MEASURING THE ADJUSTABLE FEATURES OF TREATIES (CAS SEMINAR ON RATEMAKING, MARCH, 1991

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Appendices A and B present practical approaches to pricing the expected impact of adjustable features and loss sharing provisions of reinsurance treaties. A simple quota share example is used to illustrate methods of estimating the impact of aggregate deductibles, loss ratio caps and loss corridor provisions. This example is then used to evaluate profit and sliding scale commission plans and a retrospective rating plan. Appendix C presents models used to assess the cash flow implications of alternative adjustable features under consideration in an excess-of-loss example.

Panel: Robert A. Bear
North Star Reinsurance Corporation
Appendix A: Measuring the Expected Impact via Lognormal and Collective Risk Models

Jeffrey A. Englander
Trenwick America Reinsurance Corporation
Appendix B: Measuring the Expected Impact via Simulation

Todd J. Hess
Underwriters Reinsurance Company
Appendix C: Considering the Cash Flow

SUBJECT: ADJUSTABLE FEATURES AND LOSS SHARING PROVISIONS OF REINSURANCE TREATIES.

GOAL: BROADER UNDERSTANDING OF AVAILABLE APPROACHES TO ESTIMATE IMPACT OF THESE IMPORTANT TERMS.

PLAN: USE SIMPLE EXAMPLE TO ILLUSTRATE METHODS, WITH EMPHASIS ON CONCEPTS.

BENEFITS: (1) IMPROVED UNDERSTANDING BETWEEN ACTUARIES AND NON-ACTUARIES AND BETWEEN PRIMARY COMPANIES AND REINSURERS.

(2) GREATER PRICING ACCURACY.

ADJUSTABLE PREMIUM AND COMMISSION FEATURES

PREMIUM AND COMMISSION ADJUSTMENT PLANS
WHOSE RESULTS DEPEND UPON
ACTUAL TREATY LOSS EXPERIENCE
OVER A PARTICULAR PERIOD.

EXAMPLES: RETROSPECTIVE RATING PLANS
PROFIT COMMISSION AND PROFIT-SHARING PLANS
SLIDING SCALE COMMISSION PLANS

GOAL: DETERMINE EXPECTED ADJUSTED PREMIUM RATE
OR COMMISSION RATIO FOR TREATY.

LOSS SHARING PROVISIONS (NONPROPORTIONAL COINSURANCE)

CEDING COMPANY PAYS NONPROPORTIONAL SHARE OF LOSSES.
DOES NOT RECEIVE SHARE OF REINSURANCE PREMIUM.

EXAMPLES: AGGREGATE DEDUCTIBLES
AGGREGATE LIMITS
LOSS RATIO CAPS AND LIMITED REINSTATEMENTS
LOSS CORRIDOR PROVISIONS

GOAL: ESTIMATE PROPORTION OF LOSSES OTHERWISE SUBJECT TO TREATY WHICH ARE RETAINED BY CEDANT.

THIS PERMITS ESTIMATION OF EXPECTED REINSURANCE LOSSES AFTER LOSS SHARING PROVISION.

REFERENCES

(1) "PRICING THE IMPACT OF ADJUSTABLE FEATURES AND LOSS SHARING PROVISIONS OF REINSURANCE TREATIES," R.A. BEAR AND K.J. NEMLICK, 1990 PCAS. (PRELIMINARY VERSIONS WERE PRESENTED AT 1990 DISCUSSION PAPER PROGRAM AND AT CAS CONVENTION.)

(2) "THE CALCULATION OF AGGREGATE LOSS DISTRIBUTIONS FROM CLAIM SEVERITY AND CLAIM COUNT DISTRIBUTIONS," P.E. HECKMAN AND G.G. MEYERS, 1983 PCAS.

(3) "PRICING EXCESS-OF-LOSS CASUALTY WORKING COVER REINSURANCE TREATIES," G.S. PATRIK AND R.T. JOHN, 1980 CAS DISCUSSION PAPER.

(4) "ESTIMATING PURE PREMIUMS BY LAYER," R.J. FINGER, 1976 PCAS.

BACKGROUND

1) CONCENTRATION OF W.C. EXPOSURE IN HOMOGENEOUS CLASS.

2) INSURER HAS EXCESS OF LOSS COVER ABOVE $250,000;
ALAE PART OF LOSS.

3) NET SUBJECT MATTER PREMIUM = $9,000,000 IN THIS W.C. CLASS.

4) SEEKS ADDITIONAL QUOTA SHARE COVERAGE
FOR 1991 UNDERWRITING YEAR.

ACTUARIAL ASSUMPTIONS

(1) EXPECTED CLAIM FREQUENCY = 85 CLAIMS / $1M.

(2) CLAIM SEVERITY (INDEMNITY + ALAE) IS MODELED

BY WEIBULL WITH SHAPE = .2 AND SCALE = 171.

$$F(X) = 1 - E^{-T}$$

$$\text{WHERE } T = \left(\frac{X}{171}\right)^{.2}$$

UNLIMITED MEAN SEVERITY = $20,520

MEAN LIMITED SEVERITY ($250,000) = $8,796

(1) AND (2) IMPLY EXPECTED LOSS & ALAE RATIO = 75%.

ACTUARIAL ASSUMPTIONS - CONTINUED

(3) CLASS IS HAZARD GROUP III; COUNTRYWIDE NCCI TABLE M IS EFFECTIVE WITH 1990 TABLE OF EXPECTED LOSS RANGES.

(4) ALAE IS ONLY 5% OF INDEMNITY AND A SMALL PORTION OF CLAIMS EXCEED $250,000.
HENCE, TABLE M PROVIDES A ROUGH APPROXIMATION OF EMPIRICAL INSURANCE CHARGES.

(3) AND (4) MAY BE USED OR IGNORED BY PANELISTS.

(5) PARAMETER UNCERTAINTY IS SIGNIFICANT. PANELISTS ARE ENCOURAGED TO CONSIDER AND REFLECT IT IN THEIR ANALYSES.

NON-PROPORTIONAL COINSURANCE ALTERNATIVES

CEDANT IS CONSIDERING THREE LOSS SHARING PROVISIONS.
FOR EACH, ESTIMATE EXPECTED LOSS AND ALAE RATIO TO REINSURER.

(1) AGGREGATE DEDUCTIBLE = \$5,400,000 (80% OF EXPECTED LOSS & ALAE).

(2) 90% LOSS AND ALAE RATIO CAP.

(3) CEDING COMPANY WILL PAY ALL LOSSES AND ALAE BETWEEN 75% AND 112.5% OF SUBJECT PREMIUM (LOSS CORRIDOR).

ADJUSTABLE FEATURES ALTERNATIVES

NO COINSURANCE APPLIES, SO EXPECTED LOSS AND ALAE RATIO IS 75%.

EACH OF THREE PLANS WILL BE EVALUATED BASED SOLELY ON 1991 UNDERWRITING YEAR EXPERIENCE.

(1) 50% PROFIT COMMISSION TO CEDANT AFTER 25% FOR REINSURER'S OVERHEAD AND PROFIT. WHAT IS EXPECTED PROFIT COMMISSION ?

ADJUSTABLE FEATURES - CONTINUED

(2) PROVISIONAL CEDING COMMISSION TO BE NEGOTIATED.

SUPPLEMENTAL SLIDING SCALE COMMISSION BASED ON FOLLOWING:

LOSS & ALAE RATIO INTERVAL		PERCENTAGE INCREASE IN COMMISSION RATIO PER 1% DECREASE IN LOSS & ALAE RATIO	CORRESPONDING COMMISSION RATIO INTERVAL	
LOWER BOUND	UPPER BOUND		LOWER BOUND	UPPER BOUND
75.00%	AND ABOVE	0.00%	0.00%	0.00%
60.00%	75.00%	0.50%	7.50%	0.00%
45.00%	60.00%	0.60%	16.50%	7.50%
30.00%	45.00%	0.75%	27.75%	16.50%
0.00%	30.00%	1.00%	57.75%	27.75%

WHAT IS THE EXPECTED SLIDING SCALE COMMISSION ?

ADJUSTABLE FEATURES - CONTINUED

(3) RETROSPECTIVE RATING PLAN:

QUOTA SHARE CESSION TREATED AS PROVISIONAL PREMIUM.

CEDENT WILLING TO PAY 30% MORE OR LESS BASED ON TREATY EXPERIENCE.

FORMULA:

REINSURANCE RATE = (LOSS & ALAE RATIO) + (25% MARGIN)

70% $<$ REINSURANCE RATE $<$ 130%

RETROSPECTIVE PREMIUM = (REINSURANCE RATE) x (PROVISIONAL PREMIUM)

NO DOWNWARD ADJUSTMENTS FOR 5 YEARS.

ANY PROVISIONAL COMMISSION PAID OUT OF FLAT MARGIN.

WHAT IS ULTIMATE EXPECTED REINSURANCE RATE ?

TECHNICAL REQUIREMENT:

DISTRIBUTION OF AGGREGATE TREATY LOSSES

APPROACHES:

(1) COLLECTIVE RISK MODEL

(2) LOGNORMAL MODEL

(3) TABLE M

COLLECTIVE RISK MODEL
(THE HECKMAN-MEYERS ALGORITHM)

(1) EFFICIENTLY SIMULATES AGGREGATE LOSS DISTRIBUTION BASED ON CLAIM FREQUENCY AND SEVERITY DISTRIBUTIONS.

(2) REFLECTS UNCERTAINTY IN EXPECTED CLAIM FREQUENCY THROUGH CONTAGION PARAMETER c.

c = 0 : NO PARAMETER UNCERTAINTY.
c = .05 - .10 : MODERATE PARAMETER UNCERTAINTY.
c = .25 : HIGH PARAMETER UNCERTAINTY.

(3) REFLECTS UNCERTAINTY IN AVERAGE CLAIM SEVERITY THROUGH MIXING PARAMETER b.

b = 0 : NO PARAMETER UNCERTAINTY.
b = .05 - .10 : MODERATE PARAMETER UNCERTAINTY.
b = .25 : HIGH PARAMETER UNCERTAINTY.

THE LOGNORMAL MODEL

ASSUMPTION: AGGREGATE LOSS IS PRODUCT OF LARGE NUMBER OF INDEPENDENT, IDENTICALLY* DISTRIBUTED VARIABLES.

CONCLUSION: THE LOGARITHM IS APPROXIMATELY NORMALLY DISTRIBUTED (CENTRAL LIMIT THEOREM).

IMPLICATION: AGGREGATE LOSS IS LOGNORMALLY DISTRIBUTED.

* THE STRINGENT CONDITION THAT THE FACTORS BE IDENTICALLY DISTRIBUTED MAY BE RELAXED.

REQUIREMENT OF LOGNORMAL MODEL:

COEFFICIENT OF VARIATION

$$= \frac{\text{STANDARD DEVIATION}}{\text{MEAN}}$$

$$= \frac{\sqrt{\text{VARIANCE OF AGGREGATE LOSSES}}}{\text{EXPECTED AGGREGATE LOSS}}$$

COMPONENTS COMPUTED BASED ON FREQUENCY AND SEVERITY DISTRIBUTIONS.

DEFINITIONS:

(1) EXCESS PURE PREMIUM:

EXPECTED AGGREGATE LOSSES EXCESS OF ATTACHMENT.

THE ATTACHMENT COULD BE AGGREGATE DEDUCTIBLE VALUE OR AGGREGATE LIMIT UNDER CONSIDERATION.

(2) EXCESS PURE PREMIUM RATIO:

RATIO OF EXCESS PURE PREMIUM TO EXPECTED AGGREGATE LOSS.

(3) ENTRY RATIO:

RATIO OF ATTACHMENT TO EXPECTED AGGREGATE LOSS.

IMPORTANT RESULT:

IF AGGREGATE LOSS DISTRIBUTION IS LOGNORMAL, A SIMPLE FORMULA EXISTS TO COMPUTE THE EXCESS PURE PREMIUM RATIO FOR ANY ATTACHMENT.

YOU NEED TO KNOW THE EXPECTED AGGREGATE LOSS AND THE COEFFICIENT OF VARIATION OF THE AGGREGATE LOSS DISTRIBUTION.

THE BEAR-NEMLICK PAPER SUMMARIZES TECHNICAL DETAILS AND PROVIDES TABLES OF EXCESS PURE PREMIUM RATIOS FOR COEFFICIENTS OF VARIATION BETWEEN .1 AND 5.

COMPUTATION OF EXCESS PURE PREMIUMS

(1) WITHOUT PARAMETER UNCERTAINTY

EXCESS PURE PREMIUM FOR PARTICULAR ATTACHMENT
= EXPECTED AGGREGATE LOSS X EXCESS PURE PREMIUM RATIO

(2) WITH PARAMETER UNCERTAINTY

(a) ESTIMATE EXCESS PURE PREMIUMS BASED ON ALTERNATIVE FREQUENCY AND SEVERITY ASSUMPTIONS.

(b) ASSIGN SUBJECTIVE PROBABILITIES TO EACH SCENARIO IN (a).

(c) THE UNCONDITIONAL EXCESS PURE PREMIUM IS THE WEIGHTED AVERAGE OF THE CONDITIONAL EXCESS PURE PREMIUMS IN (a), BASED ON THE WEIGHTS IN (b).

TABLE M: TABLE OF INSURANCE CHARGES
(EXCESS PURE PREMIUM RATIOS AND CORRESPONDING SAVINGS)

INSURED IS ASSIGNED TO EXPECTED LOSS GROUP BASED UPON ANNUAL EXPECTED LOSSES. ASSIGNMENTS ADJUSTED ANNUALLY.

INSURANCE CHARGES AND SAVINGS ARE GIVEN IN TABLES AS A FUNCTION OF THE EXPECTED LOSS GROUP AND ENTRY RATIO.

TABLE M IS BASED ON NCCI STUDY OF EMPIRICAL WORKER'S COMPENSATION INDIVIDUAL RISK AGGREGATE LOSS DATA.

TABLE M IS USED TO ESTIMATE NET INSURANCE CHARGES OF RETROSPECTIVE RATING PLANS.

PARAMETERS OF ALTERNATIVE APPROACHES
PRIOR TO ADJUSTABLE FEATURES AND LOSS SHARING PROVISIONS

(1) COLLECTIVE RISK MODEL

(a) EXPECTED CLAIMS = 765

(b) AVERAGE CLAIM COST = 8831
(FROM PIECEWISE LINEAR FIT TO WEIBULL CENSORED AT $250,000)

(c) CONTAGION PARAMETER c = .10

(d) MIXING PARAMETER b = .05

(2) LOGNORMAL MODEL

(a) EXPECTED AGGREGATE LOSS = .75 x $9,000,000 = $6,750,000

(b) COEFICIENT OF VARIATION = .423 (FROM COLLECTIVE RISK MODEL)

(3) TABLE M

EXPECTED LOSS GROUP = 16

AGGREGATE DEDUCTIBLES

REINSURER PAYS NOTHING UNTIL TREATY LOSSES EXCEED SPECIFIED AMOUNT ($5,400,000 IN EXAMPLE).

THE REINSURER THEN PAYS ALL LOSSES SUBJECT TO TREATY.

EXPECTED TREATY LOSSES AFTER AGGREGATE DEDUCTIBLE
= EXPECTED LOSSES x [100% - LOSS ELIMINATION RATIO]

WHERE LOSS ELIMINATION RATIO = 100% - XSPPR(D)
AND XSPPR(D) = EXCESS PURE PREMIUM RATIO CORRESPONDING
TO AGGREGATE DEDUCTIBLE.

EXPECTED TREATY LOSSES AFTER AGGREGATE DEDUCTIBLE
= EXPECTED LOSSES x XSPPR(D).

CALCULATING THE IMPACT OF THE AGGREGATE DEDUCTIBLE - EXAMPLE

(1) AGGREGATE DEDUCTIBLE IN DOLLARS = \$5,400,000

(2) EXPECTED TREATY LOSSES AND ALAE BEFORE COINSURANCE = \$6,750,000

(3) ENTRY RATIO CORRESPONDING TO AGGREGATE DEDUCTIBLE = $\frac{\$5,400,000}{\$6,750,000} = .8$

	RISK MODEL	LOGNORMAL	TABLE M
(4) EXCESS PURE PREMIUM RATIO:	27.0%	26.6%	26.2%
(5) PORTION OF TREATY LOSSES ELIMINATED:	73.0%	73.4%	73.8%
(6) EXPECTED TREATY LOSS RATIO AFTER AGGREGATE DEDUCTIBLE:	20.3%	19.9%	19.7%

LOSS RATIO CAP

REINSURER PAYS FOR ALL TREATY LOSSES UP TO LOSS RATIO CAP (90% IN EXAMPLE).

EXPECTED TREATY LOSSES AFTER LOSS RATIO CAP
= EXPECTED LOSSES x [100% - LOSS ELIMINATION RATIO]

WHERE LOSS ELIMINATION RATIO = XSPPR(C)
= EXCESS PURE PREMIUM RATIO AT LOSS RATIO CAP C

CALCULATING THE IMPACT OF THE LOSS RATIO CAP - EXAMPLE

(1) LOSS RATIO CAP IN DOLLARS = .9 x \$9,000,000 = \$8,100,000

(2) EXPECTED TREATY LOSSES AND ALAE BEFORE COINSURANCE = \$6,750,000

(3) ENTRY RATIO CORRESPONDING TO LOSS RATIO CAP = $\dfrac{\$8{,}100{,}000}{\$6{,}750{,}000}$ = 1.2

	RISK MODEL	LOGNORMAL	TABLE M
(4) EXCESS PURE PREMIUM RATIO	9.4%	9.4%	9.8%
(5) LOSS ELIMINATION RATIO	9.4%	9.4%	9.8%
(6) EXPECTED TREATY LOSS RATIO AFTER LOSS RATIO CAP	68.0%	67.9%	67.7%

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LOSS CORRIDORS

REINSURER PAYS FOR TREATY LOSSES UNTIL FIXED AMOUNT LB IS REACHED.

REINSURER STOPS PAYING LOSSES UNTIL TOTAL REACHES SECOND FIXED AMOUNT, UB.

REINSURER RESUMES PAYING LOSSES WHEN TOTAL EXCEEDS UB.

LOSS CORRIDOR = INTERVAL BETWEEN LB AND UB.

CALCULATING THE IMPACT OF THE LOSS CORRIDOR PROVISION

EXPECTED TREATY LOSSES AND ALAE AFTER LOSS CORRIDOR PROVISION = EXPECTED LOSSES AND ALAE x [100%-LOSS ELIMINATION RATIO]

WHERE LOSS ELIMINATION RATIO = XSPPR(LB) - XSPPR(UB)

AND XSPPR(LB) = EXCESS PURE PREMIUM RATIO AT LB
(75% OF SUBJECT PREMIUM IN EXAMPLE)

XSPPR(UB) = EXCESS PURE PREMIUM RATIO AT UB
(112.5% OF SUBJECT PREMIUM IN EXAMPLE)

CALCULATING THE IMPACT OF THE LOSS CORRIDOR PROVISION - EXAMPLE

(1) LOWER BOUND OF LOSS CORRIDOR = .75 x \$9,000,000 = \$6,750,000

(2) UPPER BOUND OF LOSS CORRIDOR = 1.125 x \$9,000,000 = \$10,125,000

(3) EXPECTED TREATY LOSSES AND ALAE BEFORE COINSURANCE = \$6,750,000

(4) ENTRY RATIO CORRESPONDING TO LOWER BOUND = $\dfrac{\$6{,}750{,}000}{\$6{,}750{,}000}$ = 1.0

(5) ENTRY RATIO CORRESPONDING TO UPPER BOUND = $\dfrac{\$10{,}125{,}000}{\$6{,}750{,}000}$ = 1.5

	RISK MODEL	LOGNORMAL	TABLE M
(6) EXCESS PURE PREMIUM RATIOS:			
LOWER BOUND	16.3%	16.1%	16.0%
UPPER BOUND	3.9%	4.1%	5.0%
(7) LOSS ELIMINATION RATIO	12.4%	12.0%	11.0%
(8) EXPECTED TREATY LOSS RATIO AFTER LOSS CORRIDOR	85.7%	86.0%	86.8%

PROFIT COMMISSIONS

PROFIT COMMISSION RATIO = P x [100% - LR - EXP]

WHERE P = PROPORTION OF PROFITS TO BE PAID TO CEDANT
(50% IN EXAMPLE)

LR = ACTUAL TREATY LOSS RATIO

EXP = REINSURER'S OVERHEAD PROVISION
(25% OF TREATY PREMIUM IN EXAMPLE)

THE PROFIT COMMISSION RATIO CANNOT BE NEGATIVE.
LOSS RATIOS ENTERING THE PROFIT COMMISSION FORMULA
ARE CAPPED AT BREAKEVEN LOSS RATIO.

BLR = 100% - EXP

GOAL: TO DETERMINE THE EXPECTED PROFIT COMMISSION TO BE PAID.

METHOD: DETERMINE EFFECT THAT LIMITING ACTUAL LOSS RATIOS TO THE BREAKEVEN RATIO HAS ON THE EXPECTED TREATY LOSS RATIO USED IN PROFIT COMMISSION RATIO CALCULATION.

CALCULATING THE EXPECTED PROFIT COMMISSION RATIO

FELR = EXPECTED TREATY LOSS RATIO USED IN PROFIT COMMISSION FORMULA

= EXPECTED LOSS RATIO X [100% - LOSS ELIMINATION RATIO]

LOSS ELIMINATION RATIO = XSPPR(BLR)
= EXCESS PURE PREMIUM RATIO AT BREAKEVEN LOSS RATIO

ECR = EXPECTED PROFIT COMMISSION RATIO = P x [100% - FELR - EXP]

THE EXPECTED PROFIT COMMISSION RATIO WILL ALWAYS EXCEED THAT OBTAINED BY SIMPLY PLUGGING THE EXPECTED LOSS RATIO INTO THE PROFIT COMMISSION FORMULA.

CALCULATING THE EXPECTED PROFIT COMMISSION RATIO - EXAMPLE

(1) PROPORTION OF PROFITS TO BE PAID TO CEDANT = 50%

(2) REINSURER'S OVERHEAD PROVISION = EXP = 25%

(3) EXPECTED TREATY LOSS RATIO = ELR = 75%

(4) BREAKEVEN LOSS RATIO (BLR) = 100%-EXP = 75%

(5) ENTRY RATIO CORRESPONDING TO BREAKEVEN LOSS RATIO = $\frac{\text{BLR}}{\text{ELR}} = \frac{75\%}{75\%} = 1.0$

	RISK MODEL	LOGNORMAL	TABLE M
(6) EXCESS PURE PREMIUM RATIO AT BREAKEVEN LOSS RATIO	16.3%	16.1%	16.0%
(7) LOSS ELIMINATION RATIO = XSPPR(BLR)	16.3%	16.1%	16.0%
(8) EXPECTED TREATY LOSS RATIO USED IN COMMISSION FORMULA FELR = ELR x [100% - LOSS ELIMINATION RATIO]	62.8%	62.9%	63.0%
(9) EXPECTED PROFIT COMMISSION RATIO ECR = P x [100% - FELR - EXP]	6.1%	6.0%	6.0%
(10)SIMPLISTIC PROFIT COMMISSION RATIO (PLUG ELR INTO FORMULA)	0.0%	0.0%	0.0%

SLIDING SCALE COMMISSIONS

EXAMPLE: PROVISIONAL CEDING COMMISSION TO BE NEGOTIATED.
SUPPLEMENTAL SLIDING SCALE COMMISSION BASED ON FOLLOWING PLAN:

LOSS & ALAE RATIO INTERVAL		PERCENTAGE INCREASE IN COMMISSION RATIO PER 1% DECREASE IN LOSS & ALAE RATIO	CORRESPONDING COMMISSION RATIO INTERVAL	
LOWER BOUND	UPPER BOUND		LOWER BOUND	UPPER BOUND
75%	AND ABOVE	0.00%	0.00%	0.00%
60%	75%	0.50%	7.50%	0.00%
45%	60%	0.60%	16.50%	7.50%
30%	45%	0.75%	27.75%	16.50%
0%	30%	1.00%	57.75%	27.75%

THE SLIDING SCALE COMMISSION MAY BE EXPRESSED USING PIECEWISE LINEAR FORMULA:

LOSS AND ALAE RATIO (L)	COMMISSION RATIO (C)
ABOVE 75%	0%
60% - 75%	.5 x [75% - L]
45% - 60%	7.5% + ,6 x [60% - L]
30% - 45%	16.5% + .75 x [45% - L]
0% - 30%	27.75% + 1.0 x [30% - L]

CALCULATION OF THE EXPECTED SLIDING SCALE COMMISSION

EXPECTED COMMISSION RATIO

= Cmax - EXPECTED COMMISSION REDUCTIONS

OVER ALL LOSS RATIO INTERVALS

$$= \text{Cmax} - \sum_{i=1}^{n} \text{Bi} \times [\text{EXPECTED LOSS RATIO POINTS IN i-th INTERVAL}]$$

WHERE BI = COMMISSION SLIDE ON I-th LOSS RATIO INTERVAL
(% INCREASE IN COMMISSION RATIO PER 1% DECLINE IN LOSS RATIO)
AND Cmax = MAXIMUM COMMISSION RATIO

CALCULATION OF THE EXPECTED SLIDING SCALE COMMISSION - CONTINUED

THE EXPECTED SLIDING SCALE COMMISSION RATIO EQUALS THE MAXIMUM COMMISSION RATIO LESS THE EXPECTED POINTS OF COMMISSION LOST OVER THE ENTIRE RANGE OF POSSIBLE LOSS RATIOS.

EXPECTED LOSS RATIO POINTS IN i-th INTERVAL

$$= \text{ELR} \times [\text{XSPPR}(\text{LBi}) - \text{XSPPR}(\text{UBi})]$$

WHERE XSPPR(LBi) AND XSPPR(UBi) ARE EXCESS PURE PREMIUM RATIOS CORRESPONDING TO THE LOWER AND UPPER ENDPOINTS OF i-th LOSS RATIO INTERVAL.

CALCULATION OF SLIDING SCALE COMMISSION - EXAMPLE

(1) EXPECTED LOSS AND ALAE RATIO = 75%

(2) MAXIMUM COMMISSION RATIO = 57.75%

	RISK MODEL	LOGNORMAL	TABLE M
(3) EXPECTED COMMISSION REDUCTIONS	50.85%	51.04%	51.14%
(4) EXPECTED COMMISSION RATIO	6.90%	6.71%	6.61%
(5) SIMPLISTIC SLIDING SCALE COMMISSION (PLUG ELR INTO FORMULA)	0%	0%	0%

RETROSPECTIVE RATING PLAN

FORMULA:

REINSURANCE RATE = (LOSS & ALAE RATIO) + (25% MARGIN)

RMIN = 70% < REINSURANCE RATE < 130% = RMAX

RETROSPECTIVE PREMIUM = (REINSURANCE RATE) x (PROVISIONAL PREMIUM)

CONSTRAINT ON LOSS AND ALAE RATIO (LR) USED IN RATE CALCULATION:

RMIN < LR + MARGIN < RMAX

CALCULATING THE LOSS RATIOS

CORRESPONDING TO MINIMUM AND MAXIMUM RATES

CORRESPONDING TO RMIN AND RMAX ARE

MINIMUM AND MAXIMUM LOSS RATIOS, LMIN AND LMAX.

LMIN = RMIN - MARGIN = 70% - 25% = 45%

LMAX = RMAX - MARGIN = 130% - 25% = 105%

INSURANCE CHARGES AND SAVINGS

IF LR $<$ LMIN, REINSURANCE COMPANY CHARGES FOR LMIN AND REALIZES SAVINGS DUE TO FAVORABLE LOSS EXPERIENCE.

IF LR $>$ LMAX, REINSURANCE COMPANY CHARGES FOR LMAX AND INCURS A LOSS DUE TO ADVERSE LOSS EXPERIENCE.

WE NEED TO DETERMINE EFFECT THAT LIMITING LR BETWEEN LMIN AND LMAX HAS ON THE EXPECTED LOSS RATIO USED IN THE RETROSPECTIVE RATING FORMULA.

CALCULATING THE NET INSURANCE CHARGE

NET INSURANCE CHARGE (NIC) = XSPPR(LMAX) - SAVE(LMIN)

WHERE XSPPR(LMAX) = INSURANCE CHARGE AT MAXIMUM LOSS RATIO

AND SAVE(LMIN) = INSURANCE SAVINGS AT MINIMUM LOSS RATIO

NOTE: SAVE(LMIN) = XSPPR(LMIN) + ER(LMIN) - 100%

WHERE ER(LMIN) = ENTRY RATIO AT MINIMUM LOSS RATIO

CALCULATING THE EXPECTED REINSURANCE RATE

LET AELR = ADJUSTED EXPECTED LOSS RATIO

AELR IS THE EXPECTED LOSS RATIO SUBJECT TO THE MINIMUM AND MAXIMUM LOSS RATIO CONSTRAINTS, LMIN AND LMAX.

$$AELR = ELR \times [100\% - NIC]$$

NIC IS THE LOSS ELIMINATION RATIO WHICH ARISES DUE TO LMIN AND LMAX.

EXPECTED REINSURANCE RATE = AELR + MARGIN

CALCULATING THE EXPECTED REINSURANCE RATE - EXAMPLE

(1) EXPECTED LOSS AND ALAE RATIO (ELR) = 75%

(2) REINSURER'S PROVISIONAL MARGIN = 25%

(3) MINIMUM LOSS RATIO (LMIN) = 45%

(4) ENTRY RATIO CORRESPONDING TO LMIN = .6

(5) MAXIMUM LOSS RATIO (LMAX) =105%

(6) ENTRY RATIO CORRESPONDING TO LMAX =1.4

CALCULATING THE EXPECTED REINSURANCE RATE - CONTINUED

	RISK MODEL	LOGNORMAL	TABLE M
(7) INSURANCE CHARGE AT LMAX	5.2%	5.4%	6.1%
(8) INSURANCE SAVINGS AT LMIN	1.9%	1.5%	1.3%
(9) NET INSURANCE CHARGE (NIC)	3.3%	3.9%	4.8%
(10) ADJUSTED EXPECTED LOSS RATIO	72.5%	72.1%	71.4%
AELR = ELR x [100% - NIC]			
(11) EXPECTED REINSURANCE RATE	97.5%	97.1%	96.4%
AELR + MARGIN			
(12) EXPECTED ULTIMATE MARGIN	22.5%	22.1%	21.4%

IMPORTANCE OF MODELLING PARAMETER UNCERTAINTY

EVEN IF DONE SUBJECTIVELY

THREE APPROACHES GAVE SIMILAR INDICATIONS

FOR ALL COINSURANCE AND ADJUSTABLE FEATURES ALTERNATIVES STUDIED

SIGNIFICANT PARAMETER RISK WAS REFLECTED IN COLLECTIVE RISK MODEL.

THIS WAS SIMILARLY REFLECTED IN LOGNORMAL MODEL THROUGH

SELECTION OF COEFFICIENT OF VARIATION.

IMPORTANCE OF MODELLING PARAMETER UNCERTAINTY - CONTINUED

ALTERNATIVELY, ONE COULD HAVE USED METHOD OF WEIGHTING SCENARIOS TO REFLECT PARAMETER UNCERTAINTY.

EMPIRICAL TABLE M APPROACH HAS THEORETICAL SHORTCOMINGS BUT PROVIDES REASONABILITY CHECK ON THEORETICAL METHODS.

ADDITIONAL ISSUES

USED SIMPLE EXAMPLE TO ILLUSTRATE CONCEPTS

REFER TO BEAR-NEMLICK PAPER
FOR DISCUSSION OF FOLLOWING COMPLEXITIES:

(1) VARIATION OF LAYER RETENTIONS AND LIMITS BY LINE OF BUSINESS OR OVER MULTI-YEAR RATING BLOCK.

(2) HANDLING OF ALAE.

(3) TREATIES WITH BOTH COINSURANCE PROVISIONS AND ADJUSTABLE FEATURES.

(4) TREATIES WITH SIGNIFICANT PROBABILITY OF LOSS-FREE YEAR (EG, HIGH LAYERS).

(5) CASH FLOW MODELLING.

THE PROBLEM

* Primary workers compensation quota share reinsurance cover ($250,000 limit)
* Allocated loss adjustment expenses included with losses
* Subject premium = $9,000,000
* Based upon other analysis, expected claim frequency is 85 claims per $1,000,000 subject premium, or 765 claims
* Based upon other analysis, unlimited severity distribution can be assumed to be Weibull with parameters 1/171 and .2

$$F(x) = 1 - \exp(-((x/171)^{.2}))$$

GOAL: Calculate the expected outcome to the reinsurer under several different structures involving adjustable features

ALTERNATIVE STRUCTURES

Non-Proportional Coinsurance Features:

(1) Aggregate deductible of $5,400,000

(2) 90% loss and ALAE ratio cap to reinsurer

(3) Loss corridor retained by ceding company between 75% and 112.5% loss and ALAE ratio

Retrospectively Adjustable Features:

(4) 50% profit commission after 25% reinsurer's expense allowance

(5) Contingent sliding scale commission, depending on loss & ALAE ratio:

Interval	Commission
>75%	0.0%
60%-75%	.5 x (75% - LR)
45%-60%	.6 x (60% - LR) + 7.50%
30%-45%	.75 x (45% - LR) + 16.50%
<30%	(30% - LR) + 27.75%

(6) Retrospective premium adjustment = LR + 25%
subject to min of 70% and max of 130%

KEY TO THE SOLUTION

Need to estimate the aggregate loss distribution to determine the effect of adjustable features on expected results.

It is insufficient to apply the adjustable features to the expected outcomes before adjustment, due to the effect the adjustments have on the distribution of outcomes.

SEVERAL APPROACHES

* Use an empirical aggregate loss distribution deemed to be appropriate (eg., NCCI Table M)

* Assume some form of the distribution of aggregate losses (eg., lognormal), then estimate the parameters from empirical data

* Collective Risk Model - estimate the aggregate loss distribution from the underlying claim frequency and severity distributions, using one following methods:

  - Assume some form of the distribution of aggregate losses, then estimate the moments from the moments of the frequency and severity distributions

  - Monte Carlo simulation

  - Other methods
    1. Inversion of the characteristic function of the aggregate loss distribution (Heckman-Myers)
    2. Inversion of the Laplace transform of the aggregate loss distribution (recursive method, Panjer)

MONTE CARLO SIMULATION OVERVIEW

Basic steps:

(1) Specify underlying claim frequency and severity distributions.

(2) Randomly generate a number of claims for a sample year from the assumed claim count distribution.

(3) For each claim drawn in step (2), randomly generate a claim size from the assumed claim size distribution, applying any per claim limit, if applicable.

(4) Accumulate each claim's results to get the year's total losses; use the accumulated results to determine effects of adjustable features for that year.

(5) Repeat the simulation for a large number of years, accumulating the results of each year to use in calculating overall expected effects of adjustable features.

CLAIM FREQUENCY DISTRIBUTION

Normal choices are:

- Poisson

  f(n|r) = r^n * exp(r)/n!

  where r is the expected number of claims
  mean = r, variance = r

- Negative Binomial

  f(n|m,k) = (k/(k+m))^k * (m/(k+m))^n * k(k+1)...(k+n-1)/n!

  mean = m, variance = m + m^2/k

CLAIM FREQUENCY DISTRIBUTION (Continued)

The Poisson distribution is usually thought of as a reasonable starting point for the claim process.

However, if we want to reflect parameter risk (ie., the fact that there is uncertainty in our estimate of the expected number claims), the Negative Binomial has been found to be a better model, with the parameter k used to reflect the level of parameter risk desired in the Poisson process.

While there are mathematical ways to estimate the correct k, we prefer a more intuitive approach:

Split the simulation runs into 5 equal parts. Vary the expected number of claims for each part in such a way that the average over the five parts is the desired expected number of claims. In our example, with a given expected number of claims of 765, we would reflect parameter uncertainty in the claim count distribution by using the following expected counts in each part:

765	x	0.50	=	382.50
765	x	0.75	=	573.75
765	x	1.00	=	765.00
765	x	1.25	=	956.25
765	x	1.50	=	1147.50

The spread used (.5,.75,1,1.25,1.5) is based upon a "comfort level" with repect to the underlying pricing analysis.

CLAIM FREQUENCY DISTRIBUTION (Continued)

A Negative Binomial Equivalent:

It is easy to show that the variance in the claim count distribution for all 5 parts combined is equal to the "between-group" variance plus the "within-group" variance, or:

Within-group variance	=	765.000
Between-group variance	=	73,153.125
Total variance	=	73,918.125
	=	765 + 765^2/k for a negative binomial equivalent

so that k = 8.

Alternatively, if we consider the variance of the spread (.5,.75,1,1.25,1.5), which is .125, we again have k = 1/.125 = 8

This leads to the more general statement that:

k = 1 / variance of spread

CLAIM SEVERITY DISTRIBUTION

Given: Uncapped severities can be expected to follow a Weibull distribution, with shape parameter of .2 and scale parameter of 171.

- After drawing severities from specified distribution, apply $250,000 per occurrence limit

- Parameter uncertainty ignored in the severity distribution

  * Variance of outcomes seems more sensitive to frequency
  * A little tougher to model in the severity distribution, given the curve we're using

BASE CASE

Assumptions:

1)	Subject premium	$9,000,000	
2)	Expected # of claims	765	
3)	Limit	$250,000	
4)	Expected average unlimited severity	$20,520	
4a)	Expected unlimited losses	$15,697,800	= 2 x 4
5)	Expected average limited severity	$8,796	
5a)	Expected limited losses	$6,728,940	= 2 x 5
6)	Expected loss ratio	74.77%	= 5a / 1

Simulation Results:

	With Parameter Risk	Without Parameter Risk
Iterations	10,000	10,000
Simulated average # of claims	764.8	764.7
Percent difference from expected	-0.03%	-0.03%
Simulated average unlimited severity	$20,468	$20,562
Percent difference from expected	-0.26%	0.21%
Simulated average limited severity	$8,807	$8,812
Percent difference from expected	0.12%	0.18%
Simulated average unlimited losses	$15,653,662	$15,724,844
Percent difference from expected	-0.28%	0.17%
Simulated average limited losses	$6,735,421	$6,738,819
Percent difference from expected	0.10%	0.15%
Variance-to-avg of simulated losses	994,664	157,117
Average loss ratio	74.84%	74.88%

OPTION 1 - INNER AGGREGATE DEDUCTIBLE

Assumptions:

Ceding company retains first $5,400,000 of reinsured losses

Reinsured losses = max(simulated losses - 5,400,000,0)

Simulation Results:

	With Parameter Risk	Without Parameter Risk
Average reinsured losses	$1,862,104	$1,379,146
Variance-to-avg of reinsured losses	2,116,999	667,518
Average losses eliminated by deductible	$4,873,316	$5,359,673
Loss elimination ratio	0.724	0.795
ELR to reinsurers (without credit)	20.7%	15.3%
ELR by subgroup of 2000 iterations:		
I	20.7%	15.6%
II	20.6%	15.4%
III	20.6%	15.3%
IV	21.1%	15.4%
V	20.4%	15.0%

OPTION 2 - LOSS RATIO CAP

Assumptions:

Ceding company retains all losses greater than 90% of subject premium

Reinsured losses = min(simulated losses, .9 x subject premium)

Simulation Results:

	With Parameter Risk	Without Parameter Risk
Average reinsured losses	$6,194,343	$6,687,541
Variance-to-avg of reinsured losses	584,632	129,934
Average losses eliminated by cap	$541,078	$51,278
Loss elimination ratio	0.080	0.008
ELR to reinsurers (without credit)	68.8%	74.3%
ELR by subgroup of 2000 iterations:		
I	68.5%	74.5%
II	68.8%	74.4%
III	68.8%	74.3%
IV	69.2%	74.4%
V	68.9%	74.1%

OPTION 3 - LOSS RATIO CORRIDOR

Assumptions:

Ceding company retains all losses between 75% and 112.5% of subject premium

Reinsured losses = min(simulated losses,.75 x subj prem) + max(simulated losses - 1.125 x subj prem,0)

Simulation Results:

	With Parameter Risk	Without Parameter Risk
Average reinsured losses	$5,743,502	$6,333,564
Variance-to-avg of reinsured losses	413,256	53,673
Average losses eliminated by corridor	$991,918	$405,255
Loss elimination ratio	0.147	0.060
ELR to reinsurers (without credit)	63.8%	70.4%
ELR by subgroup of 2000 iterations:		
I	63.6%	70.4%
II	63.8%	70.5%
III	63.8%	70.3%
IV	64.1%	70.5%
V	63.8%	70.2%

OPTION 4 - PROFIT COMMISSION

Assumptions:

Ceding company will be paid 50% profit commission after 25% expense allowance for reinsurer

Profit commission = max(.5 x subj prem x (1-(loss ratio + 25%)),0)

Simulation Results:

	With Parameter Risk	Without Parameter Risk
Average profit commission	$554,167	$208,331
as % of subject premium	6.16%	2.31%
Profit comm % by subgroup of 2000 iterations:		
I	6.32%	2.31%
II	6.16%	2.27%
III	6.15%	2.33%
IV	6.03%	2.27%
V	6.12%	2.40%

OPTION 5 - CONTINGENT CEDING COMMISSION

Assumptions:

Ceding company will be paid a contingent sliding scale ceding commission, depending on the loss ratio result.

Ceding commission calculated from the following table:

LR Interval	Commission
>75%	0.0%
60%-75%	.5 x (75% - LR)
45%-60%	.6 x (60% - LR) + 7.50%
30%-45%	.75 x (45% - LR) + 16.50%
<30%	(30% - LR) + 27.75%

Simulation Results:

	With Parameter Risk	Without Parameter Risk
Average contingent ceding commission	$634,598	$212,420
as % of subject premium	7.05%	2.36%

Ceding comm % by subgroup of 2000 iterations:

	With Parameter Risk	Without Parameter Risk
I	7.03%	2.44%
II	7.24%	2.36%
III	7.05%	2.31%
IV	7.05%	2.37%
V	6.88%	2.32%

OPTION 6 - RETROSPECTIVE RATING

Assumptions:

Ceding company's final premium will be determined retrospectively, based on ultimate losses under the coverage.

Retro adjustment = min(max(loss ratio + .25, .70), 1.30)

Simulation Results:

	With Parameter Risk	Without Parameter Risk
Average retro premium	$8,946,433	$8,987,088
Average retro adjustment	99.40%	99.86%

Retro adjustment by subgroup of 2000 iterations:

	With Parameter Risk	Without Parameter Risk
I	99.26%	99.58%
II	99.23%	100.09%
III	99.42%	99.93%
IV	99.36%	99.82%
V	99.75%	99.86%

OTHER CONSIDERATIONS

* Cash flow
* Risk load
* Expenses
* Market conditions

An aggregate loss model is a very important tool in analyzing adjustable features of treaties. The Lognormal and Simulation techniques presented by Bob and Jeff work well and usually provide sufficient information to make good pricing judgements. There are situations, however, where consideration of cash flow would change one's attitude towards comparable treaties. The following exhibits outline steps in helping to decide if cash flow is important.

The examples use reinsurance coverage where the cash flow will likely throw off enough investment income that it may determine the ultimate profitability or loss of the treaty. The main use of an aggregate distribution is to enable one to adjust expected loss estimates for contract terms. Based on these adjusted loss estimates, it is straightforward to compare the underwriting profitability of competing deals.

Graphing the cash flows of comparable deals may reveal whether the payment streams are different enough to compensate for expected loss differences. It is usually the case that the graphs of cash flows are sufficiently similar within a given group of terms (e.g., comparing one swing to another swing, or one profit commission plan to another profit commission plan) to make it clear that investment income differences won't affect a pricing decision.

In cases where the graph provides inconclusive evidence, two methods to reflect cash flow in an aggregate loss model may be used. The Panjer aggregate loss algorithm can be easily adjusted to reflect a payment pattern. By transforming the frequency parameter, one can get an aggregate loss distribution as of any given point in time. Investment income estimates for each year follow from each annual aggregate distribution. An alternative that is perhaps more intuitive is to reflect the payment pattern directly by simulating a payment lag for each loss as an extension to an aggregate loss simulation model.

In the end, considering cash flow seems to matter most when comparing different contract types and in measuring the value of contract terms compared with flat rating. It is not generally worth the effort when comparing similar contract types (except aggregate deductibles) or in calculating the credit for a high loss ratio cap.

Examples & Assumptions

Subject Premium:	$10 million
Expected Loss:	$1.5 million
Layer:	$500,000 xs $500,000
Severity:	Single Parameter Pareto, Q=1.5
Frequency:	Negative Binomial, V/E = 2.0
Interest:	Flat 8.0% a year

Auto example:

Long-Haul Trucking

Reporting Pattern is Exponential with 25-month average lag
Payment Pattern is Exponential with 35-month average lag

GL example:

Appliance Manufacturer

Reporting Pattern is Exponential with 45-month average lag
Payment Pattern is Exponential with 65-month average lag

Notation and Definitions of Random Variables

N – Number of Excess Loss

R_t – Reinsurance Premium net of brokerage at time t

P_t – Aggregate Paid Losses at time t

PC_t – Profit Commission at time t

C_t – Cumulative Cash Flow for the Reinsurance contract at time t

$$C_t = R_t - P_t - PC_t$$

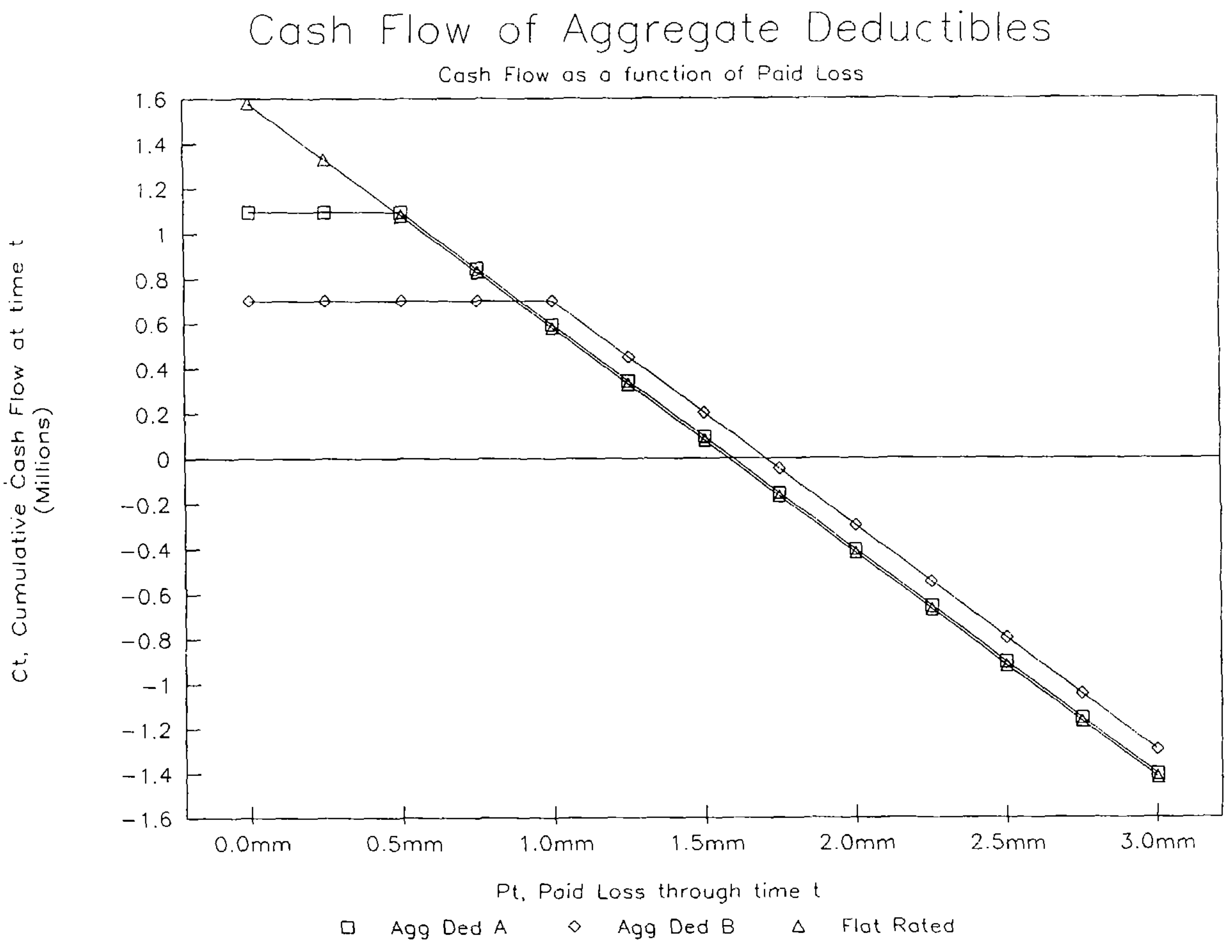
i – interest rate

V – Present Value of the net cash flow

$$V = \sum_{t=1}^{n} (C_t - C_{t-1})(1+i)^{1-t}$$

Rates Used with Graphs

Plan		Rate	Other
Flat		15.80%	
Agg Ded A		10.95%	Ded= 5.0%
Agg Ded B		7.03%	Ded= 10.0%
LR Cap A		14.74%	LR Cap= 26.0%
LR Cap B		15.62%	LR Cap= 39.0%
Prof Cmsn			
	A1	16.80%	before PC, 1st 3 yrs.
	A2	16.80%	with PC, yr. 4 & subs. (30% PC after 15% RI margin)
	B1	17.30%	before PC, 1st 3 yrs.
	B2	17.30%	with PC, yr. 4 & subs. (50% PC after 25% RI margin)
	C1	15.80%	before PC, 1st 3 yrs.
	C2	15.80%	with PC, yr. 4 & subs. (30% PC after 15% RI margin)
	D1	15.80%	before PC, 1st 3 yrs.
	D2	15.80%	with PC, yr. 4 & subs. (50% PC after 25% RI margin)
Swing A		7.5%min/21.0% max	Loss Load 100/75ths
Swing B		3.5%min/22.0% max	No Load
Swing C		7.5%min/22.0% max	Loss Load 100/80ths



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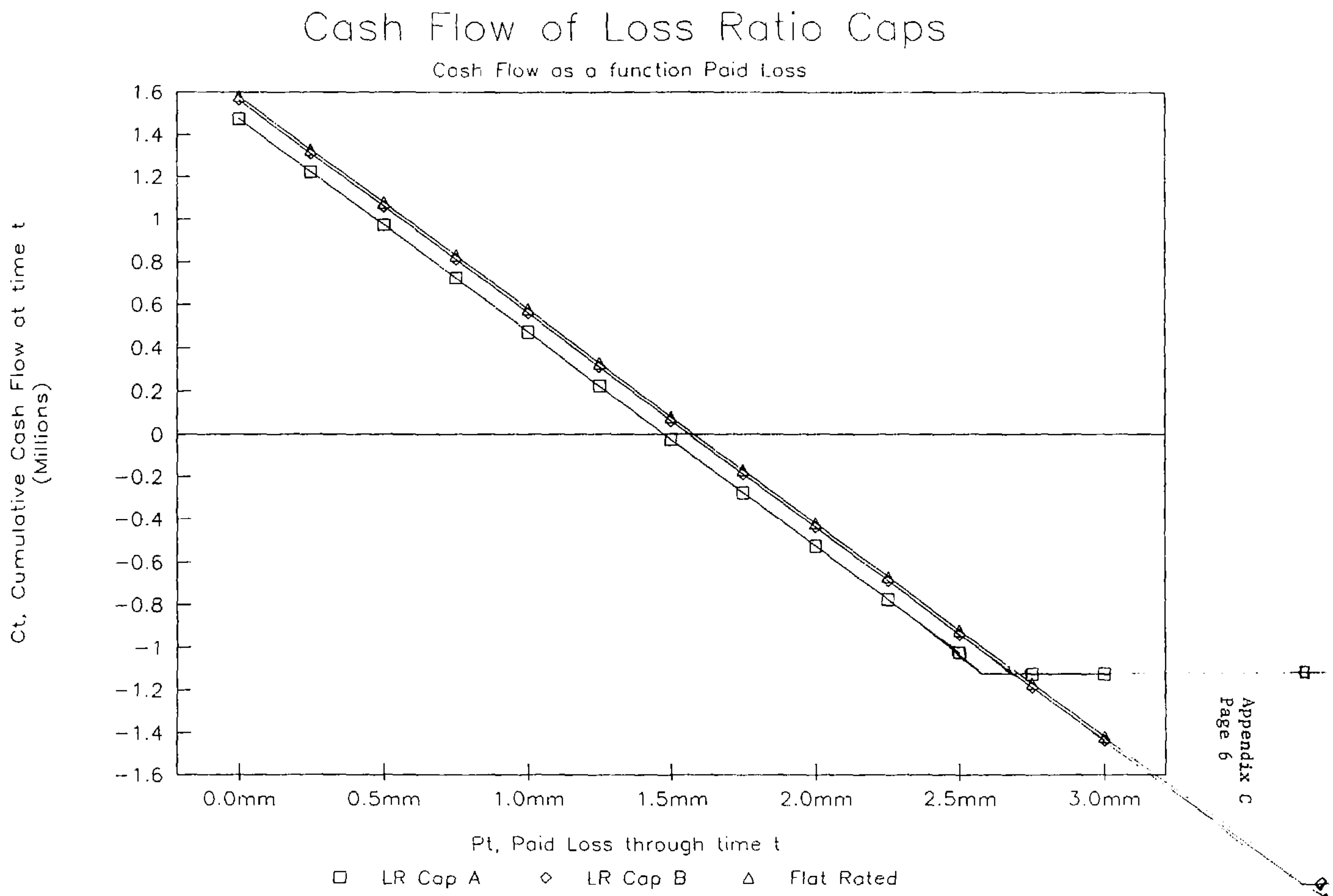
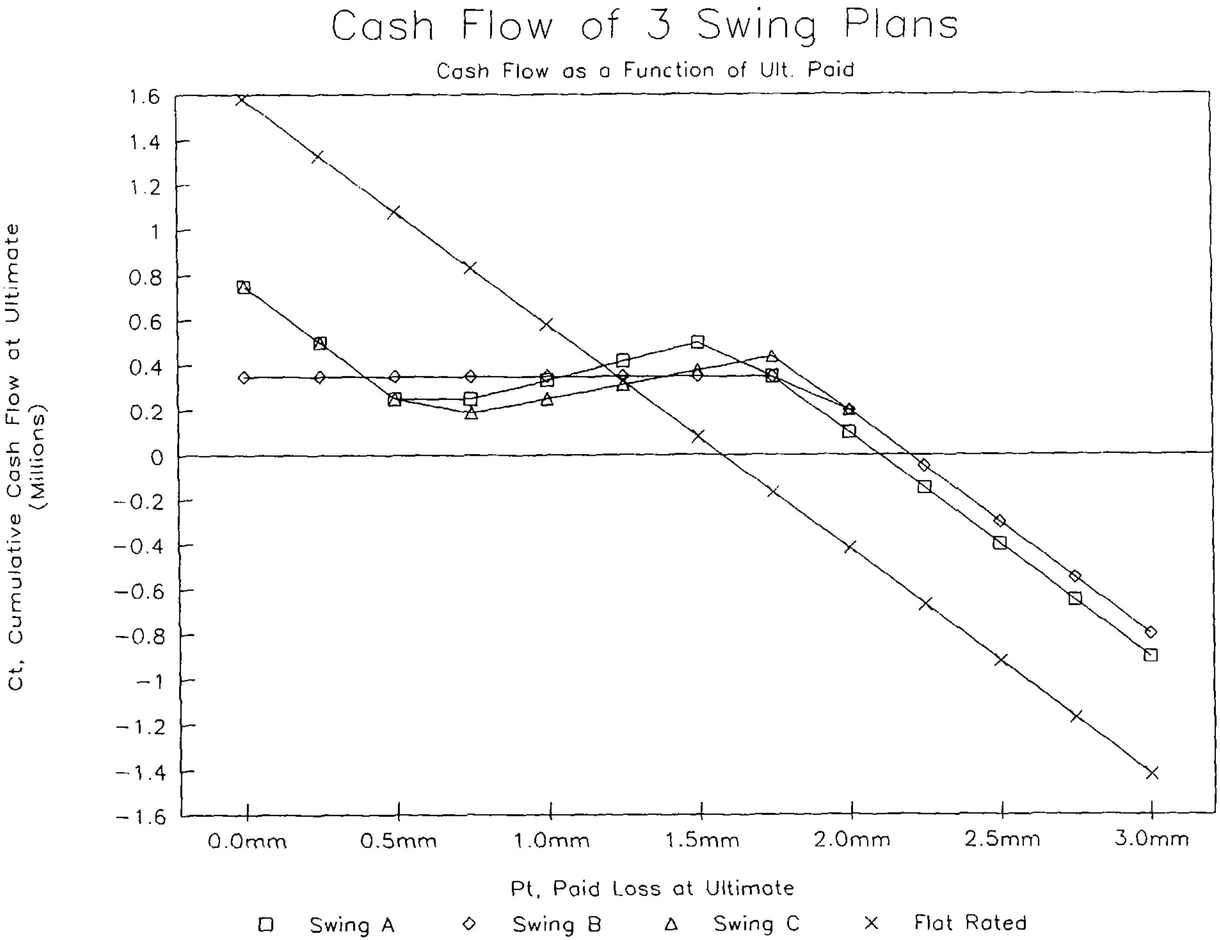
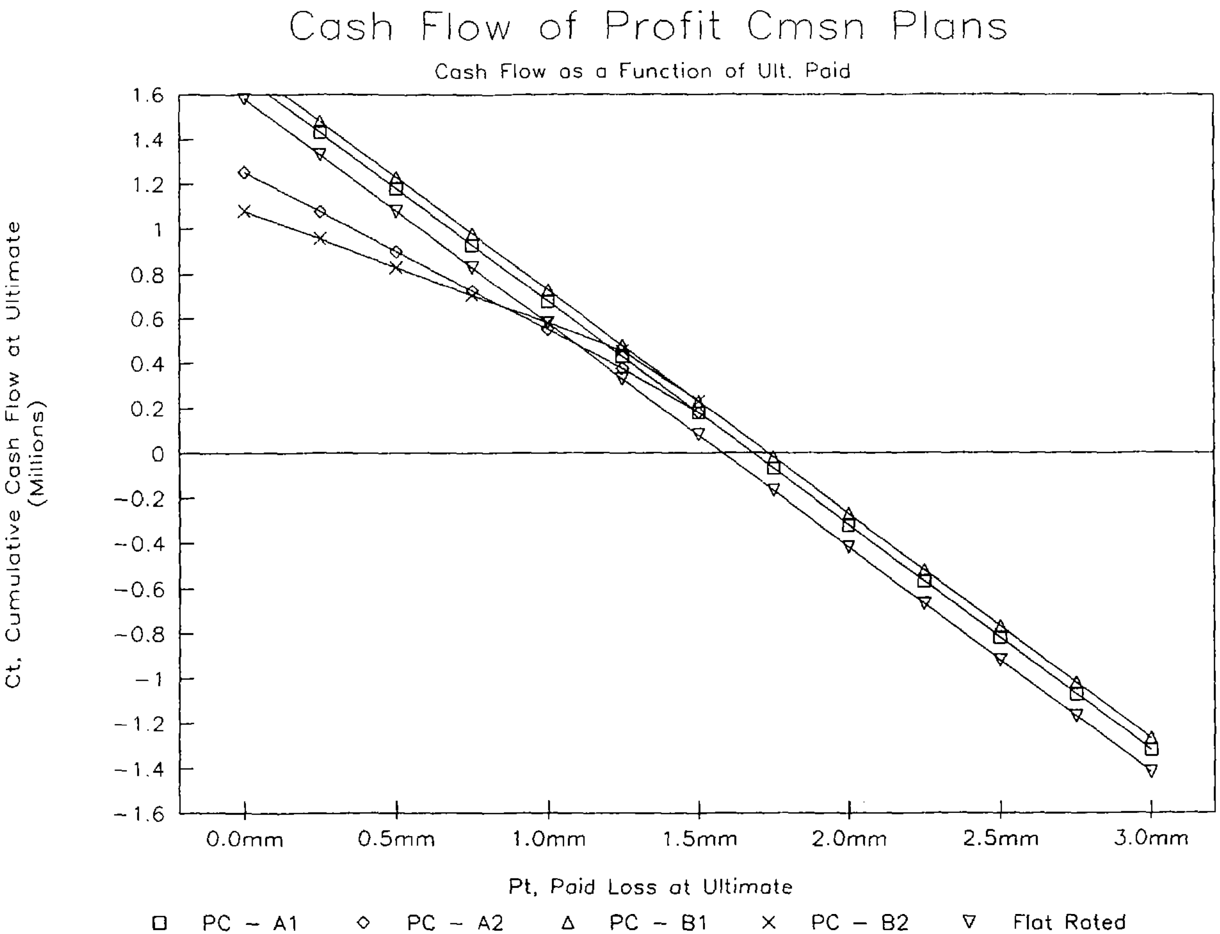

Cash Flow of Loss Ratio Caps
Cash Flow as a function Paid Loss
Ct, Cumulative Cash Flow at time t
(Millions)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
0.0mm
0.5mm
1.0mm
1.5mm
2.0mm
2.5mm
3.0mm
Pt, Paid Loss through time t
LR Cap A
LR Cap B
Flat Rated


Cash Flow of 3 Swing Plans
Cash Flow as a Function of Ult. Paid
Ct, Cumulative Cash Flow at Ultimate (Millions)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
0.0mm
0.5mm
1.0mm
1.5mm
2.0mm
2.5mm
3.0mm
Pt, Paid Loss at Ultimate
Swing A
Swing B
Swing C
Flat Rated



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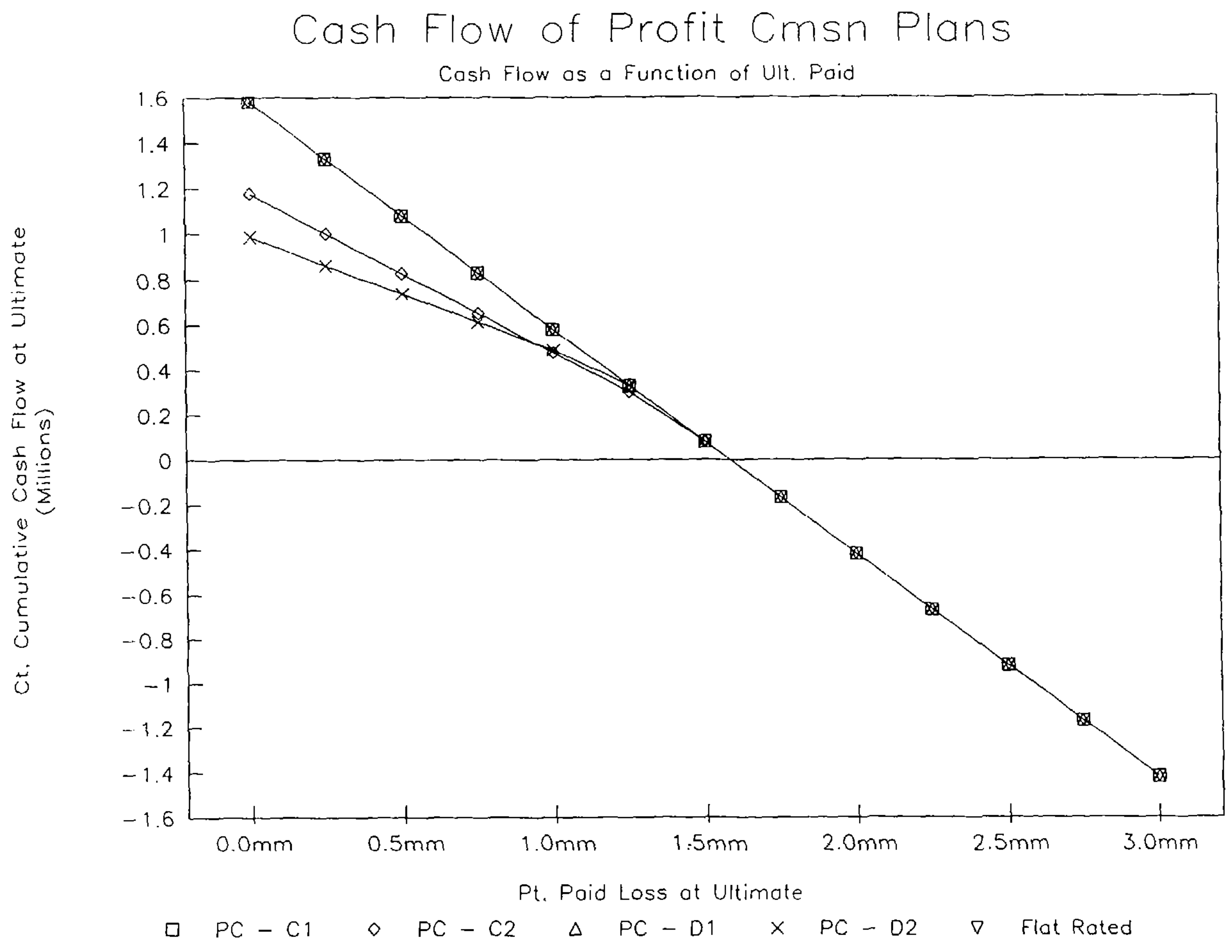

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Distribution of V

Aggregate Distribution for Excess Claims:

$$G(x) = \sum_{n=0}^{\infty} \text{Prob}[N=n]F(x)^{*n}$$

F(x) - Single Parameter Pareto

Prob[N=n] - Negative Binomial

Assumption: Individual claim reporting and payment patterns are independent of size of loss.

Observation: If M, the number of ground-up claims is Negative Binomial (α,p), then N, the number of claims excess of retention r, is also Negative Binomial with parameters (α',p') where

$$\alpha' = \alpha$$

$$\text{and} \quad p' = \frac{p}{F(t)+p(1-F(t))}$$

Distribution of V: Simulation

1. N is drawn from a negative binomial NB(α',p').

2. For each of the N claims, a paid loss amount is drawn from SPP and a payment lag is drawn from the exponential. It was assumed that claims occur mid-year and premium and loss transactions are made at mid-year.

3. The P_t values are calculated by summing total payments in the appropriate time periods using the simulated lags.

4. The reinsurance contract terms were applied to the P_t's to obtain the C_t's.

5. V is calculated $= \sum_{t=1}^{n} (C_t - C_{t-1})(1+i)^{1-t}$, then V is stored.

The above was repeated for 20,000 iterations, then E[V], Variance [V] and Probability [V>0] are calculated.

Distribution of V: Panjer's Method

Just as the number of Excess Claims is Negative Binomially distributed, so is the number of Excess Claims as of time t.

The transformation needed, is

$$\alpha_t' = \alpha'$$

$$p'_t = \frac{p'}{w(t) + p'(1-w(t))}$$

Where w(t) is the percent paid or reported as of time t.

One uses a discretized form of the severity distribution and the transformed Negative Binomial in Panjer's formula:

$$g_0 = p(0)$$

$$g_i = \sum_{j=1}^{i} (a+bj/i)f_j\, g_{i-j} \qquad i=1,2,3,\ldots$$

Using the aggregate distribution, the C_t's can be computed easily.

Aggregate Deductible

Deductible	Rate	ELR	AL NPV	GL NPV
0	15.8	95	353	517
5	10.95	95	290	426
10	7.03	95	211	310

Loss Ratio Cap

Cap	Rate	ELR	AL NPV	GL NPV
Infinite	15.8	95	353	517
250 %	15.62	95	345	507
175 %	14.74	95	317	462

Swing Rated

Swing Rate	ELR	Loss Load	AL NPV	GL NPV
15.8 Flat	95	none	353	517
7.5/12/21	95	100/75	231	320
7.5/12/22	95	100/80	222	309
3.5/12/22	95	100 +Min	212	284

Profit Commission

Profit Commission	Years No Down	Rate	ELR	Eff ILR	AL NPV	GL NPV
0		15.8	95	95	353	517
50 after 25	4	17.3	87	95	380	528
30 after 15	4	16.8	89	95	363	516
30 after 15	4	15.8	95	101	273	428
50 after 25	4	15.8	95	103	251	402

This presentation was based on:

EVALUATING THE EFFECT OF REINSURANCE CONTRACT TERMS
by James N. Stanard and Russell T. John

soon to be published in PCAS. The following references are cited in that paper:

[1] H. Buhlman and W. S. Jewell, "Optimal Risk Exchanges," *ASTIN Bulletin*, Vol. 10, Part 3, 1979.

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[8] H. H. Panjer, "Recursive Evaluation of a Family of Compound Distributions," *ASTIN Bulletin*, Vol. 12, 1981, p. 22.

[9] S. W. Philbrick, "A Practical Guide to the Single Parameter Pareto Distribution," *PCAS LXXII,* 1985, p. 44.

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[11] B. Stundt and W. S. Jewell, "Further Results on Recursive Evaluation of Compound Distributions," *ASTIN Bulletin* 12.

[12] E. W. Weissner, "Estimation of the Distribution of Report Lags by the Method of Maximum Likelihood," *PCAS LXV,* 1978, p. 1.