CASUALTY ACTUARIAL SOCIETY FORUM

SPECIAL EDITION

1993 Ratemaking Call Papers

CASUALTY ACTUARIAL SOCIETY

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RE: 1993 Ratemaking Discussion Paper Program

This special edition of the *Forum* contains 13 papers that have been prepared in response to a call for papers by the Casualty Actuarial Society. Eight of these papers will be presented and discussed at the annual CAS Ratemaking Seminar, March 4-5, 1993, in Arlington, Virginia.

We hope that these papers provide an opportunity to expand actuarial horizons and stimulate discussions.

CAS Committee on Ratemaking

NOTICE

THE FORUM—SPECIAL EDITION
1993 RATEMAKING CALL PAPERS
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A PRICING MODEL FOR NEW VEHICLE EXTENDED WARRANTIES

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ABSTRACT

In this paper, we use a pure premium approach to price a new vehicle extended warranty.

Coverage provided by a new vehicle extended warranty begins where the manufacturer's factory warranty ends. New vehicle extended warranty coverage is triggered and limited by both time and mileage. Since factory coverage is constantly being enhanced, extended warranty coverage rarely remains the same long enough for comparable statistics to develop.

Our model segregates historical claims into several major types eg. power train, non-power train, rental car and towing. The pure premium for each claim type is defined as the component pure premium.

The model utilizes claim data by type to determine the monthly component pure premiums at each stage of the warranty's life.

Exposure of an extended warranty is measured by the number of months or miles exposed to a particular claim type. By matching the proper component pure premiums with their corresponding exposure units, we can build the total pure premium of the proposed extended warranty.

Using a net discount rate of 2.5% or 3.5% p.a., the model can estimate the present value of the prospective cost of a proposed extended warranty. Both inflation and interest rate are implicitly included.

INTRODUCTION

A new vehicle extended warranty (hereinafter called an extended warranty) is usually defined by two limits, time and mileage. An extended warranty will expire when either one of the limits is reached. For example, a 5 years/60,000 miles warranty means the warranty will expire either in 5 years, or when the odometer reading reaches 60,000 miles, whichever comes first. The extended warranty for new vehicles usually does not come into effect until the coverage under the manufacturer's warranty has expired. Recently, most manufacturers offered 3 years/36,000 miles of full (bumper to bumper) coverage.

The absence of any loss statistics in the initial stage of an extended warranty makes the projection of future claim cost difficult. In this paper, we develop a model which builds the total pure premium of an extended warranty from its basic components, namely pure premium by coverage, for every contract month exposed, or every thousand miles exposed, depending on the age of the contract.

METHODOLOGY AND ASSUMPTIONS

First, the exposures (in contract months) have to be determined. Let E(j,k) be the number of exposures for a specific contract type, age month j and effective month k. For a given effective month and contract type, we can project the amount of exposure E(j,k) for each month subsequent to its effective month. We assume no lapse in our projection. For example, say there are 1,000 contracts in a 6 years/60,000 miles program with effective month in July, 1989. Then, using the above method, we would project the following exposures:

calendar month	age in month,j	exposure E(j,k)
.	.	.
.	.	.
November 1991	29	1000
December 1991	30	1000
.	.	.
.	.	.
June 1995	72	1000
July 1995	73	0

The above projection assumes that all contracts are effective on the first day of each month. For the balance of the paper, we assume there is only one type of contract.

From the data, we can estimate the monthly pure premiums by age for each contract as follows:

LET N(j,k) be the claim count in month j of the contract term for contracts with effective dates in month k.

E(.,k) be the certificate count for contracts with effective dates in month k.

A(j,k) be the ultimate claim amount in month j of the contract term for contracts with effective dates in month k.

P(j) be the average pure premium in month j of the contract term.

P(j) = frequency X average claim size.

$$= \frac{\sum_k N(j,k)}{\sum_k E(.,k)} x \frac{\sum_k A(j,k)}{\sum_k N(j,k)} \qquad \text{........ (1)}$$

$$= \frac{\sum_k A(j,k)}{\sum_k E(.,k)}$$

This is usually calculated using the last 12 or 24 calendar months of data available for each age (month j). For contracts sold recently, the data has not reached the latter part of the contract term (when claims are more likely to be made), so the pure premiums have to be estimated from the more mature contracts with similar features.

The powerful feature of the model lies in the analysis of the monthly pure premium by coverage, hereinafter called the component pure premium. An extended warranty usually provides power train protection, non-power train component protection (eg.

brakes, air conditioning, electrical systems, etc.), towing, and even rental car coverage. It is rare that the terms of any extended warranty stay the same for very long, since the manufacturer's warranty changes yearly, and that dictates what the extended warranty can offer.

It is imperative that the underlying component pure premiums be known so that the pricing model can react to changes in the manufacturer's warranty. Therefore, equation (1) can be rewritten as

$$P(j) = \sum_i P_i(j) = \sum_i \left(\sum_k A_i(j,k) / \sum_k E(.,k)\right) \qquad .... (2)$$

WHERE $P_i(j)$ is the component pure premium of a specific coverage i (eg. power train, non-power train components, etc.) in month j of the contract term,

$A_i(j,k)$ is the ultimate claim amount of a specific coverage i, in month j of the contract term, for contracts with effective dates in month k,

P(j) is the pure premium of a full coverage extended warranty in month j of the contract term,

THEN $$P(j) = \sum_{i=1}^{n} P_i(j)$$

It follows that the total pure premium of a full coverage extended warranty is given by:

$$P=\sum_{j=1}^{m} P(j)=\sum_{j=1}^{m}\sum_{i=1}^{n} P_i(j) \qquad \text{........(3)}$$

WHERE m is the length of the contract term expressed in number of months

n is the number of coverages

Data

In order to utilize this model, historical claims and sales information must be available in sufficient detail. Sales information should be available by effective month (ie., the starting point of the manufacturer's coverage). Claims amount information (related to the sales) should be available by coverage, age, effective month (ie. the starting point of the manufacturer's coverage) and odometer reading. If frequency and severity are to be analyzed separately, claim count information must also be available.

Loss Development

Among warranty insurers (and self-insurers), there are two ways of accounting for losses. One approach is to record claims only when payments are made and estimate the unpaid claims on a bulk basis. Another approach is to record a case estimate when a repair is authorized. Case estimates are usually accurate, but occasional adjustments are necessary when the actual invoices are processed.

When the second approach is used, it is usually safe to treat the recorded losses as the ultimate amount. *With the first approach, the reported payments have to be* developed to an ultimate basis by lag factors as shown below:

Lag Factors, L_e (Percentage of Ultimate Claim amount) by Report Month

	Report Month			
	0	1	2	3
Age in months j	L_0	L_1	L_2	L_3
1 to 12	.75	.90	.95	.99
13 to 24	.65	.85	.90	.98
25 to 36	.60	.80	.90	.98
37 to 48	.60	.80	.90	.98
49 to 60	.60	.80	.90	.98

Lag factors, like those displayed above, can be determined by comparing cumulative loss statistics at various reporting levels. Based on historical data, we estimate $L_e(j)$ as follows:

$$L_e(j) = \frac{\text{cumulative reported losses to report level e, for contracts at age j months}}{\text{ultimate losses for contracts at age j months}}$$

If we are using the last twelve calendar months of data, (1,...12) to estimate the $P_i(j)$'s, then the $A_i(j,k)$'s in equation (2) can be developed to an ultimate basis as follows:

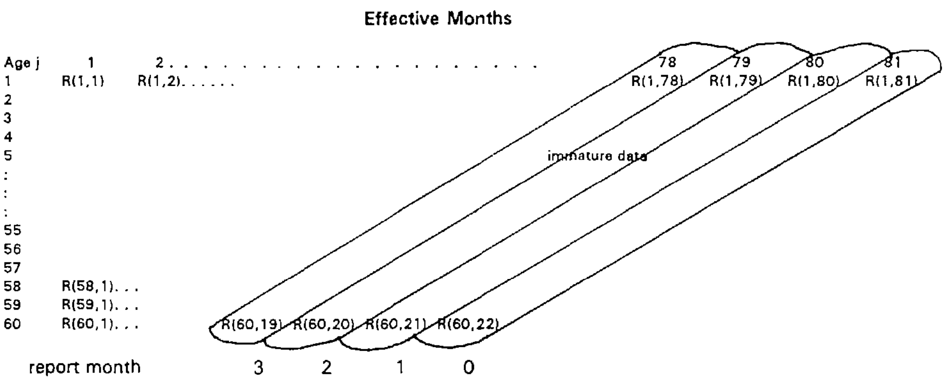
$$A_i(j,k) = R_i(j,k)/L_e$$

WHERE R_i (j,k) are the payments (up to the valuation date) for claims in month j of the contract term, for contracts with effective dates in month k.

$L_e(j) = \begin{cases} \text{lag factor applicable for claim amounts up to report level e} \\ \text{1, for report level} \geq 4 \end{cases}$

e = valuation month - k

Alternatively, all R_i (j,k) and N_i (j,k) not at the ultimate level have to be excluded in the equation. (The last few diagonals of the data triangles have to be excluded.) See the following schematic diagram:



Trending

Frequency of an extended warranty tends to increase with the age of the contract. However, for a given age, there is usually no trend. Severity also varies with the age of the contract, mainly caused by different mix of claims (eg. power train versus other types). However, inflation also plays a role. Short term severity trends (less than one year) can be estimated with some accuracy, since the mechanic's hourly rate usually changes once a year and price increases on parts can be obtained from the manufacturers in advance.

There are two components of trend, one from the experience period to the average effective date of the next rating program (t_0) and another from the average effective date to the repair date (t_j).

The first component, with a fixed trending period, can be determined from the historical average claim sizes if the volume of data is credible. Otherwise, an automotive repair index can be used to determine the first trend.

For example, the following data is available; the experience period is 1 April 1991 to 31 March 1992 and the average effective date is 1 Jan 1993.

		Average claim size	Garage rate index	Parts index	Selected
31 March 1991		\$300	100	100	
30 Sept 1991		\$308	104	102	
31 March 1992		\$312	108	104	
31 Dec 1992			110 Est	105 Est	
Indicated trend	6 mo.	312/308	108/104	104/102	
	12 mo.	312/300	108/100	104/100	
Trend from 30/9/91 to 31/12/92		$(1.04)^{1.25}$	110/104	105/102	
	t_o	1.050	1.058	1.029	1.05

Equation (2) becomes

$$P_i(j) = \sum_k A_i(j,k) * t_0 / \sum_k E(\,.\,,k) \qquad \text{............(2a)}$$

WHERE t_o *is the trend factor from the average experience date to the* average effective date of the rating program

The second trend is prospective and can cover a relatively long period. Since different makes/models can involve substantive engineering changes, it is usually not appropriate to use the past frequency trend in the second trending period. A zero trend is probably the only unbiased estimate, unless relevant quality control data about the new model is available.

Long term severity trends (over 1 year) are more related to the engineering design of new models, exchange rate (in the case of Japanese and European makes), and general wage increases. Therefore, it is difficult to estimate trends with any precision.

Since interest rate (ie. investment yield) is usually higher than the general inflation rate over a long period, the net discount rate (interest rate less annual inflation rate) should be positive, say 2.5% to 3%. The trend of pure premiums can be implicitly included by calculating claims cost at a discount rate of 2.5% or 3%. If the net discount rate is 2.5% p.a., then the present value of the selected pure premium for coverage i in month j is given by:

$$P_i(j) / [t_j]^{j/12} = P_i(j) / [(1.025)^{j/12}]$$

Credibility

Extended warranty is a high frequency and low severity coverage (a claim rarely exceeds $5,000), with the variation between loss amounts (at like ages) being quite small. As a result, loss statistics for a given age develop quickly with a great deal of stability. Although we have not developed a formal credibility procedure, we have utilized an informal one for some time with some success. Depending on the stability of frequency and severity for a given age, we either accept the indicated pure premium, or reject it. In the latter case, we use our prior selected pure premium estimate, adjusted for inflation.

Mileage Variation

Experience shows that claims increase with mileage driven. For the same type of driving, drivers who drive more per year will have their claims earlier in time. If the historical data utilized in the pure premium calculations is from a group of drivers with driving patterns similar to the population being priced, then the indicated pure premiums will correctly reflect the underlying exposure. However, if the population being priced is expected to have a much different driving pattern than the historical group, then an adjustment may be necessary. The model can readily accommodate this situation.

Up to this point, our discussion has ignored the impact of driving pattern on claim cost. In order to account for differences in driving pattern, we must limit the historical claims to certain odometer readings. If we define a "standard" driver to be someone who drives 1,000 miles per month (or any other convenient figure), then we can recast the historical claims into standard drivers experience by excluding claims whose odometer reading exceeds the term of the contract in months, times 1,000.

Suppose we have the claims experience of a 5 years/80,000 miles plan and we want to know the pure premiums of a standard driver in this plan. The true loss exposure of a standard driver is only 5 years/60,000 miles. Therefore, all claims with odometer readings exceeding 60,000 miles should be excluded in the pure premium estimation.

(Exhibit 1 shows an example of such an adjustment.)

Equation (2a) becomes

$$\hat{P}_i(j) = \sum_k \hat{A}_i(j,k) * t_0 / \sum_k E(.,k) \qquad \ldots\ldots (2b)$$

Where $\hat{A}_i$ (j,k) is the ultimate claim amount in contract month j, effective month k and odometer reading not exceeding m times 1,000 miles, m being the term of the contract in months.

$\hat{P}_i$ (j) *is the standard monthly pure premium for coverage* i *and* contract month j.

If someone drives twice the amount of the standard driver (ie. 2,000 miles per month), then his monthly pure premium should be 2 $\hat{P}_i$ (j), while his extended warranty is in-force (ie. *neither time or mileage limit has been exceeded*).

Suppose historical data (trended to the average effective date) indicates that the standard pure premium per month (or 1,000 miles) for power train coverage is about $10 per month. Further suppose that the manufacturer covers power train repairs for 5 years/60,000 miles, the extended warranty provides coverage for 6 years/72,000 miles, and we wish to estimate the cost of power train coverage for someone driving 24,000 miles per annum. Extended warranty coverage will begin after only 30 months for this type of driver (60,000/24,000 = 2.5 years), since the mileage limit of the manufacturer's warranty will have been used up. This driver's extended warranty coverage will expire after 36 months (72,000/24,000 = 3.0 years) since the mileage limit of the extended warranty will have been used up. The extended warranty in this example, provides only 6 months of coverage to this driver from month 31 to month 36.

Also, this type of driver will cost twice as much per month of coverage (ie. $20 per month) as a standard driver (ie. a driver who drives 1,000 miles per month) as long as the contract is in force. While the total power train pure premium of the standard driver and the one driving 24,000 miles per year is identical in this example, the timing of claims is much earlier in the case of the high mileage driver. The present value of claims will usually be higher for the high mileage drivers than the standard drivers, since they tend to have their claims earlier in time.

Net present value

Once the non-discounted component pure premiums are trended to the average effective date of the rating program (using equation 2b), we can project the cashflow pattern of the proposed extended warranty.

The implicit assumption in the model is that higher exposed mileage will translate into higher claim cost. Suppose a component part, by design, will fail in about 30,000 miles. Someone who drives 30,000 miles annually will probably have a claim in only 1 year while another driver who drives 10,000 miles annually will probably have a claim in 3 years.

From past claims records (showing date of repair and odometer reading) or external sources, we can roughly estimate the distribution of the annual mileage of extended warranty buyers. If $d_1, \ldots, d_y$ is the distribution of drivers by mileage driven among extended warranty buyers, and $w_1, \ldots, w_y$ are the corresponding annual mileages (expressed as multiples of a standard driver's mileage), then the weighted monthly pure premium is given by:

$$\hat{P}_i(j) * (d_1 w_1 + \ldots d_y w_y)$$

as long as the extended warranty is still inforce.

Equation (3) can be rewritten as:

$$P = \sum_{j=1}^{m} CF(j) = \sum_{j=1}^{m} \sum_{i=1}^{n} P_i(j) \qquad \ldots\ldots(3a)$$

$$P = \sum_{j=1}^{m} CF(j) = \sum_{j=1}^{m} \sum_{i=1}^{n} \hat{P}_i(j) * (d_1 w_1 + \ldots + d_y w_y) \qquad \ldots\ldots(3b)$$

WHERE $P_i(j)$ are the selected pure premiums

$\hat{P}_i(j)$ are the standard monthly pure premiums

CF(j) is cashflow in month j

If 1.025 is the net discount rate, then the net present value of the total pure premium becomes

$$PP=\sum_{j=1}^{m} CF(j) / (1.025)^{j/12} \qquad .....(4)$$

assuming payments are made at the end of each month of repair.

A NUMERICAL EXAMPLE:

A warranty company has organized its claims data in four simple coverages: power train, non-power train, towing, and rental car.

From past experience with data limited to 5 years/60,000 miles and trended to the average effective date of the new coverage, we found

the average power train monthly standard pure premium	$\hat{P}_1(n) = 10$ for $n > 24$
(See Exhibit 1 for details)	
the ave. non-power train monthly standard pure premium	$\hat{P}_2(n) = 6$ for $n > 12$
the average towing monthly standard pure premium	$\hat{P}_3(n) = .5$ for $n > 12$
the average rental car monthly standard pure premium	$\hat{P}_4(n) = .5$ for $n > 0$

During the experience period, the underlying manufacturer's warranty was 1 year/12,000 miles full coverage, 2 years/24,000 miles power train, while the extended warranty was adjusted to 5 years/60,000 miles full coverage.

Suppose the new manufacturer's warranty is enhanced to 3 years/36,000 miles full coverage (but no rental car coverage), 5 years/60,000 miles power train, and one has to price a 6 years/72,000 miles full coverage extended warranty (including rental car coverage).

The proposed extended warranty will provide one year of power train coverage, three years of non-power train coverage, three years of towing coverage, and six years of rental car coverage.

During the experience period, the extended warranty did not provide any rental car coverage. However, we estimate that the frequency of a rental car claim will be one-quarter that of a towing claim, while the severity of a rental car claim will be four times that of a towing claim. Thus, we estimate the monthly cost of rental car coverage to be about \$0.50.

Before considering the cashflow pattern, the non-discounted ultimate pure premium of a standard driver for this contract is made up of:

power train $\sum_{61}^{72} \hat{P}_1(n) = 12 \ x \ \$10 = \$120$

non-power train $\sum_{37}^{72} \hat{P}_2(n) = 36 \ x \ \$6 = \$216$

towing $\sum_{37}^{72} \hat{P}_3(n) = 36 \ x \ \$0.50 = \$18$

rental car $\sum_{1}^{72} \hat{P}_4(n) = 72 \ x \ \$0.50 = \$36$

total non-discounted standard pure premium

$$\hat{P}=\sum_{n=1}^{72}\hat{P}(n)=\sum_{n=1}^{72}\hat{P}_1(n)+\hat{P}_2(n)+\hat{P}_3(n)+\hat{P}_4(n) = \$390$$

$\hat{P}_1(n)$ = \$10 is, by design, only appropriate for someone who drives 12,000 miles annually. For someone who drives 15,000 annually, his component 1 pure premium becomes \$12.50 (10 x 15/12). However, since the contract is limited to 72,000 miles in aggregate, we would expect the latter to use up his coverage in only 57.6 months (as opposed to 72 months). His component 1 pure premium in month 58 represents only a partial month of exposure, and equals \$7.50 (0.6 x \$12.50). (See Exhibit 2 column P1 in 15,000 block 20th Qtr entry.)

Suppose the plan in question shows that 65% of drivers drive 12,000 miles per year, 25% of drivers drive 15,000 miles per year, and 10% of drivers drive 24,000 miles per year. The non-discounted pure premiums by coverage, weighted by the above driving patterns, are shown in Exhibit 2. (To facilitate the display of the results, the data has been grouped into quarters.) Next, we compute a discounted weighted pure premium reflecting claims inflation and the time value of money. We have assumed a net discount rate of 2.5% per annum and claims are paid uniformly throughout each development quarter. The discounted pure premiums are shown in Exhibit 3.

Finally, we load the discounted pure premium for expenses and profit to determine the gross rate.

$$\text{Gross Rate} = \frac{PP + FE}{1 - (VE + C)}$$

PP - discounted pure premium

FE - fixed expenses

VE - variable expenses as a % of gross premium

C - profit and contingencies load.

Exhibit 1

Actual Power Train Experience Based on 4/91 to 3/92 data limited to 5 years/60,000 miles

Age (in mos)	Frequency Per Contract Month	Average Claim Size	Monthly Pure Premium $\hat{P}_1(j)$	Quarterly Pure Premium $3 \times \hat{P}_1(j)$
1-3	0.000	0	0.00	0.00
4-6	0.000	0	0.00	0.00
7-9	0.000	0	0.00	0.00
10-12	0.001	150	0.15	0.45
13-15	0.002	140	0.28	0.84
16-18	0.003	200	0.60	1.80
19-21	0.005	210	1.05	3.15
22-24	0.007	220	1.54	4.62
25-27	0.020	280	5.60	16.80
28-30	0.030	280	8.40	25.20
31-33	0.035	260	9.10	27.30
34-36	0.040	250	10.00	30.00
37-39	0.038	250	9.50	28.50
38-42	0.040	280	11.20	33.60
43-45	0.035	275	9.63	28.89
46-48	0.030	350	10.50	31.50
49-51	0.036	300	10.80	32.40
52-54	0.035	280	9.80	29.40
55-57	0.030	290	8.70	26.10
58-60	0.025	300	7.50	22.50
Total				343.05 *

Average experience date = 1991-10-01

Average rating date = 1993-01-01

Selected trend = $1.04^{1.25} = 1.05$

Total power pure premium = $\sum_{j=1}^{60} \hat{P}_1(j) = (343.05)*1.05 = 360.20$

Power train exposure = 60 - 24 = 36 months

Average monthly pure premium = 360.20/36 = 10

* this total is three times the sum of the monthly pure premium column; each monthly pure premium entry is applicable for a three month period.

EXHIBIT 2

NON - DISCOUNTED COMPONENT PURE PREMIUMS

		12,000				24,000				15,000				Total			
Dev.		65%				10%				25%				100%			
Qtr.	P1	P2	P3	P4	P1	P2	P3	P4	P1	P2	P3	P4	P1	P2	P3	P4	P
1Q				1.50				3.00				1.875	0.00	0.00	0.00	1.74	1.74
2Q				1.50				3.00				1.875	0.00	0.00	0.00	1.74	1.74
3Q				1.50				3.00				1.875	0.00	0.00	0.00	1.74	1.74
4Q				1.50				3.00				1.875	0.00	0.00	0.00	1.74	1.74
5Q				1.50				3.00				1.875	0.00	0.00	0.00	1.74	1.74
6Q				1.50				3.00				1.875	0.00	0.00	0.00	1.74	1.74
7Q				1.50		36.00	3.00	3.00				1.875	0.00	3.60	0.30	1.74	5.64
8Q				1.50		36.00	3.00	3.00				1.875	0.00	3.60	0.30	1.74	5.64
9Q				1.50		36.00	3.00	3.00				1.875	0.00	3.60	0.30	1.74	5.64
10Q				1.50		36.00	3.00	3.00		9.00	0.75	1.875	0.00	5.85	0.49	1.74	8.08
11Q				1.50	60.00	36.00	3.00	3.00		22.50	1.88	1.875	6.00	9.23	0.77	1.74	17.74
12Q				1.50	60.00	36.00	3.00	3.00		22.50	1.88	1.875	6.00	9.23	0.77	1.74	17.74
13Q		18.00	1.50	1.50						22.50	1.88	1.875	0.00	17.33	1.44	1.44	20.21
14Q		18.00	1.50	1.50						22.50	1.88	1.875	0.00	17.33	1.44	1.44	20.21
15Q		18.00	1.50	1.50						22.50	1.88	1.875	0.00	17.33	1.44	1.44	20.21
16Q		18.00	1.50	1.50						22.50	1.88	1.875	0.00	17.33	1.44	1.44	20.21
17Q		18.00	1.50	1.50					37.50	22.50	1.88	1.875	9.38	17.33	1.44	1.44	29.59
18Q		18.00	1.50	1.50					37.50	22.50	1.88	1.875	9.38	17.33	1.44	1.44	29.59
19Q		18.00	1.50	1.50					37.50	22.50	1.88	1.875	9.38	17.33	1.44	1.44	29.59
20Q		18.00	1.50	1.50					7.50	4.50	0.38	0.380	1.88	12.83	1.07	1.07	16.84
21Q	30.00	18.00	1.50	1.50									19.50	11.70	0.98	0.98	33.15
22Q	30.00	18.00	1.50	1.50									19.50	11.70	0.98	0.98	33.15
23Q	30.00	18.00	1.50	1.50									19.50	11.70	0.98	0.98	33.15
24Q	30.00	18.00	1.50	1.50									19.50	11.70	0.98	0.98	33.15
	120.00	216.00	18.00	36.00	120.00	216.00	18.00	36.00	120.00	216.00	18.00	36.01	120.00	216.00	18.00	36.01	389.97

discount rate 2.50%

DISCOUNTED COMPONENT PURE PREMIUMS

EXHIBIT 3

	12,000				24,000				15,000				Total				
Dev.	65%				10%				25%				100%				
Qtr.	P1	P2	P3	P4	P1	P2	P3	P4	P1	P2	P3	P4	P1	P2	P3	P4	P
1Q	0.00	0.00	0.00	1.50	0.00	0.00	0.00	2.99	0.00	0.00	0.00	1.87	0.00	0.00	0.00	1.74	1.74
2Q	0.00	0.00	0.00	1.49	0.00	0.00	0.00	2.97	0.00	0.00	0.00	1.86	0.00	0.00	0.00	1.73	1.73
3Q	0.00	0.00	0.00	1.48	0.00	0.00	0.00	2.95	0.00	0.00	0.00	1.85	0.00	0.00	0.00	1.72	1.72
4Q	0.00	0.00	0.00	1.47	0.00	0.00	0.00	2.94	0.00	0.00	0.00	1.83	0.00	0.00	0.00	1.71	1.71
5Q	0.00	0.00	0.00	1.46	0.00	0.00	0.00	2.92	0.00	0.00	0.00	1.82	0.00	0.00	0.00	1.70	1.7
6Q	0.00	0.00	0.00	1.45	0.00	0.00	0.00	2.90	0.00	0.00	0.00	1.81	0.00	0.00	0.00	1.69	1.69
7Q	0.00	0.00	0.00	1.44	0.00	34.58	2.88	2.88	0.00	0.00	0.00	1.80	0.00	3.46	0.29	1.67	5.42
8Q	0.00	0.00	0.00	1.43	0.00	34.37	2.86	2.86	0.00	0.00	0.00	1.79	0.00	3.44	0.29	1.66	5.39
9Q	0.00	0.00	0.00	1.42	0.00	34.16	2.85	2.85	0.00	0.00	0.00	1.78	0.00	3.42	0.29	1.65	5.35
10Q	0.00	0.00	0.00	1.41	0.00	33.95	2.83	2.83	0.00	8.49	0.71	1.77	0.00	5.52	0.46	1.64	7.62
11Q	0.00	0.00	0.00	1.41	56.23	33.74	2.81	2.81	0.00	21.09	1.76	1.76	5.62	8.65	0.72	1.64	16.63
12Q	0.00	0.00	0.00	1.40	55.89	33.53	2.79	2.79	0.00	20.96	1.75	1.75	5.59	8.59	0.72	1.63	16.53
13Q	0.00	16.66	1.39	1.39	0.00	0.00	0.00	0.00	0.00	20.83	1.74	1.74	0.00	16.04	1.34	1.34	18.72
14Q	0.00	16.56	1.38	1.38	0.00	0.00	0.00	0.00	0.00	20.70	1.73	1.73	0.00	15.94	1.33	1.33	18.6
15Q	0.00	16.46	1.37	1.37	0.00	0.00	0.00	0.00	0.00	20.57	1.71	1.71	0.00	15.84	1.32	1.32	18.48
16Q	0.00	16.36	1.36	1.36	0.00	0.00	0.00	0.00	0.00	20.45	1.70	1.70	0.00	15.75	1.31	1.31	18.37
17Q	0.00	16.26	1.35	1.35	0.00	0.00	0.00	0.00	33.87	20.32	1.69	1.69	8.47	15.65	1.30	1.30	26.72
18Q	0.00	16.16	1.35	1.35	0.00	0.00	0.00	0.00	33.66	20.20	1.68	1.68	8.42	15.55	1.30	1.30	26.57
19Q	0.00	16.06	1.34	1.34	0.00	0.00	0.00	0.00	33.45	20.07	1.67	1.67	8.36	15.46	1.29	1.29	26.4
20Q	0.00	15.96	1.33	1.33	0.00	0.00	0.00	0.00	6.65	3.99	0.33	0.34	1.66	11.37	0.95	0.95	14.93
21Q	26.43	15.86	1.32	1.32	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	17.18	10.31	0.86	0.86	29.21
22Q	26.27	15.76	1.31	1.31	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	17.08	10.24	0.85	0.85	29.02
23Q	26.11	15.67	1.31	1.31	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	16.97	10.19	0.85	0.85	28.86
24Q	25.95	15.57	1.30	1.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	16.87	10.12	0.85	0.85	28.68
	104.76	193.34	16.11	33.47	112.12	204.33	17.02	34.69	107.63	197.67	16.47	33.95	106.22	195.53	16.30	33.72	351.79

THE USE OF SIMULATION TECHNIQUES IN ADDRESSING AUTO WARRANTY PRICING AND RESERVING ISSUES

Simon J. Noonan

Title : The Use of Simulation Techniques in Addressing Auto Warranty Pricing and Reserving Issues

Abstract : Extended warranty contracts are generally quite difficult to evaluate because the factors affecting ultimate loss emergence tend to change quite considerably over time. The actuary is forced to extrapolate from historical data to take these changes into account whatever the methodology employed, and simulation techniques provide a powerful tool to model the changes in loss exposure in a way that is easy for the actuary and layman alike to grasp.

A. Policy Coverage

The coverage generally provides mechanical breakdown protection for new and used vehicles sold by automobile dealerships. Often, the dealership is legally the policyholder of the insurance company rather that the owner of the automobile, who instead purchases a service contract from the automobile dealer. The insurance policy reimburses the dealer for expenses incurred in fulfilling his obligations under the service contract. Despite the legal form of this arrangement, the insurance company is generally obligated to fulfill the terms of the service contract with the consumer should the automobile dealer fail to meet their contractual obligations, even if this is not specified in the service contract.

Coverage is nowadays generally limited to specified mechanical failures to eliminate coverage for parts which naturally wear out (e.g. shock absorbers) and to restrict in some fashion the automobile dealer's ability to make unnecessary and expensive repairs.

For new automobiles, the policy is essentially an umbrella coverage over the manufacturer's warranty, broadening the policy form with additional coverage such as the provision of a free rental car while repairs are being made, lengthening the time for which the coverage is valid and increasing the maximum mileage that may be driven before the auto owner must pay for repairs out of his own pocket.

B. Factors Affecting Consistency of Loss Emergence

More than with most lines of insurance, the factors affecting loss emergence tend to change considerably over time. The two most important changes are generally:

☞ The manufacturer's warranties have changed dramatically over time as auto manufacturers have sought to compete more or less heavily on the basis of quality. For example, one major manufacturer has offered the following coverage in recent years:

model year	basic coverage	powertrain only coverage
1986	12/12	36/36
1987	12/12	72/60 **
1988	12/12	72/60
1989	36/50	36/50
1990	36/50	36/50
1991	36/50	36/50

** meaning coverage is provided for 72 months or 60,000 miles, whichever expires sooner.

Clearly, changes of this magnitude have a considerable effect on loss emergence.

☞ In response to the above and other changes in the marketplace, insurers writing this line have adapted the coverage they offer to the changes in the underlying warranty. This has generally meant increasing both the duration of the policy and the mileage cap on the policy.

In many cases, companies that were offering 60 month/50,000 mile policies over 12 month/12,000 mile factory warranties find themselves offering 6 or 7 year contracts with 100,000 mile caps. The fact that the bulk of the exposure for this line occurs late in the policy term exacerbates the problem by requiring that the actuary develop loss projections from loss data that stems from policy forms that are several years old.

C. Methodology Employed

The loss data are aggregated by model year and losses are then divided according to which mileage band they fall into. Based on the number of contracts originally written, a pure premium is developed. Calculations based on hypothetical data are contained in the various Exhibits. As an example of the basic structure of the loss data, loss payments that have been made as of 4/30/91 (the evaluation date) for model year 1987 with mileage on the odometer of the vehicle of between 10,000 and 20,000 miles at the time of claim would total $2,658,300 and the corresponding pure premium would be $21.79 based on 122,000 contracts written for that model year (Exhibits 3, Parts A and B).

Each of the elements of this data matrix will tend to increase over time, until either all policies in the cohort have expired, or all automobiles have been driven a distance in excess of the upper mileage band. Basic questions of pricing or loss reserving therefore boil down into how to estimate the ultimate pure premium in each cell.

Assuming the mileage on the odometer of the vehicle is captured in the claims database of the insurance company at the time of each loss (without this, the loss data cannot, of course, be produced in the requisite form), it is possible to develop estimates of the distribution of the distance driven by a typical policyholder each year, and the correlation between successive years. Armed with this information, we can estimate the following quantities using simulation techniques:

(A) the mean distance driven in each mileage band at the evaluation date while under the manufacturer's "basic" warranty.

(B) the mean distance driven in each mileage band at the evaluation date while under the manufacturer's "powertrain-only" warranty.

(C) the mean distance driven in each mileage band at the evaluation date while under the insurance company's warranty.

(D) the mean distance driven in each mileage band at the expiration of all policies while under the manufacturer's "basic" warranty.

(E) the mean distance driven in each mileage band at the expiration of all policies while under the manufacturer's "powertrain-only" warranty.

(F) the mean distance driven in the mileage band at the expiration of all policies while under the insurance company's warranty.

Exhibit 2, Parts A to F shows the estimates of these quantities where the distance driven has a lognormal distribution with mean 10,000 miles and standard deviation 5,000 miles. Coverage was assumed to be the lesser of 5 years or 50,000 miles under the extended warranty contract, the lesser of 1 year or 10,000 miles under the manufacturer's basic coverage and the lesser of 2 years or 20,000 miles under the powertrain-only coverage provided by the manufacturer. The numbers contained therein were developed by performing 500 simulations for each data cell using add-in software in conjunction with a standard computer spreadsheet, a printout of which is shown in Exhibit 1. Information on the derivation of an appropriate distribution is contained in Appendix A.

As an example of the approach outlined above, one iteration of the simulation for the distance driven at policy expiration might generate the following data:

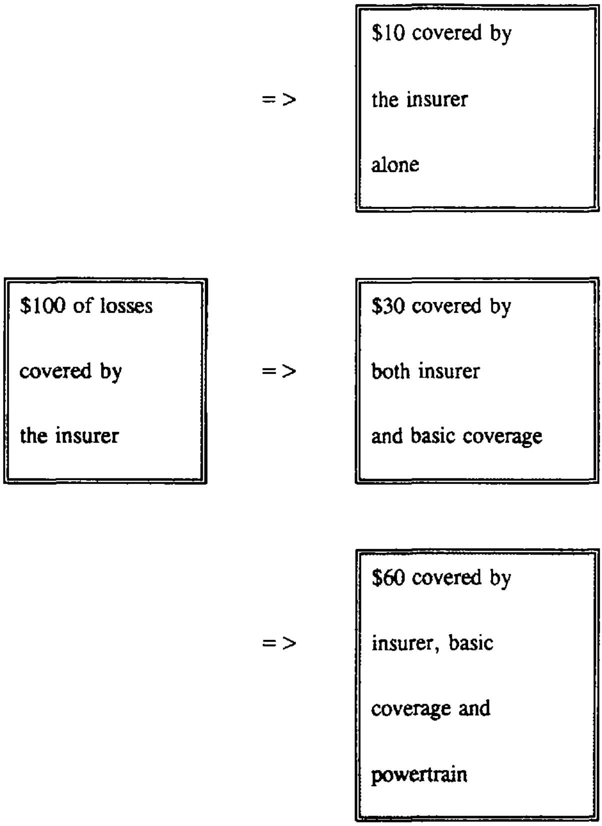
Year Driven	Mileage
1	6,000
2	12,000
3	8,000
4	20,000
5	3,000

Then the entries in the entries in Exhibit 2, Parts D, E and F would be:

Coverage	Distance Driven at Policy Expiration in Mileage Band				
	0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
"Basic" Part D	6,000				
"Powertrain" Part E	10,000	8,000			
"Insurance" Part F	10,000	10,000	10,000	10,000	9,000

For example, the coverage for the powertrain-only warranty is the lesser of 2 years and 20,000 miles. For this example, the driver covered 18,000 miles at policy expiration, which implies that the full 10,000 miles were driven in the first mileage band, but only 8,000 miles in the second band from 10,000 to 20,000 miles.

We need to examine the question of what percentage of losses that are covered under the insurance company's policy form would also be covered by either the manufacturer's "basic" coverage or by the "powertrain-only" coverage assuming that all three coverages are in force at the time of a claim. Ranked in decreasing order of coverage, the three coverages would be the insurers coverage, the manufacturers basic coverage and the powertrain only coverage. Bearing this in mind, the results of such an analysis might hypothetically be as follows:



Stated another way, while both the basic warranty and the power-train coverage are in force, the insurer is responsible for 10% of the losses. Once the basic warranty expires, the insurer is responsible for 40% of the losses; the original 10% plus the 30% that were previously covered under the basic warranty. Finally, the insurer picks up 100% of the losses once both the manufacturer's warranties expire.

The hypothetical data above might be empirically derived from a study of the cause of actual losses.

Were the loss data available, we could, of course, analyze the losses separately according to which policy form they would be covered under, dividing our single claims matrix into 3 separate matrices. In the absence of such a division, we may estimate the effective exposure to loss at the evaluation date (g.) as:

(G) = (C) - 90% * (A) - 60% * [(B) - (A)]

= 10% * (A) + 40% * [(B) - (A)] + 100% * [(C) - (B)]

The effective exposure to loss at the expiration of all policies can similarly be calculated as:

(H) = (F) - 90% * (D) - 60% * [(E) - (D)]

These exposures are tabulated in Exhibit 2, Parts G and H respectively, with the ratio:

(I) = (H) / (G)

tabulated in Exhibit 2, Part I and a smoothed version of these factors-to-ultimate shown in Exhibit 2, Part J, where the factors of close to unity, caused by random errors in the simulation process, are rounded to 1.

The derivation of estimates of ultimate pure premiums is shown in Exhibit 3. The basic loss data is shown in Part A and paid pure premiums are calculated in Part B. Ultimate pure premiums for a policyholder who drives the maximum 10,000 mile distance in the cell with no underlying manufacturer's coverage are shown in Exhibit 3, Part C. It might be thought that this step could be bypassed; if the typical policyholder has driven 2,000 miles at the evaluation date and will ultimately drive 4,000 miles then an estimate for the ultimate pure premium is two times the paid pure premium. However, the ultimate pure premium for the 10,000 mile driver is a useful quantity to know when extrapolating experience from say a 5/50 policy to a 5/100 coverage in that the underlying exposure to loss from a driver who drives more than 100,000 miles can be estimated, before the typical distance driven in the higher mileage bands is considered.

Exhibit 3, Parts D, E and F, show a regression, performed to estimate ultimate pure premiums in each mileage band by averaging the data from all model years and then trending the average for the individual model years. The normalized pure premiums shown in Part F reflect the average pure premium, assumed to relate to model year 1987.5 trended forwards or backwards for the appropriate period of time using the trend rate derived in the regression. This step is desirable in computing estimates of ultimate loss using the Bornhuetter-Ferguson[1] method, as well as in determining a trend factor from historical data for ratemaking purposes. Obviously, the methodology can be refined to separate the overall claims trend into frequency and severity components, which is generally useful, since severity tends to be amenable to estimation, even for new components, leaving only frequency as the major unknown.

In Exhibit 3, Part G, we determine the ultimate pure premium for a typical driver, rather than for one who drives the full 10,000 miles in each exposure cell. With the hypothetical numbers shown, the pure premium for the typical driver declines at the higher mileage intervals even though the pure premium for the 10,000 mile driver rises with increasing mileage -- stemming from the reduction in distance driven by the typical driver in the higher mileage bands.

Exhibit 3, Parts H and I, show the derivation of estimates of ultimate pure premiums for the typical policyholder using two different approaches.

In Part H, the paid pure premiums as of the evaluation date are increased in the same proportion as the "ultimate effective exposure" bears to the "effective exposure at the evaluation date". In an analogous fashion to most forms of loss projection using triangular methods, the approach works best for those data cells where the factor to ultimate is not excessively large.

In Part I, ultimate losses are estimated using the Bornhuetter-Ferguson methodology adapted for current purposes :

ultimate =	paid +	(ult. effective -	effective) *	normalized ult. pure premium
pure	pure	exposure	exposure at	for a 10,000 mile
premium	premium		evaluation	driver
			date	
				/ 10,000

This approach works well for the more recent model years, where little in the way of ultimate loss emergence has taken place at the evaluation date, and where expected ultimate losses are taken from trended pure premium information from the older years.

D. Advantages of the Approach Used

There are several methods that make this approach quite useful in auto warranty work:

1. The financial effects of changes in several factors that have an impact on loss emergence can easily be modelled explicitly:

   A. *Changes in Manufacturer's Warranty*

   A change in the manufacturer's warranty in any given year can be dealt with in changing the parameters of the simulation. For example, if the basic warranty increases from 1 year/10,000 miles to 3 years/36,000 miles, then one needs merely to re-run Exhibit 2, Parts A and D.

   B. *Changes in the Insurance Company's Warranty*

   As with A., changes can be made in the simulation parameters to re-run Exhibit 2, Parts C and F. In particular, were we dealing with a company that was writing contracts with longer terms than those in the data, we could explicitly reflect this by calculating the increase in "effective exposure". In an instance where the insurer was covering high mileage bands never before covered, we could use the available data for an individual who drives the full 10,000 miles in the band to extrapolate into higher mileages.

C. *Changes in Driving Habits*

There is likely to be considerable adverse selection against the insurance company if a range of policies are offered, in that policies which offer high mileage caps tend to attract high-mileage drivers. Changes in the mix of coverage written will affect the distances driven by policyholders and these can be explicitly allowed for in the computations.

D. *Bivariate Approach of the Methodology*

Both the mileage limitation of the policy and the time limitation are taken into account. In some simpler methods, such as computing the "effective" mileage limitation of the policy, this is not the case. It is obviously not appropriate to reason: "if the average driver covers 20,000 miles per year, then there is no more exposure in a 6 year/100,000 mile policy than in a 5 year/100,000 mile policy because both have an effective mileage limit of 100,000 miles".

E. *Changes in Cancellation Rates*

The approach can readily be adapted to account specifically for cancellation rates. If one can track the percentage of policies in force at a given duration (time or mileage), then a change from say 80% to 70% can be expected to reduce loss emergence by a like amount.

F. *Timing of Auto Sales*

There are frequently considerable differences between model years in the timing of new car sales, primarily because of fluctuations in the strength of the economy. These can be explicitly allowed for in the analysis.

G. *Use of Up-To-Date Data*

Unlike most forms of actuarial study, no complicated adjustments are necessary for data recorded as of a date other than the anniversary of the model year. The most up-to-date data can be readily used.

E. Difficulties With the Method

There a number of practical problems that one is likely to face in employing the approach suggested:

A. *Discounting for Investment Income*

This is generally a relatively easy exercise in the normal course of actuarial events, but in this case becomes more difficult when the loss estimates are computed by mileage band rather than time interval. The approach we use is to use the simulation model to compute the expected value of:

future miles driven	*	time to when the
in mileage band		miles are driven

If we divide each of these quantities by the expected future mileage driven, we get an estimate of the average time to payment of unpaid losses and thus the discount can be computed.

B. *Settlement Lags*

While payment lags are generally modest, there are a few weeks elapsing between the time an incident gives rise to a claim and the time when that loss has been adjusted and coded into the insurance company's system. This needs to be allowed for when selecting an evaluation date for simulation purposes, and changes in administrative procedures or claims-handling practices cause similar problems to those encountered in other books of business.

References:

1.The Actuary and IBNR, R.L.Bornhuetter and R.E.Ferguson, PCAS LIX 1972

ANALYSIS OF INSURER AND MANUFACTURER COVERAGES

Exhibit 1

	Time Cap (years)	Mileage Cap (miles)
Insurance Policy Coverage	5	50,000
Manufacturer's Basic Coverage	1	10,000
Manufacturer's Powertrain-Only Coverage	2	20,000

DISTANCE DRIVEN

Mean Distance Driven :	10,000
Standard Deviation :	5,000
Distribution :	lognormal
Correlation Between Mileage : Driven in Successive Years	0.5

INCEPTION OF POLICY

Distribution :	uniform over one year
Inception	9/30 preceding model year

EVALUATION DATE

Date at which data are collected : 1991.33 i.e. 4/30/91

SIMULATION WORKTABLE

Band Investigated	Model Year	1989
	Low Miles	0
	High Miles	10,000

Projection of Current (C) exposure as of the evaluation date or Ultimate (U) exposure. C

Projection of Exposure Under Insurer's (I) Coverage, Manufacturer's Basic (B) Coverage or Manufacturer's Powertrain (P) Coverage. B

Random Time of Policy Inception 1989.25
Time Policy in Force at Projected Date 1.00 years

Year	Distance Driven	Portion of Year Applicable	Applicable Distance Driven
1	10,000	1.00	10,000
2	10,000	0.00	0
3	10,000	0.00	0
4	10,000	0.00	0
5	10,000	0.00	0

Total Distance Driven Capped by Coverage Limit 10,000

Distance Driven in Band Under Study 10,000

Variance -- Covariance Matrix

	1	2	3	4	5
1	1	0.5	0.5	0.5	0.5
2		1	0.5	0.5	0.5
3			1	0.5	0.5
4				1	0.5
5					1

ANALYSIS OF DISTANCE DRIVEN AT THE EVALUATION DATE

Exhibit 2

Distance Driven in Each Mileage Band
at 4/30/91 While Under the
Manufacturer's "Basic" Warranty

Part A

Model Year	Mileage Band 0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	8,134	0	0	0	0
1986	8,134	0	0	0	0
1987	8,134	0	0	0	0
1988	8,132	0	0	0	0
1989	8,133	0	0	0	0
1990	7,645	0	0	0	0

Distance Driven in Each Mileage Band
at 4/30/91 While Under the
Manufacturer's "Powertrain-Only" Warranty

Part B

Model Year	Mileage Band 0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	9,876	6,530	0	0	0
1986	9,867	6,701	0	0	0
1987	9,884	6,606	0	0	0
1988	9,857	6,645	0	0	0
1989	9,860	6,348	0	0	0
1990	8,171	2,259	0	0	0

Distance Driven in Each Mileage Band
at 4/30/91 While Under the
Insurance Company Warranty

Part C

Model Year	Mileage Band 0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	10,000	9,998	9,469	7,590	5,246
1986	10,000	9,987	9,280	7,217	5,056
1987	10,000	9,896	8,431	5,706	3,369
1988	9,999	9,228	6,040	3,153	1,415
1989	9,843	6,918	2,896	942	281
1990	8,172	2,092	278	88	3

Distance Driven in Each Mileage Band
at Policy Expiration While Under the
Manufacturer's "Basic" Warranty

Part D

Model Year	Mileage Band 0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	8,134	0	0	0	0
1986	8,134	0	0	0	0
1987	8,134	0	0	0	0
1988	8,134	0	0	0	0
1989	8,134	0	0	0	0
1990	8,134	0	0	0	0

Distance Driven in Each Mileage Band
at Policy Expiration While Under the
Manufacturer's "Powertrain-Only" Warranty

Part E

Model Year	Mileage Band 0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	9,861	6,640	0	0	0
1986	9,861	6,640	0	0	0
1987	9,861	6,640	0	0	0
1988	9,861	6,640	0	0	0
1989	9,861	6,640	0	0	0
1990	9,861	6,640	0	0	0

Distance Driven in Each Mileage Band
at Policy Expiration While Under the
Insurance Company Warranty

Part F

Model Year	Mileage Band 0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	10,000	9,996	9,405	7,439	5,194
1986	10,000	9,996	9,405	7,439	5,194
1987	10,000	9,996	9,405	7,439	5,194
1988	10,000	9,996	9,405	7,439	5,194
1989	10,000	9,996	9,405	7,439	5,194
1990	10,000	9,996	9,405	7,439	5,194

Effective Exposure to Loss
at 4/30/91 in Miles
(g.) = (c.) - 90% * (a.) - 60% * [(b.) - (a.)]

Model Year	Mileage Band 0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	1,634	6,080	9,469	7,590	5,246
1986	1,640	5,966	9,280	7,217	5,056
1987	1,629	5,932	8,431	5,706	3,369
1988	1,645	5,241	6,040	3,153	1,415
1989	1,487	3,109	2,896	942	281
1990	976	737	278	88	3

Part G

Effective Exposure to Loss
at Policy Expiration
(h.) = (f.) - 90% * (d.) - 60% * [(e.) - (d.)]

Model Year	Mileage Band 0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	1,643	6,012	9,405	7,439	5,194
1986	1,643	6,012	9,405	7,439	5,194
1987	1,643	6,012	9,405	7,439	5,194
1988	1,643	6,012	9,405	7,439	5,194
1989	1,643	6,012	9,405	7,439	5,194
1990	1,643	6,012	9,405	7,439	5,194

Part H

Unsmoothed Factor to Ultimate
(i.) = (h.) / (g.)

Model Year	Mileage Band 0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	1.006	0.989	0.993	0.980	0.990
1986	1.002	1.008	1.013	1.031	1.027
1987	1.008	1.013	1.116	1.304	1.542
1988	0.999	1.147	1.557	2.359	3.671
1989	1.105	1.934	3.248	7.897	18.484
1990	1.684	8.162	33.831	84.534	1731.333

Part I

Note : in the table above, the factors have not been smoothed to remove random fluctuations in the simulation.

Smoothed Factor to Ultimate

Model Year	Mileage Band 0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	1.000	1.000	1.000	1.000	1.000
1986	1.000	1.008	1.013	1.031	1.027
1987	1.000	1.013	1.116	1.304	1.542
1988	1.000	1.147	1.557	2.359	3.671
1989	1.105	1.934	3.248	7.897	18.484
1990	1.684	8.162	33.831	84.534	1731.333

Part J

ANALYSIS OF PURE PREMIUM FOR A 10,000 MILE DRIVER

Exhibit 3

Paid Losses at 4/30/91 in $000's

Model Year	Mileage Band 0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000	Number of Contracts Originally Written
1985	507.1	2,027.4	4,013.9	3,756.7	2,893.1	102,000
1986	592.2	2,337.5	4,578.6	4,196.9	3,184.1	112,000
1987	666.7	2,658.3	4,848.3	3,903.6	2,519.1	122,000
1988	779.4	2,769.6	4,021.1	2,427.2	1,213.5	132,000
1989	818.5	1,838.3	2,157.0	842.5	277.4	142,000
1990	603.7	489.5	228.3	91.8	3.4	152,000

Part A

Paid Pure Premiums at 4/30/91 in $'s

Model Year	Mileage Band 0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000	Total
1985	$4.97	$19.88	$39.35	$36.83	$28.36	$129.39
1986	$5.29	$20.87	$40.88	$37.47	$28.43	$132.94
1987	$5.47	$21.79	$39.74	$32.00	$20.65	$119.64
1988	$5.90	$20.98	$30.46	$18.39	$9.19	$84.93
1989	$5.76	$12.95	$15.19	$5.93	$1.95	$41.79
1990	$3.97	$3.22	$1.50	$0.60	$0.02	$9.32

Part B

Ultimate Pure Premiums for a 10,000 mile driver

Model Year	Mileage Band 0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	$30.42	$32.69	$41.56	$48.53	$54.07
1986	$32.25	$34.98	$44.05	$51.92	$56.23
1987	$33.54	$36.73	$47.14	$56.08	$61.29
1988	$35.89	$40.03	$50.43	$58.32	$64.97
1989	$38.76	$41.64	$52.45	$62.98	$69.52
1990	$40.70	$43.72	$54.03	$68.65	$75.08
Average	$35.26	$38.30	$48.28	$57.75	$63.52

Part C

Ultimate Pure Premium for 10,000 mile driver =
[Paid Pure Premium at 4/30/91]
* 10,000 / [Effective Exposure at 4/30/91]

Ultimate Pure Premiums / Average for Mileage band
for the 10,000 Mile Driver

Model Year	Mileage Band 0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	0.863	0.854	0.861	0.840	0.851
1986	0.915	0.913	0.912	0.899	0.885
1987	0.951	0.959	0.976	0.971	0.965
1988	1.018	1.045	1.045	1.010	1.023
1989	1.099	1.087	1.086	1.091	1.094
1990	1.154	1.142	1.119	1.189	1.182

Part D

REGRESSION TO DETERMINE PURE PREMIUM TREND FACTOR

Exhibit 3 (cont)

Part E

Year	Ultimate /Average (U/A)	year - 1987.5	ln(U/A)
1985	0.863	-2.5	-0.147
1985	0.854	-2.5	-0.158
1985	0.861	-2.5	-0.150
1985	0.840	-2.5	-0.174
1985	0.851	-2.5	-0.161
1986	0.915	-1.5	-0.089
1986	0.913	-1.5	-0.091
1986	0.912	-1.5	-0.092
1986	0.899	-1.5	-0.106
1986	0.885	-1.5	-0.122
1987	0.951	-0.5	-0.050
1987	0.959	-0.5	-0.042
1987	0.976	-0.5	-0.024
1987	0.971	-0.5	-0.029
1987	0.965	-0.5	-0.036
1988	1.018	0.5	0.018
1988	1.045	0.5	0.044
1988	1.045	0.5	0.044
1988	1.010	0.5	0.010
1988	1.023	0.5	0.022
1989	1.099	1.5	0.095
1989	1.087	1.5	0.084
1989	1.086	1.5	0.083
1989	1.091	1.5	0.087
1989	1.094	1.5	0.090
1990	1.154	2.5	0.143
1990	1.142	2.5	0.132
1990	1.119	2.5	0.113
1990	1.189	2.5	0.173
1990	1.182	2.5	0.167

Regression Model

(U/A) = (1+t) ^ (year - 1987.5)
ln(U/A) = ln(1+t) * (year - 1987.5)

where t is the annual trend factor

Regression Output:

Constant	0	selected under model
Std Err of Y Est	0.01467163	
R Squared	0.98131894	
No. of Observations	30	
Degrees of Freedom	29	

X Coefficient(s) 0.06130442 = ln(1+t) => t= 6.3%
Std Err of Coef. 0.00156846

Normalised Ultimate Pure Premiums
for the 10,000 Mile Driver

Part F

Model Year	Mileage Band 0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000
1985	$30.25	$32.86	$41.42	$49.54	$54.50
1986	$32.16	$34.93	$44.04	$52.67	$57.94
1987	$34.20	$37.14	$46.82	$56.00	$61.61
1988	$36.36	$39.49	$49.78	$59.54	$65.50
1989	$38.66	$41.99	$52.93	$63.31	$69.64
1990	$41.10	$44.64	$56.27	$67.31	$74.05
1987.5	$35.26	$38.30	$48.28	$57.75	$63.52

Normalised Ultimate Pure Premiums
for the Typical Driver

Part G

Model Year	Mileage Band 0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000	Total
1985	$4.97	$19.75	$38.95	$36.85	$28.31	$128.84
1986	$5.28	$21.00	$41.42	$39.18	$30.10	$136.98
1987	$5.62	$22.33	$44.03	$41.66	$32.00	$145.64
1988	$5.97	$23.74	$46.82	$44.29	$34.02	$154.85
1989	$6.35	$25.24	$49.78	$47.09	$36.17	$164.64
1990	$6.75	$26.84	$52.92	$50.07	$38.46	$175.05

Ultimate Pure Premium for Typical Driver =
Pure Premium for the 10,000 Mile Driver
* Effective Exposure at Policy Expiration / 10,000

Projected Ultimate Pure Premium Using "Factor to Ultimate" Methodology

Model Year	Mileage Band 0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000	Total
1985	$4.97	$19.88	$39.35	$36.83	$28.36	$129.39
1986	$5.29	$21.03	$41.43	$38.62	$29.21	$135.58
1987	$5.47	$22.08	$44.33	$41.71	$31.83	$145.43
1988	$5.90	$24.07	$47.43	$43.38	$33.74	$154.53
1989	$6.37	$25.03	$49.33	$46.85	$36.11	$163.69
1990	$6.69	$26.28	$50.81	$51.07	$38.99	$173.85

Part H

Ultimate Pure Premium for Typical Driver =
Paid Pure Premium at 4/30/91
* Smoothed Factor to Ultimate from Exhibit 2j

Projected Ultimate Pure Premium Using Bornhuetter-Ferguson Methodology

Model Year	Mileage Band 0 to 10,000	10,000 to 20,000	20,000 to 30,000	30,000 to 40,000	40,000 to 50,000	Total
1985	$5.00	$19.65	$39.09	$36.08	$28.08	$127.90
1986	$5.30	$21.03	$41.43	$38.64	$29.23	$135.63
1987	$5.51	$22.08	$44.30	$41.70	$31.89	$145.49
1988	$5.90	$24.03	$47.21	$43.91	$33.95	$154.99
1989	$6.37	$25.13	$49.64	$47.06	$36.17	$164.37
1990	$6.71	$26.77	$52.86	$50.08	$38.46	$174.89

Part I

Ultimate Pure Premium =
Paid Pure Premium
+ (Ultimate Effective Exposure - Effective Exposure at 4/30/91)
* Ultimate Pure Premium for the 10,000 Mile Driver / 10,000

THE IMPACT OF LAW CHANGES ON RATEMAKING DATA FOR PRIVATE PASSENGER AUTOMOBILE INSURANCE

David R. Chernick

THE IMPACT OF LAW CHANGES ON RATEMAKING DATA

FOR PRIVATE PASSENGER AUTOMOBILE INSURANCE

1. Introduction

Various types of data are used in determining statewide rate level indications ("RLI") for private passenger automobile insurance. Some major types of data used in RLI's are the base data, loss development data, trend data, and data used to measure investment income. In this paper, the impacts of several hypothetical law changes on these various types of ratemaking data are analyzed. Beginning with actual Allstate data, the impact of the law change on that data is modelled, allowing the overall impact of the law change on the various types of ratemaking data to be determined.

This paper is intended primarily for students of the CAS, but also will serve as a ready reference for experienced actuaries working in a ratemaking capacity. Although the examples presented in this paper are from private passenger automobile, the applications and conclusions can be applied to other lines of business.

In an attempt to give this subject adequate coverage, yet keep it manageable, three different law changes are examined.

1. Bodily Injury liability ("BI") coverage is analyzed for a change from a tort liability system to a strong verbal threshold restricting the right to sue. A

choice no-fault option is also examined.

2. Collision coverage is examined for a law change that mandates every policy be renewed with $500 deductible. For simplicity, this paper assumes that all policies were previously written with a $200 deductible and no "buy-down" is allowed. In practice, most insureds will not exercise the option to change coverage but stay with the default coverage option.
3. Personal Injury Protection coverage is examined for a law change that mandates a $250 deductible instead of no deductible.

2. *Initial Data and Notation*

Appendix 1 contains the definition and development of the notation and general assumptions used in this paper. Appendix 2 displays the data and results of the model for BI coverage. Exhibit 1, page 1 of Appendix 2 presents accident year payments by quarter in the column labeled "Amount Paid". Also presented are the "Cumulative Amount Paid" (Column 2) and "Loss Reserves" in Column 3.

In order to shorten the length of this paper, Collision and Personal Injury Protection ("PIP") data was excluded. The exhibits for these coverages, similar to Appendix 2, are available from the author upon request.

3. Bodily Injury - Verbal Threshold

In this section all exhibits are contained in Appendix 2, except where otherwise noted. The verbal threshold will essentially eliminate small claims from the insurance system. Further assumptions regarding the BI law change are:

1. The overall reduction in pure premium due to the law change is 30%.
2. The law change is effective January 1, 1995.
3. The law change applies to all outstanding policies.

Data under a tort law is used to derive Exhibit 1, page 1. Column 1 of Exhibit 1, page 2 was created by beginning with the payment pattern on Exhibit 1, page 1 and assuming that the verbal threshold eliminates the first 30% of paid loss. Paid loss data by payment duration between accident date and payment date, and by size of loss was used to determine the amount and timing of payments eliminated by the verbal threshold. All payments under $10,000 were eliminated, along with about 90% of the losses between $10,000 and $15,000. A portion of this data is included for reference in Appendix 3.[1]

Base Data

Assume the base data used in ratemaking is accident year. Is it necessary after a law changes to adjust base data to be used in a statewide rate level calculation? The answer is maybe. In order to make that evaluation, the ratemaker must know the

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[1] The data in Appendix 3 is included with Allstate's permission and represents data for BI coverage under a tort law.

period of base data, the effective date of the law change, if the law change applied to all outstanding policies or was applied at policy renewal, and if premiums were previously adjusted. The key determining factor is whether or not the premium and loss base data match. This paper will not deal with the costing of a law change. If the law change has not yet been implemented, then it must be costed and that is beyond the scope of this paper.

Base data is 100% pre-law change

If the rates already reflect law change impact and the ratemaker is interested in a prospective rate level review, simply assuming that the previously determined price impact of the law change is proper would allow the ratemaker to proceed without adjustment of the base data. Of course, the ratemaker could adjust both losses and premiums to reflect this previously determined impact.

Base data is 100% post-law change

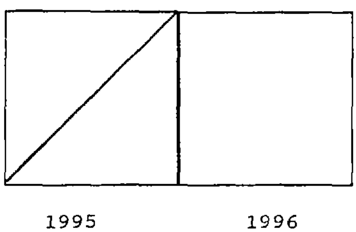
When the base data is completely reflective of the law change, no adjustments are necessary. In this paper, the base data is 100% post law change for accident years 1995 and subsequent.

Base data is a mix of pre-law change and post-law change

An adjustment may be necessary. In order to determine this, a complete understanding of how the law change was implemented from both a premium and loss perspective is necessary. If the base data is completely prior to 1995 or subsequent to 1995, the previously stated general conclusions apply. However, the case where the base data is 1995 deserves further discussion.

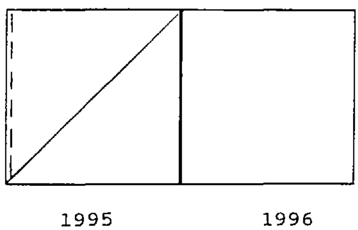
Assume that a 30% premium reduction was implemented for policies effective on or after 1/1/95 to reflect the loss reduction due solely to the law change. Assume, just for this example, that the law change does <u>not</u> apply to outstanding policies, but only applies as policies renew.

Under the assumptions of this example, no adjustment is necessary. Both the calendar year earned premium and the accident year incurred losses will be half under the old law and half under the new law. Using the hypothetical data from Exhibit 1, pages 1 and 2 accident year 1995 pre-law incurred losses would be \$50 (½ of \$100) and post-law losses would be \$35 (½ of \$70). Assuming rate adequacy and a 20% expense ratio, the pre-law and post-law earned premium would have been \$62.5 and \$43.75, respectively. Using the loss ratio method of determining an RLI and the equation: $RLI = ((EP/IL)/(1 - E)) - 1$, where EP is Earned Premium, IL is Incurred Loss, and E is the expense ratio as a percent of premium. The impact of the law change can be examined. A law change, which applies only to policies as they renew, can be represented by a diagonal line:



The pre-law RLI is ((50/62.5)/.8)-1 = 0% , and
the post-law RLI is ((35/43.75)/.8)-1 = 0% , and
combining the data, the RLI is ((85/106.25)/.8)-1 = 0%.
Thus, under these assumptions no adjustments to the base data are necessary as a result of the law change.

Instead, return to the base assumption that the law change is deemed to apply to all outstanding policies on 1/1/95, with all other assumptions unchanged including the 30% decrease in premiums as policies renew. The loss exposure can be represented by a vertical line, while the earned premium impact is still represented by a diagonal line:



The combined RLI without adjustment is:

((70/106.25)/.8)-1 = -17.6%.

However, we know that the correct prospective RLI is 0%. The discrepancy arises because the 30% premium decrease was applied upon renewal while the 30% loss reduction attributable to the law change is completely realized during accident year 1995. Thus, the premiums and losses do not match, and it would be necessary and proper to adjust the premiums completely to their post-law level.

In summary, as long as the premium and losses in the base data are equally reflective of the law change, no adjustments are necessary. If the premium and losses in the base data are not equally reflective of the law change, then some adjustment is required.

Trend Data

Since there is one exposure, the ultimate incurred pure premium for each accident year before the law change is $100. For simplicity, assume the trend data will be calendar year paid pure premium. Twelve month moving paid pure premium trend data can be developed. It is a relatively simple exercise to expand the model to severity and frequency separately, but it is not essential for the purposes of this paper.

Exhibit 2, page 1 displays twelve month moving paid pure premium data. In order to analyze the impact of the law change, the data displayed on Exhibit 2, page 1 is fit to an exponential curve. Exhibit 2, page 2 displays three examples of the calculation. The resulting annual trend for all the data evaluation periods is displayed on page 3 of Exhibit 2.

The expected pure premium trend for this data is 0% because it is assumed that there is no frequency and no severity trend. This allows the quantification of the impact of the change to a verbal threshold on BI trend data. Failure to account for the impact of the law change on trend data can result in an error of up to 10.6% on a 12 point basis and 12.4% on a 6 point basis depending on the duration between the effective date of the law change and the evaluation date. Furthermore, from this model it can be concluded that the trend data has a measurable bias for up to 8 years after the law change.

Loss Development Data

Paid loss development factors can be determined easily from column (2) of Exhibit 1. Paid loss development triangles are derived by the formula developed in Appendix 1. Pre-law and the post-law paid development triangles are displayed on pages 1 and 2 of Exhibit 3, respectively. A comparison of the indicated factors from these two exhibits (see Exhibit 3, page 4) clearly leads to the conclusion that the law change significantly changes the payment pattern and it is clearly inappropriate to apply paid loss development factors from pre-law data to base data that is post-law change. The paid development factor for 5 quarters to ultimate changes from 5.319 to 8.140. Therefore, the use of the paid loss development factors based on pre-law patterns applied to post-law change base data will understate ultimate incurred losses by almost 35% (1 - (5.319/8.140)).

When the loss development factors are based on data that is a mix of pre-law and post-law, the analysis is a bit more complicated. Assume that the base data is paid loss from accident years 1996 and 1997 evaluated as of March 15, 1998 ($P_{N,9}$ = \$ 28.6, and $P_{N,5}$ = \$ 8.6). In a loss development triangle,. accident year 1995 development from 5 to 9 quarters and from 9 to 13 quarters, and accident year 1996 development from 5 to 9 quarters would be post-law change. All other observations in the triangle would be pre-law change. This paid loss development triangle is shown on Exhibit 3, page 3. The indicated paid loss development factor would be 2.117 and 6.532 for accident years 1996 and 1997, respectively. The correct factors would be 2.448 and 8.140, respectively. The ultimate accident year losses for both these accident years are $70, because both accident years are post-law change. However, typical ratemaking procedures would develop the following estimate of ultimate incurred loss:

Accident year 1996 paid loss =	$28.6
Accident year 1997 paid loss =	$ 8.6
Paid loss development factor	
9 quarters to ultimate =	2.117
Paid loss development factor	
5 quarters to ultimate	6.532
Ultimate accident year 1995 incurred losses:	$ 60.5
Ultimate accident year 1996 incurred losses:	$ 56.2

The ultimate incurred losses in this example are understated by 13.6% and 19.7% for the two accident years, respectively.

Incurred loss development factor evaluation is more complicated because it requires assumptions on the development patterns both pre- and post-law change. If loss reserves are adequate before and after the law change, then incurred loss development factors will be 1.000 and the law change will not impact the use of incurred loss development factors.

Investment Income Data

Although much potential bias exists, ratemakers have utilized the ratio of reserves to incurred losses to estimate the amount of investment income potential that exists from the investment of premiums. The reserve to incurred ratio is not generally an accurate measure of investment income potential. This is recognized by both the actuarial and academic communities. Dr. Cummins states: "The k factor represents only a crude approximation of the discounting process that can lead to serious errors when estimating premiums (Myers and Cohn (1987)."[2] In his study note on the CAS Part 6 Examination Syllabus, Dr. Ira Robbin also recognizes the shortfall: "However, since calendar year results are an inherently retrospective summary of contributions from current and prior policy years, their applicability in prospective ratemaking could be challenged. In particular, the prior growth history and loss experience of the line could distort answers."[3]

The development of the reserve to incurred ratios for BI coverage is displayed on Exhibit 4, page 1. The incurred loss for accident year 1994 is $100, and for accident year 1995 and

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[2] Journal of Risk and Insurance, July 1991, J. David Cummins, "Statistical and Financial Models of Insurance Pricing and the Insurance Firm." pp 286-287. The k factor referred to in this quote is the reserve to premium ratio. However, the comment is equally applicable to reserve to incurred ratios.

[3] *Casualty Actuarial Society Syllabus of Examinations*, 1992, Part 6, Study Note Reading: Robbin, I. - "The Underwriting Profit Provision", p. 13.

subsequent is $70. Since accidents are equally distributed throughout the year, incurred losses for the fiscal accident years ending 3/31/95, 6/30/95, and 9/30/95 are $92.5, $85 and $77.5, respectively. The reserve to incurred ratio increases 25% from 12/31/94 to 12/31/95 (2.00 to 2.51), because incurred losses under the new law are immediately reduced while the reserves gradually reflect the new law over 25 quarters or 6¼ years. The true reserve to incurred ratio under the new law is 2.24, thus using the incurred to reserve ratio to measure investment income in an RLI can overstate the true investment income by up to 12.0% (2.51/2.24).

A superior method of measuring the investment income potential of policyholder supplied funds is a discounted cash flow of the policy transaction. For simplicity, it is assumed that all expenses are paid and all premium is collected on the policy effective date. Policy year loss payment patterns are superior because ratemaking is always done for a set of policies. Accident year patterns have already been developed and are used here for illustrative purposes. The average effective date of the policies providing coverage for losses occurring in an accident year under an annual policy is January 1 of that year. Since it is assumed in this paper that accidents and policies are equally distributed throughout the year, it is proper to discount to the average premium collection date (the beginning of the accident year).

The cash flow calculations are derived on Exhibit 4, page 2. It is assumed that the average payment date is half way through the quarter. The difference between the discounted payments and undiscounted payments yield the investment income opportunity of the loss portion the premium. Using a 6% annual yield, the investment income opportunity is 13.9% under the old law and 15.1% under the new law. Until the payment pattern data fully recognizes the new law an adjustment is necessary. Using the old payment pattern understates the investment income potential (13.9% vs. 15.1%).

Sensitivity of the Projected Payment Pattern

The original model assumes all small losses are eliminated from the system by a change from a tort system to a verbal threshold. This was done by using the distribution in Appendix 3. Two other post-law change distributions were used to test whether the results of the model were sensitive to the chosen post-law distribution. The first is based on the current distribution of another state where the data was completely under a verbal threshold. These results are shown in Appendix 4. Using the distribution of this other state removes more of the earlier payments and less of the later payments from the accident year. This makes the impact of the law change greater than under the model in Appendix 2. The impacts, however, are not significantly different.

In Appendix 5, the data from Appendix 2 is used to model what would happen under a choice no-fault system where 20% of the exposures select the pre-law system and 80% select the verbal threshold. This was accomplished by weighing Column 1 of Exhibit 1, page 1 and Column 1 of Exhibit 1, page 2 (from Appendix 2) 20% and 80%, respectively. The impact of the law change under a choice system is less by the proportion of exposures that do not convert to the verbal threshold.

Conclusion

The implementation of a verbal threshold obviously creates significant distortions in BI ratemaking data for years. This includes the base data, trend data, loss development data, and investment income data. The accident year payment pattern is a function of coverage in effect, the environment, the economy and anything else that would affect how much is paid and when. Thus, it is impossible to isolate the sole impact of a law change on a payment pattern. The model in this paper attempts to quantify the impact of the law change on the various ratemaking data. The results of the model can be used by ratemakers as a guide when confronted with ratemaking data that is impacted by a law change.

4. Collision - Mandatory Deductible roll

The same techniques used to evaluate the BI law change are used for collision coverage. The mandatory deductible roll for collision coverage eliminates the first $250 of each payment. Since, the payments for collision coverage are made relatively quickly after the accident occurs, the underlying payment pattern remains similar after the law change except that each payment is reduced by the amount of the deductible.

Base Data

The general conclusions in the BI section hold true for collision coverage under a deductible roll.

Trend Data

The impact of a deductible roll on trend data is significant. Based on the results of the model, failure to account for the impact of the law change on trend data could result in an error of up to 11.1% on a 12 point basis and 19.6% on a 6 point basis depending on the length of time between the end point of the trend data and the effective date of the law change. A significant influence from the law change remains for about three and a half years after the law change (see Exhibit 2, page 2).

Using the assumed pre-law and post-law change distributions, the trend is biased upward after a certain point because of larger subrogation recoveries under the prior law occur with a lower amount of claim payments under the new law.

Loss Development Data

The impact is minimal, because claims are paid quickly.

Investment Income Data

Again, the impact is minimal because claims are paid quickly.

Conclusion

The major influences of a deductible roll on ratemaking data for collision coverage are for base data and trend. Loss development and investment income are not significantly impacted because payments are generally made very quickly for this coverage.

5. PIP - Mandatory Deductible roll

The reason this type of law change was chosen was to contrast the impact of a deductible roll between short and long tail coverages. For a long tail coverage (PIP), the deductible roll impacts loss development and investment income data in addition to the base data and trend data.

Base Data

The general conclusions in the BI section hold true for PIP coverage under a deductible roll.

Trend Data

The impact of a deductible roll on trend data is significant. Failure to account for the impact of the law change on trend data can result in an error of up to 7.5% on a 12 point basis and 12.6% on a 6 point basis depending on the length of time between the end point of the trend data and the effective date of the law change. A significant influence from the law change remains for about three and a half years after the law change.

Loss Development Data

The impact is significant. The paid loss development factors change from 2.45 to 3.82 and from 1.58 to 1.76 for the 5 and 9 quarter evaluations, respectively.

Investment Income Data

The reserve to incurred ratios move from 2.18 to 2.59 within a year after the law change. The measurement of investment income from the discounted value of the policy transaction increases from 14.4% to 16.1%. This difference is significant enough that is must be considered by the ratemaker.

Conclusion

A deductible roll for a longer tail line also impacts paid loss development data and investment income data.

DESCRIPTION OF NOTATION

The data and model will be presented for Bodily Injury Liability ("BI") coverage in Appendix 2. The model was also used for Collision coverage and Personal Injury Protection ("PIP") coverage. Accident year paid loss patterns are presented for BI coverage in Appendix 2 on Exhibit 1, page 1. The "Amount Paid" (P) data has been derived from actual Allstate data.[1] The sum of the accident year payments pre-law change through 40 quarters of evaluation is $100, which is assumed to be the ultimate incurred loss for each accident year. For simplicity of analysis, the following assumptions hold throughout this paper:

1. There is no change in the volume of business. For simplicity of the trend data calculations it is further assumed that there is always only one exposure each year.
2. There is no frequency or severity trend.
3. Effective dates of policies are equally distributed throughout the year.
4. Accident occurrence is also equally distributed throughout the year.

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[1]With Allstate's permission, actual accident year paid loss patterns at quarterly evaluations were used to create column 1 of Exhibit 1, page 1 of each Appendix. The selected amounts as a percent of ultimate paid loss for each quarter were based on three year averages of actual data and applied to $100 to produce a payment pattern in dollars.

5. Payments made in each quarter occur such that the average payment date (based on dollars) is mid-way through the quarter.
6. All policies have an annual term.
7. All expenses vary directly with premium and are 20% of premium.

Various ratemaking data can be derived from these assumptions and the accident year payment patterns. The "Cumulative Amount Paid" (CP) is the sum of all amounts paid up to and including the end of the evaluation quarter. "Loss Reserves" (R) is 100 minus the cumulative amount paid.[2]

For purposes of this paper, let:

Subscripts:

i - represent an accident year.

j - represent an evaluation date in quarters of a year, where j=0 at the beginning of the accident year.

k,l - represent an actual evaluation date, where k is the quarter and l is the year. For example, 3,96 is the 9/30/96 evaluation.

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[2]At the 1, 2, and 3 quarter evaluations 1/4, 1/2 and 3/4 of the ultimate incurred loss are used in lieu of the ultimate incurred loss. For collision coverage the anticipated salvage and subrogation for the accident year is added to the equation for determining R, otherwise the reserves would be negative.

Variables:

1. Basic Model

$P_{i,j}$ = the payments from accident year i made during the quarter ending at the j evaluation.

$CP_{i,j}$ = the sum of all accident year i paid losses through the j quarter evaluation.

$R_{i,j}$ = the reserves from accident year i evaluated at the end of quarter j.

U = Ultimate accident year loss

Then,

$$CP_{i,j} = \sum_{n=1}^{j} P_{i,n} \text{ , and}$$

$$R_{i,j} = U - C_{i,j} \;.$$ [3]

2. Trend

$CYP_{k,l}$ = payments made during the 4 quarter moving period ending k quarter of year l.

Then, $CYP_{4,94} = P_{94,1} + P_{94,2} + P_{94,3} + P_{94,4} + P_{93,5} + P_{93,6} + P_{93,7} + P_{93,8} + P_{92,9} + \ldots + P_{85,39} + P_{85,40}$.

[3] This equation only holds for j > 3. For j = 1, 2 and 3, 1/4, 1/2, 3/4 of U are substituted for U. Also, for collision coverage anticipated salvage and subrogation needs to be added to U, otherwise the reserves would be negative.

Since there is no change in the volume of business or losses, it follows that $P_{94,j} = P_{93,j} = P_{92,j} = \ldots = P_{85,j}$, for each j.

Therefore, $CYP_{k,l} = \sum_{j=1}^{40} P_{i,j} = CP_{i,40} = 100,$

for all years (i) prior to the law change.

However, after the law change this is no longer true. The underlying assumptions make $P_{95,j} = P_{96,j} = P_{97,j} = \ldots$, but $P_{94,j}$ does not equal $P_{95,j}$.

Let V represent accident years under the verbal threshold, and T represent accident years under the tort threshold.

Then, $CYP_{k,l} = \sum_{n=1}^{j} P_{V,n} + \sum_{n=j}^{40} P_{T,n}$, where j is the number of quarters between the evaluation date k,l and 1,95.

3. Loss Development

Let, $PLDF_{j,k}$ be the paid loss development factor (also referred to as a link factor when k-j = 1) between j and k quarters of evaluation.

Then, $PLDF_{j,k} = CP_{i,k} / CP_{i,j}$.

For example, $PLDF_{5,40} = CP_{94,40} / CP_{94,5}$.

4. Investment Income

Total Reserves are derived by:

$$TR_{k,l} = \sum_{n=1}^{10} R_{l+1-n,\, k+4(n-1)}$$

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PRE-LAW CHANGE
BODILY INJURY PAYMENT PATTERNS
ACCIDENT YEAR

(j) QUARTERS OF EVALUATION	(P) (1) AMOUNT PAID	(CP) (2) CUMULATIVE AMOUNT PAID	(R) (3) LOSS RESERVES
1	0.3	0.3	24.7
2	1.9	2.2	47.8
3	4.0	6.2	68.8
4	6.0	12.2	87.8
5	6.6	18.8	81.2
6	7.4	26.2	73.8
7	7.7	33.9	66.1
8	8.2	42.1	57.9
9	6.9	49.0	51.0
10	6.9	55.9	44.1
11	6.7	62.6	37.4
12	6.4	69.0	31.0
13	5.9	74.9	25.1
14	4.9	79.8	20.2
15	4.3	84.1	15.9
16	3.2	87.3	12.7
17	3.0	90.3	9.7
18	2.3	92.6	7.4
19	1.5	94.1	5.9
20	1.2	95.3	4.7
21	0.7	96.0	4.0
22	0.5	96.5	3.5
23	0.5	97.0	3.0
24	0.5	97.5	2.5
25	0.5	98.0	2.0
26	0.3	98.3	1.7
27	0.2	98.5	1.5
28	0.2	98.7	1.3
29	0.1	98.8	1.2
30	0.1	98.9	1.1
31	0.2	99.1	0.9
32	0.1	99.2	0.8
33	0.1	99.3	0.7
34	0.2	99.5	0.5
35	0.1	99.6	0.4
36	0.1	99.7	0.3
37	0.1	99.8	0.2
38	0.0	99.8	0.2
39	0.1	99.9	0.1
40	0.1	100.0	0.0

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POST-LAW CHANGE - USING MODEL
BODILY INJURY PAYMENT PATTERNS
ACCIDENT YEAR

(j) QUARTERS OF EVALUATION	(P) (1) AMOUNT PAID	(CP) (2) CUMULATIVE AMOUNT PAID	(R) (3) LOSS RESERVES
1	0.1	0.1	17.4
2	0.9	1.0	34.0
3	1.5	2.5	50.0
4	2.7	5.2	64.8
5	3.4	8.6	61.4
6	4.5	13.1	56.9
7	5.1	18.2	51.8
8	5.5	23.7	46.3
9	4.9	28.6	41.4
10	5.3	33.9	36.1
11	5.3	39.2	30.8
12	4.9	44.1	25.9
13	4.5	48.6	21.4
14	4.0	52.6	17.4
15	3.5	56.1	13.9
16	2.7	58.8	11.2
17	2.7	61.5	8.5
18	2.0	63.5	6.5
19	1.3	64.8	5.2
20	1.1	65.9	4.1
21	0.6	66.5	3.5
22	0.3	66.8	3.2
23	0.4	67.2	2.8
24	0.4	67.6	2.4
25	0.4	68.0	2.0
26	0.3	68.3	1.7
27	0.2	68.5	1.5
28	0.2	68.7	1.3
29	0.1	68.8	1.2
30	0.1	68.9	1.1
31	0.2	69.1	0.9
32	0.1	69.2	0.8
33	0.1	69.3	0.7
34	0.2	69.5	0.5
35	0.1	69.6	0.4
36	0.1	69.7	0.3
37	0.1	69.8	0.2
38	0.0	69.8	0.2
39	0.1	69.9	0.1
40	0.1	70.0	0.0

LAW CHANGE - APPLIED TO ALL OUTSTANDING POLICIES 1/1/95
BODILY INJURY PAID PURE PREMIUMS
12 MONTH MOVING

12 MONTH MOVING ENDING		(CYP) PAID PURE PREMIUM
MARCH	1992	100.0
JUNE	1992	100.0
SEPTEMBER	1992	100.0
DECEMBER	1992	100.0
MARCH	1993	100.0
JUNE	1993	100.0
SEPTEMBER	1993	100.0
DECEMBER	1993	100.0
MARCH	1994	100.0
JUNE	1994	100.0
SEPTEMBER	1994	100.0
DECEMBER	1994	100.0
MARCH	1995	99.8
JUNE	1995	98.8
SEPTEMBER	1995	96.3
DECEMBER	1995	93.0
MARCH	1996	89.8
JUNE	1996	86.9
SEPTEMBER	1996	84.3
DECEMBER	1996	81.6
MARCH	1997	79.6
JUNE	1997	78.0
SEPTEMBER	1997	76.6
DECEMBER	1997	75.1
MARCH	1998	73.7
JUNE	1998	72.8
SEPTEMBER	1998	72.0
DECEMBER	1998	71.5
MARCH	1999	71.2
JUNE	1999	70.9
SEPTEMBER	1999	70.7
DECEMBER	1999	70.6
MARCH	2000	70.5
JUNE	2000	70.3
SEPTEMBER	2000	70.2
DECEMBER	2000	70.1
MARCH	2001	70.0
JUNE	2001	70.0
SEPTEMBER	2001	70.0
DECEMBER	2001	70.0
MARCH	2002	70.0
JUNE	2002	70.0
SEPTEMBER	2002	70.0
DECEMBER	2002	70.0
MARCH	2003	70.0
JUNE	2003	70.0
SEPTEMBER	2003	70.0
DECEMBER	2003	70.0

LAW CHANGE TREND ANALYSIS
Bodily Injury Liability
Paid Pure Premium
12 Month Moving

12 MONTH MOVING ENDING		actual data	12 pt. curve of best fit	6 pt. curve of best fit
DECEMBER	1992	100.0	100.604	
MARCH	1993	100.0	100.415	
JUNE	1993	100.0	100.226	
SEPTEMBER	1993	100.0	100.038	
DECEMBER	1993	100.0	99.850	
MARCH	1994	100.0	99.663	
JUNE	1994	100.0	99.476	100.759
SEPTEMBER	1994	100.0	99.289	100.109
DECEMBER	1994	100.0	99.103	99.462
MARCH	1995	99.8	98.917	98.820
JUNE	1995	98.8	98.731	98.182
SEPTEMBER	1995	96.3	98.546	97.548
		Average Annual % Change	-0.75%	-2.56%

12 MONTH MOVING ENDING		actual data	12 pt. curve of best fit	6 pt. curve of best fit
SEPTEMBER	1994	100.0	103.986	
DECEMBER	1994	100.0	101.354	
MARCH	1995	99.8	98.788	
JUNE	1995	98.8	96.288	
SEPTEMBER	1995	96.3	93.851	
DECEMBER	1995	93.0	91.475	
MARCH	1996	89.8	89.159	89.433
JUNE	1996	86.9	86.903	86.913
SEPTEMBER	1996	84.3	84.703	84.465
DECEMBER	1996	81.6	82.559	82.085
MARCH	1997	79.6	80.469	79.773
JUNE	1997	78.0	78.432	77.525
		Average Annual % Change	-9.75%	-10.80%

12 MONTH MOVING ENDING		actual data	12 pt. curve of best fit	6 pt. curve of best fit
SEPTEMBER	1996	84.3	82.243	
DECEMBER	1996	81.6	80.972	
MARCH	1997	79.6	79.720	
JUNE	1997	78.0	78.488	
SEPTEMBER	1997	76.6	77.276	
DECEMBER	1997	75.1	76.081	
MARCH	1998	73.7	74.906	73.398
JUNE	1998	72.8	73.748	72.840
SEPTEMBER	1998	72.0	72.608	72.286
DECEMBER	1998	71.5	71.486	71.736
MARCH	1999	71.2	70.382	71.190
JUNE	1999	70.9	69.294	70.648
		Average Annual % Change	-6.04%	-3.01%

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LAW CHANGE - APPLIED TO ALL OUTSTANDING POLICIES 1/1/95
BODILY INJURY PAID PURE PREMIUMS
SUMMARY OF 12 MONTH MOVING TRENDS

TRENDS ENDING		12 POINT	6 POINT
DECEMBER	1994	-0.0%	0.0%
MARCH	1995	-0.0%	-0.1%
JUNE	1995	-0.2%	-0.8%
SEPTEMBER	1995	-0.7%	-2.6%
DECEMBER	1995	-1.7%	-5.4%
MARCH	1996	-3.0%	-8.5%
JUNE	1996	-4.4%	-10.9%
SEPTEMBER	1996	-5.9%	-12.2%
DECEMBER	1996	-7.4%	-12.4%
MARCH	1997	-8.8%	-11.8%
JUNE	1997	-9.7%	-10.8%
SEPTEMBER	1997	-10.4%	-9.7%
DECEMBER	1997	-10.6%	-8.6%
MARCH	1998	-10.4%	-7.7%
JUNE	1998	-9.7%	-7.0%
SEPTEMBER	1998	-8.9%	-6.3%
DECEMBER	1998	-8.0%	-5.4%
MARCH	1999	-7.0%	-4.1%
JUNE	1999	-6.0%	-3.0%
SEPTEMBER	1999	-5.1%	-2.2%
DECEMBER	1999	-4.3%	-1.5%
MARCH	2000	-3.5%	-1.1%
JUNE	2000	-2.8%	-0.9%
SEPTEMBER	2000	-2.1%	-0.8%
DECEMBER	2000	-1.6%	-0.7%
MARCH	2001	-1.2%	-0.7%
JUNE	2001	-0.9%	-0.6%
SEPTEMBER	2001	-0.7%	-0.4%
DECEMBER	2001	-0.6%	-0.2%
MARCH	2002	-0.5%	-0.1%
JUNE	2002	-0.4%	0.0%
SEPTEMBER	2002	-0.3%	0.0%
DECEMBER	2002	-0.2%	0.0%
MARCH	2003	-0.1%	0.0%
JUNE	2003	-0.1%	0.0%
SEPTEMBER	2003	-0.0%	0.0%
DECEMBER	2003	0.0%	0.0%

PAID LOSS DEVELOPMENT
PRE-LAW CHANGE
BODILY INJURY LIABILITY

ACCIDENT YEAR	EVALUATION 5	9	13	17	21	25	29	33	37	40
1983	18.8	49	74.9	90.3	96	98	98.8	99.3	99.8	100
1984	18.8	49	74.9	90.3	96	98	98.8	99.3	99.8	100
1985	18.8	49	74.9	90.3	96	98	98.8	99.3	99.8	100
1986	18.8	49	74.9	90.3	96	98	98.8	99.3	99.8	
1987	18.8	49	74.9	90.3	96	98	98.8	99.3		
1988	18.8	49	74.9	90.3	96	98	98.8			
1989	18.8	49	74.9	90.3	96	98				
1990	18.8	49	74.9	90.3	96					
1991	18.8	49	74.9	90.3						
1992	18.8	49	74.9							
1993	18.8	49								
1994	18.8									

ACCIDENT YEAR	LINK FACTORS 5 TO 9	9 TO 13	13 TO 17	17 TO 21	21 TO 25	25 TO 29	29 TO 33	33 TO 37	37 TO 40	
1983	2.606	1.529	1.206	1.063	1.021	1.008	1.005	1.005	1.002	
1984	2.606	1.529	1.206	1.063	1.021	1.008	1.005	1.005	1.002	
1985	2.606	1.529	1.206	1.063	1.021	1.008	1.005	1.005	1.002	
1986	2.606	1.529	1.206	1.063	1.021	1.008	1.005	1.005		
1987	2.606	1.529	1.206	1.063	1.021	1.008	1.005			
1988	2.606	1.529	1.206	1.063	1.021	1.008				
1989	2.606	1.529	1.206	1.063	1.021					
1990	2.606	1.529	1.206	1.063						
1991	2.606	1.529	1.206							
1992	2.606	1.529								
1993	2.606									
1994	N/A									
3 YEAR AVERAGE	2.6064	1.5286	1.2056	1.0631	1.0208	1.0082	1.0051	1.0050	1.0020	
CUMMULATIVE FROM:		5	9	13	17	21	25	29	33	37	40
TO ULTIMATE		5.3191	2.0408	1.3351	1.1074	1.0417	1.0204	1.0121	1.0070	1.0020	1.0000

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PAID LOSS DEVELOPMENT
POST-LAW CHANGE
BODILY INJURY LIABILITY

ACCIDENT YEAR	EVALUATION 5	9	13	17	21	25	29	33	37	40
1995	8.6	28.6	48.6	61.5	66.5	68	68.8	69.3	69.8	70
1996	8.6	28.6	48.6	61.5	66.5	68	68.8	69.3	69.8	70
1997	8.6	28.6	48.6	61.5	66.5	68	68.8	69.3	69.8	70
1998	8.6	28.6	48.6	61.5	66.5	68	68.8	69.3	69.8	
1999	8.6	28.6	48.6	61.5	66.5	68	68.8	69.3		
2000	8.6	28.6	48.6	61.5	66.5	68	68.8			
2001	8.6	28.6	48.6	61.5	66.5	68				
2002	8.6	28.6	48.6	61.5	66.5					
2003	8.6	28.6	48.6	61.5						
2004	8.6	28.6	48.6							
2005	8.6	28.6								
2006	8.6									

ACCIDENT YEAR	LINK FACTORS 5 TO 9	9 TO 13	13 TO 17	17 TO 21	21 TO 25	25 TO 29	29 TO 33	33 TO 37	37 TO 40
1995	3.326	1.699	1.265	1.081	1.023	1.012	1.007	1.007	1.003
1996	3.326	1.699	1.265	1.081	1.023	1.012	1.007	1.007	1.003
1997	3.326	1.699	1.265	1.081	1.023	1.012	1.007	1.007	1.003
1998	3.326	1.699	1.265	1.081	1.023	1.012	1.007	1.007	
1999	3.326	1.699	1.265	1.081	1.023	1.012	1.007		
2000	3.326	1.699	1.265	1.081	1.023	1.012			
2001	3.326	1.699	1.265	1.081	1.023				
2002	3.326	1.699	1.265	1.081					
2003	3.326	1.699	1.265						
2004	3.326	1.699							
2005	3.326								
2006	N/A								
3 YEAR AVERAGE	3.3256	1.6993	1.2654	1.0813	1.0226	1.0118	1.0073	1.0072	1.0029

CUMMULATIVE										
FROM:	5	9	13	17	21	25	29	33	37	40
TO ULTIMATE	8.1395	2.4476	1.4403	1.1382	1.0526	1.0294	1.0174	1.0101	1.0029	1.0000

PAID LOSS DEVELOPMENT
MIX OF PRE-LAW AND POST-LAW CHANGE
BODILY INJURY LIABILITY

ACCIDENT YEAR	EVALUATION 5	9	13	17	21	25	29	33	37	40
1986	18.8	49	74.9	90.3	96	98	98.8	99.3	99.8	100
1987	18.8	49	74.9	90.3	96	98	98.8	99.3	99.8	100
1988	18.8	49	74.9	90.3	96	98	98.8	99.3	99.8	100
1989	18.8	49	74.9	90.3	96	98	98.8	99.3	99.8	
1990	18.8	49	74.9	90.3	96	98	98.8	99.3		
1991	18.8	49	74.9	90.3	96	98	98.8			
1992	18.8	49	74.9	90.3	96	98				
1993	18.8	49	74.9	90.3	96					
1994	18.8	49	74.9	90.3						
1995	8.6	28.6	48.6							
1996	8.6	28.6								
1997	8.6									

ACCIDENT YEAR	LINK FACTORS 5 TO 9	9 TO 13	13 TO 17	17 TO 21	21 TO 25	25 TO 29	29 TO 33	33 TO 37	37 TO 40
1986	2.606	1.529	1.206	1.063	1.021	1.008	1.005	1.005	1.002
1987	2.606	1.529	1.206	1.063	1.021	1.008	1.005	1.005	1.002
1988	2.606	1.529	1.206	1.063	1.021	1.008	1.005	1.005	1.002
1989	2.606	1.529	1.206	1.063	1.021	1.008	1.005	1.005	
1990	2.606	1.529	1.206	1.063	1.021	1.008	1.005		
1991	2.606	1.529	1.206	1.063	1.021	1.008			
1992	2.606	1.529	1.206	1.063	1.021				
1993	2.606	1.529	1.206	1.063					
1994	2.606	1.529	1.206						
1995	3.326	1.699							
1996	3.326								
1997	N/A								
3 YEAR AVERAGE	3.0858	1.5855	1.2056	1.0631	1.0208	1.0082	1.0051	1.0050	1.0020

CUMMULATIVE FROM:	5	9	13	17	21	25	29	33	37	40
TO ULTIMATE	6.5321	2.1168	1.3351	1.1074	1.0417	1.0204	1.0121	1.0070	1.0020	1.0000

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PAID LOSS DEVELOPMENT
BODILY INJURY COVERAGE
ACCIDENT YEAR

		Old Law	New Law	Example
15 months to ultimate	LDF(5,40)	5.319	8.140	6.532
27 months to ultimate	LDF(9,40)	2.041	2.448	2.117
39 months to ultimate	LDF(13,40)	1.335	1.440	1.335

BODILY INJURY COVERAGE

DEVELOPMENT OF RESERVE TO INCURRED RATIOS

ACCIDENT YEAR	AS OF: 12/93	3/94	6/94	9/94	12/94	3/95	6/95	9/95	12/95	3/96	6/96	9/96	12/96	3/97	6/97	9/97	12/97
1984	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1985	0.3	0.2	0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1986	0.8	0.7	0.5	0.4	0.3	0.2	0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1987	1.3	1.2	1.1	0.9	0.8	0.7	0.5	0.4	0.3	0.2	0.2	0.1	0.0	0.0	0.0	0.0	0.0
1988	2.5	2.0	1.7	1.5	1.3	1.2	1.1	0.9	0.8	0.7	0.5	0.4	0.3	0.2	0.2	0.1	0.0
1989	4.7	4.0	3.5	3.0	2.5	2.0	1.7	1.5	1.3	1.2	1.1	0.9	0.8	0.7	0.5	0.4	0.3
1990	12.7	9.7	7.4	5.9	4.7	4.0	3.5	3.0	2.5	2.0	1.7	1.5	1.3	1.2	1.1	0.9	0.8
1991	31.0	25.1	20.2	15.9	12.7	9.7	7.4	5.9	4.7	4.0	3.5	3.0	2.5	2.0	1.7	1.5	1.3
1992	57.9	51.0	44.1	37.4	31.0	25.1	20.2	15.9	12.7	9.7	7.4	5.9	4.7	4.0	3.5	3.0	2.5
1993	87.8	81.2	73.8	66.1	57.9	51.0	44.1	37.4	31.0	25.1	20.2	15.9	12.7	9.7	7.4	5.9	4.7
1994		24.7	47.8	68.8	87.8	81.2	73.8	66.1	57.9	51.0	44.1	37.4	31.0	25.1	20.2	15.9	12.7
1995						17.4	34.0	50.0	64.8	61.4	56.9	51.8	46.3	41.4	36.1	30.8	25.9
1996										17.4	34.0	50.0	64.8	61.4	56.9	51.8	46.3
1997														17.4	34.0	50.0	64.8
RESERVES:	198.7	199.6	200.1	199.9	199.0	192.5	166.5	181.2	176.0	172.7	169.6	166.9	164.4	163.1	161.6	160.3	159.3
INCURRED LOSSES:	100.0	100.0	100.0	100.0	100.0	92.5	85.0	77.5	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0
RESERVE TO INCURRED RATIO:	1.99	2.00	2.00	2.00	1.99	2.08	2.19	2.34	2.51	2.47	2.42	2.38	2.35	2.33	2.31	2.29	2.28

BODILY INJURY COVERAGE

DEVELOPMENT OF RESERVE TO INCURRED RATIOS

ACCIDENT YEAR	AS OF: 3/98	6/98	9/98	12/98	3/99	6/99	9/99	12/99	3/00	6/00	9/00	12/00	3/01	6/01	9/01	12/01
1987	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1988	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1989	0.2	0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1990	0.7	0.5	0.4	0.3	0.2	0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1991	1.2	1.1	0.9	0.8	0.7	0.5	0.4	0.3	0.2	0.2	0.1	0.0	0.0	0.0	0.0	0.0
1992	2.0	1.7	1.5	1.3	1.2	1.1	0.9	0.8	0.7	0.5	0.4	0.3	0.2	0.2	0.1	0.0
1993	4.0	3.5	3.0	2.5	2.0	1.7	1.5	1.3	1.2	1.1	0.9	0.8	0.7	0.5	0.4	0.3
1994	9.7	7.4	5.9	4.7	4.0	3.5	3.0	2.5	2.0	1.7	1.5	1.3	1.2	1.1	0.9	0.8
1995	21.4	17.4	13.9	11.2	8.5	6.5	5.2	4.1	3.5	3.2	2.8	2.4	2.0	1.7	1.5	1.3
1996	41.4	36.1	30.8	25.9	21.4	17.4	13.9	11.2	8.5	6.5	5.2	4.1	3.5	3.2	2.8	2.4
1997	61.4	56.9	51.8	46.3	41.4	36.1	30.8	25.9	21.4	17.4	13.9	11.2	8.5	6.5	5.2	4.1
1998	17.4	34.0	50.0	64.8	61.4	56.9	51.8	46.3	41.4	36.1	30.8	25.9	21.4	17.4	13.9	11.2
1999					17.4	34.0	50.0	64.8	61.4	56.9	51.8	46.3	41.4	36.1	30.8	25.9
2000									17.4	34.0	50.0	64.8	61.4	56.9	51.8	46.3
2001													17.4	34.0	50.0	64.8
RESERVES:	159.4	158.8	158.3	157.8	158.2	157.9	157.6	157.2	157.7	157.6	157.4	157.1	157.7	157.6	157.4	157.1
INCURRED LOSSES:	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0
RESERVE TO INCURRED RATIO:	2.28	2.27	2.26	2.25	2.26	2.26	2.25	2.25	2.25	2.25	2.25	2.24	2.25	2.25	2.25	2.24

BODILY INJURY LIABILITY
INVESTMENT INCOME MEASUREMENT
DISCOUNTED CASH FLOW OF THE POLICY TRANSACTION

EVALUATION	OLD LAW AMOUNT PAID	DISCOUNTED AMOUNT PAID	NEW LAW AMOUNT PAID	DISCOUNTED AMOUNT PAID	i = 6% DISCOUNT FACTOR
1	0.3	0.296	0.1	0.099	0.98554
2	1.9	1.845	0.9	0.874	0.97129
3	4.0	3.829	1.5	1.436	0.95724
4	6.0	5.660	2.7	2.547	0.94340
5	6.6	6.136	3.4	3.161	0.92975
6	7.4	6.781	4.5	4.123	0.91631
7	7.7	6.954	5.1	4.606	0.90306
8	8.2	7.298	5.5	4.895	0.89000
9	6.9	6.052	4.9	4.298	0.87713
10	6.9	5.965	5.3	4.582	0.86444
11	6.7	5.708	5.3	4.515	0.85194
12	6.4	5.374	4.9	4.114	0.83962
13	5.9	4.882	4.5	3.724	0.82748
14	4.9	3.996	4.0	3.262	0.81551
15	4.3	3.456	3.5	2.813	0.80372
16	3.2	2.535	2.7	2.139	0.79209
17	3.0	2.342	2.7	2.108	0.78064
18	2.3	1.770	2.0	1.539	0.76935
19	1.5	1.137	1.3	0.986	0.75822
20	1.2	0.897	1.1	0.822	0.74726
21	0.7	0.516	0.6	0.442	0.73645
22	0.5	0.363	0.3	0.218	0.72580
23	0.5	0.358	0.4	0.286	0.71531
24	0.5	0.352	0.4	0.282	0.70496
25	0.5	0.347	0.4	0.278	0.69477
26	0.3	0.205	0.3	0.205	0.68472
27	0.2	0.135	0.2	0.135	0.67482
28	0.2	0.133	0.2	0.133	0.66506
29	0.1	0.066	0.1	0.066	0.65544
30	0.1	0.065	0.1	0.065	0.64596
31	0.2	0.127	0.2	0.127	0.63662
32	0.1	0.063	0.1	0.063	0.62741
33	0.1	0.062	0.1	0.062	0.61834
34	0.2	0.122	0.2	0.122	0.60940
35	0.1	0.060	0.1	0.060	0.60058
36	0.1	0.059	0.1	0.059	0.59190
37	0.1	0.058	0.1	0.058	0.58334
38	0.0	0.000	0.0	0.000	0.57490
39	0.1	0.057	0.1	0.057	0.56659
40	0.1	0.056	0.1	0.056	0.55839
TOTAL	100	86.11	70	59.41	
PERCENT OF PREMIUM		13.89%		15.12%	

Private Passenger Auto
BODILY INJURY COVERAGE
Percent of Total Payments by Size of Loss and Time Until Payment

Time Until Payment in Months	Size of Loss Limits Lower / Upper	0 / 100	100 / 250	250 / 500	500 / 750	750 / 1,000	1,000 / 1,500	1,500 / 2,000	2,000 / 2,500	2,500 / 3,500	3,500 / 5,000	5,000 / 7,500	7,500 / 10,000	10,000 / 15,000
0 - 3		0.0000	0.0000	0.0001	0.0001	0.0001	0.0002	0.0001	0.0001	0.0002	0.0001	0.0001	0.0001	0.0004
3 - 6		0.0000	0.0000	0.0001	0.0000	0.0001	0.0002	0.0002	0.0003	0.0011	0.0017	0.0024	0.0022	0.0017
6 - 9		0.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0002	0.0003	0.0011	0.0038	0.0090	0.0055	0.0067
9 - 12		0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0003	0.0003	0.0013	0.0032	0.0116	0.0089	0.0098
12 - 15		0.0000	0.0000	0.0000	0.0000	0.0001	0.0003	0.0002	0.0003	0.0008	0.0030	0.0094	0.0091	0.0113
15 - 18		0.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0002	0.0003	0.0008	0.0031	0.0084	0.0085	0.0096
18 - 21		0.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0002	0.0002	0.0006	0.0022	0.0056	0.0078	0.0114
21 - 24		0.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0002	0.0004	0.0010	0.0030	0.0061	0.0076	0.0113
24 - 27		0.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0002	0.0001	0.0006	0.0018	0.0049	0.0053	0.0085
27 - 30		0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0002	0.0003	0.0006	0.0010	0.0034	0.0037	0.0086
30 - 33		0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0002	0.0004	0.0017	0.0034	0.0032	0.0058
33 - 36		0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0002	0.0005	0.0010	0.0031	0.0043	0.0073
36 - 39		0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0003	0.0008	0.0028	0.0041	0.0067
39 - 42		0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0002	0.0002	0.0008	0.0020	0.0021	0.0043
42 - 45		0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0004	0.0007	0.0017	0.0021	0.0038
45 - 48		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0002	0.0005	0.0010	0.0016	0.0024
48 - 51		0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0001	0.0001	0.0002	0.0009	0.0007	0.0013
51 - 54		0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0001	0.0002	0.0004	0.0003	0.0028
54 - 57		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0003	0.0004	0.0003	0.0010
57 - 60		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0002	0.0006
60 - 63		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0003	0.0003	0.0005
63 - 66		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0001	0.0005	0.0009
66 - 69		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0001	0.0001	0.0004	0.0003
69 - 72		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0000	0.0001	0.0004
72 - 75		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001
75 - 78		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0000	0.0001
78 - 81		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000
81 - 84		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0001
84 - 87		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000
87 - 90		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0003
90 - 93		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
93 - 96		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
96 - 99		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000

Time Until Payment in Months	Size of Loss Limits Lower / Upper: 0 / 100	100 / 250	250 / 500	500 / 750	750 / 1,000	1,000 / 1,500	1,500 / 2,000	2,000 / 2,500	2,500 / 3,500	3,500 / 5,000	5,000 / 7,500	7,500 / 10,000	10,000 / 15,000
99 - 102	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
102 - 105	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
105 - 108	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
108 - 111	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
111 - 114	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
114 - 117	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000
117 - 120	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
120 - 123	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000
123 - 126	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
126 - 129	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
129 - 132	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
132 - 135	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
135 - 138	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
138 - 141	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
141 - 144	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
144 - 147	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
147 - 150	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
150 - 153	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
153 - 156	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
156 - 159	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000
159 - 162	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
162 - 165	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
165 - 168	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
168 - 171	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
171 - 174	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
174 - 177	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
177 - 180	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
180 - 183	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
183 - 186	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
186 - 189	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
189 - 192	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
192 - 195	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
195 - 198	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
198 - 201	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
201 - 204	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
204 - 207	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
207 - 210	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
210 - 213	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
213 - 216	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
TOTAL	0.0000	0.0001	0.0005	0.0005	0.0011	0.0025	0.0026	0.0037	0.0106	0.0301	0.0782	0.0795	0.1186

APPENDIX 4
EXHIBIT 1
PAGE 1

POST-LAW CHANGE - TEST USING OTHER STATE
BODILY INJURY PAYMENT PATTERNS
ACCIDENT YEAR

(j) QUARTERS OF EVALUATION	(P) (1) AMOUNT PAID	(CP) (2) CUMULATIVE AMOUNT PAID	(R) (3) LOSS RESERVES
1	0.0	0.0	17.5
2	0.3	0.3	34.7
3	0.8	1.1	51.4
4	1.6	2.7	67.3
5	3.0	5.7	64.3
6	4.0	9.7	60.3
7	5.3	15.0	55.0
8	5.5	20.5	49.5
9	5.2	25.7	44.3
10	5.2	30.9	39.1
11	5.0	35.9	34.1
12	4.8	40.7	29.3
13	4.6	45.3	24.7
14	4.5	49.8	20.2
15	4.3	54.1	15.9
16	3.2	57.3	12.7
17	3.0	60.3	9.7
18	2.3	62.6	7.4
19	1.5	64.1	5.9
20	1.2	65.3	4.7
21	0.7	66.0	4.0
22	0.5	66.5	3.5
23	0.5	67.0	3.0
24	0.5	67.5	2.5
25	0.5	68.0	2.0
26	0.3	68.3	1.7
27	0.2	68.5	1.5
28	0.2	68.7	1.3
29	0.1	68.8	1.2
30	0.1	68.9	1.1
31	0.2	69.1	0.9
32	0.1	69.2	0.8
33	0.1	69.3	0.7
34	0.2	69.5	0.5
35	0.1	69.6	0.4
36	0.1	69.7	0.3
37	0.1	69.8	0.2
38	0.0	69.8	0.2
39	0.1	69.9	0.1
40	0.1	70.0	0.0

APPENDIX 4
EXHIBIT 2
PAGE 1

LAW CHANGE - APPLIED TO ALL OUTSTANDING POLICIES 1/1/95
BODILY INJURY PAID PURE PREMIUMS - TEST STATE
12 MONTH MOVING

12 MONTH MOVING ENDING		(CYP) PAID PURE PREMIUM
MARCH	1992	100.0
JUNE	1992	100.0
SEPTEMBER	1992	100.0
DECEMBER	1992	100.0
MARCH	1993	100.0
JUNE	1993	100.0
SEPTEMBER	1993	100.0
DECEMBER	1993	100.0
MARCH	1994	100.0
JUNE	1994	100.0
SEPTEMBER	1994	100.0
DECEMBER	1994	100.0
MARCH	1995	99.7
JUNE	1995	98.1
SEPTEMBER	1995	94.9
DECEMBER	1995	90.5
MARCH	1996	86.9
JUNE	1996	83.5
SEPTEMBER	1996	81.1
DECEMBER	1996	78.4
MARCH	1997	76.7
JUNE	1997	75.0
SEPTEMBER	1997	73.3
DECEMBER	1997	71.7
MARCH	1998	70.4
JUNE	1998	70.0
SEPTEMBER	1998	70.0
DECEMBER	1998	70.0
MARCH	1999	70.0
JUNE	1999	70.0
SEPTEMBER	1999	70.0
DECEMBER	1999	70.0
MARCH	2000	70.0
JUNE	2000	70.0
SEPTEMBER	2000	70.0
DECEMBER	2000	70.0
MARCH	2001	70.0
JUNE	2001	70.0
SEPTEMBER	2001	70.0
DECEMBER	2001	70.0
MARCH	2002	70.0
JUNE	2002	70.0
SEPTEMBER	2002	70.0
DECEMBER	2002	70.0
MARCH	2003	70.0
JUNE	2003	70.0
SEPTEMBER	2003	70.0
DECEMBER	2003	70.0

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LAW CHANGE - APPLIED TO ALL OUTSTANDING POLICIES 1/1/95
BODILY INJURY PAID PURE PREMIUMS - TEST STATE
SUMMARY OF 12 MONTH MOVING TRENDS

TRENDS ENDING		12 POINT	6 POINT
DECEMBER	1994	-0.0%	0.0%
MARCH	1995	-0.0%	-0.2%
JUNE	1995	-0.3%	-1.2%
SEPTEMBER	1995	-1.1%	-3.6%
DECEMBER	1995	-2.4%	-7.4%
MARCH	1996	-4.0%	-11.1%
JUNE	1996	-5.8%	-13.8%
SEPTEMBER	1996	-7.5%	-14.6%
DECEMBER	1996	-9.1%	-14.0%
MARCH	1997	-10.4%	-12.5%
JUNE	1997	-11.3%	-11.1%
SEPTEMBER	1997	-11.9%	-9.9%
DECEMBER	1997	-11.9%	-9.2%
MARCH	1998	-11.4%	-8.4%
JUNE	1998	-10.4%	-7.4%
SEPTEMBER	1998	-9.1%	-5.6%
DECEMBER	1998	-7.7%	-3.5%
MARCH	1999	-6.4%	-1.6%
JUNE	1999	-5.1%	-0.3%
SEPTEMBER	1999	-3.9%	0.0%
DECEMBER	1999	-2.9%	0.0%
MARCH	2000	-1.9%	0.0%
JUNE	2000	-1.1%	0.0%
SEPTEMBER	2000	-0.4%	0.0%
DECEMBER	2000	-0.1%	0.0%
MARCH	2001	0.0%	0.0%
JUNE	2001	0.0%	0.0%
SEPTEMBER	2001	0.0%	0.0%
DECEMBER	2001	0.0%	0.0%
MARCH	2002	0.0%	0.0%
JUNE	2002	0.0%	0.0%
SEPTEMBER	2002	0.0%	0.0%
DECEMBER	2002	0.0%	0.0%
MARCH	2003	0.0%	0.0%
JUNE	2003	0.0%	0.0%
SEPTEMBER	2003	0.0%	0.0%
DECEMBER	2003	0.0%	0.0%

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PAID LOSS DEVELOPMENT
POST-LAW CHANGE
BODILY INJURY LIABILITY - TEST STATE

ACCIDENT	EVALUATION									
YEAR	5	9	13	17	21	25	29	33	37	40
1995	5.7	25.7	45.3	60.3	66	68	68.8	69.3	69.8	70
1996	5.7	25.7	45.3	60.3	66	68	68.8	69.3	69.8	70
1997	5.7	25.7	45.3	60.3	66	68	68.8	69.3	69.8	70
1998	5.7	25.7	45.3	60.3	66	68	68.8	69.3	69.8	
1999	5.7	25.7	45.3	60.3	66	68	68.8	69.3		
2000	5.7	25.7	45.3	60.3	66	68	68.8			
2001	5.7	25.7	45.3	60.3	66	68				
2002	5.7	25.7	45.3	60.3	66					
2003	5.7	25.7	45.3	60.3						
2004	5.7	25.7	45.3							
2005	5.7	25.7								
2006	5.7									

ACCIDENT	LINK FACTORS								
YEAR	5 TO 9	9 TO 13	13 TO 17	17 TO 21	21 TO 25	25 TO 29	29 TO 33	33 TO 37	37 TO 40
1995	4.509	1.763	1.331	1.095	1.030	1.012	1.007	1.007	1.003
1996	4.509	1.763	1.331	1.095	1.030	1.012	1.007	1.007	1.003
1997	4.509	1.763	1.331	1.095	1.030	1.012	1.007	1.007	1.003
1998	4.509	1.763	1.331	1.095	1.030	1.012	1.007	1.007	
1999	4.509	1.763	1.331	1.095	1.030	1.012	1.007		
2000	4.509	1.763	1.331	1.095	1.030	1.012			
2001	4.509	1.763	1.331	1.095	1.030				
2002	4.509	1.763	1.331	1.095					
2003	4.509	1.763	1.331						
2004	4.509	1.763							
2005	4.509								
2006	N/A								
3 YEAR AVERAGE	4.5088	1.7626	1.3311	1.0945	1.0303	1.0118	1.0073	1.0072	1.0029

CUMMULATIVE										
FROM:	5	9	13	17	21	25	29	33	37	40
TO ULTIMATE	12.2807	2.7237	1.5463	1.1609	1.0606	1.0294	1.0174	1.0101	1.0029	1.0000

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PAID LOSS DEVELOPMENT
BODILY INJURY - TEST STATE
ACCIDENT YEAR

		Old Law	New Law
15 months to ultimate	LDF(5,40)	5.319	12.281
27 months to ultimate	LDF(9,40)	2.041	2.724
39 months to ultimate	LDF(13,40)	1.335	1.545

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PAGE 1

BODILY INJURY COVERAGE - TEST STATE

DEVELOPMENT OF RESERVE TO INCURRED RATIOS

ACCIDENT YEAR	AS OF 12/93	3/94	6/94	9/94	12/94	3/95	6/95	9/95	12/95	3/96	6/96	9/96	12/96	3/97	6/97	9/97	12/97
1984	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1985	0.3	0.2	0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1986	0.8	0.7	0.5	0.4	0.3	0.2	0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1987	1.3	1.2	1.1	0.9	0.8	0.7	0.5	0.4	0.3	0.2	0.2	0.1	0.0	0.0	0.0	0.0	0.0
1988	2.5	2.0	1.7	1.5	1.3	1.2	1.1	0.9	0.8	0.7	0.5	0.4	0.3	0.2	0.2	0.1	0.0
1989	4.7	4.0	3.5	3.0	2.5	2.0	1.7	1.5	1.3	1.2	1.1	0.9	0.8	0.7	0.5	0.4	0.3
1990	12.7	9.7	7.4	5.9	4.7	4.0	3.5	3.0	2.5	2.0	1.7	1.5	1.3	1.2	1.1	0.9	0.8
1991	31.0	25.1	20.2	15.9	12.7	9.7	7.4	5.9	4.7	4.0	3.5	3.0	2.5	2.0	1.7	1.5	1.3
1992	57.9	51.0	44.1	37.4	31.0	25.1	20.2	15.9	12.7	9.7	7.4	5.9	4.7	4.0	3.5	3.0	2.5
1993	87.8	81.2	73.8	66.1	57.9	51.0	44.1	37.4	31.0	25.1	20.2	15.9	12.7	9.7	7.4	5.9	4.7
1994		24.7	47.8	68.8	87.8	81.2	73.8	66.1	57.9	51.0	44.1	37.4	31.0	25.1	20.2	15.9	12.7
1995						17.5	34.7	51.4	67.3	64.3	60.3	55.0	49.5	44.3	39.1	34.1	29.3
1996										17.5	34.7	51.4	67.3	64.3	60.3	55.0	49.5
1997														17.5	34.7	51.4	67.3
RESERVES	198.7	199.6	200.1	199.9	199.0	192.6	187.2	182.6	178.5	175.7	173.7	171.5	170.1	169.0	168.7	168.2	168.4
INCURRED LOSSES:	100.0	100.0	100.0	100.0	100.0	92.5	85.0	77.5	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0
RESERVE TO INCURRED RATIO:	1.99	2.00	2.00	2.00	1.99	2.08	2.20	2.36	2.55	2.51	2.48	2.45	2.43	2.41	2.41	2.40	2.41

BODILY INJURY COVERAGE - TEST STATE

DEVELOPMENT OF RESERVE TO INCURRED RATIOS

ACCIDENT YEAR	AS OF 3/98	6/98	9/98	12/98	3/99	6/99	9/99	12/99	3/00	6/00	9/00	12/00	3/01	6/01	9/01	12/01
1987	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1988	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1989	0.2	0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1990	0.7	0.5	0.4	0.3	0.2	0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1991	1.2	1.1	0.9	0.8	0.7	0.5	0.4	0.3	0.2	0.2	0.1	0.0	0.0	0.0	0.0	0.0
1992	2.0	1.7	1.5	1.3	1.2	1.1	0.9	0.8	0.7	0.5	0.4	0.3	0.2	0.2	0.1	0.0
1993	4.0	3.5	3.0	2.5	2.0	1.7	1.5	1.3	1.2	1.1	0.9	0.8	0.7	0.5	0.4	0.3
1994	9.7	7.4	5.9	4.7	4.0	3.5	3.0	2.5	2.0	1.7	1.5	1.3	1.2	1.1	0.9	0.8
1995	24.7	20.2	15.9	12.7	9.7	7.4	5.9	4.7	4.0	3.5	3.0	2.5	2.0	1.7	1.5	1.3
1996	44.3	39.1	34.1	29.3	24.7	20.2	15.9	12.7	9.7	7.4	5.9	4.7	4.0	3.5	3.0	2.5
1997	64.3	60.3	55.0	49.5	44.3	39.1	34.1	29.3	24.7	20.2	15.9	12.7	9.7	7.4	5.9	4.7
1998	17.5	34.7	51.4	67.3	64.3	60.3	55.0	49.5	44.3	39.1	34.1	29.3	24.7	20.2	15.9	12.7
1999					17.5	34.7	51.4	67.3	64.3	60.3	55.0	49.5	44.3	39.1	34.1	29.3
2000									17.5	34.7	51.4	67.3	64.3	60.3	55.0	49.5
2001													17.5	34.7	51.4	67.3
RESERVES:	168.6	168.7	168.2	168.4	168.6	168.7	168.2	168.4	168.6	168.7	168.2	168.4	168.6	168.7	168.2	168.4
INCURRED LOSSES:	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0
RESERVE TO INCURRED RATIO:	2.41	2.41	2.40	2.41	2.41	2.41	2.40	2.41	2.41	2.41	2.40	2.41	2.41	2.41	2.40	2.41

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BODILY INJURY LIABILITY - TEST STATE
INVESTMENT INCOME MEASUREMENT
DISCOUNTED CASH FLOW OF THE POLICY TRANSACTION

EVALUATION	OLD LAW AMOUNT PAID	DISCOUNTED AMOUNT PAID	NEW LAW AMOUNT PAID	DISCOUNTED AMOUNT PAID	i = 6% DISCOUNT FACTOR
1	0.3	0.296	0.0	0.000	0.98554
2	1.9	1.845	0.3	0.291	0.97129
3	4.0	3.829	0.8	0.766	0.95724
4	6.0	5.660	1.6	1.509	0.94340
5	6.6	6.136	3.0	2.789	0.92975
6	7.4	6.781	4.0	3.665	0.91631
7	7.7	6.954	5.3	4.786	0.90306
8	8.2	7.298	5.5	4.895	0.89000
9	6.9	6.052	5.2	4.561	0.87713
10	6.9	5.965	5.2	4.495	0.86444
11	6.7	5.708	5.0	4.260	0.85194
12	6.4	5.374	4.8	4.030	0.83962
13	5.9	4.882	4.6	3.806	0.82748
14	4.9	3.996	4.5	3.670	0.81551
15	4.3	3.456	4.3	3.456	0.80372
16	3.2	2.535	3.2	2.535	0.79209
17	3.0	2.342	3.0	2.342	0.78064
18	2.3	1.770	2.3	1.770	0.76935
19	1.5	1.137	1.5	1.137	0.75822
20	1.2	0.897	1.2	0.897	0.74726
21	0.7	0.516	0.7	0.516	0.73645
22	0.5	0.363	0.5	0.363	0.72580
23	0.5	0.358	0.5	0.358	0.71531
24	0.5	0.352	0.5	0.352	0.70496
25	0.5	0.347	0.5	0.347	0.69477
26	0.3	0.205	0.3	0.205	0.68472
27	0.2	0.135	0.2	0.135	0.67482
28	0.2	0.133	0.2	0.133	0.66506
29	0.1	0.066	0.1	0.066	0.65544
30	0.1	0.065	0.1	0.065	0.64596
31	0.2	0.127	0.2	0.127	0.63662
32	0.1	0.063	0.1	0.063	0.62741
33	0.1	0.062	0.1	0.062	0.61834
34	0.2	0.122	0.2	0.122	0.60940
35	0.1	0.060	0.1	0.060	0.60058
36	0.1	0.059	0.1	0.059	0.59190
37	0.1	0.058	0.1	0.058	0.58334
38	0.0	0.000	0.0	0.000	0.57490
39	0.1	0.057	0.1	0.057	0.56659
40	0.1	0.056	0.1	0.056	0.55839
TOTAL	100	86.11	70	58.86	
PERCENT		13.89%		15.91%	

POST-LAW CHANGE - CHOICE NO-FAULT
BODILY INJURY PAYMENT PATTERNS
ACCIDENT YEAR

(j) QUARTERS OF EVALUATION	(P) (1) AMOUNT PAID	(CP) (2) CUMULATIVE AMOUNT PAID	(R) (3) LOSS RESERVES
1	0.1	0.1	18.9
2	0.6	0.7	37.3
3	1.4	2.1	54.9
4	2.5	4.6	71.4
5	3.7	8.3	67.7
6	4.7	13.0	63.0
7	5.8	18.8	57.2
8	6.0	24.8	51.2
9	5.5	30.4	45.6
10	5.5	35.9	40.1
11	5.3	41.2	34.8
12	5.1	46.4	29.6
13	4.9	51.2	24.8
14	4.6	55.8	20.2
15	4.3	60.1	15.9
16	3.2	63.3	12.7
17	3.0	66.3	9.7
18	2.3	68.6	7.4
19	1.5	70.1	5.9
20	1.2	71.3	4.7
21	0.7	72.0	4.0
22	0.5	72.5	3.5
23	0.5	73.0	3.0
24	0.5	73.5	2.5
25	0.5	74.0	2.0
26	0.3	74.3	1.7
27	0.2	74.5	1.5
28	0.2	74.7	1.3
29	0.1	74.8	1.2
30	0.1	74.9	1.1
31	0.2	75.1	0.9
32	0.1	75.2	0.8
33	0.1	75.3	0.7
34	0.2	75.5	0.5
35	0.1	75.6	0.4
36	0.1	75.7	0.3
37	0.1	75.8	0.2
38	0.0	75.8	0.2
39	0.1	75.9	0.1
40	0.1	76.0	0.0

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EXHIBIT 2
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LAW CHANGE - APPLIED TO ALL OUTSTANDING POLICIES 1/1/95
BODILY INJURY CHOICE NO-FAULT PAID PURE PREMIUMS
12 MONTH MOVING

12 MONTH MOVING ENDING		(CYP) PAID PURE PREMIUM
MARCH	1992	100.0
JUNE	1992	100.0
SEPTEMBER	1992	100.0
DECEMBER	1992	100.0
MARCH	1993	100.0
JUNE	1993	100.0
SEPTEMBER	1993	100.0
DECEMBER	1993	100.0
MARCH	1994	100.0
JUNE	1994	100.0
SEPTEMBER	1994	100.0
DECEMBER	1994	100.0
MARCH	1995	99.8
JUNE	1995	98.5
SEPTEMBER	1995	95.9
DECEMBER	1995	92.4
MARCH	1996	89.5
JUNE	1996	86.8
SEPTEMBER	1996	84.9
DECEMBER	1996	82.7
MARCH	1997	81.4
JUNE	1997	80.0
SEPTEMBER	1997	78.6
DECEMBER	1997	77.4
MARCH	1998	76.3
JUNE	1998	76.0
SEPTEMBER	1998	76.0
DECEMBER	1998	76.0
MARCH	1999	76.0
JUNE	1999	76.0
SEPTEMBER	1999	76.0
DECEMBER	1999	76.0
MARCH	2000	76.0
JUNE	2000	76.0
SEPTEMBER	2000	76.0
DECEMBER	2000	76.0
MARCH	2001	76.0
JUNE	2001	76.0
SEPTEMBER	2001	76.0
DECEMBER	2001	76.0
MARCH	2002	76.0
JUNE	2002	76.0
SEPTEMBER	2002	76.0
DECEMBER	2002	76.0
MARCH	2003	76.0
JUNE	2003	76.0
SEPTEMBER	2003	76.0
DECEMBER	2003	76.0

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PAGE 2

LAW CHANGE - APPLIED TO ALL OUTSTANDING POLICIES 1/1/95
BODILY INJURY CHOICE NO-FAULT PAID PURE PREMIUMS
SUMMARY OF 12 MONTH MOVING TRENDS

TRENDS ENDING		12 POINT	6 POINT
DECEMBER	1994	-0.0%	0.0%
MARCH	1995	-0.0%	-0.1%
JUNE	1995	-0.3%	-1.0%
SEPTEMBER	1995	-0.9%	-2.9%
DECEMBER	1995	-1.9%	-5.9%
MARCH	1996	-3.2%	-8.8%
JUNE	1996	-4.6%	-11.0%
SEPTEMBER	1996	-5.9%	-11.6%
DECEMBER	1996	-7.2%	-11.1%
MARCH	1997	-8.2%	-9.7%
JUNE	1997	-8.9%	-8.6%
SEPTEMBER	1997	-9.3%	-7.6%
DECEMBER	1997	-9.3%	-7.0%
MARCH	1998	-8.9%	-6.3%
JUNE	1998	-8.0%	-5.5%
SEPTEMBER	1998	-7.0%	-4.2%
DECEMBER	1998	-5.9%	-2.6%
MARCH	1999	-4.8%	-1.2%
JUNE	1999	-3.9%	-0.2%
SEPTEMBER	1999	-2.9%	0.0%
DECEMBER	1999	-2.1%	0.0%
MARCH	2000	-1.4%	0.0%
JUNE	2000	-0.8%	0.0%
SEPTEMBER	2000	-0.3%	0.0%
DECEMBER	2000	-0.1%	0.0%
MARCH	2001	-0.0%	0.0%
JUNE	2001	-0.0%	0.0%
SEPTEMBER	2001	-0.0%	0.0%
DECEMBER	2001	-0.0%	0.0%
MARCH	2002	-0.0%	0.0%
JUNE	2002	-0.0%	0.0%
SEPTEMBER	2002	-0.0%	0.0%
DECEMBER	2002	-0.0%	0.0%
MARCH	2003	-0.0%	0.0%
JUNE	2003	-0.0%	0.0%
SEPTEMBER	2003	-0.0%	0.0%
DECEMBER	2003	-0.0%	0.0%

PAID LOSS DEVELOPMENT
POST-LAW CHANGE
BODILY INJURY LIABILITY - CHOICE NO-FAULT

ACCIDENT YEAR	EVALUATION 5	9	13	17	21	25	29	33	37	40
1995	8.32	30.36	51.22	66.3	72	74	74.8	75.3	75.8	76
1996	8.32	30.36	51.22	66.3	72	74	74.8	75.3	75.8	76
1997	8.32	30.36	51.22	66.3	72	74	74.8	75.3	75.8	76
1998	8.32	30.36	51.22	66.3	72	74	74.8	75.3	75.8	
1999	8.32	30.36	51.22	66.3	72	74	74.8	75.3		
2000	8.32	30.36	51.22	66.3	72	74	74.8			
2001	8.32	30.36	51.22	66.3	72	74				
2002	8.32	30.36	51.22	66.3	72					
2003	8.32	30.36	51.22	66.3						
2004	8.32	30.36	51.22							
2005	8.32	30.36								
2006	8.32									

ACCIDENT YEAR	LINK FACTORS 5 TO 9	9 TO 13	13 TO 17	17 TO 21	21 TO 25	25 TO 29	29 TO 33	33 TO 37	37 TO 40
1995	3.649	1.687	1.294	1.086	1.028	1.011	1.007	1.007	1.003
1996	3.649	1.687	1.294	1.086	1.028	1.011	1.007	1.007	1.003
1997	3.649	1.687	1.294	1.086	1.028	1.011	1.007	1.007	1.003
1998	3.649	1.687	1.294	1.086	1.028	1.011	1.007	1.007	
1999	3.649	1.687	1.294	1.086	1.028	1.011	1.007		
2000	3.649	1.687	1.294	1.086	1.028	1.011			
2001	3.649	1.687	1.294	1.086	1.028				
2002	3.649	1.687	1.294	1.086					
2003	3.649	1.687	1.294						
2004	3.649	1.687							
2005	3.649								
2006	N/A								
3 YEAR AVERAGE	3.6490	1.6871	1.2944	1.0860	1.0278	1.0108	1.0067	1.0066	1.0026

CUMMULATIVE FROM:	5	9	13	17	21	25	29	33	37	40
TO ULTIMATE	9.1346	2.5033	1.4838	1.1463	1.0556	1.0270	1.0160	1.0093	1.0026	1.0000

APPENDIX 5
EXHIBIT 3
PAGE 2

PAID LOSS DEVELOPMENT
BODILY INJURY - CHOICE NO-FAULT
ACCIDENT YEAR

		Old Law	New Law
15 months to ultimate	LDF(5,40)	5.319	9.135
27 months to ultimate	LDF(9,40)	2.041	2.503
39 months to ultimate	LDF(13,40)	1.335	1.484

APPENDIX 5
EXHIBIT 4
PAGE 1

BODILY INJURY COVERAGE - CHOICE NO-FAULT
DEVELOPMENT OF RESERVE TO INCURRED RATIOS

ACCIDENT YEAR	AS OF: 12/93	3/94	6/94	9/94	12/94	3/95	6/95	9/95	12/95	3/96	6/96	9/96	12/96	3/97	6/97	9/97	12/97
1984	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1985	0.3	0.2	0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1986	0.8	0.7	0.5	0.4	0.3	0.2	0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1987	1.3	1.2	1.1	0.9	0.8	0.7	0.5	0.4	0.3	0.2	0.2	0.1	0.0	0.0	0.0	0.0	0.0
1988	2.5	2.0	1.7	1.5	1.3	1.2	1.1	0.9	0.8	0.7	0.5	0.4	0.3	0.2	0.2	0.1	0.0
1989	4.7	4.0	3.5	3.0	2.5	2.0	1.7	1.5	1.3	1.2	1.1	0.9	0.8	0.7	0.5	0.4	0.3
1990	12.7	9.7	7.4	5.9	4.7	4.0	3.5	3.0	2.5	2.0	1.7	1.5	1.3	1.2	1.1	0.9	0.8
1991	31.0	25.1	20.2	15.9	12.7	9.7	7.4	5.9	4.7	4.0	3.5	3.0	2.5	2.0	1.7	1.5	1.3
1992	57.9	51.0	44.1	37.4	31.0	25.1	20.2	15.9	12.7	9.7	7.4	5.9	4.7	4.0	3.5	3.0	2.5
1993	87.8	81.2	73.8	66.1	57.9	51.0	44.1	37.4	31.0	25.1	20.2	15.9	12.7	9.7	7.4	5.9	4.7
1994		24.7	47.8	68.8	87.8	81.2	73.8	66.1	57.9	51.0	44.1	37.4	31.0	25.1	20.2	15.9	12.7
1995						18.9	37.3	54.9	71.4	67.7	63.0	57.2	51.2	45.6	40.1	34.8	29.6
1996										18.9	37.3	54.9	71.4	67.7	63.0	57.2	51.2
1997														18.9	37.3	54.9	71.4
RESERVES:	198.7	199.6	200.1	199.9	199.0	194.0	189.8	186.1	182.6	180.5	179.0	177.2	175.9	175.2	175.0	174.6	174.5
INCURRED LOSSES:	100.0	100.0	100.0	100.0	100.0	94.0	88.0	82.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0
RESERVE TO INCURRED RATIO:	1.99	2.00	2.00	2.00	1.99	2.06	2.16	2.27	2.40	2.38	2.36	2.33	2.31	2.30	2.30	2.30	2.30

BODILY INJURY COVERAGE - CHOICE NO-FAULT
DEVELOPMENT OF RESERVE TO INCURRED RATIOS

ACCIDENT YEAR	AS OF: 3/98	6/98	9/98	12/98	3/99	6/99	9/99	12/99	3/00	6/00	9/00	12/00	3/01	6/01	9/01	12/01
1987	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1988	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1989	0.2	0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1990	0.7	0.5	0.4	0.3	0.2	0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1991	1.2	1.1	0.9	0.8	0.7	0.5	0.4	0.3	0.2	0.2	0.1	0.0	0.0	0.0	0.0	0.0
1992	2.0	1.7	1.5	1.3	1.2	1.1	0.9	0.8	0.7	0.5	0.4	0.3	0.2	0.2	0.1	0.0
1993	4.0	3.5	3.0	2.5	2.0	1.7	1.5	1.3	1.2	1.1	0.9	0.8	0.7	0.5	0.4	0.3
1994	9.7	7.4	5.9	4.7	4.0	3.5	3.0	2.5	2.0	1.7	1.5	1.3	1.2	1.1	0.9	0.8
1995	24.8	20.2	15.9	12.7	9.7	7.4	5.9	4.7	4.0	3.5	3.0	2.5	2.0	1.7	1.5	1.3
1996	45.6	40.1	34.8	29.6	24.8	20.2	15.9	12.7	9.7	7.4	5.9	4.7	4.0	3.5	3.0	2.5
1997	67.7	63.0	57.2	51.2	45.6	40.1	34.8	29.6	24.8	20.2	15.9	12.7	9.7	7.4	5.9	4.7
1998	18.9	37.3	54.9	71.4	67.7	63.0	57.2	51.2	45.6	40.1	34.8	29.6	24.8	20.2	15.9	12.7
1999					18.9	37.3	54.9	71.4	67.7	63.0	57.2	51.2	45.6	40.1	34.8	29.6
2000									18.9	37.3	54.9	71.4	67.7	63.0	57.2	51.2
2001													18.9	37.3	54.9	71.4
RESERVES:	174.8	175.0	174.6	174.5	174.8	175.0	174.6	174.5	174.8	175.0	174.6	174.5	174.8	175.0	174.6	174.5
INCURRED LOSSES:	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0	76.0
RESERVE TO INCURRED RATIO:	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30

BODILY INJURY LIABILITY - CHOICE NO-FAULT
INVESTMENT INCOME MEASUREMENT
DISCOUNTED CASH FLOW OF THE POLICY TRANSACTION

EVALUATION	OLD LAW AMOUNT PAID	DISCOUNTED AMOUNT PAID	NEW LAW AMOUNT PAID	DISCOUNTED AMOUNT PAID	i = 6% DISCOUNT FACTOR
1	0.3	0.296	0.1	0.059	0.98554
2	1.9	1.845	0.6	0.602	0.97129
3	4.0	3.829	1.4	1.378	0.95724
4	6.0	5.660	2.5	2.340	0.94340
5	6.6	6.136	3.7	3.459	0.92975
6	7.4	6.781	4.7	4.288	0.91631
7	7.7	6.954	5.8	5.220	0.90306
8	8.2	7.298	6.0	5.376	0.89000
9	6.9	6.052	5.5	4.859	0.87713
10	6.9	5.965	5.5	4.789	0.86444
11	6.7	5.708	5.3	4.549	0.85194
12	6.4	5.374	5.1	4.299	0.83962
13	5.9	4.882	4.9	4.022	0.82748
14	4.9	3.996	4.6	3.735	0.81551
15	4.3	3.456	4.3	3.456	0.80372
16	3.2	2.535	3.2	2.535	0.79209
17	3.0	2.342	3.0	2.342	0.78064
18	2.3	1.770	2.3	1.770	0.76935
19	1.5	1.137	1.5	1.137	0.75822
20	1.2	0.897	1.2	0.897	0.74726
21	0.7	0.516	0.7	0.516	0.73645
22	0.5	0.363	0.5	0.363	0.72580
23	0.5	0.358	0.5	0.358	0.71531
24	0.5	0.352	0.5	0.352	0.70496
25	0.5	0.347	0.5	0.347	0.69477
26	0.3	0.205	0.3	0.205	0.68472
27	0.2	0.135	0.2	0.135	0.67482
28	0.2	0.133	0.2	0.133	0.66506
29	0.1	0.066	0.1	0.066	0.65544
30	0.1	0.065	0.1	0.065	0.64596
31	0.2	0.127	0.2	0.127	0.63662
32	0.1	0.063	0.1	0.063	0.62741
33	0.1	0.062	0.1	0.062	0.61834
34	0.2	0.122	0.2	0.122	0.60940
35	0.1	0.060	0.1	0.060	0.60058
36	0.1	0.059	0.1	0.059	0.59190
37	0.1	0.058	0.1	0.058	0.58334
38	0.0	0.000	0.0	0.000	0.57490
39	0.1	0.057	0.1	0.057	0.56659
40	0.1	0.056	0.1	0.056	0.55839
TOTAL	100	86.11	76	64.31	
PERCENT		13.89%		15.38%	

PRICING AUTO NO-FAULT AND BODILY INJURY LIABILITY COVERAGES USING MICRO-DATA AND STATISTICAL MODELS

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Pricing Auto No-Fault and Bodily Injury Liability Coverages Using Micro-Data and Statistical Models

Abstract

Private Passenger Automobile Bodily Injury (BI) Liability Insurance, the largest subline of property-casualty insurance in the United States, has experienced during the 1980's rapidly increasing claim costs well in excess of the rate of overall inflation. The re-emergence of BI as a problem area has spotlighted traditional tort, no-fault and choice systems as competing vehicles for cost containment. Our purpose is to describe the current BI systems and to provide new methods based on micro-data and statistical models for pricing those systems. We build on the results of a major industrywide data gathering and research effort in Massachusetts. We observe that data on claimants, rather than on insureds, are critical for understanding BI systems and for supporting the least-squares, logistic and Tobit regression models for pricing alternative BI systems. The paper concludes with three applications: changing a monetary threshold, supplementing a trend factor and coordinating benefits with health insurance.

Disclaimer

The opinions expressed by the authors are solely their own and are not attributable to the Automobile Insurers Bureau of Massachusetts or its member companies.

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Pricing Auto No-Fault and Bodily Injury Liability Coverages Using Micro-Data and Statistical Models

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Pricing Auto No-Fault and Bodily Injury Liability Coverages Using Micro-Data and Statistical Models

1.0 INTRODUCTION

Private passenger automobile liability insurance, with earned premiums of $47 billion in 1990, is the largest line of property-casualty insurance in the United States.[1] Workers compensation with $31 billion ranks a distant second. Bodily injury (BI) coverage for injury to people rather than damage to vehicles accounts for about 75% of the total liability premium, still somewhat larger than workers compensation premium.[2]

By the 1960's, dissatisfaction with the cost and efficiency of the traditional tort system had led to significant reforms in many states. Variations of the no-fault concept were implemented widely between 1970 and 1976. Most BI systems have remained quite stable since that time. Recently, however, the bodily injury coverage has re-emerged as a serious problem area. Calls for cost containment and reform are increasingly being echoed (Cummins and Tennyson, 1992; Feldblum, 1990; Foppert, 1992, Maatman, 1989; Weisberg and Derrig, 1992c).

While questions about relative costs of alternative proposals will inevitably be asked of actuaries, the available analytic tools may be of limited utility. Our purpose here is to describe the current BI systems and to offer some new approaches to pricing those systems. We begin with some background on current BI reform proposals and traditional actuarial methods. We then turn to the need for data to support adequate models of the BI process. Section 2 provides

[1]Best's Aggregates and Averages, 1991 Property-Casualty Edition, p.156.

[2]Countrywide liability incurred losses for 1986-88 show about 77% BI, 23% PDL for voluntary markets (NAIC database, 1991). Earned premium can be assumed to be in approximately the same proportions.

an explanation of various BI claim processes. In Section 3 we outline a pricing methodology based upon micro-data and statistical models. We build on the results of a major industrywide data collection and research effort by the Automobile Insurers Bureau (AIB) and Correlation Research Inc. (CRI) in Massachusetts (Weisberg and Derrig (1991a, 1992a); Feldblum, 1991). The important issue of incorporating behavioral assumptions in the pricing of BI changes is described in Section 4. The methodology is illustrated with three examples from current Massachusetts experience in Section 5. Concluding remarks in Section 6 unify the perspectives addressed in the paper.

1.1 Background for Current BI System Reforms

Cummins and Tennyson (1992) point out that between 1984 and 1989 BI losses grew at an annual rate of nearly nine percent in no-fault states and eleven percent in tort states, despite annual declines of about two percent in property damage liability claims. This phenomenon, particularly pronounced in urban areas, is attributed to changes in "claiming behavior" rather than to real trends in accident frequency or severity. It appears that in some areas of the country slightly injured (or even uninjured) claimants have become increasingly willing to file claims.

The specific nature of the problem is influenced by the kind of tort system in place. Kimball (1985) provides a brief history of the legal principles underlying modern automobile accident law, starting with the Roman law of obligations. Until 1970, the automobile injury compensation system was exclusively concerned with "righting wrongs" through the tort system.[3] It was necessary for an injured plaintiff to show that a defendant was at fault, careless or negligent before compensation could be compelled. Automobile liability insurance provided a

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[3]According to Kimball, the law of torts is concerned with straightening out twisted ("tortum") relationships.

reasonably efficient mechanism to *allocate* the costs of this tort system among drivers. However, concerns with the overall high costs of the tort system, especially transaction costs in terms of legal fees and delayed payments, led to experiments that modified the tort system by relaxing the fault requirement. Of course, the tort system had been completely eliminated fifty years earlier for workplace accidents by the workers compensation insurance system.

So-called "no-fault" systems that limit the right to sue in exchange for some form of guaranteed first-party reimbursement are often justified in part as a cost-saving measure. By restricting the eligibility to file a tort claim to those whose injuries cross a specified "threshold" of severity, these systems are intended to eliminate payment of general damages (pain and suffering) for minor injuries and to reduce transaction costs. At the present time, there are fourteen states in which all drivers are covered by some form of no-fault insurance (IRC, 1990). In eight of these states, the tort threshold is defined as a monetary amount of medical expenses. In three (Florida, Michigan, and New York) the threshold is a verbal specification of what constitutes a serious injury. In addition, three states (Kentucky, New Jersey, and Pennsylvania) have adopted "choice" systems in which drivers can choose between the traditional tort system and a variant of no-fault.[4] Choice systems have been the focus of much attention since first proposed by O'Connell and Joost (1986).

Witt and Urritia (1983) have analyzed the advantages and disadvantages of various no-fault systems adopted by 24 states between 1971 (Massachusetts) and 1976 (North Dakota). Using *Best's* loss ratio data by state, 1975-1980, these researchers found that no-fault systems produced higher relative benefits per dollar of premium. Underwriting risk to the insurer, as

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[4]Since the IRC publication, Georgia has returned to a full tort system while Pennsylvania changed to a choice state (Powers, 1992).

measured by the standard deviation of state loss ratios, was higher in no-fault states but generally reflected state-specific factors other than the no-fault system of compensation.

The record of no-fault systems in *controlling*, as opposed to allocating, total costs has been mixed. Michigan and New York, with their strong (i.e., stringent) verbal thresholds, have achieved significant savings in BI costs. However, Florida's weak verbal threshold has proved relatively easy to circumvent (Maroney, Hill, and Norman, 1991), and premiums in monetary-threshold states are generally higher than in pure tort states (Cummins and Weiss, 1991).

O'Connell and Joost were motivated to suggest the choice approach primarily because of the failure of weak no-fault laws to control claim costs and the political difficulties of imposing strong verbal thresholds. In their view, the politically more palatable compromises reflected in most existing no-fault systems merely exacerbate the cost problems by creating perverse economic incentives:

> ....no-fault thresholds arguably encourage victims to inflate their claims to exceed the threshold for bringing a lawsuit. Moreover, the more medical expenses and wage losses victims accumulate, within limits, the more they can recover in tort for both economic and noneconomic losses...Permitting victims to profit from additional trips to the doctor or from staying away from work increases both no-fault and tort liability insurance rates.

From this perspective, raising the monetary threshold can lead to additional padding and further claim cost increases.

Cummins and Tennyson attribute much of the problem in both tort and low monetary threshold states to the fact that "it is simply too easy and too profitable to file bodily injury liability claims on auto insurance." Consequently, they argue, many potential claimants regard the liability system as a lottery with a high probability of payoff and a relatively low cost. Increased awareness of these potential rewards, particularly in certain urban areas, has played a major role in the cost increases that precipitate current calls for reform.

1.2 Available Tools for Pricing of BI Systems

Automobile insurance and its pricing problems were hot topics in the 1960s and 1970s. Prior to the first-in-the-nation introduction of no-fault in Massachusetts in 1970, several authors addressed the anticipated automobile liability ratemaking problems in PCAS publications.

Wittick (1963) reported on the early deliberations in Ontario regarding a proposed "compromise between the ordinary negligence system and a full workers compensation type plan". The actuarial conundrum addressed by Wittick was how to merge the available data on per-person accident and health insurance costs with per-car third party liability losses in order to price the additional costs of the hybrid no-fault/fault system.

Stern (1964) provided a comprehensive exposition of automobile liability insurance ratemaking procedures using accident year loss data gathered under a statistical plan. The reported data was, however, only the aggregate exposure, claim and loss information arising from individual claims. A particular feature of this aggregate data was that breakdowns were reported by rating classes rather than by claim characteristics. The underlying loss distribution was assumed for ratemaking purposes to remain the same for future periods except for inflation. Any changes in coverage were priced by "actuarial judgment." This classic paper survives to this day on the CAS Part 6 Syllabus.

Harwayne (1966) applied techniques similar to Wittick's to price a Basic Protection Plan for New York drivers patterned after the original no-fault plan proposed by Keeton and O'Connell (1965). Statistical plan data for bodily injury liability claims were combined with workers compensation claim data (with automobile accident proximate causes) and New York State accident statistics in order to derive estimates of frequency and severity. Interestingly, the key New York variables pertained to *claim characteristics*, such as fractures, lacerations and visible injuries, rather than to characteristics of insured drivers.

Weber (1970) called for the explicit introduction of stochastic process models for accident involvement of drivers into the pricing of auto liability insurance. The exposure unit would be a driver, not an insured vehicle. Accident rate potential would be gleaned from driver histories. Homogeneous subclasses would be established by rating territories. Research on rating based upon individual records continues to this day (Venezian, 1990).

The CAS publications have in recent years fallen silent on the subject of auto liability.[5] While the PCAS has concentrated on such standard problems as credibility and loss distributions, and such emerging concepts as rate of return methodology, solvency and financial analysis, the "500 pound gorilla" of auto insurance has continued to generate interest outside the CAS. Most notably, the Insurance Research Council (IRC) collected extensive data on a sample of automobile claims closed in 1977.[6] After their initial publication (AIRAC, 1979) the data were subsequently analyzed by researchers at the RAND Corporation (see Hammitt, 1985).

The usefulness of the data to insurers and researchers prompted the IRC to follow up with the collection of comparable data from 1987. These closed-claim studies provided an early warning about the deterioration of BI systems that had begun by the mid-1980's. For example, the percentage of Personal Injury Protection (PIP) claimants eligible for a tort claim rose dramatically from 24% in 1977 to 40% in 1987 countrywide and from 26% to 54% in Massachusetts. On a per-car basis, BI liability costs rose by a factor of 2.5 during the decade.

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[5]Venezian (1990) is one of few examples of later CAS papers treating auto insurance (see PCAS index 1964-1988, p.6-7, 46-47.)

[6]Formerly the All Industry Research Advisory Council (AIRAC).

RAND researchers used the rich countrywide data to infer the relative costs of prototypical tort and no-fault systems through specialized analyses and statistical models (Carroll et al, 1991). The IRC further alerted the industry to the nature of the evolving crisis by documenting an apparent trend in claiming behavior between 1980 to 1989 (IRC, 1990). Based on ISO fast-track data, the IRC found rising BI claim frequencies despite stable or falling accident rates.

1.3 The Need for Data and Models

A common theme in the early no-fault pricing literature was the need for insurance claim data to price major coverage changes. Richard J. Wolfrum, in a discussion of Harwayne's paper, bemoaned the lack of "proper data to evaluate a compensation system for automobile bodily injuries" He specifically cited the lack of data on the types and lengths of disability, the medical costs of each type of injury, and the economic status of the claimants. Wolfrum called for "automobile bodily injury accident tables" similar to those applied in evaluating workers compensation benefit changes and for data on the relationship between the insureds and the claimants (driver, passenger, etc.) for rating purposes. Ernest T. Berkeley, another Harwayne discussant, observed that "actuarial judgement" was exercised to a very unusual extent because of the unavailability of "studies based on individual company records."

Our brief review of the original no fault pricing dilemma highlights the essential limitation of available auto BI data. Statistical plans are designed primarily to permit efficient allocation of claim costs to classes of *insureds*. Consequently, certain relevant attributes of insured drivers and their vehicles are carefully recorded. These variables contribute valuable information about the insured's propensity to generate losses *relative to other insureds*. However, these variables tell us very little about the insured's propensity to generate losses relative to *what it would be under a different BI system*. When the system changes, especially through coverage changes such as no-fault plans, the pressing need is for data on the *characteristics of the claimant as she or he relates to the insured.* The types of injuries sustained in accidents are no less important in automobile insurance than in workers compensation.[7]

The IRC data on characteristics of BI claimants represent an important step toward Wolfrum's automobile accident injury table. Moreover, the wealth of information in those studies underscores the usefulness of this type of data. State and company specific micro-data on claim characteristics would be even better for the pricing of system alternatives. The value of detailed claim data has been demonstrated many times over by the use of workers compensation detailed claim data to evaluate reactions to changes in benefits (Butler and Worrell, 1985).

Our research efforts, described more fully below, have led us to two additional conclusions. First, a comprehensive understanding of the BI system, and its alternatives, requires data that reflect certain behavioral aspects of the system.[8] Second, once relevant data have been gathered, appropriate analytic tools are needed to distill the essential information from the mass of raw numbers. Statistical models that summarize the data and that allow for "what-if" analyses are critical if we are to gain understanding and quantification.

Policy limits, tort thresholds, legal representation, subrogation, collateral sources, and coordination of benefits are but a few of the factors that interact and are exogenous to the

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[7]Perhaps one quick meaningful innovation in current auto statistical plans would be to classify BI and PIP claims by a primary type of injury, especially strains and sprains.

[8]The accident process model of Weber foreshadows the use of behavioral variables, such as the decision to file a tort claim, and the effects of changing economic incentives that give rise to fraudulent and inflated claims.

claimant's accident and injury yet exert profound effects on insurance loss costs. The essential value of using detail claimant data comes from the fact that complexities and non-linearity of the interactions impounded in the data may not be amenable to simple aggregate data modelling. The RAND analyses (Hammitt, 1985; Carroll, 1991) used statistical models to summarize the IRC claim data, taking those claim data variables into account. Our purpose here is to elaborate further on the types of micro-data, statistical models and behavioral variables that can be used and that truly inform the pricing actuaries' judgement. We begin at the beginning, with the claiming process itself.

2.0 THE BODILY INJURY CLAIM PAYMENT PROCESS

To understand the usefulness of detailed claim data it is necessary to begin with a description of the claim payment process. The specific aspects of the process will depend upon the kind of tort system in operation. We begin with the traditional tort system. We then consider the additional components introduced by a no-fault system. Finally, we factor in the effects of subrogation between the PIP and BI coverages.

2.1 Traditional Tort System

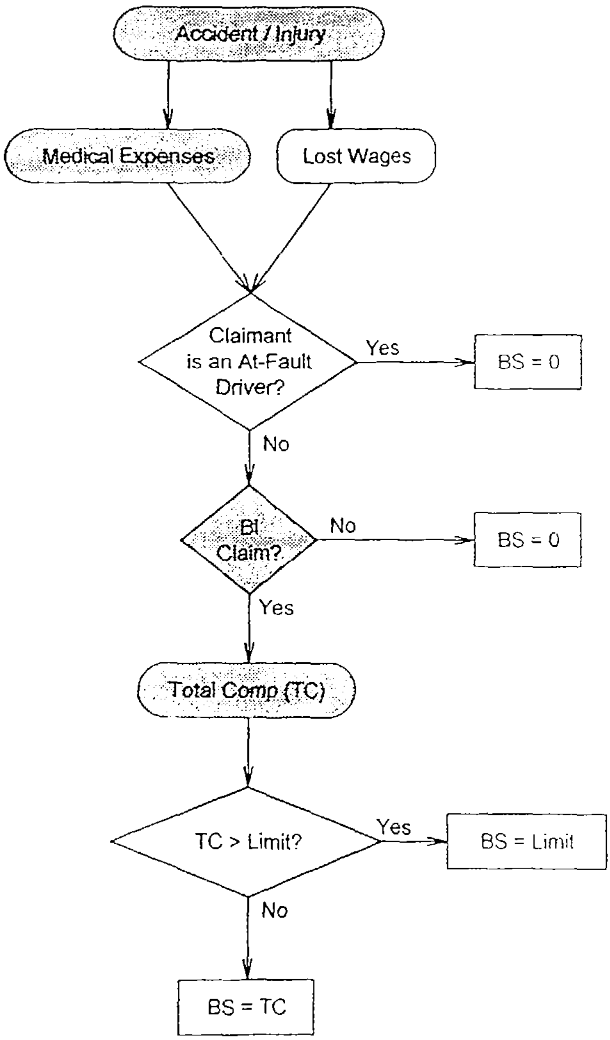
Figure 1 portrays the "case-flow" for a pure tort system in somewhat simplified form. The accident and resulting injury give rise to medical expenses and possibly lost wages. In the traditional tort environment, the victim must first establish his/her eligibility for a tort claim before proceeding further.[9] The specific negligence law of each state determines the conditions under which an accident victim is sufficiently "at-fault" to bar a potential tort recovery. For example, in Massachusetts an individual who is deemed more than 50 percent liable for the

[9] If no actual third-party can be identified, then the victim's own uninsured motorist coverage may substitute for the unavailable BI target.

FIGURE 1

Bodily Injury Liability Claim Payment Process

Tort System



LEGEND

BS = BI Settlement Value
TC = Total compensation value
(BI + PIP unconstrained by limits)

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accident cannot pursue a tort action.

An accident victim who is not at-fault must decide whether to file a BI claim. If a claim is filed, a process of negotiation with the insurer ensues, usually resulting in a settlement (but occasionally winding up in court). In some cases, the insurer might attempt to deny payment on such grounds as alleged claimant liability, lack of BI coverage, or suspected fraud. For the vast majority of claims, a payment, frequently less than the original claim, is eventually made. Theoretically, the amount of compensation received by the claimant is meant to cover the full value of both objective economic losses (also termed special damages) and of subjective pain and suffering (general damages).

The actual payment made under BI liability is constrained by the available policy limits. If the total compensation "deserved" by a claimant exceeds the available policy limit, then only the limit is paid. Moreover, if the compensation due all claimants from a single accident exceeds the aggregate accident limit, then each claimant receives a *pro rata* share of the accident limit.[10]

Note that four elements of this process have been highlighted for emphasis. Each of these represents a point at which factors exogenous to the insurance system itself can play a critical role in determining how the system operates in practice.

The *accident* and resulting *injury* to a vehicle occupant or pedestrian are the events that precipitate a potential claim under the BI liability coverage. In a majority of accidents where injuries are likely to occur, a report is filed by or with the local police and the incident becomes known to the insurance company. Under a traditional tort system, claims against the at-fault

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[10]In some cases of multiple claimants whose total damages exceed the accident limit, the shares may not be exactly pro rata due to severity or timing differences among claimants.

driver's policy can be made at any point until the time specified in the statute of limitations, usually 3 years or more from the date of the accident. Details about the accident, and any possible injuries to third parties, accumulate as potential liability claims are assessed and actual claims are investigated. While serious injuries are usually the result of easily observable serious accidents, claims for minor and non-existent injuries can arise from small "fender benders" or even staged accidents. Thus, claimant behavior prior to notification of the insurer determines the character of the claim as it moves through the system. The *amount of medical expenses* generated by the injury is the second key step and depends on the nature of the injury and the treatment (Marter and Weisberg, 1991, 1992). Treatment decisions can in turn be governed by a variety of considerations, possibly including the claimant's desire to obtain a substantial BI settlement (Weisberg and Derrig, 1992a). Patterns of medical treatment can obviously have an important bearing on the ultimate claim costs for BI liability claims.

The third critical juncture is the *decision* by the accident victim regarding *whether to file a tort claim*. What proportion of eligible individuals file claims? What systemic or individual characteristics influence the probability that a claim will be filed? In general, very little is known about claim-filing behavior, except that it varies widely by state and over time (IRC, 1990). Clearly, changes in these patterns could have a dramatic impact on BI claim costs.

The fourth highlighted element is the *valuation of total compensation deserved* by the claimant. In theory, the adjuster attempts to approximate the jury award that would result if the case went to trial. However, because so few cases actually reach the courts, there is little empirical evidence to inform this assessment. In practice, the adjuster tends to rely on guidelines that have become established over many years and have the force of strong tradition. For example, according to traditional claims adjustment lore, the amount of medical expenses is the single most important indicator of injury severity. A common rule-of-thumb is to set an initial settlement value at some multiple of total medical charges (or possibly of total economic loss). However, it is also recognized that these general rules must be modified to take account of other salient characteristics of the injury. Moreover, the effectiveness of legal representation may also affect the outcome of the settlement negotiations.

In each of the four highlighted elements, there are behavioral factors that may change as the rules and incentives of the tort system change. The "propensity to sue" in a given region or state may depend upon economic conditions, the access to specialized accident victim medical treatment, and the aggressiveness of the local plaintiff bar. Economic incentives may exist for the claimant to maximize medical treatment charges and periods of disability in order to obtain the largest settlements possible. Statutes and regulations designed to protect the consumer can also supply the opportunity for fraudulent or excessive ("built-up") claims. As a particular tort system changes in meaningful ways, these behavioral factors will change claim payments, sometimes by substantial amounts (see Section 4 below).

2.2 *Basic No-Fault System*

Figure 2 portrays the case-flow for a generic no-fault system.[11] As noted above, the specific features of the various systems in place vary significantly (PIP benefits, definition of tort threshold, etc.). However, the basic structure of all no-fault systems follows the general pattern shown in Figure 2.

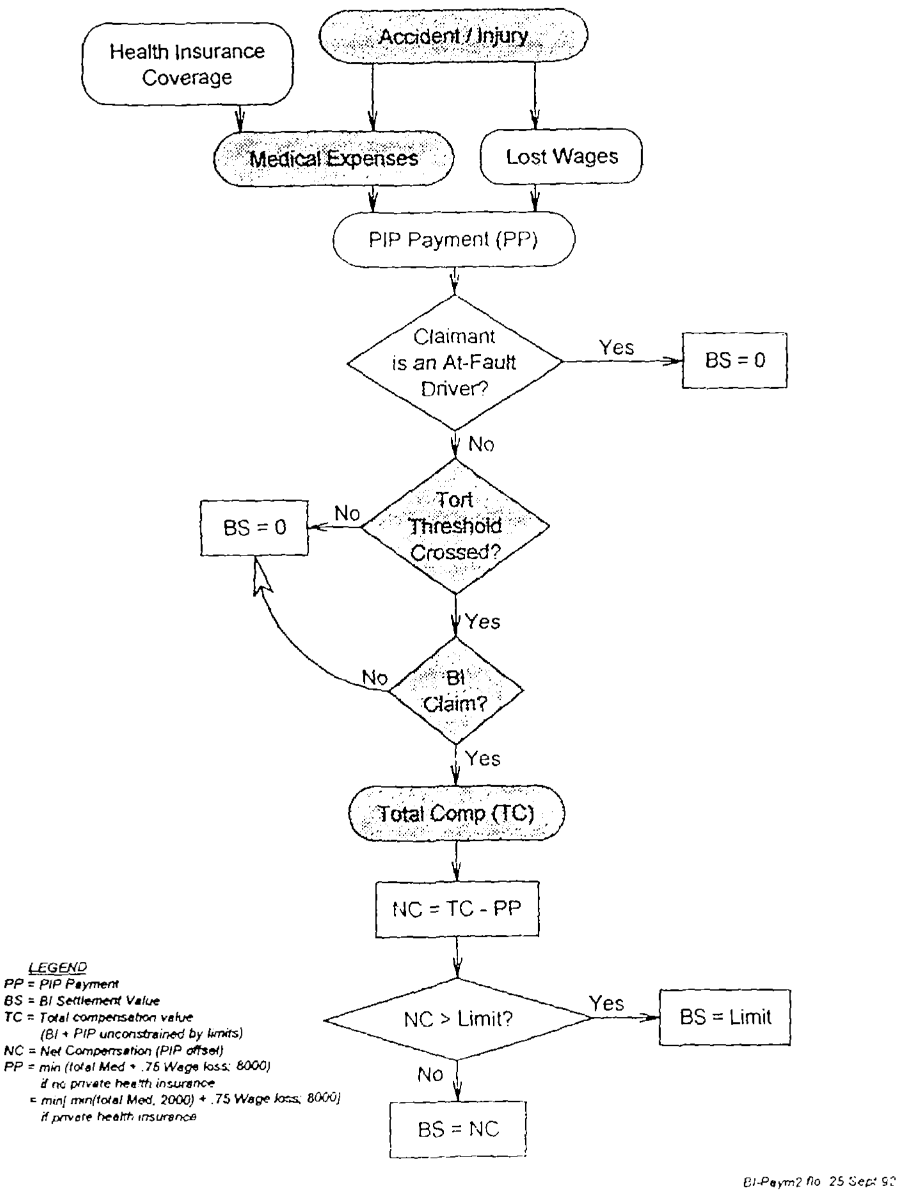
Once an accident has been alleged, real or potentially compensable injuries are assessed by company adjusters. If a claim is likely to arise, a case file and a reserve will be set up.

[11]Although a pure first party no-fault bodily injury compensation system remains a possibility, none has been implemented to date.

FIGURE 2

Bodily Injury Liability Claim Payment Process

No-Fault System



Under no-fault, at least part of the medical expenses and lost wages are reimbursed under the first-party (PIP) coverage. In Massachusetts, full medicals plus 75 percent of wages are reimbursed up to the policy limit of $8,000.[12] Note, however, that for states with coordination of benefits (COB) provisions, some or all of the medical expenses may be paid under other first-party coverage (primarily private health insurance). Thus, the effective amount of expenses for which PIP is responsible may be much smaller than the total expenses incurred. For example, in Massachusetts private health insurance is primary for all medical expenses in excess of $2,000.

The hallmark of a no-fault system is the existence of a tort threshold. The accident victim must not only qualify on the basis of liability in order to pursue a tort claim, but also must cross the tort threshold. In Massachusetts, the threshold is defined in terms of a verbal component (disfigurement, dismemberment, fracture, death, loss of sight or hearing) and a monetary component (at least $2,000 in medical expenses). Of the approximately 45% of Massachusetts PIP claimants who do cross the threshold, only 10% satisfy the verbal component.

Finally, many no-fault systems include a mechanism to preclude double payment of economic losses under the PIP and BI coverages. Typically, the amount of the PIP payment is "set off" against the BI award. That is, the claimant receives a net amount that is equal to the total compensation *reduced by the PIP amount*. In some states without such an offset provision, "double-dipping" is avoided by allowing the PIP insurer to receive reimbursement from their insured out of any BI recovery obtained. Setoffs are generally allowed when a subrogation process is in place.

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[12]Optional Medical Payments coverage can be purchased to extend in effect the PIP limit.

2.3 PIP Subrogation

In Massachusetts and several other states, there exists a further wrinkle. Under some conditions, the first-party insurer is considered to be "subrogated" to the victim's tort rights. That is, the insurer stands in the insured's place with respect to a right of action against the tortfeasor, and may seek reimbursement directly from the third-party carrier. The specific rules governing the operation of PIP subrogation in different states vary considerably. Figure 3 reflects the Massachusetts system, in which subrogation has a major effect.

If the claimant is a passenger in an at-fault vehicle or a pedestrian, then the potential BI carrier is the same as the first-party insurer of the at-fault vehicle. Therefore, subrogation is not possible. In most other situations, a potential BI target will be contacted and a request for subrogation made. Subrogation is allowed in Massachusetts regardless of whether the victim crosses the tort threshold or files a BI claim.

For a situation in which the at-fault insured driver, the "tortfeasor", is 100 percent at-fault, the amount of the subrogation request is ten percent over the PIP payment. The additional ten percent is meant to reimburse the PIP carrier for loss adjustment expenses associated with the claim. In a situation of shared liability, the amount is reduced by the claimant's proportion of fault.[13] If the claimant has filed a BI claim, then an actual subrogation payment cannot be made until after the claim has been settled, since the amount of money that remains available will depend on the BI policy limits. In Massachusetts, the entire policy limit remains available to the BI claimant, regardless of the subrogation amounts.

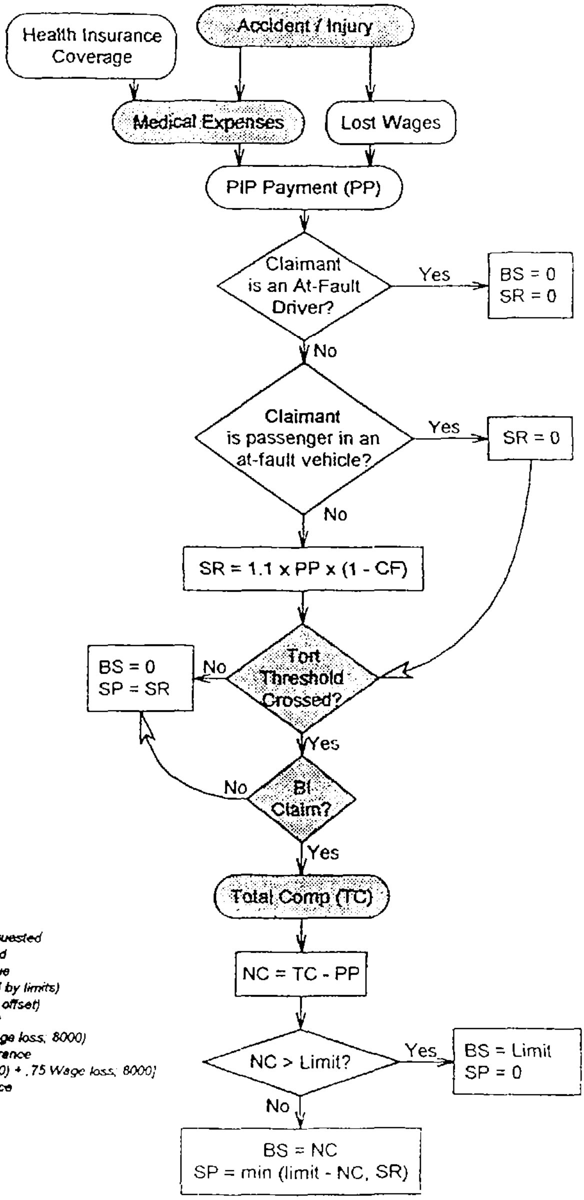
One can begin to get a flavor for the complexity and potential volatility of the claiming

[13]Of course an exact determination of fault percentage may be disputed and arbitration needed to resolve the differences.

FIGURE 3

Bodily Injury Liability Claim Payment Process

Massachusetts No-Fault



LEGEND

PP = PIP Payment
BS = BI Settlement Value
SR = Subrogation amount requested
SP = Subrogation amount paid
TC = Total compensation value
(BI + PIP unconstrained by limits)
NC = Net Compensation (PIP offset)
CF = Claimant's share of fault
PP = min (total Med + .75 Wage loss, 8000)
if no private health insurance
= min[min(total Med, 2000) + .75 Wage loss, 8000]
if private health insurance

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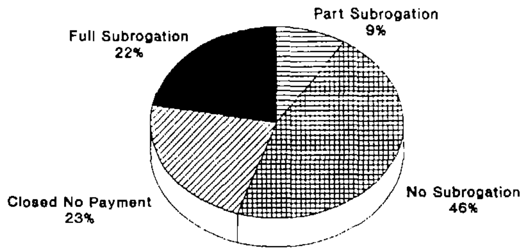
process by noting the final disposition of potential PIP claims in Figure 4. The process of subrogation creates subtle interactions which become important to recognize when alternative BI systems are considered.

Figure 4
Disposition of a PIP Claim



Source: AIB Statistical Plan Data

With a universe of all claims[14] that had positive PIP reserves set up at some time, 23% eventually were closed without any payment, 22% had PIP payments that were fully subrogated to the BI carriers, 9% were only partially subrogated[15], and 46% were pure PIP payments without any subrogation.

Table 1 provides some more detail on the extent and overlap of PIP and BI claim counts and amounts. The data are derived from a random sample of 839 PIP claims from 1989. In

[14]These percentages were derived by scanning the entire 1989 accident year statistical plan data at 42 months (June, 1992).

[15]Partial subrogations occur when the adverse party's coverage limits are exhausted or comparative negligence applies.

particular, only 28% of PIP claimants also received BI settlements; another 26% had their PIP payments fully subrogated to the BI carrier, fairly close to the 29% indicated in the statistical plan paid claim data above, while the remainder had PIP benefits fully paid by their first party PIP carrier.

Table 1
BI and PIP Claim Counts and Amounts

	(1) Number of Claims	(2) Percent of PIP	(3) Amount per Claim	(4) Amount per PIP Claim (1)x(3)/839
I. Total BI Claims*	454	54.1	7,485.70	4,051
A. Total BI Tort	237	28.2	12,569.84	3,551
B. Total PIP Subrogation	307	36.6	1,366.31	500
PIP Subro w/BI	90	10.7	2,780.96	298
II. Total PIP claims	839	100.0	1,663.63	1,664
III. BI Plus PIP Claims (IA & II)	839	100.0	5,214.34	5,214

* Includes 17 Uninsured Motorists Claims.
Source: AIB Study of 1989 PIP Claims.

3.0 A PRICING METHODOLOGY BASED UPON MICRO-DATA

In the Introduction we discussed in a general way the limitations imposed by typical statistical plan data. We now show how detailed "micro-data" can be helpful to address these problems.[16] We use the Massachusetts model (Figure 3) for illustration, but the potential for extrapolation of the basic ideas to other systems should be evident.

3.1 Changing Bodily Injury Claim Systems

Projecting future claim frequency and severity, even under a fixed tort system, is often extremely difficult. If the underlying forces driving the process (e.g., patterns of claim-filing and medical treatment) are in flux, then extrapolations of past trends based on routinely collected

[16] The term micro-data is suggested by the idea that claims are being examined as if under a microscope to reveal the fine detail that is invisible at the grosser level of ordinary statistical plan data.

statistical plan data become unreliable. Moreover, if the parameters of the system (e.g., negligence standards, tort threshold) are substantially *modified* by legislative action or judicial interpretation, then the relevance of the available data may be further attenuated.

Suppose that a monetary no-fault state like Massachusetts decided to change next year to either a verbal threshold or pure tort system (both are currently under active consideration in Massachusetts). We know qualitatively that a verbal threshold should entail a major reduction in the frequency of BI claims and an increase in severity of those claims that remain tort-eligible. A pure tort system should (in theory) generate a substantial increase in claim frequency and decrease in severity. But how could we develop a credible quantification of what will happen under the new system?

The problem faced by actuaries in either case would be to estimate the frequency and severity of an event (BI claim under new system) that is essentially different from the event about which historical data have been accumulated (BI claim under old system). Is there a way to bridge the gap between the old data and the new (anticipated) reality? The answer depends on the extent to which we can measure for the population of accidents/injuries those characteristics that determine whether a PIP or BI claim will be filed and how much compensation will be paid.

Suppose first that both the underlying distribution of accidents/injuries and the nature of individual claim-filing behavior are stable and will not be influenced significantly by the change of tort system. Assume further that we have collected detailed information for the population of accidents/injuries, or a representative sample, on the accident and injury, medical treatment and the extent of disability. In addition, we have data on whether a BI claim was filed and the amount of any BI settlement. Then it would be straightforward to calculate the expected loss distribution under various alternative scenarios.

For example. to evaluate a change from a monetary to a verbal threshold we could first calculate the loss distribution under the current system, after adjusting for any changes in claim characteristics expected to occur (e.g., economic inflation). Then, if we have measured the characteristics that define the verbal threshold (e.g., disfigurement, fracture, length of disability), we will be able to simulate the entire loss distribution that would be expected under this alternative. That is, for each accident/injury, we determine whether a BI claim will be filed and the expected BI payment. Comparing the resulting pure premiums under the two systems would provide an estimate of the rate impact of the proposed changeover. An example of this methodology for a changing monetary threshold is given below in Section 5.1.

So far, we have assumed the availability of micro-data on a sample representative of the entire population of accidents/injuries. However, the prime sources of potential data (statistical plan and claim files) pertain only to accidents/injuries that actually resulted in claims under the existing system. Thus, we may lack data on some new claims that could arise under a different system.

For a traditional tort state, virtually all potential tort claims under any contemplated system are already represented in the population of BI claims.[17] Thus, estimates of the BI loss distribution under alternative systems should be straightforward. However, estimating the number of *additional claims payable under a no-fault coverage* would require external information or theoretical assumptions.

Under an existing no-fault system, the problem is somewhat different. In theory, all potential BI claimants already file PIP claims and will thus be included in the available claim

[17]However, the proposed system may stimulate the fabrication of new fraudulent claims.

population. However, in reality there are some victims who for various reasons choose not to pursue the injury with their auto insurer. Their potential claims close without payment (Figure 4). Because some of these individuals *could* choose to file claims under an alternative system, their exclusion from the claim population might lead to an underestimation of losses.

3.2 Massachusetts BI and PIP Studies

In Massachusetts we have recently completed a series of three studies that can be used to support the kind of simulations described above. The first study was based on a representative sample of 474 BI liability claims based on accidents that occurred in 1985 and 1986. For each claim, extensive data were coded pertaining to the accident, injury, treatment, claim handling, and payment. In addition to objective information contained in the claim file, the coder's judgements regarding fraud and build-up were elicited (Weisberg and Derrig, 1991a).

The second study was a follow-up study of BI claims from accident year 1989. Claim files for a representative sample of 1154 claims were examined using a slightly revised version of the data collection instrument from the earlier baseline study. The primary purpose of the follow-up study was to assess the impact of a reform law implemented in 1989 that increased the monetary tort threshold from $500 to $2,000 (Weisberg and Derrig, 1992a)

The third study was based on a representative sample of 839 PIP claims from accident-year 1989. The primary purpose of the study was to clarify the reasons why PIP pure premiums were increasing at a much faster rate than expected, but an important secondary goal was to estimate the effects of coordination of benefits on both PIP and BI losses. We had originally hoped to obtain information on all the BI claims that arose out of the PIP claims in our sample, thus crafting a database close to the ideal described above. However, the available information on related third-party claims was not adequate. We supplemented the PIP data by searching the statistical plan data for any matches with our PIP claims. This search effort added a number of BI claims that were not evident in the PIP files. (Weisberg and Derrig, 1992b)

Finally, the PIP study also included a special sample of 387 PIP claims that could be linked with corresponding BI claims in our previous 1989 BI study. It was thought that having comprehensive data on both the PIP and BI claims would be useful for several purposes. In particular, we wished to refine the total compensation models developed on the basis of the 1989 BI data by incorporating information about health insurance. Without such information, we could not determine how much of a PIP offset had been incorporated in the BI settlement amount.

4.0 INCORPORATING ASSUMPTIONS ABOUT BEHAVIORAL FACTORS

The basic methodological approach described above assumed that the underlying dynamics of claim generation were stable and independent of the particular type of tort system. Specifically, the underlying distribution of accidents/injuries, patterns of medical treatment, claim-filing propensities, and BI claim valuation were assumed fixed. Consequently, the simulation of alternative scenarios became a mechanical exercise, providing that we could obtain detailed data on a representative sample of accidents/injuries.

The model considered so far might be termed "naive" because it ignores behavioral responses of accident victims, lawyers, and heath care providers. A more realistic model must reflect assumptions about the main behavioral factors that can influence claim losses. Even if we can only speculate about these factors, it is useful to conduct "what-if" analyses under alternative assumptions. We now consider these behavioral factors in more detail and demonstrate how statistical models based on micro-data can sometimes help to provide an empirical basis for improving upon the stable system assumption.

4.1 Profile of Accidents and Injuries

As suggested above, the profile of accidents and injuries might not be stable. For example, improvements in vehicle design or lowered speed limits might tend to decrease injury severity, while increased advertising by attorneys might engender more soft-tissue (strain and sprain) injury claims. Such factors can affect the overall frequency of claims, the distribution of claim types, or both. Pricing the PIP and BI coverages under alternative plausible scenarios regarding the impact of these factors would pose great difficulties for traditional actuarial methodology.

To account for such effects in a pricing model, we must first have some basis for hypotheses about which specific types of claims will be increasing or decreasing and how much. Then we need a way to identify the claims of these types among our sample claims. Finally, we re-weight the observations in our database to reflect the expected distribution of claims under alternative scenarios.

For example, suppose that a campaign to crack down on speeding and drunk driving is expected to reduce the frequency and severity of very serious injuries by 20 percent. If we can define a "very serious injury" in terms of claim characteristics captured in the database, then we can specify which particular claims would be subject to possible elimination. Removing a fifth of these claims from the database for purposes of analysis would then reflect the expected impact of the intervention.

4.2 Medical Expenses

The amount of total medical expenses incurred by the claimant plays a central role in the claim payment process. Under a monetary no-fault system, medical expenses can determine whether a BI claim is possible. Under all systems, the total compensation value is determined largely by total medical expenses. The decision to file a BI claim may also be influenced by the amount of medical expenses.

Suppose we have constructed a database containing micro-data on a representative sample of current accidents/injuries. We wish to simulate the distribution of outcomes (PIP and BI claim payments) that will occur under an alternative system. In theory, the treatment received for a specific type of injury should not be affected by the particular tort system in place. Thus, our simulation might assume that for each accident/injury in the population, the incurred medical expenses will remain the same (except for the effect of medical cost inflation).

It is possible, however, that patterns of medical treatment may be changing over time for a variety of reasons, including the modified tort system. We cannot necessarily assume that medical expenses incurred by future claimants, similar to those represented in the sample, will be identical to the expenses observed. For example, changing economic incentives could result in an increase in utilization of sophisticated diagnostic techniques or in the number of visits to chiropractors.

The null hypothesis of stable treatment patterns is particularly dubious when the profile of reported injuries is changing over time. For example, if increased advertising by attorneys is causing more claims of soft-tissue injuries, then simply re-weighting the observations in our database to reflect the expected increases in strains/sprains may not be adequate. We must also consider how the handling of such claims by claimants, lawyers, and health care providers might affect medical expenses. For example, marginal or fabricated injuries might tend to involve more visits to health care providers than apparently similar legitimate injuries.

Predicting changes in treatment patterns must necessarily be somewhat speculative. However, statistical models based on claim data can provide valuable insight. Our research in

Massachusetts has revealed that provider discretion appears to play a major role in determining the medical charges for injuries that involve strains or sprains, but a very minor role for injuries without a strain/sprain component. Therefore, our success in pricing any statutory modifications of Massachusetts no-fault depends in large measure on correctly anticipating the way soft-tissue injuries will be treated in the future.

A set of multiple regression models has proved particularly informative. We divided the claims in our PIP sample into three categories: pure strain/sprain, mixed, and non-strain/sprain. For each category, we found those claim characteristics that best predicted the total medical expenses. Our first set of analyses included possible predictors which were measures of accident or injury seriousness, but excluded measures of treatment utilization or lawyer involvement. Our second set of analyses included any variable that significantly improved our ability to predict medical expenses.

The results are summarized in Tables 2 and 3 respectively. For claims that involved strains or sprains, variables that reflected the seriousness of the injury explained little of the variation in medical expenses. For pure strains/sprains our model R^2 was only .04 and for mixed claims with strains/sprains and "hard" injuries, the R^2 was .21. Only for the non-strain/sprain injuries was a large proportion of the variation explained by measures of injury severity (R^2 = .62). However, when variables related to treatment utilization and claimant behavior were added in, the value of R^2 for strain/sprain claims jumped to .78 and that for mixed claims to .79, while the R^2 for non-strains/sprains increased only slightly to .68.

TABLE 2 DETERMINANTS OF MEDICAL CHARGES*									
VARIABLE	STRAIN/SPRAIN			MIXED			NON-STRAIN/SPRAIN		
	Coeff	p-value	F	Coeff	p-value	F	Coeff	p-value	F
Intercept	6.42			7.10			5.27		
Severe Collision	.58	.0004	12.9	-	-	-	.40	.02	6.1
Perm. Partial Disab.	-	-	-	1.28	.02	5.7	2.08	.003	9.3
Hospital Admission	-	-	-	1.59	<.0001	32.2	1.48	<.0001	19.3
Very Serious Injury	-	-	-	-	-	-	1.36	.0004	12.9
Serious Laceration or Fracture	-	-	-	-	-	-	.89	<.0001	20.0
Serious Trauma	-	-	-	-	-	-	.87	.0005	12.6
R^2	.04			.21			.62		

* Dependent variable = Log (Total Charges)
Independent variables all pertain to seriousness of injury only.

TABLE 3 DETERMINANTS OF MEDICAL CHARGES*									
VARIABLE	STRAIN/SPRAIN			MIXED			NON-STRAIN/SPRAIN		
	Coeff	-value	F	Coeff	p-value	F	Coeff	p-value	F
Intercept	5.18			5.72			5.04		
Log Outpatient Visits	.60	<.0001	216.9	.59	<.0001	300.5	.61	<.0001	49.5
Lawyer Involved	.46	.001	13.2	-	-	-	-	-	-
Severe Collision	.27	.001	11.2	-	-	-	.47	.003	9.6
MRI Used	.54	.003	9.3	.38	.02	5.8	-	-	-
CT Scan Used	-	-	-	.43	.001	12.0	.59	.02	6.2
Serious Visible Injury	-	-	-	-	-	-	.71	.001	13.4
At-Fault Driver	-.37	.007	7.6	-	-	-	-	-	-
Log of Hospital Days	-	-	-	.83	<.0001	76.8	1.08	<.0001	79.8
Treated by MD Only	-.42	<.0001	16.3		-	-	-	-	—
R^2	.78			.79			.68		

* Dependent variable = Log (Total Charges)

The number of outpatient visits was by far the most powerful predictor of expenses for mixed and strain/sprain claims, after adjusting for available measures of accident and injury seriousness. Therefore, assumptions regarding this aspect of treatment must be the focus of particular attention for pricing analyses, at least in Massachusetts. The statistical significance of legal representation for pure soft tissue injuries in determining medical expenses along with the indicator of whether the claimant was an at-fault driver underline the importance of behavioral factors.

4.3 Decision to File a BI Claim

It seems plausible that the propensity to file a claim will vary across victims and will depend on both individual characteristics and on the nature of the injury. Ideally, our database would contain information for each accident/injury on whether a BI claim was filed. Then to the extent that the underlying distribution of accidents and injuries remained roughly stable, our simulations of alternative scenarios could assume that the claim-filing decisions would also be the same as those observed. However, we noted above that the profile of accidents and injuries might be shifting, possibly in direct response to the tort system modifications. In such a situation, patterns of claim-filing behavior might also change.

In general, it may be difficult to obtain empirical evidence on claim-filing propensity. In traditional tort states, insurance data exist only for accident victims who filed BI claims. We do not know how many other victims could have filed but elected not to do so. In no-fault states, we can determine whether a PIP claimant was eligible to file a BI claim, but may not know whether a claim was filed. So we may have little but intuition to help frame the hypotheses about claim-filing to consider.

In Massachusetts, we were able to obtain valuable insight by developing a two-part model of the claim-filing process. First, we used logistic regression analysis of the data on PIP claims to identify factors related to crossing the monetary tort threshold. Second, we used logistic regression based on the supplementary BI data described above to identify factors related to filing a tort claim, once the threshold had been breached.

The results of the models to predict crossing the monetary threshold are summarized in Table 4. This analysis was restricted to claims that were not by at-fault drivers and did not satisfy the verbal component of the tort threshold. A stepwise regression procedure was used to select independent variables. The pool of potential variables was identical to that used in the total medical charge regression modelling with one exception. In our previous regression analyses of total charges, we found that the number of outpatient visits was a very powerful predictor. It is obvious that a large number of visits would also be correlated with exceeding the $2,000 threshold. However, our interest here was on the more subtle claim characteristics that might explain such a pattern of utilization and the resulting medical expenses. Therefore, we excluded outpatient visits as a potential predictor in this analysis.

TABLE 4
Logistic Model Describing Who Crossed the $2,000 Tort Threshold
(Excludes at-fault drivers and claims satisfying verbal component of threshold)

VARIABLE	COEFFICIENT	WALD CHI-SQUARE	P-VALUE
Intercept	**-7.48**		
Hospital Admission	4.72	17.8	<.0001
Lawyer Involved	**2.66**	**30.7**	**<.0001**
Log (Total Disability Weeks +1)	**.75**	11.6	**.0007**
Log (Partial Disability Weeks +1)	**.65**	12.2	**.0005**
Treated by MD and Chiropractor	**1.90**	11.3	**.0008**
Treated by Chiropractor Only	**2.89**	**23.5**	**<.0001**
Log (No. OP Provider + 1)	**3.56**	**22.9**	**<.0001**
Dependent Variable = Log (P / 1-P) where P = Probability of crossing threshold			

Overall, the monetary threshold was crossed by 41.5 percent of these claims. The factors that exerted the greatest impact on likelihood of crossing the threshold were admission to a hospital, presence of a lawyer, treatment by a chiropractor, and the number of outpatient providers. Other significant factors were treatment by an MD and chiropractor in combination and a lengthy period of temporary disability.

As an example, suppose that a claimant had an attorney, was treated by a chiropractor only, and was partially disabled for five weeks. Then, inserting the appropriate values in the equation, we calculate the probability (p) of filing a BI claim by:

$$\log (p/1\text{-}p) = -7.48 + 2.66 + \ln(6)\text{x}.65 + 2.89 + \ln(2)\text{x}3.56 = 1.703$$

and therefore:

$$p = .85$$

However, if the same claimant saw an MD only and did not have an attorney, we obtain:

$$\log (p/1\text{-}p) = -7.48 + \ln(6)\text{x}.65 + \ln(2)\text{x}3.56 = -3.847$$

and therefore:

$$p = .02$$

This equation supports the view that the presence of an attorney and the pattern of treatment, much more than the injury itself, determined whether the monetary tort threshold was attained. Even after accounting for possible effects of several other more direct measures of accident and injury severity, the predictive power of these variables remained strong.

The model of the decision to file a BI claim, once a claimant was tort-eligible, was much simpler. Most potential claimants (79.3 percent) chose to file a BI claim. Table 5 shows that legal representation was by far the strongest predictor, with total medical expenses also significant.

TABLE 5 LOGISTIC MODEL DESCRIBING WHO FILED A BI CLAIM (Includes only drivers who crossed the tort threshold)			
VARIABLE	COEFFICIENT	WALD CHI-SQUARE	P-VALUE
Intercept	-6.08		
Log of Total Medical Charges	.72	5.6	.02
Lawyer Involved	1.98	12.5	.0004
Dependent Variable = Log (P/1-P) where P = Probability of filing a BI claim			

An important implication of these two equations is that the presence of an attorney greatly increases the probability that a PIP claim will a) involve the necessary $2,000 to cross the threshold and b) result in the filing of a BI claim. While a direct causal interpretation may be speculative, it would seem prudent to reflect patterns of legal representation explicitly in our simulation modelling. For example, a dramatic increase in advertising by attorneys might be assumed to produce an increase in claimants, a higher percentage of represented claimants, or both.

4.4 Total Compensation Value

Ideally, our database would contain information on the amount of any BI award for each accident/injury. In our simulations of alternative systems, we could simply assume that the award would remain the same except for economic inflation. However, we noted above that the total compensation value was typically a multiple of medical expenses, modified by a variety of other considerations. If the process that determines medical expenses is changing, as discussed above, then we would expect the BI settlement to reflect these changes. For example, sharply higher medical expenses would translate into larger BI payments.[18]

[18]All of our total compensation model runs resulted in claimed medical charge elasticities of 0.50 to 0.60, significantly less than 1.0.

To incorporate such effects in our pricing analyses, we must make assumptions about the relationship between claim characteristics and total compensation value. To serve as a basis for such assumptions in Massachusetts, we have developed a regression model, with the observed total payment (PIP plus BI) as the dependent variable. In principle, we could simply have treated the sum of PIP and BI payments as a dependent variable in a regression modelling framework. However, the BI payment could have been cut off, or *censored,* by the policy limits. We have utilized a variant of regression analysis called Tobit regression (Tobin, 1958) to take into account the censoring. The final model has been summarized in Table 6.

TABLE 6
TOBIT REGRESSION MODEL FOR TOTAL BI COMPENSATION

VARIABLE	*COEFFICIENT*	*CHI-SQUARE*	*P-VALUE*
Intercept	4.74		
Log of Total Medical Charges	.52	79.0	<.0001
Log of Wages	.043	26.7	<.0001
Log of Fault Proportion	.49	17.1	<.0001
Lawyer Involved	.40	11.1	.001
Fracture Involved	.31	8.3	.004
Apparent Build-up	-.25	11.5	.001
Log of Disability Weeks	.075	7.6	.006
Serious Visible Injury	.25	5.3	.03

* Dependent variable = BI Payment + PIP Payment

As expected, the most powerful predictor of the total BI compensation was the amount of medical charges incurred. Although amount of lost wages was also highly significant, the relatively low value of the coefficient (.043) for the wage variable compared to that for medical charges (.520) suggests that the total compensation provides only for wage replacement. General damages may be unaffected by lost work time unless disability is also claimed. Other important determinants of the BI compensation were the at-fault driver's degree of fault, involvement of an attorney, and presence of a fracture injury. The number of weeks on disability also influenced the magnitude of the BI settlement, as did the fact that a serious injury was visible at the accident scene.

We hypothesized that an adjuster who suspects that medical expenses may reflect build-up will try to "compromise" the claim. To test this hypothesis we have included the perception of build-up as one of the independent variables. The highly significant negative impact on the BI award (-22%) suggests that claimants whose medical expenses appeared artificially inflated received a discounted evaluation of pain and suffering. Thus the negotiation process, and hence the final claim settlement value, is affected by the claim adjuster's perception of build-up and fraud.

5.0 SPECIFIC EXAMPLES

The considerable detail captured on each claim in the AIB Studies allowed us to "simulate" how the claim would be treated under various alternative claim environments and at different points in time. We have developed SAS computer models, where needed, to implement these simulations. For each different system and accident year of interest, we can compare the average values and other aggregate statistics of the simulated payments generated by alternative models. In this section, we summarize three examples that, although drawn from Massachusetts experience, represent a range of possible applications.

5.1 Changing a Monetary Tort Threshold

Using the Baseline Study data on 1985-86 accidents we created two models to predict the pattern of claims expected under a change in the monetary threshold from $500 to $2,000 beginning in 1989. The *naive* model embodied the assumption that treatment patterns for injuries would be unaffected by the different financial incentives implicit in the new tort system.

We assumed simply that medical costs would rise at roughly the 8.5 percent rate indicated by the Boston Medical Care Index. Moreover, the model assumed that the underlying frequency of automobile-related injuries would remain constant.[19] Under these assumptions, the model evaluated each Baseline Study claim in terms of its qualification as a potential tort claim under the new criteria. The subset of claims which remained tort-eligible formed the basis of our projections for accident-year 1989.

The fact that traditional tort settlements (or verdicts) as well as PIP subrogations are both BI payments causes a certain awkwardness of terminology. For convenience, we will refer to the BI settlement (or verdict) paid to the claimant as the (true) BI payment, although the PIP subrogation (if any) is really part of the total paid under the BI policy. The subrogation payment to the claimant's first-party insurer will be termed the PIP subrogated payment (see Table 1).

The logic of our simulation program is displayed in Figure 5. The flowchart reflects the decision-making process for each claim in the study sample. The variable denoted PIPPAY is the amount of any PIP subrogation payment generated by the model. BIPAY is the amount of any BI tort payment. VALUE represents the potential PIP payment according to the rules for the payment of PIP benefits.[20] PDPIPSUB is the amount of the actual PIP subrogation recorded in our Baseline Study data base, and CURRVAL is the BI payment for closed claims, or the ultimate estimate for open claims.[21] LIMITS represents the amount of the individual policy

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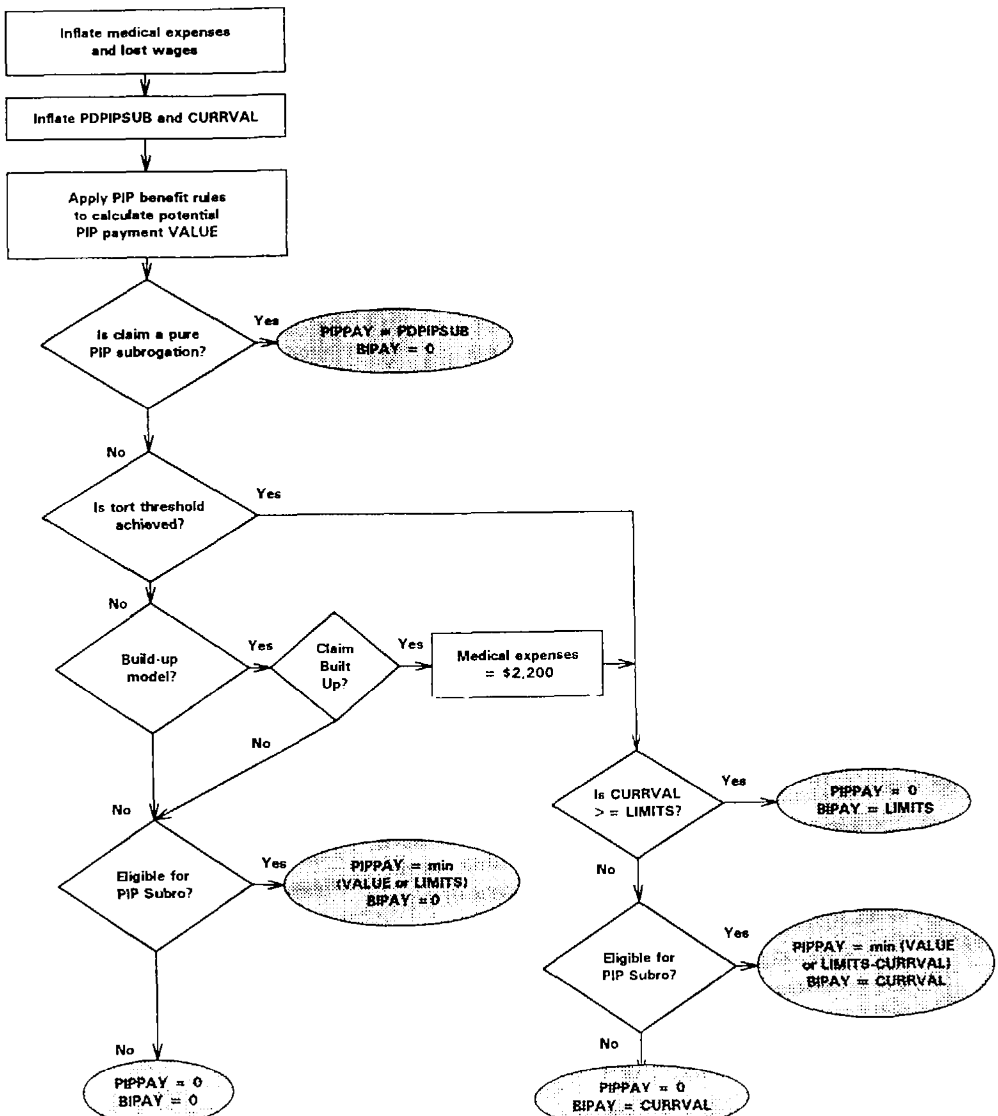
[19]For a complete description of the simulation model and the assumptions underlying its operation, see AIB Filing on Fraudulent Claims Payments for 1991 Rates (Docket G90-15), pp. 339-346.

[20]VALUE can be interpreted as the estimated total PIP payment regardless of which carrier actually ends up paying.

[21]The ultimate estimate of the BI payment was based on the last reserve maintained as of the time of coding (July, 1989).

FIGURE 5

Logic of the Simulation Model



limit.

A principal focus of the Baseline Study was suspicious claims. The coders identified cases of apparent fraud and/or build-up. Under our basic simulation model, a claim that failed to breach the threshold under any particular system was assumed to be paid under PIP. This naive model made no provision for any *more* build-up of medical expenses than that which was already reflected in the 1985-86 claims. To be more realistic, we also developed a model that reflected the hypothesis that claims similar to those that displayed apparent build-up in our Baseline Study would be further inflated (if necessary) to achieve the threshold. Our *conservative build-up* model incorporated the assumption that such claims would reach $2,200 on average in claimed medical charges.[22] The medical charges simulated under this alternative model were those expected to result from behavioral changes of claimants, physicians and lawyers.

Finally, we note that our build-up model was conservative in the sense that it reflected only build-up intended to reach the tort threshold. Build-up of claims already exceeding the threshold in order to "leverage" the general damages was not incorporated. Moreover, for claims built up over the tort threshold, we did not attempt to estimate the increased general damages that might result from the higher medical expenses claimed.[23] Furthermore, we did not

[22]Another purpose of the Baseline Study was to test the implications of alternative types of tort systems that might be considered for use in Massachusetts. For example, alternative no-fault and tort system rules were used to produce verbal threshold simulations that approximated the New York and Michigan systems.

[23]To estimate the increased general damages resulting from build-up would require a statistical model relating general damages to medical expenses. Since modelling efforts shown in Section 4 were preliminary at that time (Weisberg and Derrig, 1991b), we chose to adopt a simple inflation approach to the claim cost. The total compensation model of Section 4.4 could now be easily added.

allow for the possibility of build-up among claims that did not involve build-up under the former Massachusetts system.

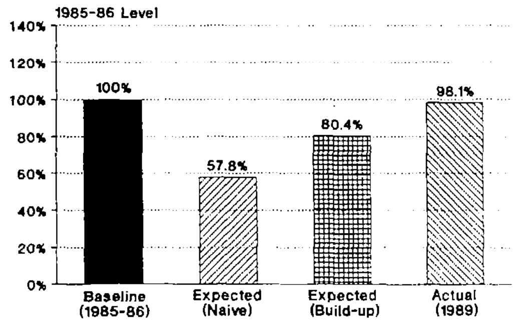
The naive model predicted that the post-reform frequency of true BI liability claims would be about half (50.8 percent) of the corresponding 1985-86 frequency. The conservative build-up model predicted that the frequency would be 70.7 percent of the 1985-86 level. Thus, both predictions were for substantial frequency changes based primarily on the claimed medical-payment data.

In order to gauge how well these two models predicted the effect of the monetary threshold change, we examined the 1989 BI results. After adjusting for the actual 13.7 percent increase in PIP frequencies, the expected frequency relative to 1985-86 would be 57.8 percent for the naive model.[24] Similarly, the conservative build-up model forecasted an adjusted relative frequency of 80.4 percent.

Figure 6 displays the predictions from the two alternative models as well as the actual results. It is evident that the theoretically expected decline in claim frequency simply failed to materialize in practice and that the build-up model was indeed conservative. Note, however that the use of behavior-modified values, based upon the expected consequences of the increased economic incentive (general damages for medical charges over $2,000), produced a predicted change with half the error of the naive model.

[24]1.137 x .508 = .578, 1.137 x .707 = .804.

Figure 6
Effect of Tort Reform
Frequency of True BI Claims Relative to Baseline



5.2 Supplementing a Trend Factor

The naive/build-up model example demonstrates the use of models to predict single year aggregate losses. Analyses of trends in annual losses can also be improved by using a simulation model, like that developed in Section 5.1, to refine the calculation of trend factors. Suppose that instead of estimating the one-time (marginal) change in a BI system, the actuary's problem is to estimate how the loss costs of a new system will continue to change over time. One simple answer would be to run the model several times, increasing medical costs and total compensation by an additional year of inflation each time.

Suppose, however, that three years of actual data are available under one system and three more years under a second system. Fitting a linear trend with a dummy variable at the system changeover point would yield a reasonable estimate for future values if the rates of change under the two BI systems were similar (equivalently, the second derivative of the time series is approximately zero). What are the chances that a BI system that involves the interaction of policy limits, subrogation, and build-up with a fixed monetary threshold will have a constant rate of change in loss costs? Probably very small. Our next example demonstrates how the micro-data and the model from Section 5.1 can help test the adequacy of simple trend models and adjust the estimated trend when those models prove inadequate.[25]

Briefly, Table 7 shows the 1986-1991 sequences of actual pure premiums and simulated BI losses at basic limits, the latter using our micro-data and the build-up model. A six-year linear trend with a dummy variable for the 1989 change in the threshold provides a projected 1993 value of $143.30 for the pure premium series.[26] Under this linear trend model, pure premiums are expected to increase 6.0% from 1991 to 1993. In other words, a linear trend factor of 1.060 is indicated by the pure premium data.

Table 7
1986-1991 BI Data

Year	*Pure Premiums*	*Simulated Losses (000's)*	*Tort Threshold*
1986	$85.73	$2,884	$ 500
1987	95.58	2,987	500
1988	102.88	3,092	500
1989	100.24	2,533	2,000
1990	112.46	2,645	2,000
1991	135.19	2,786	2,000

Our simulation model can also produce a single estimated value for the 1993 accident year. That value will take into account all the process interactions of interest (limits, inflation,

[25]The authors thank Ruy A. Cardoso for providing this example of the application of the simulation models.

[26]In this case, the use of a dummy variable effectively adjusts the old system data to the new system levels.

tort threshold, etc.). By comparing the 1993 simulation model value of $3,141 to the same type of linear trend model with a dummy variable for the 1989 change projection of $3,045 we see that the linear 1991/93 trend factor for the simulated losses must be supplemented by an additional 3.2% (3141/3045) in order to produce a correct (simulated) 1993 loss level. Thus, the sequence of simulated values is indicating that losses will accelerate (non-linearly) over time rendering linear trends inadequate. A more reasonable total pure premium trend factor might be the linear pure premium trend factor of 1.060 multiplied by the simulation model non-linear supplemental trend of 3.2% for a total trend factor of 1.094 (1.060 x 1.032).

Testing the adequacy of an exponential trend would proceed similarly. The point here is that the use of the micro-data simulation model projections can assist the actuary in choosing adequate trend factors that are based not only on a simple choice of data-fitting model (linear, exponential, etc.) but also on the expected movement in the micro-data aggregate. Moreover, the latter can be analyzed to provide the *reasons* for the changing values; the former cannot.

5.3 Coordinating with Health Insurance

One method that has been proposed to contain the rise of first-party PIP or Medical Payments claim costs is the coordination of benefits with health insurance. Total insurance system cost savings, as opposed to simple cost shifting from one insurance system to the other, can result from the elimination of double coverage and double benefit payments. Mehr and Shumate (1975) find, however, that insureds *prefer* double coverage when given a choice and will generally shun optional deductible plans designed to eliminate the double cover on the automobile side.[27] Of course, from the consumer point of view it is more economical to

[27]Less than 10% of the Massachusetts insureds have chosen PIP deductibles in the 22 years of no-fault coverage existence.

purchase the medical coverage with a pre-tax employee benefit than with after-tax disposable income. Mehr and Shumate conclude that "the strongest and only argument for making health insurance primary is the tax argument."

As mentioned above, in Massachusetts PIP is the primary coverage for the first $2,000 of medical expense. Medicals in excess of $2,000 must be covered by private health insurance, if available, up to the $8,000 PIP limit. Just how much is "saved" by the automobile insurance system using this COB provision? Could more be saved if health insurance became primary? How would increased PIP limits affect the results? The micro-data on PIP claims allowed us to estimate the savings to the PIP coverage of COB with health insurance "triggers" at zero (health primary), $2000 (current system) and $5,000.

The basic approach was to calculate for each claim the amount that would be saved by the PIP insurer under each of the six systems. When the claimant was covered by private health insurance, we first computed the expected amount that PIP would have paid in the absence of the COB provision. This expected payment was the sum of actual lost wages and medical expenses up to the PIP policy limit. We then subtracted the expected payment under COB. This payment was calculated in the same way, except that actual medical expenses were capped by the COB trigger amount (e.g. $2,000 for the current Massachusetts system). The difference between the two payment values represented the savings attributable to COB.

Table 8 shows what were at first considered surprisingly low COB savings for six alternative COB/PIP systems. Further reflection revealed that these results are quite plausible. The explanation can be found within the interactions of the claim characteristics. First, federal insurance plans like Medicare and Medicaid are by statute never primary (their costs are being contained as well). Second, a large segment of the claimant population is not currently covered by health insurance.[28]

TABLE 8 COORDINATION OF BENEFITS: SIMULATED EFFECTS OF COB WITH ALTERNATIVE SYSTEMS*				
PIP LIMIT	HEALTH INSURANCE TRIGGER	PERCENT WITH COB SAVINGS	PIP PAYMENT (NO COB)	% SAVINGS
8,000	0	44.6	1,996	38.7
25,000	0	44.9	2,304	41.0
8,000	2,000	17.8	1,996	14.2
25,000	2,000	18.5	2,304	19.4
8,000	5,000	2.9	1,996	2.2
25,000	5,000	3.6	2,304	8.1

* Assumes no medical inflation. Ignores denials and disallowances by health insurer or PIP carrier.

Finally, it is worth reporting with this example that these COB savings are not fully removed from the auto insurance system let alone the total insurance system. Typically, to avoid duplicate *automobile* insurance payments, PIP payments can be offset from total estimated BI damages to produce a lower BI payment. However, unless specifically allowed as a collateral source offset, health insurance COB payments *cannot* be similarly offset from BI damages. Thus, in the case of PIP claims that also involve a BI liability component, the BI plus PIP total auto payment is the same with or without health insurance COB. Indeed, this fact was confirmed by the lack of statistical significance of the health insurance variable in the total compensation model in Section 4.4 (Table 6).[29] Since our micro-data shows that about 68% of the current PIP savings comes from claims with a tort component, auto insurance COB savings

[28]An additional factor, the failure of some private health insurance plans (generally HMO's) to cover chiropractic treatment, was not considered in this model. A more sophisticated model could in theory be developed to account for this factor as well.

[29]The dummy variable for private health had an insignificant coefficient of -.00055 with a p-value of .9579.

are currently at the meager 5% level.

6.0 CONCLUSION

When the forces that determine no-fault and bodily injury liability losses are changing, the accurate pricing of these coverages can become a formidable challenge for actuaries. In particular, when the tort system itself undergoes a major reform, the usual statistical plan data may no longer be directly relevant. Since the impact of the change is primarily in terms of the nature of claims flowing from accidents, which may be only tangentially related to characteristics of insured drivers, detailed claim data can be extremely helpful to supplement statistical plan data.

The importance of detailed claim data for pricing the original no-fault proposals was recognized by actuaries twenty-five years ago. However, these pioneers lacked the technical and data resources necessary to exploit this insight very productively. Today, we are somewhat more fortunate. Thanks to the Insurance Research Council, we have a large national database of claims closed in 1977 and 1987, soon to be supplemented by a 1992 sample. Modern computer capabilities, coupled with sophisticated statistical modelling approaches, can enable us to identify important patterns, trends, and relationships. The kind of statistical modelling efforts undertaken by RAND researchers and our own studies in Massachusetts can serve as examples of what can be accomplished with the currently available data.

In this paper, we have demonstrated that combining the available micro-data on BI and PIP claimants with such techniques as ordinary least-squares, logistic and Tobit regression procedures can produce useful models of the BI/PIP claim payment system. The models, applied to the detailed claim data, can provide explanations for the variability in medical charges, the likelihood of crossing a monetary threshold, and the expected size of the total compensation to a claimant. Our examples show that important actuarial exercises such as estimating new aggregate loss values when the monetary threshold changes, determining the most appropriate loss trend factors under changing BI systems, and estimating the effects of coordinating claim payments with other insurance lines are all amenable to methods using micro-data and statistical models.

To extract full value from this approach, however, will require an investment in the creation of claim databases that are specific to states or companies and that address their unique circumstances. Massachusetts data and findings can be generally informative to California or New York insurers and regulators, or serve as broad guidelines, but they are obviously unacceptable for ratemaking purposes in those states.

There are two obvious approaches to obtaining the necessary data. One possibility is to amend statistical plan specifications to require the reporting of additional claim characteristics. This option may be very costly and cumbersome, but might be worth considering for a few very critical pieces of information (e.g., type of injury). An alternative would be to perform special studies based on representative samples of claim files. As in so many areas of research, a carefully designed sample will usually prove to be more cost-effective.

Finally, it has become clear that behavioral responses to the economic incentives built into a BI system cannot be ignored. Claiming behavior is no longer a "philosophical imponderable" that falls outside the scope of actuarial analysis (Harwayne/Wolfrum, 1966). Fraud and build-up are harsh realities of the present day, and attempts must be made to collect data that will allow their effects to be quantified.

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CROP-HAIL INSURANCE RATEMAKING

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CROP-HAIL INSURANCE RATEMAKING

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ABSTRACT

Monetary loss as a result of hail damage to crops is a major hazard facing farmers in many areas of the United States. Crop-hail insurance provides a means for the farmer to protect his income from the consequences of this hazard.

The authors presume that knowledge of crop-hail ratemaking procedures is not widespread among casualty actuaries. This paper will attempt to introduce the reader to the basics of crop-hail insurance and some of the ratemaking procedures currently used in the industry. The paper begins with a brief background on the crop-hail industry, the standard crop-hail policy, claims adjustment, and data collection. The central focus of the paper is upon crop-hail pure premium estimation, the development of final rates, and an analysis of the pure premium estimation procedure.

BACKGROUND

Crop Hail Statistical Profile

The United States crop-hail insurance industry provided over $9 billion of protection in 1991 for a total premium of about $350 million. Insurance was written on about 200 crops with over 95 percent of the liability on five crop groups-corn and maize, soybeans, cotton, and tobacco (in order of magnitude). Over one third of the total coverage was on corn. The insurance in force is heavily affected by crop acreage and commodity prices.

Hail insurance was written in 41 states in 1991 with a heavy concentration in the Midwest. About half of the coverage was provided in five states--Illinois, Iowa, North Dakota, Minnesota, and Nebraska. The top 17 states accounted for over 90 percent of the insurance.

Premium rates charged vary by crop, location and type of policy. For the states with most of the liability, average rates per $100 of coverage range from $9.16 (Colorado) to $1.05 (Illinois). Much of the liability is in states with an average rate of less than $2.00 (Illinois, Indiana, Wisconsin, Michigan, Ohio, and Oregon).

The average policy premium was $1,056; ranging from a high of $4,503 in Arizona to a low of $420 in Tennessee. The premium per policy in the Midwestern States averaged about $550 for Illinois and Indiana, $850 for Iowa, $1,340 for Minnesota, and $1,900 for North Dakota.

National Crop Insurance Services

For most states and crops, crop-hail rates are developed by National Crop Insurance Services ("NCIS"). NCIS' objectives are:

- Research
- Compilation of Statistics
- Ratemaking and Rate Filing
- Loss Adjustment Support
- Education

NCIS is the successor to two formerly separate organizations, National Crop Insurance Association ("NCIA"), and Crop-Hail Insurance Actuarial Association ("CHIAA"). NCIA formerly addressed the research, education and loss adjustment expense support needs of the crop-hail insurance industry. CHIAA served as the statistical, ratemaking and rate service organization for the industry.

NCIS develops rates (or loss costs) in 34 states. The frequency of rate filings in a given state is generally determined by the magnitude of the crop, and by state insurance department requirements. For large premium volume states, rates are updated every three years. Smaller volume states receive revisions less frequently.

Crop-Hail insurance statistics are gathered from the application and, in the event of a loss, from the proof of loss form. The information collected from these forms is prescribed in the Statistical Plan. This plan is designed to collect enough information to provide actuarially sound rates and to complete informative statistical reports.

Descriptions of the important data records are included as Exhibit 1. Detail premium and loss data in this format is collected from member and subscriber companies. Summary data is collected from Alternate Statistical Reporter (ASR) companies. All reports and data files discussed in this paper refer to data submitted by these companies.

Currently, about 85 percent of all U.S. crop-hail statistics are reported to NCIS in detail or summary form.

THE CROP-HAIL POLICY

Policy Form and Coverages

Appendix A contains a sample crop-hail policy.

The basic policy form is a percentage of damage contract. An insured farmer will purchase insurance for a stated amount per acre. The amount will reflect both the expected yield of the crop as well as the anticipated price at harvest. For example, if:

Expected crop yield = 100 bushels / acre

Expected price = $2.50 per bushel

the anticipated value of the crop is $250 per acre.

Under the standard policy form, indemnification for hail damage to crops will be based on the estimated percentage reduction in yield potential as a result of the damage. For example, if the adjuster determines that yield is reduced by 25%, the indemnification will be 25% of the amount insured. In the example above, if the full value of the crop ($250) is insured, the indemnification will be $62.50 per acre.

The policy is a coinsurance contract. If the farmer chooses to insure for less than the full value of the crop, the indemnification is reduced proportionately. In the above example, if the crop is insured for $125 per acre, a 25% yield reduction would result in indemnification of $31.25 per acre, or half of the estimated loss.

Other policy forms exist. Exhibit 2 identifies several of the most common, and shows how they apply.

Claim Adjustment

Because of the diversity of agriculture in the United States, crop-hail claims adjustment is a fairly involved process. Monetary losses sustained from hail damage are a function of several variables: the type of crop; the stage of crop growth; and hail intensity, both size and force of the hail. Wind damage accompanied with a hailstorm will also be an important factor.

Three principal categories of plant damage are analyzed in the claims adjustment process. These are: (1) reduction in stand or total destruction of the crop; (2) mutilation which impairs plant function; and (3) direct damage to the fruit or product of the crop.

The task of the crop-hail claims adjuster is to sufficiently sample the acreage insured to determine the overall damage to the crop. In order to establish the extent of damage to plants, the adjuster utilizes charts that translate the indicated damage to the loss in yield. All field sampling involves one or more of the above-mentioned categories, depending upon the stage of growth at the time of the storm. For most full season crops the adjustment is a prediction of future yield, in terms of percent of yield had there been no damage. For some crop areas the time of the hail season (majority of damaging storms) coincides with the maturity stage of growth (the single most vulnerable stage of growth).

An example of the Loss Instructions for corn is provided in Appendix B.

RATEMAKING METHODOLOGY

General Information

Crop-hail rates are derived using a pure premium approach. Pure premiums are called loss costs, and are calculated as the ratio of losses to exposure (insured values). Loss costs are typically expressed per $100 of exposure.

NCIS develops rates (or loss costs, in states which do not allow development of full rates) for each crop that has at least 25% of the statewide total liability. For most states, this results in two or sometimes three "base" crops.

Exhibit 3 is a summary of the crops for which separate analyses are performed in each of NCIS' 34 states.

Basic Rating Unit

The crop hail rating process is faced with a dilemma. Two fundamental concepts come into conflict in determining the appropriate rating base. On the one hand, because of meteorological influences on the hail hazard, which can vary significantly within relatively small areas, small rating areas are necessary.

On the other hand, because of the infrequency of hail losses in any specific location, larger volumes of data are needed to produce meaningful conclusions from the statistical data.

NCIS has addressed this dilemma by using the township as the basic rating unit in most states. This size unit is small enough (6 miles x 6 miles) that the rate can reflect unique meteorological influences.

The requirement for larger volumes of data is met by:

- Utilizing crop hail loss costs from 1948, and
- Incorporating broader geographic areas in the determination of the township rate. (This will be discussed in greater detail in the discussion of credibility.)

Data Conversion

As discussed above, crop hail insurance can be written on a number of policy forms. In order to increase the volume of the data used in deriving the rates, losses sustained under policy forms other than the base policy form are converted to the base policy form.

Exhibit 4 illustrates the derivation of the policy form conversion factor. Losses incurred under the basic form (Column 3) are recalculated to reflect the losses which would have been incurred under the alternative policy form (Column 4). The ratio of these two values is used to determine the conversion factor.

As Exhibit 4 illustrates, the ratio varies with the underlying rate. Presumably, this is a reflection of the fact that the low rate areas experience less severity of hail losses. Consequently the impact of a deductible in the low rate areas is greater than in the higher rated, higher severity areas.

Because of this relationship, a least squares line is fit to the actual ratios, producing the "Trend" values in Column 6.

Converted losses are then calculated as:

$$\frac{\text{Losses under alternative policy form}}{\text{Policy Form Conversion Factor}}$$

In addition to conversion of losses to allow experience from different policy forms to be included in the rate analysis, data from crops other than the base crop are also included. Crops with similar susceptibility to hail, and consequently similar loss costs, are grouped together. In most instances, data for similar crops are combined without adjustment. For a few crops, data is converted to the level of the base crop. Exhibit 5 shows the calculation of a crop conversion factor. In this illustration, wheat is the base crop, and barley is the converted crop. From the data on Exhibit 5, barley losses would be divided by 1.50 to convert to the loss cost level of the base crop (wheat). Unlike the policy form conversions there is no need to vary the factor by rate.

Catastrophe Adjustment

Despite the lengthy experience period underlying the derivation of the township loss costs (over forty years), the impact of one severe loss year can have a marked impact on a township's historical loss cost. Exhibit 6 illustrates this. The exhibit displays the loss cost history for a large township. The exhibit shows that, even after twenty years of accumulated history, changes of more than 10% in the cumulative loss cost ratio from one year to the next are not uncommon. (This is an atypical township in that losses have occurred in the majority of years. For many townships, the majority of years have no losses. For a typical township the impact of a single year on the accumulated loss costs would be more pronounced.)

In order to add stability to the township loss costs, NCIS employs a capping procedure, which is called a catastrophe adjustment. In the procedure, losses in excess of a specified catastrophe threshold are removed from the township history, and built back over a broader base. (The build back will be discussed in a later section).

The catastrophe threshold is a multiple of each township's median non-zero loss cost. The multiple which is used for a particular crop and state is determined from the ratio:

$$\frac{\text{Township Variance Eliminated by capping}}{\text{Township Losses Eliminated by capping}}$$

(Township variance refers to the variance of annual loss costs within a township. This is averaged over all townships, before and after capping, to derive the numerator of the ratio. As noted above, the losses in excess of the threshold are removed from the township loss cost and built back over a broader base.)

The value (multiple of the median) which produces the greatest value of this ratio (which is called the test statistic), is used as the catastrophe threshold. In essence, the maximum test statistic reflects the most efficient threshold, that is, the greatest variance reduction per dollar of loss eliminated. In the event that the test statistic is not maximized at levels of loss reduction greater than 1%, the multiple which produces a 1% reduction in losses is used as the default threshold.

The calculation of the test-statistic is shown on Exhibit 7. Exhibit 7a illustrates the calculation for the township data which was presented on Exhibit 6. This is for illustration only. The catastrophe procedure does not require calculation of the test statistic for individual townships.

Exhibit 7b shows the values of the test statistic as calculated on a statewide basis. The test statistic is greatest, in this instance, at a catastrophe threshold of 18.1 times the median (non-zero) loss cost. Each township's losses are thus capped at this level, with losses in excess of this threshold spread back using the distribution procedure discussed in a later section.

Credibility

Studies performed by CHIAA and NCIS have suggested that an individual township's data has little credibility. Roth's paper (see bibliography) provided the remarkable statistic that, for the largest townships in Kansas, approximately 1250 years of data would be required to achieve 95% confidence that a township's historical loss cost was within $0.50 of the true mean.

Nevertheless, as discussed earlier, meteorological differences can affect the hail hazard over relatively small areas. Consequently, NCIS has adopted a "surrounding township" approach for determining the township loss cost. Each township is aggregated with the adjacent eight townships (defined as nine-township), as well as the "next adjacent" sixteen townships (defined as twenty-five township). This can be visualized as follows:

25T	25T	25T	25T	25T
25T	9T	9T	9T	25T
25T	9T	TOWNSHIP	9T	25T
25T	9T	9T	9T	25T
25T	25T	25T	25T	25T

NCIS has examined formulae in which credibility varies with the total exposure (insured crop values) underlying each geographic entity's loss cost. The results did not produce any clear relationships between exposure and credibility. This can be explained, in part, by the fact that exposure is defined as insured crop value which is the product of the following components:

Acres insured

Yield per acre

Price per unit of production

Percentage of yield insured

The effect of the latter three components may have masked any true relationship between exposure and credibility.

As a result, credibility is generally assigned on the basis of geographic size. For most townships, "Final Average Loss Cost (FALC)" is derived as a weighted average of:

Township limited loss cost (10% weight);

Surrounding nine-township limited loss cost (15% weight);

Surrounding twenty-five township limited loss cost (75% weight).

Exceptions apply if the total exposure for any of the three geographical units falls below specified thresholds.

Exhibit 8 shows the calculation of the FALC for a number of townships.

As a final note, rates are made by township primarily in the larger volume states. In lower volume states, rates are made by county, Crop Reporting District ("CRD") or State. In the county states, the FALC is 100% of the county loss cost if the exposure (cumulative liability) is $1,250,000 or greater. For low liability counties, the CRD loss cost is used. For CRD and state rates, 100% weight is given to the geographical exposure unit.

Catastrophe Redistribution

In a previous section, we described the process used to identify catastrophe losses, which are removed from the township loss cost prior to calculation of the FALC. The catastrophe redistribution is a two level process.

The first level of redistribution is to the Crop Reporting District ("CRD"). Each state is divided into seven to ten CRD's (by the U.S. Department of Agriculture). Catastrophe losses (that is losses in excess of the catastrophe threshold discussed in Section D) are aggregated for all townships in a CRD. The CRD Redistribution Factor ("CRD-RF") is calculated as:

$$1.0+\frac{\text{Total Catastrophe losses in CRD}}{\text{Total Limited Losses in CRD}}$$

A similar calculation is performed at the statewide level.

Each township FALC (derived as in the previous section) is multiplied by the CRD-RF, with the exception that the CRD-RF is limited to:

$$1.0 + [\ (\text{Statewide RF} - 1.0) \times 2\ ]$$

The second level of redistribution applies only if the limitation to the CRD-RF comes into play. In this case, any catastrophe losses which are not redistributed in level 1 are distributed based on the following:

$$1.0+\frac{\text{Total Level 2 Catastrophe losses}}{\text{Total Limited Losses + Level 1 Cat Losses}}$$

This redistribution is illustrated on Exhibit 9. In this example, the statewide level 1 redistribution factor is 1.0986. Thus, each Crop Reporting District's level 1 redistribution factor is limited to 1.197 (1+2x(.0986)). As the exhibit illustrates, the level 1 factor for CRD 80 exceeds 1.197, and therefore this limitation applies. Level 2 losses reflect CRD 80 catastrophe losses which exceed the limit. The level 2 losses (1,746,671) represent 1.4% of the sum of the limited losses and level 1 catastrophe losses ($125,127,861). Thus, the level 2 redistribution factor is 1.014.

Each township's FALC is then multiplied by:

Level 1 Factor x 1.014

Expense Load

For those states for which NCIS publishes rates, the next step is the conversion of loss costs to rates. This is accomplished by dividing the catastrophe adjusted FALC by an Anticipated Loss Ratio (ALR).

The ALR varies by state, including provisions for loss adjustment, general, commissions and profit. ALR's ranging from 60% to 65% are common to most NCIS states.

The ALR further varies with the magnitude of the rate, with higher rated townships requiring a lower expense ratio than lower rated townships. Exhibit 10 is an example of a schedule of ALR's by rate class.

Limitations on Rate Changes

Once the rates (or loss costs) have been calculated, the final step is to limit the amount of the change from present rates. In general, three constraints are imposed on the final rate:

- Rate cannot increase or decrease by more than a fixed dollar amount;

- Rate cannot increase or decrease by more than a specified percentage;

- Rate cannot exceed a specified maximum for the state, or be less than a specified minimum.

The specific values of these constraints may vary by state and crop.

Test for BIAS in FALC

Several of the major elements of the ratemaking formula were newly implemented in 1990. In order to determine whether the changes may have introduced biases in the determination of the FALC, NCIS performed tests of the resulting loss costs, both before and after the catastrophe redistribution. A description, of the tests is presented in Appendix C, along with a summary of the results.

CONCLUSION

The process which has been described above has been generalized in a number of areas. Some of the more common variations have been described. Other less common departures from the standard approach exist for specific crops or unique situations.

Like other Property-Casualty coverages, the crop-hail ratemaking methodology has evolved over time. The methodology is monitored by NCIS, and by the crop-hail industry through company participation in National and Local Committees and industry groups.

BIBLIOGRAPHY

Brown, Philip S., "Crop-Hail Insurance", Society of Chartered Property and Casualty Underwriters (1967 publication)

Roth, Richard J., "The Rating of Crop-Hail Insurance", Proceedings of the Casualty Actuarial Society, Volume XLVII (1960), pages 108-146.

National Crop Insurance Services, "Crop-Hail Insurance Statistics and Rating Methods", 1989 Edition.

Data Elements

A list of detail data elements collected by NCIS follows. It includes all fields currently collected, and some fields which were used in the past, but are no longer obtained. See Exhibits 1 and 2 for computer record descriptions.

Acres: Acres of crop grown and insured for a loss record.

Amount of Loss: Total dollar loss for this crop.

Card: Card number. '1' used for premium record. '2' or '4' used to indicate loss record. A '2' loss record is used for percentage losses (loss is indicated as a percent of total) and a '4' is used for tonnage losses (loss is indicated in number of tons lost).

Cause of Loss: A code (peril code) used to indicate the cause of loss. The most common codes follow. These are not all peril codes, and the codes can vary by state for the lesser used peril codes.

1 - Hail
6 - Transit
7 - Fire
8 - Windshatter without hail

NCIS CPU: Year, month and day this record was received by NCIS. No statistical value.

Company: A numeric code assigned to a company per year. Will always be unique for any year/company. Usually will be unique across companies.

County: Numeric county code.

Crop: Numeric crop code. For example,

1 - Wheat
2 - Barley
3 - Rye
4 - Oats
5 - Flax
6 - Corn

For a complete list of crop codes, write NCIS.

Date of Storm: Month and day that the loss occurred.

Date Application Signed: Date the application was signed.

Discount: Discount percentage applied to the rate for any kind of premium discount, such as a cash discount.

Index: NCIS assigned sequence number to make the record key information unique, if necessary. No statistical value.

Insurance (liability): Amount of insurance from the application.

Insurance Applying to Loss: On loss records, only the amount of insurance which applied to the loss is recorded.

Insurance per Acre: Amount of liability per acre.

Interest: On tonnage loss forms, the insured's percentage interest in the crop. Used in arithmetic to compute total loss.

Item Number: Company item number, if needed.

Percent Loss: Total loss given as a percentage from the proof of loss form.

Policy Form: A code to indicate the type of coverage. These codes vary by state and year but will always be unique within state and year. For example,

Oklahoma, 1988 coverages

01 - Basic coverage form, NCIS filed rates
52 - Basic coverage form, independently filed rates
85 - 10 percent disappearing deductible form, independently filed rates (DX10 IF)
43 - 20 percent deductible, increasing payment form, NCIS filed (XS20IP)

For a complete list of policy form codes by state, write NCIS.

Policy Number: Company assigned number for a policy. This number should always be unique for a company/state/year combination.

Premium Discount: Code used to indicate percentage discount when computing premium. For example,

0 - No discount

Gross premium reported (premium dollars do not reflect the discount)

5 - 4% discount
6 - 20% discount
7 - 25% discount

Net premium reported

I - 4% discount
D - 20% discount
C - 25% discount

Premium: Premium dollars from the application.

Price per Ton: Used on tonnage loss records to compute total loss.

Range: Numeric code for the range portion of the location of the crop being covered by this policy.

Rate: Percentage rate used to compute premium, obtained from the application.

Social Security Number: Insured's social security number.

State: Two character state code. For example,

01 - Alabama
02 - Arizona

Status: System status when record received. No statistical value.

Township: township code of the location of the crop being covered.

Type: Indicates type of record received. Same usage as CARD.

Exhibit 2

CROP-HAIL INSURANCE

ILLUSTRATION OF ALTERNATIVE POLICY FORMS

Define: R = Yield Reduction (percent)
P = Amount payable

XS10 – EXCESS OVER 10% LOSS

P = (R - 10%)

DX10 -- 10% DISAPPEARING DEDUCTIBLE

		R	< 10%	P = 0%
10%	<	R	< 50%	P = 1.25 X (R - 10%)
		R	> 50%	P = R

XS101P -- EXCESS OVER 10% LOSS - INCREASING PAYMENT

		R	< 10%	P = 0%
10%	<	R	< 70%	P = (R - 10%)
		R	> 70%	P = (R - 10%) + (R - 70%)
				P </= 100%

(in this form, when yield reduction exceeds 70%, an additional 1% is paid for each percent of yield reduction in excess of 70%)

DXS5 - EXCESS OVER 5% DISAPPEARING AT 25%

		R	< 5%	P = 0%
10%	<	R	< 25%	P = (R - 5%) x 1.25
		R	> 25%	P = R

Exhibit 3 (Page 1)

State	Separately Rated Crops
Alabama	Cotton
Arizona	Cotton
Arkansas	Cotton Wheat Soybeans Rice
Colorado	Wheat Corn Potatoes
Florida	Tobacco
Georgia	Tobacco Cotton
Idaho	Wheat Barley Potatoes Peas Tree Fruit
Illinois	Corn Soybeans
Indiana	Tobacco Corn Soybeans
Iowa	Corn Soybeans
Kansas	Wheat Corn
Kentucky	Tobacco
Louisiana	Cotton

State	Separately Rated Crops
Michigan	Corn, Wheat Tree Fruit
Minnesota	Corn, Wheat Soybeans
Mississippi	Cotton
Missouri	Cotton Wheat Soybeans Corn Tobacco
Montana	Wheat Barley
Nebraska	Corn, Wheat
New Mexico	Cotton Wheat
North Carolina	Tobacco Cotton Tree Fruit
North Dakota	Wheat
Ohio	Corn, Wheat Soybeans Tobacco
Oklahoma	Wheat
Oregon	Wheat
South Carolina	Tobacco Cotton Tree Fruit

Exhibit 3 (Page 3)

State	Separately Rated Crops
South Dakota	Corn Wheat
Tennessee	Burley Tobacco Dark Tobacco
Texas	Cotton Wheat Maize
Utah	Wheat
Virginia	Tobacco
Washington	Wheat Tree Fruit
Wisconsin	Corn Potatoes Tobacco
Wyoming	Wheat

Exhibit 4

POLICY FORM COMPARISON ANALYSIS
MINNESOTA SOYBEANS 1957-1990

	Base form: BASIC		Analyzed form: XS10IP		
(1)	(2)	(3)	(4)	(5)	(6)
1991 Rate Area	Liability* (nearest $1000)	Actual Losses (nearest $1000)	Computed Losses (nearest $1000)	Policy Form Factor: Actual	Trend
6.00	5,404	1,145	727	0.63	0.58
6.50	1,920	405	253	0.62	0.59
7.00	6,982	1,530	985	0.64	0.59
7.50	5,365	812	428	0.53	0.60
8.00	10,755	2,031	1,218	0.60	0.60
8.50	6,756	1,240	727	0.59	0.61
9.00	30,558	5,436	3,143	0.58	0.62
9.50	5,120	1,002	611	0.61	0.62
10.00	17,972	3,720	2,384	0.64	0.63
10.50	7,828	1,758	1,146	0.65	0.63
11.00	28,615	6,168	3,939	0.64	0.64
11.50	14,530	2,884	1,701	0.59	0.64
12.00	21,919	4,959	3,220	0.65	0.65
12.50	23,708	5,170	3,297	0.64	0.66
13.00	46,325	11,527	7,841	0.68	0.66
13.50	31,155	7,444	4,912	0.66	0.67
14.00	36,065	8,285	5,454	0.66	0.67
14.50	26,197	6,208	4,183	0.67	0.68
15.00	42,731	10,827	7,449	0.69	0.68
15.50	24,797	6,695	4,729	0.71	0.69
16.00	47,698	12,383	8,595	0.69	0.70
17.00	40,135	10,632	7,445	0.70	0.71
18.00	23,177	7,401	5,662	0.76	0.72
19.00	27,733	8,052	5,875	0.73	0.73
STATE	533,444	127,713	85,922	0.67	

* Liability with loss

Exihibit 5

NATIONAL CROP INSURANCE SERVICES 9/7/89

CROP COMPARISON ANALYSIS
MINNESOTA 1948 - 1988

1990 RATE AREA	LIABILITY (nearest $1000) Base WHEAT	Crop 2 BARLEY	LOSS COST Base WHEAT	Crop 2 BARLEY	LOSS COST AS % OF BASE L/C Crop 2 BARLEY
2.00	43,315	12,254	0.40	0.88	220 %
2.25	9,498	2,662	0.54	1.33	246
2.50	74,888	22,041	0.87	1.07	123
2.75	49,885	20,152	1.19	2.08	175
3.00	28,033	10,381	2.58	4.01	155
3.25	62,837	22,698	2.03	3.24	160
3.50	76,069	28,203	1.67	2.55	153
3.75	38,535	12,111	1.66	3.68	222
4.00	108,518	48,539	2.02	3.00	149
4.50	106,479	43,374	2.54	3.53	139
5.00	81,573	41,009	2.90	4.26	147
5.50	56,667	26,156	3.06	4.96	162
6.00	36,989	16,122	3.31	3.78	114
6.50	41,271	18,944	4.40	4.97	113
7.00	32,436	15,083	4.15	6.22	150
7.50	20,277	12,533	5.14	5.46	106
8.00	6,557	3,799	4.56	7.25	159
8.50	13,163	6,221	3.85	5.28	137
9.00	15,888	9,277	4.68	6.60	141
TOTALS AND AVERAGES	902,878	371,559	2.36	3.64	150 %*
			INDICATED CROP FACTOR:		1.50

* Weighted by designated liability

Exhibit 6

HISTORICAL TOWNSHIP LOSS COSTS

102N 28W, FARIBAULT COUNTY, MINNESOTA

Year	Liability (000)	Loss Cost Year	Loss Cost Cumulative	Percent Change
48	11	5.99	5.99	
49	10		3.14	-91%
50	14		1.88	-67%
51	21	8.23	4.26	56%
52	20	7.40	5.09	16%
53	33		3.55	-43%
54	42		2.56	-39%
55	30	11.77	4.09	37%
56	14	7.42	4.33	6%
57	55	18.18	7.37	41%
58	105	22.82	11.94	38%
59	74	0.08	9.90	-21%
60	72	0.14	8.49	-17%
61	49	18.48	9.38	9%
62	56	1.43	8.65	-8%
63	73	25.99	10.51	18%
64	133	0.15	8.82	-19%
65	122	2.46	7.99	-10%
66	156	0.06	6.85	-17%
67	186	35.82	11.07	38%
68	224	0.70	9.52	-16%
69	273	0.79	8.18	-16%
70	196	0.77	7.44	-10%
71	231	8.07	7.51	1%
72	370	62.74	15.46	51%
73	456		13.13	-18%
74	497	0.98	11.42	-15%
75	456		10.11	-13%
76	645	3.38	9.17	-10%
77	787	0.19	7.86	-17%
78	1338	41.31	14.49	46%
79	345	4.59	14.01	-3%
80	574	19.41	14.42	3%
81	1041	21.82	15.30	6%
82	1026	1.61	13.86	-10%
83	873	46.22	16.52	16%
84	1132	1.44	15.07	-10%
85	335	13.41	15.02	-0%
86	558	5.21	14.59	-3%
87	170	3.34	14.44	-1%
88	184	3.31	14.28	-1%
89	121	10.81	14.30	0%
90	167	0.35	14.12	-1%

Exhibit 7A

	YEAR	LIABILITY (000)	LOST COST	Loss Cost Limited to 5 X Median	7.5 X Median	10 X Median
	49	10				
	50	14				
	53	33				
	54	42				
	73	456				
	75	456				
	66	156	0.06	0.06	0.06	0.06
	59	74	0.08	0.08	0.08	0.08
	60	72	0.14	0.14	0.14	0.14
	64	133	0.15	0.15	0.15	0.15
	77	787	0.19	0.19	0.19	0.19
	90	167	0.35	0.35	0.35	0.35
	68	224	0.70	0.70	0.70	0.70
	70	196	0.77	0.77	0.77	0.77
	69	273	0.79	0.79	0.79	0.79
	74	497	0.98	0.98	0.98	0.98
	62	56	1.43	1.43	1.43	1.43
	84	1132	1.44	1.44	1.44	1.44
	82	1026	1.61	1.61	1.61	1.61
	65	122	2.46	2.46	2.46	2.46
	88	184	3.31	3.31	3.31	3.31
	87	170	3.34	3.34	3.34	3.34
	76	645	3.38	3.38	3.38	3.38
	79	345	4.59	4.59	4.59	4.59
MEDIAN	86	558	5.21	5.21	5.21	5.21
	48	11	5.99	5.99	5.99	5.99
	52	20	7.40	7.40	7.40	7.40
	56	14	7.42	7.42	7.42	7.42
	71	231	8.07	8.07	8.07	8.07
	51	21	8.23	8.23	8.23	8.23
	89	121	10.81	10.81	10.81	10.81
	55	30	11.77	11.77	11.77	11.77
	85	335	13.41	13.41	13.41	13.41
	57	55	18.18	18.18	18.18	18.18
	61	49	18.48	18.48	18.48	18.48
	80	574	19.41	19.41	19.41	19.41
	81	1041	21.82	21.82	21.82	21.82
	58	105	22.82	22.82	22.82	22.82
	63	73	25.99	25.99	25.99	25.99
	67	186	35.82	26.05	35.82	35.82
	78	1338	41.31	26.05	39.34	41.31
	83	873	46.22	26.05	39.34	46.22
	72	370	62.74	26.05	39.34	52.10
	Variance of non-zero loss costs		213.45	86.86	147.55	186.82
	Limited Losses		1,868,357	1,334,169	1,695,240	1,828,989
	Variance Reduction			0.593	0.309	0.125
	Loss Reduction			0.286	0.093	0.021
	Test Statistic			2.074	3.332	5.920

Exhibit 7B

TEST STATISTICS FOR ALL MULTIPLES **1993 MINNESOTA SOYBEANS**

MULTIPLE	ACTUAL VARIANCE	NORMAL VARIANCE	% VAR. REDUCED	ACTUAL LOSSES	NORMAL LOSSES	% LOSS REDUCED	TEST STATISTIC
19.3	211.8149	191.1365	9.7625	238,353,170	229,712,094	3.6253	2.6929
19.2	211.8149	190.8559	9.8950	238,353,170	229,601,030	3.6719	2.6948
19.1	211.8149	190.5664	10.0316	238,353,170	229,482,038	3.7218	2.6953
19.0	211.8149	190.2737	10.1698	238,353,170	229,359,865	3.7731	2.6954
18.9	211.8149	189.9761	10.3103	238,353,170	229,235,171	3.8254	2.6952
18.8	211.8149	189.6724	10.4537	238,353,170	229,108,493	3.8786	2.6952
18.7	211.8149	189.3655	10.5986	238,353,170	228,981,300	3.9319	2.6955
18.6	211.8149	189.0529	10.7462	238,353,170	228,852,542	3.9859	2.6960
18.5	211.8149	188.7373	10.8951	238,353,170	228,722,103	4.0407	2.6964
18.4	211.8149	188.4185	11.0457	238,353,170	228,590,962	4.0957	2.6969
18.3	211.8149	188.0988	11.1966	238,353,170	228,458,408	4.1513	2.6971
18.2	211.8149	187.7765	11.3488	238,353,170	228,325,357	4.2071	2.6975
18.1	211.8149	187.4541	11.5010	238,353,170	228,191,033	4.2635	2.6976
18.0	211.8149	187.1305	11.6537	238,353,170	228,054,764	4.3207	2.6972
17.9	211.8149	186.8047	11.8076	238,353,170	227,915,943	4.3789	2.6965
17.8	211.8149	186.4760	11.9627	238,353,170	227,776,199	4.4375	2.6958
17.7	211.8149	186.1439	12.1195	238,353,170	227,636,303	4.4962	2.6955
17.6	211.8149	185.8095	12.2774	238,353,170	227,496,211	4.5550	2.6954
17.5	211.8149	185.4755	12.4351	238,353,170	227,355,897	4.6139	2.6952
17.4	211.8149	185.1408	12.5931	238,353,170	227,212,101	4.6742	2.6942
17.3	211.8149	184.8025	12.7528	238,353,170	227,068,061	4.7346	2.6935
17.2	211.8149	184.4610	12.9141	238,353,170	226,923,044	4.7955	2.6930
17.1	211.8149	184.1146	13.0776	238,353,170	226,775,187	4.8575	2.6923
17.0	211.8149	183.7609	13.2446	238,353,170	226,624,657	4.9206	2.6916
16.9	211.8149	183.3974	13.4162	238,353,170	226,469,458	4.9858	2.6909
16.8	211.8149	183.0246	13.5922	238,353,170	226,311,561	5.0520	2.6905
16.7	211.8149	182.6457	13.7711	238,353,170	226,153,136	5.1185	2.6905
16.6	211.8149	182.2642	13.9512	238,353,170	225,992,997	5.1857	2.6903
16.5	211.8149	181.8774	14.1338	238,353,170	225,827,806	5.2550	2.6896
16.4	211.8149	181.4780	14.3224	238,353,170	225,657,240	5.3265	2.6889

Exhibit 8

NATIONAL CROP INSURANCE SERVICES
CH510F
MINNESOTA SOYBEANS

1993
FALC ANALYSIS

BASED ON PERIOD 1948-1991

RATE YEAR	GROUP	CRD	LOCATION	LIABILITY	NORMAL LOSSES	NORMAL LOSS COSTS (CATASTROPHE REMOVED) LOC	9TWP	25TWP	CTY	CRD	FALC	FALC (WITH CATASTROPHE)
			043 FARIBAULT									
1993	011	80	101N 024W	9,563,497	1,111,590	11.62	11.08	9.85	9.66	7.70	10.21	10.93
1993	011	80	101N 025W	16,561,343	1,300,599	7.85	10.49	9.64	9.66	7.70	9.59	10.27
1993	011	80	101N 026W	12,988,682	1,101,977	8.48	9.66	9.94	9.66	7.70	9.75	10.44
1993	011	80	101N 027W	9,019,172	877,991	9.73	9.69	10.16	9.66	7.70	10.05	10.76
1993	011	80	101N 028W	12,740,136	1,146,023	9.00	10.26	9.85	9.66	7.70	9.83	10.53
1993	011	80	102N 024W	9,932,217	1,203,010	12.11	10.04	9.96	9.66	7.70	10.19	10.91
1993	011	80	102N 025W	15,087,464	2,214,799	14.68	9.94	9.66	9.66	7.70	10.20	10.92
1993	011	80	102N 026W	14,796,310	1,351,013	9.13	9.50	9.66	9.66	7.70	9.58	10.26
1993	011	80	102N 027W	10,561,707	784,427	7.43	9.83	9.63	9.66	7.70	9.44	10.11
1993	011	80	102N 028W	13,992,899	1,921,161	13.73	10.34	9.44	9.66	7.70	10.00	10.71
1993	011	80	103N 024W	11,833,624	783,674	6.62	10.48	10.29	9.66	7.70	9.95	10.66
1993	011	80	103N 025W	14,200,817	1,174,070	8.27	10.13	9.89	9.66	7.70	9.76	10.45
1993	011	80	103N 026W	15,771,457	1,754,943	11.13	9.01	9.75	9.66	7.70	9.78	10.47
1993	011	80	103N 027W	8,240,148	578,737	7.02	9.30	9.20	9.66	7.70	9.00	9.64
1993	011	80	103N 028W	9,342,278	1,043,508	11.17	9.90	8.69	9.66	7.70	9.12	9.77
1993	011	80	104N 024W	14,202,029	2,077,986	14.63	10.95	9.74	9.66	7.70	10.41	11.15

Exhibit 9

1993 MINNESOTA GRAINS

REDISTRIBUTION FACTORS

(1) CRD	(2) TOTAL LOSSES	(3) LIMITED LOSSES	(4) CATASTROPHE LOSSES	(5) UNLIMITED FACTOR	(6) LEVEL 1 FACTOR[a)]	(7) LEVEL 2 LOSSES[b)]
10	35,201,057	33,488,591	1,712,466	1.051	1.051	0
20	435,734	430,702	5,032	1.012	1.012	0
30	0	0	0	1.000	1.000	0
40	21,035,626	20,196,211	839,415	1.042	1.042	0
50	13,090,093	12,449,736	640,357	1.051	1.051	0
60	957,318	892,114	65,204	1.073	1.073	0
70	13,917,098	12,944,887	972,211	1.075	1.075	0
80	30,421,459	23,950,154	6,471,305	1.270	1.197	1,746,671
90	11,816,147	11,131,421	684,726	1.062	1.061	0
STATE	126,874,532	115,483,816	11,390,716	1.099		1,746,671

a) Column (5) limited to a maximum of 1.197

b) Column (3) x [Column (5) - Column (6)]

Exhibit 10

ANTICIPATED LOSS RATIO SCHEDULE

RATE	ALR : % FALC	% EXPENSES AND PROFIT
Under $0.99	50 %	50 %
1.00 - 1.99	52	48
2.00 - 2.99	53	47
3.00 - 3.99	54	46
4.00 - 4.99	55	45
5.00 - 5.99	56	44
6.00 - 6.99	57	43
7.00 - 7.99	58	42
8.00 - 8.99	59	41
9.00 - 9.99	60	40
10.00 - 10.99	61	39
11.00 - 11.99	62	38
12.00 - 12.99	63	37
13.00 - 13.99	64	36
14.00 - 14.99	65	35
15.00 - 15.99	66	34
16.00 - 16.99	67	33
17.00 - 17.99	68	32
18.00 - 18.99	69	31
19.00 and Over	70	30

CHIAA CROP-HAIL POLICY

The Name of Company

This policy is signed by the President and Secretary of the company. One of our authorized representatives must also countersign the policy before it is valid.

(Signature) (Signature)

Secretary President

YOUR CROP-HAIL INSURANCE POLICY

Quick Reference

Your Crop-Hail policy is composed of four parts:

1) Part I —Consists of your APPLICATION OR DECLARATION PAGE for this insurance which contains the schedule of insurance, description and location of crops insured, and binder provisions.

2) Part II —The SPECIAL PROVISIONS and ENDORSEMENTS, if any, tailor the coverage to meet the needs of the crops grown within your state and to conform to the laws and regulations of the state.

3) Part III —The following GENERAL PROVISIONS are the same for all policies written in the United States.

4) Part IV —EXPLANATION OF POLICY TERMS.

IMPORTANT: This Quick Reference is not part of the Crop-Hail Policy and does not provide coverage. *Refer to the Crop-Hail Policy itself for the actual contractual provisions.*

PLEASE READ THE CROP-HAIL POLICY CAREFULLY

EXPLANATION OF POLICY TERMS

Throughout this policy "you" and "your" refer to the "named insured" shown in the Application or Declarations, and "we", "us" and "our" refer to the Company providing this insurance. In addition, certain words and phrases are defined as follows:

1. **"Insured"** means you.

2. **"Schedule of Insurance"** is the list of crops, locations, and amounts of insurance for which you have made application.

3. **"Harvest"**: the act or process of gathering in a crop.

4. **"Replant"**: to reseed or transplant due to the condition of the original crop.

5. **"Feasible to Replant"** means that the remaining growing season is considered sufficient for a crop to reach maturity.

6. **"Insured Crop"** means a crop described in the Schedule of Insurance for which a specific amount of insurance and premium charge has been indicated.

7. **"CHIAA"**: Crop-Hail Insurance Actuarial Association.

8. **"Unit of Insurance"**: Throughout this policy the acre is the unit of insurance. This means that the limit of insurance applying to loss on any acre may not exceed the limit per acre in the Schedule of Insurance.

   This also means to the extent a crop is insured for less than its value you are self insured. As an example of how this works, assume a crop is worth $100 per acre and you insured it for only $50 per acre; assume also that there has been a yield reduction of 40% due to hail. If there is no Excess Over Loss or Deductible applying, the amount payable is 40% of $50 per acre (or $20.00 per acre), whereas the actual amount of the loss is 40% of $100 (or $40.00 per acre), and you are thus self insured for the difference of $20.00 per acre.

9. **"Crop Yield"** means the production per acre that the insured crop would reasonably be expected to produce at harvest. The production per acre is usually expressed in terms of bushels, pounds, tonnage, etc.

OPTIONAL COMPANY INFORMATION

AGREEMENT TO INSURE: We will provide the insurance described in this policy in return for the premium and *compliance with all applicable provisions.*

1. COVERAGE.
We cover the crops specified at the locations described in the schedule of insurance.

We do not cover crops that have been damaged by hail prior to signing the application.

2. INSURANCE PERIOD.
The insurance is in effect from the time the crop is clearly visible above the ground until the crop is harvested, except as follows:

a. No coverage is in effect until 12:01 a.m. following the date you signed the application.

b. For some crops there is an additional waiting period if shown in the Special Provisions or in a special crop endorsment.

c. Coverage expires on the dates shown in the Special Provisions or special crop endorsement.

d. *Increase of Existing Insurance*
Insurance added to this policy becomes effective at 12:01 a.m. following the date of the revised Schedule of Insurance or as otherwise provided in the Special Provisions or special crop endorsement.

e. *Decrease of Existing Insurance*
Reduction or cancellation of insurance will be effective at 12:01 a.m. of the date requested.

3. DUTIES AFTER LOSS.

a. *Your Duties Are:*
In case of a probable loss to crops insured under this policy you must:

(1) Give written notice to us within 10 days after the occurrence.

(2) Preserve in each damaged field of insured crop samples of the remaining damaged crop for our examination.

(3) Allow us to examine the damaged crop as often as we reasonably require.

(4) Upon our request provide a complete harvesting and marketing record of each insured crop.

(5) Upon our request submit to examination under oath.

(6) Sign a Withdrawal of Claim when our inspection of the crop determines there is no payable loss under the terms of this policy.

(7) Within 60 days after your loss, unless we extend such time in writing, submit to us a signed statement in proof of loss declaring your loss and interest in the crop.

b. *Our Duties Are:*

(1) Adjust all losses.

(2) Pay the loss within 30 days after we reach agreement with you, entry of a final judgment, or the filing of any appraisal award with us.

c. *Adjustment Procedures.*
We recognize and apply the Loss Adjustment Procedures used by the Crop Insurance Industry.

d. *Deferred Adjustment.*
At times it may be necessary for us to defer the adjustment of a covered loss until the actual loss can be determined. We will not pay for reduction of yield resulting from your failure to care for the crop during the deferral period.

4. LOSS PAYMENT.

a. The amount payable per acre will be the limit of insurance applying on the date of the loss multiplied by the percentage the crop yield is reduced because of the loss. However, the amount payable may not exceed the actual cash value of the portion of the crop destroyed by perils insured against.

b. If a crop loss is also covered by other insurance, we will pay only the proportion of the loss that our limit of insurance bears to the total amount of insurance, except that no Federal Crop Insurance policy or Multiple Peril Crop Insurance policy will be prorated with this policy.

5. REDUCTION OF INSURANCE.
The limit of insurance applying to each acre of insured crop will be reduced:

a. By the gross percentage of loss determined for each loss.

b. By the same percentage as each acre of crop is harvested.

6. APPRAISAL.
If you and we fail to agree on the percentage the yield is reduced because of the loss, the following procedure will be used:

a. One of us will demand in writing that the percentage of yield reduction be set by appraisal.

b. Each of us will select a competent appraiser and notify the other of the appraiser's identity within 10 days after receipt of the written demand.

c. The two appraisers will then select a competent, impartial umpire. If the two appraisers are unable to agree upon an umpire within 10 days, you or we can ask a judge of a court of record in the state in which the insured crop is grown to select an umpire.

d. The appraisers will then set the percentage of yield reduction. If the appraisers submit a written report of an agreement to us, the amount agreed upon will be the percentage of yield reduction.

e. If the appraisers fail to agree within a reasonable time, they will submit their difference to the umpire. Written agreement signed by any two of these three will set the percentage of yield reduction.

Each appraiser will be paid by the party selecting that appraiser. Other expenses of the appraisal and compensation of the umpire will be paid equally by you and us.

We will not be held to have waived any of our rights by any act relating to appraisal.

7. LIBERALIZATION.
If we adopt any revision which would broaden the coverage under this policy without additional premium, the broadened coverage will apply.

8. VARIATION IN ACREAGE IN CASE OF LOSS.
When the actual acreage of a crop differs from the number of acres stated by item in the Schedule of Insurance:

a. A revised Schedule of Insurance per acre will be obtained by dividing the limit of insurance by the actual acreage at the location for such item.

b. The total insurance per acre on your insured interest will not exceed the value of the crop at the time of loss.

9. WAIVER OR CHANGE OF POLICY PROVISIONS.
A waiver or change of any provision must be in writing and approved by us. Our request for an appraisal or examination will not waive any of our rights.

10. ASSIGNMENT OF INTEREST.
You may not assign your interest in this policy without our written consent.

11. ASSIGNMENT OF INDEMNITY.
You may assign to another party your right to an indemnity for the crop year only on our form and with our approval. The assignee will have the right to submit the loss notices and forms required by the policy.

12. CONCEALMENT OR FRAUD.
We do not provide coverage for any insured who has intentionally concealed or misrepresented any material fact or circumstance relating to this insurance, either before or after a loss.

13. CANCELLATION OF POLICY. (Except as provided in Special Provisions)

a. *By You:*
If you cancel or reduce coverage prior to inception of the insurance period we will refund your paid premium for the amount of insurance cancelled. If you cancel or reduce coverage during the insurance period we will not refund any premium.

b. *By Us:*
We may cancel all or any part of the insurance provided by us at any time by notifying you at least 10 days before the date and hour cancellation takes effect. Notices of cancellation may be delivered or mailed to you at your mailing address shown in the declarations. Proof of mailing will be sufficient proof of notice.

If we cancel all or any part of this policy, we will return the premium paid for the amount of insurance per acre on the portion cancelled.
(State law exceptions to the 10 days notice of cancellation, if any, are contained in the Special Provisions.)

14. EXCLUSIONS.
We do not cover:

a. Loss from any peril not insured against, even though the loss may have occurred in conjunction with a peril insured against.

b. Loss of any portion of a crop recoverable by harvesting equipment.

c. Loss due to your neglect or failure to harvest mature crops.

d. Injury or damage to the vegetative or flowering portion of any plant, tree or shrub, except to the extent that the injury results in a reduction of yield of that crop.

e. Any loss that has been contributed to by nuclear reaction, radiation, or radioactive contamination, all whether controlled or uncontrolled or however caused, or any consequence of any of these.

15. ABANDONMENT OF CROP.
We will not accept abandonment to us of any interest in any crop.

16. SUIT AGAINST US.
You cannot bring suit or action against us unless you have complied with all of the policy provisions.

If you do enter suit against us you must do so within 12 months of the occurrence causing loss or damage.
(State law exceptions to the 12 months limitation, if any, are contained in the Special Provisions.)

17. CONFORMITY TO STATUTES.
If any terms of this policy are in conflict with statutes of the state in which this policy is issued the policy will conform to such statutes.

18. SUBROGATION (Recovery of loss from a third party.)
Because you may be able to recover all or a part of your loss from someone other than us, you must do all you can to preserve any such rights. If we pay you for your loss then your right of recovery will belong to us. If we recover more than we paid you plus our expenses, the excess will be paid to you.

SPECIAL PROVISIONS

Oklahoma

1. PERILS INSURED AGAINST.

We insure for direct loss to crops described in the Schedule of Insurance caused by:

a. *Hail*

b. *Fire and Lightning*
We cover loss by fire and lightning before harvest and while crop is still in the harvester.

c. *Transit Coverage (Except Cotton)*
While the harvested crop is being transported to the first place of storage not to exceed 50 miles, this policy is extended to cover loss caused by:

(1) Fire and Lightning
(2) Windstorm
(3) Collision
(4) Overturn
(5) Collapse of bridges, docks and culverts

However, *Transit Coverage* is excess over any other valid and collectible insurance.

FIRST PLACE OF STORAGE means any drying apparatus, drying bins or storage facility of any kind.

d. *Fire Department Service Charge*
We will pay up to $250 for your obligation assumed by contract or agreement for fire department charges incurred when the fire department is called to save or protect the unharvested crop.

No Excess Over Loss or Deductible will apply to Fire, Lightning and Transit Coverage or Fire Department Service Charge.

2. MINIMUM LOSS.

We will not cover any loss until the percentage of yield reduction per acre equals 5% or more of the crop, nor any loss in addition to a paid loss until such additional reduction in yield equals 5% or more of the original crop.

3. CATASTROPHE LOSS AWARD.

When a loss exceeds 70% on any acre of the insured crop an additional amount of one-half of the percent of loss that is in excess of 70% will be paid. However:

a. the total amount payable per acre will not exceed the amount of insurance applying at the time of loss;

b. this award will not be paid if the loss is subject to any Excess Over Loss or Deductible provision which does not disappear at or less than 70% loss.

4. CANNING BEANS AND CANNING PEAS.

Insurance on canning beans and canning peas will expire 60 days after the crop is clearly visible above the ground.

5. CORN AND SORGHUM.

On corn grown for seed purposes, and on popcorn or sweet corn, the amount of any loss will be determined in the same manner as for ordinary field corn. On sorghum crops grown for seed purposes, the amount of any loss will be determined in the same manner as for ordinary field sorghum.

6. COTTON.

We do not cover cotton bolls immature at the time of a killing frost or freeze.

7. HAY, FORAGE AND GRASS CROPS.

a. For hay, forage or other crops harvested more than once each growing season, the limit of insurance per acre provided for each cutting or harvest will be determined by dividing the total insurance per acre by the number of cuttings or harvests.
b. If your schedule of insurance specifies a limit of insurance per acre for each cutting or harvest, Section (a) will not apply.
c. When hay and grass crops grown for seed are insured:

(1) The insurance will apply only to the cutting to be harvested for seed.
(2) Until the seed is set, a maximum of 25% of the insurance per acre stated in the Schedule of Insurance will apply.

8. REPLANTING DESTROYED CROPS.

When any acre of crop has been damaged by hail to the extent that replanting is necessary, and replanting to the same or a substitute crop is feasible under the growing conditions where such crop is grown, we will reimburse you for your actual expense of replanting not to exceed the following percentage of the limit of insurance applying to each acre of the insured crop, whether the crop is replanted or not.

Cotton:
Basic Form 10%
DXS10 Form 8%
XS20IP Form 7%
Other crops, all forms 20%

The limit of insurance will be reduced by the amount of the replanting award. The insurance will continue on the replanted crop if of like kind; if not of like kind, the insurance will transfer to the substitute crop at the appropriate premium upon approval by us.

9. EXPIRATION OF INSURANCE.
Coverage ceases at 12:01 a.m. on the following dates of the current year:

	Hail coverage:	Fire, Lightning and Transit coverage:
Barley		
Cimarron, Texas, and Beaver Counties . . .	July 25	July 25
All other counties.	July 15	July 15
Corn	October 15 . .	December 15
Cotton	December 15 . .	December 15
Combine maize	November 15 . .	December 15
Milo maize	November 15 . .	December 15
Oats		
Cimarron, Texas, and Beaver Counties . . .	July 25	July 25
All other counties. .	July 15	July 15
Rye		
Cimarron, Texas, and Beaver Counties . . .	July 25	July 25
All other counties. .	July 15	July 15
Sorghum crops	November 15 . .	December 15
Soybeans	November 15 . .	November 15
Wheat		
Cimarron, Texas and Beaver Counties . . .	July 25	July 25
All other counties. .	July 15	July 15
All crops not specified. .	October 15	October 15

OPTIONAL PROVISIONS

Your application and rate of premium determine whether your coverage will be amended by one of the following optional provisions.

EXCESS OVER 10% LOSS—DISAPPEARING AT 50%—PROVISION—(SYMBOL: DXS10)

We will not cover any loss until the percentage of yield reduction per acre exceeds 10%. The percentage per acre then payable will be the percent in excess of 10%, multiplied by 1.25. Once the percent of yield reduction equals or exceeds 50% this provision will no longer apply. The payable percentage may not exceed 100%.

When the percentage of yield reduction once exceeds 10%, thereafter the "Minimum Loss" provision will apply to any subsequent loss(es).

EXCESS OVER 20% LOSS—INCREASING PAYMENT PROVISION (SYMBOL: XS20IP)

We do not cover any loss until the reduction in yield per acre exceeds 20%; the percentage per acre then payable will be the percent in excess of 20%, multiplied by 1.25. The payable percentage may not exceed 100%.

When the percentage of yield reduction once exceeds 20%, thereafter the "Minimum Loss" provision will apply to any subsequent loss(es).

REPORT ON BIAS IN FALC DETERMINATION

Since the new crop hail rating method was implemented in 1990, there have been questions about how well this system works. One area of concern is whether there is any bias introduced by the Final Average Loss Cost (FALC) mix and the Catastrophe procedure.

In the new Catastrophe procedure, losses in excess of a specified amount are removed from local experience and gathered into State and Crop Reporting District loss pools. The remaining losses are called "normal" losses. The initial estimate of the FALC for each location is based on a weighted average of location normal loss costs and normal loss costs from surrounding areas. It should not consistently over- or under-estimate local normal loss costs. Normal "implied" losses are defined for each location as

NORMAL IMPLIED LOSS = FALC (w/o catastrophe) x LIABILITY.

If there is no consistent bias in the FALC calculation, then the total implied losses for the state should not deviate significantly from statewide normal losses.

After the initial FALC estimates are computed, the catastrophic losses are redistributed by means of factors applied to the FALC. The FALC with catastrophe should not consistently over- or under-estimate local loss costs. Total implied losses are calculated as

TOTAL IMPLIED LOSS = FALC (w/catastrophe) x LIABILITY,

Total implied losses for the state should not deviate significantly from statewide total losses.

Table 1 lists several of the township rated states for which a rate analysis or FALC analysis has been done using the new rating methods. Also listed is the amount by which total implied losses deviated from total losses and the percent by which implied losses deviated from normal and total losses.

Deviations from normal losses are quite small in each case. It is clear that the FALC mix does not consistently inflate or deflate losses. That the deviations from total losses don't differ much from the deviations from normal losses would indicate that the catastrophe loading procedure does not create any bias.

Areas with low liability have a different FALC mix than do areas with adequate liability. To examine the effects of the change in FALC mix, townships were separated by amount of liability. Tables 2 and 3 are examples of the results from this analysis. The amount of deviation from actual losses in the low liability areas varied considerably by crop and state. In some cases, implied losses in low liability areas differed quite a bit from actual losses. However, because the losses in these areas are so small, they have little impact overall.

DEVIATIONS OF IMPLIED LOSSES FROM ACTUAL LOSSES

RATE ANALYSIS YEAR	STATE	CROP	DEV. FROM TOTAL LOSSES	% DEV. FROM TOTAL LOSSES	% DEV. FROM NORMAL LOSSES
1990	IDAHO	BARLEY	($248,865)	-1.0%	-1.0%
1990	IDAHO	PEAS	($178,230)	-2.0%	-2.1%
1990	IDAHO	POTATOES	($48,036)	-0.4%	-0.4%
1990	IDAHO	WHEAT	($711,612)	-2.5%	-2.5%
1991	ILLINOIS	CORN	$526,560	0.7%	0.7%
1991	ILLINOIS	SOYBEANS	$859,841	0.6%	0.5%
1990	IOWA	CORN	$637,725	0.3%	0.3%
1990	IOWA	SOYBEANS	$2,432,748	0.5%	0.5%
1990	KANSAS	CORN	$46,746	0.1%	0.1%
1990	KANSAS	WHEAT	$590,628	0.3%	0.3%
1990	MINNESOTA	GRAINS	$51,511	0.0%	0.1%
1990	MINNESOTA	SOYBEANS	$2,691,539	1.3%	1.3%
1991	MONTANA	BARLEY	($132,994)	-0.4%	-0.3%
1991	MONTANA	WHEAT	($139,851)	-0.1%	-0.1%
1990	NEBRASKA	GRAINS	($363,953)	-0.1%	-0.1%
1990	N. DAKOTA	WHEAT	$783,800	0.3%	0.3%
1990	OKLAHOMA	WHEAT	$324,816	0.5%	0.5%
1991	OREGON	GRAINS	$17,994	0.2%	0.2%
1990	S. DAKOTA	CORN	$308,176	0.7%	0.7%
1990	S. DAKOTA	WHEAT	($369,777)	-0.4%	-0.4%
1991	WASHINGTON	TREE FRUIT	$21,032	0.1%	0.2%
1991	WASHINGTON	WHEAT	($459,228)	-3.4%	-3.3%

TABLE 1.

TABLE 2. 1990 IDAHO BARLEY

LIABILITY	NORMAL LOSSES	IMPLIED NORMAL LOSSES	% DEVIATION FROM NORMAL
LOW	163,047	245,595	0.3
NORMAL	24,744,793	24,415,535	-1.3
TOTALS	24,907,840	24,661,130	-1.0

LIABILITY	TOTAL LOSSES	IMPLIED TOTAL LOSSES	% DEVIATION FROM TOTAL
LOW	163,047	251,392	0.3
NORMAL	25,594,542	25,257,332	-1.3
TOTALS	25,757,589	25,508,724	-1.0

TABLE 3. 1990 IDAHO PEAS

LIABILITY	NORMAL LOSSES	IMPLIED NORMAL LOSSES	% DEVIATION FROM NORMAL
LOW	119,084	149,211	0.4
NORMAL	8,074,647	7,875,001	-2.4
TOTALS	8,193,731	8,024,212	-2.1

LIABILITY	TOTAL LOSSES	IMPLIED TOTAL LOSSES	% DEVIATION FROM TOTAL
LOW	119,084	155,991	0.4
NORMAL	8,654,480	8,439,343	-2.5
TOTALS	8,773,564	8,595,334	-2.0

AN ACTUARIAL APPROACH TO PROPERTY CATASTROPHE COVER RATING

Daniel F. Gogol

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ABSTRACT

Forty-one years of catastrophe loss data by state are used in this study to produce a model for rating catastrophe covers for insurers in any region of the Continental United States. Smooth surfaces are fitted to the data by region, and experience rating is applied in an attempt to give appropriate weight to regional departures from the smoothed results. Severity distributions and frequencies are estimated for each region and a method for applying them in pricing catastrophe covers is discussed. A method for using the experience of an insurer to produce an experience modification is also presented.

I. INTRODUCTION

United States catastrophe cover rating is an interesting problem from both practical and theoretical points of view.

On the practical side, it is an important untreated problem. No systematic attempt at using insurance loss data to produce catastrophe cover rates can be found in insurance literature, (Discussions of methods involving weather data are in Clark [4] and Friedman [6].) Catastrophe rates fluctuate greatly in the various regions of the country depending on the supply of capacity and on whether there has been a large catastrophe in the area recently. Pricing practices were not much different two decades ago when Ingrey [9] stated:

> The general yardstick is the "payback period," or, in how many years will a total loss be amortized in advance. Payback periods depend upon location, type of business written and past experience in addition to the basic ingredients of amount of capacity required, subject premium and rate. The adequacy of the initial retention is largely overlooked as are the incremental functions of exposure types, to wit, a company writing mobile homes has a much greater incremental exposure function than another insurer writing private dwellings.

Catastrophe rating is also a challenging theoretical problem. The number of large catastrophes in any region is small, so it is important to use the experience of surrounding areas as well. It is useful to examine the relationship between catastrophe

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experience and the longitude, latitude, and distance from the coast of a region. Also, the size of a region affects the probability of a catastrophe destroying more than a given percentage of property value.

By fitting a smooth surface that is a function of these variables to catastrophe loss data, it is possible to base estimates of expected losses for each region on more than just its own experience. Expected losses by region clearly have a smoother pattern than the sparse data.

An attempt can be made to estimate the appropriate credibility to be given to the actual experience of a region, as opposed to the weight given to the expected losses indicated by a fitted smooth surface. If the indications of smoothed surfaces and the actual experience of a region are credibility weighted to estimate the expected number of catastrophes for the region in various loss size intervals, a loss distribution may be fitted to the estimates in order to smooth them in a reasonable way and also to estimate tail probabilities.

II. THE MODEL

A. Data

To compare the relative destructive power of two natural catastrophes, such as windstorms, hitting different states, it is useful to consider the amount of property insurance premium in each state, as well as the amount of insured property damage in each state. The insured loss in each state will depend not only on the intensity and size of the catastrophe but also on the insured property in the area.

"Catastrophe premium," defined below, will be used as the exposure base to which loss data is related. The definition is based on Ingrey [9]. It is intended that the catastrophe premiums derived from each line of business be in roughly the same proportion to expected catastrophe losses for the line. Ingrey does not present data to support the percentages used in the formula but indicates that they were developed with the cooperation of Allen Hinkelman, Excess and Casualty Reinsurance Association; Daniel Holland, Inland Marine Insurance Bureau; Donald Kifer, New York Fire Insurance Rating Organization; and Allen Royer, Multi-Line Insurance Rating Board. Data on catastrophe losses by line will be discussed in section III.

Catastrophe premium = (10% of inland marine premium) + (10% of commercial multiple peril) + (80% of allied lines) + (10% of auto physical damage) + (20% of farmowners) + (100% of earthquake) + (20% of homeowners) + (15% of ocean marine) (1)

An estimate, for example, that the proportion of homeowners losses caused by catastrophes is twice as high as the proportion of auto physical damage losses is implicit in the formula, since the corresponding percentages of premium are 20% and 10%.

Actually, Ingrey's formula also includes 60% of mobile homes premium and 80% of difference in conditions premium, but these premiums are small and they were omitted.

Some formula for catastrophe premium is often used by underwriters in evaluating a company's catastrophe exposure. Additional insight is given by expressing the loss layer to be reinsured in terms of percentages of the catastrophe premium, for example 200% xs 20%. In this paper, layers expressed as percentages of state or regional catastrophe premium are studied. Methods of applying the study to individual company catastrophe cover rating will be discussed later.

Catastrophe covers are generally for a high enough layer so that an event must cause losses to several of a company's risks in order to produce a loss to the cover. Windstorms are the most frequent causes of losses to these covers. Other frequent causes are winter freezes, hail, and flooding. Fire is a less frequent cause.

The loss data used [11] in this study was produced by Property Claim Services (PCS) in Rahway, New Jersey and includes estimated insured loss for each United States catastrophe having an estimate of $1 million or more from 1949 through 1981 and $5 million or more from 1982 through 1989. (Note that the worst catastrophe loss year in recent history, 1989, is included in the data.) In order to be included, a loss must affect many insureds, although the exact number of insureds that must be affected has not been defined. (It is generally at least 1,000.) For each catastrophe, the estimated insured loss in each state is given. The PCS estimates are based on an extrapolation of estimates made by a set of insurers writing most of the property premium in the region of the catastrophe.

Although PCS insured loss estimates are used in the study, a loss development factor will be applied in section III, which describes the method of rating catastrophe covers.

For each of 28 overlapping regions of the continental United States, catastrophe premium was estimated for 1949-89. Gross written premium data by state from *Best's Executive Data Service*, and for older years from *The Spectator*, which is no longer published, was used to compute catastrophe premiums by state for approximately every fifth year. Exponential interpolation was used for other years, based on the computed catastrophe premiums.

For each of the 28 regions mentioned above, the estimated insured loss to the region from each catastrophe from 1949-89 was divided by the region's catastrophe premium for the year of the loss. The ratios R of individual losses to corresponding catastrophe premiums were then grouped into the somewhat arbitarily chosen intervals $8\%<R\leq16\%$, $16\%<R\leq32\%$, $32\%<R\leq64\%$, and $R>64\%$.

The number of ratios falling in each interval for each region is shown in Exhibit 1. Exhibit 2 may be helpful in connection with Exhibit 1 as well as later exhibits.

There is a theory that hurricane frequency should increase due to global warming, but no evidence of this was found in the data so no trend factor was applied. The loss trend and the premium trend are assumed to approximately cancel each other out.

EXHIBIT 1

FREQUENCIES BY REGION

Region	Interval of Ratio R			
	8% < R ≤ 16%	16% < R ≤ 32%	32% < R ≤ 64%	R > 64%
1. CA	3	1	2	0
2. AZ,NM,NV,UT,CO	10	4	1	1
3. TX	22	1	4	3
4. AL,MS,LA	14	3	5	5
5. FL	4	5	2	5
6. GA,SC,NC	8	6	4	2
7. TN,AR,OK	23	8	1	0
8. OR,WA,ID	4	1	0	1
9. ND,SD,WY,MT	4	5	1	1
10. MN,WI	13	6	5	1
11. NE,KS	22	9	4	1
12. IA,MO,IL	11	6	0	0
13. MI,IN,OH	6	2	1	1
14. KY,WV,PA	6	1	4	0
15. VA,NJ,DE,MD,DC	6	2	1	2
16. NY,VT	2	2	1	0
17. ME,NH,MA,RI,CT	7	5	0	2
18. 1,2(above)	3	3	1	0
19. 8,9	8	3	0	1
20. 3,4	8	7	2	6
21. 5,6,7	18	4	3	1
22. 10,11,12	14	4	0	0
23. 13,14	7	3	1	0
24. 15,16,17	1	2	1	2
25. 1,2,8,9	3	1	2	0
26. 3,4,7,10,11,12	11	4	3	1
27. 5,6,13,14,15,16,17	5	2	3	1
28. Continental U.S.	9	4	2	0
	252	104	54	37

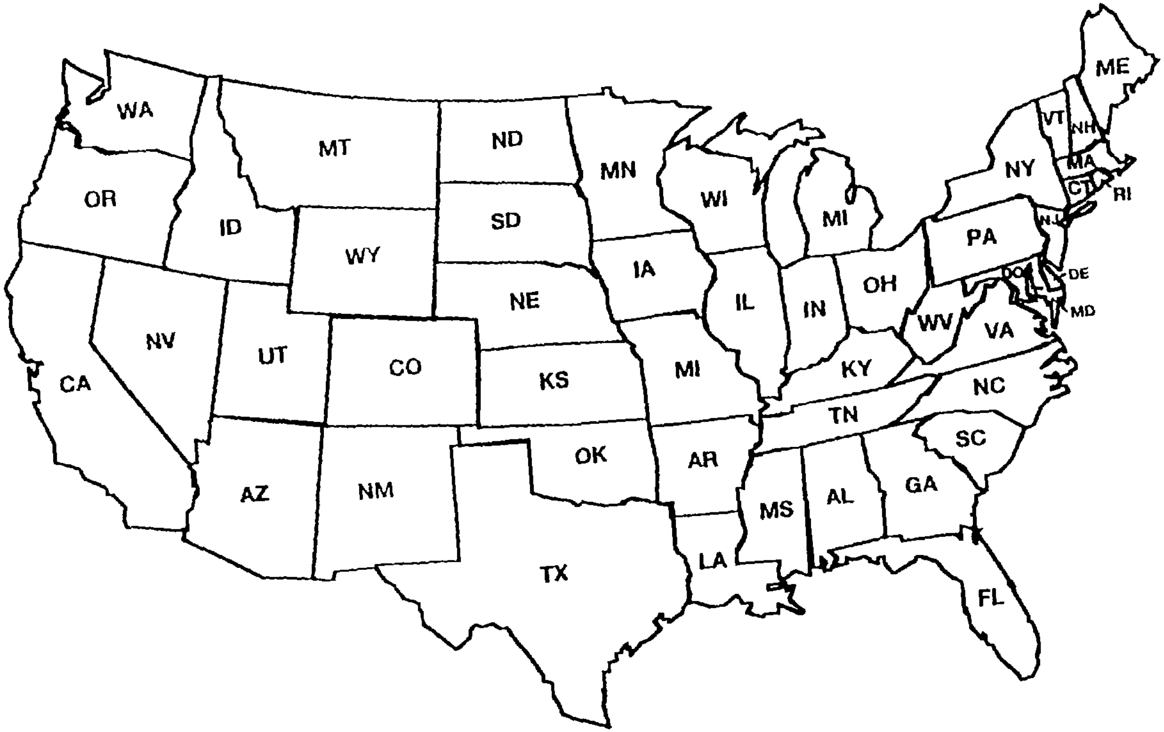

EXHIBIT 2
CONTINENTAL U.S.

B. Smoothing the Data

The expected values of frequencies in each interval vary more smoothly as a function of regions than the data in Exhibit 1, since the data includes random variation.

Most catastrophes are windstorms, and their frequency and severity is related to a region's latitude, longitude, and distance from the coast (Clark [4] and Friedman [6]). The probability distribution of the ratios of catastrophe losses to catastrophe premium is also related to the size of a region. The above facts motivate the attempt to use multiple regression for each interval of R values to fit the frequencies in Exhibit 1 to functions of the latitude, longitude, distance from the coast, and area of the 28 regions.

Multiple regression was used to relate the above variables to frequency of catastrophes in each of the intervals $8\%<R\leq16\%$, $16\%<R\leq32\%$, $32\%<R\leq64\%$, $R>64\%$, $R>32\%$, $R>16\%$, and $R>8\%$. The intervals are purposely chosen in an overlapping manner for a reason explained in section IID.

The details of the regressions are in Appendix A. A comparison of actual to fitted freqencies for four of the intervals is in Exhibit 3.

C. Experience Rating the Regions

Weights will be selected for the actual and fitted frequencies

EXHIBIT 3

COMPARISON OF ACTUAL (A) TO FITTED (F) FREQUENCIES

	Interval of Ratio R							
	8% < R ≤ 16%		16% < R ≤ 32%		32% < R ≤ 64%		R > 64%	
REGION	A	F	A	F	A	F	A	F
1	3	5.71	1	2.91	2	1.84	0	1.71
2	10	5.61	4	2.62	1	1.84	1	0.74
3	22	17.60	1	5.31	4	3.82	3	1.86
4	14	17.77	3	5.61	5	3.82	5	3.25
5	4	7.23	5	3.24	2	4.32	5	4.45
6	8	6.15	6	2.93	4	2.44	2	1.65
7	23	16.11	8	5.53	1	2.61	0	1.35
8	4	4.73	1	2.79	0	0.90	1	0.73
9	4	4.59	5	2.69	1	0.82	1	0.44
10	13	12.72	6	5.65	5	1.01	1	0.68
11	22	14.46	9	5.53	4	1.70	1	0.87
12	11	14.43	6	5.46	0	1.70	0	0.85
13	6	5.14	2	2.95	1	1.20	1	0.70
14	6	5.54	1	3.00	4	1.59	0	0.88
15	6	5.60	2	3.20	1	1.59	2	1.51
16	2	4.97	2	3.20	1	0.99	0	1.07
17	7	4.92	5	3.24	0	0.94	2	2.75
18	3	5.59	3	2.58	1	1.84	0	0.82
19	8	4.62	3	2.60	0	0.86	1	0.51
20	8	17.48	7	5.12	2	3.82	6	1.95
21	18	6.21	4	2.71	3	2.69	1	2.02
22	14	13.73	4	5.07	0	1.48	0	0.79
23	7	5.27	3	2.79	1	1.38	0	0.71
24	1	5.05	2	2.88	1	1.14	2	1.13
25	3	5.10	1	2.47	2	1.32	0	0.57
26	11	15.70	4	4.80	3	2.61	1	1.22
27	5	5.53	2	2.61	3	1.75	1	0.91
28	9	14.44	4	4.49	2	1.97	0	0.89
	252	252.00	104	103.98	54	53.99	37	37.01

in Exhibit 3 to produce estimates of expected frequencies by interval and region. The sum of the weights will be one. An explanation of the method of selecting them is as follows.

For each interval i of R values, and each region j, let the random variable $X_{i,j}$ be the frequency of catastrophes in a randomly selected 41 year period. The fitted values for interval i and region j in Exhibit 3 are estimates of the expected value of $X_{i,j}$. If each fitted value is assumed to be the mean of a probability distribution of possible expected values of $X_{i,j}$, then it can be seen that a more accurate estimate of the expected value can be produced by giving weight (credibility) to the actual frequency as well as to the fitted frequency.

The partly judgemental basis for selecting the following experience rating formula is explained in Appendix B. The number of actual catastrophes in interval i and region j is given credibility $a_{i,j}/(a_{i,j}+k_i)$ where $a_{i,j}$ is the fitted frequency for interval i and region j and

$k_i = 9$ for i = 1,2,5,6 or 7, $k_i = 6$ for i = 3 or 4 (2)

where, for each interval, i is as in Table 3 of Appendix A.

D. Nested Application of Experience Rating System

For each region, experience rating is applied to estimate expected values for the frequencies in each interval of R values. A nested process is used so that the estimates of expected frequencies for $8\%<R\leq16\%$ and $R>16\%$ are based not only on the separate experience for $8\%<R\leq16\%$ and $R>16\%$, respectively, but also on the total experience for $R>8\%$.

By applying the experience rating formula for the interval $R>8\%$, estimates A_j of the frequency in this interval are produced for each region j. The estimates B_j and C_j produced by applying the experience rating system to the intervals $8\%<R\leq16\%$ and $R>16\%$ are then multiplied by a constant D_j such that $A_j = D_j(B_j + C_j)$. The estimates D_jB_j and D_jC_j for the frequencies in region j for intervals $8\%<R\leq16\%$ and $R>16\%$, respectively, thus add up to the estimate for region j for the interval $8\%<R$ and are each in the same proportion to the estimates B_j and C_j, respectively. It is intended that D_jB_j and D_jC_j approximate the expected values of the frequencies in region j for intervals $8\%<R\leq16\%$ and $R>16\%$, respectively, given that the total of the two expected values is A_j, and that B_j and C_j are the estimates of the two expected values based on their separate data.

The weighted frequencies by region produced by directly applying the experience rating formulas for the intervals $16\%<R\leq32\%$ and $R>32\%$ are then adjusted so that their sum equals the estimate for $R>16\%$. The method is entirely similar to the method used above to adjust the estimates for $8\%<R\leq16\%$ and $R>16\%$ so that their sum equaled the estimate for $R>8\%$.

This nested process is continued until estimates are produced for each of the seven intervals. The estimates for four of the intervals are in Exhibit 4.

E. Loss Distributions by Region

The estimates of expected frequency for each region produced by the above nested application of experience rating for $8\%<R\leq16\%$, $16\%<R\leq32\%$, $32\%<R\leq64\%$, and $R>64\%$ were divided by the estimate produced for $R>8\%$ and the resulting fractions f_1, f_2, f_3, f_4 were fitted to a probability distribution. The probability distribution was then used to allocate the estimate of expected frequency for $R>8\%$ to the above four intervals. The selected yearly frequencies are the above frequencies divided by 41, since 41 years of data were used. The yearly frequencies for $R>8\%$ are in Table 1.

The single parameter Pareto distribution was used for all 28 regions. It generally was a good fit. A comparison of the estimates produced by the experience rating method in the previous section, and by the single parameter Pareto, is in Exhibit 4. The two parameter Pareto did not perform better, nor did the Burr or other distributions tested. (A study of distributions can be found in Hogg and Klugman [8].)

The single parameter Pareto was used even in regions for which another distribution fit better. This was because the generally good fit of the single parameter Pareto led to the conclusion that it was a good model for the data, and small amounts of data in particular regions were not considered credible enough to counteract this conclusion.

See Appendix C for a discussion of the method used to fit the single parameter Pareto. The parameters of the Pareto curves used are in Table 1.

EXHIBIT 4

COMPARISON OF EXPERIENCE RATED FREQUENCIES WITH FITTED PARETO FREQUENCIES

	EXPERIENCE RATED FREQUENCIES				FITTED PARETO FREQUENCIES			
REGION	8% < R ≤ 16%	16% < R ≤ 32%	32% < R ≤ 64%	R > 64%	8% < R ≤ 16%	16% < R ≤ 32%	32% < R ≤ 64%	R > 64%
1	4.42	1.91	1.41	1.00	4.40	2.18	1.08	1.07
2	7.79	3.10	1.94	0.91	7.96	3.35	1.41	1.02
3	20.42	3.51	3.80	2.08	19.35	6.79	2.39	1.29
4	15.03	4.80	4.28	3.86	13.49	6.98	3.62	3.88
5	5.70	3.44	3.35	4.68	5.84	3.85	2.54	4.93
6	7.32	4.28	3.81	2.27	8.27	4.40	2.34	2.67
7	20.85	6.63	2.11	1.09	20.70	6.74	2.19	1.06
8	4.40	1.96	0.61	0.59	4.47	1.83	0.75	0.52
9	4.57	3.57	0.97	0.55	5.68	2.34	0.96	0.67
10	13.23	7.26	1.89	0.85	15.07	5.29	1.86	1.01
11	19.89	8.52	2.78	1.11	21.27	7.26	2.48	1.29
12	11.78	5.03	1.13	0.63	12.57	4.06	1.31	0.63
13	5.66	2.62	1.31	0.82	5.82	2.57	1.13	0.90
14	5.91	2.41	2.39	0.88	6.40	2.86	1.28	1.04
15	5.99	2.75	1.55	1.70	5.78	2.99	1.55	1.66
16	3.76	2.42	0.84	0.77	4.13	1.94	0.91	0.81
17	6.19	4.24	0.58	1.78	8.17	2.95	1.06	0.60
18	4.36	2.29	1.42	0.63	4.86	2.14	0.96	0.75
19	6.09	2.67	0.71	0.52	6.34	2.32	0.85	0.49
20	10.81	6.38	3.59	3.39	11.40	6.03	3.18	3.57
21	12.48	3.36	3.03	1.92	11.48	5.14	2.30	1.86
22	13.24	3.90	0.82	0.49	13.37	3.68	1.01	0.39
23	6.15	2.71	1.35	0.65	6.42	2.63	1.07	0.74
24	3.42	2.51	0.99	1.12	3.78	2.00	1.06	1.18
25	4.09	1.82	1.16	0.42	4.45	1.81	0.73	0.50
26	12.06	4.50	2.36	1.02	12.28	4.72	1.81	1.13
27	5.36	2.46	2.09	0.94	5.79	2.70	1.26	1.10
28	10.22	3.99	1.37	0.54	10.69	3.60	1.21	0.62
	251.19	105.04	53.64	37.21	260.23	105.15	44.29	37.38

Table 1

Frequencies (F') and Parameters (P)

Region	F'	P	Region	F'	P	Region	F'	P	Region	F'	P
1	.213	1.01	8	.184	1.29	15	.292	.95	22	.450	1.86
2	.335	1.25	9	.235	1.28	16	.190	1.09	23	.265	1.29
3	.727	1.51	10	.566	1.51	17	.312	1.47	24	.196	.92
4	.682	.95	11	.788	1.55	18	.212	1.18	25	.183	1.30
5	.419	.60	12	.453	1.63	19	.244	1.45	26	.487	1.38
6	.431	.91	13	.254	1.18	20	.590	.92	27	.265	1.10
7	.749	1.62	14	.282	1.16	21	.507	1.16	28	.393	1.57

A Pareto parameter of 1 or less implies infinite expected losses for unlimited layers. For $0<P<1$, the expected losses in the layer between a and b are $(b^{1-P}-a^{1-P})/(1-P)$, which approaches infinity as b approaches infinity. In reality, catastrophe losses are limited by the total insured value, so the frequency distribution falls below a Pareto at some point. Although Pareto parameters of 1 or less were selected for some regions, they are only intended to be used in estimating expected losses for limited layers of sizes that are actually reinsured. The Pareto's overestimate of frequency far out in the tail does not have a great effect in estimating expected losses for these layers. The frequency of losses above x times the truncation point is x^{-P} times the frequency above the truncation point. Since $P>0$, this fraction x^{-P} approaches zero as x approaches infinity.

III. RATING CATASTROPHE COVERS

A. Using the Model

Rates for catastrophe covers include a risk charge, but the discussion here will be of expected losses rather than risk.

A reinsurer evaluating a catastrophe cover often receives a breakdown of the ceding company's subject property premium by state and line. The commercial multiple peril, homeowners, farmowners and auto physical damage premiums which are considered to be subject to a catastrophe treaty are sometimes only a percentage (usually approximately 65%, 90%, 90%, and 35%, respectively) of the total premiums for those lines. It is necessary to adjust for this in order to apply the catastrophe premium formula in this paper to the cedant.

If the cedant does not provide this information, estimates of catastrophe premium by state for a primary company can be made by using the company's major direct premium writings by state, and its net written premiums by line, from Best's Insurance Reports.

Based on the above type of information, and on Table 2, one of the 28 regions may be selected judgmentally as being approximately representative of the region in which the company writes.

Table 2

1988 Catastrophe Premiums by Region (in 000's)

Region	Premium	Region	Premium	Region	Premium	Region	Premium
1	1,757,793	8	365,904	15	890,083	22	1,484,958
2	473,889	9	180,551	16	973,760	23	1,793,682
3	881,629	10	238,494	17	789,209	24	2,653,051
4	521,551	11	273,418	18	2,231,681	25	2,778,136
5	668,967	12	973,046	19	546,455	26	3,366,938
6	700,932	13	1,110,098	20	1,403,180	27	5,816,632
7	478,800	14	683,584	21	1,848,699	28	11,961,706

For any region selected as representative of the company, the selected yearly frequency for catastrophe losses greater than 8% of catastrophe premium, and the selected Pareto distribution, may be found in Table 1. They may be used to compute an estimate of expected losses for any layer of a catastrophe cover by expressing the layer in terms of percentages of the company's total catastrophe premium. An example of the rating method will be given at the end of this section, but several related points will be discussed first.

The method to be used in the example is based on historical data, but due to the potential for an enormously damaging earthquake in California, and the small number of earthquakes in the historical data used, expected losses from catastrophes in California are widely believed to be greater than the estimate that would be based on historical data.

The model in this paper used gross losses, while catastrophe reinsurance covers losses net of excess reinsurance. It is assumed implicitly in the rating method presented that gross catastrophe losses are approximately the same percentage of gross premium that net catastrophe losses are of net premiums.

An adjustment will be made in the rating method for catastrophe covers to reflect the fact that the model in this paper is based on data for regions rather than for individual reinsurers. By the use of certain definitions and reasonable assumptions, the following statement could be made more precise and proven mathematically. On average, for catastrophe losses as defined by

PCS, the probability distribution of ratios of catastrophe losses to catastrophe premiums has the same mean for an insurer within a region as for the region, but greater variance.

The rating method which will be applied to individual insurers uses .9 times the Pareto parameter in Table 1 for the region selected as representative of the insurer. This is to reflect the fact that the distributions for individual insurers have greater variance, on the average, than the distribution for the region.

The expected frequencies from Table 1 will be used unadjusted for individual insurers. The expected frequency of catastrophe losses, as defined by PCS, is less for an individual insurer than for the surrounding region. However, the assumption of a smaller Pareto parameter for individual insurers implies that for some percentage P, the expected frequency for R>P% is the same for the individual insurer as for the region. The estimate that P equals 8% is implicit in the use of the expected frequencies from Table 1 for individual insurers.

The estimate that ultimate insured losses for catastrophes, on the average, are 1.15 times as great as the PCS estimates will be used in estimating expected losses for catastrophe covers. Since the PCS estimate is made within a few days of the catastrophe, it is natural to expect development. Also, the PCS estimate excludes all ocean marine and crop losses, and some inland marine and business interruption losses.

The .9 factor for Pareto parameters and the 1.15 factor for losses have the combined effect of significantly raising estimated expected losses for catastrophe covers. The resulting expected losses, as a percentage of actual premiums charged, have been found to be a reasonable match to actual loss ratios for the catastrophe cover premium of two reinsurers over a twenty year and a twelve year period respectively. This premium totaled almost $300 million and consisted of shares of a much greater amount of premium.

The application of the model to estimating expected losses for catastrophe covers is as follows.

Example

Suppose that a primary insurer, in the latest year for which data is available, had writings for which region 23 is considered the best match.

Suppose that, using cp to represent the insurer's catastrophe premium, the layer to be reinsured can be expressed as (2.00 cp) excess of (.20cp).

The selections in Table 1 for Region 23 were .265 catastrophe losses per year greater than 8% of catastrophe premium, and a Pareto parameter of 1.29. The loss development factor of 1.15 and the adjustment factor to the Pareto parameter of .9 which were discussed above are used. Therefore, .265 is the frequency for R>9.2%, and the Pareto parameter becomes 1.15. The expected losses in one year to the layer above therefore are as follows:

$$.265(.092cp)(((.20/.092)^{-.16} - (2.20/.092)^{-.16})/(.16)) \quad (3)$$

(See Philbrick [8].) This equals 4.29% of catastrophe premium.

If it is not clear which region is the best match for the primary insurer, the above method may be used for more than one region, and a final estimate may be judgementally selected.

B. Underwriting Judgement

Since the above estimate is based on data from the entire region, it may be useful to judgementally modify it if the ceding company is believed to be not typical of the region. For example, the ceding company may have a very high or low percentage of its insured property near the coast, where exposure to hurricanes is greatest.

C. The Catastrophe Premium Formula

The estimated expected catastrophe losses for individual insurers were affected by the choice of percentages by line in the catastrophe premium formula defined in section II.

If the percentages by line that were used in the formula are multiplied by the corresponding premiums in Table 4, an approximation of the relative amounts of expected catastrophe losses by line can be derived. (Although fire premium is a portion of the property premium in Table 4, it was not included in the catastrophe premium formula as it was considered to account for only a negligible portion of catastrophe losses.)

Table 4

Industry Premiums for Selected Lines - 1990

	Premiums Earned (Millions)
Fire	4,494
Allied Lines	2,097
Farmowners Multiple Peril	968
Homeowners Multiple Peril	18,116
Commercial Multiple Peril	17,626
Ocean Marine	1,169
Inland Marine	4,441
Earthquake	459
Auto Physical Damage	35,185

Some data suggests that for hurricanes a much lower percentage of losses come from auto physical damage than would be estimated based on the catastrophe premium formula. In [1], the All-Industry Research Advisory Council estimated the following percentages of losses by line for seven hurricanes in 1983-85: homeowners multiple peril 46.8%, commercial multiple peril 22.2%, auto physical damage 3.7%, all other 27.3%.

The only other data on catastrophe losses by line that the author knows of was produced by ISO for homeowners losses by individual catastrophe for 1970-78. It indicates that homeowners and dwelling extended coverage losses are 19.6% and 2.7%, respectively, of total catastrophe losses as estimated by PCS for the same catastrophes. (The ISO estimates, like the PCS estimates, are an extrapolation of total insured losses based on data from a set of insurers in the region.) The percentage of total catastrophe losses produced by homeowners is much less in the ISO data for all catastrophes combined than in the AIRAC hurricane data. Therefore, the percentage of auto physical damage losses may well be much greater for all catastrophes combined than for hurricanes.

Hurricanes produced $6.35 billion in catastrophe losses in 1981-90 as compared to $9.7 billion from hail and tornadoes and $3.7 billion from winter storms, according to PCS.

If so desired, the catastrophe cover rating method used in this paper can be applied with a catastrophe premium formula having different percentages by line from those used. Any alternative percentages used should be chosen so that, when multiplied by the premiums in Table 4, they produce the same catastrophe premium as the percentages in this paper's formula. If this is done, then Table 1 approximates the corresponding table that would have been created if the alternative catastrophe premium formula had been used in the study. Therefore, the rating method used in this paper still gives an estimate of expected losses from catastrophes if the alternative catastrophe premium formula is used.

D. Experience Rating a Catastrophe Risk

Suppose the amount of each catastrophe loss of the ceding company for a certain time period is known. The frequency of these

EXHIBIT 5

REGIONAL FREQUENCIES BY TIME PERIOD

	Interval of Ratio R							
	8% < R ≤ 16%		16% < R ≤ 32%		32% < R ≤ 64%		R > 64%	
REGION	1949-69	1970-89	1949-69	1970-89	1949-69	1970-89	1949-69	1970-89
1	1	2	1	0	1	1	0	0
2	2	8	3	1	0	1	0	1
3	10	12	0	1	1	3	1	2
4	4	10	0	3	4	1	3	2
5	2	2	1	4	1	1	5	0
6	4	4	3	3	2	2	1	1
7	9	14	4	4	0	1	0	0
8	0	4	0	1	0	0	1	0
9	2	2	3	2	1	0	1	0
10	3	10	2	4	3	2	1	0
11	9	13	3	6	3	1	1	0
12	7	4	4	2	0	0	0	0
13	4	2	2	0	1	0	0	1
14	2	4	0	1	2	2	0	0
15	3	3	2	0	0	1	2	0
16	1	1	1	1	1	0	0	0
17	1	6	4	1	0	0	2	0
18	0	3	2	1	0	1	0	0
19	3	5	2	1	0	0	1	0
20	3	5	3	4	1	1	3	3
21	7	11	3	1	3	0	0	1
22	7	7	4	0	0	0	0	0
23	4	3	3	0	0	1	0	0
24	1	0	0	2	1	0	2	0
25	1	2	1	0	1	1	0	0
26	7	4	1	3	1	2	1	0
27	2	3	1	1	2	1	1	0
28	1	8	3	1	0	2	0	0

losses in intervals expressed in terms of ratios to the company's catastrophe premium can be compared to the experience of the region selected as being representative of the company. Exhibit 5, which shows experience for 1949-69 and 1970-89 separately, may be useful for this comparison. An example of a judgmental experience rating is given below.

Example

Suppose that insurance company A had eight catastrophes greater than 9.2% (i.e. 8% times our selected development factor) of catastrophe premium in the period 1970-89 and that the region selected as corresponding to it had five catastrophes greater than 9.2% of catastrophe premium in the same period.

Suppose that the formula n/(n+9), where n is the number of catastrophes in the region in 1970-89, is the credibility assigned to the experience of Company A. (This formula is similar to one used in this paper to assign credibility to the actual frequency of catastrophes in a region.)

The credibility weighted frequency is then (5/(5+9))(8) + (9/(5+9))(5), which equals 6.07. The modifier produced by the experience rating is thus 6.07/5, i.e. 1.21. This modifier is then applied to the expected losses for the reinsured layer that are estimated as in formula (3).

IV. CONCLUSION

A model which can be used to estimate expected losses to catastrophe covers based on insured loss data has been presented. An example of the application of the model to a specific cover was given. The obstacles to using actuarial methods in catastrophe rating are not so great as has sometimes been suggested.

The application of actuarial science gives a very useful and much needed perspective in this area.

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APPENDIX A

DETAILS OF REGRESSIONS

By the center of a region is meant the point such that half the area is to the north, half to the east, half to the west, and half to the south. For each of the 28 regions, the latitude and longitude of the center of the region were estimated and were considered to be the latitude and longitude of the region. By the distance to the coast of a region is meant the length of the shortest line from the center to any ocean.

The independent variables used in the regression were x_1, x_2, x_3, and x_4, such that, for each region

x_1 = latitude of region

$x_2 = 0$ if $92 \leq$ (longitude of region) ≤ 99,

$x_2 = |\text{longitude} - 99|$ if $99 < \text{longitude} < 105$,

$x_2 = 6$ if $\text{longitude} \geq 105$,

$x_2 = |\text{longitude} - 92|$ if $86 < \text{longitude} < 92$,

$x_2 = 6$ if $\text{longitude} \leq 86$

x_3 = ln(ln(area, in thousands of miles, of region))

x_4 = ln(ln(distance, in miles, from coast of region))

The values of x_1, x_2, x_3, x_4, for the 28 regions are given in Exhibit 6. For each of the seven intervals for R, the dependent variable used in the regression for the interval was ln(frequency of catastrophes).

EXHIBIT 6

VALUES OF INDEPENDENT VARIABLES

Region	X_1	X_2	X_3	X_4
1	37	6	1.612	1.535
2	37	6	1.838	1.824
3	31.5	0	1.715	1.708
4	31.5	0	1.596	1.513
5	28	6	1.381	1.303
6	34	6	1.596	1.582
7	35.5	0	1.626	1.790
8	44.5	6	1.703	1.758
9	45.5	6	1.787	1.924
10	45.5	0	1.581	1.936
11	40	0	1.626	1.903
12	40	0	1.654	1.909
13	41.5	6	1.581	1.818
14	38.5	6	1.548	1.767
15	38.5	6	1.405	1.582
16	43.5	6	1.405	1.652
17	44	6	1.381	1.303
18	37	6	1.876	1.780
19	45	6	1.862	1.868
20	31.5	0	1.796	1.684
21	33	6	1.767	1.504
22	41.5	0	1.813	1.902
23	40	6	1.703	1.817
24	42	6	1.640	1.629
25	40.5	6	1.970	1.868
26	35.5	0	1.935	1.798
27	37.5	6	1.854	1.740
28	38.5	0	2.078	1.870

In cases that frequency was zero, ln(1/3) was judgmentally used instead of the undefined ln(0).

For each interval I_i, of R values, there is a corresponding set of frequencies by region $\{f_{i,j}\}$, j an integer from 1 to 28. Fitted values $Y_{i,j}$ were produced by regression and then the function

$$g_i(y_{i,j}) = (\exp(y_{i,j}))((\sum_{j=1}^{28} f_{i,j})/\sum_{j=1}^{28} \exp(y_{i,j})) \quad (4)$$

was used to produce values $g_i(y_{i,j})$ such that $\sum_{j=1}^{28} g_i(y_{i,j}) = \sum_{j=1}^{28} f_{i,j}$. The values $g_i(y_{i,j})$, rather than $y_{i,j}$, were used as final fitted values for the frequencies $f_{i,j}$.

Tornadoes are more prevalent in the region between latitude 92 and 99, which helps explain the motivation for the definition of the variable x_2.

The interval R>64% was the only one for which x_4 was used. It appears that distance from the coast is a useful variable for large hurricanes, but not for smaller catastrophes such as tornadoes. The variable x_4 didn't work well for intervals for which $R \leq 64\%$, possibly due to collinearity with the longitude variable. The coefficient came out only negligibly negative or even positive.

Positive coefficients for any of the variables x_1, x_2, x_3,x_4 were considered counter to the overall indications of the data and not appropriate for use in the study. For all intervals, all the variables x_1, x_2, and x_3 were used unless one of them had a positive coefficient. In these cases, a regression was done without using that variable.

In order to use certain theorems concerning the accuracy of the regressions, it would have to be true that:

1. A linear relationship exists between the independent variables used and the expected values of the dependent variables used.
2. The probability distributions of the values of the dependent variable are uncorrelated, and possess a common variance.

Neither condition is satisfied. Nothing can be done to satisfy the first condition unless a way is known to transform the variables so that they satisfy a linear relationship. Therefore, it was considered better to avoid the complication of transforming variables in an attempt to come closer to satisfying the second condition. The results of the rgression are considered to be simply a useful method of smoothing the data.

The functions resulting from the regressions are shown in Table 3.

Table 3

Interval	Function
1. $8\% < R \leq 16\%$	$-.024x_1 - .167x_2 - .083x_3 + 3.694$
2. $16\% < R \leq 32\%$	$-.00005x_1 - .108x_2 - .461x_3 + 2.312$
3. $32\% < R \leq 64\%$	$-.095x_1 - .035x_2 + 4.169$
4. $R > 64\%$	$-.030x_1 - .069x_2 - .241x_3 - 2.719x_4 + 6.457$
5. $R > 32\%$	$-.102x_1 - .002x_2 - .808x_3 + 6.150$
6. $R > 16\%$	$-.047x_1 - .987x_2 - .720x_3 + 5.172$
7. $R > 8\%$	$-.035x_1 - .119x_2 - .596x_3 + 5.393$

APPENDIX B

DERIVATION OF FORMULA (2)

In order to approximate an experience rating formula, we assume

1. Given that $g_i(y_{i,j})$ is the fitted value for interval i and region j in the smoothing method of this paper, the probability distribution of the expected value $E_{i,j}$ of the frequency of catastrophes in interval i and region j has mean $g_i(y_{i,j})$.

2. For each i, the probability distribution of $E_{i,j}$ has the same coefficient of variation C_i for each j.

It follows that, for each interval i and each region j, the Z such that

Z(actual frequency in interval i and region j) + $(1-Z)g_i(y_{i,j})$ (4)

is the best least squares estimate of the expected value of the frequency in interval i and region j is

$$g_i(y_{i,j})/(g_i(y_{i,j}) + 1/C_i^2) \qquad (5)$$

The proof is as follows. By Buhlmann's theorem (Buhlmann [3], Herzog [7]), $Z = H_{i,j}/(H_{i,j}+P_{i,j})$ where $H_{i,j}$ equals the variance of the probability distribution of the expected value of the frequency for interval i and region j, and $P_{i,j}$ equals the expected value of the variance of the frequency, given the above probability distribution for the expected value of the frequency.

For each possible value x for the expected value of the frequency, the probability distribution of actual values is Poisson and has variance x. Therefore, $P_{i,j} = g_i(y_{i,j})$.

By assumption 2, above, $H_{i,j} = (C_i g_i(y_{i,j}))^2$. Therefore,

$$z = C_i^2 g_i(y_{i,j})^2 / (C_i^2 g_i(y_{i,j})^2 + g_i(y_{i,j})) = g_i(y_{i,j}) / (g_i(y_{i,j}) + 1/C_i^2) \quad (6)$$

This completes the proof.

The estimates of the numbers C_i^2 will now be discussed.

The random variable $X_{i,j}$ represents the frequency in interval i and region j during a period of 41 years, such as the period used for the data. If assumptions 1 and 2 above are satisfied, then the expected value of $(g_i(y_{i,j}) - X_{i,j})^2$ equals the expected value of $(g_i(y_{i,j}) - E_{i,j})^2 + (E_{i,j} - X_{i,j})^2$ since the probability distributions of $g_i(y_{i,j}) - E_{i,j}$ and $E_{i,j} - X_{i,j}$ are independent and therefore the variance of the sum equals the sum of the variances.

The expected value of $\sum_{j=1}^{28} (E_{i,j} - X_{i,j})^2$ equals $\sum_{j=1}^{28} E_{i,j}$, since the frequencies are Poisson distributed. The expected value of $\sum_{j=1}^{28} E_{i,j}$ equals $\sum_{j=1}^{28} g_i(Y_{i,j})$. Therefore, the expected value of $\sum_{j=1}^{28} (g_i(y_{i,j}) - E_{i,j})^2$ equals (the expected value of $\sum_{j=1}^{28} (g_i(Y_{i,j}) - X_{i,j})^2$) $- \sum_{j=1}^{28} g_i(Y_{i,j})$. But the expected value of $\sum_{j=1}^{28} (g_i(Y_{i,j}) - E_{i,j})^2$ equals $C_i^2 \sum_{j=1}^{28} g_i(y_{i,j})^2$. So C_i^2 equals

$$((\text{the expected value of } \sum_{j=1}^{28} (g_i(Y_{i,j}) - X_{i,j})^2) - \sum_{j=1}^{28} g_i(Y_{i,j})) / \sum_{j=1}^{28} g_i(Y_{i,j})^2 \quad (7)$$

The estimate of the expected value of $\sum_{j=1}^{28} (g_i(Y_{i,j}) - X_{i,j})^2$ will depend partly on judgment and intuition, due to problems in estimating it purely mathematically.

Assume for the sake of approximation that the following two conditions are satisfied.

1. The values $g_i(y_{i,j})$ are the function values produced directly by a regression and a linear relationship with coefficients $a_{i,j}$ actually exists between the independent variables used and the expected values of the dependent variables.

2. The differences between the dependent variables and their expected values have independent probability distributions with a common variance σ. (7)

Under these conditions,

$$\left(\left(\sum_{j=1}^{28} g_i(y_{i,j}) - A_{i,j}\right)^2\right) / \text{(degrees of freedom)}, \qquad (8)$$

where $A_{i,j}$ is the actual frequency in interval i and region j, is an unbiased estimate of σ. (Draper and Smith [3]). If the values $g_i(y_{i,j})$ are not the true expected values of the frequencies in interval i and region j, then the expected value of $\sum_{j=1}^{28}(g_i(y_{i,j}) - X_{i,j})^2/28$ is greater than σ.

Assuming formula (8) is equal or less than the expected value of $\left(\sum_{j=1}^{28}(g_i(y_{i,j}) - X_{i,j})^2\right)/28$, formula (7) gives the following lower bound for C_i^2:

$$\text{(formula (8))} \; - \; \sum_{j=1}^{28}(g_i(y_{i,j})) / \sum_{j=1}^{28} g_i y_{i,j})^2 \qquad (9)$$

We now discuss an upper bound for C_i^2.

It clearly appears that the expected value of $\sum_{j=1}^{28}((g_i(y_{i,j}) - X_{i,j})^2$ is less than $\sum_{j=1}^{28}((\sum_{j=1}^{28} g_i(y_{i,j}))/28) - A_{i,j})^2$, where $A_{i,j}$ is the actual frequency in interval i and region j. The value $(\sum_{j=1}^{28} g_i(y_{i,j}))/28$ is a mere average of the values $g_i(y_{i,j})$, so the individual estimates $g_i(y_{i,j})$ intuitively appear to be better estimators for the expected values of the variables $X_{i,j}$ than $(\sum_{j=1}^{28} g_i(y_{i,j}))/28$ is. Therefore it follows, based on the above arguments and formula (7), that the following is an upper bound for C_i^2 .

$$(\sum_{j=1}^{28}((\sum_{j=1}^{28} g_i(y_{i,j}))/28) - A_{i,j})^2 - \sum_{j=1}^{28} g_i(y_{i,j}))/\sum_{j=1}^{28} g_i(y_{i,j})^2 \quad (10)$$

Thus we have (formula (9)) < C_i^2 < (formula (10)). Using the actual values of the expressions in formulas (9) and (10) for i = 1 through 7, and averaging inequalities, gives

$$.049 < ((C_1^2 + C_2^2 + C_5^2 + C_6^2 + C_7^2)/5) < .146, \text{ and} \quad (11)$$

$$.065 < ((C_3^2 + C_4^2)/2) < .215 \quad (12)$$

The reason for considering C_3 and C_4 separately from c_1, C_2, C_5, C_6, and C_7 is that the numbers $g_i(y_{i,j})$ for i = 3 and i = 4 were based on less data than for i = 1, 2, 5, 6, and 7 and thus the expectation is that they are less accurate. Therefore, it can be seen from formula (5) that C_i^2 would be expected to be greater for those intervals.

By formula (5), the choices of k_i = 9 for i = 1, 2, 5, 6 or 7 and k_i = 6 for i = 3 or 4 in formula (2) imply choices of 1/9 for each of C_1^2, C_2^2, C_5^2, C_6^2 and C_7^2, and 1/6 for C_3^2 and C_4^2. Thus the selected values for k_i are towards the low end of the range of inequalities (11) and (12). Still, the numbers $g_i(y_{i,j})$ have a much greater effect than the numbers $A_{i,j}$ on the tails of the loss distributions selected by region in section IIE.

APPENDIX C

METHOD OF FITTING PARETO

Iteration was used to find the single parameter Pareto distribution P that minimizes $\sum_{i=1}^{4}((F_i-P_i)^2/P_i^{1.5})$, where F_i is as defined in section IIE, and P_i is the corresponding fraction for the Pareto distribution.

The above method of fitting a Pareto to the numbers F_i is different, for theoretical reasons, from methods that would be used to fit a Pareto to actual frequencies. An explanation of the method is as follows.

Let the random variable X_i equal the F_i produced by performing the experiment of using the method of this paper on the data for a 41 year period. Assume that there is some Pareto distribution P* such that each P_i^*, as defined above, is the mean of X_i.

The Pareto which minimizes $\sum_{i=1}^{4}((F_i-P_i)/\sigma_i)^2$, where σ_i is the standard deviation of X_i, is an estimate of P*.

Based on the definition of X_i and the process used in computing the numbers F_i, the numbers σ_i^2, for i = 1, 2, 3, 4, are judgementally estimated to be in the same proportions to each other as the corresponding numbers $(P_i^*)^{1.5}$ are to each other. Thus the Pareto P which minimizes $\sum_{i=1}^{4}((F_i-P_i)^2/P_i^{1.5})$ is an estimate of the Pareto which minimizes $\sum_{i=1}^{4}((F_i-P_i)/\sigma_i)^2$.

WORKERS' COMPENSATION RATEMAKING
(Selected Chapters)

Sholom Feldblum

Biography

Sholom Feldblum is an Associate Actuary with the Liberty Mutual Insurance Company in Boston, Massachusetts. He was graduated from Harvard University in 1978 and spent the next two years as a visiting fellow at the Hebrew University in Jerusalem. He became a Fellow of the CAS in 1987, a CPCU in 1986, an Associate of the SOA in 1986, and a member of the American Academy of Actuaries in 1989. In 1988, while working at the Allstate Research and Planning Center in California, he served as President of the Casualty Actuaries of the Bay Area and as Vice President of Research of the Northern California Chapter of the Society of CPCU. In 1989, he served on the CAS Education and Testing Methods Task Force. He is presently a member of the CAS Syllabus Committee, the CAS Committee on Review of Papers, the Advisory Committee to the NAIC Casualty Actuarial (EX5) Task Force, and the Actuarial Advisory Committee to the NAIC Risk Based Capital Task Force, and he is the quarterly review editor for the *Actuarial Review*. Previous papers of his have appeared in *Best's Review*, the *CPCU Journal*, the *Proceedings of the Casualty Actuarial Society*, the *Actuarial Digest*, the *CAS Forum*, and the *CAS Discussion Paper Program*.

WORKERS' COMPENSATION RATEMAKING

Abstract

Workers' Compensation pricing procedures are changing rapidly for several reasons:

- The advent of open competition and the movement to bureau loss costs in several states.
- The legislative enactment of benefit and administrative reforms, often with substantial but uncertain effects on loss costs.
- The growth of involuntary pools and the deterioration of industry earnings.

Private carriers, compelled to independently set rates, improvise alternative insurance programs, and quantify the expected effects of legislative reforms, are reexamining the bureau pricing methods. This paper reviews both the traditional ratemaking procedures and the modifications now being proposed by actuaries and economists, in the following sections:

- Sections 3 through 5 define the concepts used in ratemaking and the adjustments applied to historical data.
- Sections 6 through 8 review development, trend, and adjustments to current rate and benefit levels applied to premiums and losses.
- Sections 9 and 10 discuss the direct and indirect effects of benefit reforms.
- Sections 11 through 13 deal with more specific ratemaking topics: involuntary market burdens, expense constants, premium discounts, and assessments.
- Sections 14 and 15 analyze classification systems and relativities.
- Section 16 deals with occupational diseases and cumulative injuries.
- Section 17 provides illustrative exhibits.
- Section 18 reviews current issues, such as the evolving loss costs environment and alternative insurance programs.

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Because of space constraints, we are unable to publish the full text of this paper. Complete copies may be obtained from the author. Please send requests in writing to his CAS *Yearbook* address.

WORKERS' COMPENSATION RATEMAKING

Table of Contents

Section 1: Introduction

". . . the present plan merely represents the latest stage in the gradual evolution of an ideal rate-making method . . ." – Barber [1936], page 151.

Workers' Compensation pricing procedures are changing rapidly. Until the mid-1980's, the National Council on Compensation Insurance and regional bureaus developed advisory rates, which were adopted by most carriers. Independent pricing was largely confined to uniform rate deviations or policyholder dividends.

The advent of open competition in Workers' Compensation has stimulated a renewed examination of pricing procedures. In many jurisdictions, the bureaus now provide only loss costs, not advisory rates. Carriers must independently justify the profit and contingency provisions, expense loads, and often even loss development and trend factors.

Intensifying competition compels carriers to review other components of the premium rate as well: the loss costs estimates, the experience rating modification, and the classification system. The large involuntary pool burdens and special fund assessments necessitate additional analysis of expense costs. Finally, carriers must evaluate the cost implications of the Workers' Compensation reforms now being enacted in state legislatures.

Rate making procedures were generally uniform among the various bureaus. For instance, the full credibility standards and the "three halves" partial credibility formula have little actuarial justification, yet they have been used consistently by the rating bureaus. But this uniformity is quickly disappearing. Pricing actuaries – as well as the rating bureaus – now use a variety of methods for developing and trending both losses and premiums, evaluating law amendments, and determining profit and contingency provisions.

This reading has three purposes:

- It explains the pricing procedures currently used by the rating bureaus. Some procedures are common to most lines of business; these are reviewed briefly. Others are unique to

Workers' Compensation, such as the pricing of law amendments and the determination of classification relativities; these are explained in more detail.

The bureau rate making procedures are complex. Simplified examples are included with the text to clarify the exposition. Complete exhibits from recent rate filings, with accompanying description, are included in Section 17.

- Pricing actuaries, both with rating bureaus and with private insurers, have developed alternative rate making procedures for many aspects of Workers' Compensation pricing, particularly for loss development, loss and loss ratio trends, credibility, and profit and contingency provisions. For some of these procedures, there no longer is a "standard" procedure; the NCCI even uses different loss development procedures in different states. This paper reviews several of the alternative procedures and explains the rationale for each.

- Several aspects of Workers' Compensation rate making have recently been examined by economists and financial analysts, and some recommended changes are now being used by the rating bureaus and private insurers. Foremost among these are the economic incentives of law amendments and refinements of the classification system; see Sections 10 and 14. The advent of open competition and various Workers' Compensation reforms increase the need for accurate actuarial quantification of the complex effects of law amendments and classification systems.

This introductory reading can not do justice to all aspects of Workers' Compensation rate making, particularly to the procedures that are still evolving. Rather, this paper explains the basics, and directs the interested reader to more advanced articles on each subject.

Section 2: Overview

The pricing actuary determines premium rates that suffice for anticipated losses and expenses during the future policy period and that provide the insurer with a reasonable profit. Rates may be determined in two ways:

- The *loss ratio method* quantifies the needed revision from current rates.
- The *pure premium method* quantifies the required rate per unit of exposure.

The two methods are mathematically equivalent, though each has advantages and drawbacks (Stern [1965]; McClenahan [1990], pages 36-40). Workers' Compensation rake making uses the loss ratio method for overall statewide indications and the pure premium method for classification rates.

The segmentation of data offers another dichotomy for rate making. The actuary may revise rates for the state as a whole and then allocate the revision by classification. Alternatively, he may determine either classification rates or classification relativities and combine these into a statewide revision. In the past, Workers' Compensation emphasized the statewide rate revision. The rate changes for some classifications, termed "non-reviewed," ignored their specific experience and used the overall (industry group) revision. There is now growing emphasis on classification rates – all classifications are "reviewed" to some degree.

A. Ratemaking Variety

Workers' Compensation ratemaking procedures differ among the various bureaus, carriers, and jurisdictions. The differences occur in every part of the rate review. Even basic items, such as "What experience should be used?" receive divergent treatment:

- The old NCCI method used equal weightings of the most recent two policy years and the most recent calendar year. In 1983, the NCCI changed to equal weightings of the most recent policy year and calendar/accident year (in line with the New York procedure).

- Pennsylvania uses equal weightings of three projections:
  - The most recent calendar year (incurred losses),
  - A paid loss projection from the most recent policy year, and
  - An incurred loss projection from the most recent policy year.

- Minnesota uses equal weightings of paid loss projections from the most recent policy year and the most recent calendar/accident year. As supplementary information, it shows indications from case incurred loss projections and from total incurred loss projections.

- Many private carriers examining rate adequacy use longer experience periods, since the available data are less extensive.

All ratemaking procedures must be flexible. For instance, Section 15 notes the traditional limit on classification pure premium changes:

"the statutory benefit change + 50% x the industry group change $\pm$ 25%"

This limit is arbitrary: some pricing actuaries abide by it, some do not. And rare is the pricing actuary who feels entirely constrained by it. Consideration must always be given to judgmental or underwriting factors when determining rate levels.

A comprehensive survey, noting the procedures used by each bureau and by some of the major carriers, would be ill suited for the actuarial candidate first approaching Workers' Compensation ratemaking. Instead, this reading lists the prevalent (or a prevalent) ratemaking procedure. If two or more procedures are used by different bureaus or carriers, this reading sometimes lists more than one. An emphasis on or the exclusive documentation of a single procedure, should not be interpreted as an endorsement of that procedure.

B. The Extent of the Task

> *"Present-day rate making procedure . . . is in serious danger of being overbalanced by sheer weight of complexity."* – Michelbacher [1919], page 249.

Workers' Compensation rate making procedures are more complex than those used in other lines. The complexity begins with basic terms, such as

- What earned premium should be used: manual, standard, or net? What conversions among these bases are needed, and where should they be applied?

- What exposure base should be used: total payroll, limited payroll, or man-hours? How do benefits relate to each of these? How might other pricing procedures, such as experience rating, solve some of the exposure base problems?

The complexity extends through the final aspects of the review, such as

- How should the profit provision be chosen? The 1921 NAIC formula recommended a 2.5% underwriting profit; some carriers price to a 0% provision; the NCCI uses an internal rate of return model in some jurisdictions; and the Workers' Compensation Rating Bureau of Massachusetts uses a net present value model.

- How should classification pure premiums be determined? How much weight should be given to the classification's experience, the overall statewide experience, and the countrywide experience for that classification?

This reading covers the fundamentals of Workers' Compensation manual rate making. It does not deal with individual risk rating plans, except insofar as experience rating affects the ratio of manual to standard premiums and retrospective rating affects premium development patterns. It does not deal with financial pricing models for Workers' Compensation, or with the regulatory considerations regarding open competition versus administered pricing, except insofar as these affect the work required of the pricing actuary.

C. The Structure of this Reading

Rather, this reading covers the following topics:

- Section 3 notes the complexities of experience, exposures, premiums, losses, and expenses.

- Section 4 discusses the exposure bases used in pricing (total payroll, limited payroll, and man-hours), the rationale for each, and the modifications used for certain employers.

- Section 5 explains the adjustments applied to historical data: development, trend, and statutory changes.

- Section 6 discusses premiums:

  a) Premium development, with explanation of differences between retrospectively rated and prospectively rated policies; effects of the Tax Reform Act of 1986 with its "revenue offset" provision; and the changes by many insurers to booking premium as billed.

  b) *Bringing premium to the current rate level, with the procedures needed to accommodate* the skewed distribution of Workers' Compensation effective dates.

- Section 7 discusses loss development. An incurred loss development example is provided in the text, and a paid loss development example is shown in Section 17. This section also discusses the changing development patterns in the industry and credibility weighting procedures for loss development.

- Section 8 discusses loss cost trends and loss ratio trends, along with the rationale for each. Trends may be estimated using either internal (insurance) data or external (econometric) data; the relative advantages of each are presented. This section explains the differences between (a) Workers' Compensation indemnity and medical trends, on the one hand, and (b) CPI wage and medical care inflation indices, on the other hand. It then discusses the changes in the Workers' Compensation environment and their effects on loss cost trends.

- Section 9 shows how to quantify the direct effects of statutory amendments: replacement rates, lengths of disability, waiting periods, and benefit limitations.

- Section 10 discusses the indirect "incentive" effects of statutory amendments on claim frequency and durations of disability. This section notes the types of incentive effects; the magnitude of these effects; the variations by type of injury and worker characteristics; and the effects of medical fee schedules and limits on attorney reimbursement.

- Section 11 deals with involuntary market burdens and methods of quantifying them. It presents explanations for the growth of the pools and the implications for pricing, and discusses alternative Workers' Compensation programs that alleviate the burdens.

- Section 12 deals with differences between large and small risks and the ratemaking procedures used to compensate for them, such as expense constants and loss constants. It describes the reasons for these differences: per policy expenses, economic incentives from experience rating modifications, and economies of scale.

- Section 13 shows the calculation of the overall statewide rate change, along with several factors peculiar to Workers' Compensation rate making, such as premium discounts and assesssments for special funds.

- Section 14 deals with classification systems. It shows the rationale for the current classification system, describes the differences between classification by product type and by job characteristics, and discusses alternative classification dimensions, such as

  a) age and sex of the work force.
  b) group health benefits provided by the employer,
  d) territory and claims consciousness, and
  c) financial health of the industry.

- Section 15 deals with classification rate making:

  a) industry group relativities,
  b) underlying pure premiums, state indications, countrywide indications,
  c) law differentials and experience differentials, and
  d) classification credibility procedures.

- Section 16 deals with occupational disease claims, such as asbestosis, stress claims, and psychological disorders. Of particular concern to the pricing actuary are (i) accident year or policy year effects versus (ii) report year or calendar year effects, and how these effects should be included in loss development and trend.

- Section 17 provides illustrative exhibits showing the variety of methods now used for Workers' Compensation ratemaking:

  a) Advisory exhibits from the 1991 Minnesota rate filing (a loss cost state).
  b) NCCI expense and profit exhibits from an administered pricing state.
  b) Alternative benefit trend exhibits from the California Workers' Compensation bureau.
  d) Direct and indirect (incentive) "law amendment" effects.

- Section 18 concludes this reading with current issues relevant for the Workers' Compensation pricing actuary, such as the evolving loss costs environment and alternative Workers' Compensation programs.

Section 5: Experience Adjustments

". . . the goal of the ratemaking process is to determine rates which will, when applied to the exposures underlying the risks being written, provide sufficient funds to pay expected losses and expenses; maintain an adequate margin for adverse deviation; and produce a reasonable return on (any) funds provided by investors."

– McClenahan [1990], page 33

Ratemaking is prospective. When preparing a rate review, the actuary asks: "Will premiums collected during the future policy period be sufficient to cover expected losses and expenses?" To determine the needed rates, historical experience is examined, adjusted for known or expected differences between the experience period and the future policy period.

Three types of adjustments are used in Workers' Compensation ratemaking: development, trend, and benefit changes.

A. Development

Observed data reported soon after the close of the experience period may not reflect full values. Workers' Compensation premiums are adjusted by payroll audits about three to six months after the policy expires. Loss estimates are revised as the extent of the injury becomes clearer. Some expense elements, such as contingent commissions and guarantee fund assessments, have similar lags.

Many rate making values become better known with the passage of time. For instance, ultimate loss costs are known only after all claims are settled. The observed losses depend on the valuation date. *Development* is the change in the observed values over time.[7]

Even when the observed values differ significantly from ultimate values (i.e., development is

[7] Compare Cook [1970], page 2: "A calculated past ratio of mature to immature data is called a loss development factor," or CAS [1988], page 58: "Development is defined as the change between valuation dates in the observed values of certain fundamental quantities that may be used in the loss reserve estimation process"; so also Wiser [1990], page 161). Weller [1991] says: "Often the values of observations change as we learn more about the subject that we are studying. Actuaries call such changes 'development.'"

great), the *pattern* of development may be stable. For instance, the paid losses at the end of an accident year may be only a fraction of the ultimate value. But this fraction may be stable: 20% in one year, 21% the next year, 19% the next year. The observed values plus a stable development pattern allows a good estimate of the ultimate values.

External developments may change development patterns. For instance, the 1986 federal income tax amendments caused insurers to modify their WC premium booking procedures and thereby changed premium development patterns. Similarly, statutory modifications of maximum durations of indemnity benefits change loss development patterns. The actuary must quantify the effects of these changes when estimating ultimate values (see Sections 6 and 7).

B. Trend

Inflation causes nominal values to change over time. For instance, payroll increases with wage inflation; medical benefits increase as physicians' fees rise; accident frequency changes with technological improvements in workplace safety.

Actuaries divide loss cost trends into three types: economic inflation, social inflation, and other trends. *Economic inflation* is the change over time in the purchasing power of a dollar. It is measured by econometric indices, such as a CPI index or a GNP deflator, though it will vary by benefit type (e.g., the medical inflation rate differs from the wage inflation rate). *Social inflation* is the change over time in public attitudes that affect insurance losses, such as changing claims consciousness, more liberal jury awards, and changing expectations of compensation. *Other trends*, such as frequency trends, are systematic non-monetary changes affecting insurance values, such as a decline in workplace fatalities resulting from OSHA regulations or from the movement from a manufacturing to a service economy.[8]

Trends may be estimated either from internal insurance data, such as historical claim sizes, or from external econometric data, such as CPI indices (Masterson [1968]). Internal trends are often preferred when other forces besides economic inflation affect insurance values. External

[8] The ratio of fatalities to permanent total disabilities has declined from 15 to 1 at the beginning of this century to about 1 to 1 now, reflecting greater workplace safety and better medical treatment; cf. Downey and Kelly [1918], page 261.

trends are valuable when the trend values chosen must be justified to regulators or when the expected future trend differs from the historical average.

If the exposure base is not inflation sensitive, such as car-years in Personal Auto, only loss trends are used. If the exposure base is inflation sensitive but not necessarily related to loss inflation, such as receipts in Products Liability, separate premium and loss trends are used.

In Workers' Compensation, the exposure base (payroll) is inflation sensitive and directly related to indemnity benefits. Rating bureaus use loss ratio trends. The divergences between (i) wage and medical inflation and (ii) Workers' Compensation indemnity and medical benefit trends, and the need to explain these differences to regulators, leads some pricing actuaries to prefer separate premium and loss trends (see Section 8).

C. Benefit Changes

Workers' Compensation statutory benefits are frequently modified by legislative enactments. For instance, a state may raise the weekly maximum for indemnity benefits, increase the duration of scheduled benefits, or change the administrative handling of cases.

Benefit changes have both direct and indirect effects. The direct effect considers the change in compensation, not changes in claim frequency or severity. For instance, if the indemnity benefit is raised 20%, indemnity claim costs will rise 20%. In practice, the higher benefit level may encourage greater filing of claims and longer durations of disability. The indirect "economic incentives" may raise indemnity claim costs another 10%, though the actual effect depends on the benefit structure, the characteristics of the workforce, and the economic environment (see Sections 9 and 10).

The direct effects are removed from loss and premium trends. The indirect incentive effects work more slowly and are harder to quantify. It is difficult to discern whether loss cost trends in excess of wage or medical inflation indices stem from economic incentives caused by benefit changes or from changing social expectations unrelated to statutes.

Section 8: Loss Trends and Loss Ratio Trends

Inflation raises the nominal costs of insurance premiums and losses. Accordingly, the pricing actuary adjusts historical experience with inflation trends to project future cost levels. In lines with exposure bases that are not inflation sensitive, such as Personal Auto liability, only losses are trended. In lines with exposure bases that are inflation sensitive but are not directly related to cost trends, such as General Liability, premiums and losses are trended separately.

In Workers' Compensation, the exposure base, payroll, is inflation sensitive. Indemnity benefits are a function of wages, so the indemnity loss cost trend should be similar to the exposure trend. During the 1960's, when industrial productivity increases were high and so wages rose rapidly, medical inflation was also similar to wage inflation.

The NCCI uses a loss ratio trending procedure, with credibility adjustments based on the goodness of fit of the empirical observations with a linear trend. Since inflation of wages and indemnity benefits should be similar, the complement of credibility for indemnity was originally set at "no trend." [Empirical data shows that indemnity benefits have been increasing more rapidly than wages, so the NCCI now uses the countrywide trend for the credibility complement.] Since medical inflation differs from wage inflation, the complement of credibility for medical is the countrywide medical trend, with different figures for states with an effective medical fee schedule and states with no schedule.[25]

A. Inflation and Benefit Trends

> *"When wage rates are increasing, payrolls are increased and more premiums are collected. Indemnity losses which are based on wages will increase, but not to the same extent as premiums. Therefore, rate levels as otherwise calculated should be reduced in order to avoid excessive premiums."* – Allen [1952], page 59.

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[25] Marshall [1954] and Kallop [1975] use no trend procedure; in their reviews of Kallop's paper, Gruber [1976] and Scheibl [1976] note that New York and the NCCI began using trend procedures.. NCCI [1985] describes the loss ratio trend which is now used in rate filings.

Forty years ago, Workers' Compensation pricing actuaries wondered whether premium rates should be reduced because of wage inflation. Edward Allen presented the "wage factor" procedure along with arguments for and against it. Harwayne [1953] noted that the "wage factor represents a technical adjustment to reflect recent conditions and is therefore on a par with the adjustment of experience to reflect current rate levels and current law levels" (page 28). Skelding [1953] noted the higher benefit trends than wage trends and says that "the injection of a so-called wage trend factor in the compensation rate structure would be a tragic mistake" (page 21).[26]

During the late 1970's and 1980's, loss cost trends for both medical and indemnity benefits have far exceeded wage inflation: about 14% per annum for medical, 10% for indemnity, and 6% for wage. The disparity between wage inflation and Workers' Compensation benefit trends has been increasing: although wage inflation has declined from 8% in the late 1970's to 4% in the mid-1980's, neither medical nor indemnity benefit trends have fallen as much.[27]

The disparity between wage inflation and WC benefit trends stems from several causes:

- Technological advances in medical treatment: more expensive equipment and complex therapeutic procedures.
- Increasing utilization of medical services, even for minor injuries.
- Patient "claim shifting" from employer provided health insurance plans with high

[26] Wage level factors were often used in early ratemaking analyses. For instance, 1918 Pennsylvania rate revision used an average factor of 0.92 for all classifications except coal mining (Downey and Kelly [1918], page 266). Such factors are more justified when the state has a low indemnity benefit maximum (*ibid.*, page 266-267). Gruber [1976], page 57, notes that "due to the inflationary growth of payroll and therefore the growth of premium without any compensating increase in risk, a wage factor is used to decrease the New York experience-indicated rates."

[27] On medical, indemnity, and wage trends, see Ryan and Fein [1988], pages 43-45, Hager [1991: Call for Reform], page 7, and NCCI [1991: Issues Report], page 32. Kaufmann [1990], using state data for one insurer, finds a consistently higher Workers' Compensation medical severity trend than the CPI medical costs index; see also the studies by the California WC Rating Bureau. Before the 1970's, the relationship of Workers' Compensation medical costs and wage inflation was less clear. NCCI [1991: Issues Report], p. 29, notes that "prior to [1975], wage inflation had generated enough premium to overcome indemnity and medical loss changes." [Boden and Fleischman [1989] and Victor and Fleischman [1990] note that Workers' Compensation medical benefit trends were lower than medical inflation during the early and mid-1970's but greater than medical inflation in the 1980's.] Early studies have often shown a higher trend for medical benefits than for wages (Mowbray [1919]; Greene and Roeber [1925], p. 255; Skelding [1953]).

deductibles and co-insurance payments to first-dollar Workers' Compensation benefits; physician "cost shifting" from limited reimbursement plans, such as Medicare, to higher reimbursement private insurance coverages, such as Workers' Compensation.

- Lengthening durations of disability, particularly when replacement work is not available.
- Increasing frequency/compensability of high-cost psychological injuries and occupational diseases in certain jurisdictions.
- Greater attorney involvement in Workers' Compensation claims.[28]

Loss cost trends are frequently contested in rate filings, especially if the causes of the trend are neither intuitive nor explained. The use of loss ratio trends masks these causes: it is more difficult to interpret increases in loss ratios than in average claim costs.[29]

B. Internal Data and External Indices

Trend factors can be based on either (i) observed changes in average benefit costs or (ii) econometric modeling of loss cost trends with external inflation indices, such as the CPI. When the causes of the observed trends are not well understood, observed benefit trends may be more reliable. Econometric modeling, however, separates the influences on loss cost trends into their components, such as economic inflation, utilization, durations of disability, and claim filing patterns. Similarly, analyses of attorney involvement in insurance claims may explain rises in claim frequency, average claim severity, and loss adjustment expenses. Econometric modeling and analysis of attorney involvement provide qualitative justification for Workers' Compensation trend factors.

Loss ratio trends incorporate both claim severity and claim frequency. If exposures and losses

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[28] See Appel [1989]; Boden and Fleischman [1989]; Victor and Fleischman [1990]; Borba [1989]; Pillsbury [1991]. *Appel notes several additional factors, such as (a) rising costs of medical malpractice coverage and* defensive medicine, (b) demand creation by physicians, and (c) an oversupply of physicians in urban areas. Gots [1990], pages 39-40, also emphasizes the entitlement expectations of consumers for high quality medic

[29] *Note particularly the observation by Mintel [1983], p. 167: ". . . several insurance commissioners have* rejected trending evidence based on an analysis of internal loss and expense experience presented in support of a rate filing in favor of external evidence of factors outside insurance company control that may affect future losses." Perkins [1922], page 272, also argues for separate payroll and loss projection factors.

are trended separately, both claim severity and claim frequency trends should be estimated.

In other lines of business, increases in claim frequency often stem from the addition of small, marginal claims. In Personal Auto, for example, severe injuries always led to insurance claims. The increasing claims consciousness of the public and attorney involvement in insurance claims, however, causes a higher incidence of small claims. This phenomenon depresses average claim costs (though not enough to offset economic and social inflation).

In Workers' Compensation, increases in claim frequency often result from newly mandated compensability of occupational diseases, psychological injuries, and stress claims, or from attempts to use Workers' Compensation as a substitute for early retirement. These are all high cost claims, so increases in claim frequency may raise average claim severity.

C. Loss and Exposure Trends

Exposure grows by increases in hourly wages and increases in the number of workers; only the former is needed for the trend calculation. Historical experience and future projections of average hourly wages are published by econometric consulting firms, such as DRI or Wharton.

The loss cost trend may be estimated in two ways:

- Fit average claim severities values to a curve. Average claim severities may be incurred values (case incurred losses divided by reported claims) or paid values (paid losses on closed claims divided by the number of closed claims). The observed values are usually fit to either a straight line or an exponential curve.

- Compare average incurred or paid values to an econometric index. For medical benefits, the econometric index may be the CPI medical cost index. For indemnity benefits, the index may be an average wage level index. Econometric indices are generally available only for countrywide data, though state specific figures may help to account for regional economic differences.[30]

Linear and Exponential Trends

Until recently, Workers' Compensation used linear trend factors. If the average cost of an indemnity case was $2,000 in 1992, and a 10% per annum trend was expected, the assumed average indemnity cost was $2,200 for 1993, $2,400 for 1994, $2,600 for 1995, and so forth. The expected trend was determined by fitting a linear regression (McClenahan [1990], page 51):

$$y = ax + b$$

where y is the average claim cost in each year,
a is the annual trend,
x is an index for the year, and
b is a constant.

Linear trends often underestimate future costs, since inflation is multiplicative, not additive. In the example above, with a 1992 average cost and a 10% expected trend compounded annually, the assumed future costs should be $2,200 in 1993, $2,420 in 1994, $2,662 in 1995, and so forth. The corresponding regression is

$$y = be^{ax}$$

where the parameter and variables have the same meaning.

In June 1990, the NCCI converted to an exponential trend function, as is used in other liability lines of business. To fit the exponential model, the exponential equation can be transformed into a linear equation by taking natural logarithms (McClenahan [1990], page 51):

$$\ln (y) = ax + \ln (b)$$

[30] See, for instance, DRI [1991]: "The Workers' Compensation Insurance Rating Bureau of California has asked the Cost Information Service of DRI/McGraw-Hill to develop and forecast an input price (market basket) index that measures escalation in operating costs of California hospitals. The hospital escalation projection will be used by the Bureau's Actuarial Committee in developing premiums for workers' compensation insurance" (Exhibit 2, Sheet 4), and "Over the period 1985 to 1990, the escalation rate of the California index was higher than that of the national index in every year other than 1988, reflecting the relative relationship of the corresponding wage proxies" (Exhibit 2, Sheet 3).

[Methods for solving these equations are reviewed in Wheelwright and Makridakris [1989], pages 163-170, or DeGroot [1975], p. 501. See Section 17.D.1 for a complete illustration.]

Econometric Indices

Workers' Compensation benefit trends are partially dependent on monetary inflation: indemnity benefits are linked to wage levels, and medical benefits are linked to medical inflation. Economists provide projections of future inflation indices, and expected benefit trends may be derived from these (Masterson [1968]).

Such techniques are particularly important when macro-econometric changes affect expected inflation. For instance, Workers' Compensation benefit trends were over 15% per annum in the early 1980's, when monetary inflation was high. Many actuaries expect benefit trends to be somewhat lower in the early 1990's, since monetary inflation has decreased.

During the 1980's, benefit trends have exceeded monetary inflation, since "social inflation" and "cost shifting" affect Workers' Compensation benefits. A regression of benefit trends on inflation trends yields a positive constant factor. For instance, a regression of medical benefits on the medical CPI index may yield

$$\textit{Medical benefits} = \textit{medical CPI} + 5\%.$$

Thus, a medical CPI trend of 8% one year would imply an expected Workers' Compensation medical benefits trend of 13%.

The table below illustrates this procedure, using simulated Workers' Compensation medical data and the medical CPI inflation index.

Accident Year	Incurred Medical Benefits	Medical Claim Count	Average Severity	Medical Benefit Trend	Medical CPI Trend
1979	4,714	12,405	380		
1980	5,680	12,850	442	16.3%	11.0%
1981	6,782	13,067	519	17.5	10.7
1982	7,965	12,993	613	18.1	11.6
1983	8,793	12,420	708	15.5	8.6
1984	10,919	13,365	817	15.3	6.3
1985	12,745	13,544	941	15.2	6.3
1986	15,103	13,881	1,088	15.6	7.7
1987	18,044	14,493	1,245	14.5	6.6
1988	21,926	15,650	1,401	12.5	6.5
1989	25,389	16,008	1,586	13.2	7.6
1990	29,077	16,109	1,805	13.8	9.1

The data show a spread of about 4 to 7 points between the medical benefit trend and the medical CPI trend. For a 1991 medical CPI of 8 to 9% expected in 1990, the expected 1991 medical benefit trend is about 13.5%.

D. Loss Ratio Trends

The Workers' Compensation exposure base, payroll, is inflation sensitive. Average wage changes, though, have been about 5 to 10 points below average benefit trends in many jurisdictions. Instead of using separate trends for benefits and premiums, standard bureau ratemaking procedures use a loss ratio trend.

Policy year or accident year loss ratios are formed with premium at current rate levels and losses at current benefit levels. A consistent trend in loss ratios indicates consistently different benefit and premium trends. The loss ratio trend may be applied to the developed experience period loss ratio to project expected loss ratios in the future policy period.

The observed loss ratio trends vary over time and by jurisdiction. They stem from numerous factors, as Michelbacher [1919] notes:

"Such a comparison [of loss ratios over time] measures collectively such factors as changes in wage level, amendments to the benefit schedules, greater liberality on the part of administrative claim bodies in interpreting workmen's compensation laws, a possible tendency on the part of claimants to malinger and to present fraudulent claims, the influence of immigration and emigration, variations in accident frequency and severity rates or in employment and unemployment, and, in fact, any and all influences acting upon the cost" (page 244).

The pricing actuary should investigate the probable causes of the trend, since changes in the causes affect the expected future trend. For instance,

- If the primary cause is economic incentives of statutory amendments, then the enactment of a law change should be carefully examined for its potential influence on the benefit trend (see Section 10).

- If the primary cause is a "tendency to malinger and present fraudulent claims," then the organization of an insurance fraud unit may reduce the future trend rate.

- If the primary cause is "variations in unemployment," then macroeconomic developments will influence the future benefit trend (see Section 14).

For a complete illustration of loss ratio trends, see Section 17.D.1.

Credibility for Trend

Observed benefit trends in small states fluctuate widely from year to year. The NCCI loss ratio trend procedure considers the "goodness of fit" of the observed annual trends to an exponential curve. The "squared residual," or the square of the difference between the observation and the fitted point, measures the explanatory power of the regression. The smaller the sum of the squared residuals for all policy years, the greater is the credibility accorded to the statewide trend.[31]

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[31] Scheibl [1976], page 64, notes the earlier credibility procedure: "Subsequent to the presentation of Mr. Kallop's paper, the National Council introduced loss ratio trend into its ratemaking procedure to recognize the imbalance of social and economic inflationary influences on premiums and losses. . . . Observed trends are adjusted

A variety of trend factors may be used for the complement of credibility. Originally, a trend factor of unity was used as the complement for the indemnity loss ratio trend, on the supposition that wage inflation should be about the same as indemnity benefit trends (NCCI [1985]). In October 1990, the NCCI began using the countrywide indemnity trend as the complement for the statewide trend. For medical benefits, the countrywide trend is used as the complement, though the trend figure depends on the type of medical fee schedule in the state under review. Using policy year 1985-1989 data, NCCI's countrywide trends were:

Indemnity:		+7.0%
Medical –	Jurisdictions with effective fee schedules:	3.6
	Jurisdictions without effective fee schedules:	12.5
Medical –	All Jurisdictions:	10.4

E. Length of the Trend Period

The trend period extends from the average accident date in the experience period to the average accident date in the future policy period.

- *Policy Year Experience:* A policy year considers accidents resulting from policies issued in a given time period. For instance, policy year 1992 covers accidents resulting from policies issued between January 1, 1992, and December 31, 1992. These policies are in force from 1/1/92 to 12/31/93, and the average accident date is 1/1/93.

- *Accident Year Experience:* An accident year considers accidents occurring in a given time period, so the average accident date is the midpoint of that period (assuming no change in exposures). Thus, the average accident date for accident year 1992 is 7/1/92.

for credibility using a Spearman Rank Correlation D-statistic approach." These credibility procedures are unusual. Milliman and Robertson recommend that the NCCI adopt a "Bayesian credibility [procedure] for weighting state and countrywide trend indications. . . . credibility should be based on a measure of volume, or possibly 'volume plus a constant,' instead of the current quality of the line fit." More advanced discussions of credibility procedures for trend may be found in Hachemeister [1975] and Venter [1986].

- *Calendar Year Experience:* Calendar year experience considers financial transactions occurring in a given time period. For losses, these consist of paid losses and changes in loss reserves. Since both paid losses and changes in loss reserves relate to accidents occurring the past, the average accident date for calendar year experience is often before the midpoint of the period. Since the true average accident date can not be easily quantified, the assumption of the midpoint of the calendar year is commonly used.

A rate review using experience from policy year 1989 and accident year 1990 to set rates for policy year 1992 has average accident dates of

- January 1, 1990, for policy year 1989.
- July 1, 1990, for accident year 1990.
- April 1, 1990, for the experience as a whole.
- January 1, 1993, for policy year 1992.

The length of the trend period is therefore 2.75 years: 4/1/90 to 1/1/93.

Sections 10: Law Amendments - Incentive Effects

> *"Enough experience has now developed so that we know with reasonable exactness what change in cost an amendment to the workmen's compensation law will carry with it. If the waiting period is reduced or the percentage of wages, which is the basis of compensation payments, is increased or any one of numerous changes in benefits is made, we can foretell almost with certainty just what the result will be when measured in terms of cost."* – Michelbacher [1919], page 245.

Actual loss costs have climbed far more quickly after law amendments than the traditional projections predicted, since strong but indirect economic incentives are generated by legislative enactments. In particular, statutory revisions affect the following:

1. *Claim Filing:* Greater benefits and easier access to compensation stimulate more reports.

2. *Durations of Disability:* Higher benefit levels and the removal or weakening of time limits on indemnity payments cause lengthening durations of disability.

3. *Mix of Benefits:* Changes in reimbursement levels by type of injury affect the expected mix of benefits, particularly for temporary total and permanent partial disabilities.

4. *Non-Compensation Medical Benefits:* Changes in the deductible and coinsurance provisions in governmental or group health plans affect the claim frequency of occupational injuries and diseases.

5. *Attorney Involvement:* Changes in administrative procedures may influence attorney involvement in Workers' Compensation claims, which in turn affects claim frequency and severity.

6. *Compensable Injuries and Diseases:* Changes in the definition of occupational injury and disease affect the types of claims reported.

Direct effects are immediate; indirect effects emerge slowly. The indirect effects are often hard to disentangle from loss cost or loss ratio trends, but separating indirect economic incentives from loss trends is essential for competitive pricing. For instance, suppose a statutory amendment defines certain "stress" claims as compensable. The indirect incentive effects are gradual. As workers learn what types of stress claims may be pressed, and as they see other workers receiving benefits for stress claims, there will be a steady rise in claim frequency.

If the indirect effects of law amendments are not properly priced, the increase in stress claims will appear as a loss ratio trend or as a loss cost trend. This may mislead the pricing actuary, for two reasons:

- The rate of increase in stress claims will be greatest soon after the law amendment and will taper off to zero after several years.

- The rate of increase in stress claims will vary by classification, depending on the types of stress claims deemed compensable.

A. Claim Frequency

The indirect economic effects of law amendments on claim frequency and durations of disability are quantified by econometric analyses, not by *a priori* intuition. In the early 1980's, several economists considered the effects of benefit levels on claim frequency for temporary total, major permanent partial, and minor permanent partial injuries. Butler and Appel [1983], for instance, find that both wage and benefit levels affect claim frequency: injury claims increase as wages fall and as benefits increase.

Gardner [1989], page xiii, summarizes previous studies as "A 20 percent benefit increase is estimated to have a 7 percent increase on temporary disability claims." The National Council on Compensation Insurance [1991], in an admitted understatement, uses a 1% overall indirect effect of statutory amendments. Other rating bureaus sometimes avoid quantifying the indirect effects explicitly and include them instead in the loss ratio trend (see below).

A New York Example

In 1990, New York increased the maximum benefit for temporary partial disabilities from $150 a week to $340 a week. The direct effect of this change was a 1.6% increase in temporary partial benefits.

A more complete analysis must consider several aspects of the pre-1990 New York benefits:

- Temporary partial claims were infrequent, accounting for only 1% of all benefits.

- The average weekly indemnity payment on temporary partial claims was $77.04, well below the maximum of $150. For temporary total claims, the average weekly benefit was $266.03, close to the pre-1990 maximum of $300.00.

  Two factors contribute to this disparity. First, temporary partial benefits are two thirds of the *difference* between pre-injury and post-injury wages, whereas temporary total benefits are two thirds of pre-injury wages. Second, the low maximum for temporary partial benefits induced high wage workers to avoid these claims and return to work full time.

  Both factors are important. The increase in the maximum benefit does not affect the first factor. But it removes the disincentive for filing temporary partial claims, so it will increase claim frequency. Moreover, since temporary partial claims often develop into permanent partial claims, claim frequency for all partial claims may increase.

The effect of benefit levels on claim frequency depends on the subjectivity of the injury: permanent total claims are least affected by benefit provisions and temporary partial claims are most affected (Butler and Worrall [1983]). There are no hard rules for estimating the effects, since they depend on various aspects of the benefit system. Given the low pre-1990 frequency of temporary partial claims in New York, the pricing actuary might estimate that the frequency will increase substantially. These indirect incentive effects occur gradually, so even *post hoc* tests of these presumptions are difficult.

Benefit Levels and Claim Frequency

There are several explanations for the relationship between benefit levels and claim frequency, each of which demands a different response from the pricing actuary. As benefits are increased, workers may have more incentive to file claims, less incentive to be careful on the job, or more incentive to bear additional risk on the job. Economic research on "compensating differentials" pertains to the last of these three (Dorsey [1983]; Worrall and Appel [1988]). As benefit levels increase, workers chose riskier occupations, since the economic loss from industrial accidents diminishes. Although there is some evidence for this effect, the influence on overall Workers' Compensation costs is probably minor.

Higher benefit levels may leave employees with less incentive to be careful on the job. However, employers have more control over workplace hazards. Higher benefit levels induce large employers, who are experience rated or retrospectively rated, to emphasize safety controls and loss prevention activities.[34] The employer incentives probably override the employee incentives regarding job safety. For instance, OSHA finds a continuing decline in workplace fatalities and severely disabling injuries over the past decade, though this stems from both employer safety incentives and the transition from a manufacturing to a service economy.

For claim filing, however, employee incentives generally override the employer and macroeconomic effects. Moreover, increased filing of minor claims may increase the number of major claims as well. For instance, reductions in the waiting period may stimulate numerous temporary total claims for short durations of disability. Some of these temporary total claims then develop into permanent partial claims, as accident victims become accustomed to the

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[34] Gardner [1989], page 79, summarizes several studies: "Chelius and Smith (1983) found no significant effect from less-than-full experience rating on injury rates. But Butler and Worrall (1988) found that, in larger firms, which are likely to have a higher degree of experience rating than are smaller firms, indemnity costs differ less in response to benefit differences than they do in smaller firms. Their data were observations at the establishment level in eleven risk classes in thirty-eight states for 1980 and 1981. Ruser (1985) analyzed BLS time-series data for twenty-four manufacturing industries in forty-one states from 1972 through 1979. He found the response of injury rates to benefit changes to be four times higher in small firms than in large firms. Similarly, with data in one state - South Carolina - over the long period from 1940 through 1971, Worrall and Butler (1988) also found that industries with relatively more employees per firm had smaller changes in injury rates when benefits increased than did industries with fewer employees per firm." See also Harrington [1988]; Chelius [1974; 1982; 1983]).

compensation benefits.

B. Durations of Disability

Economists have also examined the effects of benefit levels on the duration of disability. Economists often apply a "reservation wage" model derived from unemployment studies to the analysis of Workers' Compensation durations of disability. The reservation wage is the amount required to induce an individual to accept an employment offer. For injured workers, the benefit level is similar to the reservation wage: as benefit levels increase, injured workers are less likely to return to work (Butler and Worrall [1985], page 718).

Several phenomena hinder the quantification of duration effects.

- Many claims are "right-censored" in rating bureau data bases, in that the disability has not yet ended.
- The future duration of a claim may be dependent on the past duration: that is, the longer a worker has been receiving disability benefits, the less likely he may be to return to work.[35]
- The effect of benefit levels on the duration of disability varies by type of injury: it is strongest when the disability is hard to monitor, as in temporary total low back claims, and it is weakest for more severe claims.

The incentive effect of benefit levels on the duration of disability is strong. The estimated amount varies with the type of injury and the assumed dependence of future duration on past duration. A 10% rise in benefit levels appears to raise durations of disability by at least 2% (Butler and Worrall [1985], page 722; Gardner [1989], pages xiii, xv). For *temporary total*

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[35] Cf. Butler and Worrall [1985], pages 720-721: "This is a case of duration dependence – as the length of time on a claim increases, the instantaneous rate at which one changes from disability to nondisability status will decrease and expected duration will increase. Simply put, the longer one is on a claim the less likely one is to leave it to return to the work force when duration dependence is present. . . . Perhaps the length of a claim makes it increasingly difficult to return to work because of depreciation in market-oriented human capital." Quantifying duration dependence is difficult in non-homogeneous samples: "Unfortunately, in the presence of unobserved heterogeneity across claimants duration dependence may appear to characterize the sample data even if it does not exist for any of the individual observations. . . . Even if the transition rate out of Workers' Compensation is fixed to each individual, because the impact of the unobservable differences sort out higher hazard individuals first, there will appear to be some duration dependence" (page 721).

low back claims, if one assumes that the longer a worker is on disability, the less he desires to resume regular employment, a 10% rise in the benefit level may induce as much as a 9% increase in the length of disability (Butler and Worrall [1985]). (If one includes the 4% rise in claim frequency discussed above, the total loss cost increase is 25% [=10% + 9% + 4%].)[36] This phenomenon, however, is weaker for other types of injury, and other economists dispute its overall strength. The "duration elasticity" for all Workers' Compensation claims combined is probably between 10% and 40%.[37]

In incentive effects vary with the compensation system. In states with wage loss benefits for permanent disability claims, such as Florida, the award depends on the post-injury wages earned by the employee, thereby increasing incentives to stay out of work (Gardner [1989], pages xvi-xvii, 2; Brainerd [1987]). In addition, when benefit increases vary by type of injury, the mix of claims will shift towards those injury types whose benefits increase most.

Long-Term Disability Studies

Life and health actuaries have analyzed the effects of benefit provisions and economic conditions

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[36] Similarly, Gardner [1989], page xv, says: "The literature suggests that a 20 percent increase in temporary total benefits (replacement rates) to all benefit recipients would increase aggregate payments by *at least* 30 percent. This reflects the direct effect of 20 percent and an average of at least 10 percent in additional utilization. Duration would increase by at least 4 percent, while claim-filing rates would rise by about 6 percent." In a recent study of the statutory increase in the maximum weekly indemnity benefit in Connecticut from 100% to 150% of the average weekly wage, WCRI [1991: CN] found that the indirect effects were as great as the direct effects, suggesting that the previous estimates may have been understated.

Gardner [1989], page 40, also summarizes an unpublished study by Dionne and St.-Michel that differentiates between cases that are relatively easy to diagnose, in which no moral hazard component emerges, and those that are difficult to diagnose (back and spinal disorders). . . They find durations of disability to be an average of approximately 10 percent longer overall among claimants who are treated more favorably by the plan. Those claimants with difficult-to-diagnose injuries who are favorably treated under the disability plan have durations of disability about 30 percent longer than those with similar injuries who are treated less favorably; those with easily diagnosed injuries show no difference in duration from more favorable treatment under the plan."

[37] Butler and Worrall [1988] have tested the wage reservation model for the distribution of Workers' Compensation loss costs with curve fitting techniques. Indemnity costs are the product of three variables:

- the probability of filing a successful claim,
- the duration of disability, and
- the benefit level.

A pure chance generation of costs, with no effect of benefit levels on claim frequency or disability durations, would suggest a lognormal distribution of losses, whereas a reservation wage model would suggest a Weibull distribution of losses. The consistency of the reservation wage model with the observed distribution of losses is a check on the reasonableness of the economic incentives phenomenon.

on long-term disability (Kidwell et al. [1985a; 1985b]). Long-term disability termination rates dropped in the late 1970's, in response to worsening unemployment, and they rose in the early 1980's, as the economy prospered.

The effects of policy provisions are difficult to quantify in Workers' Compensation, since benefits are mandated by state statute. Long-term disability benefits vary widely among carriers as well as among policyholders, so the effects of benefit levels on the duration of disability are more easily discernable. [The new statutory disability tables published by the Society of Actuaries show these influences.] Casualty actuaries can use the health insurance results to predict the effects of statutory revisions in Workers' Compensation.

C. Claimant Characteristics

The indirect effects on claim reporting and durations of disability vary by claimant characteristics (Borba [1989]). Three groups of accident victims show the largest effects:

1. *Non-Primary Wage Earners:* If benefit levels during disability are lower than the pre-injury wage, primary wage earners often feel compelled to return to work. Secondary wage earners, such as spouses of the primary wage earner, show a greater response to economic incentives.[38]

2. *Low-Income Employees:* Lower income employees are affected by changes in maximum disability benefit levels more than higher income employees are. Moreover, they have less assets and are more dependent on current income. Benefit level changes have the greatest indirect economic effects on lower wage earners (Gardner [1989], page 58; but contrast

[38] Much of this research is from unemployment insurance studies, with the somewhat biased assumption that men are primary wage earners and women are secondary wage earners. Gardner [1989], pages xiii-xiv, notes: "A wide variety of studies document the greater labor market responses of women, especially married women, to economic incentives. An early study found that a 20 percent increase in wages would produce a 40 percent increase in work activity among women but only a 7 percent increase among men. Later studies indicate that the decisions of married women are the most sensitive, and their responsiveness grows with the size of their husband's earnings. *The responsiveness of single men exceeds that of married men.*" and page 56: ". . . *married claimants have greater* durations of disability payments. Their findings may suggest a greater willingness to file lost-time claims when there is another (actual or potential) income earned in the family."

WCRI [1991: CN], where a benefit change affecting only the highest 10% of wage earners had a large incentive effect).

3. *Older Employees:* Benefit level changes may induce older employees to use Workers' Compensation payments as "early retirement," for two reasons. First, older employees, with lower expenses, may be satisfied with disability benefits. Second, younger employees often desire regular employment, with its opportunities on promotions and advancement. Older employees, with little chance of additional work advancement, may be more content with disability payments (Gardner [1989], pages 60, 62).

Thus, the indirect effects of benefit level changes vary not only by type of injury but also by type of industry, based on the distribution of workers by age, income level, and primary versus secondary wage earners. The effects are strongest on low paying work with older employees who are secondary wage earners. The effects are weakest on high paying work with young, upwardly mobile, primary wage earners.

D. Non-Compensation Medical Benefits

Changes in non-compensation medical benefits in both public and private plans affect Workers' Compensation loss costs. For instance, a state may require that employer provided group health plans include a Health Maintenance Organization (HMO) option. Physicians employed by HMO's have an economic incentive to label injuries and diseases as "work-related." HMO physicians receive no benefit from non-occupational injuries, since they are compensated by salary for such cases. By deeming the injury or disease to be work related, they may bill the Workers' Compensation carrier directly (see Section 15).

Most group health plans have deductibles and coinsurance payments incurred by the employee. These create economic incentives for employees to consider their injuries or diseases as "work-related," since Workers' Compensation is a first dollar coverage with no employee contribution (Borba and Eisenberg-Haber [1988]). Adoption of "twenty-four" hour coverage, with similar medical benefits for occupational and non-occupational injuries and diseases, may shift some Workers' Compensation costs back to group health plans (Bateman [1991]; Bateman and

Veldman [1991].

Health actuaries, academics, and insurance research organizations have analyzed the effects of policy provisions and administrative procedures on containing medical care costs. Medical fee schedules and peer review are being used or considered in some states for Workers' Compensation.[39] The pricing actuary must quantify the likely effects of such enactments on Workers' Compensation loss costs.

E. Attorney Involvement

Workers' Compensation is intended to be a "no-fault" compensation system with little litigation or claim controversion. Attorney representation of Workers' Compensation claims has risen sharply in several states, with concomitant lengthening of disability durations and greater claim severities.

The AIRAC studies on Personal Automobile insurance suggest that attorneys cause greater "economic damages," by encouraging accident victims to stay out of work and incur large medical bills (AIRAC [1988; 1989], IRC [1990]). Similarly, Gardner [1989], page 2, finds that "incentives to remain away from work are even stronger when attorneys are negotiating [Workers' Compensation] settlements." Butler and Worrall [1985], page 719, using a multiple regression analysis, conclude that "when a lawyer represents a claimant the length of stay on Workers' Compensation will tend to increase . . ."[40]

Many states specify the reimbursement for plaintiff attorneys in Workers' Compensation cases. The 1991 Texas reform, which restricted payments for plaintiff attorneys, is expected to

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[39] Whether a state has a strong medical fee schedule affects the complement of the medical loss ratio trend in the NCCI procedure; see Section 8.

[40] *This effect is greatest when the insurance compensation is assured, such as in Personal Injury Protection or* Workers' Compensation. Under tort liability systems, claimants may be loath to incur large medical bills or income losses, since they may never be reimbursed.

reduce claim filings and claim severity (Gallagher [1990]).[41] Pricing actuaries must estimate the effects of the legislation affecting attorney involvement in insurance claims, to determine whether Workers' Compensation in particular states will be profitable.

F. Compensable Injuries and Diseases

The states vary in the statutory compensability of (i) latent diseases, (ii) diseases that are only partially work related, and (iii) stress claims. In California, for instance, stress claims are often deemed compensable and are becoming increasingly frequent (see Parry [1988], Barge [1988], Staten and Umbeck [1983], Victor [1988], Marcus [1988]).

Occupational disease claims and injuries treated by psychiatrists and psychologists have higher average severities than "traumatic" injuries (Marks [1984], Durban [1987]). Statutory amendments that encourage compensability of latent diseases and stress claims may have a great effect on overall loss costs.

Plaintiff attorneys often seek tort liability compensation for latent diseases, such as asbestosis (Millus [1987]). Workers' Compensation reimbursement generally requires physical disability and actual medical bills. Court awards under General Liability coverage are often obtained for a presumed increased likelihood of future disability or medical problems. In addition, class action suits are more common against General Liability carriers. Statutory changes that affect recoveries under tort liability will indirectly affect claim filings under Workers' Compensation.

G. Loss Cost Trends

Workers' Compensation loss cost trends and loss ratio trends are influenced by statutory amendments. Present rate making procedures adjust historical loss experience for the direct effects of statutory revisions. The indirect effects appear as part of the loss ratio trend (see Sections 8 and 17). If the historical indirect effects are included in trend factors, and indirect

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[41] The Texas reform was declared unconstitutional by a lower court. It is now in the appellate court system, and it will presumably proceed to the state Supreme Court.

effects from current statutory revisions are estimated separately, one may double count these effects. If one ignores the indirect effects of current statutory revisions, one may underestimate the short term effects. If one adjusts historical statutory amendments for the indirect effects and removes the loss ratio trends, one may overlook economic or social influences on loss costs.

Most appropriate is a complete analysis of direct and indirect effects of historical and current statutory revisions, along with a residual loss ratio trend.

H. A Caveat

The effects of benefit changes on claim frequency and severity depend on many factors, such as present benefit levels, type of injury, and the administration of the compensation system. The economists studying these effects are careful to qualify their projections, to note the types of injuries and claimant populations to which they apply. Gardner [1989] provides a list of dozens of studies on each topic with the varying results they produced. Fein [1991: Financial Crisis], pages 25-26, and Gallagher [1990] note the difficulty of predicting the effects of the Texas Senate Bill 1 (effective January 1, 1991). Flat, didactic statements about incentive effects are simply misleading.

> *"It is well documented that a 20% increase in benefits results in a 7% increase in claims and a 4% increase in duration of such claims."* – DeCarlo and Minkowitz [1991], page 445.

Section 11: Involuntary Market Burdens

Workers' Compensation risks unable to obtain coverage in the voluntary market are insured in involuntary pools, or "residual markets." The pools in most states run operating deficits, which are funded by private insurance carriers in proportion to direct written premium. The pools now constitute about 23% of countrywide business, so the "involuntary market burden" is large. Pricing actuaries generally consider the involuntary market burden as an expense element in setting voluntary market rates (NCCI [1991], pp. 38-39; Gustavson and Treischmann [1985]; Fein [1991], page 20).[42]

The involuntary market burden is the operating loss of the pools, not the underwriting loss (White [1988], page 46). One may quantify the burden by discounting cash flows for involuntary market business, by combining voluntary and involuntary market cash flows in an Internal Rate of Return model, or by calculating an investment income offset factor. The actuary must also estimate the profit or loss from servicing involuntary market business (Littmann [1990]). For servicing carriers, the involuntary market burden is the net effect of the operating loss from pool business and the profit or loss from servicing involuntary risks.

The pricing actuary has several tasks with regard to the involuntary markets:

- *Profitability:* Understand the causes of pool size and pool deficit by jurisdiction, in order to estimate the expected profitability of Workers' Compensation business.
- *Pricing:* Calculate the residual market burden, which is used as an expense element in pricing voluntary risks.
- *Strategy:* Forecast the expected residual market burden for alternative Workers' Compensation programs, such as excess coverage or large dollar deductibles, in order to devise company strategy for future business.

[42] In some jurisdictions, risks that private insurers are unwilling to service can obtain coverage from a state fund, thereby obviating the need for an involuntary market.

A. Profitability: Size of the Involuntary Markets

There are several explanations for large involuntary insurance markets. All contribute to the involuntary market problem, but each implies a different solution.

Rate Adequacy

Rate inadequacies cause the line of business to be unprofitable or only marginally profitable. In the late 1980's, for instance, as Workers' Compensation profitability declined, the involuntary markets grew rapidly. Statewide rate increases would reduce the involuntary market share.[43]

Competition

Involuntary market rates are competitive with voluntary market rates. An involuntary market risk has no incentive to seek voluntary market coverage. Involuntary market surcharges would reduce the involuntary market share.[44]

The NCCI is attempting to mitigate this phenomenon, wherever state regulation permits:

> "[The residual market] does not, and should not, guarantee that such coverage will be at a price that is competitive or lower than in the voluntary market. To eliminate this

[43] So Freeman [BRPC], page 22: "Why have so many residual market run amok? According to most observers, rate inadequacy heads the list of reasons"; see also Eisenberg and Vieweg [1987]. (McNamara [1984], page 15, gives the same explanation for automobile assigned risk plans: "The root cause of the availability problem is unquestionably the belief of underwriters that the overall rate levels, or the rates for particular classes and/or territories, are inadequate.") *Note, however, that Workers' Compensation insurers continued using rate deviations* and policyholder dividends averaging over 10% of premium through the 1980's. Voluntary risks would be profitable were there no involuntary market burden, even as the involuntary market grew. Higher manual rates may lead to increased deviations or dividends, not simply to reductions in the involuntary market share (though they have an effect).

[44] Huber [1986], page 54, provide an illustration: "In Maine, the regulatory disallowance of 'the plan managements's authority to mandate a retrospective rating plan for an account representing $4.3 million in premium resulted in the plan's forced provision of a substantially more competitive price than the voluntary market would provide. The same situation prevailed in Tennessee." Hofmann [1992: AR], page 9, notes that ". . . today's commercial insurance buyers know how to exploit bureau rates that are too low (by *voluntarily* purchasing coverage through assigned risk plans) . . ." Mintel [1983] sees competitive involuntary market rates as a major cause of the growth of certain Personal Automobile assigned risk plans.

possibility, NCCI has filed a plan change to recognize that an offer of any reasonable rating plan approved for use in a state would be considered an offer of voluntary coverage and failure to accept such an offer would exclude the risk from the residual market" (NCCI [1991: Issues report], page 38).

Hager [1991: Call for Reform; see also 1992; 1992], pages 2-3, lists five NCCI programs that should reduce the competitiveness of the pools, thereby depopulating them. The anticipated effects of such programs affect the actuary's forecast of the involuntary market load.

- Higher deposit premium requirements for involuntary risks.
- Payroll verification plans to avoid willful understatement of payrolls.
- Elimination of premium discounts for involuntary risks.
- Premium rate differentials between the involuntary and voluntary markets, ranging up to 25%.
- Two loss sensitive experience rating plans designed for involuntary risks: the Assigned Risk Adjustment Program (ARAP) and the Assigned Risk Rating Program (ARRP), which reflect more closely adverse historical experience.

Classification Refinement

Over-simplified risk classification schemes do not allow insurers to charge different rates to risks of different quality. Risks of poor quality that are not surcharged end up in involuntary markets. More accurate risk classification schemes would reduce the involuntary market share (Brunner [1985]).

Classification inefficiency in competitive markets is often used to explain large automobile involuntary markets. [Massachusetts, for instance, does not allow classification by sex, limits classification by territory, and has an involuntary market facility that insures over half the Personal Auto risks.] This explanation is particularly appropriate for Workers' Compensation, which had a rapid spread of "open competition" in the late 1980's, but retains a simple classification scheme.

Insurance Expenses

Some underwriting and administrative expenses vary more directly with the number of policies than with premium. An expense loading proportional to written premium assigns too little expenses to small risks, and the expense constants are insufficient to cover these "per policy" costs. As a result, small risks are often unable to obtain coverage from voluntary carriers and end up in the residual market.[45] Larger expense loadings for small risks would reduce the involuntary market share.

B. Pricing: Calculating the Burden

Residual market assessments vary with voluntary market writings. Thus, the operating loss on involuntary market risks may be considered an expense for voluntary market risks. To calculate the "residual market burden," the pricing actuary determines the net loss after investment income for involuntary market risks and divides this amount by voluntary market premium. There are several ways of doing this.

Investment Income Offset

The NCCI provides combined ratios by state for the involuntary market pools. An "investment income offset" is derived from Insurance Expense Exhibit data as line 11 ("Net Investment Income Gain or Loss") divided by line 2 ("Net Premiums Earned") for column 16 ("Workers'

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[45] Compare Chelius and Smith [1986], page 5: "If small businesses are not regarded as desirable clients, one can conclude that their possibly higher premiums per dollar of loss reflect higher overhead costs that are not fully recouped by insurance companies because of rigidities in the ratemaking process." They note that "small businesses are consistently and heavily over-represented in both assigned risk pools and competitive state funds. For example, the average premium paid in 1983 by those firms obtaining insurance from assigned risk pools was $1,812, while the average premium written by stock insurance companies in that same year was about $5,000" (pages 5-6). So also Huber [1986], page 52: "A review of the 20 most populous classes of the NCCI-managed reinsurance pools tells us that most accounts are small . . ." Compare also Freeman [BRPC], page 110: ". . . in workers comp . . . the carriers left in a particular market may have minimum premiums which are so excessive that smaller insureds are forced into the residual market." The NCCI, however, contests these observations: "In 1990, NCCI performed studies which refuted some common misconceptions concerning the demographics of the residual market. Although small risks account for approximately 75 percent of the residual market, they account for approximately that same percentage of the voluntary market" (NCCI [1991: Issues report], page 37). So also White [1988], page 39: "The composition of the residual market by size of insured does not differ significantly from the voluntary market except on the very high end of accounts in the million dollar range" and Fein [1990: Pricing and Profitability], page 31.

Compensation"). Industry-wide figures for 1990 give $4,172 million / $30,812 million, or 13.5% (Best's [1991: A&A]).

There are several problems with this calculation:

- The Net Investment Gain or Loss in the IEE allocated to lines of business excludes capital gains and losses, which are allocated entirely to the Capital and Surplus Account (IEE, Part II, line 11 instructions, footnote A). The 13.5% figure should be increased, perhaps by including capital gains and losses in the allocation of investment income.

- The timing of premium and loss cash flows differs between the voluntary and involuntary markets. Involuntary risks are written by servicing carriers; other member companies are charged assessments. Involuntary premiums are collected earlier, since retrospective rating plans are not used and required premium deposits are often larger than in the voluntary market. The IEE investment income offset, which is based on net loss reserves and unearned premium reserves, reflects the cash flows of all business, most of which is voluntary.

- The IEE investment income offset is based on the investment income received in the current calendar year, not the investment income expected in the future for the current policy year. The offset is distorted by changes in business growth and market interest rates (Butsic [1990]; Bingham [1992]).

- The investment income offset differs by state, since benefit provisions and loss payment patterns differ by state (see Section 7 above).

Discounted Cash Flows

Premium collections and loss payments may be discounted to the policy inception date to determine the economic loss from involuntary market risks. The premium collection and loss payment patterns should be those of the given state's involuntary market.

This approach can be used by both servicing carriers and other member companies. The servicing carrier would consider premium, loss, and expense transactions with both the policyholder and the pool. Other insurers would consider only premium and loss transactions with the pool.

Pricing considerations include:

- *Data Availability:* Some insurers do not keep the necessary records of cash flows to and from the pools by policy year, though industry statistics are compiled by the NCCI.

- *Complexity:* If the insurer does not use financial pricing models for its voluntary risks, the modeling work required may be great.

- *Discount Rate:* The actuary may select a conservative, risk free rate (e.g., Treasury bills), or an expected new money investment rate (e.g., high quality corporate bonds). Since all other values in the rate review are on a pre-tax basis, a pre-tax discount rate should be used.

Involuntary Load Illustration

There are no set procedures for calculation the involuntary market load; current methods differ by carrier and by jurisdiction. The pricing actuary must estimate

- The operating loss of the pool during the future policy period, and
- The market share of the pool during the future policy period.

Historical loss ratios for involuntary business may be obtained from the bureau managing the pool. The operating loss is either

- The undiscounted loss ratio plus an expense ratio (servicing carrier allowance) minus the investment income offset, or
- The discounted loss ratio plus an expense ratio.

For instance, the undiscounted loss ratio may be 110%, the servicing carriers allowance may be 30%, and the investment income offset may be 20%, for an operating loss of 20%.

The future market share of the pool may be estimated as the most recent market share adjusted for the anticipated effects of residual market programs. For instance, higher premium deposit amounts and the lack of premium discounts may encourage more large risks to seek coverage in the voluntary market, thereby reducing the involuntary market burden.[46] Other developments also affect the anticipated market share of the pool. For instance, factors that increase the share include

- risks leaving the voluntary market for self-insurance plans or excess coverage, and
- regulatory suppression of voluntary market rates, leading insurers to tighten underwriting restrictions.[47]

For instance, the most recent market share of the pool may be 18%, a new involuntary market experience rating plan is expected to reduce this 2 points, and the exodus of risks from the voluntary market to self-insurance and excess coverage is expected to increase this 4 points, for a projected future involuntary market share of 20%.

The market share of the involuntary pool is converted into a ratio of involuntary to voluntary premium. For instance, a 20% involuntary market share is a 25% ratio of involuntary to voluntary premium.

The involuntary market burden is the product of the pool operating loss and the ratio of involuntary to voluntary premium. Thus, a 20% operating loss times a 25% ratio of

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[46] Fein [1990: Enduring Difficult Times], page 5, estimates that "the residual market programs have reduced the burden on the voluntary market by two percentage points." Some of these programs, such as rate differentials, reduce both the involuntary market share the involuntary operating loss.

[47] In addition, not all voluntary premium is included in the residual market assessment base. For instance, carriers taking direct assignments from the pools may not receive an assessment. Countrywide, the assessment base is about 96% of the voluntary market premium, though this varies by jurisdiction (NCCI [1992: Act-92-4], Exhibit 10–2–I). The pricing actuary must also consider the effects of business growth or contraction, since direct written premium of the preceding calendar year is the assessment base for the current policy year.

involuntary to voluntary premium is a 5 point involuntary market burden.[48]

C. Strategy: Forecasting the Burden

Large involuntary market burdens are forcing insurers to leave some jurisdictions or to develop alternative insurance programs. Much insurance for large risks at lower layers of coverage is "dollar trading": the insure collects premium which it return in loss payments. Some of these expenses are a servicing charge for issuing policies and handling claims.

Alternative Workers' Compensation programs

In a jurisdiction with a large involuntary market burden, this servicing charge rises, and full coverage programs may become uneconomical. To alleviate the burden, some insurers are developing alternative programs, such as excess coverage, administrative services only (or management assistance for a self-insurance program), and large dollar deductible policies. State regulations affect the types of programs offered in each jurisdiction.

As an example, suppose an insurer has a 3% market share in a jurisdiction with a 15% involuntary market burden. Its voluntary market operating ratio is 90%, but with the involuntary market burden, its net operating ratio is 105%.

A conversion to excess coverage, by means of an assisted self-insurance program or a high deductible in the policy, with a two thirds reduction in premium, may cause the following:

- Market share drops to 1%, since premium is only one third as large.
- The insurer continues to handle all claims. The insured pays the benefit costs, and the insurer pays the loss adjustment costs. Most of the premium in some excess plans is for claims handling expenses.
- The insurer uses a larger percentage "profit and contingencies" provision to

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[48] Actual loads vary greatly state. The NCCI estimates a countrywide average of nearly 15%, though estimates by private carriers vary considerably. Jurisdictions with high involuntary market shares, such as Arizona, Florida, Kentucky, Maine, Massachusetts, and Tennessee, require large involuntary market loads, ranging from 25 to 40%. The full indicated load is not always permitted by state regulators.

accommodate the variability in the higher layers of coverage. Although the percentage provision is higher, the dollar amount is lower, since the total premium is lower. Thus, the insured's premium plus the self-funded benefit costs are lower than the premium under the full coverage policy.

- The larger percentage profit provision causes the voluntary market operating ratio to drop to 80%. With the involuntary market burden, the net operating ratio is 95%.

In sum, the cost to the insured is lower, the claims operations remain essentially unchanged, and the insurer's profitability rises.

The pricing actuary's task is complex. He or she must

- Forecast industry changes to alternative programs. If all companies switch to excess coverage in the voluntary market, the involuntary market burden increases as a percentage figure and remains constant as a dollar amount.
- Develop pricing techniques for excess layers of coverage. Workers' Compensation does not use increased limits factors. Instead, the actuary may use excess loss pricing factors from retrospective rating techniques (cf. Simon [1965]).
- Determine the appropriate profit provision for the greater variability in excess layers of coverage (cf. Miccolis [1977]).
- Quantify the anticipated effects of newly implemented involuntary market programs.

Section 12: Large vs. Small Risks

". . . the small risk does not have the same incentive to provide for efficient and extensive accident prevention work, first, because such work requires an expenditure of money and second, because it does not reduce the cost of insurance. Furthermore, it must be borne in mind that many small employers do not keep accurate and adequate payroll records and, in certain industries, are tempted to conceal and do conceal considerable portions of the payrolls actually expended. . . . The problem of premium collection is also very acute in case of a small risk where frequent changes of the insurable interests, disappearance of the assured, reluctance to pay additional premium upon audit and other similar conditions, make it well nigh impossible to collect the full premiums due. On the other hand, the expenses of handling the records of the books of the company and of preparing reports to various boards, bureaus and supervisory authorities are percentage-wise considerably higher for those risks than for risks with substantial premium volume." – Kormes [1936], page 46.

Small risks have higher average loss ratios and higher average expense ratios than large risks have. Expense constants, loss constants, premium discounts, and experience rating plans recognize these differences. This section discusses the reasons for these differences and some ratemaking techniques that adjust for them.

A. Expenses

Some underwriting expenses, such as setting up files, do not vary much by size of policy. The proportional expense loading used in Workers' Compensation ratemaking assumes that expenses are directly proportional to premium, thereby undercharging the small risk and overcharging the large risk. If no other expense component were incorporated in pricing, small risks would be unprofitable and may have difficulty obtaining coverage (Barber [1934]).

A flat "expense constant" is added to each risk's premium. The amount varies by jurisdiction and must be adjusted for inflation (Chelius and Smith [1986]). The NCCI is now using $140 in most states, though the size of the expense charge depends on regulatory approval.[49]

Expense Constants and Expense Ratios

Certain ratemaking adjustments are applicable to manual premium, not to the expense constant premium. For instance, the "on-level" procedure determines how much premium would have been collected had the policies been issued at the current rates. Rate revisions affect the manual rates, not necessarily the expense constant. The expense constant premium applicable in each year must therefore be removed at the beginning of the on-level procedure, and the current expense constant must be added at the end (cf. Kallop [1975]).

Premiums derived by extending exposures from Unit Statistical Plan data do not include expense constants. Premiums derived from financial data include the expense constants. In the past, when the expense constant differed by size of risk, removing the expense constant premium required a distribution of risks by size (cf. McConnell [1952], page 31; Marshall [1954]; Kallop [1975]). Now that the expense constant is uniform for all risks, removing the expense constant premium requires only a policy count.

Expense ratios derived from IEE data include expense constants. To avoid double counting, the pricing actuary must remove the expense constant premium from the expense loading. For instance, suppose the insurer's book of business shows

net written premium:	$45 million
average premium discount:	10%
number of policies:	2,000
expense constant:	$150 per policy

Standard premium is $45 million ÷ 0.9 = $50 million. Total expense constant premium is 2,000 x $150 = $300,000. The proportional expense loading (for general expense and other

[49] Originally, the expense constant was used only for small risks: "The loss and expense constants applied to risks producing annual premiums of less than $400 prior to July 1, 1934 and to risks producing annual premiums of less than $500 on and after July 1, 1934" (Hipp [1936], page 258). In reply, Kormes [1936], page 267, notes that ". . . the author feels that an expense constant is not necessarily attributable to small risks since if it is based on the theory that there are certain constant expenses per policy it should, in practical application, be charged as a sort of a policy fee on all risks." Marshall [1954], pages 20-21, and Kallop [1975], page 65, retain the expense constant as a charge only for small risks. Eventually, the difficulty of publicly justifying this procedure led to the present application to all policies.

acquisition costs) must therefore be reduced by $300,000 ÷ $50,000,000 = 0.6%.

The determination of the expense constant poses special problems in a loss cost environment. Many "fixed expenses," such as advertising, overhead administrative costs, and underwriting salaries, are not easily allocated to policies or premiums. It is unclear whether bureaus will continue to provide advisory expense constants in most jurisdictions, or whether company actuaries must independently select the constants.[50]

B. Losses

Loss experience is generally better on large risks than on small risks. This is evident in various ways:

- The experience rating plan generally shows a higher ratio of credit to debits for large risks than for small risks (cf. Dorweiler [1934]).
- Small risks are more likely to be assigned to involuntary markets than large risks are (Chelius and Smith [1986]; Huber [1986]).
- Independent studies of experience by premium size generally show higher loss ratios for small risks than for large risks.[51]

Two explanations of this phenomenon are often given:

- The experience rating plan does not just measure loss experience; it provides an incentive for safety procedures. Poor loss experience for a firm subject to an experience rating plan increases the cost of insurance in future years; conversely for good loss experience decreases the future cost of insurance. The more weight that is given to a firm's own experience, the greater is the employer's incentive to reduce claim costs. Since the experience of large firms receives greater credibility than the experience of small firms,

[50] Most general expenses do not vary by state. Presumably, expense constants determined for administered pricing states are reasonable for loss cost jurisdictions as well.

[51] Chelius and Smith [1986], however, find that the ratio of premiums to losses is slightly higher for small risks than for medium sized risks, suggesting that small risks have slightly better loss experience than average. Cf. also Harrington [1988].

large firms have greater incentives to reduce losses.[52]

- Safety programs require large fixed costs: installing guards on machines, replacing dangerous equipment, implementing safety programs, and hiring on-site medical personnel. The large expenditures required may be more cost-effective for large firms than for small firms.[53]

Loss Constants

Loss constants, or flat dollar premium additions either for all insureds or for small insureds, are a means of flattening the loss ratios by size of risk. Loss constants were once a standard component of the Workers' Compensation premium. They were applied only to risks below a certain size, and they varied by industry group and jurisdiction. Loss constants have been dropped in most states. In 1990, the NCCI recommended that loss constants be reinstituted in those states whose experience indicated a need. To avoid any appearance of unfair discrimination or rate redundancy, "the loss constant would be applied to all risks with a concurrent rate offset to make the program revenue-neutral" (NCCI memorandum AC-90-23).[54]

The calculation of the loss constant is illustrated below for two scenarios: one in which the loss constant is applied only to risks with annual premium less than $1,000, and one in which the loss constant is applied to all risks.

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[52] Opinions differ as to whether experience rating actually provides such an incentive effect and how great this effect is, particularly compared with the incentive effects of self-insurance. For a variety of studies, see Victor [1982; 1985]; Victor, Cohen, and Phelps [1982]; Chelius [1982; 1983]; Chelius and Smith [1983]; Ruser [1985]; Worrall and Butler [1988].

[53] Cf. Hipp [1936], page 259: "It may be that small risks are inherently more hazardous than large risks. Regardless of expense, small risks may not be readily susceptible to accident prevention methods." Cf. also Perkins [1922], pages 273-274.

Gary Venter has pointed out to me that "large and small risks may differ in off-the-books payroll that is only reported after an injury." In other words, payroll may be understated for small firms, so expense and loss ratios may be higher.

[54] The NCCI recommendation has not yet been implemented. Texas has retained its loss constant applicable to small risks only. The Delaware Compensation Rating Bureau (Circular No. 661) adopted a $45 loss constant, effective in May 1992, applicable to all risks. Loss cost systems may stimulate increasing diversity among carriers and jurisdictions.

Loss Constants Applied to Small Risks Only

Suppose the historical experience is as shown below.

Calculation of Loss Constants

Premium Range	Number Of Risks	Earned Premium	Incurred Losses	Loss Ratio	Loss Constant	Loss Cost Premium	Loss Ratio
$0 – $1,000	500	$ 300,000	$240,000	80%	$40	$20,000	75%
> $1,000	500	2,000,000	1,500,000	75	0	0	75

Loss constants will be used for risks with annual premium of $1,000 or less. Observed experience for these risks shows premium of $300,000 and incurred losses of $240,000, for a loss ratio of 80%. For risks with annual premium greater than $1,000, the total premium is $2,000,000 and incurred losses are $1,500,000, for a loss ratio of 75%. There are 500 risks in each group.

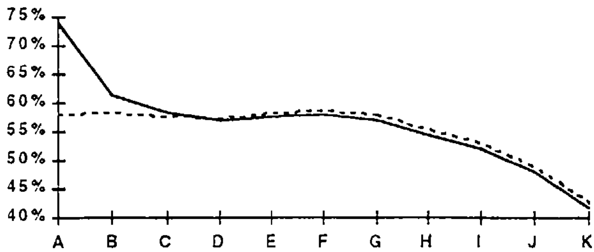
The loss constant is chosen such that the new loss ratio for risks with annual premiums of $1,000 or less becomes 75%. Since the incurred losses are $240,000, the premium must be $320,000 to produce a loss ratio of 75%. That is, an additional "loss constant" premium of $20,000 is needed. Since there are 500 risks, the loss constant must be $40.

The loss constant premium must be offset in the manual rate premium. Thus, the manual rate must be reduced by $20,000 ÷ $2,300,000, or 0.87%. Each group would have a loss ratio of 75.6% [= 75% ÷ (1 − 0.0087)].

Loss Constants Applied to All Risks

The NCCI used countrywide Unit Statistical Plan experience for 1988 through 1990 to calculate loss constants by state (NCCI memorandum Act-90-23). The experience showed steadily declining loss ratios to standard earned premium as the risk size increased, as shown by the solid line below. Use of a loss constant for all risks flattens the loss ratios for smaller risks, as shown by the dotted line.

The countrywide average indicated loss constant is $104, though this figures differs markedly by state. With an offsetting premium rate reduction of 1.78%, the average indicated loss constant is $102.15.



There are eleven premium sizes, ranging from $0 - $999 ("A") to $1 million and up ("K"). Note that the loss constants flatten the high loss ratios for small risks, but have little effect on the low loss ratios for large risks.

The pricing actuary should understand the causes of differing loss experience by size of risk. Those relating to sunk costs may be remedied by expense constants; those relating to economic incentives for safety programs may be remedied in part by varying the experience rating plan; those relating to economies of scale for safety programs can sometimes by remedied by loss control efforts provided by the insurer and by loss constants. The goal is to minimize the expected accident costs and to set a premium rate that reflects these costs.

Section 14: Classification Systems

"But the uninitiated are scarcely prepared to learn that the hazard of digging a six-foot trench and laying the pipe therein is doubled if sewage rather than water is to flow through the trench . . ."

– Downey [1915], page 12

The previous sections describe the pricing procedures for overall statewide rate revisions. But insureds are not charged "overall statewide rates." Since the risk of injury varies among insureds – for instance, miners face greater occupational hazards than retail clerks do – manual rates vary accordingly. Risk classification is the means of differentiating among insureds and aligning the premium charged with the risk of loss.

A. Industry Group and Occupation

Risk classification systems may be multidimensional or unidimensional. Personal automobile insurance uses a multidimensional system. Risks are classified by driver characteristics, use of the vehicle, territory, and driving history. Although each dimension by itself has limited explanatory power, they measure different influences on loss cost (SRI [1979]). The combination of the classification variables improves the power of the risk assessment system.

Workers' Compensation has a unidimensional classification system. Insureds are divided into three industry groups: manufacturing, contracting, and all other. Each industry group is then subdivided into classifications based on the products manufactured or the services provided. For example, the manufacturing industry group contains classifications for jewelry manufacturing, motorcycle manufacturing, and refrigerator manufacturing (see, for instance, Mowbray [1921]; NCCI [1989: Class Manual]).

Occupational injuries and diseases are related to industrial processes and operations, not necessarily to products and services. Welders face greater hazards than accountants, regardless of the industry in which they work. Some actuaries have suggested that the classification system should discriminate by occupation, not by industry.[59]

Classification by occupation entails verification problems: How many employees are welders? How many are accountants? The present Workers' Compensation classification system uses product as a *proxy* for occupation. Producers of the same product are assumed to use similar manufacturing processes, so the product produced is a rough measure of workplace hazards.[60] [Certain employees, however, such as clerical workers, draftsmen, salespersons, and drivers, are termed "standard exceptions" and are separately classified.]

This unidimensional classification system is relatively inefficient, particularly in comparison to automobile insurance classification. However, the manual rate is adjusted by a mandatory

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[59] Downey [1915] perceives the industry classification system as flawed (page 10: "The existing 'casualty' insurance classification of industries is a relict of employers' liability. . . . it is not adapted to the broader needs of compensation insurance; it is a thing of shreds and patches; it was never conceived as a whole nor based upon any reasoned principle of taxonomy"), and he presents forceful arguments for classification by occupation. The closer relationship of occupational hazard to occupation than to industry is mentioned in the text. Downey also notes that competition compels insurer to continuously refine the industry classification system until the individual classes are too small for credible rate making. Since there are far fewer industrial processes than industrial products, classification by occupation leads to more accurate pricing.

Downey has a jaundiced view of competition: "Whatever may be true of competition in service, or even in rates, competition in misclassification is an unmixed evil" (page 23). Actuarial equity in classification is similarly of little concern: "That every commodity shall bear its specific accident cost . . . is neither practically attainable nor especially important." The countervailing argument is that the industry classification system in Workers' Compensation was feasible only because of the administered pricing system and the lack of open competition.

In his discussion of Downey's paper, Gustav Michaelbacher [1915] gives a vigorous defense of classification by industry. In particular, he argues that classification by occupation would reduce safety incentives for the employer, since the rate for each occupation would be based on a diverse set of firms: "Dr. Downey's plan, if put into practical application without any modification whatsoever, would largely do away with the "Safety First" movement. If employers were to find their establishments divided by processes and grouped for insurance purposes with a resulting rate covering all of the risks in a given class, they would not be particularly interested in making their individual plant as safe as possible, for they would feel somehow that they were being assessed for accidents occurring in processes carried on in the worst possible manner and would consequently have no incentive to make their own plant as safe as it possibly could be made" (page 30). This argument seems specious. Classification by occupation would provide incentives to eliminate the more dangerous processes and operations and would thereby reduce the overall injury rate.

[60] Kallop [1975], page 63: "The fundamental concept underlying workers' compensation ratemaking and pricing is that the exposure to risk of each employer is in part a function of the business in which he is engaged. Because it is expected that each employer engaged in the same type of business would have a similar distribution of employees performing comparable functions, it follows that a single all-inclusive classification is the most practical method of determining premium." Downey [1915], page 16, takes the opposite view: "The number and character of operations, and consequently the kind and degree of hazard, differ widely as between establishments turning out the same finished product." On the practical issues, see also Black [1915], page 27: "The principle objection to process classification is the impossibility of determining the actual payrolls expended on the different processes."

experience rating plan as well as by voluntary schedule rating and retrospective rating plans. The importance of the individual risk rating plans stems from both (i) the stability of injury experience by firm and (ii) the inefficiency of the manual classification system.

B. Other Classification Dimensions

Several other classification dimensions are powerful predictors of Workers' Compensation loss costs. Important variables are

- workforce characteristics, such as age and sex,
- *group health benefits provided by the employer,*
- territory and claims consciousness, and
- the financial health of the employer and of its industry.

As open competition spreads in Workers' Compensation and carriers seek strategic advantages, classification systems will be refined.[61] The predictive power of the classification variable is the primary determinant of its usefulness. In addition, the actuary must consider issues of (i) data availability, (ii) quantification, and (iii) social acceptance of each classification variable (AAA [1990]). For instance,

- data on personal characteristics of the workforce are not now gathered by Workers' Compensation insurers, though health and disability insurers use these attributes;
- the influences of group health benefits on Workers' Compensation costs are difficult to quantify despite their importance, because employer provided group health plan provisions are so varied;
- rating by territory raises social acceptability issues, even more in Workers' Compensation than in Personal Automobile (see Section 14.E).

Rating bureaus are concerned that a proliferation of classification systems will impair the integrity of industry-wide data bases and hamper the application of a mandatory experience rating plan (AIA [1982]; Berquist, et al. [1991]). Conversely, some private insurers believe

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[61] See McNamara [1984] for the relationship of price competition and classification refinement. Cf. also Pomeroy [1990], page 26, who notes the NAIC project goal of determining whether Workers' Compensation classifications are appropriate.

that adherence to a uniform classification system and the use of a mandatory experience rating plan are impediments to true open competition (see Hofmann [1992] for a general discussion). This reading takes no position in this debate. It simply notes that underwriters, agents, and private carriers examine various risk characteristics when offering Workers' Compensation coverage. The pricing actuary must be able to quantify their effects to use them effectively in an open competition environment.

C. Workforce attributes

The distribution by age and sex of the workforce affects the expected medical and disability benefits. These distributions have long been used by health insurance actuaries for premium determination in employer provided group plans. Since many of the relationships between personal characteristics and health benefits stem from non-occupational illnesses, such as gynecological treatment for young women or cardiovascular illnesses for older individuals, the health insurance studies must be adjusted for pricing Workers' Compensation policies.

This section focuses on age, whose relationship to Workers' Compensation benefits is clear. In particular, we examine age in relationship with claim frequency, claim severity, and experience rating plan modifications.

Health care costs for *non-occupational illness* rise steeply with age, so employer provided health plans for small groups depend on the age distribution of the workforce. *Occupational injuries* are more frequent among inexperienced workers, who are generally young.[62] Durations of disability for a given injury are longer for older workers, primarily for physiological reasons but also because workers near retirement may use compensable

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[62] So Worrall, Appel, and Butler [1987: NCCI Digest], pages 7-8: ". . . younger workers are far more likely to be workers compensation claimants." The frequency of occupational *diseases*, however, often depends on the length of the exposure period. The longer an employee has worked, the greater is his or her exposure to toxic substances. Thus, disease frequency is higher for older workers, who have had more exposure.

disabilities as substitutes for early retirement.[63] Dillingham [1983], page 238, presents the following Workers' Compensation claim frequency and severity figures for New York indemnity cases in 1970:

Average Claim Frequency and Severities
New York Workers' Compensation Indemnity Cases, 1970

Age Group	Claim Frequency Per 500 Workers	Average Claim Severity	Average Loss Costs
Less than 25 Years	13.83	$ 753	$10,414
25-44 Years	9.28	1,385	12,853
45 Years & Older	9.20	1,798	16,542

One can sometimes rely on the experience rating plan to mitigate rate inequities. But this rating plan does not substitute for classification by workforce attributes, for two reasons.

- The experience rating plan has less effect on small and medium sized risks, where the age distributions of the workforce vary considerably.
- The experience rating plan aggravates the problem of varying age distributions. A small firm with many older workers will have high expected loss costs but low expected frequency. Since the experience rating plan emphasizes claim frequency, not claim severity, it may indicate a credit, not a debit. Conversely, a small firm with many young workers will have low expected loss costs but high expected frequency, and it may receive an experience rating debit instead of a credit.[64]

[63] So Worrall, Appel, and Butler [1987: NCCI Digest], page 9: "Age significantly increases the costs of medical utilization . . ." The effects on indemnity benefits are equally great. Butler and Worrall [1985], page 719, restate the "retirement" cause in more formal terms: "Since the older one is, the shorter the subsequent stream of wages upon returning to work, one would expect age to decrease the hazard rate." Their regression analysis supports this hypothesis.

As Dave Appel has pointed out to me, one must consider the effects of age on premiums as well. Older workers generally are more senior and higher paid. Their higher average loss costs may be offset by the greater payroll.

[64] The claim severity disparity between younger and older workers is most evident in serious cases. The experience rating plan divides losses into primary and excess portions, with a low cutoff point for small firms (Venter [1987]; Gillam [1991]).

D. Group health benefits

During the late 1980's, many employers increased deductibles and coinsurance payments for group health insurance plans. Workers' Compensation remains a first dollar coverage: medical losses are reimbursed in full, with no deductibles or coinsurance payments. Some accident victims file for Workers' Compensation benefits even when the injuries are not necessarily work related.[65]

Medical care practitioners have similar economic incentives to label injuries "work-related" and therefore compensable. Physicians in HMO's, for instance, receive no additional compensation for an injury or illness covered by group health plans but full reimbursements for injuries or illnesses covered by Workers' Compensation. Similarly, chiropractic treatments are covered under Workers' Compensation but may be excluded under certain group health plans.

A firm with a generous group health care plan, such as a fee for service plan with low deductibles and co-payments, will have low expected Workers' Compensation costs. Conversely, a plan with high deductibles or co-payments, or a plan emphasizing Health Maintenance Organizations or Preferred Provider Associations, may have high expected WC costs. Ducatman [1987], page 52, presents data for eight federal shipyards showing a strong correlation between the percentage of workers enrolled in HMO's and the average Workers' Compensation costs per capita. He concludes that "increases in present prepaid plan enrollments were accompanied by substantial increases in workers' compensation costs."

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[65] Ducatman [1987], page 51, summarizes this: "When individuals have access to parallel health insurance systems, they can be relied upon to use them advantageously. When one system [group health] severely constrains costs and services, and the other [Workers' Compensation] provides full access to health services without additional cost, the unconstrained system will predictably prove more popular." Hager [1991], page 9, writes: ". . . medical inflation within the workers compensation system has been running 50 percent higher than general medical inflation. . . . because compensation is the last medical insurance system that generally prohibits deductibles and coinsurance, provides for unlimited medical benefits, and makes it difficult for insurers and employers to use HMO- and PPO-type mechanisms." Borba and Eisenberg-Haber [1988] find that Workers' Compensation claims for sprains and strains (soft tissue injuries) are more common on Mondays than on other days of the week, suggesting that non-occupational injuries occurring on weekends are being reimbursed by the Workers' Compensation system. They note that "there may be economic incentives for a worker to attribute an off-the-job injury to a workplace incident. In particular, medical expense reimbursement and indemnity benefits for lost work time may be more complete under workers compensation insurance than under accident and health plans" (page 52).

HMO Enrollment and Workers Compensation Costs, Fiscal 1983					
Shipyard	% HMO Enrollment	WC Costs Per Capita	Shipyard	% HMO Enrollment	WC Costs Per Capita
A	0%	$ 347	E	53%	$ 756
B	0	370	F	53	930
C	<1	477	G	83	1,181
D	39	723	H	66	2,325

The type of group health insurance plan provided by the employer, as well as changes in the group health plan provisions, must be considered by the actuary when pricing Workers' Compensation policies. Because of the variety of group health plans and the constantly evolving nature of many provisions, an objective classification scheme may be difficult to devise. Rather, the Workers Compensation actuary must understand the qualitative influences on benefit costs and provide rough estimates of their magnitude.

E. Territory

In Personal Automobile insurance, territory is a powerful classification dimension. In the past, many actuaries presumed that traffic congestion, road conditions, and similar "physical" factors were the major influences on loss cost differences by territory. Recent studies have suggested that equally important factors are attorney involvement in insurance compensation systems and differing proclivities to file personal injury claims. For example, the AIRAC attorney involvement studies showed that claim severity was higher in urban areas than in rural areas – not because of differences in economic damages per claim (which are higher in rural areas) but because of the greater percentage of urban claims that are represented by attorneys (AIRAC [1988; 1989]). Similarly, the "BI/PD ratio" studies showed that the incidence of physical accidents was more similar across territories than the incidence or severity of Bodily Injury claims (IRC [1990]; Woll [1991]).

Workers' Compensation is a no-fault coverage, abrogating the employee's right to sue in exchange for statutory benefits. Yet attorney involvement in compensation claims is increasing rapidly, along with total benefit costs (Borba [1989], page 67). The effects of the trial bar are evident in three areas:

Claim Frequency

Many compensation claims, such as some soft-tissue injuries, stress claims, and disease claims, are of dubious validity. Oftentimes, a worker suffering from stress, moderate hearing loss, or a minor back sprain will press a compensation claim only if encouraged by an attorney.

The relationship between physical injury and insurance claim is clearest in the BI/PD studies undertaken by the Insurance Research Council [1990]. Personal Auto Property Damage (PD) claims depend primarily on physical accidents; Bodily Injury (BI) claims depend on the injured party's claims consciousness and on attorney involvement as well. The ratio of BI claims to PD claims measures the proclivity of the public to press insurance claims.

The Personal Automobile BI/PD ratio by territory is a good predictor not only of Auto loss costs but also of Workers' Compensation benefit costs. Exhibit 15.E.1 shows Insurance Service Office BI/PD ratios by Personal Auto rating territory in Florida, and Exhibit 15.E.2 shows attorneys per capita in each Florida county. Lawyers are more concentrated in the southern half of the state (e.g., Dade, Palm Beach, and Polk counties) than in the northern half (e.g., Jackson county). Similarly, the BI/PD ratios are higher in the southern territories than in the northern ones. Finally, both automobile loss costs and Workers' Compensation benefit costs are greater in the southern half of Florida than in the northern half.

Economic Damages

Attorneys raise claim costs not only by persuasive arguments in litigated cases but also by "building up" the economic damages. The All-Industry Research Advisory Council, in its 1989 study of Automobile personal injury claims, compared claims where an attorney represented the plaintiff with claims where the victim sought compensation without legal aid. The ratio of insurance payments to physical damages, about 2 to 1, was the same for each group. But the attorney-represented claimants had two to three times the average costs for medical treatment and lost workdays that the non-represented claimants had.[66]

Plaintiffs' lawyers are paid on a contingency fee basis. The greater the damages, the larger the award; the larger the award, the higher the attorney's fees. Many lawyers encourage claimants to seek repetitive medical treatment and to refrain from work. This incentive to aggravate claims is unrelated to the type of compensation system, whether liability or no-fault, Personal Automobile or Workers' Compensation. As long as the award varies with damages, the attorney benefits from increased loss costs.[67]

Medical Treatment

The type of medical treatment received by the claimant influences both economic damages and insurance compensation. Medical practitioners who deal with injuries that are difficult to objectively assess, such as psychologists, physical therapists, and chiropractors, may sometimes provide treatment primarily to collect the insurance compensation. Geographical location is often correlated with such phenomena. For instance, 1989 Personal Auto insurance claims in Lawrence, Massachusetts, were predominantly sprains and strains, treated by chiropractors, often represented by the same group of attorneys, with unusually little variance in the length of treatment or the claim medical charges – symptoms of potential fraud (Weisberg and Derrig [1991]; Marter and Weisberg [1991]). Similarly, Workers' Compensation stress claims are far more common in certain regions of California than in other areas, whether because of judicial liberality or psychological positions (Borba [1989], page 63).

In sum, territory is an important classification dimension because of social differences by region. (The use of territory is more difficult for Workers' Compensation rating than for

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66 An alternative explanation is that claimants are more likely to seek legal aid in severe cases. However, the same relationships appear even when claims are stratified by type of injury (AIRAC [1989]).

67 Butler and Worrall [1985], page 719, note that "when a lawyer represents a claimant, the length of stay on Workers' Compensation will tend to increase, since the transition rate from Workers' Compensation decreases." Similarly, NCCI [1991: issues report], page 35, attributes the increasing paid loss link ratios to greater attorney involvement in Workers' Compensation claims. Attorney involvement also increases defense fees. Pillsbury [1992] estimates that "litigation costs [in California] accounted for more than $1 billion our of $6 billion in total workers' compensation costs in1988."

automobile rating because some risks have multiple plants. However, this is no different from multi-state risks, which the rate making procedures accommodate.) The actuary must understand these influences on Workers' Compensation costs and incorporate them into pricing and marketing strategy.

F. Financial Health

Economic conditions affect Workers' Compensation claim frequency and durations of disability. Occupational injuries often stem from workers' inexperience with industrial equipment or workplace hazards. During prosperous periods, when firms hire new and less experienced workers, speed up production, and expand overtime work, claim frequency rises (NCCI [1991], page 34). Claim severity, however, is low, since employees are eager to return to work and jobs are available.

The opposite pattern occurs during recessions. Most employees are experienced, since there is little new hiring, and production is slack; claims frequencies are low. Durations of disability lengthen, however, since there are few jobs available, and alternative employment opportunities for partially disabled workers are rare.

Victor and Fleischman [1990], in a recent reanalysis of data gathered by Boden and Fleischman [1989], find a strong effect of economic conditions on average claim severity, which three attribute to three potential causes:

> "First, higher unemployment may *increase utilization* of workers' compensation income benefits as workers without jobs seek to retain income from whatever sources are available. Some of those unemployed will make claims that they would not have otherwise made, and extend the durations of the claims as long as possible or until job opportunities surface. Some who are receiving benefits will find that they no longer have jobs to which they can return. They seek to extend the duration of benefits. Some with residual disabilities find that they are especially at a competitive disadvantage in the labor market when unemployment rises. In each of these instances, workers may use more medical care in their efforts to establish entitlement or retain benefits.

"Second, when unemployment is higher, some employed workers with relatively minor injuries will be *more reluctant to file* workers' compensation claims, fearing that they may be more vulnerable to lay-off if not currently working. When some minor claims are not brought, it makes the average costs of a claim – medical as well as indemnity – appear to be *increasing, as the fraction of more serious cases rises.*

'And third, when unemployment rises, the *experience and injury mix* of employed workers changes. Less experienced workers are laid-off, and more experienced workers retained. Less experienced workers tend to be younger, and have more frequent, but less serious injuries. As a consequence, the average severity of injury and average medical costs would increase."[68]

For the individual firm, this relationship is even stronger. Impending layoffs often precipitate an increase of Workers' Compensation claims for minor injuries and latent disease claims, since disability benefits generally exceed unemployment benefits in both duration and amount.[69] Two resulting principles of Workers' Compensation pricing have been suggested, though strong empirical support is hard to produce:

- In a declining industry susceptible to disease claims, the actuary should expect rising costs.
- If a firm faces financial problems that may lead to workforce reductions, the actuary should

[68] Victor [1990: Major Challenges], page 17, summarizes these results: "Evidence is emerging that workers' compensation benefits are more heavily used in times of economic distress. The severe recession that hit Michigan saw a surge in claims by workers taking early retirement from automobile companies . . . The recession in Texas saw an increase rate of claim filing and a significant increase in the duration of lost time . . ."

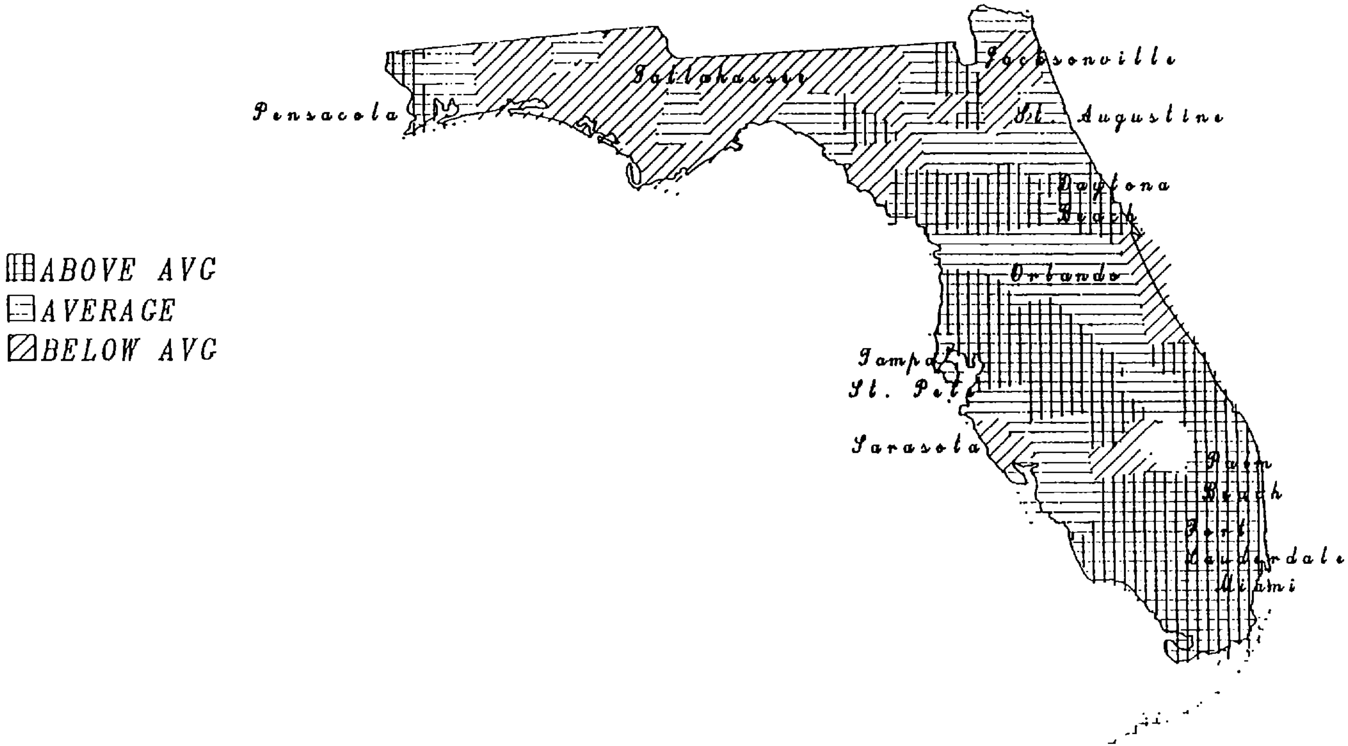
The actual effects of economic conditions on claim frequency and severity are uncertain, most evidence is anecdotal, and generalizations may be premature. Mowbray and Black [1915], p. 425, write: ". . . accident frequency per unit of exposure tends to rise and fall as production rises and falls . . ." and ". . . during times of . . . extreme depression . . . there is a slight lengthening of the average period of disability when compared with that during normal times." Greene and Roeber [1925], pages 254-255, suggest that ". . . the speeding up of industry [in 1916] due to war contracts had increased the accident rate" and that ". . . the depression of 1921-22 marked the beginning of a period of rising compensation costs." See also Whitney and Outwater [1923], pages 153-155.

[69] Cf. Marshall [1954], page 71: ". . . there are many employees working in foundries and similar dusty industries who have already contracted silicosis to some degree and need only to be thrown out of work to become a compensation claim." Marshall also notes ". . . the expected 'catastrophic' nature of the emergence of claims for dust diseases in the event of an economic depression . . ." (page 61).

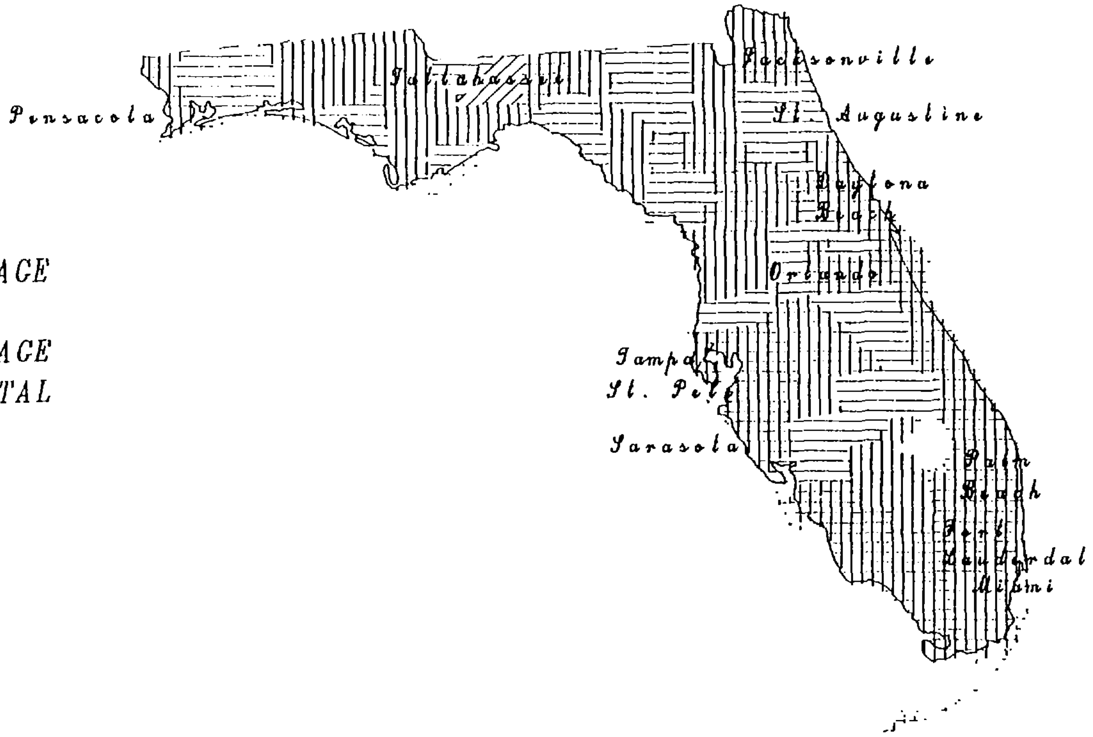
expect a higher incidence of soft-tissue claims, disease claims, and stress claims.

This section has reviewed six classification dimensions: industry, occupation, workforce attributes, group health plan provisions, territory, and financial condition. An administered pricing system requires little classification refinement, and bureau rate making procedures rely primarily on industry. In an open competition environment, however, classification efficiency is paramount. The pricing actuary must understand these influences on claim costs and how each classification variable might be used in setting policy premiums.

FLORIDA
RATIO OF BI TO PD 3 YR. CLAIM COUNTS
BY ISO TERRITORY



FLORIDA
ATTORNEYS PER 1,000 PERSONS
BY COUNTY



BELOW AVERAGE
AVERAGE
ABOVE AVERAGE
STATE CAPITAL

Section 18: Epilogue

"The greatest difficulties in insurance ratemaking do not require access to data or a knowledge of complicated mathematics, but rather the appropriate exercise of informed judgment."

– Mintel [1983], page 2

Until the 1980's, Workers' Compensation was a stable and profitable line of business. Revenues fluctuated rather mildly, crises were short-lived, insurance programs endured, and pricing techniques changed but slowly.

In the late 1970's and 1980's, some parts of the Workers' Compensation system began to unravel. Costs increased, new types of claims emerged, durations of disability lengthened, attorney involvement increased, profits declined, residual markets grew, and better risks began leaving the insurance market. Insurers and rating bureaus have responded with alternative risk management programs, changes to the involuntary pools, and cost containment measures.

As the Workers' Compensation system evolves, pricing actuaries must modify the ratemaking procedures. This section discusses the emerging issues in Workers' Compensation pricing.

A. Loss Costs

The complexities of pricing insurance products, particularly for long-tailed lines like Workers' Compensation, led to administered pricing systems and the partial antitrust exemption embodied in the McCarren-Ferguson Act. In the 1950's and 1960's, rating bureau actuaries developed rates for each line of business. Member companies generally adhered to these rates or deviated by systematic percentages across all classes. The statutory requirements for Workers' Compensation insurance, and the public policy objectives of timely and certain compensation for injured employees, led some states to require membership in rating bureaus and prior approval regulation for rate changes, even if less restrictive regulations were used in other lines.

Administered pricing system sometimes constrain innovative marketing strategies and ratemaking programs. The Personal Lines of insurance, with their large volumes of homogeneous risks, have less need for rating bureaus. Independent, low-cost carriers developed successful ratemaking strategies, and they soon dominated the profitable markets.

By the mid-1980's, pricing independence and innovation was spreading to the Commercial Lines, for several reasons:

- *Saturation:* After "skimming the cream" of the Personal Lines markets, the large direct writers entered the corresponding Commercial Lines markets: small businessowners, Commercial Automobile, CMP, and Personal Lines reinsurance.

- *Imitation:* The dominant Commercial Lines writers observed the successes of independent Personal Lines carriers and began experimenting with similar programs of their own.

- *Judicial Developments:* The right of rating bureaus to require rate adherence by their members was curtailed by the courts in the 1950's. Judicial decisions in the 1980's began chipping away at the McCarren-Ferguson partial antitrust exemption.

- *Politics:* The rising costs of insurance has encouraged some consumer activists and politicians to find inefficiencies and excessive profits in administered pricing systems.

- *Actuarial Expertise:* Casualty actuaries have become more proficient, rate making techniques have evolved, and los-cost, efficient computers have been developed. Even moderate sized carriers can now develop rates independently.

In 1989, the Insurance Services Office announced a transition from advisory rates to loss costs, and by the early 1990's, the National Council on Compensation Insurance followed suit. The coming roles of the rating bureau and company actuaries may vary by jurisdiction, depending on the loss cost system implemented in each state.

B. Elements of Loss Cost Systems

In a loss cost system, the rating bureau does not determine advisory rates. Rather, it provides historical loss data so that member companies can develop their own rates. Loss cost systems vary by jurisdiction. The following section outlines the probable roles of the rating bureau and carriers during the 1990's in loss cost jurisdictions.

Rating bureaus will provide:

- Historical exposure, pure premium, claim count, paid loss, and incurred loss data.
- Development factors, either to ultimate or to an advanced valuation.
- Cost implications of legislative or regulatory changes.
- Factors to bring pure premiums and benefits to current levels.

Member companies must determine

- Underwriting and acquisition expenses reflecting their own operations.
- Underwriting profit provisions.

Differences of opinion exist for several ratemaking procedures:

- *Loss cost trends:* Rating bureaus would like to retain authority to trend losses (Hager [1992], page 193). This is particularly true in Workers' Compensation, where the trend factors are influenced by complex social and economic developments. Some regulators and consumer activists believe that rating bureaus should provide data only. Projections about future changes in loss costs should be left to the carriers.

- *Involuntary pool burdens:* Rating bureaus administer the pools, and they have the best information for estimating their likely costs. As with trending, however, the involuntary market burdens are projections about future costs. Some analysts believe that rating bureaus should provide the needed data (e.g., market shares, pool operating margins, pool underwriting and rating programs), but member carriers should calculate the burden.

- *Assessments:* Assessment rates do not vary by carrier, so a quantification by the bureau seems efficient. However, there is no need for industry-wide data to estimate the assessment costs.

Unresolved issues with major implications for Workers' Compensation ratemaking include:

- *Experience rating plans:* Until recently, the Workers' Compensation experience rating plan was uniform among insurers and mandatory in almost all jurisdictions. Rating bureaus argue that a mandatory and uniform experience rating plan promotes equity among employers and encourages safety programs. Some insurers respond that the mandatory plan constrains innovative pricing programs; competitive markets require more flexible plans.

- *Classifications:* The most powerful competitive advantages in insurance pricing result from more efficient or more discriminating classification systems. The variety of potential classification dimensions in Workers' Compensation make classification freedom particularly enticing for some insurers. Rating bureaus are concerned, however, that the use of multiple classification systems will destroy the integrity of the Workers' Compensation database and hinder the compilation of industry-wide loss costs.

- *Economic incentives from law amendments:* The indirect incentive effects of statutory benefit changes and reforms of the compensation system are sometimes as great as the direct effects. Presently, rating bureaus quantify the direct cost effects of proposed legislation, which carriers apply to both existing and new policies. The indirect incentive effects are harder to quantify: they vary among groups of insureds and by type of compensation system. It is unclear how the indirect effects will be handled in a loss cost environment.

Some jurisdictions will leave these functions to rating bureaus; others will hand them to the individual carriers. Workers' Compensation pricing actuaries must be competent to deal with these issues as they arise.

PARTIAL LOSS DEVELOPMENT BASED ON EXPECTED LOSSES FOR WORKERS' COMPENSATION CLASS RATEMAKING

R. Michael Lamb

PARTIAL LOSS DEVELOPMENT BASED ON EXPECTED LOSSES FOR WORKERS' COMPENSATION CLASS RATEMAKING

R. Michael Lamb

Biography

Michael Lamb is the Casualty Actuary for the Oregon Department of Insurance and Finance and has served as chairman of the NAIC Casualty Actuarial Task Force since March 1989. He also chairs the NAIC Workers' Compensation Data Reporting Working Group and serves on the group which oversees NCCI compliance with recommendations of the recent NAIC examination. Over several years, he has had active roles in NAIC projects on workers' compensation, medical malpractice, product liability, and other current issues. Besides the State of Oregon, Mr. Lamb's career includes employment with Argonaut Insurance Company and Wausau Insurance Companies. He has mathematics degrees from Utah State and Purdue universities and an MBA from the University of Washington.

Michael presented a discussion paper on using closed claim data for ratemaking in 1980. He has participated on panels at CAS meetings on workers' compensation competitive rating laws (November 1982) and on casualty loss reserve opinions (May, September 1991). Mr. Lamb is on the Casualty Committee of the Actuarial Standards Board. He was one of the principal designers of the competitive workers' compensation rating law for Oregon, which is essentially the design of loss cost systems being adopted by NCCI and ISO.

Abstract

The standard multiplicative loss development factors applied to reported losses by class serve to amplify instability in partial loss data. A method of assigning loss development based on expected losses is described and tested using four years of actual class data for Oregon. The method uses payroll and "pure premium present on rate level" to estimate expected losses. Test statistics are devised to compare stability of rates calculated using this revised method and rates calculated in the standard manner. The tests are based on residuals from linear trend lines and on absolute magnitude of 1992 rate revisions by class. The tests support a conclusion that the revised method produces significantly greater rate stability eventhough credibility of indicated state experience is enhanced. There is brief discussion of other stability approaches and topics for further research in class ratemaking.

PARTIAL LOSS DEVELOPMENT BASED ON EXPECTED LOSSES FOR WORKERS' COMPENSATION CLASS RATEMAKING

Stability of premium rates by class has always been a primary objective for ratemaking methods. In recent times, actuaries have given more attention to responsiveness, which is the counterbalance to stability in ratemaking thought. The focus in this paper is exclusively on premium rate stability for workers' compensation classes.

The National Association of Insurance Commissioners (NAIC) completed an examination of the National Council on Compensation Insurance (NCCI) in 1991 which included a major review of ratemaking procedures. An important recommendation from the examination is that the National Council should use five years of experience for class ratemaking instead of only three. The purpose of this paper is to present an alternative means of enhancing class rate stability in a less haphazard manner which would not require the cost or loss of responsiveness from using additional years of data. The scope of the examination was not broad enough to include such alternatives.

The public has cause to criticize the National Council for wild swings in class rates. On the other hand, using five years of data could create ill will from the public which follows experience by selected class and is anxious to be rid of any "bad year."

A simple problem

Ratemaking procedures should not introduce instability or amplify intrinsic instabilities in the class experience data. For over a decade, regulators in some high-loss development states have believed that the customary multiplicative partial loss development factors have amplified class rate instability.

An easily understood example from Oregon is the serious indemnity loss development factor for losses at first report, which has approached 4.00 for several decades. Most "serious" injury claims take several years to emerge, usually migrating from the "non-serious" column. A serious injury on the first report in most classes is highly fortuitous. Even for large construction classes, serious losses on first report do not reliably predict ultimate losses. Nevertheless, the multiplicative loss development factor assigns all the anticipated loss development for the serious category to those classes which happen to have a serious injury on the first report. Classes which do not happen to show any serious cases get assigned no serious partial loss development.

Permanent partial disability cases are categorized as "major" (and "serious") or "minor" (and "non-serious") according to a single critical dollar amount. Whether or not this artificial distinction has a material effect on partial loss development is not addressed in this paper. This seemingly mundane topic may be a worthwhile subject for our actuarial literature.

A simple solution

The partial loss development procedure described in this paper is derived from the procedure used by the Oregon Insurance Division to adjust class rate relativities for this instability. Partial loss development is assigned to each class in proportion to partial expected losses. In that manner, the historical tendency of serious cases to eventually emerge in each class is more accurately recognized. All other mechanics and adjustments of the standard National Council class ratemaking procedure are preserved.

Partial pure premiums "present on rate level", multiplied by $100 units of payroll, determine the partial expected losses for a class. The complement of the inverse of the multiplicative partial loss development factor determines the portion of ultimate losses expected to yet emerge.

The enhanced stability of the revised loss development method means that partial credibilities can be enhanced. The Oregon Insurance Division has been using a simple classic square-root formula instead of the two-thirds root of the ratio of expected losses to the full-credibility standard used by the National Council. This concession seems to preserve a reasonable balance between stability and responsiveness.

After class rates are recalculated using the revised loss development method, balancing factors similar to the National Council test correction factors are determined by an iterative process so that class rates constrained by swing limits produce the same overall rate changes by industry group as would be achieved by the National Council rates. Such balancing procedures result in cross subsidies between classes which we should expect to diminish when systematic causes of rate instability are addressed.

DETAILS OF THE REVISED LOSS DEVELOPMENT MODEL

For the past few years, the Oregon Insurance Division has been obtaining payroll and loss data by class from the National Council. The source is described as "Report NC-235" by the NCCI and is the basis for class experience displayed in rate filings. The Oregon Insurance Division has been recreating the National Council published exhibits of class experience (Appendix B-II of NCCI filings), then recalculating partial pure premiums using the revised partial loss development method. The resulting premium rates for several dozen classes have been found to differ from National Council originally-filed rates by more than five percent and revised filings have been required. The affected classes have included several full-credibility classes. The loss development instability is not a small-credibility problem.

The partial loss development factors published by the National Council in Appendix B-I to its filings include an adjustment to the aggregate loss ratio of the latest policy year. Hence, the published factors may not precisely measure loss development. Nevertheless, the published factors have been used for this paper so the results can be replicated or similarly investigated for other states. The National Council appears to be separating the policy-year adjustment from loss development factors beginning with filings made late in 1992.

The revised method bases loss development on expected loss, using pure premiums "present on rate level" and payroll. The review of rates filed in Oregon each year has used as input for the revised method the same underlying pure premium rates as used by the National Council. These are derived from loss cost rates approved for the previous year. Hence, the review has not been a true test of the different concepts. The effect of the revised development method can only be seen when the pure premium "present on rate level" has been generated by the revised method in a succession of preceding rate revisions.

Exhibit 1 shows a comparison of the rate revision computations using the two partial loss development methods. The revisions for 1990 begin with the same set of 1989 base rates, hence this exhibit shows the actual revisions performed for this paper. The revisions for 1991 and 1992 use differing pure premium input data for the two development methods so separate worksheets were needed.

The rate revisions for Class 7600 in Exhibit 1 achieve materially different results and also illustrate the enhanced credibility formula used with the revised procedure. The NCCI credibility formula is the two-thirds root of the ratio of partial expected losses to the 100 percent standard. The Revised Procedure uses a simple square root formula (or a three-fourths root of the NCCI credibility).

The only other difference is the provision for loss development. The NCCI rate filing for 1990 displayed these loss development factors in Appendix B-I:

Policy Period	Indemnity Serious	Indemnity Non-Serious	Medical
1984	1.417	.996	1.197
1985	1.993	.990	1.348
1986	3.773	.962	1.562
Three-Year Fixed	2.394	.983	1.369

Exhibit 1 shows the payroll and losses as they would be shown in the National Council filing Appendix B-II. The losses have been developed and adjusted to current benefits, trends, and accident-year experience. The revised model simply divides these displayed losses by the partial loss development factors. Then a portion of expected losses as provision for loss development is added to the "Undeveloped Losses" and the result is labeled "Revised Losses."

Class 7600 had three serious injury cases on the first report for 1986 policies. The National Council displayed $1,731,862 losses for these cases and for anticipated development. The revised model divided this amount by 3.773, the serious indemnity development factor for 1986. The result is $459,015 "undeveloped losses" for the three cases.

The 3.773 development factor means that reported serious indemnity losses at first report should be 26.5 percent of the ultimate amount (1/3.733 = .265). Expected loss development should be 73.5 percent of expected losses. The "Revised Losses", including loss development, is computed as follows:

Pure Premium "Present on Rate Level"	1.203
Times: Payroll in $100s	435476.49
Equals: Expected serious losses	$523,878.22
Times: Expected development portion	.734959
Equals: Expected loss development	$385,029
Plus: "Undeveloped Losses"	459,015
Equals: Revised losses	$844,044

The model proceeds from there in the same manner as the National Council filings. The formula pure premium gives state credibility weight to the indicated pure premium, the national credibility weight to the pure premium "indicated by national relativity", and the remaining weight to the pure premium "present on rate level". Further adjustments for the financial data overall rate level, industry group differentials, benefit changes, changes in trends, and a test correction factor are described in NCCI filings Appendix B-III. This paper does not address the appropriateness of these elements of the class ratemaking process.

The rate for Class 7600 for 1990 is shown in Exhibit 3 as $3.06 after the balancing factors to achieve the overall and industry group averages.

Oregon has a premium adjustment program for most contracting classes. Employers in those classes that pay average wages over $15/hour and do not have debit experience rating modifications may apply for premium credits. The rates for those classes in 1991 and 1992 have been increased two percent to offset anticipated credits. No offset was needed in 1990 for Class 7600, which is in the "all other" industry group.

Balancing Factors

Exhibit 2 describes the process of balancing class rates to achieve the industry group and overall average revision for 1992. The overall revision was an 11.0 percent decrease. The percentages decreases for the manufacturing, contracting, and all other industry groups, respectively, were 11.2, 2.1, and 12.8.

The exhibit shows the current rate (1991 loss cost rate determined using the revised partial development procedure) and the formula revised rate determined from the 1992 version of the worksheet described in Exhibit 1. The "partial pure premium" columns add up to the revised rate, less any disease element. Next is a calculation of premium at the current and the revised rates applied to payroll. The sum of the differences in premium over each industry group is divided by the sum of the premium at current rates to determine the weighted average changes.

Overall, the formula revised rates only achieved a 7.9 percent decrease instead of the 11 percent objective. The column headed "RevRate Adjusted" is the product of the formula revised rate and the industry group balancing factor shown at the bottom of Exhibit 2. The worksheet then applies the swing limits again and shows the results in next column, labeled "RevRate Limited".

Finally, the premium computed using the limited revised rate is compared with premium at current rates to determine what average revision has been achieved. The desired industry group averages could not be attained exactly without loss to the overall revision. The results are within one-tenth of a percent by industry group.

COMPARISON TESTS

Comparative Test of the Partial Loss Development Methods

The test for this paper compares the revised partial loss development method with the National Council method by starting from the approved Oregon advisory loss cost rates for 1989 and calculating revised loss cost rates for 1990, 1991, and 1992 using sequential pure premium input as calculated by each loss development method. The test statistics for comparing the two methods are based on volatility of rates for each class over the four years and on the absolute magnitude of the 1992 revisions by class.

The first test statistic is computed by fitting a straight line to the rates computed for the four years for each class then summing the squares of residuals from the line. The sum is divided by the square of the sum of the four-years of rates to standardize the statistic for each class. The comparison may be more relevant if the statistic for each class is weighted by premium. This weighting is achieved by multiplying each class statistic by the latest 3-year payroll total and by the sum of the four-years of rates for the class. This four-year comparison can be seen visually in the accompanying graphs.

The second test statistic is simply the relative magnitude of the latest revision, from 1991 to 1992. The absolute difference is standardized by dividing by the sum of the 1991 and 1992 rates for the class. The premium-weighted version is computed by multiplying by three years of payroll and the sum of the rates.

The sums over all classes of these test statistics are as follows:

	TEST 1 Mean Squared Residuals		TEST 2 Latest Revision	
	Simple	Weighted	Simple	Weighted
NCCI Loss Development Method:	.5545	8767868	36.191	554808483
Revised Loss Development Method:	.3886	6568715	30.776	475227783

The lower statistics for the revised loss development method suggest greater stability.

The loss cost rates calculated by these procedures and the previously discussed comparative statistics are displayed in Exhibit 3. The comparison graphs illustrate the first test statistic. The line fitted to the four rates for each class should account for the influence of loss cost trends with the residuals representing various unstable factors.

The second test assumes that the 1992 revision is the most appropriate for comparing the methods since the pure premium input for the revised method would have resulted from the most successive applications of the revised development concept.

The test statistics do not include any classes for which rates were not available during all four years. Some classes are too new to have any experience. Some were discontinued and the payrolls and losses reassigned to other classes. These analytical impurities are part of the living classification system and a ratemaking method must be robust enough to accommodate them and still produce acceptable results.

CONCLUSION

The revised method of partial loss development improves rate stability. Because this improvement was realized while enhancing partial credibilities, it would not be proper to suggest restricting credibilities as an alternative for improving stability. An absurd indicated pure premium ratio will still be absurd when given a somewhat lower credibility weight.

Any revision to the ratemaking process which makes it more stable could be seen as assigning more credibility to years earlier than the latest. It does not follow, however, that any scheme which simply adjusts the credibility weights by year could produce optimal results. Directly addressing systematic causes of instability should be preferred before testing different credibility approaches.

TOPICS FOR FURTHER RESEARCH

Classification ratemaking is not sufficiently addressed in recent actuarial literature. Several topics have arisen during the preparation of this paper, from discussion with other actuaries, from the NAIC examination of the National Council, and from the NAIC working group overseeing the NCCI compliance with examination recommendations. Some of these topics are:

Optimal distinction between "major" and "minor" permanent partial disability cases.

Improved models for partial loss development, including migration between parts and development beyond the present statistical reporting horizon.

Bayesian credibility techniques where credibility of state class experience depends on variances in national relativity pure premium rates.

Loss limitations and swing limits for enhancing rate stability and equitable methods of balancing the effect to the overall rate level indications.

Refinements to the partial credibility scheme giving different weights to the different years.

INSPA:888

EXHIBIT 1

COMPUTATION OF REVISED PURE PREMIUM RATE
with loss development based on expected losses

Overall Revision	All Other
6.2%	Industry Group

Class: 7600 Telephone or Telegraph Co: All Other Employees & Dvrs

		Displayed Losses			Undeveloped Losses			Revised Losses		
	Payroll	Serious	Non-Ser	Medical	Serious	Non-Ser	Medical	Serious	Non-Ser	Medical
3-year	0	0	0	0	0	0	0	0	0	0
1984	42616748	393906	280841	500903	277986	281969	418465	428859	280879	505647
1985	49728462	145463	252282	480542	72987	254830	356485	371053	251631	516060
1986	43547649	1731862	237862	481927	459015	247258	308532	844044	236300	503288
	135892859	2271231	770985	1463372				1643956	768810	1524995

	NAT'L COUNCIL PROCEDURE				REVISED PROCEDURE		
	Serious	Non-Ser	Medical		Serious	Non-Ser	Medical
	1.671	0.567	1.077	Indicated Pure Premiums	1.210	0.566	1.122
	1.203	0.637	1.243	P.P. "Present on Rate Level"			
	1.287	0.917	1.769	P.P. "Ind. by Nat'l Reltvty"			
	0.59	0.78	1.00	State Credibility	0.67	0.83	1.00
Total	0.20	0.11	0.00	National Credibility	0.16	0.08	0.00
3.19	1.496	0.613	1.077	Formula Pure Premium	1.221	0.600	1.122
	1.008	1.008	1.008	Composite Factor			
	1.007	1.004	1.000	Effect of Benefit Change			
	1.092	1.092	0.975	Change in Trend Factor			
		3.39		Rounded Total		3.12	
		1.007		Ratio of Manual to Earned Premium		1.007	
		1.000		Contracting Prem Adj Program Offset		1.000	
				Specific Disease Loading			
		3.41		Calculated Pure Premium Rate		3.14	
Swing Limits:		2.86		Current Pure Premium Rate		2.86	
33% above		3.41		Swing-Limited Pure Premium Rate		3.14	
14% below		19.2%		Percentage Change		9.8%	
				Difference from Nat'l Council		-7.9%	

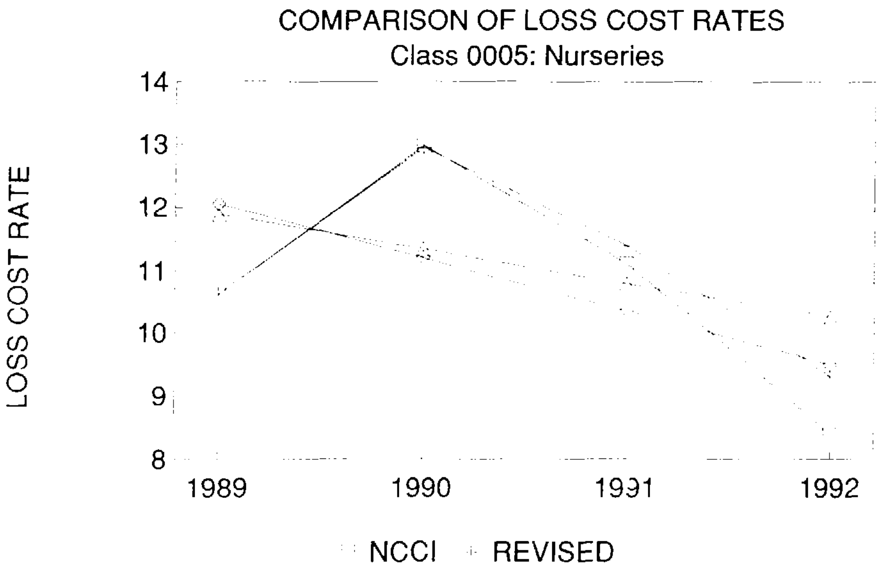
BALANCING OVERALL RATE CHANGES BY INDUSTRY GROUP

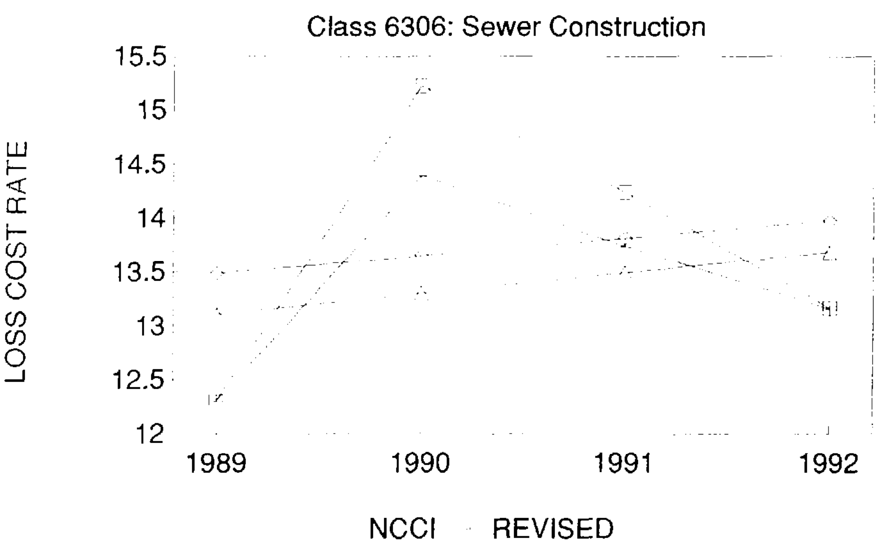
EXHIBIT 2

Oregon Loss Cost Rates for 1992 Using Revised Development Method

Ind Grp	Class	Current Rate	Revised Rate	Effect	Partial Pure Premiums REVser	REVnser	REVmed	3Pay/100	Premium@ CurrRate	Premium@ RevRate	Diffrnce	RevRate Adjusted	RevRate Limited	Adj,Ltd Diffrnce	Effect
Manufact.	1430	15.63	12.80	-18.1%	3.306	2.766	6.728	2593	40533	33194	-7339	12.21	12.21	-8869	-4.6%
Manufact.	1438	4.64	3.90	-15.9%	1.053	0.776	2.071	1515595	7032359	5910819	-1121540	3.72	3.72	-1394347	-4.6%
Manufact.	1452	4.79	4.69	-2.1%	1.332	0.972	2.386	1421	6807	6665	-142	4.48	4.48	-441	-4.5%
Manufact.	1463	9.08	8.82	-2.9%	3.985	1.510	3.325	81628	741180	719957	-21223	8.42	8.42	-53874	-4.5%
	⋮		⋮										⋮		
Contracting	50	9.17	9.53	3.9%	3.144	2.139	4.247	140401	1287480	1338025	50544	9.45	9.45	39312	-0.8%
Contracting	1322	--	--	--	--	--	--	0	0	0	0	0.00	--	0	
Contracting	2703	7.28	6.79	-6.7%	3.234	0.896	2.660	592261	4311659	4021451	-290208	6.73	6.73	-325743	-0.9%
Contracting	3365	14.66	15.91	8.5%	7.317	2.659	5.934	68199	999804	1085054	85249	15.77	15.77	75701	-0.9%
	⋮		⋮										⋮		
All Other	5	11.39	9.62	-15.5%	2.020	2.241	5.359	1541250	17554841	14826828	-2728013	9.31	9.31	-3205801	-3.2%
All Other	8	4.05	3.85	-4.9%	0.890	0.954	2.006	530906	2150169	2043988	-106181	3.72	3.72	-175199	-3.4%
All Other	16	8.99	8.42	-6.3%	2.011	2.211	4.198	850439	7645450	7160700	-484750	8.14	8.14	-722873	-3.3%
All Other	34	7.98	8.93	11.9%	2.378	1.855	4.697	203249	1621930	1815017	193087	8.64	8.64	134145	-3.2%
	⋮		⋮										⋮		

	Diffrnce	RevRate Limited	Effect
Manufacturing	-0.06976	0.9542	-0.11238
Contracting	-0.00232	0.9914	-0.02127
All Other	-0.09887	0.9673	-0.12874
	-0.07869	Balancing Factors	-0.10985


COMPARISON OF LOSS COST RATES
Class 0005: Nurseries
LOSS COST RATE
14
13
12
11
10
9
8
1989
1990
1991
1992
NCCI
REVISED


Class 6306: Sewer Construction
LOSS COST RATE
15.5
15
14.5
14
13.5
13
12.5
12
1989
1990
1991
1992
NCCI
REVISED

National Council Development Method

Class Description	1989 Base	1990	1991	1992	TEST 1 Sum(D^2)/ [Sm(y)]^2	Premium Wghted	TEST 2 |y4-y3|/ (y3+y4)	Premium Wghted
0005 FARM: Nursery Employers & Drivers	10.62	12.97	11.11	8.39	0.00378	249924.7	0.139487	9263689.0
0008 FARM: Gardening & Drivers	4.64	4.72	4.08	3.80	0.00031	2793.723	0.036532	325227.03
0016 FARM: Orchard & Drivers	9.87	9.86	8.53	8.19	0.00022	6860.211	0.020334	630352.60
0034 FARM: Poultry Producer & Drivers	8.12	9.65	8.87	9.73	0.00067	4986.792	0.046236	341789.04
0035 FARM: Florist & Drivers	8.35	9.39	8.18	7.25	0.00117	19230.07	0.060272	988344.29
0036 FARM: Dairy & Drivers	14.95	16.01	13.84	11.65	0.00099	18464.68	0.085916	1601881.7
0037 FARM: Field Crops & Drivers	12.42	12.35	10.58	9.38	0.00029	17970.32	0.060120	3747110.8
0042 Landscape, Gardening & Drivers	14.23	18.11	15.96	14.24	0.00254	34000.37	0.056953	762994.43
0050 Farm Machinery Operation & Drivers	9.49	9.99	9.54	9.93	0.00011	588.3879	0.020030	109541.17
0079 FARM: Berry or Vineyard & Drivers	10.37	12.40	11.11	6.89	0.00588	57477.39	0.234444	2291819.2
0083 FARM: Cattle Raising NOC & Drivers	15.45	17.37	15.42	14.01	0.00097	25862.59	0.047910	1282244.3
0106 Tree Pruning & Drivers	20.64	22.93	20.63	17.76	0.00111	9394.169	0.074759	632018.80
0113 FARM: Fish Hatchery & Drivers	8.41	9.25	8.92	9.46	0.00018	91.01332	0.029379	15062.643
0116 Fruit Picking, On Ground	3.17	3.70	3.51	3.36	0.00077	1309.202	0.021834	37351.413
0117 FARM: Berry Picking	1.72	1.95	1.81	1.60	0.00106	2825.251	0.061583	164868.69
0124 Reforestation or Slash Piling & Drivers	20.43	27.15	30.40	33.38	0.00033	13634.65	0.046424	1942255.3
0169 FARM: Sheep Raising & Drivers	10.03	12.59	10.84	9.32	0.00284	5528.737	0.075398	146930.11
0170 FARM: Animal Raising & Drivers	6.87	7.24	7.26	6.44	0.00047	482.0375	0.059854	60982.035
0251 Irrigation Works Operation & Drivers	6.52	7.83	7.54	5.74	0.00317	11329.27	0.135542	484735.35
0917 Domestic Service Contractor - Inside	10.66	14.16	11.28	12.36	0.00292	3737.710	0.048570	59843.474
1005 Coal Mining - Surface & Drivers	8.42	7.83	--	--	--	0	--	0
1164 Mining NOC - Underground & Drivers	13.47	11.60	9.76	8.78	0.00012	17.33007	0.052858	7494.1448
1165 Mining NOC - Surface & Drivers	5.61	6.13	5.82	6.38	0.00025	198.9203	0.045901	35751.888
1320 Oil Lease Operator - All operations & Dri	7.17	7.50	6.43	5.09	0.00111	23.54347	0.116319	2488.1374
1322 Oil or Gas Well: Cleaning Old Wells & Dr	24.64	25.48	--	--	--	0	--	0
1430 Smelting - Lead & Drivers	16.61	16.98	15.93	12.13	0.00117	186.5852	0.135424	21651.106
1438 Smelting - Metals - Not Iron or Lead - NC	6.02	5.00	4.47	3.25	0.00023	6439.255	0.158031	4488437.5
1452 Ore Milling & Drivers	5.31	4.96	4.93	4.50	0.00007	1.985197	0.045599	1276.5698
1463 Asphalt Workers & Drivers	9.82	9.72	9.59	8.62	0.00016	480.1009	0.053267	184140.80
1470 Coke Manufacturing & Drivers	9.87	10.92	--	--	--	0	--	0
1472 Distillation - Wood & Drivers	5.76	6.43	7.39	6.53	0.00118	3257.879	0.061781	169858.78
1624 Quarry NOC & Drivers	16.19	16.92	14.40	11.24	0.00119	21037.33	0.123244	2170180.2
1642 Lime Manufacturing	7.02	6.89	7.64	--	--	0	--	0
1654 Quarry - Cement Rock & Drivers	6.87	6.93	6.87	6.54	0.00005	0.796582	0.024608	370.71585
1655 Lime Mfg - Quarry - Surface & Drivers	11.64	12.88	10.55	--	--	0	--	0
1699 Rock Wool Manufacturing	3.90	4.26	4.45	3.66	0.00137	230.5908	0.097410	18380.729
1701 Cement Manufacturing	5.21	6.28	6.31	5.29	0.00205	658.3209	0.087931	28175.505
1710 Stone Crushing & Drivers	15.45	15.22	13.29	13.25	0.00020	1046.138	0.001507	7897.5641
1741 Flint Grinding & Drivers	9.11	8.08	6.95	6.35	0.00007	0.091704	0.045112	58.210682
1747 Emery Works & Drivers	6.09	5.32	4.09	3.19	0.00010	20.47378	0.123826	24954.947
1748 Abrasive Wheel Manufacturing & Drivers	6.07	6.36	6.02	5.86	0.00014	37.97688	0.013468	3623.0183
1803 Stone Cutting NOC & Drivers	15.59	15.81	14.43	10.61	0.00129	660.2071	0.152555	77954.584
1852 Asbestos Goods Manufacturing	11.05	12.54	12.56	10.32	0.00163	861.6084	0.097902	51905.183
1860 Abrasive Paper or Cloth Preparation	--	--	--	--	--	0	--	0
1924 Wire Drawing - Not Iron or Steel	--	--	--	--	--	0	--	0
1925 Die Casting Manufacturing	11.00	9.80	9.15	8.62	0.00016	339.5490	0.029825	61542.474
2001 Cracker Manufacturing	8.90	9.41	9.35	8.58	0.00249	3057.311	0.173885	213550.00
2002 Macaroni Manufacturing	10.26	10.57	11.25	11.26	0.00004	6.978904	0.000444	76.045823
2003 Bakery & Drivers, Route Supervisors	5.00	6.22	5.77	4.51	0.00341	69861.86	0.122568	2513794.6
2014 Grain Milling	8.51	8.40	7.57	8.42	0.00048	4181.691	0.053158	464140.24
2016 Breakfast Food Manufacturing	2.95	3.04	3.04	2.32	0.00143	110.2871	0.134328	10379.431
2021 Sugar Refining	3.62	3.95	4.54	4.21	0.00067	26.63918	0.037714	1498.9263
2030 Sugar Manufacturing, Beet	12.40	9.87	--	--	--	0	--	0
2039 Ice Cream Manufacturing & Drivers	5.71	5.5?	5.36	5.42	0.00003	69.97898	0.005565	11226.164
2041 Confection Manufacturing	6.34	6.82	7.07	5.05	0.00277	2275.972	0.166666	136916.05
2065 Milk Products Manufacturing NOC	5.06	5.30	5.77	5.20	0.00054	29.38529	0.051959	2839.2211
2070 Creamery & Route Supervisors, Drivers	6.67	7.71	6.78	5.98	0.00145	27941.85	0.062695	1209457.2
2086 Poultry Processors	--	--	--	--	--	0	--	0
2080 Packing House - All Operations	8.34	8.82	9.01	7.82	0.00065	4031.960	0.070707	435309.27
2095 Meat Products Manufacturing NOC	8.52	10.59	12.17	11.72	0.00092	9658.881	0.018836	197167.46
2101 Fish Curing	9.82	9.31	8.07	5.91	0.00062	95.10860	0.154506	23606.230
2104 Seafood Processors	--	--	--	--	--	0	--	0
2105 Fruit Packing	8.57	9.49	8.61	6.99	0.00147	30109.60	0.103846	2122508.4
2110 Pickle Manufacturing	13.77	17.12	19.68	12.31	0.00832	38313.78	0.230384	1060597.5
2111 Cannery NOC	7.86	6.11	5.31	5.81	0.00202	64422.70	0.044984	1433850.1
2112 Fruit Preserving	9.69	8.57	7.62	6.84	0.00003	315.5006	0.053941	630456.85
2114 Oystermen	5.35	6.16	6.49	7.22	0.00008	22.13372	0.053245	15191.309
2121 Brewery & Drivers	3.80	3.86	3.63	2.66	0.00141	4755.553	0.154213	518305.11

Revised Loss Development Method

Class Description	1989 Base	1990	1991	1992	TEST 1 Sum(D^2)/ [Sm(y)]^2	Premium Wghted	TEST 2 |y4-y3|/ (y3+y4)	Premium Wghted
0005 FARM: Nursery Employers & Drivers	10.62	12.94	11.39	9.31	0.00276	187964.9	0.100483	6854528.4
0008 FARM: Gardening & Drivers	4.64	4.72	4.05	3.72	0.00035	3143.595	0.042471	386249.43
0016 FARM: Orchard & Drivers	9.87	10.72	8.99	8.14	0.00093	29785.21	0.049620	1591756.4
0034 FARM: Poultry Producer & Drivers	8.12	8.92	7.98	8.64	0.00050	3397.634	0.039711	271679.18
0035 FARM: Florist & Drivers	8.35	9.29	7.98	7.11	0.00110	17835.57	0.057654	932871.41
0036 FARM: Dairy & Drivers	14.95	15.99	13.72	11.49	0.00103	19025.21	0.088456	1640491.5
0037 FARM: Field Crops & Drivers	12.42	13.13	11.12	9.42	0.00090	57775.91	0.082765	5315353.1
0042 Landscape, Gardening & Drivers	14.23	18.02	16.19	14.48	0.00233	31355.03	0.055754	751472.40
0050 Farm Machinery Operation & Drivers	9.49	10.02	9.17	9.45	0.00023	1217.437	0.015037	80503.811
0079 FARM: Berry or Vineyard & Drivers	10.37	11.37	9.66	7.07	0.00229	21125.61	0.154811	1427991.7
0083 FARM: Cattle Raising NOC & Drivers	15.45	16.30	14.21	12.95	0.00053	13309.44	0.046391	1174985.1
0106 Tree Pruning & Drivers	20.64	22.38	19.68	16.52	0.00108	8846.840	0.087292	713308.85
0113 FARM: Fish Hatchery & Drivers	8.41	9.57	9.01	9.76	0.00037	193.9586	0.039957	20889.232
0116 Fruit Picking, On Ground	3.17	3.63	3.33	3.11	0.00086	1418.827	0.034161	56313.248
0117 FARM: Berry Picking	1.72	1.92	1.71	1.52	0.00100	2601.738	0.058823	152806.75
0124 Reforestation or Slash Piling & Drivers	20.43	26.50	27.07	29.32	0.00058	22642.10	0.039900	1549089.5
0169 FARM: Sheep Raising & Drivers	10.03	11.72	9.79	8.86	0.00171	3138.331	0.049865	91770.365
0170 FARM: Animal Raising & Drivers	6.87	6.99	6.76	6.23	0.00015	144.2451	0.040800	40121.410
0251 Irrigation Works Operation & Drivers	6.52	6.91	6.27	5.49	0.00060	1981.965	0.066326	216254.33
0917 Domestic Service Contractor - Inside	10.66	13.92	10.75	11.65	0.00295	3653.925	0.040178	49799.859
1005 Coal Mining - Surface & Drivers	8.42	7.64	--	--	--	0	--	0
1164 Mining NOC - Underground & Drivers	13.47	11.60	9.24	8.53	0.00031	42.90526	0.039954	5564.6778
1165 Mining NOC - Surface & Drivers	5.61	5.86	5.42	5.87	0.00026	192.7207	0.039858	29514.622
1320 Oil Lease Operator - All operations & Dri	7.17	7.29	6.08	4.84	0.00085	17.45813	0.113553	2334.9209
1322 Oil or Gas Well: Cleaning Old Wells & Dr	24.64	24.96	--	--	--	0	--	0
1430 Smelting - Lead & Drivers	16.61	17.23	15.63	12.21	0.00107	171.8937	0.122844	19649.537
1438 Smelting - Metals - Not Iron or Lead - NC	6.02	5.36	4.64	3.72	0.00005	1372.787	0.110047	3292393.7
1452 Ore Milling & Drivers	5.31	5.06	4.79	4.48	0.00000	0.068568	0.033441	903.35106
1463 Asphalt Workers & Drivers	9.82	9.38	9.08	8.42	0.00002	54.71505	0.037714	112982.13
1470 Coke Manufacturing & Drivers	9.87	10.69	--	--	--	0	--	0
1472 Distillation - Wood & Drivers	5.76	6.11	6.85	6.36	0.00049	1291.543	0.037093	97958.636
1624 Quarry NOC & Drivers	16.19	17.58	15.08	12.58	0.00120	22149.23	0.090383	1664129.9
1642 Lime Manufacturing	7.02	6.86	7.28	--	--	0	--	0
1654 Quarry - Cement Rock & Drivers	6.87	6.78	6.55	6.28	0.00001	0.135585	0.021044	308.28842
1655 Lime Mfg - Quarry - Surface & Drivers	11.64	12.57	10.03	--	--	0	--	0
1699 Rock Wool Manufacturing	3.90	4.18	4.14	3.40	0.00107	171.8394	0.098143	15804.978
1701 Cement Manufacturing	5.21	6.10	5.89	4.91	0.00180	552.1240	0.090740	27841.758
1710 Stone Crushing & Drivers	15.45	15.40	13.17	11.44	0.00035	1758.400	0.070298	357089.01
1741 Flint Grinding & Drivers	9.11	7.82	6.48	5.97	0.00022	0.274863	0.040963	50.932884
1747 Emery Works & Drivers	6.09	5.14	3.83	2.97	0.00011	20.86382	0.128470	24627.585
1748 Abrasive Wheel Manufacturing & Drivers	6.07	6.03	5.48	5.55	0.00012	31.99172	0.006348	1624.3521
1803 Stone Cutting NOC & Drivers	15.59	15.35	13.51	10.07	0.00086	425.1182	0.145886	72010.546
1852 Asbestos Goods Manufacturing	11.05	12.25	11.83	9.78	0.00131	670.8270	0.094863	48605.812
1860 Abrasive Paper or Cloth Preparation	--	--	--	--	--	0	--	0
1924 Wire Drawing - Not Iron or Steel	--	--	--	--	--	0	--	0
1925 Die Casting Manufacturing	11.00	9.23	8.43	8.18	0.00043	856.0199	0.015051	29816.425
2001 Cracker Manufacturing	8.90	9.33	9.05	6.54	0.00199	2414.241	0.161000	195300.35
2002 Macaroni Manufacturing	10.26	10.98	11.32	11.15	0.00010	17.97004	0.007565	1306.1331
2003 Bakery & Drivers, Route Supervisors	5.00	5.97	5.51	4.69	0.00192	38873.81	0.080392	1623485.6
2014 Grain Milling	8.51	8.45	7.48	7.88	0.00030	2588.656	0.026041	223369.04
2016 Breakfast Food Manufacturing	2.95	2.38	2.85	2.13	0.00126	93.51901	0.144575	10738.358
2021 Sugar Refining	3.62	3.87	4.27	4.06	0.00033	12.56910	0.025210	971.25392
2030 Sugar Manufacturing, Beet	12.40	9.86	--	--	--	0	--	0
2039 Ice Cream Manufacturing & Drivers	5.71	5.48	5.13	4.87	0.00001	10.50592	0.026	50473.481
2041 Confection Manufacturing	6.34	7.01	7.05	4.99	0.00306	2522.969	0.171096	141166.61
2065 Milk Products Manufacturing NOC	5.06	5.08	5.27	4.87	0.00018	9.218835	0.039447	2049.416
2070 Creamery & Route Supervisors, Drivers	6.67	7.63	6.96	6.22	0.00112	21835.32	0.056145	1096666.0
2086 Poultry Processors	--	--	--	--	--	0	--	0
2080 Packing House - All Operations	8.34	8.50	8.45	7.64	0.00023	1377.018	0.050341	300264.93
2095 Meat Products Manufacturing NOC	8.52	10.36	11.00	11.05	0.00049	4874.684	0.002267	22592.984
2101 Fish Curing	9.82	9.20	7.71	5.71	0.00046	68.64507	0.149031	22308.954
2104 Seafood Processors	--	--	--	--	--	0	--	0
2105 Fruit Packing	8.57	9.15	8.49	7.02	0.00096	19367.21	0.094777	1912408.9
2110 Pickle Manufacturing	13.77	16.76	18.50	13.02	0.00512	23264.29	0.173857	789806.96
2111 Cannery NOC	7.86	6.95	5.39	5.70	0.00026	8743.748	0.016393	550062.43
2112 Fruit Preserving	9.69	8.86	7.66	6.83	0.00001	119.4304	0.057280	671937.10
2114 Oystermen	5.85	6.06	6.08	6.61	0.00008	22.59131	0.041765	11396.938
2121 Brewery & Drivers	3.80	3.68	3.42	2.68	0.00055	1807.504	0.121311	396909.82

National Council Development Method

Class Description	1989 Base	1990	1991	1992	TEST 1 Sum(D^2)/ [Sm(y)]^2	Premium Wghted	TEST 2 |y4-y3|/ (y3+y4)	Premium Wghted
2130 Liquor, Spirituous, Distillery	11.76	10.75	10.22	8.04	0.00034	2.436797	0.119386	849.74950
2131 Liquor, Spirituous, Bottling	5.59	5.33	4.84	5.06	0.00023	105.3181	0.022222	9967.5842
2143 Fruit Juice Manufacturing	10.62	13.21	15.19	9.51	0.00832	36433.49	0.229959	1007594.6
2150 Ice Manufacturing	18.57	18.75	18.80	15.75	0.00059	136.2353	0.088277	20372.348
2156 Bottling-Not Carbntd Liq & Rt Spvsrs, Dri	8.32	8.26	7.85	6.41	0.00052	52.56491	0.100981	10112.900
2157 Bottling NOC & Rt Spvsrs, Drivers	6.50	8.08	7.83	6.02	0.00356	56378.16	0.130685	2070157.5
2211 Cotton Batting Manufacturing	12.26	12.76	13.18	11.08	0.00082	180.7885	0.086562	19120.069
2220 Yarn Manufacturing - Cotton	5.09	5.10	--	--	--	0	--	0
2286 Wool Spinning & Weaving	6.08	6.10	5.80	6.46	0.00031	292.0389	0.053833	50947.657
2288 Felting Manufacturing	8.56	9.00	9.93	9.17	0.00044	38.95859	0.039790	3488.1279
2300 Plush Manufacturing	--	--	--	--	--	0	--	0
2302 Silk Thread or Yarn Manufacturing	--	--	--	--	--	0	--	0
2305 Textile Fiber Manufacturing	2.69	2.68	2.52	2.11	0.00040	1.550137	0.088552	338.93628
2361 Hosiery Manufacturing	3.10	3.22	--	--	--	0	--	0
2362 Knit Goods Manufacturing NOC	4.19	3.88	3.91	3.49	0.00014	199.8284	0.056756	79295.516
2380 Webbing Manufacturing	4.63	5.15	5.45	5.40	0.00019	6.443509	0.004608	153.98441
2386 Lace Manufacturing	--	--	--	--	--	0	--	0
2388 Embroidery Manufacturing	--	--	--	--	--	0	--	0
2402 Carpet Manufacturing NOC	7.46	7.96	7.88	6.86	0.00064	0.133238	0.069199	14.358942
2413 Textile Finishing	7.19	7.80	8.15	6.44	0.00172	382.0073	0.117203	25985.004
2416 Yarn Dyeing or Finishing	5.98	6.06	6.95	5.86	0.00118	2.041234	0.085089	146.63924
2417 Cloth Printing	--	--	--	--	--	0	--	0
2501 Clothing Manufacturing	5.40	5.40	4.86	4.85	0.00014	1515.514	0.001029	11464.236
2503 Dressmaking - Custom	1.45	1.43	1.35	1.32	0.00002	1.947113	0.011235	1069.6621
2534 Feather Manufacturing	4.43	4.01	4.07	3.17	0.00066	18.54236	0.124309	3513.8295
2570 Mattress Manufacturing	11.08	9.89	9.21	7.28	0.00021	600.2741	0.117040	337882.95
2576 Awning Manufacturing - Shop	6.50	7.83	8.26	7.23	0.00158	2873.694	0.068494	120674.19
2578 Bag Manufacturing - Cloth	9.53	9.41	8.37	5.71	0.00150	7391.622	0.188920	929861.20
2585 Laundry NOC & Rt Supervisors, Drivers	7.42	7.33	6.47	6.67	0.00024	3688.391	0.015220	231700.21
2586 Cleaning & Rt Supervisors, Drivers	3.71	4.61	4.85	5.28	0.00027	1349.717	0.042448	213494.26
2587 Towel Supply Co & Rt Supervisors, Driv	4.50	4.45	3.60	2.71	0.00088	120.3535	0.141045	19256.703
2600 Fur Manufacturing - Preparing Skins	--	--	--	--	--	0	--	0
2623 Tanning	9.06	8.94	8.52	6.99	0.00047	289.6334	0.098646	60553.695
2651 Shoe Stock Manufacturing	6.92	7.72	7.77	6.80	0.00092	761.0659	0.066575	54940.755
2660 Boot Manufacturing NOC	7.64	7.12	6.40	6.92	0.00047	1720.245	0.039039	141553.06
2670 Glove Manufacturing - Leather or Textile	2.87	3.24	3.59	3.60	0.00021	12.32210	0.001390	80.797222
2683 Luggage Manufacturing	--	--	--	--	--	0	--	0
2688 Leather Goods Manufacturing NOC	3.97	4.25	4.21	3.92	0.00030	173.0043	0.035670	20248.940
2697 Forest Patrollers	5.63	5.49	4.57	4.30	0.00027	410.5127	0.030439	46899.165
2702 Logging or Lumbering & Drivers	29.53	30.13	25.31	21.12	0.00067	392803.4	0.090243	52730831.
2703 Logging Equipment Mntnc & Repair	7.24	8.05	7.47	6.45	0.00103	17890.49	0.073275	1287667.9
2710 Saw Mill	12.20	11.37	9.85	8.95	0.00005	9929.221	0.047872	9805004.3
2714 Veneer Manufacturing	13.48	11.61	10.06	8.29	0.00071	20085.40	0.230581	6492010.2
2725 Logging: Mechanized Equip Operations	--	--	--	--	--	0	--	0
2731 Planing or Molding Mill	9.92	10.60	9.22	7.43	0.00120	143132.9	0.107507	12796829.
2735 Furniture Stock Manufacturing	9.23	10.05	10.46	9.54	0.00052	552.0575	0.046	49025.135
2741 Cooperage Stock Manufacturing	11.34	10.48	--	--	--	0	--	0
2747 Cooperage Assembly	5.98	7.16	--	--	--	0	--	0
2759 Box or Box Shook Manufacturing	14.55	15.05	14.68	12.08	0.00079	1965.421	0.097159	243118.39
2790 Pattern Making NOC	2.55	2.39	2.40	2.34	0.00006	61.98659	0.012658	13665.928
2802 Carpentry - Shop Only _ & Drivers	8.65	7.91	6.88	6.24	0.00003	2536.005	0.048780	4253485.9
2812 Cabinet Works - With Power Machinery	9.29	9.93	8.76	9.75	0.00058	18336.93	0.053484	1705376.0
2835 Brush or Broom Assembly	4.38	4.84	5.03	5.33	0.00004	83.32078	0.028957	67914.598
2836 Brush or Broom Manufacturing NOC	6.10	6.56	7.53	6.10	0.00190	252.5011	0.104915	13908.715
2841 Woodenware Manufacturing NOC	9.75	11.78	11.97	10.21	0.00188	7857.736	0.079350	331679.47
2881 Furniture Assembly - Wood	6.02	6.26	6.08	6.77	0.00021	818.3029	0.053696	207344.04
2883 Furniture Manufacturing - Wood - NOC	9.50	11.81	11.81	7.39	0.00704	138844.9	0.230208	4543079.1
2913 Rattan Products Manufacturing	7.31	6.56	--	--	--	0	--	0
2915 Veneer Products Manufacturing	6.02	7.48	6.77	6.38	0.00164	166921.6	0.029657	3016374.4
2916 Veneer Products Manufacturing - NO Ve	8.73	8.74	7.59	7.91	0.00034	15938.39	0.020645	968934.96
2923 Piano Manufacturing	4.67	4.39	4.36	3.38	0.00069	814.0245	0.126814	149562.10
2942 Pencil Manufacturing	5.06	5.73	--	--	--	0	--	0
2960 Wood Preserving & Drivers	8.85	10.13	9.63	8.74	0.00091	6247.649	0.048448	331487.54
3004 Iron or Steel:Mfg:Stl Mkng-Elec Fnc or C	6.36	6.61	6.66	4.41	0.00309	22583.63	0.203252	1487806.0
3018 Iron or Steel: Mfg: Rolling Mill & Drivers	7.47	9.13	9.85	8.42	0.00202	15.58263	0.078270	602.79064
3022 Pipe Manufacturing NOC & Drivers	10.91	10.47	9.60	7.08	0.00080	322.7139	0.151079	61093.421
3027 Rolling Mill NOC & Drivers	3.04	3.52	3.20	2.25	0.00355	34746.21	0.174311	1704051.4
3028 Pipe or Tube Manufacuring - Iron or Stee	7.08	7.12	6.55	7.29	0.00039	920.8764	0.053468	126173.84

Revised Loss Development Method

Class Description	1989 Base	1990	1991	1992	TEST 1 Sum(D^2)/ [Sm(y)]^2	Premium Wghted	TEST 2 |y4-y3|/ (y3+y4)	Premium Wghted
2130 Liquor, Spirituous, Distillery	11.76	10.74	9.80	7.61	0.00027	1.883897	0.125789	876.43872
2131 Liquor, Spirituous, Bottling	5.59	5.23	4.59	4.98	0.00054	239.2425	0.040752	17901.589
2143 Fruit Juice Manufacturing	10.62	12.93	14.88	10.32	0.00572	25170.29	0.180301	793269.40
2150 Ice Manufacturing	18.57	17.96	17.22	14.75	0.00021	48.54323	0.077259	16993.645
2156 Bottling-Not Carbntd Liq & Rt Spvsrs, Dri	8.32	8.04	7.32	5.94	0.00035	33.42868	0.104072	10010.113
2157 Bottling NOC & Rt Spvsrs, Drivers	6.50	7.78	7.39	6.35	0.00178	27791.93	0.075691	1181714.2
2211 Cotton Batting Manufacturing	12.26	12.47	12.32	10.45	0.00052	110.7884	0.082125	17484.871
2220 Yarn Manufacturing - Cotton	5.09	4.99	--	--	--	0	--	0
2286 Wool Spinning & Weaving	6.08	5.94	5.45	5.49	0.00009	78.98398	0.003656	3250.7540
2288 Felting Manufacturing	8.56	9.22	9.65	9.01	0.00034	30.02265	0.034297	2986.8770
2300 Plush Manufacturing	--	--	--	--	--	0	--	0
2302 Silk Thread or Yarn Manufacturing	--	--	--	--	--	0	--	0
2305 Textile Fiber Manufacturing	2.69	2.62	2.37	1.98	0.00028	1.017496	0.089655	331.48789
2361 Hosiery Manufacturing	3.10	3.15	--	--	--	0	--	0
2362 Knit Goods Manufacturing NOC	4.19	3.87	3.72	3.23	0.00009	121.7183	0.070503	95572.449
2380 Webbing Manufacturing	4.63	5.04	5.12	5.50	0.00005	1.602147	0.035781	1175.9216
2386 Lace Manufacturing	--	--	--	--	--	0	--	0
2388 Embroidery Manufacturing	--	--	--	--	--	0	--	0
2402 Carpet Manufacturing NOC	7.46	7.79	7.40	6.45	0.00049	0.097142	0.068592	13.732678
2413 Textile Finishing	7.19	7.80	7.87	6.08	0.00182	395.5108	0.128315	27833.072
2416 Yarn Dyeing or Finishing	5.98	5.93	6.54	5.52	0.00087	1.439184	0.084577	140.59418
2417 Cloth Printing	--	--	--	--	--	0	--	0
2501 Clothing Manufacturing	5.40	5.32	4.76	4.62	0.00010	1117.900	0.014925	162824.57
2503 Dressmaking - Custom	1.45	1.52	1.45	1.35	0.00024	23.27713	0.035714	3534.7720
2534 Feather Manufacturing	4.43	3.92	3.83	2.94	0.00048	13.17709	0.131462	3583.3053
2570 Mattress Manufacturing	11.08	10.10	9.10	7.11	0.00022	622.6069	0.122763	353742.61
2576 Awning Manufacturing - Shop	6.50	7.84	7.88	7.05	0.00138	2466.919	0.055592	99028.902
2578 Bag Manufacturing - Cloth	9.53	9.55	8.38	6.06	0.00122	6087.727	0.160684	802762.13
2585 Laundry NOC & Rt Supervisors, Drivers	7.42	7.36	6.48	6.40	0.00017	2591.325	0.006211	93771.224
2586 Cleaning & Rt Supervisors, Drivers	3.71	4.51	4.52	4.78	0.00041	1975.750	0.027958	133522.77
2587 Towel Supply Co & Rt Supervisors, Driv	4.50	4.31	3.39	2.54	0.00065	85.52247	0.143338	18902.901
2600 Fur Manufacturing - Preparing Skins	--	--	--	--	--	0	--	0
2623 Tanning	9.06	8.72	7.99	6.54	0.00030	177.7238	0.099793	59084.418
2651 Shoe Stock Manufacturing	6.92	7.35	7.15	6.43	0.00043	336.0124	0.053019	41716.591
2660 Boot Manufacturing NOC	7.64	7.29	6.39	6.43	0.00019	693.4797	0.003120	11180.416
2670 Glove Manufacturing - Leather or Textile	2.87	3.05	3.22	3.27	0.00003	1.700000	0.007704	417.60988
2683 Luggage Manufacturing	--	--	--	--	--	0	--	0
2688 Leather Goods Manufacturing NOC	3.97	4.25	4.04	3.72	0.00038	211.2301	0.041237	22879.287
2697 Forest Patrollers	5.63	5.23	4.22	3.94	0.00026	378.4038	0.034313	50302.615
2702 Logging or Lumbering & Drivers	29.53	31.34	26.50	22.39	0.00096	576308.3	0.084066	50820707.
2703 Logging Equipment Mntnc & Repair	7.24	7.86	7.28	6.73	0.00049	8501.821	0.039257	876830.27
2710 Saw Mill	12.20	11.64	10.20	9.14	0.00008	15883.39	0.054808	11440281.
2714 Veneer Manufacturing	13.48	12.44	11.02	7.41	0.00092	27805.20	0.195876	5902165.0
2725 Logging: Mechanized Equip Operations	--	--	--	--	--	0	--	0
2731 Planing or Molding Mill	9.92	10.39	9.33	7.87	0.00071	84952.78	0.084883	10196297.
2735 Furniture Stock Manufacturing	9.23	9.84	9.72	9.03	0.00030	304.0243	0.0388	37782.334
2741 Cooperage Stock Manufacturing	11.34	10.26	--	--	--	0	--	0
2747 Cooperage Assembly	5.98	7.01	--	--	--	0	--	0
2759 Box or Box Shook Manufacturing	14.55	14.49	13.59	11.68	0.00030	716.4051	0.076435	184236.36
2790 Pattern Making NOC	2.55	2.29	2.20	2.13	0.00012	123.4490	0.016168	16533.711
2802 Carpentry - Shop Only _ & Drivers	8.85	8.09	7.17	6.32	0.00003	2941.773	0.063009	5596030.7
2812 Cabinet Works - With Power Machinery	9.29	9.58	8.53	9.27	0.00040	12455.61	0.041573	1288330.2
2835 Brush or Broom Assembly	4.38	4.59	4.58	4.98	0.00008	186.8209	0.041841	92868.080
2836 Brush or Broom Manufacturing NOC	6.10	6.24	6.84	5.52	0.00134	166.6146	0.106796	13301.746
2841 Woodenware Manufacturing NOC	9.75	11.49	11.20	9.89	0.00133	5369.081	0.062114	251437.26
2881 Furniture Assembly - Wood	6.02	6.01	5.64	6.13	0.00024	883.9812	0.041631	152247.33
2883 Furniture Manufacturing - Wood - NOC	9.50	11.56	11.09	7.78	0.00452	68012.76	0.175410	3412105.3
2913 Rattan Products Manufacturing	7.31	6.42	--	--	--	0	--	0
2915 Veneer Products Manufacturing	6.02	7.32	6.81	6.31	0.00139	140714.8	0.038109	3848355.6
2916 Veneer Products Manufacturing - NO Ve	8.73	8.70	7.71	7.18	0.00016	7178.406	0.035594	1634227.6
2923 Piano Manufacturing	4.87	4.30	4.08	3.23	0.00033	380.1492	0.116279	133101.55
2942 Pencil Manufacturing	5.06	5.61	--	--	--	0	--	0
2960 Wood Preserving & Drivers	8.85	9.55	8.94	8.30	0.00042	2728.383	0.037122	242368.49
3004 Iron or Steel:Mfg:Stl Mkng-Elec Fnc or C	6.36	6.35	6.28	4.85	0.00104	7517.803	0.128481	932661.59
3018 Iron or Steel: Mfg: Rolling Mill & Drivers	7.47	8.88	9.12	7.82	0.00166	12.22638	0.076741	564.23602
3022 Pipe Manufacturing NOC & Drivers	10.91	10.15	8.96	6.73	0.00041	161.5636	0.142128	55495.840
3027 Rolling Mill NOC & Drivers	3.04	3.29	3.01	2.38	0.00142	13558.42	0.116883	1115043.9
3028 Pipe or Tube Manufacuring - Iron or Stee	7.08	6.84	6.15	6.69	0.00041	922.1534	0.042066	94713.211

National Council Development Method

Class Description	1989 Base	1990	1991	1992	TEST 1 Sum(D^2)/[Sm(y)]^2	Premium Wghted	TEST 2 |y4-y3|/(y3+y4)	Premium Wghted
3030 Iron or Steel; fab: I or S wks-shop struct	12.05	11.95	10.99	12.23	0.00041	4789.411	0.053402	621850.98
3040 Iron or Steel; fab: I works-Shop-Ornmntl	13.25	11.47	10.27	8.03	0.00010	966.2594	0.122404	1188859.4
3041 Iron or Steel; fab: I works-Shop-& Found	6.30	5.64	5.33	5.93	0.00077	1981.789	0.053285	137764.74
3042 Elevator Manufacturing	4.22	3.98	4.00	3.51	0.00018	0.907100	0.085246	322.66603
3064 Sign Manufacturing - Metal	8.25	8.24	8.19	6.58	0.00077	350.1455	0.109004	49365.780
3066 Sheet Metal Work - Shop	6.88	6.81	6.05	5.76	0.00012	2616.755	0.024555	526357.13
3076 Fireproof Equipment Manufacturing	5.30	4.40	3.96	4.05	0.00078	15639.94	0.011235	224718.01
3081 Foundry - Ferrous - NOC	12.12	11.28	10.61	7.86	0.00066	4275.626	0.148890	957832.67
3082 Foundry - Steel Casings	7.33	8.75	8.19	9.12	0.00059	4388.223	0.053726	397011.92
3085 Foundry - Nonferrous	12.38	10.84	10.26	7.14	0.00075	2464.687	0.179310	589681.90
3110 Forging Work	9.50	10.87	12.14	10.05	0.00195	3348.553	0.094186	162098.53
3111 Blacksmith	7.42	7.91	8.19	7.51	0.00038	722.2381	0.043312	81328.145
3113 Tool Mfg, Not Drop or Mach Forged - NC	4.08	3.57	3.35	3.11	0.00012	341.5271	0.037151	109689.23
3114 Tool Mfg, Drop/Mach Forged-NOC: Mad	5.56	5.52	5.72	5.28	0.00016	91.45738	0.04	22656.446
3118 Saw Manufacturing	5.43	6.59	8.17	8.87	0.00066	3861.864	0.053680	311908.46
3122 Cutlery Manufacturing NOC	5.55	6.61	6.82	6.60	0.00064	1019.613	0.016393	26139.321
3126 Tool Mfg	6.87	6.99	7.20	5.86	0.00111	79.74585	0.102603	7370.5250
3131 Button Manufacturing - Metal	4.58	4.41	4.20	3.49	0.00030	0.135029	0.092327	41.549901
3132 Nut Manufacturing	6.26	6.40	6.22	5.02	0.00083	507.6486	0.106761	65394.916
3145 Screw Manufacturing	4.04	4.47	4.53	4.70	0.00009	107.6292	0.018418	21951.318
3146 Hardware Manufacturing NOC	4.88	5.28	4.97	4.30	0.00077	3170.418	0.072276	295906.31
3169 Stove Manufacturing	12.69	11.91	10.76	10.80	0.00014	309.7578	0.001855	4225.6808
3175 Radiator Manufacturing	7.88	9.55	--	--	--	0	--	0
3179 Electrical Apparatus Manufacturing NOC	3.59	3.77	3.47	2.75	0.00110	10828.38	0.115755	1140497.2
3180 Electric Light Fixtures Manufacturing	5.93	6.31	6.42	5.21	0.00121	651.1438	0.104041	56174.070
3188 Plumbers' Supplies Manufacturing NOC	5.26	5.24	5.01	4.32	0.00029	27.69702	0.073954	6962.7818
3220 Can Manufacturing	4.41	4.06	4.02	4.45	0.00051	1945.380	0.050767	193488.88
3223 Lamp or Portable Lantern Mfg	7.57	7.79	8.35	7.12	0.00079	8.152693	0.079508	818.79238
3224 Agate Ware Manufacturing	7.17	7.76	8.92	7.24	0.00193	2.104160	0.103980	113.09218
3227 Aluminum Ware Manufacturing	7.07	7.15	6.55	6.10	0.00014	6.584059	0.035573	1610.6738
3240 Wire Rope Mfg - Iron or Steel	--	--	--	--	--	0	--	0
3241 Wire Drawing - Iron or Steel	8.00	8.42	--	--	--	0	--	0
3255 Wire Cloth Manufacturing	9.37	9.79	9.62	6.93	0.00204	1734.129	0.162537	138016.02
3257 Wire Goods Manufacturing NOC	5.90	6.16	6.07	4.52	0.00172	7982.830	0.148364	680668.38
3300 Bed Spring Manufacturing	11.57	11.62	11.63	9.39	0.00080	214.2868	0.106565	28698.369
3303 Spring Manufacturing	9.61	8.38	8.67	6.76	0.00072	436.8598	0.123784	74794.418
3307 Heat-Treating	7.49	7.72	7.46	6.72	0.00027	258.1823	0.052186	49475.304
3315 Brass Goods Manufacturing	6.09	5.90	--	--	--	0	--	0
3334 Tin Foil Manufacturing	2.58	2.97	--	--	--	0	--	0
3336 Type Foundry	6.07	4.72	4.18	3.17	0.00033	9753.008	0.135061	4048868.0
3365 Welding NOC & Drivers	16.57	15.90	15.25	16.50	0.00027	1166.378	0.039370	172431.91
3372 Electroplating	8.85	9.34	8.68	7.54	0.00058	4015.166	0.070283	486607.48
3373 Galvanizing	12.45	13.86	15.59	12.48	0.00215	3811.421	0.110794	185716.01
3383 Jewelry Manufacturing	2.89	3.59	3.73	3.08	0.00259	2769.022	0.095447	101884.11
3385 Watch Manufacturing	2.06	2.47	2.58	2.23	0.00167	188.5555	0.072765	8128.7728
3400 Metal Goods Manufacturing NOC	7.85	8.08	7.75	8.00	0.00006	333.8427	0.015873	81719.218
3507 Agriculture or Construction Machinery M	8.55	9.04	7.83	5.54	0.00204	112694.6	0.171278	9481583.7
3548 Printing Machine Manufacturing	2.92	3.26	3.38	2.98	0.00090	390.7320	0.062893	27329.242
3559 Confection Machine Manufacturing	6.18	6.79	6.93	5.90	0.00107	0.994287	0.080280	74.506812
3565 Typewriter Manufacturing	0.86	1.06	--	--	--	0	--	0
3574 Computing, Recording, Office Machine N	1.80	1.76	1.66	1.28	0.00074	1067.558	0.129251	186138.67
3612 Pump Manufacturing	4.08	4.55	4.05	3.44	0.00126	9464.291	0.081441	609572.06
3620 Boilermaking	7.43	9.24	10.08	7.19	0.00512	43882.16	0.167342	1432902.1
3629 Precision Machines Parts MFG, NOC	3.54	3.80	3.60	2.99	0.00098	4202.823	0.092564	398678.19
3632 Machine Shop NOC	4.56	4.82	4.04	4.07	0.00026	14100.74	0.003699	199018.45
3634 Valve Manufacturing	4.38	4.77	4.83	4.17	0.00086	2531.399	0.073333	215911.70
3635 Gear Manufacturing or Grinding	3.74	4.40	4.56	3.49	0.00296	4756.040	0.132919	213844.60
3636 Ball Bearing Manufacturing	3.81	3.99	--	--	--	0	--	0
3642 Battery Manufacturing - Dry	--	--	--	--	--	0	--	0
3643 Electric Power Equipment Manufacturing	4.78	4.33	3.87	3.89	0.00024	1442.986	0.002577	15743.364
3647 Battery Manufacturing - Storage	4.98	5.03	4.52	4.48	0.00015	795.4997	0.004444	23200.998
3648 Auto Lighting, Ign or Strtr Apparatus Mfg	8.49	7.13	6.49	5.87	0.00021	683.6021	0.050161	161462.95
3681 Television/Radio/Telephone/Telecommu	1.94	2.38	2.10	2.34	0.00113	24551.73	0.054054	1172192.2
3685 Instrument Manufacturing NOC	1.68	2.02	1.84	1.98	0.00080	6792.711	0.036849	310911.72
3719 Oil Still Erection or Repair	13.73	12.[illegible]	11.93	10.11	0.00011	15.37658	0.082577	11639.436
3724 Machinery/Equip Erection/Repair NOC &	11.33	9.84	9.56	9.58	0.00116	61699.93	0.056229	2989298.0
3726 Boiler Installation or Repair	13.46	12.46	12.75	14.28	0.00057	3876.693	0.056603	384435.79
3803 Automobile Wheel Manufacturing	--	--	--	--	--	0	--	0

Revised Loss Development Method

Class Description	1989 Base	1990	1991	1992	TEST 1 Sum(D^2)/[Sm(y)]^2	Premium Wghted	TEST 2 |y4-y3|/(y3+y4)	Premium Wghted
3030 Iron or Steel; fab: I or S wks-shop struct	12.05	11.25	10.24	10.85	0.00034	3691.399	0.028923	316620.67
3040 Iron or Steel; fab: I works-Shop-Ornmntl	13.25	11.94	10.45	8.58	0.00004	410.4880	0.098265	981035.56
3041 Iron or Steel; fab: I works-Shop-& Found	6.30	5.49	5.02	5.46	0.00082	2035.976	0.041984	104195.47
3042 Elevator Manufacturing	4.22	3.88	3.73	3.29	0.00006	0.291888	0.062678	298.32406
3064 Sign Manufacturing - Metal	8.25	7.75	7.38	5.99	0.00031	130.2981	0.103964	44236.319
3066 Sheet Metal Work - Shop	6.88	6.90	6.18	5.65	0.00018	3901.709	0.044801	964479.36
3076 Fireproof Equipment Manufacturing	5.30	4.87	4.13	3.85	0.00010	2017.640	0.035087	711260.87
3081 Foundry - Ferrous - NOC	12.12	11.64	10.63	8.05	0.00064	4187.175	0.138115	900814.84
3082 Foundry - Steel Casings	7.33	8.24	7.68	8.35	0.00038	2853.620	0.041796	292300.53
3085 Foundry - Nonferrous	12.38	10.83	9.87	7.31	0.00030	991.8698	0.149010	487262.73
3110 Forging Work	9.50	10.40	11.12	9.54	0.00107	1757.014	0.076476	125433.40
3111 Blacksmith	7.42	7.80	7.70	7.23	0.00020	383.7412	0.031480	57434.811
3113 Tool Mfg, Not Drop or Mach Forged - NC	4.08	3.59	3.25	2.99	0.00007	202.6276	0.041666	121275.80
3114 Tool Mfg, Drop/Mach Forged-NOC: Mad	5.56	5.29	5.23	4.90	0.00003	15.18619	0.032576	17532.454
3118 Saw Manufacturing	5.43	6.07	5.53	6.01	0.00047	2499.739	0.041594	222199.74
3122 Cutlery Manufacturing NOC	5.55	6.31	6.28	6.17	0.00036	539.6018	0.007240	10962.825
3126 Tool Mfg	6.87	6.93	6.83	5.47	0.00104	72.30761	0.110569	7698.9927
3131 Button Manufacturing - Metal	4.58	4.32	3.95	3.28	0.00017	0.073312	0.092669	40.328569
3132 Nut Manufacturing	6.26	6.12	5.71	4.72	0.00036	206.3463	0.094918	55489.054
3145 Screw Manufacturing	4.04	4.34	4.22	4.44	0.00010	120.1725	0.025404	29082.662
3146 Hardware Manufacturing NOC	4.88	5.10	4.69	4.15	0.00044	1756.667	0.061085	242240.90
3169 Stove Manufacturing	12.69	12.16	10.70	10.42	0.00011	255.7064	0.013257	30071.720
3175 Radiator Manufacturing	7.68	9.35	--	--	--	0	--	0
3179 Electrical Apparatus Manufacturing NOC	3.59	3.74	3.41	2.80	0.00080	7844.671	0.098228	964959.89
3180 Electric Light Fixtures Manufacturing	5.93	6.02	5.87	4.81	0.00069	352.9019	0.099250	50803.891
3188 Plumbers' Supplies Manufacturing NOC	5.26	5.01	4.57	3.94	0.00010	9.152714	0.074030	6619.8003
3220 Can Manufacturing	4.41	4.01	3.80	4.09	0.00047	1706.149	0.036755	134716.28
3223 Lamp or Portable Lantern Mfg	7.57	7.63	7.85	6.67	0.00057	5.687950	0.081267	806.76986
3224 Agate Ware Manufacturing	7.17	7.59	8.40	6.61	0.00156	1.633602	0.104538	109.62222
3227 Aluminum Ware Manufacturing	7.07	7.00	6.14	6.07	0.00018	8.003441	0.005733	253.87835
3240 Wire Rope Mfg - Iron or Steel	--	--	--	--	--	0	--	0
3241 Wire Drawing - Iron or Steel	8.00	8.24	--	--	--	0	--	0
3255 Wire Cloth Manufacturing	9.37	9.68	9.26	6.64	0.00185	1533.713	0.164779	136942.01
3257 Wire Goods Manufacturing NOC	5.90	6.02	5.74	4.47	0.00102	4642.945	0.124387	565185.52
3300 Bed Spring Manufacturing	11.57	11.70	11.25	8.87	0.00088	233.9084	0.118290	31265.114
3303 Spring Manufacturing	9.61	8.04	7.91	6.24	0.00044	253.8686	0.118021	67855.093
3307 Heat-Treating	7.49	7.38	6.89	6.33	0.00007	63.65412	0.042360	38383.246
3315 Brass Goods Manufacturing	6.09	5.77	--	--	--	0	--	0
3334 Tin Foil Manufacturing	2.58	2.91	--	--	--	0	--	0
3336 Type Foundry	6.07	5.16	4.41	3.45	0.00002	647.3794	0.122137	3857436.8
3365 Welding NOC & Drivers	16.57	16.06	14.66	15.77	0.00031	1332.696	0.036477	156850.95
3372 Electroplating	8.85	9.29	8.46	7.30	0.00060	4060.792	0.073604	502043.76
3373 Galvanizing	12.45	13.65	14.70	12.18	0.00144	2361.764	0.09375	153665.22
3383 Jewelry Manufacturing	2.89	3.51	3.44	2.93	0.00198	2027.060	0.080082	82101.601
3385 Watch Manufacturing	2.06	2.50	2.51	2.20	0.00184	182.2226	0.085817	7297.5282
3400 Metal Goods Manufacturing NOC	7.85	7.71	7.23	7.31	0.00006	283.9872	0.005502	26913.591
3507 Agriculture or Construction Machinery M	8.55	9.03	7.93	6.34	0.00111	63472.69	0.111422	6345391.8
3548 Printing Machine Manufacturing	2.92	3.17	3.15	2.84	0.00054	224.9505	0.051752	21663.517
3559 Confection Machine Manufacturing	6.16	6.65	6.51	5.54	0.00067	0.774593	0.080497	72.042423
3565 Typewriter Manufacturing	0.86	1.04	--	--	--	0	--	0
3574 Computing, Recording, Office Machine N	1.80	1.74	1.59	1.26	0.00046	645.9431	0.115789	163927.66
3612 Pump Manufacturing	4.08	4.43	3.97	3.48	0.00084	6227.170	0.068640	508021.02
3620 Boilermaking	7.43	9.04	9.11	7.45	0.00245	20432.39	0.100241	835325.04
3629 Precision Machines Parts MFG, NOC	3.54	3.76	3.48	2.86	0.00096	4027.644	0.097791	412423.82
3632 Machine Shop NOC	4.56	4.57	4.07	3.92	0.00015	7884.674	0.018773	1000106.3
3634 Valve Manufacturing	4.38	4.77	4.63	3.90	0.00100	2878.996	0.085580	245445.01
3635 Gear Manufacturing or Grinding	3.74	4.34	4.31	3.41	0.00228	3555.878	0.116580	183039.93
3636 Ball Bearing Manufacturing	3.81	3.91	--	--	--	0	--	0
3642 Battery Manufacturing - Dry	--	--	--	--	--	0	--	0
3643 Electric Power Equipment Manufacturing	4.78	4.34	3.80	3.84	0.00011	680.4456	0.021505	128950.06
3647 Battery Manufacturing - Storage	4.98	4.89	4.34	4.21	0.00012	583.1983	0.015204	76908.429
3648 Auto Lighting, Ign or Strtr Apparatus Mfg	8.49	7.40	6.46	5.74	0.00004	141.1697	0.059016	190711.27
3681 Television/Radio/Telephone/Telecommu	1.94	2.33	2.12	2.29	0.00080	17148.11	0.038548	828316.91
3685 Instrument Manufacturing NOC	1.68	1.95	1.78	1.83	0.00065	5279.094	0.013850	113124.31
3719 Oil Still Erection or Repair	13.73	12.91	11.22	9.37	0.00013	17.82710	0.089849	12302.419
3724 Machinery/Equip Erection/Repair NOC &	11.33	10.47	9.09	10.18	0.00083	46022.56	0.056564	3141758.3
3726 Boiler Installation or Repair	13.46	12.70	12.19	13.65	0.00051	3404.038	0.056501	376856.62
3803 Automobile Wheel Manufacturing	--	--	--	--	--	0	--	0

	National Council Development Method								Revised Loss Development Method							
					TEST 1		TEST 2						TEST 1		TEST 2	
Class Description	1989 Base	1990	1991	1992	Sum(D^2)/ [Sm(y)]^2	Premium Wghted	|y4-y3|/ (y3+y4)	Premium Wghted	1989 Base	1990	1991	1992	Sum(D^2)/ [Sm(y)]^2	Premium Wghted	|y4-y3|/ (y3+y4)	Premium Wghted
3807 Automobile Radiator Manufacturing	5.43	5.59	5.64	4.66	0.00081	30.96618	0.095145	3646.9068	5.43	5.35	5.14	4.26	0.00043	15.55327	0.093617	3396.4451
3808 Automobile Manufacturing or Assembly	5.68	6.12	6.02	6.58	0.00013	757.3481	0.044444	265075.60	5.68	5.89	5.64	5.97	0.00011	608.2390	0.028423	181048.84
3821 Automobile Dismantling & Drivers	7.15	8.18	7.26	6.44	0.00126	8871.029	0.059854	419861.46	7.15	7.83	6.79	6.23	0.00080	5405.733	0.043010	291005.21
3822 Auto, Bus, Truck or Tlr Body Mfg: Die Pr	10.59	13.17	13.50	10.61	0.00328	281.8504	0.119867	10285.103	10.59	12.64	12.44	9.84	0.00287	232.8767	0.126811	10299.061
3824 Auto, Bus, Truck or Tlr Body Mfg: NOC	8.73	10.55	9.67	7.86	0.00255	27654.26	0.103251	1121145.0	8.73	10.12	9.23	7.68	0.00179	18907.72	0.091861	966907.50
3826 Aircraft Engine Manufacturing	--	--	--	--	--	0	--	0	--	--	--	--	--	0	--	0
3830 Airplane Manufacturing	2.57	2.59	2.54	2.04	0.00078	5815.488	0.109170	818545.64	2.57	2.52	2.37	2.03	0.00024	1740.129	0.077272	565090.47
3851 Motorcycle Manufacturing or Assembly	4.13	4.45	4.25	--	--	0	--	0	4.13	4.38	3.99	--	--	0	--	0
3865 Baby Carriage Manufacturing	8.16	8.32	--	--	--	0	--	0	8.16	6.14	--	--	--	0	--	0
4000 Sand Digging & Drivers	10.38	13.07	12.06	7.81	0.00642	85408.45	0.213890	2844647.4	10.38	11.28	9.90	8.41	0.00102	12501.85	0.081376	998076.08
4021 Brick Manufacturing NOC & Drivers	10.73	10.52	9.60	7.72	0.00047	305.4838	0.108545	70434.008	10.73	10.16	8.94	7.18	0.00026	161.2052	0.109181	67981.308
4024 Refractory Products Manufacturing & Dri	15.23	16.35	13.87	11.98	0.00095	895.0821	0.073114	68878.514	15.23	15.90	13.29	11.32	0.00081	738.7364	0.080048	72979.754
4034 Concrete Products Manufacturing & Driv	10.96	10.65	9.51	9.04	0.00007	679.5055	0.025336	233535.28	10.96	10.66	9.51	8.93	0.00008	689.9101	0.031453	289189.74
4036 Plasterboard Manufacturing & Drivers	6.04	6.60	6.71	6.28	0.00037	113.6570	0.033102	10069.904	6.04	6.77	6.63	6.15	0.00058	176.2831	0.037558	11407.700
4038 Plaster Statuary Manufacturing	3.91	4.72	5.42	4.29	0.00324	1092.053	0.116374	39259.595	3.91	4.68	5.12	4.03	0.00298	971.4844	0.119126	38872.842
4053 Pottery Mfg: China or Tableware	5.40	5.88	5.39	4.54	0.00102	91.05096	0.085599	7608.4264	5.40	5.87	4.96	4.17	0.00079	68.67745	0.086527	7324.7394
4061 Pottery Mfg: Earthenware - Hand moldec	9.36	9.76	9.41	9.70	0.00007	61.48533	0.015175	13687.601	9.36	9.44	8.75	9.22	0.00017	143.9008	0.026154	22689.720
4062 Pottery Mfg: Porcelain Ware - Press Forr	6.26	5.80	5.22	4.35	0.00009	0.224806	0.090909	219.95743	6.26	5.66	4.90	4.07	0.00004	0.089605	0.092530	216.42852
4101 Glass Manufacturing - Polished Plate - &	4.96	4.93	4.17	4.64	0.00073	209.4179	0.053348	15345.983	4.96	4.73	3.83	4.16	0.00083	225.2583	0.041301	11232.607
4111 Glassware Mfg - No Automatic Blowing I	6.12	6.29	5.44	4.85	0.00044	1395.313	0.057337	182153.31	6.12	6.20	5.35	4.69	0.00040	1249.415	0.065737	205710.58
4112 Incandescent Lamp Manufacturing	1.70	1.95	2.24	2.07	0.00089	440.2468	0.039443	19439.166	1.70	1.93	2.11	1.96	0.00067	321.7949	0.036855	17570.353
4113 Glass Manufacturing - Cut	4.69	3.96	4.28	4.41	0.00087	16.02262	0.014959	275.30811	4.69	4.00	4.15	4.47	0.00093	17.01238	0.037122	682.00275
4114 Glassware Manufacturing NOC	6.10	6.51	6.55	6.95	0.00004	12.79147	0.029629	9688.0787	6.10	6.48	6.23	6.77	0.00018	52.49039	0.041538	13306.244
4130 Glass Merchant	8.67	10.11	9.95	6.22	0.00563	30600.27	0.230674	1253907.6	8.67	10.10	9.71	6.83	0.00418	22693.53	0.188494	1029316.7
4131 Mirror Manufacturing	9.20	10.21	9.75	7.94	0.00144	233.0736	0.102317	16504.265	9.20	10.76	10.04	8.01	0.00226	373.9365	0.112465	18586.099
4133 Cathedral Window Manufacturing	5.85	6.63	7.45	8.29	0.00000	0.829345	0.053367	38312.137	5.85	6.66	7.06	7.87	0.00004	27.28839	0.041412	28697.165
4150 Optical Goods Manufacturing NOC	1.87	2.05	2.02	1.66	0.00127	7422.187	0.097826	569662.96	1.87	2.02	1.94	1.63	0.00095	5433.335	0.088834	496343.12
4206 Pulp Manufacturing - Ground Wood	4.99	4.25	4.88	5.31	0.00123	1121.138	0.042198	38371.390	4.99	4.19	4.71	4.97	0.00114	1006.758	0.026859	23707.193
4207 Pulp Manufacturing - Chemical	3.83	3.06	2.55	1.75	0.00012	210.4838	0.186048	319441.75	3.83	3.23	2.65	1.75	0.00022	378.6472	0.204545	359678.52
4239 Paper Manufacturing	5.74	4.46	3.36	2.51	0.00018	2164.202	0.144804	1741557.7	5.74	4.46	3.74	2.73	0.00020	2440.085	0.158105	1947573.5
4240 Box Manufacturing - Set-Up Paper	7.35	6.88	7.30	7.16	0.00016	116.5186	0.009681	7018.0448	7.35	6.58	6.68	6.38	0.00018	124.7737	0.021472	14827.275
4243 Box Manufacturing - Folding Paper - NO	6.40	6.61	6.51	5.40	0.00074	3173.327	0.093198	399181.05	6.40	6.20	5.84	5.10	0.00014	549.9389	0.067841	273672.72
4244 Corrugated Container Manufacturing	9.37	8.97	8.54	6.79	0.00048	1661.052	0.114155	398945.72	9.37	8.96	8.25	6.39	0.00052	1768.475	0.127049	434776.05
4250 Paper Coating	3.45	3.31	3.26	3.61	0.00035	810.5006	0.050946	118328.05	3.45	3.17	3.04	3.23	0.00034	749.1581	0.030303	66559.827
4251 Stationery Manufacturing	5.66	5.08	4.81	4.61	0.00010	475.1825	0.021231	106191.05	5.66	4.96	4.53	4.38	0.00020	951.7021	0.016835	80802.534
4263 Fiber Goods Manufacturing	9.75	10.01	9.46	7.96	0.00056	18.87525	0.086107	2590.5837	9.75	9.74	8.87	7.40	0.00042	12.13498	0.090350	2614.4023
4273 Bag Manufacturing - Paper	8.89	9.80	9.33	6.15	0.00368	12712.27	0.205428	713992.44	8.89	9.88	9.22	6.51	0.00288	10114.18	0.172282	604577.72
4279 Paper Goods Manufacturing - NOC	6.44	6.33	6.14	6.37	0.00006	143.6092	0.018385	40757.419	6.44	6.13	5.71	5.88	0.00014	297.9924	0.014667	31075.741
4282 Dress Pattern Mfg - Paper	--	--	--	--	--	0	--	0	--	--	--	--	--	0	--	0
4283 Building/Roofing Paper/Felt Prep - No In	9.44	9.70	8.32	6.25	0.00123	5604.769	0.142072	645252.35	9.44	9.54	8.20	6.66	0.00065	2983.141	0.103633	472489.67
4299 Printing	3.48	3.40	3.00	2.79	0.00011	3134.314	0.038269	1059134.7	3.48	3.45	3.07	2.86	0.00026	7504.284	0.071553	2087838.0
4301 Wallpaper Manufacturing	5.57	8.36	--	--	--	0	--	0	5.57	8.22	--	--	--	0	--	0
4304 Newspaper Publishing	3.74	4.42	4.04	4.49	0.00089	7272.511	0.052754	557113.94	3.74	4.25	3.94	4.06	0.00045	4596.158	0.015	151762.31
4307 Bookbinding	5.35	4.99	4.56	4.56	0.00012	267.1453	0	0	5.35	4.93	4.36	4.24	0.00011	248.3688	0.013953	30502.036
4308 Linotype or Hand Composition	1.72	1.59	1.82	1.69	0.00058	4.921341	0.037037	327.07962	1.72	1.59	1.82	1.77	0.00046	4.143848	0.013927	124.43928
4312 News Carriers or Route Carriers	6.46	7.15	6.40	7.02	0.00054	1864.175	0.046199	158887.62	6.46	6.90	5.87	6.36	0.00068	2225.634	0.040065	130450.20
4350 Electrotyping	1.89	1.89	2.17	2.27	0.00019	0.154350	0.022522	17.958108	1.89	1.83	2.10	2.13	0.00029	0.222916	0.007092	5.4691489
4351 Photoengraving	1.26	1.19	1.13	1.10	0.00003	19.52474	0.013452	9092.7425	1.26	1.14	1.04	1.03	0.00016	105.8770	0.004830	3125.3510
4352 Engraving	1.97	2.32	2.58	2.76	0.00006	6.895478	0.037593	4078.4153	1.97	2.21	2.32	2.52	0.00003	3.529330	0.041322	4207.8622
4360 Motion Picture: Dev. of Negtvs & all Subs	1.36	1.30	1.38	1.32	0.00014	11.52816	0.022222	1877.5484	1.36	1.25	1.28	1.25	0.00014	11.04023	0.011857	980.73304
4361 Photographer - All Empl & Cler, Slspens	2.09	2.77	2.71	1.79	0.00731	52229.00	0.204444	1480059.2	2.09	2.70	2.57	1.91	0.00472	33369.94	0.147321	1041993.4
4362 Motion Picture: Film Exchange & Clerical	1.47	1.52	1.31	1.12	0.00062	6.894167	0.078189	864.37513	1.47	1.48	1.25	1.04	0.00056	6.025530	0.091703	980.10081
4410 Rubber Goods Manufacturing NOC	6.06	6.79	6.66	7.05	0.00018	974.5970	0.028446	157567.48	6.06	6.56	6.25	6.30	0.00019	986.5759	0.003984	20913.218
4417 Boot Manufacturing - Rubber	6.95	7.22	--	--	--	0	--	0	6.95	7.07	--	--	--	0	--	0
4420 Rubber Tire Manufacturing	7.98	8.73	8.80	9.80	0.00012	22.49057	0.053763	10380.703	7.98	9.18	9.06	9.85	0.00022	44.06523	0.041776	8239.9336
4431 Phonograph Record Manufacturing	2.23	2.29	--	--	--	0	--	0	2.23	2.24	--	--	--	0	--	0
4439 Lacquer Manufacturing	4.94	4.86	5.02	4.17	0.00083	213.4045	0.092491	31446.803	4.94	4.76	4.59	3.87	0.00027	87.33090	0.085106	27671.079
4452 Plastics Manufacturing: Fabricated Prod	11.15	10.06	8.98	6.57	0.00039	2593.499	0.154983	1033832.8	11.15	10.27	9.00	7.00	0.00023	1548.788	0.125	848793.36
4459 Plastics Manufacturing: Sheets, Rods or	5.55	5.63	6.65	7.41	0.00027	1009.633	0.054054	200302.75	5.55	6.46	6.25	6.79	0.00036	1263.315	0.041411	146610.58
4470 Cable Manufacturing - Insulated Electric	6.96	9.06	8.08	5.94	0.00318	929.2502	0.152639	44587.827	6.96	8.10	7.86	5.78	0.00318	912.2057	0.152492	44023.524
4484 Plastics Manufacturing: Molded Products	6.44	6.52	5.78	6.26	0.00040	8703.884	0.039867	874208.76	6.44	6.49	5.74	6.06	0.00032	6847.800	0.027118	588237.19
4493 Fabric Coating or Impregnating NOC	--	--	--	--	--	0	--	0	--	--	--	--	--	0	--	0
4511 Chemist, Analytical	1.69	2.06	1.92	1.51	0.00301	18804.32	0.119533	747688.50	1.69	1.93	1.77	1.52	0.00136	8173.432	0.075987	457434.24
4557 Ink Manufacturing	4.66	4.32	4.30	3.48	0.00043	615.2825	0.105398	151397.63	4.66	4.20	4.01	3.28	0.00020	270.8094	0.100137	138604.93
4558 Paint Manufacturing	4.48	4.96	4.70	3.87	0.00133	3581.558	0.096849	261368.89	4.48	4.85	4.50	3.85	0.00086	2279.206	0.077844	206230.10
4561 Varnish Manufacturing - Oleo-Resinous	4.69	3.65	--	--	--	0	--	0	4.69	3.65	--	--	--	0	--	0
4583 Fertilizer Manufacturing & Drivers	5.55	6.23	6.10	6.79	0.00022	513.2230	0.053529	125838.53	5.55	5.92	5.65	6.15	0.00019	424.3629	0.042372	93957.772
4611 Drug Preparation	2.75	3.07	3.09	2.94	0.00040	493.4484	0.024875	30741.275	2.75	3.03	2.92	2.81	0.00034	413.4326	0.019197	23043.210

National Council Development Method

Class Description	1989 Base	1990	1991	1992	TEST 1 Sum(D^2)/ [Sm(y)]^2	Premium Wghted	TEST 2 |y4-y3|/ (y3+y4)	Premium Wghted
4635 Oxygen Manufacturing & Drivers	5.13	5.80	5.54	4.68	0.00102	2610.049	0.084148	216200.17
4653 Glue Manufacturing & Drivers	5.32	5.01	4.54	3.75	0.00017	179.0105	0.096295	100448.36
4665 Rendering Works NOC & Drivers	15.31	15.66	15.65	13.87	0.00034	824.9434	0.060298	147540.09
4670 Cottonseed Oil Manufacturing - Mechani	14.31	15.38	--	–	..	0	–	0
4683 Oil Manufacturing - Vegetable - NOC	4.69	4.41	4.66	3.85	0.00063	258.4892	0.095182	38803.145
4686 Oil Manufacturing - Vegetable - Solvent I	4.86	5.49	--	--	..	0	–	0
4692 Dental Laboratory	1.13	1.20	1.25	0.95	0.00193	1380.134	0.136363	97354.059
4693 Pharmaceutical Goods Manufacturing N(	2.71	3.03	3.01	2.42	0.00168	1212.338	0.108655	78387.407
4703 Corn Products Manufacturing	5.44	5.34	5.23	4.42	0.00036	30.46103	0.083937	7122.6283
4717 Butter Substitute Manufacturing	6.83	8.49	9.54	8.08	0.00244	3142.954	0.084090	108115.89
4720 Soap Manufacturing	4.63	5.07	5.20	3.86	0.00244	1886.720	0.147902	114278.50
4740 Oil Refining & Drivers	4.64	4.63	4.42	4.85	0.00024	97.66172	0.046386	18646.588
4741 Asphalt Distilling or Refining & Drivers	9.91	10.72	9.76	7.84	0.00150	1279.819	0.121839	104175.18
4751 Rubber Manufacturing - Synthetic	6.68	6.95	5.91	5.04	0.00072	193.0710	0.079452	21334.194
4773 Explosives or Ammun Mfg: High Explsvs	12.79	13.34	13.61	13.98	0.00001	0.898120	0.013410	1783.5748
4775 Explosives or Ammun Mfg: Shell Case L	28.20	29.87	–	–	..	0	–	0
4777 Explosives Distributors & Drivers	8.02	8.09	7.62	7.97	0.00011	28.15195	0.022450	5686.5773
4825 Drug, Medicine, or Pharmaceutical Prep	--	--	..	..	..	0	–	0
4902 Sporting Goods Manufacturing NOC	4.07	4.73	4.69	5.22	0.00024	1095.849	0.053481	241738.87
4923 Photographic Supplies Manufacturing	3.18	3.02	2.96	3.29	0.00041	812.6390	0.0528	103545.78
5020 Ceiling Install, Suspended Accounstd Gr	11.76	10.03	9.90	9.78	0.00044	1385.535	0.007121	22257.028
5022 Masonry - NOC	16.08	17.49	15.70	13.70	0.00085	17789.68	0.068027	1430709.1
5037 Painting: Metal Struct - over 3 stories & c	49.39	52.37	56.28	68.66	0.00049	433.2692	0.099087	88493.805
5040 Iron or Steel: Erection - Frame Structures	32.43	34.63	34.20	32.02	0.00027	2539.485	0.032920	307156.26
5057 Iron or Steel: Erection NOC	24.17	30.06	33.66	41.06	0.00015	916.7951	0.099036	622730.60
5059 Iron or Steel: Erct Fr Strctrs not over 2 st	48.18	51.19	52.46	59.29	0.00014	321.4788	0.061118	138731.80
5069 Iron or Steel: Erect.-const dwlgs n over 2	53.51	54.18	60.07	62.87	0.00009	2.511400	0.022775	663.25541
5102 Door Erection	11.14	11.26	11.55	9.84	0.00064	3065.900	0.084225	402382.98
5146 Furniture Installation - NOC	9.39	10.72	10.63	10.15	0.00052	3582.021	0.023099	158637.81
5160 Elevator Erection or Repair	6.45	6.44	6.08	5.28	0.00027	1096.739	0.070422	290347.89
5183 Plumbing NOC & Drivers	7.38	6.88	5.82	7.10	0.00169	92378.64	0.099071	5410323.3
5188 Automatic Sprinkler Installation & Drivers	7.49	8.52	8.46	6.42	0.00251	15824.01	0.137096	864019.76
5190 Electrical Wiring - Within buildings & Driv	5.63	5.75	4.99	4.90	0.00029	14176.08	0.009100	445009.27
5191 Office Machine Installation or Repair	1.68	1.83	1.60	1.74	0.00060	7844.146	0.041916	548062.41
5192 Vending Mach.s -Install, Svc or Repr & S	4.18	4.60	4.16	4.56	0.00047	3180.230	0.045871	308965.41
5213 Concrete Construction NOC	17.25	15.53	13.44	16.39	0.00176	75869.98	0.098893	4252990.6
5215 Concrete Work - Construction of Resider	12.53	12.46	12.13	10.03	0.00051	2837.184	0.094765	522380.84
5221 Concrete Work - Floors, Driveways - & D	13.23	14.71	12.75	10.11	0.00179	77833.42	0.115485	5018009.8
5222 Concrete Construction Bridges or Culver	18.61	23.15	25.92	19.64	0.00419	61754.06	0.137840	2033447.7
5223 Swimming Pool Construction - All operat	10.15	11.39	12.69	11.79	0.00066	767.4553	0.036784	42673.923
5348 Tile Work - Inside	15.30	12.34	11.18	9.82	0.00036	1298.620	0.064761	238505.03
5402 Hothouse Erection - All operations	15.97	13.48	13.79	14.96	0.00104	180.5168	0.040695	7035.0457
5403 Carpentry NOC	22.13	24.97	21.68	18.38	0.00148	186796.8	0.081918	10307096.
5437 Carpentry - Inst. Cabinet Work Inter. Trin	9.29	9.91	9.07	8.42	0.00040	5805.918	0.037184	538541.37
5443 Lathing & Drivers	9.22	10.49	11.59	12.75	0.00000	4.658895	0.047658	75985.224
5445 Wallboard Installation & Drivers	20.02	19.77	17.07	20.82	0.00132	45671.84	0.098970	3426362.2
5462 Glazier - Away from shop & Drivers	10.01	10.75	10.37	9.33	0.00049	3850.802	0.052791	414598.47
5474 Painting or Paper Hanging NOC & Driver	16.46	17.06	14.87	18.14	0.00117	44525.41	0.099060	3765016.8
5479 Insulation Work NOC & Drivers	19.10	19.95	17.94	17.48	0.00025	3846.084	0.012987	197660.29
5480 Plastering NOC & Drivers	17.65	15.31	14.56	14.40	0.00032	2072.793	0.005524	35463.670
5491 Paperhanging & Drivers	9.15	7.36	7.73	7.42	0.00095	179.4352	0.020482	3870.3729
5506 Street or Road Const: Paving or Repavin	8.81	7.70	6.69	8.16	0.00198	77754.60	0.098989	3887154.4
5507 Street or Road Const: Subsurface work &	13.51	11.05	9.92	11.18	0.00189	17732.60	0.059715	627929.19
5508 Street or Road Const: Rock Excavation I	15.38	14.12	14.98	12.47	0.00059	775.8704	0.091438	121254.11
5511 Logging Road Construction & Maintenan	14.06	16.03	13.91	10.54	0.00253	73831.57	0.137832	4018858.7
5538 Sheet Metal Work - NOC & Drivers	9.88	12.22	10.66	8.91	0.00280	125803.6	0.089422	4010863.1
5551 Roofing - All kinds & Drivers	40.38	37.95	32.96	25.94	0.00028	14422.49	0.119185	8129886.7
5606 Contractor - Executive Spvsr Const, Sup	3.13	3.23	2.95	2.24	0.00123	14186.78	0.136801	1577241.0
5610 Cleaner - Debris Removal	8.27	8.53	8.83	9.46	0.00003	35.73586	0.034445	39439.150
5645 Carpentry - Detached Dwellings	19.27	18.95	16.56	19.15	0.00084	47452.83	0.072528	4087452.5
5651 Carpentry - Dwellings - 3 stories or less	22.70	25.20	24.24	29.57	0.00065	7226.985	0.099052	1096227.8
5703 Building Raising or Moving & Drivers	48.25	48.25	46.45	47.20	0.00003	7.551640	0.008008	2012.1382
5951 Serum Manufacturing & Drivers	3.01	3.39	3.50	3.09	0.00094	245.3756	0.062215	16186.072
6003 Pile Driving & Drivers	35.57	35.22	34.52	26.25	0.00106	2209.839	0.136086	284615.16
6005 Jetty Construction - All Operations & Driv	15.52	17.24	17.33	14.98	0.00099	325.2478	0.072732	23996.691
6017 Dam or Lock Const: Concrete Work - All	22.58	22.12	18.51	17.06	0.00026	602.1004	0.040764	95566.616
6018 Dam or Lock Const: Earth Moving - All C	11.02	10.60	10.39	11.21	0.00022	464.0345	0.037962	78719.592
6045 Levee Construction - All Ops & Drvrs	13.47	12.95	14.50	14.33	0.00025	16.39402	0.005896	394.06798

Revised Loss Development Method

Class Description	1989 Base	1990	1991	1992	TEST 1 Sum(D^2)/ [Sm(y)]^2	Premium Wghted	TEST 2 |y4-y3|/ (y3+y4)	Premium Wghted
4635 Oxygen Manufacturing & Drivers	5.13	5.31	4.97	4.53	0.00026	845.2939	0.048315	113260.55
4653 Glue Manufacturing & Drivers	5.32	4.73	4.17	3.46	0.00002	18.71394	0.093053	93133.685
4665 Rendering Works NOC & Drivers	15.31	15.43	14.81	13.48	0.00015	360.2181	0.047013	112257.22
4670 Cottonseed Oil Manufacturing - Mechani	14.31	15.05	–	–	–	0	–	0
4683 Oil Manufacturing - Vegetable - NOC	4.69	4.24	4.30	3.57	0.00037	143.4476	0.092757	36075.265
4686 Oil Manufacturing - Vegetable - Solvent I	4.86	5.37	–	–	..	0	–	0
4692 Dental Laboratory	1.13	1.18	1.17	0.90	0.00144	993.0983	0.130434	90037.788
4693 Pharmaceutical Goods Manufacturing N(	2.71	3.07	2.95	2.35	0.00188	1343.023	0.113207	81013.255
4703 Corn Products Manufacturing	5.44	5.18	4.85	4.05	0.00021	17.21404	0.089887	7287.7591
4717 Butter Substitute Manufacturing	6.83	8.31	8.66	7.63	0.00160	1961.027	0.063228	77614.057
4720 Soap Manufacturing	4.63	5.26	5.18	3.85	0.00272	2122.411	0.147286	114773.10
4740 Oil Refining & Drivers	4.64	4.63	4.25	4.82	0.00030	118.1159	0.041713	16406.514
4741 Asphalt Distilling or Refining & Drivers	9.91	10.60	9.44	7.57	0.00121	1020.704	0.109935	92736.867
4751 Rubber Manufacturing - Synthetic	6.68	6.80	5.54	4.90	0.00060	157.2864	0.061302	16018.796
4773 Explosives or Ammun Mfg: High Explsvs	12.79	12.92	12.42	12.40	0.00003	3.257029	0.000805	99.674802
4775 Explosives or Ammun Mfg: Shell Case L	28.20	29.23	--	--	..	0	–	0
4777 Explosives Distributors & Drivers	8.02	7.76	6.98	7.36	0.00029	68.85158	0.027932	6894.5763
4825 Drug, Medicine, or Pharmaceutical Prep	--	–	–	–	–	0	--	0
4902 Sporting Goods Manufacturing NOC	4.07	4.55	4.30	4.88	0.00031	1303.736	0.042318	179924.54
4923 Photographic Supplies Manufacturing	3.18	2.99	2.81	3.05	0.00038	715.3106	0.040955	77608.340
5020 Ceiling Install, Suspended Accounstd Gr	11.76	9.80	9.05	9.15	0.00068	2024.281	0.005494	16473.015
5022 Masonry - NOC	16.08	16.75	15.14	13.69	0.00037	7698.588	0.050294	1035766.5
5037 Painting: Metal Struct - over 3 stories & c	49.39	50.67	50.75	56.84	0.00019	159.0455	0.056603	46304.108
5040 Iron or Steel: Erection - Frame Structures	32.43	34.00	32.37	33.51	0.00010	967.3301	0.017304	180276.92
5057 Iron or Steel: Erection NOC	24.17	29.45	31.38	35.14	0.00013	778.9707	0.056524	331136.63
5059 Iron or Steel: Erct Fr Strctrs not over 2 st	48.18	47.64	45.18	50.60	0.00037	767.7861	0.056588	116571.78
5069 Iron or Steel: Erect.-const dwlgs n over 2	53.51	52.73	53.96	56.62	0.00006	1.732686	0.024054	658.57398
5102 Door Erection	11.14	11.16	10.82	10.05	0.00008	394.0069	0.036895	173364.67
5146 Furniture Installation - NOC	9.39	10.15	9.46	9.31	0.00028	1775.697	0.007991	51420.116
5160 Elevator Erection or Repair	6.45	6.55	6.03	5.33	0.00029	1184.264	0.061619	255206.32
5183 Plumbing NOC & Drivers	7.38	7.17	6.14	6.87	0.00072	40423.94	0.056110	3130151.6
5188 Automatic Sprinkler Installation & Drivers	7.49	8.43	7.98	6.45	0.00166	10285.38	0.106029	658541.71
5190 Electrical Wiring - Within buildings & Driv	5.63	5.71	5.07	4.93	0.00019	9321.320	0.014	686875.15
5191 Office Machine Installation or Repair	1.68	1.80	1.58	1.51	0.00049	6109.867	0.022653	284094.49
5192 Vending Mach.s -Install, Svc or Repr & S	4.18	4.53	4.05	4.38	0.00046	3022.024	0.039145	258241.17
5213 Concrete Construction NOC	17.25	16.06	14.03	15.54	0.00070	30388.65	0.051065	2205566.2
5215 Concrete Work - Construction of Resider	12.53	12.39	11.46	9.72	0.00030	1623.112	0.082152	442772.07
5221 Concrete Work - Floors, Driveways - & D	13.23	14.33	13.17	11.36	0.00061	27225.92	0.052337	2353317.8
5222 Concrete Construction Bridges or Culver	18.61	22.57	21.65	22.00	0.00072	10254.85	0.008018	114914.97
5223 Swimming Pool Construction - All operat	10.15	10.93	11.14	10.75	0.00019	200.9038	0.017816	19309.412
5348 Tile Work - Inside	15.30	13.27	11.53	9.72	0.00001	28.00072	0.085178	318603.52
5402 Hothouse Erection - All operations	15.97	13.23	12.50	13.42	0.00110	180.8124	0.035493	5811.0949
5403 Carpentry NOC	22.13	23.94	21.65	19.35	0.00087	83901.95	0.056097	7052637.2
5437 Carpentry - Inst. Cabinet Work Inter. Trin	9.29	9.90	8.90	8.58	0.00038	5152.942	0.018306	265135.12
5443 Lathing & Drivers	9.22	9.84	10.07	11.27	0.00011	158.1998	0.056232	82226.918
5445 Wallboard Installation & Drivers	20.02	20.18	17.95	19.69	0.00044	15153.03	0.048227	1603687.9
5462 Glazier - Away from shop & Drivers	10.01	10.26	9.57	9.41	0.00010	742.1452	0.008429	64224.117
5474 Painting or Paper Hanging NOC & Driver	16.46	16.37	14.42	16.15	0.00059	21289.38	0.056591	2049685.9
5479 Insulation Work NOC & Drivers	19.10	19.34	17.31	16.92	0.00016	2428.853	0.011393	169216.09
5480 Plastering NOC & Drivers	17.65	16.37	14.71	14.03	0.00005	301.4141	0.023660	153934.65
5491 Paperhanging & Drivers	9.15	7.11	6.83	6.57	0.00107	190.0441	0.019402	3438.2111
5506 Street or Road Const: Paving or Repavin	8.81	8.35	7.20	7.44	0.00034	13341.56	0.016393	652772.94
5507 Street or Road Const: Subsurface work &	13.51	11.26	9.60	10.40	0.00125	12862.37	0.04	412414.34
5508 Street or Road Const: Rock Excavation I	15.38	14.19	13.87	11.98	0.00014	181.6461	0.073945	95387.203
5511 Logging Road Construction & Maintenan	14.06	14.86	13.59	11.00	0.00101	28984.90	0.105327	3013094.5
5538 Sheet Metal Work - NOC & Drivers	9.88	11.27	10.37	9.46	0.00094	41563.96	0.045890	2024220.2
5551 Roofing - All kinds & Drivers	40.38	39.22	34.25	30.32	0.00015	8043.812	0.060864	3288547.0
5606 Contractor - Executive Spvsr Const, Sup	3.13	3.26	2.91	2.33	0.00095	11085.10	0.110687	1284995.2
5610 Cleaner - Debris Removal	8.27	9.16	7.83	8.85	0.00029	307.4320	0.049757	53432.053
5645 Carpentry - Detached Dwellings	19.27	19.08	16.77	17.53	0.00030	16499.00	0.022157	1227092.2
5651 Carpentry - Dwellings - 3 stories or less	22.70	24.01	21.81	24.42	0.00045	4546.812	0.056456	570942.33
5703 Building Raising or Moving & Drivers	48.25	46.47	41.35	41.85	0.00017	39.46195	0.006009	1412.796
5951 Serum Manufacturing & Drivers	3.01	3.42	3.41	3.04	0.00092	236.7886	0.057364	14797.616
6003 Pile Driving & Drivers	35.57	34.23	31.44	26.28	0.00023	460.0609	0.089397	181225.61
6005 Jetty Construction - All Operations & Driv	15.52	16.30	15.37	13.23	0.00059	179.9301	0.074825	22922.823
6017 Dam or Lock Const: Concrete Work - All	22.58	21.82	17.84	18.31	0.00028	652.7779	0.044802	102781.68
6018 Dam or Lock Const: Earth Moving - All C	11.02	10.48	9.70	10.43	0.00032	641.1795	0.036264	72430.931
6045 Levee Construction - All Ops & Drvrs	13.47	12.45	13.30	13.03	0.00022	13.60355	0.010254	648.08772

National Council Development Method | Revised Loss Development Method

Class Description	1989 Base	1990	1991	1992	TEST 1 Sum(D^2)/ [Sm(y)]^2	Premium Wghted	TEST 2 |y4-y3|/ (y3+y4)	Premium Wghted	1989 Base	1990	1991	1992	TEST 1 Sum(D^2)/ [Sm(y)]^2	Premium Wghted	TEST 2 |y4-y3|/ (y3+y4)	Premium Wghted
6204 Drilling NOC & Drivers	15.62	17.69	17.44	16.68	0.00048	2401.408	0.022274	112282.60	15.62	16.95	16.02	15.90	0.00024	1155.582	0.003759	18124.478
6206 Oil or Gas Well: Cementing & Drivers	10.22	10.16	10.01	9.39	0.00005	0.040614	0.031958	23.989800	10.22	9.89	8.96	8.44	0.00004	0.030199	0.029885	21.153053
6213 Oil or Gas Well: Scty Tool Op NOC-Con	13.60	13.27	--	--	--	0	--	0	13.60	13.00	--	--	--	0	--	0
6214 Oil or Gas Well: Perf of Casings - All E &	12.27	13.27	12.97	--	--	0	--	0	12.27	12.78	11.50	--	--	0	--	0
6216 Oil Lease Work NOC - By contractor & D	18.88	18.20	17.82	16.41	0.00004	20.82734	0.041191	19680.677	18.88	17.72	16.24	14.66	0.00001	4.817568	0.051132	23124.895
6217 Excavation NOC & Drivers	13.92	11.70	10.10	10.43	0.00080	33857.53	0.016074	673273.58	13.92	13.42	11.79	10.97	0.00009	3872.792	0.036028	1638227.4
6229 Irrigation System Construction & Drivers	11.37	12.61	13.55	10.47	0.00233	3182.176	0.128226	175477.15	11.37	12.26	12.21	9.87	0.00129	1683.641	0.105978	138111.43
6233 Oil or Gas Pipeline Construction & Driver	15.29	17.15	17.90	17.33	0.00032	78.32052	0.016179	3924.9070	15.29	16.42	15.88	15.63	0.00017	37.88872	0.007933	1798.1134
6235 Oil or Gas Well: Drilling or Redrilling & D	40.15	36.19	34.58	30.48	0.00006	67.80968	0.063018	72641.796	40.15	37.14	33.44	28.53	0.00005	53.62236	0.079231	89948.484
6236 Oil or Gas Well: Installation of Casing & I	47.48	51.00	51.32	45.13	0.00063	268.5439	0.064178	27152.190	47.48	49.98	46.53	41.88	0.00042	168.8172	0.054982	22154.229
6237 Oil or Gas Well: Instrmnt logng or Survey	8.96	6.88	--	--	--	0	--	0	6.96	6.74	--	--	--	0	--	0
6251 Tunneling - Not Pneumatic - All Operatio	22.52	19.53	19.06	17.82	0.00021	34.96039	0.033622	5603.6653	22.52	18.88	17.01	15.78	0.00028	43.14355	0.037511	5876.3597
6252 Shaft Sinking - All Operations	19.54	17.11	17.52	16.59	0.00029	407.3135	0.027284	38718.186	19.54	17.55	16.86	15.47	0.00006	83.83769	0.042994	59899.011
6306 Sewer Construction - All Operations & Di	12.30	15.22	14.24	13.17	0.00156	25933.27	0.039036	649105.27	12.30	14.39	13.74	13.16	0.00076	12297.34	0.021561	349776.21
6319 Gas Main Construction & Drivers	9.81	9.71	9.32	10.62	0.00044	3537.354	0.065195	523448.70	9.81	9.94	9.06	9.62	0.00023	1833.764	0.029978	234412.29
6325 Conduit Construction - For cables or wire	20.23	15.72	13.02	14.62	0.00238	25858.98	0.057887	627707.73	20.23	16.26	13.54	14.16	0.00133	14572.36	0.022382	244999.95
6400 Fence Erection - Metal	19.87	19.22	19.09	17.13	0.00012	377.8311	0.054113	164448.23	19.87	19.21	17.80	16.43	0.00003	86.54483	0.040023	118398.39
6504 Food Sundries, Manufacturing, NOC	11.63	10.33	9.02	6.39	0.00038	8292.699	0.170668	3743144.9	11.63	10.31	8.94	6.63	0.00020	4453.424	0.148362	3266110.4
6811 Boat Bldg-Wood-NOC&Drivers-State Act	6.15	7.12	--	--	--	0	--	0	6.15	6.95	--	--	--	0	--	0
6834 Boat Building or Repair & Drivers	9.95	12.50	11.87	7.60	0.00662	62957.42	0.219311	2084719.9	9.95	12.03	11.11	8.13	0.00379	35455.60	0.154885	1447717.1
6836 Marina & Drivers	5.80	7.54	7.57	8.30	0.00064	1495.576	0.045998	107616.42	5.80	7.06	6.74	7.29	0.00059	1269.324	0.039201	84430.019
6854 Boat Bldg-Iron/Steel-NOC&Drv-State Act	5.83	6.82	--	--	--	0	--	0	5.83	6.66	--	--	--	0	--	0
6878 Diving - State Act coverage	22.75	26.02	24.39	--	--	0	--	0	22.75	25.21	22.63	--	--	0	--	0
6882 Ship Repair Conversion-All Ops&D-State	7.07	3.33	--	--	--	0	--	0	7.07	8.13	--	--	--	0	--	0
6884 Painting-ship hulls-State Act	12.44	14.60	--	--	--	0	--	0	12.44	14.25	--	--	--	0	--	0
7133 Railroad Operation NOC - All Empl & Dvr	9.52	8.34	5.58	4.48	0.00068	40.15975	0.109343	6501.8158	9.52	8.20	5.34	4.25	0.00075	43.75109	0.113660	6610.8018
7219 Trucking: NOC - All Employees & Drivers	15.86	16.86	14.19	12.51	0.00082	301936.6	0.062921	23279765.	15.86	16.91	14.47	12.57	0.00084	311033.2	0.070266	26167886.
7230 Trucking: Parcel Delivery - All Empl & Dv	9.97	11.40	10.83	7.00	0.00455	4572.188	0.214806	215713.09	9.97	11.48	10.66	7.10	0.00419	4204.068	0.200450	201347.80
7231 Trucking: Mail, Parcel or Pkg Delivery - /	8.21	8.38	7.02	7.70	0.00072	4960.858	0.046195	319904.23	8.21	8.40	7.02	6.98	0.00047	3154.903	0.002857	19343.323
7360 Freight Handling NOC	10.83	13.28	12.91	10.02	0.00322	9489.052	0.126035	370983.56	10.83	13.10	12.33	9.71	0.00286	8233.612	0.118874	341946.19
7370 Taxicab Co (Gar Emplys 8385) All Other	7.12	9.16	8.31	8.06	0.00180	13555.05	0.015271	114934.63	7.12	8.58	7.82	7.60	0.00112	8047.455	0.014267	102342.09
7380 Chauffeurs NOC	6.53	6.66	5.69	6.17	0.00057	50563.07	0.040472	3609612.7	6.53	6.65	5.66	5.57	0.00038	31072.00	0.008014	696509.21
7382 Bus Co: All Other E & Drivers	8.75	9.66	3.58	8.22	0.00062	11782.67	0.021428	406289.92	8.75	9.16	7.99	7.91	0.00036	6633.081	0.005031	91604.084
7390 Beer Dealer - Wholesale & Drivers	6.75	8.35	7.24	8.00	0.00292	43008.81	0.093655	1377105.2	6.75	8.11	7.24	6.08	0.00224	32695.03	0.087087	1273292.9
7403 Aircraft or Helcptr Oper.: Sked or Suppl.	4.05	4.97	4.32	2.82	0.00571	55018.84	0.210084	2025793.1	4.05	4.79	4.22	3.09	0.00346	33339.81	0.154582	1489684.4
7405 Aircraft or Helcptr Oper.: Sked or Suppl.	0.90	1.06	1.01	0.78	0.00271	4841.808	0.128491	229806.38	0.90	1.02	0.98	0.77	0.00181	3155.588	0.109826	191186.13
7409 Aircraft or Helcptr Oper.: Aerial Applctn.	25.50	22.33	16.46	12.34	0.00021	340.2538	0.143055	235104.63	25.50	23.11	16.95	12.94	0.00039	657.9913	0.134158	225863.34
7418 Aircraft or Helcptr Oper.: Patrol, Photo:	16.61	14.31	9.57	7.63	0.00061	165.2453	0.112790	30710.791	16.81	14.31	9.31	7.66	0.00084	228.7159	0.097230	26347.477
7420 Aircraft or Helcptr Oper.: Public Exhbtn:	22.17	22.31	15.40	11.97	0.00169	568.8430	0.125319	42153.885	22.17	21.72	14.77	11.76	0.00143	471.2204	0.113456	37403.891
7421 Aircraft or Helcptr Oper.: Tsptn of Psnl:	5.25	5.65	4.71	5.00	0.00078	1032.409	0.029866	39299.810	5.25	5.44	4.50	4.78	0.00070	889.2721	0.030172	38489.966
7422 Aircraft or Helcptr Oper.: Sls or Svc Agy	10.46	13.90	15.56	13.51	0.00270	7227.066	0.070519	188431.06	10.46	13.57	13.90	11.12	0.00381	8848.357	0.111111	272555.53
7423 Aircraft or Helcptr Oper.: All Oth Empl &	3.74	3.86	3.40	3.16	0.00032	2000.324	0.036585	227849.71	3.74	3.94	3.42	3.19	0.00048	2992.580	0.034795	218693.76
7431 Aircraft or Helcptr Oper.: Air Carrier Cml	3.97	4.01	4.00	3.38	0.00053	429.2611	0.084010	68295.332	3.97	3.97	3.84	3.27	0.00038	302.5365	0.080168	63856.664
7502 Gas Co: Natural - Local Distribution & Dr	3.66	4.22	3.85	2.94	0.00255	3786.485	0.134020	199352.78	3.66	4.10	3.86	2.91	0.00180	2619.949	0.114155	165868.01
7515 Oil or Gas Pipeline Operation & Drivers	2.76	3.02	2.68	2.27	0.00110	782.8858	0.082828	59125.712	2.76	2.88	2.51	2.18	0.00061	416.4861	0.070362	48354.781
7520 Waterworks Operation & Drivers	5.13	5.48	4.68	3.40	0.00197	16245.87	0.158415	1309218.9	5.13	5.63	4.85	3.47	0.00249	21013.53	0.165865	1399389.1
7538 Electric Power Line Construction & Drive	25.00	24.11	22.06	16.82	0.00064	4615.819	0.134773	975632.79	25.00	24.52	21.78	18.61	0.00024	1794.636	0.078028	577047.05
7539 Electric Power Co NOC - All Employees	2.60	3.28	2.78	2.69	0.00213	23558.11	0.016453	181819.04	2.60	3.06	2.65	2.54	0.00127	13430.53	0.021194	223894.48
7580 Sewage Disposal Plant Operation & Driv	4.18	4.09	3.73	3.06	0.00037	1724.677	0.098674	458515.42	4.16	4.10	3.64	3.07	0.00028	1310.560	0.084947	392896.27
7590 Garbage Works	6.46	7.19	6.74	7.39	0.00034	354.4190	0.046001	48111.387	6.46	7.03	6.40	6.93	0.00039	391.4741	0.039759	40146.615
7600 Telephone or Telegraph Co: All Other Er	2.86	3.41	3.04	2.41	0.00269	40897.59	0.115596	1755642.7	2.86	3.06	2.67	2.41	0.00065	9285.563	0.051181	729569.74
7601 Telephone Line Construction & Drivers	9.88	12.29	11.74	13.38	0.00066	1345.149	0.065286	133208.75	9.88	11.99	10.81	12.10	0.00091	1780.309	0.056307	108789.74
7605 Burglar Alarm Installation or Repair & Dr	3.10	3.73	3.55	3.89	0.00054	1815.198	0.045698	154306.05	3.10	3.58	3.33	3.60	0.00048	1549.944	0.038961	125470.51
7610 Radio Brdcstng Stn-All Employees & Cle	0.71	0.85	0.77	0.63	0.00238	13372.12	0.1	561117.00	0.71	0.83	0.75	0.63	0.00184	10179.46	0.086956	481334.21
7704 Firefighters & Drivers	5.33	4.59	3.60	2.65	0.00006	1028.124	0.152	2682931.0	5.33	4.70	3.96	2.83	0.00023	4310.511	0.166421	3055558.2
7720 Police Officers & Drivers	6.67	6.07	5.18	4.41	0.00003	1409.829	0.080291	3611247.4	6.67	6.41	5.41	4.69	0.00020	9116.775	0.071287	3328288.4
7855 RR Const: Laying Tracks or Mntnc-by Co	22.14	26.75	26.75	24.57	0.00118	2326.611	0.042478	83955.988	22.14	24.41	22.68	22.74	0.00034	624.2153	0.001762	3195.6311
8001 STORE: Florist & Drivers	3.33	4.42	4.40	4.82	0.00081	5184.098	0.045553	292710.10	3.33	4.31	4.16	4.50	0.00086	5427.001	0.039260	242318.25
8002 Automobile Rental Company	2.81	3.11	3.16	3.18	0.00015	254.4042	0.003154	5478.0839	2.81	3.04	2.98	2.89	0.00021	350.2420	0.015332	25452.444
8006 STORE: Grocery, Retail	5.25	6.53	5.86	5.50	0.00176	58169.51	0.014336	456987.78	5.25	6.33	5.63	5.38	0.00135	42492.91	0.022706	712697.61
8008 STORE: Clothing, Wrng Appar. or Dry G	1.71	2.25	2.06	2.25	0.00135	35154.70	0.044083	1149207.9	1.71	1.99	1.77	1.92	0.00077	17945.05	0.040650	946947.92
8010 STORE: Hardware	2.44	2.87	2.48	2.71	0.00103	23579.79	0.044315	1013432.8	2.44	2.77	2.42	2.45	0.00077	17000.00	0.006160	135237.77
8013 STORE: Jewelry	0.90	1.15	1.15	0.74	0.00710	22901.76	0.216931	699971.89	0.90	1.14	1.11	0.77	0.00548	17586.73	0.180851	580589.88
8017 STORE: Retail, NOC	3.01	3.13	2.69	2.32	0.00064	48397.22	0.073852	5563531.0	3.01	3.18	2.75	2.36	0.00077	59120.98	0.076320	5826849.5
8018 STORE: Wholesale NOC	6.19	6.18	5.27	5.60	0.00048	30574.95	0.030358	1944038.4	6.19	6.33	5.33	5.12	0.00041	26014.93	0.020096	1271886.4
8021 STORE: Meat, Fish or Poultry Dealer - W	16.62	17.65	15.24	12.60	0.00101	54479.42	0.094827	5133181.4	16.62	17.69	15.45	12.87	0.00098	52485.78	0.091101	4972780.1
8031 STORE: Meat, Fish or Poultry - Retail	6.18	7.90	7.46	6.92	0.00184	6298.706	0.037552	128617.49	6.18	7.51	6.86	6.46	0.00137	4441.848	0.030030	97813.669
8032 STORE: Clothing, Wrng Appar. or Drygo	4.21	4.76	4.39	3.17	0.00287	10034.32	0.161375	564741.30	4.21	4.70	4.29	3.24	0.00221	7678.689	0.139442	485327.38

Class Description	National Council Development Method 1989 Base	1990	1991	1992	TEST 1 Sum(D^2)/[Sm(y)]^2	Premium Wghted	TEST 2 |y4-y3|/(y3+y4)	Premium Wghted	Revised Loss Development Method 1989 Base	1990	1991	1992	TEST 1 Sum(D^2)/[Sm(y)]^2	Premium Wghted	TEST 2 |y4-y3|/(y3+y4)	Premium Wghted
8033 STORE: Meat, Grocery & Provision Com	2.97	3.94	3.89	3.99	0.00118	73967.20	0.012690	796866.32	2.97	3.85	3.60	3.63	0.00142	84819.75	0.004149	247515.72
8044 STORE: Furniture & Drivers	4.75	6.23	5.54	4.06	0.00540	140754.9	0.154166	4021061.8	4.75	5.82	5.26	4.46	0.00236	60640.95	0.082304	2116462.8
8046 STORE: Automobile Accessory-Retail-N	3.09	4.05	3.93	2.90	0.00508	33977.85	0.150805	1008815.5	3.09	3.82	3.58	2.97	0.00258	16607.01	0.093129	600131.38
8047 STORE: Drug - Wholesale	3.16	3.52	3.50	3.08	0.00087	702.7347	0.063829	51846.035	3.16	3.32	3.21	2.88	0.00038	293.1223	0.054187	41723.483
8050 STORE: Five & Ten Cent	2.50	2.63	2.53	2.77	0.00018	292.7296	0.045283	74832.276	2.50	2.48	2.34	2.53	0.00022	340.2071	0.039014	60887.780
8058 Building Material Dealer: Store Employe	3.30	3.49	3.16	2.71	0.00069	3789.418	0.076660	419514.65	3.30	3.49	3.14	2.70	0.00069	3742.436	0.075342	411322.24
8061 Store-Grocery-Convenience-Retail	--	--	--	--	--	0	--	0	--	--	--	--	--	0	--	0
8102 Seed Merchant	6.86	8.29	7.45	6.09	0.00255	20298.88	0.100443	799405.57	6.86	8.13	7.21	6.13	0.00198	15536.81	0.080959	636254.52
8103 Wool Merchant	5.83	7.14	7.99	6.94	0.00192	435.7957	0.070328	15946.526	5.83	7.16	8.10	6.97	0.00210	479.0551	0.074983	17099.572
8105 STORE: Hide Dealer	12.56	11.57	11.20	11.16	0.00011	20.03228	0.001788	337.00093	12.56	11.23	10.48	10.43	0.00021	37.19619	0.002391	433.11862
8106 Iron Merchant & Drivers	9.79	10.78	9.48	8.58	0.00084	14820.87	0.048780	860585.10	9.79	10.15	8.80	8.26	0.00038	6421.939	0.031652	535136.21
8107 Machinery Dealer NOC & Drivers	4.72	6.27	5.82	5.31	0.00255	56251.84	0.045822	1009698.5	4.72	5.74	5.31	4.98	0.00134	27640.59	0.032069	662900.37
8111 Plumbers' Supplies Dealer & Drivers	4.62	4.91	4.59	5.04	0.00028	1182.497	0.046728	199617.37	4.62	4.77	4.37	4.68	0.00025	1037.150	0.034254	140828.52
8116 Farm Machinery Dealer - All Operations .	5.07	5.09	4.59	3.94	0.00034	1676.627	0.076201	419525.85	5.07	5.10	4.51	3.90	0.00035	1908.823	0.072532	396976.37
8203 Ice Dealer & Drivers	10.98	13.22	12.16	9.35	0.00311	1325.498	0.130636	55697.820	10.98	13.02	11.68	9.01	0.00288	1201.386	0.129047	53792.557
8204 Building Material Yard & Local Mgrs, Dri	16.14	16.59	14.72	13.63	0.00029	18.29718	0.038447	2445.0858	16.14	18.02	13.78	13.63	0.00025	15.49310	0.005472	339.41534
8209 Vegetable Packing & Drivers	7.93	7.69	6.50	5.88	0.00019	2429.228	0.050080	629069.42	7.93	7.96	6.67	5.78	0.00045	5691.054	0.071485	908845.37
8215 Hay, Grain or Feed Dealer & Local Mgrs.	10.05	9.41	7.98	7.63	0.00018	749.4415	0.022421	105513.12	10.05	9.59	7.93	7.54	0.00025	1170.521	0.025210	118771.08
8227 Construction or Erection Permanent Yar	7.49	7.93	7.05	5.34	0.00151	33091.75	0.138014	3025111.0	7.49	7.57	6.69	6.03	0.00027	5859.295	0.051886	1136068.6
8232 Lumberyard: All other Employees	7.76	9.12	6.90	5.81	0.00082	69914.57	0.085759	7294870.8	7.76	8.38	7.16	6.15	0.00101	88332.90	0.075882	6648920.0
8233 Coal Merchant & Local Mgrs, Drivers	12.24	14.08	13.01	12.15	0.00087	10.73957	0.034181	421.29546	12.24	13.58	12.11	11.51	0.00066	7.835484	0.025402	300.68394
8235 Sash Dealer & Drivers	8.37	9.95	9.37	7.08	0.00311	5500.632	0.139209	248579.41	8.37	9.66	8.79	7.03	0.00210	3621.331	0.111251	191843.64
8263 Junk Dealer & Drivers	16.80	17.11	15.96	13.68	0.00042	74.62939	0.076923	13780.035	16.80	17.18	15.67	13.58	0.00040	71.82217	0.071452	12735.667
8264 Bottle Dealer - Used & Drivers	17.79	19.54	16.80	13.28	0.00168	4844.098	0.117021	337536.29	17.79	19.15	16.33	13.46	0.00120	3415.207	0.096341	275083.05
8265 Scrap Dealers, (Iron) & Drivers	16.00	14.91	13.36	12.31	0.00001	122.3947	0.040903	344814.92	16.00	15.45	13.53	11.81	0.00014	1221.179	0.067876	574319.27
8279 Stable & Drivers	15.41	20.47	22.14	14.17	0.00852	36743.89	0.219498	946682.98	15.41	19.98	20.23	14.85	0.00500	21056.21	0.153363	645687.56
8288 Livestock Dealer & Salespersons, Driver	9.05	10.23	9.70	8.17	0.00135	2185.593	0.085818	138756.67	9.05	10.06	9.29	7.83	0.00121	1908.041	0.085280	134786.25
8291 Storage Warehouse - Cold	6.47	8.41	8.32	6.87	0.00320	15747.36	0.095457	469424.54	6.47	7.71	7.36	6.81	0.00112	5178.052	0.038814	179956.70
8292 Storage Warehouse NOC	9.30	9.85	8.60	8.97	0.00044	4990.057	0.021058	238970.77	9.30	9.89	8.64	8.55	0.00043	4804.617	0.005235	58862.876
8293 Storage Warehouse - Furniture & Dealer	16.55	17.81	15.68	12.50	0.00133	18334.67	0.112845	1385884.7	16.55	17.67	15.28	12.73	0.00101	12396.33	0.091038	1112525.5
8304 Grain Elevator Operation & Local Mgrs, I	10.44	13.02	12.39	9.84	0.00319	13170.40	0.114709	473379.61	10.44	12.15	11.23	9.34	0.00181	7068.668	0.091881	358176.12
8350 Gasoline or Oil Dealer & Drivers	8.36	7.20	6.07	4.53	0.00007	1198.786	0.145283	2603449.2	8.36	7.79	6.43	4.97	0.00029	5515.630	0.128070	2416941.4
8385 Bus Company: Garage Employees	5.91	6.36	5.68	4.46	0.00142	10436.89	0.120315	882444.42	5.91	6.33	5.58	4.59	0.00108	7890.283	0.097345	713989.61
8387 Automobile Service Station & Drivers	7.05	8.08	6.93	5.65	0.00201	168633.4	0.101748	8532111.3	7.05	7.92	6.84	5.77	0.00149	124313.8	0.084853	7081962.3
8391 Automobile Repair Shop & Parts Dpt Em	6.99	7.39	6.29	5.44	0.00080	106353.8	0.072463	9682477.9	6.99	7.58	6.55	5.64	0.00100	136238.4	0.074651	10201981.
8392 Automobile Parking Station & Drivers	4.46	5.00	4.54	4.27	0.00070	1656.754	0.030646	72172.128	4.46	4.63	4.29	3.98	0.00059	1325.525	0.037484	84844.522
8393 Automobile Body Repair	6.86	8.36	7.77	4.98	0.00588	158241.8	0.218823	5886927.8	6.86	7.55	6.87	5.31	0.00181	48215.64	0.128078	3275652.9
8411 Volunteer Municipal Personnel & Drivers	1.15	1.07	0.86	0.85	0.00043	3356.060	0.005847	45445.721	1.15	1.11	0.88	0.79	0.00039	3054.166	0.053892	418808.17
8500 Scrap Dealers (Metal) & Drivers	19.29	18.33	15.63	11.07	0.00078	1837.111	0.170786	400534.33	19.29	18.30	15.32	11.15	0.00062	1457.169	0.157536	367987.25
8601 Engineer - Consulting	1.76	1.85	1.60	1.50	0.00047	7162.556	0.032258	494866.11	1.76	1.88	1.62	1.51	0.00059	9205.508	0.035143	543736.32
8608 Geophysical Exploration - All employees	8.91	10.13	8.79	7.34	0.00168	1289.636	0.089894	68863.960	8.91	10.06	8.58	7.44	0.00143	1090.670	0.071161	54234.079
8710 Warehousing, Field Bonded - All Employ	4.55	4.31	3.11	2.20	0.00095	71.24423	0.171374	12879.036	4.55	4.17	2.94	2.08	0.00070	50.95885	0.171314	12483.838
8719 Stevedoring (Talliers or Checking Clerks	3.43	3.31	3.70	3.96	0.00027	8.687191	0.033942	1080.6523	3.43	3.23	3.58	3.81	0.00035	10.80611	0.031123	988.80445
8720 Inspection for Insurance Purposes NOC	1.95	2.59	2.72	1.95	0.00595	13970.18	0.164882	387188.23	1.95	2.53	2.47	1.97	0.00369	8392.278	0.112612	256118.27
8742 Salespersons - Outside	0.87	0.95	0.82	0.74	0.00086	87093.77	0.051282	5217314.7	0.87	0.95	0.82	0.74	0.00086	87093.77	0.051282	5217314.7
8745 News Agent - Not retail dealer - & Sales	5.02	5.32	4.57	4.03	0.00071	2054.305	0.062790	180906.28	5.02	5.23	4.44	3.97	0.00058	1652.583	0.055885	158632.39
8748 Automobile Salespersons	1.23	1.42	1.24	1.11	0.00138	24245.07	0.055319	974155.10	1.23	1.42	1.24	1.07	0.00161	28132.95	0.073593	1285586.2
8755 Labor Union - All Employees	1.34	1.22	0.97	0.90	0.00028	640.0304	0.037433	86589.390	1.34	1.27	1.01	0.87	0.00030	701.2542	0.074468	174590.87
8800 Mailing Company & Clerical	3.52	4.41	4.25	3.49	0.00281	6527.708	0.098191	227850.75	3.52	4.29	3.99	3.39	0.00216	4863.382	0.081300	182878.01
8803 Auditor - Traveling	0.42	0.43	0.37	0.27	0.00138	3465.582	0.15625	391588.96	0.42	0.44	0.37	0.29	0.00122	3120.539	0.121212	309894.44
8810 Clerical Office Employees NOC	0.53	0.57	0.50	0.46	0.00061	104860.9	0.041666	7186488.2	0.53	0.57	0.49	0.44	0.00076	129919.7	0.053763	9137845.9
8820 Attorney- All Employees, Clerical, Messn	0.48	0.53	0.47	0.49	0.00052	5560.621	0.020833	221471.83	0.48	0.53	0.47	0.45	0.00063	6570.585	0.021739	226408.64
8824 Retrmnt, Nrsing, Convlscnt Cntrs: Health	--	--	--	--	--	0	--	0	--	--	--	--	--	0	--	0
8825 Retrmnt, Nrsing, Convlscnt Cntrs: Food S	--	--	--	--	--	0	--	0	--	--	--	--	--	0	--	0
8826 Retrmnt, Nrsing, Convlscnt Cntrs: All Oth	--	--	--	--	--	0	--	0	--	--	--	--	--	0	--	0
8829 Convalescent Home - All Employees	12.39	14.81	12.71	11.68	0.00171	307885.7	0.042230	7625682.9	12.39	13.88	12.18	11.20	0.00093	161411.2	0.041916	7284313.2
8831 Hospital - Veterinary & Drivers	2.90	3.45	3.35	2.32	0.00435	23261.91	0.181657	972153.92	2.90	3.39	3.23	2.46	0.00277	14753.26	0.135325	721791.35
8832 Physician and Clerical	0.77	1.02	0.97	0.83	0.00757	244424.1	0.2125	6858568.1	0.77	0.99	0.90	0.70	0.00408	130628.0	0.125	3998748.7
8833 Hospital: Professional Employees	1.92	1.66	1.38	1.16	0.00002	1117.146	0.088614	5033491.5	1.92	1.82	1.50	1.24	0.00025	15122.85	0.094890	5838842.2
8835 Nursing - Home Health, Public and Travl	5.50	7.30	6.81	7.47	0.00125	15553.54	0.046218	575137.93	5.50	7.13	6.55	7.09	0.00123	14894.19	0.039589	477911.06
8842 Mental Health Grp Care Homes - All Em	5.70	7.14	6.71	7.13	0.00089	9428.691	0.030048	323282.29	5.70	6.90	6.25	6.29	0.00104	10466.21	0.003189	32019.210
8868 COLLEGE: Professional Employees & C	0.49	0.59	0.52	0.44	0.00225	180459.7	0.083333	5932561.8	0.49	0.57	0.50	0.44	0.00154	107833.0	0.063829	4454990.0
8901 Telephone or Telegraph Co: Office Empl	0.81	0.74	0.61	0.46	0.00024	1048.062	0.140186	600326.85	0.81	0.76	0.61	0.48	0.00033	1425.559	0.119266	518534.29
8989 Domestic Workers - Residences	6.89	6.93	5.98	5.33	0.00032	831.2093	0.057471	149176.54	6.89	6.79	5.69	4.93	0.00034	844.5107	0.071563	179619.15
9014 Buildings - Operation by Contractors	11.84	13.31	11.47	9.41	0.00170	63653.82	0.098659	3703656.2	11.84	13.38	11.61	9.77	0.00156	59171.10	0.086061	3270762.4
9015 Buildings NOC - Operation by Owner	6.89	8.12	6.95	7.62	0.00112	85404.89	0.045984	2692784.0	6.89	7.60	6.65	6.83	0.00059	32517.33	0.013353	739372.30
9016 Amusement Park Operation & Drivers	4.62	5.23	4.96	4.82	0.00050	1919.768	0.014314	55260.538	4.62	5.11	4.74	4.60	0.00044	1635.837	0.014989	56213.093
9019 Bridge Operation & Drivers	8.87	9.41	7.07	6.47	0.00014	10.95284	0.044313	3386.9040	3.87	8.11	6.64	6.04	0.00015	10.97151	0.047318	3480.4934

National Council Development Method

Class Description	1989 Base	1990	1991	1992	TEST 1 Sum(D^2)/ [Sm(y)]^2	Premium Wghted	TEST 2 |y4-y3|/ (y3+y4)	Premium Wghted
9035 Adult Community Care Facilities	9.75	9.68	8.32	8.20	0.00025	4843.125	0.007263	141786.98
9040 Hospital : All Other Employees	5.93	7.10	6.11	5.96	0.00141	52099.85	0.012427	460606.76
9052 Hotel: All other Employees, Sales & Driv	9.75	11.09	9.59	8.46	0.00135	97051.78	0.062603	4503591.0
9060 Club: Country & Clerical	3.38	4.17	3.77	3.53	0.00162	14438.27	0.032876	293751.24
9061 Club NOC & Clerical	5.17	6.15	5.55	5.67	0.00088	8880.546	0.010695	107374.92
9063 YMCA,YWCA,YMHA OR YWHA, Institut	2.48	3.29	3.18	2.41	0.00486	40697.89	0.137745	1152942.5
9079 Restaurant NOC	5.09	5.99	5.19	5.25	0.00109	303453.8	0.005747	1602243.5
9089 Billiard Hall	4.61	4.48	3.75	2.47	0.00141	39.37480	0.205787	5742.3349
9093 Bowling Lane	2.84	3.44	3.27	3.06	0.00120	2495.069	0.033175	69031.289
9101 College: All other Employees	6.62	8.30	7.11	5.84	0.00330	268454.0	0.098069	7972954.9
9102 Park NOC - All employees & Drivers	6.13	6.89	6.01	5.79	0.00082	12008.35	0.018844	273325.77
9154 Theater NOC: All other Employees	3.47	4.37	4.16	2.98	0.00482	19058.78	0.165266	652894.09
9156 Theater NOC: Players, Entertainers or M	1.75	1.98	1.72	1.88	0.00079	1791.106	0.044444	100330.14
9178 Athletic Team or Park: Non-Contact Spo	12.87	16.60	15.85	10.22	0.00710	31505.48	0.215957	957888.46
9179 Athletic Team or Park: Contact Sports	21.54	20.97	19.18	19.90	0.00017	6.041903	0.018423	666.57610
9180 Amusement Device Op NOC - Not Trave	10.59	11.35	10.11	11.09	0.00048	3224.833	0.046226	307397.93
9182 Athletic Team or Park: Operation & Driv	4.44	5.31	5.94	5.01	0.00209	1125.043	0.084931	45838.196
9186 Carnival - Traveling - All Employees & D	31.74	34.98	30.02	19.22	0.00369	16350.05	0.219333	973015.68
9220 Cemetery Operations & Drivers	8.56	8.58	7.79	6.32	0.00057	1537.934	0.104181	281484.87
9305 Cannery: Maintenance&Security Employ	--	--	--	--	--	0	--	0
9308 Bottling NOC Route Supervisors&Drivers	--	--	--	--	--	0	--	0
9310 Log Hauling & Drivers	--	--	--	--	--	0	--	0
9311 Saw Mill: Maintenance&Security Employ	--	--	--	--	--	0	--	0
9315 Planing/Molding Mill: Maintenance&Secu	--	--	--	--	--	0	--	0
9328 Trucking NOC - Garage & Dock Employe	--	--	--	--	--	0	--	0
9343 Auto Sales/Repair: Parts Department Em	--	--	--	--	--	0	--	0
9345 Nursing/Convalescent Home: Cafeteria/K	--	--	--	--	--	0	--	0
9349 School: Cafeteria/Kitchen Employees	--	--	--	--	--	0	--	0
9366 Hospital: Cafeteria/Kitchen Employees	--	--	--	--	--	0	--	0
9402 Street Cleaning & Drivers	8.06	10.16	9.85	10.05	0.00091	6010.337	0.010050	66056.477
9403 Garbage Collection & Drivers	10.65	11.12	9.51	7.90	0.00085	24292.56	0.092475	2656938.1
9410 Municipal, Township, County or State Em	3.47	3.62	3.52	2.30	0.00304	19343.91	0.209621	1332802.3
9425 City: Over 100,000 Population - Compos	3.05	3.55	--	--	--	0	--	0
9451 County: Over 300,000 Population - Comp	2.99	3.47	--	--	--	0	--	0
9470 County: Over 150,000 Population - Comp	3.04	3.44	3.03	2.86	0.00089	7030.042	0.028862	227706.70
9497 State Agencies, Higher Education	0.94	1.13	0.98	1.07	0.00114	28723.49	0.043902	1107932.4
9498 State Agencies, Administrative	1.42	1.23	0.88	0.96	0.00177	16979.61	0.043478	417711.85
9499 State Agencies, All other	4.25	4.31	3.68	2.36	0.00223	799050.7	0.218543	78184182.
9501 Painting: Shop Only & Drivers	6.83	8.49	8.77	5.62	0.00879	31232.54	0.218902	1008780.6
9505 Painting: Automobile Bodies	4.77	5.32	5.52	5.31	0.00033	185.6326	0.019390	10895.829
9519 Household Appliances-Electrical-Installa	5.83	5.77	5.08	4.23	0.00038	5070.600	0.091299	1211541.2
9521 House Furnishings Installation NOC & U	8.37	8.96	8.66	7.43	0.00074	5458.158	0.076444	562708.32
9522 Upholstering	7.95	7.59	6.86	4.73	0.00114	8317.214	0.183779	1345324.8
9534 Mobile Crane/Hoisting Contractors:NOC-		16.67	--	--	--	0	--	0
9539 Awning-Tent-Canvas Goods Erctn, Rmvl	27.60	26.70	27.23	26.75	0.00012	134.9523	0.027152	30237.915
9545 Bill Posting & Drivers	22.77	22.27	23.32	20.26	0.00041	274.7670	0.070215	46762.012
9549 Advertising Co & Drivers	17.01	15.62	14.46	16.16	0.00068	698.4226	0.055519	56762.840
9552 Sign Manufacturing - Erection & Drivers	11.72	11.23	10.57	9.25	0.00010	681.0501	0.066599	460368.02
9586 Barber Shop	1.60	2.05	1.91	2.09	0.00102	7155.520	0.045	316018.00
9600 Taxidermist	5.41	4.63	3.81	2.76	0.00007	2.868557	0.159817	6310.1968
9620 Funeral Director & Drivers	2.43	3.22	3.23	2.37	0.00538	12709.24	0.153571	362717.90
					0.55453980	8767868.	36.19057	554808483

Revised Loss Development Method

Class Description	1989 Base	1990	1991	1992	TEST 1 Sum(D^2)/ [Sm(y)]^2	Premium Wghted	TEST 2 |y4-y3|/ (y3+y4)	Premium Wghted
9035 Adult Community Care Facilities	9.75	9.85	8.33	8.09	0.00035	8774.162	0.014816	285856.49
9040 Hospital : All Other Employees	5.93	6.98	6.16	5.83	0.00122	44748.50	0.027522	1011967.8
9052 Hotel: All other Employees, Sales & Driv	9.75	11.21	9.73	8.47	0.00153	110952.7	0.069230	5014892.2
9060 Club: Country & Clerical	3.38	3.93	3.56	3.41	0.00092	7899.354	0.021520	184906.24
9061 Club NOC & Clerical	5.17	6.02	5.35	5.27	0.00092	8961.922	0.007532	73178.195
9063 YMCA,YWCA,YMHA OR YWHA, Institut	2.48	2.96	2.67	2.31	0.00185	14205.69	0.072289	554997.82
9079 Restaurant NOC	5.09	5.82	5.10	4.88	0.00095	257827.5	0.022044	5965764.2
9089 Billiard Hall	4.61	5.10	4.54	2.82	0.00419	130.4367	0.233695	7270.7280
9093 Bowling Lane	2.84	3.37	3.11	2.92	0.00111	2245.725	0.031509	63640.418
9101 College: All other Employees	6.62	7.89	6.94	6.01	0.00194	155189.5	0.071814	5752572.5
9102 Park NOC - All employees & Drivers	6.13	6.85	5.99	5.34	0.00107	15293.27	0.057369	823771.09
9154 Theater NOC: All other Employees	3.47	4.33	4.08	3.16	0.00354	14059.00	0.127071	504015.89
9156 Theater NOC: Players, Entertainers or M	1.75	1.87	1.61	1.74	0.00061	1310.976	0.038805	83299.290
9178 Athletic Team or Park: Non-Contact Spo	12.87	15.70	14.75	10.98	0.00371	18084.35	0.146521	635393.98
9179 Athletic Team or Park: Contact Sports	21.54	20.32	18.05	19.01	0.00034	11.82218	0.025903	906.54184
9180 Amusement Device Op NOC - Not Trave	10.59	11.15	9.81	9.96	0.00036	2289.826	0.007587	48547.617
9182 Athletic Team or Park: Operation & Driv	4.44	5.16	5.86	4.83	0.00221	1166.597	0.096351	50749.394
9186 Carnival - Traveling - All Employees & D	31.74	34.54	29.80	20.77	0.00260	11627.68	0.178584	798232.54
9220 Cemetery Operations & Drivers	8.56	8.32	7.33	6.30	0.00019	513.5735	0.075588	199341.78
9305 Cannery: Maintenance&Security Employ	--	--	--	--	--	0	--	0
9308 Bottling NOC Route Supervisors&Drivers	--	--	--	--	--	0	--	0
9310 Log Hauling & Drivers	--	--	--	--	--	0	--	0
9311 Saw Mill: Maintenance&Security Employ	--	--	--	--	--	0	--	0
9315 Planing/Molding Mill: Maintenance&Secu	--	--	--	--	--	0	--	0
9328 Trucking NOC - Garage & Dock Employe	--	--	--	--	--	0	--	0
9343 Auto Sales/Repair: Parts Department Em	--	--	--	--	--	0	--	0
9345 Nursing/Convalescent Home: Cafeteria/K	--	--	--	--	--	0	--	0
9349 School: Cafeteria/Kitchen Employees	--	--	--	--	--	0	--	0
9366 Hospital: Cafeteria/Kitchen Employees	--	--	--	--	--	0	--	0
9402 Street Cleaning & Drivers	8.06	9.75	9.24	8.85	0.00105	6487.187	0.021558	133446.18
9403 Garbage Collection & Drivers	10.65	11.13	9.52	8.12	0.00074	21357.62	0.079385	2294224.8
9410 Municipal, Township, County or State Em	3.47	3.44	3.21	2.49	0.00077	4812.868	0.126315	784468.25
9425 City: Over 100,000 Population - Compos	3.05	3.47	--	--	--	0	--	0
9451 County: Over 300,000 Population - Comp	2.99	3.39	--	--	--	0	--	0
9470 County: Over 150,000 Population - Comp	3.04	3.48	3.03	2.71	0.00131	10218.04	0.055749	435914.22
9497 State Agencies, Higher Education	0.94	1.07	0.94	1.02	0.00074	18005.60	0.040816	992548.76
9498 State Agencies, Administrative	1.42	1.28	1.03	1.11	0.00093	9628.769	0.037383	387150.53
9499 State Agencies, All other	4.25	4.50	3.82	2.60	0.00238	884889.9	0.190031	70838168.
9501 Painting: Shop Only & Drivers	6.83	8.31	8.25	5.67	0.00494	22208.04	0.185344	833793.51
9505 Painting: Automobile Bodies	4.77	5.14	5.11	4.98	0.00017	89.98138	0.012884	6921.3365
9519 Household Appliances-Electrical-Installa	5.83	5.72	4.93	4.38	0.00021	2759.965	0.059078	782063.92
9521 House Furnishings Installation NOC & U	8.37	8.72	8.29	7.49	0.00031	2271.779	0.050697	387037.56
9522 Upholstering	7.95	7.49	6.80	4.89	0.00055	3990.054	0.148825	1081418.1
9534 Mobile Crane/Hoisting Contractors:NOC-		--	--	--	--	0	--	0
9539 Awning-Tent-Canvas Goods Erctn, Rmvl	27.60	25.84	23.96	24.22	0.00012	128.7473	0.005396	5537.7091
9545 Bill Posting & Drivers	22.77	21.23	20.60	18.25	0.00007	46.24813	0.060489	37861.412
9549 Advertising Co & Drivers	17.01	15.49	13.46	14.90	0.00081	792.1304	0.050775	49951.446
9552 Sign Manufacturing - Erection & Drivers	11.72	11.37	10.41	9.17	0.00011	753.9382	0.063329	427257.46
9586 Barber Shop	1.60	2.02	1.85	1.96	0.00112	7644.149	0.028871	196922.07
9600 Taxidermist	5.41	4.47	3.51	2.53	0.00000	0.059726	0.162251	6140.1853
9620 Funeral Director & Drivers	2.43	2.93	2.87	2.20	0.00239	5137.809	0.096509	207276.74
					0.38862317	6588715.	30.77605	475227783

A STOCHASTIC APPROACH TO TREND AND CREDIBILITY

Joseph A. Boor

ABSTRACT

This paper contains a new approach to analyzing loss statistics which uses stochastic processes. The author views loss statistics as samples from a specific type of stochastic process. The author believes that type of process is the most consistent with the realities of insurance statistics, and he explains why. Using that mathematical framework the author develops a formula for credibility when the complement of credibility is applied to trend. The paper also contains a formula for trending data that is more consistent with the stochastic approach (and hence the realities of insurance statistics) than the trend line.

A STOCHASTIC APPROACH TO TREND AND CREDIBILITY

Joseph A. Boor

Even though insurance and econometric statistics are driven by random forces, actuaries usually treat them as deterministic. For instance, actuaries tend to assume that insurance losses follow some perfect line or exponential curve over time. Since that implies the growth in losses is a function of time alone, we are implicitly assuming that it is time alone that causes loss cost levels to change.

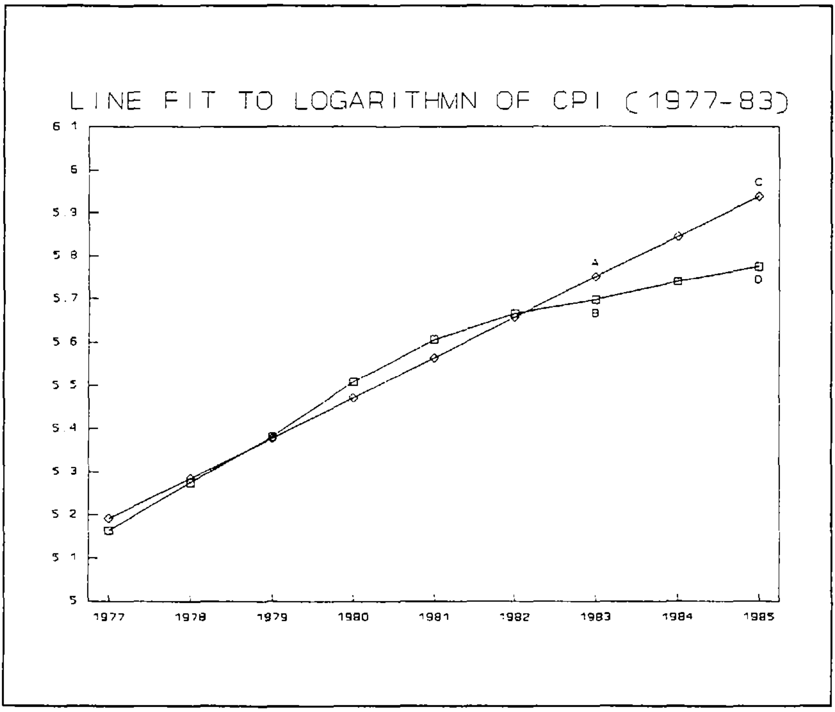
Of course, we all realize that assumption is false. But, we also recognize that we must reflect inflation and other environmental changes in ratemaking. So, in the absence of better models we use deterministic models. This paper contains a new model that reflects the randomness in econometric data.

Why the Trend Line Doesn't Work.

'I don't know where we've come from.
I don't know where we're going to.
And if all this should have a reason....
We would be the last to know.' -John Kay

Trend lines often produce unrealistic results when they are used on econometric data. Consider the United States Consumer Price Index when it began to come out of its inflationary spiral in the early 1980's. At that time a CPI prediction based on a trend line would err for two reasons: not only because the projected increase since the last actual observed point would be too high; but also because the fitted trend line value at the last observation time would be higher than the actual observed CPI at that observation time.

For example, the curve below represents that specific set of circumstances. The trend line represents a loglinear fit to the CPI during 1977-83. 'C' represents the predicted 1985 CPI log using the trend from 1977-83. 'D' is the actual recorded 1985 CPI log. The difference C-D is large because the recorded 1983-87 CPI log increase (.131) was below the trend (.374). And it is larger yet because the 1983 recorded CPI log 'B' was below the trend line value 'A'.



In this case trend line analysis works very poorly. It does so because its fundamental assumptions are contrary to the way economic systems work.

On one hand, the trend line mathematics assumes there is a straight line (or exponential curve in the case of loglinear fit) underlying the data. It assumes that the only reason the data do not lie on that straight line is that each point is imperfectly observed. In mathematic terms it assumes there is an observation error (with common variance E^2) at each point.

On the other hand, with econometric data the prediction error does not result from imperfect observation of the existing data as much as it results from year-to-year changes in the trend. There is really no logical reason for the CPI to follow a perfect exponential curve. The fact that it increased by 4% in 1984 does not mean it has to increase by exactly 4% in 1985 (although it does make it more likely). The trend line and regression have many reasonable applications in physics and chemistry; where laws of nature require that one variable be related to another by some precise formula. But at present there are no formulas that specify the behavior of econometric data. So, the author believes econometric data reflects random trend with minimal observation error rather than constant trend with significant observation errors. So, regression on econometric data may yield large errors. Some observers then conclude it is futile.

Unfortunately, the premiums and losses that are the actuary's stock in trade are econometric quantities. They inflate very much like the CPI. So actuaries need a realistic way to predict econometric quantities.

A Realistic Model

The argument above suggests we should assume that trend is random but there is no observation error. That follows from the fact that econometric data may be a series of numbers, but those numbers represent the aggregate actions of an enormous number of individuals.

For instance, the CPI is an aggregate of the buying and selling decisions of everyone in the United States. Those millions of people buy or sell independently, but their actions tend to be guided by two parameters: what others are doing (market prices) and what they see as the trend of the economy (historic inflation and other inputs). Assuming that broad econometric changes are a result of many small changes[1]; and that those changes tend to be proportional to the price level when the changes occur; results in the model below

$$y(t+\Delta)=y(t)\cdot\prod_{i=1}^{n(t,t+\Delta,\lambda)}(1+c_i(\lambda))$$

Where:

y is the econometric variable being observed (e.g. the CPI).

$(t, t+\Delta)$ is the time period over which y changes.

$n(t, t+\Delta, \lambda)$ is the number of small changes in y made between times t and $t+\Delta$. The actual number of changes, n, is random, but it is distributed around a mean of $\Delta\lambda$.

$c_i(\lambda)$ is the percentage effect on y of the 'i^{th}' change. The $c_i(\lambda)$ are random, but identically and independently distributed about some mean $C(\lambda)$.

Those bold presumptions about the pattern of y deserve further explanation. As stated earlier, econometric data represents a broad aggregate of the decisions of millions of people. If we say there are k annual exchanges between buyers and sellers; and prices agreed to by buyers and sellers change in an average of 100N% of the k exchanges; then we can expect λ=kN changes over the course of the year. So long as the k occur evenly throughout the year, $\Delta\lambda$-kN changes should occur in the interval $(t,t+\Delta)$.

Further, the changes occur with a constant frequency. And each change's occurrence is independent of the other changes. So, the number of changes $n(t, t+\Delta, \lambda)$ follows a Poisson distribution with mean $\Delta\lambda$ (see pages 21-22 in [2]).

Each time a price changes, the change only affects one of the k exchanges. So each change $c_i(\lambda)$ is very small. The size of each individual c_i is random; but the product of λ changes (the iterated product above) should average to the long-term trend of inflation 1+G. So, $E[1+c_i(\lambda)]$ should be roughly the λ'th root of 1+G. As one can see, when λ is very large and G remains fixed, $E[1+c_i(\lambda)]$ will be very close to one. So $E[c_i(\lambda)]$ will be very close to zero.

Importantly, the result of all those changes should be their product, not their sum. That is because I believe buyers and sellers consider the overall price level (y) rather than the last particular price for their exchange when the price change is determined.

Because there are so many exchanges each year, I believe λ is so large that the limit as $\lambda\to\infty$ is a close approximation to the real world. To that end, I shall define $n(t,t+\Delta,\lambda)$ to be distributed Poisson($\Delta\lambda$) (where $\lambda\to\infty$). The $c_i(\lambda)$'s should be distributed with a mean approximately equal to the λ'th root of 1+G. However, taking the Taylor's series expansion by Z of $(1+G)^z$, $\ln(1+G)/\lambda$ is a very close approximation to the λ'th root of 1+G (at least as long as $\lambda\to\infty$, so $1/\lambda\to 0$, the Taylor's series approximation works).

Of course, that suggests that the expected value of the $c_i(\lambda)$'s will be zero as $\lambda\to\infty$. But, bear in mind that as the $c_i(\lambda)$'s go to zero, $\lambda\to\infty$. So, the product averages to $(1+G)^\Delta$.

I have deliberately failed to prescribe the distribution of the $c_i(\lambda)$'s. While I have good reasons to believe the number of changes will follow a Poisson distribution, I have no such information on the distribution of change amounts. On the other hand, the central limit law suggests that the only important characteristics of their distribution are the mean and variance.

Now the mathematic framework is set, I will use the phrases 'very small' and 'very large' for the c_i and n throughout the rest of the paper. That should be taken as the case where $\lambda \to \infty$. Further, to simplify matters, I will set $\Delta=1$ and let $c_i=c_i(\lambda)$, $n=n(t,t+1,\lambda)$.

Since the year-to-year change is the limit of an interated product, it is easier to work with the natural logarithm of $y(t)$

$$x(t+1)=\ln(y(t+1))=x(t)+\sum_{i=1}^{n}\ln(1+c_i)$$

But $\ln(1+c_i)$ is very close to zero, and each c_i is very small. So, the Taylor series expansion $\ln(1+c_i)=c_i-c_i^2+2c_i^3-3c_i^4+\ldots$ will contain a small term c_i, and powers of c_i that are orders of magnitude smaller. That indicates $\ln(1+c_i)$ will be very close to c_i when λ is large and c_i is approximately the small quantity $\ln(1+G)/\lambda$. So, when $\lambda \to \infty$

$$x(t+1)=x(t)+\sum_{i=1}^{n}c_i .$$

The $y(t)$ curve was driven by a driving trend $(1+G)^t$. So, if $T=\ln(1+G)$ the expected value of Σc_i should be T. Since the sizes of the changes are independent of the number of changes (n), T must equal $\mu_n \cdot \mu_c$. Since $\mu_n=\lambda \to \infty$, μ_c must equal $(T/\lambda) \to 0$ as noted earlier. Further, the variance of each $x(t+1)-x(t)=\Sigma c_i$ is $\sigma_x^2=\lambda\sigma_c^2+\lambda\mu_c^2$ because of the formula for the collective variance of a count and amount distribution $(\lambda_n\sigma_s^2+\mu_s^2\sigma_n^2$ [3]$)$.

But there is another way to look at the variance. Since the variance generated by the combination of n and the c_i's should converge to the variance $\sigma_x^2=\mathrm{Var}(x(1)|x(0))$, we should require that $\lambda(\sigma_{c,\lambda}^2+\mu_{c,\lambda}^2)=\sigma_x^2$ for each λ. So, the limit as $\lambda \to \infty$ of the variance $\lambda(\sigma_c^2+\mu_c^2)$ must clearly be the fixed variance σ_x^2. So, even though the precise distribution of the c_i's, is undetermined; $\sigma_c^2+\mu_c^2$ must implicitly be a function of λ(the mean number of changes per unit time). Specifically,

$$\sigma_c^2+\mu_c^2=\sigma_x^2/\lambda .$$

So, the only other criterior for the $c_i(\lambda)$'s is that their variance be $(\sigma_x^2/\lambda) - (\mu_c^2/\lambda)$. As stated earlier, the central limit law will ultimately suggest that all other characteristics of the distribution of the $c_i(\lambda)$'s are irrelevant.

In fact $x(t)$ is a special form of stochastic process. Since $Var(c) \le \sigma_x^2/\lambda$ is finite, the central limit law indicates

$$\frac{1}{n}\sum_{i=1}^{n} c_i$$

is approximately a normal distribution $(\sim N(\mu_c = T/n,\ \sigma_c^2/n))$ when n is very large and fixed. But practically, since $n \sim \text{Poisson}(\lambda)$ and $\lambda \to \infty$, n has an extremely small relative standard deviation $(\sigma_n/\mu_n = \sqrt{\lambda}/\lambda = 1/\sqrt{\lambda} \to 0)$. So, n may be regarded as being nearly invariant when it is large; and for all practical purposes, the total change follows a normal distribution.

$$\Sigma\, c_i \sim N(nT/n,\ n^2\sigma_c^2/n) = N(T,\ n\sigma_c^2 = \lambda\sigma_c^2)\ .$$

These produce the seemingly contradictory results that $\sigma_x^2 = \lambda(\sigma_c^2 + \mu_c^2)$ and $\sigma_x^2 = \lambda\sigma_c^2$. But noting that $E(\Sigma c_i) = T$, μ_c^2 must equal T^2/λ^2. So, as $\lambda \to \infty$, $\mu_c^2 = T^2/\lambda^2 \to 0$ and $\sigma_c^2 = \sigma_x^2/\lambda \to 0$. That means μ_c^2 goes to zero like $1/\lambda^2$ whereas σ_c^2 only decreases like $1/\lambda$. So, the σ_c^2 term predominates and the other μ_c^2 term is functionally zero. And σ_x^2 is roughly equal to $\lambda\sigma_c^2$. In fact, at the limit as $\lambda \to \infty$, σ_x^2 is equal to $\lambda\sigma_c^2$.

Econometric Data as a Random Walk

As I stated earlier, $x(t)$ is actually a special form of stochastic process called a random walk. The expected increase between times t and s is $T(s-t)$. And T does not vary with s or t. Further, the changes over any two disjoint intervals ($x(a)-x(b)$ and $x(s)-x(t)$) are statistically independent with means proportional to the time difference. Mathematically, $E[x(a)-x(b)]=T(a-b)$ and $E[x(s)-x(t)]=T(s-t)$. In the language of stochastic processes, that means x has stationary, independent increments.

But what about the variance? Since the starting point $x(0)$ has not been defined, it does not yet make sense to talk about $\mathrm{Var}(x(t))$. But one can analyze $\mathrm{Var}(x(s)|x(t)=u)$. Consider the changes that affect x as it moves from $x(t)=u$ to $x(s)$. Since λ was the parameter used to denote the (very large) expected number of changes per unit time we expect very close to $n=\lambda(s-t)$ changes of size $c_1, \ldots, c_n$. The analysis of the previous section shows that the conditional distribution $x(s)|x(t)=u$ is a normal distribution with mean $E(n)\cdot T/\lambda=\lambda(s-t)\cdot T/\lambda=(s-t)T$ and variance $n\sigma_c^2=\lambda(s-t)\sigma_c^2$.

But that discrete model of economic change (each choice of λ and the distribution of the $c_i(\lambda)$'s) has an underlying assumption about the variance of the first year's trend. In fact, since the trend and variance are assumed to be independent of the starting value $x(0)$, one could define σ^2 by

$$\lim_{\lambda\to\infty} \lambda\sigma_{c,\lambda}^2=Var(x(1)|x(0)=u)=\sigma_u^2$$

Since the σ_u^2 are independent of u, they are all equal. So we may use the σ^2 they all equal as $\sigma_x{}^2$. And,

$$\mathrm{Var}(x(s)|x(t)=u)=n\sigma_c^2=\lambda(s-t)\sigma_c^2=(s-t)\sigma_x^2.$$

That result is entirely independent of the family of distributions $[c_i(\lambda)]$, as long as each $c_i(\lambda)$ distribution obeys the parameters imposed upon it. In other words, for any appropriate family of distributions $[C(\lambda)]$, the limiting variances will always be proportional solely to time. The above argument shows the resulting variances between times must be some constant variance parameter (σ_x^2) multiplied by the time difference.

That allows us to form some conclusions about this econometric 'random walk'.

1) The conditional distributions $x(s)|(x(t)=k)$ are normal distributions with mean and variance proportional solely to distance and starting point

$$[x(s)|(x(t)=k)] \sim N(k+(s-t)T, (s-t)\sigma^2) .$$

The variance is entirely independent of the starting point and is related solely to distance.

2) Since only the mean in 1) was influenced by the starting point $x(t)=k$. The distribution of $x(s)-x(t)$ is solely a function of the time difference $s-t$, i.e. it is $\sim N((s-t)T, (s-t)\sigma^2)$.

3) The process is piecewise continuous. Said another way, it produces piecewise continuous random walks. This is because $x(t+\Delta) \sim N(x(t)+\Delta T, \Delta\sigma^2)$ means that for any 'small' E

$$\lim_{\Delta \to 0} P(x(t+\Delta) \in (x(t)-E, x(t)+E)) = 1$$

4) The random functions $x(t)$ generated by the process, while continuous, will almost certainly be nondifferentiable (i.e. fractals). That is because the random nature of the process dictates that while $x(t+\Delta)-x(t)$ may show a slope of M; $x(t+\Delta/2)-x(t)$ being random, will show some different slope.

The above conclusions form the classic conditions for a random walk propelled by a constant force (T).

Insurance Data and Imperfect Observation

Of course the goal of most actuarial analyses is to find a better way to use historical insurance data to predict future losses. That requires recognizing both random change and observation error. There is an underlying propensity to loss $x(t)$ that results from a continuous random walk. But since insurance data only provides a random sample of the underlying propensity to loss, insurance data usually represents some $\hat{x}(t)$. The observed values $\hat{x}(t)$ differ from each $x(t)$ by some independent error variables $\epsilon(t) \sim N(0,E^2)$. So, insurance data is characterized by both random change and observation error.

With the prior analysis of econometric data switching between an exponentially trending stochastic process $y(t)$ and its linear trending cousin $x(t)=\ln(y(t))$; it is important to specify which one models insurance data. Insurance data is a reflection of a propensity to loss that is always positive and is subject to exponential inflationary pressures. So insurance data represents $y(t)$. Further, since the driving force behind the increase in $y(t)$ is severity (inflation) rather than frequency, the errors $\epsilon(t)=\hat{y}(t)-y(t)$ should be proportional to $y(t)$. Taking the log transform $x(t)=\ln(y(t)), \hat{x}=\ln(\hat{y}(t))$ yields an x subject to a linear random walk. And $\hat{x}$ is such that each $x(t)-\hat{x}(t)$ is from a set of independent, presumably identically distributed [4] $\epsilon(t) \sim N(0,E^2)$.

The insurance problem then reduces to:

> 'Given prior observations $\hat{x}(1)$, $\hat{x}(2)$ ..., $\hat{x}(n)$ of $\log(\hat{y})$, what is the best predictor of $\hat{y}(n+t)=\exp(\hat{x}(n+t))$?'

The Distribution of Future Losses - A Backward Approach

'Forward into the past'
-Firesign Theatre

Obviously, finding the best predictor of $\hat{x}(t+n)$ will involve finding the probability distribution of $x(n+t)$ given observed $\hat{x}(1), \hat{x}(2), \ldots, \hat{x}(n)$. That distribution will involve finding the reverse likelihood of $\hat{x}(1), \hat{x}(2), \ldots, \hat{x}(n)$ given $x(t+n)$. The process is complicated by the fact that each $\hat{x}(i)$ is derived from a compound process... first generating $x(i)$ using a random walk, and then generating $\hat{x}(i)$ by adding observation error $\epsilon(i) \sim N(0, E^2)$. Analyzing $x(t+n)|(\hat{x}(i), \hat{x}(j))$ will be especially difficult because the characteristics of a random walk dictate that all three observations will be highly interdependent. Unfortunately, the dependence is through the related variable $x(i)$, not direct.

That indirect dependence requires that parts of the analysis use x rather than $\hat{x}$. To do so requires creating a distribution of $x(i)|\hat{x}(i)$ rather than $\hat{x}(i)|x(i)$.

Determining that 'backward' distribution requires using both Bayes' Theorem and a uniform distribution on $(-\infty,+\infty)$ (a 'diffuse prior' distribution). Appendix I contains a 'reverse probability' theorem. That theorem shows that if the random variable A is a priori uniformly distributed on $(-\infty,+\infty)$ (i.e. each possible value is equally likely), then the density function $f(A=a|B=b)$ is proportional to B given A ($f(B=b|A=a)$). The constant of proportion is $1/\int f(B=b|A=x)\,dx$

That theorem involves the essence of this 'backward' analysis. To determine the likelihood of each potential x(n+t) ($f(x(n+t)|\hat{x}(1), \hat{x}(2), \ldots, \hat{x}(n))$) I will use $f(\hat{x}(1), \hat{x}(2), \ldots, \hat{x}(n)|x(n+t))$. Along the way, I will note that $f(x(i)|\hat{x}(i))=f(\hat{x}(i)|x(i))$ (per Appendix I).

In any event, to determine the likelihood of observing $\hat{x}(1)=\hat{x}_1,\ \hat{x}(2)=\hat{x}_2,\ \ldots$ $\ldots,\ \hat{x}(n)=\hat{x}_n$ given $x(n+t)=x_{n+t}$, it is first necessary to determine the likelihood of any $x(1)=x_1,\ x(2)=x_2$, etc. Then, going backward, while $f(x_1,\ x_2,\ \ldots,x_n|x_{n+t})$ may be complicated, $f(x_n|x_{n+t})$ is distributed $N(x_{n+1}-tT,\ t\sigma^2)$, $f(x_{n-1}|x_n)\sim N(x_n-T,\sigma^2)$, $f(x_{n-1}|x_n)\sim N(x_n-T,\sigma^2)$, $f(x_{n-2}|x_{n-1})\sim N(x_{n-1}-T,\sigma^2)$. Because the random walk has no memory those may be combined. In other words, as long as $s<u<v$, $f(x(s)=x_s|x(u)=x_u\wedge x(v)=x_v)=f(x(s)=x_s|x(u)=x_u)$, so we may multiply the adjacent conditional probabilities to obtain the overall density, $f(x_1,x_2,\ldots,x_n|x_{n+t})$.

Setting

$$f(x(1)=x_1,x(2)=x_2,\ldots x(n)=x_n|x(n+t)=x_{n+t}) = f(x_1,\ x_2,\ \ldots x_n|x_{n+t}),$$

and using the independence of the random change over time,

$$=f(x_n|x_{n+t})\cdot f(x_{n-1}|x_n)\cdot f(x_{n-2}|x_{n-1})\ \ldots\ f(x_1|x_2)$$

$$=(1/((\sqrt{2\pi}(\sqrt{t}\sigma)\ \exp\ (-(x_{n+t}-tT-x_n)^2/(2t\sigma^2))$$
$$\cdot(1/((\sqrt{2\pi}\sigma))\ \exp\ (-(x_n-T-x_{n-1})^2/(2\sigma^2))$$
$$\cdot(1/((\sqrt{2\pi}\sigma))\ \exp\ (-(x_{n-1}-T-x_{n-2})^2/(2\sigma^2))$$
$$\ldots$$
$$\cdot(1/((\sqrt{2\pi}\sigma))\ \exp\ (-(x_2-T-x_1)^2/(2\sigma^2))$$

$$=[1/((\sqrt{2\pi})^n\sigma^n\sqrt{t})]\ \exp\ [-(1/2)((x_{n+t}-tT-x_n)^2/(t\sigma^2)+(1/\sigma^2)\sum_{i=1}^{n-1}(x_i+T-x_{i+1})^2)].$$

Further, since the ϵ_i's are independent, identically distributed, and independent of the x_i's

$$f(\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n | x_1, \ldots, x_n)$$

$$= (1/((\sqrt{2\pi}E)) \exp(-(x_n - \hat{x}_n)^2/(2E^2))$$
$$\cdot (1/((\sqrt{2\pi}E)) \exp(-(x_{n-1} - \hat{x}_{n-1})^2/(2E^2))$$
$$\ldots$$
$$\cdot (1/((\sqrt{2\pi}E)) \exp(-(x_1 - \hat{x}_1)^2/(2E^2))$$

$$= [1/(\sqrt{2\pi}E)^n] \cdot \exp[-(1/2)((1/E^2) \cdot \sum_{i=1}^{n} (x_i - \hat{x}_i)^2)] .$$

So, since the ϵ's are independent of the x's

$$f(x_1, x_2, \ldots, x_n, \hat{x}_1, \ldots, \hat{x}_n | x_{n+t})$$

$$= [1/((2\pi)^n \sigma^n \sqrt{t} E^n)] \cdot \exp[-(1/2)((x_{n+t} - tT - x_n)^2/(t\sigma^2)$$
$$+ (1/\sigma^2) \sum_{i=1}^{n-1} (x_i + T - x_{i-1})^2 + (1/E^2) \sum_{i=1}^{n} (x_i - \hat{x}_i)^2)] .$$

Then, to eliminate the reliance on $x_1, \ldots, x_n$, all that is necessary is to integrate over all possible x_i's, i.e.

$$f(\hat{x}_1, \ldots, \hat{x}_n | x_{n+t})^{(5)}$$

$$= \int_{x_1} \int_{x_2} \ldots \int_{x_n} f(\hat{x}_1, \ldots, \hat{x}_n, | x_1, x_2, \ldots, x_n, x_{n+t}) \, dx_n, \ldots, dx_1$$

$$= [1/((2\pi)^n \sigma^n \sqrt{t} E^n) \cdot \int_{x_1} \int_{x_2} \ldots \int_{x_n} \exp[-(1/2)((x_{n+t} - tT - x_n)^2/(t\sigma^2)$$
$$+ (1/\sigma^2) \sum_{i=1}^{n-1} (x_i + T - x_{i+1})^2 + (1/E^2) \sum_{i=1}^{n} (x_i - \hat{x}_i)^2)] \; dx_n \ldots dx_2 \, dx_1$$

Ultimately, the best predictors of x_{n+t} will maximize that function. But since it is very unwieldy, a brief digression will illustrate what it means in concrete situations.

Two Extreme Examples

To gain some insight into the structure underlying the 'best' predictor of x_{n+t}, I will analyze two extreme examples. One is the case of 'total determinism' ($\sigma^2=0$). The other is 'perfect observation' ($E^2=0$)

'Total determinism' ($\sigma^2=0$) fulfills all the criteria needed for regression: 1) The underlying exposure $x(t)$ is a straight line; and 2) The only reason the observed data $x(t)$ do not fall on a straight line is the presence of independent, identically distributed, observation errors $\epsilon(t)$.

The fitted line $x(t)=\bar{x}+m(t-\bar{t})$ represents the regression estimate. Further, since the vectors $a_1=[1, 1, \ldots,]$ and $a_2=[-(n-1)/2, -(n-3)/2, \ldots, (n-1)/2]$ are independent, we can use them to produce the regression. Since a_1 is a 'pure constant', $\bar{x}=a_1\cdot[\hat{x}_i]/\|a_1\|^2$. And, since a_2 is pure slope, $m=a_2\cdot[\hat{x}_i]/\|a_2\|^2$. But, after some algebra, $a_2\cdot[\hat{x}_i]$ may be rewritten

$$a_2\cdot[\hat{x}_i]=[\sum_{i=1}^{n}[i-((n+1)/2)]\hat{x}_i]/\sum_{i=1}^{n}[i-((n+1)/2)].$$

Which, after some series algebra become

$$=K\sum_{i=1}^{n-1}\frac{(in-i^2)}{2}(\hat{x}_{i+1}-\hat{x}_i)$$

(where K is constant with respect to $[\hat{x}_i]$ and the $in-i^2$ are the weights used on the differences $(\hat{x}_{i+1}-\hat{x}_i)$).

So, regression is based on averaging over the observation period. The prediction keys off an average value of x - roughly its predicted value at the middle of the observation times. It adds a slope multiplied by the time elapsed since the middle of the observation times. The slope is computed by using a weighted average of year-to-year changes in $\hat{x}$. Just as the mean keys off the middle of the observation times, the weights applied to year-to-year changes place heavier weight near the middle of the observation period (Consider the shape of $in-i^2$. It is a quadratic with a maximum at $n/2$). In short, regression is oriented toward the middle of the observation period.

The 'perfect observation' case ($E^2=0$) produces estimates based largely on the latest point. Since the series has no memory, (i.e. $u<v<t$ implies $f(x(t)=x_t|x(v)=x_v) = f(x(t)=x_t|x(v)=\hat{x}_v \wedge \hat{x}(u)=x_u)$ the points prior to $\hat{x}_n=x_n$ are irrelevant except for estimating trend. In other words, the best estimate of $x(n+t)$ will be x_n+tT.

To estimate T, note that the perfect observation of the $\hat{x}_i$'s means there is no ϵ_i influencing either $\hat{x}_i-\hat{x}_{i-1}$ or $\hat{x}_{i+1}-\hat{x}_i$. Consequently, each $\hat{x}_{i+1}-\hat{x}_i$ is independent. So, each $\hat{x}_{i+1}-\hat{x}_i$ is an independent, identically distributed estimate of T. Thus, the best estimate of T is their average $T'=(1/(n-1))\sum_{i=1}^{n}\hat{x}_{i+1}-\hat{x}_i$.

Telescoping the differences produces $T'=(\hat{x}_n-\hat{x}_1)/(n-1)$. Combining the two results yields the optimum estimate for x_{n+t}

$$x_{n+t}=x_n+T(\hat{x}_n-\hat{x}_1)/(n-1) .$$

(To verify the above verbal argument, set $\hat{x}_i=x_i$ in the integral shown previously and maximize. The E^2 as a constant is superfluous.) So, the 'perfect observation' case dictates that the constant be the last observed point and the trend be an equal weighting of the observed differences.

Summarizing, the two extreme cases both key off a fixed point and a trend from the fixed point. In the case where $\sigma^2=0$ the fixed point is the mean of the observed points and the trend is a weighted average of the annual change (alternately, one could view the trended mean $\overline{x}+(n/2)T$ as the fixed point). In the perfect observation case ($E^2=0$) the trend is a straight average of the annual changes. From another perspective, when $E^2=0$ the fixed point applies 100% weight to the last observed point, and when $\sigma^2=0$ the fixed point equation applies equal weight to all the observed points.

In the typical case both E^2 and σ^2 will be non-zero. The key question is 'Where will the fixed point and trend lie between those extremes?'

The General Solution

'The only solution... isn't it amazing'

Jim Morrison

Appendix III shows the best estimator of x_{t+n} given observed $\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n$, a predetermined trend T, and a predetermined ratio E^2/σ^2. It uses a weighted average of the trended observed points for the fixed point and the trend T beyond the fixed point. The weights do not lend themselves to a closed-form formula readily, but they are easy to compute.

First, you compute the recursive values, F_i. To start, set

$$F_1=1,\ F_2=E^2+\sigma^2.$$

Then, you calculate each succeeding F_i using

$$F_{i+1}=(2E^2+\sigma^2)F_i-E^4F_{i-1}.$$

And then the best estimator of x_{n+c} is

$$x_{n+c}=cT+[\sum_{i=1}^{n}F_iE^{2(n-i)}(\hat{x}_i+(n-i)T)]/[\sum_{i=1}^{n}F_iE^{2(n-i)}].$$

(i.e. the weights for the fixed point are $F_iE^{2(n-i)}$).

Unfortunately, that estimator depends on first choosing the average trend T and the variance relationship E^2/σ^2. Appendix IV contains an estimating formula for the trend, T. The author has not yet determined the best estimator for E^2 and σ^2, but the estimating process used in appendices III and IV could be extended to produce an estimate for them as well.

In any event, the formula provides a means of assigning weights for each of the last five available years of fire experience, or each of the last three years of workers compensation class experience, etc. That alone makes it useful.

Credibility Against Straight Trend - Exponential Smoothing and Ratemaking

A useful by-product of the previous formula is a credibility formula to use when the complement of credibility is applied to straight trend.

Specifically, when the ratemaking formula is

$$ZL+(1-Z)(R+T)=R'.$$

Where L represents the rate based on raw experience, R is the existing rate, T is trend, and R' is the result of credibility. Then, the best credibility (Z) is

$$Z=[\sigma^2+\sigma\sqrt{4E^2+\sigma^2}]/[2E^2+\sigma^2+\sigma\sqrt{4E^2+\sigma^2}]$$

(where E^2 and σ^2 are as defined previously).

To prove this, first note that

$$R(i+1)=ZL(i+1)+(1-Z)(T+R(i)).$$

So,

$$R(i+1)=ZL(i+1)+(1-Z)(T+ZL(i)+(1-Z)R(i-1))$$

$$=ZL(i+1)+Z(1-Z)(L(i)+T)+(1-Z)^2(R(i-1)+T).$$

And, extending the expansion

$$R(i+1)=Z\sum_{j=0}^{i}(L(i-j)+jT)(1-Z)^j.$$

so, R is really an exponentially smoothed estimate of the loss level with smoothing parameter $(1-Z)$.

Next, I will show that the $F_iE^{2(n-i)}$ weights are also exponential in character. A theorem from numerical analysis states that the results of a recursion relation $ax_{n+1}=bx_n+cx_{n-2}$ will be $K_1r_1^n+K_2r_2^n$; where r_1 and r_2 are the roots of $ax^2-bx-c=0$.

In the case of the F_i's this means a linear combination of the form

$$F_i=K_1[(2E^2+\sigma^2+\sigma\sqrt{4E^2+\sigma^2})/2]^i + K_2[(2E^2+\sigma^2-\sigma\sqrt{4E^2+\sigma^2})/2]^i$$

But, as i gets very large, the larger root's power will grow much faster than the smaller root's. So, for large i

$$F_i\approx K_1[(2E^2+\sigma^2+\sigma\sqrt{4E^2+\sigma^2})/2]^i.$$

Now, in the estimating formula for x_{t+n}, the weights are $F_i E^{2(n-i)}$. So the smoothing parameter for successively older observed points is roughly

$$F_{i-1}E^{2(n-i+1)}/(F_iE^{2(n-i)})=E^2F_{i-1}/F_i\,,$$

or

$$2E^2/(2E^2+\sigma^2+\sigma\sqrt{4E^2+\sigma^2})\,.$$

Since $(1-Z)$ is the smoothing parameter,

$$Z = 1-[2E^2/(2E^2+\sigma^2+\sqrt{4E^2+\sigma^2})] = (\sigma^2+\sigma\sqrt{4E^2+\sigma^2})/(2E^2+\sigma^2+\sigma\sqrt{4E^2+\sigma^2})\,.$$

which is the result we seek.

Parenthetically, note that since trend is usually exponential rather than linear a logarithmic transform produces the formula $L(i)^Z\cdot(R(i)(1+T))^{1-Z}$ rather than the linear sum formula $ZL(i)+(1+Z)R(i)(1+T)$.

Summary

The random nature of most economic forces creates random behavior in econometric data, especially insurance data. So, the most effective way to project econometric series involves viewing them as a random walks. Within the general framework that imposes, the projection becomes a compromise between: 1) formula trend and random observation; and 2) random trend and error-free observation. Two of the formulas presented in this paper illustrate the 'most accurate' estimators for random walk data. The author believes those formulas to be merely the beginning. Viewing insurance data as a random walk will give actuaries many opportunities to refine our formulas and thereby make better predictions.

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DIFFUSE PRIORS AND BAYES THEOREM

Many problems seek an unknown quantity (such as the best rate to charge) which could, a priori, be any number. They can be solved through uniform distributions on infinite intervals. Those are called diffuse priors. For example, a basic problem in statistics involves the following scenario: Observed data from a normal distribution $x_1, x_2, \ldots, x_n$ are available. There are sufficient data points to give an acceptable estimate of the mean (x) and variance (σ^2), but the distribution of the true mean μ is desired. A priori, all the potential $\mu\epsilon(-\infty,\infty)$ are equally likely candidates, but obviously the μ close to $\overline{x}$ deserve greater probability.

If μ and the x_i were restricted to some finite interval (a, b) then Bayes' theorem would yield

$$f(\mu|[x_i])=f([x_i]|\mu)\cdot f(\mu)/f([x_i])=f([x_i]|\mu)(b-a)^{n-1}$$

In other words, since b-a is constant, Bayes theorem indicates the likelihood of μ given $[x_i]$ is proportional to the likelihood of those $[x_i]$ given μ.

The problem lies when the $[x_i]$ and μ, a priori, take any value in $(-\infty,\infty)$ with equal likelihood (i.e. they are uniformly distributed on $(-\infty,\infty)$). The solution involves the use of 'diffuse priors' (uniform distributions on infinite sets). The author is not familiar with whatever approaches to diffuse priors are currently used by others, but I hope to convey enough of my thinking to solve the practical problems underlying this paper.

Conceptually, one could use the infinitesimal, I, sometimes used in mathematical logic. I is a (entirely theoretical) constant that is infinitely close to zero, but non-zero. So

$$\int_{-\infty}^{\infty} \mathrm{I}\,dt=1$$

Thus, if we use the a priori distribution

$$f(u)=\mathrm{I},\quad f([x_n])=\mathrm{I}';$$

then

$$f(\mu|[x_i])=f([x_i]|\mu)\cdot\mathrm{I}/\mathrm{I}'.$$

So, the probability of μ given the observed $[x_i]$ is proportional to the probability those $[x_i]$ would be observed when μ is the underlying mean.
In the event the $[x_i]$ come from a normal $N(\mu,\sigma^2)$ distribution, σ^2 may be determined fairly accurately from the observed x_i's. So,

$$f(\mu|[x_i])=(1/(\sigma\sqrt{2\pi}))^n\exp[(-(1/2\sigma^2)\Sigma(x_i-\mu)^2]\cdot(\mathrm{I}/\mathrm{I}')$$

which probability formulas[6] reduce to a normal distribution for the mean

$$(\sqrt{n}/(\sigma\sqrt{2\pi}))\exp[-(n/(2\sigma^2))(\overline{x}-\mu)^2]\cdot K.$$

But, since

$$\int\exp[(-(n/(2\sigma^2))(\overline{x}-\mu)^2)]\,d\mu=\sigma\sqrt{2\pi}/\sqrt{n}$$

we conclude that $K=1$, and

$$\mu\sim N(\overline{x},\sigma^2/n).$$

In general, if A and B have uniform diffuse prior distributions, then $P(A=a|B=b)=P(B=b|A=a)\cdot K$. In other words, the probability of A given B is proportional to the probability of B given A.

Mathematical Niceties

At least one article [7] suggests that Bayes' original concept of a uniform distribution on $(-\infty, \infty)$ consisted of a normal distribution with infinite variance, e.g.

$$\lim_{\sigma\to\infty} N(\mu, \sigma^2).$$

Of course, that inevitably produces a specific mean and mode for the prior distribution of μ. According some specific μ that favored status makes the distribution somewhat less than uniform. But, if one were seeking to prove some $G(x)=0$ for a uniform distribution on $(-\infty, \infty)$; one could say: If

$$\lim_{\sigma\to\infty} G(x|N(\mu,\sigma^2))=0.$$

For all μ, $G(x)=0$ holds for the uniform distribution on $(-\infty, \infty)$.

The author has two alternate, but potentially mathematically equivalent, approaches. The first one involves a limit of uniform distributions. In this case the requirement is that

$$\lim_{n\to\infty} G(x|U(a_n, b_n))=0$$

($U(a_n, b_n)$ representing the uniform distribution on the interval (a_n, b_n)).

More important, that result must hold for all sequences $[a_n]$ and $[b_n]$ such that $a_n\to-\infty$ and $b_n\to\infty$.

More generally, one could require that $G(x|f_n)\to 0$ for all sequences of density functions $\{f_n\}$ with an infinite, flat limit. Specifically,

$$\lim\ (\text{non-zero domain of } f_n) = (-\infty, \infty)$$

and

$$\lim_{n\to\infty}[\max(f_n(x))/\min(f_n(x))]=1$$ [8]

Whichever definition you choose, it is clear that the formulas earlier in this paper, which use I, hold.

Pitfalls

The typical problem with diffuse priors is actually a problem with finite uniform distributions, too. There may be uncertainty over what is to be uniformly distributed. For example, when developing a prior distribution for the mean, μ, of a normal distribution it is fairly clear that μ should be uniformly distributed on $(-\infty,\infty)$. But what about the variance, σ^2? Should σ^2 be uniformly distributed on $[0,\infty)$, or should σ be uniformly distributed on $(-\infty,\infty)$? Making σ^2 uniformly distributed inherently makes 'small' σ^2 more likely than making σ uniformly distributed. So, when it is not clear what should be uniformly distributed, diffuse prior distributions are inappropriate.

Fortunately, in this paper the author has used diffuse priors solely for estimating means. So, the variance issue is moot. But, there are other situations, outside the scope of this paper, where problems may arise.

INTRODUCTORY LEMMAS

Before proceeding to prove that the $F_i E^{2(n-i)}$'s are the best weights for historical experience, it will be helpful to prove two lemmas.

Lemma 1: Weighted Squared Difference Theorem.

The weighted sum of squared differences equals the squared difference from the weighted mean plus the squared differences. Mathematically,

$$\sum_{i=1}^{n} W_i (a_i - x)^2 = (\sum_{i=1}^{n} W_i)(x - [(\Sigma W_i a_i)/\Sigma W_i])^2 + (1/\Sigma W_i) \sum_{i=1}^{n} \sum_{j \le i} W_i W_j (a_i - a_j)^2$$

Practically, this means that the estimate x which minimizes the weighted squared differences from the observed points $\{a_i\}$ is the weighted average of the a_i's. Further, the residual error after choosing that best estimate consists of squared differences between the a_i's. Each such difference is weighted by the weights of the two a_i's in the difference.

The most straightforward way to prove this involves placing the weighted mean inside the sum and using brute force.

$$\sum_{i=1}^{n} W_i (a_i - x)^2 = \sum_{i=1}^{n} W_i ([(\Sigma W_j a_j/\Sigma W_j) - x] + [a_i - (\Sigma W_j a_j/\Sigma W_j)])^2$$

Expanding the square,

$$= \sum_{i=1}^{n} W_i ([\Sigma W_j a_j/\Sigma W_j) - x]^2 + 2[(\Sigma W_j a_j/\Sigma W_j) - x][a_i - (\Sigma W_j a_j/\Sigma W_j)] + [a_i - (\Sigma W_j a_j/\Sigma W_j)]^2).$$

Then, distributing the summation across the three sums,

$$=(\Sigma W_i)\,[x-(\Sigma W_j a_j/\Sigma W_j)]^2+2[(\Sigma W_j a_j/\Sigma W_j)-x]\,[\Sigma W_i a_i-\Sigma W_j a_j]\ +\Sigma W_i\,[a_i-(\Sigma W_j a_j/\Sigma W_j)]^2.$$

Noting that $\Sigma W_i a_i=\Sigma W_j a_j$, the polynomial equals

$$1)\quad =(\Sigma W_i)\,[x-(\Sigma W_i a_i/\Sigma W_i)]^2+(\Sigma W_i)\,[a_i-(\Sigma W_j a_j/\Sigma W_j)]^2.$$

Computing the square in the last term, note that

$$\Sigma W_i\,[a_i-(\Sigma W_j a_j/\Sigma W_j)]^2$$

$$=\Sigma W_i a_i^2\ -2(\Sigma W_i a_i)\,(\Sigma W_i a_i)\,/\,(\Sigma W_i)+(\Sigma W_i)\,(\Sigma W_j a_j)^2/\,(\Sigma W_j)^2,$$

$$=(1/\Sigma W_i)\,[(\Sigma W_i a_i^2)\,(\Sigma w_i)-2(\sum_i\sum_j W_i W_j a_i a_j)+(\Sigma W_i a_i)^2],$$

$$=(1/\Sigma W_i)\,[(\Sigma W_i a_i^2)\,(\Sigma W_i)+\sum_i\sum_j W_i W_j\,(a_i a_j-2a_i a_j)],$$

$$=(1/\Sigma W_i)\,[\sum_i\sum_j W_i W_j a_i^2-\sum_i\sum_j W_i W_j a_i a_j].$$

Splitting the sums up into the cases where j is less than, equal to, or greater than i.

$$=(1/\Sigma W_i)\,[\sum_{i=2}^{n}\sum_{j=1}^{i-1} W_i W_j a_i^2\ +\ \sum_{i=1}^{n-1}\ \sum_{j=i+1}^{n} W_i W_j a_i^2$$

$$+\ \Sigma W_i^2 a_i^2\ -\ \sum_{i=2}^{n}\sum_{j=1}^{i-1} W_i W_j a_i a_j\ -\ \sum_{i=1}^{n-1}\ \sum_{j=i+1}^{n} W_i W_j a_i a_j\ -\ \Sigma W_i^2 a_i^2].$$

Subtracting the $\Sigma W_i^2 a_i^2$ terms that cancel, and interchanging i and j in two of the indices

$$=(1/\Sigma W_i)\left[\sum_{i=2}^{n}\sum_{j=1}^{i-1} W_i W_j a_i^2 + \sum_{i=2}^{n}\sum_{j=1}^{i-1} W_i W_j a_j^2 - \sum_{i=2}^{n}\sum_{j=1}^{i-1} W_i W_j a_i a_j - \sum_{i=2}^{n}\sum_{j=1}^{i-1} W_i W_j a_i a_j\right].$$

Collecting terms

$$=(1/\Sigma W_i)\sum_{i=2}^{n}\sum_{j=1}^{i-1}(W_i W_j a_i^2 + W_i W_j a_j^2 - 2W_i W_j a_i a_j),$$

$$=(1/\Sigma W_i)\sum_{i=2}^{n}\sum_{j=1}^{i-1} W_i W_j (a_i - a_j)^2.$$

Adding the case where $i=j$; $(a_i-a_j)=0$

$$=(1/\Sigma W_i)\sum_{i=1}^{n}\sum_{j\le i} W_i W_j (a_i - a_j)^2$$

Now, substituting that result back into 1) yields the lemma:

$$\Sigma W_i (a_i - x)^2 = (\Sigma W_i)(x - [\Sigma W_i a_i / \Sigma W_i])^2 + (1/\Sigma W_i)\sum_{i}\sum_{j\le i} W_i W_j (a_i - a_j)^2.$$

Exponential Integral Theorem

A textbook theorem used to analyze multivariate normal distributions states

$$\int_{-\infty}^{\infty} \exp(-(1/2)[(x-G)^2/\sigma^2 + H])\,dx = \sigma\sqrt{2\pi}\exp(-H/2).$$

The proof is comparatively simple. Exp$(-H/2)$ is constant with respect to the variable of integration (x). So

$$\int_{-\infty}^{\infty} \exp(-(1/2)[(x-G)^2/\sigma^2+H])\,dx=\exp(-H/2)\int_{-\infty}^{\infty} \exp((x-G)^2/(2\sigma^2))\,dx.$$

But up to the constant $1/(\sigma\sqrt{2\pi})$ the integral is simply the density of a normal $N(G,\sigma^2)$ distribution. So its integral is $\sigma\sqrt{2\pi}$. Thus, the theorem holds:

$$=\exp(-H/2)\cdot\sigma\sqrt{2\pi}=\sigma\sqrt{2\pi}\ \exp(-H/2).$$

Lemma 2) Integral of Weighted Squared Differences

$$\int_{-\infty}^{\infty} \exp(-(1/2)\Sigma W_i(x-a_i)^2)\,dx$$

$$=\sqrt{2\pi/\Sigma W_i}\ \exp[-(1/2)(\sum_i \sum_{j\le i} W_i W_j(a_i-a_j)^2)/(\Sigma W_i)]$$

This lemma is a straightforward combination of Lemma 1 and the exponential integral theorem.

PROOF OF THE FIXED POINT ESTIMATOR FORMULA

To prove that

1) $e_{n+t} = tT + [\sum_{1}^{n} F_i E^{2(n-i)} (\hat{x}_i + (n-i)T)] / [(\sum_{1}^{n} F_i E^{2(n-i)})]$

is the best estimator for x_{n+t}, I need to first integrate the x_{n+t} density function. Then, the formula will result from some simple algebra which proves the recursion relation.

Using a diffuse prior argument

2) $f(x_{n+t}|\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)$

$= Kf([\hat{x}_i]_1^n | x_{n+t})$,

$= K(1/[(2\pi)^n \sigma^n \sqrt{t} E^n]) \int_{x_n} \ldots \int_{x_1} \exp[-(1/2)((x_{n+t} - tT - x_n)^2/(t\sigma^2)$

$+ (1/\sigma^2)[\sum_{i=1}^{n-1}(x_i + T - x_{i+1})^2] + (1/E^2)[\sum_{i=1}^{n}(x_i - \hat{x}_i)^2])] \cdot dx_1 \ldots dx_n$.

Combining the K into K′ (a function independent of x_{n+t}) multiplied by an exponent of squared differences

3) $= K'([x_i]_1^n, E^2, \sigma^2, t, T) \cdot \exp[-(1/2) K''(E^2, \sigma^2, t) \sum_{i=1}^{n} F_i E^{2(n-i)} (\hat{x}_i + (n-i+t)T - x_{n+t})^2]$

Showing that the estimator from 1) minimizes that sum of squared differences will then suffice to show it is the best estimator of x_{n+t}

To solve the multiple integral from 2) I need to first prove a theorem

Multiple Integral Theorem

Given:

1) observed points $[\hat{x}_i]_1^n$ distributed around unknown means $[x_i]_1^n$;

2) generated by a normal stochastic process with mean increase T and variance parameter σ^2;

3) where each of the $[\hat{x}_i]$ differ from the $[x_i]$ by an independent $N(0, E^2)$ distribution;

4) and the times between valuation are t_i (so $f(x_{i+1}|x_i) \sim N(t_iT, t_i\sigma^2)$));

the integral

4) $$I(x_{n+1}) = \int_{x_n} \dots \int_{x_1} \exp(-(1/2)[(1/E^2)[\sum_{i=1}^{n}(\hat{x}_i - x_i)^2]$$

$$+[\sum_{i=1}^{n}(x_i + t_iT - x_{i+1})/(t_i\sigma^2)]])dx_1, \dots dx_n$$

$$= K([\hat{x}_i]_1^n, [t_i]_1^n, E^2, \sigma^2, T)\cdot\exp(-(1/2)[(1/F_{n+1})\sum_{i=1}^{n}F_iE^{2(n-i)}(\hat{x}_i + T(\sum_{j=i}^{n}t_j) - x_{n+1})^2]).$$

Where,

5) $F_1 = 1$

$$F_{n+1} = t_n\sigma^2(\sum_{i=1}^{n}F_iE^{2(n-i)}) + E^2F_n$$

I will prove it using mathematical induction. The proof for $n=1$ is trivial. Next, I must show the result holds for $I(x_{n+2})$ when it holds for $I(x_{n+1})$.

Note that

$$I(x_{n+2}) = \int_{x_{n+1}} \ldots \int_{x_1} \exp(-(1/2)\,[(1/E^2)\,[\sum_{i=1}^{n+1}(\hat{x}_i - x_i)^2]$$

$$+[\sum_{i=1}^{n+1}(x_i + t_i T - x_{i+1})^2/(t_i\sigma^2)]])\; dx_1, \ldots, dx_{n+1}$$

So, pulling out the terms that are constant with respect to $x_1, \ldots, x_n$

$$= \int_{x_{n+1}} \exp(-(1/2)\,[(\hat{x}_{n+1} - x_{n+1})^2/E^2 + (x_{n+1} + t_{n+1}T - x_{n+2})/(t_{n+1}\sigma^2)])$$

$$\cdot \int_{x_n} \ldots \int_{x_1} \exp(-(1/2)\Big[[(1/E^2)\,[\sum_{i=1}^{n}(\hat{x}_i - x_i)^2]$$

$$+[\sum_{i=1}^{n}(x_i + t_i T - x_{i+1})^2/(t_i\sigma^2)]\Big])\, dx_1, \ldots, dx_{n+1}.$$

Then the inner 'n' integrals may use the induction hypothesis

$$= \int_{x_{n+1}} \exp(-(1/2)\,[(\hat{x}_{n+1} - x_{n+1})^2/E^2 + (x_{n+1} + t_{n+1}T - x_{n+2})^2/(t_{n+1}\sigma^2)]) \cdot I(x_{n+1})\, dx_{n+1},$$

$$= \int_{x_{n+1}} \exp(-(1/2)\,[(\hat{x}_{n+1} - x_{n+1})^2/E^2 + (x_{n+1} + t_{n+1}T - x_{n+2})^2/(t_{n+1}\sigma^2)])$$

$$\cdot K([\hat{x}_i]_1^n, [t_i]_1^n, E^2, \sigma^2, T)$$

$$\cdot \exp(-(1/2)\,(1/F_{n+1})\,[\sum_{i=1}^{n} F_i E^{2(n-i)}(\hat{x}_i + T(\sum_{j=i}^{n} t_j) - x_{n+1})^2]) \cdot dx_{n+1}.$$

6) $= K([\hat{x}_i]_1^n, [t_i]_1^n, E^2, \sigma^2, T) \int_{x_{n+1}} \exp(-\frac{A}{2})\, dx_{n+1}$.

Where

$$A = (\hat{x}_{n+1} - x_{n+1})^2/E^2 + (x_{n+1} + t_{n+1}T - x_{n+2})^2/(t_{n+1}\sigma^2) + (1/F_{n+1}) \sum_{i=1}^{n} F_i E^{2(n-i)} (\hat{x}_i + T(\sum_{j=i}^{n} t_j) - x_{n+1})^2 .$$

Now to evaluate A, the first step is to apply the integral of weighted squared differences lemma (lemma from the previous appendix) using x_{n+1} as x. Specifically,

7) $\int_{x_{n+1}} \exp(-A/2)\, dx_{n+1}$

$$= (2\pi/[(1/E^2) + (1/(t_{n+1}\sigma^2)) + (1/F_{n+1}) \sum_{i=1}^{n} F_i E^{2(n-i)}])^{1/2}$$

$$\cdot \exp(-(1/2)\Big[[(\hat{x}_{n+1} + t_{n+1}T - x_{n+2})^2/(t_{n+1}E^2\sigma^2)]$$

$$+ [(1/(F_{n+1}E^2)) \sum_{i=1}^{n} F_i E^{2(n-i)} (\hat{x}_i + T(\sum_{j=i}^{n} t_j) - \hat{x}_{n+1})^2]$$

$$+ [(1/F_{n+1}^2) \sum_{i=1}^{n} \sum_{j \le i} F_i F_j E^{2(2n-i-j)} (\hat{x}_j + T(\sum_{k=j}^{i-1} t_k) - \hat{x}_i)^2]$$

$$+ [(1/(t_{n+1}\sigma^2 F_{n+1})) \sum_{i=1}^{n} F_i E^{2(n-i)} (\hat{x}_i + T(\sum_{j=i}^{n+1} t_j) - x_{n+2})^2]\Big]$$

$$/ [(1/E^2) + (1/(t_{n+1}\sigma^2)) + (1/F_{n+1}) \sum_{i=1}^{n} F_i E^{2(n-i)}])$$

That produces quite a long expression. But, noting that the long 'sum of the weights' term

$$(1/E)^2+(1/(t_{n+1}\sigma^2))+(1/F_{n+1})\sum_{i=1}^{n}F_iE^{2(n-i)}$$

$$=(1/(E^2t_{n+1}\sigma^2F_{n+1}))[t_{n+1}\sigma^2F_{n+1}+E^2F_{n+1}+t_{n+1}\sigma^2E^2\sum_{i=1}^{n}F_iE^{2(n-i)}];$$

and combining the $t\sigma^2$ terms

$$=(1/(t_{n+1}E^2\sigma^2F_{n+1}))[t_{n+1}\sigma^2\sum_{i=1}^{n+1}F_iE^{2(n+1-i)})+E^2F_{n+1}]$$

$$=F_{n+2}/(F_{n+1}t_{n+1}E^2\sigma^2);$$

Then, plugging that back in 7)

$$\int_{x_{n+1}}\exp(-A/2)\,dx_{n+1}$$

$$=(\sqrt{2\pi t_{n+1}F_{n+1}/F_{n+2}})E\sigma\cdot\exp(-(F_{n+1}t_{n+1}E^2\sigma^2/(2F_{n+2}))$$

$$\Big[[(\hat{x}_{n+1}+t_{n+1}T-x_{n+2})^2/(t_{n+1}E^2\sigma^2)]+[(1/(F_{n+1}E^2))\sum_{i=1}^{n}F_iE^{2(n-i)}(\hat{x}_i+T(\sum_{j=i}^{n})-\hat{x}_{n+1})^2]$$

$$+[(1/F_{n+1}^2)\sum_{i=1}^{n}\sum_{j\leq i}F_iF_jE^{2(2n-i-j)}(\hat{x}_j+T(\sum_{k=j}^{i-1}t_k)-\hat{x}_i)^2]$$

$$+[(1/(t_{n+1}\sigma^2F_{n+1}))\sum_{i=1}^{n}F_iE^{2(n-i)}(\hat{x}_i+T(\sum_{j=i}^{n+1}t_j)-x_{n+2})^2]\Big])$$

That is still quite a lengthy expression. But, part of it may be reduced immediately. Since the multiplier in front of the function and the middle two terms in the sum are constant with respect to x_{n+2},

$$= K'([\hat{x}_i]_1^{n+1}, [t_i]_1^{n+1}, E^2, \sigma^2, T) \cdot \exp\Big\{-(F_{n+1}t_{n+1}E^2\sigma^2/(2F_{n+2}))$$

$$\cdot\Big[[(\hat{x}_{n+1}+t_{n+1}T-x_{n+2})^2/(t_{n+1}E^2\sigma^2)] + [(1/(t_{n+1}\sigma^2F_{n+1}))\sum_{i=1}^{n}F_iE^{2(n-i)}(\hat{x}_i+T(\sum_{j=i}^{n+1}t_j)-x_{n+2})^2]\Big]\Big\}$$

That is reduced, but still lengthy. Applying the top of the quotient to the sums

$$= K' \exp\Big\{(-(1/2)(1/F_{n+2})\Big[[F_{n+1}(\hat{x}_{n_1}+t_{n+1}T-x_{n+2})^2]$$

$$+[E^2\sum_{i=1}^{n}F_iE^{2(n-i)}(\hat{x}_i+T(\sum_{j=i}^{n+1}t_j)-x_{n+2})^2]\Big])\Big\}$$

Adding the $n+1$ term to the sum

$$= K' \exp(-(1/2)(1/F_{n+2})\sum_{i=1}^{n+1}F_iE^{2(n+1-i)}(\hat{x}_i+T(\sum_{j=i}^{n+1})-x_{n+2})^2) .$$

Then plugging that back in the original formula in 6)

$$I(n+2) = K{\cdot}K' \exp\ (-(1/2)(1/F_{n+2})\sum_{i=1}^{n+1}F_iE^{2(n+1-i)}(\hat{x}_i+T(\sum_{j=i}^{n+1}t_j)-x_{n+2})^2) ,$$

$$= K([x_i]_1^{n+1}, [t_i]_1^{n+1}, E^2, \sigma^2, T) \cdot \exp\Big\{-(1/2)(1/F_{n+2})\sum_{i=1}^{n+1}F_iE^{2(n+1-i)}(\hat{x}_i+T(\sum_{j=i}^{n+1}t_j)-x_{n+2})^2\Big\}.$$

So the induction hypothesis is proven and the integral evaluation theorem holds.

The Best Estimator

Now that we know the density function $f(x_{n+t}|[\hat{x}_i]_1^n, E^2, \sigma^2, T)$, the next step is to show that the estimator

$$e_{n+t} = tT + [\sum_{i=1}^{n} F_i E^{2(n-i)} (\hat{x}_i + (n-i)T)] / (\sum_{i=1}^{n} F_i E^{2(n-i)})]$$

is the optimum estimator for x_{n+t}. The key is to show that the true x_{n+t} is normally distributed around e_{n+t}

$$f(x_{n+t}|[\hat{x}_i]_1^n, E^2, \sigma^2, T) \sim N(e_{n+t}, \delta^2) .$$

Then, since e_{n+t} is both the mean and the mode of the distribution, it must be the best estimator.

Plugging the results of the integration theorem into the earlier formula for $f(x_{n+t})$,

$$f(x_{n+t}|[\hat{x}_i]_1^n, t, E^2, \sigma^2, T)$$

$$= K\exp(-(1/2)(1/F'_{n+1}) \cdot \sum_{i=1}^{n} F_i E^{2(n-i)} (\hat{x}_i + (n-i+t)T - x_{n+t})^2)$$

Using the weighted sum of squares lemma (Lemma 1) from appendix II, (note $F'_{n+1} = t\sigma^2 (\sum_{i=1}^{n} F_i E^{2(n-i)}) + E^2 F_n$ instead of $\sigma^2 (\sum F_i E^{2(n-i)}) + E_2 F_n$ because of the $t\sigma^2$ in the last term)

$$= K \exp\Big\{(-(1/2)(1/F'_{n+1})\Big[[(x_{n+t} - [\sum_{i=1}^{n} F_i E^{2(n-i)} (\hat{x}_i + (n-i+t)T)]) / (\sum_{i=1}^{n} F_i E^{2(n-i)}))^2$$

$$\cdot (\sum_{i=1}^{n} F_i E^{2(n-i)}) + (1/\sum_{i=1}^{n} F_i E^{2(n-i)}) \sum_{i=1}^{n} \sum_{j<i} F_i F_j E^{2(2n-i-j)} (\hat{x}_i - (i-j)T - \hat{x}_j)^2]\Big]\Big)\Big\}$$

Noting that the second term in the sum is constant with respect to x_{n+t}, and using the definition of the F_i's in 5).

$$=K \exp\{ (-(1/2)((F'_{n+1}-E^2F_n)/(t\sigma^2F'_{n+1}))$$

$$\cdot(x_{n+t}-[\sum_{i=1}^{n} F_iE^{2(n-i)}(\hat{X}_i+(n-i+t)T)]/[\sum_{i=1}^{n} F_iE^{2(n-i)}])^2)\}$$

$$=K \exp\{ (-(1/2)[1/(t\sigma^2F'_{n+1}/(F'_{n+1}-E^2F_n))]/(x_{n+t}-e_{n+t})^2\}.$$

Since the K is merely a constant which will be adjusted to make the distribution integrate to 1.

$$f(x_{n+t})\sim N(e_{n+t}, [t\sigma^2F'_{n+1}/(F'_{n+1}-E^2F_n)])$$

Which completes the proof as soon as I show that the F_i's produced by 5) follow the recursion rule

$$F_1=1$$

$$F_2=E^2+\sigma^2$$

$$F_{k+1}=(2E^2+\sigma^2)F_k-E^4F_{k-1}$$

The proof involves fairly straightforward algebra.

$$F_{k+1}=\sigma^2\sum_{i=1}^{k}F_iE^{2(k-i)}+E^2F_k$$

$$=\sigma^2F_k+E^2\sigma^2\sum_{i=1}^{k-1}F_iE^{2(k-1-i)}+E^2F_k$$

$$=(\sigma^2+E^2)F_k+E^2(\sigma^2\sum_{i=1}^{k-1}F_iE^{2(k-1-i)})\ .$$

Applying the definition of the F_i's to the sum,

$$=(\sigma^2+E^2)F_k+E^2(F_k-E^2F_{k-1})$$

$$=(\sigma^2+2E^2)F_k-E^4F_{k-1}\ .$$

So, the F_i's fulfill the recursion rule, and thus, e_{n+t} is the best estimate.

ESTIMATING THE TREND

The best estimate of the trend is a weighted average of differences between adjacent points

$$T = [\sum_{i=1}^{n-1} W_i(\hat{x}_{i+1}-\hat{x}_i)] / \sum_{i=1}^{n-1} W_i .$$

The weights are somewhat complicated, but not overly difficult to compute.

1) $W_i = E^2 - (2E^4F_i/F_{i+1}) - (E^{2(i+1)}/F_{i+1}) + E^{-2i}[F_{i+1}-E^2F_i]G_i$

where the G_i are recursively calculated from n down, e.g.

2) $G_n = (E^{4n}+E^{2n+2}F_n)/(F_n(F_{n+1}-E^2F_n))$

$G_i = G_{i+1} + [(E^{4i}+2E^{2i+2}F_i)/(F_iF_{i+1})]$.

To prove that is the best estimate of the trend T, I will follow several steps. First, I will isolate the terms that involve T from the probability function for $\hat{x}_1, \hat{x}_x, \ldots, \hat{x}_n$. That will represent the function I must maximize. Maximizing it will involve minimizing a sum of squared differences between T and the differences between adjacent points $(\hat{x}_{i+1}-\hat{x}_i)$.

Before minimizing that function, I must show it is independent of the time (t) since the last observation. Then, I will convert it from functions of T and differences between faraway points $\hat{x}_i-\hat{x}_j$ into differences between T and differences between adjacent points $\hat{x}_{i+1}-\hat{x}_i$. That will produce a complicated set of weights for each difference $\hat{x}_{i+1}-\hat{x}_i$. Next, I will simplify those weights to show they are the weights in equations 1) and 2).

The Function to Minimize - The New Distribution of Observed Points

The previous appendix showed that the distribution of the potential observed points $\hat{x}_1, \ldots, \hat{x}_n$ given a future value $x_{n+\tau}$ was proportional to a term involving $x_{n+\tau}$ and a constant, e.g.

$$f(\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n | x_{n+\tau}, T, \sigma^2, E^2) = K_1 \exp(-K_2(e_{n+\tau} - x_{n+\tau})^2 + K_3),$$

$(K_1, K_2, K_3$ constant w.r.t. $x_{n+\tau})$

That made $e_{n+\tau}$ the best estimator of $x_{n+\tau}$. I would like to isolate T the way I isolated $x_{n+\tau}$ to produce a formula

$$f(\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n | x_{n+\tau}, T, \sigma^2, E^2) = K_1 \exp(-K_2(T' - T)^2 - K_3(e_{n+\tau} - x_{n+\tau})^2 + K_4)$$

$(K_1, K_2, K_3, K_4$ constant w.r.t. both $x_{n+\tau}$ and T).

Then, the expression T′ will represent the best (maximum likelihood) estimator of T.

The first step is to combine the terms involving T[9], e.g. to find $f(\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n | x_{n+\tau}, T, \sigma^2, E^2) = K_1 \exp(-g(T) - K_3(e_{n+\tau} - x_{n+\tau})^2 + K_4)$

Thankfully, finding g(T) is fairly easy. Simple inspection of the multiplier of $(e_{n+\tau} - x_{n+\tau})^2$ shows it is independent of T as well as $x_{n+\tau}$. The function g(T) then simply represents the terms 'cast off' as constant when integrating over the x_i's plus the x_i terms cast off when the weighted squared differences between many individual terms and $x_{n+\tau}$ were combined (at the end of appendix III).

First, let me discuss out the terms cast off when integrating over the x_l's. The terms cast off into the constant when evaluating the multiple integral over x_l were

$$\exp\left\{-(1/2)\left[(t_l\sigma^2/F'_{l+1})\sum_{j=1}^{l-1}F_jE^{2(l-1-j)}(\hat{x}_j+(l-j)T-\hat{x}_l)^2\right.\right.$$

$$\left.\left.+(t_lE^2\sigma^2/(F_lF'_{l+1}))\sum_{j=1}^{l-1}\sum_{k\le j}F_jF_kE^{2(2l-2-j-k)}(\hat{x}_k+(j-k)T-\hat{x}_j)^2\right]\right\}$$

($t_l=1$, except for $t_n=t$, and $F'_{l+1}=F_{l+1}$, except for F'_{n+1} which is $t\sigma^2\sum F_lE^{2(n-l)}+E^2F_n$.)

Which, after moving some E^2 terms outside the sums,

$$3)\quad =\exp\left\{-(1/2)\left[(t_l\sigma^2/(E^2F'_{l+1}))\sum_{j=1}^{l-1}F_jE^{2(l-j)}(\hat{x}_j+(l-j)T-\hat{x}_l)^2\right.\right.$$

$$\left.\left.+(t_l\sigma^2/(E^2F_lF'_{l+1}))\sum_{j=1}^{l-1}\sum_{k\le j}F_jF_kE^{2(2l-j-k)}(\hat{x}_k+(j-k)T-\hat{x}_j)^2\right]\right\}$$

For simplicity, let me call the first term A_l and the second B_l to get

$$4)\quad =\exp\{-(1/2)[A_l+B_l]\}$$

But there is another T term to add. When the final individual terms $(\hat{x}_l+(t+n-i)T-x_{n+t})^2$ were combined by the weighted sum of squares theorem in appendix III (to get $(e_{n+t}-x_{n+t})^2$), the following terms were 'cast off'.

$$5)\quad \exp\{-(1/2)[(1/F'_{n+1})(1/\sum_{i=1}^{n}F_iE^{2(n-i)})\cdot\sum_{l=1}^{n}\sum_{j\le l}F_lF_jE^{2(2n-l-j)}(\hat{x}_j+(l-j)T-\hat{x}_l)^2]\}$$

$$=\exp(-(C_n/2))$$

Combining all the terms involving T, I get

$$6)\quad g(T) = -(1/2)\,[C_n + \sum_{l=1}^{n} A_l + B_l]$$

Looking back at the pieces of g(T), it is much more difficult to work with than it needs to be. First, it uses t_l and F_l'-two clumsy expressions. But, as we will see later, the sum g(T) is actually independent of t.

Before proving that, I need to prove several lemmas. One will be used to prove the independence from t. The others will be used later to simplify g(T).

Lemmas

Before showing g(T) is independent of t, I need to make a brief digression. I will need several lemmas to complete the analysis. Since I need one of them to prove g(T) is independent of t, I should prove them before discussing g(T) further.

Interchange of Sum Indices Lemma.

$$7)\quad \sum_{a=1}^{n} \sum_{b=1}^{a-1} h(a,b) = \sum_{b=1}^{n-1} \sum_{a=b+1}^{n} h(a,b)$$

Proof: the indices on either side describe the case where $b<a\leq n$.

An alternate version, where $b\leq a\leq n$, is

$$8)\quad \sum_{a=1}^{n} \sum_{b=1}^{a} h(a,b) = \sum_{b=1}^{n} \sum_{a=b}^{n} h(a,b)$$

Sum of the F_l's

$$9)\quad \sum_{a=1}^{n} E^{2(n-a)} F_a = (F_{n+1} - E^2 F_n)/\sigma^2$$

Proof: Using the summation definition of the F_i's from appendix III

$$F_{n+1}=\sigma^2\left[\sum_{a=1}^{n} E^{2(n-a)}F_a\right]+E^2F_n.$$

Simple algebra produces the result.

Partial Sum of the F_i's Lemma.

$$10)\quad \sum_{a=b}^{n} E^{2(n-a)}F_a=\frac{[F_{n+1}-E^2F_n-E^{2(n-(b-1))}F_b+E^{2(n-(b-2))}F_{b-1}]}{\sigma^2}$$

Proof:

$$\sum_{a=b}^{n} E^{2(n-a)}F_a=\sum_{a=1}^{n} E^{2(n-a)}F_a-\sum_{a=1}^{b-1} E^{2(n-a)}F_a = \sum_{a=1}^{n} E^{2(n-a)}F_a-E^{2(n-(b-1))}\sum_{a=1}^{b-1} E^{2(b-1-a)}F_a$$

Using equation 9) twice produces the result.

Sum of the iF_i's Lemma.

$$11)\quad \sum_{a=1}^{n} aE^{2(n-a)}F_a=\frac{nF_{n+1}-(n-1)E^2F_n+E^{2n}}{\sigma^2}$$

Proof: Noting that $a=\sum_{b=1}^{a}$

$$\sum_{a=1}^{n} aE^{2(n-a)}F_a=\sum_{a=1}^{n}\sum_{b=1}^{a}E^{2(n-a)}F_a$$

Using the interchange of sum indices lemma 8)

$$=\sum_{b=1}^{n}\sum_{a=b}^{n}E^{2(n-a)}F_a$$

Using the formula for the partial sum (equation 10))

$$=\sum_{b=1}^{n}\frac{F_{n+1}-E^2F_n-E^{2(n-(b-1))}F_b+E^{2(n-(b-2))}F_{b-1}}{\sigma^2}$$

Distributing the sum across the addition and pulling terms constant relative to b outside the sum.

$$=\frac{nF_{n+1}-nE^2F_n-E^2[\sum_{b=1}^{n}E^{2(n-b)}F_b]+E^2[\sum_{b=0}^{n-1}E^{2(n-b)}F_b]}{\sigma^2}$$

Removing one term from the first sum

$$=\frac{nF_{n+1}-nE^2F_n-E^2F_n-E^2[\sum_{b=1}^{n-1}E^{2(n-b)}F_b]+E^2[\sum_{b=0}^{n-1}E^{2(n-b)}F_b]}{\sigma^2}$$

Now, the problem summing from $b=0$ to $n-1$ is that F_0 is undefined. Since it occurs where $b=1, F_1-E^2F_0=0$, it appears $F_0=1/E^2$ (Note that then $F_2=E^2+\sigma^2=(\sigma^2+2E^2)F_1-E^4F_0)$. And the equation is

$$=\frac{nF_{n+1}-nE^2F_n-E^2F_n-E^2\sum_{b=1}^{n-1}E^{2(n-b)}F_b+E^2\sum_{b=1}^{n-1}E^{2(n-b)}F_b+E^{2n}}{\sigma^2}$$

$$=\frac{nF_{n+1}-(n-1)E^2F_n+E^{2n}}{\sigma^2}$$

Partial Sum of the F_i's Lemma

$$12)\quad \sum_{a=b}^{n}aE^{2(n-a)}F_a=(1/\sigma^2)\{nF_{n+1}-(n-1)E^2F_n-(b-1)E^{2(n-(b-1))}F_b+(b-2)E^{2(n-(b-2))}F_{b-1}\}$$

Proof: same basic argument as equation 10).

Telescoping Sum Lemma

$$13)\quad (\hat{x}_k+(j-k)T-\hat{x}_j)^2=(j-k)\sum_{i=k}^{j-1}(\hat{x}_{i+1}-\hat{x}_i-T)^2-\sum_{i=k}^{j-1}\sum_{m\le i}(\hat{x}_{i+1}-\hat{x}_i-(\hat{x}_{m+1}-\hat{x}_m))^2$$

Proof: set

$$(\hat{x}_k+(j-k)T-\hat{x}_j)^2=(\sum_{i=k}^{j-i}(\hat{x}_{i+1}-\hat{x}_i-T))^2=(j-k)^2(T-(1/(j-k))\sum_{i=k}^{j-1}(\hat{x}_{i+1}-\hat{x}_i))^2$$

and then use the weighted sum of squares theorem from appendix II.

g(T) is Independent of t

Now that those lemmas are proven, I must show the 't' in g(T) may be replaced with '1'.

Since the trend is something reflected in the observed points $\hat{x}_1, \ldots, \hat{x}_n$, rather than something instrinsic to the length of the projection period (t), it seems that estimated trend (T') should be independent of t. That will follow from the independence of g(T) from t.

To prove g(T) is independent of t, all that is necessary is to show that the few terms in g(T) that contain a t are actually constant with respect to t. Reviewing equations, 3), 5), and 6), those are $C_n+A_n+B_n$. E.g.

$$g(T) = -(1/2)\,(K+C_n+A_n+B_n)\,,$$

where K is the terms that are obviously constant with respect to t.

First, rewrite C_n by replacing l and j with j and k to get

$$14)\quad A_n+B_n+C_n=(t\sigma^2/(E^2F'_{n+1}))\sum_{j=1}^{n-1} F_jE^{2(n-j)}(\hat{x}_j+(n-j)T-\hat{x}_n)^2$$

$$+(t\sigma^2/(E^2F_nF'_{n+1}))\sum_{j=1}^{n-1}\sum_{k\le j} F_jF_kE^{2(2n-j-k)}(\hat{x}_k+(j-k)T-\hat{x}_j)^2$$

$$+(1/F'_{n+1})(1/\sum_{i=1}^{n} F_iE^{2(n-i)})\cdot\sum_{j=1}^{n}\sum_{k\le j} F_jF_kE^{2(2n-j-k)}(\hat{x}_k+(j-k)T-\hat{x}_j)^2.$$

Then, the strategy is to convert the expression above into an expression in t times a double sum constant relative to t. Then I will show the expression in t is actually constant relative to t. The first step is to note that the first term is the case where $j=n$ for the second term (with j playing the role of k).

$$= (t\sigma^2/(E^2F_nF'_{n+1}))\sum_{j=1}^{n}\sum_{k\le j}F_jF_kE^{2(2n-j-k)}(\hat{x}_k+(j-k)T-\hat{x}_j)^2$$

$$+ (1/(F'_{n+1})(1/\sum_{j=1}^{n}F_iE^{2(n-i)})\sum_{j=1}^{n}\sum_{k\le j}F_jF_kE^{2(2n-j-k)}(\hat{x}_k+(j-k)T-\hat{x}_j)^2$$

Now the double sums in each term are identical and independent of t. So, we may set

$$= [(t\sigma^2/(E^2F_nF'_{n+1}))+(1/F'_{n+1})(1/\sum_{i=1}^{n}F_iE^{2(n-i)})]\cdot K$$

Now, all that remains is to show that is independent of t. Using the 'Sum of the F_i's Lemma' 9) (and correcting for the difference betwen the definition of F'_{n+1} and F_{n+1})

$$= [(t\sigma^2/(E^2F_nF'_{n+1})+(1/F'_{n+1})(t\sigma^2/(F'_{n+1}-E^2F_n))]\cdot K$$

$$= K\cdot\{(t\sigma^2/F'_{n+1})/[(1/(E^2F_n))+(1/(F'_{n+1}-E^2F_n))]\}$$

Performing more algebra

$$= K\cdot\{(t\sigma^2/F'_{n+1})[F'_{n+1}/(E^2F_n(F'_{n+1}-E^2F_n))]\}$$

$$= K\cdot\{t\sigma^2E^2F_n/(F'_{n+1}-E^2F_n)\}$$

$$= K\,E^2F_n/\sum_{i=1}^{n}E^{2(n-i)}F_i$$

Which is independent of t. So, in equation 3), 5) and 6) we may treat the t's as 1's and the F'_{n+1}'s as F_{n+1}.

The next step is to convert the expression involving the differences between faraway (j and k) terms to differences of adjacent terms (i and i+1).

g(T) as Differences Between Adjacent Points

g(T) can be converted to the following expression involving differences between adjacent terms.

$$15)\quad g(T) = [C_n + \sum_{l=1}^{n} A_l + B_l]/2 = -(\sigma^2/2E^2)(U(T)+V(T)) + K;$$

where K is constant with respect to T; and

$$16)\quad U(T) = \sum_{l=2}^{n-1} (1/(F_l F_{l+2})) \sum_{j=2}^{l} \sum_{k<j} E^{2(2l-j-k)} F_j F_k \cdot (j-k) \sum_{i=k}^{j-1} (\hat{x}_{i+1} - \hat{x}_i - T)^2;$$

and,

$$17)\quad V(T) = (1/(F_n(F_{n+1} - E^2 F_n))) \sum_{j=2}^{n} \sum_{k<j} E^{2(2l-j-k)} F_j F_k \cdot (j-k) \sum_{i=k}^{j-1} (\hat{x}_{i+1} - \hat{x}_i - T)^2.$$

(Notice that U and V are identical except for the terms to the left of the double sum. If the $F_{n+1} - E^2 F_n$ in V were simply F_{n+1}, V could be combined into the sum over the l's in U).

To prove that, I must state equations 3), 5) and 6) without t; perform some algebra to simplify the sums; then use the Telescoping Sum Lemma 13).

First, let me point out that when 't' is replaced by '1',

$$g(T) = -[C_n + \sum_{l=1}^{n} A_l + B_l]/2$$

18) $$= -(1/2)\Big[\{\sum_{l=1}^{n} (\sigma^2/(E^2 F_{l+1}))\sum_{j=1}^{l-1} F_j E^{2(l-j)} (\hat{x}_j + (l-j)T - \hat{x}_l)^2\}$$

$$+\{\sum_{l=1}^{n} (\sigma^2/(E^2 F_l F_{l+1}))\sum_{j=1}^{l-1} \sum_{k\le j} F_j F_k E^{2(2l-j-k)} (\hat{x}_k + (j-k)T - \hat{x}_j)^2\}$$

$$+\{(1/F_{n+1})(1/\sum_{l=1}^{n} F_l E^{2(n-l)})\sum_{l=1}^{n} \sum_{j\le l} F_l F_j E^{2(2n-l-j)} (\hat{x}_j + (l-j)T - \hat{x}_l)^2\}\Big].$$

That unwieldly expression can be simplified considerably. The first step is to note that in the first term the sum over j and the expression to the right form the case where j=l in the second term, so

$$= -(1/2)\Big[\{\sum_{l=1}^{n} (\sigma^2/(E^2 F_l F_{l+1}))\sum_{j=1}^{l} \sum_{k\le j} F_j F_k E^{2(2l-j-k)} (\hat{x}_k + (j-k)T - \hat{x}_j)^2\}$$

$$+\{(1/F_{n+1})(1/\sum_{l=1}^{n} F_l E^{2(n-l)})\sum_{j=1}^{n} \sum_{k\le j} F_j F_k E^{2(2n-j-k)} (\hat{x}_k + (j-k)T - \hat{x}_j)^2\}\Big].$$

Then, the sum in the second multiplier in the second term can receive the benefit of the 'sum of the F_l's lemma 9).

$$= -(1/2)\Big[\{\sum_{l=1}^{n} (\sigma^2/(E^2 F_l F_{l+1}))\sum_{j=1}^{l} \sum_{k\le j} F_j F_k E^{2(2l-j-k)} (\hat{x}_k + (j-k)T - \hat{x}_j)^2\}$$

$$+\{(\sigma^2/(F_{n+1}(F_{n+1} - E^2 F_n)))\sum_{j=1}^{n} \sum_{k\le j} F_j F_k E^{2(2n-j-k)} (\hat{x}_k + (j-k)T - \hat{x}_j)^2\}\Big].$$

Then, the second term may be combined with the case where l=n in the first term to get

$$=-(1/2)\Big[\{\sum_{l=1}^{n-1}(\sigma^2/(E^2F_lF_{l+1}))\sum_{j=1}^{l}\sum_{k\le j}F_jF_kE^{2(2l-j-k)}(\hat{x}_k+(j-k)T-\hat{x}_j)^2\}$$

$$+\{[(\sigma^2/(E^2F_nF_{n+1}))+(\sigma^2/(F_{n+1}(F_{n+1}-E^2F_n)))]\cdot\sum_{j=1}^{n}\sum_{k\le j}F_jF_kE^{2(2n-j-k)}(\hat{x}_k+(j-k)T-\hat{x}_j)^2\}\Big]$$

Using some algebra to simplify the multiplier in the second term

$$19)\quad =-(1/2)\Big[\{\sum_{l=1}^{n-1}(\sigma^2/(E^2F_lF_{l+1}))\sum_{j=1}^{l}\sum_{k\le j}F_jF_kE^{2(2l-j-k)}(\hat{x}_k+(j-k)T-\hat{x}_j)^2\}$$

$$+\{[(\sigma^2/(E^2F_n(F_{n+1}-E^2F_n)))\cdot\sum_{j=1}^{n}\sum_{k\le j}F_jF_kE^{2(2n-j-k)}(\hat{x}_k+(j-k)T-\hat{x}_j)^2\}\Big].$$

Then, all that remains is to use the telescoping sum lemma and cast off the $\hat{x}_{i+1}-\hat{x}_i-(\hat{x}-\hat{x}_m)$ terms (since they are constant with respect to t.

$$=-(\sigma^2/2E^2)\{\sum_{l=1}^{n-1}(1/(F_lF_{l+1}))\sum_{j=1}^{l}\sum_{k\le j}F_jF_kE^{2(2l-j-k)}\cdot(j-k)\sum_{i=k}^{j-1}(\hat{x}_{i+1}-\hat{x}_i-T)^2\}$$

$$-(\sigma^2/2E^2)\{(1/(F_nF_{n+1}))\sum_{j=1}^{n}\sum_{k\le j}F_jF_kE^{2(2n-j-k)}\cdot(j-k)\sum_{i=k}^{j-1}(\hat{x}_{i+1}-\hat{x}_i-T)^2\}$$

Noting that $j-k=0$ when $j=k$; $\sum_{j=1}^{n}\sum_{k\le j}K(j-k)=\sum_{j=2}^{n}\sum_{k<j}K(j-k)$, so

$$g(T)=-(\sigma^2/(2E^2))(U(T)+V(T))+K$$

So, g(T) may be described as weighted squared differences between T and the differences between adjacent points.

U(T) and V(T) as Sums Over Differences Between Adjacent Points

The next step is to simplify 16) by repeatedly using the 'interchange of sum indices' lemma., e.g.

$$20)\quad U(T)=\sum_{i=1}^{n-2}(\hat{x}_{i+1}-\hat{x}_i-T)^2\sum_{l=i+1}^{n-1}\sum_{j=i+1}^{l}\sum_{k=1}^{i}(j-k)E^{2(2l-j-k)}F_jF_k/(F_lF_{l+1})\ ;\ \text{and}$$

$$21)\quad V(T)=\sum_{i=1}^{n-1}(\hat{x}_{i+1}-\hat{x}_i-T)^2\sum_{j=i+1}^{n}\sum_{k=1}^{i}(j-k)E^{2(2n-j-k)}F_jF_k/(F_n(F_{n+1}-E^2F_n))\ .$$

The proof of each involves repeated and straightforward use of the two interchange of sums lemmas.

Summing the Weights Over j and k

To make the expressions for U(T) and V(T) more tractable, the last two sums should be simplified. Their sum is

$$22)\quad \sum_{j=i+1}^{l}\sum_{k=1}^{i}(j-k)E^{2(2l-j-k)}F_jF_k$$

$$=(E^{2(l-i)}/\sigma^4)\{F_{i+1}[(l-i)F_{l+1}-(l-i-1)E^2F_l+E^{2l}]$$
$$-E^2F_i[(l-i+1)F_{l+1}-(l-i)E^2F_l+E^{2l}]-E^{2i}[F_{l+1}-E^2F_l]\}$$

The proof involves using the lemmas proved earlier for the sum of the F_l's (equations 9) and 10)) and the sum of the iF_l's lemmas (equations 11) and 12)). The first step is to split the j-k term and pull the constants across the 'k' sum.

$$\sum_{j=i+1}^{l}\sum_{k=1}^{i}(j-k)E^{2(2l-j-k)}F_jF_k=\sum_{j=i+1}^{l}E^{2(2l-i-j)}F_j\{[j\sum_{k=1}^{i}E^{2(i-k)}F_k]-\sum_{k=1}^{i}kE^{2(i-k)}F_k\}$$

Using equations 9) and 11) on the two sums,

$$= \sum_{j=i+1}^{l} E^{2(2l-j-i)} F_j \{ [j(F_{l+1}-E^2F_l)/\sigma^2 - (iF_{l+1}-(i-1)E^2F_l+E^{2l})/\sigma^2 \}$$

Pulling out the terms that are constant with respect to j,

$$= (1/\sigma^2) E^{2(l-i)} \{ (F_{l+1}-E^2F_l) [\sum_{j=i+1}^{l} jE^{2(l-j)} F_j]$$

$$- (iF_{l+1}-(i-1)E^2F_l+E^{2l}) [\sum_{j=i+1}^{l} E^{2(l-j)} F_j] \}$$

Summing the 'j' sums using equations 10) and 12)

$$= (E^{2(l-i)}/\sigma^2 \Big\{ \frac{(F_{l+1}-E^2F_l)[lF_{l+1}-(l-1)E^2F_l-iE^{2(l-i)}F_{i+1}+(i-1)E^{2(l-(i-1))}F_i]}{\sigma^2}$$

$$- \frac{(iF_{l+1}-(i-1)E^2F_l+E^{2l})\cdot(F_{l+1}-E^2F_l-E^{2(l-i)}F_{i+1}+E^{2(l-(i-1))}F_i)}{\sigma^2} \Big\}$$

Multiplying those polynomials in the F's and collecting and cancelling terms produces

$$= \frac{E^{2(l-i)}}{\sigma^4} \{ F_{i+1} [(l-i)F_{l+1}-(l-i-1)E^2F_l+E^{2l}]$$

$$- E^2F_i [(l-i+1)F_{l+1}-(l-i)E^2F_l+E^{2l}] - E^{2i}[F_{l+1}-E^2F_l] \}.$$

Which is exactly equation 22).

Summing U(T) Over l

The sum over "l" in U(T) may be computed to produce

$$23)\quad U(T)=(\frac{1}{\sigma^4})\sum_{i=1}^{n-2}(\hat{x}_{i+1}-\hat{x}_i-T)^2\cdot\Big\{E^2-(n-i)E^{2(n-i)}F_{i+1}/F_n-2E^4F_i/F_{i+1}$$

$$\cdot\Big\{E^2-(n-i)E^{2(n-i)}F_{i+1}/F_n-2E^4F_i/F_{i+1}+(n-i+1)E^{2(n-i+1)}F_i/F_n-E^{2(i+1)}/F_{i+1}+E^{2n}/F_n\Big\}$$

$$+E^{-2i}(F_{i+1}-E^2F_i)\,[\sum_{l=i+1}^{n-1}(E^{4l}+2E^{2(l+1)}F_l)/(F_lF_{l+1})]\Big\}$$

Before I show that, let me note that U has become too long to be tractable. So, let me break it up into three terms. Using equations 20) and 22)

$$U(T)=\sum_{i=1}^{n-2}(\hat{x}_{i+1}-\hat{x}_i-T)^2\cdot\sum_{l=i+1}^{n-1}(E^{2(l-i)}/\sigma^4)\Big\{F_{i+1}[(l-i)F_{l+1}-(l-i-1)E^2F_l+E^{2l}]$$

$$-E^2F_i[(l-i+1)F_{l+1}-(l-i)E^2F_l+E^{2l}]-E^{2i}[F_{l+1}-E^2F_l]\Big\}/(F_lF_{l+1})\,.$$

Pulling out the constant terms and collecting coefficients produces

$$24)\quad U(T)=(1/\sigma^4)\sum_{i=1}^{n-2}(\hat{x}_{i+1}-\hat{x}_i-T)^2[A_i-B_i-C_i]\,,$$

where

$$25)\quad A_i=E^{-2i}F_{i+1}\sum_{l=i+1}^{n-1}[((l-i)E^{2l}/F_l)-((l-i-1)E^{2(l+1)}/F_{l+1})+(E^{4l}/(F_lF_{l+1}))]\,,$$

$$26)\quad B_i=E^{-2(i-1)}F_i\sum_{l=i+1}^{n-1}[((l-i+1)E^{2l}/F_l)-((l-i)E^{2(l+1)}/F_{l+1})+(E^{4l}/(F_lF_{l+1}))]\,,$$

$$27)\quad C_i=\sum_{l=i+1}^{n-1}[(E^{2l}/F_l)-(E^{2(l+1)}/F_{l+1})]\,,$$

Next, I must simplify each expression. Note that the second term within the sum of A is nearly the first term evaluated at a higher index. E.g.

$$A_i = E^{-2i}\Big\{ [\sum_{l=i+1}^{n-1} (l-i)E^{2l}/F_l]$$

$$- \sum_{l=i+1}^{n-1} (l-i-1)E^{2(l+1)}/F_{l+1}] + [\sum_{l=i+1}^{n-1} E^{4l}/(F_l F_{l+1})]\Big\}$$

$$= E^{-2i}\Big\{ [\sum_{l=i+1}^{n-1} (l-i)E^{2l}/F_l] - \sum_{l=i+2}^{n} (l-i)E^{2l}/F_l]$$

$$+ [\sum_{l=i+1}^{n-1} 2E^{2(l+1)}/(F_{l+1}] + [\sum_{l=i+1}^{n-1} E^{4l}/F_l F_{l+1})]\Big\}.$$

Then, the second and third term telescope to produce

$$= E^{-2i}F_{i+1}\Big\{ (E^{2(i+1)}/F_{i+1}) - (n-i)E^{2n}/F_n$$

$$+ [\sum_{l=i+1}^{n-1} 2E^{2(l+1)}/F_{l+1}] + [\sum_{l=i+1}^{n-1} E^{4l}/(F_l F_{l+1})]\Big\}.$$

Then, combining the last two terms, and distributing the multiplier

$$28)\quad A_i = E^2 - (n-i)E^{2(n-i)}F_{i+1}/F_n$$

$$+ E^{-2i}F_{i+1} \cdot [\sum_{l=i+1}^{n-1} (E^4 + 2E^{2(l+1)}F_l)/(F_l F_{l+1})]\ .$$

Simplifying B_i in a similar fashion produces

$$29)\quad B_i=\frac{2E^4F_i}{F_{i+1}}-\frac{(n-i+1)E^{2(n-i+1)}F_i}{F_n}$$

$$+E^{-2(i-1)}F_i\left[\sum_{l=i+1}^{n-1}\frac{(E^{4l}+2E^{2(l+1)}F_l)}{(F_lF_{l+1})}\right]$$

Simplifying C is simpler. The sums telescope to produce

$$30)\quad C_i=\left[\sum_{l=i+1}^{n-1}\frac{E^{2l}}{F_l}\right]-\left[\sum_{l=i+1}^{n-1}\frac{E^{2(l+1)}}{F_{l+1}}\right]$$

$$=\frac{E^{2(i+1)}}{F_{i+1}}-\frac{E^{2n}}{F_n}$$

Then, combining equations 28) for A_i, 29) for B_i, and 30) for C_i into equation 24)

$$U(T)=(1/\sigma^4)\sum_{i=1}^{n-2}(\hat{x}_{i+1}-\hat{x}_i-T)^2\Big\{E^2-\frac{(n-i)E^{2(n-i)}F_{i+1}}{F_n}-\frac{2E^4F_i}{F_{i+1}}+\frac{(n-i-1)E^{2(n-i-1)}F_i}{F_n}$$

$$-\frac{E^{2(i+1)}}{F_{i+1}}+\frac{E^{2n}}{F_n}+E^{-2i}[F_{i+1}-E^2F_i]\left[\sum_{l+i+1}^{n-1}\frac{(E^{4l}+2E^{2(l+1)}F_l)}{(F_lF_{l+1})}\right]\Big\}.$$

Which is exactly equation 23).

Summing V(T)

V(T) may also be summed to produce

$$31)\quad V(T) = (1/\sigma^4)\sum_{i=1}^{n-1}(\hat{x}_{i+1}-\hat{x}_i-T)^2\Big\{(n-i)E^{2(n-i)}F_{i+1}/F_n$$

$$+\frac{E^{2(n-i-1)}F_{i+1}}{(F_{n+1}-E^2F_n)}+\frac{E^{2(2n-i)}(F_{i+1}-E^2F_i)}{(F_n(F_{n+1}-E^2F_n))}$$

$$-\;(n-i-1)E^{2(n-i+1)}\left(\frac{F_i}{F_n}\right)-\frac{E^{2(n-i+2)}F_i}{(F_{n+1}-E^2F_n)}-\frac{E^{2n}}{F_n}\Big\}$$

The proof requires using the equation for the sum over j and k (22) on equation 21). Then, simple algebra produces the result.

Combining U(T) and V(T)

Now that the sums in U(T) and V(T) have been simplified, the next step is to combine them to produce the complete weights

$$32)\quad g(T) = -\left(\frac{\sigma^2}{(2E^2)}\right)(U(T)+V(T))+K$$

$$= K-\left(\frac{1}{(2E^2\sigma^2)}\right)\Big\{\sum_{i=1}^{n}(\hat{x}_{i+1}-\hat{x}_i-T)^2$$

$$\cdot[E^2-\left(\frac{2E^4F^i}{F_{i+1}}\right)-(E^{2(i+1)}/F_{i+1})+E^{-2i}(F_{i+1}-E^2F_i)G_i]\Big\}$$

To prove it, we need to combine equation 23) for U(T) and equation 31) for V(T) and simplify the result. Combining the two equations produces

$$g(T) = K - (1/(2E^2\sigma^2))\Big\{\sum_{i=1}^{n-2} (\hat{x}_{i+1}-\hat{x}_i-T)^2\Big[E^2-(n-i)E^{2(n-i)}F_{i+1}/F_n$$

$$-2E^4F_i/F_{i+1}+(n-i+1)E^{2(n-i+1)}F_i/F_n$$

$$-E^{2(i+1)}/F_{i+1}+E^{2n}/F_n+E^{-2i}(F_{i+1}-E^2F_i)\Big[\sum_{l+i+1}^{n-1}(E^{4l}+2E^{2(l+1)}F_l)/(F_lF_{l+1})\Big]\Big]\Big\}$$

$$-(1/(2E^2\sigma^2))\Big\{\sum_{i=1}^{n-1}(\hat{x}_{i+1}-\hat{x}_i-T)^2\Big[(n-i)E^{2(n-i)}F_{i+1}/F_n+E^{2(n-i+1)}F_{i+1}/(F_{n+1}-E^2F_n)$$

$$+E^{2(2n-i)}(F_{i+1}-E^2F_i)/(F_n(F_{n+1}-E^2F_n))$$

$$-(n-i+1)E^{2(n-i+1)}F_i/F_n-E^{2(n-i+2)}F_i/(F_{n+1}-E^2F_n)-E^{2n}/F_n\Big]\Big\}$$

That is an incredibly long expression. But thankfully, many of the U and V terms cancel or combine (at least for i between 1 and n-2) to produce

$$g(T) = K-(1/(2E^2\sigma^2))\Big\{\sum_{i=1}^{n-2}(\hat{x}_{i+1}-\hat{x}_i-T)^2\Big[E^2-2E^4F_i/F_{i+1}-E^{2(i+1)}/F_{i+1}$$

$$+E^{-2i}(F_{i+1}-E^2F_i)\cdot\Big[(E^{4n}+E^{2(n+1)}F_n)/(F_n(F_{n+1}-E^2F_n))$$

$$+\sum_{l=i+1}^{n-1}(E^{4l}+2E^{2(l+1)}F_l)/(F_lF_{l+1})\Big]\Big]\Big\}$$

$$-(1/(2E^2\sigma^2))(\hat{x}_n-\hat{x}_{n-1}-T)^2\Big\{E^2+E^4F_n/(F_{n+1}-E^2F_n)+E^{2(n+1)}(F_n-E^2F_{n-1})/(F_n(F_{n+1}-E^2F_n$$

$$-2E^4F_{n-1}/F_n-E^6F_{n-1}/(F_{n+1}-E^2F_n)-E^{2n}/F_n\Big\}$$

Then, noting that the definition of the G_i from equation 2), and combining some of the terms in the second product

$$g(T)=K-(1/(2E^2\sigma^2))\Big\{\sum_{i=1}^{n-2}(\hat{x}_{i+1}-\hat{x}_i-T)^2\big[E^2-2E^4F_i/F_{i+1}-E^{2(i+1)}/F_{i+1}$$

$$+E^{-2i}(F_{i+1}-E^2F_i)G_{i+1}-(1/2E^2\sigma^2))(\hat{x}_n-\hat{x}_{n-1}-T)^2\cdot\big\{E^2+E^4(F_n-E^2F_{n-1})/(F_{n+1}-E^2F_n)$$

$$-E^{2(n+1)}(F_n-E^2F_{n-1})/(F_n(F_{n+1}-E^2F_n))-2E^4F_{n-1}/F_n-E^{2n}/F_n)$$

Then, combining some of the terms applied to $(\hat{x}_n-\hat{x}_{n-1}-T)^2$

$$g(T)=K-(1/2E^2\sigma^2))\Big\{\sum_{i=1}^{n-2}(\hat{x}_{i+1}-\hat{x}_i-T)^2\big[E^2-2E^4F_i/F_{i+1}-E^{2(i+1)}/F_{i+1}+E^{-2i}(F_{i+1}-E^2F_i)G_{i+1}\big]\Big\}$$

$$-(1/2E^2\sigma^2)(\hat{x}_n-\hat{x}_{n-1}-T)^2\cdot\big\{E^2-2E^4F_{n-1}/F_n-E^{2n}/F_n+E^{-2(n-1)}(F_n-E^2F_{n-1})G_n\big\}$$

Which yields the result in 32).

$$g(T)=K-(1/(2E^2\sigma^2))\sum_{i=1}^{n-1}(\hat{x}_{i+1}-\hat{x}_i-T)^2\big[E^2-2E^4F_i/F_{i+1}-E^{i+1}/F_{i+1}+E^{-2i}(F_{i+1}-E^2F_i)G_i\big]$$

Which could be restated as

$$33)\quad g(T)=K-(1/(2E^2\sigma^2))\sum_{i=1}^{n-1}(\hat{x}_{i+1}-\hat{x}_i-T)^2W_i,$$

Where the W_i are the weights from 1) that should be the weights used to average the $(\hat{x}_{i+1}-\hat{x}_i)$'s to produce T'.

The Final Formula

Producing the final estimate is now a fairly straightforward process of using the weighted sum of squares theorem from appendix II to restate g(T), and then showing T' minimizes it.

Applying the weighted sum of squares theorem to equation 33) produces

$$g(T)=K-(1/(2E^2\sigma^2))\,(\sum_{i=1}^{n-1} W_i)\cdot(T-[(\sum_{i=1}^{n-1} W-i(\hat{x}_{i+1}-\hat{x}))/\sum_{i=1}^{n-1} W_i])^2$$

+ other terms that do not involve T.

Combining the first and last terms into the constant

$$g(T)=K_1+K_2(T-[(\sum W_i(\hat{x}_{i+1}-\hat{x}_i))/\sum W_i])^2$$

Which is clearly maximized by setting

$$34)\quad T=T'=[\sum_{i=1}^{n-1} W_i(\hat{x}_{i+1}-\hat{x}_i)]/\sum_{i=1}^{n-1} W_i$$

So, T' is the best estimator

REFERENCES/FOOTNOTES

[1] This model assumes that the average size of the changes is proportional solely to the current price level, and is independent of historical price increases. One could make an argument that sellers try to follow the current trend of the price level. That of course would lead to inflation creating more inflation. The author believes the model in this paper is a good first step, and that reflecting the current trend of the price level is a logical enhancement.

[2] Robert V. Hogg, Stuart A. Klugman, *Loss Distributions* (New York, New York: John Wiley & Sons, 1984), pp. 21-22.

[3] Casualty Actuarial Society, *Exposure Draft, Casualty Contingencies, Chapter Fourteen, Reinsurance, Retentions and Surplus*, (Atlanta, Georgia: Educational Foundation, Inc., 1975) pp. 24-25.

[4] In practice, each $\epsilon(t)$ may have a different variance, corresponding to the number of customers insured. Realistically, they are usually close to constant. Further, the model may be refined for the case where the E's vary over time.

[5] Per the definition of a marginal distribution. See, for instance pp. 66 in *Introduction to Mathematical Statistics* by Robert V. Hogg and Allen T. Craig, Fourth Edition, (New York, New York: Macmillan, 1978)

[6] This follows from the fact that when you have n samples from a $N(\mu,\sigma^2)$ distribution their mean is distributed $N(\mu,\sigma^2/n)$.

[7] Bradley Efron, 'Controversies in the Foundation of Statistics', originally printed in the *American Mathematical Monthly*, Volume 85, Number 4, April 1978, pp. 231-238. Reprinted in the *Casualty Actuarial Society Forum*, Fall 1991, pp. 259-275. See pp. 266 in the latter.

[8] Further, one could create non-uniform diffuse prior distributions by setting $\int Ih(x)\,dx=1$ when $\int h(x)\,dx=\infty$. Then $x\sim Ih(x)$ has a non-uniform diffuse prior distribution.

[9] A careful reader might point out that the function e_{n+t} is dependent on T, so I cannot isolate all the T-terms into G(T). But that is irrelevant. If I express the probability function as $K_1\exp(-K_2(T^*-T)^2-K_3(e_{n+t}-x_{n+t})^2+K_4)$ and then set $T=T^*$ and $x_{n+t}=e_{n+t}(T)$; the resulting $K_1\exp(K_4)$ is still the largest attainable probability for $\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n$.

A METHOD TO INCLUDE MULTIPLE YEARS OF DATA IN A COMPANY'S RATE INDICATION

George R. Busche

A METHOD TO INCLUDE MULTIPLE YEARS OF DATA IN A COMPANY'S RATE INDICATION

GEORGE BUSCHE

Abstract

It is the contention of this paper that the renewal retention ratio can be used in an ad hoc method to adjust indications to reflect the degree of stability. If an insurer has a stable book of business, as reflected by a high constant renewal retention ratio, the years used in the indication should be given similar weight. Unstable or low renewal retention ratios will cause older years to have less weight. In addition, as more years are added to an indication, the older years' data should have a decreasing influence on credibility. The renewal retention ratio can also measure this effect.

A METHOD TO INCLUDE MULTIPLE YEARS OF DATA IN A COMPANY'S RATE INDICATION

Introduction

Almost all rate indications can contain various weighting schemes when combining years of data to produce the indicated rate level. In addition, by adding more years of data to a state's indication, one may increase the credibility factor applied to the state indication.

This paper describes the renewal retention ratio and how it can be used to affect an actuarial indication. The first part defines the renewal retention ratio. Next is a description of two ad hoc refinements to the rate indication utilizing the renewal retention ratio of the book of business. First, the renewal retention ratio can be used in a method to assign weights to the multiple years of data that may be incorporated in the rate indication. Then, the renewal retention ratio can be used in developing the credibility factor of the experience period.

The Renewal Retention Ratio

The renewal retention ratio (RRR) is the percentage of inforce business that renewed in a given year. This ratio can vary by line of business, agency plant, geographical area, the number of years insured with the company, and the size of the account. Its complement is the lapse ratio (LR) which describes the percentage of inforce business that does not renew in a given year. That is, RRR = 1 - LR.

These ratios can be influenced by the current insurance environment, such as the underwriting cycle, the company's experience, recent rate revisions and any underwriting audits. If a company chooses to cancel or not renew a large portion of a book of business during a hard market, the renewal retention ratio would be reduced. Adverse experience, significant rate increases and underwriting audits would also tend to decrease the renewal retention ratio.

The lapse ratio or renewal retention ratio can be incorporated into the rate indication to reflect the stability of the book of business. Either premium or policy counts can be used to calculate the ratio. The preferred choice would be premium because the ratio would be applied in the weighting scheme directly to the earned premium. However, policy counts can be used to develop the ratio for the following reasons:

1) Availability. A company is more likely to possess statistics on renewal pricing by policy counts than by premium amounts.

2) Simplicity. Both renewal and nonrenewal counts have the same definition. The premium for canceled or nonrenewed policies would have to be estimated in addition to the premium for the renewed policy. This premium estimation for policies no longer inforce would require additional time and expense.

3) If one believes that the renewal retention ratio is similar across various policy size segments of the data base, the assumption could be made that the renewal retention ratio will not vary by size of risk.

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Exhibit I describes the calculation of the lapse ratio and the renewal retention ratio based on policy counts. It should be noted that column (2) includes only policies-in-force at the time of renewal for the particular effective month. That is, if a policy was canceled three months prior to renewal, it would not be included in column (2). It is assumed that midterm cancellations are few in number and usually are not influenced by the insurance environment. An example of this would be an insured who cancels his policy because of the selling of his property. In addition, midterm cancellations are a data item that is not as easily available within a company. The nonrenewal of the policies listed under column (3) can be due to either a decision of the company or the insured. The nonpayment of premium at inception would be considered under column (3). Since these nonpayments are not necessarily known until a few months after the effective date, the count for policies nonrenewing (column {3}) could increase in subsequent reports for the last few effective months.

The Application

The inclusion of the renewal retention ratio in the rate indication is intended to adjust the data for items that may produce instability. Frequently, rate indications require judgment factors. The renewal retention ratio can assist in improving the indication by applying an alternative ad hoc weighting method. The example used to highlight these refinements will be based upon a commercial fire indication. However, the adjustments can be applied to an indication for any line of business, even an indication using as few as two years of experience.

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Exhibit II displays a typical indication procedure for Company XYZ for commercial fire based on the conventional approach. The three-way credibility weighting procedure (line (13)) is different from that used in the traditional fire indication. The equation in line (13) was used for Company XYZ to reflect specific aspects of its operation and book of business. Half of the state's credibility complement was applied to industry data and the other half of the complement was applied to Company XYZ's countrywide indication for commercial fire. The industry experience is adjusted to Company XYZ's rate level and is intended to reflect the large body of risks the insurer could write as new business. The countrywide indication is intended to reflect the underwriting, marketing, and pricing philosophy unique to Company XYZ.

It should also be noted, that the credibility standard may vary by company for a line of business based upon the degree of risk, variability, and/or confidence the management of a company is willing to accept for the indication of a line of business. If less risk or variability and more confidence is required, the selected value of K would increase to possibly $25,000,000. If more risk or variability and less confidence is acceptable, K may be selected as $5,000,000.

Other than the brief explanation as to why the indication in Exhibit II may vary from a more traditional rate indication approach, this paper is not intended to discuss in detail the credibility standard or the specifics of the existing rate indication. IT NEEDS TO BE EMPHASIZED THAT THE ADJUSTMENTS USING THE RENEWAL RETENTION RATIO ARE AD HOC MODIFICATIONS TO A COMPANY'S ALREADY EXISTING RATE INDICATION PROCEDURE AND CREDIBILITY STANDARD.

As can be seen in Exhibit II, the rate indication for the state is +1.5%. The renewal retention ratio will be used to adjust this indication for stability or for the lack of stability by determining the appropriate weights to be used in column (4). In addition, the renewal retention ratio will be used to adjust the credibility factor in line (12).

It should be noted that the state's rate indication for the line of business could be developed by specifically excluding from consideration the experience of lapsed policyholders. However, it is recommended that weights and credibility be assigned to the entire body of data for the following reasons:

a) A company may not be able to segregate data for inforce policies from those that canceled or nonrenewed. Even if it was possible, it would add time and costs to the evaluation.

b) State regulators typically require the company's data that is used in a rate filing to balance to some form of financial reporting such as Page 14 of the Annual Statement. Excluding data may cause the regulators to question the validity of the indication.

c) Indications based only on the experience of inforce business could guarantee an inadequate rate level. That is, to the extent that lapsed business is worse than inforce business, the lower rate level indication may suggest and produce rate levels that are not anticipated to be unprofitable, but will likely be unprofitable.

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Weighting the Years

The number of years used in an indication is normally based upon tradition. Likewise, the weighting scheme is also based upon tradition. For example, a commercial fire indication uses five years of data weighted 10%, 15%, 20%, 25% and 30%, with the largest weight going to the most recent year. This increasing pattern implies that the more recent years are more responsive when indicating the prospective results.

The method below calculates the weighting scheme to be applied to the years of data based upon the stability of the book of business as measured by the renewal retention ratio. Equal weights would be applied to each year for a completely stable book of business. That is RRR = 1.0, meaning every policyholder renewed each year. If only a portion of the policyholders renewed each year, an increasing weighting scheme would result with the more recent years receiving the greater weights. If no policyholders renewed, or RRR = 0.0, only the latest year should be used in the state indication.

It should be noted that if the trended experience is identical for each year, then any weighting scheme would produce the same expected rate indication. The variability in the trended loss ratio experience between each year could imply that the experience from older years deserve less weight.

The weights that are applied to the years of data could also be based upon other factors besides just the renewal retention ratio. Two factors that come to mind are the trend factor and the loss development factor. Any positive trend would result in more weight to the more recent years. That is, the lower the trend factor as determined from the positive trend, the more stable the data base. Likewise, lower loss development factors would indicate data that is more stable or predictable. For long-tailed lines one would give more weight to older years than short-tailed lines. Overall, one could develop some weighting scheme that incorporates the renewal retention ratio, trend factors, and loss development factors. (See Appendix A for a possible approach.)

The renewal retention ratio can be calculated using policies on a state, branch, region, or countrywide basis. Usually, for a company's indication, a countrywide renewal retention ratio is sufficient to reflect the insurer's desire to retain its book of business for the line of business. However, adjustments to the renewal retention ratio can be made to reflect unique circumstances for a given state such as an underwriting audit. Often, actuaries have been asked to consider the effect of audits when determining a rate indication. This is usually true if the audit results in the nonrenewal of a large portion of unprofitable experience. This refinement would be a way to account for the underwriting audit and its subsequent cancellations or nonrenewals.

Exhibit III reflects three different weighting schemes based upon renewal patterns. Part I deals with a constant renewal retention ratio of 85%. Each year, 85% of all policyholders renew. Part II describes historical ratios reflecting definitive characteristics such as the underwriting cycle, rate revisions, etc. Part III is identical to Part I except that 1990 contains a reduction in the renewal retention ratio due to an underwriting audit in late 1989. As a result of the audit, the company decided to not renew a large portion of its business in 1990 due to prior unprofitable results.

For the constant, high renewal retention ratio in Part I, the indicated weights are more flat (14%, 17%, 19%, 23%, and 27%) than the traditional weights (10%, 15%, 20%, 25%, and 30%). More weight can be given to older years because of the high stable renewal retention ratio. With the historical renewal retention ratios in Part II, more weight is given to the more recent years because of the unstable and lower ratios in the earlier years. Part III, which reflects the effect of the underwriting audit, gives 54% weight to 1990 and 1991, while Part I only assigns a 50% weight to the same years. As a result, the effect of the underwriting audit and the subsequent cancellations were systematically considered in the rate indication.

Determining a Credibility Factor

Bailey and Simon have shown "that if an individual insured's chance for an accident remained constant from one year to the next and if there were no risks leaving the class or no new risks entering the class, the credibilities for experience periods of one, two and three years would be expected to vary approximately in proportion to the number of years[1]". They also demonstrated that the relative credibilities for two and three years are much less than 2.00 and 3.00 which is caused by risks entering and leaving the class. "But it can be fully accounted for only if an individual insured's chance for an accident changes from time to time within a year and from one year to the next, or if the risk distribution of individual insureds has a marked skewness reflecting varying degrees of accident proneness[2]". If this phenomenon is true for any line of business or block of business, then an older year's data should have less influence on credibility than the more recent year's data.

A way to measure the relevancy of a year's data is to use the percentage of insureds still with the company for the year being priced as calculated by the renewal retention ratio. Exhibit IV describes the calculation of the adjusted credibility Z' where P' is the five year adjusted premium. For each year, the estimated percent of insureds still with the insurer are multiplied by its current level earned premium. The result is an adjusted earned premium for each year. The total of all years equals P'. K is still the selected constant. In these examples K = 10,000,000.

All three parts produced credibility factors less than the .708 used in Exhibit II. One should expect premium from older years to have a decreasing influence on the credibility of the data. The intent of this ad hoc adjustment is to develop a methodology of combining multiple years of data. That is, a given credibility standard is being applied to the data base which consists of many years. For example, assume that full credibility is based on 683 claims. If the most recent year has 683 or more claims, that year is considered fully credible. If the data base used in the indication consists of 683 claims over 5 years, that experience should be considered fully credible only if all policyholders renewed each year. If only a portion renewed each year, the 683 claims over 5 years should not be considered fully credible. The renewal retention ratio systematically allows older years to be brought into the rate indication, but with less relevancy for older years.

Summary

Exhibit V describes the effect of the indication using the renewal retention ratio. The indication reflects the factors as calculated under Part III of both Exhibit III and Exhibit IV. As can be seen, the indication has increased from +1.5% to +4.3%.

Exhibit I

Calculation of Renewal Retention Ratio

(1) Policy Effective Month	(2) Number of Policies Eligible for Renewal	(3) Number of Policies Non-Renewing	(4) Lapse Ratio (3)/(2)
Jan.	123	20	.16
Feb.	86	10	.12
Mar.	87	12	.14
Apr.	94	8	.09
May	85	14	.16
June	63	8	.13
July	74	12	.16
Aug.	93	14	.15
Sep.	83	13	.16
Oct.	95	17	.18
Nov.	62	13	.21
Dec.	75	14	.19
Total	1,020	155	.15
		Renewal Retention Ratio	.85

COMPANY XYZ

STATEWIDE COMMERCIAL FIRE COVERAGE RATE LEVEL INDICATION

Proposed Effective Date: 04/01/92

(Reflecting Underwriting Audit and Renewal Retention Ratio)

Year	(1) Current Comm'l Fire* Earned Premiums	(2) Adjusted Comm'l Fire** Incurred Losses
1987	5,536,623	3,208,600
1988	5,201,269	3,308,180
1989	5,107,018	2,629,308
1990	4,078,421	1,645,927
1991	4,335,716	1,676,192

Year	(3) Rate Level Loss Ratio (2)/(1)	(4) Weights	(5) Loss Ratio Factor (3)x(4)
1987	.580	.10	.058
1988	.636	.15	.095
1989	.515	.20	.103
1990	.404	.25	.101
1991	.387	.30	.116

(6) Weighted Loss Ratio	=	.473
(7) Loss Ratio Including Loss Adjustment Expense (6) x 1.090	=	.516
(8) Expected Loss and Loss Adjustment Ratio	=	.531
(9) ISO Trended Loss and LAE Ratio for the State	=	.523
(10) Company's Average Deviation for the State	=	.873
(11) Company's Countrywide Indication	=	1.128
(12) State's Credibility Factor***	=	.708
(13) Credibility Weighted Loss and Loss Adjustment Expense Ratio (12) x (7) + ((1-(12))/2) x (9)/(10) + ((1-(12))/2) x (11) x (8)	=	.539
(14) Indicated Coverage Rate Change (13)/(8)	=	1.015
	or	+1.5%

*All premiums reflect current rate level.

**Incurred Losses are adjusted to current deductible and 04/01/93 cost levels.

***The credibility weight is calculated based on the formula Z = P/(P + K) where P is the five year premium and K is a constant equal to 10,000,000.

Exhibit III

CALCULATION OF WEIGHTS

I) Constant Renewal Retention Ratio

(1) Year	(2) RRR	(3) Percent Still With Company	(4) Weights (Normalized)
1987	--	.445	.14
1988	.85	.523	.17
1989	.85	.615	.19
1990	.85	.723	.23
1991	.85	.850	.27
1992*	.85	--	--
		3.156	

II) Historical Renewal Retention Ratio

(1) Year	(2) RRR	(3) Percent Still With Company	(4) Weights (Normalized)
1987	--	.211	.08
1988	.60	.352	.13
1989	.65	.542	.20
1990	.75	.723	.27
1991	.85	.850	.32
1992*	.85	--	--
		2.678	

III) Reflect Underwriting Audit

(1) Year	(2) RRR	(3) Percent Still With Company	(4) Weights (Normalized)
1987	--	.366	.13
1988	.85	.430	.15
1989	.85	.506	.18
1990	.70	.723	.25
1991	.85	.850	.29
1992*	.85	--	--
		2.875	

*Same as most recent year available which is 1991.

Exhibit IV

CREDIBILITY FACTOR

I) Constant Renewal Retention Ratio

(1) Year	(2) Percent Still With Company	(3) Current Fire Earned Premium	(4) Adjusted Fire Earned Premium (2) x (3)
1987	.445	5,536,623	2,463,797
1988	.523	5,201,269	2,720,264
1989	.615	5,107,018	3,140,816
1990	.723	4,078,421	2,948,698
1991	.850	4,335,716	3,685,359
		P' =	14,958,934
		Z' =	.599

II) Historical Renewal Retention Ratio

(1) Year	(2) Percent Still With Company	(3) Current Fire Earned Premium	(4) Adjusted Fire Earned Premium (2) x (3)
1987	.211	5,536,623	1,168,227
1988	.352	5,201,269	1,830,847
1989	.542	5,107,018	2,768,004
1990	.723	4,078,421	2,948,698
1991	.850	4,335,716	3,685,359
		P' =	12,401,135
		Z' =	.554

III) Reflect Underwriting Audit

(1) Year	(2) Percent Still With Company	(3) Current Fire Earned Premium	(4) Adjusted Fire Earned Premium (2) x (3)
1987	.366	5,536,623	2,026,404
1988	.430	5,201,269	2,236,546
1989	.506	5,107,018	2,584,151
1990	.723	4,078,421	2,948,698
1991	.850	4,335,716	3,685,359
		P' =	13,481,158
		Z' =	.574

Note: $Z' = P'/(P' + K)$
where $K = 10,000,000$

Exhibit V

COMPANY XYZ

STATEWIDE COMMERCIAL FIRE COVERAGE RATE LEVEL INDICATION

Proposed Effective Date: 04/01/92

(Reflecting Underwriting Audit and Renewal Retention Ratio)

Year	(1) Current Comm'l Fire* Earned Premiums	(2) Adjusted Comm'l Fire** Incurred Losses
1987	5,536,623	3,208,600
1988	5,201,269	3,308,180
1989	5,107,018	2,629,308
1990	4,078,421	1,645,927
1991	4,335,716	1,676,192

Year	(3) Rate Level Loss Ratio (2)/(1)	(4) Weights	(5) Loss Ratio Factor (3)x(4)
1987	.580	.13	.075
1988	.636	.15	.095
1989	.515	.18	.093
1990	.404	.25	.101
1991	.387	.29	.112

(6)	Weighted Loss Ratio	=	.476
(7)	Loss Ratio Including Loss Adjustment Expense (6) x 1.090	=	.519
(8)	Expected Loss and Loss Adjustment Ratio	=	.531
(9)	ISO Trended Loss and LAE Ratio for the State	=	.523
(10)	Company's Average Deviation for the State	=	.873
(11)	Company's Countrywide Indication	=	1.128
(12)	State's Credibility Factor***	=	.574
(13)	Credibility Weighted Loss and Loss Adjustment Expense Ratio (12) x (7) + ((1-(12))/2) x (9)/(10) + ((1-(12))/2) x (11) x (8)	=	.554
(14)	Indicated Coverage Rate Change (13)/(8)	=	1.043
		or	+4.3%

*All premiums reflect current rate level.

**Incurred Losses are adjusted to current deductible and 04/01/93 cost levels.

***The credibility weight is calculated based on the formula Z' = P'/(P' + K) where P is the five year adjusted premium and K is a constant equal to 10,000,000.

APPENDIX A
Weighing Schemes
Based on RRR, Trend, and Loss Development

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		Trend			Loss Development			Weights Average
Year	RRR (Part III)	Factor	Normalize	Offset {.2+.2-(3)}	Factor	Normalize	Offset {.2+.2-(6)}	(1)+(4)+(7) / 3
Commercial Fire								
1987	.13	1.01^5=1.051	.20	.20	1.00	.19	.21	.180
1988	.15	1.01^4=1.041	.20	.20	1.00	.19	.21	.186
1989	.18	1.01^3=1.030	.20	.20	1.00	.19	.21	.197
1990	.25	1.01^2=1.020	.20	.20	1.02	.19	.21	.220
1991	.29	1.01^1=1.010	.20	.20	1.30	.24	.16	.217
		5.152			5.32			
Medical Malpractice								
1987	.13	1.10^5=1.611	.24	.16	1.20	.12	.28	.190
1988	.15	1.10^4=1.464	.22	.18	1.30	.13	.27	.200
1989	.18	1.10^3=1.331	.20	.20	1.50	.15	.25	.210
1990	.25	1.10^2=1.210	.18	.22	1.80	.18	.22	.230
1991	.29	1.10^1=1.100	.16	.24	4.20	.42	-.02	.170
		6.716			10.00			

Footnotes

1. "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car," Robert A. Bailey and LeRoy J. Simon, P.C.A.S. XLVI, P160

2. Ibid

ASSET SHARE PRICING FOR PROPERTY-CASUALTY INSURERS

Sholom Feldblum

ASSET SHARE PRICING
FOR PROPERTY-CASUALTY INSURANCE

Biography

Sholom Feldblum is an Associate Actuary with the Liberty Mutual Insurance Company in Boston, Massachusetts. He was graduated from Harvard University in 1978 and spent the next two years as a visiting fellow at the Hebrew University in Jerusalem. He became a Fellow of the CAS in 1987, a CPCU in 1986, an Associate of the SOA in 1986, and a member of the American Academy of Actuaries in 1989. In 1988, while working at the Allstate Research and Planning Center in California, he served as President of the Casualty Actuaries of the Bay Area and as Vice President of Research of the Northern California Chapter of the Society of CPCU. In 1989, he served on the CAS Education and Testing Methods Task Force. He is presently a member of the CAS Syllabus Committee, the CAS Committee on Review of Papers, the Advisory Committee to the NAIC Casualty Actuarial (EX5) Task Force, and the Actuarial Advisory Committee to the NAIC Risk Based Capital Task Force, and he is the quarterly review editor for the *Actuarial Review*. Previous papers of his have appeared in *Best's Review*, the *CPCU Journal*, the *Proceedings of the Casualty Actuarial Society*, the *Actuarial Digest*, the *CAS Forum*, and the *CAS Discussion Paper Program*.

ASSET SHARE PRICING
FOR PROPERTY-CASUALTY INSURANCE

Abstract

Asset share pricing models are used extensively in life and health insurance premium determination. Property-Casualty rate making procedures consider only a single period of coverage. This is true for both traditional methods, such as loss ratio and pure premium rate making, and financial models, such as discounted cash flow or internal rate of return models.

This paper provides a full discussion of Property-Casualty insurance asset share pricing procedures. Section I compares life insurance to casualty insurance pricing. It notes why asset share pricing is so important for the former and how it applies to the latter as well. Section II describes the considerations essential for an asset share pricing model. Premiums, claim frequency, claim severity, expenses, and persistency rates must be examined by time since inception of the policy. Appropriate discount rates must be selected for (a) present values of the contract cash flows during each policy year and for (b) the present value of future earnings at the inception date of the policy.

Sections III through VII present four illustrations of asset share pricing:

- Section III is a general introduction.

- Section IV illustrates pricing considerations for an expanding book of business. Since both loss costs and expense costs are higher for new business than for renewal business, traditional loss ratio or pure premium pricing methods show misleading rate indications.

- Section V discusses classification relativities. Since persistency rates and coverage combinations differ by classification, the traditional relativity analyses may be erroneous.

- Section VI presents a competitive strategy illustration. Premium discounts and surcharges affect retention rates, particularly among policyholders who can obtain coverage elsewhere.

- Section VII shows how underwriting cycle movements can be incorporated into pricing strategy. Expected future profits vary with the stage of the cycle; these future earnings and losses must be considered when setting premium rates.

Section VIII discusses several types of profitability measures: returns on premium, returns on surplus or equity, internal rates of return, and the number of years until the policy becomes profitable. Traditional financial pricing models examine a single contract period and multiple loss payment periods. For asset-share pricing, these models are expanded to consider multiple contract periods. For instance, the "return on premium" is the present value of future expected profits divided by the present value of future expected premium, not the single period undiscounted amounts used for operating ratios.

Asset share models determine the long-run profitability of the insurance operations, the true task of the pricing actuary.

ASSET SHARE PRICING
FOR PROPERTY-CASUALTY INSURANCE

Table of Contents

ASSET SHARE PRICING
FOR PROPERTY-CASUALTY INSURANCE

SECTION I: INTRODUCTION

Asset share pricing models have long been used for life and health insurance premium development. These models examine the profitability of the complete insurance contract, from its inception to its final termination, including all renewals of the policy. This paper applies asset share pricing methods to Property/Casualty insurance.

Asset share pricing is especially important when cash flows and reported income vary by policy year. For instance, a whole life policy issued to a standard rated 30 year old insured shows

- High expense costs the first year (often greater than the gross premium).
- Low mortality costs the first several years.
- Higher mortality costs in later years, as the policyholder ages and the underwriting selection "wears off."
- Statutory benefit reserves that are somewhat redundant after the second or third year, because of the conservative valuation mortality tables and interest rates; during the first several years, preliminary term reserves reduce the statutory liability.[1]

In property and casualty insurance, loss ratio and pure premium rate making methods predominate. Financial pricing models are often used to set underwriting profit targets, although these methods, like the traditional Property/Casualty rate making techniques, presume an insurance contract in effect for a single policy period. Most financial models examine the duration of loss payments, but they do not consider the duration of the insurance contract (Cummins [1990]).

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[1] On asset share pricing models for life insurance, see Anderson [1959], Huffman [1978], and Atkinson [1987]; for health insurance, see Bluhm and Koppel [1988]. Menge and Fischer [1935], page 131, explain the term "asset share" as "the equitable share of the policyholders in the assets of the company."

Life versus Casualty Rate Making

The differing rate making philosophies for life and health insurance versus property and casualty insurance stem from several factors:

1. Few individual life or health insurance policies may be cancelled or non-renewed by the insurer, except for non-payment of premium. In property and casualty insurance, particularly in the Commercial Lines, the carrier has the right to terminate the policy at the renewal date and often to cancel the policy in mid-term.[2]

2. Life and health insurance claim costs vary by duration since policy inception, for two reasons:

   - Policyholder age: mortality and morbidity costs rise as the insured ages.
   - Underwriting selection: medical questionnaires and examinations for life and health insurance lead to lower average initial benefit costs for insured lives. The effects of underwriting selection "wear off" after several years (cf. Dahlman [1989], page 5).

   In property and casualty insurance, the relationship between expected losses and duration since policy inception is less apparent.

3. Expenses show a similar pattern: Whole life commission rates are high in the initial year but low for renewals (Lombardi and Wolfe [1986]). For Property-Liability carriers using the independent agency distribution system, commission rates do not differ between the first year and renewal years.

4. Much life insurance is provided by level premium contracts. The premium exceeds the anticipated benefits during the early policy years, when the insured is young and healthy. In

[2] Renewability provisions in health insurance vary among contracts, though cancellable policies are proscribed in many jurisdictions (Barnhart [1960]). Many states now proscribe mid-term cancellations of Personal Automobile policies; others, such as California or Massachusetts, prohibit even non-renewals.

later years, anticipated costs exceed the premiums and are funded by the policy reserves built up in earlier years. In contrast, property and casualty insurance rates may be revised each year. No "policy reserves" are held to shift costs among accounting periods.

Developments in Casualty Insurance

These differences are valid, and asset share pricing is therefore more common for life and health insurance premium development. But Property/Casualty insurance is taking on several of the attributes that motivate asset share pricing.

1. Most Personal Lines insurance policies are now issued by direct writers, whose commission rates are higher in the first year than in renewal years.

2. Although the insurer may have the right to cancel or non-renew the contract, it rarely does so. Profitability depends on the stability of the book of business, and carriers seek to strengthen policyholder loyalty.

3. Expected loss costs are greater for new business than for renewal business. Most actuarial studies of this phenomenon have concentrated on Personal Automobile insurance, though it is valid for most other lines of business as well.

The question faced by all insurers is the same: *"Is it profitable to write the insurance policy?"* A financially strong carrier does not focus on reported results or cash flows for the current year. Rather, it examines whether the stream of future profits, from both the original policy year and from renewal years, justifies underwriting the contract. Asset share pricing enables the actuary to provide quantitative estimates of long-term profitability.

SECTION II: ASSET-SHARE COMPONENTS

Asset share pricing is not yet common in property and casualty insurance, for several reasons:

- The data needed are not always available.
- Casualty pricing techniques are still somewhat undeveloped.
- The casualty insurance policy allows great flexibility in premiums and benefit levels.
- Liability claim costs are uncertain, both in magnitude and in timing.

This section examines the qualitative influences on the asset share pricing components, to lay the groundwork for the quantitative model that follows.

A. Premiums

Premiums for whole life policies are set at policy inception, and they continue unchanged until the termination or forfeiture of the contract. Premiums for renewable term life policies are generally guaranteed for the first several years and illustrated for an additional ten or fifteen years. Similarly, policyholder dividends on participating contracts are often illustrated for the first twenty years.[3]

Property and casualty insurance premiums may be revised each year or half-year, and insurers do not illustrate the expected future premiums. In fact, premiums fluctuate widely from year to year, for a variety of reasons.

1. Inflation raises loss costs, and premiums are adjusted accordingly. Life insurance benefits, in contrast, are fixed in nominal terms.

2. Underwriting cycles raise and lower the premiums charged, whether by manual rate

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[3] The NAIC Life Insurance Solicitation Model Regulation requires that insurers illustrate surrender cost and net payment cost indices for 10 and 20 year durations (Black and Skipper [1987]). Premiums for some newer contracts, such as indeterminate premium and universal life policies, are harder to project for future years.

revisions or individual risk rating adjustments. Underwriting cycles are not found in individual life insurance.

3. The insured's classification or exposure may change from year to year. The Personal Auto insured may marry, the Workers' Compensation insured may expand its operations, and the Commercial Property risk may install fire protection equipment.[4] The classification of the individual life policyholder generally does not change after inception of the policy.[5]

In sum, the level premiums for traditional whole life insurance policies, versus the variable premiums for casualty products, has contributed to the greater reliance of life actuaries on asset-share pricing methods.

B. Claims

Mortality rates are stable from year to year, and the influences on mortality are well documented. We may not fully understand why sex has such a strong influence on mortality, but given an individual's age, sex, and physical condition, we can provide a life expectancy (Berin, Stolnitz, and Teitlebaum [1990]). At the inception of the insurance policy, the actuary can

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[4] See, for instance, Feldblum [1990B]: ". . . average loss costs vary over the life of a policy. For example, many young unmarried men are carefree drivers, less concerned with safety than with presenting a courageous image. Once they have married, begun careers, and borne children, they feel more responsibility, both individual and financial, for their families - and their driving habits improve accordingly. When their children become adolescents and start driving the family cars, auto insurance loss costs climb rapidly. But when the children leave home and the insured retires, the automobiles may be unused except for shopping trips and weekend vacations; automobile accidents become rare. Finally, when the driver enters his or her 70's, physiological health deteriorates and reactions are slowed. If the insured continues to drive, accident frequency increases." Similarly, Whitehead [1991], page 312, writes: "Changes in inherent risk over time – the typical 'life-cycle' of an insured with respect of individual private passenger automobile insurance is for the level of inherent risk to decline as the age of the insured and his level of driving experience and competence increases (at least until a relatively advanced age)."

[5] Minor exceptions exist. For instance, a substandard rated policyholder may be re-rated after several years upon submission of evidence of insurability (Woodman [1989]). Re-entry term insurance allows reclassification at the end of each select period (Galt [1989]; Jacobs [1984]).

estimate mortality rates for the insured's lifetime. Barring major wars or epidemics, the estimates should be accurate.

Claim rates in casualty insurance are more variable and less well understood. Why do urban drivers have higher Personal Auto claim frequencies than suburban residents have? Is traffic density higher in cities than in rural areas? Are road conditions worse in urban areas? Are suburban residents, who are friendly with the neighboring children, more careful drivers? Are there more attorneys in cities, and do they encourage accident victims to file claims? Does the type and extent of medical treatment differ between urban and rural areas? Are rural residents more familiar with insurance agents and brokers and less inclined to seek compensation from "impersonal" corporations?[6]

Claim rates in Workers' Compensation vary with economic conditions and with the operations of the insured. During recessions, when layoffs or plant closings are anticipated, many employees file Workers' Compensation claims for minor, non-disabling injuries that they would ignore in more prosperous times (Borba [1989]; Butler, Worrall, and Borba [1986]). When a firm expands quickly, with young, inexperienced workers, accidental injuries are more common (Worrall, Appel, and Butler [1987]).

In the commercial liability lines (Other Liability, Products Liability, Medical Malpractice, and Professional Liability), statutory enactments and judicial precedents affect the frequency of claims. Congressional passage of the CERCLA in 1980, with strict, several, and retroactive liability, encouraged the filing of environmental impairment claims (Hamilton and Routman

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6 Casualty actuaries are just beginning to examine these issues. On traffic density in urban and suburban areas, and on the contribution of suburban drivers to urban traffic, see Brissman [1980]. The importance of attorneys can be seen by comparing claims represented by attorneys and those not represented in urban and rural areas (AIRAC [1988; 1989]). The effects of "claims consciousness," or the proclivity to file insurance claims, can be measured by the ratio of Bodily Injury claims to Property Damage claims. The frequency of PD claims is primarily determined by the incidence of physical accidents. The frequency of BI claims is affected by claims consciousness and attorney involvement as well. The ratio of BI to PD claims varies by jurisdiction, and it is higher in cities than in rural areas (IRC [1990]; Woll [1991]). The type of medical practitioner, such as physician, chiropractor, or physical therapist, affects both claim frequency and severity (Marter and Weisberg [1991; 1991]; Weisberg and Derrig [1991; 1991]).

[1988]). State legislation modifying the statute of limitations and setting caps on awards has affected the filing of Medical Malpractice claims.

The stability of life insurance benefits versus the variability of casualty insurance losses is a second reason for the greater use by life actuaries of asset-share pricing methods. However, the fundamental issue is not the predictability of losses but the relationship of losses and expenses to persistency. The asset share model examines a particular policy and asks: *"Is this risk's expected profitability above or below the average for other insureds in its class?"* To answer this question, we examine three items: relative loss costs and expenses by policy year and persistency rates by classification.

Policy Duration and Claim Frequency

Policy duration has a strong influence on claim frequency, particularly in Personal Automobile, where new insureds have higher average loss ratios than renewal policyholders. Conning and Company [1988], pages 10-11, note that "Companies have acknowledged results which show new business loss ratios varying from 10% higher to more than 30% higher, depending on the line of business and the underwriting year." Older drivers, with lower average claim frequencies and loss ratios, are more common in an insurer's renewal book than in its new business (Feldblum [1990B]). Several Personal Auto writers provide "renewal discounts," which reflect the lower loss and expense costs after the first policy year.

Inexperience, Youth, Transience, and Vehicle Acquisition

The relationship between duration of the policy and expected claim frequency results from several factors. Drivers who apply for new auto insurance policies are likely to be inexperienced, young, or "transient" insureds. Also, they have often recently acquired the automobile itself, and they may be unaccustomed with the particular hazards of the vehicle.

1. *Experience:* Good driving habits are acquired over time; safety precautions are "second nature" for the experienced driver. Many accidents result from carelessness, not recklessness, so inexperienced drivers have high claim frequencies (Bailey and Simon [1959]).

2. *Youth:* Young drivers, both male and female, have higher than average claim frequencies, even after adjusting for driving experience. Young drivers with their own residences or automobiles have relatively new auto insurance policies. [Adolescent drivers living at home may be insured on their parents' policies. Since these drivers have high average claim frequencies, they cause a temporary reversal in the generally inverse relationship of frequency with policy duration.]

3. *Transience:* Many high risk drivers, such as young males, are "transient" insureds, in that they often drop their coverage with one carrier and purchase a policy from another. Termination rates for young male drivers are as high as 20-30%, for several reasons:

    - Young male drivers are more likely to voluntarily cancel their policies, perhaps because they move to other locations, they get married and switch to their wives' insurers, or they drop their coverage after an accident.

    - Company underwriters are more likely to cancel the coverage of a young male driver than that of an adult driver, since the young male driver is more likely to have caused an accident and be considered too risky to insure.

    - Young male drivers are likely to experience financial difficulties and fail to pay the required premiums.

    - Young male drivers with high premium payments have more incentive to shop around for cheaper coverage.[7]

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[7] See Feldblum [1990A], particularly Figure 7 and the accompanying discussion. Similarly, D'Arcy and Doherty [1989], page 38, speak of "poor risks that move from insurer to insurer as their true risk exposure is discovered." D'Arcy [1988], page 28, lists four reasons for the higher loss ratios of new business: "The inability to surcharge new insureds properly since less information is available, the higher loss potential of insurance shoppers who regularly shift from insurer to insurer in search of bargain coverage, the fact that new insureds include a high proportion of risks not wanted by other insureds, and the possibility that new insureds may be individuals unfamiliar with local driving conditions."

Many low-risk insureds, such as retired drivers in their 60's and 70's, have termination rates as low as 3 or 4%. Retired drivers have less information about marketplace prices, which younger persons may hear about at the workplace.[8] These low-risk "stable" insureds reduce the claim frequencies of renewal business compared to new business.

4. The duration since the inception of the policy is correlated with the time since acquisition of the automobile. Accident frequency often decreases with time since acquisition, as the insured becomes accustomed to the hazards of the particular vehicle. For instance, the insured may have purchased a second hand vehicle during the summer, only to discover that the car skids on icy December roads.

   The age of the vehicle (not the time since acquisition) is a classification dimension for physical damage coverages, since the value of the car declines over time. The time since acquisition of the vehicle, not its age, is important for liability coverages. The two classification dimensions are the same only when the insured purchases a new automobile. Contrast (i) a recently acquired 5 year old car with (ii) a new model car bought two years ago. The two year old car would have the higher physical damage rate relativity, and the 5 year old car would have the higher liability relativity.

The relationship between loss ratios and the duration since policy inception may also be affected by the carrier's reunderwriting actions. D'Arcy and Doherty [1989] suggest that "the accumulation of private information by the contracting insurer" causes declining loss ratios as the policy ages. The importance of this private information depends on the insurer's underwriting philosophy and on power of this information to predict future loss costs.[9]

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[8] Many policy "terminations" for older drivers result from death, poor health, or other reasons that prevent them from driving, not because they find a cheaper rate with another carrier. Thus, these drivers are not "transient" insureds.

[9] "Underwriting terminations" are less important than voluntary terminations in explaining the differences between young male and adult persistency rates in Personal Automobile insurance (Feldblum [1990A], Figure 8). However, underwriting terminations weed out the particularly poor risks, and so they may have a larger effect on the relationship between loss ratios and the duration since policy inception.

In Workers' Compensation, the loss engineering services provided by the insurer, as well as its encouragement of a safe work environment, reduce claim frequency among persisting insureds. Loss control studies can be expensive, and the insurance carrier lacks the incentive to undertake them for "transient" risks. Similarly, a successful loss control program initiated by the carrier will encourage the insured employer to retain the coverage.[10]

[10] The relationship between claim frequency and "transient" risks is also applicable to Workers' Compensation. Commenting on the unprofitability of small Workers' Compensation risks, Kormes [1936], pages 49-50, says: ". . . this group of risks, which unfortunately float from carrier to carrier, has a great influence on the unsatisfactory small risk situation . . ."

Small enterprises that mushroom during prosperous years often fail when the economy sours. Since these firms lack the funds for needed workplace safety measures and their workforce often consists of inexperienced employees, their occupational injury rates are high. Those firms that fail face additional costs: Since the employee's alternative to insurance payments is unemployment, claim filings are high.

C. Expenses

Insurance expenses are greater in the year the policy is first issued than in renewal years, since underwriting and acquisition expenses are incurred predominantly at policy inception. This is true for both "per policy" expenses, such as the costs of underwriting and setting up files, and "percentage of premium" expenses, such as commissions and premium taxes.

Life Insurance Expenses

Premium determination for life insurance policies incorporates these expense differences by policy year. For instance, Jordan [1975], page 133, gives the following illustration of a gross premium calculation (see also Neill [1977], pages 53-56):

$$G \bullet \ddot{a}_x = 1005(1+i/2)A_x + .75G + .2G(\ddot{a}_{x:\overline{2}|} - \ddot{a}_{x:\overline{1}|}) + .1G(\ddot{a}_{x:\overline{6}|} - \ddot{a}_{x:\overline{2}|}) + .05G(\ddot{a}_x - \ddot{a}_{x:\overline{6}|}) + 10 + 2a_x$$

where

G is the annual gross premium for $1000 of insurance, a_x, $\ddot{a}_x$, and A_x are the standard annuity and cost of insurance functions, and expenses are as follows:

per premium: 75% of the first premium, 20% of the second premium, 10% of the third through sixth premiums, and 5% of each premium thereafter;
per amount: $10 at the beginning of the first year, and $2 at the beginning of each subsequent year per $1000 of insurance;
per claim: $5 per $1000 of insurance as the cost of settlement.

An asset share pricing model uses a table of expense rates, which might begin as follows (cf. Belth [1968], pages 22-24):

Exhibit 1: Illustrative Expense Costs for a Whole Life Policy

Policy Year	Percent of Premium: Commissions	Percent of Premium: Other	Percent of Face Value	Dollars Per Policy
1	60%	5%	2.5%	$ 200
2	10	5	0.2	50
3	10	3	0.2	25
4	5	3	0.2	25

Casualty Insurance Expenses

The loss ratio and pure premium methods that are used for casualty insurance rate making do not differentiate insurance expenses by policy year. An expected loss ratio is derived from company budgets (e.g., advertising), agency contracts (e.g., commissions), state statutes (e.g., premium taxes), or Insurance Expense Exhibit data (e.g., general expenses). The experience loss ratio, after trending, development, and similar adjustments, is compared to the expected loss ratio to determine the indicated rate change (McClenahan [1990]). This procedure treats all expenses identically, regardless of their actual incidence.

Policy Duration and Insurance Expenses

Property/casualty expense costs, like life insurance expense costs, are greater in the original year of issue than in renewal years.

1. Underwriting expenses incurred predominantly in the first year include salaries, costs of policy issuance and underwriting reports (e.g., DMV reports for automobile insurance or credit reports for Homeowners'), and expenses allocated as overhead on salaries. Renewal underwriting may be only a perfunctory review of past loss experience.

2. Loss control expenses incurred either at or before policy issuance include technical inspections (Boiler and Machinery) landfill inspections (Environmental Impairment), loss engineering services (Workers' Compensation), financial analyses (mortgage guarantee), and building inspections (Commercial Fire). Few inspections are repeated at renewal dates. Those which are, such as some workplace safety inspections for Workers' Compensation, are less comprehensive than the original underwriting inspection.

3. Acquisition expenses for direct writers are greater in the first year than in renewal years. Three types of commission schedules are used in property/casualty insurance:

    - Independent agency companies pay level commissions, such as 15% or 20% of premium, in all years. The level commission structure is needed because the agent "owns the renewals" (cf. *National Fire Insurance*, 1904). That is, the insurer may not bypass the agent when renewing the policy. Rather, the agent may place the insurance with any carrier he represents, as long as the consumer agrees. A lower commission in renewal years would induce the agent to move the policy to a competing insurer and obtain a "first year" commission.

The level commission structure does not reflect the actual incidence of acquisition expenses, since agents spend more effort writing new policies than renewing existing policies. Because of this (and other reasons), the independent agency system is inefficient.[11] In the Personal Lines of business, direct writers are steadily gaining market share, and the level commission structure is becoming less important. As the asset share pricing model shows, a level commission structure works well for risks that terminate quickly. It works poorly for risks that endure with the carrier. But the persisting risks, with lower loss ratios, are more profitable. In other words, it is inappropriate for the persisting and profitable risks.

- Many direct writers pay commissions that vary by policy year: high first year commissions (20 to 25%) and low renewal commissions (2 to 5%). Since the insurer, who is the agent's sole employer, owns the renewals, the agent has no opportunity to move the policyholder to a competing carrier.

- Some direct writers have either (i) a salaried sales force or (ii) a sales force that is compensated partly by commission and partly by salary. The acquisition costs incurred by the insurer may be determined by the actual incidence of these expenses. For instance, suppose the agent receives salary and benefits of $100,000 a year, spends 80% of his or her time obtaining $500,000 of new business a year, and 20% of his or her time servicing $2 million of renewal business. The insurer is paying the equivalent

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[11] The primary "other reasons" are the relative ease of automating a captive agency compared to an independent agency and the ability of direct writers to integrate distribution strategy with underwriting strategy. On the efficiency of insurance distribution systems, see Joskow [1973], Cummins and VanDerhei [1979], and Cummins [19__].

of a 16% commission on new business and a 1% commission on renewal business.[12]

4. Most "other acquisition expenses," such as advertising, subsidies for new agents, and development costs for expanding or automating distributions systems, are expended at or before the inception date of the policy.

Casualty actuaries often differentiate between "fixed" and "variable" expenses. Variable expenses are those that are directly proportional to premium. Fixed expenses do not vary directly with premium: some are "per policy" expenses, such as some underwriting expenses, and some are "sunk costs" related to the block of business as a whole, such as certain advertising costs. The appropriate treatment of fixed and variable expenses is discussed in Section IV below.

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[12] Formally, if "x" is the first year commission rate and "y" is the renewal commission rate, then

$$(\$500{,}000)(x) + (\$2{,}000{,}000)(y) = \$100{,}000$$

$$(0.80) + (0.20) = \{(\$500{,}000)(x)\} + \{(\$2{,}000{,}000)(y)\},$$

or $x = 16\%$ and $y = 1\%$.

D. Persistency

Persistency rates, or retention rates, are the crux of asset share pricing models. Independent insurers pay careful attention to Personal Auto retention rates, though rating bureaus have yet to incorporate them into their ratemaking procedures.

Policy Duration and Profitability

Persistency rates are most important when the net insurance income varies by duration since inception of the policy. Consider first a whole life insurance policy.

> Net insurance income = (premium collected + net investment income) – (benefits paid + increase in policy reserves + incurred expenses + federal income taxes).

The Standard Non-Forfeiture Laws of each state cause the expected value of

(premium + net investment income) – (benefits paid + increase in reserves)

to be rather level each year, whether the policyholder persists or terminates.[13]

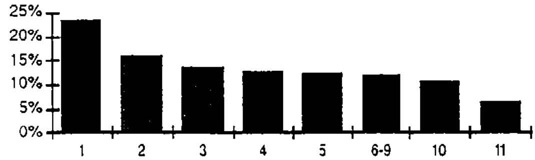
Influences on Persistency Rates

Persistency rates vary widely by company. In Personal Auto, for instance, State Farm has high retention rates, because (a) it targets a suburban and rural insured population, (b) it offers low premium rates, and (c) it provides renewal discounts. Many independent agency companies have low retention rates, (a) because the agents, who are not beholden to any particular carrier, can move the insured to whichever company offers the lowest rates, and (b) because

[13] The expected value will be level, but the actual value will vary, being lower in the year of death. Preliminary term policy reserves increase the value of net insurance income in the first policy year, though not enough to offset the higher underwriting and acquisition expenses.

these carriers use little consumer advertising.[14] The typical Personal Auto direct writer has retention rates of about 90%, ranging from under 85% in the first policy year to about 95% after 10 years. In other words, termination rates ("lapse rates") are over 15% in the first policy year and decline to about 5% after 10 years.

Persistency improves with duration since policy inception. The graph on the right shows industry-wide ordinary life insurance lapse rates (vertical axis) by policy year since inception (horizontal axis) (LIMRA [1988], Table 6, page 338; cf. Buck [1960], page 275).



There is an intuitive relationship between duration and persistency for both life and casualty insurance. In the original year of issue, many policyholders are undecided about the relative value of the policy and the required premiums. Some insureds may decide that the insurance is not worthwhile; some may be dissatisfied with their carrier's service; some may believe the premium is too high and continue shopping for a lower rate; and some may be unable to afford any insurance. Thus, voluntary termination rates during the first year are high. In casualty lines of business, moreover, where underwriting terminations are permitted, carriers often re-evaluate newly acquired risks that have had accidents in the first one or two policy years.

Once a policyholder has kept the policy for several years, it is likely that he or she will renew the contract for another year. The insured is probably satisfied with the carrier's service and finds the premiums reasonable and affordable. And unless the insured's classification changes,

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14 Life insurance shows similar variability. With regard to whole life persistency, LIMRA [1990b, page 286] notes: "Regardless of policy year, there is considerable variation in lapse experience across companies. For policy years 1-10, one quarter of the lapse rates are below 10 percent. Another quarter of the lapse rates generally exceed 20 percent." See also Anderson [1959], page 373; Winn *et al.* [1989]; Moorehead [1960], page 297; Belth [1968], page 19.

underwriting terminations are unlikely.[15]

Termination Rates and Probabilities of Termination

Persistency may be analyzed either by termination rates or by probabilities of termination. The *termination rate* is the number of terminations during a given renewal period divided by the sum of terminations during that period plus policies persisting through that period. The *probability of termination* is the number of terminations during a given renewal period divided by the number of originally issued policies in that cohort. [A cohort is a group of policies written in a given issue period.][16]

For instance, suppose an insurer writes 100 auto policies in 1990, 20 risks lapse the first year, 10 lapse the second year, and 5 lapse the third year. The termination rates are 20% [=20÷100] the first year, 12.5% [=10÷80] the second year, and 7.1% [=5÷70] the third year. The probabilities of termination are 20% [=20÷100] the first year, 10% [=10÷100] the second year, and 5% [=5÷100] the third year. Termination rates more clearly distinguish

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[15] Classification changes are common in Personal Automobile. Most changes are from higher to lower rated classifications, such as a movement from youthful to adult driver, from unmarried to married driver, or from urban to suburban resident. These changes rarely provoke underwriting terminations. Some changes are to higher rated classifications: for example, an adolescent son may turn 17 and obtain a driver's license, the use of the car may switch from "pleasure" to "drive to work," or the insured may move from a low rated territory to a higher rated territory. These changes may lead to a re-evaluation of the risk. The most common impetus for reunderwriting, though, is not classification changes but poor claim experience, as noted in the text.

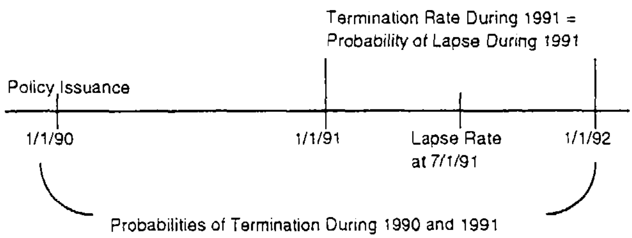
[16] Compare Huffman's distinction between asset shares and the asset fund. A_t is the "asset share per $1,000 unit of coverage *in force* at the end of policy year *t*." F_t is "the asset fund per l_0 *initially issued* units, accumulated at interest to duration *t*" (italics added). Huffman notes that "the asset share prorates funds among policyholders so that each gets its share; the asset fund does not, thereby measuring the accumulated funds held by the insurer" (Huffman [1978], pages 278-279).

persistency patterns by classification.[17] Probabilities of termination, in certain analyses, provide a better portrayal of the insurer's profitability.[18]

[17] For instance, suppose 100 policies were issued to adult drivers and 100 policies were issued to young male drivers. By the fifth renewal, 20 of the adult drivers had lapsed, and 60 of the young male drivers had lapsed, leaving 80 adult drivers and 40 young male drivers. By the next renewal, an additional 5 adult drivers and 5 young male drivers terminate their coverage. The termination rates are 5÷80, or 6.25%, for adult drivers and 5÷40, or 12.5%, for young male drivers. *The probabilities of termination, however, are 5% for both groups of insureds.*

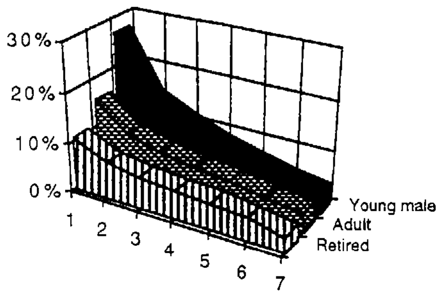
[18] The distinction between termination rates and probabilities of termination is taken from life insurance. The *mortality rate* is the annualized probability that an individual will die at a given time. The corresponding *probability* is the number of deaths at a given age divided by the number of insureds who have attained that age (Batten [1978]; Atkinson [1989], pp. 51-54).

The use of these terms here is not identical to that in life insurance. The life insurance lapse rate pertains to a given moment of time. The life insurance probability of lapse is the percent of withdrawing policyholders during the year. The termination rate as used here is equivalent to the probability of lapse. The probability of termination as used here is the percent of original policyholders who terminate in a given year. The diagram below illustrates the use of these terms.



Persistency by Classification

Persistency rates vary greatly by classification. In Personal Auto insurance, young male drivers have high termination rates, retired drivers have low termination rates, and middle aged drivers are in between. The graph on the right shows illustrative probabilities of termination for these three classifications.



The termination rate differences by classification, of course, are greater. The vertical axis in the graph above shows the probability of termination, and the horizontal axis shows the policy period since inception.[19]

Life insurance persistency patterns are analyzed by issue age, duration, interest rates, sex, rating (standard, preferred, and substandard), policy face amount, premium payment pattern (whole life versus limited payment life; annual, monthly, and payroll deduction), policy form (ordinary life, universal life, graded premium whole life, variable life, traditional term, select and ultimate term), distribution system (general agents, brokers, and branch offices),

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[19] See Feldblum [1990: EAPP; 1990: PAP]. LIMRA shows similar relationships for long-term ordinary life insurance. Lapse rates for issue ages 20-29 are about double those for issue ages 50-59 at all policy durations; see LIMRA [1990a], pages 338-339, Tables 8-10. {Add other life references for termination rates by policyholder age.}

LIMRA's most recent studies show lapse rates in the year of issue about 50 to 100% higher than those in the tenth and subsequent renewal years. Older persistency studies, such as Linton [1924], Moore [1960], and LIMRA's studies from the 1970's, show lapse rates in the year of issue about 5 times higher than those in the tenth and subsequent renewal years. (See LIMRA [1990b] page 295, Table 2, for a comparison.) Persistency patterns are sensitive to external economic and social forces, so an unexamined extrapolation from historical experience may be misleading. Similar caution should be used when extrapolating from past Personal Auto experience.

and numerous other variables.[20] Some of these dimensions are pertinent only to life insurance. For instance, if market interest rates rise faster than the credited rate on a Universal Life policy, lapse rates may increase. Other dimensions apply to casualty insurance as well. Policy duration and issue age are discussed above. The relationship between the distribution system and persistency patterns is particularly important for casualty insurance.

The dependence of persistency patterns on these dimensions warrant a careful analysis of the available experience. For an independent agency company to use persistency patterns derived from direct writers makes as much sense as for an insurer to use claim frequencies from adult drivers for young male insureds. Similarly, the persistency patterns between urban and rural territories may differ as much as loss costs differ between these territories. The termination rates used in Sections IV through VII are illustrative; only by coincidence would they be appropriate for a given company and a given block of policies.

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[20] See Atkinson [1987; 1989]. Belth [1968], page 18, notes additional dimensions, such as policyholder's income, occupation, previous ownership of life insurance, experience of the agent, and presence of policy loans.

E. Discount Rates

Asset share models examine cash flows and revenue streams over the lifetime of the policy. Future profits and losses of each policy year are discounted to the original issue date to determine present values.

Life Insurance Discount Rates

In non-participating whole life insurance contracts, both premiums and benefits are fixed at issue. Claims are paid soon after death, so there is no "settlement lag." The discount rate used to determine the present values of future premiums and benefits for statutory policy reserves is limited by the state's Standard Valuation Law. Life insurance policy reserves do not have the uncertainty of casualty insurance loss reserves, which are affected by inflation rates.

The life actuary using an asset share model begins with known quantities: premium, death benefits, and policy reserves. With appropriate assumptions for mortality and withdrawal rates, he or she can determine statutory or GAAP book profits of each year. All that is needed is a discount rate to determine the present value of future earnings.

Casualty Insurance Issues

Casualty claims are not settled immediately after the accident. Under tort liability compensation systems, claim investigation, determination of liability, and legal negotiation and adjudication may delay settlements for months or years. In the no-fault lines of business, such as Workers' Compensation and Automobile PIP, wage loss reimbursements are made only as the loss is accrued, so payments stretch out over years.

Property/Liability insurance accounting, whether statutory or GAAP, records incurred losses on an undiscounted basis, resulting either in underwriting losses or in lower underwriting profits than if discounted loss reserves were held. The investment income in the Annual Statement – which may be viewed an offset to the underwriting loss – is the present investment income from the company's financial assets, not the investment income expected in the future (Feldblum [1993]; Bingham [1990]). Property/Liability insurance accounting, both statutory and GAAP, does not match the underwriting experience on a block of policies with the investment experience for the same block of policies. This matching, though, is essential for asset share pricing models. Several methods of matching underwriting and investment experience may be used:

a. Record undiscounted incurred claims, but include an offsetting investment income account tied to the assets supporting the unpaid losses (option 3 of Salzmann [1984]).

b. Record cash transactions, not the accounting statement incurred losses. The asset share model looks like an expanded (multi-period) internal rate of return model.[21]

c. Record discounted loss reserves. The discount rates for unpaid losses may be market interest rates, risk-free rates, or "risk adjusted" rates.[22]

For simplicity, this paper uses the third method. The illustrations speak of "discounted incurred losses" without specifying the method of discounting. Note that the discount rate used to determine the present value of unpaid losses at the accident date need not be the same as the discount rate used to determine the present value of future earnings at the issue date.[23]

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[21] Internal rate of return and asset share pricing models, however, have different viewpoints. The internal rate of return model views the insurance transactions from the equityholder's perspective. It requires surplus commitment and equity flow assumptions (Feldblum [1992: IRR]). The asset share model uses the insurance company's perspective and need not consider equity flows. For instance, Anderson [1959] determines the ratio of the present value of profits to the present value of premium, not the return on investment or surplus. Thus, the asset share model is similar to a multi-period internal rate of return model in its construction, not in its perspective.

[22] Woll [1987] and Bingham [1990] use risk free rates. Fairley [1979], Myers and Cohn [1987], and Butsic [1988] use risk adjusted rates, though they determine the adjustment differently. The need for risk margins is discussed in CAS Committee on Reserves [1987; 1991] and CAS Committee on the Theory of Risk [1987; 1991]. See also D'Arcy [1987; 1988]; Lowe [1988]; FASB [1990]; and Tiller, et al. [1987].

[23] See Paquin [1987] for a life insurance discussion of different discount rates for cash inflows and outflows.

SECTION III: ASSET SHARE MODELLING – FOUR ILLUSTRATIONS

Asset share modelling is particularly valuable when differences in termination rates influence expected profits. The first three illustrations in this section show how an asset share model deals with such conditions. The fourth illustration shows how the movements of the underwriting cycle can be incorporated into policy pricing. The illustrations are as follows:

1. *Business Expansion:* When an insurer begins writing in a new territory or policyholder classification, most risks are new business, with high loss and expense ratios. Traditional rate making procedures show high combined ratios, and the pricing actuary may conclude that the business is not profitable. But this is simply the cost of building an insurance portfolio. New business is generally "unprofitable," though the "loss" may be offset by the future profits in a stable renewal book. Asset share modeling helps the actuary determine the true profitability of the insurance writings.

2. *Classification Relativities:* Traditional rate making methods determine classification relativities from loss ratios, perhaps tempered with "expense flattening" procedures. Persistency differences among classifications can cause these methods to be misleading. If persistency is ignored, then rate relativities are too low for the poorly persisting classes and too high for the long-persisting classes. The illustration shows an asset share model determination of Personal Automobile classification relativities for young male drivers.

3. *Competitive Strategy:* Traditional rate making procedures match premiums to anticipated losses and expenses. They ignore the future profits and losses from expected renewals. Moreover, they ignore the effects of rate revisions on policyholder retention and new business production. A rate increase will reduce policyholder retention, particularly among the most profitable risks, who can obtain coverage from other carriers. Competitive pricing strategy is to raise or lower rates such that the expected changes in policyholder retention, new business production, and lifetime policy profits or losses will maximize long-term income. The illustration shows how asset share modeling determines the optimal retired driver discount in Personal Automobile insurance.

4. *Underwriting Cycles:* Market share and profit objectives are the linchpins of competitive strategy. Attempts to gain market share drive the soft phase of the cycle, and attempts to restore profits drive the hard phase. It is often unclear whether market share gains during the soft phase combined with profits on these policies during the hard phase will lead to satisfactory long-term income. Asset share modeling enables the actuary to quantify the effects of different pricing strategies on overall returns.

Rate Revisions and Rates

Casualty pricing methods determine rate revisions and rate relativities, not actual rates. For instance, the actuary may determine that overall statewide rates should be increased 10%, or that the rate relativity for young male drivers should be changed from 1.750 to 1.850.

Asset share pricing determines rates, not rate revisions. Since there is no overall statewide rate, the actuary selects "pivotal" classifications for which an actual rate is determined. Interpolation and relativity analyses may be used for other (non-pivotal) classifications.

For instance, the life actuary may use an asset share model to determine whole life insurance rates for standard rated, non-smoking males at 5 year age intervals (e.g., ages 30, 35, 40). The mortality and persistency rates at these ages are derived from their own experience combined with the graduated experience for the entire insured population. Whole life insurance rates for a male aged 37 would be determine by interpolation of the rates for age 35 and age 40.

The same procedure is applicable to casualty rate making. We determine rates for pivotal classifications, such as adult married drivers in a given group of territories, or young unmarried male drivers in an urban area. To form the rates, we use the experience of these classifications as well as the graduated experience of similar classifications. We then form rates for non-pivotal classifications by interpolation and relativity analyses.

SECTION IV: ILLUSTRATION 1 - BUSINESS EXPANSION

Company growth or contraction distorts reported financial results, particularly when the expected loss and expense ratios depend on the time since inception of the policy. Even without this dependence, business growth raises the statutory combined ratio, since loss reserves are held at undiscounted values and acquisition costs are written off when incurred. Deferring acquisition expenses and adding investment income, to give a "GAAP operating ratio," does not fully resolve the problem, since the investment income received in any calendar year derives from the business insured in the past. If the insurer is growing rapidly, the investment income received is smaller than the present value of the investment income expected from the current block of business.[24]

To circumvent this problem, the following illustrations assume that all figures are restated on a fully discounted basis. For instance, the $656 of the first policy year's losses in the "business expansion" illustration does not mean statutory incurred losses of $656, but fully discounted losses of $656. Since the illustration uses a policy year model, not a calendar year model, there is no "property/casualty type" deferred acquisition cost. There is, of course, a "life insurance type" deferred acquisition cost, since underwriting and acquisition costs are higher in the original year of issue than in renewal years. The asset share pricing model incorporates this phenomenon, though without setting up an explicit asset.

Growth in a New Territory

Suppose a profitable Personal Automobile direct writer expands into a new geographic area in 1992. To ensure an accurate financial appraisal of the expansion, all statistics on the new operation are separately recorded. "Fixed" costs peculiar to the expansion, such as subsidies for new agents, construction costs for a new branch office, and extra advertising expenses during the first year, are charged to a corporate account; they are not included in these statistics.

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[24] Because premiums, losses, and insurance industry assets grew faster than after-tax investment returns during the 1970's and 1980's, statutory operating ratios were understated by about 2.2 percentage points (Feldblum [1993]).

The insurer writes 10,000 policies in 1992, at an average annual premium of $800. The company is satisfied with the new business production, and 10,000 new policies are again written in 1993. In early 1994, the policy year 1992 results are tabulated, and show a loss of $2.4 million, after full discounting of loss reserves.

The insurer accepts the $2.4 million loss as "start-up" costs in addition to what it has budgeted to the corporate account, and it continues to add 10,000 new policies a year. But when policy year 1993 results, tabulated in early 1995, reveal an additional loss of $1.9 million, company management is concerned. In early 1996, policy year 1994 results show a further loss of $1.3 million. Company management concludes that it erred by expanding too rapidly, and the growth program is curtailed. The pricing actuary tries to explain about the cost of new business but is summarily dismissed.

Has the company indeed erred? The asset share model shows that the company is earning a 19% return on surplus, despite its inexperienced sales force and lack of name recognition in this area. The error lies in curtailing a successful program. Yet actuarial generalizations do not suffice. The true return and the cause of the reported losses must be clearly presented.

Asset Share Assumptions

How can a 19% return on surplus be consistent with losses of $5.6 million in three years? Assume the following conditions for this block of business:

1. *Premiums:* The average policy premium is $800 in 1992. The loss cost trend is 10% per annum, and "fixed" expense costs are rising at 5% per annum. Regulators are not averse to insurers in this state, and the company expects average rate increases of 9% per annum.

2. *Losses:* The fully discounted loss ratio on new business is 82% in 1992, or an average of $656 a car. Loss costs are increasing at 10% per annum. The company expects the average loss costs to improve by 3% a year since policy inception, after adjusting for inflation. For example, the average loss cost for new business written in 1993 will be ($656)(1.1) =

$722. The average loss cost in 1993 for policies originally issued in 1992 will be ($722)(0.97) = $700.[25]

3. *Expenses:* A direct writer has high expense costs the first year but low expense costs in renewal years. Simulated expense costs are shown below.

Exhibit 2: Acquisition and Underwriting Expenses by Policy Year

	New Policies		Renewal Policies	
	Fixed Expense Provision	Variable Expense Provision	Fixed Expense Provision	Variable Expense Provision
Agency Commissions	0.0%	25.0%	0.0%	3.0%
Advertising and Other Acq.	5.0	0.0	0.0	0.0
General Expenses	12.0	3.0	3.0	1.0
Premium Tax	0.0	2.0	0.0	2.0
Taxes, Licenses, and Fees	0.8	0.2	0.8	0.2
Total Expenses:	17.8%	30.2%	3.8%	6.2%

Variable expenses, which vary directly with premium (such as commissions and premium taxes), increase at the same rate as premium. We assume that "fixed" expenses, such as salaries and rent, increase at 5% per annum.

4. *Persistency:* Termination rates vary by company, geographic location, class of business, and various other dimensions. The pricing actuary has chosen termination rates based on prior experience, beginning at 20% in the year the policy is originally issued and declining to 8% after 15 years.

5. *Present Values:* The company determines the present value of future earnings by discounting at its cost of capital, which is 12% in this illustration.

25 A more realistic model would show a larger effect in the first few policy years and a smaller effect in later years. For instance, the improvement in average loss costs from policyholder persistency may be 7% in the first year, 5% in the next year, 4% in the next year, and gradually decline to 1% after 10 years. There are almost no published statistics from which to model this phenomenon, though some data are provided in D'Arcy and Doherty [1990].

The Model

The asset share model is shown in Exhibit 3. The present value of current and future profits and premium is $489 and $5,012, respectively, for a return on sales of 9.7%. If the insurer has a two to one premium to surplus ratio, the return on surplus is 19.5%.[26]

Exhibit 3: Asset share model for Company Growth (Illustration I)

Policy Year (1)	Prem-ium (2)	PV of Loss (3)	Variable Expense Year 1 (4)	Variable Expense Ren (5)	Fixed Expense Year 1 (6)	Fixed Expense Ren (7)	Persis-tency Rate (8)	Cum. Persis-tency (9)	Profit (10)	Discount Factor (11)	Present Value of Profit (12)	Present Value of Premium (13)
1	800	656	242	0	142	0	1.000	1.000	-240	1.00	-240	800
2	872	700	0	54	0	33	0.850	0.850	72	1.12	64	662
3	950	747	0	59	0	35	0.860	0.731	80	1.25	64	554
4	1036	797	0	64	0	37	0.870	0.636	88	1.40	63	469
5	1129	850	0	70	0	38	0.880	0.560	95	1.57	61	402
6	1231	907	0	76	0	40	0.890	0.498	103	1.76	59	348
7	1342	968	0	83	0	42	0.900	0.448	111	1.97	56	305
8	1462	1033	0	91	0	44	0.900	0.403	119	2.21	54	267
9	1594	1102	0	99	0	47	0.910	0.367	127	2.48	51	236
10	1738	1176	0	108	0	49	0.910	0.334	135	2.77	49	209
11	1894	1255	0	117	0	51	0.920	0.307	145	3.11	47	187
12	2064	1339	0	128	0	54	0.920	0.383	154	3.48	44	168
13	2250	1428	0	140	0	57	0.920	0.260	163	3.90	42	150
14	2453	1524	0	152	0	60	0.920	0.239	172	4.36	39	135
15	2673	1626	0	166	0	62	0.920	0.220	180	4.89	37	120
Total:											482	4,963

Column (3), "Present Value of Loss," is the present value at the beginning of that policy year.
Column (9), "Cumulative Persistency," is the downward product of column (8).
Column (10), "Profit," equals column (9) times {column (2) minus the sum of columns (3, 4, 5, 6, and 7)}.
Column (11), "Discount Rate," is 12% a year compounded annually.
Column (12), "Present Value of Profit," is column (10) divided by column (11).
Column (13), "Present Value of Premium," is column (2) divided by column (11).

[26] To estimate the total return on surplus, one must consider (i) the investment return on surplus funds and (ii) federal income taxes. The investment return on surplus funds as a percentage of premiums depends on the premium to surplus ratio. Federal income taxes depend on tax loss carry-forwards and investment strategy. To avoid additional complexities, the illustrations do not incorporate these items. In this example, the effects are largely offsetting. If the investment return on surplus funds is 9% per annum, and the marginal tax rate is 34%, then the before-tax return on surplus is 19.5%+9.0% = 28.5%, and the after tax return is (66%)(28.5%) = 18.8%. In general, however, the effects are not offsetting, and these items must be considered in pricing.

Let us consider each column in Exhibit 3.

1. Column 1 shows the year since the inception of the policy. The policy in this illustration was issued in 1992. The figures in the exhibit pertain to this policy only, not to a policy issued previously or subsequently.

2. Column 2 shows the average premium: $800 a car in 1992, increasing at 9% per annum.

3. Column 3 shows the average losses. The loss ratio is 82% for new business, so 82% of $800 is $656. Losses increase at 10% per annum. At each renewal, loss experience is slightly better, because poor risks voluntarily terminate and reunderwriting efforts weed out unprofitable insureds. The illustration presumes that the average loss costs in any policy year are 3% lower than the average loss costs in the preceding policy year, after adjustment for loss cost trend. In a stable book of business, this phenomenon would not be noticed, since each policy year has a similar percentage of business by renewal year.

   In this illustration, $656 increased by 10% is $722; $722 decreased by 3% is $700. Although the aggregate loss cost trend (10%) is greater than the premium trend (9%), the loss ratio for 10 year old business (68% = 1,176 / 1,738) is lower than the loss ratio for new business (82%).

4. Columns 4 through 7 show expenses. Expenses that vary directly with premium are 30.2% of premium in the year of issue and 6.2% in renewal years. Thus, 30.2% of $800 is $242, and 6.2% of $872 is $54. Fixed expenses average 17.8% of premium in the year of issue and 3.8% of premium in the first renewal year. Thus, 17.8% of $800 is $142, and 3.8% of $872 is $33. Fixed expenses increase at 5% per annum. Thus, 105% of $33 is $35.

5. Column 8 shows the expected persistency rate. The entries indicate that 85% of new policyholders persist into the second year; 86% of second year insureds persist into the third year; and so forth. The persistency rates in this illustration are low in the year of issue (85%) and increase gradually to 92% by the fifteenth year.

6. Column 9 shows the cumulative persistency rate, or the percentage of original insureds who persist into any policy year. For instance, 85% of original policyholders persist into the second year; 73.1% [=(0.085)(0.086)] of original policyholders persist into the third year; and so forth.

7. Column 10 shows the profit in each policy year. The profit is the product of the cumulative persistency rate and the policy year income, where the income equals premiums minus discounted losses minus expenses. For instance, in the third year, policy year income is \$950 – \$747 – \$59 – \$35 = \$109. But only 73.1% of original policyholders persist into the third year, so 73.1% of \$109 is \$80.

8. Column 11 shows the discount factors for future earnings. The company's cost of capital in this illustration is 12%, so column 11 is 12% compounded annually (e.g., $1.12^2 = 1.25$).

9. Column 12 shows the present value of future earnings, or column 10 divided by column 11. Similarly, column 13 shows the present value of future premiums, or column 2 divided by column 11. The totals of columns 12 and 13 are \$489 and \$5,012, respectively. In other words, for a policy issued in 1992, the company expects to earn profits with a present value of \$489 over the next 15 years. The present value of the premiums charged this insured, during the same period and with the same discount rate, is \$5,012.

Accounting Results and Long-Term Profitability

The company reported earnings of a negative \$5.6 million for the first three policy years, even after full discounting of losses. This is the result that traditional actuarial pricing techniques would show. Calendar year statutory financial statements, which use undiscounted loss reserves and write off all underwriting and acquisition expenses when incurred, show worse results.

The dependence of loss and expense ratios on the year since the policy was first issued explains the difference between the \$5.6 million loss shown by traditional pricing analyses and the 19% return on surplus shown by the asset share model. The results by year of issue and by policy year since inception appear below.

Exhibit 4: Results by Year of Issue and Policy Year Since Inception ($000)

	Year Policies are Originally Issued			
	1992	1993	1994	Total
Policy Year of Earnings				
1992	−2,400			−2,400
1993	721	−2,625		−1,903
1994	823	738	−2,873	−1,332

The entries in the "1992" column are taken from column 10 of Exhibit 3. The entries in the "1993" column are derived from an asset share model beginning one year later. Premiums begin 9% higher, losses begin 10% higher, and "fixed" expenses begin 5% higher. The entry in the "1994" column is derived from an asset share model beginning two years later.

Federal Income Taxes

To simplify the presentation, federal income taxes are not considered in these illustrations. The simplest way of incorporating income taxes is to multiply the "profit" column in the illustrations by the marginal tax rate. Thus, the pre-tax loss of $240 in the year of issue is an after tax loss of $158 (assuming a marginal tax rate of 34%). The pre-tax profit of $72 in the second policy year is an after-tax profit of $48.

With this procedure, the discount rate used to determine the present value of losses in column 3 at the beginning of the corresponding policy year should be a before-tax discount rate appropriate for losses, and the discount rate used to determine the present value of profits at the original policy writing date in column 11 should be an after-tax discount rate. If federal income taxes are first applied to the present value of profits in column 12, then the discount rate in column 11 should be a before-tax discount rate. In addition, the federal income taxes must also be applied to the present value of premiums in column 13.

Alternatively, one could use after-tax values of premiums (revenues), losses, and expenses in columns 2 through 7. In other words, the $800 of premium in the year of issue would be replaced by an after-tax revenue of $528. If this procedure is followed, then the discount rates used in columns 3 and 11 should be after-tax discount rates.

Profitability Measures

Different measures of profitability can be incorporated in an asset share model. The illustration discounts future earnings at the company's cost of capital, implying that profits should be measured with a return on equity. To avoid the complexities of converting statutory surplus to GAAP equity, the illustration assumes that surplus equals equity and that the insurer writes at a two to one premium to surplus ratio.[27] Alternatively, one can use the premium to GAAP equity ratio for this insurer, to directly obtain a return on equity.

One could also use asset share modeling to determine the "break-even" point. The company may ask: "Is writing insurance policies more profitable than simply investing the equity in financial securities of similar risk?" Assume that securities of similar risk are yielding 10% per annum. The insurer would use a 10% discount rate in columns 3 and 11, discount losses to the same date as premiums are collected, and determine whether the present value of the total in column 12 is greater or less than zero.

One can incorporate asset share pricing into an internal rate of return model. Instead of the "present value of losses" in column 3, one would show several columns of cash transactions: losses paid, investments made, and investment income received. One would combine the cash transactions from the insurance operations with assumed equity flows and determine the internal rate of return to the equity providers (see Feldblum [1992: IRR]).

In sum, asset share pricing is not restricted to any particular measure of profitability. Rather, whatever measure is used should be applied to the entire life of the policy, not to a single policy year or a single calendar year.

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[27] In practice, GAAP equity is generally greater than statutory surplus, because of deferred acquisition costs, non-admitted statutory assets, unauthorized and "late-paying" reinsurance penalties, Schedule P penalties, and the carrying value of subsidiaries. Offsetting these are the non-recognition of deferred federal tax liabilities on unrealized capital gains and the amortization of bonds in good standing under statutory accounting. See Berthoud [1988] and AICPA [1990] for comparisons of statutory and GAAP accounting. Overall, Rosenthal [1989] estimates that average GAAP equity is 25% greater than statutory surplus for Property/Casualty insurers. The economic net worth of the insurer is greater than GAAP equity because of the unrecognized interest discount in the loss reserves.

SECTION V: ILLUSTRATION 2 – CLASSIFICATION RELATIVITIES

Traditional rate making procedures determine classification relativities by comparing relative loss ratios or pure premiums among groups of insureds (Conger [1987], Harwayne [1977]). For instance, if adult drivers (the "base" class) have average losses of $400 a year, and young male drivers have average losses of $900 a year, then young male drivers are assigned a classification relativity of 2.250. Similarly, if urban residents, with a territorial relativity of 1.500, have an average loss ratio of 70%, and the average loss ratio of all drivers in the state is 75%, then the territorial relativity for urban drivers should be reduced to 1.400 [=(1.500)(70%)+(75%)].

Expense Flattening and Persistency

Expense flattening procedures have refined classification rate making, by separating expenses into those that vary directly with premium, or "variable" expenses, and those that do not, or "fixed" expenses (ISO [n.d.]; Hunt [1978]; Childs and Currie [1980]; Wade [1973]). In the first example in the paragraph above, suppose that average losses for all drivers is $500 a year, "variable" expenses average $150 a year, and "fixed" expenses average $100 a year. Variable expenses are 150+750 (20.0%) of premium. Average losses are $400 for the base class and $900 for young male drivers, so the gross premiums are

Base class (adult drivers): premium = $400 + $100 + 20% *x* premium,
or premium = $625.

Young male drivers: premium = $900 + $100 + 20% *x* premium,
or premium = $1,250.

The classification relativity for young male drivers is 2.000 [= 1,250 + 625].

These procedures fail to incorporate differences in persistency patterns among classes of insureds, resulting in inaccurate (and either unprofitable or uncompetitive) classification relativities. In any policy year, "fixed" expenses, as a percentage of total premium, are lower for young male drivers than for adult drivers, and "variable" expenses, as a percentage of total premium, are equal for the two classes. But young male drivers have higher termination rates than adult drivers have. Because of the higher termination rates, the ratio of total expenses to total premium *over the lifetime of the policy* is greater for young male drivers.[28]

Similar considerations apply to losses. Average losses, adjusted for loss cost trends, decline as the policy matures. The "business expansion" illustration assumed that average losses (after adjustment for trend) decline by 3% in each renewal year. Insureds who terminate quickly have "new business" loss ratios, which are generally higher than "renewal business" loss ratios.[29]

The effects of persistency patterns on relative loss ratios by class depends on the type of classification system used. A simple (albeit unrealistic) example should clarify this. Suppose average losses for adult drivers [the base class] are $500 a year, average losses for 17 year old drivers are $1,000 a year, and all insureds persist for 10 years. In other words, the 17 year old drivers have twice the average loss costs of adult drivers. If all expenses vary with premium (i.e., there are no "fixed" expenses), their classification relativity should be 2.000.

But suppose that new business risks have average loss costs 25% higher than renewal business. All the 17 year old drivers are new business, but only 10% of the adult drivers are new

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[28] See Feldblum [1990A]. The generalization in the text is more applicable to direct writing insurers than to independent agency companies. Cf. also Buck [1978], page 9: "It is more expensive to handle a policy for a young, single male in a given territory than an adult policy in the same territory. This difference can be attributed to such factors as more frequent policy changes and flat cancellations in the youthful male policies." Aetna [1978], page 64, points out that the insurer "must charge policyholders for the underwriting costs of *rejecting* applications. . . . The amount charged to a policyholder would have to exceed that actual cost to compensate for the costs associated with the applications of rejected applicants, from whom the company collects no premium." Since underwriting rejections are more likely for young male applicants, more of this extra expense would be allocated to this class.

[29] The cause and effect relationships are unclear. Perhaps young male drivers, who have higher loss ratios, have poorer persistency, so higher loss ratios also appear on new business. Or perhaps persisting drivers have lower loss ratios, so young male drivers, who terminate frequently, have higher loss ratios. As Steve D'Arcy has pointed out to me, one must take care not to double count these effects. See also the following paragraphs in the text.

business.[30] The 17 year old drivers' average losses will drop to $800 during renewal years, so the 2.000 classification relativity is too high. An insurer can profit in the long-run by reducing the classification relativity for 17 year old drivers and increasing its market share.

Determinants of Rate Relativities

The correct relativity depends on the classification system, the average losses and persistency rates by classification, and the strength of loss ratio improvement by policy year.[31] Asset share pricing models enable the actuary to determine accurate and profitable relativity factors.

This illustration compares young male drivers with adult drivers to determine the classification relativity factors. We need the information listed below, of which the second and third are essential for the asset share model.

1. The dimensions of the classification system.
2. The relative average loss costs of these two groups of insureds.
3. The relative average persistency rates of these two groups of insureds.
4. The strength of loss ratio improvement by policy year for these insureds.

The Classification System

The expected losses, expenses, and the current year's premium do not depend on the shape of the classification system. Future years' premium are affected by such factors as renewal discounts

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30 Adult drivers persist for 10 years, so (in a steady state) 10% are in their first policy year, 10% in the second policy year, and so forth. This would be correct were there no switching of classifications. Since there is switching – that is, some adult drivers were first insured as young drivers – less than 10% of adult drivers are new business. If 25 is the minimum age for adult drivers, then drivers first insured below age 25 spend some renewal years in the adult classification but their first policy years as young drivers.

31 The interrelationships among these dimensions are complex. For instance, a 22 year old unmarried male driver who just completed college may have high expected losses. But if he is beginning a stable job, is engaged to be married, and is buying a house in a quiet suburb, his expected losses may drop quickly. In contrast, a 40 year old married woman may have low expected losses, but she may show no loss ratio improvement for the next 10 years.

and age boundaries between driver classes.[32]

Suppose an asset share model is being used for an 18 year old unmarried male driver. If the insurer differentiates between "males aged 25 and under" and "adult drivers," then this driver will spend 8 years in the "young male" classification. Since average losses decline rapidly between ages 17 and 25, his premium is probably too low for the next 3 or 4 years and too high for the subsequent 4 or 5 years. Termination rates are high for young male drivers but decrease with duration of the policy, so his expected termination rate will start high but decline markedly over the next 8 years. A renewal discount will improve persistency but reduce renewal gross premiums.

Ideally, the classification system should be designed from the results of an asset share model. In practice, the classification system may be a "given" for the pricing actuary. In the "classification relativities" illustration (this section), the classification system is given. In the "competitive strategy" illustration (the following section), the classification system is designed from the asset share model.

Coverage Mix

Two types of differences affect classification relativities even for single policy year costs (that is, not considering persistency effects). First, average losses for any coverage vary by classification. For instance, young male drivers have higher expected bodily injury losses than adult drivers have. Second, the coverage mix varies by classification. For instance, young male drivers are less likely to purchase physical damage coverages or excess limits for liability coverages than adult drivers are.

If the ratio of expenses to premium did not vary with the coverage mix, or with the average loss per policy, then classification relativities would be similar to loss cost relativities. But "fixed" expenses do not vary directly with premium. They remain fixed regardless of the number of coverages, limits of liability, or deductibles chosen (Childs and Currie [1980], pages 53-54).

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[32] Persistency rates, which are influenced by relative future prices between the current insurer and its peer companies, also depend on the classification system.

Policy Basis versus Coverage Basis Rate Relativities

We can use an asset share pricing model to develop rate relativities on either a policy basis or a coverage basis. The policy basis model compares losses and expenses for all coverages combined among classes of insureds. The resultant rate relativities must then be allocated to coverages. For instance, if the policy basis rate relativity for young male drivers is 2.0, and the premium volumes for liability and physical damage coverages are equal, the rate relativities by coverage might be 2.5 for liability and 1.5 for physical damage. When the coverage mix differs by classification, the allocation of the rate relativities may be complex.

The coverage basis model compares losses and expenses for an individual coverage among classes of insureds. The "fixed" expenses must be allocated to coverage before the asset share pricing model is used. Since some expenses do not vary with the number of coverages, the premiums rates are not additive: that is, there should be a "multiple coverages" discount. For instance, if the indicated rates are $500 for liability and $300 for physical damage, the correct rates might be $535 for liability alone, $325 for physical damage alone, and $780 for all coverages combined. Even when these differences are too small for practical application, the pricing actuary should know whether the rates are over- or under-stated for each classification and coverage combination.

Policy Basis Loss Cost Relativities

Policy basis loss cost differences between young male drivers and adult drivers depend on three factors:

1. *Young male driver rate relativities by coverage:* Average rate relativities for young male drivers are approximately 2.5 compared with the base classification rate (adult pleasure use). The rate relativities vary among insurers, depending on (i) the definition of young male drivers [e.g., "25 and under," "29 and under," and so forth] and (ii) the other classification dimensions, such as years of driving experience and past accident history. Some states, such as New York, require separate relativities for Comprehensive coverage, and some insurers use

separate relativities in other states as well. The total average young male driver rate relativity to that of *all* drivers is approximately 2.0.[33]

2. *Physical damage coverage by classification:* Young male drivers are more likely than other drivers to have liability coverage but no physical damage coverage, because their premiums are high, they drive less valuable automobiles, and they may be less able to afford insurance (cf. Aetna [1978], page 26).

3. *Average liability increased limits and physical damage deductibles:* Young male drivers have lower average liability limits and higher average physical damage deductibles for a given type of automobile. The higher average premiums for young male drivers, the fewer assets they have to protect, and the reluctance of company underwriters to provide high liability limits or full physical damage coverage to high risk drivers are the major reasons for this.

For the "classification rate making" illustration, we use a coverage based asset share pricing model. Since the average coverage basis rate relativities are greater than the average policy basis rate relativities (about 2.0:1 versus 1.5:1), and much of the fixed expenses relate to per policy expenses, not per coverage expenses, we must adjust the per coverage fixed expenses by classification, assigning a higher dollar amount to young male drivers than to adult drivers.[34]

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[33] See ISO [1989], pp. G-10 through G-13. ISO classifies young male drivers as (i) under 25 years of age if married or not the owner or principle operator of the vehicle and (ii) under 30 years of age if unmarried and the owner or principle operator. Rate relativities range from 1.15 for a 21 through 24 year old "good student" married male using the automobile for pleasure use to 3.75 for a 17 year old unmarried male driving his car to work and not eligible for a good student credit. Several jurisdictions, such as Massachusetts and California, prohibit classification by age, sex, or marital status {refs}. In these states, rate relativities are determined along other dimensions.

[34] An illustration should clarify this. Suppose class A purchases both liability and physical damage coverages while class B, with a similar number of insureds, purchases only liability coverage. Expected losses and variable expenses are $600 for each coverage and each classification, and per policy fixed expenses are $100 a policy.

The ratio of fixed expenses to gross premiums for the entire line of business is 10% [= 200 ÷ (600+600+600+200)]. Equivalently, fixed expenses are one ninth of losses plus variable expenses. If we used this ratio to assign fixed expenses by class, we would assign $133 [=

Persistency by Classification

An insurer selling whole life coverage expects to show an accounting loss during the first policy year. For medically underwritten risks, the acquisition and underwriting costs generally exceed the first year premium. For guaranteed issue policies, adverse selection raises first

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($600+$600)÷9] to class A and $67 [= $600÷9] to class B.

Similarly, if we first allocated fixed expenses by coverage, we would assign $133 to liability and $67 to physical damage, since liability has twice the "losses plus variable expenses" that physical damage has. Splitting the $133 equally between classes A and B gives the same result as before. The expense flattening procedure suggested by ISO [n.d.] begins with fixed expenses by coverage, so it would not solve the problem outlined here.

But this allocation is not correct. Since class A has twice the premium per policy that coverage B has, the ratio of fixed expense to premium for class B should be twice that for class A. [This is an extended "expense flattening" procedure.] Thus, (600+600)(x) + (600)(2x) = 200, or x = 8.33%. For the liability coverage, the expense loadings should be ($600)(8.33%) = $50 for class A, and ($600)(2)(8.33%) = $100 for class B. For the physical damage coverages, the expense loading should be ($600)(8.33%) = $50 (for class A).

For the example in the text, adult drivers have about four thirds [2.0 ÷ 1.5] as much coverage per policy as young male drivers have. A precise quantification of the fixed expenses by class is difficult for several reasons. First, fixed expenses are not strictly "per policy" expenses. For example, underwriting efforts are greater for a policy with both liability and physical damage coverages than for a policy with only liability coverage. Second, many fixed expenses, such as underwriting expenses, vary with the quality and type of risk. Louis E. Buck, in summarizing the findings of the Aetna Automobile Insurance Affordability Task Force for the National Association of Insurance Commissioners (Zone IV meeting, Indianapolis, Indiana, October 9, 1978), said: ". . . there are differences by classification in the cost of handling policies. It is more expensive to handle a policy for a young, single male in a given territory than an adult policy in the same territory. This difference can be attributed to such factors as more frequent policy changes and flat cancellations in the youthful male policies." His accompanying statistics show policy processing costs to be 50% to 100% higher for youthful unmarried male drivers than for adult drivers. See Aetna [1978], statement of Louis E. Buck, page 9.

There is no rigorous quantification of fixed expenses by classification in this paper. However, the dollars of fixed expenses per coverage in each policy year are higher for young male drivers in the asset share pricing model than for adult drivers. Expense flattening procedures, which are incorporated automatically in the asset share pricing model, reduce the "proportional" fixed expense loading for young male drivers in each policy year. Persistency patterns raise the lifetime "proportional" fixed expense loading for these insureds compared to adult drivers. These effects can be seen in Exhibits 6 and 7.

year benefit costs. In either case, the loss turns into a profit as the policyholder persists.

Similarly, an insurer selling Personal Automobile coverage expects an accounting loss during the first policy year, since both expenses and loss costs are higher that year. As with life insurance, the loss turns into a profit as the policyholder persists.

Expected long-term profits depend upon the policyholder persistency rates, in addition to premium, loss, and expense levels. Since persistency varies by classification, the rate relativities must consider persistency rates as well.

Classification differences may be based on either current classification or original classification. In most lines of insurance, the classification does not change: a frame building does not develop into a masonry building (Homeowners'), a retailer does not become a manufacturer (Workers' Compensation), an architect does not become a lawyer (Professional Liability). But Personal Automobile classification do change, as young drivers become adults, as urban residents move to the suburbs, and as new cars age.

Young Male Drivers

Traditional rate making procedures consider current classification. Premium rates decline when the young male marries or ages, not before. Asset share pricing models consider original classification and expected future changes: if we write a policyholder now, what is the expected long-term income?[35]

Persistency rates by duration are most easily determined for current classifications, such as the percentage of young male drivers in their fifth policy year who persist into their sixth year. But if the young male classification consists of male drivers under 25 years of age, the group considered in the previous sentence are drivers originally insured below 20 years of age.

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[35] Pricing decisions hinge on supply and demand considerations, though these factors are hard to include in traditional rate making methods. The insurer asks: "If we raise the premium, what happens to expected long-term income?" Raising premium helps the current year's income, but it lowers persistency. The next illustration, "competitive strategy," shows how asset share pricing models deal with this issue.

These drivers have different persistency rates from drivers originally insured from 22 to 24 years of age. The persistency of young male drivers in their fifth policy year does not tell us the expected fifth year persistency of young male drivers. We need persistency rates by original classification, not current classification.

Model Assumptions

For the asset share model, we begin with pivotal classifications: the adult pleasure use (the base class) and unmarried males aged 21 and 22 who drive to work. We need to know three differences by classification to form rate relativities: average loss costs, average fixed expense costs, and persistency rates. For this illustration, we assume the following differences; in actual pricing work, we would derive these from past experience:

- Average liability loss costs are $400 per annum for adults and $1,000 per annum for young male drivers. Were all expenses proportional to premium, and were persistency rates the same for both classes, the rate relativity for young male drivers would be 2.5.

- Average premium for all drivers is $550. Average first year fixed expenses are 17.8% of this, or $98. Adult drivers are less expensive to underwrite, especially per coverage. There are fewer underwriting rejections among adult drivers, and they purchase more coverages, so average fixed expenses per coverage is 10% less, or $88 per policy for the liability coverages. Conversely, young male drivers are more expensive to underwrite, especially per coverage. Underwriting rejections are more common, some applicants never remit the premiums, and many drivers purchase only basic limits liability coverages. Average fixed expenses for the liability coverages are 20% higher, or $117 per policy.[36]

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[36] Cf. Aetna [1978], page 64: "In considering how expenses should be allocated to policyholders, it must also be noted that the company must charge policyholders for the underwriting costs of *rejecting* applications. Thus, even if the actual costs of underwriting each accepted risk were known, the amount charged to a policyholder would have to exceed that actual cost to compensate for the costs associated with the applications of rejected applicants, from whom the company collects no premium."

- Retention rates are higher for adult drivers than for young male drivers. We use the simulated rates in Exhibit 5 to illustrate the asset share pricing model. Actual rates vary by insurer, distribution system, and classification plan, so these rates may not be appropriate for any given carrier.

Exhibit 5: Persistency Rates by Duration and Classification

Policy Year	1	2	3	4	5	6	7	8	9	10+
Young male	60	65	70	73	76	79	82	85	88	90
Adult	82	86	87	88	89	90	90	91	91	92

The classification plan, average loss costs, average fixed expenses, and persistency rates are given. We assume that the insurer writes at a 2:1 premium to equity ratio and desires a 15% return on equity. Thus, we use the asset share pricing model to determine a 7.5% return on premium for each class and then derive the rate relativities from the resulting premiums.

Exhibits 6 and 7 show the calculations. For each class, we select a starting gross premium and increase it 9% per annum, which determines the variable expenses in all future years. In the first year, fixed expenses are $88 for adults and $117 for young male drivers. We use the same ratio of renewal to first year fixed expenses as in the previous illustration, 3.8% to 17.8%, and increase the fixed expenses by 5% per annum. For adult drivers, $88 x 3.8% ÷ 17.8% = $19; this is then increased by 5% per annum to give all the fixed expense entries.

As before, the loss costs shown in the exhibit are discounted to the beginning of the corresponding policy year. The present values of future profits and premiums at the original policy issuance date are determined at a 12% interest rate, which is the assumed cost of capital. The original premium has been selected such that the ratio of the present value of all future profits to the present value of all future premiums is 7.5% for both classes.

Asset Share Results

The indicated premiums are $475 for adults and $1,270 for young male drivers. Note that

- The loss cost relativity is 2.50, or $1,000 ÷ $400.
- The fixed expense cost relativity is 1.33, or 1.2 ÷ 0.9 (= $117 ÷ $88).
- The rate relativity is 2.67, or $1,270 ÷ $475.

Pricing procedures used in the 1960's would have set the rate relativity equal to the loss costs relativity, or 2.50. Since the fixed expense relativity is only 1.33, expense flattening procedures would have reduced the rate relativity. But the persistency differences between the two classes show that even the loss cost relativity is too low. A premium rate relativity of 2.67 is needed to equalize the returns between these two classes.

(1) Policy Year	(2) Premium	(3) Losses	(4) Var Exp Ratio New	(5) Var Exp Ratio Renewal	(6) Fixed Exp Ratio New	(7) Fixed Exp Ratio Renewal	(8) Persistency	(9) Cumulative Persistency	(10) Profit	(11) Discount Factor	(12) PV of Profits	(13) PV of Premium
1	475	400	143	0	88	0	1.00	1.00	-157	1.00	-157	475
2	518	427	0	32	0	19	0.82	0.82	33	1.12	29	379
3	564	455	0	35	0	20	0.86	0.71	38	1.25	30	317
4	615	486	0	38	0	21	0.87	0.61	43	1.40	31	269
5	670	518	0	42	0	22	0.88	0.54	48	1.57	30	230
6	731	553	0	45	0	23	0.89	0.48	53	1.76	30	199
7	796	590	0	49	0	24	0.90	0.43	57	1.97	29	174
8	868	630	0	54	0	25	0.90	0.39	62	2.21	28	153
9	946	672	0	59	0	26	0.91	0.35	67	2.48	27	135
10	1031	717	0	64	0	28	0.91	0.32	72	2.77	26	120
11	1124	765	0	70	0	29	0.92	0.30	77	3.11	25	107
12	1225	816	0	76	0	31	0.92	0.27	83	3.48	24	96
13	1336	871	0	83	0	32	0.92	0.25	88	3.90	23	86
14	1456	929	0	90	0	34	0.92	0.23	93	4.36	21	77
15	1587	992	0	98	0	35	0.92	0.21	98	4.89	20	69
Total											216	2887

Col 2: First year premium is chosen such that the present value of profits (column 12 total) = 7.5% of the present value of premium (column 13 total). Subsequently, premiums increase 9% per annum.

Col 3: First year losses average $400; loss cost trend is +10% per annum; losses decrease 3% per annum as policy matures.

Cols 4 and 5: Variable expense ratio is 30.2% the first year and 6.2% in subsequent years.

Col 6: First year fixed expenses as $98 per policy, or 17.8% of the average premium for all drivers ($550). Fixed expenses for adult drivers are 10% lower, or $88 per policy.

Col 7: Fixed expenses in the first renewal year are $88 * 3.8% ÷ 17.8%. Subsequently, expenses increase 5% per annum.

Col 8: Assumed persistency rates for adult drivers; column 9 = downward product of column 8.

Col 10 = (column 2 − sum (columns 3 through 7)) * column 9.

Col 11: Discount factor reflecting annual 12% cost of capital, e.g., 1.25 = 1.12 * 1.12.

Col 12 = Column 10 ÷ column 11; column 13 = column 2 ÷ column 11.

Exhibit 6: Adult Pleasure Use

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Policy			Var Exp Ratio		Fixed Exp Ratio			Cumulative		Discount	PV of	PV of
Year	Premium	Losses	New	Renewal	New	Renewal	Persistency	Persistency	Profit	Factor	Profits	Premium
1	1270	1000	384	0	117	0	1.00	1.00	-231	1.00	-231	1270
2	1385	1067	0	86	0	25	0.60	0.60	124	1.12	111	742
3	1509	1138	0	94	0	26	0.65	0.39	98	1.25	78	469
4	1645	1215	0	102	0	28	0.70	0.27	82	1.40	58	320
5	1793	1296	0	111	0	29	0.73	0.20	71	1.57	45	227
6	1955	1383	0	121	0	30	0.76	0.15	64	1.76	36	168
7	2131	1476	0	132	0	32	0.79	0.12	59	1.97	30	129
8	2322	1575	0	144	0	34	0.82	0.10	56	2.21	25	103
9	2532	1680	0	157	0	35	0.99	0.10	64	2.48	26	99
10	2759	1793	0	171	0	37	0.88	0.09	65	2.77	23	85
11	3008	1913	0	186	0	39	0.90	0.08	67	3.11	21	74
12	3278	2041	0	203	0	41	0.90	0.07	68	3.48	20	65
13	3573	2178	0	222	0	43	0.90	0.06	70	3.90	18	57
14	3895	2323	0	241	0	45	0.90	0.06	72	4.36	16	50
15	4246	2479	0	263	0	47	0.90	0.05	73	4.89	15	44
Total											293	3902

Col 2: First year premium is chosen such that the present value of profits (column 12 total) = 7.5% of the present value of premium (column 13 total). Subsequently, premiums increase 9% per annum.

Col 3: First year losses average $1,000; loss cost trend is +10% per annum; losses decrease 3% per annum as policy matures.

Cols 4 and 5: Variable expense ratio is 30.2% the first year and 6.2% in subsequent years.

Col 6: First year fixed expenses as $98 per policy, or 17.8% of the average premium for all drivers ($550). Fixed expenses for young male drivers are 20% higher, or $117 per policy.

Col 7: Fixed expenses in the first renewal year are $117 * 3.8% ÷ 17.8%. Subsequently, expenses increase 5% per annum.

Col 8: Assumed persistency rates for young male drivers; column 9 = downward product of column 8.

Col 10 = {column 2 − sum (columns 3 through 7) } * column 9.

Col 11: Discount factor reflecting annual 12% cost of capital; e.g., 1.25 = 1.12 * 1.12.

Col 12 = Column 10 ÷ column 11; column 13 = column 2 ÷ column 11.

Exhibit 7: Young male drivers

SECTION VI: ILLUSTRATION 3 – COMPETITIVE STRATEGY

The illustration presented in Section IV, "business expansion," took the environment as given and asked, "Is the growth strategy profitable?" The illustration in Section V, "classification relativities," took the insured population as given and asked: "What prices are equitable?"

This is the traditional ratemaking perspective: the actuary aligns premiums with anticipated losses and expenses for a given insured population. Competitive strategy reverses the question: "How can the pricing structure create a more profitable consumer base?"

Some insurers have excelled at this task. New products, such as package policies, modifications to existing products, such as replacement cost coverage, and classification revisions, such as retired driver discounts, have spurred sustained growth for these carriers.

Two considerations should be kept in mind when seeking to change the insured population:

- Any strategy may affect new business production or retention rates. For instance, the introduction of various professional liability coverages created a new clientele, whereas the expansion of experience rating plans increases renewals among desirable insureds. [Some new products, such as universal life insurance, serve both functions: they are savings vehicles for investors otherwise uninterested in life insurance, and they are replacement vehicles for insureds who might drop inefficient whole life policies.]

- Traditional ratemaking procedures are cost-based. The pricing actuary equates premiums with anticipated losses and expenses, so economic profits are eliminated. In practice, insurers seek to optimize certain goals, such as profits or market share. The price elasticity of demand becomes a crucial determinant of optimal strategy. That is, premium rates and relativities affect consumer demand and the mix of insureds, thereby affecting insurer profitability.

Cars and Courage

> *"Although courage is a splendid attribute in its place, its place is not at the wheel of an automobile."* – Ambrose Ryder [1935]

Early classification schemes had surcharges for older drivers: reactions slow as the body ages, and senior citizens lack the quick reflexes of their sons and daughters. Insurance experience, however, eventually showed the effects of youthful intrepidity, as Ambrose Ryder notes. The physical limitations of older drivers make them less capable of escaping from dangerous situations. But their awareness of these limitations make them far less likely of entering into dangerous situations.[37]

The exposure to road hazards declines as drivers age. Older drivers, particularly after retirement, spend less time behind the wheel (Buck [1978], page 6). They less frequently drive to work, take kids to amusement parks, or attend late parties. As a result, many insurers now provide discounts for older or retired drivers.

Older drivers not only have lower expected loss costs, they also have less impetus to price shop at renewal time. Younger drivers with high premiums have incentives to find lower cost coverage, and they hear about competing rates from friends or at work. Older drivers, with

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37 Ryder [1935], page 143, says: "The next question is whether a driver is a better risk because he reacts one-fifth of a second quicker than the average. Various devices have been on the market for testing the reaction times to danger signals. I think these are all very interesting and may possibly prove of value, but generally speaking the person who is quick on the trigger and who reacts very promptly is probably a less desirable risk than the more phlegmatic person who likes to think things over two or three times before he decides to do anything. The latter type will not react as quickly to the sudden danger that presents itself to his oncoming car but on the other hand neither will he be so likely to allow himself to get into a position where any sudden danger will arise that will require a one-tenth of a second reaction. Give me my choice and I will take the man who is not so quick on the trigger in everything he does in life.

"If the individual driver is going to be measured for his reactions to danger, it is even more important that he should be measured for his willingness to keep away from danger. . . . The timid soul is a much better risk that the daring young man who has the courage to drive his car at 90 miles per hour on a slippery road. The best type of risk, therefore, is the person who is really afraid to take unnecessary chances."

lower premiums and often less information about competing carriers, have less incentive and opportunity to price shop.

This section examines the pricing of a retired driver discount. The relevant considerations for the asset-share model include

- Expected loss costs by policyholder age.
- Persistency rates by policyholder age and policy duration.
- Price elasticity of demand: that is, the effects of price on retention rates.

A Heuristic Illustration

The actual data used to price a retired driver discount are complex, though the principles are straightforward. To see their importance, let us consider a simple illustration, from both a traditional ratemaking perspective and from an asset-share pricing perspective.

Suppose an automobile insurance policy is offered, with a life of five years. That is, each insured purchases coverage for five years, though not necessarily with the same carrier each year. Cost and persistency assumptions are as follows:

- Expected loss plus expense costs, including a reasonable profit, are $100 the first year, $90 the second year, $80 the third year, $70 the fourth year, and $60 the fifth year.

- The market is competitive, and consumers are most sensitive to price at early durations. Your major competitor is offering the same product for $90 each year. If you price below the competitor's rate, your insureds will renew their policies. Moreover, you will attract 50% of your competitor's insureds in the first policy year, 25% in the second policy year, and none in subsequent policy years. If you price above your competitor's rate, you will attract none of your competitor's business, and you will lose 50% of your first year insureds and 25% of your second year insureds. If you price at the same level as your competitor, you will neither attract your competitor's insured nor lose your own business.

- You and your competitor each begin with 200 potential insureds. That is, if you charge equal rates, you will each have 200 insureds each year.

- For simplicity, there is no "time value of money." That is, interest and inflation rates are both 0%, and future events are certain. [The actual asset share pricing model, of course, determines present values of future profits and losses.]

These assumptions are summarized below.

Competitive Pricing Illustration

Policy Year	Expected Cost	Competitor's Rate	Effect of Rate Level on Retention and Production
1	$100	$90	50%
2	90	90	25
3	80	90	0
4	70	90	0
5	60	90	0

The traditional ratemaking philosophy says that premiums should correspond to expected costs: $100 the first year declining to $60 the fifth year. With these rates, you will loss 100, or 50%, of your potential insureds the first year. In subsequent years, you will neither lose nor gain insureds, since in the second policy year you and your competitor have the same rates, and in the following policy years, insureds are not price sensitive. You will earn "normal" profits on this book of 100 insureds for five years, and you will have a 50% loss of market share.

But suppose you price the policy at $85 each year.

- The first year you attract 100 of your competitor's insureds and lose $15 on each policy.
- The second year you attract 25 of your competitor's insureds and lose $5 on each policy.
- You retain these 325 policyholders for the next three years and earn $5, $15, and $25 per insured each year.

Your net profit is

(300)(–$15) + (325)(–$5) + (325)(+$5) + (325)(+$15) + (325)(+$25) = $8,500.

The factors used in this illustrations are oversimplified. For instance, the effects of rate level differences on business retention depend on the magnitude of the difference, not just on which competitor has the lower rate. But the principle is clear, and it is directly applicable to actual pricing problems: Since future profits are embedded in business renewals, long-term profits may be increased by incurring short-term losses to gain good risks.

Retired Drivers

The characteristics of this illustration are equally applicable to retired driver discounts:

- Average loss costs decrease markedly as the policyholder ages. At age 55, the insured drives to work each day and is exposed to road hazards. At age 65, the insureds makes less use of the automobile and loss costs drop.

- The price elasticity of demand, or the extent of comparison shopping, decreases as the policyholder ages. [Equivalently, "consumer loyalty" increases as the policyholder ages.] A driver is more likely to switch carriers at age 55 than at age 65 to obtain a lower rate.

Optimal pricing strategy calls for underpricing insureds in their 50's, to gain market share among this desirable group, then reap the profits when the policyholders advance into their 60's and 70's. Since expected loss costs decline when the driver retires, a level rate, or even a slightly decreasing rate, will cause the transition from losses to gains as the policyholder ages.

The pricing mechanics will be shown with an asset-share model. The task of the actuary is not simply bringing premium to current level or developing losses to ultimate, so as to estimate future costs. Rather, optimizing long-term profits requires offering a discount before short-term data seem to justify it. The actuary must determine the initial age of the retired driver discount and its optimal magnitude, based on competitor actions and market share implications:

- *Age:* The appropriate age is before actual retirement and even before any substantial decline in losses. The optimal age depends on the relationship between policyholder age and persistency and on the discounts offered by competitors, in addition to expected loss costs by age. [In the illustration above, termination rates drop from 50% in the first policy year to 0% in the third policy year. Actual termination rate differences are hardly so extreme.]

- *Magnitude:* The optimal size of the discount depends on the price elasticity of demand and the rate structures of peer companies, in addition to expected loss costs. In the illustration above, there is only one competitor, and demand is extremely elastic. In practice, you must examine the rate structures of your competitors and estimate the effects of rate differences on retention rates and new business production.

Model Assumptions

To determine the optimal age and magnitude for the retired driver discount, the asset-share pricing model requires two sets of assumptions. Some assumptions are grounded in empirical data; others must be projected by the actuary.

Loss Costs by Age of Policyholder

Many insurers examine loss costs by age of policyholder to support classification relativities. Exhibit 8 shows loss ratio relativities by policyholder age, separately for new and renewal business.[38] The relativity shows the ratio of the loss ratio in that row to the average loss ratio for all rows combined.

[38] The data are shown for all coverages combined. Actual experience differs somewhat by coverage and between frequency and severity. We use loss ratio *relativities* because absolute dollar expected loss costs vary with inflation, absolute loss ratios vary with the stage of the underwriting cycle, but loss ratio relativities are stable over time.

Exhibit 8: Loss Ratio Relativities by Policyholder Age

Policyholder Age	New Business LR Relativity	Renewal Business LR Relativity
20 - 49	1.02	1.03
50 - 54	1.00	0.98
55 - 59	0.94	0.83
60 - 64	0.84	0.72
65 - 69	0.82	0.65
70 - 74	0.98	0.76
75 & older	1.10	0.98
Total:	1.00	1.00

The loss ratio relativities are similar to those in the heuristic illustration provided earlier: about unity for drivers below age 55, but dropping as low as 65% as the policyholder ages.[39]

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[39] The loss ratio differences are more pronounced for existing policyholders than for new insureds. For new business, the loss ratio relativities never dip below 82%. The loss ratio relativities for renewal policyholders are at or below this level from age 55 through 74.

This difference makes sense, since the effects of aging differ among insureds. Some retired drivers drive less and drive more carefully; these are the best risks. Others find their responses dulled, but do not change their driving habits; these are dangerous insureds.

Why would a 65 year old driver be looking for a new auto insurance policy? Many retired persons own their own homes and have close friends in their neighborhoods. They are not inclined to move elsewhere and begin new lives or careers - the most common motive for switching insurers. Those who do move often do so because of failing health. They join retirement communities, enter old age homes, or live with their children. They are not usually seeking new auto policies.

Insurers frequently review the policies of drivers who have had recent accidents. If the insurer believes the driver is too risky, it may terminate the policy or "discourage" renewal (e.g., by indifferent customer service). Some of the retired drivers seeking new automobile insurance policies have been considered poor risks by their former insurers.

Exposure distributions by age of the principal operator for new and renewal business reflect this. Among existing policyholders, older drivers form a large percentage of the population and are generally good risks. Among new insureds, older drivers form a smaller percentage of the population. Some of these insureds are good risks; others are dangerous drivers.

For the asset share model, we will use the loss ratio relativities for renewal business. The

Persistency Rates for Older Drivers

Retention rates improve as the policy ages and as the policyholder ages. Sections IV and V show simulated persistency rates by policy duration for all drivers, adult drivers, and young male drivers. Simulated persistency rates for older drivers are shown below.

Exhibit 9: Persistency Rates by Policyholder Age

Policyholder Age	50	54	58	62	66	70	74	78
Persistency Rate	96	95	94	92	90	88	85	80

These persistency rates differ in two respects from those illustrated for adult drivers and for young male drivers in Section V. First, most insureds aged 50 and over are mature renewal business, similar to 10+ policy duration category in Exhibit 5. Thus, the rates for insureds aged 50 through 66 are high. Second, as policyholders advance into their 70's, many stop driving because of death or ill health, so persistency rates drop.

In practice, the persistency rates depend upon the premium discount that is offered. If a 60 year old driver pays $500 in premium, and a competing carrier offers the same policy for $450, the driver is unlikely to switch carriers. That is to say, price elasticity of demand is low, or policyholder loyalty is high. However, if the competing carrier's premium is also $500, but it advertises a retired driver discount of 10%, the insured is more likely to switch carriers. The qualified insured views the retired driver discount as equitable; a carrier who does not offer it is seen as unfair.

We must therefore replace the "persistency rates" in Exhibit 9 with a set of rows, showing persistency rates with no discount, with a 5% discount, with a 10% discount, and so forth. But these persistency rates depend on the discounts offered by other carriers. In other words, there are no "absolute" expected rates, since the expected rates depend on other carriers' discounts.

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indicated retired driver discounts are not necessarily appropriate for new business. The criteria for the discount should be both the age of the policyholder and the number of years since inception of the policy.

The difficulty in forecasting persistency rates highlights the importance of good assumptions. The persistency rate assumptions are subjective, at least until one develops the experience to justify them or to amend them. But they are essential for determining optimal prices.

For the asset share model, we assume two sets of persistency rates. One set, with lower rates, assumes that no premium discount is offered to older or retired drivers. The other set, with higher rates, assumes a 7.5% discount, which is the "market discount" in this illustration.

Exhibit 10: Persistency Rates by Policyholder Age

Policyholder Age	50	54	58	62	66	70	74	78
Persistency: w/ discount	98	97	96	94	92	90	85	80
Persistency: w/o discount	90	85	80	75	80	80	85	80

The persistency rates illustrated above assume that most competing carriers offer a retired (or older) driver discount to policyholders aged 60, but only some of them offer discounts to policyholders in their early or mid-50's. Thus, persistency rates in the "without discount" scenario decline as the policyholder ages from the early 50's to the mid 60's. However, if a full discount is offered even to policyholders in their 50's, few of them switch carriers.

Determining the optimal premium discount requires several runs of the asset-share pricing model, since the results depend on the actuary's assumptions. For instance, what effect does a 7.5% discount have on persistency rates? What effect does persistency rates have on average loss costs?[40] For simplicity, we use three iterations:

- No carrier offers a retired driver discount.
- Many peer companies offer the discount, but your company does not.
- Your company offers a 7.5% discount, which is the prevailing "market" discount.

[40] In life and health insurance, higher termination rates generally lead to higher mortality and morbidity costs, since insureds in poor health are more likely to retain their coverage (Bluhm [1982]).

In each case, we use a 15 year asset-share model for a cohort of insureds aged 52. We assume that persistency rates depend on the premium discount offered, but average loss costs do not.

A. No Carriers Offer Discounts

Exhibit 11 shows the asset-share model results for a cohort of 52 year old drivers, assuming the persistency patterns in Exhibit 10 and the loss ratio relativities in Exhibit 9. Note several differences from the asset-share model results in Section IV:

- The Section IV illustration models new business production, so new business expense ratios are used for the first policy year. The cohort of 52 year old drivers in this section consists of existing insureds, so only renewal business expense ratios are used.

- Average loss costs decrease sharply in the first few policy years but then level out. Section IV used a 3% decline in average loss costs per policy year; this section uses a 1% decline, since most business is mature. In addition, the loss ratio improvements by policyholder age already reflect part of the loss cost improvements as the policy ages.

The model begins with average losses of $500 in the first year and average premium of $600. Because these are existing "high-quality" insureds, with high persistency rates and declining loss costs, profitability is good. The present value of profits over the next 15 years is $1,107, and the present value of premiums is $5,505, for a return on sales of 20%. [This is not unusual. The insurer has already paid the high costs of new business production and is now earning the profits in the renewal book. Similarly, if one excludes the high first year costs in the "business expansion" illustration in Section IV, the return on sales is over 17%.]

A return on premium measure of profitability is reasonable when market shares remain steady, not when market shares are affected by the rate structure. For instance, suppose an insurer writes 10,000 risks at a premium rates of $1,000 apiece, with an average loss plus expense cost of $900 per risk. The return on premium is 10%, or $1,000,000. Suppose also that if the insurer raises rates 50%, it loses most of its business. Only 25 of the poorer risks remain, with an average loss plus expense cost of $1,300 per risk. The return on sales has improved to 13.3%, but the dollar amount of profits has declined to $500,000. The insurer's *results have deteriorated, not improved.*[41]

B. Only Competitors Offer Discounts

The profitability of this business is good, so carriers seek to increase market share by offering retired driver discounts or older driver discounts. Your company wishes to retain its high profit margin, so it offers no discount.

Persistency rates drop sharply. Your insureds see the retired driver discounts offered by other carriers, and they perceive your stance as inequitable. Exhibit 12 shows the asset-share pricing model results. The loss and expense ratios on any given policy have not changed, so the company retains the full profit margin. But retention rates are lower, as more insureds drop out each year. Although 42% of insureds persisted through the full 15 years before the rate revision, now only 8% do so. The present value of future profits has declined from $1,107 per policy to $666 per policy.[42]

C. You and Your Competitors Offer Discounts

To arrest the loss of market share, you offer a 7.5% discount to all drivers age 52 and over, which is the most common market discount (Exhibit 13). The premium discount pleases your insureds, so persistency rates are high. Expenses that are a function of premium, such as renewal commissions and premium taxes, also show a 7.5% decrease, but average loss costs and fixed expenses do not change.

The 7.5% discount can not be justified on a short term basis for drivers in their early to mid-

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[41] If the decline in market share is not offset by increases elsewhere, the insurer's return on equity has decreased. For instance, if the insurer has $5 million in equity, then the return on equity is +20% before the rate revision and +10% after the rate revision.

[42] Since insureds in their 60's are more profitable than insureds in their 50's, the reduction in persistency has a greater effect on the present value of future profits than on the present value of future premiums. Thus, the return on premium declines from 20.1% to 16.7%.

50's. In fact, you show a loss of $2 the first year and inadequate returns the next two years (4% on premium). But now 49% of insureds persist for 15 years, and the present value of future profits has increased to $797.

Other Advantages

Several other aspects of the retired driver discount have not been illustrated in the exhibits but can be incorporated into the asset-share pricing model.

1. The exhibits show only a 15 year illustration, as if all insureds terminated at age 67. But the insured can expect another 5 or 10/years of steady profits, so the difference between an 8% persistency rate in the no-discount case and a 49% persistency rate in the 7.5% discount case has a great effect on future earnings. Ideally, one should extend the pricing model until most business terminates.

2. The exhibits assume no change in the fixed expenses per policy regardless of market share. This is reasonable for premium collection costs, policy printing costs, and similar expenses. Corporate overhead expenses, however, increase as a percentage of premium (or on a per policy basis) when market share declines. Ideally, one should have three expense categories in the asset-share pricing model: variable expenses, per policy expenses, and overhead expenses.

3. Several effects of policyholder satisfaction are difficult to quantify. If policyholders perceive the discount offered at age 52 and over as equitable, there may be fewer instances of fraudulent claims. In addition, persistency may improve slightly even for policyholders younger than 52, since they expect to eventually qualify for the discount.

These items should be considered when determining the optimal premium discount. Most important, though, is a structure that examines long-term profits and market share, such as an asset-share model. Without it, the actuary is easily misled, unable to quantify the effects described in this section. With it, the actuary can project the true profitability of each risk.

(1) Policy Year	(2) Premium	(3) Losses	(4) Var Exp Ratio New	(5) Var Exp Ratio Renewal	(6) Fixed Exp Ratio New	(7) Fixed Exp Ratio Renewal	(8) Persistency	(9) Cumulative Persistency	(10) Profit	(11) Discount Factor	(12) PV of Profits	(13) PV of Premium	(14) Loss Ratio Relativity
1	600	500	0	37	0	23	1.00	1.00	40	1.00	40	600	0.98
2	654	528	0	41	0	24	0.96	0.96	59	1.12	53	561	0.95
3	713	557	0	44	0	25	0.96	0.92	80	1.25	64	524	0.92
4	777	586	0	48	0	26	0.95	0.88	102	1.40	72	484	0.89
5	847	617	0	53	0	28	0.95	0.83	124	1.57	79	448	0.86
6	923	649	0	57	0	29	0.95	0.79	149	1.76	84	414	0.83
7	1006	689	0	62	0	31	0.95	0.75	168	1.97	85	383	0.81
8	1097	732	0	68	0	32	0.95	0.71	189	2.21	85	354	0.79
9	1196	767	0	74	0	34	0.95	0.68	217	2.48	88	327	0.76
10	1303	813	0	81	0	35	0.94	0.64	238	2.77	86	299	0.74
11	1420	862	0	88	0	37	0.94	0.60	260	3.11	84	274	0.72
12	1548	912	0	96	0	39	0.93	0.56	279	3.48	80	248	0.70
13	1688	965	0	105	0	41	0.92	0.51	295	3.90	76	222	0.68
14	1839	1036	0	114	0	43	0.91	0.47	301	4.36	69	196	0.67
15	2005	1111	0	124	0	45	0.90	0.42	304	4.89	62	172	0.66
Total											1107	5505	

Col 2: First year premium is set at $600; subsequent premiums increase 9% per annum.

Col 3: First year losses average $500; loss cost trend is +10% per annum; losses decrease 1% per annum as policy matures; and losses are adjusted by the change in column 14 relativities. For instance, $528 = $500 * 1.1 * 0.99 * 0.95 ÷ 0.98.

Col 5: Variable expense ratio is 6.2% in renewal years.

Col 7: Fixed expenses in the first renewal year are $600 * 3.8% = $23. Subsequently, expenses increase 5% per annum.

Col 8: Assumed persistency rates for older drivers with mature policies; column 9 = downward product of column 8.

Col 10 = {column 2 − sum (columns 3 through 7) } * column 9.

Col 11: Discount factor reflecting annual 12% cost of capital; e.g., 1.25 = 1.12 * 1.12.

Col 12 = Column 10 ÷ column 11; column 13 = column 2 ÷ column 11.

Col 14: Loss ratio relativities by age of insured: 52 years old in first policy year shown and 66 years old in last policy year shown.

Exhibit 11: No Carriers Offer Discounts

(1) Policy Year	(2) Premium	(3) Losses	(4) Var Exp Ratio New	(5) Var Exp Ratio Renewal	(6) Fixed Exp Ratio New	(7) Fixed Exp Ratio Renewal	(8) Persistency	(9) Cumulative Persistency	(10) Profit	(11) Discount Factor	(12) PV of Profits	(13) PV of Premium	(14) Loss Ratio Relativity
1	600	500	0	37	0	23	1.00	1.00	40	1.00	40	600	0.98
2	654	528	0	41	0	24	0.96	0.96	59	1.12	53	561	0.95
3	713	557	0	44	0	25	0.94	0.90	78	1.25	62	513	0.92
4	777	586	0	48	0	26	0.92	0.83	96	1.40	69	459	0.89
5	847	617	0	53	0	28	0.90	0.75	112	1.57	71	402	0.86
6	923	649	0	57	0	29	0.88	0.66	124	1.76	70	344	0.83
7	1006	689	0	62	0	31	0.85	0.56	125	1.97	63	285	0.81
8	1097	732	0	68	0	32	0.82	0.46	121	2.21	55	227	0.79
9	1196	767	0	74	0	34	0.80	0.37	118	2.48	47	177	0.76
10	1303	813	0	81	0	35	0.77	0.28	106	2.77	38	133	0.74
11	1420	862	0	88	0	37	0.75	0.21	92	3.11	30	97	0.72
12	1548	912	0	96	0	39	0.76	0.16	81	3.48	23	72	0.70
13	1688	965	0	105	0	41	0.77	0.12	71	3.90	18	54	0.68
14	1839	1036	0	114	0	43	0.78	0.10	63	4.36	14	41	0.67
15	2005	1111	0	124	0	45	0.80	0.08	56	4.89	11	32	0.66
Total											666	3996	

Col 2: First year premium is set at $600; subsequent premiums increase 9% per annum.

Col 3: First year losses average $500; loss cost trend is +10% per annum; losses decrease 1% per annum as policy matures; and losses are adjusted by the change in column 14 relativities. For instance, $528 = $500 * 1.1 * 0.99 * 0.95 ÷ 0.98.

Col 5: Variable expense ratio is 6.2% in renewal years.

Col 7: Fixed expenses in the first renewal year are $600 * 3.8% = $23. Subsequently, expenses increase 5% per annum.

Col 8: Assumed persistency rates for older drivers with no premium discount; column 9 = downward product of column 8.

Col 10 = {column 2 − sum (columns 3 through 7) } * column 9.

Col 11: Discount factor reflecting annual 12% cost of capital; e.g., 1.25 = 1.12 * 1.12.

Col 12 = Column 10 ÷ column 11; column 13 = column 2 ÷ column 11.

Col 14: Loss ratio relativities by age of insured: 52 years old in first policy year shown and 66 years old in last policy year shown.

Exhibit 12: Only Competitiors Offer Discounts

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Policy			Var Exp Ratio		Fixed Exp Ratio			Cumulative		Discount	PV of	PV of	Loss Ratio
Year	Premium	Losses	New	Renewal	New	Renewal	Persistency	Persistency	Profit	Factor	Profits	Premium	Relativity
1	555	500	0	34	0	23	1.00	1.00	-2	1.00	-2	555	0.98
2	605	528	0	38	0	24	0.98	0.98	15	1.12	14	529	0.95
3	659	557	0	41	0	25	0.98	0.96	35	1.25	28	505	0.92
4	719	586	0	45	0	26	0.97	0.93	57	1.40	41	477	0.89
5	783	617	0	49	0	28	0.97	0.90	81	1.57	52	450	0.86
6	854	649	0	53	0	29	0.96	0.87	107	1.76	61	420	0.83
7	931	689	0	58	0	31	0.96	0.83	128	1.97	65	393	0.81
8	1015	732	0	63	0	32	0.95	0.79	148	2.21	67	363	0.79
9	1106	767	0	69	0	34	0.95	0.75	178	2.48	72	336	0.76
10	1205	813	0	75	0	35	0.94	0.71	199	2.77	72	307	0.74
11	1314	862	0	81	0	37	0.94	0.66	222	3.11	71	281	0.72
12	1432	912	0	89	0	39	0.93	0.62	242	3.48	70	254	0.70
13	1561	965	0	97	0	41	0.93	0.57	263	3.90	68	230	0.68
14	1702	1036	0	105	0	43	0.92	0.53	273	4.36	63	206	0.67
15	1855	1111	0	115	0	45	0.92	0.49	284	4.89	58	184	0.66
Total											797	5491	

Col 2: First year premium is set at $600; subsequent premiums increase 9% per annum; 7.5% discount applied to all premiums.

Col 3: First year losses average $500; loss cost trend is +10% per annum; losses decrease 1% per annum as policy matures; and losses are adjusted by the change in column 14 relativities. For instance, $528 = $500 * 1.1 * 0.99 * 0.95 ÷ 0.98.

Col 5: Variable expense ratio is 6.2% in renewal years.

Col 7: Fixed expenses in the first renewal year are $600 * 3.8% = $23. Subsequently, expenses increase 5% per annum. The 7.5% premium discount does not affect fixed expenses.

Col 8: Assumed persistency rates for older drivers with 7.5% premium discount; column 9 = downward product of column 8.

Col 10 = (column 2 − sum (columns 3 through 7)) * column 9.

Col 11: Discount factor reflecting annual 12% cost of capital; e.g., 1.25 = 1.12 * 1.12.

Col 12 = Column 10 ÷ column 11; column 13 = column 2 ÷ column 11.

Col 14: Loss ratio relativities by age of insured: 52 years old in first policy year shown and 66 years old in last policy year shown.

Exhibit 13: All Carriers Offer Discounts

SECTION VII: ILLUSTRATION 4 – UNDERWRITING CYCLES

Traditional ratemaking methods have no place for competitive pressures, marketplace prices, or consumer demand. Actuaries use volumes of data, established procedures for developing and trending losses, and careful analyses of required profit levels. Credibility formulas and actuarial judgment keep rates on a steady path, never deviating too far from either expected costs or past experience. And market prices seem to jump and skip in willful abandon.

The knowledgeable actuary does not expect market prices to adhere to rate recommendations. In a competitive industry, prices are set by the market. Actuaries tug at them, sometimes drawing them closer to costs, sometimes finding their efforts to be fruitless.

But the actuary also knows that rate recommendations must consider market prices. If competitors are charging $1,400 for a certain risk, few actuaries would recommend a rate of $1,100. If the insurer wishes to expand in this market, it might charge a rate of $1,300 and still earn profits on each risk. If the insurer believes that a rate cut will lead to matching cuts by competitors, it may continue with the $1,400 price.[43]

The actuary's rate recommendations are based on both expected costs and expected market prices. Market prices follow the course of the underwriting cycle. The future is not known with certainty, but its outline can be traced.

Indeed, its outline must be traced. Future losses are not known with certainty either, so actuaries examine past claims, observed development patterns, and projected trends to estimate future costs. Similarly, investment analysts look at historical profit cycles to project future earnings. So too must actuaries consider competitive pressures and industry structure to project future marketplace prices.

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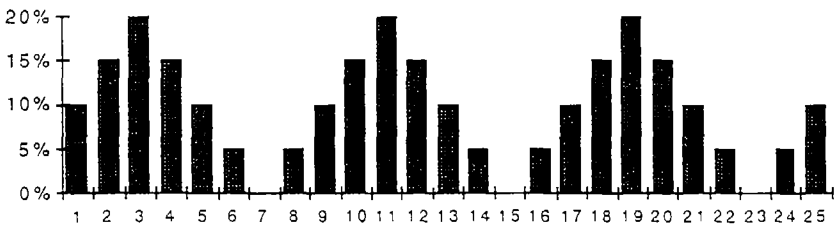
43 For the economic theory of pricing in anticipation of competitors' actions, see Tirole [1988] and Scherer [1980]. For the underlying mathematics, see Varian [1984], Waterson [1984], and Shapiro [1989]. For a general business perspective, see Porter [1980]. For an application to insurance, see Feldblum [1992B].

Let us consider several illustrations; they are all unrealistic, but they clarify the themes. Suppose first that

- Policyholder persistency is perfect: 100% retention rates each year.
- There is no time value of money; alternatively, the expected annual increase in profits exactly matches the discount rate.
- The course of the underwriting cycle is known with certainty.
- The industry alternates between soft (unprofitable) and hard (profitable) markets. The average profit exactly matches the insurer's target return.

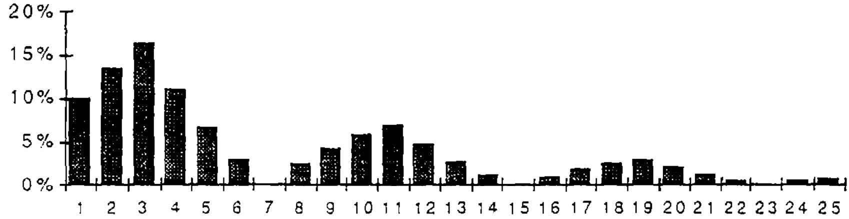
The chart below puts numbers on this illustration. The return on equity generated by this policy oscillates between 0% and 20%. The long-term return averages to 10%, regardless of when the policy is first issued.



Thus, the cycle has no effect on the insurer's underwriting decisions. The insurer will lose money in soft markets and make money in hard markets, but the long-term profits do not depend on when the policy is first written.

Let us remove the unrealistic assumptions:

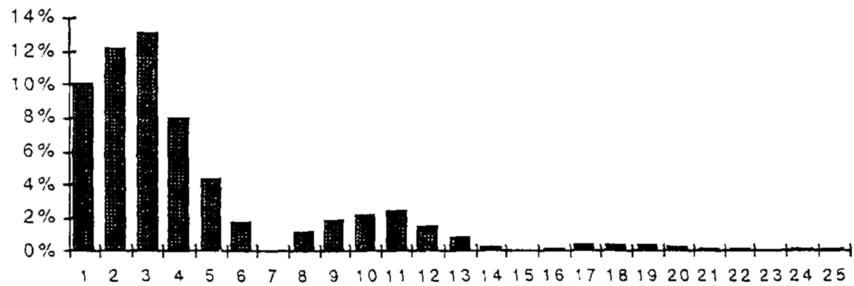
- The retention rates is 90%. Expected profits decline each year because the insured may terminate the policy. The oscillatory pattern is dampened, as shown in the chart below.



The time value of money has two parts, which must also be incorporated.

- The insurer's cost of capital exceeds the expected (inflationary) increase in profits by 5 percentage points.
- The course of the underwriting cycle is not certain. To offset the risk of uncertain future returns, the insurer discounts expected future returns by 5%.

The oscillatory pattern is further dampened.



In the latter two illustrations, the point in the underwriting cycle at which the policy is issued affects the expected long-term return. The asset-share model can be used to quantify the expected returns, using the same methods employed in the previous sections.

SECTION VIII: PROFITABILITY MEASURES

Profit measurement in insurance is difficult, and universally accepted standards do not exist. The traditional 5% of 2.5% underwriting profit provision is no longer supported even by the NAIC, though a return on premium measure is advocated by several actuaries and economists (NAIC [1984]; Woll [1987]; Stewart [1990]).

The most common life insurance asset-share profit measure is the present value of future book profits (Anderson [1959]; Griffin, Jones, Smith [1983], page 381). The rationale is that book profits determine the earnings available for stockholder dividends, so this measure is similar to financial measures of investor returns.[44]

Two differences between life and property-casualty insurers influence the optimal choice of profit measure:

- Life insurers hold discounted policy reserves, with partial adjustment for deferred acquisition costs, so their book profits are similar to economic profits. Property-casualty insurers hold full value reserves with no offset for deferred acquisition costs, so book profits may differ greatly from economic profits.

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[44] Cf. also Larner and Ryan [1991], page 448: "The definition of economic or appraisal value as the present value of future net earnings streams taken at appropriate risk discount rates is generally accepted by actuaries and others as a natural one throughout the world in our experience. . . . Modern portfolio theory and other investment work provides a theoretical basis for the suggestion that the value of a company is the present value of its future net earnings." The Actuarial Standard of Practice No. 19 concerning Actuarial Appraisals [1991], page 4, paragraph 5.2.1, explicitly notes the connection between book profits and investment returns: "Distributable Earnings – For insurance companies, statutory earnings form the basis for determining distributable earnings, since the availability of dividends to owners is constrained by the amount of accumulated earnings and minimum capital and surplus requirements, both of which must be determined on a statutory accounting basis. . . . Economic value generally is determined as the present value of future cash flows. Statutory accounting determines the earnings available to the owner. Hence, while future earnings calculated according to generally accepted accounting principles (GAAP) will often be of interest to the user of an actuarial appraisal, as may other patterns of earnings, the discounted present-value calculations contemplated within the definition of actuarial appraisal in this standard should be developed in consideration of statutory earnings, rather than some other basis."

- The life insurance patterns of cash flows, adjusted for policyholder cash values, correspond to book profits. For instance, the first year "investment," corresponding to the first year book loss, is the first year cash outflow to agents and policyholders. Thus, investor returns correspond to book profits which correspond to actual patterns of cash flows and policyholder cash values.

  Property-casualty insurance lacks this correspondence. First year cash flows are positive for the insurer. Capital to asset ratios, however, are high. The "investment" at the beginning of the insurance transaction is not simply the assets supporting the reserves, but also the investor capital "committed" to support the policy. In sum, the book profits for the insurer are not necessarily a good proxy for the implied equity transactions between the insurer and its stockholders.[45]

Measuring Rods

There are several methods of adapting asset-share profit measures for property-casualty operations:

1. Show economic profits of each year instead of book profits, by using discounted reserves. Profits may be measured as a return on surplus, using assumed premium to surplus (or reserves to surplus) leverage ratios (Butsic and Lerwick [1990]). This is the profit measure used in Section IV, the "business expansion" illustration.

2. Alternatively, profits may be measured as the net present value of premiums minus the net present value of expenditures (losses, expenses, and taxes). Surplus is relevant only for determining the taxes on investment income derived from capital (cf. Myers and Cohn

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45 In contrast, life insurance capital to asset ratios are low, and surplus is needed more for asset risk and interest rate risk than for insurance risk. In other words, there is no "commitment of surplus" to support the insurance policy.

[1987])[46]. This is similar to the dollar measure of profits in Section VI.

3. Profits may be measured by a multiperiod internal rate of return model, by showing

   - cash transaction between the insurer and policyholders or claimants,
   - investment transactions between the insurers and the financial markets, and
   - *the implied equity transactions between the insurers and its stockholders* (cf. Feldblum [1992]).

   Despite the theoretical accuracy of this procedure, its complexity may make it less suitable for practical pricing work.

4. *Some practitioners prefer simpler measures, such as the* "payback period," or the number of years until the cumulative net present value of profits is positive. In the business expansion illustration, the cumulative net present value of profits is negative for the first four years and turns to a positive $9 thousand in the fifth year. In other words, a policyholder must persist for at least five years before the transaction becomes profitable for the insurer.

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[46] Similarly, Anderson [1959] *recommends* that "the *profit objective* be *defined* by *the* criterion that the present value of the profits which will be received in the future be equal to the present value of the surplus depletion, with both present values based on a yield rate or yield rates which represent adequate return to the stockholders for the degree of risk incurred in expending surplus in the expectation of receiving future profits. That is, the present value of the entire series of profits and losses is zero" (page 356).

SECTION IX: CONCLUSION

Actuarial pricing must consider long-term profitability and market share objectives, not merely short-term accounting results. Considerations of persistency patterns, the variation of expected losses and expenses with the time since inception of the policy, and the use of a model that incorporates these effects are essential for accurate ratemaking.

This paper has presented the fundamentals of such an approach. It builds upon life insurance asset-share techniques and adapts them for personal automobile business.

Some of the specific techniques discussed above are new, but the underlying philosophy is not. Underwriters and salespersons of the major personal lines carriers base their marketing decisions upon intuitive estimates of long term results. Actuaries, seeking more accurate assessments, strive to replace the intuition with facts.

A story: At a recent management meeting of Personal Auto underwriting, actuarial, and sales executives, the underwriting SVP presented a recurring problem.

> *The company has a good, profitable risk: a married couple with two cars and no claims in the past 12 years. The couple's only son has just finished his junior year in high school and obtained a driver's license. By the company's rating rules, the premium will increase by almost a thousand dollars.*

The underwriter expects that the son will leave for college after he completes high school, and policy will then enjoy 20 profitable years. But he fears that the insured may be so incensed by the thousand dollar increase in premium that he will switch carriers.

This is the type of dilemma discussed throughout this paper. Short-term expectations say that the thousand dollar increase in premium is needed for the coming year. Long-term expectations say that this is a foolish pricing strategy.

The talents of the actuary are needed. In some cases the thousand dollar increase in premium is appropriate. [Suppose the risk has three sons, aged 13, 15, and 17, the oldest of whom just received his license, and none of whom will leave home for college.] The actuary must quantify the long-term expected profitability of each risk and then devise a classification scheme that differentiates among them. The task is difficult, but the rewards are correspondingly great.

MERIT RATING FOR DOCTOR PROFESSIONAL LIABILITY INSURANCE

Robert J. Finger

MERIT RATING FOR DOCTOR PROFESSIONAL LIABILITY INSURANCE

ROBERT J. FINGER

Abstract

Merit rating is the use of the insured's actual claim experience to predict future claim experience. This paper discusses merit rating for professional liability insurance for both individual doctors and group practices. The paper presents several different theoretical formulations for merit rating. Credibilities are stated in terms of the parameters of the risk process. The paper discusses several methods of estimating the key parameters, along with sample data. Finally, the paper discusses several practical considerations in the design of a merit rating formula.

1. INTRODUCTION

The use of an insured's past claim experience for prospective premium determination can variously be called experience rating or *merit rating.* Merit rating is common for workers' compensation and commercial liability coverages. Merit rating for individual insureds is less common, although "claim-free discounts" or accident surcharges for personal automobile insurance are widely used. Several insurers now use merit rating for doctor professional liability insurance.

After describing the general problem, this paper will restate the theoretical basis for merit rating. It then will present alternative merit rating formulations in terms of the parameters of the risk process. It then turns to methods for estimating the required parameters. It will apply these methods to actual data. Finally, it will discuss various practical problems in implementing a merit rating program. The paper will deal with two related situations: claim-free discounts and surcharges for individual doctors and merit rating for group practices.

2. GENERAL STATEMENT OF THE PROBLEM

We assume that there is some classification plan that will determine a premium for a given doctor (or group). The classification variables may include medical specialty, types of procedures, geography, and teaching or part-time status. For groups, there may also be schedule rating credits.

Why do we also need merit rating? Generally speaking, because the insured's own claim experience provides additional information that can rate the insured more accurately. We give some reasons for additional cost variations below. In a competitive environment, more accurate rates will generate greater profitability for the insurer. From the insured's point of view, more accurate rates are also fairer. Better doctors will pay less and poorer doctors will pay more. From society's point of view, merit rating (and more accurate rating, generally) will provide an incentive for loss prevention.

Merit rating must be considered in connection with the classification plan (i.e., other rating variables). The more accurate the class plan, the less meaningful individual claim experience will be, and vice versa. Assume, for example, that the presence of a particular factor makes an insured 10% more expensive. If that variable is used in the classification plan, every insured with that factor will pay 10% more. If that variable is omitted, insureds with that factor who are merit rated will pay somewhat more than those without the factor, but most likely they will not pay 10% more. This follows from the concept that most insureds will receive less than 100% credibility.

Why do individual costs differ?

Why would we expect doctors to have different loss costs? It is well recognized that different specialties have widely differing costs. This probably results from a variety of reasons. Certain specialties, such as surgeons, perform a higher percentage of procedures that can have devastating results, if done improperly. For certain specialties, such as psychiatrists, it may be very difficult to prove the causal connection between negligent practice and adverse results for the patient. For certain specialties, such as physicians versus surgeons, the average patient is much healthier and any negligence is less likely to do damage. Thus, most insurers classify doctors by specialty. For physicians, most insurers also classify by the type or amount of surgery performed.

This classification plan does not cover all possible variations in costs among doctors in the same specialty. Costs may also vary for three general reasons: (1) limitations in the class plan; (2) exposure; and (3) competence. Each will be discussed below.

Most class plans group specialties into about 10 different rate groups. In addition to specialty, the grouping may depend upon whether a doctor performs various procedures. The reason for this grouping is a lack of credibility for many specialties and procedures. That is, the number of insured doctors and the number of claims for many specialties and procedures is low. The volatility of claim experience for these low-volume categories makes it difficult to determine their cost. It is also difficult to know how many of a certain type of procedure was performed during a given year. Doctors are usually classified by whether or not they perform a procedure, not on the number of procedures.

This classification scheme can result in significant cost variation within a given rate group. For example, group 0 may have a rate relativity of 70%; group 1, 100%, and group 2, 150%. Within group 0, there may be specialties that have relativities of 50%, 60%, 70%, and 80%. Within group 1, there may be specialties with relativities of 90%, 100%, 110%, and 125%. In addition, the exposure to certain procedures may vary significantly. For example, the performance of procedure A may shift a doctor's classification from group 1 to group 2. Some doctors may perform 10 A's a year and some may perform 50 A's a year. A more exact classification plan might base the premium on the number of A procedures during the year.

The classification plan also may not consider other cost variations. Costs vary significantly from state to state. Some of this is due to differences in statutory or case law. Some of the difference may also be due to differences in the liberality of juries, the quality of the plaintiff's bar, and the claims consciousness of patients. These latter differences may exist within a state. In particular, there may be differences between urban, suburban, and rural areas.

There may also be cost differences among doctors related to differences in exposure. For example, some doctors may treat more patients or may engage in more high-risk procedures. In addition, the type of patient may be different. Some doctors may have richer or poorer clients, who may have higher or lower damages, should negligence occur. Some doctors may also undertake higher-risk patients, which could affect both the frequency and severity of loss costs.

Finally, doctors undoubtedly differ in competence, which has many aspects. Training and experience differ. Doctors vary in their adherence to continuing education and changing practice standards. Doctors vary in their dexterity, judgment, attention to detail, bedside manner, and supervisory skills. The style of practice (e.g., number of patients, number of prescribed tests) may vary. Some doctors may have alcohol, drug, or other psychological problems.

Generalized Mathematical Structure

Now that we recognize that costs can vary significantly within the classification plan, how do we structure the merit rating plan? Virtually all merit rating plans use an adjustment to the class rate. In many lines, this is called a "modification factor." The adjustment could also be a credit or surcharge, which is expressed as a percentage of the class rate.

Virtually all merit rating plans calculate the modification factor according to the following generalized formula:

$$M=Z\frac{A}{E}+1-Z$$

where: **M** is the modification factor, which is multiplied against the class rate; **Z** is the credibility factor; **A** is the insured's actual claim experience; and **E** is the average claim experience for the class. In practice, virtually always the credibility is limited to values between and including 0 and 1. Thus M is a weighted average of the insured's relative experience (to the class average) and the class rate. (We could have written the right-hand term as $(1-Z) \times 1$.)

We can express the same concept in terms of a discount or surcharge, as a percentage of the class rate. The adjustment to the class rate, as a factor of the class rate, can be calculated by subtracting 1 from M:

$$Adjustment = M-1 = \frac{A-E}{E}Z$$

When M is less than 1, the adjustment will be negative, or a discount from the class rate. When A=0, the insured has no claims. The "claim-free" discount is thus Z, the credibility. Indeed, this may often be the easiest way to measure credibility. If we have claim data for two experience periods, with a substantial number of claim-free insureds in the first period, the relative cost of these insureds in the second period, to the average cost for all insureds in the second period, is the empirical claim-free discount and the empirical credibility.

The formula for M is a linear function of the insured's actual claim experience. It would be theoretically possible for M to be some other type of function. Other functions do not seem to have been used in actual practice. Perhaps the linear function is the most intuitively reasonable function. In addition, where a linear function might not be useful, the definition of A is modified. For example, it seems unreasonable in some cases to charge the entire amount of a large claim; very often, the maximum chargeable claim size is limited in some manner. An advantage of the linear formulation comes in the estimation and interpretation of Z.

Merit rating plans differ in defining A, in calculating E, and in determining Z. The usual process is to first define A, or what data is to be used for the insured's claim experience. Once this is done, E usually can be handled in a straightforward manner; it represents the class average claim experience for the given definition of A.

The specification of Z can be done in at least three ways. First, it can be established on an ad hoc basis. For example, we could decide that 100 expected claims was "full" or 100% credibility, and partial credibility was the square root of the expected count to 100. We might inject some actuarial or statistical theory into the selection of the full credibility standard. (See, e.g., Longley-Cook [5] or Venter [14]).

Second, Z can be developed from risk theory. We can use the famous credibility formula:

$$Z=\frac{P}{P+K} \tag{1}$$

where P is a measure of exposure and K can be determined from the following equation:

$$K=\frac{\sigma^2}{\tau^2} \tag{2}$$

where σ^2 is defined as the "process variance" and τ^2 is defined as the "variance of the hypothetical means." The process variance is the variance we would expect for the class average insured's experience, given P units of exposure. The variance of the hypothetical means is the inherent variability of mean claim costs for the insureds within the given class, adjusted for P units of exposure. Depending on our definition for A, it may be possible to determine numerical equivalents for the process variance and the variance of the hypothetical means.

Third, we can estimate Z statistically from actual data. Although potentially we could use any statistical estimation procedure, the use of linear regression results in the same credibility formula and parameter explanation as the risk theory approach.

Although the risk theory and regression approaches are very similar, it should be realized that actual results may differ. The real world may differ from our theory or our theory may only approximate the real world. The theoretical approach allows us to apply knowledge from one context to another context. For example, measurement of the variance of the hypothetical means for one company, state, or line of business, may be a useful input to another company, state, or line of business. The theoretical approach also allows us to generalize actual findings. For example, we may extrapolate three-year data to a four-year experience period. We should remember, however, that the real test of merit rating is how accurately it prices insureds in practice.

Alternative Forms for Modification Factor

There are several general considerations in the design of a merit rating plan. (See, e.g., Tiller [11].) First, it should be readily understood by insureds, agents, and company personnel. Second, it should be reasonably simple to administer. Third, it should not allow for manipulation by insureds. Finally, it should strike a balance between stability and responsiveness. On the last point, any formula can be adjusted to give greater or lesser weight (i.e., credibility) to the insured's own experience. If too much weight is given, rates may fluctuate too much from year to year. If too little weight is given, the pricing system may not be as accurate as possible and loss prevention incentives are reduced.

The main decision in formulating a merit rating formula is the definition of A, the insured's actual claim experience. Choices involve the "length" of the experience period and whether to use counts or amounts. The "length" may be thought of as the number of years of experience, but could also include exposure from multiple locations or states. If the actual claim count is used, it could be defined as the reported count, the closed-paid count, or some definition of a non-nuisance claim. For example, a non-nuisance claim could be a settlement for more than $5,000 ("CP5"). If amounts are used, there may be some limitation on the maximum chargeable claim; there is also an option of including or excluding allocated loss adjustment expense, loss development, and incurred but not reported ("IBNR") claims.

In the National Council on Compensation Insurance ("NCCI") Revised Experience Rating Plan, A is defined in terms of loss amounts, usually for three policy years. A is divided into "primary" and "excess" losses, with the first $5,000 of each loss being primary and the remainder, excess. There is also a per claim limit of 2.5 times the average cost per "serious" claim, a per occurrence limit of twice the per claim limit, and a limit on the total cost of diseases. Experience generally is pooled for all NCCI states and all entities with at least 50% common ownership. E, the expected losses, is divided into primary and excess portions. E must also be adjusted for loss development and the loss limitations.

The Insurance Services Office ("ISO") has similar experience rating plans for general and automobile liability. A is limited to basic limits loss amounts. There is an additional limitation on the maximum claim size, based on premium size. A provision for IBNR, based on exposure, is added to A. E is adjusted for the loss limits and loss development.

Several insurers use merit rating for doctors. The typical plan offers an individual doctor a certain percentage discount for each claim-free year. Chargeable claims usually are limited to non-nuisance settlements (e.g., claim closed for more than $5,000). There is usually a maximum discount, which applies after five or six claim-free years. One insurer offers lower discounts for physicians than surgeons. A doctor loses the entire discount when a claim is charged; the discounts accumulate thereafter for each new claim-free year. Rules may differ according to the insurer of the claim. For example, some insurers give credit for claim-free experience with other insurers. The experience period may be actual policy experience or it may be any settlements during a given period, regardless of the occurrence or reporting date.

Several insurers offer merit rating discounts to groups of doctors, based on the following generalized formula:

$$Adjustment = M - 1 = \frac{A-E}{JE+K}$$

where E is the expected claim count, A is the actual claim count, J is a constant (e.g., 2), and K is a constant (e.g., 1). E is calculated from the number of insureds by rating class for the group; there is a separate claim frequency factor for each rating class.

Some Truisms

In workers' compensation there is the concept of the "off-balance" in the merit rating plan. That is, the average modification factor is not necessarily 1.0. The average collectible rate for a class will not necessarily be the same as the class manual rate. Thus the manual rate must be adjusted for off-balance. This concept is important for doctor professional liability insurance, particularly if we adopt a claim-free discount only approach. With only discounts and no surcharges, the average collectible rate will be less than the manual rate.

Taking another perspective, it is necessary for those who do not receive the discounts to pay for the discounts. If some insureds pay less than the average cost, some must pay more. Even if we do not call it a surcharge, the difference between the claim-free discount and the manual rate is the cost of not qualifying for the claim-free discount. For example, the claim-free discount might be 25%. A doctor who loses the discount will pay an additional 33%. Whether we call this a surcharge or the manual rate, the cost of a claim is still 33%.

Although we will estimate credibilities in a later section of the paper, it is worthwhile to consider the tradeoffs between discounts of various sizes. Exhibit I shows the required manual rate increase, given discounts of various sizes (10%, 20%, 30%, 40% and 50%). The manual rate increase is dependent upon the percentage of insureds receiving the discounts. For example, if 90% of insureds receive a discount of 10%, the manual rate must be increased 9.9%. In other words, 10% of insureds pay 109.9% of the average and 90% pay 98.9% of the average. We give a discount of 1.1% to the 90% that are claim-free and require the other 10% to pay an additional 9.9%.

3. ACTUARIAL THEORY

As we have seen, the first step in formulating a merit rating plan is to define A, the insured's actual claim experience. Once that is done, usually it is straightforward to determine E, the average claim experience for the insured's class. The most complicated and difficult part is to determine Z, the credibility to attach to the insured's experience. This section discusses various risk theory formulations for credibility. Although these formulations may not replicate the real world, they are useful in several ways. First, they provide a conceptual basis for understanding the statistical validity (i.e., credibility) of claim experience. Second, they provide a means to formulate credibilities when directly relevant claim experience is not available. Finally, they provide insight into the process of estimating credibilities.

In developing the following formulas, we will want to consider both claim counts and claim amounts. We also will want formulas for a single exposure period as well as multiple periods. There is no limit to the number and sophistication of formulas that can be developed; even so, we probably have included formulas that may be too difficult to test in practice.

The Basic Risk Process

We begin with a simple risk process and add various layers of complexity. We will develop formulas for variances. With few exceptions, the means are obvious and therefore omitted.

Assume that we have one doctor insured for one exposure unit (of time). We define **N** as a random variable for the number of claims for the period. We assume that N has a mean of λ. We assume that each claim has a claim size distribution **S**, with mean μ and coefficient of variation squared $\boldsymbol{\alpha}$. We also define **T** as the sum of individual claim amounts, or the total losses for that doctor for that exposure unit. If we assume that N and S are independent, we can calculate the variance of T from the moments of N and S.

$$Var(T) = E[N]\, Var(S) + Var(N)\, E^2[S]$$

We use the notation "E[x]" as the expected value of x. We previously defined α as $Var(S)/E^2[S]$. If we make the additional assumption that N is Poisson distributed, then $Var(N) = E[N] = \lambda$.

Thus we have a fundamental risk theory formula:

$$Var(T) = \lambda\mu^2(1+\alpha) \tag{3}$$

We can extend this formula to P exposure units. We assume that the same parameters apply to each exposure unit. Generally speaking, we can replace λ by $P\lambda$, if we assume that N is Poisson. Thus for P exposures, we have:

$$Var(T) = P\lambda\mu^2(1+\alpha)$$

There are two important assumptions in this formulation: that the count and amount distributions are independent and that the count distribution is Poisson. To the extent these are not true in practice, our use and interpretation of these formulas may be faulty. If we do not assume independence, we can still calculate the variances using covariance terms. This will be complicated, particularly when we make the formulas more complex. It seems reasonable in practice to assume independence, as long as we remove nuisance or closed-without-payment claims.

The Poisson assumption is very significant, particularly for the property that its mean equals its variance. The Poisson distribution arises from a process that satisfies three conditions: (1) events in two different time intervals are independent; (2) the number of events in an interval is dependent only on the length of the interval; and (3) the probability of more than one event occurring at the same time is zero. (See Beard [1], chapter 2). In practice, these conditions might be violated if there were some catastrophe (or contagion) or if an individual's claim frequency depended on its past history. As an example of the first case, we might have suits for breast implants or for the transmission of AIDS. As an example of the second case, we might have a plaintiff's attorney developing a series of suits against a practitioner, related to multiple incidents of unnecessary surgery or sexual misconduct with patients. For the most part, the Poisson assumption seems reasonable in practice, but we must be aware when it does not apply.

It would be possible to assume that N followed some other distribution, with two parameters. The practical consequence of this, however, would be to add one more parameter that we would need to estimate. The interpretation of this parameter likely would overlap with the interpretation of other parameters, to be explained below. In addition, the estimation of this parameter might require data from an additional time period, which might be difficult to obtain.

Heterogeneity in the Insured Population

The above formulations assumed that we knew the parameters for the given doctor. We have calculated the "process variance." By the nature of merit rating, we assume that doctors will vary in their inherent claim costs. Thus we need to expand the above formulation to add this heterogeneity. Conceivably, any of the above parameters could vary among the doctor population. We will assume that only the mean claim frequency varies among doctors; this should add sufficient complexity for practical purposes. We define a new random variable, χ, to have a mean of 1 and a variance of β. We will refer to β as the "structure variance." It is the (weighted average) variance of the insured population means (relative to the overall population mean.) For any given doctor, the mean claim frequency is assumed to be $\lambda\chi$. We can incorporate these assumptions into our formulation by using a fundamental property of conditional probabilities:

$$Var(N)=E_{\chi}[Var(N|\chi)]+Var_{\chi}(E[N|\chi])$$

If we assume a Poisson process, we have $Var(N|\chi) = \lambda\chi$. We can rewrite the last equation as:

$$Var(N)=E_{\chi}[\lambda\chi]+Var_{\chi}(\lambda\chi)$$

With the expectations taken over the variable χ, λ is a constant and can be taken outside of the operator. The variance of a scalar times a random variable is the scalar squared times the variance of the random variable. We previously defined $E[\chi]=1$ and $Var(\chi)=\beta$. Thus we can rewrite the previous equation as:

$$Var(N) = \lambda + \beta \lambda^2$$

For P exposure units, with the same parameters, we have:

$$Var(N) = P\lambda + \beta (P\lambda)^2$$

For the total amount, T, for a single exposure unit, we have:

$$Var(T) = E_\chi [Var(T|\chi)] + Var_\chi (E[T|\chi])$$

This can be written as:

$$Var(T) = E_\chi [\lambda \chi \mu^2 (1+\alpha)] + Var_\chi (\lambda \chi \mu)$$

$$Var(T) = \lambda \mu^2 (1+\alpha) + \beta (\lambda \mu)^2 \qquad (4)$$

For P exposure units with the same parameters, we have:

$$Var(T) = P\lambda \mu^2 (1+\alpha) + \beta (P\lambda \mu)^2$$

Although we used the same notation, β, for the population heterogeneity for both counts and amounts, in reality there may be a different value in the two different contexts. For example, there may be differences in the average claim size as well as in claim frequency.

For equation (4) above, we note that the first quantity is the "process variance," or the variance given one exposure unit and known parameters, from equation (3). The second quantity is the product of β, the variability in the insured population (given a mean of 1), and the square of $\lambda\mu$, which is the mean. This second quantity is the "variance of the hypothetical means." The $\lambda\mu$ term is a scalar that results from the variance calculation. Indeed, we can rewrite the first term, eliminating the square of the scalar, as:

$$\frac{(1+\alpha)}{\lambda}$$

This quantity represents the process variance relative to the mean, just as β is the structure variance, relative to the mean. We will use the term "relative variance" to be the ratio of a variance to the square of the mean. It is the coefficient of variation squared.

The Basic Credibility Formula

Using the fundamental formula for conditional probabilities, we can write Var (T) as:

$$Var(T)=E_{\chi}[Var(T|\chi)]+Var_{\chi}(E[T|\chi])$$

This is the same form as:

$$Var(T)=\sigma^2+\tau^2$$

Here σ^2 is the average process variance and τ^2 is the variance of the means of the insured population. If we define τ^2 and σ^2 in terms of one exposure unit, our credibility formula (1) becomes:

$$Z=\frac{\tau^2}{\sigma^2+\tau^2} \tag{5}$$

It is important to note that the denominator of the credibility formula is the total variance for the insured experience. Thus we have a general formula for credibility that conforms to our risk theory model of the claim process. For claim counts, we have $\sigma^2=\lambda$, and $\tau^2=\beta\lambda^2$. Dividing through by λ we have:

$$Z=\frac{\beta\lambda}{1+\beta\lambda} \tag{6}$$

If we divide through by $\beta\lambda$, we get the generalized formula, 1/(1+K), with:

$$K=\frac{1}{\beta\lambda}$$

For P exposure units, we substitute Pλ for λ above. This gives us an extra P in the τ^2 terms. By the same operations, we arrive at the generalized formula for Z = P/(P+K), with the same K as above.

It will be useful to write the credibility in terms of the expected claim count, E=Pλ. Thus we have:

$$Z=\frac{E}{E+K'} \tag{7}$$

where K'=1/β.

If A is defined in terms of amounts, then $\sigma^2=\lambda\mu^2(1+\alpha)$ and $\tau^2=\beta(\lambda\mu)^2$. Dividing through the general formula for Z by $\lambda\mu^2$ yields:

$$Z=\frac{\beta\lambda}{(1+\alpha)+\beta\lambda}$$

Dividing this through by $\beta\lambda$ leads to the formula for K:

$$K=\frac{1+\alpha}{\beta\lambda}$$

We can also see that the scalar term for the mean will appear, squared, in both the σ^2 and τ^2 terms. These items will cancel in the credibility formula. We will be left with a formula for K that is the following ratio:

$$K=\frac{(Relative)\ Process\ Variance}{(Relative)\ Structure\ Variance}$$

For counts, the numerator is $1/\lambda$ and the denominator is β. For amounts, the numerator is $(1+\alpha)/\lambda$ and the denominator, again, is β, although the numerical value β may be different for counts and amounts.

It also will be useful to analyze the total relative variance. We remember that the total variance is $\sigma^2+\tau^2$ and the relative variance is calculated by dividing the variance by the square of the mean. For the above credibility formulation, for counts, we have the following formula:

$$Total\ Relative\ Variance=\frac{1}{\lambda}+\beta$$

We know that the Poisson relative variance is $1/\lambda$. Thus the excess relative variance, for this formulation, is β.

Risk Shifting

One of the limitations mentioned in connection with the Poisson assumption was the changing of an individual's mean costs over time. This can be handled formally, by an adjustment to the credibility formula. This phenomenon has been called by various names, such as "parameter uncertainty" (see Meyers [10]) or "risk shifting" (see Mahler [6], [7] and Venezian [13].) An interesting application is presented by Meyers [10], concerning the merit rating of Canadian automobile insurance.

In effect, the basic risk theory formulation breaks down when exposure is added for a given insured. Instead of credibility increasing approximately in proportion to P, in the general credibility formula, the increase is significantly less. There is an intuitive explanation. Since the insured's mean costs may change over time, there is uncertainty that its historical mean may be the same as its future mean.

This phenomenon can be modeled in the same manner that we modeled heterogeneity among different insureds. The heterogeneity parameter, of course, should be different. Instead of reflecting the differences among the insured population, it reflects the differences for a given individual over time.

We define δ as the variance of the individual insured's mean costs over time. We should note that it may be difficult to differentiate between β and δ. Both parameters reflect the differences in individual insured experience; β reflects those differences between individuals in the same period and δ reflects differences between the same individuals in different periods. Since we do not have the opportunity to observe different experience for the same individual in the same period, there may be some ambiguity in the measurement process.

The main difference in the mathematics from the previous formulation is that the process variance is different. Instead of being λ for counts, it now becomes:

$$\sigma^2=\lambda+\delta\lambda^2$$

For amounts, the process variance is:

$$\sigma^2=\lambda\mu^2(1+\alpha)+\delta(\lambda\mu)^2$$

The formula for credibility, $\tau^2/(\sigma^2+\tau^2)$, for counts, becomes:

$$Z=\frac{\beta\lambda}{1+\delta\lambda+\beta\lambda}$$

The total relative variance is $1/\lambda+\delta+\beta$. The excess relative variance is $\delta+\beta$. Dividing through by $\beta\lambda$, we can rewrite the last equation as:

$$Z=\frac{1}{1+\frac{\delta}{\beta}+\frac{1}{\beta\lambda}} \tag{8}$$

If we let $K=1/\beta\lambda$ and we define $J=1+\delta/\beta$, then we have a general credibility formula, $Z=1/(1*J + K)$. For P exposure units, we can derive the equation:

$$Z=\frac{P}{PJ+K}$$

We can also state the credibility in terms of E, the expected claim count:

$$Z=\frac{E}{EJ+K'} \tag{9}$$

where J has the same definition as above and $K'=1/\beta$, as before in the basic credibility formulation, (7).

For amounts, we derive the credibility formula:

$$Z=\frac{1}{1+\frac{\delta}{\beta}+\frac{1+\alpha}{\beta\lambda}}$$

This has the same J as for counts, above, and the same K as for amounts in the basic credibility formulation.

We have the following changes from the basic formulation. The process variance is now larger, since there will be more variability in the individual insured's experience. The excess relative variance is the sum of δ and β. When we estimate β, we will have a smaller structure variance. Thus σ^2 is now larger and τ^2 is now smaller. The credibility will be reduced.

We should note that the maximum credibility is 1/J. In effect, we are saying that since the individual's mean cost may be different in the future than it was in the past, we may not be insuring the same risk and, hence, we will always give some credibility to the class average.

Heterogeneity within the Insured

The rationale for the next generalization in the credibility formula does not apply to individual doctor experience. It may be useful, however, in developing formulas for group experience. This generalization has been used by the NCCI. As with risk shifting, we have a situation where adding exposures does not yield as much credibility as if all exposures had had the same underlying risk parameters.

In the first credibility formulation, we developed a parameter β, which described the variance in the insured population. We now want to develop credibility for groups. If all of the doctors in the group were equally good or equally bad, we could apply the first credibility formulation, using P to represent the exposure for the number of doctors in the group. In all likelihood, however, the group will have some better doctors and some poorer doctors. Some of the underlying risk factors, such as geography, might apply to the entire group; other risk factors, such as training and experience, would be different for different members. If the composition of the group was entirely random, with respect to the insured population, we could rate each doctor individually; there would be no additional statistical validity to the group experience, apart from the individual doctor experience.

We define γ as the variance of mean costs (adjusted by class) within a given group or insured. We expect that $0 < \gamma < \beta$. In other words, the variability within the group is not as large as the insured population, but it is not zero.

The variance of the insured population means is different than before. Here the "insured population" is groups with a degree of heterogeneity. Some part of the variance will be proportional to the number of exposures (i.e., each exposure has the same parameters, for which the variances are additive) and some part will be proportional to the square of the number of exposures. We can write this as:

$$\tau^2=\lambda\gamma+(\beta-\gamma)\lambda^2$$

We know from the previous development that, for counts:

$$\sigma^2=\lambda+e\lambda^2$$

We also know that the total variance, ignoring the possibility of $\delta>0$, is $\lambda+\beta\lambda^2$. From this we can solve for $\epsilon=\gamma(\lambda-1)/\lambda$. Thus we have:

$$\sigma^2=\lambda+\gamma(\lambda-1)\lambda$$

Using the general formula for credibility and dividing by $\beta\lambda^2$, we have:

$$Z=\frac{(1-\frac{\gamma}{\beta})+\frac{\gamma}{\beta\lambda}}{1+\frac{1}{\beta\lambda}}$$

For P exposure units, we have:

$$Z=\frac{(1-\frac{\gamma}{\beta})P+\frac{\gamma}{\beta\lambda}}{P+\frac{1}{\beta\lambda}}$$

In terms of the expected count, E, we have:

$$Z=\frac{(1-\frac{\gamma}{\beta})E+\frac{\gamma}{\beta}}{E+\frac{1}{\beta}} \tag{9}$$

We can write this in a more general form:

$$Z=\frac{(1-I)E+I}{E+K'} \tag{10}$$

where $I=\gamma/\beta$ and K' has the same form as the previous formulations for E.

The interpretation of this formula depends on the specific values for the given parameters. As we will see below, this formula may produce higher credibilities than the previous two formulations, when the expected claim count is low. Excepting this situation, however, we can relate this formula to the previous formulations. We see that the (1-I) term reduces the effectiveness of additional exposures. Since the exposures within a group are heterogeneous, we would not expect to generate as much credibility per additional exposure, compared to the situation where all exposures had the same parameters. We can also see that τ^2 is generally lower than it is in the other formulations, because we have incorporated some of the population heterogeneity into the process variance for the insured.

The NCCI credibility formulation includes both risk shifting and insured heterogeneity. The credibility may be developed from the formulations for σ^2 and τ^2. As a practical matter, the sample data we used for this paper is not sufficient to separately estimate all of the required parameters.

4. PARAMETER ESTIMATION

There are several different approaches that we can take to estimate the appropriate credibility. We may estimate the credibility directly, by using claim-free discount data or a regression approach. This approach basically requires that we have data for the same insureds during at least two different experience periods. This is probably the best approach to estimating credibility, because our theoretical models may not always apply to the real world. We may also estimate credibility by estimating the parameters in the formulas that we developed above. This may be our only alternative if we do not have sufficient data. Even if we estimate credibilities directly, we may want to estimate the theoretical parameters, in order to gain more insight into the process.

Direct Estimation of Credibilities

We will define some generalized notation to simplify the estimation equations. Assume that we can measure the experience of Q insureds over two different experience periods. For each insured, **i**, we define $\mathbf{x}_i$, the *relative* cost ratio for the first period. For example, if we have 10 claims for 100 insureds, the average claim frequency is 10%. For an insured with one claim, x_i=10. For an insured with no claims, x_i=0. We define $\mathbf{y}_i$ as the *relative* cost ratio for the second period. We also define $\mathbf{w}_i$ as the weight that we will apply during the estimation process. We can think of w_i as being the relative exposure of that insured to the total group of insureds. Some of the following equations will have a special meaning where the sum of the w_i's is 1.0.

Our preference is that the x_i be defined in the same manner as A, the actual claim experience, that we are using in the modification factor formula. We want to test the predictability of the actual experience. It is possible that different definitions of x_i will give the same credibility parameters, such as β. For example, rating based on reported counts might produce the same β as rating based on CP5 counts. We would expect the level of credibility to be different, however, since the reported count frequency will be much higher than the CP5 frequency.

We can use any y_i data to test the validity of the modification factor. Since, ideally, we want to test the actual cost of insured experience, our preference might be to use insured amounts for y_i. As we saw above, however, the variability in results likely will be much higher using amounts than counts. Thus using amounts may give too much weight to outliers and render the estimation process ineffective. Thus, normally we want the x_i to reflect the definition of A and the y_i to reflect the actual costs of insurance. We can make substitutions, if we understand the limitations that this might produce.

The simplest estimate for Z is the claim-free discount. Our notation can be made simpler by grouping all insureds by their claim experience in the first period. x_0 would be the relative cost in the first period for insureds with no claims. x_1 would be the relative cost for insureds with one claim, etc. y_0 would be the second period relative cost for insureds with no claims in the first period. Similar definitions would follow for y_1, etc. The weights would represent the percentage of insureds with no claims, etc. in the first period.

The empirical claim-free discount is $1 - y_0$. This is the credibility that applies to this group of insureds.

We have assumed that the credibility is the same for all insureds in the group. The stability of our estimate will depend upon how many insureds were claim-free in the first period, as well as how volatile the claim experience is in the second period. Note that there is no particular requirement for measuring y_i in the same manner as x_i. We could try several measures of y_i, such as pure premium and different count definitions.

This formulation is somewhat limiting, however, in that we do not use the experience of non-claim-free insureds. We could expect to get a better estimate by using more information.

Least Squares Regression Formulation

A more generalized formulation uses the modification factor, M_i, to estimate the second period experience:

$$\hat{y}_i = Zx_i + (1-Z)$$

In effect, we want the most appropriate credibility, Z, to convert the insured's first period experience into a prospective rate for the second period. We can derive a mathematically appropriate Z by selecting some criteria to minimize the differences between the predicted experience (M_i) and the actual experience (y_i). Although it is not the only possible criterion, least squares minimization is commonly used to determine Z. Thus we have the following formulation:

$$\min_Z C = \sum_i w_i (\hat{y}_i - y_i)^2$$

$$C=\sum_i w_i(Zx_i+1-Z-y_i)^2$$

We can solve for Z by taking the partial derivative of C with respect to Z and setting the result equal to 0.

$$\frac{\partial C}{\partial Z}=\sum_i 2w_i(Z(x_i-1)+1-y_i)(x_i-1)$$

We can separate out the terms that have Z and those that do not.

$$\frac{\partial C}{\partial Z}=2\sum_i w_iZ(x_i-1)^2+w_i(1-y_i)(x_i-1)$$

When we set this equal to zero, the 2 drops out. We can put all the Z terms on one side of the equation and the non-Z terms on the other side. Since Z is a constant, we wind up with a ratio for Z:

$$Z=\frac{\sum_i w_i(x_i-1)(y_i-1)}{\sum_i w_i(x_i-1)^2}$$

If the sum of the w_i is 1.0, the denominator is the total relative variance and the numerator is the relative variance of the means of the insured population, the structure variance. If the w_i are the exposures for both x_i and y_i, and the sum of the w_i is 1.0, then the formula simplifies to:

$$Z=\frac{(\sum_i w_i x_i y_i) -1}{(\sum_i w_i x_i^2) -1}$$

We can also use this formula for grouped rather than individual insured data, but we must define the groups by the first period experience. For example, we might divide the data into ten groups, the first having the lowest loss ratios in the first period, etc. This approach can remove the undue impact of outliers. Strictly speaking, Z will be optimal for the selected group means, not for every insured.

Exhibit II graphically depicts the regression process. It shows the prior relative frequencies (x_i), the subsequent relative frequencies (y_i), and the modification factors (M_i), which are the fit of the regression line between the prior and subsequent experiences. The estimate based on the claim-free discount is almost the same as the regression estimate; it can be different because the regression takes into account the experience of all of the insureds.

In certain cases, we may wish to pool data together for which we know that the credibility is different for different insureds.

This formulation would be:

$$\hat{y}=Z_i x_i+1-Z_i$$

Since the Z_i vary for each insured, we cannot solve for a single value of Z. If we can formulate a reasonable function for Z_i, however, we can use the least squares approach to solve for the parameters of our Z_i function. Reasonable candidates for the credibility function can be developed from risk theory, as we showed in an earlier section. Given two periods of data, we would be limited to estimating one parameter. For example, we may assume that the appropriate credibility function is:

$$Z_i=\frac{\beta\lambda_i}{1+\beta\lambda_i} \qquad (11)$$

where λ_i is the expected (mean) frequency for class i. We may use the regression approach to solve for β. In effect, we are determining the optimal β, if credibility does indeed follow the postulated function. If the selected function is not appropriate, we may not get a reasonable estimate for β. If the credibility function is complicated, we may not be able to calculate the optimal parameter from a simple equation. We might have to resort to numerical methods.

Estimation of Credibility Parameters

The parameters λ, α, and μ can be estimated from single-period experience. In fact, we do not even need individual insured experience to estimate them. (We do need individual claim experience to estimate α, but λ and μ may be readily available from aggregate data or other projections.) If we can somehow obtain estimates for β, δ, or γ, and we also have confidence in the correct form for the credibility function, we do not need to obtain two periods of individual risk data to test the credibilities.

Estimates for the Structure Variance, β

The simplest estimate for the structure variance comes from the basic properties of the Poisson distribution. Since we know that the mean and variance of the Poisson are the same, any "excess" variance in the data can be thought of as being the structure variance.

$$\beta = \frac{Var(N) - \lambda}{\lambda^2}$$

Exhibit III displays an example. It shows the actual number of doctors with a given number of claims. It also shows the theoretical number of doctors who would have had that many claims, had the distribution been Poisson. Under some generalized assumptions, incorporating the excess variance yields a negative binomial distribution, which is also shown. We see that the actual distribution is more dispersed than the Poisson assumption. There are far more doctors with no claims, and more doctors with only one claim, than the Poisson assumption would indicate. Of course, to balance out, there are also more doctors with large numbers of claims than the Poisson assumption would indicate. The negative binomial provides a reasonably good fit to the data. It should be noted, however, that the excess variance method is greatly affected by the small number of insureds that will have very unusual experience. If we have a relatively limited sample, we would expect the excess variance estimates to be volatile.

Unfortunately, the structure variance may not be the only component of the excess variance. Other credibility formulations, such as risk shifting and within-insured heterogeneity, also affect the excess variance. We can think of the excess variance as being a combination of all of these effects. Given a reliable estimate, the excess variance is probably an upper bound on the structure variance.

We obtained another estimate for the structure variance from the numerator in the regression approach, where the sum of the w_i is 1.0:

$$\hat{\beta}=\sum_i w_i(x_i-1)(y_i-1)$$

If the w_i are the exposures, the formula simplifies to:

$$\beta = (\sum_i w_i x_i y_i) \; -1$$

This regression formulation probably is more reliable than the excess variance approach, because it is based on the predictability of actual data. This formula can be found in Woll [15] and can apply to any claim data (i.e., counts or amounts). We can also apply this formula to grouped data, although we must group by the loss experience in the first period. We also would expect the grouping process to bias the estimate on the low side, since we are taking differences of group means. We could correct for this bias by multiplying by the ratio of the total relative variance for the individual insureds to the total relative variance of the groups.

Another estimator for the structure variance is:

$$\beta = Z \, \frac{Var(T)}{E^2[T]}$$

This can be used with a variety of inputs. The estimate for Z can come from claim-free discount data. The ratio on the right is the total relative variance. This can be calculated from one-period data. We can adjust the claim experience for all insureds by the mean experience and then calculate the variance over all insureds. This estimator is based on the general credibility formula, $Z=\tau^2/(\sigma^2+\tau^2)$. It can be used for either count or amount data.

Another estimator is taken from Woll [15]. This was developed for count data where the structure function (χ) has a gamma distribution.

$$\beta=\frac{y_1-y_0}{y_0}$$

Numerical Examples

We will present various numerical calculations, based on actual data. The data was developed from the experience of one insurer in one state, for insureds that were continuously insured for seven years on an occurrence form. The "prior" period consisted of the first five years and the "subsequent" period consisted of the last two years. The evaluation date was about four years after the inception of the last policy year. For this insurer, most claims have been reported for the subsequent period, but many of these remain open. The large majority of claims from the "prior" period are closed. Data was available for the reported count, the closed-paid count, the CP5 count, and the basic limits amount, for both periods. Data was segregated for nine different class groups, based on the current classification plan by specialty. There are some rating variables that are not reflected in the class groupings.

Exhibit IV shows numerical calculations for a number of the methods described above. This data includes the experience of 153 doctors in a particular rating group. For this exhibit, we have defined "A", the actual claim experience, to be the number of CP5 claims in the five-year experience period. 91 of the doctors (59.5%) had no CP5 claims in the first period. These doctors had 13 CP5 claims in the second period, for a frequency of 14.3%. The entire class had 29 claims in the second period, for a class frequency of 19.0%. The relative frequency for the claim-free doctors is 75.4%. Thus the claim-free discount, based on CP5 count, is 24.6%. (A claim-free discount can also be calculated for the other data items, such as reported count and pure premium.)

The CP5 frequency for the group is .660 and the CP5 variance is .969. The variance for a Poisson process would be .660, thus the excess variance is .309. All of these numbers reflect the frequency of the actual data. For analysis purposes, it is easier to work with the "relative" variances, which are the actual variances divided by the square of the frequency. The total relative variance is 2.225. The Poisson relative variance is 1.515 (the reciprocal of the frequency). Thus, the excess relative variance is .710. We could also calculate the excess relative variance as the actual excess variance (.309) divided by the frequency squared (.660 * .660).

If we use the basic credibility formulation, β is the excess relative variance, or .710. This would imply a credibility of .319, from the formula: $Z=\beta\lambda/(1+\beta\lambda)$. If we use the risk-shifting credibility formulation, the excess relative variance is the sum of β and δ.

The regression method produces a credibility of .208. This estimate can be interpreted as the ratio of an estimate of β and the total relative variance, which is 2.225, as above. Based on the regression method, the estimate of β is thus .463. This might indicate that either: (1) δ is .247 or (2) the data is relatively unstable.

The claim-free discount data indicates a credibility of .246. This may imply a β of .548 (= 2.225 * .246). We can also derive another estimate of β from the relative costs of claim-free and one-claim insureds in the second period. This estimate is .556, as shown.

As can be seen, the results for this class are relatively similar among the different above methods. We also used first period reported count experience. We would expect the numerical amount of the credibilities to be different (because the frequency was different). The β estimates could be similar or different, depending upon whether the use of reported counts has the same predictability as the use of CP5 counts. For this data set, the β estimates were quite similar for both reported counts and CP5 counts. We also used claim-free discount data based on reported counts and pure premiums. As we might expect from risk theory concepts, the pure premium data was more volatile.

For some of the classes, the number of insureds was small or the actual claim experience was erratic. This raised dual questions: (1) how do we determine β for the smaller classes? and (2) does β vary by class?

Exhibit V shows the calculation of the excess relative variance by class for reported counts. Several classes have β's of about .6 or .7 and several are in the .2 to .35 range. This might indicate that the β's vary by class. Class 6, however, has the lowest excess relative variance of .215 for reported counts. We saw in Exhibit IV that its β for the CP5 count was about .5. We can also estimate the β's by the other methods.

Exhibit VI estimates β using the claim-free discount method. For two classes, the subsequent claim experience for claim-free insureds was actually **worse** than the average. This would imply a negative value for β. We also note from Exhibit VI that the claim-free discount based on CP5 counts is significantly different from the claim-free discount based on pure premiums, for several of the classes. Part of this probably is explained by the greater volatility of pure premium data. We also obtained varying β estimates by class from the regression approach.

In reviewing the individual calculations, it appears that much of the volatility is caused by the relatively low number of insureds and claims; and by the undue impact of an occasional outlier. There may be a difference in β from class to class, but it does not appear to be statistically significant.

We also pooled all of the data, for the regression and claim-free discount methods. We assumed that the credibility function was the same as equation (11), with λ_i being the expected claim frequency for the class. For the claim-free data, for insureds grouped by CP5 in the first period, the estimate of β was .54, based on CP5 counts in the second period, and .59, based on pure premiums in the second period. For insureds grouped by reported count in the first period, the estimate was .54, based on CP5 counts in the second period, and .36, based on pure premiums.

For the regression approach, for insureds grouped by CP5 in the first period, the estimate of β was .51. When insureds were grouped by the reported count in the first period, the estimate was .50.

Estimates for δ and γ

We have mentioned that all three parameters, β, δ, and γ, arise in a similar manner, to explain additional variance beyond a Poisson process. The basic formulation for δ is a shifting of parameters over time. With more years of data, it might be possible to estimate this parameter. The basic formulation for γ is heterogeneity among different doctors within the same insured group. We could estimate this parameter if we had credible data for at least several different size groups, and we assumed that the same heterogeneity applied to all size groups. In fact, the NCCI has used a similar approach to calibrate all of its credibility parameters. It divided risks into various size groups; it estimated optimal credibilities for the different groups; and it fitted these optimal credibilities to a credibility function.

We can use the above numerical example to see whether δ might be significant. If the risk-shifting formulation is correct, the total variance will *include a provision for* β and δ, as well as the usual Poisson variance. The excess variance estimate should be the sum of β and δ. The numerator of the regression credibility estimate, however, should only include β. Thus we can compare the two estimates to see if the excess variance estimate is significantly larger. Exhibit VII shows this comparison for the classes for which the individual estimates were satisfactory. In some cases the excess variance estimate is higher and in some cases it is lower! It does not appear that the excess variance estimate is consistently higher. In practical terms, this might imply that a doctor's inherent risk does not change appreciably over time.

Other Published Data

Two published papers, Ellis [2] and Venezian [12], give some estimates of credibility parameters. The Ellis data included the number of closed-paid claims against doctors in various specialties, for four years, 1980 through 1983, in New York State. It is not clear what the authors used for exposure, but it would appear to be licensed doctors. The authors published theoretical prospective mean frequencies for doctors, in a given specialty, that had various numbers of closed-paid claims within a five year experience period. Comparing the prospective frequencies for (1) doctors with no claims and (2) all doctors, yields the 5-year claim-free discount, or credibility, for the 5-year experience. Except for some minor differences, probably caused by slightly different methods of estimation, we can generate the same credibilities using the procedures outlined above. The Ellis method is equivalent to a credibility formula of $\beta\lambda/(1+\beta\lambda)$, where β is the excess relative variance and λ is the 5-year mean frequency. We have estimated the excess relative variance from the claim count distribution given in the paper. The results are shown in Exhibit VIII.

For most of the specialties, the excess relative variances are much higher than those estimated from the above data set. There are several reasons for this. First, it is not clear what exposure was used. If it was licensed doctors, which includes retired, part-time, and government-employed doctors, a substantial number of the doctors would have virtually no claim exposure; we would expect the excess variance to be higher than that for full-time doctors in private practice. Second, the exposure does not differentiate among other class variables. An insurer's premiums could vary significantly within a given specialty, due to class relativities, geographical relativities, and other rating variables. It is interesting to note that the specialties that are more likely to be grouped into one insurance class, such as anesthesiology, general surgery, neurosurgery, obstetrics, and urology, have much lower excess variances. Third, New York State could have more geographical variation in costs than the state our data was taken from. Fourth, some doctors are not insured voluntarily. These doctors may have an extreme number of claims, which would produce a much higher excess variance than an insured population. In any case, we might use this data as an upper bound on β.

The Venezian data was taken from the Pennsylvania Medical Professional Liability Catastrophe Loss Fund, which covers both excess losses (attachment points have varied over time) and late reported claims (over four years). Although this data came from insured doctors, the exposures were estimated by the authors. The excess relative variance was estimated from the data in the paper and is shown by specialty in Exhibit VIII. With one exception, the excess variances are smaller than in Ellis. Most of the above comments apply to these estimates, as well.

5. PRACTICAL CONSIDERATIONS

This section will consider several practical considerations in the design of a merit rating plan. Is it better to use counts or amounts? Is it better to use the reported count or the CP5 count? What is the best length of the experience period? Is the credibility different if we offer only discounts and have no surcharges? How do we calibrate the expected costs? What if we use non-optimal credibilities? How do we establish a formula for insured groups?

Counts or Amounts?

The NCCI and ISO use amounts, rather than counts, in their merit rating plans. The situation for doctor professional liability insurance, however, may be different. We can analyze the situation by reference to the formula for K, in the basic credibility formulation:

$$K=\frac{1+\alpha}{\beta\lambda}$$

The K for counts is similar, but a 1 replaces the $(1+\alpha)$ in the numerator.

For amounts, the K will be $(1+\alpha)$ times larger, if the β is the same. For one exposure unit, the credibility of claim amount experience will be only about $1/(1+\alpha)$ times as much. To the extent an individual's experience is relatively better or worse than the average, it will only receive credit for about $1/(1+\alpha)$ of that difference. The claim-free discount also will be only about

$1/(1+\alpha)$ as much.

It is likely that claim severity varies among insureds within the same class. If so, we would expect the β to be larger for amounts than for counts. Most likely, however, the β will not increase by as much as $(1+\alpha)$. For doctors, for basic limits of $100,000, $(1+\alpha)$ may be about 2 and for basic limits of $200,000, $(1+\alpha)$ may be about 2.5. We would expect that β for amounts would only be marginally higher than β for counts. Thus using amounts rather than counts would cut the credibility and the claim-free discounts about in half.

Which Count?

There are several choices for claim counts. We could use reported claims, closed-paid claims, or possibly some non-nuisance claim definition, such as CP5. We can analyze this situation by reference to the basic credibility formula, defined in terms of the expected count, E:

$$Z=\frac{E}{E+K}$$

where $K=1/\beta$. We note that credibilities generally will be higher for higher expected counts. We saw above that the β's for reported counts and CP5 counts tended to be about the same. This result might not be universally applicable, but we might conclude that the

β's would not increase in the same proportion. Thus reported counts would generate more credibility and higher claim-free discounts. If the β's happened to be the same, the credibility for reported count experience might be three to five times higher, depending on the claim frequency for the class and the length of the experience period.

Using reported counts, however, may cause consumer relations problems. It is common for every surgeon in the operating theater to be named in a suit, even if only one is likely to be responsible. Most claims will be closed without a payment or for a nuisance-value payment. Even if more costly doctors are sued more often (which is the logical consequence of the β's being the same), it may be difficult to charge an individual doctor more, just for being named in a suit.

On occurrence policies, in particular, charging for reported claims may also deter or delay the reporting of claims. This could have adverse consequences for both the claim settlement process and the ratemaking process.

From a pricing perspective, using reported counts probably is preferred. Practical considerations, however, may favor a CP5 program.

What Should be the Length of the Experience Period?

Both the NCCI and ISO use a three-year experience period as a standard. Claim frequency for doctors, however, is quite low, particularly when using CP5 counts. Current doctor merit rating programs typically give a certain discount for each year of claim-free experience. This is a reasonable approach, although the discount percentages should vary by specialty. Recall that the basic credibility formula is:

$$Z=\frac{\beta P\lambda}{1+\beta P\lambda}$$

for counts, for P exposure units. For each additional year of claim-free experience, the credibility will increase about $\beta\lambda$. Assuming β=.5 and λ=.02 (for one year), the claim-free discount would be about 1% per year. After 10 years, the discount would be 9.1%. For a higher-rated specialty, where λ=.1, the first year discount would be about 4.8%, the second year, an additional 4.3%, the third, 3.9%, the fourth, 3.7%, and the fifth, 3.3%, for a total of 20%.

The above credibility formulation assumes that the doctor's relative cost remains the same over time; i.e., there is no risk shifting. If there is risk shifting, and the δ parameter is relatively high compared to β, the additional discounts for additional claim-free years will decline quickly.

Discount Only Plans

Current merit rating plans for individual doctors have claim-free discounts, but no surcharges. What should the credibilities be for this type of program?

We can use the same regression formulation to select an optimal credibility. Let w_0 be the percentage of doctors with no claims in the first period and w_1 be the remaining doctors. The modification factors are 1-Z and 1, respectively. Using these modification factors, however, will lead to an "off-balance." That is, the collectible premium will be less than the manual premium. The amount of the off-balance will be w_0Z. The manual rates will be:

$$\hat{y}_0=\frac{1-Z}{1-w_0Z}$$

$$\hat{y}_1=\frac{1}{1-w_0Z}$$

We can write the optimization function as:

$$\min_Z \; C=\sum_i w_i(\hat{y}_i-y_i)^2$$

Taking the partial derivative with respect to Z and setting it equal to zero, we obtain the optimal $Z=(1-y_0)/(1-y_0w_0)$. This result can also be obtained in another manner. Since $y_0w_0 + y_1w_1 = 1$, it follows that $y_1 = (1 - y_0w_0)/(1-w_0)$. The above formula for Z makes the prospective rates proportional to the ratio of the actual second period experience, y_0/y_1.

The given credibility is optimal for the postulated pricing policy. It would be more accurate, however, to charge a higher premium for every additional claim in the experience period. The above pricing policy produces a single rate for all insureds with one or more claims. This rate will be relatively too high for the 1-claim doctors and relatively too low for the more-than-1-claim doctors.

This can be demonstrated from another perspective. When there are only discounts, and no surcharges, the loss of the claim-free discount is essentially the surcharge for one or more claims. Recalling the general modification factor formula, and assuming that the average experience period frequency for the given class is λ, the appropriate amount to surcharge for each claim is:

$$Surcharge=\frac{Z}{\lambda}$$

Given the basic credibility formula, with $Z = \beta\lambda/(1+\beta\lambda)$, the surcharge becomes $\beta/(1+\beta\lambda)$. If λ is relatively small, the surcharge will be approximately equal to β.

Calibrating the Expected Costs

Once we have defined the actual claim experience, A, we determine E, the expected claim experience, as the corresponding class average experience. If E is not calibrated to the class average, we will generate an off-balance. We briefly discuss some issues with respect to reported counts and CP5 counts.

First assume that A is defined as the reported count, for claims-made coverage, and that the insurer offers a certain fixed discount for each claim-free year. If claim frequency has changed over time, the optimal discount may be different for each year of experience. We may want to select an average frequency for the maximum number of years that credits are offered. We also may want to add an adjustment for the step of the insured policy, if we use the experience on non-mature years.

We may not have class frequencies or we may want to use our rate relativities. In this case, we should remove that part of the relativity that reflects differences in severities by class. We should also reflect other rating variables in the discounts. For example, if we give teaching doctors a 25% discount, logically their claim frequency should be about 75% of the class average and their credits should be 75% of regular doctors. The same adjustment would apply for territorial rate relativities.

We also may want to apply claim-free discounts to occurrence coverage. In this case, we should adjust for the reporting pattern of claims. Assume, for example, that 10% of claims are reported in the first year, 40% in the next year, 20% in the next year, and 10% in the fourth and fifth years. Thus the cumulative percentage of claims reported would be 10%, 50%, 70%, 80%, and 90%. We also assume that the average doctor in this class has an annual occurrence claim frequency, λ=.20, that has remained relatively constant for the past five years. The average doctor would have a reported claim frequency of .18 for the fifth prior year, .16 for the fourth prior year, and .14, .10, and .02, respectively. For the five-year experience period, the expected frequency is .60. If β=.5 and we use the basic credibility formulation, Z=23.1% for the five years of experience. If we round off and simplify, we could give a 5% discount for each claim-free year. We should note, however, that after the first year, the expected claim frequency is only .01 and the appropriate claim-free discount is only 1%. (The appropriate discounts for each successive year of claim-free experience would be 4.7%, 5.8%, 5.9%, and 5.7%).

If we define the actual claim experience, A, in terms of non-nuisance claims, such as CP5, there is an additional problem in trying to match claim experience to exposure. Even on claims-made forms, the average claim may take three years or so to be settled. On occurrence forms, the average claim may take six years to be settled. One solution is to define A as being any CP5 claim closed within the last 5 years, regardless of policy period or occurrence date. This approach would be biased in favor of newer doctors, who would not have had as much chance to have had closed claims.

Non-optimal Credibilities

For various reasons, we may design a plan that has non-optimal credibilities. For example, we may have the same discount per year for every class, even though we know that classes with higher frequencies should receive larger discounts (if their β's are the same). We may also use a discount only program.

With non-optimal credibilities, most likely there will be an off-balance. An off-balance can also arise if the book of business changes over time. (For example, those insureds that would have received stiff surcharges may move to a residual market program or another insurer.) A negative off-balance causes the class rate to be higher than the average class cost. This may cause problems in ratemaking and in analyzing claim experience. If off-balances are different by class, the ratemaking procedure for class relativities should adjust for these off-balances. Profitability analysis should focus on collectible premiums, rather than manual premiums.

Non-optimal credibilities imply an inaccuracy in pricing. This may place the insurer at a competitive disadvantage to an insurer that has more accurate pricing. An example may help to clarify this point.

Assume that the optimal credibility for claim-free insureds is 10%, that the insurer gives a 25% discount and no surcharges, that claim-free insureds comprise 80% of the class, that insureds with one claim comprise the other 20% of the class, and that all insureds have the same experience period. The insurer's off-balance would be 20% (80% of insureds receive a 25% discount), implying a manual rate of 125% (1/(1-.2)) of the average cost. The claim-free insureds would pay 93.75% (.75 x 1.25) of the average cost and the non-claim-free insureds would pay 125%.

The most accurate cost estimate for a claim-free doctor would be 90% of the manual rate. The off-balance would be 8% (80% times 10%) and the manual rate would be 108.7% (1/(1-.08)) of the average cost. The claim-free doctor would pay 97.8% of the average cost (.9 x 108.7%) and others would pay 108.7%. The optimal competitor could insure all the 1-claim doctors at a profit, while the given insurer would be left with all of the claim-free doctors, at a loss.

As a general rule, if claim-free discounts are higher than the optimal credibility, claim-free doctors will be under-priced and the non-claim-free insureds will be over-priced. The insurer will be vulnerable to price competition for the non-claim-free doctors. Another way of looking at this is as follows. When a doctor has a claim, it loses its claim-free discount and its premium increases. The additional premium is more than the insurer needs to profitably insure that doctor.

Group Formulations

Finally, we consider merit rating formulas for groups of doctors. To a large extent, the practical problems discussed above will also apply to groups. Given that the claim frequency may be much larger for groups, we may prefer a plan that looks more like the NCCI or ISO plans. We discuss the components of the merit rating formula, A, E, and Z, in turn.

The choices for the actual claim experience, A, include all of the possible choices for individual doctors, plus several more. Since groups are likely to have several experience period claims, the claim-free discount approach may not be practical. Most likely we will use a fixed experience period, of three, five, or more years. The credibility we can assign to the group's experience will increase for each additional year of experience. The amount of the increase will depend upon several factors, such as: whether there is risk-shifting among individual insureds over time, whether the composition of the group changes over time, and the extent to which there is heterogeneity within the group.

If we use claim counts for A, we may want to define them in terms of occurrences. That is, more than one member of a group may be sued for a given incident; the statistical validity of this multiple-claim single incident is probably not much different than that for a single-claim single incident.

We may want to consider using loss amounts. The reduction in credibility that we saw above, for the variability in the claim size distribution, should be more than offset by the increased number of doctors within the average group. If we use loss amounts, we might want to consider a limit on the amount of a chargeable claim, as is done in the ISO plans. The limit could be determined so that the increase in the modification factor for a maximum claim might be a given percentage (e.g., 25%). Logically, this would reduce the credibility that could be given for the group's experience, since α would be lower for lower claim limits. An adjustment also would need to be made to the expected losses, E. Both of these adjustments could be determined from claim size distribution data.

The calibration of E depends upon the definition of A. If we use reported counts for occurrence policies for a 5-year experience period, for example, we would need to adjust for the reporting pattern. The expected frequency might be calculated as the annual occurrence frequency times the number of years in the experience period times an adjustment for the reporting pattern (e.g., 60% in the above example.) If A is defined in terms of loss amounts, we need to consider loss development and IBNR.

The determination of Z is more difficult, unless we have two-period claim experience for large numbers of groups of varying sizes. There are several approaches that can be taken. First, we could use the same K that we used for individual doctors. Most likely, this is not appropriate because all of the doctors within the group will not have the same relative cost. This approach would overstate credibilities, because the heterogeneity among groups is less than the heterogeneity among individuals. (Mathematically, the τ^2 for groups is lower than the τ^2 for individuals).

Second, we could use the basic credibility formulation (e.g., (6)) and estimate the β from group experience. Since the groups (j) for which we have data most likely will have different claim frequencies (λ_j), we must use a generalized formula for Z, such as, $Z_j=(\beta\lambda_j)/(1+\beta\lambda_j)$. This approach has a few problems. If there is risk-shifting among individuals or a change in the group's composition over time, the appropriate credibility formula would have an additional term in the denominator, e.g., $\delta\lambda_j$. Thus our estimate for β may not be entirely accurate. In addition, to the extent there is risk shifting, the credibilities for very large groups should be less than those given by the basic credibility formulation. If we do not insure very many large groups and if there is reasonable homogeneity among the group, this approach may be a reasonable approximation to optimality.

Third, we could build in risk-shifting and insured heterogeneity. In order to measure the appropriate parameters, however, we will need additional data. This could be additional years of data for the same groups or a segmentation of group data by size. If we do not have the necessary data, we may make some educated guesses about the value of δ and γ.

We can compare the results we get with the three different credibility formulations, formulas (7), (9), and (11). We assumed that the excess variance was .5. For the first and third formulations, β=.5. For the second formulation, $\beta+\delta$=.5. We think there is a conceptual similarity between the δ parameter in the risk shifting formulation and the γ parameter in the insured heterogeneity formulation. We think of risk shifting as how different subsequent years of exposure are to each other. We think of insured heterogeneity as how different sub-exposures within the same experience are to each other.

We have prepared two graphs, Exhibits IX and X. The first shows the case where δ=.1, or relatively small compared to β. This would occur for groups that are relatively homogeneous. The second graph shows the case where δ=.167, or the group is less homogeneous. We see that the credibility is always lower for the risk-shifting formulation. For less homogeneous groups, the credibility will be lower. We also see that the risk heterogeneity formulation generally produces lower, though similar, credibility to the risk-shifting formulation. For very low expected counts, the risk heterogeneity formulation may produce higher credibility than the simple formulation. Exhibit XI gives the numerical credibilities for these two cases.

6. CONCLUSION

Merit rating is the use of the insured's actual claim experience to predict future losses. Merit rating modifies the otherwise applicable class rate. The modification depends on two factors: (1) how much better or worse the insured's experience is relative to the class average and (2) how credible (i.e., statistically significant) the insured's experience is. Merit rating formulas can differ in what claim experience is used. Variations include counts or amounts and different lengths of insured experience. There are several generic theoretical formulations for credibility, that have been used in insurance pricing. Given sufficient actual data, the appropriate credibility can be estimated.

Merit rating is an adjunct to rating plan. It will pick up statistically valid information that is not already reflected in other rating variables. The rest of the rating structure must be considered in calibrating and applying the merit rating plan. If the merit rating system creates a collectible premium "off-balance," class rates must be adjusted. If merit rating produces non-optimal discounts or surcharges, there will be inaccurate pricing. If claim-free discounts are too high, for example, those receiving the discounts will be relatively under-priced and those not receiving the discounts will be relatively over-priced.

The statistical validity of an insured's claim experience can be quantified by "credibility" and used in a merit rating formula.

Many formulations for credibility are available. Under virtually all formulations, credibility will increase with: (1) the increasing expected claim frequency of the insured's actual experience (λ_i) and (2) the heterogeneity of the insured population, or structure variance, β, remaining after the application of all of the other rating variables. Credibility will decrease with: (1) increasing variability in the claim size distribution, α, (2) changes in the insured's mean costs over time, or risk-shifting, δ, and (3) heterogeneity within the insured (e.g., with group practices), γ.

In practice, it is relatively easy to determine the expected claim frequency and the variability in the claim size distribution. The structure variance can be determined from single-period data (i.e., from the excess variance), but this requires the assumption that risk shifting and within-insured heterogeneity are not significant. It is better to estimate the structure variance from two-period data. That is, we must know the relative costs of insureds, within the same rating class, in two different time periods. We would expect the structure variance to be different for different insurers (because they have different underwriting standards), for different states, and for different classes.

Risk shifting and within-insured heterogeneity are important with respect to the merit rating of group practices. Since all doctors within the group will not be equally good or equally bad, credibility may not increase with additional exposure as it would for an individual doctor. For example, the credibility for one doctor's five-year experience is probably higher than the credibility of five different doctor's combined one-year experience. To measure these factors we need two-period or multi-period data for insured groups of several different sizes.

There are several practical conclusions that can be based on the general theoretical developments and the actual data presented above. Using claim count data will generate more credibility and, hence, larger discounts or surcharges, than claim amounts. Using reported count data will generate more credibility than closed-paid count data, but this may cause consumer relations and other problems. Claim-free discounts seem to be a reasonable merit rating plan for individual doctors, subject to two limitations. The amount of the discount should vary with the class expected claim frequency and, generally, the amount should decline for each successive claim-free year.

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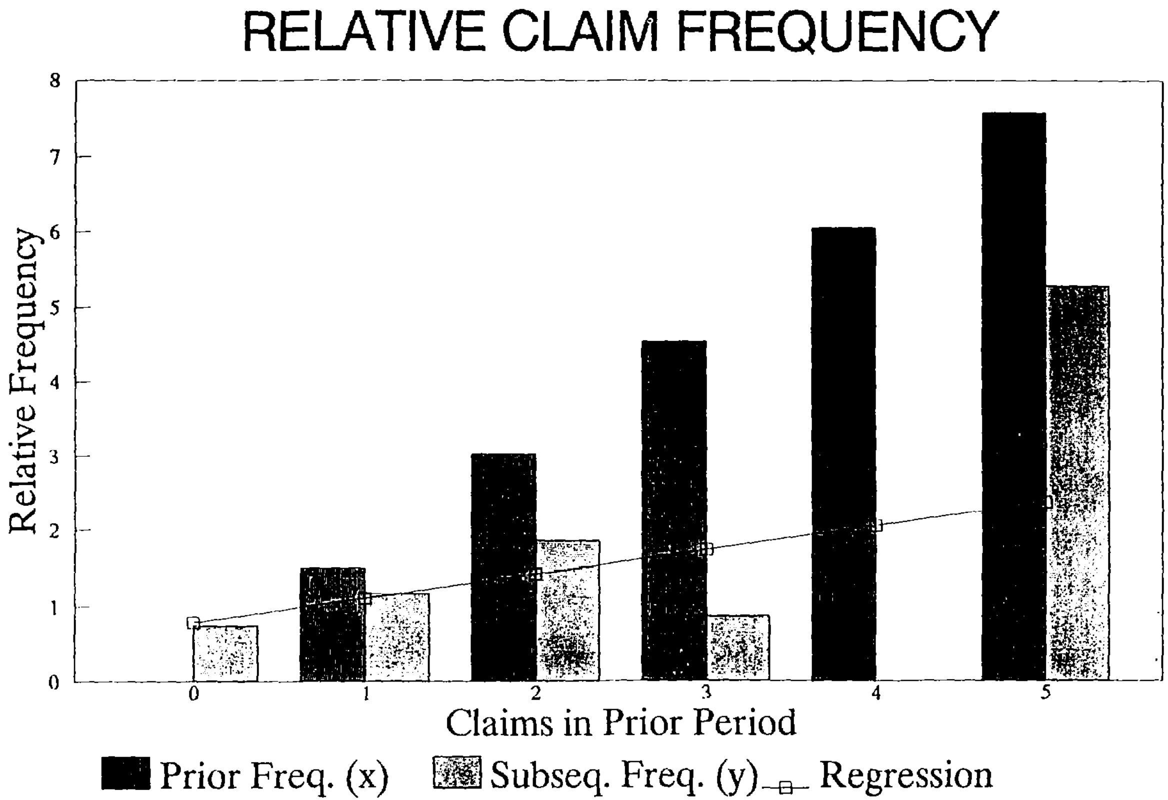
REQUIRED MANUAL RATE INCREASES FOR GIVEN CLAIM-FREE DISCOUNTS

z = Percentage Claim-free	Discounts				
po =	10%	20%	30%	40%	50%
10%	1.0%	2.0%	3.1%	4.2%	5.3%
20%	2.0%	4.2%	6.4%	8.7%	11.1%
30%	3.1%	6.4%	9.9%	13.6%	17.6%
40%	4.2%	8.7%	13.6%	19.0%	25.0%
50%	5.3%	11.1%	17.6%	25.0%	33.3%
60%	6.4%	13.6%	22.0%	31.6%	42.9%
70%	7.5%	16.3%	26.6%	38.9%	53.8%
80%	8.7%	19.0%	31.6%	47.1%	66.7%
90%	9.9%	22.0%	37.0%	56.3%	81.8%

EXHIBIT II



RELATIVE CLAIM FREQUENCY
Relative Frequency
8
7
6
5
4
3
2
1
0
0
1
2
3
4
5
Claims in Prior Period
Prior Freq. (x)
Subseq. Freq. (y)
Regression

EXHIBIT III

FREQUENCY DISTRIBUTIONS

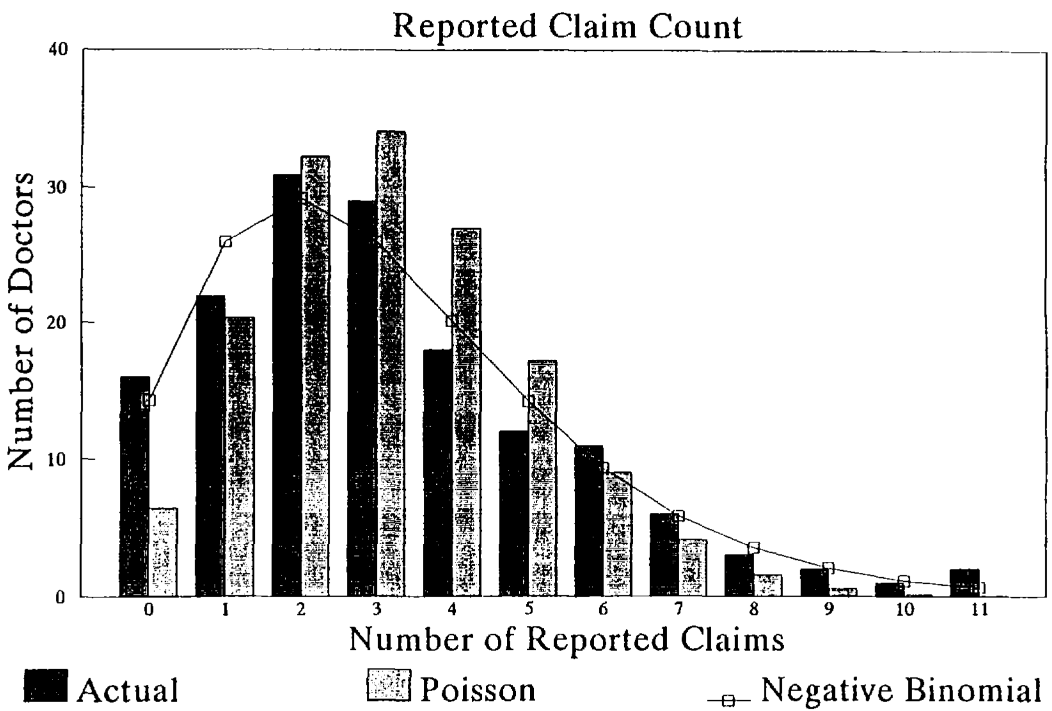


EXHIBIT IV

PARAMETER ESTIMATION EXAMPLE

	Prior Period						Subsequent Period			
Count	Doctors	Percentage of Doctors	Claims	Extension	Relative Frequency	Relative Variance	Claims	Frequency	Relative Frequency	Extension
N	P	w	NP	wNN	x	wxx	q	q/P	y	wxy
0	91	59.5%	0	0.000	0.000	0.000	13	0.143	0.754	0.000
1	36	23.5%	36	0.235	1.515	0.540	8	0.222	1.172	0.418
2	17	11.1%	34	0.444	3.030	1.020	6	0.353	1.862	0.627
3	6	3.9%	18	0.353	4.545	0.810	1	0.167	0.879	0.157
4	2	1.3%	8	0.209	6.059	0.480	0	0.000	0.000	0.000
5	1	0.7%	5	0.163	7.574	0.375	1	1.000	5.276	0.261
Total	153	100.0%	101	1.405		3.225	29			1.463
	(a)		(b)	(c)	N/(1)	(d)	(e)		q/[P(2)]	(f)
Frequency			0.660				0.190			
			(1)=(b)/(a)				(2)=(e)/(a)			

		Nominal			Relative to Mean	
		Source	Value		Source	Value
(3)	Frequency	(1)	0.660	(7)	By Def'n	1.000
(4)	Total Variance	(c)−(1)(1)	0.969	(8)	(d)−1	2.225
(5)	Poisson Variance	=(3)	0.660	(9)	1/(1)	1.515
(6)	Excess Variance	(4)−(5)	0.309	(10)	(8)−(9)	0.710

		Parameter Estimates	
	Regression:		
	Credibility,Z	[(f)−1]/(8)	0.208
	Beta	(f)−1	0.463
	Other:		
(11)	Cl−FreeD, Z	y0	0.246
	Beta	(11)(8)	0.548
	Beta	(y1−y0)/y0	0.556

EXCESS VARIANCE METHOD

Class	No. of Doctors (1)	No. of Reported Claims (2)	Frequency (3)= (2)/(1)	Total Relative Variance (4)	Poisson Relative Variance (5)= 1/(3)	Excess Relative Variance (6)= (4)−(5)
0	98	64	0.653	2.206	1.531	0.675
1	725	674	0.930	1.429	1.076	0.353
2	208	187	0.899	1.837	1.112	0.725
3	297	413	1.391	1.352	0.719	0.633
4	198	236	1.192	1.161	0.839	0.322
5	170	386	2.271	0.903	0.440	0.463
6	153	485	3.170	0.530	0.315	0.215
7	41	145	3.537	0.605	0.283	0.322
8	28	85	3.036	0.670	0.329	0.341

EXHIBIT VI

CLAIM–FREE DISCOUNT METHOD

Class	No. of Doctors (1)	No. Claim–free (2)	Class CP5 Frequency (3)	Claim–free Discount CP5 Count (4)	Total Relative Variance (5)	Beta Estimate (6)= (4)(5)	Claim–free Discount Pure Premium (7)	Beta Estimate (8)= (7)(5)
0	98	88	0.102	–11.4%	8.800	–1.003	–11.0%	–0.968
1	725	624	0.154	3.7%	6.860	0.254	3.5%	0.240
2	208	172	0.183	12.1%	5.050	0.611	3.7%	0.187
3	297	233	0.285	4.1%	5.971	0.245	44.4%	2.651
4	198	155	0.261	–1.4%	4.004	–0.056	–2.1%	–0.084
5	170	105	0.547	30.6%	2.322	0.711	31.3%	0.727
6	153	91	0.660	24.6%	2.225	0.547	16.1%	0.358
7	41	22	0.829	33.4%	1.696	0.566	20.6%	0.349
8	28	17	0.464	58.8%	1.817	1.068	52.0%	0.945
Total	1918	1507						

Notes: 1. (5)=(4)*Total Relative Variance
2. Based on CP5 count.

EXHIBIT VII

IS THERE RISK SHIFTING?

Class	Excess Relative Variance	Regression Estimate for Beta	Difference	Percentage Difference
1	0.353	0.318	0.035	9.9%
2	0.725	0.570	0.155	21.4%
3	0.633	0.868	−0.235	−37.1%
4	0.322	0.371	−0.049	−15.2%
5	0.463	0.370	0.093	20.1%
6	0.215	0.228	−0.013	−6.0%
Sum	2.711	2.725	−0.014	−0.5%

Note: Based on reported counts.

EXHIBIT VIII

OTHER DOCTOR EXPERIENCE

I. Ellis, Gallup & McGuire

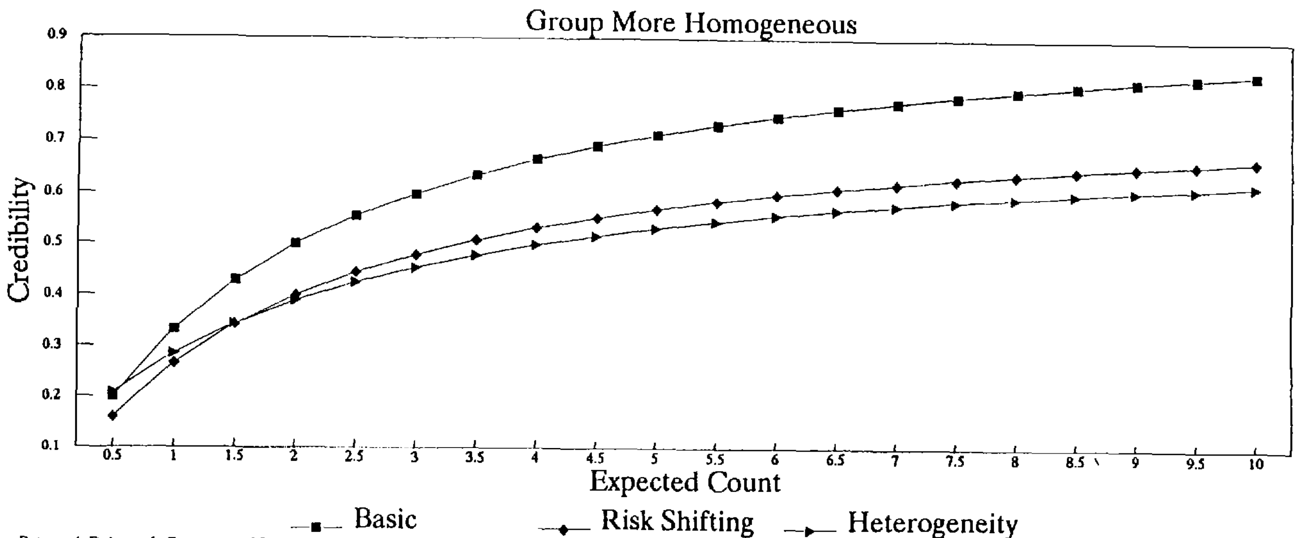
Specialty	5-Year Claim-free Discount	Excess Relative Variance	5-Year Mean Frequency
Anesthesiology	3.4%	0.20	16.3%
Dermatology	28.4%	4.04	9.2%
Family Practice	17.6%	2.88	7.1%
General Surgery	20.2%	0.90	35.2%
Internal Medicine	24.1%	3.87	8.3%
Neurosurgery	30.5%	1.07	42.8%
Obstetrics/Gynecology	29.4%	1.08	39.9%
Ophthalmology	37.0%	3.46	15.2%
Orthopedic Surgery	52.6%	4.22	26.0%
Otolaryngology	38.2%	2.64	24.5%
Pediatrics	23.6%	4.65	7.0%
Plastic Surgery	59.6%	6.78	34.2%
Psychiatry	24.2%	22.89	1.7%
Radiology	21.0%	2.92	9.1%
Urology	19.2%	1.22	15.9%
All Other	10.0%	5.22	2.5%

II. Venezian, Nye & Hofflander

Specialty	Mean Frequency	Excess Relative Variance
Anesthesiology	7.5%	0.46
General Surgery	14.4%	1.10
Internal Medicine	3.6%	0.19
Neurosurgery	50.0%	0.72
Obstetrics/Gynecology	18.7%	0.62
Ophthalmic Surgery	3.0%	5.34
Orthopedic Surgery	25.7%	1.37

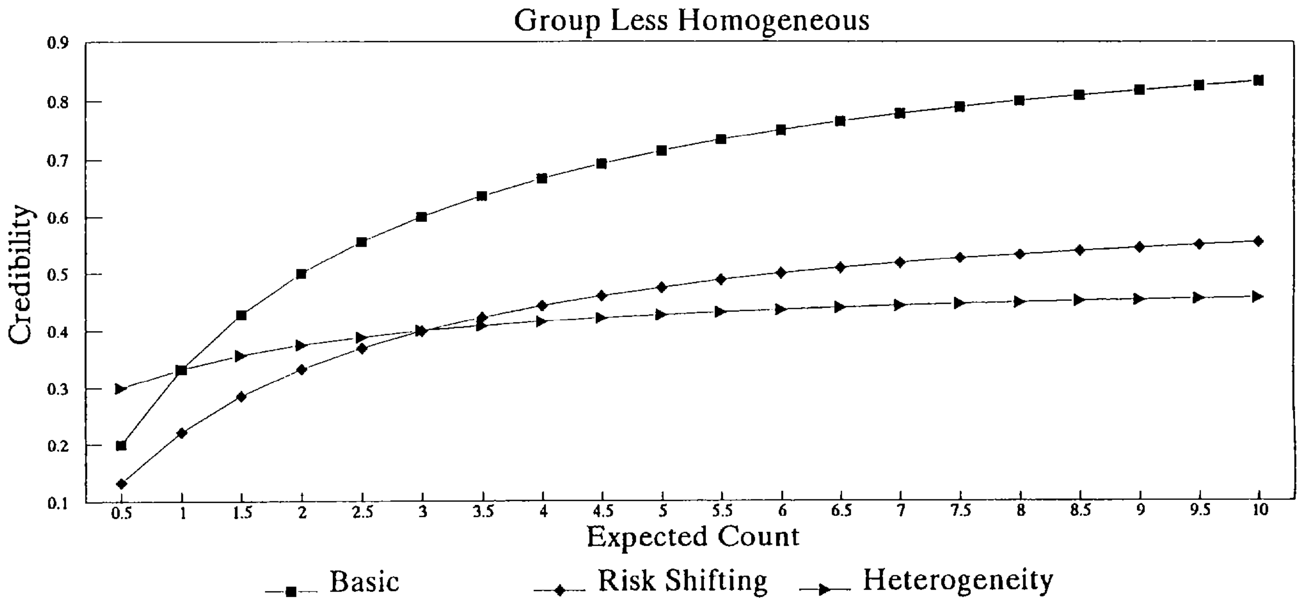
EXHIBIT IX

GROUP CREDIBILITIES FOR VARIOUS FORMULATIONS



Beta = .4, Delta = .1, Gamma = .125.

GROUP CREDIBILITIES FOR VARIOUS FORMULATIONS



Beta = .333, Delta = .167, Gamma = .25.

COMPARISON OF DIFFERENT GROUP CREDIBILITY FORMULAE

GROUP MORE HOMOGENEOUS

Beta = 0.400
Delta = 0.100
Gamma = 0.125

Expected Count	Basic (1)	Risk Shifting (2)	Heterogeneity (3)
0.5	20.0%	16.0%	20.8%
1	33.3%	26.7%	28.6%
1.5	42.9%	34.3%	34.4%
2	50.0%	40.0%	38.9%
2.5	55.6%	44.4%	42.5%
3	60.0%	48.0%	45.5%
3.5	63.6%	50.9%	47.9%
4	66.7%	53.3%	50.0%
4.5	69.2%	55.4%	51.8%
5	71.4%	57.1%	53.3%
5.5	73.3%	58.7%	54.7%
6	75.0%	60.0%	55.9%
6.5	76.5%	61.2%	56.9%
7	77.8%	62.2%	57.9%
7.5	78.9%	63.2%	58.8%
8	80.0%	64.0%	59.5%
8.5	81.0%	64.8%	60.2%
9	81.8%	65.5%	60.9%
9.5	82.6%	66.1%	61.5%
10	83.3%	66.7%	62.0%

Notes: (1) Z = E / (E + 2).
(2) Z = E / (1.25 E + 2.5).
(3) Z = (.75 E + .25) / (E + 2).

GROUP LESS HOMOGENEOUS

Beta = 0.333
Delta = 0.167
Gamma = 0.250

Expected Count	Basic (1)	Risk Shifting (2)	Heterogeneity (3)
0.5	20.0%	13.3%	30.0%
1	33.3%	22.2%	33.3%
1.5	42.9%	28.5%	35.7%
2	50.0%	33.3%	37.5%
2.5	55.6%	37.0%	38.9%
3	60.0%	40.0%	40.0%
3.5	63.6%	42.4%	40.9%
4	66.7%	44.4%	41.7%
4.5	69.2%	46.1%	42.3%
5	71.4%	47.6%	42.9%
5.5	73.3%	48.8%	43.3%
6	75.0%	50.0%	43.8%
6.5	76.5%	50.9%	44.1%
7	77.8%	51.8%	44.4%
7.5	78.9%	52.6%	44.7%
8	80.0%	53.3%	45.0%
8.5	81.0%	53.9%	45.2%
9	81.8%	54.5%	45.5%
9.5	82.6%	55.0%	45.7%
10	83.3%	55.5%	45.8%

Notes: (1) Z = E / (E + 2).
(2) Z = E / (1.5 E + 3).
(3) Z = (.5 E + .5) / (E + 2).

MONITORING TERRITORIAL RATING: A NONPARAMETRIC APPROACH

Bradford S. Gile

MONITORING TERRITORIAL RATING: A NONPARAMETRIC APPROACH

by Bradford S. Gile, FSA, MAAA

ABSTRACT

The primary concern in pricing is normally the overall adequacy of rates companywide, by state, and by territory. The primary concern of this paper, however, is the RELATIVE adequacy of rates by parts of a territory:

1. Is the rating plan for a given line or coverage in a particular territory equally "correct" in its various parts (counties, Zip Code groups, etc.)?
2. Is a particular part assigned to the right territory?

Because even whole territories often have experience of little or no credibility, traditional experience analysis is generally of little or no use. This paper circumvents the credibility problem by developing a nonparametric approach and statistical tests of the hypothesis that rates are "correct" throughout the territory under investigation. Actual applications of this process are shown.

I. THE BASIC PROBLEM

Rating territories are usually defined by the place of residence of the insured; the defining parameter is usually county or Zip Code, but other parameters are at least theoretically possible. The nature of the parameter is, for the purposes of this paper, immaterial. For this reason, we will use the parameter "county" throughout from now on, bearing in mind that we could just as easily use "Zip Code" or any other well defined parameter.

Territories, once defined, may remain unchanged for years without question. It might well seem reasonable to ask, after several years of experience, "Does our experience support our territorial definitions?" More commonly, however, the actuary will hear requests for changing the territory designation of one or more of its parts. Because experience by county is generally considered of little statistical value, the decision whether to make the change may be based solely on "judgment" (which may, unfortunately, be merely a synonym for political expediency). It is the purpose of this paper to develop a scientific approach from the experience by county in order to answer two basic questions:

QUESTION 1: Given no external information, does the experience indicate that one or more counties of a territory is improperly assigned?

This is a GLOBAL question about the territory.

QUESTION 2: Given external information about one or more counties in a particular rating territory, does the experience indicate that these counties do NOT belong in the territory to which they are assigned?

This second question is LOCAL; its focus is on one or more specified counties within a territory.

II. DEVELOPMENT OF THE STATISTICAL APPROACH

Consider the experience in a territory split up into M counties over an experience period of N years. For each year and county, suppose we have Earned Premium, losses on some uniformly consistent basis (e.g., estimated ultimate, calendar incurred, case incurred at a common age of loss), and exposure units (e.g.,policy-year, house-year, car-month,etc.). If we calculate the loss ratios by year and county we will, typically, get a matrix of loss ratios that fluctuate wildly due to lack of credibility in the individual cells. The result may appear to be meaningless. However, IF our pricing process is "correct", we would like to be able to assume the following KEY CRITERIA:

1. If in each year the loss ratios for each of the M counties are ranked by size , the M ranks are equally likely for any given county in any given year.
2. For any given county, rank in any given year is independent of the ranks that it has held in prior years.

It should be noted here that it makes no difference whether the ranking is done from low to high or high to low. Purely as a matter of personal preference, we will use the ordering 1 = lowest M =highest in this paper.

These key criteria will be satisfied if, for example, the territory is perfectly homogeneous and the M counties are all of equal size in exposure for each year. If the counties are NOT of equal size, however, we have a problem. Suppose that L(N) = losses for population size N, G = premium per unit and

$LR(N) = \text{loss ratio} = \dfrac{L(N)}{N \cdot G}$. Assuming perfect homogeneity,

the expected loss ratios will all be equal:

$$E(LR(N)) = \frac{E(L(N))}{N \cdot G} = \frac{N \cdot E(L(1))}{N \cdot G} = E(LR(1))$$

But for the loss ratio variances, we have

$$VAR(LR(N)) = \frac{VAR(L(N))}{(N \cdot G)^2} = \frac{N \cdot VAR(L(1))}{(N \cdot G)^2} = \frac{VAR(LR(1))}{N}$$

Thus, although all the expected values of county loss ratios are equal, the variances are not. Because the variance of the county loss ratio is inversely proportional to exposure, the smallest counties will have largest variance and may, therefore, be expected to have their rankings biased toward the upper and lower extremes. Because variation in exposure by county is a virtual certainty in real life, this problem must be dealt with.

The approach adopted in this paper is to substitute a set of linear transformations of the loss ratios for the loss ratios themselves. The transformed loss ratios will be called "adjusted loss ratios", and will be required to meet two conditions:

(A) The expected values of the adjusted loss ratios are equal to the expected values of the actual loss ratios.

(B) The adjusted ratios of the various counties will, in each experience year, have equal variances.

If we then rank the adjusted loss ratios, the bias due to unequal variances disappears.

Let LR(s,y) be the actual loss ratio for county s in year y, E(s,y) its exposure. We will rank ADJUSTED LOSS RATIOS ALR(s,y), $1 \le s \le M$, $1 \le y \le N$, in lieu of the actual LR(s,y):

$$(1)\ ALR(s,y) = Z(s,y) \cdot LR(s,y) + B(s,y)$$

If we write ELR(y) for the expected loss ratio in year y, (A) will require that (2) $B(s,y) = (1 - Z(s,y)) \cdot ELR(y)$. Now since

$VAR(ALR(s,y)) = Z(s,y)^2 \cdot VAR(LR(s,y))$ and the variance of the loss ratio is inversely proportional to exposure, we may write

$$(3)\ VAR(ALR(s,y)) = \frac{V(y) \cdot Z(s,y)^2}{E(s,y)},$$

where V(y) is the loss ratio variance in year y on one unit exposure. If we now take ratios in (3) for two counties s1 and s2 in year y, we get

$$(4)\ \left[\frac{Z(s1,y)}{Z(s2,y)}\right]^2 = \frac{E(s1,y)}{E(s2,y)}$$

as a necessary relationship for common variance amongst all counties in year y. Now the combination of (1) and (2) gives

(5) $ALR(s,y) = Z(s,y) \cdot LR(s,y) + (1 - Z(s,y)) \cdot ELR(y)$

which looks very much like a credibility formula. There are, of course, infinitely many ways in which the Z values may be defined to satisfy (4). This could well be a fertile area of further inquiry. The following definition of Z(s,y), however, is very appealing:

$$(6) \qquad Z(s,y) = \left[\frac{E(s,y)}{\underset{1<=j<=M}{MAX} \{ E(j,y) \}} \right]^{1/2}$$

This definition not only satisfies (4), but it also gives Z values between 0 and 1 which increase with exposure and equal 1 for the county of maximal exposure. Moreover, if all exposures ARE equal, the Z(s,y) = 1 and the adjusted loss ratios are equal to the actual loss ratios. The combination of (6) with (5) defines the adjusted loss ratios as credibility adjusted loss ratios such that the largest county is assigned full credibility and partial credibility is assigned to the other counties according to the traditional square root rule. Such adjusted loss ratios by county have the same expected values as the actual loss ratios and common variance, so that ranking of the adjusted loss ratios will not be biased due to unequal variances.

Moreover, the variances of the adjusted loss ratios satisfy

$$(7)\quad VAR(ALR(s,y)) = VAR(ALR(m,y)) = \frac{V(y)}{\max_{1<=j<=M} \{ E(j,y) \}}$$

where $E(m,y) = \max_{1<=j<=M} \{ E(j,y) \}$.

Thus, the adjusted loss ratios also have, in a sense, minimum variance.

Now suppose that we tabulate the adjusted loss ratios and rank them by size, so that the county having the lowest loss ratio gets rank 1 and the highest is rank M. We may avoid the complication of ties by viewing the adjusted loss ratio as a continuous random variable. Then, given a county, each of the possible ranks from 1 to M is equally likely. Now do this same ranking process for each of N years:

(8) R(s,y) = Rank for county s, year y; 1<=s<=M, 1<=y<=N

Each of these values has, by itself, no statistical value. However, for each county s, consider the ranksum defined by

(9) RANKSUM(s) = R(s,1) + R(s,2) + ··· + R(s,N); 1 <= s <= M

which is simply the sum of the ranks for county s over the N year period. RANKSUM(s) is identically distributed in each of the counties. The possible values are the integers from N (when s has rank 1 in every year) to M*N (when s has rank M in every year). Except when M and N are large, the exact probabilities of each possible ranksum can be calculated by brute force on a Personal Computer in a reasonably short time. A BASIC program that will do this is shown as Appendix I. Because this distribution is symmetrical with respect to its mean, a Normal approximation may be useful in cases where M*N is unduly large .

The unconditional mean and variance of the ranksum for a given county are given by

$$(10) \quad \text{MEAN} = \frac{M+1}{2} \cdot N$$

$$(11) \quad \text{VARIANCE} = \frac{M^2 - 1}{12} \cdot N$$

because when N = 1,

$$\text{MEAN} = p_1 = \frac{1}{M} \cdot \left[1 + 2 + \cdots + M \right] = \frac{M + 1}{2}$$ and the second moment is

$$p_2 = \frac{1}{M} \cdot \left[1^2 + 2^2 + \cdots + M^2 \right] = \frac{(2 \cdot M + 1) \cdot (M + 1)}{6}$$

and $\text{VARIANCE} = p_2 - \text{MEAN}^2 = \dfrac{M^2 - 1}{12}$.

Under the hypothesis that our pricing process is correct, we can determine confidence intervals for the N year rank sum for any county selected at random. We then select a confidence level of 100*p % so that

(12) Pr(a <= Ranksum <= b) = p

and we tabulate all actual ranksum values outside of that confidence interval. This should, of course, be a two tailed test, such as

$$(13)\quad \Pr(\text{Ranksum}<a) = \Pr(\text{Ranksum}>b) = \frac{1-p}{2} .$$

Now suppose we had been told in advance to watch a specific county as one which should be in a lower cost territory. For brevity, let us call the county under investigation Q. Then the a priori probability that Q's rank sum will be outside the confidence interval should be 1 - p.

If, in fact, the ranksum IS outside the interval, then we have statistical evidence (but NOT proof) that all is not well with our pricing system within the territory. We might well be willing to consider such a result to be strong evidence to support moving county Q to a lower rate territory. If, on the other hand, the ranksum for Q is not an extreme value we can only conclude that the study did not give an indication that Q's experience was unusual relative to that of the other counties. Surely, if County Q turned out to have a high extreme ranksum, indicating unusually high cost, we would reject any notion that experience supports a move of County Q to a lower rate territory!

If, on the other hand, we had been told nothing in advance of our study, we would be unable to draw any conclusions about the rating of specific counties. We can , however, still evaluate the overall hypothesis that our rating structure is correct by looking at the number of extreme ranksum values. Using Monte Carlo simulation of ranking M counties over N years, one can get an excellent approximation to the density function for the NUMBER of extreme ranksum values to be encountered, ranging from zero to M. It is clear that the ranksum values for the various counties are NOT independent of one another, because

$$(14)\quad \text{RANKSUM}(1) + \cdots + \text{RANKSUM}(M) = N \cdot \left[\frac{M \cdot (M + 1)}{2} \right]$$

We want to know the distribution of the number of extreme values to be encountered in a given year in order to get a confidence interval. Unfortunately, the probability distribution of the number of extreme values to be encountered in a given year is extremely complex. The ranksum process itself is, for a given county, equivalent to throwing an M sided die N times. The selection of extreme ranksum values is analogous to the selection of colored balls from an urn without replacement, but with the additional complication that the selected balls must meet an additional aggregate criterion (14). Fortunately, Monte Carlo simulation on a Personal Computer can give us a good approximation of the extreme value distribution. Such a program, written in BASIC is shown as Appendix II. Experimentation with Monte Carlo simulation shows that the Binomial Distribution

$$(15)\quad f(x) = \binom{M}{x} \cdot (1 - p)^{x} * p^{M - x} \quad ; x = 0, 1, 2, \ldots, M$$

where p is the probability that a given county will have a ranksum value that is NOT extreme, provides an excellent approximation to the number of extreme values distribution for the determination of confidence intervals. When M is large, the process is akin to distinguishing "extreme" balls from "non-extreme" balls among a large number of balls in an urn, so that complications of (a) non-replacement of "balls" selected and (b) the constraint that the sum of all ranksums is a constant become minor and the distribution of the number of extreme values will approach the binomial defined by (15). To demonstrate the usefulness of the binomial approximation, consider the case of 69 counties observed over a four year period so that M=69 and N=4. The four year ranksums will range from 4 to 276, inclusive. Brute force production of the rank sum distribution (Exhibit A) by computer tells us that the ranksum for a given county will range from 63 to 217, inclusive, approximately 95% of the time (exactly: 21,551,431 out of 22,667,121 possible combinations). Extreme values would thus be less than 63 or greater than 217. Exhibit B shows that Monte Carlo simulation of 1,000 four year periods resulted in generating one to seven extreme values 946 times out of 1,000. Use of the Binomial distribution with p = 21,551,431/22,667,121 = 0.950779 predicts 949.4 out of 1,000 periods will produce one to seven extreme values. This illustrates the power of the Binomial approximation in estimating confidence intervals for the number of extreme values. Thus, if we had eight or more actual extreme values, our hypothesis of "correct" pricing across counties would be considered suspect in general, without making any conclusion as to which counties were, in fact, problematical.

III. APPLICATION OF THE RANKSUM PROCESS

As has been noted, this ranksum procedure may be used to help answer the two basic questions:

QUESTION 1: Given no external information, does the experience indicate that one or more counties of a territory is improperly assigned?

QUESTION 2: Given external information about one or more counties in a particular rating territory, does the experience indicate that these counties do NOT belong in the territory to which they are assigned?

Question 1 is for routine periodic monitoring. Even if there are no requests to change territorial composition, we should still test whether our territorial composition is still reasonable. Question 2, however, is designed for queries about the appropriateness of a given county's territorial assignment, and should be asked IN CONJUNCTION with Question 1.

In Question 2, we focus on whether the particular county has an extreme value. In both cases, we start with the hypothesis that our rating system is perfect. If, as will generally be the case, the counties are of unequal size, we adjust the loss ratios by (5) and (6) for each county and year, rank the adjusted loss ratios and tabulate the ranksums for each of the counties. Using a predetermined criterion for extreme values, such as those ranksum values outside of a 95% confidence interval as defined by (3), tabulate the number of such extreme values and the identities of the counties generating such values.

In order to evaluate the overall "perfect system" hypothesis for question 1, we need only compare the actual number of observed extreme values with a confidence interval, such as 95%, for the number of extreme values one would expect under the hypothesis. Without external information, however, we can make no judgment as to which counties having extreme ranksum values are merely statistical fluctuations or are true abnormalities. The answer to that question is the subject of question 2, which requires information in advance of the analysis.

If the answer to QUESTION 1 is "yes" and the county under investigation has an extreme value, there is a strong case for the assertion that the particular county is misplaced in its rating territory. If the answer to QUESTION 1 is "yes" and the county under question does NOT have an extreme value, we are left with a need for further analysis. One approach would be to remove the experience of all counties in question and ask whether the answer to QUESTION 1 is still "yes" on the collection of all remaining counties. If it is not, there would seem to be evidence that one or more of the counties under study may be misplaced.

Now suppose that the answer to QUESTION 1 is "No". This does NOT mean that our rating process is, in fact, correct. It simply means that if it is not correct, the experience does not yet BY ITSELF expose the system's imperfections. If, in fact, we have advance external information about a county and that county does, indeed, generate an extreme value, there is then some evidence to support the assertion that the particular county is incorrectly placed and that the "perfect system" hypothesis may, on the basis of additional information, be faulty after all.

Finally, suppose that the answer to QUESTION 1 is "NO" and the counties in question do not have extreme values. In this case, the ranksum procedure fails to corroborate an assertion that the county is misplaced.

No matter what the results may be, the ultimate decision whether or not to modify the territory's composition will have to rest squarely on judgment. Unless the external information is compelling, however, it seems inappropriate to make a change unless the statistical evidence from the experience also supports such a change.

Although this paper focuses on territorial composition, it should be clear that other applications are possible. For example, one might test the hypothesis that a given state has been "correctly" rated by territory or even whether the various states themselves have been equitably treated in the rating process!

IV. THE REAL WORLD: ACTUAL APPLICATIONS

American Family Mutual Insurance Company has developed a fairly large block of health insurance business over the last 30 years; in 1991 we had $186.5 million premium written in the twelve states in which we operate. With the exception of our Medicare Supplement business, our Health rating territories are defined by county of residence. Medicare Supplement territories, on the other hand, are defined by Zip Code groupings.

The county definitions were originally set many years ago, and have been subject to periodic modification. The impetus behind such modifications has generally come from field requests. Frustrated by the absence of a rational and scientific method to apply for the evaluation of the merits of such requests, this ranksum approach was developed.

The first application is to the QUESTION 2 type problem: Is a given county improperly placed in its territory?

Over a period of two years, several requests from the field requested that a specific county in a 69 county territory be moved to a lower rated territory, with no evidence for such a move other than an unsupported assertion (which might not even be relevant!) that "our insureds in this county go to hospitals in nearby county X which is in a lower rated territory."

Whether the assertion is correct or not is really unimportant. What IS important is the empirical evidence to be found in the experience. For each of the years 1986 through 1989, earned premium, case incurred losses at age 21 months, and policy-years of exposure were tabulated by county for the 69 counties. The loss ratios were calculated, adjusted by exposure according to (5) and (6), ranked (1 = lowest, 69 = highest), and the four year ranksums tabulated.

In this case we have N = 4 and M = 69. There are 22,667,121 (69 to the fourth power) possible rank combinations. With the aid of a Personal Computer, an exact determination of the probability distribution for ranksums even in this case is not particularly tedious. Exhibit A shows the graph of this distribution and development of a 95% confidence interval for ranksum values ranging from 63 to 217, inclusive. Exhibit B then develops a 95% confidence interval of from 1 to 7, inclusive for extreme values, showing both simulation and Binomial approximation results.

We now compare the actual results with Exhibit A and Exhibit B. County number 27 is the one that we were asked to change. The 8 counties with extreme values are :

COUNTY	4 YEAR RANK SUM	1989 EXPOSURE
11	58	24
23	19	23
27	32	120
32	221	54
38	41	44
46	231	298
54	32	17
63	57	188

In this case, we have an unusual number (8) of extreme values for the territory AND the county named in advance (number 27) has one of the extreme (low) values. Moreover, county 27 is one of the larger counties in the territory.

It is interesting to note that if the correction for bias had not been made to the loss ratios before ranking, there would have been 13 extreme cases. Most of the above extrema, including county 27, would NOT have appeared among the extreme cases. Instead, the list of extreme cases was dominated by counties having trivially low exposures.

This suggests that, instead of applying this method to ALL counties in the territory, perhaps only those counties having some minimum 1989 exposure, such as 50 policy years, should be counted in the analysis. In this particular case, the number of counties would be reduced from 69 to 11. To augment credibility, we added the 1990 experience to give us five years on eleven counties. For those who like to follow actual cases from beginning to end, Appendix III shows the full detail in this shortened case.

Interestingly, if the 95% confidence standard for extreme values is maintained, County 27 is no longer extreme; in fact, county 45, which is the largest of all counties in the territory is the only extreme case at this level of confidence. If we had chosen a confidence standard of 90% rather than 95%, Counties 45, 27, 32, and 63 would have emerged as "extreme"; the occurrence of 4 extreme values at this level of confidence is highly unusual.

From these analyses, it should be reasonably clear that the questioned county, number 27, has had unusually good experience. County 27 was, in fact, moved to a lower cost territory. Because there was no external input on other counties, no other counties were moved to different territories.

The above was a "real life" answer to QUESTION 2. What about QUESTION 1? We will now look at a "real life" situation for this question.

Two years ago, it became painfully clear to us that an entire territory, Territory A in State X, had a long term history of loss results that were unacceptably poor. This territory consists of 25 rural counties, so there was no clear reason why this particular territory had by far the worst experience in the Company. We decided to determine whether the cause might be due to an abnormal number of counties whose experience might identify them as the "bad apples". Appendix 4 shows the data and analysis of this territory by county and year for accident years 1986 - 1989. In this case, we have M=25 and N=4. The ranksum values of the adjusted loss ratios exhibit only two extreme values. This number of extreme values falls within a 90% confidence interval, so we do not conclude that we have an unusual number of extreme counties. Moreover, we are dealing here with a HIGH cost territory, so we are really interested in high extreme values rather than low ones. Interestingly enough, both of the observed extreme values are low rather than high. All of this suggests that, in essence, the territory experience is uniformly " bad". The answer to QUESTION 1 is, in this case, "No".

VI. CONCLUSIONS AND OBSERVATIONS

It should be emphasized that the process set forth in this paper does NOTHING to assess the adequacy or inadequacy of rates. That is a question of absolute magnitude. The process DOES attempt to assess RELATIVE adequacy of rates by county within territory. There are, no doubt, many questions that come to the reader which have not been addressed and should probably be researched further.

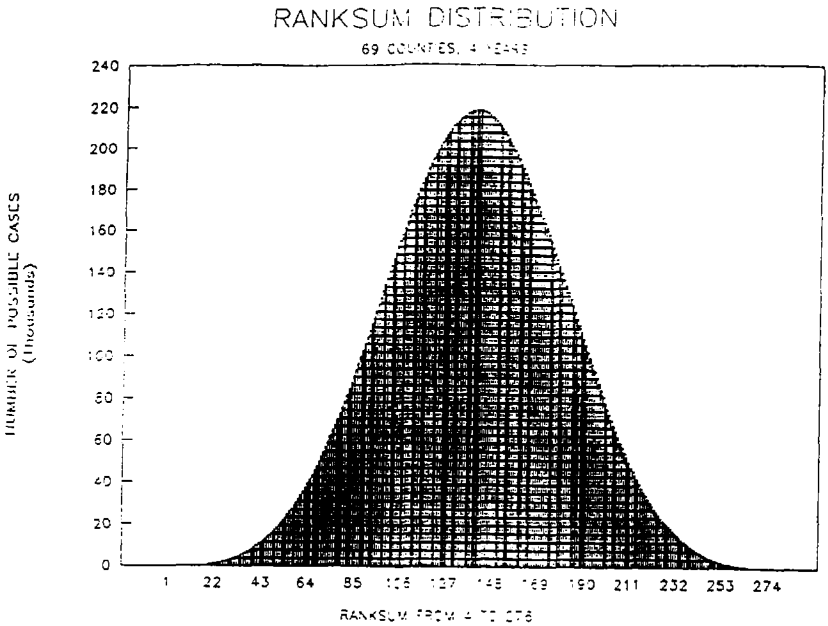
Examples that come to mind are:

1. The ranking process assumes, as part of the "correct pricing" hypothesis, that the territory is homogeneous in the sense that (1) the mean loss ratio is not changed by a population change and (2) the variance of the actual loss ratio is always inversely proportional to exposure. How much is lost with populations for which this does not hold?
2. Equation (4) defines the relationship between exposures and Z values in order that the M counties have a common variance. Although (6) turns out to be an extremely attractive choice, the possible choices are unlimited.
3. Nothing has been said about what data should be used, particularly losses. How does one deal with loss development on small populations? Are case incurred losses of equal maturity, for example, dependable as a proxy for "ultimate" losses for a long tailed coverage or line? The earned premiums for any county should, of course, be adjusted to the current territory if the county was in a different territory during part of the experience period.
4. To what extent should very small counties be removed from the analysis? What criteria should be employed?

Although the two historical examples given were in Health insurance, the methodology and principles should apply equally well to any personal line of insurance. Similarly, although the examples involved a county definition of territory, the way in which territory is defined is immaterial to the methodology.

Finally, although the problem to which this paper is addressed is territorial ratemaking, the nonparametric ranksum approach and analysis of extreme values of this paper (with particular emphasis on the use of a Personal Computer) should be applicable to an unlimited variety of actuarial questions involving comparative analysis.

EXHIBIT A

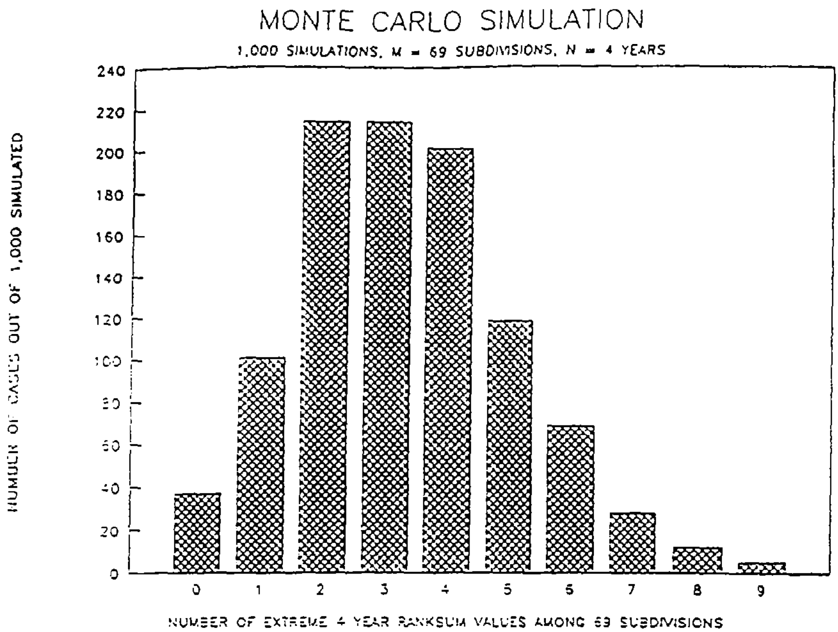


The above graph shows the exact probability density function for the ranksum values when M = 69 and N = 4. The possible ranksum values for a given subdivision range from 4 to 276, inclusive, as follows:

RANKSUM VALUES FROM	THROUGH	POSSIBLE COMBINATIONS	PERCENTAGE OF CASES IN RANGE
4	62	557,845	2.46 %
63	139	10,666,201	47.06
140	217	10,885,230	48.02
218	276	557,845	2.46
4	276	22,667,121	100.00 %

Thus, a two-tailed 95% confidence interval for the ranksum values is from 63 to 217, inclusive. Extreme values are (a) 4 to 62 and (b) 218 to 276.

EXHIBIT B



When M = 69 and N = 4, the two tailed 95% confidence interval for the ranksum of a given subdivision is from 63 to 217, inclusive. The program of Appendix II was run to simulate 1,000 four year experience periods, tabulate all ranksums and numbers of extreme values in order to approximate the distribution for number of extreme values. This result is compared with the BINOMIAL approximation

$$f(x) = \binom{M}{x} \cdot (1 - p)^{x} \cdot p^{M - x}$$

where $p = \dfrac{21{,}551{,}431}{22{,}667{,}121}$:

NUMBER OF EXTREME RANKSUMS	SIMULATION CASES OBSERVED	BINOMIAL APPROXIMATION PREDICTION
0	37	30.7
1	101	109.8
2	214	193.2
3	214	223.4
4	201	190.8
5	119	128.4
6	69	70.9
7	28	33.0
8	12	13.3
9	5	4.7
10+	0	2.0
ALL	1,000	1,000.0
1 TO 7	946	949.4

APPENDIX I: BASIC PROGRAM TO GENERATE EXACT DISTRIBUTION OF RANKSUM VALUES FOR ANY M AND N=3, 4 , 5, OR 6

(PAGE 1 OF 2)

```
MAIN:
CLS
INPUT "SUBDIVISIONS M";M
INPUT "YEARS N";N
DIM RS(M*N)
D=M^N
IF N=3 THEN GOSUB THREE
IF N=4 THEN GOSUB FOUR
IF N=5 THEN GOSUB FIVE
IF N=6 THEN GOSUB SIX
IF N>6 OR N<3 THEN GOTO MAIN
'REMARK: WE WILL CALCULATE THE TOTAL NUMBER OF WAYS OUT OF THE M^N
'        RANK COMBINATIONS THAT RANKSUM = J FOR EACH VALUE OF J
'        FROM M TO M*N. THESE VALUES WILL THEN BE WRITTEN TO A FILE
'        CALLED RESULTS.PRN.

OPEN "RESULTS.PRN" FOR OUTPUT AS 1
? #1,USING "RANKSUM DISTRIBUTION FOR ### SUBS AND ### YEARS";M;N
?  #1,"RANKSUM","NUMBER CASES"
FOR J=N TO M*N
? #1,J,RS(J)
NEXT J
RESET
?
? "FILE RESULTS.PRN  IS SET UP FOR FURTHER ANALYSIS."
END

'SUBROUTINES:
THREE:
FOR I1=1 TO M
FOR I2=1 TO M
FOR I3=1 TO M
S=I1+I2+I3
RS(S)=RS(S)+1
NEXT I3
NEXT I2
NEXT I1
RETURN

FOUR:
FOR I1=1 TO M
FOR I2=1 TO M
FOR I3=1 TO M
FOR I4=1 TO M
S=I1+I2+I3+I4
RS(S)=RS(S)+1
NEXT I4
NEXT I3
NEXT I2
NEXT I1
RETURN
```

APPENDIX I: BASIC PROGRAM TO GENERATE EXACT DISTRIBUTION OF RANKSUM VALUES FOR ANY M AND N=3, 4 , 5, OR 6
(PAGE 2 OF 2)

```
FIVE:
FOR I1=1 TO M
FOR I2=1 TO M
FOR I3=1 TO M
FOR I4=1 TO M
FOR I5=1 TO M
S=I1+I2+I3+I4+I5
RS(S)=RS(S)+1
NEXT I5
NEXT I4
NEXT I3
NEXT I2
NEXT I1
RETURN

SIX:
FOR I1=1 TO M
FOR I2=1 TO M
FOR I3=1 TO M
FOR I4=1 TO M
FOR I5=1 TO M
FOR I6=1 TO M
S=I1+I2+I3+I4+I5+I6
RS(S)=RS(S)+1
NEXT I6
NEXT I5
NEXT I4
NEXT I3
NEXT I2
NEXT I1
RETURN
```

APPENDIX II: BASIC PROGRAM FOR MONTE CARLO SIMULATION APPROXIMATION TO DISTRIBUTION OF NUMBER OF EXTREME VALUES

```
'EXTREME VALUE DISTRIBUTION GENERATOR
cls
INPUT "NUMBER SUBDIVISIONS";M
INPUT "NUMBER OF YEARS";N
INPUT "CONFIDENCE INTERVAL A,B";A,B
DIM NUMBER(M),R(M,N),RS(M)
OPEN "C:\TEMP\RESULTS.PRN" FOR OUTPUT AS 1
INPUT "TRIALS";T
T1=TIMER
RANDOMIZE TIMER
? #1,USING "#,###,### RANDOM TRIALS ON ### SUBS OVER ## YEARS";T;M;N
? #1,USING "EXTREME VALUES ARE LESS THAN ### OR GREATER THAN #,###";A;B
? #1,""
FOR TRIAL=1 TO T
FOR YEAR=1 TO N
X=RND
R(1,YEAR)=INT(M*X+1)
FOR S=2 TO M
     TEST1:
     R(S,YEAR)=INT(M*RND+1)
     FOR II=1 TO S-1
     IF R(S,YEAR)=R(II,YEAR) THEN
     'WE HAVE A DOUBLE COUNT
     GOTO TEST1
     END IF
     NEXT II
NEXT S
NEXT YEAR
W=0
FOR J=1 TO M
RS(J)=0
    FOR II=1 TO N
    RS(J)=RS(J)+R(J,II)
    NEXT II
IF RS(J)<A OR RS(J)>B THEN W=W+1
NEXT J
? #1,TRIAL,W
NEXT TRIAL
? "DONE!!!!!!"
? USING "RUN TIME ##,###.### SECONDS";TIMER-T1
? #1,USING "RUN TIME ##,###.### SECONDS";TIMER-T1
RESET
end
```

APPENDIX III: DETAILED DEVELOPMENT WHEN M = 11, N = 5
(PAGE 1 OF 5)

A. UNADJUSTED LOSS RATIOS
COUNTIES IN TERRITORY WITH 50 OR MORE POLICY YEARS EXPOSURE IN 1990

COUNTY NUMBER	UNADJUSTED LOSS RATIOS BY YEAR AND COUNTY 1990	1989	1988	1987	1986
7	58.4%	12.3%	43.0%	48.3%	77.5%
14	129.7%	34.0%	57.4%	49.3%	66.0%
27	53.6%	43.1%	34.1%	31.2%	17.3%
32	45.1%	98.8%	93.0%	70.4%	50.6%
35	140.5%	28.0%	50.6%	55.2%	156.7%
40	29.3%	92.2%	41.0%	90.4%	47.0%
46	67.2%	69.7%	67.2%	100.3%	59.1%
50	63.9%	20.0%	13.0%	63.4%	29.2%
52	101.1%	24.0%	38.9%	111.9%	35.4%
63	44.4%	51.4%	40.0%	49.3%	29.8%
67	37.4%	61.3%	52.2%	28.4%	30.6%

B. POLICY YEARS OF EXPOSURE
COUNTIES IN TERRITORY WITH 50 OR MORE POLICY YEARS EXPOSURE IN 1990

COUNTY NUMBER	POLICY YEARS OF EXPOSURE BY COUNTY AND YEAR 1990	1989	1988	1987	1986
7	79	82	62	49	30
14	156	146	134	127	122
27	151	120	97	74	47
32	52	54	44	46	45
35	79	63	51	50	51
40	82	89	89	88	101
46	297	298	273	286	273
50	53	50	46	45	51
52	71	65	64	64	71
63	198	188	189	208	188
67	125	121	125	121	115
	1,343	1,276	1,174	1,158	1,094

C. EXPOSURE ADJUSTED LOSS RATIOS
COUNTIES IN TERRITORY 6 WITH 50 OR MORE POLICY YEARS EXPOSURE IN 1990

YEAR	EXPECTED LOSS RATIO	MAXIMUM EXPOSURE
1986	69.1%	273
1987	52.4%	286
1988	50.7%	273
1989	66.8%	298
1990	50.7%	297

APPENDIX III: DETAILED DEVELOPMENT WHEN M = 11, N = 5
(PAGE 2 OF 5)

D. VALUES OF Z(COUNTY,YEAR)

COUNTY NUMBER	1990	1989	1988	1987	1986
7	0.515745	0.524564	0.476557	0.413919	0.331497
14	0.724743	0.699952	0.700602	0.666375	0.668496
27	0.713034	0.634574	0.596080	0.508666	0.414923
32	0.418431	0.425685	0.401463	0.401048	0.405999
35	0.515745	0.459793	0.432219	0.418121	0.432219
40	0.525447	0.546496	0.570971	0.554700	0.608246
46	1.000000	1.000000	1.000000	1.000000	1.000000
50	0.422435	0.409616	0.410485	0.396664	0.432219
52	0.488935	0.467034	0.484182	0.473050	0.509974
63	0.816497	0.794275	0.832050	0.852803	0.829846
67	0.648749	0.637213	0.676665	0.650444	0.649034

E. ADJUSTED LOSS RATIOS

COUNTY NUMBER	1990	1989	1988	1987	1986
7	54.7%	38.2%	47.0%	50.7%	71.9%
14	108.0%	43.9%	55.4%	50.4%	67.0%
27	52.8%	51.8%	40.8%	41.6%	47.6%
32	48.3%	80.4%	67.7%	59.6%	61.6%
35	97.0%	48.9%	50.7%	53.6%	107.0%
40	39.5%	80.6%	45.1%	73.5%	55.7%
46	67.2%	69.7%	67.2%	100.3%	59.1%
50	56.3%	47.6%	35.2%	56.8%	51.8%
52	75.3%	46.8%	45.0%	80.5%	51.9%
63	45.5%	54.5%	41.8%	49.8%	36.5%
67	42.0%	63.3%	51.7%	36.8%	44.1%

F. RANKINGS OF ADJUSTED LOSS RATIOS
(1 = LOWEST, 11 = HIGHEST)

COUNTY NUMBER	1990	1989	1988	1987	1986	RANK SUM
7	6	1	6	5	10	28
14	11	2	9	4	9	35
27	5	6	2	2	3	18
32	4	10	11	8	8	41
35	10	5	7	6	11	39
40	1	11	5	9	6	32
46	8	9	10	11	7	45
50	7	4	1	7	4	23
52	9	3	4	10	5	31
63	3	7	3	3	1	17
67	2	8	8	1	2	21

APPENDIX III: DETAILED DEVELOPMENT WHEN M = 11, N = 5
(PAGE 3 OF 5)

G. EXACT RANKSUM PROBABILITY DISTRIBUTION: M = 11, N = 5
VALUES RANGE FROM 5 TO 55
TOTAL COMBINATIONS = 11^5 = 161,051

95% CONFIDENCE INTERVAL: FROM 16 TO 43. A=16, B=43
p = PROB(A<=RANKSUM<=B) = 0.954263

RANK SUM	POSSIBLE NUMBER CASES	PROBABILITY	CUMULATIVE
5	1	0.000006209	0.000006209
6	5	0.000031046	0.000037255
7	15	0.000093138	0.000130393
8	35	0.000217322	0.000347715
9	70	0.000434644	0.000782360
10	126	0.000782360	0.001564721
11	210	0.001303934	0.002868656
12	330	0.002049040	0.004917696
13	495	0.003073560	0.007991257
14	715	0.004439587	0.012430844
15	1001	0.006215422	0.018646267
16	1360	0.00844453	0.027090797
17	1795	0.011145537	0.038236335
18	2305	0.014312236	0.052548571
19	2885	0.017913580	0.070462151
20	3526	0.021893685	0.092355837
21	4215	0.026171833	0.118527671
22	4935	0.030642467	0.149170138
23	5665	0.035175193	0.184345331
24	6380	0.039614780	0.223960112
25	7051	0.043781162	0.267741274
26	7645	0.047469435	0.315210709
27	8135	0.050511949	0.365722659
28	8500	0.052778312	0.418500971
29	8725	0.054175385	0.472676357
30	8801	0.054647285	0.527323642
31	8725	0.054175385	0.581499028
32	8500	0.052778312	0.634277340
33	8135	0.050511949	0.684789290
34	7645	0.047469435	0.732258725
35	7051	0.043781162	0.776039888
36	6380	0.039614780	0.815654668
37	5665	0.035175193	0.850829861
38	4935	0.030642467	0.881472328
39	4215	0.026171833	0.907644162
40	3526	0.021893685	0.929537848
41	2885	0.017913580	0.947451428
42	2305	0.014312236	0.961763664
43	1795	0.011145537	0.972909202
44	1360	0.008444530	0.981353732
45	1001	0.006215422	0.987569155

G. EXACT RANKSUM PROBABILITY DISTRIBUTION: M = 11, N = 5
VALUES RANGE FROM 5 TO 55
TOTAL COMBINATIONS = 11^5 = 161,051

95% CONFIDENCE INTERVAL: FROM 16 TO 43. A=16, B=43
p = PROB(A<=RANKSUM<=B) = 0.954263

RANK SUM	POSSIBLE NUMBER CASES	PROBABILITY	CUMULATIVE
46	715	0.004439587	0.992008742
47	495	0.003073560	0.995082303
48	330	0.002049040	0.997131343
49	210	0.001303934	0.998435278
50	126	0.000782360	0.999217639
51	70	0.000434644	0.999652284
52	35	0.000217322	0.999869606
53	15	0.000093138	0.999962744
54	5	0.000031046	0.999993790
55	1	0.000006209	1.000000000
	161,051		

H. DISTRIBUTION OF EXTREME VALUE COUNTS
M = 11, N = 5
BINOMIAL p = 0.954263

NUMBER OF EXTREMA	MONTE CARLO OBSERVATIONS	BINOMIAL PREDICTION
0	613	597.5
1	298	315.0
2	80	75.5
3	9	10.9
4+	0	1.1
	1,000	1,000

I. RANKSUM TESTING FOR EXTREME VALUES
95% EXTREMA: UNDER 16 OR OVER 43

COUNTY	RANKSUM	EXTREME?
7	28	NO
14	35	NO
27	18	NO
32	41	NO
35	39	NO
40	32	NO
46	45	YES
50	23	NO
52	31	NO
63	17	NO
67	21	NO

APPENDIX III: DETAILED DEVELOPMENT WHEN M = 11, N = 5
(PAGE 5 OF 5)

J. REMARKS AND OBSERVATIONS

1. In this case, there is only ONE extreme value - not an unexpected result. County 27 just slightly misses, as do counties 32 and 63. Had a 90% confidence interval been the standard here, extreme values would be less than 19 or greater than 42 and counties 27, 32, and 63 would be added to the "EXTREME" category. At the 90% confidence level, 4 extreme values is highly unusual, occurring about 1% of the time.

2. It is also interesting that county 45 is the LARGEST county in the territory and has an extreme HIGH value even at the 95% confidence level.

APPENDIX IV: TESTING OVERALL TERRITORIAL CONSISTENCY
(PAGE 1 OF 7)

I. 21 MONTH CASE INCURRED LOSS RATIOS BY COUNTY AND ACCIDENT YEAR

County	1989	1988	1987	1986
1	109.5%	94.9%	65.0%	191.4%
2	108.0%	73.4%	94.7%	76.3%
3	211.0%	69.2%	63.6%	234.5%
4	81.4%	84.9%	338.7%	34.0%
5	39.8%	132.6%	101.0%	64.2%
6	81.2%	93.5%	61.8%	43.6%
7	45.8%	41.0%	215.5%	250.6%
8	114.7%	189.7%	97.4%	53.8%
9	66.4%	77.4%	73.7%	103.9%
10	115.0%	110.7%	148.7%	22.6%
11	65.7%	58.9%	75.5%	134.3%
12	62.4%	83.3%	71.5%	58.0%
13	52.6%	73.5%	77.2%	66.3%
14	63.0%	75.2%	130.9%	61.5%
15	23.0%	120.6%	17.9%	0.0%
16	108.5%	113.1%	47.2%	49.4%
17	110.7%	63.7%	107.7%	131.2%
18	107.2%	53.7%	67.2%	100.4%
19	137.7%	100.9%	51.5%	34.9%
20	146.3%	43.7%	87.8%	254.1%
21	63.5%	61.0%	53.9%	55.4%
22	88.9%	104.3%	59.1%	52.6%
23	95.4%	44.6%	82.4%	120.0%
24	95.1%	55.4%	60.8%	38.4%
25	136.9%	74.6%	196.1%	47.9%

II. POLICY YEAR EXPOSURES BY COUNTY AND ACCIDENT YEAR

County	1989	1988	1987	1986
1	181	187	152	141
2	205	196	211	212
3	134	123	100	105
4	15	22	24	27
5	86	78	79	87
6	121	138	131	133
7	130	151	143	152
8	108	122	143	162
9	92	90	91	94
10	69	57	54	54
11	195	196	208	215
12	267	249	251	246
13	151	160	133	121
14	111	107	98	103
15	27	27	15	18
16	86	86	79	75
17	110	109	101	106
18	91	100	107	120
19	60	71	69	65

APPENDIX IV: TESTING OVERALL TERRITORIAL CONSISTENCY
(PAGE 2 OF 7)

County	1989	1988	1987	1986
20	37	43	43	41
21	147	171	177	176
22	203	213	197	197
23	66	70	67	69
24	160	185	190	173
25	74	87	90	90
MAXIMUM	267	249	251	246

II. EXPECTED LOSS RATIOS AND Z VALUES

YEAR	EXPECTED LOSS RATIO
1986	92.8%
1987	90.2%
1988	82.2%
1989	91.4%

Z COEFFICIENTS FOR ADJUSTED LOSS RATIOS

County	1989	1988	1987	1986
1	0.823348	0.866605	0.778189	0.757080
2	0.876236	0.887214	0.916863	0.928326
3	0.708430	0.702834	0.631194	0.653322
4	0.237023	0.297243	0.309221	0.331295
5	0.567536	0.559690	0.561018	0.594692
6	0.673189	0.744457	0.722435	0.735289
7	0.697776	0.778733	0.754799	0.786057
8	0.635999	0.699971	0.754799	0.811503
9	0.587000	0.601204	0.602121	0.618154
10	0.508357	0.478451	0.463831	0.468521
11	0.854598	0.887214	0.910322	0.934871
12	1.000000	1.000000	1.000000	1.000000
13	0.752026	0.801605	0.727929	0.701334
14	0.644772	0.655529	0.624851	0.647070
15	0.317999	0.329293	0.244461	0.270501
16	0.567536	0.587692	0.561018	0.552158
17	0.641861	0.661628	0.634343	0.656425
18	0.583801	0.633724	0.652913	0.698430
19	0.474045	0.533986	0.524309	0.514031
20	0.372259	0.415561	0.413902	0.408248
21	0.741999	0.828702	0.839750	0.845841
22	0.871952	0.924890	0.885924	0.894882
23	0.497183	0.530212	0.516655	0.529611
24	0.774113	0.861958	0.870041	0.838601
25	0.526454	0.591099	0.598804	0.604858

APPENDIX IV: TESTING OVERALL TERRITORIAL CONSISTENCY
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III. ADJUSTED LOSS RATIOS BY COUNTY AND ACCIDENT YEAR

County	1989	1988	1987	1986
1	106.6%	94.2%	68.8%	167.1%
2	106.2%	75.3%	93.7%	77.4%
3	176.5%	75.4%	70.4%	184.9%
4	90.1%	88.6%	161.5%	72.4%
5	62.7%	113.9%	92.7%	75.2%
6	85.0%	92.7%	67.4%	56.2%
7	60.0%	51.9%	182.8%	216.6%
8	106.7%	159.8%	93.6%	60.9%
9	77.3%	82.5%	77.1%	99.1%
10	104.1%	100.0%	113.0%	59.2%
11	69.6%	62.5%	76.1%	131.5%
12	62.4%	83.3%	71.5%	58.0%
13	62.5%	76.8%	78.5%	73.8%
14	73.6%	80.4%	112.6%	72.1%
15	70.6%	100.2%	66.4%	66.7%
16	101.7%	103.7%	62.5%	68.2%
17	104.3%	72.7%	98.3%	117.5%
18	101.2%	67.1%	72.4%	97.7%
19	114.1%	95.9%	66.1%	62.3%
20	112.7%	70.9%	84.5%	157.8%
21	71.0%	66.0%	58.4%	60.9%
22	89.4%	103.2%	61.7%	56.6%
23	94.1%	66.0%	82.3%	106.5%
24	94.5%	60.2%	63.6%	46.9%
25	116.0%	81.0%	150.4%	65.1%

IV. RANKINGS AND RANKSUMS OF ADJUSTED LOSS RATIOS

County	1989	1988	1987	1986	RANKSUM
1	20	18	8	23	69
2	19	9	19	16	63
3	25	10	9	24	68
4	12	16	24	13	65
5	4	24	17	15	60
6	10	17	7	2	36
7	1	1	25	25	52
8	21	25	18	6	70
9	9	14	13	18	54
10	17	20	22	5	64
11	5	3	12	21	41
12	2	15	10	4	31
13	3	11	14	14	42
14	8	12	21	12	53
15	6	21	6	10	43
16	16	23	3	11	53
17	18	8	20	20	66
18	15	6	11	17	49
19	23	19	5	8	55

APPENDIX IV: TESTING OVERALL TERRITORIAL CONSISTENCY
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IV. RANKINGS AND RANKSUMS OF ADJUSTED LOSS RATIOS

County	1989	1988	1987	1986	RANKSUM
20	22	7	16	22	67
21	7	5	1	7	20
22	11	22	2	3	38
23	13	4	15	19	51
24	14	2	4	1	21
25	24	13	23	9	69

V. DISTRIBUTION OF RANKSUMS WHEN M = 25, N = 4

RANKSUM VALUE	NUMBER COMBINATIONS	PROBABILITY	CUMULATIVE
4	1	0.000003	0.000003
5	4	0.000010	0.000013
6	10	0.000026	0.000038
7	20	0.000051	0.000090
8	35	0.000090	0.000179
9	56	0.000143	0.000323
10	84	0.000215	0.000538
11	120	0.000307	0.000845
12	165	0.000422	0.001267
13	220	0.000563	0.001830
14	286	0.000732	0.002563
15	364	0.000932	0.003494
16	455	0.001165	0.004659
17	560	0.001434	0.006093
18	680	0.001741	0.007834
19	816	0.002089	0.009923
20	969	0.002481	0.012403
21	1140	0.002918	0.015322
22	1330	0.003405	0.018726
23	1540	0.003942	0.022669
24	1771	0.004534	0.027203
25	2024	0.005181	0.032384
26	2300	0.005888	0.038272
27	2600	0.006656	0.044928
28	2925	0.007488	0.052416
29	3272	0.008376	0.060792
30	3638	0.009313	0.070106
31	4020	0.010291	0.080397
32	4415	0.011302	0.091699
33	4820	0.012339	0.104038
34	5232	0.013394	0.117432
35	5648	0.014459	0.131891
36	6065	0.015526	0.147418
37	6480	0.016589	0.164006
38	6890	0.017638	0.181645
39	7292	0.018668	0.200312
40	7683	0.019668	0.219981

V. *DISTRIBUTION OF RANKSUMS WHEN M = 25, N = 4*

RANKSUM VALUE	NUMBER COMBINATIONS	PROBABILITY	CUMULATIVE
41	8060	0.020634	0.240614
42	8420	0.021555	0.262170
43	8760	0.022426	0.284595
44	9077	0.023237	0.307832
45	9368	0.023982	0.331814
46	9630	0.024653	0.356467
47	9860	0.025242	0.381709
48	10055	0.025741	0.407450
49	10212	0.026143	0.433592
50	10328	0.026440	0.460032
51	10400	0.026624	0.486656
52	10425	0.026688	0.513344
53	10400	0.026624	0.539968
54	10328	0.026440	0.566408
55	10212	0.026143	0.592550
56	10055	0.025741	0.618291
57	9860	0.025242	0.643533
58	9630	0.024653	0.668186
59	9368	0.023982	0.692168
60	9077	0.023237	0.715405
61	8760	0.022426	0.737830
62	8420	0.021555	0.759386
63	8060	0.020634	0.780019
64	7683	0.019668	0.799688
65	7292	0.018668	0.818355
66	6890	0.017638	0.835994
67	6480	0.016589	0.852582
68	6065	0.015526	0.868109
69	5648	0.014459	0.882568
70	5232	0.013394	0.895962
71	4820	0.012339	0.908301
72	4415	0.011302	0.919603
73	4020	0.010291	0.929894
74	3638	0.009313	0.939208
75	3272	0.008376	0.947584
76	2925	0.007488	0.955072
77	2600	0.006656	0.961728
78	2300	0.005888	0.967616
79	2024	0.005181	0.972797
80	1771	0.004534	0.977331
81	1540	0.003942	0.981274
82	1330	0.003405	0.984678
83	1140	0.002918	0.987597
84	969	0.002481	0.990077
85	816	0.002089	0.992166
86	680	0.001741	0.993907
87	560	0.001434	0.995341

V. DISTRIBUTION OF RANKSUMS WHEN M = 25, N = 4

RANKSUM VALUE	NUMBER COMBINATIONS	PROBABILITY	CUMULATIVE
88	455	0.001165	0.996506
89	364	0.000932	0.997437
90	286	0.000732	0.998170
91	220	0.000563	0.998733
92	165	0.000422	0.999155
93	120	0.000307	0.999462
94	84	0.000215	0.999677
95	56	0.000143	0.999821
96	35	0.000090	0.999910
97	20	0.000051	0.999962
98	10	0.000026	0.999987
99	4	0.000010	0.999997
100	1	0.000003	1.000000
	390,625		

VI. DISTRIBUTION OF NUMBER OF EXTREME VALUES PER PERIOD

95% RANKSUM CONFIDENCE INTERVAL FROM 23 TO 79, INCLUSIVE

COMBINATIONS 23 TO 79	372,684
TOTAL COMBINATIONS	390,625
BINOMIAL p VALUE =	0.954071

x	BINOMIAL f(x)	PREDICTED CASES PER 1,000 TRIALS EXTREME = x	EXTREME <= x
0	0.308687	309	309
1	0.371504	372	680
2	0.214610	215	895
3	0.079207	79	974
4	0.020972	21	995
5	0.004240	4	999
6+	0.000780	1	

The binomial approximation predicts that 90% of the time, the number of extreme values is two or less, and 97.4% of the time it will be three or less.

Monte Carlo simulation of 1,000 four year periods gives the following results:

NUMBER OF EXTREME VALUES n	NUMBER OF OBSERVATIONS FOR WHICH EXTREME = n	EXTREME <= n
0	300	300
1	374	674
2	238	912
3	62	974
4	24	998
5	1	999
6	1	1,000
7+	0	
	1,000	

We thus confirm that an "unusual" number of extreme values is 3 or more at the 90% level, and 4 or more at the 97% level.

VII. OBSERVED EXTREME RANKSUM VALUES

RANKSUM IS LESS THAN 23 OR GREATER THEN 79:

COUNTY	RANKSUM
21	20
24	21

The number of extreme ranksum values is TWO, and both are at the LOW end of the range.