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Fall 1995

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NOTICE

The Casualty Actuarial Society is not responsible for statements or opinions expressed in the papers or reviews in this publication. These reports, papers, and reviews have not been reviewed by the CAS Committee on Review of Papers.

1995 CAS *Forum*, Fall 1995

To: CAS Members

This is the 1995 edition of the Casualty Actuarial Society ***Forum.*** Besides several new papers, this publication includes four reprinted papers and three committee reports. I would like to thank the other committee members:

Paul Lacko	Kimberly Ward
James Wilson	Gerald Yeung

I hope you enjoy this publication.

Sincerely yours,

George R. Busche
CAS ***Forum,*** Chairperson

Credits

"Using Expected Loss Ratios in Reserving," by Daniel Gogol is reprinted from *Insurance Mathematics and Economics,* Vol. 12 (1993), ppp. 297-299, with kind permission from Elsevier Science B.V.. Amsterdam, The Netherlands.

"Which Stochastic Model is Underlying the Chain J-adder Method?" was originally published in *Insurance: Mathematics ond Economics.* It was awarded the first ever CAS Charles Hachemeister Prize for the best 1992-1993 ASTIN paper, and was presented at the ASTIN Colloquium 1993 in Cambridge (U.K.).

"When the Wind Blows," by D.E.A. Sanders was presented at the ASTM Colloquium 1993 in Cambridge (U.K.).

"A Simulation Procedure for Comparing Different Claims Reserving Methods," by Teivo Pentikäinen and Jukka Rantala was originally published in the *ASTIN Bulletin.*

Apology

It has been brought to my attention that the paper "Duration, Hiding in a Taylor Series" by Keith Holler, which was published in the Summer 1994 Casualty Actuarial Society *Forum* had some similarities to the paper "Asset/Liability Matching (Five Moments)," by Robert K. Bender, published in the *1993* CAS *Proceedings.*

We apologize if the timing of the publications caused any confusion or embarrassment to the authors, and the CAS membership in general. The *Forum* Committee has put procedures in place to do a better job in coordinating the two publications.

Sincerely yours,

Joel Kleinman
Past Chairperson
The Casualty Actuarial Society *Forum* Committee

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Report of the Travel Time Working Group
by the Travel Time Working Group

REPORT OF THE TRAVEL TIME WORKING GROUP

FEBRUARY, 1995

Working Group Members:

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G.G. Meyers
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TRAVEL TIME WORKING GROUP REPORT

Preamble

The following report of the Travel Time Working Group represents the culmination of a 2 year effort to establish the information needs of the CAS necessary to monitor travel time, ensure that the CAS database contains the requisite information, define the criteria by which travel time should be monitored and draw preliminary conclusions regarding the impact of exam partitions on travel time, if possible.

The Executive Council and Board of Directors discussed this report at several meetings during the third and fourth quarters of 1994 and the first quarter of 1995. At its February, 1995 meeting the Board of Directors adopted the data collection and data monitoring recommendations in the report and authorized distribution of the report to the CAS membership through publication in the CAS Forum.

In discussing the report, the Board acknowledged that the statistics included in the report are inconclusive at this time vis-à-vis travel time. Even if travel time was changing, the Board agreed that it might not be possible to isolate the effect of exam partitioning on travel time. Finally, the Board re-affirmed its earlier decision to take no further partitioning steps at this time.

I. Backaround

Partitioning of the examinations required for membership in the Casualty Actuarial Society began with Part 3 in May, 1987 in coordination with the implementation by the Society of Actuaries of their Flexible Education System (FES). Subsequently, the CAS Board of Directors requested that the Education Policy Committee address the issue of whether the CAS should adopt a Flexible Education System.

The Education Policy Committee report was presented to the Board of Directors in September, 1988. That report, in the form of a "White Paper" was distributed to the membership in March, 1989 and was accompanied by a letter from the President requesting that the membership carefully consider the contents of the "White Paper" and provide comments on the recommendations contained therein.

Subsequent to the September, 1988 Board meeting, the Partitioned Exam Task Force (PETF) was created to determine whether an implementation plan could be developed which would address the issues contained in the "White Paper". The PETF submitted its report to the Education Policy Committee in October, 1990.

The recommendations of the Education Policy Committee and a supplemental analysis made by the Vice President - Admissions was presented to the Board of Directors at its November 11, 1990 meeting. After consideration of the reports of the PETF and Vice President - Admissions, and substantial discussion and debate the Board decided to partition Part 4 effective with the May, 1992 exam administration and Part 5 effective with the November, 1993 administration. Each of these exams began being offered twice a year coincident with their partitioning. The Board also decided that Parts 6 and 7 would not be partitioned and consideration of 'partitioning of the Fellowship exams would be deferred for at least three years. The EPC "White Paper", PETF Report and various letters to the membership on partitioning are contained in the Winter 1991 edition of the CAS Forum on pages 189-467.

The Travel Time Working Group was created in February, 1993 in response to the Board of Directors' desire to ensure that the database structure, reports and analytical tools necessary to monitor the impact of partitioning on travel time would be established before post-partitioning candidate performance information became available. The assignment included:

- A determination of the information required to monitor travel time.

- An opinion regarding the sufficiency of the CAS database to evaluate the impact of partitioning on travel time.

- Conclusions, if any, which can be drawn at this time regarding the impact of partitioning on travel time.

The working group did not consider its charge to include, nor did it examine the question of whether partitioning has been successful in better educating actuaries.

II. Workina Group Deliberations

In the course of its deliberations, the working group met via teleconference on March 26, August 30, October 7 and October 21, 1993 and March 17, 1994 as well as at meetings at the CAS office on April 22, 1993 and June 6, 1994. During the course of those meetings, the group identified a number of key concepts for monitoring travel time and additional information that needed to be included in the CAS database in order to develop the necessary statistics.

The Working Group realized that it is not possible to separate partitioning from other factors affecting travel time. Any evaluation of travel time includes the impact of both partitioning and all other factors. Changes in the frequency of exam administration, the number of candidates entering the system, candidate taking CAS exams for SoA and CIA credit and the passing standards set by the

Examination Committee are all examples of phenomenon that can affect travel time. Therefore, it may be impossible to isolate the impact of partitioning on travel time.

III. Recommendations for Monitoring Travel Time

The Working Group makes the following recommendations regarding the monitoring of the effect of partitioning on travel time.

- Assign primary responsibility for monitoring and interpreting travel time statistics to the Education Policy Committee. See Section IX.
- Identify candidate cohorts so that travel time can be compared from one group of candidates to the next. See Section V.
- Establish historical baselines before drawing any conclusions regarding the impact of partitioning *on* travel time.
- Define travel time to membership as the number of years from the first time any exam (or part thereof) in the sequence 3 through 7 is passed through the attainment of Associateship.

- In order to gain early insight into any impact of partitioning on travel time, monitor travel time for the exam sequence 3 through 5.

- Consider monitoring travel time from ACAS to FCAS, although this is not relevant to partitioning.

- Start with the May, 1987 cohort. See Section V.

- If a longer historical period is needed, evaluate the cost/benefit of obtaining the necessary information from the SoA. See Section VI.

- Using cohort success information as displayed in Appendix A, focus on changes in the time necessary for a common success level to be reached. The Working Group believes success levels of 20%, 35% and 50% are useful benchmarks. See Section V.

- Do not monitor travel time for an individual exam. With the advent of partitioning the entire dynamic interplay between various exams has changed. Travel time through individual exams could lengthen while total travel time does not. See Appendix C.

- Monitor student exam strategy and performance on partitioned exams It is important to know if students are taking fewer exams and how their performance is related to exam load. See Sections V and VIII.

IV. Initial Observations

The Working Group made the following initial observations:

- Travel time appears to have been increasing prior to partitioning of Parts 4 and 5. The exception appears to be with the early percentiles (1 0%, 20%) for the 3 through 7 exam group. See Appendix A for an example of the success level of 35%.

- Although it is too early to make a definitive statement, it appears that travel time has increased subsequent to partitioning of Parts 4 and 5.

- A significant percentage of candidates are opting to take just a single exam subpart even though there are other subparts which they still need. See Appendix B. As a group, these candidates are not as successful (as measured by passing percentages) as the candidates who take a fuller exam load.

V. Kev Concepts Identified

Four key concepts emerged:

- Measurement bv Cohort: Candidates must be grouped so that performance comparisons can be made. The working group defined a cohort as the set of candidates in an exam period who first took any exam in a group of exams for which travel time is to be measured. For example, if travel time from Part 3 through ACAS is to be measured, the cohort for each exam session would be those candidates who first took any exam in the sequence 3 through 7 during that exam session. Candidates who first took any exams in the 3 through 7 sequence in May, 1990, would be members of the May, 1990 cohort and so on. Once a candidate is assigned to a cohort he or she remains in that cohort.

- Establishment of Base-Line: In order to evaluate whether partitioning is having any impact on travel time it is necessary to know what the trend in travel time was prior to partitioning. In other words, a baseline, or history, would have to be created. Because of the possible impact on travel time of changes in the number of exams in the early 1970s and the difficulty in obtaining candidate registration information on Part 3 prior to 1987, the working group decided that any baseline evaluation should start with the May, 1987 cohort if Part 3 is to be included in the analysis. If Part 3 is to be excluded, the baseline evaluation should start with the November, 1982 cohort, which is when Part 4 became a CAS only exam.

. <u>Measurement of Travel Time</u>: The working group believes that travel time should be measured and changes in travel time monitored from the perspective of the number of years it takes cohorts to reach various completion levels for the same series of exams. Since most cohorts do not attain 100% completion, measurements of travel time cannot be made on that basis. Consequently, the Working Group examined travel time at various percentile completion points and concluded that 20%, 35% and 50% are useful benchmarks.

<u>Student Exam Strategy</u>: While not directly related to the measurement of travel time, observing candidate exam load (partial exams vs. full exams) for each sitting and the relative success of students under different strategies can provide an indication of why any change in travel time is occurring. The working group developed a report (see Appendix B) which provides information on what parts or sub-parts students are taking and their success on these parts of sub-parts.

VI. Database Enhancements

During the course of the Working Group's deliberations, needs for enhancements to the database were identified. These included:

- Update of Exam Histories: For pre-1991 Fellows, the exam history on the database was incomplete in that the record would contain the fact that an exam had been passed but not the date that it had been passed. In addition, the dates for passing or failing jointly sponsored exams were neither included in the paper records nor the database. Since the date of passing an exam was critical for measuring travel time and failure dates for Part 3 are also needed to assess candidate exam strategies the database records had to be updated for the missing dates.

  Because the SoA has electronic records back only to 1987, the database could not easily be updated for Part 3 prior to 1987. Information on earlier administrations are contained in paper files maintained by candidate, not exam administration. In order for the CAS to obtain the necessary information, a manual review of these files would be required, which could prove to be costly and time consuming with no guarantee of complete accuracy.

- Attainment of ACAS Status: Records for many Fellows did not include the date (month/year) that ACAS status was attained. In order to establish a historical baseline for travel time to ACAS, this information was obtained and entered.

- Attainment of FCAS Status: Records for many Fellows included only the year of fellowship. In order to establish a baseline for travel time to fellowship, the month is required as well. This information was obtained and entered. These tasks required a painstaking search of old yearbooks, SoA pass lists and other paper records. The necessary information has been obtained and recorded by the CAS office staff.

VII. Adjustments to Cohorts

Once the concept of cohorts had been defined the Working Group was concerned with changes in the ultimate success rate caused by:

- SoA credit being granted for some CAS exams.

- SoA members who took joint exams many years ago and are now returning to pursue membership in the CAS.

In order to minimize the impact of these phenomena, edits were instituted to remove from the database any candidates who could be identified as fitting into either of these groups.

The Working Group also considered removing from the database candidates who ceased taking exams before reaching ACAS status or took exams intermittently. The database was edited to remove candidates who:

- Did not sit for a CAS exam in the most recent exam period (11/93);

- Did not sit for a CAS exam in at least 50% of the exam periods since the cohort was formed and who had not yet attained ACAS status.

The Working Group rejected these adjustments because they would eliminate different segments of the original cohort depending on the maturity of the cohort For example, if we are trying to evaluate if there has been a change in what percentage of a cohort had successfully completed a series of exams within 3 years of the cohort being defined, then the database adjustments that were rejected by the Working Group would impact a cohort that had matured 6 years since its formation much more than a cohort that had just reached 3 years maturity.

Appendix D summarizes the number of candidates from each cohort that would be removed by each of these edits as of the 11193 exam administration as well as the number that would be eliminated by the application of both criteria (the intersection of both sets).

VIII. Modeling of Travel Time

What should be expected to be seen in the travel time charts? To address this, the working group considered the negative binomial model. The negative binomial distribution gives the probability of k failures before n successes. From this distribution it is possible to get the distribution of travel time by dividing the sum of k and n by the number of exams taken per year

In Appendix E, we show that a consequence of the negative binomial model is that we should not expect an increase in travel time if exams are partitioned and students continue to take “full exams” at the same rate.

However, if students pass all but one of the subparts in a range of exams, they cannot take the remaining exam at the same rate, although, in reality, they could be taking exams outside the range. In Appendix E we show that this effect could add as much as 0.75 years to the observed travel time. We call this effect the “last exam effect”.

We now turn to actual results. Appendix A, Sheets 3 & 4 gives the most recent travel time plots for Parts 3 through 5. As a point of information, it should be noted that the partitioning of Part 3 was introduced in 1987, the partitioning of Part 4 was introduced in 1992 and the partitioning of Part 5 was introduced in 1993. It is clear that lower passing percentages are happening after the introduction of partitioning. How much of this can be attributed to the last exam effect is not clear from this exhibit.

Additional preliminary observations can be made by examining exam-taking patterns. Appendix B, Sheet 1 provides a summary of exam-taking patterns for recent exams through May, 1994. As can be seen from this exhibit, the number of students that take only one part of a partitioned exam has been on an upward trend. Also, Appendix B suggests that students who take just a partial exam do not perform any better than the rest of the student population, as measured by pass ratio.

IX. Ongoing Monitoring of Travel Time

Travel Time is a diagnostic concept relative to the admission of members to associateship and fellowship status in the Casualty Actuarial Society. As such, it demands the awareness of CAS general management and is a specific responsibility of the Vice President -Admissions. While the concept is simple, measuring the time and effort it takes to get through actuarial exams, fact gathering and interpretation is an elusive endeavor.

One thing is clear, basic tracking information must be available in a form that allows consistent time series performance observations of the candidate universe. The CAS office has established a data base and has begun to support this performance observation process. The CAS office should retain this responsibility and make whatever changes are deemed appropriate by the leadership of the CAS.

The Vice President -Admissions delegates various responsibilities to the Education Policy Committee, Syllabus Committee and Examination Committee. The Syllabus Committee is responsible for determining the content, depth, breadth and jurisdictional flavor of the learning materials on which candidates are to be tested. The Examination Committee has direct control over the amount of material reflected in each exam, the difficulty of questions to be answered and exam specific measurement of candidate performance. The Education Policy Committee is responsible for the practice emphasis, education techniques (exams vs. papers or academic work, on the job training vs. formal, continuing education vs. on time qualification), alternative qualifications and educational liaison with other actuarial bodies throughout the world. The Education Policy Committee must also deal with the general motivation and preparedness of the candidate universe.

While each of the Admissions Committees has an impact on the travel time of candidates, the primary responsibility for monitoring and interpreting travel time statistics should rest with the Education Policy Committee which can draw on the expertise of the other Admissions Committees for assistance.

APPENDIX A

APPENDIX A

Important Note

The Parts 3-7 and 3-5 exhibits were produced from the database at different points in time. In the time interval between the production of these exhibits the database was updated for the results of the May, 1994 exam administration as well as the results of the ongoing project to complete the exam histories of past exam takers. Therefore, the two exhibits may be inconsistent with regard to the identification of cohort membership.

Appendix A
Sheet 1

Jul-94

Steady State Test Parts 3-7: Cumulative Percentage

	Unedited Pop.	Edited Pop.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
May 77	60	60	0.00%	0.00%	3.33%	21.67%	31.67%	35.00%	40.00%	45.00%	53.33%	56.67%	63.33%	65.00%	65.00%	66.67%	70.00%	70.00%	70.00%	70.00%	70.00%	70.00%	70.00%	70.00%	71.67%
Nov 77	49	48	0.00%	0.00%	0.00%	18.75%	20.83%	35.42%	37.50%	52.08%	52.08%	58.33%	58.33%	60.42%	60.42%	64.58%	66.67%	68.75%	68.75%	70.83%	70.83%	72.92%	75.00%	75.00%	75.00%
May 78	77	77	0.00%	0.00%	0.00%	3.90%	12.99%	15.58%	23.38%	28.57%	36.36%	36.36%	46.75%	48.05%	51.95%	51.95%	57.14%	59.74%	64.94%	64.94%	67.53%	67.53%	71.43%	71.43%	71.43%
Nov 78	61	61	0.00%	0.00%	0.00%	8.20%	9.84%	22.95%	27.87%	42.62%	44.26%	49.18%	49.18%	55.74%	55.74%	60.66%	60.66%	60.66%	60.66%	62.30%	62.30%	63.93%	63.93%	63.93%	63.93%
May 79	62	62	0.00%	0.00%	1.61%	3.23%	8.06%	11.29%	22.58%	22.58%	33.87%	33.87%	41.94%	41.94%	46.77%	50.00%	51.61%	51.61%	56.45%	56.45%	56.45%	56.45%	59.68%	59.68%	59.68%
Nov 79	108	108	0.00%	0.00%	0.00%	5.56%	8.33%	12.96%	14.81%	24.07%	24.07%	34.26%	35.19%	47.22%	47.22%	51.85%	52.78%	53.70%	53.70%	56.48%	56.48%	58.33%	59.26%	60.19%	60.19%
May 80	72	72	0.00%	0.00%	1.39%	1.39%	12.50%	12.50%	25.00%	26.39%	33.33%	36.11%	52.78%	52.78%	65.28%	65.28%	66.67%	66.67%	66.67%	66.67%	68.06%	69.44%	70.83%	70.83%	73.61%
Nov 80	66	64	0.00%	0.00%	0.00%	7.81%	7.81%	17.19%	18.75%	28.13%	29.69%	40.63%	42.19%	50.00%	53.13%	62.50%	64.06%	65.63%	65.63%	67.19%	67.19%	70.31%	70.31%	70.31%	70.31%
May 81	70	69	0.00%	0.00%	1.45%	2.90%	7.25%	11.59%	24.64%	24.64%	34.78%	36.23%	37.68%	39.13%	43.48%	43.48%	50.72%	52.17%	57.97%	59.42%	60.87%	63.77%	65.22%	65.22%	65.22%
Nov 81	79	79	0.00%	0.00%	0.00%	3.80%	3.80%	13.92%	13.92%	16.46%	21.52%	24.05%	24.05%	31.65%	31.65%	40.51%	40.51%	44.30%	45.57%	45.57%	46.84%	46.84%	46.84%	48.10%	48.10%
May 82	79	79	0.00%	0.00%	0.00%	0.00%	8.86%	11.39%	20.25%	24.05%	29.11%	30.38%	32.91%	34.18%	39.24%	45.57%	48.10%	48.10%	49.37%	49.37%	49.37%	49.37%	50.63%	50.63%	51.90%
Nov 82	102	101	0.00%	0.00%	0.00%	0.00%	0.00%	3.96%	6.93%	14.85%	16.83%	21.78%	21.78%	23.76%	23.76%	24.75%	25.74%	27.72%	28.71%	29.70%	30.69%	30.69%	30.69%	32.67%	32.67%
May 83	94	93	0.00%	0.00%	0.00%	1.08%	10.75%	12.90%	22.58%	27.96%	37.63%	41.94%	48.39%	50.54%	50.54%	51.61%	52.69%	52.69%	52.69%	52.69%	54.84%	54.84%	56.99%	58.06%	
Nov 83	72	70	0.00%	0.00%	0.00%	4.29%	7.14%	11.43%	17.14%	20.00%	21.43%	28.57%	30.00%	37.14%	37.14%	37.14%	40.00%	42.86%	45.71%	45.71%	45.71%	47.14%	47.14%		
May 84	89	88	0.00%	0.00%	0.00%	1.14%	4.55%	6.82%	11.36%	17.05%	21.59%	22.73%	26.14%	28.41%	32.95%	34.09%	36.36%	40.91%	42.05%	43.18%	43.18%	44.32%			
Nov 84	84	84	0.00%	0.00%	1.19%	5.95%	8.33%	10.71%	10.71%	15.48%	19.05%	26.19%	30.95%	33.33%	36.90%	40.48%	40.48%	40.48%	42.86%	44.05%	44.05%				
May 85	75	71	0.00%	0.00%	4.23%	8.45%	15.49%	21.13%	23.94%	26.76%	35.21%	40.85%	42.25%	43.66%	46.48%	49.30%	50.70%	52.11%	53.52%	54.93%					
Nov 85	92	91	0.00%	0.00%	0.00%	3.30%	5.49%	9.89%	14.29%	18.68%	20.88%	27.47%	32.97%	39.56%	40.66%	46.15%	46.15%	52.75%	52.75%						
May 86	118	116	0.00%	0.00%	0.00%	2.59%	9.48%	12.07%	20.69%	26.72%	31.90%	37.07%	40.52%	43.10%	43.97%	45.69%	49.14%	49.14%							
Nov 86	99	96	0.00%	0.00%	0.00%	3.13%	6.25%	18.75%	30.21%	31.25%	34.38%	39.58%	40.63%	43.75%	43.75%	47.92%	48.96%								
May 87	239	231	0.00%	0.00%	0.87%	4.33%	8.23%	11.26%	16.45%	20.35%	27.27%	30.30%	33.77%	36.80%	38.96%	41.13%									
Nov 87	198	190	0.00%	0.00%	0.00%	4.74%	6.32%	13.68%	17.37%	21.58%	23.16%	26.84%	28.42%	33.68%	37.89%										
May 88	275	262	0.00%	0.00%	0.00%	2.29%	6.49%	8.40%	14.50%	16.79%	23.28%	26.34%	33.21%	37.79%											
Nov 88	207	195	0.00%	0.00%	0.00%	4.62%	9.23%	13.33%	13.33%	15.90%	17.95%	22.56%	24.62%												
May 89	300	273	0.00%	0.00%	0.00%	1.10%	2.93%	4.76%	9.89%	12.45%	17.58%	20.51%													
Nov 89	261	238	0.00%	0.00%	0.42%	3.78%	4.62%	11.34%	13.03%	16.39%	18.91%														
May 90	383	332	0.00%	0.00%	0.60%	1.81%	4.22%	6.33%	10.54%	12.95%															
Nov 90	341	297	0.00%	0.00%	0.00%	1.35%	3.70%	9.43%	11.11%																
May 91	376	308	0.00%	0.00%	0.00%	1.30%	4.22%	7.14%																	
Nov 91	405	316	0.00%	0.00%	0.00%	5.06%	6.01%																		
May 92	364	244	0.00%	0.00%	0.41%	0.41%																			
Nov 92	313	199	0.00%	0.00%	0.50%																				
May 93	378	193	0.00%	0.00%																					
Nov 93	283	183	0.00%																						
May 94	237	122																							

*Candidates who have requested credit with the SoA for CAS Parts 4A and/or 4B have been excluded from the population in any given cohort

Appendix A
Sheet 2

Jul-94

	Unedited Pop.	Edited Pop.	24	25	26	27	28	29	30	31	32	33	34
May 77	60	60	71.67%	73.33%	73.33%	73.33%	73.33%	73.33%	73.33%	73.33%	73.33%	75.00%	76.67%
Nov 77	49	48	77.08%	77.08%	77.08%	77.08%	77.08%	77.08%	77.08%	79.17%	81.25%	81.25%	
May 78	77	77	72.73%	72.73%	72.73%	72.73%	72.73%	74.03%	75.32%	76.62%	76.62%		
Nov 78	61	61	63.93%	65.57%	67.21%	67.21%	67.21%	67.21%	68.85%	68.85%			
May 79	62	62	61.29%	61.29%	61.29%	61.29%	61.29%	62.90%	64.52%				
Nov 79	108	108	61.11%	61.11%	61.11%	61.11%	62.96%	63.89%					
May 80	72	72	73.61%	75.00%	75.00%	75.00%	75.00%						
Nov 80	66	64	70.31%	70.31%	70.31%	70.31%							
May 81	70	69	65.22%	65.22%	65.22%								
Nov,81	79	79	48.10%	48.10%									
May 82	79	79	51.90%										

Appendix A
Sheet 3

06-Jan-95

Steady State Test Parts 3-5: Cumulative Percentage

	No. of Candidate	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
May 77	68	0.00%	4.41%	23.53%	35.29%	42.65%	57.35%	57.35%	69.12%	69.12%	72.06%	72.06%	72.06%	73.53%	75.00%	75.00%	75.00%	75 00%	75.00%	75.00%	76.47%	76.47%	76.47%
Nov 77	70	0.00%	0.00%	30.00%	32.86%	45.71%	48.57%	57.14%	57.14%	64.29%	65.71%	70.00%	71.43%	72.86%	72.86%	75.71%	75.71%	75.71%	75 71%	78.57%	78.57%	78.57%	78.57%
May 78	89	0.00%	0.00%	16.85%	26.97%	30.34%	46 07%	46.07%	55.06%	55.06%	62.92%	62.92%	65.17%	65.17%	69.66%	70.79%	75.28%	75 28%	76 40%	76.40%	76.40%	76.40%	77.53%
Nov 78	69	0.00%	0.00%	14.49%	20.29%	44.93%	44.93%	55.07%	56.52%	59.42%	60.87%	62.32%	62 32%	65.22%	65.22%	66.67%	66.67%	66 67%	68.12%	68.12%	68.12%	69.57%	69.57%
May 79	92	0.00%	0.00%	5.43%	26.09%	27.17%	41.30%	41.30%	47.83%	47.83%	53.26%	53 26%	58.70%	58.70%	61.96%	63.04%	64.13%	65.22%	66 30%	66.30%	68.48%	68.48%	68.48%
Nov 79	127	0.00%	0.00%	14.96%	14 96%	28.35%	29.13%	44.09%	44.88%	49.61%	51.18%	55.91%	55.91%	59.84%	59.84%	62.99%	64.57%	66 14%	66.14%	67.72%	67.72%	68.50%	69.29%
May 80	74	0.00%	0.00%	5.41%	24.32%	28.38%	44.59%	45 95%	50.00%	51.35%	58.11%	58.11%	62.16%	64.86%	71.62%	71.62%	77.03%	77.03%	77 03%	77.03%	78.38%	78.38%	78.38%
Nov 80	85	0.00%	0.00%	18.82%	20.00%	40.00%	41.18%	49.41%	50 59%	63.53%	64.71%	69.41%	70.59%	77.65%	80.00%	82.35%	82.35%	84.71%	84.71%	84.71%	84.71%	84.71%	84.71%
May 81	91	0.00%	0.00%	4.40%	24.18%	26.37%	34.07%	37.36%	50.55%	50.55%	57.14%	57.14%	63.74%	64.84%	65.93%	68.13%	71.43%	72.53%	75 82%	75.82%	76.92%	79.12%	81.32%
Nov 81	134	0.00%	0.00%	11.94%	13.43%	22 39%	26.87%	40.30%	40.30%	45.52%	47.01%	52.24%	53.73%	54.48%	54.48%	56.72%	56.72%	56 72%	57.46%	57.46%	58.21%	58.21%	58.96%
May 82	104	0.00%	0.00%	2 88%	25.00%	26.92%	36.54%	39.42%	47.12%	48.08%	50.96%	50.96%	55.77%	56 73%	59.62%	62.50%	65 38%	65 38%	65.38%	65.38%	65.38%	65.38%	65.38%
Nov 82	151	0.00%	0.00%	3 31%	4.64%	9.93%	11.92%	22 52%	23.84%	30.46%	30.46%	33.77%	34.44%	36.42%	38.41%	40.40%	40 40%	40.40%	40 40%	40.40%	40.40%	40.40%	40.40%
May 83	104	0.00%	0.00%	8.65%	18.27%	24.04%	33.65%	40.38%	44.23%	46.15%	51.92%	57.69%	58.65%	59.62%	59.62%	59.62%	59.62%	59 62%	60.58%	61.54%	61.54%	61.54%	62.50%
Nov 83	90	0.00%	0 00%	11 11%	21.11%	25.56%	30.00%	32.22%	35.56%	37.78%	38.89%	38.89%	40 00%	40.00%	41.11%	42.22%	42.22%	43.33%	45.56%	45.56%	45.56%	45.56%	45.56%
May 84	110	0 00%	0 00%	3.64%	9.09%	17.27%	24.55%	26.36%	30.91%	36.36%	39.09%	42.73%	43.64%	44.55%	45.45%	45.45%	46.36%	49.09%	50.00%	50.00%	50.00%	50.91%	
Nov 84	90	0.00%	0.00%	8 89%	15.56%	18.89%	27.78%	32.22%	33.33%	38.89%	42.22%	44.44%	50.00%	51.11%	54.44%	54.44%	55.56%	56.67%	56.67%	56.67%	56.67%		
May 85	94	0 00%	0.00%	14 89%	22.34%	35.11%	36.17%	36.17%	38.30%	42.55%	50.00%	51.06%	51 06%	52.13%	53.19%	54.26%	55.32%	56.38%	57.45%	57.45%			
Nov 85	108	0.00%	0.00%	8.33%	12.04%	25.00%	28.70%	37.04%	40.74%	45.37%	52.78%	53.70%	53.70%	56.48%	58.33%	59.26%	60.19%	60.19%	60.19%				
May 86	200	0.00%	0.00%	12.00%	25.50%	29.00%	40.00%	45.00%	50.00%	52.50%	53.50%	55.00%	57.00%	59.00%	60.00%	60.50%	61.50%	62.00%					
Nov 86	147	0.00%	0.00%	16.33%	27.21%	36.05%	42.86%	50 34%	55.78%	57.14%	59.86%	63.95%	66.67%	66.67%	67.35%	68.03%	69.39%						
May 87	309	0.00%	0.00%	5.50%	8.74%	16.50%	24.60%	33.01%	40.78%	43 69%	47.90%	51.46%	52.75%	54.37%	55.02%	56.63%							
Nov 87	228	0.00%	0.00%	7.02%	12.72%	20.61%	26.32%	30.70%	35 96%	39.91%	40.35%	42.54%	44.30%	48 68%	52.63%								
May 88	280	0.00%	0.00%	3.93%	11.79%	17.86%	23 57%	30.36%	35.71%	38.93%	44.29%	47.14%	48.93%	51.43%									
Nov 88	220	0.00%	0.00%	9 09%	14.55%	15.91%	19.55%	23.18%	25.91%	31.36%	34.09%	39.09%	40.91%										
May 89	320	0.00%	0.00%	2.50%	4.69%	8.13%	16.25%	18.44%	23.44%	27.19%	30.31%	33.44%											
Nov 89	264	0.00%	0.00%	3.41%	6.44%	12 12%	16.29%	22.73%	26.14%	29.55%	32.58%												
May 90	389	0.00%	0.00%	2.31%	5.66%	9.00%	14.14%	15.68%	19.54%	23 91%													
Nov 90	372	0.00%	0.00%	2.15%	4.84%	9.14%	11 02%	14.52%	16.40%														
May 91	396	0.00%	0.00%	1 26%	5.81%	7.32%	13.89%	16.92%															
Nov 91	424	0 00%	0 00%	4.01%	4.48%	9.43%	12.03%																
May 92	393	0.00%	0.00%	0.76%	1.53%	4.33%																	
Nov 92	321	0.00%	0.00%	0.62%	1.25%																		
May 93	399	0.00%	0.00%	0.25%																			
Nov 93	291	0.00%	0.00%																				
May 94	217	0.00%																					

Appendix A
Sheet 4

23	24	25	26	27	28	29	30	31	32	33	34	35
76.47%	79.41%	79.41%	79.41%	79.41%	79.41%	79.41%	79.41%	82.35%	82.35%	82.35%	82.35%	82.35%
78.57%	78.57%	78.57%	78.57%	78.57%	78.57%	78.57%	78.57%	78.57%	78.57%	78.67%	78.57%	
77.53%	77.53%	77.53%	77.53%	77.53%	78.65%	78.65%	78.65%	78.65%	78.65%	78.65%		
71.01%	71.01%	71.01%	72.46%	72.46%	72.46%	72.46%	72.46%	72.46%	72.46%			
69.57%	69.57%	69.57%	69.57%	69.57%	69.57%	69.57%	69.57%	69.57%				
69.29%	69.29%	70.08%	70.08%	70.08%	70 08%	70.87%	71.65%					
78.38%	78.38%	78.38%	78.38%	78.38%	78.38%	78.38%						
84.71%	84.71%	84.71%	84.71%	84.71%	84.71%							
81.32%	81.32%	81.32%	81.32%	81.32%								
58.96%	58.96%	58.96%	58.96%									
65.38%	65.38%	65.38%										
40.40%	40.40%											
62.50%												

APPENDIX B

Appendix B
Sheet 1

Summary of Candidate Exam Strategy

Percentage of Candidates Taking a Single Subpart But Needing Additional Subparts

	May 1992	November 1992	May 1993	November 1993# U	November 1993# A	May 1994
Part 3B	27.6%	28 0%	27.2%	35.1%	41.0%	45.4%
Part 4A	25.6%	23.9%	34.5%	29.1%	33 7%	43.4%
Part 4B	19.5%	29 4%	26.4%	43.0%	48.2%	45.4%
Pan 5A				17.8%	22.0%	23.1%
Pan 5B				14.9%	18.0%	24.5%

First administration of Part 5 as a partitioned exam. For consistency with previous exam administrations, the percentages are displayed unadjusted (U) and adjusted (A) for ineffective candidates. All subsequent exam administrations will reflect the adjustment for ineffective candidates.

Appendix B
Sheet 2

May 1992

1. Success on 3B

Multiple Exam Takers	104/268 = 38.8%
Single Exam Takers	
- Needed only 3B	0/0 =
- Needed other sub-parts	43/102 = 42.2%
Total	43/102 = 42.2%

2. Success on 4A

Multiple Exam Takers	2431664 = 36.6%
Single Exam Takers	
- Needed only 4A	010 =
- Needed other sub-parts	72/228 = 31.6%
Total	72/228 = 31.6%

3. Success on 4B

Multiple Exam Takers	302/707 = 42.1%
Single Exam Takers	
- Needed only 4B	010 =
- Needed other sub-pans	37/171 = 21.6%
Total	37/171 = 21.6%

Appendix B
Sheet 3

November 1992

1. Success on 3B

Multiple Exam Takers	1691364 = 46.4%
Single Exam Takers	
- Needed only 3B	32/43 = 74.4%
- Needed other sub-parts	45/158 = 28.5%
Total	77/201 = 38.3%

2. Success on 4A

Multiple Exam Takers	1311394 = 33.2%
Single Exam Takers	
- Needed only 4A	5/30 = 16.7%
- Needed other sub-parts	28/133 = 21.1%
Total	33/163 = 20.2%

3. Success on 4B

Multiple Exam Takers	1561386 = 40.4%
Single Exam Takers	
- Needed only 4B	38/57 = 66.7%
- Needed other sub-parts	69/180 = 38.3%
Total	107/237 = 45.1%

Appendix B
Sheet 4

May 1993

1. Success on 3B

Multiple Exam Takers	1481324 = 45.7%
Single Exam Takers	
- Needed only 3B	38/64 = 59.4%
- Needed other sub-parts	37/145 = 25.5%
Total	75/209 = 35.9%

2. Success on 4A

Multiple Exam Takers	124/395 = 31.4%
Single Exam Takers	
- Needed only 4A	30/66 = 45.5%
- Needed other sub-parts	89/243 = 36.6%
Total	119/309 = 38.5%

3. Success on 4B

Multiple Exam Takers	111/445 = 24.9%
Single Exam Takers	
- Needed only 4B	46/157 = 29.3%
- Needed other sub-parts	101/196 = ***51.5%***
Total	147/353 = 41.6%

November, 1993
(Unadjusted for ineffective candidates)

1. Success on 3B	
Multiple Exam Takers	121/299 = 40.5%
Single Exam Takers	
- Needed only 3B	20/24 = 83.3%
- Needed other sub-parts	69/175 = 394%
Total	89/199 = 44.7%
2. Success on 4A	
Multiple Exam Takers	1601349 = 45.8%
Single Exam Takers	
- Needed only 4A	19/21 = 90.5%
- Needed other sub-parts	57/152 = 37.5%
Total	76/173 = 43.9%
3. Success on 4B	
Multiple Exam Takers	115/363 = 31.7%
Single Exam Takers	
- Needed only 4B	18/33 = 54.5%
- Needed other sub-parts	981299 = 32.8%
Total	116/332 = 34.9%
4. Success on 5A	
Multiple Exam Takers	127/359 = 35.4%
Single Exam Takers	
- Needed only 5A	13/29 = 44.8%
- Needed other sub-parts	12/84 = 14.3%
Total	25/113 = 22.1%
5. Success on 5B	
Multiple Exam Takers	112/331 = 33.8%
Single Exam Takers	
- Needed only 5B	
- Needed other sub-parts	16/58 = 27.6%
Total	16/58 = 27.6%

November, 1993
(Adjusted for ineffective candidates)

1. Success on 3B	
Multiple Exam Takers	***1081222 = 48.1%***
Single Exam Takers	
- Needed only 3B	**23/30 = *16.1%***
- Needed other sub-parts	***791204 = 38.7%***
Total	***1021234 = 43.6%***
2. Success on 4A	
Multiple Exam Takers	***1391250 = 55.6%***
Single Exam Takers	
- Needed only 4A	**18/24 = *75.0%***
- Needed other sub-parts	**79/176** = 44.9%
Total	971200 = 48.5%
3. Success on **4B**	
Multiple Exam Takers	***1031250 = 41.2%***
Single Exam Takers	
- Needed only 4B	**17/36** = 41.2%
- Needed other sub-parts	**108/335** = 32.2%
Total	***1251371 = 33.7%***
4. Success on **5A**	
Multiple Exam Takers	**119/273** = 43.6%
Single Exam Takers	
- Needed only 5A	**10/29** = 34.5%
- Needed other sub-parts	**23/104** = 22.1%
Total	**33/133** = 24.8%
5. Success on 5B	
Multiple Exam Takers	**98/260** = 31.7%
Single Exam Takers	
- Needed only **5B**	**2/3** = 66.7%
- Needed other sub-parts	**28/70** = 40.0%
Total	**30/73** = 41.1%

Appendix B
Sheet 7

May, 1994
(Adjusted for ineffective candidates)

1. Success on 3B	
Multiple Exam Takers	**102/196 = *52.0%***
Single Exam Takers	
- Needed only 3B	16/45 = *35.6%*
- Needed other sub-parts	*681200 = 34.0%*
Total	*841245 = 34.3%*
2. Success on 4A	
Multiple Exam Takers	**130/221 = *58.8%***
Single Exam Takers	
- Needed only 4A	**15/54 = *27.8%***
- Needed other sub-parts	66/211 = 31.3%
Total	81/265 = 30.6%
3. Success on 4B	
Multiple Exam Takers	142/322 = 44.1%
Single Exam Takers	
- Needed only 4B	**41/163 = *25.2%***
- Needed other sub-parts	128/403 = 31.8%
Total	1691566 = 29.9%
4. Success on 5A	
Multiple Exam Takers	**85/187 = *45.5%***
Single Exam Takers	
- Needed only **5A**	19/89 = 21.3%
- Needed other sub-parts	31/83 = 31.3%
Total	50/172 = 29.1%
5. Success on 5B	
Multiple Exam Takers	89/216 = 41.2%
Single Exam Takers	
- Needed only 5B	12/43 = 27.9%
- Needed other sub-parts	**25/84 = *29.8%***
Total	37/127 = 29.1%

APPENDIX C

DO NOT FOCUS ON TRAVEL TIME FOR INDIVIDUAL EXAMS

The question was raised as to how has travel time changed on Part 4 subsequent to partitioning of the exam in May, 1992. While it is possible to calculate any change in travel time for Part 4 using the concepts of cohorts and travel time developed by the Working Group, we could not understand the relevance of such a calculation.

The ultimate goal of the CAS is to educate actuaries as measured by the successful completion of a series of exams. With the relationship between individual exams radically altered by partitioning, focusing on individual exams could lead to the conclusion that travel time is expanding while what is actually happening is that travel time through the entire series of exams is unchanged.

An example may prove to be illuminating. On Exhibit 1 attached are the examination records for two hypothetical candidates. By looking at the entire exam sequence we see that both candidates took 10 exam sessions to make it through the 5 exam sequence. But because of the way Candidate 2 could mix and combine exam subparts, take a partial exam for the first time while repeating another partial exam, the travel time for individual exams is markedly different.

Despite the Working Group's conclusion that focusing on travel time for individual exams is not appropriate, Exhibit 2 attached sets forth the results for Part 4.

Examination Records for Hypothetical Candidates

Candidate 1	Candidate 2
5105 Fail 3	11191 Pass 3A,C
1 1/85 Pass 3	5/92 Fail 4A
5/86 Fail 4	1 1/92 Fail 4A, Fail 4B
11/86 Fail 5	5/93 Pass 4A, Fail 4B
5/87 Fail 4	11193 Pass 4B, Pass 3B
11187 Pass 5	5/94 Pass 5A, Fail 5B
5/88 Pass 4	11194 Pass 5B, Fail 7
11/88 Fail 7	5/95 Fail 6
5/89 Pass 6	11195 Pass 7
11/89 Pass 7	5196 Pass 6

Travel Time

	Candidate 1	Candidate 2
Part 3	2 exam sessions	5 exam sessions
Part 4	5 exam sessions	4 exam sessions
Part 5	3 exam sessions	3 exam sessions
Part 6	1 exam session	3 exam sessions
Part 7	3 exam sessions	3 exam sessions
Parts 3-7	10 exam sessions	10 exam sessions

Appendix C
Exhibit 2
Sheet 1

Travel Time
Part 4 Only

	Candidates	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
May 77	63	49.21%	49.21%	73.02%	73.02%	84.13%	85.71%	90.48%	90.48%	92.06%	92.06%	92.06%	92.06%	92.06%	92.06%	95.24%	95.24%	95.24%	95.24%
Nov 77	0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
May 78	105	47.62%	47.62%	76.19%	79.05%	80.95%	83.81%	83.81%	84.76%	86.67%	87.62%	89.52%	89.52%	89.52%	89.52%	89.52%	89.52%	89.52%	89.52%
Nov 78	0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
May 79	105	57.14%	63.81%	70.48%	77.14%	80.00%	80.95%	80.95%	83.81%	85.71%	85.71%	88.57%	88.57%	88.57%	88.57%	88.57%	88.57%	88.57%	88.57%
Nov 79	28		*				*	*				*	*	*	*	*			
May 80	42	*	*		*		*	*		*		*	*	*	*	*			
Nov 80	52				*		*	*						*	*		*	*	*
May 81	62	*	*				*	*			*			*	*	*			
Nov 81	42		*	*			*	*		*	*	*		*	*	*			*
May 82	66	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
Nov 82	161	34.16%	51.55%	51.55%	60.25%	60.25%	65.22%	65.22%	67.70%	67.70%	68.94%	68.94%	70.81%	70.81%	72.67%	72.67%	73.91%	73.91%	75.16%
May 83	106	23.58%	23.58%	43.40%	43.40%	55.66%	55.66%	59.43%	59.43%	60.38%	60.38%	62.26%	62.26%	62.26%	62.26%	63.21%	63.21%	65.09%	65.09%
Nov 83	0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
May 84	153	27.45%	27.45%	46.41%	46.41%	58.17%	58.17%	61.44%	61.44%	65.36%	65.36%	65.36%	65.36%	66.01%	66.01%	66.67%	66.67%	67.97%	67.97%
Nov 84	0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
May 85	132	25.76%	25.76%	40.91%	40.91%	54.55%	54.55%	58.33%	58.33%	62.88%	62.88%	65.15%	65.15%	67.42%	67.42%	69.70%	69.70%	70.45%	71.21%
Nov 85	0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	
May 86	152	28.95%	28.95%	60.53%	60.53%	66.45%	66.45%	74.34%	74.34%	78.95%	78.95%	79.61%	79.61%	80.26%	81.58%	82.24%	83.55%		
Nov 86	0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%			
May 87	188	29.79%	29.79%	53.72%	53.72%	63.83%	63.83%	70.21%	70.21%	73.40%	73.40%	74.47%	75.00%	75.00%	75.00%				
Nov 87	0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%					
May 88	265	27.17%	27.17%	50.57%	50.57%	64.15%	64.15%	71.70%	71.70%	73.21%	75.09%	76.98%	78.87%						
Nov 88	0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%							
May 89	381	28.08%	28.08%	53.28%	53.28%	61.42%	61.42%	64.30%	67.45%	70.60%	72.70%								
Nov 89	0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%									
May 90	414	19.81%	19.81%	45.41%	45.41%	49.76%	55.56%	58.94%	61.84%										
Nov 90	0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%											
May 91	348	26.72%	26.72%	37.64%	45.98%	54.60%	59.48%												
Nov 91	0	0.00%	0.00%	0.00%	0.00%	0.00%													
May 92	732	9.02%	20.49%	29.23%	38.52%														
Nov 92	261	4.21%	14.56%	23.37%															
May 93	442	3.17%	11.76%																
Nov 93	302	2.65%																	

* Part 4 was jointly administered from Nov. 1979 until May 1982. Therefore, only pass information was recorded by the CAS.

Appendix C
Exhibit 2
Sheet 2

19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
96.83%	96.83%	96.83%	96.83%	96.83%	96.83%	96.83%	96.83%	96.83%	96.83%	96.83%	96.83%	96.83%	96.83%	96.83%	96.83%
0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	
89.52%	89.52%	89.52%	89.52%	89.52%	89.52%	89.52%	89.52%	89.52%	89.52%	89.52%	89.52%	89.52%	89.52%		
0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%			
88.57%	88.57%	88.57%	88.57%	88.57%	88.57%	88.57%	88.57%	88.57%	88.57%	88.57%	88.57%				
•	•	•	•	•	•	•	•	•	•	•					
•	•	•	•	•	•	•	•	•	•						
•	•	•	•	•	•	•	•	•							
•	•	•		•	•	•	•								
•	•	•	•	•	•	•									
•	•	•		•	•										
75.16%	75.16%	75.16%	75.16%	75.16%											
65.09%	65.09%	65.09%	65.09%												
0.00%	0.00%	0.00%													
67.97%	67.97%														
0.00%															

APPENDIX D

CANDIDATES ELIMINATED

Cohort	Did not sit for at Least 1/2	Did not sit in November '93	Both Cases
May 77	2		0
Nov 77	3	3	0
May 78	5	2	
Nov 78	6	6	0
May 79	12	10	
Nov 79	5	6	
May 80	5	3	2
Nov 80	17	IO	3
May 81	7		0
Nov 81	29	17	
May 82	9	8	2
Nov 82	28	50	11
May 83	34	22	
Nov 83	33	26	16
May 84	39	28	11
Nov 84	31	26	9
May 85	25	20	9
Nov 85	29	25	15
May 86	30	17	8
Nov 86	25	23	12
May 87	35	27	9
Nov 87	23	24	6
May 88	33	33	8
Nov 88	32	43	11
May 89	87	59	19
Nov 89	42	57	19
May 90	56	79	23
Nov 90	35	59	10
May 91	26	48	12
Nov 91	20	38	16
May 92	42	81	22
Nov 92	24	54	14
May 93	0	43	0
Nov 93	0	0	0

APPENDIX E

Appendix E - Prior Expectations of the Effect of Partitioning *on* Travel Time

The Negative Binomial Model

The working group considered the question as to how one should expect travel time to change as exam partitioning is introduced. To address this, the negative binomial model was introduced.

Let: n = the number of subparts to be passed;

K = the number of failures before passing n subparts (random);

p = the probability of passing a given subpart; and

m = the number of subparts taken in a year.

Then K has a negative binomial distribution with:

$$\Pr[K = k] = \binom{n+k-1}{n-1} p^n (1-p)^k$$

We then have: $E[K] = \frac{n(1-p)}{p}$ and $Var[K] = \frac{n(1-p)}{p^2}$

We define the travel time, T, as: $T = \frac{K+n}{m}$

Then: $E[T] = \frac{n}{mp}$ and $Var[T] = \frac{n(1-p)}{m^2 p^2}$

If we use the negative binomial **distribution** as a model for the effect of partitioning, we obtain the following consequence. *There should be no* increase *in the expected* travel *time due to* ***partitioning*** *if the student takes* all *exam subparts* a ***rate*** *corresponding to the original* ***pre-partitioned*** *fate.*

To see this, suppose that n parts are partitioned into a•n subparts. Suppose also that the student takes a•m subparts per year. Then:

$$\text{New Expected Travel Time} = \frac{an}{amp} = \frac{n}{mp} = \text{Old expected Travel Time.}$$

However, the negative binomial model is not a perfect analogy to the actual exam process. Consider, for example, the student who takes all three subparts of part 3 on the first sitting. Suppose the student passes 3A and 3B. It will be impossible to take the equivalent of "one part" on the next sitting. It will have to take either 5/6 of a part (3C and 4A) or 4/3 of a part (3C, 4A and 4B).

The Last Exam Effect

Suppose the student can keep up a reasonable approximation to taking "one part per sitting." In the actual exam process, we *should expect to observe* a *small increase in the* travel *time in the Travel Time Charts of Appendix A.*

To see this, consider a travel time chart for Parts 3 to 5. Suppose the student takes exams at a rate of two subparts per sitting (i.e. 4 subparts per year). Once the student has passed all but Part 5B, it can only continue taking Part 5B at a rate of two subparts per year. While the student *may* actually be taking Part 6 or 7, we will observe a longer travel time for passing Parts 3 to 5. This effect is called the "last exam effect."

The increase in travel time due the last exam effect can be estimated. Suppose the student takes two subparts per sitting, or four subparts per year. Suppose further that the student's pass probability is 0.40. According to the negative binomial model, the student's expected travel time for the seven subparts is:

$$\frac{7}{4 \bullet 0.40} = 4.375 \text{ years.}$$

If instead, the student were to take six subparts at a rate of four per year, and one additional subpart at a rate of two per year, the student's expected travel time would be:

$$\frac{6}{4 \bullet 0.40} + \frac{1}{2 \bullet 0.40} = 5.00 \text{ years}$$

The second travel time estimate is an overestimate of the expected travel time of the actual exam process since it ignores the possibility that the student might have passed the last two subparts on a single sitting. From this example, it would appear that 0.625 years is a reasonable upper bound for the last exam effect.

It should be noted that the last exam effect is a function of the student's probability of passing, p. Since $p=0.40$ is an overall average probability, we should check to see if the last exam effect holds when the student population is diverse. To do this suppose that the student population consists of students with $p=0.20$, 0.40 and 0.60 in equal proportions. One can then work out an expected last exam effect of 0.764 years.

If the effect of partitioning is measured on the Part 3 through the Part 5 range, it is most likely that the last exam effect holds when the student is finishing Parts 4 or 5. Thus it seems reasonable to expect a raise no larger than 0.75 years.

What this all means is that we should not conclude that the travel time is increasing due to partitioning of exams 3 to 5 unless we observe an increase in the mean travel time of (conservatively) greater than 0.75 *years.* This is due to the last exam effect, which is a property of the way we measure travel time.

Report of the CAS Long Range Planning Committee
by the CAS Long Range Planning Committee

CASUALTY ACTUARIAL SOCIETY

Long-Range Planning Committee

REPORT TO THE BOARD

September 1994
(As revised based on Board input in 2/95)

REPORT OF THE CAS LONG-RANGE PLANNING COMMITTEE

Abstract

The CAS Long-Range Planning Committee prepares a report to the CAS Board each year regarding issues the Committee believes will be of importance to the evolution of the CAS over the next several years. This report was originally prepared in 1994 but reflects some changes based on input from the Board at its February, 1995 meeting. The recommendations are those of the Committee and have not been adopted by the Board at this time.

LONG **RANGE** PLANNING **COMMITTEE**
1994 REPORT TO TEE BOARD
(As revised based on Board input in **2/95)**

PROCESS

The discussions and recommendations contained in this report represent the collective efforts of the 1994 Long Range Planning Committee **(LRPC)** to identify those issues which **will** be of critical importance to the evolution of the CAS during the **next decade.**

In order to assure as broad a context as possible for our deliberations, we retied on the following sources of information:

1) An historical review of prior LRPC activity;

2) Minutes of the 1994 Committee Chairpersons Meeting **regarding** the topic of CAS Long Range Planning;

3) 1993 CAS Membership Survey;

4) Personal discussions with non-actuarial professionals both within and outside the **Property/Casualty** insurance **area,** and;

5) Informal discussions at LRPC meetings with prominent industry figures.

CONCLUSIONS

The remaining portion of this report summarizes our comments and recommendations regarding the following issues:

A. Key Long-Term Issues (without prioritization)

- Dynamic **Financial** Analysis
- Health Care Delivery Costs
- **Mega Risk**
- Coordination With Other U.S. **and** Canadian Actuarial Organizations
- International Activity
- Data Reporting
- Actuarial Input to Public Policy Issues
- Accounting Principles and Practices
- Basic and **Continuing** Education
- Committee Structure and Management

B. Other Important Considerations

A. KEY LONG TERM ISSUES

DYNAMIC FINANCIAL ANALYSIS

In the 1993 Long Range Planning Committee Report to the Board, "Solvency" was identified as one of our highest priority issues. It was our belief that the CAS has a meaningful role to play in the measurement and maintenance of solvency for both traditional insurers and alternative risk transfer mechanisms.

Since that report, the NAIC has approved a risk-based capital formula to be applied to property and casualty insurers beginning in 1995. Continued high levels of property catastrophe losses have called into question the solidity of some insurers which, only a few years ago, were considered models of efficiency and strong capitalization. Some states (notably Florida and Hawaii) have formed alternative risk transfer mechanisms to deal with the inevitable lack of availability of essential catastrophe coverages. Also, legislation authorizing the formation of a federal disaster insurance fund as recommended by the Natural Disaster Coalition is slowly generating congressional support.

Over a five to ten year planning horizon, we believe that this issue will continue to be among the highest priorities of the actuarial profession. Financial data alone cannot provide the definitive answer to the question of insurer solvency since no financial reporting requirement captures the range of potential dynamic variables affecting solvency. The insuring public and insurance regulators at both the state and federal level have become increasingly strident in their criticisms of the industry for the absence of meaningful progress toward a credible solvency monitoring standard. Industry analysts will continue to probe and criticize the industry for failing to provide leading indicators of solvency impairment for weak insurers.

Against this backdrop, the CAS has a number of efforts which collectively address many of these concerns. Through our Appointed Actuary Advisory Committee, we have monitored initiatives in solvency regulation, guaranty fund reform, catastrophe exposure funding, and dynamic solvency testing.

From a broad perspective, the CAS, the SOA, the CIA and other actuarial groups have been working in the area of the Valuation Actuary and the Appointed Actuary. The discussions on this topic have ranged from a narrow focus on requirements that the "Actuary" opine on the continued viabiity of an insurer to the broader oversight of management performance. The CAS committees that have been working in this area have focused on "Dynamic Financial Analysis" (DFA) as a title more descriptive of this field. We believe that this reference is much more in line with the more expansive financial management roles which actuaries will perform in the near future. It is our opinion that a more uniform use of the term "Dynamic Financial Analysis" in place of "Appointed Actuary" will be more descriptive of the type of activity we wish to promote.

Recommendations

- Establish DFA as a preferred approach for our clients. To accomplish this task, the CAS cannot rely solely on intra-actuarial publications to establish recognition and acceptance of actuarial DFA work. The actuarial profession as a whole must aggressively establish leadership roles in the insurance, self-insurance and risk management industries in order to solidify a position of expertise for DFA types of analysis and the evaluation of the financial implications of risk decisions.

- An important first step in this initiative would be to change the name of the Appointed Actuary Advisory Committee to the Dynamic Financial Analysis Committee. As is the case today, this group would be charged with coordinating all pertinent CAS activities, maintaining a close working relationship with other organizations (both actuarial and non-actuarial) and providing regular reports to the CAS membership.

HEALTH CARE DELIVERY COSTS

In 1991 the CAS issued a Health Issues Policy Statement largely in response to external pressures on health costs. This statement focused primarily on health issues as they relate to casualty coverages. Also in 1991, the CAS LRPC expressed concern that the syllabus, as then constructed, was not adequate for future FCASs to be sufficiently qualified to address evolving issues such as 24-hour coverage and other managed care products.

Now, in 1994, we see that these concerns were on target. The importance of this issue was dramatically reinforced by a recent ISO study which showed that over 20% ($29 Billion) of the U.S. Property/Casualty industry's losses were Health Care related. It is especially interesting to note that almost 50% of Workers' Compensation losses arise from medical costs and that this percentage has been rising over time.

The syllabus has been updated in the last few years to include more relevant and modem readings on health insurance. However, additional efforts are needed in other areas if the future FCAS is to have a significant impact on the Health Industry. While this may not have been a major consideration in the past, as the lines between casualty insurance and accident and health insurance become blurred we are faced with the alternatives of either widening our scope and expertise or being left in the wake of market/coverage realignments.

Recommendations

- CAS Continuing Education and Program Committees should see that meeting content reflects the impact of these changes on our members.

- The CAS should take the necessary steps to ensure that casualty actuaries are full participants in the AAA working groups studying various aspects of health care reform.

MEGA RISKS

The property-casualty industry continues to be reminded of the actual and potential impact of Mega Risks on the financial strength of individual companies and the entire industry. Natural catastrophes and mass tort liabilities pose as much a threat to solvency as do underpricing, under-reserving or poor management.

While they caused huge losses and the insolvency of several companies, it is obvious that neither Hurricane Andrew nor the Northridge earthquake represents the maximum magnitudes of loss which could result from these types of catastrophic events. Similarly, the emerged costs of asbestos, environmental, and other mass tort claims are believed to be only a portion of the ultimate losses. Recently, A.M. Best published an analysis of asbestos and environmental liability costs which suggested that the range of ultimate costs from these perils would endanger the solvency of many sizable carriers and may even exceed the capital of the insurance industry as a whole. Other mass torts, such as lead paint, electromagnetic radiation and tobacco claims could total many billions of dollars as well.

Audiences such as the SEC, state and federal regulators, shareholders, rating agencies and the accounting profession are urging insurers and insureds to quantify the potential risks of these events as well as the liabilities they may have already incurred. There is a distinct possibility that others will dictate how these liabilities must be quantified, if the actuarial profession does not take a leadership role in establishing appropriate methodologies and standards.

The actuarial ramifications of these catastrophic risks are many, including dynamic financial analysis, pricing, and reserving, as well as the public policy issues to which the actuarial profession should contribute. It is, therefore, not surprising that the Long Range Planning Committee again includes "Mega Risks" as one of the CAS's key issues.

Recommendations

- The Committees on Reserves and Reinsurance Research should stimulate papers on appropriate methodologies for primary and reinsurance companies to use in quantifying their mass tort liabilities.

- The Committee on Ratemaking should continue to encourage research on methods of pricing natural catastrophe risks on both a macro and a micro level.

- The Appointed Actuary Advisory Committee should assure that mega risks be given sufficient recognition within model actuarial reports on dynamic financial analysis.

- The CAS leadership must work with the AAA and its Casualty Practice Council to assure that the ASB completes its Standards of Practice on catastrophic loss provisions and unquantifiable liabilities in a timely fashion.

COORDINATION WITH OTHER U.S. AND CANADIAN ACTUARIAL ORGANIZATIONS

American Academy of Actuaries

The 'Role of the AAA" has been a "monitoring" issue of the LRPC for the past three years. LRPC was concerned that AAA efforts have not been adequate (1993) and observed that CAS members expect that the CAS will monitor the AAA efforts on their behalf.

The CAS needs to take steps to be sure that the Academy is accomplishing the important objectives of CAS members and is functioning appropriately for other disciplines. If the Academy fails to achieve its objectives for other disciplines, it could weaken the Academy's role as a coordiiting body among US actuarial organizations.

Canadian Institute of Actuaries

As the public voice for all actuaries in Canada, the CIA has been actively promoting actuarial standards and issues. The CAS needs to ensure that the current formal and informal links between the CIA and CAS continue to address the educational needs of Canadian members in a timely and effective manner.

Society of Actuaries

The SGA is subject to increasing stresses of various types. The market for health benefit systems actuaries is very strong but the market for pension actuaries is declining. The number of students taking SOA examinations is falling, and this will create budget pressures.

Since the SOA is so large, it is difficult for the CAS to avoid the effect of SOA efforts. The CAS needs to monitor the areas of CAS/SOA coordination to be sure that activities are consistent with overall CAS objectives. Some of the areas of importance are the following:

1. Research - general principles
2. Education - casualty content on SOA exams.
3. Continuing Education - coordination on asset and finance education.
4. Standards - dealing with standards that cross boundaries.
5. Health Coverages - monitor developments that affect casualty coverages.

Recommendations

- The CAS President and/or President-Elect should monitor the overall performance of the Academy and provide regular reports to the Board.
- SOA areas cross CAS function (VP) boundaries so monitoring the overall consistency of our approach can be difficult. The Executive Council should assign the responsibility of developing and maintaining a list of CAS/SOA areas of interaction to one individual or committee.

INTERNATIONAL ACTIVITY

Given the broad scope of this topic, it would prove helpful to review the issue from an historical perspective.

1991

In 1991 the LRPC identified international activity as a high priority issue based upon four distinct opportunities: (1) the growth of P/C business outside North America, (2) lack of general insurance requirements in actuarial education outside the U.S., (3) the European Community need for consistent practice and (4) projected demand for actuaries in the rapidly developing Pacific Rim and the privatization of Eastern Europe.

The LRPC identified six obstacles to CAS worldwide involvement as follows: (1) university education, rather than examinations, is a very common qualification route, (2) national actuarial organizations might resist CAS involvement, (3) the Institute of Actuaries is the role mode for most English speaking countries, (4) the CAS is perceived by its members and others as a U.S./Canadian organization, (5) CAS education and practice are U.S./Canada oriented and (6) our practices are based on data collection procedures not common in other countries.

In its report, the LRPC recommended the following:

1. The CAS Board evaluate whether the CAS should establish goals such as the following:

   Short Term: Actively assist in providing education to aspiring general insurance actuaries worldwide.

   Mid-Term: Be recognized as the leading source of general insurance basic and continuing education.

   Long Term: General insurance actuaries worldwide should aspire to Fellowship in the CAS in addition to satisfying national accreditation requirements.

2. CAS staff/committees should compile a compendium of information on actuarial practice for major countries such as the following: (1) nature of actuarial education, (2) degree of general insurance education, (3) existing organizations and membership requirements, (4) number of actuaries (total and general insurance), and (5) size and growth of local general insurance market and number of insurers.

1992

1992 the CAS Board discussed international policy alternatives prepared by a CAS Task Force. The Board's conclusions were the following:

1. The CAS should not view itself as solely a North American organization and should move beyond the status quo of limited international involvement.

2. The CAS should take the **following** steps:

   a. Move forward in establishing diplomatic relations with other **organizations**.

   b. The Syllabus Committee should explore the availability of international materials for adding international content to the syllabus.

   c. The Continuing Education Committee should invite overseas actuaries to CAS seminars and look into joint sponsorship of seminars on general insurance topics.

   d. The **CAS** should **continue** high level counterpart discussions.

   e. The CAS should **continue** to explore methods to involve itselfworldwide.

1993 - Present

Since that time the CAS activities include the following:

1. **CAS** Presidents-Elect continued their annual visits to the general insurance study group of the Institute of Actuaries.

2. CAS Presidents have made-other visits to English speaking organizations in the UK and Ireland.

3. CAS Presidents and Presidents-Elect have been involved with the **"McCrossan** Group" over the past several years.

4. The Working Agreement Task Force including the CAS President-Elect has been involved in elements of the NAFTA implementation process.

5. The International Relations Committee established the Hachmeister **ASTIN** Prize.

6. The Syllabus Committee and the Continuing Education Committee have been pursuing the Board directives. The Syllabus Committee work **remains unfinished**. CAS seminars and programs have had regular international participation but there have been no efforts at international joint sponsorship of seminars on general insurance topics.

7. Exam waiver programs have been developed with the Institute of Actuaries for UK and **Australian** actuaries. In addition, an exam waiver policy for university education was approved by the Board of Directors.

8. An exchange of publications program has been expanded to twenty-one countries.

In 1993 the **LRPC** observed the CAS activity in examination waiver policies and **"McCrossan** Croup" efforts related to standards of practice and codes of conduct. The LRPC identified the need to (1) focus on education and research in international interactions and (2) relate pro-actively to Eastern Europe and developing nation education needs.

Perspective o n

With the passage of time we have additional perspectives on potential CAS roles outside of the U.S. and Canada.

Special Development

A casualty (general) discipline has developed in the U.S., UK and Canada where (1) the actuarial community overall is reasonably large and (2) the general insurance market is large enough to support a critical mass of general insurance specialists. The general insurance issues in those countries are relatively similar. Japan has a general insurance committee within its actuarial organization.

Actuarial "Priorities"

An actuary% professional focus might typically be prioritized as follows: (1) national organizations, (2) specialty (pension, general life, finance, health, etc.) and (3) type of employer (primary insurer, reinsurer, consultant, other). Alternatively, first priority might go to accrediting organization, generally the national organization (the CAS being an exception) and second priority to practice area.

Actuarial Qualifications

Examinations are the standard route of qualification in the English-speaking world and in parts of Asia. The university degree is a common form of professional training in Europe and Mexico. In some countries the organization is simply a voluntary association, with or without a method for demonstrating competence.

The two primary models for examination - based education are the Institute of Actuaries program (which includes all actuarial disciplines) and the Society of Actuaries program (which includes all but general insurance material).

There is also some use of mixed university/examination qualification processes

Trends

The following trends for the future are suggested as reasonable possibilities:

1. Emerging countries are looking for an actuarial professional model for their countries. They are likely to choose an examination process to supplement university education.

   No emerging country is likely to adopt the U.S. model of separate organizations for different specialties. The separation represents an inefficient use of their resources and is not responsive to their current market needs.

   The SOA program is at a disadvantage relative to the Institute of Actuaries program because the SOA program does not include a general insurance segment.

2. General insurance specialty groups will develop as required by national market places. The

general insurance study group within the UK now numbers approximately 300 people and has grown to that **size** at a growth rate faster than the **CAS** growth rate. Japan has a general insurance sub-group.

3. Combination of university training and professional examinations may become more and more common,

CAS Interests

The CAS member interest in international developments might be summarized as **follows** (m order of importance):

1. Recognition of value in FCAS designation to:

    a. U.S. and Canadian "employers" and regulators

    b. Foreign owners of U.S. and Canadian companies

    c. Foreign **(non-U.S**. and non-Canadian) regulators and government bodies involved with U.S. and Canadian companies.

2. Easy to obtain recognition of **qualification** to work in near-by countries

    Canada/U.S.
    Bermuda
    Mexico

3. Avoiding rules that preclude work in any country.

4. **Recognition** of general insurance as specialty of actuarial work requiring some specific technical knowledge.

5. Good "image" of actuaries worldwide.

Recommendations

The CAS international activities that would support these interests include the following:

Functional Directions

1. **Maintain** our strong U.S. and Canadian role.

2. Cooperate in research and continuing education with general insurance specialty subgroups of **non-U.S**. actuarial organizations.

    This **includes** both **(1)** invitiig non-U.S. help on issues of U.S. importance, for example, the loss reserve uncertainty Theory of Risk project and (2) offering to provide CAS-member assistance, through committees or otherwise, on non-U.S. issues.

3. Cooperate with the SOA in its efforts to integrate casualty material in a “complete” education program suitable for countries without a casualty specialty.

4. Seek methods of cooperating with other non-U.S./Canadian organizations to strengthen basic and continuing, education of general insurance specialists.

Organizational

1. Continue to develop and strengthen high level contacts between the CAS and the general insurance groups in actuarial organizations in other countries.

2. Strengthen the role of the International Relations Committee (IRC) so it can participate and monitor these efforts. The IRC Committee chair should be a past-officer (President or Vice President) or recognized as a senior international actuary.

3. Identify a CAS Officer/Committee chair to monitor and report on all CAS international activities--research, admissions, continuing education, programs, etc.

DATA REPORTING

The issue of data reliability for use by regulators has recently become a major concern. Some believe that since the data is compiled by statistical agents controlled by the insurance industry, it is necessarily suspect. In addition, wncerns have been expressed that insurance data is insufficient to examine certain public policy issues such as availability of insurance in urban areas. Congress is considering legislation that requires potentially costly data compilation by insurers.

Actuaries are uniquely qualified to provide the expertise that is demanded in these debates. They are trained to compile and analyze insurance statistics and can advise regulators on data quality and usefulness of information for the purposes intended.

The profession has a key role to play in this area and the means to do it through the work of the CAS Data Management and Information Committee, the American Academy of Actuaries, the Canadian Institute of Actuaries and the Insurance Data Management Association (IDMA).

The issues should be carefully monitored by the CAS since reliable data is an essential resource for the casualty actuary.

Recommendations

- The Committee on Management Data and Information should monitor developments in this area.
- The CAS could consider serving as a repository of data where traditional mechanisms are not able to function (e.g., Alternative Market). It would be prudent to wait for a specific opportunity or need arises before considering involvement of the CAS in calls for data.

ACTUARIAL INPUT TO PUBLIC POLICY ISSUES

Today's insurance industry is one that is constantly in the public eye. Consumer issues of affordability and availability wmbiied with a high level of interest at the Federal level will hold the nation's interest, at least for the foreseeable future. Actuaries will be required to speak out and explain insurance phenomena and trends. As these issues become more complex, actuaries will be increasingly called upon for their expertise.

Our responsibilities include not only quantifying and projecting system costs, but also identifying the causes and 'drivers" behind these costs. Pressures on insurance costs include the economy, the legal environment, the regulatory climate and judicial decisions. Specific examples may include changes in interest rates, fraud, the imposition of Federal regulation and health care reform legislation. Cost drivers can impact on the frequency and/or severity component of losses and on investment yields which could have an impact on ultimate costs. As such we see that cost drivers may either increase or decrease total losses and costs. We cannot hope for a single, unique solution to the quantigcation of cost drivers since their sources and impacts can vary by many factors, such as: line of insurance, individual company procedures, and state/geographic location of risks. In addition, even if one is able to quantify the cause it is not necessarily controllable.

The abiity of the private sector to adequately address broad social policy issues is generally limited. The CAS, in a public policy role, should consider whether it wishes to be proactive in this area. Among the topics to be addressed are health care reform, pay at the pump auto insurance, mandatory insurance coverages, private sector subsidies (e.g., assigned risk plan), public sector subsidies (e.g.; insurance stamps), urban enterprise zones to encourage reinvestment in urban areas. and stripped down policies, among others.

Recommendations:

- The CAS should assist the AAA Casualty Practice Council to make sure the current mechanism for public policy involvement works.
- The CAS, through the Continuing Education Committee, should encourage the submission of papers on this topic, including possibly a bibliography of data sources.
- The Program Planning Committee should provide sessions on this issue at CAS functions.
- The CAS should coordinate with the regional affiliates to include this in their programs.

ACCOUNTING PRINCIPLES AND PRACTICES

In the last decade, actuaries have become increasingly influenced by accounting rules and practices. In certain situations actuaries have taken strong positions on such issues as discounting and risk transfer, while in other cams the actuarial profession has remained relatively silent on issues that are ambiious such as the use of a range of results. From time to time a CAS task force or committee has responded to the NAIC, FASB, SEC, AICPA and other groups that deal with accounting. In particular, the Academy Committee on Property and Liability Financial Reporting is charged with this responsibility.

The implications of various accounting principles and practices have had a greater impact on actuarial work than was anticipated. In many cases the actuarial point of view has not been sufficiently considered in accounting. The consequence of this has been confusion and, in some cases, misuse in financial reporting.

Because of the structure of the rule-making bodies for accounting and financial reporting, it is difficult for the actuarial profession to become proactive and influential. Whether or not the CAS can change the status of our profession in the accounting realm, it has become clear that the actuarial profession is not generally recognized as being an integral part of the designing of accounting practices or principles in the areas normally associated with actuaries.

Recommendations

- The CAS should promote activities that give actuaries a stronger voice in both the accounting rules that are established and the interpretations and guides for compliance as they impact areas of actuarial import.
- The CAS should direct its research activities to identify problems both within and outside the insurance arena where actuarial approaches could be used to solve or manage such problems.
- The CAS should direct the appropriate committees to identify current accounting or financial reporting rules or practices that are ambiguous, vague or difficult to comply with from an actuarial viewpoint and pursue the development of solutions in those areas.

BASIC AND CONTINUING EDUCATION

Two of the primary responsibilities of the CAS are to provide basic actuarial education to its students in order to qualify them as Fellows of the CAS, and to provide sufficient continuing education opportunities to its members to meet professional standards, keep knowledge current and provide for individual growth.

In order to meet these responsibiities aggressive measures have been taken over the past several years. The basic education function has been under intense scrutiny as it relates to the effect of partitioning of exams and the ongoing review of the syllabus.

The Executive Committee was charged with developing parameters measuring the success or failure of partitioning for a decision in 1994. A conclusion to the study of this issue should be reached in 1994-5.

The syllabus wmmittee has been reviewing the input from the 1993 membership survey and should be very responsive to member comments.

The wntinuing education program has been expanding each year. The post-fellowship course, “Principles of Finance in Property Casualty Insurance”, received high praise and is a step towards alternative methods of providing education to Fellows of the CAS. More opportunities for education of members in the areas of asset management and dynamic solvency testing should be pursued. The 1995 syllabus will address these topics for fellowship candidates.

The CAS is at a point in its development where a distinction must be made between "core" and “specialized” education. The next ten years will likely introduce more heterogeneity into our professional lives and we need to establish a flexible, yet identifiable, basic educational curriculum.

R[redacted]dations

- Partitioning should continue to be studied and preliminary conclusions and recommendations made in 1995.

- The CAS should formally study the definition of the core learning necessary to become an FCAS and relegate other topics to continuing education.

- The role of universities in the educational process, including using universities as a source for continuing education opportunities, should be explored.

- The CAS through liaison with regional affiliates should take a more active role in supporting exam preparatory courses.

COMMITTEE STRUCTURE AND MANAGEMENT

The CAS has **always** taken great pride in the active involvement of its members. The willingness of the members to volunteer large amounts of their time has enabled the organization to **staff an** ever growing number of committees and to carry out its administrative functions, thereby **allowing** the CAS to keep its dues at a very low level. Indeed it is only in the last few years that the emerging **administrative** burden **resulting** from the burgeoning membership has dictated the need for a full time Executive Director and a larger, though still modest-sized, office staff

New CAS members still are imbued with a feeling of responsibility to their profession and to the CAS, which they express by giving of themselves in volunteering their time to the CAS and its committees. At this point there is little or no evidence of a shortage of volunteers although any change in this would greatly impact the **CAS's** ability to carry out its activities. In addition, there is a question as to whether certain administrative activities **could** be better handled by professionals in the CAS office and thereby ease the burden on some committees and make volunteering somewhat more attractive. The CAS **office** has already done an outstanding job in this regard for the Examination and a number of other wmmittees.

As the number of issues facing the CAS grows, so do the number of committees and subcommittees. While the great majority of committees do **an** excellent job, the key to the effectiveness of a **committee** remains the Chairpersons, and those individuals are increasingly burdened by the number of issues with which they must deal and the number of subcommittees they must manage. Other approaches must be considered to **organizing** and managing the Committee work while still fostering the valuable culture of volunteerism which has played a significant role in the success of the CAS.

Recommendations

- The Executive Director should study the committee workings and recommend additional areas in which the CAS office could take over or support the non-technical responsibilities of the committee.

- The CAS Executive Council should continue to monitor the supply of volunteers to identify any emerging shortfalls.

- The CAS should consider utilizing Working Croups of interested members to undertake specific studies or issues. These Working Croups would cease to exist when their assignment is completed.

B. OTHER IMPORTANT CONSIDERATIONS

Many external forces have had, and will continue to have. a profound influence on CAS members and their careers.

- The development of **Financial Products** (such as Options, Derivatives and **Financial** Insurance) has caused many professionals both inside and outside **the P/C industry** to **re-evaluate** the purpose and effectiveness of traditional insurance coverage. It is clearly within the **actuarial** domain to play a major role in the creation and pricing of these instruments as **well** as providing valuable input to the appropriate regulatory and accounting authorities. We encourage the Continuing Education Committee to keep the membership abreast of these issues.

- The general **Consolidation** of the P/C Industry, in **concert** with the significant growth of **Alternative Market** mechanisms has **dramatically** changed the **career** opportunities available to our members. The CAS must carefully monitor these developments and provide information which will **allow** for prudent and professional decisions on both a corporate and personal level.

From an **organizational perspective,** the **LRPC** felt that the CAS should continue to monitor a number of **significant factors**:

- Although there has been noticeable improvement recently, the general characterization of an actuary is still that of a technician who lacks- and an appreciation of a **Business Perspective** Our abiity to lead our profession into the next century will largely be based on the credibility we earn as **communicators** who can blend technical knowledge with business instinct. The recent Call Paper topic on this issue was a positive sign that the problem has been identified. However, the CAS must continue to promote these qualities through seminars, regional **affiliate** meetings and general programs.

- The **CAS** membership witnessed its **first** public disciplinary action this year. As unfortunate an event as this **was,** it should promote a higher awareness of our Principles, Standards of Practice and the Discipline process. As we seek to gain more prestige among our peers in the next century, the CAS must demonstrate a high level of Professionalism. In this regard the **LRPC** believes it is appropriate to create a Committee on Professionalism. Not only would this Committee assume responsibility for the practical aspects of the Course on Professionalism, but it would also coordinate the distribution of all educational material to the membership in a uniform and focused manner.

- **Minority Recruiting** and **Academic Participation** will serve as two crucial sources of intellectual resources to the CAS during the next ten years. Role models and **successful** experiences **will** invariably lead to greater involvement of each of these groups.

Causes of Reserve Deficiency Among Property/Casualty Insurers: A Survey

by the American Academy of Actuaries Committee on Property-Liability Financial Reporting

Causes of Reserve Deficiency

Among Property-Casualty Insurers:

A Survey

Prepared by the

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ABSTRACT

In 1994, the Committee on Property and Liability Financial Reporting of the American Academy of Actuaries (COPLFR) surveyed actuaries representing 26 property-casualty insurance companies to determine what factors contributed to adverse reserve development in individual companies' total loss and loss adjustment expense reserves. The survey results indicated that the major causes of adverse reserve development during the period covered by the survey were: (1) environmental and asbestos liabilities; (2) loss development tail factors; (3) involuntary pool reserves; and (4) unwinding of discount.

COPLFR concluded that some recently adopted changes to the annual statement and other regulatory initiatives under consideration can help identify and/or reduce the impacts of some of these elements. However, COPLFR also concluded that the actuarial profession needs to engage in further work on the appropriate treatment of reserves for environmental and asbestos losses and possibly in the estimation of loss development tail factors.

The American Academy of Actuaries is the public policy voice of the actuarial profession, providing the actuarial profession's expertise to policy makers. This report was produced under the direction of Jean K. Rosales, Assistant Director of Public Policy.

Causes of Reserve Deficiencies among Property-Casualty Insurers: A Survey

INTRODUCTION

It is the appointed actuary's job to evaluate a company's claims reserves. The Statement of Actuarial Opinion (SAO), signed by the appointed actuary, is the document that attests to the reasonableness of the company's reserves.

"Adverse reserve development" indicates that the company did not set aside sufficient reserves to meet its claims.

Adverse reserve development in any one year does not indicate that a company is in financial trouble. Nonetheless, repeated problems with adverse reserve development could signal the beginnings of financial distress. It is important, therefore, for the financial health of the company that the analysis and evaluation of reserves in the SAO be as accurate and dependable as possible.

SURVEY BACKGROUND

In the summer of 1994, the Committee on Property and Liability Financial Reporting of the American Academy of Actuaries (COPLFR) undertook a survey of 52 property-casualty insurance companies to better understand the causes of companies' adverse reserve development in the three-year period beginning year-end 1990 and ending year-end 1993. The thought was that a greater understanding of the causes of adverse'reserve development would help determine where improvements could be made. Possible areas of improvement might include enhancements to the SAO or more education of actuaries in the areas causing adverse reserve development.

The survey was initiated because COPLFR observed that some industry analysts concluded that industry reserves were deficient by 10%-15% (on an undiscounted basis) despite the fact that few companies received adverse SAOs. Beginning at year-end 1990, most companies bad SAOs signed by qualified actuaries (members of the American Academy of Actuaries and the Casualty Actuarial Society). Thus, concern developed that the overall reserve deficiency of the property-casualty insurance industry asserted by some industry analysts might indicate a credibility gap for actuaries signing SAOs.

It was not the intent of the COPLFR survey to test or validate studies of reserves by industry analysts, nor were those observers' conclusions accepted as fact by COPLFR. However, the initial premise was to accept those conclusions and determine whether the observations that industry reserves were deficient could be consistent with non-adverse SAOs for the vast majority of companies. It was considered possible that both could be right and that the adverse reserve development might be related to items outside the purview of the SAO. Should that be true, the.recommendation might be to expand the areas covered by the SAO.

Alternatively, if the adverse reserve development were determined to be related to items already within the purview of the opining actuary, the recommended solution might be to improve the training and education of the opining actuary. Courses of action might include recommending changes to the opinion instructions and developing an explanatory article for outside audiences.

DESCRIPTION OF COMPANIES SURVEYED

Attached as Appendix A is the survey form used by COPLFR, which was mailed to 52 selected companies. Of these, 40 were chosen because they had the greatest adverse reserve development in the industry during the three years, 1991 through 1993, measured as a percent of surplus, percent of reserves, or dollar amount. Twelve companies that did not demonstrate adverse reserve development were also included in the survey. Their size or other unique characteristics led COPLFR to believe that their responses to questions on reserve ranges and level of analysis, as well as their ideas on improving the SAO, would be of value to tbe study.

As shown in Table 1, of the 26 survey responses, 20 came from the 40 companies that had demonstrated adverse reserve development in the three-year period. Six of the twelve companies selected for the other reasons responded. As shown in Table 2, the 26 companies responding held 61% of the total reserves at year-end 1993 for the 52 surveyed companies. Those 52 companies, in turn, accounted for 69% of 1993 total industry reserves. Thus, 42% of total industry reserves were represented by respondents to the survey. Fifty-seven percent of the total 1993 year-end reserves held by survey respondents were attributable to

Table 1

Response Rates of Companies Surveyed

	Companies Responding	Companies Surveyed
Adverse Reserve Development	20	40
Other Companies	6	12
TOTAL	26	52

Tahle 2

1993 Reserves of Respondents Compared to
1993 Reserves of Companies Surveyed & Industry

Reserves of Companies Surveyed as a Percent of Total Industry Reserves	69%
Respondents' Reserves as a Percent of Reserves of Companies Surveyed	61%
Respondents' Reserves as a Percent of Total Industry Reserves	42%

the 20 companies with adverse reserve development; the six other companies represented 43%.

Of the 40 companies surveyed that had adverse reserve development, 19 had 1990 SAOs signed by consultants and 21 had SAOs signed by company employees. The ratio of responses from consultants to those from company employees parallels that of companies surveyed overall. The consultant/non-consultant split is shown in Table 3

Table 3

Use of Consultants & Non-Consultants by Companies

	Adverse Reserve Development		Other Companies	
	Respondents	Surveyed	Respondents	Surveyed
Consultants	9	19	1	4
Other Opiners	11	21	5	8

Survey results follow, grouped by topic in the same order as the survey itself. Causes of adverse reserve development are discussed first, followed by reserve ranges, cash flow testing, and general respondent comments.

CAUSES OF ADVERSE RESERVE DEVELOPMENT

Of particular interest to COPLFR was the identification of the causes of adverse reserve development. If causes could be identified, it might be possible to analyze the treatment of those factors in SAOs and to consider whether current reserving techniques are adequate or whether further research is needed in this area.

Section l/Sheet 2 of the survey listed 24 possible causes of adverse reserve development and asked respondents to allocate by percent (adding to 100%) the major causes of their firms' adverse reserve development. Nineteen of the 20 survey respondents with adverse reserve development responded to this part of the survey.

Table 4 summarizes the responses to Section l/Sheet 2; Appendix B provides a more detailed summary. Even though the total industry adverse reserve development from year-end 1990 to year-end 1993 was approximately $9 billion, the 40 surveyed companies that demonstrated adverse reserve development had over $14 billion of adverse reserve development in the three-year period studied. Favorable reserve development exists for many companies which caused the total adverse reserve development for the selected companies to be greater than the industry total. The 19 companies responding to this question had $7 billion of adverse reserve development in that period.

It should be noted that the survey focused on causes of adverse reserve development over a three-year period. Should year-end 1990 reserves evaluated as of December 1993

Table 4

Major Causes of Adverse Reserve Development

Cause	Number of Companies Identifying Cause	Percent of Development
Pollution, environmental, asbestos, toxic materials and other similar items	12	70%
Loss development tail factor underestimation	9	10%
Involuntary pool reserve strengthening	8	8%
Unwinding of disclosed discount	8	6%
All other listed causes	N/A	17%
Write-ins	9	14%
All beneficial development	8	-25%

still not represent ultimate costs, further adverse reserve development might ensue.

Table 4 shows the major causes of adverse reserve development as identified by survey respondents. Twelve of the 19 companies listed pollution, environmental, asbestos, toxic materials, and other similar items as a major cause of adverse reserve development. This category accounted for 70% of the total adverse reserve development for the 19 companies responding to this question.

The second greatest contributor to adverse reserve development was underestimation of loss development tail factors. This cause, identified by nine of the 19 companies, represented 10% of total adverse reserve development.

The thud largest category, identified by eight companies, was reserve strengthening in involuntary pools and associations and represented 8% of total survey development. Unwinding of disclosed discount was the fourth largest category, noted by eight companies, representing 6% of total development.

Nine companies used the "write-in" line to identify other sources of adverse reserve development. Emergence of construction defect losses was identified by three companies. Other areas mentioned as causes by one or two companies were changes in economic conditions, poor stratification of data, and the impact of court or regulatory actions. Another cause noted was booking reserves at the low end of a reserve range.

Seventeen of the 19 companies experiencing adverse reserve development responded to this question with one or more lines of business identified as the source of adverse reserve development. The two lines of business most frequently identified with adverse reserve

Table 5

Companies' Lines of Business

Line of Business	Number of Companies Identifying
Workers' Compensation (apparently only 4 are WC only)	13
General Liability (including products and treaty casualty excess)	12
Medical Malpractice (specialty company)	I
No line of business identified	2

development were Workers' Compensation and General Liability. Thirteen companies listed Workers' Compensation as a major source of development, Twelve listed General Liability, Summary information on adverse reserve development by line of business is shown in Table 5.

Summary

Although COPLFR presented respondents with a list of 24 possible sources of adverse reserve development, the companies surveyed demonstrated substantial consistency in identifying what had led to this outcome. The causes most frequently mentioned were pollution, environmental, asbestos, toxic materials and other similar items; loss development tail factors; reserve strengthening in involuntary pools and associations; and unwinding of disclosed discount. A discussion of ways the actuarial profession can follow up on this information appears in the "Concluding Observations and Recommendations" section below.

LEVEL OF RESERVE ANALYSIS

Section 2/Sheet 1 of the survey asked respondents to identify the level of actuarial analysis performed for the reserves established in December 1993. Appendix C summarizes the responses from all 26 survey respondents on the level of analysis of company reserves. As shown in Table 6, 88% of reserves for all companies surveyed, and 82% of reserves for the 20 companies with adverse reserve development were analyzed using

Table 6

Percentage of Reserves Analyzed Using Standard Actuarial Techniques

All Respondents	88%
Adverse Reserve Development Respondents	82%

standard actuarial techniques. The only other major categories of levels of analysis mentioned were "involuntary pools" (7% of reserves) and "inestimable" (5% of reserves). Responses for "other" are shown in Appendix D.

The identification of "involuntary pools" as amounts not subjected to standard actuarial techniques is of interest, since eight companies identified this as a source of adverse reserve development. Similarly, the "inestimable" amounts may relate to other items — such as environmental and asbestos claims and the impact of court or regulatory actions — mentioned in the previous section as causes of adverse reserve development.

RESERVE RANGES

In performing their reserve analysis, actuaries may elect to develop a range of estimates for reserves. Section 2/Sheet 2 asked respondents to identify whether they used a range and to provide details on their use of ranges.

Appendix E summarizes the responses to the questions on range methodology and cash flow analysis. Table 7 shows that, of the 26 survey respondents, 15 estimated ranges as part of their

Table 7

Use of Ranges

	Adverse Development Companies	Other Companies
Number using ranges	12	3
Percentage of respondents:		
Straight average	60%	50%
Weighted' average	77%	26%

Weighted based on held reserves by company.

Table 8

Range Methodology Used by Companies

	Range Size/ Reserves	Range Size/ Surplus	Held Reserve Percentile Within Range
Number of respondents	13	12	12
Straight average	14%	44%	49%
'Weighted' average - adverse development companies	16%	64%	47%
'Weighted' average - all companies	16%	61%	44%

'Weighted based on held reserves by company

reserve analysis. Most of the companies with larger adverse reserve development used ranges: 60% of these companies used ranges, representing 77% of the carried reserves

On average , the reserve width for the respondents was 16% of carried reserves, and 61% of surplus (Table 8). On average, carried reserves were between the 40th and 50th percentile of the reserve range. Table 9 shows the stratification of reserve range widths. Seven of the respondents had a range width representing 10% of carried reserves, two had a range width representing 11% of carried reserves, one was 15 % of carried reserves, one was 16%. one was 23%, and one was 30%. Many of the 13 companies appear to be using a probabilistic criterion in their analysis rather than developing ranges based on alternative methods.

Table 9

Number of Companies by Range to Carried Reserves

Range/ Carried Reserve	Number of Companies
10%	7
11%	2
15%	1
16%	1
23%	1
30%	1

Table 8 also includes a column representing the range as a percent of surplus for the 12 companies that responded to this question. For them, the reserve range as a percent of surplus went from a low of 7% to a high of **122%**, with a straight average of 44% and a weighted average of **61%**, indicating that larger companies have a larger range relative to surplus. Most company-carried reserves are near the middle of the range, perhaps because the range was established around a selected point estimate. One company indicated that its carried reserves were 32% above the top of the reserve range.

Section **2/Sheet** 2 asked respondents whether actuaries should be required to include a range in the SAO. Most respondents felt that including a range in the reserve opinion would be more harmful than helpful, fearing misuse or lack of understanding on the part of the reader and concern that the range might be used as a warranty or guaranty that actual results won't develop outside the range. Further, respondents felt that there is at present a lack of standards on the use of reserve ranges. They also believe that more research needs to be done by the actuarial profession regarding the determination and understanding of a reserve range.

Respondents also identified benefits of including a range in the reserve opinion including: (1) publicizing the issue of the uncertainty in reserve estimates, (2) highlighting the relative strength of the carried reserves, and (3) possibly leading to more adequate reserves.

CASH FLOW TESTING

In reviewing the written responses to the questions on cash flow testing, COPLFR members could not draw many conclusions. Only nine of the 26 respondents indicated that they do some form of cash flow testing. Some respondents felt it was only an issue if a company discounted reserves. A better definition of cash flow testing, or clearer phrasing in the survey questions, was needed. (Perhaps this can be addressed in any future surveys.)

COMMENTS BY SURVEY RESPONDENTS

Section 3 of the survey form requested suggestions for strengthening the SAO, for better educating actuaries, and other items. The responses to these questions provided useful information to COPLFR. Summarizing and analyzing these responses is beyond the scope of this report. Members of COPLFR have compiled the written responses and will be communicating them to the Board of Directors of the Casualty Actuarial Society for their use in furthering the education of casualty actuaries.

CONCLUDING OBSERVATIONS AND RECOMMENDATIONS

The treatment of discounting, involuntary pools, and environmental and asbestos liabilities within the SAO appear to be the major areas that account for the differences between industry analysts' perceptions of deficiencies in industry reserves and the generally favorable SAOs issued by actuaries. Some recently adopted changes in these areas to the annual statement and other regulatory initiatives under consideration can help identify and/or reduce these differences.

After studying the responses to its survey of the causes of reserve deficiency, members of COPLFR identified the following observations and recommendations:

Pollution, Environmental, Asbestos, Toxic Materials, and other similar items. This item was cited most frequently as the cause of adverse reserve development. Estimating required reserves for environmental and asbestos exposures is a major challenge for the actuarial profession. Such exposures will likely continue as major contributors of adverse development unless there are significant changes in federal or state legislation. Members of COPLFR recommend that research efforts in estimating such reserves continue to be a priority for the Casualty Actuarial Society.

Loss Development Tail Factors. A second cause of adverse reserve development was underestimation of loss development tail factors. More focus on methods for estimating loss development tail factors estimation may be useful, as would surveys of historical data. This should be considered by the Casualty Actuarial Society Loss Reserve Committee and would be an appropriate topic at the Casualty Loss Reserve Seminar. Discussion paper programs could include tail factor estimation, and other industry studies and educational possibilities for this topic area should be encouraged. The American Academy of Actuaries may wish to consider developing a practice note on tail factor estimation methodology and testing.

Pools and Associations. The fact that strengthening of reserves of pools and associations was cited as a cause of adverse reserve development leads members of COPLFR to conclude that statements of actuarial opinion on reserves for pools and associations would be helpful. Some major pools have recently begun developing SAOs and providing them to members, but this is not required of most pools. However, COPLFR is working with the

Casualty Actuarial Task Force of the National Association of Insurance Commissioners on developing instructions for opinions for voluntary and involuntary pools and it is possible that such opinions will become more common in the future.

Unwinding of disclosed discount. Although the unwinding of disclosed discount was mentioned fairly frequently by survey respondents as a cause of adverse reserve development, its impact on one important data source will be eliminated by the recent change to record Schedule P - parts 2 and 4 gross of all discount, both tabular and non-tabular.

Use of ranges. Review of the wide variation in use of ranges among survey respondents and analysis of respondents' comments regarding the use of ranges leads members of COPLFR to conclude that development of definitions, procedures and practice standards regarding range methodologies may be needed.

COPLFR wishes to thank the staff of the American Academy of Actuaries for their help in putting the survey results together, and the respondents themselves for their time and effort in responding to the survey.

AMERICAN ACADEMY of ACTUARIES

August 18, 1994

TO: Survey Recipients

The American Academy of Actuaries Committee on Property and Liability Financial Reporting (COPLFR) has previously surveyed signers of statements of actuarial opinion for large insurers, seeking to determine the causes of runoff and suggestions for improvement to the statement of opinion. A summary of the findings appeared in the May 1991 issue of The Actuarial Review.

The Committee is again performing this survey and we are seeking your help. This survey has three goals: (1) determine the causes of runoff of the 1990 reserves; (2) determine the degree of analysis applied by actuaries in establishing reserves; and (3) obtain suggestions to improve the loss reserve opinion document and other general suggestions to aid in the establishment of reasonable reserves.

The Committee's motivation for this survey results from the potential impact of the following factors on actuarial credibility:

1. Industry analysts estimate that reserves were deficient by 10% - 15% as of 12/90 and 12/91.

2. Since 1990, in most cases, loss reserve opinions must be signed by qualified actuaries. Most of these opinions have been interpreted as unqualified.

The Committee is attempting to determine (Section 1) the causes of past runoff, believing this would help explain the perceived deficiency in recent reserves. Also, the Committee wants to identify areas where current procedures and requirements can be improved (Sections 2 and 3).

Companies were selected to participate in this survey in two ways.

Using the NAIC data base, the Committee identified forty company groups that had adverse runoff, after 12/90, which was a large dollar amount or large percent of carried reserves. Actuaries forthese 40 company groups are being asked to complete all three sections of the survey. The Committee decided to send the survey to the signer of the 1993 Opinion believing this individual would best understand what has occurred since 1990 to cause the runoff. In completing Section 1, it could be helpful to discuss this with the signer of the 1990 Opinion, if different.

Additionally, twelve large national company groups, small companies and specialty companies, whose runoff did not fit the above criteria were selected. These companies are being asked to complete the latter two sections. These companies were selected to assure a broad sample of the industry was included.

Attached are the 1990 reserves and runoff for your pooled companies and the companies the NAIC data base includes in the group. Similarly, attached are the 1993 reserves. Please verify that the data is correct. If not, please explain in Attachment 2 the likely reasons for the difference.

We do want to hear from you so that we can further improve the statement of actuarial opinion, improve actuarial procedures and enhance the credibility of actuaries. Our findings will not identify a company or individual. Responses will be kept confidential and will be destroyed after the results are tabulated. Attachment 1 explains the procedure the Committee will use to collect information, respect confidentiality and provide for contact of respondents if needed.

If you wish to discuss any portion of the survey, please feel free to contact David Bryant (AAA staff) or me.

We are asking that the survey be completed by September 15, 1994.

Sincerely,

Patrick J. Grannan /DB

Patrick J. Grannan
COPLFR Chairperson

Milliman & Robertson, Inc.
259 Radnor-Chester Road
Suite 300
Radnor, PA 19087
Phone (215) 975-8026
Fax (215) 687-4236

Return *Survey* To:

David Bryant
American Academy of Actuaries
1100 Seventeenth Street, NW
7th Floor
Washington, D.C. 20036
Phone (202) 223-8196
Fax (202) 872-1948

Survey Procedure

1. The survey will be returned to the American Academy of Actuaries office.

2. Each response will be assigned a code and entered onto a master list. The master list will be retained in the AAA office. The AAA office will also retain Attachment 2, the company group 12/90 Reserves, Runoff as of 12/93 and 12/93 Reserves.

3. Company names, logos, addresses, and other identification will be deleted from the response. The response will then be forwarded to the Committee on Property and Liability Financial Reporting (COPLFR) for review.

4. If the Committee has questions regarding a response, AAA staff will relay the questions to the respondent. Respondents can discuss these questions with AAA staff, or with the Committee chairperson, on a confidential basis.

5. Summarization of company responses (determining averages for all companies) will be done in the AAA office.

6. On December 15, 1994, approximately 3 months after receipt, the AAA will destroy all survey forms submitted to them.

Attachment 2

(From NAIC Data Base)

Company Group Name ______________________

Code (AAA use) ______________________

12/90 Reserves ______________________

Runoff as of 12/93 ______________________

12/93 Reserves ______________________

The amount of reserves and runoff have been determined from the NAIC data base. Explain if the NAIC numbers are incorrect and write in the correct amounts.

Section 1
Sheet 1

Section 1: Contribution to Runoff Since 1990 (for Accident Years 1990 and prior).

Instructions: Identify the sources of the adverse runoff for accident years 1990 and prior which has occurred since 12/90. The amount of reserves carried at 12/90 and runoff have been determined from the NAIC data base and are shown on a separate sheet. It is likely that portions of runoff are caused by two or more factors (such as Involuntary Pool Strengthening and Unwinding of Discount within the Pool). Select the predominant cause. Include in the Comment section whether any portion of the runoff could have been identified at 12/90 if current types of data bases and procedures were available at 12/90. Please quantify the percent of total runoff to the extent possible and provide your best judgment where not quantifiable.

Section 2
Sheet 2

Company Code (AAA to complete) ________

1. Is a range of reasonable estimates determined for the total carried reserves? If no, go to question 6.

2. How wide is the range (from low point to high point) as a percent of carried reserves?

3. How wide is the range as a percent of surplus?

4. Where in the range are the carried reserves at 12/93?

5. Would it be helpful/harmful to require a range to be shown in the loss reserve opinion (and why)?

6. a. Do you perform cash flow testing? Yes ____ No ____

   b. If yes, how are the results used in the actuarial opinion process, specifically in determining whether or not the opinion is qualified?

Section 2
Sheet 1

Company Code (AAA to complete) ____________

Section 2: Identify the level of analysis performed for the reserves established at 12/93. In the following, the term standard techniques includes development of losses, lae, counts and average amounts, Bornhuetter-Ferguson or other methods you apply on a regular basis. Attachment 2 provides the 12/93 reserves shown in the NAIC data base.

Level of Analysis	% of Reserves
1. Reviewed by an actuary but ultimate liability deemed to be inestimable.	______
2. Not analyzed by the actuarial area as too variable or liability is in litigation.	______
3. Not analyzed with standard techniques as volume is too low.	______
4. Not analyzed with standard techniques as line of business is new.	______
5. Amounts assigned by Involuntary Pools and not analyzed.	______
6. Amounts assigned by Voluntary Pools and not analyzed.	______
7. Foreign exposure and not reviewed or limited review.	______
8. Analyzed with standard actuarial techniques.	______
9. Other (describe) ______	
Total (should add to 100%)	______

Company Code (AAA to complete) ____________

Section 3: In completing section 3, please consider the causes of runoff you may have identified in Section 1.

Please provide suggestions to a) improve the statement of actuarial opinion, b) to aid in the establishment of reasonable reserves and c) to improve actuarial knowledge and procedures. (Please suggest areas in which you would like more guidance from the Actuarial Standards Board.)

a) ____________

APPENDIX A

Company Code (AAA to complete) ________ Section 1
Sheet 2

Cause of Runoff	% of Total Runoff
1. Pollution, Environmental, Asbestos, toxic materials or other similar items.	____
2. Other long tail, shock type situations such as landmark court decisions or new area of liability.	____
3. Reinsurance (Commutation or Insolvency).	____
4. Involuntary Pools strengthening.	____
5. Timing of the release of Involuntary Pool information.	____
6. Voluntary Pools strengthening.	____
7. Timing of the release of Voluntary Pool information.	____
8. Unwinding of disclosed discount (including tabular).	____
9. Unwinding of undisclosed discount.	____
10. Result of loss responsive programs where future premiums were netted against future losses.	____
11. Management or Company Reorganization (other than Claims Department). (explain) ________	____
12. Claims Department reorganization or changes in practice.	____
13. Result of financial pressures.	____
14. Change in reserve procedures.	____
15. Data base detail deficient or incomplete.	____
16. Data base error.	____
17. Other system problems. (explain) ________	____
18. New area, where insufficient historical information was available.	____
19. Low volume line, where estimation was difficult.	____
20. Catastrophic line (umbrella, excess) - too variable.	____
21. Area was not reviewed.	____
22. Tail factors were too low.	____
23. Other (explain). ________	____
________	____
24. All beneficial runoff.	____

Total (should add to 100%) ____

Which lines contributed the most to the adverse runoff? ________

Other Comments: ________

RESPONSES TO SURVEY SECTION 1 SHEET 2 — CAUSE OF RUNOFF ANALYSIS — APPENDIX

COMPANY CODES

ITEM #	13	15	18	19	20	22	24	25	27	28	30	34	35	37	39	41	49	50	52
1			101%	45%	75%	72%	149%	10%		2%	17%	64%		250%	87%	40%			
2				10%						2%					34%			20%	
3																			
4			5%	13%	17%	5%				19%	9%				30%	15%			
5						10%				10%	7%					15%			
6		5%		16%						14%	15%								
7																			
8			12%	13%		13%			90%	8%	9%				14%	10%			
9													12%			10%			
10						2%				10%		6%							
11													50%						
12																15%	15%	20%	20%
13																10%			
14																			
15																		25%	40%
16																			
17																			
18								25%										15%	40%
19						-6%		25%											
20								15%		10%									
21						2%													
22	20%				40%	2%	10%		10%			7%		10%		5%		20%	
23	80%	95%		21%				25%		25%	50%	23%	38%				85%		
24			-18%	-18%	-32%		-59%				-7%			-160%	-65%	-20%			
SUM	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
LINES NOTED	WC	WC	GL	GL,WC	GL,WC	PROD, GL,WC SP LB	TRTY CAS XS		WC	COMM.	WC, GL, CMP		WC	WC, PROD	REIN, WC,GL	WC,GL	MED MAL	GL,WC, COMM AUTO	CMP, GL

ITEM #	COUNT	AVG 90	AVG R/O	AVG 93
1	12	72%	70%	74%
2	4	2%	1%	2%
3	0	0%	0%	0%
4	8	9%	8%	9%
5	4	4%	4%	4%
6	4	2%	2%	2%
7	0	0%	0%	0%
8	8	7%	6%	8%
9	2	2%	2%	2%
10	3	1%	1%	1%
11	1	0%	0%	0%
12	4	3%	3%	3%
13	1	2%	2%	2%
14	0	0%	0%	0%
15	2	0%	1%	0%
16	0	0%	0%	0%
17	0	0%	0%	0%
18	3	0%	1%	0%
19	2	0%	0%	0%
20	2	0%	0%	0%
21	1	0%	0%	0%
22	9	11%	10%	10%
23	9	11%	14%	12%
24	8	-27%	-25%	-29%
SUM	19	100%	100%	100%
Percent Responding	48%	52%	45%	48%

Number of Companies -LOB Analysis

LOB	Number	Notes
WC	13	Four apparently WC only, one company had (4)-invol pool strengthening and (8)-unwinding of discount w/o WC noted.
GL	12	Included products and treaty cas xs
Med Mal	1	Specialty company
None ID	2	

RESPONSES TO SURVEY SECTION 2 SHEET 1 APPENDIX C

LEVEL OF ANALYSIS

COMPANY NUMBER	CATEGORY NUMBER 1	2	3	4	5	6	7	8	9	SUM
1	1%							99%		100%
2								90%	10%	100%
3		2%	1%		2%			95%		100%
4								100%		100%
5								100%		100%
6	2%			2%	5.5%		0.5%	90%		100%
7								100%		100%
8						5%		95%		100%
9		7%			2%	2%		89%		100%
10			10%		5%	10%		75%		100%
11	4%				12%	1%		83%		100%
12	3%	2%		0.5%	9%	0.5%		85%		100%
13			5%					95%		100%
14	10%		10%					80%		100%
15						5%		95%		100%
16	3%			2%	10%	3%		79%	3%	100%
17				1%	9%	2%		88%		100%
18	8%				8%	1%		75%	7%	100%
19					2%			48%	50%	100%
20	23%				2%	2%		73%		100%
21								100%		100%
22	7%		1%		13%	2%		77%		100%
23			1%		6%	9%		84%		100%
24								100%		100%
25								100%		100%
26								100%		100%

											% Responding RUNOFF	ALL
COUNT	9	3	6	4	13	12	1	26	4	26	50%	50%
AVG 90	5%	1%	1%	0%	7%	2%	0%	83%	1%	100%	53%	53%
AVG R/O	5%	1%	1%	0%	7%	2%	0%	82%	2%	100%	41%	47%
AVG 93	3%	1%	1%	0%	5%	1%	0%	88%	1%	100%	50%	61%

APPENDIX D

RESPONSES TO SURVEY SECTION 2/SHEET 1

LEVEL OF ANALYSIS - CATEGORY 9 DETAIL

1. A loss ratio method technique was used, not considered to be a standard actuarial technique by the respondent;

2. The reserve for asbestos was set by reserving at policy limits with a reduction for the probability of not exhausting high layers and including a provision for expense outside limits;

3. A non-standard technique was used for some areas including a limits available method or a limits exposed method;

4. A method was used for ULAE other than standard techniques, known as the "Wendy Johnson technique";

5. Reserves were analyzed using other techniques due to substantial case reserve strengthening in the most recent two years.

RESPONSES TO SURVEY SECTION 2 SHEET 2

APPENDIX E

RANGE AND CASH FLOW ANALYSIS

(IN COLUMN 2 ORDER)

COMPANY LETTER	ITEM NUMBER 1	2	3	4	5	6	COMMENT ON 6
A	1	30%	122%	49%	-	1	Not in opin.,in dynamic solvncy
B	1	23%	84%	65%	+	0	
C	1	16%	22%	50%	+	1	Reserves undisc, no impact
D	1	15%	56%	47%	+/-	0	
E	1	11%	35%	2%	+/-	0	
F	1	11%	28%	30%	-/+	0	
G	1	10%	50%	50%	+?	0	
H	1	10%	40%	50%	-/+	1	WC discount
I	1	10%	35%			1	
J	1	10%	33%	50%	no?	0	
K	1	10%	16%	49%	+/-	0	
L	1	10%	7%	132%		0	
M	1	10%		20%		1	As a check
N	1				-	0	
0	1				-	0	
P	0					1	At beginning stage
Q	0					0	
R	0					0	
S	0				-	0	
T	0					0	
U	0				-	1	
V	0					1	Disclosed disc. in opinion
W	0					0	
X	0					0	
Y	0					0	
Z	0					1	Nlot used.

	1	2	3	4	6	PERCENTAGE RESPONDING RUNOFF	ALL
STR. AVG	0.58	0.14	0.44	0.49	0.35	50%	50%
COUNT	15	13	12	12	9		
AVG 90	72%	17%	66%	45%	30%	53%	53%
AVG R/O	77%	16%	64%	47%	29%	47%	47%
AVG 93	52%	16%	61%	44%	22%	50%	61%

/

Dynamic Financial Models of Property/Casualty Insurers
by the Subcommittee on Dynamic Financial Models of the
CAS Committee on Valuation and Financial Analysis

DYNAMIC FINANCIAL MODELS OF PROPERTY-CASUALTY INSURERS

Prepared by
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of the Casualty Actuarial Society's
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CHAPTER 1
PURPOSE

The purpose of this report is to discuss and provide guidance on the important issues and considerations that confront actuaries when designing, building or selecting dynamic financial models of property-casualty risks. It has been prepared by the Subcommittee on Dynamic Financial Models of the Casualty Actuarial Society's Valuation and Financial Analysis Committee. It constitutes part of the Society's ongoing educational efforts on issues affecting actuaries responsible for the strategic and dynamic financial analysis of insurers.[1]

In writing this report, the Subcommittee has intentionally avoided placing requirements on actuaries or the models used by actuaries. These requirements have been and will continue to be addressed by the Actuarial Standards Board.

[1] Other sources of information regarding dynamic financial models is included in Appendix A.

CHAPTER 2
INTRODUCTION AND BACKGROUND

What is Dynamic Financial Analysis?

One of the early references to dynamic financial analysis comes from Jay W. Forrester in Industrial Dynamics. He defines it as ". . a way of studying the behavior of industrial systems to show how policies, decisions, structure, and delays are interrelated to influence growth and stability. It integrates the separate functional areas of management -- marketing, investment, research, personnel, production and accounting. Each of these functions is reduced to a common basis by recognizing that any economic or corporate activity consists of flows of money, orders, materials, personnel, and capital equipment. These five flows are integrated by an information network."[2]

Models are the key tools in dynamic financial analysis. Such models are ". . , a systematic way to express our wealth of descriptive knowledge about industrial activity. The model tells us how the behavior of the system results from the interactions of its component parts."[3]

The Actuary's Need for Dynamic Financial Models

Historically, casualty actuaries have focussed primarily on rates and loss and loss adjustment expense reserves. As the portion of insurers' liabilities arising from casualty insurance has increased, their use of reinsurance has decreased and actuarial valuations of liabilities have become increasingly important. Property-casualty actuaries, and in particular members of the Casualty Actuarial Society, have had increasing responsibility to provide opinions on the loss and

[2] MIT Press, 1961, p. vii.

[3] Ibid.

loss adjustment expense reserves of property-casualty insurance companies in the U.S. since 1980, when the first state introduced such requirements.

In more recent years, regulatory and competitive pressures, as well as the desire for a more holistic understanding of the insurance process, have led and continue to lead to expansions of the casualty actuary's role. It is anticipated that, with the emergence of the Appointed Actuary concept for property-casualty insurers in the United States, actuaries will be responsible for understanding insurance company assets, cash flows and investment risks as well as liabilities. Property-casualty actuaries are expected to place increased reliance on dynamic financial models similar to the mandated use of dynamic financial models in emerging standards for life insurers. As such, it is becoming increasingly important that casualty actuaries become familiar with dynamic financial models.

Concurrent with the changing role of the property-casualty actuary have been changes in computer power and software ease that have made the use of dynamic financial models more practical. Specifically, models that would have taken days to code a decade ago can now be implemented in minutes and results can now be. expressed graphically using standard software, easing interpretation.

Why Use Dynamic Financial Models?

Dynamic financial models generally reflect the interplay between assets and liabilities and the resultant effects on income and cash flows. This explicit recognition of all of the insurer's' operations gives dynamic financial models the power to illustrate the links between strategies and results. Therefore these models make a unique contribution to the actuary's set of tools for financial analysis.

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[4] Throughout this report, the application of dynamic financial models to insurers is discussed. These models are equally useful for captives, risk retention groups, self-insurance pools and large self-insureds.

Dynamic financial models are characterized by the projection of both income and cash flows over a period of time. The time delays between occurence of claims and their payment in the property-casualty insurance business make it difficult or impossible to evaluate strategies and decisions without explicit consideration of their effects on flows of funds. Indeed, the results of management decisions or the effects of outside forces may often be counter-intuitive unless a dynamic financial model has clarified the situation.

A scenario is a set of assumptions about the environment in which the insurer's operations will take place. Scenarios are used to illustrate the implications of strategies and decisions in the context of information about the risks that confront the insurer. The explicit consideration of scenarios gives a dynamic financial model a unique role in helping management in identifying profit opportunities and encouraging investment in the company. Such explicit consideration also assists regulators in understanding problems before they grow to crisis size. Management can often identify potential problems earlier, and regulators can distinguish short-term problems that do not warrant intervention from long-term problems that require action.

CHAPTER 3

USES, USERS AND RESOURCES

The design and/or selection of a dynamic financial model will depend heavily upon the purpose of the engagement (use of the model), the users of the model and its results, and the available resources.’

Uses

Dynamic financial models have many uses, including:

Determining the value of an insurance company or block of policies to a potential buyer or seller.

- Assessing how an insurer might fare in a range of future economic environments.
- Strategic planning, including asset-liability management, tax planning, reinsurance planning and costing, and market strategy.
- Feasibility studies.
- Tactical decision-making, including product pricing. (Although dynamic financial models are not yet widely used to price property-casualty insurance products today, they are already widely used to price life insurance products.)

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[5] These considerations, along with the others identified in this report, are summarized in Appendix B.

- Identifying the kinds of risks that most threaten the solvency of the insurer.

The use of the model will be a key determinant of many of its requirements. Examples include:

(1) Model input and output depend on the use. For example, if modelling a worst-case scenario for solvency testing, a complex tax module is not important because the insurer is unlikely to pay substantial income taxes, at least under current federal income tax laws,

(2) The use of the model helps determine the time frame and accounting basis. For example, if regulators ask the actuary to model solvency over a two-year time horizon and ensure that risk-based capital requirements are met, then a minimum of two years of future statutory accounting statements is required.

(3) The use of the model may determine whether a deterministic or stochastic model is more appropriate. This decision in turn will greatly affect the resources and data needed. the model structure and the form that output will take. As an example, if the goal is to develop probability distributions of results, then an actuary will be more likely to use a stochastic model.

Users

Future Appointed Actuaries and insurers that wish to anticipate the results of the Appointed Actuary's work are the users who are driving the CAS's educational efforts on dynamic financial analysis. Other users of dynamic financial models include consulting firms and insurers that employ such models as tools for tactical and strategic decision-making, including pricing decisions. Third-party users of the results of dynamic financial models can include regulators, reinsurers, investment bankers, financial intermediaries, institutional investors, securities rating organizations, and financial analysts.

The intended users' needs are a consideration in designing and selecting the model. The type of model used and its structure depend on customers (users) and their needs. As an example, regulators may focus mainly on the insurer in total. Company management may focus on the total corporation or on each individual product.

Resources

The choice of dynamic financial model will depend on the available resources, whether these resources are people available for system design and programming, data from which to derive assumptions and with which to initialize the model, money available to purchase an existing software package, or computer architecture.

Detailed dynamic financial models require a significant investment of time for research to determine assumptions, as well as for maintenance to keep the model's logic current and to revise assumptions in light of new data. Such models also require a significant expenditure of time in interpreting the results.

The choice of computer architecture is often determined by the purpose of the analysis and the level of detail of the projections. A simple spreadsheet might be appropriate if the purpose of the study is to highlight the effects on financial results of one particular risk, such as adverse development of loss reserves. At the other extreme, complex, report-generating software with a user-friendly front-end and efficient coding of the detailed calculations might be appropriate if the model is intended to cope with a wide range of different problems and be used by a wide range of users.

CHAPTER 4

RISKS OF THE PROPERTY-CASUALTY INSURANCE BUSINESS

The evaluation of risk is the focus of dynamic financial models. The relative importance of each type of risk will determine the detail of assumptions and analyses built into any model. In this chapter, the risks of the propertycasualty industry are described and the related modelling considerations addressed.

Property-casualty insurance risks can be divided into many categories. In this paper, we will follow the definitions originated by the Committee on Valuation and Related Matters of the Society of Actuaries and will discuss these risks in the following four categories:

- C-l risk - Uncertainty surrounding cash flows from invested assets other than from uncertainty regarding interest rate risk.
- C-2 risk - Uncertainty surrounding cash flows from the obligation or underwriting aspects of an insurance company.
- C-3 risk - Uncertainty surrounding cash flows from interest rate fluctuations in the presence of a mismatch of assets and liabilities and the risk of disintermediation caused by embedded options that are sensitive to changes in interest rate.
- C-4 risk - Uncertainty emanating from mismanagement, i.e., making incorrect or fraudulent actions in light of the available information.

As do many discussions of insurance risks, this paper will focus on the first three of these risks. At present, measuring the risk of mismanagement is beyond the scope of most actuarial engagements.

Asset Risk

Asset risk, also known as C-l risk, is the risk that the amount or timing of items of cash flow connected with assets will differ from expectations or assumptions as of the valuation date for reasons other than a change in interest rates. It encompasses uncertainty regarding:

- Default rates.
- Future'market value of equity assets.
- Liquidity of assets.

In addition to these inherent asset risks, there is also the risk that the character of the assets will not be evident from their general descriptions. This problem is increasing as capital markets develop a greater range of non-equity investments that have many of the risk characteristics of equity investments.

Appropriate data and methods are critical to the development of ranges of assumptions to reflect asset risk in the projected performance of the insurer. Historical data developed for investment managers is readily available, including time series of default rates of various classes of assets as a function of age.

Dynamic financial models can be used to estimate the effects of these risks alone on the projected performance of the insurer and can also be used to estimate the interrelationships between these risks and other risks. In modelling, asset risks may be assumed to correlate with inflation or some other variable or to be autoregressive.

Obligation Risk

Obligation risk, also known as C-2 risk, is the risk that the amount or timing of items of cash flow connected with the obligations considered will differ from expectations or assumptions. For property casualty companies, obligation risk encompasses:

- Reserve risk - the risk that the actual cost of losses for obligations incurred before the valuation date will differ from expectations or assumptions.

- Premium risk - the risk that premium for future obligations will differ from expectations or assumptions.

- Loss projection risk - the uncertainty regarding assumptions about future loss costs.

- Catastrophe risk - the uncertainty regarding the costs of natural disasters and other catastrophes.

- Reinsurance risk - the uncertainty regarding the cost, value, availability and collectibility of reinsurance.

- Expense risk - the risk that expenses and taxes will differ from those projected.

Dynamic financial models can be used to estimate the effects of these risks individually on the projected performance of the insurer and to evaluate the interrelationships between these risks and other risks.

Reserve risk may be a function of:

- Inflation in medical costs and other determinants of claims costs.
- The legal environment in which claims will be resolved, including the environment in which claims are pursued by policyholders or third parties.
- The possibility of a breakdown in some basic premise underlying the reserves for a particular coverage (such as has occurred with environmental impairment liability).
- Past patterns of pricing adequacy which affect case reserves or financial reserves.
- Corporate culture, training, and incentives that affect the payment of claims or the adequacy of case reserves.
- Currency fluctuations which affect the costs of losses when expressed in local currency.
- The randomness of the claims process itself.[6]
- Incompleteness of data bases.

[6] The randomness of the claims process itself can be studied by modelling the patterns of loss development or by more detailed analysis of the claims process. Inevitably, however, data for such models always include the effects of other factors affecting the claims process.

Premium risk may be a function of:

- Competitive pressures that do not allow the insurer to achieve assumed levels of exposure and/or rate adequacy.

- Regulatory intervention that restrains premium increases or decreases or that requires business to be underwritten that would not be underwritten in the absence of such intervention.

- Premiums for involuntary business underwritten at premium rates and in volumes that differ from assumptions.

- Retrospective premiums or dividends that differ from assumptions.

- Amounts collectible from agents that differ from assumptions.

If premium risk is expected to arise from a cyclical pattern of premium adequacy in the competitive market, a cyclical component could be incorporated into the model or into the premium adequacy assumptions.

Loss projection risk is a function of the factors that affect reserve risk and also of the uncertainty regarding:

- Changes in loss costs and exposures from the historical experience period.

- Loss costs for the mix of new policies being underwritten, including the effect of adverse selection.

- Loss adjustment practices in the future that may differ from those in the past.

Catastrophe **risk** can **be** considered a component of loss projection risk. It is a function of:

- The coverage being written.

- The concentration of insured values in specific geographic areas or legal jurisdictions.

- Uncertainty regarding the frequency, seventy and nature of catastrophic events.

Computerized models of the damage arising out of certain types of catastrophes are available which may be of value in determining assumptions about the probabilities and sizes of catastrophic losses. Output from these catastrophe models may be used as input to a dynamic financial model or a link between the models may be established so as to include the impact of catastrophe risk in dynamic financial models.

Reinsurance risk is a function of changes in the price and availability of desired reinsurance, and of uncertainty regarding the collectibility of reinsurance recoverables arising from the financial condition of the reinsurer or ambiguity about the coverage provided. Reinsurance risk exists in each of the four obligation risks identified thus far. In many models, projections are made on a net of reinsurance basis. Such projections incorporate implicit assumptions regarding reinsurance risks, whereas projections made on a gross of reinsurance basis require explicit instructions regarding the reinsurance mechanism. Reinsurance risk recognizes how reinsurance responds under stress, such as a large catastrophe or other strain on collectibility, aggregates, reinstatements and other reinsurance parameters.

Expense risks, those associated with expenses (other than loss adjustment expenses) and taxes, include uncertainty regarding:

- Contingent commissions to agents.
- Marginal expenses of adding new business.
- Overhead costs, including the risk that overhead costs will be changed by regulatory intervention and the risk that there may be periods of changing premium during which overhead costs will not change in proportion to premium.
- Assigned risk overburdens, second injury funds and other assessments.
- Federal and local income taxes, both in interpreting the current Internal Revenue Code and in anticipating changes in the code.

These lists of uncertainties regarding the major components of obligation risk are illustrative. Other factors may affect obligation risk.

Interest Rate Risk

Interest rate risk, also known as C-3 risk, is the risk that net cash flows will depart from expectations or assumptions as the result of interest rate fluctuations. Interest rate risk encompasses uncertainty regarding cash flows from assets, including bond yields, mortgage interest rates, real estate income, and dividends on equity investments. It also encompasses uncertainty regarding cash flows related to borrowing, such as the interest rate on any loans taken out by the company or cost of capital.

Reinvestment risk is uncertainty regarding yields that will be. available on reinvestment of proceeds from investments that are currently held. In many dynamic financial models, a set of assumptions must be made about the yields that will be earned on future investments. Often in practice the apparent solvency or insolvency of the enterprise will be sensitive to the choice of interest rate ("reinvestment rate").

Another component of C-3 risk is the uncertainty regarding the market value of any fixed-income assets that must be sold prior to maturity to meet cash flow needs. C-3 risk includes market value uncertainty related only to changes in interest rates; market value uncertainty related to changes in perceived credit or default risk is a component of C-l risk. The reinvestment rates, discussed above as being determinants of reinvestment risk, also determine market value risk for fixed-income assets. Thus, the reinvestment rate can have a significant impact on the results of the model, resulting in an under- or over-statement of risks because of an inexact choice of reinvestment rate.

CHAPTER 5
RISKS INHERENT IN THE MODELLING PROCESS

Once the risks to be incorporated in the model have been identified and the model built, there are a number of risks inherent in the modelling process to consider, including:

- The range of scenarios may not reflect the user's intent.
- The model may be incorrectly or incompletely specified for the intended purpose.
- The model's results may be inappropriately interpreted.

Importance of Scenario Testing and Selection of Assumptions

Proper use of a model depends on the selection of appropriate scenarios to evaluate and the development of consistent assumptions within each scenario. The purpose of the model will influence the data and methods used to provide assumptions for understanding the projected performance of the insurer. Scenarios permit links between assumptions for various parts of the model. For example, a high interest rate scenario might include assumptions of high bond yields, low common stock values with high dividends, high inflation in medical costs, and a low level of unemployment.

Scenarios provide an especially relevant tool for determining the implications of risks on the projected performance of an insurer. Observing the results for a variety of scenarios yields information about the company's response to risk. Careful selection of scenarios is essential.

One of the reasons for using dynamic financial models is that they can provide information about the interactions among risks. Dynamic financial models can indicate the extent to which components of the company interact with one another. Depending on the purpose of the model, the actuary may have a responsibility to describe the ways in which several components appear to be interacting, particularly if they alter the risk that arises from uncertainty about the assumptions or logic for a single component.

In many situations, the actuary will be constrained with respect to the choice of scenarios. At this time, life insurance regulatory authorities specify certain scenarios to be modeled by the actuary, at a minimum. Similarly, Canadian regulations provide general guidance regarding the choice of scenarios. This kind of regulatory requirement may expand to U. S. property-casualty actuarial work in the future. Sometimes the scenarios to be studied will be specified by company management rather than by the actuary. However the range of scenarios is selected, its choice will impact the results that the model produces. It may be appropriate to observe the model under scenarios other than those specified by regulators or management to adequately understand the implications of the scenarios that were specified.

When the range of scenarios has been selected using only retrospective tests as a guide, the model may be prone to be over-determined. For example, the risk that the probability distributions in a stochastic model are incorrectly specified can be minimized by choosing probability distributions that have greater uncertainty (central tendency, dispersion, and skewness) than historical data.

Model Specification

The risk that a model is incorrectly or insufficiently specified can be minimized by validation, i.e., matching the model to the insurer's own history over some period of time. A well-specified model will reasonably reproduce past actual results. Actual results varying from projections may not be an indication of a poor model. Rather, it is generally appropriate to investigate such differences and reconcile the model's results with the actual results. This process of reconciliation may identify weaknesses in the model, or it may clarify ways in which the enterprise's activities departed from what would have been reasonably expected (e.g., writing more, rather than less, unprofitable business to cover up poor experience).

Interpreting the Results for a Range of Scenarios

Summarizing a range of outcomes includes development of measures of the performance of the insurer, as well as description and explanation of anomalous results. Measures of performance include:

- Risk-adjusted present value of future cash flows.
- Management-defined objectives.
- Probability or cost of "ruin."
- Option-adjusted pricing.

Other measures may also be appropriate. The method of summarizing results will depend on the purpose of the model.

Under the first approach, value is calculated as the risk-adjusted present value of the future cash flows. Calculations of risk-adjusted present value may include separate risk adjustments for stochastic or process risk (random variation) and scenario or parameter risk (variation among scenarios). This approach allows for specific consideration of the cost of risk, Similar results may be obtained by observing the model's results under a set of assumptions that are conservative in light of the uncertainties indicated by the model and computing the present value of the resulting flows of funds at a risk-free rate.

An insurer's modeled performance may also be measured in terms of objectives defined by company management. For example, management may set objectives such as maintaining acceptable risk-based capital results, failing no more than two IRIS tests or maintaining a combined ratio less than 100%. The insurer's performance relative to these benchmarks can be measured by using a model that calculates these statistics.

In the third of these measures, the probability and expected value of each outcome is estimated. The actuary may decide on a threshold characterized as "ruin," and use a stochastic model to estimate the probability of ruin for a given set of assumptions. Alternatively, the actuary may establish a cost of ruin (and perhaps establish nominal values for certain other types of outcomes as well), and compute an average of the adjusted financial outcomes over a range of assumptions. The actuary may also select a threshold much closer to the current financial condition, such as a decline in financial rating by one level, and estimate the probability of such an outcome.

Under the fourth measure, the total value of the insurer is summarized as the current market price of a set of investments available in the capital markets which has the same risk characteristics as the model indicates for the insurer. Such a set of investments almost always includes a large proportion of options because the insurer's cash flows are typically inflows first and outflows second, so the resulting value is called the option-adjusted price of the insurer's assets and liabilities. This value reflects the insurer's strategies for investment and for handling unexpected shortages of cash, at least as far as those strategies are reflected in the model.

There is an ongoing dialogue among actuaries about the appropriate basis for summarizing the results of a model. The Combination of Risks Task Force of the Society of Actuaries' Committee on Valuation and Related Problems concluded that the appropriate basis for summarizing the results of a dynamic financial model is the cash basis.[7] According to this school of thought, the other accounting bases (statutory, GAAP, and tax) are important only insofar as they serve to identify constraints on the enterprise's operations (e.g., tax payments).

On the other hand, the Actuarial Standard of Practice for Appraisals, promulgated by the Actuarial Standards Board, suggests that statutory accounting is the appropriate basis for measuring financial results. In this school of thought, the statutory and tax accounting rules place real constraints on the cash flows that can be realized by the investor.

[7] Transactions of the Society of Actuaries, 1991-92 Reports, p. 451.

Depending on the purpose of the model, the actuary may need to describe anomalous results indicated by the model. The results of the model may suggest that-either some assumptions are incorrect (in which case the assumptions will likely be revised before results are presented) or that the insurer's strategies could be improved. As an example of the latter, the results of the model may suggest that the insurer is particularly at risk due to one or more sources of risk.

The risk of inappropriate interpretation can be minimized by communicating the limited extent of variation among scenarios compared to the potential range of variation in the results of the insurer's operations.

CHAPTER 6

OTHER MODEL CHARACTERISTICS

However simple or sophisticated, a model is no more than a metaphor for the insurer. Dynamic financial models differ in the types of risks they are effective at measuring. A key consideration in the selection of a dynamic financial model is its ability to evaluate material sources of solvency risk for the case at hand.

Generalized vs. Tailor-Made

Generalized models, such as those developed by several consulting firms and software vendors, usually permit the user to specify several different types of insurance products, or lines of business, and a range of different investments. Other models are often tailor-made, such as one that addresses the unique characteristics of a company or because a simple model is sufficient. If a generalized model is used, it is important to consider whether results may be distorted by features inapplicable to a particular application or because a characteristic of the particular company is not addressed. For example, if a general purpose model is used for an insurer that plans to invest only in bonds and cash equivalents. the model does nor need to include a strategy that involves investment in other assets. If it does, the ramifications of that logic should not distort the projections.

Logic vs. Input

Whatever computer hardware and software may be used to implement the model, there are always tradeoffs between the coding of logic versus the selection of parameters. Dynamic financial models differ in the choices the developers make about which assumptions will be represented by variables and which will be fixed by the software or hardware. Also, the user will be able to determine the values of certain variables used by the model, whereas others will have been pre-set by the developer. The mix between input and logic will be determined in part by the users of the model (both the operator and the decision-maker), as models with extremely large numbers of variables can be daunting to use and difficult to interpret.

In selecting or building a dynamic financial model, decisions must also be made about the level of detail to be captured about the insurance coverage (by broad product group, statutory line of business, individual form, etc.), the factual context (including the level of detail about accounting and tax rules), and the precision with which strategies will be defined.

Strategies are inevitably a part of the logic of a model. The strategies incorporated in the model should be reasonably consistent with its purpose. Some software allows the user to build in explicit recognition of management strategies. Other software assumes certain strategies, even to the extent of letting presumptions about strategies affect the architecture or modular design of the model.

Time Frame

The time frame for the analysis is an important consideration in the choice and design of dynamic financial models. For example, it may be appropriate to use a time frame of 24 months to evaluate strategies for a property insurer (although a longer time frame may be needed to address recovery from a large catastrophe), whereas a time frame of 24 years may be more appropriate to evaluate the solvency of an underwriter of products' liability. The choice of time frame will also be a reflection of whether the model includes only the run-off of current business, a going concern for some stated period, or a going concern in a long-range projection valuation.

In addition to the time horizon of the model, the model also reflects a choice about the length of time intervals under study. While annual time intervals may be appropriate for some purposes, quarterly or even monthly time intervals might be appropriate for other purposes.

Basis of Accounting

Comprehensive dynamic financial models of the corporate insurance enterprise usually include accounts on at least four bases simultaneously: cash, statutory, GAAP, and tax[8]. Doing so is the only way to reflect the details of the interrelationships among constraints imposed by investment opportunities, underwriting commitments, laws and regulations, generally accepted accounting principles, and income tax laws. Less comprehensive models may be appropriate, however, for some purposes.

Relationship between Parent and Subsidiaries

Parents and subsidiaries have a number of different effects on an enterprise. A consolidated model of the entire organization can be developed, or the existence of the parent and subsidiaries might simply show up as assumptions about flows of funds, tax calculations, and income. A model may explicitly reflect a range of scenarios regarding the availability of or drain on surplus due to external influences. Alternatively, each entity may be modeled separately, with output from one model serving as input for other models.

Feedback Loops

Dynamic financial models may employ feedback loops (automatic conditional decisions) which are algorithms that make calculations for each modeled time period dependent on values calculated for earlier periods. Feedback loops provide for reactions to specific conditions. Models without feedback loops may be underdetermined, showing excessive income under favorable scenarios and excessive loss under unfavorable scenarios. Models with feedback loops, however, may be overdetermined, showing little risk regardless of the scenario because the model builder often assumes that management will respond quickly to increased risk with appropriate strategic or operational responses.

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[8] Financial reporting, and therefore modelling, may be more complex for international users.

stochastic Risk

Purely random fluctuations in the modelled variables may be important for a particular application. Stochastic features surrounding input assumptions can be added to a model. Random fluctuations around projected losses, for example, may be incorporated into a model by introducing probability distributions about loss costs or loss ratios, by modelling the collective risk process, or by modelling the underlying claim settlement process.

A simple model of the collective risk process may assume, only a probability distribution for the frequency of losses as a function of some assumptions and a probability distribution of the sizes of losses as a function of other assumptions. A more complicated model of the collective risk process may include estimates of the uncertainty of the parameters of frequency and size-of-loss, and may include a number of different kinds of losses, each with its own frequency and size-of-loss assumptions. A model of the underlying claim settlement process may be a multi-state Markov chain model or some other appropriate model.

The importance of identifying and modelling the interactions among risks increases when stochastic models are used. When assumptions are stochastically generated, a model that does not reflect these interactions may produce meaningless results in certain scenarios. At best, the results of such models would be difficult to interpret.

CHAPTER 7

FORCES FOR CHANGE

Thus far, this report has focused on the state of art and practice at this time. There are sweeping changes underway that may affect modelling in the future.

Proliferation of Insurance Products

Although regulation and custom tend to slow the creation of insurance products by entrepreneurs, changes in the markets served by insurance enterprises constantly press for new products and services. Dynamic financial models may need to be refined to adapt to these changes.

Competitive Pressures

In the past, pressures were perceived to arise from competition at the point of sale of the insurance product. From at least as long ago as 1970, competitive pressure has increasingly come to mean competition at the point that capital is being raised. Dynamic financial models are playing an increasingly visible role in corporate decisions regarding purchases and sales of business units, means to tap capital markets, and trade-offs between capital and reinsurance. This trend might reasonably be expected to continue.

Innovation in Assets

Recent innovations in asset design make it difficult to understand the riskiness of many investments by looking at their designations for accounting purposes. For example, some bonds have the risks of stock investments or mortgages and mortgages are backed **by** a wide range of security. Existing accounting classifications may be misleading to tabulate information about assets for input into dynamic financial models.

New types of asset classes are emerging, some with purposes other than purely generation of investment returns. For example, some assets, such as catastrophe futures, can hedge risks undertaken by the insurer's underwriting activities. More innovation can be expected, along with the need to model these kinds of investments.

Regulatory, Accounting, and Tax Requirements

Dynamic financial models may need to be revised from time to time to reflect the latest developments in regulation. Such changes may be as simple as adding a set of calculations, or they may require modelling of the corporate response to the impact of the regulations (e.g., a shift in marketing or investment strategy to accommodate surplus constraints of risk-based capital). Projections of cash flow may react to changes in these constraints differently from projections of statutory results, and dynamic financial models with feedback loops may react differently from static models.

Hardware

Although changes in computer hardware over the past twenty years have in some ways increased the speed with which tasks get done, they have had a fundamental and irreversible effect on the kinds of problems that people address. For example, before data processing was available that could prepare an extensive Schedule D (details of assets of insurers), regulators simply prohibited and restricted investments outside a few narrow categories; today, they attempt to monitor insurers' investments. Models of corporate financial results were not considered to be important tools for actuaries until computer hardware existed on which such models could be run, The actuary can expect that the changes in hardware will transform both the problems the actuary will be expected to address and the nature of actuarial work.

One major change on the horizon is distributed processing. In the future, the actuarial tool kit may consist of essentially instant communication with a large number of models of a given insurance enterprise, each being updated with new information essentially in real time. Between that future and to&y lies a time of rapid change in the power and distributions of hardware, software, and data.

Life Insurance Models

Dynamic financial models of a high degree of sophistication now exist for life insurance enterprises. These models are being used for product pricing and corporate valuation as well as for strategic and tactical (e.g. tax) planning. These models, and the experiences of their users, may have an important effect on the direction of development of models of property-casualty insurance companies. Life insurance models affect the perceptions and expectations of regulators, many of whom have responsibility for both life insurance regulation and property-casualty insurance regulation.

Other Countries

The increasing degree of globalization of the national economies, and the long-standing trend to lower economic borders between countries, suggest that actuarial work in the United States will be affected by innovations developed outside the United States and vice versa.

For example, Canada recently introduced solvency regulation for property-casualty insurance companies. All companies are required to designate an appointed actuary who is a Fellow of the Canadian Institute of Actuaries (CIA). In addition to performing the valuation of loss reserves, unearned premium reserves and deferred acquisition expenses for a company, the appointed actuary is required to report to the Board of Directors at least once a year on the current and expected future solvency of the company. To make this report, the appointed actuary is expected to perform dynamic solvency testing in conformance with the standards of practice set by the CIA. In cases in which a company is thought to be in difficulty, federal regulators can require that the appointed actuary submit a report on the results of a dynamic solvency test of the company's business plan over a planning horizon of one year.

APPENDIX A

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APPENDIX B
CHECKLIST OF CONSIDERATIONS

1. Is the model appropriate for the intended use?

2. Are the model and related communications appropriate for the expected users of its results?

3. Can the model be developed, purchased, maintained and/or used within the personnel, time, hardware, software and budget resources available?

4. Does the model contain input, output and processing regarding each of the risks to be evaluated in appropriate detail? Are the available historical data regarding these risks sufficient to use to derive the assumptions needed by the model? These risks include:

   - Asset risk
   - Obligation risk
     - Reserve risk
     - Pricing risk
     - Loss projection risk
     - Catastrophe risk
     - Reinsurance risk
     - Expense risk
   - Interest rate risk

5. Is the range of scenarios broad enough to reasonably address the questions at hand?

6. Is the model specification accurate and appropriately complete?

7. Are the measures used to summarize and interpret the range of results reasonable for the application?

8. Have the limitations of the model and range of scenarios been communicated clearly to reduce the risk of misinterpretation?

9. Is a generalized model reasonable for the application or would a tailor-made model better address specific issues?

10. Does the model have a reasonable balance between input assumptions and hard-coded logic?

11. Is the model's time horizon appropriate to the application?

12. Are the accounting bases upon which the model makes forecasts of appropriate breadth to the application?

13. Does the model provide sufficient detail (input and output) with respect to interactions with parents, subsidiaries and affiliates?

14. Will the value of the model results be enhanced enough by the presence of feedback loops (automatic conditional decisions) to warrant a model with such features?

15. Is a deterministic or stochastic model better suited for the application?

A Simulation Procedure for Comparing Different Claims Reserving Methods

by Teivo Pentikäinen and Jukka Rantala

A SIMULATION PROCEDURE FOR COMPARING DIFFERENT CLAIMS RESERVING METHODS

Teivo Pent&linen and Jukka Rantala

Abstract

The estimation of outstanding claims is one of the important aspects in the management of the insurance business. Various methods have been widely dealt with in the actuarial literature. Exploration of the inaccuracies involved is traditionally based on a *post-facto comparison* of the estimates against the actual outcomes of the settled claims. However, until recent years it has not been usual to consider the inaccuracies inherent in claims reserving in the context of more comprehensive (risk theoretical) models, the purpose of which is to analyse the insurer as a whole. Important parts of the technique which will be outlined in this paper can be incorporated into over-all risk theory models to introduce the uncertainty involved with technical reserves as one of the components in solvency and other analyses (PENTIKÄINEN et al (1989)).

The idea in this paper is to describe a procedure by which one can explore how various reserving methods react to fictitious variations, fluctuations. trends, etc. which might influence the claims process, and, what is most important, how they reflect on the variables indicating the financial position of the insurer. For this purpose, a claims process is first postulated and claims are simulated and ordered to correspond to an actual handling of the observed claims of a fictitious insurer. Next, the simulation program will 'mime' an actuary who is calculating the claims reserve on the basis of these 'observed' claims data. Finally, the simulation is further continued thus generating the settlement of the reserved claims. The difference between reserved amounts and settled amounts gives the reserving (run-off) error in this particular simulated case By repeating the simulation numerous times (Monte Carlo method) the distribution of the error can be estimated as well as its effect on the total outcome of the insurer:

By varying the assumptions which control the claims process the sensitivity of the reserving method visa-à-vis the assumed phenomena can be tested. By applying the procedure to several reserving methods in parallel a conception of their properties can be gained, in particular, how robust they are against various variations and irregularities in the claims process.

It is useful to recognize and classify error sources which give rise to the reserving inaccuracies (cf. PENTIKÄINEN et al (1989) item 2.4b):

1) Tbe model (often simply called reserving rule or formula or method) can be only a more or less idealized description of the real world and of the actual claims settlements; the deviations give rise to what can be termed *model errors*.

2) The parameters used in calculations are subject to *parameter* errors owing to the fact that they are to be estimated from various data statistics or found from other more or less uncertain sources.

3) The actual claims and claims settlements are subject to stochastic fluctuations causing

deviations from the estimates, *stochastic* errors, even in those (theoretical) cases where the model and its parameters would be precisely correct.

The above procedure enables us to examine the effects of all these three errors, in fact, it is very general, not being restricted to any specific reserving model or assumptions on the claims process. It is intended for studies of the properties of the reserving methods on ***a general level.*** However, it is not meant for post-facto analyses, i.e. in the investigation and estimation of the inaccuracies in reserves in particular concrete cases, for those purposes well-known actuarial and statistical approaches are needed.

It is still worth noting that the approach can find application to other estimations as well. We have, for instance, also treated premiunis in an analogous way, although limited to simple examples in this paper.

After having first described our method in general terms a number of numerical examples will be given to illustrate some of its relevant features. They are based on some well-known elementary reserving rules and simple assumptions on the claims process. Also some conclusions on the properties of the reserving rules are derived therefrom. They should be understood merely as examples of the use of our model, not as any real analyses of the reserving methods. Even though our method is aimed at making such conclusions and comparisons between methods, their pertinent performance would require quite extensive studies. Such have been fully beyond the possibilities in this context.

KEYWORDS

Claims reserving. run-off errors, chain ladder, model errors, parameter errors, simulation

1. Basic concepts

1.1. References to related works. A summary of the ***c/aim reserving techniques*** was compiled by VAN EEGHEN (1981). Furthermore, the monograph by TAYLOR (1986) is referred to as is the recent Claims Reserving Manual (1989) of the UK Institute of Actuaries. Enhanced methods for analyses, among others regarding the above listed sources of errors, have been recently proposed, for example, by ASHE (1986), NORBERG (1986), SUNDT (1990) and WRIGHT (1990).

The run-of-errors, as a source of uncertainty in solvency considerations, were dealt with by the British Solvency Working Party in a series of reports: DAYKIN & al (1984). . . . . DAYKIN and HEY (1990). STANARD (1986), RENSHAW (1989), VERRALL (1989), (1990) have analysed the properties of the chain ladder method.

The stochastic claim run-off error was analysed by PENTIKÄINEN and RANTALA (1986) to which this paper is a continuation. The results were incorporated as a suhmodel into the application-orientated risk theoretical over-all model in PENTIKÄINEN et al (1989).

We are going to use, as far as possible, the notations and concepts used in the above-referred papers. However, the terminology adopted in the Manual of IA (1989) is also taken into account. For the convenience of the reader the main features are recapitulated.

1.2. Claims cohorts. In order to clarify the terminology and the notation it is useful to note that the claim process includes the following elements:

1) the event (accident) which causes a claim in year t.

2) The claim is reported to the insurer in year t *or* later.

3) The claim is settled in year $t+s$ $(s \geq 0)$ or possibly in several parts in years $t+s_1$, $t+s_2$, . .

4) If the claim is reported by the end of the accounting year but not yet fully settled, it is called open and a provision is made to meet the outstanding liability either as a case estimate or by using some statistical technique.

5) The claims which are incurred but not yet repotted by the end of the accounting year are "IBNR-claims".

Following the terminology of Manual of IA (1989) (A 5.1) outstanding claims is an umbrella concept for open and IBNR claims.

It is appropriate to group the claims originating in the same accident year, t, as a "cohort". The year t is also called the *year* of origin. Fig.1.1 illustrates the structure of a cohort and its *development.*

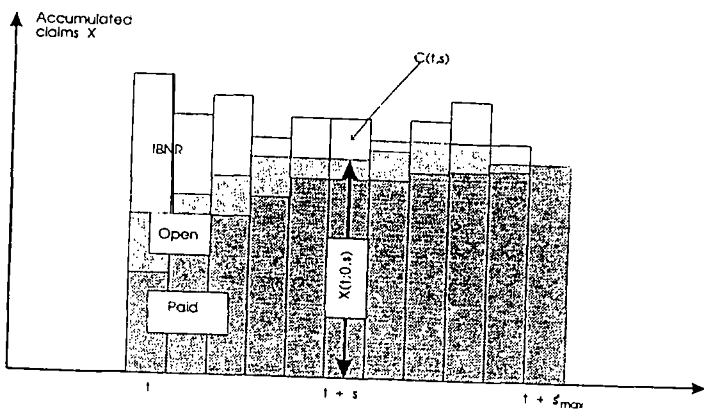


FIG. I. 1. The development of a claims cohort.

The accumulated amount of settled claims from development years t, t+ l, t+2,...,t+s supplemented by the provision of the open claims at the end of year t+s is called, still following the terminology of Manual of IA (1989), p. A5.2, the incurred loss and is denoted by

(1.1) $X(t;0,s)$ = claims originating from year t and paid in years $t,t+1,\dots,t+s$ on settled or partially settled claims ***plus*** reserve held for the open claims at the end of year $t+s$.

A notation for the increments of X is also needed:

$$(1.2) \quad X(t;s_1,s_2) = X(t;0,s_2) - X(t;0,s_1-1)$$

and especially

$$(1.3) \quad X(t;s,s) = X(t;0,s) - X(t;0,s-1)$$

which is the increment in the development year t+s (by convention, $X(t;0,-1)=0$).

It is assumed that after some period s_{max} all claims of the origin year t are settled. The parameter s_{max} characterizes a feature of the portfolio which is called the length of the run-off tail. Hence, the development time variable s can have values $0,1,\dots,s_{max}$, and,

(1.4) $X(t;0,s_{max})$ = is the final total amount of claims of the cohort t.

It is also called the loss related to the cohort

1.3. The reserve for IBNR claims of the cohort t at the end of year t+s is defined as:

(1.5) $C(t,s)$ = Estimate for $\{X(t;s+1,s_{max})\}$.

Various methods, 'reserving rules', can be applied in this estimation. The purpose of this paper is to find ***methods and measures for the evaluation of the uncertainty*** involved with the rules.

Concept (1.5) is in conformity with the "London market" definition presented in the Manual of IA (1989), p. A5.1 where the IBNR-reserve is defined to be equal to the estimated ultimate loss on *all* outstanding claims *less* the reserve at the accounting date for open claims. Hence, the uncertainty in the reserve of open claims is included within that of the IBNR-reserve, as thus defined. As stated in the next paragraph, this type of definition seems to be convenient in this context, because it allows the collective handling of all kinds of uncertainties in claims process. Note that this definition is different from the common accounting practice according to which the provisions for both the open claims and IBNR's are included in the claims reserve. No safety margins are assumed to he included in the reserve.

1.4. Claims process. The model lo be employed is based on the fact that tie increment $X(t;s,s)$ 'is made up of the sum of changes in the status of individual claims. It is helpful to classify "*change-causing events" as* follows:

1) A claim is reported and added to the paid and/or open claims.

2) An open claim, k, is fully or partly settled in year $t+s$, the amount being $S_k(t,s)$. For it (possibly) a reserve (case estimate) $C_k(t,s-1)$ was made at the end of the preceding year and now can be released. Then

$$X_k(t,s) = S_k(t,s) - C_k(t,s-1) \qquad (s \geq 1) \tag{1.6}$$

contributes to the change of the cohort's aggregate loss $X(t;s,s)$. If C_k were exactly correct, then X_k would, of course, be zero, but in practice it will often be non-zero $(\pm)$.

3) The provision C_k for an *open* claim is changed (possibly without any payment action), for instance, if new information has been obtained.

Both 1) the number *of events* and 2) the *amount of the changes* involved in, $X_k(t,s)$ above, are random variables. Our techniques, both simulation and others (PENTIKÄINEN and RANTALA (1986)). are based on utilizing probability distributions for both of them. Note that the approach is analogous to that of risk theory. Thanks to FILIP LUNDBERG, HARALD CRAMER and others the *collective* approach replaced the earlier "individual risk theory". The number of claims and their size are handled as a "risk process" without reference to the tiles of the individual policies which actually are behind the claims. The philosophy proved enormously fruitful notwithstanding that the theory can also be built on the individual bases.

As in general collective risk theory and even still more in the context of claims cohort processes it is crucially important to account for the correlations between the development cells of the cohorts as well as the correlations between consecutive cohorts.

Furthermore, note that the claim size variable X_k may also be negative. This can be the case particularly in classes 2) and 3) above. This feature should be kept in mind when the risk theory formulae and distributions are built up (cf. BEARD et al (1984), Section 1.3 p. 7).

For illustration of the approach numerical examples will be exhibited in section 4, therefore, some basic features of the claims process *need* to be specified. This is done in the Appendix. We recall that irrespective of which approach is applied in defining the concept of claim development the technique we are going to present can, with obvious modifications, also be applied to claims processes defined otherwise than the collective one. For example, the procedure allows for the use of the bootstrapping technique for claims simulation (as was remarked by one of the referees of this paper),

1.5. The aggregate loss process related to the whole business of the insurer consists of a the sum of the cohort variables X.

Following the practice adopted by NORBERG (1986) a diagram of the Lexis type is constructed in Fig. 1.2. The data array representing a cohort develops as an ascending diagonal. The information which the actuary, or in our simulation the computer, has available for the reserve calculation is in the "run-off trapezium" (in the diagram the vertical pillar at the accounting year t and the area left therefrom). The claims to be estimated for the reserve for outstandings are inherent in all of the still open cohorts and are located in the "triangle of outstandings" right from the column at t:

$$(1.7) \qquad C(t) = \sum_{s \le t} C(t-s,s).$$

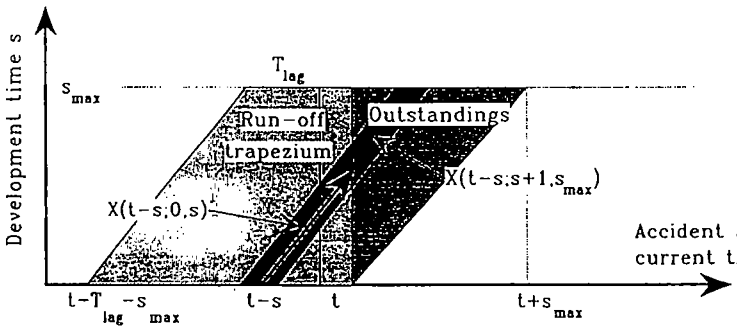


FIG. I .2. Claims process as a sum of cohorts. The current accounting year is denoted by t and the cohort originating in the accident year t-s is represented by an ascending diagonal.

NOTE. The problem in premium rating is basically the same as is the claims reserving. An estimate for the amount of claims of *future* cohorts is required. The difference in the claims reserving is that only present and past cohorts are considered and that a number of the earliest notified claims are already known and the estimation is focused to the remaining ones only. It is a bit surprising that the methods developed for premium rating are only little utilized in claims reserving.

2. Run-off error

2.1. Run-off error, break-up consideration. The run-off error is the remainder (±) which is left of the reserve C(t) when all the outstanding claims are ultimately settled:

$$R(t) = C(t) - \sum_{s=0}^{s_{max}-1} X(t-s;s+1,s_{max}). \tag{2.1}$$

In practice, of course, R can be determined only when the settlement of (practically) all of the outstandings is completed. Our approach is to compute it by continuing the simulation until all of the terms of the sum in (2.1) are obtained.

2.2. Going-concern consideration. Further, the effect of the runoff error on the ***aggregate loss*** X(t) is examined. This variable is the conventional entry for the total amount of the claims in the profit and loss accounts of the standard annual reports. In the terms of the definitions and notations introduced in item 1.3 it is

$$X(t) = \sum_{s=0}^{s_{max}-1} X(t-s;s,s) + C(t) - C(t-1). \tag{2.2}$$

As was noted in item 1.3. in our considerations the provision for open claims is included in the X terms, not in C, notwithstanding that this does not accord with the common accounting practice.

2.3. Properties of a good reserving method. For the appreciation of the efficiency of the reserving methods a great variety of optimality criteria are proposed in actuarial literature. From the point of view of the company's management the following features might be the most important:

(1) Probability of insufficiency of the reserve should be small (ε), more exactly

$$\text{Prob}\{R + L < 0\} \le \varepsilon \tag{2.3}$$

where L is a safety loading. (In practice it can either be included in the claims reserve C(t) in addition to the unbiased estimate (1.5) as an extra margin or e.g. as an equalisation provision or it can be available otherwise as a part of the insurer's solvency margin).

(2) The safety loading L should be as small as possible.

(3) The variation of the aggregate claims in the profit and loss accountshould be as small as possible (particularly in the going-concern approach).

In the next item some potential measures will be proposed for the comparison of different reserving methods having regard for the above criteria (1) - (3).

2.4. Measures of uncertainty. The runoff error **R** and its impact on X depend self-evidently on the reserving method. This dependence varies with the different claims processes. We shall use as primary measures in describing these effects both the direct values of R and X and their ratios and the standard deviations σ_R and σ_X of these variables together with the ratios

(2.4) σ_R/C, σ_R/P, σ_X/P and σ_X/σ_0

where P is the premium income corresponding to the relevant X (more in item 3.3). Furthermore, σ_0 is the standard deviation of $X_0(t)$ which is the incurred aggregate loss from which the runoff error is removed. This is obtained from the simulated data, in terms of our notations, $X_0(t) = X(t;0,s_{max})$ (= the total loss related to the cohort t). Hence, the difference $\sigma_X-\sigma_0$ is to be credited to run-off error.

Let us also recall that indicators based on the distribution of extreme deviations or confidence intervals, are good candidates as measures (cf. PENTIKÄINEN and RANTALA (1986)). but at this stage of work we mainly used the standard deviations. They need less simulations, but involve the drawback that the effect of skewness of the distributions is partly lost.

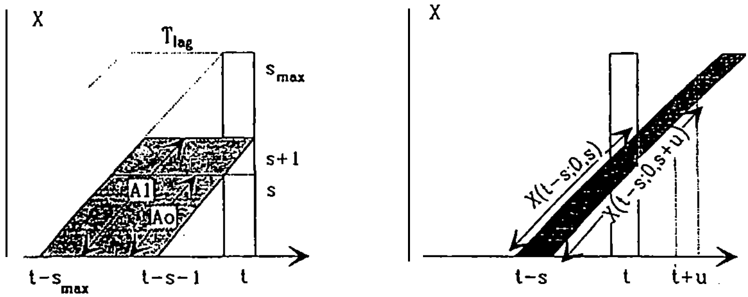
Note that when we in the following illustrate the comparison of two or more reserving rules, the very *same claim pattern* X(t;s,s) is used for all of them. Therefore, it can be expected that the differences revealed in results can be credited to the differing structures of the rules. This is still further verified by repeating the test after a change of the seed of the random generator.

3. Reserving methods used in the case studies

3.1. Chnin Ladder method. This well-known method is chosen as the first of our test examples. It operates auxiliary development coefficients

(3.1) d(s) = A,(s)/&(s).

Where the A's represent the sums of all X(t-u;v,v) located in the areas marked by the same symbols in Fig.3. la.



'a) A,(s) is the parallelogram shaded in the diagram, and A,,(s) is obtained by removing the top-most row from A,.

b) Development of a cohort.

FIG.3. I. Derivation of the Chain Ladder rule.

The claim sums to be estimated for the remaining parts of the cohorts are now obtained by assuming that the cohorts grow in the same proportion as the parallelograms A, i.e.

$$X(t\text{-}s;0,s+1) = X(t\text{-}s;0,s)\cdot d(s)$$

$$X(t\text{-}s;0,s+2) = X(t\text{-}s;0,s+1)\cdot d(s+1) = X(t\text{-}s;0,s)\cdot d(s)\cdot d(s+1)$$

etc. Hence. the claims reserve for the cohort t-s is

(3.2) $$C(t\text{-}s,s) = X(t\text{-}s;0,s)\cdot c_{c\text{-}l}(s),$$

where

(3.3) $$c_{c\text{-}l}(s) = \prod_{u=0}^{s_{max}-1} d(s+u) - 1$$

and the total claims reserve at the end of the accounting year t is

(3.4) $$C(t) = \sum_{s=0}^{s_{max}-1} C(t\text{-}s,s)$$

Note that c,,(s) should be recalculated in each accounting year t (hence, a notation $c_{c\text{-}l}(t,s)$ would, perhaps, be more advisable).

The Chain Ladder rule is at its best in the cases where the so-called structural (also called mixing) variation is large. This is a well-known feature and is again confirmed by the numerical example to be set out later as well as also in PENTIKÄINEN and RANTALA (1986. Appendix I). This is because the Chain Ladder assumes that the structure variation affects the total claims amount of each cohort but no longer how these claims are distributed during the runoff of the cohort for consecutive development years.

3.2. A variant. The chain ladder method can be amended by broadening the "runoff triangle" to a trapezium from which the parallelograms A are cut, if this is available. The dotted line associated with a "broadening parameter" T_{bg} (see Fig. 1.2 and 3. 1) refers to this variant. Its effect will be tested in Section 4.4 below.

3.3. The premium-based method is chosen as the second example for testing:

$$C(t-s,s) = P(t-s) \cdot c_p(t,s) \qquad (3.5)$$

where P is the unloaded net premium applied for the cohort and the coefficient c_p is an estimate for the ratio of the still outstanding IBNR claims of the cohort to the total amount of the claims. This rule theoretically is suitable for pure Poisson claims processes (see PENTIKÄINEN and RANTALA (1986), Appendix 1).

The premium income P(t-s) in our simulation example was calculated by a simple formula of the moving average type, determining P on the basis of the latest settled and open claims which are known at the year of origin of the cohort t-s:

$$P(t-s) = \sum_A X^\star / T_A \qquad (3.6)$$

where the sum stands for all of the simulated claims amounts $X^\star$ located in the rectangle A shaded in Fig.3.2, and the amounts $X^\star$ are the claims increment variables $X(t;s,s)$, (see (1.3)). transformed to match the value of money and business volume of the accounting year t having regard for the simulated inflation and presumed growth of the business volume (details in Appendix).

In practice, the coefficients c_p can either be fixed in advance or be derived from the pattern of the known claims. We used a simple rule defining these coefficients as the ratios of the simulated sums of the above $X^\star$ located in the rectangles B and A in Fig.3.2:

$$c_p(t,s) = \sum_B X^\star / \sum_A X^\star. \qquad (3.7)$$

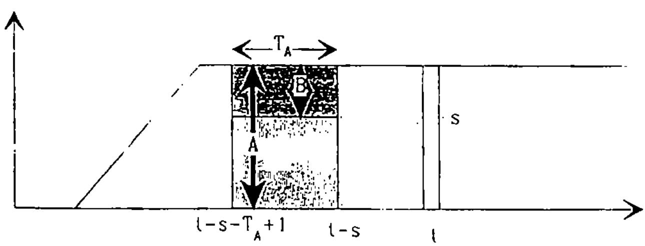


FIG.3.2. Derivation of the Premium-based reserving formula.

3.4. The mixed method is constructed as a combination of the Chain Ladder and the Premium-based reserves:

$$(3.8) \qquad C(t) = \sum_{s} [z(t,s) \cdot C_{prem}(t-s,s) + (1-z(t,s)) \cdot C_{c-l}(t-s,s)]$$

The idea is to assign to the coefficients $z(t,s)$ such values that the premium-based C_{prem} is predominant at the beginning of the runoff of the cohort (s small) and later, when s is approaching s,, the weight moves to the chain ladder rule.

The intended purpose can be achieved by taking z to be the same as the premium-based coefficient in (3.7):

$$(3.9) \qquad z(t,s) = c_p(t,s).$$

This formula was proposed by BENKTANDER (1976). The logic is analogous to the BORNHUETTER-FERGUSON (1972) approach, but it is applied to a different variable.

An alternative formula for $z(t,s)$ could be derived by using credibility considerations (see PENTIKÄINEN and RANTALA (1986), p. 127).

In order to keep the paper within reasonable limits we have restricted the application examples to these simple rules, the more so because our purpose is to describe the test and comparison method, not to arrive at any analysis of the reserving rules and their properties.

4. Numerical examples

4.1. Single realisations. We used the same numerical basic data as in PENTIKÄINEN and RANTALA (1986). For convenience of reading they are recapitulated in the Appendix. The run-off tail s_{max} is alternatively either 12 (long) or 3 (short) years (cf. Section 3.4 of the referred paper).

The model is programmed to give outputs both in tabular and graphic forms. Table 4.1 provides an example. The long-tailed claims pattern is simulated for 25 consecutive accounting years t by using, in parallel, the three reserve methods specified above (C-L=Chain Ladder, Pr=Premium-based, Mix=Mixed Method, formulae (3.8) and (3.9)).

TABLE 4. 1. Simulated run-off errors R and the aggregate losses X.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
t	P	Xo/P%	X-r-o	C(t)			R(t)			R(t)/C(t)%			X(t)/P%		
				C-L	Pr	Mix	C-L	Pr	Mix	C-L	Pr	Mix	C-L	Pr	Mix
0	65.7	83.8	175.6	184.4	189.8	188.4	8.8	14.2	12.8	4.8	7.5	6.0	73.9	94.1	77.0
1	68.8	98.6	187.3	196.1	197.3	194 2	8.8	10 0	6.9	4.5	5 1	3.6	98.6	92.5	90.0
2	71.0	89.6	195.0	186.4	205.2	19J.2	-8.6	10.2	..·1.8	-4.6	5.0	.9	65.1	89.9	77.1
3	73.4	99.2	206.7	183.3	213.1	198.0	·23.4	6.4	-8.6	-12.8	3.0	-4.4	79.1	94.0	89.8
4	75.8	107 6	225.3	209.8	221.0	214.5	-15.4	-4.3	·10.8	-7.3	1 .9	-5.0	118.1	93.5	104.8
5	80.4	120.4	2'8.9	237.6	230.8	227.1	-11.2	·18.1	-21.2	-4.7	-7.8	-9.3	125.6	103.2	107.4
6	85.0	104.5	257.5	260.0	241.5	243.3	2.6	-16.0	-14.2	1.0	-6.6	-5.8	120.8	107.0	112.7
7	90.4	117.9	284.0	273.8	254.7	258.8	-10.1	-29.2	-25.2	-3.7	-11.5	-9.7	103.9	103.3	105.8
8	96.6	100.1	288.2	291.6	270.1	279.8	3 . 3	-18.1	-8.4	1.1	-6.7	-3.0	114.1	111.6	117.'
9	104.6	117.1	317.4	328.6	288.9	300.0	11.2	-28.6	-17.5	3.4	-9.9	-5.8	124.5	107.1	108.4
10	114.5	114.1	350.0	360.4	311.8	329.0	IO.,	-38.2	-21.0	2.9'	-12.3	-6.1	113.4	105.7	111.0
11	121.7	95.9	361.1	364.1	335.3	349.5	3.1	-25.8	-11.6	.8	-7.7	-3.3	89.9	106.1	103.6
12	130.2	101.9	388.5	365.8	360.3	370.9	-22.7	-28.3	-17.6	-6.2	-7.8	-4.7	82.1	100.0	97.3
13	143.2	101.3	'15.9	365.9	390.4	,a,.&	-50.1	-25.6	-32.5	-13.7	-6.5	-8.5	82.2	103.2	90.9
14	151.2	98.8	435.7	431.4	423.8	432.3	-4.3	-11.9	-3.4	-1.0	-2.8	-.a	127.9	107.5	117.3
15	169.2	98.2	467.9	c.9.1	459.4	464.3	-8.9	-8.6	-3.6	-1.9	-1.9	-.8	95.5	100.2	98.1
16	,a,.,	93.4	497.7	'29.1	499.3	485.5	-68.6	1.6	-12.2	-16.0	.3	-2.5	60.9	98.9	88.7
17	190.0	95.9	532.7	502.2	541.0	531.7	-30.4	8.4	.9	-6.1	1.6	-.2	115.1	99.3	101.5
18	212.2	96.3	571.4	572.1	585.0	586.8	.7	13.6	15.4	.1	2.3	2.6	113.0	100.8	106.0
19	220.9	104.7	634.6	631.7	626.7	633.5	-2.9	-8.0	-1.2	-.5	-1.3	-.2	103.1	95.0	97.3
20	233.9	0.0	652.0	742.8	666.6	699.8	110 8	34.7	67.8	14.9	5.2	9.7	136.7	106.3	117.5
21	246.9	95.8	681.3	693.3	708.3	711.1	12.0	27.0	29.8	1.7	,.a	4.2	55.8	92.7	80.4
22	256.7	89.3	695.8	713.1	7L5.9	742.9	11.3	50.1	47.1	2 4	6.7	6.3	91.3	98.3	96.0
23	275.2	100.6	744 3	727.9	788.0	754.7	·16.4	'3.7	10.4·	-2.3	5.5	1.'	88.3	98.3	87.3
24	286.0	88.4	762.8	764.6	828.8	793.8	1.8	66.0	31.0	.2	8.0	3.9	94.8	96.2	95.6
25	295.3	92.9	786.3	817.0	865.0	822.3	30.7	78.7	36.0	3.8	9.1	4.4	102.7	97.2	PC.6

The variables P, R, X and C are given in monetary units ($\approx$ \$ million) and the ratios as percentages. The growth of premium income P and other monetary quantities is due to inflation (average 5%) and real growth (1%). Claims pattern is long-tailed. X-r-o is the "true" value of the outstandings, i.e. the simulated sum term in (2. I).

The loss ratios of columns 3 and 14 are plotted in Fig. 4.1. as well as the ratio R/P corresponding to col. I 1 (Chain Ladder method) but expressed as a ratio to premium P.

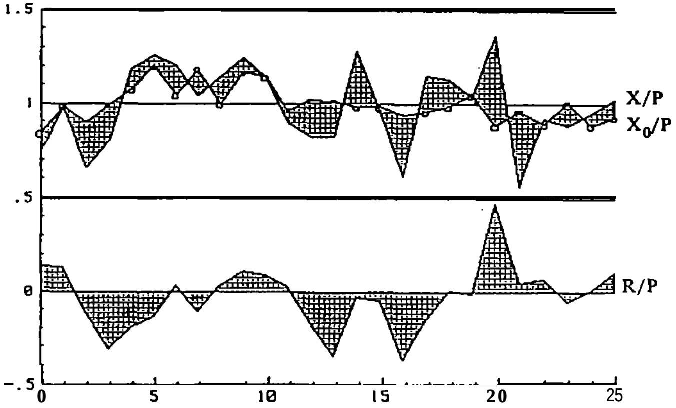


FIG. 4. 1. The ratios X_0/P (-o-), X/P (--) and R/P. Chain Ladder rule.

The ratio R/P and the deviation of X/P from X_0/P are shaded in order to show the strong correlation between them. When R is increasing, it worsens (increases) the loss ratio and vice versa. Note that X/P fluctuates more than 'original' X_0/P.

Fig.4.2 depicts the premium income P and the aggregate "no-run-off affected" loss X_0 from which P is derived according to (3.6) as a moving average with the range 10 years and with a necessary time lag. For clarity, the effect of inflation and growth is stripped away from the time series by operating the variables in the initial value of money and volume (at $t=0$).

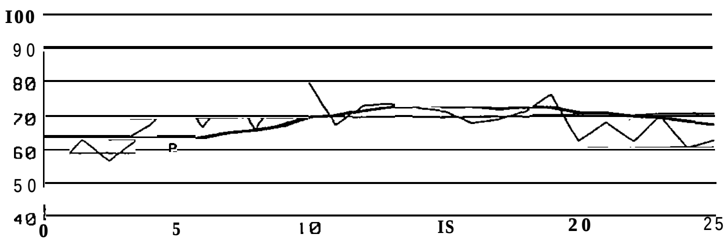


FIG. 4.2. The premium income P, deflated into the monetary value of the initial time point, as a (delayed) moving average of the loss X_0.

All the loss ratios X/P of Tahle 4. 1 and the ratios R/P are plotted in Fig.4.3.

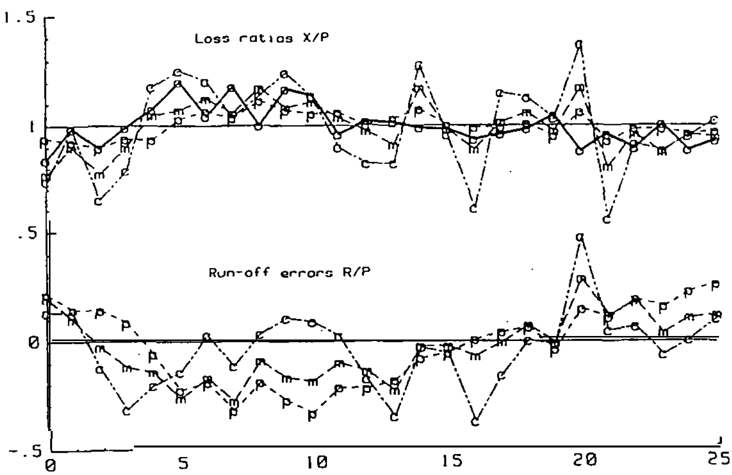


FIG. 4.3. Loss ratios X/P and RIP calculated hy using Chain Ladder (marked hy c), Premium-based (p) and Mixed (m) methods, respectively. The thick line represents X_0/P.

A smoother flow of X/P can be achieved at the expense of larger reserve errors R/P.

Simulations confirm the well-known fact (STANARD (1986) and ZEHNWIRT'S article in the Manual of the IA (1989), Vol. If) that the Chain Ladder method has a tendency to show a greater volatility than the other rules compared.

4.2. Monte Carlo simulations. In order IO get hroader insights into the hehaviour of the target variables the simulations exemplified in Figures 4. I and 4.3 were repeated 50 times for each of the three rules. "A stochastic bundle" is generated in this way in Fig. 4.4.

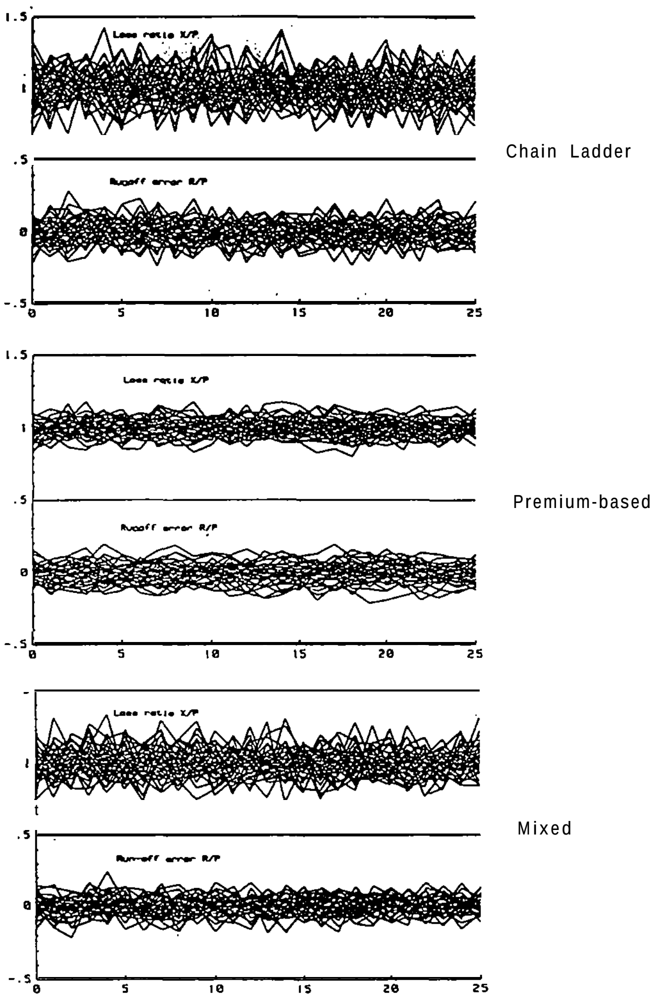


FIG.4.4. Monte Carlo simulation of loss ratios X/P and run-off errors d/P for the three reserve rules. Short tail ($S_{max}=3$). Premium rule stochastic moving average (3.3 above). Sample size 50.

The breadth of the bundle of the simulated realisations gives an idea of the volatility involved with the reserving methods.

A useful observation, seen in Fig.4.4, is that the bundles are stabilized at about a state of equilibrium, i.e. the breadth of the bundles is approximately constant. This feature appeared to be common in those cases we experimented with apart from some extreme situations (premiums defined deterministically and kept unchanged for a long period), where the bundle could have some tendency to diverge. If a reasonably satisfactory attainment of the equilibrium state can be achieved, then it is possible to record the values of the relevant variables, X/P, etc. at *each* time point t of the run, and to compute the required standard deviations as "steady-state" characteristics from the set of all of them. This procedure greatly reduces the number of simulations needed compared with approaches which might require a new simulation for each variable value. Table 4.2 is obtained from Fig.4.4 in this way.

TABLE 4.2. Standard deviations of the simulated ratios.

	Chain Ladder	Premium-based	Mixed
σ_X/P	0.126	0.061	0.102
σ_X/σ_0	1.749	0.851	1.414
σ_R/P	0.079	0.062	0.066

Similar data will be given for a long-tail pattern in the next item. Therein the obviously characteristic features of the methods are more clearly seen.

4.3. Error distributions. The X/P and R/P values simulated, as shown in Fig.4.4, can he recorded and plotted, as is exhibited in Fig.4.5a and in Fig.4.5b which set out the critical tails of distributions.

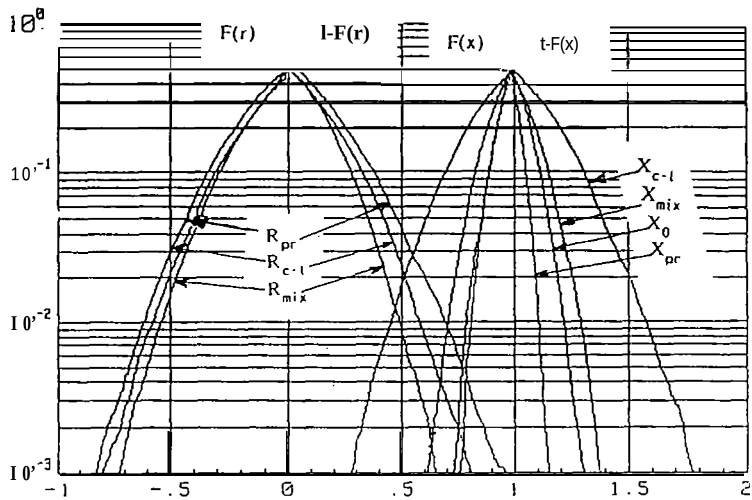


FIG. 4.5a. The cumulative distributions F(x) and F(r) for the ratios x=X/P and r=R/P, respectively, are obtained from the simulated patterns of these ratios. For clarity, F is plotted for the left-hand tail of the distributions and I-F for the right-hand tails in a semi-logarithmic scale. The number of sample points is 15600 for each curve. Long tail s_{max}= 12. Premium method stochastic.

Confidence limits can be directly read from the picture. For instance, the limit which the Chain Ladder ratio X/P exceeds by 1 % probability, is 1.57. Similarly, the limit, which the Premium-based R/P falls below by 1% probability, is -.58.

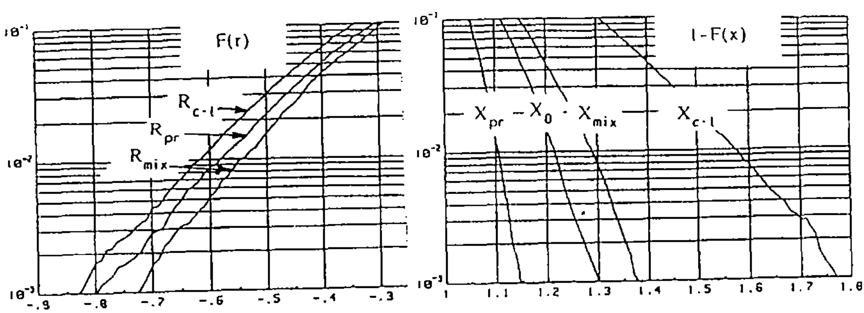


FIG.4.5b. The tails of the distributionsof Fig.4.5a

Note that the distributions exhibited in Fig.4.5 are based on the development tail of 12 years which is rather long and, on the other hand, on the portfolio which is relatively small, the average number of claims being 10000.

For a comparison of the exemplified reserving methods, the standard deviations derived from the same simulation as Fig.4.5 are shown in Table 4.3.

TABLE 4.3. The basic characteristics related to the distributions of Fig.4.5

Variable	Mean	St.dev	Rel.st.d. σ/σ_0
Xo;--/P			
X;c-l/P	1.0031.001.087.2402.7591.000		
X;pre/P	.980	.065	.745
X;mix/P	.993	.125	1.431
R;c-l/P	-.002	.259	2.979
R;pre/P	.039	.267	3.066
R;mix/P	.004	.221	2.534

The ***mean*** values are shown in the table to verify that they are, as they theoretically should be, close to unity for X/P's and zero for R/P's (in order to check that the simulation variahility and programming are under control).

In extreme cases the ***skewness*** of the distribution may be considerable and might suggest that it should be seriously regarded in order to avoid the caveat of understating the run-off risk. Some tests (not set out in this paper) also indicated rather great volatility in the development of the tails. We had to leave further studies on this problem for later work.

A feature of interest is ***the smoothing effect of rhe premium-based*** rule. The Premium-based rule, in fact, reduces the range of fluctuation of the loss ratio X/P compared with the case X_0/P from which the run-off error is eliminated. This happens, of course, at the expense of larger run-off errors R/P, as seen in Figures 4.3 - 4.5 and Table 4.3 when comparing the premium based rule to the mixed one. The adverse tops of the fluctuation of X are spread over a lengthy period.

As expected, the performance of the chain ladder in these examples proved to be rather poor in regard to both the loss ratio and run-off error.

4.4. Stability profiles. The tools developed in the preceding sections are now readily available for the comparison of different reserving methods. We exemplify the idea by applying it to the three methods which were specified in Section 3. For the purpose, the standard deviations σ_X, σ_R and σ_0 are calculated in parallel. Fig.4.6 exhibits an example. The relevant indicators are plotted as columns in order to provide a clear view of their magnitudes. Various patterns of the claim process are simulated for all the three reserving methods. They are constructed from the standard data by allowing options and inserted special variations, as explained in the captions of the figures. The standards are the same as we had in PENTIKÄINEN and RANTALA (1986) and a summary is given 'in the Appendix below.

The left-hand displays of Fig.4.6 represent the relevant standard deviations as ratios to the premium income P. In order to show more clearly the role of the run-off inaccuracy the σ_X's are also given as ratios to the "no-runoff standard deviation σ_0 in the right-hand section of the figure.

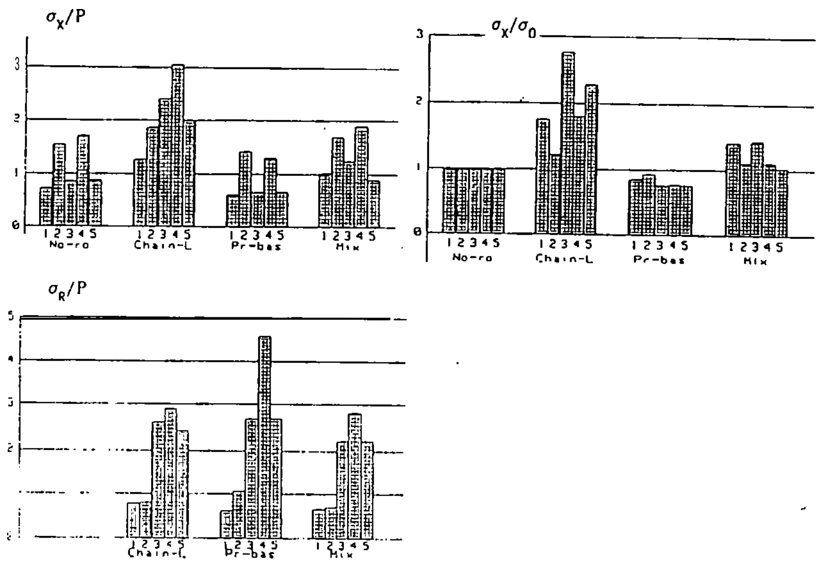


FIG. 4.6. Stability profiles. The numbered claims process options processed in parallel are as follows:

1) Short tail, stochastic premium rule (the same as Fig.4.4 and Table 4.2)
2) Short tail, deterministic premium rule
3) Long tail, stochastic premium rule
4) Long tail, deterministic premium rule
5) Long tail, stochastic premium rule. Chain Ladder with trapezium $T_{bg}=5$ (see Fig.I.2 and 3.1a).

Fig. 4.6 gives rise to the following observations and comments:

* As expected, the short-tail portfolios (I and 2) are less vulnerable IO run-off inaccuracies than are the long-tail patterns.

*The premium-hased rule reduces the fluctuations in the loss ratio below even that level which would prevail if the run-off errors were stripped away, i.e. from the level which is shown by the "no-ro" columns in the figure. But this may happen at the expense of the run-off error being buried in the loss reserve (in particular the option 4 in the figure!).

* The use of a stochastic premium basis reduces the volatility, especially, for the premium method as seen in comparing the option 1 against 2 and the option 3 against 4 in the left-hand displays. The remarkable differences in the magnitudes of these outcomes indicate that the premium calculation basis is likely of primary concern and possibly its effect often outpaces that of the run-off inaccuracies inherent in the reserving method itself.

* The extension of the conventional runoff triangle of the Chain Ladder methods to a trapezium, as expected, improved the stability, as seen by comparing the options 3 and 5 of the Chain Ladder and Mixed columns.

* Note that in the cases 1, 3 and 5 the stochastic variation of the premium income also is involved.

4.5. Sensitivity testings. The effects of various impulses, shocks and disturbances on these processes can be studied by the same model outlined above.

As an example of these kinds of sensitivity testing an extra increment was given to the structure variable q(t) in accounting years 3 and 4 as shown in Fig.4.7. The outcomes are simulated as 'single shots", first without this extra increment, and then with it. The changes in the relevant variables are shown by shading the area between the original and changed curves.

Both the ratios X/P and R/P are plotted for the three reserving methods as depicted in Figures 4.7 and 4.8. The effect is channeled in two ways: 1) via the premium income P, which was simulated to be the moving average (3.6) and 2) via the reserve calculations. The change in X_0, of course, wholly arises via the premium channel and the continued effect after the cease of the impulse at $t=4$ is due to the moving average rule of P which is based on a retroactive account for claims from a lengthy period preceding the accounting year t.

Note that expectedly the q-impulse has (nearly) no effect on in R(t) in the case of the Chain Ladder method. This is due to the well-known fact that the changes in both terms of the run-off error formula (3.1) offset each other, i.e. the Chain Ladder method automatically adjusts for the change in the level of X.

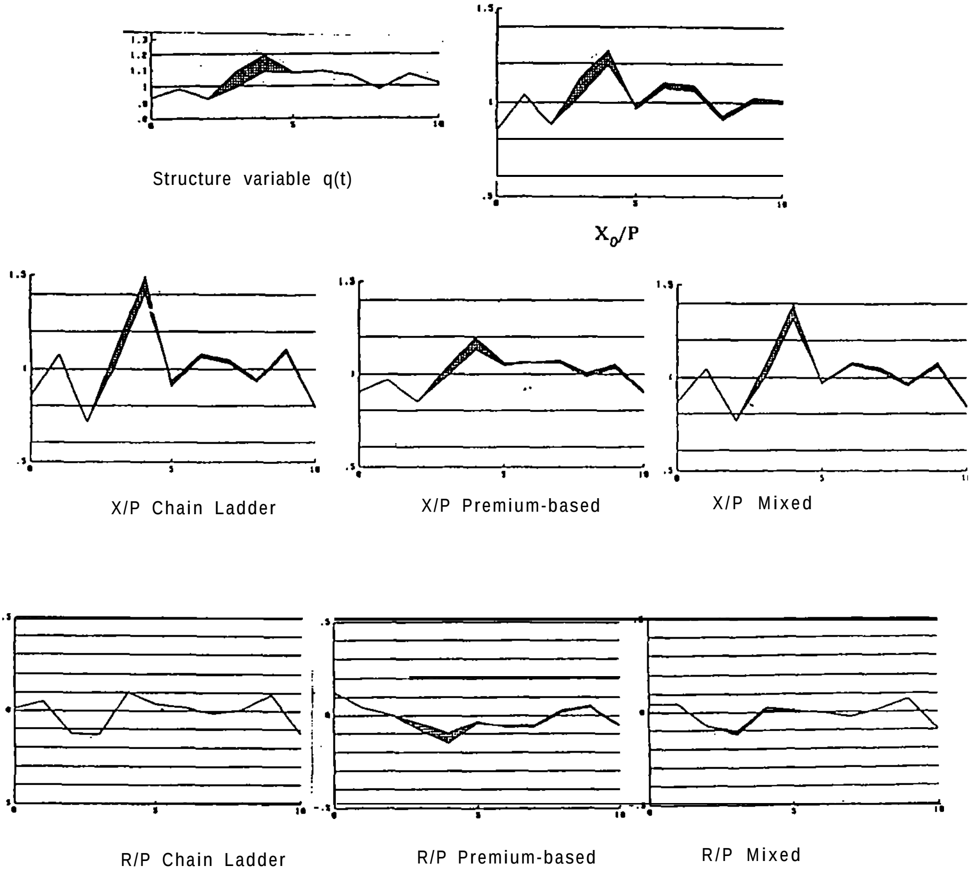


FIG.4.7. The effects provoked by an impulse of magnitude 0.1 exerted on the structure function q(r) in years 3 and 4.

Fig.4.8 displays the effects which are brought about when an extra shock is given to the simulated flow of inflation, represented hy variable I,(t) The technique is the same as in Fig. 4.7.

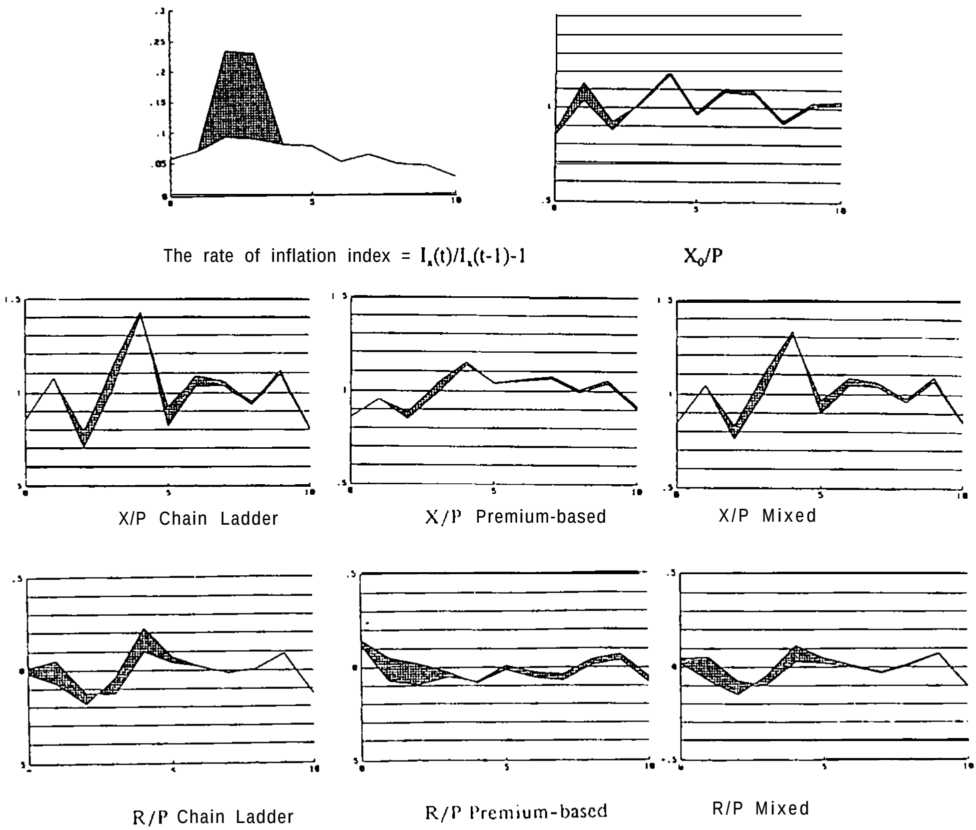


FIG.4.8. The effects provoked by an extra impulse of magnitude 0.14 exerted on the simulated rate of inflation in years 2 and 3.

5. Discussion

5.1. Reservation. Let us recall that this paper is intended to dcscrihe a simulation-based approach of how to analyse the various kinds of unccrtainries which are involved with claims reserving methods. The numerical examples are only intended to illustrate the method and do not claim to have universal validity in the evaluation of the merits and demerits nut even of the exemplified rules, though some observations can be made on the particular portfolios studied. However, we hope that the ideas outlined above might prove useful and inspire further research efforts in acquiring insights into the properties of the most common and often sophisticated reserving methods and, perhaps, to find guidance for their future development.

5.2. Our primary **appraisal of the applicability** of the outlined testing **procedure is** positive. Here, as quite commonly in many other contexts, the simulation approach seems to be flexible and susceptible to extension also into the realm of very complex problems and models which otherwise are beyond the tractability of conventional (rigorous) treatment. Obviously the simulation method can compliment the conventional practices which are based on the post-facto recording and analyzing of the observed runoff errors. This approach provides possibilities to *separately* reveal the effects of specified background factors, such as inflation, catastrophes, changes in the portfolio, claims handling, legislation, etc. Even circumstantial irregular impulses can easily be examined. These are useful additional features to the conventional methods which are fully or, at least to a great degree, restricted to deal with the data of total loss as a bulk, and seldom occurring events or combinations of events may not appear at all.

5.3. The purpose of the **procedure (when further** experience on its usefulness is acquired) may be to test the commonly used or proposed reserving techniques and qualify such ones which prove to be reasonably immune against variations in the structures of background factors, for instance, in claims process, inflation, etc. and against the three sorts of errors referred to above. Possibly a roughly scaled measure to rate the quality of the reserving methods can be found? Furthermore, the testings can provide advance knowledge about reactions of the methods to adverse impulses such as, for example, abruptly increasing inflation.

5.4. Discounting of the fulure claims settlements is another feature to be incorporated into the analyses. It introduces the effects of the fluctuations and risks related to the investment income, which can be substantial particularly if the business is long-tailed (see DAYKIN et al(1987b)).

5.5. Effects to be credited to human bebaviour A comment, sometimes heard, is that the reserves may have a tendency to excessive growth during the profitable phase of business cycles and, on the other hand, to be largely reduced in years when the profitability is poor (see for example Hewitt (1986)). Self-evidently, such kinds of "fluctuations" are beyond the scope of our testing methods which presume a strict and consequent application of some specified reserving formula. However, the possibility of the "human behaviour fluctuations" should be kept in mind as one of the potential determinants of observed phenomena for instance in the cases where actual reserve inaccuracies have been discovered.

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Appendix: Technical details

Abreviation: P&R = PENTIKÄINEN and RANTALA (1986)

1. Definitions and assumptions. We first simulate the "actual" claims in the areas depicted in Fig. I .2. A random number representing the increment variable (cf. (I .3)) $X(t;s,s)$ is generated for each cell, i.e. for all relevant pairs oft and s values.

The random number generator is the same as is represented in BEARD et al (1984). Section 6.8.3, however, using instead of the NP-generator (BEARD et al (1984), item 6.8.3b) the so-called WH-(WILSON-HILFERTY) generator, which is described in P&R, section 5.6. The generator is built up on the assumption that the variable X to be simulated is of the (conditional) compound Poisson type. It requires as input parameters the mean, standard deviation and skewness of $X(t;s,s)$. They can be computed when the mean claim number and the lowest moments (not necessarily the whole specified distribution function) of the individual claims are available, for instance, as estimates from observed data or being suitably assumed. Though, in the cases where the number of claims is very small both the number of claims as well as their individual sixes preferably can be directly generated. For brevity, the formulae of mean value only are outlined in what follows, because they reveal the most relevant background factors and their formulation.

The mean of the increment $X(t;s,s)$ is defined, as in P&R, as the product of mean claim number and mean claim size:

$$E\{X(t;s,s)\} = n(t;s,s) \cdot m(t;s,s) \qquad \text{(A1)}$$

The first factor on the RHS stands for the expected number of the claims in the target cell:

$$n(t;s,s) = n \cdot I_n(t) \cdot q(t) \cdot g_n(s) \qquad \text{(A2)}$$

where

- n is the mean claim number at the initial time $t=0$,
- I, is a function representing the growth $(\pm)$ of the business volume,
- q is the structure (mixing) variable introducing into the model the stochastic fluctuation of the mean claim number controlled by a (first order) time series (see (A4) below), and
- g_n distributes n(t) to the development years t, tt $1,\dots,t+s_{max}$, n(t) being the mean of the total claim number of the cohort obtained as the product of the first three factors in (A2).

The mean claim size, the second factor in (Al), is ohtained from

$$m(t;s,s) = m \cdot I_x(t+s) \cdot g_x(s) \qquad \text{(A3)}$$

where

- m is the mean claim size at $t=0$,

- I, an index representing the changes of the mean claim sizes owing to inflation and possibly also to other reasons. It is calibrated to be = 1 at $t=0$.
- Finally, g_m allows the possibility to take into account changes in claim sizes which cannot be explained by the index I,, for instance, if it is observed that the average value of delayed claims (s large) has a tendency to differ from that of early paid claims.

Note: Instead of employing two development distributors g_n and g_m an alternative approach is to build the model on the basis of their product $g_x=g_n \cdot g_m$ which represents the distribution of the total claim *sums* betweenthe cohort cells (cf. P&R, Section 1.7).

2.Specifications.

Portfolio parameters: Expected annual number of claims n= 10000 (see eq.(A2))
Claim size distribution: the lowest moments about zero $a_1=0.006$, $a_2=0.001$, $a_3=0.0001$ (Unit suitably $million, then the average claim size is $6000).

Structure function (also called mixing function):

$$\text{(A4)} \quad q(t) = a_q q(t-1) + \sigma_q \varepsilon(t)$$

where $a_q=0.6$, $\sigma_q=0.05$ and ε is a normally distributed (0,1) random number (white noise).

The rate of inflation:

$$\text{(A5)} \quad i_x(t) = I_x(t)/I_x(t-1)-1 = i_0 + a_i(i_x(t-1)-i_0) + \sigma_i \varepsilon(t) \geq \tfrac{1}{2} i_0$$

+ (an optional manually inserted) "shock"

where $i_0= 0.05$, $a_i=0.7$ and $\sigma_i=0.015$.

Real growth of the portfolio $I_n(t) = (1 + i_n)^t$ with $i_n = 0.01$.

Developmetu distribution $g_n(s)$ for $s=0, 1, 2,\ldots$ (see eq. (AS) and P&R, Section 3.4)

Short tail 0.6, 0.2, 0.15, 0.05

Long tail 0.15, 0.25, 0. 15, 0.15, 0. IO, 0.05, 0.05, 0.02, 0.02. 0.02, 0.02, 0.01. 0.01.

Formulae of the basic characteristics, see P&R, Section 5.1.

Random number generator is described in P&R, Section 5.6 and Pentikäinen et al (1989), Appendix A

The transformed amount of loss (claims) in a development cell s of the cohort of the origin t-s (Item ***3.3, eq. (3.6)*** and (3.7)).

(A6) $X^*(t,s) = X(t-s;s,s) \cdot V(0)/V(t-s)$

where **V** is an auxiliary variable representing the volume of the business with reference to simulated inflation and assumed real growth of the portfolio:

(A7) $V(t) = I_x(t) \cdot I_n(t).$

3. Discussion. The following features of our numerical simulation might be worth some special comments:

* Parameter n introduces into the model allowance for changes in ***business volume.***

* The structure variable q is stochastic and is generated as a first order time series (see Appendix). Hence, the *n*-values obtained for consecutive years are not independent (contrary to what is mostly the case in the traditional risk theory). This correlation is one of the factors which can crucially affect the range of fluctuations (cf. PENTIKÄINEN et al (1989), 2.2).

* *Inflation* is stochastic and generated by using first order time series (AS).

* Also ***other backgound processes*** as the structure variation and inflation could be assumed to be *stochastic,* in particular, the return on investments.

* The model can be extended by introducing return on investments and ***discounting*** of the future payments. Then a new component of stochasticity is incorporated into the model probably having a significant effect in long-tailed business. However, we had to postpone this to later works. Hence, in what follows, discounting is not performed.

* The portfolio of general insurers mostly consists of numerous lines and sublines, and reserves need to be made up for all of them. This feature is not dealt with in this paper, the approaches, which are described, handle the claims as one single block which can either be any of the lines separately or two or more of them combined. The multi-line problem is considered in PENTIKÄINEN et al (1989), Section 3.1.1a, p.27 and BEARD et al (1984) Section 3.7.

When the Wind Blows:
An Introduction to Catastrophe Excess of Loss Reinsurance

by D.E.A. Sanders

THE STORY OF A TILE

On 25th January 1990 a tile blew off my house - luckily I managed to get a handyman in who replaced it - for £75.00 This may be exorbitant but they were busy and, in any case, insurers were paying claims up to £ 1,000 without question.

I put in an insurance claim, and received £75.00. By this time the insurer - my own company - had breached their deductible. They themselves put in a claim totalling £67.50 (10% of the risk was retained). This cover was placed with over 100 reinsurance companies, including Munich Re, M & G Re and Syndicates with Lloyds. By this time these reinsurers had breached their limits and were passing their excess (£60.75) to their reinsurers. The trail is now more difficult to follow. This £60.75 was passed from Reinsurer to Reinsurer (including Eagle Star's own reinsurance operation) time and time again.

For convenience I will assume it went 10 times round the system, and generated some £500 in transaction. It then ended up at a Whole Account protection programme and went into the Marine market as an "incidental non-marine loss". This went round the system yet again - and is still moving. My tile has been involved in over 20 financial transactions, with total amounts in excess of £1,000.

If that storm happened today, the situation would be different - there would possibly be only two transactions since the secondary market has completely disappeared. The challenge for the Actuary is to estimate the total cost of this simple transaction and to assist in the pricing of the products. As the old age dies, and a new one arises, I hope it is useful to put down some of the methods used in the past to solve the problem of tracking the claim.

THE POLICY

Excess of Loss Policies are split into two distinct types - Risk XL's or working covers and CATXL or catastrophe covers.

A Risk XL covers the cost of individual losses above a certain specified sum up to a maximum amount. The lower level is the deductible and the difference between the lower level and the maximum amount is the cover or line. Cover is sometimes expressed as a number of lines which equals cover/deductible, but this is more appropriate to surplus treaties. The losses may be unlimited in amount or limited by aggregate amount. Generally today policies have limited aggregate amounts, i.e. a reinsurers exposure is limited.

CATXL's covers the cost of the aggregate claims (after deduction of other reinsurance recoveries) in excess of a specific amount, up to a maximum. The type of risk and cover is specified. For example the policy may cover losses in excess of £5 million up to £25 million. The cover is called into play, and the insured may receive up to £20 million. This may be achieved by one loss of £25 million or 20 losses of £6 million.

In the event of a loss, the cover is normally reinstated on a pro-rata basis by the payment of a reinstatement premium. (The calculation may also be pro-temp I.e. related to remaining exposure period). Thus, in our example, a loss of £10 million will mean a £5 million payout, less a reinstatement premium of 5/20 x initial premium.

In general in Non-Marine Insurance one reinstatement is given, and in Marine Insurance two reinstatements are given. In effect, the aggregate covers are two and three times the stated cover. The policy may be specific to the type of risk (e.g. UK windstorm) or general. (All losses world-wide).

Other specific considerations are two loss warranties (i.e. for the cover to come into force there must be two losses). Thus a single vessel sinking may be excluded.

Another important feature is the "hours clauses". Under this, in respect of most losses, an event is defined as a 72 hour period. Thus as a hurricane hits one part of the US causing damage, and then another part four days later, this is categorised as two catastrophe losses and hence two deductibles apply. However, if two separate events occur within a specific 72 hour period, each event is separate, despite the hours clause, and two deductibles apply.

The exception is winter freeze losses which apply over a 156 hour period. The art form in this case is to pick the 7 days which maximises the loss - and hence the reinsurance recoverable.

In 1990, it was difficult to differentiate the losses arising from two storms on 25th January and 27th January. The market took a pragmatic view of this.

THE PLACING OF CATASTROPHE REINSURANCE

Catastrophe Reinsurance is generally placed by Brokers in the National and International Reinsurance Market via a slip system. Under a slip system a specific percentage of the risk is underwritten. For example, if the risk is for £10 million in excess of £2 million (i.e. to cover losses above £2 million up to an aggregate of £ 10 million) an Underwriter may place a line of 10%. This gives him an initial exposure of £ 1 million (excluding reinstatement).

The Broker aims to try and place more than 100% of the risk. In the Non-Marine market, the insured normally retains 10% of the risk - but for the purpose of what follows this will be ignored. For Marine risks 100% can still sometimes be placed.

If a Broker writes so the total "signings" exceed 100%, then the slip is signed down. In the case of the Broker placing 125%, the 10% line is signed down to 8%, and the exposure is reduced to £800,000.

If the Broker places 75% of the risk, there is no increasing the line - the reinsurers' limits are set and the residual 25% is unplaced and hence retained by the insured. Brokers like continuity, in that they always aim to place more than 100% of the risk, and the renewal business is always given to the existing reinsurers as a first refusal. An example of a slip, with the stamps and lines is attached as Appendix 1.

Now consider a major UK insurer. The exposure to property is astronomical. The reinsurance it wishes to purchase is £175 million in excess of £25 million. It is extremely difficult - indeed impossible - to place such a risk in one tranche. The largest reinsurer would only want a small (2.5%) line, and the very smallest would be writing decimal point lines. Note in the real slip some individuals are writing only 0.15% of 95% of $25 million.

A Broker would spend an eternity trying to place the risk. What happens is that the reinsurance is structured into a placeable programme. The £175 million over £25 million could be structured into, say, four separate categories:-

(i)	£25	million	xs	£ 25 million
(ii)	£25	million	xs	£ 50 million
(iii)	£50	million	xs	£ 75 million
(iv)	£75	million	xs	£ 125 million

The consequences of this are three fold:-

a) The business has a greater possibility of being placed. The smaller company which only wants an exposure of £250,000 can write a 1% line on programme (i) or (ii).

b) Different reinsurers like different types of risk. Specialists can be identified for each contract.

c) The cost of the programme theoretically reduces.

A simple example will explain this last point (again reinstatements are ignored). Let us consider a company with the following loss:-

(i)	1 Loss of £60 million	(A)
(ii)	1 Loss of £40 million	(B)
(iii)	3 Losses of £ 30 million	(C), (D) and (E)

Under the one policy structure the insurer received £35 million from A, £ 15 million from B and £5 million each from C, D and E - a total of £65 million. Under the new structure he receives £35 million from A, (£25 million from the first policy and £ 10 million from the second) and nothing from B, C, D and E. If one reinstatement is allowed, he will also receive £ 15 million from B, £5 million from C and D and nothing from E! As the expected receipt is lower, so should the theoretical premium.

The consequences of the above restructuring lead to innovative products which increase the exposure of the actual programme. These include cascade programmes and top and drop, where unused parts of the vertical programme (i.e. the higher value programme) is used to cover a horizontal exposure (more losses of lower value). Under the example, an insurers cover (say £ 50 m xs £ 150 m) can be used to cover the losses in (iii).

The important issue to note is that the price for CATXL has changed radically in the last three years. This is due to recent major losses. Losses in the CATXL market are usually given a name (e.g. Hurricane Andrew) or a CAT code (e.g. 87J). This is the 'J'th event of year 1987. This storm is .the event of 15th October when Michael Fish, the Weatherman, got it all wrong! Illustrations of how, for example, Sevenoaks became one oak can be found in [6].

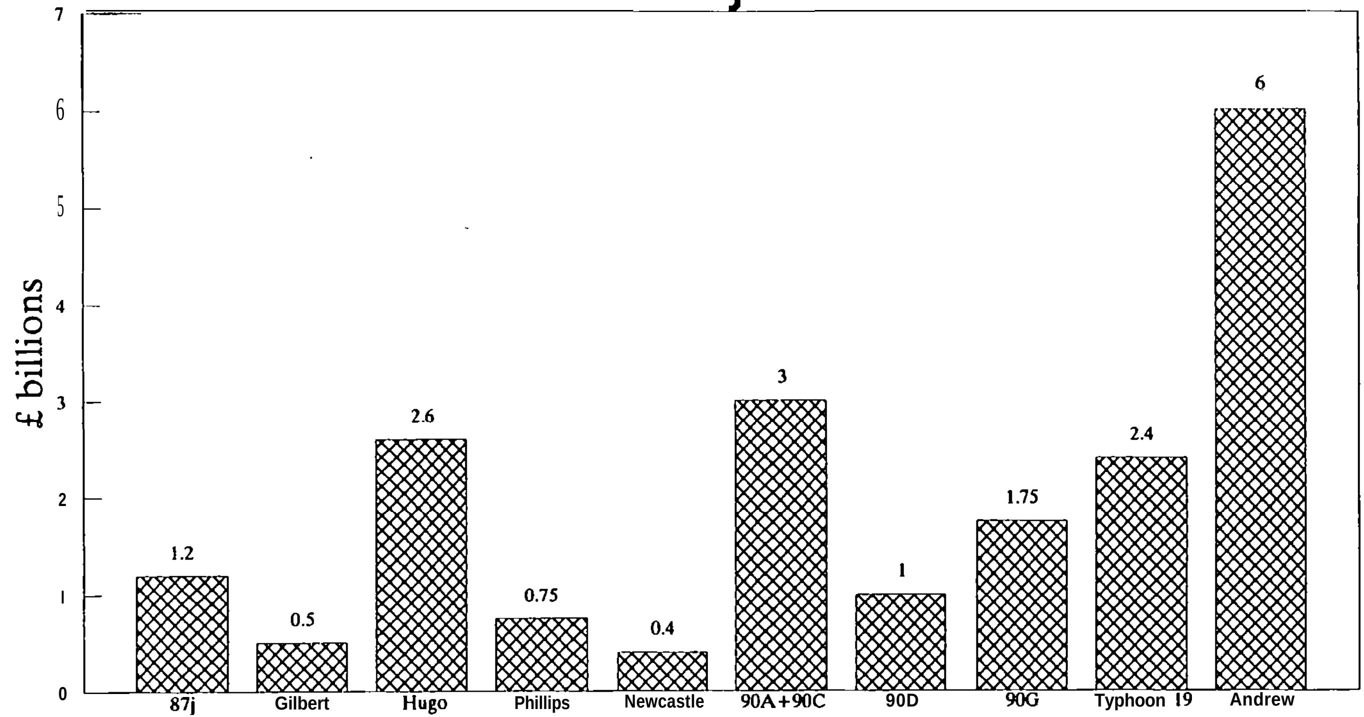
The storm of 1990 on 25 January is 90A. This is followed by 90D and 90G - 90B was an aviation loss. Recent losses are given in the graphs attached to this section. Catastrophe cover costs have jumped by a factor of nearly 4.

The policy is rated on Premium Income i.e. as a percentage of premium income of the cedant company. There is normally a Minimum and Deposit premium which relates to the expected premium income of the cedant. However, this premium is usually expressed as a Rate on Line, the Line being the exposure. The graphs following this section illustrate the point. In the rating section the issues will be explained in greater depth. The following graphs indicate the cost as a mid point in a spread of layers, and indicate how the cover, expressed as a percentage of premium income, has changed.

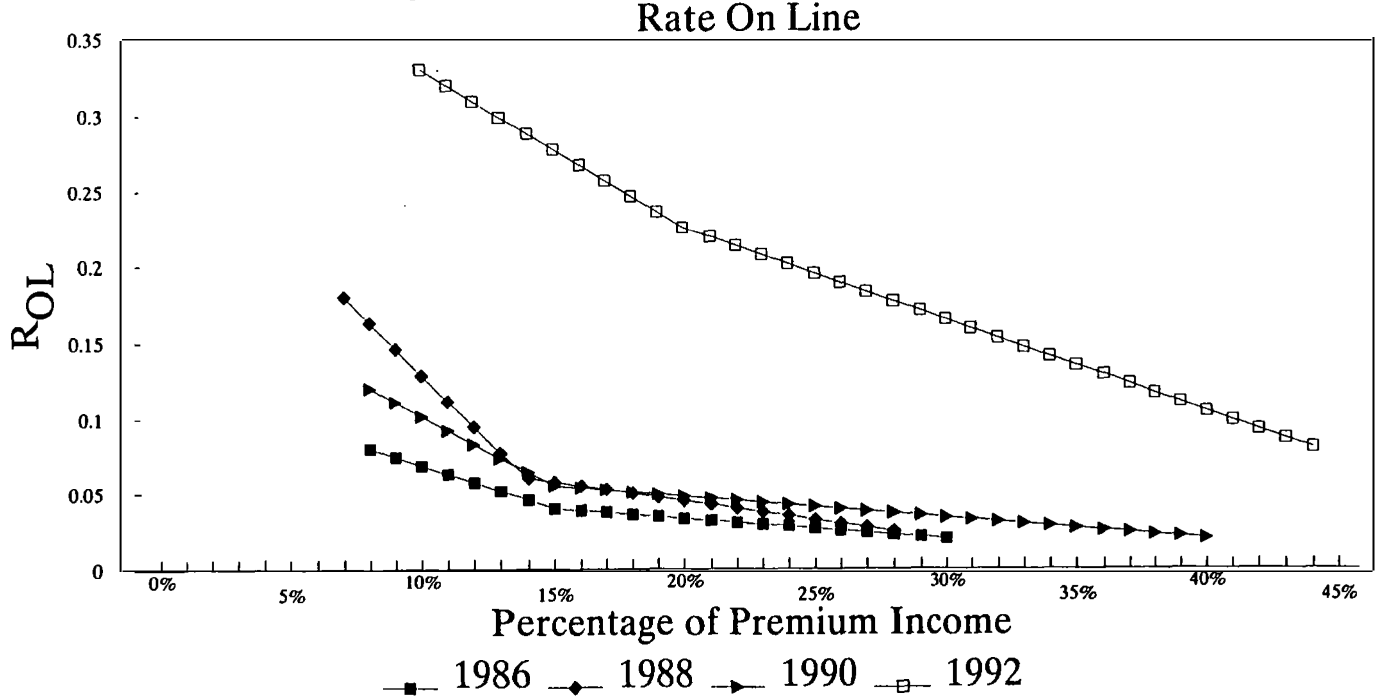
A company with a premium income of £ 100 million wanting cover from £10 million to £30 million would, therefore, expect to pay a price above the 20% of premium income on this graph. In 1990 this would have been about 5% (5% x £20 million line gives £ 1 million). In 1992 this would be 25% on £ 5 million.

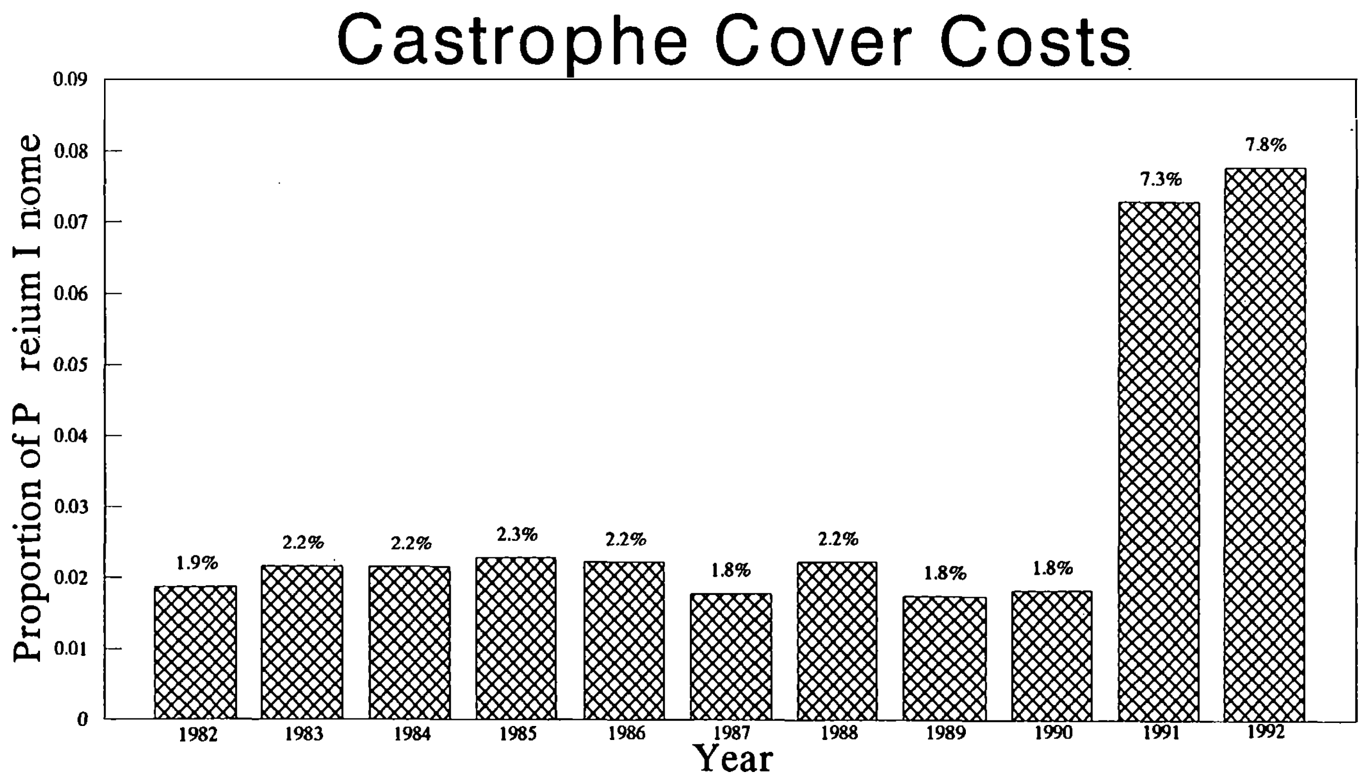
This massive increase in rates has created new problems for insurers. When rates were cheap the philosophy was to place as much as you can. Why have rates increased substantially?

Recent Major Losses



UK Composite Insurance Companies




Castrophe Cover Costs
Proportion of P reium I nome
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
1.9%
2.2%
2.2%
2.3%
2.2%
1.8%
2.2%
1.8%
1.8%
7.3%
7.8%
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
Year

THE **RETROCESSION** MARKET AND THE SPIRAL

Although pronounced dead, the spiral and retrocessionary (reinsurance of reinsurance) markets are just alive - prices have increased tenfold. The key phrase is LMX; which is Excess of Loss placed on London Market Excess of Loss business. The principles of writing this business are simple.

I have a series of risks for which I received a premium of (say) £ 100. If I can place these risks with someone for (say) £98, I will have a guaranteed profit of £2! Also, in direct reinsurance, the higher up the programme the cheaper per unit the risk. It was thought that the same applied to Retrocessionary market, this led to considerable mispricing. As long as I could sell my book of business cheaper than I bought it, the basic reinsurance product itself was being priced too cheaply.

Take two reinsurers. Let us assume both have £10 million of inwards reinsurance exposure. Insurer A reinsures its whole portfolio with B and vice versa. Both now, individually, have £20 million of gross exposure of which £ 10 million is reinsured. (The first program is £ 10 million xs £0 million). They then place this second level (£ 10 million xs £10 million) with each other.

Their individual total exposure is £30 million of which £20 million is reinsured. We continue this for, say 10 times, giving us a comfortable £ 110 million exposure of which £ 100 million is reinsured. Of course, the higher levels of reinsurance are more remote for the loss and accordingly are cheaper! The Broker takes 10% of each placing as brokerage.

A loss of £ 10 million occurs to each insurer. Insurer A passes £ 10 million to Insurer B. A has £ 10 million loss which he recovers. B has £ 20 million loss, which he recovers from A; A has £30 million loss, £ 10 million of which is recovered, so he asks B for £20 million and so on. An initial loss of £10 million for each company produces payments for A of £110 million - and a net loss of £ 10 million.

This example is simplified. In practice there were hundreds of companies and Lloyds syndicates playing the game.

The rules of the game were quite simple - understand the total aggregate exposure and make sure you had more reinsurance than your rival. For example, if A had written one more reinsurance its exposure would be £ 110 million with reinsurance of £ 110 million, and B would be £ 120 million with reinsurance of £ 100 million. In the case of no loss B would be the winner - the premium from A would be its profit. In the event of a claim, however A would be the winner. Several syndicates at Lloyds were the B players - reporting profit to names. Since the top layer was mispriced, when a catastrophe occurred the results for company B would be bankruptcy.

How would a prudent reinsurer have behaved in the Spiral market? I will assume the aggregate exposure is £100 million (i.e. the total of all reinsurance written). It would be inefficient/impossible to reinsure the total exposure. A prudent reinsurer should have purchased £60 million excess of £5 million. This would have cost a considerable amount of the incoming premium.

This gives a perceived retention of £ 5 million and a "hidden" retention of £35 million (£ 100-£60-£5). In practice what was happening was that either insurers were not aware of their aggregate exposure or were being imprudent. They were reinsuring f25 million excess of f2 million. The hidden retention was f73 million (i.e. an unreinsured exposure of f73 million). A series of losses would devastate the market - which turned out to be the case. A lot of the criticisms by Lloyds have been the lack of understanding of aggregate.

The turning events for the market were the following losses:-

(1) <u>Piper Alpha</u>

Press reports regarding major professional reinsurers indicate how they got their reserves and recoveries wrong.

(2) <u>1999 Losses</u>

Hugo, Exxon Valdez, Phillips Petroleum and Arco Platform. Their losses are not yet fully developed.

1989 was also hit by smaller losses such the San Francisco Earthquake (17.10.89) and Newcastle (Australian) Earthquake (28.12.89).

(3) <u>The European Storms of 1990</u>

For further details of this topic see either the "C.A.S. Loss Reserving Talk" [3] or read Cathy Gunn's excellent book "Nightmare on Lime Street" [11].

RATING

There are three basic methods of assessing ratios for the risks:-

(1) Some form of simulation relating storms to a portfolio of risks. The risks are usually categorised by type (Household, Property, Shops, Offices etc.) by value and by postal code. Old storms or hypothetical new storms are then simulated on the portfolio.

Examples of this type of estimation may be found in the GISG paper "Storm Rating in the Nineties" (8). This type of method is often revealing about the area by area exposure, but the estimation of losses is extremely subjective. A windstorm loss may vary between 0.5% to 2% of Sum Insured and the uncertainty is enormous. Key factors are often excluded from the databases, for example, construction type. On ordinary household policies, no account is taken of the square footage and number of stories. We rate policies by Sum Insured (a linear type rating), yet Danish experience indicates storm exposure increases with increased square footage (square footage is a rating factor in Danish household policies).

The information given by such simulations should not, however, be discounted.

(2) **Burning Cost Rating**

Under Burning Cost Rating actual losses incurred are used to determine the cost. The keys to assessing these rates are:-

(a) **Loss Frequency**

A burning cost method is only suitable if there are a sufficient number of losses to obtain a suitable loss frequency.

(b) **Indexation**

Losses should be revalued into current terms. This involves both inflation and the increase in number of policies. A suitable index could be premium income adjusted for any rate changes.

(c) **Changes in Policy Conditions**

(d) Changes in Retentions

(3) **Exposure Rating**

Simulation is one form of Exposure Rating. Normally, exposure rating is intended to provide a comparison with the burning cost rate - particularly if changes to the portfolio have taken place.

Exposure rating is used to rate areas and covers with little or no loss experience. There are three stages:-

(1) Establish a Catastrophe Estimated Maximum Loss (E.M.L.).

(2) Establish a Catastrophe Premium - this is normally From The Ground Up - (F.G.U.).

(3) Establish a suitable Loss Distribution Curve. In the example I will use a Pareto type distribution.

As an alternative to this type of approach, formula could be used. In my ASTIN paper, I use formulae from Financial Mathematics and Option Pricing (Black-Scholes) to derive consistent price rating for certain classes of loss. This involves the estimation of three parameters, the return period if an event being one of them and implied volatility is another. A similar approach is made by using Pareto formulae. These methods involve difficult mathematics and are beyond the scope of this paper.

Set out below is an example of a calculation for a UK direct writer requiring a quote of £25 million excess of £50 million. Reinstatements and brokerage are ignored.

The estimated Gross Premium income for 1992 is £ 230 million and the data is as follows:-

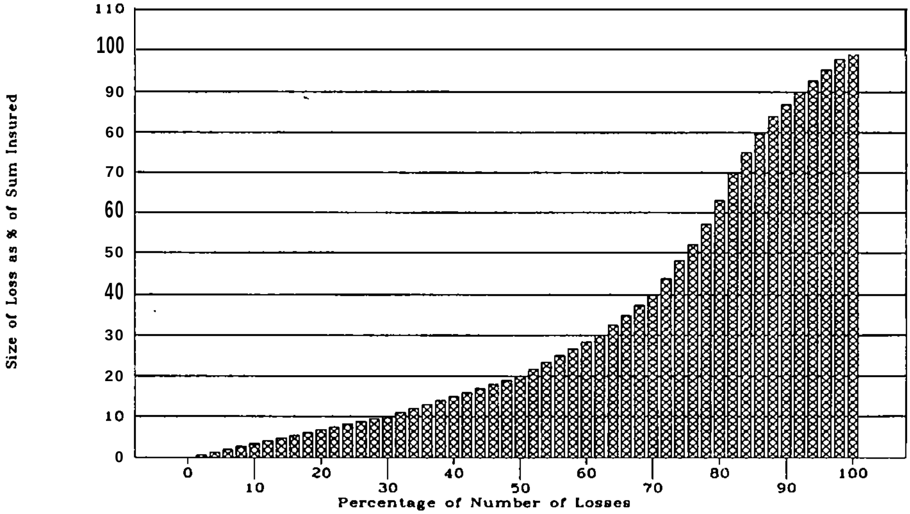
	Premium	Losses F.G.U.		Indexed
1991	220,000,000	Nil		Nil
1990	200,000,000	95,000,000 22,000,000	(90A) (90G)	109,250,000 25,300,000
1989	180,000,000	Nil		Nil
1988	170,000,000	Nil		Nil
1987	160,000,000	65,000,000	(87J)	96,451,612
1986	155,000,000	Nil		Nil
1985	150,000,000	Nil		Nil
1984	145,000,000	6,500,000		10,310,344
1983	120,000,000	Nil		Nil
1982	100,000,000	Nil		Nil

We first calculate the Maximum Possible loss. This is taken as twice the 90A Loss Indexed i.e. £220 million (2 x 109.250). This is the current market practice.

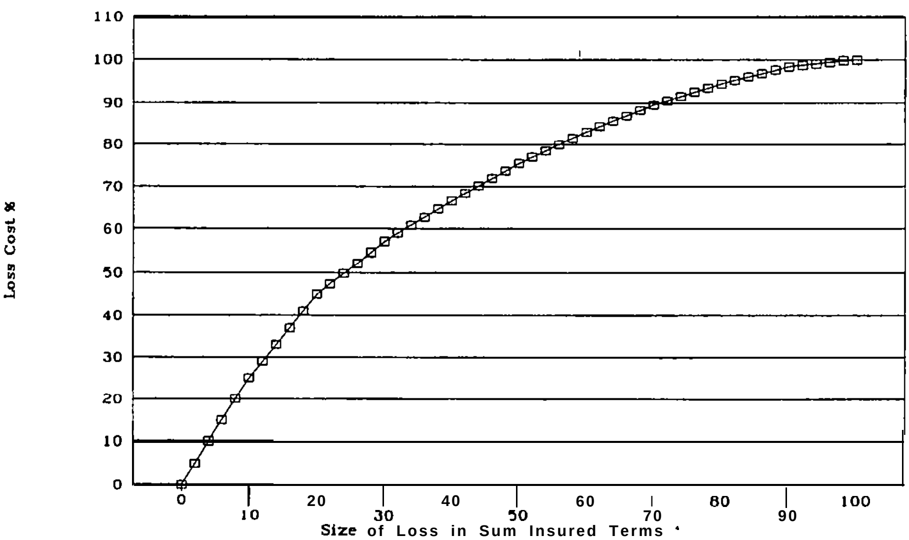
Next, we calculate a loss for a specific layer. I use 90% xs of 10% of the largest loss (109,250,000) say £ 90 million xs £ 10 million.

The losses are larger and in this treaty today would be £90 million + £15.3 million + £86.451 million + £0.310 million = £ 192.151 million. (This is similar to the burning cost). The average cost is £19.215 million per annum.

Loss Distribution Pattern



Loss Distribution Pattern



This cost, from the Pareto curve, represents about 50% of the total cost. This is taken from the size of loss curve looking at the size of loss of 10 (giving 20%) and 50 giving 70%). Therefore, the total catastrophe programme should cost £38.42 million.

The £50 million point represents about 22.5% of the E.M.L. of £220,000,000 and £75 million (i.e. £25 million xs of £50 million) is about 34% of E.M.L. Using the lower graph 22.5% is about 45% of loss cost, 34% is 60% of loss cost and so the premium is 15% of the total cost of £38.42 million or £5.73 million (before expense, commission and safety loading).

The basic problem is that the market is not applying this type of rating, and reinsurance costs are substantially higher than those derived by the above calculations or any pure exposure basis. They are trying to recover the rest of the early losses to re-establish capital.

The Capacity of Reinsurance has been devastated. Lloyds names have ceased to be members of syndicates and Reinsurers have ceased to trade. Accordingly, premium rates are substantially above the theoretical calculated rate, due to demand exceeding supply and the absence of any real retrocession or spiral market.

Let us consider the need. I will relate everything to 90A as this is the market norm (remember PML is 2x Indexed 90A loss).

I will consider nine companies, A-I. These are all UK composite insurers. In the first graph 90A losses are expressed as a proportion of Premium Income. Thus for Company A, 90A loss F.G.U. represents 40% of its total property premium income.

The next graph represents the deductible as a proportion of premium. The average deductible is about 10% of property premium, although there is wide fluctuation.

Finally, I give the cover purchased From The Ground Up. Thus Company A purchased reinsurance between about 12.5% and 87.5% of its premium income, 90A accounted for about 40% of its premium income, so in an event which is twice as damaging it should still have protection. Company B, however is only purchasing up to its 90A cover and it is, therefore, more exposed to possibly higher losses. The rate on Line, as a Proportion of 90A, is given for 1992 reinsurance costs.

In the example I calculate a premium for £25 million xs £50 million at £5.73 million or about 23% rate on line.

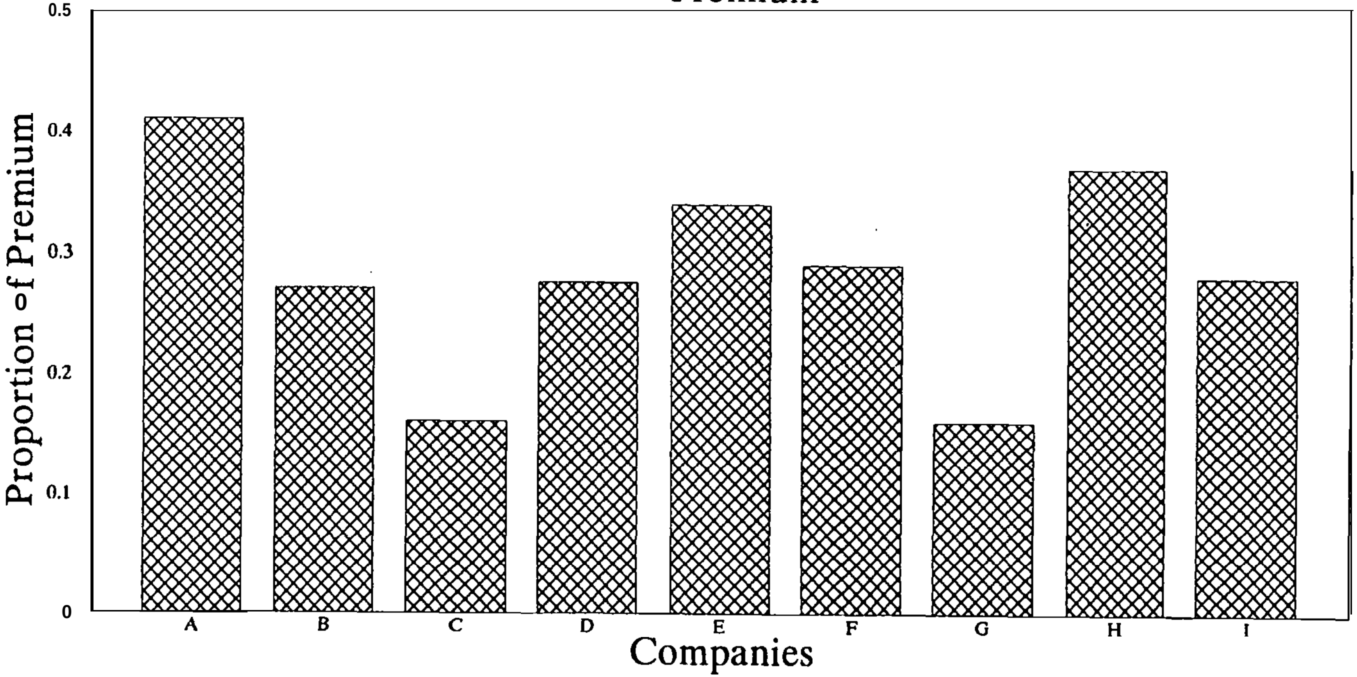
Based on this, we have exposure from 45.5% (50/109.25) to 68.6% (75/109.25). This has an average of 57.2. From the graph for 1992, the Market would be charging a rate on Line of slightly more than 30% or £7.5 million.

There are clearly many considerations that need to be taken into account:-

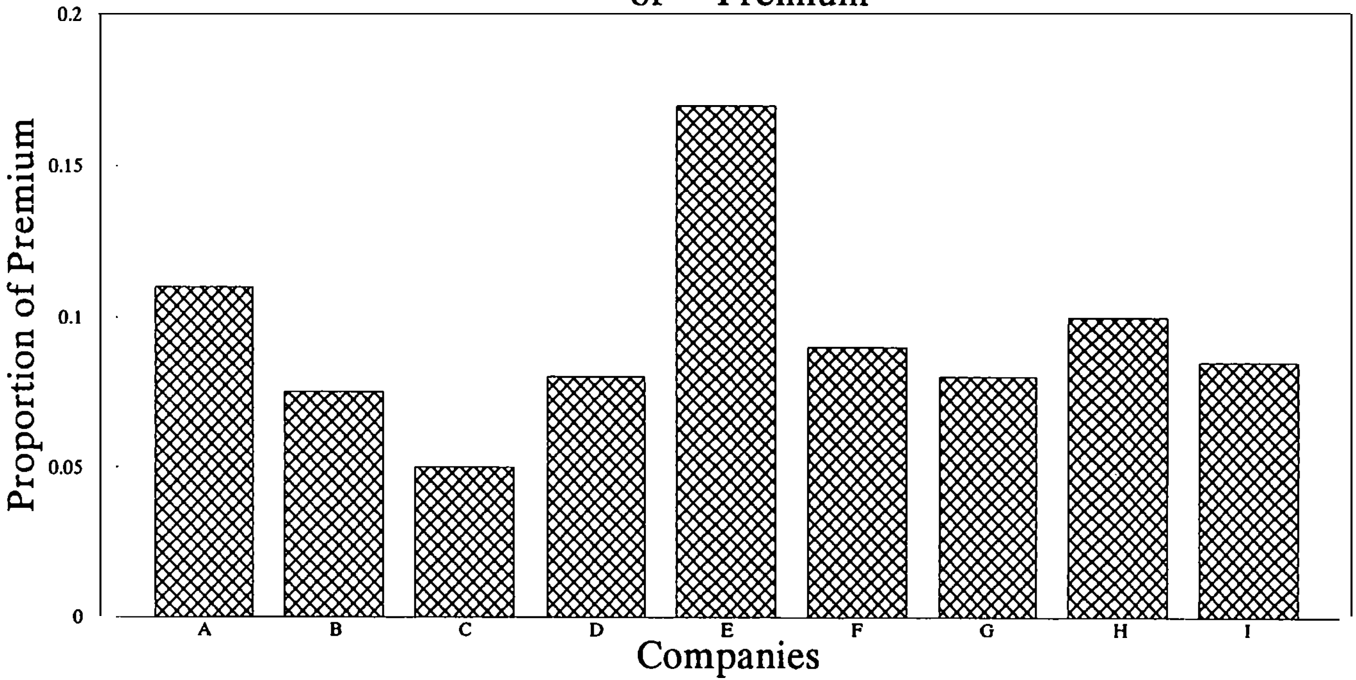
(a) If the actual price is loaded by 25% to 40% over expected values should the cover be bought? The answer to this depends on the shareholders resources and/or future employment prospects for the Managers. Should an event occur what would be the impact on the P & L account.

(b) What should be done about the retention? If only 75% of the business is placed, how should the reinsurance of the 25% be planned for. Losses need to be financed. Should the "loaded" or "real" premium be transferred to the Internal

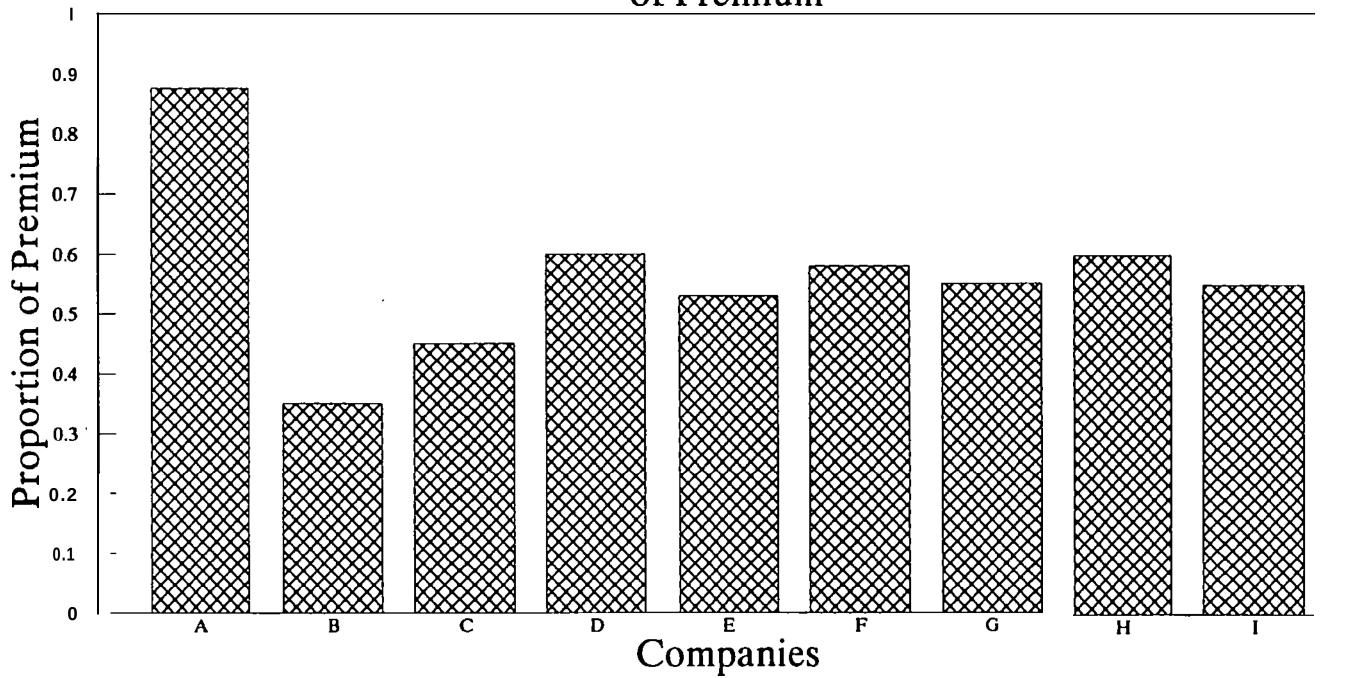
90A as a Proportion of Premium



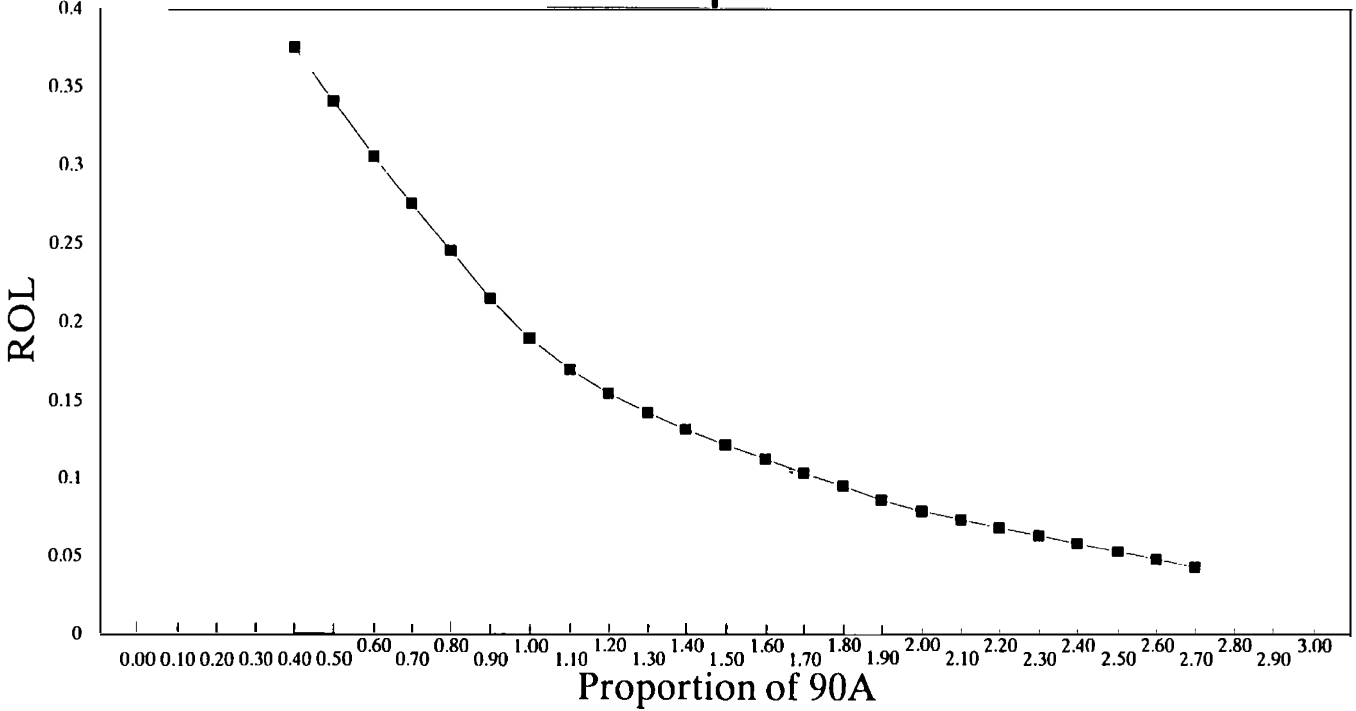
Deductible as Proportion of Premium



Cover FGU as Proportion of Premium



1992 Cost Comparison With 90A



Funding mechanism, if that route is chosen. The loading represents brokerage (10%) and safety margins (15%).

(c) What about losses below the retention? In previous years retentions were set as low as 2%-3% of premium income. Freeze and other losses were reinsured as part of the overall programme. How should they be financed or planned?

In simulations made for the ASTIN paper it is not unusual to find the catastrophe attrition losses (i.e. those below the deductible) to be, on average, a factor of between 100% and 150% of the deductible. The reasons for this are as follows:-

(i) We have a considerable number of small losses (e.g. floods, freeze etc.) below the catastrophe. The recent 1993 January storms and floods have cost many insurers £10 million or more.

(ii) When the big catastrophe hits, a prior charge of the deductible is made before any reinsurance can be recovered.

These issues need careful planning.

Finally, pre 1990, the cost of reinsurance for the UK property account was small compared with the premium income and deductibles were considerably lower. Premiums were based on gross experience, and profit made on reinsurance. Nowadays, the cost of catastrophe claims via catastrophe premium, deductible, retained percentage of programme and so on is considerably higher.

The basis for premium rates should be the larger of:-

(i) Gross premium.

(ii) Net premium plus catastrophe costs.

I believe the rating basis has switched i.e. (ii) is larger than (i); yet the insurance market has not reacted. I also believe that the UK property account could be suffering because the market has not addressed this problem. The reinsurance or catastrophe costs are not yet fully costed in the premium basis.

RESERVING FOR CATASTROPHES

It is normal to review a book of Excess of Loss Reinsurance Business in two parts:-

1. The attrition losses arising from working covers.

2. The individual (main) catastrophes separately.

For the catastrophe, the losses can be reviewed either in aggregate or the cover to which they relate (Reinsurance, Retrocession business, Spiral business, Specific, International, Whole Account).

The purpose of reserving is two-fold:-

1. To ensure adequate reserves are placed, and the account is not under or over reserved.

2. To provide management information at specific points of time.

   This management information may be used to purchase additional reinsurance cover.

The method I use is curve fitting a three parameter curve to the paid and incurred claims:-

$$Y = A\ (1\text{-}EXP\ (-t/B)^{C})$$

This is a monatonic increasing curve.

The parameters are:-

A	=	Anticipated ultimate loss.
B	=	Parameter for slope of the curve.
C	=	Parameter for the shape of the curve.
t	=	Period (in days).

For pre 1992 catastrophes B was in general about 600 and C = 2. For modern catastrophes (Typhoon 19 and Hurricane Andrew) B is much lower.

Reserving is not just curve fitting. Several other factors need to be taken into account

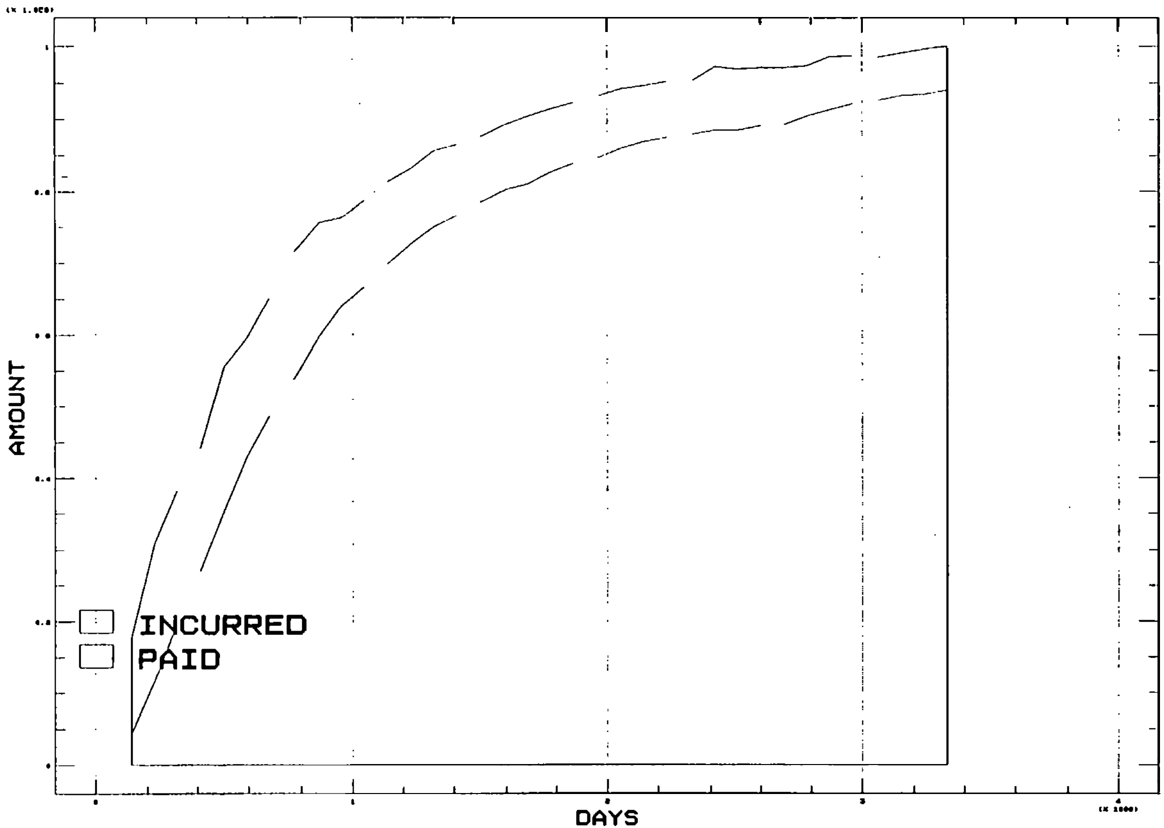
(i) Estimation is based on Paid Claims and Incurred (i.e. Paid plus Reported Outstanding Claims).

In most catastrophes there is a gap between these paid and incurred. The first three graphs attached to this section show the gaps for Hurricanes ALICIA, GLORIA and GILBERT. The amounts have been normalised so that today's incurred claims are £100,000,000.

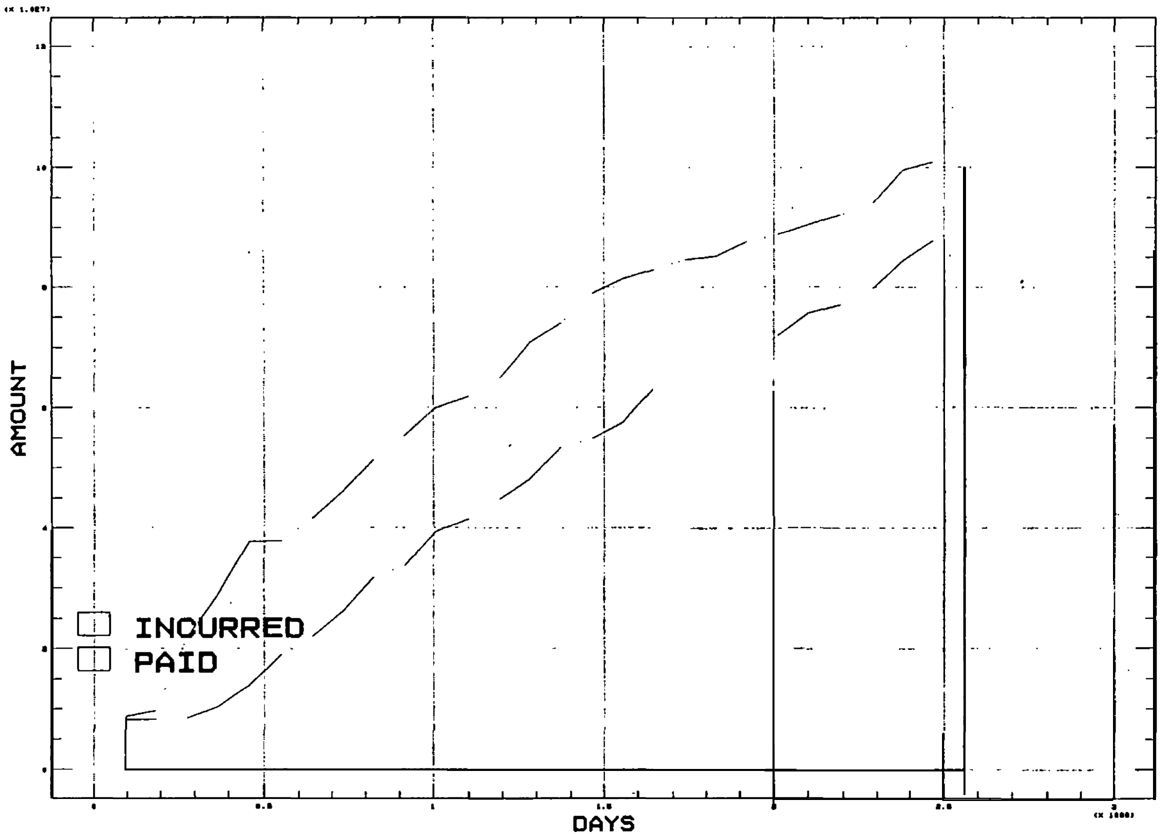
The most developed is ALICIA when a gap of about £10,000,000 has been apparent for a number of years. The possible explanation is that there are a residual amount of outstanding losses reported by Brokers, which have not been released as the catastrophe claims are made. These are possibly redundant.

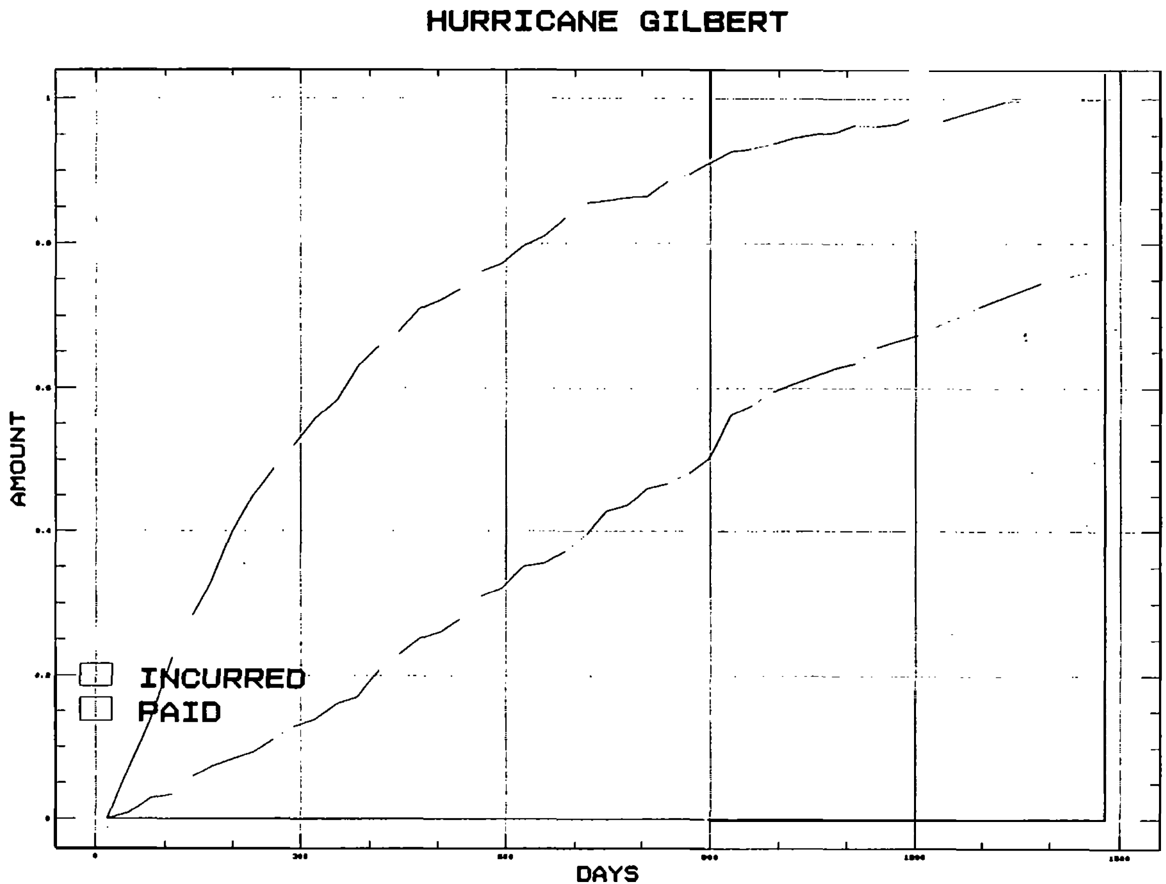
When reserving, one needs to be aware of this 5%-10% gap. The incurred position should unwind as these reserves are released. ALICIA occurred in 1983; GLORIA in 1985 and GILBERT in 1987. Gilbert is primarily a Jamaican loss and reporting standards for Caribbean countries may reflect the wider gap. All the losses are expressed in one currency.

HURRICANE ALICIA



HURRICANE GLORIA




HURRICANE GILBERT
AMOUNT
DAYS
INCURRED
PAID

(ii) Curve fitting is statistical by nature, and one should be aware of standard errors. The best fit curve may give an Ultimate below the current paid or incurred. This feature should be taken into account when undertaking the reserves Whereas incurred unwinds, paid claims increase.

(iii) The use of a single curve may not be appropriate. Certain loss payments come in two distinct surges. The first is normally the physical damage (Loss of Rig - Piper Alpha; Loss of Aircraft - Japanese 747; Earthquake - San Francisco - Plant Destruction - Phillips).

This is followed by liability or business interruption losses:-

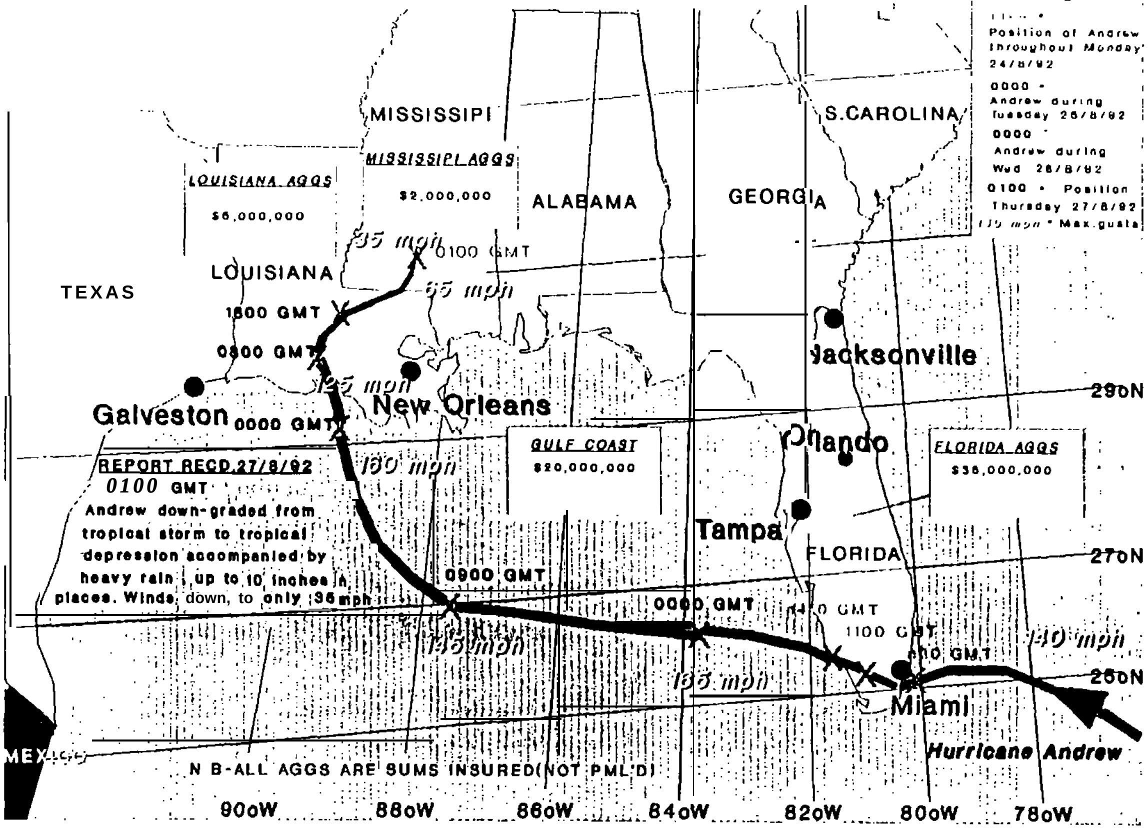
Employers Liability	-	Piper Alpha
Passengers Liability	-	Japanese 747
Architects Liability		San Francisco Earthquake
Business Interruption	-	Phillips

It may be appropriate to superimpose a second (later) curve for this final surge. Examples are clearer in the development curves at the end of this section.

(iii) Underwriters judgement and exposures should be taken into account. although based on crude estimates, the exposure multiplied by a probable maximum loss (80% say) may be the only guidelines available.

Attached is a typical exposure for Hurricane Andrew. (Amounts are artificial).

Hurricane Andrew/Aggregates in exposed states



(iv) **The difference between Marine and Non-Marine Losses**

In general a Non-Marine loss such as Hurricane Hugo will rise rather rapidly in the Non-Marine account. As the Non-Marine Specific reinsurance is absorbed the Whole Account protections (with associated spiral) come into play. Non-Marine losses are normally settled first and the CAT developments reach a stable position fairly early. Marine Excess of Loss and Whole Account claims then take up.

My estimation for parameter B for Hugo is 232 days Non-Marine and 744 days Marine.

Marine Gross Losses also tend to be substantially higher than Non-Marine Gross Losses. This is due to the more effective spiral (no 10% retention). A 30 times spiral (i.e. gross to net) is not unusual.

(v) **The Special Impact of 1989**

In 1989 there were a number of losses which have had a substantial impact on the CATXL market - particularly the Marine market. There are only three large losses allowed for on most treaties - yet we have four major losses - Hurricane Hugo, Exxon Valdez, Phillips Petroleum (an explosion) and Arco Platform (a drilling rig). For a large number of reinsurers one of these three is redundant - and the smallest is Arco Platform.

To put these figures into perspective the Marine Market losses: Hurricane Hugo (total $4 billion of which about $2.4 million is non Marine and the Marine losses are likely to be $1.6 billion) $1 billion Exxon Valdez, $1 billion Phillips and $0.4 billion Arco Platform. A consequence of this is that in the book of incurred claims there is likely to be some double counting (i.e. the sum of all the notified losses per cedant is likely to exceed the aggregate exposure). The paid losses are controlled by physical checks on amounts recovered under treaties, but aggregate exposures are not. As a result the smallest losses are likely to have higher than average redundancy as the incurred position unwinds.

Secondly, Phillips Petroleum is a very confusing loss in that it is one of the few losses which the model fails to fit. The reason is that it is , in reality, three different types of loss which behave differently - namely a material damage loss, a business interruption loss and a US liability loss. It is, in practice slower to develop than its peer losses.

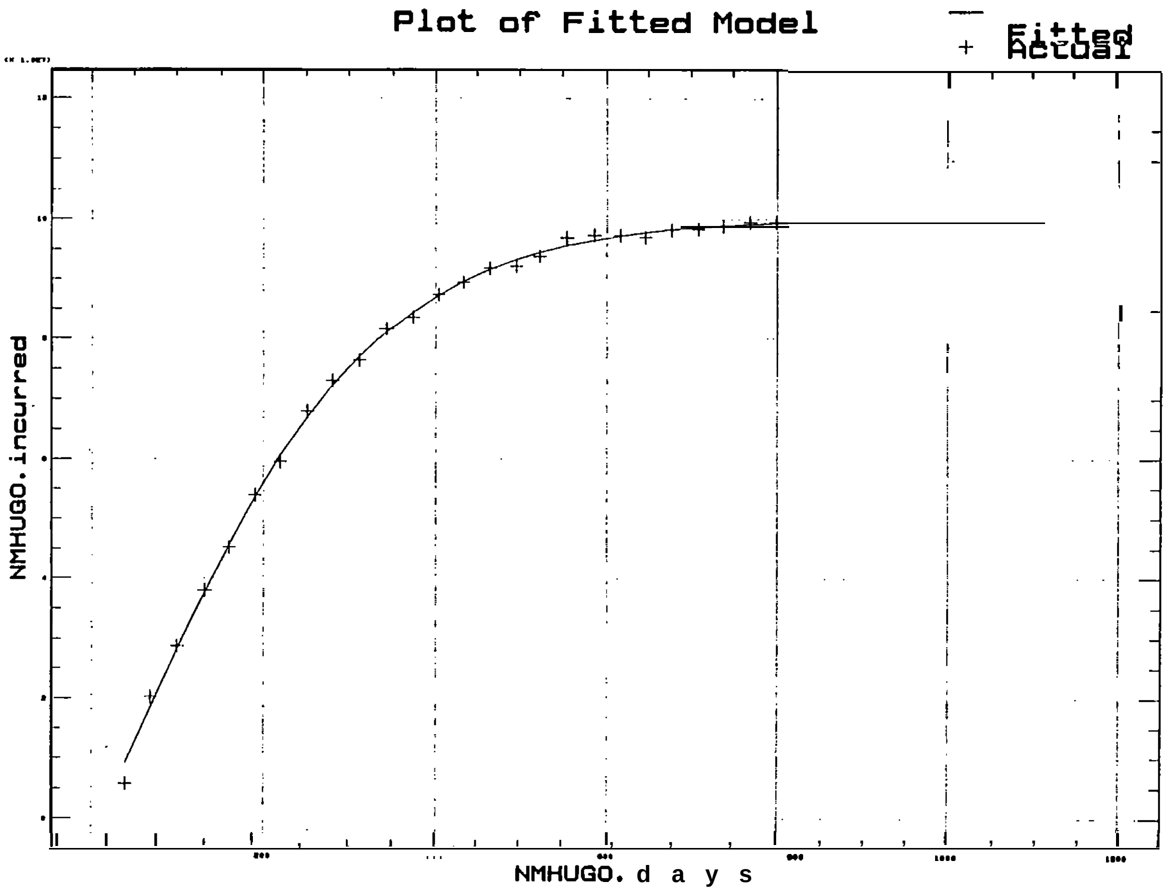
On the attached sheets I calculate the factors for these losses. I have normalised the losses so that today's incurred losses are £100 million.

Note that Non-Marine Hugo has stopped and Marine Hugo has nearly completed its development, and Arco and Exxon are near complete development. Considerable uncertainty surrounds Phillips so an alternative method may be required.

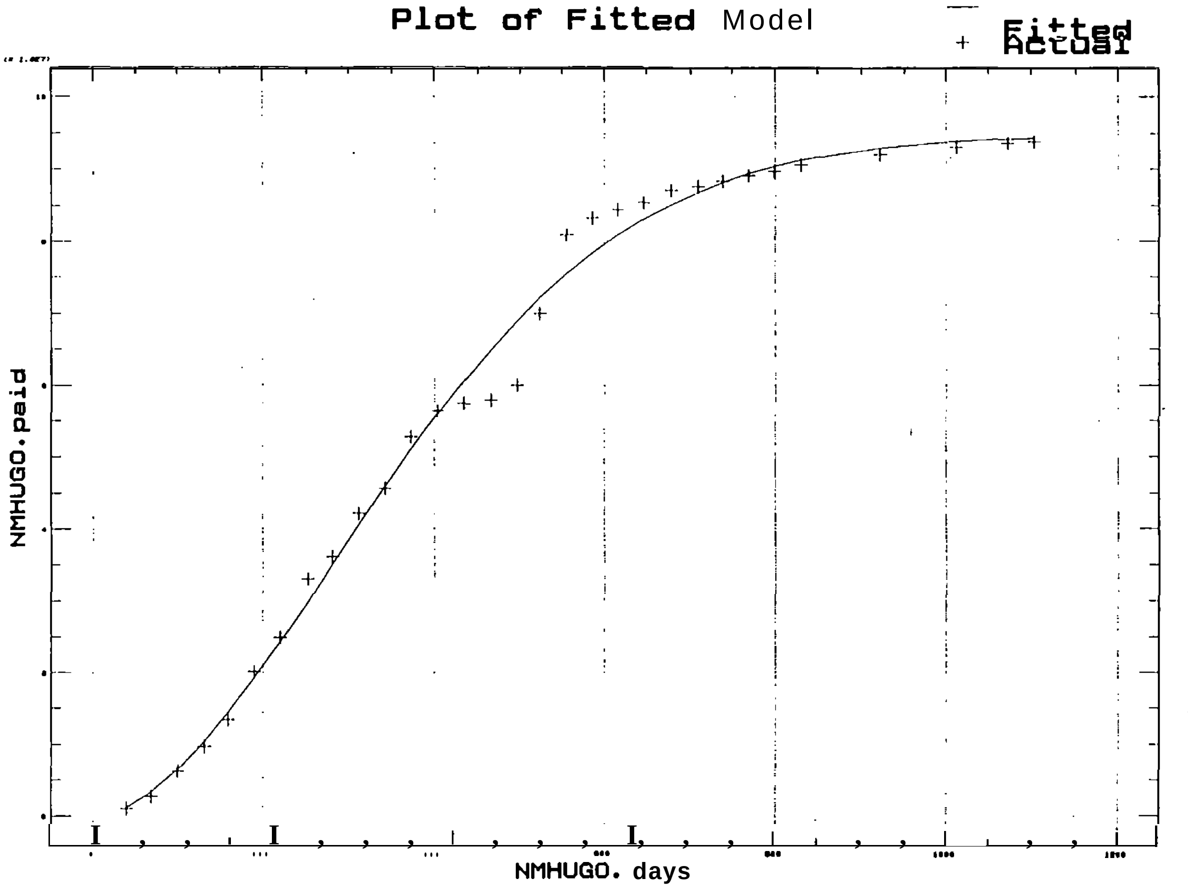
The figure in brackets is the standard error.

CATS OF 1989

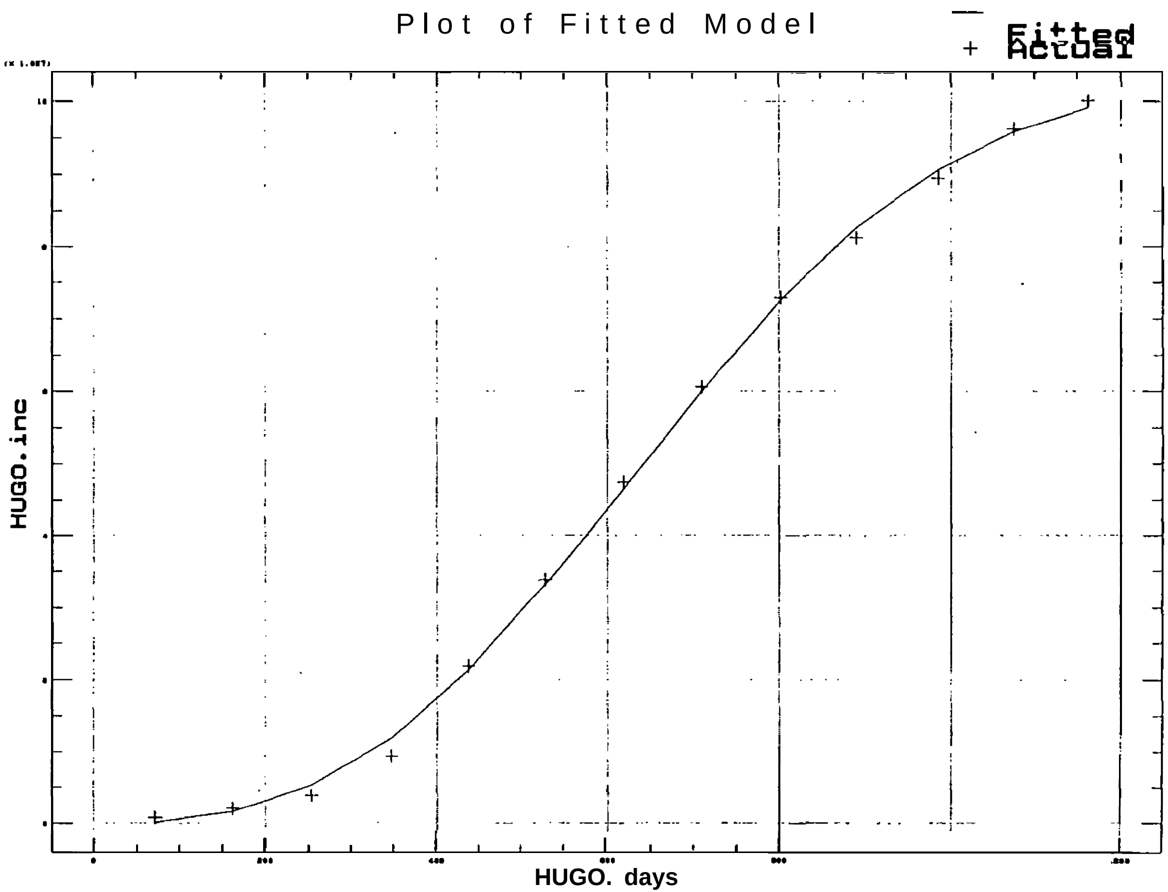
CATASTROPHE			BASIS	A		B		C	
HUGO	NON-MARINE	(NMHUGO)	INCURRED	100.050	(0.323)	232	(2.03)	1	(0.19)
HUGO	NON-MARINE	(NMHUGO)	PAID	94.763	(1.532)	429	(10.58)	1.8	(0.08)
HUGO	MARINE	(HUGO)	INCURRED	102.508	(1.721)	744	(11.08)	3	(0.08)
HUGO	MARINE	(HUGO)	PAID	90.833	(1.055)	786	(6.81)	3.4	(0.08)
ARCO	MARINE	(ARCO)	INCURRED	105.419	(3.259)	960	(21.44)	3.0	(0.14)
ARCO	MARINE	(ARCO)	PAID	80.514	(1.887)	933	(15.5)	3.4	(0.15)
EXXON	MARINE	(EXXON)	INCURRED	108.97	(5.628)	897	(43.19)	2.0	(0.15)
EXXON	MARINE	(EXXON)	PAID	83.93	(7.284)	988	(62.21)	2.9	(0.30)
PHILLIPS	MARINE	(PHIL)	INCURRED	211.421	(9.678)	1,341	(404.80)	2.0	(0.2)
PHILLIPS	MARINE	(PHIL)	PAID	95.57	(3.610)	995	(22.73)	3.0	(0.7)

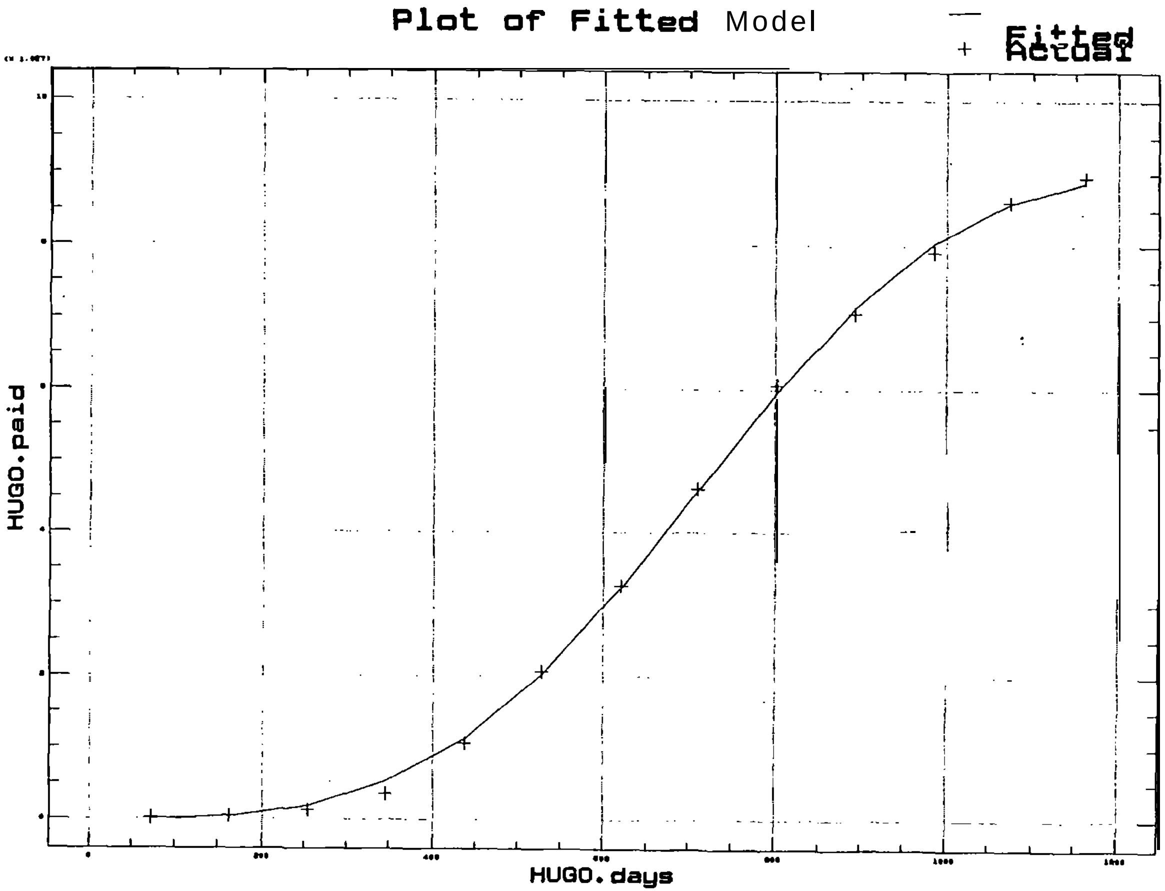


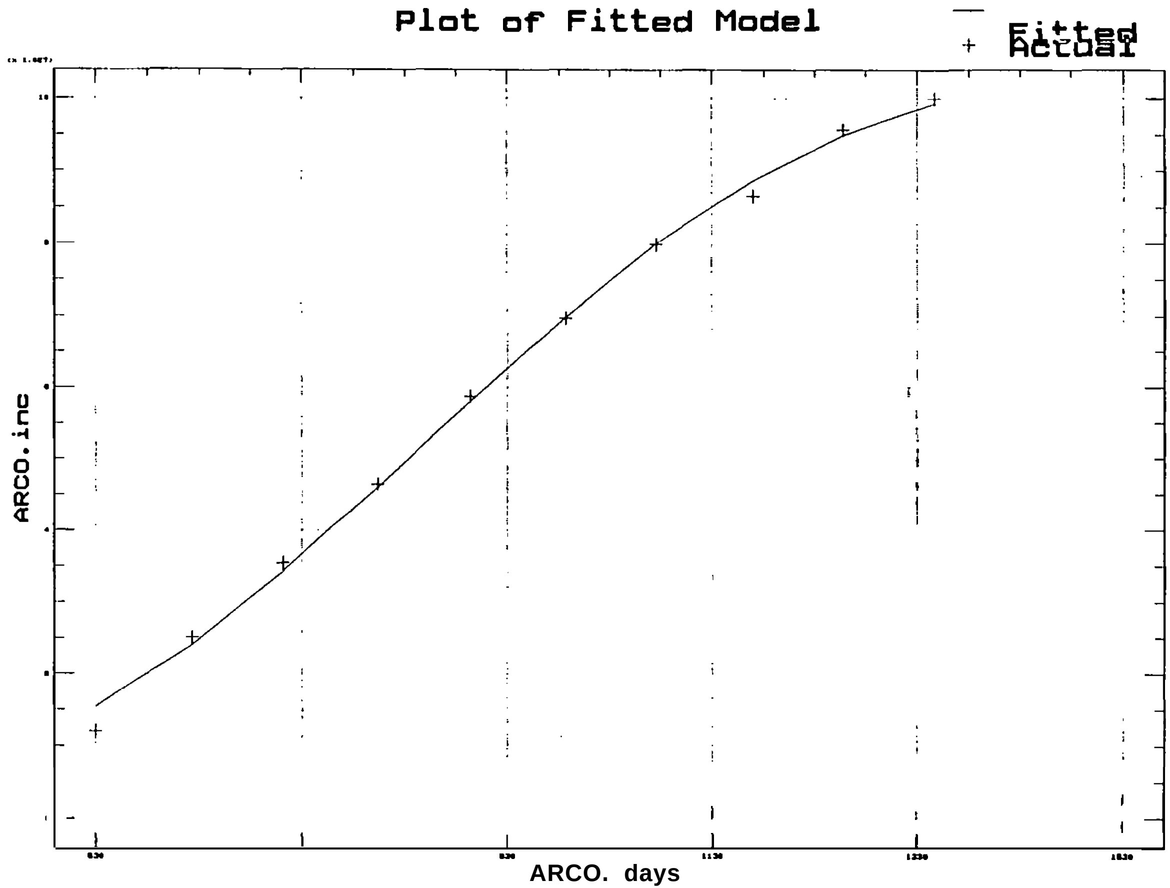
Plot of Fitted Model
Fitted
Actual
NMHUGO.incurred
NMHUGO. d a y s



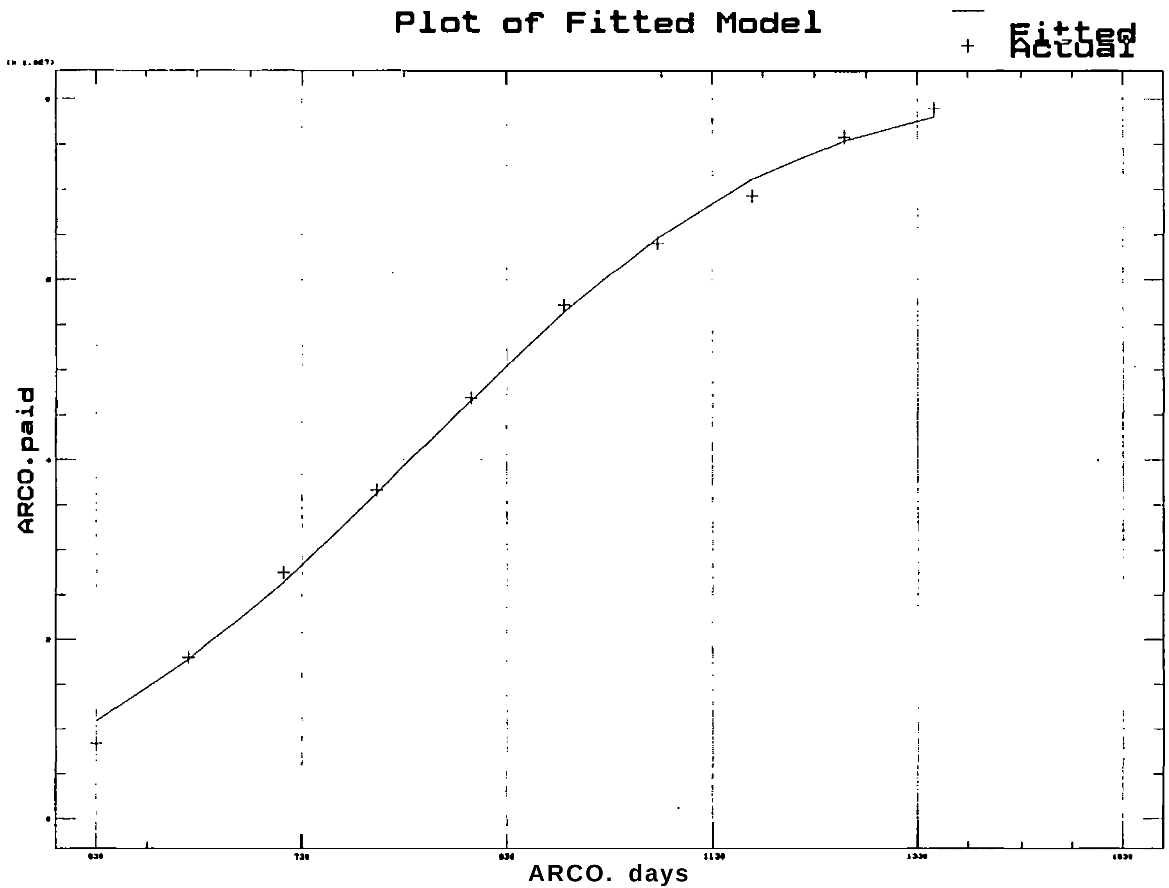
Plot of Fitted Model
Fitted
Actual
NMHUGO.paid
NMHUGO. days


Plot of Fitted Model
Fitted
Actual
HUGO.inc
HUGO. days

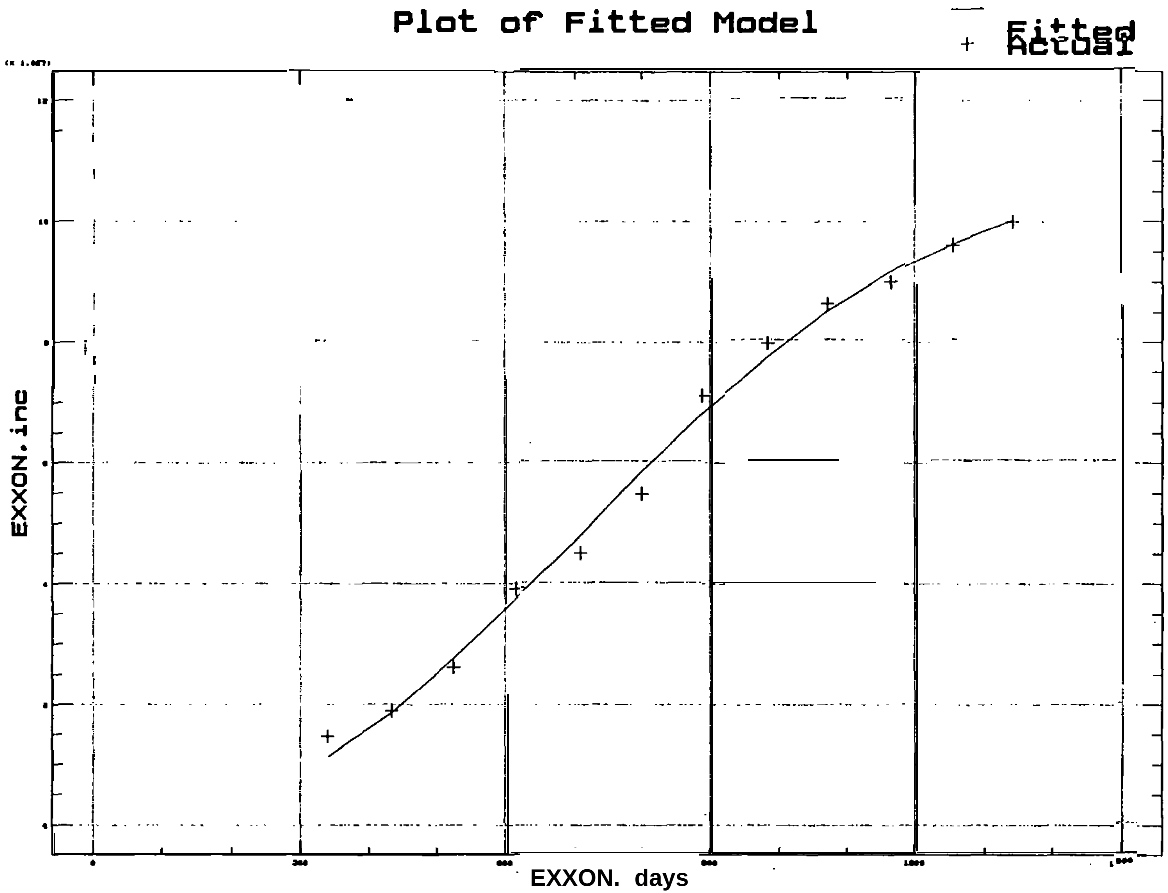

Plot of Fitted Model
Fitted
Actual
HUGO.paid
HUGO.days

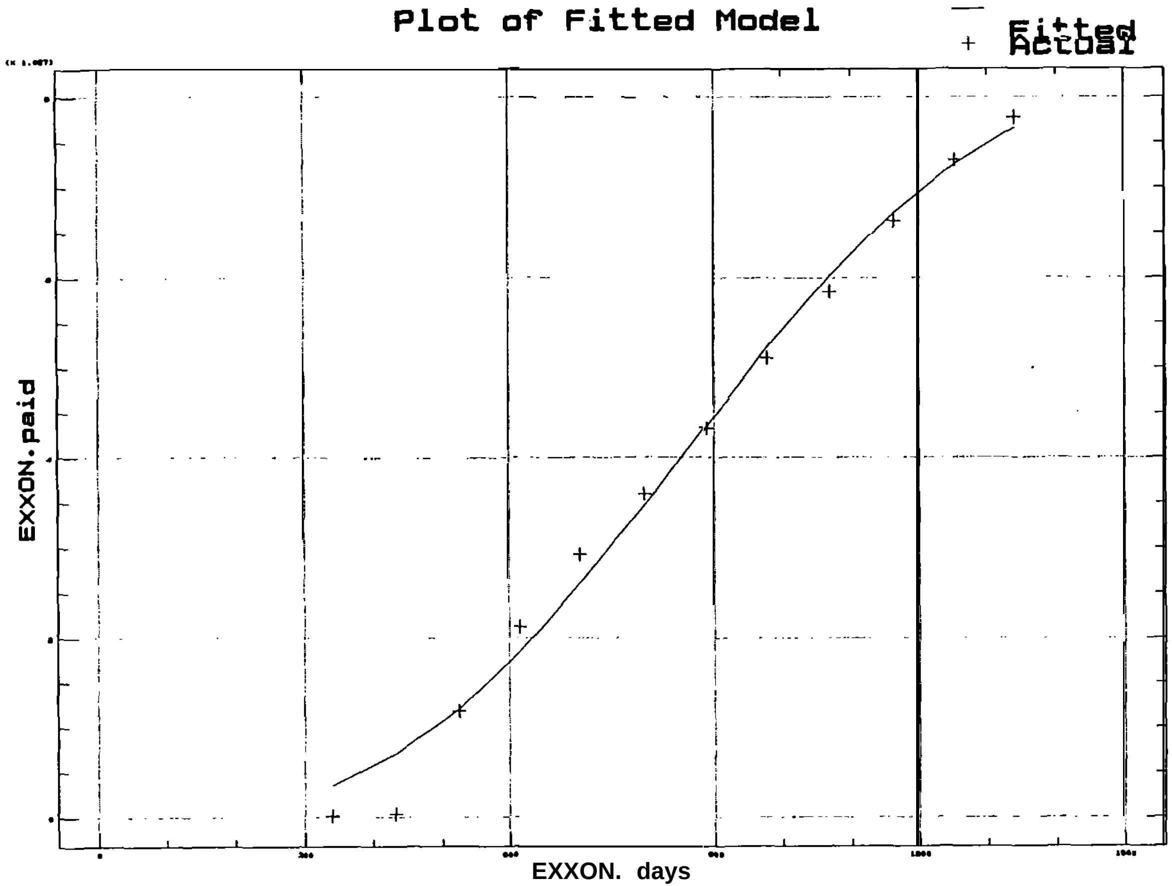


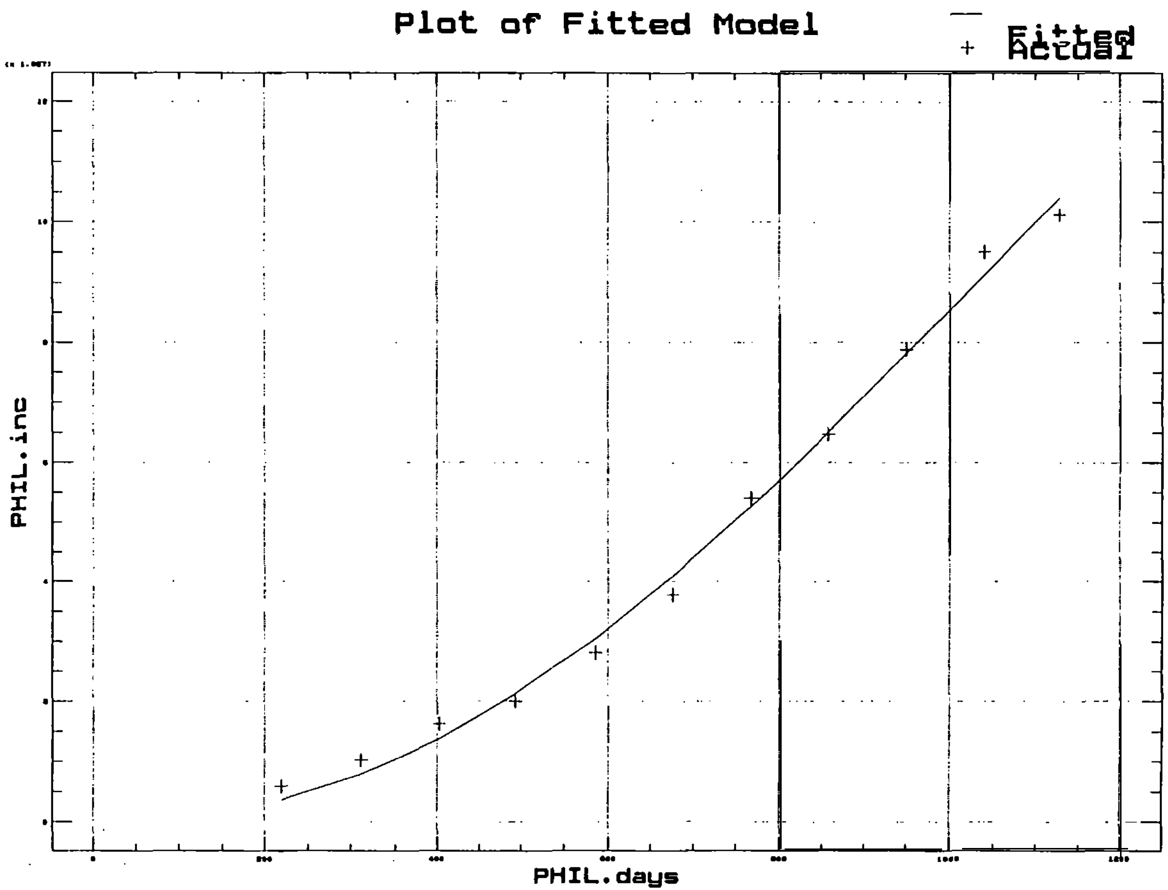
Plot of Fitted Model
Fitted
Actual
ARCO.inc
ARCO. days

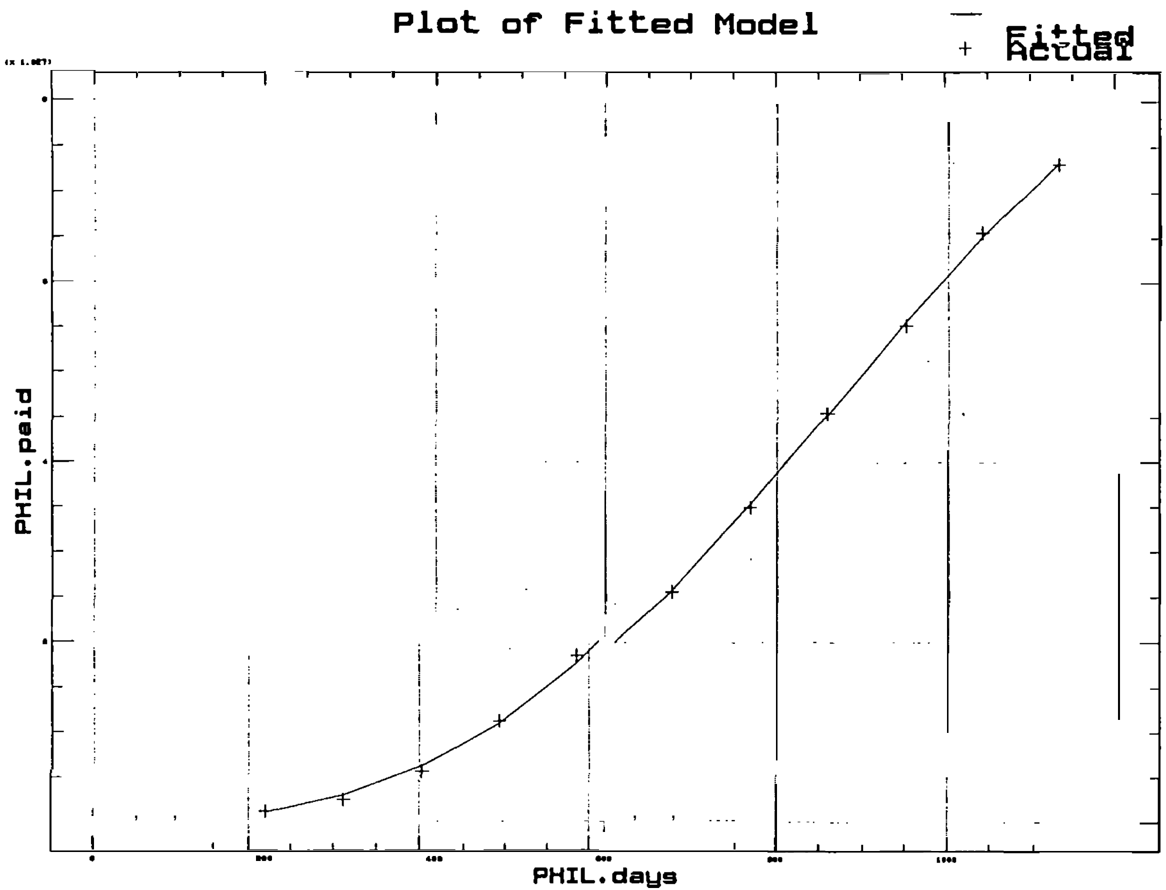

Plot of Fitted Model
Fitted
Actual
ARCO.paid
ARCO. days

Plot of Fitted Model




Plot of Fitted Model
Fitted
Actual
EXXON.paid
EXXON. days


Plot of Fitted Model
Fitted
Actual
PHIL.inc
PHIL.days


Plot of Fitted Model
Fitted
Actual
PHIL.paid
PHIL.days

Finally, I set out some further examples of Windstorm Losses. Note how different the Development of Typhoon 19 (Merielle) is when compared with the other losses.

Hurricane Andrew also has the same features. The amounts in the brackets are standard errors to the parameter estimation.

Several points need to be noted:-

(i) In Lloyds and many London Market Companies Reserves are only reviewed annually. This leads to a lack of on-going data. Furthermore, accounts are not finalised until three years' losses have occurred. The lower the number of data points, the less information is available. This leads to a large error potential in the parameter estimations. Frequent data points are needed for better estimations.

(ii) The estimation process is only the first stage of establishing the reserves. The estimate may exceed the aggregate exposure and special features may need to be brought into consideration.

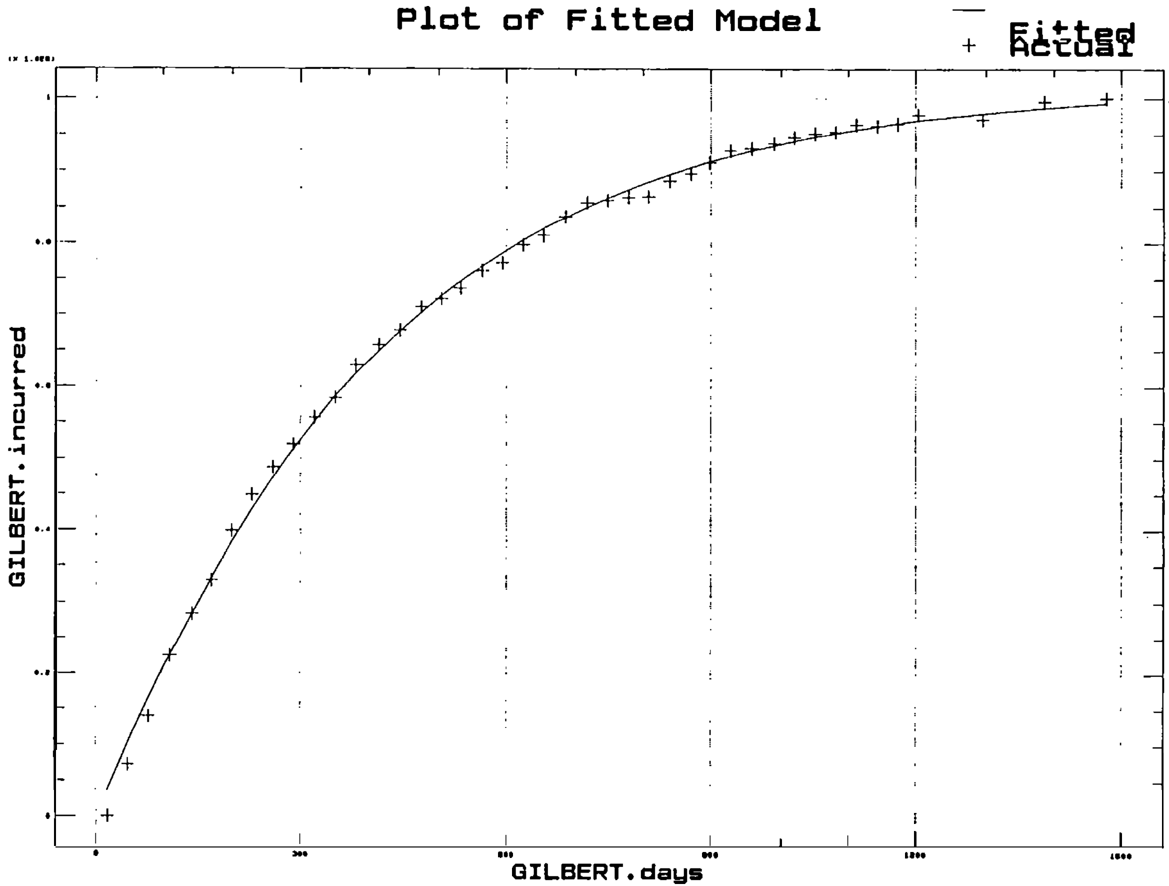
(iii) The reserves are gross reserves. Net reserves are calculated by super-imposing the reinsurance programme on anticipated ultimate loss to obtain the net reserves.

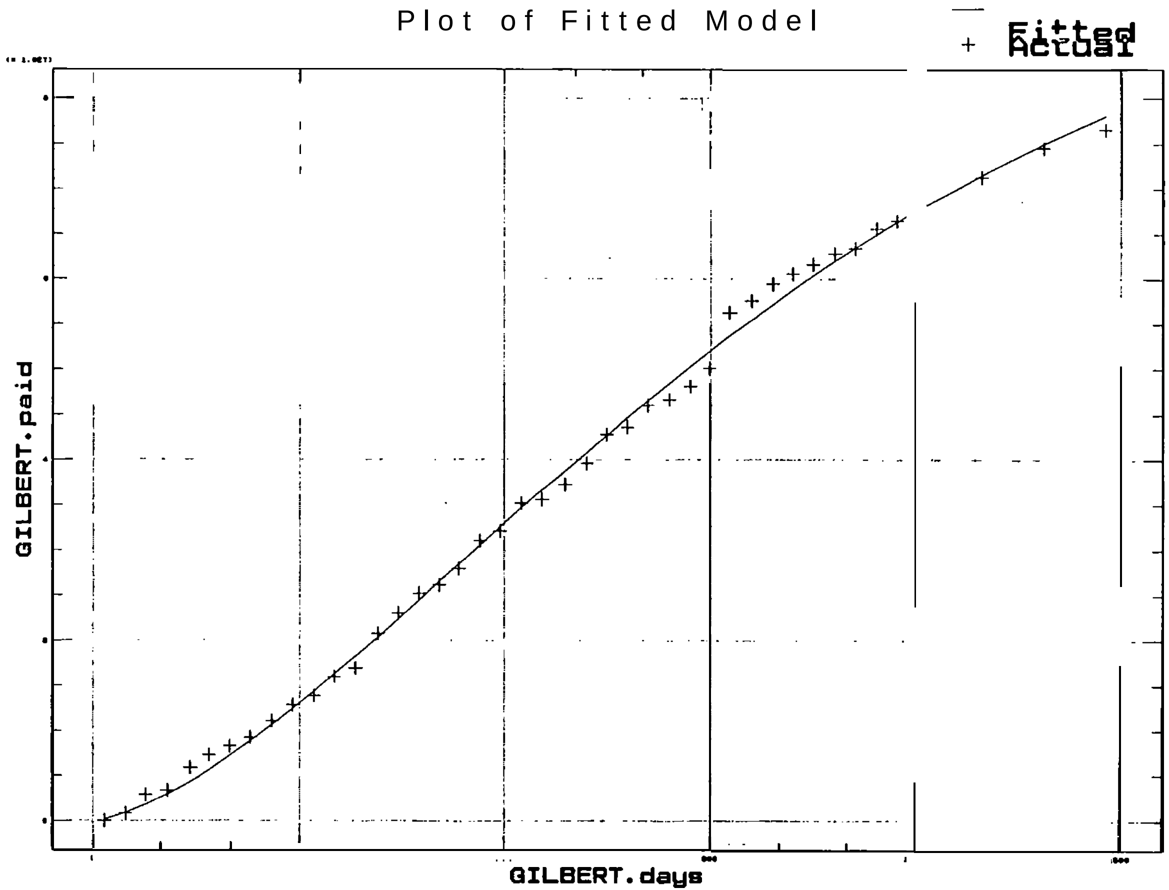
(iv) There is no need to fit the curve over the whole period. Recent developments can also be fitted to highlight any local short term variation in the data. Errors may occur due to information not being put in the database in a uniform manner which can distort the picture.

LOSSES

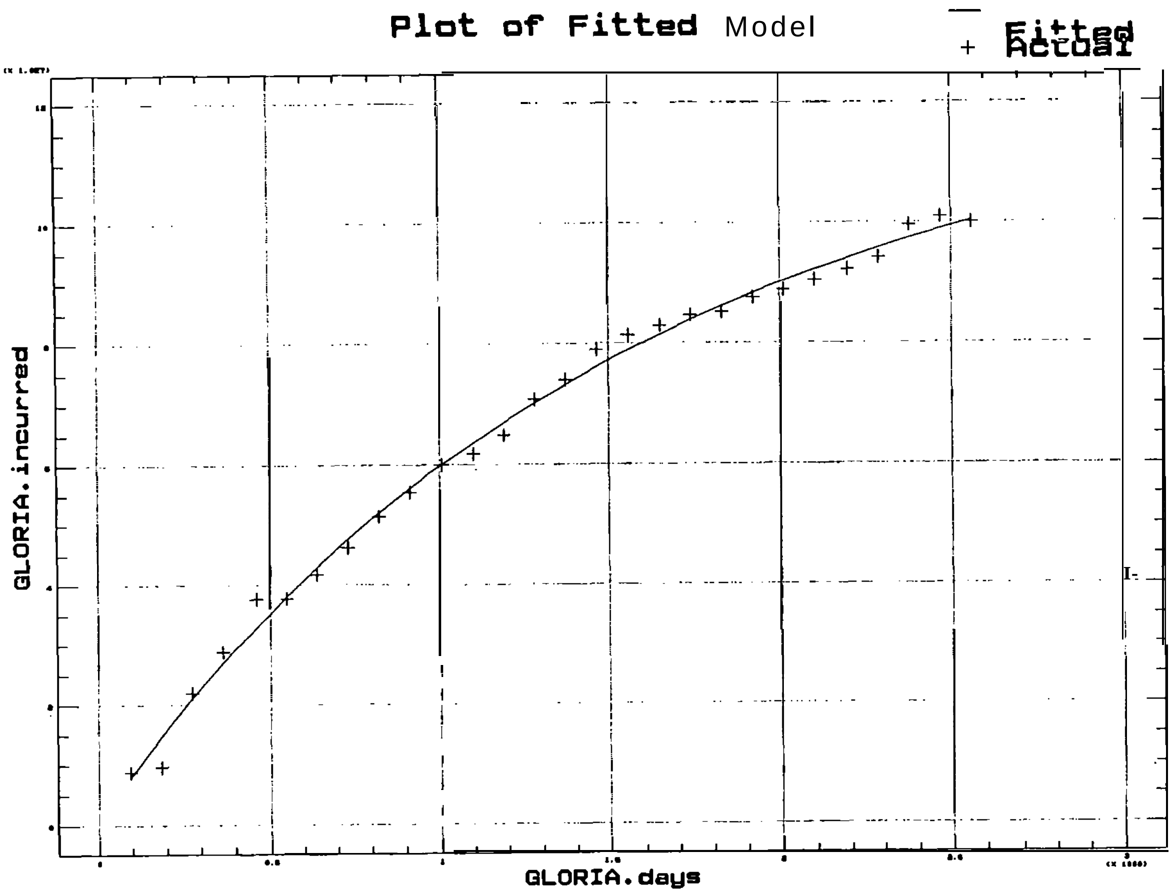
CATASTROPHE		MARINE/ NON MARINE	BASIS	A		B		C	
GILBERT	(1987)	(NM)	INCURRED	101.363	(0.781)	405	(7.08)	1	(0.019)
GILBERT	(1992)	(NM)	PAID	96.537	(4.110)	1063.2	(53.04)	1.5	(0.04)
GLORIA	(1988)	(NM)	INCURRED	124.837	(7.822)	1555	(184.6)	1	(0.05)
GLORIA	(1989)	(NM)	PAID	161.726	(31.326)	3091	(777.6)	1	(0.1)
MERIELLE	(1991)	(NM)	INCURRED	97.204	(1.359)	762	(2.59)	3.1	(0.40)
MERIELLE	(1991)	(NM)	PAID	93.717	(1.059)	81.2	(1.90)	3.7	(0.39)
STORM	90A	(M)	INCURRED (ST90M)	106.823	(6.163)	810	(26.78)	4.0	(0.28)
STORM	90A	(M)	PAID (ST90A)	91.079	(2.217)	841.7	(9.79)	4.7	(0.15)
STORM	90D	(M)	INCURRED (ST90D)	113.690	(6.156)	464	(46.42)	1.0	(0.07)
STORM	90D	(M)	PAID (ST90D)	69.796	(1.092)	521.8	8.21	2.8	(0.12)
STORM	90G	(M)	INCURRED (ST90G)	110.001	(4.456)	567	(29.0)	2.0	(0.07)

STORM	90G	(M)	PAID (ST90G)	85.566	(9.231)	798.9	(66.39)	2.3	(0.13)
STORM	87J	(NM)	INCURRED	96.516	(0.422)	320.1	(4.39)	1.4	(0.04)
STORM	87J	(NM)	PAID	89.377	(0.045)	512.1	(11.15)	1.6	(0.06)
STORM	90A	(NM)	INCURRED	100.163	(0.815)	331	(4.44)	2.0	(0.06)
STORM	90A	(NM)	PAID	89.267	(1.721)	439	(8.92)	3.3	(0.02)
STORM	90D	(NM)	INCURRED	100.163	(3.211)	402	(22.64)	1	(0.08)
STORM	90D	(NM)	PAID	68.593	(1.055)	529	(8.43)	2.8	(0.12)
STORM	90G	(NM)	INCURRED	110.513	(4.317)	589	(29.42)	2	(0.06)
STORM	90G	(NM)	PAID	83.248	(6.994)	799	(52.83)	2.3	(0.11)


Plot of Fitted Model
Fitted
Actual
GILBERT.incurred
GILBERT.days



Plot of Fitted Model
Fitted
Actual
GILBERT.paid
GILBERT.days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Fitted
Actual
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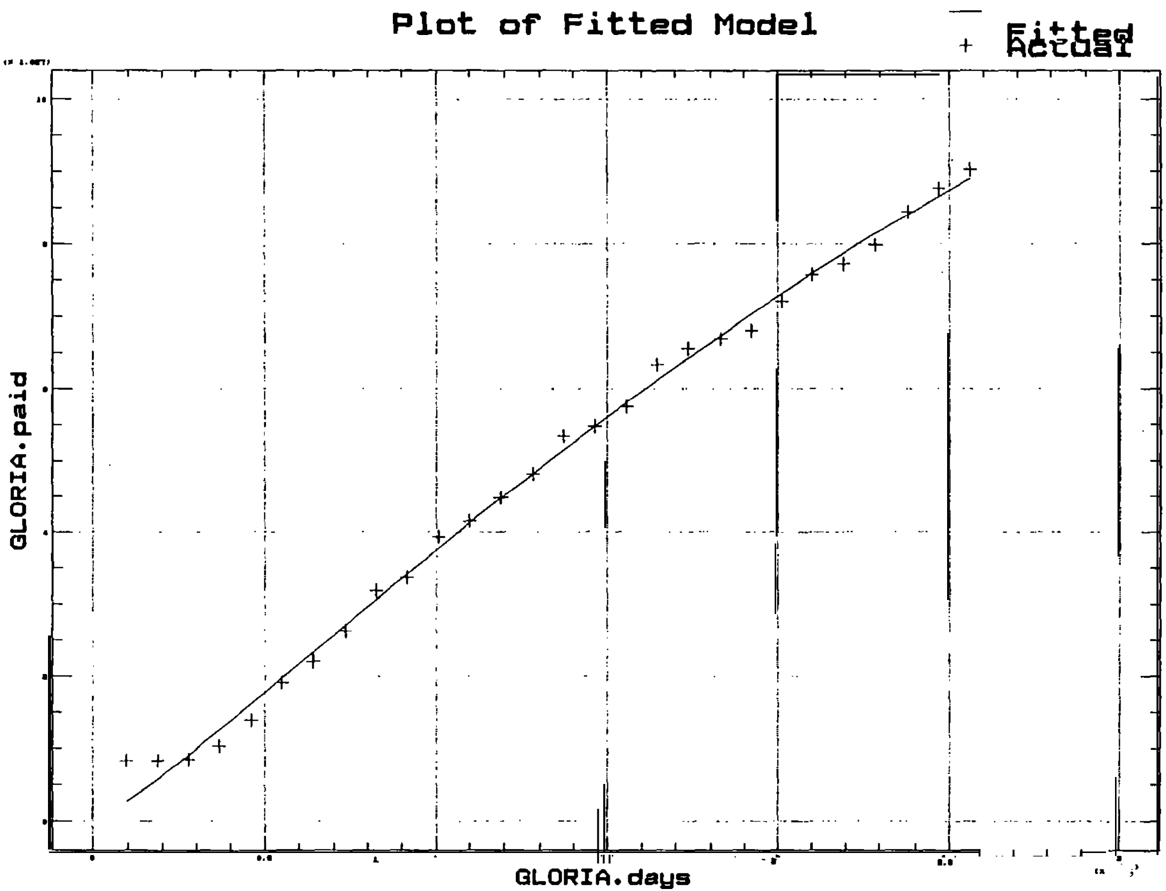
GLORIA.days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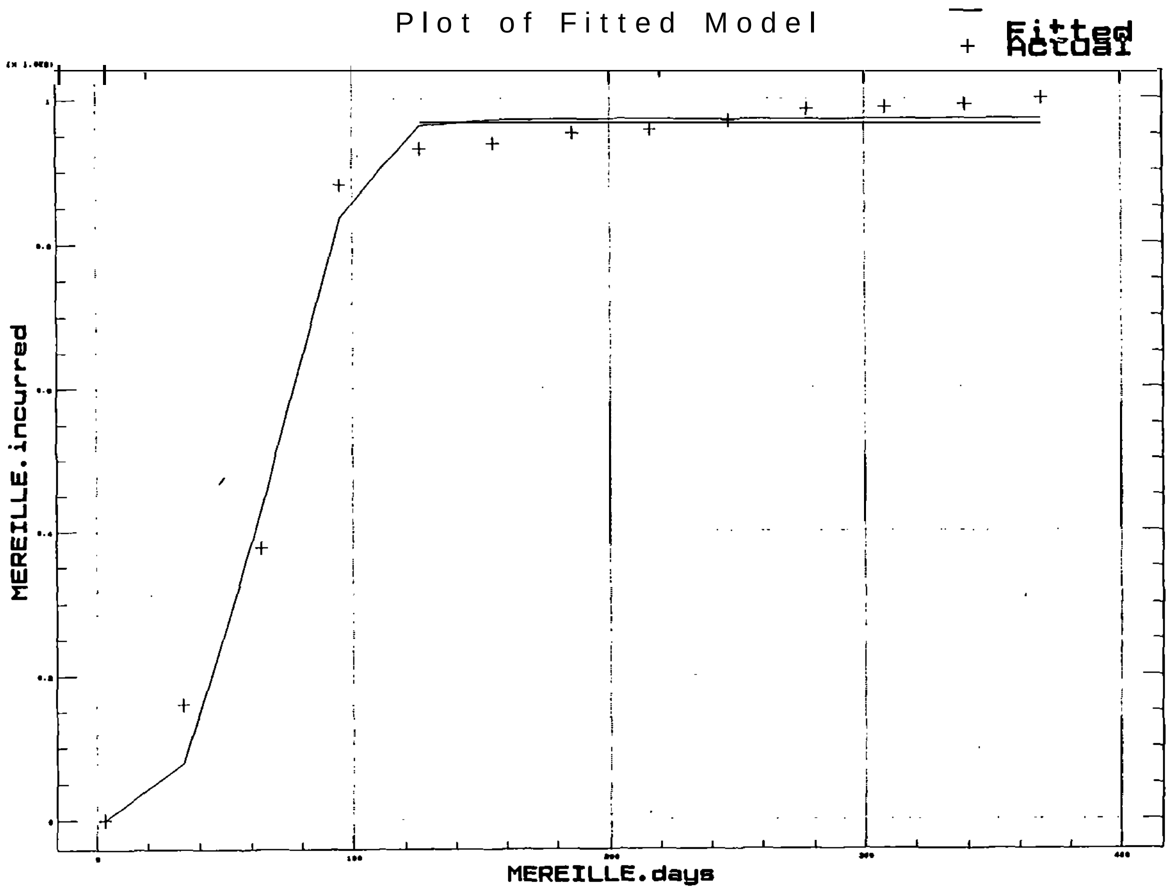
Fitted
Actual
GLORIA.paid
GLORIA.days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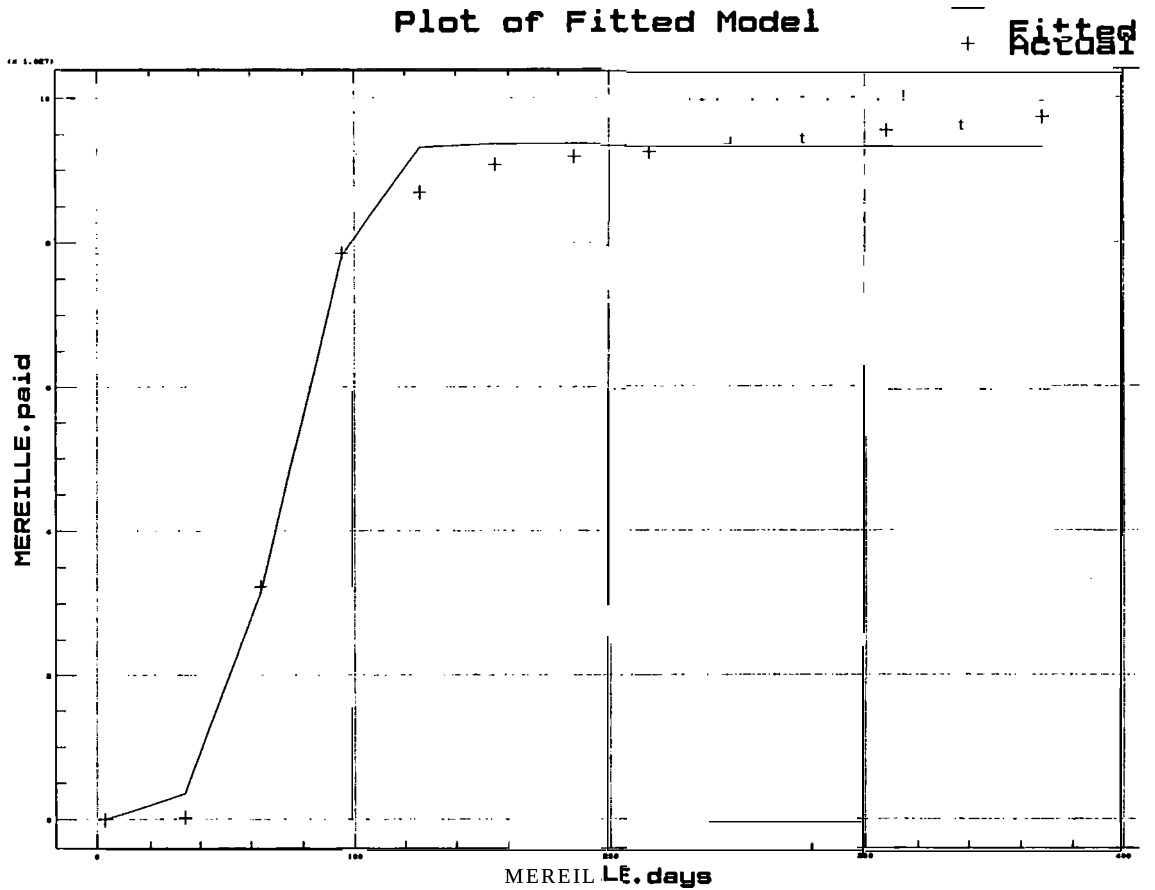
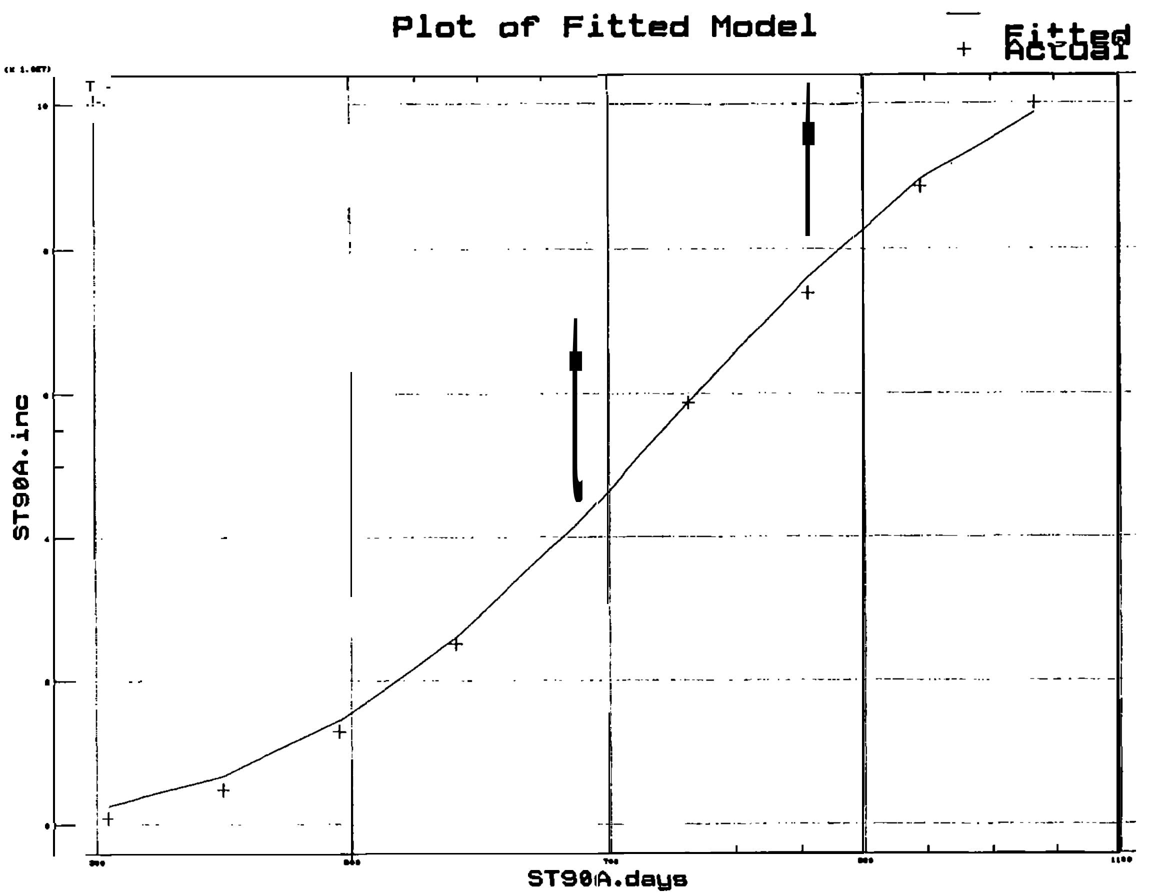
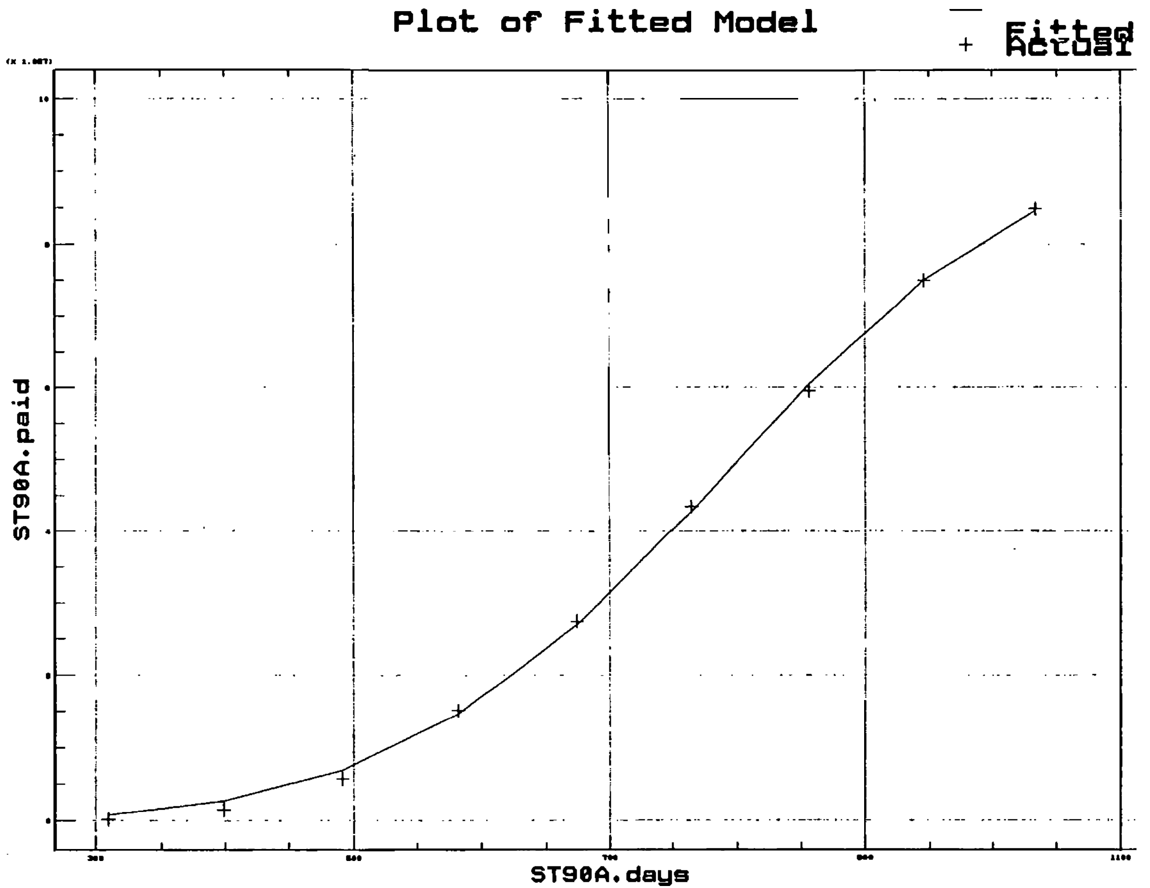
Fitted
Actual
MEREILLE.incurred
MEREILLE.days



Plot of Fitted Model
Fitted
Actual
MEREILLE.paid
MEREILLE.days



Plot of Fitted Model
Fitted
Actual
ST90A.inc
ST90A.days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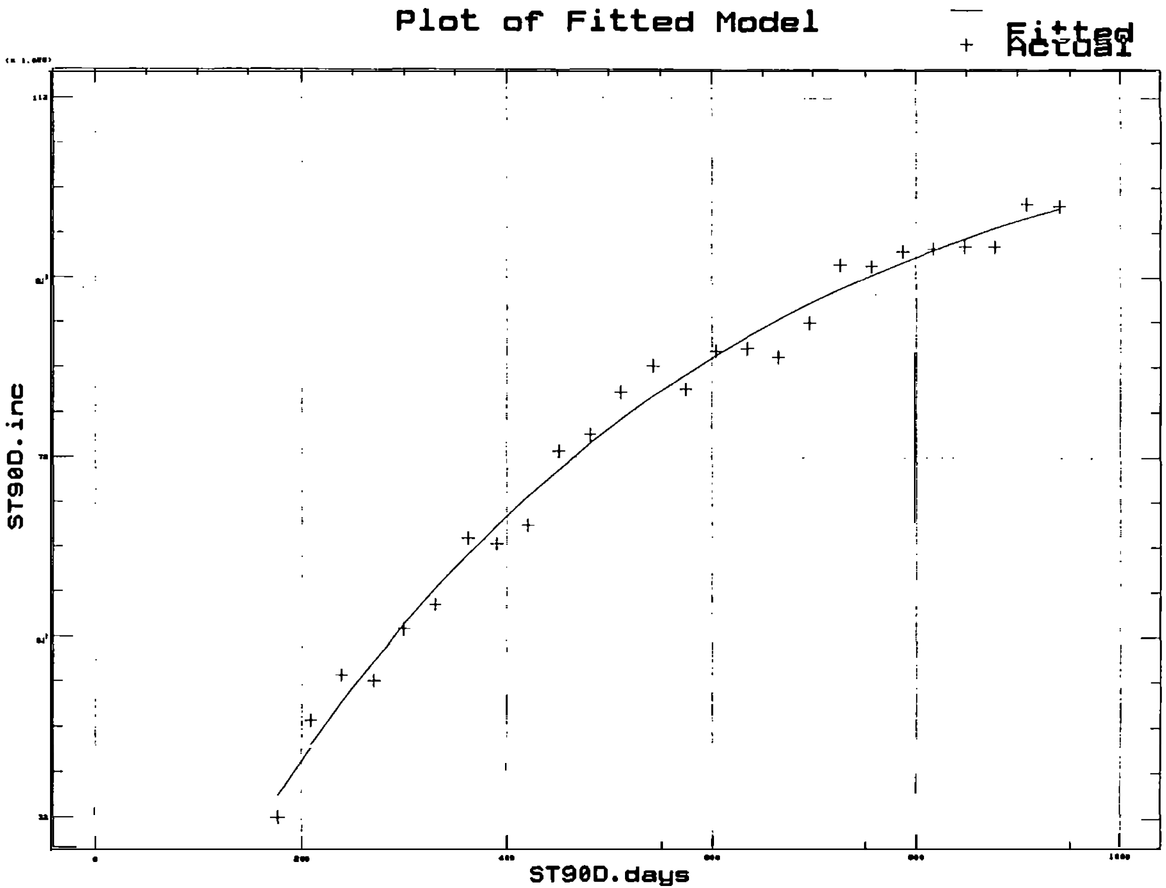
Fitted
Actual
ST90A.paid
ST90A.days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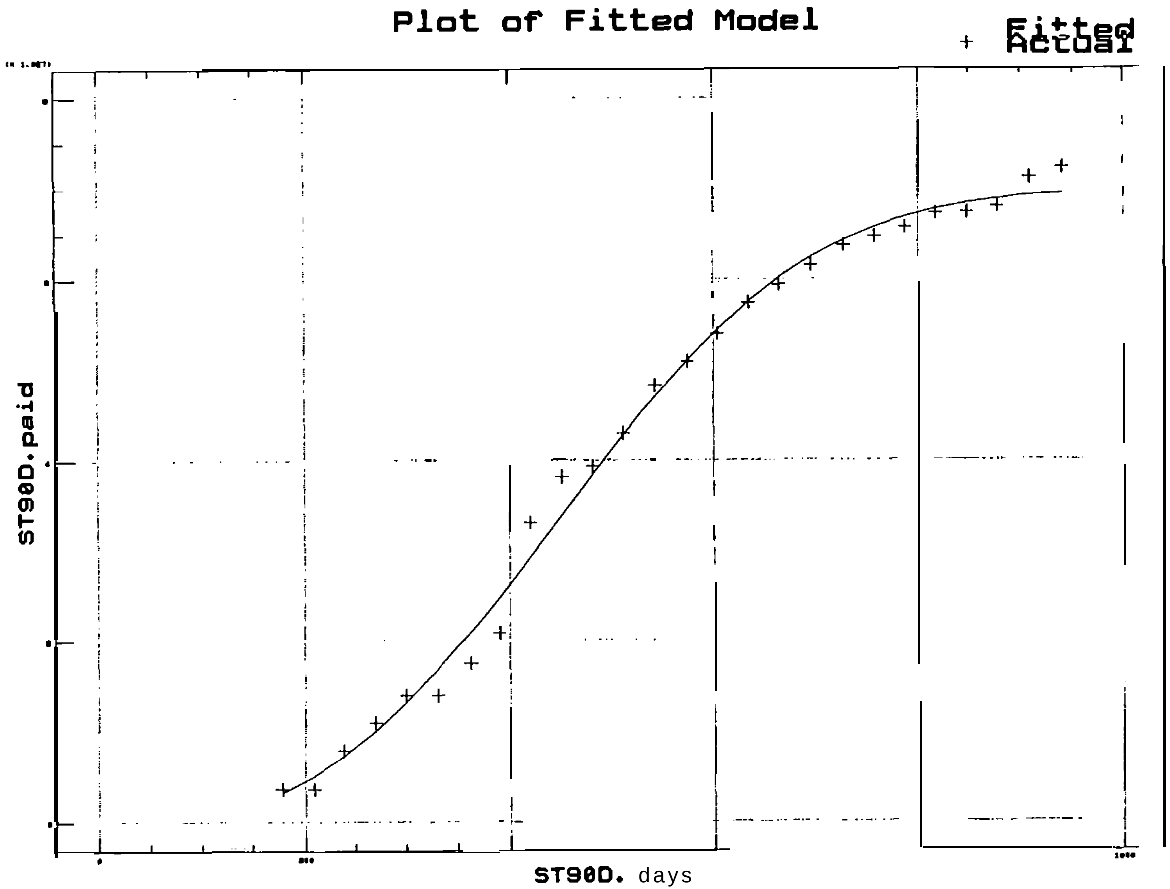
Fitted
Actual
ST90D.inc
ST90D.days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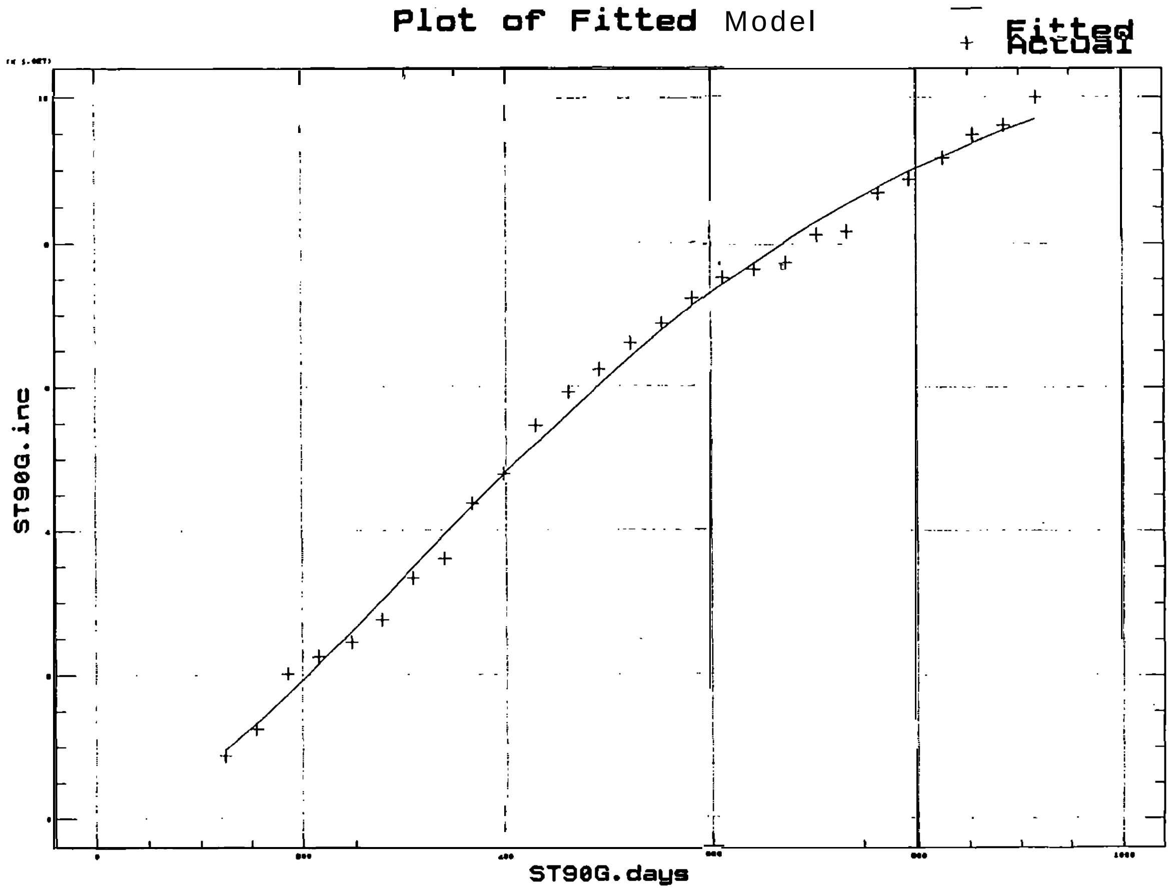
ST90D. days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Actual
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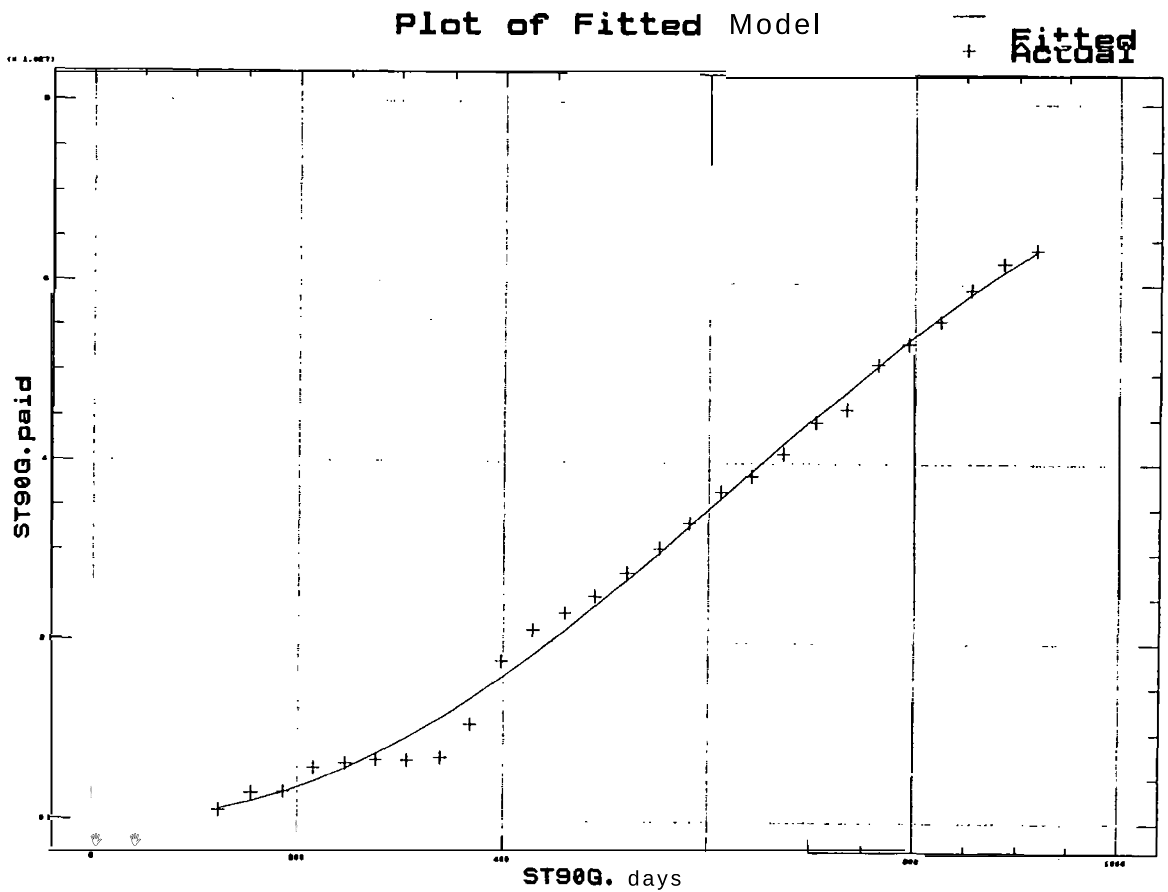
ST90G.days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Fitted
Actual
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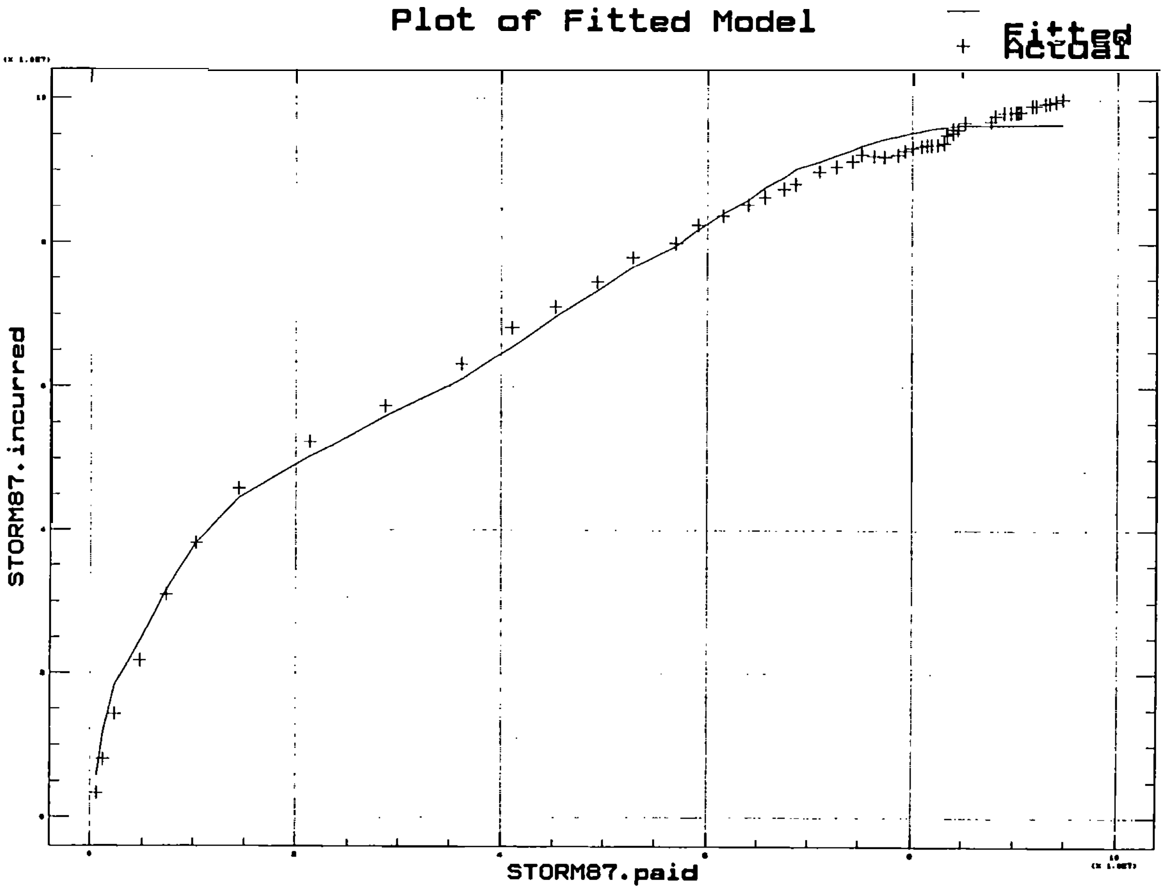
ST90G. days



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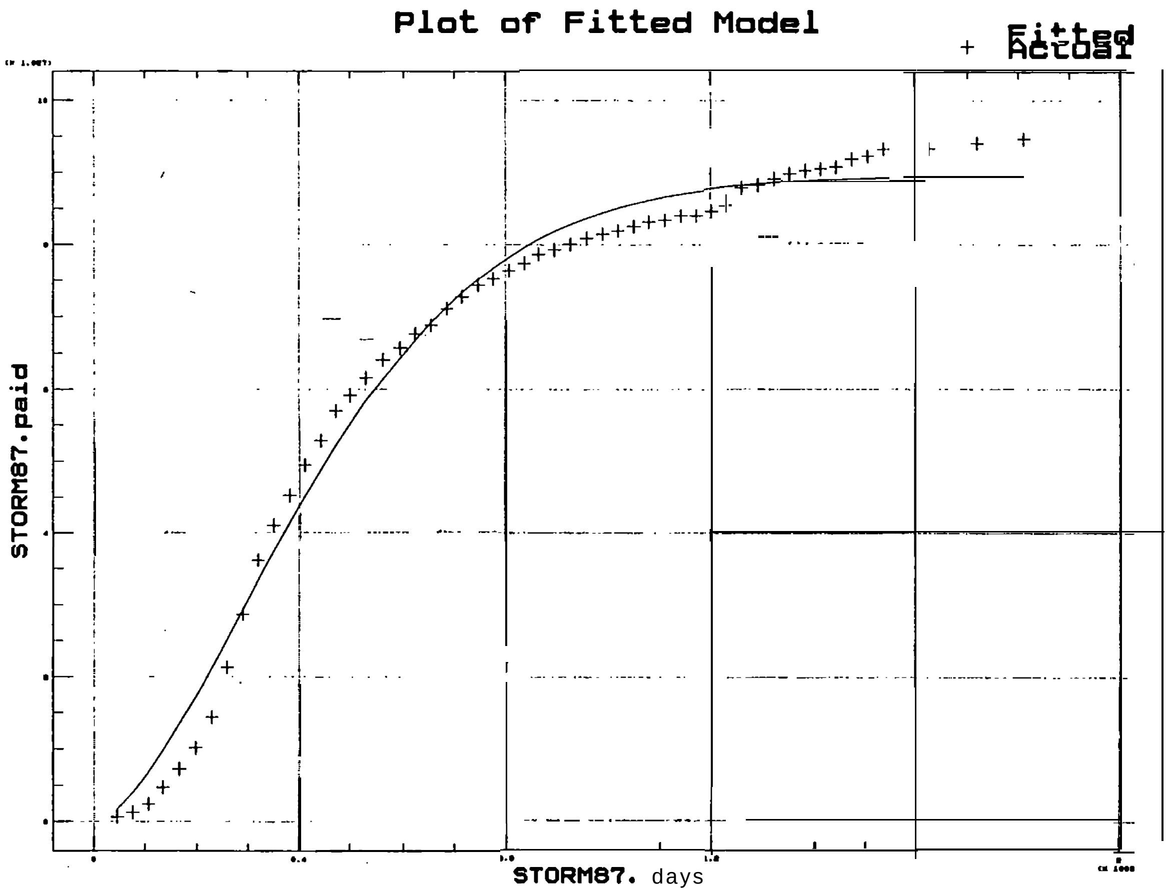
Fitted
Actual
STORM87.incurred
STORM87.paid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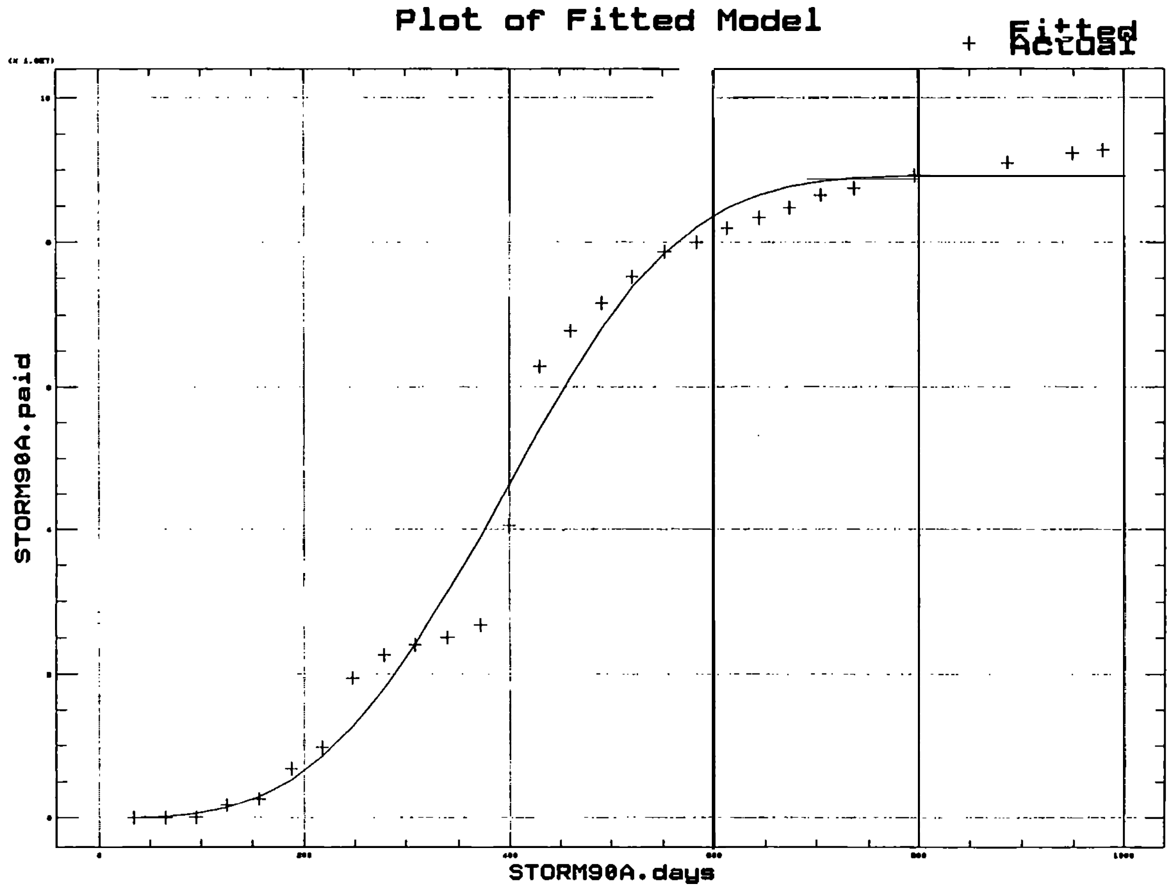
Fitted
Actual
STORM87.paid
STORM87. days



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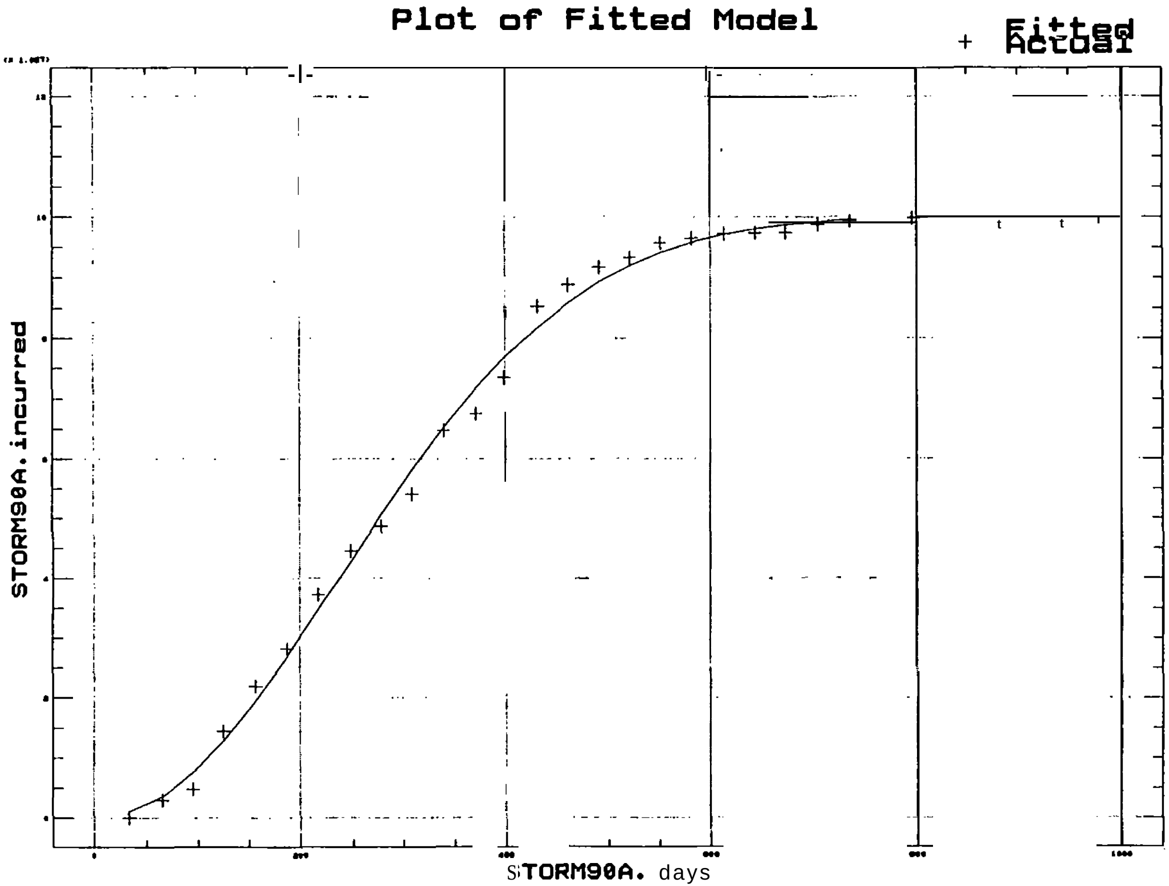
Fitted
Actual
STORM90A.paid
STORM90A.days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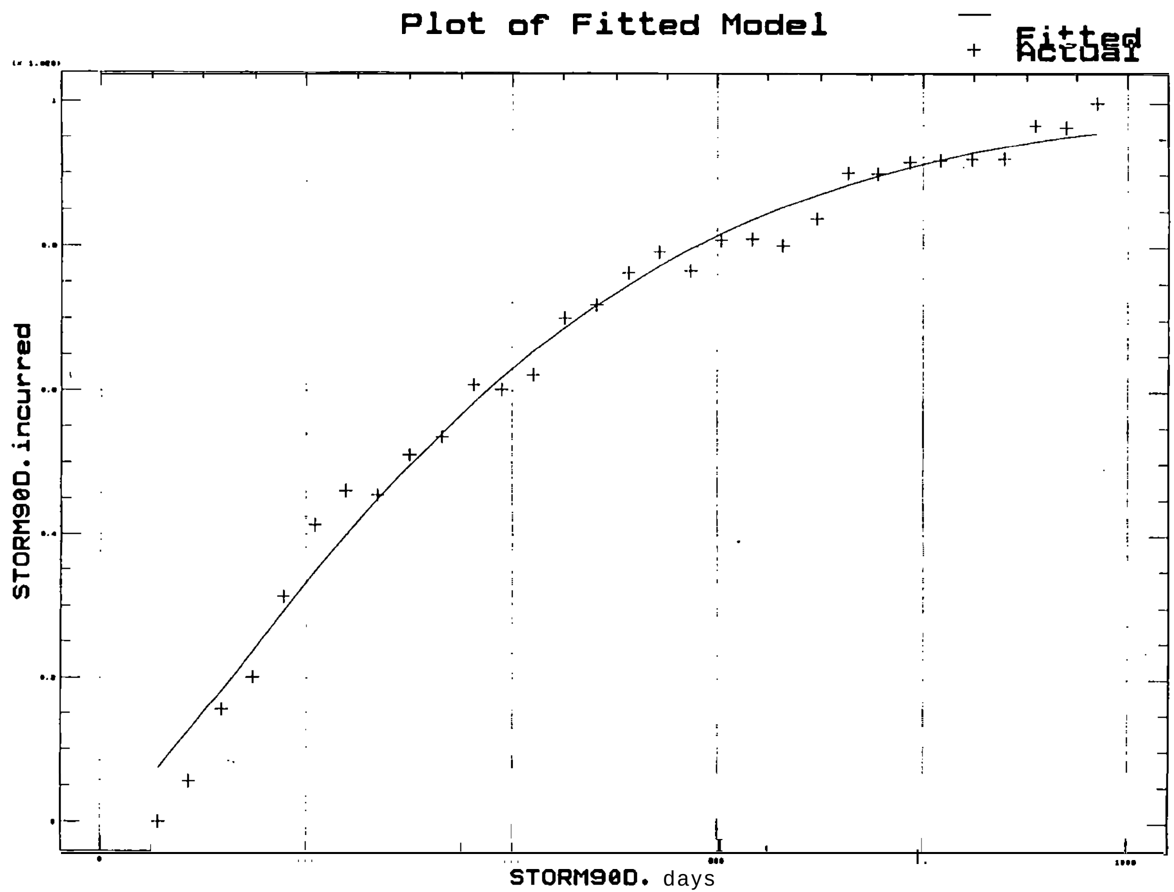
Fitted
+ Actual
STORM90A.incurred
STORM90A. days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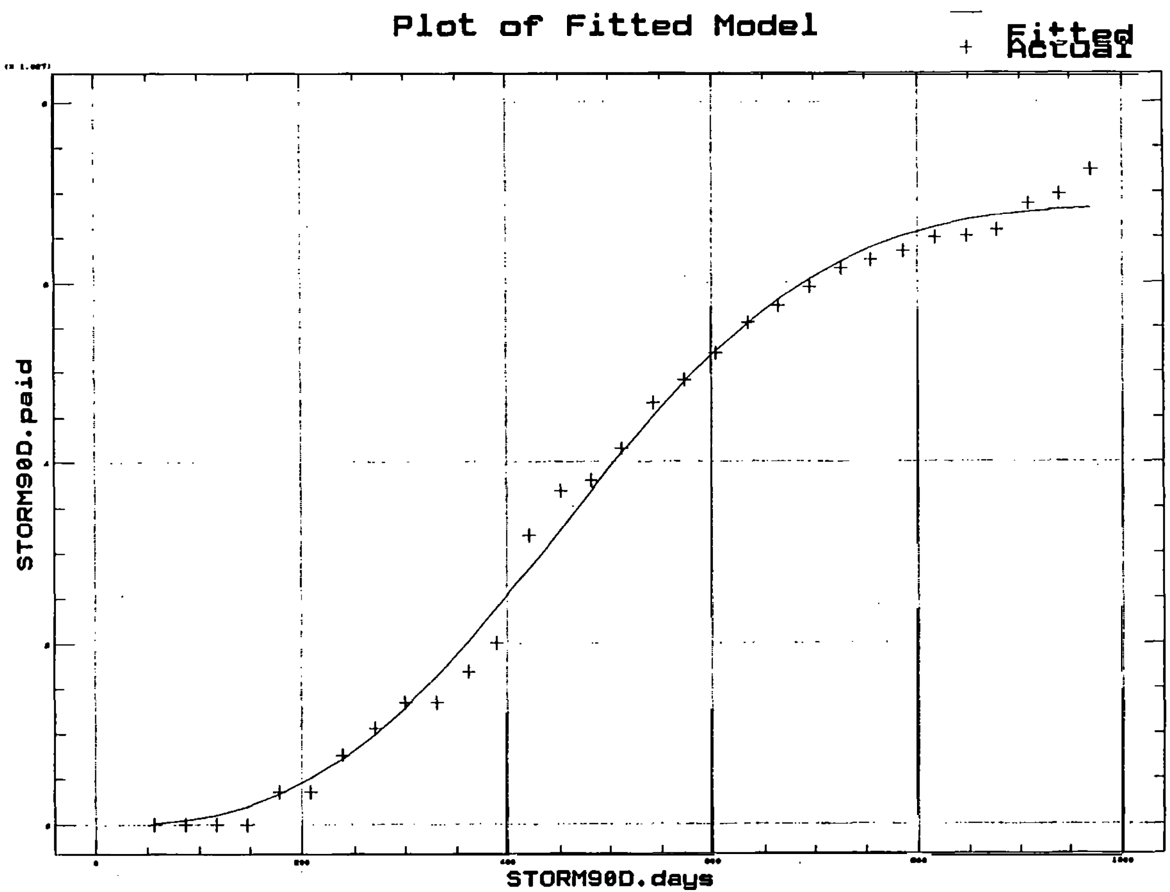
Fitted
Actual
STORM90D.incurred
STORM90D. days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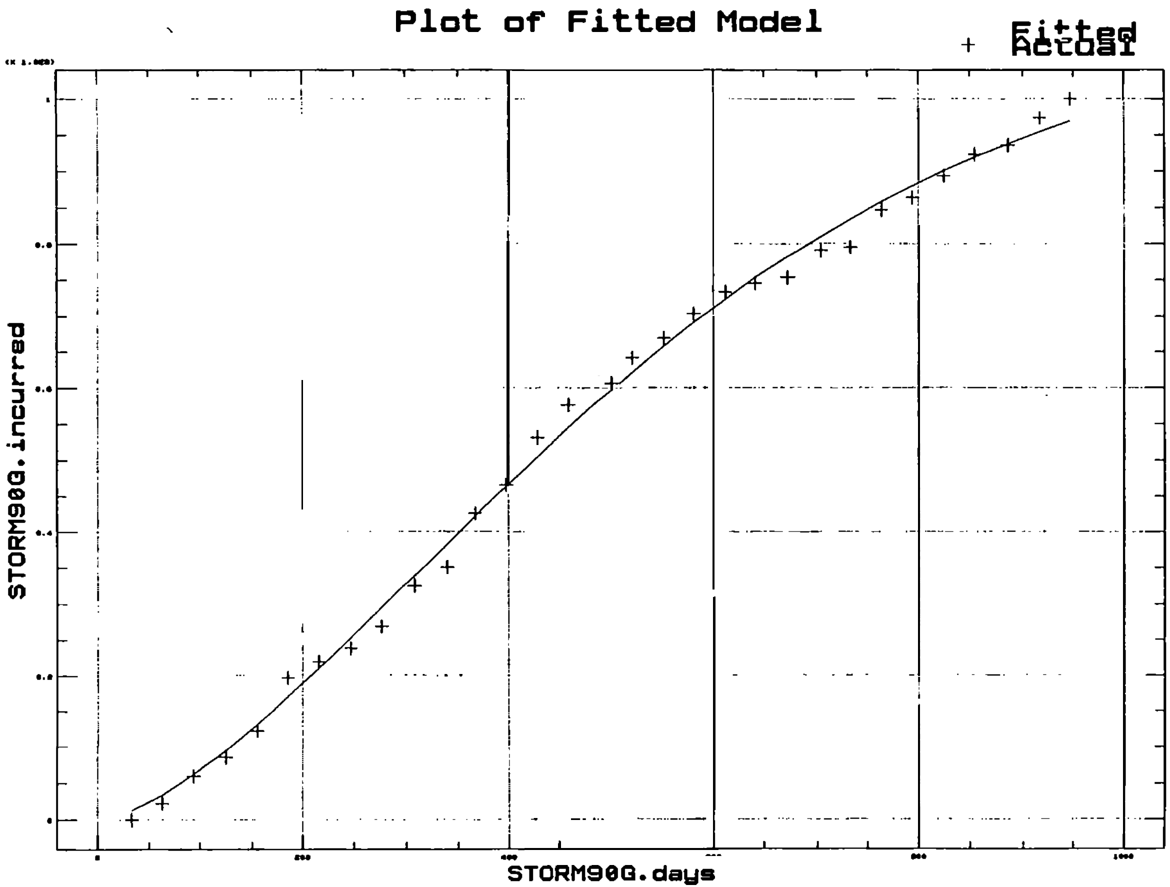
Fitted
Actual
STORM90D.paid
STORM90D.days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Fitted
Actual
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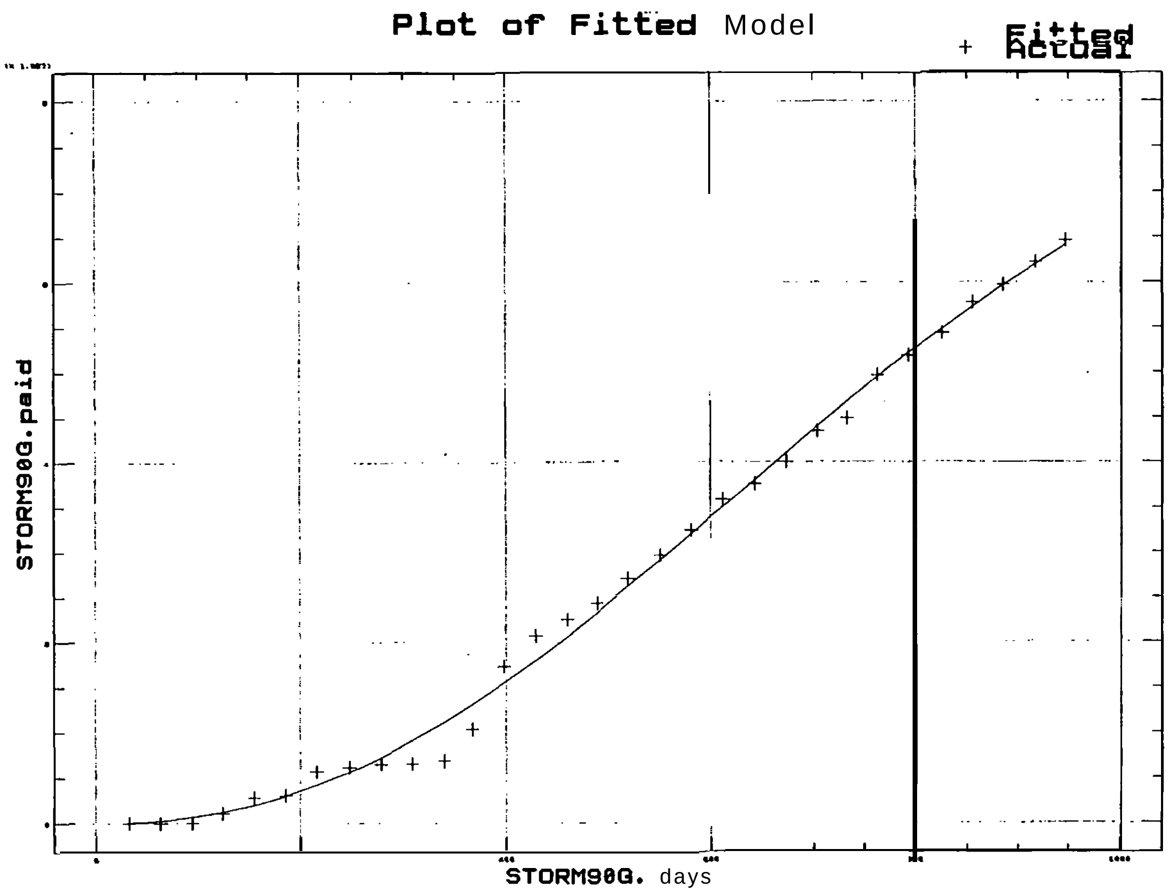

Plot of Fitted Model
Fitted
+ Actual
STORM90G.paid
STORM90G. days

SOME THOUGHTS ON THE FUTURE:

What type of losses should we look for in the future? There is a time bomb of potential losses out there, and I will try and give an indication of the magnitude:-

a) Meteorite Hit

These events are not rare. It is possible that once every 65 million years a meteorite large enough hits the earth and causes mass extinctions. A large meteor, big enough to devastate a substantial part of Europe is expected once every million years. We have no recent experience of such events. An underwriter said that they gave the cover for free!

b) Earthquake

The potential for "big ones" are:-

Tokyo - due any time.

Los Angeles

San Francisco/Hayward Fault

Central Europe - about one every 10,000 years

The Market has not had a significant earthquake in recent times. The Loma Prieta (San Francisco) earthquake insurance was largely retained in the US and very little found its way to London. A Tokyo earthquake on the scale of the one in 1923 is anticipated to cost $400 billion and reduce world GNP. The Japanese have insured for this event by buying assets outside Japan (e.g. Manhattan) and the realisation of these assets and the impact on the Yen are difficult to assess [see 12].

A Californian earthquake will not be as expensive, the main factor of loss being the wind speed and direction at the time and its effect on the fires. The maximum cost is of the order of $60 billion. California has tried to create an earthquake fund to finance this cost, but realised that the cost of payments would break the State if any event should occur.

A Central/North European earthquake would be devastating because construction standards do not take into account earthquake exposures.

c) Hurricanes

Saffir - Simpson Hurricane **Scale:-**

Index 0	Winds	less than 74 m.p.h.
Index 1	Winds	74-95 m.p.h.
Index 2	Winds	96-110 m.p.h.
Index 3	Winds	111-130 m.p.h.
Index 4	Winds	131-155 m.p.h.
Index 5	Winds	over 155 m.p.h.

All measurements are standard anemometer elevations.

Whilst the number of storms seems to be fairly consistent, the number of powerful Hurricanes and Windstorms has increased. On the graphs appended to this section I set out details on an annual basis, of the number of Storm and Hurricanes per annum over period of 120 and 105 years respectively. Details are found in [9]. These indicate a steady number of storms, but a cyclic frequency (80 year cycle) in Hurricanes. Local fluctuation could possibly be attributed to E1 Nino events.

We are seeing an increase in storm intensity. Hurricanes Hugo and Andrew were given Index 5 (although the Andrew damage seemed to indicate it was about Index 3.5). Index 5 storms are due to occur only once in 100 years. In the UK we have seen our once in 300 year storm twice in the past few years. The actual number of storms appear to be constant (see [8]). Is this the impact of Global Warming? Has the new volcanic dust from Mount Pinatoba affected weather for a short period - particularly as it came with an E1 Nino event. Have we been lucky? Certainly if Andrew had struck Florida 10 miles further North, the cost of the loss is estimated to have been $40 billion as opposed to the current estimate of $12 billion (and rising!).

The cost of such storm damage has been increased by two factors:-

(i) The inflationary value of property.

(ii) The population wishing to live in more exposed areas (e.g. sea fronts).

Buildings have been constructed to inadequate standards for the newer weather patterns' energy.

For more details see [7], [8] and [10].

Flood

If the Thames barrier fails, what would be the consequence?

If the Thames barrier doesn't fail, what happens to Essex?!

The Future

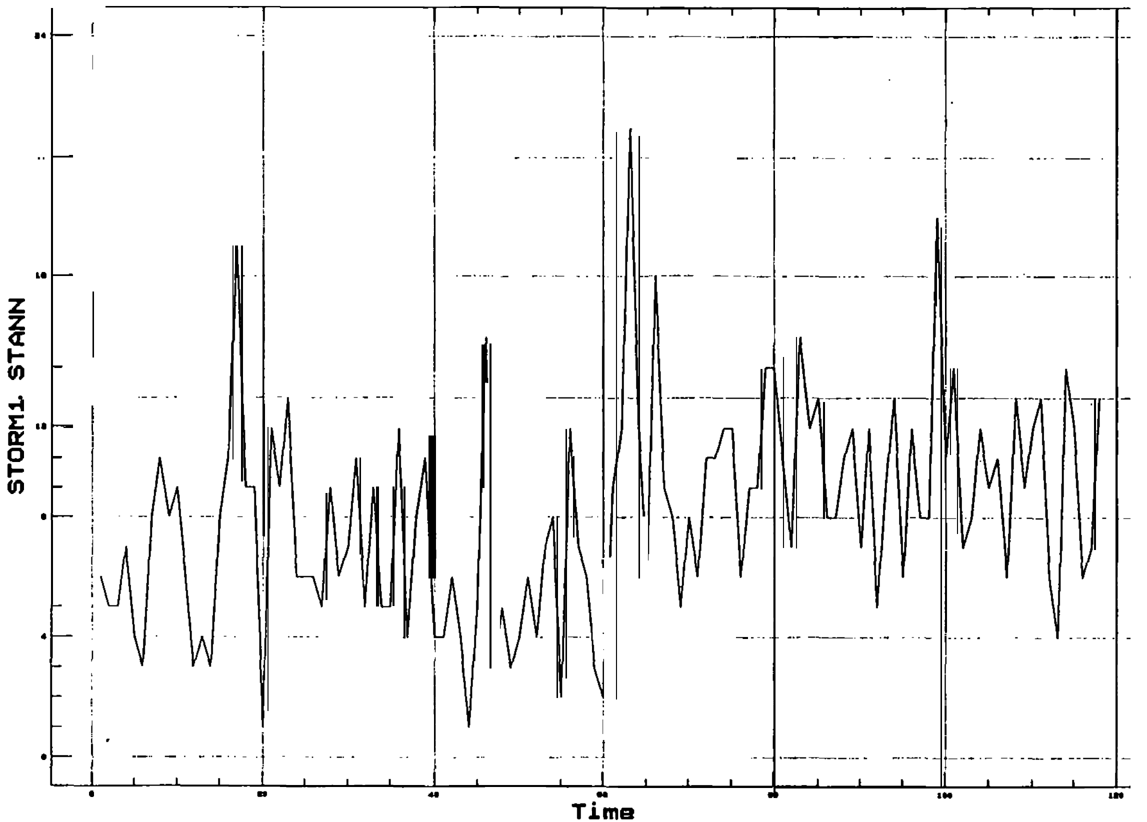
It is clear from the above that reserves need to be built out of current income to provide for the cost of these events. The Revenue puts the UK Market at a potential disadvantage to its European competitors by taxing such reserves.

CATXL is accordingly becoming more and more difficult to purchase. Alternative forms of insurance are being introduced to meet the shortfall. These fall into the stable of Financial (or Finite) Reinsurance. A classic example is a "spread loss" contact when losses from one event are spread forward over many years. Actuaries are becoming more involved with such contracts because of the need to get future cash flows correct to minimise loss. How long will it be before such contracts are traded and a "spread loss" spiral is created?

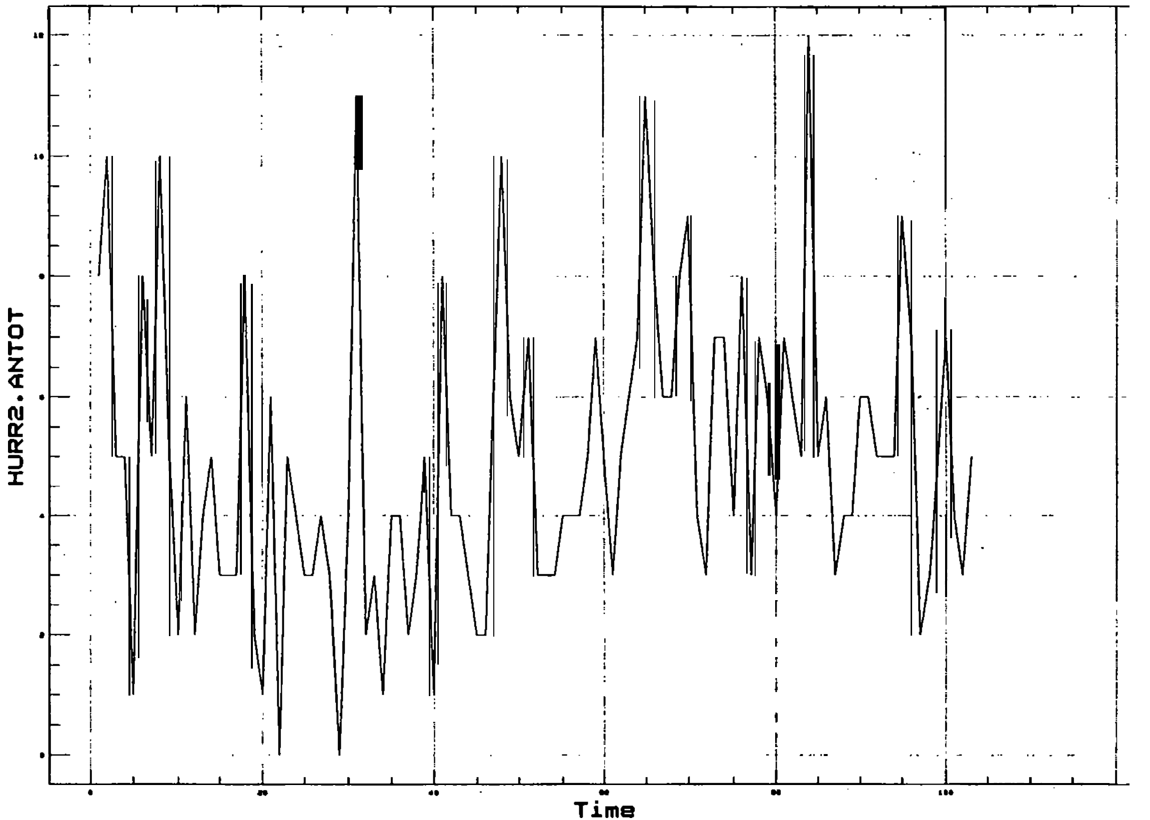
Other insurers are using quota share as a form of catastrophe cover. The Proportional Treaty Reinsurer is waking up to this.

Actuaries will become more involved with Catastrophe Reinsurance as a result of the new alternative.

Time Sequence Plot



Time Sequence **Plot**



CONCLUSIONS

The Catastrophe XL Market is one of the most interesting and stimulating markets open to Actuaries. This paper briefly touches the surface of many of the issues involved. The greater challenge is to find methods of managing the uncertainty and profitability of a market where demand exceeds supply, and where profits, though great, can be just as easily blown away with the wind.

I have kept this paper brief for two reasons. The first is a personal one in that I have no intention of giving all my secrets away. The second is to stimulate interest in the expanding role of the Actuary in Non-life Insurance.

Next time a major catastrophe event occurs, many UK insurers may be exposed to considerable loss. The challenge is to find methods of managing and funding for these potential losses. If the tile should fall today, the claim paid by the direct insurer is going to impact more substantially on the Profit and Loss Account. In addition, the cost to the individual can only increase as the impact of storm damage is felt by UK. insurers.

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[1] The Potential Effect of Climate Changes in the United Kingdom.
HMSO

[2] Option Pricing and Reinsurance -ASTIN Colloquium, Montreux.
D E A SANDERS

[3] Speech or Casualty Loss Reserve Seminar 1991.
D E A SANDERS

[4] Reinsurance (Kluwer and M & G Re).
R L CARTER

[5] Catastrophes (Private Note at GISG 1992).
D H CRAIGHEAD

[6] In the Wake of the Hurricane (Froglets).
OGLEY

[7] US Hurricanes and Windstorms (Risk Research Group).
D G FRIEDMAN

[8] Storm Rating in the Nineties (GISG Conference 1992).
CHRISTOFIDES ET AL

[9] Tropical Cyclones of the North Atlantic Ocean 1871 to 1986.
US DEPARTMENT OF COMMERCE

[10] Historic Storms of the North Sea, British Isles and North West Europe (Cambridge 1991).
LAMB

[11] "Nightmare on Lime Street".
C GUNN

[12] Sixty Seconds that will change the World - The Coming Tokyo Earthquake (Sidgwick & Jackson).
P HADFIELD

APPENDIX 1

A Slip

225

NO.		REF. NO	
REGISTRATION 920653005	VAT	TOC TRIBUNAL	
O T I. CODE 4	REGISTRATION CATEGORY	YEAR	MONTH
REINSURED/ACCOUNT		ADJUST SCHEME YES	X
COUNTRY OF ORIGIN U.S.A.	MARINE XP	NON-MARINE	AVIATION
US$ / C$ / MUS	OVERSEAS BROKER		
CURRENCY	SIGNED LINE	GROSS PREMIUM	
TOTAL			
LLOYD'S			
ILU			
PSAC			
COMPANIES			

AGENT/CLIENT

AGENT/ADDR CODE		REINSURED	
CLASS CODE		W.P. ID	
TOTAL WRITTEN ALL SLIPS	ORDER 41.557		PREMIUM ENTRY
MICRO NO. DATE			
X	Y	Z	
L P S O Treaty No		I L U Treaty No	

REINSURED : XYZ Insurance Company

PERIOD : 12 months at 1st January, 1992
Losses occurring during basis.

TYPE : SECOND PROPERTY CATASTROPHE EXCESS OF LOSS REINSURANCE.

CLASS : All Property insurance and reinsurance, classified by the Reinsured as Fire, Allied Lines, Inland Marine, Homeowners, Automobile Physical Damage (excluding Collision) Multiple Peril and Casualty (with respect to Glass business only).

TERRITORIAL SCOPE : U.S.A. and/or Canada and/or . . original.

LIMIT : 95% of $25,000,000 each and every loss occurrence
IN EXCESS OF AN ULTIMATE NET LOSS OF
$27,500,000 each and every lam occurrence.

CO-REINSURANCE : 5% retained net by the Reinsured.

REINSTATEMENT : One reinstatement for all perils at ~~pro rata~~ 100% additional premium. as to time pro rata as to limit

PREMIUM : Deposit Premium $2.612.000
payable in four equal instalments in advance and adjustable at 0.825 % of the Reinsured's subject matter Gross Net Earned Premium Income.
Minimum Premium $2.089.000. 10/4/92.

DEDUCTIONS : Federal Excise Tax as required under applicable law or regulation and 15% Brokerage (Nil for Reinstatement).

LOSS RESERVE : Non-admitted Reinsurers hereon agree Letters of Credit (Citibank N.A. &/or Chase Scheme) In respect of known and reported losses only but Cash O.C.As. for Canadian Dollars, as required by the Reinsured (Excluding I.B.N.R.).

REC'D	SETT DUE DATE		
	31/3/92	30	120 adj

HEREON

WRITTEN LINES PERCENTAGES PART OF $25.000.000

SIGNED LINES PERCENTAGES PART OF

Underwriters agree to authorise L.P.S.O., P.S.A.C. and I.L.U. to sign book slips and to take down premium A.P's and R.P's and to settle claims and returns on a balance of account basis irrespective of discrepancies in the accounts subject to these being rectified in the next account.
Off slips and signing slips t.b.a. (if required) by L/UR only.
I.L.U. authorised to sign and seal treaty wording as agreed by leading I.L.U. Coy

Subject otherwise to the terms of the C.C.S.A. 1980. Companies hereon agree that authorisation forms in respect of this policy shall not be required, it being understood that Closing instructions or addenda will be sent to Companies together with a form for return (if necessary) within 7 days of receipt showing any objection or change of administrative details, any amended reference to be quoted thereafter.

By signing this slip signatories to the C.C.S.A. 1980 authorise the Leading C.C.S.A. 1980 Company to arrange for P.S.A.C. to sign the Policy on its behalf and accept that such signing will be a valid signing for all the purposes of the C.C.S.A. 1980.

226

GENERAL CONDITIONS :

- Intermediary Clause.
- Net Loss Clause.
- Net Retained Lines Clause.
- Excess of Loss Reinsurance Clause.
- Loss Settlement and Notice Of Loss Clause.
- Hours Clauses as per NMA 2244.
- Service of Suit Clause (NMA 1998).
- Arbitration Clause.
- Access to Records Claude.
- Insolvency Funds Exclusion.
- Insolvency Clause as per wording.
- Nuclear Incident Exclusion Clause(s) as per wording.
- Pools, Associations and Syndicates Exclusion Clause.
- War Exclusion.
- Seepage and Pollution Exclusion as per wording.

WORDING : As expiring as far as applicable, to be agreed L/U only.

INFORMATION : As per submission of 22 pages dated 9.10.91.

INTERNAL ARRANGEMENTS

Reinsurers hereon agree to accept all special agreements and acceptance8 relating to this contract made prior to the inception of this slip.

Loss Reserve - Admitted Reinsurers hereon will provide Letters of Credit (Citibank N.A. Scheme) In respect of known and reported losses only but cash O.C.A.'s for Canadian Dollars, where they consider circumstances warrant It.

Lloyd's Underwriters hereon agree that claims are to be agreed by, leading Lloyd's Underwriter and LUNCO only, including LOCs as applicable.

Reinsurers hereon authorise LPSO/LIRMA to take down adjustments/reinstatements without sight of client account subject to checking prior to submission and rectifying any errors as soon as possible after discovery.

Loss advices: Where the reserve from the ground up on an individual loss is 50% of the retention or less, claim advices will not be shown to the market.

2½% CIRU 1168

PGR 561

STN 566

1¼%

2 [redacted]

31.10.91.

2% 017567935 01 /41

NWRe

3% G0104SH92 NCAD 4/11/91

0.15% C0369D492 NCAD 25/11/91

0.3% D0097NQ92 NCAD 4/11/91

COPY OF

ANT 51

CAA82XA01333

227

RCB 512

0006G6E9 AXEX

1¼%

WLM 807

A85592833 15D

1·5%

P0625908

C4009	LIPMA	NCAD

Excluding LOC'S/OCA'S/IBNR

PER H.G.S. 15/11/91

12

ZURICH RE (UK)

2	6	2	0	0	3	9	0	3	7	9	2

Z4508	LIRMA	NCAD

EXCL. LOC'S/OCA'S, I/O I.B.N.R.

3/4

Caudle BFC 780

B011 83HBA5111X

5.25

0·30%

DDM 606

09M0124 66 10

THE TOA-RE (UK)

0.5%

92 X 00 [illegible]

EXCLUDING LOC'S/OCA'S/IBNR

SVT 1007

0.75%

FB7BBJ92A46

C. W. SPRECKLEY

EVERNDEN PAR 1177

A0392C92A154

0.40%

A1

0.7% 0.50%

JHV 376

800N00180 A LA

0.35% O/A

LLOYD-ROBERTS GLR 55

BA1905 92AAC1

3

0.25%

ROS 227

800S7JCAAH2X

0.25%

CJ456992A480

0.50%

DPM 435

15/11/91

603232

NVB 1114

0.2%

0341009 2AANN

BURTON

R. E. BROWN & ORS REB 702

0.70%

P60002X2XC

Terra Nova

1½%

0	9	2	B	P	0	7	1	6	7TA
1	4	P	1	1	A	7		3	1

T3902	LIRMA

Ex LOC IBNR

2/8

7113 F11 82 160 L

AFB

0.4 T0031N 92 ANCC

EPT
$ 316,555,000

228

0.3 % EAGLE STAR

3	8	9	2	9	5	5	0	2	0	0
U	S	A	U	S	1	S	F	1	F	X

E1102 LIRMA

MH

NO. 920653005

REINSURED

TYPE

SECOND PROP. CAT. X/L REINSURANCE

RENEWING	BABCLM	CLIENTS REF
~~910653005~~		

RELATED SLIPS

LESS 10%

ORDER 41.55 %

TOTAL WRITTEN HEREON

NO. of SLIPS 1

BROKER	C/N DATED	PHOTOGRAPH
DS / FGS.		
REPORTS YES/NO		

FOR LPSO USE

FOR ILU USE

FOR PSAC USE

Which Stochastic Model is Underlying the Chain Ladder Method?

by Thomas Mack, Ph.D.

WHICH STOCHASTIC MODEL IS UNDERLYING THE CHAIN LADDER METHOD?

BY THOMAS MACK, PH.D., MUNICH RE

Editor's Note: This paper was presented to the XXIV ASTIN Colloquim, in Cambridge in 1993. Also, *this paper was awarded the first-ever CAS Charles A. Hachemeister Prize in November 1994.*

Abstract:

The usual chain ladder method is a deterministic claims reserving method. In the last years, a stochastic loglinear approximation to the chain ladder method has been used by several authors especially in order to quantify the variability of the estimated claims reserves. Although the reserves estimated by both methods are *clearly different, the loglinear approximation has been called "chain ladder," too, by these authors.*

In this note, we show that a different distribution-free stochastic model is underlying the chain ladder method; i.e. yields exactly the same claims reserves as the usual chain ladder method. Moreover, a comparison of this stochastic model with the above-mentioned loglinear approximation reveals that the two models rely on different philosophies on the claims process. Because of these fundamental differences the loglinear approximation deviates from the usual chain ladder method in a decisive way and should therefore not be called "chain ladder" any more.

Finally, in the appendix it is shown that the loglinear approximation is much more volatile than the usual chain ladder method.

1. *The usual deterministic chain ladder method*

Let C_{ik} denote the accumulated claims amount of accident year i, $1 \le i \le n$, either paid or incurred up to development year k, $1 \le k \le n$. The values of C_{ik} for $i + k \le n + 1$ are known to us (run-off triangle) and we want to estimate the values of C_{ik} for $i + k > n + 1$, in particular the ultimate claims amount C_{in} of each accident year $i = 2, ..., n$.

The chain ladder method consists of estimating the unknown amounts C_{ik}, $i + k > n + 1$, by

(1) $$\hat{C}_{ik} = C_{i,n+1-i}\hat{f}_{n+1-i} \times \ldots \times \hat{f}_{k-1}, \quad i + k > n + 1,$$

where

(2) $$\hat{f}_k = \sum_{j=1}^{n-k} C_{j,k+1} \Big/ \sum_{j=1}^{n-k} C_{jk}, \quad 1 \le k \le n-1.$$

For many years this has been used as a self-explaining deterministic algorithm which was not derived from a stochastic model. In order to quantify the variability of the estimated ultimate claims amounts, there have been several attempts to find a stochastic model underlying the chain ladder method. Some of these will be reviewed in the following chapter.

2. *Some stochastic models related to the chain ladder method*

In order to find a stochastic model underlying the chain ladder method we have to cast the central equation (1) of the chain ladder method into stochastic terms. One way of doing this runs along the following lines: We conclude from (l) that

$$\hat{C}_{i,k+1} = \hat{C}_{ik}\,\hat{f}_k, \quad k > n+1-i.$$

This is generalized to the stochastic model

(3) $$E(C_{i,k+1}) = E(C_{ik})\,f_k, \quad 1 \le k \le n-1,$$

where all C_{ik} are considered to be random variables and $f_1, \ldots, f_{n-1}$ to be unknown parameters.

Introducing the incremental amounts

$$S_{ik} = C_{ik} - C_{i,k-1}, \quad 1 \le i, k \le n,$$

with the convention $C_{i0} = 0$, one can show that model (3) is equivalent to the following model for S_{ik} :

(4) $$E(S_{ik}) = x_i y_k, \quad 1 \le i,\ k \le n,$$

with unknown parameters x_i, $1 \le i \le n$, and y_k, $1 \le k \le n$, with $y_1 + \ldots + y_n = 1$.

Proof of the equivalence of (3) and (4):

(3)==> (4): Successive application of (3) yields

$$E(C_{in}) = E(C_{ik}) f_k \times \ldots \times f_{n-1}$$

Because

$$E(S_{ik}) = E(C_{i,k}) - E(C_{i,k-1})$$

$$= E(C_{in})\big((f_k \times \ldots \times f_{n-1})^{-1} - (f_{k-1} \times \ldots \times f_{n-1})^{-1}\big)$$

we obtain (4) by defining

$$x_i = \mathrm{E}(C_{in}), \quad 1 \le i \le n,$$

$$y_i = (f_1 \times \ldots \times f_{n-1})^{-1}$$

$$y_k = (f_k \times \ldots \times f_{n-1})^{-1} - (f_{k-1} \times \ldots \times f_{n-1})^{-1}, \quad 2 \le k \le n-1,$$

$$y_n = 1 - (f_{n-1})^{-1}.$$

This definition fulfills $y_1 + . + y_n = 1$.

(4) ==> (3): we have

$$\mathrm{E}(C_{ik}) = \mathrm{E}(S_{i1}) + \ldots + \mathrm{E}(S_{ik})$$

$$= x_i (y_1 + \quad + y_k)$$

and therefore

$$\frac{\mathrm{E}(C_{i,k+1})}{\mathrm{E}(C_{ik})} = \frac{y_1 + \ldots + y_k + y_{k+1}}{y_1 + \ldots + y_k} =: f_k, \quad 1 \le k \le n-1.$$

The stochastic model (4) clearly has $2n-1$ free parameters x_i, y_k. Due to the equivalence of (3) and (4) one concludes that also model (3) must have 2n $-$ 1 parameters. One immediately sees $n - 1$ parameters $f_1, \ldots, f_{n-1}$. The other n parameters become visible if we look at the proof (3) ==> (4). It shows that the level of each accident year i, here measured by $x_i = E(C_{in})$, has to be considered a parameter, too.

Now, one additionally assumes that the variables S_{ik}, $1 \le i, k \le n$, arc independent. Then the parameters x_i, y_k of model (4) can be estimated (e.g. by the method of maximum likelihood) if we assume any distribution function for S_{ik}; e.g.. a one-parametric one with expected value $x_i y_k$ or a twoparametric one with the second parameter being constant over all cells (i,k). For example, we can take one of the following possibilities:

(4a) $$S_{ik} \propto \text{Normal } (x_i y_k, \sigma^2)$$

(4b) $$S_{ik} \propto \text{Exponential } (1/(x_i y_k))$$

(4c) $$S_{ik} \propto \text{Lognormal } (x_i + y_k, \sigma^2)$$

(Observe that (4a) and (4c) introduce even a further parameter σ^2). Possibility (4a) has been introduced into the literature by de Vylder 1978 using least squares estimation of the parameters. The fact that claims variables are usually skewed to the right is taken into account by possibilities (4b) and (4c) but at the price that all incremental variables S_{ik} must be positive (which is not the case with the original chain ladder method and often restricts the use of (4b) and (4c) to triangles of paid amounts).

Possibility (4b) has been used by Mack 1991. Possibility (4c) was introduced by Kremer 1982 and extended by Zehnwirth 1989 and 1991. Renshaw 1989, Christofides 1990. Verrall 1990 and 1991. It has the advantage that it leads to a linear model for $\log(S_{ik})$, namely to a two-way analysis of variance, and that the parameters can therefore be estimated using ordinary regression analysis.

Although model (4c) seems to be the most popular possibility of model class (4). we want to emphasize that it is only one of many different ways of stochastifying model (4). Moreover, possibilities (4a), (4b), (4c), yield different estimators for the parameters x_i, y_k, and for the claims reserves and all of these arc different from the result of the original chain ladder method. Therefore this author finds it to be misleading that in the papers by Zehnwirth 1989 and 1991, Renshaw 1989. Christofides 1990. Verrall 1990 and 1991 model (4c) explicitly or implicitly is called "*the* scholastic model underlying the chain ladder" or even directly "chain ladder model." In fact, it is something different. In order to not efface this difference, model (4c) should better Ix called "loglinear cross-classified claims reserving method." In the next chapter we show that this difference does not only rely on a different parametric assumption or on different estimators but stems from a different underlying philosophy.

3. A distribution-free stochastic model for the original chain ladder method

The stochastic models (4a). (4b), (4c) described in the last chapter did not lead us to a model which yields the same reserve formula as the original chain ladder method. But we will now develop such a model.

If we compare model (3) with the chain ladder projection (1), we may get the impression that the transition

(A) $$\hat{C}_{i,n+2-i} = C_{i,n+1-i}\hat{f}_{n+1-i}$$

in (1) from the most recent observed amount $C_{i,n+1-i}$ to the estimator for the first unknown amount $C_{i,n+2-i}$ has not been captured very well by model (3) which uses

(B) $$\hat{C}_{i,n+2-i} = E(C_{i,n+1-i})\, f_{n+1-i}.$$

The crucial difference between (A) and (B) is the fact that (A) uses the actual observation $C_{i,n+1-i}$ itself as basis for the projection whereas (B) takes its expected value. This means that the chain ladder method implicitly must use an assumption which states that the information contained in the most recent observation $C_{i,n+1-i}$ is more relevant than that of the average $E(C_{i,n+1-i})$ This is duly taken into account by the model

(5) $$E(C_{i,k+1}|C_{i1},\ldots,C_{ik}) = C_{ik}\, f_k, \quad 1 \le i \le n, \quad 1 \le k \le n-1$$

which is (due to the iterative rule for expectations) more restrictive than (3). Moreover, using (5) we ate able to calculate the conditional expectation $E(C_{ik}|D)$, $i + k > n + 1$, given the data

$$D = \{C_{ik} \mid i + k \leq n + 1\}$$

observed so far, and knowing this conditional expectation is more useful than knowing the unconditional expectation $E(C_{ik})$ which ignores the observation D. Finally, the following theorem shows that using (5) we additionally need only to assume the independence of the accident years, i.e. to assume that

(6) $$\{C_{i1}, \ldots, C_{in}\}, \{C_{j1}, \ldots, C_{jn}\}, i \neq j,$$

are independent, whereas under (4a), (4b). (4c) we had to assume the independence of both. the accident years and the development year increments.

Theorem: Under assumptions (5) and (6) we have fork $> n + 1 - i$

(7) $$E(C_{ik} | D) = C_{i,n+1-i} \, f_{n+1-i} \times \ldots \times f_{k-1}.$$

Proof: Using the abbreviation

$$E_i(X) = E(X | C_{i1}, \ldots, C_{i,n+1-i})$$

we have due to (6) and by repeated application of (5)

$$\begin{aligned} E(C_{ik}|D) &= E_i(C_{ik}) \\ &= E_i(E(C_{ik}|C_{i1}, \ldots, C_{i,k-1})) \\ &= E_i(C_{i,k-1}) \, f_{k-1} \\ &= \text{etc.} \\ &= E_i(C_{i,n+2-i}) \, f_{n+2-i} \times \ldots \times f_{k-1} \\ &= C_{i,n+1-i} \, f_{n+1-i} \times \ldots \times f_{k-1}. \end{aligned}$$

The theorem shows that the stochastic model (5) produces exactly the same reserves as the original chain ladder method if we estimate the model parameters f_k by (2). Moreover. wc see that the projection basis $C_{i,n+1-i}$ in formulae (7) and (1) is not an estimator of the parameter $E(C_{i,n+1-i})$ but stems from working on condition of the data observed so far. Altogcthcr. model (5) employs only n-l parameters $f_1, \ldots, f_{n-1}$. The price for having less parameters than models (3) or (4) is the fact that in model (5) we do not have a good estimator for $E(C_{in})$ which are the additional parameters of models (3) and (4).

But even models (4) do not use $E(C_{in})$ as estimator for the ultimate claims amount because this would not be meaningful in view of the fact that the knowledge of $E(C_{1n})$ is completely useless (because we already know C_{1n} exactly) and that one might have $E(C_{in}) < C_{i,n+1-i}$ (e.g. for $i = 2$) which would lead to a negative claims reserve even if that is not possible. Instead models (4) estimate the ultimate claims amount by estimating

$$C_{i,n+1-i} + E(S_{i,n+2-i} + \dots + S_{in}),$$

i.e. they estimate the claims reserve $R_i = C_{in} - C_{i,n+1-i} = S_{i,n+2-i} + + S_{in}$ by estimating

$$E(R_i) = E(S_{i,n+2-i} + \dots + S_{in}).$$

If we assume that we know the true parameters x_i, y_k of model (4) and f_k of model (5). we can clarify the essential difference between both models in the following way: The claims reserve for model (4) would then be

$$E(R_i) = x_i(y_{n+2-i} + \dots + y_n)$$

independently of the observed data D, i.e. it will not change if we simulate different data sets D from the underlying distribution. On the other hand, due to the above theorem, model (5) will each time yield a different claims reserve

$$E(R_i \mid D) = C_{i,n+1-i}\ (f_{n+1-i} \times \dots \times f_{n-1} - 1)$$

as $C_{i,n+1-i}$ changes from one simulation to the next.

For the practice, this means that we should use the chain ladder method (1) or (5) if we believe that the deviation

$$C_{i,n+1-i} - E(C_{i,n+1-i})$$

is indicative for the future development of the claims. If not, we can think on applying a model (4) although doubling the number of parameters is a high price and may lead to high instability of the estimated reserves as is shown in the appendix.

4. Final Remark

The aim of this note was to show that the loglinear cross-classified model (4c) used by Renshaw. Christofides. Verrall and Zehnwirth is *not* a model underlying the usual chain ladder method because it requires independent and strictly positive increments and produces different reserves. We have also shown that model (5) *is* a stochastic model underlying the chain ladder method. Moreover, model (5) has only $n - 1$ parameters-as opposed to 2, - 1 (or even $2n$) in case of model (4c)—and is therefore more robust than model (4c).

Finally. one might argue that one advantage of the loglinear model (4c) is the factthatit allows to calculate the standard errors of the reserve estimators as has been done by Renshaw 1989. Christofides 1990 and Verrall 1991. But this is possible for model (5). too, as is shown in a separate paper (Mack 1993).

Acknowledgement

I first saw the decisive idea to base the stochastic model for the chain ladder method on conditional expectations in Schnieper 1991.

APPENDIX

NUMERAL EXAMPLE WHICH SHOWS THAT THE LOGLINEAR MODEL (4C) IS MORE VOLATILE THAN THE USUAL CHAIN LADDER METHOD

The data for the following example are taken from the "Historical Loss Development Study," 1991 Edition, published by the Reinsurance Association of America (RAA). There, we fmd on page 96 the following run-off triangle of Automatic Facultative business in General Liability (excluding Asbestos & Environmental):

	C_{i1}	C_{i2}	C_{i3}	C_{i4}	C_{i5}	C_{i6}	C_{i7}	C_{i8}	C_{i9}	GO
$i=1$	5012	8269	10907	11805	13539	16181	18009	18608	18662	18834
$i=2$	106	4285	5396	10666	13782	15599	15496	16169	16704	
$i=3$	3410	8992	I3873	16141	18735	22214	22863	23466		
$i=4$	5655	11555	15766	21266	23425	26083	27067			
$i=5$	1092	9565	15836	22169	25955	26180				
$i=6$	1513	6445	11702	12935	15852					
$i=7$	557	4020	10946	12314						
$i=8$	1351	6947	13112							
$i=9$	3133	5395								
$i=10$	2063									

The above figures are cumulative incurrcd case losses in $ 1000. Wc have taken the accident years from 1981 ($i=1$) to 1990 ($i=10$). The following table shows the corresponding incremental amounts $S_{ik} = C_{ik} - C_{i,k-1}$:

	S_{i1}	S_{i2}	S_{i3}	S_{i4}	S_{i5}	S_{i6}	S_{i7}	S_{i8}	S_{i9}	S_{i10}
$i=1$	5012	3257	2638	898	1734	2642	1828	599	54	172
$i=2$	106	4179	1111	5270	3116	1817	-103	673	535	
$i=3$	3410	5582	488 I	2268	2594	3479	649	603		
$i=4$	5655	5900	4211	5500	2159	2658	984			
$i=5$	1092	8473	6271	6333	3786	225				
$i=6$	1513	4932	5257	1233	2917					
$i=7$	557	3463	6926	1368						
$i=8$	1351	5596	6165							
$i=9$	3133	2262								
$i=10$	2063									

Note that in development year 7 of accident year 2 we have a negative increment $S_{2,7} = C_{2,7} - C_{2,6} = -103$. Because model (4c) works with logarithms of the incremental amounts S_{ik} it cannot handle the negative increments $S_{2,7}$. In order to apply model (4c), we therefore must change $S_{2,7}$ artificially or leave it out. We have tried the following possibilities:

(a) $S_{2,7} = 1$, i.e. $C_{2,7} = 15496 + 104 = 15600$, $C_{2,8} = 16169 + 104$

$= 16273$, $C_{2,9} = 16704 + 104 = 16808$

(b_1) $C_{2,7} = 16000$, i.e. $S_{2,7} = 401$, $S_{2,8} = 169$

(b_2) $S_{2,7}$ = missing value, i.e. $C_{2,7}$ = missing value

When estimating the reserves for these possibilities and looking at the residuals for model (4c), we will identify $S_{2,1} = C_{2,1} = 106$ as an outlier. We have therefore also tried:

C_1 like (b_1) but additionally $S_{2,1} = C_{2,1} = 1500$, i.e. all $C_{2,k}$ are augmentedby $1500 - 106 = 1394$

C_2 like (b_2) butadditionally $S_{2,1} = C_{2,1}$ = missing value.

This yields the following results (the calculations for model (4c) were done using Ben Zehnwirth's ICRFS, version 6.1):

	Total Estimated Reserves	
Possibility	Chain Ladder	Loglinear Model (4C)
unchanged data	52,135	not possible
(a)	52,274	190,754
(b_1)	51,523	102,065
(b_2)	52,963	107,354
(c_1)	49,720	69,999
(c_2)	51,834	70,032

This comparison clearly shows that the two methods are completely different and that the usual chain ladder method is much less volatile than the loglinear cross-classified method (4c).

For the sake of completeness, the following two tables give the results for the above calculations per accident year:

CHAIN LADDER METHOD—ESTIMATED RESERVES PER ACCIDENT YEAR

Acc. Year	Unchanged	(a)	(b_1)	(b_2)	(c_1)	(c_2)
1981	0	0	0	0	0	0
1982	154	155	154	154	167	154
1983	617	616	617	617	602	617
1984	1,636	1,633	1.382	1,529	I ,348	1529
1985	2,747	2.780	2,664	2.964	2.606	2.964
1986	3.649	3.671	3593	3.795	3,526	3.795
1987	5.435	5,455	5.384	5568	5,286	5,568
1988	10.907	10,935	10.838	11,087	10.622	11,087
1989	10.650	10.668	10.604	10,770	10,322	10.770
1990	16.339	16360	16287	16,477	IS,242	15349
1981-90	52.135	52374	51523	52.963	49.720	51,834

LOGLINEAR METHOD—ESTIMATED RESERVES PER ACCIDENT YEAR

Acc. Year	(a)	(b_1)	(b_2)	(c_1)	(c_2)
1981	0	0	0	0	0
1982	309	249	313	282	387
1983	2.088	949	893	749	674
1984	6.114	2,139	2.683	I.675	1.993
1985	3.773	2,649	3.286	2,086	2.602
1986	6.917	4.658	5,263	3,684	4,097
1987	9.648	6,312	6.780	4.968	5.188
1988	24.790	15,648	16.468	12,000	12.174
1989	36.374	21.429	22.213	15,545	15.343
1990	100,739	48,033	49,454	29,010	27,575
1981-90	190.754	102.065	107.354	69,999	70.032

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Using Expected Loss Ratios in Reserving

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Using expected loss ratios in reserving

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Abstract: The required loss reserve for a recent time period is estimated by using the recent loss experience plus two probability distributions. One distribution is of ultimate losses for the recent period, based on prior experience and rate adequacy changes. The other distribution is of the ratio of the estimator based on recent experience to the true ultimate loss.

Keywords: Loss reserving; Expected loss ratio.

1. Introduction

This paper presents a method of using expected loss ratios, together with prior and posterior distributions, in order to estimate loss reserves. This Bayesian method is especially useful for recent accident years and for lines of business with slow development. It incorporates, in a rigorous way, the degree of reliability of the expected loss ratio and of the loss development factors. Estimates of ultimate loss ratios for recent accident years can be important factors in underwriting decisions.

A method of using expected loss ratios which is now well-known was presented by Bornhuetter and Ferguson (1972). The ultimate losses of an accident year are estimated by using the prior expectation of ultimate losses (expected losses) as well as the reported losses and the selected development factor to ultimate. The ultimate losses are estimated as

$$\text{reported losses} + (1 - z)(\text{expected lows}), \quad (1)$$

where z is the reciprocal of the development factor to ultimate.

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It is implicit in this method of estimation that the expecred development for an accident year in each future year is independent of the reported losses.

If 'developed losses' is defined as the product of the reported losses and the development factor to ultimate, then formula (1) can be expressed as

$$z(\text{developed losses}) + (1 - z)(\text{expected losses}). \quad (2)$$

Bornhuetter–Ferguson and Bayesian estimates of loss reserves will be compared in an example later in this paper.

2. The model

In a Bayesian approach, the prior expectation of ultimate losses for an exposure period E may be an estimate made several years after the beginning of E. If ultimate loss ratios are estimated for the same line of business for the insurer for previous periods, and industry-wide data as well as the insurer's changes in premium adequacy are taken into account, an estimate of the ultimate loss ratio for the period E can be made prior to considering the reported losses for E.

The following direct application of Bayes' theorem is basic to this discussion. Let $f(x)$ be the probability density function of the distribution of ultimate losses for exposure period E prior to considering the losses for E. Let $g(y \mid x)$ be the probability density function of the distribution of y, the developed losses defined previously, for E as of t months, given that the ultimate losses are x. Assume that this distribution has mean x. Let $h(x \mid y)$ be the probability density function of the distribution of the ultimate losses given that the developed losses are y. Then

$$h(x|y) = g(y|x)f(x) \Big/ \int_0^\infty g(y|x)f(x)\,\mathrm{d}x. \quad (3)$$

In order to use the above proposition, it is necessary to estimate $g(y \mid x)$ and f(x). The mean of the distribution given by $h(x \mid y)$ will be the estimate of ultimate losses.

The variance of the distribution given by $g(y|x)$ can be estimated from a study of the historical variability of developed loss ratios at different stages of development. The variance of the distribution given by $f(x)$ can be estimated from the differences between prior expectations of ultimate losses for previous periods, based on the current method of predicting, and the latest developed losses for those periods. The estimated variances between the latest developed losses and the ultimate losses for those periods will also be considered. Historical data of the above types should be supplemented by judgement, experience, and related data.

If a method other than development factors is used for projecting the loss data to ultimate, Bayes' theorem can still be applied as above with $g(y|x)$ defined as necessary.

In order to apply Bayes' theorem to a set of accident years, a single development factor to ultimate for the period can be selected as follows. Estimate the ratios between the ultimate losses for each accident year by using the premium and the estimated relative rate adequacy for each year. Then use the reciprocal of the development factor for each year to estimate the ratio of the total ultimate losses for the period to the expected losses for the period at the stage of development. See Bühlmann's Cape Cod method [Schnieper (1991), Straub (1988)].

Bühlmann's (1967) formula for the least squares line estimate of the Bayesian estimates could be used to estimate the credibility of the actual developed losses. [This credibility approximation is exact Bayesian in certain useful cases. In the proof of formula (4), below, we use a special case of Jewell's result that credible means are exact Bayesian for exponential families. See Jewell (1974, 1975).] This method has the advantage of simplicity since it does not require the choice of particular distributions.

3. Lognormal distributions

Let $f(x)$, $g(y|x)$, and $h(x|y)$ be defined as for formula (3). For certain choices of $f(x)$ and $g(y|x)$, an explicit formula for the mean of $h(x|y)$ is known. An important example is the case in which $f(x)$ and $g(y|x)$ represent lognormal distributions. This is a reasonably good fit in many cases.

Suppose that the prior probability distribution of logs of ultimate losses has mean μ and variance ν^2. Suppose that for all x, the distribution, given ultimate losses x, of logs of actual developed losses has variance σ^2. Note that if x is the mean of a lognormal distribution and m and s^2 are the mean and variance of the distribution of the logs, then $\log x = m + s^2/2$. Therefore, for all x the distribution of logs of actual developed losses has mean $\log x - \sigma^2/2$. Then the mean of the distribution given by $h(x|y)$ (and thus the estimate of ultimate losses) is

$$\exp(\mu_1 + \nu_1^2/2), \tag{4}$$

where

$$\mu_1 = (1-z)\mu + z(\log y + \sigma^2/2), \tag{5}$$

$$\nu_1^2 = \sigma^2 z, \tag{6}$$

$$z = \nu^2/(\sigma^2 + \nu^2). \tag{7}$$

The derivation is given in the appendix.

Example. Assume that, based on historical experience as described previously, the prior distribution for an insurer's overall ultimate loss ratio for 1987–91 for medical malpractice has a mean of 0.90 (i.e. 90%) and a variance of 0.16. Suppose the selected development factor to ultimate for 1987-91 reported losses as of 12/31/91 is 2.065 and the probability distribution for the ratio of the developed losses to the ultimate losses has a variance of 0.075.

If both of the above distributions are lognormal, then μ, ν^2 and σ^2 in equations (5) and (6) can be found by solving the following equations for the mean and variance of lognormal distributions:

$$0.90 = \exp(\mu + \nu^2/2), \tag{8}$$

$$0.16 = \exp(2\mu + \nu^2)(\exp(\nu^2) - 1), \tag{9}$$

$$1.00 = \exp(m + \sigma^2/2), \tag{10}$$

$$0.075 = \exp(2m + \sigma^2)(\exp(\sigma^2) - 1). \tag{11}$$

By squaring both sides of equation (8) and then dividing by the corresponding sides of equation (9), we get

$$(0.90)^2/0.16 = 1/(\exp(\nu^2) - 1). \tag{12}$$

Table 1
Comparison of methods of estimation.

Actual developed loss ratio	Bayesian estimate of ultimate loss ratio	Bornhuetter–Ferguson estimate of ultimate loss ratio
20%	32%	56%
40%	52%	66%
80%	85%	85%
160%	139%	124%
320%	229%	201%

Solving for ν^2 and μ is then immediate. The same method can be used for σ^2 and m. The solutions are 0.180, -0.195, and 0.072, respectively, for ν^2, μ, and σ^2, so formula 4 becomes $\exp(-0.004 + 0.714 \log y)$. So, if y = 20%. for example, the estimated ultimate loss ratio is 32%. Table 1 compares three methods of estimation.

Appendix: Derivation of formula (4)

The following lemma will be used.

Lemma. *Suppose that an element is chosen at random from a normal distribution for which the value of the mean θ is unknown $(-\infty < \theta < \infty)$ and the value of the variance σ^2 is known $(\sigma^2 > 0)$. Suppose also that the prior distribution of θ is a normal distribution with given values of the mean μ and the variance ν^2. Then the posterior distribution of θ, given that the element chosen equals x_1, is a normal distribution for which the mean μ_1 and the variance ν_1^2 are as follows:*

$$\mu_1 = (\sigma^2\mu + \nu^2 x_1)/(\sigma^2 + \nu^2), \tag{A.1}$$

$$\nu_1^2 = (\sigma^2\nu^2)/(\sigma^2 + \nu^2). \tag{A.2}$$

See DeGroot (1986) for the-proof of the above.

Proof of formula (4). The mean and variance of the distribution, given ultimate losses x, of $\sigma^2/2$ + log(developed losses). are $\log x$ and σ^2, respectively. The prior distribution of log(ultimate losses) has mean μ and variance ν^2. Therefore, the posterior distribution of log(ultimate losses), given $\sigma^2/2$ + log(developed losses) = x_1, has mean μ_1 and variance ν_1^2 given in the Lemma, where $x_1 = \sigma^2/2$ + log(developed losses). Therefore, the distribution of ultimate losses has mean $\exp(\mu_1 + \nu_1^2/2)$.

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Expected Loss Development: A Shift in Credibility

Christopher J. Poteet

Expected Loss Development: A Shift in Credibility

Christopher J. Poteet

This paper is a commentary on the previously published paper "Partial Loss Development Based On Expected Losses For Workers' Compensation Class Ratemaking", Casualty Actuarial Society Forum. Special Edition. 1993 Ratemaking Call Papers.

This paper shows that expected loss development is equivalent to adjusting the full credibility standard and applying credibility by policy period.

Expected Loss Development: A Shift in Credibility

Concerns with the current loss development method used in Workers' Compensation class ratemaking have been raised. If a class has zero losses at a first report, using a first to ultimate loss development factor produces zero ultimate losses as well. One possible solution that has been proposed is to use expected loss development. To simplify the illustration, assume that all losses are at the same benefit level etc., so as to only look at loss development. The other factors can easily be taken into account later. Also for simplicity assume that there is only one policy period used and national pure premiums are not used. The following arguments will then be extended to include more policy periods and the use of national pure premiums.

Workers' compensation classification ratemaking relies on several estimates of class pure premiums. One estimate is based on the latest available data for the class and state. This is called the indicated pure premium. Another estimate is the pure premium underlying current rates brought up to the level of the indicated pure premiums. This estimate is called the present on rate level pure premium. A third estimate is a national pure premium which includes data from other states adjusted to reflect conditions in the reviewed state. A formula pure premium to be used in calculating rates, is obtained by credibility weighting these estimates.

Here is a brief description of expected loss development. Initially, expected losses E (present on rate level pure premium times payroll in hundreds) is the estimate of ultimate losses used to calculate the indicated pure premium. At a first report the actual losses A which have emerged at that point can replace the losses that were expected to have emerged by then, namely (1/D)E, where D is the first to ultimate loss development factor. This method relies less on actual losses and more on expected losses than the current method. It is important to note that if the development factor is less than one, the estimate of ultimate, losses might be negative.

Credibility weighting produces the losses used in the formula pure premium:

Expected Loss Development: Losses used in Formula Pure Premium

$$Z[A+(1-\frac{1}{D})E] + (1-Z)E$$

$$= ZA + ZE - \frac{Z}{D}E + E - ZE$$

$$= ZA - \frac{Z}{D}E + E$$

$$= \frac{Z}{D}AD + \left(1-\frac{Z}{D}\right)E$$

Current Method: Losses used in Formula Pure Premium

$$ZAD + (1-Z)E$$

These two formulas are equivalent where Z/D is substituted for Z. Using Z/D instead of Z is equivalent to changing the full credibility standard which already limits fluctuations of formula pure premiums to a desired amount. For example. if $Z=(n/n_f)^{1/2}$ and $D=3$, then $Z/D=(n/9n_f)^{1/2}$. The expected loss development method implicitly lowers credibility by $1/D$, when D> 1. Expected loss development is a shift in credibility, giving less weight to actual losses and more weight to expected losses.

The equation which shows that expected loss development is equivalent to changing the full credibility standard can be expanded to include more policy periods and the use of national pure premiums. The relationship holds if the credibility of indicated data is calculated by policy period and the national credibility is allowed to remain unchanged as one switches from one method to the other.

Attached is a detailed algebraic proof of the equivalence relationship (Attachment 1). The proof shows that the serious (or nonserious or medical) formula pure premium calculated using expected loss development is equal to the serious (or nonserious or medical) formula pure premium calculated by using credibility by policy period. where the credibility one would normally use is divided by the policy period's development to ultimate factor and multiplied by a factor reflecting the contribution of the policy period's exposure to the total. These individual credibilities are then used as weights for the indicted pure premiums calculated separately for each individual policy period.

Also attached is a specific illustration (Attachment 2) of the equivalence relationship which uses the example from exhibit 1 of the paper "Partial Loss Development Based On Expected Losses For Workers' Compensation Class Ratemaking". Casualty Actuarial Society Forum. Special Edition. 1993 Ratemakine Call Papers, as well as the development factors listed in the paper on page 321 (See attachment 3). Note that, as a separate issue, the state credibilities in the paper are calculated using a square root rule instead of NCCI's old two thirds rule so that the serious state credibility of .67 is equal to .59 to the three fourths power $[.67=(.59^{3/2})^{1/2}]$.

The illustration focuses on the calculation of the serious formula pure premium. More recent years have higher development factors so credibility is lowered more for them. This could be considered a reliability factor. Each year's credibility also gets multiplied by a weight equal to the year's proportion of exposure to the total of all years. This could be considered a relevance factor since more recent years would tend to have higher exposures due to wage inflation, all else being constant.

Expected loss development can be thought of as a shift in credibility from the indicated pure premiums to the present on rate level pure premium (See table below). Note that expected loss development relies heavily on the present on rate level pure premium to the extent that the indicated is not considered credible, whereas the new NCCI full credibility standard and partial credibility formula give equal weight to the present on rate level pure premium and the national pure premium.

NCCI now uses higher full credibility standards and a .4 power partial credibility formula to recognize the need for stability. Note that the credibility given to the indicated data using the new NCCI standard and formula is about the same as the credibility for expected loss development, therefore limiting fluctuations by about the same amount as expected loss development, An advantage to the expected loss development scheme is the consideration of different credibilities by policy period.

Credibilities - Class 7600

Serious Pure Prem	Indicated	National	PORL
Current Loss Development	.67	.16	.17
Expected Loss Development	.33	.16	.51
New NCCI Standard And Formula	.38	.31	.31

A_1 = actual first report losses, A_2 = second report, A_3 = third report
D_1 = first to ultimate loss development factor, D_2 = second to ultimate, D_3 = third to ultimate
E_1 = ultimate expected losses for first report, E_2 = second report, E_3 = third report
$E = E_1 + E_2 + E_3$
P_1 = first report payroll in hundreds, P_2 = second report, P_3 = third report
$P = P_1 + P_2 + P_3$
Z = state indicated credibility
Z_n = national credibility
N/P = national pure premium
E/P = present on rate level pure premium
$E_1 = (E/P)P_1$, $E_2 = (E/P)P_2$, $E_3 = (E/P)P_3$

Expected Loss Development: Formula Pure Premium

$$Z\left[\frac{(A_1+(1-\frac{1}{D_1})E_1)+(A_2+(1-\frac{1}{D_2})E_2)+(A_3+(1-\frac{1}{D_3})E_3)}{P}\right]+(1-Z-Z_n)\left[\frac{E}{P}\right]+Z_n\left[\frac{N}{P}\right]$$

$$=\frac{Z(A_1+A_2+A_3)+Z(E_1+E_2+E_3)-\frac{Z}{D_1}E_1-\frac{Z}{D_2}E_2-\frac{Z}{D_3}E_3+E-ZE-Z_nE+Z_nN}{P}$$

$$=\frac{\frac{Z}{D_1}A_1D_1+\frac{Z}{D_2}A_2D_2+\frac{Z}{D_3}A_3D_3+ZE-\frac{Z}{D_1}E_1-\frac{Z}{D_2}E_2-\frac{Z}{D_3}E_3+(E_1+E_2+E_3)-ZE-Z_nE+Z_nN}{P}$$

$$=\frac{[\frac{Z}{D_1}A_1D_1+(1-\frac{Z}{D_1})E_1]+[\frac{Z}{D_2}A_2D_2+(1-\frac{Z}{D_2})E_2]+[\frac{Z}{D_3}A_3D_3+(1-\frac{Z}{D_3})E_3]-Z_nE+Z_nN}{P}$$

$$=(\frac{Z}{D_1})(\frac{P_1}{P}\frac{A_1D_1}{P_1})+(\frac{Z}{D_2})(\frac{P_2}{P}\frac{A_2D_2}{P_2})+(\frac{Z}{D_3})(\frac{P_3}{P}\frac{A_3D_3}{P_3})$$

$$+[(1-\frac{Z}{D_1})\frac{P_1}{P}+(1-\frac{Z}{D_2})\frac{P_2}{P}+(1-\frac{Z}{D_3})\frac{P_3}{P}](\frac{E}{P})-Z_n(\frac{E}{P})+Z_n(\frac{N}{P})$$

$$= \left(\frac{Z}{D_1}\frac{P_1}{P}\right)\left(\frac{A_1 D_1}{P_1}\right) + \left(\frac{Z}{D_2}\frac{P_2}{P}\right)\left(\frac{A_2 D_2}{P_2}\right) + \left(\frac{Z}{D_3}\frac{P_3}{P}\right)\left(\frac{A_3 D_3}{P_3}\right)$$

$$+ \left[1 - \left(\frac{Z}{D_1}\frac{P_1}{P}\right) - \left(\frac{Z}{D_2}\frac{P_2}{P}\right) - \left(\frac{Z}{D_3}\frac{P_3}{P}\right) - Z_n\right]\left(\frac{E}{P}\right) + Z_n\left(\frac{N}{P}\right)$$

Current Method: Formula Pure Premium

$$Z\left[\frac{A_1 D_1 + A_2 D_2 + A_3 D_3}{P}\right] + (1 - Z - Z_n)\left[\frac{E}{P}\right] + Z_n\left[\frac{N}{P}\right]$$

Serious pure premium - class 7600

st cred	3rd rpt pay		3rd rpt dev loss	
0.67	42,616,748	3rd rpt cred	393,906	3rd rpt ind pp
------------- *	--------------- =	0.15	-------------------- =	0.924
1.417	135,892,859		42,616,748/100	
3rd-ult dev	total pay		3rd rpt pay	

st cred	2nd rpt pay		2nd rpt dev loss	
0.67	49.728.462	2nd rpt cred	145,463	2nd rpt ind pp
------------- *	--------------- =	0.12	-------------------- =	0.293
1.993	135,892,859		49,728,462/100	
2nd-ult dev	total pay		2nd rpt pay	

st cred	1st rpt pay		1st rpt dev loss	
0.67	43,547,649	1st rpt cred	1,731,862	1st rpt ind pp
------------- *	--------------- =	0.06	-------------------- =	3.977
3.773	135,892,859		43,547,649/100	
1st-ult dev	total pay		1st rpt pay	

nat cred	nat pure prem
0.16	1.287

remaining cred	porl pure prem
0.51	1.203

form pure prem

0.15*0.924+0.12*0.293+0.06*3.977+0.16*1.287+0.51*1.203= 1.221

(float from the start to eliminate rounding difference)

CASUALTY ACTUARIAL SOCIETY FORUM

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NOTICE

The Casualty Actuarial Society is not responsible for statements or opinions expressed in the papers or reviews in this publication. These reports, papers, and reviews have not been reviewed by the CAS Committee on Review of Papers.

1995 CAS *Forum*, Fall 1995

To: CAS Members

This is the 1995 edition of the Casualty Actuarial Society ***Forum.*** Besides several new papers, this publication includes four reprinted papers and three committee reports. I would like to thank the other committee members:

Paul Lacko	Kimberly Ward
James Wilson	Gerald Yeung

I hope you enjoy this publication.

Sincerely yours,

George R. Busche
CAS ***Forum,*** Chairperson

Credits

"Using Expected Loss Ratios in Reserving," by Daniel Gogol is reprinted from ***Insurance Mathematics and Economics,*** Vol. 12 (1993), ppp. 297-299, with kind permission from Elsevier Sci-*ence* B.V.. Amsterdam, The Netherlands.

"Which Stochastic Model is Underlying the Chain J-adder Method?" was originally published in ***Insurance: Mathematics ond Economics.*** It was awarded the first ever CAS Charles Hachemeister Prize for the best 1992-1993 ASTIN paper, and was presented at the ASTIN Colloquium 1993 in Cambridge (U.K.).

"When the Wind Blows," by D.E.A. Sanders was presented at the ASTM Colloquium 1993 in Cambridge (U.K.).

"A Simulation Procedure for Comparing Different Claims Reserving Methods," by Teivo Pentikäinen and Jukka Rantala was originally published in the ***ASTIN Bulletin.***

Apology

It has been brought to my attention that the paper "Duration, Hiding in a Taylor Series" by Keith Holler, which was published in the Summer 1994 Casualty Actuarial Society ***Forum*** had some similarities to the paper "Asset/Liability Matching (Five Moments)," by Robert K. Bender, published in the ***1993*** CAS ***Proceedings.***

We apologize if the timing of the publications caused any confusion or embarrassment to the authors, and the CAS membership in general. The ***Forum*** Committee has put procedures in place to do a better job in coordinating the two publications.

Sincerely yours,

Joel Kleinman
Past Chairperson
The Casualty Actuarial Society ***Forum*** Committee

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Report of the Travel Time Working Group
by the Travel Time Working Group

REPORT OF THE TRAVEL TIME WORKING GROUP

FEBRUARY, 1995

Working Group Members:

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TRAVEL TIME WORKING GROUP REPORT

Preamble

The following report of the Travel Time Working Group represents the culmination of a 2 year effort to establish the information needs of the CAS necessary to monitor travel time, ensure that the CAS database contains the requisite information, define the criteria by which travel time should be monitored and draw preliminary conclusions regarding the impact of exam partitions on travel time, if possible.

The Executive Council and Board of Directors discussed this report at several meetings during the third and fourth quarters of 1994 and the first quarter of 1995. At its February, 1995 meeting the Board of Directors adopted the data collection and data monitoring recommendations in the report and authorized distribution of the report to the CAS membership through publication in the CAS Forum.

In discussing the report, the Board acknowledged that the statistics included in the report are inconclusive at this time vis-à-vis travel time. Even if travel time was changing, the Board agreed that it might not be possible to isolate the effect of exam partitioning on travel time. Finally, the Board re-affirmed its earlier decision to take no further partitioning steps at this time.

I. Backaround

Partitioning of the examinations required for membership in the Casualty Actuarial Society began with Part 3 in May, 1987 in coordination with the implementation by the Society of Actuaries of their Flexible Education System (FES). Subsequently, the CAS Board of Directors requested that the Education Policy Committee address the issue of whether the CAS should adopt a Flexible Education System.

The Education Policy Committee report was presented to the Board of Directors in September, 1988. That report, in the form of a "White Paper" was distributed to the membership in March, 1989 and was accompanied by a letter from the President requesting that the membership carefully consider the contents of the "White Paper" and provide comments on the recommendations contained therein.

Subsequent to the September, 1988 Board meeting, the Partitioned Exam Task Force (PETF) was created to determine whether an implementation plan could be developed which would address the issues contained in the "White Paper". The PETF submitted its report to the Education Policy Committee in October, 1990.

The recommendations of the Education Policy Committee and a supplemental analysis made by the Vice President - Admissions was presented to the Board of Directors at its November 11, 1990 meeting. After consideration of the reports of the PETF and Vice President - Admissions, and substantial discussion and debate the Board decided to partition Part 4 effective with the May, 1992 exam administration and Part 5 effective with the November, 1993 administration. Each of these exams began being offered twice a year coincident with their partitioning. The Board also decided that Parts 6 and 7 would not be partitioned and consideration of 'partitioning of the Fellowship exams would be deferred for at least three years. The EPC "White Paper", PETF Report and various letters to the membership on partitioning are contained in the Winter 1991 edition of the CAS Forum on pages 189-467.

The Travel Time Working Group was created in February, 1993 in response to the Board of Directors' desire to ensure that the database structure, reports and analytical tools necessary to monitor the impact of partitioning on travel time would be established before post-partitioning candidate performance information became available. The assignment included:

- A determination of the information required to monitor travel time.

- An opinion regarding the sufficiency of the CAS database to evaluate the impact of partitioning on travel time.

- Conclusions, if any, which can be drawn at this time regarding the impact of partitioning on travel time.

The working group did not consider its charge to include, nor did it examine the question of whether partitioning has been successful in better educating actuaries.

II. Workina Group Deliberations

In the course of its deliberations, the working group met via teleconference on March 26, August 30, October 7 and October 21, 1993 and March 17, 1994 as well as at meetings at the CAS office on April 22, 1993 and June 6, 1994. During the course of those meetings, the group identified a number of key concepts for monitoring travel time and additional information that needed to be included in the CAS database in order to develop the necessary statistics.

The Working Group realized that it is not possible to separate partitioning from other factors affecting travel time. Any evaluation of travel time includes the impact of both partitioning and all other factors. Changes in the frequency of exam administration, the number of candidates entering the system, candidate taking CAS exams for SoA and CIA credit and the passing standards set by the

Examination Committee are all examples of phenomenon that can affect travel time. Therefore, it may be impossible to isolate the impact of partitioning on travel time.

III. Recommendations for Monitoring Travel Time

The Working Group makes the following recommendations regarding the monitoring of the effect of partitioning on travel time.

- Assign primary responsibility for monitoring and interpreting travel time statistics to the Education Policy Committee. See Section IX.

- Identify candidate cohorts so that travel time can be compared from one group of candidates to the next. See Section V.

- Establish historical baselines before drawing any conclusions regarding the impact of partitioning *on* travel time.

- Define travel time to membership as the number of years from the first time any exam (or part thereof) in the sequence 3 through 7 is passed through the attainment of Associateship.

- In order to gain early insight into any impact of partitioning on travel time, monitor travel time for the exam sequence 3 through 5.

- Consider monitoring travel time from ACAS to FCAS, although this is not relevant to partitioning.

- Start with the May, 1987 cohort. See Section V.

- If a longer historical period is needed, evaluate the cost/benefit of obtaining the necessary information from the SoA. See Section VI.

- Using cohort success information as displayed in Appendix A, focus on changes in the time necessary for a common success level to be reached. The Working Group believes success levels of 20%, 35% and 50% are useful benchmarks. See Section V.

- Do not monitor travel time for an individual exam. With the advent of partitioning the entire dynamic interplay between various exams has changed. Travel time through individual exams could lengthen while total travel time does not. See Appendix C.

- Monitor student exam strategy and performance on partitioned exams It is important to know if students are taking fewer exams and how their performance is related to exam load. See Sections V and VIII.

IV. Initial Observations

The Working Group made the following initial observations:

- Travel time appears to have been increasing prior to partitioning of Parts 4 and 5. The exception appears to be with the early percentiles (1 0%, 20%) for the 3 through 7 exam group. See Appendix A for an example of the success level of 35%.

- Although it is too early to make a definitive statement, it appears that travel time has increased subsequent to partitioning of Parts 4 and 5.

- A significant percentage of candidates are opting to take just a single exam subpart even though there are other subparts which they still need. See Appendix B. As a group, these candidates are not as successful (as measured by passing percentages) as the candidates who take a fuller exam load.

V. Kev Concepts Identified

Four key concepts emerged:

- Measurement bv Cohort: Candidates must be grouped so that performance comparisons can be made. The working group defined a cohort as the set of candidates in an exam period who first took any exam in a group of exams for which travel time is to be measured. For example, if travel time from Part 3 through ACAS is to be measured, the cohort for each exam session would be those candidates who first took any exam in the sequence 3 through 7 during that exam session. Candidates who first took any exams in the 3 through 7 sequence in May, 1990, would be members of the May, 1990 cohort and so on. Once a candidate is assigned to a cohort he or she remains in that cohort.

- Establishment of Base-Line: In order to evaluate whether partitioning is having any impact on travel time it is necessary to know what the trend in travel time was prior to partitioning. In other words, a baseline, or history, would have to be created. Because of the possible impact on travel time of changes in the number of exams in the early 1970s and the difficulty in obtaining candidate registration information on Part 3 prior to 1987, the working group decided that any baseline evaluation should start with the May, 1987 cohort if Part 3 is to be included in the analysis. If Part 3 is to be excluded, the baseline evaluation should start with the November, 1982 cohort, which is when Part 4 became a CAS only exam.

- Measurement of Travel Time: The working group believes that travel time should be measured and changes in travel time monitored from the perspective of the number of years it takes cohorts to reach various completion levels for the same series of exams. Since most cohorts do not attain 100% completion, measurements of travel time cannot be made on that basis. Consequently, the Working Group examined travel time at various percentile completion points and concluded that 20%, 35% and 50% are useful benchmarks.

Student Exam Strategy: While not directly related to the measurement of travel time, observing candidate exam load (partial exams vs. full exams) for each sitting and the relative success of students under different strategies can provide an indication of why any change in travel time is occurring. The working group developed a report (see Appendix B) which provides information on what parts or sub-parts students are taking and their success on these parts of sub-parts.

VI. Database Enhancements

During the course of the Working Group's deliberations, needs for enhancements to the database were identified. These included:

- Update of Exam Histories: For pre-1991 Fellows, the exam history on the database was incomplete in that the record would contain the fact that an exam had been passed but not the date that it had been passed. In addition, the dates for passing or failing jointly sponsored exams were neither included in the paper records nor the database. Since the date of passing an exam was critical for measuring travel time and failure dates for Part 3 are also needed to assess candidate exam strategies the database records had to be updated for the missing dates.

  Because the SoA has electronic records back only to 1987, the database could not easily be updated for Part 3 prior to 1987. Information on earlier administrations are contained in paper files maintained by candidate, not exam administration. In order for the CAS to obtain the necessary information, a manual review of these files would be required, which could prove to be costly and time consuming with no guarantee of complete accuracy.

- Attainment of ACAS Status: Records for many Fellows did not include the date (month/year) that ACAS status was attained. In order to establish a historical baseline for travel time to ACAS, this information was obtained and entered.

- Attainment of FCAS Status: Records for many Fellows included only the year of fellowship. In order to establish a baseline for travel time to fellowship, the month is required as well. This information was obtained and entered. These tasks required a painstaking search of old yearbooks, SoA pass lists and other paper records. The necessary information has been obtained and recorded by the CAS office staff.

VII. Adjustments to Cohorts

Once the concept of cohorts had been defined the Working Group was concerned with changes in the ultimate success rate caused by:

- SoA credit being granted for some CAS exams.

- SoA members who took joint exams many years ago and are now returning to pursue membership in the CAS.

In order to minimize the impact of these phenomena, edits were instituted to remove from the database any candidates who could be identified as fitting into either of these groups.

The Working Group also considered removing from the database candidates who ceased taking exams before reaching ACAS status or took exams intermittently. The database was edited to remove candidates who:

- Did not sit for a CAS exam in the most recent exam period (11/93);

- Did not sit for a CAS exam in at least 50% of the exam periods since the cohort was formed and who had not yet attained ACAS status.

The Working Group rejected these adjustments because they would eliminate different segments of the original cohort depending on the maturity of the cohort For example, if we are trying to evaluate if there has been a change in what percentage of a cohort had successfully completed a series of exams within 3 years of the cohort being defined, then the database adjustments that were rejected by the Working Group would impact a cohort that had matured 6 years since its formation much more than a cohort that had just reached 3 years maturity.

Appendix D summarizes the number of candidates from each cohort that would be removed by each of these edits as of the 11193 exam administration as well as the number that would be eliminated by the application of both criteria (the intersection of both sets).

VIII. Modeling of Travel Time

What should be expected to be seen in the travel time charts? To address this, the working group considered the negative binomial model. The negative binomial distribution gives the probability of k failures before n successes. From this distribution it is possible to get the distribution of travel time by dividing the sum of k and n by the number of exams taken per year

In Appendix E, we show that a consequence of the negative binomial model is that we should not expect an increase in travel time if exams are partitioned and students continue to take "full exams" at the same rate.

However, if students pass all but one of the subparts in a range of exams, they cannot take the remaining exam at the same rate, although, in reality, they could be taking exams outside the range. In Appendix E we show that this effect could add as much as 0.75 years to the observed travel time. We call this effect the "last exam effect".

We now turn to actual results. Appendix A, Sheets 3 & 4 gives the most recent travel time plots for Parts 3 through 5. As a point of information, it should be noted that the partitioning of Part 3 was introduced in 1987, the partitioning of Part 4 was introduced in 1992 and the partitioning of Part 5 was introduced in 1993. It is clear that lower passing percentages are happening after the introduction of partitioning. How much of this can be attributed to the last exam effect is not clear from this exhibit.

Additional preliminary observations can be made by examining exam-taking patterns. Appendix B, Sheet 1 provides a summary of exam-taking patterns for recent exams through May, 1994. As can be seen from this exhibit, the number of students that take only one part of a partitioned exam has been on an upward trend. Also, Appendix B suggests that students who take just a partial exam do not perform any better than the rest of the student population, as measured by pass ratio.

IX. Ongoing Monitoring of Travel Time

Travel Time is a diagnostic concept relative to the admission of members to associateship and fellowship status in the Casualty Actuarial Society. As such, it demands the awareness of CAS general management and is a specific responsibility of the Vice President -Admissions. While the concept is simple, measuring the time and effort it takes to get through actuarial exams, fact gathering and interpretation is an elusive endeavor.

One thing is clear, basic tracking information must be available in a form that allows consistent time series performance observations of the candidate universe. The CAS office has established a data base and has begun to support this performance observation process. The CAS office should retain this responsibility and make whatever changes are deemed appropriate by the leadership of the CAS.

The Vice President -Admissions delegates various responsibilities to the Education Policy Committee, Syllabus Committee and Examination Committee. The Syllabus Committee is responsible for determining the content, depth, breadth and jurisdictional flavor of the learning materials on which candidates are to be tested. The Examination Committee has direct control over the amount of material reflected in each exam, the difficulty of questions to be answered and exam specific measurement of candidate performance. The Education Policy Committee is responsible for the practice emphasis, education techniques (exams vs. papers or academic work, on the job training vs. formal, continuing education vs. on time qualification), alternative qualifications and educational liaison with other actuarial bodies throughout the world. The Education Policy Committee must also deal with the general motivation and preparedness of the candidate universe.

While each of the Admissions Committees has an impact on the travel time of candidates, the primary responsibility for monitoring and interpreting travel time statistics should rest with the Education Policy Committee which can draw on the expertise of the other Admissions Committees for assistance.

APPENDIX A

APPENDIX A

Important Note

The Parts 3-7 and 3-5 exhibits were produced from the database at different points in time. In the time interval between the production of these exhibits the database was updated for the results of the May, 1994 exam administration as well as the results of the ongoing project to complete the exam histories of past exam takers. Therefore, the two exhibits may be inconsistent with regard to the identification of cohort membership.

Jul-94

Steady State Test Parts 3-7: Cumulative Percentage

	Unedited Pop.	*Edited* Pop.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
May 77	60	60	0.00%	0.00%	3.33%	21.67%	31.67%	35.00%	40.00%	45.00%	53.33%	56.67%	63.33%	65.00%	65.00%	66.67%	70.00%	70.00%	70.00%	70.00%	70.00%	70.00%	70.00%	70.00%	71.67%
Nov 77	49	48	0.00%	0.00%	0.00%	18.75%	20.83%	35.42%	37.50%	52.08%	52.08%	58.33%	58.33%	60.42%	60.42%	64.58%	66.67%	68.75%	68.75%	70.83%	70.83%	72.92%	75.00%	75.00%	75.00%
May 78	77	77	0.00%	0.00%	0.00%	3.90%	12.99%	15.58%	23.38%	28.57%	36.36%	36.36%	46.75%	48.05%	51.95%	51.95%	57.14%	59.74%	64.94%	64.94%	67.53%	67.53%	71.43%	71.43%	71.43%
Nov 78	61	61	0.00%	0.00%	0.00%	8.20%	9.84%	22.95%	27.87%	42.62%	44.26%	49.18%	49.18%	55.74%	55.74%	60.66%	60.66%	60.66%	60.66%	62.30%	62.30%	63.93%	63.93%	63.93%	63.93%
May 79	62	62	0.00%	0.00%	1.61%	3.23%	8.06%	11.29%	22.58%	22.58%	33.87%	33.87%	41.94%	41.94%	46.77%	50.00%	51.61%	51.61%	56.45%	56.45%	56.45%	56.45%	59.68%	59.68%	59.68%
Nov 79	108	108	0.00%	0.00%	0.00%	5.56%	8.33%	12.96%	14.81%	24.07%	24.07%	34.26%	35.19%	47.22%	47.22%	51.85%	52.78%	53.70%	53.70%	56.48%	56.48%	58.33%	59.26%	60.19%	60.19%
May 80	72	72	0.00%	0.00%	1.39%	1.39%	12.50%	12.50%	25.00%	26.39%	33.33%	36.11%	52.78%	52.78%	65.28%	65.28%	66.67%	66.67%	66.67%	66.67%	68.06%	69.44%	70.83%	70.83%	73.61%
Nov 80	66	64	0.00%	0.00%	0.00%	7.81%	7.81%	17.19%	18.75%	28.13%	29.69%	40.63%	42.19%	50.00%	53.13%	62.50%	64.06%	65.63%	65.63%	67.19%	67.19%	70.31%	70.31%	70.31%	70.31%
May 81	70	69	0.00%	0.00%	1.45%	2.90%	7.25%	11.59%	24.64%	24.64%	34.78%	36.23%	37.68%	39.13%	43.48%	43.48%	50.72%	52.17%	57.97%	59.42%	60.87%	63.77%	65.22%	65.22%	65.22%
Nov 81	79	79	0.00%	0.00%	0.00%	3.80%	3.80%	13.92%	13.92%	16.46%	21.52%	24.05%	24.05%	31.65%	31.65%	40.51%	40.51%	44.30%	45.57%	45.57%	46.84%	46.84%	46.84%	48.10%	48.10%
May 82	79	79	0.00%	0.00%	0.00%	0.00%	8.86%	11.39%	20.25%	24.05%	29.11%	30.38%	32.91%	34.18%	39.24%	45.57%	48.10%	48.10%	49.37%	49.37%	49.37%	49.37%	50.63%	50.63%	51.90%
Nov 82	102	101	0.00%	0.00%	0.00%	0.00%	0.00%	3.96%	6.93%	14.85%	16.83%	21.78%	21.78%	23.76%	23.76%	24.75%	25.74%	27.72%	28.71%	29.70%	30.69%	30.69%	30.69%	32.67%	32.67%
May 83	94	93	0.00%	0.00%	0.00%	1.08%	10.75%	12.90%	22.58%	27.96%	37.63%	41.94%	48.39%	50.54%	50.54%	51.61%	52.69%	52.69%	52.69%	52.69%	54.84%	54.84%	56.99%	58.06%	
Nov 83	72	70	0.00%	0.00%	0.00%	4.29%	7.14%	11.43%	17.14%	20.00%	21.43%	28.57%	30.00%	37.14%	37.14%	37.14%	40.00%	42.86%	45.71%	45.71%	45.71%	47.14%	47.14%		
May 84	89	88	0.00%	0.00%	0.00%	1.14%	4.55%	6.82%	11.36%	17.05%	21.59%	22.73%	26.14%	28.41%	32.95%	34.09%	36.36%	40.91%	42.05%	43.18%	43.18%	44.32%			
Nov 84	84	84	0.00%	0.00%	1.19%	5.95%	8.33%	10.71%	10.71%	15.48%	19.05%	26.19%	30.95%	33.33%	36.90%	40.48%	40.48%	40.48%	42.86%	44.05%	44.05%				
May 85	75	71	0.00%	0.00%	4.23%	8.45%	15.49%	21.13%	23.94%	26.76%	35.21%	40.85%	42.25%	43.66%	46.48%	49.30%	50.70%	52.11%	53.52%	54.93%					
Nov 85	92	91	0.00%	0.00%	0.00%	3.30%	5.49%	9.89%	14.29%	18.68%	20.88%	27.47%	32.97%	39.56%	40.66%	46.15%	46.15%	52.75%	52.75%						
May 86	118	116	0.00%	0.00%	0.00%	2.59%	9.48%	12.07%	20.69%	26.72%	31.90%	37.07%	40.52%	43.10%	43.97%	45.69%	49.14%	49.14%							
Nov 86	99	96	0.00%	0.00%	0.00%	3.13%	6.25%	18.75%	30.21%	31.25%	34.38%	39.58%	40.63%	43.75%	43.75%	47.92%	48.96%								
May 87	239	231	0.00%	0.00%	0.87%	4.33%	8.23%	11.26%	16.45%	20.35%	27.27%	30.30%	33.77%	36.80%	38.96%	41.13%									
Nov 87	198	190	0.00%	0.00%	0.00%	4.74%	6.32%	13.68%	17.37%	21.58%	23.16%	26.84%	28.42%	33.68%	37.89%										
May 88	275	262	0.00%	0.00%	0.00%	2.29%	6.49%	8.40%	14.50%	16.79%	23.28%	26.34%	33.21%	37.79%											
Nov 88	207	195	0.00%	0.00%	0.00%	4.62%	9.23%	13.33%	13.33%	15.90%	17.95%	22.56%	24.62%												
May 89	300	273	0.00%	0.00%	0.00%	1.10%	2.93%	4.76%	9.89%	12.45%	17.58%	20.51%													
Nov 89	261	238	0.00%	0.00%	0.42%	3.78%	4.62%	11.34%	13.03%	16.39%	18.91%														
May 90	383	332	0.00%	0.00%	0.60%	1.81%	4.22%	6.33%	10.54%	12.95%															
Nov 90	341	297	0.00%	0.00%	0.00%	1.35%	3.70%	9.43%	11.11%																
May 91	376	308	0.00%	0.00%	0.00%	1.30%	4.22%	7.14%																	
Nov 91	405	316	0.00%	0.00%	0.00%	5.06%	6.01%																		
May 92	364	244	0.00%	0.00%	0.41%	0.41%																			
Nov 92	313	199	0.00%	0.00%	0.50%																				
May 93	378	193	0.00%	0.00%																					
Nov 93	283	183	0.00%																						
May 94	237	122																							

*Candidates who have requested credit with the SoA for CAS Parts 4A and/or 4B have been excluded from the population in any given cohort

Appendix A
Sheet 2

Jul-94

	Unedited Pop.	Edited Pop.	24	25	26	27	28	29	30	31	32	33	34
May 77	60	60	71.67%	73 33%	73.33%	73.33%	73.33%	73.33%	73.33%	73.33%	73.33%	75.00%	76.67%
Nov 77	49	48	77.08%	77.08%	77.08%	77.08%	77.08%	77.08%	77.08%	79.17%	81 25%	81.25%	
May 78	77	77	72.73%	72.73%	72.73%	72.73%	72.73%	74 03%	75.32%	76.62%	76 62%		
Nov 78	61	61	63.93%	65.57%	67.21%	67.21%	67.21%	67.21%	68.85%	68.85%			
May 79	67	62	61.29%	61 29%	61.29%	61.29%	61.29%	62.90%	64.52%				
Nov 79	108	108	61 11%	61.11%	61.11%	61.11%	62.96%	63 89%					
May 80	72	72	73.61%	75.00%	75.00%	75.00%	75 00%						
Nov 80	66	64	70.31%	70.31%	70.31%	70 31%							
May 81	70	69	65.22%	65.22%	65.22%								
Nov,81	79	79	48.10%	48 10%									
May 82	79	79	51 90%										

Appendix A
Sheet 3

06-Jan-95

Steady State Test Parts 3-5: Cumulative Percentage

	No. of Candidate	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
May 77	68	0.00%	4.41%	23.53%	35.29%	42.65%	57.35%	57.35%	69.12%	69.12%	72.06%	72.06%	72.06%	73.53%	75.00%	75.00%	75.00%	75 00%	75.00%	75.00%	76.47%	76.47%	76.47%
Nov 77	70	0.00%	0.00%	30.00%	32.86%	45.71%	48.57%	57.14%	57.14%	64.29%	65.71%	70.00%	71.43%	72.86%	72.86%	75.71%	75.71%	75.71%	75 71%	78.57%	78.57%	78.57%	78.57%
May 78	89	0.00%	0.00%	16.85%	26.97%	30.34%	46 07%	46.07%	55.06%	55.06%	62.92%	62.92%	65.17%	65.17%	69.66%	70.79%	75.28%	75 28%	76 40%	76.40%	76.40%	76.40%	77.53%
Nov 78	69	0.00%	0.00%	14.49%	20.29%	44.93%	44.93%	55.07%	56.52%	59.42%	60.87%	62.32%	62 32%	65.22%	65.22%	66.67%	66.67%	66 67%	68.12%	68.12%	68.12%	69.57%	69.57%
May 79	92	0.00%	0.00%	5.43%	26.09%	27.17%	41.30%	41.30%	47.83%	47.83%	53.26%	53 26%	58.70%	58.70%	61.96%	63.04%	64.13%	65.22%	66 30%	66.30%	68.48%	68.48%	68.48%
Nov 79	127	0.00%	0.00%	14.96%	14 96%	28.35%	29.13%	44.09%	44.88%	49.61%	51.18%	55.91%	55.91%	59.84%	59.84%	62.99%	64.57%	66 14%	66.14%	67.72%	67.72%	68.50%	69.29%
May 80	74	0.00%	0.00%	5.41%	24.32%	28.38%	44.59%	45 95%	50.00%	51.35%	58.11%	58.11%	62.16%	64.86%	71.62%	71.62%	77.03%	77.03%	77 03%	77.03%	78.38%	78.38%	78.38%
Nov 80	85	0.00%	0.00%	18.82%	20.00%	40.00%	41.18%	49.41%	50 59%	63.53%	64.71%	69.41%	70.59%	77.65%	80.00%	82.35%	82.35%	84.71%	84.71%	84.71%	84.71%	84.71%	84.71%
May 81	91	0.00%	0.00%	4.40%	24.18%	26.37%	34.07%	37.36%	50.55%	50.55%	57.14%	57.14%	63.74%	64.84%	65.93%	68.13%	71.43%	72.53%	75 82%	75.82%	76.92%	79.12%	81.32%
Nov 81	134	0.00%	0.00%	11.94%	13.43%	22 39%	26.87%	40.30%	40.30%	45.52%	47.01%	52.24%	53.73%	54.48%	54.48%	56.72%	56.72%	56 72%	57.46%	57.46%	58.21%	58.21%	58.96%
May 82	104	0.00%	0.00%	2 88%	25.00%	26.92%	36.54%	39.42%	47.12%	48.08%	50.96%	50.96%	55.77%	56 73%	59.62%	62.50%	65 38%	65 38%	65.38%	65.38%	65.38%	65.38%	65.38%
Nov 82	151	0.00%	0.00%	3 31%	4.64%	9.93%	11.92%	22 52%	23.84%	30.46%	30.46%	33.77%	34.44%	36.42%	38.41%	40.40%	40 40%	40.40%	40 40%	40.40%	40.40%	40.40%	40.40%
May 83	104	0.00%	0.00%	8.65%	18.27%	24.04%	33.65%	40.38%	44.23%	46.15%	51.92%	57.69%	58.65%	59.62%	59.62%	59.62%	59.62%	59 62%	60.58%	61.54%	61.54%	61.54%	62.50%
Nov 83	90	0.00%	0 00%	11 11%	21.11%	25.56%	30.00%	32.22%	35.56%	37.78%	38.89%	38.89%	40 00%	40.00%	41.11%	42.22%	42.22%	43.33%	45.56%	45.56%	45.56%	45.56%	45.56%
May 84	110	0 00%	0 00%	3.64%	9.09%	17.27%	24.55%	26.36%	30.91%	36.36%	39.09%	42.73%	43.64%	44.55%	45.45%	45.45%	46.36%	49.09%	50.00%	50.00%	50.00%	50.91%	
Nov 84	90	0.00%	0.00%	8 89%	15.56%	18.89%	27.78%	32.22%	33.33%	38.89%	42.22%	44.44%	50.00%	51.11%	54.44%	54.44%	55.56%	56.67%	56.67%	56.67%	56.67%		
May 85	94	0 00%	0.00%	14 89%	22.34%	35.11%	36.17%	36.17%	38.30%	42.55%	50.00%	51.06%	51 06%	52.13%	53.19%	54.26%	55.32%	56.38%	57.45%	57.45%			
Nov 85	108	0.00%	0.00%	8.33%	12.04%	25.00%	28.70%	37.04%	40.74%	45.37%	52.78%	53.70%	53.70%	56.48%	58.33%	59.26%	60.19%	60.19%	60.19%				
May 86	200	0.00%	0.00%	12.00%	25.50%	29.00%	40.00%	45.00%	50.00%	52.50%	53.50%	55.00%	57.00%	59.00%	60.00%	60.50%	61.50%	62.00%					
Nov 86	147	0.00%	0.00%	16.33%	27.21%	36.05%	42.86%	50 34%	55.78%	57.14%	59.86%	63.95%	66.67%	66.67%	67.35%	68.03%	69.39%						
May 87	309	0.00%	0.00%	5.50%	8.74%	16.50%	24.60%	33.01%	40.78%	43 69%	47.90%	51.46%	52.75%	54.37%	55.02%	56.63%							
Nov 87	228	0.00%	0.00%	7.02%	12.72%	20.61%	26.32%	30.70%	35 96%	39.91%	40.35%	42.54%	44.30%	48 68%	52.63%								
May 88	280	0.00%	0.00%	3.93%	11.79%	17.86%	23 57%	30.36%	35.71%	38.93%	44.29%	47.14%	48.93%	51.43%									
Nov 88	220	0.00%	0.00%	9 09%	14.55%	15.91%	19.55%	23.18%	25.91%	31.36%	34.09%	39.09%	40.91%										
May 89	320	0.00%	0.00%	2.50%	4.69%	8.13%	16.25%	18.44%	23.44%	27.19%	30.31%	33.44%											
Nov 89	264	0.00%	0.00%	3.41%	6.44%	12 12%	16.29%	22.73%	26.14%	29.55%	32.58%												
May 90	389	0.00%	0.00%	2.31%	5.66%	9.00%	14.14%	15.68%	19.54%	23 91%													
Nov 90	372	0.00%	0.00%	2.15%	4.84%	9.14%	11 02%	14.52%	16.40%														
May 91	396	0.00%	0.00%	1 26%	5.81%	7.32%	13.89%	16.92%															
Nov 91	424	0 00%	0 00%	4.01%	4.48%	9.43%	12.03%																
May 92	393	0.00%	0.00%	0.76%	1.53%	4.33%																	
Nov 92	321	0.00%	0.00%	0.62%	1.25%																		
May 93	399	0.00%	0.00%	0.25%																			
Nov 93	291	0.00%	0.00%																				
May 94	217	0.00%																					

Appendix A
Sheet 4

23	24	25	26	27	28	29	30	31	32	33	34	35
76.47%	79.41%	79.41%	79.41%	79.41%	79.41%	79.41%	79.41%	82.35%	82.35%	82.35%	82.35%	82.35%
78.57%	78.57%	78.57%	78.57%	78.57%	78.57%	78.57%	78.57%	78.57%	78.57%	78.67%	78.57%	
77.53%	77.53%	77.53%	77.53%	77.53%	78.65%	78.65%	78.65%	78.65%	78.65%	78.65%		
71.01%	71.01%	71.01%	72.46%	72.46%	72.46%	72.46%	72.46%	72.46%	72.46%			
69.57%	69.57%	69.57%	69.57%	69.57%	69.57%	69.57%	69.57%	69.57%				
69.29%	69.29%	70.08%	70.08%	70.08%	70 08%	70.87%	71.65%					
78.38%	78.38%	78.38%	78.38%	78.38%	78.38%	78.38%						
84.71%	84.71%	84.71%	84.71%	84.71%	84.71%							
81.32%	81.32%	81.32%	81.32%	81.32%								
58.96%	58.96%	58.96%	58.96%									
65.38%	65.38%	65.38%										
40.40%	40.40%											
62.50%												

APPENDIX B

Appendix B
Sheet 1

Summary of Candidate Exam Strategy

Percentage of Candidates Taking a Single Subpart But Needing Additional Subparts

	Mav 1992	November 1992	May 1993	November 1993#		Mav 1994
				U	A	
Part 3B	27.6%	28 0%	27.2%	35.1%	41.0%	45.4%
Part 4A	25.6%	23.9%	34.5%	29.1%	33 7%	43.4%
Part 4B	19.5%	29 4%	26.4%	43.0%	48.2%	45.4%
Pan 5A				17.8%	22.0%	23.1%
Pan 5B				14.9%	18.0%	24.5%

First administration of Part 5 as a partitioned exam. For consistency with previous exam administrations, the percentages are displayed unadjusted (U) and adjusted (A) for ineffective candidates. All subsequent exam administrations will reflect the adjustment for ineffective candidates.

Appendix B
Sheet 2

May 1992

1. Success on 3B

Multiple Exam Takers	104/268 = 38.8%
Single Exam Takers	
- Needed only 3B	0/0 =
- Needed other sub-parts	43/102 = 42.2%
Total	43/102 = 42.2%

2. Success on 4A

Multiple Exam Takers	2431664 = 36.6%
Single Exam Takers	
- Needed only 4A	010 =
- Needed other sub-parts	72/228 = 31.6%
Total	72/228 = 31.6%

3. Success on 4B

Multiple Exam Takers	302/707 = 42.1%
Single Exam Takers	
- Needed only 4B	010 =
- Needed other sub-pans	37/171 = 21.6%
Total	37/171 = 21.6%

Appendix B
Sheet 3

November 1992

1. Success on 3B

Multiple Exam Takers	1691364 = 46.4%
Single Exam Takers	
- Needed only 3B	32/43 = 74.4%
- Needed other sub-parts	45/158 = 28.5%
Total	77/201 = 38.3%

2. Success on 4A

Multiple Exam Takers	1311394 = 33.2%
Single Exam Takers	
- Needed only 4A	5/30 = 16.7%
- Needed other sub-parts	28/133 = 21.1%
Total	33/163 = 20.2%

3. Success on 4B

Multiple Exam Takers	1561386 = 40.4%
Single Exam Takers	
- Needed only 4B	38/57 = 66.7%
- Needed other sub-parts	69/180 = 38.3%
Total	107/237 = 45.1%

Appendix B
Sheet 4

May 1993

1. Success on 3B

Multiple Exam Takers	1481324 = 45.7%
Single Exam Takers	
- Needed only 3B	38/64 = 59.4%
- Needed other sub-parts	37/145 = 25.5%
Total	75/209 = 35.9%

2. Success on 4A

Multiple Exam Takers	124/395 = 31.4%
Single Exam Takers	
- Needed only 4A	30/66 = 45.5%
- Needed other sub-parts	89/243 = 36.6%
Total	119/309 = 38.5%

3. Success on 4B

Multiple Exam Takers	111/445 = 24.9%
Single Exam Takers	
- Needed only 4B	46/157 = 29.3%
- Needed other sub-parts	101/196 = *51.5%*
Total	147/353 = 41.6%

November, 1993
(Unadjusted for ineffective candidates)

1. Success on 3B	
Multiple Exam Takers	121/299 = 40.5%
Single Exam Takers	
- Needed only 3B	20/24 = 83.3%
- Needed other sub-parts	69/175 = 394%
Total	89/199 = 44.7%
2. Success on 4A	
Multiple Exam Takers	1601349 = 45.8%
Single Exam Takers	
- Needed only 4A	19/21 = 90.5%
- Needed other sub-parts	57/152 = 37.5%
Total	76/173 = 43.9%
3. Success on 4B	
Multiple Exam Takers	115/363 = 31.7%
Single Exam Takers	
- Needed only 4B	18/33 = 54.5%
- Needed other sub-parts	981299 = 32.8%
Total	116/332 = 34.9%
4. Success on 5A	
Multiple Exam Takers	127/359 = 35.4%
Single Exam Takers	
- Needed only 5A	13/29 = 44.8%
- Needed other sub-parts	12/84 = 14.3%
Total	25/113 = 22.1%
5. Success on 5B	
Multiple Exam Takers	112/331 = 33.8%
Single Exam Takers	
- Needed only 5B	
- Needed other sub-parts	16/58 = 27.6%
Total	16/58 = 27.6%

November, 1993
(Adjusted for ineffective candidates)

1. Success on 3B	
Multiple Exam Takers	***1081222 = 48.1%***
Single Exam Takers	
- Needed only 3B	**23/30 = *16.1%***
- Needed other sub-parts	***791204 = 38.7%***
Total	***1021234 = 43.6%***
2. Success on 4A	
Multiple Exam Takers	***1391250 = 55.6%***
Single Exam Takers	
- Needed only 4A	**18/24 = *75.0%***
- Needed other sub-parts	79/176 = 44.9%
Total	971200 = 48.5%
3. Success on 4B	
Multiple Exam Takers	***1031250 = 41.2%***
Single Exam Takers	
- Needed only 4B	17/36 = 41.2%
- Needed other sub-parts	108/335 = 32.2%
Total	***1251371 = 33.7%***
4. Success on 5A	
Multiple Exam Takers	119/273 = 43.6%
Single Exam Takers	
- Needed only 5A	10/29 = 34.5%
- Needed other sub-parts	23/104 = 22.1%
Total	33/133 = 24.8%
5. Success on 5B	
Multiple Exam Takers	98/260 = 31.7%
Single Exam Takers	
- Needed only 5B	2/3 = 66.7%
- Needed other sub-parts	28/70 = 40.0%
Total	30/73 = 41.1%

Appendix B
Sheet 7

May, 1994
(Adjusted for ineffective candidates)

1. Success on 3B	
Multiple Exam Takers	102/196 = *52.0%*
Single Exam Takers	
- Needed only 3B	16/45 = *35.6%*
- Needed other sub-parts	*681200 = 34.0%*
Total	*841245 = 34.3%*
2. Success on 4A	
Multiple Exam Takers	130/221 = *58.8%*
Single Exam Takers	
- Needed only 4A	15/54 = *27.8%*
- Needed other sub-parts	66/211 ≐ 31.3%
Total	81/265 = 30.6%
3. Success on 4B	
Multiple Exam Takers	142/322 = 44.1%
Single Exam Takers	
- Needed only 4B	41/163 = *25.2%*
- Needed other sub-parts	128/403 = 31.8%
Total	1691566 = 29.9%
4. Success on 5A	
Multiple Exam Takers	85/187 = *45.5%*
Single Exam Takers	
- Needed only 5A	19/89 = 21.3%
- Needed other sub-parts	31/83 = 31.3%
Total	50/172 = 29.1%
5. Success on 5B	
Multiple Exam Takers	89/216 = 41.2%
Single Exam Takers	
- Needed only 5B	12/43 = 27.9%
- Needed other sub-parts	25/84 = *29.8%*
Total	37/127 = 29.1%

APPENDIX C

DO NOT FOCUS ON TRAVEL TIME FOR INDIVIDUAL EXAMS

The question was raised as to how has travel time changed on Part 4 subsequent to partitioning of the exam in May, 1992. While it is possible to calculate any change in travel time for Part 4 using the concepts of cohorts and travel time developed by the Working Group, we could not understand the relevance of such a calculation.

The ultimate goal of the CAS is to educate actuaries as measured by the successful completion of a series of exams. With the relationship between individual exams radically altered by partitioning, focusing on individual exams could lead to the conclusion that travel time is expanding while what is actually happening is that travel time through the entire series of exams is unchanged.

An example may prove to be illuminating. On Exhibit 1 attached are the examination records for two hypothetical candidates. By looking at the entire exam sequence we see that both candidates took 10 exam sessions to make it through the 5 exam sequence. But because of the way Candidate 2 could mix and combine exam subparts, take a partial exam for the first time while repeating another partial exam, the travel time for individual exams is markedly different.

Despite the Working Group's conclusion that focusing on travel time for individual exams is not appropriate, Exhibit 2 attached sets forth the results for Part 4.

Examination Records for Hypothetical Candidates

Candidate 1	Candidate 2
5105 Fail 3	11191 Pass 3A,C
1 1/85 Pass 3	5/92 Fail 4A
5/86 Fail 4	1 1/92 Fail 4A, Fail 4B
11/86 Fail 5	5/93 Pass 4A, Fail 4B
5/87 Fail 4	11193 Pass 4B, Pass 3B
11187 Pass 5	5/94 Pass 5A, Fail 5B
5/88 Pass 4	11194 Pass 5B, Fail 7
11/88 Fail 7	5/95 Fail 6
5/89 Pass 6	11195 Pass 7
11/89 Pass 7	5196 Pass 6

Travel Time

Part 3	2 exam sessions	5 exam sessions
Part 4	5 exam sessions	4 exam sessions
Part 5	3 exam sessions	3 exam sessions
Part 6	1 exam session	3 exam sessions
Part 7	3 exam sessions	3 exam sessions
Parts 3-7	10 exam sessions	10 exam sessions

Appendix C
Exhibit 2
Sheet 1

Travel Time
Part 4 Only

	Candidates	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
May 77	63	49.21%	49.21%	73.02%	73.02%	84.13%	85.71%	90.48%	90.48%	92.06%	92.06%	92.06%	92.06%	92 06%	92.06%	95.24%	95.24%	95.24%	95.24%
Nov 77	0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
May 78	105	47.62%	47.62%	76.19%	79.05%	80.95%	83.81%	83.81%	84.76%	86.67%	87.62%	89.52%	89.52%	89.52%	89.52%	89.52%	89.52%	89.52%	89.52%
Nov 78	0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
May 79	105	57.14%	63.81%	70 48%	77.14%	80.00%	80.95%	80.95%	83.81%	85.71%	85.71%	88.57%	88.57%	88.57%	88.57%	88.57%	88.57%	88.57%	88.57%
Nov 79	28		*				*	*				*	*	*	*	*			
May 80	42	*	*		*		*	*		*		*	*	*	*	*			
Nov 80	52				*		*	*						*	*		*	*	*
May 81	62	*	*				*	*			*			*	*	*			
Nov 81	42		*	*			*	*		*	*	*		*	*	*			*
May 82	66	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
Nov 82	161	34.16%	51.55%	51.55%	60.25%	60.25%	65.22%	65.22%	67.70%	67.70%	68.94%	68.94%	70.81%	70.81%	72.67%	72.67%	73.91%	73.91%	75.16%
May 83	106	23.58%	23.58%	43.40%	43.40%	55.66%	55.66%	59.43%	59.43%	60.38%	60.38%	62.26%	62.26%	62.26%	62.26%	63.21%	63.21%	65.09%	65.09%
Nov 83	0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
May 84	153	27.45%	27.45%	46 41%	46.41%	58.17%	58.17%	61.44%	61.44%	65.36%	65.36%	65.36%	65.36%	66.01%	66.01%	66.67%	66.67%	67.97%	67.97%
Nov 84	0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0 00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0 00%
May 85	132	25.76%	25.76%	40.91%	40.91%	54.55%	54.55%	58.33%	58.33%	62.88%	62.88%	65.15%	65.15%	67.42%	67.42%	69.70%	69.70%	70.45%	71.21%
Nov 85	0	0.00%	0.00%	0.00%	0 00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	
May 86	152	28.95%	28.95%	60.53%	60.53%	66.45%	66.45%	74.34%	74.34%	78.95%	78.95%	79.61%	79.61%	80.26%	81.58%	82.24%	83.55%		
Nov 86	0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0 00%	0.00%	0.00%	0.00%	0.00%	0.00%			
May 87	188	29.79%	29.79%	53.72%	53.72%	63.83%	63.83%	70.21%	70.21%	73.40%	73.40%	74.47%	75 00%	75.00%	75 00%				
Nov 87	0	0.00%	0.00%	0.00%	0.00%	0.00%	0 00%	0.00%	0 00%	0.00%	0.00%	0.00%	0.00%	0.00%					
May 88	265	27.17%	27.17%	50.57%	50.57%	64.15%	64.15%	71.70%	71.70%	73.21%	75.09%	76 98%	78.87%						
Nov 88	0	0 00%	0.00%	0.00%	0.00%	0.00%	0.00%	0 00%	0.00%	0.00%	0.00%	0.00%							
May 89	381	28.08%	28.08%	53.28%	53.28%	61.42%	61.42%	64.30%	67.45%	70 60%	72.70%								
Nov 89	0	0.00%	0 00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%									
May 90	414	19.81%	19.81%	45.41%	45.41%	49 76%	55.56%	58.94%	61.84%										
Nov 90	0	0.00%	0 00%	0.00%	0.00%	0.00%	0 00%	0.00%											
May 91	348	26.72%	26.72%	37.64%	45.98%	54.60%	59.48%												
Nov 91	0	0.00%	0.00%	0.00%	0.00%	0.00%													
May 92	732	9 02%	20.49%	29.23%	38.52%														
Nov 92	261	4.21%	14.56%	23.37%															
May 93	442	3.17%	11.76%																
Nov 93	302	2.65%																	

* Part 4 was jointly administered from Nov. 1979 until May 1982. Therefore, only pass information was recorded by the CAS.

Appendix C
Exhibit 2
Sheet 2

19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
96.83%	96.83%	96.83%	96.83%	96.83%	96.83%	96.83%	96.83%	96.83%	96.83%	96.83%	96.83%	96.83%	96.83%	96.83%	96.83%
0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	
89.52%	89.52%	89.52%	89.52%	89.52%	89.52%	89.52%	89.52%	89.52%	89.52%	89.52%	89.52%	89.52%	89.52%		
0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%			
88.57%	88.57%	88.57%	88.57%	88.57%	88.57%	88.57%	88.57%	88.57%	88.57%	88.57%	88.57%				
•	•	•	•	•	•	•	•	•	•	•					
•	•	•	•	•	•	•	•	•	•						
•	•	•	•	•	•	•	•	•							
•	•	•		•	•	•	•								
•	•	•	•	•	•	•									
•	•	•		•	•										
75.16%	75.16%	75.16%	75.16%	75.16%											
65.09%	65.09%	65.09%	65.09%												
0.00%	0.00%	0.00%													
67.97%	67.97%														
0.00%															

APPENDIX D

Appendix D

CANDIDATES ELIMINATED

Cohort	Did not sit for at 1/2 Least	Did not sit in November '93	Both Cases
May 77	2		0
Nov 77	3	3	0
May 78	5	2	
Nov 78	6	6	0
May 79	12	10	
Nov 79	5	6	
May 80	5	3	2
Nov 80	17	IO	3
May 81	7		0
Nov 81	29	17	
May 82	9	8	2
Nov 82	28	50	11
May 83	34	22	
Nov 83	33	26	16
May 84	39	28	11
Nov 84	31	26	9
May 85	25	20	9
Nov 85	29	25	15
May 86	30	17	8
Nov 86	25	23	12
May 87	35	27	9
Nov 87	23	24	6
May 88	33	33	8
Nov 88	32	43	11
May 89	87	59	19
Nov 89	42	57	19
May 90	56	79	23
Nov 90	35	59	10
May 91	26	48	12
Nov 91	20	38	16
May 92	42	81	22
Nov 92	24	54	14
May 93	0	43	0
Nov 93	0	0	0

APPENDIX E

Appendix E - Prior Expectations of the Effect of Partitioning *on* Travel Time

The Negative Binomial Model

The working group considered the question as to how one should expect travel time to change as exam partitioning is introduced. To address this, the negative binomial model was introduced.

Let: n = the number of subparts to be passed;

K = the number of failures before passing n subparts (random);

p = the probability of passing a given subpart; and

m = the number of subparts taken in a year.

Then K has a negative binomial distribution with:

$$\Pr[K = k] = \binom{n+k-1}{n-1} p^n (1-p)^k$$

We then have: $E[K] = \frac{n(1-p)}{p}$ and $\mathrm{Var}[K] = \frac{n(1-p)}{p^2}$

We define the travel time, T, as: $T = \frac{K+n}{m}$

Then: $E[T] = \frac{n}{mp}$ and $\mathrm{Var}[T] = \frac{n(1-p)}{m^2 p^2}$

If we use the negative binomial distribution as a model for the effect of partitioning, we obtain the following consequence. *There should be no* increase *in the expected* travel *time due to partitioning if the student takes* all *exam subparts* a *rate corresponding to the original pre-partitioned fate.*

To see this, suppose that n parts are partitioned into a•n subparts. Suppose also that the student takes a•m subparts per year. Then:

$$\text{New Expected Travel Time} = \frac{an}{amp} = \frac{n}{mp} = \text{Old expected Travel Time.}$$

However, the negative binomial model is not a perfect analogy to the actual exam process. Consider, for example, the student who takes all three subparts of part 3 on the first sitting. Suppose the student passes 3A and 3B. It will be impossible to take the equivalent of "one part" on the next sitting. It will have to take either 5/6 of a part (3C and 4A) or 4/3 of a part (3C, 4A and 4B).

The Last Exam Effect

Suppose the student can keep up a reasonable approximation to taking "one part per sitting." In the actual exam process, we *should expect to observe* a *small increase in the* travel *time in the Travel Time Charts of Appendix A.*

To see this, consider a travel time chart for Parts 3 to 5. Suppose the student takes exams at a rate of two subparts per sitting (i.e. 4 subparts per year). Once the student has passed all but Part 5B, it can only continue taking Part 5B at a rate of two subparts per year. While the student *may* actually be taking Part 6 or 7, we will observe a longer travel time for passing Parts 3 to 5. This effect is called the "last exam effect."

The increase in travel time due the last exam effect can be estimated. Suppose the student takes two subparts per sitting, or four subparts per year. Suppose further that the student's pass probability is 0.40. According to the negative binomial model, the student's expected travel time for the seven subparts is:

$$\frac{7}{4 \bullet 0.40} = 4.375 \text{ years.}$$

If instead, the student were to take six subparts at a rate of four per year, and one additional subpart at a rate of two per year, the student's expected travel time would be:

$$\frac{6}{4 \bullet 0.40} + \frac{1}{2 \bullet 0.40} = 5.00 \text{ years}$$

The second travel time estimate is an overestimate of the expected travel time of the actual exam process since it ignores the possibility that the student might have passed the last two subparts on a single sitting. From this example, it would appear that 0.625 years is a reasonable upper bound for the last exam effect.

It should be noted that the last exam effect is a function of the student's probability of passing, p. Since $p=0.40$ is an overall average probability, we should check to see if the last exam effect holds when the student population is diverse. To do this suppose that the student population consists of students with $p=0.20$, 0.40 and 0.60 in equal proportions. One can then work out an expected last exam effect of 0.764 years.

If the effect of partitioning is measured on the Part 3 through the Part 5 range, it is most likely that the last exam effect holds when the student is finishing Parts 4 or 5. Thus it seems reasonable to expect a raise no larger than 0.75 years.

What this all means is that we should not conclude that the travel time is increasing due to partitioning of exams 3 to 5 unless we observe an increase in the mean travel time of (conservatively) greater than 0.75 *years.* This is due to the last exam effect, which is a property of the way we measure travel time.

Report of the CAS Long Range Planning Committee
by the CAS Long Range Planning Committee

CASUALTY ACTUARIAL SOCIETY

Long-Range Planning Committee

REPORT TO THE BOARD

September 1994
(As revised based on Board input in 2/95)

REPORT OF THE CAS LONG-RANGE PLANNING COMMITTEE

Abstract

The CAS Long-Range Planning Committee prepares a report to the CAS Board each year regarding issues the Committee believes will be of importance to the evolution of the CAS over the next several years. This report was originally prepared in 1994 but reflects some changes based on input from the Board at its February, 1995 meeting. The recommendations are those of the Committee and have not been adopted by the Board at this time.

LONG **RANGE** PLANNING **COMMITTEE**
1994 REPORT TO TEE BOARD
(As revised based on Board input in **2/95)**

PROCESS

The discussions and recommendations contained in this report represent the collective efforts of the 1994 Long Range Planning Committee **(LRPC)** to identify those issues which **will** be of critical importance to the evolution of the CAS during the **next decade.**

In order to assure as broad a context as possible for our deliberations, we retied on the following sources of information:

1) An historical review of prior LRPC activity;

2) Minutes of the 1994 Committee Chairpersons Meeting **regarding** the topic of CAS Long Range Planning;

3) **1993 CAS Membership Survey;**

4) Personal discussions with non-actuarial professionals both within and outside the **Property/Casualty** insurance **area,** and;

5) Informal discussions at LRPC meetings with prominent industry figures.

CONCLUSIONS

The remaining portion of this report summarizes our comments and recommendations regarding the following issues:

A. Key Long-Term Issues (without prioritization)

- Dynamic **Financial** Analysis
- Health Care Delivery Costs
- **Mega Risk**
- Coordination With Other U.S. **and** Canadian Actuarial Organizations
- International Activity
- Data Reporting
- Actuarial Input to Public Policy Issues
- Accounting Principles and Practices
- Basic and **Continuing** Education
- Committee Structure and Management

B. Other Important Considerations

A. KEY LONG TERM ISSUES

DYNAMIC FINANCIAL ANALYSIS

In the 1993 Long Range Planning Committee Report to the Board, "Solvency" was identified as one of our highest priority issues. It was our belief that the CAS has a meaningful role to play in the measurement and maintenance of solvency for both traditional insurers and alternative risk transfer mechanisms.

Since that report, the NAIC has approved a risk-based capital formula to be applied to property and casualty insurers beginning in 1995. Continued high levels of property catastrophe losses have called into question the solidity of some insurers which, only a few years ago, were considered models of efficiency and strong capitalization. Some states (notably Florida and Hawaii) have formed alternative risk transfer mechanisms to deal with the inevitable lack of availability of essential catastrophe coverages. Also, legislation authorizing the formation of a federal disaster insurance fund as recommended by the Natural Disaster Coalition is slowly generating congressional support.

Over a five to ten year planning horizon, we believe that this issue will continue to be among the highest priorities of the actuarial profession. Financial data alone cannot provide the definitive answer to the question of insurer solvency since no financial reporting requirement captures the range of potential dynamic variables affecting solvency. The insuring public and insurance regulators at both the state and federal level have become increasingly strident in their criticisms of the industry for the absence of meaningful progress toward a credible solvency monitoring standard. Industry analysts will continue to probe and criticize the industry for failing to provide leading indicators of solvency impairment for weak insurers.

Against this backdrop, the CAS has a number of efforts which collectively address many of these concerns. Through our Appointed Actuary Advisory Committee, we have monitored initiatives in solvency regulation, guaranty fund reform, catastrophe exposure funding, and dynamic solvency testing.

From a broad perspective, the CAS, the SOA, the CIA and other actuarial groups have been working in the area of the Valuation Actuary and the Appointed Actuary. The discussions on this topic have ranged from a narrow focus on requirements that the "Actuary" opine on the continued viabiity of an insurer to the broader oversight of management performance. The CAS committees that have been working in this area have focused on "Dynamic Financial Analysis" (DFA) as a title more descriptive of this field. We believe that this reference is much more in line with the more expansive financial management roles which actuaries will perform in the near future. It is our opinion that a more uniform use of the term "Dynamic Financial Analysis" in place of "Appointed Actuary" will be more descriptive of the type of activity we wish to promote.

Recommendations

- Establish DFA as a preferred approach for our clients. To accomplish this task, the CAS cannot rely solely on **intra-actuarial** publications to establish recognition and acceptance of actuarial DFA work. The actuarial profession as a whole must aggressively establish leadership roles in the insurance, self-insurance and risk management industries in order to **solidify** a position of expertise for DFA types of analysis and the evaluation of the financial implications of risk decisions.

- An important first step in this initiative would be to change the name of the Appointed Actuary Advisory Committee to the Dynamic Financial Analysis Committee. As is the case today, this group would be charged with **coordinating** all pertinent **CAS** activities, maintaining a close working relationship with other organizations **(both** actuarial and non-actuarial) and providing regular reports to **the CAS** membership.

HEALTH CARE DELIVERY COSTS

In 1991 the CAS issued a Health Issues Policy Statement largely in response to external pressures on health costs. This statement focused primarily on health issues as they relate to casualty coverages. Also in 1991, the CAS LRPC expressed concern that the syllabus, as then constructed, was not adequate for future FCASs to be sufficiently qualified to address evolving issues such as 24-hour coverage and other managed care products.

Now, in 1994, we see that these concerns were on target. The importance of this issue was dramatically reinforced by a recent ISO study which showed that over 20% ($29 Billion) of the U.S. Property/Casualty industry's losses were Health Care related. It is especially interesting to note that almost 50% of Workers' Compensation losses arise from medical costs and that this percentage has been rising over time.

The syllabus has been updated in the last few years to include more relevant and modem readings on health insurance. However, additional efforts are needed in other areas if the future FCAS is to have a significant impact on the Health Industry. While this may not have been a major consideration in the past, as the lines between casualty insurance and accident and health insurance become blurred we are faced with the alternatives of either widening our scope and expertise or being left in the wake of market/coverage realignments.

Recommendations

- CAS Continuing Education and Program Committees should see that meeting content reflects the impact of these changes on our members.
- The CAS should take the necessary steps to ensure that casualty actuaries are full participants in the AAA working groups studying various aspects of health care reform.

MEGA RISKS

The property-casualty industry continues to be reminded of the actual and potential impact of Mega Risks on the financial strength of individual companies and the entire industry. Natural catastrophes and mass tort liabilities pose as much a threat to solvency as do underpricing, under-reserving or poor management.

While they caused huge losses and the insolvency of several companies, it is obvious that neither Hurricane Andrew nor the Northridge earthquake represents the maximum magnitudes of loss which could result from these types of catastrophic events. Similarly, the emerged costs of asbestos, environmental, and other mass tort claims are believed to be only a portion of the ultimate losses. Recently, A.M. Best published an analysis of asbestos and environmental liability costs which suggested that the range of ultimate costs from these perils would endanger the solvency of many sizable carriers and may even exceed the capital of the insurance industry as a whole. Other mass torts, such as lead paint, electromagnetic radiation and tobacco claims could total many billions of dollars as well.

Audiences such as the SEC, state and federal regulators, shareholders, rating agencies and the accounting profession are urging insurers and insureds to quantify the potential risks of these events as well as the liabilities they may have already incurred. There is a distinct possibility that others will dictate how these liabilities must be quantified, if the actuarial profession does not take a leadership role in establishing appropriate methodologies and standards.

The actuarial ramifications of these catastrophic risks are many, including dynamic financial analysis, pricing, and reserving, as well as the public policy issues to which the actuarial profession should contribute. It is, therefore, not surprising that the Long Range Planning Committee again includes "Mega Risks" as one of the CAS's key issues.

Recommendations

- The Committees on Reserves and Reinsurance Research should stimulate papers on appropriate methodologies for primary and reinsurance companies to use in quantifying their mass tort liabilities.

- The Committee on Ratemaking should continue to encourage research on methods of pricing natural catastrophe risks on both a macro and a micro level.

- The Appointed Actuary Advisory Committee should assure that mega risks be given sufficient recognition within model actuarial reports on dynamic financial analysis.

- The CAS leadership must work with the AAA and its Casualty Practice Council to assure that the ASB completes its Standards of Practice on catastrophic loss provisions and unquantifiable liabilities in a timely fashion.

COORDINATION WITH OTHER U.S. AND CANADIAN ACTUARIAL ORGANIZATIONS

American Academy of Actuaries

The 'Role of the AAA" has been a "monitoring" issue of the **LRPC** for the past three years. **LRPC** was concerned that **AAA** efforts have not been adequate (1993) and observed that CAS members expect that the CAS **will** monitor the **AAA** efforts on their behalf.

The CAS needs to take steps to be sure that the Academy is accomplishing the important objectives of CAS members and is functioning **appropriately** for other disciplines. If the Academy **fails** to achieve its objectives for other disciplines, it could **weaken** the **Academy's** role as a coordiiting body among US actuarial organizations.

Canadian Institute of Actuaries

As the public voice for all actuaries in Canada, the CIA has been actively promoting actuarial standards and issues. The CAS needs to ensure that the current formal and informal links between the CIA and CAS **continue** to address the educational needs of Canadian members in a timely and **effective** manner.

Society of Actuaries

The SGA is subject to increasing stresses of various types. The market for health benefit systems actuaries is very strong but the market for pension actuaries is declining. The number of students taking **SOA** examinations is falling, and this will create budget pressures.

Since the **SOA** is so large, it is difficult for the CAS to avoid the effect of **SOA** efforts. The CAS needs to monitor the areas of **CAS/SOA** coordination to be sure that activities are consistent with overall CAS objectives. Some of the areas of importance are the following:

1. Research - general principles

2. Education - casualty content on **SOA** exams.

3. Continuing Education - coordination on asset and **finance** education.

4. Standards - dealing **with** standards that cross boundaries.

5. Health Coverages - monitor developments that affect casualty coverages.

Recommendations

- The CAS President and/or President-Elect should monitor the overall performance of the Academy and provide regular reports to the Board.

- **SOA** areas cross CAS function (VP) boundaries so monitoring the overall consistency of our approach can be **difficult**. The Executive Council should assign the responsibility of developing and maintaining a **list** of **CAS/SOA areas** of interaction to one individual or committee.

INTERNATIONAL ACTIVITY

Given the broad scope of this topic, it would prove helpful to review the issue from an historical perspective.

1991

In 1991 the LRPC identified international activity as a high priority issue based upon four distinct opportunities: (1) the growth of P/C business outside North America, (2) lack of general insurance requirements in actuarial education outside the U.S., (3) the European Community need for consistent practice and (4) projected demand for actuaries in the rapidly developing Pacific Rim and the privatization of Eastern Europe.

The LRPC identified six obstacles to CAS worldwide involvement as follows: (1) university education, rather than examinations, is a very common qualification route, (2) national actuarial organizations might resist CAS involvement, (3) the Institute of Actuaries is the role mode for most English speaking countries, (4) the CAS is perceived by its members and others as a U.S./Canadian organization, (5) CAS education and practice are U.S./Canada oriented and (6) our practices are based on data collection procedures not common in other countries.

In its report, the LRPC recommended the following:

1. The CAS Board evaluate whether the CAS should establish goals such as the following:

   Short Term: Actively assist in providing education to aspiring general insurance actuaries worldwide.

   Mid-Term: Be recognized as the leading source of general insurance basic and continuing education.

   Long Term: General insurance actuaries worldwide should aspire to Fellowship in the CAS in addition to satisfying national accreditation requirements.

2. CAS staff/committees should compile a compendium of information on actuarial practice for major countries such as the following: (1) nature of actuarial education, (2) degree of general insurance education, (3) existing organizations and membership requirements, (4) number of actuaries (total and general insurance), and (5) size and growth of local general insurance market and number of insurers.

1992

1992 the CAS Board discussed international policy alternatives prepared by a CAS Task Force. The Board's conclusions were the following:

1. The CAS should not view itself as solely a North American organization and should move beyond the status quo of limited international involvement.

2. The CAS should take the **following** steps:

   a. Move forward in establishing diplomatic relations with other **organizations**.

   b. The Syllabus Committee should explore the availability of international materials for adding international content to the syllabus.

   c. The Continuing Education Committee should invite overseas actuaries to CAS seminars and look into joint sponsorship of seminars on general insurance topics.

   d. The **CAS** should **continue** high level counterpart discussions.

   e. The CAS should **continue** to explore methods to involve itselfworldwide.

<u>1993 - Present</u>

Since that time the CAS activities include the following:

1. **CAS** Presidents-Elect continued their annual visits to the general insurance study group of the Institute of Actuaries.

2. CAS Presidents have made-other visits to English speaking organizations in the UK and Ireland.

3. CAS Presidents and Presidents-Elect have been involved with the **"McCrossan** Group" over the past several years.

4. The Working Agreement Task Force including the CAS President-Elect has been involved in elements of the NAFTA implementation process.

5. The International Relations Committee established the Hachmeister **ASTIN** Prize.

6. The Syllabus Committee and the Continuing Education Committee have been pursuing the Board directives. The Syllabus Committee work **remains unfinished**. CAS seminars and programs have had regular international participation but there have been no efforts at international joint sponsorship of seminars on general insurance topics.

7. Exam waiver programs have been developed with the Institute of Actuaries for UK and **Australian** actuaries. In addition, an exam waiver policy for university education was approved by the Board of Directors.

8. An exchange of publications program has been expanded to twenty-one countries.

In 1993 the **LRPC** observed the CAS activity in examination waiver policies and **"McCrossan** Croup" efforts related to standards of practice and codes of conduct. The LRPC identified the need to (1) focus on education and research in international interactions and (2) relate pro-actively to Eastern Europe and developing nation education needs.

Perspective o n

With the passage of time we have additional perspectives on potential CAS roles outside of the U.S. and Canada.

Special Development

A casualty (general) discipline has developed in the U.S., UK and Canada where (1) the actuarial community overall is reasonably large and (2) the general insurance market is large enough to support a critical mass of general insurance specialists. The general insurance issues in those countries are relatively similar. Japan has a general insurance committee within its actuarial organization.

Actuarial "Priorities"

An actuary% professional focus might typically be prioritized as follows: (1) national organizations, (2) specialty (pension, general life, finance, health, etc.) and (3) type of employer (primary insurer, reinsurer, consultant, other). Alternatively, first priority might go to accrediting organization, generally the national organization (the CAS being an exception) and second priority to practice area.

Actuarial Qualifications

Examinations are the standard route of qualification in the English-speaking world and in parts of Asia. The university degree is a common form of professional training in Europe and Mexico. In some countries the organization is simply a voluntary association, with or without a method for demonstrating competence.

The two primary models for examination - based education are the Institute of Actuaries program (which includes all actuarial disciplines) and the Society of Actuaries program (which includes all but general insurance material).

There is also some use of mixed university/examination qualification processes

Trends

The following trends for the future are suggested as reasonable possibilities:

1. Emerging countries are looking for an actuarial professional model for their countries. They are likely to choose an examination process to supplement university education.

   No emerging country is likely to adopt the U.S. model of separate organizations for different specialties. The separation represents an inefficient use of their resources and is not responsive to their current market needs.

   The SOA program is at a disadvantage relative to the Institute of Actuaries program because the SOA program does not include a general insurance segment.

2. General insurance specialty groups will develop as required by national market places. The

general insurance study group within the UK now numbers approximately 300 people and has grown to that **size** at a growth rate faster than the **CAS** growth rate. Japan has a general insurance sub-group.

3. Combination of university training and professional examinations may become more and more common,

CAS Interests

The CAS member interest in international developments might be summarized as **follows** (m order of importance):

1. Recognition of value in FCAS designation to:

   a. U.S. and Canadian "employers" and regulators

   b. Foreign owners of U.S. and Canadian companies

   c. Foreign **(non-U.S**. and non-Canadian) regulators and government bodies involved with U.S. and Canadian companies.

2. Easy to obtain recognition of **qualification** to work in near-by countries

   Canada/U.S.
   Bermuda
   Mexico

3. Avoiding rules that preclude work in any country.

4. **Recognition** of general insurance as specialty of actuarial work requiring some specific technical knowledge.

5. Good "image" of actuaries worldwide.

Recommendations

The CAS international activities that would support these interests include the following:

Functional Directions

1. **Maintain** our strong U.S. and Canadian role.

2. Cooperate in research and continuing education with general insurance specialty subgroups of **non-U.S**. actuarial organizations.

   This **includes** both **(1)** invitiig non-U.S. help on issues of U.S. importance, for example, the loss reserve uncertainty Theory of Risk project and (2) offering to provide CAS-member assistance, through committees or otherwise, on non-U.S. issues.

3. Cooperate with the SOA in its efforts to integrate casualty material in a "complete" education program suitable for countries without a casualty specialty.

4. Seek methods of cooperating with other non-U.S./Canadian organizations to strengthen basic and continuing, education of general insurance specialists.

<u>Organizational</u>

1. Continue to develop and strengthen high level contacts between the CAS and the general insurance groups in actuarial organizations in other countries.

2. Strengthen the role of the International Relations Committee (IRC) so it can participate and monitor these efforts. The IRC Committee chair should be a past-officer (President or Vice President) or recognized as a senior international actuary.

3. Identify a CAS Officer/Committee chair to monitor and report on all CAS international activities--research, admissions, continuing education, programs, etc.

DATA REPORTING

The issue of data reliability for use by regulators has recently become a major concern. Some believe that since the data is compiled by statistical agents controlled by the insurance industry, it is necessarily suspect. In addition, wncerns have been expressed that insurance data is insufficient to examine certain public policy issues such as availability of insurance in urban areas. Congress is considering legislation that requires potentially costly data compilation by insurers.

Actuaries are uniquely qualified to provide the expertise that is demanded in these debates. They are trained to compile and analyze insurance statistics and can advise regulators on data quality and usefulness of information for the purposes intended.

The profession has a key role to play in this area and the means to do it through the work of the CAS Data Management and Information Committee, the American Academy of Actuaries, the Canadian Institute of Actuaries and the Insurance Data Management Association (IDMA).

The issues should be carefully monitored by the CAS since reliable data is an essential resource for the casualty actuary.

Recommendations

- The Committee on Management Data and Information should monitor developments in this area.
- The CAS could consider serving as a repository of data where traditional mechanisms are not able to function (e.g., Alternative Market). It would be prudent to wait for a specific opportunity or need arises before considering involvement of the CAS in calls for data.

ACTUARIAL INPUT TO PUBLIC POLICY ISSUES

Today's insurance industry is one that is constantly in the public eye. Consumer issues of affordability and availability wmbiied with a high level of interest at the Federal level will hold the nation's interest, at least for the foreseeable future. Actuaries will be required to speak out and explain insurance phenomena and trends. As these issues become more complex, actuaries will be increasingly called upon for their expertise.

Our responsibilities include not only quantifying and projecting system costs, but also identifying the causes and 'drivers" behind these costs. Pressures on insurance costs include the economy, the legal environment, the regulatory climate and judicial decisions. Specific examples may include changes in interest rates, fraud, the imposition of Federal regulation and health care reform legislation. Cost drivers can impact on the frequency and/or severity component of losses and on investment yields which could have an impact on ultimate costs. As such we see that cost drivers may either increase or decrease total losses and costs. We cannot hope for a single, unique solution to the quantigcation of cost drivers since their sources and impacts can vary by many factors, such as: line of insurance, individual company procedures, and state/geographic location of risks. In addition, even if one is able to quantify the cause it is not necessarily controllable.

The abiity of the private sector to adequately address broad social policy issues is generally limited. The CAS, in a public policy role, should consider whether it wishes to be proactive in this area. Among the topics to be addressed are health care reform, pay at the pump auto insurance, mandatory insurance coverages, private sector subsidies (e.g., assigned risk plan), public sector subsidies (e.g.; insurance stamps), urban enterprise zones to encourage reinvestment in urban areas. and stripped down policies, among others.

Recommendations:

- The CAS should assist the AAA Casualty Practice Council to make sure the current mechanism for public policy involvement works.
- The CAS, through the Continuing Education Committee, should encourage the submission of papers on this topic, including possibly a bibliography of data sources.
- The Program Planning Committee should provide sessions on this issue at CAS functions.
- The CAS should coordinate with the regional affiliates to include this in their programs.

ACCOUNTING PRINCIPLES AND PRACTICES

In the last decade, actuaries have **become** increasingly **influenced** by accounting rules and practices. In certain situations actuaries have taken strong positions on such issues as discounting and risk transfer, while in other cams the actuarial profession has remained relatively silent on issues that are ambiious such as the use of a range of results. From time to time a **CAS** task force or committee has responded to the NAIC, FASB, SEC, AICPA and other groups that deal with accounting. In **particular,** the Academy Committee on Property and **Liability Financial** Reporting is charged with this **responsibility.**

The implications of various accounting principles and practices have had a greater impact on **actuarial** work than was anticipated. In many cases the actuarial point of view has not been **sufficiently** considered in accounting. The **consequence** of this has been **confusion** and, in some cases, misuse in **financial** reporting.

Because of the **structure** of the rule-making bodies for **accounting** and financial reporting, it is **difficult** for the **actuarial profession** to become proactive and influential. Whether or not the CAS can change the status of our profession in the accounting realm, it has become clear that the actuarial profession is not generally recognized as being an integral part of the designing of accounting practices or principles in the areas normally associated with actuaries.

Recommendations

- The CAS should promote activities that give actuaries a stronger voice in both the accounting rules that are established and the interpretations and guides for compliance as they impact areas of actuarial import.

- The CAS should direct its research activities to identify problems both within and outside the insurance arena where actuarial approaches **could** be used to solve or manage such problems.

- The CAS should **direct** the appropriate **committees** to identify current accounting or financial reporting rules or practices that are ambiguous, vague or **difficult** to comply with **from** an actuarial viewpoint and pursue the development of solutions in those areas.

BASIC AND CONTINUING EDUCATION

Two of the primary **responsibilities** of the CAS are to provide **basic** actuarial education to its students in order to **qualify** them as Fellows of the CAS, and to provide sufficient continuing education opportunities to its members to meet professional **standards,** keep knowledge current and provide for individual growth.

In order to meet these responsibiities aggressive measures have been taken over the past several years. The basic education **function** has been under intense scrutiny as it relates to the effect of partitioning of exams and the ongoing review of the syllabus.

The Executive Committee was charged with developing parameters measuring the success or failure of partitioning for a decision in 1994. A conclusion to the study of this issue should be reached in 1994-5.

The syllabus wmmittee has been reviewing the input **from** the 1993 membership survey and should be very responsive to member comments.

The wntinuing education program has been expanding each year. The post-fellowship course, "Principles of Finance in Property Casualty Insurance", received high praise and is a step towards alternative methods of providing education to Fellows of the CAS. More opportunities for education of members in the areas of asset management and dynamic solvency testing should be pursued. The 1995 **syllabus** will address these topics for fellowship candidates.

The **CAS** is at a point in its development where a distinction must be made between **"core"** and "specialized" education. The next ten years will likely introduce more heterogeneity into our professional lives and we need to establish a flexible, yet identifiable, basic educational curriculum.

R[redacted]dations

- Partitioning should continue to be studied and preliminary conclusions and recommendations made in 1995.
- **The CAS** should formally study the definition of the **core** learning necessary to become an FCAS and relegate other topics to continuing education.
- The role of universities in the educational process, including using universities as a source for continuing education opportunities, should be explored.
- The CAS through liaison with regional **affiliates** should take a more active role in supporting **exam** preparatory courses.

COMMITTEE STRUCTURE AND MANAGEMENT

The CAS has **always** taken great pride in the active involvement of its members. The willingness of the members to volunteer large amounts of their time has enabled the organization to **staff an** ever growing number of committees and to carry out its administrative functions, thereby **allowing** the CAS to keep its dues at a very low level. Indeed it is only in the last few years that the emerging **administrative** burden **resulting** from the burgeoning membership has dictated the need for a full time Executive Director and a larger, though still modest-sized, office staff

New CAS members still are imbued with a feeling of responsibility to their profession and to the CAS, which they express by giving of themselves in volunteering their time to the CAS and its committees. At this point there is little or no evidence of a shortage of volunteers although any change in this would greatly impact the **CAS's** ability to carry out its activities. In addition, there is a question as to whether certain administrative activities **could** be better handled by professionals in the CAS office and thereby ease the burden on some committees and make volunteering somewhat more attractive. The CAS **office** has already done an outstanding job in this regard for the Examination and a number of other wmmittees.

As the number of issues facing the CAS grows, so do the number of committees and subcommittees. While the great majority of committees do **an** excellent job, the key to the effectiveness of a **committee** remains the Chairpersons, and those individuals are increasingly burdened by the number of issues with which they must deal and the number of subcommittees they must manage. Other approaches must be considered to **organizing** and managing the Committee work while still fostering the valuable culture of volunteerism which has played a significant role in the success of the CAS.

Recommendations

- The Executive Director should study the committee workings and recommend additional areas in which the CAS office could take over or support the non-technical responsibilities of the committee.

- The CAS Executive Council should continue to monitor the supply of volunteers to identify any emerging shortfalls.

- The CAS should consider utilizing Working Croups of interested members to undertake specific studies or issues. These Working Croups would cease to exist when their assignment is completed.

B. OTHER IMPORTANT CONSIDERATIONS

Many **external** forces have had, and will continue to have. a profound influence on CAS members and their careers.

- The development of **Financial Products** (such as Options, Derivatives and **Financial** Insurance) has caused many professionals both inside and outside **the P/C industry** to **re-evaluate** the purpose and effectiveness of traditional insurance coverage. It is clearly within the **actuarial** domain to play a major role in the creation and pricing of these instruments as **well** as providing valuable input to the appropriate regulatory and accounting authorities. We encourage the Continuing Education Committee to keep the membership abreast of these issues.

- The general **Consolidation** of the P/C Industry, in **concert** with the significant growth of **Alternative Market** mechanisms has **dramatically** changed the **career** opportunities available to our members. The CAS must carefully monitor these developments and provide information which will **allow** for prudent and professional decisions on both a corporate and personal level.

From an **organizational** **perspective,** the **LRPC** felt that the CAS should continue to monitor a number of **significant factors**:

- Although there has been noticeable improvement recently, the general characterization of an actuary is still that of a technician who lacks- and an appreciation of a **Business Perspective** Our abiity to lead our profession into the next century will largely be based on the credibility we earn as **communicators** who can blend technical knowledge with business instinct. The recent Call Paper topic on this issue was a positive sign that the problem has been identified. However, the CAS must continue to promote these qualities through seminars, regional **affiliate** meetings and general programs.

- The **CAS** membership witnessed its **first** public disciplinary action this year. As unfortunate an event as this **was,** it should promote a higher awareness of our Principles, Standards of Practice and the Discipline process. As we seek to gain more prestige among our peers in the next century, the CAS must demonstrate a high level of Professionalism. In this regard the **LRPC** believes it is appropriate to create a Committee on Professionalism. Not only would this Committee assume responsibility for the practical aspects of the Course on Professionalism, but it would also coordinate the distribution of all educational material to the membership in a uniform and focused manner.

- **Minority Recruiting** and **Academic Participation** will serve as two crucial sources of intellectual resources to the CAS during the next ten years. Role models and **successful** experiences **will** invariably lead to greater involvement of each of these groups.

Causes of Reserve Deficiency Among Property/Casualty Insurers: A Survey

by the American Academy of Actuaries Committee on Property-Liability Financial Reporting

Causes of Reserve Deficiency

Among Property-Casualty Insurers:

A Survey

Prepared by the

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ABSTRACT

In 1994, the Committee on Property and Liability Financial Reporting of the American Academy of Actuaries (COPLFR) surveyed actuaries representing 26 property-casualty insurance companies to determine what factors contributed to adverse reserve development in individual companies' total loss and loss adjustment expense reserves. The survey results indicated that the major causes of adverse reserve development during the period covered by the survey were: (1) environmental and asbestos liabilities; (2) loss development tail factors; (3) involuntary pool reserves; and (4) unwinding of discount.

COPLFR concluded that some recently adopted changes to the annual statement and other regulatory initiatives under consideration can help identify and/or reduce the impacts of some of these elements. However, COPLFR also concluded that the actuarial profession needs to engage in further work on the appropriate treatment of reserves for environmental and asbestos losses and possibly in the estimation of loss development tail factors.

The American Academy of Actuaries is the public policy voice of the actuarial profession, providing the actuarial profession's expertise to policy makers. This report was produced under the direction of Jean K. Rosales, Assistant Director of Public Policy.

Causes of Reserve Deficiencies among Property-Casualty Insurers: A Survey

INTRODUCTION

It is the appointed actuary's job to evaluate a company's claims reserves. The Statement of Actuarial Opinion (SAO), signed by the appointed actuary, is the document that attests to the reasonableness of the company's reserves.

"Adverse reserve development" indicates that the company did not set aside sufficient reserves to meet its claims.

Adverse reserve development in any one year does not indicate that a company is in financial trouble. Nonetheless, repeated problems with adverse reserve development could signal the beginnings of financial distress. It is important, therefore, for the financial health of the company that the analysis and evaluation of reserves in the SAO be as accurate and dependable as possible.

SURVEY BACKGROUND

In the summer of 1994, the Committee on Property and Liability Financial Reporting of the American Academy of Actuaries (COPLFR) undertook a survey of 52 property-casualty insurance companies to better understand the causes of companies' adverse reserve development in the three-year period beginning year-end 1990 and ending year-end 1993. The thought was that a greater understanding of the causes of adverse'reserve development would help determine where improvements could be made. Possible areas of improvement might include enhancements to the SAO or more education of actuaries in the areas causing adverse reserve development.

The survey was initiated because COPLFR observed that some industry analysts concluded that industry reserves were deficient by 10%-15% (on an undiscounted basis) despite the fact that few companies received adverse SAOs. Beginning at year-end 1990, most companies bad SAOs signed by qualified actuaries (members of the American Academy of Actuaries and the Casualty Actuarial Society). Thus, concern developed that the overall reserve deficiency of the property-casualty insurance industry asserted by some industry analysts might indicate a credibility gap for actuaries signing SAOs.

It was not the intent of the COPLFR survey to test or validate studies of reserves by industry analysts, nor were those observers' conclusions accepted as fact by COPLFR. However, the initial premise was to accept those conclusions and determine whether the observations that industry reserves were deficient could be consistent with non-adverse SAOs for the vast majority of companies. It was considered possible that both could be right and that the adverse reserve development might be related to items outside the purview of the SAO. Should that be true, the.recommendation might be to expand the areas covered by the SAO.

Alternatively, if the adverse reserve development were determined to be related to items already within the purview of the opining actuary, the recommended solution might be to improve the training and education of the opining actuary. Courses of action might include recommending changes to the opinion instructions and developing an explanatory article for outside audiences.

DESCRIPTION OF COMPANIES SURVEYED

Attached as Appendix A is the survey form used by COPLFR, which was mailed to 52 selected companies. Of these, 40 were chosen because they had the greatest adverse reserve development in the industry during the three years, 1991 through 1993, measured as a percent of surplus, percent of reserves, or dollar amount. Twelve companies that did not demonstrate adverse reserve development were also included in the survey. Their size or other unique characteristics led COPLFR to believe that their responses to questions on reserve ranges and level of analysis, as well as their ideas on improving the SAO, would be of value to tbe study.

As shown in Table 1, of the 26 survey responses, 20 came from the 40 companies that had demonstrated adverse reserve development in the three-year period. Six of the twelve companies selected for the other reasons responded. As shown in Table 2, the 26 companies responding held 61% of the total reserves at year-end 1993 for the 52 surveyed companies. Those 52 companies, in turn, accounted for 69% of 1993 total industry reserves. Thus, 42% of total industry reserves were represented by respondents to the survey. Fifty-seven percent of the total 1993 year-end reserves held by survey respondents were attributable to

Table 1

Response Rates of Companies Surveyed

	Companies Responding	Companies Surveyed
Adverse Reserve Development	20	40
Other Companies	6	12
TOTAL	26	52

Tahle 2

1993 Reserves of Respondents Compared to
1993 Reserves of Companies Surveyed & Industry

Reserves of Companies Surveyed as a Percent of Total Industry Reserves	69%
Respondents' Reserves as a Percent of Reserves of Companies Surveyed	61%
Respondents' Reserves as a Percent of Total Industry Reserves	42%

the 20 companies with adverse reserve development; the six other companies represented 43%.

Of the 40 companies surveyed that had adverse reserve development, 19 had 1990 SAOs signed by consultants and 21 had SAOs signed by company employees. The ratio of responses from consultants to those from company employees parallels that of companies surveyed overall. The consultant/non-consultant split is shown in Table 3

Table 3

Use of Consultants & Non-Consultants by Companies

	Adverse Reserve Development		Other Companies	
	Respondents	Surveyed	Respondents	Surveyed
Consultants	9	19	1	4
Other Opiners	11	21	5	8

Survey results follow, grouped by topic in the same order as the survey itself. Causes of adverse reserve development are discussed first, followed by reserve ranges, cash flow testing, and general respondent comments.

CAUSES OF ADVERSE RESERVE DEVELOPMENT

Of particular interest to COPLFR was the identification of the causes of adverse reserve development. If causes could be identified, it might be possible to analyze the treatment of those factors in SAOs and to consider whether current reserving techniques are adequate or whether further research is needed in this area.

Section l/Sheet 2 of the survey listed 24 possible causes of adverse reserve development and asked respondents to allocate by percent (adding to 100%) the major causes of their firms' adverse reserve development. Nineteen of the 20 survey respondents with adverse reserve development responded to this part of the survey.

Table 4 summarizes the responses to Section l/Sheet 2; Appendix B provides a more detailed summary. Even though the total industry adverse reserve development from year-end 1990 to year-end 1993 was approximately $9 billion, the 40 surveyed companies that demonstrated adverse reserve development had over $14 billion of adverse reserve development in the three-year period studied. Favorable reserve development exists for many companies which caused the total adverse reserve development for the selected companies to be greater than the industry total. The 19 companies responding to this question had $7 billion of adverse reserve development in that period.

It should be noted that the survey focused on causes of adverse reserve development over a three-year period. Should year-end 1990 reserves evaluated as of December 1993

Table 4

Major Causes of Adverse Reserve Development

Cause	Number of Companies Identifying Cause	Percent of Development
Pollution, environmental, asbestos, toxic materials and other similar items	12	70%
Loss development tail factor underestimation	9	10%
Involuntary pool reserve strengthening	8	8%
Unwinding of disclosed discount	8	6%
All other listed causes	N/A	17%
Write-ins	9	14%
All beneficial development	8	-25%

still not represent ultimate costs, further adverse reserve development might ensue.

Table 4 shows the major causes of adverse reserve development as identified by survey respondents. Twelve of the 19 companies listed pollution, environmental, asbestos, toxic materials, and other similar items as a major cause of adverse reserve development. This category accounted for 70% of the total adverse reserve development for the 19 companies responding to this question.

The second greatest contributor to adverse reserve development was underestimation of loss development tail factors. This cause, identified by nine of the 19 companies, represented 10% of total adverse reserve development.

The thud largest category, identified by eight companies, was reserve strengthening in involuntary pools and associations and represented 8% of total survey development. Unwinding of disclosed discount was the fourth largest category, noted by eight companies, representing 6% of total development.

Nine companies used the "write-in" line to identify other sources of adverse reserve development. Emergence of construction defect losses was identified by three companies. Other areas mentioned as causes by one or two companies were changes in economic conditions, poor stratification of data, and the impact of court or regulatory actions. Another cause noted was booking reserves at the low end of a reserve range.

Seventeen of the 19 companies experiencing adverse reserve development responded to this question with one or more lines of business identified as the source of adverse reserve development. The two lines of business most frequently identified with adverse reserve

Table 5

Companies' Lines of Business

Line of Business	Number of Companies Identifying
Workers' Compensation (apparently only 4 are WC only)	13
General Liability (including products and treaty casualty excess)	12
Medical Malpractice (specialty company)	I
No line of business identified	2

development were Workers' Compensation and General Liability. Thirteen companies listed Workers' Compensation as a major source of development, Twelve listed General Liability, Summary information on adverse reserve development by line of business is shown in Table 5.

Summary

Although COPLFR presented respondents with a list of 24 possible sources of adverse reserve development, the companies surveyed demonstrated substantial consistency in identifying what had led to this outcome. The causes most frequently mentioned were pollution, environmental, asbestos, toxic materials and other similar items; loss development tail factors; reserve strengthening in involuntary pools and associations; and unwinding of disclosed discount. A discussion of ways the actuarial profession can follow up on this information appears in the "Concluding Observations and Recommendations" section below.

LEVEL OF RESERVE ANALYSIS

Section 2/Sheet 1 of the survey asked respondents to identify the level of actuarial analysis performed for the reserves established in December 1993. Appendix C summarizes the responses from all 26 survey respondents on the level of analysis of company reserves. As shown in Table 6, 88% of reserves for all companies surveyed, and 82% of reserves for the 20 companies with adverse reserve development were analyzed using

Table 6

Percentage of Reserves Analyzed Using Standard Actuarial Techniques

All Respondents	88%
Adverse Reserve Development Respondents	82%

standard actuarial techniques. The only other major categories of levels of analysis mentioned were "involuntary pools" (7% of reserves) and "inestimable" (5% of reserves). Responses for "other" are shown in Appendix D.

The identification of "**involuntary** pools" as amounts not subjected to standard actuarial techniques is of interest, since eight companies identified this as a source of adverse reserve development. Similarly, the "inestimable" amounts may relate to other items — such as environmental and asbestos claims and the impact of court or regulatory actions — mentioned in the previous section as causes of adverse reserve development.

RESERVE RANGES

In performing their reserve analysis, actuaries may elect to develop a range of estimates for reserves. Section **2/Sheet** 2 asked respondents to identify whether they used a range and to provide details on their use of ranges.

Appendix E summarizes the responses to the questions on range methodology and cash flow analysis. Table 7 shows that, of the 26 survey respondents, 15 estimated ranges as part of their

Table 7

Use of Ranges

	Adverse Development Companies	Other Companies
Number using ranges	12	3
Percentage of respondents:		
Straight average	60%	50%
weighted' average	77%	26%

Weighted based on held reserves by company.

Table 8

Range Methodology Used by Companies

	Range Size/ Reserves	Range Size/ Surplus	Held Reserve Percentile Within Range
Number of respondents	13	12	12
Straight average	14%	44%	49%
'Weighted' average - adverse development companies	16%	64%	47%
'Weighted' average - all companies	16%	61%	44%

'Weighted based on held reserves by company

reserve analysis. Most of the companies with larger adverse reserve development used ranges: 60% of these companies used ranges, representing 77% of the carried reserves

On average , the reserve width for the respondents was 16% of carried reserves, and 61% of surplus (Table 8). On average, carried reserves were between the 40th and 50th percentile of the reserve range. Table 9 shows the stratification of reserve range widths. Seven of the respondents had a range width representing 10% of carried reserves, two had a range width representing 11% of carried reserves, one was 15 % of carried reserves, one was 16%. one was 23%, and one was 30%. Many of the 13 companies appear to be using a probabilistic criterion in their analysis rather than developing ranges based on alternative methods.

Table 8 also includes a column representing the range as a percent of surplus for the 12 companies that responded to this question. For them, the reserve range as a percent of surplus went from a low of 7% to a high of **122%**, with a straight average of 44% and a weighted average of **61%**, indicating that larger companies have a larger range relative to surplus. Most company-carried reserves are near the middle of the range, perhaps because the range was established around a selected point estimate. One company indicated that its carried reserves were 32% above the top of the reserve range.

Table 9

Number of Companies by Range to Carried Reserves

Range/ Carried Reserve	Number of Companies
10%	7
11%	2
15%	1
16%	1
23%	1
30%	1

Section **2/Sheet** 2 asked respondents whether actuaries should be required to include a range in the SAO. Most respondents felt that including a range in the reserve opinion would be more harmful than helpful, fearing misuse or lack of understanding on the part of the reader and concern that the range might be used as a warranty or guaranty that actual results won't develop outside the range. Further, respondents felt that there is at present a lack of standards on the use of reserve ranges. They also believe that more research needs to be done by the actuarial profession regarding the determination and understanding of a reserve range.

Respondents also identified benefits of including a range in the reserve opinion including: (1) publicizing the issue of the uncertainty in reserve estimates, (2) highlighting the relative strength of the carried reserves, and (3) possibly leading to more adequate reserves.

CASH FLOW TESTING

In reviewing the written responses to the questions on cash flow testing, COPLFR members could not draw many conclusions. Only nine of the 26 respondents indicated that they do some form of cash flow testing. Some respondents felt it was only an issue if a company discounted reserves. A better definition of cash flow testing, or clearer phrasing in the survey questions, was needed. (Perhaps this can be addressed in any future surveys.)

COMMENTS BY SURVEY RESPONDENTS

Section 3 of the survey form requested suggestions for strengthening the SAO, for better educating actuaries, and other items. The responses to these questions provided useful information to COPLFR. Summarizing and analyzing these responses is beyond the scope of this report. Members of COPLFR have compiled the written responses and will be communicating them to the Board of Directors of the Casualty Actuarial Society for their use in furthering the education of casualty actuaries.

CONCLUDING OBSERVATIONS AND RECOMMENDATIONS

The treatment of discounting, involuntary pools, and environmental and asbestos liabilities within the SAO appear to be the major areas that account for the differences between industry analysts' perceptions of deficiencies in industry reserves and the generally favorable SAOs issued by actuaries. Some recently adopted changes in these areas to the annual statement and other regulatory initiatives under consideration can help identify and/or reduce these differences.

After studying the responses to its survey of the causes of reserve deficiency, members of COPLFR identified the following observations and recommendations:

Pollution, Environmental, Asbestos, Toxic Materials, and other similar items. This item was cited most frequently as the cause of adverse reserve development. Estimating required reserves for environmental and asbestos exposures is a major challenge for the actuarial profession. Such exposures will likely continue as major contributors of adverse development unless there are significant changes in federal or state legislation. Members of COPLFR recommend that research efforts in estimating such reserves continue to be a priority for the Casualty Actuarial Society.

Loss Development Tail Factors. A second cause of adverse reserve development was underestimation of loss development tail factors. More focus on methods for estimating loss development tail factors estimation may be useful, as would surveys of historical data. This should be considered by the Casualty Actuarial Society Loss Reserve Committee and would be an appropriate topic at the Casualty Loss Reserve Seminar. Discussion paper programs could include tail factor estimation, and other industry studies and educational possibilities for this topic area should be encouraged. The American Academy of Actuaries may wish to consider developing a practice note on tail factor estimation methodology and testing.

Pools and Associations. The fact that strengthening of reserves of pools and associations was cited as a cause of adverse reserve development leads members of COPLFR to conclude that statements of actuarial opinion on reserves for pools and associations would be helpful. Some major pools have recently begun developing SAOs and providing them to members, but this is not required of most pools. However, COPLFR is working with the

Casualty Actuarial Task Force of the National Association of Insurance Commissioners on developing instructions for opinions for voluntary and involuntary pools and it is possible that such opinions will become more common in the future.

Unwinding of disclosed discount. Although the unwinding of disclosed discount was mentioned fairly frequently by survey respondents as a cause of adverse reserve development, its impact on one important data source will be eliminated by the recent change to record Schedule P - parts 2 and 4 gross of all discount, both tabular and non-tabular.

Use of ranges. Review of the wide variation in use of ranges among survey respondents and analysis of respondents' comments regarding the use of ranges leads members of COPLFR to conclude that development of definitions, procedures and practice standards regarding range methodologies may be needed.

COPLFR wishes to thank the staff of the American Academy of Actuaries for their help in putting the survey results together, and the respondents themselves for their time and effort in responding to the survey.

AMERICAN ACADEMY of ACTUARIES

August 18, 1994

TO: Survey Recipients

The American Academy of Actuaries Committee on Property and Liability Financial Reporting (COPLFR) has previously surveyed signers of statements of actuarial opinion for large insurers, seeking to determine the causes of runoff and suggestions for improvement to the statement of opinion. A summary of the findings appeared in the May 1991 issue of The Actuarial Review.

The Committee is again performing this survey and we are seeking your help. This survey has three goals: (1) determine the causes of runoff of the 1990 reserves; (2) determine the degree of analysis applied by actuaries in establishing reserves; and (3) obtain suggestions to improve the loss reserve opinion document and other general suggestions to aid in the establishment of reasonable reserves.

The Committee's motivation for this survey results from the potential impact of the following factors on actuarial credibility:

1. Industry analysts estimate that reserves were deficient by 10% - 15% as of 12/90 and 12/91.

2. Since 1990, in most cases, loss reserve opinions must be signed by qualified actuaries. Most of these opinions have been interpreted as unqualified.

The Committee is attempting to determine (Section 1) the causes of past runoff, believing this would help explain the perceived deficiency in recent reserves. Also, the Committee wants to identify areas where current procedures and requirements can be improved (Sections 2 and 3).

Companies were selected to participate in this survey in two ways.

Using the NAIC data base, the Committee identified forty company groups that had adverse runoff, after 12/90, which was a large dollar amount or large percent of carried reserves. Actuaries forthese 40 company groups are being asked to complete all three sections of the survey. The Committee decided to send the survey to the signer of the 1993 Opinion believing this individual would best understand what has occurred since 1990 to cause the runoff. In completing Section 1, it could be helpful to discuss this with the signer of the 1990 Opinion, if different.

Additionally, twelve large national company groups, small companies and specialty companies, whose runoff did not fit the above criteria were selected. These companies are being asked to complete the latter two sections. These companies were selected to assure a broad sample of the industry was included.

Attached are the 1990 reserves and runoff for your pooled companies and the companies the NAIC data base includes in the group. Similarly, attached are the 1993 reserves. Please verify that the data is correct. If not, please explain in Attachment 2 the likely reasons for the difference.

We do want to hear from you so that we can further improve the statement of actuarial opinion, improve actuarial procedures and enhance the credibility of actuaries. Our findings will not identify a company or individual. Responses will be kept confidential and will be destroyed after the results are tabulated. Attachment 1 explains the procedure the Committee will use to collect information, respect confidentiality and provide for contact of respondents if needed.

If you wish to discuss any portion of the survey, please feel free to contact David Bryant (AAA staff) or me.

We are asking that the survey be completed by September 15, 1994.

Sincerely,

Patrick J. Grannan /DB

Patrick J. Grannan
COPLFR Chairperson

Milliman & Robertson, Inc.
259 Radnor-Chester Road
Suite 300
Radnor, PA 19087
Phone (215) 975-8026
Fax (215) 687-4236

Return *Survey* To:

David Bryant
American Academy of Actuaries
1100 Seventeenth Street, NW
7th Floor
Washington, D.C. 20036
Phone (202) 223-8196
Fax (202) 872-1948

Survey Procedure

1. The survey will be returned to the American Academy of Actuaries office.

2. Each response will be assigned a code and entered onto a master list. The master list will be retained in the AAA office. The AAA office will also retain Attachment 2, the company group 12/90 Reserves, Runoff as of 12/93 and 12/93 Reserves.

3. Company names, logos, addresses, and other identification will be deleted from the response. The response will then be forwarded to the Committee on Property and Liability Financial Reporting (COPLFR) for review.

4. If the Committee has questions regarding a response, AAA staff will relay the questions to the respondent. Respondents can discuss these questions with AAA staff, or with the Committee chairperson, on a confidential basis.

5. Summarization of company responses (determining averages for all companies) will be done in the AAA office.

6. On December 15, 1994, approximately 3 months after receipt, the AAA will destroy all survey forms submitted to them.

Attachment 2

(From NAIC Data Base)

Company Group
Name ______________________

Code (AAA use) ______________________

12/90 Reserves ______________________

Runoff as of 12/93 ______________________

12/93 Reserves ______________________

The amount of reserves and runoff have been determined from the NAIC data base. Explain if the NAIC numbers are incorrect and write in the correct amounts.

APPENDIX A

Section 1
Sheet 1

Section 1: Contribution to Runoff Since 1990 (for Accident Years 1990 and prior).

Instructions: Identify the sources of the adverse runoff for accident years 1990 and prior which has occurred since 12/90. The amount of reserves carried at 12/90 and runoff have been determined from the NAIC data base and are shown on a separate sheet. It is likely that portions of runoff are caused by two or more factors (such as Involuntary Pool Strengthening and Unwinding of Discount within the Pool). Select the predominant cause. Include in the Comment section whether any portion of the runoff could have been identified at 12/90 if current types of data bases and procedures were available at 12/90. Please quantify the percent of total runoff to the extent possible and provide your best judgment where not quantifiable.

Section 2
Sheet 2

Company Code (AAA to complete) _______

1. Is a range of reasonable estimates determined for the total carried reserves? If no, go to question 6.

2. How wide is the range (from low point to high point) as a percent of carried reserves?

3. How wide is the range as a percent of surplus?

4. Where in the range are the carried reserves at 12/93?

5. Would it be helpful/harmful to require a range to be shown in the loss reserve opinion (and why)?

6. a. Do you perform cash flow testing? Yes ____ No ____

   b. If yes, how are the results used in the actuarial opinion process, specifically in determining whether or not the opinion is qualified?

Section 2
Sheet 1

Company Code (AAA to complete) ____________

Section 2: Identify the level of analysis performed for the reserves established at 12/93. In the following, the term standard techniques includes development of losses, lae, counts and average amounts, Bornhuetter-Ferguson or other methods you apply on a regular basis. Attachment 2 provides the 12/93 reserves shown in the NAIC data base.

Level of Analysis	% of Reserves
1. Reviewed by an actuary but ultimate liability deemed to be inestimable.	______
2. Not analyzed by the actuarial area as too variable or liability is in litigation.	______
3. Not analyzed with standard techniques as volume is too low.	______
4. Not analyzed with standard techniques as line of business is new.	______
5. Amounts assigned by Involuntary Pools and not analyzed.	______
6. Amounts assigned by Voluntary Pools and not analyzed.	______
7. Foreign exposure and not reviewed or limited review.	______
8. Analyzed with standard actuarial techniques.	______
9. Other (describe) ______________________	
Total (should add to 100%)	______

Company Code (AAA to complete) ____________

Section 3: In completing section 3, please consider the causes of runoff you may have identified in Section 1.

Please provide suggestions to a) improve the statement of actuarial opinion, b) to aid in the establishment of reasonable reserves and c) to improve actuarial knowledge and procedures. (Please suggest areas in which you would like more guidance from the Actuarial Standards Board.)

a) ____

APPENDIX A

Company Code (AAA to complete) ________ Section 1
Sheet 2

Cause of Runoff	% of Total Runoff
1. Pollution, Environmental, Asbestos, toxic materials or other similar items.	____
2. Other long tail, shock type situations such as landmark court decisions or new area of liability.	____
3. Reinsurance (Commutation or Insolvency).	____
4. Involuntary Pools strengthening.	____
5. Timing of the release of Involuntary Pool information.	____
6. Voluntary Pools strengthening.	____
7. Timing of the release of Voluntary Pool information.	____
8. Unwinding of disclosed discount (including tabular).	____
9. Unwinding of undisclosed discount.	____
10. Result of loss responsive programs where future premiums were netted against future losses.	____
11. Management or Company Reorganization (other than Claims Department). (explain) ________	____
12. Claims Department reorganization or changes in practice.	____
13. Result of financial pressures.	____
14. Change in reserve procedures.	____
15. Data base detail deficient or incomplete.	____
16. Data base error.	____
17. Other system problems. (explain) ________	____
18. New area, where insufficient historical information was available.	____
19. Low volume line, where estimation was difficult.	____
20. Catastrophic line (umbrella, excess) - too variable.	____
21. Area was not reviewed.	____
22. Tail factors were too low.	____
23. Other (explain). ________ ________	____
24. All beneficial runoff.	____
Total (should add to 100%)	____

Which lines contributed the most to the adverse runoff? ________

Other Comments: ________

RESPONSES TO SURVEY SECTION 1 SHEET 2 — CAUSE OF RUNOFF ANALYSIS — APPENDIX

COMPANY CODES

ITEM #	13	15	18	19	20	22	24	25	27	28	30	34	35	37	39	41	49	50	52
1			101%	45%	75%	72%	149%	10%		2%	17%	64%		250%	87%	40%			
2				10%						2%					34%			20%	
3																			
4			5%	13%	17%	5%				19%	9%				30%	15%			
5						10%				10%	7%					15%			
6		5%		16%						14%	15%								
7																			
8			12%	13%		13%			90%	8%	9%				14%	10%			
9													12%			10%			
10						2%				10%		6%							
11													50%						
12																15%	15%	20%	20%
13																10%			
14																			
15																		25%	40%
16																			
17																			
18								25%										15%	40%
19						-6%		25%											
20								15%		10%									
21						2%													
22	20%				40%	2%	10%		10%			7%		10%		5%		20%	
23	80%	95%		21%				25%		25%	50%	23%	38%				85%		
24			-18%	-18%	-32%		-59%				-7%			-160%	-65%	-20%			
SUM	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
LINES NOTED	WC	WC	GL	GL,WC	GL,WC	PROD, GL,WC SP LB	TRTY CAS XS		WC	COMM.	WC, GL, CMP		WC	WC, PROD	REIN, WC,GL	WC,GL	MED MAL	GL,WC, COMM AUTO	CMP, GL

COUNT	AVG 90	AVG R/O	AVG 93
12	72%	70%	74%
4	2%	1%	2%
0	0%	0%	0%
8	9%	8%	9%
4	4%	4%	4%
4	2%	2%	2%
0	0%	0%	0%
8	7%	6%	8%
2	2%	2%	2%
3	1%	1%	1%
1	0%	0%	0%
4	3%	3%	3%
1	2%	2%	2%
0	0%	0%	0%
2	0%	1%	0%
0	0%	0%	0%
0	0%	0%	0%
3	0%	1%	0%
2	0%	0%	0%
2	0%	0%	0%
1	0%	0%	0%
9	11%	10%	10%
9	11%	14%	12%
8	-27%	-25%	-29%
19	100%	100%	100%

Percent Responding

48%	52%	45%	48%

Number of Companies -LOB Analysis

LOB	Number	Notes
WC	13	Four apparently WC only, one company had (4)-invol pool strengthening and (8)-unwinding of discount w/o WC noted.
GL	12	Included products and treaty cas xs
Med Mal	1	Specialty company
None ID	2	

RESPONSES TO SURVEY SECTION 2 SHEET 1 APPENDIX C

LEVEL OF ANALYSIS

COMPANY NUMBER	CATEGORY NUMBER 1	2	3	4	5	6	7	8	9	SUM
1	1%							99%		100%
2								90%	10%	100%
3		2%	1%		2%			95%		100%
4								100%		100%
5								100%		100%
6	2%			2%	5.5%		0.5%	90%		100%
7								100%		100%
8						5%		95%		100%
9		7%			2%	2%		89%		100%
10			10%		5%	10%		75%		100%
11	4%				12%	1%		83%		100%
12	3%	2%		0.5%	9%	0.5%		85%		100%
13			5%					95%		100%
14	10%		10%					80%		100%
15						5%		95%		100%
16	3%			2%	10%	3%		79%	3%	100%
17				1%	9%	2%		88%		100%
18	8%				8%	1%		75%	7%	100%
19					2%			48%	50%	100%
20	23%				2%	2%		73%		100%
21								100%		100%
22	7%		1%		13%	2%		77%		100%
23			1%		6%	9%		84%		100%
24								100%		100%
25								100%		100%
26								100%		100%

	1	2	3	4	5	6	7	8	9	SUM	% Responding RUNOFF	% Responding ALL
COUNT	9	3	6	4	13	12	1	26	4	26	50%	50%
AVG 90	5%	1%	1%	0%	7%	2%	0%	83%	1%	100%	53%	53%
AVG R/O	5%	1%	1%	0%	7%	2%	0%	82%	2%	100%	41%	47%
AVG 93	3%	1%	1%	0%	5%	1%	0%	88%	1%	100%	50%	61%

APPENDIX D

RESPONSES TO SURVEY SECTION 2/SHEET 1

LEVEL OF ANALYSIS - CATEGORY 9 DETAIL

1. A loss ratio method technique was used, not considered to be a standard actuarial technique by the respondent;

2. The reserve for asbestos was set by reserving at policy limits with a reduction for the probability of not exhausting high layers and including a provision for expense outside limits;

3. A non-standard technique was used for some areas including a limits available method or a limits exposed method;

4. A method was used for ULAE other than standard techniques, known as the "Wendy Johnson technique";

5. Reserves were analyzed using other techniques due to substantial case reserve strengthening in the most recent two years.

RESPONSES TO SURVEY SECTION 2 SHEET 2

APPENDIX E

RANGE AND CASH FLOW ANALYSIS

(IN COLUMN 2 ORDER)

COMPANY LETTER	ITEM NUMBER 1	2	3	4	5	6	COMMENT ON 6
A	1	30%	122%	49%	-	1	Not in opin.,in dynamic solvncy
B	1	23%	84%	65%	+	0	
C	1	16%	22%	50%	+	1	Reserves undisc, no impact
D	1	15%	56%	47%	+/-	0	
E	1	11%	35%	2%	+/-	0	
F	1	11%	28%	30%	-/+	0	
G	1	10%	50%	50%	+?	0	
H	1	10%	40%	50%	-/+	1	WC discount
I	1	10%	35%			1	
J	1	10%	33%	50%	no?	0	
K	1	10%	16%	49%	+/-	0	
L	1	10%	7%	132%		0	
M	1	10%		20%		1	As a check
N	1				-	0	
0	1				-	0	
P	0					1	At beginning stage
Q	0					0	
R	0					0	
S	0				-	0	
T	0					0	
U	0				-	1	
V	0					1	Disclosed disc. in opinion
W	0					0	
X	0					0	
Y	0					0	
Z	0					1	Nlot used.

PERCENTAGE RESPONDING

	1	2	3	4	6	RUNOFF	ALL
STR. AVG	0.58	0.14	0.44	0.49	0.35	50%	50%
COUNT	15	13	12	12	9		
AVG 90	72%	17%	66%	45%	30%	53%	53%
AVG R/O	77%	16%	64%	47%	29%	47%	47%
AVG 93	52%	16%	61%	44%	22%	50%	61%

Dynamic Financial Models of Property/Casualty Insurers
by the Subcommittee on Dynamic Financial Models of the
CAS Committee on Valuation and Financial Analysis

DYNAMIC FINANCIAL MODELS OF PROPERTY-CASUALTY INSURERS

Prepared by
The Subcommittee on Dynamic Financial Models
of the Casualty Actuarial Society's
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September 22, 1995

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CHAPTER 1
PURPOSE

The purpose of this report is to discuss and provide guidance on the important issues and considerations that confront actuaries when designing, building or selecting dynamic financial models of property-casualty risks. It has been prepared by the Subcommittee on Dynamic Financial Models of the Casualty Actuarial Society's Valuation and Financial Analysis Committee. It constitutes part of the Society's ongoing educational efforts on issues affecting actuaries responsible for the strategic and dynamic financial analysis of insurers.[1]

In writing this report, the Subcommittee has intentionally avoided placing requirements on actuaries or the models used by actuaries. These requirements have been and will continue to be addressed by the Actuarial Standards Board.

[1] Other sources of information regarding dynamic financial models is included in Appendix A.

CHAPTER 2
INTRODUCTION AND BACKGROUND

What is Dynamic Financial Analysis?

One of the early references to dynamic financial analysis comes from Jay W. Forrester in Industrial Dynamics. He defines it as ". . a way of studying the behavior of industrial systems to show how policies, decisions, structure, and delays are interrelated to influence growth and stability. It integrates the separate functional areas of management -- marketing, investment, research, personnel, production and accounting. Each of these functions is reduced to a common basis by recognizing that any economic or corporate activity consists of flows of money, orders, materials, personnel, and capital equipment. These five flows are integrated by an information network."[2]

Models are the key tools in dynamic financial analysis. Such models are ". . , a systematic way to express our wealth of descriptive knowledge about industrial activity. The model tells us how the behavior of the system results from the interactions of its component parts."[3]

The Actuary's Need for Dynamic Financial Models

Historically, casualty actuaries have focussed primarily on rates and loss and loss adjustment expense reserves. As the portion of insurers' liabilities arising from casualty insurance has increased, their use of reinsurance has decreased and actuarial valuations of liabilities have become increasingly important. Property-casualty actuaries, and in particular members of the Casualty Actuarial Society, have had increasing responsibility to provide opinions on the loss and

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[2] MIT Press, 1961, p. vii.

[3] Ibid.

loss adjustment expense reserves of property-casualty insurance companies in the U.S. since 1980, when the first state introduced such requirements.

In more recent years, regulatory and competitive pressures, as well as the desire for a more holistic understanding of the insurance process, have led and continue to lead to expansions of the casualty actuary's role. It is anticipated that, with the emergence of the Appointed Actuary concept for property-casualty insurers in the United States, actuaries will be responsible for understanding insurance company assets, cash flows and investment risks as well as liabilities. Property-casualty actuaries are expected to place increased reliance on dynamic financial models similar to the mandated use of dynamic financial models in emerging standards for life insurers. As such, it is becoming increasingly important that casualty actuaries become familiar with dynamic financial models.

Concurrent with the changing role of the property-casualty actuary have been changes in computer power and software ease that have made the use of dynamic financial models more practical. Specifically, models that would have taken days to code a decade ago can now be implemented in minutes and results can now be. expressed graphically using standard software, easing interpretation.

Why Use Dynamic Financial Models?

Dynamic financial models generally reflect the interplay between assets and liabilities and the resultant effects on income and cash flows. This explicit recognition of all of the insurer's' operations gives dynamic financial models the power to illustrate the links between strategies and results. Therefore these models make a unique contribution to the actuary's set of tools for financial analysis.

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[4] Throughout this report, the application of dynamic financial models to insurers is discussed. These models are equally useful for captives, risk retention groups, self-insurance pools and large self-insureds.

Dynamic financial models are characterized by the projection of both income and cash flows over a period of time. The time delays between occurence of claims and their payment in the property-casualty insurance business make it difficult or impossible to evaluate strategies and decisions without explicit consideration of their effects on flows of funds. Indeed, the results of management decisions or the effects of outside forces may often be counter-intuitive unless a dynamic financial model has clarified the situation.

A scenario is a set of assumptions about the environment in which the insurer's operations will take place. Scenarios are used to illustrate the implications of strategies and decisions in the context of information about the risks that confront the insurer. The explicit consideration of scenarios gives a dynamic financial model a unique role in helping management in identifying profit opportunities and encouraging investment in the company. Such explicit consideration also assists regulators in understanding problems before they grow to crisis size. Management can often identify potential problems earlier, and regulators can distinguish short-term problems that do not warrant intervention from long-term problems that require action.

CHAPTER 3

USES, USERS AND RESOURCES

The design and/or selection of a dynamic financial model will depend heavily upon the purpose of the engagement (use of the model), the users of the model and its results, and the available resources.[5]

Uses

Dynamic financial models have many uses, including:

- Determining the value of an insurance company or block of policies to a potential buyer or seller.

- Assessing how an insurer might fare in a range of future economic environments.

- Strategic planning, including asset-liability management, tax planning, reinsurance planning and costing, and market strategy.

- Feasibility studies.

- Tactical decision-making, including product pricing. (Although dynamic financial models are not yet widely used to price property-casualty insurance products today, they are already widely used to price life insurance products.)

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[5] These considerations, along with the others identified in this report, are summarized in Appendix B.

- Identifying the kinds of risks that most threaten the solvency of the insurer.

The use of the model will be a key determinant of many of its requirements. Examples include:

(1) Model input and output depend on the use. For example, if modelling a worst-case scenario for solvency testing, a complex tax module is not important because the insurer is unlikely to pay substantial income taxes, at least under current federal income tax laws,

(2) The use of the model helps determine the time frame and accounting basis. For example, if regulators ask the actuary to model solvency over a two-year time horizon and ensure that risk-based capital requirements are met, then a minimum of two years of future statutory accounting statements is required.

(3) The use of the model may determine whether a deterministic or stochastic model is more appropriate. This decision in turn will greatly affect the resources and data needed. the model structure and the form that output will take. As an example, if the goal is to develop probability distributions of results, then an actuary will be more likely to use a stochastic model.

Users

Future Appointed Actuaries and insurers that wish to anticipate the results of the Appointed Actuary's work are the users who are driving the CAS's educational efforts on dynamic financial analysis. Other users of dynamic financial models include consulting firms and insurers that employ such models as tools for tactical and strategic decision-making, including pricing decisions. Third-party users of the results of dynamic financial models can include regulators, reinsurers, investment bankers, financial intermediaries, institutional investors, securities rating organizations, and financial analysts.

The intended users' needs are a consideration in designing and selecting the model. The type of model used and its structure depend on customers (users) and their needs. As an example, regulators may focus mainly on the insurer in total. Company management may focus on the total corporation or on each individual product.

Resources

The choice of dynamic financial model will depend on the available resources, whether these resources are people available for system design and programming, data from which to derive assumptions and with which to initialize the model, money available to purchase an existing software package, or computer architecture.

Detailed dynamic financial models require a significant investment of time for research to determine assumptions, as well as for maintenance to keep the model's logic current and to revise assumptions in light of new data. Such models also require a significant expenditure of time in interpreting the results.

The choice of computer architecture is often determined by the purpose of the analysis and the level of detail of the projections. A simple spreadsheet might be appropriate if the purpose of the study is to highlight the effects on financial results of one particular risk, such as adverse development of loss reserves. At the other extreme, complex, report-generating software with a user-friendly front-end and efficient coding of the detailed calculations might be appropriate if the model is intended to cope with a wide range of different problems and be used by a wide range of users.

CHAPTER 4
RISKS OF THE PROPERTY-CASUALTY INSURANCE BUSINESS

The evaluation of risk is the focus of dynamic financial models. The relative importance of each type of risk will determine the detail of assumptions and analyses built into any model. In this chapter, the risks of the propertycasualty industry are described and the related modelling considerations addressed.

Property-casualty insurance risks can be divided into many categories. In this paper, we will follow the definitions originated by the Committee on Valuation and Related Matters of the Society of Actuaries and will discuss these risks in the following four categories:

- C-l risk - Uncertainty surrounding cash flows from invested assets other than from uncertainty regarding interest rate risk.

- C-2 risk - Uncertainty surrounding cash flows from the obligation or underwriting aspects of an insurance company.

- C-3 risk - Uncertainty surrounding cash flows from interest rate fluctuations in the presence of a mismatch of assets and liabilities and the risk of disintermediation caused by embedded options that are sensitive to changes in interest rate.

- C-4 risk - Uncertainty emanating from mismanagement, i.e., making incorrect or fraudulent actions in light of the available information.

As do many discussions of insurance risks, this paper will focus on the first three of these risks. At present, measuring the risk of mismanagement is beyond the scope of most actuarial engagements.

Asset Risk

Asset risk, also known as C-l risk, is the risk that the amount or timing of items of cash flow connected with assets will differ from expectations or assumptions as of the valuation date for reasons other than a change in interest rates. It encompasses uncertainty regarding:

- Default rates.
- Future'market value of equity assets.
- Liquidity of assets.

In addition to these inherent asset risks, there is also the risk that the character of the assets will not be evident from their general descriptions. This problem is increasing as capital markets develop a greater range of non-equity investments that have many of the risk characteristics of equity investments.

Appropriate data and methods are critical to the development of ranges of assumptions to reflect asset risk in the projected performance of the insurer. Historical data developed for investment managers is readily available, including time series of default rates of various classes of assets as a function of age.

Dynamic financial models can be used to estimate the effects of these risks alone on the projected performance of the insurer and can also be used to estimate the interrelationships between these risks and other risks. In modelling, asset risks may be assumed to correlate with inflation or some other variable or to be autoregressive.

Obligation Risk

Obligation risk, also known as C-2 risk, is the risk that the amount or timing of items of cash flow connected with the obligations considered will differ from expectations or assumptions. For property casualty companies, obligation risk encompasses:

- Reserve risk - the risk that the actual cost of losses for obligations incurred before the valuation date will differ from expectations or assumptions.

- Premium risk - the risk that premium for future obligations will differ from expectations or assumptions.

- Loss projection risk - the uncertainty regarding assumptions about future loss costs.

- Catastrophe risk - the uncertainty regarding the costs of natural disasters and other catastrophes.

- Reinsurance risk - the uncertainty regarding the cost, value, availability and collectibility of reinsurance.

- Expense risk - the risk that expenses and taxes will differ from those projected.

Dynamic financial models can be used to estimate the effects of these risks individually on the projected performance of the insurer and to evaluate the interrelationships between these risks and other risks.

Reserve risk may be a function of:

- Inflation in medical costs and other determinants of claims costs.
- The legal environment in which claims will be resolved, including the environment in which claims are pursued by policyholders or third parties.
- The possibility of a breakdown in some basic premise underlying the reserves for a particular coverage (such as has occurred with environmental impairment liability).
- Past patterns of pricing adequacy which affect case reserves or financial reserves.
- Corporate culture, training, and incentives that affect the payment of claims or the adequacy of case reserves.
- Currency fluctuations which affect the costs of losses when expressed in local currency.
- The randomness of the claims process itself.[6]
- Incompleteness of data bases.

[6] The randomness of the claims process itself can be studied by modelling the patterns of loss development or by more detailed analysis of the claims process. Inevitably, however, data for such models always include the effects of other factors affecting the claims process.

Premium risk may be a function of:

- Competitive pressures that do not allow the insurer to achieve assumed levels of exposure and/or rate adequacy.

- Regulatory intervention that restrains premium increases or decreases or that requires business to be underwritten that would not be underwritten in the absence of such intervention.

- Premiums for involuntary business underwritten at premium rates and in volumes that differ from assumptions.

- Retrospective premiums or dividends that differ from assumptions.

- Amounts collectible from agents that differ from assumptions.

If premium risk is expected to arise from a cyclical pattern of premium adequacy in the competitive market, a cyclical component could be incorporated into the model or into the premium adequacy assumptions.

Loss projection risk is a function of the factors that affect reserve risk and also of the uncertainty regarding:

- Changes in loss costs and exposures from the historical experience period.

- Loss costs for the mix of new policies being underwritten, including the effect of adverse selection.

- Loss adjustment practices in the future that may differ from those in the past.

Catastrophe risk can be considered a component of loss projection risk. It is a function of:

- The coverage being written.

- The concentration of insured values in specific geographic areas or legal jurisdictions.

- Uncertainty regarding the frequency, seventy and nature of catastrophic events.

Computerized models of the damage arising out of certain types of catastrophes are available which may be of value in determining assumptions about the probabilities and sizes of catastrophic losses. Output from these catastrophe models may be used as input to a dynamic financial model or a link between the models may be established so as to include the impact of catastrophe risk in dynamic financial models.

Reinsurance risk is a function of changes in the price and availability of desired reinsurance, and of uncertainty regarding the collectibility of reinsurance recoverables arising from the financial condition of the reinsurer or ambiguity about the coverage provided. Reinsurance risk exists in each of the four obligation risks identified thus far. In many models, projections are made on a net of reinsurance basis. Such projections incorporate implicit assumptions regarding reinsurance risks, whereas projections made on a gross of reinsurance basis require explicit instructions regarding the reinsurance mechanism. Reinsurance risk recognizes how reinsurance responds under stress, such as a large catastrophe or other strain on collectibility, aggregates, reinstatements and other reinsurance parameters.

Expense risks, those associated with expenses (other than loss adjustment expenses) and taxes, include uncertainty regarding:

- Contingent commissions to agents.
- Marginal expenses of adding new business.
- Overhead costs, including the risk that overhead costs will be changed by regulatory intervention and the risk that there may be periods of changing premium during which overhead costs will not change in proportion to premium.
- Assigned risk overburdens, second injury funds and other assessments.
- Federal and local income taxes, both in interpreting the current Internal Revenue Code and in anticipating changes in the code.

These lists of uncertainties regarding the major components of obligation risk are illustrative. Other factors may affect obligation risk.

Interest Rate Risk

Interest rate risk, also known as C-3 risk, is the risk that net cash flows will depart from expectations or assumptions as the result of interest rate fluctuations. Interest rate risk encompasses uncertainty regarding cash flows from assets, including bond yields, mortgage interest rates, real estate income, and dividends on equity investments. It also encompasses uncertainty regarding cash flows related to borrowing, such as the interest rate on any loans taken out by the company or cost of capital.

Reinvestment risk is uncertainty regarding yields that will be. available on reinvestment of proceeds from investments that are currently held. In many dynamic financial models, a set of assumptions must be made about the yields that will be earned on future investments. Often in practice the apparent solvency or insolvency of the enterprise will be sensitive to the choice of interest rate ("reinvestment rate").

Another component of C-3 risk is the uncertainty regarding the market value of any fixed-income assets that must be sold prior to maturity to meet cash flow needs. C-3 risk includes market value uncertainty related only to changes in interest rates; market value uncertainty related to changes in perceived credit or default risk is a component of C-l risk. The reinvestment rates, discussed above as being determinants of reinvestment risk, also determine market value risk for fixed-income assets. Thus, the reinvestment rate can have a significant impact on the results of the model, resulting in an under- or over-statement of risks because of an inexact choice of reinvestment rate.

CHAPTER 5
RISKS INHERENT IN THE MODELLING PROCESS

Once the risks to be incorporated in the model have been identified and the model built, there are a number of risks inherent in the modelling process to consider, including:

- The range of scenarios may not reflect the user's intent.
- The model may be incorrectly or incompletely specified for the intended purpose.
- The model's results may be inappropriately interpreted.

Importance of Scenario Testing and Selection of Assumptions

Proper use of a model depends on the selection of appropriate scenarios to evaluate and the development of consistent assumptions within each scenario. The purpose of the model will influence the data and methods used to provide assumptions for understanding the projected performance of the insurer. Scenarios permit links between assumptions for various parts of the model. For example, a high interest rate scenario might include assumptions of high bond yields, low common stock values with high dividends, high inflation in medical costs, and a low level of unemployment.

Scenarios provide an especially relevant tool for determining the implications of risks on the projected performance of an insurer. Observing the results for a variety of scenarios yields information about the company's response to risk. Careful selection of scenarios is essential.

One of the reasons for using dynamic financial models is that they can provide information about the interactions among risks. Dynamic financial models can indicate the extent to which components of the company interact with one another. Depending on the purpose of the model, the actuary may have a responsibility to describe the ways in which several components appear to be interacting, particularly if they alter the risk that arises from uncertainty about the assumptions or logic for a single component.

In many situations, the actuary will be constrained with respect to the choice of scenarios. At this time, life insurance regulatory authorities specify certain scenarios to be modeled by the actuary, at a minimum. Similarly, Canadian regulations provide general guidance regarding the choice of scenarios. This kind of regulatory requirement may expand to U. S. property-casualty actuarial work in the future. Sometimes the scenarios to be studied will be specified by company management rather than by the actuary. However the range of scenarios is selected, its choice will impact the results that the model produces. It may be appropriate to observe the model under scenarios other than those specified by regulators or management to adequately understand the implications of the scenarios that were specified.

When the range of scenarios has been selected using only retrospective tests as a guide, the model may be prone to be over-determined. For example, the risk that the probability distributions in a stochastic model are incorrectly specified can be minimized by choosing probability distributions that have greater uncertainty (central tendency, dispersion, and skewness) than historical data.

Model Specification

The risk that a model is incorrectly or insufficiently specified can be minimized by validation, i.e., matching the model to the insurer's own history over some period of time. A well-specified model will reasonably reproduce past actual results. Actual results varying from projections may not be an indication of a poor model. Rather, it is generally appropriate to investigate such differences and reconcile the model's results with the actual results. This process of reconciliation may identify weaknesses in the model, or it may clarify ways in which the enterprise's activities departed from what would have been reasonably expected (e.g., writing more, rather than less, unprofitable business to cover up poor experience).

Interpreting the Results for a Range of Scenarios

Summarizing a range of outcomes includes development of measures of the performance of the insurer, as well as description and explanation of anomalous results. Measures of performance include:

- Risk-adjusted present value of future cash flows.
- Management-defined objectives.
- Probability or cost of "ruin."
- Option-adjusted pricing.

Other measures may also be appropriate. The method of summarizing results will depend on the purpose of the model.

Under the first approach, value is calculated as the risk-adjusted present value of the future cash flows. Calculations of risk-adjusted present value may include separate risk adjustments for stochastic or process risk (random variation) and scenario or parameter risk (variation among scenarios). This approach allows for specific consideration of the cost of risk, Similar results may be obtained by observing the model's results under a set of assumptions that are conservative in light of the uncertainties indicated by the model and computing the present value of the resulting flows of funds at a risk-free rate.

An insurer's modeled performance may also be measured in terms of objectives defined by company management. For example, management may set objectives such as maintaining acceptable risk-based capital results, failing no more than two IRIS tests or maintaining a combined ratio less than 100%. The insurer's performance relative to these benchmarks can be measured by using a model that calculates these statistics.

In the third of these measures, the probability and expected value of each outcome is estimated. The actuary may decide on a threshold characterized as "ruin," and use a stochastic model to estimate the probability of ruin for a given set of assumptions. Alternatively, the actuary may establish a cost of ruin (and perhaps establish nominal values for certain other types of outcomes as well), and compute an average of the adjusted financial outcomes over a range of assumptions. The actuary may also select a threshold much closer to the current financial condition, such as a decline in financial rating by one level, and estimate the probability of such an outcome.

Under the fourth measure, the total value of the insurer is summarized as the current market price of a set of investments available in the capital markets which has the same risk characteristics as the model indicates for the insurer. Such a set of investments almost always includes a large proportion of options because the insurer's cash flows are typically inflows first and outflows second, so the resulting value is called the option-adjusted price of the insurer's assets and liabilities. This value reflects the insurer's strategies for investment and for handling unexpected shortages of cash, at least as far as those strategies are reflected in the model.

There is an ongoing dialogue among actuaries about the appropriate basis for summarizing the results of a model. The Combination of Risks Task Force of the Society of Actuaries' Committee on Valuation and Related Problems concluded that the appropriate basis for summarizing the results of a dynamic financial model is the cash basis.[7] According to this school of thought, the other accounting bases (statutory, GAAP, and tax) are important only insofar as they serve to identify constraints on the enterprise's operations (e.g., tax payments).

On the other hand, the Actuarial Standard of Practice for Appraisals, promulgated by the Actuarial Standards Board, suggests that statutory accounting is the appropriate basis for measuring financial results. In this school of thought, the statutory and tax accounting rules place real constraints on the cash flows that can be realized by the investor.

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[7] Transactions of the Society of Actuaries, 1991-92 Reports, p. 451.

Depending on the purpose of the model, the actuary may need to describe anomalous results indicated by the model. The results of the model may suggest that-either some assumptions are incorrect (in which case the assumptions will likely be revised before results are presented) or that the insurer's strategies could be improved. As an example of the latter, the results of the model may suggest that the insurer is particularly at risk due to one or more sources of risk.

The risk of inappropriate interpretation can be minimized by communicating the limited extent of variation among scenarios compared to the potential range of variation in the results of the insurer's operations.

CHAPTER 6

OTHER MODEL CHARACTERISTICS

However simple or sophisticated, a model is no more than a metaphor for the insurer. Dynamic financial models differ in the types of risks they are effective at measuring. A key consideration in the selection of a dynamic financial model is its ability to evaluate material sources of solvency risk for the case at hand.

Generalized vs. Tailor-Made

Generalized models, such as those developed by several consulting firms and software vendors, usually permit the user to specify several different types of insurance products, or lines of business, and a range of different investments. Other models are often tailor-made, such as one that addresses the unique characteristics of a company or because a simple model is sufficient. If a generalized model is used, it is important to consider whether results may be distorted by features inapplicable to a particular application or because a characteristic of the particular company is not addressed. For example, if a general purpose model is used for an insurer that plans to invest only in bonds and cash equivalents. the model does nor need to include a strategy that involves investment in other assets. If it does, the ramifications of that logic should not distort the projections.

Logic vs. Input

Whatever computer hardware and software may be used to implement the model, there are always tradeoffs between the coding of logic versus the selection of parameters. Dynamic financial models differ in the choices the developers make about which assumptions will be represented by variables and which will be fixed by the software or hardware. Also, the user will be able to determine the values of certain variables used by the model, whereas others will have been pre-set by the developer. The mix between input and logic will be determined in part by the users of the model (both the operator and the decision-maker), as models with extremely large numbers of variables can be daunting to use and difficult to interpret.

In selecting or building a dynamic financial model, decisions must also be made about the level of detail to be captured about the insurance coverage (by broad product group, statutory line of business, individual form, etc.), the factual context (including the level of detail about accounting and tax rules), and the precision with which strategies will be defined.

Strategies are inevitably a part of the logic of a model. The strategies incorporated in the model should be reasonably consistent with its purpose. Some software allows the user to build in explicit recognition of management strategies. Other software assumes certain strategies, even to the extent of letting presumptions about strategies affect the architecture or modular design of the model.

Time Frame

The time frame for the analysis is an important consideration in the choice and design of dynamic financial models. For example, it may be appropriate to use a time frame of 24 months to evaluate strategies for a property insurer (although a longer time frame may be needed to address recovery from a large catastrophe), whereas a time frame of 24 years may be more appropriate to evaluate the solvency of an underwriter of products' liability. The choice of time frame will also be a reflection of whether the model includes only the run-off of current business, a going concern for some stated period, or a going concern in a long-range projection valuation.

In addition to the time horizon of the model, the model also reflects a choice about the length of time intervals under study. While annual time intervals may be appropriate for some purposes, quarterly or even monthly time intervals might be appropriate for other purposes.

Basis of Accounting

Comprehensive dynamic financial models of the corporate insurance enterprise usually include accounts on at least four bases simultaneously: cash, statutory, GAAP, and tax[8]. Doing so is the only way to reflect the details of the interrelationships among constraints imposed by investment opportunities, underwriting commitments, laws and regulations, generally accepted accounting principles, and income tax laws. Less comprehensive models may be appropriate, however, for some purposes.

Relationship between Parent and Subsidiaries

Parents and subsidiaries have a number of different effects on an enterprise. A consolidated model of the entire organization can be developed, or the existence of the parent and subsidiaries might simply show up as assumptions about flows of funds, tax calculations, and income. A model may explicitly reflect a range of scenarios regarding the availability of or drain on surplus due to external influences. Alternatively, each entity may be modeled separately, with output from one model serving as input for other models.

Feedback Loops

Dynamic financial models may employ feedback loops (automatic conditional decisions) which are algorithms that make calculations for each modeled time period dependent on values calculated for earlier periods. Feedback loops provide for reactions to specific conditions. Models without feedback loops may be underdetermined, showing excessive income under favorable scenarios and excessive loss under unfavorable scenarios. Models with feedback loops, however, may be overdetermined, showing little risk regardless of the scenario because the model builder often assumes that management will respond quickly to increased risk with appropriate strategic or operational responses.

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[8] Financial reporting, and therefore modelling, may be more complex for international users.

stochastic Risk

Purely random fluctuations in the modelled variables may be important for a particular application. Stochastic features surrounding input assumptions can be added to a model. Random fluctuations around projected losses, for example, may be incorporated into a model by introducing probability distributions about loss costs or loss ratios, by modelling the collective risk process, or by modelling the underlying claim settlement process.

A simple model of the collective risk process may assume, only a probability distribution for the frequency of losses as a function of some assumptions and a probability distribution of the sizes of losses as a function of other assumptions. A more complicated model of the collective risk process may include estimates of the uncertainty of the parameters of frequency and size-of-loss, and may include a number of different kinds of losses, each with its own frequency and size-of-loss assumptions. A model of the underlying claim settlement process may be a multi-state Markov chain model or some other appropriate model.

The importance of identifying and modelling the interactions among risks increases when stochastic models are used. When assumptions are stochastically generated, a model that does not reflect these interactions may produce meaningless results in certain scenarios. At best, the results of such models would be difficult to interpret.

CHAPTER 7
FORCES FOR CHANGE

Thus far, this report has focused on the state of art and practice at this time. There are sweeping changes underway that may affect modelling in the future.

Proliferation of Insurance Products

Although regulation and custom tend to slow the creation of insurance products by entrepreneurs, changes in the markets served by insurance enterprises constantly press for new products and services. Dynamic financial models may need to be refined to adapt to these changes.

Competitive Pressures

In the past, pressures were perceived to arise from competition at the point of sale of the insurance product. From at least as long ago as 1970, competitive pressure has increasingly come to mean competition at the point that capital is being raised. Dynamic financial models are playing an increasingly visible role in corporate decisions regarding purchases and sales of business units, means to tap capital markets, and trade-offs between capital and reinsurance. This trend might reasonably be expected to continue.

Innovation in Assets

Recent innovations in asset design make it difficult to understand the riskiness of many investments by looking at their designations for accounting purposes. For example, some bonds have the risks of stock investments or mortgages and mortgages are backed by a wide range of security. Existing accounting classifications may be misleading to tabulate information about assets for input into dynamic financial models.

New types of asset classes are emerging, some with purposes other than purely generation of investment returns. For example, some assets, such as catastrophe futures, can hedge risks undertaken by the insurer's underwriting activities. More innovation can be expected, along with the need to model these kinds of investments.

Regulatory, Accounting, and Tax Requirements

Dynamic financial models may need to be revised from time to time to reflect the latest developments in regulation. Such changes may be as simple as adding a set of calculations, or they may require modelling of the corporate response to the impact of the regulations (e.g., a shift in marketing or investment strategy to accommodate surplus constraints of risk-based capital). Projections of cash flow may react to changes in these constraints differently from projections of statutory results, and dynamic financial models with feedback loops may react differently from static models.

Hardware

Although changes in computer hardware over the past twenty years have in some ways increased the speed with which tasks get done, they have had a fundamental and irreversible effect on the kinds of problems that people address. For example, before data processing was available that could prepare an extensive Schedule D (details of assets of insurers), regulators simply prohibited and restricted investments outside a few narrow categories; today, they attempt to monitor insurers' investments. Models of corporate financial results were not considered to be important tools for actuaries until computer hardware existed on which such models could be run, The actuary can expect that the changes in hardware will transform both the problems the actuary will be expected to address and the nature of actuarial work.

One major change on the horizon is distributed processing. In the future, the actuarial tool kit may consist of essentially instant communication with a large number of models of a given insurance enterprise, each being updated with new information essentially in real time. Between that future and to&y lies a time of rapid change in the power and distributions of hardware, software, and data.

Life Insurance Models

Dynamic financial models of a high degree of sophistication now exist for life insurance enterprises. These models are being used for product pricing and corporate valuation as well as for strategic and tactical (e.g. tax) planning. These models, and the experiences of their users, may have an important effect on the direction of development of models of property-casualty insurance companies. Life insurance models affect the perceptions and expectations of regulators, many of whom have responsibility for both life insurance regulation and property-casualty insurance regulation.

Other Countries

The increasing degree of globalization of the national economies, and the long-standing trend to lower economic borders between countries, suggest that actuarial work in the United States will be affected by innovations developed outside the United States and vice versa.

For example, Canada recently introduced solvency regulation for property-casualty insurance companies. All companies are required to designate an appointed actuary who is a Fellow of the Canadian Institute of Actuaries (CIA). In addition to performing the valuation of loss reserves, unearned premium reserves and deferred acquisition expenses for a company, the appointed actuary is required to report to the Board of Directors at least once a year on the current and expected future solvency of the company. To make this report, the appointed actuary is expected to perform dynamic solvency testing in conformance with the standards of practice set by the CIA. In cases in which a company is thought to be in difficulty, federal regulators can require that the appointed actuary submit a report on the results of a dynamic solvency test of the company's business plan over a planning horizon of one year.

APPENDIX A

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APPENDIX B
CHECKLIST OF CONSIDERATIONS

1. Is the model appropriate for the intended use?

2. Are the model and related communications appropriate for the expected users of its results?

3. Can the model be developed, purchased, maintained and/or used within the personnel, time, hardware, software and budget resources available?

4. Does the model contain input, output and processing regarding each of the risks to be evaluated in appropriate detail? Are the available historical data regarding these risks sufficient to use to derive the assumptions needed by the model? These risks include:

    - Asset risk
    - Obligation risk
        - Reserve risk
        - Pricing risk
        - Loss projection risk
        - Catastrophe risk
        - Reinsurance risk
        - Expense risk
    - Interest rate risk

5. Is the range of scenarios broad enough to reasonably address the questions at hand?

6. Is the model specification accurate and appropriately complete?

7. Are the measures used to summarize and interpret the range of results reasonable for the application?

8. Have the limitations of the model and range of scenarios been communicated clearly to reduce the risk of misinterpretation?

9. Is a generalized model reasonable for the application or would a tailor-made model better address specific issues?

10. Does the model have a reasonable balance between input assumptions and hard-coded logic?

11. Is the model's time horizon appropriate to the application?

12. Are the accounting bases upon which the model makes forecasts of appropriate breadth to the application?

13. Does the model provide sufficient detail (input and output) with respect to interactions with parents, subsidiaries and affiliates?

14. Will the value of the model results be enhanced enough by the presence of feedback loops (automatic conditional decisions) to warrant a model with such features?

15. Is a deterministic or stochastic model better suited for the application?

A Simulation Procedure for Comparing Different Claims Reserving Methods

by Teivo Pentikäinen and Jukka Rantala

A SIMULATION PROCEDURE FOR COMPARING DIFFERENT CLAIMS RESERVING METHODS

Teivo Pent&linen and Jukka Rantala

Abstract

The estimation of outstanding claims is one of the important aspects in the management of the insurance business. Various methods have been widely dealt with in the actuarial literature. Exploration of the inaccuracies involved is traditionally based on a *post-facto comparison* of the estimates against the actual outcomes of the settled claims. However, until recent years it has not been usual to consider the inaccuracies inherent in claims reserving in the context of more comprehensive (risk theoretical) models, the purpose of which is to analyse the insurer as a whole. Important parts of the technique which will be outlined in this paper can be incorporated into over-all risk theory models to introduce the uncertainty involved with technical reserves as one of the components in solvency and other analyses (PENTIKÄINEN et al (1989)).

The idea in this paper is to describe a procedure by which one can explore how various reserving methods react to fictitious variations, fluctuations. trends, etc. which might influence the claims process, and, what is most important, how they reflect on the variables indicating the financial position of the insurer. For this purpose, a claims process is first postulated and claims are simulated and ordered to correspond to an actual handling of the observed claims of a fictitious insurer. Next, the simulation program will 'mime' an actuary who is calculating the claims reserve on the basis of these 'observed' claims data. Finally, the simulation is further continued thus generating the settlement of the reserved claims. The difference between reserved amounts and settled amounts gives the reserving (run-off) error in this particular simulated case By repeating the simulation numerous times (Monte Carlo method) the distribution of the error can be estimated as well as its effect on the total outcome of the insurer:

By varying the assumptions which control the claims process the sensitivity of the reserving method visa-à-vis the assumed phenomena can be tested. By applying the procedure to several reserving methods in parallel a conception of their properties can be gained, in particular, how robust they are against various variations and irregularities in the claims process.

It is useful to recognize and classify error sources which give rise to the reserving inaccuracies (cf. PENTIKÄINEN et al (1989) item 2.4b):

1) Tbe model (often simply called reserving rule or formula or method) can be only a more or less idealized description of the real world and of the actual claims settlements; the deviations give rise to what can be termed *model errors*.

2) The parameters used in calculations are subject to *parameter* errors owing to the fact that they are to be estimated from various data statistics or found from other more or less uncertain sources.

3) The actual claims and claims settlements are subject to stochastic fluctuations causing

deviations from the estimates, *stochastic* errors, even in those (theoretical) cases where the model and its parameters would be precisely correct.

The above procedure enables us to examine the effects of all these three errors, in fact, it is very general, not being restricted to any specific reserving model or assumptions on the claims process. It is intended for studies of the properties of the reserving methods on ***a general level.*** However, it is not meant for post-facto analyses, i.e. in the investigation and estimation of the inaccuracies in reserves in particular concrete cases, for those purposes well-known actuarial and statistical approaches are needed.

It is still worth noting that the approach can find application to other estimations as well. We have, for instance, also treated premiunis in an analogous way, although limited to simple examples in this paper.

After having first described our method in general terms a number of numerical examples will be given to illustrate some of its relevant features. They are based on some well-known elementary reserving rules and simple assumptions on the claims process. Also some conclusions on the properties of the reserving rules are derived therefrom. They should be understood merely as examples of the use of our model, not as any real analyses of the reserving methods. Even though our method is aimed at making such conclusions and comparisons between methods, their pertinent performance would require quite extensive studies. Such have been fully beyond the possibilities in this context.

KEYWORDS
Claims reserving. run-off errors, chain ladder, model errors, parameter errors, simulation

1. Basic concepts

1.1. References to related works. A summary of the ***c/aim reserving techniques*** was compiled by VAN EEGHEN (1981). Furthermore, the monograph by TAYLOR (1986) is referred to as is the recent Claims Reserving Manual (1989) of the UK Institute of Actuaries. Enhanced methods for analyses, among others regarding the above listed sources of errors, have been recently proposed, for example, by ASHE (1986), NORBERG (1986), SUNDT (1990) and WRIGHT (1990).

The run-of-errors, as a source of uncertainty in solvency considerations, were dealt with by the British Solvency Working Party in a series of reports: DAYKIN & al (1984). . . . . DAYKIN and HEY (1990). STANARD (1986), RENSHAW (1989), VERRALL (1989), (1990) have analysed the properties of the chain ladder method.

The stochastic claim run-off error was analysed by PENTIKÄINEN and RANTALA (1986) to which this paper is a continuation. The results were incorporated as a suhmodel into the application-orientated risk theoretical over-all model in PENTIKÄINEN et al (1989).

We are going to use, as far as possible, the notations and concepts used in the above-referred papers. However, the terminology adopted in the Manual of IA (1989) is also taken into account. For the convenience of the reader the main features are recapitulated.

1.2. Claims cohorts. In order to clarify the terminology and the notation it is useful to note that the claim process includes the following elements:

1) the event (accident) which causes a claim in year t.

2) The claim is reported to the insurer in year t *or* later.

3) The claim is settled in year $t+s$ $(s \geq 0)$ or possibly in several parts in years $t+s_1$, $t+s_2$, . .

4) If the claim is reported by the end of the accounting year but not yet fully settled, it is called open and a provision is made to meet the outstanding liability either as a case estimate or by using some statistical technique.

5) The claims which are incurred but not yet repotted by the end of the accounting year are "IBNR-claims".

Following the terminology of Manual of IA (1989) (A 5.1) outstanding claims is an umbrella concept for open and IBNR claims.

It is appropriate to group the claims originating in the same accident year, t, as a "cohort". The year t is also called the *year* of origin. Fig.1.1 illustrates the structure of a cohort and its *development.*

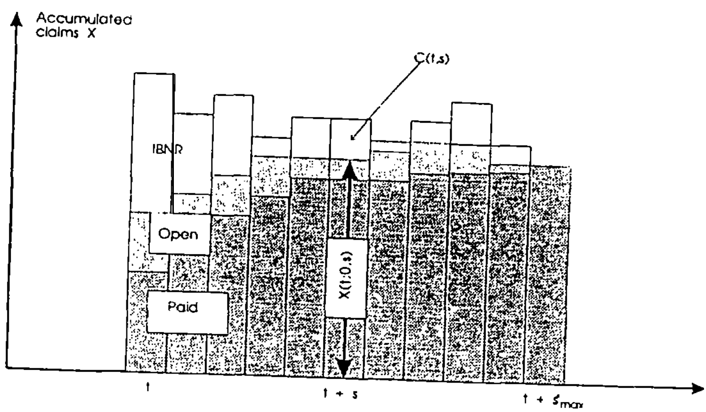


FIG. I. 1. The development of a claims cohort.

The accumulated amount of settled claims from development years t, t+ l, $t+2,\ldots,t+s$ supplemented by the provision of the open claims at the end of year $t+s$ is called, still following the terminology of Manual of IA (1989), p. A5.2, the incurred loss and is denoted by

(1.1) $X(t;0,s)$ = claims originating from year t and paid in years $t,t+1,\ldots,t+s$ on settled or partially settled claims ***plus*** reserve held for the open claims at the end of year $t+s$.

A notation for the increments of X is also needed:

(1.2) $$X(t;s_1,s_2) = X(t;0,s_2) - X(t;0,s_1-1)$$

and especially

(1.3) $$X(t;s,s) = X(t;0,s) - X(t;0,s-1)$$

which is the increment in the development year t+s (by convention, $X(t;0,-1)=0$).

It is assumed that after some period s_{max} all claims of the origin year t are settled. The parameter s_{max} characterizes a feature of the portfolio which is called the length of the run-off tail. Hence, the development time variable s can have values $0,1,\ldots,s_{max}$, and,

(1.4) $X(t;0,s_{max})$ = is the final total amount of claims of the cohort t.

It is also called the loss related to the cohort

1.3. The reserve for IBNR claims of the cohort t at the end of year t+s is defined as:

(1.5) $C(t,s)=$ Estimate for $\{X(t;s+1,s_{max})\}$.

Various methods, 'reserving rules', can be applied in this estimation. The purpose of this paper is to find ***methods and measures for the evaluation of the uncertainty*** involved with the rules.

Concept (1.5) is in conformity with the "London market" definition presented in the Manual of IA (1989), p. A5.1 where the IBNR-reserve is defined to be equal to the estimated ultimate loss on *all* outstanding claims *less* the reserve at the accounting date for open claims. Hence, the uncertainty in the reserve of open claims is included within that of the IBNR-reserve, as thus defined. As stated in the next paragraph, this type of definition seems to be convenient in this context, because it allows the collective handling of all kinds of uncertainties in claims process. Note that this definition is different from the common accounting practice according to which the provisions for both the open claims and IBNR's are included in the claims reserve. No safety margins are assumed to he included in the reserve.

1.4. Claims process. The model lo be employed is based on the fact that tie increment $X(t;s,s)$ 'is made up of the sum of changes in the status of individual claims. It is helpful to classify "*change-causing events" as* follows:

1) A claim is reported and added to the paid and/or open claims.

2) An open claim, k, is fully or partly settled in year $t+s$, the amount being $S_k(t,s)$. For it (possibly) a reserve (case estimate) $C_k(t,s-1)$ was made at the end of the preceding year and now can be released. Then

$$X_k(t,s) = S_k(t,s) - C_k(t,s-1) \qquad (s \geq 1) \tag{1.6}$$

contributes to the change of the cohort's aggregate loss $X(t;s,s)$. If C_k were exactly correct, then X_k would, of course, be zero, but in practice it will often be non-zero ($\pm$).

3) The provision C_k for an *open* claim is changed (possibly without any payment action), for instance, if new information has been obtained.

Both 1) the number *of events* and 2) the *amount of the changes* involved in, $X_k(t,s)$ above, are random variables. Our techniques, both simulation and others (PENTIKÄINEN and RANTALA (1986)). are based on utilizing probability distributions for both of them. Note that the approach is analogous to that of risk theory. Thanks to FILIP LUNDBERG, HARALD CRAMER and others the *collective* approach replaced the earlier "individual risk theory". The number of claims and their size are handled as a "risk process" without reference to the tiles of the individual policies which actually are behind the claims. The philosophy proved enormously fruitful notwithstanding that the theory can also be built on the individual bases.

As in general collective risk theory and even still more in the context of claims cohort processes it is crucially important to account for the correlations between the development cells of the cohorts as well as the correlations between consecutive cohorts.

Furthermore, note that the claim size variable X_k may also be negative. This can be the case particularly in classes 2) and 3) above. This feature should be kept in mind when the risk theory formulae and distributions are built up (cf. BEARD et al (1984), Section 1.3 p. 7).

For illustration of the approach numerical examples will be exhibited in section 4, therefore, some basic features of the claims process *need* to be specified. This is done in the Appendix. We recall that irrespective of which approach is applied in defining the concept of claim development the technique we are going to present can, with obvious modifications, also be applied to claims processes defined otherwise than the collective one. For example, the procedure allows for the use of the bootstrapping technique for claims simulation (as was remarked by one of the referees of this paper),

1.5. The aggregate loss process related to the whole business of the insurer consists of a the sum of the cohort variables X.

Following the practice adopted by NORBERG (1986) a diagram of the Lexis type is constructed in Fig. 1.2. The data array representing a cohort develops as an ascending diagonal. The information which the actuary, or in our simulation the computer, has available for the reserve calculation is in the “run-off trapezium” (in the diagram the vertical pillar at the accounting year t and the area left therefrom). The claims to be estimated for the reserve for outstandings are inherent in all of the still open cohorts and are located in the “triangle of outstandings” right from the column at t:

$$(1.7) \qquad C(t) = \sum_{s \le t} C(t-s,s).$$

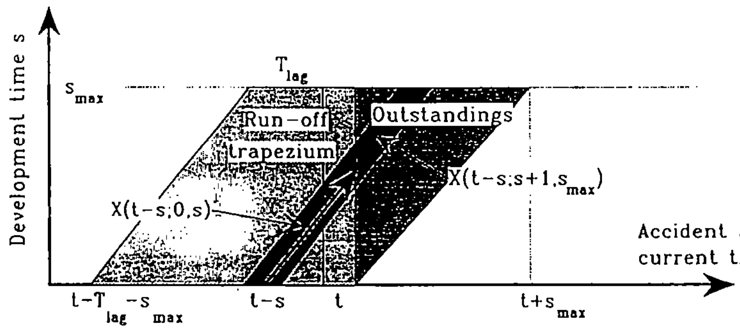


FIG. 1.2. Claims process as a sum of cohorts. The current accounting year is denoted by t and the cohort originating in the accident year t-s is represented by an ascending diagonal.

NOTE. The problem in premium rating is basically the same as is the claims reserving. An estimate for the amount of claims of *future* cohorts is required. The difference in the claims reserving is that only present and past cohorts are considered and that a number of the earliest notified claims are already known and the estimation is focused to the remaining ones only. It is a bit surprising that the methods developed for premium rating are only little utilized in claims reserving.

2. Run-off error

2.1. Run-off error, break-up consideration. The run-off error is the remainder ($\pm$) which is left of the reserve C(t) when all the outstanding claims are ultimately settled:

$$(2.1) \qquad R(t) = C(t) - \sum_{s=0}^{s_{max}-1} X(t-s;s+1,s_{max}).$$

In practice, of course, R can be determined only when the settlement of (practically) all of the outstandings is completed. Our approach is to compute it by continuing the simulation until all of the terms of the sum in (2.1) are obtained.

2.2. Going-concern consideration. Further, the effect of the runoff error on the ***aggregate loss*** X(t) is examined. This variable is the conventional entry for the total amount of the claims in the profit and loss accounts of the standard annual reports. In the terms of the definitions and notations introduced in item 1.3 it is

$$(2.2) \qquad X(t) = \sum_{s=0}^{s_{max}-1} X(t-s;s,s) + C(t) - C(t-1).$$

As was noted in item 1.3. in our considerations the provision for open claims is included in the X terms, not in C, notwithstanding that this does not accord with the common accounting practice.

2.3. Properties of a good reserving method. For the appreciation of the efficiency of the reserving methods a great variety of optimality criteria are proposed in actuarial literature. From the point of view of the company's management the following features might be the most important:

(I) Probability of insufficiency of the reserve should be small (E), more exactly

$$(2.3) \qquad \text{Prob}\{R + L < 0\} \leq \varepsilon$$

where L is a safety loading. (In practice it can either be included in the claims reserve C(t) in addition to the unbiased estimate (1.5) as an extra margin or e.g. as an equalisation provision or it can be available otherwise as a part of the insurer's solvency margin).

(2) The safety loading L should be as small as possible.

(3) The variation of the aggregate claims in the profit and loss accountshould be as small as possible (particularly in the going-concern approach).

In the next item some potential measures will be proposed for the comparison of different reserving methods having regard for the above criteria (I) - (3).

2.4. Measures of uncertainty. The runoff error **R** and its impact on X depend self-evidently on the reserving method. This dependence varies with the different claims processes. We shall use as primary measures in describing these effects both the direct values of R and X and their ratios and the standard deviations σ_R and σ_X of these variables together with the ratios

(2.4) σ_R/C, σ_R/P, σ_X/P and σ_X/σ_0

where P is the premium income corresponding to the relevant X (more in item 3.3). Furthermore, σ_0 is the standard deviation of $X_0(t)$ which is the incurred aggregate loss from which the runoff error is removed. This is obtained from the simulated data, in terms of our notations, $X_0(t) = X(t;0,s_{max})$ (= the total loss related to the cohort t). Hence, the difference σ_X-σ_0 is to be credited to run-off error.

Let us also recall that indicators based on the distribution of extreme deviations or confidence intervals, are good candidates as measures (cf. PENTIKÄINEN and RANTALA (1986)). but at this stage of work we mainly used the standard deviations. They need less simulations, but involve the drawback that the effect of skewness of the distributions is partly lost.

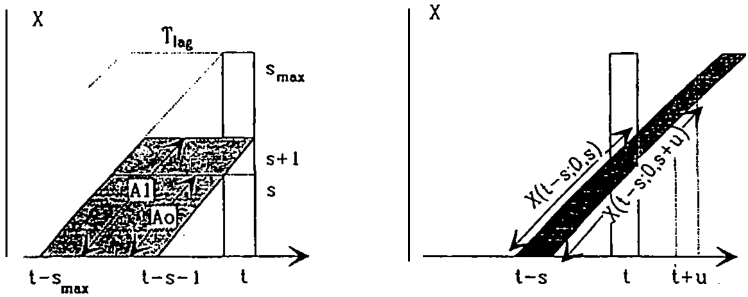
Note that when we in the following illustrate the comparison of two or more reserving rules, the very *same claim pattern* $X(t;s,s)$ is used for all of them. Therefore, it can be expected that the differences revealed in results can be credited to the differing structures of the rules. This is still further verified by repeating the test after a change of the seed of the random generator.

3. Reserving methods used in the case studies

3.1. Chnin Ladder method. This well-known method is chosen as the first of our test examples. It operates auxiliary development coefficients

(3.1) d(s) = A,(s)/&(s).

Where the A's represent the sums of all $X(t-u;v,v)$ located in the areas marked by the same symbols in Fig.3. la.



a) $A_1(s)$ is the parallelogram shaded in the diagram, and $A_0(s)$ is obtained by removing the top-most row from A_1.

b) Development of a cohort.

FIG. 3.1. Derivation of the Chain Ladder rule.

The claim sums to be estimated for the remaining parts of the cohorts are now obtained by assuming that the cohorts grow in the same proportion as the parallelograms A, i.e.

$$X(t\text{-}s;0,s+1) = X(t\text{-}s;0,s)\cdot d(s)$$

$$X(t\text{-}s;0,s+2) = X(t\text{-}s;0,s+1)\cdot d(s+1) = X(t\text{-}s;0,s)\cdot d(s)\cdot d(s+1)$$

etc. Hence. the claims reserve for the cohort t-s is

(3.2) $$C(t\text{-}s,s) = X(t\text{-}s;0,s)\cdot c_{c\text{-}l}(s),$$

where

(3.3) $$c_{c\text{-}l}(s) = \prod_{u=0}^{s_{max}-1} d(s+u) - 1$$

and the total claims reserve at the end of the accounting year t is

(3.4) $$C(t) = \sum_{s=0}^{s_{max}-1} C(t\text{-}s,s)$$

Note that $c_{c\text{-}l}(s)$ should be recalculated in each accounting year t (hence, a notation $c_{c\text{-}l}(t,s)$ would, perhaps, be more advisable).

The Chain Ladder rule is at its best in the cases where the so-called structural (also called mixing) variation is large. This is a well-known feature and is again confirmed by the numerical example to be set out later as well as also in PENTIKÄINEN and RANTALA (1986. Appendix I). This is because the Chain Ladder assumes that the structure variation affects the total claims amount of each cohort but no longer how these claims are distributed during the runoff of the cohort for consecutive development years.

3.2. A variant. The chain ladder method can be amended by broadening the "runoff triangle" to a trapezium from which the parallelograms A are cut, if this is available. The dotted line associated with a "broadening parameter" T_{bg} (see Fig. 1.2 and 3. 1) refers to this variant. Its effect will be tested in Section 4.4 below.

3.3. The premium-based method is chosen as the second example for testing:

$$C(t-s,s) = P(t-s) \cdot c_p(t,s) \tag{3.5}$$

where P is the unloaded net premium applied for the cohort and the coefficient c_p is an estimate for the ratio of the still outstanding IBNR claims of the cohort to the total amount of the claims. This rule theoretically is suitable for pure Poisson claims processes (see PENTIKÄINEN and RANTALA (1986), Appendix 1).

The premium income P(t-s) in our simulation example was calculated by a simple formula of the moving average type, determining P on the basis of the latest settled and open claims which are known at the year of origin of the cohort t-s:

$$P(t-s) = \sum_A X^* / T_A \tag{3.6}$$

where the sum stands for all of the simulated claims amounts X^* located in the rectangle **A** shaded in Fig.3.2, and the amounts X^* are the claims increment variables X(t;s,s), (see (1.3)). transformed to match the value of money and business volume of the accounting year t having regard for the simulated inflation and presumed growth of the business volume (details in Appendix).

In practice, the coefficients c_p can either be fixed in advance or be derived from the pattern of the known claims. We used a simple rule defining these coefficients as the ratios of the simulated sums of the above X^* located in the rectangles B and A in Fig.3.2:

$$c_p(t,s) = \sum_B X^* / \sum_A X^*. \tag{3.7}$$

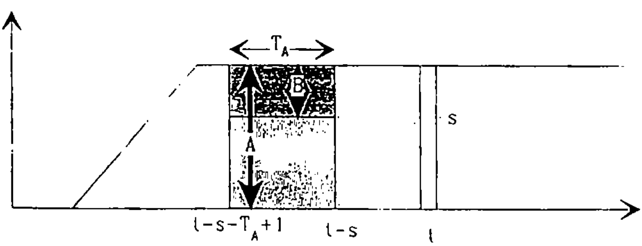


FIG.3.2. Derivation of the Premium-based reserving formula.

3.4. The mixed method is constructed as a combination of the Chain Ladder and the Premium-based reserves:

$$(3.8) \qquad C(t) = \sum_{s} [z(t,s) \cdot C_{prem}(t-s,s) + (1 - z(t,s)) \cdot C_{c-l}(t-s,s)]$$

The idea is to assign to the coefficients $z(t,s)$ such values that the premium-based C_{prem} is predominant at the beginning of the runoff of the cohort (s small) and later, when s is approaching s_{max}, the weight moves to the chain ladder rule.

The intended purpose can be achieved by taking z to be the same as the premium-based coefficient in (3.7):

$$(3.9) \qquad z(t,s) = c_p(t,s).$$

This formula was proposed by BENKTANDER (1976). The logic is analogous to the BORNHUETTER-FERGUSON (1972) approach, but it is applied to a different variable.

An alternative formula for $z(t,s)$ could be derived by using credibility considerations (see PENTIKÄINEN and RANTALA (1986), p. 127).

In order to keep the paper within reasonable limits we have restricted the application examples to these simple rules, the more so because our purpose is to describe the test and comparison method, not to arrive at any analysis of the reserving rules and their properties.

4. Numerical examples

4.1. Single realisations. We used the same numerical basic data as in PENTIKÄINEN and RANTALA (1986). For convenience of reading they are recapitulated in the Appendix. The run-off tail s_{max} is alternatively either 12 (long) or 3 (short) years (cf. Section 3.4 of the referred paper).

The model is programmed to give outputs both in tabular and graphic forms. Table 4.1 provides an example. The long-tailed claims pattern is simulated for 25 consecutive accounting years t by using, in parallel, the three reserve methods specified above (C-L=Chain Ladder, Pr=Premium-based, Mix=Mixed Method, formulae (3.8) and (3.9)).

TABLE 4. 1. Simulated run-off errors R and the aggregate losses X.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
t	P	Xo/P%	X-r-o	C(t)			R(t)			R(t)/C(t)%			X(t)/P%		
				C-L	Pr	Mix	C-L	Pr	Mix	C-L	Pr	Mix	C-L	Pr	Mix
0	65.7	83.8	175.6	184.4	189.8	188.4	8.8	14.2	12.8	4.8	7.5	6.0	73.9	94.1	77.0
1	68.8	98.6	187.3	196.1	197.3	194 2	8.8	10 0	6.9	4.5	5 1	3.6	98.6	92.5	90.0
2	71.0	89.6	195.0	186.4	205.2	19J.2	-8.6	10.2	..·1.8	-4.6	5.0	.9	65.1	89.9	77.1
3	73.4	99.2	206.7	183.3	213.1	198.0	-23.4	6.4	-8.6	-12.8	3.0	-4.4	79.1	94.0	89.8
4	75.8	107 6	225.3	209.8	221.0	214.5	-15.4	-4.3	-10.8	-7.3	1 .9	-5.0	118.1	93.5	104.8
5	80.4	120.4	2'8.9	237.6	230.8	227.1	-11.2	-18.1	-21.2	-4.7	-7.8	-9.3	125.6	103.2	107.4
6	85.0	104.5	257.5	260.0	241.5	243.3	2.6	-16.0	-14.2	1.0	-6.6	-5.8	120.8	107.0	112.7
7	90.4	117.9	284.0	273.8	254.7	258.8	-10.1	-29.2	-25.2	-3.7	-11.5	-9.7	103.9	103.3	105.8
8	96.6	100.1	288.2	291.6	270.1	279.8	3 . 3	-18.1	-8.4	1.1	-6.7	-3.0	114.1	111.6	117.'
9	104.6	117.1	317.4	328.6	288.9	300.0	11.2	-28.6	-17.5	3.4	-9.9	-5.8	124.5	107.1	108.4
10	114.5	114.1	350.0	360.4	311.8	329.0	IO.,	-38.2	-21.0	2.9'	-12.3	-6.1	113.4	105.7	111.0
11	121.7	95.9	361.1	364.1	335.3	349.5	3.1	-25.8	-11.6	.8	-7.7	-3.3	89.9	106.1	103.6
12	130.2	101.9	388.5	365.8	360.3	370.9	-22.7	-28.3	-17.6	-6.2	-7.8	-4.7	82.1	100.0	97.3
13	143.2	101.3	'15.9	365.9	390.4	,a,.&	-50.1	-25.6	-32.5	-13.7	-6.5	-8.5	82.2	103.2	90.9
14	151.2	98.8	435.7	431.4	423.8	432.3	-4.3	-11.9	-3.4	-1.0	-2.8	-.a	127.9	107.5	117.3
15	169.2	98.2	467.9	c.9.1	459.4	464.3	-8.9	-8.6	-3.6	-1.9	-1.9	-.8	95.5	100.2	98.1
16	,a,.,	93.4	497.7	'29.1	499.3	485.5	-68.6	1.6	-12.2	-16.0	.3	-2.5	60.9	98.9	88.7
17	190.0	95.9	532.7	502.2	541.0	531.7	-30.4	8.4	.9	-6.1	1.6	-.2	115.1	99.3	101.5
18	212.2	96.3	571.4	572.1	585.0	586.8	.7	13.6	15.4	.1	2.3	2.6	113.0	100.8	106.0
19	220.9	104.7	634.6	631.7	626.7	633.5	-2.9	-8.0	-1.2	-.5	-1.3	-.2	103.1	95.0	97.3
20	233.9	0.0	652.0	742.8	666.6	699.8	110 8	34.7	67.8	14.9	5.2	9.7	136.7	106.3	117.5
21	246.9	95.8	681.3	693.3	708.3	711.1	12.0	27.0	29.8	1.7	,.a	4.2	55.8	92.7	80.4
22	256.7	89.3	695.8	713.1	7L5.9	742.9	11.3	50.1	47.1	2 4	6.7	6.3	91.3	98.3	96.0
23	275.2	100.6	744 3	727.9	788.0	754.7	-16.4	'3.7	10.4·	-2.3	5.5	1.'	88.3	98.3	87.3
24	286.0	88.4	762.8	764.6	828.8	793.8	1.8	66.0	31.0	.2	8.0	3.9	94.8	96.2	95.6
25	295.3	92.9	786.3	817.0	865.0	822.3	30.7	78.7	36.0	3.8	9.1	4.4	102.7	97.2	PC.6

The variables P, R, X and C are given in monetary units ($\approx$ \$ million) and the ratios as percentages. The growth of premium income P and other monetary quantities is due to inflation (average 5%) and real growth (1%). Claims pattern is long-tailed. X-r-o is the "true" value of the outstandings, i.e. the simulated sum term in (2. I).

The loss ratios of columns 3 and 14 are plotted in Fig. 4.1. as well as the ratio R/P corresponding to col. I 1 (Chain Ladder method) but expressed as a ratio to premium P.

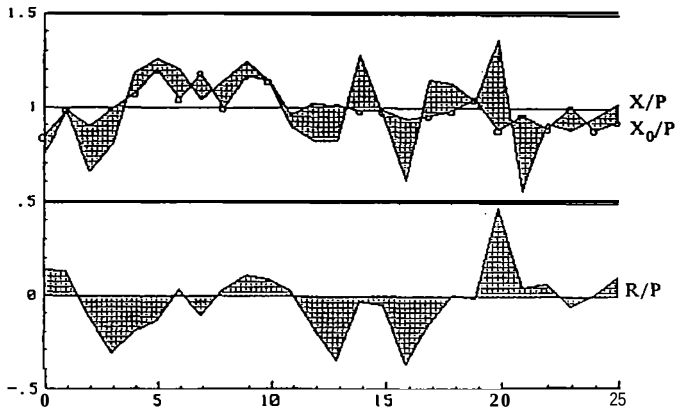


FIG. 4. 1. The ratios X_0/P (-o-), X/P (--) and R/P. Chain Ladder rule.

The ratio R/P and the deviation of X/P from X_0/P are shaded in order to show the strong correlation between them. When R is increasing, it worsens (increases) the loss ratio and vice versa. Note that X/P fluctuates more than 'original' X_0/P.

Fig.4.2 depicts the premium income P and the aggregate "no-run-off affected" loss X_0 from which P is derived according to (3.6) as a moving average with the range 10 years and with a necessary time lag. For clarity, the effect of inflation and growth is stripped away from the time series by operating the variables in the initial value of money and volume (at $t=0$).

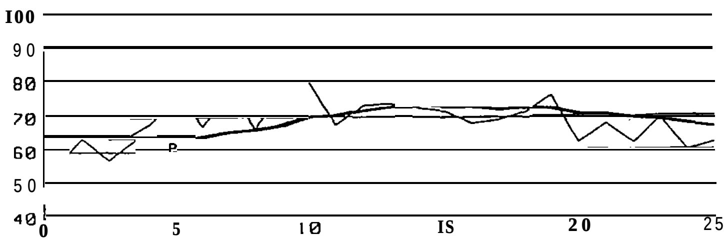


FIG. 4.2. The premium income P, deflated into the monetary value of the initial time point, as a (delayed) moving average of the loss X_0.

All the loss ratios X/P of Tahle 4. 1 and the ratios R/P are plotted in Fig.4.3.

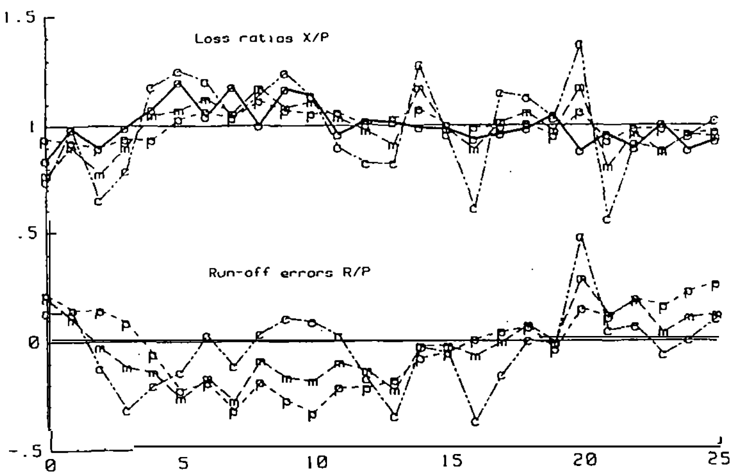


FIG. 4.3. Loss ratios X/P and RIP calculated hy using Chain Ladder (marked hy c), Premium-based (p) and Mixed (m) methods, respectively. The thick line represents X_0/P.

A smoother flow of X/P can be achieved at the expense of larger reserve errors R/P.

Simulations confirm the well-known fact (STANARD (1986) and ZEHNWIRT'S article in the Manual of the IA (1989), Vol. If) that the Chain Ladder method has a tendency to show a greater volatility than the other rules compared.

4.2. Monte Carlo simulations. In order IO get hroader insights into the hehaviour of the target variables the simulations exemplified in Figures 4. I and 4.3 were repeated 50 times for each of the three rules. "A stochastic bundle" is generated in this way in Fig. 4.4.

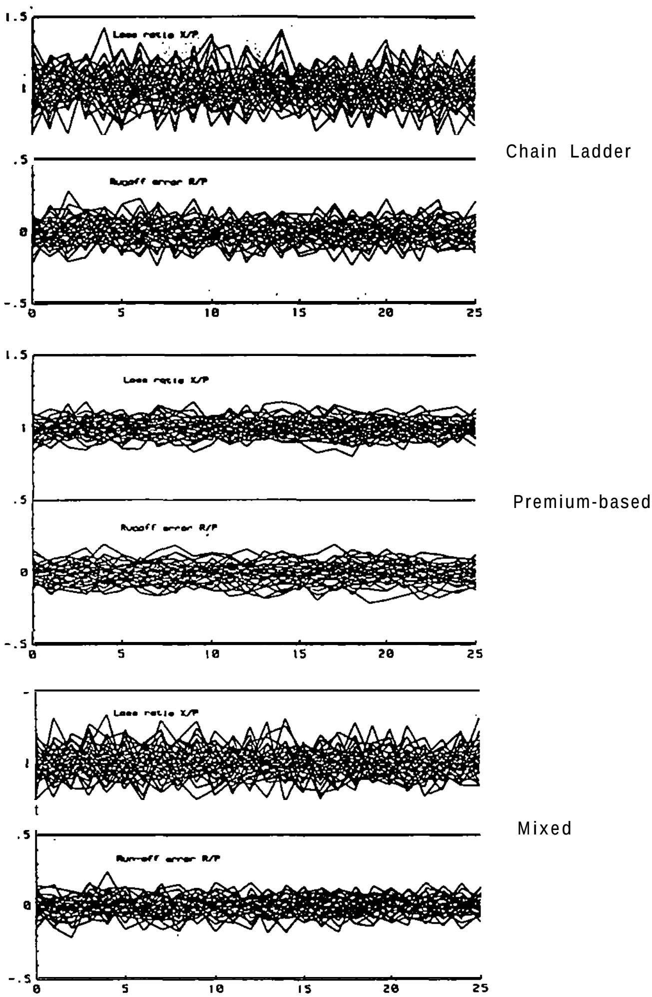


FIG.4.4. Monte Carlo simulation of loss ratios X/P and run-off errors d/P for the three reserve rules. Short tail ($S_{max}=3$). Premium rule stochastic moving average (3.3 above). Sample size 50.

The breadth of the bundle of the simulated realisations gives an idea of the volatility involved with the reserving methods.

A useful observation, seen in Fig.4.4, is that the bundles are stabilized at about a state of equilibrium, i.e. the breadth of the bundles is approximately constant. This feature appeared to be common in those cases we experimented with apart from some extreme situations (premiums defined deterministically and kept unchanged for a long period), where the bundle could have some tendency to diverge. If a reasonably satisfactory attainment of the equilibrium state can be achieved, then it is possible to record the values of the relevant variables, X/P, etc. at *each* time point t of the run, and to compute the required standard deviations as "steady-state" characteristics from the set of all of them. This procedure greatly reduces the number of simulations needed compared with approaches which might require a new simulation for each variable value. Table 4.2 is obtained from Fig.4.4 in this way.

TABLE 4.2. Standard deviations of the simulated ratios.

	Chain Ladder	Premium-based	Mixed
σ_X/P	0.126	0.061	0.102
σ_X/σ_0	I.749	0.851	I.414
σ_R/P	0.079	0.062	0.066

Similar data will be given for a long-tail pattern in the next item. Therein the obviously characteristic features of the methods are more clearly seen.

4.3. Error distributions. The X/P and R/P values simulated, as shown in Fig.4.4, can he recorded and plotted, as is exhibited in Fig.4.5a and in Fig.4.5b which set out the critical tails of distributions.

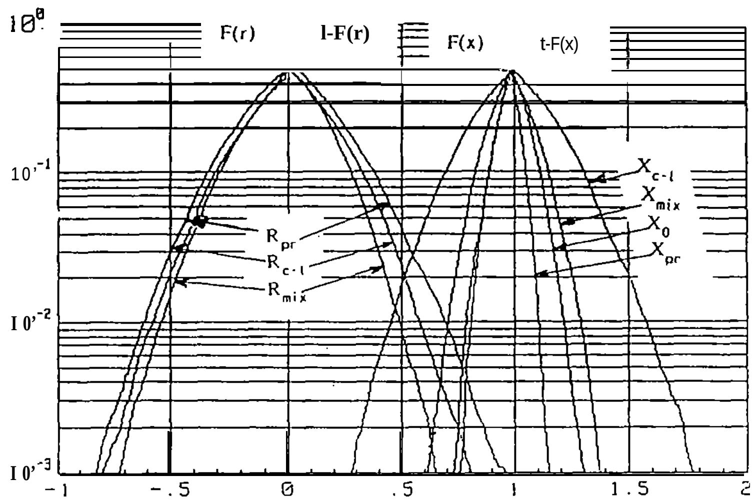


FIG. 4.5a. The cumulative distributions F(x) and F(r) for the ratios x=X/P and r=R/P, respectively, are obtained from the simulated patterns of these ratios. For clarity, F is plotted for the left-hand tail of the distributions and l-F for the right-hand tails in a semi-logarithmic scale. The number of sample points is 15600 for each curve. Long tail s_{max}= 12. Premium method stochastic.

Confidence limits can be directly read from the picture. For instance, the limit which the Chain Ladder ratio X/P exceeds by 1 % probability, is 1.57. Similarly, the limit, which the Premium-based R/P falls below by 1% probability, is -.58.

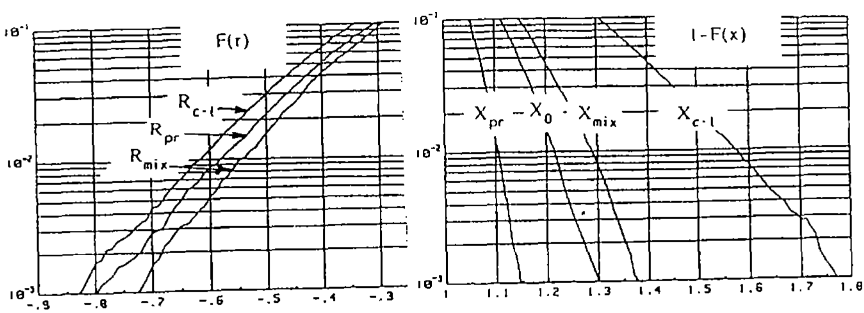


FIG.4.5b. The tails of the distributionsof Fig.4.5a

Note that the distributions exhibited in Fig.4.5 are based on the development tail of 12 years which is rather long and, on the other hand, on the portfolio which is relatively small, the average number of claims being 10000.

For a comparison of the exemplified reserving methods, the standard deviations derived from the same simulation as Fig.4.5 are shown in Table 4.3.

TABLE 4.3. The basic characteristics related to the distributions of Fig.4.5

Variable	Mean	St.dev	Rel.st.d. σ/σ_0
Xo;--/P			
X;c-l/P	1.0031.001.087.2402.7591.000		
X;pre/P	.980	.065	.745
X;mix/P	.993	.125	1.431
R;c-l/P	-.002	.259	2.979
R;pre/P	.039	.267	3.066
R;mix/P	.004	.221	2.534

The ***mean*** values are shown in the table to verify that they are, as they theoretically should be, close to unity for X/P's and zero for R/P's (in order to check that the simulation variahility and programming are under control).

In extreme cases the ***skewness*** of the distribution may be considerable and might suggest that it should be seriously regarded in order to avoid the caveat of understating the run-off risk. Some tests (not set out in this paper) also indicated rather great volatility in the development of the tails. We had to leave further studies on this problem for later work.

A feature of interest is ***the smoothing effect of rhe premium-based*** rule. The Premium-based rule, in fact, reduces the range of fluctuation of the loss ratio X/P compared with the case X_0/P from which the run-off error is eliminated. This happens, of course, at the expense of larger run-off errors R/P, as seen in Figures 4.3 - 4.5 and Table 4.3 when comparing the premium based rule to the mixed one. The adverse tops of the fluctuation of X are spread over a lengthy period.

As expected, the performance of the chain ladder in these examples proved to be rather poor in regard to both the loss ratio and run-off error.

4.4. Stability profiles. The tools developed in the preceding sections are now readily available for the comparison of different reserving methods. We exemplify the idea by applying it to the three methods which were specified in Section 3. For the purpose, the standard deviations σ_X, σ_R and σ_0 are calculated in parallel. Fig.4.6 exhibits an example. The relevant indicators are plotted as columns in order to provide a clear view of their magnitudes. Various patterns of the claim process are simulated for all the three reserving methods. They are constructed from the standard data by allowing options and inserted special variations, as explained in the captions of the figures. The standards are the same as we had in PENTIKÄINEN and RANTALA (1986) and a summary is given 'in the Appendix below.

The left-hand displays of Fig.4.6 represent the relevant standard deviations as ratios to the premium income P. In order to show more clearly the role of the run-off inaccuracy the σ_X's are also given as ratios to the "no-runoff standard deviation σ_0 in the right-hand section of the figure.

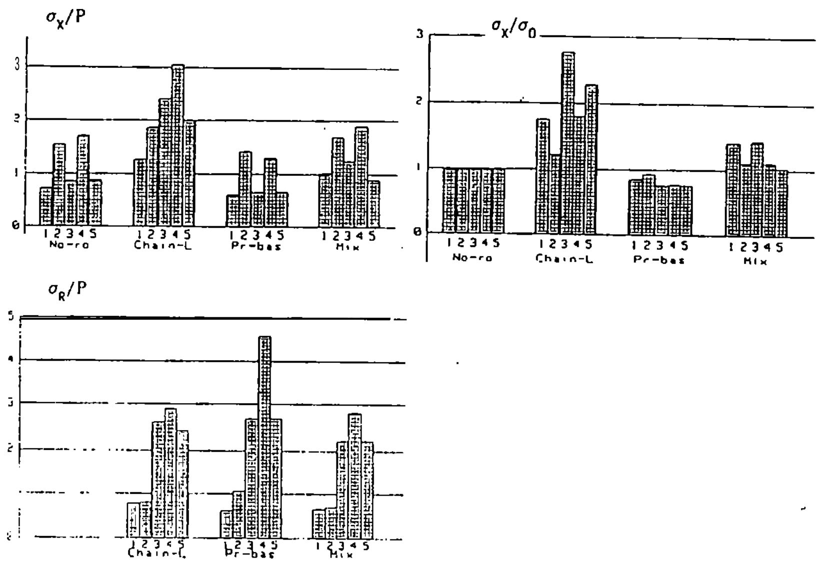


FIG. 4.6. Stability profiles. The numbered claims process options processed in parallel are as follows:

1) Short tail, stochastic premium rule (the same as Fig.4.4 and Table 4.2)
2) Short tail, deterministic premium rule
3) Long tail, stochastic premium rule
4) Long tail, deterministic premium rule
5) Long tail, stochastic premium rule. Chain Ladder with trapezium $T_{bg}=5$ (see Fig.I.2 and 3.1a).

Fig. 4.6 gives rise to the following observations and comments:

* As expected, the short-tail portfolios (I and 2) are less vulnerable IO run-off inaccuracies than are the long-tail patterns.

*The premium-hased rule reduces the fluctuations in the loss ratio below even that level which would prevail if the run-off errors were stripped away, i.e. from the level which is shown by the "no-ro" columns in the figure. But this may happen at the expense of the run-off error being buried in the loss reserve (in particular the option 4 in the figure!).

* The use of a stochastic premium basis reduces the volatility, especially, for the premium method as seen in comparing the option 1 against 2 and the option 3 against 4 in the left-hand displays. The remarkable differences in the magnitudes of these outcomes indicate that the premium calculation basis is likely of primary concern and possibly its effect often outpaces that of the run-off inaccuracies inherent in the reserving method itself.

* The extension of the conventional runoff triangle of the Chain Ladder methods to a trapezium, as expected, improved the stability, as seen by comparing the options 3 and 5 of the Chain Ladder and Mixed columns.

* Note that in the cases 1, 3 and 5 the stochastic variation of the premium income also is involved.

4.5. Sensitivity testings. The effects of various impulses, shocks and disturbances on these processes can be studied by the same model outlined above.

As an example of these kinds of sensitivity testing an extra increment was given to the structure variable q(t) in accounting years 3 and 4 as shown in Fig.4.7. The outcomes are simulated as 'single shots", first without this extra increment, and then with it. The changes in the relevant variables are shown by shading the area between the original and changed curves.

Both the ratios X/P and R/P are plotted for the three reserving methods as depicted in Figures 4.7 and 4.8. The effect is channeled in two ways: 1) via the premium income P, which was simulated to be the moving average (3.6) and 2) via the reserve calculations. The change in X_0, of course, wholly arises via the premium channel and the continued effect after the cease of the impulse at $t=4$ is due to the moving average rule of P which is based on a retroactive account for claims from a lengthy period preceding the accounting year t.

Note that expectedly the q-impulse has (nearly) no effect on in R(t) in the case of the Chain Ladder method. This is due to the well-known fact that the changes in both terms of the run-off error formula (3.1) offset each other, i.e. the Chain Ladder method automatically adjusts for the change in the level of X.

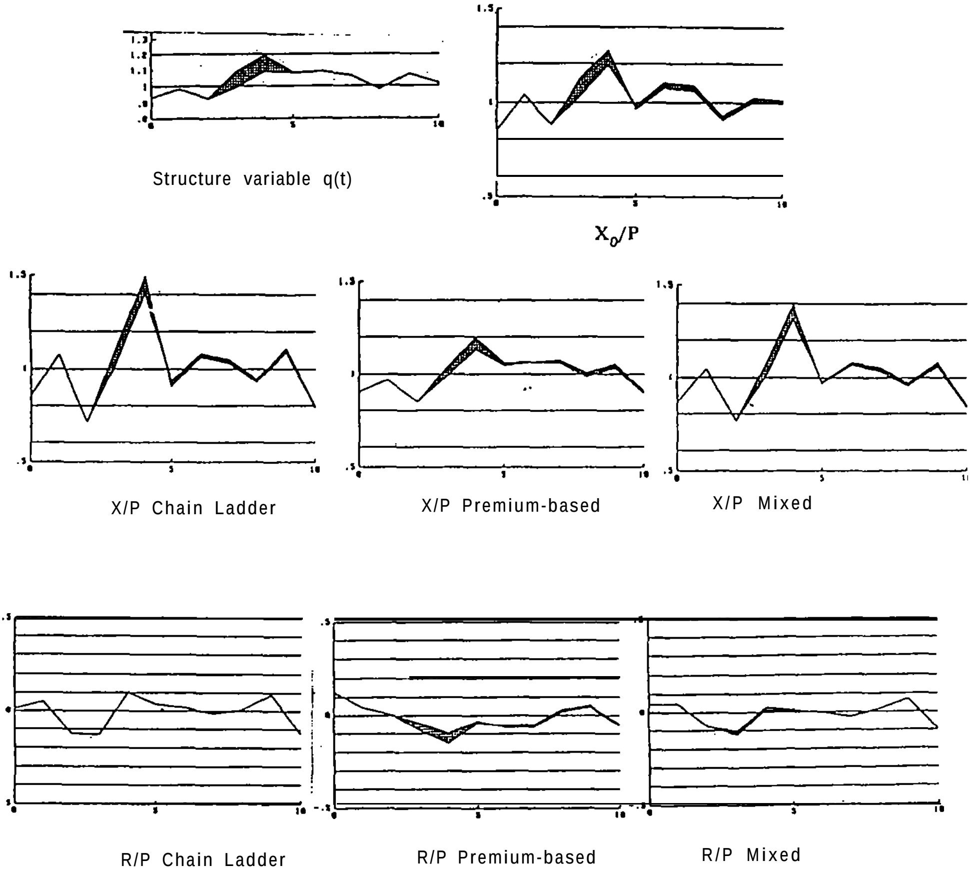


FIG.4.7. The effects provoked by an impulse of magnitude 0.1 exerted on the structure function q(r) in years 3 and 4.

Fig.4.8 displays the effects which are brought about when an extra shock is given to the simulated flow of inflation, represented hy variable I,(t) The technique is the same as in Fig. 4.7.

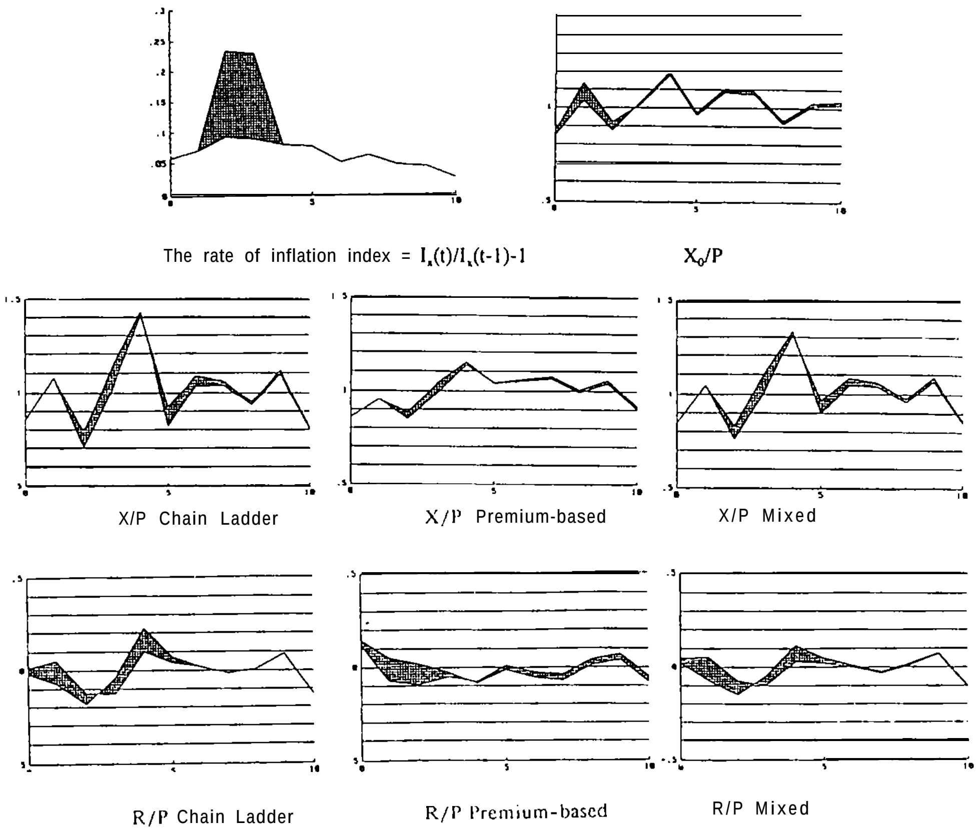


FIG.4.8. The effects provoked by an extra impulse of magnitude 0.14 exerted on the simulated rate of inflation in years 2 and 3.

5. Discussion

5.1. Reservation. Let us recall that this paper is intended to dcscrihe a simulation-based approach of how to analyse the various kinds of unccrtainries which are involved with claims reserving methods. The numerical examples are only intended to illustrate the method and do not claim to have universal validity in the evaluation of the merits and demerits nut even of the exemplified rules, though some observations can be made on the particular portfolios studied. However, we hope that the ideas outlined above might prove useful and inspire further research efforts in acquiring insights into the properties of the most common and often sophisticated reserving methods and, perhaps, to find guidance for their future development.

5.2. Our primary **appraisal of the applicability** of the outlined testing **procedure is** positive. Here, as quite commonly in many other contexts, the simulation approach seems to be flexible and susceptible to extension also into the realm of very complex problems and models which otherwise are beyond the tractability of conventional (rigorous) treatment. Obviously the simulation method can compliment the conventional practices which are based on the post-facto recording and analyzing of the observed runoff errors. This approach provides possibilities to *separately* reveal the effects of specified background factors, such as inflation, catastrophes, changes in the portfolio, claims handling, legislation, etc. Even circumstantial irregular impulses can easily be examined. These are useful additional features to the conventional methods which are fully or, at least to a great degree, restricted to deal with the data of total loss as a bulk, and seldom occurring events or combinations of events may not appear at all.

5.3. The purpose of the **procedure (when further** experience on its usefulness is acquired) may be to test the commonly used or proposed reserving techniques and qualify such ones which prove to be reasonably immune against variations in the structures of background factors, for instance, in claims process, inflation, etc. and against the three sorts of errors referred to above. Possibly a roughly scaled measure to rate the quality of the reserving methods can be found? Furthermore, the testings can provide advance knowledge about reactions of the methods to adverse impulses such as, for example, abruptly increasing inflation.

5.4. Discounting of the fulure claims settlements is another feature to be incorporated into the analyses. It introduces the effects of the fluctuations and risks related to the investment income, which can be substantial particularly if the business is long-tailed (see DAYKIN et al(1987b)).

5.5. Effects to be credited to human bebaviour A comment, sometimes heard, is that the reserves may have a tendency to excessive growth during the profitable phase of business cycles and, on the other hand, to be largely reduced in years when the profitability is poor (see for example Hewitt (1986)). Self-evidently, such kinds of “fluctuations” are beyond the scope of our testing methods which presume a strict and consequent application of some specified reserving formula. However, the possibility of the “human behaviour fluctuations” should be kept in mind as one of the potential determinants of observed phenomena for instance in the cases where actual reserve inaccuracies have been discovered.

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Appendix: Technical details

Abreviation: P&R = PENTIKÄINEN and RANTALA (1986)

1. Definitions and assumptions. We first simulate the "actual" claims in the areas depicted in Fig. I .2. A random number representing the increment variable (cf. (I .3)) $X(t;s,s)$ is generated for each cell, i.e. for all relevant pairs oft and s values.

The random number generator is the same as is represented in BEARD et al (1984). Section 6.8.3, however, using instead of the NP-generator (BEARD et al (1984), item 6.8.3b) the so-called WH-(WILSON-HILFERTY) generator, which is described in P&R, section 5.6. The generator is built up on the assumption that the variable X to be simulated is of the (conditional) compound Poisson type. It requires as input parameters the mean, standard deviation and skewness of $X(t;s,s)$. They can be computed when the mean claim number and the lowest moments (not necessarily the whole specified distribution function) of the individual claims are available, for instance, as estimates from observed data or being suitably assumed. Though, in the cases where the number of claims is very small both the number of claims as well as their individual sixes preferably can be directly generated. For brevity, the formulae of mean value only are outlined in what follows, because they reveal the most relevant background factors and their formulation.

The mean of the increment $X(t;s,s)$ is defined, as in P&R, as the product of mean claim number and mean claim size:

$$E\{X(t;s,s)\} = n(t;s,s) \cdot m(t;s,s) \qquad \text{(A1)}$$

The first factor on the RHS stands for the expected number of the claims in the target cell:

$$n(t;s,s) = n \cdot I_n(t) \cdot q(t) \cdot g_n(s) \qquad \text{(A2)}$$

where

- n is the mean claim number at the initial time $t=0$,
- I, is a function representing the growth ($\pm$) of the business volume,
- q is the structure (mixing) variable introducing into the model the stochastic fluctuation of the mean claim number controlled by a (first order) time series (see (A4) below), and
- g_n distributes n(t) to the development years t, tt $1,\ldots,t+s_{max}$, n(t) being the mean of the total claim number of the cohort obtained as the product of the first three factors in (A2).

The mean claim size, the second factor in (Al), is ohtained from

$$m(t;s,s) = m \cdot I_x(t+s) \cdot g_{,}(s) \qquad \text{(A3)}$$

where

- m is the mean claim size at $t=0$,

- I, an index representing the changes of the mean claim sizes owing to inflation and possibly also to other reasons. It is calibrated to be = 1 at $t=0$.
- Finally, g_m allows the possibility to take into account changes in claim sizes which cannot be explained by the index I,, for instance, if it is observed that the average value of delayed claims (s large) has a tendency to differ from that of early paid claims.

Note: Instead of employing two development distributors g_n and g_m an alternative approach is to build the model on the basis of their product $g_x=g_n \cdot g_m$ which represents the distribution of the total claim *sums* betweenthe cohort cells (cf. P&R, Section 1.7).

2.Specifications.

Portfolio parameters: Expected annual number of claims n= 10000 (see eq.(A2))
Claim size distribution: the lowest moments about zero $a_1=0.006$, $a_2=0.001$, $a_3=0.0001$ (Unit suitably $million, then the average claim size is $6000).

Structure function (also called mixing function):

$$\text{(A4)} \quad q(t) = a_q q(t-1) + \sigma_q \varepsilon(t)$$

where $a_q=0.6$, $\sigma_q=0.05$ and ε is a normally distributed (0,1) random number (white noise).

The rate of inflation:

$$\text{(A5)} \quad i_x(t) = I_x(t)/I_x(t-1)-1 = i_0 + a_i(i_x(t-1)-i_0) + \sigma_i\varepsilon(t) \geq \tfrac{1}{2}i_0$$
+ (an optional manually inserted) "shock"

where $i_0=$ 0.05, $a_i=0.7$ and $\sigma_i=0.015$.

Real growth of the portfolio $I_n(t) = (1 + i_n)^t$ with i_n = 0.0 I.

Developmetu distribution g,(s) for $s=0, 1, 2,...$ (see eq. (AS) and P&R, Section 3.4)
Short tail 0.6, 0.2, 0.15, 0.05
Long tail 0.15, 0.25, 0. 15, 0.15, 0. IO, 0.05, 0.05, 0.02, 0.02. 0.02, 0.02, 0.01. 0.01.

Formulae of the basic characteristics, see P&R, Section 5.1.

Random number generator is described in P&R, Section 5.6 and Pentikäinen et al (1989), Appendix A

The transformed amount of loss (claims) in a development cell s of the cohort of the origin t-s (Item ***3.3, eq. (3.6)*** and (3.7)).

(A6) $X^*(t,s) = X(t\text{-}s;s,s) \cdot V(0)/V(t\text{-}s)$

where **V** is an auxiliary variable representing the volume of the business with reference to simulated inflation and assumed real growth of the portfolio:

(A7) $V(t) = I_x(t) \cdot I_n(t).$

3. Discussion. The following features of our numerical simulation might be worth some special comments:

* Parameter **n** introduces into the model allowance for changes in ***business volume.***

* The structure variable q is stochastic and is generated as a first order time series (see Appendix). Hence, the *n*-values obtained for consecutive years are not independent (contrary to what is mostly the case in the traditional risk theory). This correlation is one of the factors which can crucially affect the range of fluctuations (cf. PENTIKÄINEN et al (1989), 2.2).

* *Inflation* is stochastic and generated by using first order time series (AS).

* Also ***other backgound processes*** as the structure variation and inflation could be assumed to be *stochastic,* in particular, the return on investments.

* The model can be extended by introducing return on investments and ***discounting*** of the future payments. Then a new component of stochasticity is incorporated into the model probably having a significant effect in long-tailed business. However, we had to postpone this to later works. Hence, in what follows, discounting is not performed.

* The portfolio of general insurers mostly consists of numerous lines and sublines, and reserves need to be made up for all of them. This feature is not dealt with in this paper, the approaches, which are described, handle the claims as one single block which can either be any of the lines separately or two or more of them combined. The multi-line problem is considered in PENTIKÄINEN et al (1989), Section 3.1.1a, p.27 and BEARD et al (1984) Section 3.7.

When the Wind Blows:
An Introduction to Catastrophe Excess of Loss Reinsurance

by D.E.A. Sanders

THE STORY OF A TILE

On 25th January 1990 a tile blew off my house - luckily I managed to get a handyman in who replaced it - for £75.00 This may be exorbitant but they were busy and, in any case, insurers were paying claims up to £ 1,000 without question.

I put in an insurance claim, and received £75.00. By this time the insurer - my own company - had breached their deductible. They themselves put in a claim totalling £67.50 (10% of the risk was retained). This cover was placed with over 100 reinsurance companies, including Munich Re, M & G Re and Syndicates with Lloyds. By this time these reinsurers had breached their limits and were passing their excess (£60.75) to their reinsurers. The trail is now more difficult to follow. This £60.75 was passed from Reinsurer to Reinsurer (including Eagle Star's own reinsurance operation) time and time again.

For convenience I will assume it went 10 times round the system, and generated some £500 in transaction. It then ended up at a Whole Account protection programme and went into the Marine market as an "incidental non-marine loss". This went round the system yet again - and is still moving. My tile has been involved in over 20 financial transactions, with total amounts in excess of £1,000.

If that storm happened today, the situation would be different - there would possibly be only two transactions since the secondary market has completely disappeared. The challenge for the Actuary is to estimate the total cost of this simple transaction and to assist in the pricing of the products. As the old age dies, and a new one arises, I hope it is useful to put down some of the methods used in the past to solve the problem of tracking the claim.

THE POLICY

Excess of Loss Policies are split into two distinct types - Risk XL's or working covers and CATXL or catastrophe covers.

A Risk XL covers the cost of individual losses above a certain specified sum up to a maximum amount. The lower level is the deductible and the difference between the lower level and the maximum amount is the cover or line. Cover is sometimes expressed as a number of lines which equals cover/deductible, but this is more appropriate to surplus treaties. The losses may be unlimited in amount or limited by aggregate amount. Generally today policies have limited aggregate amounts, i.e. a reinsurers exposure is limited.

CATXL's covers the cost of the aggregate claims (after deduction of other reinsurance recoveries) in excess of a specific amount, up to a maximum. The type of risk and cover is specified. For example the policy may cover losses in excess of £5 million up to £25 million. The cover is called into play, and the insured may receive up to £20 million. This may be achieved by one loss of £25 million or 20 losses of £6 million.

In the event of a loss, the cover is normally reinstated on a pro-rata basis by the payment of a reinstatement premium. (The calculation may also be pro-temp I.e. related to remaining exposure period). Thus, in our example, a loss of £10 million will mean a £5 million payout, less a reinstatement premium of 5/20 x initial premium.

In general in Non-Marine Insurance one reinstatement is given, and in Marine Insurance two reinstatements are given. In effect, the aggregate covers are two and three times the stated cover. The policy may be specific to the type of risk (e.g. UK windstorm) or general. (All losses world-wide).

Other specific considerations are two loss warranties (i.e. for the cover to come into force there must be two losses). Thus a single vessel sinking may be excluded.

Another important feature is the "hours clauses". Under this, in respect of most losses, an event is defined as a 72 hour period. Thus as a hurricane hits one part of the US causing damage, and then another part four days later, this is categorised as two catastrophe losses and hence two deductibles apply. However, if two separate events occur within a specific 72 hour period, each event is separate, despite the hours clause, and two deductibles apply.

The exception is winter freeze losses which apply over a 156 hour period. The art form in this case is to pick the 7 days which maximises the loss - and hence the reinsurance recoverable.

In 1990, it was difficult to differentiate the losses arising from two storms on 25th January and 27th January. The market took a pragmatic view of this.

THE PLACING OF CATASTROPHE REINSURANCE

Catastrophe Reinsurance is generally placed by Brokers in the National and International Reinsurance Market via a slip system. Under a slip system a specific percentage of the risk is underwritten. For example, if the risk is for £10 million in excess of £2 million (i.e. to cover losses above £2 million up to an aggregate of £ 10 million) an Underwriter may place a line of 10%. This gives him an initial exposure of £ 1 million (excluding reinstatement).

The Broker aims to try and place more than 100% of the risk. In the Non-Marine market, the insured normally retains 10% of the risk - but for the purpose of what follows this will be ignored. For Marine risks 100% can still sometimes be placed.

If a Broker writes so the total "signings" exceed 100%, then the slip is signed down. In the case of the Broker placing 125%, the 10% line is signed down to 8%, and the exposure is reduced to £800,000.

If the Broker places 75% of the risk, there is no increasing the line - the reinsurers' limits are set and the residual 25% is unplaced and hence retained by the insured. Brokers like continuity, in that they always aim to place more than 100% of the risk, and the renewal business is always given to the existing reinsurers as a first refusal. An example of a slip, with the stamps and lines is attached as Appendix 1.

Now consider a major UK insurer. The exposure to property is astronomical. The reinsurance it wishes to purchase is £175 million in excess of £25 million. It is extremely difficult - indeed impossible - to place such a risk in one tranche. The largest reinsurer would only want a small (2.5%) line, and the very smallest would be writing decimal point lines. Note in the real slip some individuals are writing only 0.15% of 95% of $25 million.

A Broker would spend an eternity trying to place the risk. What happens is that the reinsurance is structured into a placeable programme. The £175 million over £25 million could be structured into, say, four separate categories:-

(i)	£25	million	xs	£ 25 million
(ii)	£25	million	xs	£ 50 million
(iii)	£50	million	xs	£ 75 million
(iv)	£75	million	xs	£ 125 million

The consequences of this are three fold:-

a) The business has a greater possibility of being placed. The smaller company which only wants an exposure of £250,000 can write a 1% line on programme (i) or (ii).

b) Different reinsurers like different types of risk. Specialists can be identified for each contract.

c) The cost of the programme theoretically reduces.

A simple example will explain this last point (again reinstatements are ignored). Let us consider a company with the following loss:-

(i)	1 Loss of £60 million	(A)
(ii)	1 Loss of £40 million	(B)
(iii)	3 Losses of £ 30 million	(C), (D) and (E)

Under the one policy structure the insurer received £35 million from A, £ 15 million from B and £5 million each from C, D and E - a total of £65 million. Under the new structure he receives £35 million from A, (£25 million from the first policy and £ 10 million from the second) and nothing from B, C, D and E. If one reinstatement is allowed, he will also receive £ 15 million from B, £5 million from C and D and nothing from E! As the expected receipt is lower, so should the theoretical premium.

The consequences of the above restructuring lead to innovative products which increase the exposure of the actual programme. These include cascade programmes and top and drop, where unused parts of the vertical programme (i.e. the higher value programme) is used to cover a horizontal exposure (more losses of lower value). Under the example, an insurers cover (say £ 50 m xs £ 150 m) can be used to cover the losses in (iii).

The important issue to note is that the price for CATXL has changed radically in the last three years. This is due to recent major losses. Losses in the CATXL market are usually given a name (e.g. Hurricane Andrew) or a CAT code (e.g. 87J). This is the 'J'th event of year 1987. This storm is .the event of 15th October when Michael Fish, the Weatherman, got it all wrong! Illustrations of how, for example, Sevenoaks became one oak can be found in [6].

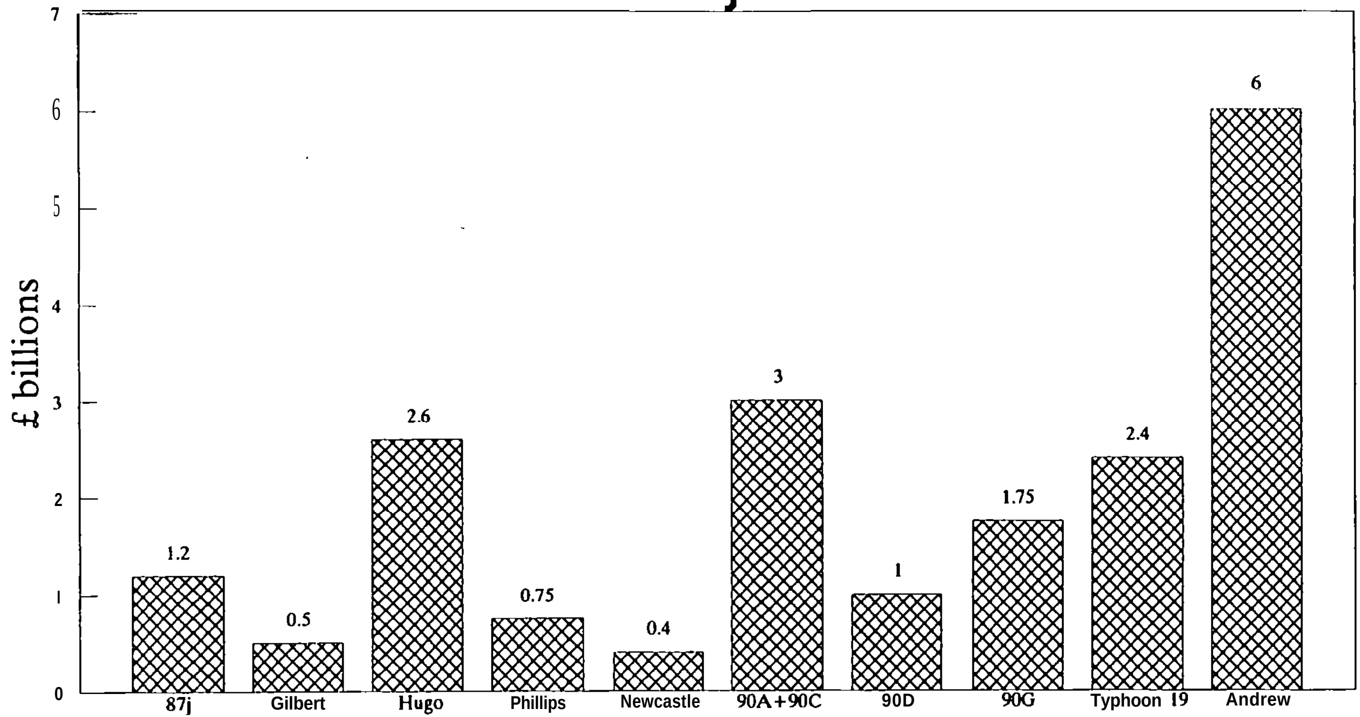
The storm of 1990 on 25 January is 90A. This is followed by 90D and 90G - 90B was an aviation loss. Recent losses are given in the graphs attached to this section. Catastrophe cover costs have jumped by a factor of nearly 4.

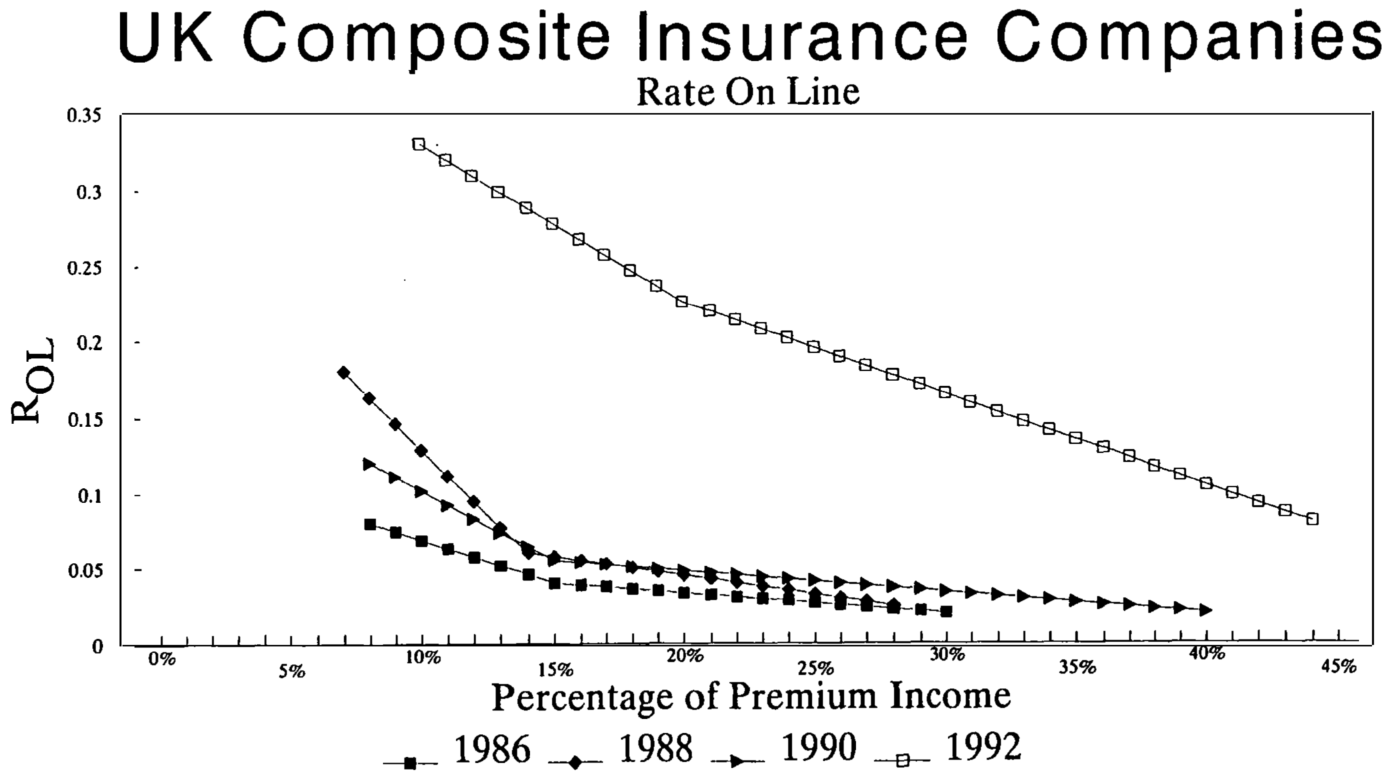
The policy is rated on Premium Income i.e. as a percentage of premium income of the cedant company. There is normally a Minimum and Deposit premium which relates to the expected premium income of the cedant. However, this premium is usually expressed as a Rate on Line, the Line being the exposure. The graphs following this section illustrate the point. In the rating section the issues will be explained in greater depth. The following graphs indicate the cost as a mid point in a spread of layers, and indicate how the cover, expressed as a percentage of premium income, has changed.

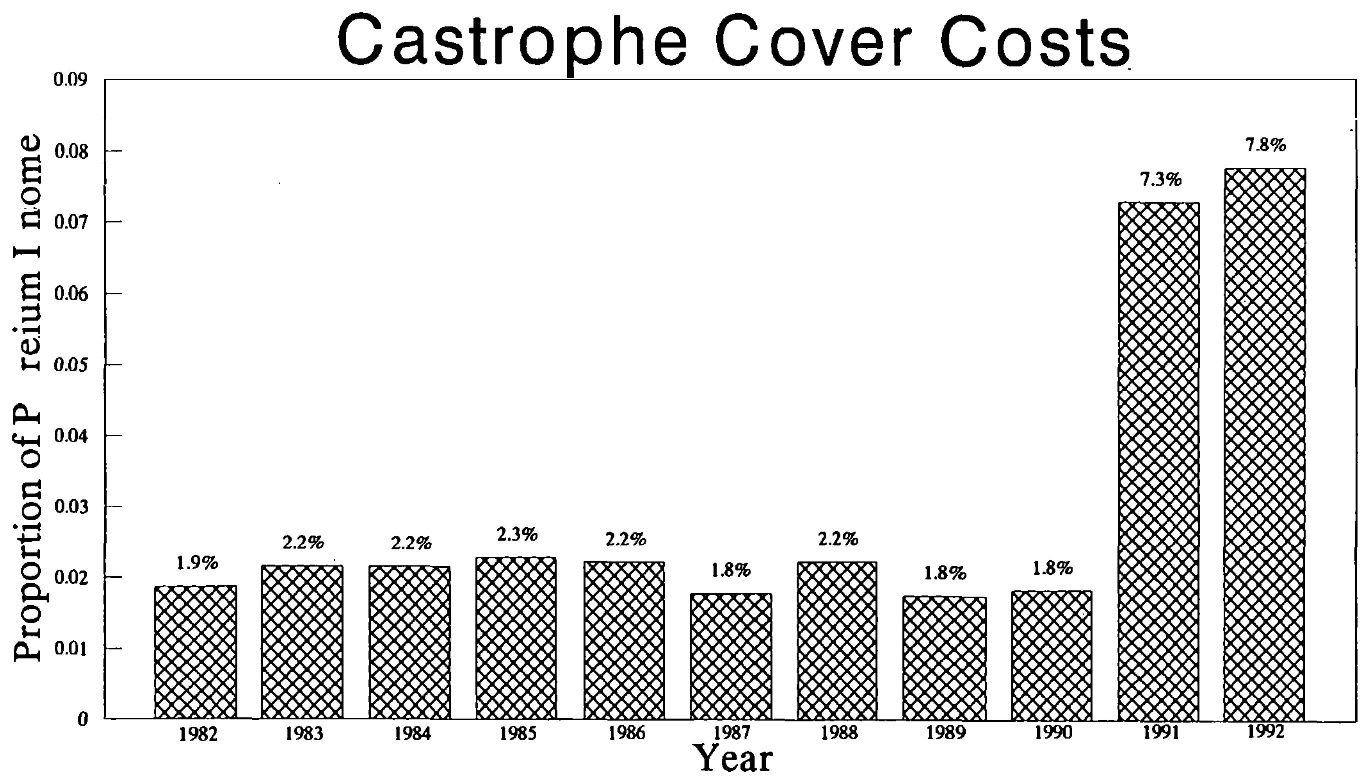
A company with a premium income of £ 100 million wanting cover from £10 million to £30 million would, therefore, expect to pay a price above the 20% of premium income on this graph. In 1990 this would have been about 5% (5% x £20 million line gives £ 1 million). In 1992 this would be 25% on £ 5 million.

This massive increase in rates has created new problems for insurers. When rates were cheap the philosophy was to place as much as you can. Why have rates increased substantially?

Recent Major Losses




UK Composite Insurance Companies
Rate On Line
R_{OL}
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0%
5%
10%
15%
20%
25%
30%
35%
40%
45%
Percentage of Premium Income
1986
1988
1990
1992


Castrophe Cover Costs
Proportion of P reium I nome
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
1.9%
2.2%
2.2%
2.3%
2.2%
1.8%
2.2%
1.8%
1.8%
7.3%
7.8%
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
Year

THE **RETROCESSION** MARKET AND THE SPIRAL

Although pronounced dead, the spiral and retrocessionary (reinsurance of reinsurance) markets are just alive - prices have increased tenfold. The key phrase is LMX; which is Excess of Loss placed on London Market Excess of Loss business. The principles of writing this business are simple.

I have a series of risks for which I received a premium of (say) £ 100. If I can place these risks with someone for (say) £98, I will have a guaranteed profit of £2! Also, in direct reinsurance, the higher up the programme the cheaper per unit the risk. It was thought that the same applied to Retrocessionary market, this led to considerable mispricing. As long as I could sell my book of business cheaper than I bought it, the basic reinsurance product itself was being priced too cheaply.

Take two reinsurers. Let us assume both have £10 million of inwards reinsurance exposure. Insurer A reinsures its whole portfolio with B and vice versa. Both now, individually, have £20 million of gross exposure of which £ 10 million is reinsured. (The first program is £ 10 million xs £0 million). They then place this second level (£ 10 million xs £10 million) with each other.

Their individual total exposure is £30 million of which £20 million is reinsured. We continue this for, say 10 times, giving us a comfortable £ 110 million exposure of which £ 100 million is reinsured. Of course, the higher levels of reinsurance are more remote for the loss and accordingly are cheaper! The Broker takes 10% of each placing as brokerage.

A loss of £ 10 million occurs to each insurer. Insurer A passes £ 10 million to Insurer B. A has £ 10 million loss which he recovers. B has £ 20 million loss, which he recovers from A; A has £30 million loss, £ 10 million of which is recovered, so he asks B for £20 million and so on. An initial loss of £10 million for each company produces payments for A of £110 million - and a net loss of £ 10 million.

This example is simplified. In practice there were hundreds of companies and Lloyds syndicates playing the game.

The rules of the game were quite simple - understand the total aggregate exposure and make sure you had more reinsurance than your rival. For example, if A had written one more reinsurance its exposure would be £ 110 million with reinsurance of £ 110 million, and B would be £ 120 million with reinsurance of £ 100 million. In the case of no loss B would be the winner - the premium from A would be its profit. In the event of a claim, however A would be the winner. Several syndicates at Lloyds were the B players - reporting profit to names. Since the top layer was mispriced, when a catastrophe occurred the results for company B would be bankruptcy.

How would a prudent reinsurer have behaved in the Spiral market? I will assume the aggregate exposure is £100 million (i.e. the total of all reinsurance written). It would be inefficient/impossible to reinsure the total exposure. A prudent reinsurer should have purchased £60 million excess of £5 million. This would have cost a considerable amount of the incoming premium.

This gives a perceived retention of £ 5 million and a "hidden" retention of £35 million (£ 100-£60-£5). In practice what was happening was that either insurers were not aware of their aggregate exposure or were being imprudent. They were reinsuring f25 million excess of f2 million. The hidden retention was f73 million (i.e. an unreinsured exposure of f73 million). A series of losses would devastate the market - which turned out to be the case. A lot of the criticisms by Lloyds have been the lack of understanding of aggregate.

The turning events for the market were the following losses:-

(1) Piper Alpha

Press reports regarding major professional reinsurers indicate how they got their reserves and recoveries wrong.

(2) 1999 Losses

Hugo, Exxon Valdez, Phillips Petroleum and Arco Platform. Their losses are not yet fully developed.

1989 was also hit by smaller losses such the San Francisco Earthquake (17.10.89) and Newcastle (Australian) Earthquake (28.12.89).

(3) The European Storms of 1990

For further details of this topic see either the "C.A.S. Loss Reserving Talk" [3] or read Cathy Gunn's excellent book "Nightmare on Lime Street" [11].

RATING

There are three basic methods of assessing ratios for the risks:-

(1) Some form of simulation relating storms to a portfolio of risks. The risks are usually categorised by type (Household, Property, Shops, Offices etc.) by value and by postal code. Old storms or hypothetical new storms are then simulated on the portfolio.

Examples of this type of estimation may be found in the GISG paper "Storm Rating in the Nineties" (8). This type of method is often revealing about the area by area exposure, but the estimation of losses is extremely subjective. A windstorm loss may vary between 0.5% to 2% of Sum Insured and the uncertainty is enormous. Key factors are often excluded from the databases, for example, construction type. On ordinary household policies, no account is taken of the square footage and number of stories. We rate policies by Sum Insured (a linear type rating), yet Danish experience indicates storm exposure increases with increased square footage (square footage is a rating factor in Danish household policies).

The information given by such simulations should not, however, be discounted.

(2) **Burning Cost Rating**

Under Burning Cost Rating actual losses incurred are used to determine the cost. The keys to assessing these rates are:-

(a) **Loss Frequency**

A burning cost method is only suitable if there are a sufficient number of losses to obtain a suitable loss frequency.

(b) **Indexation**

Losses should be revalued into current terms. This involves both inflation and the increase in number of policies. A suitable index could be premium income adjusted for any rate changes.

(c) **Changes in Policy Conditions**

(d) **Changes in Retentions**

(3) **Exposure Rating**

Simulation is one form of Exposure Rating. Normally, exposure rating is intended to provide a comparison with the burning cost rate - particularly if changes to the portfolio have taken place.

Exposure rating is used to rate areas and covers with little or no loss experience. There are three stages:-

(1) Establish a Catastrophe Estimated Maximum Loss (E.M.L.).

(2) Establish a Catastrophe Premium - this is normally From The Ground Up - (F.G.U.).

(3) Establish a suitable Loss Distribution Curve. In the example I will use a Pareto type distribution.

As an alternative to this type of approach, formula could be used. In my ASTIN paper, I use formulae from Financial Mathematics and Option Pricing (Black-Scholes) to derive consistent price rating for certain classes of loss. This involves the estimation of three parameters, the return period if an event being one of them and implied volatility is another. A similar approach is made by using Pareto formulae. These methods involve difficult mathematics and are beyond the scope of this paper.

Set out below is an example of a calculation for a UK direct writer requiring a quote of £25 million excess of £50 million. Reinstatements and brokerage are ignored.

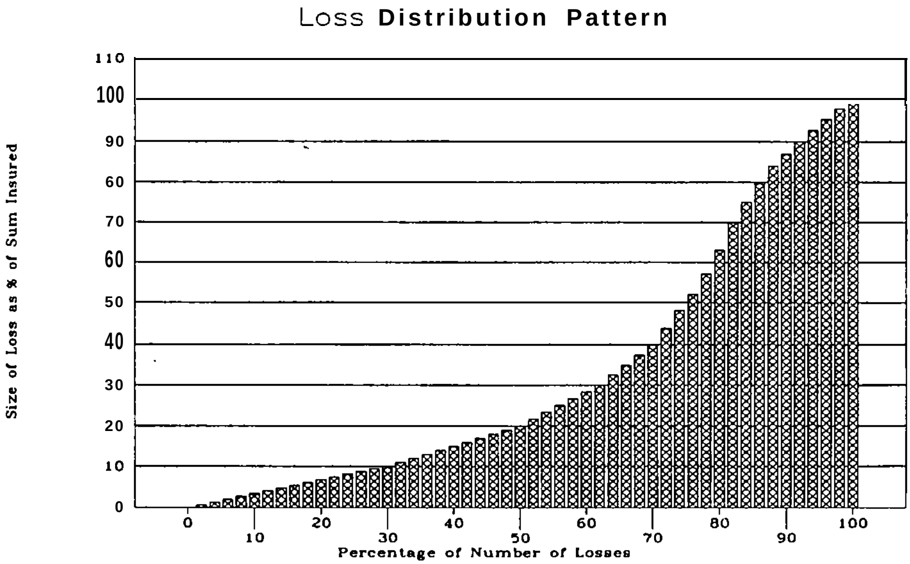
The estimated Gross Premium income for 1992 is £ 230 million and the data is as follows:-

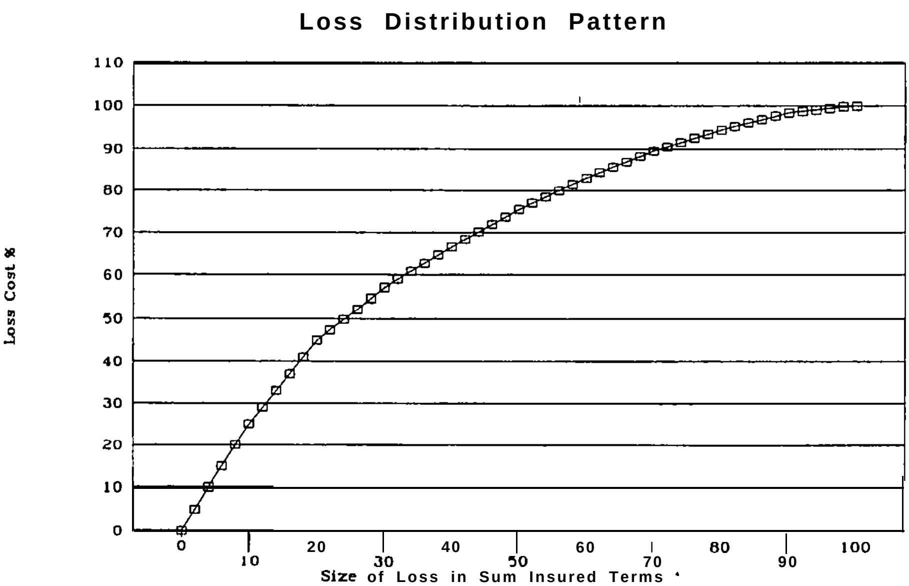
	Premium	Losses F.G.U.		Indexed
1991	220,000,000	Nil		Nil
1990	200,000,000	95,000,000	(90A)	109,250,000
		22,000,000	(90G)	25,300,000
1989	180,000,000	Nil		Nil
1988	170,000,000	Nil		Nil
1987	160,000,000	65,000,000	(87J)	96,451,612
1986	155,000,000	Nil		Nil
1985	150,000,000	Nil		Nil
1984	145,000,000	6,500,000		10,310,344
1983	120,000,000	Nil		Nil
1982	100,000,000	Nil		Nil

We first calculate the Maximum Possible loss. This is taken as twice the 90A Loss Indexed i.e. £220 million (2 x 109.250). This is the current market practice.

Next, we calculate a loss for a specific layer. I use 90% xs of 10% of the largest loss (109,250,000) say £ 90 million xs £ 10 million.

The losses are larger and in this treaty today would be £90 million + £15.3 million + £86.451 million + £0.310 million = £ 192.151 million. (This is similar to the burning cost). The average cost is £19.215 million per annum.


Loss Distribution Pattern
Size of Loss as % of Sum Insured
110
100
90
60
70
60
50
40
30
20
10
0
0
10
20
30
40
50
60
70
80
90
100
Percentage of Number of Losses


Loss Distribution Pattern
Loss Cost %
110
100
90
80
70
60
50
40
30
20
10
0
0
10
20
30
40
50
60
70
80
90
100
Size of Loss in Sum Insured Terms

This cost, from the Pareto curve, represents about 50% of the total cost. This is taken from the size of loss curve looking at the size of loss of 10 (giving 20%) and 50 giving 70%). Therefore, the total catastrophe programme should cost £38.42 million.

The £50 million point represents about 22.5% of the E.M.L. of £220,000,000 and £75 million (i.e. £25 million xs of £50 million) is about 34% of E.M.L. Using the lower graph 22.5% is about 45% of loss cost, 34% is 60% of loss cost and so the premium is 15% of the total cost of £38.42 million or £5.73 million (before expense, commission and safety loading).

The basic problem is that the market is not applying this type of rating, and reinsurance costs are substantially higher than those derived by the above calculations or any pure exposure basis. They are trying to recover the rest of the early losses to re-establish capital.

The Capacity of Reinsurance has been devastated. Lloyds names have ceased to be members of syndicates and Reinsurers have ceased to trade. Accordingly, premium rates are substantially above the theoretical calculated rate, due to demand exceeding supply and the absence of any real retrocession or spiral market.

Let us consider the need. I will relate everything to 90A as this is the market norm (remember PML is 2x Indexed 90A loss).

I will consider nine companies, A-I. These are all UK composite insurers. In the first graph 90A losses are expressed as a proportion of Premium Income. Thus for Company A, 90A loss F.G.U. represents 40% of its total property premium income.

The next graph represents the deductible as a proportion of premium. The average deductible is about 10% of property premium, although there is wide fluctuation.

Finally, I give the cover purchased From The Ground Up. Thus Company A purchased reinsurance between about 12.5% and 87.5% of its premium income, 90A accounted for about 40% of its premium income, so in an event which is twice as damaging it should still have protection. Company B, however is only purchasing up to its 90A cover and it is, therefore, more exposed to possibly higher losses. The rate on Line, as a Proportion of 90A, is given for 1992 reinsurance costs.

In the example I calculate a premium for £25 million xs £50 million at £5.73 million or about 23% rate on line.

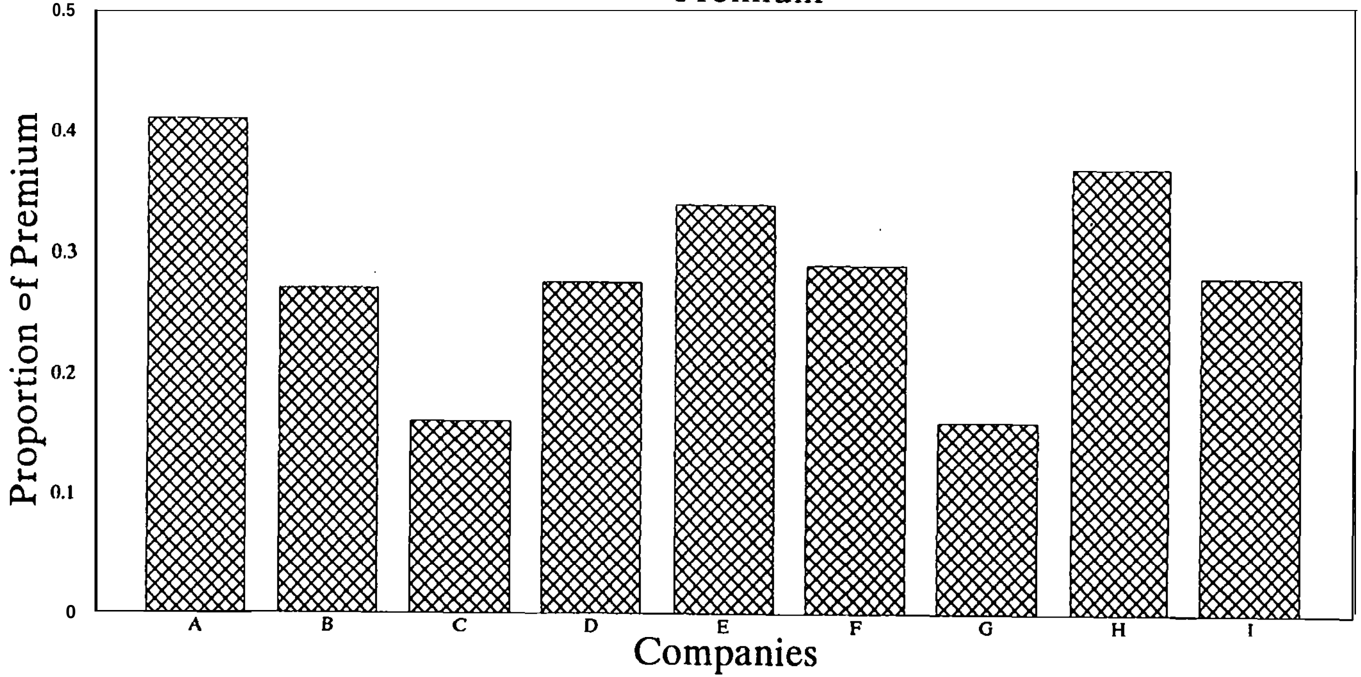
Based on this, we have exposure from 45.5% (50/109.25) to 68.6% (75/109.25). This has an average of 57.2. From the graph for 1992, the Market would be charging a rate on Line of slightly more than 30% or £7.5 million.

There are clearly many considerations that need to be taken into account:-

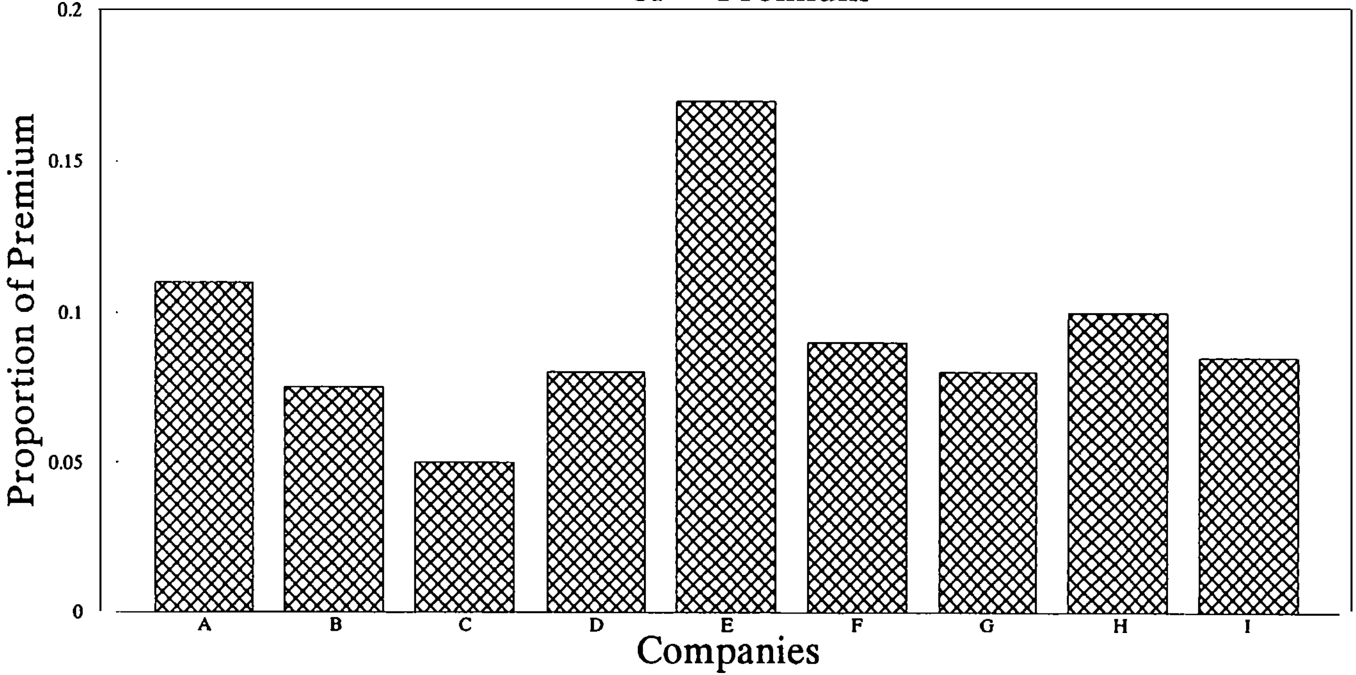
(a) If the actual price is loaded by 25% to 40% over expected values should the cover be bought? The answer to this depends on the shareholders resources and/or future employment prospects for the Managers. Should an event occur what would be the impact on the P & L account.

(b) What should be done about the retention? If only 75% of the business is placed, how should the reinsurance of the 25% be planned for. Losses need to be financed. Should the "loaded" or "real" premium be transferred to the Internal

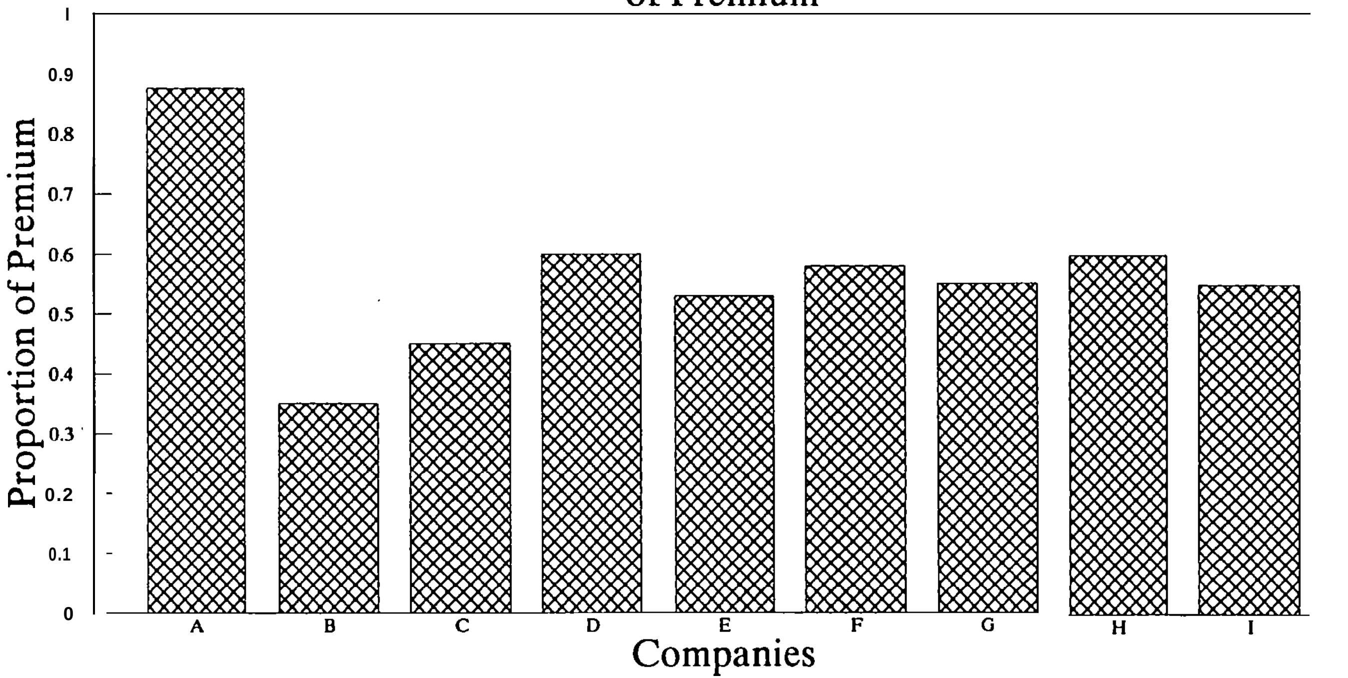
90A as a Proportion
of Premium

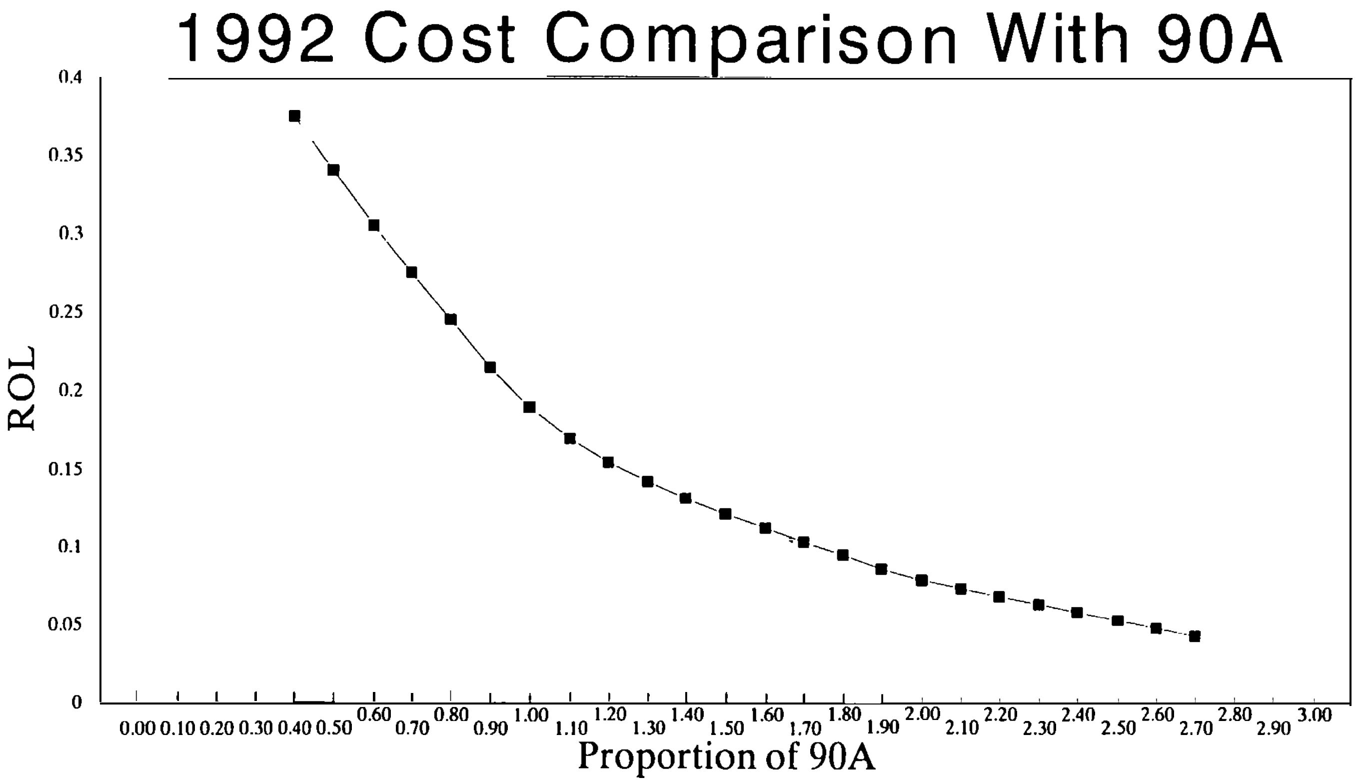


Deductible as Proportion of Premium



Cover FGU as Proportion of Premium




1992 Cost Comparison With 90A
ROL
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.00 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00 1.10 1.20 1.30 1.40 1.50 1.60 1.70 1.80 1.90 2.00 2.10 2.20 2.30 2.40 2.50 2.60 2.70 2.80 2.90 3.00
Proportion of 90A

Funding mechanism, if that route is chosen. The loading represents brokerage (10%) and safety margins (15%).

(c) What about losses below the retention? In previous years retentions were set as low as 2%-3% of premium income. Freeze and other losses were reinsured as part of the overall programme. How should they be financed or planned?

In simulations made for the ASTIN paper it is not unusual to find the catastrophe attrition losses (i.e. those below the deductible) to be, on average, a factor of between 100% and 150% of the deductible. The reasons for this are as follows:-

(i) We have a considerable number of small losses (e.g. floods, freeze etc.) below the catastrophe. The recent 1993 January storms and floods have cost many insurers £10 million or more.

(ii) When the big catastrophe hits, a prior charge of the deductible is made before any reinsurance can be recovered.

These issues need careful planning.

Finally, pre 1990, the cost of reinsurance for the UK property account was small compared with the premium income and deductibles were considerably lower. Premiums were based on gross experience, and profit made on reinsurance. Nowadays, the cost of catastrophe claims via catastrophe premium, deductible, retained percentage of programme and so on is considerably higher.

The basis for premium rates should be the larger of:-

(i) Gross premium.

(ii) Net premium plus catastrophe costs.

I believe the rating basis has switched i.e. (ii) is larger than (i); yet the insurance market has not reacted. I also believe that the UK property account could be suffering because the market has not addressed this problem. The reinsurance or catastrophe costs are not yet fully costed in the premium basis.

RESERVING FOR CATASTROPHES

It is normal to review a book of Excess of Loss Reinsurance Business in two parts:-

1. The attrition losses arising from working covers.

2. The individual (main) catastrophes separately.

For the catastrophe, the losses can be reviewed either in aggregate or the cover to which they relate (Reinsurance, Retrocession business, Spiral business, Specific, International, Whole Account).

The purpose of reserving is two-fold:-

1. To ensure adequate reserves are placed, and the account is not under or over reserved.

2. To provide management information at specific points of time.

   This management information may be used to purchase additional reinsurance cover.

The method I use is curve fitting a three parameter curve to the paid and incurred claims:-

$$Y = A\ (1\text{-}EXP\ (\text{-}t/B)\ ^{C})$$

This is a monatonic increasing curve.

The parameters are:-

A	=	Anticipated ultimate loss.
B	=	Parameter for slope of the curve.
C	=	Parameter for the shape of the curve.
t	=	Period (in days).

For pre 1992 catastrophes B was in general about 600 and C = 2. For modern catastrophes (Typhoon 19 and Hurricane Andrew) B is much lower.

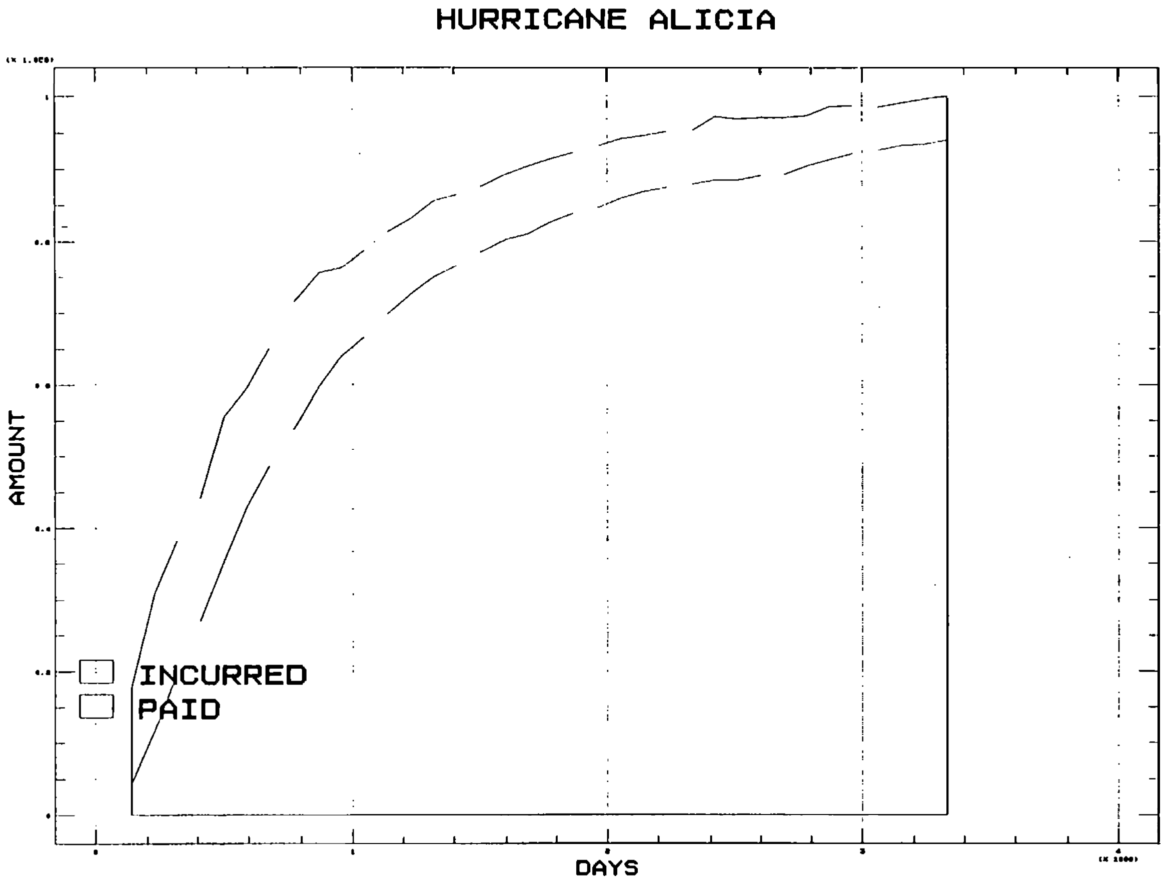
Reserving is not just curve fitting. Several other factors need to be taken into account

(i) Estimation is based on Paid Claims and Incurred (i.e. Paid plus Reported Outstanding Claims).

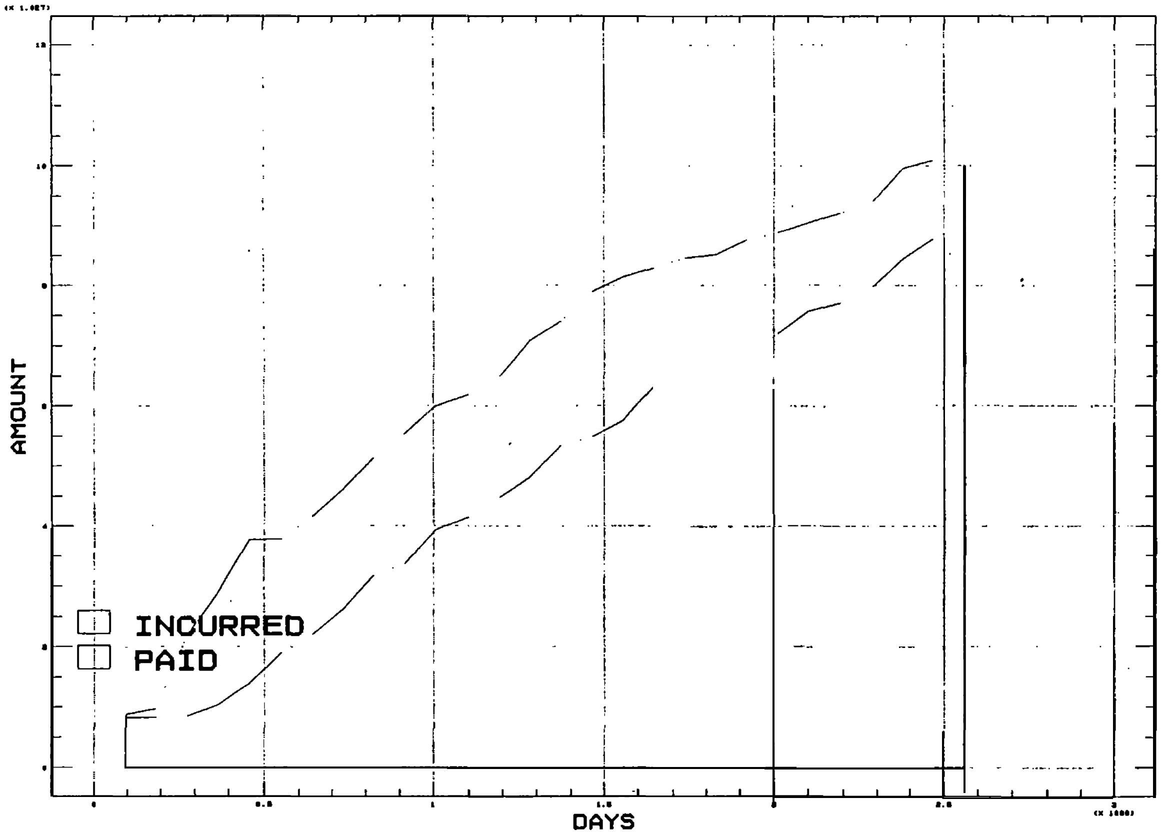
In most catastrophes there is a gap between these paid and incurred. The first three graphs attached to this section show the gaps for Hurricanes ALICIA, GLORIA and GILBERT. The amounts have been normalised so that today's incurred claims are £100,000,000.

The most developed is ALICIA when a gap of about £10,000,000 has been apparent for a number of years. The possible explanation is that there are a residual amount of outstanding losses reported by Brokers, which have not been released as the catastrophe claims are made. These are possibly redundant.

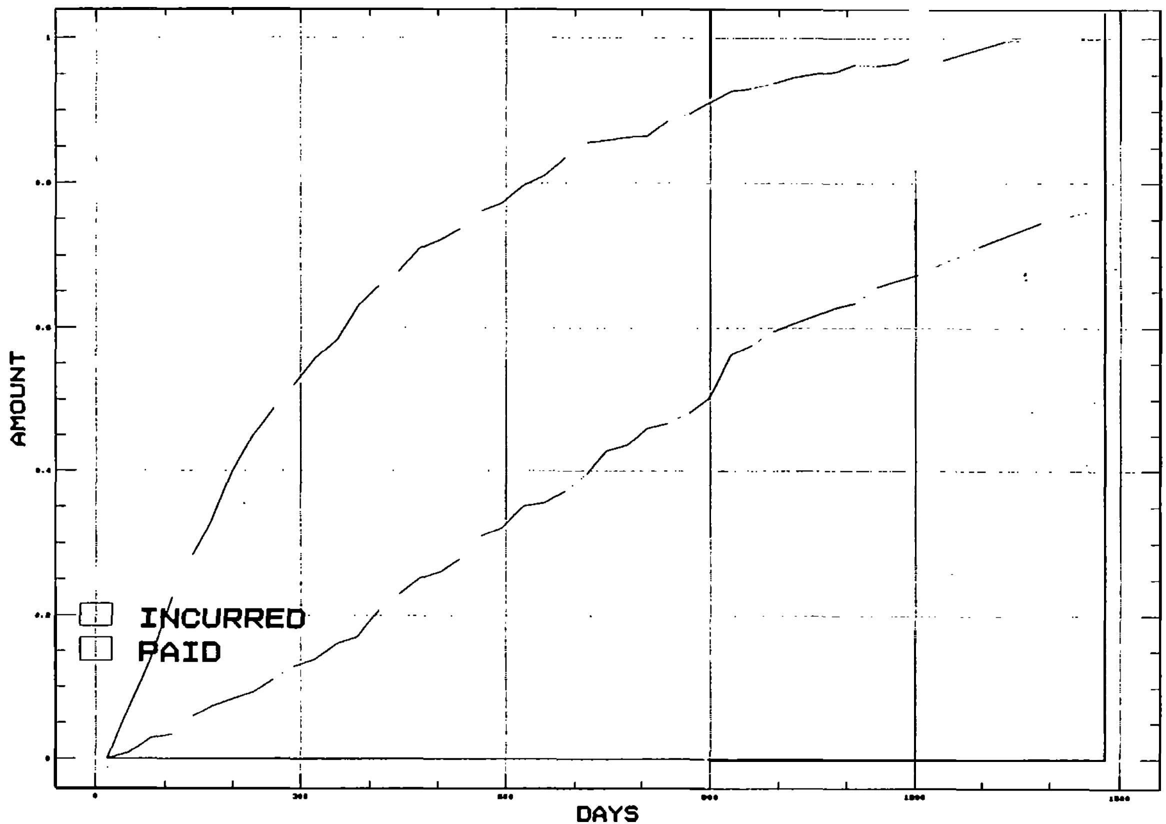
When reserving, one needs to be aware of this 5%-10% gap. The incurred position should unwind as these reserves are released. ALICIA occurred in 1983; GLORIA in 1985 and GILBERT in 1987. Gilbert is primarily a Jamaican loss and reporting standards for Caribbean countries may reflect the wider gap. All the losses are expressed in one currency.


HURRICANE ALICIA
AMOUNT
DAYS
INCURRED
PAID

HURRICANE GLORIA



HURRICANE GILBERT



(ii) Curve fitting is statistical by nature, and one should be aware of standard errors. The best fit curve may give an Ultimate below the current paid or incurred. This feature should be taken into account when undertaking the reserves Whereas incurred unwinds, paid claims increase.

(iii) The use of a single curve may not be appropriate. Certain loss payments come in two distinct surges. The first is normally the physical damage (Loss of Rig - Piper Alpha; Loss of Aircraft - Japanese 747; Earthquake - San Francisco - Plant Destruction - Phillips).

This is followed by liability or business interruption losses:-

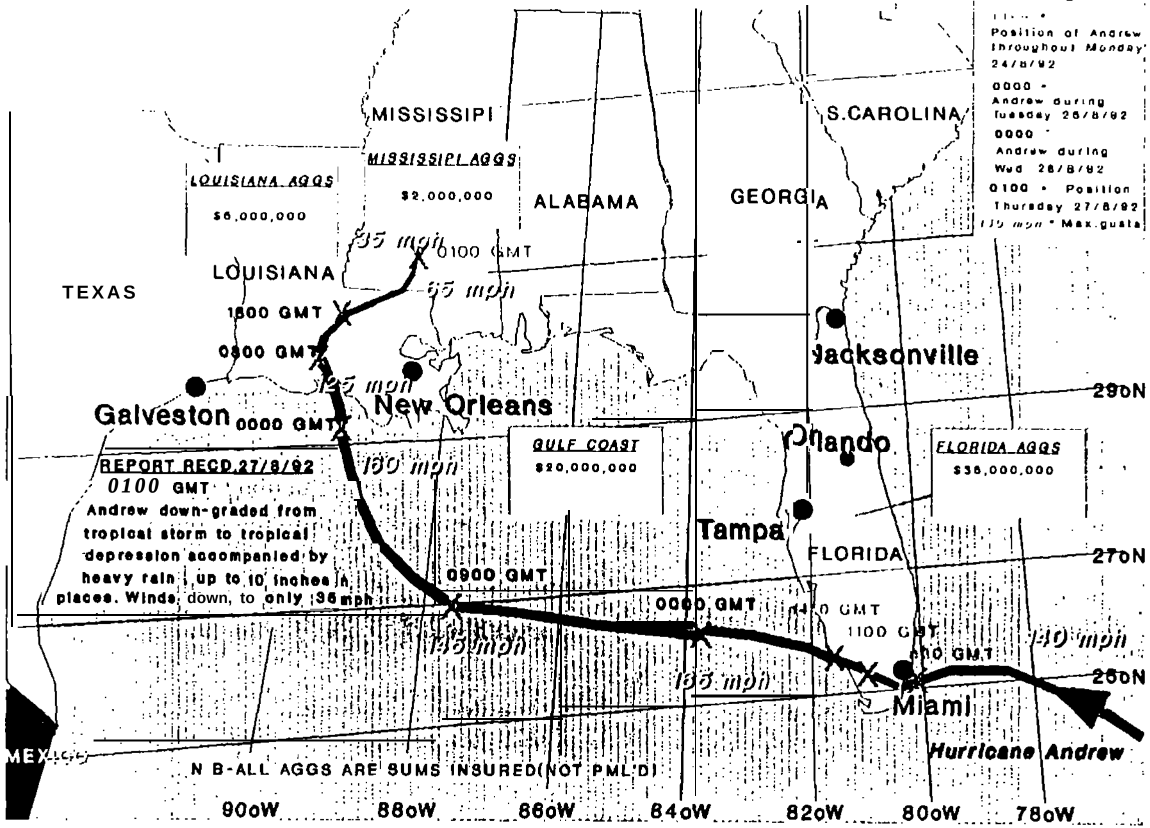
Employers Liability	-	Piper Alpha
Passengers Liability	-	Japanese 747
Architects Liability		San Francisco Earthquake
Business Interruption	-	Phillips

It may be appropriate to superimpose a second (later) curve for this final surge. Examples are clearer in the development curves at the end of this section.

(iii) Underwriters judgement and exposures should be taken into account. although based on crude estimates, the exposure multiplied by a probable maximum loss (80% say) may be the only guidelines available.

Attached is a typical exposure for Hurricane Andrew. (Amounts are artificial).

Hurricane Andrew/Aggregates in exposed states



(iv) **The difference between Marine and Non-Marine Losses**

In general a Non-Marine loss such as Hurricane Hugo will rise rather rapidly in the Non-Marine account. As the Non-Marine Specific reinsurance is absorbed the Whole Account protections (with associated spiral) come into play. Non-Marine losses are normally settled first and the CAT developments reach a stable position fairly early. Marine Excess of Loss and Whole Account claims then take up.

My estimation for parameter B for Hugo is 232 days Non-Marine and 744 days Marine.

Marine Gross Losses also tend to be substantially higher than Non-Marine Gross Losses. This is due to the more effective spiral (no 10% retention). A 30 times spiral (i.e. gross to net) is not unusual.

(v) **The Special Impact of 1989**

In 1989 there were a number of losses which have had a substantial impact on the CATXL market - particularly the Marine market. There are only three large losses allowed for on most treaties - yet we have four major losses - Hurricane Hugo, Exxon Valdez, Phillips Petroleum (an explosion) and Arco Platform (a drilling rig). For a large number of reinsurers one of these three is redundant - and the smallest is Arco Platform.

To put these figures into perspective the Marine Market losses: Hurricane Hugo (total $4 billion of which about $2.4 million is non Marine and the Marine losses are likely to be $1.6 billion) $1 billion Exxon Valdez, $1 billion Phillips and $0.4 billion Arco Platform. A consequence of this is that in the book of incurred claims there is likely to be some double counting (i.e. the sum of all the notified losses per cedant is likely to exceed the aggregate exposure). The paid losses are controlled by physical checks on amounts recovered under treaties, but aggregate exposures are not. As a result the smallest losses are likely to have higher than average redundancy as the incurred position unwinds.

Secondly, Phillips Petroleum is a very confusing loss in that it is one of the few losses which the model fails to fit. The reason is that it is , in reality, three different types of loss which behave differently - namely a material damage loss, a business interruption loss and a US liability loss. It is, in practice slower to develop than its peer losses.

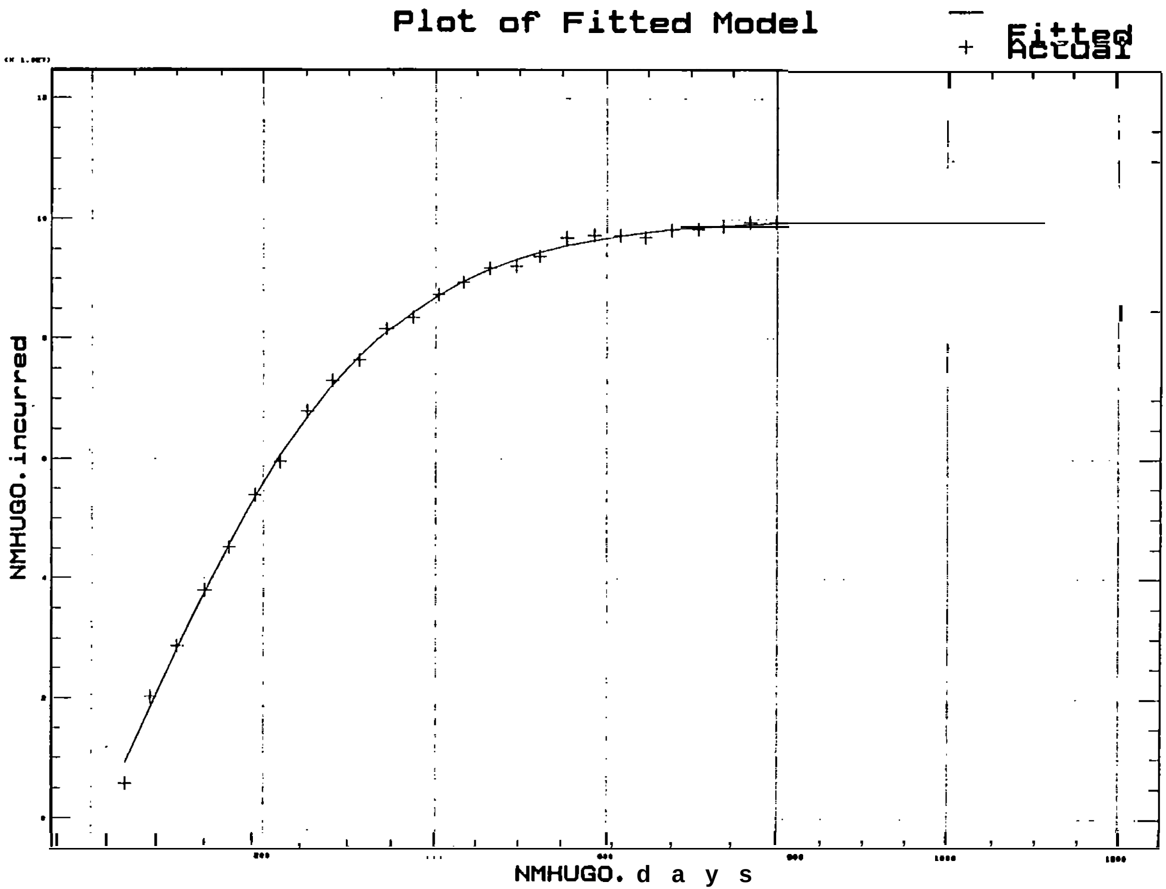
On the attached sheets I calculate the factors for these losses. I have normalised the losses so that today's incurred losses are £100 million.

Note that Non-Marine Hugo has stopped and Marine Hugo has nearly completed its development, and Arco and Exxon are near complete development. Considerable uncertainty surrounds Phillips so an alternative method may be required.

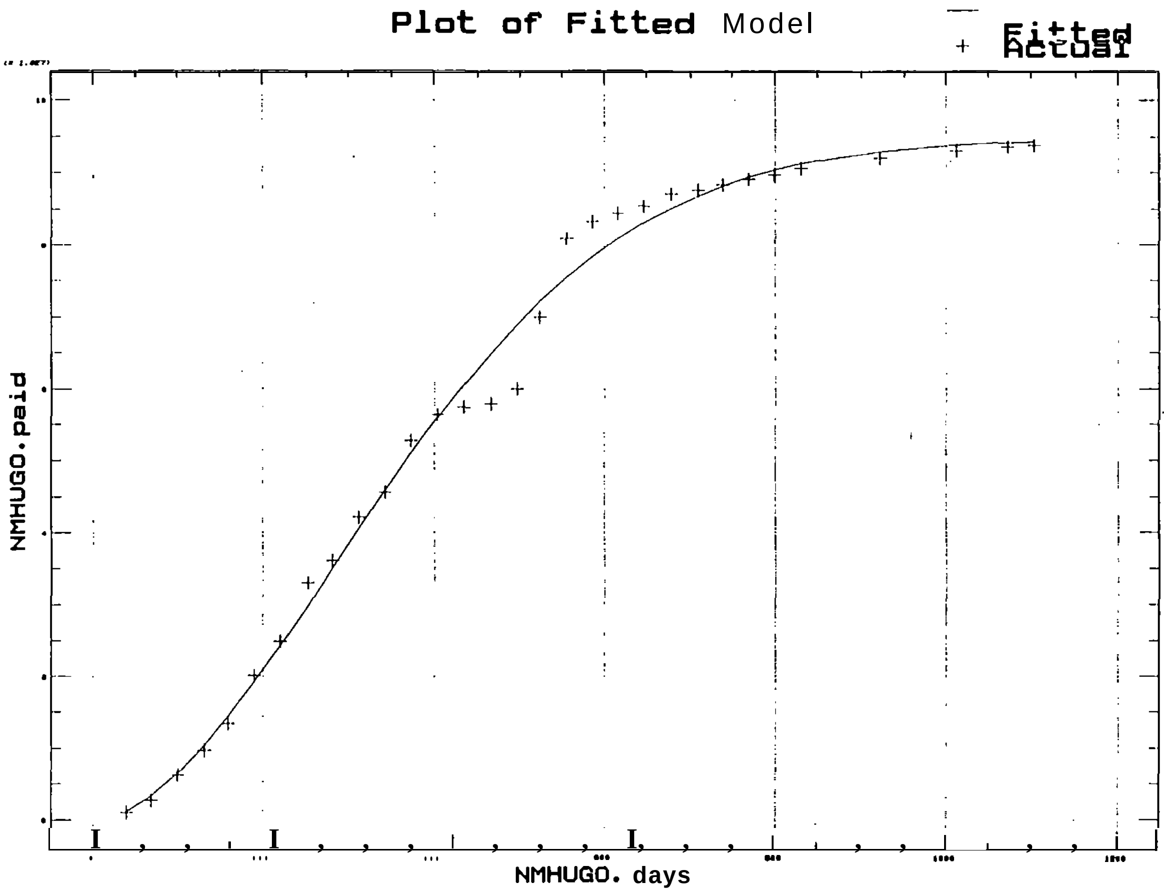
The figure in brackets is the standard error.

CATS OF 1989

CATASTROPHE			BASIS	A		B		C	
HUGO	NON-MARINE	(NMHUGO)	INCURRED	100.050	(0.323)	232	(2.03)	1	(0.19)
HUGO	NON-MARINE	(NMHUGO)	PAID	94.763	(1.532)	429	(10.58)	1.8	(0.08)
HUGO	MARINE	(HUGO)	INCURRED	102.508	(1.721)	744	(11.08)	3	(0.08)
HUGO	MARINE	(HUGO)	PAID	90.833	(1.055)	786	(6.81)	3.4	(0.08)
ARCO	MARINE	(ARCO)	INCURRED	105.419	(3.259)	960	(21.44)	3.0	(0.14)
ARCO	MARINE	(ARCO)	PAID	80.514	(1.887)	933	(15.5)	3.4	(0.15)
EXXON	MARINE	(EXXON)	INCURRED	108.97	(5.628)	897	(43.19)	2.0	(0.15)
EXXON	MARINE	(EXXON)	PAID	83.93	(7.284)	988	(62.21)	2.9	(0.30)
PHILLIPS	MARINE	(PHIL)	INCURRED	211.421	(9.678)	1,341	(404.80)	2.0	(0.2)
PHILLIPS	MARINE	(PHIL)	PAID	95.57	(3.610)	995	(22.73)	3.0	(0.7)



Plot of Fitted Model
Fitted
Actual
NMHUGO.incurred
NMHUGO.days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Actual
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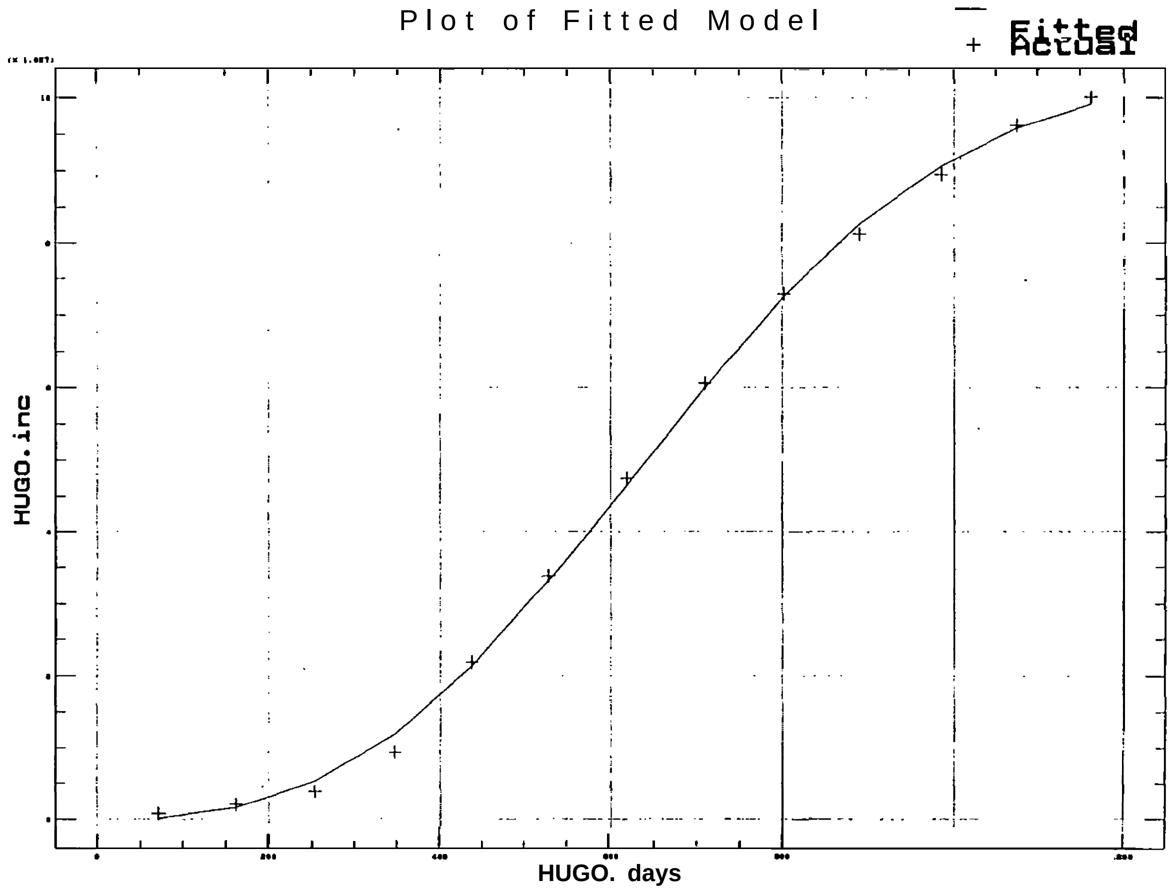
NMHUGO. days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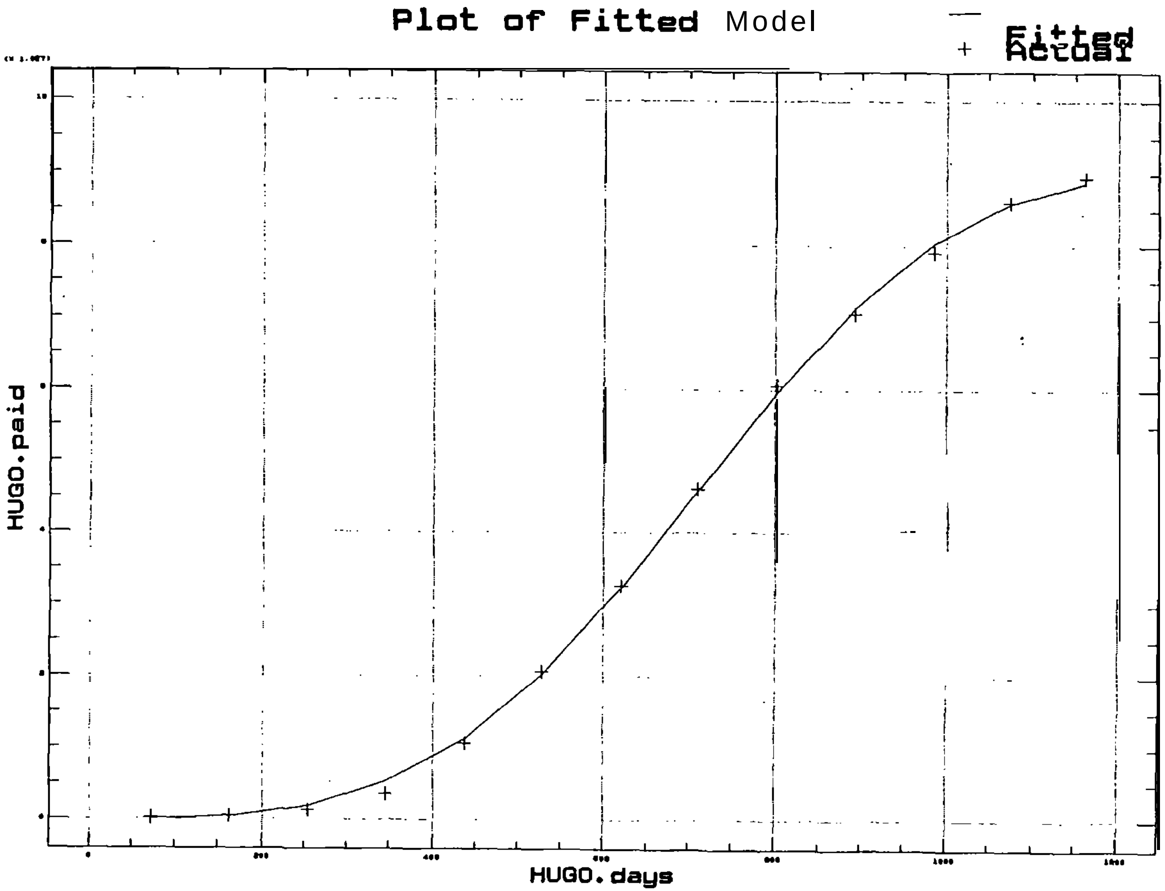
Actual
HUGO.inc
HUGO. days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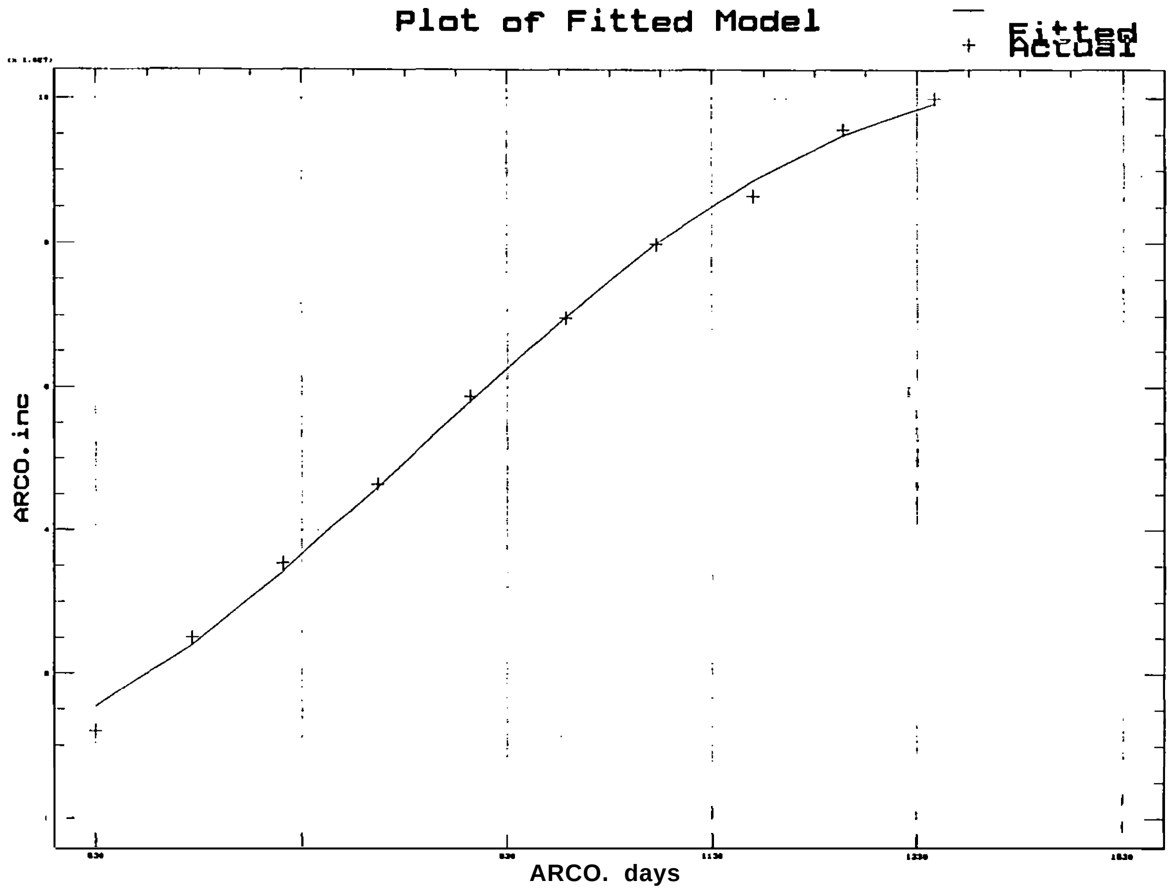
Actual
HUGO.paid
HUGO.days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Actual
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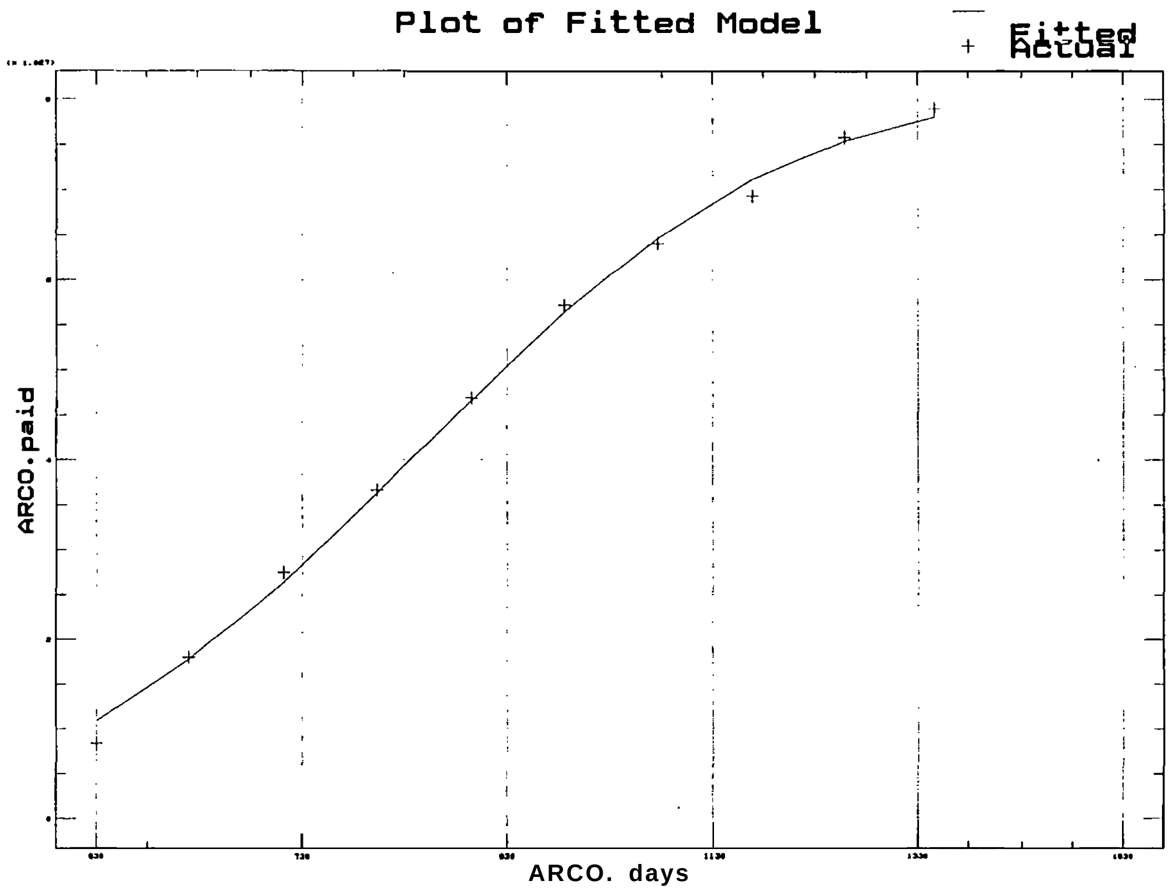
ARCO. days



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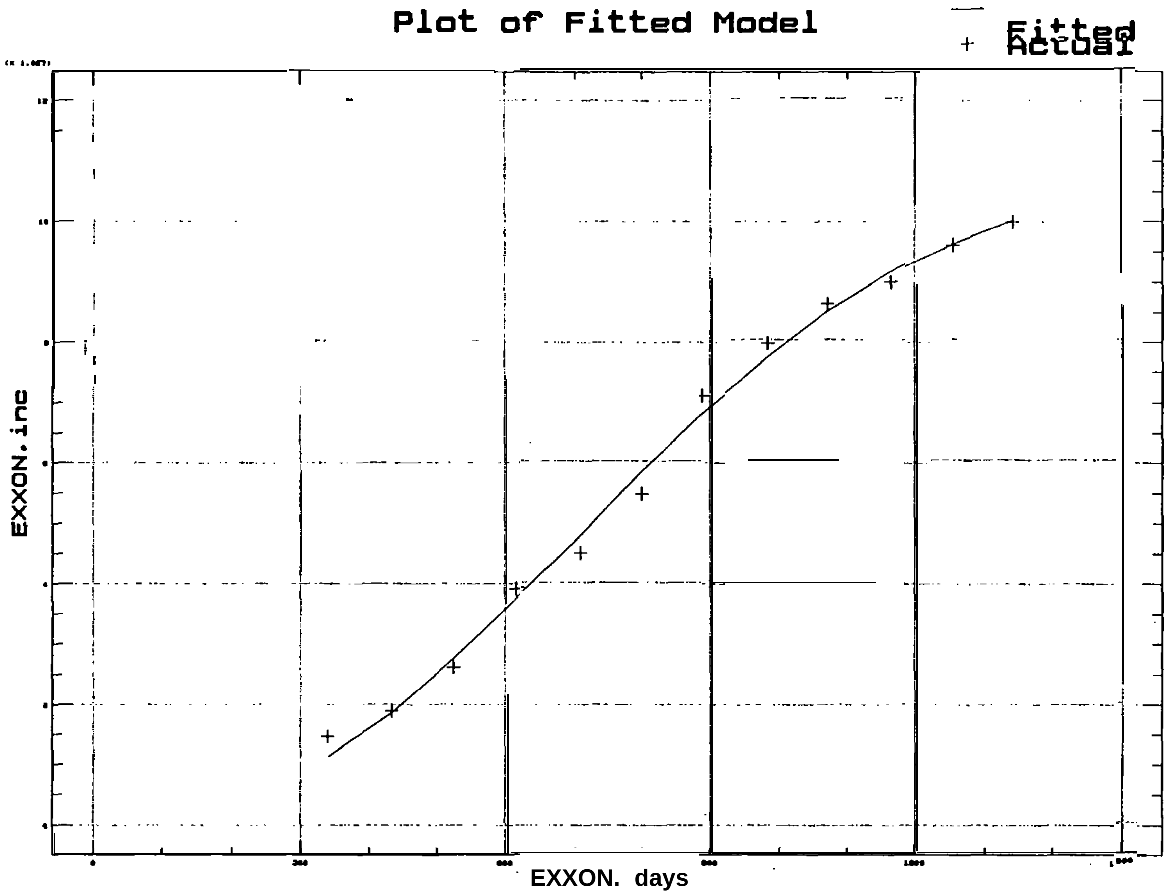
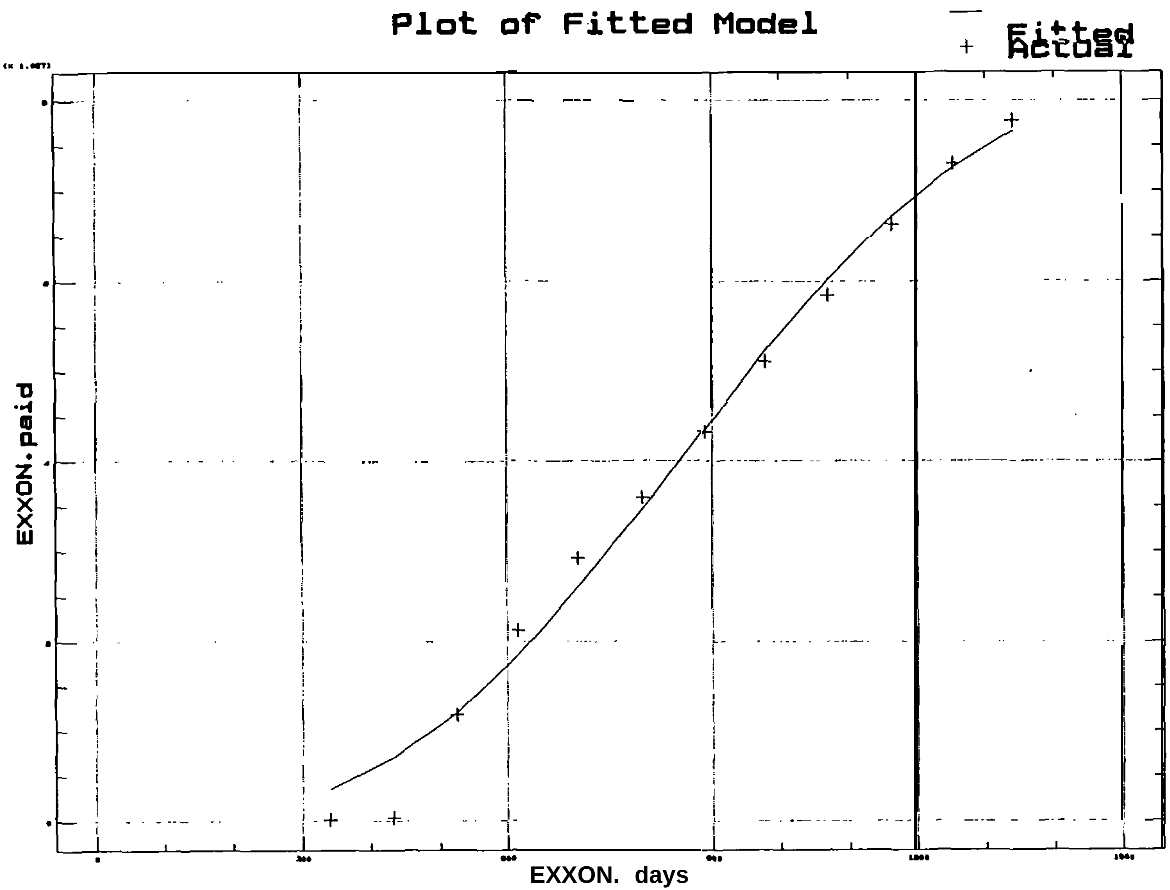
Fitted
Actual
ARCO.paid
ARCO. days



Plot of Fitted Model
Fitted
Actual
EXXON.inc
EXXON. days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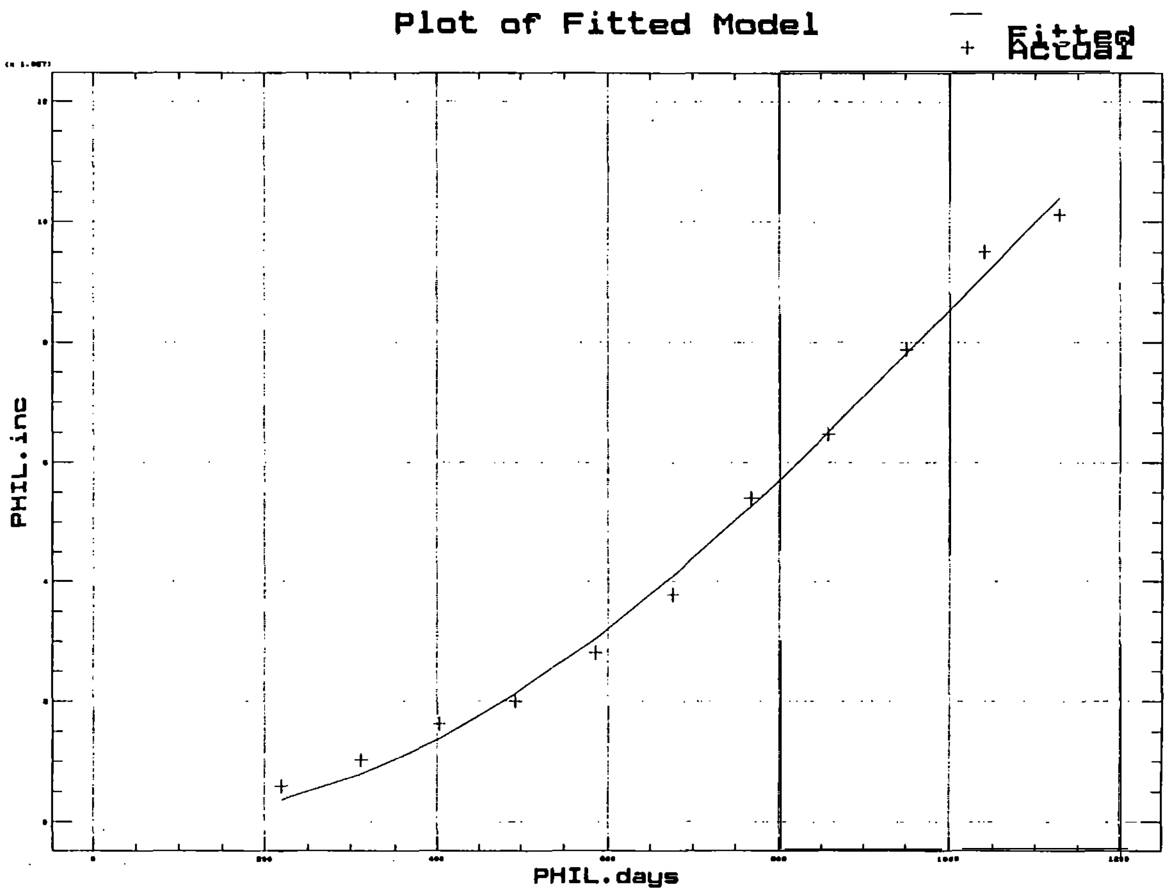
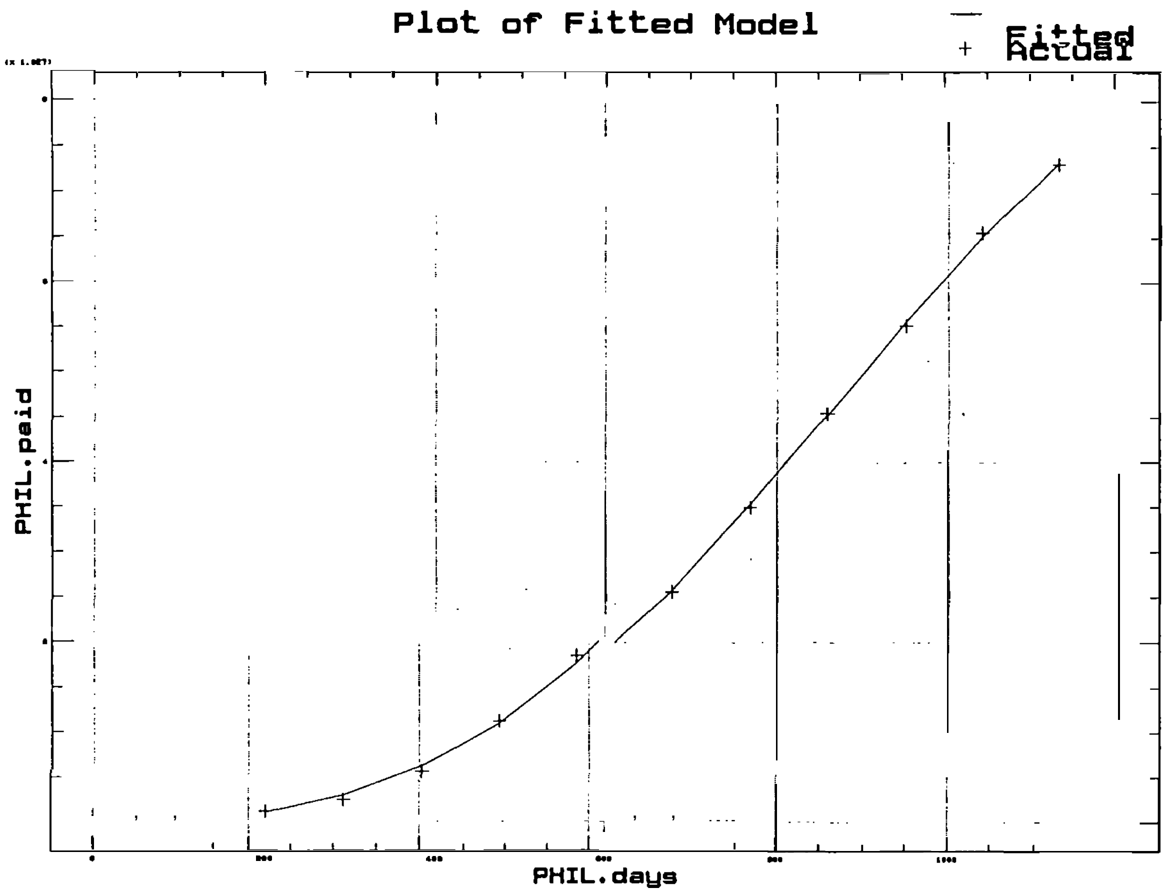
Fitted
Actual
EXXON.paid
EXXON. days



Plot of Fitted Model
Fitted
Actual
PHIL.inc
PHIL.days


Plot of Fitted Model
Fitted
Actual
PHIL.paid
PHIL.days

Finally, I set out some further examples of Windstorm Losses. Note how different the Development of Typhoon 19 (Merielle) is when compared with the other losses.

Hurricane Andrew also has the same features. The amounts in the brackets are standard errors to the parameter estimation.

Several points need to be noted:-

(i) In Lloyds and many London Market Companies Reserves are only reviewed annually. This leads to a lack of on-going data. Furthermore, accounts are not finalised until three years' losses have occurred. The lower the number of data points, the less information is available. This leads to a large error potential in the parameter estimations. Frequent data points are needed for better estimations.

(ii) The estimation process is only the first stage of establishing the reserves. The estimate may exceed the aggregate exposure and special features may need to be brought into consideration.

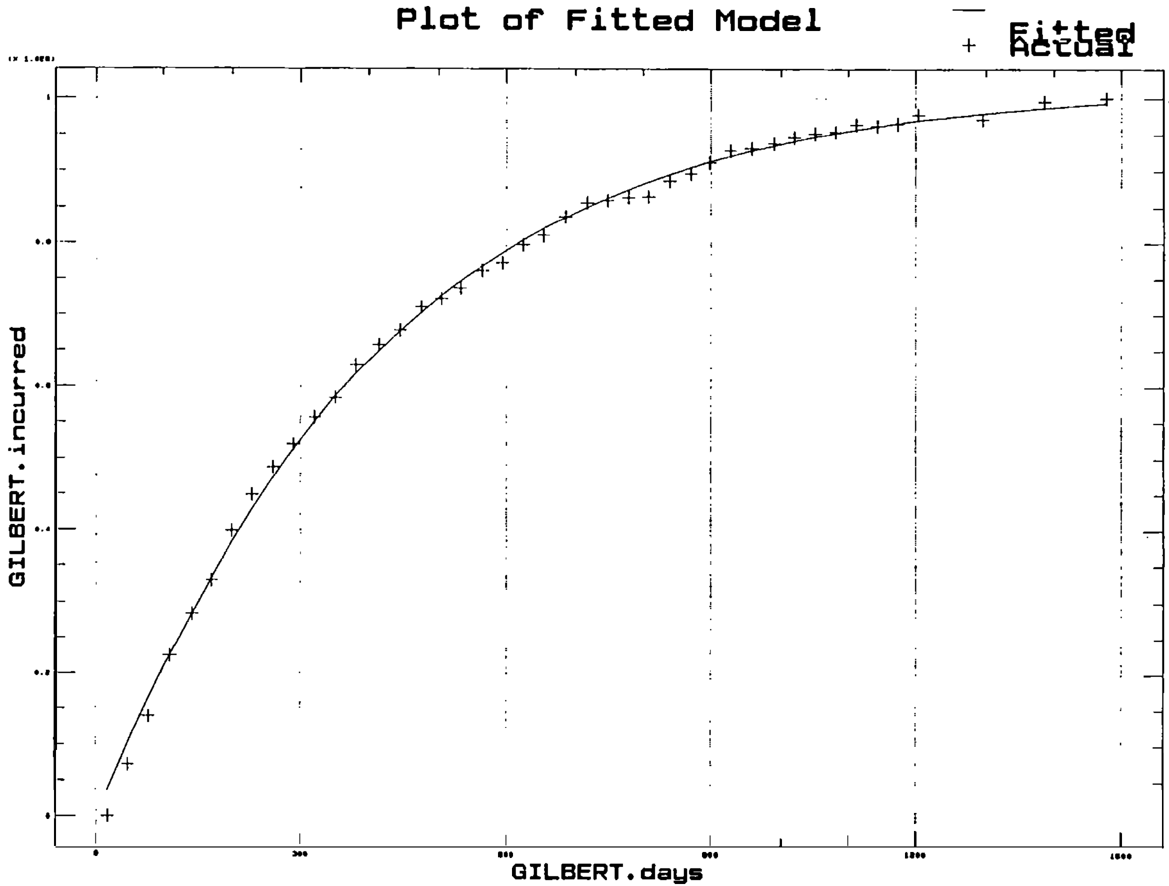
(iii) The reserves are gross reserves. Net reserves are calculated by super-imposing the reinsurance programme on anticipated ultimate loss to obtain the net reserves.

(iv) There is no need to fit the curve over the whole period. Recent developments can also be fitted to highlight any local short term variation in the data. Errors may occur due to information not being put in the database in a uniform manner which can distort the picture.

LOSSES

CATASTROPHE		MARINE/ NON MARINE	BASIS	A		B		C	
GILBERT	(1987)	(NM)	INCURRED	101.363	(0.781)	405	(7.08)	1	(0.019)
GILBERT	(1992)	(NM)	PAID	96.537	(4.110)	1063.2	(53.04)	1.5	(0.04)
GLORIA	(1988)	(NM)	INCURRED	124.837	(7.822)	1555	(184.6)	1	(0.05)
GLORIA	(1989)	(NM)	PAID	161.726	(31.326)	3091	(777.6)	1	(0.1)
MERIELLE	(1991)	(NM)	INCURRED	97.204	(1.359)	762	(2.59)	3.1	(0.40)
MERIELLE	(1991)	(NM)	PAID	93.717	(1.059)	81.2	(1.90)	3.7	(0.39)
STORM	90A	(M)	INCURRED (ST90M)	106.823	(6.163)	810	(26.78)	4.0	(0.28)
STORM	90A	(M)	PAID (ST90A)	91.079	(2.217)	841.7	(9.79)	4.7	(0.15)
STORM	90D	(M)	INCURRED (ST90D)	113.690	(6.156)	464	(46.42)	1.0	(0.07)
STORM	90D	(M)	PAID (ST90D)	69.796	(1.092)	521.8	8.21	2.8	(0.12)
STORM	90G	(M)	INCURRED (ST90G)	110.001	(4.456)	567	(29.0)	2.0	(0.07)

STORM	90G	(M)	PAID (ST90G)	85.566	(9.231)	798.9	(66.39)	2.3	(0.13)
STORM	87J	(NM)	INCURRED	96.516	(0.422)	320.1	(4.39)	1.4	(0.04)
STORM	87J	(NM)	PAID	89.377	(0.045)	512.1	(11.15)	1.6	(0.06)
STORM	90A	(NM)	INCURRED	100.163	(0.815)	331	(4.44)	2.0	(0.06)
STORM	90A	(NM)	PAID	89.267	(1.721)	439	(8.92)	3.3	(0.02)
STORM	90D	(NM)	INCURRED	100.163	(3.211)	402	(22.64)	1	(0.08)
STORM	90D	(NM)	PAID	68.593	(1.055)	529	(8.43)	2.8	(0.12)
STORM	90G	(NM)	INCURRED	110.513	(4.317)	589	(29.42)	2	(0.06)
STORM	90G	(NM)	PAID	83.248	(6.994)	799	(52.83)	2.3	(0.11)


Plot of Fitted Model
Fitted
Actual
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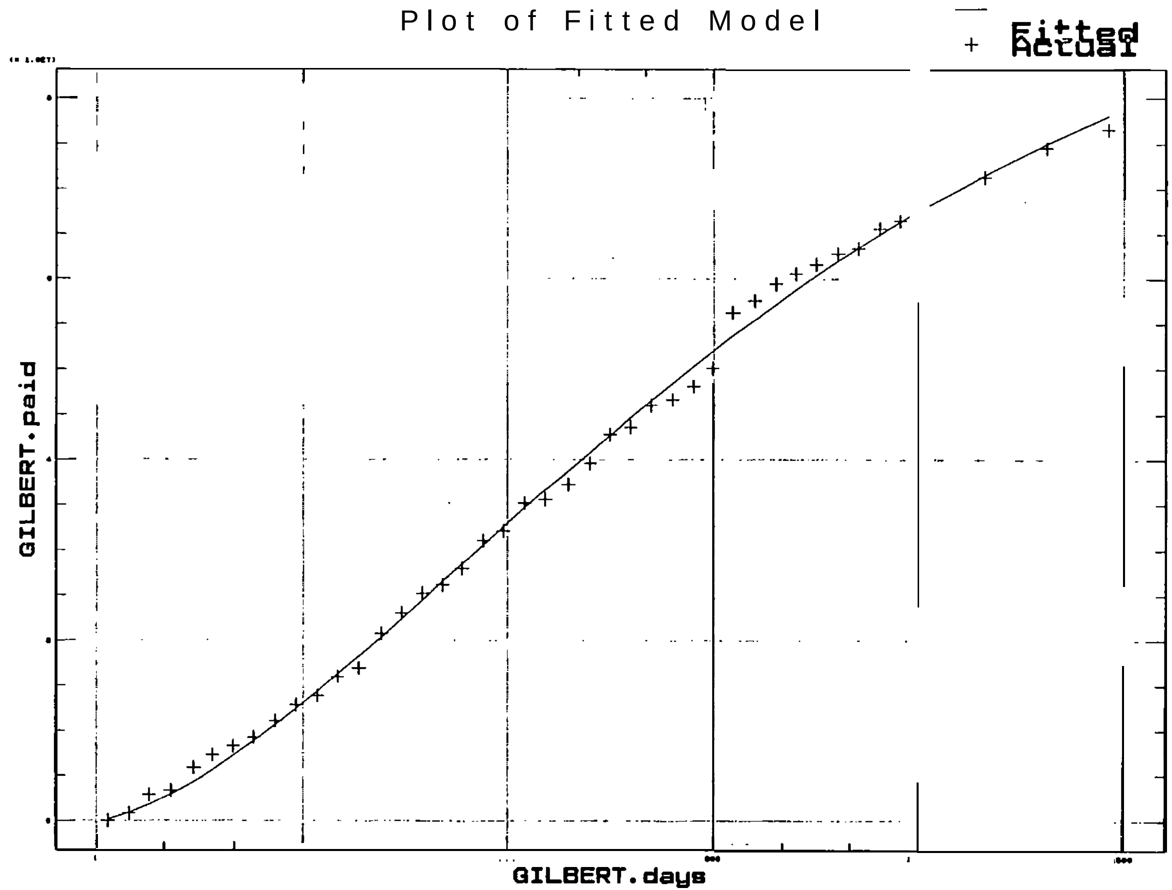
GILBERT.days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Actual
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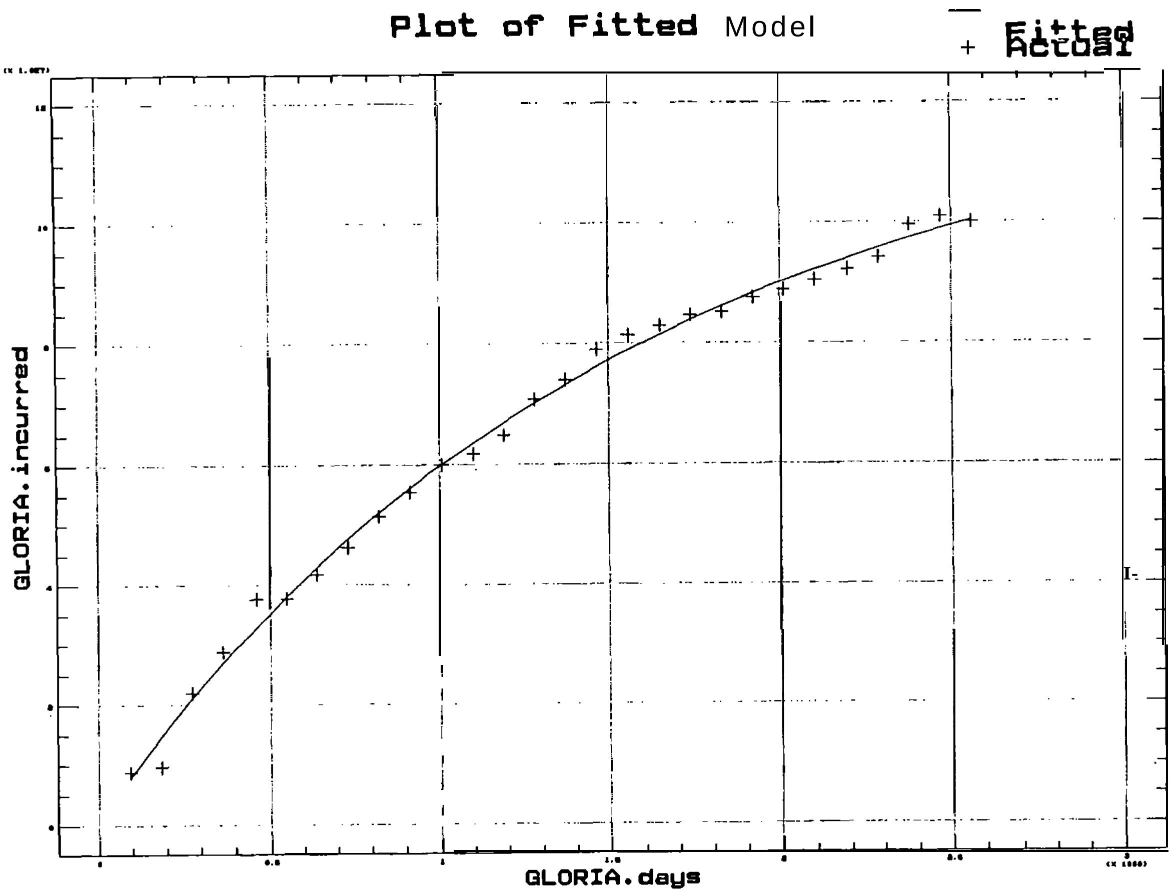
GILBERT.days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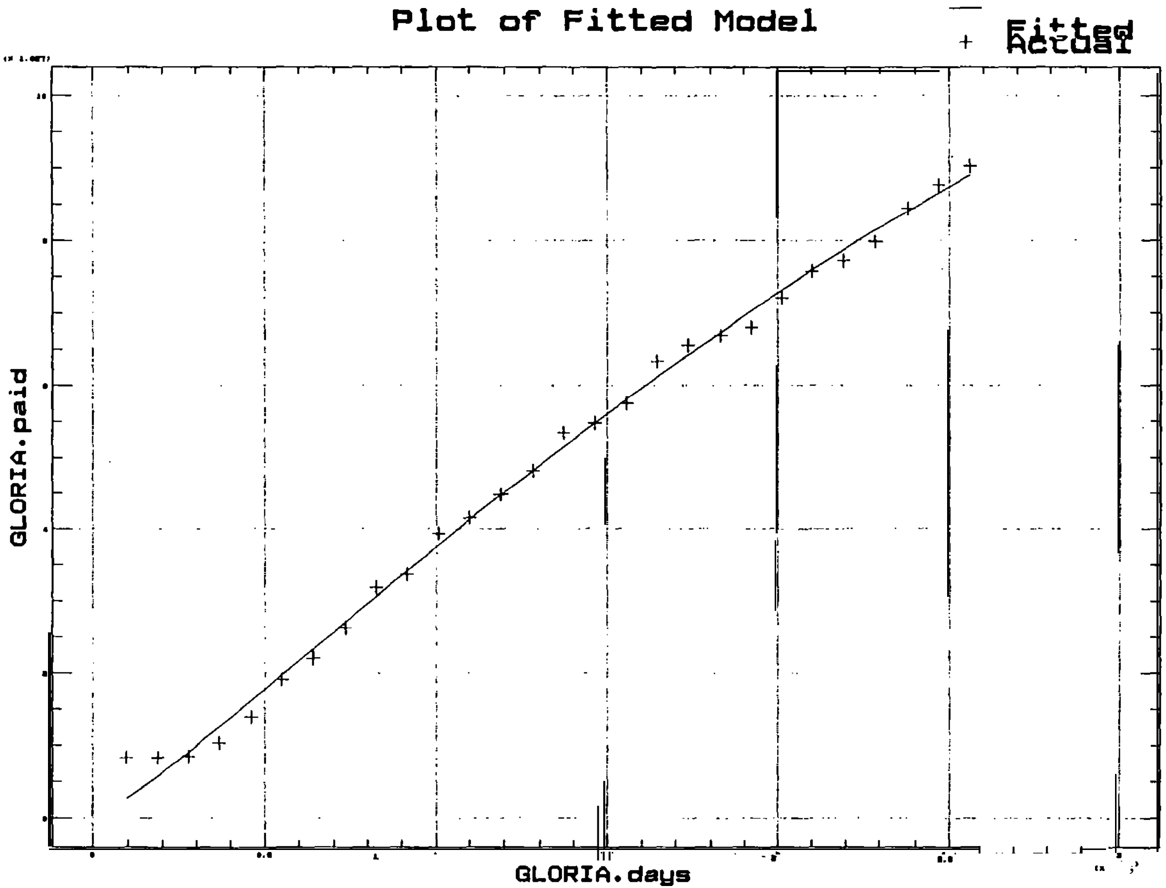
GLORIA.days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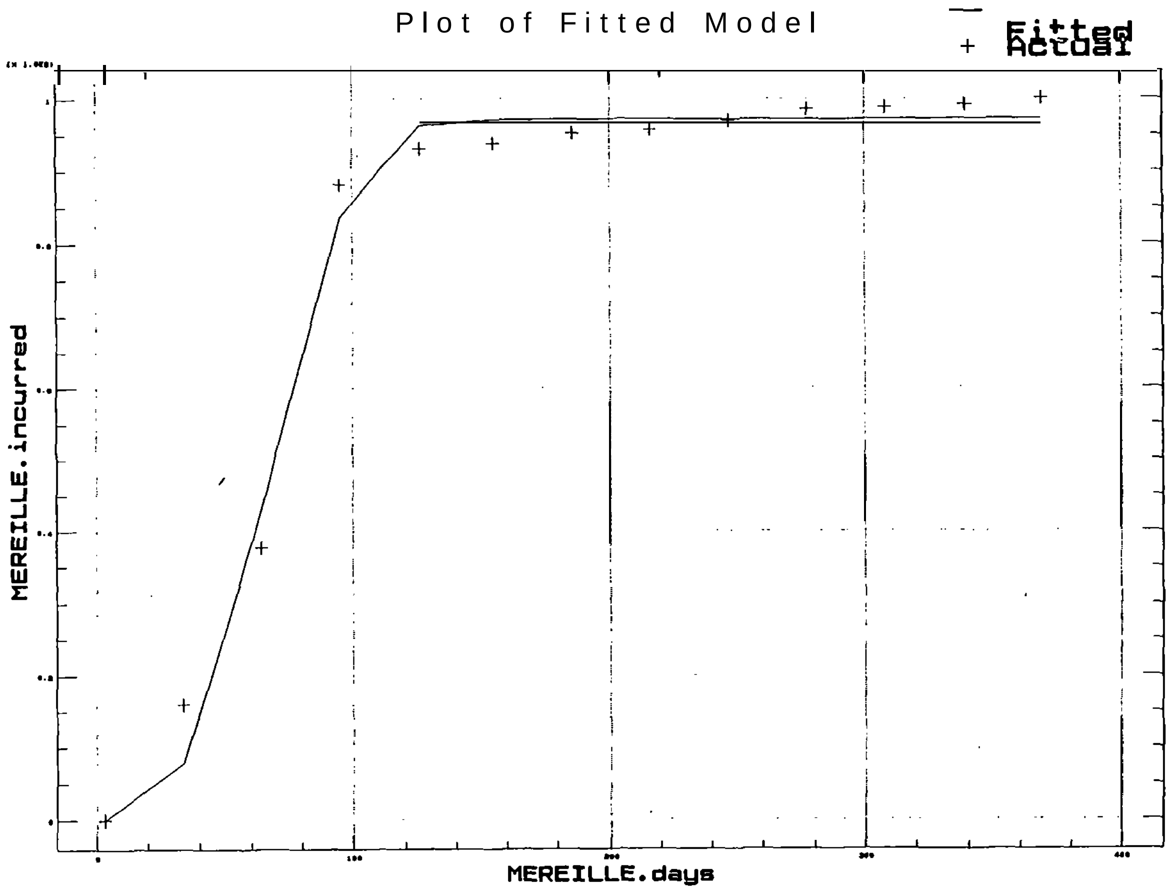
Fitted
Actual
GLORIA.paid
GLORIA.days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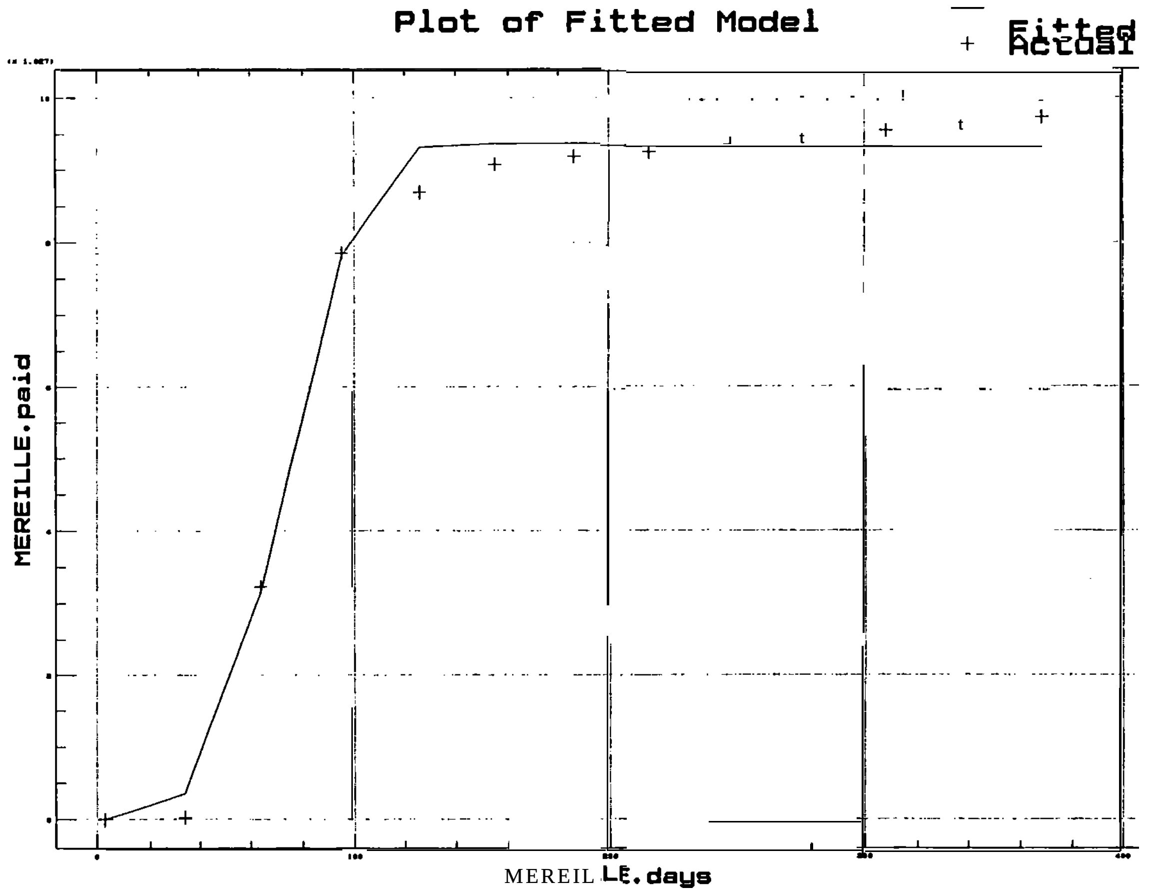
Fitted
Actual
MEREILLE.incurred
MEREILLE.days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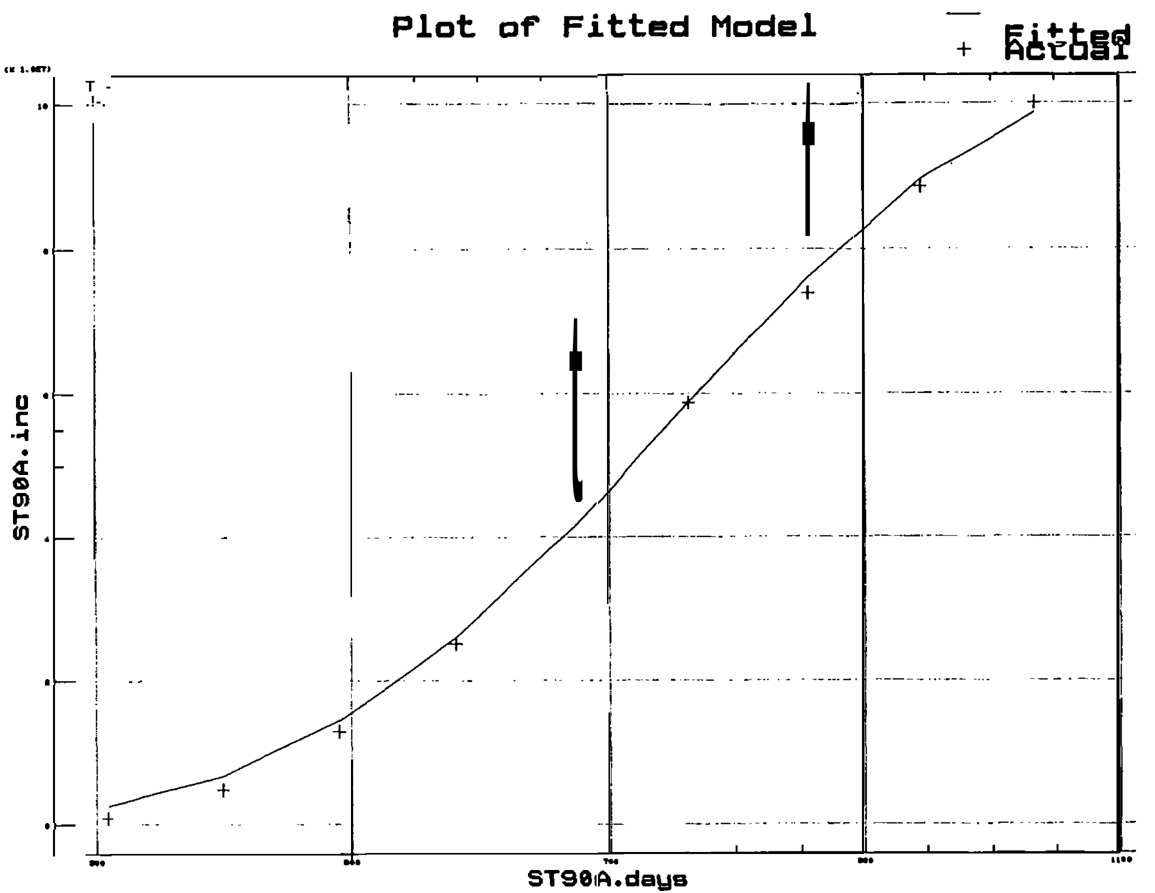
Fitted
Actual
MEREILLE.paid
MEREILLE.days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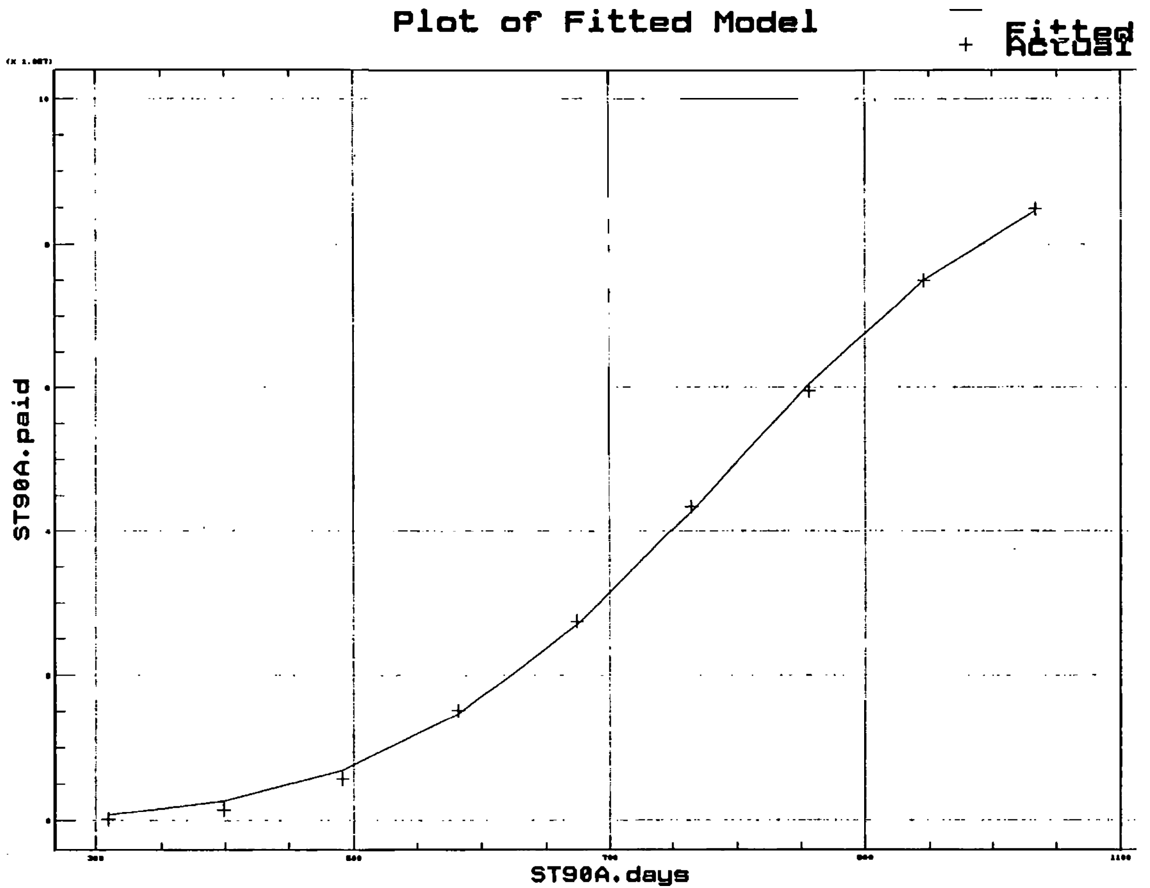
Actual
ST90A.inc
ST90A.days



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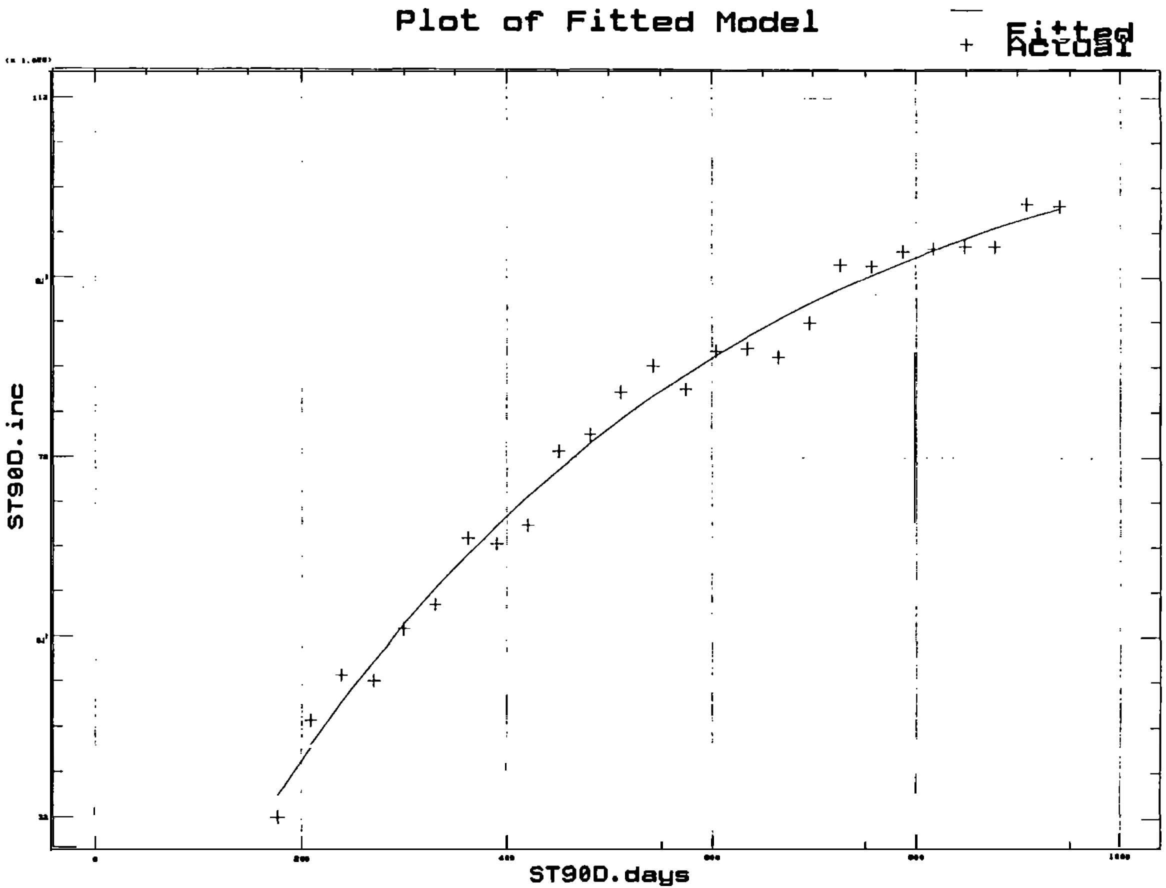
ST90A.days



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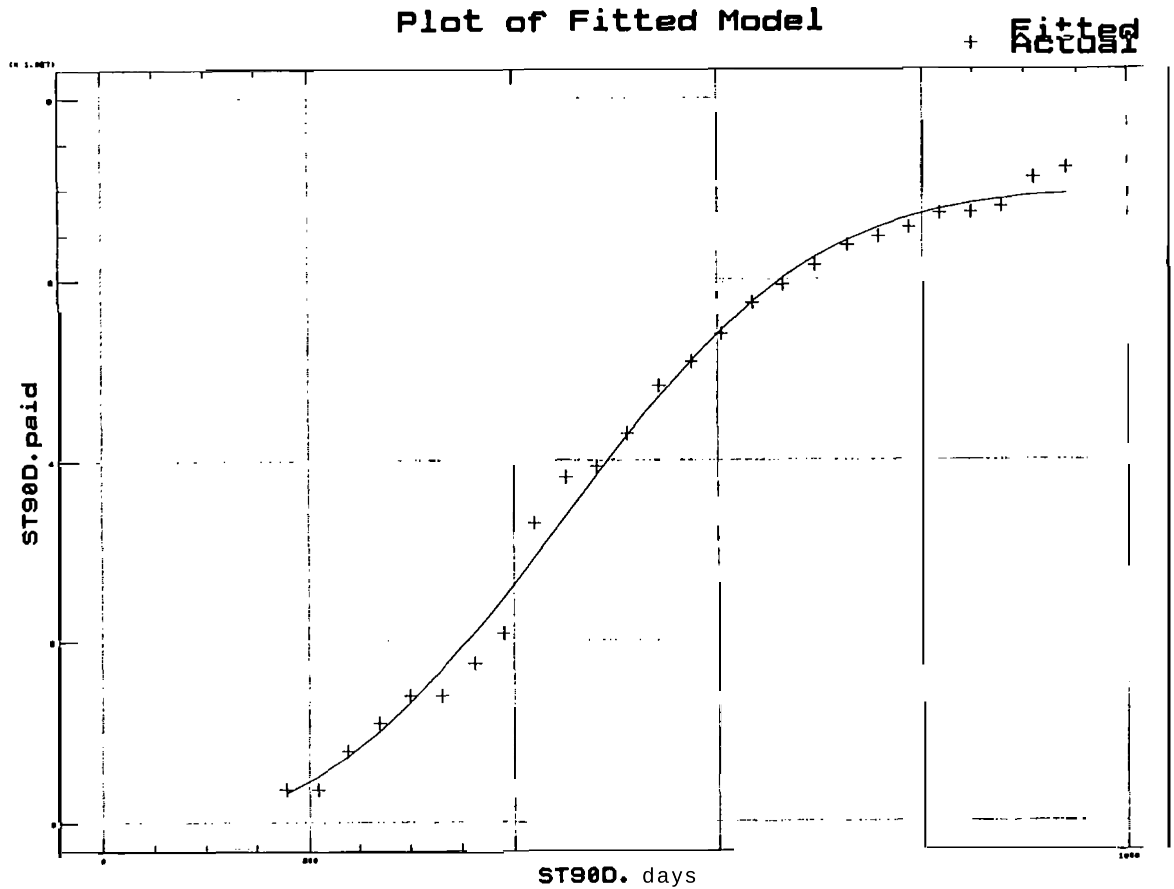
Fitted
Actual
ST90D.inc
ST90D.days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Fitted
Actual
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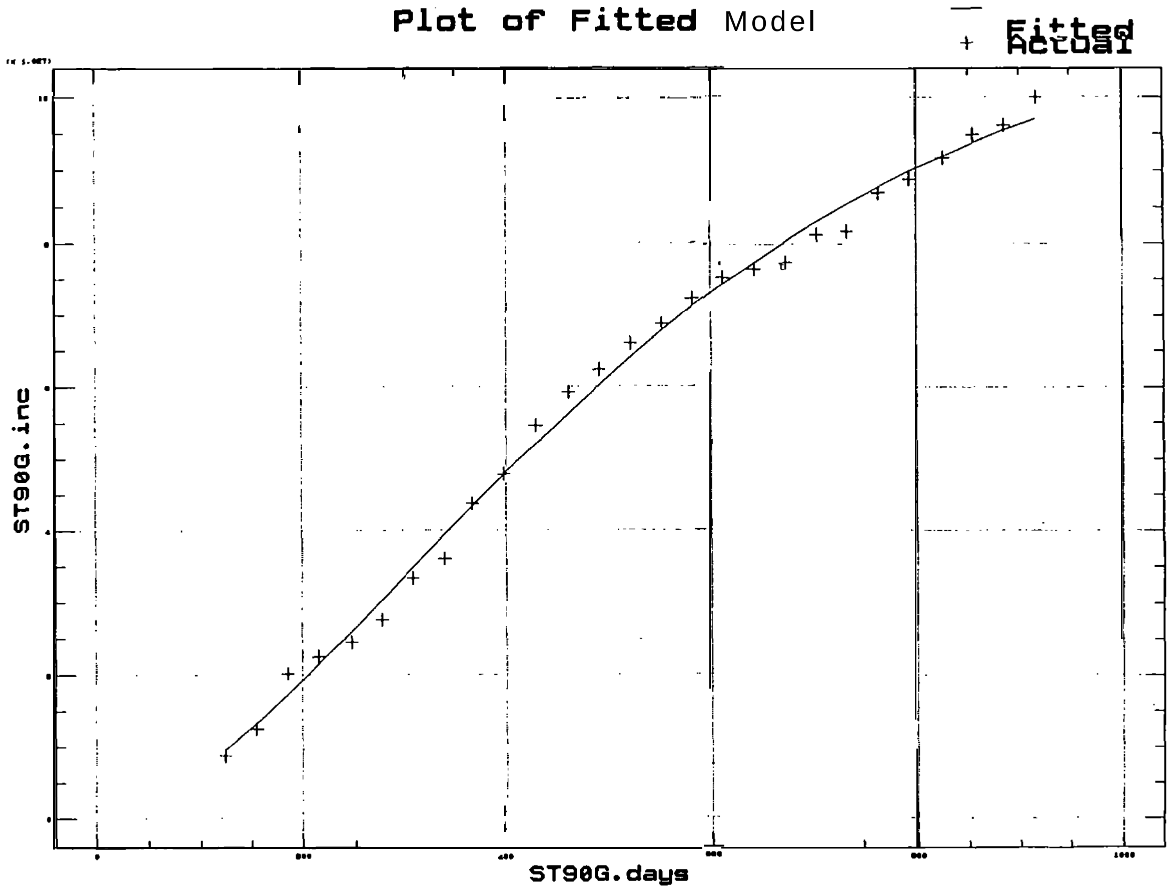
ST90D. days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Fitted
Actual
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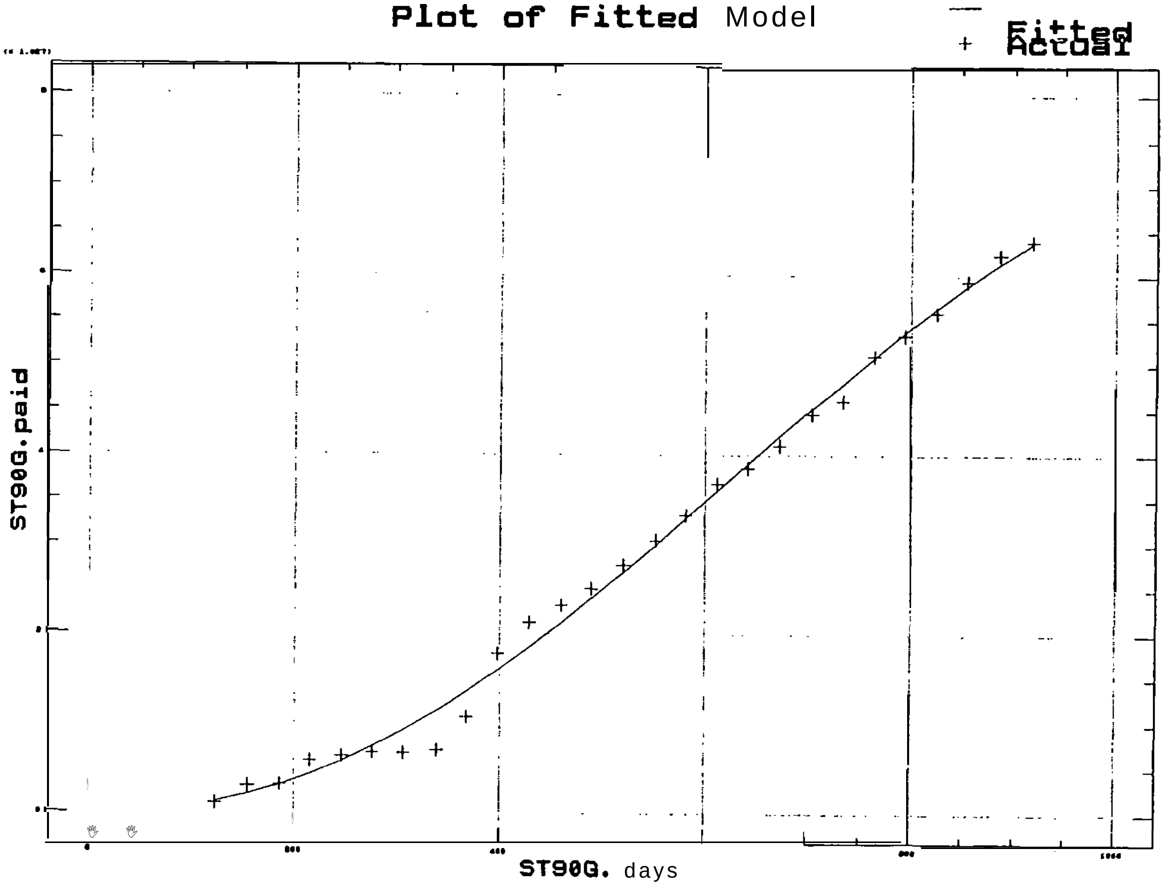
ST90G.days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Fitted
Actual
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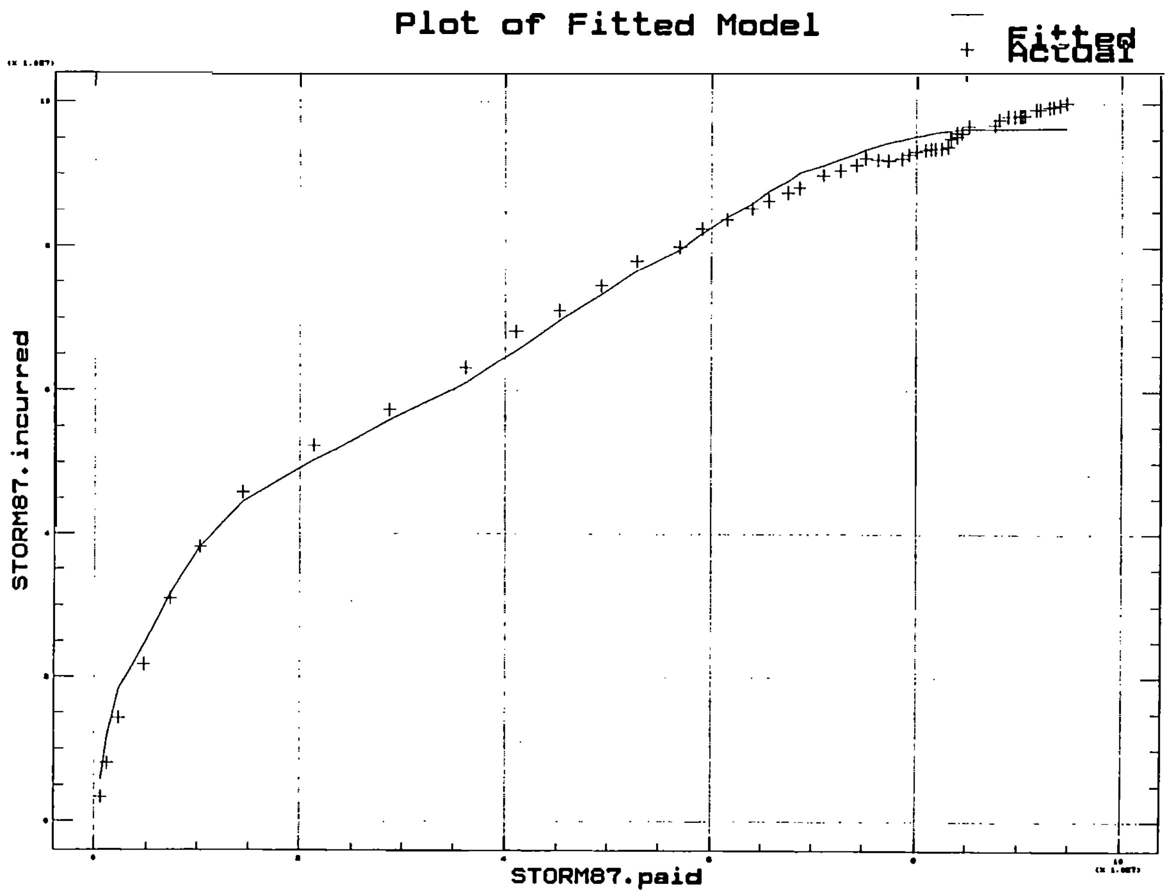
ST90G. days



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Actual
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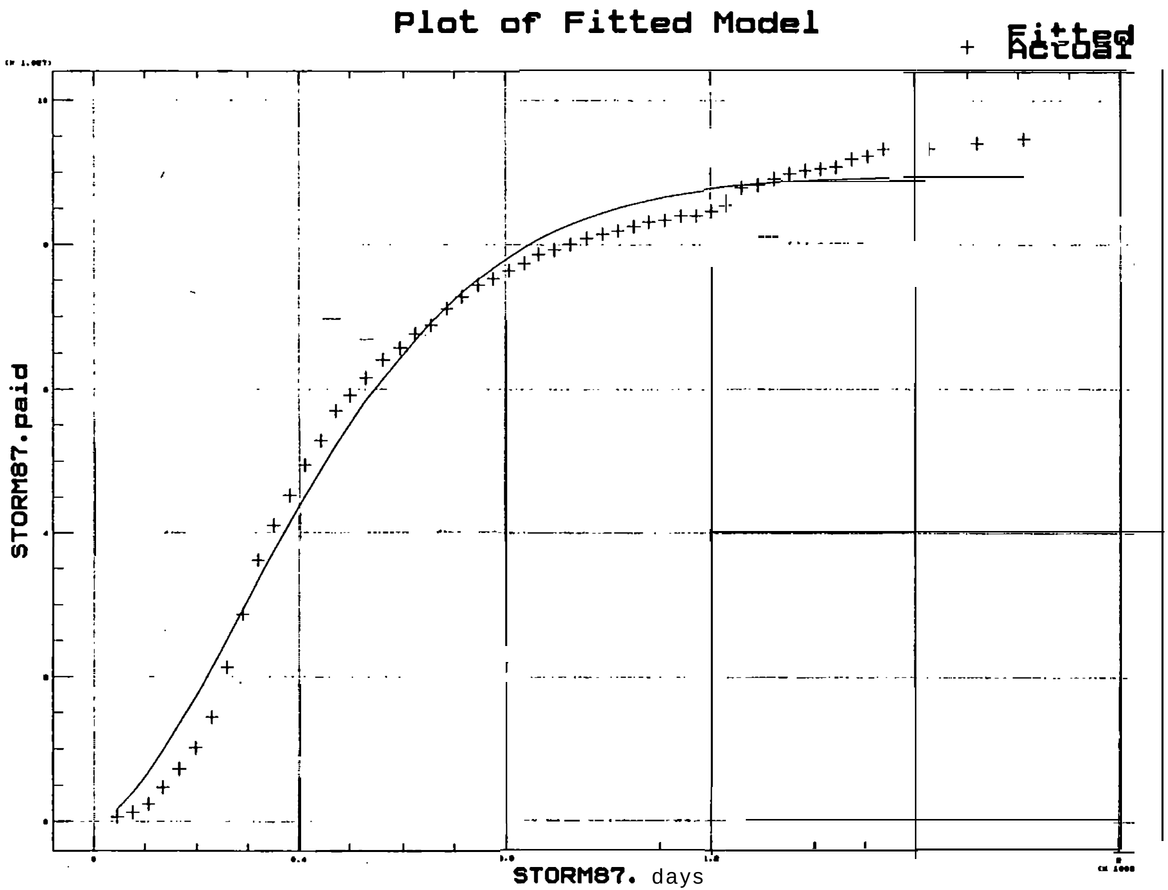
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Actual
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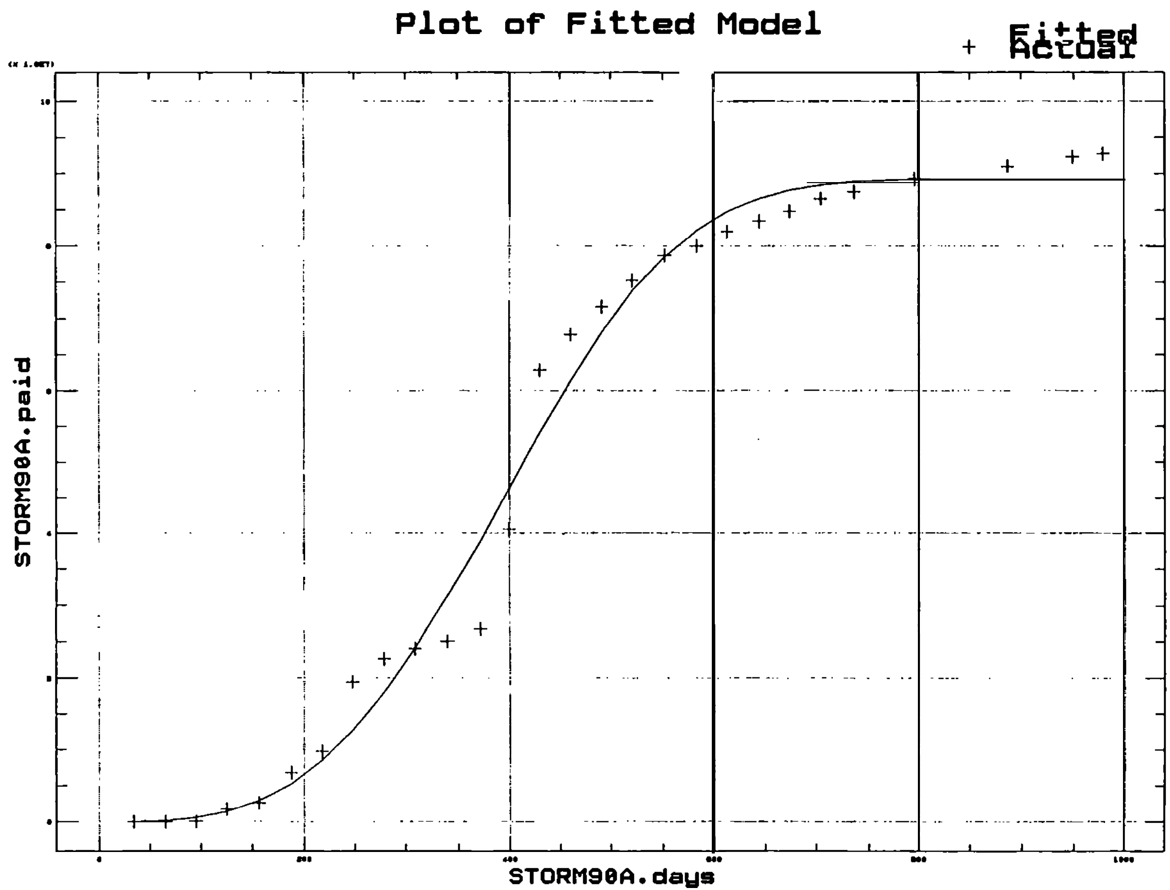
STORM87. days



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Fitted
Actual
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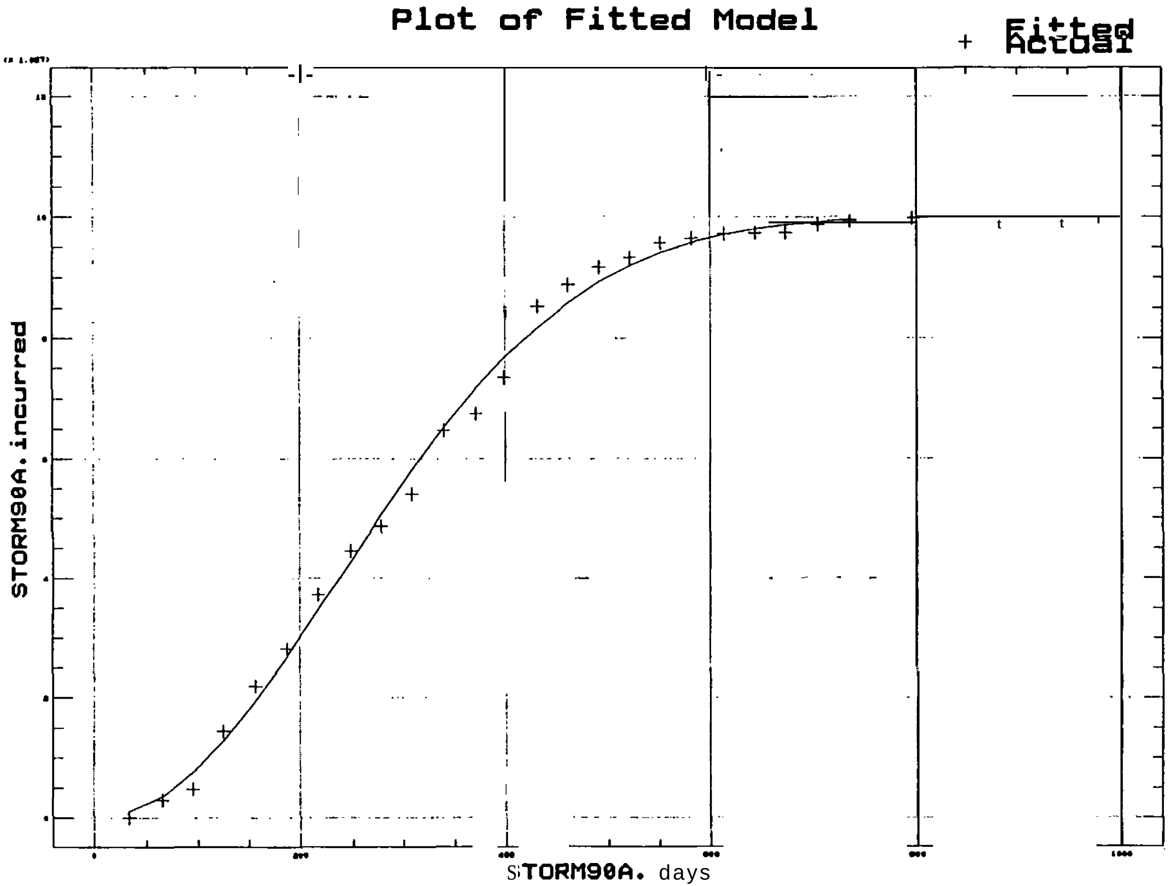
STORM90A.days



Plot of Fitted Model
Fitted
Actual
STORM90A.incurred
STORM90A. days

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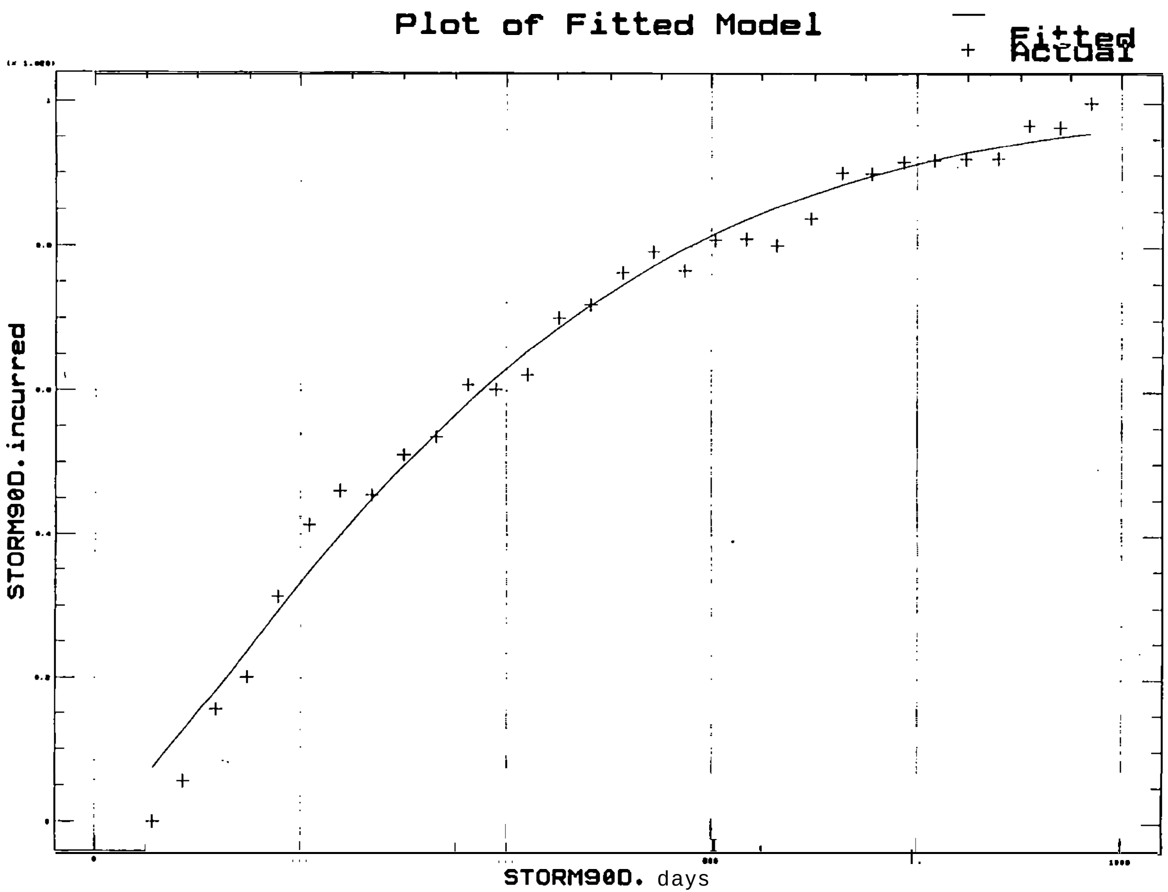
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Fitted
Actual
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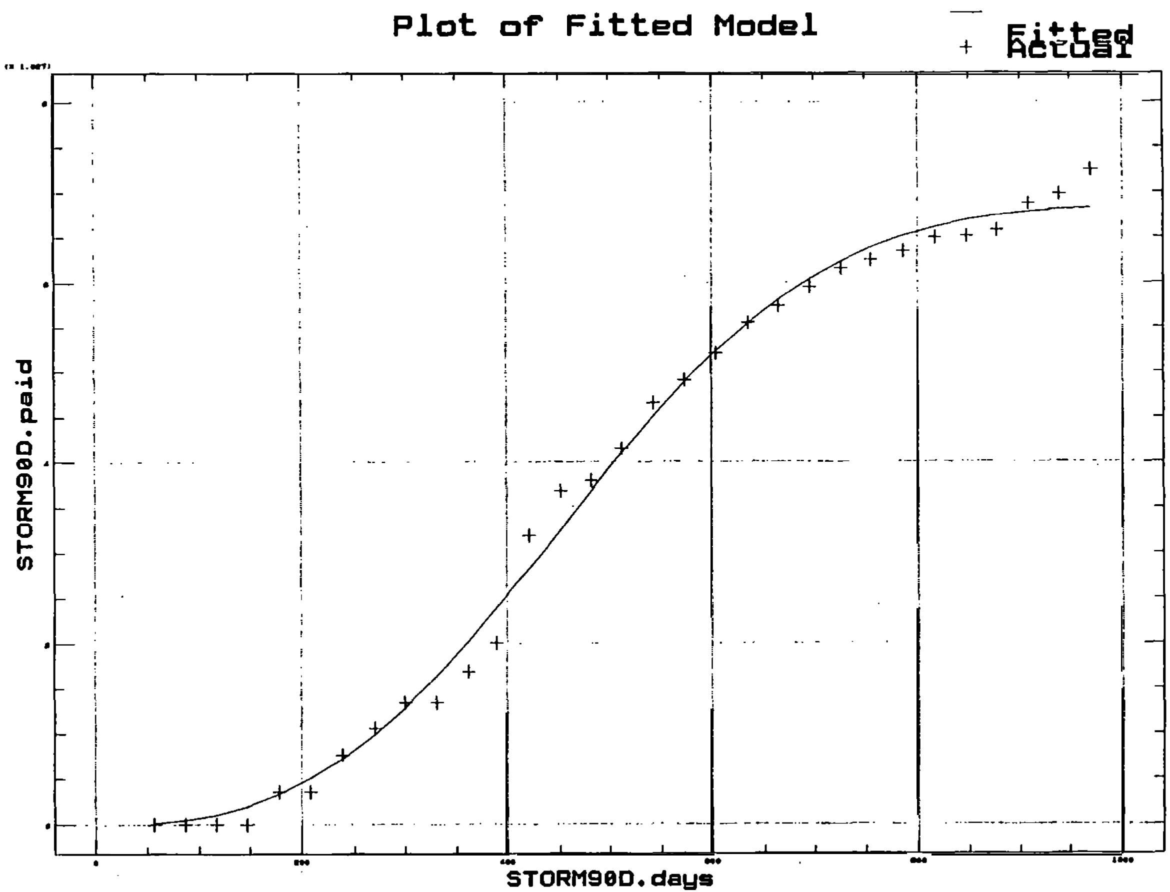
STORM90D. days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Fitted
Actual
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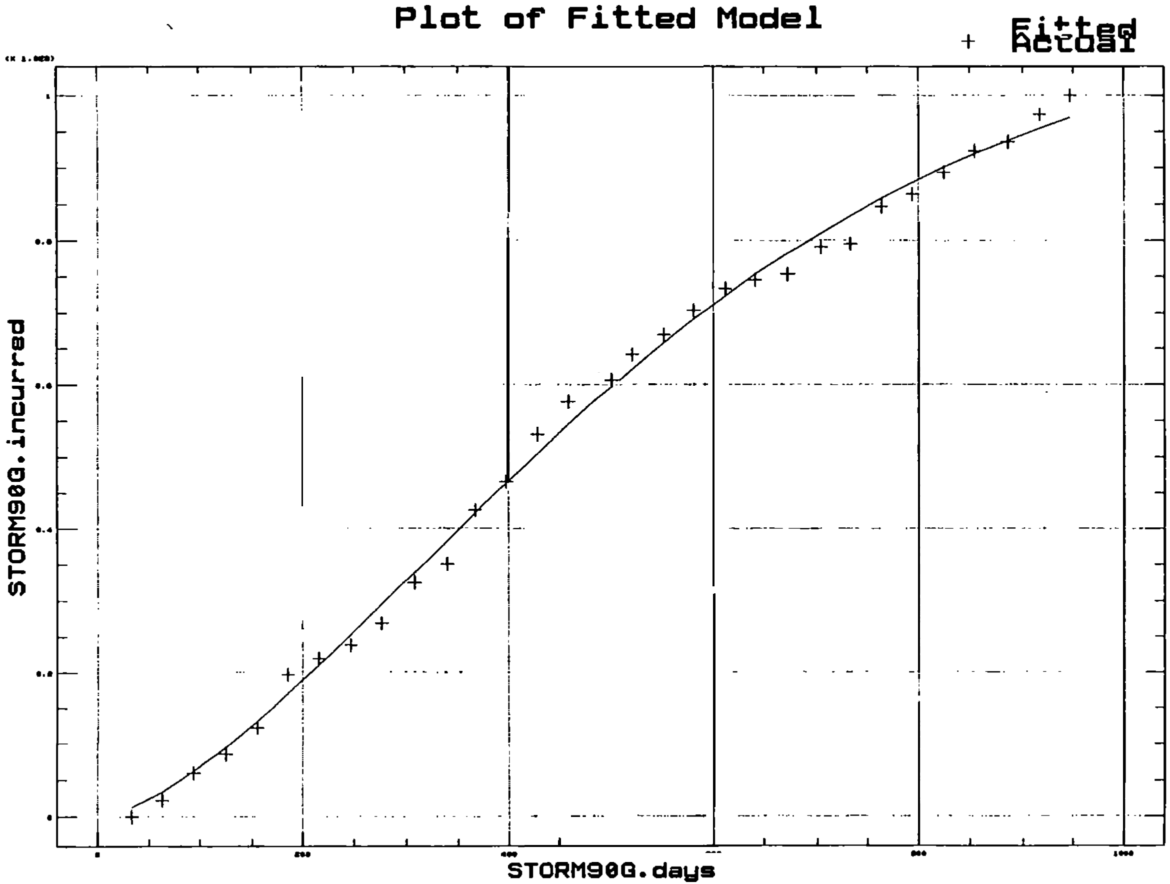

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Fitted
Actual
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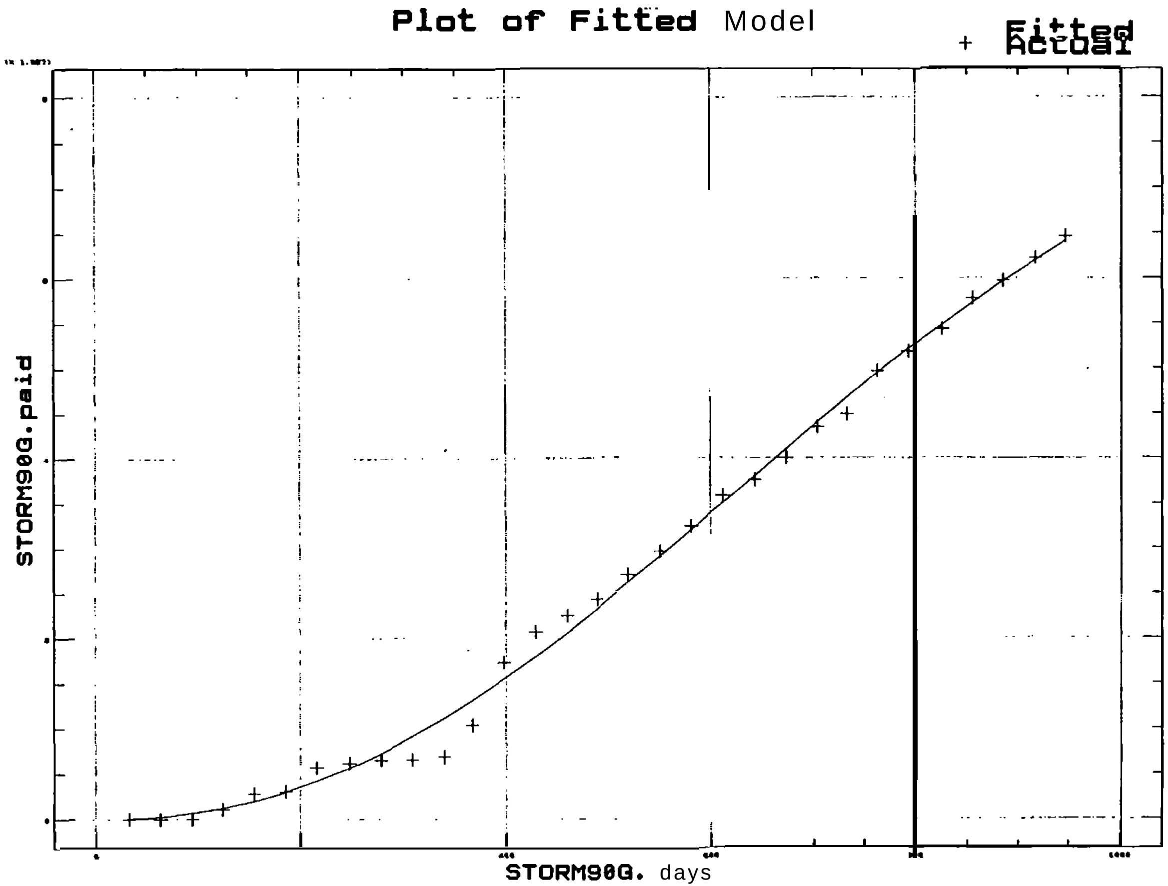
STORM90G.days


Plot of Fitted Model
Fitted
+ Actual
STORM90G.paid
STORM90G. days

SOME THOUGHTS ON THE FUTURE:

What type of losses should we look for in the future? There is a time bomb of potential losses out there, and I will try and give an indication of the magnitude:-

a) Meteorite Hit

These events are not rare. It is possible that once every 65 million years a meteorite large enough hits the earth and causes mass extinctions. A large meteor, big enough to devastate a substantial part of Europe is expected once every million years. We have no recent experience of such events. An underwriter said that they gave the cover for free!

b) Earthquake

The potential for "big ones" are:-

Tokyo - due any time.

Los Angeles

San Francisco/Hayward Fault

Central Europe - about one every 10,000 years

The Market has not had a significant earthquake in recent times. The Loma Prieta (San Francisco) earthquake insurance was largely retained in the US and very little found its way to London. A Tokyo earthquake on the scale of the one in 1923 is anticipated to cost $400 billion and reduce world GNP. The Japanese have insured for this event by buying assets outside Japan (e.g. Manhattan) and the realisation of these assets and the impact on the Yen are difficult to assess [see 12].

A Californian earthquake will not be as expensive, the main factor of loss being the wind speed and direction at the time and its effect on the fires. The maximum cost is of the order of $60 billion. California has tried to create an earthquake fund to finance this cost, but realised that the cost of payments would break the State if any event *should occur.*

A Central/North European earthquake would be devastating because construction standards do not take into account earthquake exposures.

c) Hurricanes

Saffir - Simpson Hurricane **Scale:-**

Index 0	Winds	less than 74 m.p.h.
Index 1	Winds	74-95 m.p.h.
Index 2	Winds	96-110 m.p.h.
Index 3	Winds	111-130 m.p.h.
Index 4	Winds	131-155 m.p.h.
Index 5	Winds	over 155 m.p.h.

All measurements are standard anemometer elevations.

Whilst the number of storms seems to be fairly consistent, the number of powerful Hurricanes and Windstorms has increased. On the graphs appended to this section I set out details on an annual basis, of the number of Storm and Hurricanes per annum over period of 120 and 105 years respectively. Details are found in [9]. These indicate a steady number of storms, but a cyclic frequency (80 year cycle) in Hurricanes. Local fluctuation could possibly be attributed to E1 Nino events.

We are seeing an increase in storm intensity. Hurricanes Hugo and Andrew were given Index 5 (although the Andrew damage seemed to indicate it was about Index 3.5). Index 5 storms are due to occur only once in 100 years. In the UK we have seen our once in 300 year storm twice in the past few years. The actual number of storms appear to be constant (see [8]). Is this the impact of Global Warming? Has the new volcanic dust from Mount Pinatoba affected weather for a short period - particularly as it came with an E1 Nino event. Have we been lucky? Certainly if Andrew had struck Florida 10 miles further North, the cost of the loss is estimated to have been $40 billion as opposed to the current estimate of $12 billion (and rising!).

The cost of such storm damage has been increased by two factors:-

(i) The inflationary value of property.

(ii) The population wishing to live in more exposed areas (e.g. sea fronts).

Buildings have been constructed to inadequate standards for the newer weather patterns' energy.

For more details see [7], [8] and [10].

Flood

If the Thames barrier fails, what would be the consequence?

If the Thames barrier doesn't fail, what happens to Essex?!

The Future

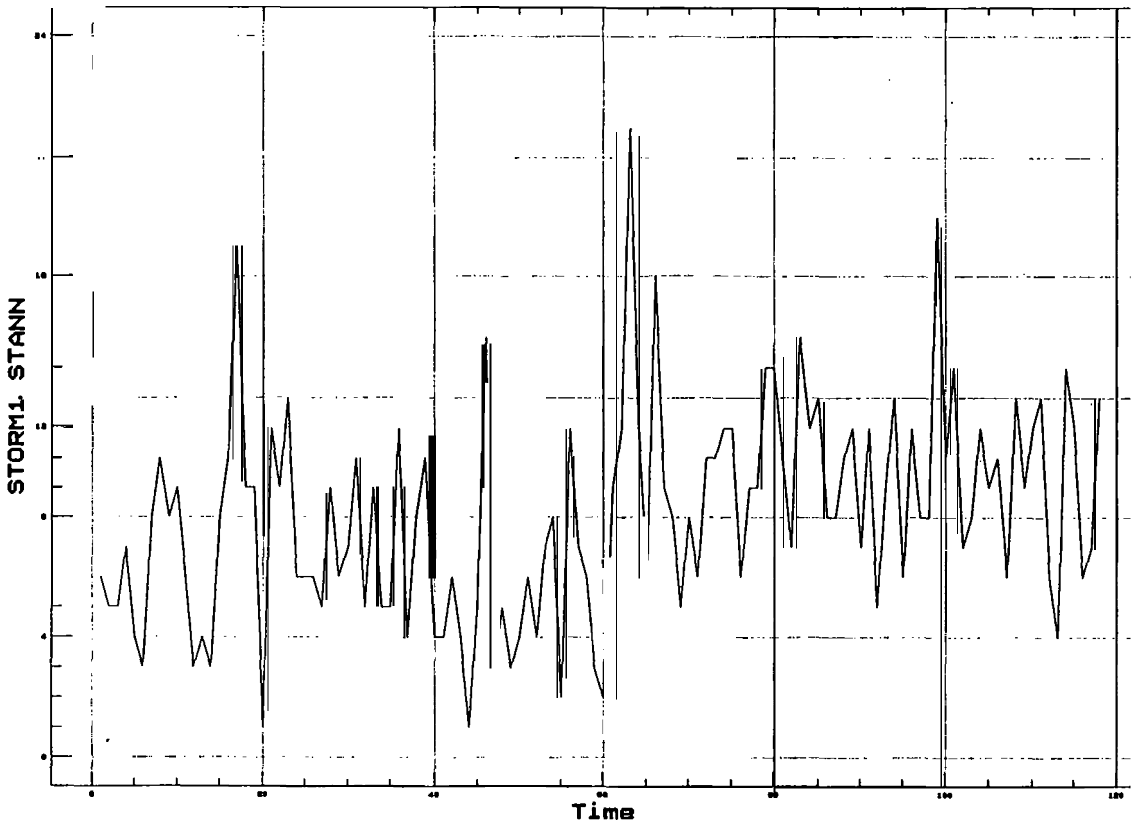
It is clear from the above that reserves need to be built out of current income to provide for the cost of these events. The Revenue puts the UK Market at a potential disadvantage to its European competitors by taxing such reserves.

CATXL is accordingly becoming more and more difficult to purchase. Alternative forms of insurance are being introduced to meet the shortfall. These fall into the stable of Financial (or Finite) Reinsurance. A classic example is a "spread loss" contact when losses from one event are spread forward over many years. Actuaries are becoming more involved with such contracts because of the need to get future cash flows correct to minimise loss. How long will it be before such contracts are traded and a "spread loss" spiral is created?

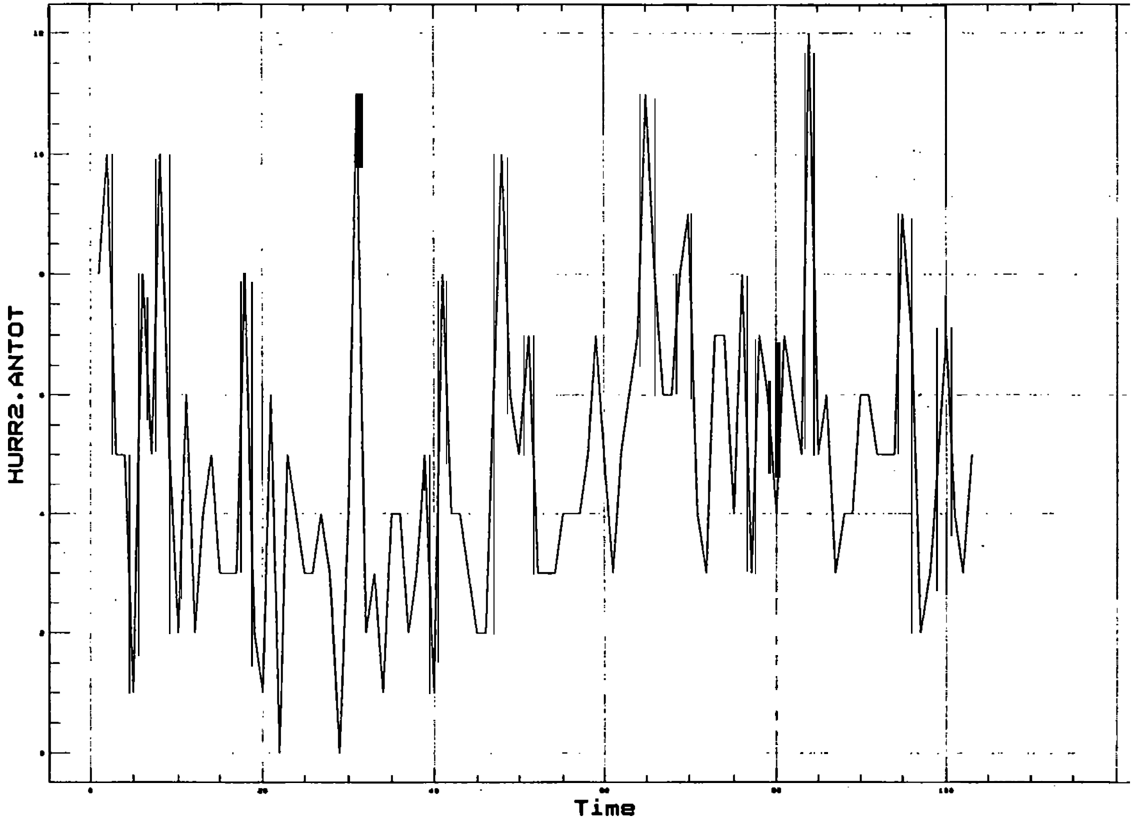
Other insurers are using quota share as a form of catastrophe cover. The Proportional Treaty Reinsurer is waking up to this.

Actuaries will become more involved with Catastrophe Reinsurance as a result of the new alternative.

Time Sequence Plot



Time Sequence Plot



CONCLUSIONS

The Catastrophe XL Market is one of the most interesting and stimulating markets open to Actuaries. This paper briefly touches the surface of many of the issues involved. The greater challenge is to find methods of managing the uncertainty and profitability of a market where demand exceeds supply, and where profits, though great, can be just as easily blown away with the wind.

I have kept this paper brief for two reasons. The first is a personal one in that I have no intention of giving all my secrets away. The second is to stimulate interest in the expanding role of the Actuary in Non-life Insurance.

Next time a major catastrophe event occurs, many UK insurers may be exposed to considerable loss. The challenge is to find methods of managing and funding for these potential losses. If the tile should fall today, the claim paid by the direct insurer is going to impact more substantially on the Profit and Loss Account. In addition, the cost to the individual can only increase as the impact of storm damage is felt by UK. insurers.

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LAMB

[11] "Nightmare on Lime Street".
C GUNN

[12] Sixty Seconds that will change the World - The Coming Tokyo Earthquake (Sidgwick & Jackson).
P HADFIELD

APPENDIX 1

A Slip

NO.	REF. NO
REGISTRATION 920653005	VAT % · TOC TRIBUNAL
D.T.I. CODE 4	REGISTRATION CATEGORY · YEAR · MONTH
REINSURED/ACCOUNT	ADJUST SCHEME · YES · X
COUNTRY OF ORIGIN U.S.A.	MARINE XP · NON-MARINE · AVIATION
US$ · C$ · NUS	OVERSEAS BROKER

CURRENCY	SIGNED LINE	GROSS PREMIUM
TOTAL		
LLOYD'S		
ILU		
PSAC		
COMPANIES		

AGENT/CLIENT

AGENT/ADDR CODE	REINSURED
CLASS CODE	W.P. ID

TOTAL WRITTEN ALL SLIPS · ORDER 41.557 · PREMIUM ENTRY

MICRO NO. DATE

X · Y · Z

L.P.S.O. Treaty No · I.L.U. Treaty No

225

REINSURED : XYZ Insurance Company

PERIOD : 12 months at 1st January, 1992
Losses occurring during basis.

TYPE : SECOND PROPERTY CATASTROPHE EXCESS OF LOSS REINSURANCE.

CLASS : All Property insurance and reinsurance, classified by the Reinsured as Fire, Allied Lines, Inland Marine, Homeowners, Automobile Physical Damage (excluding Collision) Multiple Peril and Casualty (with respect to Glass business only).

TERRITORIAL SCOPE : U.S.A. and/or Canada and/or . . original.

LIMIT : 95% of $25,000,000 each and every loss occurrence
IN EXCESS OF AN ULTIMATE NET LOSS OF
$27,500,000 each and every lam occurrence.

CO-REINSURANCE : 5% retained net by the Reinsured.

REINSTATEMENT : One reinstatement for all perils at ~~pro rata~~ 100% additional premium. as to time pro rata as to limit

PREMIUM : Deposit Premium $2.612.000 payable in four equal instalments in advance and adjustable at 0.825 % of the Reinsured's subject matter Gross Net Earned Premium Income.
Minimum Premium $2.089.000.

10/4/92.

DEDUCTIONS : Federal Excise Tax as required under applicable law or regulation and 15% Brokerage (Nil for Reinstatement).

LOSS RESERVE : Non-admitted Reinsurers hereon agree Letters of Credit (Citibank N.A. &/or Chase Scheme) In respect of known and reported losses only but Cash O.C.As. for Canadian Dollars, as required by the Reinsured (Excluding I.B.N.R.).

REC'D	SETT DUE DATE	
	31/3/92	30

120 adj

HEREON

WRITTEN LINES PERCENTAGES PART OF $25.000.000

SIGNED LINES PERCENTAGES PART OF

Underwriters agree to authorise L.P.S.O., P.S.A.C. and I.L.U. to sign book slips and to take down premium A.P.s and R.P.s and to settle claims and returns on a balance of account basis irrespective of discrepancies in the accounts subject to these being rectified in the next account.
Off slips and signing slips l.b.a. (if required) by L/UR only.
I.L.U. authorised to sign and seal treaty wording as agreed by leading I.L.U. Coy.

Subject otherwise to the terms of the C.C.S.A. 1980. Companies hereon agree that authorisation forms in respect of this policy shall not be required, it being understood that Closing instructions or addenda will be sent to Companies together with a form for return (if necessary) within 7 days of receipt showing any objection or change of administrative details, any amended reference to be quoted thereafter.

By signing this slip signatories to the C.C.S.A. 1980 authorise the Leading C.C.S.A. 1980 Company to arrange for P.S.A.C. to sign the Policy on its behalf and accept that such signing will be a valid signing for all the purposes of the C.C.S.A. 1980.

GENERAL CONDITIONS : Intermediary Clause.
Net Loss Clause.
Net Retained Lines Clause.
Excess of Loss Reinsurance Clause.
Loss Settlement and Notice Of Loss Clause.
Hours Clauses as per NMA 2244.
Service of Suit Clause (NMA 1998).
Arbitration Clause.
Access to Records Claude.
Insolvency Funds Exclusion.
Insolvency Clause as per wording.
Nuclear Incident Exclusion Clause(s) as per wording.
Pools, Associations and Syndicates Exclusion Clause.
War Exclusion.
Seepage and Pollution Exclusion as per wording.

WORDING : As expiring as far as applicable, to be agreed L/U only.

INFORMATION : As per submission of 22 pages dated 9.10.91.

INTERNAL ARRANGEMENTS

Reinsurers hereon agree to accept all special agreements and acceptance8 relating to this contract made prior to the inception of this slip.

Loss Reserve - Admitted Reinsurers hereon will provide Letters of Credit (Citibank N.A. Scheme) In respect of known and reported losses only but cash O.C.A.'s for Canadian Dollars, where they consider circumstances warrant It.

Lloyd's Underwriters hereon agree that claims are to be agreed by, leading Lloyd's Underwriter and LUNCO only, including LOCs as applicable.

Reinsurers hereon authorise LPSO/LIRMA to take down adjustments/reinstatements without sight of client account subject to checking prior to submission and rectifying any errors as soon as possible after discovery.

Loss advices: Where the reserve from the ground up on an individual loss is 50% of the retention or less, claim advices will not be shown to the market.

227

RCB
512

000 6 6 E 9 A X E X

WLW
307

A 8 5 5 9 2 8 3 3 1 5 D

1·5%

P 0 6 2 5 9 0 B

C4009	LIPMA	NC

Excluding LOC'S for IBNR

PER H.G.S. 15/11/91

ZURICH RE (UK)

2 6 2 0 0 3 9 0 3 7 9 2

Z4508	LIRMA	NCAD

EXCL. LOC'S/OCA'S, ICO I.B.N.R.

BFC
780

B 0 1 1 8 3 H B A 5 1 1 X

5.25

0·30%

DJM
606

0 9 M 0 1 2 4 6 6 1 0

THE TOA-RE (UK)

92 X 00 40

EXCLUDING LOC'S/OCA'S/IBNR

0·5%

SVT
1007

0·75%

F B 7 B B J 9 2 A 4 6 0

C. W. SPRECKLEY

EVERNDEN

PAR
1177

A 0 3 9 C 9 2 A 1 5 4

0·40%

A1

0·7% 0·50%

JHV
376

8 0 0 N 0 0 1 8 0 A LA

0·35% 0/A

LLOYD-ROBERTS

GLR
55

B A 1 9 0 J 9 2 A A C 1

3

0·25%

ROS 227

8 0 0 5 7 J C A A H 2 X

0·25%

C J 4 5 6 9 9 2 A 4 8 0

0·50%

DPM
435

15/11/91

6 0 3 2 3 2 0 6

NVB
1114

0·2%

0 3 4 1 0 C 9 2 A A N N

BURTON

10.12.90

R. E. BROWN & ORS

REB
702

0·70%

P 6 0 C U 9 2 X C 2 X C

Terra Nova

0 9 2 B P 0 7 1 6 7 T A
1 4 P 1 1 A 7 3 1

T3902	LIRMA

Ex LOC IBNR

2.18%

7 11 3 4 1 1 8 2 6 0 1 0

AFB

0.4% T 0 0 3 1 1 N 9 2 ANCC

EPI

$ 316,555,000

0.3 % EAGLE STAR

3 8 9 2 9 5 5 0 2 0 0

U S A U S 1 5 F 1 F X

E1102 LIRMA

MH

228

NO. 920653005

REINSURED

TYPE

SECOND PROP. CAT. X/L REINSURANCE

RENEWING		CLIENTS REF
~~910653005~~	BABCLM	
RELATED SLIPS	LESS 10%	
	ORDER 41.55 %	TOTAL WRITTEN HEREON
		NO. of SLIPS 1
BROKER DS / FGS.	C/N DATED	PHOTOGRAPH
REPORTS YES/NO		

FOR LPSO USE

FOR ILU USE

FOR PSAC USE

Which Stochastic Model is Underlying the Chain Ladder Method?

by Thomas Mack, Ph.D.

WHICH STOCHASTIC MODEL IS UNDERLYING THE CHAIN LADDER METHOD?

BY THOMAS MACK, PH.D., MUNICH RE

Editor's Note: This paper was presented to the XXIV ASTIN Colloquim, in Cambridge in 1993. Also, *this paper was awarded the first-ever CAS Charles A. Hachemeister Prize in November 1994.*

Abstract:

The usual chain ladder method is a deterministic claims reserving method. In the last years, a stochastic loglinear approximation to the chain ladder method has been used by several authors especially in order to quantify the variability of the estimated claims reserves. Although the reserves estimated by both methods are *clearly different, the loglinear approximation has been called "chain ladder," too, by these authors.*

In this note, we show that a different distribution-free stochastic model is underlying the chain ladder method; i.e. yields exactly the same claims reserves as the usual chain ladder method. Moreover, a comparison of this stochastic model with the above-mentioned loglinear approximation reveals that the two models rely on different philosophies on the claims process. Because of these fundamental differences the loglinear approximation deviates from the usual chain ladder method in a decisive way and should therefore not be called "chain ladder" any more.

Finally, in the appendix it is shown that the loglinear approximation is much more volatile than the usual chain ladder method.

1. *The usual deterministic chain ladder method*

Let C_{ik} denote the accumulated claims amount of accident year i, $1 \le i \le n$, either paid or incurred up to development year k, $1 \le k \le n$. The values of C_{ik} for $i + k \le n + 1$ are known to us (run-off triangle) and we want to estimate the values of C_{ik} for $i + k > n + 1$, in particular the ultimate claims amount C_{in} of each accident year $i = 2, ..., n$.

The chain ladder method consists of estimating the unknown amounts C_{ik}, $i + k > n + 1$, by

$$\hat{C}_{ik} = C_{i,n+1-i}\hat{f}_{n+1-i} \times ... \times \hat{f}_{k-1}, \quad i + k > n + 1, \tag{1}$$

where

$$\hat{f}_k = \sum_{j=1}^{n-k} C_{j,k+1} / \sum_{j=1}^{n-k} C_{jk}, \quad 1 \le k \le n-1. \tag{2}$$

For many years this has been used as a self-explaining deterministic algorithm which was not derived from a stochastic model. In order to quantify the variability of the estimated ultimate claims amounts, there have been several attempts to find a stochastic model underlying the chain ladder method. Some of these will be reviewed in the following chapter.

2. Some stochastic models related to the chain ladder method

In order to find a stochastic model underlying the chain ladder method we have to cast the central equation (1) of the chain ladder method into stochastic terms. One way of doing this runs along the following lines: We conclude from (1) that

$$\hat{C}_{i,k+1} = \hat{C}_{ik}\hat{f}_k, \quad k > n+1-i.$$

This is generalized to the stochastic model

$$E(C_{i,k+1}) = E(C_{ik})f_k, \quad 1 \le k \le n-1, \tag{3}$$

where all C_{ik} are considered to be random variables and $f_1, \ldots, f_{n-1}$ to be unknown parameters.

Introducing the incremental amounts

$$S_{ik} = C_{ik} - C_{i,k-1}, \quad 1 \le i, k \le n,$$

with the convention $C_{io} = 0$, one can show that model (3) is equivalent to the following model for S_{ik}:

$$E(S_{ik}) = x_i y_k, \quad 1 \le i, k \le n, \tag{4}$$

with unknown parameters x_i, $1 \le i \le n$, and y_k, $1 \le k \le n$, with $y_1 + \ldots + y_n = 1$.

Proof of the equivalence of (3) and (4):

(3)==> (4): Successive application of (3) yields

$$E(C_{in}) = E(C_{ik})f_k \times \ldots \times f_{n-1}$$

Because

$$E(S_{ik}) = E(C_{i,k}) - E(C_{i,k-1})$$

$$= E(C_{in})((f_k \times \ldots \times f_{n-1})^{-1} - (f_{k-1} \times \ldots \times f_{n-1})^{-1})$$

we obtain (4) by defining

$$x_i = \mathrm{E}(C_{in}), \quad 1 \le i \le n,$$

$$y_i = (f_1 \times \ldots \times f_{n-1})^{-1}$$

$$y_k = (f_k \times \ldots \times f_{n-1})^{-1} - (f_{k-1} \times \ldots \times f_{n-1})^{-1}, \quad 2 \le k \le n-1,$$

$$y_n = 1 - (f_{n-1})^{-1}.$$

This definition fulfills $y_1 + . + y_n = 1$.

(4) ==> (3): we have

$$\mathrm{E}(C_{ik}) = \mathrm{E}(S_{i1}) + \ldots + \mathrm{E}(S_{ik})$$

$$= x_i (y_1 + \quad + y_k)$$

and therefore

$$\frac{\mathrm{E}(C_{i,k+1})}{\mathrm{E}(C_{ik})} = \frac{y_1 + \ldots + y_k + y_{k+1}}{y_1 + \ldots + y_k} =: f_k, \quad 1 \le k \le n-1.$$

The stochastic model (4) clearly has $2n-1$ free parameters x_i, y_k. Due to the equivalence of (3) and (4) one concludes that also model (3) must have 2n $-$ 1 parameters. One immediately sees $n-1$ parameters $f_1, \ldots, f_{n-1}$. The other n parameters become visible if we look at the proof (3) ==> (4). It shows that the level of each accident year i, here measured by $x_i = E\,(C_{in})$, has to be considered a parameter, too.

Now, one additionally assumes that the variables S_{ik}, $1 \le i, k \le n$, arc independent. Then the parameters x_i, y_k of model (4) can be estimated (e.g. by the method of maximum likelihood) if we assume any distribution function for S_{ik}; e.g.. a one-parametric one with expected value $x_i y_k$ or a twoparametric one with the second parameter being constant over all cells (i,k). For example, we can take one of the following possibilities:

(4a) $$S_{ik} \propto \text{Normal}\ (x_i y_k, \sigma^2)$$

(4b) $$S_{ik} \propto \text{Exponential}\ (1/(x_i y_k))$$

(4c) $$S_{ik} \propto \text{Lognormal}\ (x_i + y_k, \sigma^2)$$

(Observe that (4a) and (4c) introduce even a further parameter σ^2). Possibility (4a) has been introduced into the literature by de Vylder 1978 using least squares estimation of the parameters. The fact that claims variables are usually skewed to the right is taken into account by possibilities (4b) and (4c) but at the price that all incremental variables S_{ik} must be positive (which is not the case with the original chain ladder method and often restricts the use of (4b) and (4c) to triangles of paid amounts).

Possibility (4b) has been used by Mack 1991. Possibility (4c) was introduced by Kremer 1982 and extended by Zehnwirth 1989 and 1991. Renshaw 1989, Christofides 1990. Verrall 1990 and 1991. It has the advantage that it leads to a linear model for $\log(S_{ik})$, namely to a two-way analysis of variance, and that the parameters can therefore be estimated using ordinary regression analysis.

Although model (4c) seems to be the most popular possibility of model class (4). we want to emphasize that it is only one of many different ways of stochastifying model (4). Moreover, possibilities (4a), (4b), (4c), yield different estimators for the parameters x_i, y_k, and for the claims reserves and all of these arc different from the result of the original chain ladder method. Therefore this author finds it to be misleading that in the papers by Zehnwirth 1989 and 1991, Renshaw 1989. Christofides 1990. Verrall 1990 and 1991 model (4c) explicitly or implicitly is called "*the* scholastic model underlying the chain ladder" or even directly "chain ladder model." In fact, it is something different. In order to not efface this difference, model (4c) should better Ix called "loglinear cross-classified claims reserving method." In the next chapter we show that this difference does not only rely on a different parametric assumption or on different estimators but stems from a different underlying philosophy.

3. A distribution-free stochastic model for the original chain ladder method

The stochastic models (4a). (4b), (4c) described in the last chapter did not lead us to a model which yields the same reserve formula as the original chain ladder method. But we will now develop such a model.

If we compare model (3) with the chain ladder projection (1), we may get the impression that the transition

(A) $$\hat{C}_{i,n+2-i} = C_{i,n+1-i}\hat{f}_{n+1-i}$$

in (1) from the most recent observed amount $C_{i,n+1-i}$ to the estimator for the first unknown amount $C_{i,n+2-i}$ has not been captured very well by model (3) which uses

(B) $$\hat{C}_{i,n+2-i} = E(C_{i,n+1-i})\, f_{n+1-i}.$$

The crucial difference between (A) and (B) is the fact that (A) uses the actual observation $C_{i,n+1-i}$ itself as basis for the projection whereas (B) takes its expected value. This means that the chain ladder method implicitly must use an assumption which states that the information contained in the most recent observation $C_{i,n+1-i}$ is more relevant than that of the average $E(C_{i,n+1-i})$ This is duly taken into account by the model

(5) $$E(C_{i,k+1}|C_{i1},\dots,C_{ik}) = C_{ik}\, f_k, \quad 1 \le i \le n, \quad 1 \le k \le n-1$$

which is (due to the iterative rule for expectations) more restrictive than (3). Moreover, using (5) we ate able to calculate the conditional expectation $E(C_{ik}|D)$, $i + k > n + 1$, given the data

$$D = \{C_{ik} \mid i + k \leq n + 1\}$$

observed so far, and knowing this conditional expectation is more useful than knowing the unconditional expectation $E(C_{ik})$ which ignores the observation D. Finally, the following theorem shows that using (5) we additionally need only to assume the independence of the accident years, i.e. to assume that

$$\{C_{i1}, \ldots, C_{in}\}, \{C_{j1}, \ldots, C_{jn}\}, i \neq j, \tag{6}$$

are independent, whereas under (4a), (4b). (4c) we had to assume the independence of both. the accident years and the development year increments.

Theorem: Under assumptions (5) and (6) we have fork $> n + 1 - i$

$$E(C_{ik} \mid D) = C_{i,n+1-i} f_{n+1-i} \times \ldots \times f_{k-1}. \tag{7}$$

Proof: Using the abbreviation

$$E_i(X) = E(X \mid C_{i1}, \ldots, C_{i,n+1-i})$$

we have due to (6) and by repeated application of (5)

$$E(C_{ik}|D) = E_i(C_{ik})$$

$$= E_i(E(C_{ik}|C_{i1}, \ldots, C_{i,k-1}))$$

$$= E_i(C_{i,k-1}) f_{k-1}$$

$$= \text{etc.}$$

$$= E_i(C_{i,n+2-i}) f_{n+2-i} \times \ldots \times f_{k-1}$$

$$= C_{i,n+1-i} f_{n+1-i} \times \ldots \times f_{k-1}.$$

The theorem shows that the stochastic model (5) produces exactly the same reserves as the original chain ladder method if we estimate the model parameters f_k by (2). Moreover. wc see that the projection basis $C_{i,n+1-i}$ in formulae (7) and (1) is not an estimator of the parameter $E(C_{i,n+1-i})$ but stems from working on condition of the data observed so far. Altogcthcr. model (5) employs only n-l parameters $f_1, \ldots, f_{n-1}$. The price for having less parameters than models (3) or (4) is the fact that in model (5) we do not have a good estimator for $E(C_{in})$ which are the additional parameters of models (3) and (4).

But even models (4) do not use $E(C_{in})$ as estimator for the ultimate claims amount because this would not be meaningful in view of the fact that the knowledge of $E(C_{1n})$ is completely useless (because we already know C_{1n} exactly) and that one might have $E(C_{in}) < C_{i,n+1-i}$ (e.g. for $i = 2$) which would lead to a negative claims reserve even if that is not possible. Instead models (4) estimate the ultimate claims amount by estimating

$$C_{i,n+1-i} + E(S_{i,n+2-i} + \dots + S_{in}),$$

i.e. they estimate the claims reserve $R_i = C_{in} - C_{i,n+1-i} = S_{i,n+2-i} + + S_{in}$ by estimating

$$E(R_i) = E(S_{i,n+2-i} + \dots + S_{in}).$$

If we assume that we know the true parameters x_i, y_k of model (4) and f_k of model (5). we can clarify the essential difference between both models in the following way: The claims reserve for model (4) would then be

$$E(R_i) = x_i(y_{n+2-i} + \dots + y_n)$$

independently of the observed data D, i.e. it will not change if we simulate different data sets D from the underlying distribution. On the other hand, due to the above theorem, model (5) will each time yield a different claims reserve

$$E(R_i \mid D) = C_{i,n+1-i}\ (f_{n+1-i} \times \dots \times f_{n-1} - 1)$$

as $C_{i,n+1-i}$ changes from one simulation to the next.

For the practice, this means that we should use the chain ladder method (1) or (5) if we believe that the deviation

$$C_{i,n+1-i} - E(C_{i,n+1-i})$$

is indicative for the future development of the claims. If not, we can think on applying a model (4) although doubling the number of parameters is a high price and may lead to high instability of the estimated reserves as is shown in the appendix.

4. Final Remark

The aim of this note was to show that the loglinear cross-classified model (4c) used by Renshaw. Christofides. Verrall and Zehnwirth is *not* a model underlying the usual chain ladder method because it requires independent and strictly positive increments and produces different reserves. We have also shown that model (5) *is* a stochastic model underlying the chain ladder method. Moreover, model (5) has only $n - 1$ parameters-as opposed to $2n - 1$ (or even $2n$) in case of model (4c)—and is therefore more robust than model (4c).

Finally. one might argue that one advantage of the loglinear model (4c) is the factthatit allows to calculate the standard errors of the reserve estimators as has been done by Renshaw 1989. Christofides 1990 and Verrall 1991. But this is possible for model (5). too, as is shown in a separate paper (Mack 1993).

Acknowledgement

I first saw the decisive idea to base the stochastic model for the chain ladder method on conditional expectations in Schnieper 1991.

APPENDIX

NUMERAL EXAMPLE WHICH SHOWS THAT THE LOGLINEAR MODEL (4C) IS MORE VOLATILE THAN THE USUAL CHAIN LADDER METHOD

The data for the following example are taken from the "Historical Loss Development Study," 1991 Edition, published by the Reinsurance Association of America (RAA). There, we fmd on page 96 the following run-off triangle of Automatic Facultative business in General Liability (excluding Asbestos & Environmental):

	C_{i1}	C_{i2}	C_{i3}	C_{i4}	C_{i5}	C_{i6}	C_{i7}	C_{i8}	C_{i9}	GO
$i=1$	5012	8269	10907	11805	13539	16181	18009	18608	18662	18834
$i=2$	106	4285	5396	10666	13782	15599	15496	16169	16704	
$i=3$	3410	8992	I3873	16141	18735	22214	22863	23466		
$i=4$	5655	11555	15766	21266	23425	26083	27067			
$i=5$	1092	9565	15836	22169	25955	26180				
$i=6$	1513	6445	11702	12935	15852					
$i=7$	557	4020	10946	12314						
$i=8$	1351	6947	13112							
$i=9$	3133	5395								
$i=10$	2063									

The above figures are cumulative incurrcd case losses in $ 1000. Wc have taken the accident years from 1981 ($i=1$) to 1990 ($i=10$). The following table shows the corrcsponding incremental amounts $S_{ik}=C_{ik}-C_{i,k-1}$:

	S_{i1}	S_{i2}	S_{i3}	S_{i4}	S_{i5}	S_{i6}	S_{i7}	S_{i8}	S_{i9}	S_{i10}
$i=1$	5012	3257	2638	898	1734	2642	1828	599	54	172
$i=2$	106	4179	1111	5270	3116	1817	-103	673	535	
$i=3$	3410	5582	488 I	2268	2594	3479	649	603		
$i=4$	5655	5900	4211	5500	2159	2658	984			
$i=5$	1092	8473	6271	6333	3786	225				
$i=6$	1513	4932	5257	1233	2917					
$i=7$	557	3463	6926	1368						
$i=8$	1351	5596	6165							
$i=9$	3133	2262								
$i=10$	2063									

Note that in development year 7 of accident year 2 we have a negative increment $S_{2,7}=C_{2,7}-C_{2,6}=-103$. Because model (4c) works with logarithms of the incremental amounts S_{ik} it cannot handle the negative increments $S_{2,7}$. In order to apply model (4c), we therefore must change $S_{2,7}$ artificially or leave it out. We have tried the following possibilities:

(a) $S_{2,7} = 1$, i.e. $C_{2,7} = 15496 + 104 = 15600$, $C_{2,8} = 16169 + 104$

$= 16273$, $C_{2,9} = 16704 + 104 = 16808$

(b_1) $C_{2,7} = 16000$, i.e. $S_{2,7} = 401$, $S_{2,8} = 169$

(b_2) $S_{2,7}$ = missing value, i.e. $C_{2,7}$ = missing value

When estimating the reserves for these possibilities and looking at the residuals for model (4c), we will identify $S_{2,1} = C_{2,1} = 106$ as an outlier. We have therefore also tried:

C_1 like (b_1) but additionally $S_{2,1} = C_{2,1} = 1500$, i.e. all $C_{2,k}$ are augmentedby $1500 - 106 = 1394$

C_2 like (b_2) butadditionally $S_{2,1} = C_{2,1}$ = missing value.

This yields the following results (the calculations for model (4c) were done using Ben Zehnwirth's ICRFS, version 6.1):

	Total Estimated Reserves	
Possibility	Chain Ladder	Loglinear Model (4C)
unchanged data	52,135	not possible
(a)	52,274	190,754
(b_1)	51,523	102,065
(b_2)	52,963	107,354
(c_1)	49,720	69,999
(c_2)	51,834	70,032

This comparison clearly shows that the two methods are completely different and that the usual chain ladder method is much less volatile than the loglinear cross-classified method (4c).

For the sake of completeness, the following two tables give the results for the above calculations per accident year:

CHAIN LADDER METHOD—ESTIMATED RESERVES PER ACCIDENT YEAR

Acc. Year	Unchanged	(a)	(b_1)	(b_2)	(c_1)	(c_2)
1981	0	0	0	0	0	0
1982	154	155	154	154	167	154
1983	617	616	617	617	602	617
1984	1,636	1,633	1.382	1,529	I ,348	1529
1985	2,747	2.780	2,664	2.964	2.606	2.964
1986	3.649	3.671	3593	3.795	3,526	3.795
1987	5.435	5,455	5.384	5568	5,286	5,568
1988	10.907	10,935	10.838	11,087	10.622	11,087
1989	10.650	10.668	10.604	10,770	10,322	10.770
1990	16.339	16,360	16,287	16,477	IS,242	15,349
1981-90	52.135	52374	51523	52.963	49.720	51,834

LOGLINEAR METHOD—ESTIMATED RESERVES PER ACCIDENT YEAR

Acc. Year	(a)	(b_1)	(b_2)	(c_1)	(c_2)
1981	0	0	0	0	0
1982	309	249	313	282	387
1983	2.088	949	893	749	674
1984	6.114	2,139	2.683	I.675	1.993
1985	3.773	2,649	3.286	2,086	2.602
1986	6.917	4.658	5,263	3,684	4,097
1987	9.648	6,312	6.780	4.968	5.188
1988	24.790	15,648	16.468	12,000	12.174
1989	36.374	21.429	22.213	15,545	15.343
1990	100,739	48,033	49,454	29,010	27,575
1981-90	190.754	102.065	107.354	69,999	70.032

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Using Expected Loss Ratios in Reserving

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Using expected loss ratios in reserving

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Abstract: The required loss reserve for a recent time period is estimated by using the recent loss experience plus two probability distributions. One distribution is of ultimate losses for the recent period, based on prior experience and rate adequacy changes. The other distribution is of the ratio of the estimator based on recent experience to the true ultimate loss.

Keywords: Loss reserving; Expected loss ratio.

1. Introduction

This paper presents a method of using expected loss ratios, together with prior and posterior distributions, in order to estimate loss reserves. This Bayesian method is especially useful for recent accident years and for lines of business with slow development. It incorporates, in a rigorous way, the degree of reliability of the expected loss ratio and of the loss development factors. Estimates of ultimate loss ratios for recent accident years can be important factors in underwriting decisions.

A method of using expected loss ratios which is now well-known was presented by Bornhuetter and Ferguson (1972). The ultimate losses of an accident year are estimated by using the prior expectation of ultimate losses (expected losses) as well as the reported losses and the selected development factor to ultimate. The ultimate losses are estimated as

$$\text{reported losses} + (1 - z)(\text{expected lows}), \quad (1)$$

where z is the reciprocal of the development factor to ultimate.

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It is implicit in this method of estimation that the expecred development for an accident year in each future year is independent of the reported losses.

If 'developed losses' is defined as the product of the reported losses and the development factor to ultimate, then formula (1) can be expressed as

$$z(\text{developed losses}) + (1 - z)(\text{expected losses}). \quad (2)$$

Bornhuetter–Ferguson and Bayesian estimates of loss reserves will be compared in an example later in this paper.

2. The model

In a Bayesian approach, the prior expectation of ultimate losses for an exposure period E may be an estimate made several years after the beginning of E. If ultimate loss ratios are estimated for the same line of business for the insurer for previous periods, and industry-wide data as well as the insurer's changes in premium adequacy are taken into account, an estimate of the ultimate loss ratio for the period E can be made prior to considering the reported losses for E.

The following direct application of Bayes' theorem is basic to this discussion. Let $f(x)$ be the probability density function of the distribution of ultimate losses for exposure period E prior to considering the losses for E. Let $g(y \mid x)$ be the probability density function of the distribution of y, the developed losses defined previously, for E as of t months, given that the ultimate losses are x. Assume that this distribution has mean x. Let $h(x \mid y)$ be the probability density function of the distribution of the ultimate losses given that the developed losses are y. Then

$$h(x|y) = g(y|x)f(x) \Big/ \int_0^\infty g(y|x)f(x)\,dx. \quad (3)$$

In order to use the above proposition, it is necessary to estimate $g(y \mid x)$ and f(x). The mean of the distribution given by $h(x \mid y)$ will be the estimate of ultimate losses.

The variance of the distribution given by $g(y \mid x)$ can be estimated from a study of the historical variability of developed loss ratios at different stages of development. The variance of the distribution given by $f(x)$ can be estimated from the differences between prior expectations of ultimate losses for previous periods, based on the current method of predicting, and the latest developed losses for those periods. The estimated variances between the latest developed losses and the ultimate losses for those periods will also be considered. Historical data of the above types should be supplemented by judgement, experience, and related data.

If a method other than development factors is used for projecting the loss data to ultimate, Bayes' theorem can still be applied as above with $g(y \mid x)$ defined as necessary.

In order to apply Bayes' theorem to a set of accident years, a single development factor to ultimate for the period can be selected as follows. Estimate the ratios between the ultimate losses for each accident year by using the premium and the estimated relative rate adequacy for each year. Then use the reciprocal of the development factor for each year to estimate the ratio of the total ultimate losses for the period to the expected losses for the period at the stage of development. See Biihlmann's Cape Cod method [Schnieper (1991), Straub (1988)].

Biihlmann's (1967) formula for the least squares line estimate of the Bayesian estimates could be used to estimate the credibility of the actual developed losses. [This credibility approximation is exact Bayesian in certain useful cases. In the proof of formula (4), below, we use a special case of Jewell's result that credible means are exact Bayesian for exponential families. See Jewell (1974, 1975).] This method has the advantage of simplicity since 'it does not require the choice of particular distributions.

3. Lognormal distributions

Let $f(x)$, $g(y \mid x)$, and $h(x \mid y)$ be defined as for formula (3). For certain choices of $f(x)$ and $g(y \mid x)$, an explicit formula for the mean of $h(x \mid y)$ is known. An important example is the case in which $f(x)$ and $g(y \mid x)$ represent lognormal distributions. This is a reasonably good fit in many cases.

Suppose that the prior probability distribution of logs of ultimate losses has mean μ and variance ν^2. Suppose that for all x, the distribution, given ultimate losses x, of logs of actual developed losses has variance σ^2. Note that if x is the mean of a lognormal distribution and m and s^2 are the mean and variance of the distribution of the logs, then $\log x = m + s^2/2$. Therefore, for all x the distribution of logs of actual developed losses has mean $\log x - \sigma^2/2$. Then the mean of the distribution given by $h(x \mid y)$ (and thus the estimate of ultimate losses) is

$$\exp(\mu_1 + \nu_1^2/2), \tag{4}$$

where

$$\mu_1 = (1-z)\mu + z(\log y + \sigma^2/2), \tag{5}$$

$$\nu_1^2 = \sigma^2 z, \tag{6}$$

$$z = \nu^2/(\sigma^2 + \nu^2). \tag{7}$$

The derivation is given in the appendix.

Example. Assume that, based on historical experience as described previously, the prior distribution for an insurer's overall ultimate loss ratio for 1987–91 for medical malpractice has a mean of 0.90 (i.e. 90%) and a variance of 0.16. Suppose the selected development factor to ultimate for 1987-91 reported losses as of 12/31/91 is 2.065 and the probability distribution for the ratio of the developed losses to the ultimate losses has a variance of 0.075.

If both of the above distributions are lognormal, then μ, ν^2 and σ^2 in equations (5) and (6) can be found by solving the following equations for the mean and variance of lognormal distributions:

$$0.90 = \exp(\mu + \nu^2/2), \tag{8}$$

$$0.16 = \exp(2\mu + \nu^2)(\exp(\nu^2) - 1), \tag{9}$$

$$1.00 = \exp(m + \sigma^2/2), \tag{10}$$

$$0.075 = \exp(2m + \sigma^2)(\exp(\sigma^2) - 1). \tag{11}$$

By squaring both sides of equation (8) and then dividing by the corresponding sides of equation (9), we get

$$(0.90)^2/0.16 = 1/(\exp(\nu^2) - 1). \tag{12}$$

Table 1
Comparison of methods of estimation.

Actual developed loss ratio	Bayesian estimate of ultimate loss ratio	Bornhuetter–Ferguson estimate of ultimate loss ratio
20%	32%	56%
40%	52%	66%
80%	85%	85%
160%	139%	124%
320%	229%	201%

Solving for ν^2 and μ is then immediate. The same method can be used for σ^2 and m. The solutions are 0.180, -0.195, and 0.072, respectively, for ν^2, μ, and σ^2, so formula 4 becomes $\exp(-0.004 + 0.714 \log y)$. So, if y = 20%. for example, the estimated ultimate loss ratio is 32%. Table 1 compares three methods of estimation.

Appendix: Derivation of formula (4)

The following lemma will be used.

Lemma. *Suppose that an element is chosen at random from a normal distribution for which the value of the mean θ is unknown $(-\infty < \theta < \infty)$ and the value of the variance σ^2 is known $(\sigma^2 > 0)$. Suppose also that the prior distribution of θ is a normal distribution with given values of the mean μ and the variance ν^2. Then the posterior distribution of θ, given that the element chosen equals x_1, is a normal distribution for which the mean μ_1 and the variance ν_1^2 are as follows:*

$$\mu_1 = (\sigma^2\mu + \nu^2 x_1)/(\sigma^2 + \nu^2), \tag{A.1}$$

$$\nu_1^2 = (\sigma^2\nu^2)/(\sigma^2 + \nu^2). \tag{A.2}$$

See DeGroot (1986) for the-proof of the above.

Proof of formula (4). The mean and variance of the distribution, given ultimate losses x, of $\sigma^2/2$ + log(developed losses). are $\log x$ and σ^2, respectively. The prior distribution of log(ultimate losses) has mean μ and variance ν^2. Therefore, the posterior distribution of log(ultimate losses), given $\sigma^2/2$ + log(developed losses) = x_1, has mean μ_1 and variance ν_1^2 given in the Lemma, where $x_1 = \sigma^2/2$ + log(developed losses). Therefore, the distribution of ultimate losses has mean $\exp(\mu_1 + \nu_1^2/2)$.

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Expected Loss Development: A Shift in Credibility

Christopher J. Poteet

Expected Loss Development: A Shift in Credibility

Christopher J. Poteet

This paper is a commentary on the previously published paper "Partial Loss Development Based On Expected Losses For Workers' Compensation Class Ratemaking", Casualty Actuarial Society Forum. Special Edition. 1993 Ratemaking Call Papers.

This paper shows that expected loss development is equivalent to adjusting the full credibility standard and applying credibility by policy period.

Expected Loss Development: A Shift in Credibility

Concerns with the current loss development method used in Workers' Compensation class ratemaking have been raised. If a class has zero losses at a first report, using a first to ultimate loss development factor produces zero ultimate losses as well. One possible solution that has been proposed is to use expected loss development. To simplify the illustration, assume that all losses are at the same benefit level etc., so as to only look at loss development. The other factors can easily be taken into account later. Also for simplicity assume that there is only one policy period used and national pure premiums are not used. The following arguments will then be extended to include more policy periods and the use of national pure premiums.

Workers' compensation classification ratemaking relies on several estimates of class pure premiums. One estimate is based on the latest available data for the class and state. This is called the indicated pure premium. Another estimate is the pure premium underlying current rates brought up to the level of the indicated pure premiums. This estimate is called the present on rate level pure premium. A third estimate is a national pure premium which includes data from other states adjusted to reflect conditions in the reviewed state. A formula pure premium to be used in calculating rates, is obtained by credibility weighting these estimates.

Here is a brief description of expected loss development. Initially, expected losses E (present on rate level pure premium times payroll in hundreds) is the estimate of ultimate losses used to calculate the indicated pure premium. At a first report the actual losses A which have emerged at that point can replace the losses that were expected to have emerged by then, namely (1/D)E, where D is the first to ultimate loss development factor. This method relies less on actual losses and more on expected losses than the current method. It is important to note that if the development factor is less than one, the estimate of ultimate, losses might be negative.

Credibility weighting produces the losses used in the formula pure premium:

Expected Loss Development: Losses used in Formula Pure Premium

$$Z[A+(1-\frac{1}{D})E] + (1-Z)E$$

$$= ZA + ZE - \frac{Z}{D}E + E - ZE$$

$$= ZA - \frac{Z}{D}E + E$$

$$= \frac{Z}{D}AD + (1-\frac{Z}{D})E$$

Current Method: Losses used in Formula Pure Premium

$$ZAD + (1-Z)E$$

These two formulas are equivalent where Z/D is substituted for Z. Using Z/D instead of Z is equivalent to changing the full credibility standard which already limits fluctuations of formula pure premiums to a desired amount. For example. if $Z=(n/n_f)^{1/2}$ and $D=3$, then $Z/D=(n/9n_f)^{1/2}$. The expected loss development method implicitly lowers credibility by $1/D$, when D> 1. Expected loss development is a shift in credibility, giving less weight to actual losses and more weight to expected losses.

The equation which shows that expected loss development is equivalent to changing the full credibility standard can be expanded to include more policy periods and the use of national pure premiums. The relationship holds if the credibility of indicated data is calculated by policy period and the national credibility is allowed to remain unchanged as one switches from one method to the other.

Attached is a detailed algebraic proof of the equivalence relationship (Attachment 1). The proof shows that the serious (or nonserious or medical) formula pure premium calculated using expected loss development is equal to the serious (or nonserious or medical) formula pure premium calculated by using credibility by policy period. where the credibility one would normally use is divided by the policy period's development to ultimate factor and multiplied by a factor reflecting the contribution of the policy period's exposure to the total. These individual credibilities are then used as weights for the indicted pure premiums calculated separately for each individual policy period.

Also attached is a specific illustration (Attachment 2) of the equivalence relationship which uses the example from exhibit 1 of the paper "Partial Loss Development Based On Expected Losses For Workers' Compensation Class Ratemaking". Casualty Actuarial Society Forum. Special Edition. 1993 Ratemakine Call Papers, as well as the development factors listed in the paper on page 321 (See attachment 3). Note that, as a separate issue, the state credibilities in the paper are calculated using a square root rule instead of NCCI's old two thirds rule so that the serious state credibility of .67 is equal to .59 to the three fourths power $[.67=(.59^{3/2})^{1/2}]$.

The illustration focuses on the calculation of the serious formula pure premium. More recent years have higher development factors so credibility is lowered more for them. This could be considered a reliability factor. Each year's credibility also gets multiplied by a weight equal to the year's proportion of exposure to the total of all years. This could be considered a relevance factor since more recent years would tend to have higher exposures due to wage inflation, all else being constant.

Expected loss development can be thought of as a shift in credibility from the indicated pure premiums to the present on rate level pure premium (See table below). Note that expected loss development relies heavily on the present on rate level pure premium to the extent that the indicated is not considered credible, whereas the new NCCI full credibility standard and partial credibility formula give equal weight to the present on rate level pure premium and the national pure premium.

NCCI now uses higher full credibility standards and a .4 power partial credibility formula to recognize the need for stability. Note that the credibility given to the indicated data using the new NCCI standard and formula is about the same as the credibility for expected loss development, therefore limiting fluctuations by about the same amount as expected loss development, An advantage to the expected loss development scheme is the consideration of different credibilities by policy period.

Credibilities - Class 7600

Serious Pure Prem	Indicated	National	PORL
Current Loss Development	.67	.16	.17
Expected Loss Development	.33	.16	.51
New NCCI Standard And Formula	.38	.31	.31

A_1 = **actual** first report losses, A_2 = **second** report, A_3 = **third** report
D_1 = **first** to ultimate loss development factor, D_2 = **second** to ultimate, D_3 = **third** to ultimate
E_1 = **ultimate** expected losses for **first** report, E_2 = **second** report, E_3 = **third** report
$E = E_1 + E_2 + E_3$
P_1 = **first** report payroll in hundreds, P_2 = **second** report, P_3 = **third report**
$P = P_1 + P_2 + P_3$
Z = state indicated credibility
Z_n = **national** credibility
N/P = **national** pure premium
E/P = present on rate level pure premium
$E_1 = (E/P)P_1$, $E_2 = (E/P)P_2$, $E_3 = (E/P)P_3$

Expected Loss Development: Formula Pure Premium

$$Z\left[\frac{\left(A_1+\left(1-\frac{1}{D_1}\right)E_1\right)+\left(A_2+\left(1-\frac{1}{D_2}\right)E_2\right)+\left(A_3+\left(1-\frac{1}{D_3}\right)E_3\right)}{P}\right]+(1-Z-Z_n)\left[\frac{E}{P}\right]+Z_n\left[\frac{N}{P}\right]$$

$$=\frac{Z(A_1+A_2+A_3)+Z(E_1+E_2+E_3)-\frac{Z}{D_1}E_1-\frac{Z}{D_2}E_2-\frac{Z}{D_3}E_3+E-ZE-Z_nE+Z_nN}{P}$$

$$=\frac{\frac{Z}{D_1}A_1D_1+\frac{Z}{D_2}A_2D_2+\frac{Z}{D_3}A_3D_3+ZE-\frac{Z}{D_1}E_1-\frac{Z}{D_2}E_2-\frac{Z}{D_3}E_3+(E_1+E_2+E_3)-ZE-Z_nE+Z_nN}{P}$$

$$=\frac{\left[\frac{Z}{D_1}A_1D_1+\left(1-\frac{Z}{D_1}\right)E_1\right]+\left[\frac{Z}{D_2}A_2D_2+\left(1-\frac{Z}{D_2}\right)E_2\right]+\left[\frac{Z}{D_3}A_3D_3+\left(1-\frac{Z}{D_3}\right)E_3\right]-Z_nE+Z_nN}{P}$$

$$=\left(\frac{Z}{D_1}\right)\left(\frac{P_1}{P}\frac{A_1D_1}{P_1}\right)+\left(\frac{Z}{D_2}\right)\left(\frac{P_2}{P}\frac{A_2D_2}{P_2}\right)+\left(\frac{Z}{D_3}\right)\left(\frac{P_3}{P}\frac{A_3D_3}{P_3}\right)$$

$$+\left[\left(1-\frac{Z}{D_1}\right)\frac{P_1}{P}+\left(1-\frac{Z}{D_2}\right)\frac{P_2}{P}+\left(1-\frac{Z}{D_3}\right)\frac{P_3}{P}\right]\left(\frac{E}{P}\right)-Z_n\left(\frac{E}{P}\right)+Z_n\left(\frac{N}{P}\right)$$

$$= \left(\frac{Z}{D_1}\frac{P_1}{P}\right)\left(\frac{A_1D_1}{P_1}\right) + \left(\frac{Z}{D_2}\frac{P_2}{P}\right)\left(\frac{A_2D_2}{P_2}\right) + \left(\frac{Z}{D_3}\frac{P_3}{P}\right)\left(\frac{A_3D_3}{P_3}\right)$$

$$+ \left[1 - \left(\frac{Z}{D_1}\frac{P_1}{P}\right) - \left(\frac{Z}{D_2}\frac{P_2}{P}\right) - \left(\frac{Z}{D_3}\frac{P_3}{P}\right) - Z_n\right]\left(\frac{E}{P}\right) + Z_n\left(\frac{N}{P}\right)$$

Current Method: Formula Pure Premium

$$Z\left[\frac{A_1D_1 + A_2D_2 + A_3D_3}{P}\right] + (1 - Z - Z_n)\left[\frac{E}{P}\right] + Z_n\left[\frac{N}{P}\right]$$

Serious pure premium - class 7600

$$\frac{0.67}{1.417} * \frac{42{,}616{,}748}{135{,}892{,}859} = 0.15 \qquad \frac{393{,}906}{42{,}616{,}748/100} = 0.924$$

st cred / 3rd-ult dev * 3rd rpt pay / total pay = 3rd rpt cred; 3rd rpt dev loss / 3rd rpt pay = 3rd rpt ind pp

$$\frac{0.67}{1.993} * \frac{49.728.462}{135{,}892{,}859} = 0.12 \qquad \frac{145{,}463}{49{,}728{,}462/100} = 0.293$$

st cred / 2nd-ult dev * 2nd rpt pay / total pay = 2nd rpt cred; 2nd rpt dev loss / 2nd rpt pay = 2nd rpt ind pp

$$\frac{0.67}{3.773} * \frac{43{,}547{,}649}{135{,}892{,}859} = 0.06 \qquad \frac{1{,}731{,}862}{43{,}547{,}649/100} = 3.977$$

st cred / Ist-ult dev * 1st rpt pay / total pay = 1st rpt cred; 1st rpt dev loss / 1st rpt pay = 1st rpt ind pp

nat cred	nat pure prem
0.16	1.287

remaining cred	porl pure prem
0.51	1.203

form pure prem

$$0.15*0.924+0.12*0.293+0.06*3.977+0.16*1.287+0.51*1.203= \quad 1.221$$

(float from the start to eliminate rounding difference)

Attachment 3

EXHIBIT 1

COMPUTATION OF REVISED PURE PREMIUM RATE
with loss development based on expected losses

Overall Revision	All Other
6.2%	Industry Group

Class: 7600 Telephone or Telegraph Co: All Other Employees & Dvrs

		Displayed Losses			Undeveloped Losses			Revised Losses		
	Payroll	Serious	Non-Ser	Medical	Serious	Non-Ser	Medical	Serious	Non-Ser	Medical
3-year	0	0	0	0	0	0	0	0	0	0
1984	42616748	393906	280841	500903	277986	281969	418465	428859	280879	505647
1985	49728462	145463	252282	480542	72987	254830	356485	371053	251631	516060
1986	43547649	1731862	237862	481927	459015	247258	308532	844044	236300	503288
	135892859	2271231	770985	1463372				1643956	768810	1524995

	NAT'L COUNCIL PROCEDURE				REVISED PROCEDURE		
	Serious	Non-Ser	Medical		Serious	Non-Ser	Medical
	1.671	0.567	1.077	**Indicated Pure Premiums**	1.210	0.566	1.122
	1.203	0.637	1.243	**P.P. "Present on Rate Level"**			
	1.287	0.917	1.769	**P.P. "Ind. by Nat'l Reltvty"**			
	0.59 *	0.78	1.00	**State Credibility**	0.67	0.83	1.00
Total	0.20	0.11	0.00	**National Credibility**	0.16	0.08	0.00
3.19	1.496	0.613	1.077	**Formula Pure Premium**	1.221	0.600	1.122
	1.008	1.008	1.008	**Composite Factor**			
	1.007	1.004	1.000	**Effect of Benefit Change**			
	1.092	1.092	0.975	**Change in Trend Factor**			
		3.39		**Rounded Total**		3.12	
		1.007		**Ratio of Manual to Earned Premium**		1.007	
		1.000		**Contracting Prem Adj Program Offset**		1.000	
				Specific Disease Loading			
		3.41		**Calculated Pure Premium Rate**		3.14	
Swing		2.86		**Current Pure Premium Rate**		2.86	
Limits:							
33%	above	3.41		**Swing-Limited Pure Premium Rate**		3.14	
14%	below	19.2%		**Percentage Change**		9.8%	
				Difference from Nat'l Council		-7.9%	

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$*(.59)^{3/4} = .67$

performed for this paper. The revisions for 1991 and 1992 use differing pure premium input data for the two development methods so separate worksheets were needed.

The rate revisions for Class 7600 in Exhibit 1 achieve materially different results and also illustrate the enhanced credibility formula used with the revised procedure. The NCCI credibility formula is the two-thirds root of the ratio of partial expected losses to the 100 percent standard. The Revised Procedure uses a simple square root formula (or a three-fourths root of the NCCI credibility).

The only other difference is the provision for loss development. The NCCI rate filing for 1990 displayed these loss development factors in Appendix B-I:

Policy Period	Indemnity		Medical
	Serious	Non-Serious	
1984	1.417	.996	1.197
1985	1.993	.990	1.348
1986	1.773	.962	1.562
Three-Year Fixed	2.394	.983	1.369

Exhibit 1 shows the payroll and losses as they would be shown in the National Council filing Appendix B-II. The losses have been developed and adjusted to current benefits, trends, and accident-year experience. The revised model simply divides these displayed losses by the partial loss development

An Algebraic Reserving Method for Paid Loss Data

by Alfred O. Weller

AN ALGEBRAIC RESERVING METHOD

FOR PAID LOSS DATA

by Alfred O. Weller

Sooner or later a casualty actuary is confronted by the question, "Given a history of paid loss amounts by calendar year, what should reserves be?" Often, it is not possible to accurately gather and analyze additional data within the time constraints for the reserving decision. The algebraic reserving method presented in this paper offers one approach to rapidly addressing this problem. The paper consists of four sections - General Considerations, Formulas, Examples, and Conclusion.

General Considerations

In general, reserve estimates will prove more accurate to the extent that they reflect information from a variety of sources and several actuarial methods. In any reserving situation, available data and information is limited by practical constraints (e.g., design of systems) and time constraints (e.g., financial reporting deadlines). In addition, the question of whether the benefits of better actuarial estimates are worth the costs of gathering better information is implicit in any reserving situation.

The situation to which the algebraic reserving method applies is one in which available information is paid losses by calendar year and there is some basis on which the actuary can assess the annual change in the level of incurred losses by accident year.

For example, a history of earned premiums might be used to create an index of loss levels by accident year. Or, the assumption regarding loss levels by accident year might even be weaker. For example, losses for similar business might have increased an average of 10% per year for the period for which paid losses are available.

The information on loss levels need not be detailed to afford an algebraic solution to the reserving problem. However, in general the more accurate the assumed relative loss levels, the more accurate the estimated reserves will be.

In addition to requiring an assumption regarding relative loss levels by accident year, the method assumes that there is a stable development pattern across all accident years. Thus, as the number of calendar years increases or the numbers of claims whose payments comprise calendar year paid amounts decreases, the possibility of fluctuations in actual payment patterns becomes more important in evaluating the results of the algebraic method.

The information on paid losses should cover all calendar years from the inception of the program. Otherwise, the method cannot estimate reserves without ad hoc adjustments. For example, if data started with the third year of the program, the method would estimate the portion of accident year losses paid through 36 months maturity instead of 12 months. Since the most recent accident year would be at 12 months maturity, the estimate through 36 months would have to be allocated to the maturities 0-12 months, 12-24 months, and 24-36 months using other techniques in order to derive reserve estimates for all accident years.

Tail factors are beyond the scope of the algebraic method. For n calendar year periods, the algebraic method derives development through maturity n years and leaves the tail factor to further analysis. Unless a parameterized payment pattern is assumed and the structure of the equations changed, the tail factor will require separate actuarial analysis.

Finally, the method is called the "algebraic method" because it is based on the algebraic solution of n linear equations in n unknowns. Thus, for any set of assumed relative loss levels, there is a unique solution for unpaid (unreported) losses that will be paid (reported) on or before accident years attain maturity n years. Reserve estimates based on successful mathematical solutions of the equations may differ from reasonable actuarial estimates. The algebraic method can provide useful input into actuarial decisions on appropriate reserves, but should not be used as an algorithm without professional scrutiny.

Formulas

The following equations define the "algebraic method."

$$I_j = \text{Incurred amount for accident year } j. \tag{1}$$

$$\begin{aligned} n &= \text{Number of calendar years for which data is available} \\ &= \text{Number of accident years affecting data} \end{aligned} \tag{2}$$

$$f_i = \text{Fraction of accident year loss paid during year } i \text{ after start of accident year.} \quad (3)$$

Because the algebraic method estimates development through maturity n and because the sum of the fractions of losses at maturity n paid in each calendar year must total unity (i.e., 100%),

$$f_n = 1 - \sum_{i=1}^{n-1} f_i \quad (4)$$

Calendar year payments can now be expressed in terms of accident year components.

$$P_j = \text{Amount paid during calendar year } j \text{ for all accident years}$$

$$= \sum_{i=1}^{j} f_i \, I_{j+1-i} \quad (5)$$

$$\text{so that, if } j=n, \; P_j = \sum_{i=1}^{n-1} f_i \, I_{n+1-i} + \left(1 - \sum_{i=1}^{n-1} f_i\right) I_1$$

Introducing loss level indices facilitates solving equation (5). We define indices as follows:

$$g_j = \text{Index for accident year } j \text{ loss level}$$

$$= \frac{I_j}{I_1} \quad (6)$$

$$\text{so that } g_1 = 1.000$$

$$g_j = g_2^{(j-1)} \text{ for uniform growth}$$

Equation (5) can now be rewritten as:

$$P_j = \sum_{i=1}^{j} f_i \, g_{j+1-i} \, I_1 \qquad \text{if } j<n$$

$$P_j = I_1 + \sum_{i=1}^{n-1} f_i \, (g_{n-i}-1) \, I_1 \quad \text{if } j=n$$

In order to generate n linear equations in n unknowns, we introduce a variable equal to the reciprocal of incurred losses.

$$R_j = \frac{1}{I_j} \tag{8}$$
$$= \textit{Reciprocal of incurred loss for accident year } j$$

The resulting n linear equations are:

$$0 = -P_j R_1 + \sum_{i=1}^{j} f_i \, g_{j+1-i} \qquad \text{if } j<n$$

$$-1 = -P_j R_1 + \sum_{i=1}^{n-1} f_i \, (g_{n-i}-1) \qquad \text{if } j=n$$

Thus, the algebraic reserving method solves the n equations

$$\begin{array}{rclllll}
0 & = & -P_1R_1 + & 1 \; f_1 + & 0 \; f_2 + & 0 \; f_3 + \ldots + & 0 \; f_{n-1} \\
0 & = & -P_2R_1 + & g_2 \; f_1 + & 1 \; f_2 + & 0 \; f_3 + \ldots + & 0 \; f_{n-1} \\
0 & = & -P_3R_1 + & g_3 \; f_1 + & g_2 \; f_2 + & 1 \; f_3 + \ldots + & 0 \; f_{n-1} \quad (10) \\
\ldots & = & \ldots & & & & \\
-1 & = & -P_nR_1 + & (g_n-1) f_1 + & (g_{n-1}-1) f_2 + & (g_{n-3}-1) f_3 + \ldots + & (g_2-1) f_{n-1}
\end{array}$$

for R_1, f_1, ..., f_{n-1}.

Examples

In the attached exhibits, data for private passenger automobile liability/medical from pages 63 and 79 of the 1993 edition of Best's Aggregates & Averages is used to illustrate the algebraic method. For convenience, loss and allocated loss adjustment expense is called "loss" in this discussion.

Exhibit I presents a link ratio approach to establish a benchmark for comparison to the results of the algebraic method. Weighted three point average development factors are employed. Other link ratio calculations are possible, but only one is used for comparison purposes in this paper. Exhibit I-1 presents raw data. Exhibit I-2 derives development factors. Exhibit I-3 derives reserve estimates using the development pattern from Exhibit I-2.

Exhibit II derives values for use in subsequent algebraic method calculations. Exhibit II-1 derives calendar year paid loss as if accident year 1983 were the first year of a program. Exhibit II-2 uses earned premiums to estimate loss level indices. Distinct indices by year are used in Exhibit III and a rough average annual growth rate is used in Exhibit IV.

Exhibit III applies the algebraic method using distinct indices by year. Exhibit III-1 presents the matrix defining the simultaneous equations. Exhibit III-2 presents the inverted matrix and the estimated parameters R_1, f_1, ..., f_{n-1}. Exhibit III-3 compares the paid amounts based on the parameters to the actual paid amounts by accident year component as well as by calendar year total. Exhibit III-4 adjusts the development pattern for negative values and derives corresponding reserve estimates.

Negative values might be attributable to several causes (e.g., influence of particular large claims, shifts in development patterns over time). Consideration of alternative possible adjustments will vary with available data and reserving context, and is, therefore, beyond the scope of this paper.

Exhibits IV are organized identically to Exhibits III. The difference is that a uniform annual change in loss level is used in lieu of individual annual indices.

Following Exhibits IV are four graphs. Graph 1 presents the three cumulative development patterns fit using the above techniques. Graph 2 presents the same development patterns on an interval basis. Graph 3 compares reserves estimates by accident year. Graph 4 presents the components of accident year losses using the three methods.

Conclusion

For the data used in the example, the algebraic method presented above produced reserve estimates quite close (within 10%) to reserve estimates based on a link ratio method. Therefore, it might prove useful in situations in which detailed data is unavailable. In particular, it might prove useful in reserving situations for which only calendar year paid loss data is available.

For the example, the method required elimination of some negative values from the development pattern. Also, the algebraic reserving method is quite sensitive to the selection of loss level indices. Therefore, although it can prove useful in particular

situations, it is not well suited to use as an algorithm without professional scrutiny by a casualty actuary.

ALGEBRAIC RESERVING

EXHIBIT I-1

Link Ratios - Amounts* by Maturity

Year	1	2	3	4	5	6	7	8	9	10
1983	6,336,136	12,087,849	14,818,118	16,318,063	17,116,020	17,537,521	17,709,879	17,801,459	17,869,922	17,914,459
1984	7,115,534	13,753,038	16,936,543	18,711,542	19,694,316	20,188,931	20,439,114	20,585,826	20,663,474	xx
1985	7,816,829	15,437,173	19,146,806	21,281,679	22,406,157	22,974,802	23,280,050	23,433,065	xx	xx
1986	8,701,618	17,269,667	21,541,709	23,930,252	25,211,125	25,858,777	26,169,144	xx	xx	xx
1987	9,697,467	19,400,683	24,147,746	26,805,181	28,826,913	28,910,978	xx	xx	xx	xx
1988	10,916,881	21,763,312	26,980,241	29,807,907	31,280,692	xx	xx	xx	xx	xx
1989	12,051,811	24,101,585	29,805,169	32,850,611	xx	xx	xx	xx	xx	xx
1990	13,320,847	26,043,240	32,085,138	xx	xx	xx	xx	xx	xx	xx
1991	13,320,110	25,851,612	xx	xx	xx	xx	xx	xx	xx	xx
1992	14,400,031	xx	xx	xx	xx	xx	xx	xx	xx	xx
Total	103,677,264	175,708,159	185,461,470	169,705,235	144,535,223	115,471,009	87,598,187	61,820,350	38,533,396	17,914,459
3 Pt Num	75,996,437	88,870,548	89,463,699	85,318,730	77,744,557	69,888,308	61,820,350	38,533,396	17,914,459	xx
3 Pt Den	38,692,768	71,908,137	80,933,156	80,543,340	76,444,195	69,022,510	61,429,043	38,387,285	17,869,922	xx

* 1993 Edition of Best's "Aggregates & Averages" Page 79

Link Ratios - Development

Year	1	2	3	4	5	6	7	8	9	10
1983	1.908	1.226	1.101	1.049	1.025	1.010	1.005	1.004	1.002	xx
1984	1.933	1.231	1.105	1.053	1.025	1.012	1.007	1.004	xx	xx
1985	1.975	1.240	1.112	1.053	1.025	1.013	1.007	xx	xx	xx
1986	1.985	1.247	1.111	1.054	1.026	1.012	xx	xx	xx	xx
1987	2.001	1.245	1.110	1.075	1.003	xx	xx	xx	xx	xx
1988	1.994	1.240	1.105	1.049	xx	xx	xx	xx	xx	xx
1989	2.000	1.237	1.102	xx	xx	xx	xx	xx	xx	xx
1990	1.955	1.232	xx	xx	xx	xx	xx	xx	xx	xx
1991	1.941	xx	xx	xx	xx	xx	xx	xx	xx	xx
1992	xx	xx	xx	xx	xx	xx	xx	xx	xx	xx
Average	1.966	1.237	1.106	1.055	1.021	1.012	1.006	1.004	1.002	xx
3 Pt Avg	1.965	1.236	1.106	1.059	1.018	1.013	1.006	1.004	1.002	xx
3 Pt Wtd	1.964	1.236	1.105	1.059	1.017	1.013	1.006	1.004	1.002	xx
Selected	1.964	1.236	1.105	1.059	1.017	1.013	1.006	1.004	1.002	xx
Cum Factor	2.964	1.509	1.221	1.105	1.043	1.025	1.013	1.006	1.002	1.000
Cum %	33.74%	66.26%	81.89%	90.52%	95.89%	97.52%	98.74%	99.37%	99.75%	100.00%
Int %	33.74%	32.52%	15.63%	8.63%	5.37%	1.63%	1.22%	0.63%	0.38%	0.25%

Link Ratios - Reserve Estimates

Year	Paid Loss	Factor	Est Inc	Est Res
1983	17,914,459	1.000	17,914,459	0
1984	20,663,474	1.002	20,714,973	51,499
1985	23,433,065	1.006	23,580,881	147,816
1986	26,169,144	1.013	26,501,970	332,826
1987	28,910,978	1.025	29,645,938	734,960
1988	31,280,692	1.043	32,621,524	1,340,832
1989	32,850,611	1.105	36,289,927	3,439,316
1990	32,085,138	1.221	39,180,226	7,095,088
1991	25,851,612	1.509	39,014,898	13,163,286
1992	14,400,031	2.964	42,684,451	28,284,420
Total	253,559,204	1.215	308,149,247	54,590,043

Calendar Year Paid Losses by Accident Year Components

Year	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
1983	6,336,136	5,751,713	2,730,269	1,499,945	797,957	421,501	172,358	91,580	68,463	44,537
1984	xx	7,115,534	6,637,504	3,183,505	1,774,999	982,774	494,615	250,183	146,712	77,648
1985	xx	xx	7,816,829	7,620,344	3,709,633	2,134,873	1,124,478	568,645	305,248	153,015
1986	xx	xx	xx	8,701,618	8,568,049	4,272,042	2,388,543	1,280,873	647,652	310,367
1987	xx	xx	xx	xx	9,697,467	9,703,216	4,747,063	2,657,435	2,021,732	84,065
1988	xx	xx	xx	xx	xx	10,916,881	10,846,431	5,216,929	2,827,666	1,472,785
1989	xx	xx	xx	xx	xx	xx	12,051,811	12,049,774	5,703,584	3,045,442
1990	xx	xx	xx	xx	xx	xx	xx	13,320,847	12,722,393	6,041,898
1991	xx	xx	xx	xx	xx	xx	xx	xx	13,320,110	12,531,502
1992	xx	xx	xx	xx	xx	xx	xx	xx	xx	14,400,031
Total	6,336,136	12,867,247	17,184,602	21,005,412	24,548,105	28,431,287	31,825,299	35,436,266	37,763,560	38,161,290

Thus,

P1 = 6,336,136
P2 = 12,867,247
P3 = 17,184,602
P4 = 21,005,412
P5 = 24,548,105
P6 = 28,431,287
P7 = 31,825,299
P8 = 35,436,266
P9 = 37,763,560
P10 = 38,161,290

Loss Level Indices based on Earned Premiums

Loss level indices by year

Year	Earned Premium	Index
1983	22,382,780	1.0000
1984	23,968,070	1.0708
1985	26,608,441	1.1888
1986	31,360,994	1.4011
1987	35,801,570	1.5995
1988	39,732,848	1.7752
1989	43,038,375	1.9228
1990	46,899,296	2.0953
1991	50,069,836	2.2370
1992	54,197,133	2.4214

Loss level assuming uniform annual rate

1 Earned premium for 1992	54,197,133
2 Earned premium for 1983	22,382,780
3 Ratio: (1)/(2)	2.4214
4 g2 = Ninth root of (3)	1.1032
5 Indices by year	
1983	1.0000
1984	1.1032
1985	1.2172
1986	1.3428
1987	1.4815
1988	1.6344
1989	1.8032
1990	1.9894
1991	2.1948
1992	2.4214

Algebraic Method using Yearly Indices - Simultaneous Equation Matrix

Coefficients of

R1	f1	f2	f3	f4	f5	f6	f7	f8	f9	=	Value
-6336136	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	=	0.0000
-12867247	1.0708	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	=	0.0000
-17184602	1.1888	1.0708	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	=	0.0000
-21005412	1.4011	1.1888	1.0708	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	=	0.0000
-24548105	1.5995	1.4011	1.1888	1.0708	1.0000	0.0000	0.0000	0.0000	0.0000	=	0.0000
-28431287	1.7752	1.5995	1.4011	1.1888	1.0708	1.0000	0.0000	0.0000	0.0000	=	0.0000
-31825299	1.9228	1.7752	1.5995	1.4011	1.1888	1.0708	1.0000	0.0000	0.0000	=	0.0000
-35436266	2.0953	1.9228	1.7752	1.5995	1.4011	1.1888	1.0708	1.0000	0.0000	=	0.0000
-37763560	2.2370	2.0953	1.9228	1.7752	1.5995	1.4011	1.1888	1.0708	1.0000	=	0.0000
-38161290	1.4214	1.2370	1.0953	0.9228	0.7752	0.5995	0.4011	0.1888	0.0708	=	-1.0000

Algebraic Method using Yearly Indices - Inverse Matrix and Solutions

R1	f1	f2	f3	f4	f5	f6	f7	f8	f9		Solution
0.0000000038	0.0000000010	0.0000000035	0.0000000029	0.0000000066	0.0000000099	0.0000000124	0.0000000072	0.0000000045	-0.0000000634	R1 =	6.337091E-08
1.0243871066	0.0064850583	0.0219796232	0.0182600681	0.0415287133	0.0626983083	0.0786896308	0.0453516440	0.0284386579	-0.4015266761	f1 =	40.15%
-1.0474160274	1.0062253031	0.0210992423	0.0175286718	0.0398653051	0.0601869643	0.0755377637	0.0435351105	0.0272995642	-0.3854437605	f2 =	38.54%
-0.0300395396	-1.0676134154	1.0108894151	0.0090466274	0.0205746657	0.0310627667	0.0389853843	0.0224686690	0.0140894295	-0.1989292822	f3 =	15.89%
-0.0771176366	-0.0405499867	-1.0654993606	1.0044254940	0.0100648624	0.0151955068	0.0190711497	0.0109913846	0.0068923681	-0.0973136520	f4 =	9.73%
0.0418089213	-0.0825008529	-0.0403348031	-1.0693418180	1.0033761961	0.0050972392	0.0063972997	0.0036869922	0.0023120024	-0.0326432651	f5 =	3.26%
0.0553379117	0.0405217265	-0.0806710901	-0.0401635819	-1.0663729923	1.0067234513	0.0084382803	0.0048632823	0.0030496187	-0.0430577017	f6 =	4.31%
0.0592292651	0.0530628298	0.0409788867	-0.0820705381	-0.0399439834	-1.0675384980	1.0041263906	0.0023781863	0.0014912894	-0.0210555812	f7 =	2.11%
-0.0071439575	0.0585140748	0.0546330919	0.0414133519	-0.0794186593	-0.0366723564	-1.0639871140	1.0039416802	0.0024717096	-0.0348981783	f8 =	3.49%
0.0359820107	-0.0099568210	0.0556006888	0.0507706570	0.0353866483	-0.0897308763	-0.0505340641	-1.0756746687	0.9969597474	0.0429254613	f9 =	-4.29%
										f10 =	-17.19%

Algebraic Method using Yearly Indices - Estimated Calendar Year Paid Amounts by Accident Year Components

Year	Est Inc	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
1983	15,780,112	6,336,136	6,082,346	3,139,126	1,535,620	515,114	679,455	332,259	550,697	(677,369)	(2,713,274)
1984	16,897,760	xx	6,784,901	6,513,136	3,361,459	1,644,383	551,598	727,579	355,792	589,701	(725,344)
1985	18,759,251	xx	xx	7,532,340	7,230,636	3,731,764	1,825,531	612,363	807,730	394,987	654,664
1986	22,109,854	xx	xx	xx	8,877,696	8,522,105	4,398,297	2,151,591	721,738	952,000	465,536
1987	25,240,510	xx	xx	xx	xx	10,134,738	9,728,797	5,021,077	2,456,246	823,933	1,086,798
1988	28,012,106	xx	xx	xx	xx	xx	11,247,608	10,797,091	5,572,428	2,725,960	914,407
1989	30,342,540	xx	xx	xx	xx	xx	xx	12,183,339	11,695,343	6,036,020	2,952,743
1990	33,064,532	xx	xx	xx	xx	xx	xx	xx	13,276,292	12,744,518	6,577,504
1991	35,299,799	xx	xx	xx	xx	xx	xx	xx	xx	14,173,811	13,606,087
1992	38,209,590	xx	xx	xx	xx	xx	xx	xx	xx	xx	15,342,170
Total	xx	6,336,136	12,867,247	17,184,602	21,005,412	24,548,105	28,431,287	31,825,299	35,436,266	37,763,560	38,161,290

Differences - Estimated less Actual

Year	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
1983	0	330633	408857	35675	-282843	257954	159901	459117	-745832	-2757811
1984	xx	-330633	-124368	177954	-130616	-431176	232964	105609	442989	-802992
1985	xx	xx	-284489	-389708	22131	-309342	-512115	239085	89739	501649
1986	xx	xx	xx	176078	-45944	126255	-236952	-559135	304348	155169
1987	xx	xx	xx	xx	437271	25581	274014	-201189	-1197799	1002733
1988	xx	xx	xx	xx	xx	330727	-49340	355499	-101706	-558378
1989	xx	xx	xx	xx	xx	xx	131528	-354431	332436	-92699
1990	xx	xx	xx	xx	xx	xx	xx	-44555	22125	535606
1991	xx	xx	xx	xx	xx	xx	xx	xx	853701	1074585
1992	xx	xx	xx	xx	xx	xx	xx	xx	xx	942139
Total	0	0	-0	-0	-0	0	-0	0	-0	0

Algebraic Method using Yearly Indices - Development Pattern and Reserve Estimates

Maturity	Pattern	Neg Val	Mod Patt	Cum Patt
1	40.15%	0.00%	33.05%	33.05%
2	38.54%	0.00%	31.73%	64.78%
3	19.89%	0.00%	16.37%	81.15%
4	9.73%	0.00%	8.01%	89.16%
5	3.26%	0.00%	2.69%	91.85%
6	4.31%	0.00%	3.54%	95.39%
7	2.11%	0.00%	1.73%	97.13%
8	3.49%	0.00%	2.87%	100.00%
9	-4.29%	4.29%	0.00%	100.00%
10	-17.19%	17.19%	0.00%	100.00%
Total	100.00%	21.49%	100.00%	xx

Year	Est Inc	% Unpaid	Est Unpaid
1983	15,780,112	0.00%	0
1984	16,897,760	0.00%	0
1985	18,759,251	0.00%	0
1986	22,109,854	2.87%	635,125
1987	25,240,510	4.61%	1,162,514
1988	28,012,106	8.15%	2,282,980
1989	30,342,540	10.84%	3,288,208
1990	33,064,532	18.85%	6,231,732
1991	35,299,799	35.22%	12,433,203
1992	38,209,590	66.95%	25,580,919
Total	263,716,053	19.57%	51,614,681

Algebraic Method using Uniform Annual Rate - Simulteneous Equation Matrix

Coefficients of R1	f1	f2	f3	f4	f5	f6	f7	f8	f9	=	Value
-6336136	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	=	0.0000
-12867247	1.1032	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	=	0.0000
-17184602	1.2172	1.1032	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	=	0.0000
-21005412	1.3428	1.2172	1.1032	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	=	0.0000
-24548105	1.4815	1.3428	1.2172	1.1032	1.0000	0.0000	0.0000	0.0000	0.0000	=	0.0000
-28431287	1.6344	1.4815	1.3428	1.2172	1.1032	1.0000	0.0000	0.0000	0.0000	=	0.0000
-31825299	1.8032	1.6344	1.4815	1.3428	1.2172	1.1032	1.0000	0.0000	0.0000	=	0.0000
-35436266	1.9894	1.8032	1.6344	1.4815	1.3428	1.2172	1.1032	1.0000	0.0000	=	0.0000
-37763560	2.1948	1.9894	1.8032	1.6344	1.4815	1.3428	1.2172	1.1032	1.0000	=	0.0000
-38161290	1.4214	1.1948	0.9894	0.8032	0.6344	0.4815	0.3428	0.2172	0.1032	=	-1.0000

Algebraic Method using Uniform Annual Rate - Inverse Matrix and Solutions

R1	f1	f2	f3	f4	f5	f6	f7	f8	f9		Solution
0.0000000065	0.0000000065	0.0000000065	0.0000000065	0.0000000065	0.0000000065	0.0000000065	0.0000000065	0.0000000065	-0.0000000628	R1 =	6.280758E-08
1.0410887432	0.0410887432	0.0410887432	0.0410887432	0.0410887432	0.0410887432	0.0410887432	0.0410887432	0.0410887432	-0.3979573429	f1 =	39.80%
-1.0651383684	1.0381107459	0.0381107459	0.0381107459	0.0381107459	0.0381107459	0.0381107459	0.0381107459	0.0381107459	-0.3691145068	f2 =	36.91%
0.0193819998	-1.0838671145	1.0193819998	0.0193819998	0.0193819998	0.0193819998	0.0193819998	0.0193819998	0.0193819998	-0.1877207365	f3 =	18.77%
0.0132712953	0.0132712953	-1.0899778190	1.0132712953	0.0132712953	0.0132712953	0.0132712953	0.0132712953	0.0132712953	-0.1285366503	f4 =	12.85%
0.0089095215	0.0089095215	0.0089095215	-1.0943395928	1.0089095215	0.0089095215	0.0089095215	0.0089095215	0.0089095215	-0.0862915054	f5 =	8.63%
0.0087455143	0.0087455143	0.0087455143	0.0087455143	-1.0945036000	1.0087455143	0.0087455143	0.0087455143	0.0087455143	-0.0847030444	f6 =	8.47%
0.0029733371	0.0029733371	0.0029733371	0.0029733371	0.0029733371	-1.1002757772	1.0029733371	0.0029733371	0.0029733371	-0.0287977006	f7 =	2.88%
0.0021077831	0.0021077831	0.0021077831	0.0021077831	0.0021077831	0.0021077831	-1.1011413312	1.0021077831	0.0021077831	-0.0204145390	f8 =	2.04%
-0.0086343461	-0.0086343461	-0.0086343461	-0.0086343461	-0.0086343461	-0.0086343461	-0.0086343461	-1.1118834604	0.9913656539	0.0836263452	f9 =	-8.36%
										f10 =	-21.99%

Algebraic Method using Uniform Annual Rate - Estimated Calendar Year Paid Amounts by Accident Year Components

Year	Est Inc	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
1983	15,921,646	6,336,136	5,876,911	2,988,823	2,046,515	1,373,903	1,348,612	458,507	325,033	(1,331,469)	(3,501,324)
1984	17,565,542	xx	6,990,336	6,483,696	3,297,416	2,257,816	1,515,757	1,487,855	505,847	358,592	(1,468,942)
1985	19,379,169	xx	xx	7,712,082	7,153,132	3,637,872	2,490,933	1,672,258	1,641,475	558,075	395,617
1986	21,380,051	xx	xx	xx	8,508,348	7,891,687	4,013,479	2,748,120	1,844,917	1,810,955	615,696
1987	23,587,522	xx	xx	xx	xx	9,386,828	8,706,497	4,427,867	3,031,861	2,035,403	1,997,935
1988	26,022,913	xx	xx	xx	xx	xx	10,356,009	9,605,435	4,885,040	3,344,898	2,245,556
1989	28,709,755	xx	xx	xx	xx	xx	xx	11,425,258	10,597,187	5,389,416	3,690,256
1990	31,674,012	xx	xx	xx	xx	xx	xx	xx	12,604,906	11,691,337	5,945,869
1991	34,944,326	xx	xx	xx	xx	xx	xx	xx	xx	13,906,351	12,898,458
1992	38,552,297	xx	xx	xx	xx	xx	xx	xx	xx	xx	15,342,170
Total	xx	6,336,136	12,867,247	17,184,602	21,005,412	24,548,105	28,431,287	31,825,299	35,436,266	37,763,560	38,161,290

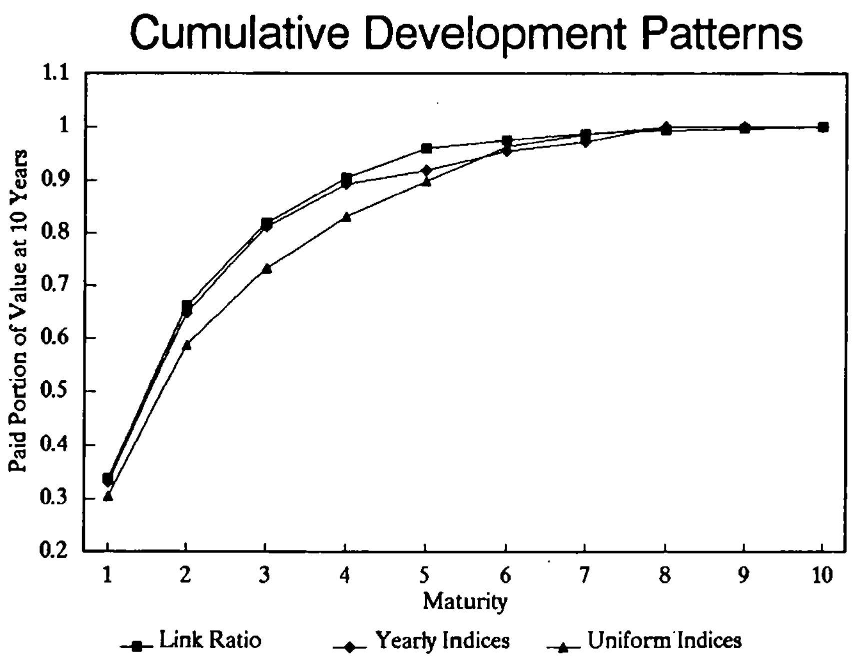
Differences - Estimated less Actual

Year	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
1983	0	5876911	2988823	2046515	1373903	1348612	458507	325033	-1331469	-3501324
1984	xx	6990335	6483696	3297416	2257816	1515757	1487855	505847	358593	-1468942
1985	xx	xx	7712081	7153132	3637872	2490933	1672258	1641475	558076	395617
1986	xx	xx	xx	8508347	7891687	4013479	2748120	1844917	1810955	615696
1987	xx	xx	xx	xx	9386827	8706497	4427867	3031861	2035403	1997935
1988	xx	xx	xx	xx	xx	10356008	9605435	4885040	3344898	2245556
1989	xx	xx	xx	xx	xx	xx	11425257	10597187	5389416	3690256
1990	xx	xx	xx	xx	xx	xx	xx	12604905	11691337	5945869
1991	xx	xx	xx	xx	xx	xx	xx	xx	13906351	12898458
1992	xx	xx	xx	xx	xx	xx	xx	xx	xx	15342170
Total	0	12867246	17184601	21005411	24548104	28431286	31825298	35436265	37763561	38161290

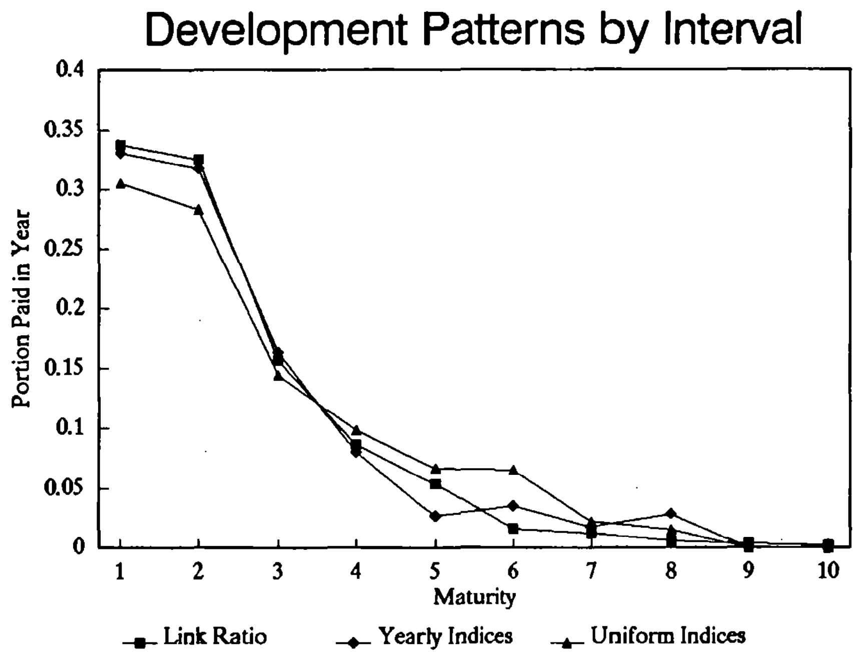
Algebraic Method using Uniform Annual Rate - Development Pattern and Reserve Estimates

Maturity	Pattern	Neg Val	Mod Patt	Cum Patt
1	39.80%	0.00%	30.53%	30.53%
2	36.91%	0.00%	28.32%	58.85%
3	18.77%	0.00%	14.40%	73.25%
4	12.85%	0.00%	9.86%	83.11%
5	8.63%	0.00%	6.62%	89.73%
6	8.47%	0.00%	6.50%	96.22%
7	2.88%	0.00%	2.21%	98.43%
8	2.04%	0.00%	1.57%	100.00%
9	-8.36%	8.36%	0.00%	100.00%
10	-21.99%	21.99%	0.00%	100.00%
Total	100.00%	30.35%	100.00%	xx

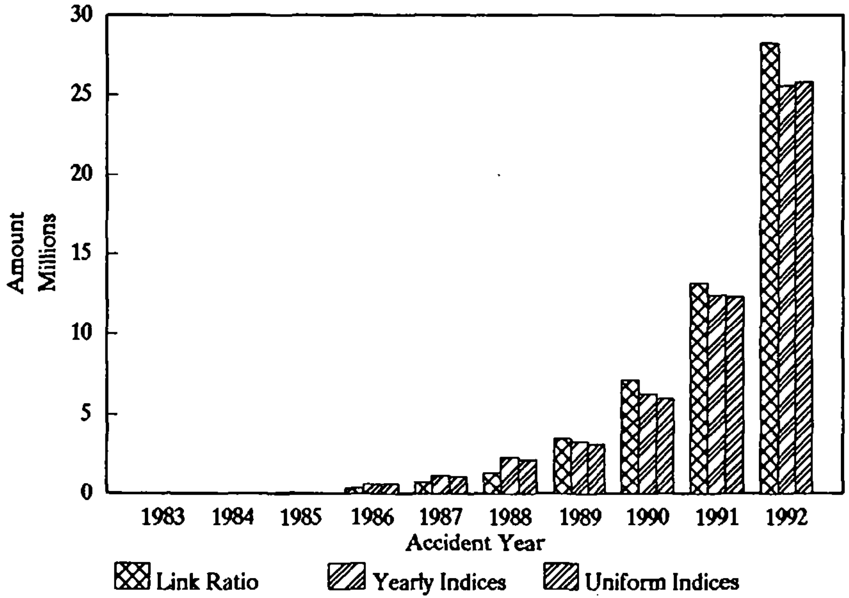
Year	Est Inc	% Unpaid	Est Unpaid
1983	15,921,646	0.00%	0
1984	17,565,542	0.00%	0
1985	19,379,169	0.00%	0
1986	21,380,051	2.87%	614,161
1987	23,587,522	4.61%	1,086,382
1988	26,022,913	8.15%	2,120,861
1989	28,709,755	10.84%	3,111,264
1990	31,674,012	18.85%	5,969,658
1991	34,944,326	35.22%	12,307,999
1992	38,552,297	66.95%	25,810,357
Total	257,737,233	19.80%	51,020,682


Cumulative Development Patterns
Paid Portion of Value at 10 Years
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
1
2
3
4
5
6
7
8
9
10
Maturity
Link Ratio
Yearly Indices
Uniform Indices

GRAPH 2

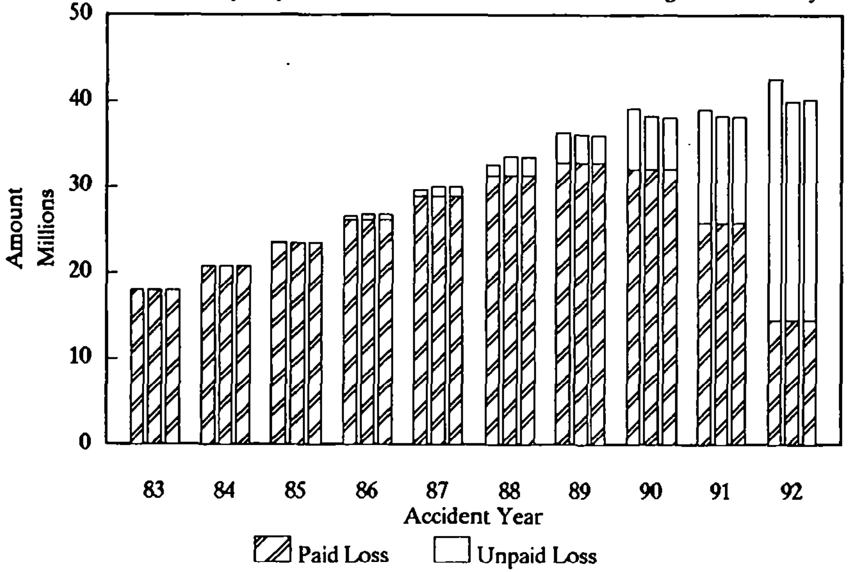

Development Patterns by Interval
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Portion Paid in Year
1
2
3
4
5
6
7
8
9
10
Maturity
Link Ratio
Yearly Indices
Uniform Indices

Reserve Estimates



Incurred Loss Estimates

(Link ratio to left, yearly indices above, and uniform indices to right of accident year.)



Credibility for Hiawatha
by Oakley E. Van Slyke

Hiawatha Designs An Experiment

by Maurice G. Kendall

1. Hiawatha, mighty hunter
He could shoot ten arrows upwards
Shoot them with such strength and swiftness
That the last had left the bowstring
Ere the first to earth descended.
This was commonly regarded
As a feat of skill and cunning.

2. One or two sarcastic spirits
Pointed out to him, however,
That it might be much more useful
If he sometimes hit the target.
Why not shoot a little straighter
And employ a smaller sample?

3. Hiawatha, who at college
Majored in applied statistics
Consequently felt entitled
To instruct his fellow men on
Any subject whatsoever
Waxed exceedingly indignant
Talked about the law of error,
Talked about truncated normals,
Talked about loss of information,
Talked about his lack of bias
Pointed out that in the long run
Independent observations
Even though they missed the target
Had an average point of impact
Very near the spot he aimed at
(With the possible exception
Of a set of measure zero.)

4. This, they said, was rather doubtful.
Anyway, it didn't matter
What resulted in the long run;
Either he must hit the target
Much more often than at present
Or himself would have to pay for
All the arrows that he wasted.

5. Hiawatha, in a temper
Quoted parts of R.A. Fisher
Quoted Yates and quoted Finney
Quoted yards of Oscar Kempthorne
Quoted reams of Cox and Cochran
Quoted Anderson and Bancroft
Practically in extenso
Trying to impress upon them
That what really mattered
Was to estimate the error.

6. One or two of them admitted
Such a thing might have its uses
Still, they said, he might do better
If he shot a little straighter.

7. Hiawatha, to convince them,
Organized a shooting contest
Laid out in proper manner
Of designs experimental
Recommended in the textbooks
(Mainly used for tasting tea, but
Sometimes used in other cases)
Randomized his shooting order
In factoral arrangements
Used in the theory of Galois
Fields of ideal polynomials
Got a nicely balanced layout
And successfully confounded
Second-order interaction.

8. All the other tribal marksmen
Ignorant, benighted creatures,
Of experimental set-ups
Spent their time of preparation
Putting in a lot of practice
Merely shooting at a target.

9. Thus it happened in the contest
That the scores were most impressive
With one solitary exception.
This (I hate to have to say it)
Was the score of Hiawatha,
Who, as usual, shot his arrows
Shot them with great strength and swiftness
Managing to be unbiased
Not, however, with his salvo
Managing to hit the target.

10. There, they said to Hiawatha,
That is what we all expected.

11. Hiawatha, nothing daunted,
Called for pen and called for paper
Did analyses of variance
Finally produced the figures
Showing beyond preadventure
Everybody else was biased
And the variance components
Did not differ from each other
Or from Hiawatha's
(This last point, one should acknowledge
Might have been much more convincing
If he hadn't been compelled to
Estimate his own component
From experimental plots in
Which the values all were missing.
Still, they didn't understand it
So they couldn't raise objections.
This is what often happens
With analyses of variance.)

12. All the same, his fellow tribesmen
Ignorant, benighted heathens,
Took away his bow and arrows,
Said that though my Hiawatha
Was a brilliant statistician
He was useless as a bowman,
As for variance components
Several of the more outspoken
Made primeval observations
Hurtful to the finer feelings
Even of a statistician.

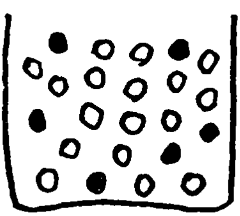
13. In a corner of the forest
Dwells alone my Hiawatha
Permanently cogitating
On the normal law of error
Wondering in idle moments
Whether an increased precision
Might perhaps be rather better
Even at the risk of bias
If thereby one, now and then, could
Register upon the target.

Introduction

This paper is called credibility for Hiawatha because it is about *expected value* ratemaking. Like Hiawatha, and unlike users of "classical" credibility, we are concerned today with making good estimates, that is, minimum variance unbiased estimates of the expected value of the outcome of a stochastic process. Our first point is that Bayesian credibility is *always* better than classical credibility if the goal is to estimate future loss costs.

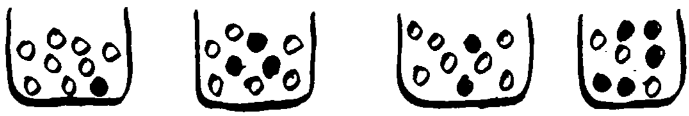
Nonetheless, like Hiawatha, we must consider that: "... an increased precision / Might perhaps be rather better / Even at the risk of bias / If thereby one, now and then, could / Register upon the target." Our second point is that there are tricks you can use to make Bayesian credibility computations easily. Each trick introduces a little bias, but the tricks improve the precision of your estimates as well as making them easy to calculate.

Definition of the Problem



Consider a big urn with an unknown number of red and white balls. Each red ball has a number on it (called a "loss amount").

A sprite has drawn balls at random from the big urn and put them into four small urns. The sprite may have put a greater proportion of red balls into some urns, and a lesser proportion into others.



The small urns correspond to various classes, various territories, various years, or *any other way* the universe of risks is divided into experience groups.

Problem 1:

Examine the entire contents of each of the four small urns.

Estimate $\overline{R}_i$, the expected rate of loss per draw, for each small urn.

		Urn 1	Urn 2	Urn 3	Urn 4
Number of balls	N_i	10	20	30	40
Total losses	L_i	10	40	90	160
$\overline{R}_i = \frac{L_i}{N_i}$		1	2	3	4

Problem 2:

Examine the entire contents of each of the four small urns. Estimate $\overline{R}_{\bullet}$, the expected rate of draw for the big urn.

		Big Urn
Number of balls	ΣN_i	100
Total losses	ΣL_i	300
$\overline{R}_{\bullet}=\frac{\Sigma L_i}{\Sigma N_i}$		3.0

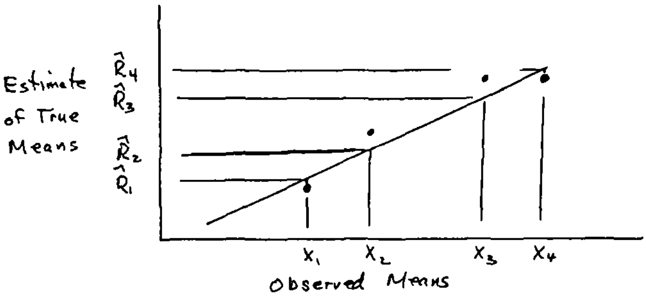
This is equivalent to $\frac{\Sigma \overline{R}_i N_i}{\Sigma N_i}$. That is, the various R_i's are weighted according to their number of balls (their sample sizes).

Problem 3:

Sample the contents of each small urn N_i times with replacement. Estimate $\overline{R}_i$, the expected rate of loss per draw from each small urn, as a linear combination of the observed rates of loss.

This is the Bayesian formulation of the insurance rating problem. It represents the insurance rating problem because the observed accidents are a random sample of the accidents that might have happened.

We use a linear combination of the observed rates of loss because this is the simplest way to reflect the information about the means of the various classes. Graphically, the process is as follows:



Historically, the general results have been in use since they were first published in 1914.

Hans Bühlmann showed that the best linear unbiased estimate of $\bar{R}_i$ is a weighted average of:

1. The observed average, X_i, with weight:

$$\frac{e_i}{var[S]}$$

(the reciprocal of the process variance for the urn)
where:
e_i = number of units of exposure in the observed average
$var[S]$ = variance of the claims process for one unit of exposure

2. The estimated grand average, $\hat{R}_\bullet$, with weight

$$\frac{1}{var[\mu]}$$

(the reciprocal of the variance of the distribution of means among the urns).

These weights correspond to the "number of balls" in Problem 2 because the number of balls is proportional to the reciprocal of the estimated variance of the estimate of $\hat{R}_i$.

Bühlmann showed this result in terms of credibility:

BAYESIAN ESTIMATES OF LOSS RATES

$$E[\overline{R}_i|X_i] = X_i\left[\frac{e_i}{e_i+K}\right] + \hat{R}_\bullet\left[1 - \frac{e_i}{e_i+K}\right]$$

$$\hat{R}_\bullet = \frac{\sum \frac{e_i}{e_i+K} X_i}{\sum \frac{e_i}{e_i+K}}$$

The constant K is the expected value of the process variance for one draw divided by the variance of the means among the small urns.

The quantity $\frac{e_i}{e_i+K}$ is called the credibility of X_i. It is often denoted Z_i.

The expected loss rate for the average of all classes for Problem 3 is the expected loss rate for the big urn. The formula says that when we only observe a sample from each small urn, the best estimate of the loss rate for the big urn is a weighted average of the observed averages of the small urns, with weights equal to the credibility of each urn's average loss rate. The complement of the credibility goes to the credibility-weighted average of the observed average loss rates.

Bayesian, or expected-value, credibility says that K depends on the expected value of the process variance for one unit of exposure and the variance of unknown class means.

"Classical" credibility says that K is a function of the process variance, the choice of a tolerable percentage error, such as ± 5%, and the choice of a tolerable probability of unacceptable error.

Therefore classical credibility theory will only be correct when the percentage error and probability of error are *chosen* to yield the same credibility value as expected-value credibility. In all other cases, classical credibility theory will give the wrong credibility weight, if the objective is to estimate the expected loss rates.

Implicit Assumptions:

1. All classes have some process variance per unit of exposure. That is, all classes have measures of process variance and exposure, and process variance decreases as exposure increases.
2. The underlying mean for any particular class is a random variable from a certain process, and that process is applicable to all classes. (I.e., don't credibility-weight malpractice loss rates with homeowners loss rates.)

A Practical Understanding of K

1. The Three Components of K

The "credibility constant", K, is the number of times we must sample from a small urn to have enough experience to give the observed average, X_i, a credibility of 50%. That is, when

$$E[\overline{R}_i|X_i] = \frac{1}{2}X_i + \frac{1}{2}\hat{R}_{\bullet} ,$$

then:

$$\frac{e_i}{e_i+K} = \frac{1}{2}$$

and:

$$K = e_i$$

The purpose of this presentation is to show that K is the product of the average exposure per claim and two dimensionless quantities that reflect the predictability of claim sizes and the relevance of the grand mean to the prediction of individual means.

$$K = \frac{var[N]\,(E[Y])^2 + E[N]\,var[Y]}{var[\mu]}$$

$$= \frac{1}{E[N]} \cdot \frac{\dfrac{var[N]}{E[N]} + \dfrac{var[Y]}{(E[Y])^2}}{\dfrac{var[\mu]}{(E[N]\,E[Y])^2}}$$

$$= \frac{1}{E[N]} \cdot \frac{1+\beta+CV_y^2}{CV_\mu^2}$$

$$= \textit{Avg Exposure per Claim} \cdot \frac{\textit{Dispersion of the Loss Process}}{\textit{Dispersion of the Unknown Means}}$$

For a Poisson frequency distribution:

$$K = \frac{1}{E[N]} \cdot \frac{1+CV_y^2}{CV_\mu^2}$$

The credibility of frequency relativities is:

$$K_N = \frac{1}{E[N]} \cdot \frac{1+\beta}{CV_\mu^2}$$

Notation:

$E[N]$, $var[N]$:		Claim frequency process
$E[Y]$, $var[Y]$:		Claim severity process
$E[N]E[Y]$	=	Expected value of class means
$\frac{1}{E[N]}$	=	Average exposure per claim
β	=	Dispersion of the claim frequencies among risks within the classes. (The coefficient of variation, or CV, of a probability distribution is the ratio of its standard deviation to its mean.)
CV_Y^2	=	Dispersion of claim sizes

$CV_\mu{}'$ = Dispersion of mean loss rates

This result is exact only if frequency and severity are independent. In practice, a lack of independence is usually a negative correlation between frequency and severity and can be reflected by increasing K slightly.

2. Average Exposure per Claim

The starting point for determining K is the average exposure per claim. This could be:

Auto Insurance	60 car-years in a given class
Worker's Compensation	$120,000 of payroll in a given class.
Property, wind exposure, claims over $10 million	100 billion dollar-years of insured value.

The average exposure per claim is defined by the problem. It is easily determined from loss experience, and it is known with considerable accuracy, even if the expected claim frequency is small.

One quality of a good choice of exposure unit is that both expected loss costs and expected process variance increase in proportion to the number of units of exposure. Alternatively, the average loss rate is unaffected by the volume of exposure, and the variance of the observed loss rate decreases in proportion to the exposure.

3. Estimating the Dispersion of Claim Sizes

The next point is the determination of the dimensionless quality reflecting the volatility of the claims process. The dispersion of frequencies, β, is usually small and can usually be ignored. The quantity $(1 + CV_Y{}^2)$ can be computed from claim size data. For a group of claims valued at Y_i, $i = 1 \ldots n$, and n sufficiently large, this can be estimated from:

$$1 + CV_y^2 \doteq \frac{\sum\left(\frac{Y_i}{\overline{Y}}\right)^2}{n}$$

$$= \frac{\sum Y_i^2}{n(\sum Y_i)^2}$$

Another estimate, more stable for claim size distributions that are highly skewed (including but not limited to the lognormal) is e^{σ^2}, where σ^2 is the variance of the logs of the claim sizes. The value of $(1 + CV_Y^2)$ may be any number greater than 1, but it is usually between 5 (for claims that are not widely dispersed) and 35 (for claims that are very widely dispersed).

The following table shows values from my experience. It also shows the effect on K of truncating various claim sizes. Truncating really unusual claims sizes reduces $(1 + CV_Y^2)$ and K, but truncating more common values, such as worker's compensation claims between \$25,000 and \$100,000, has little effect.

Examples of Dispersion of Claim Sizes

Line of Business	Approximate Value $(1 + CV_Y^2)$
California School District	
Worker's Compensation	
Unlimited	40
Limited to $100,000	15
Limited to $25,000	10
Private Passenger Automobile Collision	3
Commercial Truck Liability	
Limited to $250,000	15
Limited to $600,000	25
Limited to $1,000,000	35
Hospital Professional Liability	
Unlimited	45
California Municipal Liability	
24 mm. Excess of 1 mm.	5
Physician Medical Malpractice	
Limited to $250,000	3
Limited to $2,500,000	10
Automobile Products Liability	
Unlimited	80

4. Estimating CV_μ^2

Estimating CV_μ^2 from Data

One way to estimate CV_μ^2 is to use the data from the problem at hand. If there are enough "urns" (classes, regions, etc.), and a sufficient number of them have a credible volume of experience, then a value of CV_μ^2 can be found by trial and error which gives estimates of z_i, $\hat{R}_\bullet$, and

$$CV_\mu^2 = \frac{1}{\sum Z_i^2}\left[\sum Z_i^2\left(\frac{\overline{X}_i - \hat{R}_\bullet}{\hat{R}_\bullet}\right)^2 - \frac{(1+\beta+CV_Y^2)}{e_i E[N]}\right]$$

When these simultaneous equations are solved, the credibilities are underestimated because of the dependence upon data for estimates of unkown intermediate quantities. For k classes, the unbiased estimates of the z_i are:

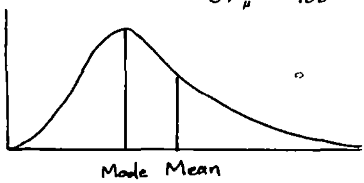
$$Z_i \leftarrow 1 - \frac{k-3}{k}(1-Z_i)$$

Often in practice the X_i are by chance close to $\hat{R}_\bullet$ and this formula gives an unreliable estimate of CV_μ^2. This is particularly a problem if there are few classes or the classes have low credibility.

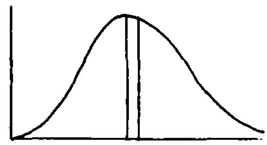
Estimating CV_μ^2 from the Relationship of Mean to Mode

In most cases of actuarial interest, the various class means must be greater than zero. As a result, the mean class mean, or grand average, is greater than the modal class mean, or most common class average. The greater the variance of the distribution, the greater the ratio of the mean to the mode.

$CV_\mu^2 = .33$



$CV_\mu^2 = .06$



Although your understanding of the classification process and the resulting means might be sketchy, you might be able to make your estimate of the CV_μ^2 consistent with your understanding of the extent by which the mean class mean exceeds the mode.

For example, for a gamma distribution of unknown class means, the results are as follows. A gamma is a reasonable choice because of its genesis as mixture of exponentials.

Gamma Distribution of Unknown Means

Ratio of Mean to Mode	CV_μ^2	$\frac{1}{CV_\mu^2}$
2.00	.50	2
1.50	.33	3
1.25	.2	5
1.11	.1	10
1.06	.06	18
1.03	.03	34

The reciprocal of CV_μ^2 is usually between 2 and 25. A tabulation of data that divides good risks from bad will lead to a high ratio of the mean to the mode and a CV_μ^2 of as much as 0.50. A tabulation that does not meaningfully distinguish one group from another (such as tabulation by accident year of on-level premiums and losses) will lead to a low ratio of mean to mode and a low CV_μ^2 of 0.03 or less.

5. Introduction of Bias

These estimates of loss rates are biased (in statistical terms) because they rely on outside data. This is unimportant. In practice, the gain in accuracy more than makes up for the bias that is introduced. Like Hiawatha's tribesmen, we are introducing some bias in order to hit the target more often. Even more important, we are aiming our arrows at the target of expected-value estimation.

Credibility of Claim-Free Experience

A simple example of the usefulness of Bayesian credibility is the calculation of the credit for claim-free experience for a particular risk. One such credit is offered by reinsurers whose risks present seven years of claim-free experience. Another such credit is offered by auto insurers who give lower rates to claim-free drivers.

$$E[\mathit{loss\ rate},\ 0\ \mathit{claims}] = \hat{R}_{\bullet}\left(1 - \frac{e_i}{e_i + K}\right)$$

where $\hat{R}_{\bullet}$ = a priori estimate of expected rate

= 1.0, for determining a credit for claim-free experience.

Charge $=$ $\left(1-\frac{e_i}{e_i+K}\right)$

Credit $=$ $\frac{e_i}{e_i+K}$

We define our exposure period to be 1.0 units. Then:

Credit $=$ $\frac{1}{1+K}$

$$= \frac{E[N]\cdot CV_\mu^2}{1+\beta+E[N]\cdot CV_\mu^2}$$

Where:

$E[N]$ = The expected number of claims in the exposure period.

CV_μ^2 = The dispersion of means of claim frequencies from risk to risk.

Examples

The credit is:

	ß:	0		0.3	
$E[N]$	CV_μ^2:	.05	.50	.05	.50
0.10		.0050	.048	.0038	.037
1.00		.048	.33	.037	.28
10.0		.33	.83	.28	.79

If the risks are believed to be different from one another the credit is more than if the risks are believed to be similar to one another. The greater the number of expected claims, the greater the credit for claim-free experience.

The Valuation of a Pure Risk Element

by David Ruhm

The Valuation of a Pure Risk Element

Abstract: It is a generally accepted principle of finanacial theory that an assumption of risk entitles the assuming party to a higher expected return on investment. This is paralleled in property/casualty insurance by the concept of a risk/contingency loading, or underwriting profit margin, which varies directly with the riskiness of the business written. A risky liability can be separated into two distinct components: a fixed liability, and a pure-risk element which is neither an asset nor a liability, but which negatively impacts net worth. It is demonstrated that, under certain assumptions: 1) the dollar value of a given risky liability is inversely related to the net capitalization of the entity assuming or retaining it, and 2) the transfer of risk from a lower-capitalized entity to a higher-capitalized entity for an appropriate premium results in gain for both parties, allowing them to achieve higher rates of return than would otherwise be available. This implies that insurance offered at an appropriate premium creates net economic value for both parties, aside from the value created by the "pooling" of risks. A fair premium is defined to be the premium which equalizes the gains to both parties.

Introduction

Insurance is a transaction which is designed to generate economic value for both parties, the customer and the insurer. Implicit to an insurance transaction's creation of value is that the economic cost of the risk being transferred is greater to the insured than to the insurer. This is in contrast to a fixed, riskless liability which has the same discounted cost to all parties. In analyzing the cost of a risky liability, it is useful to separately identify the portion of the liability which is pure risk, apart from the portion which is a definite, fixed liability. We will present an analysis of risk value and a method which calculates the difference in risk values between the customer and the insurer, apart from diversification or risk-pooling considerations.

Measuring the Economic Value Created by an Insurance Transaction

Suppose that, at 1/1/95, Party C (Customer) has cash assets of \$600, and Party N (Insurer) has cash assets of \$1,000. In addition, C has a risk which will come to fruition at 1/1/96 (no timing risk involved). The risk consists of either a gain of \$100 or a loss of \$100, each with probability 50%. For this example, N has assets only, with no risks. Assume the risk-free rate is constant at 8% per year. Then, if no transfer of risk takes place, N will have assets of \$1,080 on 1/1/96, and C will have assets of either \$748 or \$548, each with probability 50%. The rates of return are 8.00%, 24.67%, and -8.67%, respectively.

If an investor invests \$100,000 for two years, at 5% for one year and 9% for the other year, he will end up with (\$100,000)(1.05)(1.09) = \$114,450. If instead the invetment had been at 7% for both years, the result would be (\$100,000)(1.07)(1.07) = \$114,490. Thus, there is a loss of income associated with deviation from the arithmetic mean. If the investor makes a long-term investment in which the annual return is either 5% or 9% each year, always with 50% probability, and dividends are reinvested for compounding, then the effective annual rate will approach:

$$\sqrt{(1.0500)(1.0900)} - 1 = 6.98\%$$

with near certainty. This is just the principle that the geometric mean is less than the arithmetic mean, and recognition of the fact that compound interest is geometric. (A fuller technical discussion of this aspect of the model appears in Appendix 1.)

If we apply the above principle to Party C's position, we get a long-term effective annual rate of:

$$\sqrt{(1.2467)(0.9133)} - 1 = 6.71\%$$

a 6.71% return-including-risk. Receiving an effective 6.71% return when the available risk-free rate is 8.00% is equivalent to the Customer's having **risk-discounted capital** of:

$$(600.00)(1.0671)/(1.0800) = \$592.81$$

Thus, the value of ceding the risk from C's standpoint is \$600.00 - \$592.81 = \$7.19. By ceding the risk for a premium that is less than \$7.19, C will be able to invest the remaining assets (net of premium), and obtain a better return than by retaining the risk. If C cedes the risk for a premium of \$6.10, the long-term effective rate of return will be:

$$(600.00-6.10)(1.0800)/(592.81) = 8.20\%.$$

Suppose N assumes the risk on 1/1/95 for a premium of \$6.10. Then, N invests \$1,006.10 at 8% and has assets of either \$986.59 or \$1186.59 at 1/1/96, for a long-term effective rate of return of:

$$\sqrt{(0.98659)(1.18659)} - 1 = 8.20\%$$

This is an improvement over the 8.00% rate available without assuming any risk. To the two decimal places calculated, the returns to both parties are equal - therefore, the premium is "fair". The transaction has made it possible for **both** parties to realize a higher effective rate of return than would be available without the risk transfer.

Each party's net economic gain can be calculated directly. Customer C has ceded a risk worth \$7.19 for a premium of \$6.10, realizing a gain of \$1.09, or +0.18% on initial risk-discounted capital. Insurer N began with assets of \$1,000.00 and no liabilities. After the transaction , N has assets of \$1,006.10 and the assumed risky liability, for \$1,001.83 in risk-discounted capital, a gain of \$1.83, or +0.18%. Note that the parties' gains are equal as ratios to their respective pre-transaction risk-discounted capital. This **transaction gain** combined with the annual yield of

8.00% gives the post-transaction rate-of-return of 8.20% calculated earlier.

One point of this analysis is that risk should be evaluated in terms of rate-of-return on capital against which the risk is assumed, rather than independently of supporting capital. The point carries with it the concept that the value of risk is *relative* to the net capitalization of the party assuming or retaining the risk, and does not have a fixed value independent of supporting capital. This model does not use a discounting of future liabilities with an adjusted discount rate in order to valuate risk. The "risk element" liability in the example above has expected value of zero, and therefore cannot be assigned a nonzero value via the application of a discount factor, yet it has a negative value since a rational investor would not accept it without an accompanying premium.

The economic cost of the risk element in the above example can be calculated for both the customer and the insurer. It has already been shown that, for the customer, the risk element has a cost, or negative value, of $7.19 . For the insurer, the negative value of the risk element following the transaction is:

$$\$1,006.10 - \$1,001.83 = \$4.27$$

The difference between the customer's and insurer's risk values is $7.19 - $4.27 = $2.92 . It is this **risk-value gap** that allows the insurance transaction to generate net value by the transfer of risk. If the "fair premium" condition is met, the risk-value gap is split between the customer and the insurer, in proportion to the pre-transaction risk-discounted net capital of each; thus the economic gains as percentages of risk-discounted capital are equalized.

Theoretical Considerations

Any risky loss liability can be split into two parts: 1) a definite **fixed liability**, equal to the expected loss amount (discounted at the risk-free rate for time value), and 2) a **pure-risk element** with expected value of zero, equal to the loss distribution (also discounted for time value) minus the expected loss amount. For example: a loss liability with possible values $100 and $300, each with probability 50%, is equivalent to a fixed $200 liability (the expected value of the loss) combined with the zero-expected-value pure risk element from the above example. The same principles apply - the "fair" premium is calculated so as to equalize transaction gains for customer and insurer. The results are shown on Exhibit 1. After the premium calculation, the premium amount can be allocated into two parts: the discounted $200 fixed liability, and the premium for the risk element. The value of the discounted fixed liability is 200.00/1.0800 = $185.19. If C begins with assets of $785.19 ($600.00 as before, plus $185.19 to offset the fixed liability), the risk element's values and the fair risk premium are the same as calculated in the earlier example. This equivalent example is shown in Exhibit 2. The case is identical to the prior pure-risk case, with the addition of a fixed liability and offsetting assets to be transferred along with the pure-risk element.

Mathematical Development

Notation: We begin by defining the **risk-discounted assets** of an entity, X, as:

$$\Pi_X = \pi(x, <\$_j, p_j>) = \prod_j (x - \$_j)^{p_j}$$

where:

x = assets of entity X (assume no asset risk for simplicity)
j = index for loss distribution
$ = discounted loss amount
p = probability of loss amount
<$, p> = *loss distribution*

This is the general formula associated with the above analysis. Next, define **liability value** by:

$$L_X = x - \Pi_X$$

Liability value is the difference between unadjusted capital and risk-discounted capital. It is also the sum of the **fixed component** of liability, F=expected loss amount, and the **risk value**, V:

$$L_x = F_x + V_x, \text{ where } F_x = \sum p_j \$_j$$

If P represents the premium paid for the transfer of the liability, then the **net transaction gain** as a percentage of risk-discounted assets can be calculated for both C (customer) and N (insurer):

$$k_C = \frac{c-P}{\prod_C} - 1,\ k_N = \frac{\prod_N}{n} - 1, \text{ where } \prod_N = \pi(n+P, <\$_j, p_j>)$$

Note that Π_N is a post-transaction value.

The "fair premium" as defined above is the value of P for which $k_C = k_N$. When this is the case, solving the k_C equation for P yields:

$$P = c - (1+k)\prod_C$$

It can be shown that:

$$k = \frac{\Delta V}{n + \Pi_C}, \text{ where } \Delta V = V_C - V_N$$

ΔV is the **risk-value gap** - the difference in risk values between the insurer and the customer. The risk-value gap actually quantifies the total net value generated by the risk-transfer transaction. It can be shown (Appendix 2) that ΔV is closely approximated by:

$$\Delta V \approx \pi(n+L_C, <\$_j, p_j>) - n = \pi(n+V_C, <r_j, p_j>) - n, \text{ where } r_j = \$_j - F_C. \ (Note: \sum p_j r_j = 0.)$$

The distribution R = $<r_j, p_j>$ represent the **pure risk element** associated with the discounted loss distribution $<\$_j, p_j>$. As noted above, $E[R] = \Sigma r_j p_j = 0$.

Both P and k can be calculated by iteration, but close approximations can be obtained without iteration by using the above three formulae. Substituting from the earlier example,

$$\Delta V \approx (1{,}000.00 + 7.19 + \frac{100.00}{1.0800})^{1/2} \bullet (1{,}000.00 + 7.19 + \frac{100.00}{1.0800})^{1/2} - 1{,}000.00 = \$2.92$$

Then,

$$k \approx \frac{2.92}{1{,}000.00 + 592.81} = 0.18\%, \quad P \approx 600.00 - 592.81(1.0018) = \$6.12$$

closely approximating the exact answers obtained by iteration.

Generalization of Model

The geometric function used to evaluate risk comes from a model which assumes: 1) at any given time, a party will assume or acquire risk which is proportional in magnitude to the party's assets, and 2) the shape of the pure risk density function is always the same. The model does not consider any externally generated infusions of capital. This is the simplest model for performing calculations and illustrating the concepts being presented. Other functions based on alternative modeling assumptions can be used to value risk as a function of capitalization.

To generalize the mathematics above, let $V^R(x)$ be a **risk-value function** which maps capitalization net of discounted expected losses to risk value, for a given pure risk element (R). Thus V is parameterized by R, and:

$$V^R(x) = x - x',$$

where x' = risk-discounted value of net capital x combined with pure risk R.

Two basic tenets of this paper are: 1) $V^R(x) > 0$ for all x and all R, and 2) $V^R(x)$ is a decreasing function of x for any fixed R. The formulas given earlier for the geometric model generalize to:

$$k_C = \frac{V^R(c) - P}{c - V^R(c)}$$

$$k_N = \frac{P - V^R(n + P)}{n}$$

$$P = V^R(c) - kc'$$

$$k = \frac{\Delta V}{n + c'}$$

$$\Delta V = V^R(c) - V^R(n + P) \quad (by\ definition)$$

$$\Delta V \approx V^R(c) - V^R[n + V^R(c)]$$

The last formula is significant, because it gives the approximate value of an insurance transaction using only the capitalization of both parties, the risk-value function V, and the risk element R.

Let $V = V^R(c)$ and $W = V^R(n + P)$. Then, from the above definition, $\Delta V = V - W$. Also, c' = c - V. Combining the above formulas for k and P and reintroducing discounted expected losses (F), algebra yields:

$$P = \frac{Vn + Wc'}{n + c'} + F$$

This is the general premium formula for pricing a risky liability, based on estimates of the pure risk element's respective values to the customer and insurer (V and W), capitalizations net of discounted expected losses (c and n), and the amount of discounted expected losses being transferred (F). The fraction term is the risk load. It is a cross-weighted average of the customer's and insurer's risk values, each weighted on the other party's net capitalization. This creates the even split of the insurance transaction gain.

Example: Consider the following situation:

Customer Assets	= 2,500
Discounted Exp.'d Losses (F)	= 500
Customer Net Assets (c)	= 2,500 - 500 = 2,000
Insurer Assets (n)	= 10,000
Risk Value to Customer (V)	= 400
Risk Value to Insurer (W)	= 250

Then c' = c - V = 1,600, and:

$$P = \frac{(400)(10,000) + (250)(1,600)}{10,000 + 1,600} + 500$$

P = \$879.31

Risk Load = \$879.31 - \$500.00 = \$379.31

Customer Transaction Gain = \$400.00 - \$379.31 = \$20.69 = 1.29% of \$1,600

Insurer Transaction Gain = \$379.31 - \$250.00 = \$129.31 = 1.29% of \$10,000

Premium vs. Transaction Gain

For the customer, the transaction gain is inversely related to the premium paid. For the insurer, the transaction gain is directly related to the premium received. This relationship is depicted in Exhibit 3, which shows gain vs. premium for both the customer (the downsloping curve) and the insurer (upsloping curve). The point of intersection indicates the "fair" premium P and gain k described earlier. A mutually profitable transaction can occur at any premium in the range between the two points where the curves intersect the x-axis. This **competitive premium range** varies with the two parties' capitalizations. The width of the competitive premium range equals the risk-value gap. The plot of the customer and insurer gain-vs.-premium curves provides a picture of the risk-value gap.

Capitalization and Price Competition

As insurer capitalization increases, the cost to the insurer of assuming a given risk decreases, creating a larger risk value gap. At first, it appears that higher insurer capitalization would imply greater potential for total net (customer plus insurer) transaction gain, and therefore a lower fair premium. In fact, this is true only up to a point. As Exhibit 4 shows, there is an amount of insurer capitalization which minimizes the fair premium. This is because the insurer's transaction gain is equal to premium minus assumed risk value, divided by initial assets. Therefore, a larger insurer must receive more premium net of assumed risk value to achieve the same return as a smaller insurer. Up to a certain capitalization, the risk value decreases at a rate which is fast enough to provide additional return to both the customer and the insurer. Beyond that point, the decrease in risk value is not sufficient to support the increased insurer capitalization, and additional premium must be collected.

Exhibit 5 shows the gain vs. premium curves for two insurers, L (low capitalized) and H (high capitalized). For higher premium values, L has an advantage since the same amount of premium received produces a larger percentage return on capital. For lower premium values, H shows an advantage over L due to the lower risk value. There is a range of premiums for which H shows a gain whereas L shows a loss. By offering a premium in this range, H provides the customer with a lower premium than L can afford to offer. This advantage is present whenever H's capital exceeds L's capital. To exercise this competitive pricing advantage, H might have to accept a smaller transaction gain than the fair premium would give. The conclusion is that the dynamic of capacity and demand in the competitive marketplace may preclude equal transaction gains. The transaction can still be financially beneficial to both parties, and can be considered "fair" from a supply-and-demand perspective.

Reducing Risk by Combining Independent Risks

The discussion so far has focused exclusively on the value generated by a single insurance transaction between two parties, and the value generated by transferring risk from a party with less capital to a party with more capital, for an appropriate premium. It is also possible for value to be generated by an insurance transaction through the combining of two or more independent risk elements. When the resulting combined risk element is supported by capitalization equal to the total of the original capitalizations that supported the individual risk elements, the risk value will be less than the sum of the individual elements' risk values, creating a risk-value gap.

Example: Consider two identical customers A and B. Each customer has assets of $2,000.00 and a risky liability which will cost $1,000.00 (discounted to present value) with probability 50%, or $0.00 with probability 50%. The liabilities' future outcomes are independent of each other. Then, each liability has a fixed component of $500.00 in expected losses and a pure risk element of:

+$500.00 @ 50%
-$500.00 @ 50%

Calculation of the pure risk element's value relative to either customer yields V=$85.79 per liability, or $171.58 in total for both.

Suppose N is an insurer with assets of $2,870.38 and no liabilities. A and B both agree to transfer their liabilities to N along with a premium of $564.81 each - $500.00 for the fixed component, and $64.81 for the risk element. After the transaction, N has assets of $2,870.38 + 2($564.81), or exactly $4,000.00. This is equal to the combined original assets of the two customers. Thus, N is in the same position that would have been created if A and B had merged their assets and risky liabilities into one entity. The new pure risk element is:

+$1,000.00 @ 25%
$0.00 @ 50%
-$1,000.00 @ 25%

The $1,000.00 amount equals the sum of the original risk elements' amounts, but at half of their original 50% probability. Qualitatively, half of the original risk has been replaced by a 50% probability of no change. This is one advantage of viewing the loss distribution as a fixed liability plus a pure risk element - different pure risk elements can be compared to each other directly for qualitative evaluation. The risk value for N is $87.05, which is about half of the original total risk value. The transaction gain in this example is equal for all parties, at 1.48%.

Alternatively, A and B could have agreed to reinsure each other for 50% of their respective liabilities. No premium would be involved in this symmetric liability "swap". The result is approximately the same - the risk elements are reduced similarly and the risk value is reduced by about half.

If the insurer in this example had less capital, the gain from combining independent risks would have been partially or completely offset by the reduction in total capital supporting the total risk. At some point, the capital limitations of an insurer create a "diminishing returns" effect with regard to the assumption of additional pure risk, even when the additional risk is independent of all risk currently retained by the insurer.

Conclusion

An uncertain future liability can be viewed as the sum of two components: a fixed component equal to discounted expected losses, and a pure risk component that is a distribution of outcomes with expected value of zero. Since the pure risk element has an expected value of zero, it is neither a true asset nor a true liability, but it has a discounting effect on net capital (assets minus liabilities). The cost associated with a risk element is a function of the amount of net capital against which the risk is retained or assumed.

The cost of a given risk element is inversely related to the net capital of the entity. Transferring risk from a lesser-capitalized entity to a greater-capitalized entity by an insurance transaction generates economic value equal to the difference in the values of the risk to the parties. This total gain, the "risk-value gap", is divided between the parties via the risk load in the premium. An equitable premium sets the risk load so that the parties' transaction gains (as percentages of initial net capitalization) are equal.

A risk value function is a function parameterized by a pure risk element, which maps net capital to risk value. A risk value function should be a decreasing, positive function of net capital for any given pure risk element parameter. There are an unlimited number of possible candidates for risk value functions, one of which is the geometric mean function presented in this paper.

Appendix 1 : Technical Discussion of Geometric-Mean Model

Let i_k represent an outcome (expressed as a percentage of initial capital) obtained by a risk-retaining entity in year k. Assume the i_k are independent and identically distributed. Then, define the equivalent level return j by:

$$(1+j)^n = \prod_{k=1}^{n} (1+i_k) - 1$$

The variable j represents the effective annual return that is achieved by the risk-retaining entity over the time period k=1 to n. It follows that:

$$\log(1+j) = \frac{1}{n}\sum_{k=1}^{n} \log(1+i_k) \sim Normal(\log(1+g), \sigma^2)$$

where 1+g represents the geometric mean of the $\{1+i_k\}$. Therefore, the distribution of 1+j is approximately lognormal for large n, with mode equal to 1+g.

The mean of the lognormal is higher than the mode because the lognormal is skewed, and the mean includes extreme, low-probability results in the tail. As n gets large, the standard deviation σ decreases, and the probability of actual results falling within a fixed interval around the mode 1+g increases toward unity. Therefore, the mode is most representative of the anticipated long-term return for a single entity. In particular, it is more representative than the mean which is distorted by extreme values that are unlikely to be realized. The mean would be representative of average aggregate results for many independent entities (e.g., an entire industry), because a small but positive percentage of the entities could reasonably be expected to obtain the extreme results in the distribution's tail. Also, a single entity which can obtain new capital after a negative or low result could be expected to achieve an overall result closer to the mean, since such capital infusions would effectively generate repeated trials.

An Alternative Risk-Value Function for Shorter Term Results

The model presented in this paper assumes that the hypothetical insurer will always follow a consistent policy of assuming risk in constant proportion to current capitalization. So, if the insurer's capital increases, it will assume more risk, and if capital declines, less risk will be assumed. While somewhat simplified, this assumption probably corresponds in some degree to actual conditions, since writings are dependent on surplus.

In the model, if the insurer never places 100% of capital at risk, it can never go broke. However, if the insurer does risk 100% of capital, even with a very small probability of total loss, the geometric model discounts insurer capital to zero, because long-term pursuit of such a policy would inevitably lead to ruin.

Suppose a customer has a 99% probability of no loss for a single time period, and a 1% chance of total loss. According to the geometric model, the customer has risk-discounted assets of zero. This is counterintuitive, since the customer's assets combined with the low-probability risk should have positive value. Moreover, they should have more value than if the probability of total loss were higher (say 80%). Therefore, for a shorter-term scenario such as this one, an alternative risk-value function would be appropriate. One possibility is:

$$V^R(c) = c - [tg + (1 - t)m]$$

where m is the arithmetic mean, and t is a weighting parameter between 0 and 1. The probability of severe or total loss is incorporated in this formula by including the arithmetic mean, m. One way to calculate a value for the t parameter is to exclude the extreme upper and lower tails (selecting a percential cutoff point) of a lognormal distribution, take the mean M of the resulting distribution, set M equal to the expression in brackets in the above equation, and solve for t.

To the extent that a given insurer risks insolvency, the insurance transaction includes an implicit assumption of credit risk by the customer. Additionally, insurers assume and retain risk on a regular and controlled basis over many years. This is in contrast to customers, who are exposed to risk in what may be a more volatile or temporary manner. It is therefore reasonable for a risk-value model to severely discount insurer capital when a significant risk of insolvency exists, in relation to the probability of insolvency risk.

Appendix 2 : Derivation of Risk-Value Gap Approximation Formula

By definition, $V_C = L_C - F_C$, $V_N = n + P - \Pi_N - F_C$, $\Pi_N = \pi(n + P, <\$_j p_j>)$, $\Delta V = V_C - V_N$.

Approximate P in the V_N and Π_N formulas by $P \approx L_C = c - \Pi_C$. Then,

$$V_N \approx n + L_C - \pi(n + L_C, <\$_j, p_j>) - F_C$$

$$\Delta V \approx V_C - [n + L_C - \pi(n + L_C, <\$_j, p_j>) - F_C]$$

and since $V_C = L_C - F_C$ the result follows:

$$\Delta V = \pi(n + L_C, <\$_j, p_j>) - n .$$

Exhibit 1

Fair Premium and Risk Values Calculation

CUSTOMER POSITION
Pre-Transaction

Assets: $600.00

	Amount	Probab'lty
Liability:	$100.00	50%
	$300.00	50%

Risk-Disct'd Assets: $404.35

INSURER POSITION
Pre-Transaction

Assets: $1,000.00

Liability: (None)

Risk-Disct'd Assets: $1,000.00

Fair Premium:	$193.86
Fixed Liability Component (Disct'd):	$185.19
Fair Risk Premium Component :	$8.67

CUSTOMER POSITION
Post-Transaction

Assets @ 1/1/95: $406.14
Interest Rate: 8.00%
Assets @ 1/1/96: $438.63

Liability: (None)

Risk-Disct'd Assets: $438.63
Rate of Return: 8.48%
Transaction Gain: 0.44%

INSURER POSITION
Post-Transaction

Assets @ 1/1/95: $1,193.86
Interest Rate: 8.00%
Assets @ 1/1/96: $1,289.37

	Amount	Probab'lty
Liability:	$100.00	50%
	$300.00	50%

Risk-Disct'd Assets: $1,084.77
Rate of Return: 8.48%
Transaction Gain: 0.44%

Exhibit 2

Fair Premium and Risk Values Calculation

CUSTOMER POSITION
Pre-Transaction

Assets:	$785.19	
	Amount	Probab'lty
Liability:	$100.00	50%
	$300.00	50%
Risk-Disc'td Assets:	$592.81	

INSURER POSITION
Pre-Transaction

Assets:	$1,000.00
Liability:	(None)
Risk-Disc'td Assets:	$1,000.00

Fair Premium:	$191.29
Fixed Liability Component (Disct'd):	$185.19
Fair Risk Premium Component :	$6.10

CUSTOMER POSITION
Post-Transaction

Assets @ 1/1/95:	$593 90
Interest Rate:	8.00%
Assets @ 1/1/96:	$641.41
Liability:	(None)
Risk-Disc'td Assets:	$641.41
Rate of Return:	8.20%
Transaction Gain:	0.18%

INSURER POSITION
Post-Transaction

Assets @ 1/1/95:	$1,191.29	
Interest Rate:	8.00%	
Assets @ 1/1/96:	$1,286.59	
	Amount	Probab'lty
Liability:	$100.00	50%
	$300.00	50%
Risk-Disc'td Assets:	$1,081.98	
Rate of Return:	8.20%	
Transaction Gain:	0.18%	

Exhibit 3

Gain vs. Premium for Customer and Insurer

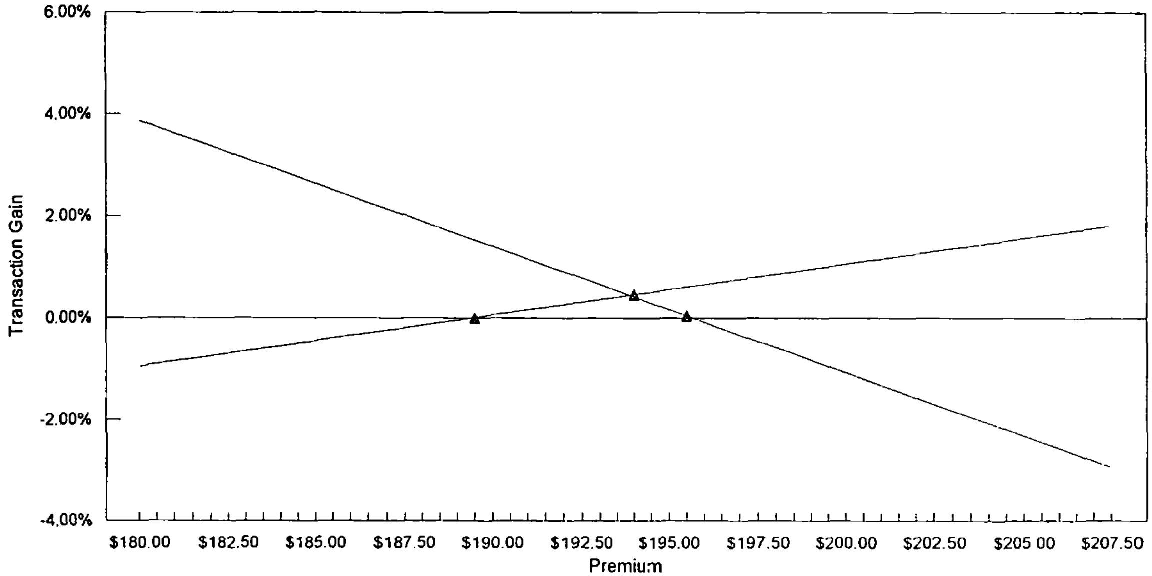


Exhibit 4

Fair Premium vs. Insurer Capitalization

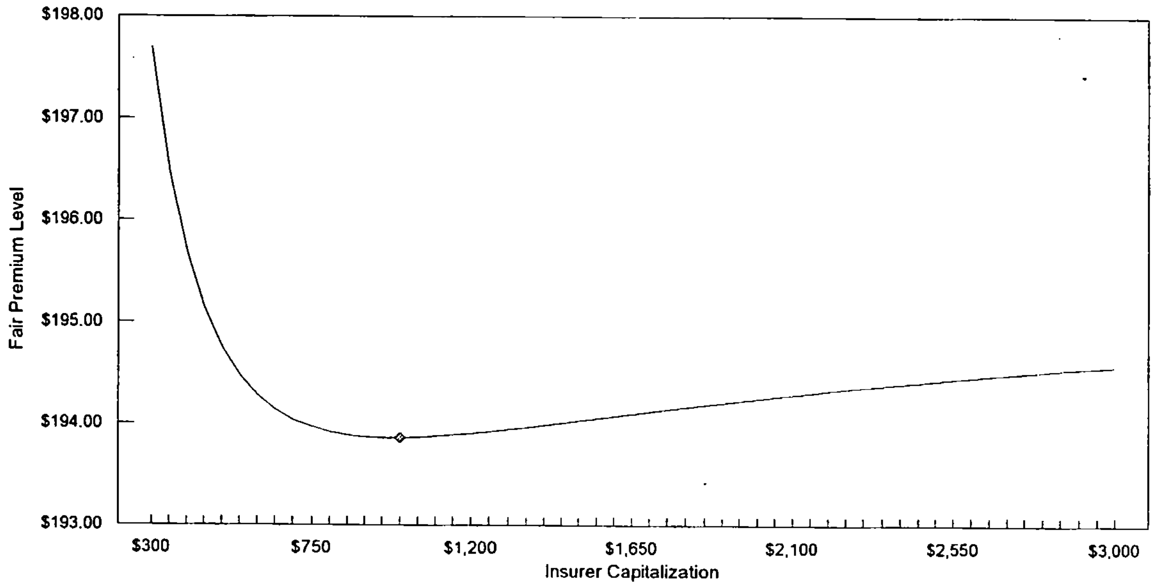
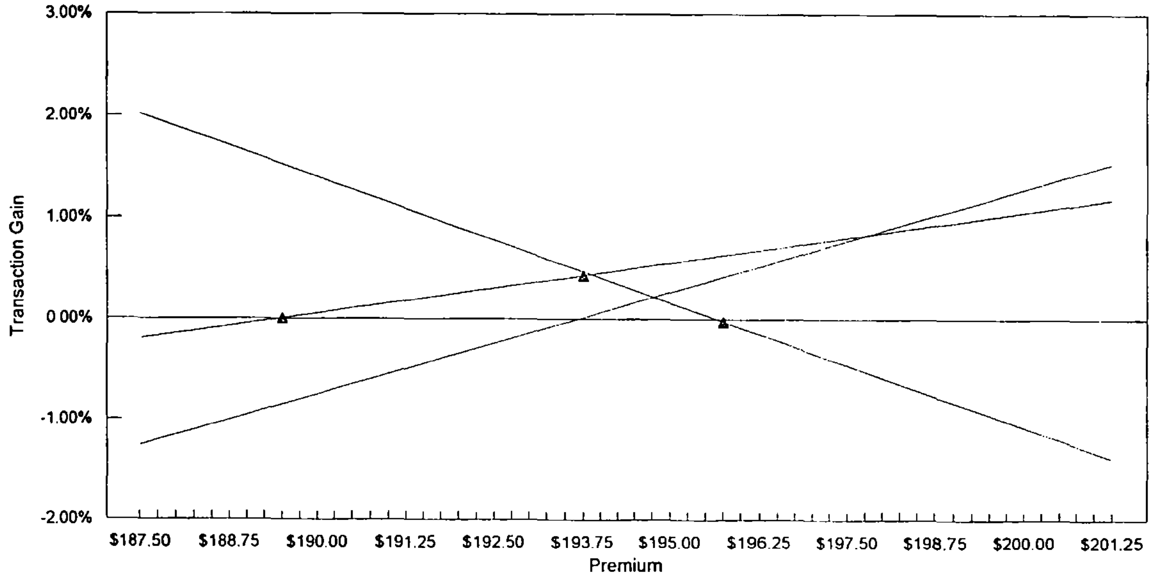


Exhibit 5

Gain vs. Premium, Customer and 2 Insurers



Post-Reform Ratemaking: Adjustment of Pre-Reform to Post-Reform Loss Development Patterns

by Mujtaba Datoo

Objective Ratemaking data, particularly loss development patterns, for a state which has enacted major workers compensation reform is not available for a number of years following reform. As a result adjustment, or actuarial judgment, needs to be applied to historical pre-reform data to reflect expected post-reform loss development patterns. The adjusted pattern can then be incorporated into traditional ratemaking methodolgies.This paper offers a model to calculate actuarially appropriate adjustments for this situation.

Background Colorado enacted workers compensation reform SB 218 effective July 1, 1991. This reform resulted in a savings of 32.8% in indemnity loss costs based on the initial pricing by National Council On Compensation Insurance (NCCI). The bulk of the quantified savings came from Permanent Total and Permanent Partial benefits. The law memo in the June 27, 1991 Colorado filing outlines the details of SB 218.

Data

- Ratemaking utilizes aggregate data from annual financial calls to derive rate level (loss costs) adequacy. These calls, while comprised of more recent data, do not provide detailed breakdown of benefits by injury type. Financial data is currently reported to a thirteenth report.
- Unit Statistical Plan (USP) data, which lags financial data in reporting, contains claim counts and incurred (paid+case) losses by benefit type. The benefit types are Fatal (F), Permanent Total (PT), Permanent Partial - major (PP-major), Permanent Partial - minor (PP-minor) and Temporary Total (TT).

The USP data is reported from a first to fifth reporting.

Assumptions under SB 218 From the actuarial law memo analyzing SB 218, the following assumptions are incorporated into the model :

1) Fatal: No impact.
2) Permanent Total: Tightened definition of PTs, hence severity not impacted, but frequency reduced by 75%.
   The claims that used to be PT under pre-reform will now shift to PP-major.
3) Permanent Partial - major: These are considered Non-Scheduled benefits. SB 218 impacted both severity and frequency.
   Some PP-major claims would shift to PP-minor (Scheduled) benefits.
4) Permanent Partial - minor: These are considered Scheduled benefits. There is a frequency increase from PP-major, but no severity change.
5) Physician Choice: Reduces PP and TT severity by 1.4%.
6) Overall claim counts do not change.
7) There is no reform impact on development (for paid+case outstading reserves) beyond fifth report. It is assumed that the cases are adequately reserved beyond this report.

Methodology The derivation of adjustments is accomplished in five steps. The model is outlined on Exhibit A.

Step One: *Compilation of data.*

USP data was compiled for the latest five policy periods ending with period 3/89-2/90. All of this data is pre-reform and includes paid plus case losses.

From this data ultimate claim counts (frequency) and severity were calculated for policy period 3/89-2/90. (Note that the severity reconciles with that in the 1993 Annual Statistical Bulletin, Exhibit XI, page 282.)

The data was tabulated by benefit type and reporting age, i.e. at first report, second report, etc. The pre-reform claim counts represent the three-year average excluding high/low data points from the latest five periods of data.

Step Two: *Incorporation of SB 218 assumptions.*

The impact of SB 218 by type of benefit is applied to pre-reform frequency and severity to obtain corresponding post-reform frequency and severity.

Exhibit A displays the assumptions and procedure.

Based on the assumptions stated, PT frequency is reduced by 75%.

The PP-major frequency is reduced by 35%. Extracted from Exhibit IV of the law memo, this figure is derived from the number of claims shifting from Non-Scheduled (193 claims) to Scheduled (363 claims). This results in a decrease in frequency of 35% [0.65 = 363/(363+193)].

These PP-major claims shift to PP-minor thus resulting in an increase in frequency of 35% (from 3,450 PP-minor claims to 4,639 claims. This difference comes from the 35% decrease of PP-major claims (0.35 x 3,397 = 1,189).

The overall impact on all benefits is 32.8%. The impact on combined PP (major + minor) is 26.3%. Thus the missing piece, the severity component of PP-major, is determined by a trial-and-error approach to ensure that the overall savings of 32.8% and about 26% of PP savings are obtained. A decrease in severity of 6% yields these desired impacts.

Step Three: *Pre-reform loss development.*

Claim counts by type of benefit at each report are then multiplied by pre-reform severity. This produces the amount of losses at that particular report. For example, the 54 Fatal claims at first report are multiplied by the Fatal severity of $220,780 amounting to $11,920,000 (rounded to the nearest thousand) of Fatal losses at first report. Likewise, at second report the Fatal losses amount to $13,688,000 (62 x $220,780).

The losses by benefit are aggregated at each report. This produces *pre-reform* report-to-report loss development factors, i.e. from 1st-to-2nd (1:2), 2:3, 3:4 and 4:5.

Step Four: *Post-reform loss development.*

(a) Claim counts from pre-reform are adjusted by the SB 218 impacts. These adjusted claims are then multiplied by the adjusted (post-reform) severity amount to obtain the losses at each report by benefit type.

For example, PT claims were reduced by 75%, with no severity impact. Thus, at first report, the 35 pre-reform PT claims are reduced by 75% from 35 to 9. Likewise, at second report the 90 pre-reform PT claims are reduced by 75% to 23 claims, etc. These claims are then multiplied by PT post-reform severity (unchanged) of $327,791 generating PT losses of $2,950,000 (9 x $327,791) and $7,539,000 (23 x $327,791) at first and second reports, respectively. This process is continued for the other reportings, up to the fifth report.

For PP-major, the 2,179 claims at first report are adjusted by the 35% decrease in frequency (0.65 x 2,179 = 1,416) and the shifted PT claims (75% of 35 = 26) are added to obtain post-reform claims of 1,442 (1416 + 26). These 1,442 PP-major claims are finally multiplied by the post-reform severity of $73,222 producing PP-major losses at first report of $105,586,000.

(b) The losses by benefit are aggregated at each report. This produces *post-reform* report-to-report loss development factors, i.e. from 1st-to-2nd (1:2), 2:3, 3:4 and 4:5.

Step Five: *Pre- and post-reform loss development comparison.*

As can be seen on Exhibit A, the pre-reform and post-reform loss development factors (LDF) at each report can now be compared. Post-reform development patterns can now be derived by adjusting the 1st-to-2nd loss development factor (1:2 LDF) by -6.8%, 2:3 LDF by -2.8%, 3:4 LDF by -1.1%, and 4:5 LDF by 0.6%. The resultant report-to-ultimate adjustments to pre-reform LDF are -10.4% for 1:ULT, -3.9% for 2:ULT, -1.1% for 3:ULT and 0.6% for 4:ULT. By assumption (7), there is no impact on development beyond fifth report.

Application of LDF adjustments

The report-to-report loss development factors (LDF) adjustments are applied to pre-reform loss development patterns from the aggregate financial calls. Loss development factors are as follows:

Indemnity Paid+Case Loss Development Factors

Policy Year	1:2	2:3	3:4	4:5	5:ULT
1984				1.036	
1985			1.057	1.052	
1986		1.149	1.069	1.058	
1987	1.307	1.162	1.090	1.038	
1988	1.312	1.155	1.061	1.016	
1989	1.310	1.137	1.045		
1990	1.281	1.139			
1991	1.235				
5-year average	1.289	1.148	1.064	1.040	1.130
5-year ex hi/lo average	1.299	1.148	1.062	1.042	1.130
latest 2-year average	1.258	1.138	1.053	1.027	1.123
Pre-reform LDF selected	1.300	1.145	1.060	1.040	1.125
Adjustment Factor	0.932	0.972	0.983	1.006	1.000
Post-reform LDF	1.122	1.113	1.042	1.046	1.125

Report-to-ultimate	1:ULT	2:ULT	3:ULT	4:ULT	5:ULT
Pre-reform	1.844	1.419	1.240	1.170	1.125
Post-reform	1.651	1.363	1.225	1.176	1.125
% change	-10.3%	-3.9%	-1.2%	0.5%	0%

Conclusion

As experience unfolds under the post-reform environment, assumptions underlying the model and the original pricing can be tested and re-evaluated. So far, these assumptions have proven valid, or have not been conclusively disproven, by special aggregate financial calls collected to monitor this reform.

While actuarial judgment, supported by claim adjusters' impressions, can be substituted to establish post-reform development patterns, this model can be employed, in addition to actuarial judgment, to determine a more statistically and actuarially appropriate pattern.

LOSS DEVELOPMENT FACTOR ADJUSTMENT MODEL : PRE-REFORM TO POST-REFORM

assumptions:
[1] frequency: no change in total claim counts; PT claims shift to PP-major; PP-major claims shift toPP-minor.
[2] severity: no change for PT; -.1.4% for PP, TT due to choice of physician; PP-major decreases by -30.3% (and -1.4% for physician choice)
[3] no LDF adjustment beyond 5th report

type of benefit	pre-reform claims	pre-reform severity	impact of reform freq	impact of reform severity	post-reform claims	post-reform severity	total cost = freq x severity (thousands) pre-reform	post-reform	impact	
Fatal	66	220,780	0.0%	0.0%	66	220,780	14,571	14,571	0.0%	
Permanent Total	267	327,791	-75.0%	0.0%	67	327,791	87,520	21,962	-74.9%	
Permanent Partial - major	3,397	77,896	-35.0%	-6.0%	2,408	73,222	264,613	176,319	-33.4%)	-25.7%
Permanent Partial - minor	3,450	10,127	34.5%	-1.4%	4,639	9,985	34,938	46,320	32.6%)	
Temporary Total	19,334	1,765	0.0%	-1 4%	19,334	1,740	34,125	33,641	-1.4%	
total	26,514	19,131	0.0%	-32.8%	26,514	12,856	435,767	292,814	-32.8%	

	claim counts ==> pre-reform @1st	@2nd	@3rd	@4th	@5th	total losses (thousands) ==> pre-reform @1st	@2nd	@3rd	@4th	@5th
Fatal	54	62	66	72	79	11,922	13,688	14,571	15,896	17,442
Permanent Total	35	90	162	219	235	11,473	29,501	53,102	71,786	77,031
Permanent Partial - major	2,179	3,012	3,172	3,062	2,913	169,735	234,623	247,086	238,518	226,911
Permanent Partial - minor	3,666	3,265	3,207	3,084	3,271	37,126	33,065	32,477	31,232	33,125
Temporary Total	20,014	21,070	22,785	23,556	24,719	35,325	37,189	40,216	41,576	43,629
total	25,948	27,499	29,392	29,993	31,217	265,580	348,066	387,453	399,008	398,138

	claim counts ==> post-reform @1st	@2nd	@3rd	@4th	@5th	total losses (thousands) ==> post-reform @1st	@2nd	@3rd	@4th	@5th
Fatal	54	62	66	72	79	11,922	13,688	14,571	15,896	17,442
Permanent Total	9	23	41	55	59	2,950	7,539	13,439	18,029	19,340
Permanent Partial - major	1,442	2,025	2,183	2,154	2,069	105,586	148,275	159,844	157,720	151,496
Permanent Partial - minor	4,929	4,390	4,312	4,147	4,398	49,216	43,834	43,055	41,408	43,914
Temporary Total	20,014	21,070	22,785	23,556	24,719	34,824	36,662	39,646	40,987	43,011
total	26,448	27,570	29,387	29,984	31,324	204,499	249,998	270,556	274,040	275,203

comparison of LDF (total)	1:2	2:3	3:4	4:5	1:ULT	2:ULT	3:ULT	4:ULT
pre-reform	1.311	1.113	1.030	0.998	1.500	1.144	1.028	0.998
post-reform	1.222	1.082	1.013	1.004	1.344	1.100	1.017	1.004
adjustment to pre-SB 218 LDF	0.932	0.972	0.983	1.006	0.896	0.961	0.989	1.006
	-6.8%	-2.8%	-1.7%	0.6%	-10.4%	-3.9%	-1.1%	0.6%

EXHIBIT A

The Complement of Credibility

by Joseph A. Boor

ABSTRACT

THE COMPLEMENT OF CREDIBILITY

This paper explains the most commonly used complements of credibility and offers a comparison of the effectiveness of the various methods. It includes numerous examples. It covers credibility complements used in excess ratemaking as well as those used in first dollar ratemaking. It also offers six criteria for judging the effectiveness of various credibility complements. One criterion, statistical independence, has not previously been covered in the actuarial literature. This paper should explain all the common credibility complements to the actuarial student.

THE COMPLEMENT OF CREDIBILITY

Many actuarial papers discuss credibility. Actuaries use credibility when data is sparse and lacks statistical reliability. Specifically, actuaries use it when historical losses have a large error around the underlying expected losses (average of the distribution of potential loss costs) the actuary is estimating. In thōse circumstances, the statistic that receives the remainder of the credibility can be more important than the credibility attached to the data. For example, if the ratemaking statistic varies around the true expected losses with a standard deviation equal to its mean, it will probably receive a very low credibility. So, the vast majority of the rate (in this context, expected loss estimate) will come from whatever statistic receives the complement of credibility. So, it is very important to use an effective statistic for the complement of credibility. This paper will discuss fundamental principles to use in choosing the complement. And, it will discuss several methods actuaries use regularly.

I. Fundamental Principles- What Should The Actuary Consider?

There are four types of issues that any actuary must consider when choosing the complement: practical issues; competitive market issues; regulatory issues; and, statistical issues.

A. Practical Issues

The easiest statistic to use is one that is readily available. For example, the best possible statistic is next year's loss costs. Unfortunately, that statistic is not available (otherwise, companies would not need actuaries). The actuary must choose from the statistics that are available to him. Since some statistics require more complicated programming or expensive processing than others, some statistics are more readily available than others.

Ease of computation is another factor to consider. If a statistic is easy to compute, it is often easier to explain to management and customers. Since few actuaries have unlimited budgets, they usually weigh the time involved in computing a very accurate statistic against the accuracy improvement it generates. Also, when computations are easy to do there is less chance of error.

B. Competitive Market Issues

Rates are rarely made in a vacuum. Generally, whatever rate the actuary produces will be subject to market competition. If the rate is too high, competitors can undercut the rate and still make a profit. That will cost the actuary's employer customers and profit opportunities. If the rate is too low, the employer will lose money. So, in mathematical terms, the rate should be unbiased (neither too high nor too low over a large number of loss cost estimates) and accurate (the rate should have as low an error variance as possible around the future expected losses being estimated). Hence the complement of the credibility should help make the rate as unbiased and accurate as possible.

C. Regulatory Issues

Usually, rates require some level of approval from insurance regulators. The classic rate regulatory law requires that rates be 'not inadequate, not excessive, and not unfairly discriminatory.' The principles of adequacy and non-excessiveness imply that rates should be as unbiased as possible.

Those principles could be stretched to imply that rates should be accurate. The argument goes as follows. Highly inaccurate rates create a much greater risk of insolvency through random inadequacies. The law is concerned with inadequacy because it seeks to prevent insolvencies. So, law suggests rates should be as accurate as possible. For most purposes, actuaries interpret 'unfairly discriminatory' in the ratemaking context as 'unbiased'. Many believe that if a rate truly reflects a class's probable loss experience, it is fair by definition.

The actuary can mitigate regulatory concerns by choosing a complement that has some logical relationship to the loss costs of the class or individual being rated. That means it is easier to explain a high rate for a class or individual in light of the related loss costs.

D. Statistical Issues

Clearly, the actuary must attempt to produce the most accurate rate that is practical. If the complement of the credibility is accurate in its own right and relatively independent of the base statistic (which receives the credibility), the resulting rate will be more accurate. The rationale involves statistical properties of credibility-weighted estimates. As Appendix A shows, if the optimum credibility for two unbiased statistics is used, then the prediction error of the credibility-weighted estimate is

$$\frac{\tau_1^2\tau_2^2(1-\rho^2)}{\tau_1^2+\tau_2^2-2\rho\tau_1\tau_2},$$

where

τ_1^2 is the average squared error (inaccuracy) of the base statistic as a stand-alone predictor of next years' loss costs;
τ_2^2 is the average squared error (inaccuracy) of the complement of the credibility as a stand-alone predictor of next year's mean loss costs;
ρ is the correlation (interdependence) between the first statistic's prediction error (error in predicting next year's mean loss costs) and the second statistic's prediction error.

Reviewing that error expression shows that greater inaccuracy in either the base statistic *or* the complement of credibility will yield greater inaccuracy in the resulting prediction. The expression is symmetric in the two errors. So, the accuracy of the complement of credibility is just as important as the accuracy of the base statistic.

The benefits of independence are more subtle. As it turns out, independence is most important when credibility is most important. That is independence is most important for the intermediate credibilities (Z between 10% and 90%). Following Appendix B, that occurs when the largest standard predicting error ($\sqrt{\text{inaccuracy}} = \tau$) is within two to three times[1] the smaller error. Consider the following graphs of the total prediction error by correlation for τ_2 = one, two, and three times τ_1.

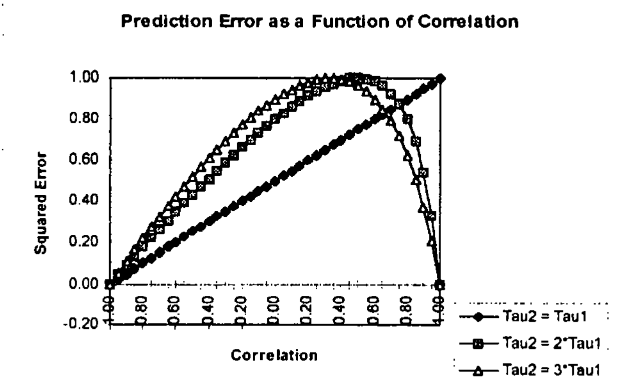


Figure 1

As you can see, the predictions are generally best when there is actually a negative correlation between the two errors (that is, they offset). But, that rarely occurs in practice. Generally, the complement of credibility will have some weak correlation with the base statistic. In that range the prediction error is clearly lowest as the correlation is smaller. Further, the graph beyond the maximum error (correlations near unity) is misleading. Appendix B shows that the downward slope near unity brings negative credibilities. Those negative credibilities are clearly outside the general actuarial philosophy of credibility. So, a complement of credibility is best when it is statistically independent (that is, not related to) the base statistic.

[1] Since Boor[1] shows that credibility is roughly proportional to the relative τ^2's, these examples cover credibilities between 10% and 90%. That range covers mosts instances where credibility matters most.

E. Summary of Desirable Qualities

The previous sections show six desirable qualities for a complement of credibility:

- Accuracy as a predictor of next year's mean loss costs;

- Unbiasedness as a predictor of next year's mean subject expected losses;

- Independence from the base statistic;

- Availability of data;

- Ease of computation; and

- Explainable relationship to the subject loss costs.

II. First Dollar Ratemaking

First dollar (that is, not pricing losses excess above a very high deductible) ratemaking credibility complements are affected by a common characteristic of first dollar ratemaking. First dollar ratemaking generally uses historical loss data for the base statistic. And, in first dollar ratemaking the historical losses are usually roughly the same magnitude as the true expected losses. The regulatory quality of an explainable relationship to the subject loss costs is more important for first dollar than excess ratemaking.

There are a wide variety of techniques actuaries use to develop credibility complements. The following pages discuss some of the major methods in use.

A. Loss Costs of a Larger Group Including the Class -- Classic Bayesian Credibility

The most basic credibility complement comes from the most classic casualty actuarial technique ... Bayesian credibility. In Bayesian credibility actuaries are typically either making rates for a large group of classes or making rates for a number of large insureds that belong to a single class. The classes (or individual insureds) do not contain enough exposure units for their historical loss data to reliably predict next year's mean loss costs. So, actuaries supplement the class's historical loss data by credibility weighting them with the loss costs of the entire group.

In mathematical terms, we use

$$Z(L_c/E_c) + (1-Z)(\sum_i L_i / \sum_i E_i);$$

where

L_c is the historical loss costs for the subject class, c;

E_c is the historical exposure units for class c;

L_i is the historical loss costs for the ith class in the group;

E_i is the historical exposure units for the ith class in the group; and

Z is the credibility.

(For the rest of this paper P_c will denote the historical loss rate for class c (L_c/E_c). P_g will do the same for the group's historical loss cost rate.)

Complement's Qualities

This complement has problems in two areas, accuracy and unbiasedness. The group mean loss costs may be the best available substitute. And they may be unbiased with respect to all the information the actuary has when making the rate (e.g., historical loss data -- the real means remain unknown). But, the actuary should believe that the true expected class losses will take a different value than the group expected losses. So, this method contains an intrinsic bias and inaccuracy that is unknown.

This complement generally has some independence from the base statistic. As long as the base class does not predominate in the whole group, the process errors of all the other classes should be independent from that of the base class. And the error created by using the group mean instead of the class mean is independent of the base class process variance (error). To the extent that the actuary uses the same loss development, trend, and current level factors on the class and group, the error from those factors is interdependent between the class and group loss costs. But, you could view the ratemaking process as first estimating undeveloped, untrended historical expected losses at previous rates; then applying adjustment factors. In the first part of that process, the predicting errors are nearly independent.

This complement performs well on availability and ease of computation. Generally, actuaries compute the group mean and group rate indication as the first stage of the pricing process for the entire line of business.

As long as all the classes in the group have something in common that puts them in the group, that forms the logical connection between the class's loss costs and those of the group. However, that does not totally eliminate controversy from this credibility complement. Customers may often complain that they are treated 'just like everyone else' when their historical losses are below average. Overall, this has an average degree of relationship to the expected subject losses.

Choosing the Larger Group

An actuary should be careful when choosing which larger group to use. For example, given a choice between using same class data from other states (provinces) or other class data from the same state, the actuary should consider: Whether the differences by state in loss levels are more significant than the differences between class costs in the same state? (Usually, class differences are larger); Can the other state's class data be adjusted to reflect the base state loss levels? (reducing bias); Is there a group of classes in the state that the actuary would expect to have about the same loss costs? (small bias.) All those factors merit consideration. The actuary should attempt to find the larger group statistic that has the least expected bias.

Example

Consider the data table below.

Data for Bayesian Credibility Complement

		Last Year's Data			Last Three Year's Data		
Rate Group	Class	Exposures	Losses	Pure Premium	Exposures	Losses	Pure Premium
A	1	100	5000	$50	250	16000	$64
	2	300	20000	$67	850	55000	$65
	3	400	19000	$48	1100	55000	$50
	Subtotal	800	44000	$55	2200	126000	$57
B	Subtotal	600	29000	$48	1700	55000	$32
C	Subtotal	500	36000	$72	1400	120000	$86
D	Subtotal	800	75000	$94	2300	200000	$87
Total		2700	184000	$68	7600	501000	$66

Table 1

If one is making rates for class 1 in rate group A, one must first consider whether to use the one year or three year historical losses. One must consider that the three year pure premiums will be less affected by process variance (year-to-year fluctuations in experience due to small samples from the distribution of potential claims). On the other hand, sometimes the exposure base is large enough to minimize process variance and societal events are causing pure premiums to change (changes in the potential losses one is sampling from). In that situation the one year pure premiums are preferable.

Suppose one chooses the one year pure premium ($50) for historical data. Then using the three year pure premiums of the class ($64) for the complement would be inappropriate. That is because the three year pure premiums are heavily interdependent with the one year class pure premium. Also, presumably the actuary has already decided that the three year data is biased because of changes in loss cost levels. So, the actuary believes the three year data does not add accuracy to the prediction. For the same reasons, the three year rate group and grand total pure premiums would be inappropriate complements.

The next decision is between the rate group and grand total pure premiums. The choice between these involves a tradeoff between bias reduction and process variance reduction. The rate group data should reflect risks that are more similar to class 1. So, it should have less bias. On the other hand, the grand total data is spread over more risks, so it has less process variance. This example makes the choice difficult. The one year and three year rate group pure premiums are very similar ($55 versus $57). But the other rate groups show more pronounced inconsistencies (i.e. $32 versus $48 for rate group B). The grand total shows it has little process variance. But it appears to contain roughly $15 of bias. The one year rate group A pure premium ($55) is probably the best choice.

One could also consider using the three year pure premium for historical losses. That does not preclude using the one year rate group data as a complement. Using the one year rate group A pure premium would simply assume that the entire rate group A exposures were sufficient to minimize process variance. So, it may be appropriate to use one year data as a complement to three year data.

B. Loss Costs of a Larger Related Class

Actuaries sometimes use the loss costs of a larger, but related class for the complement of credibility. For example, if a company writes very few picture framing stores but writes a large number of art stores, the actuary may choose to use the art store loss costs for the framing store complement of credibility. He may or may not make some adjustments to the art store loss costs to make them more applicable to framing stores. For example, he may wish to adjust for the minor woodworking exposure. Actuaries pricing General Liability often use this 'base class' (meaning the larger related class in this context) approach.

Complement's Qualities

This approach has qualities similar to the large group complement. It is biased (though the bias and its direction are unknown) and so it is inaccurate. The more the actuary adjusts the related class loss data to match the loss exposure in the subject base class; the more the bias is reduced. The independence may be slightly less if the factor relating the classes generates high losses for the two classes simultaneously. But the actuary must be careful that this seeming independence is not just a simultaneous shift in the expected losses (which is not prediction error, it is an increase in expected losses). It is usually the latter.

This complement does not fare quite as well as the group mean in other categories. Data is not as readily available for this complement as the group mean. But, if the company writes some related class, data should be available and already computed for that class's rates.

The computations involved in adjusting related class data may be more difficult. Any loss cost adjustments will require some extra work. Since there is some relationship between the base class and the related class (they must be related some way by definition), explaining this complement may be easier than explaining the larger group complement.

Example

Consider the case of the framing stores. Suppose the actuary wishes to estimate a fire rate for framing stores and already has a well-established rate for art stores. Perhaps the actuary sees that the only visible difference in exposure is the presence of substantial wood and sawdust. So he might choose to add a judgmental 10% of the excess of the fire rate for lumberyards over the fire rate for art stores.

C. Harwayne's Method

Harwayne's method[3] uses a specific type of data from a related class. Usually it is also a case of using loss costs from the larger group. In Harwayne's method actuaries use countrywide (excepting the base state being reviewed) class data to supplement the loss cost data for each class. But we adjust countrywide data to remove overall loss cost differences between states (or provinces).

The process is as follows. First we determine what the total countrywide average pure premium would be if the countrywide data had the same percentage mixture of classes (class distribution) as the base state. The result reflects the base state class distribution but probably reflects the differences in overall loss costs differences between states.

Next, actuaries use that difference in overall loss costs to adjust the countrywide class data to match the base state overall loss cost levels. We determine the ratio of overall state loss costs to overall (all classes) adjusted countrywide loss costs. Then we multiply that ratio times the countrywide base class loss costs to get the complement of credibility.

That is Harwayne's basic method. In a variant form, actuaries may adjust each state's loss costs individually to the base state level to eliminate biases due to different state distributions between classes (Harwayne used this variant). Then, actuaries compute the average class complement by weighting the individual state's adjusted loss costs. In another variant, actuaries adjust other state's historical loss ratios by class to match the base state's overall loss ratio. In either variant, the basic principles are the same.

Formulas

The simplified formula for Harwayne's method is as follows. Let

$L_{c,s}$ denote the historical losses for class c in the base state s;
$E_{c,s}$ denote the associated exposure units;
$P_{c,s}$ denote the state pure premium for class c
$L_{i,j}$ denote the historical losses for an arbitrary class 'i' in some state j; and
$E_{i,j}$ will denote the associated exposure units; and
$P_{i,j}$ will denote the state j pure premium for class i.

First, actuaries compute the countrywide pure premium adjusted to the state class distribution. The first step is to compute the "state s" average pure premium (rate)

$$P_s = \sum_i L_{i,s} / \sum_i E_{i,s}.$$

The next step is to compute the countrywide rates by class

$$P_i = \sum_{j \neq s} L_{i,j} / \sum_{j \neq s} E_{i,j}.$$

Then, actuaries compute the countrywide rate using the state s distribution of exposures

$$\overline{P} = \sum_i E_{i,s} P_i / \sum_i E_{i,s}.$$

So, the overall pure premium adjustment factor is

$$F = P_s / \overline{P}.$$

And the complement of the credibility for class c is assigned to $F \times P_c$.

Harwayne's more complicated (and more accurate) formula replaces the overall adjustments to countrywide data with separate adjustments for each state. That is, actuaries compute state overall means with the base state ("s") class distribution.

$$\overline{P}_j = \frac{\sum_m E_{m,s} P_{m,j}}{\sum_m E_{m,s}}$$

Then, we compute individual state adjustment factors

$$F_j = P_s / \overline{P}_j .$$

And then we adjust each state's class c historical rates using the F_j's. That is, we compute the adjusted "state j" rates

$$P'_{c,j} = F_j P_{c,j}.$$

and then we weight them with the countrywide distribution between states

$$\text{Complement} = C = \frac{\sum_j E_{c,j} P'_{c,j}}{\sum_j E_{c,j}}.$$

The result is Harwayne's more complicated complement of credibility.

<u>Complement's Qualities</u>

This complement has very high statistical quality. Because Harwayne's method uses data from the same class in other states and attempts to adjust for state-to-state differences, it is very unbiased. It is also reasonably accurate as long as there is sufficient countrywide data to minimize process variance. Since the loss costs are from other states, their prediction errors (remaining bias) should be fairly independent of the base class process error in the base state. One exception might be where there is an across-the-board jump in all class's loss costs in state s that alter the adjustment to the state experience level. But, across-the-board jumps usually flow through into the next year's expected losses, so they are rarely prediction errors.

This complement has a mixed performance on the less mathematical qualities. Data are usually available for this process. But the computations do take time and are complicated. Thankfully, they do bear a much more logical relationship to class loss costs in individual states than unadjusted countrywide statistics. On the other hand, this may be harder to explain because of complexity.

Example

Consider the data below. It is for Harwayne's method on class 1 in state S.

Data for Harwayne's Method

State "s"	Class "c"	Exposure "E"	Losses "L"	Pure Premium "P"
S	1	100	200	2.00
	2	180	600	3.33
	Subtotal	280	800	2.86
T	1	150	550	3.67
	2	300	1200	4.00
	Subtotal	450	1750	3.89
U	1	90	200	2.22
	2	220	900	4.09
	Subtotal	310	1100	3.55
All	1	340	950	2.79
	2	700	2700	3.86
	Total	1040	3650	3.51

Table 2

For Harwayne's full method, one first computes

$$\overline{P}_T = \frac{100 \times 3.67 + 180 \times 4.00}{100 + 180} = 3.88 \text{, And}$$

$$\overline{P}_U = \frac{100 \times 2.22 + 180 \times 4.09}{100 + 180} = 3.42 \text{.}$$

Then, one computes the state adjustment factors $F_T = 2.86/3.88 = .737$ and $F_U = 2.86/3.42 = .836$. The next step is to compute the other state's adjusted class 1 rates $P'_{1,T} = .737 \times 3.67 = 2.70$ and $P'_{1,U} = .836 \times 2.22 = 1.86$. The last step is to weight the two state's adjusted rates with their class 1 exposures to produce

$$C = \frac{2.70 \times 150 + 1.86 \times 90}{150 + 90} = 2.39.$$

That is Harwayne's complement of the credibility.

D. Trended Present Rates

In some cases, most notably countrywide rate indications, there is no larger group to use for the complement. So, actuaries use present rates adjusted for inflation (trend) since the last rate change. If there was a difference between the last actuarial indication and the charged rate, we build that in too. Essentially, this test allows some credibility procedure to dampen swings in the historical loss data yet still forces the manual rates to keep up with inflation.

Formula for the Complement

The formula for this complement of credibility is

$T^t \times R_L \times P_L \div P_C$, where

T is the annual trend factor, expressed as one plus the rate of inflation (this will usually be the same as the trend factor in the base indication);

t is the number of years between the original target effective date of the current rates (not necessarily the date they actually went into effect) and the target effective date of the new rates (This will often be different than the number of years in the base class trend. It is also usually different than the number of years between the experience period and the effective date of the new rates);

R_L represents the loss costs presently in the rate manual;

P_L represents the last indicated pure premium (loss costs); and

P_C represents the pure premiums actually being charged in the current manual. This may differ from R_L because P_L and P_C may be taken over a broader group.

Complement's Qualities

This complement is not as desirable as the previous complements but sometimes it may be the only alternative. It is less accurate for loss costs with high process variance. That is because that process variance is presumably reflected in last year's rate. That is why it is primarily used for countrywide indications or state indications with voluminous data. It is unbiased in the sense that pure trended loss costs (i.e., with no updating for more current loss costs) are unbiased. Since it includes no process variance, it is fairly independent from the base statistic.

On the less mathematical side, this statistic performs fairly well. Everything an actuary needs to compute it is already in the base rate filing. So, it is available and easy. There is one exception to this. Should you wish to analyze the effects of rate changes the company did not achieve at the level of individual classes, this may require more data than companies typically maintain. This statistic is also very logically related to the loss costs being analyzed. After all, the present rates are based on this complement.

Example

Consider the following data for 1996 policy rates:

Present pure premium rate -- $120;

Annual inflation (trend) -- 10%;

Amount requested in last rate change -- +20%;

Effective date requested for last rate change -- 1/1/94;

Amount approved by state regulators -- +15%;

Effective date actually implemented -- 3/1/94.

The complement of the credibility would be

$$C = \$120 \times 1.1^2 \times \frac{1.20}{1.15} = \$152.$$

E. Rate Change from the Larger Group Applied to Present Rates

This complement is very similar to the Bayesian complement. But it does not have the substantial (though unknown) bias of the Bayesian complement. That is because the true class expected losses may be very different from the large group expected losses. This larger group test uses the large group rate change applied to present rates instead of the large group historical loss data (Bayesian complement). Presumably, present rates are an unbiased predictor of the prior (i.e., before changes reflected in current ratemaking data) loss costs. And, as long as both rates need reasonably small changes, any bias in the overall larger group rate change as a predictor of the class rate change should be small. Also, using large group rate changes instead of straight trend allows the rate to mirror broad changes in loss cost levels that may not be reflected in trend.

Example

An example may help to illustrate how eliminating bias improves rate accuracy over time. In the graph below the group experience was simulated by successively applying N(10%,0.25%) (normal distribution with a mean of 10% and a standard deviation of $\sqrt{0.25\%} = 5\%$) trends to a value starting at one. The true class expected losses were set at exactly half the group expected losses each and every year (a slightly unrealistic assumption). The historical class losses have a standard deviation of one-third the true expected losses for the class. A detailed chart of the values actually simulated is in Appendix C.

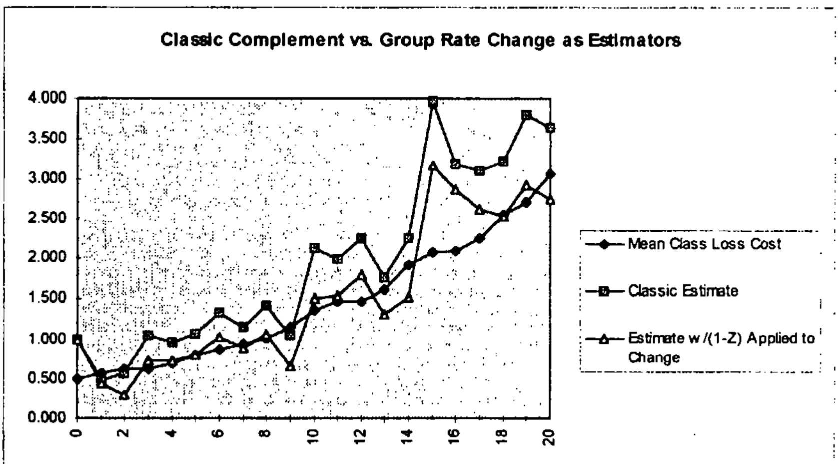


Figure 2

As the graph shows, the classic complement results in rates with consistent bias above the true expected losses. The complement based on applying group changes to present rates starts too high but very quickly becomes unbiased. It is almost always a better estimate.

Formula

This complement has a fairly straightforward formula. It is

$$R_c \times \left\{1 + \frac{(P_g - R_g)}{R_g}\right\}, \text{ where}$$

R_c is the present manual loss cost rate for class c;

P_g is the present indicated loss cost rate for the entire group of classes; and

R_g is the present average loss cost rate for the entire group.

Complement's Qualities

This is a significant improvement over the Bayesian complement. It is largely unbiased. If the year-to-year changes are fairly small, it is very accurate over the long term (though often not as accurate as Harwayne's complement in practice). And since the complement is based on group variance, it is fairly independent. Since this requires a group rate change that must be calculated anyway, it is both available and easy to compute. Since it includes the present rate, it has a logical relationship to the class loss costs.

Numerical Example

Consider the following data.

Data for Applying Group Rate Change to Class Data

Class	Exposure	Losses	Indicated Pure Premium	Present Pure Premium	Underlying Losses
1	100	$70,000	$700	$750	$75,000
2	200	$180,000	$900	$920	$184,000
3	300	$200,000	$667	$700	$210,000
Total	600	$450,000	$750	$782	$469,000

Notes : -Both indicated and present pure premiums are at current cost levels.
- Underlying losses are extension of exposures by present premiums
-Total present premium is ratio of total underlying to total exposures.

Table 3

Using this data, the complement for class 1 would be

$$\$750 \times \left(1 + (\$750 - \$782) / \$782\right) = \$719 .$$

F. Competitor's Rates

New companies and companies with small volumes of data often find their own data too unreliable for ratemaking. So their actuaries use competitor's rates for the complement of credibility. They rationalize that if the competitor has a much larger number of exposures, the competitor's statistics have less process error. An actuary in this situation must consider that manual rates reflect marketing considerations, judgment, and the effects of the regulatory process as well as loss cost statistics. So competitor's rates have significant inaccuracies. They are also affected by differences in underwriting and claim practices between the subject company and its competitors. So, competitor's rates probably have systematic bias as well. The actuary will often attempt to correct for those differences by using judgment. But those corrections and their size and direction may generate controversy. However, using competitor's rates may be the best viable alternative in some situations.

Complement's Qualities

Competitor's rates generally have prediction errors that are independent of the subject class loss costs. That is because their errors stem more from inter-company differences that are unrelated to subject company loss cost errors. They are often available from regulators, although the process may take some work. They are harder to use since they usually must be posted manually.

Regulators may complain that competitor's rates are unrelated to the subject company's own loss costs. But, if the company's own data is too unreliable, competitor's rates may be the only alternative.

Example

Consider a competitor's rate of $100. Suppose a Schedule P analysis suggests the competitor will run a 75% loss ratio. Further, suppose one's own company has less underwriting expertise. So, one's company expects 10% more losses per exposure than the competitor. The complement would be $\$100 \times .75 \times 1.1 = \$83$.

G. Loss Ratio Methods

This paper discussed all the previous complements in terms of pure premium ratemaking. But all the methods except the loss costs from a larger related class and competitor's rates also work with loss ratio methods. All the actuary needs to do is consider earned premium to be the exposure base. Replacing the exposure units with earned premium yields usable formulas.

III. Specific Excess Ratemaking

Complements for excess ratemaking are structured around the special problems of excess ratemaking. Since specific excess policies only cover losses that exceed a very high per claim deductible (attachment point), there usually are very few actual claims in the historical loss data. So, actuaries will try to predict the volume of excess loss costs using the loss costs below the attachment point. For liability coverages, the loss development of excess claims may be very slow. That accentuates the sparsity of ratemaking data. Also, the inflation inherent in excess layers is different (usually higher) than that of total limits losses (see [2]). Since the 'burning cost' (historical loss data) is an unreliable predictor, the statistic that receives the complement of credibility is especially important.

A. Increased Limits Factor

When loss costs for the first dollar coverage up to the insurer's limit of liability are available, actuaries may use an increased limits factor approach. Actuaries multiply the 'capped' loss costs by the increased limits factor for the attachment point plus the limit of liability. Then, we divide the result by the increased limits factor for the attachment point. That produces an estimate of loss costs from the first dollar up to the limit of liability. Then we subtract the loss costs below the original attachment point. The remainder estimates the expected losses in the specific excess layer.

Actuaries use a variety of sources for increased limits factors. The Insurance Services Office publishes tables of estimated increased limits factors for products, completed operations, premises and operations liability, and manufacturers and contractors liability. The National Council on Compensation Insurance publishes excess loss pure premium factors that allow actuaries to compute increased limits factors for workers compensation. The Proceedings of the Casualty Actuarial Society may contain tables of property losses by ratio to probable maximum loss. Those can be converted to increased limits factors by using the factors for the ratio of the attachment point to the probable maximum loss (and the ratio of the attachment point plus the limit of liability to the probable maximum loss). Actuaries may compute increased limits factor tables using a company's own data (if the company sells enough specific excess). Actuaries may modify industry tables to reflect their company's loss cost history. Competitor prices may allow actuaries to estimate increased limits factors for obscure coverages. We would consider the ratios between competitor prices for various limits of liability.

Formula

The formula is as follows:

$$\left(P_A \times ILF_{A+L} \div ILF_A\right) - P_A \quad \text{or} \quad P_A \times \left(\frac{ILF_{A+L}}{ILF_A} - 1\right).$$

And in this case

P_A is the loss costs capped at the attachment point (A) (by convention, it usually premium capped at the attachment point multiplied by the loss ratio the actuary projects.);

ILF_{A+L} is the increased limits factor for the sum of the attachment point and the limit of liability(L); and

ILF_A is the increased limits factor for the attachment point.

Complement's Qualities

As long as the insured being rated has a different loss severity distribution than the norm, this complement contains bias. In that fairly likely event, it is also inaccurate. But, actuaries must weigh those facts against the greater inaccuracy of burning cost statistics. When pricing specific excess insurance, actuaries must usually settle for less accurate and potentially biased estimators. That is because there are few highly accurate estimators available.

This complement's error is fairly independent of the burning cost error. This complement tends to contain a systematic (parameter-type) error rather than the process error inherent in burning cost. It is dependent on burning cost only to the extent that both are highly related to the losses below the attachment point.

Very few specific excess statistics are readily available or easy to compute. Considering the alternatives, the availability of industry increased limits tables (in the United States) makes this the easiest specific excess complement to compute. Also, the data for this test is available as long as premiums or loss costs capped at the attachment point are available.

The excess loss cost estimates this complement produces are more logically related to the losses below the attachment point than those above. That can be controversial with customers. But that is a common problem with excess insurance pricing. However, burning cost is unreliable in isolation. And that problem is common to all excess complements.

Example

Consider the following table of increased limits factors.

Increased Limits Factors

Limit of Liability	Increased Limits Factor
$50,000	1.00
$100,000	1.50
$250,000	1.90
$500,000	2.50
$1,000,000	3.50

Table 4

Suppose one wishes to estimate the layer between \$500,000 and \$1,000,000 given losses capped at \$500,000 of \$2,000,000. The complement using increased limits would be

$$C = \$2{,}000{,}000 \times \left(\frac{3.5}{2.5} - 1\right) = \$800{,}000.$$

B. Lower Limits Analysis

Sometimes the historical losses near the attachment point may be too sparse to be reliable. So an actuary may wish to base his complement on basic limits losses, where the basic limit is some fairly low loss cap. In this case the formula is almost exactly the same as that of the previous analysis. The actuary simply multiplies the historical basic limits losses by a difference of increased limits factors. Specifically, he multiplies basic limits losses by the difference between the increased limits factor for the attachment point plus the limit of liability and the increased limits factor for the attachment point. The result is the complement of credibility.

Formula

The formula is

$P_b \times (ILF_{A+L} - ILF_A)$; where

P_b represents the historical loss data with each loss capped at the basic limit (b); and

ILF_{A+L} and ILF_A are as before.

Alternately, the actuary might choose to use a low capping limit (d) that is different from the basic limit underlying the increased limits table. Then, the formula would be

$$P_d \times \left(\frac{ILF_{A+L}}{ILF_d} - \frac{ILF_A}{ILF_d}\right).$$

Complement's Qualities

Actuaries must usually use judgment to decide whether loss costs capped at the attachment point or some lower limit are more accurate and unbiased predictors of the excess loss. Estimates made using the lower cap are more prone to bias. That is because using losses far below the attachment point accentuates the impact of variations in loss severity distributions. But, when there are few losses near the attachment point, historical losses limited to the attachment point may be unreliable and inaccurate predictors of future losses. So, using higher loss caps may produce even more inaccurate predictors of excess losses.

By an argument similar to that of the previous test, this complement's errors are fairly independent of those of burning cost.

Generally, this complement features more available statistics and a slightly greater complexity. Basic limits losses may need to be coded for statistical reporting. So, they may be readily available for this complement. On the other hand, since insureds and reinsureds may place a higher priority on accounting for the total losses they retain, they are not as available as losses limited to the attachment point. The calculations are no more complicated for basic limits analysis than retained limits (attachment point) analysis. The only exception would where actuaries must manually compute the loss costs between basic limits and the attachment point from a claims list.

As with the straight increased limits factor approach, this complement may generate controversy with customers because it is not based on actual burning cost.

<u>Example</u>

Suppose an actuary is estimating the losses between \$500,000 and \$1,000,000 and the actuary feels he can only rely on historical losses limited to \$100,000. The estimated historical losses limited to \$100,000 are \$1,000,000. Then, using the increased limits factors from Table 4, he would calculate the complement at

$$C = \$1{,}000{,}000 \times \left(\frac{3.5}{1.5} - \frac{2.5}{1.5}\right) = \$666{,}667.$$

<u>C. Limits Analysis</u>

The previous approaches work well when losses limited to a single capping point are available, but sometimes they are not. Reinsurance customers generally sell policies with a wide variety of policy limits. Some of the policy limits will fall below (not expose) the attachment point. Some limits may extend beyond the sum of the attachment point and the reinsurer's limit of liability. In any event, each subject (first dollar) policy limit will require its own increased limits factor.

So, actuaries analyze each limit of coverage separately. Generally, we assume that all the limits will experience the same loss ratio. So, we multiply the all limits combined (total limits) first dollar loss ratio times the premium in each first dollar limit to estimate the loss costs for that limit. Then, we perform an increased limits factor analysis on each first dollar limit's loss costs separately. The formula is as follows:

Formula

$$LR_T \times \sum_{d \geq A} W_d \frac{\left(ILF_{\min(d, A+L)} - ILF_A\right)}{ILF_d}$$; where

LR_T is the estimated total limits loss ratio;

The 'd' are all the policy limits the customer sells that exceed the attachment point ($\geq A$); and

each W_d is the premium volume the customer sells with policy limits of 'd'.

The ILF's have the same meaning as previously.

Complement's Qualities

Actuaries use this approach because it may be all that is available. Reinsureds may be unable to split their historical losses any more finely than losses that would have pierced the cover in the past versus all other losses. Since the total limits loss costs (which are almost always available, at least as an estimate) may include claims beyond the layer, it may be impossible to calculate losses limited to the attachment point. In any event, if some of the reinsured's policy limits are below the attachment point, they do not expose the layer and should be excluded from an increased limits factor calculation. So, this may the only available complement with low bias.

It is biased and inaccurate to the same extent that the previous increased limits factor-based complements were biased or inaccurate. It is more time-consuming to compute (unless the alternative is computing limited claims from claims lists). And it generates the same controversy as the other methods since it is not the same as the actual burning cost.

Example

Suppose an actuary is estimating the losses in a layer between $250,000 and $500,000. Breakdowns of losses by size are unavailable. But, the actuary believes the loss ratio of the customer's entire business to be 70%. He does have a breakdown of premiums by limit of liability. Using that breakdown and the increased limits factors from Table 4, he computes the losses in the layer below.

Limits Analysis for Layer Between $250,000 and $500,000

Limit of Liability	Premium	Times 70% Loss Ratio	Increased Limits Factor	% in Layer	Loss in Layer
$250,000	$ 600,000	$ 420,000	1.9	0.00%	$ -
$500,000	$ 300,000	$ 210,000	2.5	24.00%	$ 50,400
$1,000,000	$ 300,000	$ 210,000	3.5	17.14%	$ 36,000
Total	$ 1,200,000	$ 840,000			$ 86,400

Table 5

So, he estimates the losses in the layer at $86,400.

D. Fitted Curves

The problem with most of the previous complements is that they do not give special attention to the claims above or near the attachment point. So, they miss differences in loss severity distributions between insureds. But of course that must be counterbalanced against the fact that individual insured's large claims histories usually lack credibility.

By fitting a family of loss severity curves to the distribution, actuaries make the most of the large claim data that is available. If the loss history shows no claims beyond the attachment point but many claims that are very near to the attachment point, a fitted curve will usually reflect that and project high loss costs in the subject layer. On the other hand, if there are few large claims close to the attachment point, the fitted curve will project low loss costs for the layer.

The details of how to fit curves are beyond the scope of this paper(see [4]), but it will provide an outline of how to use fitted curves in practice. After fitting and trending the curve, an actuary will use the curve to estimate what percentage of the curve's total loss costs lie in the subject layer. He may do this by evaluating the difference between the limited mean function $\int_{-\infty}^{L} xf(x)dx + (1 - F(L))L$ at the attachment point and the attachment point plus the limit of liability. He would then divide the result by the total mean (or the mean when claims are capped at the typical policy limit) to get the percentage of the total loss costs that lie in the layer. Multiplying that percentage by the total claims cost yields the estimate of claim costs in the layer (for details, see [4]).

Of course, excess values from curve fits need extensive loss development just like burning costs. Actuaries may use excess loss development factors such as those published by the Reinsurance Association of America, or they may triangulate the fitted loss costs.

Complement's Qualities

This method is generally unbiased (except for concerns that the general shape of a family of curves may predispose the results for the family to estimated costs in particular layers that are either too high or too low.) When there are few large claims, it is more accurate than burning cost. It is often more accurate than increased limits factors simply because it does a better job reflecting any general tendency towards large or small claims. On the hand, fitting curves forces data into a mold that may not fit the data. The actual loss severity distribution will almost certainly look very different from all the members of the family of curves. This 'super-parameter' risk introduces error of its own. The 'super-parameter' risk is totally distinct from process risk, and that makes the complement fairly independent. On the other hand, the presence or absence of burning cost claims in the layer can influence the curve fit heavily. So, this complement has somewhat more dependent (relative to burning cost) errors than the increased limits approaches.

Data availability and computational complexity are problems here. To fit a loss severity curve an actuary must either use a detailed breakdown of all the claims into size ranges or use a listing of every single claim. Usually, that data is not readily available. Further, the processing required to fit curves requires fairly complex mathematical calculations. Besides the fact that complex calculations require special personnel, the complexity makes the results difficult to explain to lay people.

On one hand, this complement uses more of the insured's own data in and near the layer than any other excess complement. On the other hand, its complexity may make that fact difficult to communicate.

IV. Summary

The complement of the credibility deserves at least as much actuarial attention as the base statistic (historical loss data). Actuaries owe special attention to its unbiasedness and accuracy. In some cases, interdependence must be avoided. And any actuarial method must be implemented using reasonable labor on available statistics. Meeting those qualities may require statistics that make less explainable sense to lay people, but explainability must be considered, too.

This paper has detailed several statistics that are commonly used for the complement of credibility. Their use improves many actuarial projections considerably.

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THE ERROR IN CREDIBILITY ESTIMATES

This appendix will show that the error in an optimum credibility weighted estimate is

$$\Phi(x_1, x_2) = \frac{\tau_1^2 \tau_2^2 (1-\rho^2)}{\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2}.$$

The proof involves three equations from Boor[1]:

(1) $\Phi(x_1, x_2) = Z\tau_1^2 + (1-Z)\tau_2^2 + (Z^2 - Z)\delta_{1.2}^2$ (p.182, the simplified error of the credibility-weighted estimate);

(2) $Z = \dfrac{\tau_2^2 - \tau_1^2 + \delta_{1.2}^2}{2\delta_{1.2}^2}$ (p.183, the formula for the optimum credibility); and

(3) $\delta_{1.2}^2 = \tau_1^2 + \tau_2^2 - 2\mathrm{Cov}(x_1, x_2)$ (p.179, the formula relating $\delta_{1.2}^2$ to the correlation).

In this case τ_1, τ_2, and ρ are the same as they were in the body of the paper (the prediction errors of burning cost and the credibility complement and their correlation); $\Phi(x_1, x_2)$ is the minimum possible average squared prediction error from credibility weighting burning cost (x_1) and the credibility complement (x_2); and $\delta_{1.2}^2$ is the average squared difference between burning cost and the credibility complement.

Simple algebra on (1) allows one to pull out several terms that will create the numerator of (2).

$$\Phi(x_1, x_2) = -Z(\tau_2^2 - \tau_1^2 + \delta_{1.2}^2) + \tau_2^2 + Z^2\delta_{1.2}^2;$$

$$= -Z^2 2\delta_{1.2}^2 + \tau_2^2 + Z^2\delta_{1.2}^2 = \tau_2^2 - Z^2\delta_{1.2}^2.$$

Using the definition of Z (equation (2)) once again with some algebra gives

$$= \tau_2^2 - \frac{(\tau_2^2 - \tau_1^2 + \delta_{1,2}^2)^2}{4\delta_{1,2}^2}.$$

Using (3) and the relationship between the covariance and correlation gives

$$= \tau_2^2 - \frac{(\tau_2^2 - \tau_1^2 + \tau_1^2 + \tau_2^2 - 2\mathrm{Cov}(x_1, x_2))^2}{4\delta_{1,2}^2};$$

$$= \tau_2^2 - \frac{(\tau_2^2 - \tau_1^2 + \tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2)^2}{4\delta_{1,2}^2};$$

$$= \tau_2^2 - \frac{(2\tau_2^2 - 2\rho\tau_1\tau_2)^2}{4\delta_{1,2}^2};$$

$$= \tau_2^2 - \frac{(\tau_2^2 - \rho\tau_1\tau_2)^2}{\delta_{1,2}^2}.$$

Then, more algebra gives

$$= \tau_2^2\left(1 - \frac{(\tau_2 - \rho\tau_1)^2}{\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2}\right);$$

$$= \frac{\tau_2^2}{\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2} \times (\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2 - \tau_2^2 + 2\rho\tau_1\tau_2 - \rho^2\tau_1^2);$$

$$= \frac{\tau_2^2}{\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2} \times (\tau_1^2 - \rho^2\tau_1^2);$$

$$= \frac{\tau_2^2\tau_1^2(1 - \rho^2)}{\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2};$$

and that is the error formula we sought to prove.

FOR CORRELATIONS NEAR UNITY, CREDIBILITY IS NEGATIVE

This appendix will show that whenever the correlation exceeds the point of maximum error, the credibility of one statistic is negative. To explain this principle, reviewing the graph of error by correlation will help.

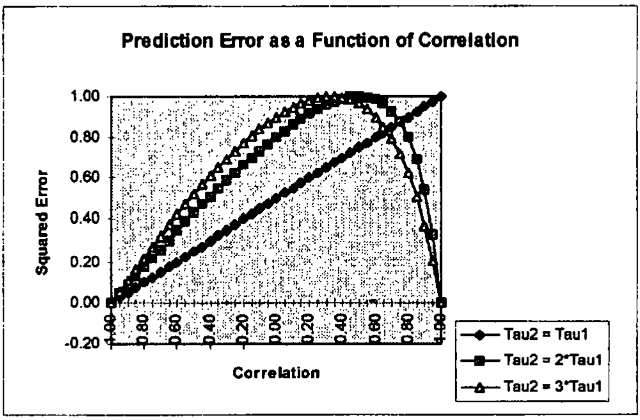


Fig. 1 (reprinted)

As one can see, the prediction error is initially minimized when the correlation is negative. Then it increases until the error is maximized. Then the error decreases again beyond that maximum point. This section will show that the one credibility is actually negative beyond that maximum point.

As it happens, when $\tau_2 \geq \tau_1$, that maximum point is where $\rho = \tau_1/\tau_2$. And all correlations beyond that yield negative credibility for the complement. Alternately, when $\tau_1 \geq \tau_2$, $\rho = \tau_2/\tau_1 \leq 1$ is the point of maximum prediction error. Beyond that, the burning cost's credibility will be negative. But, this appendix must prove that.

It is easy to show that Φ has a maximum where $\rho = \tau_1/\tau_2$. One need only note that the function $\Phi(\rho)$ has a maximum where

$$0 = \frac{\partial\Phi}{\partial\rho} = \frac{2\rho(\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2) - 2\tau_1\tau_2(1-\rho^2)}{(\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2)^2}$$

(using the definition of $\Phi(\rho)$ from appendix I). Using some algebra, that is equivalent to

$0 = 2\rho\tau_1^2 + 2\rho\tau_2^2 - 4\rho^2\tau_1\tau_2 - 2\tau_1\tau_2 + 2\rho^2\tau_1\tau_2$; or

$0 = (\tau_1 - \rho\tau_2)(\tau_2 - \rho\tau_1)$.

So, the maximum is at τ_1/τ_2 or τ_2/τ_1, whichever is less than one.

To show that correlations beyond that maximum point result in negative credibilities, it suffices to show that they fulfill Boor's condition for negative credibility ([1], p.183)

$\tau_2^2 \geq \tau_1^2 + \delta_{1,2}^2$.

But that follows directly from Boor's equation relating the credibility and covariance ([1], p. 179). That is, since

$\delta_{1,2}^2 \ = \ \tau_1^2 + \tau_2^2 - 2\text{Cov}(x_1, x_2) \ = \ \tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2$,

Boor's condition is equivalent to

$\tau_2^2 \geq \tau_1^2 + \tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2$.

Or,

$\rho \geq \dfrac{\tau_1}{\tau_2}$;

that is, Boor's condition for negative credibility is fulfilled and fulfilled only for ρ beyond the point of maximum error. So, the correlations near unity yield negative credibilities.

DATA FOR EXAMPLE APPLYING COMPLEMENT TO GROUP RATE CHANGE

(a) Year	(b) Group N(.1,.0025) Trend	(c) Group Loss Cost	(d) Mean Class Loss Cost	(e) Class with Process Variance N(0,((d)/3)^2)	(f) Classic Z	(g) Classic Estimate	(h) Estimate w/(1-Z) Applied to Change
0	0.115	1.000	0.500	0.188	0.692	1.000	1.000
1	0.101	1.115	0.558	0.256	0.692	0.481	0.438
2	0.021	1.228	0.614	0.825	0.692	0.572	0.306
3	0.107	1.254	0.627	0.695	0.692	1.044	0.724
4	0.137	1.389	0.694	0.782	0.692	0.954	0.731
5	0.091	1.579	0.790	1.037	0.692	1.065	0.792
6	0.082	1.723	0.862	0.747	0.692	1.324	1.025
7	0 082	1.865	0.932	1.034	0.692	1.153	0.885
8	0.143	2.017	1.009	0.468	0.692	1.418	1.056
9	0.188	2.305	1.153	1.759	0.692	1.039	0.659
10	0.075	2.739	1.369	1.393	0.692	2.119	1.498
11	0.000	2.945	1.472	1.653	0.692	1.988	1.545
12	0.093	2.946	1.473	0.992	0.692	2.256	1.782
13	0.192	3.220	1.610	1.516	0.692	1.753	1.315
14	0.075	3.839	1.919	3.501	0.692	2.244	1.527
15	0.009	4.128	2.064	2.358	0.692	3 966	3.162
16	0.077	4.167	2.083	2.213	0.692	3.193	2.862
17	0.136	4.487	2.244	2.225	0.692	3.096	2 616
18	0.062	5.096	2.548	2.733	0.692	3.214	2 525
19	0.133	5.411	2.705	2.394	0.692	3.806	2.917
20	0.093	6.128	3.064	2.819	0.692	3.654	2.752

Notes

- Column (g) is [(f)*(previous column (e) + (1-(f))(previous column (c)] * (1+10% trend)
- Column (h) is {(f)*[previous column (e) - previous column (h)] + (1-(f))*[previous column (b) *previous column (c) -1.1*previous column (c)] + previous column (h)}*(1 +10% trend)

Portfolio Optimization and the Capital Asset Pricing Model: A Matrix Approach

by Leigh J. Halliwell

Portfolio Optimization and the Capital Asset Pricing Model: A Matrix Approach

Leigh J. Halliwell

Abstract

Actuaries are acquainted with the basic ideas of Modern Portfolio Theory and the Capital Asset Pricing Model (CAPM). Briefly, portfolios are formed by weighting risky assets with varying means, variances, and covariances. Each portfolio can be plotted in the X-Y plane by its total return, with the standard deviation as the x-coordinate and the mean as the y-coordinate. It is plausibly asserted that the resulting subspace of returns has an envelope, which is called the efficient frontier. The efficient frontier contains the returns which offer the greatest mean for a given standard deviation, or the least standard deviation for a given mean, and therefore would correspond to portfolios chosen by perfectly informed and rational investors. However, when a riskless asset is introduced, represented by $R_f = (0, \mu_f)$, *one point on the efficient frontier becomes preferable to the others, the point at which a line through* R_f *becomes tangent to the efficient frontier. Since this point is optimal, it will be chosen by all informed and rational investors, which is to say that it will correspond to the portfolio of an efficient market. This market point,* R_m, *is the point* (σ_m, μ_m); *and the CAPM equation for the* i^{th} *asset is readily derived:* $\mu_i = \mu_f + \beta_i(\mu_m - \mu_f)$, *where* $\beta_i = \frac{\text{cov}(R_i, R_m)}{\text{var}(R_m)}$. *This article shows how the aforementioned argument can be made rigorous through fairly simple matrix algebra, which will foster a deeper understanding of and appreciation for the theory. Moreover, the article offers an easy method for determining the optimal, or market, portfolio. Finally, there will be a few remarks as to why CAPM theory may falter under empirical testing.*

I. Portfolio Optimization and the CAPM in Theory

Consider a universe of n risky assets. The return of the i^{th} asset, denoted R_i, is a random variable with $E(R_i) = \mu_i$ and $Var(R_i) = \sigma_i^2$, for i = 1, 2, ... , n. Let $Cov(R_i, R_j) = \sigma_{ij}$, which implies that $\sigma_{ii} = \sigma_i^2$. Now, instead of regarding the R_i's individually as random scalars, consider the (n x 1) column vector whose elements are the R_i's. Let us call this random vector **R**, using bold type for vectors and matrices; and let us represent it by writing a typical element within matrix brackets. So $\mathbf{R} = [\ R_{i1}\ ]$, or just $[\ R_i\ ]$. [1]

Define the expectation of a matrix as the matrix of the expectations, or $E(\mathbf{X}) = [\ E(X_{ij})\ ]$. Therefore, $E(\mathbf{R}) = [\ E(R_i)\ ] = [\ \mu_i\ ] = \mathbf{M}$. Also, if **X** and **Y** are two column vectors, define $Cov(\mathbf{X}, \mathbf{Y}) = E(\ (\mathbf{X}-E(\mathbf{X}))\ (\mathbf{Y}-E(\mathbf{Y}))'\ ) = [\ E\{\ (X_i - E(X_i)\ )\ (Y_j - E(Y_j)\ )\ \}\ ]$, where the prime (') is the operator for matrix transposition. If **X** is (n x 1) and **Y** is (m x 1), then their covariance is an (n x m) matrix. So $Var(\mathbf{R}) = [\ E(\ (R_i - \mu_i)\ (R_j - \mu_j)\ )\ ] = [\ \sigma_{ij}\ ] = \Sigma$. Obviously, variances of column vectors are symmetric matrices. We will write $\mathbf{R} \sim [\mathbf{M}, \Sigma]$ as shorthand for saying that **R** is distributed with mean **M** and variance Σ.

If **A** is a non-stochastic matrix conformable with **X**, so that $\mathbf{Y} = \mathbf{AX}$ is defined, then $E(\mathbf{Y}) = [\ E(Y_{ij})\ ] = [\ E(\ \Sigma\ a_{ik}\ X_{kj}\ )\ ] = [\ \Sigma\ a_{ik}\ E(X_{kj})\ ] = \mathbf{A}\ E(\mathbf{X})$. Similarly, if **XA** is defined, then $E(\mathbf{XA}) = E(\mathbf{X})\ \mathbf{A}$. Therefore, given the meaning of Cov(**X**, **Y**) above, if **AX** and **BY** are defined, then

$$
\begin{aligned}
Cov(\mathbf{AX}, \mathbf{BY}) &= E(\ (\mathbf{AX}-E(\mathbf{AX}))\ (\mathbf{BY}-E(\mathbf{BY}))'\ ) \\
&= E(\ \mathbf{A}(\mathbf{X}-E(\mathbf{X}))\ (\mathbf{B}(\mathbf{Y}-E(\mathbf{Y})))'\ ) \\
&= E(\ \mathbf{A}(\mathbf{X}-E(\mathbf{X}))\ (\mathbf{Y}-E(\mathbf{Y}))'\ \mathbf{B}') \\
&= \mathbf{A}\ E(\ (\mathbf{X}-E(\mathbf{X}))\ (\mathbf{Y}-E(\mathbf{Y}))'\ )\ \mathbf{B}' \\
&= \mathbf{A}\ Cov(\mathbf{X}, \mathbf{Y})\ \mathbf{B}'.
\end{aligned}
$$

Therefore, if a non-stochastic matrix Ω' is conformable with **R**, then $\Omega'\mathbf{R}$ has mean $\Omega'\mathbf{M}$ and variance $\Omega' Var(\mathbf{R})(\Omega')' = \Omega'\Sigma\Omega$, or $\Omega'\mathbf{R} \sim [\Omega'\mathbf{M}, \Omega'\Sigma\Omega]$.

The goal of portfolio optimization is to find an (n x 1) vector $\boldsymbol{\Omega}^*$, given $\mathbf{R} \sim [\mathbf{M}, \Sigma]$, such that $\boldsymbol{\Omega}^{*\prime}\mathbf{R}$ offers the greatest ratio of expected return in excess of the risk-free return μ_0 to its standard deviation. Let $\mathbf{R}_0 \sim [\Sigma_0 = \mathbf{0}, \mathbf{M}_0]$ denote the risk-free return, which is a trivial (1 x 1) random vector. $\Sigma_0 = \mathbf{0} = [0]$ and $\mathbf{M}_0 = [\mu_0]$ are (1 x 1) matrices. To be precise, a (1 x 1) matrix is not the same as a scalar, since a scalar can multiply any matrix, whereas a (1 x 1) matrix can only premultiply a (1 x n) matrix or postmultiply an (n x 1) matrix.

Let **J** be the (n x 1) vector all of whose elements are ones. Then $\mathbf{R} - \mathbf{J}\mathbf{R}_0 = [\, R_i - \mu_0 \,]$, which represents the return in excess of the risk-free return. The optimization problem is thus to maximize $(E(\boldsymbol{\Omega}'\mathbf{R}) - \boldsymbol{\Omega}'\mathbf{J}\mathbf{R}_0)\,(\mathrm{Var}(\boldsymbol{\Omega}'\mathbf{R}))^{-1/2}$, or equivalently, $E(\boldsymbol{\Omega}'(\mathbf{R} - \mathbf{J}\mathbf{R}_0))\,\mathrm{Var}(\boldsymbol{\Omega}'(\mathbf{R} - \mathbf{J}\mathbf{R}_0))^{-1/2}$, for some $\boldsymbol{\Omega} = \boldsymbol{\Omega}^*$. To simplify further calculations, we may relativize μ_0 as 0, which is in effect to substitute $\mathbf{R} + \mathbf{J}\mathbf{R}_0$ for $\mathbf{R}$. This will not affect the maximization, and later we can convert our results back into absolute form by substituting $\mathbf{R} - \mathbf{J}\mathbf{R}_0$ for $\mathbf{R}$.

So, in relative form, we wish to maximize $E(\boldsymbol{\Omega}'\mathbf{R})\,(\mathrm{Var}(\boldsymbol{\Omega}'\mathbf{R}))^{-1/2}$. Now $\mathrm{Var}(\boldsymbol{\Omega}'\mathbf{R}) = \boldsymbol{\Omega}'\Sigma\boldsymbol{\Omega}$ is a (1 x 1) matrix, whose only element must be nonnegative since it represents the variance of a scalar random variable. In matrix theory, Σ is said to be nonnegative definite. A symmetric matrix Σ such that $\boldsymbol{\Omega}'\Sigma\boldsymbol{\Omega} > [0]$ for any non-zero column vector $\boldsymbol{\Omega}$ is said to be positive definite. We make the assumption that Σ is positive definite; otherwise, our universe of risky assets would not be risky in some combination. Texts in elementary matrix theory show the proof that if Σ is positive definite, then Σ^{-1} exists and is also positive definite. The other assumption which we will make is that $(\Sigma^{-1}\mathbf{M})'\,\mathbf{J}$ is non-zero, which implies that **M** is non-zero. The purpose of the second assumption will become apparent below.

Therefore, for all non-zero Ω, $Var(\Omega'R) > [0]$, and $(Var(\Omega'R))^{-1/2}$ exists. We will make one more modification by seeking optimize the square: $E(\Omega'R)^2 (Var(\Omega'R))^{-1}$. One might think that this would lead to the worst Ω^* if $E(\Omega^{*\prime}R) < [0]$; however, it will turn out that in such a case the optimal investment will be negative, or a disinvestment. Hence, the goal is to maximize some function of Ω, $\Phi(\Omega) = E(\Omega'R)^2 (Var(\Omega'R))^{-1} = (\Omega'M)^2 (\Omega'\Sigma\Omega)^{-1}$.

Although the derivation is too involved to be presented here, an optimal Ω^* is $\Sigma^{-1}M$. Now Σ^{-1} must exist since Σ is positive definite. Furthermore, $E(\Omega^{*\prime}R) = (\Sigma^{-1}M)' M = M'(\Sigma^{-1})'M = M'\Sigma^{-1}M$. And $Var(\Omega^{*\prime}R) = (\Sigma^{-1}M)'\Sigma(\Sigma^{-1}M) = (M'\Sigma^{-1})\Sigma(\Sigma^{-1}M) = M'\Sigma^{-1}M = E(\Omega^{*\prime}R)$. Since $M'\Sigma^{-1}M > [0]$ for our non-zero M, $Var(\Omega^{*\prime}R)^{-1}$ exists. Therefore, $\Phi(\Omega^*) = M'\Sigma^{-1}M$. Also note that $E(\Omega^{*\prime}R) > [0]$, irrespective of how many negative elements M contains. However, negative elements in M are likely to produce negative investment elements in Ω^*.

Now consider:
$$\begin{aligned}\Phi(\Omega) &= (\Omega'M)^2 (\Omega'\Sigma\Omega)^{-1}\\ &= (\Omega'M)^2 (\Omega'\Sigma\Omega)^{-1} (M'\Sigma^{-1}M)^{-1} (M'\Sigma^{-1}M)\\ &= (\Omega'M)^2\, Var(\Omega'R)^{-1}\, Var(\Omega^{*\prime}R)^{-1}\, \Phi(\Omega^*)\\ &= (\Omega'\Sigma\Sigma^{-1}M)^2\, Var(\Omega'R)^{-1}\, Var(\Omega^{*\prime}R)^{-1}\, \Phi(\Omega^*)\\ &= (\Omega'\Sigma\Omega^*)^2\, Var(\Omega'R)^{-1}\, Var(\Omega^{*\prime}R)^{-1}\, \Phi(\Omega^*)\\ &= Cov(\Omega'R, \Omega^{*\prime}R)^2\, Var(\Omega'R)^{-1}\, Var(\Omega^{*\prime}R)^{-1}\, \Phi(\Omega^*)\\ &= \rho(\Omega'R, \Omega^{*\prime}R)^2\, \Phi(\Omega^*),\end{aligned}$$
which is less than or equal to $\Phi(\Omega^*)$, since $[0] <= \rho^2 <= [1]$. Thus there is no investment strategy superior to Ω^*.

$\Sigma^{-1}\mathbf{M}$ is not the only optimal value, since $\Phi(k\Omega) = \Phi(\Omega)$ for any non-zero scalar k. This shows that the optimization is not affected by the total amount of the investment, which in matrix terms is $k\Omega^{*\prime}\mathbf{J} = k\mathbf{M}'\Sigma^{-1}\mathbf{J}$. Since a return is relative to the initial investment, we may define the optimal return $\mathbf{R}^*$ as $\Omega^{*\prime}\mathbf{R}\,(\Omega^{*\prime}\mathbf{J})^{-1}$. By our second assumption, $\Omega^{*\prime}\mathbf{J} = (\Sigma^{-1}\mathbf{M})'\,\mathbf{J}$ is nonsingular, so the inverse exists.

Before investigating the properties of $\mathbf{R}^*$ we should ask about the practicality of a singular $\Omega^{*\prime}\mathbf{J}$, i.e., what if $\mathbf{M}'\Sigma^{-1}\mathbf{J} = [0]$? If this were the case, then the optimal return would be attained by a total investment of zero (dollars, or other units of money), whether this meant that zero would be invested in every asset or that positive and negative investments would net to zero. Either way, each investor would have a net position in the market of zero, which means that the value of the whole market of risky assets would be zero. Because this is unrealistic, we may assume $\Omega^{*\prime}\mathbf{J}$ to be nonsingular.

Since $\mathbf{R}^* = \Omega^{*\prime}\mathbf{R}\,(\Omega^{*\prime}\mathbf{J})^{-1}$, $\mathbf{R}^* \sim [\mathbf{M}'\Sigma^{-1}\mathbf{M}(\Omega^{*\prime}\mathbf{J})^{-1}, \mathbf{M}'\Sigma^{-1}\mathbf{M}(\Omega^{*\prime}\mathbf{J})^{-2}]$. Notice that $\mathrm{Var}(\mathbf{R}^*) = \mathrm{E}(\mathbf{R}^*)\,(\Omega^{*\prime}\mathbf{J})^{-1}$. Also, $\mathrm{Cov}(\mathbf{R}, \mathbf{R}^*) = \mathrm{Cov}(\mathbf{R}, \Omega^{*\prime}\mathbf{R}\,(\Omega^{*\prime}\mathbf{J})^{-1}) =$ $\mathrm{Cov}(\mathbf{R}, \mathbf{R})\,\Omega^*\,(\Omega^{*\prime}\mathbf{J})^{-1} = \Sigma\,\Omega^*\,(\Omega^{*\prime}\mathbf{J})^{-1} = \Sigma\,\Sigma^{-1}\mathbf{M}\,(\Omega^{*\prime}\mathbf{J})^{-1} = \mathbf{M}\,(\Omega^{*\prime}\mathbf{J})^{-1} =$ $\mathrm{E}(\mathbf{R})\,(\Omega^{*\prime}\mathbf{J})^{-1}$.

As an (n x1) vector we may write the CAPM beta as follows:

$$\begin{aligned}\mathbf{B} &= \mathrm{Cov}(\mathbf{R}, \mathbf{R}^*)\,(\mathrm{Var}(\mathbf{R}^*))^{-1}\\ &= \mathrm{E}(\mathbf{R})\,(\Omega^{*\prime}\mathbf{J})^{-1}\,(\mathrm{E}(\mathbf{R}^*)\,(\Omega^{*\prime}\mathbf{J})^{-1})^{-1}\\ &= \mathrm{E}(\mathbf{R})\,(\Omega^{*\prime}\mathbf{J})^{-1}\,((\Omega^{*\prime}\mathbf{J})^{-1})^{-1}\,(\mathrm{E}(\mathbf{R}^*))^{-1}\\ &= \mathrm{E}(\mathbf{R})\,\mathrm{E}(\mathbf{R}^*)^{-1}.\end{aligned}$$

Therefore, $\mathrm{E}(\mathbf{R}) = \mathbf{B}\,\mathrm{E}(\mathbf{R}^*)$, which is the CAPM equation in relative form. As mentioned earlier, the absolute form of the equation is obtained by substituting $\mathbf{R} - \mathbf{J}\mathbf{R}_0$ for $\mathbf{R}$. So $\mathrm{E}(\mathbf{R} - \mathbf{J}\mathbf{R}_0) = \mathbf{B}\,\mathrm{E}(\Omega^{*\prime}\{\mathbf{R} - \mathbf{J}\mathbf{R}_0\}(\Omega^{*\prime}\mathbf{J})^{-1}) = \mathbf{B}\,(\mathrm{E}(\mathbf{R}^*) - \Omega^{*\prime}\mathbf{J}\mathbf{R}_0(\Omega^{*\prime}\mathbf{J})^{-1})$

$= \mathbf{B}\,(E(\mathbf{R}^*) - \mathbf{R}_0)$. Therefore, $E(\mathbf{R}) = \mathbf{J}\mathbf{R}_0 + \mathbf{B}\,(E(\mathbf{R}^*) - \mathbf{R}_0)$.

Although $\mathbf{R}^*$ has been called the optimal return, it is also represents the *market* return. The argument for this is theoretical: namely, that if every investor is fully informed and rational, then every investor will combine assets proportionately to $\mathbf{\Omega}^*$. This means that the whole market itself of risky assets will allocate value according to $\mathbf{\Omega}^*$, and will have the return characteristics of $\mathbf{R}^*$. Reasons why this may not happen in practice will be presented later.

In concluding this section, let us derive the familiar theorem that the beta of a portfolio is the weighted average of the betas of the portfolio's assets. Letting $\mathbf{\Omega}$ be the asset allocation, the portfolio's beta is $\mathrm{Cov}(\mathbf{\Omega}'\mathbf{R}, \mathbf{R}^*)(\mathrm{Var}(\mathbf{R}^*))^{-1} = \mathbf{\Omega}'\mathrm{Cov}(\mathbf{R}, \mathbf{R}^*)\,(\mathrm{Var}(\mathbf{R}^*))^{-1}$ $= \mathbf{\Omega}'\mathbf{B}$.

II. An Illustration of the Theory

If the author's argument has not been clear enough, perhaps an example will be of help. Consider the simple case of a two-asset universe. Suppose asset A to be priced so as to have an expected return of 0.08, or 8 percent. We regard return as a dimensionless number: $\frac{x_1}{x_0} - 1$, where x_0 and x_1 represent initial and terminal wealth respectively. *Rate* of return is return per time, and has units of $(\text{time})^{-1}$. It makes no difference to the example whether we deal with returns or with rates of return; however, actuaries should ensure the dimensional consistency of their formulae. Suppose that the variance of asset A's return is 0.10. Next, let asset B have an expected return of 0.02 and a variance of

0.04. And let the covariance of A and B be -0.06. Finally, suppose the risk-free return to be 0.04.

The returns in excess of the risk-free return are 0.04 and -0.02 respectively. One might wonder why asset B with its substandard return would exist in the market. The answer lies in the negative covariance. Asset B has value not in itself, but in its tendency to cancel out the variance of asset A. Using notation from above, we write:

$$\mathbf{R} \sim \left[\mathrm{M} = \begin{bmatrix} 0.04 \\ -0.02 \end{bmatrix}, \Sigma = \begin{bmatrix} 0.10 & -0.06 \\ -0.06 & 0.04 \end{bmatrix} \right].$$

M is expressed in relative form; Σ is a positive definite matrix. The numbers were chosen so that the example would not be cluttered with fractions or repeating decimals:

$$\Sigma^{-1} = \begin{bmatrix} 100 & 150 \\ 150 & 250 \end{bmatrix}, \text{ and}$$

$$\Omega^* = \Sigma^{-1}\mathrm{M} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \propto \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}.$$

Therefore, the market, in order to obtain the optimal return, will allocate value among assets A and B in equal proportions. Hence, the optimal return is:

$$\begin{aligned} \mathbf{R}^* &= \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \mathbf{R} \\ &\sim \left[\begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \mathrm{M}, \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \Sigma \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \right] \\ &\sim \left[[0.01], [0.005] \right]. \end{aligned}$$

$$\begin{aligned}\text{Also, } \operatorname{cov}(\mathbf{R},\mathbf{R}^*) &= \operatorname{cov}(\mathbf{R},[0.5 \quad 0.5]\mathbf{R}) \\ &= \operatorname{cov}(\mathbf{R},\mathbf{R})\begin{bmatrix}0.5\\0.5\end{bmatrix} \\ &= \Sigma\begin{bmatrix}0.5\\0.5\end{bmatrix} \\ &= \begin{bmatrix}0.02\\-0.01\end{bmatrix}.\end{aligned}$$

Therefore, $\mathbf{B} = \operatorname{cov}(\mathbf{R}, \mathbf{R}^*)\operatorname{var}(\mathbf{R}^*)^{-1} = \begin{bmatrix}0.02\\-0.01\end{bmatrix}[0.005]^{-1} = \begin{bmatrix}4.0\\-2.0\end{bmatrix}$. So, the CAPM equation in relative form is true: $E(\mathbf{R}) = \begin{bmatrix}0.04\\-0.02\end{bmatrix} = \begin{bmatrix}4.0\\-2.0\end{bmatrix}[0.01] = \mathbf{B}E(\mathbf{R}^*)$, as well as the equation in absolute form:

$$\begin{aligned}E(\mathbf{R}) &= \begin{bmatrix}0.08\\0.02\end{bmatrix} \\ &= \begin{bmatrix}0.04\\0.04\end{bmatrix} + \begin{bmatrix}4.0\\-2.0\end{bmatrix}([0.05]-[0.04]) \\ &= \begin{bmatrix}1\\1\end{bmatrix}[0.04] + \begin{bmatrix}4.0\\-2.0\end{bmatrix}([0.05]-[0.04]) \\ &= \mathbf{J}\mathbf{R}_0 + \mathbf{B}(E(\mathbf{R}^*)-\mathbf{R}_0).\end{aligned}$$

Also, note that the market-weighted beta is $[0.5 \quad 0.5]\begin{bmatrix}4.0\\-2.0\end{bmatrix} = [1.0]$, as expected.[2]

The econometric material in the CAS part 10 exam induced the author to study matrix theory from an econometric perspective.[3] This effort has repaid me with a generous dividend, and I hope that many readers will have their appetites whetted to undertake similar studies. In my first draft of this article I had not seen the matrix application, and was tediously proving just the two-asset case of the CAPM by considering the efficient frontier as a parametric equation in one parameter.[4] I am convinced that matrix theory is a powerful tool in its own right, rather than just a convenient shorthand, and that in statistics the econometricians are far ahead of us actuaries precisely because of their matrix approach to this subject.

III. Portfolio Optimization and the CAPM in Practice

Throughout the article we have been speaking of a perfectly informed and rational investor. However, we know that no two investors have the same beliefs about the future, and no two have the same utilities. For example, a socially conscious investor who refuses to purchase tobacco stocks, or South African gold stocks, is undoubtedly shaving from the optimal return. However, the loss is compensated by his perceived loyalty to virtue. No two investors are alike; and perhaps the perfectly informed and rational investor is a far-away ideal.

Furthermore, we cannot obtain the needed M and Σ matrices. Indeed, the first problem is to define what belongs to the universe of assets. In the standard applications of the CAPM "the market" is proxied by the S&P 500 index. Granted that the S&P 500 makes up about two-thirds of the market value of US. stocks, what about the stocks of the rest of the world? And what about the other risky assets of the world, which is just about everything except US. treasury securities? What about real estate? And perhaps commodities, such as wheat, oil, and gold, should be included -- perhaps even collectibles, such as rare coins and art. In other words, although we speak glibly of "the market," no one really knows its extent. Anything that can traded, perhaps even insurance loss portfolios, might be part of the market. So probably our proxies are rather bad ones, and partly responsible for the mixed results of CAPM tests.

And even with a limited universe of say 500 stocks, there remains a problem in estimating 500 betas and one equity risk premium (r_m-r_f). The problem is well known to actuaries as the dilemma between stability and responsiveness, viz., that by the time you have enough observations to perform a good estimation, the underlying parameters have more or less drifted. So the CAPM might be perfectly corroborated, if only we knew the current parameters, rather than the outdated ones. Perhaps "the market" has some great collective intuition, which transcends the knowledge of individual investors. The logical positivist would balk at such a statement, which is more or less the capitalist's credo. However, the notion that there really is an "invisible hand" in human affairs which directs toward the greatest good is somewhat reasonable, even if difficult to verify -- as difficult to verify as the CAPM itself.

The CAPM is of one piece with the efficient market hypothesis. It is of no help in the selection of stocks or of any other asset. In fact, it dictates that every investor's portfolio be a microcosm of the whole market. If the market really were the S&P 500, for example, then the CAPM would have everyone invested in a mutual fund indexed to the S&P 500, which is called passive investing. Herein lies a parting conundrum: although passive investing should be optimal, the market needs to be winnowed and sifted by active investors endeavoring to outperform it.

Notes

1 It is presumed that the reader has some familiarity with matrix algebra. Therefore, some of the steps in the derivations may involve the application of multiple matrix theorems. Some of the basic properties of matricies are stated here, and may be of help if the reader is puzzled by a derivation:

A. Matrix multiplication is associative: A(BC)=(AB)C.
B. Matrix multiplication is not commutative; however, (1 x 1) matrices commute.
C. Matrix multiplication is distributive: A(B+C)=AB+AC.
D. Transposition of a product behaves thus: (AB)' = B'A'.
E. Similarly, with matrix inversion, $(AB)^{-1}=B^{-1}A^{-1}$, if A and B are nonsingular.
F. By definition, A is symmetric if and only if A'=A.
G. Every (1 x 1) matrix is symmetric.
H. If A is nonsingular, then $(A^{-1})^{-1} = A$. Also, $(A^{-1})'=(A')^{-1}$.

2 For those who wonder if the example might be contrived in that the optimal combination of assets was 50/50, we modify the example by changing the risk-free return from 0.04 to 0.03. The reader can verify:

$$\mathbf{R} \sim \left[M = \begin{bmatrix} 0.05 \\ -0.01 \end{bmatrix}, \Sigma = \begin{bmatrix} 0.10 & -0.06 \\ -0.06 & 0.04 \end{bmatrix} \right].$$

$$\Omega^* = \Sigma^{-1}M = \begin{bmatrix} 3.5 \\ 5.0 \end{bmatrix} \propto \begin{bmatrix} 7/17 \\ 10/17 \end{bmatrix}$$

$$\begin{aligned} \mathbf{R}^* &= [7/17 \quad 10/17]\mathbf{R} \\ &\sim [[1/68],[1/578]]. \end{aligned}$$

$$\begin{aligned} \text{cov}(\mathbf{R},\mathbf{R}^*) &= \text{cov}(\mathbf{R},[7/17 \quad 10/17]\mathbf{R}) \\ &= \text{cov}(\mathbf{R},\mathbf{R})\begin{bmatrix} 7/17 \\ 10/17 \end{bmatrix} \\ &= \Sigma\begin{bmatrix} 7/17 \\ 10/17 \end{bmatrix} \\ &= \begin{bmatrix} 1/170 \\ -1/850 \end{bmatrix}. \end{aligned}$$

So $\mathbf{B} = \text{cov}(\mathbf{R}, \mathbf{R^*})\,\text{var}(\mathbf{R^*})^{-1} = \begin{bmatrix} 1/170 \\ -1/850 \end{bmatrix} \begin{bmatrix} 1/578 \end{bmatrix}^{-1} = \begin{bmatrix} 17/5 \\ -17/25 \end{bmatrix}$. The CAPM equation in relative form checks: $\text{E}(\mathbf{R}) = \begin{bmatrix} 0.05 \\ -0.01 \end{bmatrix} = \begin{bmatrix} 17/5 \\ -17/25 \end{bmatrix} [1/68] = \mathbf{B}\,\text{E}(\mathbf{R^*})$. Also, the market-weighted beta is $[7/17 \quad 10/17] \begin{bmatrix} 17/5 \\ -17/25 \end{bmatrix} = [1.0]$.

3 For those interested in studying econometrics, the author recommends Introduction to the Theory and Practice of Econometrics, 2nd edition, by G. G. Judge, R. C. Hill, *et al.* (New York: John Wiley & Sons, 1988). The seventy-five page appendix on matrix theory alone makes the book worth reading.

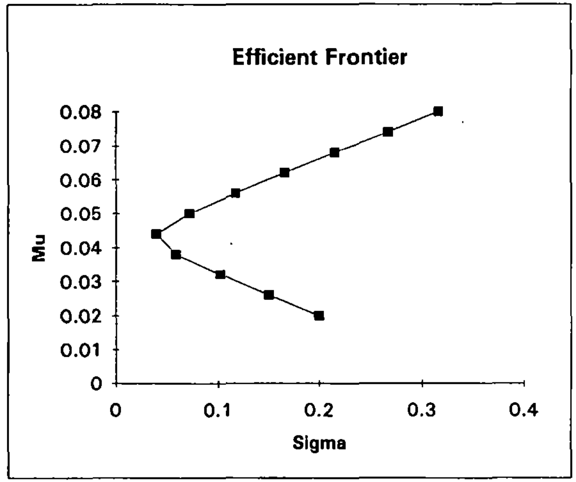
4 See the following Appendix for a speadsheet of the two-asset example.

APPENDIX

CAPM Illustration showing Optimal Mix at 50/50

	Mu	Sigma^2	Cov(A,B)
Asset A	0.08	0.1	-0.06
Asset B	0.02	0.04	
Risk-free	0.04		

Wgt(A)	Wgt(B)	Sigma	Mu	(Mu-0.04)/ Sigma
0%	100%	0.2	0.02	-0.1
10%	90%	0.150333	0.026	-0.09313
20%	80%	0.10198	0.032	-0.07845
30%	70%	0.05831	0.038	-0.0343
40%	60%	0.04	0.044	0.1
50%	50%	0.070711	0.05	0.141421
60%	40%	0.116619	0.056	0.137199
70%	30%	0.165529	0.062	0.132907
80%	20%	0.215407	0.068	0.129987
90%	10%	0.265707	0.074	0.127961
100%	0%	0.316228	0.08	0.126491



Mu = Wgt(A)*Mu(A) + Wgt(B)*Mu(B)
Sigma = SQRT{Wgt(A)^2*Sigma(A)^2 + 2*Wgt(A)*Wgt(B)*Cov(A, B) + Wgt(B)^2*Sigma(B)^2}

Ratemaking 1993: A Play "Not Ready for a Stable Market"
by Nolan E. Asch

DRAFT

In April, 1993, the "Not Ready For A Stable Market Players" (Dave Skurnick, Jerry Tuttle, Helen Exarhos, Nolan Asch) presented a 3 Act Play at a CAS Special Interest Seminar in Raleigh/Durham, North Carolina. It looked at 3 Mythical Companies; Mindless Mutual, Global Galactic and Cowboy Casualty. It concentrated on explaining some of the behavioral forces that might influence the UW Cycle and how they interact with certain Actuarial factors.

I repeated the 1989 Play and then revisited the same three fimrs in 1993 with a topical update. Perhaps four years later (or sooner) the Author will try to update the Play. There seems to be a need, in my opinion, for Actuaries to test the accuracy and appropriateness of their models and assumptions in the real world over the long run.

The Play

We are revisiting the three firms we looked in on in 1989 again in 1993. We (The "Not Ready For A Stable Market Players") feel there are serious causative factors for the Underwriting Cycles being addressed between the lines of this exercise. Note that we have tried to be consistent in our second incarnation of our 3 act play. Note that we are using actual industry statistics in many of the 1993 updates slides. There are sections meant to reflect what the person is really thinking, but not saying. The speaker will turn to the audience and preface those remarks with "well folks." We will visit each of our three mythical firms first, in 1989, to repeat the drama they faced then - And then their updated 1993 situation. Of course, we are describing mythical firms and not advocating any particular course of action for any firm today.

ACT I (1989)

GLOBAL GALACTIC

CAST

Nolan Asch. . . . . .CHAIRMAN

Jerome Tuttle . . . . .PLANNER

ACT I (1989)

GLOBAL GALACTIC

PLANNER: (J. Tuttle) ...As you can clearly see -- the trend in pricing for all lines is clear via our monthly monitoring systems.

Price Levels
See Chart 1

(Slide 89-1-1)

June 1984 June 1986

The decline continues ...although at a less severe slope this month...

CHAIRMAN: (N. Asch) I know all this -- what I must know is where the break-even profit position for these rates is -- I am the chairman and the final strategic decision must be mine.

PLANNER: (J. Tuttle) Break-even levels are, as you know, a result of many factors -- the payment pattern and loss ratio outcomes, investment returns --

CHAIR: (N. Asch) Yes, I know all this. It's clear the June 1984 rates were ruinously low and the trend had to change. In 1986, rates peaked out at high profit margins, and rates have plummeted ever since. -- My actuary keeps telling me about claims cost inflation, "shock" awards, the next "pollution fiasco" -- while my marketing VP keeps telling me about the market share and anti-selection. But what I want to know is...

PLANNER: (J. Tuttle) Yes - I know - you want to know which strategy will have the better impact on long-term Earning Per Share.

CHAIR: (N. Asch) And Short-term EPS.

PLANNER: (J. Tuttle) Well, here I can maintain a simple position. Given our large casualty distribution of business, the easiest way to improve short term earnings is--

CHAIR: (N. Asch) I know - maximize current premium volume. The losses cannot appear immediately, but the premiums do. Let's look at those premium numbers again.

PLANNER: (J. Tuttle) (SLIDE 89-1-2)

As you know, premiums exploded from 1985 thru mid-1987, due to price increase. As you can see, (SHOW 89-1-1) our commitment to high standards led to flat premiums through 1988 and signs of premium shrinkage in 1989.

However, our actuarial analysis shows clearly, that on the "1985 standards basis," the percentage of premiums written to that standard has dropped consistently -- from 1985 - 100%.

To 1987 - Jan. 90% Dec. 70% (SLIDE 89-1-3)

1988 - July 50% Dec. 25%

In other words - only.

CHAIR: (N. Asch) Yes, I know --

PLANNER: (J. Tuttle) Don't interrupt!

CHAIR: (N. Asch)	Damn those actuaries, their logic is irrefutable. They're like my conscience! So... the only certain way to achieve the desired EPS increase is to increase premiums - by writing more business whose rates, terms and conditions today are marginal and appear to be still deteriorating.
PLANNER: (J. Tuttle)	We don't have to kow-tow to Wall Street. We're a Top Ten firm in this industry and we have credibility with most on Wall Street.
CHAIR: (N. Asch)	It's not just Wall Street I'm worried about... It's our parent company. The cereal people.
PLANNER: (J. Tuttle)	I thought they said ...
CHAIR: (N. Asch)	Yes -- I have their total confidence. Since they bought us in 1984, I showed them nothing but massive earnings increase in 1985 and 1986. In 1987, they saw that EPS was increasing, but at a much slower rate. In 1988, they didn't like flat earnings, with several "down" quarters, AT ALL. Now, I'm afraid, if 1989 isn't up to expectations they'll be eating me for breakfast. They don't totally understand all the technical nuances of this business -- like we do. I'm afraid if EPS doesn't move up, I'll be replaced. Aside from ego and selfish motives, replacing me with a less responsible or less competent CEO will be bad for the whole industry ... and the public. What should I do?

Act One (1993)
GLOBAL GALACTIC

PLANNER: (J. Tuttle) ... As you can clearly see -- the trend in pricing for all Commercial Lines is clear via our monthly monitoring systems. (SLIDE 93-1-1)
we maintain price stability.....

CHAIRMAN: (N. Asch) I know all this --- what I need to know is how long we can continue viably in this environment. We made the hard choice in 1989 to maximize current premium volume, focus on rate of return rather than targeting an underwriting profit in every pricing exercise (Well Folks that's why I'm still here) but our ROEs have plummeted.
SHOW Exhibit 93-1-2 ROE in industry
We never thought the downturn would be this sharp or this long. We never planned on Cat losses like Andrew. We had secure Cat Reinsurance for that one but if it happens again we do not have that level of coverage. Also, this year we mitigated our Andrew losses though taking capital gains, but that's a one-time thing!
OH MY GOD! NOT AGAIN!

PLANNER: (J. Tuttle) Yes and the Actuaries are getting more vocal about it!

CHAIRMAN: (N. Asch) Can't you get a room deodorizer?

PLANNER: (J. Tuttle) You know how the wind gets a hold of those Asbestos and Pollution claim files in the basement. They have to go down there to pay some of the Asbestos claims and that stirs the air down there. All those $1 Environmental claim files there must be thousands of them...

CHAIRMAN: (N. Asch) 11,857 by the latest weekly count. If only the smell were the only consequence. We have been forthright. We have established a fund and begun to build. We have made the appropriate caveats in our opinions....

PLANNER: (J. Tuttle) And we have reams of studies that show our reserve problem here is less than our three major competitors...

CHAIR: (N. Asch) Yes. We're under reserved by 30% of our surplus while they average 60% of their surplus. Sometimes I wish we didn't know so much about it. Perhaps we would have been better off with a "pay as you go" approach? When will this mad competitive cycle end? When we're all technically bankrupt?

PLANNER: (J. Tuttle) Perhaps. You see here a retrospective test of the industry's surplus at 12/84 if our current best estimate of needed 1984 reserves is used. As you can see the reinsurance industry (by this measure) was technically insolvent and the primary industry was close. Slides 93-1-3, 93-1-4

According to an ISO analysis of 12/91 industry loss reserves they estimate $50 Billion of under reserving on a $160 Billion surplus base, up from a $36 Billion estimate last year.

CHAIR: (N. Asch) Thanks for your advice. I will see ya later.

PLANNER: (J. Tuttle) Yeah. See ya later.

CHAIR: (N. Asch) Well folks, what do I do now? Come clean about our reserve problems? Tell everyone that our current pricing implies to me even worse true levels of current profitability than we're reporting or --- slog ahead, and muddle through, putting the best possible public face on all this? What would you do?

ACT II (1989)

COWBOY CASUALTY

CAST

Nolan Asch. . . . . .CHAIRMAN

Jerome Tuttle . . . . .PLANNER & STAFF MAN

Helen Exarhos . . . . .STAFF PERSON 2

Dave Skurnick . . . . .ACTUARY

ACT II (1989)

CAFETERIA OF COWBOY CASUALTY

(THE CHAIRMAN IS HOLDING ONE OF HIS "KITCHEN CABINETS" WITH SEVERAL KEY EXECUTIVES)

CHAIR: (N. Asch) You know... We have a motto here at Cowboy Casualty -- "No one has a job here unless somebody out there makes a sale." It's taken us from a medium-sized regional insurer to a major national insurance company in less than 5 years. We have had a compound premium growth rate of over 30% a year throughout the period. (SHOW SLIDE 89-2-1)

STAFF: (J. Tuttle) But to continue that growth rate we'd need to become a $450 Million company in 1993. (SHOW SLIDE 89-2-2)

CHAIR: (N. Asch) Why not? It's just perpetuating the same growth rate of the last 4 years.

STAFF: (J. Tuttle) Because, sooner or later there are limits to our size. We can't write almost every risk. And by continuing to cut rates we are helping to reduce the total Industry Premium pie every year.

CHAIR: (N. Asch) I know you worry about our recent rate reductions -- but let's look at the "big picture" (SHOW SLIDE 89-1-1 AGAIN ON IND RATES). Even though rates are declining. They are still well above 1983/84 rate levels. ... Also, you forget our 3 secret weapons ...

STAFF: (J. Tuttle) I know

CHAIR: (N. Asch) But do you really believe? We have a saying here at Cowboy Casualty ...

STAFF: (J. Tuttle) I know .. "Knowledge without belief is a barren tree."

CHAIR: (N. Asch) Well -- Let's review our 3 weapons:
#1 - you no longer need underwriting profits to realize a profit on business. Our investment department has consistently earned returns 2 to 3 points better than the industry.

STAFF: (J. Tuttle) Only over 5 years, after investing in riskier instruments than our competitors.

CHAIR: (N. Asch) But you agree we've been earning 10% per annum. Our average payout is 3 years after premium collection. That means we can break even at a 133% combined ratio. (SLIDE 89-2-3)

STAFF: (J. Tuttle) If the 10% holds up. Also, you're ignoring the new tax law and the fact that at 20% commission you only earn interest on 80%, and you are <u>not always</u> going to earn investment income faster than loss payments materialize. (SLIDE 89-2-4)

CHAIR: (N. Asch) Your 80% point is well taken ... (SLIDE 89-2-5) But we still break even at 1.0648 - .80 = .2648 + 1 = 126.48%. Also, our new plan is write even longer-tail business to increase our investment leverage.

Our second weapon is our superior portfolio. We have had a clientele of smaller, loyal risks in rural locales. Their frequency characteristics have always been superior to industry averages. And we avoid anti-selection by being the lowest priced market in each of our target sectors.

STAFF: (J. Tuttle) This weapon is eroding. We're now a national company with a slightly less select book and our target sectors now cover 50% of our premium volume ... not 10% as when we started the program. Also our rate is eroding.

CHAIR: (N. Asch) How are we going to lose money on people who never file claims? My claims-free discount system has been praised by many industry experts.

STAFF: (J. Tuttle) Giving a 5% discount on renewal to a claims-free risk the first year is fine, even for a 2nd or 3rd year -- but extending it up to 10 years for a maximal 50% discount!!! It didn't matter in the early years when no one had earned many discounts -- but we're now in year 4 and 90% of those policyholders have earned a 20% discount.

CHAIR: (N. Asch) That's great! We've kept them loss free <u>and</u> <u>with</u> <u>us</u> for 4 years! 90% claims-free!!! Just imagine if 10% or 20% more had left us?! We'd have lost all that clean premium ! These people are going to think twice about leaving us, or filing any small claims to forfeit their claims free discount!

STAFF 2: (H. Exarhos) Mr. Chairman - we've got a large risk new business submission that needs your immediate attention.

CHAIR: (N. Asch) YA HOO - There's nothing like new business.

STAFF 2: (H. Exarhos) It's a fairly large firm. The key to the risk is their products liability for automobile parts. (SHOW 89-1-1) As you can see -- with loss development, their rate per exposure has been climbing slowly. (SLIDE 89-2-6)
With current trends, it seems next year's ultimate net loss cost should be $322,000 grossed up for 25% Expenses by 100/75ths; (SLIDE 89-2-7) that's a $430,000 Premium. That's probably not enough since their latest loss control report from their existing carrier has caused them to quote a renewal rate higher than this designed to lose the renewal.

CHAIR: (N. Asch) Maybe -- Maybe not. Also, what's the policy limit and policy aggregate? Let's see, with a 5-year average payout at 10% ... that's a 161% combined to break-even. So -- We don't need $430,000. We need 430/1.61 = $267,000.
(SLIDE 89-2-8)

STAFF 2: (H. Exarhos) It's a $1M occurrence policy with a $2M general policy aggregate but the LAE is in addition to limits. (SLIDE 89-2-9) The 5-year average indication is $326,000 not $430,000 but the risk manager is looking for a premium of around $150,000. Last year, they paid $250,000 and Mindless Mutual is competing also.

CHAIR: (N. Asch) (TO STAFF 1) We haven't yet factored in our 3rd and strongest secret weapon ... (PAUSE)

STAFF 1: (J. Tuttle) What's that?

CHAIR: (N. Asch)	RICKETTY RE If memory serves me well, we have a 750 xs 250 treaty with Ricketty Re and a 1M xs 1M treaty. We pay a rate of 10% for both covers combined. Aggregate excess <u>is</u> included for products. That means we are writing a policy with a $250,000 Net Aggregate loss-limit and 5-year average pay-out lag.
STAFF 1: (J. Tuttle)	But -- I've told you how shaky Ricketty Re is getting. Also, we <u>know</u> we'll suffer that full 250K loss for certain -- and the payout pattern for us will be far shorter than 5 years, since we're paying the first losses -- our reinsurer will be paying the later losses. We can't just assume 10% interest rates.
CHAIR: (N. Asch)	Hmm - This sounds like a tough one -- well -- Let's call our actuary in on this one. Why don't you both go get him. (BOTH STAFF PEOPLE GET ACTUARY WHEEL HIM OUT AND UNTIE HIM) (ACTUARY IS WHEELED OUT -- BOUND AND GAGGED) (CHAIR SPEAKS WHILE STAFF UNTIES ACTUARY) Let's summarize -- let him look at all the data on this risk -- then give him 3 minutes to speak. As I see it, it's a golden opportunity. This is precisely the kind of longer tail business we now want to write. With our reinsurance arrangements at a $150,000 Premium and a 10% treaty cost ... (well folks, that's the price the risk manager wanted) That's $135,000 left and 1.61 for investment income, that's $217,000 to pay a maximum loss of $250,000. That's good odds to me. (SLIDE 89-2-10)

ACTUARY: (D. Skurnick) This is nonsense! You need to subtract at least 25% for commissions, taxes and expenses up front! Even using all your assumptions that generates (217) times (.75) = $163. Not 217. (SLIDE 89-2-11) The 250 is expected to be paid every year. Remember our reinsurance does not include LAE! Also, there is generally 40 cents of LAE for every dollar of loss - (SLIDE 89-2-7, again) so expect 322 x .40 = $129,000 of LAE per annum to fund. That yields an ultimate loss and LAE of $451,000 per annum to pay for. Our payout pattern is going to be shorter than 5 years! Most importantly -- my security review of Ricketty Re finds them very Ricketty indeed.

CHAIR: (N. Asch) That's enough. I'm beginning not to like you -- Boy. Ricketty Re is solid! Highly regarded by all the rating agencies.

ACTUARY: (D. Skurnick) They're growing too fast in relation to their surplus! They're at 2.5 to 1! Their loss reserving is consistently testing inadequate.

CHAIR: (N. Asch) Hell! That's what everybody's whispering about us -- Growing to fast!! Over leveraged! We've got positive cash flow up our ying-yang!!! See you later!

(ACTUARY IS REBOUND AND REGAGGED)

CHAIR: (N. Asch) (ALONE) Well folks, that actuary is a smart guy. Stands up to me. I like that. Got to think about that angle. Still -- these technicians just somehow cannot grasp the BIG PICTURE.

ACT TWO (1993)

CAFETERIA OF COWBOY CASUALTY

CHAIRMAN: (N. Asch) My isn't that satisfying?! Way back in 1989,We planned to be at $450 Mil in 1992 and here we are, in black and white, right on the money! And you said it couldn't be done!

STAFF: (J. Tuttle) I did not say that. I said there were risks involved in this type of rapid growth. We have failed to even approach an underwriting profit in any year. Look at this!
SHOW slide 93-2-1 Industry (c lines) UW ratios

CHAIR: (N. Asch) I've been telling you for years you don't need UW profits! Look at our rate of return!
SHOW slide 93-2-2 Industry ROE

STAFF: (J. Tuttle) That rate of return line is nothing to write home about, especially in recent years.

CHAIR: (N. Asch) There's that smell again! Didn't we figure out what it was?

STAFF: (J. Tuttle) It's just those old claims files in the basement.

CHAIR: (N. Asch) That MGA was a nice guy. How could his business smell so bad when it looked so good in the years he wrote it?

STAFF: (J. Tuttle) Don't worry about that! We have more troubles coming from the state of Despair.

CHAIR: (N. Asch) — Not that state again! They're the real thieves! Didn't the courts just rule on our industry appeal?

STAFF: (J. Tuttle) — Yes. They ruled in our favor on confiscatory rate levels but upheld them on the residual market issue.

CHAIR: (N. Asch) — That's not good! But, those actuaries must be full of it again! How could we lose $40 mill on just $10 mill of voluntary writings in the state! We shrunk by 50% that year in that state! You know how that goes against my grain but I fully agreed to it! Wheel that Actuary in!
(Actuary is wheeled in, bound and gagged.)

ACTUARY: (D. Skurnick) — That's better! You know, we need an employee health plan for dental work!

CHAIR: (N. Asch) — Enough of that! How could your projections be right?

ACTUARY: (D. Skurnick) — Well sir, in fact, they are wrong!

CHAIR: (N. Asch) — I knew it!

ACTUARY: (D. Skurnick) — We won't lose $40M, we'll lose $80M!

CHAIR: (N. Asch) — What!

ACTUARY: (D. Skurnick)	The actual size of the residual market turned out to be 80% of the market on audit, not 60%. That means the voluntary market is only 20% ($20M) and not 40% ($40M) as originally thought. That means that the $160 Mill residual market loss goes 50% to us, since we wrote $10M (10%) rather than only 25% to us as we thought. And it gets worse!
CHAIR: (N. Asch)	WORSE!
ACTUARY: (D. Skurnick)	Yes. The auditors are finding irregularities. I predict the ultimate loss will be far greater than $160M and we will assume 50% of the loss no matter how large it is!
CHAIRMAN: (N. Asch)	How bad do you think it will get?
ACTUARY: (D. Skurnick)	Honestly?
CHAIR: (N. Asch)	Yes, honestly. This talk about my temper is exaggerated. You know that you've been here 15 years!
ACTUARY: (D. Skurnick)	Only 7 years!
CHAIR: (N. Asch)	Seems like 15 to me!
ACTUARY: (D. Skurnick)	In addition to all these problems, we have the problems related to reinsurance recoverables.

CHAIRMAN: (N. Asch)	What reinsurer are we worried about?
ACTUARY: (D. Skurnick)	Ricketty Re --- We have $150 million of recoverable from Ricketty on paid losses over 90 days and I don't think we will ever collect that money. By the way, that is equal to our total statutory surplus. We also have another $300 million of reinsurance recoverable from Ricketty Re on unpaid losses, and I don't think we're going to collect that money either.
CHAIRMAN: (N. Asch)	Wait a second! What are they rated by the rating agencies for security.
STAFF: (J. Tuttle)	You don't want to know!
CHAIRMAN: (N. Asch)	You better tell me.
STAFF: (J. Tuttle)	The Best's rating has just been reduced from C- to NA7. Below minimum standards.
ACTUARY: (D. Skurnick)	The Insurance Department has just secretly started an audit of Ricketty Re - and it's not a normal tri-annual examination!
ACTUARY: (D. Skurnick)	Whew! What's that smell?
CHAIR: (N. Asch)	Oh nothing!

CHAIRMAN: (N. Asch)	We do have a problem here. How quickly can we begin negotiations for a commutation of all our reinsurance treaties with Ricketty Re?
STAFF: (J. Tuttle)	Immediately, Boss!
ACTUARY: (D. Skurnick)	The tougher question is, how many pennies on the dollar can we realistically expect to negotiate out of Ricketty Re.
CHAIRMAN: (N. Asch)	OK - You raised the question - give me a range!
ACTUARY: (D. Skurnick)	Given their circumstances and condition, I honestly feel that it could not be any better than 40¢ on the dollar and could be as bad as 10¢ on the dollar, if we can get <u>any</u> commutation at all.
CHAIRMAN: (N. Asch)	Do you know what that means to our solvency?!!!
ACTUARY: (D. Skurnick)	Of course, we would be anywhere from $120 million to $255 million in the hole!
CHAIRMAN: (N. Asch)	What! Who asked you! Tie him up again! Get him out! ACTUARY IS THEN WHEELED AWAY.
STAFF: (J. Tuttle)	Mr. Chairman, Mr. Chairman! Boss! There is perhaps another way out of this mess!
CHAIRMAN: (N. Asch)	Right now, I'm willing to listen to ANYTHING!

STAFF: (J. Tuttle)	I have heard rumors that Ricketty Re is for sale!
CHAIRMAN: (N. Asch)	That's great! Maybe some highly solvent organization will buy them and we will still get full recovery! Make sure that pessimistic Actuary doesn't get too eager in any commutation negotiations! 10 cents on the dollar my foot!
CHAIR: (N. Asch)	By the way, you don't think he realized where that smell came from?
STAFF: (J. Tuttle)	No. But he has to review the reserves just like every year.
CHAIR: (N. Asch)	Couldn't you sign that loss reserve opinion this year?
STAFF: (J. Tuttle)	No. It must be signed by a qualified Actuary. Further, subject to new rules you went to the Board and made him the Appointed Actuary this year. (WHEW!) (Well folks - that's good news for me!)
CHAIR: (N. Asch)	Why the hell did I do that?!
STAFF: (J. Tuttle)	You had to appoint someone and we did not think we could find another Actuary loyal or reliable enough.

CHAIR: (N. Asch) Why not shop around for a nice compliant consulting actuary? Some other CEOs have given me a couple of names...

Well, let's go back to looking up our daily stock price. Wall St. knows quality! Look at those numbers!

ACT III (1989)

MINDLESS MUTUAL

CAST

Nolan Asch. . . . . .CHAIRMAN

David Skurnick. . . . . .ACTUARY

Jerome Tuttle .SAM SALES

Helen Exarhos .NEW PLAYER

ACT III
MINDLESS MUTUAL

Last, but not least of our 3 outstanding insurance organizations is the firm of Mindless Mutual. We will first repeat our 1989 dramatization and then visit them again in 1993.

CHAIRMAN: (N. Asch) Well, I can see here that premiums are not meeting our growth plans.

ACTUARY: (D. Skurnick) I told you that accepting the sales department's proposal of a 20% rate decrease would generate less premium rather than more ---.

CHAIRMAN: (N. Asch) But they guaranteed us a 50% increase in policies in-force at those rates to create 20% premium growth.

ACTUARY: (D. Skurnick) And once again they failed us all -- And -- the analysis shows us that they only wrote more business in the "preferred category" -- where rates are down 40%, and less business than ever in the one-third of the former portfolio with no rate change. So the original plan was as follows:

CHART 1 (SLIDE 89-3-1)

	TERRTY 1	TERRTY 2	TERRTY 3	AVERAGE
Old Weight	1/3	1/3	1/3	
Rate Change	-40% (.60)	-20% (.80)	0% (1.00)	-20% (.80)
Planned PIF	1.5	1.5	1.5	1.5
Planned New Weight	1/3	1/3	1/3	
Premium Volume Change				+20.0%

WHAT WE GOT LAST YEAR WAS THIS

CHART 2 (SLIDE 89-3-2)

	TERRTY 1	TERRTY 2	TERRTY 3	AVERAGE
Old Weight	1/3	1/3	1/3	
Rate Change	-40% (.60)	-20% (.80)	0% (1.00)	-20% (.80)
Act. PIF Change	+20%	+0%	-20%	
Premium Volume Change				-23%

A 23.2% PREMIUM DECREASE WITH SAME POLICY COUNT AND EXPOSURE LEVEL

SAM SALES: (J. Tuttle) Hello everyone

OTHERS: Hello Sam!!!

SAM SALES: (J. Tuttle) Still trying to brainwash our chairman against the "tried and true" techniques that this firm has used for 30 years.

ACTUARY: (D. Skurnick) And should have stopped using 30 years ago ---

SAM: (J. Tuttle) When Charlie's dad founded this firm 70 years ago -- its intent was to supply low cost and reliable insurance to people no one else would insure. We're not a greedy stock firm -- a prisoner of Wall Street's expectations. We are not in existence for greed and profit. We represent a way of life.

ACTUARY: (D. Skurnick) Yes -- we all know -- THE MINDLESS WAY

SAM: (J. Tuttle) Well -- I know the 23% premium drop was a disappointment to us all. Our sales reps worked like mad last year -- but as I told you last year -- even with that measly 20% rate decrease, our rates are still not competitive. Our high rate levels cause only the poorer risks to stay with us and the good ones to leave -- perpetuating poor loss ratios that justify more rate increases that drive away more "good" business.

ACTUARY: (N. Asch) This is ridiculous! We took a rate decrease -- not a rate increase. Not competitive!!! With whom?!

SAM: (J. Tuttle) I'm glad you asked -- Look at these figures -- You can see we're never the lowest rated. Podunk Mutual is beating our brains out in most places --

SLIDE 89-3-3 PREMIUM COMPARISON

	TER'TY 1	TER'TY 2	TER'TY 3	AVG
Podunk Mutual	100	80	80	96
Global Galactic	80	110	80	104
Cowboy Casualty	60	60	60	60
Mindless Mtl - Before	100	100	100	100
Mindless Mtl - After	60	80	100	80
Actuarially Indicated	100	100	100	100
Weight	1/3	1/3	1/3	
Policy Count Change	+20%	0	-2-%	(100)

ACTUARY: (N. Asch) — We've been through all this -- These three firms; Podunk Mutual, Global and Cowboy, only represent 20% of the market. Our tables always use the 5 largest firms in the market for comparison. Global Galactic has 80% of their portfolio in Territory 2 so their average rate is (110) (.80) + (.2) (80) = 88 + 16 = 104. (SLIDE 89-3-3) Podunk Mutual writes 80% in Territory 1 -- so they come to (100) (.8) + (.2) (80) = 96.

SAM: (J. Tuttle) — What about Cowboy Casualty? They're the "hot market," -- They're big and getting bigger fast! They beat us everywhere. Also -- rumor has it that even Global Galactic is about to get more competitive. Their field offices get so many mixed signals from their Home Office -- everyone's dizzy.

ACTUARY: (D. Skurnick) — Cowboy Casualty will be bankrupt within 5 years --

SAM: (J. Tuttle) — Says you -- They're A-rated and surplus goes up every year --

ACTUARY: (N. Asch) — Yeah -- much faster than their absurdly understated loss reserves!

SAM: (J. Tuttle) — So emotional! By the way, Charlie -- How's the golf game?

CHAIRMAN: (N. Asch) — Fine -- We really need to get together soon. You know I love to play with you.

ACTUARY: (D. Skurnick) — Let's go back to business.

CHAIR: (N. Asch) — Must we?! It's a lovely day.

ACTUARY: (D. Skurnick) — Look at the situation we've put ourselves in! Our average rate is only 80 now! Our premium is dropping! Our loss ratios are booming!

CHAIR: (N. Asch) — You know -- you really should take up golf. You're far too emotional and serious about all this. We've gotten by for 70 years without all this advanced Actuarial analysis. It was my idea -- over Sam's objections, to start Actuarial 5 years ago. How are you going to get us the sales we need?

ACTUARY: (D. Skurnick) — What! Sam's the sales VP, not me! I've already bent over backwards to accommodate him.

NEW PLAYER: (H. Exarhos) — (TIMIDLY) Excuse me -- I thought it important to show you a new business proposition just in from Fearless Freddie.

SAM: (J. Tuttle) — See -- Sales once again can save the day.
(SAM READS THE NEW BUSINESS PROPOSAL)
We're up against Cowboy Casualty on this one -- It will be tough. However, we've had the property insurance on this account for 20 years! It has had a 30% loss ratio at $100,000 per year. That's 2 Million in Premium with a profit of (30% + 30% Exp = 60% $800,000. If Cowboy gets the Casualty the Property will be next. We need to defend this core account.

ACTUARY: (D. Skurnick) — Don't get emotional! Why don't you go to your normal office at the golf course.

SAM: (J. Tuttle) It can be done! We can quote $100,000 and use our Property profits on the risk to make it profitable on a joint basis.

(EVERYONE LEAVES BUT THE CEO)

CEO: (N. Asch) What should I do? Well folks, Sam has been with the firm for ever. The Actuaries appear to be so smart, with all their logic and numbers. I'm going to have to make a policy decision, sooner or later. The status quo or this new "scientific" Actuarial approach to pricing?

ACT III (1993)
MINDLESS MUTUAL

CHAIRMAN: (N. Asch)	Well, I can see here that premiums are not meeting our growth plans.
ACTUARY: (D. Skurnick)	I told you before, in a market as soft and unprofitable as this one. We should not have any growth plans.
CHAIRMAN: (N. Asch)	Yes, I know only survival plans
ACTUARY: (D. Skurnick)	What's that smell!!??
CHAIRMAN: (N. Asch)	I don't know. Every once in a while it seems to come here from the general direction of the Claims Department. Charlie's been in charge there for 30 years. You know dad hired him. He tells me there's nothing to worry about. Anyway, I have called you here to hear the solution to our premium problem...
ACTUARY: (D. Skurnick)	We don't have a premium problem!!!
CHAIRMAN: (N. Asch)	A distinguished reinsurance broker will be showing us ways of massively increasing our premium volume with the assumption of virtually no risk. It relates to these new Financial lines of business and some other things.
BROKER: (J. Tuttle)	HELLO!!! It is an honor to meet such an exalted insurance executive!!

CHAIR: (N. Asch) — The pleasure is mine! I have invited my Chief Actuary to join us....

BROKER: (J. Tuttle) — Is that really necessary? These technical types often do not understand the big picture strategic considerations...

CHAIR: (N. Asch) — But you told me these were highly technical product lines so I have my best technician with me.

BROKER: (J. Tuttle) — All right! The basic concept of these products is really quite simple although the mechanics can be complex. You will be using your statutory surplus in transactions that will look like you are losing money, while, in reality, you are earning a very high rate of return with almost no risk. The profit margins on each deal are very small but the risk you assume is even smaller. Of course, on paper it has to look like you are assuming a lot of risk (risk transfer) so the limits are very large but the aggregate loss scenarios it would take to trigger these events would be so massive...

ACTUARY: (D. Skurnick) — You mean, like Hurricane Andrew...

BROKER: (J. Tuttle) — Well, sometimes the unexpected does happen.

CHAIR: (N. Asch) — But I really don't understand....

BROKER: (J. Tuttle) — (TO AUDIENCE) Well folks, that's what I'm counting on! I can see they don't call this place "mindless" for nothing!

BROKER: (J. Tuttle)	It's really quite simple you are involved in a disguised banking arrangement.
CHAIR: (N. Asch)	That's why there's a 99% profit commission - Doesn't that mean in all outcomes favorable to us we realize almost no underwriting income.
BROKER: (J. Tuttle)	Yes -- but you get to keep 10% of all the accumulated investment income!
ACTUARY: (D. Skurnick)	But, if the 500 million aggregate deductible is pierced, we could lose 100 million in real money, while our upside potential in the best case is only $2 million.
BROKER: (J. Tuttle)	I calculate it to be $4 Million.
CHAIRMAN: (N. Asch)	But can we book this transaction as $100 Million in premium volume?
ACTUARY: (D. Skurnick)	That depends on the accounting treatment ... with the new FASB pronouncements I doubt ...
BROKER: (J. Tuttle)	Let's leave that to the CPAs. Where is SAM SALES? He'd know what to do! I must have your answer within 29 hours or this deal will be placed with someone else?
CHAIR: (N. Asch)	SAM's in the hospital. He had a serious coronary at the 16th hole tee! Well --- Thank you! Why don't you both leave me. I will give you my decision tomorrow.

CHAIR: (N. Asch)

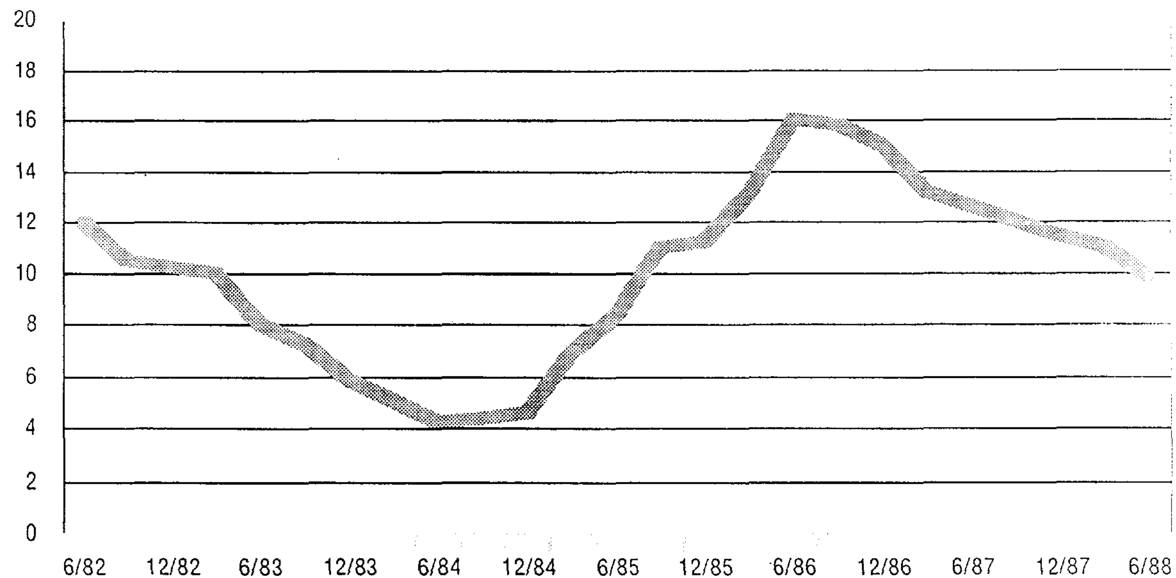
Well folks - what should I do? After Andrew, we've had a major surplus hit and we must reduce our Property business accumulations. I agree with our casualty underwriters that prospects and prices there today are abysmal. I don't totally understand these new finite products but there's a lot of premiums there with very little need for additional staffing or expense. Everybody seems to be doing these "deals." I know growth in all my traditional insurance lines is ill advised. What would you do?

NARRATOR: (N. Asch)

Well - we will have to wait a few years to learn what decisions our mythical CEOs made in 1993. I do know decisions and corporate actions like these will influence the future course of the underwriting cycle.

Slide 89–1–1

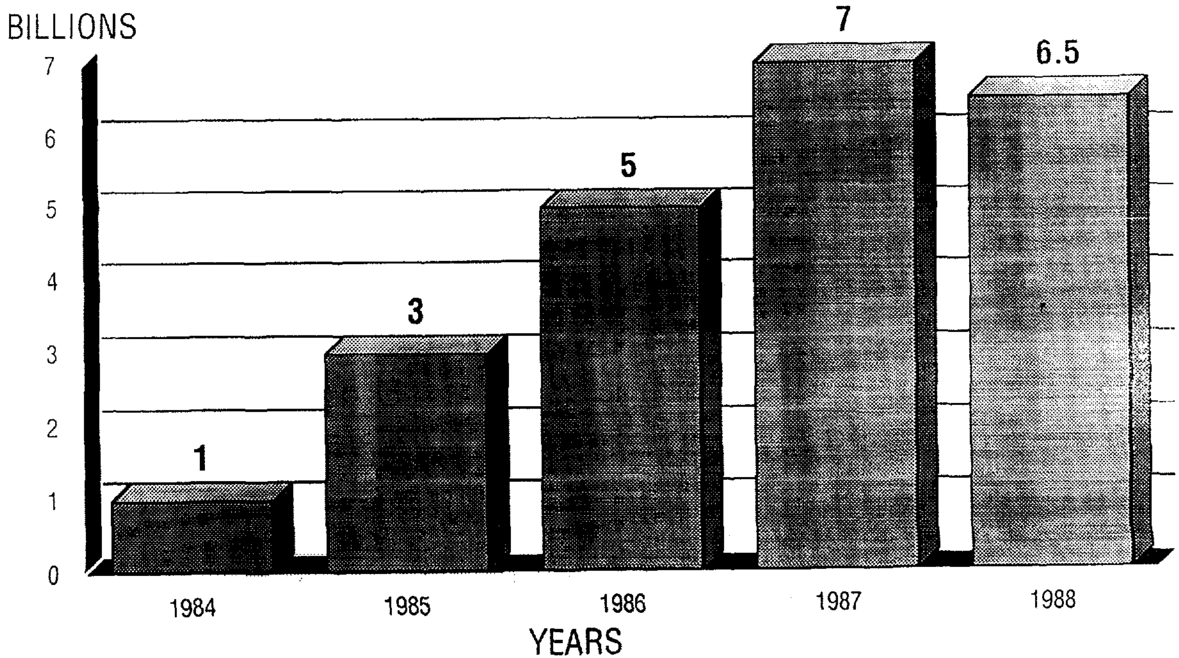
PRICE LEVELS



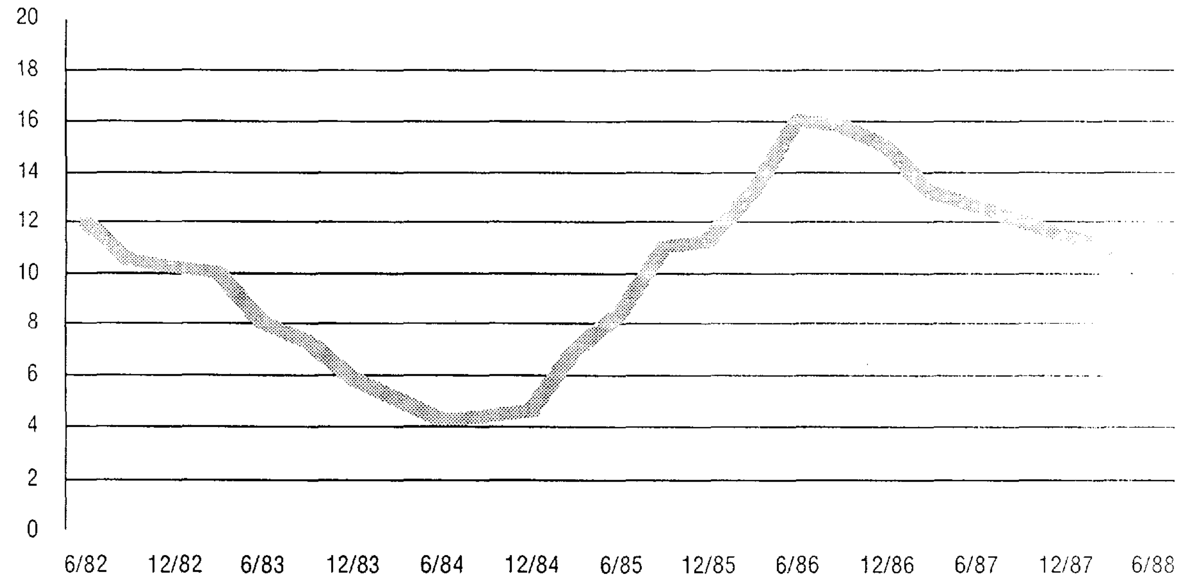
Slide 89–1–2

GLOBAL GALACTIC

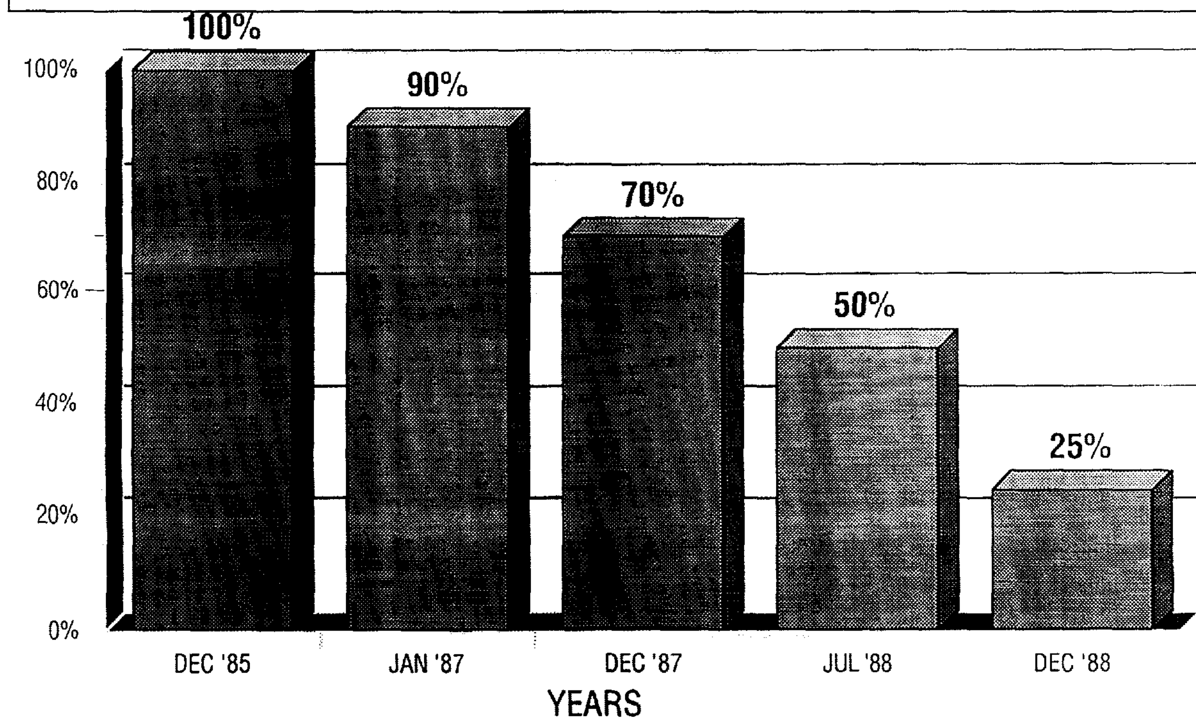
Written Premiums



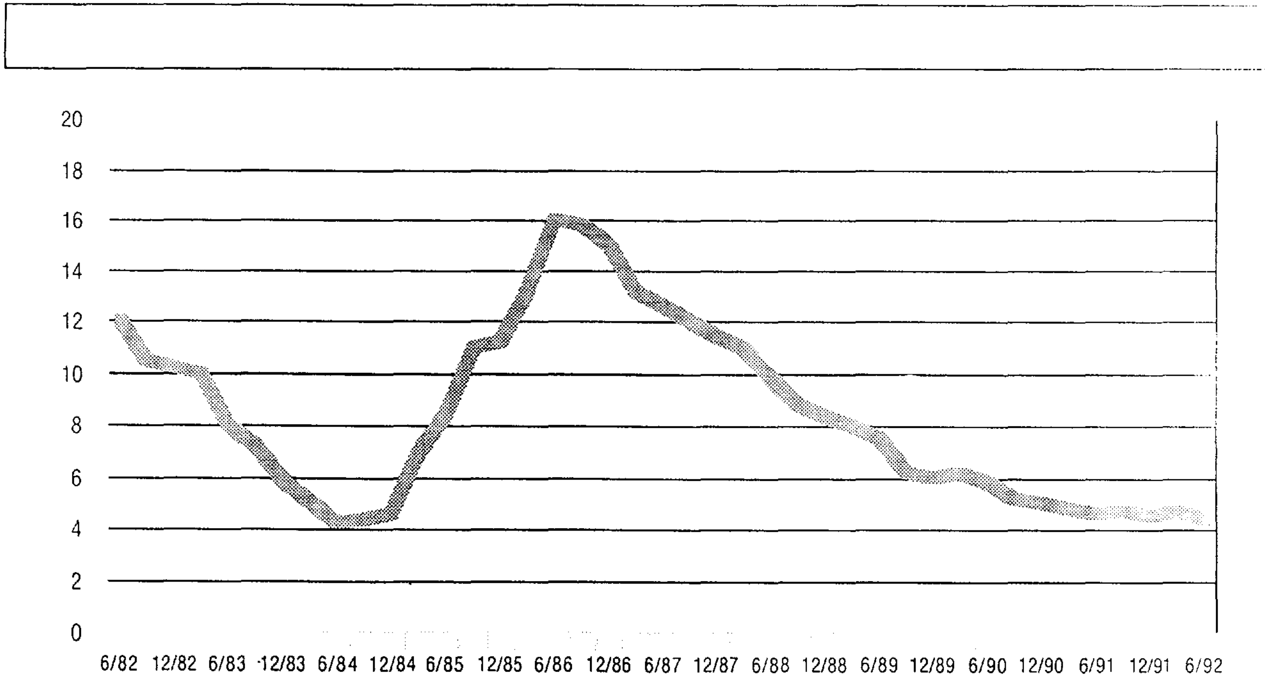
PRICE LEVELS



% WRITTEN PREMIUMS USING 1985 STANDARDS

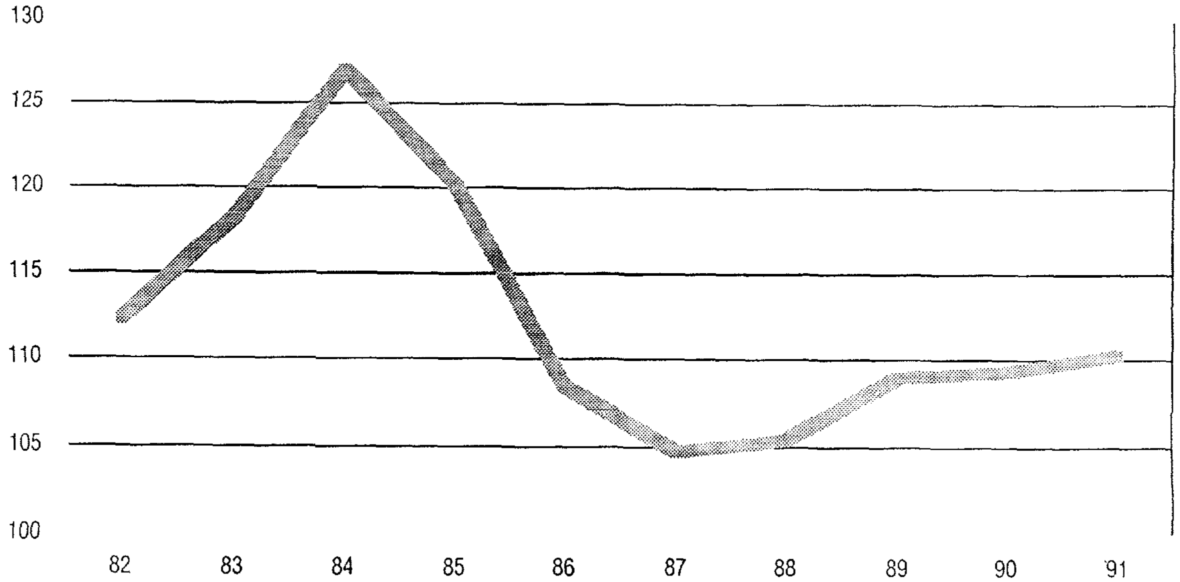


PRICE LEVELS



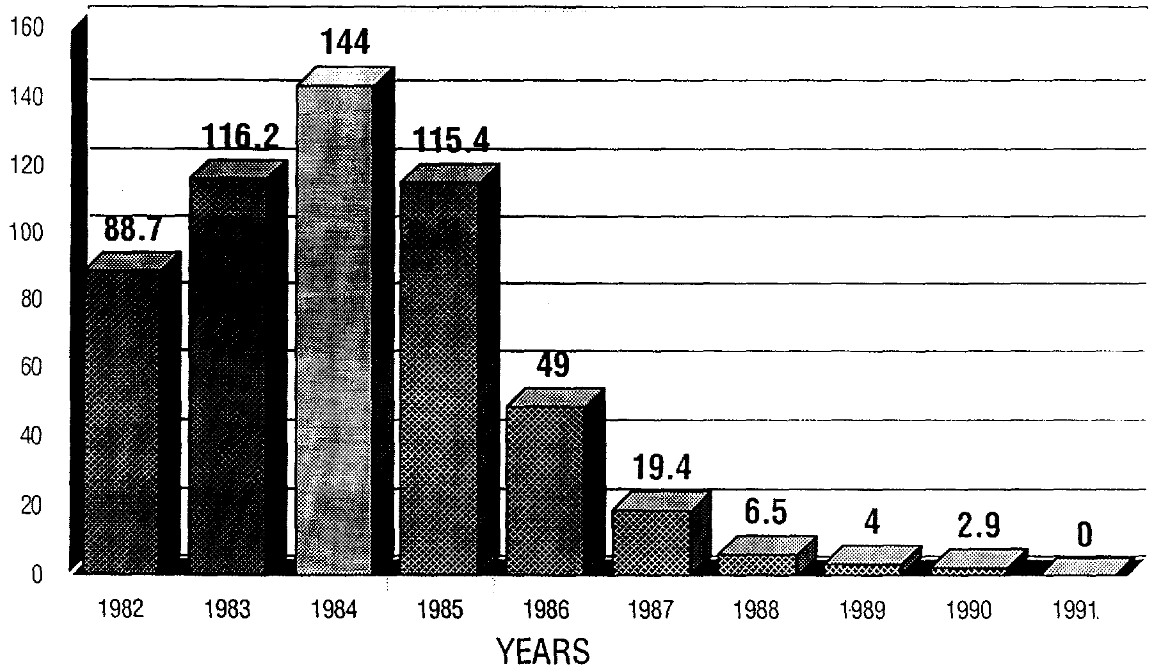
INDUSTRY (COMMERCIAL LINES) UW RATIOS

(Combined Ratios After Dividends)



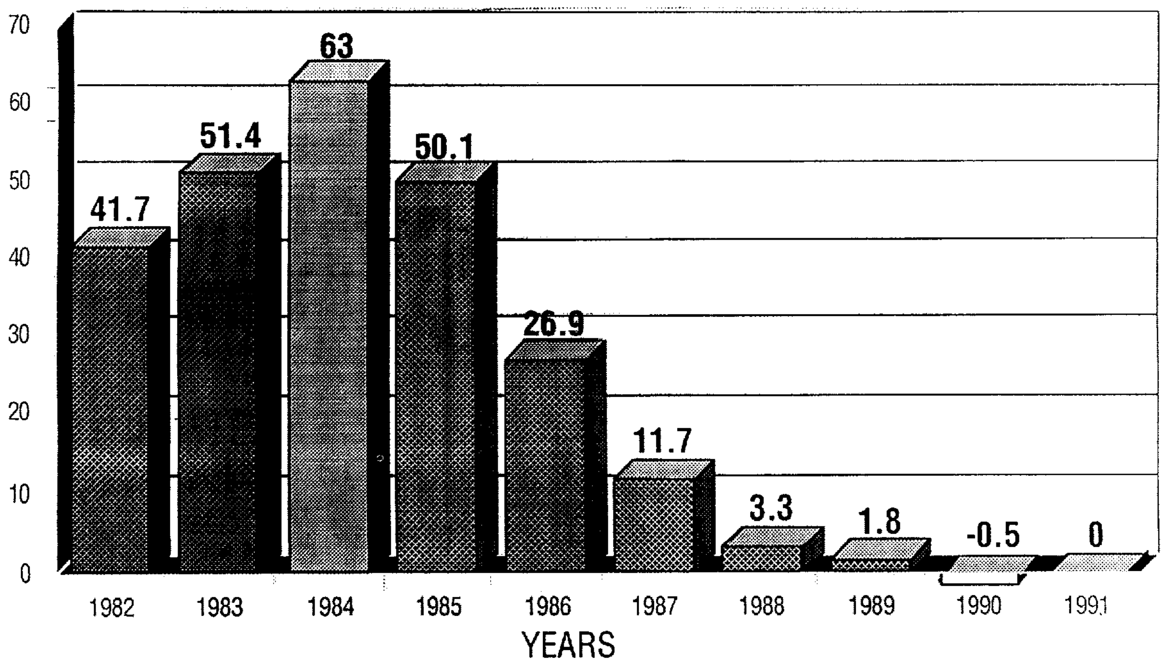
REINSURANCE-LOSS RESERVE ANALYSIS

(Developed Reserves to PHS for 1991)



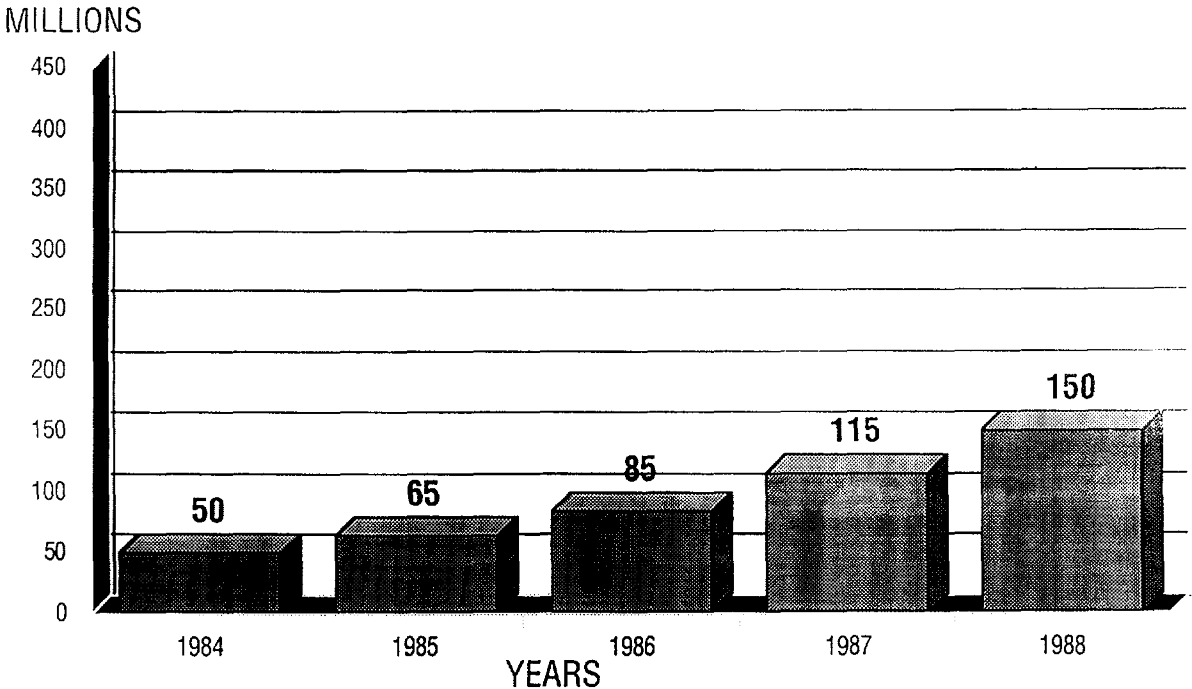
INSURANCE-LOSS RESERVE ANALYSIS

(Developed Reserves to PHS for 1991)



COWBOY CASUALTY COMPANY

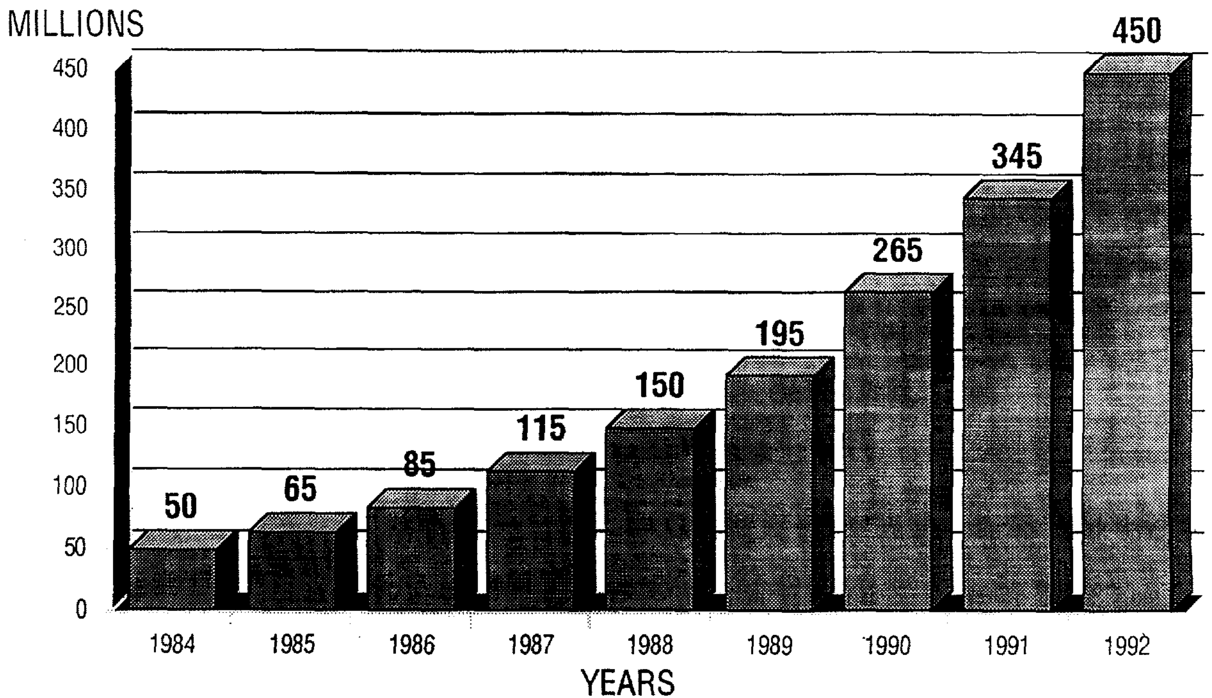
Written Premium Volume



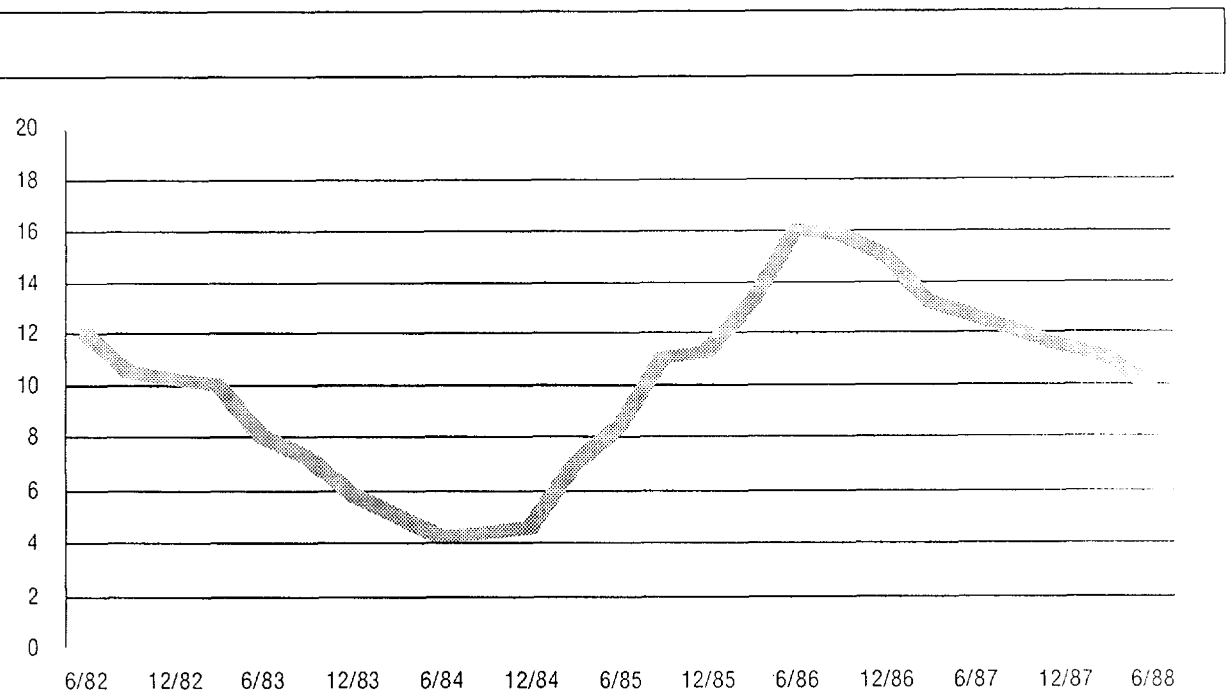
Slide 89–2–2

COWBOY CASUALTY COMPANY

Written Premium Volume



PRICE LEVELS

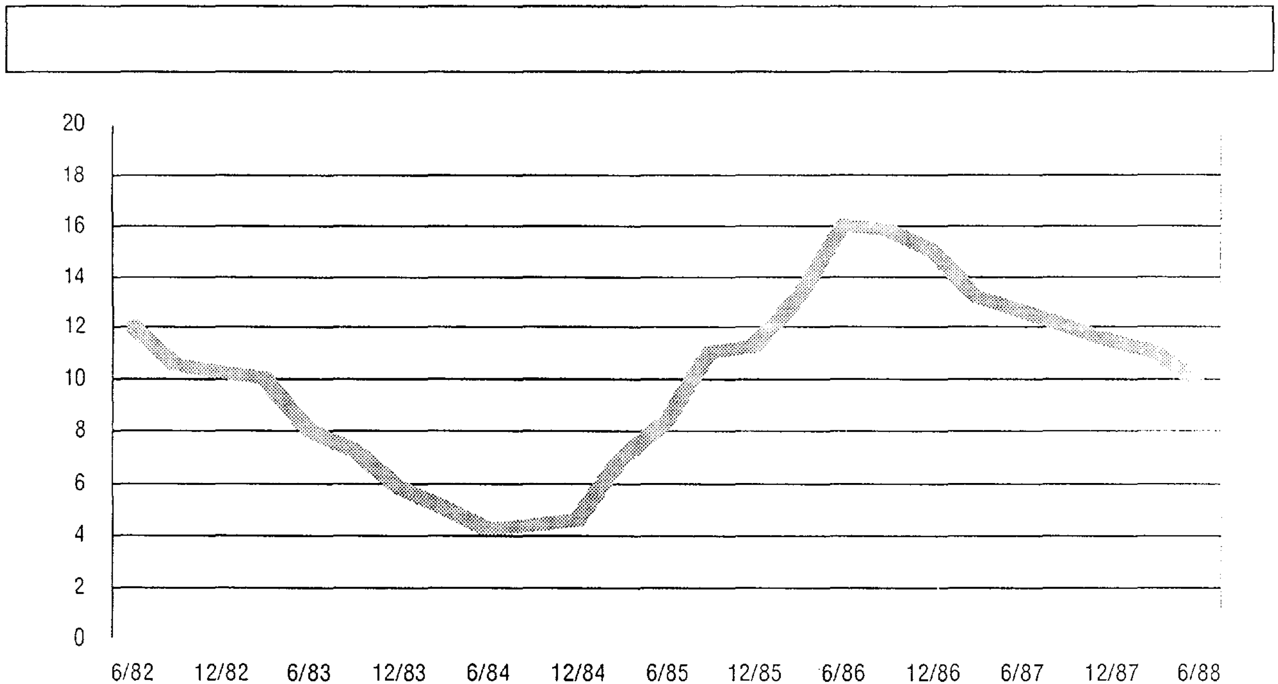


$$(1.1)^3 = 1.331$$

$$(.8)(1.1)^3 = 1.0648$$

$$1.0648 - .80 = 0.2648 + 1$$
$$= 126.48\%$$

PRICE LEVELS



SUBMISSION

XYZ Auto Parts

Year	Exposures	Ultimate Loss Costs	Ultimate Cost Per Exposure	Estimated Average Payout
1982	1,000	200	200	3.0 Years
1983	1,000	220	220	3.5 Years
1984	1,000	242	242	4.0 Years
1985	1,000	266	266	4.0 Years
1986	1,000	293	293	4.5 Years

5 YEAR AVERAGE: 244

SUBMISSION

XYZ Auto Parts

Year	Exposure	Ultimate Loss Cost	Ultimate Cost Per Exposure	Estimated Average Payout
1982	1,000	200	200	3.0 Years
1983	1,000	220	220	3.5 Years
1984	1,000	242	242	4.0 Years
1985	1,000	266	266	4.0 Years
1986	1,000	293	293	4.5 Years

EXPECTED '87
VIA TREND ANALYSIS: 322

$322,000 X (100/75ths) = $430,000

Slide 89–2–8

SUBMISSION

XYZ Auto Parts

Year	Exposures	Ultimate Loss Costs	Ultimate Cost Per Exposure	Estimated Average Payout
1982	1,000	200	200	3.0 Years
1983	1,000	220	220	3.5 Years
1984	1,000	242	242	4.0 Years
1985	1,000	266	266	4.0 Years
1986	1,000	293	293	4.5 Years

EXPECTED '87 VIA TREND ANALYSIS: 322

$$\$322{,}000 \text{ X } (100/75\text{ths}) = \$430{,}000$$

$$(1.1)^5 = 1.61051 \qquad \frac{\$430{,}000}{1.61051} = \$267{,}000$$

SUBMISSION

XYZ Auto Parts

Year	Exposures	Ultimate Loss Costs	Ultimate Cost Per Exposure	Estimated Average Payout
1982	1,000	200	200	3.0 Years
1983	1,000	220	220	3.5 Years
1984	1,000	242	242	4.0 Years
1985	1,000	266	266	4.0 Years
1986	1,000	293	293	4.5 Years

5 YEAR AVERAGE: 244

$244,000 X (100/75ths) = $326,000

RICKETTY RE

Written Premium		$150,000
Treaty Cost		10%
Net Investable Funds		$135,000
5 Yr Compounded Interest Income		1.61
Cumulative Fund After 5 Years	=	$217,000

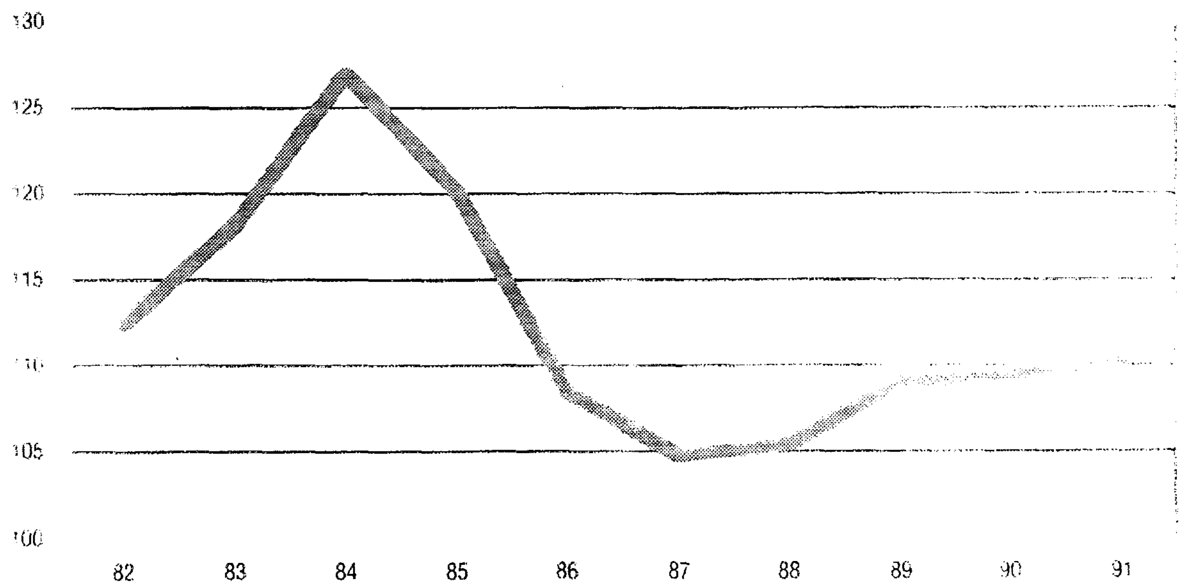
RICKETTY RE

Actuarial Analysis

Commissions, Taxes, & Expenses	25%
Adjusted Cumulative Fund After 5 Yrs	\$217,000 (.75) = \$163,000
Expected Ultimate Losses	\$322,000
Expected LAE Per Annum	40%
Expected LAE Amount Per Annum	\$129,000
Total Expected Losses	\$451,000

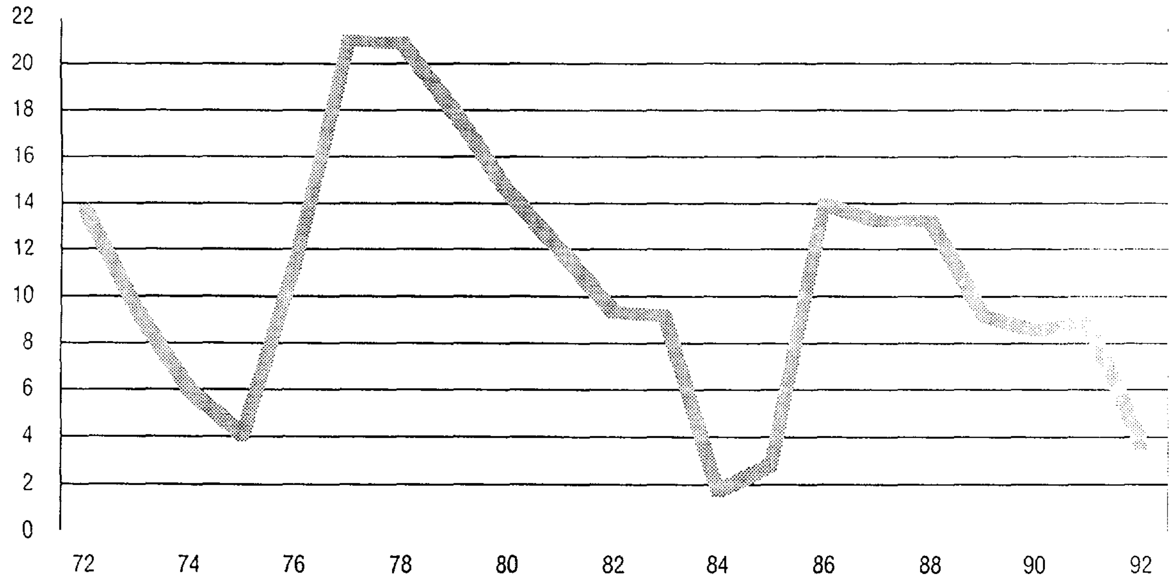
INDUSTRY (COMMERCIAL LINES) UW RATIOS

(Combined Ratios After Dividends)



AVERAGE ANNUAL RATES OF RETURN ON NET INCOME AFTER TAXES

as % of Net Worth for the Property-Casualty Industry



MINDLESS MUTUAL

Chart 1

	TERR 1	TERR 2	TERR 3	AVERAGE
Old Weight	1/3	1/3	1/3	
Rate Change	-40%(.60)	-20% (.80)	0%(1.00)	-20%(.80)
Planned PIF Change	1.5	1.5	1.5	1.5
Planned New Weight	1/3	1/3	1/3	

PREMIUM VOLUME CHANGE ⟶ +20.0%

MINDLESS MUTUAL

Chart 2

	TERR 1	TERR 2	TERR 3	AVERAGE
Old Weight	1/3	1/3	1/3	
Rate Change	-40%(.60)	-20% (.80)	0%(1.00)	-20%(.80)
Actual PIF Change	+20%	+0%	-20%	

PREMIUM VOLUME CHANGE ⟶ -23%

PREMIUM COMPARISON

	TERR 1	TERR 2	TERR 3	AVERAGE
Podunk Mutual	100	80	80	96
Global Galactic	80	110	80	104
Cowboy Casualty	60	60	60	60
Mindless Mutual-Before Rate Change	100	100	100	100
Mindless Mutual-After Rate Change	60	80	100	80
Actuarially Indicated	100	100	100	100
Weight	1/3	1/3	1/3	1/3
PIF Change	+20%	0%	-20%	-110%