CASUALTY ACTUARIAL SOCIETY FORUM

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CASUALTY ACTUARIAL SOCIETY

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CASUALTY ACTUARIAL SOCIETY

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To: CAS Readership

Re: ***The Forum***

The Spring 1992 issue of *The Forum* continues to provide an outlet for the publication of research on a non-refereed basis, communication of Casualty Actuarial Society committee work, and information from other actuarial societies. This issue includes papers from each of these areas ranging from a paper on credibility and a paper on risk provisions to guidance notes from the Institute of Actuaries and the Faculty of Actuaries.

Through its publication of *The Forum*, the CAS provides an alternate source for continuing education, since up to six hours of continuing education each year can come from sources other than attendance at formal seminars and meetings according to the Qualification Standards for Public Statements of Actuarial Opinion of the American Academy of Actuaries.

The publication in *The Forum* does not constitute recognition by the CAS of the contents of any article because the contributions are not subject to review by a panel of expert referees.

Many thanks to our authors for their continued contributions to our readership. We continue to encourage you to contribute to this publication. Please submit your papers or articles for the next issue no later than September 30, 1992 to Joel Kleinman at his yearbook address.

Yours very truly,

Irene K. Bass

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Vice President - Continuing Education

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Richard J. Roth, Jr.

OBSERVATIONS ON THE CALIFORNIA PROPOSITION 103 DEBATE OVER PROFITABILITY AND SURPLUS

RICHARD J. ROTH, JR.

1. Introduction

In November, 1988, the voters of California narrowly passed Proposition 103, which requires the California Insurance Department to approve certain insurance rates, primarily homeowners, automobile, and the commercial coverages. There is also a provision requiring an immediate 20% rollback in these rates; however, the California Supreme Court made this rollback requirement subject to an insurer's right to earn a "fair and reasonable" rate of return. Premiums on the affected coverages amounted to $25 billion in 1989, probably the largest single property-casualty market in the world.

In order to implement Proposition 103, two issues had to be addressed: (1) what is a "fair and reasonable" rate of return, and (2) what are the appropriate criteria for the prior approval of rates. Lawsuits were filed and hearings were held, while the world insurance, investment, and academic communities watched hoping to see a stimulating intellectual inquiry into the issues and a leadership in the advancement of knowledge and theory on the issues of required profitability and required risk-based capital and surplus. However, after two years of public hearings, the result has been no discernable resolution of the issues, hours of indeterminate, unproductive, and excruciatingly boring attorney controlled proceedings, and huge legal and consulting fees. History will show that this was a golden opportunity to advance the science of insurance regulation and it was lost.

There are a number of specific reasons why these hearings failed:

(1) The group of attorneys and staff who were put in charge of the hearings knew surprisingly little about insurance, but they did not let that fact interfere with their work or inhibit them in the drafting of regulations.

(2) There seemed to be an insistence on ignoring whatever work that has been done in the past on these issues. The issues of profitability, investment return, and required capital and surplus are issues which have been explored by many state insurance departments in the years since World War II. In addition, financial

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economists, actuaries, and academicians around the world have done much work on these issues. Instead of making an effort to review this work, there was an insistence on addressing these issues from scratch, with the result that proposed regulations were constantly being revised.

(3) Even though some of the country's leading economists, investment experts, and actuaries were called to testify and, in many cases, submitted lengthy written documents, what emerged was a perception that no unified theory has yet been worked out which would connect the insurer's need to raise capital with the regulator's duty to approve or disapprove a specific insurance rate. Casualty actuaries estimate required rates based on an individual insurer's losses, claims inflation, and frequency trends. Financial economists deal with such issues as optimizing investment strategies, the pricing of assets, the relationship between profit and risk, solvency, and ruin probabilities. The casualty actuaries and the financial economists need to get together and exchange business cards.

(4) A decision was made to use a methodology for estimating profitability which has long since proven to be defective and unworkable. The methodology involves taking an insurer's national figures for expenses, investment income, capital gains or losses, and federal taxes and to allocate these figures proportionately to get by line by state results, which are then combined with the state loss experience to get a profit or rate of return as a percent of premium. The insurer's surplus was then imputed by line by state using estimated premium to surplus ratios, called "leverage ratios", to get a rate of return by line for California as a percent of surplus (or net worth). The leverage ratios would vary by line of insurance depending on the perceived risk, such as a 2.5 ratio of premiums to surplus for homeowners insurance, but a 1.0 ratio for medical malpractice. This approach has long since proven to be defective and unworkable and was so characterized by many witnesses. The problem is that the insurance business involves a wide range of risks from underwriting and investment to catastrophe and credit, some of which are unrelated to the premium volume in a given year. The result is that the true premium to surplus ratios can vary widely between insurers writing the same lines of business. A simple example would be two insurers currently writing the same premium volume in automobile liability insurance, where one insurer has large loss reserves from business written in prior years and another, new insurer which has practically no loss reserves from prior years. Clearly, the required risk-based surplus would be different for these two insurers. In any event, the problem with the methodology is that it requires the choice of some arbitrary assumptions, which then inevitably lead to strange results. It is a mathematical based procedure, while this paper will suggest an economic based procedure.

The purpose of this paper is to describe some of the serious misconceptions about insurance which have dominated these Proposition 103 hearings and to demonstrate that the issues of "fair and reasonable return", the criteria for prior approval of rates, and the proper measure of return should all be analyzed using the general principles of economics, combined with an actuarial analysis of the structure and trends in the insurance industry.

2. What is income?

There has been endless testimony on what constitutes income in calculating rates of return. A common assertion is that "total rate of return" should be used. However, when this term is explained, it is revealed that many items of income are omitted, especially either realized or unrealized capital gains and losses.

This issue of "what is income?" has a long history, and, surprisingly, disagreement at the Proposition 103 hearings was widespread.

In 1921, the National Association of Insurance Commissioners adopted the so-called 1921 Profit Formula, which provided that (see NAIC (1922), NAIC (1970)):

(1) a reasonable underwriting profit is 5% of premiums plus 3% for conflagrations, and

(2) no items of profit or loss connected with the so-called banking end of the business should be taken into consideration.

This remained the standard meaning of income until 1970. In that year, the NAIC published a 233 page study of the issue prepared by the NAIC Central Office. The study for the most part was only a discussion of insurance accounting and a discussion of numerous approaches and techniques that have been proposed to measure profitability. Such approaches included use of investment earnings on unearned premiums and/or loss reserves, including or excluding realized and/or unrealized capital gains. There are also discussions of premium to surplus leverage, proper level of surplus, policyholder versus stockholder surplus, and the need to attract capital. Also, the study noted that income can be measured against sales, net worth, or total investable funds, each with its advantages and disadvantages. The proper base against which income is compared is as important an issue as the issue of what is income.

The most important result of this study is that it repudiated the 1921 Profit Formula. The study recommended that income from all sources be ascertained and considered, including income on capital funds. However, the study reported that it could not conclude how much capital was required nor the proper base against which to measure rate of return. The study concluded that income should be determined from an investor's perspective.

The issue was not raised again by the NAIC until 1984, when the "NAIC Study of Investment Income" was published as a supplement to Volume II of the 1984 NAIC Proceedings. By 1984, interest rates, and therefore investment income, had risen so high that now investment income has become the dominant, if not the only, source of net income for insurers. The study easily reaffirmed the repudiation of the 1921 Profit Formula. The study concluded that the "total return approach" was most appropriate in regulating property/casualty insurance rates. However, the approach suggested in the study contained the same defects, intractable problems, and dead ends that were to visit the Proposition 103 hearings later. Namely,

(1) despite the use of the term "total return approach", significant items of income are excluded, such as unrealized capital gains, policy fees and sometimes even realized capital gains.

(2) the approach suggested relied on an allocation of surplus by line by state in order to set a by line by state rate of return. Modern risk theory has conclusively shown

that it is not meaningful to do this. Curiously, the study eventually recognized this, but did not attempt to suggest a solution or an alternative.

(3) there is an implication that the proper rate of return is a constant to be determined. In fact, it is a dynamic target, requiring econometric expertise to determine.

Most of the reason for the controversy lies in the question, "whose income is it?" The claim that income on stockholder invested funds belongs to stockholders and income on policyholder invested funds belongs to policyholders only distracts from the proper analysis.

In economic terms, annual income is the annual increase in net worth of the business. This is the only correct meaning of "total return" and the only meaning which conforms to the vision of an investor. Specifically, if an insurer's annual statement for 1989 reports the following figures:

Surplus at 12/31/88	$10,000,000
Surplus at 12/31/89	$12,000,000
Stockholders dividends	$500,000
Additional paid-in capital	$1,000,000

Then the income of the insurer based on the business conducted in 1989 is:

Income = ($12,000,00 - $10,000,000) + $500,000 - $1,000,000 = $1,500,000

In other words, if there were no dividends or capital paid-in, then the business earned $1,500,000, or 15% of $10,000,000, the initial net worth, which we call surplus. Thus, the insurer earned a 15% rate of return.

Let **S** equal the beginning statutory surplus of the insurer. Let **dS** equal the increase in surplus over the year, including stockholder dividends and excluding additional paid-in capital. Then the term **dS/S** is the total rate of return.

The calculation of **dS** is shown in detail on page 4 of every insurer's Annual Statement. For 1989 the industry results were:

dS = net underwriting gain or loss	-$16,895m
+ net investment income	31,207
+ net realized capital gains or losses	4,649
+ other income	-1,228
- dividends to policyholders	2,713
- federal taxes	2,802
+ net unrealized capital gains or losses	8,035
+ change in non-admitted assets	43
+ change in liability for reinsurance	-702
+ change in foreign exchange	29
+ change in excess statutory reserves	195
+ other write-in items	-645
= total economic income	$19,173m

Therefore, **dS/S** = 19,173/117,935 = 16.2% for 1989, since the surplus of the industry was $117,935 million at the beginning of 1989.

To my knowledge, no one in the Proposition 103 hearings ever advocated that the definition of income should be expanded to be defined in terms of change in surplus, yet this is the only true definition of economic income and the only definition which includes <u>all</u> sources of income. Note the importance of net unrealized capital gains in 1989.

If instead surplus is measured on a generally accepted accounting principle (GAAP) basis, then we get GAAP net worth from statutory surplus as follows:

To statutory surplus (SAP)
Add: unauthorized reinsurance
excess statutory reserves
prepaid expenses
non-admitted assets
special reserves
Less: tax on prepaid expenses
tax on unrealized capital gains
Equals GAAP net worth.

It turns out that GAAP net worth is equal to about 1.15-1.20 times SAP surplus. Since prepaid expenses are by far the dominant item and since prepaid expenses are proportional to premiums, which in turn, are proportional to surplus, it is often assumed that GAAP net worth is proportional to SAP surplus by a fixed factor, such as 1.15 or 1.20. In that event, dS/S is the same whether S is based on GAAP or SAP. dS/S has the property that any change in the accounting definition of surplus will affect both the numerator and the denominator.

3. Surplus and Risk

In the 1984 NAIC Study of Investment Income, the chosen base for measuring profitability was surplus (or net worth). It is also the base used in the Proposition 103 hearings as proposed by the Department of Insurance and others. It is the correct base. However, in order to get a by line by state measure of profitability, the 1984 NAIC Study indicated that an insurer's surplus could be allocated by line by state in proportion to either premiums, reserves, or a combination of premiums and reserves. Alternatively, the Proposition 103 hearings imputed surplus by line by state using leverage ratios. Both methods have the same theoretical faults.

For a given multi-line, multi-state insurer, there is an appropriate level of risk-based surplus. This level of surplus is based on the sources of risk, which include:

(1) underwriting risk - the adequacy of the premium to pay losses and expenses.
(2) investment risk - whether or not the expected investment performance is realized.
(3) financial risk - the leverage of total assets to surplus, particularly with respect to fluctuations in invested asset values.
(4) reserve risk - the leverage of total liabilities to surplus, particularly the loss and expense reserves.
(5) specific sources of risk - such as inflation, changes in the law, deficiency of reinsurance recoveries, and changes in claim frequency.
(6) catastrophe risk - the whole of an insurer's surplus is at risk for a catastrophe in any one state or line of insurance.

The risk-based surplus must increase each year to support the annual inflation rate, the increase in new business, and any change in risk leverage ratios.

The appropriate level of risk-based surplus is determined for the insurer as a whole and will vary between insurers of the same size. If for each state and each line, the appropriate risk-based surplus were determined separately, then the aggregate surplus would be too great; that is, there would be an inefficient use of capital.

This point has been proven with great rigor and completeness in the 1989 book entitled, Insurance Solvency and Financial Strength, by Pentikainen, Bonsdorff, Pesonen, Rantala, and Ruohonen. These Finnish authors are the world's leading theoreticians on the subject of risk and solvency. The conclusion of their work is that an appropriate aggregate surplus is unique to each insurer depending on all of the sources of risk. These sources of risk interact. The result is that the premium to surplus ratios of insurers may vary widely. A result of their analysis is that an appropriate aggregate surplus once determined cannot be subdivided or allocated by line by state, nor by year. Furthermore, even if premium to surplus ratios could be determined by line by state for each insurer, they would not be the same between insurers.

Thus, only two quantities are meaningful: (a) the required surplus of the insurer group and, (b) the required marginal surplus for a specified change in assets, liabilities, or premiums. Therefore, there are no fixed premium to surplus ratios by line which are appropriate for all insurers.

As mentioned, the lengthy 1984 NAIC study relies heavily on the efficacy of allocating surplus by line. However, an interesting aside is made on page 44 of the study, in which an admission is made that allocating surplus by either premiums or liabilities is not producing satisfactory results. Then the following statement is made:

> "Whether target returns should vary for each line of insurance is a final consideration in analyzing the variations between lines. The risk of the industry as a whole can be estimated, but any effort to determine the risk for each line will meet with the same problem faced in allocating surplus. No definitive answer is apparent."

In other words, the authors of the 1984 NAIC study intuitively realized that it is not proper to subdivide surplus or risk. The Proposition 103 hearings are also showing that you get strange results when you attempt to subdivide surplus. The high point of absurdity was reached in the Proposition 103 hearings when the California Insurance Department published a proposed allocation of surplus for earthquake insurance using a one dollar of premium to one dollar of surplus ratio. In fact, for a portfolio of dwellings in one earthquake zone, as much as seventy-five dollars of surplus may be required for each dollar of premium, which is why earthquake insurance can only really be sold by a multi-line insurer. The earthquake coverage is a clear example of a situation in which the required surplus is so great that the whole of the insurer's surplus is at stake. This is true of any catastrophe potential, and one of the fundamental reasons why reinsurance is used to protect the insurer's surplus against catastrophic losses.

Myers and Cohn prepared a famous paper for the 1982 Massachusetts automobile rate hearings (published in Fair Rate of Return in Property - Liability Insurance). The paper is famous because it outlines a discounted cash flow model using risk-based discount rates derived from the capital asset pricing model. The paper contains this sentence (p.68): "The premiums-to-surplus ratio is assumed to be given exogenously - e.g., by the regulator." The Proposition 103 hearings are showing the world that regulators are not up to doing that correctly.

The theory that it is not proper to subdivide surplus or risk is subject to some qualification. First, the insurer may write only one or two lines, in which case a reasonable allocation of surplus by state by line might be possible. Second, the application of risk theory may justify imputing a required surplus for the purposes of establishing a rate of return, regardless of what the actual surplus might be. This approach is discussed, under certain risk limiting conditions, by Richard A. Derrig in his paper in Financial Models of Insurance Solvency.

4. "Fair and Reasonable Rate of Return"

That a regulated industry is entitled to earn a fair (or just) and reasonable rate of return was affirmed in the U. S. Supreme Court case, Hope Natural Gas. When Proposition 103 passed, the insurance industry immediately sued over the provision requiring a 20% rollback. In the resulting case, Calfarm Insurance Company, the California Supreme Court referred to Hope Natural Gas to affirm the fair and reasonable rate of return standard for insurers under Proposition 103.

In this famous case, the U.S. Supreme Court enunciated the test that income or return to the equity owner should:

(1) be commensurate with returns on investments in other enterprises having corresponding risks, and

(2) be sufficient to attract capital and maintain credit.

This test is also what is meant by the term, "fair and reasonable return". The only definition of income which can be used in the application of this test is the change in net worth.

For the past two years, the Proposition 103 hearings have been trying to put a number on "fair and reasonable" rate of return. Someone looked at the industry figures for the 15 year period 1973-87 and estimated that the average after tax "rate of return on equity" was 11.2%. No other justification has been given for this figure. Apparently, statutory net income divided by average surplus was used as "rate of return on equity". This definition excludes unrealized capital gains. The insurance industry's expert witnesses have been vigorous in condemning this figure as arbitrary and too low. While they are certainly correct in that it is arbitrary, the expert witnesses have not been successful in establishing an alternative figure and there may be a good reason for this.

A "fair and reasonable" rate of return is not necessarily something which can be measured. Like the concept of "competition", it can only be described. That is, we can only determine whether the rate of return is adequate or inadequate in the present economic environment, but we can't give it a number, such as 11.2%. For instance, the rate of return is adequate if

- the industry attracts capital
- new companies are being formed

and inadequate if

- stockholder dividends exceed the in-flow of capital
- little competition exists or companies are withdrawing.

The problem is that the cost of capital is not static, it depends on perceived, prospective returns, not past returns.

In his book, The Economics of Regulation, Alfred E. Kahn makes this point when he explains that the cost of capital depends on the moment in time, the volatility of the stock market, the concept of "comparable earnings", and the need to create incentives for efficiency and innovation. So, there is no objective, unequivocal method of ascertaining the cost of capital, even for a particular regulated company at a particular time and place. Thus, it is impossible to measure a fair and reasonable rate of return precisely. (Volume I, pp. 43-54)

The law does not require a fair and reasonable rate of return, but only the fair and reasonable opportunity to make a fair and reasonable rate of return. This distinction is very important in the regulatory rate approval process. The issue is whether average expenses, actual expenses, or capped expenses should be allowed. Inefficient insurers should not be protected, nor should efficient insurers be penalized. Similarly, heavily capitalized insurers should not be forced to give up the additional investment income. That the rate approval process is not intended to guarantee a fair and reasonable rate of return was emphasized in the 1984 NAIC study (page 24). It was also stated in the Hope Natural Gas Case (320 U.S. 591, 603).

Table 1 shows the historical rate of return for the period 1977 to 1989, which covers a complete underwriting cycle. The rate of return is defined in terms of **dS/S**, defined above, using data from A. M. Best and Co. Table 1 shows that:

(1) the industry paid dividends to stockholders each year, and
(2) the industry attracted capital (paid-in surplus) each year, even in 1984 when the industry lost money.

From this we can draw the conclusion that during this time period the U. S. insurance industry earned at least a fair and reasonable rate of return. While it is true that the actual return ranged from -3.1% to 23.5%, the perception existed that a fair and reasonable rate of return was obtainable.

The insurance industry is very unusual among industries in that about 35% of the business is conducted by mutual insurers, owned by the policyholders. Unlike stock insurers, mutual insurers cannot raise capital, nor do they pay stockholder dividends. Table 2 shows a comparison of stock insurers versus mutual insurers. If mutual insurers don't pay stockholder dividends and cannot attract capital, how can the fair and reasonable test be applied to the rates of these insurers? The answer lies as follows.

After adjusting for inflation, Table 3 shows that surplus, premiums and reserves have each been increasing annually in deflated terms. This growth represents the growth in the demand for insurance and the growth in the need for surplus to support the growth in reserves of the insurance business. Note that the ratio of reserves to premiums has increased from .80 to 1.29, reflecting the increasing importance of workers' compensation insurance and liability insurance. This has caused the premium to surplus ratio to decline over the years, as surplus has increased to support the increase in reserves.

From 1975 to 1989, the industry appears to have tried to maintain a level reserve to surplus ratio of around 2.00, but this constancy is only a coincidence, since the theoretical risk-based reserve to surplus ratio varies significantly by line of insurance and the mix of lines of insurance changes over time. In fact, the ratio is significantly higher for most insurers as seen in Table 4, where the reserve to surplus ratio for most insurers is about 2.2 - 2.3. Table 4 breaks out State Farm Mutual Automobile Insurance Company separately to show the effect of its large size. It is the largest writer of private passenger automobile and homeowners insurance in the U.S.

Tables 4 and 5 are shown in order to point out some important differences between stock insurers and mutual insurers. In Table 5, stock insurers tend to concentrate on the commercial lines which require larger loss and expense reserves, such as Workers' Compensation and Other Liability. On the other hand, mutual insurers tend to concentrate on the personal lines which require smaller loss and expense reserves, such as Auto Liability and Auto Physical Damage. This is seen in Table 4, line (4), where the reserve to earned premium ratio is highest for stock insurers.

Table 1
Historical Insurance Industry Rate of Return
(in billion dollars, unless a %)

	1977	1978	1979	1980	1981
1. Beginning Surplus (S)	$24.7b	$29.4b	$35.5b	$42.5b	$51.0b
2. Ending Surplus	29.4	35.5	42.5	51.0	54.0
3. Increase in Surplus	4.7	6.1	7.0	8.5	3.0
4. Stockholders Dividends	1.1	1.4	1.8	2.2	2.4
5. Surplus Paid-In	1.0	.6	.6	.7	.6
6. Surplus Change (dS)	4.8	6.9	8.2	10.0	4.8
7. dS/S	19.4%	23.5%	23.1%	23.5%	9.4%

	1982	1983	1984	1985	1986
1. Beginning Surplus (S)	$54.0b	$61.0b	$65.4b	$63.7b	$76.4b
2. Ending Surplus	61.0	65.4	63.7	76.4	94.8
3. Increase in Surplus	7.0	4.4	-1.7	12.7	18.4
4. Stockholders Dividends	2.7	3.0	2.5	2.7	2.8
5. Surplus Paid-in	1.5	1.1	2.8	7.7	6.8
6. Surplus Change (dS)	8.2	6.3	-2.0	7.7	14.4
7. dS/S	15.2%	10.3%	-3.1%	12.1%	18.8%

	1987	1988	1989
1. Beginning Surplus (S)	$94.8b	$105.0b	$117.9b
2. Ending Surplus	105.0	117.9	133.9
3. Increase in Surplus	10.2	12.9	16.0
4. Stockholders Dividends	4.4	4.9	5.5
5. Surplus Paid-In	4.0	1.7	2.4
6. Surplus Change (dS)	10.6	16.1	19.1
7. dS/S	11.2%	15.3%	16.2%

Note: line (6) = line (3) + line (4) - line (5)

Source: A.M. Best & Co., Aggregates & Averages, respective years.

Table 2
Stock Insurers vs. Mutual Insurers
(in billion dollars, unless a %)

Stock Insurers	1984 B$	1984 %S	1985 B$	1985 %S	1986 B$	1986 %S
(1) Beginning Surplus (S)	$40.1	100.0%	$36.4	100.0%	$45.8	100.0%
(2) Ending Surplus	36.4		45.8		57.7	
(3) Increase in Surplus	-3.7	-9.2%	9.4	25.8%	11.9	25.9%
(4) Stockholders Dividends	2.5	6.2%	2.7	7.4%	2.8	6.1%
(5) Surplus Paid In	2.8	7.0%	7.7	21.1%	6.8	14.8%
(6) Return on Surplus, dS	-4.0	-10.0%	4.4	12.1%	7.9	17.2%

Note: (6) = (3) + (4) - (5)

	1987 B$	1987 %S	1988 B$	1988 %S	1989 B$	1989 %S
(1) Beginning Surplus (S)	$57.7	100.0%	$63.8	100.0%	$72.5	100.0%
(2) Ending Surplus	63.8		72.5		82.4	
(3) Increase in Surplus	6.1	10.6%	8.7	13.6%	9.9	13.7%
(4) Stockholders Dividends	4.4	7.6%	4.9	7.7%	5.5	7.5%
(5) Surplus Paid In	4.0	6.9%	1.7	2.6%	2.4	3.3%
(6) Return on Surplus, dS	6.5	11.3%	11.9	18.7%	13.0	17.9%

Note: (6) = (3) + (4) - (5)

Mutual Insurers	1984 B$	1984 %S	1985 B$	1985 %S	1986 B$	1986 %S
(1) Beginning Surplus (S)	$21.6	100.0%	$23.3	100.0%	$26.0	100.0%
(2) Ending Surplus	23.3		26.0		31.0	
(3) Increase in Surplus	1.7	7.9%	2.7	11.6%	5.0	19.2%
(4) Stockholders Dividends	0.0		0.0		0.0	
(5) Surplus Paid In	0.0		0.0		0.0	
(6) Return on Surplus, dS	1.7	7.9%	2.7	11.6%	5.0	19.2%

Note: (6) = (3) + (4) - (5)

	1987 B$	1987 %S	1988 B$	1988 %S	1989 B$	1989 %S
(1) Beginning Surplus (S)	$31.0	100.0%	$35.2	100.0%	$38.5	100.0%
(2) Ending Surplus	35.2		38.5		43.4	
(3) Increase in Surplus	4.2	13.5%	3.3	9.4%	4.9	12.7%
(4) Stockholders Dividends	0.0		0.0		0.0	
(5) Surplus Paid In	0.0		0.0		0.0	
(6) Return on Surplus, dS	4.2	13.5%	3.3	9.4%	4.9	12.7%

Note: (6) = (3) + (4) - (5)

Source: A. M. Best & Co., Aggregates and Averages, respective years.

Table 3
Inflation Adjusted Times Series and Ratios

	Value of $ vs 1967	Policyholders' Surplus Actual	Policyholders' Surplus in 1967 $	Net Premiums Written Actual	Net Premiums Written in 1967 $	Loss & Expense Reserves Actual	Loss & Expense Reserves in 1967 $	Ratio Premiums to Surplus	Ratio Reserves to Premiums	Ratio Reserves to Surplus
1975	.62	19,712	12,228	49,605	30,772	39,513	24,512	2.51	.80	2.00
1976	.59	24,631	14,446	60,439	35,448	47,105	27,628	2.45	.78	1.91
1977	.55	29,300	16,143	72,406	39,893	56,970	31,388	2.47	.79	1.94
1978	.51	35,379	18,106	81,699	41,811	68,767	35,193	2.31	.84	1.94
1979	.46	42,395	19,501	90,169	41,476	81,113	37,310	2.13	.90	1.91
1980	.41	52,174	21,140	95,702	38,777	92,493	37,477	1.83	.97	1.77
1981	.37	53,805	19,752	99,373	36,480	102,422	37,600	1.85	1.03	1.90
1982	.35	60,395	20,891	104,038	35,987	111,959	38,727	1.72	1.08	1.85
1983	.34	65,606	21,986	109,247	36,611	122,715	41,124	1.67	1.12	1.87
1984	.32	63,809	20,511	118,591	38,120	134,926	43,371	1.86	1.14	2.11
1985	.31	75,511	23,436	144,860	44,960	154,425	47,928	1.92	1.07	2.05
1986	.30	94,288	28,720	176,993	53,912	184,577	56,222	1.88	1.04	1.96
1987	.29	103,996	30,551	193,689	56,900	217,646	63,938	1.86	1.12	2.09
1988	.28	118,195	33,370	202,285	57,110	241,692	68,236	1.71	1.19	2.04
1989	.27	133,972	36,092	208,834	56,259	269,294	72,547	1.56	1.29	2.01
Annual Change	6.1%	14.7%	8.0%	10.8%	4.4%	14.7%	8.0%			

Source: 1990 Best's Aggregrates and Averages, page 94, consolidated figures.

Table 4
Comparison of Dividend and Leverage Ratios
(in billion dollars, unless a ratio)

	Stock Insurers 1988	Stock Insurers 1989	State Farm Mutual 1988	State Farm Mutual 1989
(1) Ending Surplus	$72.5b	$82.4b	$15.9b	$18.4b
(2) Loss & Expense Reserve	$166.6b	$183.3b	$12.1b	$14.4b
Ratio to Surplus	2.30	2.22	.76	.78
(3) Policyholders Dividends	$1.1b	$1.3b	$ .2b	$0.0b
Ratio to Surplus	.015	.016	.013	.0
(4) Earned Premiums	$124.2b	$126.4b	$19.6b	$21.5b
Ratio Reserve to EP (2)/(4)	1.34	1.45	.62	.67

	Other Mutual Insurers 1988	Other Mutual Insurers 1989	Reciprocal & Lloyds 1988	Reciprocal & Lloyds 1989
(1) Ending Surplus	$22.6b	$25.0b	$7.2b	$8.1b
(2) Loss & Expense Reserve	$51.7b	$56.2b	$11.3b	$15.4b
Ratio to Surplus	2.29	2.25	1.57	1.90
(3) Policyholders Dividends	$1.1b	$1.0b	$ .4b	$ .4b
Ratio to Surplus	.049	.040	.056	.049
(4) Earned Premiums	$42.7b	$44.3b	$13.5b	$14.5b
Ratio Reserve to EP (2)/(4)	1.21	1.27	.84	1.06

Source: Best's Aggregates and Averages, respective years, consolidated figures

Table 5
Percent Comparison of Lines Written - 1989

Lines	Stock Insurers	Mutual Insurers
Workers' Compensation	14.87%	12.61%
Commercial multi-peril	10.24%	4.55%
Other Liability	11.17%	4.28%
Auto Liability	22.24%	35.35%
Auto Physical Damage	13.55%	22.95%
Other Lines	27.93%	20.26%
Total	100.00%	100.00%

Source: 1990 Best's Aggregates and Averages, pages 125-127.

In Table 4, line (2) shows the ratio of loss and expense reserves to surplus for mutual insurers (reciprocals are like mutuals) and for stock insurers. Generally, mutual insurers (including reciprocals) are more conservative in that they put aside more surplus for each dollar of loss and expense reserves than stock insurers do. This is clear looking at State Farm Mutual Automobile Insurance Company and looking at the reciprocals (in California, the insurers affiliated with the northern and southern auto clubs are reciprocals and are major auto insurers in the state.) It is also true in general, but this is not clear from Table 4, line (2) for Other Mutual Insurers, since this group contains a large number of medical malpractice mutuals which have ratios of reserves to surplus on the order of 3 or 4 to 1.

Therefore, mutual insurers not only concentrate on those lines which require smaller loss and expense reserves, but often they put up more surplus for the loss and expense reserves which they do have to provide. This follows as a natural consequence of their inability to raise capital: They must take a risk adverse strategy. The medical malpractice mutuals do have large loss and expense reserves and a high ratio of reserves to surplus, but these mutuals were created to satisfy an unwanted market which the stock insurers largely found too uncertain. As another conservative approach, mutual insurers pay higher policyholder dividends than stock insurers do. This is conservative because policyholder dividends act as a cushion against adversely high losses, since they aren't paid if the losses are high. In California, medical malpractice mutuals rely heavily on dividends.

Any insurance enterprise must make enough money and increase surplus enough this year to support the insurance enterprise the following year. Since certain risk to surplus relationships must be maintained and since any increased risk must be supported by additional surplus, the profit provision (or new capital) must provide for:

(1) expense and claims inflation
(2) increase in the aggregate reserves
(3) increase in the demand for insurance
(4) dividends to stockholders

In general economic terms, surplus must increase each year in order to support the business next year in terms of projected inflation and new business. For a stock insurer, the profit provision must provide a sufficient return to pay stockholder dividends and a return on capital sufficient to attract additional capital to fund the increase in liabilities, inflation, and the increase in demand for insurance.

This brings us back to the rates which mutual insurers must charge. Table 6 shows the approximate rate of return components which mutual and stock insurers needed in 1989. Table 6 also shows where the need for the rate of return (dS/S) arises. Back in Table 2, for 1989, it is shown that stock insurers earned 17.9% rate of return on surplus, and mutual insurers earned 12.7%. Table 6 is a breakdown of these rates of return, using information obtained from the A. M. Best time series in Table 3. The inclusion of State Farm Mutual Automobile Insurance Company does not distort Table 6 nor affect the conclusions.

Table 6
Rate of Return Components - 1989
(as a percent of surplus (S))

	Stock Insurers	Mutual Insurers
Required surplus change:		
(1) expense and claims inflation	5.7%	4.7%
(2) increase in demand for insurance	4.4%	4.4%
(3) increase in reserves	3.6%	3.6%
Total	13.7%	12.7%
Actual surplus change:		
(4) retained return on capital	10.4%	12.7%
(5) surplus paid in	3.3%	0.0%
Total	13.7%	12.7%
Rate of return (dS/S):		
(6) dividends to stockholders	7.5%	0.0%
(7) retained return on capital	10.4%	12.7%
Total (dS/S)	17.9%	12.7%

Source: based on data from Tables 2 and 3

Note: If the shares of a stock insurer are selling for twice "book value" or surplus, then the dividend yield on the stock would be 7.5%/2 = 3.75% and the total return per share at market value would be 17.9%/2 = 8.95% (or a price/earnings ratio of 11.2). This is the way to compare insurance companies and non-insurance companies. In other words, you need to know the ratio of market value to book value.

The details of Table 6 are explained as follows:

(1) The general inflation rate in the United States in 1989 was about 4.0%. However, the inflation rate for medical expenses was higher. Furthermore, in insurance claims, particularly workers' compensation and auto liability, there has been an increasing claims frequency as well as severity inflation. Therefore, 5.7% for stock insurers is a reasonable estimate of the additional surplus required in 1990 to support the same volume of risks that were insured in 1989. A lower value of 4.7% is reasonable for mutual insurers, which sell homeowners and auto physical damage.

(2) The demand for insurance coverages increases each year, as the population increases and as the desire to protect property and business increases. The surplus of the industry must expand to support this additional demand for insurance. An estimate of the long term growth in this demand is given by the average annual increase in net premiums written (deflated), which is shown to be 4.4% in Table 3.

(3) Table 3 also shows that the loss and expense reserves have been growing faster than net written premiums, due mainly to increased litigation, increased delay in resolving disputes, and increased demand for the liability coverages. The average annual increase in the deflated reserves was 8.0%, less 4.4% for the increasing demand for insurance leaves 3.6% for the annual increase in reserves. This increase each year must be supported by a proportional increase in surplus.

(4) For stock insurers, Table 2 shows that, for 1989, surplus paid in was $2.4 billion or 3.3% of beginning surplus. The actual surplus change was $9.9 billion, or 13.7% of beginning surplus, which implies that the retained return on capital must have been 10.4%.

(5) The rate of return for mutual insurers of 12.7% was exactly the right amount to cover inflation and the increase in surplus necessary to support the increase in demand for insurance and the increase in reserves.

(6) For stock insurers, the story is different. They can only obtain the surplus required to support inflation, the additional demand, and the increase in reserves by attracting new capital. They needed 13.7% (5.7% + 4.4% + 3.6%) and did this with 3.3% for capital paid in and 10.4% from retained return on capital to give the required 13.7%. To attract and retain this capital, the stock insurers had to pay 7.5% back in stockholder dividends.

The point of table 6 is to show that even though the profit provisions for stock insurers and mutuals are quite different, the profit provisions, and therefore the fair and reasonable rate of return, can be determined by examining the financial economics of the business of insurance. It also shows that a fair and reasonable rate of return may vary by type of insurer, depending on stock or mutual, and even by the lines of business which the insurer writes.

If stock insurers require a higher rate of return, how can they compete against mutual insurers? The answer lies in market segmentation as seen in Table 5, where it is shown that mutual insurers focus on the lower risk personal lines and the unwanted market, while stock insurers focus on the higher risk commercial lines.

A reviewer of this paper asked two important questions about Table 6 which should be answered here:

(1) What if the demand for insurance suddenly increased to a 20% annual rate, shouldn't the rate of return to stockholders remain the same? The answer is yes. In Table 6, if the 4.4% increase in demand became 20%, then the required surplus change would be 29.3%. This could be met by increasing the surplus paid in from 3.3% to 18.9% by selling shares of stock. The rate of return of 17.9% would not need to change (which is now on a much larger surplus base). Note: mutual insurers probably could not grow 20%, because <u>their</u> rate of return would have to increase to 28.3% to fund the growth, and this could only be accomplished by premium rate increases.

(2) If the investor is only receiving 7.5% (the amount of the dividends to stockholders), why is the investor investing in the risks of the insurance business? The investor is actually

receiving 7.5% in cash dividends and 10.4% in growth in value of the stock, for a total of 17.9%. If the insurance needs stop growing, then the retained return on capital would drop and the dividends to stockholders would rise. The dividends to stockholders, the retained return on capital, and the surplus paid in are all continually adjusting to maintain the competitive equilibrium rate of return.

Perhaps the most common method advanced by economists at the Proposition 103 hearings for determining the proper rate of return was a method based on a discounted cost flow (DCF) model. The numerical results of these models give a rate of return in the 16-18% range for publicly traded stock insurers, in agreement with Table 6. Since the models are formulated in terms of an annual change in the investment of investors, the resulting rate of return is actually equivalent to **dS/S**. Furthermore, most models include an estimate of the growth in earnings per share, which is equivalent to recognizing that some return on capital is being retained for the increase in demand for insurance. However, these models do not include all of the dynamics of the insurance industry, nor do they explain the rate of return requirements for mutual insurers. Also, these models offer no procedure for setting rates or rollbacks by line by state for a particular insurer, other than by assuming constant leverage or risk.

It has now been shown that the proper measure of the required rate of return is **dS/S**, which will vary between stock and mutual insurers and vary depending on inflation and the dynamics of the insurance business.

5. Competition and Loss Ratios

At the national level, there is little doubt that the property/casualty insurance industry is highly competitive and getting more so as insurance and reinsurance become more international. The issue of competition has been a subject of study since the NAIC All Industry Model Laws were proposed in 1946 and adopted in some form by all states by 1951. California was one of the few states which chose an open competition rating law, relying entirely on competition. This open competition rating law remained in effect until the passage of Proposition 103.

In 1974, the NAIC produced a major study of the issue of competition and published a 767 page supplement to the 1974 NAIC Proceedings. The NAIC used these tests of competition: (1) structural indices such as concentration ratios and product differentiation, (2) performance indices such as price differentiation and solvency, and (3) conduct, meaning the degree of independent behavior. The NAIC did not find a failure of competition and did find that the type of rating law utilized by a state did not seem to have a great impact upon the structure of the market in that state. In other words, when a state adopts an open rating law, there does not seem to be a movement towards a non-competitive structure.

In 1989, Dr. Robert Klein of the NAIC staff wrote a report to the NAIC Personal Lines (c) Committee entitled, "Competition in Private Passenger Automobile Insurance". He concluded that from readily available evidence on traditional structural and performance measures of competition, the market for private passenger auto insurance is competitive, at least at the national level.

Therefore, without questioning the issue further, let us assume that, at the national level, the property/casualty insurance industry is competitive. It turns out that if we can accept this one conclusion, which certainly seems to be true, then a number of useful conclusions follow:

Conclusion #1: If the property/casualty insurance industry is competitive at the national level, then it expects to earn a fair and reasonable rate of return at the national level.

This conclusion derives from the necessary condition for competition that there be ease of entry and exit. Therefore, the return on capital for the national insurance industry must be neither excessive nor inadequate.

Conclusion #2: If the by line loss ratios for a particular state match the by line loss ratios for the nation, then the insurance market in that state is competitive.

There is no way to prove this statement other than to demonstrate that it is true. National insurers move capital and marketing efforts among the states to maximize profit, with the result that profit opportunities between the states are about equal and equal to the profitability of the national account figures. There are certain obvious exceptions to this conclusion, namely automobile insurance in certain states. However, if this conclusion can be established, then the national account figures can be used to establish the test for fair and reasonable rate of return in a particular state.

This conclusion was the central assumption (if not conclusion) in the 1969 New York Insurance Department Report on measuring insurer profitability. The report concluded that since both California and New York have loss ratios near country-wide median values, that the rates in these states are neither excessive nor inadequate.

Table 7 shows a comparison of California versus national loss ratios by line. These loss ratios are "calendar year" loss ratios, meaning that they include adjustments for policies written in past years, so there is some volatility in the ratios. Also, some of the differences are explainable by catastrophes or changes in the law in the California. In any event, Table 7 shows a general similarity in the loss ratios by line.

Why loss ratios?

The premium rates vary significantly by state and even within a state, but the loss ratios tend to be the same by line of insurance. It is not surprising to actuaries that the loss ratios would be the same, because actuaries determine the premium rates by making a percentage loading to the losses. It turns out that despite significant differences in corporate form between stock insurers and mutual insurers, the loss ratios between these two types of insurers tend to be the same. See Table 8.

If the loss ratios are low, there is a lack of competition and the premium rates are too high. If the loss ratios are high, the insurance industry is losing money, probably because the state insurance department is refusing to grant rate increases or has a rate freeze on that particular line (most likely automobile).

Table 9 shows the loss ratios by state for automobile liability and physical damage insurance written by State Farm Mutual Automobile Insurance Company, the country's largest insurer. In the major states, the loss ratios consistently centered around 75%. In three states, Michigan, New York, and Texas, the situations were special. Both Tables 7 and 10 show that the loss ratios vary significantly by line of insurance.

Where competition exists or the rate approval process works efficiently, we can conclude that:

(1) the loss ratios do not vary by type of insurer (Table 8), nor by state (Table 9), but

(2) the loss ratios do vary by line of insurance (Tables 7 and 10) and do vary over time (Table 7).

Conclusion #3: If the insurance market in the state is competitive, then the rate for a particular insurer for a specific line set such that the permissible loss ratio is equal to the national loss ratio is a rate which is neither excessive nor inadequate. Equivalently, the rate will enable the insurer the opportunity to make a fair and reasonable rate of return.

Naturally, the actual rate of return will depend on the actual losses, the actual expenses, and the investment income earned. In actual practice, the experience of insurers will vary widely, but the average return will be a fair and reasonable rate of return.

During the Proposition 103 hearings, no one actively advocated using loss ratios as a standard for approving rates, or even as a measure of fair and reasonable rate of return. However, the California Insurance Department has been using this loss ratio approach to approve workers' compensation rates for 75 years. The standard has been a 65% loss ratio for years, which would approximate the 78.1% national loss ratio after policyholder dividends. In fact, no specific estimate of the return on surplus is shown in any workers' compensation filing, only a general discussion of average expense provisions, investment income, and policyholders dividends.

Many insurance departments have been prior approving rates for years and most use a loss ratio approach or, equivalently, assume a proper expense provision and use a combined ratio standard.

Table 7
Comparison of National and California Loss Ratios

	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
Homeowners Multiple Peril													
National	53.8	53.3	60.4	66.4	63.0	64.5	63.7	66.4	70.7	61.9	56.0	59.1	70.9
California	45.5	52.1	79.8	64.7	58.9	62.1	75.2	67.6	75.1	66.4	53.4	57.6	54.4
Commercial Multiple Peril													
National	45.9	43.5	50.6	53.0	58.5	64.8	69.6	81.2	72.2	51.3	44.8	45.5	53.0
California	40.4	43.5	45.5	52.0	61.0	67.7	84.2	88.2	78.7	56.6	50.2	50.0	51.1
Private Auto Liability													
National	64.3	63.4	65.7	67.1	72.7	73.5	74.5	77.2	82.7	82.3	80.8	80.0	80.7
California	56.9	59.7	62.3	65.0	70.1	70.7	75.8	83.5	84.6	85.4	86.8	81.9	74.8
Private Auto Physical Damage													
National	61.3	64.4	68.6	64.8	66.1	68.5	63.9	68.2	67.5	62.7	59.3	61.2	64.4
California	60.0	68.6	69.8	65.5	64.9	67.7	68.6	67.9	61.9	59.4	59.8	62.4	59.5
Commercial Auto Liability													
National	62.6	62.9	66.4	68.8	74.6	81.0	86.6	96.7	87.1	75.1	69.5	69.3	70.4
California	53.0	58.3	64.3	67.1	72.6	83.8	96.3	128.3	93.8	74.5	69.9	69.6	80.7
Commercial Auto Physical Damage													
National	54.9	56.0	59.8	60.3	61.9	66.2	65.2	71.8	61.5	49.4	44.7	46.1	50.2
California	47.4	53.2	59.3	62.3	62.9	62.2	70.9	80.0	58.1	42.7	41.2	44.0	49.1
TOTAL ALL LINES													
National	61.6	61.1	63.9	65.4	66.8	69.4	70.7	77.1	77.0	70.2	66.6	66.4	69.2
California	52.7	58.1	58.5	59.7	61.8	69.9	74.3	78.5	78.8	70.8	69.6	66.1	66.7

Source: California figures - Aggregates of Annual Statements, page 14, respective years
National figures - Best's Aggregates and Averages, respective years

Table 8
Showing the Similarity Between Stock and Mutual Insurers
Loss and Adjustment Expense Ratios (as a %)

Year	Homeowners Stock	Homeowners Mutual	Auto Liability Stock	Auto Liability Mutual
1979	68.2	65.4	76.1	79.1
1980	74.0	75.2	78.1	80.4
1981	70.8	70.6	84.9	86.8
1982	72.4	75.5	87.5	86.5
1983	72.1	69.5	89.3	87.1
1984	74.9	75.0	94.4	92.7
1985	80.0	76.3	95.9	98.2
1986	71.4	69.6	91.9	94.0
1987	64.7	65.5	88.7	92.5
1988	68.7	67.0	88.3	93.6
Average	71.5	70.8	88.3	90.4

Source: Best's Aggregates and Averages (figures reported include loss adjustment expenses)

Table 9
Showing the Consistency in Loss Ratios by State
for Automobile Insurance Written by State Farm Mutual Automobile Insurance Company

State	Loss Ratio		State	Loss Ratio
Major States				
Alabama	74.6%		Indiana	78.4%
Arizona	77.6		Iowa	77.0
California	74.3		Kansas	74.1
Colorado	72.9		Missouri	70.2
Florida	73.7		North Carolina	77.3
Georgia	74.8		Ohio	74.2
Illinois	74.8		Pennsylvania	75.7
Exceptions				
Michigan	92.0	no fault state		
New York	86.2	no fault state		
Texas	85.0	state sets rates		

Comments

The lower volume states have volatile loss ratios.
These loss ratios include both liability and physical damage coverages and no expenses.

Source: 1989 Annual Statement, Schedule T.

Table 10
Showing the Significant Differences between Loss Ratios by Line

Line	Loss Ratio
(1) Boiler and Machinery	40.9%
(2) Inland Marine	53.3
(3) Fire	54.8
(4) Commercial Auto Physical Damage	55.5
(5) Commercial Multi Peril	56.3
(6) Homeowners Multi Peril	64.1
(7) Private Auto Physical Damage	64.1
(8) Other Liability	67.6
(9) Commercial Auto Liability	75.8
(10) Workers' Compensation	78.1
(11) Private Auto Liability	78.2
(12) Medical malpractice	83.5

Source: Best's Aggregates & Averages, 1990, pp. 108-109, ten year average for the industry.

6. Actual Prior-Approval Procedure

With the passage in the 1940's of state laws regulating the business of insurance pursuant to the federal McCarran-Ferguson Act, most states adopted a prior approval rate provision. Thus, state departments of insurance have been in the business of prior approving rates for many years.

In general, the prior approval process has been working as follows: There is a small staff of 6 to 10; if possible, supervised by an actuary. There is no standard format for the filings, since the coverages and lines of insurance vary so much and can vary significantly from insurer to insurer. The insurers are required to show loss and expense statistics and to explain the loss development, inflation, and frequency trend factors. In the last few years, there has been increasing attention given to investment income. The insurance department staff look for completeness and reasonableness in the filing. Generally, if the increase requested is in line with known loss and inflation trends in the state, the requested increase is routinely approved. More attention is given to the personal lines, especially private passenger automobile.

As a practical matter, it is almost impossible to prior approve commercial rates effectively. The commercial premium for a risk is determined by the choice of debits and credits, as well as a choice of a rating base (such as number of customers or gross receipts). Since there is so much room for manipulation and since no two commercial risks are really the same, there is no assurance that the rates will be applied in the manner that they were approved.

Another aspect of prior approval ratemaking that is not commonly mentioned is underwriting, or the criteria used to decide whether or not to insure a risk at all. When rates are approved, the assumption is that the underwriting criteria will remain unchanged. However, approval of the rates does not include approval of the underwriting criteria. Therefore, for a given set of approved rates, the insurer can significantly affect its profitability by loosening or tightening its underwriting criteria.

Hence, state insurance departments usually use a pragmatic, judgment-based approach to prior approving rates. Also, since the loss development, inflation, and frequency trend assumptions are so influential in the ratemaking calculations, very little attention is usually given to the issue of rate of return.

The approach can change dramatically when there is a contested public hearing. In that event, the pragmatic approach gives way to the theoretical approach demanded by the consumer groups and the lawyers. Now, what is a fair and reasonable rate of return becomes the all consuming issue. Financial economists and actuaries need to master the issue of fair and reasonable rate of return if only to restore a proper perspective as to what is really involved in ratemaking. This is not to down play the importance of the issue of fair and reasonable rate of return in a broader context. The issue of fair and reasonable rate of return involves a necessary financial and economic analysis of the industry which is basic in the work of legislators, regulators, investment analysts, and insurance management in their efforts to monitor and manage the industry.

Solvency, not the prior approval of rates, is the primary responsibility of state insurance departments. Regulating solvency involves monitoring both surplus and profitability. So, the issue of rate of return is important to regulating solvency as well as to prior approving rates. Ideally, a financial analysis of the insurers should always precede an approval of the rates, but this seems to happen rarely.

Now, how can the results in the last sections be applied in an actual ratemaking situation? Suppose a multi-state, multi-line insurer makes a rate filing in California for private passenger automobile bodily injury liability coverage. The filing includes:

(1) loss statistics for California, including loss development, inflation, and frequency trend factors.

(2) the latest annual statement, showing expenses, investments, and surplus for the insurer as a whole on a national account basis.

In reviewing the filing, the insurance department is subject to two formidable constraints:

(1) the following items are only available on a national account basis:

(a) surplus
(b) invested assets
(c) investment income
(d) realized capital gains
(e) unrealized capital gains
(f) general expenses
(g) federal income taxes
(h) other income and surplus adjustments

(2) the surplus cannot be subdivided by line by state and be meaningful, since the surplus supports a complex array of asset, liability, premium and coverage risks.

The discussions in the preceding sections argued that the way to get around this problem was to use loss ratios, provided that the conditions of competition exist. Table 7 shows a comparison of calendar year loss ratios for California and nationally. In most cases, the two loss ratios are remarkably close, considering that calendar year loss ratios include adjustments from prior years. Many of the cases in which differences occur can be readily explained. For instance, in 1989, for homeowners insurance, the national loss ratio jumped to 70.9 from 59.1 because of natural catastrophes, mainly hurricane Hugo, which was far more costly than the Loma Prieta earthquake in California. For private passenger auto liability, the low California loss ratio of 74.8% for 1989 could possibly be explained by rate increases taken before passage of Proposition 103. Therefore, Conclusions 2 and 3 in Section 5 hold.

For private passenger automobile liability, the national loss ratios were:

	1987	1988	1989	Average
Auto liability	80.8%	80.0%	80.7%	80.5%

Therefore, the filing should be approved for a permissible loss ratio of 80.5%. As an example, if the insurer files in 1990 a projected loss ratio at current rates of 91.0% for business to be written in 1991, then a 13% rate increase should be approved (.910/.805 = 13% increase). By the arguments presented in Section 5, the average insurer with this loss ratio and average expense and investment income will earn a fair and reasonable rate of return. Note that it is not considered what this particular insurer's actual expenses, taxes, and investment income are or will be.

The same approach could be used for the rollback. If in 1989, an insurer had a loss ratio of 75% for private passenger auto liability compared to a national loss ratio of 80.7%, then the insurer should have to refund 7% of its 1989 premium in order to bring the loss ratio up to 80.7%.

This example ignores the effects of the insurance cycle, which will be discussed in a following section. It also ignores the possible argument that auto liability is a loss leader for auto physical damage (note the much lower loss ratio for auto physical damage).

7. Advantage of Using a Loss Ratio Approach

The advantage of using a loss ratio approach is that it overcomes the disadvantages of using an approach based on a by line by state apportionment of surplus, expenses, and investment income, as used in the Proposition 103 hearings. Specifically, the approach used by the California Insurance Department lawyers in the Proposition 103 hearings had these unfavorable characteristics:

(1) Heavy reliance was placed on by line premium to surplus ratios (called "leveraged norms"), which were outdated, artificial, and based only on intuitive judgment and not on a recognized risk analysis.

(2) By using an insurer's actual expenses, inefficient (high cost) insurers are favored over efficient (low cost) insurers.

(3) Overcapitalized insurers will get higher rates approved than undercapitalized insurers. This is so because the permissible rate of return was applied to the actual surplus, and the permissible rate of return is greater than the investment rate of return.

(4) An attempt was made to solve the overcapitalization problem by excluding "surplus-surplus". This would be a hopelessly complex project to do correctly. This effort gets even more complex when it is realized that if surplus-surplus is excluded, then the investment income earned and taxes paid on surplus-surplus must also be excluded.

(5) Most proposals excluded unrealized capital gains in measuring "total income" in the calculation of rate of return. This encourages the postponing of realized capital gains in order to make the insurer appear less profitable than it really is. Of course, excluding unrealized capital gains understates "total income" in the first place.

(6) The approach requires an artificial allocation of national accounts (such as overhead expenses, investment income, and federal taxes) based on premiums, reserves, or invested assets. This allocation creates the appearance that actual California experience is being used when in fact it is only a pro rata apportionment of the national experience.

(7) The 11.2% permissible rate of return was chosen arbitrarily without any serious economic analysis. In fact, the correct economic rate of return required may vary from year to year depending on changes in inflation. Furthermore, the 11.2% was based on a restricted definition of net income which makes it non-comparable with any of the other measures of rate of return.

(8) There are no ordinary premium to surplus guidelines for surety (where the risk is fully collateralized), or boiler and machinery (where the insurance policy is basically an inspection service contract), or earthquake (where the whole of the insurer's surplus is at stake).

(9) There is no easy way to assign investment income or an investment yield to capital.

(10) There is no easy way to allocate federal taxes by line by state, since federal taxes are paid at the holding company level and often include non-insurance business with substantial depreciation charges.

(11) There is the issue of whether to base the rate of return on GAAP net worth, SAP surplus market value of stock, or economic value (discounted).

Using the loss ratio approach avoids all of these issues.

8. Approving Rates in a Cyclical Business

Whether called underwriting, business, or economic cycles, cycles are a fact of economic life. Cycles are characterized by high and low periods of profitability for an industry or an economy. They have a whole range of causes; namely, changes in interest rates, changes in inflation, changes in demand, social changes, political changes, even catastrophes and weather. Just changes in collective optimism and pessimism can cause business cycles. No two cycles are usually the same.

The business of insurance is greatly affected by cycles in the national economy, particularly with respect to inflation and interest rates. While consideration of economic cycles greatly increases the complexity of the rate approval process, economic cycles must be recognized. In California workers' compensation rate approval hearings, the economic cycle is recognized by requiring the filing to show an econometric projection for the following year of the:

(1) workers' wage inflation in California, since the premium income is a function of wage levels.
(2) hospital and medical inflation in California.
(3) investment yield.

In general, these factors are affected by the economic cycle:

(1) expense and claims inflation.
(2) demand for insurance.
(3) leverage of reserves.
(4) investment yield (including interest rates and required return to stockholders).

All of these factors affect the rate of return analysis in Table 6, and, therefore, the required rate of return (dS/S) is not a fixed number, but a number which varies with the economic cycle. In order to project the required rate of return for a succeeding year, these factors must be individually projected. For mutual insurers, only the first three factors and interest rates need to be projected to get the required rate of return.

As can be seen in Table 7, the loss ratios by line vary in a wave pattern with the economic cycle. Also, Table 1 shows that the rate of return (dS/S) of the insurance industry can be volatile. In fact, the rate of return was negative in 1984, the bottom of the underwriting cycle in the 1980's. On the other hand, the rate of return for years 1978, 1979, and 1980 could be considered excessive.

Even though the national insurance industry is competitive and is therefore earning a fair and reasonable rate of return, it cannot completely deflect the vicissitudes of a national economic cycle, and, therefore, will not earn a fair and reasonable rate of return each year.

As a regulator, a decision has to be made whether, in the prior approval process, to ride with the cycle or to try to counteract it. Counteracting a cycle will mean mandating rate increases and denying rate decreases, not easy things to do. However, by monitoring the rate of return (dS/S), the regulator has, through the prior approval process, a lot of power to dampen the sudden changes in insurance rates which often occur at some point in an economic cycle.

When the national loss ratios reached an unreasonable high level (as they did in 1984-85), the regulator must adjust the loss ratios downward to achieve the proper dS/S.

Changes in claims cost inflation are taken care of automatically in projecting the losses by established actuarial methods. However, the impact of changes in inflation on premium (such as is the case in workers' compensation) and the impact of changes in interest rates on investment income have not necessarily been worked out by actuaries and must be examined by the regulator.

In the last decade, there has been a wealth of new research started on the subject of modelling cyclical behavior and emerging cost analysis. See the First and Second International Conferences on Insurer Solvency and the work of the British Solvency Working Party (1990).

In particular, the work of Derrig and Woll is very important, because their work is based on a discounted cash flow analysis. Richard Woll assumes a leverage ratio and calculates a target loss ratio based on a discounted cash flow analysis. This approach could be used to get the implied leverage ratio, and therefore the particular risk based rate of return, given the target (permissible) loss ratio. Furthermore, by projecting changes in interest rates during the business cycle, the change in permissible loss ratio could be determined. Both Woll and Derrig generalize their work to include risk based discount rates.

9. General Comments on Insurance Rate Regulation

The differences between insurance industry regulation and public utility regulation should be made clear:

(a) Public utility regulation

- high fixed costs, low marginal costs
- cost minimization
- homogeneous product
- ineffective competition
- barriers to entry

(b) Insurance industry regulation

- low fixed costs, high marginal costs
- profit maximization
- heterogeneous products
- effective competition
- low barriers to entry

There is really nothing about the theory of utility rate regulation which is transferable to insurance rate regulation. Fortunately, insurance regulators can rely heavily on the benefits of national and international competition and don't have to worry about such issues as cost of capital replacement and depreciation.

Prior approval will not in general produce lower insurance rates. Prior approval will not increase the availability of insurance coverages; if anything, the requirements of the prior approval process will reduce availability slightly. However, prior approval can stabilize rates, particularly in the liability coverages, where rates in California have shown wide swings with the insurance economic cycle.

Some of the weaknesses of the prior approval process:

(1) If done properly, the rate approval process requires a full actuarial analysis of the loss and expense reserves.
(2) It is very difficult to regulate commercial rates.
(3) The approval process is slow to react to rapid changes, such as rapid increases in auto bodily injury frequency in Los Angeles.

General regulatory issues:

(1) The regulator should be mainly interested in the percent of the premium which is returned to the policyholder, i.e., the loss ratio.
(2) Policyholder dividends should be encouraged, especially for mutual insurers. This promotes insurer economic stability and enables insureds with low loss histories to be rewarded.
(3) Insurers should have the opportunity to be innovative and flexible in developing new coverages and new markets.

What does it mean then to ask if insurers are earning excessive profits? Insurers are making excessive profits if the profits that they are earning are greater than is necessary to support the business the following year. If, in Table 6, the assumed projected rates are all correct, then any profit level greater than 17.9% or 12.7% would be excessive. In economic theory, excessive profits can only occur if the industry has at least some monopolistic characteristics. As already discussed, there is strong evidence that the insurance industry is highly competitive, at least at the national level.

10. Conclusion

The California Proposition 103 hearings have revealed that the theory of rate of return for the insurance industry has not been satisfactorily worked out by financial economists and actuaries. Allocating national account financials (such as surplus, assets, liabilities, investment income, expenses, taxes) in order to get by line by state rates of return has been producing strange and unworkable results, mainly because the procedure is essentially arbitrary.

Modern risk theory has shown rigorously that the optimum surplus of an insurer cannot be subdivided by line by state, since the risks which the surplus supports cannot be subdivided. Therefore, any procedure based on the allocation of surplus by line by state is academically as well as realistically invalid.

How, then, can the regulator approve rates subject to the legal requirement that the insurer be able to earn a fair and reasonable rate of return, that is, the rates are adequate, but not excessive? The proposed procedure is based on the conclusion that, at the national level, the insurance industry is competitive and therefore is earning a fair and reasonable rate of return. The argument is made that if an insurer's loss ratio by line is set equal to the national loss ratio by line, then the insurer will have the opportunity to earn a fair and reasonable rate of return and the rates so set are adequate and not excessive.

Practically every prior approval state relies on loss ratios (or, equivalently, a combined ratio of losses and expenses). The California workers' compensation rates have been set based on a target loss ratio for 75 years. This paper presents an economic justification for using loss ratios to approve rates and presents an economic analysis of the components that make up the required rate of return which the insurance industry must have in order to remain economically viable.

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SOME UNIFYING REMARKS ON RISK LOAD

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Some Unifying Remarks on Risk Load

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ABSTRACT

The purpose of this note is to point out the connections between the Marginal Surplus and Competitive Market Equilibrium approaches to calculating risk loads and to show that these methods incorporate and unify several other conceptual approaches to risk loading.

Some Unifying Remarks on Risk Load

The casualty actuarial literature has of late provided a forum for very active debate and discussion on the subject of risk load. In this note, we shall study two promising approaches to the problem and show that they are intimately connected and mutually illuminating. The two are the Marginal Surplus approach, as expounded by Rodney Kreps (1), and the Competitive Market Equilibrium approach of Glenn Meyers (2).

Marginal Surplus

This concept has been mentioned in several sources but treated concretely by Kreps in the context of reinsurance. The central idea is the following: any piece of business should be priced in such a way that , after deducting expected losses and expenses, there remains a contribution to the company's surplus which leaves the company in the same risk position as before the business was written. Aside from being the source of profit over the long run, this risk load satisfies the company's fiduciary obligation to maintain product quality - primarily, reasonable assurance of the ability to pay claims - for current and former policyholders and their potential beneficiaries.

Kreps quantifies this notion in a way that is fraught with implications. Suppose that we can treat the insurer's net worth as a random variable with finite second moment. Hence we can define a standard deviation, Σ, and express the insurer's actual surplus, S, as a multiple of this quantity:

$$S = Z\,\Sigma. \qquad (1)$$

So far this is only a definition, but we may argue that countervailing pressures of the insurance and the capital markets will tend to confine Z within a fairly narrow equilibrium range.

To see why this is so, consider that Σ typifies the scale of variability on the distribution of aggregate net worth. The multiple, Z, then, should map directly to a value for the insurer's probability of ruin, the probability that present assets are insufficient to satisfy present liabilities so that present liabilities will have to be subsidized from future earnings. Such subsidies are routine in the public sector, but in the private sector they pose serious problems of equity and are often frowned upon. In particular the future earnings may prove insufficient to provide the subsidy to cover past mistakes, and actual insolvency may result. Hence there will be pressure from the insurance market (and from regulatory authorities) to keep the effective Z value reasonably high since the stability and reliability thus achieved is the central determinant of insurance product quality.

On the other hand, increasing Z to very high values will have little effect on the insurer's actual risk position; but it will be penalized by the capital markets since it ties up funds better utilized elsewhere. (Remember that the policyholder also participates in the capitalization of the insurer but receives actual equity only in mutual companies.) This is clear because each successive increment of Z has a smaller effect on the probability of ruin. Suppose that Z maps to a probability of .001 (For the normal distribution, $Z \cong 3$.) This means that, of a thousand companies in a similar position, only one, on average, would prove deficient in runoff. Clearly there will be little reward from the market for cutting the probability further to .0001. There are not even 10,000 insurers in the entire market.

Let us suppose that we have an equilibrium Z value, a market consensus. It then makes sense to price new business in a way that keeps Z constant. That means we must examine the insurer's risk position before and after the transaction. Before the transaction, the standard deviation of the insurer's net worth is

$$\Sigma_0 = \Sigma. \tag{2}$$

After the transaction, it is

$$\Sigma_1 = \sqrt{\Sigma^2 + \sigma^2 + 2\rho\Sigma\sigma}, \tag{3}$$

where σ is the standard deviation of the net present value (NPV) of the accepted risk, and ρ is the coefficient of correlation between the risk's NPV and that of the existing book. With this notation, the required surplus contribution is

$$\Delta S = Z\,(\Sigma_1 - \Sigma_0) = \\ = Z\frac{2\rho\Sigma\sigma + \sigma^2}{\Sigma_1 + \Sigma_0}. \tag{4}$$

In the limiting (and usual) case, where the added risk is only a small fraction of that of the existing book, (4) can be approximated

$$\Delta S \cong Z\rho\sigma, \tag{5}$$

where we have neglected terms of second and higher order in σ. Note that this is independent of the total standard deviation and depends only on the risk's standard deviation, its correlation with the rest of the book, and the product quality factor, Z. In the next section, we shall see this relation take on more concrete form in terms of explicit stochastic models.

Competitive Market Equilibrium Model

This approach was invented to deal with some long-standing problems of increased limit ratemaking where risk load has long been an explicit issue; but it addresses the general problem; and we shall review it in that context.

In addressing the problem of risk load, it is important to recognize that our deliberations are of little use unless we have a plausible way of estimating the relevant variances and covariances. This Meyers treats at considerable length in the context of the Collective Risk Model (3). This model treats aggregate loss payments as a random sum of variates from a known loss size distribution with a multiplicity drawn from a known claim count distribution. The conceptual framework is flexible enough to accommodate parameter uncertainty since the distribution parameters can themselves be drawn from specified prior distributions. This is very important because parameter uncertainty is the prime determinant of the risk load in a consistent and market-viable scheme. Along with the catastrophe hazard, it is a chief source of the correlation discussed in the previous section. Many of the causes of parameter uncertainty act market-wide and are a reflection of the climates, legal, political, seismic, and meteorological, which determine the fortunes of the industry as a whole.

Meyers' discourse on quantifying these risks is a solid demonstration that it is feasible to put actual numbers in the place of all the Greek letters and to reduce the problem to calculation. This capability is an extremely important one and can be expected eventually to have a profound effect on the industry and on the way it manages itself. We will not dwell here on the details but cite instead the form of the final result. This is expressed in matrix notation wherein the aggregate variance of an insurer's net worth can be written

$$\Sigma^2 = A + \mathbf{n}^T \cdot U + \mathbf{n}^T \cdot V \cdot \mathbf{n}, \qquad (6)$$

where

n is a vector of exposures in force by class,
U is a vector of positive elements giving process variance per unit of exposure,
V is a symmetric, positive definite matrix describing parameter and catastrophe risk,
A is a scalar - our addition - which quantifies all other sources of variation independent of exposures currently in force,
T as superscript denotes the transpose, which interchanges rows and columns.

Reference (2) allows for the case where V is singular, but we will not consider that here.

The next step in the derivation is to pose and solve the optimization problem to determine what combination of exposure in various lines and classes and at various limits a carrier in a given risk position will choose to write. There are two distinct, but equivalent, approaches to this problem. One is to maximize profit at a fixed level of risk, quantified as the variance given in (6). The other is to minimize risk at a fixed profit quota. We state the latter explicitly by way of illustration:

Minimize

$$A + \mathbf{n}^T \cdot \mathbf{U} + \mathbf{n}^T \cdot V \cdot \mathbf{n} + \mu \, (P - \mathbf{n}^T \cdot \mathbf{R}) \tag{7}$$

on **n** and μ, where

μ is the Lagrange Multiplier,

P is the target profit,

R is the vector of risk loads per unit of exposure, by class, supposed given.

Either approach leads to a spectrum of solutions on an "efficient frontier" in the space of profit vs. risk. The Lagrange Multiplier can be thought of as quantifying the relative importance of profit maximization and risk minimization, a tradeoff which must be decided by management.

At this stage, supposing that **R** is known, the solution can be expressed as

$$\mathbf{n} = \tfrac{1}{2} V^{-1} \cdot (\mu \mathbf{R} - \mathbf{U}). \tag{8}$$

Note that this is not guaranteed positive and will only be so for appropriate values of **R** and μ. The model, in fact, describes underwriting shutdown in the riskier classes as μ becomes smaller. The correct procedure when this happens is to exclude those classes (n = 0) and to solve the reduced problem for the remaining components of **n**.

To this point, **R** has been assumed known. The final step is to find what risk loads are needed to allow the market to clear. This is determined by equating industry supply with total market demand and solving for **R**. This is equivalent to using in place of **n**, the industry average exposure spectrum, $\bar{\mathbf{n}}$, and introducing an industry average profit requirement, $\bar{P}$. The answer that emerges is

$$\mathbf{R} = \bar{P} \frac{U + 2V \cdot \bar{\mathbf{n}}}{\bar{\mathbf{n}}^T \cdot U + 2\bar{\mathbf{n}}^T \cdot V \cdot \bar{\mathbf{n}}} \tag{9}$$

Note that $\bar{P}$ remains to be determined and may, in fact, depend on $\bar{\mathbf{n}}$.

The model assumes tacitly that insurance pricing is supply-driven - that is to say, that capital committed to insurance enterprises is a scarce good. The existence of a slack market in the trough of every underwriting cycle gives testimony that this is not always the case. Because it is much easier to enter the insurance market than it is to withdraw, the presence of excess underwriting capacity has a profound effect on insurance pricing, very much like any other commodity. In fact, pricing behavior in the property/casualty industry bears a striking similarity to agricultural commodity pricing. In agriculture the excess capacity problem is "solved" by government subsidies and price supports. In the insurance industry, attempts at administered pricing have seldom had enduring success. Rather the *de facto* "solution" for excess capacity is, in effect, to pay insureds for accepting coverage until the excess capital has been dissipated and the market has moved back towards equilibrium.

Such behavior sounds bizarre; but it is, in fact, rational in the context of available information. The farmer deciding what and how much to plant and the developer deciding to initiate a new office development are in similar positions. The only remedy for underwriting cycles and the inefficiencies they cause is better and more timely information. The hope in introducing models such as the present one, with its statistical underpinnings, is to provide such information so that the industry's risk position and capital needs are definite quantities rather than vague notions. Insurance data are notoriously noisy - prone to large fluctuations and distortions, especially in the short term. Few people can be found in the industry who pay serious attention to monthly data, and few more who attend to quarterly results.

The reason for this is simple: to be interpretable, noisy data must be filtered. The design of the filter is all-important. The only noise filter in general use in insurance - and most other industries - is the device of averaging over a sufficiently long time interval. This imposes an unavoidable delay time and exposes the industry to the kinds of cycles that are observed. It is not widely recognized that the decisive advantage of statistical quality control methods is that they provide a real-time noise filter, allowing managers to discern the conditions which most urgently require action without waiting forever for the averages to settle down and without risking ill-advised interventions which will only amplify the noise and magnify the problem. The required information about process dispersion is very hard to come by. In insurance, it can only be obtained by a risk analysis: breaking the process down into components and reconstructing the variability without waiting around for things to happen. What Meyers has done here is a convincing first step in that direction. The ultimate goal is to make market capacity manageable in real time rather than in a feedback loop with a three-year time lag. If this is achieved, potential new entrants will not have to find out the hard way whether or not their contributions to the market were superfluous.

Connections

It is most illuminating to address the marginal surplus problem using the form for aggregate variance deduced in the context of the CME Model. Combining the two notations and using (1) and (6), we find for the needed surplus

$$S = Z \sqrt{A + \mathbf{n}^T \cdot \mathbf{U} + \mathbf{n}^T \cdot V \cdot \mathbf{n}}\,. \tag{10}$$

Then the risk load vector giving the contribution to surplus needed to write an additional unit of exposure in any class is

$$\begin{aligned} \mathbf{R} &= \partial S / \partial \mathbf{n}^T \\ &= Z \frac{\tfrac{1}{2}\mathbf{U} + V \cdot \mathbf{n}}{\sqrt{A + \mathbf{n}^T \cdot \mathbf{U} + \mathbf{n}^T \cdot V \cdot \mathbf{n}}}\,. \end{aligned} \tag{11}$$

If we substitute the industry-average exposure spectrum, then (11) gives the same distribution of risk load by class as (9) as well as a way of re-expressing in terms of the other variables.

Even though it is seldom achieved in practice, the limit of large exposures is still instructive. The only terms that remain in this limit are those involving the systematic components of risk, parameter uncertainty and catastrophe hazard, as quantified in the matrix V:

$$\mathbf{R} \cong \frac{Z\, V \cdot \mathbf{n}}{\sqrt{\mathbf{n}^T \cdot V \cdot \mathbf{n}}} \tag{12}$$

In this limit, **R** depends only on how exposures are distributed among lines, classes, and limits and not on the total amount of exposure. This equation (12) is nothing more nor less than the expression of (5) in different notation and in the limit of large exposure.

To see this in closer detail, suppose that the exposure of the risk being insured is described by the vector **e** so that

$$\sigma^2 = \mathbf{e}^T \cdot \mathbf{U} + \mathbf{e}^T \cdot V \cdot \mathbf{e}; \tag{13}$$

$$\Sigma^2 = A + \mathbf{n}^T \cdot \mathbf{U} + \mathbf{n}^T \cdot V \cdot \mathbf{n}; \tag{14}$$

$$\begin{aligned} \Sigma^2 + \sigma^2 + 2\rho\Sigma\sigma &= A + (\mathbf{n}^T + \mathbf{e}^T) \cdot \mathbf{U} + \\ &\quad (\mathbf{n}^T + \mathbf{e}^T) \cdot V \cdot (\mathbf{n} + \mathbf{e}); \end{aligned} \tag{15}$$

$$\begin{aligned} \Sigma^2 + \sigma^2 + 2\rho\Sigma\sigma &= A + \mathbf{n}^T \cdot \mathbf{U} + \mathbf{n}^T \cdot V \cdot \mathbf{n} \\ &\quad + \mathbf{e}^T \cdot \mathbf{U} + \mathbf{e}^T \cdot V \cdot \mathbf{e} \\ &\quad + 2\mathbf{n}^T \cdot V \cdot \mathbf{e}. \end{aligned} \tag{15a}$$

From this we identify

$$\rho\sigma = \frac{\mathbf{n}^T \cdot \mathbf{V} \cdot \mathbf{e}}{\sqrt{A + \mathbf{n}^T \cdot \mathbf{U} + \mathbf{n}^T \cdot \mathbf{V} \cdot \mathbf{n}}} . \tag{16}$$

In the CME approach, the covariance identified above is essentially that of the individual risk with the overall market. This may be taken as a point of connection with the Capital Asset Pricing Model (CAPM), advocated by Feldblum (4) as a basis for risk loading. CAPM implies that assumption of "diversifiable" risk (process risk) cannot be compensated reliably in the marketplace whereas assumption of systematic risk (parameter uncertainty) can be and is regularly compensated. More accurately, compensation for assuming diversifiable risk is strongly dependent on market conditions. Furthermore, diversifiability is a relative notion, not an absolute one. If no actual opportunities for diversification exist, then diversifiability is merely an academic concept. Changing market conditions can have a radical effect on diversifiability of risk in insurance. For instance, the incipient market in insurance futures may prove to have just such an effect by broadening the opportunities for diversification.

Conclusions

At this point, it should be clear that we have a very striking convergence of different theoretical approaches, many of which have been thought to be mutually independent, or incompatible, or diametrically opposed. It may be helpful to draw up a thematic list of the concepts which have emerged in this inquiry and fitted themselves together like the pieces of a jigsaw puzzle:

- Marginal Cost Pricing
- Probability of Ruin
- Standard Deviation Principle
- Product Quality Assurance
- Variance Principle
- Market Equilibrium
- Constrained Optimization, Lagrange Multipliers, Efficient Frontiers
- Underwriting Cycles
- Capital Asset Pricing Model.

The only major concepts which have not surfaced are utility theory and the option pricing approach (5) although this does not deny their relevance. The point is that a viable theoretical approach to risk loading depends on the convergence of many ideas, the more the better. And anyone who says that the correct approach is one thing and not another is probably off the mark. The relevant question is how the pieces should fit together in a unified, convincing whole.

There is no doubting that the game is worth the candle because there is no lack of instances where a reliable and flexible pricing formula has significantly influenced market behavior - even to the point of creating markets where none existed previously. The most famous example of this is the Black-Scholes option pricing formula, which had enough descriptive power and conceptual plausibility to allow confident pricing of financial instruments which previously were traded sparsely and tentatively, if at all. This success, like success in any such endeavor, depended on gaining an understanding of the dispersion in the underlying process, on gaining control of the variability.

The insurance industry has traditionally been content to price to the mean and to rely on rules of thumb for governing variability - the Kenney Rule and the five percent underwriting margin are the best-known examples. The cost of this reliance is readily apparent because these rules of thumb leave yawning chasms of uncertainty in our knowledge of the industry's financial condition, its true underwriting capacity, and the quality of insurance products being offered to the public. Discounts are offered in slack markets with no reliable yardstick to gauge their financial consequences. This uncertainty is very costly indeed because it allows injection of excess capacity into a market that is already slack, although no one knows it yet. Further, capital committed to an insurance enterprise is not easily withdrawn. Capital injected into a slack market is likely as not to be consumed in subsidizing a superfluous, and practically irreversible market presence. Few entrepreneurs would fall into such a trap if they had adequate information; and, here, adequate information means quantitative estimates of risk, as well as cost, which accrue the uncertainties inhering in future events to the present and provide a basis for informed decision making. We have not merely to consider the variability of individual balance sheet items, but their covariability as well; we must learn to do accounting for variance if we are to bring the insurance process under control.

This is the goal, as we remarked earlier: real-time risk and capacity management. Without such a capability, the industry will remain locked in the predicament of having to relearn the same lessons every six years or so. The goal implies a challenge to the actuarial profession - for who will attain it if we do not? We must become as adept at characterizing, quantifying, and controlling variability as we have traditionally been at estimating expected values. If we do so, we will find ourselves doing both things better and more reliably.

Note also that, in discussing insurance risk, we have implied rather little, and said even less, about asset risk. Clearly, a "net present value" , and its variance, must involve the variability of assets as well as the countervailing liabilities. Only by considering both, can we get to the bottom line - and a useful result. Recent events in the industry have made it impossible to ignore asset risk. We may find ourselves broadening the purview of "actuaries of the third kind" to include the characterization and control of this bottom line variability.

The signs point hopefully to an early realization of these goals. The authors cited herein have done a great deal to hasten the day, and more yet if their works are examined together. It has been a pleasure to review their work and to underline its significance.

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HOMEOWNERS EXCESS WIND LOADS: AUGMENTING THE ISO WIND PROCEDURE

John Bradshaw & Mark Homan

Homeowners Excess Wind Loads:

Augmenting the ISO Wind Procedure

BY JOHN BRADSHAW & MARK J. HOMAN

The ISO excess wind procedure is widely used by many companies. However, it has one major flaw. It depends on the loss history in the state to provide a true representation of the future expected wind experience. The procedure presented here removes this flaw. Modeling is used to augment history to yield more accurate wind expectations. The procedure has the added side benefit of providing a means to reflect different wind loadings by territory.

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Overview

The ISO Excess Wind Procedure is a popular procedure that is in use by many companies. The procedure relies on the past history, currently about thirty years, to be a representative sample of true long term wind experience. This assumption is not valid in many cases. Most experts have stated that the past thirty years of experience in Florida have had much less hurricane activity than any other thirty year period. South Carolina's experience now includes Hurricane Hugo. Hugo is treated as if it will recur once every thirty years by the ISO procedure. However, experts feel that Hugo is more likely a one in one hundred year event, if not less frequent.

The procedure outlined in this paper uses modeling to determine the expected wind experience over a longer period of time. In this case, it is a 50 year time period. The procedure augments the scant history in a state like Florida and makes adjustments to allow removal of events like Hurricane Hugo in South Carolina. It still rests primarily on the ISO procedure.

It should be noted that the ISO procedure has been criticized in other ways and other procedures have been developed. [1] However, most companies lack sufficient data to use these other procedures. We are looking for ways to improve the ISO procedure without requiring historical data which may be unobtainable.

ISO Excess Wind Procedure

We will start by explaining the ISO excess wind procedure briefly. As the name implies, the procedure only makes adjustments for excess wind losses. It makes no adjustment for non-wind catastrophes that occur, such as freezing in the South. The procedure determines which losses should be considered excess and removed from an experience period and calculates a long-term load to replace the excluded losses by spreading them over a longer time period.

Currently, the history period used in the ISO procedure in most states is about 30 years. This corresponds to the introduction of the Homeowners policy. History before that period is difficult to use since the coverages were not the same.

Exhibit I shows the calculation of the excess wind threshold and the long term load for a sample state. The procedure starts by breaking down the losses into wind and non-wind categories. The ratio of wind to non-wind is then calculated. The median wind/non-wind ratio is calculated to determine the excess wind threshold.

The excess wind threshold is the greater of 1.5 times the median or 0.25. By using a threshold that is greater than the median, adjustments are only made for the truly unusual wind years rather than for some fairly common events. The use of 0.25 as a minimum threshold eliminates the need to make adjustments in states where the wind experience is relatively light.

Each wind/non-wind ratio is tested against the threshold to determine whether it is an excess year. If the ratio is greater than the threshold, it is an excess year and the excess portion is calculated. The excess ratio is the portion of the wind/non-wind ratio greater than the median. The excess losses are then calculated by taking the excess ratio multiplied by the non-wind losses. The non-excess losses are then calculated by subtracting the excess losses from the total losses.

The excess wind load is calculated by taking the average excess ratio multiplied by the average non-excess ratio.

Modeling

Modeling is used to project expected losses from a fifty year event. A fifty year event is a storm that is expected to occur once every fifty years. A storm of fifty year intensity is determined by the expected wind speeds. The fifty year event differs from area to area due to storm expectations in the area.

The model used to develop this paper is one that was developed at the Hartford Re Management Company. Other reinsurers and reinsurance brokers have developed similar models. The model will not be discussed in detail but a brief outline is needed.

The model uses projected storm tracks through a state or group of states. The storm track includes average wind speeds as the storm moves along the track and a damage matrix based on these wind speeds and the distance from the track. The model applies this information against the distribution of business in a company's book to determine expected losses from the storm.

The expected losses are output by area and in total. We take several possible storm tracks through a state and then average them. Exhibits II and III are the output from the model for the projected storm tracks through New York and Connecticut.

Adding "History"

The average projected losses that we get from the model represent the losses expected from a storm of fifty year severity. In order to include this as "history" in the ISO procedure, we must act as if we have 50 years of data.

Exhibit IV shows how we make this adjustment. We start with the 29 years of data that we already have. Since none of the events in the 29 year period are more severe than the 50 year projection, we do not eliminate any years. We then insert a year to represent the 50 year event.

The non-wind losses used are a projection from the level of losses in the most recent years of data. The company losses should be used for this projection to match the modelled wind losses even though ISO data may be used for the history. The excess calculation continues as before. However, the averages are now weighted averages using the 29 years of history to represent 49 years and the projection from the model to represent the fiftieth year. The median wind/non-wind ratio is not adjusted since it is assumed that one extreme year should have no impact on the median.

The final wind load is used in the same way as the typical ISO wind load. No further adjustments are necessary.

In a case like South Carolina, one additional step would be needed in the above process. A year that was more severe than the 50 year event should be eliminated. In South Carolina, for example, the year of Hurricane Hugo (1989) would be dropped from the 29 year history. We recommend totally eliminating it and using only the remaining years of history, with the addition of the 50 year event from the model. One could also consider replacing 1989 with a "typical" year. Given the difficulty in determining a typical year, we do not recommend this alternative.

Territorial Loadings

An additional benefit of this modeling is that you get information on the distribution of the storm losses by area within the state. This data can be used to develop territorial wind loadings to be used in ratemaking rather than merely using statewide loadings.

To use the model output, you start by taking averages of the losses by area across the various storm tracks modeled as shown in Exhibit III. The expected wind losses by area from the model are then divided by the non-excess losses in the area. This gives a wind to non-excess ratio for each area. The territorial ratio is divided by the statewide ratio to determine a relativity for each area. These indices by area are multiplied by the statewide wind load to determine a wind load for each area. These adjusted wind loads are then applied to the territories that comprise the area when calculating new territorial relativities for ratemaking.

Exhibit V shows this calculation using 5 year incurred losses and 5 year earned premiums at current rates. The loss ratio relativities before the loading show the results that would occur using a typical statewide loading. The relativities after the loading show the more accurate results.

One variation on this procedure that we recommend is using the current in-force amount of insurance by territory instead of non-wind losses. By dividing the wind losses from the model by the exposures, one obtains a damage potential for each territory. Since the exposures form the base for the model, using exposures will be slightly more accurate. The additional accuracy results from removing the variation due to changes in distribution and the random variation in the actual losses.

Conclusion

The ISO procedure has its flaws. However, due to the difficulty in obtaining a sufficient volume of credible data for any other method, it remains the most widely used method. The adjustment outlined in this paper allows for the elimination of one of the major flaws in the ISO procedure, namely its reliance on past history as a representative sample of possible losses. We recognize that not every company has a wind loss model in their company. However, several reinsurance companies and brokers do have these models and contract for their use.

An additional shortcoming of the ISO procedure is that it fails to adjust for demographic shifts. In particular it does not consider the increase in coastal exposures. The adjustment of the model reflects the current distribution of a company's book and can be updated periodically to reflect any shifts. This does not eliminate the ISO shortfalls since many of the years are still based purely on history. However, the additional year from the model will dampen this problem with the ISO procedure.

Finally, the more accurate territorial indications that result allow a company to more accurately charge for the additional exposure in the wind territories.

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[1] See the 1990 Pricing Discussion Paper titled "Pricing the Catastrophe Exposure" by David H. Hays and W. Scott Farris, Vol. II pp. 559-603.

Exhibit 1

CONNECTICUT

HOMEOWNERS INSURANCE - FORMS 1,2,3&5
DERIVATION OF EXCESS WIND FACTOR

Year	HO Wind Losses	HO Total Losses	Non-Wind Losses	Wind-to-Non-Wind	Excess Years*	Excess Ratio	Excess Losses	Non-Excess Losses	Non-Wind/ Non-Excess
1961	39180	421841	382661	0.102	0.000	0.000	0	421841	0.907
1962	57857	525788	467931	0.124	0.000	0.000	0	525788	0.890
1963	38690	579712	541022	0.072	0.000	0.000	0	579712	0.933
1964	24077	483403	459326	0.052	0.000	0.000	0	483403	0.950
1965	22309	721579	699270	0.032	0.000	0.000	0	721579	0.969
1966	22428	750139	727711	0.031	0.000	0.000	0	750139	0.970
1967	44329	922439	878110	0.050	0.000	0.000	0	922439	0.952
1968	52551	1064312	1011761	0.052	0.000	0.000	0	1064312	0.951
1969	54499	1276897	1222398	0.045	0.000	0.000	0	1276897	0.957
1970	49047	1493849	1444802	0.034	0.000	0.000	0	1493849	0.967
1971	128182	1639387	1511205	0.085	0.000	0.000	0	1639387	0.922
1972	120507	1871461	1750954	0.069	0.000	0.000	0	1871461	0.936
1973	103326	2653614	2550288	0.041	0.000	0.000	0	2653614	0.961
1974	222439	2854392	2631953	0.085	0.000	0.000	0	2854392	0.922
1975	91049	2679652	2588603	0.035	0.000	0.000	0	2679652	0.966
1976	112610	2618827	2506217	0.045	0.000	0.000	0	2618827	0.957
1977	43872	2309037	2265165	0.019	0.000	0.000	0	2309037	0.981
1978	198862	2160841	1961979	0.101	0.000	0.000	0	2160841	0.908
1979	523824	2899303	2375479	0.221	0.000	0.000	0	2899303	0.819
1980	152170	3088639	2936469	0.052	0.000	0.000	0	3088639	0.951
1981	125697	4422524	4296827	0.029	0.000	0.000	0	4422524	0.972
1982	143262	4229727	4086465	0.035	0.000	0.000	0	4229727	0.966
1983	206742	4414828	4208086	0.049	0.000	0.000	0	4414828	0.953
1984	367046	5290981	4923935	0.075	0.000	0.000	0	5290981	0.931
1985	2772884	8654450	5881566	0.471	0.471	0.420	2468097	6186353	0.951
1986	412685	5954039	5541354	0.074	0.000	0.000	0	5954039	0.931
1987	415849	9040467	8624618	0.048	0.000	0.000	0	9040467	0.954
1988	161040	9480386	9319346	0.017	0.000	0.000	0	9480386	0.983
1989	2310963	12857786	10546823	0.219	0.000	0.000	0	12857786	0.820
Total	9017976	97360300	88342324	2.364		0.420	2468097	94892203	27.230
Average						0.014			0.939
			Median	0.052					
		Excess Wind Factor		1.014		[1 + (0.014 * 0.939)]			

*The ratio for a year must be > 1.5M and at least .250 for that year to qualify as an excess year.

Exhibit IV

CONNECTICUT

HOMEOWNERS INSURANCE - FORMS 1, 2, 3 & 5
DERIVATION OF EXCESS WIND FACTOR

Year	HO Wind Losses	HO Total Losses	Non-Wind Losses	Wind-to-Non-Wind	Excess Years*	Excess Ratio	Excess Losses	Non-Excess Losses	Non-Wind/ Non-Excess
1961	39,180	421,841	382,661	0.102	0.000	0.000	0	421841	0.907
1962	57,857	525,788	467,931	0.124	0.000	0.000	0	525788	0.890
1963	38,690	579,712	541,022	0.072	0.000	0.000	0	579712	0.933
1964	24,077	483,403	459,326	0.052	0.000	0.000	0	483403	0.950
1965	22,309	721,579	699,270	0.032	0.000	0.000	0	721579	0.969
1966	22,428	750,139	727,711	0.031	0.000	0.000	0	750139	0.970
1967	44,329	922,439	878,110	0.050	0.000	0.000	0	922439	0.952
1968	52,551	1,064,312	1,011,761	0.052	0.000	0.000	0	1064312	0.951
1969	54,499	1,276,897	1,222,398	0.045	0.000	0.000	0	1276897	0.957
1970	49,047	1,493,849	1,444,802	0.034	0.000	0.000	0	1493849	0.967
1971	128,182	1,639,387	1,511,205	0.085	0.000	0.000	0	1639387	0.922
1972	120,507	1,871,461	1,750,954	0.069	0.000	0.000	0	1871461	0.936
1973	103,326	2,653,614	2,550,288	0.041	0.000	0.000	0	2653614	0.961
1974	222,439	2,854,392	2,631,953	0.085	0.000	0.000	0	2854392	0.922
1975	91,049	2,679,652	2,588,603	0.035	0.000	0.000	0	2679652	0.966
1976	112,610	2,618,827	2,506,217	0.045	0.000	0.000	0	2618827	0.957
1977	43,872	2,309,037	2,265,165	0.019	0.000	0.000	0	2309037	0.981
1978	198,862	2,160,841	1,961,979	0.101	0.000	0.000	0	2160841	0.908
1979	523,824	2,899,303	2,375,479	0.221	0.000	0.000	0	2899303	0.819
1980	152,170	3,088,639	2,936,469	0.052	0.000	0.000	0	3088639	0.951
1981	125,697	4,422,524	4,296,827	0.029	0.000	0.000	0	4422524	0.972
1982	143,262	4,229,727	4,086,465	0.035	0.000	0.000	0	4229727	0.966
1983	206,742	4,414,828	4,208,086	0.049	0.000	0.000	0	4414828	0.953
1984	367,046	5,290,981	4,923,935	0.075	0.000	0.000	0	5290981	0.931
1985	2,772,884	8,654,450	5,881,566	0.471	0.471	0.420	2468097	6186353	0.951
1986	412,685	5,954,039	5,541,354	0.074	0.000	0.000	0	5954039	0.931
1987	415,849	9,040,467	8,624,618	0.048	0.000	0.000	0	9040467	0.954
1988	161,040	9,480,386	9,319,346	0.017	0.000	0.000	0	9480386	0.983
1989	2,310,963	12,857,786	10,546,823	0.219	0.000	0.000	0	12857786	0.820
Total	9,017,976	97,360,300	88,342,324	2.364		0.420	2468097	94892203	27.230
Average						0.014			0.939
50 Year	15,119,000	26,119,000	11,000,000	1.374	1.374	1.323	14548972	11570028	0.951
Average						0.041			0.939
			Median	0.052					
			Excess Wind Factor	1.038		[1 + (0.041 * 0.939)]			

*The ratio for a year must be > 1.5M and at least .250 for that year to qualify as an excess year.

Exhibit V

HOMEOWNERS TERRITORIAL EXPERIENCE

CONNECTICUT — TERRITORIAL EXCESS WIND FACTORS

Zone	Adjusted Earned Premium	Non-Excess Incurred Losses	Loss Ratio	Loss Ratio Relativity	Territorial Excess Wind Factor	Adjusted Incurred Losses	Loss Ratio	Loss Ratio Relativity
28	1,368,915	672,307	49.1%	**1.047**	1.059	711,743	52.0%	**1.068**
29	2,231,951	1,410,928	63.2%	**1.348**	1.059	1,493,688	66.9%	**1.375**
31	17,377,565	7,866,176	45.3%	**0.965**	1.059	8,327,578	47.9%	**0.985**
32	1,544,439	682,356	44.2%	**0.942**	1.073	732,222	47.4%	**0.974**
33	478,717	381,935	79.8%	**1.702**	1.073	409,847	85.6%	**1.759**
34	7,623,692	4,195,286	55.0%	**1.174**	1.073	4,501,877	59.1%	**1.213**
35	1,587,717	718,700	45.3%	**0.965**	1.010	725,980	45.7%	**0.939**
36	3,514,166	1,316,946	37.5%	**0.799**	1.010	1,330,284	37.9%	**0.778**
37	991,207	404,694	40.8%	**0.871**	1.010	408,793	41.2%	**0.847**
38	22,875,106	10,647,978	46.5%	**0.993**	1.010	10,755,826	47.0%	**0.966**
39	3,793,237	1,818,060	47.9%	**1.022**	1.079	1,962,300	51.7%	**1.063**
40	3,399,010	1,478,268	43.5%	**0.928**	1.071	1,582,994	46.6%	**0.957**
41	6,164,932	2,632,560	42.7%	**0.911**	1.005	2,646,143	42.9%	**0.882**
42	4,753,070	2,207,787	46.4%	**0.991**	1.010	2,229,199	46.9%	**0.964**
Total	**77,703,724**	**36,433,981**	**46.9%**	**1.000**	**1.038**	**37,818,472**	**48.7%**	**1.000**

Zones	County	Non-Excess Incurred Losses	50 Year Model Wind Losses	Wind/ Non-Excess Ratio	Wind/ Non-Excess Relativity	Excess Wind Factor
28,29,31	Fairfield	9,949,411	6,373,167	0.641	1.544	1.059
35-38	Hartford	13,088,318	1,447,667	0.111	0.267	1.010
41	Litchfield	2,632,560	148,333	0.056	0.136	1.005
40	Middlesex	1,478,268	1,143,667	0.774	1.864	1.071
32-34	New Haven	5,259,577	4,197,500	0.798	1.923	1.073
39	New London	1,818,060	1,575,167	0.866	2.088	1.079
42	Tolland & Windham	2,207,787	233,833	0.106	0.255	1.010
	Total	**36,433,981**	**15,119,333**	**0.415**	**1.000**	**1.038**

CARe RESEARCH COMMITTEE – MINUTES FOR MEETING OF AUGUST 23, 1990 – THE REVISIONS TO ISO's INCREASED LIMITS PROCEDURE

CARe Research Committee

CARE RESEARCH COMMITTEE
MINUTES FOR THE MEETING OF AUGUST 23, 1990

Attendee	Company
Bear, Robert	North Star Reinsurance
Cellars, Ralph	North American Reinsurance
Gaydos, Eugene	ISO
Francis, Louise	Tillinghast
Giambo, Robert	Trenwick Reinsurance
Grady, David	Prudential Reinsurance
Handte, Malcolm	Zurich Reinsurance
Hughes, Brian	Skandia America Group
Iafrate, Anthony	General Reinsurance
Krakowski, Israel	CNA Insurance
Licht, Peter	ISO
Mahon, John B.	American Reinsurance
Mashitz, Isaac	North American Reinsurance
Meyers, Glenn	ISO
Moller, Karl	Home Insurance
Newville, Benjamin S.	U.S. International Reinsurance
Norton, Jonathan	Guy Carpenter
Patrik, Gary	North American Reinsurance
Robbins, Ira	CIGNA Insurance
Speigler, David	American Reinsurance
Weissner, Edward	Prudential Reinsurance

CARE RESEARCH COMMITTEE
MINUTES FOR THE MEETING OF AUGUST 23, 1990

INTRODUCTION:

Gary Patrik introduced the meeting, outlined the topic and presented the agenda (Attachment 1).

SHORT-TERM CHANGES:

ISO distributed a handout entitled Pilot Increased Limits Ratemaking Procedure (Attachment 2). Glenn Meyers outlined the current ISO ILF procedure and noted the more significant changes which will be made.

1. Four Parameter Mixed Pareto Distribution:

Introduction: The intent of ISO is to use a mixed distribution fit to settled claims (paid claims) to estimate the severity distribution underlying the ILF's.

Using a mixed distribution would eliminate the problem of selecting a truncation point T. It has been shown that the selection of T under the current procedure can significantly affect the ILF's, particularly at higher policy limits (Attachment 2 Page 4). With the mixed distribution, the selection of the mixing parameter p is estimated through the maximum likelihood estimation process.

Using settled claims (paid claims) eliminates the current incurred claim development procedure (Attachment 2 Page 3). Mixed distributions tested by ISO fit equally well for settled data as for incurred data.

However for 1991, ISO does not expect to have this procedure in place. Instead ISO intends to use incurred loss data (indemnity occurrences), with the current development procedure, to fit a mixed Pareto distribution (Attachment 2 Page 5) for Commercial Auto, Premises/Operations and Products/Completed Operations.

Discussion: ISO is proposing a mixed Pareto distribution i.e. two different Paretos F(x:b1,q) and F(x:b2, q+2) with the mixing parameter p. If the idea is that small claims have a less severe distribution, then why use two Paretos? Why not use one distribution with a less severe tail? As an example, why not use an

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MINUTES FOR THE MEETING OF AUGUST 23, 1990

Exponential and a Pareto? Why are the shape parameters of the Pareto distribution q and q+2?

ISO tried various other pairs of distribution on Products Liability settlement data, e.g. Exponential/Pareto and Pareto/Pareto with shape parameter pairs q,q+1; q,q+3; and q,q+4. ISO's conclusion was that the proposed mixed Pareto distribution resulted in the best fit. ISO noted however that they have not finalized their decision and that testing is still being done. ISO intends to fit the mixed Pareto distribution to all lines of business, not just Products, and test the results before any ILF's will be published using this model. ISO also encourages others to try different models. It was noted that similar type of fitting is being tried at Wharton and that ISO is not aware of any better results.

Did ISO try using distributions with more than two parameters? Yes, but the results were not satisfactory.

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How sensitive is the fit of large claims to the selected fit on the small claims? Because of the large volume of small claims, it is not difficult to get a model to fit well for smaller claims, but how well does the model fit for larger claims? How many claims are there above $1 million?

Because the mixing parameter is estimated from the maximum likelihood estimation, the fit for large claims should not be unduly affected by the fit to small claims.

By graphing the two Pareto distributions and noting the intersection of the curves, an intuitive judgement as to the correct "split" of the distributions can be made.

Because there is not much data in the ISO data base above $1 million, the fit to large claims is somewhat an extrapolation process. It is believed that significant large claim data exists in other lines, such as D&O liability (data outside the ISO data base)

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Professional Liability, and that the model should be tested on such lines.

Under the current ISO ILF procedure, there is a problem with the truncation point drastically changing from review to review. Is it possible that the mixing parameter will drastically change from review to review?

The mixing parameter is expected to be stable from review to review. For each accident year (currently using accident years 1973-1986) at any evaluation age the same shape parameter q (and consequently q+2) will be used to fit the data. The scale parameter b is expected to increase by accident year and will be investigated for trend. The mixing parameter will be required to be the same for each accident year.

Further the number of accident years used to fit the mixed Pareto will be stable. Currently fourteen (14) accident years are used. Subsequent reviews will add additional accident years while dropping a minimal number of the oldest accident years (possible none).

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Consequently the parameter constraints combined with a stable data base should result in stable mixing parameters from review to review.

What type of statistical testing is being done to judge the fitted distribution?

General statistical tests such as Kolmogorov Smirnov or Chi- square tests do not work well on insurance data. ISO uses a set of diagnostic tests including a comparison of limited average severity (LAS) to judge the goodness of fit.

2. Risk Load:

Introduction: Originally ISO used a variance based risk load in the ILF's. This resulted in too large a risk load for higher limits with consequential inconsistencies between limits. ISO changed and is currently using a standard deviation risk load. This has resulted in apparent inconsistencies in risk load between lines of business and/or ILF tables within a line.

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ISO is proposing a Commercial Market Equilibrium Risk Load (CMERL) procedure which incorporates both process risk and parameter risk.

Discussion: Two views emerged concerning CMERL. One view is that although there are problems with the variance and standard deviation based risk loads, it is clear how these risk loads are being calculated and what they measure. It is not clear what CMERL is. The correct risk load needs to be defined and estimated to measure how far CMERL differs from it.

Small insurance companies will use the ILF's blindly, so the best estimate of the correct risk load should be used.

Furthermore, ISO previously tried to build a model of the insurance market. It is a very difficult task and the model did not fit well. Why does ISO think it can build a better model now?

The alternative view is that no one knows what the correct risk load is, but ISO is moving in the right direction. That is, risk load is market driven.

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In spite of this affirmation of CMERL, some concerns with the ISO model are that it does not include the effects of the reinsurance market, the flow of capital in and out of the industry, insurance transaction costs, or investment income.

Conclusion: Even in light of ISO's decision to move away from providing rates to providing loss costs, ISO still intends to provide ILF's with risk load. That eventually will mean CMERL.

ISO also proposes to provide computer software to allow companies to compute ILF's with risk load based on the company's own selected parameters.

3. Composite Rated Risks/U.E.C.F.

Introduction: Composite Rated Risk (CRR) claims cannot be identified by class code, so CRR claims cannot be matched to ILF table, for example Premises/Operations Table 1, 2 or 3. Hence severity distributions for Tables 1, 2 or 3 do not include CRR experience. The Uniform Excess Change Factor (U.E.C.F.) is selected to reflect the effect of CRR claims on the ILF tables, by

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comparing severities fit separately to all Tables with and without CRR claims. The U.E.C.F. is the same for each table within a subline. While this results in ILF's which reflect CRR data, the underlying severity distribution for the tables do not. There was strong argument that the U.E.C.F. method be eliminated and that the final ISO ILF tables should each be based upon an underlying probability distribution for claim severity.

ISO intends to change the procedure it uses on CRR claims to produce severity distributions by table, which reflect CRR claims.

LONG-TERM CHANGES:

1. Pareto Soup Model:

Introduction: ISO gave a handout (Attachment 3) which depicted a Pareto Soup model with 43 parameters. This model is typical of other Pareto Soup models.

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MINUTES FOR THE MEETING OF AUGUST 23, 1990

In this example, nine different four parameter mixed Pareto distributions are fit to accident year 1974 paid claims at settlement lags 1 through 9. Trends (S, T1, T2, T3, T4 and T5) are used to adjust the nine mixed Paretos to fit different accident year settlement lag cells.

The parameters for the mixed Pareto distributions, the trends and the mixing parameters are all simultaneously estimated via maximum likelihood techniques.

Discussion: It is difficult to comprehend a model with 43 or more parameters. It is important that the parameters satisfy intuitive opinions on how they should behave. It is especially important that the asymptotic behavior of the patterns be checked as settlement lags increase.

In the example given for AY 1974 it is not intuitively clear why the trend parameter S=0.8865 is less than 1.00 (Attachment 3 Page 2), nor why the mixing parameter P(J) does not decrease to zero as the settlement lag increases. For longer settlement lags, small settled claims should have less effect.

ISO is currently investigating a technique to treat the Q(J) parameters as a function of the settlement lag which would require the Q's to decrease with increasing lag. Possibly a similar approach could be used on the P(J) parameters. The intuitive progression of the B(J) parameters is not as easily identified because each B(J) is associated with a different Q(J) parameter.

Once the various Pareto distributions are estimated, how can they be combined into one distribution? Using a settlement distribution W(J), the various mixed Paretos are weighed together by the proportion of occurrences in each settlement period.

Isn't the settlement distribution effected by partial payments? It probably has a minor effect. In fact, the settlement distribution is fit to average per occurrence settlement dates and not actual settlement dates.

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How is the model tested for settlement lags of 30 years? In the example given, the B2(30) parameter trends to 145. Is this reasonable? ISO is developing a set of diagnostic tests, including diagnostics based on incurred loss, to be used in testing the Pareto Soup model particularly for long settlement lags. The reasonableness of these diagnostics will strongly impact the final model selected.

It is expected that a model with a large number of parameters should result in a good model. How much predictive improvement is gained by a model with such a large number of parameters? Can the model be reduced to a simpler format for others to use?

Parsimony is a nice objective, but ISO has a lot of data so even when the data is subdivided into many accident year settlement lag cells there is still sufficient data in each cell to get good fits. The final model can be described as in the example by a matrix of parameters (Attachment 3 Page 2) which can then be used by others.

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The Pareto Soup model doesn't reflect policy limits. Isn't there a correlation between the size of loss and the size of the policy limit? ISO has tested and found that for a fixed settlement lag, the size of the settled losses is independent of the policy limit. That is, it appears that the settlement lag reflects policy limits.

Doesn't ALAE vary by policy limits. In preliminary tests ISO also found the ALAE is independent of policy limit for a fixed settlement lag. Further tests will be done.

For reinsurers, however, settlement lags are hard to get from ceding companies, but policy limit distributions are easier to obtain. Couldn't ISO build a similar model reflecting policy limits instead of settlement lags?

Possibly ISO could relate settlement lags to the more common policy limits. A problem with this might be what policy limit is reported. For example, if an insured has an umbrella policy over its primary policy the settlement of the loss may be affected by the

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umbrella limit even though only the primary policy limit is reported to ISO.

In the example, data from accident years 1973 through 1986 are used to project accident year 1991. A rhetorical question was asked whether the lag between the end of the data and the projection date could be shortened?

2. Paid Versus Reported Loss Data?

Introduction: ISO has found in examining inconsistencies in reported data that most inconsistencies involve open claims. There is less of a problem with reporting actual paid loss. Furthermore, paid claims lead open claims with respect to major changes in claims settlement practices. For example, stacking of UM/UIM had to result in a settled claim against an insurer before open claim reserves were increased to reflect stacking.

Discussion: For lines of business with long settlement lags, there aren't many large claims, e.g. excess of $1 million, that are likely to settle quickly enough to be

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included in the settled claim experience. This seems to be a high cost to pay for somewhat cleaner data.

While it may be true that paid claims lead open claims in reflecting major changes in claim settlement practices, the impact of the change is delayed if only settled claims are used. The increased reserves on open claims will not enter the data until the claims are settled. Valuable information will not be incorporated as quickly as it should.

By use of diagnostic tests on open claims the ISO results based on settled claims should indicate whether the settled claim data is failing to reflect the open claim reserves correctly. Also the delay in incorporating changes in claim settlement practices will vary by company. ISO data is reported from many different companies all with different claim reserving practices. It is more difficult for ISO to adjust open claim reserving practices for all the different companies reporting to ISO than to reflect such practices for one company.

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ISO has not yet finalized the ILF methodology using settled claim data. The diagnostics tests are still evolving. If the methodology using settled claims fails, the incurred loss methodology is still available.

3. Discounted Increased Limit Factors

Introduction: Discounted limited average severities (LAS) can be calculated by settlement lag for a fixed interest rate (interest rates may vary by settlement lag). Weighing together the LAS, the discounted LAS can be calculated. The discounted LAS can then be used to calculate discounted ILF's.

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Discussion: Many concerns were raised. Will the discounted LAS be used in the risk load calculations? Will variation in interest rates be considered? Will discounted ILF's be used in filings? If ILF's will reflect investment income on loss payments shouldn't they also reflect other expenses such as overhead or commissions?

Many of these concerns have yet to be addressed by ISO. ISO has no intention of filing discounted ILF's. Using discounted LAS to calculate ILF's was noted as a point of information only.

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AGENDA

ISO INCREASED LIMITS PROCEDURE

Introduction:

9:30 - Overview of short-term and longer term changes

Short-Term Changes:

10:00 - 1. severity model (4-parameter Pareto)

10:40 - 2. risk load

11:40 - 3. composite rated risk data and uniform excess change

12:00 - LUNCH

Longer-Term Changes:

1:00 - 1. Pareto Soup Model (36 or 43 or more parameters)

2:45 - 2. paid versus reported loss data

3:15 - 3. discounted increased limit factors

Closing:

3:45 - Summarization and wrap-up

4:00 - Adjournment

Pilot Increased Limits Ratemaking Procedure

- Developed by ISO staff and Actuarial Research Committee

- Significant new features

\

1. Pareto "Soup"

2. Distribution fit to settled (paid) occurrences

3. Explicit loss development model

4. Empirical testing procedures

5. New risk load formula

   - Derived from economic equilibrium assumptions

   - Explicit recognition of parameter uncertainty

Current Increased Limit Procedure

- Truncated Pareto distribution
- Development of number of occurrences by layer
- Risk load based on standard deviation of loss

Short Term Changes

- Mixed Pareto distribution ???
- Development of number of occurrences by layer
- Competitive Market Equilibrium Risk Load Formula

PRODUCT
ACCIDENT YEAR

INTERVAL ENDPOINT	27-ULT	39-ULT	51-ULT	63-ULT	75-ULT	87-ULT	99-ULT
27866	1.22714	0.94869	0.90311	0.93339	0.95704	0.96580	0.97279
28677	1.23678	0.95290	0.90520	0.93521	0.95855	0.96741	0.97441
29532	1.24461	0.95722	0.90718	0.93687	0.95987	0.96885	0.97581
30433	1.25439	0.96147	0.90909	0.93841	0.96106	0.97009	0.97706
31387	1.26461	0.96576	0.91097	0.93988	0.96213	0.97123	0.97817
32396	1.27692	0.97015	0.91286	0.94129	0.96314	0.97228	[illegible]
33448	1.28748	0.97466	0.91478	0.94268	0.96409	0.97326	[illegible]
34408	1.29885	0.97933	0.91675	0.94406	0.96501	0.97419	0.98099
35823	1.31050	0.98419	0.91882	0.94546	0.96592	0.97509	0.98182
37122	1.32269	0.98929	0.92098	0.94690	0.96683	0.97597	0.98261
38515	1.33549	0.99465	0.92329	0.94839	0.96776	0.97684	0.98339
40012	1.34898	1.00051	0.92574	0.94995	0.96872	0.97772	0.98416
41627	1.36324	1.00632	0.92839	0.95159	0.96973	0.97862	0.98492
43376	1.37838	1.01272	0.93124	0.95335	0.97080	0.97954	0.98570
45276	1.39448	1.01957	0.93433	0.95522	0.97195	0.98053	0.98650
47351	1.41169	1.02691	0.93770	0.95725	0.97318	0.98155	0.98732
49627	1.43015	1.03481	0.94137	0.95940	0.97450	0.98262	0.98818
52130	1.44995	1.04333	0.94537	0.96173	0.97592	0.98376	0.98906
54924	1.47131	1.05253	0.94973	0.96421	0.97741	0.98493	0.98996
58059	1.49433	1.06243	0.95442	0.96680	0.97892	0.98610	0.99085
61547	1.52087	1.07427	0.96051	0.97054	0.98146	0.98804	0.99228
65556	1.55114	1.08807	0.96796	0.97331	0.98497	0.99064	0.99418
70128	1.58471	1.10341	0.97628	0.98058	0.98862	0.99345	0.99617
75485	1.62199	1.12038	0.98547	0.98627	0.99290	0.99632	0.99810
81760	1.66320	1.13894	0.99557	0.99209	0.99690	0.99888	0.99960
89582	1.71000	1.15994	1.00655	0.99847	1.00120	1.00148	1.00118
98710	1.77492	1.19139	1.02547	1.02179	1.01212	1.01032	1.00825
110890	1.84942	1.22682	1.04670	1.02572	1.02320	1.01956	1.01562
126525	1.94234	1.27112	1.07325	1.04328	1.03725	1.03122	1.02490
134328	2.02388	1.30921	1.09549	1.05725	1.04807	1.03974	1.03167
148555	2.09089	1.34026	1.11381	1.06804	1.05715	1.04692	1.03737
165187	2.16845	1.37703	1.13561	1.08265	1.06805	1.05556	1.04421
180027	2.26306	1.42136	1.16196	1.09953	1.08127	1.06604	1.05249
206948	2.38851	1.47625	1.19460	1.11971	1.09764	1.07901	1.06272
241852	2.53208	1.54670	1.23646	1.14616	1.11856	1.09556	1.07574
298190	2.73934	1.64232	1.29508	1.18132	1.14662	1.11769	1.09509
389929	3.05089	1.78438	1.37647	1.23247	1.18740	1.14973	1.11809
419085	3.32198	1.90580	1.44687	1.27488	1.22111	1.17609	1.13855
454006	3.44352	1.95994	1.47806	1.29358	1.23390	1.18761	1.14747
497134	3.58634	2.02310	1.51430	1.31500	1.25295	1.20087	1.15770
551905	3.75779	2.09859	1.55724	1.34028	1.27296	1.21640	1.16967
624327	3.96988	2.19061	1.60947	1.37073	1.29702	1.23502	1.18397
725727	4.24174	2.30789	1.67532	1.40874	1.32695	1.25810	1.20164
880617	4.61257	2.46555	1.76284	1.45855	1.36603	1.28811	1.22451
1155571	5.16734	2.69754	1.88974	1.52942	1.42137	1.33035	1.25653
1835234	6.17024	3.10417	2.10821	1.64814	1.51332	1.39992	1.30883
2909430	7.80630	3.74613	2.43816	1.82001	1.64476	1.49806	1.38173
7.9E+08	13.6127	5.92292	3.51535	2.35115	2.04336	1.78880	1.59239

Policy Limit	Current T=6000	Current T=12000	Pilot 87 Call	Pilot 88 Call
$25,000	$4,077	$4,049	$5,381	$5,177
50,000	5,459	5,381	7,243	7,016
100,000	7,096	6,898	9,407	9,109
300,000	10,244	9,518	12,931	12,606
500,000	11,959	10,785	14,489	14,193
1,000,000	14,573	12,532	16,462	16,254
2,000,000	17,573	14,298	18,265	18,200
5,000,000	22,137	16,651	20,402	20,626

Pareto Distribution:

$$F(x: b,q) = 1 - \left[\frac{b}{x + b}\right]^q$$

Mixed Pareto Distribution:

$$G(x) = (1\text{-}p) \cdot F(x: b_1,q) + p \cdot F(x: b_2,q+2)$$

Long Tail Short Tail

Trending and Developing the Occurrence Severity Distribution

Delay in Settlement

		0	1	2	3	4	...	S	
P	0	o	o	o	o	o	o	o	x →
o	1	o	o	o	o	o	o	x	x →
l	2	o	o	o	o	o	x	x	x →
	3	o	o	o	o	x	x	x	x →
Y	4	o	o	o	x	x	x	x	x →
e	...	o	o	x	x	x	x	x	x →
a	S	o	x	x	x	x	x	x	x →
r		x	x	x	x	x	x	x	x →
		↓	↓	↓	↓	↓	↓	↓	↓

o - observed occurrence severity distribution

x - projected occurrence severity distribution.

Trended Mixed Pareto Distribution

$$G_{y,d}(x) = (1-p_d)\cdot F(x: b_{1d}\cdot t_d^y , q_d) + p_d\cdot F(x: b_{2d}\cdot t_d^y , q_d+2)$$

y = Accident Year
d = Delay in Settlement

Relationship between parameters

1. Trend factors, t_d's, are equal for selected d's.

2. Shape parameters, q_d's and p_d's, are equal for selected d's.

3. Scale parameters, b_{id}'s are equal for selected d's.

$$\text{Likelihood} = \prod_{y=0}^{S} \prod_{d=0}^{y} \prod_{i=1}^{130} \left(G_{y,d}(L_i) - G_{y,d}(L_{i-1})\right)^{n_{y,d,i}}$$

The final claim severity distribution for year S+1:

$$\sum_{d=0}^{\infty} w_d G_{S+1,d}(x)$$

w_d = proportion of occurrences in settlement period d.

w_d's are estimated by maximum likelihood.

We assume w_d's have an exponential tail.

Note

The final occurrence severity distribution is a mixture of Pareto distributions. The proportion of each Pareto is determined by the w_d's and the p_d's. Hence the term:

Pareto Soup

Fitting Diagnostics

Compare Case Reserves with Projected Future Settlements

PRODUCTS CGL TABLE B AY 73 OPEN

POLICY LIMIT	SAMPLE L.A.S.	MODEL L.A.S.	% DIFF
$25,000	7,346	9,669	31.62%
$50,000	10,541	13,972	32.55%
$100,000	15,274	18,893	23.69%
$300,000	21,739	27,208	25.16%
$500,000	25,210	31,135	23.50%
$1,000,000	28,412	36,382	28.05%
$2,000,000	32,264	41,527	28.71%
$5,000,000	38,571	48,118	24.75%
# OF OCCS.	194	313	

PRODUCTS CGL TABLE B AY 74 OPEN

POLICY LIMIT	SAMPLE L.A.S.	MODEL L.A.S.	% DIFF
$25,000	8,237	9,751	18.38%
$50,000	12,824	14,167	10.48%
$100,000	18,059	19,194	6.29%
$300,000	21,511	27,739	28.95%
$500,000	23,289	31,749	36.33%
$1,000,000	25,702	37,164	44.59%
$2,000,000	26,582	42,441	59.66%
$5,000,000	27,358	49,205	79.86%
# OF OCCS.	360	390	

PRODUCTS CGL TABLE B AY 76 OPEN

POLICY LIMIT	SAMPLE L.A.S.	MODEL L.A.S.	% DIFF
$25,000	5,488	9,972	81.70%
$50,000	7,583	14,542	91.77%
$100,000	10,017	19,825	97.91%
$300,000	14,857	28,786	93.76%
$500,000	17,469	33,030	89.08%
$1,000,000	20,267	38,720	91.05%
$2,000,000	22,433	44,303	97.49%
$5,000,000	25,655	51,456	100.57%
# OF OCCS.	544	721	

PRODUCTS CGL TABLE B AY 78 OPEN

POLICY LIMIT	SAMPLE L.A.S.	MODEL L.A.S.	% DIFF
$25,000	8,150	10,180	24.91%
$50,000	11,316	14,945	32.07%
$100,000	14,406	20,446	41.93%
$300,000	18,871	29,880	58.34%
$500,000	20,481	34,331	67.63%
$1,000,000	21,821	40,340	84.87%
$2,000,000	22,115	46,225	109.02%
$5,000,000	22,305	53,786	141.14%
# OF OCCS.	819	1,118	

PRODUCTS CGL TABLE B AY 80 OPEN

POLICY LIMIT	SAMPLE L.A.S.	MODEL L.A.S.	% DIFF
$25,000	7,470	10,321	38.16%
$50,000	10,092	15,107	49.69%
$100,000	12,762	20,615	61.53%
$300,000	16,703	30,050	79.91%
$500,000	18,435	34,495	87.12%
$1,000,000	20,191	40,469	100.43%
$2,000,000	22,158	46,317	109.03%
$5,000,000	25,859	53,797	108.04%
# OF OCCS.	2,122	2,990	

PRODUCTS CGL TABLE B AY 82 OPEN

POLICY LIMIT	SAMPLE L.A.S.	MODEL L.A.S.	% DIFF
$25,000	11,893	10,595	-10.91%
$50,000	16,758	15,423	-7.97%
$100,000	21,490	20,907	-2.71%
$300,000	28,194	29,973	6.31%
$500,000	30,446	34,107	12.02%
$1,000,000	32,448	39,529	21.82%
$2,000,000	33,939	44,696	31.70%
$5,000,000	35,378	51,141	44.55%
# OF OCCS.	2,438	4,370	

Parameter Uncertainty - Severity

$$b_{id} \text{-------}> B_y \cdot b_{id}$$

y+1973	B_y	
1973	1.000	(by definition)
1974	1.053	
1975	1.016	
1976	0.964	
1977	1.013	
1978	1.013	
1979	0.990	
1980	1.001	
1981	1.014	
1982	1.103	
1983	0.982	
1984	1.060	
1985	0.975	
1986	0.987	

The distribution of B_y is estimated in the maximum likelihood equation.

Parameter Uncertainty - Occurrence Count

Let n = expected claim count for an insurance company

$$n \dashrightarrow C_y \cdot n$$

$E[C_y] = 1$

$Var[C_y] = c$

Poisson - No Parameter Uncertainty

Negative Binomial - Parameter Uncertainty

$$c = (\text{Coefficient of Variation of Gamma Prior})^2$$

c is estimated by maximum likelihood.

Risk Load

Goals of the Risk Load Formula

The risk load should be sufficient to attract an adequate supply of coverage for all desired policy limits.

The risk load should reflect stable, yet competitive, market conditions. It should not reflect such effects as the underwriting cycle.

The risk load should reflect the risks faced by the insurer in estimating the price of its product. It should recognize parameter uncertainty.

Risk Load

Insurance Market Assumptions

The insurance market is highly competitive. The risk load cannot be influenced by the actions of a single insurer.

Insurers can decide how much insurance to write in each line of business and policy limit.

Insurers will write line/limit combinations in such a way as to maximize the risk load subject to a constraint on the variance of its total insurance portfolio.

The result of all insurers competing for business as described above will result in an equilibrium characterized by the supply of insurance equaling the demand for insurance for each line/limit combination.

Risk Load

Characterization of Equilibrium

Technical note: vectors and matricies will have cells corresponding to each line/limit combination.

Define

m - number of insurance companies

n(k) - vector of expected occurrence counts for the kth company

$\bar{n}$ - average $n(k) = \frac{1}{m} \cdot \sum_{k=1}^{m} n(k)$

U - vector quantifying process risk

V - covariance matrix quantifying parameter risk

L - constant of proportionality

R - vector for risk load per expected occurrence

Then $$R = L \cdot (U + 2 \cdot V \cdot \bar{n})$$

Risk Load

Outline of Derivation of Risk Load Formula

Step 1

For a given risk load vector, R, each insurance company decides how much insurance it will write in each line and policy limit by solving the constrained optimization problem.

> Maximize total risk load subject to the constraint on total variance of its insurance portfolio. This is a standard Lagrange multiplier problem.

This exercise will tell how much insurance will be supplied at each line and policy limit as a function of the risk load vector, R.

Step 2

Do a market survey to determine how much is demanded for each line and policy limit.

Step 3

Select the risk load vector, R, that will cause the total supply equal to the total demand for each line and policy limit.

Risk Load

Sample Calculations

Limit (000)	Severity	Process Risk	Parameter Risk	ILF w/o RL	ILF w RL
25	12032	44	708	1.000	1.000
50	14082	109	965	1.170	1.186
100	16387	257	1252	1.362	1.400
300	20140	859	1723	1.674	1.777
500	21799	1431	1931	1.812	1.968
1000	23901	2763	2194	1.986	2.257
2000	25821	5195	2434	2.146	2.617
5000	28097	11716	2720	2.335	3.327

Risk Load

Risk Reduction by Layering

Common Practice - Calculate the ILF for an excess layer by subtracting the ILF for the lower limit from the ILF for the upper limit.

Sample Calculations

Limit (000)	Severity	Process Risk	Parameter Risk	Total Risk	ILF w RL
1000	23901	2763	2194	4957	2.257
2000	25821	5195	2434	7629	2.617
Diff	1920	2432	240	2762	0.359

Which would an insurer rather sell?

1. A ground up $2,000,000 policy limit, or

2. A ground up $1,000,000 policy limit to one insured, and a $1,000,000 over $1,000,000 policy limit to a second insured.

Concluding Remarks on Risk Load

Our goal is to provide a generic risk load formula which accounts for basic economic conditions.

This risk load formula is, at best, an approximation. It should be judged on its usefulness.

It is up to insurers to make whatever modifications they feel should be made. It is ISO's goal to make common changes easy.

Risk Load

Note that the "subtraction" method implies indifference between the two options.

However, the risk load expression, $R = L \cdot (U + 2 \cdot V \cdot \bar{n})$, implies preference for separate layers.

Sample Calculations

Limit (000)	Severity	Process Risk	Parameter Risk	Total Risk	ILF w RL
1000	23901	2763	2194	4957	2.257
2000	25821	5195	2434	7629	2.617
Diff	1920	2432	240	2762	0.359
RL Eqn	1920	737	240	977	0.227

Note that the subtraction method works for parameter risk but not for process risk.

OSD PRODUCTS TABLE B SEVERITY MODEL

ACCIDENT YEAR	LAG 1	2	3	4	5	6	7	8	9	10	11	12	13	14
1974	Q1(1)	Q1(2)	Q1(3)	Q1(4)	Q1(5)	Q1(6)	Q1(7)	Q1(8)	Q1(9)	Q1(9)	Q1(9)	Q1(9)	Q1(9)	Q1(9)
	Q2(1)	Q2(2)	Q2(3)	Q2(4)	Q2(5)	Q2(6)	Q2(7)	Q2(8)	Q2(9)	Q2(9)	Q2(9)	Q2(9)	Q2(9)	Q2(9)
	B1(1)	B1(2)	B1(3)	B1(4)	B1(5)	B1(6)	B1(7)	B1(8)	B1(9)	B1(9)*S	B1(9)*S**2	B1(9)*S**3	B1(9)*S**4	B1(9)*S**5
	B2(1)	B2(2)	B2(3)	B2(4)	B2(5)	B2(6)	B2(7)	B2(8)	B2(9)	B2(9)*S	B2(9)*S**2	B2(9)*S**3	B2(9)*S**4	B2(9)*S**5
	P(1)	P(2)	P(3)	P(4)	P(5)	P(6)	P(7)	P(8)	P(9)	P(9)	P(9)	P(9)	P(9)	P(9)
1975	Q1(1)	Q1(2)	Q1(3)	Q1(4)	Q1(5)	Q1(6)	Q1(7)	Q1(8)	Q1(9)	Q1(9)	Q1(9)	Q1(9)	Q1(9)	Q1(9)
	Q2(1)	Q2(2)	Q2(3)	Q2(4)	Q2(5)	Q2(6)	Q2(7)	Q2(8)	Q2(9)	Q2(9)	Q2(9)	Q2(9)	Q2(9)	Q2(9)
	B1(1)*T1	B1(2)*T2	B1(3)*T3	B1(4)*T4	B1(5)*T5	B1(6)*T5	B1(7)*T5	B1(8)*T5	B1(9)*T5	B1(9)*T5*S	B1(9)*T5*S**2	B1(9)*T5*S**3	B1(9)*T5*S**4	B1(9)*T5*S**5
	B2(1)*T1	B2(2)*T2	B2(3)*T3	B2(4)*T4	B2(5)*T5	B2(6)*T5	B2(7)*T5	B2(8)*T5	B2(9)*T5	B2(9)*T5*S	B2(9)*T5*S**2	B2(9)*T5*S**3	B2(9)*T5*S**4	B2(9)*T5*S**5
	P(1)	P(2)	P(3)	P(4)	P(5)	P(6)	P(7)	P(8).	P(9)	P(9)	P(9)	P(9)	P(9)	P(9)

ETC.

NOTE: Q2 = Q1 + 2

Attachment 3
Page 1

PARAMETERS FROM THE FULL 14X14 TRIANGLE MODEL WITH SEVERITY TREND
BASED ON EDITED 1988 CALL PRODUCTS CGL TABLE B OSD DATA
FOR ACCIDENT YEARS 1974 TO 1987

LAG(J)	Q1(J)	Q2(J)	B1(J)	B2(J)	P(J)	T(J)	W(J)
1	2.1730	4.1730	2,155	665	0.8513	1.0889	0.4057
2	1.5905	3.5905	2,057	800	0.7520	1.1044	0.2669
3	1.2644	3.2644	5,096	2,047	0.7028	1.1235	0.0753
4	1.2748	3.2748	8,181	3,082	0.6007	1.1185	0.0552
5	1.3772	3.3772	18,460	6,527	0.5540	1.0518	0.0436
6	1.2196	3.2196	12,963	4,893	0.3843	1.0518	0.0284
7	1.3469	3.3469	15,993	4,312	0.3269	1.0518	0.0215
8	0.8381	2.8381	3,635	54	0.0398	1.0518	0.0176
9	0.9456	2.9456	10,491	1,818	0.3386	1.0518	0.0146
10	0.9456	2.9456	9,300	1,612	0.3386	1.0518	0.0121
11	0.9456	2.9456	8,245	1,429	0.3386	1.0518	0.0101
12	0.9456	2.9456	7,309	1,267	0.3386	1.0518	0.0084
13	0.9456	2.9456	6,480	1,123	0.3386	1.0518	0.0069
14	0.9456	2.9456	5,744	996	0.3386	1.0518	0.0058

S = 0.8865 SIGMA = 0.0387

LIMITED AVERAGE SEVERITY ANALYSIS FOR PRODUCTS TABLE B
MODEL INCLUDING TREND ACROSS LAGS FOR EDITED 1988 CALL DATA

PRODUCTS CGL TABLE 2 AY 74 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	2167	1915	-11.64
50000	2824	2425	-14.13
100000	3523	2954	-16.15
300000	4330	3779	-12.73
500000	4614	4149	-10.10
1000000	4967	4637	-6.63
2000000	5409	5116	-5.42
5000000	5816	5740	-1.30

NUMBER OF OCCURRENCES = 5525

PRODUCTS CGL TABLE 2 AY 75 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	1810	1771	-2.14
50000	2302	2215	-3.77
100000	2818	2671	-5.22
300000	3568	3368	-5.61
500000	3753	3675	-2.07
1000000	3919	4077	4.03
2000000	4073	4463	9.59
5000000	4261	4961	16.41

NUMBER OF OCCURRENCES = 7181

PRODUCTS CGL TABLE 2 AY 76 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	1983	2187	10.28
50000	2447	2796	14.25
100000	2957	3431	16.01
300000	3738	4419	18.24
500000	4020	4860	20.89
1000000	4361	5437	24.69
2000000	4651	5997	28.94
5000000	5040	6721	33.36

NUMBER OF OCCURRENCES = 7764

PRODUCTS CGL TABLE 2 AY 77 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	2301	2297	-0.16
50000	2969	2922	-1.57
100000	3708	3567	-3.81
300000	4728	4547	-3.84
500000	5124	4972	-2.97
1000000	5471	5519	0.88
2000000	5813	6039	3.88
5000000	6264	6699	6.94

NUMBER OF OCCURRENCES = 9637

PRODUCTS CGL TABLE 2 AY 78 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	2491	2495	0.14
50000	3168	3204	1.13
100000	3946	3939	-0.16
300000	5015	5055	0.80
500000	5363	5537	3.24
1000000	5654	6151	8.79
2000000	5848	6729	15.06
5000000	6082	7453	22.53

NUMBER OF OCCURRENCES = 8680

PRODUCTS CGL TABLE 2 AY 79 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	2870	2850	-0.71
50000	3678	3660	-0.50
100000	4512	4495	-0.38
300000	5742	5746	0.08
500000	6158	6278	1.95
1000000	6550	6949	6.08
2000000	6836	7570	10.74
5000000	7145	8338	16.70

NUMBER OF OCCURRENCES = 15123

PRODUCTS CGL TABLE 2 AY 80 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	2797	2671	-4.52
50000	3568	3397	-4.80
100000	4388	4137	-5.71
300000	5702	5217	-8.51
500000	6222	5659	-9.04
1000000	6677	6199	-7.16
2000000	6976	6678	-4.27
5000000	7290	7241	-0.67

NUMBER OF OCCURRENCES = 19612

PRODUCTS CGL TABLE 2 AY 81 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	2756	2658	-3.52
50000	3492	3364	-3.67
100000	4302	4076	-5.25
300000	5583	5088	-8.86
500000	6066	5486	-9.57
1000000	6596	5949	-9.81
2000000	6938	6333	-8.72
5000000	7200	6738	-6.43

NUMBER OF OCCURRENCES = 20940

LIMITED AVERAGE SEVERITY ANALYSIS FOR PRODUCTS TABLE B
MODEL INCLUDING TREND ACROSS LAGS FOR EDITED 1988 CALL DATA

PRODUCTS CGL TABLE 2 AY 82 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	2833	2682	-5.30
50000	3588	3380	-5.79
100000	4378	4089	-6.59
300000	5477	5106	-6.76
500000	5876	5509	-6.25
1000000	6170	5981	-3.07
2000000	6391	6374	-0.27
5000000	6620	6792	2.60

NUMBER OF OCCURRENCES = 20619

PRODUCTS CGL TABLE 2 AY 83 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	2761	2603	-5.71
50000	3459	3226	-6.71
100000	4247	3851	-9.31
300000	5381	4736	-12.00
500000	5739	5080	-11.48
1000000	6183	5479	-11.37
2000000	6494	5807	-10.58
5000000	6810	6147	-9.74

NUMBER OF OCCURRENCES = 19304

PRODUCTS CGL TABLE 2 AY 84 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	2526	2445	-3.17
50000	3050	2974	-2.47
100000	3579	3497	-2.32
300000	4360	4237	-2.82
500000	4701	4530	-3.64
1000000	5081	4873	-4.09
2000000	5358	5160	-3.69
5000000	5652	5466	-3.30

NUMBER OF OCCURRENCES = 18696

PRODUCTS CGL TABLE 2 AY 85 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	2161	2025	-6.28
50000	2550	2347	-7.96
100000	2947	2636	-10.55
300000	3518	3015	-14.31
500000	3766	3157	-16.16
1000000	4034	3320	-17.70
2000000	4248	3454	-18.70
5000000	4462	3594	-19.45

NUMBER OF OCCURRENCES = 14921

PRODUCTS CGL TABLE 2 AY 86 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	1631	1653	1.33
50000	1775	1815	2.24
100000	1867	1930	3.38
300000	1962	2041	4.04
500000	2007	2073	3.27
1000000	2027	2103	3.71
2000000	2027	2122	4.69
5000000	2027	2138	5.48

NUMBER OF OCCURRENCES = 9955

PRODUCTS CGL TABLE 2 AY 87 ALL LAGS

Policy Limit	Sample L.A.S.	Model L.A.S.	% Diff
25000	1127	1236	9.69
50000	1207	1300	7.73
100000	1265	1335	5.52
300000	1316	1357	3.08
500000	1344	1361	1.24
1000000	1346	1364	1.32
2000000	1347	1365	1.36
5000000	1347	1366	1.38

NUMBER OF OCCURRENCES = 5170

1988 CALL EDITED PRODUCTS TABLE B DATA — MODEL INCLUDING TREND ACROSS LAGS — LIMIT = 25,000

MODEL DEVIATIONS FROM SAMPLE

AY/LAG	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1974	4.08%	-4.60%	-0.47%	-2.82%	7.59%	-8.07%	10.54%	-20.93%	-30.48%	-39.00%	-51.81%	-4.36%	23.93%	-25.27%
1975	-5.47%	-7.10%	-3.66%	5.20%	8.87%	6.11%	-5.53%	-23.52%	9.84%	-20.19%	-4.55%	52.37%	-13.36%	
1976	5.86%	1.85%	0.51%	-3.85%	22.29%	0.12%	-0.42%	4.46%	36.50%	-0.11%	125.64%	50.66%		
1977	1.48%	-1.97%	7.46%	4.46%	-11.67%	5.08%	-4.99%	-18.64%	15.63%	38.57%	-11.74%			
1978	2.08%	11.72%	5.40%	-8.66%	5.18%	-11.21%	0.01%	1.16%	-8.54%	2.61%				
1979	-4.22%	5.09%	-3.36%	0.98%	-1.54%	-6.27%	-1.41%	5.46%	-1.88%					
1980	-3.69%	-6.47%	-1.05%	-5.27%	-2.43%	-9.18%	0.45%	-9.82%						
1981	-3.97%	-5.96%	-2.32%	1.84%	-9.06%	-2.15%	-1.80%							
1982	-13.43%	-6.85%	-8.04%	-5.19%	-2.34%	6.11%								
1983	-5.17%	-8.12%	-4.68%	-4.55%	-5.34%									
1984	-4.96%	-3.28%	-0.73%	-4.01%										
1985	-5.03%	-5.94%	-7.79%											
1986	-0.21%	2.50%												
1987	9.69%													

1988 CALL EDITED PRODUCTS TABLE B DATA — MODEL INCLUDING TREND ACROSS LAGS — LIMIT = 100,000

MODEL DEVIATIONS FROM SAMPLE

AY/LAG	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1974	-0.20%	-9.26%	-5.03%	-12.63%	21.20%	-12.91%	8.55%	0.77%	-12.51%	-44.28%	-62.31%	18.77%	-4.54%	-22.46%
1975	-19.18%	-7.21%	0.09%	13.13%	20.69%	7.79%	-11.66%	-33.61%	27.75%	-39.09%	-2.72%	9.60%	-17.13%	
1976	6.57%	4.10%	-2.31%	5.36%	34.69%	12.42%	0.29%	7.15%	34.64%	-6.73%	137.58%	57.14%		
1977	3.85%	-1.94%	2.62%	8.04%	-18.83%	9.25%	-10.74%	-21.88%	19.74%	38.97%	-30.13%			
1978	4.89%	13.66%	6.19%	-14.61%	4.45%	-14.71%	8.53%	5.80%	-11.53%	11.21%				
1979	-7.26%	4.64%	-4.74%	3.29%	0.91%	-4.66%	-5.45%	6.94%	2.99%					
1980	-3.68%	-7.75%	-0.65%	-9.85%	2.74%	-13.33%	-2.16%	-13.36%						
1981	-4.90%	-4.87%	-2.65%	0.96%	-10.56%	-5.63%	-9.64%							
1982	-14.71%	-11.80%	-11.17%	-6.46%	-5.93%	8.14%								
1983	-7.24%	-13.34%	-6.21%	-7.57%	-11.34%									
1984	-8.04%	-1.10%	1.22%	-3.42%										
1985	-9.71%	-8.13%	-13.22%											
1986	0.47%	5.35%												
1987	5.52%													

1988 CALL EDITED PRODUCTS TABLE B DATA　　MODEL INCLUDING TREND ACROSS LAGS　　LIMIT = 500,000

MODEL DEVIATIONS FROM SAMPLE

AY/LAG	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1974	-20.06%	-5.69%	-8.90%	-12.00%	43.81%	-0.40%	15.46%	41.84%	28.43%	-36.09%	-58.62%	-4.85%	-12.46%	10.00%
1975	-18.68%	-2.84%	-15.56%	15.29%	37.53%	1.43%	3.24%	-22.76%	100.33%	-45.26%	20.34%	-3.50%	2.68%	
1976	7.30%	5.39%	-4.14%	17.06%	52.42%	23.60%	13.62%	38.11%	10.30%	-8.32%	121.00%	26.06%		
1977	6.63%	-1.21%	5.09%	11.94%	-25.84%	16.85%	-7.11%	-9.62%	7.03%	48.58%	-32.96%			
1978	5.75%	15.26%	-0.11%	-21.25%	4.61%	-7.01%	12.83%	22.88%	-6.30%	50.66%				
1979	-9.52%	-0.41%	6.49%	10.13%	2.00%	-12.52%	3.81%	10.68%	16.66%					
1980	-4.72%	-9.91%	-3.96%	-16.17%	6.64%	-24.30%	-6.43%	-7.68%						
1981	-4.59%	-9.70%	-5.78%	-2.00%	-15.60%	-11.47%	-14.75%							
1982	-14.09%	-18.07%	-12.17%	-1.05%	-9.78%	11.68%								
1983	-6.99%	-20.27%	-1.16%	-8.08%	-17.86%									
1984	-9.01%	-2.80%	5.08%	-8.51%										
1985	-14.03%	-12.91%	-19.31%											
1986	0.87%	4.73%												
1987	1.24%													

1988 CALL EDITED PRODUCTS TABLE B DATA　　MODEL INCLUDING TREND ACROSS LAGS　　LIMIT = 2,000,000

MODEL DEVIATIONS FROM SAMPLE

AY/LAG	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1974	-20.00%	-4.43%	-6.62%	-7.96%	57.11%	13.45%	28.14%	73.65%	58.41%	-29.04%	-53.89%	30.70%	-54.46%	41.21%
1975	-18.61%	-1.01%	-24.56%	17.15%	56.74%	10.33%	19.82%	12.65%	181.77%	-32.26%	57.76%	4.53%	17.59%	
1976	7.40%	7.49%	5.61%	33.45%	61.36%	37.52%	28.75%	101.85%	0.43%	-4.96%	103.07%	32.90%		
1977	4.74%	-0.30%	10.04%	28.55%	-15.17%	24.30%	-4.12%	9.28%	-8.04%	37.56%	-21.45%			
1978	5.88%	17.69%	5.70%	-22.12%	16.53%	2.85%	24.98%	67.74%	7.48%	99.21%				
1979	-9.40%	-3.15%	17.52%	13.05%	12.08%	-6.90%	13.42%	28.30%	46.85%					
1980	-4.58%	-8.61%	4.12%	-9.27%	12.63%	-25.58%	-2.10%	7.84%						
1981	-4.48%	-13.35%	-9.91%	5.90%	-16.45%	-7.55%	-14.29%							
1982	-13.94%	-19.37%	-11.11%	11.66%	-6.25%	27.04%								
1983	-6.93%	-24.24%	7.13%	-5.16%	-19.90%									
1984	-9.01%	-3.72%	6.32%	-9.00%										
1985	-13.91%	-19.68%	-19.71%											
1986	1.13%	6.83%												
1987	1.36%													

AGENDA
ACTUARIAL RESEARCH COMMITTEE
MEETING OF JANUARY 23, 1990

ARC 89-4C — OCCURRENCE SETTLEMENT PATTERNS

REFERENCES

ARC 89-4, Agenda & Minutes for Meeting of March 14, 1989
ARC 89-4A, Agenda & Minutes for Meeting of June 28, 1989
ARC 89-12, Agenda & Minutes for Meeting of June 28, 1989
ARC 89-13, Agenda & Minutes for Meeting of June 28, 1989
ARC 89-4B, Agenda & Minutes for Meeting of September 26, 1989

BACKGROUND

The increased limits procedure being developed is based on a model which separates data by year into "time of settlement" periods or lags for which severity distributions, trend parameters, and ultimately fitted trended curves are developed. ARC 89-4 began the analysis of the distribution of occurrences by settlement period on data organized by accident year, rather than by policy year.

At the June 28, 1989 meeting, results of fitting the full triangle with roof function models, that is, exponential models having piecewise linear mixing distributions (see ARC 89-4A) and of fitting individual years with mixed Cauchy models (see ARC 89-12) were presented.

At the September 26, 1989 meeting, results of fitting the full triangle with various mixed distribution models (see ARC 89-4B) were presented. The committee suggested using simpler actuarial techniques or models for fitting the available data and an exponential decay curve for the tail.

SIMPLE MODELS

Two simple models were tested: a three-year average link ratio model and a maximum likelihood estimation (MLE) of lag probabilities model (see ARC 89-4B and ARC 89-13). Staff then focused on testing various ways of splicing an exponential tail derived from the pre-1979 data to the available data for earlier lags.

RESULTS

Attachment I summarizes the results of staff's analysis of occurrence settlement patterns including results of the other attachments to the current item. This attachment exhibits the loss distribution by lag resulting from the occurrence settlement pattern obtained with the currently recommended procedure and the severity model. Attachment II gives the results of fitting an exponential tail to the available data for earlier lags. Attachment III presents the key results using the currently recommended settlement pattern procedure for the revised Products CGL Table 2 data.

The MLE model had a lower chi-squared total over all settled cells than the link ratio model. Analysis suggested a difference between the GLSP-data (pre-1979) and the CSP-data (post-1979). The MLE approach was applied to obtain separate

AGENDA
ACTUARIAL RESEARCH COMMITTEE
MEETING OF JANUARY 23, 1990

ARC 89-4C — OCCURRENCE SETTLEMENT PATTERNS

RESULTS (CONTINUED)

fits for the time spans 1973-1978 and 1980-1986. The combined results were the best achieved so far. Exponentials were fit to various tails of the earlier time span. The fit to six lags and beyond did best. But, when only the relativities for lags eight and on from this exponential tail were spliced to the MLE-derived relativities for the first seven lags, the fits were improved. When the exponential was used to project the open cases for 1973 before deriving the MLE lag probabilities, the fits were further improved.

STAFF RECOMMENDATION

THAT the Committee discuss this item and offer guidance for further investigations.

ATTACHMENTS TO AGENDA

I. Occurrence Settlement Patterns.

II. Exponential Tail Fit to Settlement Patterns.

III. Settlement Patterns and Exponential Tails for Revised Products CGL Table 2 Data.

MODEL BASED ON MLE OF EXPONENTIAL FIT
USING REVISED PRODUCTS CGL TABLE 2 DATA FROM ACCIDENT YEARS 1973-1978, LAGS 6-14

ACTUAL VALUES

YR/LAG	1	2	3	4	5	6	7	8	9	10	11	12	13	14	OPEN	TOT STL 6-LAST	TOTAL 6 & UP
1973	3,017	1,706	431	257	221	128	121	58	36	108	134	144	87	38	194	854	1,048
1974	2,991	2,119	473	303	259	189	138	113	91	86	88	40	42		360	787	1,147
1975	4,463	2,551	506	416	263	247	146	115	100	124	91	160			1,784	983	2,767
1976	4,302	2,619	627	411	402	292	203	156	148	97	124				544	1,020	1,564
1977	4,520	3,061	776	488	420	355	241	199	122	140					456	1,057	1,513
1978	5,151	2,929	761	524	489	487	419	234	239						819	1,379	2,198
MLE WT																	
73-78	0.4302	0.2637	0.0629	0.0422	0.0362	0.0299	0.0223	0.0154	0.0130	0.0125	0.0131	0.0147	0.0093	0.0057	0.0290	0.1357	0.1648
80-86	0.4072	0.2747	0.0749	0.0526	0.0434	0.0279	0.0248								0.0944	0.0527	0.1472

EXPONENTIAL FITTED VALUES

YR/LAG	1	2	3	4	5	6	7	8	9	10	11	12	13	14	OPEN	TOT STL 6-LAST	TOTAL 6 & UP
1973						159	137	118	102	88	76	66	57	49	313	854	1,167
1974						155	134	116	100	87	75	65	56		354	787	1,141
1975						209	180	156	135	116	100	87			552	983	1,535
1976						238	205	177	153	132	114				727	1,020	1,747
1977						277	240	207	179	154					982	1,057	2,039
1978						424	366	316	273						1,736	1,379	3,115
EXPO WT						0.0224	0.0194	0.0167	0.0145	0.0125	0.0108	0.0093	0.0081	0.0070	0.0442	0.1206	0.1648

CHI-SQUARED CONTRIBUTIONS (SIGNED)

YR/LAG	1	2	3	4	5	6	7	8	9	10	11	12	13	14	OPEN
1973						+6	+2	+31	+43	-4	-43	-92	-16	+3	+45
1974						-7	-0	+0	+1	+0	-2	+9	+3		-0
1975						-7	+7	+11	+9	-1	+1	-62			-2,754
1976						-12	+0	+3	+0	+9	-1				+46
1977						-22	-0	+0	+18	+1					+281
1978						-9	-8	+21	+4						+484

CHI-SQUARED CONTRIBUTIONS

YR/LAG	1	2	3	4	5	6	7	8	9	10	11	12	13	14	OPEN	TOT STL 6-LAST	TOTAL 6 & UP
1973						6	2	31	43	4	43	92	16	3	45	240	285
1974						7	0	0	1	0	2	9	3		0	23	24
1975						7.	7	11	9	1	1	62			2,754	96	2,850
1976						12	0	3	0	9	1				46	26	71
1977						22	0	0	18	1					281	42	323
1978						9	8	21	4						484	43	527
TOTAL															3,611	469	4,080

ALL-YEARS CURVE FIT

USING REVISED PRODUC[illegible] GL TABLE 2 DATA FROM ACCIDENT YEARS 1[illegible]-78 & 1980-86

Attachment 4
Page 4

MODEL APPLYING LAG-6 EXPONENTIAL TAIL TO LAG 8 & BEYOND

ACTUAL VALUES

ACCIDENT YEAR	1	2	3	4	5	6	7	8	9	10	11	12	13	14	TOTAL PAID	OUTSTANDING
1973	3,017	1,706	431	257	221	128	121	58	36	108	134	144	87	38	6,486	194
1974	2,991	2,119	473	303	259	189	138	113	91	86	88	40	42		6,932	360
1975	4,463	2,551	506	416	263	247	146	115	100	124	91	160			9,182	1,784
1976	4,302	2,619	627	411	402	292	203	156	148	97	124				9,381	544
1977	4,520	3,061	776	488	420	355	241	199	122	140					10,322	456
1978	5,151	2,929	761	524	489	487	419	234	239						11,233	819
1979	5,794	5,396	1,424	1,264	819	722	481	700							16,600	1,864
1980	8,851	6,360	1,689	1,232	1,060	597	557								20,346	2,122
1981	9,742	6,558	1,712	1,298	888	693									20,891	1,813
1982	9,958	6,144	1,663	1,216	1,080										20,061	2,438
1983	10,774	7,536	1,796	1,316											21,422	3,713
1984	9,324	6,121	2,082												17,527	4,594
1985	8,795	6,028													14,823	4,624
1986	6,388														6,388	5,817

FITTED VALUES

ACCIDENT YEAR	1	2	3	4	5	6	7	8	9	10	11	12	13	14	TOTAL PAID	OUTSTANDING
1973	2,874	1,762	420	282	241	200	149	118	102	88	76	66	57	49	6,486	313
1974	3,095	1,897	453	304	260	215	161	128	110	95	82	71	61		6,932	390
1975	4,136	2,536	605	406	348	287	215	170	147	127	110	95			9,182	604
1976	4,270	2,618	624	419	359	297	222	176	152	131	114				9,381	721
1977	4,756	2,916	695	467	400	330	247	196	169	146					10,322	930
1978	5,250	3,218	768	515	441	365	272	216	187						11,233	1,188
1979	7,318	4,936	1,347	946	781	502	446	325							16,600	2,066
1980	9,149	6,171	1,684	1,183	976	628	557								20,346	2,990
1981	9,658	6,515	1,777	1,248	1,030	662									20,891	3,744
1982	9,578	6,461	1,763	1,238	1,022										20,061	4,370
1983	10,777	7,269	1,983	1,393											21,422	6,067
1984	9,431	6,361	1,735												17,527	6,528
1985	8,852	5,971													14,823	7,756
1986	6,388														6,388	9,906

CHI-SQUARED CONTRIBUTIONS

ACCIDENT YEAR	1	2	3	4	5	6	7	8	9	10	11	12	13	14	OUTSTANDING
1973	-7	+2	-0	+2	+2	+26	+5	+31	+43	-4	-43	-92	-16	+3	+45
1974	+3	-26	-1	+0	+0	+3	+3	+2	+3	+1	-0	+14	+6		+2
1975	-26	-0	+16	-0	+21	+6	+22	+18	+15	+0	+3	-44			-2308
1976	-0	-0	-0	+0	-5	+0	+2	+2	+0	+9	-1				+44
1977	+12	-7	-9	-1	-1	-2	+0	-0	+13	+0					+241
1978	+2	+26	+0	-0	-5	-41	-79	-1	-14						+115
1979	+317	-43	-4	-107	-2	-96	-3	-432							+20
1980	+10	-6	-0	-2	-7	+1									+252
1981	-1	-0	+2	-2	+20	-1									+996
1982	-15	+16	+6	+0	-3										+854
1983	+0	-10	+18	+4											+913
1984	+1	+9	-69												+573
1985	+0	-1													+1265
1986	0														+1688

ALL-YEARS CURVE FIT
USING REVISED PRODUCTS CGL TABLE 2 DATA FROM ACCIDENT YEARS 1973-78 & 1980-86

MODEL APPLYING LAG-6 EXPONENTIAL TAIL TO LAG 8 & BEYOND

CHI-SQUARED CONTRIBUTIONS

ACCIDENT YEAR	1	2	3	4	5	6	7	8	9	10	11	12	13	14	TOT CHI SQ	STL OUTSTANDING
1973	7	2	0	2	2	26	5	31	43	4	43	92	16	3	276	45
1974	3	26	1	0	0	3	3	2	3	1	0	14	6		63	2
1975	26	0	16	0	21	6	22	18	15	0	3	44			171	2,308
1976	0	0	0	0	5	0	2	2	0	9	1				20	44
1977	12	7	9	1	1	2	0	0	13	0					46	241
1978	2	26	0	0	5	41	79	1	14						169	115
1979	317	43	4	107	2	96	3	432							1,005	20
1980	10	6	0	2	7	1	0								26	252
1981	1	0	2	2	20	1									26	996
1982	15	16	6	0	3										40	854
1983	0	10	18	4											32	913
1984	1	9	69												79	573
1985	0	1													1	1,265
1986	0														0	1,688
TOTALS: 73-86															1,954	9,316
W/O 79															950	9,296
W/O 75&79															778	6,988

LOSS DISTRIBUTION BY LAG RESULTING FROM
OCCURRENCE SETTLEMENT PATTERN AND SEVERITY MODELS
FOR PRODUCTS CGL TABLE 2 DATA

LAG J	OCCURRENCE SETTLEMENT DISTRIBUTION W(J)	LIMITED AVG SEVERITY (LIMIT=$500K) LAS(J)	LOSS DISTRIBUTION LD(J)	CUMULATIVE LOSS DISTRIBUTION CLD(J)
1	0.3920	1,981	5.39%	5.39%
2	0.2644	5,070	9.30%	14.68%
3	0.0721	22,814	11.41%	26.09%
4	0.0507	39,491	13.89%	39.98%
5	0.0418	29,828	8.65%	48.62%
6	0.0269	36,917	6.89%	55.51%
7	0.0239	36,297	6.02%	61.53%
8	0.0174	34,507	4.16%	65.69%
9	0.0151	44,970	4.71%	70.40%
10	0.0130	45,250	4.08%	74.48%
11	0.0112	45,532	3.54%	78.02%
12	0.0097	45,815	3.08%	81.10%
13	0.0084	46,100	2.69%	83.79%
14	0.0072	46,386	2.32%	86.10%
15	0.0063	46,675	2.04%	88.14%
16	0.0054	46,965	1.76%	89.90%
17	0.0047	47,256	1.54%	91.44%
18	0.0040	47,549	1.32%	92.76%
19	0.0035	47,844	1.16%	93.92%
20	0.0030	48,139	1.00%	94.92%
21	0.0026	48,438	0.87%	95.80%
22	0.0023	48,736	0.78%	96.58%
23	0.0019	49,037	0.65%	97.22%
24	0.0017	49,340	0.58%	97.80%
25	0.0015	49,643	0.52%	98.32%
26	0.0013	49,948	0.45%	98.77%
27	0.0011	50,255	0.38%	99.15%
28	0.0009	50,563	0.32%	99.47%
29	0.0008	50,874	0.28%	99.75%
30	0.0007	51,186	0.25%	100.00%
	0.9955	14,484	100.00%	

LOSS DISTRIBUTION BY LAG RESULTING FROM
OCCURRENCE SETTLEMENT PATTERN AND SEVERITY MODELS
FOR PRODUCTS CGL TABLE 2 DATA

LAG J	OCCURRENCE SETTLEMENT DISTRIBUTION W(J)	LIMITED AVG SEVERITY (LIMIT=$1M) LAS(J)	LOSS DISTRIBUTION LD(J)	CUMULATIVE LOSS DISTRIBUTION CLD(J)
1	0.3920	1,987	4.75%	4.75%
2	0.2644	5,219	8.42%	13.18%
3	0.0721	25,468	11.21%	24.39%
4	0.0507	45,437	14.06%	38.45%
5	0.0418	33,234	8.48%	46.93%
6	0.0269	41,799	6.86%	53.80%
7	0.0239	40,130	5.86%	59.65%
8	0.0174	39,651	4.21%	63.86%
9	0.0151	53,691	4.95%	68.81%
10	0.0130	54,047	4.29%	73.10%
11	0.0112	54,406	3.72%	76.82%
12	0.0097	54,768	3.24%	80.06%
13	0.0084	55,131	2.83%	82.89%
14	0.0072	55,496	2.44%	85.33%
15	0.0063	55,866	2.15%	87.48%
16	0.0054	56,236	1.85%	89.33%
17	0.0047	56,609	1.62%	90.96%
18	0.0040	56,984	1.39%	92.35%
19	0.0035	57,362	1.23%	93.57%
20	0.0030	57,741	1.06%	94.63%
21	0.0026	58,123	0.92%	95.55%
22	0.0023	58,507	0.82%	96.38%
23	0.0019	58,892	0.68%	97.06%
24	0.0017	59,282	0.62%	97.67%
25	0.0015	59,673	0.55%	98.22%
26	0.0013	60,066	0.48%	98.70%
27	0.0011	60,461	0.41%	99.10%
28	0.0009	60,859	0.33%	99.44%
29	0.0008	61,260	0.30%	99.74%
30	0.0007	61,662	0.26%	100.00%
	0.9955	16,455	100.00%	

MEASURING THE ADJUSTABLE FEATURES OF TREATIES (CAS SEMINAR ON RATEMAKING, MARCH, 1991

Robert A. Bear,
Jeffrey A. Englander, and
Todd J. Hess

MEASURING THE ADJUSTABLE FEATURES OF TREATIES

CAS SEMINAR ON RATEMAKING

MARCH 14-15, 1991

Appendices A and B present practical approaches to pricing the expected impact of adjustable features and loss sharing provisions of reinsurance treaties. A simple quota share example is used to illustrate methods of estimating the impact of aggregate deductibles, loss ratio caps and loss corridor provisions. This example is then used to evaluate profit and sliding scale commission plans and a retrospective rating plan. Appendix C presents models used to assess the cash flow implications of alternative adjustable features under consideration in an excess-of-loss example.

Panel: Robert A. Bear
North Star Reinsurance Corporation
Appendix A: Measuring the Expected Impact via Lognormal and Collective Risk Models

Jeffrey A. Englander
Trenwick America Reinsurance Corporation
Appendix B: Measuring the Expected Impact via Simulation

Todd J. Hess
Underwriters Reinsurance Company
Appendix C: Considering the Cash Flow

SUBJECT: ADJUSTABLE FEATURES AND LOSS SHARING PROVISIONS OF REINSURANCE TREATIES.

GOAL: BROADER UNDERSTANDING OF AVAILABLE APPROACHES TO ESTIMATE IMPACT OF THESE IMPORTANT TERMS.

PLAN: USE SIMPLE EXAMPLE TO ILLUSTRATE METHODS, WITH EMPHASIS ON CONCEPTS.

BENEFITS: (1) IMPROVED UNDERSTANDING BETWEEN ACTUARIES AND NON-ACTUARIES AND BETWEEN PRIMARY COMPANIES AND REINSURERS.

(2) GREATER PRICING ACCURACY.

ADJUSTABLE PREMIUM AND COMMISSION FEATURES

PREMIUM AND COMMISSION ADJUSTMENT PLANS WHOSE RESULTS DEPEND UPON ACTUAL TREATY LOSS EXPERIENCE OVER A PARTICULAR PERIOD.

EXAMPLES: RETROSPECTIVE RATING PLANS
PROFIT COMMISSION AND PROFIT-SHARING PLANS
SLIDING SCALE COMMISSION PLANS

GOAL: DETERMINE EXPECTED ADJUSTED PREMIUM RATE OR COMMISSION RATIO FOR TREATY.

LOSS SHARING PROVISIONS (NONPROPORTIONAL COINSURANCE)

CEDING COMPANY PAYS NONPROPORTIONAL SHARE OF LOSSES.
DOES NOT RECEIVE SHARE OF REINSURANCE PREMIUM.

EXAMPLES: AGGREGATE DEDUCTIBLES
AGGREGATE LIMITS
LOSS RATIO CAPS AND LIMITED REINSTATEMENTS
LOSS CORRIDOR PROVISIONS

GOAL: ESTIMATE PROPORTION OF LOSSES OTHERWISE SUBJECT TO TREATY WHICH ARE RETAINED BY CEDANT.

THIS PERMITS ESTIMATION OF EXPECTED REINSURANCE LOSSES AFTER LOSS SHARING PROVISION.

REFERENCES

(1) "PRICING THE IMPACT OF ADJUSTABLE FEATURES AND LOSS SHARING PROVISIONS OF REINSURANCE TREATIES," R.A. BEAR AND K.J. NEMLICK, 1990 PCAS. (PRELIMINARY VERSIONS WERE PRESENTED AT 1990 DISCUSSION PAPER PROGRAM AND AT CAS CONVENTION.)

(2) "THE CALCULATION OF AGGREGATE LOSS DISTRIBUTIONS FROM CLAIM SEVERITY AND CLAIM COUNT DISTRIBUTIONS," P.E. HECKMAN AND G.G. MEYERS, 1983 PCAS.

(3) "PRICING *EXCESS-OF-LOSS* CASUALTY WORKING COVER REINSURANCE TREATIES," G.S. PATRIK AND R.T. JOHN, 1980 CAS DISCUSSION PAPER.

(4) "ESTIMATING PURE PREMIUMS BY LAYER," R.J. FINGER, 1976 PCAS.

BACKGROUND

1) CONCENTRATION OF W.C. EXPOSURE IN HOMOGENEOUS CLASS.

2) INSURER HAS EXCESS OF LOSS COVER ABOVE $250,000;
ALAE PART OF LOSS.

3) NET SUBJECT MATTER PREMIUM = $9,000,000 IN THIS W.C. CLASS.

4) SEEKS ADDITIONAL QUOTA SHARE COVERAGE
FOR 1991 UNDERWRITING YEAR.

ACTUARIAL ASSUMPTIONS

(1) EXPECTED CLAIM FREQUENCY = 85 CLAIMS / $1M.

(2) CLAIM SEVERITY (INDEMNITY + ALAE) IS MODELED

BY WEIBULL WITH SHAPE = .2 AND SCALE = 171.

$$F(X) = 1 - E^{-T}$$

$$\text{WHERE } T = \left(\frac{X}{171}\right)^{.2}$$

UNLIMITED MEAN SEVERITY = $20,520

MEAN LIMITED SEVERITY ($250,000) = $8,796

(1) AND (2) IMPLY EXPECTED LOSS & ALAE RATIO = 75%.

ACTUARIAL ASSUMPTIONS - CONTINUED

(3) CLASS IS HAZARD GROUP III; COUNTRYWIDE NCCI TABLE M IS EFFECTIVE WITH 1990 TABLE OF EXPECTED LOSS RANGES.

(4) ALAE IS ONLY 5% OF INDEMNITY AND A SMALL PORTION OF CLAIMS EXCEED $250,000.
HENCE, TABLE M PROVIDES A ROUGH APPROXIMATION OF EMPIRICAL INSURANCE CHARGES.

(3) AND (4) MAY BE USED OR IGNORED BY PANELISTS.

(5) PARAMETER UNCERTAINTY IS SIGNIFICANT. PANELISTS ARE ENCOURAGED TO CONSIDER AND REFLECT IT IN THEIR ANALYSES.

NON-PROPORTIONAL COINSURANCE ALTERNATIVES

CEDANT IS CONSIDERING THREE LOSS SHARING PROVISIONS.
FOR EACH, ESTIMATE EXPECTED LOSS AND ALAE RATIO TO REINSURER.

(1) AGGREGATE DEDUCTIBLE = $5,400,000 (80% OF EXPECTED LOSS & ALAE).

(2) 90% LOSS AND ALAE RATIO CAP.

(3) CEDING COMPANY WILL PAY ALL LOSSES AND ALAE BETWEEN 75% AND 112.5% OF SUBJECT PREMIUM (LOSS CORRIDOR).

ADJUSTABLE FEATURES ALTERNATIVES

NO COINSURANCE APPLIES, SO EXPECTED LOSS AND ALAE RATIO IS 75%.

EACH OF THREE PLANS WILL BE EVALUATED BASED SOLELY ON 1991 UNDERWRITING YEAR EXPERIENCE.

(1) 50% PROFIT COMMISSION TO CEDANT AFTER 25% FOR REINSURER'S OVERHEAD AND PROFIT. WHAT IS EXPECTED PROFIT COMMISSION ?

ADJUSTABLE FEATURES - CONTINUED

(2) PROVISIONAL CEDING COMMISSION TO BE NEGOTIATED.
SUPPLEMENTAL SLIDING SCALE COMMISSION BASED ON FOLLOWING:

LOSS & ALAE RATIO INTERVAL LOWER BOUND	LOSS & ALAE RATIO INTERVAL UPPER BOUND	PERCENTAGE INCREASE IN COMMISSION RATIO PER 1% DECREASE IN LOSS & ALAE RATIO	CORRESPONDING COMMISSION RATIO INTERVAL LOWER BOUND	CORRESPONDING COMMISSION RATIO INTERVAL UPPER BOUND
75.00%	AND ABOVE	0.00%	0.00%	0.00%
60.00%	75.00%	0.50%	7.50%	0.00%
45.00%	60.00%	0.60%	16.50%	7.50%
30.00%	45.00%	0.75%	27.75%	16.50%
0.00%	30.00%	1.00%	57.75%	27.75%

WHAT IS THE EXPECTED SLIDING SCALE COMMISSION ?

ADJUSTABLE FEATURES - CONTINUED

(3) RETROSPECTIVE RATING PLAN:

QUOTA SHARE CESSION TREATED AS PROVISIONAL PREMIUM.

CEDENT WILLING TO PAY 30% MORE OR LESS BASED ON TREATY EXPERIENCE.

FORMULA:

REINSURANCE RATE = (LOSS & ALAE RATIO) + (25% MARGIN)

70% < REINSURANCE RATE < 130%

RETROSPECTIVE PREMIUM = (REINSURANCE RATE) x (PROVISIONAL PREMIUM)

NO DOWNWARD ADJUSTMENTS FOR 5 YEARS.

ANY PROVISIONAL COMMISSION PAID OUT OF FLAT MARGIN.

WHAT IS ULTIMATE EXPECTED REINSURANCE RATE ?

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TECHNICAL REQUIREMENT:

DISTRIBUTION OF AGGREGATE TREATY LOSSES

APPROACHES:

(1) COLLECTIVE RISK MODEL

(2) LOGNORMAL MODEL

(3) TABLE M

COLLECTIVE RISK MODEL
(THE HECKMAN-MEYERS ALGORITHM)

(1) EFFICIENTLY SIMULATES AGGREGATE LOSS DISTRIBUTION BASED ON CLAIM FREQUENCY AND SEVERITY DISTRIBUTIONS.

(2) REFLECTS UNCERTAINTY IN EXPECTED CLAIM FREQUENCY THROUGH CONTAGION PARAMETER c.

c = 0 : NO PARAMETER UNCERTAINTY.
c = .05 - .10 : MODERATE PARAMETER UNCERTAINTY.
c = .25 : HIGH PARAMETER UNCERTAINTY.

(3) REFLECTS UNCERTAINTY IN AVERAGE CLAIM SEVERITY THROUGH MIXING PARAMETER b.

b = 0 : NO PARAMETER UNCERTAINTY.
b = .05 - .10 : MODERATE PARAMETER UNCERTAINTY.
b = .25 : HIGH PARAMETER UNCERTAINTY.

THE LOGNORMAL MODEL

ASSUMPTION: AGGREGATE LOSS IS PRODUCT OF LARGE NUMBER OF INDEPENDENT, IDENTICALLY* DISTRIBUTED VARIABLES.

CONCLUSION: THE LOGARITHM IS APPROXIMATELY NORMALLY DISTRIBUTED (CENTRAL LIMIT THEOREM).

IMPLICATION: AGGREGATE LOSS IS LOGNORMALLY DISTRIBUTED.

* THE STRINGENT CONDITION THAT THE FACTORS BE IDENTICALLY DISTRIBUTED MAY BE RELAXED.

REQUIREMENT OF LOGNORMAL MODEL:

COEFFICIENT OF VARIATION

$$= \frac{\text{STANDARD DEVIATION}}{\text{MEAN}}$$

$$= \frac{\sqrt{\text{VARIANCE OF AGGREGATE LOSSES}}}{\text{EXPECTED AGGREGATE LOSS}}$$

COMPONENTS COMPUTED BASED ON FREQUENCY AND SEVERITY DISTRIBUTIONS.

DEFINITIONS:

(1) EXCESS PURE PREMIUM:

EXPECTED AGGREGATE LOSSES EXCESS OF ATTACHMENT.

THE ATTACHMENT COULD BE AGGREGATE DEDUCTIBLE VALUE OR AGGREGATE LIMIT UNDER CONSIDERATION.

(2) EXCESS PURE PREMIUM RATIO:

RATIO OF EXCESS PURE PREMIUM TO EXPECTED AGGREGATE LOSS.

(3) ENTRY RATIO:

RATIO OF ATTACHMENT TO EXPECTED AGGREGATE LOSS.

IMPORTANT RESULT:

IF AGGREGATE LOSS DISTRIBUTION IS LOGNORMAL, A SIMPLE FORMULA EXISTS TO COMPUTE THE EXCESS PURE PREMIUM RATIO FOR ANY ATTACHMENT.

YOU NEED TO KNOW THE EXPECTED AGGREGATE LOSS AND THE COEFFICIENT OF VARIATION OF THE AGGREGATE LOSS DISTRIBUTION.

THE BEAR-NEMLICK PAPER SUMMARIZES TECHNICAL DETAILS AND PROVIDES TABLES OF EXCESS PURE PREMIUM RATIOS FOR COEFFICIENTS OF VARIATION BETWEEN .1 AND 5.

COMPUTATION OF EXCESS PURE PREMIUMS

(1) WITHOUT PARAMETER UNCERTAINTY

EXCESS PURE PREMIUM FOR PARTICULAR ATTACHMENT
= EXPECTED AGGREGATE LOSS X EXCESS PURE PREMIUM RATIO

(2) WITH PARAMETER UNCERTAINTY

(a) ESTIMATE EXCESS PURE PREMIUMS BASED ON ALTERNATIVE FREQUENCY AND SEVERITY ASSUMPTIONS.

(b) ASSIGN SUBJECTIVE PROBABILITIES TO EACH SCENARIO IN (a).

(c) THE UNCONDITIONAL EXCESS PURE PREMIUM IS THE WEIGHTED AVERAGE OF THE CONDITIONAL EXCESS PURE PREMIUMS IN (a), BASED ON THE WEIGHTS IN (b).

TABLE M: TABLE OF INSURANCE CHARGES
(EXCESS PURE PREMIUM RATIOS AND CORRESPONDING SAVINGS)

INSURED IS ASSIGNED TO EXPECTED LOSS GROUP BASED UPON ANNUAL EXPECTED LOSSES. ASSIGNMENTS ADJUSTED ANNUALLY.

INSURANCE CHARGES AND SAVINGS ARE GIVEN IN TABLES AS A FUNCTION OF THE EXPECTED LOSS GROUP AND ENTRY RATIO.

TABLE M IS BASED ON NCCI STUDY OF EMPIRICAL WORKER'S COMPENSATION INDIVIDUAL RISK AGGREGATE LOSS DATA.

TABLE M IS USED TO ESTIMATE NET INSURANCE CHARGES OF RETROSPECTIVE RATING PLANS.

PARAMETERS OF ALTERNATIVE APPROACHES
PRIOR TO ADJUSTABLE FEATURES AND LOSS SHARING PROVISIONS

(1) COLLECTIVE RISK MODEL

(a) EXPECTED CLAIMS = 765

(b) AVERAGE CLAIM COST = 8831
(FROM PIECEWISE LINEAR FIT TO WEIBULL CENSORED AT $250,000)

(c) CONTAGION PARAMETER c = .10

(d) MIXING PARAMETER b = .05

(2) LOGNORMAL MODEL

(a) EXPECTED AGGREGATE LOSS = .75 x $9,000,000 = $6,750,000

(b) COEFICIENT OF VARIATION = .423 (FROM COLLECTIVE RISK MODEL)

(3) TABLE M

EXPECTED LOSS GROUP = 16

AGGREGATE DEDUCTIBLES

REINSURER PAYS NOTHING UNTIL TREATY LOSSES EXCEED SPECIFIED AMOUNT ($5,400,000 IN EXAMPLE).

THE REINSURER THEN PAYS ALL LOSSES SUBJECT TO TREATY.

EXPECTED TREATY LOSSES AFTER AGGREGATE DEDUCTIBLE
= EXPECTED LOSSES x [100% - LOSS ELIMINATION RATIO]

WHERE LOSS ELIMINATION RATIO = 100% - XSPPR(D)
AND XSPPR(D) = EXCESS PURE PREMIUM RATIO CORRESPONDING TO AGGREGATE DEDUCTIBLE.

EXPECTED TREATY LOSSES AFTER AGGREGATE DEDUCTIBLE
= EXPECTED LOSSES x XSPPR(D).

CALCULATING THE IMPACT OF THE AGGREGATE DEDUCTIBLE - EXAMPLE

(1) AGGREGATE DEDUCTIBLE IN DOLLARS = $5,400,000

(2) EXPECTED TREATY LOSSES AND ALAE BEFORE COINSURANCE = $6,750,000

(3) ENTRY RATIO CORRESPONDING TO AGGREGATE DEDUCTIBLE = $\frac{\$5,400,000}{\$6,750,000}$ = .8

	RISK MODEL	LOGNORMAL	TABLE M
(4) EXCESS PURE PREMIUM RATIO:	27.0%	26.6%	26.2%
(5) PORTION OF TREATY LOSSES ELIMINATED:	73.0%	73.4%	73.8%
(6) EXPECTED TREATY LOSS RATIO AFTER AGGREGATE DEDUCTIBLE:	20.3%	19.9%	19.7%

LOSS RATIO CAP

REINSURER PAYS FOR ALL TREATY LOSSES UP TO LOSS RATIO CAP (90% IN EXAMPLE).

EXPECTED TREATY LOSSES AFTER LOSS RATIO CAP
= EXPECTED LOSSES x [100% - LOSS ELIMINATION RATIO]

WHERE LOSS ELIMINATION RATIO = XSPPR(C)
= EXCESS PURE PREMIUM RATIO AT LOSS RATIO CAP C

CALCULATING THE IMPACT OF THE LOSS RATIO CAP - EXAMPLE

(1) LOSS RATIO CAP IN DOLLARS = .9 x \$9,000,000 = \$8,100,000

(2) EXPECTED TREATY LOSSES AND ALAE BEFORE COINSURANCE = \$6,750,000

(3) ENTRY RATIO CORRESPONDING TO LOSS RATIO CAP = $\frac{\$8,100,000}{\$6,750,000}$ = 1.2

	RISK MODEL	LOGNORMAL	TABLE M
(4) EXCESS PURE PREMIUM RATIO	9.4%	9.4%	9.8%
(5) LOSS ELIMINATION RATIO	9.4%	9.4%	9.8%
(6) EXPECTED TREATY LOSS RATIO AFTER LOSS RATIO CAP	68.0%	67.9%	67.7%

LOSS CORRIDORS

REINSURER PAYS FOR TREATY LOSSES UNTIL FIXED AMOUNT LB IS REACHED.

REINSURER STOPS PAYING LOSSES UNTIL TOTAL REACHES SECOND FIXED AMOUNT, UB.

REINSURER RESUMES PAYING LOSSES WHEN TOTAL EXCEEDS UB.

LOSS CORRIDOR = INTERVAL BETWEEN LB AND UB.

CALCULATING THE IMPACT OF THE LOSS CORRIDOR PROVISION

EXPECTED TREATY LOSSES AND ALAE AFTER LOSS CORRIDOR PROVISION =
EXPECTED LOSSES AND ALAE x [100%-LOSS ELIMINATION RATIO]

WHERE LOSS ELIMINATION RATIO = XSPPR(LB) - XSPPR(UB)

AND XSPPR(LB) = EXCESS PURE PREMIUM RATIO AT LB
(75% OF SUBJECT PREMIUM IN EXAMPLE)

XSPPR(UB) = EXCESS PURE PREMIUM RATIO AT UB
(112.5% OF SUBJECT PREMIUM IN EXAMPLE)

CALCULATING THE IMPACT OF THE LOSS CORRIDOR PROVISION - EXAMPLE

(1) LOWER BOUND OF LOSS CORRIDOR = .75 x $9,000,000 = $6,750,000

(2) UPPER BOUND OF LOSS CORRIDOR = 1.125 x $9,000,000 = $10,125,000

(3) EXPECTED TREATY LOSSES AND ALAE BEFORE COINSURANCE = $6,750,000

(4) ENTRY RATIO CORRESPONDING TO LOWER BOUND = $\frac{\$6,750,000}{\$6,750,000}$ = 1.0

(5) ENTRY RATIO CORRESPONDING TO UPPER BOUND = $\frac{\$10,125,000}{\$6,750,000}$ = 1.5

	RISK MODEL	LOGNORMAL	TABLE M
(6) EXCESS PURE PREMIUM RATIOS:			
LOWER BOUND	16.3%	16.1%	16.0%
UPPER BOUND	3.5%	4.1%	5.0%
(7) LOSS ELIMINATION RATIO	12.4%	12.0%	11.0%
(8) EXPECTED TREATY LOSS RATIO AFTER LOSS CORRIDOR	85.7%	86.0%	86.8%

PROFIT COMMISSIONS

PROFIT COMMISSION RATIO = P x [100% - LR - EXP]

WHERE P = PROPORTION OF PROFITS TO BE PAID TO CEDANT
(50% IN EXAMPLE)
LR = ACTUAL TREATY LOSS RATIO

EXP = REINSURER'S OVERHEAD PROVISION
(25% OF TREATY PREMIUM IN EXAMPLE)

THE PROFIT COMMISSION RATIO CANNOT BE NEGATIVE.
LOSS RATIOS ENTERING THE PROFIT COMMISSION FORMULA ARE CAPPED AT BREAKEVEN LOSS RATIO.

BLR = 100% - EXP

GOAL: TO DETERMINE THE EXPECTED PROFIT COMMISSION TO BE PAID.

METHOD: DETERMINE EFFECT THAT LIMITING ACTUAL LOSS RATIOS TO THE BREAKEVEN RATIO HAS ON THE EXPECTED TREATY LOSS RATIO USED IN PROFIT COMMISSION RATIO CALCULATION.

CALCULATING THE EXPECTED PROFIT COMMISSION RATIO

FELR = EXPECTED TREATY LOSS RATIO USED IN PROFIT COMMISSION FORMULA

= EXPECTED LOSS RATIO X [100% - LOSS ELIMINATION RATIO]

LOSS ELIMINATION RATIO = XSPPR(BLR)
= EXCESS PURE PREMIUM RATIO AT BREAKEVEN LOSS RATIO

ECR = EXPECTED PROFIT COMMISSION RATIO = P x [100% - FELR - EXP]

THE EXPECTED PROFIT COMMISSION RATIO WILL ALWAYS EXCEED THAT OBTAINED BY SIMPLY PLUGGING THE EXPECTED LOSS RATIO INTO THE PROFIT COMMISSION FORMULA.

CALCULATING THE EXPECTED PROFIT COMMISSION RATIO - EXAMPLE

(1) PROPORTION OF PROFITS TO BE PAID TO CEDANT = 50%

(2) REINSURER'S OVERHEAD PROVISION = EXP = 25%

(3) EXPECTED TREATY LOSS RATIO = ELR = 75%

(4) BREAKEVEN LOSS RATIO (BLR) = 100%-EXP = 75%

(5) ENTRY RATIO CORRESPONDING TO BREAKEVEN LOSS RATIO = $\frac{\text{BLR}}{\text{ELR}} = \frac{75\%}{75\%} = 1.0$

	RISK MODEL	LOGNORMAL	TABLE M
(6) EXCESS PURE PREMIUM RATIO AT BREAKEVEN LOSS RATIO	16.3%	16.1%	16.0%
(7) LOSS ELIMINATION RATIO = XSPPR(BLR)	16.3%	16.1%	16.0%
(8) EXPECTED TREATY LOSS RATIO USED IN COMMISSION FORMULA FELR = ELR x [100% - LOSS ELIMINATION RATIO]	62.8%	62.9%	63.0%
(9) EXPECTED PROFIT COMMISSION RATIO ECR = P x [100% - FELR - EXP]	6.1%	6.0%	6.0%
(10) SIMPLISTIC PROFIT COMMISSION RATIO (PLUG ELR INTO FORMULA)	0.0%	0.0%	0.0%

SLIDING SCALE COMMISSIONS

EXAMPLE: PROVISIONAL CEDING COMMISSION TO BE NEGOTIATED.
SUPPLEMENTAL SLIDING SCALE COMMISSION BASED ON FOLLOWING PLAN:

LOSS & ALAE RATIO INTERVAL		PERCENTAGE INCREASE IN COMMISSION RATIO PER 1% DECREASE IN LOSS & ALAE RATIO	CORRESPONDING COMMISSION RATIO INTERVAL	
LOWER BOUND	UPPER BOUND		LOWER BOUND	UPPER BOUND
75%	AND ABOVE	0.00%	0.00%	0.00%
60%	75%	0.50%	7.50%	0.00%
45%	60%	0.60%	16.50%	7.50%
30%	45%	0.75%	27.75%	16.50%
0%	30%	1.00%	57.75%	27.75%

THE SLIDING SCALE COMMISSION MAY BE EXPRESSED USING PIECEWISE LINEAR FORMULA:

LOSS AND ALAE RATIO (L)	COMMISSION RATIO (C)
ABOVE 75%	0%
60% - 75%	.5 x [75% - L]
45% - 60%	7.5% + ,6 x [60% - L]
30% - 45%	16.5% + .75 x [45% - L]
0% - 30%	27.75% + 1.0 x [30% - L]

CALCULATION OF THE EXPECTED SLIDING SCALE COMMISSION

EXPECTED COMMISSION RATIO

= Cmax - EXPECTED COMMISSION REDUCTIONS

OVER ALL LOSS RATIO INTERVALS

$$= \text{Cmax} - \sum_{i=1}^{n} \text{Bi} \times [\text{EXPECTED LOSS RATIO POINTS IN i-th INTERVAL}]$$

WHERE BI = COMMISSION SLIDE ON I-th LOSS RATIO INTERVAL
(% INCREASE IN COMMISSION RATIO PER 1% DECLINE IN LOSS RATIO)
AND Cmax = MAXIMUM COMMISSION RATIO

CALCULATION OF THE EXPECTED SLIDING SCALE COMMISSION - CONTINUED

THE EXPECTED SLIDING SCALE COMMISSION RATIO EQUALS THE MAXIMUM COMMISSION RATIO LESS THE EXPECTED POINTS OF COMMISSION LOST OVER THE ENTIRE RANGE OF POSSIBLE LOSS RATIOS.

EXPECTED LOSS RATIO POINTS IN i-th INTERVAL

$$= \text{ELR} \times [\text{XSPPR}(\text{LBi}) - \text{XSPPR}(\text{UBi})]$$

WHERE XSPPR(LBi) AND XSPPR(UBi) ARE EXCESS PURE PREMIUM RATIOS CORRESPONDING TO THE LOWER AND UPPER ENDPOINTS OF i-th LOSS RATIO INTERVAL.

CALCULATION OF SLIDING SCALE COMMISSION - EXAMPLE

(1) EXPECTED LOSS AND ALAE RATIO = 75%

(2) MAXIMUM COMMISSION RATIO = 57.75%

	RISK MODEL	LOGNORMAL	TABLE M
(3) EXPECTED COMMISSION REDUCTIONS	50.85%	51.04%	51.14%
(4) EXPECTED COMMISSION RATIO	6.90%	6.71%	6.61%
(5) SIMPLISTIC SLIDING SCALE COMMISSION (PLUG ELR INTO FORMULA)	0%	0%	0%

RETROSPECTIVE RATING PLAN

FORMULA:

REINSURANCE RATE = (LOSS & ALAE RATIO) + (25% MARGIN)

RMIN = 70% < REINSURANCE RATE < 130% = RMAX

RETROSPECTIVE PREMIUM = (REINSURANCE RATE) x (PROVISIONAL PREMIUM)

CONSTRAINT ON LOSS AND ALAE RATIO (LR) USED IN RATE CALCULATION:

RMIN < LR + MARGIN < RMAX

CALCULATING THE LOSS RATIOS

CORRESPONDING TO MINIMUM AND MAXIMUM RATES

CORRESPONDING TO RMIN AND RMAX ARE

MINIMUM AND MAXIMUM LOSS RATIOS, LMIN AND LMAX.

LMIN = RMIN - MARGIN = 70% - 25% = 45%

LMAX = RMAX - MARGIN = 130% - 25% = 105%

INSURANCE CHARGES AND SAVINGS

IF LR < LMIN, REINSURANCE COMPANY CHARGES FOR LMIN AND REALIZES SAVINGS DUE TO FAVORABLE LOSS EXPERIENCE.

IF LR > LMAX, REINSURANCE COMPANY CHARGES FOR LMAX AND INCURS A LOSS DUE TO ADVERSE LOSS EXPERIENCE.

WE NEED TO DETERMINE EFFECT THAT LIMITING LR BETWEEN LMIN AND LMAX HAS ON THE EXPECTED LOSS RATIO USED IN THE RETROSPECTIVE RATING FORMULA.

CALCULATING THE NET INSURANCE CHARGE

NET INSURANCE CHARGE (NIC) = XSPPR(LMAX) - SAVE(LMIN)

WHERE XSPPR(LMAX) = INSURANCE CHARGE AT MAXIMUM LOSS RATIO

AND SAVE(LMIN) = INSURANCE SAVINGS AT MINIMUM LOSS RATIO

NOTE: SAVE(LMIN) = XSPPR(LMIN) + ER(LMIN) - 100%

WHERE ER(LMIN) = ENTRY RATIO AT MINIMUM LOSS RATIO

CALCULATING THE EXPECTED REINSURANCE RATE

LET AELR = ADJUSTED EXPECTED LOSS RATIO

AELR IS THE EXPECTED LOSS RATIO SUBJECT TO THE MINIMUM AND MAXIMUM LOSS RATIO CONSTRAINTS, LMIN AND LMAX.

$$AELR = ELR \times [100\% - NIC]$$

NIC IS THE LOSS ELIMINATION RATIO WHICH ARISES DUE TO LMIN AND LMAX.

EXPECTED REINSURANCE RATE = AELR + MARGIN

CALCULATING THE EXPECTED REINSURANCE RATE - EXAMPLE

(1) EXPECTED LOSS AND ALAE RATIO (ELR) = 75%

(2) REINSURER'S PROVISIONAL MARGIN = 25%

(3) MINIMUM LOSS RATIO (LMIN) = 45%

(4) ENTRY RATIO CORRESPONDING TO LMIN = .6

(5) MAXIMUM LOSS RATIO (LMAX) = 105%

(6) ENTRY RATIO CORRESPONDING TO LMAX = 1.4

CALCULATING THE EXPECTED REINSURANCE RATE - CONTINUED

	RISK MODEL	LOGNORMAL	TABLE M
(7) INSURANCE CHARGE AT LMAX	5.2%	5.4%	6.1%
(8) INSURANCE SAVINGS AT LMIN	1.9%	1.5%	1.3%
(9) NET INSURANCE CHARGE (NIC)	3.3%	3.9%	4.8%
(10) ADJUSTED EXPECTED LOSS RATIO	72.5%	72.1%	71.4%
AELR = ELR x [100% - NIC]			
(11) EXPECTED REINSURANCE RATE	97.5%	97.1%	96.4%
AELR + MARGIN			
(12) EXPECTED ULTIMATE MARGIN	22.5%	22.1%	21.4%

IMPORTANCE OF MODELLING PARAMETER UNCERTAINTY

EVEN IF DONE SUBJECTIVELY

THREE APPROACHES GAVE SIMILAR INDICATIONS

FOR ALL COINSURANCE AND ADJUSTABLE FEATURES ALTERNATIVES STUDIED

SIGNIFICANT PARAMETER RISK WAS REFLECTED IN COLLECTIVE RISK MODEL.

THIS WAS SIMILARLY REFLECTED IN LOGNORMAL MODEL THROUGH

SELECTION OF COEFFICIENT OF VARIATION.

IMPORTANCE OF MODELLING PARAMETER UNCERTAINTY - CONTINUED

ALTERNATIVELY, ONE COULD HAVE USED METHOD OF

WEIGHTING SCENARIOS TO REFLECT PARAMETER UNCERTAINTY.

EMPIRICAL TABLE M APPROACH HAS THEORETICAL SHORTCOMINGS

BUT PROVIDES REASONABILITY CHECK ON THEORETICAL METHODS.

ADDITIONAL ISSUES

USED SIMPLE EXAMPLE TO ILLUSTRATE CONCEPTS

REFER TO BEAR-NEMLICK PAPER
FOR DISCUSSION OF FOLLOWING COMPLEXITIES:

(1) VARIATION OF LAYER RETENTIONS AND LIMITS BY LINE OF BUSINESS OR OVER MULTI-YEAR RATING BLOCK.

(2) HANDLING OF ALAE.

(3) TREATIES WITH BOTH COINSURANCE PROVISIONS AND ADJUSTABLE FEATURES.

(4) TREATIES WITH SIGNIFICANT PROBABILITY OF LOSS-FREE YEAR (EG, HIGH LAYERS).

(5) CASH FLOW MODELLING.

THE PROBLEM

* Primary workers compensation quota share reinsurance cover ($250,000 limit)

* Allocated loss adjustment expenses included with losses

* Subject premium = $9,000,000

* Based upon other analysis, expected claim frequency is 85 claims per $1,000,000 subject premium, or 765 claims

* Based upon other analysis, unlimited severity distribution can be assumed to be Weibull with parameters 1/171 and .2

$$F(x) = 1 - \exp(-((x/171)^{.2}))$$

GOAL: Calculate the expected outcome to the reinsurer under several different structures involving adjustable features

ALTERNATIVE STRUCTURES

Non-Proportional Coinsurance Features:

(1) Aggregate deductible of $5,400,000

(2) 90% loss and ALAE ratio cap to reinsurer

(3) Loss corridor retained by ceding company between 75% and 112.5% loss and ALAE ratio

Retrospectively Adjustable Features:

(4) 50% profit commission after 25% reinsurer's expense allowance

(5) Contingent sliding scale commission, depending on loss & ALAE ratio:

Interval	Commission
>75%	0.0%
60%-75%	.5 x (75% - LR)
45%-60%	.6 x (60% - LR) + 7.50%
30%-45%	.75 x (45% - LR) + 16.50%
<30%	(30% - LR) + 27.75%

(6) Retrospective premium adjustment = LR + 25%
subject to min of 70% and max of 130%

KEY TO THE SOLUTION

Need to estimate the aggregate loss distribution to determine the effect of adjustable features on expected results.

It is insufficient to apply the adjustable features to the expected outcomes before adjustment, due to the effect the adjustments have on the distribution of outcomes.

SEVERAL APPROACHES

* Use an empirical aggregate loss distribution deemed to be appropriate (eg., NCCI Table M)

* Assume some form of the distribution of aggregate losses (eg., lognormal), then estimate the parameters from empirical data

* Collective Risk Model - estimate the aggregate loss distribution from the underlying claim frequency and severity distributions, using one following methods:

  - Assume some form of the distribution of aggregate losses, then estimate the moments from the moments of the frequency and severity distributions

  - Monte Carlo simulation

  - Other methods
    1. Inversion of the characteristic function of the aggregate loss distribution (Heckman-Myers)
    2. Inversion of the Laplace transform of the aggregate loss distribution (recursive method, Panjer)

MONTE CARLO SIMULATION OVERVIEW

Basic steps:

(1) Specify underlying claim frequency and severity distributions.

(2) Randomly generate a number of claims for a sample year from the assumed claim count distribution.

(3) For each claim drawn in step (2), randomly generate a claim size from the assumed claim size distribution, applying any per claim limit, if applicable.

(4) Accumulate each claim's results to get the year's total losses; use the accumulated results to determine effects of adjustable features for that year.

(5) Repeat the simulation for a large number of years, accumulating the results of each year to use in calculating overall expected effects of adjustable features.

CLAIM FREQUENCY DISTRIBUTION

Normal choices are:

- Poisson

  f(n|r) = r^n * exp(r)/n!

  where r is the expected number of claims
  mean = r, variance = r

- Negative Binomial

  f(n|m,k) = (k/(k+m))^k * (m/(k+m))^n * k(k+1)...(k+n-1)/n!

  mean = m, variance = m + m^2/k

CLAIM FREQUENCY DISTRIBUTION (Continued)

The Poisson distribution is usually thought of as a reasonable starting point for the claim process.

However, if we want to reflect parameter risk (ie., the fact that there is uncertainty in our estimate of the expected number claims), the Negative Binomial has been found to be a better model, with the parameter k used to reflect the level of parameter risk desired in the Poisson process.

While there are mathematical ways to estimate the correct k, we prefer a more intuitive approach:

Split the simulation runs into 5 equal parts. Vary the expected number of claims for each part in such a way that the average over the five parts is the desired expected number of claims. In our example, with a given expected number of claims of 765, we would reflect parameter uncertainty in the claim count distribution by using the following expected counts in each part:

765	x	0.50	=	382.50
765	x	0.75	=	573.75
765	x	1.00	=	765.00
765	x	1.25	=	956.25
765	x	1.50	=	1147.50

The spread used (.5,.75,1,1.25,1.5) is based upon a "comfort level" with repect to the underlying pricing analysis.

CLAIM FREQUENCY DISTRIBUTION (Continued)

A Negative Binomial Equivalent:

It is easy to show that the variance in the claim count distribution for all 5 parts combined is equal to the "between-group" variance plus the "within-group" variance, or:

Within-group variance	=	765.000
Between-group variance	=	73,153.125
Total variance	=	73,918.125
	=	765 + 765^2/k for a negative binomial equivalent

so that k = 8.

Alternatively, if we consider the variance of the spread (.5,.75,1,1.25,1.5), which is .125, we again have k = 1/.125 = 8

This leads to the more general statement that:

k = 1 / variance of spread

CLAIM SEVERITY DISTRIBUTION

Given: Uncapped severities can be expected to follow a Weibull distribution, with shape parameter of .2 and scale parameter of 171.

- After drawing severities from specified distribution, apply $250,000 per occurrence limit

- Parameter uncertainty ignored in the severity distribution

  * Variance of outcomes seems more sensitive to frequency
  * A little tougher to model in the severity distribution, given the curve we're using

BASE CASE

Assumptions:

1)	Subject premium	$9,000,000	
2)	Expected # of claims	765	
3)	Limit	$250,000	
4)	Expected average unlimited severity	$20,520	
4a)	Expected unlimited losses	$15,697,800	= 2 x 4
5)	Expected average limited severity	$8,796	
5a)	Expected limited losses	$6,728,940	= 2 x 5
6)	Expected loss ratio	74.77%	= 5a / 1

Simulation Results:

	With Parameter Risk	Without Parameter Risk
Iterations	10,000	10,000
Simulated average # of claims	764.8	764.7
Percent difference from expected	-0.03%	-0.03%
Simulated average unlimited severity	$20,468	$20,562
Percent difference from expected	-0.26%	0.21%
Simulated average limited severity	$8,807	$8,812
Percent difference from expected	0.12%	0.18%
Simulated average unlimited losses	$15,653,662	$15,724,844
Percent difference from expected	-0.28%	0.17%
Simulated average limited losses	$6,735,421	$6,738,819
Percent difference from expected	0.10%	0.15%
Variance-to-avg of simulated losses	994,664	157,117
Average loss ratio	74.84%	74.88%

OPTION 1 - INNER AGGREGATE DEDUCTIBLE

Assumptions:

Ceding company retains first $5,400,000 of reinsured losses

Reinsured losses = max(simulated losses - 5,400,000,0)

Simulation Results:

	With Parameter Risk	Without Parameter Risk
Average reinsured losses	$1,862,104	$1,379,146
Variance-to-avg of reinsured losses	2,116,999	667,518
Average losses eliminated by deductible	$4,873,316	$5,359,673
Loss elimination ratio	0.724	0.795
ELR to reinsurers (without credit)	20.7%	15.3%
ELR by subgroup of 2000 iterations:		
I	20.7%	15.6%
II	20.6%	15.4%
III	20.6%	15.3%
IV	21.1%	15.4%
V	20.4%	15.0%

OPTION 2 - LOSS RATIO CAP

Assumptions:

Ceding company retains all losses greater than 90% of subject premium

Reinsured losses = min(simulated losses, .9 x subject premium)

Simulation Results:

	With Parameter Risk	Without Parameter Risk
Average reinsured losses	$6,194,343	$6,687,541
Variance-to-avg of reinsured losses	584,632	129,934
Average losses eliminated by cap	$541,078	$51,278
Loss elimination ratio	0.080	0.008
ELR to reinsurers (without credit)	68.8%	74.3%
ELR by subgroup of 2000 iterations:		
I	68.5%	74.5%
II	68.8%	74.4%
III	68.8%	74.3%
IV	69.2%	74.4%
V	68.9%	74.1%

OPTION 3 - LOSS RATIO CORRIDOR

Assumptions:

Ceding company retains all losses between 75% and 112.5% of subject premium

Reinsured losses = min(simulated losses,.75 x subj prem) + max(simulated losses - 1.125 x subj prem,0)

Simulation Results:

	With Parameter Risk	Without Parameter Risk
Average reinsured losses	$5,743,502	$6,333,564
Variance-to-avg of reinsured losses	413,256	53,673
Average losses eliminated by corridor	$991,918	$405,255
Loss elimination ratio	0.147	0.060
ELR to reinsurers (without credit)	63.8%	70.4%
ELR by subgroup of 2000 iterations:		
I	63.6%	70.4%
II	63.8%	70.5%
III	63.8%	70.3%
IV	64.1%	70.5%
V	63.8%	70.2%

OPTION 4 - PROFIT COMMISSION

Assumptions:

Ceding company will be paid 50% profit commission after 25% expense allowance for reinsurer

Profit commission = max(.5 x subj prem x (1-(loss ratio + 25%)),0)

Simulation Results:

	With Parameter Risk	Without Parameter Risk
Average profit commission	$554,167	$208,331
as % of subject premium	6.16%	2.31%
Profit comm % by subgroup of 2000 iterations:		
I	6.32%	2.31%
II	6.16%	2.27%
III	6.15%	2.33%
IV	6.03%	2.27%
V	6.12%	2.40%

OPTION 5 - CONTINGENT CEDING COMMISSION

Assumptions:

Ceding company will be paid a contingent sliding scale ceding commission, depending on the loss ratio result.

Ceding commission calculated from the following table:

LR Interval	Commission
>75%	0.0%
60%-75%	.5 x (75% - LR)
45%-60%	.6 x (60% - LR) + 7.50%
30%-45%	.75 x (45% - LR) + 16.50%
<30%	(30% - LR) + 27.75%

Simulation Results:

	With Parameter Risk	Without Parameter Risk
Average contingent ceding commission	$634,598	$212,420
as % of subject premium	7.05%	2.36%

Ceding comm % by subgroup of 2000 iterations:

	With Parameter Risk	Without Parameter Risk
I	7.03%	2.44%
II	7.24%	2.36%
III	7.05%	2.31%
IV	7.05%	2.37%
V	6.88%	2.32%

OPTION 6 - RETROSPECTIVE RATING

Assumptions:

Ceding company's final premium will be determined retrospectively, based on ultimate losses under the coverage.

Retro adjustment = min(max(loss ratio + .25, .70), 1.30)

Simulation Results:

		With Parameter Risk	Without Parameter Risk
Average retro premium		$8,946,433	$8,987,088
Average retro adjustment		99.40%	99.86%
Retro adjustment by subgroup of 2000 iterations:			
	I	99.26%	99.58%
	II	99.23%	100.09%
	III	99.42%	99.93%
	IV	99.36%	99.82%
	V	99.75%	99.86%

OTHER CONSIDERATIONS

* Cash flow
* Risk load
* Expenses
* Market conditions

An aggregate loss model is a very important tool in analyzing adjustable features of treaties. The Lognormal and Simulation techniques presented by Bob and Jeff work well and usually provide sufficient information to make good pricing judgements. There are situations, however, where consideration of cash flow would change one's attitude towards comparable treaties. The following exhibits outline steps in helping to decide if cash flow is important.

The examples use reinsurance coverage where the cash flow will likely throw off enough investment income that it may determine the ultimate profitability or loss of the treaty. The main use of an aggregate distribution is to enable one to adjust expected loss estimates for contract terms. Based on these adjusted loss estimates, it is straightforward to compare the underwriting profitability of competing deals.

Graphing the cash flows of comparable deals may reveal whether the payment streams are different enough to compensate for expected loss differences. It is usually the case that the graphs of cash flows are sufficiently similar within a given group of terms (e.g., comparing one swing to another swing, or one profit commission plan to another profit commission plan) to make it clear that investment income differences won't affect a pricing decision.

In cases where the graph provides inconclusive evidence, two methods to reflect cash flow in an aggregate loss model may be used. The Panjer aggregate loss algorithm can be easily adjusted to reflect a payment pattern. By transforming the frequency parameter, one can get an aggregate loss distribution as of any given point in time. Investment income estimates for each year follow from each annual aggregate distribution. An alternative that is perhaps more intuitive is to reflect the payment pattern directly by simulating a payment lag for each loss as an extension to an aggregate loss simulation model.

In the end, considering cash flow seems to matter most when comparing different contract types and in measuring the value of contract terms compared with flat rating. It is not generally worth the effort when comparing similar contract types (except aggregate deductibles) or in calculating the credit for a high loss ratio cap.

Examples & Assumptions

Subject Premium: $10 million

Expected Loss: $1.5 million

Layer: $500,000 xs $500,000

Severity: Single Parameter Pareto, Q=1.5

Frequency: Negative Binomial, V/E = 2.0

Interest: Flat 8.0% a year

Auto example:

Long-Haul Trucking

Reporting Pattern is Exponential with 25-month average lag
Payment Pattern is Exponential with 35-month average lag

GL example:

Appliance Manufacturer

Reporting Pattern is Exponential with 45-month average lag
Payment Pattern is Exponential with 65-month average lag

Notation and Definitions of Random Variables

N – Number of Excess Loss

R_t – Reinsurance Premium net of brokerage at time t

P_t – Aggregate Paid Losses at time t

PC_t – Profit Commission at time t

C_t – Cumulative Cash Flow for the Reinsurance contract at time t

$$C_t = R_t - P_t - PC_t$$

i – interest rate

V – Present Value of the net cash flow

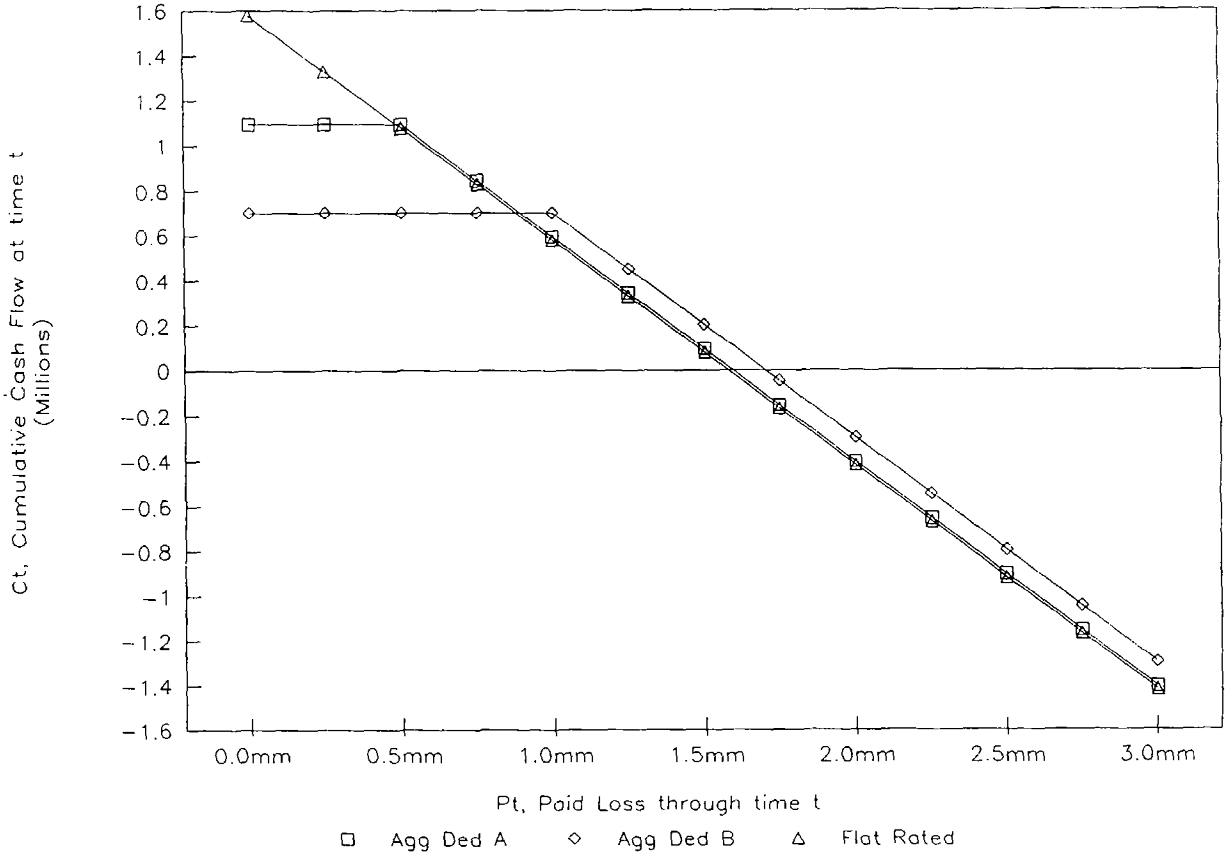
$$V = \sum_{t=1}^{n} (C_t - C_{t-1})(1 + i)^{1-t}$$

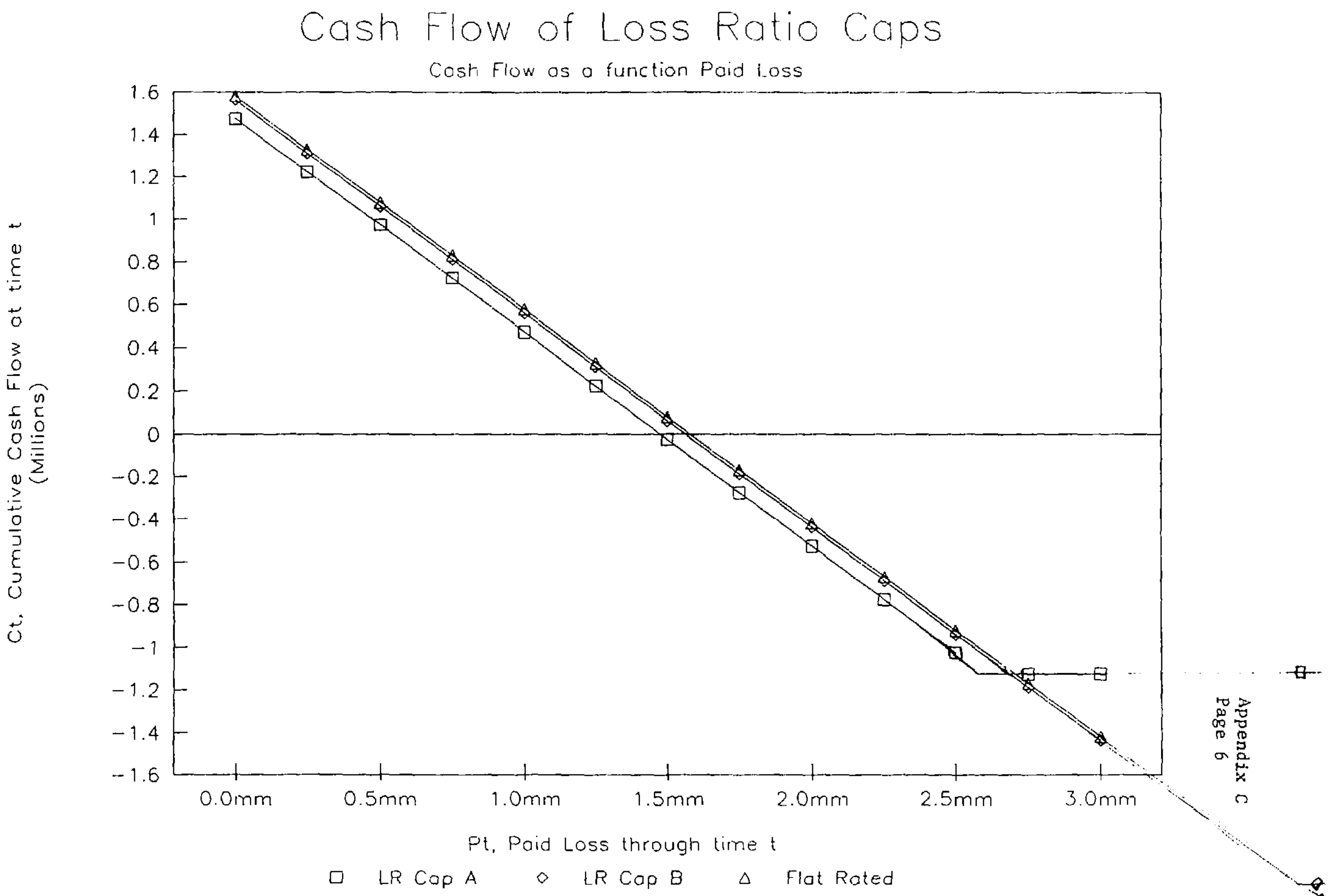
Rates Used with Graphs

Plan		Rate	Other
Flat		15.80%	
Agg Ded A		10.95%	Ded= 5.0%
Agg Ded B		7.03%	Ded= 10.0%
LR Cap A		14.74%	LR Cap= 26.0%
LR Cap B		15.62%	LR Cap= 39.0%
Prof Cmsn			
	A1	16.80%	before PC, 1st 3 yrs.
	A2	16.80%	with PC, yr. 4 & subs. (30% PC after 15% RI margin)
	B1	17.30%	before PC, 1st 3 yrs.
	B2	17.30%	with PC, yr. 4 & subs. (50% PC after 25% RI margin)
	C1	15.80%	before PC, 1st 3 yrs.
	C2	15.80%	with PC, yr. 4 & subs. (30% PC after 15% RI margin)
	D1	15.80%	before PC, 1st 3 yrs.
	D2	15.80%	with PC, yr. 4 & subs. (50% PC after 25% RI margin)
Swing A		7.5%min/21.0% max	Loss Load 100/75ths
Swing B		3.5%min/22.0% max	No Load
Swing C		7.5%min/22.0% max	Loss Load 100/80ths

Cash Flow of Aggregate Deductibles

Cash Flow as a function of Paid Loss

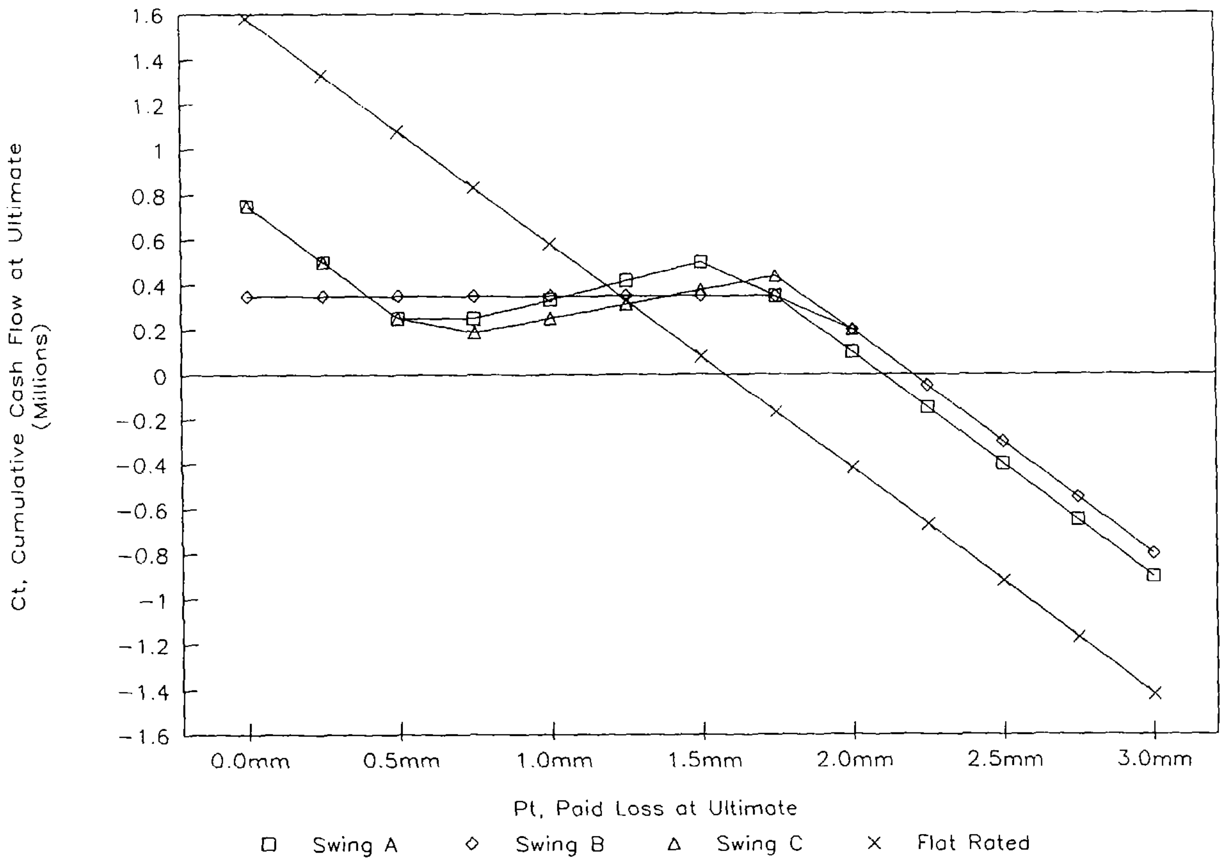




Appendix C
Page 6

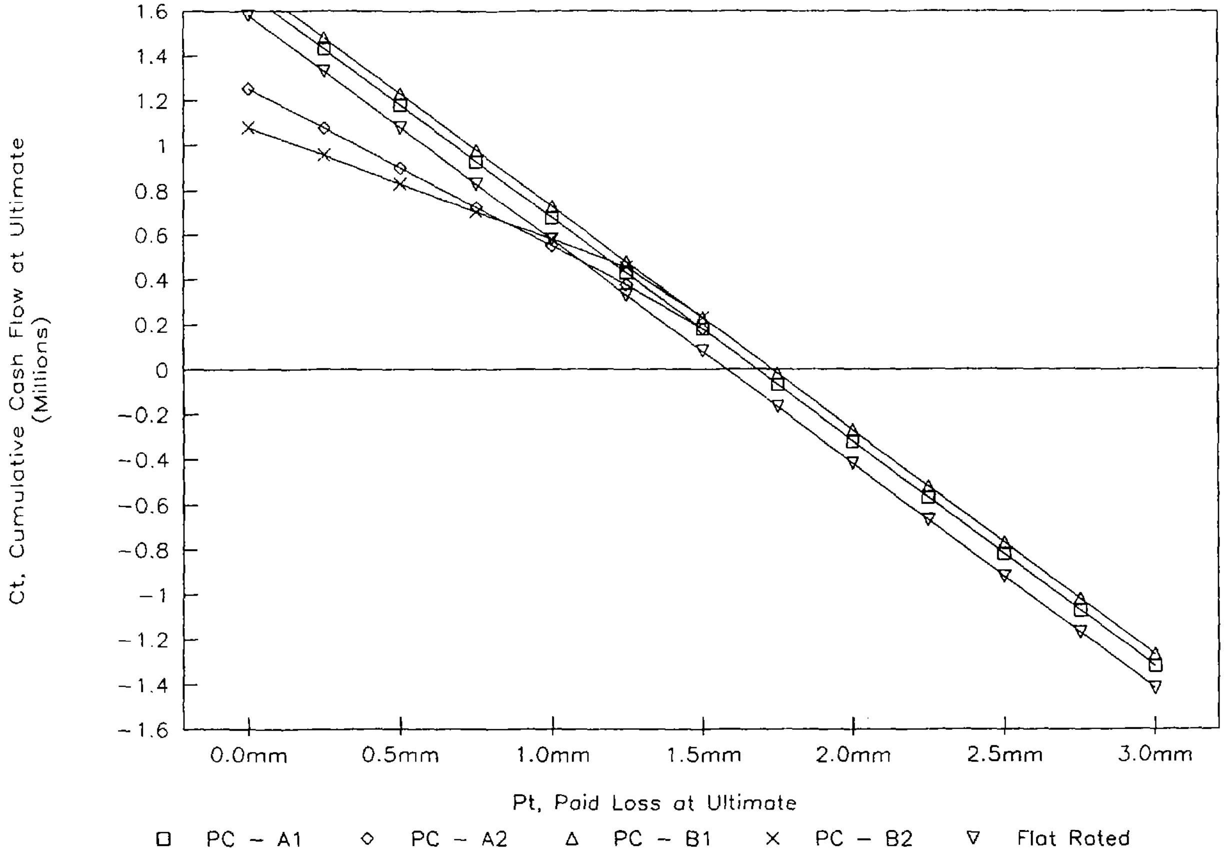
Cash Flow of 3 Swing Plans

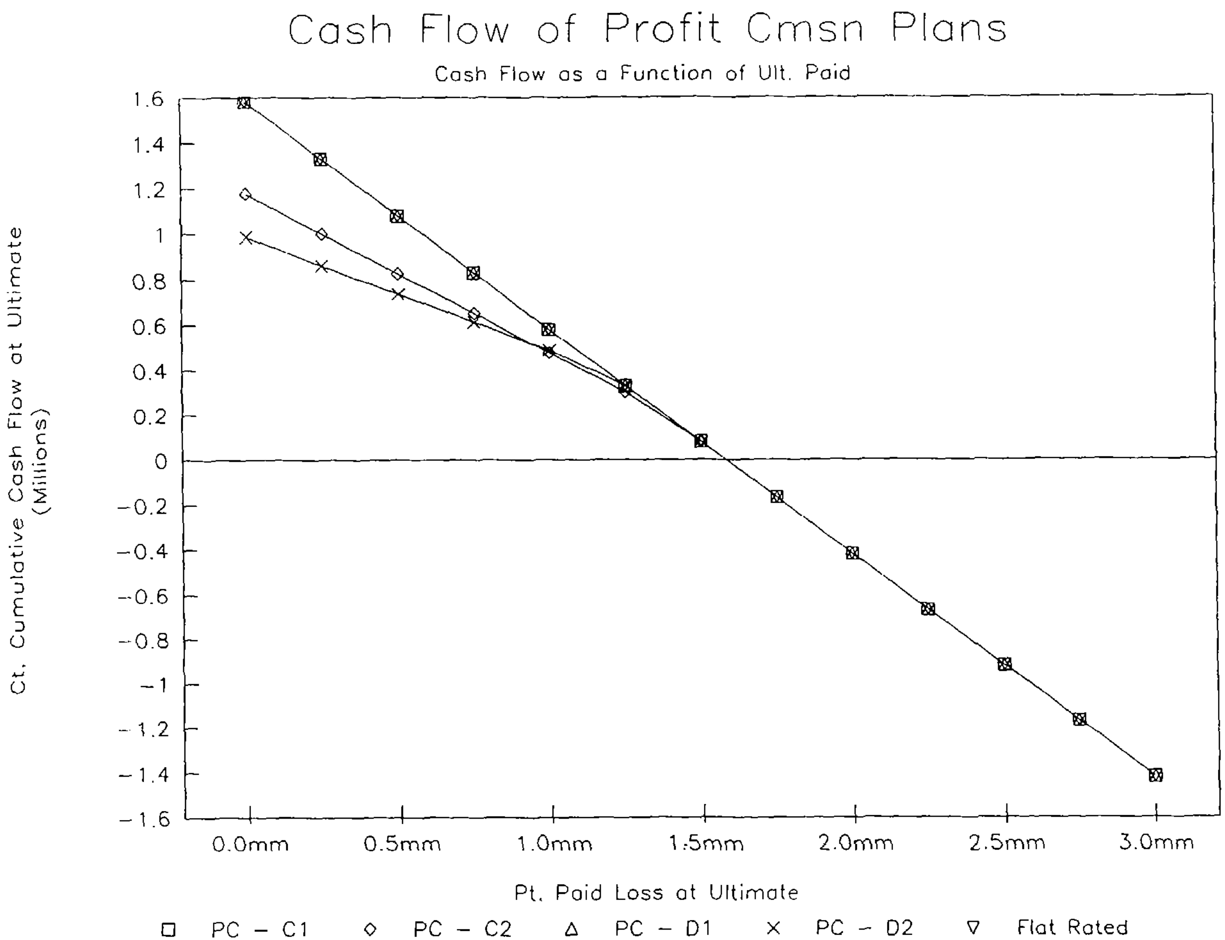
Cash Flow as a Function of Ult. Paid



Cash Flow of Profit Cmsn Plans

Cash Flow as a Function of Ult. Paid




Cash Flow of Profit Cmsn Plans
Cash Flow as a Function of Ult. Paid
Ct. Cumulative Cash Flow at Ultimate
(Millions)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
0.0mm
0.5mm
1.0mm
1.5mm
2.0mm
2.5mm
3.0mm
Pt. Paid Loss at Ultimate
PC - C1
PC - C2
PC - D1
PC - D2
Flat Rated

Distribution of V

Aggregate Distribution for Excess Claims:

$$G(x) = \sum_{n=0}^{\infty} \text{Prob}[N=n]F(x)^{*n}$$

F(x) - Single Parameter Pareto

Prob[N=n] - Negative Binomial

Assumption: Individual claim reporting and payment patterns are independent of size of loss.

Observation: If M, the number of ground-up claims is Negative Binomial (α,p), then N, the number of claims excess of retention r, is also Negative Binomial with parameters (α',p') where

$$\alpha' = \alpha$$

and
$$p' = \frac{p}{F(t)+p(1-F(t))}$$

Distribution of V: Simulation

1. N is drawn from a negative binomial NB(α',p').

2. For each of the N claims, a paid loss amount is drawn from SPP and a payment lag is drawn from the exponential. It was assumed that claims occur mid-year and premium and loss transactions are made at mid-year.

3. The P_t values are calculated by summing total payments in the appropriate time periods using the simulated lags.

4. The reinsurance contract terms were applied to the P_t's to obtain the C_t's.

5. V is calculated $= \sum_{t=1}^{n} (C_t - C_{t-1})(1+i)^{1-t}$, then V is stored.

The above was repeated for 20,000 iterations, then E[V], Variance [V] and Probability [V>0] are calculated.

Distribution of V: Panjer's Method

Just as the number of Excess Claims is Negative Binomially distributed, so is the number of Excess Claims as of time t.

The transformation needed, is

$$\alpha_t' = \alpha'$$

$$p'_t = \frac{p'}{w(t) + p'(1-w(t))}$$

Where w(t) is the percent paid or reported as of time t.

One uses a discretized form of the severity distribution and the transformed Negative Binomial in Panjer's formula:

$$g_0 = p(0)$$

$$g_i = \sum_{j=1}^{i} (a+bj/i) f_j \, g_{i-j} \qquad i=1,2,3,\ldots$$

Using the aggregate distribution, the C_t's can be computed easily.

Aggregate Deductible

Deductible	Rate	ELR	AL NPV	GL NPV
0	15.8	95	353	517
5	10.95	95	290	426
10	7.03	95	211	310

Loss Ratio Cap

Cap	Rate	ELR	AL NPV	GL NPV
Infinite	15.8	95	353	517
250 %	15.62	95	345	507
175 %	14.74	95	317	462

Swing Rated

Swing Rate	ELR	Loss Load	AL NPV	GL NPV
15.8 Flat	95	none	353	517
7.5/12/21	95	100/75	231	320
7.5/12/22	95	100/80	222	309
3.5/12/22	95	100 +Min	212	284

Profit Commission

Profit Commission	Years No Down	Rate	ELR	Eff ILR	AL NPV	GL NPV
0		15.8	95	95	353	517
50 after 25	4	17.3	87	95	380	528
30 after 15	4	16.8	89	95	363	516
30 after 15	4	15.8	95	101	273	428
50 after 25	4	15.8	95	103	251	402

This presentation was based on:

EVALUATING THE EFFECT OF REINSURANCE CONTRACT TERMS
by James N. Stanard and Russell T. John

soon to be published in PCAS. The following references are cited in that paper:

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THE WORKERS' COMPENSATION CRISIS: ADDRESSING THE REAL PROBLEM (REBUTTAL TO RICHARD A. HOFMANN'S ARTICLE IN THE FALL 1991 *FORUM*)

William D. Hager

THE WORKERS COMPENSATION CRISIS: ADDRESSING THE REAL PROBLEM

By William D. Hager
President, National Council on Compensation Insurance

For seven years workers compensation insurance has been in a state of crisis, with combined ratios averaging nearly 120%, a residual market share that has grown from less than 10% to 24% of the total market, and a number of state systems teetering on the verge of catastrophe. Only adequate rates, workers compensation system reforms resulting in cost reductions, or a combination of both will restore this system.

Early in 1991 the workers compensation insurance industry described activities which will identify specific causes for rising workers compensation costs in 12 initial target states and the development of legislative or administrative cost containment proposals to stem the rise in workers compensation costs. I strongly believe that this approach when coupled with adequate rates, will provide the real solution to the current problems of the workers compensation system.

Some proposals and criticisms of the insurance industry miss the mark by addressing only the **symptoms** of a troubled workers compensation system rather than its root causes. Mandated rate reductions and interference in the ratemaking process are short sighted and destructive. Such actions will not resolve the workers compensation crisis. Insurers must be free to charge adequate rates for employers with both good and bad safety records and the voluntary market subsidy of the residual market should be reduced. The formation of self-insurers and group self-insurers will not solve rate inadequacy problems--it merely shifts those costs to other people. Such proposals evade the real problem in workers compensation by creating the appearance that something is being done to bring down costs and rates. There are many reasons, which vary from state to state, for the current workers compensation insurance crisis. The problems and solutions must be addressed by each state individually.

THE RESIDUAL MARKET

The workers compensation reinsurance pools operated by NCCI now provide coverage to over 600,000 policyholders who are unable to find coverage in the voluntary market. The residual, or involuntary, market now accounts for 24% of the workers compensation insurance market - - - - up from less than 10% only six years ago. This unprecedented growth has put a strain on the entire system. NCCI and its members have been laboring to correct the problem of the residual market by implementing higher carrier performance standards, providing more information and assistance to insurance agents who use the residual market, reducing fraud, and tightening rules and procedures.

I believe that **consumers are best served in a competitive voluntary market where they may choose among carriers competing to provide the programs, prices and services they need.** This can best be accomplished by the adoption of adequate rates in both the voluntary and residual markets. The residual market must also be redefined as the true market of last resort.

Criticism of the current residual market attacks a mechanism that is not the problem; the growing residual market is merely a reflection of the problem. Changing the delivery system for the product is mere "window-dressing" and a costly one at that. The problem of the residual market is that it has grown far beyond what is healthy and manageable. The goal of reform should not be the building of a residual market mechanism that can service 600,000 employers. Rather, efforts must be directed to depopulate the residual market to a point where efficient and effective service is assured and rates that cover actual costs are allowed to be charged.

Proposals, such as group self-insurance plans, purport to offer the public a simple solution to the woes of the system. However, we are seldom told how such groups would be any more successful than the current system when inundated by 600,000 employers. The most significant flaw in such a proposal, however, is the basic premise that another mechanism would allow the regulator to approve a rate sufficient for the market to be self-supporting. In 1990, in most jurisdictions, the indicated rate differential needed to obtain a self-supporting residual market was between 40-50%. Very few regulators would or could approve a rate increase of that magnitude in today's economy. Thus, solutions must emphasize depopulation, loss control, cost containment, and adequate residual market rates.

Efforts are underway at NCCI to eliminate fraud in the system, provide a "single stream" application process, and develop standards and programs that will permit servicing carriers to aggressively manage and reduce the residual market loss costs.

We are already seeing results from these efforts. During the first quarter of 1991, NCCI introduced assigned risk investigative units to provide greater scrutiny of applications meeting specific criteria based on premium size, complexity of risk, or on suspected misrepresentation of application information. These units detect understated premium upon application and identify those employers who do not possess good faith entitlement to coverage. After only three months of this additional review, over 14.6 Million Dollars in annual premium has been identified as of July 1, 1991. The residual market uncollectible unit has generated an additional 1.5 Million Dollars in collected premium by refusing to settle open accounts without extensive review and negotiation with the client. These represent just a few of the administrative changes already under way within NCCI which have focused on employers who abandon their responsibilities to bear their fair share of the costs due to their own poor experience.

NCCI continues to review the application process to arrive at a more efficient and streamlined system to better address the concerns of consumers, agents and servicing carriers while also providing the necessary information and verification to assure eligibility for coverage under the Plan. NCCI has developed system specifications and identified automation enhancements, including image processing, designed to streamline the current system. We expect to conclude the necessary programming changes this year. We have also sponsored educational workshops to improve communication channels with insurance agents who frequently use the plan.

Equally important, NCCI recognizes that if the residual market is to survive, the underlying costs of the system must be aggressively managed and controlled. We are reviewing such programs as fraud detection in claim handling, disability management, and medical and legal cost management. All are critical to the survival of the market because they directly impact the health of the overall system.

Our current activities acknowledge that the residual market can be improved. However, the growth and expansion of the residual market is only a symptom of a much larger crisis. Once this fact is accepted, the weaknesses in our critics' arguments are exposed. Alternatives to the residual market delivery system only address the symptom and ignore the depth and complexity of a troubled workers compensation system. Regulators, legislators and consumers must recognize that there are no easy solutions to the problems facing the workers compensation system. On the other hand, these problems are not insurmountable either. The insurance industry is focusing its extensive efforts on a multi-faceted reform package to bring the workers compensation system back into balance.

LOSS COSTS RATING

Some industry critics advocate deregulating workers compensation pricing and prohibiting collective pricing activities. This issue is under review by the National Association of Insurance Commissioners, which is expected to approve model workers compensation loss costs legislation for introduction in 1994. Under a loss cost rating system, the rating organization prepares prospective loss costs for workers compensation classifications. These loss costs use data on historical losses and loss adjustment expenses, developed to an ultimate basis and trended. Provisions for other carrier expenses and profit are not included. Insurance companies factor in anticipated expenses and profit to develop their own final rates. Many states have already adopted loss cost rating programs. NCCI's Board of Directors pledged in 1990 to assist states that wish to move to a loss cost approach or to respond to any alternative rating approach that is legislatively or regulatorily selected.

However, regulators, legislators and consumers must be reminded that loss cost rating does not have any effect on rapidly rising system costs. As a result, loss costs rating will not directly address the industry's severe rate inadequacy problem.

THE QUEST FOR WORKERS COMPENSATION DATA

Everyone wants to know what's driving workers compensation costs. Some have suggested that a new data base be created and administered by a new state statistical data collection agency. It has been suggested that such a new data base would better monitor the performance of workers compensation systems, allow legislatures to quantify expected cost impacts of reforms sooner, improve the performance of the systems, and help produce long-term cost savings for the benefit delivery systems.

The intention of these recommendations are desirable. However, advocates of "new" data systems may overlook the wealth of workers compensation statistics which already exists. The collection of workers compensation data is a costly, complicated and time-consuming task. Several disciplines, including claims evaluation, computer programming, statistical analysis and data base management are required to collect, collate and analyze this information.

While rating organizations such as NCCI do not currently collect self-insurance data (because self-insurers are not members), with few exceptions, 100% of the workers compensation insurance company experience is reported to the authorized rating organization. This data base, therefore, provides a highly credible source of statistics for ratemaking, law evaluations, and research. In addition, every state has an industrial commission, bureau or accident board that administers, monitors and collects data on its workers compensation system.

Several different kinds of data are now collected and assembled into NCCI's vast statistical data base. For determining the overall change in state rate level, financial experience in the form of policy year aggregates, calendar year aggregates and calendar/accident year aggregates are reported for each state.

Another important and critical form of data received by NCCI on a policy-by-policy basis is Unit Statistical Plan data. Here, detailed information on all transactions on insured employers are reported. These data are used to determine the classification relativities by state for approximately 600 classes, to calculate the Experience Rating Plan modifier for individual employers, and to evaluate the price or cost effect of workers compensation law changes.

A third form of data collected by NCCI, is Detailed Claim Information (DCI). The DCI system obtains data on a sampling basis for research purposes. The DCI data base, with oversight by the National Association of Insurance Commissioners, is being expanded to cover all states in 1992. Clearly there are sufficient data being collected (presently at a significant cost) that can identify underlying system costs without the expense and unknown value of a new data base. What is important, is that system reform must occur now, (with information we now have), to bring the system back in balance.

CONCLUSION

I invite regulators, legislators, and consumers to join with the workers compensation insurance industry in resolving the real problems of the workers compensation system. I know that rate increases are difficult to approve, and accept in today's economy . However, I also know that only rate adequacy coupled with effective cost containment efforts will restore a long overdue balance in the workers compensation system.

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REVIEW OF "THE MATHEMATICS OF EXCESS OF LOSS COVERAGES AND RETROSPECTIVE RATING – A GRAPHICAL APPROACH", PCAS, 1988

Keith Holler

Introduction

The first exposure that many children have to numbers in school is through the use of a number line, or a picture displaying bundles of sticks. In high school the Pythagorean theorem is often proven by comparing the areas of the triangle to a surrounding rectangle. While teaching calculus, many instructors depict areas and volumes through involved diagrams. But, later courses in advanced mathematics often lead to more obscure illustrations or more often no illustration at all. I would like to thank the author for reminding us of the value of pictures. This review contains a description of how pictures were used to produce a practical solution to an insurance problem.

The Setting

As the allocated loss adjustment expense (ALAE) portion of the premium dollar has become much more significant recently, new variations of defense options have arisen. Defense costs are as major a concern as loss costs in many of the general liability lines for the insurer and insured alike. The traditional policy where defense costs are supplemental to the policy limit is more or less priced by loading the unlimited average defense costs into the basic limits rate. Supplemental defense costs are paid by the insurer in full and do not erode the policy limit. With new policy options whereby defense may be included within the limit, and more frequent use of self insured retentions (SIR's) with liability policies, it becomes necessary to modify the ratemaking techniques associated with defense costs. When defense costs are included in the limit they are usually combined with the loss dollars before the policy limit is applied to determine the insurer's liability.

Ideally one would like to model the joint distribution of loss and defense costs and use this model to estimate any costs of factors associated with different policy defense options. Perhaps the simplest fashion in which one may try to include defense costs is either as a flat percentage of loss or a flat dollar amount per loss. The following solution is somewhat of a middle ground between these two extremes.

The setting under which the following solution arose is as follows. A software package that had already encoded a loss distribution and readily calculated limited expected losses was available. Further, this package allowed one to manipulate the parameters in order to account for inflation. The package had some other features that were useful and would have been rather tedious to program. Time and money were constrained in such a way as to make the calculation of the ideal joint distribution infeasible. It was decided to use the software package and alter it in such a fashion as to hopefully reflect reality with respect to the joint distribution.

This alteration took the form of a combination of a fixed dollar piece of allocated expense and a piece that is a fixed percentage of the individual loss. The fixed percentage piece will be referred to as the variable piece of the defense cost.

ALAE = Fixed plus Variable

Intuitively one may view this combination as fitting the small claims mostly through the fixed piece and the large claims through the variable piece. If only a percentage of loss were used to estimate ALAE, many smaller claims that incur ALAE as a larger percentage of loss would be incorrectly represented. As an extreme example consider a claim that settles for $1. If an ALAE to loss ratio of 40% is used then this would suggest that 40 cents covers the ALAE. It seems more reasonable to assume these smaller claims incur some fixed costs. For the larger claims the variable portion may become the dominant portion of the ALAE estimate. If the fixed piece of the average unlimited ALAE is $16,000 and the variable piece is 8% of loss then a two million dollar claim would incur $16,000 plus $160,000 of ALAE.

The choice of the amounts of the fixed and variable pieces was solved in another expeditious manner. The software package contained an expected unlimited ALAE amount that was judged to be reasonable for use. To apportion this estimate into fixed and variable pieces a simple linear regression was performed on a file of individual closed claim and ALAE amounts. The dependent variable was the ALAE amount and the independent variable was the loss amount.

$$ALAE = a + b * Loss$$

The fixed portion was determined as "a" divided by the average ALAE of the closed claim and ALAE file. The complement of this was the variable portion. For example, if the constant is $14,400 and the average ALAE is $18,000, the fixed portion is 80% and the variable portion is 20%. It is interesting to note that if representing the ALAE as entirely fixed or entirely variable was truly "better", in the least squares sense of the word, than a mix of the two, one of the fitted parameters of the regression would have been close to zero. This was not the case.

Let us return to the software package with the allocated 80/20 split in hand. Assume that the unlimited expected ALAE from the software package is $20,000 and the unlimited expected loss from the software package is $50,000. The fixed part of the ALAE is $16,000. The variable part as a percentage of loss is 8%. Using the inflation adjusting capabilities of the package, the distribution was simply increased 8% to account for the variable piece of the ALAE.

Graph 1 displays the cumulative distribution function (CDF) for loss and the CDF for loss and variable allocated. The latter distribution is the same the loss distribution adjusted for inflation.

Graph 2 incorporates the fixed defense costs. Note the area marked "variable defense" is the average variable defense cost, $4,000. The area marked "fixed defense" is the average fixed defense cost, $16,000. The area marked "loss" is the average loss cost, $50,000. Graph 2 is essentially graph 1 placed atop the fixed costs.

For clarity, names are assigned to three of the four random variables whose distributions are illustrated in graphs 1 and 2. Let X be the random variable of loss size only, the lower function in graph 1. Let Y be the random variable for loss plus variable defense, the upper function in graph 1. Let Z be the random variable for loss plus all defense, both fixed and variable, the upper function in graph 2. Note in the example Y = 1.08X and Z = 1.08X + 16000.

Suppose we want a rate for a policy with a limit of $100,000 per occurrence with defense included in the limit. Using the notation of Hogg and Klugman and ignoring risk loads and ULAE, if the rate for a basic limits policy with a limit of $25,000 with defense costs supplemental to the policy limit is $5, then the rate for the first policy is:

$$\frac{5 * E[Z;100{,}000]}{E[X;25{,}000] + 16{,}000 + 4{,}000}$$

Graph 3 depicts E[Z;P] for some P as the heavily shaded area under the horizontal at P plus the lightly shaded rectangle representing the average fixed expense (fd). Graph 4 depicts E[Y;P-fd]. It is readily apparent that E[Z;P] = E[Y;P-fd] + fd. The software package readily calculates limited expected values for Y, hence for Z. The fact that E[Y;P-fd] = 1.08 * E[X;(P-fd)/1.08] could have been used if the package was not able to model Y so readily.

As one last illustration suppose we want a rate for a policy with a limit of $1 million per occurrence with defense included excess of a SIR of $50,000 per occurrence with defense included. The rate for this policy is:

$$\frac{5 * (E[Z;1{,}050{,}000] - E[Z;50{,}000])}{E[X;25{,}000] + 16{,}000 + 4{,}000}$$

Which is now readily calculable.

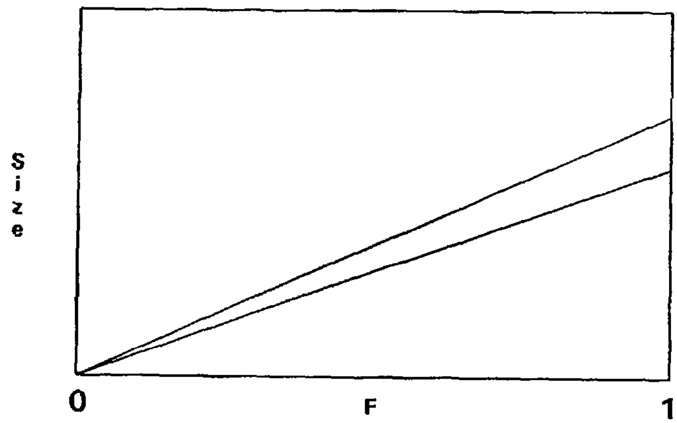
Considerations

There were several considerations that arose while this procedure was being devised. Most fundamental of them all was the nature of the ALAE separation. A line necessarily implies a decreasing percentage of ALAE to loss. The closed claim file used was the subject of several questions concerning maturity and policy limits contained. Finally, the software package had some distributional implications that had to be thought through. These conceptual problems were wrestled with and accounted for where possible and necessary. The determining criteria was reasonableness.

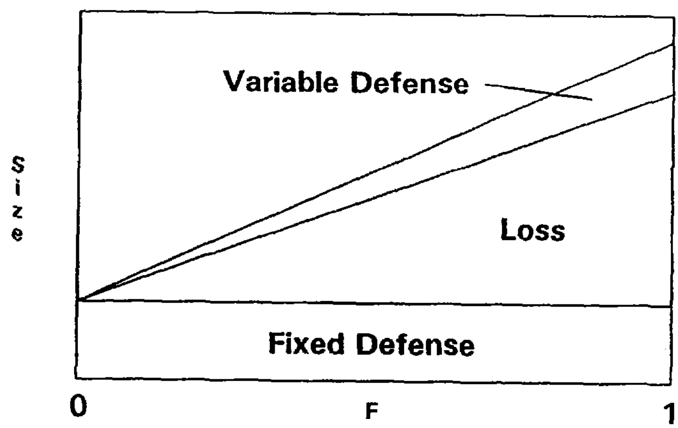
Conclusion

Pictures are a very useful tool that the actuary should keep ready in his or her toolbox. The concepts conveyed through a picture are often so much simpler to grasp than the sometimes tedious algebra that accompanies them. Understanding the concept often makes the algebra that much more palatable. I welcome any tool that aids in my understanding. Once again, I thank the author for reminding me of the usefulness of pictures.

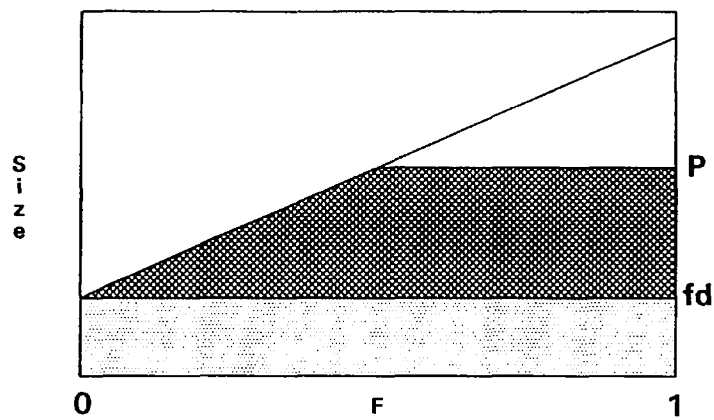
Graph 1
CDF for Loss and Loss + Var ALAE



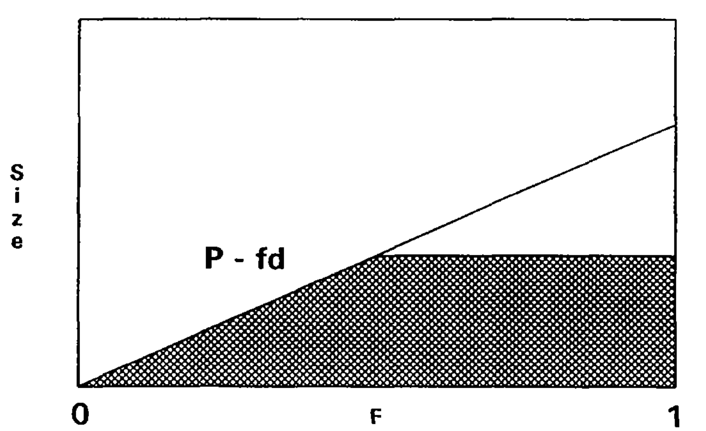
Graph 2
CDF for L + All ALAE and L + Fixed ALAE



Graph 3
CDF for Loss plus all ALAE



Graph 4
CDF for Loss plus Variable ALAE



PROPERTY-CASUALTY RISK-BASED CAPITAL REQUIREMENT—A CONCEPTUAL FRAMEWORK

Actuarial Advisory Committee to the NAIC Property & Casualty Risk-Based Capital Working Group

PROPERTY-CASUALTY RISK-BASED CAPITAL REQUIREMENT

A CONCEPTUAL FRAMEWORK

Prepared by the
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February, 1992

PROPERTY-CASUALTY RISK-BASED CAPITAL REQUIREMENT

A CONCEPTUAL FRAMEWORK

FOREWORD

This paper develops a conceptual framework for a risk-based capital requirement for property-casualty insurance companies. It has been written to assist the National Association of Insurance Commissioners (NAIC) as they work on developing appropriate risk measurements in the context of a series of initiatives designed to improve solvency regulation. We believe the NAIC will find this paper useful.

Risk-based capital is the theoretical amount of capital needed to absorb the risks involved in the operation of a business. Different companies face different risks and, therefore, should have different levels of capital based on those different risks, rather than on some arbitrary basis. The major areas of risk facing a property-casualty insurance company include asset risk, reserve risk, pricing risk and credit risk.

State regulators of property-casualty insurance companies have had two tools with which to monitor required capital. One is a statutory minimum capital and surplus requirement which has been characterized as unrealistic and archaic, and the other is a premium-to-surplus rule-of-thumb, which does not effectively reflect relative riskiness. Many regulators feel they lack the statutory authority to require a company to increase their capital until the company's surplus falls below the statutory minimum. A risk-based capital requirement would help raise that safety net up off the floor and could apply uniformly in all states as a threshold capital requirement.

Table of Contents

Appendices

I. Statement of Purpose of a Risk-Based Capital Requirement for Property-Casualty Companies

There are two main purposes of a risk-based capital requirement:

1. **Permitting Regulatory Attention**

   The risk-based capital requirement should help regulators to meaningfully discriminate between those companies needing regulatory attention due to potential capital inadequacy and those which do not require such attention.

2. **Changing Company Behavior**

   The requirement will likely also lead company managements to modify their behavior so as to carry sufficient capital to avoid such regulatory attention.

It should be noted there is no simple way a formula can accurately discriminate under all circumstances. It must be used in conjunction with other regulatory tools and be subject to judgmental interpretation. It should also be remembered that meeting the risk-based capital requirement is not a guarantee of solvency.

Since the risk-based capital requirement will affect behavior, as explained in subsequent sections, care must be taken to assure that unintended changes in behavior do not occur. Implementation of any requirement will have broad ramifications with subtle potential consequences. The requirement is *not* intended to provide a capital base to be used as a measure of return on equity for rate regulation, nor for rating insurance companies.

It is in the public interest for the promises made by insurance companies to be fulfilled. Implementation of this new requirement should enhance that goal as well.

Given the above goals are met, a risk-based capital requirement represents a potentially significant improvement over current capital requirements, which do not effectively respond to the changing riskiness of an insurance company.

II. Historical Perspective of Risk-Based Capital

The NAIC Solvency Policing Agenda for 1990 as adopted in December 1989 has five main components:

- Financial regulation standards for effective solvency regulation
- Improved reinsurance evaluation
- More effective examinations
- Improved solvency analysis support
- Risk-based capital requirements

Through these initiatives, state regulators hope to enhance their ability to protect insurance consumers from the financial trauma of insurer insolvency.

In early September, 1990, the Examination Oversight Task Force concluded that risk-based capital requirements are preferable to the generally prevailing system of minimum capital and surplus requirements (summarized by state in Appendix 1). The current statutory minimum capital and surplus requirements provide very little help to regulators in regulating for solvency. While the specific minimums vary from state to state, they typically require companies to maintain only two to three million dollars of capital and surplus. Companies that meet these requirements can seek licenses in all jurisdictions. It is difficult for regulators to legally intervene in the affairs of a company once it is licensed until its capital and surplus falls below these minimums.

At their December, 1990, meeting, the NAIC charged two working groups (one life and one property-casualty) to develop risk-based capital formulas and to develop model laws to make the risk-based capital requirement operational. In addition to the formulas, there would be established a legal mechanism for regulatory intervention when capital and surplus falls below a threshold that is meaningfully related to the amounts and types of exposure faced by the individual company.

Here in the U.S., the regulators of the banking and thrift industries have recently begun phasing in a risk-based capital measure as one component of a new set of supervisory ratios which will be used to assess capital adequacy. The new standards are based on a framework, referred to by some as the Basle Accord, developed by an international group of bank regulators. (See Appendix 2 for a more detailed discussion of the development of and description of the new banking standards.)

The concept of risk-based capital has been considered for many years. More than 20 years ago the concept was discussed in the book *Insurance, Government, and Social Policy* edited by Spencer Kimball and Herbert Denenberg. Some European countries have had risk-based capital requirements for their domestic insurers for more than 20 years.

III. Bank/Thrift Comparisons to Insurance

Both banks and insurance companies collect money from customers and assume liabilities; this process creates pools of assets that they must invest and safeguard. In fact, banks and life insurers are viewed as competitors for some products. However, the differences in the liabilities assumed create some fundamental differences in these types of financial institutions. The differences between property-casualty insurers and other financial institutions may create the need for alternative approaches to risk-based capital.

Banks/thrifts and life insurers both assume liabilities that are reasonably definite in nature (e.g., deposits and death benefits). Both make their money by investing the funds they generate at rates higher than their cost. Both have customers with the option to withdraw funds. As a result, the principal risk faced by banks and life insurers stems from uncertainty in their ability to maintain investment spreads and in the potential to suffer disintermediation in times of changing interest rates.

This similarity is reflected in their financial structure:

- both have similar capital/asset ratios - 6.5% for commercial banks and 6.6% for life insurers (with MSVR as a liability for life insurers equal to 1.0% of assets) as of 12-31-90;

- both have liabilities that are interest sensitive;

- competitive pressures have forced both to increase their investment in riskier assets: high yield bonds, stocks, mortgage loans.

In contrast, the liabilities assumed by property-casualty insurers are indefinite. They make their money by careful risk selection and effective management of their claim liabilities, as well as effectively managing their assets. The principal risk faced by property-casualty insurers is usually considered to be the inherent uncertainty of the liabilities assumed.

The fundamental difference is also reflected in the property-casualty insurer's financial structure:

- property-casualty insurers have a higher capital to asset ratio than the other two types of institutions: 25% as of 12-31-90.

- property-casualty insurers have a different mix of assets than do life insurers and banks. At 12-31-90, property-casualty insurers held 60% of their assets in bonds (with 98% of these being investment grade), 15% in stocks and 2% in mortgages and real estate. Life insurers held 50% of their assets in bonds (including junk bonds), 4% in stocks and 21% in mortgages and real estate.

Asset Distribution of Insurers

	Property-Casualty	Life
Bonds	60%	50%
Stocks	15	4
Mortgages and Real Estate	2	21
Other	23	25
	100%	100%

Some banking industry regulators view their new risk-based capital requirements as being deficient because a measurement of interest rate risk is excluded. (In fact, thrift regulators are currently developing an interest rate component). This same reason would also make them deficient for application to the life insurance industry. Further, because the banks' risk-based capital measurements are slanted towards asset and off-balance sheet risks, there are some who believe that the banking industry's standards do not reflect fundamental differences in the operations of banks and insurers. Nor could they be properly applied to the property-casualty industry because they fail to measure certain significant risks which are unique to that industry.

IV. Underlying Principles

This section outlines a set of principles that should govern the development of any risk-based capital formula. These principles can be used to evaluate any proposed formula, and should apply to the final formula that is adopted. As a practical matter, the formula should come as close as is possible to satisfying all of them simultaneously.

The principles have been grouped into three areas: a) those relating to formula mechanics, b) those relating to behavior induced by the formula, and c) those relating to economic consequences of the formula.

The following set of criteria should apply to the final adopted risk-based capital formula.

A. Formula Mechanics

1. *The formula should be subjected to extensive testing that demonstrates its discriminatory value.*

   When it is implemented, the formula is likely to identify some companies as being near or below whatever regulatory thresholds are selected. Thus, the formula will inevitably generate some controversy when it is introduced. Such controversy can only be dealt with effectively if the record reflects diligent testing and careful study designed to assure the formula's discriminatory value. Ideally, the formula should neither identify companies as weak when they are not, nor fail to identify companies as weak when they are. The latter is more serious to policyholders; however, the former will be very serious to shareholders and employees to the extent that it undermines public confidence in the company. While it may not be possible to validate every aspect of a proposed formula using historical data, the formula should produce results consistent with the historical experience of the industry.

   Also, it would probably be wise to have initial formula testing done privately by regulators, prior to the formal public exposure period for any tentative formula. Given the sensitivity of the results, it would be counter-productive to generate public controversy over the formula prior to the completion of this testing.

Additional comments on testing are included in Appendix 3.

2. *The formula should reflect individual company circumstance to the fullest extent practical.*

   Companies differ considerably as to the types and volumes of exposures written, their experience with those exposures, their reliance on reinsurers to help them manage those exposures, and the type, quality and duration of the assets held to discharge the liabilities created by those exposures. While it will never be possible to reflect all of the nuances of these differences, it is important that the formula be responsive to those differences that are material. It is unlikely that the formula will have much real discriminatory power if it does not.

3. *The formula must be practical; users will need to recognize its limitations.*

   The formula should focus on the major risk elements, recognizing that the benefits of measuring minor or very unusual risks may not be cost effective. Even the measurements of the major risks will only be approximate, again striking an appropriate balance between cost and benefit.

   Users will need to recognize these limitations, and, after qualitative discussions with each company's management, be prepared to apply informed judgment in interpreting the results.

4. *The formula must be simple to explain.*

   While details and mechanics of the formula may need to be somewhat complex, the basic formula must be intuitively sensible and simple. Company managements, regulators, and others must have a clear sense of why a particular formula result has occurred and what it means. Since the formula will never be able to take into account a company's particular circumstance perfectly, some interpretation and judgment will be necessary.

5. *The formula should be evolutionary.*

   To effectively serve its purpose, the formula and its accompanying parameters will need to be constantly reviewed, updated, and revised, if necessary, to reflect changing industry circumstance.

   In addition, as risk concepts are more fully developed by the actuarial profession and others, they can be incorporated into the formula as innovations.

6. *The formula should produce reasonably consistent results from year to year, both for the industry in total and for an individual company.*

   The formula will presumably be applied to company financial data at each year end. It is desirable that the indicated risk-based capital rise and fall with changing circumstance, both for the company and the industry as a whole. However, it is clearly undesirable for risk-based capital to change abruptly due to some discontinuity in the financial database to which the formula is applied. Stated simply, the turn of the page in a calendar from one year to the next should not cause an abrupt change in the amount of risk-based capital. It is also desirable to have risk-based capital levels respond *appropriately* to the underwriting cycle (i.e., risk-based capital should not fall just because rate levels decline and vice versa).

B. Induced Behavior

1. *The formula should motivate companies to "do the right thing."*

   Solvency regulation tests can often have undesirable side-effects, due to their influence on company behavior. Sometimes, they create powerful disincentives for management to deal with financial problems in a forthright manner. For example, a company that needs to strengthen its loss reserves knows that the strengthening may trigger several IRIS test failures. Rather than draw attention to itself, the company might choose to strengthen its reserves gradually in a manner that does not cause it to fail the IRIS tests, or enter into an uneconomic reinsurance transaction that masks the strengthening entirely. To the fullest extent possible, the risk-based capital formula should not encourage uneconomic activity or financial irresponsibility.

2. *The formula should not be susceptible to manipulation by changes in financial statement presentation.*

Differences in risk-based capital requirements should reflect meaningful differences in company circumstances, but should not differ merely because of different accounting treatment of items or different corporate structures. Differences which are not meaningful can occur because of flexibility in accounting practices (e.g., retrospective additional premiums) or extraordinary transactions (balance sheet reinsurance).

3. *Care must be taken to prevent the abuse of the risk-based capital formula.*

Concerns have already emerged that some groups may misuse the formula to serve their own agendas. For example, if the formula produces a *minimum* capital requirement, it is possible that some rate regulations may be proposed that allow only a return on that capital, thereby denying a fair rate of return on the capital above the minimum. The formula's intended application must be clearly stated to minimize potential abuses.

For example, if capital is denied a fair rate of return, that capital may exit the industry, reducing policyholder security, thus defeating the purpose of a risk-based capital requirement.

C. Economic Consequences

1. *The formula should recognize economic realities.*

Legislating the capital requirements of insurers, like legislating the price of bread, cannot be done without due consideration of the economic forces of supply and demand. Capital will only flow into the insurance industry if those supplying the capital perceive the opportunity to earn a fair return.

Establishing capital requirements at higher than existing levels will not cause capital to magically flow into the insurance industry. It must be recognized that requiring higher levels of capital than currently exist (either for a particular line or in total for the industry) will necessitate higher prices to produce returns that attract that additional capital, or will restrict the availability of insurance.

This issue should not be underestimated. Some believe that the current credit crunch (an availability problem) is directly attributable to the introduction of risk-based capital requirements in the banking industry. They argue that the high capital requirements for loans have caused banks to invest more of their funds in other ways.

2. *The formula should maintain a "level playing field."*

As noted earlier, capital requirements have implications for prices and competitive position. Care must therefore be taken to assure that any risk-based capital formula does not create undesirable distortions in the marketplace. First, any formula should not place U.S. property-casualty insurers at an unfair competitive disadvantage with foreign insurers. As the insurance industry becomes increasingly global, this issue becomes very significant.

Similarly, any formula should not place the insurance industry at an unfair competitive disadvantage with alternative risk transfer mechanisms, nor should any formula unfairly disadvantage one segment of the industry over another: stock vs. mutual, primary vs. reinsurer, national vs. regional, small vs. large, multiline vs. specialty, new vs. established, etc.

Additionally, any formula should not produce differences between insurers due to organizational structure (e.g., holding companies, subsidiaries, etc.) which do not affect risk characteristics.

Finally, the level playing field issue extends to individual companies. Any formula should produce a result, and be compared to a base, that is consistently and equitably calculated for each insurer.

3. *The formula should measure risk consistently between the various components of the formula.*

   The amount of risk-based capital for each source of risk (e.g., underwriting, investment, or credit) should be such that the risk of insolvency (or other applicable impairment) is directly proportional to that source of risk. For example, the amount of risk-based capital for asset risk should not be double the underwriting risk amount if their respective underlying risks are not related in that proportion. The allocation of risk-based capital should reflect reality. Failure to recognize the consistency of risk measurement may produce unintended market displacements, such as reduced product availability.

4. *Finally, those designing and using the formula should do so with the understanding that the intent is to minimize insolvencies (in the sense of insufficient assets to meet liabilities), not to prevent "failures".*

   In a competitive market it is necessary (and desirable) that inefficient companies be driven from the market by competitors that are more efficient, innovative, and better managed. Companies that are forced to withdraw from the business are "failures". Because the current minimum capital requirements are so low, too many of these forced withdrawals are insolvencies. In essence, a risk-based capital requirement would raise the regulatory safety net off the floor, and place it at a level where intervention can occur prior to insolvency. Companies then would be forced to withdraw when their capital fell below the minimum, rather than at the point of insolvency. This would seem to represent a potentially significant improvement over the current system, which doesn't provide much room for anything other than a "hard landing".

   The formula, and its regulatory implementation, should not attempt to prevent failures from occurring. Rather they should focus on minimizing the economic and social consequences of such failures when they occur.

V. Discussion of Risk and Risk-Based Capital

Definition of Risk

Risk is the possibility of suffering unexpected harm or loss. For financial statement items, risk is present when the realization of an asset or liability can produce a value different from its expected amount. Risk also exists if future events can lead to unexpected operating losses on new and renewal business not already reflected on the balance sheet. Financial statements contain elements that are either directly measurable (payroll taxes due and unpaid, for example) or estimates (e.g., loss reserves). Loss reserves may develop unfavorably, for instance, while payroll taxes remain fixed when reexamined. Stocks and bonds may fluctuate in value due to market conditions and provide less cash than expected if the company needed to sell those assets unexpectedly. Liabilities may be paid faster or slower than expected. Bonds may be called when interest rates fall, reducing expected investment income.

Bonds and real estate may similarly fluctuate in market value, even if accounting convention keeps their financial statement values constant. Conversely, change in an accounting value *per se* does not indicate risk; rather, it is the uncertainty in the actual realized value of the asset itself (represented by the accounting value) that conveys risk. For example, the ultimate value of a discounted unpaid loss may be known with certainty, but although its accounting measure will change (increase) through time, there is no risk present. On the other hand, an unpaid loss with a 50% chance of either a $1,000 payment or no payment might carry a constant $500 reserve for several years until the uncertainty is resolved.

Generally, the greater the spread of possible realizable values (in financial statement values or future operation) subsequent to the current valuation, the greater the risk.

Definition of Risk-Based Capital

Risk-based capital is the theoretical amount of capital needed to absorb the risks involved in the operation of a business. A higher risk business requires more capital than does one of lower risk. More specifically, it is the amount of capital necessary to insure that the business has an acceptably low expectation of becoming financially impaired (the standard for this low expectation will be addressed later).

Measurement Bias

For financial statement items, measurement bias occurs when the recorded value differs from the anticipated realizable value. Two insurers may carry an identical financial statement element (usually an estimated item) at different amounts. For example, one insurer may record its loss reserves including a margin for adverse deviation, while another may discount its loss reserves to reflect the time value of money. Because different valuation standards may be used, it is useful to specify an *expected present value* benchmark: for an estimated financial statement quantity, expected present value is the mathematical average of the present value of all possible realizable values, weighted by the probability of each value occurring.

The difference between the carried and expected present value is a measure of the *bias*. In general, bias does *not affect the risk* of a financial item, because the *spread* of potential subsequent values does not depend on the valuation basis for the original estimate. However, if an item has a known bias, it may be necessary to adjust the financial statement value to remove the bias.

Bias may exist because 1) the valuation standard is conservative/liberal (e.g., ignoring salvage or income tax liability), or 2) the estimation process consistently overstates or understates the realizable value (e.g., reserves are set using a faulty method).

As an example, suppose two insurers with the same recorded total liabilities (including capital) have identical unpaid loss obligations: $5,000 with 50% probability and $15,000 with 50% probability; the expected value is $10,000. Thus their ability to pay the loss would also be identical. However, the first insurer carries the reserve at $11,000 and the second at $9,000. The risk of adverse development relative to the unbiased $10,000 reserve is identical for both insurers, but the second insurer would *appear* to have $2,000 more capital to withstand the adverse development, while in fact it would not. Therefore, recognizing the bias is an important issue in setting risk-based capital.

Removal of Bias

Since the issue of bias is critically important in valuation, when establishing a risk-based capital formula designed for general application, financial statements should first be adjusted to remove any clearly identifiable bias. Then the application of a formula or other risk-based capital technique could assume that the relevant financial statement is free from bias.

The risk-based capital formula itself need not carry the burden of correcting any current deficiencies or conservativeness of statutory accounting or of any perceived weaknesses of a particular insurer's Annual Statement. However, there may be instances where the collective financial statement values of individual insurers are biased (for example, some analysts maintain that the property-liability industry's total reserves are chronically understated) but it is difficult to ascertain whether an individual company has a bias. In this circumstance, it might be proper to correct the bias using the risk-based capital formula.

Whether the risk-based capital formula should address these matters is an open issue. One view is that any changes to statutory accounting rules should be made separately, outside the formula. Weaknesses in individual insurers' reserving practices, for example, are to be addressed by the statement of actuarial opinion requirement on the loss reserves. Another perspective is that, as a practical matter, the effectiveness of reserve opinions has not been tested, and meaningful changes to statutory accounting rules may occur only in the distant future. Thus, it might be better to use the risk-based capital formula to help address financial reporting deficiencies, a core solvency concern.

When considering a risk-based capital requirement, one of the items to be considered is the Casualty Actuarial Society's *Statement of Principles Regarding Property and Casualty Valuations* (see Appendix 4). Also, the Actuarial Standard of Practice of the Actuarial Standards Board, *Performing Cash Flow Testing for Insurers*, adopted in July 1991, should be considered (see Appendix 5).

Risk Measurement

In simplest terms, a risk-based capital requirement must consider the size of an insolvency as well as its probability. The combination of these factors is the anticipated cost of the insolvency, or the *expected policyholder deficit*. By relating this amount to the anticipated claims against the company's insureds, a consistent measure of risk can be maintained for all risk-producing items, either assets or liabilities. Risk measurement for single risk elements, along with the effect of combining risk elements, is addressed more fully in Appendix 6.

VI. Elements of Risk for Property-Casualty Insurance Companies

The generally recognized risk areas specific to property-casualty companies are shown in Table 1 below.

Table 1
Risk Areas

Risk Area	Type of Risk
Loss and LAE reserves	Chance of under-valuation (over-valuation) of liabilities from past business.
Pricing (Profitability)	Income (including investment income) from future business will be inadequate to cover claims and expenses because of catastrophes (hurricanes, earthquakes) or inadequate prices. This includes business already written but not earned.
Credit risk (ceded reinsurance, retro premiums due, etc.)	Defaults on amounts due from reinsurers, policyholders, etc.; over-estimates of amounts due.
Asset risks	Default of principal or interest, calls on bonds, fluctuation in market value.

Company characteristics that tend to affect these risks are shown in Table 2 below.

Table 2
Company Characteristics That Modify Risk

Characteristics	Discussion
Rapid growth	Loss of control. Increased pricing and reserve risk. Historical data shows that rapid growth is related to company failures.

Small size	Lack of credible experience; Greater effect of random fluctuation.
New company	Historical data shows that new companies fail more frequently than mature companies.
Asset/liability mismatch	Company is vulnerable to changes in interest rates.
Concentration/diversification	Increased (decreased) exposure to natural catastrophes (earthquake, hurricane, tornado, etc.) and pricing errors, regulatory or court decisions, etc.
Net retention	Higher retentions increase risk due to catastrophe or large claims. Lower retentions increase reliance on reinsurers; need for reinsurer profits to be included in prices.

General Business Risks

Listed below are examples of general business risks. These risks can be very significant, but the relative importance will vary widely from company to company. It may not be feasible to include all (or any) of them in a risk-based capital formula.

General Business Risks

1. Competitors will win customers away through superior service or lower prices.

2. Suits (EEOC, bad faith, etc.) will be initiated against the company.

3. Lease obligations will exceed future needs.

4. Pension and other post-retirement obligations will cost more than anticipated.

5. Legislative actions, court decisions or regulatory rulings will alter markets and/or competitive abilities or create or expand coverage.

6. Mismanagement or fraud will damage the company.

7. Taxes and other governmental levies will rise.

8. Economic and/or social conditions will change in a manner detrimental to the company.

History of Failures

The A.M. Best Company recently completed a study of 302 insolvencies, which occurred from 1969 through 1990, in which they identified the principal cause of each failure. Those causes can be roughly cast into the risk framework above as follows.

		Number of Companies
A.	Policyholder Obligations Larger than Anticipated	
	Deficient Loss Reserves/Inadequate Pricing	86
	Rapid growth	64
	Significant Change in Business	26
	Reinsurance Failure	21
	Catastrophe Losses	17
		214
B.	Asset Deterioration	
	Overstated Assets	30
		30
C.	Other	
	Alleged Fraud	30
	Miscellaneous	28
		58
Total		302

Risk Measurement

Measurement issues common to many of the generally recognized risk areas listed in Table 1 are the following:

1. Accuracy of financial statement estimates

2. Variation by line of insurance

3. Measurement base

4. Individual company experience

5. Usefulness of historical experience vs. need for judgment of the future

Accuracy of Financial Statement Estimates

Many of the most important financial statement values, loss reserves in particular, are based on estimates. The risk-based capital approach might involve steps which replace the company estimates by alternative estimates. Alternatively, the risk-based capital calculation might begin with the financial statement estimates prepared by company management. Revisions to those estimates, if appropriate, might derive from other forms of regulatory oversight.

Variation by Line of Insurance

Some risks, loss reserve adequacy and profitability for example, vary by type of company and by line of insurance. In those cases, the risk-based capital calculation should reflect that variation. For other risks, asset values for example, the risks do not vary by line of insurance. Those risks should be measured in the same way for all companies on a companywide basis.

Measurement Base

The risk-based capital factors should be applied to an appropriate base for the asset or liability whose risks are being measured. In many situations the measurement base will be the annual statement value of an item; bonds at amortized value, for example.

In some cases another measurement base might be considered. The measurement base for the loss reserve risk might be the held reserve or an alternative calculation of expected claim payout such as loss reserve plus Schedule P reserve (annual statement excess of statutory reserves over statement reserves), or loss reserve plus an alternative Schedule P-type reserve.

Individual Company Experience

To the extent credible, individual company experience should be considered when the risk-based capital calculation involves estimates of bias, for example in loss reserve or profitability risks. To be considered credible, a company's past experience must be demonstrably related to future experience during times of greatest risk.

Historical Experience vs. Judgment

It is desirable to give significant weight to historical experience to develop the risk-based capital factors. For reserve and profitability risk, historical experience can provide significant guidance depending on current and future conditions. However, the risk-based capital factors also need to consider data outside of the property-casualty insurance industry and leave room for informed judgments.

VII. SAP, GAAP and Risk-Based Capital

Once the risk-based capital amount has been calculated, the amount must be compared to a consistently calculated base. Statutory surplus might be that base, but may need adjustment.

To some degree, statutory accounting principles (SAP) can be viewed as a form of risk-based capital. SAP include conservatism which should be considered in the risk-based capital calculation to prevent "double counting" risk in certain areas, once by reductions in statutory surplus and once by a charge in the risk-based capital calculation. Five such areas are the following: 1) discounting of loss reserves, 2) statutory write-offs of recoverables, 3) deferred policy acquisition costs, 4) valuation of subsidiaries, and 5) salvage and subrogation.

These areas are discussed below:

Loss Reserve Discounting

SAP reserves are intended to be computed on a basis closer to nominal, than to present value basis. This creates an implicit capital margin equal to the difference between the two.

In recognizing the time value of money, the risk-based capital calculation should consider, among other things, the following: 1) the interest rates available during the experience period analyzed for purposes of selecting the risk-based capital factors, 2) the interest rates likely to be available in "normal" times, 3) differences in interest rate potential between short-tail lines of insurance and long-tail lines of insurance, and 4) risk adjustments to interest rates to reflect uncertainty about timing and amounts of loss payments.

SAP Treatment of Reinsurance Recoverables and Premium Receivables

Some ceded reinsurance is written-off for SAP purposes for non-collateralized unauthorized reinsurance and for reinsurers indicating late payment of recoverables. The risk-based capital charge for ceded reinsurance recoverable should not double-count these write-offs.

Similarly, premium receivables over 90 days past due are written-off for SAP and should not be double counted in any risk-based capital calculation.

Capitalization of Policy Acquisition Costs

SAP accounting does not allow acquisition expenses to be capitalized and amortized over the life of the policy. Stated differently, the unearned premium liability is gross, rather than net, of these expenses.

Valuation of Subsidiaries

Insurance subsidiaries are valued at SAP value rather than Generally Accepted Accounting Principles (GAAP) value or market value. The difference might be considered in evaluating risk-based capital risk charges for those types of assets.

Salvage and Subrogation

SAP accounting practices do not currently allow reduction of loss reserves for anticipated salvage and subrogation, while GAAP does. Recently it has been learned that some companies actually do reduce their statutory loss reserves for anticipated salvage and subrogation. The surplus of all companies should be stated consistently as possible--either all net of salvage and subrogation or all gross.

Differences in SAP by Company

SAP asset and liability values are not necessarily uniform among companies. For risk-based capital purposes, areas of significant differences should be removed.

First, some companies record reserves discounted for future investment income. This is particularly true of medical malpractice and workers' compensation lines of insurance. The amount of discount must be determined and SAP surplus should be reduced (or risk-based capital increased) by the amount of the discount. In many cases the discount amount is clearly disclosed in the Annual Statement. In some cases, the disclosure is not completely clear in the statement. Proper application of the risk-based capital rules may require

Annual Statement changes to make the disclosure of discounting uniform by company.

Second, the degree of conservatism included in loss reserves varies among companies. Actuarial opinions and increased regulatory attention to reserves may reduce the degree of variation among companies. Still there will be certain unquantifiable areas of exposure for the property-casualty industry, such as pollution claims reserves. Those steps are unlikely to eliminate the variation, some of which may represent legitimate management discretion.

Third, as noted above, some companies net anticipated salvage and subrogation out of reserves.

While it might be desirable to replace the held reserve with a standardized reserve, this may not be practical. There is no magic formula which will mechanically produce the correct reserve. However, the present Schedule P statutory reserve, or an improved Schedule P statutory reserve, could be used to help assure that reserves include a minimum level of conservatism. The risk-based capital charge might best be applied to the held reserve plus the applicable Schedule P statutory reserve.

Future of SAP

A major rationale for the use of SAP for insurance companies rather than GAAP is that SAP deliberately introduces conservatism into insurance accounting. SAP is sometimes characterized as "liquidation basis" accounting while GAAP is characterized as "going-concern" accounting.

The introduction of the risk-based capital process into statutory reporting provides the opportunity to re-think the use of dual accounting principles. The existing differences between SAP and GAAP could still be accommodated by recording those differences as required capital. For example, consider deferred acquisition expenses. SAP requires a 100% write-off of prepaid expenses. GAAP permits those expenses to be amortized if the business is projected to be sufficiently profitable. The SAP result could be achieved by requiring some risk-based capital charge for deferred acquisition expenses.

The ramifications of this concept cannot be fully explored in this white paper. The material above was presented merely to identify an area that might warrant further consideration.

The Canadian Model

The Canadian statutory annual statement is prepared on a GAAP basis (with bonds at amortized values and loss reserves at nominal values). The Blank includes what is called a "minimum asset test", which effectively is a minimum surplus test. This test compares assets available for test purposes (carried assets after some adjustments) to assets required for test purposes, which are the sum of carried liabilities (after some adjustments) plus the greater of:

1. 15% of loss reserves, or
2. 15% of written premium, or
3. 22% of claims incurred in the last year.

This summary oversimplifies the calculation. The details are contained in Appendix 7.

As a rule of thumb, as long as the margin is greater than 10% of the assets required for test purposes, the company does not receive extra regulatory attention. Perhaps a simplified model, such as this one which is used effectively in Canada, would be appropriate in the United States.

VIII. Ramifications of the Level of Risk-Based Capital

There are several approaches that could be followed in establishing risk-based capital standards from which the threshold for additional regulator attention for a company could be determined. Three of those approaches and their potential ramifications are as follows:

Capital Standard	Hypothetical % of Companies Meeting Standard	Description
Minimum Minimally Acceptable (shut-down level)	95-99%	Least amount allowable; regulatory control of company below this level
Prudent Margin Prudently Managed (normal level)	70-90%	No regulatory intervention required if company exceeds capital standard; long-term industry average surplus meets standard
Triple A Disaster-Proof (top-quality level)	1-5%	Able to withstand all reasonable worst-case scenarios; more than this amount rarely needed

The "Minimum" Capital Standard

This approach would set as the standard the absolute minimum capital a company could carry to be allowed to continue to conduct its business without imposed changes. Ideally, a very small percentage of all companies would fall below this minimum. This standard would have the advantage of reducing the potential impact on the public's perception regarding the industry's strength.

However, there are some potential disadvantages of such a standard, for example:

- A minimum standard could cause the industry's actual capital to gravitate downwards towards that standard, reducing the margins that companies would retain to cushion against unexpected events. Such an outcome might actually increase the potential for insolvencies.

  This concern has also arisen in the banking industry. While a risk-based capital standard has been introduced for that industry which has been characterized as a "minimum", there are signs it is becoming, in effect, a target.

- Since insurers might not be allowed a reasonable rate of return on capital carried in excess of the standard, downward regulatory and consumer pressure on rates could result from any misperception that a company may have excess capital. This could in turn diminish the willingness of the industry to commit capital in excess of the published minimum standard.

The "Prudent Margin" Capital Standard

This approach would publish as the standard the indicated amount of capital necessary for a company to be permitted to operate its business free of regulatory intervention. This amount, at a reasonable margin above the minimum required to remain in business, could be considered a target amount. A majority of companies would likely currently exceed this level. The prudent margin standard would focus on the capital the average company should carry to minimize its long term risk of insolvency, provided that adverse outcomes are not substantially beyond what a prudent manager could reasonable anticipate.

A potential disadvantage of this standard could arise if an undue perception of the formula's precision is created, in turn creating an excessive reliance upon a pure formula approach for determining the capital needs of companies.

The "Triple A" Capital Standard

This approach would publish as the standard an indicated amount of capital sufficient to absorb financial impacts from a scenario that is so pessimistic as to be considered highly unlikely. The indicated capital should be something obtainable, but would be at a level at which very few companies are currently capitalized. At its extreme, this standard might require a level of capital that would only be needed if very pessimistic assumptions were ultimately realized for essentially all of the insurer's assets and liabilities. If all companies were required to carry this much capital, and if the industry in general could secure such capital, then the risk of future insolvencies might be virtually eliminated.

However, the requirement that the capital markets provide sufficient capital for an environment free from all risk of insolvency is economically unrealistic. Further, since insurers and their investors will seek a reasonable return on each dollar of capital invested, upward pressure on rates will result to achieve the desired return. However, if public policy issues prevent the requested rate increases, the markets will be unwilling to provide the required capital. Ironically, an excessive capital requirement relative to the expected return could reduce the amount of capital invested in the industry, reducing availability.

Companies that are actually stronger than the formula suggests could find their public image severely tarnished. This may impair their ability to write profitable business and actually increase the risk of insolvency.

Use of the Capital Standard

Whichever standard is chosen, regulatory responses may be triggered when the capital a company falls below a given threshold, which may be stated in terms relative to specified percentages of the standard. For example, if the prudent margin standard is used and a company's capital falls below a threshold of, say, 75% of the standard, the regulator could establish on-site monitoring, more frequent financial analyses, and other measures as deemed appropriate. If the company's capital fell below 50%, stronger action, including a cease and desist order, could be indicated. In a similar fashion, if the minimum standard is used and a company's capital is at perhaps 110% of the standard's indication, the regulator could establish on-site monitoring, etc. Note that the percentages given are for illustration only.

If regulatory responses are triggered by set percentages of the standard, then the economic impact on the industry will depend on the combined effect of the standard and the percentages used. Conceptually, the combined effect of the standard and percentages used should generate the same regulatory response for a given level of capital regardless of which standard is chosen. Therefore, the decision regarding which standard is chosen should depend on which standard provides the best tool for regulatory purposes and which also minimizes the potential for public misperceptions arising from the standard published.

Consistency Considerations

Whatever standard is selected, there are significant advantages to having a comparable standard to that adopted by the Life Risk-Based Capital Working Group and consistency with similar standards for property-casualty companies worldwide. For example, consider the possible consequences if the formula for property-casualty companies applies a 10% factor to common stock investments in calculating risk-based capital but the formula for life companies applies a 5% factor. This would tend to cause a parent company with both life and property-casualty operations to shift all stock holdings to the life subsidiaries. This would reduce risk-based capital for the parent as a whole even though this shift in assets would not change the parent's fundamental risk characteristics.

CAPITAL AND SURPLUS REQUIREMENTS FOR MULTI-LINE PROPERTY-CASUALTY INSURERS

(Excluding Title, Mortgage Guaranty, Home Protection, Legal Expenses, Residual Value, Credit Unemployment and Health Care Services Contractors Lines)

As of 11-1-91

STATE	CAPITAL REQUIREMENT		SURPLUS REQUIREMENT	
Alabama	$ 500,000		$ 500,000	(See Note 1)
Alaska	$3,000,000		$3,000,000 $2,250,000	(Initial) (Maintained)
Arizona	$1,000,000	(See Note 2)	$ 500,000	
Arkansas	$1,250,000	(See Note 3)	$1,250,000	(See Note 3)
California	$2,600,000		$2,600,000	
Colorado	$2,000,000	(Total Capital and Surplus)		(See Note 4)
Connecticut	$2,000,000		$2,000,000	
Delaware	$ 500,000		$ 250,000	
District of Columbia	$ 300,000		$ 300,000	
Florida	$2,500,000	(Total Capital and Surplus)		(Initial)
	Total maintained capital and surplus must be at least:			
	$1,300,000 $1,500,000	(Until 12-31-92) (Thereafter)		(See Note 5) (See Note 5)
Georgia	$1,500,000		$1,500,000	(or 50% of Capital)
Hawaii	$2,500,000		$1,250,000	(See Note 6)
Idaho	$ 650,000		$ 650,000	

STATE	CAPITAL REQUIREMENT		SURPLUS REQUIREMENT	
Illinois	$1,000,000		$1,000,000 $ 500,000	(Initial) (Maintained)
Indiana	$1,000,000		$1,000,000 $ 250,000	(Initial) (Maintained)
Iowa	$2,500,000		$2,500,000	
Kansas	$ 900,000		$ 600,000	
Kentucky	$1,000,000		$2,000,000	
Louisiana	$ 650,000		$1,350,000 $1,000,000	(Minimum Surplus) (Operating Surplus)
Maine	$2,500,000		$2,500,000	
Maryland	$1,500,000	(See Note 7)	$2,250,000	(See Note 8)
Massachusetts	$2,100,000		$4,200,000	
Michigan	$1,000,000	(See Note 9)	$ 500,000	(See Note 9)
Minnesota	$1,000,000		$1,000,000 $ 500,000	(Initial) (Maintained)
Mississippi	$ 600,000		$ 900,000	
Missouri	$1,200,000		$1,200,000	
Montana	$ 800,000		$ 800,000	
Nebraska	$2,000,000	(Total Initial Capital and Surplus)	$2,000,000	(Maintained)
Nevada	$ 500,000		$1,000,000	
New Hampshire	$ 400,000		$ 400,000	
New Jersey	$2,000,000		$1,000,000	

STATE	CAPITAL REQUIREMENT		SURPLUS REQUIREMENT	
New Mexico	$ 700,000		$ 700,000	(Initial)
	Insurer shall maintain an aggregate of capital and surplus of $2,400,000 if earned or received premium volume in previous calendar year was $5 million to $10 million Insurer shall maintain an aggregate of capital and surplus of $2,700,000 if earned or received premium volume in previous calendar year was $10 million to $25 million Insurer shall maintain an aggregate of capital and surplus of $3,000,000 if earned or received premium volume in previous calendar year was over $25 million			
New York	A multiple-line P&C company must maintain paid-in capital of $1,000,000 and surplus to policyholders of $3,200,000			(See Note 10)
North Carolina	$1,800,000		$2,700,000 $ 450,000	(Initial) (Maintained)
North Dakota	$ 500,000		$ 500,000	
Ohio	(A) $2,500,000 (Total Capital and Surplus)			(See Note 11)
	(B) $5,000,000 (Total Capital and Surplus)			(See Note 12)
Oklahoma	$ 500,000	(Aggregate Capital and Surplus)	At time of initial authorization, shall also possess expendable surplus of not less than $250,000	
	Workers' Compensation:			
	$5,000,000	(Minimum Aggregate Capital and Surplus)		
Oregon	$1,000,000	(Total Capital and Surplus)		(See Note 13)
	Workers' Compensation:			
	$3,000,000	(Total Capital and Surplus)		
Pennsylvania	$2,350,000	(See Note 14)	$1,175,000	(See Note 14)
Rhode Island	$1,000,000		$2,000,000	
South Carolina	$1,500,000	(See Notes 15 and 16)	$1,500,000 $ 375,000	(Initial) (Maintained) (See Notes 15 and 16)

STATE	CAPITAL REQUIREMENT		SURPLUS REQUIREMENT	
South Dakota	$ 400,000		$ 400,000	
Tennessee	$1,000,000		$1,000,000	
Texas	$1,000,000	(See Note 17)	$1,000,000	(See Note 17)
Utah	$2,000,000		The greater of: (a) $1,500,000 or (b) net total of $.50 per $1,000 life insurance amount at risk, plus 10% earned disability premiums, plus 15% net workers' compensation and other liability premiums earned, plus 20% medical malpractice premiums earned, plus 10% of net premiums earned on lines of insurance not set forth, plus 5% admitted value of common stocks and real estate, plus 2% admitted value of all other invested assets, less any mandatory security valuation reserve being maintained, and less minimum required capital	
Vermont	$2,000,000	(See Note 18)	$3,000,000	(See Note 18)
Virginia	$1,000,000		$3,000,000	
Washington	$3,000,000	(See Note 19)	$3,000,000	(See Note 19)
West Virginia	$1,000,000	(See Note 20)	$1,000,000	(See Note 20)
Wisconsin	$2,000,000	(See Note 21)	$1,000,000	(See Note 21)
Wyoming	$2,000,000		$2,000,000	

NOTES

Note 1 - If insurer has not transacted business for five years, it is required to maintain surplus of $750,000.

Note 2 - Director may require additional capital based on type, volume and nature of business conducted.

Note 3 - Commissioner may require insurer to possess and maintain additional capital and surplus in addition to that required, based on types, volume or nature of business transacted by insurer.

Note 4 - Companies licensed prior to 7-1-91 shall have until 12-31-92 to increase their total capital and surplus to this amount.

Note 5 - Or alternative calculation, based on liabilities.

Note 6 - Additional amount required of new insurers after 7-1-88, and of all insurers after 7-1-93.

Note 7 - Applies to insurers commencing business on or after 7-1-91. On or after 7-1-2001, any insurer qualified to engage in business prior to 7-1-91 shall possess and maintain paid-in capital in an amount not less than 150% of that required of insurers commencing business on 6-30-91.

Note 8 - Vehicle liability insurers commencing business prior to 7-1-66 also must maintain $300,000 additional surplus.

Note 9 - Amounts are the minimums required for an initial certificate of authority. The insurance department has the authority to require additional surplus. After licensure, $1,000,000 must remain unimpaired.

Note 10 - Applies to insurers writing the following lines: Miscellaneous property; water damage; burglary and theft; glass; boiler and machinery; elevator; animal; collision; personal injury liability; property damage liability; workers' compensation/employer liability; fidelity and surety; credit; title; motor vehicle and aircraft physical damage; marine protection and indemnity; fire; and marine and inland marine.

Note 11 - For insurers writing fire; allied lines; farmowners' multiple peril; homeowners' multiple peril; ocean marine; inland marine; earthquake; group accident and health; credit accident and health; accident and health; auto liability; auto physical damage; aircraft; glass, burglary and theft; boiler and machinery; and credit, not less than $1,000,000 shall be paid-in capital and not less than $1,000,000 shall be contributed surplus.

Note 12 - For insurers writing commercial multiple peril; financial guaranty; medical malpractice; workers' compensation; other liability; fidelity; surety; and any other risk other than life insurance, not less than $2,000,000 shall be paid-in capital and not less than $2,000,000 shall be contributed surplus.

Note 13 - A domestic insurer applying for its original certificate of authority in this state shall possess at the time of authorization additional capital and/or surplus of not less than $500,000.

Note 14 - The Insurance Commissioner has discretion to require additional amounts. Because Section 503 of Pennsylvania's Insurance Department Act requires insurers to maintain the minimum required capital and surplus unimpaired at all times, the Insurance Commissioner will require newly incorporated insurers to demonstrate possession of surplus over the statutory minimum amount. The exact amount of additional surplus will be dependent upon the financial forecasts included in the insurer's business plan.

Note 15 - The Insurance Commission may require additional initial capital and surplus based on type or nature of business transacted.

Note 16 - An insurer licensed prior to 7-1-91 which does not meet minimum requirements shown must maintain at least the capital shown on its 1990 annual statement and surplus in an amount of at least 25% of such capital.

Note 17 - The Texas Board of Insurance may adopt rules, regulations and guidelines requiring an insurer to maintain capital and surplus levels in excess of the required statutory levels, based upon nature, type and volume of risks, company's portfolio and company's reserves.

Note 18 - The Commissioner may prescribe additional capital or surplus for all insurers, based upon type, volume and nature of insurance transacted.

Note 19 - Applies to insurers authorized on or after 7-1-91.

Note 20 - Insurers are required to maintain $2,000,000 statutory surplus.

Note 21 - The Commissioner may reduce required amount.

Risk-Based Capital Requirements in the Banking Industry

The Basle Capital Framework

U.S. banking agencies first issued a risk-based capital proposal in 1986. While initial reaction was favorable, many reviewers felt that, without similar requirements for foreign competitors, the proposed requirements would put U.S. banks at a competitive disadvantage. In light of these concerns, the U.S. banking agencies began working with the Bank of England on the development of a common approach. A joint proposal was published in 1987. The Committee on Banking Regulations and Supervisory Practices (frequently referred to as the "Cooke Committee" or the "Basle Supervisors Committee") subsequently took the U.S./U.K. proposal under consideration and addressed the possibility of expanding the agreement to include all 12 of the countries represented on the Committee.

Although the banking industry had had various forms of capital standards for many years (usually measured by capital to asset ratios), the regulators were concerned about decreasing capital ratios and, in the case of U.S. thrifts, hundreds of insolvencies. Their objective was to strengthen the soundness of the international banking system and to encourage the establishment of uniform minimum capital standards among the major industrial countries.

The initial standards were amended in July 1988 and were then endorsed by the Group of Ten Central Bank Governors (representatives of the major industrialized countries). They have become known as the Basle Capital Framework or the Basle Accord.

United States Regulatory Standards

United States banking and thrift regulators soon adopted risk-based capital standards which were consistent with the Basle Capital Framework. The Federal

Reserve Board (the Fed), which regulates state member banks and bank holding companies, issued guidelines for banking organizations. The Office of the Controller of the Currency (OCC) issued guidelines for nationally chartered banks. The Office of Thrift Supervision (OTS) issued guidelines for thrift (savings and loan) institutions. The Federal Deposit Insurance Corporation (FDIC) issued guidelines for non-federal member banks and state chartered institutions. All four sets of guidelines are very similar.

In adopting risk-based capital measures, the Fed, the OCC, the OTS and the FDIC shared the objective of the Basle Committee for more consistency in worldwide capital adequacy standards. They also wanted to make regulatory capital requirements more sensitive to differences in risk profiles among banks, to factor off-balance sheet exposures into the assessment of capital adequacy and to minimize disincentives to holding liquid, low-risk assets.

The "interim final" rules for thrifts were put into place on 12-7-89; interim minimum requirements for banking organizations became effective on 12-31-90, with final measures for the entire industry to be in place by 12-31-92.

The risk-based capital requirement is just one measure in a new set of capital standards. The new requirements contain two components for banks and three for thrifts. The components which are applicable to both banks and thrifts are:

Leverage Ratio Standard: Tier 1 (or Core) capital must be at least 4% (for thrifts) or 6% (for banks) of adjusted total assets.

Risk-Based Capital Standard: Tier 1 plus Tier 2 (Supplementary) capital must be at least 8% of *risk-adjusted* assets.

In both of these standards, lower percentage requirements are being utilized prior to 12-31-92.

The third component, which applies to thrifts only, is:

> Tangible Capital Standard: Tangible capital must be at least 1.5% of adjusted total assets.

Risk-adjusted assets for the second component are computed by assigning weights, ranging from 0% to 100% for banks and from 0% to 200% for thrifts, to the various categories of assets and off-balance sheet items. The dollar amount of each asset is then multiplied by the risk weight, and the resulting weighted values are summed to arrive at total weighted-risk assets.

As described above, the risk-based capital requirements include risk from assets and off-balance sheet items (such as letters of credit) but exclude such items as interest rate risk, liquidity risk, market risk, and operational risk. For this reason, the calculated risk-based capital requirement is treated as a minimum and banks are expected to maintain capital positions above the minimum ratio. An institution which does not meet the minimum, or whose capital is otherwise considered inadequate, is expected to develop and implement a plan, acceptable to its regulator, for achieving adequate capital within a reasonable time frame.

Outlook for Banks

The risk-based capital requirements have just recently been enacted and they are being gradually phased in. Therefore, it is premature to predict what their ultimate impact will be. Early indications (from a *Brookings* study) are that the majority of banks will meet the requirements. However, recent articles in *The Wall Street Journal* indicate that the new requirements have already caused many banks to change their investment policies by shifting assets out of corporate loans (which carry the maximum 100% risk weight) and into government securities (which require little or no capital under the formula). It is speculated that banks' growing investment in government securities has helped to keep interest rates low. On the other hand, the Federal Reserve Board and the Bush Administration have become concerned that the decrease in bank lending is contributing to the current credit crunch. They have begun discussing the possibility of easing some of the capital requirements in order to quicken the economic recovery. (Under the risk-based capital guidelines, the Fed may modify the rules in order to reflect significant changes in the economy, financial markets, banking practices, etc.)

Outlook for Thrifts

The outlook for thrifts is different, however. A study appearing in the Fall 1990 edition of the *FDIC Banking Review* indicated that a third of all thrifts which were not already in conservatorship would fail to meet the interim standards and 46% of non-conservatorship thrifts would not meet the final ultimate requirements. (If these companies which failed the requirements were measured by assets, rather than by number, the percentages would increase to 44% and 70%, respectively). Thrifts which fail to meet the standards will come under significant regulatory pressure to increase capital by selling or securitizing high-risk assets, attracting outside capital, or merging with healthier thrifts. Such activities will put them into competition with the Resolution Trust Corporation (RTC), which is attempting to do the same things for thrifts which are already in conservatorship, and could make the RTC's job more difficult.

Further Study

The regulators of the banking and securities industries have held informal talks over the last two years regarding minimum capital standards for debt and equity securities. Because traditional distinctions between banks and securities firms are quickly eroding, some feel there is a need for common standards. Securities regulators have made a formal proposal to the Basle Supervisors Committee, and it is expected that discussion of the proposal will begin in November. The main impact of the plan on banks would be to address the effect of market risk factors on capital standards.

Testing of the Formula

When it is implemented, the risk-based capital formula is likely to identify some companies as being near or below whatever regulatory thresholds are selected. As a result, such companies may be faced with the difficult task of raising capital to avoid forced reductions in business or perhaps even being forced out of business altogether. Thus, the formula will inevitably generate some controversy when introduced. Such controversy is likely to focus on several issues including whether the formula has identified the appropriate companies as being undercapitalized, and whether the degree of under-capitalization is correct. The controversy would likely be intense at both a state and federal level if the impact on the industry were to be so major as to cause significant market dislocations, and if numerous policyholders were to be affected through sharply increased prices and the unavailability of necessary coverages. Such controversy can only be dealt with effectively if the record reflects diligent testing and careful study by experienced professionals to assure the formula's effectiveness.

In structuring the testing approach to be used, the following guidelines should be applied.

1. *The predictive capability of the formula should be tested.*

   One of the main purposes of a risk-based capital requirement is to help regulators to meaningfully discriminate between companies needing regulatory attention due to potential capital inadequacy and those companies which do not require such attention. Accordingly, the formula should be retrospectively applied to a large group of companies (perhaps all) wherever possible to past annual statements to evaluate how effective and how timely it would have been at predicting the insolvencies that have actually occurred in prior years. The formula's responsiveness to the various phases of the underwriting cycle should also be evaluated. Should the formula fail to indicate a problem at least one year before an insolvency occurred, or if such a potential was identified but masked by numerous other companies being incorrectly identified as well, then adjustments to the formula will be indicated. Should data regarding prior company failures prove insufficient for adequate testing, simulation techniques to allow scenario testing should be applied to evaluate how the formula responds to changing conditions. For example, illustrative companies could be generated with high growth rates and a developing reserve inadequacy problem for the more recent accident years to see how quickly the formula responds in detecting an emerging problem.

Test Procedure 1

A. Identify companies which have failed or merged in anticipation of failure during the period of 1980-1990.

B. Based upon historical results, identify how effectively the formula detected emerging problems.

Data may need to be extracted from a variety of sources to accomplish this task in addition to standard NAIC materials. Perhaps companies should be requested to supply supplementary data on diskettes.

These steps will identify the ability of the formula to predict "false positives" as well as "true positives."

C. Based upon a random, credible sample of companies which have remained solvent in the test period, identify how effectively the formula detected solvent and strong companies.

Use NAIC and/or A.M. Best data tapes with request to individual company to correct any errors that may be revealed in the data.

2. *The formula should be tested for reasonable consistency in results from year to year, both for the industry in total and for an individual company.*

The formula will be significantly reduced in value as a regulatory tool if it produces results that fluctuate wildly from one year to the next and for reasons not clearly associated with changes in risk. For a given company, the indicated capital requirement should only change dramatically in one year's time if there has been an identifiable and material change in the company's financial condition, size, mix of business, or mode of operation during that year. Accordingly, the formula should be retrospectively tested to evaluate its stability over time, and to judge whether changes in results are reasonable in light of changes in conditions including the effects of the underwriting cycle. Such retrospective testing should at a minimum be applied to three successive prior years' statements.

Test Procedure 2

A. Calculate the ratio of risk-based capital to statutory capital and surplus for each of the three year-ends. Rank companies on the difference between the highest and lowest values of these three ratios. Use NAIC or A.M. Best data tapes to analyze variation in both absolute and relative terms.

B. In case of unusual year-to-year variations, identify what changes in the formula would be needed to reduce or eliminate them.

3. *Results of the formula should be evaluated in private prior to its implementation to gauge whether the results among industry peers are reasonable.*

No formula can remove the need for applying informed judgment when reviewing the capital needs of property-casualty insurance companies. Scrutiny should be performed of the formula's results for groups of companies that are deemed similar in operation and risk to evaluate whether the comparative results by company appear reasonable given all available empirical and subjective information. Groupings could be small vs. large, multi-line vs. specialty, national vs. regional, stock vs. mutual, primary vs. reinsurer, new vs. established, etc. If the results appear counter-intuitive, adjustments to the formula may be indicated.

Test Procedure 3

A. Compare the historical ratings by Best's and other rating agencies, IRIS test results, etc., to the companies identified in Section 1 and determine if this comparison identifies any additional risk factors not incorporated in the formula.

B. Review differences developed in 1 and determine if they are reasonable. If unreasonable, identify what changes in the formula would be needed to eliminate unreasonable differences.

4. *The formula should be evaluated critically as to its possible economic effects on the industry and how it might shape company behavior.*

The manner in which the risk-based capital formula determines capital requirements will likely affect behavior that may vary by type of insurer. One result may be that companies will adjust financial statement items where flexibility exists in order to minimize capital requirements. Testing should include a thorough evaluation of ways in which a company could attempt to manipulate the results of the formula, with adjustments then being made to the formula where indicated. It should also evaluate the extent to which equal risks result in equal capital requirements and whether there are any differential effects by industry segment.

Test Procedure 4

A. Designate a group of professionals to evaluate, both qualitatively and to the extent possible quantitatively, the effects across segments of the property-casualty industry and how individual companies may react.

B. For each significant effect identified, the group would develop any modifications in the formula necessary to eliminate or minimize undesirable effects or incentives that might arise.

Other Considerations

Since the implications of the formula's results will be both important and sensitive, extensive testing should be performed in private and the results communicated to affected companies before the formula's parameters and its results become public knowledge. Further, since objectivity in testing is crucial, the testing should be performed by an independent body of insurance professionals that would work with both Actuarial and Accounting Advisory Committees to the NAIC Working Group, as well as with a designated group of regulators.

As the testing is in process, and results are reviewed, more tests will become evident. At the conclusion of each thorough round of testing, changes to the risk-based capital formula may be indicated. It is likely that such changes could be major after the first round of testing, with subsequent rounds demonstrating a decreasing number of indicated adjustments. While such a process may become time-consuming, its importance in avoiding undesirable effects on the industry should not be overlooked.

Final Test

Using the tentative final formula as confirmed by the NAIC Working Group leader, project the distribution of companies that would result from its application. Set minimum, prudent and strong company thresholds based upon findings.

STATEMENT OF PRINCIPLES REGARDING PROPERTY AND CASUALTY VALUATIONS

(AS ADOPTED SEPTEMBER 22, 1989)

The purpose of this Statement is to identify and describe principles applicable to property and casualty valuations. The Statement establishes fundamental concepts for research and education regarding valuation techniques. The principles in this Statement provide the foundation for actuarial procedures and standards of practice regarding valuations. These principles apply to valuations regarding any risk bearer of property and casualty contingencies.

This Statement consists of three parts:

I. Definitions

II. Principles

III. Discussion

I. *Definitions*

Valuation is the process of determining and comparing, for the purpose of assessing a risk bearer's financial condition as of a given date, called the valuation date, the values of part or all of a risk bearer's obligations and the assets and considerations designated as supporting those obligations.

A valuation is carried out in accordance with specified rules or assumptions selected or prescribed in accordance with the purpose of the valuation.

A **risk bearer** is a person or other entity that is exposed to the risk of financial losses that may arise out of specified contingent events during a specified period of exposure.

Cash flows are receipts or disbursements of cash.

An **asset** is cash held or any other resource that can generate receipts or reduce disbursements.

An **obligation** is a commitment by or requirement of a risk bearer to make disbursements with respect to financial losses arising out of specified contingent events or with respect to any type of other expense or investment commitment.

A **consideration** is a receipt or a reduction in disbursements in exchange for accepting the risk of financial losses that may arise out of specified contingent events during a specified period of exposure.

II. *Principles*

1. Every obligation, consideration or asset, with the exception of cash held, is associated with one or more items of cash flow.
2. The value of every item of cash flow depends upon the following valuation variables, each of which may involve uncertainty:
   a. the occurrence of the item of cash flow,
   b. the amount of the item of cash flow,

c. the interval of time between the valuation date and the date of occurrence of the item of cash flow, and

d. a rate of interest related to the interval of time between the valuation date and the date of occurrence of the cash flow.

3. The degree of uncertainty affecting each valuation variable for any item of cash flow associated with a given asset, obligation or consideration depends upon:

a. the nature of the asset, obligation or consideration,

b. the various environments (e.g. regulatory, judicial, social, financial and economic environments) within which the valuation is being performed, and

c. the predictive value of the data used to estimate the valuation variables associated with each item of cash flow.

4. In general, the values of items of cash flow associated with a given asset, obligation or consideration, and the values of assets, obligations and considerations themselves are not only uncertain, they are also not independent of each other. Consequently, the degree of uncertainty relative to the combined value of items of cash flow or of assets, obligations and considerations reflects the uncertainties affecting the underlying valuation variables and arising out of the interaction of those variables in the process of combination.

5. The value of an asset, obligation or consideration is equal to the combined values of its constituent items of cash flow.

6. The result of a valuation is the combined value of the assets, obligations and considerations involved in the valuation with due recognition of the offsetting characteristics of receipts and disbursements.

7. These valuation principles apply to any valuation whether it involves a risk bearer's total assets, obligations and considerations as of a given valuation date or only identified segments of the risk bearer's assets, obligations and considerations including:

a. commitments made on or before the valuation date, or

b. the commitments in (a) and commitments projected to be made after the valuation date, or

c. only those commitments projected to be made after the valuation date.

III. *Discussion*

Although no valuation methodology is appropriate in all situations, a number of considerations commonly apply. Some of these considerations are discussed in this section. These discussions are intended to provide a foundation for the development of actuarial procedures and standards of practice.

Data

Data to be used in valuation include descriptions of the characteristics of the risk bearer's assets, obligations and considerations. The descriptions should be sufficiently detailed to permit reasonable projections of cash flows from these assets, obligations and considerations.

VALUATION PRINCIPLES

The actuary may use a risk bearer's own experience relative to its assets, obligations and considerations if this provides a basis for developing a reasonable indication of the future. Moreover, the actuary may use external data drawn from relevant experience of the insurance industry, other financial institutions or surrounding environments.

Organization of Data

Organization of data for valuation is affected by the characteristics of the assets, obligations and considerations involved and the characteristics of the valuation variables connected with them.

Much of the data organizational work relative to obligations and considerations begins with data used in connection with the reserving and ratemaking processes. However, it may be necessary to adjust the results of those processes so as to take into account differences between cash flow dates and the various dates used in those processes. It may also be necessary to identify any relevant expenses that fall outside the data used in the reserving and ratemaking processes and reflect them in the valuation process. It is important, too, to identify potential adjustments to considerations like retrospective premiums or audit premiums that may be received or paid in the future.

If a valuation deals with detailed analyses of cash flows, data organization relative to assets involves principally the work of classifying the assets and developing projections of contractual or anticipated cash flows from them. It is also often necessary to divide assets into classes of investment by such things as time to maturity or quality and to project flows of anticipated receipts into particular classes of investment in accordance with an assumed investment strategy.

Homogeneity

Valuation accuracy is often improved by dividing the data on assets, obligations and considerations into groups exhibiting similar characteristics. Homogeneous groupings recognize, when appropriate, the interrelationships between those assets, obligations and considerations.

Credibility

Credibility is a measure of the predictive value attached to a body of data. Credibility is increased by defining groups of assets, obligations or considerations so as to increase their homogeneity or to increase the volume of data relative to the groups. Increasing homogeneity may fragment the groups to such an extent that their predictive value is reduced to an unacceptable level. Each situation requires balancing homogeneity and the volume of data.

Operating Conditions

Operating conditions should be reflected in valuation. Operating conditions include mix of business, underwriting, claims handling, marketing, accounting, premium processing, portfolio of investments, investment strategy, and reinsurance programs.

Environmental Conditions

Environmental conditions should be reflected in valuation. The regulatory, judicial, social, financial and economic environments are some of the major ones to be considered.

Losses and Loss Adjustment Expenses

The major obligations of a risk bearer are usually those relating to the future payment of losses and loss adjustment expenses. When these obligations are estimated for purposes of a valuation, their future development may be a factor for consideration. Development of losses and loss adjustment expenses is defined in the Casualty Actuarial Society's Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expense Reserves.

Rules and Assumptions

The objective of a valuation is to produce an assessment of a risk bearer's financial condition that will be useful for the purpose for which the valuation is performed. The purpose of the valuation affects the rules and assumptions used.

Cash flow analyses produce projections of receipts and disbursements. These analyses are conceptually the most fundamental of the forms of valuation. The other forms of valuation can be derived from cash flow analysis by suitable selection of rules and assumptions relative to the valuation variables.

Balance sheets and income statements are often produced internally by a risk bearer using rules and assumptions established by its management to assess financial strength and earning performance.

Appraisals are intended to help determine the value of all or a part of a risk bearer's assets, obligations and considerations related to property and casualty contingencies, taking into account not only financial statement items but also off-balance-sheet items such as investment in staff, leases and so on. Appraisals are usually made in connection with mergers and acquisitions and the sale of parts of a risk bearer's business.

GAAP accounting rules or assumptions are intended to produce financial statements that the financial community believes are useful for assessing a risk bearer's earning capacity.

Statutory accounting rules or assumptions are intended to produce financial statements that regulators believe are useful for assessing whether an insurer's financial condition warrants its being allowed to write insurance.

The value of any of the valuation variables with respect to a given set of items of cash flow may be determined on the basis of any set of rules and assumptions that is appropriate to the purpose of the valuation. Rules and assumptions relative to different classes of assets, obligations or considerations need not necessarily be consistent with each other as long as the differences are consistent with the purpose of the valuation, or the effect of the inconsistencies is not great enough to invalidate the valuation.

Assumptions are based on a reasonable review of whatever appropriate facts are available supplemented by the actuary's experience and judgment as necessary. Rules are helpful to the assurance of appropriately consistent treatment of facts and assumptions in valuation. Both rules and assumptions can be helpful to achieving a result with a degree of refinement consistent with the purpose of the valuation. Anticipated changes in operating and environmental conditions should be reflected in the rules and assumptions applied to a valuation.

Valuation Variables

The valuation variables of occurrence, amount, interval of time and rate of interest describe the quantitative characteristics of all cash flows for purposes of financial analysis. All of the valuation variables are conceptually involved in the determination of the values of all assets, obligations and considerations. The roles of the valuation variables in the determination of values may be limited by the selection of rules or assumptions.

The value of any item of cash flow changes with the passage of time. This implies that valuations of the same sets of items of cash flow performed at different valuation dates will in general produce different results. It further implies that a valuation of one set of items of cash

i. simply because of a change in the interest environment, or

ii. because a change in the interest environment brings about a change from expected experience as to the occurrence, amount or timing of items of cash flow connected with assets, obligations or considerations.

There are several factors that affect interest risk:

a. Mismatch of asset and obligation cash flows—this factor relates to the development of an excess of a risk bearer's receipts over its required disbursements or vice versa.

If an excess of receipts over required disbursements develops, the risk bearer may not be able to invest the excess cash at yields that will produce future cash flows large enough to meet its obligations as they mature. This is "reinvestment" risk.

If an excess of required disbursements over receipts develops, the risk bearer may have to borrow or liquidate assets with yields below then current market rates to make up the difference. Borrowing at a relatively high interest rate, or inability to invest the difference at then current market rates produces a reduction in the risk bearer's future profits. This is "market" risk.

b. Changes in the timing of receipts and disbursements—this factor relates to the preference of borrowers to prepay debt carrying high rates of interest when rates go down and to defer repayments of debt carrying low rates of interest when rates go up. For risk bearers of property and casualty contingencies, this risk affects mainly their assets.

c. General economy—this factor relates to the way in which things such as liquidity, inflation, demand for cash to fund expansion, government debt, trade imbalances and distortions in the yield curve affect the general level of interest rates.

d. Trends—this factor relates to changes over time in the interest valuation variable and in the degree of uncertainty affecting it and how those changes affect the other asset and obligation valuation variables.

Interaction with Other Professionals

The uncertainties that affect other actuarial fields, such as ratemaking and reserving, also affect valuation. In addition, valuation is affected by uncertainties met in other fields, such as marketing, underwriting, finance, regulation, risk management and so on. This implies that professionals working in other fields can be helpful in gathering information and developing rules and assumptions to be used in valuation.

Actuarial Judgment

It is important to apply actuarial judgment based on education and experience in selecting and organizing data and making rules and assumptions to be used in the valuation process and in assessing the reasonableness of the results.

ACTUARIAL STANDARD OF PRACTICE NO. 7

(Revised)

PERFORMING CASH FLOW TESTING FOR INSURERS

PREAMBLE

Section 1. Purpose, Scope, and Effective Date

1.1 Purpose - This standard of practice sets out recommended practices and considerations that bear on the actuary's professional work in the area of cash flow testing, also referred to as cash flow analysis, whenever projections and comparisons of cash flows are performed for an insurer.

1.2 Scope - This standard applies to cash flow testing for life, health, property, or casualty insurers. Cash flow testing may be part of many types of analyses, such as:

- Determination of reserve adequacy
- Pricing studies
- Evaluations of investment strategy
- Financial projections or forecasts
- Actuarial appraisals
- Testing of future charges or benefits that may vary at the discretion of the company (e.g., policyholder dividend scales and other non-guaranteed elements of insurance and annuity contracts)

Elements of cash flow testing include asset cash flows, obligation cash flows, and the economic and operating assumptions affecting cash flows.

1.3 Effective Date - This standard of practice is effective October 17, 1991.

Section 2. Definitions

2.1 Asset - Any tangible or intangible resource that can generate receipts or reduce disbursements.

2.2 Asset Risk - The risk that the amount or timing of items of cash flow connected with assets will differ from expectations or assumptions as of the valuation date for reasons other than a change in investment rates of return. Asset risk includes delayed collectibility, default, or other financial nonperformance.

2.3 Cash Flow Testing - The process of projecting and comparing, as of a given date called the valuation date, the timing and amount of asset and obligation cash flows after the valuation date.

2.4 Cash Flow - Any receipt or disbursement of cash.

2.5 Insurer - An entity that accepts the risk of financial losses or, for a specified time period, guarantees stated benefits upon the occurrence of specific contingent events.

2.6 Investment-Rate-of-Return Risk - The risk that investment rates of return will depart from expectations or assumptions as of the valuation date, causing a change in the amount or timing of asset or obligation cash flows.

2.7 Obligation - Any tangible or intangible commitment by, requirement of, or liability of an insurer that can reduce receipts or generate disbursements.

2.8 Obligation Risk - The risk that the amount or timing of items of cash flow connected with obligations will differ from expectations or assumptions as of the valuation date, for reasons other than a change in investment rates of return or a change in asset cash flows.

2.9 Scenario - A set of economic and operating assumptions on the basis of which cash flow testing is performed.

Section 3. Background and Historical Issues

Actuaries have been performing financial projections for many years. Various cash flow elements have often been an integral part of these projections. The large increase in the level and volatility of investment rates of return that occurred in the 1970s and 1980s caused significant swings in asset values, as well as changes in cash flow expectations. In addition, fluctuating operating results have led to increased attention to improving the measurement of the financial security of insurers. As a result of these changes, cash flow testing has become an increasingly important aspect of actuarial work.

Section 4. Current Practices and Alternatives

Some states require comparison of asset and obligation cash flows related to items contained in the statutory financial statement. Other instances where cash flow testing is used include internal financial or investment planning, rate of return calculations, and assessments of an insurer's ability to meet its obligations as they come due.

Common approaches to cash flow testing typically follow these steps:

- Identify which assets and obligations are to be included in the cash flow test
- Select and validate models for assets and obligations
- Select an appropriate scenario or set of scenarios, either deterministic or stochastic
- Project the cash flows of the selected assets and obligations
- Develop conclusions based on analysis of the cash flow projections

There are variations on this process. For example, if cash flow testing is used to test the effects of changes in investment strategy, specific assets may not be identified in the initial step of the process. It may be sufficient instead to test on the basis of variations in asset portfolio characteristics such as yield and duration.

STANDARD OF PRACTICE

Section 5. Analysis of Issues and Recommended Practices

5.1 Scope of Cash Flow Test - A cash flow test may involve part or all of an insurer's obligations that are outstanding as of the valuation date or come into existence subsequently. The obligations and the assets to be included in the cash flow test should be specifically identified.

5.2 Allocation of Assets - In the case of a cash flow test involving only a portion of the assets or a portion of the obligations, the actuary should disclose whether the adequacy of any remaining assets to support the remaining obligations has been examined and if not, why not.

The actuary should be satisfied that the same block of assets is not being improperly used to support different blocks of obligations, either within the cash flow test being performed or in that test and one or more contemporaneous tests.

5.3 Scenarios - The scenario is a key element of cash flow testing. Often, more than one scenario will be analyzed. Scenarios may be generated by either deterministic or stochastic methods.

5.3.1 Range of Scenarios Consistent with Purpose of Test - In some situations, the scenario(s) to be tested may be specified by the client or employer, or by regulation. In other situations, the actuary may develop the scenario(s). In all cases, the actuary should be satisfied that the scenario testing reflects a range of conditions that is consistent with the purpose of the cash flow test.

5.3.2 Number of Scenarios - In determining the number of scenarios that will reflect a range of conditions that is consistent with the purpose of the cash flow test, the actuary should consider the relative importance of the investment-rate-of-return risk, asset risk, and obligation risk.

5.3.3 Disclosure of Limitations - When the actuary draws conclusions from the cash flow test, any limitations due to the number, types, or likelihood of scenarios used should be disclosed.

5.4 Projection of Asset Cash Flows - In order to project an insurer's asset cash flow, the actuary should consider the assets' characteristics as well as the insurer's investment strategy. The actuary should be satisfied that the model used to reflect these considerations produces reasonable estimates of expected asset cash flows.

5.4.1 Asset Characteristics - The characteristics of an asset affect the timing and amounts of its cash flow items. The cash flows of some assets are relatively immune to external factors and can be predicted on the basis of asset structure alone (e.g., high-quality non-callable bonds). The cash flows of other assets (e.g., callable bonds, mortgage-backed securities, common stocks, or premium receivables) are highly influenced by external events, and their analysis must be based on a combination of their structure and external factors. The actuary should consider the following issues in making cash flow projections:

a. Variation - The extent to which the expected cash flows vary due to changes in the scenarios

b. Quality - The asset quality rating as it relates to the risk of delayed collectibility, default, or other financial nonperformance

c. Associated Costs - The costs of maintaining the assets or of converting the assets into cash

d. Experience - The historical experience of similar assets, to the extent such experience is credible and relevant to the projection of future cash flows

e. Other Factors - Other factors that have a material effect on asset cash flows, particularly those factors that have an effect on asset risk or investment-rate-of-return risk.

5.4.2 Investment Strategy - The actuary should consider the insurer's strategy concerning asset management and the effect that this strategy will have on the projection of asset cash flows. Strategy considerations that might affect the projection include use of positive cash flows, funding of negative cash flows, policies and practices relative to the sale of assets prior to maturity and the disposal of assets with declining values, and receivable collection practices.

5.5 Projection of Obligation Cash Flows - In order to project an insurer's expected obligation cash flow, the actuary should consider the obligations' characteristics as well as the insurer's policies concerning the management of its obligations. The actuary should be satisfied that the model used to reflect these considerations produces reasonable estimates of expected obligation cash flows.

5.5.1 Obligation Characteristics - The characteristics of an obligation affect the timing and amounts of its cash flow items. The actuary should consider the following factors in the cash flow projection:

a. Variation - The extent to which the expected cash flows vary due to changes in the scenarios

b. Nonperformance Risks - The risk of reinsurer insolvency or other nonperformance by reinsurers; if it is not practical to model these risks, they should be disclosed if the potential risks could be material

c. Experience - The historical experience of similar obligations, to the extent such experience is credible and relevant to the projection of future cash flows

d. Other Factors - Other factors that have a material effect on obligation cash flows, particularly those factors that have an effect on asset risk, obligation risk, or investment-rate-of-return risk.

5.5.2 Management Policy - The actuary should consider management policy concerning the settlement or payment of obligations, and the effect that this policy will have on the projection of obligation cash flows. Considerations that might affect the projection include claim settlement and benefit payment practices, expense-control strategies, company philosophy relative to the determination of policyholder dividends and charges or benefits that vary at the discretion of the company, as well as any relationships between management policy and the scenarios.

5.6 Determination of Assumptions - No model can fully take into account all the uncertainties and interdependencies affecting an insurer's future cash flows. This implies the need to make simplifying assumptions in developing the specifications of a cash flow testing model.

5.6.1 Sensitivity Testing - The actuary should consider the sensitivity of the model to the effect of variations in key assumptions, and should be satisfied that the issue of sensitivity testing has been adequately addressed. In determining whether sensitivity testing has been adequately addressed, the actuary should consider the intended purpose and use of the testing and whether the results reflect a reasonable range of variation in the key assumptions, consistent with that intended purpose and use.

5.6.2 Internal Consistency - The actuary should analyze the assumptions with regard to the interrelationships between the scenarios and other assumptions to assure internal consistency.

5.6.3 External Requirements - The actuary should consider how laws, regulations, and other external requirements relating to such things as financial statements and operating ratios, federal income taxes, insurer capitalization, and distribution of an insurer's earnings to policyholders or shareholders may affect future cash flows or constrain the range of possible scenarios. These factors should be appropriately reflected in the model.

5.7 Development of Conclusions - The cash flow test is the combination and analysis of the asset and obligation cash flow projections. This analysis may involve the discounting or accumulating of cash flows or a year-by-year comparison. Generally, cash flow projections are performed for a given time period. The actuary should consider the possible effect of cash flows beyond such a time period in analyzing results.

In developing conclusions, the actuary should be satisfied that the results of cash flow testing are reasonable. In determining whether the results are reasonable, the actuary should consider the intended purpose and use of the cash flow testing and the degree of uncertainty in the cash flow projections due to asset, obligation, and investment rate-of-return risks.

Any material limitations of the conclusions presented by the actuary should be described.

Section 6. Communications and Disclosures

6.1 Reliance on Another - The actuary may not be qualified to measure the expected cash flows of all assets and obligations. In such instances, the actuary may make use of another person's work, or of other information provided by another person. The actuary should be guided by Interpretative Opinion 3(a)(4), "Reliance on Another," of the Guides and Interpretative Opinions as to Professional Conduct of the American Academy of Actuaries.

6.2 Actuarial Report - A written actuarial report is recommended as a means of documenting the assumptions, techniques, and conclusions reached when providing a professional recommendation or opinion.

6.3 Special Communications and Disclosures - The actuary's report relative to the results of the cash flow test should contain the following:

a. Specific identification of the insurer's obligations that are to be involved in the test and the assets that are to be dedicated to financing those obligations

b. The scenario(s) used, the likelihood of the scenario(s), and the rationale behind the methodology used to develop the scenario(s)

c. Description of the model used in the cash flow test, including the sources of the data and the key assumptions

d. Conclusions related to sensitivity testing

e. Disclosure of the source of or basis for any material assumption on which the actuary expresses no opinion as to appropriateness. The actuary should be guided by Interpretative Opinion 3(c)(1), "Conflict with Professional Judgment."

6.4 Deviation from Standard - An actuary who uses a procedure which differs from this standard must include, in any actuarial communication disclosing the result of the procedure, an appropriate and explicit statement with respect to the nature, rationale, and effect of such use.

Using the Expected Policyholder Deficit Risk Measure to Determine Risk-Based Capital Factors

The expected policyholder deficit (EPD) risk measure can be used to consistently assess insolvency risk in such a way that a standard level of protection is provided to all classes of policyholder and insurers. The EPD measure can apply equally to all risk elements, whether assets or liabilities.

To illustrate, suppose that an insurer has the following balance sheet:

Assets		*Liabilities*	
Investments	$13,000	Loss Reserve	$10,000
		Capital	$3,000

The realizable value of the investments is $13,000, known with certainty. However, the unpaid loss can be one of three different values, each with a particular probability:

Loss Amount	Probability
2,000	.2
10,000	.6
18,000	.2

The expected value of the loss is $10,000. This is the amount that would be recorded as an unbiased reserve. Therefore, the capital of this company would be assets minus the reserve, or $3,000. The expected policyholder deficit can be readily calculated:

Insurer A

	Asset Amount	Loss Amount	Probability	Claim Payment	Deficit
	13,000	2,000	.2	2,000	0
	13,000	10,000	.6	10,000	0
	13,000	18,000	.2	13,000	5,000
Expected Value	13,000	10,000		9,000	1,000

Capital: 3,000 (= Assets - Expected Loss)
EPD/Expected Loss: .10
Capital/Expected Loss: .30

If the loss is $2,000 or $10,000, the assets are sufficient to pay the claim. However, if the loss is $18,000 (which happens 20% of the time), the deficit is $5,000. Its expected value is .20 x 5,000 = $1,000, which is 10% of the expected loss.

The 30% ratio of capital to expected loss is the relevant factor for a risk-based capital program whose aim is to provide policyholder security equal to a 10% expected deficit. Another insurer with a different amount of losses, but having the same probability distribution, would still require capital equal to 30% of expected losses in order to provide the same 10% level of protection.

Insurer B

	Asset Amount	Loss Amount	Probability	Claim Payment	Deficit
	1,300	200	.2	200	0
	1,300	1,000	.6	1,000	0
	1,300	1,800	.2	1,300	500
Expected Value	1,300	1,000		900	100

Capital: 300 (= Assets - Expected Loss)
EPD/Expected Loss: .10
Capital/Expected Loss: .30

Let's extend the preceding numerical example to assets. Insurer C has a known loss of $5,000 about to be paid, but its $6,000 of assets are risky:

Insurer C

	Asset Amount	Loss Amount	Probability	Claim Payment	Deficit
	12,000	5,000	.1	5,000	0
	6,000	5,000	.8	5,000	0
	0	5,000	.1	0	5,000
Expected Value	6,000	5,000		4,500	500

Capital: 1,000 (= Assets - Expected Loss)
EPD/Expected Loss: .100
Capital/Assets: .167

Here the policyholders will come up short the 10% of the time when assets turn out to be worth nothing. The deficit in this case is $5,000, giving an EPD of $500. Here the ratio of capital to assets needed to provide the 10% EPD/Expected Loss is 16.7%. This is less than the capital factor for losses in the Insurer B example because the assets are not as risky as the losses.

Effect of Combining Risk Elements

When two risk elements are combined, the risk-based capital equals the sum of the separate risk-based capital amounts only if their realizable values are *positively correlated* (in fact, the correlation must be perfect). For example, suppose the losses for insurers A and B are actually separate lines of business for another insurer (Insurer D). Assume that if Insurer A has a $2,000 loss then Insurer B has a $200 loss. Similarly, the $10,000 and $1,000 losses are matched, as well as the $18,000 and $1,800 losses. The risk-based capital needed for a 10% EPD/Expected Loss is calculated below:

Insurer A + B

	Asset Amount	Loss Amount	Probability	Claim Payment	Deficit
	14,300	2,200	.2	2,200	0
	14,300	11,000	.6	11,000	0
	14,300	19,800	.2	14,300	5,500
Expected Value	14,300	11,000		9,900	1,000

Capital:	3,300	(= Assets - Expected Loss)
EPD/Expected Loss:	.10	
Capital/Expected Loss:	.30	

The $3,300 of capital equals the sum of the separate risk-based capital amounts of $3,000 and $300.

Combining the risk elements will *reduce* the risk-based capital if the elements are *independent*. For example, suppose that the value of the loss for Line A does not depend on the value for Line B. Then we have the following possible total losses with their associated probabilities:

Loss Amount			Probability		
A	B	Combined	A	B	Combined
2,000	200	2,200	0.20	0.20	0.04
2,000	1,000	3,000	0.20	0.60	0.12
2,000	1,800	3,800	0.20	0.20	0.04
10,000	200	10,200	0.60	0.20	0.12
10,000	1,000	11,000	0.60	0.60	0.36
10,000	1,800	11,800	0.60	0.20	0.12
18,000	200	18,200	0.20	0.20	0.04
18,000	1,000	19,000	0.20	0.60	0.12
18,000	1,800	19,800	0.20	0.20	0.04

Adding the $13,000 and the $1,300 asset amounts and using the above combined losses and probabilities, we can determine the expected policyholder deficit for the total of the two lines:

	Asset Amount	Loss Amount	Probability	Claim Payment	Deficit
	14,300	2,200	0.04	2,200	0
	14,300	3,000	0.12	3,000	0
	14,300	3,800	0.04	3,800	0
	14,300	10,200	0.12	10,200	0
	14,300	11,000	0.36	11,000	0
	14,300	11,800	0.12	11,800	0
	14,300	18,200	0.04	14,300	3,900
	14,300	19,000	0.12	14,300	4,700
	14,300	19,800	0.04	14,300	5,500
Expected Value	14,300	11,000		10,060	940

Capital	3,300
EPD/Loss	0.300
Capital/Loss	0.085

Notice that the $940 expected deficit for the combined lines is less than the sum of the individual expected deficits ($1,100). This produces an 8.5% EPD/Expected Loss protection level, compared to the 10% value for the separate pieces. To reach the same 10% level as before, we do not need the amount ($3,300) of capital obtained by adding the separate amounts of risk-based capital.

	Asset Amount	Loss Amount	Probability	Claim Payment	Deficit
	13,500	2,200	0.04	2,200	0
	13,500	3,000	0.12	3,000	0
	13,500	3,800	0.04	3,800	0
	13,500	10,200	0.12	10,200	0
	13,500	11,000	0.36	11,000	0
	13,500	11,800	0.12	11,800	0
	13,500	18,200	0.04	13,500	4,700
	13,500	19,000	0.12	13,500	5,500
	13,500	19,800	0.04	13,500	6,300
Expected Value	13,500	11,000		9,900	1,100
Capital	2,500				
EPD/Loss	0.100				
Capital/Loss	0.227				

As shown here, we only need $2,500 in capital, which is 22.7% of expected losses. This compares to the 30% factor required for the losses taken separately.

Using a similar analysis, it can be easily shown that if assets and liabilities are independent, the risk-based capital factor for their combination will also be less than the sum of the separate risk capital amounts. In general, risk-based capital cannot be properly determined unless we know whether risk elements are independent or whether they are correlated.

1990

Insurer Year

MINIMUM ASSET TEST

Reference Page		Current Year (01)	Prior Year (02)
		$'000	$'000
	1. Assets Available for Test Purposes		
05	Total Assets 01	413,068	378,808
08	Less: Non-Admitted Assets 02	11,506	6,990
08	Investment Valuation Reserve and Reserve for Foreign Exchange fluctuations 03	4,922	1,108
	Non-admitted portion of shares of property & casualty insurers (attach details) 04	0	0
05	Deferred Policy Acquisition Expenses 05	19,414	19,400
	 06	0	0
70	Plus: Excess of Market Value over Book Value (page 70, line 08, column 03) 07	0	0
	 08	0	0
	Equals: Assets Available for Test Purposes 09	377,226	351,310
	2. Liabilities for Test Purposes		
06	Total Liabilites 10	300,512	279,937
08	Plus: Hail Insurance Surplus Fund 11	0	0
06	Reserve for negative non-cancellable accident and sickness liabilities 12	0	0
08 50	Reserve for Reinsurance Ceded to Unregistered Insurers (page 50, line 99, column 10) 13	2,926	2,982
	 14	0	0
	 15	0	0
	Equals: Liabilities for Test Purposes 16	303,438	282,919
	3. Reinsurance Ratio (Accident and Sickness claims are excluded from the calculation)		
40	Gross claims incurred during preceding 12 months (page 40, line 19, column 08 plus column 09) 17	100,527	107,595
40	Portion of line 17 in respect of reinsurance ceded during preceding 12 months (page 40, line 19, column 10) 18	6,886	27,173
	Reinsurance Ratio: lesser of (line 18 / line 17 x 100) and 50% 19	7.00	25.00
	4. Margins Required for Test Purposes		
	(a) Accident and Sickness Policies		
45	Margin on claims (15% of unpaid claims and adjustment expenses other than those in respect of instalment claims) 20	15	28
43	Plus: Margin on unearned premiums (15% of unearned premiums other than those in respect of non-cancellable policies or, if applicable, page 69, line 15, column 05) 21	0	24
	Equals: Margin required for Accident and Sickness policies 22	15	52

1990

Insurer | Year

MINIMUM ASSET TEST - Continued

Reference Page		Current Year (01)	Prior Year (02)
		$'000	$'000
	4. Margins Required for Test Purposes (cont'd)		
	(b) Policies other than Accident and Sickness Policies		
	(i) Unpaid Claims and Unearned Premiums		
36	Margin on claims(15% of unpaid claims and adjustment expenses) 01	26,793	23,716
39	Plus: Margin on unearned premiums (15% of unearned premiums or, if applicable, page 69, line 14, column 05) 02	7	231
50	Excess of "Required Coverage" over "Reserve" for Reinsurance Ceded to Unregistered Insurers (page 50, line 99, column 11 minus column 10) 03	805	817
	Equals: Margin required for Unpaid Claims and Unearned Premiums 04	27,605	24,764
	(ii) Premiums Written		
40	Basic margin (15% of gross premiums written during preceding 12 months) 05	30,761	30,213
	Plus: Supplementary margin on gross premiums (lesser of 5% of gross premiums written during preceding 12 months and $500,000) 06	500	500
	Equals: Gross margin 07	31,261	30,713
	Less: Margin reduction for reinsurance (Gross margin x Reinsurance Ratio) 08	2,188	7,678
	Equals: Margin required for premiums written 09	29,073	23,035
	(iii) Claims Incurred		
16	Basic margin (22% of average annual gross claims incurred during preceding 36 months) 10	22,895	23,700
	Plus: Supplementary margin (lesser of 7% of average annual gross claims incurred during preceding 36 months and $500,000) 11	500	500
	Equals: Gross margin 12	23,395	24,200
	Less: Margin reduction for reinsurance (Gross margin x Reinsurance Ratio) 13	1,638	6,050
	Equals: Margin required for claims incurred 14	21,757	18,150
	5. Excess of Assets Available Over Assets Required		
67	Liabilities for test purposes (page 67, line 16) 15	303,438	282,919
67	Plus: Margin required for Accident and Sickness policies (page 67, line 22) 16	15	52
	Margin required for policies other than Accident and Sickness policies(the greatest of above, lines 04,09 and 14) 17	29,073	24,764
	Equals: Assets required for test purposes 18	332,526	307,735
67	Assets available for test purposes (page 67, line 09) 19	377,226	351,310
	Excess of Assets Available over Assets Required for Test Purposes 20	44,700	43,575
	Authorized Adjustments 21	0	0
	Adjusted Margin 22	44,700	43,575

GUIDANCE NOTES GN12, GN14 AND GN18

Institute of Actuaries and Faculty of Actuaries

GN12: GENERAL INSURANCE BUSINESS: ACTUARIAL REPORTS

Classification (see APC)
This Guidance Note is classified in relation to the code of professional conduct as *best practice*.

Scope
World-wide.

Application
Any actuary preparing a formal report on the reserves or on the financial soundness of a general insurance undertaking, including a Lloyd's syndicate, whether as a consultant or as an employee. This Guidance Note does not cover other aspects of general insurance, such as rate-making.

Legislation or Authority
There is no United Kingdom legislation specifically relating to actuarial reporting on general insurance business. There are separate Guidance Notes for actuaries appointed in terms of Lloyd's Byelaw No. 17 of 1989 (GN14) and actuaries signing certificates for submission to the Non-Admitted Insurers Information Office in the United States of America (GN18).

First issued
August 1987.

Revised
July 1991.

1. INTRODUCTION

1.1 This Guidance Note has been produced to assist actuaries working in the field of general insurance, including the general insurance business of Lloyd's syndicates, whether as consultants or as employees. It is restricted to general points which should be taken into account when making a formal report (as distinct from a brief statement or opinion) on the reserves or on the financial soundness of a general insurance undertaking. The Guidance Note does not cover other matters on which an actuary may report, such as rate-making.

1.2 It is recognized that there may be circumstances in which some of the guidance given below will not be applicable. The actuary's report should indicate indicate any areas which are inconsistent with the guidance.

1.3 It is important that the nature and scope of the brief given to the actuary and the capacity in which the actuary is reporting should be clearly defined. Examples of briefs which could be given to an actuary are listed below:

(i) A report commissioned by the management of an insurance company or a Lloyd's syndicate to recommend the level of reserves to be established in the insurer's accounts and/or statutory returns. The brief may be limited to outstanding claim reserves or it may cover the totality of technical reserves.
(ii) A report commissioned by management or by shareholders to provide an independent check on the amount and adequacy of the reserves. For this purpose the actuary will normally need to have full access to the insurer's data.
(iii) A report commissioned by an insurer to provide supporting evidence for outside bodies, such as tax authorities, supervisory authorities or potential purchasers.
(iv) A report commissioned by an outside body, such as a supervisor, a potential purchaser or an investment analyst, to provide an opinion on the strength of the reserves, without the knowledge of the insurer concerned. Typically such a report will be based on published information.
(v) A report commissioned by management or shareholders, or by a supervisor or other outside body, on the financial strength of the insurer. Such a report may require the actuary to recommend the amount, if any, of additional capital necessary to establish the insurer as being financially sound.

1.4 Where the report relates to business written in another country, or has been commissioned by a supervisory authority or other body in another country, the actuary should be familiar with the relevant legislation, local conditions and, where applicable, any professional code of practice in the country concerned.

1.5 Section 2 of this Guidance Note sets out general points which an actuary should take into account. Sections 3 and 4 are concerned specifically with reporting on reserves and financial soundness respectively.

2. GENERAL POINTS

2.1 The report should state:

(i) who has commissioned the report and, if different, the addressee(s) of the report;
(ii) the purpose of the report or the terms of reference given;
(iii) the extent, if any, to which the report falls short of, or goes beyond, its stated purpose;
(iv) the name of the actuary, his professional qualification and the capacity in which he has prepared the report; and
(v) whether it is in accordance with this Guidance Note or, if not, any material areas where the Guidance Note has not been applied.

2.2 It would be normal practice for the actuary to comment on:

(i) the methodology used and the key assumptions contained therein;

(ii) any changes made in the methodology and key assumptions as compared with the last similar report; and
(iii) the extent of any reliance on the opinions of others, for example in regard to certifying the accuracy of the data.

2.3 The report should indicate, where appropriate, how the following issues have been addressed

(i) the nature, accuracy and interpretation of the data;
(ii) the grouping of the data by class of business, category of risk and currency;
(iii) comparisons of actual experience with that expected under the assumptions made in the previous report;
(iv) the effect of underwriting, claim reporting and settlement, data processing and accounting procedures, with particular reference to any significant known changes therein;
(v) the nature and spread of the reinsurance arrangements, with particular reference to any significant changes therein;
(vi) potential exhaustion of the reinsurance coverage and the possibility of non-performance of reinsurance;
(vii) the effect of any significant known changes in the legal and social environments;
(viii) future claim handling expenses, both direct external costs and internal costs;
(ix) the treatment of any abnormal types of claim; and
(x) the treatment of future premiums in and out (including reinstatement premiums), profit commission and portfolio transfers.

Significant issues emanating from the above list might be identified from discussions with underwriting or claim personnel, from inspection of the data, or from the actuary's wider experience of the business being projected.

2.4 Where the report is being prepared in regard to technical reserves for statutory accounts or returns to a supervisory authority, the actuary should be aware of, and give due recognition to, any relevant accounting principles or statutory requirements. In the United Kingdom, for example, a Statement of Recommended Practice produced by the Association of British Insurers comments on such matters as reporting of gross and net reserves, claim handling expenses and deferred acquisition costs and recommends that there should be no cross-funding or implicit discounting.

2.5 General insurance terminology includes a number of words and phrases which, although commonly encountered, are not accepted universally or are capable of different interpretations. The actuary should seek to ensure as far as possible that any such words or phrases in a report will not be misunderstood. The following are common examples:

(i) The word 'solvency' is capable of a number of interpretations and should not be used without further clarification. For example, if the criterion for 'solvency' is that a company satisfies the minimum statutory solvency requirements, this should be stated in the report.

(ii) It is common for actuaries to use the term 'reserves' when referring to the value placed on an insurer's liabilities and this term is also used in the United Kingdom Insurance Companies Acts and Regulations. Accountants, on the other hand, use the word 'provisions' for the amount held in the accounts to meet specific liabilities and attach a rather different meaning to 'reserves'. It is acceptable for actuaries to use the words 'provisions' and 'reserves' interchangeably provided the meaning is clear from the context of the report. Actuaries should, however, have regard to the definitions incorporated in any relevant legislation.

(iii) The term 'IBNR' can be used in two senses. The first just covers claims which have been 'incurred but not reported'. The second extends the first definition to include development (positive or negative) on notified claims.

(iv) Particular care should be taken when using terms such as 'best estimate', 'adequate', 'cautious', 'prudent', etc. which, although imprecise in their meaning, are nevertheless intended to provide an indication of the strength of the reserves. In the United Kingdom, the word 'adequate' in connexion with reserves usually suggests that there is a more than even chance, but not much more, that they will prove large enough to meet the liabilities. The words 'cautious' or 'prudent' usually imply a rather higher probability and the word 'sufficient' implies a very high probability that the liabilities will be met. However, even these imprecise definitions are not universally adopted and it is always possible that the meaning attached to any of these words by recipients of the report may differ from that intended by the actuary. In the United States, for example, the phrase 'good and sufficient' has generally been taken to have the same meaning as that attached to 'adequate' in the United Kingdom. For these reasons it is strongly recommended that the actuary provides additional comment if there is any possibility of misunderstanding. A report might explain, for example, that 'adequate' indicates that there is little more than an even chance that the reserves will be large enough to meet the liabilities, or 'cautious' implies that the reserves incorporate some margins for caution. If the word 'prudent' is used, it should be made clear whether it is intended in the actuarial sense indicated above or in the accounting sense of being rather more likely to give rise to a subsequent release of profit than a need to recognize a loss.

When any of the terms in (iv) is used, it should be made clear whether it is used purely in relation to an estimate of the ultimate cost of claims or in relation to an estimate of a reserve which takes account of other factors such as investment income or currency matching.

2.6 Reports on reserves or on capital requirements may be produced in terms of either point estimates or ranges of acceptability. With some types of business the conclusions will often be subject to margins of error which may be large. Notwithstanding such uncertainty it is acceptable for the actuary to give positive opinions and provide estimates of the liabilities. The report should draw attention to the uncertainty, making it clear that the eventual outcome will almost certainly differ from any projections made; the actuary may wish to draw attention to particular unquantifiable contingent liabilities for which no explicit allowance has been made.

2.7 Where appropriate to the purpose of the report, the actuary should indicate the degree to which cross-funding exists, i.e. where the reserves are adequate in the aggregate but one or more parts are deficient, for example:

(i) a deficiency in the reserve for unexpired risks offset by some redundancy in the outstanding claim reserve, or vice versa;
(ii) a deficiency in the combined reserve for one cohort year offset by some redundancy in that for another year;
(iii) a deficiency in the reserve for one class of business offset by some redundancy in that for another class.

3. REPORTING ON RESERVES

3.1 The reserves may be calculated either as net reserves or as gross reserves with a separate offset for the effect of reinsurance. In either case the actuary should describe the methods and assumptions used to allow for reinsurance.

3.2 Consideration should be given separately to the liabilities in respect of outstanding claims and unexpired risks, unless the business is accounted for on a funded basis, in which case a combined reserve may be considered.

3.3 Outstanding claim reserves should cover, unless specifically excluded:

—reported outstanding claims (estimated ultimate cost);
—claims incurred but not reported (IBNR);
—reopened claims; and
—future expenses of handling these claims.

Each of these reserves may be calculated and reported explicitly or any two or more of them may be aggregated together.

3.4 Any reserve for future claim handling expenses should be consistent with the reporting objectives. When reporting on the business as a going concern, this reserve should cover only the costs of the claim function. If the business is being run off, expenses might rise significantly and might include areas other than claims, such as general management.

3.5 The choice of method for the estimation of claim reserves depends on the class or nature of the business and the form and quality of the data. It is for the actuary to select the method(s) appropriate in the circumstances. Particular points to consider include:

(i) lack of homogeneity or changes in the mix of the data;
(ii) the effect of large claims, including catastrophe claims and aggregations from a single event;
(iii) cyclical characteristics or temporal trends, including the effect of inflation;
(iv) patterns of claims paid or settled; and
(v) the effect of reinsurance.

3.6 When applying statistical methods of estimation, the actuary should be aware that, in addition to the effect of random variation, there may be significant sources of error associated with the choice of model or its parameters. The actuary should consider how these uncertainties should be communicated to the recipients of the report.

3.7 In estimating future payments on reported claims, the actuary should consider the effect of future escalation of claim costs. Where no explic allowance is made for inflation, the actuary should indicate how allowance ha been made.

3.8 The reserve for unexpired risks comprises:

—the unearned premium reserve (UPR); and
—any adjustment considered necessary to cover future outgo, including future claim handling expenses, arising from unexpired periods of exposure to risk existing at the accounting date.

3.9 The actuary should consider the appropriateness of any approximations underlying the method of calculation of the UPR, in particular those relating to:

(i) the incidence of risk over the policy term;
(ii) the grouping of base dates, e.g. daily, monthly, quarterly, or at mid-year;
(iii) the treatment of non-annual premiums; and
(iv) the choice of base date, e.g. debit of premium, policy inception.

Where unbooked premiums and lapses have been ignored, the actuary should consider whether it would be prudent to establish additional reserves.

3.10 The UPR may be net of an allowance for deferred acquisition costs or it may be gross with these costs shown separately as an asset.

3.11 The actuary should state whether or not allowance has been made for future investment income and, if applicable, how such allowance has been made and the rate of discount used. If allowance for future investment income has been made, attention should be paid to the nature, term and value of the assets backing the technical reserves. Consideration should be given to the effects of possible future changes in the value of the assets on their adequacy to cover the liabilities and, where necessary, provision should be made for such effects.

3.12 In the case of business accounted for on a fund-accounting basis, the factors to be taken into account may be especially complex in regard to the nature of the business, the accounting methods and the associated administrative procedures. The actuary must have regard to the particular features of the business and should pay particular attention to:

(i) the definition of the cohort; and
(ii) the duration at which a profit is first allowed to emerge, i.e. the point of first closure.

3.13 The calculation of the estimated outstanding amount at the point of closure may cover not only outstanding claims, whether notified or not, but also outstanding claim handling expenses, premiums and commissions. The actuary should consider whether each of these items requires a separate calculation or whether one aggregate figure will suffice. The term 'IBNR' may be used but the definition should be made clear (see 2.5 (iii)).

3.14 The basis for the calculation of open-year funds is reported premium income less paid claims, expenses and exchange adjustments, augmented by any additional amount considered necessary to ensure that the amount of the fund is prudent, having regard to the potential net liabilities. The nature of funded business means that information for making a satisfactory estimate is often not available. However, the actuary should make his best assessment in the circumstances.

4. REPORTING ON FINANCIAL SOUNDNESS

4.1 An actuary may be asked to report on whether a company has satisfied statutory solvency requirements. Such a report would refer to an assessment of the adequacy of the technical reserves as well as to a check on whether the shareholders' capital and reserves are sufficient to comply with the statutory solvency requirements. Where the actuary is reporting on the continuing solvency, i.e. the financial soundness, of an insurer over a period, aspects to be considered would include:

(i) the expected volume, nature and profitability of new/renewed business;
(ii) fluctuations in the claims experience, including the effect of inflation;
(iii) the nature, term and value of the assets;
(iv) fluctuations in investment income;
(v) fluctuations in and the ability to realize asset values;
(vi) the suitability and security of the reinsurance arrangements; and
(vii) the insurer's ability to withstand adverse deviations, including catastrophe claims.

4.2 There may be factors which are relevant to the insurer's financial condition but which are not necessarily within the actuary's brief. These could include, for example, political risks, the adverse consequences of bad management or fraud. It would be appropriate to draw attention to such factors where they may be material.

GN14: ACTUARIAL REPORTING ON LLOYD'S RUN-OFF YEARS OF ACCOUNT

Classification (see APC)
This Guidance Note is classified in relation to the code of professional conduct as *mandatory*.

Scope
United Kingdom.

Application
Actuaries appointed by Lloyd's managing agents to provide the required reports.

Legislation or Authority
This Guidance Note is written with specific reference to actuaries appointed in terms of Lloyd's Byelaw No. 17 of 1989 which forms part of the regulatory control of the Lloyd's insurance market, under the Lloyd's Act 1981. Actuaries appointed by Lloyd's managing agents to report on run-off years of account are expected to interpret this Note with reference to Byelaw No. 17.

Date of issue
April 1990.

1. INTRODUCTION

1.1 This Guidance Note is written with specific reference to actuaries instructed by Lloyd's managing agencies to report on the run-off years of account of syndicates under their management, in accordance with their duties under Lloyd's Byelaw No. 17 of 1989. This Byelaw forms a part of the regulatory control of the Lloyd's insurance market, under the Lloyd's Act 1982.

Syndicate Accounting Byelaw No. 11 of 1987, *inter alia*, lays down the framework under which the managing agent can close an underwriting year of account of a syndicate by reinsuring the outstanding liabilities into the open years of a Lloyd's syndicate. This is normally into a later year of account of the same syndicate. The premium for this transaction, known as the reinsurance to close, is required, under the Syndicate Accounting Byelaw, to be equitable as between the two generations of Names. It is in the circumstances in which the managing agent feels unable to determine such a premium that the Byelaw No. 17 of 1989 takes effect.

1.2 This Guidance Note supplements the requirements of that Byelaw. In addition, it supplements the provisions of any other relevant guidance given by the Institute or Faculty of Actuaries which remains generally applicable.

1.3 It should be noted that Byelaw No. 17 of 1989 refers to the Insurance Companies Act 1982. The effect of this is that for the purpose of the Byelaw an actuary must be a Fellow of the Institute of Actuaries or of the Faculty of Actuaries who has attained the age of 30. It should also be noted that an 'independent' actuary for the purposes of this Byelaw shall not be under a contract of service with the managing agent who commissioned the report in question, or with a company which is a related company with that managing agent.

1.4 The nature of the report required under Lloyd's Byelaw No. 17 of 1989 is such as to place a high level of responsibility on the profession. Any actuary, before signing such a document, must consider carefully, in the light of his previous experience and work, whether doing so would be in line with proper professional behaviour and standards.

Of prime importance to this consideration will be the extent of his experience of work on Lloyd's syndicates; it is the duty of any actuary who is in doubt as to his proper course of action to seek help from another actuary with relevant experience or from an Honorary Secretary of the Institute or Faculty. It is emphasized, however, that the responsibility for signing the opinion and report is his and his alone. The profession's rules of conduct make it clear that every actuary, in his professional capacity, whether remunerated by salary or fee, has a duty to his profession, and his responsibility to his employer or client must be consistent with this.

1.5 If the actuary is concerned that he might not be impartial, or that it would be difficult for outsiders to believe he was impartial (as might, for example, be the case if he were a Name on one of the ceding or accepting years of account of the syndicate) then he should arrange for another actuary to produce and sign the report.

1.6 An actuary carrying out work for a Lloyd's syndicate which does not involve an instruction to report on the run-off years in accordance with Byelaw No. 17 of 1989 is not required to consider or comment on whether it is reasonable to close or keep open an underwriting year.

2. REPORTING REQUIREMENTS

2.1 The actuary accepting an assignment under the terms of the Byelaw is required to provide for the managing agent a report containing an opinion on the issues in paragraph 7 (b). This is the Report of the Independent Actuary and hereafter in this Guidance Note will be referred to as the full report. The full report will be incorporated in the managing agent's report and will effectively be in the public domain.

2.2 The Appendix to this Guidance Note illustrates an acceptable form of words for the full report containing a clean opinion. Whilst there is scope for extending the report, the opinion must follow explicitly the wording in the Byelaw except as stated in paragraph 2.3 below.

2.3 In relation to the accounting records, the word 'adequate' from the Byelaw has been replaced by the phrase 'adequate in the sense that they are reasonable in the circumstances'. This is illustrated in the Appendix and has been agreed with Lloyd's. This has been done in view of the differing understanding of the word 'adequate' between members of the actuarial profession and other interested parties.

It should also be noted that the accounting records on which an opinion is required is restricted to that relevant to the reserving process. The accounting records relevant to the reserving process would be considered reasonable if no practical and cost-effective enhancement would materially reduce the uncertainty of the reserve estimate. Materiality is assessed against the 'normal uncertainty' as perceived by the actuary using the definition in section 5 of this Guidance Note. However, for the purposes of this paragraph 'normal uncertainty' should exclude any items, such as pollution, which of themselves are dominating the 'normal uncertainty', even if they have not been identified by the managing agent as a reason for keeping the year open. The actuary may refer, in the full report, to normal market practice in relation to data availability.

2.4 The actuary may need to supplement the full report by a separate management report amplifying certain issues. This will depend on how much, if any, additional detail has been inserted in the full report. The management report is outwith the terms of the Byelaw.

The purposes of the management report would be to provide any necessary explanation of the opinion given in the full report, and offer any ancillary recommendations. It would have the additional advantages of reducing the amount of unpublished working papers which the actuary will need to retain and of reducing the degree of publication of commercially sensitive information regarding the syndicate's business. No reference should be made to the management report in the full report, which is, as stated, publicly available; the management report would only be available on the same basis as other records which are confidential to management.

2.5 It is not, under the terms of the Byelaw, necessary for the actuary to provide an estimate of the syndicate's liabilities (but see also the second paragraph of paragraph 4.1). There is, however, the possibility of a reserve or premium estimate being requested as a supplementary assignment, and the actuary should not accept the original brief unless he is prepared to extend it in this way.

2.6 Under the terms of paragraph 7(h) of the Byelaw, the actuary is required to produce a short report if the full report cannot be prepared within the very tight time-scale involved. The short report will contain only that part of the opinion wording required under section 7(b) (iii) and 7(b) (iv) of the Byelaw. A report is normally required in time to enable the solvency return to be completed by the solvency deadline of each calendar year following failure to close the account at the normal time and at the end of each year thereafter. Lloyd's normally advises the date of the solvency deadline a few months in advance.

There may be circumstances in which it is impossible to comment on 7(b) (iii) and 7(b) (iv) in isolation, in which case the full report will have to be provided. If a short report is produced it should be recognized that this is likely to preclude the syndicate auditor finalizing his work. In addition, a full report will be required as soon as possible.

2.7 In cases where the actuary is able to give an opinion that the managing agent has acted reasonably in proposing to keep the year of account open owing to material uncertainties, it may still be that the methods and assumptions used in estimating the future liabilities are not considered reasonable. In these circumstances, the actuary is likely to propose alternatives. Provision of numerical estimates is, however, considered to be outside the terms of the Byelaw and would, therefore, be subject to the managing agent requesting the actuary to carry out a supplementary assignment.

3. RELATIONSHIPS

3.1 Under paragraph 7 of the Byelaw, the managing agent has to provide access to whatever available data and information the actuary considers relevant; these should include information as to the recent involvement of any other actuaries, together with copies of any relevant reports which they have produced. This process may, of course, involve a series of discussions, as further questions arise from the actuary's work and findings to date.

3.2 The actuary should make it clear to the managing agent, at the outset, that he may require access to the board of directors or partners of the managing agency.

3.3 The actuary should inform the Council of Lloyd's of his having taken on the assignment.

3.4 The actuary should liaise with the syndicate auditor to enable them both to have a proper understanding of their respective responsibilities and to avoid, as far as practicable, duplication of effort in areas such as accounting records and data. The actuary should ascertain the extent of the work to be done by the auditor and consider whether this is sufficient for his own purposes. If not, he may elect to carry out additional data checking himself, or request the auditor to extend his planned work. Conversely, the auditor will wish to obtain an understanding of the actuary's approach and the basis for his opinion; this may be assisted if the report to management deals comprehensively with these matters.

3.5 When the actuary has all the data which he needs for the production of the full report, he is recommended to prepare, for the managing agent to sign, a 'letter of representation' outlining the basic facts which have resulted in the managing agent coming to the view that he may have to leave the year of account open. Once signed, a copy of this 'letter of representation' should be supplied to the syndicate auditor.

3.6 The reports of the actuary, auditor and managing agent need to dovetail with one another. The actuary should not sign his report until final agreed drafts from the other parties are available.

3.7 The managing agent is the actuary's principal, and it would be improper to disclose the opinion, report or findings to any third party other than the syndicate auditor. Under the Byelaw, it is the duty of the managing agent to distribute copies of the report to other parties.

3.8 Under the Byelaw, the actuary will be required to attend a meeting of members' agents who have placed Names on the syndicate years of account concerned. At this meeting, he may be required to provide some explanation of the reasons for reaching his opinion.

4. SCENARIOS

4.1 The managing agent is required, under the Byelaw, to attempt to obtain an outside quotation for the reinsurance to close premium before the requirement for an actuarial opinion.

If an outside quotation has been obtained and rejected, it is likely that the actuary will need to carry out his own calculations as to the quantum of the syndicate's liabilities.

Three situations are envisaged:

(i) the actuary has already concluded that the managing agent has acted unreasonably in not closing the year of account. In this case, the rejection of the quotation becomes irrelevant to the actuary's opinion;
(ii) the rejection of the quotation is clearly unreasonable. In this case, the actuary should give an opinion that the managing agent has acted unreasonably in rejecting the quotation; and
(iii) in other cases, which may be the majority, the actuary is recommended to given an opinion that the managing agent has acted reasonably, but that acceptance of the quotation may, nonetheless, be attractive to certain Names on the syndicate. A specimen wording is shown in the Appendix.

In coming to the decision regarding into which of the categories a particular case falls, the actuary should bear in mind the criteria outlined in section 5 of this Guidance Note.

4.2 In the event of the actuary's arriving at an adverse opinion (i.e. one which states that the actuary believes that the managing agent has acted unreasonably), he should liaise with the managing agent and the syndicate auditor. In such a case, the actuary is likely to be given a supplementary assignment to produce an estimate of the reinsurance to close premium. Such a supplementary assignment would not be a regulatory requirement, and would not, therefore, be subject to this Guidance Note.

4.3 It is possible that as a result of the actuary's giving, or proposing to give, an adverse opinion, the managing agent will decide to close the year of account. In these circumstances, the Byelaw requirements fall away and any report confirming the advice given would be outside the Byelaw.

4.4 In the event of the actuary's arriving at an opinion that the managing agent has acted reasonably, he should make it clear that whilst the methods, data and assumptions are all reasonable, the final reserve remains that of the managing agent.

5. MATERIALITY

5.1 The reinsurance to close involves the payment of a premium, the setting of which is an underwriting decision. An actuary accepting an assignment under the terms of Lloyd's Byelaw No. 17 of 1989 is not required to make the decision, but to make specific comment on the reasonableness of the decision made by the managing agent that he is unable to determine an equitable premium. It is therefore necessary to have regard to the managing agent's duties in this respect.

5.2 Where the premium is payable to a later year of account of the same syndicate, or, in fact, to any other syndicate managed by the managing agent, the decision should consider the interests of both ceding and accepting Names. A decision not to close is equivalent to concluding that a willing buyer/willing seller price cannot be determined owing to the extent of the uncertainty involved in the liabilities.

5.3 The decision as to materiality is a matter for the actuary's judgement, to be made in conjunction with the overall policy of the syndicate regarding reinsurance to close. The following factors should be included among those taken into account in making this decision:

(*a*) the 'normal uncertainty', which is that which would obtain in the absence of the factors identified by the managing agent as responsible for keeping the year open. This can be assessed by considering the level of risk and uncertainty to the accepting Names having regard to the normal nature of the syndicate's liabilities (including any reinsurance to close which would normally be accepted without any specific problems), the nature and volatility of the business written by the syndicate in the accepting year of account, and the underwriter's attitude to change in portfolio mix;

(*b*) the absolute size of the transferred liabilities in relation to the size of the business portfolio of the accepting syndicate; and
(*c*) the timing of the likely resolution of, or substantial reduction in, the uncertainty involved in (for example) a pending law suit; greater expected speed of resolution of a factor is likely to make it a more material consideration, as seen from the perspective of the accepting Names.

5.4 The managing agent implicitly has to consider utility functions for the Names on whose behalf he is acting as agent. The utility function for the ceding Names will relate the cost to the risk reinsured; that for the accepting Names will require the premium to be adequate compensation for any excess risks. These utility functions will not be known to the actuary, but he will need to obtain some understanding of them by consideration of:

(*a*) current market standards and practices;
(*b*) current underwriting philosophy of the syndicate as well as the terms of past reinsurance to close premiums (given the information available at the time); and
(*c*) any claim that the managing agent would close the relevant year of account in the absence of the specific factors identified.

5.5 The actuary's decision as to materiality should include a combination of his actuarial knowledge with the criteria outlined in 5.3 and 5.4 above. In this way, the degree of reasonableness of the managing agent's decision can be gauged from its consistency or otherwise with past decisions.

6. OTHER ASPECTS

6.1 The actuary is expected to follow normal practice and to take due cognizance of any other relevant guidance given by the Institute or Faculty of Actuaries in relation to the sections of his report dealing with the data, methods and assumptions used by the managing agent. It is necessary for him to take due account of the characteristics of the syndicate and its portfolio of business in assessing data requirements and the practicalities involved. It is also important to recognize the emphasis placed on reinsurance protection by many Lloyd's syndicates, including Time and Distance policies. This may result in the need for evaluation of the adequacy, security and timing of the reinsurance programme.

6.2 In accordance with normal practice, the actuary is advised to make it clear in his report that the findings and conclusions are based on the current state of knowledge as to methodology and the external world and in particular make clear the impossibility of guaranteeing the outcome of outstanding or future litigation. This would be particularly important in environmental pollution liability but also applies more generally.

6.3 The assets side of the syndicate's finances will probably be outside the terms of reference of the investigation, since the basic requirement is for an opinion as to the reasonableness of a decision not to close an account, rather than any assessment of the overall financial state of the syndicate.

6.4 In checking the reasonableness of the managing agent's assumptions, the actuary should watch out for situations where a whole range of assumptions each tend to be on the low side, with the result that the final answer is likely to be unduly optimistic and hence unreasonable.

Appendix

REPORT OF THE INDEPENDENT ACTUARY

To the Directors (Partners) of the ABC Managing Agency:

In accordance with your instructions under Lloyd's Byelaw No. 17 of 1989, I submit the following report in respect of the year of account 19XX of the PQR syndicate. This report has been prepared in accordance with the relevant guidance of the Institute of Actuaries and the Faculty of Actuaries.

The following factors have been identified by the managing agent as the reason(s) for leaving the year open:

(List of statements, such as, 'the uncertainty caused by the outstanding dispute with LMN syndicate'.)

In my opinion:

(i) the accounting records kept by the managing agent pursuant to the Syndicate Accounting Byelaw are adequate, in the sense that they are reasonable in the circumstances, as a basis for the determination which has been made by it of the amount to be retained to meet all known and unknown liabilities;
(ii) the methods and assumptions used by the managing agent, in order to make the determination, are reasonable;
(iii) the factors taken into account by the managing agent in forming the view that it is or may not be possible to close the year of account are material; and
(iv) the managing agent has acted reasonably in forming that view. [I would, however, point out that this has involved declining an external quotation. Whilst the quote seems high, it is possible for the liabilities to exceed this premium and there may be some members who would pay such a premium to reinsure their liabilities.]

Under the Byelaw, I am not expected to provide numerical estimates, and hence this report does not constitute confirmation of the adequacy of the reserve.

GN18: CERTIFICATION OF LOSS RESERVES FOR THE NON-ADMITTED INSURERS INFORMATION OFFICE

Classification (see APC)
This Guidance Note is classified in relation to the code of professional conduct as *best practice.*

Scope
United Kingdom.

Application
Any actuary signing a certificate of loss reserves for submission to the Non-Admitted Insurers Information Office (NAIIO) in the United States of America.

Legislation or Authority
This Guidance Note applies specifically to certificates of loss reserves required by the NAIIO.

Date of issue
July 1991.

1. United Kingdom insurance companies authorized to write excess and surplus lines non-life insurance in the United States of America are required by the Non-Admitted Insurers Information Office (NAIIO) to certify the adequacy of their reserves each year. This Guidance Note applies to any actuary who is involved in signing such a certificate. It is expected that the actuary will be familiar with the latest version of the instructions issued by the NAIIO for this purpose.

2. The certificate takes the form of a statement of opinion and is therefore much briefer than the type of reserve reporting covered by GN12. However, it is expected that the actuary signing the certificate will previously have prepared a report in accordance with GN12 or will have access to sufficient information from which such a report could have been prepared.

3. A specimen certificate is provided as an Appendix to this Guidance Note. Whilst it is expected that this will be used as a model by actuaries signing NAIIO certificates, modifications may be necessary to suit particular cases.

3.1 In the normal case the reserves covered by the certificate will be those relating to claims in Form 15 of the return to the Department of Trade and Industry for the year in question. These are worldwide reserves and are net of reinsurance. There may, however, be circumstances where the worldwide reserves are not contained within a single return. It is essential that the actuary has examined all the reserves covered by the certificate; if this is not the case the certificate should be modified so as to identify clearly which reserves are covered by the opinion.

3.2 If none of the reserves have been discounted the relevant sentence should be abbreviated accordingly.

3.3 The sentence regarding the bad debts provision may be modified or omitted if this provision has been examined by the actuary and/or it is considered to be immaterial.

3.4 Care must be taken to avoid giving the impression that pollution and asbestos property claims are covered, unless this is the case, since there is a danger that such an impression could be used as evidence of admission of liability. This is particularly important while such claims are being contested. The specimen certificate incorporates a suggested paragraph which has been drafted with this point in mind. The argument as to whether insurers are liable for asbestos and pollution losses is, however, a developing one and the actuary may wish to amend or extend the relevant paragraph to reflect the latest position. For example, if a provision is being held for such risks, perhaps on legal advice and without admitting liability, the actuary may wish to indicate this in the certificate. The relevant paragraph may be omitted if it is either not applicable or not material.

3.5 The word 'reasonable' in the final sub-paragraph of the specimen certificate is, of course, central to the opinion. It is intended to indicate that the reserves do not necessarily contain any significant margins for caution. In the United States, the phrase 'good and sufficient' was commonly used with the same meaning, although the word 'reasonable' is now accepted. In the United Kingdom, 'good and sufficient' is generally taken to imply a significantly stronger reserving basis and the use of this phrase is therefore not recommended.

Appendix

To: The Non-Admitted Insurers Information Office

CERTIFICATE OF LOSS RESERVES

I, ..., am an actuary employed by the .... Insurance Company (the Company) and a Fellow of the Institute/Faculty of Actuaries with experience of loss reserving.

I have examined the assumptions and methods used in determining the reserves listed below, as shown in the annual returns of the Company prepared for submission to the Department of Trade and Industry in respect of the year ended 31 December 19.......

	£000	\$000 (at \$ =£1)
Claims outstanding: reported claims		
Claims outstanding: IBNR		
Expenses for settling outstanding claims		
Funds		
Claims equalization		
Total		

The above reserves are not discounted for the time value of money, except for those in respect of .... business, which are discounted and are included above at their discounted values.

'Funds' include unpaid losses, unpaid loss adjustment expenses and unearned premiums received.

I have relied upon data prepared by the responsible employees of the Company. I have also relied upon the provision for bad debts, as estimated by the responsible employees of the Company, as being a reasonable provision for the risks of non-performance of outwards reinsurance and other non-recovery of debts. In other respects my examination included such review of the assumptions and methods used and such tests of the calculations made as I considered necessary.

The opinion given below is based on the view of the Company that, in general, claims arising from the removal of asbestos from buildings and cleaning up of hazardous waste sites are not covered by insurance and that the only obligations in respect of such claims will be those arising from the Company's own legal expenses.

In my opinion, subject to the above comments, the reserves identified above:

(i) are computed in accordance with accepted loss reserving standards and are fairly stated in accordance with sound loss reserving principles;
(ii) are based on factors relevant to policy provisions;
(iii) meet the requirements of the insurance laws of the United Kingdom; and
(iv) make a reasonable provision for the unpaid loss obligations and allocated loss adjustment expenses of the Company as at 31 December 19....... under the terms of its policies and agreements.

Signed ....

Fellow of the Institute/Faculty of Actuaries

Date ....

CREDIBILITY FOR REGRESSION MODELS WITH APPLICATION TO TREND (REPRINT)

Charles A. Hachemeister,
with discussion by Al Quirin

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Credibility for Regression Models with Application to Trend

Charles A. Hachemeister
Prudential Reinsurance Company

Introduction

Inflation has moved from a minor annoyance to a major element in Casualty insurance rate making. Twenty years ago it was sufficient to adjust automobile rate levels without any trend of loss severity or frequency. Presently, this minor annoyance has become a major element in the rate making process. This development has led to the necessity of estimating these trends by state. However, no standards have been specifically developed for evaluating credibility of state trend line versus country wide trend lines.

Standards for developing credibility adjusted state trend lines are developed in this paper. The general approach is a direct extension of the Bühlmann & Straub (1970), "Credibility for Loss Ratios." The results obtained apply to much more general models than simple linear trend. In fact, credibility standards have been developed for arbitrary linear regression models.

Expected Severity Over Time

To put our thoughts into perspective, let us consider a concrete example of estimating expected severity over time for total private passenger BI total limits severity.[1]

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[1]The Automobile Bodily Injury data in this paper has been supplied by the Insurance Services Office.

FIGURE 1

State #1
Private Passenger
Bodily Injury
Total Limits Severities

Time Period	t	# of Claims P_{t1}	Observed Severity x_{t1}
7-9/70	12	7861	1738
10-12/70	11	9251	1642
1-3/71	10	8706	1794
4-6/71	9	8575	2051
7-9/71	8	7917	2079
10-12/71	7	8263	2234
1-3/72	6	9456	2032
4-6/72	5	8003	2035
7-9/72	4	7365	2115
10-12/72	3	7832	2262
1-3/73	2	7849	2267
4-6/73	1	9077	2517

Figure 1 shows Private Passenger Automobile data from a particular state giving a number of claims in each calendar quarter along with the observed severity. Time is denoted by an index, t, for which observations are available from time n to time 1. Time runs backwards for reasons of computational ease below. In figure 1, we also introduce notation P_{ts} as the number of claims, and x_{ts} as the observed severity in time period t and state s.

It is our objective to estimate the expected value of x over time given s:

$$E(x_{ts}) = \mu_{ts}$$

Two competing choices for a model to estimate μ_{ts} are time series analysis, where the major emphasis lies on the interdependence of the x_{ij} for various i and j, and the regression model, where μ_{ts} is considered a linear combination of other observed variables. These two approaches are not entirely independent since it is possible to create a model which contains both the elements of interdependence of the x_{ij} and also a mean value μ_{ts} which is dependent upon observed values of other variables. The problem of dealing with such a model is the practical one of producing estimates of the auto-covariance function of the x_{ij} for different i and j at the same time as estimating the regression coefficients. However, the results of the analysis below will follow in large measure for either choice of model.

The Classical Trend and Regression Model

We will make the particular choice to model this expected value as a linear trend:

$$\mu_{st} = a_s + b_s t$$

If we introduce the two column matrices,

$$\beta_s = \begin{pmatrix} a_s \\ b_s \end{pmatrix}; \quad Y_{ts} = \begin{pmatrix} 1 \\ t \end{pmatrix}$$

then we will be able to write the expected value of x_{ts} in matrix form,

$$\mu_{ts} = Y'_{ts}\beta_s$$

Notice that this matrix formulation of μ_{ts} is not limited to a simple trend, but would apply also for models where

$$\mu_{ts} = \sum_{i=1}^{r} \beta_{si} y_{sti}$$

In this case,

$$\beta_s = \begin{pmatrix} \beta_{s1} \\ \beta_{s2} \\ \vdots \\ \beta_{sr} \end{pmatrix}$$

and the r by 1 matrix of independent variables is

$$Y_{ts} = \begin{pmatrix} y_{st1} \\ y_{st2} \\ \vdots \\ y_{str} \end{pmatrix} .$$

While we will only discuss the trend model in the numerical example given below, all the theoretical results follow for this more general model.

For development of the classical regression results, it will be necessary to deal with our data in matrix formulation. We will refer to the column matrix of severities for a given state as

$$X_s = \begin{pmatrix} x_{ns} \\ x_{n-1,s} \\ \vdots \\ x_{1s} \end{pmatrix} .$$

For each state we will also refer to the n by r matrix of independent variable observations over time as

$$Y_s = \begin{pmatrix} Y'_{ns} \\ Y'_{n-1,s} \\ \vdots \\ Y'_{1,s} \end{pmatrix} .$$

For our trend model this is a 12 by 2 matrix. The first column of which is all 1's; the second column of which has entries which go from 12 to 1.

With regard to the number of claims, it will be valuable to introduce an n by n square matrix with zeros in the nondiagonal elements and with the number of claims for each time period going down the main diagonal:

$$P_s = \begin{pmatrix} P_{ns} & & & \bigcirc \\ & P_{n-1,s} & & \\ & & \ddots & \\ \bigcirc & & & P_{1s} \end{pmatrix}$$

We will also find it necessary to refer to the mean value of the process for various time periods for a given state,

$$\mu_s = \begin{pmatrix} \mu_{ns} \\ \mu_{n-1,s} \\ \vdots \\ \mu_{1s} \end{pmatrix}$$

for which

$$\mu_s = Y_s \beta_s$$

now follows.

<u>Time Series Implications</u>

In a time series model one does not usually consider that the mean value μ_{ts} as dependent upon other variables, Y_{ts}. The direction of the investigation in such models is concerned with the n by n autocovariance matrix

$$C_s = E[X_s X_s'] - \mu_s \mu_s'$$

It is not the intention of this paper to pursue the time series direction of analysis. However, the results developed in this paper hold in large measure with an arbitrary autocovariance matrix.

We will follow the Bühlmann, Straub formulation in which the variance of x_{ts} is proportional to the number of claims:

$$E(x_{ts}^2) - \mu_{ts}^2 = \frac{\sigma_s^2}{P_{ts}}$$

and the severity x_{ts} is independent from time period to time period:[2]

$$E(x_{is}x_{js}) - \mu_{is}\mu_{js} = 0 \qquad i \neq j$$

This is, of course, an over simplification of the real world. With these assumptions we find the $n \times n$ auto-covariance matrix in terms of matrix P_s, defined above, as

$$C_s = \sigma_s^2 P_s^{-1}$$

Basic Summary Statistics

There will be certain statistics which will arise frequently in our discussion of the trend example. Figure 2 defines the summary statistics that we will need below. Note, of course, that only those statistics which involve x_{ts} are random variables.

FIGURE 2

Basic Summary Statistics

$$P_{.s} = \sum_{t=1}^{n} P_{ts} \qquad P_{..} = \sum_{s=1}^{N} P_{.s}$$

$$\bar{t}_s = \sum_{t=1}^{n} P_{ts}t/P_{.s} \qquad \bar{t} = \sum_{s=1}^{N} P_{.s}\bar{t}_s/P_{..}$$

$$\overline{t_s^2} = \sum_{t=1}^{n} P_{ts}t^2/P_{.s} \qquad \overline{t^2} = \sum_{s=1}^{N} P_{.s}\overline{t^2}/P_{..}$$

$$\bar{x}_s = \sum_{t=1}^{n} P_{ts}x_{ts}/P_{.s} \qquad \bar{x} = \sum_{s=1}^{N} P_{.s}\bar{x}_s/P_{..}$$

$$\overline{xt}_s = \sum_{t=1}^{n} P_{ts}tx_{ts}/P_{.s} \qquad \overline{xt} = \sum_{s=1}^{N} P_{.s}\overline{xt}_s/P_{..}$$

[2]Note particularly that this last assumption implies that there are no seasonal factors affecting the data.

FIGURE 2 (continued)

$$\sigma^2_{ts} = \overline{t^2_s} - \bar{t}^2_s \qquad \sigma^2_t = \overline{t^2} - \bar{t}^2$$

$$\sigma_{txs} = \overline{xt}_s - \bar{x}_s \bar{t}_s \qquad \sigma_{tx} = \overline{xt} - \bar{x}\,\bar{t}$$

$$\sigma^2_{xs} = \overline{x^2_s} - \bar{x}^2_s \qquad \sigma^2_x = \overline{x^2} - \bar{x}^2$$

State Wide Full Credibility Trend Estimates

Were we to follow the classical generalized least squares estimation procedures for β_s, we would find in terms of the matrices defined above

$$\hat{\beta}_s = (Y'_s C_s^{-1} Y_s)^{-1} Y'_s C_s^{-1} X_s$$

For our particular trend example these results become:

$$\hat{a}_s = \bar{x}_s - \bar{t}_s \hat{b}_s$$

and

$$\hat{b}_s = \sigma_{txs}/\sigma^2_{ts}$$

Pooled Data

Figure 3 compares the private passenger BI severity experience from state to state. Figure 4 contains the values for the summary statistics needed to calculate the estimates of slopes and intercepts contained on Figure 3. For our purposes we will consider that these five states make up the entire country. However, the analysis can be generalized to any number of states. Accordingly, we will refer below to N states. The right-hand two columns of this figure show the pooled data being the sum of the data elements from the five states for comparable time periods.

FIGURE 3

Private Passenger
Bodily Injury
Total Limits Severities
by State

Time Period	t	1: # of Claims P_{t1}	1: Severity x_{t1}	2: # of Claims P_{t2}	2: Severity x_{t2}	3: # of Claims P_{t3}	3: Severity x_{t3}	4: # of Claims P_{t4}	4: Severity x_{t4}	5: # of Claims P_{t5}	5: Severity x_{t5}	"Countrywide": # of Claims $P_{t.}$	"Countrywide": Severity $x_{t.}$
7-9/70	12	7861	1738	1622	1364	1147	1759	407	1223	2902	1456	13939	1623
10-12/70	11	9251	1642	1742	1408	1357	1685	396	1146	3172	1499	15918	1579
1-3/71	10	8706	1794	1523	1597	1329	1479	348	1010	3046	1609	14952	1690
4-6/71	9	8575	2051	1515	1444	1204	1763	341	1257	3068	1741	14703	1882
7-9/71	8	7917	2079	1622	1342	998	1674	315	1426	2693	1482	13545	1827
10-12/71	7	8263	2234	1602	1675	1077	2103	328	1532	2910	1572	14180	2009
1-3/72	6	9456	2032	1964	1470	1277	1502	352	1953	3275	1606	16324	1836
4-6/72	5	8003	2035	1515	1448	1218	1622	331	1123	2697	1735	13764	1853
7-9/72	4	7365	2115	1527	1464	896	1828	287	1343	2663	1607	12738	1893
10-12/72	3	7832	2262	1748	1831	1003	2155	384	1243	3017	1573	13984	2024
1-3/73	2	7849	2267	1654	1612	1108	2233	321	1762	3242	1613	14174	2027
4-6/73	1	9077	2517	1861	1471	1121	2059	342	1306	3425	1690	15826	2157
Intercept	$\hat{a}_s$		2470		1621		2096		1538		1676		2148
Slope	$\hat{b}_s$		- 62.39		- 17.14		- 43.32		- 27.81		- 11.87		- 43.35

FIGURE 4

Values of Summary Statistics by State

State:	1	2	3	4	5	"Countrywide"
$P_{.s}$	100,155	19,895	13,735	4,152	36,110	174,047
$\bar{t}_s$	6.54972	6.41171	6.69982	6.66089	6.43725	6.52511
$\overline{t^2}_s$	54.88889	53.22398	56.91824	56.79143	53.75876	54.66964
$\bar{x}_s$	2,060.92	1,511.22	1,805.84	1,352.98	1,599.83	1,865.40
$\overline{xt}_s$	12,750.36	9,481.90	11,577.80	8,666.54	10,152.19	11,647.75
σ^2_{ts}	11.99009	12.11393	12.03068	12.42402	12.32061	12.09264
σ_{txs}	-748.09102	-207.62975	-521.01641	-345.04749	-146.30085	-524.21257
σ^2_{xs}	55,881.	18,725.	60,776.	68,275.	7,573.	99807

Just as we have a need to be able to refer to all the data within a state in a concise fashion, we will have a need to refer to all of the data country wide in a concise fashion. To this end for severities we define the $n \times N$ by 1 column of severities as

$$X = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{pmatrix},$$

the $n \times N$ by r matrix of independent variable observations as

$$Y = \begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_N \end{pmatrix},$$

and the super matrix of numbers of claims matrices as the $n \times N$ matrix

$$P = \begin{pmatrix} P_1 & & & \bigcirc \\ & P_2 & & \\ & & \ddots & \\ \bigcirc & & & P_N \end{pmatrix}.$$

Also, we will consider the $n \times N$ by 1 column matrix of mean values:

$$E(X) = \mu = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_N \end{pmatrix}$$

It will also be necessary for us to use the autocovariance matrix of all of the severities country wide:

$$E[XX'] - \mu\mu' = C = \begin{pmatrix} C_1 & & & \bigcirc \\ & C_2 & & \\ & & \ddots & \\ \bigcirc & & & C_N \end{pmatrix}$$

It is important to note that since this "super" autocovariance matrix is made up of the state autocovariance matrices down the super diagonal with zero elements elsewhere, this model specifically considers that the observations from one state are independent of those from another state.

In terms of these super matrices, the pooled "country wide" estimates of β become:

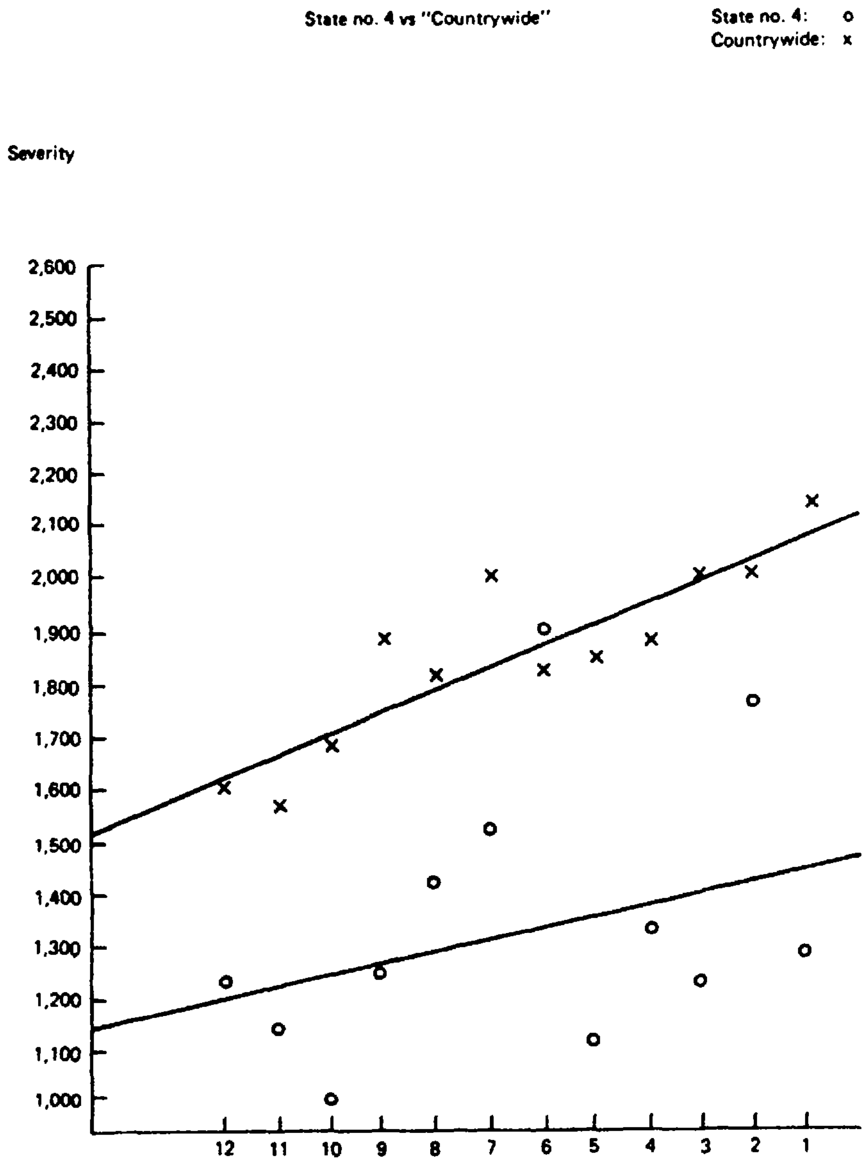
$$\hat{\beta} = (Y'C^{-1}Y)^{-1}Y'C^{-1}X$$

State Versus "Countrywide" Trend

The estimates of the intercept and slope of the trend line shown on figure 3 vary substantially from state to state. Without credibility the only two alternatives available to the decision maker is whether to consider the data from the other states to be from the same basic population as the state in question, and therefore use the country wide estimate; or to consider that the state data was sufficiently different, and therefore throw out the data from other states using only the state estimate. Figure 5 compares the country wide severity data with that of state #4. Notice that the country wide data lies more closely about the least squares trend line, although the country wide line lies substantially above the state line. One is not exactly happy with the trend line estimate for the state because of the very wide variation of the data points about that line. In this instance, one might be more ready to accept the country wide versus the state trend.

Figure 5

State no. 4 vs "Countrywide"



However, state versus country wide are not the only two choices. If one were to believe that the distribution of x_{ts} varied from state to state and had to choose an optimal decision over all of the states, a compound decision problem, then it is not clear whether the choice should be a state wide or a country wide trend. The exact solution of this problem, produces a credibility weighting between the two trends, as will be seen below.

Alternatively, if one is only making a single decision for one state but if it is believed that the distribution of x is a random pick from some set of distributions governed by an index, say θ_s, then the result is the same as the compound decision.

Figure 6 contains the estimated trend lines for each of our five states and the heavier line as that for country wide. It is clear from looking at this figure that the slopes and intercepts vary from state to state. In the compound problem of trying to choose a set of trend lines for all of the states to optimize the total trend choice, one should act as if the slopes and intercepts do have a distribution which is reflected in these differences.

With the introduction of an index θ_s to describe these distributions, we need to reformulate the state data in terms of this index. First of all, the β_s become functions of θ_s

$$\beta_s = B(\theta_s)$$

as does the expected value of x_{ts} given θ_s

$$E[x_{ts}|\theta_s] = \mu_{ts}(\theta_s) = Y'_{ts}B(\theta_s)$$

The autocovariance matrix is in general a matrix function of θ_s

$$C_s = C_s(\theta_s)$$

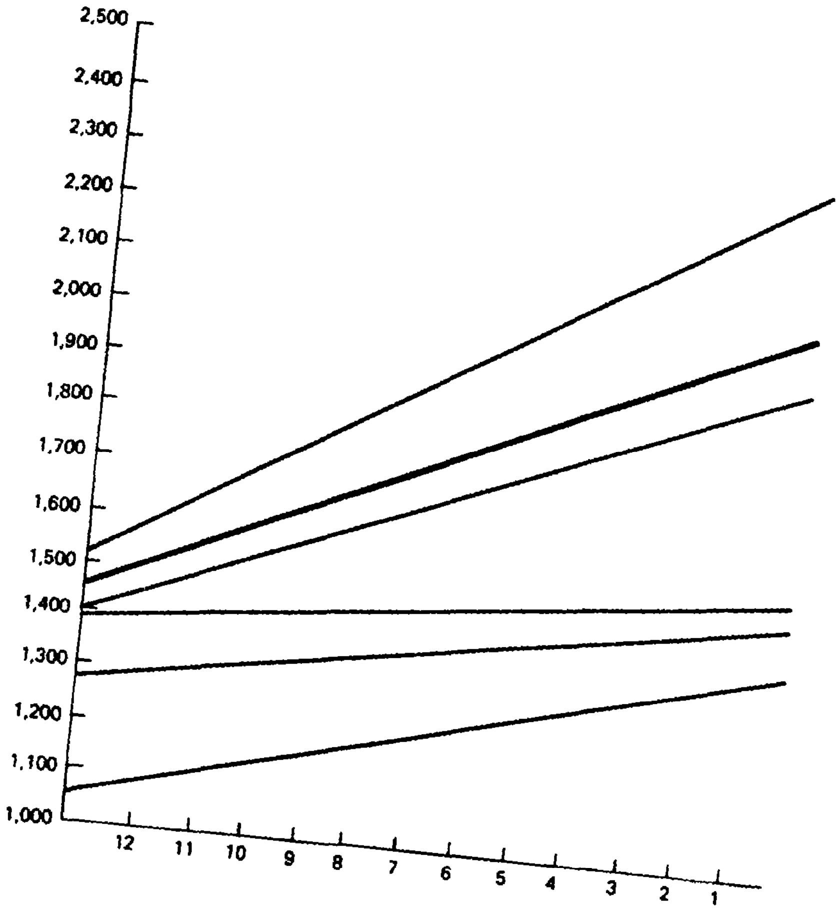


Figure 6

Comparison of Observed State Trends

In this paper we will pursue the case of where the autocovariance matrix is known up to a scalar multiplier, the variance of x_{ts} which is a function of θ_s:

$$C_s(\theta_s) = \sigma^2(\theta_s)P_s^{-1}$$

<u>Expected Values Over θ</u>

It will be necessary below to take expected values of various functions of θ.

$B(\theta)$:

The expected value of the column matrix B is equal to a column matrix β without subscripts

$$E[B(\theta)] = \beta \ .$$

The covariance matrix of the $B(\theta)$ will be denoted by the r by r matrix:

$$E[B(\theta_s)B'(\theta_s)] - \beta\beta' = \Gamma_{rxr} \ .$$

μ:

The expected value of μ_{ts} is now:

$$E[\mu_{ts}(\theta_s)] = Y'_{ts}\beta$$

With a natural extension to the column matrix μ_s within a state and then country wide to μ as:

$$E[\mu_s(\theta_s)] = Y_s\beta \quad \text{and} \quad E[\mu(\theta_1,\ldots,\theta_N)] = Y\beta$$

We will also find it necessary to refer below to the column matrix of autocovariances between a particular mean value and that of all other mean values:

$$E[\mu\mu_{tk}(\theta_k)] - Y_h\beta\beta' Y_{tk} = \begin{pmatrix} Y_1\Gamma Y_{tk}\delta_{1k} \\ Y_2\Gamma Y_{tk}\delta_{2k} \\ \vdots \\ Y_N\Gamma Y_{tk}\delta_{Nk} \end{pmatrix}$$

where δ_{ij} is the Kronecker delta:

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

The autocovariance matrix of the mean values is a super matrix of $n \times n$ matrices down the super diagonal with zero elements elsewhere:

$$E[\mu\mu'] - E[\mu]E[\mu'] = \begin{pmatrix} Y_1\Gamma Y_1' & & & \bigcirc \\ & Y_2\Gamma Y_2' & & \\ & & \ddots & \\ \bigcirc & & & Y_N\Gamma Y_N' \end{pmatrix}$$

$\sigma_s^2(\theta_s)$:

The state variance is also a variable now, which depends upon θ_s. The expected value of the autocovariance matrix for a given state is denoted by:

$$E[C_s(\theta_s)] = V_s$$

However, in our case we will take:

$$V_s = \sigma^2 P_s^{-1}$$

The extension of this to the country wide autocovariance matrix is:

$$E[C] = V = \begin{pmatrix} V_1 & & & \bigcirc \\ & V_2 & & \\ & & \ddots & \\ \bigcirc & & & V_N \end{pmatrix} = \sigma^2 P^{-1}$$

<u>Estimation of</u> $\mu_{ij}(\theta_j)$

With this preliminary background, it is now possible to consider estimates of the mean value of the trend line at any point of time. We take the usual conditions of unbiasedness and minimum variance:

$$E\hat{\mu}_{ij} = E\mu_{ij}(\theta_j)$$
$$E\{(\hat{\mu}^*_{ij} - \mu_{ij}(\theta))^2\} \leq E\{\hat{\mu}_{ij} - \mu_{ij}(\theta))^2\} \qquad (1)$$

where we will accept the estimator μ^*_{ij} as the optimal estimator, if (1) holds for all possible estimators $\hat{\mu}_{ij}$.

Following Bühlmann and Straub, we will consider estimators of the form:

$$\hat{\mu}_{ij} = \alpha_0 + \sum_{s=1}^{N} \sum_{t=1}^{n} \alpha_{ts} x_{ts} = \alpha_0 + X' A$$

Where we introduce the column vector of coefficients for state and country wide as

$$A_s = \begin{pmatrix} \alpha_{1s} \\ \alpha_{2s} \\ \vdots \\ \alpha_{ns} \end{pmatrix} \quad \text{and} \quad A = \begin{pmatrix} A_1 \\ A_2 \\ \vdots \\ A_N \end{pmatrix}$$

While we require our estimator to be unbiased, this will happen automatically because of the inclusion of the additive constant of α_0 in the estimator. Accordingly, to determine our estimator we will minimize:

$$\Phi_{ij} = E[(\alpha_0 + X' A - \mu_{ij}(\theta_j))^2]$$

To do this, we take the partial derivative of Φ_{ij} with respect to α_0 set to 0

$$\frac{\partial \Phi_{ij}}{\alpha_0} = 2E[\alpha_0 + X' A - \mu_{ij}(\theta_j)] = 0$$

to find:

$$\alpha_0 = E[\mu_{ij}(\theta_j)] - E[\mu']A = \beta' [Y_{ij} - Y' A]$$

The column vector of partial derivatives of Φ_{ij} with respect to A is set equal to 0,

$$\frac{\partial \Phi_{ij}}{\partial A} = 2E[XX' A + X(\alpha_0 - \mu_{ij}(\theta_j))] = 0$$

finding:

$$E[(C + \mu\mu')A + \mu\alpha_0] = E[\mu\mu_{ij}(\theta_j)]$$

after taking conditional expectations holding the θ_s for $s = 1$ to N, constant and rearranging terms. Carrying out the expectation over the θ_s, we find:

$$[V + E(\mu\mu') - E(\mu)E(\mu')]A = E[\mu\mu_{ij}(\theta_j)] - E[\mu]E[\mu_{ij}(\theta_j)]$$

To this point the analysis has been quite general without depending upon the form of V or of the form of the autocovariance matrix of the μ. To proceed it is necessary for us to assume V and the autocovariance matrix of μ to be comprised of n by n matrices of state data down the super diagonal with zeros elsewhere. If this is the case for each state, we may now write:

$$(V_s + Y_s \Gamma Y_s')A_s = Y_s \Gamma Y_{ij} \delta_{sj} \tag{2}$$

which immediately indicates that

$$A_s = 0 \quad \text{for} \quad s \neq j$$

If we premultiply (2) for state j by $Y_j' V_j^{-1}$, we find:

$$(I + Y_j' V_j^{-1} Y_j \Gamma) Y_j' A_j = Y_j' V_j^{-1} Y_j \Gamma Y_{ij}$$

Anticipating later results, let us pause for a moment to define:

$$K_j = P_{.j} (Y_j' V_j^{-1} Y_j \Gamma)^{-1}$$

and the credibility matrix:[3]

$$Z_j = P_{.j} (P_{.j} I + K_j)^{-1}$$

[3]The K_j matrix only exists if Γ is positive definite. However, the Z_j matrix always exists even when K_j does not; and may be written in the form:

$$Z_j = Y_j' V_j^{-1} Y_j \Gamma (I + Y_j' V_j^{-1} Y_j \Gamma)^{-1}$$

This immediately yields:

$$Y'_j A_j = Z_j Y_{1j}$$

Combining this with (2), we now find:

$$A_j = V_j^{-1} Y_j \Gamma[I - Z_j] Y_{1j}$$

Premultiplying this by X'_j and rearranging terms, since

$$Y'_j V_j^{-1} Y_j \Gamma[I - Z_j] = Z_j$$

we find:

$$X'_j A_j = \hat{\beta}'_j Z_j Y_{1j}$$

for the case where C_j is known up to a scalar multiplier[4] which depends upon θ_j. Recall that in the case of greatest interest to us $C_j = \sigma^2(\theta_j)P_j^{-1}$. Now since

$$\alpha_0 = \beta'[I - Z_j]Y_{1j}$$

we can finally write our estimator as:

$$\hat{\mu}_{1j} = [\hat{\beta}'_j Z_j + \beta'(I - Z_j)]Y_{1j}$$

It is particularly interesting and satisfying to note that this estimator holds for any Y_{1j}. In other words, we have credibility adjusted the regression coefficients.

<u>Relation to the Bühlmann, Straub Model</u>

The form of the estimator in the Bühlmann, Straub model was:

$$\hat{\mu}_{1j} = X'A$$

---

[4] If C_j is some more complex function of θ_j, $\hat{\beta}_j$ becomes a function of θ_j such that in general

$$E\hat{\beta}_j \neq (Y'_j V_j^{-1} Y_j)^{-1} Y'_j V_j^{-1} X_j$$

without an additive constant. If this model were followed through for the regression case, one would find:

$$\hat{\mu}_{ij} = [\beta_j' Z_j + d\beta' (I - Z_j)]Y_{ij}$$

which is the same as the estimator above, except for d, which is equal to the expression:

$$d = \sum_{s=1}^{N} \hat{\beta}_s' Z_s \Gamma^{-1} \beta / \sum_{s=1}^{N} \beta' Z_s \Gamma^{-1} \beta .$$

In the univariate case of Bühlmann and Straub the parameter equivalent of β cancelled entirely from the estimator. However, in the multivariate case, this is not so; so that there is no benefit to using the estimator without the additive constant.

Parameter Estimation

To apply our credibility model to real data, we need to be in a position to estimate the various elements which are not directly observable within it. Up to this point we have been able to be very general in the form of the autocovariance matrix within a given state. At this point, we sacrifice this generality to be able to produce unbiased estimators of the parameters in question. The easiest parameter to deal with is the column matrix β. The least squares estimate of β, using pooled data, is unbiased:

$$E(\hat{\beta}) = E[(Y'PY)^{-1}Y'PX] = \beta$$

For an estimator of expected value of the state variance σ^2, let us consider the mean square error for a given state:

$$\hat{\sigma}_s^2 = \frac{1}{n - r} \sum_{t=1}^{n} P_{ts}(x_{ts} - \hat{\mu}_{ts})^2 .$$

In matrix terms this becomes:

$$\hat{\sigma}_s^2 = \frac{1}{n - r} (X_s' P_s X_s - X_s' P_s Y_s (Y_s' P_s Y_s)^{-1} Y_s' P_s X_s)$$

Following the classical evaluation of the expected value of the mean square error as outlined in Goldberger,[5] we note that the above matrix is a 1 by 1 matrix and further that the trace of any two matrices is independent of the order of multiplication:

$$tr(AB) = tr(BA)$$

so that we may evaluate the expected value of $\hat{\sigma}_s^2$ as:

$$(n - r)E(\hat{\sigma}_s^2) = E\ tr[P_s(I - Y_s(Y_s'P_sY_s)^{-1}Y_s'P_s)X_sX_s']$$

since

$$I - Y_s(Y_s'P_sY_s)^{-1}Y_s'P_s \quad \text{annihilates} \quad Y_sB(\theta_s)B'(\theta_s)Y_s'$$

this becomes:

$$(n - r)E(\hat{\sigma}_s^2) = tr[P_s(I - Y_s(Y_s'P_sY_s)^{-1}Y_s'P_s)V_s]$$

or $$E(\hat{\sigma}_s^2) = \frac{1}{n - r}\{trI_{nxn} - tr\ I_{rxr}\}\sigma^2 ,$$

so that $\hat{\sigma}_s^2$ is an unbiased estimator of σ^2. We shall take the unweighted average of these state mean square errors as our overall estimator of σ^2:

$$\hat{\sigma}^2 = \frac{1}{N}\sum_{s=1}^{N}\hat{\sigma}_s^2$$

which is clearly unbiased.

The estimator of the covariance matrix of the $B(\theta)$ is somewhat more difficult to find an estimator for. First of all, consider:

$$G = \sum_{s=1}^{N}(Y'PY)^{-1}(Y_s'P_sY_s)(\hat{\beta}_s - \hat{\beta})(\hat{\beta}_s - \hat{\beta})' .$$

To evaluate the expected value of G, let us first consider expected values of matrices of estimators of the $\hat{\beta}_s$. In

[5]"Econometric Theory"; John Wiley & Sons, Inc. - Page 166

particular, we note:

$$\hat{\beta}_j\hat{\beta}'_s = (Y_jP_jY_j)^{-1}Y'_jP_jX_jX'P_sY_s(Y'_sP_sY_s)^{-1} ,$$

so that:

$$E(\hat{\beta}_j\hat{\beta}'_s) = \beta\beta' + [\Gamma + \sigma^2(Y'_sP_sY_s)^{-1}]\delta_{js} .$$

At this point we now wish to consider the expected value of $\hat{\hat{\beta}}\hat{\beta}'_s$. To evaluate this expected value, we will assume:

$$\hat{\hat{\beta}}\hat{\beta}'_s = \sum_{j=1}^{N} (Y'PY)^{-1}(Y'_jP_jY_j)\hat{\beta}_j\hat{\beta}'_s$$

Using this relationship, we find:

$$E(\hat{\hat{\beta}}\hat{\beta}'_s) = \beta\beta' + (Y'PY)^{-1}(Y'_sP_sY_s)\Gamma + (Y'PY)^{-1}\sigma^2$$

Using a similar analysis for $\hat{\hat{\beta}}\hat{\hat{\beta}}'$ yields:

$$\hat{\hat{\beta}}\hat{\hat{\beta}}' = \sum_{j=1}^{N} \hat{\hat{\beta}}\hat{\beta}'_j(Y'_jP_jY_j)(Y'PY)^{-1} \quad \text{and}$$

$$E(\hat{\hat{\beta}}\hat{\hat{\beta}}') = \beta\beta' + \sum_{j=1}^{N} (Y'PY)^{-1}(Y'_jP_jY_j)\Gamma(Y'_jP_jY_j)(Y'PY)^{-1} + (Y'PY)^{-1}\sigma^2$$

Combining our results we find:

$$E(G) = \left[I - \sum_{s=1}^{N} (Y'PY)^{-1}(Y'_sP_sY_s)(Y'PY)^{-1}(Y'_sP_sY_s)\right]\Gamma + (N-1)(Y'PY)^{-1}\sigma^2$$

If we introduce the r by r matrix

$$\Pi = I - \sum_{s=1}^{N} (Y'PY)^{-1}(Y'_sP_sY_s)(Y'PY)^{-1}(Y'_sP_sY_s) ,$$

an unbiased estimator for Γ is

$$H = \Pi^{-1}(G - (N-1)(Y'PY)^{-1}\hat{\sigma}^2) .$$

However, since Γ is symmetric we will take our estimator as

$$\hat{\Gamma} = \tfrac{1}{2}(H + H')$$

Form of the Estimators for the Trend Example

To put the above theoretical results into perspective, let us translate them into the trend example. The 2 by 2 matrix of weighted independent variables becomes:

$$Y_s' P_s Y_s = P_{.s}\begin{pmatrix} 1 & \bar{t}_s \\ \bar{t}_s & \overline{t^2_s} \end{pmatrix}$$

The slope and intercept are:

$$\hat{\beta}_s = \begin{pmatrix} \hat{a}_s \\ \hat{b}_s \end{pmatrix} = \begin{pmatrix} \bar{x}_s - \bar{t}_s\, \sigma_{txs}/\sigma^2_{ts} \\ \sigma_{txs}/\sigma^2_{ts} \end{pmatrix}$$

The estimate of average variance is:

$$\hat{\sigma}^2 = \frac{1}{N(n-2)} \sum_{s=1}^{N} P_{.s}(\sigma^2_{xs} - \sigma^2_{txs}/\sigma^2_{ts})$$

The elements of $\hat{\Gamma}$ are denoted as:

$$\hat{\Gamma} = \begin{pmatrix} \hat{\sigma}^2_a & \hat{\sigma}_{ab} \\ \hat{\sigma}_{ab} & \hat{\sigma}^2_b \end{pmatrix}$$

The K matrix within the credibility form then becomes:

$$\hat{K}_s = \begin{pmatrix} \hat{k}_{s11} & \hat{k}_{s12} \\ \hat{k}_{s21} & \hat{k}_{s22} \end{pmatrix}$$

$$= \frac{\hat{\sigma}^2}{\sigma^2_{ts}(\hat{\sigma}^2_a\hat{\sigma}^2_b - \hat{\sigma}^2_{ab})}\begin{pmatrix} \hat{\sigma}^2_b\overline{t^2_s} + \hat{\sigma}_{ab}\bar{t} & -\hat{\sigma}^2_b\bar{t} - \hat{\sigma}_{ab} \\ \hat{\sigma}_{ab}\overline{t^2} - \hat{\sigma}^2_a\bar{t} & \hat{\sigma}_{ab}\bar{t} + \hat{\sigma}^2_a \end{pmatrix}$$

Thus the credibility formula becomes:

$$Z_s = \frac{P_{.s}}{P_{.s}^2 + (\hat{k}_{s11} + \hat{k}_{s22})P_{.s} + \hat{k}_{s11}\hat{k}_{s22} - \hat{k}_{s12}\hat{k}_{s21}} \times \begin{pmatrix} P_{.s} + \hat{k}_{s22} & -\hat{k}_{s12} \\ -\hat{k}_{s21} & P_{.s} + \hat{k}_{s11} \end{pmatrix}$$

Using the data shown in figure 4 these estimators take on the values as shown in figure 7.

Figure 7

Numerical Value of the estimates

$$\Pi = \begin{pmatrix} .61017 & -.00468 \\ -.00066 & .60537 \end{pmatrix}$$

$$\hat{\Gamma} = \begin{pmatrix} 241,550 & -13,819 \\ -13,819 & 805 \end{pmatrix}$$

$$K_1 = \begin{pmatrix} -49,179 & 9,073 \\ -874,219 & 160,327 \end{pmatrix}$$

$$K_2 = \begin{pmatrix} -48,080 & 9,097 \\ -854,430 & 160,691 \end{pmatrix}$$

$$K_3 = \begin{pmatrix} -49,479 & 8,914 \\ -879,957 & 157,592 \end{pmatrix}$$

$$K_4 = \begin{pmatrix} -47,466 & 8,664 \\ -844,260 & 153,154 \end{pmatrix}$$

$$K_5 = \begin{pmatrix} -47,194 & 8,923 \\ -838,835 & 157,632 \end{pmatrix}$$

$$G - (N-1)(Y'PY)^{-1}\hat{\sigma}^2 =$$

$$= \begin{pmatrix} 147,451 & -8,415.88 \\ -8,544.26 & 496.3438 \end{pmatrix}$$

$$\hat{\sigma}^2 = 44,057,744$$

$$Z_1 = \begin{pmatrix} 1.2489 & -.0435 \\ 4.0219 & .2444 \end{pmatrix}$$

$$Z_2 = \begin{pmatrix} 1.3871 & -.0699 \\ 6.4852 & -.2165 \end{pmatrix}$$

$$Z_3 = \begin{pmatrix} 1.3680 & -.0712 \\ 7.0261 & -.2854 \end{pmatrix}$$

$$Z_4 = \begin{pmatrix} 1.1083 & -.0610 \\ 6.0202 & -.3052 \end{pmatrix}$$

$$Z_5 = \begin{pmatrix} 1.2376 & -.0570 \\ 5.5842 & -.0708 \end{pmatrix}$$

Using these numerical values, we find the credibility adjusted slopes and intercepts. These are compared with the state and country wide slopes and intercepts on figure 8.

FIGURE 8

State			State Data	Credibility Adjusted Data	Countrywide Data
1	Intercept:	a	2470	2473	2148
	Slope:	b	-62.39	-61.98	-43.35
2	Intercept:	a	1621	1587	2148
	Slope:	b	-17.14	-12.19	-43.35
3	Intercept:	a	2096	2077	2148
	Slope:	b	-43.31	-39.64	-43.35
4	Intercept:	a	1538	1566	2148
	Slope:	b	-27.81	-10.85	-43.35
5	Intercept:	a	1676	1740	2148
	Slope:	b	-11.87	-18.68	-43.35

Figure 9 compares the state trend line denoted by S and the country wide trend line denoted by C with the credibility adjusted trend line denoted by A. In all of the states, except state # 4, the credibility adjusted trend line is virtually the same as the state trend line. However, in state #4, with a smaller claim volume, the credibility adjusted trend line is much different from the state trend line. State #4 trend lines clearly point out a distressing aspect of the credibility adjusted trend line. The credibility adjusted trend line has a lower trend than both the country wide and state trend lines. In fact, a closer examination of the other state trend line graphs will show that the credibility adjusted trend for state #2 is also lower than both state and country wide. In state #1 the credibility adjusted slope is less than for the state but the credibility adjusted trend line lies above both the state and country wide lines for the

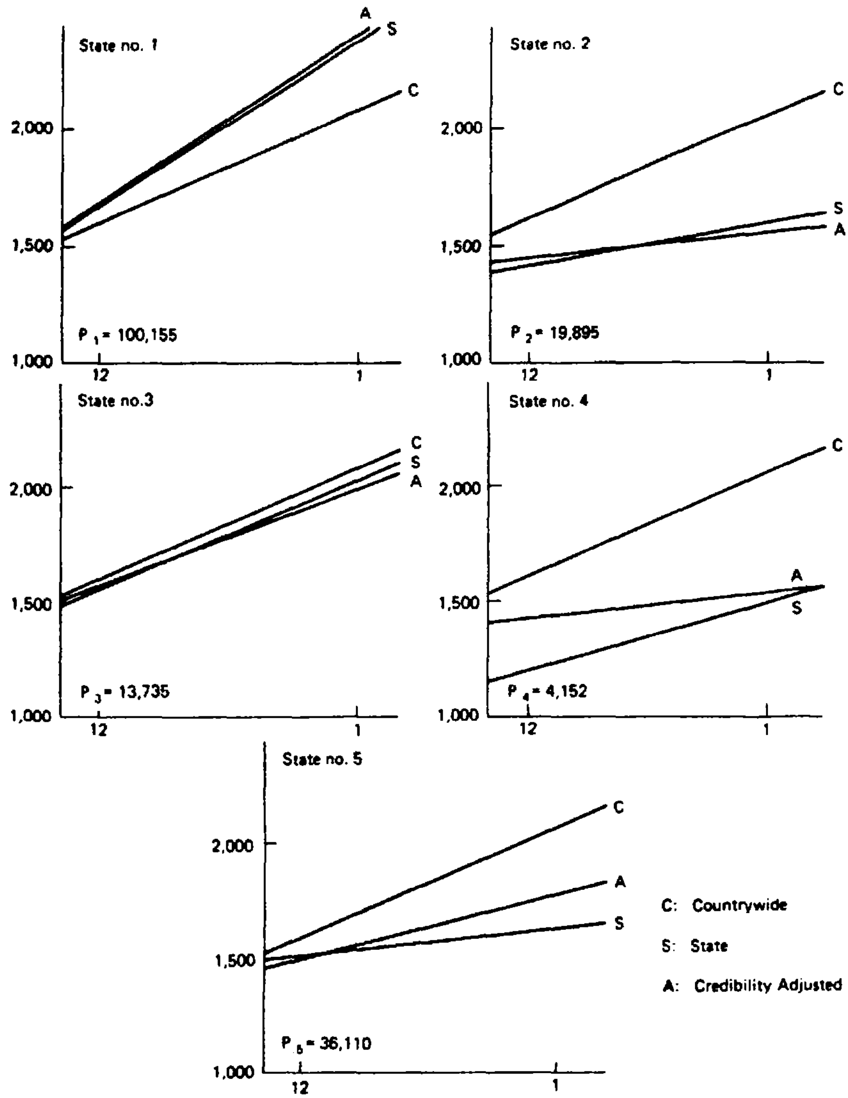


Figure 9

Comparison of
Credibility Adjusted Trend Lines
with State and Countrywide Lines

time period from our observed values were taken.

These strange results arise from our choice of model. That is, we have assumed that not only can the trend for a given state be considered as being a pick from a distribution of trends, but also that the level of severity for a random pick over some distribution of average severity levels. However, if we were to reflect upon what a proper model for trend would be, it is fairly easy to conclude that the level of severity as embodied by the intercept, a_s in the trend line, is distinctly different from state to state and should not be credibility adjusted for.

It is possible to alleviate this defect by changing the basic credibility model. In order to more adequately discuss this, it is necessary for us to first discuss the effect of linear transformations of the independent variables on our credibility estimate, $\hat{\mu}_{ij}$.

<u>Invariance of $\hat{\mu}_{ij}$ Under Transformations of the Independent Variables</u>

The column matrix Y_{ts} describes the values of r variables which are observed at time t. Such that

$$\mu_{ts} = Y'_{ts}\beta_s$$

This mean value could just as well be described by a linear combination of transformed variables Y^*_{ts}

$$\mu_{ts} = Y^{*\prime}_{ts}\beta^*_s$$

The easiest example of this is simple scaling and translation of each of the independent variables. In our case we would define time about an origin and with a scale such that the weighted average of the scaled times was zero and the sample variance of the scaled times was equal to one. This transformation would be accomplished by a matrix:

$$T_s = \begin{pmatrix} 1 & 0 \\ -\bar{t}_s/\sigma_{ts} & 1/\sigma_{ts} \end{pmatrix}$$

This matrix can be considered a mapping of Y_{ts} to Y_{ts}^*:

$$Y_{ts} = \begin{pmatrix} 1 \\ t \end{pmatrix} \xrightarrow[T_s]{} Y_{ts}^* = \begin{pmatrix} 1 \\ \dfrac{t - \bar{t}_s}{\sigma_{ts}} \end{pmatrix}$$

However, it is not necessary to merely consider simple locations scaling transformations; but any arbitrary linear transformation on Y_{ts} will not affect the credibility estimate $\hat{\mu}_{ts}$.

An arbitrary transformation T_s will generate:

$$Y_{ts}^* = T_s Y_{ts}$$

from which

$$Y_s^* = Y_s T_s'$$

and

$$Y_s^{*\prime} P_s Y_s^* = T_s Y_s' P_s Y_s T_s'$$

follow immediately.

In order that the mean value estimate still holds, the inverse transformation must be applied to β_s

$$\mu_{ts} = Y_{ts}' \beta_s = Y_{ts}^{*\prime} \beta_s^* \Rightarrow \beta_s^* = T_s'^{-1} \beta_s$$

Similarly, if the mean value were to hold using the countrywide β, this same transformation needs to be applied:

$$\beta_s^* = T_s'^{-1} \beta$$

With regard to the transformed estimates of β_s, it follows from the above that:

$$\hat{\beta}_s^* = T_s'^{-1} \hat{\beta}_s$$

With regard to the countrywide estimates $\hat{\beta}$, a transformed estimate will be denoted as:

$$\hat{\beta}_s^{*} = T_s'^{-1}\hat{\beta}$$

The transformed β_s will now generate a transformed Γ matrix which varies by state, denoted by:

$$\Gamma_s^{*} = T_s'^{-1}\Gamma T_s^{-1}$$

This will lead to a transformed credibility matrix:

$$Z_s^{*} = T_s Z_s T_s^{-1}$$

combining these elements to find the transformed estimate:

$$\mu_{ts}^{*} = [\hat{\beta}_s^{*} Z_s^{*} + \hat{\beta}_s^{*}(I - Z_s^{*})]Y_{ts}^{*}$$

It is immediately clear that this estimate is identical with the original untransformed estimate.

<u>Origin and Scale Transformations for the Trend Model</u>

One of the immediate implications of the above results is that the credibility results found above would have been the same if our time data had been transformed to have zero mean and unit variance. Using the result of this transformation

$$Y_{ts}^{*} = \begin{pmatrix} 1 \\ \dfrac{t - \bar{t}_s}{\sigma_{ts}} \end{pmatrix}$$

simplifies the credibility form since

$$Y_s^{*\prime} P_s Y_s^{*} = P_{.s} I$$

However, now the Γ matrix varies from state to state. Explicitly

$$\Gamma_s^* = \begin{pmatrix} \sigma_{as}^2 & \sigma_{abs} \\ \sigma_{abs} & \sigma_{bs}^2 \end{pmatrix}$$

$$= \begin{pmatrix} \sigma_a^2 + 2\sigma ab\bar{t}_s + \bar{t}_s^2\sigma b^2 & \sigma_{ts}(\sigma_{ab} + \bar{t}_s\sigma b^2 \\ \sigma_{ts}(\sigma_{ab} + \bar{t}_s\sigma b^2) & \sigma_{ts}^2\sigma b^2 \end{pmatrix}$$

The transformed credibility constant K_s^* now takes on the simple form:

$$K_s^* = \sigma^2\Gamma_s^{*-1}$$

The transformed credibility matrix:

$$Z_s^* = P_{.s}(P_{.s}I + K_s^*)^{-1}$$

still has the same general form as in the untransformed case. The $\hat{\beta}_s^*$, $\hat{\beta}_s^*$ and estimated values of Γ_s^*, K_s^* and Z_s^* are shown in figure 10 by state for the scale and location transformation.

Mixed Models

The upsetting results for the credibility adjusted trend line shown above in figure 9 came about because the mean value μ_{ts} is modeled in the same fashion for each state, specifically assuming that both slopes and intercepts were distributed about some mean value slope b and mean value intercept a. If we were to pause for a moment to think about our personal model of the trend situation; we would be more inclined to believe that while the avarage dollar at any point and time would vary substantially from state to state, the rate of change in the average dollar would tend to be the same from state to state. The modeling implication of this is, first of all, not to use a trend line; but to use an exponential trend. We will not pursue this direction in this paper. However, this analysis will be carried out in further research on this subject.

FIGURE 10

Estimates for Scaled t

State	Transformed State Coefficients	Transformed Countrywide Coefficients	Transformed Γ	Transformed K	Transformed Credibility Matrix
s	$\hat{\beta}^*_s$	$\hat{\hat{\beta}}^*_s$	Γ^*_s	K^*_s	Z^*_s
1	$\begin{pmatrix} 2{,}061 \\ -216.04 \end{pmatrix}$	$\begin{pmatrix} 1{,}864 \\ -150.11 \end{pmatrix}$	$\begin{pmatrix} 95{,}058 & -29{,}596 \\ -29{,}596 & 9{,}651 \end{pmatrix}$	$\begin{pmatrix} 10{,}244 & 31{,}415 \\ 31{,}415 & 100{,}904 \end{pmatrix}$	$\begin{pmatrix} .9494 & -.1483 \\ -.1483 & .5213 \end{pmatrix}$
2	$\begin{pmatrix} 1{,}511 \\ -59.66 \end{pmatrix}$	$\begin{pmatrix} 1{,}870 \\ -150.88 \end{pmatrix}$	$\begin{pmatrix} 97{,}433 & -30{,}135 \\ -30{,}135 & 9{,}751 \end{pmatrix}$	$\begin{pmatrix} 10{,}244 & 31{,}661 \\ 31{,}661 & 102{,}367 \end{pmatrix}$	$\begin{pmatrix} .9068 & -.2348 \\ -.2348 & .2235 \end{pmatrix}$
3	$\begin{pmatrix} 1{,}806 \\ -150.21 \end{pmatrix}$	$\begin{pmatrix} 1{,}858 \\ -150.36 \end{pmatrix}$	$\begin{pmatrix} 92{,}511 & -29{,}227 \\ -29{,}227 & 9{,}684 \end{pmatrix}$	$\begin{pmatrix} 10{,}244 & 30{,}919 \\ 30{,}919 & 97{,}868 \end{pmatrix}$	$\begin{pmatrix} .8911 & -.2469 \\ -.2469 & .1915 \end{pmatrix}$
4	$\begin{pmatrix} 1{,}353 \\ -98.01 \end{pmatrix}$	$\begin{pmatrix} 1{,}860 \\ -152.80 \end{pmatrix}$	$\begin{pmatrix} 93{,}168 & -29{,}811 \\ -29{,}811 & 10{,}000 \end{pmatrix}$	$\begin{pmatrix} 10{,}244 & 30{,}539 \\ 30{,}539 & 95{,}443 \end{pmatrix}$	$\begin{pmatrix} .8251 & -.2530 \\ -.2530 & .1193 \end{pmatrix}$
5	$\begin{pmatrix} 1{,}600 \\ -41.68 \end{pmatrix}$	$\begin{pmatrix} 1{,}869 \\ -152.16 \end{pmatrix}$	$\begin{pmatrix} 96{,}991 & -30{,}318 \\ -30{,}318 & 9{,}917 \end{pmatrix}$	$\begin{pmatrix} 10{,}244 & 31{,}320 \\ 31{,}320 & 100{,}194 \end{pmatrix}$	$\begin{pmatrix} .9222 & -.2119 \\ -.2119 & .3136 \end{pmatrix}$

Restricting our thinking to the trend line model, the credibility model which is most meaningful would be one in which only the slope is considered to be a variable from state to state, but where the intercept is a constant:

$$\mu_{ts}(\theta_s) = a_s + b(\theta_s)t$$

This sort of model is directly analogous to the Bühlmann, Straub introduction of treaty conditions in their paper, which allow the severities to be modified by some function before entering the credibility formula.

We have shown above that scale and translation formulation will not affect our final credibility estimate. For ease of exposition in this section, we will assume that the time values in our trend line have been chosen so that the weighted average of observed times is zero and the weighted sample variance is equal to one. The modifications to our basic credibility model, because of the constant values a_s within the mean value μ_{ts} formula, are fairly simple. For the regular credibility model β_s was the same function of θ_s for all states. In our mixed model this function varies from state to state:

$$B_s(\theta_s) = \begin{pmatrix} a_s \\ b(\theta_s) \end{pmatrix}$$

The expected value of this function varies from state to state:

$$E[B_s(\theta_s)] = {}_s\beta = \begin{pmatrix} a_s \\ b \end{pmatrix}$$

We have chosen to denote this expected value as ${}_s\beta$ to avoid confusion with the function of θ, β_s. The covariance matrix Γ_s^* is:

$$E[B_s(\theta_s)B'(\theta_s)] - {}_s\beta_s\beta' = \Gamma_s^* = \begin{pmatrix} 0 & 0 \\ 0 & \sigma_{bs}^2 \end{pmatrix}$$

with the only non-zero entry being σ_b^2.

If we introduce for state j

$$K_{bj} = \sigma^2/\sigma_{bj}^2$$

to define:

$$Z_{bj} = P_{.j}/(P_{.j} + K_{bj})$$

The credibility matrix for our mixed model becomes:

$$Z_j = \begin{pmatrix} 0 & 0 \\ 0 & Z_{bj} \end{pmatrix}$$

Using the same theoretical development as in the regular credibility model, for the mixed model leads to:

$$\hat{\mu}_{ij} = [\hat{\beta}_j' Z_j + {}_j\beta'(I - Z_j)]Y_{ij}$$

The only difference is this estimate is that ${}_j\beta'$ replaces β'.[6] This estimate may be written for the trend case without recourse to matrices simply as:

$$\hat{\mu}_{ij} = a_j + [\hat{b}_j Z_{bj} + \hat{b}(1 - Z_{bj})]i$$

Using the formulas for the mixed model, the constant K, the credibility and finally the credibility adjusted slopes are shown on figure 11. For this mixed model, our credibility results are much more pleasing since the credibility adjusted

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[6] It is important to note that this result holds for any mixed model, not just for out trend case. The most general mixed model, of course, allows arbitrary elements of β_s to be considered independent of θ_s.

FIGURE 11.

Credibility Adjusted Slopes
Without Intercept Adjustments

State	Number of Claims Over 3 Years		Credibility	Transformed State Slope	Transformed Credibility Adjusted Slope	Transformed Countrywide Slope
s	$P_{.s}$	K_{bs}	Z_{bs}	$\hat{b}^*$	$\tilde{b}^*$	$\hat{\hat{b}}^*$
1	100,155	4,565	.9564	-216.04	-213.17	-150.11
2	19,895	4,518	.8149	- 59.66	- 76.54	-150.88
3	13,735	4,550	.7512	-150.21	-150.21	-150.36
4	4,152	4,406	.4852	- 98.01	-126.22	-152.80
5	36,110	4,443	.8904	- 41.68	- 53.79	-152.16

slope must lie between the state and countrywide slopes. Further, some general observations can be made concerning the relative size of credibility to be given to state data. With this five state base as countrywide for most states, the number of claims that are observed show extremely high credibility. Only for the smallest state #4, with 4,152 claims observed over three years is credibility lower than .5. Of course, for practical application, the credibility standard should be developed using all of the states not just five.

Discussion by Al Quirin of Credibility for Regression Models with Application to Trend

This paper considers an arbitrary linear regression model, incorporates the Bühlmann Straub formulation of the model, extends the estimator form considered in the Bühlmann Straub model, exhibits the relationship between the least squares estimators, and finally derives computational results involving simple linear trend.

Arbitrary Linear Regression Model Considered	$E(x_{ts}) = u_{ts} = y'_{ts}\beta_s$	(1)
Bühlmann-Straub Formulation Incorporated	$E(x^2_{ts}) - u^2_{ts} = \sigma^2_s/\rho_{ts}$	(2)
	$E(x_{is}x_{js}) - u_{is}u_{js} = 0,\ i \neq j$	(3)
Bühlmann-Straub Estimator Form	$\hat{u}_{ts} = x'A$	(4a)
Extended to	$\hat{u}_{ts} = \alpha_0 + x'A$	(4b)

Relationship of Least Squares Estimators

using (4b) $\quad \hat{u}_{ts} = [\hat{\beta}'_s z_s + \beta'(I - z_2)]y_{ts}$

using (4a) $\quad \hat{u}_{ts} = [\hat{\beta}'_s z_s + \beta'(I - z_2)]y_{ts}$

$$\text{Where } d = \frac{\sum_s \beta'_s z_s \Gamma^{-1}\beta}{\sum_s \beta' z_s \Gamma^{-1}\beta}$$

Adequate accountability for inflation has become the single most important need in Property and Casualty insurance ratemaking today. In response to this need, Mr. Hachemeister's paper developing credibility standards for arbitrary linear regression models and in particular, developing credibility adjusted state trend lines, should prove to be invaluable.

In his Introduction, the author mentions that "no standards have been specifically developed for evaluating (the) credibility of state trend lines vs. countrywide trend lines." Although not specifically developed for analyzing trend, a credibility procedure has been used for some time by the Insurance Services Office (ISO) in their trend calculations, at least in private passenger automobile insurance. In each state, the determination of the average annual change in paid claim costs and claim frequencies is accomplished by credibility weighing the state and countrywide average annual changes. These average annual changes are taken from linear and exponential least squares trend lines for paid claim costs and claim frequencies, respectively. The credibility weights assigned are based on the latest year ending number of claims. Unfortunately, the theoretical justification for this approach is no deeper than assuming the number of claims has a Poisson distribution, and approximating probabilities by the use of the normal distribution. The standard for full credibility is 10,623 claims and reflects a probability of .99 that the number of claims will be within ±2.5% of the expected number of claims (on the assumption that the mean is equal to the variance). Partial credibilities are obtained using the formula $Z^2 = \frac{P}{10,623}$, where P is the latest year ending number of claims needed for partial credibility Z. The theoretical soundness of this procedure has been proven deficient by several authors, but up until this point in time, the theoretical advantages of alternative procedures do not seem to outweigh the practical advantage of simplicity (both in explanation to state insurance departments and in mathematical computation) present in the current procedure. From my own point of view, even though I feel that simplicity is a much overrated virtue in the very technical business of insurance ratemaking and that theoretical soundness should be of primary importance, I am convinced that <u>any alternative</u> credibility procedure will face the rather strict test of practical expediency before being implemented by those in the business of pricing insurance. With regard to Mr. Hachemeister's paper, it is precisely its simplicity in practical application (as well as its theoretical validity) which leads me to believe that it will someday soon become extensively utilized in calculating trend.

In the first half of the paper, the author states the problem of state vs. countrywide trend, introduces notation, displays data for a computational example, presents basic summary statistics, and reviews the classical and generalized linear regression model. Although the author has made mention to the point, it should be reiterated that even though the form of the estimator

$$\hat{\beta}_s = (y_s' c_s^{-1} y_s)^{-1} y_s' c_s^{-1} x_s$$

follows that obtained in classical <u>generalized</u> least squares estimation and that the theoretical results hold in general for the positive-definite matrix C_s, the assumption made regarding autocorrelation in deriving numerical results is not that of generalized least squares. In particular, recall that the classical generalized least squares formulation of the state s trend model is

i) $E(x_{ts}) = u_{ts} = a_s + b_s t \qquad t = 1,\ldots,n$

ii) $E(x_{ts}^2) - u_{ts}^2 = c_{ts} = \sigma_s^2/\rho_{ts} \qquad s = 1,\ldots,N$

The $n \times n$ positive definite matrix C_s allows for both heteroscedasticity and autocorrelation, i.e., for both

iii) $E(x_{ts}^2) - u_{ts}^2 = \sigma_s^2, \qquad \forall t$

and

iv) $E(x_{is}x_{js}) - u_{is}u_{js} = 0, \quad i \neq j$

not holding. However, in deriving numerical results, Hachemeister disallows autocorrelation by assuming that iv) holds. In other words, should these problem be found to occur in trend data, further computational refinements will become necessary in practical application.

An approach to the solution of the problem of state vs. countrywide trend, is then formulated as a compound decision problem. In particular, the mean value μ_{ts} of a "credibility adjusted state s trend line" is modeled as

v) $\mu_{ts}(\theta_s) = a_t(\theta_s) + b_t(\theta_s)t$

where for each state s and each time period t, one acts as if the slopes and intercepts were distributed about some mean slope $E[b_t(\theta_s)]$ and some mean intercept $E[a_t(\theta_s)]$. Best linear unbiased estimators (BLUE) are then considered of the form

vi) $\hat{\mu}_{ts} = \alpha_0 + \sum_{s=1}^{N} \sum_{t=1}^{n} \alpha_{ts}x_{ts} = \alpha_0 + x'A$

and are found to be

vii) $\hat{\mu}_{ts} = [\hat{\beta}_s z_s + \beta'(I - z_s)]y_{ts}$

The application of this result to real data requires that estimates be made of various parameters not directly observable within the credibility model (e.g. z_s in vii) is a function of K_s which in turn depends on estimates of σ^2, v, and Γ). Because of the need for these estimates, assumptions iii) and iv) are made to simplify the derivation of numerical results.

The invariance property of $\hat{\mu}_{ts}$ for <u>any linear transformation</u> of the independent variables follows in a straightforward manner. Using this result, Hachemeister performs a scaling and translation on the linear trend model so that the weighted average (using # claims as weights) of scaled times in zero and the sample variance of scaled times is equal to unity. Finally, a mixed model is employed, to avoid the distressing results obtained when state intercepts are credibility adjusted, so that the final model chosen is

$$\mu_{ts}(\theta_s) = a_s + b(\theta_s)t \ .$$

Note that in this model the intercept varies by state but is assumed constant over all time periods. For each state s and each time period t the slope is still considered to be distributed about some mean slope. The effect of the estimated form in this mixed model is that slopes are credibility adjusted while intercepts are not.

To investigate the credibility standards developed and to evaluate the procedure finally decided upon in credibility adjusting state trend lines, consider the transformed simple linear trend model which credibility adjusts slopes without intercept adjustments.

Here,

$$u_{ts}(\theta_s) = a_s + b(\theta_s)t$$

where

$$\beta_s(\theta_s) = \begin{pmatrix} a_s \\ b(\theta_s) \end{pmatrix}$$

and

$$E[\beta_s(\theta_s)] = {}_s\beta = \begin{pmatrix} a_s \\ b \end{pmatrix} \ .$$

The estimator becomes for state s

$$\hat{u}_{ts} = [\hat{\beta}'_s z_s + {}_s\beta' (I - z_s)]y_{ts}$$

$$= a_s + [\hat{b}_s z_{bs} + \hat{b}(1 - z_{bs})]t$$

where

$$z_{bs} = P_{.s}/(P_{.s} + K_{bs})$$

and

$$K_{bs} = \sigma^2/\sigma^2_{bs} .$$

Note that the credibility parameter K_{bs} satisfies the general definition demonstrated by Bühlmann that it be equal to

$$\frac{\text{expected value of process variance } (= \sigma^2)}{\text{variance of the hypothetical means } (= \sigma^2_{bs})} .$$

The K_{bs}'s vary by state but a single constant K value could be adopted should the K_{bs}'s developed for all states show the same stability (centered around 4,500) as those developed for the five selected states.