 9.11	$ 9.11	$ 9.11	$ 91.96		$ 119.29
NET						$357.85	$257.85	$ 157.85	$ 0.00	$ 0.00	
BOND 3	10.0%	$ 143.50	$158.84	3	2.749	$ 14.35	$ 14.35	$ 157.85			$ 186.55
NET						$343.50	$243.50	$ 0.00	$ 0.00	$ 0.00	
BOND 4	9.0%	$ 223.39	$235.69	2	1.920	$ 20.11	$243.50				$ 263.61
NET						$323.39	$ 0.00	$ 0.00	$ 0.00	$ 0.00	
BOND 5	0.0%	$ 323.39	$305.08	1	1.000	$323.39					$ 323.39
NET						$ 0.00	$ 0.00	$ 0.00	$ 0.00	$ 0.00	
TOTALS FOR BONDS		$ 840.09	$880.71		2.136						$1,000.00

DURATIONS MATCH

ASSUMPTIONS:

1. LIABILITY PAYOUT STREAM IS ASSUMED TO BE: 37.5%, 27.5%, 17.5%, 10.0% AND 7.5%.
2. ALL PAYMENTS AT END OF YEAR
3. BOTH LIABILITIES AND BONDS ARE DISCOUNTED AT 6%
4. SINCE DISCOUNT RATES ARE EQUAL, BOTH MACAULAY AND MODIFIED DURATIONS ARE EQUAL FOR LIABILITIES AND BONDS

CHART 4
PAGE 1

Surplus Duration

Example

Assets = Liability + Surplus

$PV_A * D_A = PV_L * D_L + PV_S * D_S$

Assume

$PV_A = \$100$

$PV_L = \$90$

$PV_S = \$10$

$D_A = 5.15$

$D_L = 3.50$

$100 * 5.15 = 90 * 3.5 + 10 * D_s$

$10D_s = 100 * 5.15 - 90 * 3.5$

$D_s = 20.0$ years

if interest rates rise 1%, surplus drops 20%:

Revised PV_A =	\$100 * .9485	= \$94.85
Revised PV_L =	\$90 * .9650	= \$86.85
Revised PV_S =		\$8.00

Surplus Duration

Use amounts from prior example but $D_A = D_L$

$$(100)(3.5) = (90)(3.5) + 10 * D_s$$
$$350 = 315 + 10D_s$$
$$1.0D_s = 35$$
$$D_s = 3.5$$

The assets that support liabilities should be matched

Dynamic Financial Analysis Issues in Investment Portfolio Management

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Abstract

This paper will discuss issues that arise when using dynamic financial models to assist in the management of a property/casualty insurer's investment portfolio. There are three areas covered in this paper. The first discusses how much detail should be included on the asset side of a dynamic financial model in order to make it useful in making investment decisions. The second section applies a dynamic financial analysis to more accurately determine the optimal after-tax income for an insurer. The third area offers some suggested approaches to summarizing and conveying the results of a dynamic financial model.

Detail to be Included in the Asset Side of a Dynamic Financial Model

Financial models have many uses in the property/casualty insurance industry. A few examples are solvency evaluation, tax and investment planning, evaluation of reinsurance agreements, and pricing. The purpose for which the model will be used will determine the amount of detail (or complexity) needed in each area of the financial model. If the primary intent of the model is to estimate variations in loss reserves and future loss costs, then a simplified investment model may be appropriate. However, when the financial model is to be used for tax and investment planning, a more robust investment section is necessary.

The following are elements that we feel a model must address to be of practical use to an asset manager in order to make coordinated investment decisions. These elements can be viewed as a minimum level of detail needed to handle the majority of decisions that enter the property/casualty insurer's investment process.

1. **Cash flows from invested assets and operations**

Accurately modeled cash flows are important for the proper calculation of income that will be earned on reinvestment of those cash flows. Additionally, investment decisions for a property/casualty insurer should be made to enhance the operational underwriting side of the business. One of the major areas where they interact is in the use of cash flows. A rapidly growing insurer would be generating a significant amount of positive cash flow and the investment manager's strategy should look to take advantage of that. On the other hand, an insurer whose premium volume is shrinking may look to its investment portfolio for cash to support its operations. The investment manager in this scenario should have an investment strategy that can provide ready cash. The ability to forecast the needs and opportunities in these scenarios depends on the accuracy of the projected cash flows produced by the financial model.

2. **Income generated by invested assets**

Insurance companies are taxed on the book income generated by their fixed income portfolio, not on cash flows. In some portfolios these numbers can be materially different. Therefore it is important to track book income as well cash flow, particularly for tax planning and the generation of income statements. Book yields and book value are important not only for the income they produce, but also because realized gains and losses are based on comparisons of market to book value.

3. **Varying interest rates**

Varying underlying interest rates and therefore varying the market values of the fixed income portfolio has many uses. They allow the manager to assess the risk that the market value of surplus will vary beyond acceptable bounds. It also allows the manager to test different investment strategies (for example long duration versus short duration) given his future expectations of interest rates. The manager can then evaluate the risk and rewards to the company if those expectations do not come to fruition.

It is useful to have a model that varies interest rates in several different ways. The first way varies interest rates completely randomly according to a random interest rate generator (for example an autoregressive stochastic model). This can be useful for evaluating the effect of different investment strategies under uncertain future interest rate scenarios. There are, however, some shortcomings to this method. First, there is no guarantee that your model will accurately represent future interest rate changes. Second, it does not allow the manager to test scenarios given his (or her) expectations for the future. Third, the number of future scenarios can become so large that it becomes difficult to pull useful management information from them.

A second method is to allow the model to run a fixed set of scenarios that incorporate the major factors of what the investment manager is trying to analyze. For example, a manager may keep his portfolio at a short duration in the expectation that interest rates will rise and he will be able to invest at higher yields than are currently available. The trade off is that by currently being short on the yield curve he is giving up current investment income. This short term decrease is expected to be made up later by an increase in interest rates. Different interest rate scenarios can be run to evaluate how long the manager can wait for interest rates to rise before decreased investment income from the short portfolio cannot be recouped.

It is important to do these evaluations in the context of the insurance company's entire operations, since many companies have minimum income constraints needed to meet objectives such as policyholder dividends and minimum return on equity.

4. **Subclasses of invested assets**

The major decisions to made in this area are how many subclasses are needed and how much information needs to be entered for each subclass.

An advantage to having a large number of subclasses for the invested assets is that it allows the person doing the modeling to accurately capture the particular nuances of each type of security. An example of a necessary refinement is the need to differentiate between taxable and tax-exempt income for tax purposes. A more exact refinement, which may or may not be necessary depending on the use of the model, would be to subdivide bonds according to their call features. A simple model would price the market value of the portfolio simply according to interest rate changes. For many bonds the redemption date of the bond is actually dependent on current interest rate levels. If interest rates drop 100 basis points, the price increase will be much greater for non-callable bonds than for callable bonds. This is because interest rate decreases cause bonds to be called, which in turn shortens their duration, which leads to a smaller price change relative to interest rates. If the insurer has many callable bonds in its portfolio and the model varies interest rates but does not account for call features, errors in the projection will result. In particular, market value will be overstated and there will be a misallocation of cash flows from the maturing of these bonds.

A second advantage of more detail is that it allows for more accurate asset allocation strategies. A common approach in investing is to move between different "sectors" depending on the manager's feeling on how well they will perform after-tax and the needs of the insurance company. Sectors can be broad - taxable versus tax exempt securities; or they can be narrow - corporate bonds could be divided into bank & finance, industrial, and telephone & utilities. The refinement necessary would depend on the investment manager's style and the purposes for which the model will be used.

The major disadvantage of a highly refined model is the time it would add to the modeling process. More refinement adds more time up front. That is, there will be more detail that needs to be entered before the model is run. There is also more time added on to the back end. More data types results in more possible variations that can occur and need to be analyzed. There is also an increasing parameter risk. More variables mean there are more distributions and correlations to determine. With more variables it becomes more likely that the modeller will not be able to produce accurate estimates of these variables. A simpler model, combined with a modeller who understands the model's weaknesses can often produce more accurate answers than an overspecified model.

The following are some suggested subclasses of invested assets for a basic financial model:

4.1. ***Fixed Income*** (Note: For a good discussion of the characteristics of fixed income securities, an invaluable reference is "The Handbook of Fixed Income Securities" by Frank J. Fabozzi.[1])

4.1.1. *U.S Government Treasury and Agency Securities*

U.S Government securities make up the core portion of many insurers portfolios. These bonds are distinguished by their fixed cash flows from coupon payments, their taxable status, and by their lack of credit risk.

4.1.2. *Corporate Bonds*

These bonds are similar to U.S. Government securities in that they have fixed cash flows and are taxable. Corporate bonds add an extra dimension of credit risk. To account for credit risk, some probability of default needs to be built into the model. Subclasses of corporates should be created to attempt to create homogeneous groups with similar default characteristics. A simple categorization would be by the Standard and Poor's or Moody's ratings. At a minimum the classes should at least be divided into investment grade vs. below investment grade.

4.1.3. *Tax Exempts*

Tax-exempt bonds generally have fixed cash flows from coupon payments that are 85% tax-exempt for a property/casualty insurer. Most tax-exempt bonds can be classified into one of four categories: general obligation, revenue, prerefunded, and insured. These classifications are one way to group these bonds.

A second method of grouping would be by credit risk in a manner similar to that suggested for corporates. An approximate order for creditworthiness would be prerefunded, insured, revenue, and general obligation. Prerefunded bonds are backed by U.S. Treasuries and are generally Triple A rated. Insured bonds are usually rated according to their insurer but are also generally Triple A rated. General Obligation are generally more credit worthy than revenue bonds, although there is significant overlap. A simple grouping would place prefundeds, insured, and investment grade general obligation and revenue in one group and everything else in another.

4.1.4. *Mortgage Backed*

Mortgage backed and other similar loan backed securities are generally taxable and may have some credit risk. Their most distinguishing feature is that their cash flows are not fixed and can vary widely depending on the current interest rate environment. For life insurer modeling, this can be a major issue because not only is their cash flow from mortgage backeds affected by interest rates but also their premium inflow.

A general rule for a property/casualty insurer is that the complexity of the mortgage backed modeling should increase with the extent that they are part of the insurer's portfolio. For many smaller insurers, the lack of fixed cash flow from mortgage backed securities makes them unattractive and therefore they only compose a small part of their portfolio. If an insurer plans to make these a major part of their investment strategy, they need to have a good model to understand the interest rate risk they are assuming.

At a minimum, mortgage backeds should be put into as homogeneous groups as possible. One way to do this is by subdividing by expected prepayment pattern. The expected pre-payment patterns should be built into the model. If changes in interest rates are part of the model, then any change in interest rates must have some corresponding change in the prepayment pattern. In general, declining interest rate speed up pre-payments and higher interest rates slow them down.

4.2. *Cash*

Cash is generally completely liquid and is often invested in some type of money market fund. Fixed income maturities of less than one year can either be grouped with cash or with the longer term assets depending on the preference of the modeller. Some interest rate needs to be entered into the model for cash and should be distinguished between taxable and tax-exempt investments.

4.3. ***Equities***

After fixed income securities, equities are the next largest group of invested assets for property/casualty insurers. At a minimum, price changes and dividend level information for the equity investments need to be built into the model. For the more complex modeller who believes in CAPM theory, equities could be grouped according to their beta and varied accordingly with some underlying market changes built into the model.

4.4. *Real Estate*

For many companies, real estate constitutes a minor portion of their invested portfolio. If a company does have significant holdings in real estate, it should be segregated out from the rest of the invested assets. The ability to model future price changes and income levels from real estate should be included in the model.

4.5. *Other Invested Assets*

The remaining invested assets can be grouped together and most of the time will total to an insignificant amount. The ability to model price changes and income from these assets should be included in the model.

5. Timing of cash flows

For short term planning the timing of cash flows and maturities from the assets is very important. For long term planning it may be enough to assume the average cash flow occurs in the middle of the year. But for making actual decisions about when to make shifts in the portfolio, a greater level of detail is necessary. The best approach would be to have cash flows and maturities summarized quarterly for at least the first two projection years of a financial model. For the following years annual cash flows will suffice.

6. **Tax calculation**

A model that can incorporate all of the nuances of the tax laws and accurately calculate taxes is invaluable. Without accurate tax calculations, many of the uses of a financial model from a management point of view disappear. All investment decisions should be evaluated on an after tax basis. Unfortunately, the tax position for an insurance company is not always that easy to evaluate. The combined impact of discount rates, changes in loss reserves, varying underwriting results, and carrybacks and carryforwards, make a simple evaluation of the final effects on taxes extremely difficult. A good tax model is important because it can perform the "black box" function of churning through the numbers to get to the after tax results. The investment manager can use this to evaluate the returns under different investment strategies given a variety of future scenarios. Without a good model to evaluate the tax consequences, the correct strategies on an after tax basis are not at all obvious.

Tax Optimization

It has been documented that after-tax income can be increased through the optimal mix of taxable and tax-exempt investments (the rest of this discussion will assume an understanding of the basic dynamics of tax optimization. For a discussion of the fundamental issues, see Almagro and Ghezzi[2]). A problem with many types of tax optimization analyses is that they assume the investment portfolio is either all cash (or totally liquid) and can be moved around to achieve any desired taxable / tax-exempt mix. This is not generally true. There is an optimal mix for each one year horizon, but given where the company's portfolio currently stands it may not actually add value to sell bonds to reach the optimal point. If the portfolio was all cash, shifts in the portfolio would be frictionless. But a real portfolio has certain characteristics such as a maturity schedule, realized gains and losses, and imbedded yields that will affect the company's taxes and future income depending on what shifts are made.

It is useful to use a dynamic financial analysis to evaluate different optimization strategies under different scenarios. In reality, management expects the bond portfolio to produce certain results or puts certain limitations on the characteristics of the portfolio. Some examples of these expectations and limitations are:

- Restrictions on realizing capital gains and losses (and the accompanying effect on statutory surplus)
- Stability in investment income
- Duration constraints
- Credit constraints
- Maximum amount of AMT carryforwards allowed
- Imbedded yield of portfolio
- Market value of portfolio

Additionally, in trying to meet management's objective there are a number of variables that will affect future results. From the perspective of an investment manager, some of the these future variables are:

- Interest rates
- Ratio of taxable to tax-exempt interest rates
- Performance of stock portfolio (if included)

- Underwriting results
- Cash flow

We will use a financial model to examine two issues in particular. First, how does the choice of a time horizon affect the results of a tax optimization analysis. Second, we will undertake an evaluation of optimizing under scenarios of stochastic underwriting results.

The first example we will examine is an insurance company that at the end of 1995 is projecting to have too much tax-exempt income for 1996. This "excess" tax-exempt income would put them into AMT in 1996 and would imply a need to sell tax-exempts and buy taxable bonds. Additionally, assume that their entire bond portfolio is at an unrealized gain (This was a very common situation for companies at the end of 1995). Since realized gains are taxed as regular income, any movement towards the optimal point has two effects which must be considered.

First, selling bonds will add a one time boost to taxable income in 1996 (due to the booking of realized capital gains) which will not be there for 1997 and forward. Second, the effective tax rate on the income from tax-exempt bonds is 5.25%. By taking gains in the tax exempt portfolio the company is essentially increasing the tax rate on those bonds from 5.25% to 35%. We will show how multi -year modeling will produce different strategies based on the time horizon over which the company chooses to optimize.

The second example will take the same company and evaluate its possible optimization based on variable future underwriting results. Issues to be addressed include how variance of underwriting results effect an optimal portfolio mix, the magnitude of that possible effect, and the implications of those effects.

For purposes of illustration, the financial model will be somewhat simplified. The most significant simplifications are regarding future cash flow into the investment portfolio and the loss reserve tax discount. We are assuming no future cash flows into the investment portfolio other than reinvestment of coupons. In other words, net cash from operations equals zero. Additionally, when we vary calendar year underwriting results, we will assume that there was no effect on the tax discount of the loss reserves. These are important variables when doing tax planning and should be considered. However, for the purposes of demonstrating our conclusions, they are not needed.

Example 1 - Tax Optimization on a Multi-Year Horizon

The following is assumed for the company being modeled:

- The company has $300 million in taxable securities with a market yield of 6% and a book yield of 7%. This implies an unrealized gain of $12.8 million dollars.
- The company has $700 million in tax-exempt securities with a market yield of 4.8% and a book yield of 5.6%. This implies an unrealized gain of $24.6 million dollars.
- The company owns no other invested assets.
- All bonds bought and sold mature at the end of the year 2000. Therefore, there are no issues of unrealized gains or losses in the portfolio at the end of the evaluation period.
- The company is expecting to take a one year prior year reserve hit (increase) of $37 million which will cause it enter AMT in 1996.

- For the calendar years 1997 to 2000 the company expects its underwriting results to return to a constant profit of $5 million per year.

The company is considering three strategies:

- **Strategy 1**: Do nothing. In the scenario constructed the company will go into AMT in 1996 by $4.9 million. It will exactly recoup all of the AMT carryforwards at the end of the year 2000.
- **Strategy 2**: Sell tax-exempt bonds and buy taxables so that the company will reach the "optimal" point in 1996. This is the point at which the regular tax and alternative minimum tax are equal. Continue to sell taxable or tax-exempt bonds to optimize on a one year basis for each year as needed. This is the traditional optimization strategy.
- **Strategy 3**: Sell and buy back taxable bonds to realize the capital gains and generate taxable income in 1996. This will lower the AMT carryforwards to $3.3 million at the end of 1996. The carryforwards will be exactly recouped at the end of the year 2000.

Table 1 outlines the portfolio transactions involved under the three strategies:

Table 1

	Strategy 1: Do Nothing		Strategy 2: "Optimize Each Year"		Strategy 3: Sell Taxables in 1996	
Year	**Trade**	**Realized Gains**	**Trade**	**Realized Gains**	**Trade**	**Realized Gains**
1996	None	$0	Sell 34% of Tax-Exempt Buy Taxables	$8.5 million	Sell 100% of Taxables Buy Taxables	$12.8 million
1997	None	$0	Sell 54% of Taxables Buy Tax-Exempts	$5.7 million	None	$0
1998	None	$0	Sell 3% of Tax-Exempts Buy Taxables	$0.3 million	None	$0
1999	None	$0	Sell 9% of Taxables Buy Tax-Exempts	$0.2 million	None	$0
2000	None	$0	Sell 8% of Taxables Buy Tax-Exempts	$0.1 million	None	$0

The three strategies lead to the following after-tax income results:

Table 2

In $000s	1996	1997	1998	1999	2000
Strategy 1: Do Nothing					
Taxable Investment Income Earned	22,680	26,138	29,801	33,681	37,790
Tax-Exempt Investment Income Earned	39,301	39,509	39,730	39,964	40,211
Realized Capital Gains	0	0	0	0	0
After-tax Income	24,815	58,357	61,473	64,774	68,270
Cumulative Income	24,815	83,172	144,646	209,420	277,690
Cumulative AMT Carryforwards	4,895	4,492	3,564	2,078	2
Strategy 2: "Optimize" Each Year Strategy					
Taxable Investment Income Earned	37,699	19,808	24,576	25,852	27,468
Tax-Exempt Investment Income Earned	25,799	42,010	41,081	43,110	45,039
Realized Capital Gains	8,486	5,686	274	231	96
After-tax Income	32,195	59,906	58,607	61,331	64,121
Cumulative Income	32,195	92,100	150,707	212,038	276,159
Cumulative AMT Carryforwards	0	0	0	0	0
Strategy 3: Sell Taxables in 1996 Strategy					
Taxable Investment Income Earned	20,385	23,715	27,243	30,979	34,936
Tax-Exempt Investment Income Earned	39,297	39,498	39,710	39,935	40,174
Realized Capital Gains	12,795	0	0	0	0
After-tax Income	33,212	56,409	59,410	62,588	65,951
Cumulative Income	33,212	89,622	149,032	211,620	277,571
Cumulative AMT Carryforwards	3,319	3,279	2,732	1,649	0

More complete tax calculation exhibits can be found in Appendix B, Exhibits 1-3.

Based on the results of the financial model under the three chosen strategies, the following conclusions can be drawn:

1) ***In trying to optimize after-tax investment income, the choice of a time period over which to optimize will affect the choice of the optimal strategy.***

Strategy 2 is the strategy that is often implemented by insurance companies. When it appears financial results will put a company into AMT the immediate reaction is to sell tax-exempt securities. If those bonds are at a gain, it is considered to be a bonus since realizing gains will add to statutory surplus. Unfortunately, by selling those bonds the company will take an income stream that would have been taxed at 5.25% and increase the effective tax rate to 35%. This effect will not show up in a one year financial model. It is only when viewed from a multi-year horizon that the negative effect on after-tax investment income begins to emerge. In Strategy 2 the company continues to optimize until the end of the year 2000. Its cumulative net income over this period is $266.2 million. This is $1.5 million dollars less than Strategy 1.

Although Strategy 1 is labeled the "Do Nothing" strategy, that is not really accurate. What Strategy 1 really is a strategy that optimizes after-tax income on a multi-year horizon. The advantage of tax optimizing over multiple years is that it allows the full after-tax income effects of portfolio transactions to emerge and also takes into account future underwriting expectations.

One additional note on comparing Strategies 1 and 2. The observer might look at the cumulative income amounts and say $1.5 million on about $277 million in income is a small variation. There are two points we would make in response to this.

First, the $1.5 million was actually lost when the gains were taken in 1996. It was only as the bonds began to mature that it showed up in income. Additionally, consider that $1.5 million is not an unrealistic amount for an outside manager to charge for a portfolio of that size. By simply optimizing over a multi-year horizon, the fees would have been paid for the year.

The second point is that the company may have been under the impression that the transaction in Strategy 2 was actually adding value to the bottom line. When viewed on a one year horizon this would appear so. In order to implement Strategy 2, the company had to turnover 34% of their tax-exempt portfolio. The income lost in this transaction is significant when you consider that doing nothing would have added more income.

2) Optimizing on a one year horizon adds significant turnover into the portfolio strategy.

In Strategy 2, the company had to sell 34% of their tax-exempt securities in 1996. Since the poor calendar year results in 1996 were due to a one time increase in prior years' reserves, their underwriting results were expected to improve in 1997. This would call for a shift back into tax-exempt securities. In the model, 54% of the taxable securities had to be sold in 1997 to return to the optimal point. This turnover can be contrary to other operational and investment objectives. Taking gains in taxables or tax-exempts when viewed on a cash flow basis simply accelerates tax payment and often costs the company money on a horizon analysis (The Prime Advisor, "Evaluating Bond Swaps"[3]). Realized losses directly reduce statutory surplus which may not be acceptable to the company at that time. Furthermore, the portfolio manager may be involved in a sector strategy that involves waiting for a price shift before selling the current securities. Optimizing over a multi-year horizon allows the smoothing of these shifts in the portfolio for better overall management.

3) ***Realizing gains will lower the income stream going forward.***

We have already described the penalizing effects of realizing gains in the tax-exempt portfolio. But there is a more subtle effect that affects both taxable and tax-exempt securities that is worth mentioning. When estimating the effects of taking gains in a portfolio, many managers assume that if you realize the gain and simply buy back the same bonds, the income generated by those bonds going forward will be unaffected by realizing the gain. It is true that on a pre-tax market value basis the economic value a holding or selling the bonds is the same, but that does not mean after-tax income is unchanged.

In Strategy 3, the company realizes all of its gains in its taxable portfolio in 1996 and buys back the same taxable bonds. Other than the realizing of the gains, this is the same as Strategy 1. The two strategies cumulative after-tax income in the year 2000 is very similar. The difference in the two numbers is due to cash flow affects from realizing the gains and the different amount of AMT carryforwards in the two strategies. Although the cumulative after-tax income is similar, the way that income is achieved is not.

Strategy 3 has realized gains from the taxable portfolio of $12.8 million in 1996. This realized gain is simply the acceleration of future income. So now going forward for the next five years investment income is lowered by about $2.5 million per year, when compared to Strategy 1.

What does this mean? Investment income has been lowered going forward and more instability has been added to that income steam. This can adversely affect an insurer in several ways. For example, more stress will be put on a company's ability to pay its policyholder or stockholder dividends, since they are based on expected amount of income each year. Operating ratios will have more volatility and will decrease going forward, even if underwriting results remain constant. Regulators and rating agencies are often more concerned with a consistent income stream than realized gains, which they consider to be a one time deal. Additionally, the value for the NAIC IRIS test for investment yield will be decreased.

Example 2 - Tax Optimization with Stochastic Underwriting Scenarios

When trying to determine an optimal tax mix, one of the inputs into the process is the expected underwriting results. Of course for a property/casualty insurer, future calendar year results are uncertain (or else why would there be reinsurance?). Attempting to understand tax optimization with uncertain underwriting results can be accomplished with dynamic financial modeling. This analysis involves running the model for different mixes of taxable and tax-exempt securities in an environment where the underwriting results are determined by a probability distribution.

The model used is similar to that in the prior section. The following are the significant changes in the assumptions:

- The company has $1 billion in combined taxable and tax-exempt securities where the unrealized gain or loss equals zero. Therefore this company is able to switch to any mix of taxables and tax-exempts without the implications of realized gains and losses.
- The company's expected underwriting loss is a constant $13 million each year.
- If the underwriting results were certain, the optimal mix would be 50% taxables and 50% tax-exempts.

The goal of this analysis will be to optimize after-tax income over a two year period. Each strategy consists of a specific mix of taxable and tax-exempt bonds. This varies from $320 million in taxables and $680 million in tax-exempts and shifts by $20 million until the mix is from $680 million in taxables and $320 million in tax-exempts (19 strategies). For each strategy, 5000 simulations are run selecting the varying underwriting result from a given distribution. The strategies were tested using four different distributions (these were chosen for illustrative purposes):

- **Scenario 1: Fixed Underwriting**

  Expected Underwriting Gain or Loss = E(x) = -$13 million

  Mass: p(x) = 1 if x = -$13 million

  p(x) = 0 otherwise

  This is the deterministic model assuming underwriting results are known.

- **Scenario 2: Uniform Underwriting**

  E(x) = -$13 million

  Density: f(x) = 1/ 42,000,000 if -$34 million <= x <= $8 million

  f(x) = 0 otherwise

  This is could be interpreted as the projections for a company that has an idea of the range of its results (due to reinsurance, policy limits, etc.) but does not know the relative likelihood of any value within that range.

- **Scenario 3: Skewed Left**

  E(x) = -$13 million

  Density: f(x) = .3/ 23,800,000 if -$34 million <= x <= -$10.2 million

  f(x) = .7/ 2,200,000 if -$10.2 million < x <= -$8 million

$f(x) = 0$	otherwise

This is could be interpreted as the projections for a company that expects it underwriting results to come in within a narrow range that is slightly better than the mean. But when results are outside this range, they have the potential for becoming much worse than normal.

- **Scenario 4: Skewed Right**

  E(x) = -$13 million

Density:		
	$f(x) = .7/\ 2{,}200{,}000$	if -$18 million <= x <= -$15.8 million
	$f(x) = .3/\ 23{,}800{,}000$	if -$15.8 million < x <= $8 million
	$f(x) = 0$	otherwise

  This scenario is the reverse of Scenario 3.

Note that for each of the four scenarios above, the expected value of the underwriting results are the same. It is the effects of the form of the distribution we are trying to estimate, not the expected value. Graphs of the probability density functions Scenarios 2-4 can be found in Appendix C, Exhibits 1-3. Appendix C, Exhibit 4 display the summary statistics for the different combinations of optimization strategies and underwriting scenarios. Chart 1 summarizes the two year after-tax income for the various combinations.

Chart 1
Summary of Tax Optimization Strategies

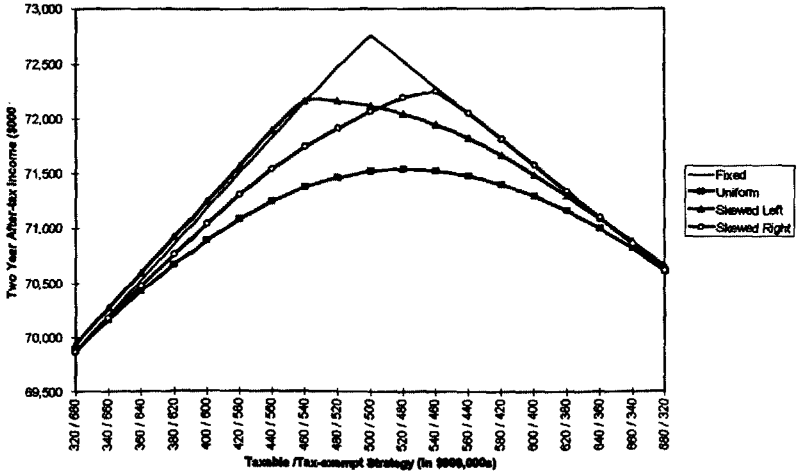

Table 3 summarizes the optimal strategies.

Table 3
In ($000s)

Underwriting Scenario	Optimal Tax Mix		Two Year After-tax Income
	Taxables	Tax-exempts	
Fixed	500,000	500,000	72,764
Uniform	520,000	480,000	71,539
Skew Left	480,000	520,000	72,162
Skew Right	540,000	460,000	72,254

Based on this information, the following conclusions can be drawn:

1) ***As the variance of the underwriting results increase, the expected value of after-tax income at the optimal mix decreases.***

This conclusion has several implications. First is that as underwriting variance increases, the penalty for missing the optimal point becomes less. Therefore, there is a broader range of acceptable portfolio mixes that will be within an acceptable range of optimal. This also means that the value added through tax optimization becomes less with increased variance. A company may want to undertake an analysis of this type to better understand the value that can be added through tax optimization. With increasing variance of underwriting results, it may be determined that there are other areas of the investment process through which income can be increased more effectively.

Secondly, if a company does decide that it wants to pursue a strategy of tax optimization, it must also take the time to understand its underwriting. As the ability to accurately estimate the expected results and the likelihood of variance from the expectations increases, so does the value added through tax optimization.

These results lead to the following question: As calendar year results emerge, can a company increase after-tax income by re-optimizing mid year? The answer is both yes and no. Yes, the company may be able to more accurately hit the optimal point for that year as the results become known. But no, that strategy may not add any more value to the company. Income will be earned as the year begins to emerge. In order to optimize for that year, larger shifts will have to take place in the portfolio to reach the optimal point to compensate for the income already earned. This will most likely put the company in a position where it will be far off optimal for the next year. This implies another large shift in the portfolio to re-optimize and so on and so on. As described in the previous section, this large turnover in the portfolio to chase a one year optimal strategy may hurt the company in terms of after-tax yields, managing realized gains and losses, and implementing portfolio strategies.

*2) **Skewed distributions will shift the optimal mix.***

Intuitively, the results from the skewed distributions make sense. If the distribution is skewed left, there is a greater likelihood that the underwriting results will come in better than the mean. Therefore more tax-exempts are need than in the fixed underwriting scenario. For a skewed right distribution, the reverse is true. For property companies who are exposed to occasional catastrophic loss or companies with particularly limiting reinsurance agreements, it may be wise to understand the variation of the underlying net losses when undertaking tax optimization.

These conclusions intuitively make sense and will hold in general. The magnitude of the conclusions will vary by company. With the increased interest in dynamic financial modeling, one practical application should be to help companies better understand the risks and rewards involved in different portfolio strategies such as tax optimization. As demonstrated, stochastic modeling improves a company's understanding of the different strategies it undertakes better than simpler deterministic models.

Data Analysis and Presentation

In the previous modeling, some simplifying assumption were made. Additionally we only varied three variables - the amount of bonds to sell, underwriting results, and time period of evaluation. The need to simplify the financial model highlights the paradoxical nature of a good financial model. An advantage of a good financial model is that the flows from different areas and the calculations of an insurer's taxes are too difficult to track and calculate without such a model. So many of the variables are interdependent, that it is often difficult to get an intuitive feel for what is the appropriate management decision. Thus a financial model can be an invaluable tool for decision making.

However, this ability to evaluate different strategies under varying scenarios also leads to a disadvantage. The model may be evaluating so many variables, times so many years of evaluation, times so many model runs, that the amount of output data produced can be overwhelming. This enormous amount of data may itself become too much to summarize and explain to management. Thus, limiting its effectiveness as a decision making tool.

There are at least two issues to be dealt with when confronted with this large amount of output. The first area is electronic data processing issues. Where will you find the computer space to store all of the data? Also, what software will you use to effective manipulate and sort the data? The second issue is interpretation. What techniques can be used to understand the results? Also, how can these results be presented in a way that is understandable to others? We will briefly discuss this second issue below.

One issue that affects the ease of understanding the model's results is the choice between stochastic and deterministic variables. Each variable that is stochastically varied increases the range of possible outputs exponentially. When building a model the careful selection of stochastic variables is very important.

One approach to assist in this selection is through the use of sensitivity testing. An initial model may built with many stochastic variables. After some initial runs have been completed, it is useful to summarize the results of the output results you are tracking relative to the underlying stochastic variables. For example, a company may want to see how cash flow from operations is affected by changes in written premium, future loss ratios, loss payment speed, and adverse development of loss reserves. If the initial results show that cash flow only decreases when either written premium decreases or the payment pattern speeds up, it may be helpful to eliminate the other stochastic variables. This gives a priority order for which variables the model must most accurately reflect the true underlying distributions.

There is another tool in helping to understand the results of a dynamic model which may seem obvious, but often is hard to remember when the modeller is faced with the results of 100 variables for 100 scenarios for 10 years of projections. This is to simply take a step back and ask, "Do these results make sense?" Often a model is confirming what a manager already knows but can't quantify. If it feels wrong, an understanding of how the model produced that answer should be determined before the results are accepted and further work is done.

Results that differ from expectations usually follow from either of two possibilities (assuming there are no hardware or software errors). The first is that an assumption was made that was wrong or oversimplified which caused the model to run incorrectly. The second possibility is that the model produced new information that wasn't previously apparent. This is one of the most beneficial uses of a financial model. Its ability to take into account all of the different interrelationships of an insurance company that can not be easily understood otherwise.

Once the model has incorporated all of the important factors and the results are accepted as reasonable, there is one last step. This is how to present results to the appropriate audience. After all of the data has been compiled and some information has been gleamed from it, there is often a feeling that the task is completed. But in reality, this is usually only the halfway point. One of the strengths of actuaries is their ability to understand numbers and make decisions based on those numbers. But others in insurance company management may not share that same ability. Even in summarized form, the amount of numbers in a report of a dynamic financial model can be intimidating and confusing. One solution to this is an increased use of color graphs and charts.

Often making an effort to create good summary charts may seem like a superfluous effort that can be very time consuming. If it is not analytical, it may not be considered "real" work. But if a manager is not able to make a decision based on the results of the data, all of the effort put into creating a good financial model was for naught. In the current world of computers and software, this has never been easier. There are numerous software programs available that can be used to create clear and attractive tables, charts, and presentations with relative ease. With access to color printers becoming more and more the norm, the use of contrast in color in a presentation can make a point much more quickly and effectively than words or rows of numbers ever could.

Conclusions

Dynamic financial models can be a important tool for helping an investment manager to assess risk and increase returns for a property/causality insurer's portfolio. Although historically much of the actuary's work has been on the liability side of the balance sheet, there is a great opportunity for actuaries to add value in the investment area. With respect to financial modeling, this can be accomplished by first making sure that enough attention is given to the development of the asset side of the financial models. The next step is to then use those models to develop new and useful analytical techniques. Finally, these techniques must be presented in a way so that are understood and accepted into a company's strategic investment planning methodology.

Appendix A

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[2] Almagro, M., and Ghezzi, T.L., "Federal Income Taxes - Provisions Affecting Property/Casualty Insurers," PCAS LXXV, 1988, pp. 95-162.

[3] "Evaluating Bond Swaps", The Prime Advisor, 1995 Series, Issue 3.

Appendix B

Appendix B **Exhibit 1**

TAX ABLE INCOME CALCULATIONS

(In $000s)

		1996	1997	1998	1999	2000
	Statutory Income Calculation					
(1)	Net Underwriting Gain / (Loss)	(32,000)	5,000	5,000	5,000	5,000
(2)	Taxable Investment Income Earned	22,680	26,138	29,801	33,681	37,790
(3)	Tax-Exempt Investment Income Earned	39,301	39,509	39,730	39,964	40,211
(4)	Realized Capital Gains	0	0	0	0	0
(5)	TOTAL STATUTORY INCOME	29,981	70,647	74,531	78,645	83,001
	Regular Tax Adjustments to Statutory Income					
(6)	85% Tax Exempt Interest Income	33,406	33,583	33,770	33,969	34,179
(7)	20% Change in UEPR	200	200	200	200	200
(8)	Loss Reserve Discount	4,000	(1,000)	(1,000)	(1,000)	(1,000)
(9)	Total Adjustments	(29,206)	(34,383)	(34,570)	(34,769)	(34,979)
(10)	REGULAR TAXABLE INCOME	775	36,265	39,961	43,876	48,022
	AMT Adjustments to Regular Taxable Income					
(11)	85% Tax Exempt Interest Income	33,406	33,583	33,770	33,969	34,179
(12)	Tax Preferred Ratio	0.75	0.75	0.75	0.75	0.75
(13)	Total AMT Adjustment	25,054	25,187	25,328	25,477	25,635
(14)	AMT INCOME	25,829	61,452	65,289	69,352	73,657
	Net Income					
(15)	Regular Tax	271	12,693	13,986	15,356	16,808
(16)	Alternative Minimum Tax	5,166	12,290	13,058	13,870	14,731
(17)	AMT Carryforward Used	0	402	929	1,486	2,076
(18)	Federal Income Tax Incurred	5,166	12,290	13,058	13,870	14,731
(19)	AMT Carryforward Incurred	4,895	0	0	0	0
(20)	NET INCOME	24,815	58,357	61,473	64,774	68,270
	Cumulative Totals					
(21)	Cumulative Net Income	24,815	83,172	144,646	209,420	277,690
(22)	Cumulative AMT Carryforwards	4,895	4,492	3,564	2,078	2

TAX ABLE INCOME CALCULATIONS
(In $000s)

	1996	1997	1998	1999	2000
Statutory Income Calculation					
(1) Net Underwriting Gain / (Loss)	(32,000)	5,000	5,000	5,000	5,000
(2) Taxable Investment Income Earned	37,699	19,808	24,576	25,852	27,468
(3) Tax-Exempt Investment Income Earned	25,799	42,010	41,081	43,110	45,039
(4) Realized Capital Gains	8,486	5,686	274	231	96
(5) TOTAL STATUTORY INCOME	39,984	72,504	70,931	74,193	77,603
Regular Tax Adjustments to Statutory Income					
(6) 85% Tax Exempt Interest Income	21,929	35,708	34,919	36,643	38,283
(7) 20% Change in UEPR	200	200	200	200	200
(8) Loss Reserve Discount	4,000	(1,000)	(1,000)	(1,000)	(1,000)
(9) Total Adjustments	(17,729)	(36,508)	(35,719)	(37,443)	(39,083)
(10) REGULAR TAXABLE INCOME	22,255	35,996	35,212	36,750	38,520
AMT Adjustments to Regular Taxable Income					
(11) 85% Tax Exempt Interest Income	21,929	35,708	34,919	36,643	38,283
(12) Tax Preferred Ratio	0.75	0.75	0.75	0.75	0.75
(13) Total AMT Adjustment	16,447	26,781	26,189	27,483	28,712
(14) AMT INCOME	38,702	62,777	61,401	64,232	67,233
Net Income					
(15) Regular Tax	7,789	12,598	12,324	12,862	13,482
(16) Alternative Minimum Tax	7,740	12,555	12,280	12,846	13,447
(17) AMT Carryforward Used	0	0	0	0	0
(18) Federal Income Tax Incurred	7,789	12,598	12,324	12,862	13,482
(19) AMT Carryforward Incurred	0	0	0	0	0
(20) NET INCOME	32,195	59,906	58,607	61,331	64,121
Cumulative Totals					
(21) Cumulative Net Income	32,195	92,100	150,707	212,038	276,159
(22) Cumulative AMT Carryforwards	0	0	0	0	0

Appendix B **Exhibit 3**

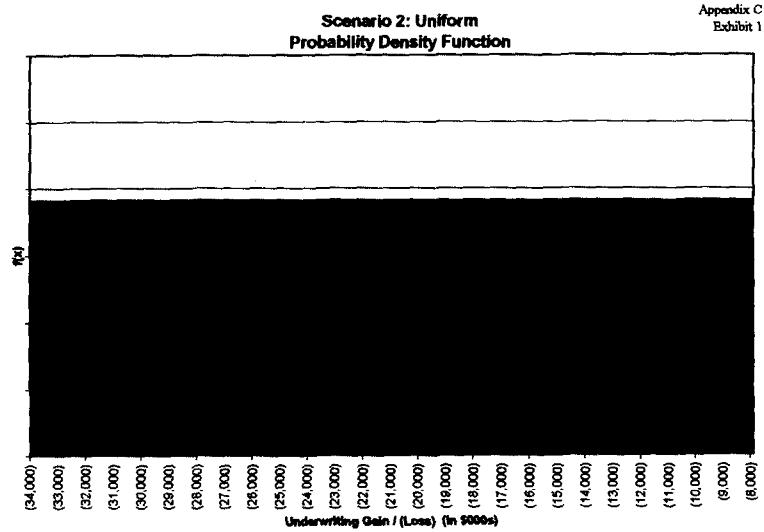
TAX ABLE INCOME CALCULATIONS

(In $000s)

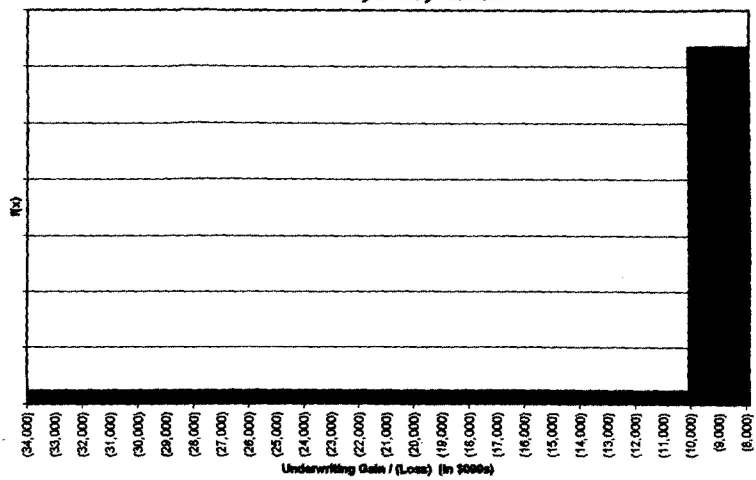
	1996	1997	1998	1999	2000
Statutory Income Calculation					
(1) Net Underwriting Gain / (Loss)	(32,000)	5,000	5,000	5,000	5,000
(2) Taxable Investment Income Earned	20,385	23,715	27,243	30,979	34,936
(3) Tax-Exempt Investment Income Earned	39,297	39,498	39,710	39,935	40,174
(4) Realized Capital Gains	12,795	0	0	0	0
(5) TOTAL STATUTORY INCOME	40,478	68,213	71,953	75,914	80,109
Regular Tax Adjustments to Statutory Income					
(6) 85% Tax Exempt Interest Income	33,403	33,573	33,754	33,945	34,148
(7) 20% Change in UEPR	200	200	200	200	200
(8) Loss Reserve Discount	4,000	(1,000)	(1,000)	(1,000)	(1,000)
(9) Total Adjustments	(29,203)	(34,373)	(34,554)	(34,745)	(34,948)
(10) REGULAR TAXABLE INCOME	11,275	33,840	37,399	41,169	45,162
AMT Adjustments to Regular Taxable Income					
(11) 85% Tax Exempt Interest Income	33,403	33,573	33,754	33,945	34,148
(12) Tax Preferred Ratio	0.75	0.75	0.75	0.75	0.75
(13) Total AMT Adjustment	25,052	25,180	25,315	25,459	25,611
(14) AMT INCOME	36,327	59,020	62,714	66,628	70,772
Net Income					
(15) Regular Tax	3,946	11,844	13,090	14,409	15,807
(16) Alternative Minimum Tax	7,265	11,804	12,543	13,326	14,154
(17) AMT Carryforward Used	0	40	547	1,084	1,649
(18) Federal Income Tax Incurred	7,265	11,804	12,543	13,326	14,158
(19) AMT Carryforward Incurred	3,319	0	0	0	0
(20) NET INCOME	33,212	56,409	59,410	62,588	65,951
Cumulative Totals					
(21) Cumulative Net Income	33,212	89,622	149,032	211,620	277,571
(22) Cumulative AMT Carryforwards	3,319	3,279	2,732	1,649	0

Appendix C

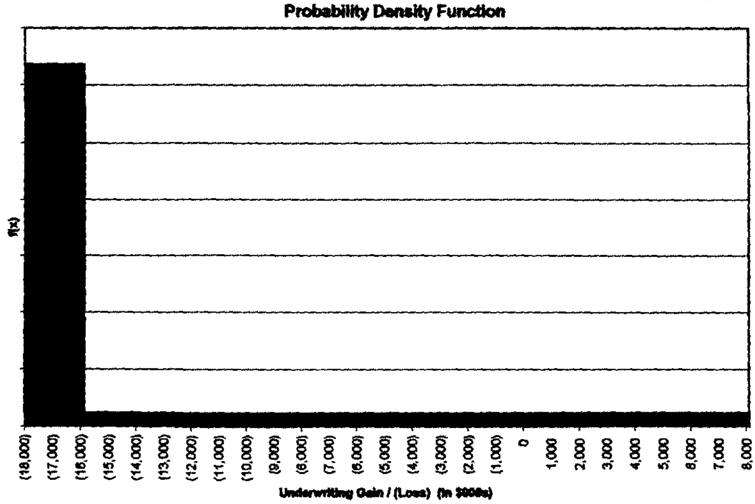
Appendix C
Exhibit 1



Scenario 3: Skewed Left
Probability Density Function



Scenario 4: Skewed Right
Probability Density Function



Tax Optimization: Summary of Two Year After-tax Income
(In $000s)

Appendix C
Exhibit 4

Strategy	Scenario 1: Fixed Underwriting				Scenario 2: Uniform				Scenario 3: Skewed Left				Scenario 4: Skewed Right			
Taxables / Tax-exempts	Mean	Standard Deviation	10th Percentile	90th Percentile	Mean	Standard Deviation	10th Percentile	90th Percentile	Mean	Standard Deviation	10th Percentile	90th Percentile	Mean	Standard Deviation	10th Percentile	90th Percentile
320 / 680	69,885	0	NA	NA	69,882	13,616	51,240	88,399	69,950	7,988	58,499	76,793	69,874	7,964	63,011	81,633
340 / 660	70,209	0	NA	NA	70,165	13,565	51,564	88,683	70,274	7,988	58,823	77,117	70,180	7,928	63,334	81,823
360 / 640	70,533	0	NA	NA	70,429	13,489	51,888	88,927	70,597	7,988	59,147	77,441	70,477	7,875	63,658	82,012
380 / 620	70,857	0	NA	NA	70,670	13,390	52,212	89,040	70,921	7,988	59,471	77,765	70,763	7,805	63,982	82,149
400 / 600	71,181	0	NA	NA	70,885	13,265	52,536	88,922	71,245	7,988	59,795	78,089	71,040	7,720	64,306	82,323
420 / 580	71,505	0	NA	NA	71,074	13,121	52,860	88,734	71,569	7,988	60,119	78,413	71,301	7,614	64,630	82,523
440 / 560	71,828	0	NA	NA	71,237	12,959	53,184	88,528	71,893	7,988	60,443	78,737	71,534	7,469	64,954	82,488
460 / 540	72,152	0	NA	NA	71,369	12,780	53,508	88,317	72,157	7,946	60,761	78,854	71,739	7,297	65,278	82,361
480 / 520	72,476	0	NA	NA	71,464	12,588	53,832	88,078	72,162	7,763	60,951	78,615	71,917	7,105	65,602	82,282
500 / 500	72,764	0	NA	NA	71,521	12,388	54,156	87,840	72,119	7,571	61,127	78,376	72,070	6,904	65,926	82,182
520 / 480	72,525	0	NA	NA	71,539	12,188	54,480	87,601	72,045	7,379	61,295	78,138	72,195	6,703	66,250	82,066
540 / 460	72,286	0	NA	NA	71,523	11,992	54,804	87,362	71,946	7,195	61,485	77,899	72,254	6,521	66,574	81,900
560 / 440	72,048	0	NA	NA	71,475	11,807	55,107	87,124	71,819	7,029	61,632	77,660	72,050	6,495	66,462	81,669
580 / 420	71,809	0	NA	NA	71,396	11,636	55,427	86,885	71,664	6,892	61,822	77,422	71,811	6,495	66,223	81,430
600 / 400	71,570	0	NA	NA	71,289	11,486	55,582	86,646	71,483	6,787	61,713	77,183	71,573	6,495	65,985	81,191
620 / 380	71,332	0	NA	NA	71,155	11,358	55,793	86,408	71,289	6,699	61,638	76,944	71,334	6,495	65,746	80,953
640 / 360	71,093	0	NA	NA	70,996	11,257	55,765	86,169	71,086	6,626	61,515	76,706	71,096	6,495	65,507	80,714
660 / 340	70,854	0	NA	NA	70,814	11,182	55,644	85,930	70,875	6,565	61,443	76,467	70,857	6,495	65,269	80,475
680 / 320	70,616	0	NA	NA	70,612	11,131	55,447	85,692	70,655	6,521	61,305	76,229	70,618	6,495	65,030	80,237

Interpreting Model Output—
The California Earthquake Authority and the Cost of Capital of the Reinsurance Layer

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Interpreting Model Output

The California Earthquake Authority and the Cost of Capital of the Reinsurance Layer

Introduction

Actuaries and other financial analysts had had difficulty interpreting the voluminous data that is typically output by a dynamic financial model. This paper illustrates the use of the decision-theoretic approach of Borch (1962) and Van Slyke (1995) to produce a simple illustration of the meaning of the results of 10,000 simulations of the financial results of a reinsurance program. The illustration in Figure 1 relates the model's results to the cost of capital in international financial markets.

The California Earthquake Authority

Earthquakes in California have accounted for large losses to homeowner insurers in the past ten years. The Loma Prieta earthquake in San Francisco in 1989 accounted for $1 billion of insured commercial and residential losses and the Northridge earthquake in Los Angeles in 1993 accounted for an additional $8.2 billion of insured residential losses. There are predictions from many experts that these regions are due for another large earthquake in the near future. These large losses to insurers have created a crisis in California. These losses suggest limiting underwriting. However, state statute requires that all companies selling homeowners insurance in California offer earthquake insurance. In the present market, earthquakes in California have become an uninsurable risk.

In response to this problem, California legislators have proposed two significant changes to earthquake insurance issued in California. First, AB 1366 proposes changes in the minimum coverage to earthquake policies issued in California, known as the "mini-earthquake policy." Coverage A (Structure) has an increased deductible of 15%; it includes the basic structure, foundations and walkways but excludes any external masonry veneer. The coverage is on a replacement cost basis and requires insurance to be purchased to value. Coverage B (Contents) limits loss to $5,000 at replacement cost excluding obvious items such as computers, glassware, securities and money, etc. Coverage C (Loss of Use) limits losses to $1,500 with no deductible. There are similar changes to the condominium and renters form of the insurance. Mini-earthquake policies will provide significant reductions in loss exposure; estimates show Northridge earthquake losses would have total $4.3 billion under mini-earthquake policies.

The second, and more significant change, is the introduction of AB 13. AB 13 proposes to establish an agency of the State of California known as the California Earthquake Authority (CEA). This agency will be the insuring mechanism for all mini-earthquake policies sold in California by *participating* members. This authority is unique in that it relies on various sources for insurance coverage: the insurance market, state government, and the financial markets. If an insurance company joins the CEA, the CEA will sell all mini-earthquake policies for this insurer. At 100% participation, the CEA will have sufficient capacity to pay $10.5 billion in earthquake losses in the first year. All capacities will be prorated downward based on the percentage of participation in the CEA. If an insurer does not join the CEA, the insurer must offer, at a minimum, the mini-earthquake policy along with all home insurance policies sold in California.

The CEA capacity consists of seven different layers of coverage. The first layer of coverage comprises of initial industry capital contributions. At 100% participation, the total coverage amount for this layer is $1 billion with each participant contributing an amount proportion to their total market share. This contribution layer is not reinstated after a loss. The second layer is comprised of retained earnings. This layer will increase the paying capacity of the agency in the first two years. If earthquake losses are low enough to lead to retained earnings in these years, the CEA will use this layer to reduce the coverage exposure in the first and second assessment layer. The next $3 billion of capacity is the first industry assessment layer. Participants in the CEA must pay an amount proportional to their market share if losses penetrate this layer. Any portion of this layer used to pay for losses or reduced by the favorable increases in the retained earnings layer cannot be reinstated. The next $2 billion dollars of losses is covered by a reinsurance layer. For the first two years of the CEA, this reinsurance is basically $2 billion coverage excess of $4 billion plus retained earnings. This study note discusses this layer in great detail, including comparing the risk/reward payoff of this layer to other risky investments. The remaining three layers to do not affect the reinsurance layer in any way but we will discuss them for completeness. The fifth layer is comprised of up to $1 billion of general revenue bonds issued by the State of California. If penetrated, the state legislator can reinstate this layer through the passage of subsequent legislation. It will be repaid by a surcharge to mini-earthquake policyholders of up to 20%. The sixth layer is also a very interesting and unique layer. It comprises of $1.5 billion of coverage through the sale of Cat Bonds to the financial markets. Only the interest paid on these bonds is contingent; any losses penetrating this layer will reduce proportionally the payment of interest to these bonds. If the entire layer is used to pay for losses, no future interest payments will be made on these bonds. The bonds are to be paid back 10 years after the inception of the CEA, whether or not a loss penetrates this layer. Finally, the remaining $2 billion of coverage is the second industry assessment layer. This layer is identical to the first industry assessment layer except that it is reduced only if the assets of the CEA exceed $6 billion for at least 6 months.

CEA Reinsurance Layer

One of the arguments against the formation of the California Earthquake Authority is the extraordinary cost of the reinsurance layer. The goal of this study is to quantify the cost of this risk and compare it to comparable investments in the financial markets. In particular, we will compare risks and rewards of underwriting this reinsurance layer to the risks and rewards of investments in Baa subordinate bonds and investments in the S&P 500 index.

At 100% participation level, the reinsurance layer is basically $2 billion dollars of coverage excess of $4 billion less retained earnings. This coverage will cost the CEA a total of $575 million in premium over two years. Since the CEA will underwrite new mini-earthquake policies as homeowner policies are renewed throughout the year, we assumed the exposure of loss to the CEA would increase proportionally in the first year of exposure, starting from no exposure at inception to full exposure of loss at the end of the first year. At this premium level, our calculations show a loss ratio of 33% to 42%, depending on the uncertainty one has with EQECAT loss estimates.

EQECAT Catastrophe Management is the firm responsible for estimating the catastrophic earthquake losses for the CEA (EQECAT Report No. 710003.001, 12/95.) They modeled twelve scenario earthquake events; these simulations examined earthquake losses in highly populated urban areas in California. They calibrated their model using the San Francisco earthquake 1906 and the Northridge earthquake as benchmarks. Their estimate of total expected annual loss for California was $742 million. We simulated the earthquake losses for this study using the annual loss distribution estimated by EQECAT.

The Cost of Capital for CEA Reinsurance

Appendix 1 lists the assumptions used in the model of the CEA. We performed simulation runs using different levels of market participation and found the results to be insensitive to this assumption. For a realistic model, we settled on a market participation level of 80%. This reduces the total premium to $460 million and the maximum loss to $1,600 million.

Reinsurance underwriters do not know whether the EQECAT earthquake loss estimates reflect the true probabilities of losses to the CEA. By assuming that the yearly earthquake loss incurred by the CEA is the product of the earthquake loss estimated by EQECAT and a random variable representing uncertainty about the EQECAT estimates. (We assume this random variable is lognormal with a mode, or most likely value of 1.0, and uncertainty measured by the parameter σ.) When the uncertainty parameter equals 0.0, the CEA earthquake losses are equivalent to losses generated by the EQECAT model. When the uncertainty parameter equals 1.0, the mean earthquake loss of the simulation model is 65% greater than the mean earthquake loss of the EQECAT estimate.

We choose values of σ from 0.0 to 1.0 because this range reflects a wide variation in uncertainty. As we noted, $\sigma = 0.0$ denotes no uncertainty in EQECAT estimates. An uncertainty of 1.0 is about the same uncertainty as in auto collision claims about the average for a given make and model of car. That is, $\sigma = 0.0$ is like being told "pay the average collision loss of $2,500"; $\sigma = 1.0$ is like being told "pay the actual collision loss, the average is $2,500."

Let R denote a random variable representing the total ceded loss over the two years of exposure. Similar to the CEA earthquake losses, ceded losses also increase with increases in σ but in a more complicated manner.

Table 1 displays the results of the simulations for various values of σ. As a percentage of premium, the expected loss to the reinsurer ranges from 33% with no parameter uncertainty to 42% when $\sigma = 1$. At first glance, it may seem that this expected loss ratio is too low but note the other statistics in Table 1. In particular, if the reinsurer does incur a loss, the expected loss is more than two and one half times the $460 million dollars of premium. Furthermore, there is approximately a 50% probability that if a loss occurs, the reinsurer will have to pay the probable maximum loss, $1,600 million.

σ	0.0	1.0
Probability of no loss	85.3%	83.7%
Probability of total loss	6.6%	8.9%
Probability of total loss, given a loss occurs	44.8%	54.4%
Expected loss to reinsurer, given a loss occurs (millions)	1,043	1,172
Expected loss to reinsurer, given a loss occurs but it is not a full loss (millions)	591	660
Expected loss to reinsurer (millions)	153	192
Expected loss ratio	33%	42%

Table 1: Earthquake Loss Distribution of CEA Reinsurance Layer

Clearly, the distribution of R is not the typical normal random variable that is familiar to everyone (including the critics). In particular, there are two point masses associated with the distribution of R; a large point mass at \$0 and a second probability of loss at the probable maximum loss of \$1,600 million. For R between \$0 and \$1,600 million, we found the truncated exponential distribution to fit well. For further details, we refer the reader to Appendix 3.

To quantify the cost of risk in this coverage, we used a risk-adjusted value (RAV_c) characterized by an exponential utility function. Some literature refers to the "economic value added" of a transaction as the change in risk-adjusted value. Appendix 2 contains a thorough discussion of these functions. There are many appealing properties of this approach including the specification of risk capacity c and a measure of cost associated with this value. Some other important properties of RAV are:

- First, if we know the ceded loss with certainty then the risk-adjusted value of this loss is simply the loss itself, regardless of the insurer's commitment of risk capacity.

- Second, if two risks are independent then we can separately determine their risk-adjusted values.

- Third, reinsurance losses can be pooled. If one underwrites $1/k$ of the total, one would expect to get $1/k$ of the premium. Furthermore, the reinsurer's risk-adjusted value of a loss is proportional to the capacity it puts up to bear the risk. In other words, if the total ceded exposure is too large for an individual reinsurer with a specified risk capacity c, we can divide the risk amongst k reinsurers, each with capacity c; each reinsurer is responsible to an amount R/k. This property is precisely what allows the California Earthquake Authority to obtain a large reinsurance layer of coverage. There is no one reinsurer that is willing to allocate all the capacity needed to underwrite the entire ceded loss. Rather, the ceded loss exposure is divided amongst a large number of reinsurers with varying commitments of capacity; each reinsurer is willing to underwrite a small portion of the total loss relative to their committed risk capacity.

- Fourth, the risk-adjusted value of an uncertain loss is always greater than the pure premium. Furthermore, the risk capacity is critical to the value of the risk-adjusted value. In particular, the more capacity the insurer chooses to apply to underwriting a random loss exposure R, the less is the risk-adjusted value charged for this exposure. That is,

Risk Premium at Capacity c

= Risk-Adjusted Value at Capacity c – Expected Losses

or

Estimate Risk Premium = Observed Market Price – Estimated Pure Premium

We refer to the difference between the risk-adjusted value and the pure premium as the risk premium. In this model, the measure of risk to reward is not the absolute size of the risk premium, or the absolute size of the risk capacity, but rather the amount of risk premium one receives relative to one's risk capacity.

- Fifth, in a competitive market in rough equilibrium, the amount of risk capacity will rise as the risk premium increases.

The concept of risk-adjusted value and risk premium is not limited to insurance problems. We may apply this model to any form of risk including investments in stocks and bonds. One can compare directly the percentage of risk premium to risk capacity for various investments to determine if the risk/reward payoffs are sufficient.

Figure 1 displays risk premium curves for the CEA reinsurance layer as a function of risk capacity. This figure displays two risk premium curves; one curve associated with $\sigma = 0.0$ and the second curve associated with $\sigma = 1.0$. Each of these curves shows the intersection of the observed risk premium with the risk premium curve. The solid line between these two points displays the location of similar points for values of σ between 0.0 and 1.0. Notice that as uncertainty increases, the risk premium curves shift upward and to the right. The farther these curves are from the origin, the higher the risk. Recall from the discussion above that although this graph is for the whole reinsurance layer, one can obtain the corresponding graph for an individual reinsurer by scaling the figure downward according to the level of the reinsurer's participation in the CEA.

From the individual reinsurer's perspective of underwriting a risk, one must decide whether the risk premium one receives for bearing the risk is sufficient in light of the amount of committed capacity. If we know the premium the reinsurer is requesting for the risk, we can determine an *implied* capacity that corresponds to the intersection of the observed risk premium with the risk premium curve. If the implied capacity is less than the total capacity the reinsurer is willing to commit, given the premium, then the risk is acceptable.

Because the premium for the reinsurance layer is fixed at $460 million, the observed risk premium decreases with increases in uncertainty associated with EQECAT loss estimates. Furthermore, because the risk premium curves are also shifting upward and to the right with increases in uncertainty, the implied risk capacity of the reinsurer increases with increases in uncertainty. Intuitively, this graph says that with the premium fixed at $460 million, the more one is uncertain of EQECAT loss estimates, the less return one will receive from this investment. Table 2 displays the results of the risk-adjusted value calculations for the CEA. The percentage of risk premium to risk capacity ranges between 53% with no parameter uncertainty and 38% with large parameter uncertainty.

σ	0.0	1.0
Observed Risk Premium	307	268
Implied Risk Capacity	575	713
Ratio of Risk Premium to Risk Capacity	53%	38%

Table 2: Risk-adjusted Value Analysis of CEA Reinsurance Layer

Is a ratio of risk premium to risk capacity of between 38% and 53% too high as opponents to the CEA would argue? To answer this question, let us compare these values to other investment alternatives in the market.

The Cost of Capital for Stocks and Bonds

Figure 2 displays the risk premium curve versus risk capacity for an investment of \$5 million in the S&P 500 composite index. We assume the S&P 500 follows a log-normal random process with drift parameter μ_S and volatility σ_S. Changes in the drift parameter do not result in any significant shifts in the risk premium curve but do affect the observed risk premium. Changes in the volatility do not affect the observed risk premium but they do affect the risk premium curve. The region displayed in the graph is the intersection of the observed risk premium with the risk premium curve for various combinations of μ_S and σ_S; the actual combination for a particular investment in the S&P 500 index depends on market timing. This region also does not display all possible values of μ_S and σ_S; it only represents values that we believe are realistic in today's investment environment. Specifically, we let μ_S vary between 9.5% and 10.5% with the associated σ_S varying between 0.5% and 1.0% above μ_S.

Figure 3 shows the risk premium versus risk capacity for a \$2 million investment in Baa subordinate debt. The displayed curve assumes a coupon rate of 7.5% and a time to maturity of 20 years. We used default rates and severity estimates from Moody's. By varying the assumption about the coupon rate from 6.5% to 8.5% and the time to maturity from 15 to 20 years, one obtains a risk premium -- risk capacity combination somewhere in the circled region.

Allocation of Capital and Return on Investment

The level of risk depends on the amount one is willing to invest in each of these instruments. We measure this risk level by the distance from the origin to the point on the risk premium curve that corresponds the intersection of the observed risk premium and risk capacity. For example, a $1 million reinsurance limit in the CEA reinsurance layer has an associated risk capacity of approximately $400,000 and provides a risk premium of approximately $160,000. This is the same level of risk and reward as an investment of approximately $1.7 million in an issue of a Baa subordinate bond or an investment of approximately $3.6 million in the S&P 500 index.

The $160,000 of risk premium is not the total return on the investment. It is only the premium for the *risk* associated with the investment. The total return on each investment is the expected return plus the risk premium. For example, for the investment in an issue of Baa debt, the total return is the risk-free return of about 6.2% plus an additional return of about 1.1% for the cost of capital. (The cost of defaults accounts for the balance of the quoted yield on the Baa bond.) In contrast, allocating $750,000 of capital together with a premium of $287,500 to secure a commitment to a $1 million limit of CEA reinsurance, the risk premium of approximately $160,000 over a two-year period would be 10.2% per year in addition to the risk-free return.

These numbers are approximate since that actual level depends on the uncertainty about EQECAT loss estimates. If one believes there is no uncertainty then the required investment amount is lower whereas if one believes a high level of uncertainty then the required investment amount is higher.

Is CEA Reinsurance Overpriced?

Comparing the CEA reinsurance risk to the S&P 500 and Baa subordinate bonds, one sees that the percentage of risk premium to risk capacity remains between 35% and 55%. This suggests that the level of risk/reward associated with the CEA reinsurance layer is no smaller and no greater than investments in the stock market or the bond market.

It is also interesting that as little as a $75 million increase or decrease in premium would have a significant effect on these conclusions. Figures 4 and 5 show that decreasing the premium by $75 million would undercharge this risk exposure whereas increasing the premium by $75 million would overcharge this risk exposure.

Conclusion

This study measures the risks and rewards associated with the CEA reinsurance layer for the first two years of the agency. It quantifies the uncertainty one has with the EQECAT loss data and associates a measure of risk and reward with this uncertainty. We show that there is no measurable difference between the premium paid for this reinsurance layer and premiums paid for other risky investments including Baa subordinate bonds and S&P 500 index.

One can use this model to evaluate the risk/reward relationship for other loss exposures including future extensions of the CEA reinsurance coverage and investments in the Capital Market Layer.

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Figure 1: Risk Premium vs Risk Capacity
CEA Reinsurance Layer with Premium of $460 Million

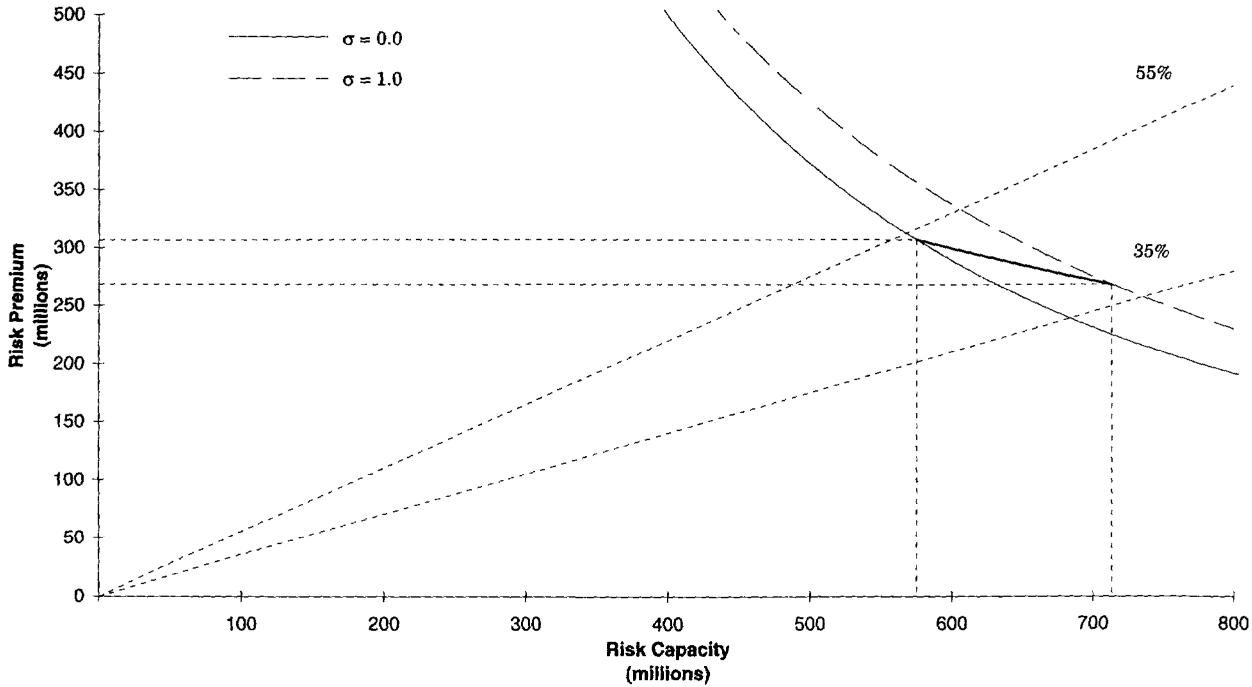

Figure 1 displays risk premium curves for the CEA reinsurance layer as a function of risk capacity. Risk premium is the amount by which premium exceeds expected losses and expenses. This figure displays two risk premium curves; one curve associated with $\sigma = 0.0$ and the second curve associated with $\sigma = 1.0$. Each of these curves shows the intersection of the observed risk premium with the risk premium curve. The percentage of risk premium to risk capacity ranges between 53% for $\sigma = 0.0$ and 38% for $\sigma = 1.0$. This graph assumes an 80% participation in the CEA.

Figure 2: Risk Premium vs Risk Capacity
$5 Million Investment in S&P 500 Index

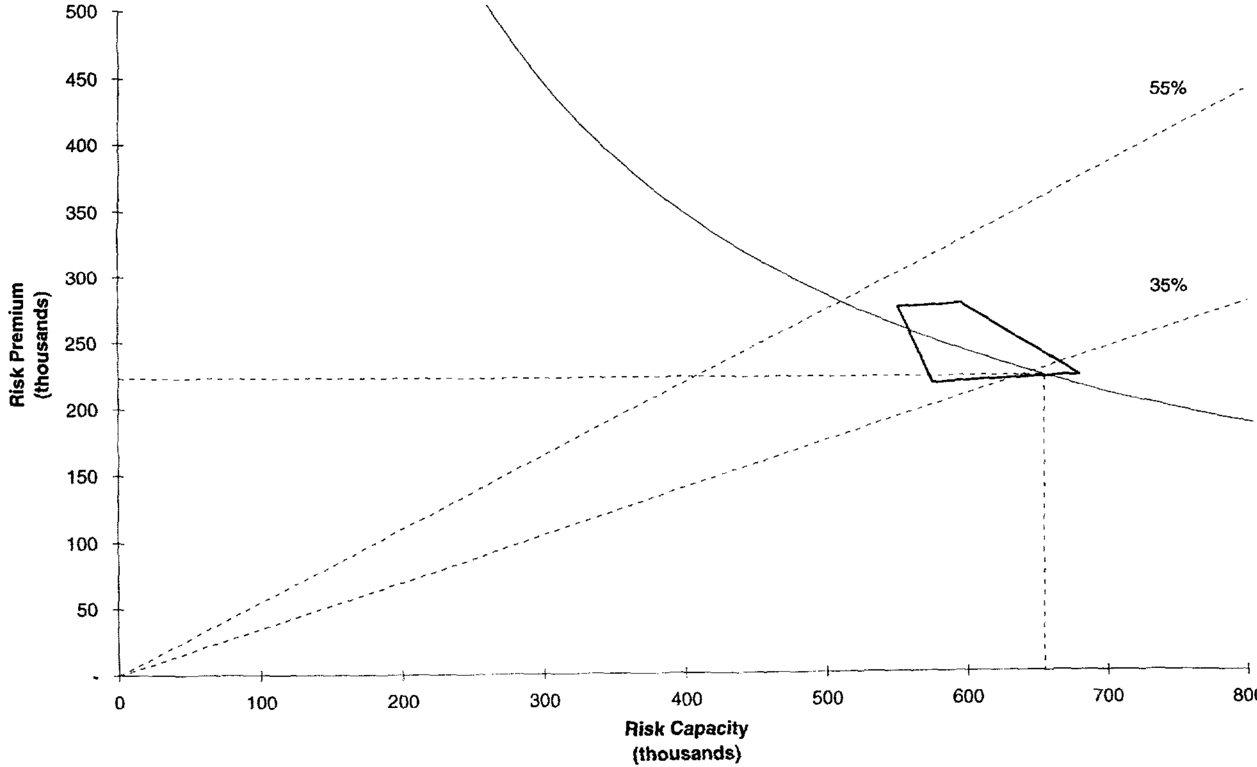

Figure 2 displays the risk premium curve versus risk capacity for an investment of $5 million in the S&P 500 composite index. We assume the S&P 500 follows a log-normal random process with drift parameter μ_S and volatility σ_S. Changes in the drift parameter do not result in any significant shifts in the risk premium curve but do affect the observed risk premium. Changes in the volatility do not affect the observed risk premium but they do affect the risk premium curve. The region displayed in the graph is the intersection of the observed risk premium with the risk premium curve for various combinations of μ_S and σ_S; the actual combination for a particular investment in the S&P 500 index depends on market timing.

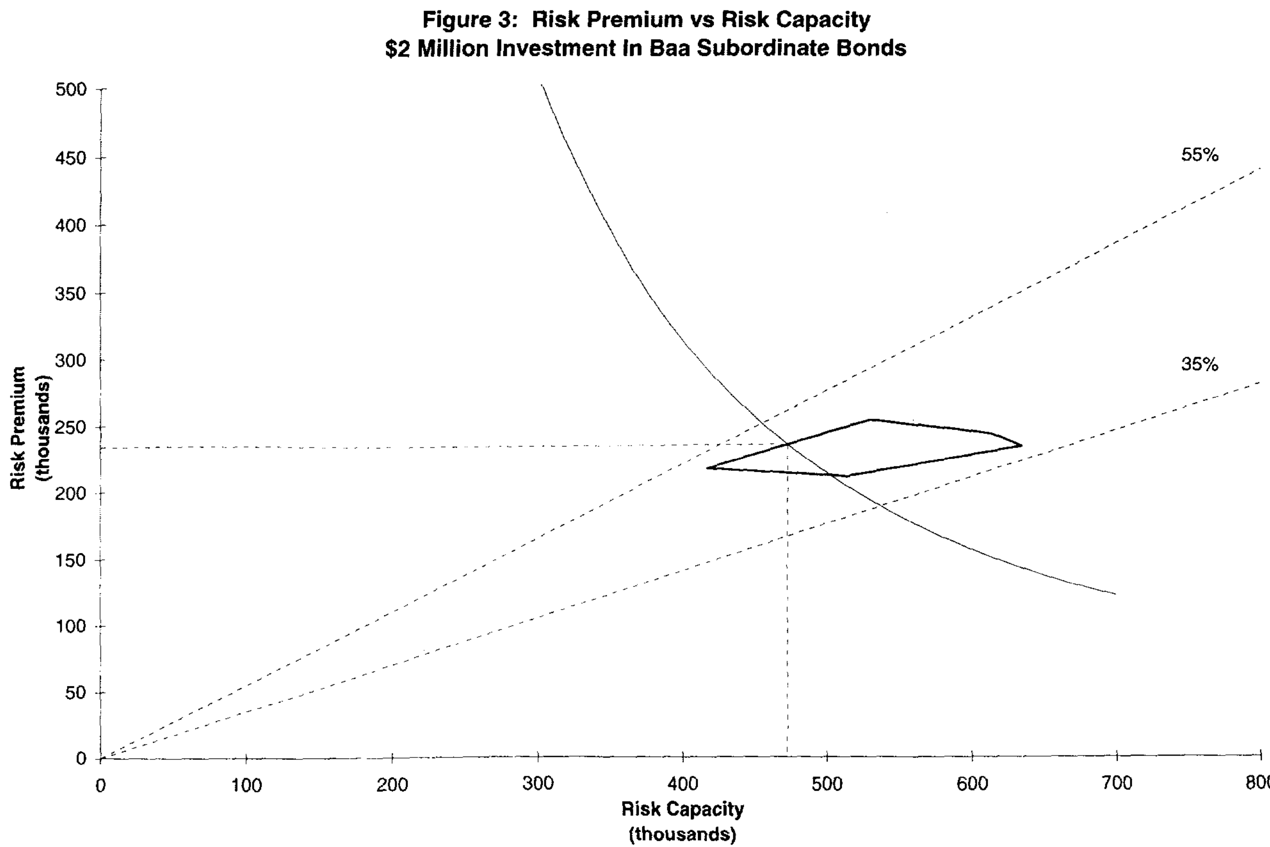

Figure 3 shows the risk premium versus risk capacity for a $2 million investment in Baa subordinate debt. The displayed curve assumes a coupon rate of 7.5% and a time to maturity of 20 years. We used default rates and severity estimates from Moody's. By varying the assumption about the coupon rate from 6.5% to 8.5% and the time to maturity from 15 to 20 years, one obtains a risk premium -- risk capacity combination somewhere in the circled region.

Figure 4: Risk Premium vs Risk Capacity
CEA Reinsurance Layer with Premium of $385 Million

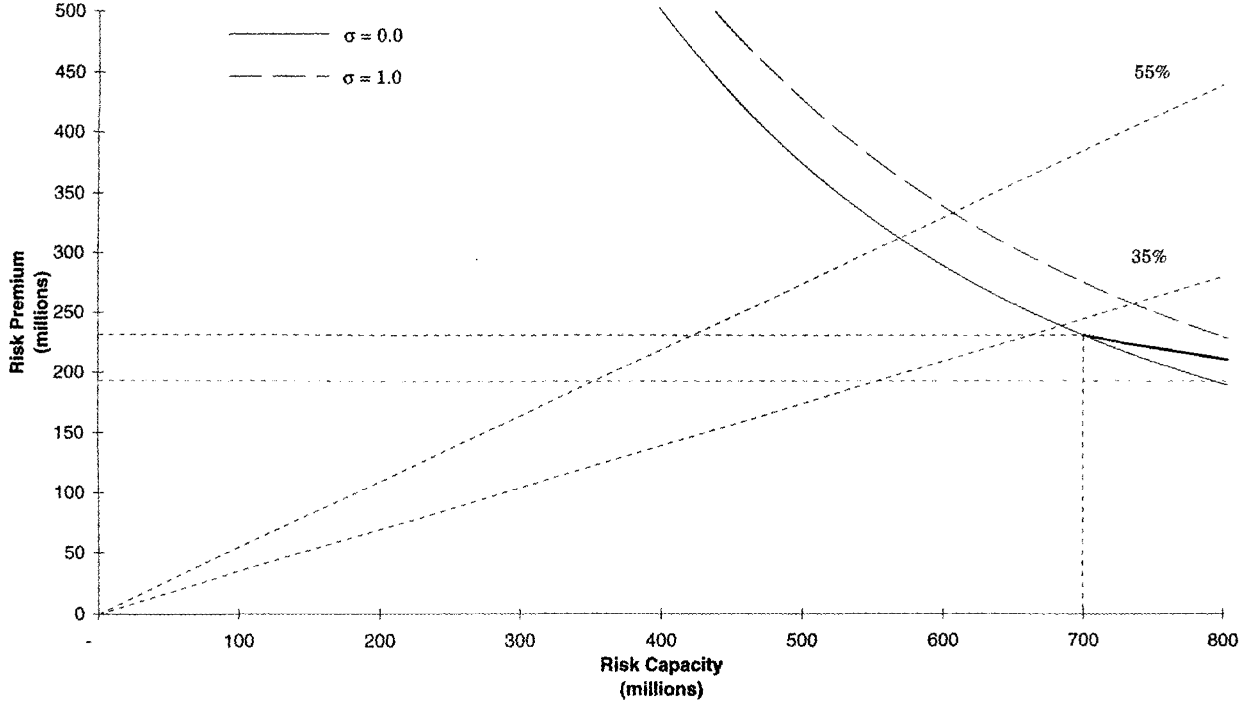

Figure 4 displays the effect a decrease in premium of about 15%, or $75 million, has on the risk premium and the implied capacity displayed in Figure 1. Since the ratio of observed risk premium to implied risk capacity is below the 35% to 55% interval, the risk/reward associated with this premium is lower than other comparable investments in the financial markets. This suggests that this premium is too low compared to the associated risk. This graph assumes an 80% participation in the CEA.

Figure 5: Risk Premium vs Risk Capacity
CEA Reinsurance Layer with Premium of $535 Million

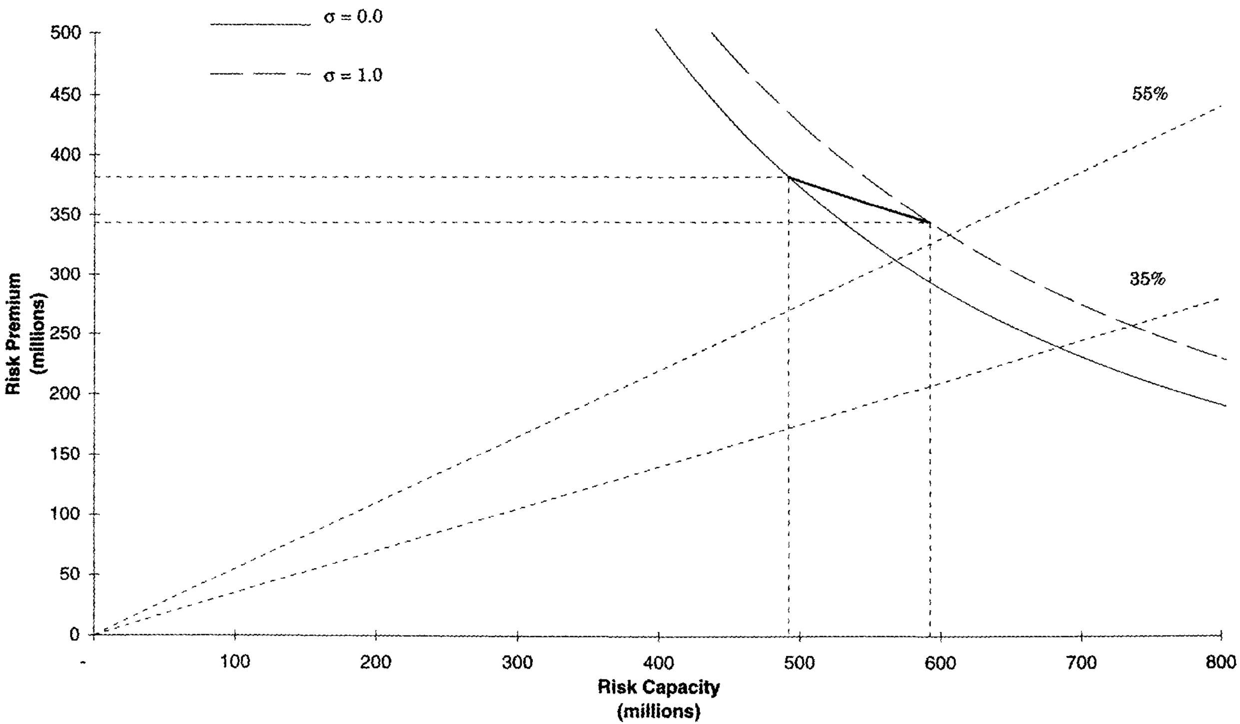

Figure 5 displays the effect a increase in premium of about 15%, or $75 million, has on the risk premium and the implied capacity displayed in Figure 1. Since the ratio of observed risk premium to implied risk capacity is above the 35% to 55% interval, the risk/reward associated with this premium is higher than other comparable investments in the financial markets. This suggests that this premium is too high compared to the associated risk. Th[illegible] an 80% participation in the CEA.

Appendix 1: Simulation Assumptions

Number of simulations	10,000
Percentage Market Participation	80.0%
Layer Assumptions at 100% Participation (millions)	
Initial Capital Layer	1,000
1st IAL Layer	3,000
Reinsurance Layer	2,000
Bond Layer	1,000
Capital Market Layer	1,500
2nd IAL Layer	2,000
Premium Assumptions	
Total Annual Premium at 100% Participation	1,000
Annual Growth in Premium	0.0%
Expense Description	
Commissions Per Premium	10.0%
General Expenses Per Premium	5.0%
Interest Assumptions	
Yield on Cash and Investments	6%
Capital Markets Cat Notes Rate	8%
Capital Markets Term Notes Rate	11%
California State Bond Rate	5%

Exposure to losses rose proportionally in the first year from 0% at inception to 100% at the end of the first year.

Appendix 2: Risk-adjusted Value

We use a risk-adjusted value formula derived from the following set of axioms:

1. There is a frontier of opportunities that are optimal for the firm. The role of the decision-maker is to quantify and identify this frontier.

2. The decision-maker should be risk averse.

3. There are no riskless arbitrage opportunities. That is, the decision-maker would never pay more to avoid a loss than the amount of the loss. In turn, no individual would be able to sell insurance for a premium greater than the amount of the exposure.

4. The evaluation of an alternative is robust with respect to the input data. That is, a small change in an input parameter should not lead to a large change in the evaluation of an alternative.

5. The evaluation of an alternative is robust with respect to the analytical process one is using. For example, making small refinements to a particular scenario should not drastically change the evaluation of a particular alternative.

6. The evaluation of an alternative is robust to changes in the time scale. For example, changing the time intervals of the analysis from quarterly to monthly should not have a significant change in the evaluation of an alternative.

7. If there is no risk, one can determine the present value of a stream of future cash flows by discount factors derived from the term structure of interest rates.

These axioms imply that the firm, or decision-maker, must base his or her decisions using an exponential utility function. This conclusion does not say that either the individual making the decisions for the firm or the firm itself has an exponential utility function. It only says that if a firm would want to make decisions consistent with the above axioms then they must evaluate the alternatives assuming the firm has an exponential utility function.

This approach to evaluating risky investments satisfies the following properties:

1. $RAV_c(R) = R$, if R is known with certainty,

2. $RAV_c(R_1 + R_2) = RAV_c(R_1) + RAV_c(R_2)$, if R_1 and R_2 are independent,

3. $RAV_{kc}(kR) = k\, RAV_c(R)$,

4. $RAV_c(R)$ is equivalent to a variance load if R is distributed Normal,

5. $RAV_c(R) > E(R)$ for all c,

6. RAV_c is a decreasing function in c with $RAV_c(R) \to \infty$ as $c \to 0$ and $RAV_c(R) \to E(R)$ as $c \to \infty$.

One can use this theory to derive the risk-adjusted value corresponding to the CEA reinsurance layer. Let p_0 represent the probability of incurring no loss, p_1 represents the probability of incurring a total loss of $1,600 million, and λ the parameter of the truncated exponential distribution. Then, the risk-adjusted value of the reinsurance layer is

$$RAV_c(R) = c\ln\left[E\left(e^{\frac{R}{c}}\right)\right], \text{ where } R \text{ is the ceded loss}$$

$$= c\ln\left[p_0 + p_1 e^{\frac{1600}{c}} + (1 - p_0 - p_1)\left(\frac{\lambda}{\lambda - c^{-1}}\right)\left(\frac{1 - e^{-1600(\lambda - c^{-1})}}{1 - e^{-1600\lambda}}\right)\right]$$

Appendix 3: Modeling the Earthquake Loss Exposure

Simulated Earthquake Losses

The yearly earthquake loss, X, of the CEA simulation model is the product of two random variables:

$$X = YL \tag{1}$$

where Y is the random loss sampled from EQECAT earthquake loss estimates and L is a random variable representing uncertainty about the EQECAT estimates. We assume L to be log-normal with $\mu = 0$ and σ measuring the level of uncertainty. The mean increase in X over Y due to this uncertainty is

$$E(X) = E(Y)e^{\frac{1}{2}\sigma^2}$$

Reinsurer's Loss Distribution

There are three parts to the fitted model of earthquake loss to the reinsurer. First, we provided the distribution with point masses at the two end points of the loss exposure interval, namely R = \$0 and R = \$1,600 million. For losses between these two endpoints, we assumed a one parameter truncated exponential distribution. Table 3 shows the Kolmogorov-Smirnov goodness of fit statistics for four different fits of σ. Figures A-1 and A-2 plot the empirical and fitted distribution for $\sigma = 0.0$ and $\sigma = 1.0$ respectively. Although this study only discusses the results for two values of σ, we also performed these calculations on two other values of σ: $\sigma = 0.7$ and $\sigma = 0.9$. For $\sigma = 0.7$, $\sigma = 0.9$ and $\sigma = 1.0$, there is no evidence to suggest that there is any difference between the empirical distribution and the fitted distribution at the 5% significance level. For $\sigma = 0.0$, the fitted distribution is rejected at the 5% level. In Figure A-1, one can see that between losses of \$0 and \$700, the empirical distribution jumps up erratically four different times suggesting that there is no simple parametric distribution that fits this data. Since we wanted to keep the model simple, we used the truncated exponential distribution for all values of σ.

σ	Parameter Estimate	Kolmogorov-Smirnov statistic	5% critical value
0.0	1.63	0.0755	0.0477
0.7	1.34	0.0229	0.0483
0.9	1.15	0.0276	0.0490
1.0	1.07	0.0382	0.0498

Table 3: Fit of Truncated Exponential Distribution

Table 4 displays the fitted parameters of the ceded loss distribution using maximum likelihood approach. The fitted and simulated mean losses to the reinsurer, given that losses occurs, are very similar under all four scenarios.

σ	Probability of No Loss p_0	Probability of Complete Loss p_1	Estimate of Trunc. Exp. Parameter λ	Simulated Average Loss, Given Loss > \$0	Fitted Average Loss, Given Loss > \$0
0.0	85.3%	6.6%	1.632	1,043.60	1,043.29
0.7	85.1%	7.0%	1.340	1,084.24	1,084.20
0.9	84.2%	8.1%	1.148	1,137.15	1,137.14
1.0	83.7%	8.9%	1.073	1,171.52	1,171.52

Table 4: Estimated Parameters of Loss Distribution

Figure A-1: Fitted vs Simulated - Losses Between $0 and $1,600 Million

σ = 0.0

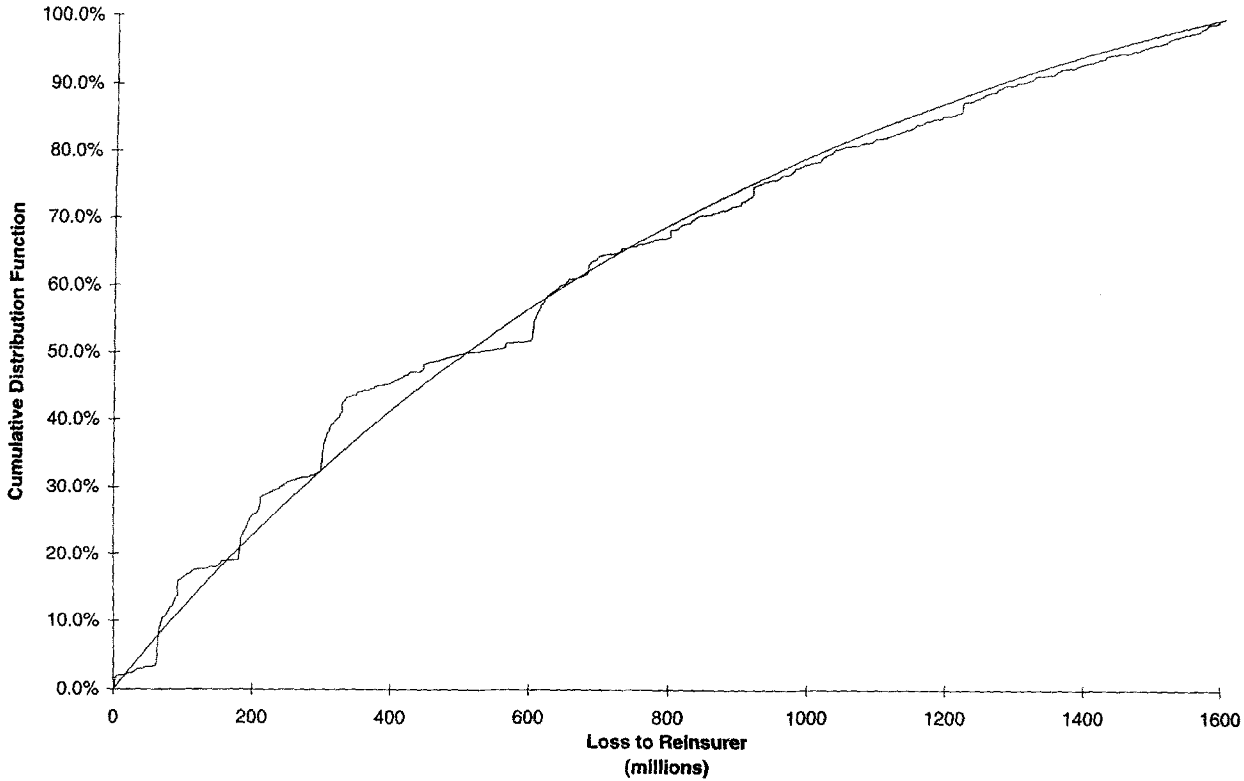

Figure A-1 plots the empirical and fitted ceded loss distribution for σ = 0.0. Using Kolmogorov-Smirnov goodness of fit test, this fitted distribution is rejected at the 5% significance level. One can see that between losses of $0 and $700, the empirical distribution jumps up erratically four different times suggesting that there is no simple parametric distribution that fits this data. This graph assumes an 80% participation in the CEA.

Figure A-2: Fitted vs Simulated - Losses Between $0 and $1,600 Million
σ = 1.0

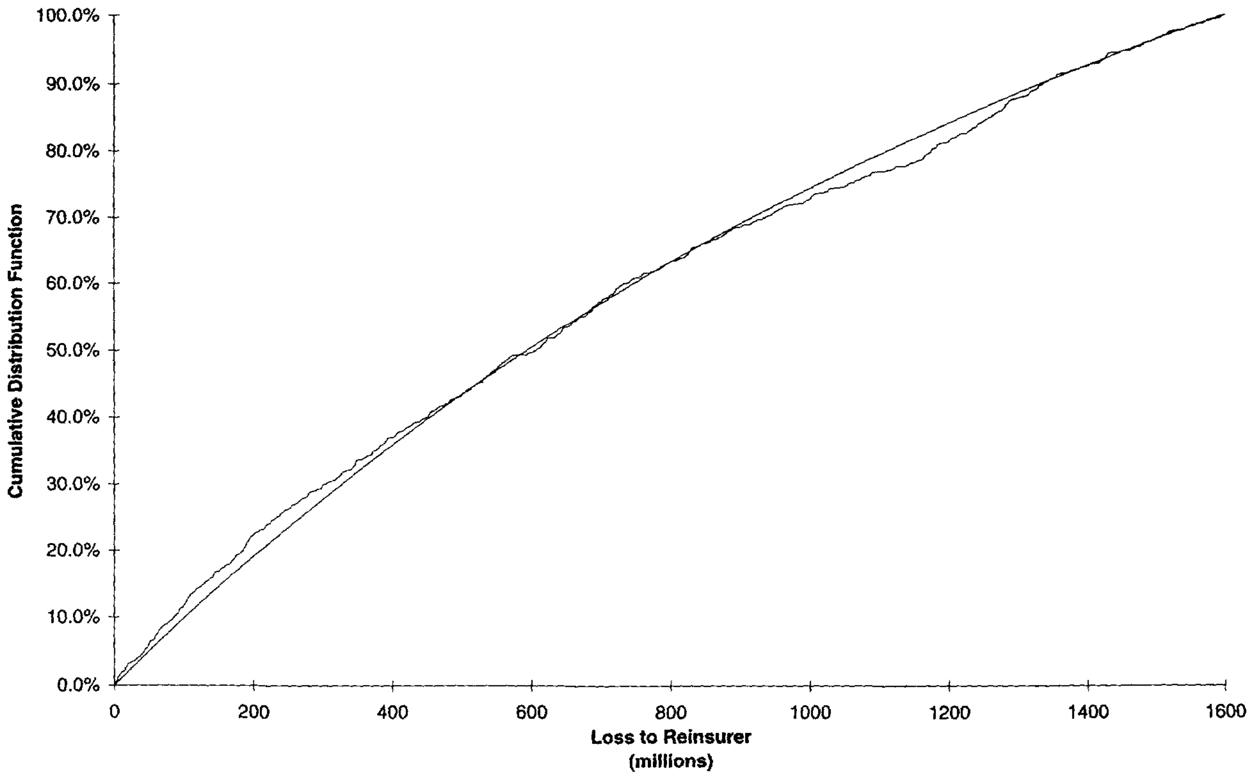

Figure A-2 plots the empirical and fitted ceded loss distribution for σ = 1.0. This fitted distribution cannot be rejected using Kolmogorov-Smirnov goodness of fit test at the 5% significance level. This graph assumes an 80% participation in the CEA.

Simulation Models for Self-Insurance

by Trent R. Vaughn

Simulation Models for Self-Insurance

Trent R. Vaughn, FCAS

Abstract

Actuaries are increasingly utilizing simulation models in a variety of practical applications. Most of these applications have focused on the future operating results and financial condition of an insurance company. These simulation models are also applicable in the risk management field, focusing on the financial consequences of self-insurance. This paper discusses the special considerations and applications of applying simulations to self-insurance. The author also discusses the integration of these results with the long-term and short-term financial plan of the self-insured.

1. INTRODUCTION

Many large commercial insureds choose to finance pure risk exposures with a combination of a self-insured retention (SIR) and excess insurance protection. The insured's retention is usually stated as a given dollar amount per claim (or per occurrence). *Specific excess* insurance provides coverage against individual losses in excess of this retention, up to the insurer's policy limit.[1] *Aggregate excess* insurance provides coverage against aggregate retained losses exceeding a given amount.[2] This aggregate excess insurance only covers losses arising from the insured's SIR; individual large losses above the specific excess policy limit are not covered by the aggregate policy.[3]

The insured will consider many issues before deciding on the appropriate combination of SIR and excess insurance coverage. Most insureds will forecast the expected cost of a risk financing program over several years, incorporating this cost into the pro forma financial statements. However, the diligent financial manager is not interested only in the forecasted cost of the risk financing program, but also in the variability of this cost.

From this standpoint, one may perform a simulation of the costs of the program over several years. This simulation should incorporate the stochastic nature of all the relevant

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[1]Donald S. Malecki et al., *Commercial Liability Risk Management and Insurance*, vol. II (Malvern, Pa: American Institute for Property and Liability Underwriters, 1986), 342.

[2]Ibid., 342-343.

[3]Peter W. Rice, *The Risk Funding and Self-Insurance Bulletins*, vol. I (Cincinnati: National Underwriter Co., 1992), Ec-6.

variables, including aggregate retained losses, insurance premiums, and administrative expenses. A realistic simulation model should also include advanced features, such as a stochastic model of inflation, business cycles, and dynamic control.

This paper considers the unique aspects of applying simulation models to self-insurance situations, as well as potential uses of these models. Section 2 outlines the simulation of aggregate retained losses. Section 3 discusses the simulation of administrative expenses and excess insurance premiums. Section 4 details applications of the model.

2. TECHNIQUES FOR SIMULATING AGGREGATE LOSSES

Simulating aggregate losses for self-insureds presents special difficulties. Aggregate retained losses usually represent the largest portion of each year's total risk financing cost. However, excess insurance premiums can be subject to profit share and experience rating arrangements. Thus, in addition to aggregate retained losses, the model must also simulate aggregate excess (insured) losses for each year.

Current Methods

The simplest and most-familiar method of simulating an aggregate loss distribution is straightforward simulation.[4] This method proceeds by first simulating the number of claims, then producing the dollar amount of each claim. The advantage of straightforward simulation is that the loss amount is available for each individual claim. Thus, when we are dealing with insurance contracts which attach on a claim-by-claim basis, this method can easily determine the gross and

[4]James N. Stanard, "A Simulation Test of Prediction Errors of Loss Reserve Estimation Techniques," *PCAS LXXII*, 1985, 124-148.

ceded (insured) aggregate loss amounts. However, Daykin, et al., point out that straightforward simulation is "only applicable in the (rare) cases where the number of claims is fairly small and the claim size d.f. is easy to handle."[5]

In contrast to straightforward simulation, Daykin, et al., offer a "short cut" method, utilizing the Wilson-Hilferty (WH) formula.[6] Unfortunately, the inaccuracy of the WH formula increases dramatically as the skewness of the aggregate loss distribution increases.[7] Most simulation models circumvent this problem by defining both the individual claim size and aggregate claim amount variables net of claim-by-claim insurance (or reinsurance) arrangements. This convention significantly reduces the skewness of the aggregate claim amount distribution, allowing the WH formula to provide an acceptable approximation. Yet, by defining the relevant distributions net of insurance, we lose valuable information regarding individual large losses.

Proposed Hybrid Method

This section presents a hybrid simulation method, which combines the advantages of both current methods, while avoiding the inherent drawbacks. The model is briefly described, with mathematical details contained in Appendix A.

First, simulate the claim number variable, **k**=k, which is often assumed to have a mixed Poisson distribution. Next, utilizing the (gross) claim size distribution, determine the probability, p_e, that an individual claim will exceed the insured's retention. Then, generate the number of

[5]C.D. Daykin, T. Pentikainen, and M. Pesonen, *Practical Risk Theory for Actuaries* (London: Chapman & Hall, 1994), 143.

[6]Ibid., 144-145.

[7]Ibid., 132.

excess claims (k_e=k_c) as a random draw from a binomial distribution with parameters n=k and p=p_e. The number of claims that are strictly less than the retention is then given by $k_p = k - k_e$.

The aggregate losses pertaining to the k_p primary claims can then be simulated by means of the conditional WH generator, utilizing the censored (at the insured's retention) claim size distribution. The conditional WH generator slightly modifies the WH procedure, providing the aggregate loss amount when the number of claims is known (k_p in this case).[8] Lastly, the dollar amount of each of the k_e excess claims can then be simulated utilizing the conditional (given a claim amount in excess of the retention) claim size distribution and straightforward simulation.

The portion of each excess claim which exceeds the policy limit of the specific excess insurance protection is considered part of the retained (by the insured) loss. Thus, aggregate retained losses are equal to the sum of the following: (1) aggregate losses pertaining to the k_p primary claims, (2) the number of excess claims (k_e) multiplied by the insured's retention, and (3) the portion of any excess claim which exceeds the policy limit of the specific excess. Aggregate losses from sources (1) and (2) may also be subject to an aggregate excess policy.

The portion of each excess claim within the policy limit of the insurance is attributed to the loss experience of the specific excess contract. Hence, the aggregate losses attributed to the specific excess policy are easily determined by summing the amount of each excess claim between the insured's retention and the policy limit.

Provided that the insured's SIR is reasonably high, the number of excess claims which must be individually simulated (k_e) is sufficiently small. Moreover, the conditional claim size distribution (given that the claim is larger than the SIR) can be easily determined for claim size d.f.'s that are expressed in either analytic or tabular form. In fact, even in cases where the tabular

[8]Ibid., 146.

method is used for the claim size d.f., the tail of the distribution is still often expressed in analytic form.[9]

Finally, note that the aggregate losses for the k_p primary claims are simulated by means of the conditional WH-generator. The skewness of this aggregate loss distribution is usually low enough for the WH formula to provide an acceptable level of accuracy, since the tail of the claim size distribution has been removed by specific excess insurance.

Treatment of Allocated Loss Adjustment Expenses

Specific excess insurance contracts are usually designed to handle allocated loss adjustment expenses (ALAE) in one of two ways. First, the ALAE amount can be added to the indemnity loss prior to the application of the insured's SIR and policy limit. If this treatment of ALAE is employed, the individual size of loss distribution utilized in the simulation should be based on the total amount of indemnity loss and ALAE per claim.

Alternatively, the ALAE for each claim is prorated according to each party's share of the indemnity loss. In this case, the claim size distribution pertains only to indemnity loss; ALAE must be simulated separately--for both the k_p primary claims and the k_e excess claims.

For the k_p primary claims, the entire ALAE amount is attributed to the insured. This ALAE amount is dependent on both the number and the aggregate dollar amount of primary claims. Additionally, ALAE is incurred on claims which close with no payment; this expense will exhibit much year-to-year variation.

For the k_e excess claims, the ALAE amount must be simulated for each individual claim, then prorated according to the corresponding indemnity amount. This requires a bivariate model,

[9]Ibid., 75.

where the probability distribution of ALAE is dependent on the size of the indemnity loss. Details are provided in Hogg and Klugman.[10]

Determining the Payment Pattern

The simulation model discussed above is easily modified to produce aggregate retained losses by accident year and payment year. The expected claim count for each accident year is first allocated by payment year based on the estimated probability of a claim settling in the nth payment year. Aggregate losses for each combination of accident year/payment year are then simulated separately. This procedure is discussed in detail in Daykin, et al.[11]

A necessary refinement to the Daykin procedure is to allow the individual claim amount distribution to vary with the payment year. For instance, the mean and variance of the claim severity distribution will usually increase as the payment year (relative to the accident year) increases.[12]

The ALAE amount for each combination of accident year/payment year is simulated as discussed above.

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[10]Robert V. Hogg and Stuart A. Klugman, *Loss Distributions* (New York: John Wiley & Sons, 1984), 167-177.

[11]Daykin, Pentikainen, and Pesonen, 296-298.

[12]Emanuel Pinto and Daniel F. Gogol, "An Analysis of Excess Loss Development," *PCAS LXXIV*, 1987, 245-247.

ulation of Losses for Multiple Accident Years

For most applications, the simulation will extend over a period of several accident years. n analyzing multiple accident years, we must consider both changes in the claim severity distribution and the claim number distribution.

Changes in the claim severity distribution are elicited by claims inflation. In order to create a realistic model of uncertainty, we need to introduce a stochastic model of claims inflation. The loss severity distribution for each accident year/payment year is adjusted by the simulated claims inflation rate, as discussed in Daykin, et al.[13]

This stochastic model of inflation is a crucial submodel in the self-insurance context; great care should be exercised in accurately modeling inflation's future variability. Many insureds will find that exposures which are subject to extremely volatile inflation are not well suited for self-insurance.[14]

Also, in extending the model for several years, the number of claims will be affected by the growth in the self-insured portfolio. In this context, the number of claims by accident year is best modeled as the product of the claim frequency and the number of exposure units. The actual claim frequency (claims per exposure unit) will depend on trends, cycles and short-term (stochastic) fluctuation.[15] The number of exposure units is largely determined by the business plan of the insured, and may be treated as a deterministic variable. With this treatment, we are modeling the variation in risk financing costs for a given business plan.

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[13]Daykin, Pentikainen, and Pesonen, 282-283.

[14]Rice, Ar-10.

[15]Daykin, Pentikainen, and Pesonen, 40-41.

3. SIMULATION OF ADMINISTRATIVE EXPENSES AND EXCESS INSURANCE PREMIUMS

Administrative Expenses

For self-insureds, administrative expenses can be a substantial portion of overall risk financing cost. These administrative expenses include the cost of services such as MIS, loss control, actuarial, and claims handling expenses that are not allocable to a specific claim. Usually, allocated loss adjustment expenses are included in the aggregate loss distribution, as the amount of these expenses is closely related to the dollar amount of indemnity loss (Section 2). A realistic long-term simulation should consider the stochastic behavior of these administrative expenses.

The model should separate in-house expenses from the cost of services purchased externally. In-house expenses, which largely consist of salaries, fringe benefits, and overhead items, may increase through time broadly in line with general earnings inflation and the growth of the self-insured portfolio. Other in-house expenses, such as the cost of MIS equipment, are most accurately modeled as step-increase expenses,[16] with the number of incurred claims determining the position of the steps. For example, a certain number of claims may exceed the capacity of the current management information system, prompting additional investment.

External costs may exhibit much more fluctuation, as they are affected by the specific forces of supply and demand for these types of services. These costs should also increase broadly in line with earnings inflation and growth of the portfolio, but with larger year-to-year variations.

Additionally, the model should include the cost of assessments, fees and taxes which are often levied on self-insurers. For example, firms which self-insure workers compensation

[16]Ibid., 323.

posures are often required to contribute to second-injury funds.[17] A detailed discussion of the ature and extent of these assessments is beyond the scope of this paper.

Lastly, one can also monitor varying levels of loss control efforts. For example, the model may specify that if aggregate retained losses (insured losses may also be included) exceed a certain threshold, this will trigger a strategic impact variable,[18] representing the cost of improved loss control efforts. Presumably, this loss control effort would reduce the future frequency and/or severity of retained losses. Such a feature requires careful consideration, as not all exposures will respond to loss control efforts.

Excess Premiums and Business Cycles

Many articles and publications have dealt extensively with the issue of the insurance underwriting cycle. Empirical evidence suggests that the underwriting results of the insurance industry, measured by the combined ratio, fluctuate in an irregular cyclical fashion.[19] While self-insurance offers some protection against the underwriting cycle, self-insureds and captives are still exposed to the risk of a hard market in the area of excess insurance or reinsurance.

In the short-term, the commercial premium for excess coverage is easily determined. For a guaranteed cost policy, the amount and timing of the payments are fixed in advance, subject only to possible audit adjustments. However, over the long-term, the price and availability of excess coverage are subject to the vagaries of the insurance underwriting cycle. In fact, evidence

[17]Rice, vol. II, Da-1 through Dwy-1.

[18]Daykin, Pentikainen, and Pesonen, 323-324.

[19]Steven P. D'Arcy, "Investment Issues in Property-Liability Insurance," *Foundations of Casualty Actuarial Science* (New York: Casualty Actuarial Society, 1990), 510-514.

exists, in the context of reinsurance, that the underwriting cycle is more severe for excess business.[20] Moreover, this excess insurance cycle will not necessary coincide with the underwriting cycle in the primary market.

A realistic long-term model should incorporate the cyclical aspects of excess coverage premiums. Methods for accomplishing this are discussed in Daykin, et al.[21] In particular, the model should consider the potentially harsh consequences of a hard market occurring early in the life of a self-insurance program, possibly leaving the insured without excess protection at reasonable prices and terms.

Excess Premiums and Experience Rating

Many insureds purchase numerous layers of specific excess coverage, each from a different insurer. In this manner, the insured is not financially reliant entirely on one carrier. However, the disadvantage of such an approach is that conflicts often arise between the various insurers.[22] Moreover, the total insurance coverage may be more costly, as the insurers are duplicating certain underwriting functions.

A reliable long-term model should recognize that future premiums for lower level coverage will be affected by the loss experience incurred under these contracts, in addition to the premium effects caused by the market cycle. Possible mechanisms for accomplishing this are

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[20]R.C. Reinarz et al., *Reinsurance Practices*, vol. II (Malvern, Pa.: Insurance Institute of America, 1990), 90-91.

[21]Daykin, Pentikainen, and Pesonen, 327-356.

[22]Rice, vol. I, Ec-4.

xperience rating and exponential smoothing,[23] which can be incorporated into the model. The higher level excess contracts may be regarded as more of a pure risk situation; insurers will not usually require "payback" of a large loss, nor should the insured expect a refund from favorable experience.[24]

Since the simulation model outlined in Appendix A produces individual information on each excess claim, determining the experience of each insured layer is possible. However, in long-term analysis, one should consider the effect of delayed payment of claims ("run-off error"). Generally, the experience under excess contracts takes much more time to develop and assess profitability. In fact, the insurer may not acquire reliable knowledge regarding the true profitability of the contract until many years after it has expired. Thus, the time lag between the loss experience and its reflection in premium rates is usually longer for excess coverage.

The model builder may also incorporate the risk that very adverse loss experience will leave the insured without coverage at any price, forcing complete self-insurance.

Dynamic Control - Modifying the Level of Excess Insurance Protection over Time

Generally, the insured will gradually increase the retention and policy limit of the specific excess roughly in lockstep with inflation. However, the amount of excess protection desired and available is also dependent on market conditions and the insured's loss experience.

In practice, the management of self-insured firms is constantly fine tuning the risk financing strategy. When the market is soft, many insureds purchase additional insurance

[23]Daykin, Pentikainen, and Pesonen, 180-183.

[24]Richard F. Gilmore, "Planning and Managing a Reinsurance Program," *Reinsurance*, ed. R.W. Strain (New York: College of Insurance, 1980), 384-385.

coverage (through higher limits and/or reduced deductibles). Likewise, insureds often rely more heavily on self-insurance when the market hardens, out of desire or necessity.

As discussed above, the premium for lower level excess contracts is also influenced by the developing loss experience. Unfavorable results will prompt the excess carrier to increase the required premium, possibly with a time lag. If this premium becomes too high, the insured may choose to increase the attachment point of the coverage.

The final model may allow the attachment point (and possibly policy limit) of the excess coverage to respond to market conditions and loss experience. However, it is important to recognize the limitations on this flexibility. A significant self-insurance program, once in place, is not easy to abandon; staff and resources have been dedicated and may not be easily transferable to other areas.

4. APPLICATIONS

Presentation of Results

For each year of the planning horizon, the simulation model will produce a probability distribution for the total risk financing cost, on both a cash basis and an accounting basis. The primary distinction between the cash model and the accounting model is in the treatment of retained losses. In the cash model, the risk financing cost for each year includes only the paid retained losses; in the accounting model, the risk financing cost includes the total retained losses from claims occurring any time during the year, regardless of when the payment is made ("incurred losses"). Daykin, et al., discuss the relationship between the two methods of defining losses, including the treatment of run-off error and discounting in the accounting model.[25]

This distinction between paid and incurred losses is crucial, as most self-insureds are only permitted to deduct paid losses from each year's taxable income. When an insured initially switches from fully insured status to a self-insurance program, federal income taxes can be expected to increase, at least in the first few years of the program. This is especially true for self-insurance of long-tailed exposures, where the payment date may extend several years past the occurrence date.

The cash model is most useful for the applications discussed below. The accounting model is not discussed in detail, but it can be useful for demonstrating the potential impact of a risk financing program on future financial accounting statements.

Comparison of Alternative Programs

The simulation model is well suited for comparing the relative cost savings and cost stability of alternative risk financing solutions, over a period of several years. A fully insured strategy is a good "base case" scenario from which to compare alternatives.

A common criticism of simulation routines is that the results for alternative scenarios may be dramatically influenced by sampling error.[26] Daykin, et al., respond that "the impact of the distortion resulting from sampling errors can be greatly reduced by using the same sequence

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[25]Daykin, Pentikainen, and Pesonen, 9-10.

[26]Sholom Feldblum, "Forecasting the Future: Stochastic Simulation and Scenario Testing," *Incorporating Risk Factors in Dynamic Financial Analysis* (Landover, Md: Casualty Actuarial Society, 1995), 156.

of random numbers for each of the concurrent simulations (i.e. always starting from the same specified seed of the primary random number generator)."[27]

Applications to Long-Term Financial Planning

Most business firms rely on a combination of debt and equity for long-term financing needs. The trade-off theory of capital structure proposes that business firms determine the optimal debt-equity ratio as a trade-off between interest tax shields and the costs of financial distress.[28] The risk financing decision interacts with this trade-off theory by affecting the taxes and business risk of the firm.

For instance, most financial economists agree that there is a moderate tax advantage to corporate borrowing for firms which are reasonably confident that they will be in a taxpaying position.[29] The cash basis simulation model displays the probability distribution of tax deductible risk financing costs for each year in the planning horizon. This distribution can be used to calculate the probability of these costs exceeding a given threshold at which the firm would not be able to fully utilize tax deductible interest payments during any given year in the future. If this probability is significant, the tax advantage of the debt is decreased.

Secondly, for a given level of debt, the probability of financial distress increases as the business risk of the firm increases. The cash basis simulation model can be used to estimate the probability of risk financing costs exceeding a given threshold, causing the firm to default (or

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[27]Daykin, Pentikainen, and Pesonen, 154.

[28]Richard A. Brealey and Stewart C. Myers, *Principles of Corporate Finance* (New York: McGraw-Hill Book Co., 1991), 434.

[29]Ibid., 447.

have difficulty meeting) its principal and interest obligations during any year in the future. As this probability of financial distress increases, the expected cost of financial distress also increases.

The distribution of risk financing costs during each year of the planning horizon will also influence the timing of debt and equity issues. For example, if an insured switches from fully insured status to self-insurance, next year's expected cash outflow may be dramatically reduced. Depending on the variability of these risk financing costs, the insured may want to invest part of the savings in an increased level of net working capital. However, the balance can be invested in operations or utilized to reduce long-term debt or equity.

Applications to Short-Term Financial Planning

The cash basis model can also be utilized to demonstrate the impact of the risk financing program on short-term cash planning. Although most simulation models are designed to display results at annual intervals, a simulation can be created to demonstrate the variability of risk financing costs during the next 12 months. This model can be extremely useful to insureds in determining net working capital requirements. For example, the model may reveal the significant probability of a required cash outflow which would exceed the insured's cash balance, line of credit, and other sources of short-term borrowing (such as "stretching payables").[30] This may prompt the insured that an increase in net working capital is required.

This discussion exposes an often-overlooked cost of self-insurance. Self-insurance increases the short-term variability in cash outflow, often increasing net working capital requirements.

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[30]Ibid., 736.

Relationship to Insured's Business Plan

The discussion above analyzes the variability in risk financing costs, relative to the insured's projected cash flows net of risk financing costs. However, the insured's cash flow from operations (net of risk financing costs) is dependent on the risk financing costs. For example, if risk financing costs are exaggerated by an unusually high rate of claims inflation, this high inflation will also affect other areas of the insured's operation (for example, revenues and cost of goods sold).

If the insured utilizes a Monte Carlo simulation model for other aspects of the business, this model can be integrated with the risk financing model. The overall model should recognize that claims inflation reflects general price inflation as well as social inflation.

If the insured does not utilize a Monte Carlo simulation, the results of the risk financing model are still valuable in demonstrating the variability involved in alternative risk financing solutions.

5. CONCLUSION

Actuaries are increasingly utilizing simulation routines for a variety of practical applications; most of these applications focus on the future operating results and financial performance of insurance companies. Simulation models are also applicable in the risk management field, focusing on the financial consequences of self-insurance of pure risk exposures.

The application of simulation models to self-insurance presents unique challenges. Excess insurance will protect the insured from a catastrophic frequency or severity of losses, but premiums for this excess coverage are subject to outside influences. Also, many insureds are capable of mitigating their loss exposures through loss control programs. A realistic model should reflect management's ability to respond to changing conditions in loss experience and in the insurance marketplace.

These simulation models will assist the risk manager in evaluating the cost and cash flow variability inherent in alternative risk financing strategies. These results can be incorporated into the overall financial plan of the insured. In this manner, simulation models will serve as a useful planning tool for risk managers, and as an important marketing tool for insurers.

Appendix A - Hybrid Simulation Model for Aggregate Losses

Step 1: Simulate random value of **q**=q, the mixing variable.

Step 2: Simulate random value of **k**=k, the number of claims from a mixed Poisson distribution with mean nq.

Step 3: Determine the probability, p_e, that a given claim will exceed the insured's retention, M. This probability is given by 1 - S(M), where S(x) is the (gross) claim size distribution.

Step 4: Simulate the number of excess claims, $\mathbf{k_e}$=k_e as a random draw from a Binomial distribution with parameters n = k and p = p_e.

Step 5: For each of the k_e claims, simulate the amount of the claim utilizing the conditional claim size d.f., given that a claim exceeds M. This conditional d.f. is given by:

[S(x) - S(M)] divided by [1 - S(M)] for x greater than or equal to M

Step 6: Simulate the aggregate amount of loss with respect to claims that are strictly less than the retention. The conditional claim size distribution, given that a claim is less than the retention, is given by:

S(x) divided by S(M) for x less than or equal to M

Next, calculate the moments of this conditional distribution. These moments, as well as the number of claims less than M, k_p = k - k_e, are inputs in the conditional WH generator. This conditional WH generator will produce a simulated aggregate amount of losses for claims which are strictly less than M.

Step 7: The aggregate amount of losses under M is given by the aggregate losses from step 6 plus k_e x M.

Step 8: The aggregate amount of losses in excess of M is given by the sum of the k_e loss amounts minus k_e x M.

Dynamic Financial Modeling—Issues and Approaches

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Daykin, Pentikainen, and Pesonen (as well as other authors) have described the elements that can comprise a dynamic financial model of a property/casualty insurer and have discussed options for several important elements where different approaches are reasonable. This paper further describes the issues which arise in designing and implementing a dynamic financial model and discusses possible methods of analyzing and interpreting the results of DFA analyses. Implementing such a model raises a number of difficult conceptual and technical issues. In this paper we discuss a number of important model elements and the related implementation issues.

1 Introduction

1.1 Background And Context

Historically, actuaries have opined on the adequacy of loss reserves for insurance companies and self-insuring non-insurance entities. Recently, two concerns have arisen in the actuarial profession. First, opinions regarding loss reserves do not speak fully to the greater issue of solvency. Second, loss reserve opinions are point estimates and do not address the variability which is characteristic of insurance. A technique that could address these two concerns is Dynamic Financial Analysis, where a company's complete operations are modeled and simulated.

Historically, the actuary's role focused on some well-defined technical aspects of a company's operations: pricing products; designing risk classification structures; designing and applying experience rating plans; and estimating loss reserve requirements. As the profession evolved, actuaries were called upon to apply their analytical skills and industry knowledge to a broader array of problems, including claims management, underwriting, and strategic and operations planning.

At the same time that the actuary's role was expanding to encompass these additional problems, regulators became increasingly concerned over their ability to monitor and safeguard insurers' solvency in an increasingly volatile economy and competitive business. Regulators' concerns over more effective solvency monitoring tools led to the implementation of dynamic solvency testing in Canada and risk-based capital solvency monitoring in the United States.

The expansion of the actuary's role, and the continuing development of solvency monitoring tools, have converged under the category of Dynamic Financial Analysis, or DFA. The CAS has defined Dynamic Financial Analysis as:

"... the process by which an actuary analyzes the financial condition of an insurance enterprise. Financial condition refers to the ability of the company's capital and surplus to adequately support the company's future operations through an unknown future environment." [12]

Dynamic Financial Analysis can be a tool which can provide much of the information necessary for insurance company managements, investors, regulators and others with an interest in the insurance business to:

- Evaluate an insurer's financial strength, including the likelihood of insolvency and the economic and operating scenarios which would impair surplus;
- Evaluate and measure an insurer's operating risks;
- Evaluate alternative operating strategies and tactics relative to different management objectives, including maximizing the value of the firm subject to risk constraints;

- Determine the value of an insurer.

The range of these analyses implies the use of different types of data and outcome performance measures.

Given these desired capabilities for DFA, we feel it is appropriate to define DFA, and the models which implement such analyses, as having the following distinguishing characteristics:

- DFA incorporates both the liability and asset processes;
- DFA takes into account the stochastic nature of an insurer's operations;
- DFA treats the company as an ongoing operation, and extends over a time frame defined by the transaction lifetimes of the insurance products involved, although it may also have the flexibility to evaluate a company in run-off.

1.2 Objectives

Our objectives in this paper are:

- To discuss some of the issues which arise in designing and implementing a dynamic financial analysis computer simulation model;
- To suggest possible ways to approach the modeling of some of the more important components of the insurance process;
- To suggest directions and tools for the analysis of the output of a DFA model.

Although we have considered and experimented with many of the concepts discussed in this paper, we have not completed the extensive research necessary to specify all of the interactions that may be included in a DFA model, or to estimate all of the required parameters. We feel, however, that by describing our conception of the interactions and parameters involved in modeling the insurance process, based upon our understanding of the insurance business and the potential uses of DFA, we can assist others in designing, applying, and interpreting the results of DFA models.

The process of defining a valid and useful DFA model will be an iterative process consisting of:

- Hypothesizing relationships among the relevant variables;
- Research to parameterize those relationships;
- Experiments via a series of simulations to analyze the results for reasonableness.

The research phases of this process will produce a better understanding of the insurance process being modeled, such as the mechanics of the competitive market for insurance products or the correlations of experience between lines of business. The experimental phases will test the new understandings produced by research and may also identify and rank the priority of future research efforts.

2 Modeling Insurance

2.1 Introduction

Existing models of insurance range from complex representations which attempt to describe insurance buyers' preferences and decisions as they select among different policies and non-insurance alternatives to preserve and protect their wealth, to relatively straightforward accounting representations of company experience. Where, in this range, is the appropriate level to build a DFA model? What level of detail and specific components should be included in a DFA model?

The business of insurance is complex. When an insurer enters a given product market, it must design and price the product, market it, underwrite the risks who apply for coverage, issue the policies, maintain and service the insureds, and adjust and pay claims as necessary. In addition, the company must manage its business (control its expenses; invest its assets; set reserves; control exposure concentrations; and devise and implement strategic and operating plans). And this list is far from exhaustive!

From this capsule description of the insurance business, it is apparent that its reality is too complex to be replicated in detail in a single model. Parsimony is a desirable feature of any model. One attractive aspect of modeling is precisely the reduction of complex problems to manageable proportions. A model builder therefore usually selects only the salient features of the process to be modeled, based upon hypotheses, prior knowledge of the subject real world process, or existing models of similar processes. By salient features, we mean the features of the real world process which have relatively greater effects on the process outcomes.

In this paper, where one of our objectives is to discuss the issues which arise in the design and implementation of DFA models, we consider details of the insurance process and introduce variables (well beyond what even Tolstoy could consider parsimonious) which might be considered unimportant in other contexts. The importance or unimportance of some of the features mentioned will ultimately only be determined by the implementation and application of actual DFA models incorporating those features.

2.2 The Insurance Process

The insurance process (the interactions of insurance product buyers and sellers in more or less competitive markets) exists because of the existence of insurable risks and the desire of individuals and corporations to reduce the financial effects of those risks. For the purpose of modeling the insurance process we can conceptualize and describe insurable risks in terms of their corresponding insurance products. Each insurance product can be defined according to that product's attributes. In describing these attributes, we again adopt the perspective that a conceptual representation of an insurance product can be quite detailed. We recognize that not all the details dealt with below will be relevant to all DFA analyses, but may indeed be relevant to particular analyses for which DFA techniques are applicable. In our description of each product attribute, we attempt to provide some brief discussion of its relevance in a DFA modeling application.

The product attributes are grouped in terms of:

- Policy characteristics;
- Market characteristics;
- Operational characteristics; and
- Loss characteristics.

2.3 Policy Characteristics

2.3.1 Name

The product name serves as an identifier so that the experience of a particular product can be separated from the output as necessary.

2.3.2 State

The state is that state associated with some, or all, of the principal locational characteristics of the product (exposure location; regulatory jurisdiction; applicable residual market(s), etc.). In the United States, most aspects of the insurance business are regulated at the state level. The state which has regulatory jurisdiction, which is often also the venue of the product sale, and the subject exposure location, is therefore a relevant and convenient descriptor for insurance products from a management viewpoint. One can also associate some of the differences in the competitive environment with the product state.

2.3.3 Territory

The territory identifies the exposure location in terms of its rating or statistical territory as defined by the insurer's operating experience information system's data structure. Insurers commonly identify and price their products according to the corresponding exposure location in terms of its rating or statistical territory. Operating tactics are often defined in terms of product territories. Accurate evaluation of the risk posed by catastrophes typically requires information on the distribution of exposures by location by small geographic units, where territory may be interpreted in terms of the exposure location zip code. The differentiation among products by state and territory may also affect the product pricing and growth or retraction decision rules used in a DFA model.

2.3.4 Annual Statement Line

Because much of the insurer's internal, regulatory, and financial rating bureau reporting and evaluation uses the structure of the Annual Statement, it is necessary to allocate each of the products defined to its appropriate Annual Statement line. In addition, the Annual Statement line descriptor for a product can help define the covered losses for that product and explain the applicability of other product characteristics.

2.3.5 Exposure

Exposure specifies the amount of exposure written by the modeled company. Exposure is well defined for pricing purposes for all lines of business. The exposure bases defined for pricing purposes may not, however, be the most appropriate, or easiest to use, for financial modeling purposes.[13] If this is the case, one can equate exposure to policy counts, insured limits, or another suitable measure.

2.3.6 Price

A dynamic financial model which takes into account competitive market interactions will need to recognize more than one price for a given insurance product in the insurance process.

- Price as a result of the pricing algorithm - A starting point for the determination of the price charged by an insurer in a competitive market is typically a price determined by a formulaic algorithm which usually takes into account past and expected loss, expense, and investment experience and an appropriate profit and contingency load.
- Market price - depending upon current competitive market conditions, the insurer's current strategic plan and objectives, and financial condition, the price actually charged in a competitive market may differ from that indicated by the pricing algorithm.

The distinction, and changing relationship during the simulation period, between these two variables is an important and difficult aspect of designing and implementing a DFA model. There is little research presently available which defines and specifies the interaction between these two variables as market conditions change, yet it is difficult to see how a realistic and meaningful model can be constructed without incorporating this interaction.

2.3.7 Policy Term

The term of the policy coverage for a product has an effect on the (unearned premium and retrospective rating adjustments) reserves held by the company and the rate at which some pricing changes can be reflected in the collected premium levels.

2.3.8 Coverage Terms

Insurers have some flexibility in controlling loss costs by managing policy coverage terms (deductibles, co–payments, limits, coverage wording). Two of the more important coverage limitations are the coverage limit (the maximum amount payable for indemnity under this product) and the coverage deductible (the portion of the direct loss borne by the policyholder). Coverage limitations are a potentially important response to the risks presented by concentrations of exposure subject to catastrophes and therefore may be an important variable in DFA.

2.3.9 Expense Structure

An insurer's expense structure affects both its financial results, and its relative competitive position. The treatment of insurer expenses can recognize both loss adjustment and non-loss adjustment expenses, and different specifications may be appropriate for these two components (fixed and variable, a function of premium and/or loss). Changes over time in both general and/or claim specific inflation can be incorporated in the expense structure.

2.4 Market Characteristics

2.4.1 Operating Environment

The operating results for nominally identical products offered by the same insurer can be materially different depending upon the operating environment in which they are offered. Within the insurance industry, regulatory practices, competition and the corresponding operating environment are commonly perceived to vary significantly by jurisdiction, with some believed to present very difficult operating environments, and others as more favorable. Factors that may be used in measuring the "friendliness" of a jurisdiction could include rate regulation, residual markets, and market competitiveness. The operating environment for a particular product can also be affected by industrywide events, such as the occurrence of a catastrophe, or a change of law or regulation.

The evaluation of the operating environment can be taken account of in the model through the decision rules for growth, and in the setting of the market product price.

2.4.2 Target Competitive Position

The target competitive position, in an insurance product market which represents a population of insureds with differing risk characteristics, defines a percentile in the distribution of risks, and premiums, where the insurer wants to attract and retain business.

A given insurer is assumed to position itself in its product markets after evaluating its competitive strengths and weaknesses. Companies with, for example, material expense efficiencies can exploit that advantage by offerring lower premiums than its competitors to obtain lower risk insureds at each price level.

In addition to its expense structure, the competitive profile of a company is defined by the quality of its product (the coverage and service delivered to its insureds), its financial solidity and ability to honor its obligations, and its underwriting selectivity. Of these factors, underwriting selectivity is easiest for the insurer's management to vary in the short run.

As we note elsewhere, the competitive market for an insurance product is segmented, and defined by a distribution of prices. Given otherwise identical insurers, their position in the competitive market will be determined by their chosen underwriting selectivity. Having selected a level of underwriting selectivity, and assuming the other factors defining the company's competitive profile are determined, the company's competitive market product price is determined. Thus the cmpany's target competitive position specifies where in the distribution of prices a modeled company intends to compete (the target percentile of its market price).

Having selected a given target competitive position, a company may not be able to realize that target because:

- The market as whole may price irrationally in the short run, either above or below its equilibrium price;
- The company's expense structure, or other aspects of its competitive profile, prevent it from profitably competing at its target level. That is, the price at which the company can cover its costs and earn an adequate return on capital is uncompetitive in the market at the target percentile.

These interactions are an important component of the operating risk of an insurer, and should be reflected in a DFA model.

2.4.3 Underwriting Selectivity

Underwriting selectivity identifies the relative selectivity with which the modeled insurer underwrites risks for a given product. Underwriting selectivity is a critical element in an insurer's success. Unlike other products, the ultimate cost of insurance products is affected by who purchases them. Each potential purchaser has a risk characteristic with respect to the perils covered by the insurance product. In total, the market of potential insureds represents a distribution of risk characteristics. If an insurer is successful in selecting lower risk insureds, the loss cost component of its product cost will be lower.

Modeling competitive market interactions requires explicit consideration of the level of underwriting selectivity. For example, attempts to grow at a rate exceeding the growth rate of the market as a whole, assuming all prices fixed, requires that a company relax its underwriting standards. Because of the interaction between underwriting selectivity and loss cost, changes in selectivity will affect prices as indicated by the pricing algorithm, and thereby, the insurer's financial results.

2.4.4 Target Growth (Reduction)

A fundamental aspect of the insurer's operating plans is the amount of growth in volume desired by product, which can be defined in terms of exposure, premium, or market share. The actual growth can be represented stochastically around the target with interactions with product pricing variables.

In a competitive market, the ability of one company to grow will be constrained by the actions of its competitors. It is certainly possible, using a financial model which does not incorporate the effects of competitive interactions, to create misleadingly optimistic scenarios by coupling unattainable growth rates with profitable prices. We therefore conceptualize growth within a DFA model as two components: a target or planned growth; and the actual growth achieved taking into account the rate of growth for the overall market, the company's relative competitive price position; and relative competitive underwriting selectivity.

2.4.5 Target Underwriting Profit And Contingency Margin

The target underwriting profit and contingency margin, as a percentage of premium, is the underwriting profit and contingency margin entering the pricing algorithm for this product. We noted earlier that the market price, which is derived from the price produced by the pricing algorithm, need not be equal to pricing algorithm price, but, depending upon competitive market conditions, may be higher or lower than the algorithm price.

2.4.6 Residual Market Costs

Residual market costs represent the costs imposed on insurers by law or regulation for the provision of coverage to insureds who, from a regulatory viewpoint, do not otherwise have sufficient access to coverage. Residual market costs can be modeled as a charge per voluntary exposure and/or a percentage charge per dollar of voluntary direct written premium. These charges represent the costs imposed by law or regulation on insurers conducting business in a state and/or line of business to maintain a residual market for coverage and should include assessments by second injury or guaranty funds.

2.4.7 New Versus Renewal Differential

In experience analyses it has been shown that there is a material difference in the experience of newly acquired versus seasoned business. This difference can be critical for companies whose books are changing rapidly, and should therefore be incorporated as a variable in a DFA model. This difference can be reflected in a model as a differential in claim frequency between new and seasoned business, although in actuality it may arise from differences in both claim frequency and severity.

2.5 Operational Characteristics

2.5.1 Reinsurance Protection(s)

Reinsurance is one of the more accessible tools available to management to control operating risk or achieve some limited short run financial effects. Reinsurance protections can be flexibly designed to meet a wide range of needs. Because of the potential use of reinsurance to control some sources of operating risk, evaluating reinsurance contracts is likely to be a prime application of DFA models.

In order to accurately model the effects of different reinsurance protections, a DFA model must include a relatively large parameter set describing the coverage terms and sequence of application of the reinsurance protections (if any) applicable to this product. The reinsurance coverages should be defined, as appropriate, in terms of their applicable retentions, limits, quota share percentage, coreinsurance participations, per occurrence limit, loss adjustment expense treatment, minimum, deposit, and maximum rates, ceding commission terms, reinstatement provisions, reinsurer's margin, ceded premium calculation algorithm, and sequence and scope of application.

2.5.2 Pricing Algorithm

This is the algorithm the model uses to estimate the price required to cover the costs incurred from the sale of the product and produce a competitive return to the company's owners. Since DFA models will often be applied over extended time frames, the model will require an algorithm to determine prices from period to period in the simulation. Without such a reactive pricing algorithm, responding to changes in the modeled experience over time, the output of the analysis will be far from realistic.

There are many algorithms available to determine insurance product prices. It may not be feasible to incorporate all the possible options in a given model. However, in the application of a DFA model to a particular insurer, it is important that the modeled pricing algorithm replicate, as accurately as possible given a limited number of options, the essence of the modeled company's actual pricing process.

2.5.3 Reserving Algorithm

For most property-casualty insurance products, the ultimate cost of that product to the insurer can only be estimated for some period of time after the product sale. The product reserving algorithm, like the product pricing algorithm, defines the components and calculations which the model uses to estimate the ultimate cost of claims associated with this product, and thereby, the reserves which are carried in the insurer's annual statement.

As with the pricing algorithm, there are many reasonable alternatives for the reserving algorithm and we cannot incorporate them all in a given DFA model. The selected algorithm can be deterministic (three year average of previous development) or stochastic (normally distributed around a mean level) and adverse development can also be imbedded in either approach. The resulting reserve should generally not be that implied by the ultimate losses generated by the stochastic claim process, although the algorithm can certainly use them as input. For a variety of reasons, the modeler may want to incorporate the ability to carry reserves in the modeled financial statements which are different from the reserves indicated by the selected reserving algorithm.

As with the pricing algorithm, the selected reserving algorithm should match as closely as possible the essence of the modeled company's actual reserve estimation process.

2.6 Loss Characteristics

2.6.1 Claim Frequency

In order to take into account the stochastic nature of an insurer's experience, a DFA model must incorporate some mechanism to generate claim experience from a random process. The claim process can be modeled at an individual claim level, or at an aggregated level. The trade-offs between these two options are important:

- Modeling at the individual claim level may simply not be feasible from a computation viewpoint given the presently available computer capability. Modeling at this level does, however, allow analysis of the effects of individual claim characteristics without resort to approximation. Modeling changes in coverage (deductibles, coverage limits, per risk reinsurance terms) is straightforward at the individual claim level, but requires more complicated and careful modeling at the aggregated claim or loss ratio level.
- Modeling at an aggregated level allows the use of computationally efficient approximation techniques, which, in many circumstances, are quite accurate. It is not yet clear, however, that aggregated level models will allow modeling of, for example, the interactions between premiums, exposures and losses involved in a competitive market.

2.6.2 Claim Severity

As for claim frequency, this element of the stochastic claim process can be taken into account and modeled at the individual claim level, or at an aggregated level, with the same trade-off issues.

In modeling claim severity, it may also be necessary to model allocated loss adjustment expense separately from losses, if allocated expense treatment has a material impact on the outcomes for the questions being explored by the particular application of DFA techniques. Such treatment may be easier to implement when claim severity is modeled at the individual claim level.

2.6.3 Loss Reporting Pattern And Distribution

The loss reporting pattern specifies the expected sequence with which aggregated losses for this product arising from a given occurrence period will be reported over time, from the date of occurrence. Since variability in this pattern will have a financial effect on the insurer, a DFA model should also consider the distribution around the expected pattern. The randomization of the reporting pattern need not affect the ultimate amount of losses reported, but only the sequence in which the amount determined by the other processes is reported.

2.6.4 Loss Payment Pattern And Distribution

This is the loss payment analog of the loss reporting pattern, the expected sequence with which aggregated losses for this product arising from a given occurrence period will be paid out over time, from the date of occurrence, assuming a corresponding sequence of claim inflation. Variation in the sequence of loss payments is a source of operating risk to insurers and therefore should be reflected in a DFA model. Note that there is a potential interaction between the effect of inflation on claims, and the sequence with which claims are paid. Depending upon the claim inflation model selected, the payment pattern of losses will change over time if the rate of claim inflation changes and this interaction is another potential component of a DFA model.

2.6.5 Premium Collection Pattern And Distribution

These are the premium collection pattern analogs of the loss payment pattern and distribution.The modeling approach to loss reporting, loss payment, and premium collection should be conceptually similar in a given model implementation.

Because of the importance of its loss experience on an insurer's results, the approach taken in modeling the loss process is a defining characteristic of a DFA model. Losses can be modeled at the individual claim level, on an aggregate basis, or (in conjunction with premium) as loss ratios.

The obvious trade-off between individual claim versus aggregate modeling is accuracy for simplicity. This trade-off should be considered carefully, as evaluating reinsurance contracts, specific sub-books of business, or the effectiveness of mitigating strategies on catastrophic losses (such as increased deductibles) will be made much more difficult if losses are modeled at an aggregated level. However, regardless of which model implementation approach is taken (aggregate or individual claims), some careful analysis of the underlying frequency and severity processes, payment and reporting patterns, and their interactions with other relevant variables is vital to the reasonableness of the resulting model.

Loss payment and reporting patterns and their variability is another difficult modeling problem. Most actuaries feel that there is some dependence between the columns (representing common valuation or maturity dates) of a paid or reported loss triangle. Unfortunately, there is no consensus as to the direction, much less the magnitude, of this dependence. This complicates the possible approaches to generating random observations of payment patterns. One possible solution is to develop a concept of "path variance", where the variability of the pattern is associated with the payment or reporting pattern as whole, rather than its incremental components. Another approach is to find a way of restating patterns such that the dependence between successive evaluations is removed.

2.7 Summary

It is dauntingly obvious that a DFA model which takes all of the features mentioned above into account will be extremely complex, in terms of:

- The multiple interactions defined;
- The number of parameters requiring estimation;
- The potential for unstable results;
- The potential for subtle errors (either in defining the array of interactions and relationships among variables, or in programming those relationships).

Our objective in discussing the rather detailed representation of the insurance process above is not to imply that all the features mentioned must be included in a DFA model, but to assist in the analysis of the relative importance of those features. When conducting a DFA analysis, the actuary should carefully consider if the particular modeling context, or objective, is one in which, at the extreme, all of the various product attributes should be recognized in the modeling process, or, alternatively, there are some which can be safely ignored.

3 Insurance Process - Assets

One of the primary concerns with loss reserve opinions is the failure of the opinion to address the ability of the associated asset portfolio to support the liabilities. A DFA model can provide a means of analyzing an asset portfolio as the fund from which an associated reserve will be paid over time. In order to perform this analysis, a DFA model must incorporate the ability to model asset values and their returns over the simulation period, as economic conditions change.

Asset modeling is complex enough to be the topic of many textbooks and articles in professional journals, including those of the actuarial societies. Given that level of complexity, we only attempt to give a brief discussion of asset modeling.

Assets may be defined by the following attributes:

- Yield rates or market interest rate spreads;
- Default rates;
- Term;
- Investment expense;
- Prepayment functions;
- SFAS classification.

Assets can be modeled in a number of ways. The most common modeling technique involves using a "cascade" structure, where first one time series (such as short-term interest rates) is determined, and then those values are used to calculate subsequent series (such as inflation and stock yields). Other time series methods include the use of cointegration or transfer functions. Wilkie provides an instructive, though somewhat long example of the application of this type of modeling.[3]

Alternatively, one can use a model for economy-wide inflation (as measured by the GDP deflator), market values of cash equivalents, Treasury bonds, and large capitalization equities. From these basic components, other asset types can be directly modeled through the use of regression. A key consideration is that for whichever measure of inflation and interest exists in the model selected or developed, the relationships with other operational variables, assets and liabilities, will need to be developed using time series of those indices.

4 Insurance Process - Capital

Insurers require capital, in order to comply with minimum capital requirements imposed by regulation, but more fundamentally, to ensure their ability to survive the inevitable fluctuations in operating results which characterize the insurance business (and the need for DFA). This capital can come from alternative sources (debt, equity, retained earnings), some of which are unique to insurance (surplus notes).

The capital structure of an insurer is constrained by specific laws and regulations on the makeup of its balance sheet as well as by external financial criteria (Risk-Based Capital requirements, insurer rating bureau leverage analyses).

A realistic dynamic financial model should incorporate these constraints and the management responses invoked when the insurer's actual or anticipated operating results bring its financial profile too close to those constraints. For example, growth scenarios should be curtailed when leverage exceeds acceptable levels. This can be effectively handled through the use of decision rules.

5 Insurance Process - Inflationary Effects

Inflation appears to be an unavoidable feature of modern economies and has powerful effects on insurers' operating results. DFA models must therefore specify how inflation is measured and considered. Inflation is generally measured as the change in a price or wage index representing a broad sample of goods or workers. Defining inflation for modeling purposes raises a number of issues:

- Time Horizon - Will inflation be modeled monthly? Quarterly? Annually? This is certainly dependent on how the corresponding asset and liability cash flows are modeled;
- Model - Will the model be of the index or the change in the index?
- Index - Which index will be used? Or will an average of indices be modeled? Will the index representing a time period be that for the end of the period or an average over the period?

Once a times series model of inflation is in place, a model of claim inflation must be developed. While time series or other statistical approaches can be used to analyze the relationship between general economic and insurance claim inflation, there are additional questions whose answers will aid in developing a claims inflation model and its incorporation in a DFA model:

- How does a measure of claim inflation for a specific insurance product relate to the general economic inflation measure? Should this relationship be based on the level of economic inflation or its deviation from some "expected" level (a la Rational Expectations school)?
- Given a measure of claim inflation for a specific insurance product, an index, for example, what specific model of claim inflation should the model incorporate? That is, how does the cost of a specific claim, or group of claims, within the model vary with the claim inflation index?
- (How) does claim inflation affect claim payment patterns and their variability?
- Should the claim inflation effect assumed in the model be uniform by claim size?
- What other effects caused by, or related to, general economic inflation should be incorporated in the model? How should those effects be modeled? For example, is inflation a proxy for other economic events that may affect claim frequency? Market conditions? Reserve adequacy? Etc.

One example of work analyzing the issues raised above is by Butsic (*The Effect of Inflation on Losses and Premiums for Property-Liability Insurers*). In this work, he develops a unified model between two paradigms - the payment date model and the accident date model. Each of the two paradigms relates the period in which the inflation is relevant to the final value of a claim. Butsic designs a model that is a weighted average of the two paradigms, implying that the inflation rate at both the date of loss and the date of payment are relevant. Unfortunately, we have found that the method suggested for calculating the weight is very unstable, depending greatly on the particular data values.

6 Insurance Process - Competition

The risks arising from an insurer's participation in a competitive business are significant. The CAS Dynamic Financial Handbook identifies five major classes of risk to a typical property/casualty insurance company:

- Inappropriate pricing - generally underpricing and often coupled with excessive growth;
- Inappropriate business plan - generally (excessive) growth in areas with significant underpricing or areas for which there is little data or limited company expertise;
- Inappropriate reserving - underreserving due to lack of data, inadequate techniques, and/or management pressure, often coupled with underpricing;
- Inappropriate reinsurance program - a company retains too much risk relative to surplus, or over-relies on a small number of reinsurers who subsequently experience financial difficulty;
- Inappropriate investment portfolio - the company invests too much of its portfolio in asset classes that are overly volatile, poorly understood, or is overly concentrated with a few issuers who subsequently experience financial difficulty. Also, the portfolio could be severely mismatched relative to the cash flow demand of the liabilities during a time when the portfolio is weak.[12]

The first two categories of risk identified are directly related to the competitive market environment in which the insurer operates. The ability of an insurer to price at an adequate level, or to grow at a prudent rate, may be constrained by the actions of its competitors. If the competitive market price for a product does not cover costs, as has happened in some highly competitive markets, it will be difficult or impossible for an individual company to maintain an adequate price for that product.

Pricing below cost is irrational and self-destructive in the long run, but can be seen as a necessary response in the short run during corrections to an imbalance in supply and demand. Versions of the following dynamic have been offered as an explanation of the causes of the underwriting cycle.[8] [29]

- Capital, and capacity, flow into and out of the insurance business as operating loss shocks occur and investor's expectations of returns in insurance compared to other opportunities change over time;
- When capacity is higher, competition is greater, and companies attempt to retain business by following the market to lower prices to avoid the costs of losing, and then attempting to regain, seasoned business (business where the insurer has accumulated knowledge of the insured, the risk he presents, his ability to pay the premiums, and his integrity as a claimant and customer);

- When the cost of selling below cost exceeds the cost of losing and regaining seasoned business, and returns from insurance drop relative to other investments, capacity and prices stabilize;
- This all takes place in a market where information flows are imperfect (insurers' and their investors' estimates of expected returns on existing and future business are uncertain; insurance purchasers do not have complete information about the quality and price of all the products available to them in the market). Imperfect information flows may contribute to the cyclical nature of underwriting returns, and to the persistence of pricing below cost behavior in a down-cycle.

Because the risk of being forced by competitive pressures to price below cost are real, and the effects potentially disastrous, a DFA model whose output is used in formulating an insurer's strategic plan should consider the insurer's competitive environment.

Unfortunately, there are difficulties in modeling the insurance market:

- Insurance product markets are inherently segmented. The competitive equilibrium price for an insurance product, unlike other products, depends in part on the risk characteristics of the product purchasers. For otherwise identical products, the loss cost, and therefore price, for a product sold subject to stringent underwriting standards will be lower than the price of the same product sold under less stringent standards. Equilibrium in an insurance product market is therefore defined by: the level of demand for the product; the cost structures of the participating insurers; and by their underwriting selectivities. To the extent that underwriting selectivity differs among the insurers in a given product market, that market is segmented, and the single competitive equilibrium price of traditional economic theory applied to other types of products becomes a distribution of equilibrium prices corresponding to different underwriting selectivities;
- There may be an interaction between changes in an insurer's relative competitive price position and its underwriting results. That is, lower (more competitive) prices may give an insurer the opportunity to attract lower risk (cost) business, and vice-versa. The experience of the insured group, with the addition of the lower risk insureds, may be better than anticipated (based upon the former insured group), and vice-versa. In other industries, a producer's relative competitive price position has no material effect on its cost structure;
- The cost of the product (a contingent promise to pay a variable amount) is uncertain;
- The cost of capital by product is difficult to determine;[13]
- Relevant variables, such as the elasticity of demand, may not be estimable;
- The competitive environment of insurers may be subject to a cycle. While a definitive consensus on the existence and causes of the cycle has not developed, there is agreement that, if underwriting results are affected by an underlying cycle of hard and soft markets, such a cycle will have an important effect on insurers' results, and therefore should be included in a DFA model.[31]

The relevant model implementation questions are:

- What are the mechanics of the transition from cost plus pricing to below cost pricing in insurance markets? That is, what conditions trigger the descent from a competitive equilibrium where prices produce adequate returns to a hyper-competitive market where coverage is offered at below cost prices? What are the mechanics of the recovery from a hyper-competitive market?
- What statistic(s) should be used to describe the competitive environment?
- How can the elasticity of demand for the modeled insurer's product be defined?
- Should factors such as the ease of price changes be incorporated? How?
- Is the modeled insurer's market share an important competitive market model variable?
- In general, how can we model competitive markets at the product level?
- How will the degree of departure from perfect competition affect the model?

7 Insurance Process - Catastrophic Losses

Recent actual experience has demonstrated the importance of the risk of catastrophes as a factor in insurer strategic planning. Relatively new catastrophe exposure analysis models are now available to help quantify the potential frequency and cost of catastrophes, but the problem of incorporating that information in a financial simulation model remain. One important consideration in reflecting catastrophic risk is that the modeler must be very open-minded in considering what belongs in this module of the model. Catastrophes should not just be considered earthquakes and hurricanes, but rather potential shocks to underwriting losses. This includes potential "casualty catastrophes" such as environmental and asbestos, and breast implant claims.

There are three alternatives to modeling an insurer's catastrophe risk within a DFA model.

- Build the capability to model the effect of natural catastrophes into the DFA model itself;
- Provide the results of a separate catastrophe exposure analysis to the DFA model as part of its input, defining the frequency and severity of catastrophes which could affect the company;
- Build a component into the stochastic loss process that "shocks" the frequency and/or severity for catastrophic events.

As most natural catastrophe models analyze data at the zip code or county level, incorporating a catastrophe exposure model within the DFA model will complicate the model in terms of its input requirements, run time, organization and size. However, it also gives the model user the capability to analyze the effects of changes in the company's catastrophe exposure on its overall financial condition, rather than as an isolated element.

The results of an independent catastrophe exposure analysis reflect a particular input (exposure portfolio). Because the effects of catastrophes can vary significantly with the composition of the exposure portfolio (risk characteristics by location, coverage terms and construction), it may not be possible to accurately reflect changes in the exposure portfolio during the simulation time period. Thus the catastrophe risk derived from an analysis independent of a DFA model, used as input to a DFA model, might not truly reflect the catastrophe potential for the company at a given point in the simulation. However, as simulation output is just a sample of potential outcomes, and these outcomes can be adjusted to reflect exposure growth, use of an independent catastrophe model may be a reasonable approach compared to the other alternatives.

The third approach is the most practical when there do not exist any external models and little is known about the potential effects of a given peril. However, it is important when developing this component that events beyond the range of those that have occurred historically are considered as possible.

Finally, if a given DFA model incorporates a component which models competitive market interactions, it will require a method to extrapolate from catastrophe effects on the market as whole to the modeled company, or from the company to the market, since catastrophes can clearly disrupt markets in the affected products and regions.

To summarize, the questions involved in implementing a catastrophe risk component in a DFA model are:

- What events describe the risk of catastrophes in a DFA model? How should they be parameterized?;
- The risk of catastrophes varies by the event and the affected exposure characteristics. How detailed should the partition of events and exposure characteristics be to accurately model the risk of catastrophes and still represent a practical model implementation?
- What is the appropriate relationship between a modeled individual company's catastrophe experience and that of the industry?
- How does one take into account the impact of a catastrophe on the competitive market environment and reinsurance environment?

8 Insurance Process - Decision Rules

We have defined dynamic financial analysis in this paper as modeling an insurance operation over a future time period against a range of potential future operating environments. In that framework, it is relatively easy to devise future operating scenarios which lead to unreasonable financial outcomes during the simulation time period. In the real world, an insurer's management can react to its current and anticipated financial condition and operating environment, revising strategies as necessary, rather than passively continuing an operating plan devised at an earlier time which is now inappropriate in light of current conditions.

A dynamic financial model which does not incorporate reactive management decisions rules is therefore unrealistic. Incorporating such reactive decision rules, however, represents a significant complication in specifying the model. In addition to the specific decision rules, the modeler must also specify what the modeled insurer management is presumed to know about its current and expected future operating environment at a given point in the modeled time frame (management's state of knowledge).

For example, should we assume that management has retained a gifted actuarial seer who can accurately predict future claims inflation so that his predictions should be included in the company's pricing decisions? Alternatively, is it reasonable to assume that the modeled management makes no attempt to estimate its future operating environment given the available past experience?

Once the modeled management's state of knowledge and decision rules are specified in the model, the process of dynamic financial modeling becomes not only an analysis of the financial condition of an insurer in uncertain future operating environments given a defined initial asset, liability (product) and capital structure, but also an analysis of the effects of the modeled management's operating strategies and set of decision rules as incorporated in the model.

Examples of decision rules which could be identified as relevant are:

- Product pricing - In practice, insurance product pricing can involve multiple processes. Rating bureaus can establish prices based on the combined experience of companies. Those bureau prices can serve as benchmarks for individual companies. Pricing at the individual insurer level may involve assorted combinations of bureau- and company-derived rates and relativities as well as individual risk rating modifications. In each of these components there may be both an empirical algorithm (where the available experience data are analyzed as previously discussed) and a managerial decision process (where the results of computational processes are combined with other relevant factors not explicitly reflected in the computation) to arrive at an ultimate pricing decision. To be realistic, a DFA model should incorporate product pricing processes similar to those actually used by the modeled company. While it may not possible to capture the detail of most actual pricing processes, the pricing decision rules reflected in a model should at least reflect the essence of the pricing process of the modeled company, particularly how that process responds to recent past experience and the current competitive environment.

- Investment - If the cash flow from operations is positive, the company can invest the excess over that required for current cash obligations. If cash flow is negative, the company must obtain sufficient cash from the liquidation of some assets, or other sources, to satisfy its current payment obligations. The model therefore requires decision rules to determine how assets are to be invested and liquidated as cash needs change. In addition, investment strategies can potentially vary according to the company's perception of its likely future economic environment, operating results, and tax position.
- Growth/retraction - An insurer's operating objectives can include growth, either as increases in the volume of business for existing products, or expansion into additional products and/or jurisdictions. Growth as an operating objective is an aspect of the broader operating objective of maximizing the value of the firm. Assuming that each business segment (product) can be ranked based on its expected future contribution to the firm's value, management can continuously adjust its investment (allocation of surplus and other resources) among products, growing the business in the more profitable products, and retracting in the less profitable products. A dynamic financial model therefore requires a statement of the rules governing these resource allocation decisions. Because of the potential and actual interactions among products in terms of production, marketing, law and regulation and current commercial practice, the rules governing growth and retraction at the product level can become complex. We also note that operating plans which affect the volume of business affect, and are affected by, the company's expenses, and these interactions must be accounted for (production profit sharing plans and commission levels, for example).
- Loss and loss expense reserve estimation - Insurance is characterized, in part, by the uncertainty in the costs of its products from the time of sale until the settlement of the associated claims. There are available many well-defined actuarial techniques to estimate the ultimate cost of claims after the sale of the product, but prior to ultimate claim settlement. Like pricing, the results of the reserving algorithm may influence management's perception of the company's financial position at any given point in time, but so will other factors that affect the company's operating objectives and strategy. A dynamic financial model should therefore incorporate decision rules for loss and loss expense reserve estimation which replicate the essence of the decision-making process actually used by the modeled company to set its reserves.
- Reinsurance protections - Reinsurance is one of the more accessible, flexible, and potentially effective management tools for an insurer. As the financial condition, size, and product mix of the insurer changes, and/or the reinsurance product market varies, different reinsurance protections become appropriate. The analysis of alternative reinsurance programs is therefore likely to be one of common uses of DFA models. Because of the flexibility of design available for reinsurance programs, it may not be possible to incorporate decision rules describing the company's

reinsurance responses to different operating results, operating environments, and financial conditions. However, alternative reinsurance programs can be tested as sets of input to the model. To the extent that reasonable and realistic rules for the employment of reinsurance can be defined for a given analysis, it may be appropriate to incorporate them in the model structure.

- Dividends - Insurers can issue dividends both to owners of their stock and to policyholders of participating policies. Models can require decision rules to determine the timing and amount of such dividends.

In any application of the model for management decisionmaking, the decision rules modeled should be defined by the management of the insurer being modeled to ensure their appropriateness.

9 Insurance Process - Dependencies

A significant complicating factor in modeling an insurer is modeling the dependencies between variables.[5] That is, changes in the value of a given asset may be related to changes in the value of another asset. Similarly, the experience of one insurance product may vary with the experience of another product. And, finally, the changes in the value of an asset may vary in concert with insurance product experience. This raises some subtle issues. In particular:

- Which variables should be considered dependent? It is possible that all pairs of variables arising from the insurance process will show some dependence as measured by their covariance, based upon a representative set of data. However, the observed dependence may only be a statistical artifact, with no corresponding underlying dependent processes. That is, the variables may be unrelated in terms of the events and causes which determine their ultimate values, but the particular values which entered the data we used to measure their covariance may indicate a statistical dependence. On the other hand, it may be the case that there is some other phenomenon which affects both variables, such that the causal link is indirect, but real nonetheless. A DFA model should incorporate the significant dependencies between assets and liabilities which arise out of direct and indirect causal connections, while separating and ignoring the other spurious dependencies using our understanding of the underlying processes;
- What measure of dependence, or association, among dependent variables is appropriate for implementation in a DFA model? More than one statistical measure of dependence are available. Different measures may be more or less appropriate in the context of a DFA model;
- Having selected an appropriate measure of dependence, how do we impose the dependencies on the variables recognized in the model? This can be done through the explicit modeling of the relationship or through the imposition of a covariance structure. Here, perhaps more than elsewhere, the trade-off between theoretical accuracy and practicability rears its ugly head.

9.1 Correlation

Before venturing down the road of interrelationships, it is worthwhile to discuss the topic of correlation. This is because, although correlation is a commonly used tool to measure the existence of relationships between variables, it is also one that is generally not fully understood.

There are a number of measures of correlation, such as Spearman's rank correlation, Pearson's product moment correlation, and Kendall's tau. These each make different assumptions regarding the underlying distributions of the variables and they each measure different things.

For example, correlation can mean a specific computational result which quantifies an attribute of the relationship between two variables in a sample of model input. In this context, the

Pearson product moment correlation is a measure of the concentration of the input sample about a line fitted to the ordered pairs of that sample.

Correlation can also be interpreted as a measure of the degree of relatedness, or association, between two variables. That is, given the value of one variable, how closely determined is the value of the second by the value of the first?

Correlation can also be interpreted as the tendency of one variable to take on a value above or below its mean value, depending upon the value of a second variable, relative to its mean.[14]

There are a number of methods for imposing a covariance structure on a set of variables. While it is beyond the scope of this paper to fully describe these, we will list a few so the interested reader can research them in an appropriate statistics reference. These methods include normal transformations, the Iman-Conover process, the Cario-Nelson method, as well as time-series approaches like transfer functions, cointegrated series, and, of course, direct modeling.

9.2 Dependencies Among Asset Values

There are many ways to model dependent variables. However, in the exercise of modeling inflation, asset values, and their interdependencies, time-series approaches are a very natural tool to apply. If one uses a cascade model, the first step is to select the independent variable (inflation, for example), not unlike regression analysis. However, rather than regressing, we model this variable on its own, probably using some class of ARIMA (or AR or MA or ...) model. This provides the tools to project future values of inflation.

Once inflation has been modeled, additional series can be modeled using time series containing inflation as some of its terms. Alternatively, inflation can be used to transform the variables (for example, determining real rates of return given the corresponding inflation) or as a regressor to determine a direct relationship. These additional time series and/or functions can then generate, simultaneously with the inflation series, consistent future values. This approach avoids some potentially messy approaches mentioned in the previous section.

9.3 Dependencies Among Liabilities

The experience of different insurance products may be affected by common factors such that the products' experiences tend to vary in concert. The financial impact of deteriorating operating results in several lines together is clearly much greater than the impact of similar, but independent, fluctuations in results by product.[10] Defining and modeling the dependencies among products' experience is therefore an important consideration in a DFA model (see Appendix 7 of Property-Casualty Risk-Based Capital Requirement, A Conceptual Framework, by the Actuarial Advisory Committee to the NAIC Property & Casualty Risk-Based Capital Working Group[26]). Some examples of these dependencies are between:

- Claim frequency by product;
- Claim severity by product and general inflation;
- Recent past industry experience and the level of current competitiveness;
- Recent past reinsurer experience and the price and availability of reinsurance coverage.

With respect to the second example above, the relationship between claim inflation and general economic inflation may vary by insurance product, having the same general behavior for all products and all resulting in greater or lesser dependence between general economic and individual product claim inflation. This would support the hypothesis that there is some correlation between products' experiences caused by the common dependence upon the general economic inflation time series.

In addition, there may be direct relationships between products. This could be a result of geographic concentration, niche marketing, or social/legislative/judicial change. These correlations will need to be modeled directly or externally imposed if they are to be considered in a DFA model. As any assumption of independence between products reduces the variability in potential underwriting results, such an assumption should be analyzed carefully.

9.4 Dependencies Between Assets and Liabilities

At least as important as the dependencies discussed above, the dependencies between assets and liabilities (insurance products) should be considered in a DFA model. As mentioned previously, one can use a general economic time series to derive a corresponding time series of claim inflation. Since assets are related to general inflation and liabilities are related to general inflation, interactions should be revealed through the use of a common source. However, the dependency of assets and insurance product experience arising from inflation is not the only possible relationship between assets and insurance liabilities. There may be other causes, such as a relationship between frequency and inflation, which require research to find, define, and model.

10 Parameter Risk

In modeling, it is generally accepted that there are two types of risk - process risk and parameter (or model) risk. Process risk is the risk that the actual outcome differs from the initial estimate due to random fluctuations. This is akin to the result of two dice being rolled. The expected value is seven, and any observation that is realized which is different from seven (5/6 of the time) is due to process risk. Parameter risk, however, is the risk that the initial estimate of the process is inaccurate. This would come about if the dice were loaded, causing the underlying expectation to not be seven at all. In mathematical terms, process and parameter risk can be described as:

$$\begin{aligned} ProcessRisk &= (X - E) \\ ParameterRisk &= (E - M) \\ TotalRisk &= (X - M) \end{aligned}$$

Where X is the observation, E is the estimate of the process mean, and M is the true mean.

These descriptions may look familiar as they have widespread use in statistics, including the definitions of variance, explanatory power of models, and ANOVA.

Process risk is recognized in DFA models through the use of stochastic variables. However, the modeling of parameter risk is more problematic, perhaps even paradoxical. If one can model the degree and direction that the initial estimate of parameters varies from the true value, why would this error be modeled as parameter risk rather than by revising the parameter estimates? Especially in situations where values are characterized as "best estimates"? In addition, does one need to model the parameter risk of the parameter risk model? And so on?

While it is important to be aware of potential misspecification and/or misparameterization of a model, it is up to the modeler to determine how these potential errors should be addressed. One should also realize that, of the many aspects of the insurance process which can have significant effects on a company's financial results, there may be some which we cannot yet model due to a lack of understanding or data to define their relationships to other model components. For example, it is possible that future reinterpretations of the laws and regulations applicable to insurance will change an insurer's obligations under policies already issued or expired (as has happened with respect to environmental coverage) or that there may be some unforeseen widespread tort activity due to newly discovered health hazards. At this time, we can only recognize that these risks exist. To the extent that we can define the likelihood and effect of such additional risks, they should be recognized in an analysis of an insurer's financial risk. Where they cannot be defined, we can only acknowledge their existence and qualify our analysis appropriately.

11 Miscellaneous Topics

In addition to the aspects of the insurance process considered above, there are some additional issues.

11.1 Stochastic Variables

Recognition of the stochastic features of insurers' operations is a defining characteristic of a DFA model. This can be done by implementing some of the important model variables as observations drawn from distributions that are reasonable representations of those variables. It is ultimately a matter of judgement and computing limitations to designate how many of the variables recognized in the model should be modeled stochastically. For model validation and sensitivity testing purposes it will be desirable to turn off the stochasticity, defaulting the variable to its mean value.

Based upon the previous discussion, stochastic variability can be incorporated in a DFA model via the following variables:

- Variation of planned versus actual growth;
- Competitive market price distribution and the relationship of the competitive market price at any given time to a market equilibrium price which covers costs and provides an adequate return on investment;
- Asset returns;
- General economic and claim inflation;
- Claim frequency and severity by product;
- Loss payment and reporting patterns by product, including run-off of initial reserves;
- The amount of allocated loss adjustment expense incurred for individual claims;
- Expense payment pattern;
- Premium collection pattern;
- The frequency and severity of catastrophic losses for different products and locations.

This list is representative, but by no means exhaustive.

11.2 Accounting and Taxes

Statutory accounting has unique features. Inter-company comparisons and historical analyses are easier to perform when the financial representation of an insurer has been cast in statutory accounting terms. The Federal income tax calculation for insurers proceeds from the statutory accounting results. There are, however, other uses of insurer financial analyses for which other accounting representations are appropriate. The model should therefore be capable of presenting results under statutory,GAAP, and market value accounting. In addition, Federal income tax calculations for insurers are complicated and should be part of the resulting dynamic financial model..

12 Model Output

12.1 Introduction

The most important aspect of model design is to make sure that the model tells you what you need to know. In other words, the specification of the model must include the calculation of output that will be useful to the decision-making process. This is particularly challenging for DFA modelers in that:

- There are many types of management problems to which DFA can be applied;
- The level of output detail necessary to examine all such types may be extensive.

The volume of data potentially generated from a highly detailed model such as the types discussed in this paper is overwhelming. The task of analyzing the data may be easier if we define some types of analyses in the absence of context and then discuss them in detail:

- Expectation and distribution of the objective variable;
- Identification and categorization of scenarios which resulted in extreme values;
- Determination of explanatory variables with respect to the level of the objective variable;
- Evaluation of decision rules, reinsurance programs, etc. relative to the objective variable;
- "Good versus bad" analysis.

12.2 Expectation And Distribution

Given the inclusion of stochastic elements in a DFA model, a primary output of the model should be the distributions of the relevant decision variables. Of interest is not only the expectation and shape of these distributions, but also particular values of the density and/or cumulative distribution functions. Some examples of these include:

- Ending surplus and the associated probability of ruin;
- Operating ratios and the probability of losing money;
- A. M. Best ratings and probability of a ratings downgrade.

Each of these distributions, in addition to its shape and mean, have other interesting features. The scenarios generated by the model can be sorted based on any carried variable to obtain information regarding the variable's probability distribution, and, of course, its moments.

12.3 Identification And Categorization Of Scenarios

It is very difficult to examine thousands of scenarios and dozens of variables for patterns. However, it is not unlike another situation in which we have many observations and need to determine the interactions of the associated variables - classification rating.

Using iterative methods or generalized linear models, one analyzes the objective variable (loss cost, or surplus, for example) trying to determine the effect of individual variables (rating variables, operational variables) on the objective. By using a tool already in the actuarial toolkit and applying it to a different problem, we may have a better grasp of how to approach this challenge.

12.4 Determination Of Explanatory Variables

One possible approach to this analysis is very familiar - multiple regression. This technique allows variables to be fit with coefficients and computes confidence intervals around them. In addition, levels of explanatory power, such as R^2, can be determined. However, it is very important to be aware of the assumptions underlying this technique (such as independent variables) and to interpret the results in the light of informed judgement.

12.5 Evaluation Of Decision Rules, Reinsurance, Etc.

We can evaluate the effect of these model elements in a binary fashion - the rule was either on or off, the reinsurance protection was purchased or not, etc. By separating the scenarios into two sets and comparing the relevant output variable distributions, perhaps through graphical superimposition, differences may be readily apparent. However, it is also important to determine the significance level of these differences and there exist tools, such as the Chi-squared test, to assist the modeler in that task.

12.6 "Good Versus Bad" Analysis

Another desirable feature of DFA is to be able to analyze trade-offs between decisions or scenarios. This can be done graphically with great effect and one of the more well-known applications of this technique is the risk-return plot. This concept can be extended to growth versus loss ratio, reinsurance costs versus ending surplus, or pricing decisions versus market share, to name a few. The graphical approach allows for easy determination of dominated strategies and leaves management with only the decision of where on the boundary of possible strategies they should strive to be.

13 Conclusion

Actuaries have taken on a difficult task in defining and building dynamic financial models of property-casualty insurers. The research and experiments necessary to create such models will, however, benefit our understanding of the insurance business and enable us to contribute more effectively to the management processes of our employers as well as to the regulatory process.

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