CASUALTY ACTUARIAL SOCIETY FORUM

Spring, 1989 Edition

CASUALTY ACTUARIAL SOCIETY
ORGANIZED 1914

75 YEARS

ALTY ACTUARIAL SOCIETY

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ident-Development

May 1, 1989

To: CAS Members

From: Charles A. Bryan

Re: Fourth Issue of CAS Forum

Dear CAS Members:

This fourth issue of the CAS Forum represents the fruition of a concept that we had when we started the Forum in 1987. At that time we hoped the availability of the Forum would stimulate many of our members to submit informal papers that could be quickly distributed to the members. We have succeeded!

This CAS Forum is almost entirely devoted to new papers. I congratulate the authors for their willingness to share their broad knowledge and experience with their fellow actuaries.

The broad range of topics reflects the expanding practice of casualty actuaries. No doubt, several of these papers will eventually appear in the Proceedings. Authors who would like to submit articles for the next Forum should submit these articles to me for the Fall 1989 issue by September 1st.

We have also included the 1959 Presidential address. From time to time the CAS Forum will publish items of historical interest. The 1959 Presidential Address is particularly interesting in view of Proposition 103 and the pressures expected in the personal lines of insurance during 1989 and 1990.

Yours truly,

Charles A. Bryan

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TABLE OF CONTENTS

1959 Presidential Address

VOL. XLVI, Part II No. 86

PROCEEDINGS

November 19-20, 1959

ST. VITUS'S DANCE

PRESIDENTIAL ADDRESS BY DUDLEY M. PRUITT

"After all, those in the (insurance) business who do other than routine work, are paid about half for what they do and half for what they endure."

Kenneth O. Force in The National Underwriter
October 2, 1959

On a dismal morning of the year 1374 in the German town of Aachen hundreds of men and women came together on the streets and, forming circles, hand in hand, danced hysterically for hours on hours till they fell exhausted and insensible on the cobblestones to be left where they fell or pushed aside to make way for others who were still bouncing and jiggling like puppets on a string. It was thought that these poor people were possessed of demons and prayers were said by the holy for their healing. This is the picture given of one incident in the dancing mania which afflicted Germany during the years following the Black Death.

The years of the Black Death had taken their frightful toll, wiping out half the population of Europe and then, at the very moment of release, when the grip of the plague was at last relaxing, the hysteria of the dancing mania took hold. It was born out of physical and spiritual exhaustion, out of an emotional desperation that had benumbed the wits and depraved the reason. It was "catching," as any mass hysteria is "catching," propagated by the sight of the sufferers, like a demoniacal epidemic. Here was the origin of the name St. Vitus's Dance, for St. Vitus it was who had been granted specific powers for the healing of the mania.

We are, I suspect, witnessing in the automobile insurance business a parallel to the dancing mania of Germany in the Middle Ages. For several long years our business has had the insurance equivalent of the plague; many of us have been suffering severe underwriting losses in the private passenger lines; we have appealed to higher authority for rate relief over and over again and been rebuffed; and just as the picture begins to look clearer, just as the rate situation seems to be brightening, we begin whirling and jiggling, hopping and prancing in a maniacal, competitive dance of policy forms and rating methods.

The picture, of course, was not really becoming clearer with the increased rates approved and the reduced commissions being allowed during the past year. One might almost imagine that these necessary corrective measures have aggravated the situation. The underlying disease, whatever it has been, is not cured. Quite recently the Bureau received considerable criticism to the effect that a major reason for the success of the direct writers in getting the preferred business is because the Bureau rates favored the youthful driver at the expense of Class 1 business. The Bureau vehemently denied such favoritism and implied that a company charging Bureau rates should be just as glad to write insurance on a car owned by a youth as on a car owned by a man of mature years; should be, in fact, happier to do so because, with the larger premium involved, there would be more dollars in the profit allowance. The trouble has been that, although the Bureau was undoubtedly right actuarially, the carriers have continued to show an irrational resistance to youthful drivers and have shown a keen preference for Class 1 business despite its smaller average premium. The direct writers have, in fact, been willing to make those premiums still smaller.

Every year a prominent analyst makes a careful study of the underwriting results of the big four direct writers: Allstate, the State Farm, the Nationwide, and the Farmers Exchange. His annual conclusion is that so long as these four carriers can continue to pay their producers less they can charge less for their wares. This gives them a competitive advantage and makes it possible for them to be very selective, resulting, of course, in lower loss ratios, which allows them to charge still less for their wares, and so on round and round. And to rub salt into the wounds, the salesmen, who get paid less in percentage, earn a good living on volume.

Some years ago a method of operation was devised within the American Agency System which was calculated to save the agent and his carrier before the big four got all the business. This is frequently referred to as the Safeco plan in recognition of the originating carrier. The agents have not generally relished this form of salvation, preferring, if possible, to live in original sin. The plan attempts to meet the big four competition by adapting their methods to independent agency operation. It involves a signed application, giving improved control over selection and classification assignment, and such money saving devices as a lower-than-normal commission rate and the requirement that the premium be paid in advance of effective date. In order to make the commission reduction more palatable the plan also includes automatic machine renewal and direct collection of the renewal premium by the carrier. One of the advertised inducements has been that the agent could take his smaller commissions and devote his energies to new production confident that the company machinery would keep the renewal certificates endlessly flowing to the assured with the cash flowing back, and the direct writers vanquished. It really has worked.

The lower rates produced the same sort of competitive leverage for the Safeco plan companies as it has for the direct writers. In the last two or three years there has been a great burgeoning of "economy plans" competing vigorously for the best of this low premium business that has theoretically less profit built into it than the big fat youthful driver business. There has been, however, no observable indication that the big four felt the slightest jolt from the growth of the Safeco plan. The two chief sufferers have been the so-called tariff or Bureau companies and the assigned risk plans, the former out of dearth of business and the latter out of surfeit.

Shakespeare once wrote, "The smallest worm will turn, being trodden on," and in this case the victim of the treading was hardly a small worm. Every indication today points to the certainty that the Bureau companies have had enough. They are doing something about it besides cutting commissions.

The new merit rating plan jointly sponsored by the National Bureau and the N.A.U.A. has now been introduced into several states. It is an attempt to meet what a Bureau spokesman described as "a public demand of long standing for a safe driver insurance plan which will produce a substantial difference in the price paid by insureds who are not accident prone vs. those who are. . . . The plan is designed to produce more competitive rates for the better classes of risks so that bureau companies will not be faced with an ever worsening cross-section of business."

With this move on the part of the bureaus the Black Death was ended and the dancing mania began. Every day has brought its new manifestations. When the Travelers withdrew from the National Bureau the insurance world was as shocked as the average American would be if ex-President Truman were to withdraw from the Democratic Party. Several other outstanding company groups also withdrew in order to be free to try out their own individual steps. The independents, with their various "economy" plans, found overnight their happiness gone, their complacency shattered, and were seized with an acute realization that new ideas were needed fast. The automobile insurance industry is in a competitive struggle of titans. I see no reason why Senators Kefauver and O'Mahoney need fear for the freedom of the automobile insurance enterprise at the moment. For the past several months the insurance page of the Journal of Commerce has carried daily stories of new plans and projects. I quote a few headlines:

"NBCU FIRMS TO PUSH NEW AUTO PLANS"

"TRAVELERS LAUNCHES NEW AUTO PLAN IN NEBRASKA"

"AMERICAN CASUALTY HAS NEW AUTO COVER PROGRAM"

"ST. PAUL VETOES NBCU AUTO PLAN"

"MICH. STUDIES MERIT RATING"

"MARYLAND AGENTS REQUEST ADOPTION OF MERIT RATING"

"LOSSES SEEN AS MAIN COST DIFFERENTIAL"

"ALLSTATE SAYS 'REVOLUTION' BENEFITS INSURING PUBLIC"

And about the same day that three major plans were announced on one page, there was also displayed a large want ad by one of America's larger fire and casualty fleets, appealing persuasively for an actuary of mature experience and judgment capable of assuming the duties of vice president.

I wish I had space here to deal also with that other dementia that has seized our industry, the rate and coverage evolution taking place in the homeowners' business. Where are those wise men who promoted the multiple line approach on the theory that when the casualty lines went bad the property lines would save us by being good, and vice versa? Suffice it to say that many of the points discussed here in connection with automobile apply equally to homeowners'.

There has been considerable favorable comment in the local press as the merit plans have been introduced in various states. It is a very popular concept that "good drivers" should not have to share in the losses caused by the "poor drivers who have the accidents." The lower rate for the better record seems reasonable and just to most people. A fairly representative reaction as expressed by an insurance commissioner was to the effect that merit rating supplies what our young people have wanted for a long time—to be treated as individuals and not as a group of helter-skelter irresponsible undesirables. In fact there has been so much demand for the new plan that the Bureau has been constrained to ask for time. The plan is frankly experimental and needs maturing.

There are also those who take a very dim view of all automobile merit rating plans. At one time the Bureau was not nearly so sanguine about the practice as it seems to be today. In Best's Insurance News of January, 1952, a paper entitled "Merit Auto Rating" appeared, sponsored by the National Bureau of Casualty Underwriters and the Mutual Insurance Rating Bureau in collaboration. This paper cites twenty "administrative and rating difficulties to be encountered" in any program of merit rating for private passenger automobiles in 1952 and closes with the profoundly actuarial statement that "the extremely small exposure in a single private passenger car risk does not lend itself to self-analysis in terms of rate making as the element of chance overshadows a credibility expectancy." Perhaps it is unfortunate that the paper has this year been republished as a part of the *Readings in Property and Casualty Insurance,* edited by H. Wayne Snider, for it would seem that most of the twenty difficulties to be encountered in 1952 are still difficulties in 1959 with a few more added by the processes of time and the specific characteristics of the current plans.

A serious current criticism of the new plans is that they are far too difficult to administer, depending as they do on information that must be obtained from state motor-vehicle departments and that this difficulty brings the plans into direct conflict with the rapidly developing mechanization of automobile risk rating and policy issuance. This may turn out to be a decisive factor.

Another and widely held criticism attacks the plan at its actuarial foundations. This stands on the principle that insurance is a pooling of potential losses, that there must, of course, be some separation of risks into reasonable rating classifications, which are usually interpreted to mean the present classifications by use, and that any attempt to separate those who have accidents from those who do not breaks down the pooling principle and thus does violence to the "mathematical science of insurance." This has been a rather common charge and one that has the surface look of truth. It has, however, serious actuarial blemishes.

While insurance is certainly based on the principles of pooling, it has none of the elements of charity and very little in common with Marxism. The maxim, "From each according to his abilities, to each according to his needs," is from Karl Marx and not from Lloyds of London. The mathematical science underlying the insurance business, it has always seemed to me, is the science of finding mathematical measures of hazard, of determining the bounds of reasonable probability of an occurrence, including, to the extent practicable, a quantitative differentiation of such circumstances as who, where, and when. You have all heard the story of the horse-and-rabbit stew—"one rabbit, one horse." Such a stew has none of the elements of mathematical averaging. Surely, if you are certain to have no loss and I am certain to have a loss I can hardly expect you to pool your insurance with me on an equal, or in fact on any, basis. The impossibility of effecting a workable private flood insurance program is a clear illustration of this principle. And certainly if it can be demonstrated that you are less apt to have a loss than I, you would be the giver of pure charity and I would be the taker if we pooled our hazards on a fifty-fifty basis. As much as is possible the predisposition to loss is a proper subject for fair discrimination; only the operations of chance are the proper subject for averaging.

We are apt to cry discrimination rather readily in the insurance business, saying quite properly that it is unfair to discriminate between risks of essentially the same hazard. But over two thousand years ago Plato pointed out the other side of discrimination very clearly when he said that the greater injustice is to treat unequal causes equally. A single automobile liability rate for the entire state of Massachusetts, as has been at times politically proposed, would be unfairly discriminatory in the extreme. There is no need to labor this point further. I think we will all agree, when we look at the subject objectively, that insurance rates should be related as nearly as is feasible to the hazard of the risk, and that a proliferation of classifications which actually do measure hazard, though it may complicate the business of insurance, does no more violence to its mathematical principles than does the betting on different odds do violence to the mathematics of gambling. In fact the pari mutuel is the very essence of mathematics.

Classifications can, however, complicate the business, and I suspect that most of the noncompetitive complaint within the industry about the merit plan is based on this problem of administration. There is always the compromise between the proper and the feasible. One of my friends, an eminent psychologist, has assured me that he could quite certainly discover the accident-prone and rather accurately measure the degree of proneness if he were permitted to examine personally all my company's applicants for automobile insurance. I believe he is essentially correct; but it is not out of perversity that the insurance carriers have failed to replace their underwriters and actuaries with psychologists; there would seem to be no feasible way to bring together car drivers and psychologists. This does, however, suggest the possibility of the carriers' employing a reasonable number of these learned men to help devise some less exact and possibly less drastic method of discovering the accident-prone. Perhaps with every application for insurance we should also demand a signed interpretation of one of Rorschach's ink blots.

What the Industry so desperately seeks is a simply-manipulated device for determining and mathematically evaluating the risk of accident inherent in a motor vehicle owner or operator. That this has not yet been found I believe even the promoters of the various merit rate plans will agree. That it ever can be found is extremely doubtful. It seems to me clear that in this area, as in so many others, simplicity and accuracy are mutually antagonistic. To the degree that we require a mathematical and clearly defined accuracy we must perforce sacrifice simplicity and ease of operation. There is no harder task than to make the intangible tangible.

Here I believe is the crux of the problem. Our statutes say, quite properly, that rates shall not be excessive, inadequate or unfairly discriminatory. In the abstract these three principles are ideal; in their specific administration, however, we find them far too broad and indeterminate. It is a bit like legislating that men shall not be niggardly, over-generous, or unfairly prejudiced. How can anyone know truly that a given rate for a given risk is neither excessive, nor inadequate, nor unfairly discriminatory? We are justly proud that ours is a government of laws and not of men, but an excess of zeal for legal safeguards beyond the needs of the circumstance can destroy the effectiveness of such natural safeguards as judgment and self-discipline.

Because we are so firmly committed to the regulation of rates rather than the supervision of their administration, we find ourselves taking an unrealistic and essentially Procrustean approach to rating philosophy. We imagine that all risks can be fitted into a limited number of specific classifications, subject to exact definition, and that by the mere fact of fitting risks to a definition which describes their tangible attributes we can make them homogeneous. This is at times in direct conflict with the clear evidence of experience and judgment. I recommend to all insurance men a rereading of that immortal classic "Pigs is Pigs," by Ellis Parker Butler. There is something profoundly prophetic about Mr. Flannery's position:

> "Pigs is pigs. Guinea-pigs or dago pigs or Irish pigs is all the same to the Interurban Express Company an' to Mike Flannery. Th' nationality of the pig creates no differentiality in the rate."

Of course this has worked badly. Procrustes found that all men did not, after all, fit his standard-sized bed, and was forced to resort to stretching some and lopping others. I have a theory, which I shall call the Procrustean Law of Classification Stability, that classifications tend to produce their expected experience; in other words, the experience of any class accommodates itself to the pure premium for that class. While this may be partly due to the effect of underwriting selection as it adjusts to the adequacy of the rate, there also seems to be a tendency in our business, which will probably be honestly denied by all concerned, to let the risk's inherent hazard, arrived at intangibly, determine the assigned classification. Since the results are relatively reasonable and uniform, I must conclude that this reprehensible practice does in fact produce a less unfair discrimination as to risks of the same hazard than could prevail by a careful adherence to the definitions.

Our classifications are broad bands of hazard; each one with a wide spectrum of good and bad risks. They overlap to the point where the best of the worst classification produces a lower loss cost than the worst of the best classification. Under our present rating concept the only discrimination allowed a carrier between the best and the worst within the same rate group, or even between the better and the worse, is by selection. If accepted they must be charged the same rate. Under discrimination by selection the risk, which because of the intangibles should fall into a worse classification than is indicated by the tangibles, has difficulty obtaining any insurance at all and will finally have to pay a higher rate either from a non-preferred risk carrier or through the assigned risk plan. That this selection is valid is vouched for by the experience of the so-called "clean" risks in the assigned risk plans, which has been found to be as bad as or even at times worse than the surcharged business at the same rate level. In the main it is pure underwriting selection on the basis of intangibles that places a risk without accident or conviction record in the assigned risk plan. (Incidentally this clean assigned risk experience could cast some doubt on the complete validity of the various merit plans currently competing in the market place.)

I can see no fundamental reason why discrimination by selection should be considered socially preferable to discrimination by rate. Both can be fair and both can be unfair. Discrimination by rate has the one advantage that it keeps the market open, and, in general because of competition, causes each risk to pay a premium fairly commensurate with its hazard. Discrimination by selection, besides being a thoroughly annoying practice to the public, is the basic cause of the assigned risk program, that great Procrustean leveler where all risks are treated in a most unfairly nondiscriminating manner.

Because our present regulatory system has grown up gradually and because we have all breathed this atmosphere from our beginnings in the business, we accept it as appropriate and inevitable. Our friends in Great Britain, however, have grown up in a somewhat more liberal insurance rating atmosphere. Apparently they place more trust than we do in competition and sane judgment. The following rather amusing letter was published in the Manchester (England) Guardian Weekly for July 30, 1959:

> "Your article on car insurance contains one statement which calls for correction. Your correspondent says that insurance companies accept possession of a valid driving license as the only qualification necessary for the granting of 3rd party insurance.
>
> "As an actor I have found that this is not so. Although I have had a clear driving license for 5 years I have on several occasions found that my proposal for 3rd party insurance has been refused outright because of my occupation. I finally obtained 3rd party only, passenger liability excluded, and the premium 'loaded.' Comprehensive, I was told, was out of the question except for a fantastic premium.
>
> "Even worse, in my opinion, is the state of affairs described to me by an actor who owns a self-drive car hire firm. In the first place insurance cover for his business was difficult enough to get because he was connected with the 'entertainment industry,' but further, he was required not to hire his cars to, among others, actors, publicans, jockeys, pilots, ice cream vendors, and log merchants!
>
> "And most absurd of all are the car dealers who are keen to sell you their cars and who are also insurance agents. They find it necessary to suggest as I have had done to me, that if I describe myself as, perhaps, an 'interpreter' (dramatically, I suppose) or 'Commercial Artist' all will be well. No doubt it would be until the first court case, when such falsification might leave the customer uninsured and criminally guilty and the agent untouched.
>
> "Surely, if the law requires us to have 3rd party insurance it should be available to all on equal terms with our fellow-motorists."
>
> "Yours &c
> "Paul Whitsun-Jones
> "12 Flask Walk
> "London NW 3"

In this country, though his premium would not be arbitrarily "loaded", Mr. Whitsun-Jones would be in the assigned risk plan and even more unhappy. I quote this letter merely to show that automobile insurance can be operated on a different plan from ours. It is my impression that, in spite of Mr. Whitsun-Jones's dissatisfaction, there is more justice and less turmoil in British insurance than in ours.

I believe the insuring public would be better served, the premiums charged would be more equitable (by which I mean more nearly commensurate with the inherent hazard involved), and there would be a much more open and healthy competitive atmsophere in the private passenger automobile insurance market if carriers were permitted the exercise of some judgment in individual risk rate determination within the framework of over-all state supervision of rating administration.

There remains still the fear that unregulated rates in the face of keen competition will be inadequate rates from the point of view of company solvency, thus endangering the very security of our system. Under today's operating procedures, however, the safety of a carrier is irretrievably given over to the judgment of its underwriting organization through the authority to accept and reject. A company can sink into insolvency with tragic speed through bad risk selection even with every rate charged strictly according to manual. Why should we expect our staffs, which we trust to exercise adequate restraint in risk selection, to cast that restraint to the winds if given some limited discretion in rate assignment?

Some will accuse me at this point of selling my actuarial profession down the river. I plead "not guilty." It has always seemed to me that when the law is too pervasive the atmosphere breeds shysterism. The present regulatory climate makes actuarial shysterism a distinct, though, I hope, as yet an unrealized, possibility. When the rating laws or their administration in any state is unrealistic or pettifogging the temptation is very strong for the actuary to forget his professional obligation which is to seek the best estimate of a future rate and instead, to become the protagonist who uses his skill to argue his client's cause regardless of merit. In the three-cornered contest produced by current conditions, with the carriers, the agents, and the insurance departments all employing actuaries to interpret and promote their parochial points of view, the temptation has at times become well-nigh irresistible.

There would be adequate place for the actuary in a freer rating climate. The freedom I suggest does exist in the life insurance business where the actuary seems to do very well, and, although the actuarial problems in life insurance differ materially from those in fire and casualty insurance, there is a common need in both fields for rational analysis and the tempering of what is competitively wished for by what has a reasonable hope for success. The actuary can and does supply technical skill and logical perspective to the solution of problems involving insurance rating and risk evaluation. These attributes grow in usefulness as the carriers gain in freedom.

But let us never make the basic mistake of considering the actuary a brake on competition. The current automobile situation clearly demonstrates that competition is alive in our business and that the actuary should be in the thick of it. With his analytical training, his interest in discovering relationships, and his familiarity with the substantive data of the business, he is uniquely placed for the exercise of creative imagination. He should be the source of new ideas and of new approaches to old ones. Such talents are much in demand in a free competitive system and the freer the system the greater should be the demand. In the current automobile dancing mania the Bureau actuaries have come in for a great deal of criticism from both the fearful and the offended. Some day in the future we shall all know just how good or bad this latest creation of theirs has turned out to be. Certainly I am no prophet. But one thing I know: their action has been in the best actuarial tradition; it has been logically developed, honestly presented, and saturated with the competitive spirit. I salute them for it.

New Papers Submitted
To The Forum

EXPOSURE BASES REVISITED:

The author would like to thank Marshall Auck, Scott Bradley, Lisa Chanzit, Jenni Ermisch, Mike Levin, Jim Morrow, Debbie Moyer, Deborah Rennie, Bill Safreed, and Susan Woerner, all of whom read drafts of the paper. Special thanks to Rich Hofmann, who shredded the second draft and greatly improved the final product.

EXPOSURE BASES REVISITED

By Amy Bouska

ABSTRACT

The paper has many purposes. They are: (1) to review the definition and selection of an exposure base and to clarify the distinction between the exposure base and variables which are used in classification; (2) to review the exposure bases currently in use for manually rated risks, and to note how the manual exposure base becomes less important as the risk size increases; (3) to highlight problems in the determination of an exposure base (including temporal mismatch, interpretive mismatch, and complexity of hazard); and (4) to discuss both the current controversy regarding the use of payroll as the exposure base for workers' compensation and the recent change in the exposure bases for general liability.

INTRODUCTION

The business of insurance presumes an exposure to loss: if there is no possibility of a loss, there is no need for insurance. However, if an entity does have an exposure to loss, it is desirable that the cost of transferring that loss to another party be proportional to the expected loss, which is assumed to vary with the size of the exposure base. Thus, the selection of an exposure base, which quantifies and proxies for the exposure, is a fundamental step in the insurance process.

The following discussion is limited to the property and casualty lines of insurance in the United States and is not intended to address the life, pension, or accident and health lines or foreign business; nor is it intended to be an exhaustive survey of all exposure bases or rating plans used by individual companies.

DEFINITION

The classic definitions of exposure and premium bases were supplied by Paul Dorweiler in his 1929 paper "Notes on Exposures and Premium Bases."[1] In that paper, he wrote that "when critical conditions and injurable objects exist in such relationship that accidents may result there is said to be exposure"[2] and "...premium funds are accumulated from charges called the rate collected per unit exposure. The exposure medium selected as the basis for the charge of the premium is known as the premium basis."[3]

He notes that the premium basis cannot be selected arbitrarily: "Obviously, the premiums collected are to be proportional to the

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[1]Dorweiler, P., "Notes on Exposures and Premium Bases," PCAS XVI, 1929, p. 319; reprinted: PCAS LVIII, 1971, p. 59.

[2]Dorweiler, p. 59.

[3]Dorweiler, p. 60.

hazard which is measured by the losses. .... The medium most desirable as a premium basis is the one possessing a combination of these two qualifications in the largest degree: 1. Magnitude of the Medium should vary with hazard. ... 2. The Medium should be practical and preferably already in use."[4]

Although the premium basis is somewhat less accurately referred to as the exposure base today, the definition and requirements are as correct and pertinent now as they were sixty years ago.

In their text Insurance Company Operations,[5] Webb et al. expanded on Dorweiler's requirement of "practicality" by stating that "A good exposure base should have three characteristics. First and foremost, of course, it should be an accurate measure of the exposure to loss. Second, it should be easy for the insurer to determine. Finally, it should be difficult for the insured to manipulate."[6] Adding one more level of cynicism (or realism, as the case may be), we should also require that the exposure base be immune to manipulation by underwriters.

Underlying all of these definitions are two themes: the relatively simple and reliable development of correct premiums

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[4]Dorweiler, p. 61.

[5]Webb, B.L., Launie, J.J., Rokes, W.P., Baglini, N.A., Insurance Company Operations, Volume II, American Institute for Property and Liability Underwriters, 1978.

[6]Webb et al., p. 25

for the insurers (i.e., it should accurately reflect the overall exposure to loss, be simple to compile, and not be subject to manipulation) and equitable distribution of those premiums among the insureds (i.e., it should accurately reflect differences in exposure to loss). It is not surprising that some historically appropriate exposure bases are showing signs of failing to satisfy these two conditions. The bases may have functioned well--or at least without controversy--in a world where the risks were relatively well understood, the insured commercial population was regulated, the economic and social structures were stable, and the insurers used bureau rates. Changes in these external conditions and internal weaknesses in the underlying insurance structure are causing exposure base problems.

SELECTION OF AN EXPOSURE BASE

Before considering the impact of the changing environment, however, it is important to pause and consider the process involved in selecting an exposure base for a line of insurance.

The first step is to analyze the coverage offered and the coverage trigger to determine what factors influence the expected losses. Some of these factors will not be usable in the determination of premiums (see the Comments later in this section). Those which are usable will be divided into two groups: the first group, consisting of one factor, will be the exposure base, and the second group will be the rating variables, which influence the projected expected losses indirectly by affecting the rate.

This division is based on the simple theoretical equation:

A. (number of exposure units) x (loss cost per exposure unit) = expected losses.

This is derived from the equation we define to be true:

A1. f(exposure) = expected losses.

As will be discussed later, the true exposure is complex and changing, so we must simplify by selecting a proxy for the true exposure. This is the exposure base. The theoretical model is then quantified to become:

B. (number of exposure base units) x (loss cost per exposure base unit) = expected losses.

Once the exposure base has been selected, projection of the loss cost per exposure base unit (usually by projection of frequency and severity) is the core of the ratemaking process. The loss cost generally varies with different combinations of the other factors. These combinations are known as the rating variables or class plan, and they may affect the loss cost through either the frequency or the severity or both. Equation B can also be written as:

B1. (number of exposure base units) x (expected number of losses per exposure base unit) x (expected dollars per loss) = expected losses, or

B2. (number of exposure base units) x (frequency) x (severity) = expected losses.

The final step in the manual ratemaking process is the inclusion of expenses, which leads to the equation:

C. (number of premium base units) x (rate per premium base unit) = manual premium.

In practice, the exposure base unit in equation B and the premium base unit in equation C are always the same and the terms are used interchangeably.

Thus, expected losses (and premium) do not vary only with the exposure base, but also with many other factors which are built into the rating variables. Any factor which affects the losses but has not been quantified in either the exposure base or the class plan will allow the company which recognizes it in underwriting to "skim the cream" of the business. In this way, simple classification plans provide the opportunity for sophisticated companies to make profits by accepting only the better risks within a class.

In general, the factor selected as the exposure base should have a uniform multiplicative relationship with all of the expected loss costs and rates, i.e., within any rating class, the same rate will be used for one unit or fifty units (as opposed to requiring a higher or lower rate with increasing volume). Thus, a policy covering two physicians practicing the same specialty in the same territory will use the same rate but multiply it by two, producing twice the premium.[7]

It is also desirable that the factor selected as the exposure base be simple and have an obvious relationship to losses. In addition to making the plan easier to use, simplicity is likely to enhance its perceived equity, even if the technical accuracy is not improved.

It is important to make note of two things which exposure bases are <u>not</u>. First, the exposure base is not the true exposure. The exposure base is a proxy for the true exposure, which we are unable to know, both because it is constantly changing and because it is generally a function of a large number of

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[7]This simple multiplicative relationship is occasionally modified later in the calculation of the premium, either to reflect some exposure effect or to recognize the decrease in unit expenses associated with larger policies. Examples include (1) the multi-car discount in private passenger auto, which reflects the reduced usage and improved loss experience on policies covering multiple cars, and (2) premium discount plans in workers' compensation and other commercial lines, which reflect the decreased percentage of the premium required to cover fixed expenses for large premium policies.

variables. For example, the collision exposure of a private passenger auto is effectively zero when it is parked in a secure garage, somewhat higher when it is being driven on an isolated highway by an alert and competent driver, and substantially higher on a crowded street with a drunk driver. The exposure base (car-month) recognizes the average situation rather than these fluctuations in the true exposure to loss. As is noted later, there are even situations where the exposure base is zero but a significant exposure still exists. The best way to keep this distinction clearly in mind is to think of the exposure base as the "units" designator (square footage, payroll, etc.) of a blank to be filled in on the premium calculation worksheet.

Second, the exposure base is not a rating variable, although the dividing line between the two is somewhat arbitrary at times. In order to determine the correct manual premium for a risk, it is first necessary to classify the risk based on whatever the rating variables are for the risk under consideration. Once the risk's classification is known, the rate for that classification is multiplied by the number of exposure units to produce the premium. As is noted above, the use of a variable in the exposure base implies a uniform and continuous multiplicative relationship between the variable and the expected losses; use as a rating element implies a discrete, nonlinear relationship. For example, physican-month is an exposure base, and coverage for two physician-months costs twice as much as the coverage for one physician-month. On the other hand, age is a rating variable, and coverage for Driver A who is twice as old as Driver B does not (usually) cost twice as much.

Comments

It is important to remember that, for most lines of business, the exposure to loss varies with a substantial number of factors. Some of these cannot be used in determining the premium because they are either indeterminate, too subjective, or fluctuate too rapidly. An example of such a factor would be the mood of an automobile driver--while it could be argued that a person who is angry (either momentarily or on average) is more likely to have an accident, this is not used in any rating scheme.

Some factors may have a demonstrable or assumed correlation with losses but may be socially unacceptable as a rating variable or exposure base. Foremost among these are race and religion; age and gender are still used in many private passenger automobile rating plans but are being attacked (and defended) on social equity grounds.

Other factors which are observable but not quantifiable are allowed to influence commercial lines rates through the individual risk rating plans. Schedule rating plans for commercial general liability, for example, allow modification of the rate based on upkeep of the premises and management attitude.

The variables which are left--those which are socially acceptable, quantifiable, and demonstrably related to the level of losses--may be used directly in determining the premium. The one with the most uniform relationship to the losses will be the exposure base. The others can be used in the classification plan.

A nonexhaustive list of the factors affecting the final premium for some of the major lines of business includes:

Property: construction, occupancy, location (territory), external hazards (technically called "exposure" but not in the same sense as the topic of this paper), internal protection (sprinklers, smoke alarms), external protection (local fire department and police), amount of insurance;

Automobile liability: driver's age, gender, marital status, driving record, and school record; business or pleasure use; mileage or distance to work; radius of operation; location (territory of principal garaging); truck weight; insurance limit; number of vehicles; claims experience (safe driving credit (personal) or experience modification (commercial));

Automobile physical damage: car make, model and year for private passenger auto or vehicle age and original cost new for commercial autos; number of vehicles; territory; deductible; claims experience;

Workers' compensation: location (territory), occupation, claims experience (experience modification), payroll; and

General liability: classification; territory; insurance limit; type of coverage (claims-made or occurrence); claims experience; square footage or acreage, payroll or receipts; new/discontinued businesses.

Some of these factors--notably territory--are proxies for more basic influences on the level of losses, such as cost of medical care, traffic density and tendency to litigate.

As these lists make clear, many factors affect the expected losses (and, therefore, the premium) in any given line or subline of insurance, but only one becomes the exposure base.

A SUMMARY OF THE MAJOR LINES OF INSURANCE AND THEIR EXPOSURE BASES

Property Coverages (Annual Statement Lines 1, 2, 12 & 25)

Glass coverage is rated on the square footage; all other coverages are based on the limit of insurance in hundreds of dollars, which is assumed to be related to the value of the property insured.

Homeowners and Farmowners Multiperil (Annual Statement Lines 3 & 4)

The property and crime sections of these policies generally use the insured value (in hundreds or thousands of dollars) as an exposure base. The liability section has an implicit exposure base of one household.

Ocean and Inland Marine (Annual Statement Lines 8 & 9)

These lines are essentially property coverages and are generally based on the insured value in whole dollars. However, there are numerous exceptions, since "inland marine" covers a multitude of sins.

Aircraft -- All Perils (Annual Statement Line 22)

Aircraft hull coverage is rated on the insured value (in thousands of dollars); liability is based on revenue-passenger miles (or kilometers).

Burglary and Theft (Crime) (Annual Statement Line 26)

The crime coverages are rated on the insured value in thousands of dollars.

Boiler and Machinery (Annual Statement Line 27)

Boiler and machinery coverage uses the number of objects as its exposure base.

Credit (Annual Statement Line 28)

Credit coverage is based on the dollars of indebtedness.

Fidelity and Surety (Annual Statement Lines 23 & 24)

Fidelity coverages are rated on the number of persons; surety, on the amount of coverage (contract cost) in thousands of dollars.

Automobile (Annual Statement Lines 19 & 21)

All private passenger and commercial liability, no-fault, and physical damage coverage is based on the number of car-months.

Workers' Compensation (Annual Statement Line 16)

There has been a great deal of discussion about the exposure base for workers' compensation, but it remains payroll (limited payroll for officers and sole proprietors and partners) in every state except Washington.

Medical Malpractice (Annual Statement Line 11)

Hospitals and other health care facilities are rated on occupied beds and outpatient visits; premiums for health care providers (physicians & surgeons, dentists, optometrists, etc.) are based on provider-months.

General Liability (Annual Statement Line 17)

The exposures bases for the various general liability sublines and classes used to range from mundane (square footage) to mercenary (payroll) to morbid (number of bodies). Since the introduction of the Insurance Services Office (ISO) Simplification Program in 1986, most classes are now rated on either gross sales or payroll, although apartment exposures use the number of units, and rates for offices and lessors are based on area. There are numerous other exceptions, such as the use of number of tanks for underground tank pollution liability rating.

Reinsurance (Annual Statement Line 30)

Facultative reinsurance has as many different exposure bases as does primary insurance; treaty reinsurance is generally rated as a percentage of the underlying premium.

LARGE RISKS

Large risks are an exception to almost all of the above because they are frequently subject to either composite or loss rating plans which modify the usual exposure bases.

Under a composite rating plan, the risk's premium is calculated normally and then divided by a proxy exposure base, such as mileage or receipts for long-haul trucking firms. This gives a rate per proxy unit. When the policy expires, the firm's records are audited in order to determine the actual receipts (or mileage), and this is used to calculate the final premium.

The intention is to simplify the rating for insureds with hundreds of vehicles in their auto fleets or many insured locations. The proxy base should have at least some reasonable relationship to the expected losses, but it does not usually reflect the detail of the underlying exposure bases and classification systems.

If a large risk is loss-rated, the premium is calculated directly from its historical losses without any reference to the standard rating plans. In this case it is correct to say that the exposure base is the risk itself and the rate is its expected losses. If, in addition, a composite rating procedure is used in order to reflect changes during the year, then a proxy base is introduced.

Recall equation C:

C. (number of premium base units) x (rate per premium base unit) = manual premium

In this equation, the rate is a classification or manual rate (the subject of Part 6). Such a manual premium is used directly only for small risks. The premium for a medium-sized risk is frequently modified by schedule rating and expense modifiers, which reflect characteristics of the individual risk, and experience modifications and dividends, both of which give some recognition to the risk's own experience. This changes equation C to give:

C-medium: (number of premium base units) x (rate per premium base unit) x (schedule modifiers) x (experience modifiers) = manual premium x modifiers = charged premium.

If the risk is composite-rated, this equation is continued to:

C-comp: "charged" premium = (number of expected proxy units) x (rate per proxy unit.)

At the final audit, the actual number of proxy units is determined and multiplied by the rate derived above to give the final premium.

As the size of the risk increases, more and more weight is put on the individual risk, diminishing the importance of the manual rate. In the case of a very large, loss-rated risk, the normal underlying exposure base and class plan disappear, leaving:

C-large: expected losses + expense load = charged premium.

THE CHANGING ENVIRONMENT

There is a pervasive feeling that accurately forecasting losses in some lines of insurance has become impossible. The problem is frequently attributed to the degradation of the tort system, an increase in litigiousness, and the search for "deep pockets." These have clearly made it very difficult to accurately estimate the future frequency and severity of losses. However, in some cases, it may be more correct to say that we have not been able to identify an exposure base which successfully reflects these and other changes.

As we will see, many of the problems of mismatch between exposure bases and the underlying exposures for which they are proxies arise from the exchange of a steady-state universe for one subject to abrupt changes. Determining the expected losses is easy when all factors are constant; the demands become somewhat greater but are still generally manageable if constant change, such as a constant rate of growth, is introduced into the system (see, for example, Steve Philbrick's paper "Implications of Sales as an Exposure Base for Products

Liability"[8]). In recent years, these changes include emerging theories of liability, economic inflation, social inflation, changing insurance requirements and preferences, new products and services, increased tendencies towards acquisitions and divestitures, deregulation of industries such as trucking, technological advances, and the emergence of long-tail exposures. When severe discontinuities appear, the underlying correspondence between the expected losses and the exposure base can be disrupted beyond correction. The following is a discussion of three types of problems in the selection of the exposure base: temporal mismatch, interpretive mismatch, and complexity of hazard.

These problems should not be confused with the ever-present ratemaking problem of future shock. A failure to accurately predict the frequency and/or severity of future losses is usually a problem with our crystal balls (or other ratemaking tools), not the sign of a failing exposure base. For example, medical malpractice occurrence rates have been historically inadequate in spite of having a coverage trigger which is rarely a matter of dispute.

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[8]Philbrick, S., "Implications of Sales as an Exposure Base for Product Liability," PCAS LXVII, 1980, p. 181.

Problem: Temporal Mismatch

As the tail of liability losses lengthens and coverage triggers are changed in order to ease pricing and reserving problems, the possibility of a temporal mismatch between expected losses and an otherwise acceptable exposure base arises. The two outstanding examples of this are claims-made policies and products liability.

Claims-made policies are triggered by the notice of a claim but rated on the normal occurrence exposure base, a physician-month in medical malpractice, for example. If the practice of medicine for a year causes a number of claims, some of them will generally be filed after the policy expires, giving rise to a loss under an occurrence policy but not under a claims-made policy. No other candidate for the exposure base of a claims-made policy has been identified and the problem has been solved by the incorporation of a rating step to recognize the number of years since the retroactive date (i.e., the year in claims-made). The calculation of this modification is thoroughly discussed in "Rating Claims-Made Insurance Policies" by J.O. Marker and F.J. Mohl.[9]

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[9]Marker, J.O. and Mohl, F.J., "Rating Claims-Made Insurance Policies," Pricing Property and Casualty Insurance Products, CAS Discussion Paper Program, 1980, p. 265.

Careful evaluation of the trigger is necessary when making the adjustment, since, for example, the new CGL claims-made form is triggered when notification has been received and recorded by any insured or by the insurer. This may be a relatively long time before a formal claim is filed with the insurance company.

Products liability coverage is triggered by the injury but the exposure base is sales (with the exception of the few classes where products coverage is included with the premises and operations coverage). If the trigger were based on the date of manufacture or if the product has a short lifespan, it appears that sales would be a reasonable exposure base (ignoring for a moment ratemaking problems arising from the long tail, social inflation, etc.). However, triggering coverage on the date of injury gives rise to a mismatch. The problem is most easily illustrated by the case of a manufacturer who has gone out of business and therefore has no sales but whose products are still being used and producing injuries. The situation is frequently encountered in the case of the acquisition of a company with a discontinued product line which is still in use or the evaluation of a conglomerate which has actively acquired and disposed of subsidiaries over the years.

One possible solution to this mismatch would be to change the exposure base to "products in use during the year." Unfortunately, while more precise in its reflection of the exposure, this is not an easily available figure and it therefore fails the second test of a potential exposure base, namely that it be easily available and not subject to manipulation.

A more acceptable answer has been proposed by Steve Philbrick in his paper "Implications of Sales as an Exposure Base for Products Liability."[10] In this article, he also develops the adjustment methodology which could be used as an input to schedule rating to correct for the mismatch.

In general, the temporal mismatch problem can be solved, although the solution is likely to be inexact.

Problems: Interpretive Mismatch

The exposure base selected must be compatible with policy language which is sufficiently precise so that mismatch does not arise through deliberate or accidental misinterpretation of the coverage trigger. For example, a pollution policy meant to cover losses arising out of disposal activities starting after policy inception could be rated on tons of waste produced (or disposed of, if there is a lag between production and disposal). This is a reasonable prospective exposure base, but

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[10]Philbrick, p. 181.

the policy language must be precise and enforceable or there is a possibility that courts will find coverage for losses from past disposal activities, for which a different exposure base would be necessary.

Without commenting on the appropriateness of the asbestos coverage theories used to date and ignoring the fact that products liability is already subject to temporal mismatch, the fact that it is possible for injury to one person to trigger many policies indicates that interpretive mismatch is also a problem for the affected products policies. Even if these policies had been rated on "products in use during the year," coverage would not have been expected from the policies triggered after the asbestos work stopped (the "injury in residence" and "manifestation" triggers).

Problems: Complexity of Hazard

In some cases, the problems are much more basic than those mentioned previously. The difficulty frequently lies in the first step of determining the exposure base, i.e., making a complete list of all factors affecting the level of losses. What, for instance, would be contained in such a list for directors and officers (D&O) insurance? Obvious candidates include:

- the number of directors and officers
- business activities

- (change in) revenues
- (change in) profits
- (change in) assets
- number of stockholders
- number of employees
- hiring/firing policies
- (change in) overall financial condition as rated by S&P
- (change in) stock price
- attractiveness as an acquisition
- responses to past acquisition offers (e.g., "poison pills")
- state of domicile
- response to any recent emergencies (accidents, etc.)
- recent changes in management
- ....??

All of these are believed to have some bearing on the likelihood or size of D&O claims, which have been known to arise from abrupt changes in a company's stock price, resistance on the part of the directors to being acquired, and wrongful termination of employees. But is the list complete? Probably not. Even if it is, the numerical relationship of the factors to the loss level is unclear even for the most obvious candidate for the exposure base: does a company with twice as many directors have twice the exposure to loss? Probably not.

It could be argued that the general reluctance of the industry to offer this coverage is an outgrowth of our inability to determine a meaningful exposure base for it. It is to be hoped that when (if?) we are able to correlate the losses with some other measurable factor, the "D&O crisis" will abate.

THE INTERNAL ENVIRONMENT

In fairness to the world at large, it must be admitted that not all problems with exposure bases arise outside of the insurance industry. Two serious problems are based on insurance company practices themselves: (1) exposure estimates can be (and are) manipulated in response to the competitive situation, and (2) even when the policy premium is based on the correct exposures, the coding of the exposure information into the computer records is often poor, with whole dollars frequently switched with "per hundreds" or "per thousands."

Mechanical rating and direct production of the statistical records from the policy rating files will solve the second problem, but control of the first is likely to be more elusive. Most companies track their average premium per policy rather than the average premium per exposure unit so that good exposure data is not considered necessary. In addition, competitive pressures tend to degrade the exposure data. In a very competitive (soft) insurance market, a low price can be produced in a variety of ways, a number of which are legitimate but frequently require documentation, such as the aggressive use of schedule rating. In some instances, it is easier for the underwriter to "low-ball" the exposure estimate. In theory, such "errors" will be corrected when the policy is audited, but that is usually eighteen months in the future (and after the renewal). Under the calendar/accident year ratemaking used for many lines, audit premiums are reported and fully earned in the calendar year of the audit, not the calendar year(s) when the policy premium was earned. Thus, even in the case of perfectly correct audits, a severe mismatch between the premiums and losses can be introduced by low exposure estimates. In a steady state, the rates eventually respond to a systematic underestimation of the exposures, but when the insurance cycle changes quickly and the "low-balling" stops abruptly, the problem of excessive rates appears.

Thus, some of the practical mismatch between exposures and exposure bases can be attributed to the pricing practices of the industry as a whole rather than a more esoteric theoretical failure.

CHANGING EXPOSURE BASES:
CAUSES AND CONTROVERSY

Once established, the exposure base for a line of insurance tends to acquire an aura of sanctity. It is very difficult and very expensive to change the exposure base for a widely written line: difficult, because the historical data uses the old base, but the new rates must refer to the new base; and expensive, because data on both bases must be collected for at least one year prior to the change or all insureds must be contacted to determine their new exposure and then all policies must be rerated and reissued.

So why change? In theory, change could be caused by a better understanding of the nature of the exposure. In practice, this does not seem to be the case, either because a line does not become widely written until the exposure is reasonably well understood, or because the marginal gain is less than the cost, or because inertia is stronger than the profit motive. Thus, the two recent exposure base controversies have been forced on the industry by changes in the world which is being insured. One of these--in workers' compensation--was caused by increasing discontent among insureds over inequities in the rating mechanism; the other--in general liability--was the result of both the industry's difficulty in keeping rates current and the increasing automation of commercial lines.

It should be noted that the frequent discussions regarding the use of driving record in place of age, gender and/or marital status in determining private passenger auto premiums concern only the rating plan, not the exposure base. To date, there has been very little discussion of the use of car-months, although

Andrew Tobias in his book The Invisible Bankers[11] suggested a plan based on fuel consumption. However, as the workers' compensation changes illustrate, the line between the exposure base and the rating plan is very fine, and a discussion which begins on one side of that line may well finish on the other.

Workers' Compensation (WC): Hours-worked vs. Payroll

The problem is simple: consider two construction firms, one of which is unionized and one of which is not. Assume they have the same number of employees, do the same type of work, and have the same expected number and type of losses. If the unionized company pays more per hour, it will have a higher payroll and, therefore, pay more for its workers' compensation coverage. To the extent that its indemnity losses (based on lost wages) are higher, this premium difference is correct; however, to the extent that the losses arise from medical payments or are capped by the maximum benefits payable under state law, the difference is not justified in terms of expected losses. Obviously, there is no problem if the work is sufficiently different that separate classifications are used.

For many years, limited payroll--reflecting the limited

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[11]Tobias, Andrew, The Invisible Bankers, Washington Square Press, 1982, in the section "Pay-As-You-Drive: How God Would Restructure Automobile Insurance" (p. 230).

benefits--was the exposure base for WC in all states other than Washington which used and still uses work-hours. In the early 1980s, the payroll limitation was removed. This change obviously made the problem worse.

In 1984-85, the perceived inequity became a matter of national debate between the National Council on Compensation Insurance (NCCI) on the one hand and insureds (both labor and management) on the other. It was caused not only by union/nonunion differentials, but also by the varying wage scales which appeared as a result of deregulation in many industries. Based on these differences, the insureds proposed both hours-worked and mixed hours-worked/payroll as exposure bases, while the NCCI preferred to retain unlimited payroll because it was easy to verify and it reduces the size of the annual rate revisions needed. Regulators were concerned that, whatever program resulted, it should be fair and encourage workplace safety.

Because wage level and unionization status are not recorded in the standard WC data, insurance records at NCCI and insurance companies could not resolve the question. Therefore, the state of Oregon did a special "Study of Premium Equity by Employer Groups." Obviously, the issue was not important to very large employers whose experience is fully credible, so the study addressed primarily the small (nonexperience-rated) and medium (experience-rated but not fully credible) employers.

NCCI's analysis of the Oregon data found no bias against either union or high wage paying employers among the small employers, but it did show that high wage paying and union employers in the medium-sized group developed lower loss costs per premium dollar (11% and 12% less, respectively). This result appears somewhat counter-intuitive, since one would expect _a priori_ that the availability of experience rating would reduce the bias.

Among others, the Florida Labor/Management Council proposed a mixed rating base, using both payroll (for wage-related benefits) and worker-hours (for medical-related benefits).

Payroll won out in the exposure base arena, but concessions were made on the classification side: in California, each of six construction classes were split into two new classifications (high and low wage rates); in Florida, a table of credits based on wage rates was implemented for all contracting classes; in Oregon, the legislature authorized the collection of worker-hour data by the NCCI and the Oregon workers compensation division; and the NCCI-proposed Loss Ratio Adjustment Program (LRAP) was put into place in Oregon, Illinois, Maryland and Nebraska, although the approved version differed by state.

LRAP is a modification to the WC experience rating plan for the specific construction classifications shown to have problems. Its effect is to make the experience rating plan more responsive to the individual employer's three-year loss ratio. NCCI favored this response because it was problem-specific (i.e., did not affect other classifications), did not require an overall rate change, and encouraged workplace safety.

Thus, what began as an exposure base question was addressed by changes to various other parts (classification and experience modification) of the rating system.

General Liability: Area vs. Receipts

Virtually all of the public attention to the ISO's Commercial Lines Policy and Rating Simplification Project was focused on the expansion of the claims-made coverage form to all sublines of general liability (GL) and, to a lesser extent, the changes to the pollution coverage. However, this program, which became effective in 1986 and 1987, also encompassed a massive revision of the exposure bases for GL in addition to substantial revisions to the forms, classification plans, and coverages of nearly all ISO lines (i.e., WC and surety were not affected because ISO is not the primary bureau for these lines; although it is an ISO line, professional liability was not revised).

In terms of the impact on insureds and insurers, the changes to the forms and exposure bases were much more important than the expansion of the claims-made form. This was partly true because the softening market in 1986 and 1987 meant that insurers and reinsurers were more willing to write occurrence coverage, so that the usage of claims-made was much more restricted than was originally thought. However, even if the hard market had continued, many insureds--and, in all likelihood, many smaller insurance companies--would have continued on occurrence policies, but no one escaped the other changes.

Each of the three major GL industry groups was brought to a single exposure base for all of their sublines and coverages.

Thus:

		Prior	
Group	Current	Prem/Ops	Prod/Comp Ops
Mercantile	Gross Sales	Area	Receipts
Manufacturing	Gross Sales	Payroll	Receipts
Contracting	Payroll	Payroll	Receipts

Some exceptions to the above remain. The most major of these are apartments, which were rated on area but changed to units, and office buildings, which were and are based on area.

The short diagram above conceals the true extent of the simplification. In order to calculate the premium for a small contractor before simplification, for example, the underwriter needed to know (1) the payroll ... for the M&C coverage; (2) receipts ... for the products/completed ops coverage; (3) total contract cost ... for the contractual liability; (4) the building's fire rate ... for fire damage legal liability; (5) the M&C property damage rate ... for broad form property damage coverage; and the M&C bodily injury rate ... for personal and advertising injury. Under the new structure, all of these coverages are based on payroll.

These changes were implemented for a variety of reasons, including (1) simplification of rating, both manual and mechanized, (2) sensitivity to inflation, and (3) sensitivity to economic cycles. It is, of course, very desirable to have an exposure base which incorporates inflation, fully or partially, since this reduces the need for frequent and relatively large rate filings.

The changeover was not easy for many reasons. Among the most important of the difficulties were the premium swings caused by the change of exposure bases.

ISO realized that the change from area to receipts (gross sales) would cause large premium swings for some insureds and filed a transition program along with the new policies. The transition program was meant to cap the premium effect of only the exposure base change. Using Dun & Bradstreet data, ISO calculated the average ratio of receipts to area for each class, territory and state and used this to convert the current area-based rates to the new receipts base. If an insured had a higher-than-average ratio of receipts to area, this would cause its premium to increase substantially. The increase (and decrease) was capped by the establishment of maximum and minimum ratios for each class, territory and state. The caps increased over five years to bring the insureds to their new premium gradually.

ISO's preliminary investigations indicated that the manufacturing and contracting classes did not have as much variability in their exposure base ratios, so no transition program was developed for these classes. However, as companies began to implement the simplified policies, it quickly became apparent that there was a problem. This was exacerbated by the effects of the change to a combined single limit and the inclusion of other coverages in the base rate. ISO responded by filing a transition program for other than mercantile risks, but it used countrywide caps rather than varying them by state and territory.

On the whole, the expanded transition program was successful, but it was given very little credit. In many cases, the first renewal on the simplified forms followed the hardening of the market. This meant that premium increases due to changes in companies' rates and deviations were frequently blamed on the exposure base change. Premium increases from changes in the increased limits tables (also part of the simplification program) made this problem worse.

From the companies' viewpoint, the transition program was a mixed blessing. On the negative side, it represented another training and programming hurdle; it introduced another step in the rating process which will persist for five years for many risks; and it was difficult to explain to insureds. On the positive side, once it was expanded, it did what it was designed to do, and it provided a convenient scapegoat for rate increases.

One long-term result of the exposure base change which has been given relatively little consideration is the effect of using an audited exposure base for many risks which were previously rated on area. This increases expenses somewhat for the insurer (many of these risks have products coverage, for which an audit was already required) and increases uncertainty for the insured, since the final premium is not known until after the policy expires. Of course, many smaller risks will be audited by mail or by telephone, but this increases the opportunity for manipulation of the premium while decreasing the audit cost.

In light of the expense and confusion surrounding the change of exposure bases, it is reasonable to ask whether the insurance community--both insureds and insurers--is in a better long-term position than it was before the change. It is clear that the simplification program as a whole eliminated many inconsistencies in the rating process and vastly simplified policy rating. This could not have been accomplished without changing the exposure bases. To the extent that the automation of the commercial lines has been accelerated, the program also decreased expenses. The price of these improvements is short-term upheaval and a possible long-term increase in audit costs.

The above points may well have been sufficient cause for the change, but it is also reasonable to ask whether receipts are a better exposure base than area for most mercantile risks. Recall that this should be judged on the basis of (1) ease of collectibility, (2) difficulty of manipulation, and (3) correct reflection of the underlying losses. To the extent, that the fringe coverages, such as contractual liability and fire legal liability, are rated more fairly (i.e., with greater precision) on other exposure bases, the simplification may have reduced the correct reflection of these underlying losses.

Since receipts are used for other purposes, most notably tax calculations, it is easy to collect the data. However, the use of receipts requires a post-expiration audit unless the insurer decides to forego the possible change in premium. While the risk may well have already required an audit for its products coverage, the change does mean that the premium for two coverages must now be checked. On the whole, it is difficult to say that there has been a net improvement on this point over area, which is relatively easily available (although requiring a detailed definition) and not does not require an audit.

It has been amply demonstrated over the course of the last insurance cycle that both area and receipts can be manipulated by both the insured and the underwriter. It has been argued that the introduction of the audit step, especially if it is done by telephone and relies on the insured's reporting, increases the number of opportunities for manipulation.

With no clear advantage to either exposure base on the first two criteria, the question becomes one of correlation with losses. If the traffic of customers and suppliers through a mercantile establishment can be assumed to be correlated with the loss exposure, then receipts may be more closely correlated with losses. Thus, an establishment with a thriving business has more customers, more loss exposure, higher receipts and a higher premium. On the other hand, one must consider the effect of price on receipts: a store selling expensive imported shoes may have the same total receipts as a mass-market store but far fewer clients and a lower exposure to loss (unless "upscale" clients are more prone to sue).

Time will judge the appropriateness of the exposure bases. Any inequities between classes of business will be erased as the rates adjust to the information passed into the ratemaking process. The real long-term test will be within classes: whether a stronger correlation between a risk's exposure and its expected losses exists for receipts or area. Of course, even if receipts should fail this test, it may be easier to adjust the class plan in some way than to change the exposure base again.

CONCLUSION

The exposure base is a fundamental part of the distribution of loss costs among insureds, i.e., of the premium calculation. The tests that it must meet are relatively simple and clear, but changes in external environment and problems in the internal environment have made it more difficult to satisfy those tests. In addition, insurance coverages for which the exposure base is not immediately obvious have been developed or are more in demand. The insurance industry has reacted differently in the two cases where change was forced by outside conditions: adapting the classification and individual risk modification system in one case, and completely revising the exposure base and rating system in the other. The ISO Simplification was an example of some of the problems and responses to be expected in the course of a changeover, which can be studied as a prototype of the changes which are undoubtedly to come.

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1988 CAS RATEMAKING SEMINAR

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The following is the material presented at the 1988 CAS Ratemaking Seminar. Although the actual presentation was given from an outline and notecards, this would be approximately the script that would have been used.

INTRODUCTION

Many of you may be wondering why the topic of Involuntary Markets is on the schedule for the Ratemaking Seminar this year. One only need look at the trade press to see an article on the Massachusetts involuntary auto market and its problems, the Maine workers compensation pool or the New Jersey personal auto JUA. The problems that the industry faces from any of these situations could likely be a seminar topic in itself.

I will be addressing the Commercial Automobile Involuntary Markets. The results for fiscal year 1985 have a $29 million operating loss with $140 million of written premium and $93 million of earned premium. Fiscal year 1986 has a $74 million operating loss with $561 million of written premium and $386 of earned premium. The significant growth in written premium seen from 1985 to 1986 is also seen from 1986 to 1987.

DIFFERENT MECHANISMS

There are seven different mechanisms currently being used for Commercial Automobile involuntary markets in the 51 jurisdictions (the 50 states plus the District of Columbia). See Exhibits I and Ia.

The most predominant pool type is the CAIP (Commercial Automobile Insurance Procedure). It is used in 39 jurisdictions. CAIPs operate with a limited number of servicing carriers. The results are accounted as 100% ceded business with the results being shared by all companies. The reserves are held by the participating companies. CAIP reserves are the sum of the reserves determined and reported by the servicing carriers. Some of the CAIPs are called "limited CAIPs" because they don't include the light and medium trucks which are part of the private passenger automobile involuntary market.

There are 3 states with SRDP (Special Risk Distribution Program) for their residual market. Again, these operate with a limited number of servicing carriers and their results are shared by all companies. For most purposes, SRDP can be considered a CAIP with a different name.

There are 4 JUAs (Joint Underwriting Associations) in operation. Again, the JUA has a limited number of servicing carriers and the results are shared by all companies. The reserves are held by the plan and the participation results include the investment income earned from the reserves. In truth, both CAIP and SRDP are considered modified JUAs.

There is one state fund in operation in Maryland. Private insurers are required to subsidize any losses and are permitted to charge back the cost against their own policyholders.

There is also one reinsurance facility in operation. Each insurer must provide coverage and service to any applicant - "take-all-comers". Each carrier is permitted to cede a certain percentage of their writings to the facility. The profit or loss is shared among all licensed companies. The participation is based on market share <u>and</u> use of the facility.

There are 2 AIPs (Automobile Insurance Plans). These are the true assigned risk mechanisms which will be explained further in the personal auto segment of this session. AIPs operate with CAIP or SRDP in additional states for the non-fleet and personal lines risks. Large fleets usually are not subject to AIP (except in these two states) because of the large amount of money at risk for a single policy.

Finally, there is one plan mechanism called "other". This is CAR (Commonwealth Automobile Reinsurers) in operation here in Massachusetts. CAR is a modified JUA with a limited number of servicing carriers. Participation in CAR is based on voluntary market share.

The base for participation in results or assignments, or assessment base, is typically the voluntary market share (excluding involuntary writings) from two years prior. The 2 year lag means that 1988 results are based on 1986 market share. The exceptions as noted above are Massachusetts CAR and the South Carolina Reinsurance Facility.

RATEMAKING IMPLICATIONS

Given the seven different mechanisms, how do we reflect the differences in ratemaking? For the most part, we don't treat them differently. The difference in treatment for ratemaking comes from the size of the involuntary load. Only in the largest few states do we actually look at the mechanics of the plan while doing ratemaking. In the states of Massachusetts, New Jersey and South Carolina, we have to more accurately forecast the involuntary load since these states have the largest pools.

PLAN SIZES

Given this, let's take a look at the pool sizes in the 51 jurisdictions. The pool size is expressed as a market share for the involuntary markets as a percent of the total market. Exhibits II and IIa show the total market share (liability and physical damage). Exhibits III and IIIa show the same information for liability only since many of the pools write only liability and do not write physical damage. The pool sizes break down as follows:

	Number of Jurisdictions	
Pool Size	Total	Liability
1%	15	10
1- 3%	15	15
3- 5%	7	8
5-10%	9	10
10-15%	3	5
|15%	2	3

The three states with liability shares greater than 15% all have different plan mechanisms, so this is not the cause for the large pool size. The plan size is a result of the voluntary rate adequacy - the perceived long-term voluntary rate adequacy. If the insurers perceive the rates to be inadequate they will refuse to write risks voluntarily and they will end up in the involuntary market. Many times the inadequacy is real. For example, in New Jersey the latest ISO rate change was 11.6% below the original indication and the New Jersey CAIP has been about 70% of ISO until recently when it increased to 80% of ISO. The New Jersey CAIP has consistently had lower rates than ISO. In South Carolina, the latest ISO increase was 62.6% short of the indication. And as for Massachusetts, the disagreements on rate adequacy are perennially in the press.

COMPANY ASSESSMENTS

The company assessment is the result of three items. The pool size and the pool operating ratio determine the total profit or loss to be shared by the participating companies. The company share is determined by the company participating ratio. Each of the methods for reflecting involuntary results in ratemaking must estimate these three items.

INVOLUNTARY COSTS

The standard method at my company for determining involuntary costs is a quick and easy method that uses readily available financial data. "Financial" is emphasized because it leads to the shortfalls of this method. Similar to the use of calendar year ratemaking rather than accident year, this procedure assumes a stable scenario in order to be accurate. More refined methods eliminate the need for these stability assumptions but take more time.

First, the involuntary loss for the past three years is determined. This information comes from the AIPSO participation reports or from company financial reports. Next, the voluntary written premium for the past three years is determined. This is Page 14 written premium minus any direct involuntary premium (from assigned risk or servicing business). The involuntary cost is the loss divided by the voluntary written premium. This procedure uses a three year average of these involuntary costs.

Contained within this method are certain assumptions regarding the three basic elements. For the operating ratio, the use of calendar year operating result rather than policy year assumes stability. The pool size is assumed to be relatively constant over time. The company's participation ratio is also assumed constant.

Obviously, these assumptions are hardly valid. In the most recent hard market, the involuntary markets grew tremendously. The mix in the residual markets also changed significantly with more truckers, etc. in the pools. Thus the stability assumptions are not accurate.

However, we still use this method since in most states the load is so small that the final indication is not sensitive to the accuracy of the load. There are a few cases where refinements are necessary because the size of the load requires the additional accuracy and the additional work is justified.

Refinements

The first refinement is to do a more accurate job of estimating the pool size. To start, express the pool as a percent of the total market. If the percent is stable over time, pool growth is due solely to market growth and/or rate changes. Then the future share for the pool is projected as a percentage of the total market. When projecting the pool size one must be conscious of the rate adequacy of the pool relative to the voluntary market and at what point in the insurance cycle you are projecting to and from, since the cycle has a tremendous effect on pool size.

The second area of refinement is in projecting the operating ratio for the pool. Start by adjusting history to an ultimate basis. AIPSO releases quarterly participation reports which can be used to derive loss development factors. Although IBNR is on the reports, I have found that due to the extreme growth in the CAIPs and their newness that there is still development that is unaccounted for in the reported IBNR. These loss ratios should then be adjusted for any projected change in rate adequacy for the pool. Also, an adjustment should be made for the impact that the change in pool size will have on the quality of business in the pool. Large growth implies that better quality business is flowing into the pool and large decline implies the opposite.

Finally, a more precise estimate of future participation ratios can be made. AIPSO releases participation data but it is not available very far in advance of the year it is used for. Another source is the A.M. Best A-7 reports. From these one must subtract the involuntary writings for your company and the industry. Or one can project future writings for the company and the industry.

Methods of Refining Load in Final Indication

There are two basic methods of how to reflect the involuntary cost (assesment as a % of voluntary written premium) in indications. The implicit method uses voluntary and involuntary combined for the experience. The implicit method is valid when both the voluntary and the involuntary get the same rate level and the mix and differential are assumed constant. This can only be used when direct experience is available.

The preferable method is the explicit method. The voluntary only experience is used and then a load for the involuntary cost is added. This is really the only valid method in CAIP states or other states using a modified JUA. The explicit method will always be valid when the implicit method is, but not vice-versa.

Examples of Involuntary loads in indications

Exhibits IV and V illustrate some examples of the explicit method. All examples assume that the involuntary cost, expressed as a % of voluntary premium is known with complete accuracy. The first set of examples, (Exhibit IV) is for a small involuntary cost with either no voluntary indication (Case I) or a moderate increase (Case II). The second set of examples (Exhibit V) is for a large involuntary cost. These examples illustrate the sensitivity of the methods to size of involuntary cost and size of the underlying voluntary rate need.

HIG Procedure

The first procedure is the one we use most often. This method adjusts the voluntary indication. The involuntary load is the involuntary cost divided by (1 - variable expenses). This is multiplied by the voluntary indications to get the final indication. The voluntary indication is derived by the standard ELR method. This method falls short of producing the desired 5% profit as the involuntary cost increases but tends to produce a higher profit as the underlying indication increases. This is only adequate when the involuntary cost is low and the indications are moderate. Its benefits are expediency.

The second procedure treats all expenses as variable and the involuntary cost is considered an additional "tax" item. Again this procedure yields a larger profit when the underlying indication is larger but also as the load increases. This method typically will overstate the indication and is therefore not desirable.

The most accurate method is to split the expenses into fixed and variable portions and treating the involuntary cost as a variable "tax item. The problem is in determining which expenses are fixed and which are variable.

Thank you for your time.

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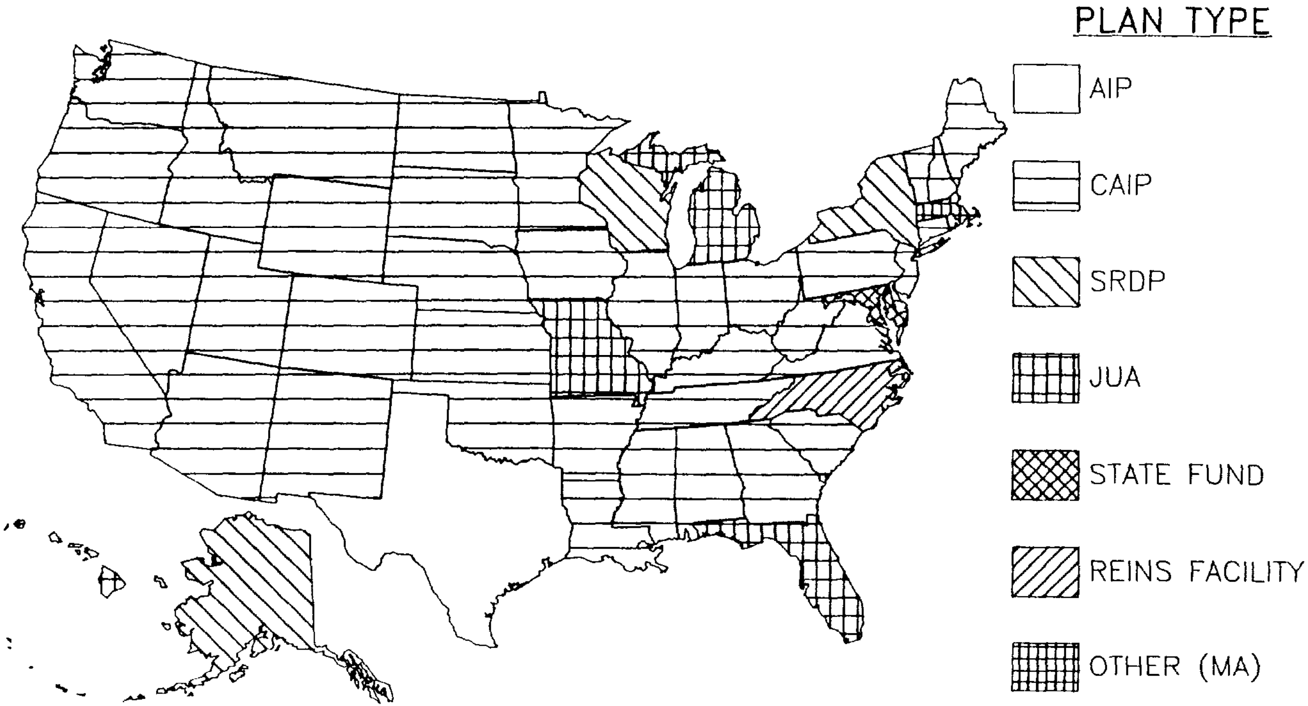
COMMERCIAL AUTOMOBILE

INVOLUNTARY MARKETS



COMMERCIAL AUTOMOBILE

INVOLUNTARY MARKETS

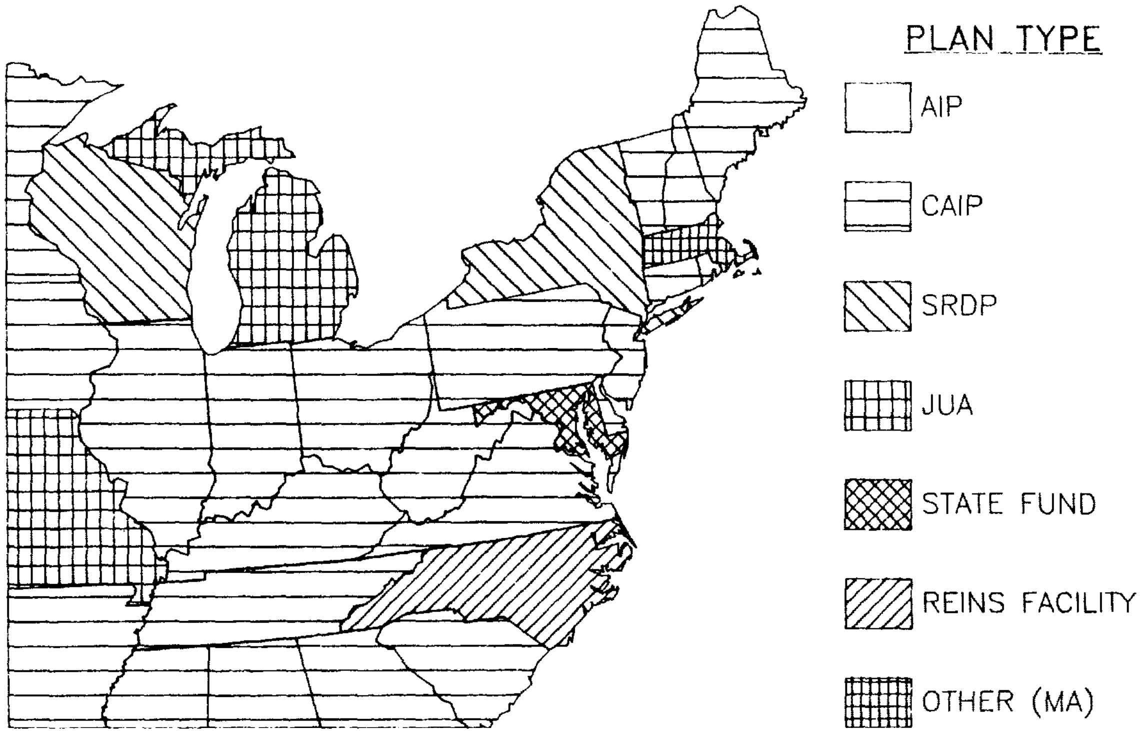


Exhibit Ic

COMMERCIAL AUTOMOBILE

INVOLUNTARY MARKETS

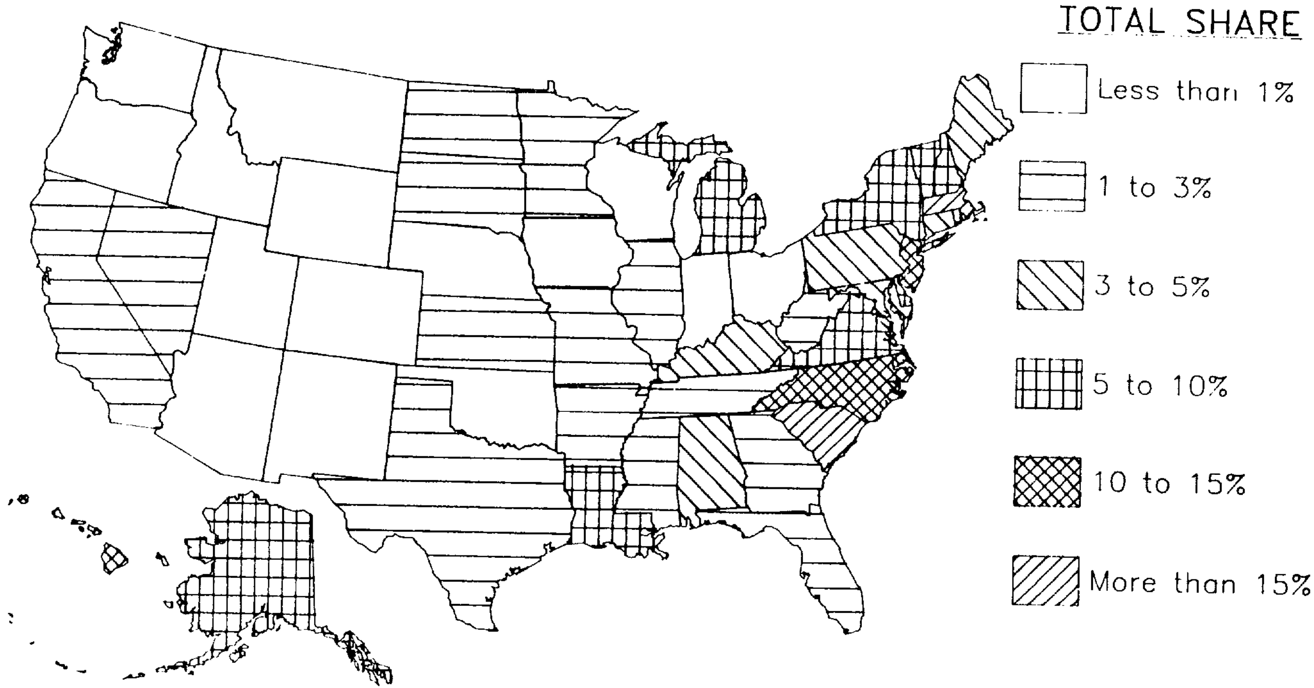


Exhibit II

COMMERCIAL AUTOMOBILE
INVOLUNTARY MARKETS

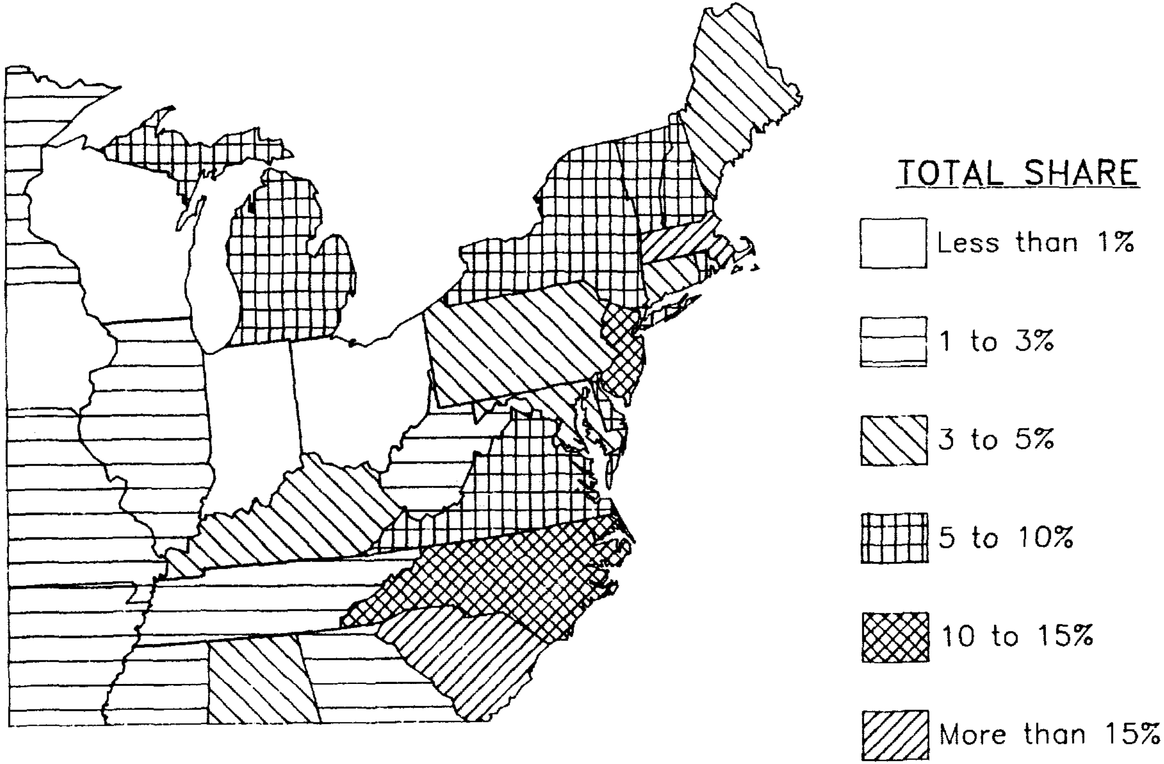


Exhibit II

COMMERCIAL AUTOMOBILE

INVOLUNTARY MARKETS

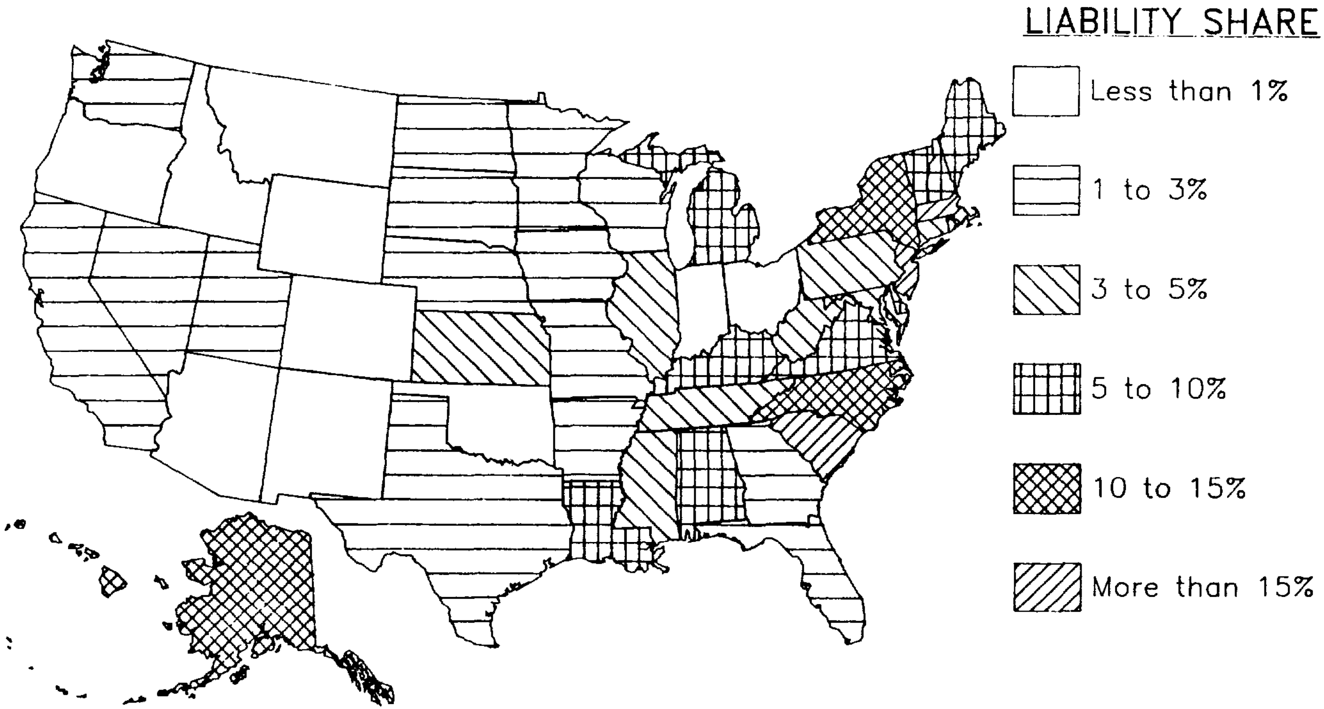
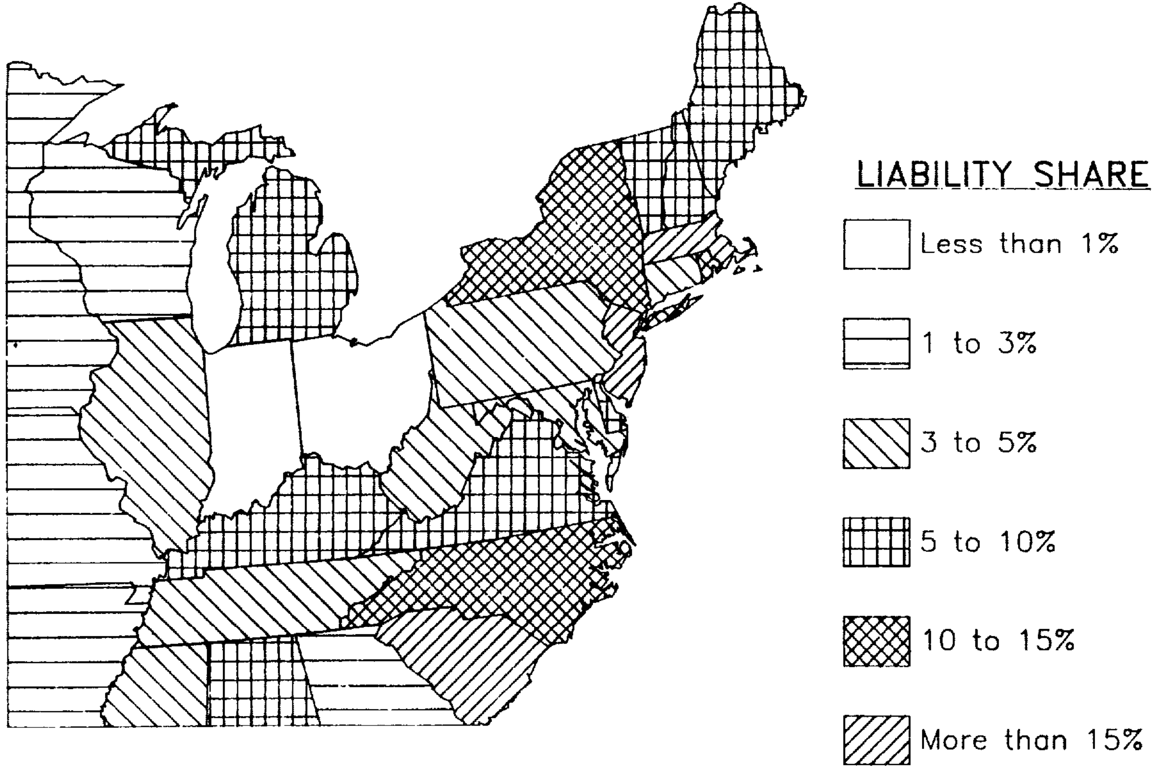


Exhibit II

COMMERCIAL AUTOMOBILE
INVOLUNTARY MARKETS



Exhibit

ASSUMPTIONS

	CASE I	CASE II
LOSS RATIO	65.0%	72.0%
EXPENSES		
TAXES	3.4%	3.4%
COMMISSIONS	15.1%	15.1%
OTHER ACQ	2.9%	2.9%
PROFIT	5.0%	5.0%
VARIABLE-SUBTOTAL	26.4%	26.4%
GENERAL (FIXED)	8.6%	8.6%

COMPANY PROCEDURE

	CASE I	CASE II
INVOLUNTARY COST	-0.8%	-0.8%
INVOLUNTARY LOAD	-1.09%	-1.09%
[INVOLUNTARY COST/(1 - VARIABLE EXPENSES)]		
VOLUNTARY INDICATION (ELR)	0.0%	+10.8%
FINAL INDICATION	+1.09%	+11.97%
(1 + VOLUNTARY INDICATION) * (1 - INVOLUNTARY LOAD)		
RECONCILIATION		
PREMIUM	$101.09	$111.97
LOSSES	$ 65.00	$ 72.00
TAXES	$ 3.44	$ 3.81
COMMISSIONS	$ 15.26	$ 16.91
OTHER ACQ	$ 2.93	$ 3.25
GENERAL	$ 8.60	$ 8.60
INVOLUNTARY	$ 0.81	$ 0.90
PROFIT	$ 5.05	$ 6.52
AS % OF PREMIUM	5.0%	5.8%

Exhibit IV
Page 2

ALL EXPENSES AS VARIABLE

	CASE I	CASE II
INVOLUNTARY COST	-0.8%	-0.8%
FINAL INDICATION	+1.25%	+12.15%
LOSS RATIO/(1 - EXPENSES - INVOLUNTARY COST)		
RECONCILIATION		
PREMIUM	$101.25	$112.15
LOSSES	$ 65.00	$ 72.00
TAXES	$ 3.44	$ 3.81
COMMISSIONS	$ 15.29	$ 16.93
OTHER ACQ	$ 2.94	$ 3.25
GENERAL	$ 8.60	$ 8.60
INVOLUNTARY	$ 0.81	$ 0.90
PROFIT	$ 5.17	$ 6.65
AS % OF PREMIUM	5.1%	5.9%

Exhibit II
Page 3

FIXED & VARIABLE EXPENSES

	CASE I	CASE II
INVOLUNTARY COST	-0.8%	-0.8%
FINAL INDICATION	+1.10%	+10.71%

(LOSS RATIO + FIXED EXPENSES)/(1 - VARIABLE EXPENSES - INVOLUNTARY COST)

RECONCILIATION

	CASE I	CASE II
PREMIUM	$101.10	$110.71
LOSSES	$ 65.00	$ 72.00
TAXES	$ 3.44	$ 3.76
COMMISSIONS	$ 15.27	$ 16.72
OTHER ACQ	$ 2.93	$ 3.21
GENERAL	$ 8.60	$ 8.60
INVOLUNTARY	$ 0.81	$ 0.89
PROFIT	$ 5.05	$ 5.54
AS % OF PREMIUM	5.0%	5.0%

Exhibit —
Page

COMPANY PROCEDURE

	CASE I	CASE II
INVOLUNTARY COST	-5.0%	-5.0%
INVOLUNTARY LOAD	-6.79%	-6.79%
[INVOLUNTARY COST/(1 - VARIABLE EXPENSES)]		
VOLUNTARY INDICATION (ELR)	0.0%	+10.8%
FINAL INDICATION	6.79%	+18.29%
(1 + VOLUNTARY INDICATION) * (1 - INVOLUNTARY LOAD)		

RECONCILIATION

	CASE I	CASE II
PREMIUM	$106.79	$118.29
LOSSES	$ 65.00	$ 72.00
TAXES	$ 3.63	$ 4.02
COMMISSIONS	$ 16.13	$ 17.86
OTHER ACQ	$ 3.10	$ 3.43
GENERAL	$ 8.60	$ 8.60
INVOLUNTARY	$ 5.34	$ 5.91
PROFIT	$ 5.00	$ 6.46
AS % OF PREMIUM	4.7%	5.5%

ALL EXPENSES AS VARIABLE

	CASE I	CASE II
INVOLUNTARY COST	-5.0%	-5.0%
FIANL INDICATION	+8.33%	+20.00%
LOSS RATIO/(1 - EXPENSES - INVOLUNTARY COST)		
RECONCILIATION		
PREMIUM	$108.33	$120.00
LOSSES	$ 65.00	$ 72.00
TAXES	$ 3.68	$ 4.08
COMMISSIONS	$ 16.36	$ 18.12
OTHER ACQ	$ 3.14	$ 3.48
GENERAL	$ 8.60	$ 8.60
INVOLUNTARY	$ 5.42	$ 6.00
PROFIT	$ 6.13	$ 7.72
AS % OF PREMIUM	5.7%	6.4%

FIXED & VARIABLE EXPENSES

	CASE I	CASE II
INVOLUNTARY COST	-5.0%	-5.0%
FINAL INDICATION	7.29%	17.49%

(LOSS RATIO + FIXED EXPENSES)/(1 - VARIABLE EXPENSES - INVOLUNTARY COST)

RECONCILIATION

	CASE I	CASE II
PREMIUM	$107.29	$117.49
LOSSES	$ 65.00	$ 72.00
TAXES	$ 3.65	$ 3.99
COMMISSIONS	$ 16.20	$ 17.74
OTHER ACQ	$ 3.11	$ 3.41
GENERAL	$ 8.60	$ 8.60
INVOLUNTARY	$ 5.36	$ 5.87
PROFIT	$ 5.36	$ 5.87
AS % OF PREMIUM	5.0%	5.0%

CREDIBILITY — AN AMERICAN IDEA

CREDIBILITY - AN AMERICAN IDEA

by

Charles C. Hewitt, Jr.

PROLOGUE

Setarcos: What may we say about the basis for human decision and human action?

Student: I assume you mean conscious decision and action as opposed to instinct; all animals possess instinct to some degree. If so, we may say that human decisions (and actions) are based upon a set of beliefs - we may call this knowledge.

Setarcos: Of what does knowledge consist?

Student: Knowledge may consist of a set of suppositions or ideas, many of which exist in the absence of corroborating evidence. Also, knowledge consists of a set of facts accumulated throughout a lifetime.

Setarcos: How does a human being obtain knowledge?

Student: In the instance of ideas, these are obtained from many sources - parents, teachers, friends, and, importantly, from the creativity of the human mind. In the case of facts, these are obtained from personal observation and the reported observations of others.

Setarcos: What if there is a conflict between one's suppositions and the facts?

Student: Of course, we must first dispose of the situations where there are either no facts or no suppositions. Where there are no facts, the supposition prevails; where there is no basis for supposition, facts constitute the only knowledge.

< < < Credibility - An American Idea > > >

Setarcos: Very well, but suppose a set of facts contradicts a set of ideas?

Student: The wise person must then weigh the ideas against the evidence and make a judgment. The judgment may be to alter the ideas to fit the facts or, alternatively, to question the facts themselves. Of course, one may compromise by giving partial weight to each.

Setarcos: But what weight do we assign to supposition and what weight do we assign to the evidence?

Student: That's the real question! Perhaps some day there will be a mathematical answer.

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BAYESIAN ANALYSIS

In 1763 Thomas Bayes proposed that 'a priori' probabilities could be assigned to the several hypotheses (suppositions). Next, calculate the probability of each outcome for each hypothesis. After making an observation, or a series of observations, the 'a posteriori' probabilities of the hypotheses could be calculated. Hence, an improved knowledge of the underlying (but unknown) probabilities could be obtained. This approach is generally referred to as Bayesian analysis (sometimes as inverse or, where the judgment is mostly intuitive, subjective probability).

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CREDIBILITY - AN ANSWER TO AN INSURANCE PROBLEM

The Casualty Actuarial Society was formed in 1914 - in part to deal with the problem of making rates for a new (to the United States) line of insurance - Workmens' Compenation. (Many of the charter members of the CAS were also members of the Actuarial Society of America.) Workmens' Compensation had been available in Europe for some time but there was almost no data which might represent American experience, except for Employer's Liability Insurance which Workmens' Compensation Insurance was designed to replace.

As data became available for the new line of insurance the question arose as to how to revise the hypothetical rates to admit this new evidence in ratemaking. A group of casualty actuaries, whose spokesperson was Albert Whitney (12), conceived the idea of assigning weights in the linear expression:

$$\text{New Rate} = \text{Credibility} \times \text{Observed Rate} + (1 - \text{Credibility}) \times \text{Old Rate}$$

The underlying mathematics which determined the value of the term 'Credibility' was implicitly Bayesian but not generalized. Rather, it dealt with a specific 'a priori' distribution and a specific random process. It produced the expession:

$$\text{Credibility} = \frac{\text{Number of Observations (Exposure)}}{\text{Number of Observations (Exposure)} + K}$$

where "K" was a positive constant to be determined from the underlying factors.

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CLASSICAL STATISTICS - THE DARK AGES FOR CREDIBILITY

As will be shown later, the development of "K" required a Bayesian approach. However, in the time of Whitney, and for many years to come, the theory underlying mathematical statistics refused to admit that element of Bayesian analysis which required a subjective approach to the assignment of 'a priori' probabilities. In the Neyman-Pearson school of classical statistics, each hypothesis was something to be either accepted or rejected, but only after an examination of some data which admitted or denied the hypothesis. In Bayesian analysis the original set of hypotheses is not totally rejected, but, rather, the 'a priori' probabilities of the hypotheses may be changed ('a posteriori') to adjust to the new evidence!

Thus, it was to come to pass that casualty actuaries were to "sell their birthright for a mess of pottage". They "copped out" on calculating "K" on the basis required by Whitney's mathematics and settled for an arbitrary assignment of a value chosen by means most convenient. Having yielded to the demands of the then popular approach of statisticians, they capitulated even further by yielding to the demand by insurance buyers and insurance marketing people for 'full' credibility.

The 'Credibility Formula' as set forth above does not admit of 'full' credibility being assigned to the observations - the expression may approach unity as the number of observations (or exposure) increases, but only asymptotically. Insurance buyers with better-than-average experience wanted full recognition in their rates. Since these buyers were, more often than not, the larger customers and also, a fortiori, the preferred risks, their wishes had to be respected. Once again an arbitrary assignment was made - the point at which exposures were sufficient to admit of 'full' credibility - and, of course, on the basis of convenience.

The actuarial literature during this period - Dark Ages - was almost exclusively in the Proceedings of the Casualty Actuarial Society and dealt with questions of determining standards for 'full' credibility and with interesting, but rather baroque, approaches as to how to make the transition from the 'Whitney' formula to 'full' credibility as the size of the risks being rated increased.

There were, however, notable exceptions. A paper by Ralph Keffer (1929) in the Transactions of the Actuarial Society of America (5)* suggested a Bayesian approach in group life insurance. In 1940 Ove Lundberg (6) in Sweden presented a - now classic - paper, using a Bayesian approach to ratemaking for health insurance. And in 1950 Arthur Bailey (1) rediscovered the original approach of Whitney and his colleagues, using subjective methods in probability theory rather than the classical approach. In so doing Bailey** reaffirmed the underlying strength of the Bayesian roots of Credibility Theory.

Following this "Renaissance" of Bayesian Credibility, successive papers by Dropkin (4), Robert Bailey (2) - son of Arthur Bailey - and

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* Keffer provides an interesting footnote (p. 135) identifying the Poisson Distribution as the "Bortkewitsch Law of Small Numbers". Readers familiar with the history of the use of the Poisson Distribution may readily infer the identity of (Oberst) Bortkewitsch.

** Many persons, including Matt Rodermund and Ben Zehnwirth, regard Arthur Bailey as the "Father of Modern Credibilty Theory".

Simon (11)* applied specialized Bayesian approaches to the experience rating of private passenger automobile insurance.

It remained for two individuals in the 1960's to reawaken and restore interest in the true nature of the credibility concept, Hans Buhlmann in Switzerland and Allen Mayerson in the United States. The latter individual provided a paper on Bayesian Credibility (7) using an approach which was not generalized, but, rather, used two specific cognate processes, the Beta-Binomial and the Gamma-Poisson.

Meanwhile in Europe, Hans Buhlmann took the more general approach to the problem. In a paper to ASTIN (3) - the non-life section of the International Actuarial Association, Buhlmann demonstrated that the "K" in the much earlier Whitney formula was:

$$K = \frac{\text{Variance of the Processes}}{\text{Variance of the Hypotheses}}$$

These 'Renaissance' protagonists recognized that the straight line produced by the Credibility formula represented a least-squares fit to the 'process means' generated by the differing hypotheses (with weight given to each point on the basis of the 'a priori' probability of each hypothesis).

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* The brief history in my paper omits reference to many works in this field. It is deliberately intended to highlight American actuarial efforts, with reference to foreign contributions where appropriate. A fairly complete, if not up-to-date, historical bibliography may be folund on pp. 6-7 of the Simon paper (11).

CURRENT DEVELOPMENTS

With the increased acceptance of subjective probabilities - see Savage (Yale) (9) and Raiffa and Schlaiffer (Harvard) (8) & (10) - the new/old approach to credibility has flowered - at least theoretically. Many latter-day members of the Casualty Actuarial Society have written on Credibility - Philbrick, Van Slyke and Venter - just to name a few.

Professor William Jewell at the University of California (Berkeley) has been prolific in his research and publication. In Switzerland, Buhlmann and his pupils, Hans Gerber and Erwin Straub, have contributed, and, in Belgium, Fl. DeVylder. In Australia, Gregory Taylor has published frequently on Credibilty.

Credibility may be thought of as a form of 'short-hand' for the more descriptive Bayesian analysis. When Credibility Theory was first developed there were no modern high-speed computers; hence a simplified, two-dimensional linear substitute for the more complex Bayesian analysis was eminently desirable and acceptable.

Modern Credibility Theory has expanded to include non-linear and multi-dimensional approaches. Ultimately, with the advent of high-speed methods of calculation, Credibility, as a form of 'short-hand', may yield to full Bayesian analysis. In the meantime, practical application of theoretical credibility has lagged far behind the state-of-the-art. The Neyman-Pearson training of many mathematical statisticians seems to produce a revulsion for any subjective approach. However, the ideas are there!

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POSTLOGUE

Sometime in the 1920's Sinclair Lewis, the author, was to receive an award as a distinguished alumnus of Yale University. In accepting the award, Lewis took the occasion to assert his known atheism.

"I do not believe there is a God," said Lewis (in substance). "If, in fact, there is one, let him strike me down here and now!" And, of course, nothing happened. However, several days later the noted newspaper columnist, Arthur Brisbane took Lewis to task.

"Lewis, you poor misguided fool," wrote Brisbane (in substance). "You remind me of the ants who lived along the right-of-way of the Atchison, Topeka and Santa Fe Railroad. This colony of ants depended for its existence upon the crumbs thrown from the dining cars of the railroad trains as they passed by.

"It came to pass that the ant colony fell upon hard times because - through chance - no crumbs were thrown out near its particular place along the right-of-way. The situation became desperate and the colony decided to hold a meeting. It was suggested that they all pray to the President of the Atchison, Topeka and Santa Fe Railroad to send more dining cars so that crumbs would be thrown off in their area.

"So they did pray and the following day they waited, but no crumbs were thrown off where they lived. So the ants concluded that there was no such person as the President of Atchison, Topeka and Santa Fe Railroad"

+ + + NUMBER OF OBSERVATIONS + + +

* * * * *

** Four boys decided to play "hooky" from school, because they knew there was to be a test that morning. About 11 A. M. their consciences got the better of them and they decided to show up at school after all. Upon reaching their classroom, they explained to their teacher that they had been on their way to school in a car, but the car had a flat tire. This made them late because they had to have the flat tire fixed.

"No problem!", said the teacher. "Just come back here during the lunch hour and I'll give you a make-up test." At lunch time, when they reported back to the classroom, the teacher instructed the four boys to take seats in opposite corners of the room.

"Now," said the teacher, "there is only one question on this make-up test. Which tire was flat?"

+ + + VARIANCE OF THE PROCESSES + + +

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< < < Credibility - An American Idea > > >

*** Television interviewer: "Do you believe in miracles?"

Guest: "Of course."

Television interviewer: "Have you ever seen a miracle?"

Guest: "No."

Television interviewer: "Do you know any one who has actually seen a miracle?"

Guest: "No, but that doesn't prove anything!"

+ + + VARIANCE OF THE HYPOTHESES + + +

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BIBLIOGRAPHY

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(1)	Bailey, Arthur L.	Credibility Procedures (Shortened title)	PCAS XXXVII (1950)
(2)	Bailey, Robert	Experience Rating Reassesed	PCAS XLVIII (1961)
(3)	Buhlmann, Hans	Experience Rating and Credibility	IV ASTIN Bulletin (1967)
(4)	Dropkin, Lester	Automobile Merit Rating and Inverse Probabilities	PCAS XLVII (1960)
(5)	Keffer, Ralph	An Experience Rating Formula	TASA XXX (1929)
(6)	Lundberg, Ove	On Random Processes and Their Application to Sickness and Accident Statistics	Uppsala (1940)
(7)	Mayerson, Allen L.	A Bayesian View of Credibility	PCAS LI (1964)
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HOSPITAL SELF-INSURANCE FUNDING: A MONTE CARLO APPROACH

By Dave Bickerstaff

ABSTRACT

The common theme which appears to have evolved in the actuarial methodology for determining self-insurance funding contributions can be described in basic terms as a two-step process: (a) estimating expected retained losses for the self-insured entity and (b) estimating a safety margin or risk loading to maintain funding at a selected high level of confidence. Variations on this general theme abound. Using Hospital Professional Liability as an example, this paper sets forth a simulation technique which approximates the aggregate loss distribution and the distribution of required funding to cover losses, focusing on the interaction of several variables. Special emphasis is placed on treating the run-off of the fund's prior year losses and the prospective target year losses simultaneously in determining the required funding on a year-by-year basis.

HOSPITAL SELF-INSURANCE FUNDING: A MONTE CARLO APPROACH

The establishment of self-insured trust funds has become, over the last 10-12 years, a widely accepted response by U.S. hospitals to an increasingly constricted liability and workers' compensation market. Accurate estimates relating to the total aggregate hospital funding for self-insurance may be difficult if not impossible to come by, but it seems apparent that here in the late 1980's the larger hospitals (say, 500 beds and up) who self insure a significant first layer for at least their professional liability exposure are the rule rather than the exception.

The determination of appropriate funding levels for self-insured funds calls for the careful application of a special chapter from the property-casualty actuarial repertoire. It appears from this author's perspective that, for the most part, actuarial input of one kind or another has been solicited and delivered as an integral part of the hospital self-insurance planning process. (It should be acknowledged, of course, that no small factor in the prominence and high visibility of this actuarial input was the inclusion by the HEW Department of required actuarial "certification" of self-insured funding levels in their original funding guidelines for Medicare reimbursement purposes in the mid 1970's. No attempts will be made by this author to chronicle the evolution of the Medicare guidelines. The concepts discussed in this paper are intended to be more generic, concentrating on the intrinsic risk encountered by a self-insuring entity and the funding required to retain and sustain that risk -- irrespective of government guidelines.)

Basic Principles

Any practicing actuary searching for a standard "cookbook" or "generally recognized" approach to calculating funding levels for self-insured funds will probably end up waving the white flag. It seems that there have evolved over the past decade or so several (dozens, maybe) methodologies or families of methodologies which represent variations on a general theme. Despite all the variations, it appears that the common denominator among all self-insurance funding procedures can be described in general terms as follows: An *expected* annual retained loss is estimated for the hospital, using the hospital's own loss experience to the extent deemed credible (and outside data otherwise), and, to supplement this expected level, a *safety margin* or risk loading is included in the funding, based in one way or another on some measurement of the distribution of aggregate retained losses and defining confidence intervals from that distribution. Beyond this simple theme, though, variations of all shapes and forms (which may be equally defensible) abound.

In the process of walking through the development of a self-insurance procedure, or one "variation" on the common theme, this author found that the first major building block beyond the central theme is the treatment of the funding calculation for the first year of a fund contrasted with the "renewal" funding for each year thereafter. For the initial funding calculation one is concerned only with the prospective expected retained losses for the target year and with the confidence levels around that expected level. For each year thereafter, the funding level would logically be predicated on the amount required to run off the losses from prior years as well as the amount required for the prospective target year. With these dual objectives in mind, we can set forth our first general expression for determining a self-insured funding level:

Indicated funding for year N
\+ Current Assets of fund
= present value of losses from prior years paid in years N forward
\+ Present value of losses incurred in year N.

When N=1, of course, the current assets = 0 and there is no runoff from prior years. For N > 1, however, the remaining unpaid losses from prior years and the projected losses for the next target year are treated simultaneously in determining what additional funding, when combined with the current assets which were generated from prior funding and the interest earned thereon, will be necessary to cover all future losses.

The two loss categories in the above general expression are, of course, treated as random variables and thus the value solved for -- the required year N funding level -- is also defined as a random variable. After the probability distribution of this random variable is approximated, a funding level is then determined corresponding to a desired confidence level. It is safe to assume that most of the actuarial effort expended over the past few years in the self-insurance field has been in the determination of this probability distribution, given all of the necessary parameters. It is also pretty safe to assume that it is in this phase of the actuarial exercise that most of the wide variations on the theme occur.

As the above basic formula implies, the annual funding amount is continually self-correcting, based on each new year's experience. Not unlike pension plan funding, the actuarial "gains" or "deficits" from prior years, represented in the formula by the present value of the runoff of prior years' losses less the current assets, are built into the formula to determine the indicated level of funding for the next year. Rather than treating the funding of each new year independently of the prior years and thereby stacking single year safety margins on top of prior single year safety margins, all years are treated collectively to determine the safety margin to cover all losses.

Expected Losses for Hospital

This paper will not dwell on the details of analyzing the loss and exposure data of a particular hospital and the loss experience from pertinent "global" sources to supplement the hospital-specific data. It would seem that the choice of which loss reserving techniques to use to analyze the data and project expected loss costs per exposure unit to a target year would depend largely on the size of the hospital, the availability of loss data, and the judgement of the actuary doing the analysis. Conceivably, for large hospitals with as many as 8-10 years of accessible loss data, one could construct historical loss development triangles, including paid and open claim counts and amounts, and determine historical development patterns based on the hospital's data itself. For middle-sized hospitals, the actual claim data might be used, but for purposes of loss development and trending, more global (e.g., statewide or countrywide) indications would probably be required. Finally, for the small hospitals, the loss experience of the entity itself would rarely, if ever, be used and the expected loss costs might be derived exclusively from the global sources.

Even for the larger hospitals, the final selection of the expected loss cost might be based to some extent on a credibility-weighted average of the hospital-specific data and the statewide average. Rather than being a slave to some dogmatic credibility standard (all together now, 682 claims = 100% credibility, etc.), it would seem that a great deal of actuarial judgement should be exercised in arriving at the final selections, particularly since, from a subjective standpoint, there may well be some unique risk characteristics for the hospital in question (types of procedures, etc.) which should be reflected irrespective of its sheer size and statistical credibility.

Given the projected expected loss cost for the hospital, the second task at hand in the procedure to determine funding levels is to approximate the probability distributions around the expected values from which confidence levels can be defined. To accomplish this task this author has developed a Monte Carlo simulation model to "sample" the experience of a fund, as defined by certain parameters, over a large number of trials (usually 1,000). Accordingly, no matter how the final weighted average expected loss cost is derived for the hospital, it will be necessary to break out this loss cost into a few key components, for purposes of generating the probability distribution. As a first step, an average claim cost (at some limit) and a total claim frequency would be determined, the product of which will equal the pure loss cost per exposure unit. The average claim cost can then be subdivided into: (a) average indemnity cost and (b) average claim expense, while frequency can be subdivided into (c) percentage of claims closed with indemnity, (d) percentage of claims closed with claim expense only, and (e) percentage of claims closed with no payment.

A quick preview of the key distributions which will need to be developed in the Monte Carlo model is as follows:

(a) Distribution of indemnity amounts
(b) Distribution of ALAE amounts
(c) Claim frequency distribution
(d) Claim reporting and claim payout distributions
(e) For renewal funding: distribution of the number of IBNR's from prior years, given the expected number of IBNR's

The key distributions used in the model will now be explored in some detail.

THE INDEMNITY SIZE OF LOSS DISTRIBUTION

The NAIC Closed Claim Studies

Perhaps the most critical component in our procedure to approximate the confidence levels of self insurance funding is the distribution of indemnity amounts (from ground up, with no limit) for one accident year. Using medical professional liability as the line of business in question, we referred to the NAIC closed claim study.[1] For this study, some 75,000 claims closed during the period 1975-78 were recorded. Among many other items of information, the accident dates, report dates, closed dates, and indemnity and ALAE amounts were included.

It has been shown by many researchers[2] that, in order for any calendar year closed claim distribution to accurately represent the claim-size distribution applicable to an accident year, some trending adjustments are necessary for both claim frequency and claim severity. For this author's model claim-size distribution, we first devised annual indices of claim severity and frequency (both accident year) from available national data covering a period of about 20 years

[1]National Association of Insurance Commissioners, **NAIC Malpractice Claims**, 1980.

[2]See, for example, Archer McWhorter, Jr., "Drawing Inferences from Medical Malpractice Closed Claim Studies", **The Journal of Risk and Insurance**, XLV, no. 1 (March, 1978) and Michael R. Lamb, "Uses of Closed Claim Data for Pricing," **Pricing Property and Casualty Insurance Products**, 1980 C.A.S. Discussion Paper Program, p. 219.

up to calendar year 1978 (the final closing year of the study). The frequency and severity indices for each year were then expressed in terms of the 1978 index equal to 1.0. Then to each detail claim record,[3] based on the accident date, we applied the reciprocal of the frequency index to the claim count (1 per record, initially) and the reciprocals of both the frequency and severity indices to the indemnity and ALAE amounts. As a result of this exercise, we produced a claim size distribution adjusted to represent the *accident year* 1978.

A printout of the trend-adjusted claim size distribution (indemnity) is shown in Appendix A, page 1. The brackets of indemnity size are set up on logarithmic (geometric) scale, with the end point of each bracket a constant factor (about 1.3335) times the end point of the previous bracket. A plot of the histogram for the non-zero members of this adjusted distribution is displayed on page 2 of Appendix A. The cumulative distribution ogive is then plotted on page 3. But the most revealing and useful plot of this accident-year adjusted distribution is shown on pages 4-5, on which we have plotted the cumulative distribution on lognormal probability graph paper, the grids of which are constructed so that the cumulative distribution ogive of a lognormal probability distribution is a straight line.

The lognormal model has been used extensively to represent claim size distributions in property and casualty lines.[4] Finger, in particular, used the lognormal model to determine implied increased limit factors for medical professional liability. It would follow, then, that the lognormal would be a good candidate to investigate for modelling self-insured losses.

On the first page of our cumulative distribution graph (claims up to $100,000), the lognormal fit -- a straight line drawn though the points strictly by sight -- clearly is good enough to represent the actual data. On the continuation of the distribution on page 5, it can be noted that for values above about $500,000 the actual data points veer out above the hand-selected lognormal line. There is a very plausible explanation for this. If the lognormal model does in fact provide a good representation of the claim size distribution *with no limit*, then the imposition of policy limits on the bigger claims in the data base itself would have had a dampening effect on the relative frequency of these claims in the higher, potentially excess, layers. It can be approximated from the graph, for example, that the extension of the

[3]In addition to referring to the hard-copy NAIC report, we also purchased the detail data tape from the association.

[4]See, for example, Charles C. Hewitt, Jr., "Credibility for Severity," PCAS, LVII (1970), p. 148; David R. Bickerstaff, "Automobile Collision Deductibles and Repair Cost Groups: the Lognormal Model," PCAS, LIX (1972), p. 68; and Robert J. Finger, "Estimating Pure Premiums by Layer -- an Approach," PCAS, LXIII (1976), p. 34.

lognormal line would indicate a frequency of claims in the $2 million plus range about 4 to 5 times greater than the actual data points would indicate. For this reason, more than any other, this author disdained any idea of walking through a rigorous, analytical curve-fitting choreography, which would have generated a "best fitting" line that understates the potential for big claims.

The selected lognormal parameters for indemnity

We estimated a mean and variance from our fitted lognormal claim size distribution by marking off the median and standard deviation directly from the graph, using the 50 percentile and +1 standard deviation marks on the vertical scale, as follows:

Observed median $= e^{\mu} = 10650$.
Observed $\sigma = \log_e(68000) - \log_e(10650) = 1.853$

Our final selected value for the mean is, then

$$\exp(\log_e(10650)+(1.853)^2/2) = 59300 .$$

The coefficient of variation (standard deviation divided by the mean) of the fitted distribution is calculated as follows:

$$(CV)^2 = e^{\sigma^2} - 1$$
$$= 29.988 .$$

Thus, for future modelling purposes, we set the CV value $= \sqrt{30}$.

Working Size of Loss Model for Indemnity

The absolute values of the 1978 NAIC closed claim distribution, even after adjusting for frequency and severity trends, are not particularly important to us - especially in 1989. The *shape* of the adjusted, fitted distribution is the key parameter, measured by the CV. We believe that it is reasonable to assume that as the average unlimited indemnity increases over time or from one territory to another, the $(CV)^2$ should remain relatively constant. This also implies that as the *average* unlimited claim increases k percent from one point in time to another, it is reasonable to expect that the entire distribution of claims moves up about k per cent. Put another way, an $800,000 claim has about the same relative niche in a distribution whose unlimited mean is $100,000 as a $400,000 claim in a distribution with half the unlimited mean.

Our working indemnity distribution can, then, be represented by a lognormal distribution whose unlimited mean is 1.0 and whose $(CV)^2$ is 30, as shown in page 6 of Appendix A. The top line represents the basic distribution of claims by size and the bottom line depicts the *first moment* distribution.[5] To illustrate how this graph is read, from the top line one can note that about 82.5% of all claims are less than or equal to the mean and about 96.5% of the claims are less than or equal to five times the mean. From the bottom line, one can further note that about 18% of the total *dollars* in the distribution come from claims which are less than or equal to the mean and about 47% of the dollars from claims below five times the mean.

Generation of random claim amounts from lognormal model

To tabulate sample claims from the lognormal distribution, our Monte Carlo model employs a random number generator which generates *normal* random numbers.[6] The sample random claim size (indemnity) is determined from the following formula:

$$X = \exp(\mu + N\sigma)$$

where μ = mean of the logs of the distribution
= S.D. of " " " "
N = normal random number (mean 0, var. 1) .

From the basic relationships of the lognormal distribution,

$$M = \exp(\mu + \sigma^2/2)$$

where M = mean of the distribution .

Then we have

$$\mu = \log_e(M) - \sigma^2/2$$

[5]For a discussion of moment distributions and other attributes of the lognormal distribution, see J. Aitchison and J. A. C. Brown, **The Lognormal Distribution**, (Cambridge University Press, 1969).

[6]A full discussion of random number generation is beyond the scope of this paper. For further reference, we recommend G. S. Fishman, **Principles of Discrete Event Simulation** (New York: John Wiley & Sons, 1978), chap. 8-9.

and then the sample claim would be generated with

$$X = \exp(\log_e(M) - \sigma^2/2 + N\sigma) .$$

REPORT YEAR / CALENDAR YEAR STRATIFICATION OF ACCIDENT YEAR

In our basic funding formula, it will be recalled that we were looking for the *present value* of future losses or the distribution of the present value of losses. Furthermore, the self insured retention (SIR) limit per claim, as regards the terms with an excess carrier, may be *indexed*, i.e., the excess attachment point would be increased a specified amount (or percentage) each year based on the *calendar year* that first payment is made on the claim. Thus, we are interested not only in the distribution of claims by size but also the distribution of claims by lag time to settlement. Because of the well-recognized correlation between payment lag and payment size, we have introduced a form of stratification in the sampling of medical professional claim amounts. To accomplish this, we first set forth some basic relationships between report year and calendar year severities, within the accident year:

Let $R(i)$ = Frequency of claims reported in report year i of acc. year, relative to total accident year

$C(j)$ = Freq. of claims of one rep. year paid in cal. year j, relative to total report year

S_i = Severity of claims of report year i, relative to total accident year severity

T_j = Severity of claims of calendar year j, relative to total severity of report year

n = total report years in accident year

m = total calendar years for each report year

Then you have

$$\sum_{i=1}^{n} R(i) = 1$$

$$\sum_{j=1}^{m} C(j) = 1$$

and, by definition,

$$\sum_{i=1}^{n} S_i R(i) = 1$$

$$\sum_{j=1}^{m} T_j C(j) = 1 \quad .$$

The total accident year can then be stratified into n*m report year/calendar year cells. The cell identified by the ith report year and the jth relative calendar year in that report year would have a claim frequency of R(i)*C(j) times the total accident year frequency and a severity of $S_i{}^*T_j$ relative to the total accident year severity. It also holds that the mean severity over all n*m cells is

$$\sum_{i=1}^{n} \sum_{j=1}^{m} S_i T_j C(j) R(i) = 1$$

Since the above mean = 1, the coefficient of variation squared over all n*m cells is:

$$C^2 = \sum_{i=1}^{n} \sum_{j=1}^{m} [S_i T_j]^2 C(j) R(i) \quad - 1 \ .$$

Modified CV's for stratified sampling

We earlier developed a model indemnity size-of-loss distribution for an entire accident year, with a $(CV)^2$ of 30. But instead of simply sampling indemnity amounts from the entire accident year distribution, our Monte Carlo model will first select (randomly) a report year and then a calendar year paid for each random claim and then, based on the relative severity levels discussed above, sample from an indemnity distribution the mean of which has been adjusted to the levels corresponding to that report year and relative calendar year. Consequently, it becomes necessary to modify the CV applicable to each RY/CY stratum so that when you combine the sampled claims from the various RY/CY cells, you achieve the desired composite accident year $(CV)^2 = 30$.

To accomplish the desired approximation of the modified CV applicable to each RY/CY cell, we used a method first advanced by Hewitt.[7] He demonstrated that, if (a) a random variable Y were stratified into groups and (b) the means of the groups were lognormally distributed and (c) the variance of the logs of the means were S^2, and (d) if the variance of the logs of each group were $(\sigma_Y)^2$, a constant, then the variance of the logs of the combined distribution of all groups would be $S^2 + (\sigma_Y)^2$. The "spread parameter" S^2 over the n*m report year/calendar cells can be determined directly from the C^2, calculated above:

$$C^2 = e^{S^2} - 1$$

$$S^2 = \log(C^2 + 1)\ .$$

Thus,

$$\log(C^2 + 1) + (\sigma_Y)^2 = \mathrm{Log}(31)$$

and

$$(\sigma_Y)^2 = \mathrm{Log}(31) - \log(C^2 + 1)\ .$$

It should be emphasized that the above expression is an "approximation" of the modified variance (of the logs) to be used in the stratified sampling, since some of Hewitt's prerequisites are not necessarily met. Therefore, it is appropriate to perform a test of the stratified sampling, using sample values of R(i), C(j), S_i, and T_j, to determine if the overall accident year CV is achieved within an acceptable tolerance.

Testing the stratified sampling parameters

To determine appropriate values for the distributions of R(i),C(j),S_i, and T_j, we referred again to the NAIC closed claim studies. Using the detail NAIC data base, after the frequency and severity trend adjustments, we constructed a report year/calendar year matrix as shown in

[7]Hewitt, *op. cit.*, Appendix A, p. 167.

Appendix B. The entire claim data base, now adjusted to represent an accident year, was stratified into cells defined by ten report years and 16 calendar years (relative to the accident year). Each cell contains the (adjusted) claim counts, amounts, and averages. From the totals by report year, we derived the percentages of total claims by report year and the relative severity for each report year. On pages 5-6 of that same Appendix we determined relative severity values for calendar years, relative to report years. The values from this matrix will, then, be a starting point to determine the R(i),C(j),S_i, and T_j values for a specific case (it should be pointed out that the actual historical report year and calendar year patterns for a given jurisdiction and self-insured entity, to the extent that they are credible, should be given more weight than the NAIC numbers).

For this paper's case study, we have selected the report year and calendar year distributions shown in page 7 and 8 of Appendix B. We have used a total of seven report years (n = 7) and nine relative calendar years (m = 9). The relative severity factors have been selected (roughly from the NAIC matrix) and then adjusted so that the sum of the products of the frequency times the relative severities is 1.0. The $(CV)^2$ of the cell means,

$$C^2 = \sum_{i=1}^{n} \sum_{j=1}^{m} [S_i T_j]^2 C(j) R(i) \quad - 1$$

$$= .2607 .$$

Thus,

$$(\sigma_Y)^2 = \text{Log}(31) - \log(1.2607)$$

$$= 3.20232$$

and $\sigma_Y = 1.7895$.

Thus, while the standard deviation (of the logs) of the entire accident year is $\sqrt{\log(31)}$ = 1.8531, the standard deviation applicable to each cell will be reduced to 1.7895.

The results of our test of the stratified sampling versus unstratified is summarized in Appendix C. Rather than sampling from the lognormal distribution with no limit, we sampled successively from distributions with limits of $50,000, $100,000, $500,000, $1,000,000, $10,000,000, and $25,000,000. In each case, the unlimited mean was $100,000. For each limit, we (a) calculated the mean and CV directly,[8] (b) generated a sample mean and CV from the unstratified distribution, and (c) generated a sample mean and CV using the RY/CY strata with the adjusted means and appropriately reduced variance. To make sure we covered a full spectrum of possibilities, we used three values for $(CV)^2$: 10, 20, and 30. The report year and calendar year distributions were similar, but not identical, to those in pages 7-8 of Appendix B. For each combination, 100,000 claims were sampled.

The test samples demonstrated that the composite means and CV's derived from the stratified process were a good approximation to the direct calculation, within an acceptable tolerance.

THE ALAE COMPONENT IN THE MODEL

The ALAE-Indemnity relationship

Most excess policies written over a self-insured's SIR provide that ALAE on a claim (occurrence) is recoverable "pro rata," i.e., the percentage of the ALAE in a claim which is covered by the excess policy is the same as the percentage of the gross indemnity amount which is covered. Some contracts (relatively infrequent) set forth a retention level based on the *sum* of the indemnity and ALAE for one claim. In any case, the interaction between ALAE and indemnity would be an important consideration in any self-insured risk model.

It should be emphasized that in our self insured funding model the ALAE for the sampled claim is not treated as a constant factor related to the indemnity size (like tax and gratuity), but rather the *expected* ALAE (mean value of a separate ALAE distribution) is established, given the sample observed value of the indemnity. To treat ALAE otherwise would result in an understatement in the overall variability of the aggregate loss distribution.

To determine the functional relationship (if indeed a measurable relationship exists) between ALAE size and indemnity size for medical professional liability claims, we turned again to the NAIC Closed Claim Study.[9] As shown in Appendix D, Page 1, the *average* ALAE was calculated for each of several brackets of indemnity size. After plotting the average ALAE in each bracket against the corresponding average indemnity for the bracket, using logarithmic X and Y axes (see Appendix D, page 2), it was observed that a reasonably good straight line

[8] The calculation of the moments of a lognormal distribution limited (censored) by some limit L is fairly straightforward but is not covered here.

[9] NAIC, *op. cit.*

fit was obtainable, implying that the ALAE-indemnity relationship was representable by a member of the "power" curve family, $Y=AX^B$.

The equation used to regress the ALAE means against the indemnity values (grouped into brackets) is:

$$Log_e(Y) = A + B^*Log_e(X).$$

The weighted least squares best fit coefficients, using the number of claims in each indemnity bracket as weights, were

$$A = 3.66331$$

$$B = .482945$$

From the same data base which was used to develop this relationship between average ALAE and indemnity, it was also determined that the average indemnity was $53,363. Thus,

Let I = average indemnity = 53363.

Then restate the regression formula above by expressing both ALAE and indemnity as a ratio to the average indemnity over the entire distribution, as follows:

$$Y'=Y/I$$

$$X'=X/I$$

Then the restated expression becomes:

$$Log_e(I^*Y') = B^*Log_e(I^*X') + A .$$

Simplifying, you get

$$Log_e(Y') = B^*Log_e(X') + B^*Log_e(I) + A - Log_e(I)$$

$$= B^*Log_e(X') + (B-1)^*Log_e(I) + A .$$

Then let

$$C = (B-1)*\text{Log}_e(I) + A = -1.964768 .$$

You then have

$$\text{Log}_e(Y') = B*\text{Log}_e(X') + C$$

and

$$Y' = e^C X'^B = .1401884 * X'^{.482945}$$

For future reference, we call

$$D = e^C .$$

From the above expression, it can be noted that, in approximate terms, the expected ALAE varies in proportion to the square root of the sample indemnity.

Distribution of ALAE per claim, independent of indemnity

The next step of our treatment of ALAE in the model is to examine the distribution of ALAE per claim (defendant), irrespective of indemnity amounts. To do this, we again investigated the NAIC closed claim study.[10] The distribution is graphed in Appendix D, page 3. Using lognormal probability graph paper, the near straight line plot of the cumulative distribution function suggests that, just as was the case for the distribution of indemnity values by size, the ALAE amounts also can be represented quite adequately by the lognormal model.

We determined a mean and variance for the ALAE distribution two ways: first, we calculated the mean and variance directly from the data and then we followed the same procedure used for the indemnity graph. After drawing a straight line fit for the cumulative distribution function on the lognormal probability graph paper (the plotted points from the actual data were close enough to a straight line to allow us to simply draw the fitted line free-hand), we "picked off" the median and standard deviation directly from the graph, using the 50 percentile and +1

[10]For this distribution, we chose, for the sake of conservatism, the earlier 1975 version of the NAIC study, since the plotted CV was higher than that of the 1978 release.

standard deviation marks on the vertical scale, as follows:

Observed median $= e^{\mu} = 1355$.
Observed $\sigma = \log_e(5200) - \log_e(1355) = 1.345$

Our final selected value for the mean is, then

$$\exp(\log_e(1355)+(1.345)^2/2) = 3348 .$$

Of more importance, as will become clear later, our selected value for the variance was $(1.345)^2$, or 1.809.

Parameters for conditional ALAE distribution

We established earlier that, for purposes of sampling ALAE for any Monte Carlo simulation model, the *expected* ALAE in the distribution sampled from will be dependent on the sample indemnity value, or

$$E[Y|X] = DX^B ,$$

where

Y = random variable ALAE, conditional on value of indemnity, X
D = .1401884
B = .482945

and both Y and X are expressed relative to the unlimited mean indemnity.

Aitchison and Brown[11] have shown that if the random variable X is lognormally distributed with parameters μ and σ^2, then DX^B is also lognormally distributed with parameters log(D) + $B\mu$ and $B^2\sigma^2$. The parameters are the mean and variance, respectively, of the *logs* of the random variables.

We now let

S^2 = variance of the logs of ALAE means E[Y|X], conditional on sample indemnity values
$= B^2\sigma^2$

[11]*op. cit.*, p. 11.

$= (.482956)^2(1.8531)^2$
$= .8009$

Again employing Hewitt's method of isolating the "spread parameter",[12] we can solve for the variance applicable to each ALAE "group", $(\sigma_Y)^2$, defined as the sample ALAE *given* the sample indemnity mean:

We earlier derived an approximation for the combined variance

$$S^2 + (\sigma_Y)^2 = 1.809$$

then

$$(\sigma_Y)^2 = 1.809 - .8009$$
$$= 1 \text{ (approx.)}$$

In a word summary, then, we have established that the sample ALAE (relative to the unlimited mean indemnity) would be drawn from a lognormal distribution whose mean is .1401884 $X^{.482945}$ and the variance of whose logs is 1.0, where X represents the sample indemnity, relative to the unlimited mean indemnity.

Testing the sampled ALAE values, conditional on sample indemnity

Using the parameters estimated above, a test was set up to randomly sample 100,000 claims to make sure that the resulting overall ALAE sample moments were sufficiently close to those from direct calculations. For all ALAE combined, the coefficient of variation $(CV)_a$ is determined:

$$(CV_a)^2 = e^{S^2 + \sigma^2} - 1$$
$$= 5.104$$
$$CV_a = 2.259\ .$$

From our sample of 100,000 claims, the sample CV for ALAE was 2.24363.

THE MONTE CARLO MODEL

Having highlighted the key actuarial considerations in approximating the probability

[12]Hewitt, *loc. cit.*

distribution of self insured losses, we are now ready to describe the Monte Carlo model in some detail. The use of Monte Carlo models shows up with increasing regularity in the actuarial literature.[13] But despite the general agreement, in risk theory circles, that Monte Carlo models are an acceptable technique for approximating these distributions, this author perceives that any number of the direct approximation methods[14] are considered superior, assuming that the mean and variance of the distribution can be calculated directly and precisely.

Given all of the interactions between the many variables discussed above -- e.g., the calendar year payout and the present value calculation and the indexed retention and the ALAE-indemnity relationship -- plus the necessity of treating the runoff of prior years' losses and the target prospective year simultaneously, this author is hard pressed to identify any direct approximation formula from any risk theory text which will yield adequate results for the defined problem. The use of a Monte Carlo model, in which all of the interactions can be adequately defined and programmed into one composite risk process, would appear to be the only satisfactory approach.

A full description of our self-insurance Monte Carlo risk model is included in Appendix E. In the first section, we have listed the miscellaneous assumptions, the input parameters, and the various distributions from which samples are made. For our selected case study (which we will call "XYZ Hospital"), the initial target year is accident year 1989. A second run, made one year later, considers the run off from the 1989 year and the 1990 prospective losses. In the second section of the outline the actual simulation process for one trial (normally, at least 1,000 trials are run for a given case study) is outlined in pseudo code. Tracking the program flow through this pseudo code will reveal how the many variables interact with each other.

Parameter variance

Over the past few years there have been welcome additions to the body of actuarial literature dealing with *parameter variance*, as it relates to simulation models to approximate aggregate loss distributions. We will not attempt in this paper to supply another textbook treatment of *parameter variance* and *process variance*. Suffice it to say that it would be hard to imagine deriving any valid results from a Monte Carlo risk model which did not incorporate some kind of parameter variance -- particularly for a line with as much uncertainty surrounding the

[13]See, for example, P. E. Heckman and G. G. Meyers, "The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions," PCAS, LXX (1983), p. 22.

[14]No attempt will be made to provide a list of these methods here.

"universe" means (frequency and severity) as medical professional liability.

The key point is that the parameter variance is the same over all size of hospital risks. The vagaries of the business -- the social, economic, and legal dynamics which dictate that we do not deal from the same 52-card deck from one year to the next -- apply equally to all sizes of risks. Thus, while the process variance may play the lead role in driving the overall aggregate loss distribution for small hospitals, the parameter variance is predominate in models of larger risks, for which the process variance, or the pure statistical sampling error, has been reduced simply by virtue of the larger volume.

Rather than mathematically rolling the parameter and process variances into one combined variance for simulation purposes, this author chose to incorporate the two variances into the model as separate routines, in step-wise fashion. For a given trial, the first step is to randomly select the "universe" frequency and severity (average unlimited indemnity) from distributions the means of which represent our best estimate of these two parameters, based on the data which is available (statewide, countrywide, the hospital itself). The standard deviations of these distributions of the frequency and severity universe means are judgmentally selected to represent the "uncertainty" surrounding these means, resulting from many forces. This author is not aware of any successful attempts to quantify these factors, if, indeed, all of them have identified.

After the universe mean frequency and severity have been selected, the second step is to select the *sample* frequency (or total claim count) and then, for each claim, the *sample* claim amount. For sampling the frequency distribution, we use a Poisson process, unless the "universe" mean, selected in the first step, is greater than 15, in which case the model uses the normal approximation. As developed earlier in some detail, the lognormal distribution is used in the sampling of the individual claim amounts.

IBNR Distribution

Treating the runoff from prior years as a random process in our model requires not only simulating the payoff of reported and unpaid claims but also determining the *expected* IBNR from those years and the distribution around that expected value. As was shown in the description of the model and the "pseudo-code", the open cases are treated separately from the IBNR's. For our model, the expected number of IBNR's is determined by selecting an *a priori* total ultimate claim count for each of the prior years, and multiplying times the reporting percentages taken from our assumed reporting distribution. The actual *sample* number of

IBNR's for a particular trial is then determined by randomly selecting an ultimate number of claims for the prior year in question and then for each of these claims randomly selecting the report year (again from the report year distribution). If the report year thus selected is prior to the current year (thus indicating the claim would have already been reported) the claim is not counted as an IBNR and the loop continues to the next claim.

DERIVATION OF THE PROBABILITY DISTRIBUTION OF REQUIRED CONTRIBUTIONS

The Simulated Loss Distribution

With the Monte Carlo model loaded up with the appropriate input parameters and distributions, we can now make the run for our selected case study. For case 1, the initial year of the fund, the resulting printout of the distribution, generated from 1,000 trials of the model, is shown in Appendix F, page 2. A printout of the input parameters is on page 1 of that appendix. The results of the 1,000 trials have been tabulated and summarized into 31 intervals of retained losses (at present value), including the number of "hits" in each bracket and also the total retained losses in each bracket.

For case 2, performed on the same hospital at the beginning of the second year of the fund, the input parameters are shown on page 3 of Appendix F. For this case, the current assets become part of the input variables, as well as the assumed expected average indemnity on unpaid claims from the prior year. The resulting distribution of required *additional* funding for year 2 is shown on page 4. It can be noted that in over half of the trials no additional funds would have been required. In other words, the assets of the fund after one year (the first year's contribution plus earned interest less the losses paid) would have carried forward sufficient safety margin to cover not only the run-off from year one but also a second year's incurred losses. However, in order to continue to maintain funding at a high level of confidence for year 2, additional funding is required.

The histogram of the simulated distribution and the cumulative distribution ogive for cases 1 and 2 are shown on pages 5 and 6, for the first and second year funding. These plots display a fairly smooth and regular contour -- so much so that, with enough effort and with an appropriate set of parameters, someone could undoubtedly uncover some exotic probability density function which would supply an acceptable "fit" to this curve. But what purpose would this serve? It would be unlikely that such a curve, or even a member of its immediate family, would adequately fit another case defined by an entirely different set of initial variables (retentions, unlimited means, report-year/calendar-year payouts, etc.). Thus, the final estimated loss and required contribution distributions in Appendix F, generated solely for this one particular situation, initial funding and second year renewal funding, are simply what they are. They need no name.

From the final simulated distributions of required funding, one needs only to make a few simple interpolations to approximate the indicated funding levels at selected confidence levels. For this example we chose to display the 90%, 95%, and 99% confidence levels. These interpolations are shown at the end of the printouts on pages 2 and 4. Thus, the indicated funding levels for the two years would be as follows:

	Confidence Level		
	90%	95%	99%
Year 1	$2,340,000	$2,734,000	$3,594,000
Year 2	1,457,000	1,968,000	2,980,000

The second year funding indication depends, of course, on which funding level was selected for year one (corresponding to a selected level of confidence) and what the assets were at the beginning of year 2. For our case study, we assumed that the assets, after the first year's contribution, one year's interest earnings on the funds, less the disbursements (paid losses) were $2,950,000. We further assumed that there were seven claims reported and unpaid from year 1 at the beginning of year 2, with an average reserve of $130,000.

SUMMARY AND CONCLUSIONS

In this paper we have developed a procedure to determine the required funding for hospitals which self insure some layer of their professional liability exposure. The method would apply equally to workers' compensation. To derive indicated funding at various confidence levels, a probability distribution is approximated which combines the runoff of losses from prior years with the prospective losses of the target year. This distribution is approximated with a Monte Carlo simulation model, incorporating the interaction of many variables. The model is designed to be run on an annual basis, and at each renewal it calculates the distribution of additional contributions required which, when combined with the current assets, will cover the present value of all losses.

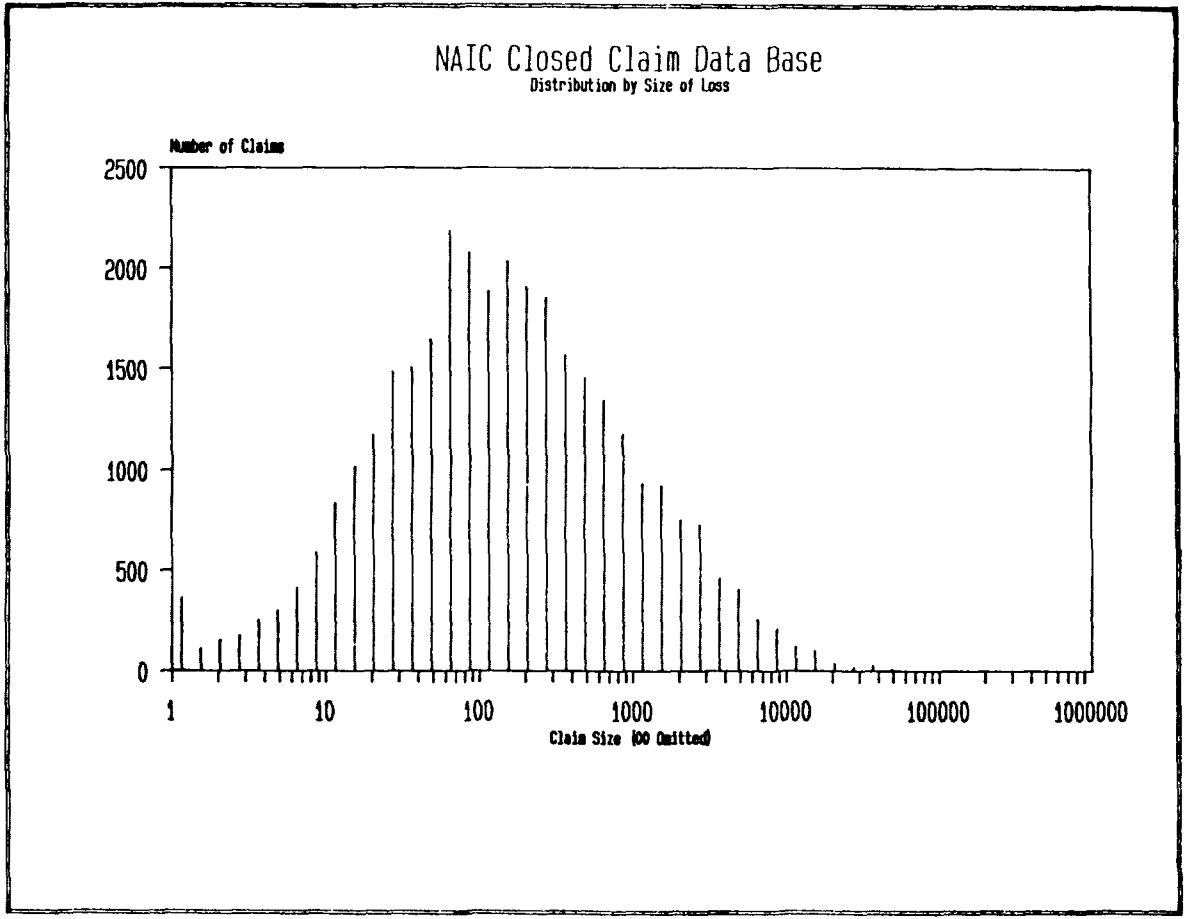
NAIC CLOSED CLAIM DATA BASE - ADJUSTED FOR FREQUENCY/SEVERITY INDICES

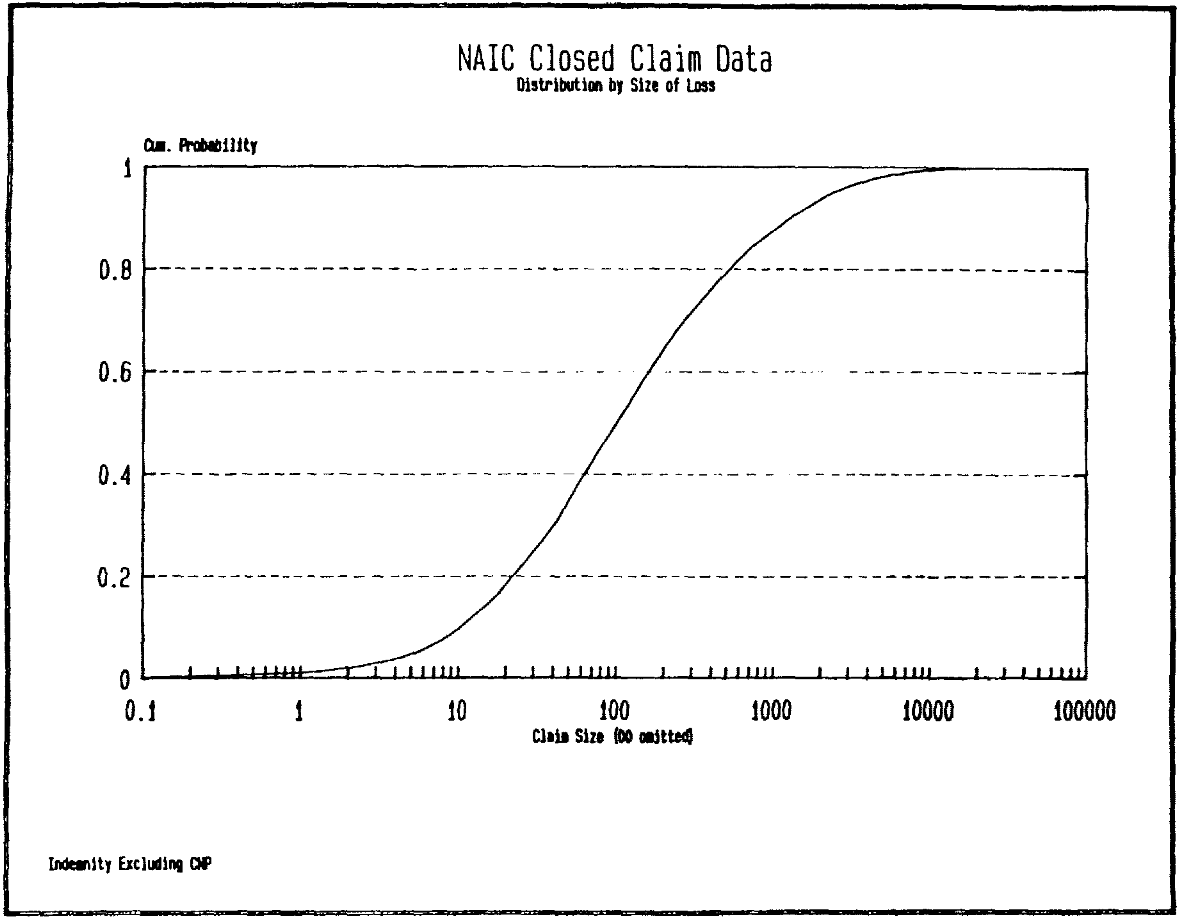
Distribution by Size of Loss

All Claims Combined

Bracket#	# Claims	Cum. # Claims	Indem. Amount	Avg. Indem.	Exp.Amount	Avg. Expense
0	51607.8	51607.8	0	0	133432000	2586
100	358.3	51966.1	18105	51	82011	229
133	103.2	52069.3	11821	115	28022	272
178	145.3	52214.6	22401	154	24138	166
237	167.7	52382.3	34386	205	65789	392
316	242.8	52625.1	67813	279	127607	526
422	292.9	52918.0	108852	372	120612	412
562	411.8	53329.8	201463	489	306647	745
750	581.2	53911.0	379945	654	409760	705
1000	828.3	54739.3	720464	870	767031	926
1334	1015.0	55754.3	1167310	1150	1408230	1387
1778	1170.2	56924.5	1831020	1565	1483850	1268
2371	1477.1	58401.6	3059210	2071	2794090	1892
3162	1499.5	59901.1	4177710	2786	2815350	1878
4217	1640.8	61541.9	6069360	3699	3594630	2191
5623	2180.2	63722.1	10755100	4933	5663140	2598
7499	2071.1	65793.2	13590200	6562	6580210	3177
10000	1884.5	67677.7	16401600	8703	5619610	2982
13335	2029.0	69706.7	23358300	11512	7190910	3544
17783	1906.4	71613.1	29460500	15453	9797740	5139
23714	1848.9	73462.0	37950200	20526	8096010	4379
31623	1564.3	75026.3	42906200	27428	8307880	5311
42170	1448.2	76474.5	53156900	36705	8734200	6031
56234	1340.3	77814.8	65590800	48937	9357350	6982
74989	1171.7	78986.5	76561700	65342	9231510	7879
100000	926.5	79913.0	79771100	86099	7090310	7653
133352	917.8	80830.8	105277000	114706	8637350	9411
177828	746.2	81577.0	114798000	153843	10081600	13511
237137	722.3	82299.3	148033000	204947	10681500	14788
316228	456.1	82755.4	124647000	273289	6077140	13324
421697	402.6	83158.0	145920000	362444	7202570	17890
562341	247.9	83405.9	120768000	487164	4983840	20104
749894	199.7	83605.6	129525000	648598	7204110	36075
1000000	112.6	83718.2	97909200	869531	2094480	18601
1333520	93.3	83811.5	106538000	1141890	2284480	24485
1778280	34.0	83845.5	50086600	1473140	1177000	34618
2371370	15.1	83860.6	30357800	2010450	434327	28763
3162280	22.4	83883.0	62135900	2773920	978374	43677
4216970	4.9	83887.9	19205700	3919530	206093	42060
5623410	0.0	83887.9	0	0	0	0
7498940	0.0	83887.9	0	0	0	0
10000000	0.0	83887.9	0	0	0	0
TOTALS		83887.9	1722570000	20534	295171000	3519
TOTAL, EXCL. CNP's		32280.1	1722570000	53363	161739000	5011

#End point of interval of indemnity amount

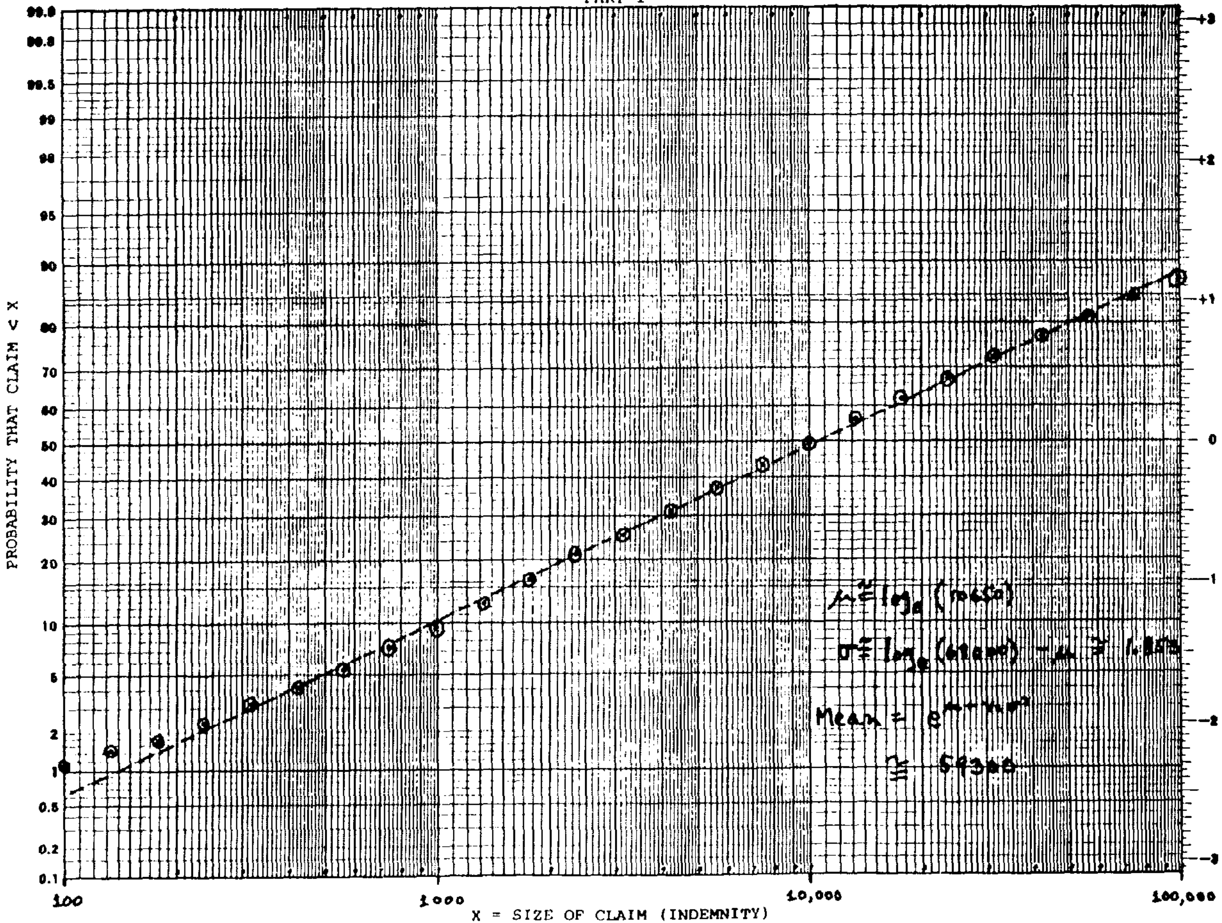

NAIC Closed Claim Data Base
Distribution by Size of Loss
Number of Claims
2500
2000
1500
1000
500
0
1
10
100
1000
10000
100000
1000000
Claim Size (00 Omitted)


NAIC Closed Claim Data
Distribution by Size of Loss
Cum. Probability
1
0.8
0.6
0.4
0.2
0
0.1
1
10
100
1000
10000
100000
Claim Size (00 omitted)
Indemnity Excluding CNP

NAIC CLOSED CLAIM STUDY

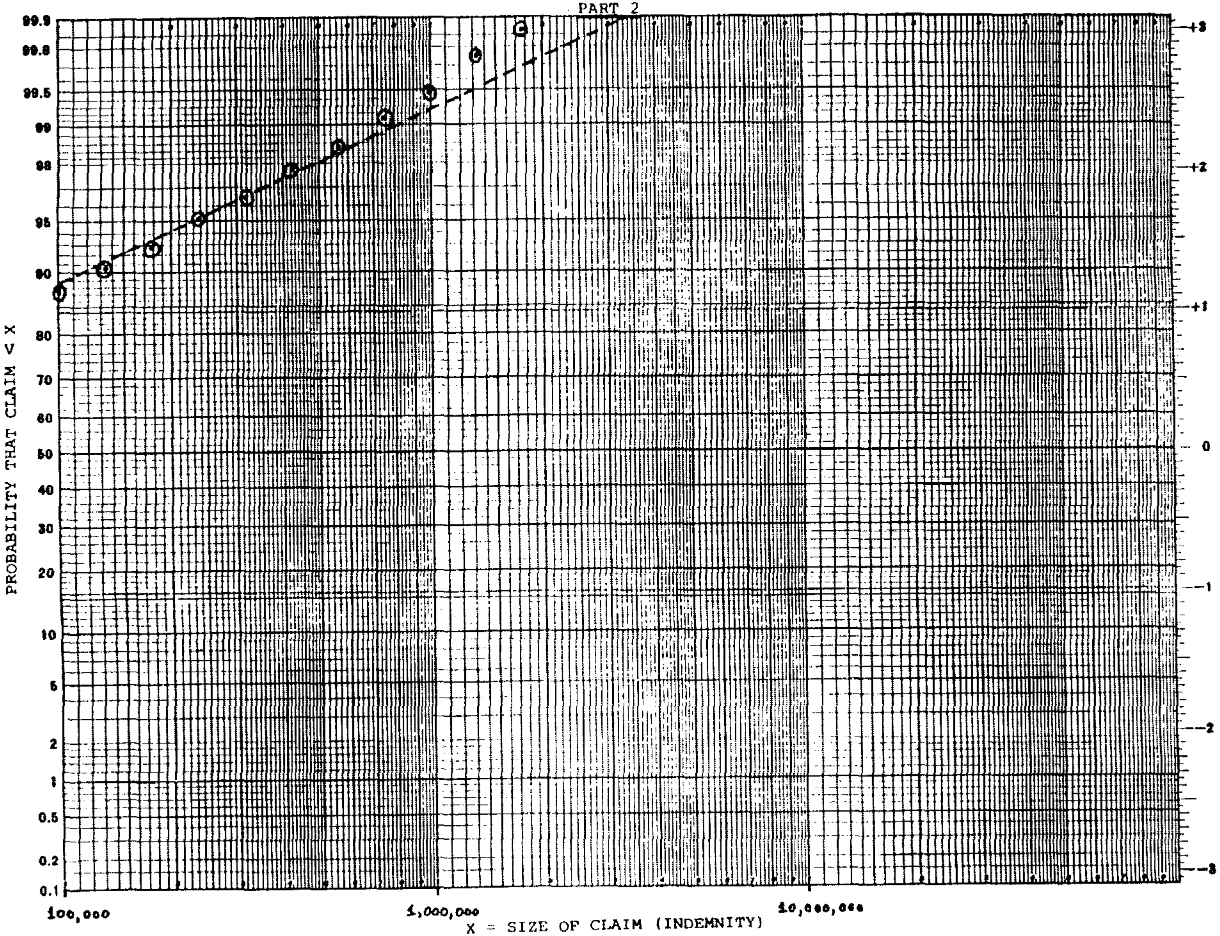
CUMULATIVE DISTRIBUTION BY SIZE OF LOSS

PART 1



Appendix A

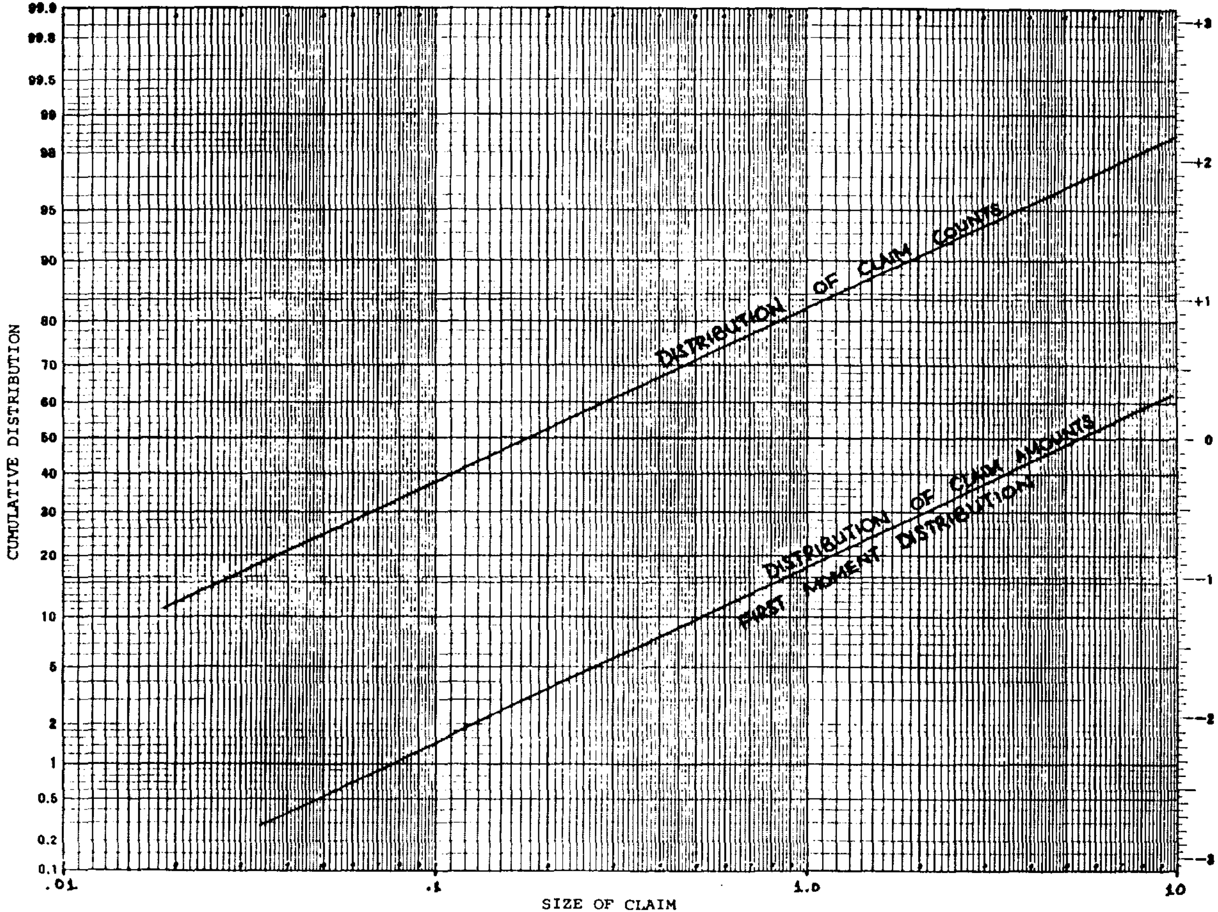
NAIC CLOSED CLAIM STUDY
CUMULATIVE DISTRIBUTION BY SIZE OF LOSS
PART 2



Appendix A

MEDICAL PROFESSIONAL LIABILITY - MODEL CLAIM SIZE DISTRIBUTION

(MEAN = 1)



Appendix A

NAIC CLOSED MEDICAL LIABILITY CLAIMS - ADJUSTED FOR FREQUENCY/SEVERITY TRENDS

REPORT YEAR/CALENDAR YEAR MATRIX FOR LOSSES OF ONE ACCIDENT YEAR

	Report Year										
Cal.Year	1	2	3	4	5	6	7	8	9	10+	Total CY
1:											
#CWI	4218.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4218.5
INDEM	32999300	0	0	0	0	0	0	0	0	0	32999300
#CWI/CWE	2822.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2822.5
ALAE	1985220	0	0	0	0	0	0	0	0	0	1985220
#CNP	11648.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	11648.7
AVG.INDEM	7823	0	0	0	0	0	0	0	0	0	7823
AVG.ALAE	703	0	0	0	0	0	0	0	0	0	703
2:											
#CWI	3400.8	998.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4399.5
INDEM	97259700	15218500	0	0	0	0	0	0	0	0	112478000
#CWI/CWE	4685.5	1305.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5990.8
ALAE	11119700	2209980	0	0	0	0	0	0	0	0	13329700
#CNP	6591.1	2076.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	8667.3
AVG.INDEM	28599	15238	0	0	0	0	0	0	0	0	25566
AVG.ALAE	2373	1693	0	0	0	0	0	0	0	0	2225
3:											
#CWI	2771.4	2013.0	659.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5443.8
INDEM	114001000	63847100	11433500	0	0	0	0	0	0	0	189282000
#CWI/CWE	4473.3	4075.9	1418.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	9967.7
ALAE	18193800	10811900	2336750	0	0	0	0	0	0	0	31342400
#CNP	1754.2	2141.0	966.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4862.1
AVG.INDEM	41135	31717	17339	0	0	0	0	0	0	0	34770
AVG.ALAE	4067	2653	1647	0	0	0	0	0	0	0	3144
4:											
#CWI	2059.9	2065.8	1183.4	196.7	0.0	0.0	0.0	0.0	0.0	0.0	5505.8
INDEM	119169000	97840500	37162100	5590990	0	0	0	0	0	0	259762000
#CWI/CWE	3439.6	3882.1	2687.6	425.0	0.0	0.0	0.0	0.0	0.0	0.0	10434.3
ALAE	22287000	22019200	9143340	750406	0	0	0	0	0	0	54200000
#CNP	512.5	700.7	945.5	367.6	0.0	0.0	0.0	0.0	0.0	0.0	2526.3
AVG.INDEM	57852	47362	31403	28424	0	0	0	0	0	0	47180
AVG.ALAE	6480	5672	3402	1766	0	0	0	0	0	0	5194
5:											
#CWI	1287.5	1494.5	1353.5	365.7	100.6	0.0	0.0	0.0	0.0	0.0	4601.8
INDEM	90294500	99669400	60844300	19196000	4988230	0	0	0	0	0	274992000
#CWI/CWE	2012.5	2716.1	2502.3	852.4	165.1	0.0	0.0	0.0	0.0	0.0	8248.4
ALAE	15079500	18122900	12210800	2898220	360409	0	0	0	0	0	48671900
#CNP	221.8	359.9	411.7	357.0	175.5	0.0	0.0	0.0	0.0	0.0	1525.9
AVG.INDEM	70132	66691	44953	52491	49585	0	0	0	0	0	59758
AVG.ALAE	7493	6672	4880	3400	2183	0	0	0	0	0	5901

NAIC CLOSED MEDICAL LIABILITY CLAIMS - ADJUSTED FOR FREQUENCY/SEVERITY TRENDS

REPORT YEAR/CALENDAR YEAR MATRIX FOR LOSSES OF ONE ACCIDENT YEAR

	Report Year										
Cal.Year	1	2	3	4	5	6	7	8	9	10+	Total CY
6:											
#CWI	637.0	954.0	936.5	448.5	175.6	40.5	0.0	0.0	0.0	0.0	3192.1
INDEM	62810500	68229600	51776400	29412800	7451200	2176270	0	0	0	0	221857000
#CWI/CWE	1047.0	1611.6	1645.2	822.7	354.8	72.2	0.0	0.0	0.0	0.0	5553.5
ALAE	8918380	15749100	11532300	5062490	1394870	124000	0	0	0	0	42781100
#CNP	114.7	234.5	257.7	144.7	186.3	89.3	0.0	0.0	0.0	0.0	1027.2
AVG.INDEM	98604	71520	55287	65580	42433	53735	0	0	0	0	69502
AVG.ALAE	8518	9772	7010	6154	3931	1717	0	0	0	0	7703
7:											
#CWI	312.0	457.1	501.4	288.3	159.3	91.9	35.5	0.0	0.0	0.0	1845.5
INDEM	25695300	43347500	36378300	28472600	7354830	7186140	1840770	0	0	0	150276000
#CWI/CWE	519.9	842.0	850.2	588.1	311.2	186.9	64.2	0.0	0.0	0.0	3362.5
ALAE	5202720	7814970	7501700	5747590	1468480	847159	119039	0	0	0	28701700
#CNP	63.4	118.7	161.5	64.2	72.1	118.8	76.6	0.0	0.0	0.0	675.3
AVG.INDEM	82357	94832	72553	98760	46170	78195	51853	0	0	0	81428
AVG.ALAE	10007	9281	8823	9773	4719	4533	1854	0	0	0	8536
8:											
#CWI	166.6	169.9	246.1	154.4	107.2	86.1	69.5	48.6	0.0	0.0	1048.4
INDEM	22553100	20196800	24585500	23101900	14556600	8527560	15314600	3321850	0	0	132158000
#CWI/CWE	261.4	343.5	508.7	319.7	174.7	138.9	138.0	52.6	0.0	0.0	1937.5
ALAE	2390770	3691680	5584570	2777160	1436690	1067770	561138	106096	0	0	17615900
#CNP	53.6	57.0	60.5	24.4	33.7	32.2	78.4	56.9	0.0	0.0	396.7
AVG.INDEM	135373	118875	99900	149624	135789	99043	220354	68351	0	0	126057
AVG.ALAE	9146	10747	10978	8687	8224	7687	4066	2017	0	0	9092
9:											
#CWI	91.9	125.3	138.1	87.3	55.5	76.6	56.4	45.1	12.6	0.0	688.8
INDEM	16032600	16306000	28059500	18619100	4322220	16250000	3333660	6686580	553704	0	110163000
#CWI/CWE	178.4	208.7	264.1	197.0	108.4	104.9	95.6	99.4	17.8	0.0	1274.3
ALAE	2408700	2365090	4652560	2499220	1299700	798795	524145	559968	17809	0	15126000
#CNP	17.6	27.6	46.9	22.8	16.0	21.9	50.8	61.2	31.2	0.0	296.0
AVG.INDEM	174457	130136	203182	213277	77878	212141	59107	148261	43945	0	159935
AVG.ALAE	13502	11332	17617	12686	11990	7615	5483	5633	1001	0	11870
10:											
#CWI	38.2	59.6	102.1	64.4	30.8	33.5	33.0	46.3	40.9	9.4	458.2
INDEM	6153010	10470000	25091200	11632900	7919250	9263940	6371450	7106910	6234870	550632	90794100
#CWI/CWE	77.4	99.1	153.7	100.6	70.3	59.8	61.3	70.1	72.5	21.1	785.9
ALAE	767007	1593940	2390640	1606360	1013300	495524	477845	302084	354016	791526	9792240
#CNP	7.0	12.2	30.3	19.0	2.2	0.0	9.8	7.4	30.5	7.1	125.5
AVG.INDEM	161074	175671	245751	180635	257119	276536	193074	153497	152442	58578	198154
AVG.ALAE	9910	16084	15554	15968	14414	8286	7795	4309	4883	37513	12460

NAIC CLOSED MEDICAL LIABILITY CLAIMS - ADJUSTED FOR FREQUENCY/SEVERITY TRENDS

REPORT YEAR/CALENDAR YEAR MATRIX FOR LOSSES OF ONE ACCIDENT YEAR

page 3 of 4

Cal.Year	1	2	3	4	5	6	7	8	9	10+	Total CY
11:											
#CWI	32.4	28.4	50.5	27.5	10.6	12.8	40.6	20.3	30.8	49.0	302.9
INDEM	4386450	4233490	5834620	5565890	1339760	765109	3187830	3816980	1481460	4948140	35559700
#CWI/CWE	47.1	55.4	80.9	50.7	23.5	14.9	67.9	56.2	59.2	54.5	510.3
ALAE	747815	842896	1031990	859194	329762	109646	626445	510024	462201	185338	5705310
#CNP	10.5	12.1	10.4	4.8	0.0	7.6	5.3	2.6	17.6	46.0	116.9
AVG.INDEM	135384	149067	115537	202396	126393	59774	78518	188029	48099	100982	117398
AVG.ALAE	15877	15215	12756	16947	14032	7359	9226	9075	7807	3401	11180
12:											
#CWI	8.3	24.1	24.2	16.0	12.8	16.5	13.4	26.0	28.1	57.6	227.0
INDEM	1117480	6275840	4763150	2770500	2870580	4038150	1583540	2216640	5530560	6412130	37578600
#CWI/CWE	32.7	51.7	37.0	19.0	18.1	19.3	29.8	28.3	44.1	109.9	389.9
ALAE	545520	764081	426788	214047	146696	109696	443067	434754	1042200	13770900	17897800
#CNP	8.0	0.0	10.8	10.7	5.2	0.0	0.0	2.7	0.0	47.1	84.5
AVG.INDEM	134636	260408	196824	173156	224264	244736	118175	85255	196817	111322	165544
AVG.ALAE	16683	14779	11535	11266	8105	5684	14868	15362	23633	125304	45904
13:											
#CWI	5.4	5.3	22.4	2.8	5.3	2.8	5.2	8.9	24.8	57.5	140.4
INDEM	1103330	1078500	6978730	137336	560984	610383	1687220	10049400	5933220	6842250	34981300
#CWI/CWE	8.2	16.5	30.6	5.4	13.9	2.7	13.6	8.9	24.8	98.2	222.8
ALAE	117986	235890	551016	97325	50023	29515	212639	367939	349327	571351	2583010
#CNP	2.8	2.6	2.7	0.0	0.0	2.8	2.6	8.5	13.6	24.8	60.4
AVG.INDEM	204320	203491	311550	49049	105846	217994	324464	1129140	239243	118996	249155
AVG.ALAE	14389	14296	18007	18023	3599	10931	15635	41341	14086	5818	11593
14:											
#CWI	8.8	5.8	14.4	0.0	0.0	5.5	0.0	2.8	5.7	66.7	109.7
INDEM	162381	519054	1590490	0	0	615932	0	2136340	385388	14013500	19423100
#CWI/CWE	9.0	8.8	23.0	5.7	0.0	5.5	0.0	2.8	14.3	89.6	158.7
ALAE	27520	178291	246741	26013	0	60947	0	1896	525605	805173	1872190
#CNP	5.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	17.8	23.6
AVG.INDEM	18452	89492	110451	0	0	111988	0	762979	67612	210098	177056
AVG.ALAE	3058	20260	10728	4564	0	11081	0	677	36756	8986	11797
15:											
#CWI	5.7	3.1	0.0	0.0	0.0	0.0	0.0	3.1	0.0	14.8	26.7
INDEM	1076180	274815	0	0	0	0	0	1544170	0	1588750	4483920
#CWI/CWE	14.5	3.1	0.0	3.0	0.0	0.0	0.0	3.1	3.0	43.9	70.6
ALAE	221683	99595	0	80071	0	0	0	609400	29850	473084	1513680
#CNP	0.0	0.0	0.0	0.0	0.0	0.0	3.0	0.0	0.0	5.8	8.8
AVG.INDEM	188804	88650	0	0	0	0	0	498121	0	107348	167937
AVG.ALAE	15288	32127	0	26690	0	0	0	196581	9950	10776	21440

NAIC CLOSED MEDICAL LIABILITY CLAIMS - ADJUSTED FOR FREQUENCY/SEVERITY TRENDS

REPORT YEAR/CALENDAR YEAR MATRIX FOR LOSSES OF ONE ACCIDENT YEAR

Cal.Year	Reoort Year 1	2	3	4	5	6	7	8	9	10+	Total CY
16:											
#CWI	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	70.4	70.4
INDEM	0	0	0	0	0	0	0	0	0	15784300	15784300
#CWI/CWE	0.0	0.0	0.0	3.1	0.0	3.1	0.0	0.0	0.0	80.3	86.5
ALAE	0	0	0	16284	0	65949	0	0	0	1951080	2033320
#CWP	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	21.4	21.4
AVG.INDEM	0	0	0	0	0	0	0	0	0	224209	224209
AVG.ALAE	0	0	0	5253	0	21274	0	0	0	24297	23507
Total Reo.Year											
#CWI	15044.4	8404.6	5232.0	1651.6	657.7	366.2	253.6	201.1	142.9	325.4	32279.5
INDEM	594814000	447507000	294498000	144500000	51363700	49433500	33319100	36878800	20119200	50139700	1722570000
#CWI/CWE	19629.0	15219.8	10201.8	3392.4	1240.0	608.2	470.4	321.4	235.7	497.5	51816.2
ALAE	90013400	86499500	57609200	22634400	7499930	3709000	2964320	2892160	2781010	18548500	295151000
#CWP	21011.7	5742.5	2904.9	1015.2	491.0	272.6	226.5	139.3	92.9	170.0	32066.6
AVG.INDEM	39537	53246	56288	87491	78096	134990	131384	183386	140792	154086	53364
AVG.ALAE	4586	5683	5647	6672	6048	6098	6302	8999	11799	37283	5696
Ratio, avg. indemnitv to total acc.yr.	.74	1.00	1.05	1.64	1.46	2.53	2.46	3.44	2.64	2.89	
"Smoothed" avg. indem. ratio	.74	1.00	1.20	1.40	1.60	2.00	2.35	2.70	2.845	3.00	
Ratio, total # claims to total acc.yr.	.484	.250	.156	.053	.021	.011	.008	.005	.004	.008	

Source: NAIC Maloractice Claims: Medical Maloractice Closed Claims, 1975-78, National Association of Insurance Commissioners. 1980.
Adjustments for frequencv/severitv trends performed bv the author on the detail data tape purchased from NAIC. Accordingly, the conclusions drawn from the adjusted data are those of the author and not necessarily those of the NAIC.

MAIC CLOSED MEDICAL LIABILITY CLAIMS - ADJUSTED FOR FREQUENCY/SEVERITY TRENDS

REPORT YEAR/CALENDAR YEAR MATRIX FOR LOSSES OF ONE ACCIDENT YEAR

Average Indemnity by Calendar Year Components of Report Year
Page 1 of 2

Cal. Year	Report Year 1	2	3	4	5	6	7	8	9	10+	
1:	7823	<---------- Average indemnity									
	0.198	<---------- Ratio, avg. indemnity to avg.ind.,total report year									
2:	28599	15238									
	0.723	0.286									
3:	41135	31717	17339								
	1.040	0.596	0.308								
4:	57852	47362	31403	28424							
	1.463	0.890	0.558	0.325							
5:	70132	66691	44953	52491	49585						
	1.774	1.253	0.799	0.600	0.635						
6:	98604	71520	55287	65580	42433	53735					
	2.494	1.343	0.982	0.750	0.543	0.398					
7:	82357	94832	72553	98760	46170	78195	51853				
	2.083	1.781	1.289	1.129	0.591	0.579	0.395				
8:	135373	118875	99900	149624	135789	99043	220354	68351			
	3.424	2.233	1.775	1.710	1.739	0.734	1.677	0.373			
9:	174457	130136	203182	213277	77878	212141	59107	148261	43945		
	4.412	2.444	3.610	2.438	0.997	1.572	0.450	0.808	0.312		
10:	161074	175671	245751	180635	257119	276536	193074	153497	152442	58578	
	4.074	3.299	4.366	2.065	3.292	2.049	1.470	0.837	1.083	0.380	Rel. CY 1
11:	135384	149067	115537	202396	126392	59774	78518	188029	48099	100982	
	3.424	2.800	2.053	2.313	1.618	0.443	0.598	1.025	0.342	0.655	Rel. CY 2
12:	134636	260408	196824	173156	224264	244736	118175	85255	196817	111322	
	3.405	4.891	3.497	1.979	2.872	1.813	0.899	0.465	1.398	0.722	Rel. CY 3
13:	204320	203491	311550	49049	105846	217994	324465	1129150	239243	118996	
	5.168	3.822	5.535	0.561	1.355	1.615	2.470	6.157	1.699	0.772	etc.
14:	18452	89492	110451	0	0	111988	0	762979	67612	210097	
	0.467	1.681	1.962	0.000	0.000	0.830	0.000	4.161	0.480	1.364	
15:	188804	88650	0	0	0	0	0	498119	0	107348	
	4.775	1.665	0.000	0.000	0.000	0.000	0.000	2.716	0.000	0.697	
16:	0	0	0	0	0	0	0	0	0	224209	
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.455	
Total	39537	53246	56288	87491	78096	134990	131384	183386	140792	154086	
rep.yr	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	

NAIC CLOSED MEDICAL LIABILITY CLAIMS - ADJUSTED FOR FREQUENCY/SEVERITY TRENDS

REPORT YEAR/CALENDAR YEAR MATRIX FOR LOSSES OF ONE ACCIDENT YEAR

Average Indemnity by Calendar Year Components of Report Year
Page 2 of 2

Composite Average Indemnity by Relative Calendar Year Cells

		Smoothed
relative cal. year 1 avg. =	0.233	.25
relative cal. year 2 avg. =	0.669	.67
relative cal. year 3 avg. =	0.891	.89
relative cal. year 4 avg. =	1.295	1.30
relative cal. year 5 avg. =	1.531	1.53
relative cal. year 6 avg. =	2.125	2.13
relative cal. year 7 avg. =	2.623	2.60
relative cal. year 8 avg. =	3.173	2.80
relative cal. year 9 avg. =	2.972	3.00

XYZ HOSPITAL

Assumed Distribution of Claims by Report Year

For Claims Incurred in One Accident Year

Report Year	(1) Ratio, Number of Claims Reported to Total Accident Year Claims	(2) Ratio, Average Indemnity to Average for Entire Accident Year	(3) Ratio, Amount of Indemnity to Total Accident Year = (1) x (2)
1	.387	.73873	.28589
2	.300	.98498	.29549
3	.201	1.18197	.23758
4	.066	1.37897	.09101
5	.025	1.67446	.04186
6	.012	2.16695	.02600
7	.009	2.46245	.02216
Total	1.0000		1.000

XYZ HOSPITAL

Assumed Distribution of Claims by Calendar Year of Payment

For Claims Incurred in One Report Year

Calendar Year	(1) Ratio, Number of Claims Paid to Total Report Year	(2) Ratio, Average Indemnity to Average for Entire Report Year	(3)* Ratio, Amount of Indemnity to Total Report Year = (1) x (2)
1	.25742	.26416	.068
2	.18505	.70794	.131
3	.25840	.94040	.243
4	.13104	1.37362	.180
5	.07175	1.61664	.116
6	.03110	2.25062	.070
7	.02403	2.74724	.066
8	.02197	2.95857	.065
9	.01924	3.16989	.061
Total	1.0000		1.000

* Column (1) x Column (2)

Note: Distribution includes *all* claims from ground up

MEDICAL PROFESSIONAL LIABILITY CLAIM SIZE DISTRIBUTION

TEST OF SAMPLED MEANS AND CV'S, STRATIFIED AND UNSTRATIFIED COMPARED TO DIRECT CALCULATIONS, WITH VARIOUS POLICY LIMITS

Lognormal distribution with Unlimited mean = 100,000

Each sample = 100,000 random trials

	Unlim. CV^2=10		Unlim. CV^2=20		Unlim. CV^2=30	
	Limited Mean	Limited CV	Limited Mean	Limited CV	Limited Mean	Limited CV
Limit=50,000						
Direct Calc.	29686	0.6361	26076	0.7511	24185	0.8173
Sample, unstrat.	29716	0.6352	26119	0.7502	24231	0.8164
Sample, strat.	29242	0.6525	25717	0.7655	23861	0.8309
Limit=100,000						
Direct Calc.	43878	0.8464	38297	0.9696	35416	1.0413
Sample, unstrat.	43960	0.8453	38370	0.9681	35476	1.0395
Sample, strat.	43245	0.8614	37723	0.9831	34868	1.0544
Limit=500,000						
Direct Calc.	77888	1.4981	70163	1.6635	65847	1.7595
Sample, unstrat.	77742	1.4948	69996	1.6605	65667	1.7566
Sample, strat.	77020	1.5166	69251	1.6829	64935	1.7796
Limit=1,000,000						
Direct Calc.	88071	1.8412	81451	2.0531	77437	2.1725
Sample, unstrat.	87797	1.8374	81158	2.0508	77136	2.1711
Sample, strat.	87386	1.8657	80648	2.0786	76594	2.1988
Limit=10,000,000						
Direct Calc.	99499	2.8548	98364	3.5134	97273	3.8728
Sample, unstrat.	98367	2.7628	96964	3.4215	95784	3.7946
Sample, strat.	99335	2.9231	98250	3.5966	97164	3.9585
Limit=25,000,000						
Direct Calc.	99916	3.0473	99582	3.9620	99169	4.4987
Sample, unstrat.	98575	2.8535	97794	3.7435	97141	4.2810
Sample, strat.	99619	3.0336	99436	3.9946	99192	4.5895

Notes:

The objective of this test is to establish the reliability of the Monte Carlo simulation process in sampling indemnity amounts, both stratified and unstratified. The stratified process samples from distributions for assigned report year/calendar year subsets of an accident year. Prior to each RY/CY sampling, the report year and calendar year are selected randomly from RY/CY distributions. For the selected subset, the mean has been adjusted by report year and calendar year severity relativity factors and the variance has been adjusted downward from the variance for the entire accident year, so that the total sample variance for all subsets combined will approximate that of the overall accident year. The unstratified sampling bypasses the partitioning of the accident year into report year/calendar cells and simply samples from the total accident year distribution, using the accident year mean and overall variance.

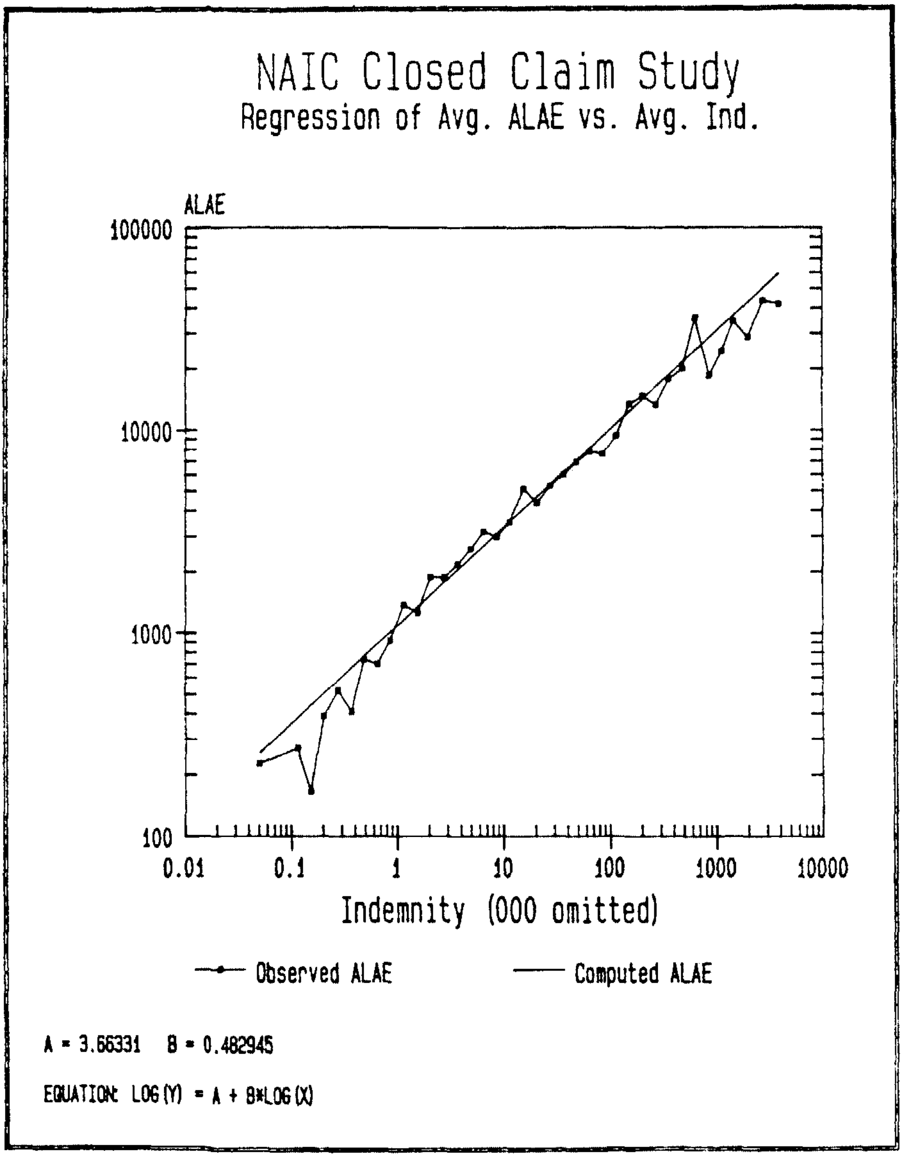
NAIC CLOSED CLAIM STUDY

Regression of Avg. Expense Versus Avg. Indemnity

X = Average Indemnity Bracket	Y = Average ALAE In Bracket	Weight (Number of Claims)	Computed Y
51	229	358.3	259.2
115	272	103.2	384.9
154	166	145.3	444.3
205	392	167.7	509.9
279	526	242.8	591.9
372	412	292.9	679.5
489	745	411.8	776.0
654	705	581.2	892.6
870	926	828.3	1024.6
1150	1387	1015.0	1172.5
1565	1268	1170.2	1360.5
2071	1892	1477.1	1557.7
2786	1878	1499.5	1797.6
3699	2191	1640.8	2061.3
4933	2598	2180.2	2368.8
6562	3177	2071.1	2718.7
8703	2982	1884.5	3116.1
11512	3544	2029.0	3566.7
15453	5139	1906.4	4111.7
20526	4379	1848.9	4715.8
27428	5311	1564.3	5424.5
36706	6031	1448.2	6244.1
48937	6982	1340.3	7174.5
65342	7879	1171.7	8249.5
86099	7653	926.5	9425.2
114706	9411	917.8	10825.7
153844	13511	746.2	12474.7
204947	14788	722.3	14328.0
273289	13324	456.1	16464.3
362444	17890	402.6	18869.6
487164	20104	247.9	21766.5
648598	36075	199.7	24993.1
869532	18601	112.6	28794.1
1141890	24485	93.3	32843.9
1473140	34618	34.0	37143.0
2010450	28763	15.1	43161.8
2773930	43677	22.4	50421.5
3919530	42060	4.9	59583.4

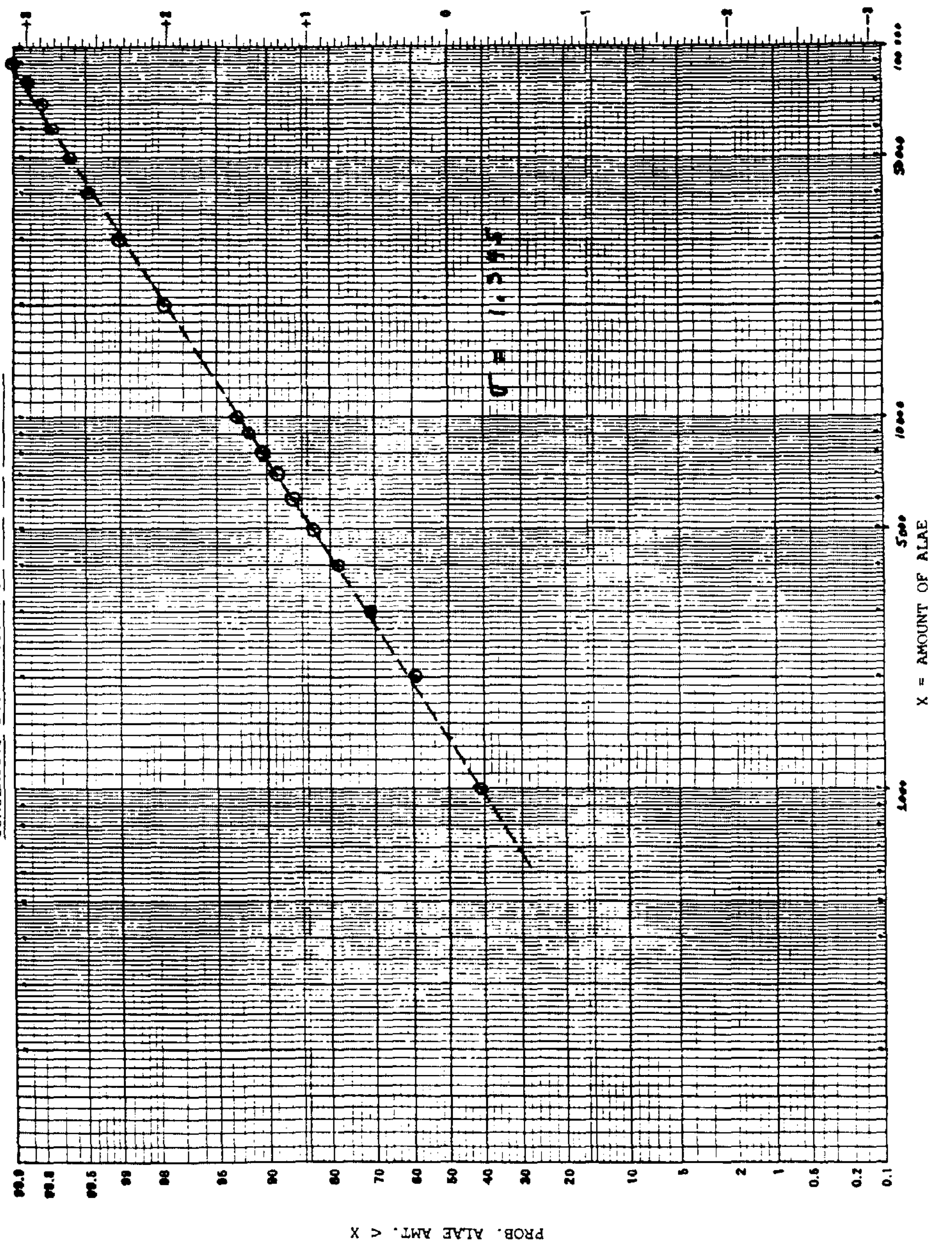
B = 0.48294500 A= 3.66331000

EQUATION: Log (Y) = A+B*LOG(X)


NAIC Closed Claim Study
Regression of Avg. ALAE vs. Avg. Ind.
ALAE
100000
10000
1000
100
0.01
0.1
1
10
100
1000
10000
Indemnity (000 omitted)
Observed ALAE
Computed ALAE
A = 3.66331 B = 0.482945
EQUATION: LOG(Y) = A + B*LOG(X)

NAIC CLOSED CLAIM STUDY

CUMULATIVE DISTRIBUTION OF ALAE PER DEFENDANT



DESCRIPTION OF MONTE CARLO MODEL TO GENERATE PROBABILITY DISTRIBUTION OF REQUIRED SELF INSURANCE CONTRIBUTION

I. Miscellaneous Assumptions, Input parameters, and Distributions

(a) Report year distribution of accident year losses, with relative severity factors by report year - see Appendix B, page 7.

(b) Calendar year distribution of report year losses, with relative severity factors by calendar year - see Appendix B, page 8.

(c) Distribution of claims (indemnity) by size - see Appendix A, page 6.

Note: the basic distribution applies to all claims of one accident year, using the overall mean value for the entire year. The model stratifies the claims first in 63 report year/calendar year cells, each with a modified mean value from (a) and (b) above. Accordingly the variance applicable to each cell has been reduced from the overall variance for random selection purposes, such that the combined sample variance over all 63 cells will approximate the entire accident year distribution.

(d) Average unlimited indemnity by year - used as the parameter in the size of loss distribution for each accident year:

year 1: $200,000
year 2: $225,000

(e) Average claim expense by year. Based on the functional relationship derived between expected average ALAE and the sample indemnity value (see Appendix D), the sample ALAE is SELECTED from a distribution the *mean* of which is determined as a function of the sample indemnity. The starting values for the average ALAE for the entire accident year, over all indemnity values, are:

year 1: $12,000
year 2: $13,000

(f) Total *expected* number of claims by accident year, including claims closed with indemnity (CWI) and claims closed with expense only (CWE):

year 1: 20

year 2: 21
(claims closed with no payment are excluded)

(g) Percentages for claims disposed, all years:
CWI: 60%
CWE: 40%

(h) Self insured retention, all years:
per claim: $1,000,000
annual aggregate: $5,000,000

(i) Parameter variance (uncertainty factor). These values are expressed in relation to the expected population frequency and severity, which are input. In this case study we are assuming a "standard error" of .15 for frequency and .18 for severity, both expressed relative to the expected values.

II. The Monte Carlo Simulation Process (in Pseudo Code)

Accumulators set up:

(1) Aggregate retained loss brackets (31) for all trials combined (probability distribution), less current assets. One accumulator for counts (number of trials falling into bracket) and another for total loss dollars.

(2) Total retained by policy year. To be compared with aggregate SIR. Reinitialized for each trial.

Input:

(1) Uncertainty factors for population mean frequency and severity (parameter variance).

(2) Retentions by policy year and index amount (if applicable). Per claim and aggregate

(3) Current assets

(4) Number of claims open for all prior years and assumed average indemnity for these open claims by year.

(5) Assumed average unlimited indemnity, claim frequency, and average ALAE for next (target) year.

(6) Assumed rate of return for present value discounting

(7) Number of trials to sample

(8) Present (target) year of coverage [Y1]. For initial funding Y1=1.

(9) Percentage of claims closed with expense only (CWE). Note: claims closed with no payment ignored.

** Main trial loop

For each trial

If Y1= 1 then skip to Routine for current year

For each prior year 1 to Y1-1

If Number of claims open for year = 0 then skip to [next year]

For each open claim for year

(1) Determine year reported (from actual, if available, else by randomizing from report year distr.)

(2) Establish mean indemnity for year from input values for open claims for that year.

(3) SELECT calendar year paid, relative to report year, and modify mean indemnity by calendar year severity factor

(4) Establish retention per claim applicable to calendar year, including index, if applicable.

(5) SELECT mode of closure (CWI or CWE). If CWE, SELECT ALAE amount only and then skip to next claim.

(6) SELECT gross (unlimited) indemnity from size of loss distribution, the mean of which was adjusted by calendar year severity factors from (3).

(7) Limit indemnity to per-claim retention for that year, as necessary.

(8) If claim burns through remaining annual aggregate SIR, then limit claim accordingly.

(9) Based on indemnity amount, adjust *expected* ALAE, and SELECT sample gross ALAE from distribution.

(10) Add retained indemnity and pro-rata retained ALAE to retained accumulator for calendar year of payment selected in (3).

(11) Add retained indemnity to the aggregate losses for that year.

Next claim

Next year

** Now do loop for prior year's IBNR's and/or current year's losses

For each year 1 to Y1

SELECT "universe" mean frequency and severity, drawing from expected and using the parameter variances (input).

SELECT sample number of claims for year, drawing from "universe". If expected number < 15, use Poisson, else use normal distribution.

For each claim

(1) Determine year reported (from report year distr.). If claim already reported (report year < Y1), then branch to next claim. Thus, IBNR claims from prior years are included.

(2) Establish mean indemnity from input value for that year and modify with report year severity factor.

(3) SELECT calendar year paid, relative to report year, and further modify mean indemnity by calendar year severity factor.

(4) Establish retention per claim applicable to calendar year, including index, if applicable.

(5) SELECT mode of closure (CWI or CWE). If CWE, SELECT ALAE amount only and then skip to next claim.

(6) SELECT gross (unlimited) indemnity from size of loss distribution, the mean of which was adjusted by report year and calendar year severity factors from (2) and (3).

(7) Limit indemnity to per-claim retention for that year, as necessary.

(8) If claim burns through remaining annual aggregate SIR, then limit claim accordingly.

(9) Based on indemnity amount, adjust *expected* ALAE, and SELECT sample gross ALAE from distribution.

(10) Add retained indemnity and pro-rata retained ALAE to retained accumulator for the calendar year of payment from (3).

(11) Add retained indemnity to the aggregate losses for that year.

Next claim

Next year

** Tally section for this trial

Determine present value of all retained losses from accumulator by calendar year and deduct current assets to get required funding for this trial (if < 0 then make it 0).

Determine which one of the 31 brackets of aggregate retained losses this trial falls in and bump the corresponding accumulators for counts (1) and total retained dollars.

Reinitialize all accumulators, except aggregate loss brackets.

Next trial

Print out probability distribution

NOTE: Each time the word "SELECT" is used in the above process, the program randomly samples from the appropriate distribution described in Part I, using a random number generator.

```
RUN
SELFRISB      SUN, FEB 26 1989          13:23:56

Report year distribution:
RY  Cum.     Rel.
    counts   Sev.
-- ------- -------
1   .38700   .73873
2   .68700   .98498
3   .88800  1.18197
4   .95400  1.37897
5   .97900  1.67446
6   .99100  2.16695
7  1.00000  2.46245

Cal. Year distribution:
CY  Cum.     Rel.
   Counts    Sev.
-- ------  -------
1  .25742   .26416
2  .44247   .70794
3  .70087   .94040
4  .83191  1.37362
5  .90366  1.61664
6  .93476  2.25062
7  .95879  2.74724
8  .98076  2.95857
9 1.00000  3.16989

INPUT RATE OF RETURN (X.XX) ?1.07
S1=.9999999999998            [mean of ry*cy severities]
S2=1.632776340059            [second moment of ry*cy severities]
NET S2=.4902818419985        [log(S2)]
ADJUSTED S=1.715722985358    [  sqrt[log(31) - log(S2)]    ]
INPUT NO. TRIALS ? 10000
INPUT PERCENT CLAIMS CLOSED EXPENSE ONLY ?.4
INPUT UNLIMITED SEVERITY TREND (X.XX) ?1.12
INPUT ALAE TREND (x.xx) ?1.08
INPUT FREQUENCY TREND (X.XX) ?1.04
INPUT CLIENT NAME
?XYZ HOSPITAL
INPUT PRESENT YEAR OF COVERAGE
?1
INPUT LIMIT PER CLAIM FOR THIS YEAR FORWARD
?1000000
INPUT AGGREGATE LIMIT FOR THIS YEAR FORWARD
?5000000
INPUT AVERAGE INDEMNITY WITH NO LIMIT FOR THIS YEAR
?200000
AVERAGE ALLOCATED CLAIMS EXPENSE FIRST YEAR 12000
ALAE ADJ. FACTOR =.65711657248
INPUT EXPECTED TOTAL CLAIM COUNT FOR THIS YEAR
?20
INPUT NET EXPECTED RETAINED LOSSES FOR THIS YEAR
?1000000
UNCERTAINTY FACTORS FOR POPULATION MEAN FREQUENCY AND SEVERITY (.XX,.XX)?.15,.18
STARTING
```

XYZ HOSPITAL

ANNUAL BREAKEVEN CONTRIBUTION FOR SELF-INSURANCE TRUST

INTERVAL END POINT	NUMBER OF TRIALS	CUMULATIVE NUMBER TRIALS	TOTAL AMOUNT	CUMULATIVE TOTAL AMOUNT
0	0	0	0	0
100000	20	20	1567150	1567150
117210	9	29	988079	2555229
137382	23	52	2979852	5535081
161026	31	83	4660393	10195474
188739	39	122	6842149	17037623
221222	71	193	14544679	31582303
259294	98	291	23550079	55132382
303920	128	419	36079809	91212191
356225	202	621	66547061	157759252
417532	237	858	91816354	249575605
489390	324	1182	146497576	396073181
573615	384	1566	204715603	600788784
672336	512	2078	318352411	919141195
788046	647	2725	473392789	1392533984
923671	770	3495	659660990	2052194975
1082637	941	4436	943185973	2995380948
1268961	965	5401	1131134839	4126515787
1487352	1048	6449	1441612233	5568128020
1743329	1004	7453	1614047558	7182175578
2043360	957	8410	1806550499	8988726077
2395027	691	9101	1529247998	10517974074
2807216	479	9580	1238595136	11756569210
3290345	246	9826	737560999	12494130209
3856621	133	9959	465756993	12959887202
4520354	40	9999	162642492	13122529694
5298317	1	10000	4539766	13127069460
6210170	0	10000	0	13127069460
7278954	0	10000	0	13127069460
8531679	0	10000	0	13127069460
10000000	0	10000	0	13127069460

Interpolated values for selected confidence levels:
(geometric interpolation)

2340077	9000
2733743	9500
3594291	9900

```
>RUN
SELFRIS8      SUN, FEB 26 1989          15:26:11

Report year distribution.
RY  Cum.      Rel.
    counts    Sev.
-- ------- -------
1  .38700   .73873
2  .68700   .98498
3  .88800  1.18197
4  .95400  1.37897
5  .97900  1.67446
6  .99100  2.16695
7 1.00000  2.46245

Cal. Year distribution:
CY  Cum.      Rel.
   Counts     Sev.
-- ------   -------
1  .25742   .26416
2  .44247   .70794
3  .70087   .94040
4  .83191  1.37362
5  .90366  1.61664
6  .93476  2.25062
7  .95879  2.74724
8  .98076  2.95857
9 1.00000  3.16989

INPUT RATE OF RETURN (X.XX) ?1.07
S1=.9999999999998            [mean of ry*cy severities]
S2=1.632776340059            [second moment of ry*cy severities]
NET S2=.4902818419985        [log(S2)]
ADJUSTED S=1.715722985358    [  sqrt[log(31) - log(S2)]   ]
INPUT NO. TRIALS ? 10000
INPUT PERCENT CLAIMS CLOSED EXPENSE ONLY ?.4
INPUT UNLIMITED SEVERITY TREND (X.XX) ?1.12
INPUT ALAE TREND (x.xx) ?1.08
INPUT FREQUENCY TREND (X.XX) ?1.04
INPUT CLIENT NAME
?XYZ HOSPITAL
INPUT PRESENT YEAR OF COVERAGE
?2
INPUT PRESENT FUND ASSETS
?2950000
INPUT NUMBER CLAIMS OUTSTANDING FOR EACH OF THE FIRST 1 YEARS OF COVERAGE
YEAR :
?7
INPUT ULTIMATE AVERAGE (UNLIMITED) INDEMNITY RESERVE FOR OPEN CLAIM FOR EACH OF
THE FIRST 1 YEARS OF COVERAGE
YEAR 1
?130000
INPUT LIMIT PER CLAIM FOR EACH OF THE FIRST 1 YEARS OF COVERAGE
YEAR 1
?1000000
INPUT TARGET AGGREGATE EACH OF THE FIRST 1YEARS OF COVERAGE
YEAR 1
?4960000
INPUT LIMIT PER CLAIM FOR THIS YEAR FORWARD
?1000000
INPUT AGGREGATE LIMIT FOR THIS YEAR FORWARD
?5000000
INPUT AVERAGE INDEMNITY WITH NO LIMIT FOR THIS YEAR
?225000
AVERAGE ALLOCATED CLAIMS EXPENSE FIRST YEAR 13000
ALAE ADJ. FACTOR =.6327789216474
INPUT EXPECTED TOTAL CLAIM COUNT FOR THIS YEAR
?21
INPUT NET EXPECTED RETAINED LOSSES FOR THIS YEAR
?1200000
UNCERTAINTY FACTORS FOR POPULATION MEAN FREQUENCY AND SEVERITY (.XX,.XX)?.15,.18
STARTING
```

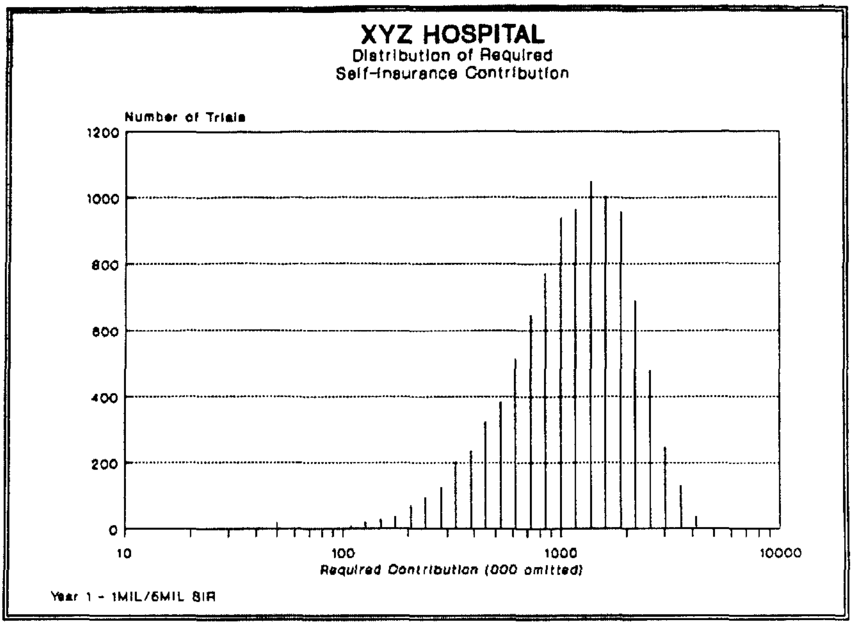
XYZ HOSPITAL

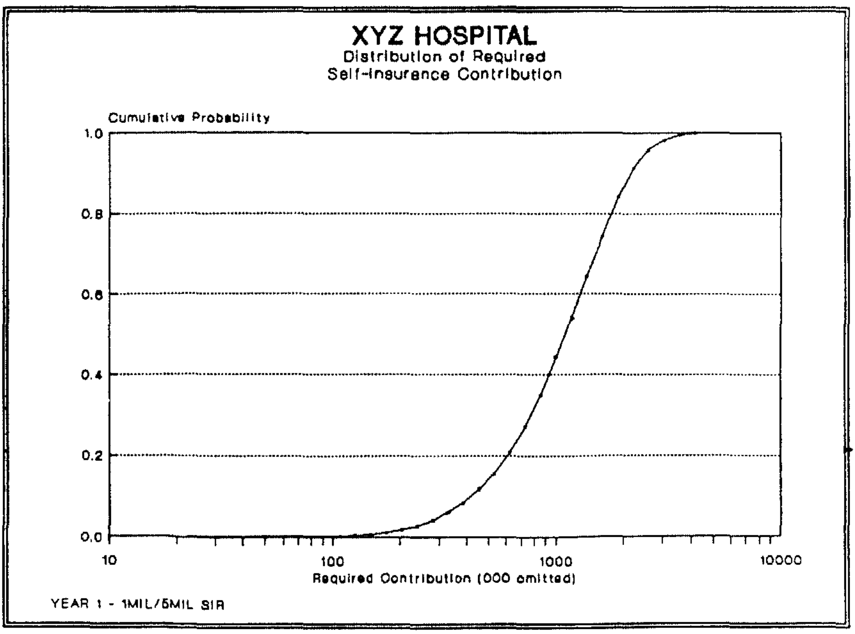
ANNUAL BREAKEVEN CONTRIBUTION FOR SELF-INSURANCE TRUST

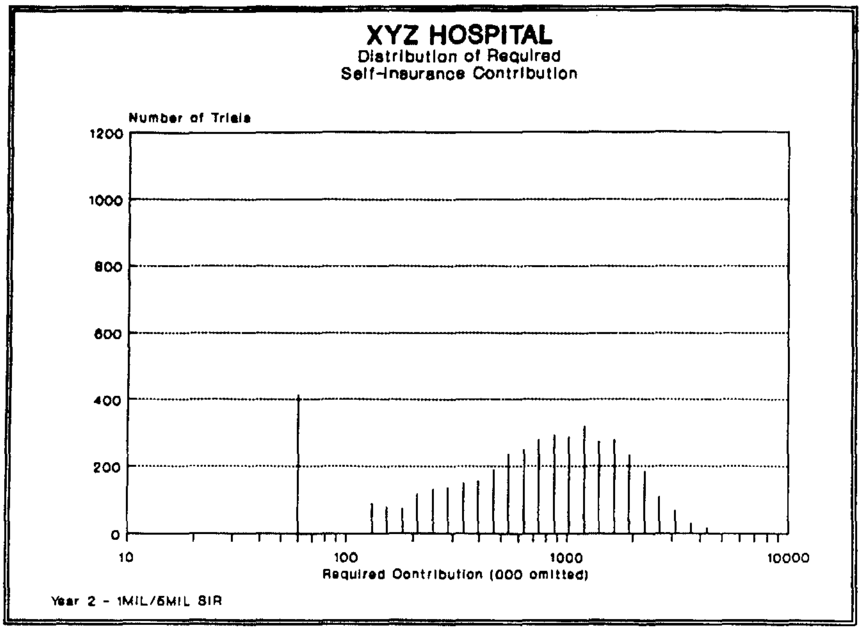
INTERVAL END POINT	NUMBER OF TRIALS	CUMULATIVE NUMBER TRIALS	TOTAL AMOUNT	CUMULATIVE TOTAL AMOUNT
0	5583	5583	0	0
120000	414	5997	24648222	24648222
140652	91	6088	11830798	36479020
164859	81	6169	12307073	48786092
193231	76	6245	13571310	62357402
226487	118	6363	24706476	87063878
265466	133	6496	32901574	119965452
311153	138	6634	39861648	159827100
364703	151	6785	50702085	210529184
427470	158	6943	62734682	273263867
501038	189	7132	88302392	361566259
587268	235	7367	127689223	489255482
688338	251	7618	159438697	648694178
806803	281	7899	209627406	858321584
945656	293	8192	257166219	1115487803
1108405	289	8481	296198263	1411686066
1299164	321	8802	386577962	1798264028
1522753	274	9076	384974193	2183238221
1784823	281	9357	462473361	2645711581
2091995	233	9590	449359285	3095070867
2452032	185	9775	418087611	3513158478
2874032	109	9884	286821719	3799980197
3368660	70	9954	215857101	4015837298
3948414	30	9984	107482615	4123319913
4627945	16	10000	68141691	4191461605
5424425	0	10000	0	4191461605
6357980	0	10000	0	4191461605
7452203	0	10000	0	4191461605
8734745	0	10000	0	4191461605
10238015	0	10000	0	4191461605
12000000	0	10000	0	4191461605

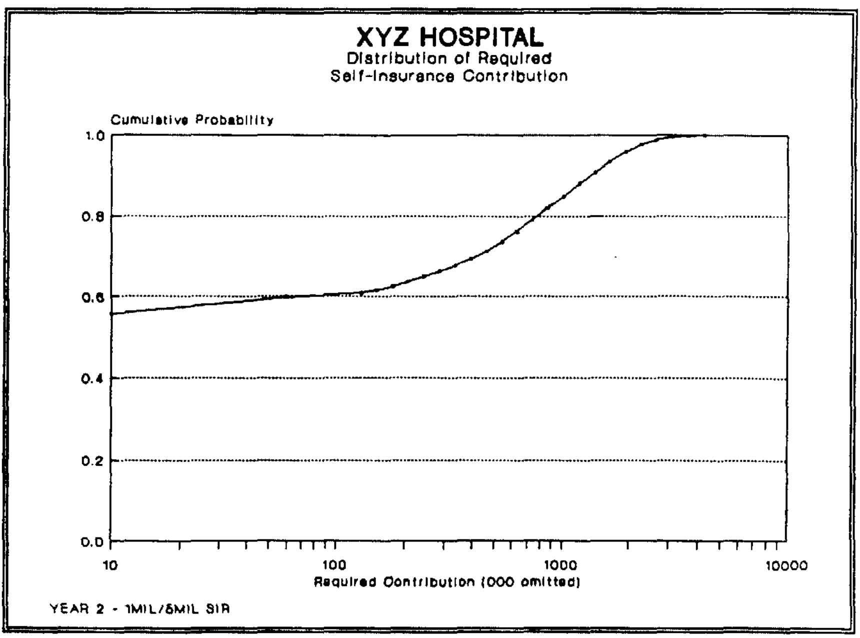
Interpolated values for selected confidence levels:
(geometric interpolation)

1457137	9000
1967531	9500
2980267	9900


XYZ HOSPITAL
Distribution of Required
Self-Insurance Contribution
Number of Trials
1200
1000
800
600
400
200
0
10
100
1000
10000
Required Contribution (000 omitted)
Year 1 - 1MIL/5MIL SIR


XYZ HOSPITAL
Distribution of Required
Self-Insurance Contribution
Cumulative Probability
1.0
0.8
0.6
0.4
0.2
0.0
10
100
1000
10000
Required Contribution (000 omitted)
YEAR 1 - 1MIL/5MIL SIR


XYZ HOSPITAL
Distribution of Required
Self-Insurance Contribution
Number of Trials
1200
1000
800
600
400
200
0
10
100
1000
10000
Required Contribution (000 omitted)
Year 2 - 1MIL/5MIL SIR


XYZ HOSPITAL
Distribution of Required
Self-Insurance Contribution
Cumulative Probability
1.0
0.8
0.6
0.4
0.2
0.0
10
100
1000
10000
Required Contribution (000 omitted)
YEAR 2 - 1MIL/5MIL SIR

PRICING CASUALTY EXCESS REINSURANCE: DISCOUNTING EXPECTED L&LE
Draft

Steven Neyer

In developing an estimated price for casualty excess of loss reinsurance contracts, it is not uncommon to adjust the expected loss component of the rate to reflect the estimated value of investment income on funds held to pay outstanding loss reserves. The discount rate is generally a function of 1) a projected payment pattern for losses and loss adjustment expenses (L&LE), and 2) a specified interest rate. While, for many excess reinsurance contracts, it may be difficult to accurately project L&LE payments over time, the mechanics of the technique are fairly straightforward. An example of its application to a workers compensation excess program is provided in Appendix A.

The technique has been criticized for several reasons. The purpose of this draft is to investigate two of the most common objections:

1) a reinsurer's objective should be to price business to an underwriting profit, and

2) the Tax Reform Act of 1986 (TRA) has offset to a large extent the investment income imputed to the reinsurer in the above adjustment.

In view of its intuitive reasonableness, the principal focus was on the second objection. A model was developed to quantify the magnitude (if any) of the offset necessary due to the provisions of the TRA, and to adjust the discount rate accordingly. Our preliminary conclusions, as well as the assumptions and methodology, are discussed in the paragraphs that follow.

Our principal conclusion is as follows:

- Counter to intuitive expectations, a significant upward adjustment in the discount rate is not necessitated by the provisions of the TRA.

We would appreciate comments and suggestions with respect to the analysis presented in the subsequent paragraphs.

BACKGROUND

The principal components of a reinsurer's economic return are income from 1) underwriting and 2) investments, and both of these need not be simultaneously positive in order for the reinsurer to earn an adequate return. The principal criticism of the traditional actuarial pricing method is that it fails to explicitly recognize the investment component of this equation. For example, the traditional profit margin of 5% of premiums has historically been applied across all lines of business, ignoring the relatively greater magnitude of investment earnings generated by the "long-tail" lines of business (although in many states and for some lines of business, explicit recognition of investment income is now required in the ratemaking methodology).

In considering the merits of explictly considering investment income in reinsurance pricing, two qualifications must first be noted:

1) Implicit recognition has been given to investment income in the actual market price of insurance (and reinsurance), and there are some arguments for continuing to give investment income only indirect attention in the ratemaking process. These arguments are presented in Appendix B.

2) It is also recognized that excess reinsurance is more volatile (and risky) than primary insurance; if a 5% underwriting margin provides an adequate return for a stable, non-volatile line of business, then clearly a higher margin should be required for a more

volatile line of business.

These qualifications notwithstanding, we believe that it is appropriate and beneficial for both the ceding company and the reinsurer if the reinsurer explicitly recognizes investment income in its pricing. It better enables both parties to isolate the various components of the quoted price and determine whether the rate is reasonable, inadequate, or excessive.

To provide a simple illustration, assume that the expected ceded losses and allocated loss adjustment expenses (hereinafter abbreviated as L&LE) for a given reinsurance contract are $1,000 and the reinsurer's loading for expenses, profit, and contingencies is 100/75th. The various components of the final price, related to premium, are as follows:

Component	$	Pct.
L&LE	$1,000	75%
Op. Expenses	133	10%
Brokerage	67	5%
Profit & Cont.	133	10%
Total	$1,333	100%

However, if L&LE are paid out evenly over a ten year timeframe, and the reinsurer earns investment income on the funds held in the interim at an annual rate of 6%, then $432 of investment income will be generated over the ten year period. The present value of this investment income (discounted at 6%) is $242, or 18% of the premium.

It can be argued that 6% is not the appropriate interest rate to use for discounting and that a risk-adjusted rate should be used; however, this issue will be addressed later. For purposes of this illustration, it is assumed that 6% is the appropriate rate. Thus, the total return, as a function of premium, is 28% (10% profit and contingency margin plus 18%).

This is not necessarily to imply that 28% is an excessive return; rather it is to demonstrate that the reinsurer's return, as it relates to premium, is far in excess of the 10% underwriting profit margin contemplated in the rate.

Discounting the expected L&LE effectively provides a credit to the ceding company for the anticipated investment income on funds held to pay L&LE, thus limiting the reinsurer's expected profit to the margin contemplated in its loading. In this example, if the expected L&LE are discounted to $758 to reflect the anticipated investment income, then the reinsurer's return would be limited to the 10% margin cited in the table above.

METHODOLOGY

The basic methodology was to construct a simplified income and cash flow statement for a single accident year cohort, assuming that expected L&LE of $1,000 are paid out over a period of sixteen years, according to the payment pattern presented in Appendix A. Investment income is earned at a specified rate on the funds held to pay the L&LE and is accumulated in a cash balance.

Each scenario modeled consists of two illustrations. For example, in the first illustration in Exhibit A (top of Exhibit), the reinsurer collects 100% of the $1,000 and accumulates a positive cash balance over the sixteen year timeframe (the line entitled "Cumulative"). In the second illustration in Exhibit A (bottom of Exhibit) the $1,000 is discounted to $718 (71.8% of the undiscounted pure premium). This reduction reflects the expected investment income that will be earned until all L&LE are paid. The reinsurer's final cash balance for the sixteen year period is $0.00.

This does not mean that the expected profit is being driven to zero. Rather, it demonstrates the point that we made earlier, i.e., that the effect of discounting L&LE is to strip out any profit margin from the loss component of the reinsurance price. Note that there is no provision in this example for the reinsurer's expenses, profit, and contingencies. It is assumed that whatever loading is added to the $1,000 pure premium would be sufficient to provide for these items.

The base case scenario in Exhibit A excludes any consideration of taxes. A fundamental goal of this exercise was to develop a method for deriving an appropriate discount factor that reflects tax considerations. Consequently, in subsequent scenarios we tested the impact of taxes under the terms of: 1) the old tax law (pre-1987) and 2) the new tax law (post-1987). These results are presented in Exibits B through G and are discussed below.

RESULTS UNDER THE OLD TAX LAW
(Exhibits B and C)

As indicated earlier, one objection to the method used to derive a discount rate is that it is overly simplistic, because it fails to consider the ramifications of the TRA. However, since the method ignores taxes entirely, a logical hypothesis is that it was theoretically simplistic even before the tax law was changed. Hence, we decided that it would be beneficial to first determine the appropriate discount factor under the old tax law, before deriving an appropriate discount factor reflecting the new tax law.

Exhibit B

We initially assumed that the interest income earned at a 6% interest rate was attributed to tax-exempt bonds. Hence in the first illustration (before discounting), there is no taxable underwriting or investment income and no taxes. As a result, the cumulative cash balance is identical to Exhibit A (no taxes).

In the second illustration (after discounting), the pure premium has been discounted to $502, creating an underwriting loss of $498 ($502 - $1,000) in Year 1. From a tax standpoint, this creates a tax credit of $229 (46% of $498) that could theoretically be applied to offset taxable income from other operations. The value of this credit is treated as a cash contribution in this Exhibit.

Because of this tax credit, the final discount factor of 50.2% is <u>lower</u> then in Exhibit A, and the premium required to fund the expected L&LE payments is $502 (versus $718).

Exhibit C

A critical assumption underlying the projections in Exhibit B is that the reinsurer has other operations with taxable income which can utilize the tax credit arising from the underwriting loss created by the discounting of the pure premium in this particular transaction. A possible problem with this assumption is that if the reinsurer priced every contract on this basis, it may not have sufficient income to use up the tax credits generated by the underwriting losses.

In Exhibit C, no credit has been allowed for the underwriting loss created by discounting.

However, at the same time, it has been assumed that the reinsurer is able to shift its investment portfolio mix in Year 1 to taxable bonds, in order to offset at least some of the tax credit that is otherwise lost. As a result, while the discount factor and discounted premium are greater than they were in Exhibit B (69% and $699, versus 50.2% and $502), it is stll less than the discounted rate derived under the no-tax scenario.

Conclusion - Our hypothesis was confirmed (i.e., the method that we originally used to discount losses is a simplification because it ignores taxes); however, it appears that our original calculation was a conservative simplification (from the reinsurer's viewpoint), at least in light of the old tax law.

Hence, the obvious question is whether this conclusion is also applicable under the provisions of the TRA.

RESULTS UNDER THE NEW TAX LAW (TRA)
(Exhibits D and E)

The principal provisions of the TRA that we have attempted to reflect in our revised model are as follows:

1) Tax Rate - the marginal tax rate was changed from 46% to 34%,

2) L&LE Reserves - Reserves on unpaid L&LE are discounted for purposes of calculating taxable income according to methods prescribed by the Treasury

Department. A firm has the option of using factors based on its own experience or industry experience, subject to certain restrictions. For purposes of these examples, we used Schedule P Composite industry experience for 1985 (use of Schedule P Composite for business reported as Reinsurance in the Annual Statement is mandated by the Treasury). Alternatively, industry experience for the statutory Workers Compensation line would be used, if the firm reports its premiums in this line.

3) Proration - Under the TRA, 15% of otherwise tax-exempt interest income is taxable. The effective tax rate is 5.1% (15% x 34%).

For simplification purposes revenue offset has been ignored. Possible implications of the Alternative Minimum Tax (AMT) will be addressed separately.

Exhibit D

The assumptions underlying Exhibit D correspond to Exhibit B, i.e., all invested funds are in tax-exempt bonds and any tax credit is treated as a cash contribution. Two items to note with respect to the tax calculation are:

1) the discounting of L&LE reserves creates a timing difference. It does not create additional taxable income, but it does defer part of the deduction for L&LE,

2) the proration of tax-exempt income is a permanent difference; it creates additional taxable income. The total regular tax of $35 in the first illustration can be attributed solely to the proration provision.

The effect of adjusting the pure premium of $1,000 to reflect investment income in the second illustration is the same as it was in Exhibit B, although the magnitude is not as great. The adjustment creates a tax benefit due to the increase in the underwriting loss. This benefit is mitigated due to the discounting of L&LE reserves for tax purposes; however, the final discount (adjustment) factor derived (62.83%) is still lower than the discount (adjustment) factor that we derived in the no-tax scenario (71.81% - Exhibit A).

Exhibit E

The assumptions in Exhibit E correspond to Exhibit C, i.e., no tax credit is allowed for the underwriting loss and investable funds are invested in taxable bonds. The result is similar: the discount (adjustment) factor derived in the second illustration (70.37%) is not as low the the factor derived in Exhibit D but is still comparable to the discount factor derived excluding taxes (71.81%).

Conclusion - The provisions of the TRA do not appear to necessitate an upward adjustment in our original adjustment factor (excluding taxes). In fact, in light of the examples cited above, it can argued that the original factor that we derived is conservative.

ALTERNATIVE MINIMUM TAX (AMT)
(Exhibits F and G)

An additional complexity arising out the new tax law is the provision for the Alternative Minimum Tax. Basically, this provision was created because of the fact that many major corporations with substantial reported income were effectively paying no taxes. Congress decided that this was inappropriate from a public policy standpoint. Under the new law, every corporation must pay at least the

Alternative Minimum Tax, which is based on AMT income as prescribed by the Treasury Department.

The AMT provisions have effectively created a separate and parallel set of rules for calculating taxes which are somewhat complex. One complication is that separate sets of rules apply for 1987-90 and 1990 and beyond.

For simplification purposes, we ignored the AMT in Exhibits D and E. In Exhibits F and G, we estimated the AMT (using the rules that apply for 1990 and beyond) on a simplified basis and substituted the AMT for the regular tax in all years. While the adjustment factors derived in these exhibits were somewhat higher than those derived in Exhibits D and E (regular tax), the results do not necessitate any change in our previously-stated conclusion.

A table listing all of the premium adjustment factors derived in Exhibits A through G is presented as follows:

Base Case (No Taxes)	71.81%		
	Old Tax Law	New Tax Law	AMT
Tax Credit treated as Cash Contribution	50.19%	62.83%	69.66%
No tax credit allowed, Change in Inv. Mix	69.94%	70.37%	72.17%

ADDITIONAL ISSUES

Two issues not addressed in the paragraphs above that merit further investigation are as follows.

Timing and Interest Rate Risks

For purposes of the analysis above it was assumed that L&LE would be paid out at a specified pattern and that interest income would be earned at a specified rate. In actuality, it is highly unlikely that the L&LE will pay out at the assumed pattern or that the reinsurer's investment portfolio will earn interest income at the rate specified. The variances from the expected case can work in the reinsurer's favor or to its detriment. In other words, there are timing investment risks that need to be considered.

While it is possible to apply quantitative techniques in measuring these risks, their magnitude is to a large extent a subjective consideration, and qualitative judgment may be no less reliable than quantitative techniques. In the original discount factor derivation (Appendix A), we attempted to subjectively account for the timing and investment uncertainty by making what we believed were conservative assumptions at every step in the calculation.

To provide another illustration of an adjustment for these risks, it is useful to recall the simple example discussed earlier. Expected ceded incurred L&LE for a given reinsurance contract are $1,000 and the reinsurer's loading for expenses, profit, and contingencies is 100/75th. The various components of the final price, related to premium, are as follows:

Component	$	Pct.
L&LE	$1,000	75%
Op. Expenses	133	10%
Brokerage	67	5%
Profit & Cont.	133	10%
Total	$1,333	100%

It is anticipated that L&LE will be paid out evenly over a ten year timeframe, and the reinsurer will earn investment income on the funds held in the interim at an annual rate of 6%. The present value of the $432 of investment income earned over the ten year period (discounted at 6%) is $242, or 18% of the premium.

However, to reflect the uncertainty with respect to both the timing of the actual payments and the actual rate at which the reinsurer's investments will earn interest, the reinsurer may apply a higher (risk-adjusted) discount rate to the $432 of investment income cited above. Assuming that this rate is 10% (instead of 6%), then the present value of the $432 of investment income is only $167, or 13.4% of premium.

Estimating the Reinsurer's ROI

An issue related to the one above is best described by two fundamental questions - 1) what exactly is the reinsurer's expected return on its investment in the various examples above, and 2) which expected returns satisfy his target criteria.

In order to address the first question, it would be useful to enhance the model used for this analysis to incorporate 1) assumptions with respect to surplus requirements and 2) net present value and/or internal rate of return estimates at various risk-adjusted discount rates. We have constructed at least one other model that incorporates these features and believe that the analysis presented in this memo can be readily extended to quantify the reinsurer's expected return on investment.

CONCLUSION

We believe that the examples presented in Exhibits A through G illustrate that it is not inappropriate to discount expected incurred L&LE to reflect investment income, regardless of the provisions of the tax law.

However, it is intuitively apparent that the provisions of the TRA will affect reinsurers more adversely than primary insurers, due to the heavier discounting of reinsurers' L&LE reserves. Possibly, this warrants higher loadings in the rates for some reinsurance contracts. Extending the model used in this analysis to include an internal rate of return measure will enable us to quantify to some extent what the magnitude of this loading should be.

As indicated earlier, any comments, and/or suggestions would be greatly appreciated.

RATE ADJUSTMENT TO REFLECT INV. INCOME

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
WITH NO TAXES																	
Premium	1000																1000
Incurred L&LE 100%	1000																
Paid L&LE	68	108	196	92	49	57	40	50	40	40	40	50	40	40	50	40	1000
Chg. in Reserve	932	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	0
Und. Profit	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Cash From Und.	932	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	0
Inv. Income @ 6.0%	58	56	50	45	43	43	42	42	42	42	42	42	42	42	42	42	715
Total Cash	990	-52	-146	-47	-6	-14	2	-8	2	2	2	-8	2	2	-8	2	715
Cumulative	990	938	792	745	739	725	727	719	721	723	725	717	719	721	713	715	
Discounted Paid L&LE	66	99	169	75	38	41	27	32	24	23	22	26	19	18	21	16	716

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
Discount Factor:	71.81%																
Adjusted Premium	718																
Paid L&LE	68	108	196	92	49	57	40	50	40	40	40	50	40	40	50	40	1000
Und. Expenses	0																
Cash from Und.	650	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	-282
Inv. Income @	41	38	31	25	22	20	18	17	15	14	12	10	8	6	4	1	282
Total Cash	691	-70	-165	-67	-27	-37	-22	-33	-25	-26	-28	-40	-32	-34	-46	-39	0
Cumulative	691	621	456	389	362	325	303	270	245	219	191	151	119	85	39	0	

08-Dec-88

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
OLD TAX LAW																		
Premium		1000																1000
Incurred L&LE	100%	1000																
Paid L&LE		68	108	196	92	49	57	40	50	40	40	40	50	40	40	50	40	1000
Chg. in Reserve		932	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	0
Und. Profit		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Cash From Und.		932	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	0
Inv. Income:																		
Taxable @	9.0%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Tax-exempt @	6.0%	58	56	50	45	43	43	42	42	42	42	42	42	42	42	42	42	715
Taxes @	46%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Total Cash		990	-52	-146	-47	-6	-14	2	-8	2	2	2	-8	2	2	-8	2	715
Cumulative		990	938	792	745	739	725	727	719	721	723	725	717	719	721	713	715	
Discount Factor:		71.81%																
Tax Adjustment		69.900%																
Net Factor		50.195%																
Adjusted Premium		502																
Paid L&LE		68	108	196	92	49	57	40	50	40	40	40	50	40	40	50	40	1000
Und. Profit		-498	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-498
Cash from Und.		434	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	-498
Inv. Income:																		
Taxable @	9.0%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Tax-exempt @	6.0%	28	38	31	25	22	20	18	17	15	14	12	10	8	6	4	1	269
Taxes @	46%	-229	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-229
Total Cash		691	-70	-165	-67	-27	-37	-22	-33	-25	-26	-28	-40	-32	-34	-46	-39	0
Cumulative		691	621	456	389	362	325	303	270	245	219	191	151	119	85	39	0	

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
ASSUMPTIONS																	
(Old Tax Law)																	
Payout Pattern	0.068	0.108	0.196	0.092	0.049	0.057	0.04	0.05	0.04	0.04	0.04	0.05	0.04	0.04	0.05	0.04	1
Period	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5	15.5	
Weights	0.066	0.099	0.1694	0.075	0.0377	0.0414	0.0274	0.0323	0.0244	0.023	0.0217	0.0256	0.0193	0.0182	0.0215	0.0162	
Discount Factor	71.81%																
Credit for Tax Benefit (1,0)	1																
Investment Mix																	
Taxable	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	
Tax-exempt	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	
Adjusted Investment Mix																	
Taxable	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	
Tax-exempt	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	

RATE ADJUSTMENT TO REFLECT INV. INCOME

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
OLD TAX LAW																		
Premium		1000																1000
Incurred L&LE	100%	1000																
Paid L&LE		68	108	196	92	49	57	40	50	40	40	40	50	40	40	50	40	1000
Chg. in Reserve		932	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	0
Und. Profit		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Cash From Und.		932	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	0
Inv. Income:																		
Taxable @	9.0%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Tax-exempt @	6.0%	58	56	50	45	43	43	42	42	42	42	42	42	42	42	42	42	715
Taxes @	46%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Total Cash		990	-52	-146	-47	-6	-14	2	-8	2	2	2	-8	2	2	-8	2	715
Cumulative		990	938	792	745	739	725	727	719	721	723	725	717	719	721	713	715	
Discount Factor:		71.81%																
Tax Adjustment		97.400%																
Net Factor		69.943%																
Adjusted Premium		699																
Paid L&LE		68	108	196	92	49	57	40	50	40	40	40	50	40	40	50	40	1000
Und. Profit		-301	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-301
Cash from Und.		631	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	-301
Inv. Income:																		
Taxable @	9.0%	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	60
Tax-exempt @	6.0%	0	38	31	25	22	20	18	17	15	14	12	10	8	6	4	1	241
Taxes @	46%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Total Cash		691	-70	-165	-67	-27	-37	-22	-33	-25	-26	-28	-40	-32	-34	-46	-39	0
Cumulative		691	621	456	389	362	325	303	270	245	219	191	151	119	85	39	0	

RATE ADJUSTMENT TO REFLECT INV. INCOME

EXHIBIT C

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
ASSUMPTIONS (Old Tax Law)																	
Payout Pattern	0.068	0.108	0.196	0.092	0.049	0.057	0.04	0.05	0.04	0.04	0.04	0.05	0.04	0.04	0.05	0.04	1
Period	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5	15.5	
Weights	0.066	0.099	0.1694	0.075	0.0377	0.0414	0.0274	0.0323	0.0244	0.023	0.0217	0.0256	0.0193	0.0182	0.0215	0.0162	
Discount Factor	71.81%																
Credit for Tax Benefit (1,0)	0																
Investment Mix																	
Taxable	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	
Tax-exempt	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	
Adjusted Investment Mix																	
Taxable	100.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	
Tax-exempt	0.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	

RATE ADJUSTMENT TO REFLECT INV. INCOME

EXHIBIT D

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
NEW TAX LAW																		
Premium		1000																1000
Incurred L&LE	100%	1000																
Paid L&LE		68	108	196	92	49	57	40	50	40	40	40	50	40	40	50	40	1000
Chg. in Reserve		932	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	0
Und. Profit		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Cash From Und.		932	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	0
Inv. Income:																		
Taxable @	9.0%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Tax-exempt @	6.0%	58	53	47	41	39	39	38	37	37	37	37	37	37	37	36	36	646
Taxes		54	5	-6	0	1	2	0	-3	-2	-5	-4	-4	-2	-1	-1	1	35
Total Cash		936	-60	-143	-51	-11	-20	-2	-10	-1	2	1	-9	-1	-2	-13	-5	611
Cumulative		936	876	733	682	671	651	649	639	638	640	641	632	631	629	616	611	
Discount Factor:		71.81%																
Tax Adjustment		87.50%																
Net Factor		62.83%																
Adjusted Premium		628																628
Paid L&LE		68	108	196	92	49	57	40	50	40	40	40	50	40	40	50	40	1000
Und. Profit		-372	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-372
Cash from Und.		560	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	-372
Inv. Income:																		
Taxable @	9.0%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Tax-exempt @	6.0%	36	37	30	23	21	19	17	15	14	12	11	9	7	6	4	1	262
Taxes @	0%	-74	4	-7	-1	0	1	-1	-4	-3	-6	-5	-6	-4	-3	-2	0	-111
Total Cash		670	-75	-159	-68	-28	-39	-22	-31	-23	-22	-24	-35	-29	-31	-44	-39	1
Cumulative		670	595	436	368	340	301	279	248	225	203	179	144	115	84	40	1	

08-Dec-88

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
New Tax Law																	
Tax Payout Pattern																	
Schedule P	0.34314	0.26722	0.12541	0.08113	0.04904	0.03699	0.01953	0.01320	0.00980	0.00400	0.00400	0.00400	0.00400	0.00400	0.00400	0.03043	1
Workers Comp.	0.2592	0.2861	0.1333	0.0774	0.0447	0.035	0.0188	0.0173	0.015	0.0062	0.0062	0.0062	0.0062	0.0062	0.0062	0.0758	0.9998
Selected	0.34314	0.26722	0.12541	0.08113	0.04904	0.03699	0.01953	0.01320	0.00980	0.00400	0.00400	0.00400	0.00400	0.00400	0.00400	0.03043	1
Period	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5	15.5	
Tax Disc. Factors																	
Int. Factors 7.5%	0.9645	0.8972	0.8346	0.7764	0.7222	0.6718	0.6249	0.5813	0.5408	0.5031	0.4680	0.4353	0.4049	0.3767	0.3504	0.3260	
Discount Factors	0.83953	0.81037	0.79251	0.77002	0.75127	0.72024	0.70809	0.70457	0.70712	0.73819	0.77257	0.81103	0.85466	0.90502	0.96448	1	
Loss Reserves:																	
Undiscounted	932	824	628	536	487	430	390	340	300	260	220	170	130	90	40	0	
Chg.	932	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	0
Discounted	782	668	498	413	366	310	276	240	212	192	170	138	111	81	39	0	
Chg.	782	-114	-170	-85	-47	-56	-34	-36	-28	-20	-22	-32	-27	-30	-42	-39	0
Difference in Chgs.	150	6	-26	-7	-2	-1	-6	-14	-12	-20	-18	-18	-13	-10	-8	-1	0

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
Before Adjustment																		
Und. Income		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Proration:		9	8	7	6	6	6	6	6	6	6	6	6	6	6	5	5	100
Disc. LR Impact		150	6	-26	-7	-2	-1	-6	-14	-12	-20	-18	-18	-13	-10	-8	-1	0
Inv. Income		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Taxable Income		159	14	-19	-1	4	5	0	-8	-6	-14	-12	-12	-7	-4	-3	4	100
Regular Tax @	34%	54	5	-6	0	1	2	0	-3	-2	-5	-4	-4	-2	-1	-1	1	35
AMT																		
Adjusted Earnings		-101	39	66	42	35	34	38	45	43	51	49	49	44	41	39	32	546
Inclusion %		0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	
AMT Income		83	43	31	31	30	31	29	26	26	24	25	25	26	27	26	28	511
AMT @	20%	17	9	6	6	6	6	6	5	5	5	5	5	5	5	5	6	102
Selected Tax	1	54	5	-6	0	1	2	0	-3	-2	-5	-4	-4	-2	-1	-1	1	35
After Adjustment																		
Und. Income		-372	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-372
Proration:		5	6	5	3	3	3	3	2	2	2	2	1	1	1	1	0	40
Disc. LR Impact		150	6	-26	-7	-2	-1	-6	-14	-12	-20	-18	-18	-13	-10	-8	-1	0
Inv. Income		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Taxable Income		-217	12	-21	-4	1	2	-3	-12	-10	-18	-16	-17	-12	-9	-7	-1	-332
Regular Tax @	34%	-74	4	-7	-1	0	1	-1	-4	-3	-6	-5	-6	-4	-3	-2	0	-111
AMT																		
Adjusted Earnings		-119	25	51	27	20	17	20	27	24	30	27	26	19	15	11	2	222
Inclusion %		0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	
AMT Income		-306	31	17	16	16	15	12	8	8	5	4	3	2	2	1	1	-165
AMT @	20%	-61	6	3	3	3	3	2	2	2	1	1	1	0	0	0	0	-34
Selected Tax	1	-74	4	-7	-1	0	1	-1	-4	-3	-6	-5	-6	-4	-3	-2	0	-111

RATE ADJUSTMENT TO REFLECT INV. INCOME

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
Credit for																	
Tax Benefit	1																
Investment Mix																	
Taxable	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	
Tax-exempt	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	
Adjusted																	
Investment Mix																	
Taxable	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	
Tax-exempt	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
NEW TAX LAW																		
Premium		1000																1000
Incurred L&LE	100%	1000																
Paid L&LE		68	108	196	92	49	57	40	50	40	40	40	50	40	40	50	40	1000
Chg. in Reserve		932	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	0
Und. Profit		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Cash From Und.		932	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	0
Inv. Income:																		
Taxable @	9.0%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Tax-exempt @	6.0%	58	53	47	41	39	39	38	37	37	37	37	37	37	37	36	36	646
Taxes		54	5	-6	0	1	2	0	-3	-2	-5	-4	-4	-2	-1	-1	1	35
Total Cash		936	-60	-143	-51	-11	-20	-2	-10	-1	2	1	-9	-1	-2	-13	-5	611
Cumulative		936	876	733	682	671	651	649	639	638	640	641	632	631	629	616	611	

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
Discount Factor:		71.81%																
Tax Adjustment		98.00%																
Net Factor		70.37%																
Adjusted Premium		704																704
Paid L&LE		68	108	196	92	49	57	40	50	40	40	40	50	40	40	50	40	1000
Und. Profit		-296	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-296
Cash from Und.		636	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	-296
Inv. Income:																		
Taxable @	9.0%	60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	60
Tax-exempt @	6.0%	0	39	32	25	22	20	18	17	15	14	12	10	8	6	4	1	243
Taxes @	0%	0	4	0	0	0	1	0	0	0	0	0	0	0	0	0	0	5
Total Cash		696	-73	-164	-67	-27	-38	-22	-33	-25	-26	-28	-40	-32	-34	-46	-39	2
Cumulative		696	623	459	392	365	327	305	272	247	221	193	153	121	87	41	2	

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
New Tax Law																	
Tax Payout Pattern																	
Schedule P	0.34314	0.26722	0.12541	0.08113	0.04904	0.03699	0.01953	0.01320	0.00980	0.00400	0.00400	0.00400	0.00400	0.00400	0.00400	0.03043	1
Workers Comp.	0.2592	0.2861	0.1333	0.0774	0.0447	0.035	0.0188	0.0173	0.015	0.0062	0.0062	0.0062	0.0062	0.0062	0.0062	0.0758	0.9998
Selected	0.34314	0.26722	0.12541	0.08113	0.04904	0.03699	0.01953	0.01320	0.00980	0.00400	0.00400	0.00400	0.00400	0.00400	0.00400	0.03043	1
Period	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5	15.5	
Tax Disc. Factors																	
Int. Factors 7.5%	0.9645	0.8972	0.8346	0.7764	0.7222	0.6718	0.6249	0.5813	0.5408	0.5031	0.4680	0.4353	0.4049	0.3767	0.3504	0.3260	
Discount Factors	0.83953	0.81037	0.79251	0.77002	0.75127	0.72024	0.70809	0.70457	0.70712	0.73819	0.77257	0.81103	0.85466	0.90502	0.96448	1	
Loss Reserves:																	
Undiscounted	932	824	628	536	487	430	390	340	300	260	220	170	130	90	40	0	
Chg.	932	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	0
Discounted	782	668	498	413	366	310	276	240	212	192	170	138	111	81	39	0	
Chg.	782	-114	-170	-85	-47	-56	-34	-36	-28	-20	-22	-32	-27	-30	-42	-39	0
Difference in Chgs.	150	6	-26	-7	-2	-1	-6	-14	-12	-20	-18	-18	-13	-10	-8	-1	0

08-Dec-88

RATE ADJUSTMENT TO REFLECT INV. INCOME

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
Before Adjustment																		
Und. Income		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Proration:		9	8	7	6	6	6	6	6	6	6	6	6	6	6	5	5	100
Disc. LR Impact		150	6	-26	-7	-2	-1	-6	-14	-12	-20	-18	-18	-13	-10	-8	-1	0
Inv. Income		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Taxable Income		159	14	-19	-1	4	5	0	-8	-6	-14	-12	-12	-7	-4	-3	4	100
Regular Tax @	34%	54	5	-6	0	1	2	0	-3	-2	-5	-4	-4	-2	-1	-1	1	35
AMT																		
Adjusted Earnings		-101	39	66	42	35	34	38	45	43	51	49	49	44	41	39	32	546
Inclusion %		0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	
AMT Income		83	43	31	31	30	31	29	26	26	24	25	25	26	27	26	28	511
AMT @	20%	17	9	6	6	6	6	6	5	5	5	5	5	5	5	5	6	102
Selected Tax	1	54	5	-6	0	1	2	0	-3	-2	-5	-4	-4	-2	-1	-1	1	35
After Adjustment																		
Und. Income		-296	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-296
Proration:		0	6	5	4	3	3	3	3	2	2	2	2	1	1	1	0	38
Disc. LR Impact		150	6	-26	-7	-2	-1	-6	-14	-12	-20	-18	-18	-13	-10	-8	-1	0
Inv. Income		60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	60
Taxable Income		-86	12	-21	-3	1	2	-3	-11	-10	-18	-16	-16	-12	-9	-7	-1	-198
Regular Tax @	34%	-29	4	-7	-1	0	1	-1	-4	-3	-6	-5	-5	-4	-3	-2	0	-65
AMT																		
Adjusted Earnings		-150	27	53	28	21	18	21	28	25	32	28	26	20	15	11	2	205
Inclusion %		0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	
AMT Income		-199	32	19	18	17	16	13	10	9	6	5	4	3	2	1	1	-43
AMT @	20%	-40	6	4	4	3	3	3	2	2	1	1	1	1	0	0	0	-9
Selected Tax	1	-29	4	-7	-1	0	1	-1	-4	-3	-6	-5	-5	-4	-3	-2	0	-65

RATE ADJUSTMENT TO REFLECT INV. INCOME

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
Credit for Tax Benefit	0																
Investment Mix																	
Taxable	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	
Tax-exempt	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	
Adjusted Investment Mix																	
Taxable	100.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	
Tax-exempt	0.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	

RATE ADJUSTMENT TO REFLECT INV. INCOME

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
NEW TAX LAW																		
Premium		1000																1000
Incurred L&LE	100%	1000																
Paid L&LE		68	108	196	92	49	57	40	50	40	40	40	50	40	40	50	40	1000
Chg. in Reserve		932	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	0
Und. Profit		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Cash From Und.		932	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	0
Inv. Income:																		
Taxable @	9.0%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Tax-exempt @	6.0%	58	55	49	43	41	40	39	38	37	37	36	36	35	34	33	32	643
Taxes		17	9	6	6	6	6	6	5	5	5	5	5	5	5	5	5	101
Total Cash		973	-62	-153	-55	-14	-23	-7	-17	-8	-8	-9	-19	-10	-11	-22	-13	542
Cumulative		973	911	758	703	689	666	659	642	634	626	617	598	588	577	555	542	
Discount Factor:		71.81%																
Tax Adjustment		97.00%																
Net Factor		69.66%																
Adjusted Premium		697																697
Paid L&LE		68	108	196	92	49	57	40	50	40	40	40	50	40	40	50	40	1000
Und. Profit		-303	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-303
Cash from Und.		629	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	-303
Inv. Income:																		
Taxable @	9.0%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Tax-exempt @	6.0%	40	40	33	26	23	21	19	17	15	14	12	10	8	6	4	1	289
Taxes @	0%	-47	7	4	4	4	3	3	2	2	1	1	1	1	0	0	0	-14
Total Cash		716	-75	-167	-70	-30	-39	-24	-35	-27	-27	-29	-41	-33	-34	-46	-39	0
Cumulative		716	641	474	404	374	335	311	276	249	222	193	152	119	85	39	0	

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
New Tax Law																	
Tax Payout Pattern																	
Schedule P	0.34314	0.26722	0.12541	0.08113	0.04904	0.03699	0.01953	0.01320	0.00980	0.00400	0.00400	0.00400	0.00400	0.00400	0.00400	0.03043	1
Workers Comp.	0.2592	0.2861	0.1333	0.0774	0.0447	0.035	0.0188	0.0173	0.015	0.0062	0.0062	0.0062	0.0062	0.0062	0.0062	0.0758	0.9998
Selected	0.34314	0.26722	0.12541	0.08113	0.04904	0.03699	0.01953	0.01320	0.00980	0.00400	0.00400	0.00400	0.00400	0.00400	0.00400	0.03043	1
Period	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5	15.5	
Tax Disc. Factors																	
Int. Factors 7.5%	0.9645	0.8972	0.8346	0.7764	0.7222	0.6718	0.6249	0.5813	0.5408	0.5031	0.4680	0.4353	0.4049	0.3767	0.3504	0.3260	
Discount Factors	0.83953	0.81037	0.79251	0.77002	0.75127	0.72024	0.70809	0.70457	0.70712	0.73819	0.77257	0.81103	0.85466	0.90502	0.96448	1	
Loss Reserves:																	
Undiscounted	932	824	628	536	487	430	390	340	300	260	220	170	130	90	40	0	
Chg.	932	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	0
Discounted	782	668	498	413	366	310	276	240	212	192	170	138	111	81	39	0	
Chg.	782	-114	-170	-85	-47	-56	-34	-36	-28	-20	-22	-32	-27	-30	-42	-39	0
Difference in Chgs.	150	6	-26	-7	-2	-1	-6	-14	-12	-20	-18	-18	-13	-10	-8	-1	0

08-Dec-88

RATE ADJUSTMENT TO REFLECT INV. INCOME

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
Before Adjustment																		
Und. Income		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Proration:		9	8	7	6	6	6	6	6	6	6	5	5	5	5	5	5	96
Disc. LR Impact		150	6	-26	-7	-2	-1	-6	-14	-12	-20	-18	-18	-13	-10	-8	-1	0
Inv. Income		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Taxable Income		159	14	-19	-1	4	5	0	-8	-6	-14	-13	-13	-8	-5	-3	4	96
Regular Tax @	34%	54	5	-6	0	1	2	0	-3	-2	-5	-4	-4	-3	-2	-1	1	33
AMT																		
Adjusted Earnings		-101	41	68	44	37	35	39	46	43	51	49	49	43	39	36	28	547
Inclusion %		0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	
AMT Income		83	45	32	32	32	31	29	27	26	24	24	24	24	24	24	25	506
AMT @	20%	17	9	6	6	6	6	6	5	5	5	5	5	5	5	5	5	101
Selected Tax	2	17	9	6	6	6	6	6	5	5	5	5	5	5	5	5	5	101
After Adjustment																		
Und. Income		-303	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-303
Proration:		6	6	5	4	3	3	3	3	2	2	2	2	1	1	1	0	44
Disc. LR Impact		150	6	-26	-7	-2	-1	-6	-14	-12	-20	-18	-18	-13	-10	-8	-1	0
Inv. Income		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Taxable Income		-147	12	-21	-3	1	2	-3	-11	-10	-18	-16	-16	-12	-9	-7	-1	-259
Regular Tax @	34%	-50	4	-7	-1	0	1	-1	-4	-3	-6	-5	-5	-4	-3	-2	0	-86
AMT																		
Adjusted Earnings		-116	28	54	29	22	19	22	28	25	32	28	26	20	15	11	2	245
Inclusion %		0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	
AMT Income		-234	33	20	19	18	16	14	10	9	6	5	4	3	2	1	1	-73
AMT @	20%	-47	7	4	4	4	3	3	2	2	1	1	1	1	0	0	0	-14
Selected Tax	2	-47	7	4	4	4	3	3	2	2	1	1	1	1	0	0	0	-14

08-Dec-88

RATE ADJUSTMENT TO REFLECT INV. INCOME

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
Credit for																	
Tax Benefit	1																
Investment Mix																	
Taxable	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	
Tax-exempt	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	
Adjusted																	
Investment Mix																	
Taxable	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	
Tax-exempt	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	

RATE ADJUSTMENT TO REFLECT INV. INCOME

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
NEW TAX LAW																		
Premium		1000																1000
Incurred L&LE	100%	1000																
Paid L&LE		68	108	196	92	49	57	40	50	40	40	40	50	40	40	50	40	1000
Chg. in Reserve		932	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	0
Und. Profit		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Cash From Und.		932	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	0
Inv. Income:																		
Taxable @	9.0%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Tax-exempt @	6.0%	58	55	49	43	41	40	39	38	37	37	36	36	35	34	33	32	643
Taxes		17	9	6	6	6	6	6	5	5	5	5	5	5	5	5	5	101
Total Cash		973	-62	-153	-55	-14	-23	-7	-17	-8	-8	-9	-19	-10	-11	-22	-13	542
Cumulative		973	911	758	703	689	666	659	642	634	626	617	598	588	577	555	542	
Discount Factor:		71.81%																
Tax Adjustment		100.50%																
Net Factor		72.17%																
Adjusted Premium		722																722
Paid L&LE		68	108	196	92	49	57	40	50	40	40	40	50	40	40	50	40	1000
Und. Profit		-278	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-278
Cash from Und.		654	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	-278
Inv. Income:																		
Taxable @	9.0%	62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	62
Tax-exempt @	6.0%	0	40	33	26	23	21	19	17	15	14	12	10	8	6	4	1	249
Taxes @	0%	0	7	4	4	4	3	3	2	2	1	1	1	1	0	0	0	33
Total Cash		716	-75	-167	-70	-30	-39	-24	-35	-27	-27	-29	-41	-33	-34	-46	-39	0
Cumulative		716	641	474	404	374	335	311	276	249	222	193	152	119	85	39	0	

08-Dec-88

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
New Tax Law																	
Tax Payout Pattern																	
Schedule P	0.34314	0.26722	0.12541	0.08113	0.04904	0.03699	0.01953	0.01320	0.00980	0.00400	0.00400	0.00400	0.00400	0.00400	0.00400	0.03043	1
Workers Comp.	0.2592	0.2861	0.1333	0.0774	0.0447	0.035	0.0188	0.0173	0.015	0.0062	0.0062	0.0062	0.0062	0.0062	0.0062	0.0758	0.9998
Selected	0.34314	0.26722	0.12541	0.08113	0.04904	0.03699	0.01953	0.01320	0.00980	0.00400	0.00400	0.00400	0.00400	0.00400	0.00400	0.03043	1
Period	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5	15.5	
Tax Disc. Factors																	
Int. Factors 7.5%	0.9645	0.8972	0.8346	0.7764	0.7222	0.6718	0.6249	0.5813	0.5408	0.5031	0.4680	0.4353	0.4049	0.3767	0.3504	0.3260	
Discount Factors	0.83953	0.81037	0.79251	0.77002	0.75127	0.72024	0.70809	0.70457	0.70712	0.73819	0.77257	0.81103	0.85466	0.90502	0.96448	1	
Loss Reserves:																	
Undiscounted	932	824	628	536	487	430	390	340	300	260	220	170	130	90	40	0	
Chg.	932	-108	-196	-92	-49	-57	-40	-50	-40	-40	-40	-50	-40	-40	-50	-40	0
Discounted	782	668	498	413	366	310	276	240	212	192	170	138	111	81	39	0	
Chg.	782	-114	-170	-85	-47	-56	-34	-36	-28	-20	-22	-32	-27	-30	-42	-39	0
Difference in Chgs.	150	6	-26	-7	-2	-1	-6	-14	-12	-20	-18	-18	-13	-10	-8	-1	0

RATE ADJUSTMENT TO REFLECT INV. INCOME

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
Before Adjustment																		
Und. Income		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Proration:		9	8	7	6	6	6	6	6	6	6	5	5	5	5	5	5	96
Disc. LR Impact		150	6	-26	-7	-2	-1	-6	-14	-12	-20	-18	-18	-13	-10	-8	-1	0
Inv. Income		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Taxable Income		159	14	-19	-1	4	5	0	-8	-6	-14	-13	-13	-8	-5	-3	4	96
Regular Tax @	34%	54	5	-6	0	1	2	0	-3	-2	-5	-4	-4	-3	-2	-1	1	33
AMT																		
Adjusted Earnings		-101	41	68	44	37	35	39	46	43	51	49	49	43	39	36	28	547
Inclusion %		0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	
AMT Income		83	45	32	32	32	31	29	27	26	24	24	24	24	24	24	25	506
AMT @	20%	17	9	6	6	6	6	6	5	5	5	5	5	5	5	5	5	101
Selected Tax	2	17	9	6	6	6	6	6	5	5	5	5	5	5	5	5	5	101
After Adjustment																		
Und. Income		-278	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-278
Proration:		0	6	5	4	3	3	3	3	2	2	2	2	1	1	1	0	38
Disc. LR Impact		150	6	-26	-7	-2	-1	-6	-14	-12	-20	-18	-18	-13	-10	-8	-1	0
Inv. Income		62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	62
Taxable Income		-66	12	-21	-3	1	2	-3	-11	-10	-18	-16	-16	-12	-9	-7	-1	-178
Regular Tax @	34%	-22	4	-7	-1	0	1	-1	-4	-3	-6	-5	-5	-4	-3	-2	0	-58
AMT																		
Adjusted Earnings		-150	28	54	29	22	19	22	28	25	32	28	26	20	15	11	2	211
Inclusion %		0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	
AMT Income		-179	33	20	19	18	16	14	10	9	6	5	4	3	2	1	1	-18
AMT @	20%	-36	7	4	4	4	3	3	2	2	1	1	1	1	0	0	0	-3
Selected Tax	2	-36	7	4	4	4	3	3	2	2	1	1	1	1	0	0	0	-3

RATE ADJUSTMENT TO REFLECT INV. INCOME

EXHIBIT G

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
Credit for																	
Tax Benefit	0																
Investment Mix																	
Taxable	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	
Tax-exempt	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	
Adjusted																	
Investment Mix																	
Taxable	100.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	
Tax-exempt	0.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	100.000%	

APPENDIX A

This is to the document the methodology supporting the discount factors attached. Essentially, the discount factors are a function of an estimated (or assumed) loss payment pattern and a projected interest rate.

In this example, the basis for our estimated loss payment pattern are the workers compensation age-to-ultimate paid factors for Best's Selected Reinsurance Companies (Exhibits prepared by ABC Re). The cumulative paid factor for a given period is the reciprocal of the paid-to-ultimate factor.

Best's factors were selected simply because we had no other industry data to use. RAA does not publish paid-to-ultimate factors. Even though the Best factors are from reinsurance companies, it is possible that they could include some primary experience as well as excess; hence, we believe that the figures are somewhat conservative.

Unfortunately, the paid-to-ultimate factors produced negative payment factors in certain periods, due to the methodology on which they were based. We judgmentally adjusted the cumulative payment pattern by plotting the points of the Best Data and drawing a smooth curve (actually, it turned out to be a straight line) beginning at the point where the the Best data turned negative. We also elected to truncate the payment pattern at 16 years. We believe that both of these adjustments would tend to produce more conservative results.

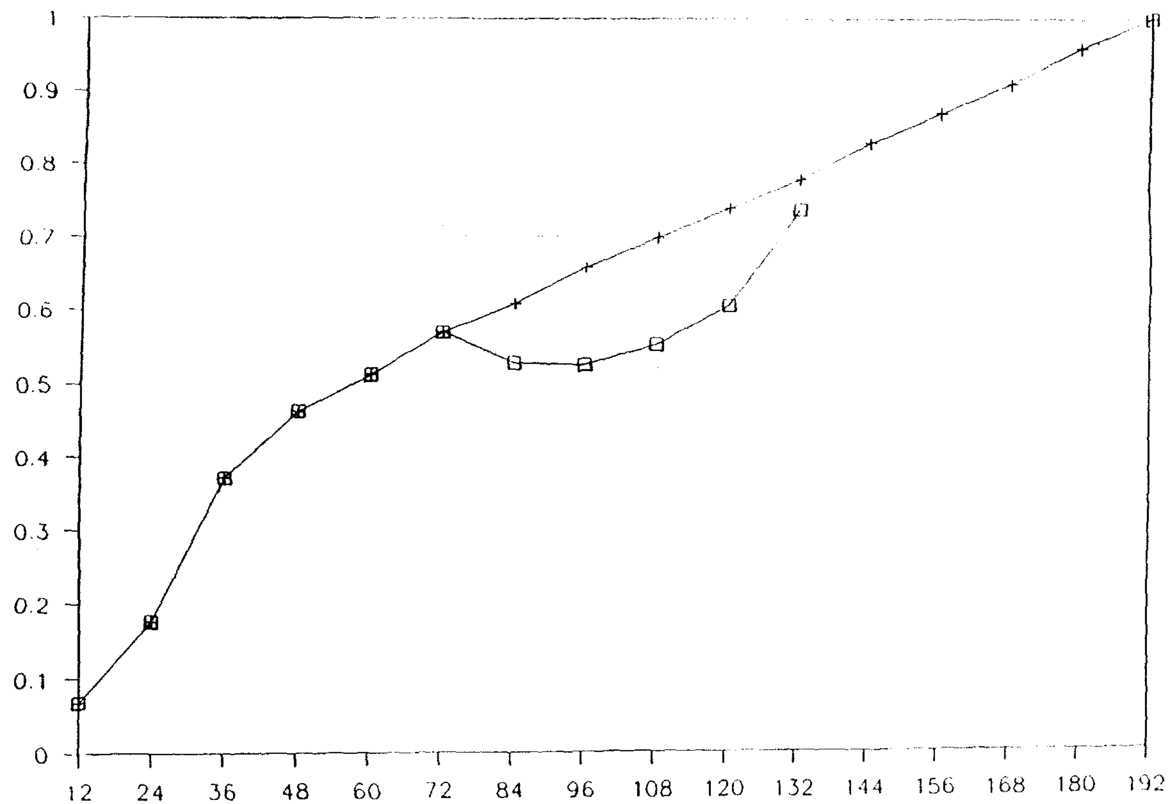
The discount factors calculated at interest rates of 5%, 7%, and 9% are 75.45%, 68.49%, and 62.65% respectively. These factors would be applied to the loss component of the reinsurance rate to adjust the rate for investment income on loss reserves.

WORKERS COMPENSATION XS

	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192
Best's Selected ATU Factors	14.684	5.667	2.691	2.155	1.949	1.753	1.89	1.9	1.808	1.648	1.355					
Cumulative Paid	0.068101	0.176460	0.371609	0.464037	0.513083	0.570450	0.529100	0.526315	0.553097	0.606796	0.738007					
Period Paid	0.068101	0.108358	0.195148	0.092428	0.049046	0.057367	-0.04135	-0.00278	0.026781	0.053698	0.131211					
Selected Cumulative Paid	0.068	0.176	0.372	0.464	0.513	0.57	0.61	0.66	0.7	0.74	0.78	0.83	0.87	0.91	0.96	1
Period Paid	0.068	0.108	0.196	0.092	0.049	0.057	0.04	0.05	0.04	0.04	0.04	0.05	0.04	0.04	0.05	0.04

Period	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5	15.5
Discount Factor @ Int. Rates:																
5.0% 75.446%	0.066361	0.100378	0.173493	0.077557	0.039340	0.043584	0.029129	0.034677	0.026421	0.025162	0.023964	0.028529	0.021736	0.020701	0.024644	0.018776
7.0% 68.485%	0.065738	0.097577	0.165499	0.072601	0.036138	0.039288	0.025767	0.030101	0.022505	0.021033	0.019657	0.022964	0.017169	0.016046	0.018745	0.014015
9.0% 62.645%	0.065132	0.094903	0.158011	0.068044	0.033248	0.035483	0.022844	0.026198	0.019228	0.017640	0.016183	0.018559	0.013621	0.012496	0.014331	0.010518

29-Jan-89 jsn


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
12 24 36 48 60 72 84 96 108 120 132 144 156 168 180 192

APPENDIX B

The following is an exerpt from FOUNDATIONS OF CASUALTY ACTUARIAL SCIENCE, Chapter 8 - Special Issues (draft version published in the CAS Forum series), authored by Steve D'Arcy.

> ..... The various methodologies for including investment income in the determination of an allowable underwriting profit margin have the advantage of producing specific indications which can be used to establish rates. However, each method is subject to criticism for ignoring certain circumstances or requiring a value to be estimated that is difficult or impossible to obtain. An alternative school argues that investment income should be given indirect consideration, rather than be attempted to be included directly in the ratemaking process. The arguments in favor of this position are:
>
> 1. No formula approach is recognized as producing the correct results in all situations,
>
> 2. The effect of competition on insurance prices is ignored in ratemaking formulae, but is crucial to the ability of an insurer to charge a particular rate level,
>
> 3. If rates in a particular market are producing an excessive rate of return for insurers in total then new entry will drive the price down to the proper level,
>
> 4. If rate levels are inadequate to produce an acceptable rate of return in total then insurers will exit from the market until price levels increase to the acceptable level,
>
> 5. Analysis of the difference in rate levels in prior approval and competition states indicates that there are no significant differences in profitability over any extended time.
>
> The conclusion of these observations is that financial and insurance markets will work to produce the proper total rate of return for insurers, without the need for complicated formula adjustments. .....

SPLITTING ALLOCATED LOSS ADJUSTMENT EXPENSE

Splitting Allocated Loss Adjustment Expense

by

LeRoy J. Simon*

Facultative casualty reinsurance certificates and working layer casualty excess of loss reinsurance treaties will often provide that the primary company and its reinsurer are to share Allocated Loss Adjustment Expense (ALAE) in proportion to their respective amounts of the indemnity loss. This works well in most cases and can be properly priced by the reinsurer and evaluated by the primary company before entering into the reinsurance contract.

A conflict may occur when a subrogation opportunity arises, however. The reinsurance will usually provide that the apportionment is based on the indemnity loss payments as determined after the subrogation is finalized. Unless the expected value of the primary company's loss plus ALAE after subrogation is less than beforehand, there is no incentive to pursue the matter. However, the reinsurer would be anxious to do so in most cases and here lies an opportunity for some actuarial help to both parties.

A careful analysis of the situation may help each party focus more accurately on the implied probability distributions and thus more accurately evaluate the expected values. It may also show the way for two parties with different probability distributions to come closer together in their agreement on a common distribution. Failing to reach agreement on a course of action, the mediator/actuary might show the parties how to fashion a division of the ALAE that will bring them into agreement on how to proceed.

A simple case for illustrative purposes might be a casualty excess of loss treaty (or a facultative casualty certificate if you prefer) where the primary company's retention (R) is 5 (all figures can be thought of as being in millions of US$). A loss (L), which is covered by reinsurers through one or more layers, has been paid for 25 with an ALAE (A) of 0.2. It is now proposed to spend an additional amount of ALAE (b) to recover an uncertain amount of subrogation (x) which has a probability distribution f(x). Capital letters are used for those values which are known when the analysis is to be made and lower case letters are used for those which must be estimated or are variables in the solution.

Now let us consider the simple probability distribution case (see Table 1 for a convenient summary of the equations involved) where there are only two possible outcomes: (1) to win and get a total recovery with probability p, or (2) to lose and recover nothing with probability 1 - p. Under (1) the cost to the primary company will be (A + b). Under (2) the cost will be R*(1 + (A + b)/L). Hence the expected value of the cost to the primary company is:

$$(A + b)*(p) + (R + R*(A + b)/L)*(1 - p)$$

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* Presented during the Speaker's Corner of the March 29, 1989 meeting of the Casualty Actuaries of Greater New York.

If this value is less than the cost of not pursuing the subrogation, then the primary company will be interested in pursuing it. That cost is:

$$R*(1 + A/L)$$

A little algebra shows that the no-go situation exists for the primary company when the following decision function, PCDF, is negative:

$$PCDF_1 = R*(p - b/L) - p*(A + b)*(1 - R/L)$$

When the function is positive, it is in the primary company's best economic interest to pursue the subrogation. Remember, however, that we are making the (1) and (2) outcome assumptions above.

TABLE 1

Summary of Equations for the Simple Case

	Primary Company	Reinsurance Company	Third Party	TOTAL
If win (p)	A + b	0	L	L + A + b
If Lose (1-p)	R*(1 + (A + b)/L)	(L - R)*(1 + (A + b)/L)	0	L + A + b
If No Action	R*(1 + A/L)	(L - R)*(1 + A/L)	0	L + A
Decision Function	R*(p - b/L) - p*(A + b)*(1 - R/L)	(1 - R/L)*(p*(L + A + b) - b)	-	p*L - b
Critical Probability	b/(L + (A + b)*(1 - L/R))	b/(L + A + b)	-	-

In our numeric example, suppose b is estimated to be 4 and p is ½. The $PCDF_1$ evaluates to 0.02. Therefore the primary company would have an interest in pursuing the subrogation. The reinsurers (with 20 of indemnity loss at stake and a 50-50 chance of winning) no doubt would also, but we will investigate that below.

If the probability of winning were just slightly different, say p = .40, the primary company decision function would evaluate to -0.144 and it would not be interested in going forward. In fact, under the assumptions made, the critical probability value is .4878 (that is, b/(L + (A + b)*(1 - L/R))); above that the primary company is willing to pursue subrogation; below it, it is unwilling.

Now let's look at things from the reinsurer's standpoint. Under the same (1) and (2) assumptions, the expected value of the cost to the reinsurer under (1) is zero and under

(2) is (L - R)*(1 + (A + b)/L)*(1 - p). If this value is less than the cost of not pursuing the subrogation, then the reinsurer will be interested in pursuing it. That cost is:

$$(L - R)*(1 + A/L)$$

Again, the reinsurer's no-go situation exists when the following decision function, $RCDF_1$ is negative:

$$RCDF_1 = (1 - R/L)*(p*(L + A + b) - b)$$

When the function is positive, it is in the reinsurer's best economic interest to pursue the subrogation. Since (L + A) will ordinarily be quite large, it takes a combination of a large b and a small p to make this function negative; hence the reinsurer's preference will most likely be to pursue subrogation.

In our numeric example, b = 4 and p = ½, $RCDF_1$ evaluates to 8.48 and thus a go situation for the reinsurer as well.

If the probability of winning were quite low, say p = .10, the reinsurer's decision function would evaluate to -0.864 and it would not be interested in going forward. Under the assumptions made, the critical probability value for the reinsurer is .1370 (that is, b/(L + A + b)); below that it is unwilling to pursue subrogation; above it, it is willing.

Hence, in this illustrative case, even if the two parties could agree on the estimate of additional ALAE to pursue subrogation (b) and upon the probability of success (p), if b was 4 and p fell between .1370 and .4878, they would reach opposite conclusions.

Certain conclusions are quite clear from the decision functions. In the $PCDF_1$, for example, a very small retention makes it unlikely that the primary company will be interested in pursuing subrogation. In such treaties you should have a split of ALAE which is a better incentive to both parties. The reinsurer who writes an entire layer and considers the $RCDF_1$ will find it compelling in most cases to want to purse subrogation. However, if the reinsurer were to only write a small piece of a layer or the entire amount of a narrow ribbon of a layer, it would be much easier to rationalize the waiving of the subrogation which the primary company was unlikely to wish to pursue anyway.

Reinsurers will also want to consider carefully the situation where the primary company is in liquidation -- the liquidator may have a different viewpoint from that of an ongoing company. He might be quite amenable to, and much more flexible in, negotiating the split of the additional ALAE to pursue the subrogation. Also note how much easier it will be for a reinsurer to negotiate if it has a single, substantial commitment instead of being a small part of various covers spread over an extended set of layers.

As a final exercise, consider the situation where all results between no recovery and full recovery of the indemnity loss are considered to be equally likely (see Table 2 for a convenient summary of the equations involved). In that event the PCDF would again come in two parts: (3) to recover more than the reinsurer's interest (that is, bring the loss down under the retention) in which case the cost to the primary company would be (L - x + A + b) and (4) to recover less than the reinsurer's interest in which case the cost would be R*(1 + (A + b)/(L - x)).

If this value is less than the cost of not pursuing the subrogation then the primary company will not be interesting in pursuing it. That cost continues to be:

$$R*(1 + A/L)$$

We now integrate over the respective ranges of (3) and (4):

$$\int_{L-R}^{L} (L - x + A + b)\, dx \;+\; \int_{0}^{L-R} R*(1 + (A + b)/(L - x))\, dx$$

We then evaluate, divide by the integral of dx over the range 0 to L, and deduct the cost of not pursing subrogation to arrive at the decision function under the new probability function assumption:

$$PCDF_2 = R*(R/2 - b - (A + b)*\ln(L/R))/L$$

where ln is the natural logarithm.

When the function is positive, it is in the primary company's best economic interest to pursue the subrogation.

TABLE 2

Summary of Equations for the Equal Probability Case

	If Recovery is Greater than reinsurer's interest (Range = L-R to L)	If Recovery is Less than reinsurer's interest (Range = 0 to L-R)	If No Action
Primary Company	L - x + A + b	R*(1 + (A + b)/(L - x))	R*(1 + A/L)
Reinsurer	0	(L - x - R)*(1 + (A + b)/(L - x))	(L - R)*(1 + A/L)
Third Party	x	x	0
TOTAL after integration over the range	R*(L + A + b)	(L - R)*(L + A + b)	L + A

If we keep all the values of our previous example but now use the new probability function, $PCDF_2$ evaluates to -1.65; that is, do not pursue.

Now let's again look at this situation from the reinsurer's standpoint. Under (3) its expected value of the cost is zero and under (4) it is (L - x - R)*(1 + (A + b)/(L - x)).

If this value is less than the cost of not pursuing the subrogation then the reinsurer will not be interesting in pursuing it. That cost continues to be:

$$(L - R)*(1 + A/L)$$

We now integrate (4) over the range:

$$\int_0^{L-R} (L - x - R)*(1 + (A + b)/(L - x))\, dx$$

We then evaluate, divide by the integral of dx over the range 0 to L, and deduct the cost of not pursing subrogation to arrive at the decision function under the new probability function assumption:

$$RCDF_2 = (R*((A + b)*\ln L/R) - (L - R)*(b - L/2 - R/2))/L$$

When the function is positive, it is in the reinsurer's best economic interest to pursue the subrogation.

In our numeric example, $RCDF_2$ evaluates to 10.15; that is, pursue. The reinsurer and primary company reach opposite conclusions.

This analysis has focused entirely on the expected value of the decision to be made and has not considered factors such as the working relationship between the primary company and its reinsurer. That bond may be strong enough to override an expected value calculation because of long term -- past or future -- values. Neither have we considered the effect of retrospective or prospective experience rating on treaties. This may again cause a sufficient effect on the primary company's total expected value in the long run that it would reach a different conclusion in some instances. Further sophistication could introduce present value concepts since pursing subrogation can sometimes be a time consuming process.

Note that the unallocated loss expenses of the primary company and of the reinsurer are not considered here at all. ALAE for the Third Party is not mentioned, of course, since it is not relevant to the decision process although it does contribute to the global cost of the entire system.

In closing, it would be of interest to increase the number of discrete probabilities in our first approach and study the situation then. It would also be of interest to have the second approach assume probability curves which were quite optimistic of success or quite pessimistic and have the reinsurer and the primary company choose them in the four possible combinations. Finally it might be worthwhile to study a very different contractual agreement on splitting allocated loss adjustment expense to see if the conflict presented by this proportional method could be avoided.

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A METHOD TO CALCULATE AGGREGATE EXCESS LOSS DISTRIBUTIONS

A Method to Calculate Aggregate Excess Loss Distributions

by Joseph R. Schumi

Abstract
The purpose of the paper is to develop a method of calculating the aggregate loss distribution of excess claims based on a formula described in the book Risk Theory by Beard, Pentikainenen, and Pesonen. This formula requires that the claim frequency distribution satisfy a certain recursive relationship.

The first part of the paper shows that a claim frequency distributions of excess claims derived from a claim frequency distribution satisfying the recursive relationship also has that recursive property.

The second part describes a simple Pascal program that implements the calculation of aggregate loss distributions using these formulas.

Introduction

Like many actuarial departments, we have been using a variety of tools to determine the distribution of aggregate losses assuming we know something about the underlying frequency and severity parameters.

We have an analytical model based on the Fortran program described in the the article by Glenn Meyers and Phil Heckman. Through the work of our own staff we also have stochastic simulations. While these models have proven to be very useful, they do not work in all situations. The analytic models often don't behave well when there is a large probability spike in the severity distribution or a fractional expected number of claims. The effectiveness of simulation models may be questioned because of concerns about the "randomness" of the random number generators, at least for some of the PC based versions.

In their book, *Risk Theory*, Beard etal describe a method of calculating aggregate loss distributions for Compound Poisson processes when the claim frequency can be expressed in a particular recursive form and the claim severity distribution is discrete on uniformly spaced points. It turns out that the family of claim distributions satisfying the recursive relationship includes both the Poisson and Negative Binomial distributions, both of which are familiar to actuaries.

It is also true that any reasonable claim size distribution can be approximated to any desired degree of accuracy by an equally spaced discrete probability distribution, at least over a finite interval. Though, to suitably approximate many of the standard claim size distributions, it can require a relatively large number of grid points. On the other hand, the probability distribution of excess claims usually exhibits a fairly simple pattern and thus can be approximated reasonably well by a relatively small number of points.

The advantage of this method is that the computation does not have any of the convergence problems of the analytic approaches based on Fourier Series either when the number of claims is small or there is a point mass at the upper end of the interval. Of course, you do need to pay attention to the quality of the claim severity approximation.

To implement this approach there were two things that needed to be done.

The first was to answer a purely statistical question - given the probability distribution of first dollar claims, what is known about the probability distribution of excess claims. In particular, if you assumed that the starting distribution was Poisson or Negative Binomial with a given set of parameters, could you assume that the distribution of claims in excess of some loss amount was still Poisson or Negative Binomial and determine the new parameters? More generally, if the form of the original distribution satisfied the recursive relationship, will the resulting excess claim distribution?

The second was to write a computer program to carry out the calculations. In the second section I will describe an elementary version of such a program written in Turbo Pascal for the Macintosh.

The Frequency of Excess Claims
In this section, I consider the recursive formula described in ***Risk Theory***. (esp. Sections 2.9 and 3.8) The intent is to show that the distribution of excess claims derived from such a recursive claim distribution is also recursive. The basic procedure I follow is to consider the probability of no excess claims and the probability of n excess claims.

Let Π_n stand for the probability of n excess claims, and let π stand for the probability that a claim is an excess claim and let $\theta = (1-\pi)$. Recall that $({}_iC_j)$ stands for the binomial coefficient $(i(i-1)...(i-j+1))/j!$

Make whatever assumptions about independence are necessary and assume that the claim frequency distribution satisfies the recursive relationship

$$P_{n+1} = [a+b/(n+1)]\, P_n ,$$

where P_n, for $n \geq 0$, is the probability that the number of claims equals n. Since $\Sigma P_n = 1$, the parameter a must be less than one, and in the following assume that the parameter a is positive.

Then the probability of no excess claims can be expressed as

$$\Pi_0 = P_0 + P_1\theta + P_2\theta^2 + ... + P_n\theta^n + ...$$

and by using the recursive relationship

$$\Pi_0 = P_0 + (a+b)P_0\theta + (a+b/2)(a+b)P_0\theta^2 + ... + (a+b/n)...(a+b)P_0\theta^n + ...$$

Factoring out P_0, we have

$$\Pi_0 = P_0 \{1 + (a+b)\theta + (a+b/2)(a+b)\theta^2 + ... + (a+b/n) ... (a+b)\theta^n + ... \}.$$

By rearranging the terms, we obtain

$$\Pi_0 = P_0 \{1 + [(b/a)+1](a\theta) + [(b/a)+1][(b/a)+2](a\theta)^2/2 + ... + [(b/a)+1] ... [(b/a)+n](a\theta)^n/n! + ... \}.$$

Since $a\theta$ is less than one, the series inside the brackets converges and is equal to

$$\{1 - a\theta\}^{-[(b/a)+1]},$$

or alternatively

$$\{1 - a\theta\}^{-[(a+b)/a]},$$

and

(1) $$\Pi_0 = P_0/\{1 - a\theta\}^{[(a+b)/a]}.$$

For the probability of n excess claims

$$\Pi_n = P_n \pi^n + P_{n+1} ({}_{n+1}C_1)\theta \pi^n + P_{n+2} ({}_{n+2}C_2)\theta^2 \pi^n + ... + P_{n+q} ({}_{n+q}C_q) \pi^n\theta^q \pi^n + ...$$

Again using the recursive relationship, we have

$$\Pi_n = P_n \pi^n + [a+b/(n+1)] P_n ({}_{n+1}C_1)\theta \pi^n + [a+b/(n+2)][a+b/(n+1)] P_n ({}_{n+2}C_2) \theta^2 \pi^n + ... + [a+b/(n+q)] ... [a+b/(n+1)] P_n ({}_{n+q}C_q) \theta^q \pi^n + ...$$

Factoring out $P_n \pi^n$, we have

$$\Pi_n = P_n \pi^n \{1 + [a+b/(n+1)] ({}_{n+1}C_1)\theta + [a+b/(n+2)][a+b/(n+1)] ({}_{n+2}C_2)\theta^2 + ... + [a+b/(n+q)] ... [a+b/(n+1)] ({}_{n+q}C_q)\theta^q + ... \}.$$

Rearranging terms this becomes

$$\Pi_n = P_n \pi^n \{1 + [(b/a)+n+1] (a\theta) + [(b/a)+n+1][(b/a)+n+2](a\theta)^2/2 + ... +[(b/a)+n+1] ... [(b/a)+n+q](a\theta)^q/n! + ... \}.$$

As before, since $a\theta$ is less than one, the series converges and is equal to

$$\{1 - a\theta\}^{-[(b/a)+n+1]},$$

or alternatively

$$\{1 - a\theta\}^{-[(a+b)/a]} \{1 - a\theta\}^{-n}$$

Thus

$$\Pi_n = P_n \{1 - a\theta\}^{-[(a+b)/a]}[\pi /\{1 - a\theta\}]^n$$

and

$$\Pi_{n+1} = [\pi/(1 - a\theta)][a+b/(n+1)] \Pi_n.$$

Thus Π_n satisfies the recursive relationship

(2) $$\Pi_{n+1} = [A+B/(n+1)] \Pi_n$$

with

(3)
$$A = [\pi/(1 - a\theta)] a$$
$$B = [\pi/(1 - a\theta)] b.$$

For the case where $a = 0$, the derivations above can be modified slightly to yield

$$(4) \qquad \Pi_0 = P_0\, e^{b\theta}$$

$$\Pi_n = P_n\, \pi^n\, e^{b\theta} \ .$$

These can be seen to satisfy equation (2) with $A = 0$ and $B = \pi b$.

In summary, excess losses generated from the family of distributions characterized by the recursive formula are also in the family of such recursive distributions, with parameters scaled by $\pi/(1 - a\theta)$.

As noted above, this family of distributions includes the Poisson and Negative Binomial distributions.

For the Poisson distribution, with expected value λ, we have for $n \geq 0$

$$P_n = e^{-\lambda}\, (\lambda^n/n!).$$

In this case, the recursive parameters are $a = 0$ and $b = \lambda$, and by (4) the excess distribution satisfies

$$\Pi_0 = P_0\, e^{b\theta} = e^{-\lambda}\, e^{\lambda\theta} = e^{-\lambda} e^{\lambda - \pi\lambda} = e^{-\pi\lambda}$$

$$\Pi_n = P_n\, \pi^n\, e^{b\theta} = e^{-\lambda}\, (\lambda^n/n!)\, \pi^n\, e^{\lambda - \pi\lambda} = e^{-\pi\lambda}\, [(\lambda\pi)^n/n!]$$

Thus, the excess distribution will be a Poisson distribution with parameter $\pi\lambda$, which is, of course, what you would expect.

For the Negative Binomial with parameters h and n, we have for $k \geq 0$

$$P_k = ({}_{h+k-1}C_k)\ [h/(n+h)]^h\, [n/(n+h)]^k ,$$

The recursion parameters for this distribution are

$$a = n/(n+h),$$

$$b = n(h-1)/(n+h) \ .$$

And the mean and variance are given by

$$\mu = n$$

$$\sigma^2 = n + n^2/h.$$

Note that $(a+b)/a = h$ and $\{1 - a\theta\} = \{1 - n(1-\pi)/(n+h)\} = (n\pi+h)/(n+h)$, so from equation (1) we have

$$\begin{aligned}\Pi_0 &= P_0/\{1 - a\theta\}[(a+b)/a] \\ &= [h/(n+h)]^h/[(n\pi+h)/(n+h)]^h \\ &= [h/(n\pi+h)]^h\end{aligned}$$

And using (3) we see that

$$\begin{aligned}A &= [\pi/(1 - a\theta)]\, a \\ &= [\pi/(1 - \{n/(n+h)\}\theta)][n/(n+h)] \\ &= [(\pi n)/(n+h)][(n+h)/(n+h-n\theta) \\ &= [(\pi n)/\{n+h-n(1-\pi)\}] \\ &= (\pi n)/[(\pi n)+ h] .\end{aligned}$$

And

$$\begin{aligned}B &= [\pi/(1 - a\theta)]\, b \\ &= [\pi/(1 - a\theta)][n(h-1)/(n+h)]. \\ &= [\pi/(1 - \{n/(n+h)\}\theta)][n(h-1)/(n+h)] \\ &= [(\pi n)(h-1)/(n+h)][(n+h)/(n+h-n\theta)] \\ &= (\pi n)(h-1)/[(\pi n) + h] .\end{aligned}$$

Thus the excess distribution is again a Negative Binomial distribution with parameters πn and h. In particular, the mean of the excess distribution is πn and the variance is $\pi n + (\pi n)^2/h$.

Implications for the Negative Binomial

In the case of the Negative Binomial it is instructive to look at the relationship between means and variances for the original distribution and the excess distribution.

The following table summarizes the statistics for the two distributions.

	Original	Excess
Mean	n	πn
Variance	$n + n^2/h$	$\pi n + (\pi n)^2/h$
Ratio	$1 + n/h$	$1 + \pi\ (n/h)$.

Thus the ratio of the mean and variance becomes closer to unity as the size of the excess claim increases. Since the probabilities of an excess claim are often on the order of 10^{-3} or less, for most distributions the variance to mean ratio will be very close to unity. In some sense the derived claim frequency becomes nearly Poisson.

This observation would seem to run counter to the "common sense" view that the further out you are in the tail, the more volatile the claim distribution becomes. Of course, since this is all predicated on having perfect knowledge of the claim severity distribution with no questions about trend or loss development maybe it isn't so surprising.

The Program

The program basically implements the formulas shown in section 3.8 of Risk Theory by Beard etal.

The **parameter file** contains the name of the distribution, the mean and variance of the distribution of first dollar claims - either Poisson or Negative Binomial; the parameters of the Pareto claim size distribution, B, Q, P, T, and S; the upper and lower limit of the excess interval under consideration; the number of grid points to use for the claim size approximation and the Stopping Probability.

Following is an example of the parameter file:

```
Test Data                         {Label}
10000 15000                       {Expected Number and Variance of Claim Process}
2000 1.25   0.5   500   1000      {B, Q, P, S, T of Pareto Severity Dist}
1000000  5000000                  {Attachment Point, Limit}
20  0.99999                       {# of Pts in Sev Dist, Cum Prob stopping value}
```

The **output file** displays a variety of summary information to assess the quality of the estimation as well as the aggregate distribution itself.

Following is a copy of the summary data based on the Test Data parameters.

```
   Test Data
The Expected Number and Variance of First Dollar Claims
   10000           15000
The Pareto Parameters are
B  2000  Q 1.25  P 0.5000  S   500  T  1000
The AttachPt is    1000000  The Limit is    5000000
The Number of GridPts is 20, The Prob Stopping Value is 0.99990000

Prob of an XS Claim is 0.000350  Prob of a Limits Claim is 0.000037
The Prob of a Limits Claim given an excess claim 0.106712
Parameter a equals   0.000175, Parameter b equals   3.500974

The Mean and Variance of the Claim Frequency Distribution is:
          3.501762            3.502375

The Mean and Standard Deviation of the Actual Severity Distribution is
          1446050             1607771

The Mean and Standard Deviation of the Discrete Severity Distribution is:
          1477067             1583653

The Mean and Standard Deviation of the Actual Aggregate Distribution is:
          5063723             4046662
```

The Discrete Claim Severity Distribution is
0.328 0.127 0.089 0.066 0.051 0.040 0.032 0.026 0.022 0.019
0.016 0.014 0.012 0.011 0.009 0.008 0.008 0.007 0.006 0.110

The Cumulative Probability is 0.99990328
The Total Number of AggPts calculated is 112

The Mean and Standard deviation of the ESTIMATED Aggregate Distribution is:
5169444 4045234

This data allows one to assess the quality of the modeling of the severity distribution - compare the means and standard deviations of the actual and discrete severity distributions. If the fit is inadequate, the number of gridpoints, which is the number or points used to approximate the severity distribution, should be increased.

There is also a corresponding comparison for the aggregate distribution. Using the well known formula for mean and variance of a Compound Poisson process in terms of corresponding statistics for the frequency and severity distributions, the program computes The Mean and Standard deviation of the ESTIMATED Aggregate Distribution. This can be compared to The Mean and Standard Deviation of the Actual Aggregate Distribution, which is computed from the statistics of the claims frequency and actual severity distributions. If this comparison is not satisfactory, an increase in the Stopping Probability is indicated.

If you are satisfied with these estimates you can use the information written to the output file. This data set displays summary statistics and the aggregate distribution itself.

Following is an example of the output data set.

Test Data
The Mean and Variance of the Claim Frequency Distribution is:
3.501762 3.502375

The Mean and Standard Deviation of the Discrete Severity Distribution is:
1477067 1583653

The Mean and Standard Deviation of the ESTIMATED Aggregate Distribution is:
5169444 4045234

loss amount	probability	cum probability
0	0.030153453	0.030153453
250000	0.034619700	0.064773154
500000	0.033230037	0.098003191
750000	0.032365086	0.030153453
1000000	0.031735804	0.162104080
1250000	0.031000556	0.193104636
1500000	0.030124671	0.223229307
1750000	0.029144564	0.252373871
2000000	0.028098459	0.280472330
2250000	0.027016611	0.307488942
2500000	0.025921784	0.333410726
...	...	...

The first numbers are a restatement of the key statistics of the frequency, severity and aggregate distribution followed by the probabilities of the aggregate distribution.

In its present form, the program has limitations due mainly to my limited programming experience. The constraint is a limit on the amount of memory that can be specified in a single program unit. There are ways around it with more sophisticated memory management techniques, but that would have taken me too far afield.

Practically, the limitation means that the program is limited to evaluating the aggregate distribution at about 1000 points. If you describe the claim distribution with 25 points this means that you are limited to 40 claims. Not much of a constraint if the expected number of claims is 0.25.

However, since my main goal was to show that the calculations would work, the program appears to have accomplished those goals and could now be transferred to a less constrained environment or more talented programmer.

If anyone is interested in a program listing I would be glad to provide one with the understanding that it was not intended to be a finished product, is not warranted to be free of defects and comes with no technical support.

Reference

Beard, R.E., Pentikainenen, T. and Pesonen, E., *Risk Theory: The Stochastic Basis of Insurance*, (Third Edition), 1984.

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THE EDUCATIONAL CONTENT OF THE CAS SYLLABUS

THE EDUCATIONAL CONTENT OF THE CAS SYLLABUS

This report has been prepared by the Long Range Planning Subcommittee of the Syllabus Committee at the request of the Board of Directors of the CAS. The purpose is to evaluate the educational content of the syllabus from the standpoint of the educational needs of future actuaries. The subcommittee has been reviewing Syllabus content for several years in terms of a long range comprehensive plan for educating actuaries. The subcommittee was expanded for this project to include a representative of the Educational Policy Committee. In addition, the Report reflects comments from the full Syllabus Committee.

The report draws upon the results of the CAS Membership Survey in evaluating the current Syllabus and subjects which were identified by the survey for inclusion in the Syllabus. The subcommittee is also indebted to Michael Walters and John Muetterties for their thoughtful discussions of future educational directions as presented in recent articles in the Actuarial Review and The Actuarial Update, respectively. The committee has incorporated some ideas from recent discussions of "the actuary of the future".

The Actuary of the Future

For many years, the actuary's focus has been on the liability side of the balance sheet. It seems clear that the actuary of the future will increasingly be required to look at the asset side as well. The subject of matching of assets and liabilities in terms of duration and suitability has already received increased attention and several articles on this subject have now been added to the Syllabus. Future actuaries will be increasingly involved in the investment side of the business. This focus is not necessarily restricted to insurance companies but would also apply to self-insurance and other funding mechanisms.

A related area in which the actuary is already becoming more involved is Finance. In recent years, more emphasis in the insurance pricing arena has been placed on rate of return and the cost of capital. Questions posed include the relative risk of the insurance industry as compared to other industries, the financial structure of the industry (leverage ratios, etc.), the profitability of the industry, the need for a contingency margin and the allocation of surplus to line and state. Various methods have been presented for determining the cost of capital (Discounted Cash Flow, Internal Rate of Return, the Capital Asset Pricing Model, etc.). Finance is an appropriate area for further actuarial involvement.

In an increasingly complex and socially conscious era, legislators, regulators, and others are looking for actuarial advice to understand the cost implications of relevant social and political proposals. To the extent that the actuary is viewed as professional and objective, that advice will be sought and valued. This suggests several important qualifications for the actuary of the future. The first is communication skills, for without these skills the result of the actuary's work will go unheeded or given little weight. Secondly, the actuary must be viewed as a professional. Hence, integrity, discipline, professional standards of practice and guides to professional conduct are very important.

More and more, employers are looking for actuaries with a broader perspective, going beyond the traditional technical skills. They are looking for actuaries who possess communication and management skills. These employers are looking for greater competence in such skills as problem identification, dealing with unstructured situations, applying interdisciplinary approaches, conceptualization, and creativity.

Actuaries will be called on to play a larger role in the arena of regulation. Actuarial certification is currently expanding into new areas. Regulators want actuaries to certify reserves and other balance sheet items to assure that these have been estimated in a professional, objective manner and fairly stated. Actuaries also may be called on to take a larger role in attesting to ratemaking procedures and results.

We will see broader application of actuarial techniques. Actuaries have demonstrated the ability to quantify difficult problems using a variety of methods and models. These techniques can and will be applied to other areas than the traditional insurance problems. Initially, these areas will likely be offshoots of insurance and risk management fields.

Actuaries will become more international. As financial institutions and businesses become multi-national, actuarial work will likely expand. Actuaries will need to become more knowledgeable about the insurance and risk management systems in other countries.

The actuary of the future will be a problem solver but with a broader perspective, an expert at quantifying difficult problems with a variety of scenarios. The actuary will play a greater role in areas such as economics, finance and government. Due to this increased role, he or she will play a more active part in dealing with many difficult social problems and must be able to communicate. He or she must be viewed as a professional, with professional standards of practice and conduct.

Intellectual Core of the Profession

Is there a common core to all of this? What distinguishes actuaries from practitioners of related disciplines such as economics, accounting and general mathematics? What makes the actuary unique?

Jim Hickman recently referred to the "intellectual roots" of the actuarial profession. We believe that the core upon which the casualty actuarial profession is built is a thorough grounding in the following as applied to property/casualty risks:

--Applied Mathematics
--Economics
--Finance
--Risk Theory

All of these disciplines come together in evaluating current financial implications of future contingent events; this has been advanced as the definition of an actuary and may be the best definition possible.

As we strive to educate actuaries of the future, we must begin by instilling a strong base of knowledge in these core areas. In effect, we will be molding an actuary who begins as a generalist, trained in the fundamentals and principles of applied mathematics, economics, finance, and risk theory as applied to property/casualty risks. As the actuary's knowledge and experience base expands, specialization will occur. However, as actuaries move forward in various career directions, this intellectual core will travel with them and will provide them with a firm base from which to grow.

Educational Issues

With this backdrop, we need to examine our current educational process to see whether it is providing the necessary training in the intellectual core for the actuary of the future. We have identified the following issues as important to the educational process:

--Balance of Mathematical vs. Non-Mathematical Subject Matter
--Sequencing of Subject Matter
--Conceptual vs. Factual Material
--Canadian Content
--Associateship vs. Fellowship Examinations
--Communication Skills
--Management Skills
--Professional Standards
--Development of Syllabus Materials

Each of these issues will be discussed in more detail in the following paragraphs.

Balance

Actuaries must master both mathematical and non-mathematical subject matter. If we "over-focus" on mathematical topics, we may not provide the broad background desired to allow actuaries to solve problems creatively, using different disciplines. In addition, there is a real danger that an "overkill" on heavy mathematics will discourage non-mathematicians from entering the profession. These non-mathematicians may possess the desired communications, management, and general business problem-solving abilities. Given that we want to retain our mathematical roots within the core of our profession, we must strive to find a proper balance on the Syllabus between mathematical and non-mathematical subject matter.

Sequencing of Subject Matter

There has been some discussion in recent years about whether all of the mathematical subjects should be kept to the early examinations. We do not believe that as soon as an actuary achieves the associateship designation, he or she loses the ability to handle mathematics. In fact, it would be impossible to properly cover advanced ratemaking and excess pricing without significant mathematical content. Since we have cited applied mathematics as a part of the core of our profession, we believe that "mathematical subjects" should be presented through all levels of the educational process for the actuary.

By the same token, we feel that non-mathematical content could be introduced earlier in the examinations (for example, at the Part 3 level). This would allow students to study material early on in the examinations which would be more relevant to their work than pure mathematics.

Another issue in sequencing is whether or not certain examinations or subjects should be prerequesites for later exams. We don't see the need for this under the current Syllabus, although it should be further considered under the proposed flexible education system.

Conceptual vs. Factual

Given our mathematical foundation, how do we focus the educational process on the problem-solving skills for which we are looking? This concern also appeared on the Membership Survey from a slightly different perspective. Specifically, do examination questions focus enough attention on understanding and problem-solving as compared to list recitation? While these are related issues, we believe that the latter question is not primarily a question of educational content. It, instead, is an issue that should be addressed by the Examination Committee or the new Task Force on Educational and Testing Methods. As to the first question, we believe that the Syllabus needs to be broad enough to give the student both factual information as well as conceptual perspectives. Although practical applications of the theory and principles should be included, we need to emphasize that these applications are not important in and of themselves, but as examples of the application of problem-solving techniques to particular problems. In addition, as a general principle, readings that emhasize problem solving are desirable and should be sought.

Canadian Content

In general, insurance concept material probably does not fall into the category of being nation-specific. A general insurance concept should be applicable regardless of country and, therefore, would not pose a problem in either testing or development of material for the Syllabus. In developing this kind of material, the committee would consider any published material, regardless of the country of publication, as long as the material satisfies the criterion of generality of concepts. However, as stated above, we believe that the presentation and testing of factual material should also be part of the educational process, especially when the facts serve as examples which help make a general concept more specific. Thus, it is both feasible and proper to improve the balance, throughout the whole Syllabus, between U.S. and Canadian examples of general insurance concepts.

We also believe that some fact teaching and testing is necessary as we educate future actuaries. Rather than including a separate track with Canadian content for Canadian students, we believe that all casualty actuaries should study the same material and take the same set of examinations which would qualify them to practice in North America. All that would be required from a practical point of view would be the establishment of a minimum amount of factual information which would satisfy Canadian needs and U.S. needs and the development of that material to the extent that current materials are insufficient.

THE EDUCATIONAL CONTENT OF THE CAS SYLLABUS
(continued)

Associateship vs. Fellowship Examinations

The Syllabus for Associateship should have the general objective of developing an ACAS who is competent technically to practice ratemaking and reserving. This would include the following Syllabus subjects:

--mathematical foundations
--property/casualty coverages and operations
--risk theory
--economics
--finance
--ratemaking
--individual risk rating
--reserving
--accounting
--insurance rate regulation

The following subjects would be covered in the Fellowship Syllabus:

--more advanced treatment of certain of the above subjects
--statutory insurance
--reinsurance
--valuation
--insurance law and regulation

The Syllabus should have the general objective of developing an FCAS who has a solid knowledge of the core areas of the profession.

Communication Skills

Communication skills have been identified as vital to the success of future actuaries. The application of these skills will run the gamut from inter-office memoranda to presentations before Congress. While the importance of communication skills has been recognized for many years and discussed numerous times at Syllabus Committee meetings, the consensus has been that this subject is not suitable for the examination process with its emphasis on self-study.

Reexamination of the subject in the light of the actuary of the future has convinced us that this subject may be too important to be left to individual development. We therefore recommend a three-pronged approach to the subject.

(1) The Syllabus Committee should investigate the feasibility of adding to the exam syllabus the subject of Argumentation: the presentation of ideas in a logical and persuasive manner.

(2) Exam questions offer an opportunity to test actuaries' communication skills. The Examination Committee should be encouraged to develop essay questions which not only test facts and concepts, but the ability to present these ideas clearly. A statement to that effect could be put in the Syllabus.

We would also raise the question as to whether our examination process in general, with its emphasis on short answer and multiple choice questions, is effective in emphasizing communication skills. This question is more properly a question for the Examination Committee and the new task force on Educational and Testing Methods.

(3) The Continuing Education Committee should consider the feasibility of offering a regular seminar on communication skills. Ideally, this would be taught by experts in the field, who would structure the seminar towards presentation of technical subjects such as actuarial analyses.

Management Skills

We have identified these skills as important to many future actuaries. However, given the volume limitations of the Syllabus, it is impossible to cover everything that would be valuable for actuaries. There has to be a prioritization. In addition, the Syllabus may not be the best place to teach this subject. There are a variety of courses already available on the subject which are taught by experts in the field. Many employers offer such courses to their actuaries as well as other employees who are potential management candidates. Given an already crowded Syllabus, with indicated new material on finance and other subjects, we have concluded that this subject should not be added to the Syllabus at this time. However, due to its importance to future actuaries, we recommend that it be considered by the Continuing Education Committee as soon as possible.

Professional Standards

Should the educational process for the actuary of the future include the topic of professional standards, including ethics? We believe that an emphasis should be placed on the need for professionalism. Therefore, we believe that no actuary should attain the Associateship designation without studying the Guides to Professional Conduct. This topic need not be a direct part of the examination process. A special seminar or a session at the CAS meeting are two possible arenas for presentation. In addition, as Statements of Principles and/or Standards of Practice are promulgated, we believe they should be strongly considered for addition to the Syllabus. To the extent they are not added to the Syllabus, they should be considered as part of the special seminar for new Associates.

Development of Syllabus Materials

A review of educational issues would not be complete without discussion of the quality of the educational materials currently on the Syllabus and the development of readings in the future. While the quality of much of the current educational material is excellent, it is recognized that the material is voluminous and repetitive in many areas. An obvious solution is the replacement of outdated and general readings with study notes or articles specifically tailored for CAS students. We have tried for several years to encourage the writing of these study notes and have, together with the Committee on Research, published lists of topics for which papers are needed. To date, we have not met with much success in this area. An alternative, which will be attempted in 1989 will be to form a subcommittee of the Syllabus Committee which will identify specific materials needing updates or replacements, in priority order, and attempt to identify and encourage prospective authors. In addition, with the success of the CAS Textbook Committee on Foundations of Casualty Actuarial Science, this committee could be continued to work on some of the most important papers.

Summary

In summary, the actuary of the future will have a broader role, both within the property/casualty industry, and will expand into other areas outside the traditional insurance industry. To be prepared to meet these increased challenges, we must establish a solid educational program which provides the actuary of the future with an intellectual core of knowledge in applied mathematics, economics, finance, risk theory, and the property/casualty insurance business.

The educational process which will be required in order to achieve the necessary results should:

- --include both mathematical and non-mathematical subject matter throughout the entire educational process
- --provide the student with both factual information as well as conceptual perspectives
- --endeavor to include one track of study material and examinations which would provide the training needed for both Canadian and U.S. actuaries to practice in North America
- --include communications skills through a combination of Syllabus material, examination structure and Continuing Education Seminars
- --present management topics in a continuing education format
- --include the study of all published Statements of Principles and Standards of Practice; and
- --require the study of the Guides to Professional Conduct as a prerequisite for the Associateship designation
- --strive to maintain a Syllabus of updated readings tailored for CAS students

In addition, there must be greater coordination of the Educational and Examination Committees in order to effectively meet these challenges. By meeting all of these goals, the actuary of the future will be prepared to face the ever broadening role that we foresee.

The Appendix, which follows, provides a detailed description of the subjects that are currently on the Syllabus that should be included or deleted in the future, as well as any new subjects which should also be included. There is also a brief discussion of subjects considered for inclusion but not recommended.

THE EDUCATIONAL CONTENT OF THE CAS SYLLABUS
(continued)

Recommendations

Based on the report and the detailed Appendix, we recommend the following actions be taken:

(1) The subjects for inclusion and deletion to the Syllabus as set forth in the Appendix should be evaluated and incorporated into the Syllabus by the Syllabus Committee over time in an orderly fashion.

In particular, the following major changes are recommended:

a) Elimination of Operations Research as a separate examination part.

b) Earlier examination treatment of Property/Casualty coverages, operations of insurance companies and introductory basic material on ratemaking and reserving.

c) Move Forecasting to an Associateship topic e.g. inclusion with Applied Statistics.

d) Addition of Finance

(2) The Continuing Education Committee should be asked to consider communication skills and management skills as subjects for seminars, meeting topics, or other appropriate forums.

(3) The Examination Committee should consider the question of how communication skills can be most effectively tested in the examination process and evaluate whether our current examination structure places appropriate emphasis on these skills.

(4) The Syllabus Committee should investigate the feasibility of adding to the exam Syllabus the subject of Argumentation: the presentation of ideas in a logical and persuasive manner.

(5) A seminar or a session at a CAS meeting on the Guides to Professional Conduct should be added to the requirements for an Associate of the Society.

(6) Additional Canadian content should be added to the Syllabus to provide a balanced single track of examinations.

(7) The Syllabus Committee should continue its efforts to develop additional study notes and other materials specifically tailored for actuarial students. The Syllabus Committee would identify needed material and work with the VP--Development to get the necessary material produced.

Appendix

SYLLABUS REVIEW

I. Subjects for Inclusion

A. Mathematical Foundations

1. Calculus--The equivalent of a one year calculus course should be required of all CAS students.

2. Linear Algebra--Students should know enough matrix algebra to solve systems of linear equations and understand the matrix formulation of multiple regression analysis.

3,4. Probability and Statistics--The level of the current Part 2 is appropriate. It should be noted that relevant statistical evaluation is part of the later exams.

5. Forecasting--The student should master simple and multiple linear and non-linear regression, as well as some time series methods and Delphi methods. Fitting of models, testing goodness of fit, testing for failure of regression assumptions, and measuring possible deviations from forecast values should all be mastered. We recommend that a joint CAS/SA Committee reevaluate the Part 3 Intermediate Business Statistics course to see if forecasting could be included.

6. Numerical Analysis--Numerical methods of integration, minimization, graduation and curve fitting, and solving systems of non-linear equations should be learned. We believe this Part 3 exam should also be reevaluated by a joint committee to make it more relevant for actuaries.

7. Theory of Interest--A basic introduction: simple and compound interest; present value and discount; force of interest; perpetuities; continuous and varying annuities; unknown rate and time, is enough for the initial exam on this topic. More advanced material, such as internal rate of return and the valuation of bonds, stocks, and options should be part of the new Finance subject.

8. Credibility Theory--A thorough grounding in credibility theory should be included. In addition, students should learn how to estimate credibilities from data, including the case of unequal cell sizes and model testing.

9. Loss Distributions--Frequency, severity, and aggregate loss distributions should be studied, including estimation of parameters by maximum likelihood in cases of complete and limited (grouped, truncated, censored) data, estimation of confidence bands for the probabilities using the information matrix, calculation of aggregate moments and probabilities from frequency and severity distributions by practical computer methods, and calculation of excess probabilities and costs.

I. Subjects for Inclusion (continued)

B. Property/Casualty Coverages and Operations

1. Coverages--Before analyzing a body of data (whether for ratemaking, loss reserving, etc.), the actuary must know the parameters defining that body of data. A key parameter is the coverage applicable. The characteristics of the data may vary substantially depending upon the line of insurance and the coverage provided by the insurance contract. It is critical that the actuary understand the different coverages and the exposures to loss that these coverages were designed to address.

2. Operations (Underwriting, Marketing, Claims)--In order to understand the insurance business, the actuary must have a knowledge of the underwriting, marketing and claims functions. The actuary must understand how changes in these functions may impact data used for ratemaking, loss reserving and other analyses.

C. Risk Theory

As risk (uncertainty) is an integral part of ratemaking, reserving, etc., the actuary must be able to apply appropriate techniques for addressing risk. Furthermore, the actuary as a member or observer of a corporate structure must appreciate financial risk. In general, the actuary should know the kinds of risk and the ways in which they can be handled for property/casualty insurance.

Specific subjects that should be covered include: frequency and severity distributions; Poisson processes, compound Poisson processes, and diffusion processes; calculation of aggregate loss moments and probabilities; calculation of excess loss percentages; probabilities of adverse deviation over a time period; premium calculation principles.

D. Economics

In projecting or developing data, the actuary must estimate the impact of inflation and other economic factors on the data. Consequently, the actuary should understand the principles of economics and how a competitive economy functions, particularly its impact upon the insurance industry. The specific topics which should be included in the Syllabus are:

--supply and demand (on a macroeconomic and microeconomic basis)
--price, utility, costs and competition
--consumption, investment, fiscal policy and inflation
--money, interest rates and deficits
--exchange rates and international finance

E. Finance

A subcommittee of the Syllabus Committee has identified several Finance topics which are appropriate for inclusion in the Syllabus. These include: present value, opportunity cost of capital, risk and return, internal rate of return, capital asset pricing model, options, analyzing financial performance, valuing risky debt, mergers, and international financial management.

Basic finance material at the Associateship level will provide a background for more advanced Finance material at the Fellowship level and prepare the actuary to pursue continuing education in financial theory. Material on valuation will likely be enhanced by the study of Finance topics.

I. Subjects for Inclusion (continued)

F. Ratemaking and Individual Risk Rating

The Syllabus should include both basic and advanced material which deal with the topics of ratemaking and individual risk rating. The papers should present comparisons of various ratemaking and individual risk rating techniques in order to help the candidiate learn how to evaluate and select appropriate techniques for a given problem. In addition, there should be more technical material which should prepare the student to deal with a wide range of problems, including those for which there are not generally recognized solutions.

The topics which should be part of the Syllabus should include:

1. Ratemaking

--general principles of ratemaking
--loss development, trend, credibility
--classification ratemaking, including risk classification
--excess and deductible ratemaking, and
--data for ratemaking

2. Individual Risk Rating

--experience rating
--retrospective rating
--schedule rating, and
--composite rating
--merit rating, dividend plans, loss rating

G. Reserving

An actuary may be expected to design and test reserving methods, should be familiar with the general principles of reserving, should be able to complete Schedules O and P of the Annual Statement, and should be prepared to discuss such topics as:

1. The selection and evaluation of a loss reserving method appropriate to a given line of insurance:

   a) For known claims
   b) For IBNR claims
   c) For all incurred claims

2. Testing of adequacy of previous loss and loss expense reserve levels.

3. Evaluating the adequacy of current loss and loss expense reserve levels.

4. The identification of, and correction for, effects on loss reserves stemming from:

   a) Changes in the loss climate
   b) Changes in a company's handling of claims
   c) Data problems

I. <u>Subjects for Inclusion</u> (continued)

G. <u>Reserving</u> (continued)

5. Special reserving problems in a line arising from:

   a) Catastrophe losses
   b) Reopened claims
   c) Policies on a claims-made rather than occurrence basis
   d) Fidelity and Surety IBNR
   e) Credit insurance
   f) Late recorded premiums (earned and unearned)

6. The unearned premium reserve:

   a) Improving its accuracy
   b) The reserve for retrospective returns
   c) Policies on a claims-made basis
   d) Deposit premium policies

7. Allocated loss expense reserves.

8. Unallocated loss expense reserves.

9. Statutory Annual Statement reserves.

To support these goals, techniques other than chain ladder need to be covered. Chain ladder on paid, incurred, accident year, report year, etc. is one technique, not several. Testing the strengths and weaknesses of various methods also needs to be covered. Exposure based methods, credibility methods, curve fitting, regression methods, and fitting of lag distributions are examples of various methods that could be included. Estimation of confidence intervals for the loss liability should also be covered. It is appropriate to emphasize adjustments needed to react to changes in data and testing of reserving assumptions. The actuary should also be familiar with the subject of discounting of reserves and with the concepts of deferred acquisition expenses, premium deficiency reserves, dividend reserves and deferred tax liabilities.

H. <u>Accounting, Expense Analysis, and Published Data</u>

1. <u>Accounting</u>

Students should learn and be tested on accounting concepts, income statements and balance sheets, and the need for and methods of maintaining audit trails in computer based systems, and tax accounting. Statutory and GAAP insurance accounting should be covered in depth, and FASB rulings relevant to insurance issues should be studied as well. All Associates should know the U.S. Annual Statement blank, as well as major differences between the U.S. and the Canadian statements. Also, valuation differences and solvency standards in European statements should be studied, in part to understand how else it can be done, and in part to be able to analyze foreign reinsurance and insurance.

2. <u>Expense Analysis</u>

The current emphasis is appropriate, but a more up to date study of expenses by size of risk is needed. We also need a paper on expense flattening and/or other expense topics. These latter items are really more in the ratemaking area.

I. Subjects for Inclusion (continued)

H. Accounting, Expense Analysis, and Published Data (continued)

3. Published Data

More rationale is needed for this to be on the examinations than for students to know it is there. Periodically sending CAS members a detailed reference list of these data sources, including what is on each, should be considered as an alternative to keeping this section.

I. Statutory Insurance

This topic includes insurance required by the government and insurance provided by the government. It is appropriate for CAS students to know the different forms that both have taken in various jurisdictions, and the reasons for them. This should probably be restricted to Property and Casualty lines broadly construed. Students should be exposed to enough material on social security to evaluate the interaction between social security and Workers' Compensation, first party Medical benefits, and third party liability settlements. Both U.S. and Canadian systems should be studied by all students, as examples of the diversity employed. Systems from other countries should also be reviewed and included to the extent that they illustrate alternative perspectives.

J. Insurance Law

This subject provides a background and basic understanding of how tort law underlies and affects the insurance contract. In addition, the actuary should be knowledgeable about the various state and provincial laws which affect ratemaking, reserving, and other actuarial issues.

K. Insurance Regulation

The actuary needs to have an understanding of the system of regulatory controls within which the insurance business operates. The basis for insurance regulation is the law of the particular jurisdiction, either state, provincial or federal. The actuary should be knowledgeable about the purposes of regulation, types of rate regulation in use, and issues of concern to regulators.

L. Reinsurance

This topic continues to rate very high on interest surveys of actuaries. Reinsurance is crucial to almost any insurance or self-insurance program.

The topic of excess rating, or the mathematics of reinsurance pricing, is covered in another section. The Syllabus should contain sufficient material on the general subject of reinsurance concepts so that actuaries who are responsible for either assumed or ceded reinsurance operations are familiar with the functions of reinsurance and its purpose.

Reinsurance plays a large role in the management of the solidity of insurance systems through the use of risk reduction. Actuaries should understand this relationship and how the various forms of reinsurance are designed to enhance the viability of primary insurance programs. This subject can also be related to material on regulation.

The Syllabus should not provide all material which may be of interest to actuaries directly working in reinsurance but, rather, should provide a sufficient understanding of reinsurance concepts and the development of a well-managed reinsurance program.

I. Subjects for Inclusion (continued)

M. Valuation

The past few years have seen increased interest in acquisitions and mergers in the insurance industry. This has given rise to requests for valuations of insurance companies.

In addition, regulators have expressed more interest in the relationship of assets and loss reserves with respect to loss reserve opinions. There is a growing awareness of the need for actuaries to consider the appropriateness and the valuation of the assets as well as the proper estimation and statement of liabilities.

A new section has been added to the Syllabus dealing with this topic. Future Syllabus material should be kept current with developments in this area.

N. Investments

While most of the major functions of an insurer pertain to the business of insurance, the investment of policyholder surplus and premiums is necessary to meet future liabilities and ultimately critical to the solvency and profitability of the insurer. The actuary must understand the importance of asset and liability matching and alternative investment instruments including their risk maturity, expected yield, and tax characteristics.

O. Professional Principles and Standards

The actuary should have a thorough understanding of the principles, professional standards, and guides to professional conduct of the profession and the discipline process which enforces them. The Syllabus should include study of the principles and, perhaps, standards of practice of property/casualty actuaries.

P. Communication Skills

As summarized in the body of this report, the subject of argumentation or presentation of ideas in a logical and persuasive fashion should be considered for inclusion in the Syllabus.

Q. Health and Group Insurance

Topics covering basic ratemaking for health insurance and group insurance are currently included in the CAS Syllabus. Some actuaries give a very low ranking to the importance of ratemaking of health and group insurance. However, a number of casualty actuaries do practice in health related areas and it is felt that a basic understanding of the fundamentals of ratemaking for health and group insurance should be obtained.

The articles that are on the current syllabus which deal with health and group insurance are very outdated (1962 and 1965). It is believed that if more current material were found which gave a better presentation of the fundamentals of health and group insurance ratemaking, then these topics should continue to be a part of the education of a casualty actuary.

II. Subjects for Deletion or Reduction

A. Operations Research

Operations Research material is and will be, less relevant to a casualty actuary than deemed in the past. Topics of limited relevance or applicability include project scheduling, dynamic programming, integer programming, and queuing heory. Therefore, a separate examination part on Operations Research should be deleted from the Syllabus. However, topics such as simulation and decision analysis are felt to be of continued relevance and should continue to be included on the CAS Syllabus.

B. Nuclear Risk

Material on nuclear risk, including the Price-Anderson Act, have minimal applicability for a casualty actuary. Regulatory issues for coverages mandated by statute or regulation are covered elsewhere in the Syllabus with more relevant presentations of regulation for automobile insurance, workers compensation, and social security. Thus, specific readings pertaining to Nuclear Risk should be considered for deletion from the Syllabus.

C. New York Insurance Law/Canadian Provincial Acts

While the New York Insurance Law has been deemed typical of state insurance laws, it is felt that studying the actual statutes does not add significantly to the casualty actuary's understanding of the fundamentals of regulation with one exception. The exception is the rating statute which should be continued because of the importance of these statutes to actuarial work. With that exception, more value will be achieved by including material which helps the candidate understand the fundamentals of regulation. The inclusion of specific state laws should be reduced or deleted from the Syllabus. Similarly, the details of the various Canadian provincial acts should not be studied except as examples of the regulatory principles. A Canadian ratemaking statute similar to New York would be appropriate.

D. Life Contingencies

The casualty actuary should have a working knowledge of the mathematics of life contingencies. However, the scope of the material presently included on the Syllabus is broader than what is necessary for a casualty actuary. Thus, some life contingency topics should be considered for reduction on the Syllabus, e.g. life insurance reserves.

III. Subjects Considered for Inclusion but Not Recommended

Many subjects have been suggested through various channels (e.g. Syllabus and Education Committees, presidential addresses, regional actuarial clubs, etc.) for inclusion on a Casualty Actuarial Society Syllabus. Some of these subjects and topics include

--General Business Management
--Life Insurance and Pension Plans
--Public Relations

As FCAS's proceeds through the various directions of their careers, business management and skills are usually required; public relations skills are needed by some actuaries; and knowledge of life insurance and pension plans is useful to a smaller number of casualty actuaries. However, these subjects are not required in the development of qualified professionals in the field of casualty actuarial science. Instead, these subjects may be better presented as part of a continuing education program offered by the CAS.