CASUALTY ACTUARIAL SOCIETY

Fall, 1988 Edition

CASUALTY ACTUARIAL SOCIETY
ORGANIZED 1914

CASUALTY ACTUARIAL SOCIETY

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October 1, 1988

TO: CAS Members

RE: Third Issue of the CAS Forum

Dear CAS Members:

The third issue of the CAS Forum provides an extensive collection of articles on ratemaking and a preview of chapters from the CAS textbook. We also have a new paper and an interesting historical presidential address.

We have reprinted the Presidential Address from Francis Perryman in May of 1939. It is titled "The Casualty Actuary." In this time of discussions on the strengthening of the actuarial profession, I think you will find reading this address provides considerable historical perspective.

We have one new paper entitled "Varying Trend Factors by Size of Loss" by Sholom Feldblum. Sholom is becoming a regular contributor to the Forum. I encourage any member who has information to share in article form to submit these articles to me at my Yearbook address.

We have included a number of presentations from the very successful Ratemaking Seminar held last Spring. As you know, the Ratemaking Seminar will now become a permanent fixture within the CAS and will probably have its own Proceedings issued, similar to CLRS.

We are also including two papers which will appear in the 1988 Proceedings. The first was presented to the May, 1988 CAS meeting. The second is a paper on reserving in the London reinsurance market. Early publication will allow an opportunity for members to review these papers and provide comments to the author or discussions to the Committee on Review of Papers.

We have included the remaining five chapters of the CAS textbook. It is very important that you read these chapters and provide comments to the authors. The chapters are written to be included in a basic textbook and your comments should be directed towards content appropriate to that level.

We hope you enjoy this third issue of the Actuarial Forum. Please send any new articles for the Spring, 1989 issue to me by February 1.

Yours truly,

CHARLES A. BRYAN

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THE CASUALTY ACTUARY

PRESIDENTIAL ADDRESS BY FRANCIS S. PERRYMAN

Last November the Casualty Actuarial Society did me the great honor of electing me its President. I want to take this, my first, opportunity of expressing to the Society my appreciation of the high honor and of stating that the responsibilities of the office, of which I am fully conscious, I shall do my best to undertake and that the interests of the Society will always be looked after by me to the fullest extent of my powers—and I trust that my efforts will be a worthy continuation of the high standards set by my predecessors.

When I came to prepare this address, the first question that raised itself was "What is the object of the President's address?" Is it to fill up part of each session or is it merely to continue a tradition that got started? The answer to each of these suggestions is obviously "no," for if the President's address has no value in itself then something more valuable should be found to fill up the sessions and the tradition is not worthy of being continued. Neither does it seem that the reason for such an address should be to make some valuable contribution to actuarial science, for first of all it is impossible to expect that the President for the time being shall be able to make such a contribution twice a year and secondly if he has such a contribution to make he can always present a paper containing it. Upon reflection, I think it will be plain that the object of the address is rather to review the status of the profession either as a whole or from some particular angle, or else to point out some outstanding or new aspect of our science to which the attention of the profession should be drawn, for it is the privilege and duty of the President, when necessary, to urge some new point of view on the profession or to indicate some new direction in which actuarial activity should be extended.

The President's address could even, and perhaps should from time to time, be the occasion of the taking of stock of the Society and if the President finds that certain things that are not being done should be done, or that are being done should not be done, he should call on the Society to consider these things. With these ideas in mind, I took the occasion to reread a good many of the former addresses of your Presidents and found, as you will find if you also reread those addresses (and incidentally I believe it would be a good idea for you to do so from time to time) that all the aspects that I have just mentioned of a review or a commentary have been adopted in some form or other by my predecessors in the presidential office. Incidentally, I might say I have been considerably concerned at the task of maintaining the high standard of previous presidential addresses.

In accordance with the foregoing, which leaves a wide latitude to your President, I have not found it hard to think of a great deal to talk about as regards various aspects of the profession. Therefore, in this address I am not going to present any valuable contribution to our science but I am going to touch on various general aspects of the profession and my remarks will be quite general and will, I hope, perhaps furnish the occasion for the members of this Society to think a little more deeply than we usually have time to do about the fundamentals of our professional activities which should always be in our thoughts at least subconsciously if not consciously.

To start such a general review I should, I suppose, talk briefly about what constitutes an actuary, and particularly a casualty actuary, and discuss his evolution. However, into the early history of actuarial science and of the growth of the body of men with specialized knowledge and training (now known as actuaries) I will not go at length. In many of the presidential addresses of Actuarial Societies, both life and casualty, you will find traced the derivation of the name actuary, and an account of the development of the modern actuary. I was recently reading the address to the Institute of Actuaries in England of its new President, Colonel Oakley. In this he aptly characterizes actuaries as "dealers in futures." He pointed out that actuaries are concerned with future mortality, future rates of interest, future expenses, future margins. He was thinking, of course, principally of actuaries dealing with life insurance but, nevertheless, his criterion of an actuary is a true one for all actuaries, including actuaries in the casualty business, actuaries dealing with social insurance and actuaries dealing with any other branch of human endeavor, as well as life insurance actuaries. It was because of this essential concern of actuaries with the future that when the casualty business started on its meteoric career in the second decade of this century, it was recognized that actuarial training would be helpful if not requisite to deal with the problems that were cropping up in every direction. There was at that time available very little actual experience and what there was seemed ill adapted to elucidate the future that was looming ahead. What were the correct answers to the questions on rates, reserves, etc.? What in other words was to be done as regards the future? Here was a situation that obviously called for actuaries and hence was born the Casualty Actuarial Society. It happens that in this country those actuaries who were called on to deal with these new and growing problems decided that their interests and those of the business would best be served by a separate forum where they could exchange views and help one another and so this separate forum was set up in the shape of the Casualty Actuarial Society. In most other countries, such a separation was not effected, perhaps because of different local conditions, perhaps because elsewhere the problems did not arise so fast and the existing Actuarial Societies could find time to give them adequate attention; nevertheless, the fact remains that in this country a separate society was set up and this has without doubt been of considerable aid to the actuaries, called on to deal with casualty problems, as well as to the casualty business as a whole. Those of us who are acquainted with the casualty business in parts of the world other than the North American continent realize that the technical handling of casualty business is far more advanced here than elsewhere and often think that elsewhere more progress would have been made if separate societies had been established to foster the purely casualty end of actuarial science.

In the early days of our Society, compensation insurance presented the greatest number of new and pressing problems. There were lots of them. It is difficult for us now-a-days to realize just what it must have been like for the actuaries who had to deal with these new questions. The actuaries of those days had no precedents to go by, no established methods, no statistical information—in fact, they had practically nothing to go by except their wits and training; so they energetically set to work to find solutions to their problems. The younger members of the Society may realize to some degree what these pioneers were up against if they reread some of the earlier numbers of the *Proceedings*, bearing in mind that what is in the *Proceedings* is the finished product (if I may so term it), and that there does not appear in the *Proceedings* any record of the hours of thought and toil that the pioneer actuary went through before he was able to arrive at what eventually appeared in the *Proceedings*. It may seem to the younger actuary, rereading the earlier numbers, that the pioneers were rather perverse at times, paying a lot of attention to what we now know are, or at any rate regard as, trivial points, while on the other hand the pioneer skipped lightly over what we now believe are the essentials of the questions. However, this is always the way with pioneering. A high-school student now-a-days does mathematics, which would have baffled a first-class mathematician of a few centuries ago; and college students take in their stride whole fields of thought unknown to leading thinkers of a generation or so ago.

How did these pioneer casualty actuaries and their successors make out? On the whole they did quite well—the casualty business has been growing rapidly and very complexly, and but for the work of its actuaries, even its present state (however far from perfect it may be) would not have been attained. This brings me to another observation. When evaluating the degree of success of actuaries in the casualty field we must recognize the quite considerable differences between actuarial problems in the casualty business and in the original field of actuarial effort, namely life insurance. In essence, life business involves much more technical work, that is to say more purely actuarial work. The deals in futures, that I referred to above, are spread over a long period of time and it has been possible and desirable for the life actuary to use more margins in his calculations. Casualty business involves less technical or mathematical work and essentially deals more with what I may term "humanities" and quicker results are looked for—particularly as the economic and social factors that casualty business deals with change quite rapidly and have particularly done so during the last twenty-five years. Perhaps under different conditions, for instance a more stable world, or had the temperament of the inhabitants of this continent been less energetic, results similar to those attained in casualty insurance could have been reached by slower and longer range methods. Something of this sort has been experienced in other parts of the world, for instance in England, where while the growth of casualty business has been almost as rapid as here, it has so far been along more conservative lines; coverages and rates and conditions generally have not changed as rapidly as in this country. Actuaries here might have preferred some such more steady development but they were not the choosers. They have had to grapple with the problems as they arose; they have had to endeavor to make rates or reserves or what else to fit the present or the near future in times of great flux. They have, of course, tried to influence insurance opinions to take the more reasonable road, and not without success. The really technical aspects of Casualty Actuarial work have been naturally dealt with more successfully than have the social problems to which the business has given rise, but that, of course, is true of other actuarial fields and indeed of all modern life. Man has achieved great technical accomplishments; he can build bridges, battleships, airplanes, successfully, but he has not displayed the same ability in managing himself, as is evidenced by the present state of the world. To come nearer home, we find in life insurance that actuaries have had least success when they have had to deal with the aforesaid "humanities"—for instance, look at disability coverage, which has been wrecked by its impact with factors not at all unfamiliar to casualty actuaries (I hope our friends—the life actuaries—will not resent this reference. Some of casualty's ventures into similar fields—e.g. non-cancellable accident and health business—have had similar unfortunate results). So the casualty actuary should not be too discouraged at what he may consider his apparent lack of success in dealing with the social and economic aspects of his work. As a matter of fact, the casualty actuary's training in dealing with such problems is precisely what is needed in the modern world. I think that serious thinkers will agree that ultimately the complex problems of social and government planning, that is politics in its true sense, will have to be handled on more scientific lines. Proper plans will have to be made to deal with the intricacies of our civilized life. These plans will take the place of the existing lack of method consisting too often of day-to-day expedients foisted on a restive population by so-called politicians with none of the required technical training and ability. And who is there more fitted than the actuary to make such proper plans? His will be the privilege of using his knowledge and experience, his actuarial tools and methods, so as to solve our modern social problems, our problems of living together in harmony and cooperativeness; for this is sure, that such problems will be solved and they can be dealt with only by scientific methods that are in essence those we use and know as our actuarial ones; and if actuaries do not make it their business to take a fuller part in the life of the country along the lines of true scientific planning, some other body of men will and they will attain success only by the use of actuarial methods; in other words, only by becoming actuaries.

However, this is perhaps going rather too far into the future. What is the more immediate prospect? Up to now, despite the efforts of some of us to diversify our proceedings, Workmen's Compensation problems have occupied a large proportion of our time—that is of the Society's time. I think that other kinds of insurance will and should claim an increasing share of our attention. Recent developments in lines like Automobile insurance indicate the need of actuarial methods there and perhaps reflect our lack of attention to this field. Most of the other casualty lines—Miscellaneous Liability and Property Damage, Burglary, Bonding, Boiler and Machinery, will and should receive an evergrowing amount of actuarial assistance. All of which must result in continual improvement in the status of the actuary in casualty business. And as to that, here again let me say—don't let us be discouraged about that status. The standing of actuaries in this country is progressively getting better. In life insurance, while actuaries have not perhaps yet attained the preeminent position they occupy in some other countries, they are getting there. Casualty actuaries have been engaged in their particular branch of the profession for a comparatively short period of years during which the size of the business has expanded enormously and where a lot of emphasis, perhaps too much emphasis, has been placed on other aspects such as the "selling end." During this period, as I have mentioned above, many things have been done that perhaps the actuaries would have preferred should not have been done—at any rate, the business finds itself with still a large number of problems on its hands, many of which we know the actuary will eventually have to solve. Let him, therefore—this casualty actuary about whom I have been talking—continue to grapple with these problems, knowing full well that he has an enormous advantage in the possession of a scientific mind and of scientific methods; with these he will, on his merits, be called on to play a larger and most responsible part in the business of casualty insurance.

VARYING TREND FACTORS BY SIZE OF LOSS

Varying Trend Factors by Size of Loss

By Sholom Feldblum

Average loss costs per claim have risen faster than the associated economic inflation indices for most lines of business, indicating that economic inflation is not the only cause of increasing insurance prices. "Social inflation," meaning the heightened claims consciousness of the public, the evolution of new causes of action in lawsuits, and the increasing liberality of jury awards, accelerates claim cost trends beyond what might be expected from economic inflation. Social inflation is most clearly evident in exorbitant judgments in large liability cases. Several actuaries have therefore suggested that social inflation has a stronger effect on larger claims, and so claim cost trend factors should vary with the size of the loss.

In their 1981 paper, "Adjusting Size of Loss Distributions for Trend" (in Inflation Implications for Property-Casualty Insurance, 1981 Casualty Actuarial Society Discussion Paper Program, p. 458), Sheldon Rosenberg and Aaron Halpert present methods for determining whether claim cost trends differ by size of loss and for quantifying this difference. Their second method, which has received wide acceptance, is to (1) construct the loss distribution functions in two or more years, and then to (2) compare the loss sizes which have equivalent cumulative probabilities in these years. For example, suppose that in 1985, 20% of losses are less than $5,000 apiece and 80% of losses are less than $40,000 apiece; in 1986, 20% of losses are less than $5,500 apiece and 80% of losses are less than $50,000 apiece. For losses of $5,000 in 1985, loss cost inflation is +10%, but for losses of $40,000 in 1985, loss cost inflation is +25%. The example used by Rosenberg and Halpert, using actual Products Liability Bodily Injury data collected by ISO for policy years 1973 and 1977, is shown in Figure 1.

Figure 1: Loss Cost Trend Varying by Size of Claim:
Products BI data for policy years 1973 and 1977

(1) 1973 Value	(2) 1977 Value	(3) (2)/(1)	(4) Annual Trend: (3)**0.25 - 1
$ 10,000	$ 21,929	2.193	+21.7%
50,000	116,355	2.327	+23.5
100,000	255,310	2.553	+26.4
200,000	571,995	2.860	+30.0
500,000	1,692,052	3.384	+35.6
1,000,000	3,872,216	3.872	+40.3

Trends that vary by size of loss are particularly important for determining increased limits factors. Such factors increase if there is a positive trend, since losses that already exceed the basic limit show all their inflation increase in the excess layers, while losses that are just below the basic limit break this boundary because of inflation and increase the frequency of excess limits losses. If loss cost trend factors increase with the size of the claim, the increased limits factors climb even more steeply, since small claims have only a minor inflationary increase, but large claims have a more severe inflationary increase.

ISO has begun using loss cost trends that vary by size of loss in its general liability and increased limits reviews, using trend factors suggested by the Actuarial Research Committee. Figure 2 shows the results from the Hospital loss experience used in the 1986 increased limits review.

Figure 2: Loss Cost Trend Varying by Size of Claim:
Hospital loss experience used in ISO 1986 increase limits review

Loss Size	Actual Trend	Loss Size	Actual Trend	Loss Size	Actual Trend
$ 49,500	+ 2.9%	$ 593,800	+12.8%	$1,420,000	+17.9%
85,800	4.3	665,800	13.4	1,514,500	18.3
127,200	5.6	740,300	14.0	1,610,900	18.7
173,100	6.8	817,400	14.6	1,709,100	19.1
223,100	7.9	896,800	15.1	1,809,300	19.4
276,800	8.9	978,600	15.6	1,911,200	19.8
334,000	9.8	1,062,600	16.1	2,015,000	20.1
394,500	10.6	1,148,800	16.6	2,120,500	20.5
458,100	11.4	1,237,100	17.0	2,227,700	20.8
524,500	12.1	1,327,600	17.6		

Something is amiss here. The actual trends say that for losses below $100,000, hospital cost inflation has been about 3 or 4% per annum. In fact, for all loss sizes below $250,000, the trend factors seem unreasonably low.

Moreover, the actual trends by size of loss form an almost perfectly smooth progression. But social inflation affects losses in an erratic fashion, and one would hardly expect it to cause such a smooth increase in trend factors.

Finally, the ISO Surgeons and Premises/Operations experience data show **decreasing** trend factors as the size of loss increases. It hardly seems logical to suppose that social inflation affects small losses more than large losses.

Social inflation definitely increases loss frequency, but does it also increase the average loss size per claim? People often assume that "pain and suffering" awards are causing the escalation of private passenger automobile bodily injury claim costs. In truth, compensation for medical costs now form a higher percentage of total losses than 10 years ago, while the percentage formed by general damages has decreased slightly. (See the discussion in the forthcoming All-Industry Research Advisory Council automobile personal injury closed claim study for the data supporting this.) In sum, the Rosenberg-Halpert method of determining trend factors by size of loss warrants re-examination.

The Rosenberg-Halpert method is valid only if the loss frequency distribution does not change. If a change does occur, such as an increased frequency of small nuisance claims, a decrease in small claims due to a more widespread use of deductibles, an increase in large claim frequency due to higher reinsurance retentions, or any other such change, then there may be an apparent varying trend by loss size even when inflation affects all losses equally. This has a crucial effect on the determination of increased limits factors, as well as on various other business decisions.

Suppose an insurer records four claims during 1985, for $10,000, $20,000, $30,000 and $40,000. The same four events occur the next year as well, but economic inflation of +100% per annum affects all claims equally and causes the loss sizes to be $20,000, $40,000, $60,000 and $80,000. In addition, a new small claim of $10,000 is also recorded.

Cumulative probability values for small, discrete samples can be tricky. We use a particularly simple method for the illustration; any other method would produce similar results, though with slightly different figures. We match the endpoints of the 1985 distribution with the endpoints of the 1986 distributions. Since there are three intervals in the 1985 data, and four intervals in the 1986 data, each 1985 interval is equivalent to one and one third 1986 intervals. For instance, the $20,000 1985 loss should be matched with a weighted average of the $20,000 and $40,000 1986 losses, with the weights being 2/3 and 1/3, respectively. The loss cost trends by size of claim are shown in Figure 3.

Figure 3: Loss Cost Trend Varying by Size of Claim:
Effect of Increasing Frequency of Small Claims

1985 Claims	1986 Claims	Matched 1986 Claim Sizes	Annual Trend
	10,000		
10,000		10,000	+ 0.0%
	20,000		
20,000		26,667	33.3
	40,000		
30,000		53,333	77.8
	60,000		
40,000		80,000	100.0
	80,000		

Conversely, suppose the insurer introduced a $20,000 deductible in its 1986 policies. Then only three of the 1986 losses are recorded by the insurer: the $40,000, $60,000, and $80,000 claims. Using the "ground-up" figures, not the actual insurer payments, for the size of loss distribution, the loss cost trends by size of claim are shown in Figure 4.

Figure 4: Loss Cost Trend Varying by Size of Claim
Effect of Increasing Use of Deductibles

1985 Claims	1986 Claims	Matched 1986 Claim Sizes	Annual Trend
10,000		40,000	+300.0%
	40,000		
20,000		53,333	166.7
	60,000		
30,000		66,667	122.2
	80,000		
40,000		80,000	100.0

One may ask: "In any case, the shape of the loss distribution is changing over time. What difference does it make whether it is due to loss cost trends varying by size of claim or to changing loss frequency distributions by size of claims?" There are many differences: consider first the effect on increased limits factor calculations.

Suppose the basic limit is $25,000 per claim, and one must calculate factors for the $100,000 increased limit. Thus, for 1985, the factor is (10+20+30+40)/(10+20+25+25) = 1.250. Suppose also that the trend factors by size of loss indicated by the Rosenberg-Halpert procedure are as shown in Figure 3 above.

If there was indeed a varying trend by size of loss that produced the trend factors shown in Figure 3, then the 1986 loss sizes must have been $10,000, $26,667, $53,333 and $80,000. The total loss is $170,000, for an overall trend of +70%. Had each loss increased by 70%, the individual loss sizes would have been $17,000, $34,000, $51,000, and $68,000, and the increased limits factor would have been (17+34+51+68)/(17+25+25+25) = 1.848. Using the actual 1986 loss sizes ($10,000, $26,667, $53,333 and $80,0000) indicated by the varying trend, the increased limits factor for 1986 is (10+26.7+53.3+80)/(10+25+25+25) = 2.000. In other words, if the loss cost trend increases with the size of the claim, then the indicated increased limits factor is higher.

But suppose that the varying trend factors shown in Figure 3 were due to the addition of a small claim. Economic inflation is +100% per annum, and were this the only influence on the loss distribution, the increased limits factor would be (20+40+50+80)/(20+25+25+25) = 2.105. But the actual increased limits factor for 1986 should be (10+20+40+60+80)/(10+20+25+25+25) = 2.000. In other words, a higher frequency of small claims will also cause an apparent varying trend by size of loss but will indicate lower increased limits factors.

Of course, the final increased limits factors for the two cases are identical - because they required differing underlying inflation rates. If the numbers in Figure 3 are due to varying trends by size of loss, then the underlying inflation rate that would be measured **by an external index** is +70% per annum. This inflation rate would produce an increased limits factor of 1.848, but the varying trend by size of loss increases this to 2.000. If the numbers in Figure 3 are due to a change in loss frequency distribution by size of loss, then the underlying inflation rate that would be measured **by an external index** is +100% per annum. This inflation rate would produce an increased limits factor of 2.105, but the changed loss frequency distribution decreases this to 2.000.

In other words, if we expect the overall trend to be X% per annum, but the Rosenberg-Halpert method shows an apparent varying trend by size of loss, should the change in the increased limits factors be greater than or less than that indicated by a uniform X% trend? The answer depends upon the cause of the varying trend by size of loss.

Conversely, a loss cost trend that decreases with the size of the claim produces a smaller change in increased limits factors than would be indicated by a uniform trend. But if the decreasing varying trend is due to a more widespread use of deductibles, then a larger increased limits factor is required.

This paper does not argue that loss cost trend factors are uniform for all loss sizes. Rather, the varying trend by size of loss noted by many actuaries may be due simply to an increase in small nuisance claims, a more widespread use of deductibles, different reinsurance retention levels, or any other cause of a changing loss frequency distribution. An apparent decrease in trend by size of loss is not anomalous: it may be due to an increasing use of deductibles or a change in reinsurance retention levels, not the effects of social inflation.

A practical implication is in target marketing. If an increase in trend factors by size of loss is due to social inflation, then the low frequency high severity risks will become progressively less profitable than the high frequency low severity risks. But if the cause is an increase in small nuisance claims, then the low frequency high severity risks will become more profitable than the high frequency low severity risks.

Another practical implication deals with responses to the claim cost problem. If social inflation causes the varying trend by size of loss, then a change to a compensation system not based on tort liability may be warranted (as in Workers' Compensation and automobile no-fault insurance). If an increase in small nuisance claims is causing the varying trend by size of loss, then a change to a no-fault compensation system may accelerate this increase.

Clearly, one must distinguish the effects of social inflation and of changes in the loss frequency distribution. Three methods of doing so are suggested below.

First, social inflation affects primarily personal injury claims; nuisance claims and deductibles affect both personal injury and property damage claims. For example, both Rosenberg-Halpert and ISO find varying trend factors for Premises/Operations Bodily Injury. One should use the Property Damage coverage from the same body of data to see whether similar varying trends show up there as well. Changes in loss frequency distributions would account for any varying trends in the latter data, since social inflation has little effect.

Second, nuisance claims and deductibles affect the loss frequency distributions primarily for small claims. If one truncates from below the loss distribution of each experience year, one can remove most of the effect of nuisance claims and deductibles. The truncation point must be indexed: if the overall loss cost trend is +10% per annum, the truncation point may be $5,000 in 1985, $5,500 in 1986, $6,050 in 1987, and so forth.

Ideally, the indexed truncation point should be chosen such that the overall loss frequency ratio remains constant from year to year. This is not always possible, as the loss frequency ratio may be changing at all loss sizes. A non-uniform change in the loss frequency distribution at **any** loss size will cause a varying trend.

Third, one should examine loss cost trends by size of claim, where the claim size is **not** based on a dollar figure. For example, one may subdivide the personal injury claims by the number of days the claimant spent in a hospital: 0 days, 1-3 days, 4-7 days, and so forth. For each cell, one may determine the loss cost trend factor. The effect of changes in loss frequency distribution has been removed, but social inflation would still cause a varying trend by size of loss.

No matter what procedure is used, the data from the different experience years must be comparable. If one uses experience from different carriers for 1985 and 1986, the loss frequency distributions will probably differ, and varying trend factors are expected from the Rosenberg-Halpert test, regardless of whether they are truly present. This is a problem particularly for rating bureaus, which have different members by year, (sometimes) different statistical plans, and little control over deductible and marketing changes.

Dr. Glenn Meyers, a research actuary at the Insurance Services Office, has suggested another explanation for the apparent loss cost trends that vary by size of claim. Large claims have a longer average time to settlement than small claims do. If economic inflation affects loss payments between the accident date and the settlement date, as seems reasonable for General Liability claims, then different inflation rates affect large and small claims. A period of rising inflation rates would show loss cost trends increasing with the size of the claim, and a period of declining inflation rates would show loss cost trends decreasing with the size of the claim.

A simplified example should clarify this. Suppose a line of business has only two types of claims: small claims with a 1984 present value of $1,000 and large claims with a 1984 present value of $10,000. All small claims are paid one year after the accident date, and large claims are paid two years after the accident date. Loss cost inflation, affecting all claim sizes equally, is +10% from 1984 to 1985, +20% from 1985 to 1986, +30% from 1986 to 1987, and +20% from 1987 to 1988. Needless to say, these inflation rates are purely illustrative, and are not meant to reflect actual inflation in the U.S. during these years.

Figure 5 shows payments for large and small claims in each accident and settlement year. Small claims incurred in 1984, with a present value of $1,000, and paid in 1985 for $1,100. Similarly, small claims incurred in 1985 are paid in 1986 for $1,320, and small claims incurred in 1986 are paid in 1987 for $1,716. Large claim incurred in 1984 for a present value of $10,000 are paid in 1986 for $13,200. Similarly, large claims incurred in 1985 are paid in 1987 for $17,160, and large claims incurred in 1986 are paid in 1988 for $20,592.

Figure 5: Loss Cost Trend Varying by Size of Claim
Effect of Differing Inflation Rates by Year

Size of Claim	Accident Date	Present Value	Settlement Date	Paid Loss	Apparent Inflation
Small	1984	$1,000	1985	$1,100	
	1985	1,100	1986	1,320	+20%
	1986	1,320	1987	1,716	+30%
Large	1984	10,000	1986	13,200	
	1985	11,000	1987	17,160	+30%
	1986	13,200	1988	20,592	+20%

Assumed inflation rates:
1984-85: +10%; 1985-86: +20%; 1986-87: +30%; 1987-88: +20%.

In this illustration, inflation affects all losses equally. But between accident years 1984 and 1985, small claims show an apparent loss cost trend of +20%, and large claims show an apparent loss cost trend of +30%. Conversely, the apparent trends from accident years 1985 to 1986 are +30% for small claims and +20% for large claims.

As Dr. Meyers points out, the varying trend indications during the historical period provide no information about expected trends by size of claim during the forecast period. Although we can (and we must) quantify estimated inflation during the coming year or two, it is almost impossible to predict whether inflation rates will be increasing or decreasing in the future. Moreover, a higher inflation rate for claim liabilities generally corresponds to higher investment income rates for the assets supporting those liabilities (for a full discussion of this, see Robert P. Butsic, "The Effect of Inflation on Losses and Premiums for Property Liability Insurers," _Inflation Implications for Property-Casualty Insurance_, 1981 Casualty Actuarial Society

Discussion Paper Program, p. 58.) In the illustration above, the present values of the large and small claims at occurrence date do not show differing inflation rates. Thus, even if one knows that inflation would be increasing or decreasing in the future, and that large and small claims had different times to settlement, using differing loss cost trends by size of claim is inappropriate.

Frank Sullivan, of the ISO Actuarial Research Committee staff, has examined varying loss cost trend factors for the Products property damage (PD) line of business. Social inflation should have little or no varying effect on property damage losses by size of claim; rather, loss cost trends should be uniform for all claim sizes. Yet the Rosenberg-Halpert method produces just the opposite conclusion. The 1987 ISO General Liability Actuarial Committee (GLAC) indications for Products PD showed a varying trend increasing from +11% per annum at claim sizes below $2,000 to +21% per annum at claim sizes of $100,000 and +29% per annum at claim sizes of $900,000. The 1987 loss cost trends increased more steeply by size of claim for Products PD than for the bodily injury lines of business - a remarkable result.

However, Frank found that truncating the loss cost distribution from below with an indexed truncation point had little effect on the apparent loss cost trends varying with size of claim. First, he "purified" the ISO data by eliminating Composite Rated Risks. Then he obtained Rosenberg-Halpert indications for both the full distribution and for the truncated distribution. For the truncation points, he used $3,000 for policy year 1980 and a +5.1% per annum overall loss cost trend to give indexed points of $3,153 for 1981, $3,314 for 1982, and $3,660 for 1983. Both the unadjusted and the truncated distributions showed loss cost trends increasing from about 3% per annum at a claim size of $5,000 to +11% per annum at a claim size of $100,000.

During the 1980's, economic inflation rates have not varied significantly by year. Moreover, PD payment lags, unlike BI payment lags, do not differ that greatly between small and large claims; in other words, the hypothesis suggested by Dr. Glenn Meyers should have no effect. Thus, the explanation of the varying loss cost trends by size of claim for Products PD is unclear. As casualty actuaries, we have the ability to work with these figures and trends, and it behooves us to uncover the causes of these indications.

Innovations in actuarial science follow a strange course. Pure actuaries write theoretical papers; were it not for them, no changes in our procedures would emerge. Practical actuaries use the results, but their major concern about the procedures is simply that they be correct; were it not for them, no changes in our procedures would be required. Most needed, however, are researchers like the ISO Actuarial Research Committee Staff, who take the theoretical concepts and apply them to actual data. Were it not for them, the actuarial innovations would never find their way into the insurance world.

* * * * *

Numerous people contributed the ideas in this paper. The ISO Actuarial Research Committee Staff, Dan Crifo, Mayer Riff, Noson Kopel, and Frank Sullivan, produced the varying loss cost trend analyses by line of business. Frank analyzed the most recent data, saw the anomalies in the results, and noted the problems of inconsistent data. Gary Koupf showed how a more

widespread use of deductibles could cause a decreasing varying trend, and Lee Steeneck suggested other causes of under-reporting of small losses that would have the same effect. Gary and Isaac Mashitz suggested used an indexed truncation point for the size of loss distributions to remove much of the effect of differing loss frequency distributions. Dr. Glenn Meyers suggested the alternative explanation in the text, different inflation rates by year, and he intends to empirically test this on the ISO General Liability data. Richard Woll first suggested to me that the standard explanations for the varying trend factors phenomenon may not be correct, and he encouraged me to examine the data for other possible causes.

SELECTED PRESENTATIONS FROM THE 1988 RATEMAKING SEMINAR

RECENT DEVELOPMENTS IN RESERVING FOR LOSSES IN THE LONDON REINSURANCE MARKET

RECENT DEVELOPMENTS IN RESERVING FOR LOSSES IN THE LONDON REINSURANCE MARKET

BY HAROLD CLARKE

Abstract

The paper describes in detail a new method which can be applied by any insurance company to its own data to set reserves for outstanding losses (including IBNR) and to calculate a confidence interval for these reserves. The method has also opened up a whole range of interesting ways of looking at data. Although the method can be applied to any sort of business it is particularly helpful in looking at long tail business, such as that written by reinsurers, for which other methods have proved less satisfactory. The methodology can also be applied by a supervisory authority to establish minimum reserving standards for companies where global general market data on run-offs for different classes of business is available. A new method of setting minimum reserves for individual syndicates based on the methodology in the paper is currently being tested by Lloyd's of London. This work is briefly described in the final section of the paper.

1. INTRODUCTION

This paper describes a system which our firm has developed and refined over the last 5 years to enable us to comment on reserves set up for outstanding and IBNR claims by companies writing marine, aviation, liability and reinsurance accounts or alternatively to advise on such reserves. The companies we have advised have been operating in the London Market in the UK of which Lloyd's is the centre. The London Market underwrites a significant part of the world's insurance and in particular its reinsurance and is a dominating influence on insurance world-wide. Although the system described is particularly suitable for reserving for reinsurance accounts it is also applicable to all other types of casualty business. The system is fully operational on our main frame computer. It has been used many times and it is stable.

In the London Market details of numbers of claims are generally not available or not relevant. Data is usually available for each "account year", i.e. for all risks written in a particular accounting year which is usually a calendar year. The items normally available are:

(i) Premiums paid to date

(ii) Claims paid to date

(iii) Claims outstanding, i.e. the case estimates as notified by the brokers to the companies for outstanding claims.

Further details of the constraints and problems posed by the data are given in Section 2.

The system had to be able to generate estimates of the reserves from this limited amount of data. The method works by estimating the Ultimate Loss Ratio ("ULR") for each account year, from which the necessary reserve is easily derived. An important innovation of the method is that a confidence interval is produced for the ULR and hence for the reserves. An outline of the method is given in Section 3, a detailed worked example in Section 4 and some further problems and considerations are discussed in section 5. The method is very graphical and easy to see and present to actuaries and non-actuaries.

In the final section of the paper, Section 6, we describe an application of the method to setting minimum reserves at Lloyd's which is currently being tested. The method can also be used in that way to set minimum reserves for companies operating in any insurance market where industry wide statistics are available.

The method starts from an idea put forward in a paper by D.H. Craighead (1) to the Institute of Actuaries. Inside our firm we have considerably refined and extended this idea. A detailed description of the potential use of the method by Lloyd's together with an outline of the general method is given in the paper by my colleagues S. Benjamin and L.M. Eagles (2) to the Institute of Actuaries. In this paper the emphasis is reversed with considerably greater detail being given about the general method. We also wish to thank A.B. English for the programming and application of the curve fitting algorithm and for much other programming.

2. DATA

As previously mentioned the data available for setting reserves in the London Market is sparser than that usually available from companies writing mainly domestic risks. The reasons for that are outlined below.

For risks written in the London Market cover is usually given for one year. The premiums are received over a period of typically three years. This delay can be due, for instance, to excess of loss treaties being rated on a burning cost basis or to delays in monies being forwarded by brokers. The incidents which take place during the year of cover give rise to claims which may not be reported for many years and then may take several years to settle. The main reason for this delay is that the London market tends to deal in reinsurance where the information is "second-hand" in the sense that it comes from a primary insurer which may itself be subject to delays of information. For instance suppose you are writing a catastrophe excess of loss treaty covering property damage exceeding $10 million in aggregate for any one incident for a Californian company. The reinsurer may not hear anything from the Californian company until its own claims reach the agreed limit. The final outcome for the reinsurer in the London Market may then take a long time to become fully known. Further, as this example illustrates, the concept of number of claims is not meaningful in this market.

Also the risk will often be placed on a coinsurance basis, often with 20 or 30 different underwriters. Detailed data may be available to the leading underwriter, but that detailed information may not be available to others on the risk and will not be recorded centrally. Statistics have in fact tended to be subordinate to accounting data, which is therefore the only data commonly available. This also has the problem that if an error is discovered in the statistics (e.g. an outstanding claim has been notified in Italian lire rather than US dollars) it will be corrected from discovery, but the history will be left unchanged so that the statistics still reconcile with the published accounts.

The data is usually available for each account year. Thus, the method described in this paper will be presented for data collected on that basis. However as will become clear the method is equally applicable to data collected on an accident year basis. It is common for the data to be missing for early account years or early years of development, often due to computerisation of the accounting function taking place at that point.

In the case of Lloyd's, further problems arise from the use of very broad risk categories which cannot be assumed to be homogeneous over time. The classic example of this is Non-marine All Other which can include marine business written by non-marine syndicates. Further the data collected centrally consist only of premiums received and claims paid, both net of reinsurance. After the end of the third year of development of an account year future premiums received are set off against future claim payments in the statistics.

More information on the operation of the London Market in general and Lloyd's in particular is given in the paper by D.H. Craighead (1).

The techniques described in the paper can be applied to gross data, net data, paid losses, paid plus outstanding losses. That is why we have not defined closely the basis of the data.

3. SYSTEM REQUIREMENTS AND OUTLINE OF METHOD

For the data described in the previous section most of the reserving methods commonly in use break down. We needed a method which:

(i) Was able to cope with long tail business.

(ii) Would use only information on premiums, paid claims and claims outstanding as notified.

(iii) Could provide estimates where there were missing items of information from the run-off triangle.

(iv) Could handle multi currency portfolios. Most of the companies whose reserving we examine write substantial US dollar business even though they report in pounds sterling.

(v) Would enable us to set a range of values within which reserves would be acceptable. After all, no single estimate can be correct unless we have business which has completely run off. We would expect in the early years of development of an account year that the range would be relatively wide and should reduce as development increases.

(vi) Where necessary would use market information or information from other similar businesses to establish reserves for a particular insurer.

It is vital that any system should be able to cope with all the preliminary data handling and be able to accept data in a variety of formats. In particular the system needs to be flexible enough to deal with the following variations:

(a) The data can be either cumulative or incremental.

(b) Claims data can show paid claims and claims outstanding either separately or summed, and can be expressed either as loss ratios or cash.

(c) Development time intervals can be either quarterly, half-yearly or annual.

(d) The data may be presented in a number of different currencies which the system must be able to combine at the user's discretion. (When currencies are combined uniform exchange rates are assumed to apply for all periods of origin and development).

(e) The data may be provided for a large number of separate categories in a variety of currencies. Again at user option the system must be able to combine any or all of the categories.

The system needs to be able to accommodate a variety of currencies because the London Reinsurance Market writes business internationally. It therefore accepts business in a wide variety of currencies. It is possible for a company to keep separate statistics for each of the currencies in which it does business. In practice it is usual for a company to keep statistics in three currencies, US dollars, Canadian dollars and sterling. In this case currencies other than the first two are converted into sterling at the exchange rates applying at the date of the relevant transaction.

A major consideration underlying our whole approach is that for the classes of business we are considering, standard assumptions, e.g. homogeneous account from year to year, standard pay out pattern, no change in speed of claims advice, etc., would almost certainly all be violated. This suggested as a basic starting point that we examine the run-off of each account year separately. It also suggested that we look at the development of loss ratios rather than losses. Empirical considerations suggested that if we were seeking a smooth curve to fit the shape of the loss ratio at development time t, plotted against t, that curve would have a negative exponential shape.

In the remainder of this section we outline the reserving method we have developed to meet the above criteria. A worked example of the method is then given in section 4 to expand on the outline.

(a) Run-off triangles are drawn up for as many account years as possible showing the development year by year (or quarter by quarter) of premiums and claims.

(b) An estimate of the ultimate premiums receivable is made for each account year. If we have to calculate the estimate then we simply apply development factors calculated from the data without smoothing. Other methods could be used in appropriate circumstances. Often we use the underwriters' estimates since they have a better feel for the way, in practice, policies are being signed down.

(c) The estimates of ultimate premiums are divided into the relevant claims to give a run-off triangle of loss ratios.

(d) Separately for each account year for which there is sufficient development (this depends on the length of the tail of the business) a curve of negative exponential form is fitted to the loss ratio development for that account year. From this curve a preliminary estimate of the ULR for that account year can be made. In certain cases we can fix some of the parameters in the negative exponential curve from our knowledge of the values of the parameters for the same class of business in other companies, or on an industry wide basis. In the remainder of the paper this part of the process is referred to as "curve fitting".

(e) For each year of development, e.g. year r, we then combine the results obtained in (d) to give a table of the loss ratios at the end of year r and the corresponding estimated ULR's. A line is fitted to these points by standard linear regression techniques. Then given the loss ratio at the end of development year r a best estimate of the ULR for that account year can be obtained from the fitted line. Further a confidence limit for the ULR can also be obtained. In the remainder of the paper this part of the process is referred to as "line of best fit".

For an account year which is well developed the estimate of the ULR is obtained from (d) so no range is quoted, or usually needed. For a year with little development the ULR and accompanying confidence interval from (e) is quoted. For intermediate years the method depends on one's judgement.

4. WORKED EXAMPLE TO ILLUSTRATE METHOD

The approach outlined in the previous section is illustrated below by means of an example based on typical medium tail data. The data is available for account years by quarters of development up to 1st July 1985. This is the date as at which the reserves for outstanding claims are being calculated. For early years of development for the earlier account years the data is missing. It will be seen that this does not cause a problem to the system. Appendix 1 contains computer produced tables and graphs for the example. These are typical of the output produced by the computer system.

Estimating Ultimate Premiums

In this example we assume that no premiums are received after the end of development year 5. We thus need to estimate the ultimate premiums to be received for account years 1981 to 1984 (1985 is omitted from our consideration since half way through the year is too early to establish reserves using this method). The estimates of ultimate premiums are given in Table 1.1 of Appendix 1. The numbers above the dotted line are the cumulative premiums to date. The numbers below the dotted line are the estimates of cumulative premiums for future development years estimated by development factors. Thus for each account year the last number in the column of data for that year is the estimate of total premiums receivable that we intend to use for that year.

Triangle of Loss Ratios

The estimates of total premiums are then divided into the cumulative development of incurred claims (i.e. claims paid plus notified claims outstanding) to generate the cumulative incurred loss ratios, based on ultimate premiums. Details of the loss ratios are given in Table 1.2 of Appendix 1.

Estimation of ULR by Curve Fitting

We now make a first estimate of the ULR's for each account year by fitting a suitable curve to the loss ratio development for that account year. Over the years we have tried a number of different families of curves for this purpose. The family of curves should satisfy the two criteria:

(i) For an account year where the ULR is already known with a fair degree of certainty the curve must level out at a value near that loss ratio.

(ii) For later account years the curve must fit the known data well and also allow for a reasonable amount of future development. In most cases this will mean a development period similar to the more fully developed years.

The curve we have found most suitable is:

$$L_t = A \times [1 - \exp(-[t/B]^C)]$$

where t is the development period and L_t the loss ratio for that development period. There are 3 parameters A, B and C. A determines the ULR while B and C determine the length of the tail and the way in which it approaches the ULR. The curve was originally suggested in a paper by D.H. Craighead (1). In Appendix 2 we give examples of the effect on the shape of the curve of changing the parameters B and C. These illustrate the wide variety of run off shapes which can be fitted by this curve.

This family of curves is used to give estimates of ULR's for account years 1971 to 1981. For later years, not enough development has yet taken place for a satisfactory curve to be fitted. In Figures 1.3 to 1.13 of Appendix 1 we give the graphs of the curves fitted (the solid curves) in this example together with the developed loss ratios. Each loss ratio is represented by a vertical line, with the dotted line joining up the developed loss ratios. The quality of the goodness of fit can be tested by eye by comparing the closeness of the dotted and solid curves. The comparison should obviously concentrate on the later years of development. At the bottom of each curve we give the values of A, B and C fitted together with the mean squared error. In this particular example C was set equal to 1.5 and only A and B were fitted. We discuss the selection of the parameters to be fitted and the choice of the developed loss ratios to be included in the fitting in Section 5. The graphs need not be studied in detail but should just be looked through quickly to see how well, in general, the curves fit the data.

On occasions we have found that the graph produced by the computer does not suggest a smooth curve. Particularly when looking at incurred loss ratios we have found that the development can oscillate violently. An advantage of the system is that since it presents this in visual form it can be discussed with the underwriter. The most common explanations we have found for odd patterns are:

(a) Miscoding of data either by currency or category

(b) Data corrections that have not been carried back to the beginning of the account year

(c) Delays in reinsurance recoveries.

Thus the system is acting as a powerful check on the data.

In the particular example being used it appears likely that initially some claims for 1978 development year 7 and 1980 development year 5 have been coded as 1979 development year 6 with the error not being fully corrected retrospectively.

Estimation of ULR's by "line of best fit"

We have so far analysed the run-off of one account year at a time. We now analyse the run-off by examining one development year at a time for all account years together. Thus we use all the information in the run-off triangle.

For example at development year 3 we have the following data:

Account year	Loss ratio at development year 3	Estimated ULR from previous curve fitting
	%	%
1973	53.1	91.0
1974	65.8	92.1
1975	50.3	75.7
1976	43.6	70.2
1977	46.2	70.0
1979	73.5	103.8
1980	40.4	69.6
1981	39.1	72.2

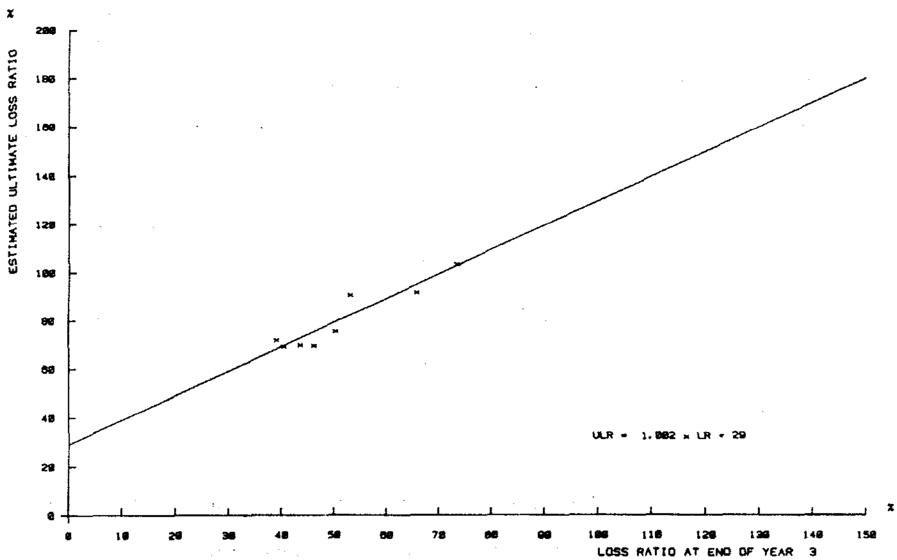
Account years 1971 and 1972 are omitted because the loss ratios for early development years are missing and 1978 is omitted because the run-off curve for that year seems to be a different shape from the other years.

The points are then plotted and the plot is examined to see if there is a statistically significant relationship between the loss ratio at development year 3 and the ULR. The method we use is to fit a regression line and test whether the gradient is significantly different from zero.

In this case the regression line is:

Estimated ULR = 1.002 x Year 3 Loss Ratio + 29.00%.

The fitted line is shown below, together with the 8 points to which it was fitted.



Account years fitted: 73, 74, 75, 76, 77, 79, 80, 81

To test if the gradient is significantly different from zero we use a t-test, with 2 degrees of freedom less than the number of points fitted. In this case we have $t_6 = 6.55$ which is significant at the 99% level. Thus the line is a good fit and the gradient is non zero, which supports the evidence available from inspection of the fitted line. As a general rule as well as applying the t-test one should also look at the graph of the relevant regression line to check that it appears reasonable to assume that the shape is significantly different from zero.

From the fitted line we can estimate the ULR for 1983 (where development year 3 is the latest known loss ratio) as:

$$1983 \text{ ULR} = 1.002 \times 39.57 + 29.00$$
$$= 68.65\%.$$

Since we have fitted a regression line we can also construct a confidence interval for this estimate of the ULR. There are two alternative methods, one empirical and the other mathematical.

The empirical method is to take the historical point furthest from the regression line and state that the true result for the year is unlikely to fall outside the historical maximum. This gives a likely variation of the result of $\pm$ 8.8% in this particular case.

The mathematical method is to derive the statistical confidence interval from the regression line fit. We have found that a 90% confidence interval does the right job for our analyses of individual portfolios. This gives a confidence interval in the example of $\pm$ 10.9%. Obviously the width of the confidence interval depends on where the point lies on the regression line.

The choice of method is a matter of personal preference. The advantage of the maximum deviation is the ease of presentation to the underwriter of the rationale for the range. The advantage of the second method is that it is statistically based and does allow properly for the number of points to which the line is fitted. It should be noted that underlying the second method as well as the t-test is the assumption that the underwriting results for different account years are independent identically distributed random variables. Such limited investigations as we have carried out suggest that this is a reasonable assumption.

We have found in a number of cases that the gradient of the regression line is found to be not significantly different from zero. This is particularly likely to be true for the most recent years of account. This implies that there is no correlation between the loss ratios at year 3 (say) and the ULR. In this case we would estimate the ULR as the average of the historic ULR's and obtain a confidence interval using the maximum deviation. In such a case it would obviously be desirable to adjust the ULR's to allow for changes in premium rates that may have taken place. However in the London Reinsurance Market the effects of changes in limits of cover etc. make this a very difficult exercise to carry out. The fact that no correlation exists also tells us something very useful about the data for that account year. It says that effectively there is no information in the data showing the development of the account year so far to indicate how the year will turn out ultimately in practice. Although this is a negative statement we feel that it is a fact that is often not fully appreciated by management, particularly with regard to long tail business. However in these cases it can usually be clearly demonstrated by the plots of loss ratios against ULR's that there is no relationship between the position at the end of the particular year of development and the ultimate outcome.

For our example the regression lines fitted for development years 2 to 10 together with the account years for which they are fitted are shown in Figures 1.14 to 1.22 of Appendix 1. Looking through the regression lines you will see how the fit gets better as the development year increases. When we reach the year of development where the "tail" of claims has effectively run off the loss ratio will equal the ULR. The regression line will pass through the origin of the graph and the slope of the line will be "1 in 1" i.e. 45%. You will see from Figure 1.22 that for the class of business being used for the example this position has almost been reached by the end of year 10. A summary of the lines fitted and the statistics is given in Table 1.23 of Appendix 1. From this you will see that for 1984 it is not appropriate to fit a regression line, since the t-test statistic is not significant at the 95% level. Thus for this year an average ULR was used as described above. It will be seen from Table 1.23 that the slopes of the regression lines range from about 0.7 to 1.5. The value of the slope can be interpreted as an indication of the "gearing" between the loss ratio at a particular development year and its ultimate value. Thus if the slope is greater than 1 this means that if you have a "bad" loss ratio at a particular point the year will ultimately be proportionately much worse than if you had a "good" loss ratio at the same point. If the slope is less than 1 the converse holds.

Final estimates of ULR.

In this example we consider that the estimates of ULR obtained from the curve fits are the appropriate ones to use for account years 1971 to 1977. Clearly for the early account years one can not use the regression lines to estimate the ULR's since the lines would be based on too few data items to be credible. For account years 1979 to 1984 the results from the line of best fit seem most suitable. As previously stated for 1978 the position is difficult because the shape is different from the other account years and we have therefore used the curve fit. Although no confidence interval can be calculated for this year it is obvious from looking at the curve fit that in order to convey the correct information to management that one should be quoted. This has arbitrarily been taken to be the same as 1979. We have on this occasion used 90% confidence intervals rather than maximum deviation intervals.

The final results of the analysis are set out in Table 1.24 in Appendix 1.

Further considerations

We have already mentioned how this approach suggests how much information about the ULR is contained in the development to date of the relevant account year. The other useful thing that we find comes out of this approach is that it shows senior management that the estimate of the ULR is just that - an estimate. Thus the actual result will be better or worse than that estimate. The confidence intervals give an indication to senior management of the range in which the result will in fact lie. If the reserving model is correctly specified then the confidence intervals will be accurate. In practice the model is probably not specified exactly correctly so the confidence intervals only give an indication of the likely range of possible outcomes. Despite this proviso the confidence intervals do enable the management to assess the implications of establishing reserves based on particular estimates of the ULR. The closer to the upper limit of the ULR that the reserve is established the more likely it is that in practice the reserve will turn out to be more than adequate and the excess may be released as a profit in the future. The nearer to the lower limit of the range of the ULR that the reserve is established the more likely that the reserve will turn out to be inadequate. That would mean that additional cash would have to be found in the future either by restricting dividend payments or raising new capital.

5. FURTHER DETAIL ON THE RESERVING METHOD

In this section we consider some of the practical problems that arise from using the approach to reserving discussed in the preceding two sections and describe some of the methods we have used to overcome these problems. Although a few of these problems and solutions were mentioned in the previous section we have covered all of these in this section for completeness.

Problems encountered with curve fitting

The exponential curve we fit has 3 parameters A, B and C. Initially for each account year we fit the curve allowing all 3 parameters to vary. This is because a free fit allows the curve to reflect the data as accurately as possible given the constraints of the curve. Further a free fit permits the curves to reflect any lack of homogenity in the data. Sometimes where there is an error in the data, or some other reason, one can find that for particular account years the fit to the early years of development is satisfactory but it is rather less good to the later years of development. In such cases we fix either B or C in order to try and make the curve fit the later years of development better at the expense of a worse fit in the earlier years of development. We prefer to fit C as this allows more freedom in the shape of the curve than fitting B. If we have to fix a parameter for a particular account year then if most of the other account years are fitting well on a free fit we would take the values of the parameters of those other years into account when deciding on the values of the parameters to be fixed. If the parameters B or C all take similar values then it is clear that all the account years are fairly homogenous so the choice of B or C is straightforward. In other cases it is less clear cut. If there is an obvious trend in the parameters then that can be reflected in the choice of the values for the parameters for the account years for which the parameters have to be fixed. If there is no obvious trend then it may be possible to obtain from the underwriter an indication of the relative lengths of the tails of the various account years. That judgement can then be incorporated in fixing a value of B or C for a particular account year. Alternatively, we would take into account the values of the parameters we have found suitable for similar classes of business either for other companies or on an industry wide basis. The point to be emphasised is that by fixing or not fixing some of the parameters as considered appropriate one can allow for any homogenity or lack of it in the data and also incorporate additional outside information.

As already mentioned we do not fit curves to recent account years since for such years there is insufficient development to permit a curve to be fitted. For longer tail categories we usually omit the first 8 to 12 quarters of data in fitting the curve to ensure that the fit is reasonable to the later development. This also solves the problem that for some of the earlier account years this early development can be missing from the data. Finally we sometimes find that the curve is approaching the value of A slowly so that A is probably too high an estimate of the ULR. In such cases we assume that the development is completed after a reasonable period, say 15 to 20 years for the longer tail classes, and take the value of L_t for that development period as the estimate of the ULR.

Problems encountered with "line of best fit"

One important problem that is often encountered is where a particular account year has a significantly different speed of development from all the other account years for that class. This may be due for example to writing a peculiar treaty or treaties in that year. That such a thing is happening is usually clear from the graphs of the curves and the reason can often be found from discussion with the underwriter. In these cases that account year is omitted from the calculation of the line of best fit. A good example of this was the omission of account year 1978 from the calculation of the lines of best fit in the previous section.

Another problem is where the data is very variable particularly in the early years of development so that there are significant random fluctuations on top of the basic run off pattern. In this case we have found that it is better to use the developed loss ratios obtained from the fitted curves rather than the actual values. This smooths out the random fluctuations which one may consider are not being repeated in the account year for which one is using the line of best fit to calculate a ULR. Alternatively the data for early years of development for some account years may be missing and using the modelled data will permit the inclusion of those years in the calculation of the line of best fit. Because of the smoothing that takes place with modelled data it will be found that the confidence intervals are narrower than those brought out by using the unadjusted data. They should therefore either be quoted with a cautionary note that they underestimate the true amount of fluctuation or not quoted at all.

It is interesting comparing the line of best fit approach with the approach using development factors. The development factor approach is equivalent to fitting a line for ULR against developed loss ratio that passes through the origin. Our experience is that for early years of development the lines of best fit often miss the origin by a wide margin. However as one progresses to the lines of best fit for the later development years they become closer and closer to lines through the origin. If in looking at some lines of best fit we do not see this pattern then this suggests that something is awry. The most probable reason is an error in the data.

As will be apparent from the example and the above discussion the method is not an automatic method for setting loss reserves. It requires one to use one's judgement at all stages of the process. In particular we have found that a careful study of the graphs of the curve fits and the linear regressions is very important in deciding upon an appropriate best estimate of the ULR and the accompanying confidence intervals. Although the method described uses a curve fitting approach to obtain the initial estimates of ULR's there is no reason why alternative methods, as for example described in the paper by J.R. Berquist and R.E. Sherman (3), should not be used to obtain these initial estimates. However we would emphasise that in practice we have found the curve fitting approach to be very flexible and more than adequate for calculating values of ULR's to use in the line of best fit. The alternative methods are found to be more necessary to assist in estimating the ULR's for the early account years where the line of best fit is not going to be used as part of the estimating process.

6. APPLICATION OF METHOD TO LLOYD'S

One important use of the method we have developed, and in fact one of the reasons for developing it, was to provide a new method for calculating the minimum reserves to be established by Lloyd's syndicates. This is described in considerable detail in the paper by S. Benjamin and L.M. Eagles (2) and we shall therefore give only a brief outline of the method for setting minimum reserves here.

The syndicates in Lloyd's are the bodies in Lloyd's equivalent to companies that underwrite the risks. Collectively the syndicates comprise Lloyd's. The syndicates each maintain their own statistics and also certain statistics are collected centrally. Among other things the central statistics are used to help set minimum levels of the reserves for each account year to be established by the syndicates.

The current method of setting minimum reserves is by the use of the "Lloyd's audit percentages" which are set by Lloyd's centrally. Under this present method percentages are supplied for use as at the end of each calendar year separately for each class of business and each account year in which business was written. The minimum reserve for claims outstanding and IBNR at the end of that calendar year for the class of business and account year is the premium advised to date multiplied by the relevant percentage. Thus the minimum level for the total claims expected to be paid by the syndicate is the claims paid to date plus the minimum reserve. Suppose under the present method the paid loss ratio to date is, say 10% and the audit percentage for the minimum reserve is 78%. Then under the present method we have

Paid Loss Ratio	=	10%
Reserve (Audit Percentage)	=	78%
(Implied) ULR	=	88%

It will be clear that this method does not reflect the progress of the individual syndicates.

Under the proposed new method two figures are used instead of one. In this particular case instead of 78% the two figures are 3.4 and 33% and the calculation is as follows:

ULR	=	3.4 x Paid Loss Ratio + 33%		
	=	3.4 x 10% + 33%	=	67%
Paid Loss Ratio	=			10%
(Implied) Reserve	=			57%

Thus two figures are provided for each class of business and account year for which currently one audit percentage is provided. The proposed new method has been tried on a limited experimental basis for three years. The evidence so far is favourable and the experiment is currently being widened to cover the whole market.

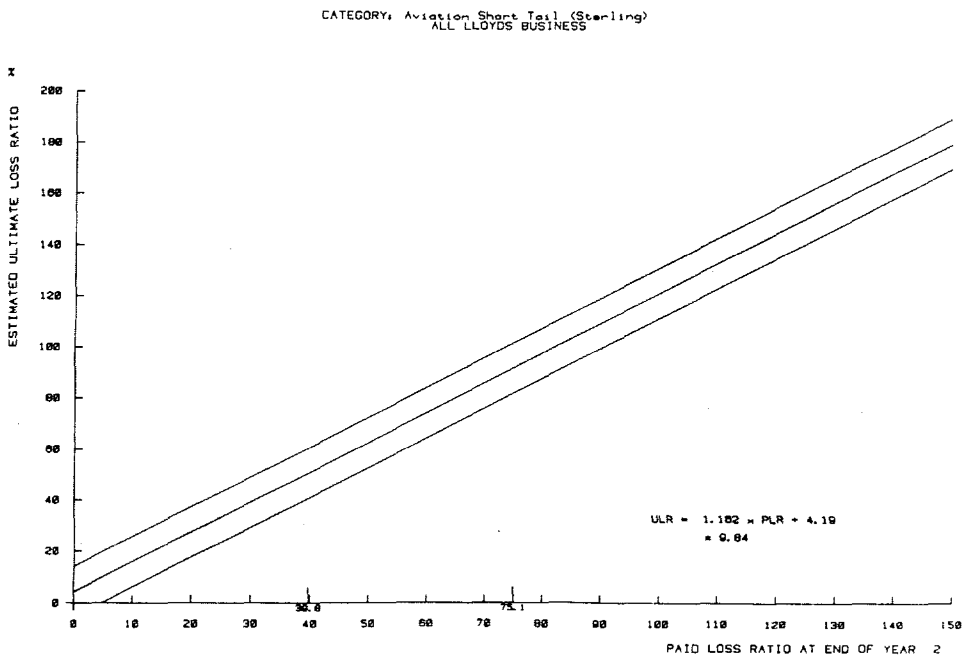
The two figures under the new method are calculated by applying the general method described in the preceding sections to the data collected centrally at Lloyd's for each class of business. For each class of business if one carries out that process one produces for each account year or year of development a line of best fit, together with an associated confidence interval, based on the point furthest from the line of best fit. The two numbers under the proposed method are the parameters that define the line of best fit. Thus in the example 3.4 gives the slope of the line and 33% its intercept on the vertical axis. There was considerable discussion inside the working party which reported to the Audit Committee as to whether the line of best fit or one of the other lines should be used to set minimum reserves. In the end the upper edge of the confidence interval seemed too high, the lower too low. The use of the line of best fit as a minimum allowed one to say that the total reserves set up in Lloyd's were at least as great as the average indicated by past experience, which seemed to be a useful statement to make. Underlying this approach to setting reserves is the assumption that for any class of business the business written by a syndicate will be similar to that "written" by all of Lloyd's combined. Incorporating the paid loss ratio in the calculation of the ULR in the way proposed then allows the quality of the business written by a particular syndicate to be reflected in the ULR in what appears intuitively to be a reasonable way. Also the new method would be easy to implement requiring very little change by individual syndicates in the work they carry out.

In addition to being provided with the new figures for calculating the minimum reserves the syndicates are also provided with graphs for each class of business and year of development showing:

(i) The lines of best fit together with the lines based on the point furthest from the line of best fit

(ii) The historic range of paid loss ratios.

Thus the syndicates are provided with graphs looking like this



The syndicates are being encouraged to plot their own data on the graphs to see how their experience compares with that of all of Lloyd's combined. It is hoped that as a result they will obtain useful information about their experience. For example if a syndicate's own path was narrow and different from the all-Lloyd's path then that would demonstrate in a very vivid way that it was writing a different class of business.

Clearly this approach can be adopted by any supervisory authority which wishes to set reserving standards for companies where global general market data of run-offs for the different classes of business is available.

REFERENCES

(1) CRAIGHEAD D.H. (1979) "Some aspects of the London reinsurance market in worldwide short-term business" J.I.A. Vol. 106 Part III

(2) BENJAMIN S. and EAGLES L.M. (1986) "Reserves in Lloyd's and the London Market" J.I.A. Vol. 113 Part II

(3) BERQUIST J.R. and SHERMAN R.E. (1978) "Loss Reserve Adequacy Testing: A Comprehensive, Systematic Approach" P.C.A.S. Vol. LXV

APPENDIX 1

DATA AND OUTPUT FOR WORKED EXAMPLE

Table 1.1

Estimation of Ultimate Premiums

Development Quarter 2

Development Year	Account Year: 1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984
1				2,706	3,714	3,751	5,550	6,580	6,774	9,098	12,214	11,611	15,541	20,082
2			3,524	4,489	5,869	6,439	8,475	9,712	9,797	13,173	17,839	17,901	23,250	29,902
3		3,355	3,924	4,821	6,393	7,109	8,800	10,083	10,670	14,613	19,927	19,322	25,602	32,928
4	5,189	3,373	4,040	4,876	6,473	7,067	8,894	10,142	10,978	15,123	20,570	19,967	26,457	34,027
5	5,240	3,415	3,999	4,928	6,521	7,081	8,942	10,161	11,035	15,356	20,887	20,275	26,865	34,551
6	5,126	3,432	4,027	4,894	6,557	7,065	8,981	10,250	11,147					
7	5,279	3,449	4,024	4,911	6,570	7,091	9,006	10,329						
8	5,297	3,446	4,040	4,917	6,592	7,046	9,030							
9	5,300	3,452	4,035	4,896	6,580	7,070								
10	5,301	3,454	4,036	4,898	6,585									
11	5,288	3,455	4,037	4,894										
12	5,286	3,476	4,027											
13	5,284	3,474												
14	5,284													

Table 1.2

Loss Ratios

Quarter of Development	Account Year: 1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984
1					0.1	-0.6	1.2	0.1	1.0	0.0	0.1	0.0	0.1	0.9
2					0.8	-0.4	2.3	0.3	2.1	1.0	0.4	3.0	2.0	2.3
3					2.9	4.5	5.0	1.8	7.1	4.2	1.4	7.0	5.0	5.0
4				7.7	7.0	5.1	8.1	3.4	9.6	12.1	6.5	8.3	12.2	11.6
5				14.5	18.7	12.0	12.4	7.3	17.7	2.0	9.3	8.6	17.1	12.1
6				25.5	22.5	15.7	21.1	12.5	21.4	10.1	17.2	11.3	20.7	23.0
7				40.0	30.8	22.6	30.0	19.6	31.2	20.7	25.1	17.4	25.2	
8			41.6	48.2	39.1	37.1	35.6	26.0	63.1	29.9	30.7	25.4	29.6	
9			47.7	60.8	45.6	38.5	42.0	38.8	64.6	32.9	36.5	3.08	34.5	
10			53.1	65.8	50.3	43.6	46.2	40.7	73.5	40.4	39.1	33.5	39.6	
11			59.9	71.6	55.9	48.2	49.2	40.8	77.3	47.2	45.2	40.3		
12		55.9	64.9	76.0	61.5	52.3	51.0	46.8	75.5	44.2	47.8	43.8		
13		59.4	67.8	76.7	64.2	56.4	51.3	39.9	89.6	47.7	49.9	46.1		
14		61.9	73.5	79.1	67.4	58.3	52.9	41.9	87.2	50.5	53.2	47.5		
15		63.7	76.4	82.4	70.2	59.7	55.8	43.3	92.2	53.3	55.4			
16	63.2	65.2	78.3	83.4	71.4	60.9	60.7	43.6	92.6	55.7	56.6			
17	64.4	69.5	80.7	86.6	72.7	61.5	63.3	46.6	95.9	56.3	58.5			
18	65.5	69.1	81.4	85.0	72.6	64.3	65.4	48.8	96.2	58.2	60.6			
19	66.0	70.3	82.7	86.2	74.7	65.6	67.3	50.4	99.0	59.8				
20	66.5	71.1	84.1	86.6	75.5	67.3	68.7	51.7	107.3	60.5				
21	67.9	71.8	84.7	89.0	73.6	66.5	67.8	52.3	111.1	62.4				
22	68.4	72.4	87.2	89.2	72.9	67.2	69.5	58.1	117.7	63.8				
23	68.7	73.9	88.2	89.7	73.3	67.3	70.1	61.4	96.7					
24	68.0	74.1	88.2	90.5	73.0	67.9	71.3	57.9	96.0					
25	68.4	75.4	88.9	91.6	72.8	68.6	72.4	56.1	92.4					
26	68.6	75.6	89.8	92.5	73.8	69.1	72.1	55.0	94.0					
27	68.8	75.6	90.4	92.7	73.8	70.2	70.2	79.9						
28	69.9	75.6	91.0	93.1	74.0	71.4	69.7	82.0						
29	68.8	75.2	91.0	93.6	75.3	70.5	69.0	82.6						
30	69.3	75.2	91.6	93.8	75.7	72.0	68.5	86.3						
31	69.3	75.8	91.9	94.5	75.8	71.5	69.1							
32	69.2	75.9	91.7	94.1	76.2	70.6	68.6							
33	69.5	76.7	91.5	92.3	75.0	68.4	66.2							
34	69.8	78.2	91.7	92.8	75.3	68.6	65.0							
35	69.9	78.4	91.6	92.9	76.2	68.9								
36	69.6	78.6	91.0	92.3	75.5	68.8								
37	69.1	77.7	91.6	91.9	74.4	68.7								
38	68.9	77.8	91.6	91.9	74.4	69.8								
39	69.1	77.9	91.9	91.6	74.5									
40	69.1	78.6	91.6	91.6	75.1									
41	68.6	79.6	90.4	90.6	75.7									
42	68.7	79.9	90.4	90.6	75.7									
43	68.7	79.9	90.5	90.7										
44	68.6	79.9	90.6	90.7										
45	68.6	79.6	90.3	90.7										
46	69.3	79.6	90.3	90.7										
47	69.3	79.6	90.3											
48	69.5	79.6	90.0											
49	69.3	80.2	88.8											
50	69.3	81.1	88.8											
51	69.4	81.1												
52	69.4	81.3												
53	69.3	80.8												
54	69.5	80.8												
55	69.4													
56	69.4													
57	69.3													
58	69.4													

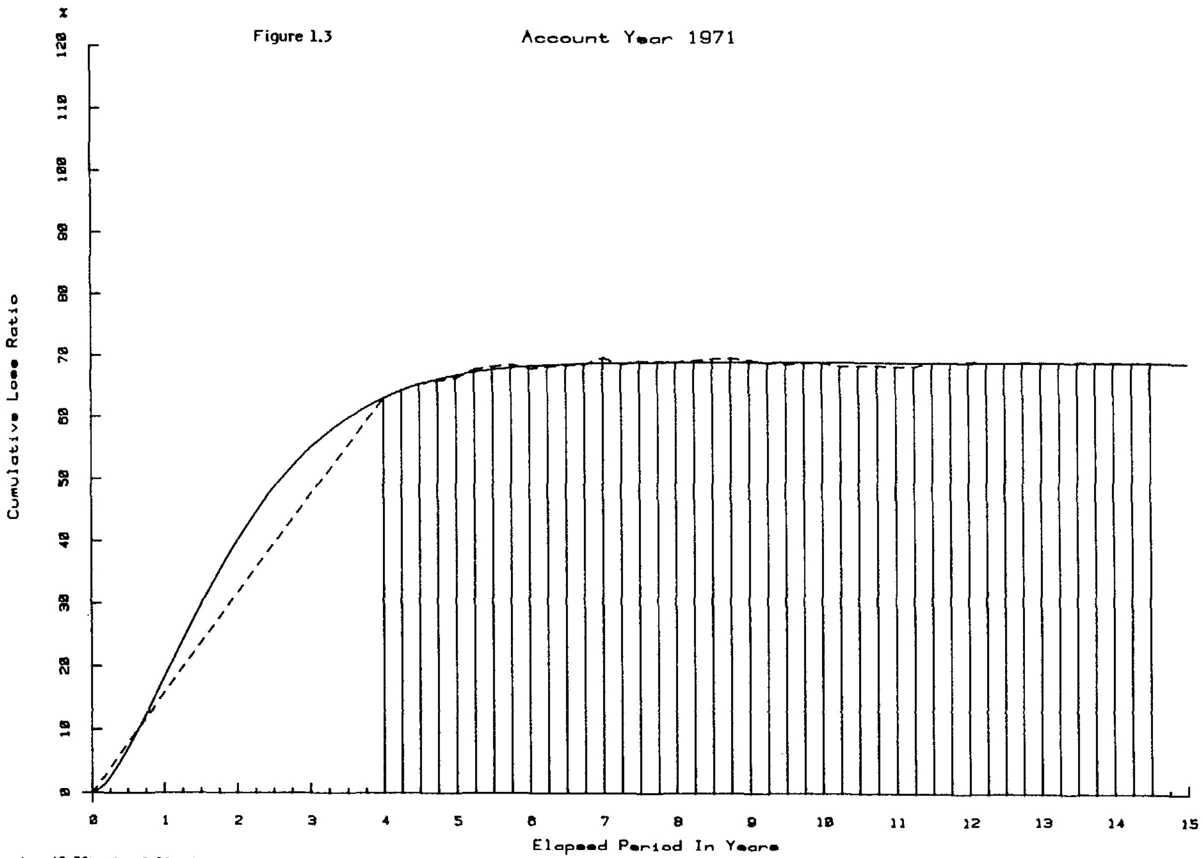
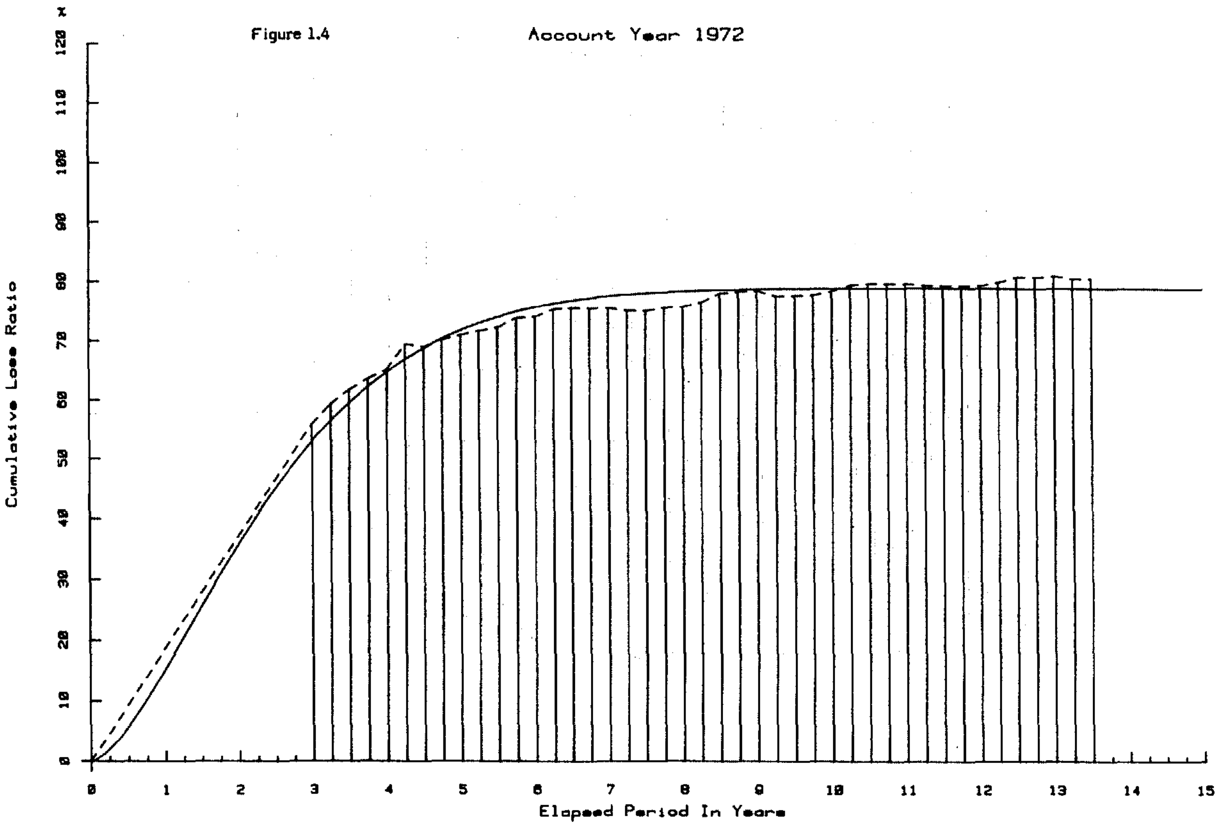


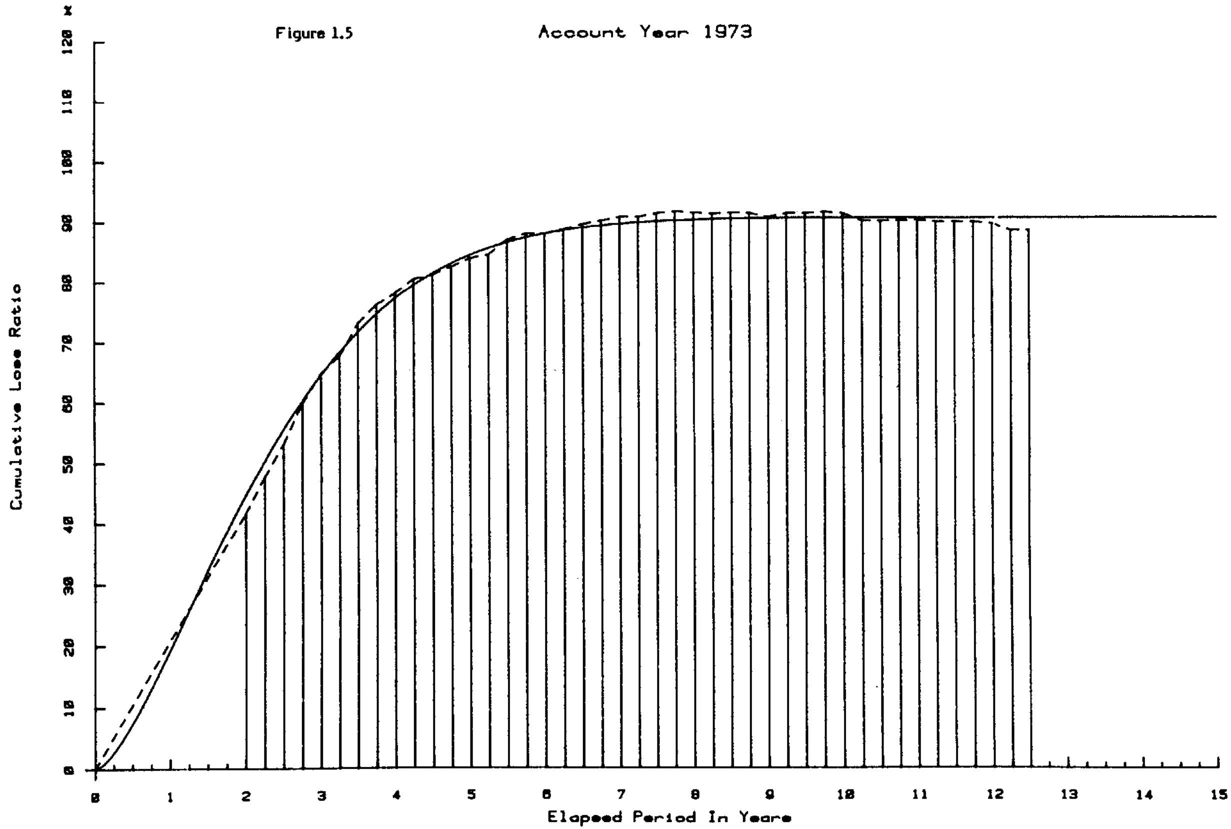
Figure 1.3

A = 69.3%, B = 2.21, C = 1.50, Mean squared error = 1.1

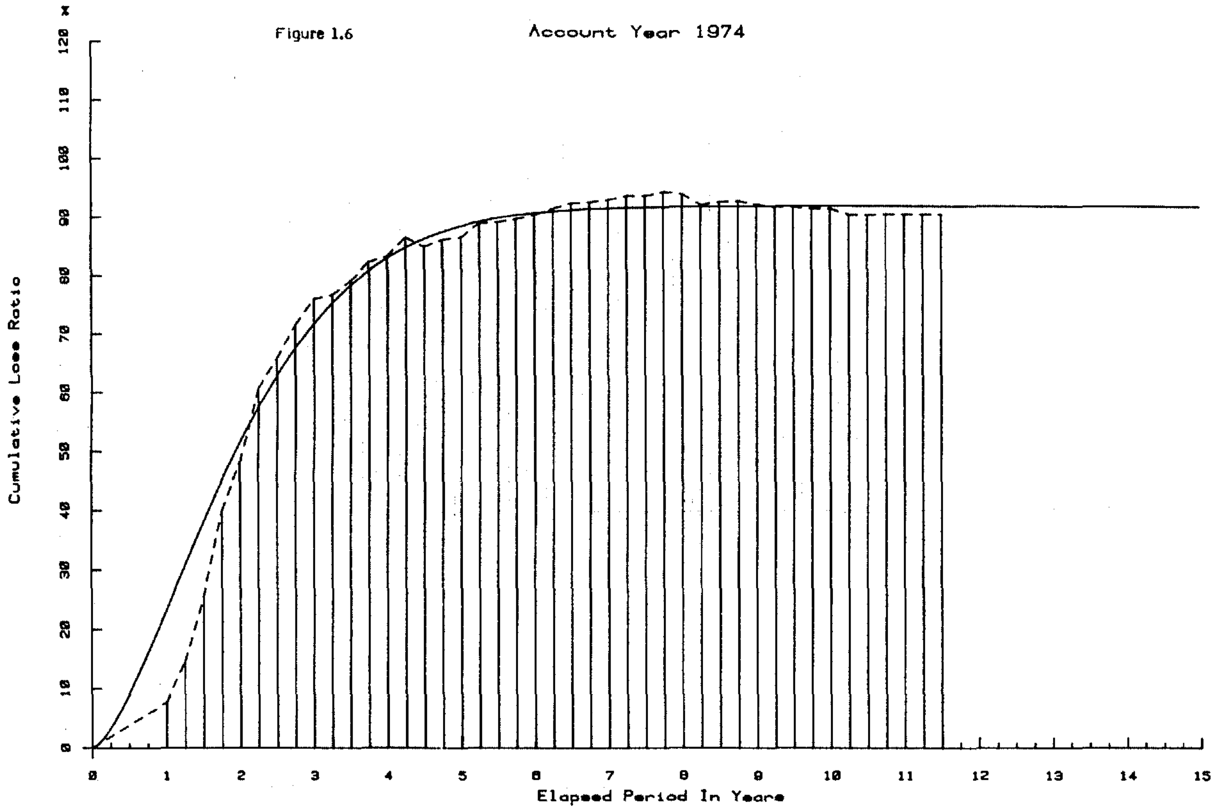


A = 79.2%, B = 2.80, C = 1.50, Mean squared error = 18.5

Figure 1.5

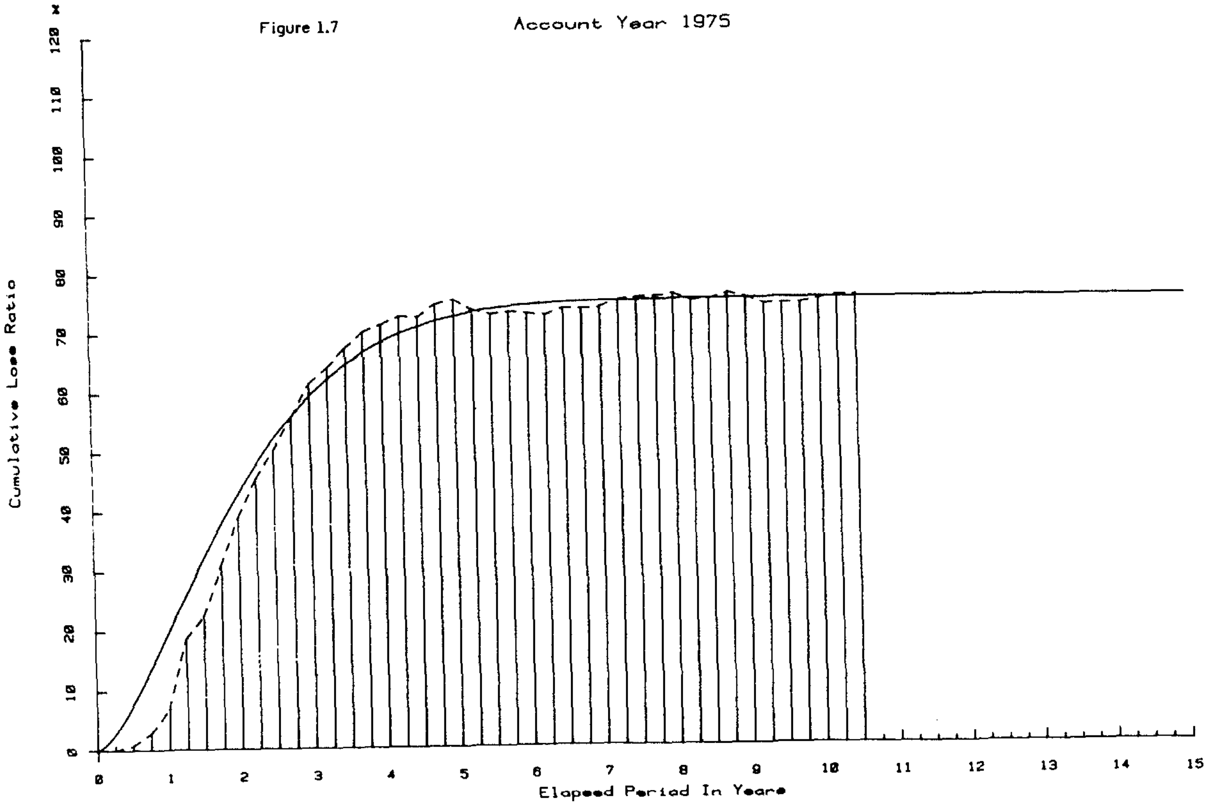


A = 91.0%, B = 2.60, C = 1.50, Mean squared error = 8.1

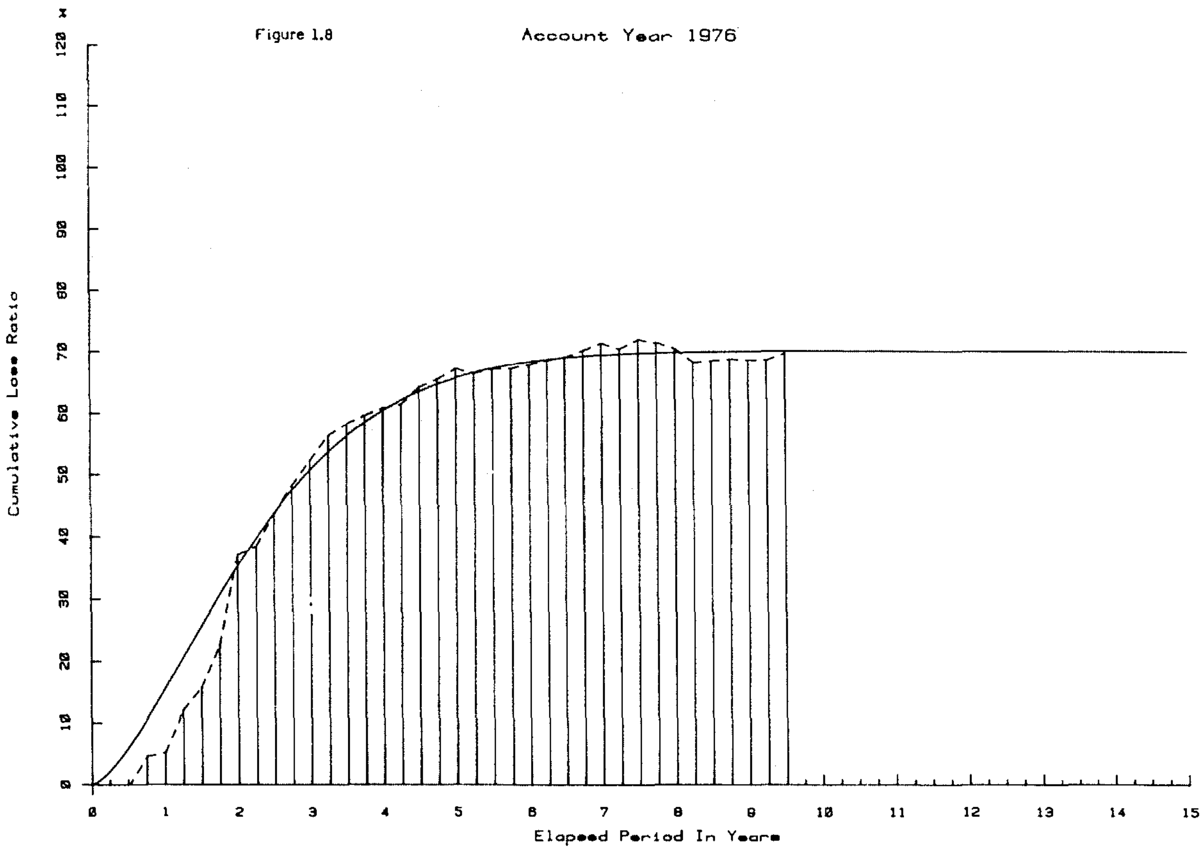


A = 92.1%, B = 2.28, C = 1.50, Mean squared error = 28.0

Figure 1.7

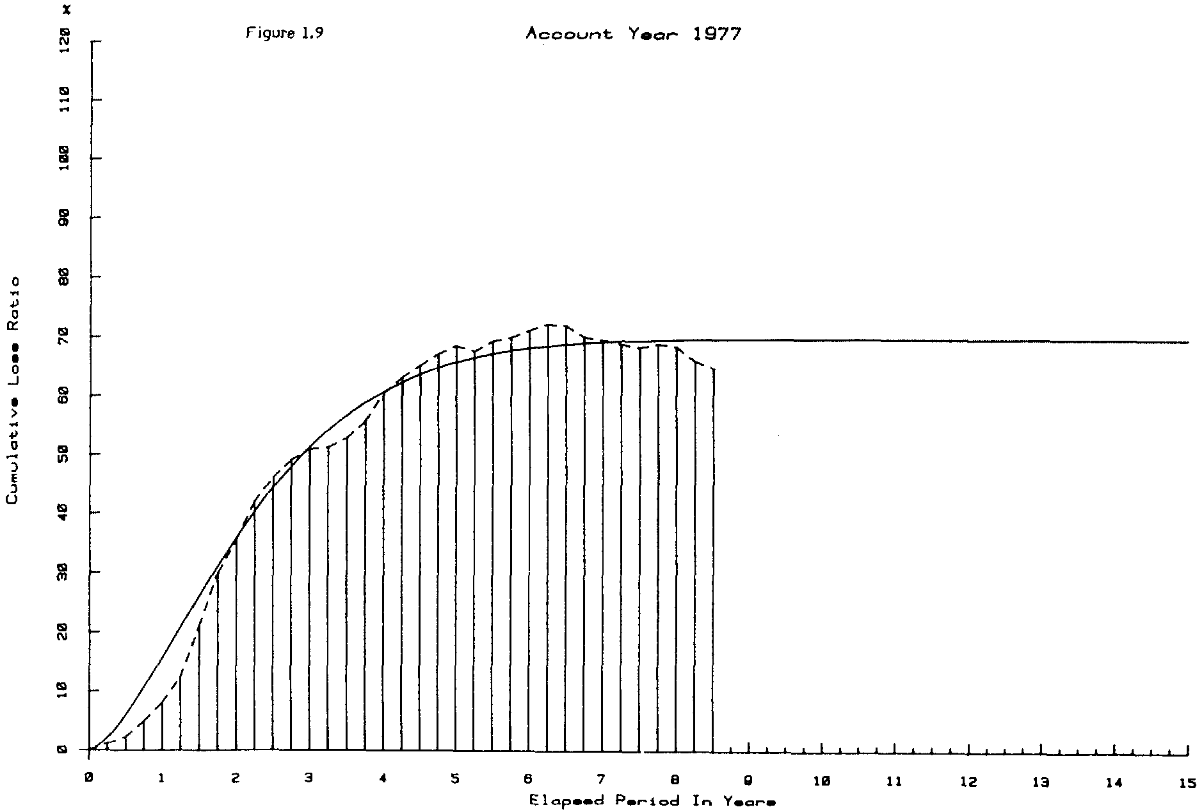


A = 75.4%, B = 2.24, C = 1.50, Mean squared error = 18.2



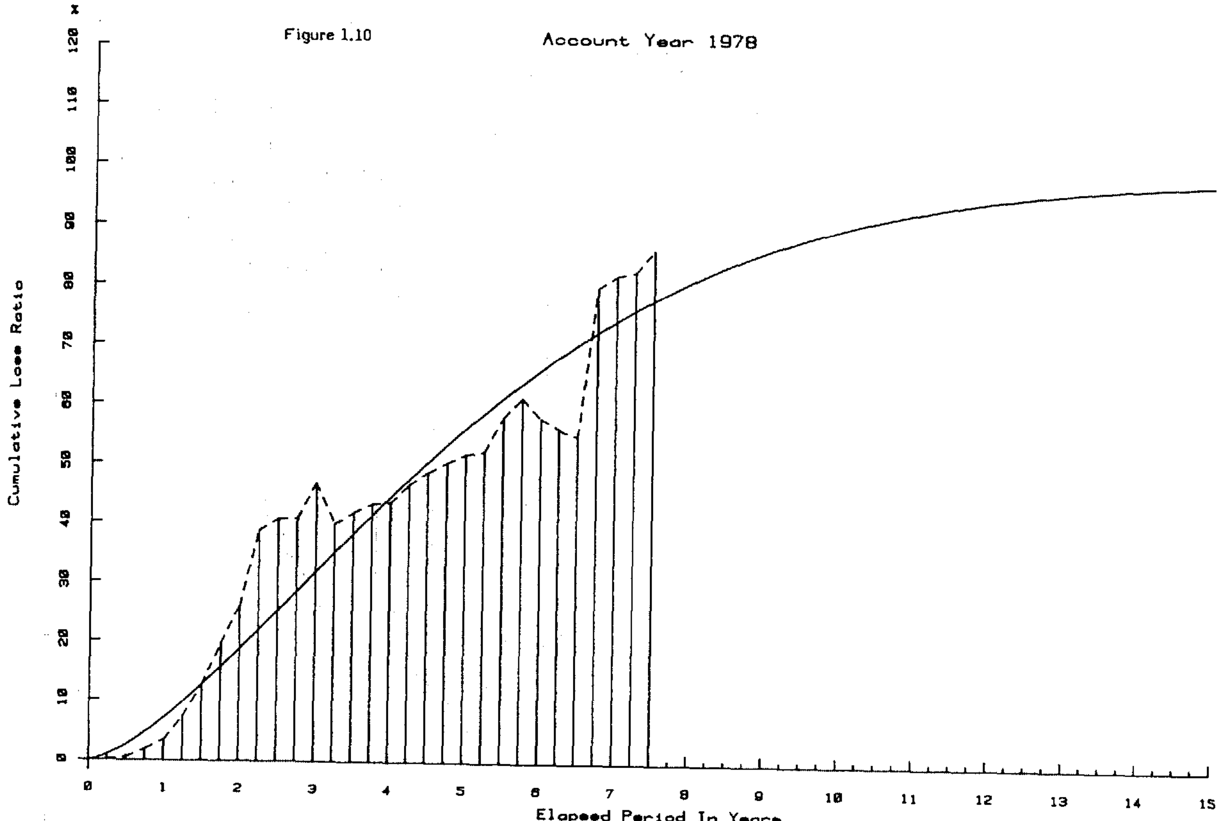
A = 70.2%, B = 2.54, C = 1.50, Mean squared error = 18.2

Figure 1.9 Account Year 1977

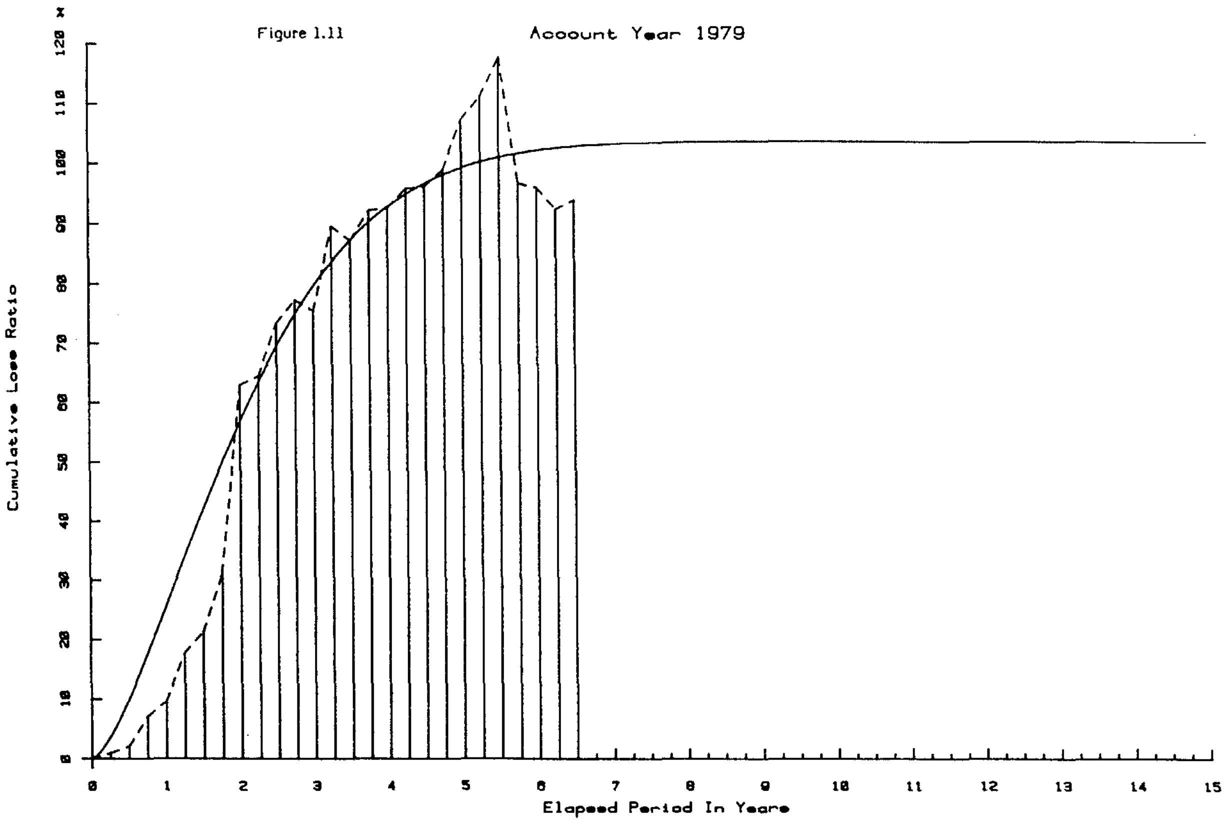


A = 70.0%, B = 2.49, C = 1.50, Mean squared error = 31.8

Figure 1.10

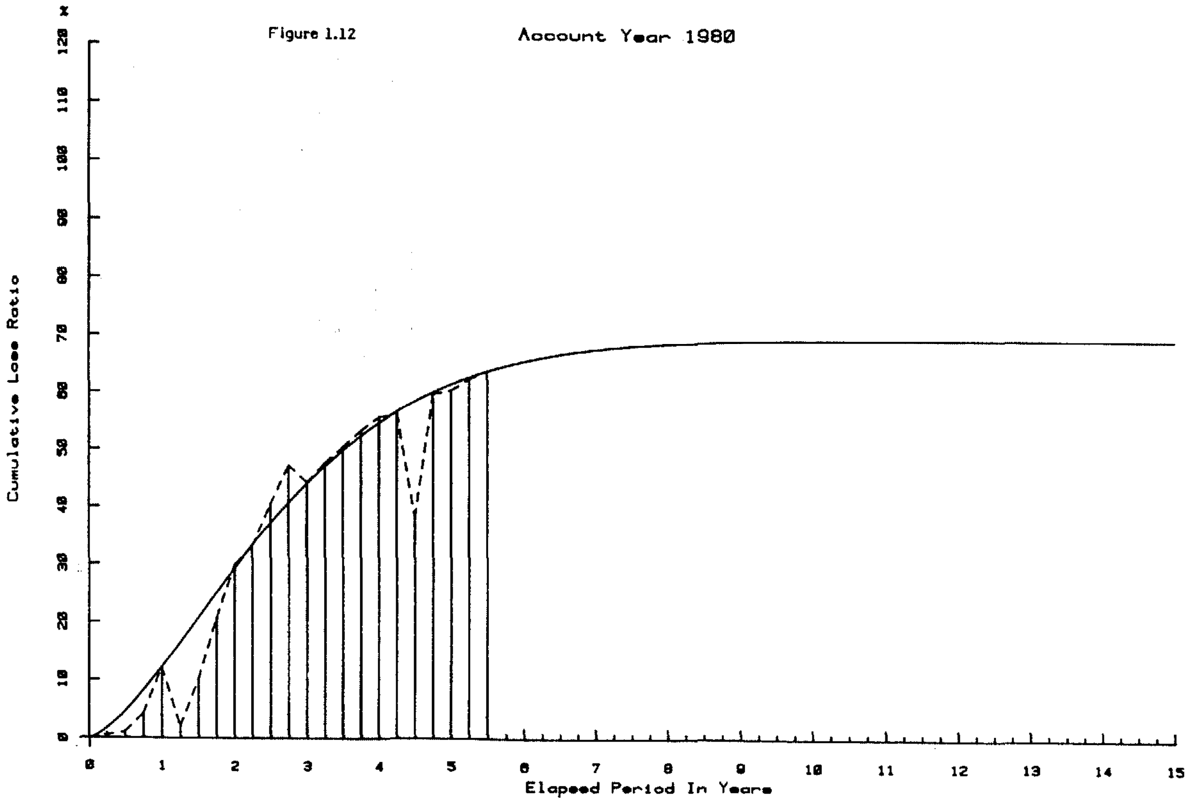


A = 99.9%, B = 5.69, C = 1.50, Mean squared error = 289.6



A = 103.8%, B = 2.33, C = 1.50, Mean squared error = 265.3

Figure 1.12 Account Year 1980



A = 69.6%, B = 3.00, C = 1.50, Mean squared error = 34.7

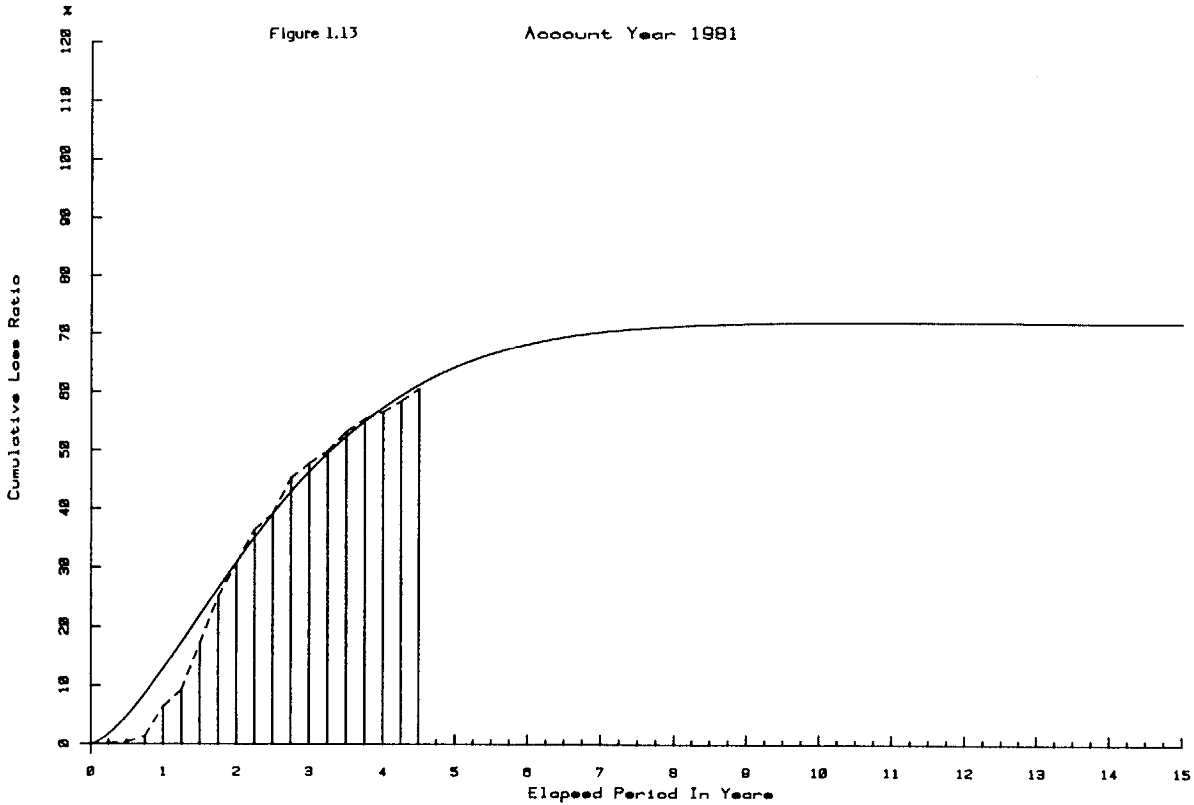
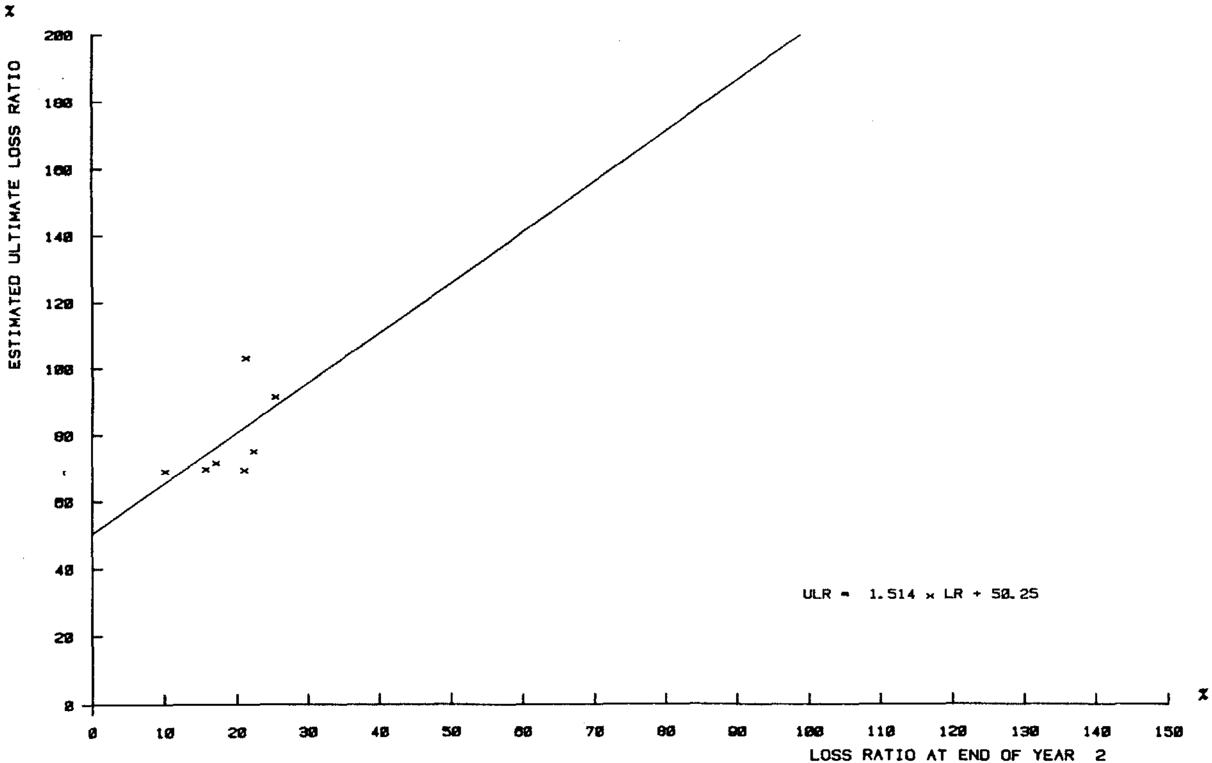


Figure 1.13 Account Year 1981

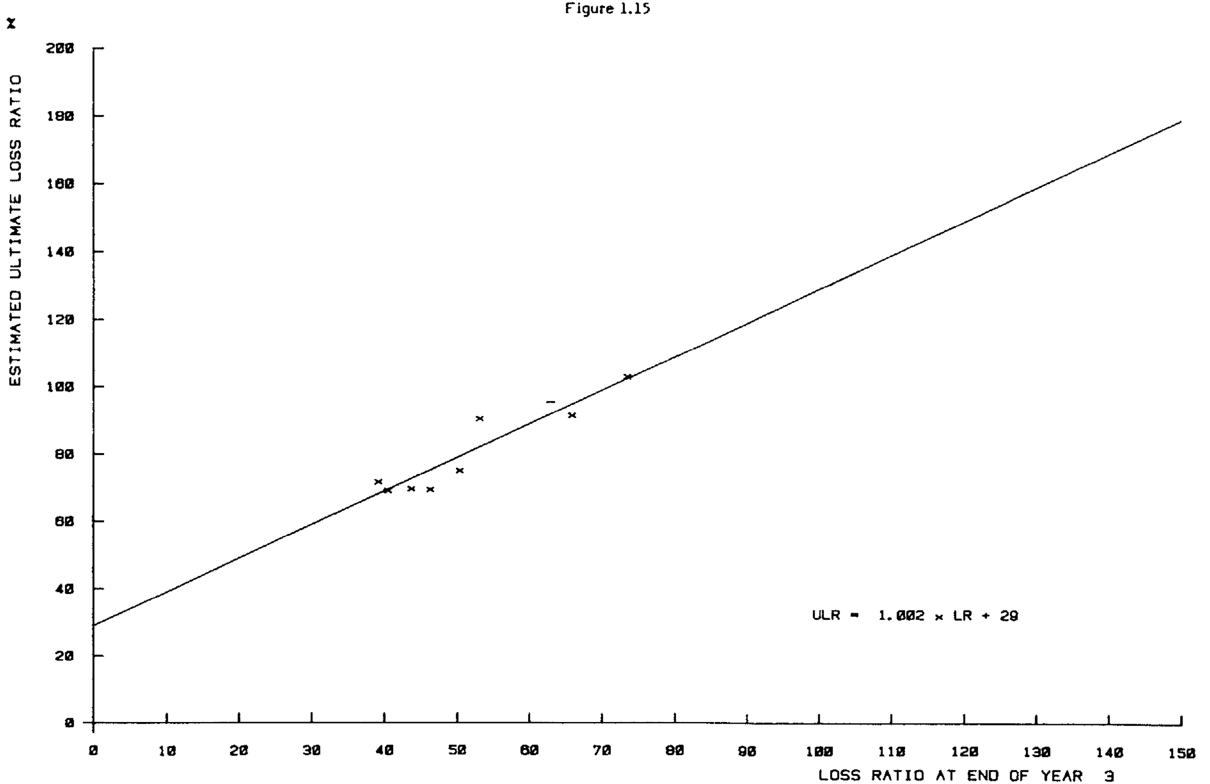
A = 72.2%, B = 2.95, C = 1.50, Mean squared error = 11.2

Figure 1.14



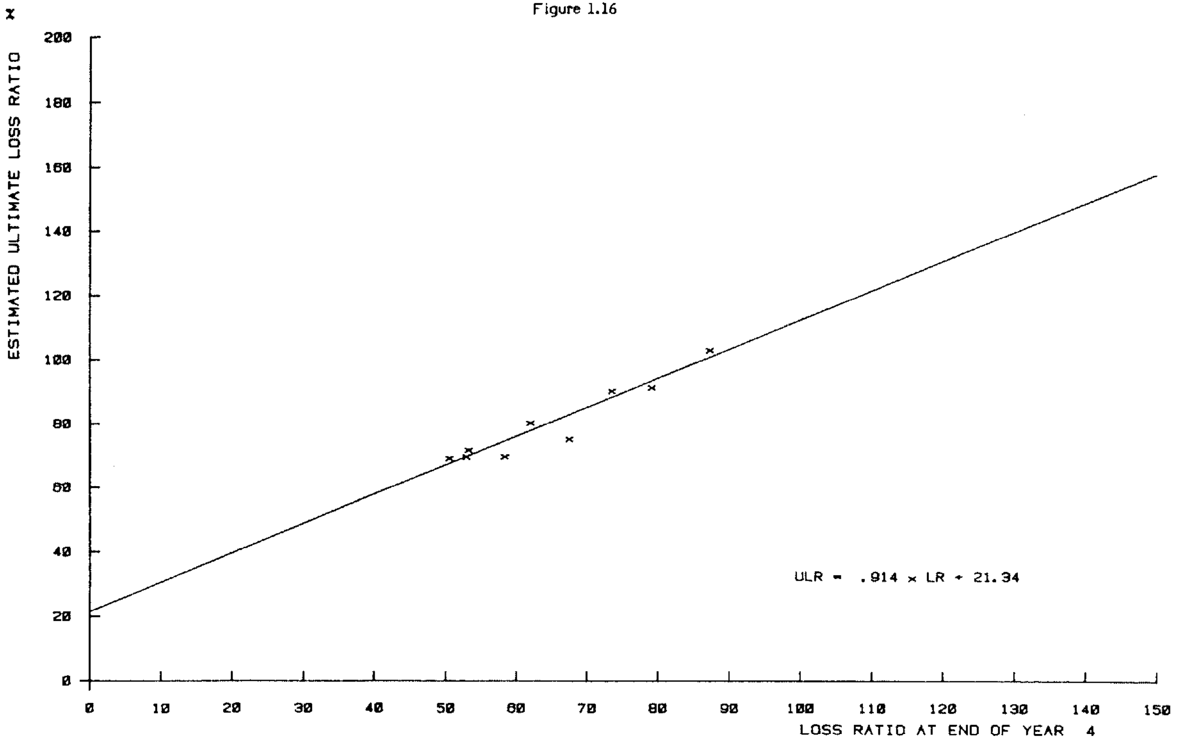
Account years fitted: 74, 75, 76, 77,79, 80, 81

Figure 1.15



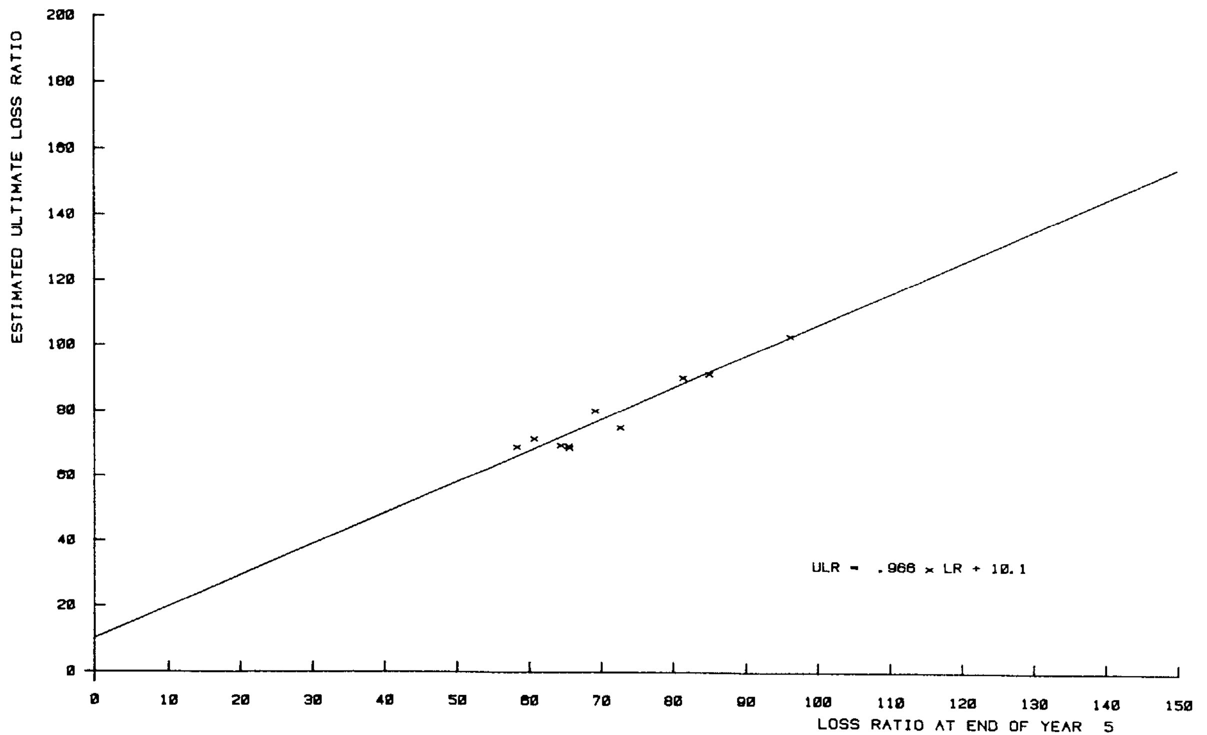
Account years fitted: 73, 74, 75, 76, 77, 79, 80, 81

Figure 1.16



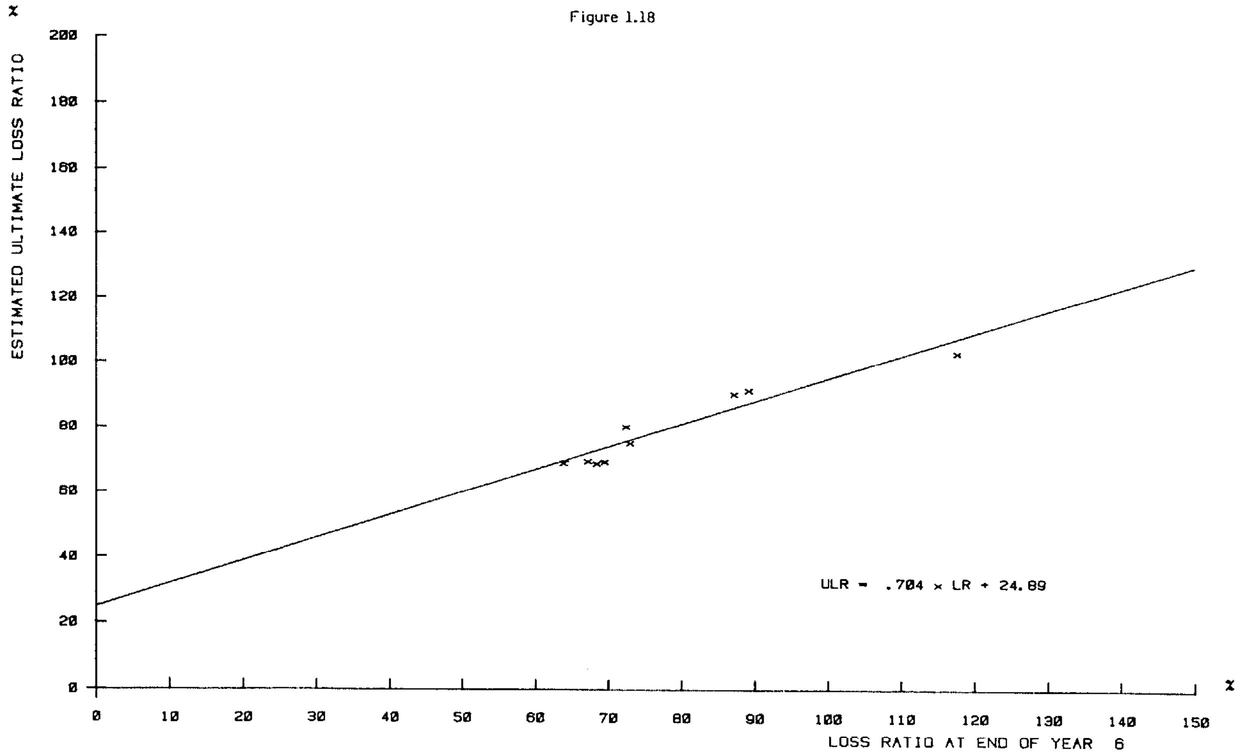
Account years fitted: 72, 73, 74, 75, 76, 77, 79, 80, 81

Figure 1.17



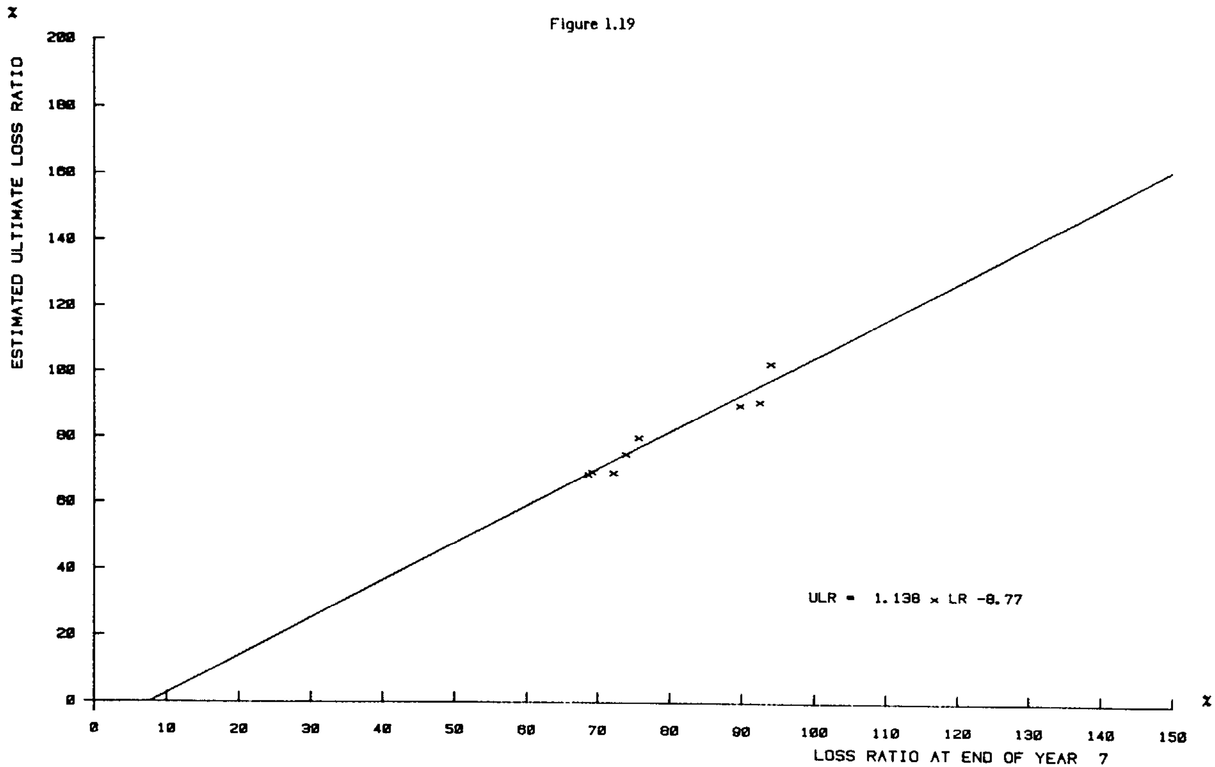
Account years fitted: 71, 72, 73, 74, 75, 76, 77, 79, 80, 81

Figure 1.18



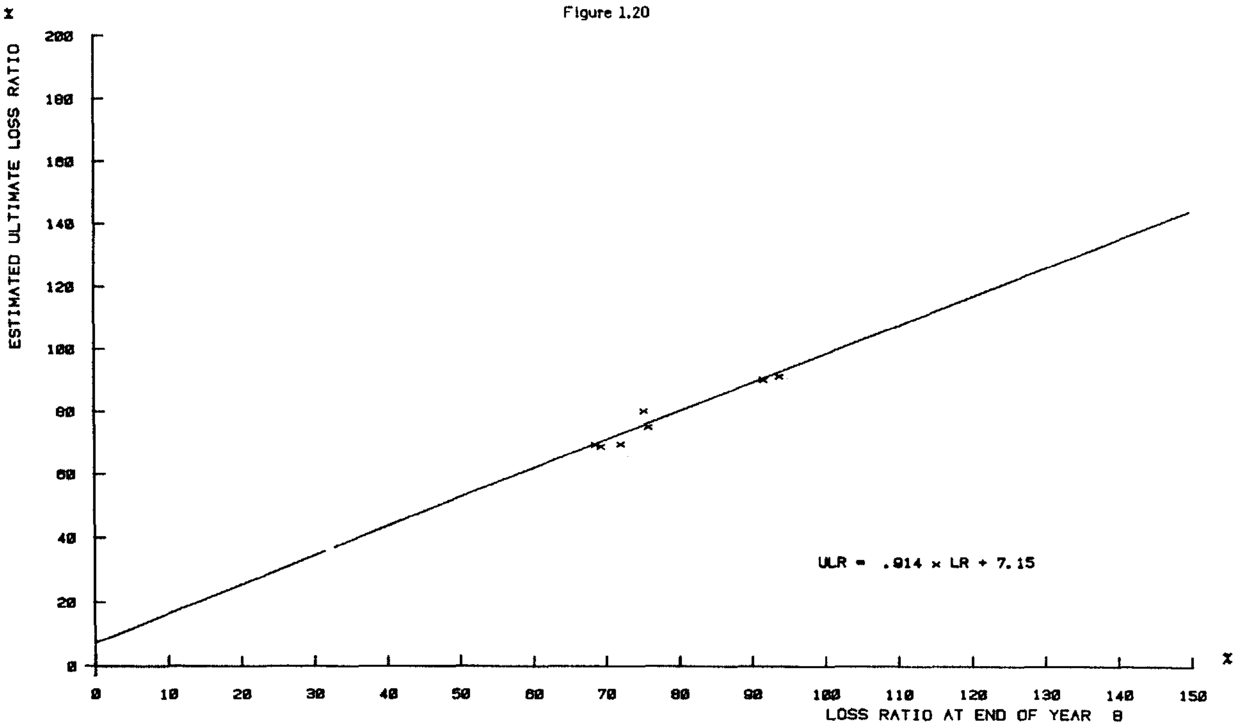
Account years fitted: 71, 72, 73, 74, 75, 76, 77, 79, 80

Figure 1.19



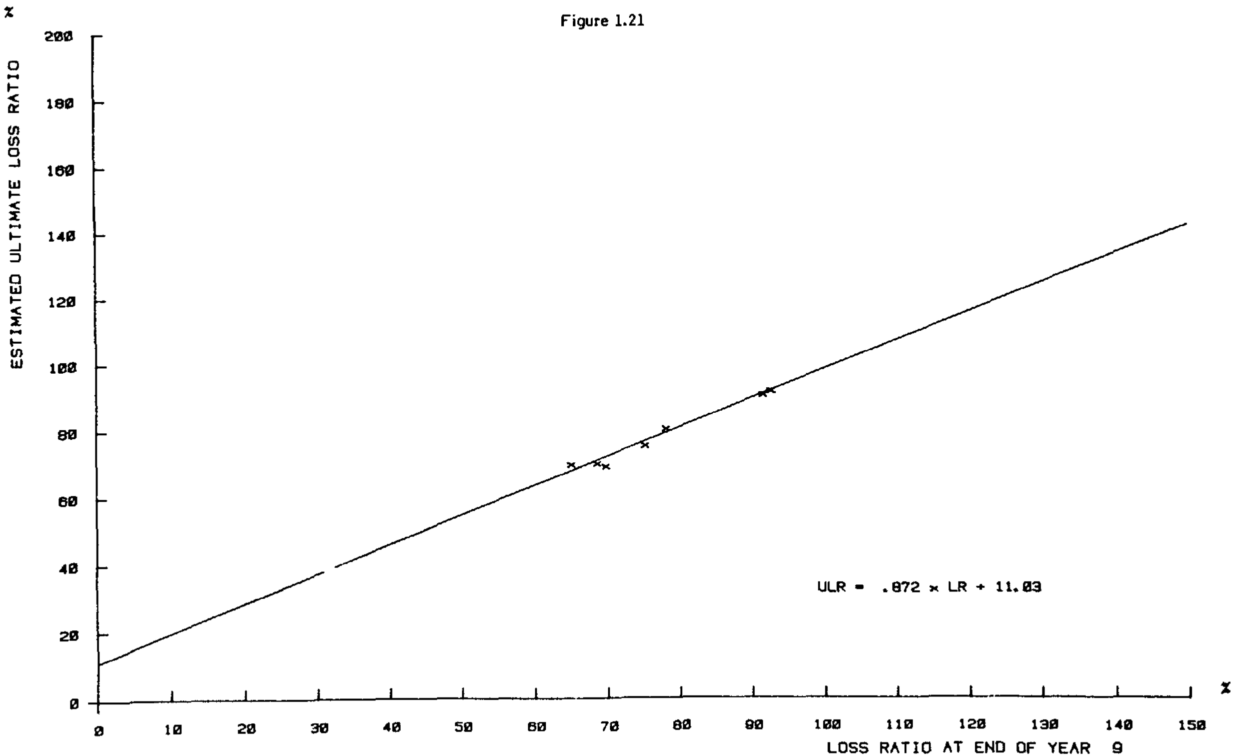
Account years fitted: 71, 72, 73, 74, 75, 76, 77, 79

Figure 1.20



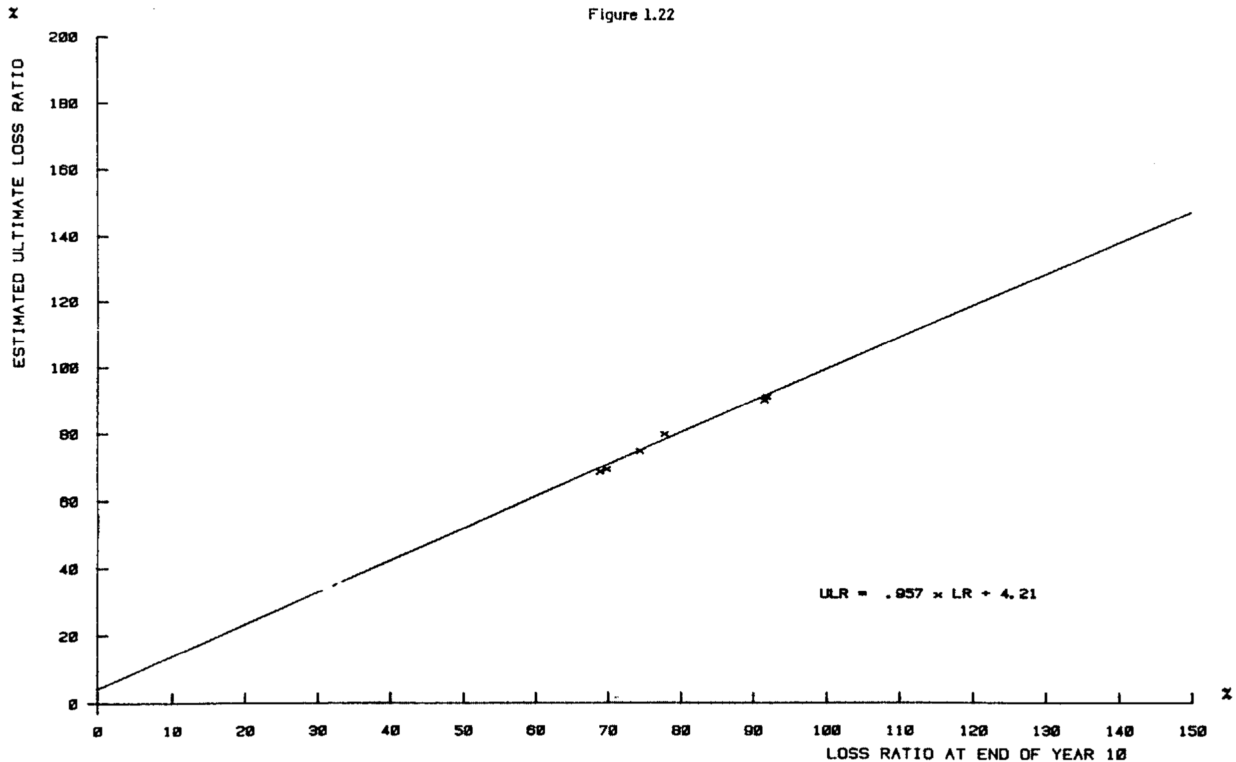
Account years fitted: 71, 72, 73, 74, 75, 76, 77

Figure 1.21



Account years fitted: 71, 72, 73, 74, 75, 76, 77

Figure 1.22



Account years fitted: 71, 72, 73, 74, 75, 76

Table 1.23

Summary of regression lines fitted

Account year	Corresponding development year	Regression line: Slope	Constant	t-test statistic: Value	Degrees of freedom
1984	2	1.514	50.25	1.58	5
1983	3	1.002	29.00	6.55	6
1982	4	.914	21.34	8.54	7
1981	5	.966	10.10	10.11	8
1980	6	.704	24.89	8.41	7
1979	7	1.138	-8.77	8.85	6
1978	8	.914	7.15	9.00	5
1977	9	.872	11.03	13.41	5
1976	10	.957	4.21	16.95	4

Account year	Latest loss ratio	Estimated ULR	Maximum deviation	90% confidence interval
	%	%	%	%
1984	23.05	85.15	21.15	27.07
1983	39.57	68.65	8.75	10.86
1982	47.48	64.74	7.23	8.54
1981	60.63	68.67	4.55	7.05
1980	63.75	69.77	4.98	8.41
1979	93.97	98.17	5.63	8.40
1978	86.30	86.03	4.89	5.83
1977	64.96	67.68	2.48	4.14
1976	69.84	71.05	2.17	3.19

Table 1.24

Recommended estimates of ULR

Account year	Loss ratio to date	Estimated ULR	Confidence interval (+ or -)
	%	%	%
1971	69.4	69.4	-
1972	80.8	80.8	-
1973	88.8	91.0	-
1974	90.7	92.1	-
1975	75.7	75.7	-
1976	69.8	70.2	-
1977	65.0	70.0	-
1978	86.3	103.8	8.4
1979	94.0	98.2	8.4
1980	63.8	69.8	8.4
1981	60.6	68.7	7.0
1982	47.5	64.7	8.5
1983	39.6	68.6	10.9
1984	23.0	85.1	27.1

APPENDIX 2

EFFECT ON SHAPE OF CURVE $L_t = A \times [1 - \exp(-[t/B]^C)]$

OF CHANGING VALUES OF PARAMETERS B AND C

Figure 2.1

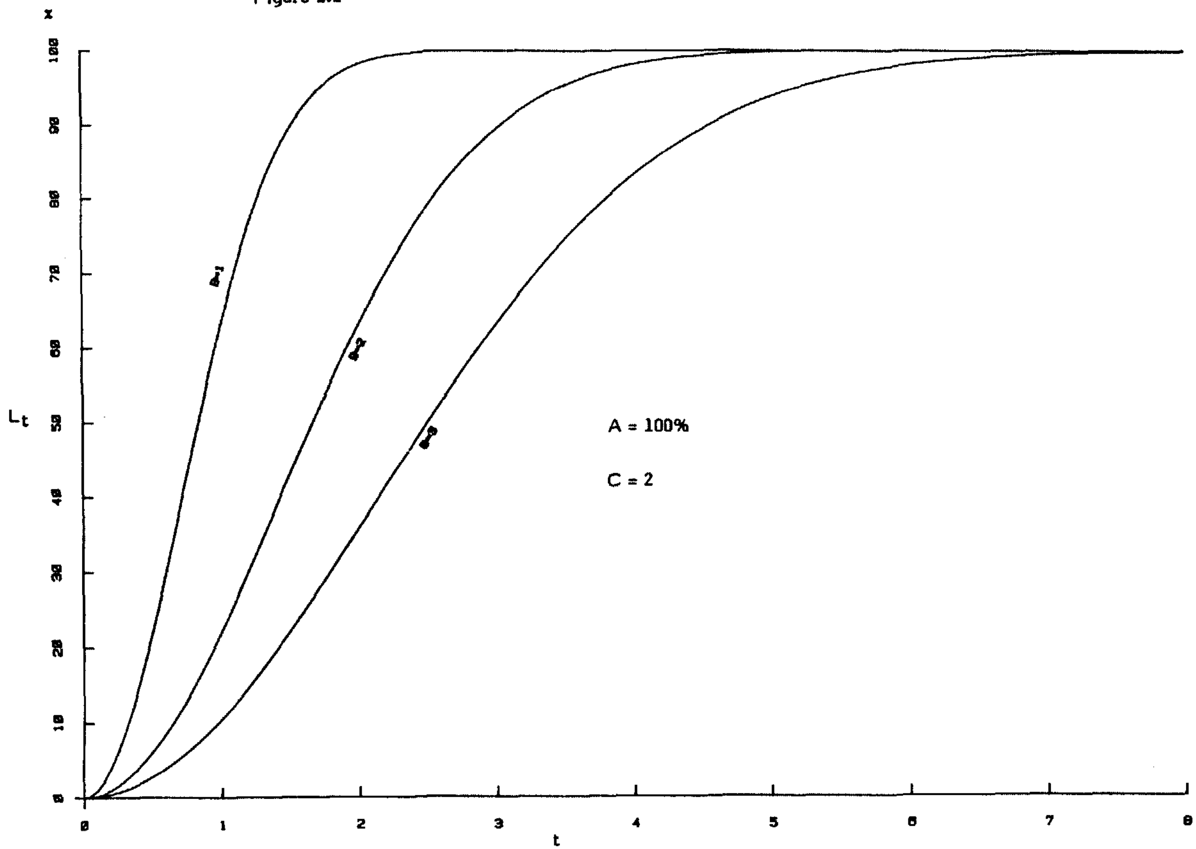
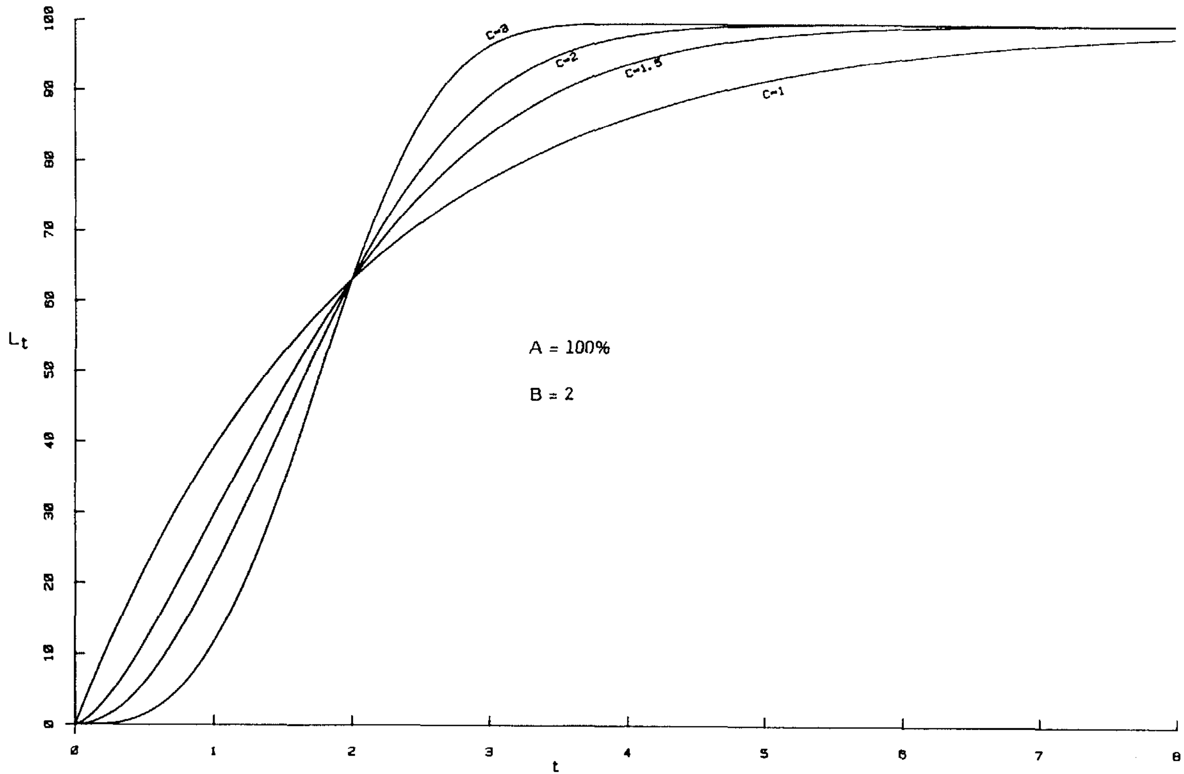


Figure 2.2



THE MATHEMATICS OF EXCESS OF LOSS COVERAGES AND RETROSPECTIVE RATING — A GRAPHICAL APPROACH

The Mathematics of Excess of Loss Coverages and Retrospective Rating - A Graphical Approach

Yoong-Sin Lee

Abstract

The mathematics of excess of loss coverages and retrospective rating involves heavy algebra, mainly because the indemnity payment under such contracts assumes different functional forms in different parts of the loss size. This paper presents a graphical approach to the theory, in which the indemnity payment under various conditions is represented by the areas of regions in a graph described by the cumulative distribution function of size of loss. Many intricate formulas and relations occurring in the two subjects, some expressible algebraically only in very complicated forms, can be understood simply and clearly through the pictures. Treated visually in this paper are many mathematical relations and results included in the examination syllabus.

1. INTRODUCTION

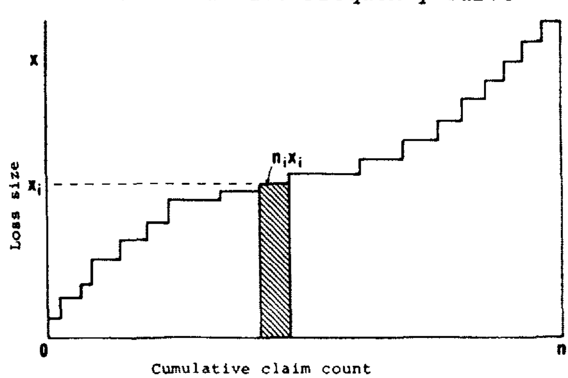
The theory of excess of loss coverages and retrospective rating involves rather complicated mathematics. The underlying ideas in most cases are relatively simple, but the heavy algebra is often a great mental burden to the actuary and the student. This paper applies a graphical technique to excess of loss coverages and retrospective rating. Most of the algebraic results on these topics are capable of being interpreted in terms of the graphs. The advantages of this approach are that the results so derived are, for most people, easier to understand and that formulas can be easily remembered and written down.

Graphical methods are widely used in mathematics and statistics to present visually ideas which would otherwise be abstruse. Many mathematical ideas have geometric as well as symbolic interpretation. For example, the integral of a positive-valued function can be regarded as the area under the curve representing the function as well as the antiderivative of the function. The use of diagrams and graphs to present numerical information in statistics is more well known. Graphs in statistics are also used to explain ideas such as density functions and cumulative distribution functions. In actuarial science graphical methods have not been extensively utilized. The graphical device we are going to present is for the explanation of the underlying mathematical ideas. It will not only provide powerful insight into the abstract relations, but also make the mathematical procedure much easier to follow compared with algebraic manipulations. For those who always prefer algebra, it will serve at least as a very useful supplement to the predominantly algebraic treatment that has been given to the subject in the literature.

To start with, consider a large number of losses, of sizes $x_1, x_2, \ldots, x_k$, occurring $n_1, n_2, \ldots, n_k$ times, respectively, with $n = n_1+\ldots+n_k$. In Figure 1 we represent these losses by means of a cumulative frequency curve, in which the abscissa represents the loss size, and the ordinate represents the cumulative loss $c_i = n_1+\ldots+n_i$, $i \leq k$. This representation is different from the usual form in statistical textbooks, where the abscissa and ordinate are reversed, but agrees with the representation in Snader. See also Philbrick (1985).

Figure 1

A Cumulative Frequency Curve



The curve is a step function (with argument along the vertical axis) which has a jump of n_i at the point x_i. Consider the shaded vertical strip in the graph. It has an area equal to $n_i x_i$. Summing all such vertical strips we have

$$\text{Total amount of loss} = n_1x_1+\ldots+n_kx_k$$

We may therefore interpret the area of the vertical strip corresponding to x_i as the amount of loss of size x_i, and the total enclosed area below the cumulative frequency curve as the total amount of loss. In fact, we have a new way of viewing the cumulative frequency function curve. This curve can be constructed by arranging the losses in ascending order of magnitude, and laying them from left to right with each loss occupying a unit horizontal length.

Now let X be a random variable representing the amount of loss incurred by a risk. Define the cumulative distribution function (cdf) $F(x)$ as

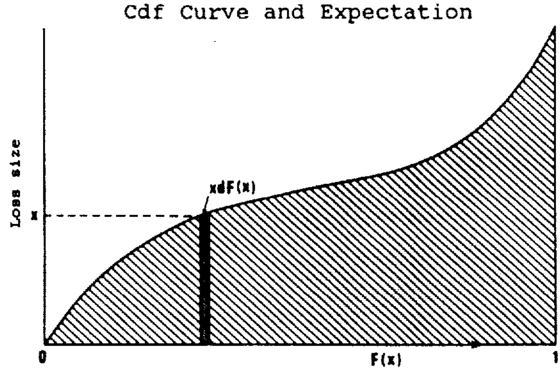
$$F(x) = \text{Prob}(X \leq x).$$

Figure 2 shows the graph of a continuous cdf. Consider the vertical strip in the graph, with area $xdF(x)$. If we sum up all these strips, we will obtain the expected value of X, i.e.

$$E(X) = \int_0^\infty xdF(x),$$

Figure 2

Cdf Curve and Expectation

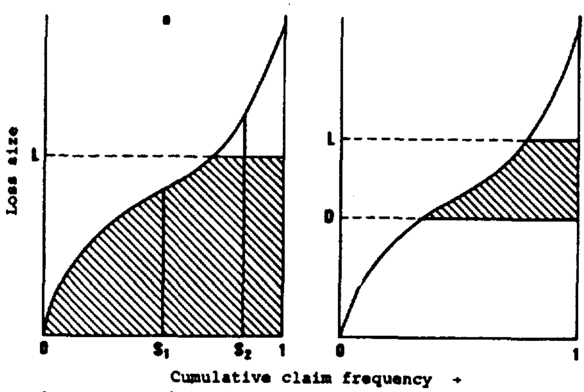


which is represented by the enclosed area below the cdf curve (the shaded area in the graph). We may interpret the expected loss as composed of losses of different sizes, and the strip $xdF(x)$ as the contribution from losses of size between x and $x+dx$. Throughout this paper, an expression such as $E(X)$ represents the expected value of a random variable X.

<u>Limited payments</u>. As an immediate application consider a coverage which pays for losses up to a limit L only. Figure 3(a) shows that a loss of size not more than L, such as S_1, is paid in full, while a loss of size S_2, which is greater than L, is paid only an amount L. By summing up vertical strips as before, except that strips with length greater than L are limited to length L, we obtain the expected payment per loss under such a coverage as the shaded area in Figure 3(a).

Figure 3

Expected Loss with (a) Limit and (b) Deductible



Deductibles. Likewise a coverage which pays for losses subject to a flat deductible D and up to limit L has expected payment per loss represented by the shaded area in Figure 3(b).

Size and Layer. As another application we first derive an integration identity. Consider Figure 4(a). The vertical strip has area $xdF(x)$ and the horizontal strip, $G(x)dx$, where

$$G(x) = 1 - F(x).$$

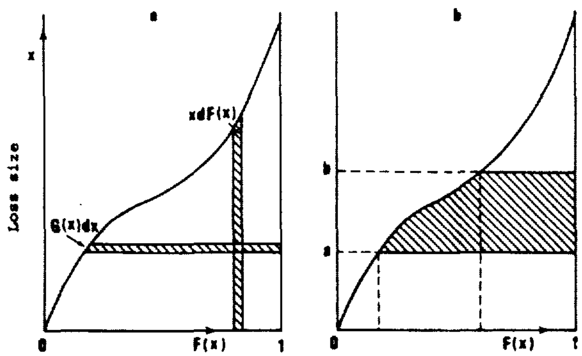
Summing up the vertical strips and the horizontal strips separately we have

$$\int_0^\infty x dF(x) = \int_0^\infty G(x)dx = E(X),$$

because each of the integrals is equal to the enclosed area below the cdf curve, which, as we have seen, also represents the expected loss E(X). The equality can also be algebraically derived via integration by parts.

Figure 4.

Size and Layer Views of Losses



The two modes of summation correspond, in fact, to two views of the losses. The vertical strips group losses by size, whereas the horizontal strips group the loss amounts by layer. We may therefore call them the size method and the layer method. It is often more convenient to evaluate the expected loss in a layer by the layer fashion, i.e. summing horizontal strips, than by the size method, i.e. summing vertical strips. For example, consider the layer of loss between a and b in Figure 4(b). The expected loss in this layer is represented by the shaded area. The layer method of summation gives simply

$$\int_a^b G(x)\,dx.$$

To express this integral by the size method is more difficult. A moment's reflection, with the help of Figure 4(b), yields the following expression for the integral:

$$\int_a^b x\,dF(x) \quad + \quad bG(b) \quad - \quad aG(a).$$

Again, the equality of the two expressions can be established via integration by parts.

The more complicated expression derived from the size method is the form commonly found in the literature. This is because, although the integral associated with the layer method is simple in form, $G(x)$ is a function that is generally more difficult to integrate. This disadvantage disappears, however, when the distribution is given numerically, as, for example, when actual experience is used. The retrospective rating Table M and Table L have been constructed by the layer method; see Simon (1965) and Skurnik (1975). We shall give the graphical interpretation later.

2. EXPECTED VALUE PREMIUM

Generally, given a loss X, a coverage would pay an amount depending on the value of X. We may represent this function by $g(X)$. The expected payment per loss is

$$E(g(X)) = \int_0^{\infty} g(x)dF(x).$$

The number of losses incurred by a risk in a policy period is a random variable, N, so that the total loss payment is

$$Y = \sum_{i=1}^{N} g(X_i) ,$$

which is the sum of a random number of random variables. It is customarily assumed that the loss severity X is distributed independently of the loss frequency N. With this assumption it can be shown that the expected payment in a policy period is

$$E(Y) = E(N).E(g(X)),$$

which says that the expected value pure premium of a risk is the product of average frequency of loss and the average severity. See for example Miccolis (1977).

<u>Increased Limits Coverage</u>. A liability insurance coverage is generally written to cover a loss in full up to a specified maximum dollar amount for any one loss. Let k be such a policy limit. We can express the payment function $g(X; k)$ of a loss X as

$$g(X;\ k) = \begin{cases} X, & 0 < X \leq k \\ k & k < X. \end{cases}$$

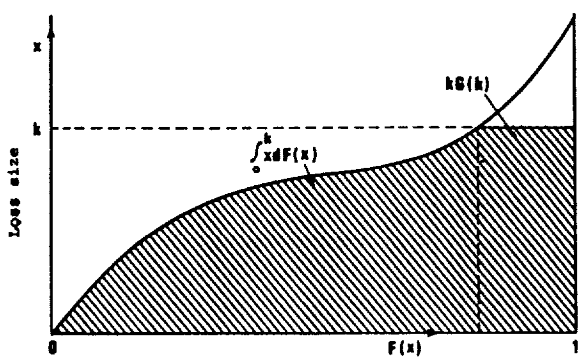
The expected payment per loss under this coverage can be expressed as

$$E(g(X;\ k)) = \int_0^k x dF(x) + kG(k).$$

The formula is demonstrated graphically in Figure 5, where integral on the right is represented by the shaded area below the broken vertical line, while the term kG(k) is represented simply by the rectangle above the line.

Figure 5

Losses with Indemnity Limited to k



Rates are generally published for some standard limit called the basic limit; let this be b, say. Increased limits rates are expressed as a factor, I(k), called the increased limits factor, to be applied to the basic limit pure premium rate. Thus

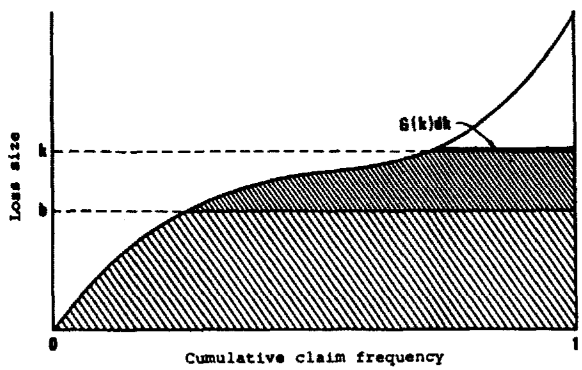
$$I(k) = [E\{g(X;\ k)\}.E\{N\}] \ / \ [E\{g(X;\ b)\}.E\{N\}]$$
$$= E\{g(X;\ k)\} \ / \ E\{g(X;\ b)\},$$

which depends on the distribution of size of loss only; see Miccolis (1977). The situation is demonstrated in Figure 6, where the increased limits factor is the ratio of the area of the shaded area up to k, to the shaded area up to b. The picture also displays another property of the increased limits factor. Miccolis (1977) shows that the derivative of I(k) can be expressed as

$$I'(k) = G(k) \ / \ E\{g(X;\ b)\}.$$

Figure 6

Increased Limits Factor

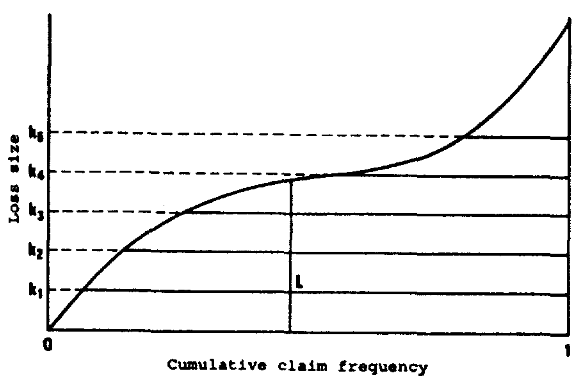


The picture shows that when k is increased by dk, the area representing the expected payment is increased by G(k)dk. Hence the result shown above.

Miccolis (1977) also discusses a consistency test for increased limits factors. A picture will provide much better insight into this question. In Figure 7 the enclosed region below the *cdf* curve is divided into horizontal panels which, for convenience of exposition, have equal width. The horizontal lines serve to subdivide a loss, such as L, into layers. With layers of equal width, the picture makes it quite plain that the expected payment in any layer is less than that in a preceding layer. If the layers are of different widths, this property holds between the layers for the expected payment per unit coverage. Hence the increased limits factor must increase at a decreasing rate as the increased limit increases. This is the consistency test. Actually Figure 7 also shows that this is a common sense argument: a loss must have penetrated a lower layer before it reaches an upper layer.

Figure 7

Consistency of Increased Limit Factor



Excess of Loss Coverage. An excess of loss contract generally covers losses in excess of a retention R, subject to a maximum limit L. The payment under such a contract may be expressed as a function of the loss X:

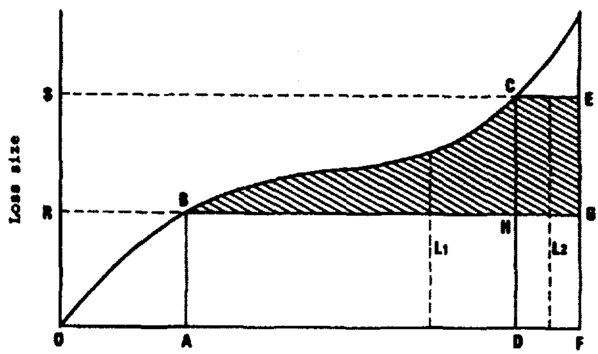
$$h(X;\ R,\ L) = \begin{cases} 0, & 0 < X \leq R \\ X - R, & R < X \leq S \\ L, & S < X, \end{cases}$$

where

$$S = R + L.$$

Figure 8

Losses with Retention and Limit



The situation may be described by means of the graph in Figure 8. For a loss such as represented by the line L_1 or L_2, the payment is represented by that portion of the line which falls inside the shaded region BGEC. The expected payment under such contract has been derived in the literature by the size method, and can be expressed in many different forms; the following are given in Miccolis (1977).

$$E\{h(X;\ R,\ L)\} = \int_R^S (x - R)dF(x) + LG(S)$$

$$= \int_R^S xdF(x) - r\ [F(S) - F(R)] + LG(S)$$

$$= \int_R^S xdF(x) + SG(S) - RG(R).$$

Figure 8 gives a simple graphical explanation of these integration results. They can be expressed in terms of the areas of the various regions shown in the graph, respectively as follows.

$$E\{h(X;\ R,\ L)\} = BHC + HGEC$$

$$= ADCB - ADHB + HGEC$$

$$= ADCB + DFEC - AFGB.$$

Each of these is equal to the shaded area in the graph.

It is, of course, much easier to express the expected payment of such an excess of loss contract by the layer method:

$$E\{h(X;\ R,\ L)\} = \int_S^R G(x)dx.$$

The result is plain from Figure 8; it can also be derived from the integral expressions given above via integration by parts.

Relations in the mathematics of excess of loss coverages could take on very complicated algebraic form, sometimes concealing the simplicity of the underlying idea. For example, Patrik (1978) gives an expression for the expected loss excess of R subject to a upper limit of L in terms of E(X)-R and other quantities. This is

$$E(X)-R + \text{Prob}(X \leq R).(R-E(X|X \leq R))$$
$$- \text{Prob}(X \geq R+L).[E(X|X \geq R+L) - (R+L)].$$

This can be demonstrated by the graph in Figure 9 where A, B, C, D, represent areas of the respective regions. The above relation says simply that

$$B = (A + B + C) - (A + D) + D - C,$$

because

B = expected excess loss

$A + B + C = E(X)$, i.e. expected loss

$A + D = R$

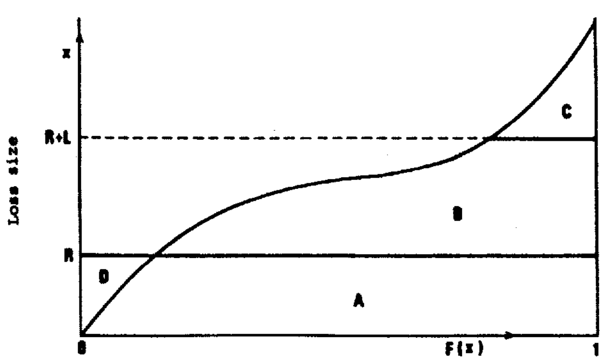
$D = \text{Prob}(X \leq R) . (R - E(X|X \leq R))$

$C = \text{Prob}(X \geq R+L) [E(X|X \geq R+L) - (R+L)]$

as is clear from the picture.

Figure 9

Excess of Loss Coverage



3. TREND

The effects of economic and social inflationary trends are to increase the size of losses. These effects act differently on the first dollar and the excess of loss coverages. Suppose the effect of inflation is, after a period of time, to change a loss of size x to a loss of size x', such that

$$x' = \alpha(x).$$

Assume that $\alpha(x)$ is a monotonic function, and let $F_1(x')$ be the cdf of x', i.e. the cdf after inflation. Then

$$F_1(x') = F(x),$$

and

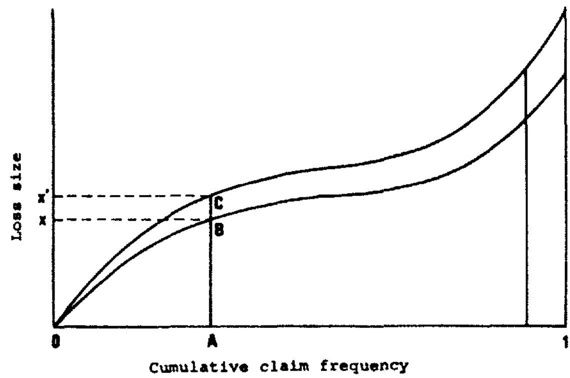
$$F_1(\alpha(x)) = F(x).$$

The effect of inflation is demonstrated in Figure 10, where the lower curve represents the cdf before inflation, and the upper curve represents the cdf after inflation. The graph shows that a loss AB of size x becomes a loss AC of size x'. When, starting from the cdf curve $F(x)$, each size of loss, as represented by the vertical distance from the horizontal axis to the curve $F(x)$, is extended according to the function $x' = \alpha(x)$, we obtain the cdf curve after inflation. A simple case of inflation is one in which the loss is increased by a uniform multiplicative factor a, so that

$$x' = ax$$

Figure 10

Effect of Inflation

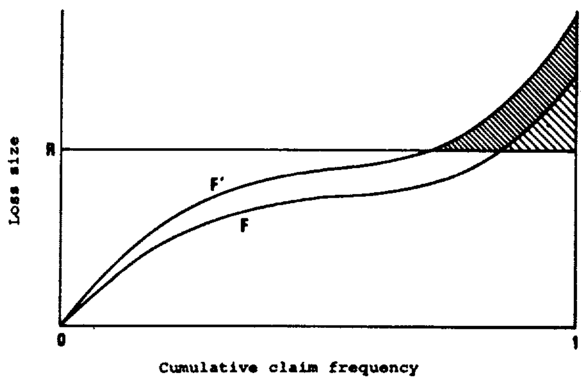


In this case the cdf curve after inflation, F'(x'), is obtained by extending each loss before inflation by a constant factor a-1.

It is well known that an excess of loss coverage is more seriously affected by inflation (assuming, for example, a uniform rate for all loss sizes); see, for example, Ferguson (1975). Figure 11 gives a dramatic demonstration of the leveraged effect of inflation on the excess of loss coverage. Let the rate of inflation be uniform for all sizes of loss, and the cdf curve after inflation be constructed from the curve before inflation as described above. The additional amount of loss resulting from inflation is shown in Figure 11 as the more heavily shaded region. If the retention R remains fixed, the expected excess loss payment is increased proportionally much more than indicated by the general rate of inflation.

Figure 11

Effect of Inflation on Excess Losses



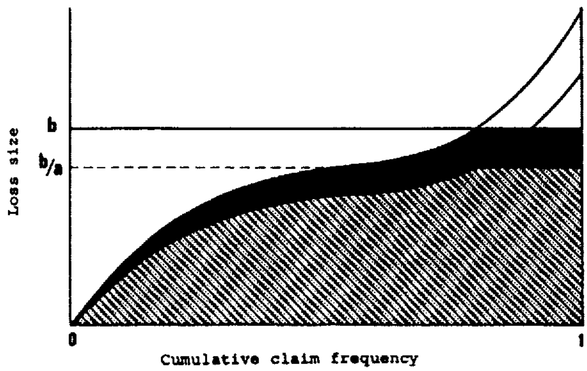
Since the total increase by inflation is divided between the basic limit loss and the excess loss, the basic limit loss is expected to incur an inflationary increase at a lower rate than the total limit rate. This topic has been treated in Finger (1976). Figure 12 gives a graphical demonstration of this effect and also shows the following algebraic result (see, for example, Miccolis, 1977):

$$E\{g(X'; b)\} = a\,E\{g(X; b/a)\}.$$

The picture says that the new expected basic limits loss, represented by the shaded area, is equal to the old expected loss up to the limit b/a, represented by the dotted area, extended by a factor a-1. A vertical line through the two-tone shaded region in Figure 12 bears this proportionality.

Figure 12

Effect of Inflation on Basic Limit Losses



The study of the effect of inflation on excess of loss coverages can lead to rather complicated algebraic expressions. For example, Ferguson (1975) relates the pure premium of an excess of loss coverage with indexing to the pure premium of one without indexing, the difference being expressed as a discount on the coverage without indexing. In an excess of loss coverage with indexing, the retention increase with inflation. A moment's reflection shows that the discount can be determined by comparing the expected loss under one contract with that under another. Let $\bar{X}$ be the average excess loss trended and indexed, R be the retention, $a-1$ be the proportional increase due to inflationary trend, Δ' be excess cost (per claim) on claims that exceed the retention as a result of inflation, and k be the multiplying factor which is equal to $G(R)$. Then Figure 13 shows that

$$E(L_O) = k\bar{X} + k(a-1)R + k\Delta' ,$$

$$E(L_I) = k\bar{X} ,$$

where $E(L_O)$ is the expected excess loss without indexing and $E(L_I)$ the expected excess loss with indexing. Thus

$$D = 1 - \frac{E(L_I)}{E(L_O)}$$

$$= 1 - \frac{1}{1 + R(a-1)/\bar{X} + {}'/\bar{X}}.$$

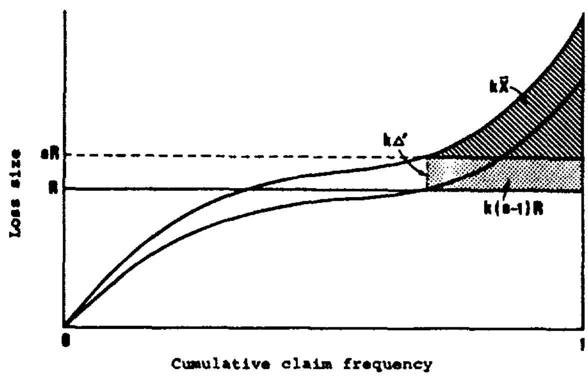
Or,

$$D = 1 - \frac{1}{1 + R(a-1)/\bar{X}}$$

as proposed by Ferguson (1975), neglecting the relatively small term involving '.

Figure 13

Indexing Excess of Loss Coverage



4. RETROSPECTIVE RATING

<u>The Excess Pure Premium Ratio</u>. We first consider the mathematics of the excess pure premium ratio, commonly denoted by $\phi(r)$. This is defined to be a risk's average amount of loss in excess of r times its expected loss, divided by the expected loss. It is also known as the table M charge, while the table M savings at the entry ratio r (meaning r times the expected loss) is defined as the expected amount by which the risk's actual loss falls short of r times the expected loss, divided by the expected loss. More precisely, let

A = actual loss of the risk;

E = $E(A)$, the expected loss;

Y = A/E, actual loss in units of expected loss; and

$F(.)$ = the cumulative distribution function of Y.

Then

$$\phi(r) = \int_r^\infty (y - r)dF(y)$$

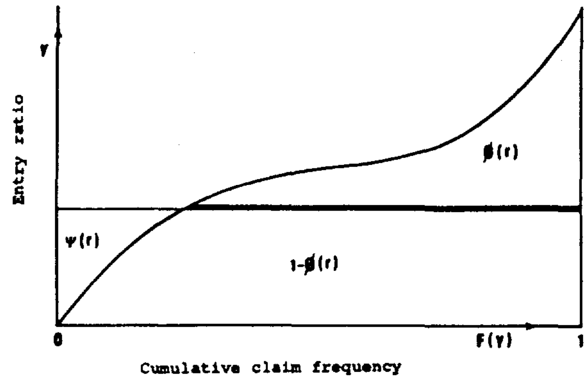
and

$$\psi(r) = \int_0^r (r - y)dF(y).$$

These functions are illustrated in Figure 14, where the cdf $F(y)$ is graphed against the entry ratio y. The functions $\phi(r)$ and $\psi(r)$ are represented by the areas indicated in the graph. A number of mathematical properties are now clearly demonstrated.

Figure 14

Functions in Retrospective Rating



(1) By definition, the bounded area below the F(x) curve is equal to 1. Hence $\phi(0) = 1$.

(2) $\phi(r)$ is a decreasing function of r, and $\phi(r) \to 0$ as $r \to \infty$.

(3) $\psi(r)$ is an increasing function of r; its value is unbounded as $r \to \infty$.

(4) Consider the samll strip at $y = r$ in the graph. This shows that an increment dr from r will yield a decrease $G(r)dr$ in $\phi(r)$. Hence

$$\phi'(r) = (d/dr)\,\phi(r) = -G(r).$$

A second differentiation yields

$$\phi''(r) = f(r),$$

where $f(r)$ is the density function of the entry ratio, a result well known in the literature (Valerius 1942). Similarly, we may deduce from Figure 14 that

$$\psi'(r) = (d/dr)\,\psi(r) = F(r)$$

and

$$\psi''(r) = f(r).$$

(5) Consider the area of the rectangle on the interval from 0 to r in Figure 14. This gives the relation

$$r = [1 - \phi(r)] + \psi(r),$$

or

$$\psi(r) = \phi(r) + r - 1;$$

this is a fundamental relation connecting $\psi(r)$ and $\phi(r)$.

A result more general than (5) above can also be obtained quite easily from Figure 15. Let

$$L = \begin{cases} r_1E & \text{if } A \le r_1E \\ A & \text{if } r_1E < A \le r_2E \\ r_2E & \text{if } r_2E < A. \end{cases}$$

Then the cdf of L/E can be represented by the solid line in Figure 15. The shaded area represents the quantity $E(L)/E$ and we have

$$E(L)/E - \psi(r_1) + \emptyset(r_2) = 1,$$

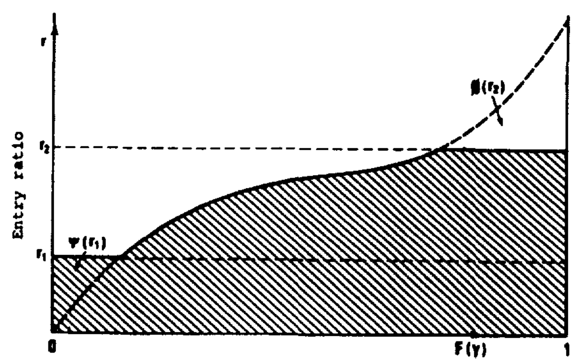
or

$$E(L)/E = 1 + \psi(r_1) - \emptyset(r_2).$$

see Skurnick (1974).

Figure 15

Expectation of L in Retrospective Rating



<u>Retrospective Rating</u>. In the Workers' Compensation Retrospective Rating Plan, the retrospective premium R is given by

$$R = b + CA,$$

subject to a maximum premium G and a minimum premium H, where b is the basic premium and C is the loss conversion factor (LCF), and where b is alternatively represented by

$$b = BP,$$

with P as the standard premium (before any applicable expense gradation) and B as the basic premium ratio. Let L_G be actual loss that will produce the maximum premium:

$$G = b + CL_G$$

and let

$$r_G = L_G/E.$$

Similarly, define L_H to be

$$H = b + CL_H,$$

$$r_H = L_H/E.$$

Further, let

$$L = \begin{cases} L_H & \text{if } A \le L_H \\ A & \text{if } L_H < A \le L_G \\ L_G & \text{if } L_G < A. \end{cases}$$

Then the retrospective premium can be represented by

$$R = b + CL.$$

For ease of exposition, we ignore the tax factor. If we identity r_H and r_G with r_1 and r_2 respectively, then Figure 16 shows the quantity E(L) as the area of the shaded region OFDCBA. It then follows that

$$E(L) = E - \emptyset(r_G)E + \psi(r_H)E$$
$$= E - I,$$

where

$$I = E[\emptyset(r_G) - \psi(r_H)]$$

is called the net insurance charge of Table M. If the plan is to be balanced, the expected retrospective premium must be equal to the sum of the total expenses, e, and the expected loss, E:

$$E(R) = e + E.$$

On the other hand, it also follows from the above that

$$E(R) = b + C(E - I).$$

Equating these two quantities we obtain the basic premium in terms of the expense, expected loss, and the net insurance charge:

$$b + C(E - I) = e + E$$

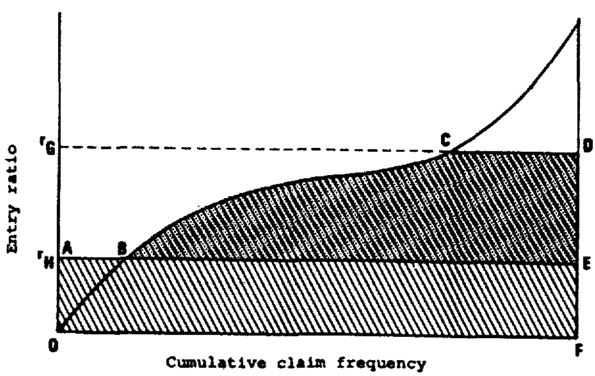
or

$$b = e - (C - 1)E + I.$$

A formula relating the charge difference to the minimum premium, expected loss and expense provision has been used to facilitate the determination of retrospective rating values from specified maximum and minimum premiums. This formula can be derived with the help of Figure 16.

Figure 16

Retrospective Rating Premium



Consider the equation

$$R = b + CL$$

Taking expectation and representing the expectation $E\{L\}$ by the shaded area of Figure 16 we have

$$e + E = b + CE\,[OFDCBA].$$

On the other hand, we have for the minimum premium H:

$$H = b + Cr_H$$
$$= b + CE\ [OFEA].$$

Taking the difference on both sides of the two equations above we have

$$(e + E) - H = CE\ [BEDC]$$
$$= CE\ [\emptyset(r_H) - \emptyset(r_G)].$$

This formula, together with the formula

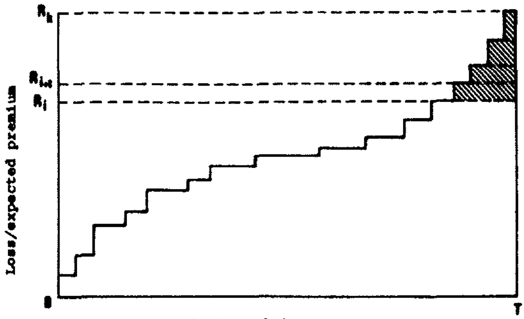
$$G - H = CE(r_G - r_H),$$

which is much easier to derive, can be used to determine the rating values given the maximum and minimum premiums. One may interpret the difference in charge, $\emptyset(r_H) - \emptyset(r_G)$, as indicated by the dotted area in Figure 16, to be the difference between the expected retrospective premium and the minimum premium, apart from a conversion factor CE.

<u>Construction of Table M</u>. A Table M has been constructed by Simon (1965); see also Skurnick (1974). The algebra involved in the construction procedure appears to be rather complicated. Actually the idea is very simple when this is expressed in a graph. Figure 17 shows a cumulative frequency curve constructed from observed data on risks within a premium group. Let the loss ratios be arranged in ascending order: $R_1, R_2, \ldots, R_k$, with R_i

Figure 17

Table M Construction



occurring N_i times. Also let the total number of claims be $T = N_1+\ldots+N_k$. The cumulative frequency up to R_i, i.e. $T_i = N_1+\ldots+N_i$ is plotted against R_i for each i so as to form a step function whose abscissa in the interval (R_i, R_{i+1}) is the cumulative frequency T_i, as shown in Figure 17. We may think of this graph as a rescaled version of the cdf curve plotted against the entry ratio. It now appears quite clearly that the value of ϕ for the entry ratio corresponding to R_i is simple the shaded area in Figure 17 divided by the total enclosed area below the cumulative frequency curve. The entry ratio corresponding to R_i is simple R_i divided by the average loss ratio $\Sigma N_i R_i/T$.

A convenient procedure to construct a Table M is to sum the horizontal strips downward, cumulatively, starting from the strip corresponding to (R_{k-1}, R_k), down to the strip corresponding to $(0, R_1)$. It is convenient also to sum the frequencies downward, cumulatively, because the cumulative sum of such frequencies down to and including N_{i+1} is the length of the strip corresponding to the interval (R_i, R_{i+1}). Thus let

$$S_{1,i} = \sum_{j=1}^{i} N_j,$$

which is represented by the length of the strip on (R_i, R_{i+1}), and

$$S_{2,i} = S_{2,i+1} + S_{1,i+1} (R_{i+1} - R_i),$$

which describes the fact that the sum of the strips above R_i is obtained by adding the strip on (R_i, R_{i+1}) to the sum of the strips above R_{i+1}. The value of ϕ at the entry ratio corresponding to R_i is then $S_{2,i}/S_{2,0}$, with $S_{2,0}$ equal to the total area of all the strips. The entry ratio corresponding to R_i is obtained by normalization:

$$r_i = R_i \Big/ \left(\frac{S_{2,0}}{S_{1,0}} \right).$$

We may think of R_i as loss expressed in an arbitrary unit and the denominator as the expected loss in this unit. The procedure is described in algebraic form by Skurnick. It is easy to see that this is a layer approach.

Table L. A retrospective rating plan may provide for a per accident limit on losses. The table of charges which incorporates this per accident limitation is called the Table L, which has been described by Skurnick (1974). Let A be the actual unlimited loss, as before, A^* be the actual limited loss, and $F^*(.)$ be the cdf of $Y^* = A^*/E$. Then the Table L charge is defined as (Skurnick, 1974)

$$\phi^*(r) = \int_r^\infty (y - r)dF^*(y) + k,$$

where k is the loss elimination ratio

$$k = [E - A^*]/E$$

Further, the Table L savings are defined as

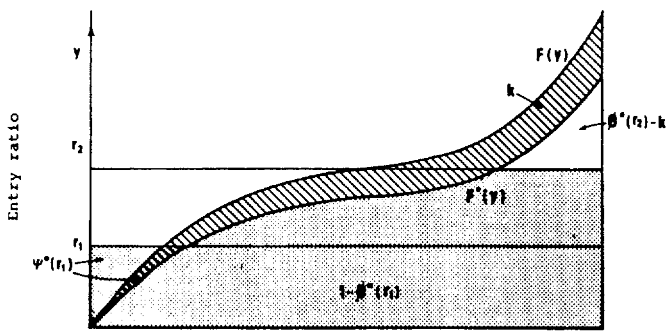
$$\psi^* = \int_0^r (r - y)dF^*(y).$$

In Figure 18 the curves for $F(y)$ and $F^*(y)$ are plotted against the entry ratio $r = A/E$. $F(y)$ is necessarily situated above $F^*(y)$, and by the definition of r, the enclosed area below the $F(y)$ curve is equal to 1, while the enclosed area below the $F^*(y)$ curve is $1 - k$. The area of the shaded belt is equal to the loss elimination ratio k. Many of the properties of the Table L charges, as presented by Skurnick (1974), can be easily obtained from the graph. For example, consider the limited loss

$$L = \begin{cases} r_1E & \text{if } A^* \le r_1E \\ A^* & \text{if } r_1E < A^* \le r_2E \\ r_2E & \text{if } r_2E < A^*. \end{cases}$$

Figure 18

Table L Functions



Then $E\{L^*\}/E$ is represented by the dotted area in Figure 18. We deduce that

$$E\{L^*\}/E - \psi^*(r_1) + [\emptyset^*(r_2) - k] = 1 - k$$

and hence

$$E\{L^*\}/E = 1 + \psi^*(r_1) - \emptyset^*(r_2),$$

as in Skurnick (1974). As another example, identify r_1 and r_2, respectively, with r_H and r_G as defined before. Also let

$$R^* = b^* + CL^*$$

be the retrospective premium with per accident limitation. Then, combining the equation

$$E(R^*) = e + E = b^* + CEr_H + CE[\phi^*(r_H) - \phi^*(r_G)],$$

which follows from the fact that the expected retrospective premium is b^* plus the dotted area (converted), with the equation

$$H = b^* + CEr_H,$$

we have the Table L version of a familiar formula

$$e + E - H = CE\,[\phi^*(r_H) - \phi^*(r_G)],$$

the last factor on the right being represented by the dotted area between $r_1 = r_H$ and $r_2 = r_G$ in Figure 18. As a final example of the use of Figure 18, one may consider the constructions of Table L. This can be done in a manner similar to the construction of Table M, except that the cumulative frequency function of the limited loss is used, and the final result has to be adjusted for the loss elimination factor k.

Asymptotic Behavior. As the premium size becomes large, the limiting form of the charge takes on a simple function. The graphs in Figure 19 help us to understand the asymptotic behavior. Consider the case with no per loss limitation.

Figure 19

Limiting Case in Retrospective Rating

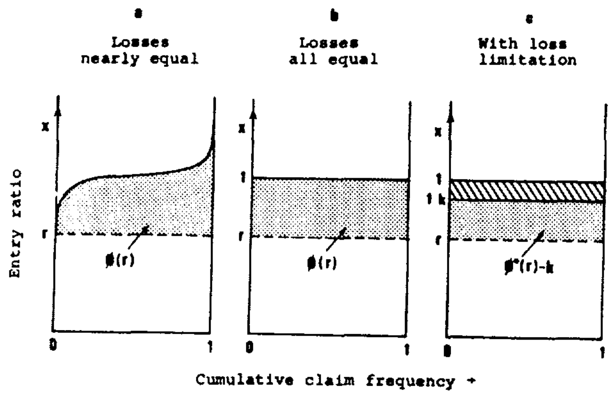


Figure 19(a) shows a cdf curve for losses which are nearly equal; here the $\phi(r)$ region almost forms an rectangle. When all losses are equal, the cdf $F(x)$ is a step function with a single jump at $x = 1$, as shown in Figure 19(b). The Table M charge $\phi(r)$ at the entry point r is represented by the area of the rectangle between r and 1. Hence

$$\phi(r) = \begin{cases} 1 - r & r \leq 1 \\ 0 & 1 < r. \end{cases}$$

The limiting case with per loss limitation is shown in Figure 19(c). Here the cdf $F^*(x)$ is shown as the horizontal line $x=1-k$, where it has its single jump. The Table L charge $\phi^*(r)$ is the area of the rectangle between r and 1-k, plus the loss elimination ratio k. Thus

$$\phi^*(r) = \begin{cases} 1 - r & r < 1-k \\ k & 1-k \leq r. \end{cases}$$

Other Applications. There are other interesting mathematical relations in the mathematics of retrospective rating, and many such intricate relations are presented in Carlson (1941). It is a great burden to follow the algebra of the many complicated relations presented there. Most of these, however, become much clearer if we make use of the graphical approach adopted here. Rather than go through the numerous equations and formulas in Carlson (1941), we present a particular example to illustrate the power of our graphical method. Let us pick, almost at random, equations (15a) in Carlson, which can be explained as follows. Let the minimum premium be greater than the basic premium, and the maximum premium be equal to the standard premium:

$$H > B, \quad G = P.$$

Then, in Carlson's notation,

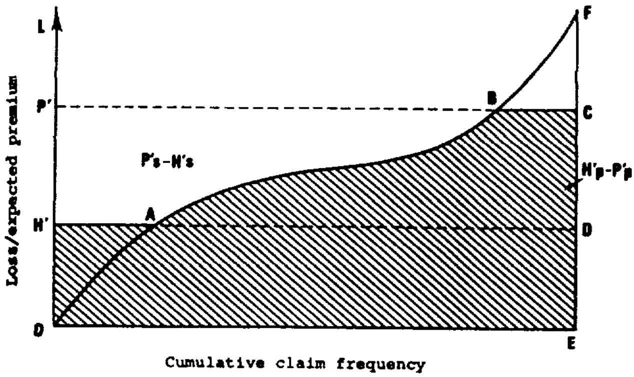
$$P - Rv = C(P's - H's)$$
$$= C(P' - H') - C(H'p - P'p).$$

These equations follow immediately from Figure 20 with the following interpretation of Carlson's notations:

$$P = b + CP'$$
$$Rv = \text{expected retrospective premium}$$
$$= b + C[OECBAH']$$

Figure 20

Relationships in Retrospective Rating



P's = OBP'

H's = OAH'

H'p = ADF

P'p = BCF.

5. CONCLUSION

This paper presents a graphical approach to the mathematics of excess of loss coverages and related topics. The graphs serve to simplify and clarify much of the complicated algebra which has hitherto been the sole vehicle to express the mathematical ideas involved. We hope this will become a useful addition to the actuarial tool box of the student and the practicing casualty actuary alike. This technique has been used in explaining the principles of coinsurance and its many properties (Lee, 1985). Philbrick (1985) uses the same idea to describe size of loss distributions.

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RATE FILING UNDER THE FLEX RATING SYSTEM

RATE FILING UNDER THE FLEX RATING SYSTEM

RATEMAKING SEMINAR

CASUALTY ACTUARIAL SOCIETY

MARCH 17, 1988

BY DEBORAH M. ROSENBERG

The topic that I am going to speak about deals with making a rate filing under the flexible rating system. Flex-rating is a concept that was introduced with the enactment in New York of the Omnibus Insurance Legislation of 1986. Prior to this, there were two filing methods in New York - prior approval for private passenger automobile, workers' compensation and most types of medical malpractice, and open competition, i.e., file and use, for the remaining lines. As a result of the liability crisis, and a reluctance to impose prior approval for lines not previously so subject, the compromise position of flex-rating was created. Under this system, rates for certain commercial liability markets may be changed without prior approval, as long as the resulting rate falls within a specified range, called a flex-band, applicable to that insurance market.

Webster's defines the word "flexible" as, "capable of responding or conforming to changing or new situations." I am sure that many of you feel that the title of this new method of regulation is a total misnomer. After all, lines of business previously free of almost all state scrutiny and modifiable almost at whim, are suddenly subject to a whole array of new guidelines and regulations. How could the imposition of additional requirements be described as flexible? Part of my job here today is to reassure you that, in fact, the Department's goal in implementing flex-rating was to preserve as much of the spirit of competition as possible while at the same time enabling it to effectively monitor the insurance market place.

How does the system really work?

Flex-bands are defined for the various lines of insurance for both overall rate level change and for individual insureds' rate changes. Overall rate changes without prior approval can vary from ±10% to ±30% within a twelve month period, depending on the line. These bands will be periodically reviewed by the Superintendent. Individual rates, in addition to any average change, could reflect revised class, territory and/or increased limit relativities. An individual rate change cannot exceed the overall change by more than 20% in either direction. There is a maximum of three rate changes per twelve month period, which if they remain within the flex-band would not be subject to prior approval. If a filing does not exceed the flex-band then it may be used before the Department passes judgment on it. This does not mean that the Department does not review the filing or follow through with any questions or concerns it may have vis-a-vis the filing. If a filing exceeds the band then it must be approved by the Department before it can be implemented by the insurer. This is not to say that the filing will not be approved, just that the rates cannot be used until they are. During 1987, the first full year that flex-rating was in effect, the Department received 961 rate filings which were subject to the provisions of flex-rating. Over 850 of the filings were within their flex-band; of the 108 that exceeded the flex-band, 80 were approved as filed, 15 disapproved and 13 were still pending at year's end.

If the great majority of filings get approved anyway what does flex-rating accomplish? First, it enables the Department to concentrate on those filings that appear to require "large" rate changes. One of the purposes of flex-rating was to smooth out the peaks and troughs of the underwriting cycle. Monitoring those insurers whose needs appear to exceed the expected, assists in smoothing the cycle. Secondly, companies will be encouraged to carefully consider their rate request and to determine what their real rating needs are before they file for huge revisions.

What to include in Rate Filings

1. -Investment Income: Section 2303 of the New York Insurance Law requires consideration of investment income in determination of all rates, those subject to prior approval and those subject to file and use. While the question of how investment income should be included in ratemaking is a subject of great controversy, the law requires that it should be reflected. In general the Department does not specify methodologies to be used when making a rate a filing, and is willing to review any innovative approaches.

2. -Trend Factors: Trend factors should be coverage appropriate. If a company is proposing to use a trend that is significantly different from the industry average, then supporting information is necessary. When individual company data are not credible, industry data or other economic indicators should be used.

3. -Loss Development Factors: Ideally individual company loss development factors for New York State should be submitted along with the filing. However, if credibility is an issue, country-wide data or even industry data for New York could be substituted.

4. -Underlying Data: The data accompanying the filing are the basis for the Department's evaluation. Therefore, the data should be as complete as possible. Premiums should be on current rate level. Losses should be developed and trended. Calendar year data should be avoided for long tail lines. Whenever possible, basic limits data should be used as opposed to total limits. Individual large losses should be identified. Data prior to August, 1986, should reflect the impact of Tort Reform.

Conclusion:

Flex-rating is scheduled to expire June 30, 1988. Legislation has been proposed to extend this system until 1994. At this point it is still too early to tell whether flex-rating has indeed stabilized the insurance marketplace. However, the Department hopes that over the course of the next few years flex-rating will assist in providing affordable and available insurance products for New York consumers.

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INVESTMENT INCOME, UNDERWRITING PROFIT INCLUDING THE TAX REFORM ACT OF 1986 AND CONTINGENCIES: FINANCIAL MODELS

INVESTMENT INCOME, UNDERWRITING PROFIT AND CONTINGENCIES: FINANCIAL MODELS

BY RICHARD A. DERRIG

INTRODUCTION

THE ESSENCE OF AN INSURANCE POLICY IS THE PROMISE BY THE INSURER TO PAY ALL CLAIMS OF THE INSURED THAT ARE COVERED BY THE POLICY. IN RETURN FOR THE INSURER'S PROMISE, THE INSURED PAYS THE POLICY PREMIUM. THE INSURER CAN BE PARTIALLY DESCRIBED FINANCIALLY OR ECONOMICALLY BY THE SET OF ALL THESE POLICIES. BEFORE PROCEEDING WITH A DISCUSSION OF THE USE OF INVESTMENT INCOME IN RATEMAKING, ALLOW ME TO FIT OUR INSURANCE TRANSACTION INTO A GENERAL ECONOMIC OR FINANCIAL PICTURE BY A TRANSLATION TABLE OF THE KEY WORDS IN THE DESCRIPTION OF THE INSURANCE POLICY.

KEY WORDS

INSURANCE	ECONOMICS
POLICY	CONTRACT
PROMISE	
ALL	GOODS & SERVICES
CLAIMS	
PREMIUM	PRICE

I WANT TO DESCRIBE RATEMAKING IN THIS CONTEXT AS THE METHOD FOR DETERMINING THE (LIST) PRICE TO BE CHARGED FOR EACH HOMOGENEOUS SUBSET OF INSURANCE CONTRACTS. WHAT MAKES THE INSURANCE TRANSACTION ESSENTIALLY DIFFERENT FROM SOME OTHER TRANSACTIONS IN THE ECONOMY, AND THEREFORE INTERESTING TO US, IS THAT THE PAYMENT OF THE PRICE (PREMIUM) AND THE DELIVERY OF THE GOODS AND SERVICES (PROMISE TO PAY

ALL CLAIMS) DO NOT OCCUR SIMULTANEOUSLY, BUT RATHER THEY CAN OCCUR WITH A LONG TIME GAP BETWEEN PREMIUM AND CLAIM PAYMENTS. THIS MAKES THE INSURANCE CONTRACT RISKY. INDEED, THE INSURANCE CONTRACT IS RISKY FOR BOTH THE INSURED AND THE INSURER.[1] THIS TIME GAP IS ALSO PRESENT IN OTHER FINANCIAL INTERMEDIARY TRANSACTIONS SUCH AS STOCK AND BOND ISSUES, MORTGAGE CONTRACTS, AS WELL AS OPTIONS AND FUTURE CONTRACTS. THE PRICING OF THOSE RISKY FINANCIAL CONTRACTS ARE GENERALLY ACCOMPLISHED IN OPEN COMPETITIVE MARKETS FOR CAPITAL.

INSURANCE RATEMAKING SHOULD RECOGNIZE THAT IT MUST COEXIST WITH THE COMPETITIVE MARKET PRICING OF OTHER FINANCIAL INTERMEDIARY PRODUCTS AND OTHER GOODS AND SERVICES IN GENERAL. FOR INSURANCE POLICIES IN A COMPETITIVE MARKET WE MIGHT FURTHER STRIKE AN ANALOG WITH PRICES IN THE GENERAL ECONOMY.

PREMIUM	PRICE
ACTUARIAL	LIST
MARKET	SALE

[1]IF YOU CAN'T IMAGINE THAT YOUR OWN PERSONAL AUTO POLICY IS RISKY TO YOU AS THE INSURED THEN THINK OF YOUR COMPANY AS AN INSURED WHEN IT REINSURES SOME OF ITS DIRECT BUSINESS. THE RISK TO YOUR COMPANY IS IN WHETHER THE REINSURERS WILL PAY, A VERY REAL PROBLEM IN TODAY'S MARKETS.

INVESTMENT INCOME, UNDERWRITING PROFIT AND CONTINGENCIES: FINANCIAL MODELS

BY THE ACTUARIAL PREMIUM, I MEAN THE RESULT OF PROVIDING THE BEST CURRENT VALUE ESTIMATE OF ALL THE COMPONENTS OF THE POLICY CONTRACT BY MEANS OF THE INSURER'S ANALYTIC PROCESS. IN A REAL SENSE, THE ACTUARIAL PREMIUM IS ONLY THE LIST PRICE FOR THE INSURANCE CONTRACT.

BY THE MARKET PREMIUM, I MEAN THE POLICY PREMIUM THAT RESULTS FROM THE ACTUARIAL PREMIUM AFTER DIVIDENDS, SCHEDULE RATING AND ALL OTHER MARKETING DEVICES HAVE HAD THEIR INFLUENCE ON THE ACTUARIAL PRICE IN ORDER TO MATCH THE COMPETITIVE MARKET SOLUTION OF THE SALE PRICE. SOME OF THOSE PRESENT AT THIS SEMINAR MAY WANT TO FORECAST THE DAY WHEN THOSE TWO INSURANCE CONTRACT PRICES, ACTUARIAL AND MARKET, ARE EQUAL; OTHERS WILL BE MORE REALISTIC AND RECOGNIZE THE EVER-PRESENCE OF SALE PRICES FOR INSURANCE POLICIES. (CAN ANYONE FORGET THE FABULOUS 1983-85 GOING-OUT-OF-BUSINESS SALE BY MISSION INSURANCE OF WORKERS' COMPENSATION AND REINSURANCE CONTRACTS?)

WITH THIS GENERAL CONTEXT IN MIND, LET ME PROVIDE YOU WITH A VERY BRIEF SUMMARY OF SOME OF THE ACTUARIAL PRICING MODELS OF TWO BASIC TYPES: MARKUP MODELS AND FINANCIAL MODELS. I WILL THEN CONCENTRATE ON A FEW DETAILS OF THE FINANCIAL MODELS ACTUALLY USED TO SET MASSACHUSETTS AUTOMOBILE AND WORKERS' COMPENSATION RATES. REFERENCES FOR FURTHER READING ARE PROVIDED AT THE END OF THE DISCUSSION.

INVESTMENT INCOME, UNDERWRITING PROFIT AND CONTINGENCIES: FINANCIAL MODELS

MARKUP MODELS

BY A MARKUP MODEL I MEAN SIMPLY THAT THE OTHERWISE DETERMINED ACTUARIAL ESTIMATE OF LOSSES AND EXPENSES EXPECTED TO BE INCURRED UNDER THE TERMS OF THE INSURANCE CONTRACT IS LOADED OR MARKED-UP FOR AN UNDERWRITING PROFIT TO GET THE ACTUARIAL PREMIUM. THE NOTION HERE IS THAT JUST AS THE SUPERMARKET MARKS-UP THE PRICE OF TOMATOES FOR PROFIT, INSURANCE CONTRACTS TOO CAN BE PRICED USING SOME FIXED PROFIT MARGIN.

STATED IN WORDS,

PREMIUM = (LOSSES + EXPENSES) X (1 + PROFIT)

STATED SOMEWHAT MORE FORMALLY,

$$P = \frac{L + E}{1 - \mu}$$

WHERE:

P = PREMIUM (ACTUARIAL)

L = LOSSES

E = EXPENSES[2]

μ = UNDERWRITING PROFIT PERCENT OF PREMIUM

[2]FOR THIS PURPOSE, EXPENSES ARE ASSUMED NOT TO VARY WITH PREMIUM.

INVESTMENT INCOME, UNDERWRITING PROFIT AND CONTINGENCIES: FINANCIAL MODELS

OVER TIME, THERE HAVE BEEN SOME TRADITIONAL UNDERWRITING PROFIT MARK-UPS FOR PROPERTY-LIABILITY INSURANCE CONTRACTS. YOU MAY HAVE HEARD THAT 5% WOULD BE APPROPRIATE FOR THE PROPERTY-LIABILITY LINES[3] AND 2.5% FOR THE WORKERS' COMPENSATION LINE. HISTORY SHOWS US QUITE CLEARLY THAT IF TRADITIONAL MARKUPS HAD BEEN USED TO SET ACTUARIAL PREMIUMS, THEN THE OBSERVED MARKET PREMIUMS HAVE DEVIATED DOWNWARD FROM THOSE ACTUARIAL PREMIUMS.[4] SURPRISED?

FOR THE SAME REASON AMERICANS FLOCK TO COMPETITIVE 20%-40% OFF SALES, WE SHOULD EXPECT THE COMPETITIVE MARKET TO PROVIDE ITS DETERMINATION OF THE (PRESENT) VALUES OF LOSSES AND EXPENSES, THEREBY GIVING A MARKET-DRIVEN NET PREMIUM OR SALE PRICE FOR INSURANCE CONTRACTS.[5] THE ATTACHED GRAPH, LABELED COUNTRYWIDE ACTUARIAL VS NET PREMIUM, ILLUSTRATES THIS PHENOMENON BY DISPLAYING A PLANE OF POSSIBLE NET

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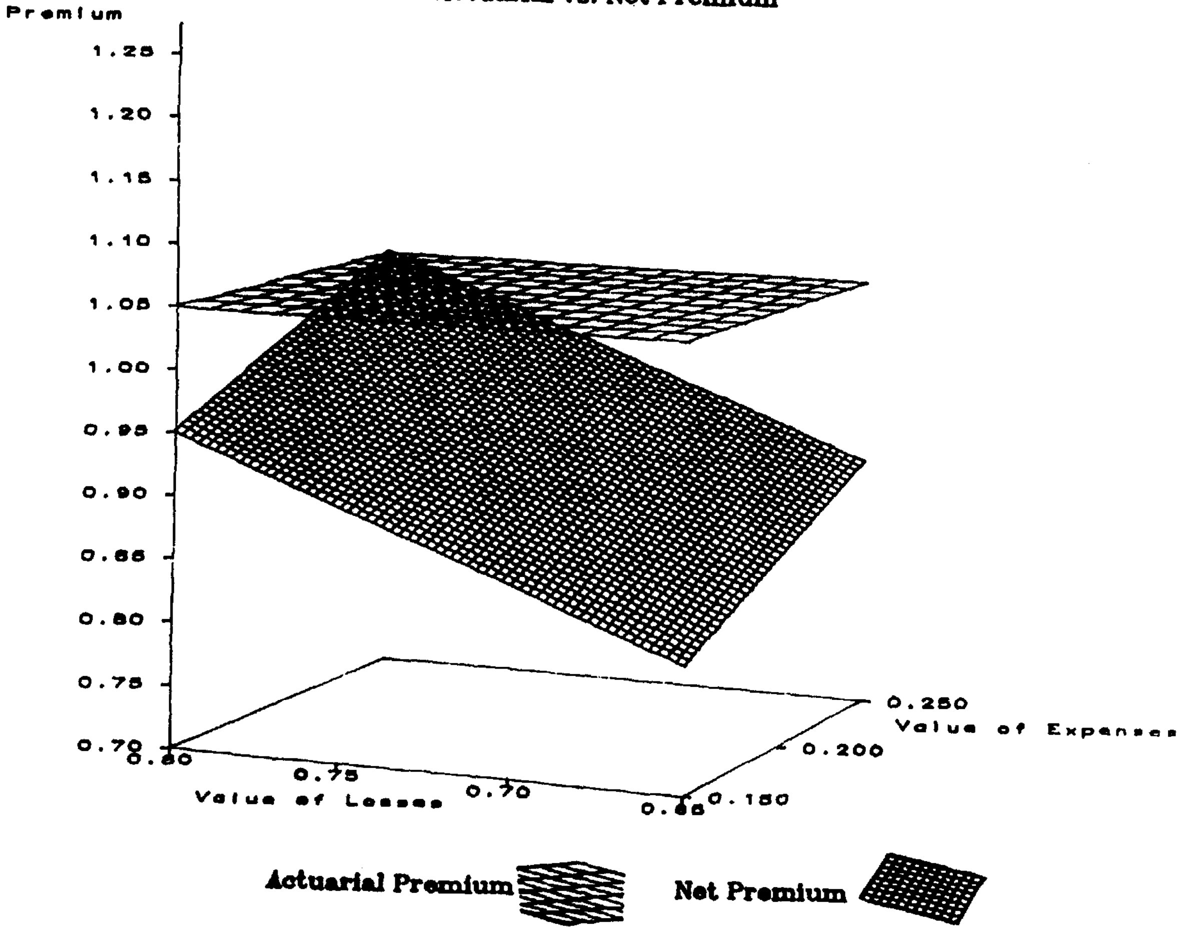
[3]THIS IS THE SO-CALLED 1921 PROFIT FORMULA. IT ACTUALLY SUGGESTED AN ADDITIONAL 3% FOR "CONFLAGRATION", REDUCED IN THE LATE 1940'S TO 1%. SEE NAIC [13, VOL. I, PAGE 28]. [] REFERS TO REFERENCES AT THE END OF THIS DISCUSSION.

[4]THIS DOWNWARD DEVIATION PHENOMENON HAS BEEN ILLUSTRATED MOST RECENTLY, AT THE RATE OF RETURN LEVEL, BY INDUSTRYWIDE RETURNS ON NET WORTH FROM 1968-1984. SEE ATTACHED GRAPH FROM A 1986 ARTICLE BY DAVID ELEY. OTHERWISE THIS IS THE SO-CALLED "SHORTFALL" PHENOMENON (SEE FAIRLEY (1), P.20 IN *CUMMINS AND HARRINGTON* [3]).

[5]SEE APPEL AND GEROFSKY [1] FOR THE WORKERS' COMPENSATION CASE.

Countrywide
Actuarial Vs. Net Premium



PREMIUMS GIVEN MARKET VALUATIONS OF LOSSES AND EXPENSES AND A TRADITIONAL 5% PROFIT LOADING IN THE ACTUARIAL PREMIUM.

MOST OF THE INSURANCE PRICING MODELS I KNOW ABOUT USE AN UNDERWRITING PROFIT MARKUP MODEL.[6] THE DIFFERENCES AMONG MODELS CAN BE FOUND IN HOW THE VALUE OF THE MARKUP IS DETERMINED. ONE FEATURE IS, OR SHOULD BE, COMMON TO THEM ALL; NAMELY, THAT SOUND AND UNBIASED ACTUARIAL TECHNIQUES ARE BROUGHT TO BEAR ON THE DIFFICULT PROBLEM OF FORECASTING LOSSES AND EXPENSES EXPECTED DURING THE POLICY CONTRACT.

FINANCIAL MODELS

BY A FINANCIAL MODEL I MEAN SIMPLY THAT SOME PRINCIPLES OF FINANCE (RATES OF RETURN, RISK, PRESENT VALUES) ARE USED TO SUPPORT THE CHOICE OF THE VALUE OF THE UNDERWRITING PROFIT LOADING. FINANCIAL MODELS OF EVER-INCREASING COMPLEXITY HAVE BEEN USED TO SET AUTOMOBILE AND WORKERS' COMPENSATION RATES IN MASSACHUSETTS SINCE 1976. THESE HAVE BEEN DOCUMENTED THROUGH 1983 IN MY ARTICLE[7] IN THE RECENTLY PUBLISHED BOOK, FAIR RATE OF RETURN IN PROPERTY-LIABILITY INSURANCE, EDITED BY DAVE CUMMINS AND SCOTT HARRINGTON, BOTH AT THE WHARTON SCHOOL, UNIVERSITY OF PENNSYLVANIA.

TWO KINDS OF FINANCIAL MODELS HAVE BEEN USED IN MASSACHUSETTS, RATE OF RETURN AND PRESENT VALUE MODELS. A

[6] AN EXCEPTION IS FOUND IN ROSS AND KRAUS (3) IN CUMMINS AND HARRINGTON [3].

[7] SEE DERRIG (6) IN CUMMINS AND HARRINGTON [3].

INVESTMENT INCOME, UNDERWRITING PROFIT AND CONTINGENCIES: FINANCIAL MODELS

RATE OF RETURN MODEL SEEKS TO DETERMINE THE RATE OF RETURN ON THOSE INSURANCE CONTRACTS (THE UNDERWRITING PROFIT) AS THAT RESIDUAL PROFIT NEEDED IN ORDER THAT THE RATE OF RETURN ON INVESTMENTS PLUS THE UNDERWRITING PROFIT EQUAL AN APPROPRIATE RATE OF RETURN ON THE EQUITY INVESTED TO UNDERWRITE THOSE CONTRACTS. RATE OF RETURN MODELS ARE MOST NATURALLY APPLICABLE IN A ONE-PERIOD CONTEXT WITH THE CENTRAL VALUATION TAKING PLACE AT THE END OF THE PERIOD. FOR ACTUARIAL PRICING PURPOSES, SINCE MOST INSURANCE CONTRACTS EXPECT MULTIPERIOD PAYMENTS OF CLAIMS, THE SIMPLE RATE OF RETURN MODEL MUST BE RESET WITHIN THE MULTIPERIOD CONTEXT TO BE PRACTICAL. THESE ARE NECESSARILY APPROXIMATE METHODS. THEY HAVE BEEN IMPLEMENTED BY STONE AND FAIRLEY [3,(1)] AND MODIGLIANI AND HILL [3,(2)] AND DISCUSSED EXTENSIVELY AS TO FORM BY MAHLER [11]. OF NOTE, IS THE FACT THAT THE FAIRLEY MODEL COMBINES THE GENERAL RATE OF RETURN APPROACH WITH A SPECIFIC FINANCIAL RATE OF RETURN MODEL CALLED THE CAPITAL ASSET PRICING MODEL (CAPM). THIS RESULTS IN A WORKABLE EQUILIBRIUM SOLUTION MATCHING THE INVESTOR'S EXPECTED RETURN ON EQUITY WITH THE INSURANCE COMPANY'S EXPECTED RETURN ON OPERATIONS. THE UNDERWRITING PROFIT MARGIN IS A RESIDUAL. A VERSION OF THE FAIRLEY MODEL WAS USED IN MASSACHUSETTS FROM 1978 TO 1981. ALTERNATIVELY, INTERNAL RATE OF RETURN MODELS CAN BE DEVISED WITH MULTIPERIOD CASH FLOWS AS EXEMPLIFIED IN RECENT FILINGS BY THE N.Y. COMPENSATION INSURANCE RATING BOARD AS WELL AS IN NCCI

FILINGS AROUND THE COUNTRY. A BRIEF EXHIBIT HIGHLIGHTING THE ESSENTIALS OF THE RATE OF RETURN MODELS IS ATTACHED.

A PRESENT VALUE MODEL, ON THE OTHER HAND, DEALS DIRECTLY WITH THE MULTIPERIOD CONTEXT BY SIMPLY EQUATING THE PRESENT VALUE OF THE PREMIUM PAYMENTS WITH THE PRESENT VALUE OF ALL LOSS, EXPENSE AND TAX PAYMENTS. THE PRESENT VALUE MODEL DEVELOPED FOR MASSACHUSETTS BY PROFESSORS MYERS AND COHN[8], AND ADOPTED FOR RATEMAKING IN 1981, HIGHLIGHTED TWO ADDITIONAL REQUIREMENTS FOR INSURANCE CONTRACTS. FIRST, THE PRESENT VALUE OF LOSSES AND EXPENSES MUST BE CALCULATED USING A DISCOUNT RATE ADJUSTED FOR RISK. THIS RESULTS IN USING A DISCOUNT RATE SOMEWHAT HIGHER THAN THE PREVAILING RISK-FREE RATE IN ORDER TO LOAD A POSITIVE EXPECTED PROFIT. SECOND, THE ACTUARIAL PREMIUM MUST CONTAIN A PROVISION FOR THE PRESENT VALUE OF ALL FEDERAL INCOME TAXES, TAXES ON BOTH INVESTMENT AND UNDERWRITING INCOME. THE INCLUSION OF TAXES IS OF THE UTMOST IMPORTANCE FOR REAL APPLICATIONS OF THESE MODELS.[9]

ALTHOUGH TIME LIMITATIONS DO NOT ALLOW ME TO COVER DETAILS, PERMIT ME TO INCLUDE WITH THIS DISCUSSION PAPER RECYCLED COPIES OF EXHIBITS ON THIS SUBJECT FROM A PRESENTATION TO THE CASUALTY ACTUARIES OF NEW ENGLAND

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[8]SEE MYERS AND COHN (3) IN CUMMINS AND HARRINGTON [3].

[9]SEE DERRIG [6].

(CANE). ALSO INCLUDED IS A SAMPLE CAR COMPANY CALCULATION OF AN UNDERWRITING PROFIT PROVISION USING THE MYERS-COHN MODEL, TOGETHER WITH A TEMPLATE FOR THOSE WISHING TO TEST THEIR OWN CALCULATION SKILLS. THE KEY IS NOT SO MUCH IN THE ARITHMETIC BUT RATHER IN UNDERSTANDING THE CONCEPTS AND ASSUMPTIONS WHICH UNDERLIE THOSE DECEPTIVELY-SIMPLE CALCULATIONS. LET ME TURN, IF I HAVE TIME, TO SOME OF THOSE ISSUES.

ISSUES

THE IMPLEMENTATION OF ANY OF THESE APPROACHES TO DETERMINING AN ACTUARIAL PREMIUM, WHICH PRESUMABLY IN "EQUILIBRIUM" WILL BE THE MARKET PREMIUM, FORCES THE ACTUARY TO CONFRONT MANY ISSUES DIRECTLY. THE CANE WORKSHOP EXHIBIT REPRODUCED HERE, LISTS WHAT I BELIEVE ARE THE MAJOR CONSIDERATIONS WHICH ENTER INTO THE SKILLFUL USE OF ANY OF THESE PROFIT MODELS. ALTHOUGH WE COULD SPEND DAYS ON EACH ONE, WE DO THAT IN MASSACHUSETTS RATE HEARINGS[10], I WOULD LIKE TO HIGHLIGHT ONE CURRENT ISSUE -- THE PRICING OF THE TAX REFORM ACT OF 1986 AND ONE GENERAL ISSUE -- PRECISELY THE ONE ACTUARIES MUST PAY STRICT ATTENTION TO -- LOSS AND EXPENSE BIAS.

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[10]THE HEARING ON 1987 PRIVATE PASSENGER AUTOMOBILE RATES CONCLUDED AFTER A RECORD-BREAKING 86 HEARING DAYS STRETCHING MORE OR LESS CONTINUOUSLY FROM SEPTEMBER 1986 TO FEBRUARY 1987.

PRICING THE TAX REFORM ACT OF 1986

THE TAX REFORM ACT (TRA) WAS SIGNED BY PRESIDENT REAGAN ON OCTOBER 22, 1986. IT HAS SET IN MOTION CHANGES TO A GREAT MANY PARTS OF THE FEDERAL TAX CODE. AN ANALYSIS OF THE TEXT OF THE NEW TAX LAW, EXAMPLES OF HOW THE TAX BURDEN WILL BE CALCULATED, AND AN ANALYSIS OF INVESTMENT STRATEGIES WERE ALL COVERED NICELY IN A MAY, 1987 CAS DISCUSSION PAPER BY OWEN GLEESON AND GERALD LENROW [9]. MY SUMMARY ANALYSIS WILL DEAL WITH THE MOST IMPORTANT ASPECTS OF THE TAX CHANGES AS THEY WILL AFFECT PROPERTY-CASUALTY INSURERS, ESPECIALLY HOW THEY WILL AFFECT THE ACTUARIAL PRICING OF THE PROPERTY-CASUALTY INSURANCE CONTRACT.[11] THE PRICING EFFECTS OF THE CHANGES WILL ALL BE FELT IN THE CALCULATION OF THE UNDERWRITING PROFIT PROVISION, A CALCULATION NOT NECESSARILY LEFT TO THE ACTUARY.

THE SUM OF THE EFFECTS OF THE TAX CODE CHANGES ON MASSACHUSETTS PRIVATE PASSENGER AUTOMOBILE INSURANCE IN 1988 WAS TO RAISE THE OTHERWISE-DETERMINED OVERALL UNDERWRITING PROFIT PROVISION FROM -7.8% TO -6.3%. THIS INCREASE OF 1.5% RESULTS FROM THE DIRECT INCORPORATION OF THE REFORM ACT PROVISIONS RELATING TO (1) THE INCLUSION IN TAXABLE INCOME OF A PORTION OF THE UNEARNED PREMIUM RESERVE, THE SO-CALLED "REVENUE OFFSET", (2) THE INCLUSION OF LOSS RESERVE

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[11] FULL DETAILS OF THE CALCULATIONS ARE AVAILABLE UPON REQUEST.

DISCOUNTING FOR INCURRED LOSSES AND EXPENSES, AND (3) THE CORPORATE TAX RATE CHANGE TO 34% FOR TAXABLE YEARS BEGINNING JULY 1, 1987. THE CHANGES TO THE DEDUCTIBILITY, FOR REGULAR TAX PURPOSES, OF STOCK DIVIDENDS AND TAX-EXEMPT INCOME, SO-CALLED "PRORATION", IS INCLUDED IN THE CALCULATION OF THE INVESTMENT TAX RATE.

UNDER THE "REVENUE OFFSET" PROVISION, A PORTION OF THE UNEARNED PREMIUM RESERVE IS INCLUDED IN TAXABLE INCOME. ONE-SIXTH OF 20% OF THE 1986 YEAR-END UNEARNED PREMIUM RESERVE IS INCLUDED BY CALCULATING ITS PROPORTIONAL RELATIONSHIP TO 1988 WRITTEN PREMIUM. ONE FIFTH OF THE CHANGE IN THE UNEARNED PREMIUM RESERVES, 1987 TO 1988, IS INCLUDED BY ESTIMATING THE EXPECTED GROWTH RATE IN THE UNEARNED PREMIUM RESERVE.[12] THE NET EFFECT OF EACH OF THE TWO PARTS OF THE "RESERVE OFFSET" PROVISION IS TO RAISE THE PROFIT PREMIUM BY ABOUT 0.4% FOR A COMBINED EFFECT OF 0.8%.

THE EFFECT ON THE UNDERWRITING TAX LOSS FLOW OF THE DISCOUNTING OF LOSS RESERVES CAN BE CALCULATED FROM AVAILABLE IRS AND MASSACHUSETTS DATA. INDUSTRY DISCOUNT FACTORS ARE APPLIED TO PROSPECTIVE MASSACHUSETTS LOSS FLOWS IN ORDER TO PRODUCE THE EXPECTED TIME PATTERN OF DEDUCTIONS FOR LOSS RESERVES EMANATING FROM AN AVERAGE POLICY. THE NET EFFECT OF THIS PROVISION OF TRA IS TO DELAY THE DEDUCTION FOR

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[12]IN THE ABSENCE OF GROWTH, THIS EFFECT WOULD BE ZERO.

INCURRED LOSSES RELATIVE TO THE TIMING UNDER THE PRE-TRA TAX CODE AND, THEREFORE, INCREASE THE PRESENT VALUE OF THE OVERALL TAX LIABILITY. THE EFFECTS VARY BY SUBLINE FROM NO EFFECT FOR THE PHYSICAL DAMAGE COVERAGES TO INCREASES OF 0.1% FOR PROPERTY DAMAGE LIABILITY AND 0.5% FOR BODILY INJURY LIABILITY COVERAGES. THE OVERALL EFFECT IS AN INCREASE OF 0.2% IN THE NEEDED PROFIT PROVISIONS.

THE EFFECTS OF THE CHANGE IN THE MARGINAL RATE FROM 46% TO 34% AND THE CHANGE IN DEDUCTIBILITY OF TAX-EXEMPT, DIVIDEND AND CAPITAL GAIN INCOME ARE ALL INCORPORATED IN THE CALCULATION OF AN EFFECTIVE[13] INVESTMENT TAX RATE FOR THE AVERAGE U.S. PROPERTY-CASUALTY COMPANY ASSET PORTFOLIO. THE CALCULATION OF AN EFFECTIVE INVESTMENT TAX RATE OF 24.1% FOR 1988 REFLECTS THE TAX ADVANTAGES OF TAX-EXEMPT BOND AND STOCK DIVIDEND INCOME APPLIED TO AN INVESTMENT PORTFOLIO OF 43% TAXABLE BONDS, 33% TAX-EXEMPT BONDS, 23% STOCK AND THE REMAINDER IN MISCELLANEOUS INCOME PRODUCING ASSETS. THE EFFECTIVE TAX RATE UNDER THE PRE-TRA TAX RATES WOULD HAVE BEEN 28.9%.

THE VALUE OF THE EFFECT OF THE CHANGE IN TAX RATES VARIES DRAMATICALLY BY THE LENGTH OF THE LOSS PAYOUT

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[13]THE EFFECTIVE INVESTMENT TAX RATE INCLUDES AN ESTIMATE OF AN IMPLICIT TAX OF 20.7% ON TAX-EXEMPT SECURITIES AND 0.1% FOR THE TAX DUE ON TAX-EXEMPT INCOME UNDER THE PRORATION PROVISIONS. THE ACTUAL INVESTMENT TAX RATE IS ABOUT 20%.

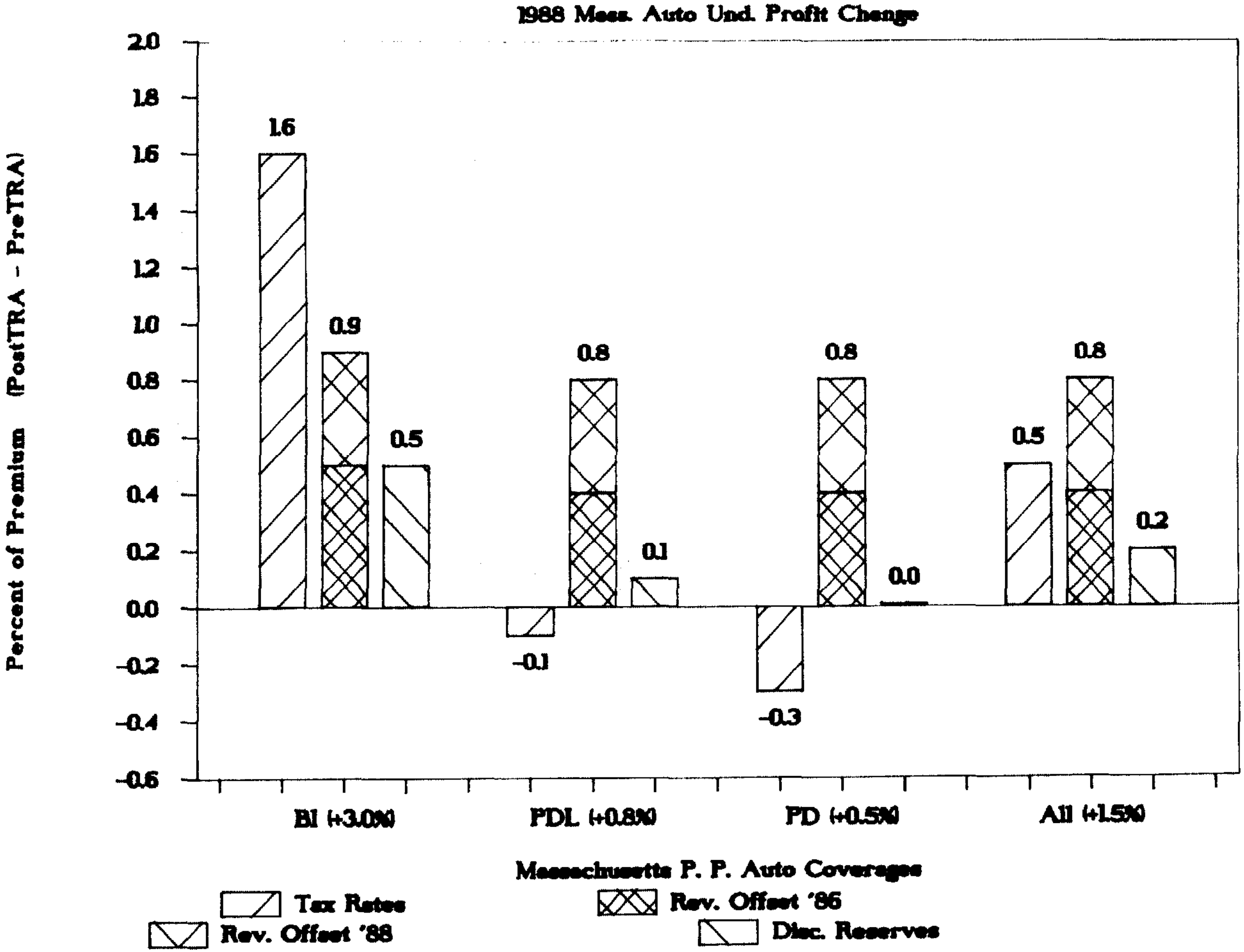
PATTERN. THE LONG BODILY INJURY LIABILITY LINE PROFIT PROVISION INCREASES 1.6% WHILE THE PROPERTY DAMAGE LIABILITY PROVISION DECREASES BY 0.1% AND THE PHYSICAL DAMAGE PROVISION DECREASES BY 0.3%. THE REASON FOR THESE EFFECTS IS TWO-FOLD. FIRST, THE TAX SHIELD GENERATED BY THE DEDUCTION FOR AN UNDERWRITING LOSS HAS DROPPED FROM 46% TO 34% OF THE LOSS. THAT TENDS TO RAISE THE TAX LIABILITY SUBSTANTIALLY ON LONG PAYOUT LINES.[14] SECOND, THE EFFECTIVE INVESTMENT TAX RATES DROP AS WELL, FROM 28.9% TO 24.1%, BUT NOT AS MUCH AS THE DROP IN THE MARGINAL RATE FOR THE TAX SHIELD. THUS, THE LONG LINES ARE AFFECTED BY BOTH CHANGES, IN PROPORTION TO THE LENGTH OF THE LINE, WHILE THE SHORT LINES SUCH AS PHYSICAL DAMAGE, ARE AFFECTED PRIMARILY BY THE DECREASE IN THE INVESTMENT TAX RATE. ON AN ALL AUTO BASIS, THE CHANGE IN THE TAX RATES AND DEDUCTIBILITY OF TAXABLE INCOME RAISES THE NEEDED PROFIT PROVISION BY 0.5%.

ON AN OVERALL BASIS, THE PROFIT PROVISION REQUIRED UNDER TRA HAS INCREASED BY 1.5% FROM WHAT IT WOULD HAVE BEEN UNDER THE PRIOR TAX CODE. THE VALUES OF EACH OF THE EFFECTS BY SUBLINE ARE SHOWN IN THE FOLLOWING GRAPH. IN SUM, THE EFFECTS ARE:

1.	TAX RATE CHANGES	+0.5%
2.	DISCOUNTING RESERVES	+0.2%
3.	RESERVE OFFSET (UPR)	+0.8%
4.	TOTAL	+1.5%

[14] THIS EFFECT IS IN ADDITION TO THE USE OF DISCOUNTED LOSS RESERVES TO CALCULATE THE ACTUAL TAX DEDUCTION.

1986 Tax Reform Act Effect



THE MASSACHUSETTS WORKERS COMPENSATION LINE OFFERS THE OPPORTUNITY TO UNDERSTAND HOW LARGE THE EFFECT MIGHT BE FOR THE CHANGE TO DISCOUNTED LOSS RESERVES FOR TAX PURPOSES.[15] NEW RATES WENT INTO EFFECT ON 1/1/88 THAT INCORPORATED BOTH THE CHANGES IN THE TAX LAW AND IN THE MASSACHUSETTS WORKERS' COMPENSATION LAW. THE RELEVANT FEATURE OF THE NEW MASSACHUSETTS LAW FOR THIS DISCUSSION WAS THE LARGE EXPANSION OF ESCALATED BENEFITS. THAT EXPANSION, TOGETHER WITH ALL OTHER CHANGES, IS EXPECTED TO PRODUCE A LOSS PAYOUT PATTERN IN WHICH THE AVERAGE PAID DOLLAR OCCURS NEARLY FIVE YEARS FROM THE EFFECTIVE DATE OF THE POLICY, ABOUT ONE AND ONE-THIRD YEARS LATER THAN THE AVERAGE FOR WORKERS' COMPENSATION COUNTRYWIDE. THE TABLE BELOW COMPARES THE AVERAGE PAYDATES FOR THE AUTOMOBILE LINES AND THE WORKERS' COMPENSATION LINES FOR MASSACHUSETTS AND COUNTRYWIDE. THE COUNTRYWIDE FLOWS ARE TAKEN FROM THE ACTUAL IRS ACCIDENT YEAR PAYOUT PERCENTAGES USED TO CALCULATE THE LOSS RESERVE DISCOUNT FACTORS.

[15]THE EFFECT OF DISCOUNTED LOSS RESERVES HERE APPLIES FOR NEW POLICY YEARS AFTER 1/1/87 AND, THEREFORE, IS INDEPENDENT OF ANY BENEFITS OF THE SO-CALLED "FRESH START" PROVISION FOR DISCOUNTING RESERVES FOR ACCIDENT YEARS PRIOR TO 1987.

	AVERAGE PAYDATE	
	MASSACHUSETTS (1/1/88 RATES)	COUNTRYWIDE (IRS REV. RULING)
AUTO PHS. DAM.	0.52 YEARS	0.69 YEARS
AUTO PD LIAB.	1.04	-
AUTO BI LIAB.	2.60	-
AUTO LIAB.	2.10	1.98
WORKERS' COMP.	4.88	3.56

THE FOLLOWING GRAPH OF THE CUMULATIVE PAYOUT PATTERNS FOR THESE MASSACHUSETTS AND COUNTRYWIDE LINES SHOW HOW DIFFERENT THEY ACTUALLY ARE BY YEAR.

THE LOSS RESERVE DISCOUNTING EFFECT BY LINE CAN ALSO BE GAUGED TO SOME DEGREE BY COMPARING THE IRS PROMULGATED DISCOUNT FACTORS FOR USE IN DISCOUNTING THE LOSS RESERVES AT THE END OF EACH ACCIDENT YEAR. SINCE THE IMPLICIT DISCOUNT FACTOR UNDER THE PRE-TRA TAX LAW WAS 1.0000, THE SIZE OF THE DISCOUNT FACTOR FOR THE END OF THE ACCIDENT YEAR BY LINE IS IN SOME WAY A MEASURE OF HOW GREAT THE CHANGE WILL BE IN THE TAX LIABILITY.

IRS REVENUE RULING[16]

YEAR ZERO DISCOUNT FACTORS (%)

AUTO PHYSICAL DAMAGE	95.9640
AUTO LIABILITY	89.1776
COMPOSITE SCHEDULE P	84.4514
WORKERS COMPENSATION	81.0030
OTHER LIABILITY	76.7789
MEDICAL MALPRACTICE	68.8804

AS AN EXAMPLE, WE CAN CALCULATE THE EFFECT OF THE LOSS RESERVE DISCOUNTING WITHIN THE OVERALL EFFECT FOR THE MASSACHUSETTS WORKERS' COMPENSATION LINE. USING THE SAME METHOD TO CALCULATE THE EFFECTIVE INVESTMENT TAX RATE AS THE COMMISSIONER USED IN THE 1/1/88 MASSACHUSETTS AUTOMOBILE RATES,[17] THE VALUES OF THE VARIOUS TAX EFFECTS ARE:

1.	TAX RATE CHANGES	+1.5%
2.	DISCOUNTING RESERVES	+2.7%
3.	REVENUE OFFSET	+0.8%
4.	TOTAL	+5.0%

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[16] IRS REVENUE RULING 87-34, IRS BULLETIN 1987-17, 4/27/87.

[17] A DIFFERENT METHOD FOR CALCULATING AN EFFECTIVE INVESTMENT TAX RATE APPROPRIATE FOR POLICYHOLDERS WAS USED IN THE ACTUAL APPROVED RATES. SEE DERRIG [6].

THE RESULTS ARE QUITE DIFFERENT FROM THE AUTO BODILY INJURY LIABILITY LINE, DUE SOLELY TO THE DIFFERENCE FROM THE EFFECT OF DISCOUNTING RESERVES. THE FOLLOWING GRAPH ILLUSTRATES THE COMPARATIVE AUTOMOBILE AND WORKERS' COMPENSATION EFFECTS.

IT MIGHT BE USEFUL TO TRANSLATE THE ABOVE CALCULATION INTO DOLLARS TO GIVE SOME FEEL FOR THE MAGNITUDE OF THE EFFECT OF TRA. PRIOR TO ANY FAVORABLE REMAND DECISION[18], MASSACHUSETTS PRIVATE PASSENGER AUTOMOBILE 1988 PREMIUM IS EXPECTED TO BE ABOUT $2.2 BILLION. THIS MEANS THAT THE PREMIUM VALUE OF TRA IS ABOUT $33 MILLION. COUNTRYWIDE, THE 1987 PRIVATE PASSENGER AUTO NET PREMIUM WRITTEN WAS ABOUT $64 BILLION[19] WHICH, WITH GROWTH, SHOULD PUT THE COUNTRYWIDE PRIVATE PASSENGER AUTO TRA VALUE AT ABOUT $1 BILLION.

IF WE FURTHER ASSUME THAT THE TOTAL INDUSTRY ANNUAL TRA BILL WILL BE THE SAME AS OUR MASSACHUSETTS PRIVATE PASSENGER AUTOMOBILE AT ABOUT 1.5% OF PREMIUMS (PROBABLY AN UNDERESTIMATE, GIVEN RESULTS FOR THE MASSACHUSETTS WORKERS COMPENSATION LINE), THEN FOR A 1987 NET WRITTEN PREMIUM VALUE OF ABOUT $192 BILLION, THE COUNTRYWIDE ALL LINES TRA VALUE WOULD BE ABOUT $3 BILLION. THAT NUMBER IS CLOSE TO RECENT

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[18]THE SUPREME JUDICIAL COURT REMANDED BOTH 1987 AND 1988 RATES TO THE COMMISSIONER FOR REVIEW. RETROACTIVE RATE INCREASES OF 8.3% AND 7.7% WERE GRANTED ON MARCH 10 FOR 1987 AND 1988 POLICY YEARS RESPECTIVELY.

[19]BEST'S MANAGEMENT REPORTS, 12/28/87.

REPORTS OF THE ESTIMATED INDUSTRY TAX BILL FOR 1987 OF $2.8 BILLION. OTHER ANALYSTS[20] MIGHT APPROACH THIS PRICING PROBLEM IN DIFFERENT WAYS BUT I BELIEVE THAT THIS LITTLE IMPRECISE EXERCISE SHOWS QUITE CLEARLY THAT (1) THE PRICING CHANGE DUE TO TRA '86 IS NON-TRIVIAL AND ONE WHICH SHOULD BE OF GENUINE CONCERN TO RATEMAKING ACTUARIES AND (2) THE TRA '86 TAX BILL IS PROBABLY GOING TO BE MUCH LARGER THAN THE POPULAR PRESS ACCOUNT OF $7 BILLION OVER 1987-92 WHEN TRA WAS PASSED.

NO PROVISION FOR THE EFFECT OF THE ALTERNATIVE MINIMUM TAX (AMT) PROVISIONS ON COMPANIES HAS BEEN INCLUDED. THE AMT IS DESIGNED TO PRODUCE A LONG-RUN MINIMUM TAX RATE ON ALL INVESTMENT AND UNDERWRITING INCOME OF AT LEAST 10%. THE AMT CAN ONLY INCREASE THE FEDERAL INCOME TAXES THAT COMPANIES MUST PAY, SO THAT OMISSION OF CONSIDERATION OF THIS SUBJECT HAS THE EFFECT OF POSSIBLY UNDERESTIMATING THE NEEDED PROFIT PROVISIONS.

BIASED LOSS ESTIMATES

I HAVE INCLUDED THE RATING BUREAU'S MOST RECENT COMPILATION OF THE TRACK RECORDS OF ESTIMATING LOSS PURE PREMIUMS IN MASSACHUSETTS AUTOMOBILE FOR THE MAJOR PARTIES IN THE RATE HEARING PROCESS. IT SHOWS THAT, DESPITE THE

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[20]FOR EXAMPLE, THE RECENT ISO ANALYSIS PROJECTS SURPLUS IMPAIRMENTS OF ABOUT $3 BILLION PER YEAR IN THE ABSENCE OF PRICING CHANGES TO REFLECT THE INCREASED TAX LIABILITY OF TRA.

VENEER OF A REGULATORY PROCESS BENT ON DETERMINING THAT MARKET AND ACTUARIAL PRICES WILL BE THE SAME, THE PARTIES CONSISTENTLY UNDERESTIMATE LOSS COSTS. MOREOVER, THIS UNDERESTIMATION IS AT LEVELS FAR IN EXCESS OF THE EXPECTED TOTAL OPERATING PROFIT OF ABOUT 2% OF PREMIUM THAT THIS COMPLEX MACHINERY EXPLICITLY PUTS INTO THE RATES AS A RISK PREMIUM LOADING.

LOSS COST PREDICTIONS
PREDICTION ERRORS
1978-1986

	AVERAGE ERROR	AVERAGE ABSOLUTE ERROR[21]
MARB	-5	7%
AG*	-12	12
DECISION	-12	12
SRB	-14	14

* 1980-1986 ONLY

BY USING ACTUARIALLY BIASED ESTIMATES, THE MASSACHUSETTS AUTOMOBILE INSURANCE RATE SETTING PROCESS HAS STOOD THE TRADITIONAL MARKUP/COMPETITIVE PROCESS MODEL ON ITS HEAD. THIS IS ILLUSTRATED BY THE ATTACHED GRAPH, A TWIN TO THE PREVIOUS GRAPH, LABELED MASSACHUSETTS ACTUARIAL VS NET PREMIUM. UNFORTUNATELY, INSURANCE COMPANIES IN MASSACHUSETTS CANNOT DEVIATE UPWARD FROM STATE-SET RATES OR EQUIVALENTLY RUN (-20%)

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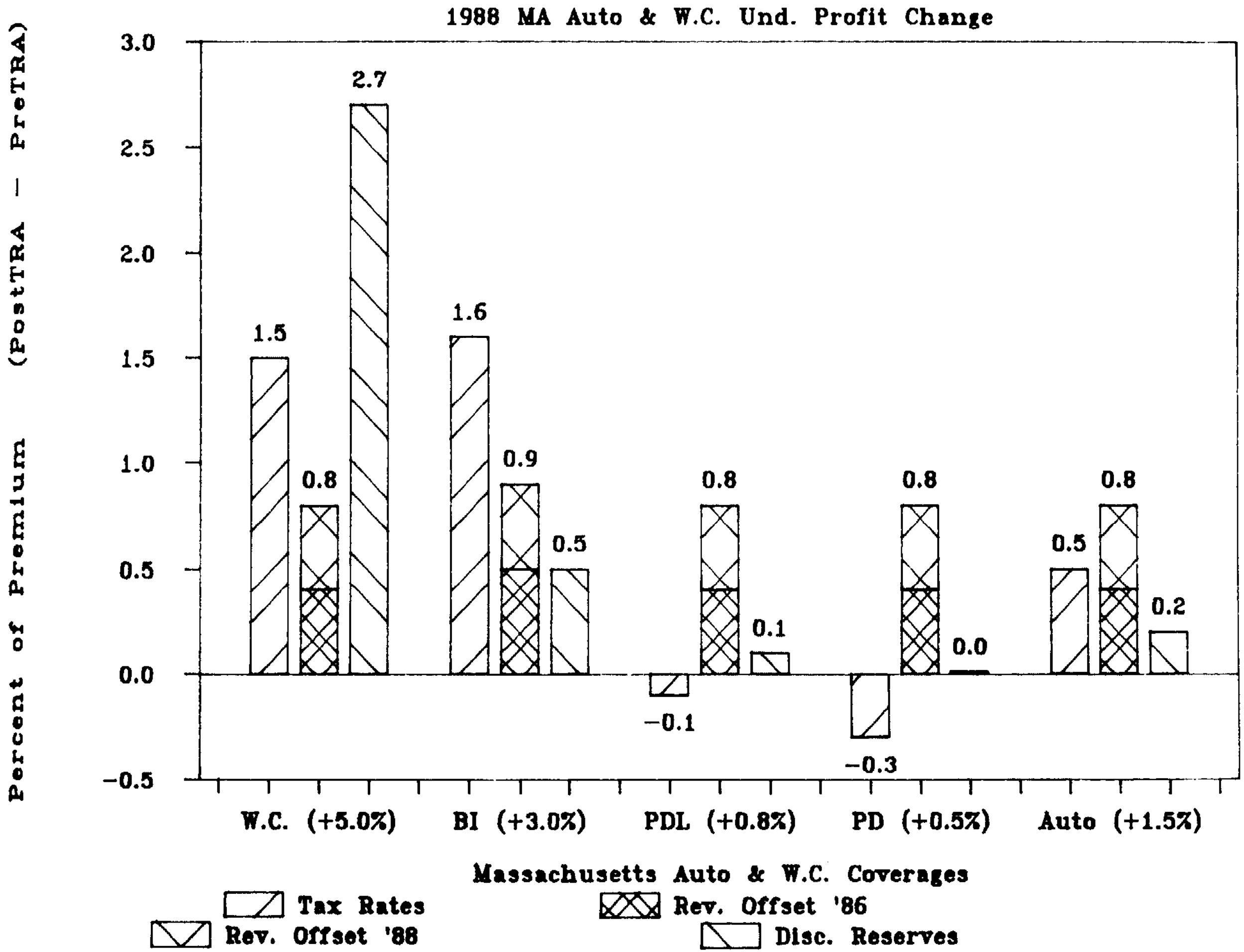
[21] IT IS NO ACCIDENT THAT, EXCEPT FOR MARB, THE AVERAGE ERRORS AND AVERAGE ABSOLUTE ERRORS ARE THE SAME.

1986 Tax Reform Act Effect



OR (-40%) OFF SALES. THE MARCH 10, 1988 REMAND DECISION INCREASING 1987 AND 1988 AUTOMOBILE RATES BY AN ADDITIONAL 8% PER YEAR GOES A LONG WAY TOWARD RECTIFYING THIS BIAS WHILE PROMISING A FULL REVIEW OF LOSS TRENDING METHODOLOGIES FOR 1989 RATES. STAY TUNED TO SEE WHAT HAPPENS.

THANK YOU FOR THE INVITATION TO BE AT THIS SECOND CAS RATEMAKING SEMINAR.

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    2. Generalized Cash Flow Models
    3. Solvency and Risk (OPTIONS)
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    5. Capital Market Considerations
    6. Reserving and Ruin

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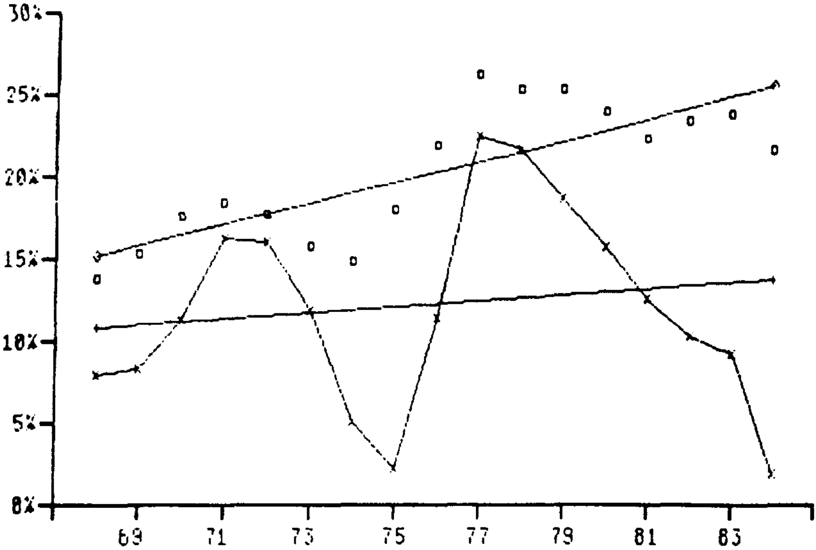
FIGURE IV

Property/Casualty Industry Returns on Net Worth, 1968-1984 and A Line-of-Best-Fit Reflecting Median Returns on Net Worth for All Industries, 1968-1984

Also Returns on Net Worth Effectively Allowed Under Traditional Approaches to Ratemaking, Shown Both as Individual Annual Returns and On A Line-of-Best-Fit Basis



Connected points reflect Property/Casualty industry returns on net worth as calculated by the Texas State Board of Insurance using data provided by the A.M. Best Company.

Lower line-of-best-fit reflects annual median returns on net worth for all industries as reported by *Fortune* magazine.

Unconnected points and upper line-of-best-fit reflect returns on net worth effectively allowed under traditional approaches to ratemaking, as calculated by the Texas State Board of Insurance.

SOURCE: DAVID ELEY 1986, "INVESTMENT INCOME IN RATEMAKING", <u>JOURNAL OF INSURANCE REGULATION</u>, 5:186.

MARK-UP MODELS

PREMIUM = (LOSSES + EXPENSES) x (1 + PROFIT)

FORMALLY,

$$P = \frac{L + E}{1 - \mu}$$

WHERE:

P = PREMIUM (ACTUARIAL)

L = LOSSES

E = EXPENSES

μ = UNDERWRITING PROFIT PERCENT OF PREMIUM

FINANCIAL MODELS

I. RATE OF RETURN MODEL

TOTAL RETURN = INVESTMENT RETURN + UNDERWRITING RETURN

$$R_E \text{ (EQUITY)} = R_A \text{ (EQUITY + RESERVES)} + R_U \text{ (PREMIUMS)}$$

(AFTER TAXES)

$$R_E = (1 - T_A)\ R_A\ (1 + KS) + (1 - T_U)\ R_U\ (S)$$

A. STONE MODEL

$$R_E = (1 - T)\ [R_F + (KS)(1 - R_U) + R_U\ (S)]$$

B. FAIRLEY CAPM MODEL

UNDERWRITING PROFIT = LOAN INTEREST + RISK LOAD + TAXES

$$R_U = -K\ R_F - KB_L\ [E(R_M) - R_F] + [\frac{T}{(1 - T)S}]\ R_F$$

C. INTERNAL RATE OF RETURN MODEL

NCCI & N.Y. COMP. BOARD

FINANCIAL MODELS

II. PRESENT VALUE MODELS

MYERS-COHN VERSION FOR MULTIPERIOD MODEL

1. PREMIUM = LOSSES + EXPENSES + FEDERAL TAXES ON ALL INCOME (AT PRESENT VALUES)

2. PV(PREMIUM) = PV(LOSSES + EXPENSES)
   + PV(FEDERAL TAXES ON INVESTMENTS)
   + PV(FEDERAL TAXES ON UNDERWRITING)

$$K_2P = K_1 (L + E) + T_2R_FK_3P + T_1 [K_4P - K_5 (L + E)]$$

K_1 to K_5 = PRESENT VALUE FACTORS

R_F = RISK-FREE RATE

T_1 = UNDERWRITING TAX RATE

T_2 = INVESTMENT TAX RATE

MASSACHUSETTS AUTO 1982 TO 1984

3. NUMERICAL EXAMPLE (1983 AUTO BI)

P	=	(L + E)	- (INV)	+ (TAX INV + TAX UND)	+ (OP PROF)
\$82.07	=	\$100	- 17.55	+ (5.53 - 7.59)	+ 1.68

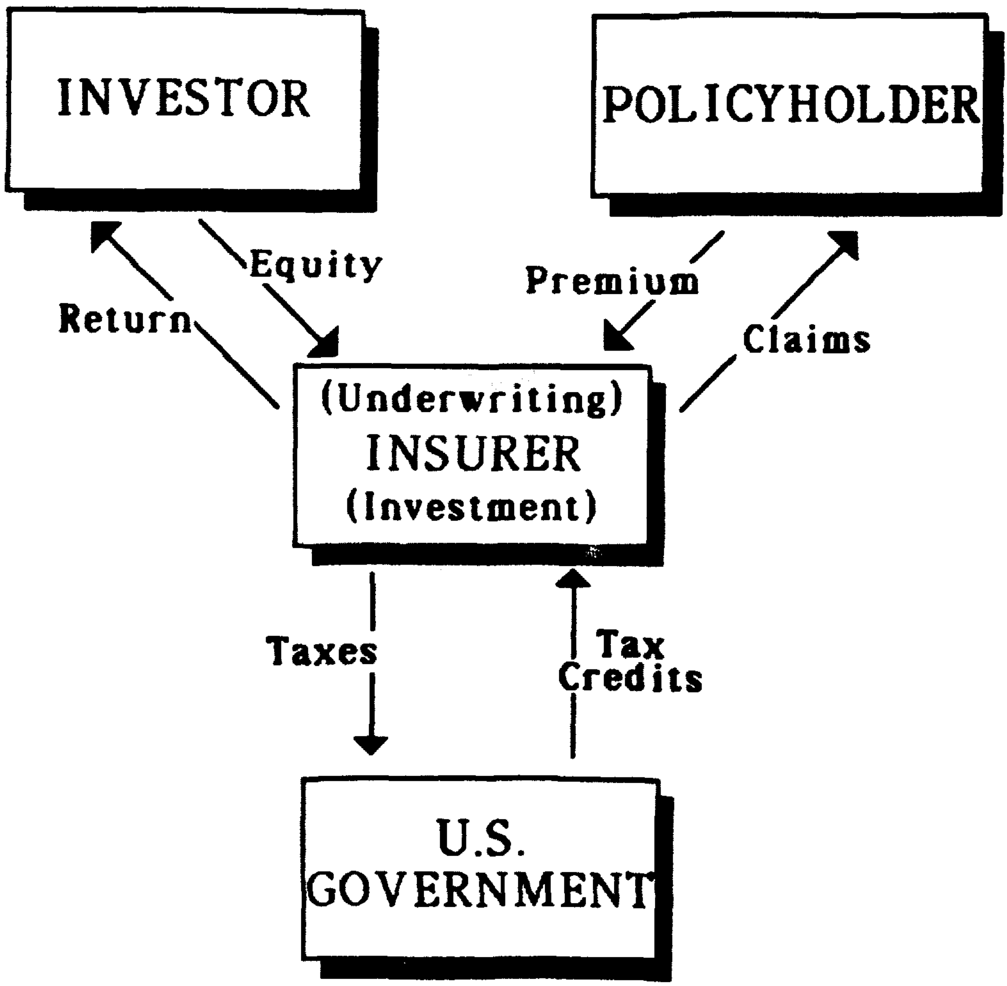
R_F = 14.2%, T_1 = 28%, T_2 = 46%

CANE WORKSHOP

RATEMAKING METHODS FOR EXPLICIT RECOGNITION OF INVESTMENT INCOME

SEPTEMBER 18, 1984
STURBRIDGE, MASSACHUSETTS

CASH FLOW PARTICIPANTS



CANE WORKSHOP

BASIC RATE OF RETURN MODEL

I. METHODOLOGY - A SIMPLIFIED VERSION WITH NO TAXES

1. INSURANCE COMPANY RETURN

TOTAL RETURN = INVESTMENT RETURN + UNDERWRITING RETURN

$$R_E \text{ (EQUITY)} = R_A \text{ (EQUITY + RESERVES)} + R_U \text{ (PREMIUMS)}$$

$$R_E = R_A (1 + KS) + R_U (S)$$

K = RESERVES/PREMIUM

S = PREMIUM/EQUITY

RESERVES = (PREM + LOSS + LAE) RESERVES

2. INVESTOR RETURN

TOTAL RETURN = INVESTED EQUITY RETURN +
INSURANCE OPERATION RETURN

$$R_E \text{ (EQUITY)} = R_A \text{ (EQUITY)} + R_O \text{ (EQUITY)}$$

$$R_E = R_A + [R_A (KS) + R_U (S)]$$

CANE WORKSHOP

BASIC RATE OF RETURN MODEL

II. METHODOLOGY - A SIMPLIFIED VERSION WITH TAXES

1. INSURANCE COMPANY RETURN

TOTAL RETURN = INVESTMENT RETURN + UNDERWRITING RETURN (AFTER TAXES)

$$R_E = (1 - T_A)\ R_A\ (1 + KS) + (1 - T_U)\ R_U\ (S)$$

2. INVESTOR RETURN

TOTAL RETURN = INVESTED EQUITY RETURN + INSURANCE OPERATION RETURN

$$R_E = (1 - T_A)\ R_A + [(1 - T_A)\ R_A\ (KS) + (1 - T_U)\ R_U\ (S)]$$

$$= R_A + [R_A\ (KS) + R_U\ (S) - T_A R_A\ (1 + KS) - T_U R_U\ (S)]$$

WHERE ASSET RETURN = RISK-FREE RETURN + RISK PREMIUM

$$R_A = R_F + R_P$$

3. KEY PARAMETERS

R_E, R_A;	S, K, KS;	T_A, T_U
YIELD;	LEVERAGE;	TAXES

CANE WORKSHOP

BASIC RATE OF RETURN MODEL

III. METHODOLOGY - REGULATORY COMPANY RETURN

A. FOR REGULATORY PURPOSES WE CAN ASSUME A RISK-FREE RETURN ON ASSETS:

TOTAL RETURN = RISK-FREE EQUITY RETURN + INSURANCE OPERATION RETURN

$$R_E = R_F + [R_F (KS) + R_U (S) - T_F R_F (1 + KS) - T_U R_U (S)]$$

MASSACHUSETTS AUTO 1976-1978, 1980-1982

B. FOR REGULATORY PURPOSES BY LINE N WE CAN ESTIMATE THE INSURANCE OPERATING RETURN USING A CAPITAL ASSET PRICING MODEL (CAPM) BETA (B) AND MARKET RISK PREMIUM (M) TOGETHER WITH RESERVES/PREMIUM BY LINE (K_N)

$$R = R_F + BMK_N S$$

MASS. STATE RATING BUREAU 1981-1984

NAIC MODEL A

CANE WORKSHOP

INCLUSION OF INVESTMENT INCOME IN RATES

I. METHODOLOGY - A SIMPLIFIED VERSION FOR SINGLE PERIOD MODEL

1. PREMIUMS = LOSSES + EXPENSES - INVESTMENT INCOME ON CASH FLOW
   + FEDERAL TAXES ON ALL INCOME
   + OPERATING PROFIT

2. PREMIUMS = LOSSES + EXPENSES - RESERVES x INVESTMENT RATE
   + INVESTMENT TAX RATE X SURPLUS X INVESTMENT RATE
   + INVESTMENT TAX RATE X RESERVES X INVESTMENT RATE
   + UNDERWRITING TAX RATE X UNDERWRITING PROFIT/LOSS
   + OPERATING PROFIT

CANE WORKSHOP

INCLUSION OF INVESTMENT INCOME IN RATES

II. METHODOLOGY - MYERS-COHN VERSION FOR MULTIPERIOD MODEL

1. PREMIUM = LOSSES + EXPENSES + FEDERAL TAXES ON ALL INCOM
   (AT PRESENT VALUES)

2. PV(PREMIUM) = PV(LOSSES + EXPENSES)
   + PV(FEDERAL TAXES ON INVESTMENTS)
   + PV(FEDERAL TAXES ON UNDERWRITING)

$$K_2P = K_1 (L + E) + T_2R_FK_3P + T_1 (K_4P - K_5 (L + E)]$$

K_1 to K_5 = PRESENT VALUE FACTORS

R_F = RISK-FREE RATE

T_1 = UNDERWRITING TAX RATE

T_2 = INVESTMENT TAX RATE

MASSACHUSETTS AUTO 1982 TO PRESENT

3. NUMERICAL EXAMPLE (1983 AUTO BI)

P =	(L + E)	- (INV)	+ (TAX INV + TAX UND)	+	(OP PROF)
\$82.07 =	\$100	- 17.55 +	(5.53 - 7.59)	+	1.68

R_F = 14.2%, T_1 = 28%, T_2 = 46%

CANE WORKSHOP

ACTUARIAL ISSUES

1. CHOICE OF MODEL
   A. ECONOMIC VALUE AND DATA
   B. ACCOUNTING VALUE AND DATA
2. AMOUNT OF RESERVES FOR INVESTMENT
   A. TIMING OF CASH FLOWS
   B. ACCOUNTING DATA
3. AMOUNT OF EQUITY (SURPLUS) FOR INVESTMENT
   A. TOTAL
   B. BY LINE
4. INVESTMENT INCOME
   A. YIELD RATE
   B. TAX RATE
5. REWARD FOR RISK-BEARING
   A. DIRECT - TARGET RATE OF RETURN (DCF)
   B. INDIRECT - OPERATING PROFIT (BETA)
6. CONTINGENCY FACTOR
   A. LOSS AND EXPENSE BIAS
   B. UNKNOWN CONTINGENCIES AND BIASES

CANE WORKSHOP

Ratemaking Methods for Explicit Recognition of Investment Income

Massachusetts Method
Car Company Examples

© Richard A. Derrig
Massachusetts Rating Bureaus
Boston, Massachusetts 02109
September 18, 1984

#2579

CANE WORKSHOP

Extended Car Insurance Company Example
Data

1. **Cash Flows**

Loss + Expense/Year	0	0.5	1	1.5	2.5	3.5	4.5	Sum
a. Average Line	0	.6535	0	.2376	.0792	.0198	.0099	1
b. Long Line	0	.3310	0	.2611	.1985	.1433	.0661	1
Premium/Year								
a. Pre-Paid	1	0	0	0	0	0	0	1
b. Installments	.3000	.6000	.1000	0	0	0	0	1

2. **Capital Flow/Prem/Year**

	0	0.5	1	1.5	2.5	3.5	4.5	Sum
a. Policy Year: (Block @ 2 to 1)	.5000	.5000	0	0	0	0	0	1
b. Liabilities Avg:	.5000	.1733	.1733	.0545	.0149	.0050	0	.9210
(Flow @ 2 to 1) Long:	.5000	.3345	.3345	.2040	.1047	.0331	0	1.5108

3. **Tax Flows**

Underwriting/Year	0	0.5	1	1.5	2.5	3.5	4.5	Sum
a. Economic			Same as Loss and Expense					1
b. Accounting	0	.5000	.5000	0	0	0	0	1

Investment/Prem/Year

Cumulative Premium - Cumulative Loss and Expense + Capital, all advanced one period when income is received and taxes paid.

4. **Investment Yield**
   - a. Low Yield 6.00%
   - b. Medium Yield 10.00%
   - c. High Yield 14.00%

Investment Tax Rate
   - i. Low Tax Rate 28%
   - ii. High Tax Rate 46%

Underwriting Tax Rate
46%

5. **Underwriting Risk Adjustment For Investment Yield**
   - a. No Risk 0%
   - b. Low Risk -1.5%
   - c. High Risk -4.5%

CANE WORKSHOP

Calculation of Underwriting Profit Provision

Myers-Cohn Present Value Model

	0	0.5	1	1.5	2.5	3.5	4.5	Sum	Variable
1. Discount Factors/Year									
a. Risk-Free 10.0%	1	.9535	.9091	.8668	.7880	.7163	.6512	-	-
b. Risk-Adj. 8.5%	1	.9600	.9217	.8848	.8155	.7516	.6927	-	-
2. Loss and Expense/Year									
a. Flow	0	.6535	0	.2376	.0792	.0198	.0099	1	-
b. Disc. Risk-Free (2a x 1a)	0	.6231	0	.2060	.0624	.0142	.0064	.9121	-
c. Disc. Risk-Adj. (2a x 1b)	0	.6274	0	.2102	.0646	.0149	.0069	.9240	κ_1
3. Premium/Year									
a. Flow	1	0	0	0	0	0	0	1	-
b. Disc. Risk-Free (3a x 1a)	1	0	0	0	0	0	0	1	κ_2
4. Invest Tax/Prem/Year									
a. Cum Prem	1	1	1	1	1	1	1	-	-
b. Cum Loss & Exp	0	.6535	.6535	.8911	.9703	.9901	1	-	-
c. Capital	.5000	.5000	0	0	0	0	-	-	
d. Inv. Bal (4a - b + c)	1.500	.8465	.3465	.1089	.0297	.0099	0	2.8415	-
e. Yield 10.0%	.0488	.0488	.0488	.1000	.1000	.1000	.1000	-	-
f. Tax Rate 46.0%	.46	.46	.46	.46	.46	.46	.46	-	-
g. Inv. Bal Tax (4d x e x f)/(Year-1)	0	.0337	.0190	.0078	.0050	.0014	.0005	-	-
h. Disc. Inv. Bal Tax (4g x 1a)	0	.0321	.0173	.0067	.0039	.0010	.0003	.0613	$\pi\kappa_3$
5. Underwriting Tax/Year									
a. Und. Tax Prem	0	.5	.5	0	0	0	0	1	-
b. Und. Tax Loss & Exp.	0	.5	.5	0	0	0	0	1	-
c. Disc. Und. Tx. Prem. (5a x 1a)	0	.4768	.4546	0	0	0	0	.9314	κ_4
d. Disc. Und. Tx. L & E (5b x 1b)	0	.4800	.4609	0	0	0	0	.9409	κ_5

6. Underwriting Profit

a. P/(L + E) $\qquad (\kappa_1 - \tau\kappa_5)/(\kappa_2 - \pi\kappa_3 - \tau\kappa_4)$

b. Und. Profit (2c,3b,4h,5c,5d) $\qquad (.9240 - .46(.9409))/(1 - .0613 - .46(.9314)) = .9626$

c. Und. Profit (1 - 1/6b) $\qquad 1 - (.9626)^{-1} = -.0389 = \underline{-3.9\%}$

Massachusetts Method

CANE WORKSHOP

Calculation of Underwriting Profit Provision

Myers-Cohn Present Value Model

	0	0.5	1	1.5	2.5	3.5	4.5	Sum	Variable
1. Discount Factors/Year									
a. Risk-Free ___%									
b. Risk-Adj. ___%									
2. Loss and Expense/Year									
a. Flow									
b. Disc. Risk-Free (2a x 1a)									
c. Disc. Risk-Adj. (2a x 1b)									κ_1
3. Premium/Year									
a. Flow									
b. Disc. Risk-Free (3a x 1a)									κ_2
4. Invest Tax/Prem/Year									
a. Cum Prem									
b. Cum Loss + Exp									
c. Capital									
d. Inv. Bal (4a - b + c)									
e. Yield _____%									
f. Tax Rate _____%									
g. Inv. Bal Tax (4d x e x f)/(Year-1)									
h. Disc. Inv. Bal Tax (4g x 1a)									$r\tau_2\kappa_3$
5. Underwriting Tax/Year									
a. Prem Flow									
b. Loss + Exp Flow									
c. Disc. Prem (5a x 1a)									κ_4
d. Disc. Loss + Exp (5b x 1b)									κ_5
6. Underwriting Profit									
a. P/L + E				$(\kappa_1 - \tau_1\kappa_5)/(\kappa_2 - r\tau_2\kappa_3 - \tau_1\kappa_4)$					
b. Und. Profit (2c,3b,4h,5c,5d)									
c. Und. Profit (1 - 1/6b)									

CANE WORKSHOP

Ratemaking Methods for Explicit Recognition of Investment Income

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September 18, 1984

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INVESTMENT INCOME, UNDERWRITING PROFIT AND CONTINGENCIES; FUTURE DEVELOPMENTS

INVESTMENT INCOME, UNDERWRITING PROFIT & CONTINGENCIES: FUTURE DEVELOPMENTS

By James R. Garven

1. Introduction

In this session's previous presentations, Steve Lehman and Rich Derrig have rather capably set forth the logic underlying the use of financial models in ratemaking as opposed to more traditional "markup" models. Since insurance firms exist in an economic environment within which they must compete for capital with other insurers, financial intermediaries, and even nonfinancial firms, they must therefore concern themselves with delivering competitive rates of return to capital in order to prosper. Furthermore, since the cost of producing insurance services is jointly determined by the firm's investment and underwriting activities, any ratemaking model which ignores the role of investment income will only coincidentally produce a realistic estimate of the actual cost of doing business.

A number of different types of financial ratemaking models have been developed within the last decade. Although these models differ widely in terms of underlying assumptions, parameter specifications, and methods of calculation, they are generally organized around the basic principle that certain targets must be met so as to justify continued or even further allocation of capital to a particular set of insurance activities. The models of Fairley [9], Hill [10], and Hill and Modigliani [11] in particular address this issue by applying the Capital Asset Pricing Model (CAPM) to derive the risk-adjusted rate of return on equity that capital markets require of property-liability insurers.

Unfortunately, CAPM-based ratemaking models suffer from a number of non-trivial problems. First, there are some peculiar difficulties related to parameter estimation.[1] Second, these models do not address the effect of the risk of insolvency on the return to shareholders despite the attention given to this prospect by actuaries and regulators. Third, in spite of the fact that the underwriting and investment activities of

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[1]In order to determine appropriate risk premium loadings on policies, the CAPM approach requires that an "underwriting beta" be estimated. However, as Cummins and Harrington [3] have shown, underwriting betas are extremely difficult, if not impossible to calculate with any degree of accuracy.

property–liability firms often result in underutilized tax shields (this is especially true in recent years), these models typically either ignore taxation altogether or implicitly assume that, once realized, tax shields are always fully utilized.

The purpose of my presentation will be to summarize some recent developments in the theory of ratemaking. In particular, I will focus upon the use of option pricing theory as an alternative to the CAPM in calculating the underwriting profit margin. As Neil Doherty and I have shown elsewhere (see Doherty and Garven [8]), the rationale for applying the theory of option pricing to ratemaking is that the values of the claims held by shareholders, policyholders, and the government are contingent upon the amount of investment income earned by the insurance firm. In addition to its intuitive appeal, the option pricing approach also has several practical advantages over the CAPM. Most importantly, it is not plagued by the problems noted to exist for the CAPM. Furthermore, I will show that CAPM–based ratemaking models can be characterized as special cases of option–based models.

With this general framework in mind, let me provide you with a brief summary of the intuition underlying the option pricing approach to ratemaking. I also plan to provide numerical illustrations in which I compare CAPM–based with option–based underwriting profit margin calculations using workers compensation insurance data. Interested readers can refer to the appendix for further details on the mathematical structure of both the CAPM and the option models. Also, references for further reading are included at the end of this paper.

2. The Option Pricing Approach to Ratemaking

2.1. Some General Comments on Options

Before I demonstrate the option pricing approach to ratemaking, some general comments regarding options are clearly in order. First of all, I will define what an option is. An option is a financial contract which endows its holder with the privilege to either buy or sell a particular asset at a given price within a specified period of time. It is not an obligation to buy or sell, but a choice which may be exercised at the option of the holder. Call options derive value from the possibility that the underlying asset can be purchased at some point in time for a price which is less than the market price, thus securing a profit to the holder. Similarly, put options derive value from the possibility that the underlying asset can be sold at some point in time for a price which exceeds its market price.

Next, consider the source of value from holding an option. To keep the analysis as simple as possible, I will consider the case of a European call option. The holder of a European call option is endowed with the right to buy a security at a future date for a price agreed upon now. The future date is known as the expiration date, and the agreed upon price is the exercise, or striking, price. To clarify the example, we will insert values. Suppose the current price of the underlying stock, P_0, is $95, the exercise price, X, is $100 and the expiration date is 6 months from now. When the option is purchased, the buyer and seller of the option do not know what the price of the stock will be at expiration. The unknown terminal value of the underlying stock is denoted P_t. If the price at expiration is less than the exercise price of $100, the holder of the option would allow the option to expire worthless, since it would not be rational to purchase an asset for a price in excess of its market value. But if the price at expiration exceeds the exercise price of $100, the holder will find it worthwhile to exercise his option and purchase the stock at a price less than the market price. The difference between the terminal market price of the stock and the exercise price represents pure profit to the holder. Thus, the holder is in the enviable position of holding an security that yields nonnegative payoffs at maturity; viz., there is only upside potential. Such a "no lose" position has value, and in a competitive market, the option will trade for this value.

The payoff to the option I just described can be written in the following manner:

$$\text{Payoff on Call Option} = \text{MAX}[P_t - X, 0]. \tag{1}$$

Figure 1 depicts equation (1) graphically.

2.2. Payoffs to Insurance Claimholders

Next, I will show how the limited liability rule as well as the existence of underutilized tax shields cause the payoffs to the claimholders of the insurance firm to resemble options. I start by identifying the principle cash flow to and from the insurance firm. Imagine that the insurance firm is set up at one point in time (e.g., at the beginning of the year, subsequently referred to as t_0) and operated until the end of the period (subsequently referred to as t_1), at which time all liabilities are either discharged or reserved. At t_0, the insurer receives surplus (equity) and premiums and pays its marketing and production expenses. Thus the initial cash flow is

$$Y_0 = S_0 + P_0, \tag{2}$$

Where:

S_0 = the initial surplus;

P_0 = the premiums (net of expenses).

At t_1, allowing for the accumulation of investment income at a rate r_i, the insurer's assets will assume the following value:

$$Y_1 = S_0 + P_0 + (S_0 + kP_0)r_i. \tag{3}$$

The term k is commonly referred to as the funds generating coefficient. This parameter represents the average time delay between premium receipts and claims payments. While this aspect of the model is a somewhat crude correction for the multiperiod nature of claims payments, it is nevertheless a feature common to most financial models, including the CAPM.

2.2.1. The No-Tax Case

Next, consider the manner in which Y_1 would be allocated in the absence of taxes. By issuing insurance policies at t_0, the shareholders are essentially selling the firm's assets to the policyholders in exchange for premium income plus a call option to repurchase these assets at t_1. This call option has an exercise price which is equal to the claims costs (L) which are realized at t_1. Consequently, the terminal payoffs to shareholders and policyholders, S_1 and P_1, can be written

$$S_1 = MAX[Y_1-L,0], \text{ and} \tag{4}$$

$$P_1 = Y_1 - MAX[Y_1-L,0] = MIN[L,Y_1]. \tag{5}$$

Should the firm fare poorly (i.e., if $Y_1-L<0$), then shareholders will rationally choose to not exercise their option to repurchase the firm's assets from the policyholders; consequently, the firm will now be owned by the policyholders. However, should things go well (i.e., if $Y_1-L>0$), then shareholders will find it worthwhile to exercise their option to repurchase the firm's assets by making good on the policies. These payoffs are depicted in Figure 2.

Before considering the effect of taxes, it is worthwhile to reflect for a moment on the relationship between the option model and a CAPM model without taxes, such as that of Fairley. Fairley's no–tax version of the CAPM ratemaking model is a essentially a special case of the option model described here. The primary difference is due to the CAPM's implicit assumption that either the function $MAX[Y_1-L,0]$ is always positive, or that shareholders have unlimited liability. Consequently, under the CAPM model, the terminal payoffs to shareholders and policyholders, S_1 and P_1, are written

$$S_1 = Y_1-L, \text{ and} \tag{6}$$

$$P_1 = L \tag{7}$$

for all possible values of Y_1 and L.

2.2.2. The Effect of Taxes

The analysis is complicated somewhat by taxes. Tax shields are created whenever the insurer incurs losses from either its underwriting or investment activities. Furthermore, it is common practice for insurers to shelter at least a portion of their investment incomes from taxation by purchasing tax-favored financial assets such as municipal bonds and common stocks. Therefore, depending upon how well or poorly the insurer fares, it is possible for some of these tax shields to be underutilized. Although insurers are able to make use of the tax loss carryback/carryforward provision in the tax code, the net effect of tax shield underutilization is to increase the burden of the corporate tax on the insurer as compared to a tax system which would allow for the complete and contemporaneous realization of tax rebates as well as liabilities.

For the sake of simplicity and in the interest of determining an upper bound for the effect of underutilized tax shields on insurance rates, we will assume that a tax liability is incurred if and only if the terminal asset value of the firm (Y_1) exceeds the terminal value of the firm's tax shields (TS).[2] Consequently, the government can be characterized as holding a fractional position in a call option on Y_1, the exercise price of which is equal to TS. The payoff to this option (T_1) is given in equation (8), and depicted in Figure 3; viz.,

$$T_1 = \tau \mathrm{MAX}[Y_1 - TS, 0], \tag{8}$$

[2]By using a single period model, I have implicitly assumed away the possibility of the insurer making use of tax loss carrybacks and carryforwards (CB–CF) which could be introduced in a multi-period framework. However, the effect of the CB–CF provision can nevertheless be readily inferred. Since tax shield underutilization effectively increases the burden of the corporate tax on the insurer, this burden will be passed on to policyholders in the guise of higher insurance prices and underwriting profit margins, everything else the same. However, since the effect of the CB–CF provision is to reduce this tax burden, lower insurance prices and underwriting profit margins would be implied than are predicted by the option model presented here. Interested readers are referred to the recent paper by Majd and Myers [13] which numerically simulates the valuation effects of the CB–CF provision in a multi-period option pricing framework.

where τ is the statutory corporate income tax rate.

In Figure 4, I show the payoffs to the policyholders, shareholders, and the government. The effect of taxes is to decrease the payoff received by shareholders whenever Y_1 exceeds TS. Although taxes do not affect policyholders' terminal payoffs, the burden of taxes falls squarely on the policyholders in the guise of higher premiums than would be the case in the absence of a corporate tax.

As Rich Derrig noted in his presentation, there is much concern in ratemaking over the effects of the Tax Reform Act of 1986. The option pricing model presented here is capable of accomodating all of the effects which he addressed (specifically, tax rate changes, discounting reserves, and unearned premium reserve offsets). Furthermore, the alternative minimum tax (AMT) could be incorporated by changing the specification of the payoff to the tax option shown in equation (8) to the payoff given in equation (9):

$$T_1^{AMT} = MAX[\tau(Y_1 - TS), AMT, 0]. \tag{9}$$

The effect of the AMT on the tax option is shown in Figure 5. Since this provision can only increase the corporate tax burden, the AMT is therefore likely to give rise to even higher insurance prices and underwriting profit margins.

It is interesting to note that the after-tax versions of the CAPM (e.g., see Fairley and Hill and Modigliani) are special cases of the after-tax version of the option model. The primary difference is due to the CAPM's implicit assumption that either the function $MAX[Y_1-TS,0]$ is always positive, or that the tax system allows for the complete and contemporaneous realization of tax rebates as well as liabilities. Consequently, under the CAPM model, the terminal payoff to the government, T_1, is written

$$T_1 = \tau(Y_1 - TS) \tag{10}$$

for all possible values of Y_1 and TS.

2.3. Using Option Pricing Theory to Value Insurance Payoffs

Now that the terminal payoffs to the insurer's claimholders have been determined, it is a fairly simple matter to value them. By applying the appropriate valuation functions to the payoffs given in equations (4), (5), and (8), the current (t_0) values of the claims held by the policyholders (P_0), the government (T_0) and the shareholders (V_e) can be written

$$P_0 = V(Y_1) - C(Y_1;L), \text{ and} \tag{11}$$

$$T_0 = \tau C[Y_1;TS], \tag{12}$$

$$V_e = C[Y_1;L] - \tau C[Y_1;TS]. \tag{13}$$

In the above equations, the function $V(Y_1)$ represents the t_0 value of the cash flow Y_1, $C[Y_1;L]$ represents the t_0 value of the shareholder's option to repurchase the firm's assets from the policyholders at t_1, and $\tau C[Y_1;TS]$ represents the t_0 value of the government's tax option. It is worthwhile noting that the sum $(P_0 + T_0 + V_e)$ is equal to $V(Y_1)$.

2.4. Determining the Competitive Insurance Price and Underwriting Profit Margin

Given the values for P_0, T_0, and V_e as determined by equations (11)–(13), the ratemaking problem is to price the insurance policies such that the shareholders receive a competitive rate of return on their investment in the insurance firm. Such a return would be made for shareholders if the current value of their future payoff is equal to the value of the capital they invest in the firm; viz.,

$$V_e = C[Y_1(P_0^*);L] - \tau C[Y_1(P_0^*);TS(P_0^*)] = S_0. \tag{14}$$

This is an implicit solution to the competitive insurance price. Among other things, the values of the two call options $C[Y_1;L]$ and $C[Y_1;TS]$ depend upon the premiums charged to policyholders. The premiums affect the value of the underlying asset against which these call options are written, as well as the exercise price of the tax option. Thus the solution requires that a level of premiums P_0^* be chosen such that equation (14) is satisfied.

Before equation (14) can be solved for P_0^*, an explicit pricing model must be implemented. Doherty and Garven provide two such models, both of which are summarized in the appendix. Their first pricing model requires assuming that the insurer's claims costs and investment returns are jointly normally distributed, while their second model requires joint lognormality. Furthermore, each model requires further assumptions regarding the nature of investors' risk preferences. Although neither option model provides a closed form solution for P_0^*, P_0^* can be solved for numerically by implementing appropriately parameterized versions of equation (14). Furthermore, P_0^* may be translated into the underwriting profit margin by the routine solution of equation (15):

$$UPM^* = \frac{P_0^* - E(L)}{P_0^*}, \tag{15}$$

where E(L) is the expected value of the insurer's claims costs.

3. Numerical Illustration

In this section I provide a numerical example which illustrates the points of comparison between option–based and CAPM–based ratemaking models. The option—based models were solved iteratively from equations (A–2) and (A–4), whereas the corresponding CAPM models were solved from equation (A–1) and (A–3).[3] The solutions were derived from a set of parameters presented in Table I which are intended as a crude representation of a typical workers compensation insurance business. Table II and Figures 6–11 show the underwriting profit margins required to deliver a competitive rate of return on equity over different ranges of values for the model parameters. Furthermore, I also show the implied probabilities of insolvency and tax shield underutilization for the option–based models in Table II.

The points of interest include the following. In general, the option–based models provide higher underwriting profit margins than the CAPM. The most useful comparison is between the CAPM results and those produced under the normal option pricing model. Since the distributional assumptions are comparable, the differences in underwriting profit margins are explained by the attention paid in the option pricing model to the probabilities of insolvency and underutilized tax shields. The results of the simulations generally reveal the following set of relationships between UPM^* and the model parameters:[4]

$$UPM^* = f(\underset{+}{S_0},\ \underset{-}{k},\ \underset{+,-}{\sigma_i},\ \underset{+,-}{\sigma_L},\ \underset{-}{r_f},\ \underset{+}{\theta}) \qquad (16)$$

[3]Since the results obtained with the lognormal CAPM do not differ materially from the results obtained with the normal CAPM, only the latter model's results are presented here.

[4]See the appendix for definitions of the parameters shown in equation (16).

4. Summary and Discussion of the Relative Merits of the Option Pricing Model

The option–based ratemaking model discussed in this paper is based upon straightforward principles. The insurance firm must discharge a sequence of liabilities to policyholders, the tax authorities and to its shareholders. The values of these respective claims are contingent upon the terminal value of the insurer's assets. Therefore, the option model presented here values the various claims as options written on the insurer's asset portfolio. The competitive price for insurance is derived by choosing the premium such that the present value of the shareholders' claim is equal to the value of their equity (surplus) investment in the firm.

I will conclude my presentation by comparing the features of the option pricing model with CAPM–based models. As my analysis demonstrated, CAPM–based ratemaking models can generally be characterized as special cases of option–based models. Not only are the option–based models more general; they also have several important practical advantages over earlier CAPM models. First, the option model gets around some peculiar difficulties related to parameter estimation. Second, the option model explicitly accounts for the risk of insolvency and will therefore yield an estimate of the probability of ruin which is implicit in the calculation of the competitive insurance price. Third, the option model explicitly models the effects of underutilized tax shields. My numerical calculations reveal that this can have a major impact on the results.

Because of its practical advantages and greater generality, my expectation is that the option pricing approach is likely to do a better job of approximating competitive insurance prices than will the CAPM and other previous, more *ad hoc* models.[5]

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[5] I hope to provide some empirical support for this expectation fairly soon. Steve D'Arcy and I are currently working on a paper which will examine the goodness of fit for several pricing techniques (including target underwriting profit margin, total rate of return, discounted cash flow analysis, the CAPM, and the option pricing model) by comparing predicted model values with actual property–liability insurance industry experience over

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Appendix

In this appendix, I present the analytics which underly the normal and lognormal CAPM and option pricing solutions to the competitive underwriting profit margin as derived originally in Doherty and Garven [8]. I start by assuming that 1) the insurer's claims costs and investment returns are jointly normally distributed, and 2) investors' utility functions exhibit constant absolute risk aversion (CARA). Under these assumptions, the CAPM solution can be written

$$UPM^{*} = -\frac{(1-\theta\tau)}{(1-\tau)}kr_f + (V_e/P_0)\frac{\theta\tau}{(1-\tau)}r_f + \lambda COV(r_u,r_m), \qquad (A\text{-}1)$$

Where:

θ = proportion of investment income that is taxable ($\theta\in[0,1]$);
r_f = rate of return on a riskless asset;
r_u = rate of return on underwriting;
λ = the market price of risk;
= $[E(r_m) - r_f]/\sigma_m^2$.

Hill and Modigliani derive a comparable expression for UPM^{*}, and a similar relationship is derived by Fairley.

Assuming CARA preferences and jointly normally distributed investment returns and claims costs, the functional form of equation (13) is written

$$V_e = (1+r_f)^{-1}\Big[\hat{E}(X)N[\hat{E}(X)/\sigma_X] - \tau\hat{E}(W)N[\hat{E}(W)/\sigma_W] + \sigma_X n[\hat{E}(X)/\sigma_X] - \tau\sigma_W n[\hat{E}(W)/\sigma_W]\Big] \qquad (A\text{-}2)$$

Where:

$\hat{E}(\cdot)$ = the certainty–equivalent expectation operator;[6]
$\hat{E}(X) = \hat{E}(Y_1) - \hat{E}(L) = S_0 + (S_0+kP_0)r_f + P_0 - \hat{E}(L)$;
$\sigma_X^2 = (S_0+kP_0)^2\sigma_i^2 + \sigma_L^2 - 2(S_0+kP_0)COV(L,r_i)$;
$\hat{E}(W) = \theta(S_0+kP_0)r_f + P_0 - \hat{E}(L)$;
$\sigma_W^2 = (S_0+kP_0)^2\theta^2\sigma_i^2 + \sigma_L^2 - 2(S_0+kP_0)\theta COV(L,r_i)$;
$N[\cdot]$ = the standard normal distribution function;
$n[\cdot]$ = the standard normal density function.

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[6]Mathematically, a certainty–equivalent expectation of cash flow is equal to the difference between the expected value of cash flow and an appropriate risk premium as implied by the capital asset pricing model.

As I discuss in the paper, equation (A–2) can be solved iteratively for P_0^*. Once P_0^* is known, UPM^* can be determined by applying equation (15).

Next, I present the lognormal CAPM and option pricing solutions to the competitive underwriting profit margin. By assuming that 1) the insurer's claims costs and investment returns are jointly lognormally distributed, and 2) investors' utility functions exhibit constant relative risk aversion (CRRA), the CAPM solution to the competitive underwriting profit margin is given by equation (A–3):

$$UPM^* = 1 - \left[1+\frac{(1-\theta\tau)}{(1-\tau)}kr_f-(V_e/P_0)\frac{\theta\tau}{(1-\tau)}r_f\right]\exp\{\psi COV[\ln L,\ln R_m]\}, \quad \text{(A–3)}$$

Where:

$$\psi = \text{the market price of risk}$$
$$= \frac{E(\ln R_m) - \ln R_f}{VAR(\ln R_m)} + \frac{1}{2};$$
$$R_m = 1 + r_m;$$
$$R_f = 1 + r_f.$$

Assuming CRRA preferences and jointly lognormally distributed investment returns and claims costs, the functional form of equation (13) is written

$$V_e = V_0^U N(d_1^U) - \tau V_0^T N(d_1^T) - R_f^{-1}P_0\left[N(d_2^U) - \tau N(d_2^T)\right], \quad \text{(A–4)}$$

Where:

$$V_0^U = \text{the contemporaneous value of the claim U}$$
$$= V_0^Y - V_0^L + R_f^{-1}P_0 = S_0 + R_f^{-1}P_0(2+kr_f) - V_0^L;$$
$$V_0^L = R_f^{-1}\hat{E}(L)$$
$$= R_f^{-1}E(L)\exp\{-\psi COV[\ln L,\ln R_m]\};$$
$$d_1^U = \frac{\ln(V_0^U/P_0) + \ln R_f + \sigma_u^2/2}{\sigma_u};$$
$$d_2^U = d_1^U - \sigma_u;$$
$$\sigma_u = \text{the standard deviation of the natural logarithm of U}$$
$$= [\sigma_y^2 + \sigma_l^2 - 2COV(\ln Y,\ln L)]^{1/2};$$
$$\sigma_y = \text{the standard deviation of the natural logarithm of } Y_1;$$
$$\sigma_l = \text{the standard deviation of the natural logarithm of L};$$

V_0^T = the contemporaneous value of the claim T

$= R_f^{-1}[\theta(S_0+kP_0)r_f + 2P_0] - V_0^L;$

$$d_1^T = \frac{\ln\ (V_0^T/P_0) + \ln R_f + \sigma_t^2/2}{\sigma_t};$$

$d_2^T = d_1^T - \sigma_t;$

σ_t = the standard deviation of the natural logarithm of T

$= [\sigma_{\theta\Delta y}^2 + \sigma_1^2 - 2COV(\ln[\theta(Y_1-Y_0)],\ln L)]^{1/2};$

$\sigma_{\theta\Delta y}$ = the standard deviation of the natural logarithm of $\theta(Y_1-Y_0)$.

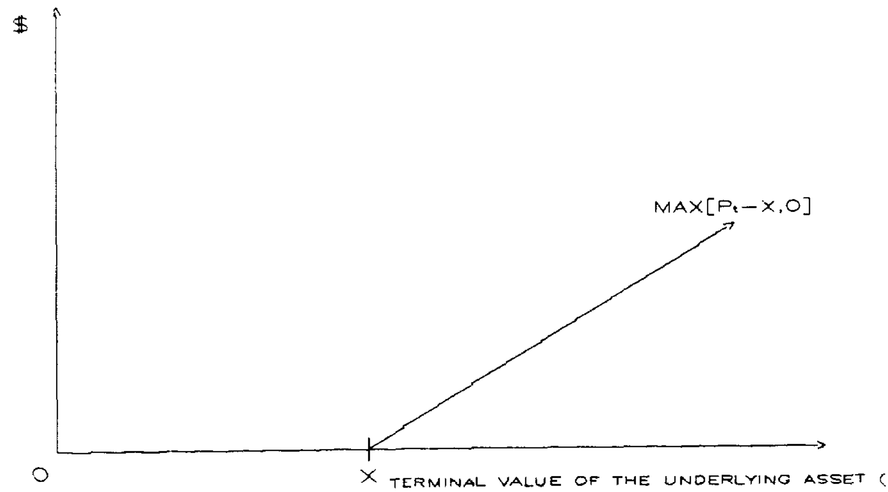


Figure 1: Terminal Payoff on a European Call Option

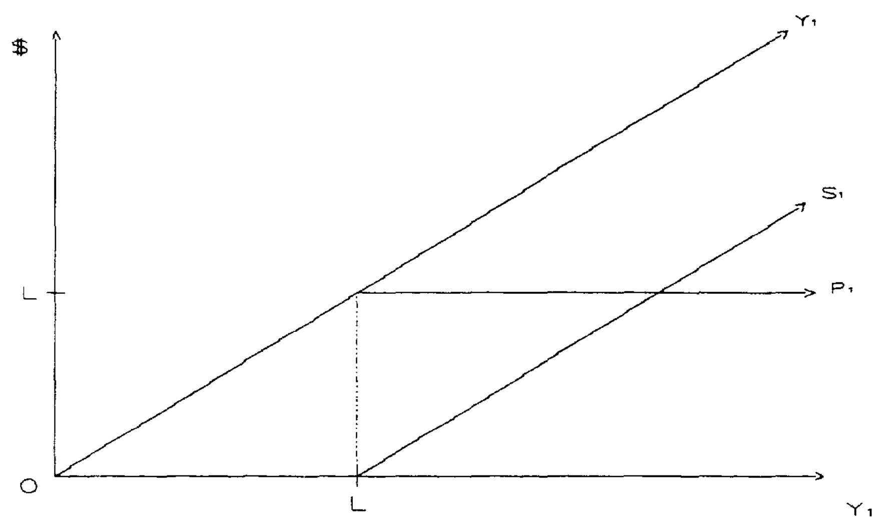


Figure 2: Payoffs to Policyholders (P_1) & Shareholders (S_1) in the Absence of Taxes

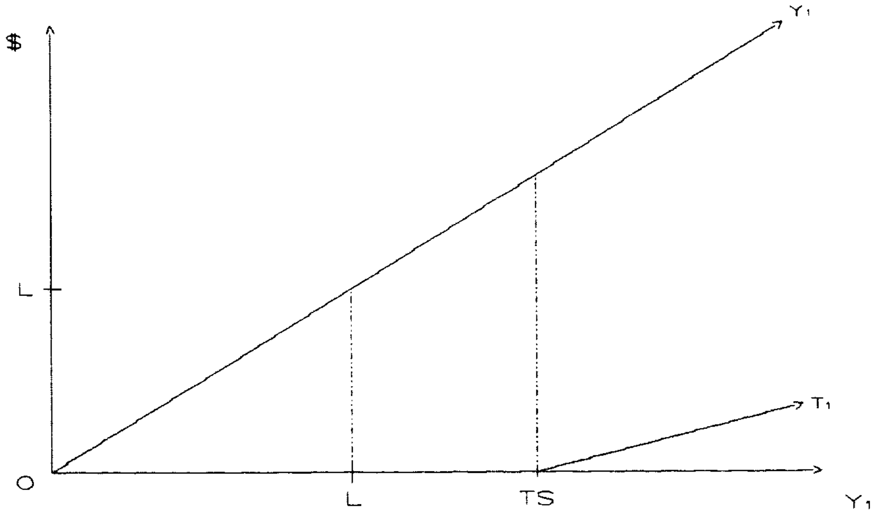


Figure 3: Payoff to the Government (T_1)

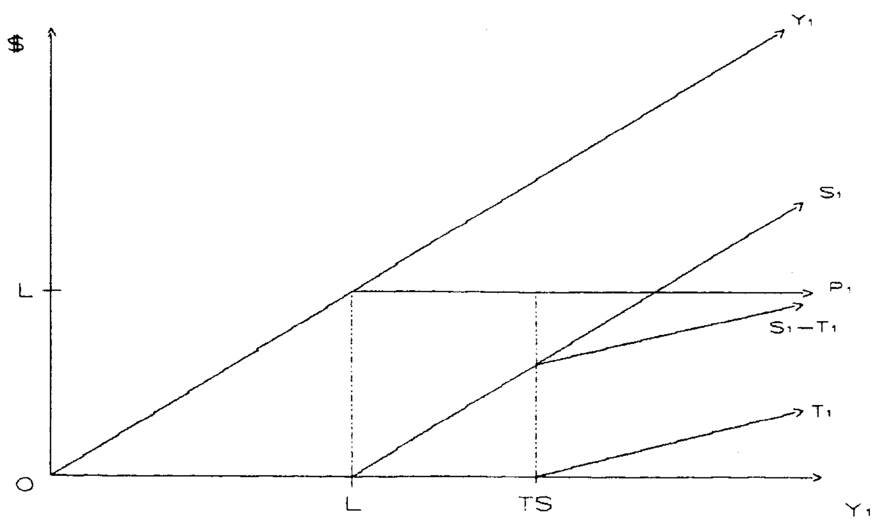


Figure 4: Payoffs to Policyholders (P_1), Shareholders (S_1) and the Government (T_1)

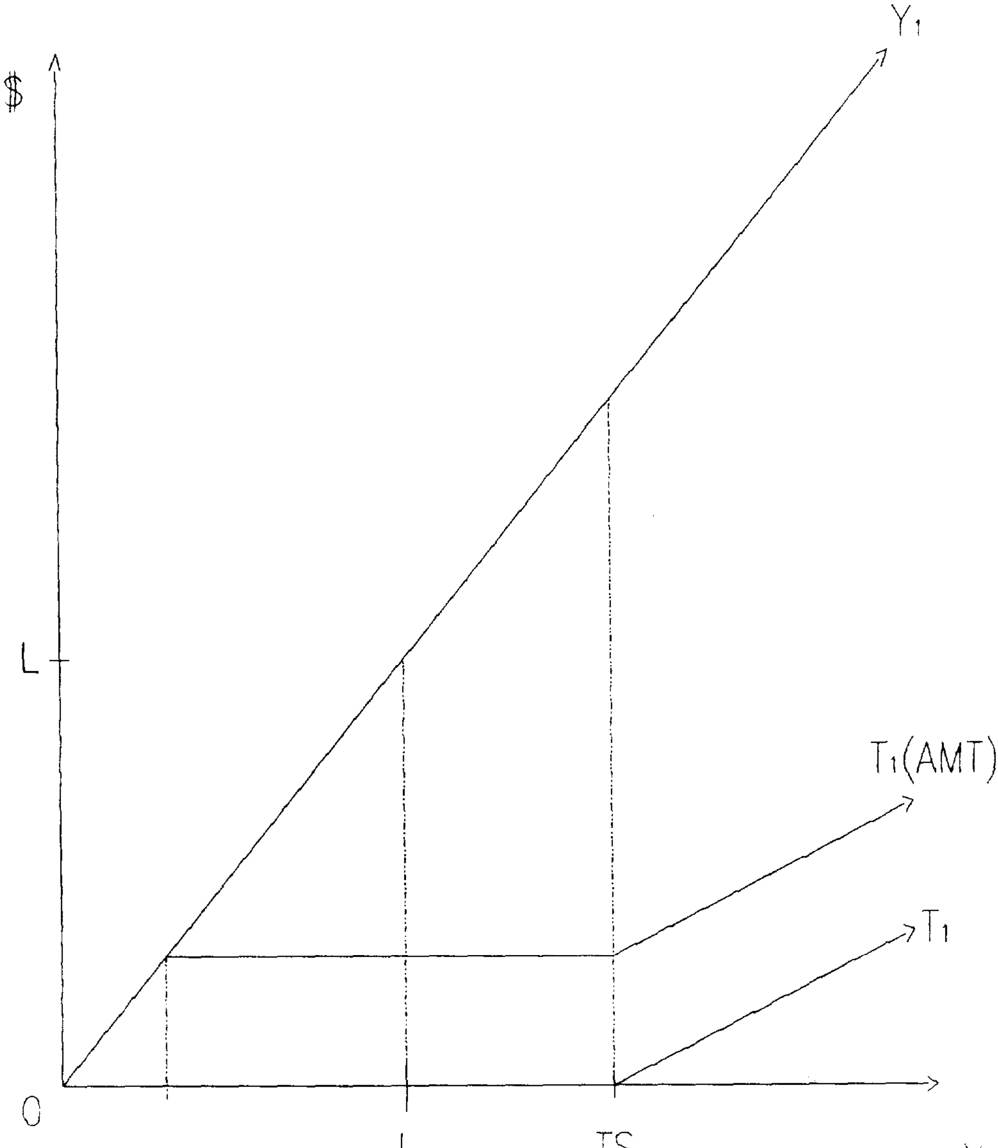


Figure 5: Effect of the Alternative Minimum Tax on the Payoff to the Government

Table I
Model Parameterization: The Base Case

Initial Equity (S_0)	1.00
Funds-Generating Coefficient (k)	2.00
Standard Deviation of Investment Returns (σ_i)	0.0427
Expected Claims Costs (E(L))	1.80
Standard Deviation of Claims Costs (σ_L)	0.142
Correlation Between Investment Returns/Claims Costs (ρ_{iL})	0.114
Riskless Rate of Interest (r_f)	0.07
Statutory Tax Rate (τ)	0.34
Tax Adjustment Parameter (θ)	0.60
Beta of Investment Portfolio (β_i)	0.20
Expected Return on the Market ($E(r_m)$)	0.15
Standard Deviation of Market Return (σ_m)	0.2137

Table II
Effects of Variations in Model Parameters Upon the Equilibrium Rate of Return on Underwriting

Panel A: Effects of Variations in Initial Equity (S_0)

	CAPM	OPM (Normal)			OPM (Lognormal)		
S_0	UPM	UPM	P(default)	P(no tax)	UPM	P(default)	P(no tax)
0.25	−0.1653	−0.1409	0.0469	0.6161	−0.1381	0.0873	0.6430
0.50	−0.1619	−0.1352	0.0013	0.5614	−0.1301	0.0053	0.5884
0.75	−0.1584	−0.1338	0.0000	0.5239	−0.1284	0.0002	0.5575
1.00	−0.1550	−0.1324	0.0000	0.4876	−0.1268	0.0000	0.5287
1.50	−0.1481	−0.1292	0.0000	0.4187	−0.1234	0.0000	0.4741
2.00	−0.1414	−0.1256	0.0000	0.3566	−0.1197	0.0000	0.4241

Panel B: Effects of Variations in the Funds Generating Coefficient (k)

	CAPM	OPM (Normal)			OPM (Lognormal)		
k	UPM	UPM	P(default)	P(no tax)	UPM	P(default)	P(no tax)
0.50	−0.0298	−0.0223	0.0000	0.3449	−0.0167	0.0000	0.3371
1.00	−0.0715	−0.0596	0.0000	0.4005	−0.0538	0.0000	0.4089
2.00	−0.1550	−0.1324	0.0000	0.4875	−0.1268	0.0000	0.5287
3.00	−0.2384	−0.2036	0.0000	0.5453	−0.1988	0.0000	0.6189
4.00	−0.3218	−0.2741	0.0000	0.5836	−0.2704	0.0000	0.6877
5.00	−0.4052	−0.3443	0.0002	0.6098	−0.3416	0.0001	0.7416
6.00	−0.4887	−0.4144	0.0007	0.6287	−0.4127	0.0001	0.7852

Table II (continued)
Effects of Variations in Model Parameters Upon the Equilibrium Rate of Return on Underwriting

Panel C: Effects of Variations in Investment Risk (σ_i)

	CAPM	OPM (Normal)			OPM (Lognormal)		
σ_i	UPM	UPM	P(default)	P(no tax)	UPM	P(default)	P(no tax)
0.00	−0.1550	−0.1311	0.0000	0.4797	−0.1323	0.0000	0.5518
0.20	−0.1550	−0.1389	0.0846	0.5039	−0.1052	0.0606	0.5364
0.40	−0.1550	−0.2949	0.2712	0.5941	−0.1404	0.2530	0.6197
0.60	−0.1550	−0.5582	0.3730	0.6555	−0.2042	0.3868	0.6883

Panel D: Effects of Variations in Underwriting Risk (σ_L)

	CAPM	OPM (Normal)			OPM (Lognormal)		
σ_L	UPM	UPM	P(default)	P(no tax)	UPM	P(default)	P(no tax)
0.25	−0.1550	−0.1261	0.0000	0.4770	−0.1182	0.0011	0.5176
0.50	−0.1550	−0.1107	0.0103	0.4685	−0.1044	0.0495	0.5326
0.75	−0.1550	−0.1057	0.0560	0.4746	−0.1115	0.1469	0.5721
1.00	−0.1550	−0.1177	0.1193	0.4886	−0.1347	0.2399	0.6140
1.50	−0.1550	−0.1950	0.2402	0.5232	−0.1987	0.3778	0.6835
2.00	−0.1550	−0.3555	0.3359	0.5566	−0.2633	0.4680	0.7331

Panel E: Effects of Variations in the Riskless Rate of Interest (r_f)

	CAPM	OPM (Normal)			OPM (Lognormal)		
r_f	UPM	UPM	P(default)	P(no tax)	UPM	P(default)	P(no tax)
0.05	−0.1111	−0.0913	0.0000	0.3370	−0.0854	0.0000	0.4304
0.07	−0.1550	−0.1324	0.0000	0.4875	−0.1268	0.0000	0.5287
0.09	−0.1986	−0.1733	0.0000	0.6316	−0.1680	0.0000	0.6212
0.11	−0.2419	−0.2140	0.0000	0.7528	−0.2091	0.0000	0.7037
0.13	−0.2849	−0.2546	0.0000	0.8442	−0.2500	0.0000	0.7738

Panel F: Effects of Variations in the Tax Parameter Theta (θ)

	CAPM	OPM (Normal)			OPM (Lognormal)		
θ	UPM	UPM	P(default)	P(no tax)	UPM	P(default)	P(no tax)
0.00	−0.1977	−0.1430	0.0000	0.9436	−0.1394	0.0000	0.9602
0.20	−0.1835	−0.1418	0.0000	0.8573	−0.1381	0.0000	0.8585
0.40	−0.1692	−0.1387	0.0000	0.6923	−0.1342	0.0000	0.6975
0.60	−0.1550	−0.1324	0.0000	0.4875	−0.1270	0.0000	0.5286
0.80	−0.1407	−0.1224	0.0000	0.3122	−0.1166	0.0000	0.3869
1.00	−0.1265	−0.1095	0.0000	0.1940	−0.1037	0.0000	0.2807

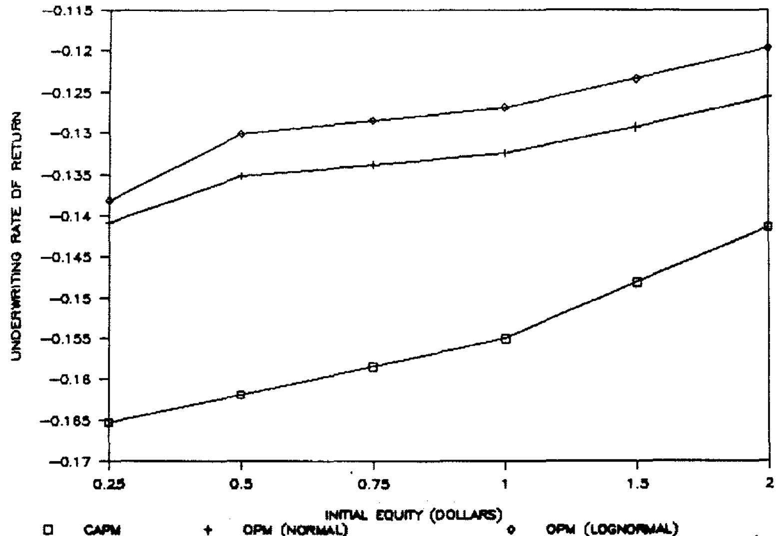


Figure 6: Effect of Variations in Initial Equity (S_0) on UPM*

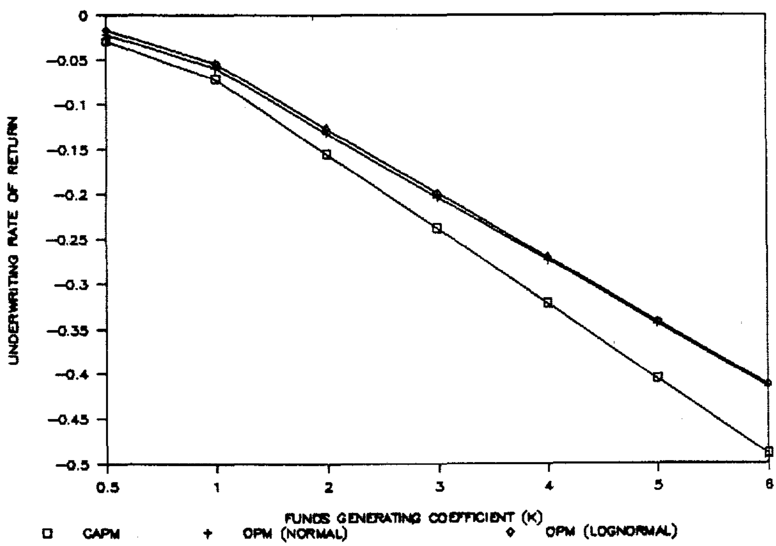


Figure 7: Effect of Variations in the Funds Generating Coefficient (k) on UPM*

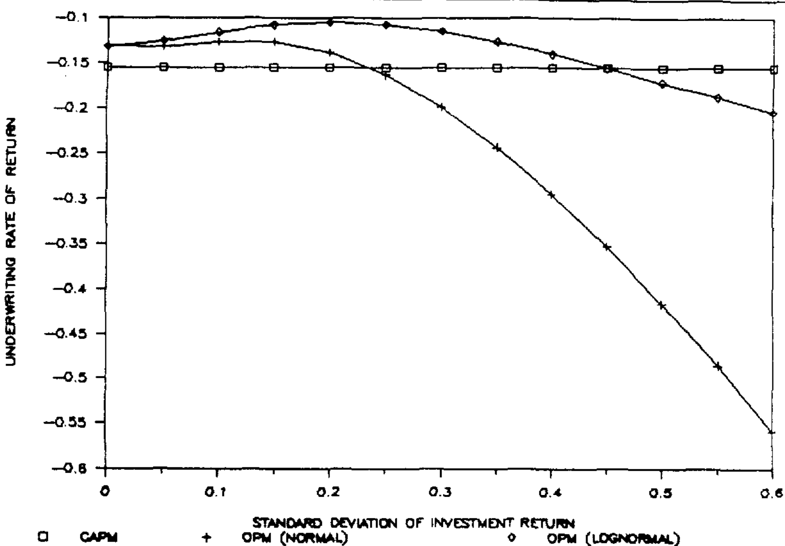


Figure 8: Effect of Variations in Investment Risk (σ_i) on UPM*

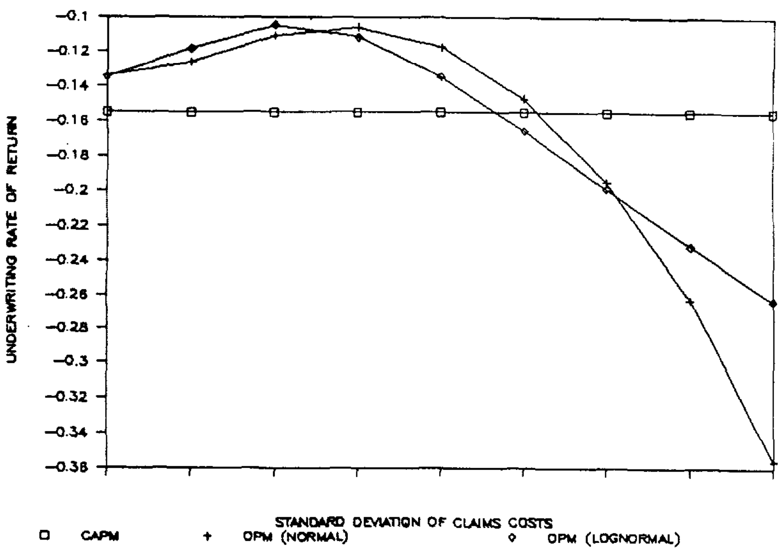


Figure 9: Effect of Variations in Underwriting Risk (σ_L) on UPM*

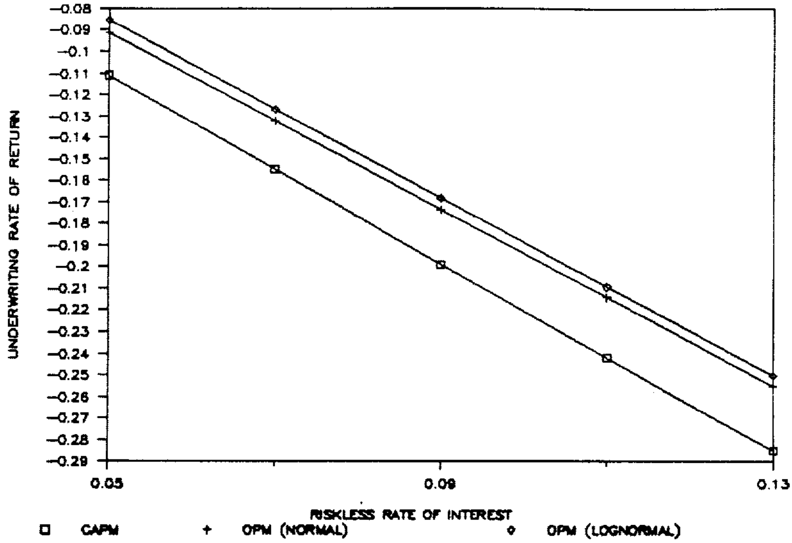


Figure 10: Effect of Variations in the Riskless Rate of Interest (r_f) on UPM*

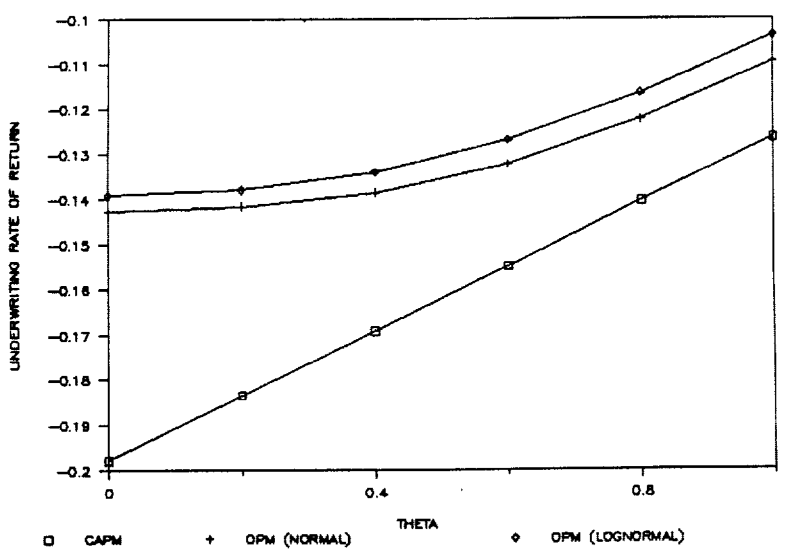


Figure 11: Effect of Variations in the Tax Parameter Theta (θ) on UPM*

1988 CASUALTY RATEMAKING SEMINAR

1988 CASUALTY RATEMAKING SEMINAR

PRICING EXCESS OF LOSS TREATIES

"WHAT SHOULD BE INCLUDED IN A SUBMISSION"
(INTRODUCTORY COMMENTS)

BY MICHAEL PINTER

From the period between 1980 thru 1984, the insurance industry in the United States was in the midst of a period of extreme price competition. This competition, was fueled by unusually high interest rates which suggested that market share and cash flow would more than overcome any statutory price inadequacies. Unfortunately, the drive for market share and cash flow took on a life of itself, resulting in radical price discounts, which time has proven that even the originally anticipated high interest rates could not sustain. To make matters worse, interest rates fell dramatically which quickly spotlighted the folly of the market share/cash flow principle, at least for commercial lines of business.

Meanwhile, the reinsurance marketplace in the United States was emerging from its adolescence in 1980. Results for reinsurers thru the 1970's, apart from natural catastrophe years, were quite good, or at least they appeared to be. As a result, more capacity came into the market, much of it coming from non-traditional sources. Reinsurance was viewed as an easy way to enter the insurance marketplace, which did not require large staffs, major distribution facilities or huge support systems, in order to generate cash flow to take advantage of the interest rate climate. Traditional markets ultimately got caught up as well. All this provided further fuel to the competitive cycle in the primary insurance industry.

There were other contributing factors too. The result, as we all know, led to the overall market reaction of 1985. At that time, reinsureds found that reinsurers, rebounding from a series of unacceptable results, were no longer willing to rely merely on hand-shake agreements without proper examination of exposure. Since that time, the marketplace has stabilized, but reinsurers continue to be selective in their acceptances and require more information from prospective reinsureds. While it's true that a more competitive environment exists in the marketplace today than we have seen in quite some time, it is also true that reinsurers have become more sophisticated. Current technology and analytical methods provide reinsurers with tools to more adequately assess the underwriting risk and hopefully resist the temptations of investment return.

Ultimately, both parties to the reinsurance agreement benefit when the reinsurer is provided with proper and sufficient information to adequately price the reinsurance product. Lack of proper information leads to improper pricing. Extrapolation of improper pricing at the reinsurance level to the entire property and casualty insurance industry leads to one inevitable conclusion: irresponsible competition and repetition of the disaster of the early 80's.

We need to focus on exactly what type of information is necessary for fair and honest pricing of the reinsurance product. We should not forget that the most sophisticated pricing technique is useless, unless the required underlying information is made available.

Reinsurance underwriters today take much more comfort in reinsuring a company which demonstrates a knowledge and ability to price the whole risk from the ground -- Why? Because not only can a good quantitative assessment be made from the underlying information and data available but also a good qualitative assessment can be made on the depth and breath of this information. This is certainly preferable over the company from whom this information is either unavailable or extracted only with great difficulty.

Insurers and reinsurers alike load their pure risk transfer costs for expenses, contingencies and profit. The contingency loading in any pricing -- is designed to be reflective of the degree of uncertainty of the expected outcome. This uncertainty only increases when information is limited or incomplete. Getting a handle on the basic underlying information should increase the efficiency of excess of loss pricing, which in turn should reduce the effects of swings in cycles and lay the foundations for more solid relationships between reinsureds and reinsurers.

OUTLINE OF DATA AND INFORMATIONAL REQUIREMENTS

I. Brief but thorough background on the reinsured and on the business to be reinsured.
   A. History of the company
   B. Types of business written
   C. Business to be reinsured

II. Supporting Reports
   A. Most recent Statutory Report
   B. Most recent Annual Report to stockholders
   C. CPA Audit Opinion
   D. Actuarial Reserve Certification
   E. Actuarial reports (either internal or by consultants) on the gross pricing of the underlying business. Actuarial reports on the associated loss reserving.

III. Underlying Underwriting
   A. Risk selection guidelines
   B. Rating -- Independent of Bureau?
   C. Rate Deviations
      1. Deviations from bureau rates
      2. Schedule Credits deviating from Company's manual rate
      3. Field deviations
      4. Overall deviation impact on rates: magnitude and frequency of deviations

IV. Underlying Business
   A. Policy Limits Profile: Within each category to be reinsured, the distribution of business by each policy limit issued.
      1. Is the distribution on a sample or the universe?
      2. Are the limits gross or net of facultative or other inuring reinsurance?
      3. Are limits expressed on a "from ground up basis" or are they excess of a deductible or retention?
      4. Are the limits on a risk, occurrence, or aggregate basis?
      5. Is allocated loss expense inside the limit?
      6. Exactly to what underlying policy limits is the reinsurance policy exposed?
   B. Composition of the business
      1. By line
      2. Personal vs. Commercial
      3. Geographic Distribution
      4. Tort vs. No-Fualt
      5. General Liability Exposures by type
         a. Form: OL&T, M&C, CGL Premises & Operations, Products
         b. Severity: Low, Medium, High
      6. Worker's Compensation Mix
         a. Distribution by State
         b. Within State, distribution by hazard group
         c. Excess Comp policies? How does the underlying retention affect the reinsurer's exposure?

OUTLINE OF DATA AND INFORMATIONAL REQUIREMENTS

C. Rates
   1. How are the underlying basic limits and excess rates determined?
   2. What is the expected gross loss ratio under the current rating scheme? Does the company rate for their gross line or their net line?

D. Experience
   1. Complete development history (evaluated at equal intervals) on all losses (separately for paid, outstanding, and alae) in excess of one-half the primary retention for the past 5 to 10 years. Did the policy limits censor these losses and to what extent?
   2. Corresponding history of subject premium and projections for the immediate future periods. Has the definition of subject premium been unchanged? Can the subject premiums be restated on current rate level?
   3. Has the company performed any analysis of frequency and severity trends?

V. Proposed Reinsurance Program
   A. Reinsurance Slip
   B. *Contractual Considerations*
      1. How will the reinsurance attach? On a risk, occurrence, or aggregate basis? How is each defined? Risk Attaching or Loss Occurring basis?
      2. Will allocated loss expense be shared and how?
      3. What is the ceding commission and brokerage fee?
      4. Is there a swing plan? What are the parameters? Has the company done any study of the gross aggregate loss distribution? Is there any data available for a loss distributional analysis.
      5. Is the definition of subject premium clear? How will the subject premium be determined? If subject premium is any other than gross underlying premium, how will the gross premium be allocated under the definition?
      6. How does the existence of a primary policy aggregate effect the attachment of per risk or per occurrence reinsurance?

VI. Other Miscellaneous Underwriting Considerations

FINANCIAL GUARANTY INSURANCE

FINANCIAL GUARANTY INSURANCE

INVOLVES

UNCONDITIONAL GUARANTEES

OF DEBT OBLIGATIONS

AND

PROVIDES

GREATER SECURITY

TO THE INVESTOR

BY CLIFTON BROWN

CHARACTERISTICS

. SECURED Vs. NON SECURED

. SHORT Vs. INTERMEDIATE Vs. LONG TERM

. COMMERCIAL Vs. PERSONAL Vs. MUNICIPAL

. RECOURSE Vs. NON RECOURSE

PURPOSE

. CREDIT ENHANCEMENT

. GREATER SECURITY

. LOWER BORROWING COST

. INCREASED MARKETABILITY OF THE DEBT OBLIGATION

BENEFITS TO THE INSURER/REINSURER

. UNDERWRITING PROFITS

. STABILITY OF EARNINGS

. CASH FLOW

MARKET TRENDS

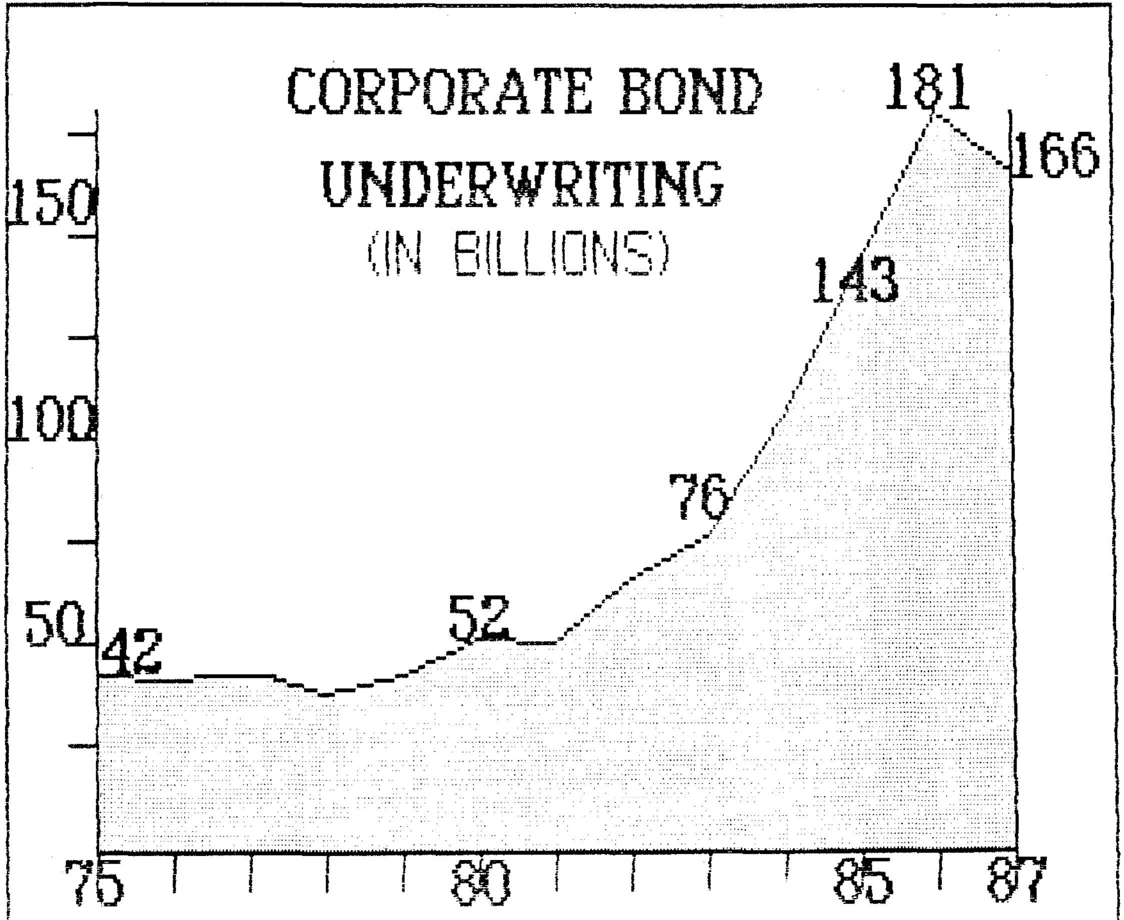
THE INVESTING PUBLIC'S DESIRE FOR GREATER SECURITY

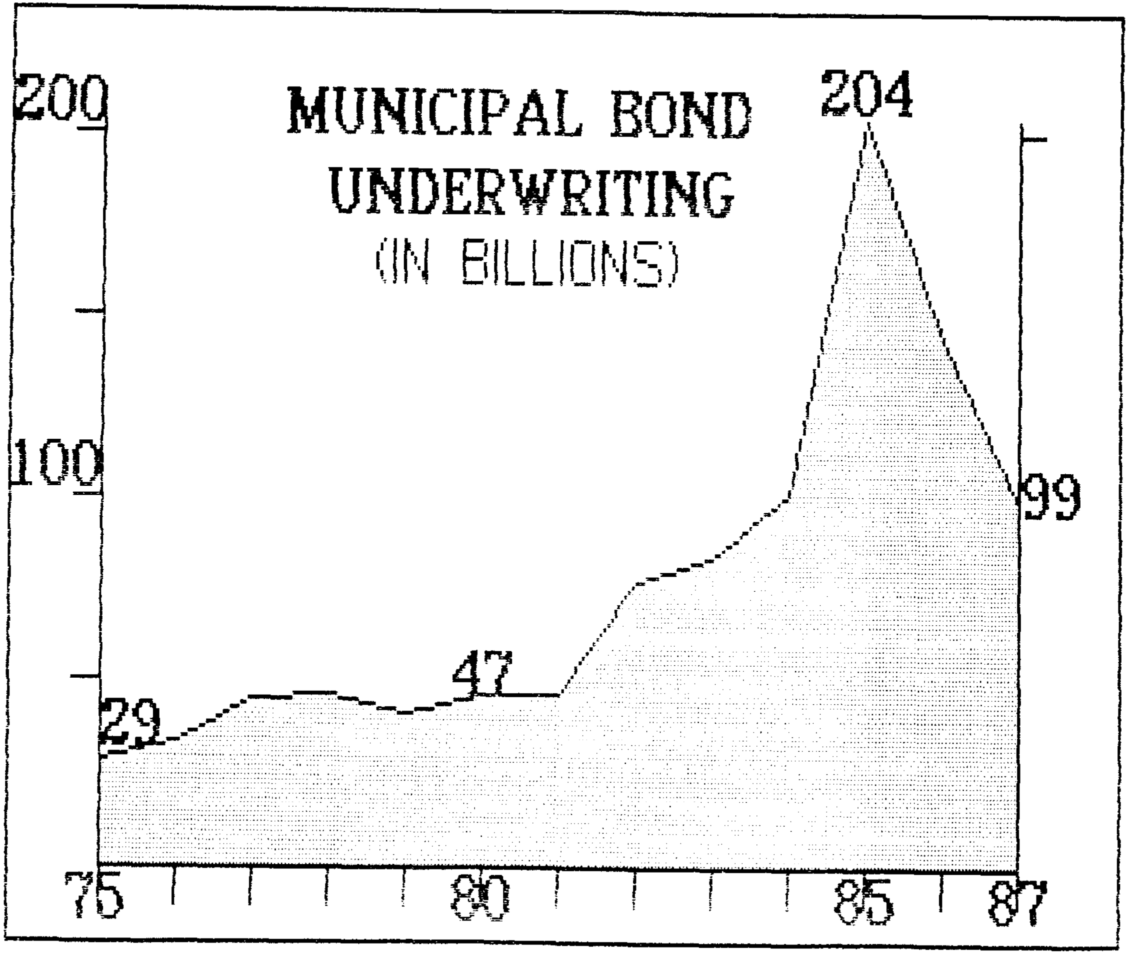
BROKERS AND AGENTS ARE ACTIVELY SEARCHING FOR WAYS TO PROVIDE NEW INSURANCE PRODUCTS AND FINANCIAL SERVICES TO THEIR CLIENTS.

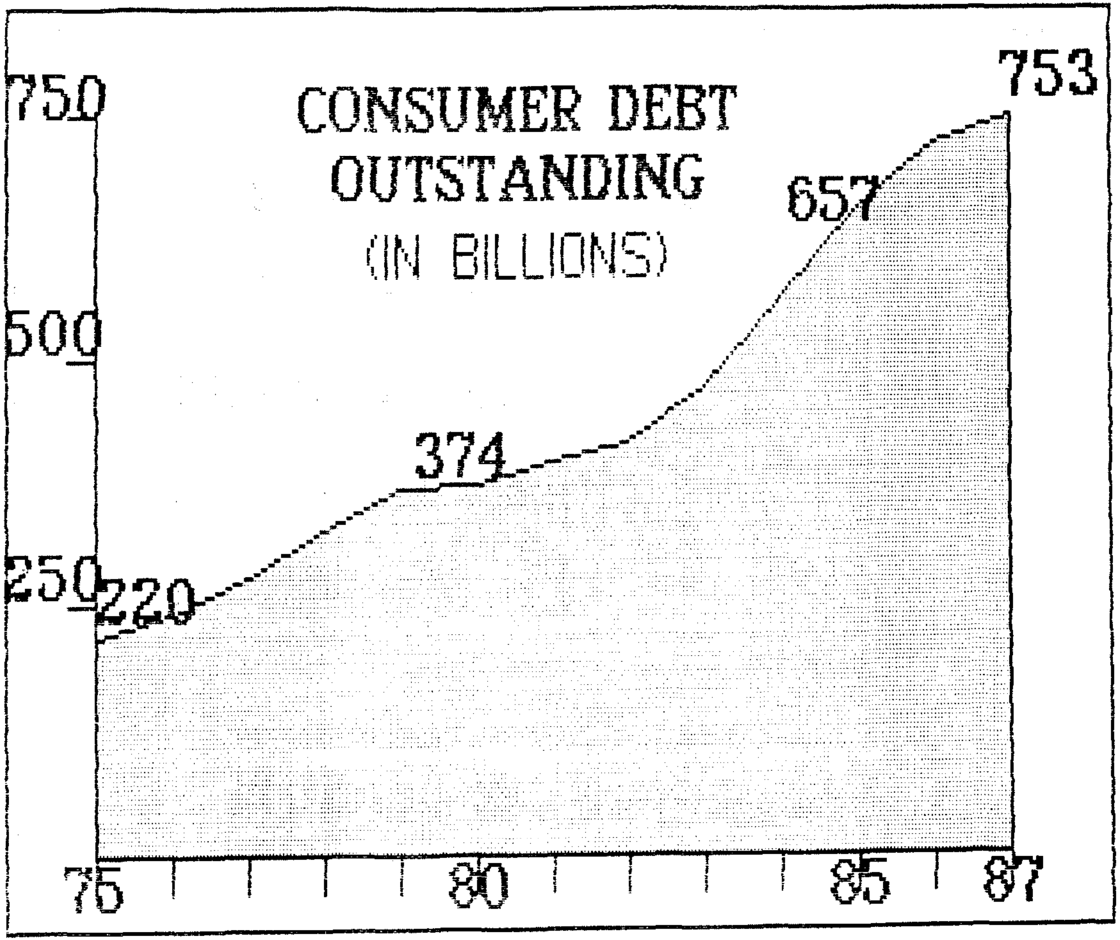
BANKS, PARTICULARLY THE LARGER ONES, ARE REACHING THEIR LIMITS FOR AMOUNTS OF THEIR LETTERS OF CREDIT OUTSTANDING.

MARKET DEBT

1975 - 1987


CORPORATE BOND
UNDERWRITING
(IN BILLIONS)
150
100
50
42
52
76
143
181
166
75
80
85
87


MUNICIPAL BOND
UNDERWRITING
(IN BILLIONS)
200
100
204
99
47
29
75
80
85
87


CONSUMER DEBT
OUTSTANDING
(IN BILLIONS)
750
500
250
220
374
657
753
75
80
85
87

MARKETING STRATEGY

. PROGRAM APPROACH

. SUBSTANTIAL PREMIUM

. REPEAT DEMAND

. SPREAD OF RISK

. THOROUGH RESEARCH AND DEVELOPMENT

TYPES OF TRANSACTIONS

ZERO LOSS

STRUCTURED

LAW OF LARGE NUMBERS

ZERO LOSS BUSINESS

. MINIMAL RISK

. LARGE LIMITS

. LOW PREMIUM

. INVESTMENT GRADE CREDITS

. EXAMPLES : MUNICIPAL BONDS
COMMERCIAL PAPER

STRUCTURED BUSINESS

. MULTIPLE LEVELS OF PROTECTION (DEDUCTIBLES)

. RESERVES WELL IN EXCESS OF EXPECTED LOSSES

. COLLATERAL

. ANY ACCELERATION IS OPTIONAL FOR THE GUARANTOR

LAW OF LARGE NUMBERS BUSINESS

. INDIVIDUAL RISKS ARE SMALL

. LOSSES EXPECTED, BUT PREDICTABLE

. NO CATASTROPHE

. CREDIT ANALYSIS PER RISK IS MINIMAL

. EXAMPLE: CONSUMER LOANS

UNDERWRITING CONSIDERATIONS

- NATURE OF THE OBLIGATION
- PURPOSE OF THE FINANCIAL GUARANTEE
- OBLIGOR - CHARACTER, CAPITAL, CAPACITY
- OTHER PARTIES TO THE TRANSACTION
- STRUCTURE - COLLATERAL, DEDUCTIBLES/ RESERVES, RECOURSE, LEGAL
- PROJECT PROPERTY/ASSETS

PRICING

. ANALYSIS OF PAST DEFAULTS

. RATE – A MULTIPLE OF HIGHEST HISTORICAL DEFAULT RATE

. INTEREST SAVINGS FROM THE GUARANTY

. COST OF THE FINANCIAL GUARANTY

. INCOME STATEMENT/RETURN ON EQUITY

PRODUCTS OFFERED

CORPORATE BOND INSURANCE

LIMITED PARTNERSHIP INVESTOR BOND COVERAGE

EDUCATIONAL LOANS

INVESTMENT CAPITAL INSURANCE

CORPORATE BOND INSURANCE

GUARANTEES THE PROMPT PAYMENT OF PRINCIPAL AND INTEREST ON BONDS FOR COMMERCIAL AND INDUSTRIAL PROJECTS.

CORPORATE BOND INSURANCE

. FIRST MORTGAGE

. DEBT SERVICE RESERVE

. MULTI USE PROPERTY

. RECOURSE

LIMITED PARTNERSHIP INVESTOR BOND

GAURANTEES THAT AN INVESTOR IN A PARTNERSHIP WILL MEET THE FUTURE OBLIGATIONS TO THE PARTNERSHIP.

LIMITED PARTNERSHIP INVESTOR BOND

- . SYNDICATOR AND PLACEMENT AGENT
- . PROPERTY
- . CREDITWORTHY INVESTORS

EDUCATION LOANS

GAURANTEES TIMELY PAYMENT OF PRINCIPAL AND INTEREST IN THE EVENT OF A BORROWER DEFAULT ON EDUCATION LOANS TO FINANCE UNDERGRADUATE OR POST-GRADUATE COLLEGE EDUCATION.

EDUCATION LOANS

. CREDITWORTHY BORROWERS (USUALLY PARENTS)

. ACCEPTABLE ORIGINATORS AND COLLEGES

. FIRST LOSS POSITION

INVESTMENT CAPITAL INSURANCE

INSURES A FRACTION OF THE UNDERLYING VALUE OF EQUITY REAL ESTATE PORTFOLIOS DEVELOPED BY MAJOR REAL ESTATE COMPANIES THAT ARE HELD FOR A FIXED PERIOD OF UP TO TWELVE YEARS.

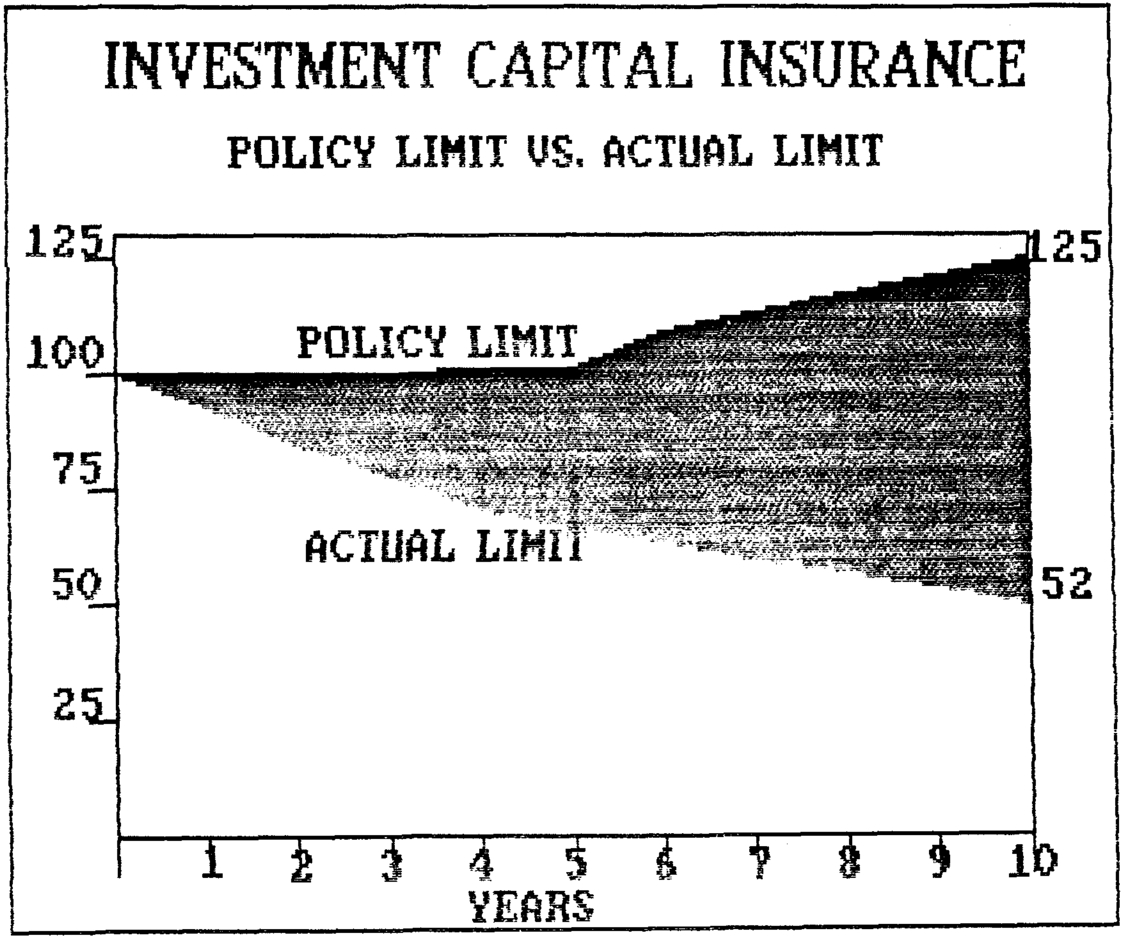
INVESTMENT CAPITAL INSURANCE

. MAJOR SYNDICATORS

. PROVEN REAL ESTATE PROPERTIES

. CROSS COLLATERALIZED

. SPREAD OF RISK BY
LOCATION AND TYPE



INVESTMENT CAPITAL INSURANCE
POLICY LIMIT VS. ACTUAL LIMIT
125
100
75
50
25
125
52
POLICY LIMIT
ACTUAL LIMIT
1
2
3
4
5
6
7
8
9
10
YEARS

THE ROLE OF UNDERWRITING AND MARKETING IN PRICING

THE ROLE OF UNDERWRITING AND MARKETING IN PRICING

BY IRENE K. BASS

GENERAL COMMENTS

Pricing in the global sense means the actuarial, marketing and underwriting process by which manual premiums are determined. Just as pricing is not restricted to the members of the actuarial department, so too are marketing and underwriting efforts not restricted to the employees in those departments. All employees of an insurance company should direct their efforts toward selling, for without a sale, there is no company but the emphasis must be towards selling at a profitable level. Likewise, underwriting cannot be divorced from the pricing process, because prices are not constructed in a vacuum, but rather with specific kinds of insureds in mind.

Rather than just talking about the roles of underwriting and marketing in the personal lines pricing process, I'd like to outline the various phases that can occur while pricing personal lines in a "typical" company and to explore with you along the way, the role of the actuary in all of it.

THE PROCESS OF PRICING

Only part of the pricing process is pure actuarial ratemaking, and often the subtleties of that are not largely appreciated in departments outside of actuarial (Are we the Rodney Dangerfields of insurance, we don't get no respect?). To see how this process works, let's go through what many companies may engage in as the extended pricing process.

Often the process begins in the actuarial department with the preparation of an analysis which includes some information on the rate level indication. Let's focus first on the calculation and presentation of the rate level indication as prepared in the actuarial department. Most readers of the actuarial analysis inside the insurance company probably consider the production of the actuarial rate level indication to be the objective, formula-driven part of the pricing process. However, things may not be as straightforward as one might think.

Does the actuary prepare a rate review with indications that derive straight from some formula which includes a 5% under-writing profit margin, largely ignoring the impact of investment income? Or does she reflect the underwriting profit margin that management would find "acceptable" thus covertly reflecting investment income? Does the actuary reflect budgeted expense provisions? Or does she reflect regional differences in expenses? Or does she rely on the coming year's planned expense provisions instead of relying on the last three year's average from the IEE? How is loss development selected -- is it consistent with that under-lying the company's financial loss reserves, or does it derive from some other system which may not operate in parallel?

All of this suggests that the actuary has an incredible amount of influence before this so-called "objective" document is even released from the actuarial department. Some companies may even go so far as to produce two sets of indications, one reflecting the most they can hope to get by the regulators and one reflecting what management might be happy to settle for under the pressures of regulation, the goals of the marketing department and the financial outlook they are committed to.

Once the actuarial rate level indications are determined, they are usually included in a package with other relevant statistics such as renewal rates, growth statistics, and a general profile of the current insureds with respect to rating characteristics.

The actuarial department prepares these indications and all of the relevant statistics to serve as a LENS through which all of the activities of the company can come into focus for one purpose, projecting them into the future and thereby selecting the appropriate premium. Sometimes, it's like the "ghost of Christmas past". Rarely are future changes in marketing thrust, underwriting criteria, expense control, or general management reflected directly in the actuarial document. And I'm not suggesting that they should be, for they are largely unquantifiable.

The rate level indication in the strictest sense is an estimate of the needed rate change in order to attain the target profit underlying the calculation assuming that there is no change in the way the company currently operates. It is a static picture as just described.

The second part of a typical rate review package includes some type of rate comparison with the perceived competition. I say "perceived", since that company whose rates are lowest is usually perceived to be the current competition. Often the rate level indications are reviewed cursorily by those involved in making the pricing decision, and then all attention is focused on the market comparisons. I am assuming here that it is more than the actuarial department who is involved in making the price-setting decision, regardless of where ultimate authority may rest.

It is an important part of the pricing process to consider the market conditions. This seminar teaches cost-based pricing in several of its sessions, but a free economy tells us that if the cost-based price of the product is $700 and responsible companies are selling the same product at $600, it really doesn't matter what the actuarial indications are. The problem cannot be solved by setting the price at the $700 premium and trying hard to sell all of those "intangibles" that are peddled -- such as better policyholder service, better claims service, account billing (as if get-ting one single, outrageously large bill is better than two separate moderately large bills), readable policies (who reads them at the point of sale anyway??). After all, a $100 difference in price is $100!

The personal lines insurance market is characterized by a lack of product differentiation and by ease of entry into the market. This means that price competition is keen and the buyer will often comparison shop. This is especially true in states such as Texas where the state mandates the maximum rate and the product is a standard one. The buyer of insurance need only know how much the insurer deviates from the state rate to determine which policy to buy.

Getting back to the $700 indicated premium versus the $600 marketplace premium -- something must be done in this situation in order to make the product saleable. The actuary has to work with the underwriters and marketers in reaching logical solutions which will allow the product to be sold at market or near-market rates. And in that process the actuary must be faithful in telling the underwriters and marketers the expected effect on the profitability of the company if certain actions are taken.

Let's first consider the role of the underwriter in this process. Generally speaking, the personal lines underwriter does not have the individual pricing discretion available that the commercial lines underwriter has at his disposal. He must either accept or reject a risk according to a list of underwriting criteria and cannot change the rate in the manual. In the sense that there is this (sometimes unwritten) list of underwriting criteria by which the underwriter either accepts or rejects or places the risk in the preferred, standard or non-standard company, the under-writing criteria are definite extensions of the classification ratemaking that actuaries engage in. The actuary creates a class rate for, say, all drivers who are over 25 years old. The underwriter perceives that this class is not homogeneous and further imposes his judgment as to whether a given risk belonging to this group is better or worse than the average of the group. Criteria such as occupation, length of time in the current job, marital status, number of

speeding tickets, become further sub-classes into which he subdivides the classification.

Needless to say, if the underwriting selection criteria have changed since the time of the gathering of the data under-lying the actuarial ratemaking calculation, then something must be done to put the two in sync. Because the actuarial indication derives from historical experience and is based on the kinds of insureds written in times past, it will apply effectively in the future only if future risks have the same expected underlying costs as in the past. If suddenly this changes, the actuarial indications could be worthless. It is extremely important that this relationship be kept uppermost in the minds of all who are involved with pricing. The underwriting department is often known as the sales-prevention department, and it is in the business of declining risks. For this reason the underwriter tends to be the advocate for higher premium levels.

Speaking of advocates for a higher rate, let's do a 180 degree turn and speak about the marketing department.
These are the people who must actually go out there and try to market the product either to the agents who will in turn sell to the insurance-buying public or who have direct responsibility for selling to the public. Their jobs are a lot easier if rates are lower and since many of them are judged on number of units of sales rather than on a properly constructed loss ratio in conjunction with unit sales, the pressure is on. And their concerns about price are very real. If the actuarial indication is for $700 and the market is operating at $600, what are they to do? The marketplace drives the price in a free economy. And yet the actuarial rate must be attained if the company is to make a profit.

While recognizing that the concerns are real, we actuaries cannot be too eager to believe their arguments for why the future will not be like the past. How many of these arguments have you heard from the marketing folks about why the future will be better? If you have never heard them before, you better prepare yourself with some answers, because you will probably hear them soon if you are involved with personal lines pricing.

* "We're not writing that kind of business anymore. The quality of the business we are going to write will be much better." Somehow the empirical evidence that we have compiled over the years which shows that the loss ratio of new business is worse than the loss ratio of aged business gets lost in this argument.

* "We've changed our emphasis in sales to writing the more expensive home, and more expensive homes are simply better risks. We used to write tar-paper shacks, but now we write only mansions." What they mean here is that there exists such a thing as the objectively "good" risk and that it is totally unrelated to price. At some price every risk is a good risk and at some price, even the best risk is a bad deal. They forget, too, that when the tar-paper shack burned, it didn't cost a lot to replace it. The mansions that burns costs millions.

* "We just appointed a lot of new agents and they are going to give us much better business than our current agents." This is a slight variation of the first example I gave, aimed here

at the producer rather than the insured him-self. The rationale here is usually based on the loss ratios of the prospective agents and comes from their experience with carriers usually already in the agent's office. It's impossible to tell if the new company will get the same business as the current companies or if it is being slated for the left-over business. Another problem is that no mention is generally about the agents remaining with the company. They don't just disappear in general.

* "Except for the two large losses two years ago the experience of this state would be good. You can't let that determine the price level." This kind of comment illustrates that there is a lack of understanding that single, large losses in fact don't drive the rate level indication. But I have never heard the obverse of this statement, namely, "Gee, we were lucky last year that there were no large losses, so I guess the rates should be increased a little to reflect that".

* "We need to keep the homeowners rates lower so that the higher prices we charge for auto will produce a combined price that is competitive." This is the parallel argument to "We better keep the auto rate low so that the combined package with homeowner added in will be competitive." Not a bad argument, but not be applied concurrently!

* We cancelled all our bad agents and so the business we are

going to get will be better". This is similar to the first argument of getting rid of all the bad business, but now they just concentrate on the bad agent. This argument is often used as a reason for adjusting the indications prior to filing them with the regulatory authority. This way, the decrease in rates will be actuarially justified and the regulator will not question the solvency of the company and the adequacy of rates. Of course, carrying this concept through to its ultimate conclusion, I'd like to suggest that the regulator might be just a bit upset that so many agents were cancelled.

* "Our sales reps are better trained this year and are more capable of focusing on the service aspects of our product rather than just the price. And our marketplace is now for the upper income people who don't care so much about the price and are more interested in service". Upper income people didn't get to be upper income people by not caring about how much things cost. Especially one that is undifferentiated in their minds and kind of a pain to have to buy.

* "How can the actuarial indication be so high for homeowners insurance. I just looked at the last 8 months of producer calendar year experience and the loss ratio was great. This isn't reflected in your indications which are all outdated". This argument suggests that the marketing staff needs more education in the area of understanding actuarial indications.

* My own personal favorite was always, "You actuaries live in an ivory tower and so of course you have no concern for the problems we face out there in the real world." Where do we actuaries live? In hyper-space?

And I'm sure that many of you could add to this list. I don't want to make light of these issues. Nor do I wish to trash the marketing departments in general. They have real issues, real concerns in the pricing of their products. It is the actuary's challenge to use her available resources to help in the solution of the problem, not contribute to it.

Before I end this presentation, I'd like to say that there are a couple more aspects to pricing that we have to deal with besides the marketing and underwriting concerns. One could be the planning department which often has made plans or forecasts without the advice of the actuarial department about attainable rate levels for the coming year and the attendant effects on the unit sales and hence the written premium. There can occur a problem if the "plan" becomes ensconced as a part of the culture and worshipped and begins to drive the process.

Another, often overlooked area within the insurance company that has an incredible affect on the pricing posture of a company is the claims department. The actuary must make the claims department aware that the actions they take today will be reflected in tomorrow's prices, and any change they make in procedures should be communicated to the actuarial department so they can be quantified prior to development of a premium level indication.

The other challenge comes from regulators, who must rely on the objective standards of actuarial ratemaking and who must make decisions when sometimes faced with requests for decreases when increases are actually indicated. (This is always the dilemma of the pricing actuary. The indication, usually for a territory, is for an increase and the marketing department wants to decrease the rates. But for another territory with similar indications they want to raise the rates sky-high. How on earth do you accomplish they and still maintain integrity in the pricing system?) Clearly the public would like to have lower insurance premiums, but the solvency of the insurance companies must be preserved or the low premiums will do them no good.

Whatever the source of the pressure, be it underwriting, marketing, planning, or regulation, the actuary must attempt in his role not to be the advocate of anything except the TRUTH.

In conclusion, let me leave you with this thought about the marketing departments of many insurance companies. They often entertain and serve wine at their receptions. Do you know that the favorite wine (whine) in the marketing department is:
"Why do the rates have to be so high?"

COMMERCIAL LINES PRICING: UNDERWRITING AND MARKETING CONSIDERATIONS

COMMERCIAL LINES PRICING: UNDERWRITING & MARKETING CONSIDERATION

BY A. E. KALISKI

DURING THE NEXT FEW MINUTES, I'D LIKE TO TALK ABOUT COMMERCIAL LINES PRICING, PAYING PARTICULAR ATTENTION TO THE VARIOUS UNDERWRITING/MARKETING AND OTHER CONSIDERATIONS THAT INFLUENCE THE FINAL PRICING DECISIONS THAT COMPANIES LIVE AND DIE BY. I WILL BE FOCUSING SPECIFICALLY ON WHAT I FEEL IS THE ACTUARY'S ROLE AND WILL BE RELATING MUCH OF IT TO MY PERSONAL EXPERIENCES AT THE ROYAL (AS WELL AS AREAS WHERE I BELIEVE WE AT THE ROYAL CAN DO A BETTER JOB). ALSO, I WILL BE TALKING ABOUT BUSINESS THAT IS MORE OF A CONVENTIONAL NATURE RATHER THAN THE VERY LARGE, JUMBO NATIONAL COMMERCIAL ACCOUNTS.

WHEN WE SPEAK OF COMMERCIAL LINES PRICING, THERE ARE AT LEAST 2 DIFFERENT SITUATIONS THAT I THINK EACH MERIT SEPARATE DISCUSSION BECAUSE THE ACTIVITIES ARE ENTIRELY DIFFERENT:

1. JUDGMENTAL RATING - THIS IS WHERE WIDE LATITUDE EXISTS FOR THE UNDERWRITER TO CREDIT/DEBIT MANUAL RATES SO THAT HE/SHE CAN COME UP WITH VIRTUALLY ANY PRICE HE/SHE DESIRES.

2. CLASS RATES - THIS IS WHERE MANUAL RATES ARE FILED AND WHERE THERE IS NO (OR MINIMAL) FLEXIBILITY TO CHARGE OTHER THAN THE MANUAL RATE. EXAMPLES ARE BUSINESS OWNER POLICIES, NONFLEET AUTOMOBILE ACCOUNTS.

LET'S FIRST TALK ABOUT THE FIRST CASE, THAT IS WHERE THE PRICE IS ACTUALLY DETERMINED BY THE DESK UNDERWRITER WHO HAS A GREAT DEAL OF FLEXIBILITY VIA INDIVIDUAL RISK RATING PLANS TO CREDIT/DEBIT MANUAL RATES. THIS BUSINESS REPRESENTS A LARGE PART OF THE COMMERCIAL ARENA AND HENCE MERITS A GOOD DEAL OF OUR ATTENTION FOR THIS BUSINESS. WE WORK DIRECTLY WITH THE FIELD UNDERWRITING MANAGERS TO SET OVERALL PRICING GUIDELINES. THESE GUIDELINES CAN BE EITHER IN THE FORM OF CREDITS OFF MANUAL OR PRICE CHANGES ON RENEWAL BUSINESS. LATELY, WE HAVE BEEN GEARING THE GUIDELINE TO TARGET RENEWAL PRICE CHANGES, AS WE FEEL THAT THE MANUAL RATES AND HENCE CREDITS OFF MANUAL HAVE BECOME SOMEWHAT VOLATILE AS A RESULT OF THE CONVERSION TO ISO SIMPLIFIED; RENEWAL PRICE CHANGES, ON THE OTHER HAND, ARE MUCH MORE OF A CONSTANT IN THAT THEY DEAL WITH DOLLARS ON RENEWALS VS. DOLLARS ON EXPIRING POLICIES.

THE FIRST THING WE DO IS TO LOOK AT SOME PAST EXPERIENCE, MAKING ACTUARIAL ADJUSTMENTS TO PROJECT THE LOSS COSTS INTO THE FUTURE AND THE PREMIUMS TO CURRENT PRICE LEVELS. FROM THIS, WE DETERMINE WHAT THE ACTUARIALLY INDICATED ADEQUACY IS OF THE CURRENT PRICE LEVELS, OFTEN EXPRESSING THIS AS A RANGE RECOGNIZING CREDIBILITY, AND OTHER CONSTRAINTS THAT EFFECT THE PREDICTABILITY.

AT THIS POINT, WE BEGIN THE DISCUSSIONS WITH OUR FIELD MANAGERS AND TAKE INTO ACCOUNT THE REAL WORLD PRACTICAL IMPLICATIONS OF THE MARKETPLACE AS WELL AS VARIOUS UNDERWRITING/MARKETING ISSUES. THE FIRST QUESTION WE ASK IS: WHAT IS OUR STRATEGY IN THE GEOGRAPHIC AREA WE ARE TALKING ABOUT - I.E., IS IT AN AREA WHERE WE HAVE ASPIRATIONS TO INCREASE OUR MARKET SHARE AND SEE LONG TERM PROFITABILITY OR IS IT AN AREA WHERE WE ARE CONTENT TO HOLD OR GO DOWN SLIGHTLY. WE THEN LOOK AT VARIOUS PRODUCTION MEASURES TO SEE HOW WE ARE DOING. WE CONSIDER THESE TO BE A GOOD MEASURE OF THE RELATIVE COMPETITIVENESS OF THE PRICING, ALTHOUGH THERE ARE ADMITTEDLY OTHER PHENOMENA THAT ALSO AFFECT PRODUCTION. A KEY INDICATOR WE LOOK AT IS RENEWAL RETENTION (LAST YEAR WE DEVELOPED AND IMPLEMENTED, THROUGH THE PRICING ACTUARIAL DEPARTMENT, A NEAT SYSTEM THAT ALLOWS THE FIELD TO EFFICIENTLY TRACK RENEWAL RETENTION AND SUMMARIZE IT IN A NUMBER OF MEANINGFUL WAYS). WE LOOK FOR 75-80% AS BEING A REASONABLE RENEWAL RETENTION RATIO - ANYTHING OVER 80% IS EXCELLENT AND SUGGESTS OUR PRICING (AND OTHER RELATIONSHIPS) IS COMPETITIVE - ANYTHING LESS THAN 70% SUGGESTS WE HAVE PROBLEMS. WE ALSO LOOK AT NEW BUSINESS WRITINGS. UNDER NORMAL CONDITIONS, NB SHOULD BE ABOUT 25% OF ALL THE BUSINESS WRITTEN SO AS TO BACKFILL NORMAL RENEWAL ATTRITION. IF WE ARE DOING SIGNIFICANTLY MORE OR LESS, IT TELLS US SOMETHING ABOUT

THE COMPETITIVENESS ON THIS BUSINESS. I SHOULD NOTE THAT IN ALL OF THIS, THERE CAN BE ABERRATIONS IN THE PRODUCTION NUMBERS DUE TO LARGE ACCOUNTS THAT ARE WRITTEN OR LOST, DELIBERATE UNDERWRITING ACTIONS, ETC. THIS IS WHERE FIELD MANAGEMENT NEEDS TO HAVE INPUT INTO THE ANALYSIS. FINALLY, RECENT PRODUCTION MEASURES ARE A REFLECTION OF RECENT MARKETPLACE CONDITIONS - IN AGREEING TO A GUIDELINE, THERE MUST BE A CONSCIOUS ASSUMPTION ABOUT WHAT THE MARKET WILL DO IN THE NEAR FUTURE FOR WHICH WE AR SETTING GUIDELINES. AGAIN, FIELD HAS GOOD FEEL FOR THIS, ALTHOUGH THE ACTUARY'S FAMILIARITY WITH THE MACRO SITUATION, HAVING DONE THE PRICING EXERCISE WITH MANY OFFICES, OFTEN IS INFLUENTIAL IN THIS PART OF THE DIALOGUE WITH THE FIELD MANAGER.

SO, TO SUMMARIZE, FROM DISCUSSIONS REGARDING ACTUARIAL INDICATIONS, FROM OUR STRATEGY, FROM OUR RECENT COMPETITIVENESS (PER PRODUCTION MEASURES), WE AGREE TO AN OVERALL GUIDELINE FOR THE TERRITORY. THE TERRITORY WILL THEN TAKE THIS AND REAPPORTION IT IN A VARIETY OF WAYS: BY AREA WITHIN THE TERRITORY, BY CLASS, BY PRODUCER SOURCE, ETC.

A FEW FINAL POINTS ON THIS PART OF THE TALK.

1) IT IS CRITICAL TO HAVE GOOD MONITORS IN PLACE TO TRACK ACTUAL PRICING PERFORMANCE VS GUIDELINE - NO MONITOR IMPLIES NO PROGRAM.

2) FIELD MANAGEMENT, AFTER YOU EARN THEIR CONFIDENCE, REALLY WELCOME THIS TYPE OF PRICING DISCIPLINE. THEY WANT TO BE GUIDED TOWARDS HOW TO PROPERLY PRICE THEIR BUSINESS AND WANT ALL THE HELP THEY CAN GET. IN THE PROCESS, HOWEVER, THE ACTUARY HAS TO BE PRAGMATIC AND PRACTICAL, WILLING TO LISTEN AND UNDERSTAND THE FIELD'S POSITION AND THE DYNAMICS OF WHAT HAPPENS OUT THERE - ALSO, NEED TO BE RIGHT MOST OF THE TIME!

A SECOND MAJOR ACTIVITY RELATIVE TO PRICING IS WITH REGARD TO MANUAL RATES. THESE MANUAL RATES ACTUALLY SERVE AS THE PRICE ON THAT BUSINESS WHERE JUDGMENTAL SCHEDULE RATING IS NOT APPLICABLE - ALSO, AS RESPECTS RISKS WHERE SCHEDULE RATING FLEXIBILITY EXISTS, THESE MANUAL RATES ARE THE STARTING POINTS FROM WHICH THE UNDERWRITER APPLIES THE VARIOUS INDIVIDUAL RISK RATING PLANS. AS RESPECTS THE LATTER, I USED TO THINK THE MANUAL RATES WERE SOMEWHAT ACADEMIC AS THE UNDERWRITER COULD GET WHATEVER PRICE HE/SHE WANTED, GIVEN THE WIDE LATITUDE OF FLEXIBILITY AVAILABLE TO SCHEDULE CREDIT/DEBIT. I HAVE CHANGED MY THINKING SOMEWHAT ON THIS, HOWEVER, AS I TEND TO THINK THAT INCREASES/DECREASES TO MANUAL RATES WILL GENERALLY FLOW INTO THE FINAL PRICES AS THERE IS A GENERAL MINDSET AMONG SOME UNDERWRITERS ABOUT CREDITS. IN OTHER WORDS, A PARTICULAR AN UNDERWRITER MAY HAVE A MINDSET THAT HE SHOULD BE UNWILLING TO DISCOUNT A MANUAL RATE BY MORE THAN 50% OR FEEL THAT 25-30% REPRESENTS LEVELS THAT SHOULD BE AVERAGED OVER THE BOOK - RIGHTLY OR WRONGLY,THIS MINDSET GENERALLY EXISTS WITH RESPECT TO WHATEVER THE MANUAL RATES ARE, BE THEY BUREAU OR BUREAU LESS 20%. SO, IN TERMS OF THE PRICING DISCIPLINE, WHERE THE MANUAL RATES ARE SET IS AN IMPORTANT PART OF THE WHOLE PROCESS.

FOR EACH OF THE COMMERCIAL LINES OTHER THAN WC, ISO RATES ARE GENERALLY A REFERENCE POINT. AS RESPECTS THE PRELIMINARY ANALYSIS, WE GENERALLY DO TWO THINGS. FIRST, WE LOOK AT THE COMPONENTS OF THE ISO INDICATION AND MAKE OUR OWN EVALUATION AS TO WHETHER OR NOT THE INDICATION IS APPROPRIATE. FOR EXAMPLE, FOR SOME LINES, WE LOOK AT THE BASE ISO RATEMAKING EXPERIENCE AND USING PROJECTION FACTORS THAT SEEM MORE TYPICAL OF OUR INTERNAL EXPERIENCE, EVALUATE IF THERE ARE MARGINS IN THE RATE. ALSO, IN STATES WHERE WE HAVE REASONABLE CREDIBILITY, WE CONDUCT A TRADITIONAL ACTUARIAL REVIEW USING OUR OWN DATA. FROM THIS, WE ATTEMPT TO ESTABLISH IF ANY MARGINS EXIST.

A SECOND PART OF THE EXERCISE IS A COMPETITIVE ANALYSIS. GENERALLY, THIS IS DONE BY OUR FIELD OFFICES. THE COMPETITIVE ANALYSIS IS GENERALLY A SPREADSHEET OF MAJOR COMPETITORS AND WHERE THEY ARE RELATIVE TO ISO (I.E., AT ISO, DEFERRED LAST CHANGE, ADOPTED WITH A 15% DEVIATION, ETC.).

ALSO, THE GEOGRAPHICAL AREA, THE STRATEGY FOR THE AREA AND THE LONG TERM PROFIT POTENTIAL COME INTO PLAY AS WELL, SO THAT THE DECISION OF WHETHER TO DEVIATE OR NOT IS A BALANCED ONE BASED ON PROFITABILITY AS WELL AS MARKETPLACE CONDITIONS AND LONGER-TERM STRATEGIES.

ANOTHER POINT - IF A DEVIATION IS INTRODUCED, I THINK IT MAKES SENSE TO PERIODICALLY DIARY (SAY EVERY 3-6 MONTHS) AND RE-SURVEY THE MARKETPLACE. A DANGER THAT EXISTS IS THE FILING OF A DEVIATION AT SOME POINT IN TIME AND LEAVING IT IN PLACE WHEN MARKETPLACE CONDITIONS AND REQUIREMENTS MAY HAVE CHANGED, CAUSING THE DEVIATION TO BE OUTDATED.

IN ALL OF THE PROCESS DESCRIBED, THE ACTUARY IS THE ONE IN OUR COMPANY WHO IS THE FOCAL POINT TO THE ENTIRE PROCESS AND AS SUCH HAS A KEY ROLE.

I'D LIKE TO MAKE A BRIEF COMMENT ON WC, WHICH IS ESSENTIALLY A LINE WHERE ADMINISTERED BUREAU PRICING EXISTS. ON WC, A MAJOR VEHICLE BY WHICH COMPANIES COMPETE IS IN THE PARTICIPATING DIVIDEND PROGRAMS. I FEEL THE ACTUARY IS UNIQUELY QUALIFIED AND

SHOULD BE AT THE FOREFRONT IN THE DEVELOPMENT OF THESE PROGRAMS. SPECIFICALLY, ACTUARIES UNDERSTAND LOSS RATIO DISTRIBUTIONS, INSURANCE CHARGES AND THE NEED TO HOLD BACK A PORTION OF ANY PROFITS GENERATED BY RISKS THAT GENERATE A PROFIT IN ORDER TO FUND FOR UNRECOUPABLE LOSSES ON THOSE RISKS WHICH ARE EXPECTEDLY GOING TO GENERATE LOSSES. I BELIEVE THAT ACTUARIES HAVE THE DISCIPLINE TO PRODUCE A REALISTIC DIVIDEND PLAN, WHEREAS OTHER DISCIPLINES WILL NATURALLY TEND TO BE OVERLY OPTIMISTIC AS RESPECTS THE TREATMENT OF INSURANCE CHARGES.

SO AS NOT TO RAMBLE ON, I'D LIKE TO MAKE JUST A FEW MORE REMARKS AT THIS POINT.

- ACTUARIES RESPONSIBLE FOR PRICING SHOULD GET THEMSELVES INVOLVED, EITHER DIRECTLY OR INDIRECTLY, IN THE PLANNING PROCESS. IN OTHER WORDS, ANY PREMIUM VOLUME TARGETS THAT SERVE AS A FOUNDATION AROUND WHICH COMPANIES PREPARE THEIR BUDGETS, SHOULD BE REALISTIC AS RESPECTS ASSUMPTIONS RELATIVE TO PRICING. ORGANIZATIONS ARE NOT WELL SERVED BY SETTING OVERLY AMBITIOUS PREMIUM TARGETS BECAUSE OF OVERLY AMBITIOUS PRICING ASSUMPTIONS - MID-WAY THROUGH THE YEAR, THEY WILL FIND THAT THE ONLY WAY TO ACHIEVE THE PREMIUM TARGET WILL BE TO CUT PRICES FURTHER SO AS TO WRITE MORE BUSINESS - THIS DAMAGES THE COMPANY IN THE LONGER TERM WHEN THE UNPROFITABILITY FROM THE LOW PRICES EMERGES, AS IT INEVITABLY WILL.

- RECOGNIZE THAT UNDERWRITING CYCLES AND CHANGING MARKETPLACE CONDITIONS ARE A FACT OF LIFE. IN TERMS OF THE PRICING EXERCISE, DON'T BE NAIVE AND ASSUME THAT ACTUARIES CAN CHANGE THE MARKET - WORK TO OPTIMIZE THE PRICING OVER TIME, BALANCING ALL OF THE OTHER RELEVANT FACTORS IN ADDITION TO THE PURE ACTUARIAL INDICATIONS.

- VISIT SOME FIELD OFFICES AND/OR TALK TO FIELD MANAGEMENT AND UNDERWRITERS TO UNDERSTAND THE DYNAMICS OF WHAT HAPPENS AT THE DESK. THESE ARE THE FOLKS WHO HAVE TO SELL THE PRODUCTS AND PRICES.

- OUR PROFESSION IS UNIQUELY QUALIFIED TO OVERSEE THE PRICING. ALSO, WITH THE DEBACLE OF THE LAST CYCLE STILL FRESH, I BELIEVE SENIOR MANAGEMENTS WILL BE LOOKING MORE THAN EVER TO YOU FOLKS TO KEEP THE BALANCE DURING THE NEXT CYCLE. I BELIEVE THIS CREATES OPPORTUNITIES FOR ALL OF US, AS ACTUARIES, TO FURTHER ENHANCE THE CONTRIBUTION OF OUR PROFESSION TO OUR ORGANIZATIONS.

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PREFACE
(TEXTBOOK CHAPTER DRAFT)

FIVE NEW TEXTBOOK CHAPTERS

In the first edition of the Actuarial Forum we published the draft textbook chapter "Credibility." In the second edition, we published two more chapters, "Principles of Ratemaking" by Charlie McClenahan and "Special Issues" by Steve D'Arcy.

In this issue, we publish the remaining chapters of the CAS textbook. It is very important that CAS members review these chapters and provide comments to the authors. The Textbook Steering Committee, under the leadership of Irene Bass, has the responsibility for assuring that each chapter addresses its subject matter properly in a way understandable to beginning level actuaries. Each CAS member has the responsibility for providing input to the authors and an opinion as to how well they have succeeded. Address your comments directly to the authors.

With the completion of this Forum, we will have published in draft form all of the chapters of the textbook. The textbook is currently scheduled to be published in 1989.

FOUNDATIONS OF CASUALTY ACTUARIAL SCIENCE

BY MATTHEW RODERMUND

PREFACE

What It's All About

If it is agreed that an actuary is one who analyzes the current financial implications of future contingent events, then it might also be agreed that actuarial science concerns, first, the realistic perception of such contingent events and, second, the critical study of their current financial implications.

The foregoing definitions of the actuary and of actuarial science apply to all types of actuaries -- life, health, pension, and casualty and property -- but in different degrees and clothed in different perspectives. The future contingent events for life and pension actuaries involve, mostly, mortality, but life actuaries study the current financial implications of dying, and pension actuaries the financial implications of continued living. For health actuaries, the future contingent events are sickness and disability (with death as the extreme case), and they ponder the current financial implications of the need (by individuals and by the social order) for medical treatment and rehabilitation.

Surely, much can be written about the actuarial science of disciplines other than casualty and property insurance, but this book is about casualty actuarial science; to that discipline we will confine our remarks.

For casualty actuarial science (we will omit the word "property" for convenience, just as we do in the name of the Casualty Actuarial Society), the future contingent events are so widely varied that they cannot be described in a phrase. They may best be characterized by Murphy's Law: If it can happen, it will. The current financial implications of such events defy precise measurement. And classical probability procedures haven't helped much. That is why casualty actuaries must embrace a priori, or even intuitive, probabilities, in addition to experience indications, if they are to get on with their jobs.

The mention of probabilities reminds us to state the obvious, that probability theory (whether classical or Bayesian) forms the basis of actuarial science. If the actuaries hadn't had it, they would have had to invent it. In "An Introduction to Credibility Theory" [1], Laurence H. Longley-Cook quotes a statement by E. W. Phillips, from

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"Biometry of the Measurement of Mortality" [2], which is interesting because in 1935 it forecast as destiny for actuaries what was already rooted into their lives, but it also foretold their future concerns:

> "The calculus of probability is a fascinating subject, and one which is destined to play a large part in actuarial science; and a day may come when it can truly be said of the actuary that he has fused together the theories of finance and probability."

The Beginnings

It all began with the advent of workmen's (now <u>workers</u>) compensation. That statement holds for casualty actuarial science, and it holds for the Casualty Actuarial and Statistical Society of America (CASSA), which later became the Casualty Actuarial Society. The first constitutionally acceptable state workmen's compensation law, passed in Wisconsin in 1911, began to excite interest among scattered members of both the Actuarial Society of America (mostly in the East) and the American Institute of Actuaries (mostly in the Midwest). Even before the New York Workmen's Compensation Act was passed, in 1914, the interested actuaries (plus many people whose interest was not actuarial but either statistical or social or both -- including, among the latter, I. M. Rubinow, the founder and first president of the Society) had realized the need to establish a technically sound basis for this new "social" insurance. Out of this interest came the professional society that we have inherited.

Considerable work in ratemaking for employers' liability insurance had been done in the late 1890s; it depended largely on loss ratio comparisons, and these were studied for about eight industrial classifications in each of several regions of the country. In 1909, a conference on workmen's compensation was held (in Atlantic City) at which papers by future charter members of CASSA were among those presented. In his book, Social Insurance [3], published in 1913, I. M. Rubinow included a section on industrial accidents. In 1914, Albert H. Mowbray, who was to be one of the charter members of CASSA, presented to the Actuarial Society of America a paper on the criteria for testing the adequacy of rates for workmen's compensation insurance [4]. At the same meeting, Harwood E. Ryan, also to be a CASSA charter member, delivered "A Method of Determining Pure Premiums for Workmen's Compensation Insurance" [5].

On November 7 that same year, CASSA was born.

The new society tackled the workmen's compensation problems directly. Among the first (and it has ever been thus) was the question of how to use relatively scanty experience to make justifiable rates. The second paper in Volume I of the Proceedings, by Albert Mowbray (clearly one of the giants at that time), was "How Extensive a Payroll Exposure Is Necessary To Give a Dependable Pure Premium?" [6]. That paper represented the first formal introduction to the concept of credibility, the concept that the volume of past experience of a risk or class of risks is a considerable factor in the weight, or "credibility," to be given such experience in using it for ratemaking.

It is the concept of credibility that has been the casualty actuaries' most important and most enduring contribution to casualty actuarial science. Any list of the great contributors to casualty actuarial science would also be a list of those who developed and implemented the theories of credibility: Albert H. Mowbray, Albert W. Whitney, G. F. Michelbacher, Winfield W. Greene, Francis S. Perryman, Paul Dorweiler, Thomas O. Carlson, Arthur L. Bailey, Laurence H. Longley-Cook, Robert A. Bailey (Arthur Bailey's son), LeRoy J. Simon, Frank Harwayne, Lester B. Dropkin, Allen L. Mayerson, Charles C. Hewitt Jr., Hans Buhlmann (a Swiss actuary). If we have omitted names of others who have made comparable contributions, we are sorry. The foregoing are the ones who stand out in our memory.

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Days To Remember

One of the memorable moments in the development of casualty actuarial science came apparently in 1917, at a meeting of the Actuarial Section of the National Reference Committee on Workmen's Compensation Insurance. The event is described in Albert W. Whitney's famous paper, "The Theory of Experience Rating" [7], presented at the May 1918 meeting of the Society. According to Whitney, the committee -- Winfield W. Greene, chairman; Albert H. Mowbray; Benedict D. Flynn; George D. Moore; and Joseph H. Woodward; all charter members and future presidents of the Society -- was seeking to formulate a plan of experience rating of workmen's compensation risks.

The problem of experience rating, Whitney wrote, "arises out of the necessity of striking a balance between class-experience on the one hand and risk-experience on the other." Whitney's paper traced and analyzed verbally and mathematically the general line of reasoning pursued by the committee, which apparently had struggled at some length with the problem of the weight to be given risk experience, examining and rejecting many suggestions and assumptions. The committee used the term "credibility" and the notation "Z" to express this "weight," and sought to quantify it. Then Win Greene suggested that the relatively complicated second term of the denominator of an equation that the committee agreed summed up its thinking (No. 22 in Whitney's exposition) be taken as a constant. The development of his suggestion resulted in

$$Z = \frac{P}{P + K} .$$

Voila!

That formula (where P is exposure and K a constant), which underlies most of the credibility studies since then, has generally been attributed to Albert Whitney, because it first appeared and was analyzed in his paper (referred to above), but apparently it sprang out of the deliberations of the special actuarial committee on workmen's compensation, and, specifically, was one of Win Greene's suggestions.

We feel safe in saying that casualty actuarial science was born at that moment. The concept of credibility clearly has fascinated the casualty actuarial profession, and, later on, some of the life actuaries, who took it up mainly for group insurance.

In his 1918 paper on the theory of experience rating, Whitney explored the implications of the credibility concept contained in the statement of the Z formula. He recognized, for instance, that reasonable values of K would have to be determined by judgment. depending on underlying factors.

Such judgment considerations were treated by Michelbacher in "The Practice of Experience Rating" [8], presented at the same 1918 CASSA meeting as the Whitney paper. It was quite a day for actuarial science. Michelbacher's paper complemented Whitney's, setting forth the development of a practical plan from the theoretical principles discussed by Whitney. In the plan greater credibility was given to a greater amount of observable data. Workmen's compensation loss experience was divided into two groups -- death and permanent total disability losses in one, and all other losses in the second. Credibility factors were calculated separately for each group. In later years the losses were divided into three groups -- serious, non-serious, and medical.

Whitney had assumed that inherent hazards differed among classifications of risks, and he assumed a knowledge of the distribution of such hazards; but in his mathematical development he, in effect, reversed his assumptions and fell back on Bayes's Rule, which, prior to Laplace's generalization, declared that, a priori, all possible events were equally likely. Whitney's efforts were criticized, but he was aware that the casualty actuaries had practical problems of statistical estimation to attend to -- specifically, reliable and marketable ratemaking where classical statistics didn't provide acceptable answers -- and he pursued his own line of study.

Among the other practical problems confronting actuaries was finding a way to establish full credibility. The Z formula didn't allow full credibility, but there were many buyers of workmen's compensation insurance who insisted that they should be rated solely on their own experience. How this problem has been resolved over the years, in many lines of insurance, in many kinds of experience and retrospective rating, and in classification rating, is one of the great stories in casualty actuarial science, and is covered in the various chapters of this book.

Retrospective Rating

Albert Whitney, who developed the theory of experience rating, had also shown an early interest in retrospective rating, and he passed along his interest to Paul Dorweiler, his understudy in the National Workmen's Compensation Service Bureau. Retrospective rating, which was explored, described, and refined by Dorweiler [9] in the 1920s and 1930s, and by other well-known actuaries in the 1940s, was the next -- after experience rating -- important contribution to the methodologies of casualty actuarial science by members of the Society.

It was a rating scheme applied on top of experience rating, and it permitted workmen's compensation risks whose estimated premiums were greater than certain specified minimums to limit their final retrospective premiums, depending on losses, to amounts between preselected maximum and minimum percentages of the audited premiums. Obviously, risks whose loss records were better than average could save on their workmen's compensation costs, first prospectively through experience rating, then at policy expiration through retrospective rating. Insurance charges in the retro plan protected the insurance company against the probability that the risk, because of high losses, would exceed the preselected maximum premium; and there was a saving from the excess charge to recognize the probability that, because of low losses, the calculated retrospective premium would be less than the minimum.

Again the credibility concept, in which the measurement of risk is related to the volume of experience, came into play, because the charges and savings were higher for small premium accounts than for large. Originally there were several tabular retrospective rating plans whose maximum and minimum premium percentages were specified for varying sizes of risk, the range between the maximum and minimum percentages being less for small risks than for large. Another plan (Plan D, so-called) was created for risks that preferred to select their own maximums and minimums. Moreover, Plan D made it possible to combine large workmen's compensation and liability insurance risks in a single rating scheme, which applied also on an interstate basis. The table from which insurance companies calculated the excess charges and the savings for Plan D, once called "Table "M" -- now "The Table of Insurance Charges" -- is one of the more esoteric features of the casualty insurance rating scheme.

At any rate, retrospective rating was not widely used in the 1930s and early 1940s. One of the reasons was that large mutual carriers were strongly opposed to it because they had their own dividend schemes to reward better than average risks. The use of retrospective rating expanded during the war years, especially in war related industries. Although stock companies were the principal writers, the mutuals, which insured many large workmen's compensation risks, became more receptive. In December 1948, a plan developed by the New York Compensation Insurance Rating Board (accepted by mutual companies as well as stock companies) was approved by the New York Superintendent of Insurance. Since then retrospective rating has played a major role countrywide in workmen's compensation and liability insurance. And Paul Dorweiler is still considered its actuarial father.

An Actuary To Remember

It would not be proper, in a preface to a book on casualty actuarial science, to fail to give recognition to the contributions of Arthur L. Bailey. His papers in the Proceedings, from 1942 through 1950, give such a solid foundation to casualty actuarial concepts that today they underlie all other sources of basic reading required of those who aspire to the actuarial profession. His 1950 paper, even in its hefty title, "Credibility Procedures -- Laplace's Generalization of Bayes' Rule and the Combination of Collateral Knowledge with Observed Data" [10], goes to the heart of the casualty actuarial endeavor. Be warned, however, that in its technical context the paper is not necessarily easy reading.

In addition to his mathematical brilliance -- and in spite of it -- Arthur Bailey had a way of presenting ideas so lucidly that even lay people could get his message. Some of Bailey's words about the actuary and his work are remarkably simple and direct. For example, in his 1942 paper, "Sampling Theory in Casualty Insurance" [1], he said:

> "Thus the losses paid by an insurer never actually reflect the hazard covered, but are always an isolated sample of all possible amounts

> of losses which might have been incurred. It is this condition, of never being able to determine, even from hindsight, what the exact value of the inherent coverage was, that has brought the actuary into being."

Again, from "Credibility Procedures," Bailey's 1950 paper cited above [10], speaking of the need for different schedules of credibility for different components, and even for different intervals, of workmen's compensation losses:

> "The trained statistician cries 'Absurd! Directly contrary to any of the accepted theories of statistical estimation.' The actuaries themselves have to admit that they have gone beyond anything that has been proven mathematically, that all of the values involved are still selected on the basis of judgment, and that the only demonstration they can make is that, in actual practice, it works. Let us not forget, however, that they have made this demonstration many times. It does work!"

In a discussion in the <u>Journal of the American Teachers of Insurance</u>, 1950, Arthur Bailey stated, on the difference in philosophy of the casualty actuary and the classical statistician:

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"First, there is the belief of casualty underwriters that they are not devoid of knowledge before they have acquired any statistics. This belief is probably held by operating personnel in all businesses. When a new form of insurance is initiated or a new classification or territory established, there may be a considerable variety in the opinion of individual underwriters as to what the rates should be; but the consensus of opinion invariably produces a rate. This rate soon becomes embedded in the minds of the underwriters as the 'right' rate. Later, when statistics as to the actual losses under the new coverage, classification or territory, finally are acquired, the problem is not 'what should the rate have been?' but 'how much should the existing rate be changed as a result of the factors observed?' In revisions of rates for regular coverages, classes and territories, this is always the question.

"The statistical methods, developed by the mathematicians and available in the standard textbooks on statistical procedures, deal with the evaluation of the indications of a group of observations, but under the tacit or implicit assumption that no knowledge existed prior to the making of those particular observations. The credibility procedures, used in the revisions of casualty rates, have been developed by casualty actuaries to give consistent weightings to additional knowledge in its combination with already existing knowledge.". [12]

Writing of such clarity does not often appear in our literature. Would that it did, because the casualty actuary needs to relate ever more closely to the public he or she serves.

Tom Carlson, another eminent actuary, in a 1964 presentation to the International Congress of Actuaries, said that Arthur Bailey was "probably the most profound contributor to casualty actuarial theory the United States has produced." [13] Whether or not Arthur Bailey is mentioned specifically in the diverse chapters of this book, it is certain that much of the thought expressed in those chapters will have its foundation in his contributions.

Credibility and the Private Passenger Car

Roughly ten years after Arthur Bailey's studies, interest grew in the possibility of rating private passenger automobile policies on the basis of individual driving records. In 1959, Robert Bailey and LeRoy Simon presented a paper -- "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car" [14] -- that affected actuarial science profoundly, not only in the United States, but also in Europe. In the same year, Frank Harwayne wrote "Merit Rating in Private Passenger Automobile Liability Insurance and the California Driver Record Study" [15], in which he suggested that the negative binomial distribution is preferable to the Poisson as a description of risk distributions by numbers of accidents. Thereupon, Lester Dropkin prepared "Some Considerations on Automobile Rating Systems Utilizing Individual Driving Records" [16], expanding and developing the advantages of the negative binomial as a tool in rating private passenger cars.

The following year, LeRoy Simon's paper, "The Negative Binomial and Poisson Distributions Compared" [17], indicated that the Poisson distribution underestimates the probability of the number of accidents that will be experienced by one car or a fleet of cars. It turned out that the negative binomial concept, for all its actuarial brilliance, was an enigma to all but a handful of CAS members; and Harwayne, Dropkin, Simon, and Charlie Hewitt (whose 1960 paper applied the negative binomial to Canadian auto experience [18]), and Tom Carlson (whose "Negative Binomial Rationale" [19] appeared in 1962) were regarded with awe -- and amused toleration.

In the same year as the Simon paper, Dropkin wrote "Automobile Merit Rating and Inverse Probabilities" [20], in which he developed a general expression for the probability of y accidents in subsequent years, knowing that a specified number of accidents had occurred in a given time period. Inverse probability -- a priori probability -- was the key, and thus the solution was afforded through Bayes's theorem.

All seven of the papers cited lean heavily on the theories of credibility that had been developed in previous years. To that extent the influence of the credibility concept in casualty actuarial science was again demonstrated. Moreover, in private passenger car rating, exposure and loss frequencies were small compared to those available in workmen's compensation and liability experience rating and classification rating. The utility of the credibility concept was greatly expanded.

A Scientific Basis

In 1962, a great boon was afforded to members and students of the Casualty Actuarial Society when Laurence H. Longley-Cook presented his monograph, "An Introduction to Credibility Theory" [1]. In relatively simple and concise terms, Longley-Cook brought together the essentials of the concept of credibility that had been developed since Whitney. The treatise unraveled a lot of the mystery that had troubled many actuaries, who realized how vital the subject was but hadn't taken the time, or lacked the capacity, to pore through the profound and technically challenging writings that had accumulated in the Proceedings.

But Longley-Cook had a reassuring word for those who worried that their relative inexpertise in credibility theory might adversely affect their own actuarial skills:

> "It is perhaps necessary to stress that credibility procedures are not a substitute for informed judgment, but an aid thereto. Of necessity so many practical considerations must enter into any actuarial work that the student cannot substitute the blind application of a credibility formula for the careful consideration of all aspects of an actuarial problem."

Closing the Credibility Gap

We have seen that the concept of credibility was used in casualty actuarial ratemaking procedures for both commercial and personal insurance. But the theoretical justification for such use differed considerably from the basic tenets of classical statistical theory. Quoting Arthur Bailey's 1950 paper again [10]"

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> "There have been rare instances of rebellion against this [classical statistical] philosophy by practical statisticians who have insisted that they actually had a considerable store of knowledge apart from the specific observations being analyzed. Philosophers have recently discussed the credibilities to be given to various elements of knowledge, thus undermining the accepted philosophy of the statisticians. However, it appears to be only in the actuarial field that there has been an organized revolt against discarding all prior knowledge when an estimate is to be made using newly acquired data."

Allen L. Mayerson, in his 1964 paper, "A Bayesian View of Credibility [21], bridged this gap between casualty actuarial practice and statistical theory. He pointed, first, to outstanding books by Savage in 1954 [22], Schlaifer in 1959 [23], and Raiffa and Schlaifer in 1961 [24], which in effect rebelled against the classical approach and saluted a trend toward the use of prior knowledge for statistical inference. And he referred to advances that had been made in probability and stochastic processes. Those advances resulted in mathematical techniques that lend themselves to the solution of actuarial problems -- techniques that can more easily be used by actuaries. In his paper, Mayerson's purpose was "to attempt to continue the work started 15 years ago by Bailey, and, using modern probability concepts, try to develop a theory of credibility which will bridge the gap that now separates the actuarial from the statistical world." And "to summarize the Bayesian point of view, to show its relevance to credibility theory, and to express credibility concepts in terms which are meaningful to a mathematical statistician."

Mayerson proceeded to do exactly what he promised. In the conclusion of a review of this paper in the 1965 PCAS, Charles C. Hewitt Jr. said:

> "This is one of the most significant papers presented to this Society in many years and, happily, should produce much controversy and further thought in this important area. European actuaries have outstripped us in the classical 'theory of risk.' Professor Mayerson has distilled the essence of American achievement in the areas of credibility and the Bayesian approach." [25]

Credibility and Severity

Neither Mayerson nor Hewitt was totally satisfied. The credibility studies of Albert Whitney and Arthur Bailey had involved the distribution of the number of claims alone, ignoring the distribution of claim amounts. Bailey had said in his 1950 paper [10]:

> "In casualty insurance, the inherent hazard of an insured, or of a classification of insureds, is the product of an inherent frequency of loss occurrence and an inherent average amount of loss. . . . At the present time there is little or no knowledge as to the correlation between frequencies of loss and average loss amounts in casualty insurance. It is the hope of this writer that someone with a knowledge of the statistical behavior of [such] products will undertake the development of the appropriate procedure."

Of course Mayerson and Hewitt were aware that Francis Perryman, as far back as 1932 [26], had developed a formula for full credibility of the pure premium (the pure premium is the product of the claim frequency and the average claim amount), and that in 1962 Longley-Cook [1] had developed the same formula in slightly different form. But Perryman and Longley-Cook had assumed that both the claim frequency and the average claim cost are normally distributed. Mayerson and Hewitt believed that such an assumption is not necessary, that credibility tables can be derived from actual data. So they accepted Bailey's challenge. In 1968 Mayerson, with the collaboration of Donald A. Jones and Newton L. Bowers Jr., fellow professors of mathematics at the University of Michigan, presented "On the Credibility of the Pure Premium" [27], and in 1970 Hewitt presented "Credibility for Severity" [28].

Mayerson et al developed formulas that indicated that the number of claims required for full credibility of the pure premium is considerably more than the number required for full credibility of the claim frequency. The paper "attempted to supply a basis for more accurate and scientific credibility tables. . . . If losses by size data were available for various coverages, both countrywide and state, it would be possible to calculate the full credibility point for each coverage and state."

Hewitt's paper brought to attention the enormous contribution that Hans Buhlmann had made to basic credibility theory [29]. In addition, Hewitt employed a clever analogy to a casualty insurance situation by using the roll of two dice to indicate frequency and two six-slot spinners to indicate severity. With those he was able to apply credibility concepts to combinations of frequency, severity, and their product -- pure premium. The paper concluded that, as expected, "Credibility is greatest when severity is ignored entirely (as has been the case in the past)." When the sizes of claims are introduced, a degree of credibility is obtained by limiting the values at which losses enter the rating; but as the limits are increased and more of the value of the individual claims enters the rating, the credibility decreases. It reaches a fixed value when all loss amounts are included.

Reserves

The concept of credibility is fundamental to all aspects of casualty actuarial science. In ratemaking, the credibility of loss experience is of course basic; but the experience includes reserves for unpaid losses, and in the 1970s and 1980s the credibility of the reserves alone became a matter of increasing concern. Such concern was directed only partly at the effect of the reserves on the reasonableness of the rates; it was also directed quite pointedly at the effect of the reserves on the financial position of the company issuing the rates.

Here the practitioner of casualty actuarial science walks a fine line indeed. His competence is questioned by the marketplace if it believes he sets the reserves so high that rates are unaffordable; by his own production department if rates are seen as unsalable; by his CFO if profits are unreachable; by the IRS if taxes turn out to be negligible. On the other hand, the actuary's competence is also questioned if his CEO, or his own auditors, or the state auditors, believe the reserves are too low to discharge future contingent liabilities. The actuary's lot is not a happy one.

Happy or not, he must evaluate the credibility of the development of losses and of all the available ancillary information that affects the reserves. He will draw on whatever casualty actuarial science he has mastered. Of course, his knowledge of Bayesian principles and other actuarial techniques is not his only tool in the reserving process. Mastery of casualty actuarial science implies not only familiarity, but also limited expertise, in economics, finance, demographics, engineering, law, medicine, ecology, what else -- it sounds frightening. But if he is indeed one who analyzes the current financial implications of future contingent events, he can't afford to be solely a mathematician.

Reinsurance

It was the 1960s before casualty actuaries became involved in reinsurance, and it was the 1970s before reinsurance companies in large numbers decided it was useful to have actuaries around. But the services that actuaries have rendered in reinsurance have been more visible in financial areas than in underwriting and rating. The sharp inflation in the United States in the 1970s caused reinsurance managers serious concern about their reserves. In excess of loss reinsurance (which predominates in the casualty business), most of the losses are big ones and of types that take a long time to settle. Inflation made any book of casualty excess of loss reserves suspect, and actuaries were brought in to employ whatever scientific methods they had available to establish adequate reserves.

Unfortunately, in many instances the actuarial calculations suggested such large increases in reserves that their use would have seriously threatened the policyholders surplus of the reinsurers. That in turn would have brought low ratings by A. M. Best, the financial watchdog of the industry. Reinsurers cannot compete in the marketplace with low ratings from Best. The result was that actuarially produced reserves were not always accepted by management. Many reinsurance operations failed during the late 1970s and early 1980s. Those that survived were aided considerably by large investment earnings from the high interest rates that accompanied the inflation. The companies that used the reserves their actuaries recommended showed huge underwriting losses, but investment earnings kept them in business.

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Casualty actuarial science has not been employed to any great extent (at least until recently) in reinsurance rating. In 1952, long before actuaries became part of the reinsurance picture, L. H. Longley-Cook prepared for the CAS Proceedings "A Statistical Study of Large Fire Losses with Application to a Problem in Catastrophe Insurance" [30]. Catastrophe reinsurance in property lines is usually on an excess of loss basis, but covers losses by wind (tornado, hurricane) as well as losses by fire. Longley-Cook's paper was not applicable to wind losses, but it should have been useful to fire underwriters at the time, because it gave them a guide to determining a premium for a broad cover when the premium for some other cover (a lower layer, e.g.) had been established.

The principal reason that actuarial science has had little impact in pricing reinsurance contracts is that the pricing process usually consists of a bargaining session between the reinsurance underwriter and the client. All the participants are generally well-informed, they are favored with a high degree of integrity, and their negotiations are conducted in a spirit of free and open competition. In a paper presented in 1972, "Actuarial Applications in Catastrophe Reinsurance" [31], LeRoy Simon suggested that "One of the important contributions that the actuary can make to the reinsurance field is the maintenance of logical consistency among the various alternatives that may be considered at different stages of the negotiation process." He mentioned contract modifications that might be discussed, such as altering the retention, changing the thickness of the layer, or subdividing the layer. In his paper he set up mathematical models that implied the actuarial relationships among such alternatives.

Another notable contribution to actuarial science in reinsurance was the 1977 paper by Robert S. Miccolis, "On the Theory of Increased Limits and Excess of Loss Pricing [32]. Like Simon, Miccolis created a mathematical model that he hoped would be helpful in making pricing judgments or evaluating such judgments. Sometimes the evaluation of reinsurance pricing judgments is as close as the actuary gets to the nitty-gritty of the reinsurance business.

Pro rata reinsurance has little need for actuarial input, principally because the main element of judgment concerns the commission rate, and that usually turns out to be à compromise of the difference between the expense loadings of the client and those of the reinsurer. Frequently the parties agree to a sliding-scale commission rate, which is inversely related to loss ratio. Actuarial expertise is not customarily needed for such an agreement.

Actuaries also get involved in deep financial analyses of ceding companies, as well as of reinsurers. Here, elements of ruin theory come into play, an area in which European actuaries have specialized. In periods of high inflation, especially, mathematical analyses of the risks involved in various underwriting and financial commitments are vital for the well-being, not only of the insurers and reinsurers, but also of the general public, which relies on their services.

Nevertheless, in spite of the limited efforts that actuaries can make in the reinsurance business, it's the uncertainties of the business that are fascinating, that stir the imagination to greater exploration. Whether or not new discoveries are to be made, it's a fun game.

Classifications -- and Politics

In the 1980s, the problems of reserving have almost dwarfed the problems of ratemaking. Company actuaries and board and bureau actuaries today have a good idea how to produce rates for primary coverages, assuming the possession of experience of some degree of credibility. But, aside from the reserving, a new factor, one that actuaries used not to worry about unduly, has entered ratemaking considerations: classification distinctions. Actuaries used to take it for granted that fairness in ratemaking demanded homogeneous classifications with similar risk characteristics. It seemed reasonable, for example, that since women as a class live longer than men, their life insurance rates should be lower, their annuity rates higher. If young unmarried female drivers under a certain age have significantly better accident records than unmarried males of the same age, then the automobile insurance rates for the young women should be lower. If the mistakes of surgeons and obstetricians and anaesthetists cause more physical disability than those of internists and dermatologists, higher malpractice insurance premiums for the more serious offenders are reasonable.

Today the casualty actuarial scientist must add considerations of politics and sociology to the areas of necessary knowledge we have mentioned. Yet, such considerations are really not new. Many years ago, life insurance actuaries agreed that black men and women would not be rated higher than whites, even though mortality studies clearly showed lower longevity for black people as a class. Politically, rating according to experience indications, in this situation, was not feasible. Underwriters and actuaries eventually recognized that the different mortality indications resulted probably from economic conditions, so that the indicated experience distinctions were not really related to color, but rather to relative affluence. Presumably mortality for whites is also affected by relative affluence. Thus, although the decision not to differentiate between blacks and whites was political, it actually reflected an inability to classify all lives according to more realistic guidelines.

Many of the classification problems in life and automobile insurance are now political and social. Apparently some feminists are willing to pay higher insurance costs in order to eliminate what they regard as sexist discrimination. For actuaries, the resolution of the argument simply involves combining classifications that they had formerly regarded as independently ratable. The inherent factors that truly cause the female experience to be different from the male are not easily classifiable. Actuaries have been unable to set up new risk classifications that could be equally applicable to women and men.

The political-social dilemma for actuaries (principally for life and health actuaries) has been sharply exemplified in the AIDS epidemic. The natural inclination is to establish a separate classification for those who are found to have the AIDS virus. For many reasons, however, some politicians and segments of the public have resisted insurance company use of the AIDS blood test. Actuarial science, in what looks like an actuarial problem, has not been a strong factor.

The problems in malpractice and other liability lines are not only political, but also legal and economic and social. Here the solutions are not as easy as for the female-male issue. Society needs the protection from liability losses that insurance once afforded but no longer affords willingly -- at least in some areas of coverage. The necessary resolution of this troubling situation will come probably from the combined efforts of government, industry (insurance and other), and the legal and medical professions. For our purposes, the important fact is that casualty actuarial science must henceforth include political and social realities in its already widening scope.

The Need in Other Fields

The ever-widening scope of casualty actuarial science requires that its rationales be part of the equipment of any individual organization or institution whose successful operation depends on a realistic evaluàtion of "future contingent events." Clearly, no insurance organization can afford to be unaware of the concept of credibility, and the same should be true of the growing profession of risk management, now that risk retention legislation is on the scene. Dependence on data that does not necessarily indicate what its compilers claim for it is one of the hazards of risk management, as it is of insurance. A little research would surely uncover instances of failure -- or near failure -- of a venture mainly because of a careless evaluation of future contingent events. For example, a more cautious evaluation of future events in the oil industry might have spared the Continental Illinois Bank its troubles in 1984, might have limited its lending ventures.

Actuarial techniques are being applied in the investment business to a greater extent than ever before.

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The employment of actuaries by investment houses for the analysis of companies as investment prospects is becoming more common. And although governmental institutions in their regulatory capacity may only infrequently apply scientific techniques, most regulators, in the exercise of their oversight responsibilities, certainly are aware that the techniques exist.

Is That All It Is?

The opening paragraphs of this preface are headed "What It's All About." Readers who have come this far may conclude from what they have read that casualty actuarial science is the study and application of the theory of credibility, and that's all. Is it all? One might ask, what about the theory of probability? Probability, of course, is part of classical statistical theory, and the record shows that classical statistical theory alone has not provided the tools necessary adequately to measure risk in any line of insurance, including life. Bayes, and then Laplace, added a different dimension to the classical theory. Out of that dimension came the basis for the theory of credibility.

Insurance is a risk business, which is required to price its product before it knows what its costs will be. The theory of credibility has made the most significant contribution to quantifying the risk aspects of the insurance product, and thus qualifies as the dominant factor in casualty actuarial science.

How about the theory of risk? European actuaries, and many professors of mathematics and statistics in the United States, have been studying and writing about the theory of risk for years; but only recently have American actuaries actively pursued it. The Casualty Actuarial Society established a Committee on Theory of Risk in 1977; and the committee has been presenting risk theoretic issues in forums and panel discussions, recommending readings on risk theory for the Syllabus, and compiling a bibliography. So far, relatively few papers on risk theoretic issues have been presented to the Society. Although risk theory, by definition, connotes the idea of insurance, its students have not, to any great extent, offered ideas -- actuarial or otherwise -- for implementing its principles. It seems as though, in a practical sense, risk theory still stands on the shoulders of the theory of credibility.

Casualty actuarial science may also be thought to include the development of the methodologies of classification rating, experience rating, retrospective rating, and other rating schemes; but, in our opinion, the fundamentals of ratemaking stem from the theory of credibility, and the methodologies are a matter of applying the agreed-upon principles to the data at hand. We point to the various tables used in rating -- credibility tables for classification rating, tables of primary and excess losses, and other implements of experience and retrospective rating, such as D ratios, K values, B values, W values, Table M; their creation is a skilled actuarial function, but we're not sure it is properly includible as a basic aspect of casualty actuarial science. Rather, it is an operation that puts to practical use the theoretical concepts of actuarial science.

Closing Thoughts

Thus, casualty actuarial science has come a long way from its tentative beginnings in workmen's compensation insurance in 1914. Not only has the technology expanded, but the need for the technology has expanded. It is hoped this textbook will contribute to the comprehension of actuarial science by members and students of the Casualty Actuarial Society, and by outsiders who are being drawn into the actuarial world whether they planned it or not.

Quotations from the presidential address of Francis S. Perryman in 1939 are appropriate to close an introduction to this casualty actuarial science textbook. The first was used by Laurence H. Longley-Cook at the conclusion of his monograph on credibility theory in 1962 [1]; and the second was quoted by Dudley M. Pruitt at the end of his monumental paper, "The First Fifty Years," in 1964 [33]. We unhesitatingly re-use them here, because more than twenty years later we find Mr. Perryman's words to be even wiser and more prescient than the Messrs. Longley-Cook and Pruitt probably imagined:

> " -- the business finds itself with still a large number of problems on its hands, many of which we know the actuary will eventually have to solve. Let him, therefore -- the casualty actuary about whom I have been talking -- continue to grapple with these problems, knowing full well that he has an enormous advantage in the possession of a scientific mind and of scientific methods; with these he will, on his merits, be called on to play a larger and most responsible part in the business of casualty insurance." [34]

> "His (the actuary's) will be the privilege of using his knowledge and experience, his actuarial tools and methods, so as to solve our modern social problems, our problems of living together in harmony and cooperativeness; for this is sure,

that such problems will be solved and they can be dealt with only by scientific methods that are in essence those we use and know as our actuarial ones." [35]

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REINSURANCE
(TEXTBOOK CHAPTER DRAFT)

FUNDAMENTALS OF CASUALTY ACTUARIAL SCIENCE

BY GARY S. PATRIK

CHAPTER 5: REINSURANCE

5.1 INTRODUCTION

This introduction is only a brief review of basic reinsurance concepts and terminology. The interested reader can further pursue this through the general reinsurance texts listed in the bibliography to this section.

A. What is reinsurance?

Reinsurance is a form of insurance. A reinsurance contract is legally an insurance contract. Under a reinsurance contract, the reinsurer agrees to indemnify the cedent insurer for a specified share of the insurance losses paid by the cedent for a single insurance policy or a designated set of policies. The terminology used is that the reinsurer assumes the liability ceded. The cession, or share of losses to be paid by the reinsurer, may be defined as a percentage share of losses or on some other basis.

The nature and purpose of insurance is to reduce the financial impact upon individuals and corporations from the potential occurrence of specific kinds of contingent events. An insurance company sells many policies which for fixed or bounded (e.g., retro-rated plans) prices guarantee the policyholders that the insurer will indemnify them for part of their financial losses arising from these events. This pooling of liabilities allows the insurer's total losses to be more predictable than is the case for each individual insured.

Insurance enables individuals and corporations to perform task: and manufacture products which might be too risky for one enti This increases competition and efficiency in a capitalistic marketplace.

The nature and purpose of reinsurance is to reduce the financia impact upon insurance companies of insurance claims; thus furth enhancing competition and efficiency in the marketplace. The cession of shares of liability spreads risk further throughout the insurance system. Just as an individual may purchase an insurance policy from a large insurer, a small insurance compan may purchase fairly comprehensive reinsurance from a large reinsurer. And a large insurer or reinsurer may spread its assumed insurance risk by purchasing reinsurance coverages from many other reinsurers, both domestically and worldwide. A cession from one reinsurer to another of some part of assumed reinsurance liability is called a retrocession.

Reinsurers write business either directly through their own employed account executives or through reinsurance intermediaries. More than 50% of U.S. reinsurance is estimated to be placed through intermediaries.

A reinsurance contract is a contract of indemnification; the reinsurer agrees to compensate the cedent for a specified share of insurance payments made on certain of the cedent's insured policies. Except for special cases (e.g., cut-through endorsements), the cedent's policyholders are not parties to the contract, and thus have no direct legal recourse against the reinsurer.

The form and wording of reinsurance contracts are not as closely regulated as insurance contracts and there is no rate regulation of reinsurance between private companies. Reinsurance contracts are often manuscript contracts setting forth the unique agreement of the two parties. Because of the many special cases and exceptions in reinsurance, it is extremely difficult to make correct generalizations. Thus whenever analyzing reinsurance experience, one should be careful that when comparing data, the coverages producing the data are reasonably similar. We will be encountering this problem often throughout this chapter.

B. <u>The functions of reinsurance</u>

Reinsurance does not change the basic nature of an insurance coverage; on a long-term basis, it cannot be expected to make ba business good. But it does provide certain direct assistance to the cedent:

1. Capacity

With reinsurance, the cedent can write larger policy limits. By ceding shares of all policies or just of larger policies, the ne retained loss exposure per individual policy or in total can be kept in line with the cedent's surplus. Thus smaller insurers can compete with larger insurers, and policies beyond the capacity of any single insurer can be written.

2. Stabilization

Reinsurance can help stabilize the cedent's underwriting and financial results over time and help protect the cedent's surplus against shocks due to especially large and infrequent losses. Reinsurance can be written so that the cedent keeps smaller predictable losses, but shares larger infrequent losses. It can also be written to provide protection against a larger than predicted accumulation of claims either from one catastrophic event or from many. Thus the financial effects of large losses or large accumulations of loss are spread out over many years. This decreases the cedent's probability of financial ruin.

3. Financial results management

Reinsurance can alter the timing of income, enhance statutory and/or GAAP surplus and improve various financial ratios by which insurers are judged. We will see this as we discuss the effects of various covers below.

4. Management advice

Many professional reinsurers have the knowledge and ability to provide an informal consulting service for their cedents regarding underwriting, claims handling, marketing, pricing, reserving, investments, loss prevention and personnel. Enlightened self-interest forces the reinsurer to critically review the cedent's operation and be in a position to offer advice. The reinsurer probably has more expertise in the pricing of high limits policies and in the handling of large and rare claims. Also, through its contact with many similar cedent companies, the reinsurer might be able to provide an overview of general trends.

C. The forms of reinsurance

1. Facultative certificates

A facultative certificate reinsures just one primary policy. Its main function is to provide additional capacity. It is used to cover exposure in excess of or in addition to that covered by the cedent's treaties. A cedent may also use facultative certificates for certain large or especially hazardous policies or exposures to limit their potential impact upon his ongoing treaty results. The reinsurer underwrites and accepts each certificate individually; it is very similar to primary insuran large risk underwriting.

Most property certificate coverage is on a proportional basis wherein the reinsurer reimburses a fixed percentage of each cla on the subject policy. Most casualty certificate coverage is o an excess basis wherein the reinsurer reimburses a share (up to some limit) of the part of each claim on the subject policy whi lies above some fixed retention (deductible).

2. Facultative automatic programs

A facultative automatic agreement reinsures many policies, usually with the reinsurer taking a proportional share of each policy ceded. It may also be written on an excess basis. It is like a treaty except that either the cedent may not be required to cede or the reinsurer may not be required to assume every single policy of a certain type. It is usually written for new or special programs marketed by the cedent, and the reinsurer is usually very much involved in the primary pricing of the policies. For example, a facultative automatic agreement may cover 90% of the personal umbrella business written by the cedent, and the reinsurer may help the cedent establish underwriting guidelines and rates.

3. Treaty proportional covers

A treaty reinsures a share of a certain part of the loss exposure for a whole set of insurance policies for a certain time period. The set of policies are those of a specified type written during the term of the treaty, but may also include those policies in effect at inception. The subject exposure is usually defined by Annual Statement line of business or some subsets thereof. The treaty may be exposed only to those losses occurring during the term of the treaty on subject policies, or may include coverage for losses occurring after termination upon policies in force at termination (run-off exposure). One benefit of a treaty is that it creates a close working relationship between the parties so that the expertise and services of the reinsurer are available to the cedent.

A quota share treaty reinsures a fixed percentage of each subject policy. Its main functions are capacity and financial results management. Quota share treaties may assume inforce exposure at inception and return unearned premium at termination. This cession of unearned premium reserve creates a financing effect because of the ceding commission thereon. Quota share treaties sometimes attach net of all other reinsurance covers in order to cede an amount of premium necessary to protect the cedent's premium-to-surplus ratio. However, the term quota share is sometimes used improperly when there is a cession of a proportional share of an excess layer.

A surplus share treaty also reinsures a fixed percentage of eac subject policy, but the share varies by policy according to the relation between the limit of the policy and the cedent's net retention. Its main function is capacity, but it also provides some stabilization and financing. Surplus share treaties also may assume inforce exposure at inception and return unearned premium at termination. They are used for property coverage an(only rarely used for casualty.

4. Treaty excess covers

An excess treaty reinsures, up to a limit, a share of the part c loss in excess of some specified cedent retention (attachment point of the treaty). Its main functions are capacity and stabilization. An excess treaty usually covers exposure earned during its inforce term on a losses-occurring basis with no run-off. The definition of "loss" is important.

For a per-risk excess treaty, a loss is defined to be that loss occurring on one policy for one event. Per-risk excess is used for property exposures to provide protection net of facultative coverage and possibly also net of proportional treaties. It is used for casualty less often than per-occurrence coverage.

For a per-occurrence excess treaty, a loss is defined to be all losses arising from one loss event or occurrence for all subject policies. Per-occurrence excess is used for casualty covers to protect a cedent all the way up from working layers through clas layers. A working layer is loosely defined as a layer for which a number of losses are expected each year. A higher exposed layer is above the working layer(s), but within some policy limits. A clash layer usually attaches above policy limits and is only exposed by extra-contractual-obligations or excess-of-policy-limit damages (if covered), by catastrophic workers' compensation accidents and by the "clash" of losses from multiple coverages or policies stemming from one loss event. The main purpose of a clash layer is catastrophe protection.

A per-occurrence excess treaty used for property exposure is called a catastrophe cover. It is used to protect the net position of the cedent against the accumulation of claims stemming from a single large natural event. It is usual to stipulate that two or more insureds must be involved before coverage attaches. The cessions are usually 90% to 95% of layers excess of the maximum retention the cedent can absorb or can afford.

For an aggregate excess treaty, a loss is the accumulation of all subject losses during one time period, usually one year. It usually covers the net retention of the cedent, either property or casualty or both. It protects net results, providing very strong stabilization and catastrophe protection (unless catastrophes are excluded from coverage).

5. Nontraditional covers

These are almost always treaties whose main, and sometimes only, purpose is financial. The reinsurer's risk is reduced by various contractual conditions. And the reinsurer's expected margin is reduced to reflect this.

A financial proportional cover usually has a ceding commission which varies within some range inversely to the subject loss ratio. The ceded loss share may also decrease somewhat if the loss ratio exceeds some maximum, or the loss share may be fixed at some percentage less than the premium share. The cover may also have some kind of funding mechanism wherein the aggregate limit of coverage is based upon the fund (net cash position less margin of the reinsurer) plus, of course, some risk layer at least at the beginning of the contract.

A loss portfolio transfer is a cession of some part of the loss liability of the cedent as of some accounting date. It may be a cession of the total liability or, more usually, some aggregate excess layer. It is almost always subject to a limit, and may have sublimits upon payment timing. The retention may be stated in terms of dollars and/or time. A loss portfolio transfer may be a pure risk cover, but usually it is essentially a present value funding of liabilities. It may include profit commissions to be paid to the cedent if the actual loss experience is better than originally anticipated.

A funded aggregate excess cover is, as one might suspect, an aggregate excess cover for which the losses are essentially funded. It is analogous to a funded loss portfolio transfer except that it covers future occurring losses. Besides financing, it may provide strong stabilization.

D. A typical reinsurance program

Of course there is no such thing as a typical reinsurance program. Every company is in a unique situation regarding loss exposure, financial solidity, management culture and future plans. Thus each company needs a unique reinsurance program, a combination of ceded reinsurance covers tailor-made for that company.

Nevertheless, Table 5.1.1 displays a reinsurance program for a medium sized insurance company that we might regard as being typical:

(5.1.1)

A "TYPICAL" REINSURANCE PROGRAM

FOR A MEDIUM SIZED COMPANY

Lines of Business	Type of Reinsurance
A. Fire and Allied Lines HO Section I SMP Section I	1. Proportional facultative certificates to bring each individual policy's net exposure down to $1,000,000
	2. Surplus share of 4 lines not to exceed $800,000; maximum cedent retention of $200,000
	3. Per risk excess working layer $100,000 excess of $100,000
	4. Catastrophe covers:
	a) 95% of $3,000,000 excess of $2,000,000
	b) 95% of $5,000,000 excess of $5,000,000
	c) 95% of $5,000,000 excess of $10,000,000
	d) 95% of $5,000,000 excess of $15,000,000
B. Casualty Lines excluding Medical Malpractice and Umbrella	1. Facultative certificates for primary per policy coverage exces of $1,000,000
	2. Working layer excess: $700,000 excess of $300,000
	3. Clash layers:
	a) $4,000,000 excess of $1,000,00
	b) $5,000,000 excess of $5,000,00
	c) $10,000,000 excess of $10,000,000
C. Personal Umbrellas	1. 90% share facultative automatic program

If the company writes Surety, Fidelity, Marine, Medical Malpractice or other special business, other similar reinsurance covers would be purchased. If the company is entering a new market (e.g., a new territory or type of business), it may purchase quota share coverage to lessen the financial impact of the new premium volume (the ceding commissions on unearned premium) and to obtain the reinsurer's assistance. If the company is exiting a market, it may purchase a loss portfolio transfer to cover the run-off of loss payments.

E. The cost of reinsurance to the cedent

1. The reinsurer's margin

The reinsurer charges a margin over and above ceded loss expectation, commissions and brokerage fees (to the intermediary, if any). It is usually stated as a percentage of the reinsurance premium and is theoretically based upon the reinsurer's expenses, the degree of risk transfer and the magnitude of capacity and financial support, but it is practically influenced by competition in the reinsurance market. The actual resulting margin can differ greatly from that anticipated because of the stochasticity of the loss liability and cash flow transferred.

2. Lost investment income

By transferring premium funds (net of ceding commission) to the reinsurer, the cedent naturally loses the use of those funds until returned as loss payments or as profit commissions, and the reinsurer theoretically keeps a margin and the intermediary, if any, keeps a fee. On the surface, this loss may be diminished if the reinsurer agrees to allow the cedent to withhold funds and keep an account of the funds withheld. But of course the reinsurer will charge a higher margin for this. The actual lost investment income depends upon the actual cash flow on the cover; as with (1), this may be highly stochastic.

3. Additional cedent expenses

The cedent incurs various expenses for ceding reinsurance. These include the cost of negotiation, the cost of a financial analysis of the reinsurer, accounting and reporting costs, etc.. If an intermediary is involved, the fee covers some of these services to the cedent. In general, facultative is more expensive than treaty because of individual policy negotiation, accounting and loss cessions.

4. Reciprocity

In order to cede reinsurance, the cedent might be required to assume some liability from the reinsurer. If this assumption is unprofitable, the loss should be considered in the cost of the cession. Reciprocity is more prevalent outside the U.S.

F. Balancing costs and benefits

In balancing the costs and benefits of a reinsurance cover or of a whole reinsurance program, the cedent should consider not only the direct loss coverage benefit and functions. A major consideration is the reinsurer's financial solidity: will the reinsurer be around to pay late-settled claims many years from now? Also important may be the reinsurer's services, including underwriting, marketing, investment, claims, loss prevention, actuarial and personnel advice and assistance.

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CHAPTER 5: REINSURANCE

5.2 REINSURANCE PRICING

A. General considerations

In general, reinsurance pricing is more uncertain than primary pricing. Coverage terms can be highly individualized, especially for treaties. These terms determine the coverage period, premium and loss payment timing, commission arrangements, application of limits, etc. It is often difficult and sometimes impossible to get meaningful and credible loss experience pertinent to the cover being evaluated. Often the data are not as it first appears, so one must continually ask questions in order to discover their true nature. Because of these problems of coverage definition and of the meaning of loss and exposure statistics, the degree of risk relative to premium volume is usually much greater for reinsurance business.

Additional risk arises from the low claim frequency/high severity nature of many reinsurance coverages, from the lengthy time delays between the occurrence, reporting and settlement of covered loss events, and also from the leveraged effect of inflation upon excess claims. In general, the lower the expected loss frequency, the higher the variance of results relative to expectation, and thus the higher the risk level. Also, IBNR emergence and case reserve development are severe problems for casualty excess business. Development beyond 10 years can be large, highly variant and extremely difficult to evaluate. Concomitant is the increased uncertainty for asset/liability matching because of the very long tail and extreme variability of the loss payout timing. Future predictability is decreased by greater uncertainty affecting loss severity inflation above excess cover attachment points. All these elements create a situation where the variance (and higher moments) of the loss process and its estimation are much more important relative to the expected value than is the case for primary coverage. For some reinsurance covers, the higher moments (or at least the underwriter/actuary's beliefs regarding fluctuation potential) determine the price.

There are many ways to price reinsurance covers. For any given situation, there is no one right way. In this section, we will discuss a few reasonable methods. As in most actuarial work, one should try as many reasonable methods as time permits (and recon-reconcile the answers, if possible).

B. Pricing facultative certificates

Since a facultative certificate covers a share of a single insurance policy, the individual insured can be underwritten and priced. The exposure of the individual insured can be evaluated and manual rates and rating factors can be used. However, since most facultative certificates are written on larger or more hazardous exposures, manual rates and rating factors may not exist or must often be modified. Thus individual loss experience and a great deal of underwriting judgment are important.

To the extent that actuaries are involved with facultative certificate business, they can be useful in the following ways:

1. Be sure that the facultative underwriters are provided with and know how to use the most current and accurate manual rates and rating factors, e.g., increased limits factors, loss development factors, trend factors, actuarial opinions on rate adequacy by exposure type and by territory (state), etc.

2. Work with the underwriters to design and maintain good pricing methodologies, perhaps in the form of interactive computer programs.

3. Work with the underwriters to design and maintain good portfolio monitoring systems for meaningful categories of their business, both for relative price level and for the monitoring of loss experience.

4. Work with the underwriters to evaluate and determine which lines of business and which exposure layers to concentrate upon as market/pricing conditions change.

In contemplating any form of facultative coverage, the underwriter first evaluates the exposure to decide if the risk is acceptable, and then evaluates the rate used by the cedent to decide if it is adequate. The underwriter also determines if the ceding commission fairly covers the cedent's expenses, but does not put the cedent into a greatly more advantageous situation than the reinsurer.

Property certificate coverage on a proportional share basis usually needs little further actuarial assistance. However, the actuary should be involved in the corporate discussion and evaluation of catastrophe accumulation potential.

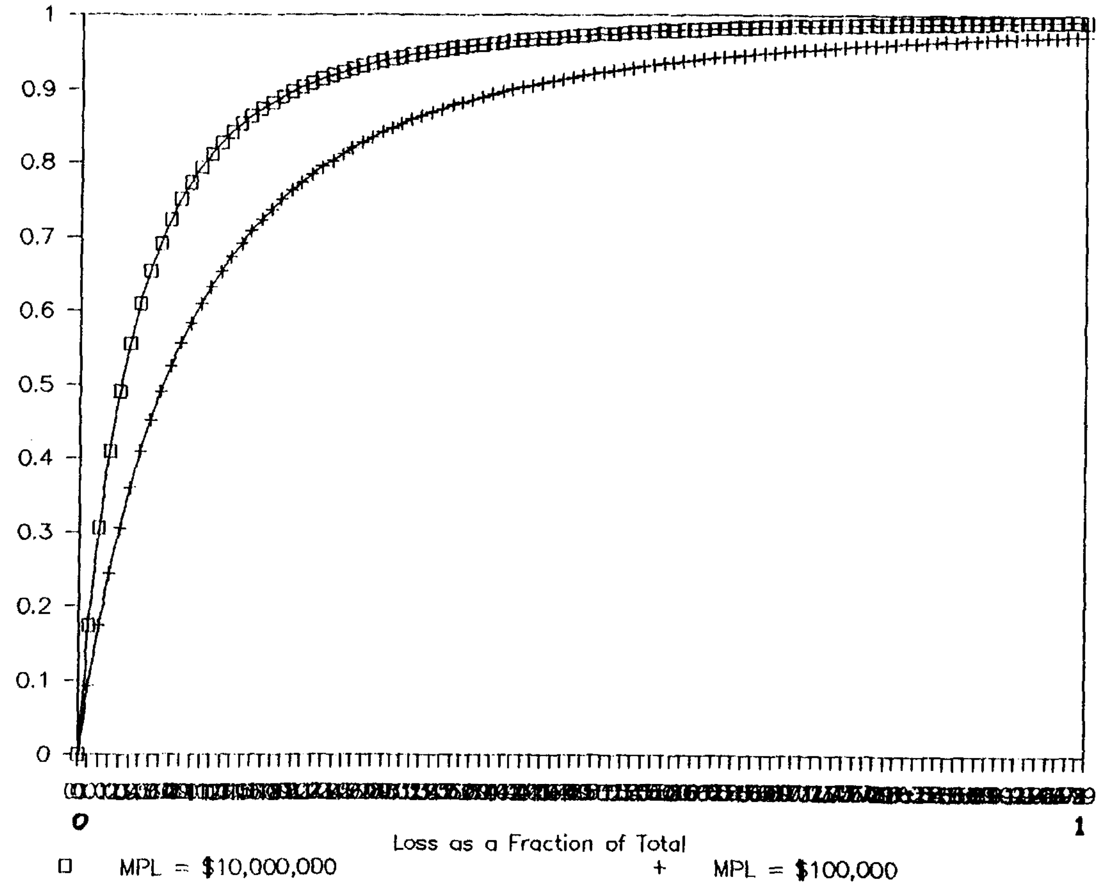
Evaluating and pricing property certificate coverage on an excess basis is more difficult. There exist very little reliable published information on the rating of property excess coverage. Some underwriters use so-called Lloyds Scales, tables of excess loss factors determining the average excess loss as part of the total according to the relationship of excess attachment point to the MPL (maximum possible loss). The MPL, sometimes also called "amount subject", is a very conservative estimate by the individual underwriter of the maximum loss possible on the policy. It includes the maximum full value of contiguous buildings together with contents and also reflects maximal time element (e.g.,business interruption) coverage. The actuarial basis for the Lloyds Scales, if any, is lost in the murky remembrance of post-war (World War II) London. I know of no published actuarially sound tables for rating property per-risk excess coverage.

One actuarially sound concept for developing a table of property per risk excess rating factors would be to express the excess loss cost for coverage above an attachment point up to the MPL as a percent of the total loss cost. The curve would depend upon the class of business (its severity potential) and upon the size of the MPL, and also upon the relative size of the PML (probable maximum loss). The PML is a less conservative estimate of the

(5.2.1)

CLAIM SEVERITY (FICTIONAL EXAMPLE)

CUMULATIVE DISTRIBUTION FUNCTIONS



largest loss, assuming for example, that the sprinkler system works, that the contents are normal, etc. The difference between MPL and PML is illustrated by considering an office building: the MPL is the total value; the PML is usually thought to be three to five floors. The MPL and PML affect the shape of the loss cost curve because one expects, for example, very different loss severity distributions for an insured with a $100,000 MPL and PML versus an insured with a $10,000,000 MPL and $5,000,000 PML. This is illustrated by the accompanying graph (5.2.1).

Appropriate risk loadings could be incorporated in the table or could be recommended as additional loading factors.

An appropriate pricing formula for an excess cover is as follows:

$$RR = \frac{ELCF * PPLR * RCF}{(1 - CR - BF)*(1 - IXL)*(1 - TER)} \qquad (5.2.2)$$

where

RR = reinsurance rate (as a percent of total premium)

ELCF = excess loss cost factor (from the table; as a percent of total loss cost)

PPLR = primary company permissible loss ratio

RCF = rate correction factor (for adequacy of primary rate)

CR = reinsurance ceding commission rate

BF = reinsurance brokerage fee (if any)

IXL = reinsurer's internal expense loading (as a percent of premium net of CR and BR)

TER = reinsurer's target economic return (as a percent of pure risk premium, net of CR, BF and IXL)

The reinsurance rate is applied to the primary total premium to determine the reinsurance premium. If the reinsurer wishes to reflect the investment income to be earned on the contract, then the ELCF would include an appropriate loss discount factor. We will see this later for casualty coverage. To maintain consistent terminology with casualty pricing where investment income is more likely to be reflected, we use the term "target economic return" instead of simply "risk (and profit) loading".

For example, suppose we have the following situation:

(5.2.3) EXAMPLE

Facts:

1. Primary total premium = $100,000.

2. MPL = PML = $10,000,000.

3. Attachment point = $1,000,000.

4. Reinsurance limit = $4,000,000.

5. PPLR = 65%

6. CR = 30%

7. BF = 0% (no broker)

8. IXL = 8%

Suppose that for this class of business and for this layer ($4 million excess of $1 million), we (the reinsurer) want to price to a TER of 10%. Also suppose that we believe that the cedent's rate is inadequate by 5%; thus we believe the total expected loss cost to be $100,000 * .65 * 1.05 = $68,250.

Now assume that we believe that the loss severity for this class of business and this MPL is given by a censored (at MPL) Pareto distribution of the following form:

(5.2.4)

$$1 - F(x) = \text{Prob}[X > x] = \begin{cases} (b/(b + x))^q & \text{for } x < 1 \\ 0 & \text{for } x = 1 \end{cases}$$

where the loss size X is expressed as a percent of MPL. (Properties of the Pareto are outlined in Appendix A.)

Suppose that the parameters are given by b = .1 and q = 2.

The reader can verify the following facts:

9. $\text{Prob}[X = 1] = (b/(b + 1))^q = .008$

(probability of a loss, if it occurs, hitting the MPL)

10. $E[X;c] = (b/(q - 1)) * \{1 - (b/(b + c))^{q-1}\}$

(expected loss cost up to any censor c <= 1)

11. E[X;1] = .091 (as a percent of MPL)

(Thus, if a loss occurs, its average size is $910,000)

12. $E[X-.1 | (.1,.5)] =$

$$(b/(q - 1)) * \{(b/(b + .1))^{q-1} - (b/(b + .5))^{q-1}\}$$

$= .033$ (as a percent of MPL)

(per-occurrence expected loss cost in the reinsured layer)

13. ELCF = (12)/(11) = .033/.091 = .367

14. RR = .432

15. Reinsurance gross premium

= .432 * $100,000 = $43,200

The reader can also verify the following facts:

16. E[N(excess)] = (68,250/910,000) * Prob[x > .1] = .01

(expected number of claims excess of $1,000,000)

17. The reinsurance gross premium for the layer $5 million excess of $5 million, with a 15% TER, is $10,400.

Of course, quite often the pricing situation is much more complicated, with multiple locations and coverages. The underwriter/pricer generally determines a price for each location, each coverage, and adds. Instead of working directly with an estimated underlying loss severity distribution like this Pareto the ELCF (13) might be obtained from a table such as the Lloyds Scale.

Clearly, a pricing procedure such as this can be easily programmed into an interactive PC package for the underwriters. The package would contain all the appropriate rates and rating factors to be called upon by the user. It would ask most of the pertinent questions of the user and would document the decision trail for each submission seriously contemplated by the underwriters.

For facultative certificate property coverage as with any reinsurance business segment, the pricing cycle is very severe. This is mainly due to naive capital flowing into the market because of easy access, but also due to the short-term nature of most peoples' memories. Thus it is very important to monitor the results very closely. Renewal pricing and rate competition in the marketplace should be watched monthly; perhaps summaries derived from the aforementioned pricing system would be appropriate. Quarterly underwriting results in appropriate business segment detail are very important.

Evaluating and pricing facultative certificate casualty covers is even trickier, due mainly to the uncertainty arising from delayed loss reporting and settlement. Because of this increased uncertainty, the actuary's role can be more important in pricing and monitoring the results.

Analogously to property excess, a cover may be exposure rated via manual rates and increased limits factors, together with exposure evaluation and underwriting judgement. The same formula (5.2.2) may be used except that the ELCF will be based upon increased limits loss cost tables and the RCF may be determined both by a judgement regarding the cedent's basic limits rate level and by a judgement regarding the cedent's increased limits factors.

Since most companies use Insurance Services Office (ISO) increased limits factors for casualty pricing (especially for commercial lines), it is very important that the actuaries very closely monitor ISO factors and understand their meaning.

> **NOTE**: Most increased limits factors, including those published by ISO, have no provision for allocated loss adjustment expense (ALAE) outside of the basic limit. ALAE is usually covered on an excess basis either 1) proportional to the indemnity loss share of the excess cover vis-a-vis the total or 2) by adding the ALAE to the indemnity loss before applying the attachment point and limit. Thus ELCFs based upon increased limits factors must be adjusted to cover the reinsurer's share of ALAE.

Since policies subject to facultative coverage are larger than usual, experience rating often comes into play. The simplest method of experience rating is to first experience rate the basic limits experience or the experience below the proposed excess attachment point, if it is not too high, to get an experience base rate. This rate may be used together with the reinsurer's ELCF table to determine an excess rate.

For a buffer layer of coverage where the likelihood of loss penetration is significant, a more difficult to estimate but perhaps more relevant excess experience rate may be determined directly from a careful analysis of the large loss experience of the insured. To see this let us consider the following example:

(5.2.5) EXAMPLE

Facts:

1. Estimated 1990 basic limits premium ($25,000 limit) = $100,000
2. Policy limit = $1,000,000, no aggregate
3. Estimated 1990 total limits premium = $260,000
4. PPLR = 75%
5. Attachment point = $250,000
6. Reinsurance limit = $750,000
7. ALAE covered pro rata
8. General liability premises/operations exposure
9. CR = 20%
10. BF = 5%
11. IXL = 10%
12. Pricing for 1990 policy period
13. Have exposure and loss experience for policy years 1984 through 1988, consisting of exposures, basic and total limits premiums, current evaluation of basic limits losses and a detailed history for each known claim larger than $25,000

Suppose that for this class of business and for this layer ($750,000 excess of $250,000), we (the reinsurer) want to price to an undiscounted TER of 10%. Also suppose that the cedent's basic limits premium was determined from a standard experience and schedule rating plan which we believe to be adequate, and the cedent uses the appropriate ISO increased limits factors which the reinsurer believes to be adequate, but which include the ISO risk loading but no ALAE provision for the layer. Suppose the ISO increased limits factors for this exposure are as follows:

(5.2.6) (Fictional) ISO Increased Limits Factors

Limit	Published Factor	Factor without risk load
$100,000	1.80	1.769
$250,000	2.25	2.154
$1,000,000	2.60	2.429

Suppose that the cedent is offering us a manual difference excess premium of $35,000 calculated by:

14. Manual difference excess premium

    = $100,000 * (2.60 - 2.25) = $35,000

Suppose that, based upon a study of the relationship of ALAE to claim size for this type of exposure, we believe that an appropriate loading for pro rata ALAE is 10% of indemnity loss cost for this layer. Then if we believe the ISO increased limits factors to be adequate for this exposure, the reinsurance rate as a percentage of the basic limits premium would be calculated from formula (5.2.2) with ELCF = 1.10*(2.429 - 2.154) = 0.3025.

It is left to the reader to check that this exposure rate premium would be:

15. Reinsurance premium 1 = $37,346

Now suppose that we are willing to price to a discounted loss basis. Suppose that we have an estimated expected loss payout pattern for this type of exposure and this layer. And suppose that the corresponding discount factor, using current U.S. Treasury rates (risk-free) timed to the payout pattern and reflecting the implications of the current Tax Act, is .80. Assume that with the reflection of investment income in the pricing, we wish to increase the TER to 20%. Then the reader can check that the new price is:

16. Reinsurance premium 2 = \$33,611

In this case, the offered \$35,000 premium looks adequate. But what about the large loss experience?

Assume that we believe that the ISO claim severity distribution is reasonably accurate for this insured's large loss exposure and that the ISO distribution can be used to compute probabilities of loss for points above \$25,000 and also to compute severity moments. Assume for convenience that this distribution is a Pareto of form (5.2.3) with parameters b = 50,000 and q = 2 (this is consistent with (5.2.6) with a 30% loading for ALAE in the basic limit rate).

The reader can verify the following facts:

17. $\text{Prob}[X > 250{,}000 \mid X > 25{,}000] = 0.0625$

(Use Formula (5.2.4.)

18. E[X25; 975] = $69,643

(expected claim severity in the excess layer $975,000 excess $25,000 where X25 notates the excess claim size for claims strictly greater than $25,000 - see Appendix A(10))

19. E[X250; 750] = $214,286

Suppose that we believe, based upon ISO and other industry information, that the large loss severity trend from 1984 to 1990 is about 13% per annum and the ground-up frequency trend is 2% per annum. (For simplicity, assume constant trends.) And suppose that we believe, based upon the claim severity model, that the severity and frequency trends translate into a 12.2% frequency trend excess of $25,000. (NOTE: This can be seen by deflating the Pareto parameter <u>b</u> by 13% per annum back for four years and measuring the exponential effect on the probability Prob[x > 25,000]. Combine this exponentially smoothed annual change with the 2% ground-up frequency trend to get the 12.2% frequency trend excess of $25,000.) Suppose that for accident year 1984-1988, there are three claims known as of June 30, 1989 whose indemnity values are greater than $25,000. We will use the trended frequency excess of $25,000 to price the layer $750,000 excess $250,000.

Suppose that we expect that, based upon reinsurance data for this type of business, claims will be reported over time in a pattern defined by a lognormal distributional with mean 3 and coefficient of variation 1.311 (u = 0.6, s=1 in the usual parameterization - see Appendix B), with time measured from the midpoint of the accident year. Note that this means that 5% of the claims are expected to be reported beyond 10 years after the beginning of the accident year. Suppose that for this cedent, for this insured, we expect claim emergence above $25,000 to be no different from the portfolio information. Thus American-style chain ladder development may be used to estimate the ultimate developed claims. Note that the basic limits premiums are adjusted to 1990 rate level:

(5.2.7) DATA AS OF 6/30/89 AND DEVELOPMENT

(1) Year	(2) Adjusted BL Prem (in $000's)	(3) # Claims >$25,000	(4) Expected Report Lag	(5) # Devel. (3)/(4)
1984	75	1	.844	1.185
1985	80	0	.784	0
1986	85	1	.691	1.447
1987	90	1	.537	1.862
1988	95	0	.274	0
Total	425	3	na	4.494

We may further adjust the claims for the assumed 12.2% per annum trend excess of $25,000:

(5.2.8) ADJUSTED DATA ($ IN 1,000'S)

(1) Year	(2) Adjusted BL Prem	(3) # Devel.	(4) Trend Factor	(5) (3)*(4)	(6) Frequency 100*(5)/(2)
1984	75	1.185	1.995	2.364	3.152
1985	80	0	1.778	0	0
1986	85	1.447	1.585	2.293	2.698
1987	90	1.862	1.412	2.629	2.921
1988	95	0	1.259	0	0
Total	425	4.494	na	7.287	1.715
1984-87	330	4.494	na	7.287	2.208

Most actuaries would discard the 1988 data as being too immature. Thus this experience rating indicates an expected frequency per $100,000 of basic limits premium of 2.208 excess of $25,000, and

2.208 * Prob [X > 250,000 X > 25,000]

= 2.208 * 0.0625 = 0.138 excess of $250,000.

Since the premium for 1990 is $100,000, the indicated expected number of claims excess of $250,000 is the same 0.138. We can combine this with the expected excess claim size (19) of $214,286 and the 10% ALAE loading to compute undiscounted and discounted excess expected losses (including ALAE) and reinsurance premiums via Formula (5.2.9), which is a slight generalization consistent with (5.2.2):

(5.2.9)

$$RP = \frac{RLC}{(1-CR-BF)*(1-IXL)*(1-TER)}$$

where RP = reinsurance premium

RLC = reinsurance loss cost (expected aggregate loss, either discounted or undiscounted)

CR, BF, IXL, TER as defined in (5.2.2)

The reader can verify the following facts:

20. Reinsurance expected loss = $29,571

21. Discounted (20) = $23,657 (discount factor = .80)

22. Reinsurance premium 3 = $48,676

    (Use (20) in Formula (5.2.9.))

23. Reinsurance premium 4 = $43,809

    (Use (21) in Formula (5.2.9.))

Now it's time for the underwriter to sharpen his pencil. Is the RP4 = $43,809 significant? That is, how certain is this experience rate? Let us look first at the excess $250,000 frequency indication of 0.138 per $100,000 of basic limits premium. This is based upon a developed loss count of 7.287 excess of $25,000 and upon the ISO loss severity distribution. If the offered reinsurance premium of $35,000 were correct on a discounted loss basis and the loss severity curve were correct for this insured, then the expected frequency excess of $25,000 would be 1.764 (= 2.208 * (35,000/43,809)), and the expected number of developed claims excess of $25,000 would be 5.822 (= 7.287 * (35,000/43,809)). Under the assumption that the excess claims are distributed Poisson with mean 5.822, the probability of seeing a number 7.287 or more is approximately 27%. Thus our observed 7.287 is not too unlikely.

The underwriter might also consider the average known loss size excess of $25,000 to see if it is significantly different, after considering development and trend, than the expected excess size of $69,643. The underwriter must now ponder these facts, together with all his other knowledge of the particular insured's exposure and general rate adequacy/inadequacy for this class of business in order to make a decision. He may require at least $38,500 (adjusting the offered manual difference $35,000 to cover ALAE prorata); or he may want $45,000 in light of RP3 and RP4 (remember the total premium is $260,000); or he may decide not to write the cover at all in fear of the large loss exposure indicated by the experience.

As with property excess, it is clear that this rating method can be programmed into an interactive PC package for underwriters. Also, as with property coverage, it is very important to monitor relative rate level and results in appropriate business segment detail. The actuarial evaluations and opinions regarding future case reserve development and IBNR emergence should be very important to the underwriters.

C. Pricing facultative automatic programs

These large multi-insured programs are very similar to treaties. One difference however is that the reinsurance premium for a facultative automatic excess cover is usually computed on a policy-by-policy basis using agreed upon excess rates, instead of as a rate times total subject premium. Thus the reinsurance premium may be more responsive to the individual exposures ceded to the reinsurer. The risk of anti-selection against the reinsurer on a non-obligatory contract should be evaluated by the underwriter.

D. Pricing reinsurance treaties in general

Since a treaty covers a share of an indeterminant (at the beginning) set of insurance policies, insureds are rarely individually underwritten and priced by the reinsurer. Instead the reinsurance underwriter/pricer considers the whole set (book of business) of subject policies. To do this, the reinsurer evaluates first the management of the potential cedent. What is their management philosophy and ability? Are they honest, fair-dealing? Do they know what they are doing? Is the company financially solid? What are their business plans? Why do they want, why do they need reinsurance?

Once the reinsurance underwriter is satisfied that this is a company and these are people he would like to deal with on a long-term basis, he can then evaluate their underwriting, primary pricing, marketing and claims handling ability. Since individual insureds are not underwritten by the reinsurer except on an exception basis, he must be satisfied with the cedent's underwriting expertise and pricing for the exposure he may assume. For any treaty, he must understand the cedent's insurance exposures, rate level and limits sold. Many direct-marketing reinsurers will send a team of marketing and underwriting people to perform a pre-quote audit, and will also send claimspeople to review the company's claims handling and reserving practices.

The reinsurer also reviews the structure of the cedent's reinsurance program, that is, how all the reinsurance contracts, facultative and treaty, fit together to provide benefits to the cedent. Lastly, he evaluates the particular reinsurance treaties and suggested rates if offered, or he creates a program and rates to offer to the cedent company.

Actuaries can provide extremely useful, and often necessary, technical support for treaty business. Besides the list of four items mentioned for the support of facultative certificate business, for treaty (and facultative automatic) business they can also get involved in the technical evaluation and pricing of individual large and/or difficult treaties. Experience rating is much more important for treaties, so the actuarial tools of data analysis and loss modeling can be critical to a reinsurer's ability to write difficult exposures, especially for casualty exposures where long tail loss development and IBNR are critical factors.

E. Pricing proportional treaties

A traditional quota share treaty covers a share of the cedent's net retention after all other reinsurance covers. The cedent's historical experience net of all other reinsurance must be evaluated. If the cedent's other reinsurance covers have been approximately the same for many years, then Schedules O and P of the cedent's Annual Statement may be used for this evaluation. If the other covers have changed significantly so that the remaining net exposure to be covered differs from the cedent's past net, then the reinsurer must request historical data which can be recast to the proper net exposure. The underwriter/actuary must be careful that the data includes an adequate provision for reported case reserve development and IBNR.

The reinsurer's evaluation of the cedent's net historical experience should not only consider averages, but should reflect the effects of the underwriting/pricing cycle and of random fluctuations. And this history should be adjusted to the future coverage period by the reinsurer's estimates of relative rate lev (including the underwriting cycle).

Proportional treaties often have contingent or sliding scale cedi commissions. In each case, the reinsurer pays the cedent a provisional commission on the reinsurance gross written premium a it is transferred to the reinsurer. At suitable dates (often quarterly), the cumulative experience on the treaty (usually from the beginning if there is a deficit carryforward; or over some period such as three years) is reviewed. If it is profitable, the reinsurer pays the cedent an additional commission; if it is unprofitable, the cedent returns some of the provisional commissic to the reinsurer. An example should clarify this.

(5.2.10) EXAMPLE

Facts:

1. 25% quota share on various property lines
2. Cumulative subject written premium = $34,000,000
3. Cumulative subject earned premium = $30,000,000
4. Provisional commission = 35%
5. Commission slides 0.5% for each loss ratio 1%

6. Minimum commission = 30%

7. Reinsurer provisional expense and profit margin = 10% (at 55% loss ratio)

8. Cumulative subject incurred loss = $19,800,000

The reader can verify the following facts:

9. Subject loss ratio = 66%

10. Indicated cumulative reinsurance commission

= 35% + 0.5*(55% - 66%) = 29.5%

11. Cumulative commission adjustment = - 5% (minimum)

12. Reinsurance written premium = $11,250,000

13. Reinsurance earned premium = $10,000,000

14. Return commission (to reinsurer) = $500,000

(5% of earned premium; some part may have already been adjusted at previous evaluation dates)

To properly evaluate the historical results on this treaty, the reinsurer must be sure that appropriate loss development is accounted for, and, if the reinsurer wishes to evaluate the bottomline result, then appropriate investment income must be assigned. Also, the reinsurer must consider the long-term required economic return (RER) for this type of treaty and this type of exposure. For each type of cover, each type of exposure, the RER is some fraction of the reinsurer's TER defined earlier. The fraction may be less than one if the reinsurer is willing to be satisfied with a long-term return lower than the pricing formula target.

A simplified evaluation formula parallels the pricing formulas (5.2.2) and (5.2.9) we saw earlier:

(5.2.11) EVALUATION FORMULA

AER - RER (Evaluation formula)

where

RER = required economic return (on pure premium)

AER = actual economic return (on pure premium)

$$= \frac{(1 - CR - BF)*(1 - IXL)*(1 + UPRF)*REP - DF*RIL}{(1 - CR - BF)*(1 - IXL)*REP}$$

UPRF = unearned premium reserve factor = 0.5*UPRR*UPIF

UPRR = average ratio of unearned premium reserve to earned premium (for each year)

UPIF = unearned premium reserve investment return facto

REP = reinsurer earned premium

DF = loss discount factor

RIL = reinsurer incurred loss

CR, BF and IXL as defined in (5.2.2)

Suppose that with respect to a conservative risk-free interest rate, the cash flow for this type of contract (or for this contract in particular, if the cash flow is known) allows a loss discount factor of .96 on losses and a 7% short-term investment rate on unearned premium reserve balances held by the reinsurer. Suppose that UPRR = 40% for the contract in Example (5.2.11), and the reinsurer needs IXL = 5%.

The reader can verify that, with respect to the minimum 30% commission, the actual economic return on pure premium has been:

15. AER = 6.12%

The reinsurer's required economic return should be based upon the degree of risk transferred and upon the statutory surplus relief arising from the ceding commission on the unearned premium reserve. The surplus effect arises from the fact that the cedent's unearned premium liability decreases by the amount of gross unearned premium ceded, while assets decrease only by the amount of the cash transfer, premium net of provisional commission. Since the subject unearned premium is currently $4,000,000, the reader can verify that the current surplus relief is:

16. Cedent's surplus relief = $350,000.

This is, in effect, a statutory surplus loan; which is why a reinsurer will charge an increment on top of the usual risk margin. Suppose in this case, that the reinsurer wants a 7% return on the surplus loan. To keep this simple, suppose that the unearned premium reserve has been constant over time, so that the surplus relief has been constant. From the assumption that UPRR = 40%, the reader can verify the following facts:

17. One year reinsurance earned premium = $2,500,000

18. One year required surplus loan return = $24,500

19. Surplus loan return stated as part of RER on earned pure premium (with CR = 35% and IXL = 5%) = 1.59%

If the reinsurer needs a minimum 5% risk RER (with respect to pure premium) for this treaty plus a 1.59% surplus loan return for a total RER of 6.59%, then the 6.12% AER might prompt the reinsurer to consider nonrenewal unless the future profitability looks better or the minimum ceding commission can be negotiated downward. The reader can verify the following:

20. If the loss ratio is 65% (CR = 30%), then the reinsurer's 5% margin on gross ceded premium translates into a 7.57% AER.

21. If the loss ratio is 55%, then the reinsurer's 10% margin on gross ceded premium translates into a 15.89% AER.

The evaluation of a (true ground-up net retention) quota share on casualty exposure would be similar except that the reinsurer would have to be very careful about loss development. And because of the additional uncertainty arising from loss development, most likely the RER would be higher.

A property surplus share treaty is somewhat more difficult to evaluate. Since the reinsurer does not provide coverage for small insureds and covers larger insureds in proportion to their size above some fixed retention, the reinsurer must be more concerned with the cedent's pricing of larger insureds. An example should clarify this.

(5.2.12) EXAMPLE

Facts:

1. Four line first surplus not to exceed $800,000

2. Maximum cedent retention = $200,000

Then the following statements are true:

3. Maximum reinsurance limit per risk = $800,000

4. For a policy with limit <= $200,000, the reinsurer receives no premium and pays no losses.

5. For a policy with limit = $500,000, the reinsurer receives 60% of the policy's premium less ceding commission and brokerage fee and pays 60% of the policy's losses.

6. For a policy with limit = $1,000,000, the reinsurer receives 80% of the policy's premium less ceding commission and brokerage fee and pays 80% of the policy's losses.

7. For a policy with limit = $2,000,000, the reinsurer receives 40% of the policy's premium less ceding commission and brokerage fee and pays 40% of the policy's losses.

It is easy to see that, given this complicated proportional structure depending upon the limit of each policy, the premium an loss accounting for a surplus share treaty is somewhat complex. Despite this, surplus share treaties are popular because they provide more large loss protection than a quota share and are muc easier for the reinsurer to evaluate and price (usually only the ceding commission and slide is the subject of negotiations) than excess treaty.

A surplus share treaty is generally riskier than a simple quota share. So the reinsurer will charge a correspondingly higher margin for risk assumption.

F. Pricing working cover excess treaties

A working cover is an excess layer where losses are expected. The reinsurer will consider the cedent's policy limits distribution by line of business and will want to examine the historical gross large loss experience in order to determine the types of losses generated by the exposure and to study the development patterns. As we discussed for facultative certificates, an excess cover is usually riskier than a proportional cover. So the reinsurer will be more mindful of the predictive error and fluctuation potential, and will charge a higher margin for assuming this risk.

If losses are covered per-occurrence, then the reinsurer is exposed by policy limits below the attachment point because of the "clash" of losses on different policies or coverages arising from the same occurrence. If ALAE is added to individual claims in order to determine the reinsurer's excess share, then losses from some policy limits below the attachment point will bleed into the excess layer.

The reinsurance premium is usually specified by a reinsurance rate times subject premium as we saw in formula (5.2.2). However, for liability coverage, it may be on an increased limits premium collected basis. Here the total reinsurance premum is the sum of the individually-calculated reinsurance premiums for each policy, as we saw for the premium offered in example (5.2.5).

In either case, ideally the reinsurance pricing consists of both an exposure rating and an experience rating, and a reconciliation of the two rates. The exposure rating differs from facultative certificate pricing in that the reinsurer deals with broad classes of business instead of individual insureds. The reinsurer considers manual rate relativities to bureau rates and/or to other companies writing the same exposure and evaluates the cedent's experience and schedule rating plans and pricing abilities. The increased limits factors used by the cedent for liability coverages are especially important. The same formulas (5.2.2) and (5.2.9) can be used except that the rate correction factor RCF now adjusts for both basic limits and increased limits (in)adequacy. If the coverage is per occurrence, the reinsurer must load the manual difference rate for the clash exposure. If the coverage includes

ALAE, the reinsurer must adjust the manual increased limits factors to account for this additional exposure.

As with the facultative example we saw earlier, the reinsurer must adjust the historical experience to the estimated level of the proposed coverage period by trend factors for the losses and rate on-level factors for the premiums. Working cover treaties are often large enough so that many of the risk parameters can be determined either directly from the exposure and loss history or by a credibility weighting of the history with more general information.

For working covers, the provisional reinsurance premium is often subject to retrospective rating, with the final premium over certain coverage periods, such as three years, being adjusted according to the actual loss experience. A simple example should clarify this.

(5.2.13) EXAMPLE

Facts:

1. Proposed attachment point = $300,000
2. Proposed reinsurance limit = $700,000
3. Coverage is per-occurrence
4. Coverage is on an accident year basis (losses occurring on or after the effective date up through the termination date)

5. ALAE added to indemnity for each claim

6. All liability and workers compensation exposure for a medium-sized primary company

7. Cedent wants a proposal for a three-year retrospective rated treaty incepting Jan. 1, 1990

8. CR = 0%

9. BF = 0%

10. Estimated 1990 subject premium = $100,000,000

11. Possible reinsurance premium range to $10,000,000

12. An underwriting review has been performed

13. A claims review has been performed

14. Have Annual Statements and Annual Reports for last five years; a more detailed breakdown of premiums, deviations from bureau manual rates, limits profiles, increased limits factors, basic limits premiums, total subject premiums and basic and total subject losses as of June 30, 1989 by subline for last five years plus predictions for 1990; a detailed history for each known claim larger then $25,000 occurring in the last ten years.

15. Have the names of contact people at the ceding company; in particular, an actuary to talk with.

The exposure consists of private passenger and commercial automobile liability, premises/operations general liability with incidental products coverage, homeowners and SMP section II, and workers compensation. The cedent writes limits up to $10,000,000, but purchases facultative coverage for coverage excess of $1,000,000. The cedent also purchases facultative coverage above $300,000 for any difficult exposures on the reinsurer's exclusion list, and a 90% facultative automatic cover for his umbrella programs.

Before getting into the complications arising from a retrospective rating plan, let us first consider how to go about determining a flat (fixed) rate.

NOTE: A traditional excess rating methodology prevalent among reinsurers is the "burning cost" method. To compute a burning cost rate, the underwriter divides the sum of known losses in the excess layer occurring over some time period, usually five years, by the cedent's subject premium for the same time period. This ratio is then multiplied by a selected loss development factor, perhaps multiplied by some selected trend factor, loaded by some "free cover" factor and divided by a permissable loss ratio (1 - CR - BF - IXL) to get a rate. Clearly, a problem with this summary approach is that it does not allow one to carefully take into account underlying exposure changes, rate changes, policy limits changes, true excess IBNR emergence and development, true excess claim frequency growth and severity growth and the aggregate excess loss fluctuation potential. I would argue that burning cost rating is not very informative even for property excess covers for which it was designed. Unfortunately, it has been misapplied to the pricing of casualty covers. For more on this topic, see Ferguson (1978) and Patrik's review.

Let us return to the example. For a full discussion of the pricing of casualty working covers, see Patrik and John(1980). We will only sketch an outline of the procedure and add a few improvements developed since then.

STEP 1: The first step is to reconcile with the cedent's audited financial reports as best as possible all the exposure and loss data received from the cedent. This is an ongoing process as we ask for and receive more data.

STEP 2: The second step is to segregate the main types of underlying exposure for separate consideration. In this case, we might want to consider the following breakdown:

(5.2.14) EXPOSURE CATEGORIES

Private passenger automobile

Commercial automobile

Premises/operations

Homeowners Section II

SMP Section II

Workers compensation

Umbrella

These categories can or must be further broken down as desirable or feasible. If we can, it is desirable to split the underlying exposure at least according to applicable increased limits table and by policy limit.

STEP 3: The next step is to perform an exposure rating. This is best done by estimating the aggregate excess loss cost for 1990 based upon the estimated 1990 exposure and general pricing information. The overall exposure loss cost would be the sum of the exposure loss costs for the individual exposure categories. The exposure loss cost for each category would be determined as in Example (5.2.5), items 15 and 16, leaving out the loading factors CR, BF, IXL and TER.

For example, suppose that the company writes commercial automobile light exposure and the following policy limits distribution is estimated for 1990 for a group of states with the same 10/20 basic limit and the same increased limits tables and, for simplicity, adequate basic rates:

(5.2.15) COMMERCIAL AUTOMOBILE LIABILITY

(1) Policy Limit	**(2) Estimated Total Subject Premium**	**(3) Cedent's Inc. Limits Factor**	**(4) Adequate ELCF**
$100,000	$3,000,000	2.10	NA
$300,000	2,000,000	2.40	.0346
$500,000	2,000,000	2.51	.1070
$750,000	1,000,000	2.60	.1449
$1,000,000 or more	2,000,000	2.66	.1526
Total	10,000,000	2.38	NA

The Adequate ELCFs are stated as fractions of 10/20 basic limits losses including all ALAE; but they allocate to the layer \$700,000 excess of \$300,000 the expected loss plus ALAE per accident arising from the policy limit. The reinsurer has tables of Adequate ELCFs for each type of coverage, attachment point and rate jurisdiction or has an exposure rating computer program to compute them. These are based upon claim severity distributions, as are increased limits factors.

Assuming that the cedent's permissable loss ratio for this business is 65%, the reader can verify:

16. Manual difference pure premium = \$234,040
    (From (5.2.15), columns 2 and 3: remember that the subject premium is for coverage up to a \$1,000,000 limit)

Suppose we believe that the basic limits loss costs implied by the basic limits premiums and permissable loss ratio are adequate. The reader can then verify:

17. Expected loss based upon the ELCFs = \$184,964
    (From (5.2.15), columns 2 and 4)

Note that this is not yet loaded for clash exposure or for risk.

An even better way of estimating an exposure loss cost is to break the estimation down to an estimate of the number of excess claims and an estimate of their sizes. For example, suppose we believe that the indemnity loss distribution is Pareto with b = 25,000 and q = 2 and that the distribution of the sum of indemnity loss and ALAE per claim is Pareto with b = 30,000 and q = 2. Then the reader can verify the entries in Table (5.2.16), where the Table (5.2.15) implied excess loss costs are assumed and we simplistically assume that the effect of adding ALAE to each claim increases the effective policy limit by 20% (along with the parameter change):

(5.2.16) EXCESS EXPECTED LOSS, CLAIM SEVERITY AND COUNT

(1) Policy Limit	(2) Effective Policy Limit	(3) Expected Loss Cost	(4) Expected Claim Severity	(5) Expected Claim Count (3)/(4)
100	120	$ 0	$ 0	0
300	360	18,742	50,769	.3692
500	600	55,418	157,143	.3527
750	900	36,225	212,903	.1701
1000	1000	74,579	224,272	.3325
Total		184,964	151,053	1.2245

Now we have exposure-determined estimates of excess expected claim count and severity. Why this added complication? The answer is that with a few mild assumptions regarding second (and perhaps third) moments for the claim count and claim amount distributions, we can use a standard risk theoretic model to estimate the distribution of the aggregate excess loss, the loss the reinsurer will cover. This standard model writes the aggregate loss naturally as the sum of the individual claims (events) as follows:

(5.2.17) $L = X(1) + X(2) + \ldots + X(N)$

where L = rv (random variable) for aggregate loss

N = rv for number of claims (occurrences, events)

X(i) = rv for the dollar size of the i<u>th</u> claim

For us, N and X(i) refer to the excess number of claims and the excess amount of the ith excess claim respectively. The model relates the distributions of L, N and the X(i)'s. In particular, under reasonable assumptions, the kth moment of L is completely determined by the first k moments of N and the X(i)'s. The model is outlined in Appendix C. Here we will only discuss its usage.

Underlying Table (5.2.16) is an assumption that the claim severities follow a censored (at policy limits) Pareto distribution. With this model, or with any other reasonable and tested model, we can easily add another column to the table consisting of the individual claim variance for each policy limit. See Patrik (1980) or Hogg and Klugman (1984). Likewise, if we assume that the total ground-up claim count follows either a Poisson or a Negative Binomial distribution, then so do the excess claim counts. So a column of claim count variance for each policy limit can be added.

We can assemble these moments into estimates of the variance of the aggregate loss L for each policy limit. For this case where the parameters for N and the X(i)'s are assumed to be known, the L's are independent and their variances can be added to estimate the overall variance for the excess losses arising from this Commercial

Automobile exposure category. Likewise, third and higher moments can be handled. For the more interesting and realistic case wher the parameters themselves are uncertain, the L's are no longer independent from the point of view of the observer/actuary, so th second and higher moments do not add. This is covered thoroughly in Patrik and John (1980).

From the first, second and third moments of L, we can get a prett good idea of its distribution, and riskiness. The really ambitio reader can verify the following:

18. Assuming that the excess claim counts are Poisson, then f the case with known parameters, the standard deviation of the aggregate excess loss arising from the Table (5.2.15) exposure is approximately $170,000.

The estimation of excess exposure for other categories would be similar. For workers compensation, for example, excess loss fact differences would be weighted by estimated subject premium by hazard group, by major state grouping.

We assemble our estimates of the excess claim count, claim severi and aggregate excess loss from each exposure category and perform the appropriate additions to get total expectations and higher moments. The total expectations should be loaded for clash exposure arising from more than one policy or coverage for the sa occurrence. If we assume that, based upon the cedent's book of business, the loading should be 5%, I would prefer to increase th claim count (and thus aggregate) expectation by 5%. This has the effect of increasing the variance of aggregate excess loss by approximately 5% (see Appendix C, item 3).

STEP 4: For a working cover such as this, the next step is an experience rating. Since we have a detailed reserving and payment history for each claim over $25,000 for the past ten years (14), we should work with these data. As with the earlier facultative example, we would use the data for each exposure category to evaluate claim count and claim severity excess of an attachment point lower than the proposed $300,000. For example, if the ground-up claim severity inflation has been less than 13% per annum for the last ten years (and into 1990), then we can evaluate a $100,000 attachment point. The preferred method is to inflate the individual claim values to a common 1990 level and work from there.

Suppose we now have ten-year development triangles for excess of $100,000 inflated claim counts and average sizes for the Commercial Automobile category we were exposure rating. A very delicate issue in the trending is that of policy limits. If a 1980 claim on a policy with limit less than $100,000 inflates to above $100,000, would the policy sold in 1990 be greater than $100,000, so to allow this excess claim? Of course, the counter-argument is that policy limits are not increasing as rapidly as claim severity inflation. If possible, information on the cedent's policy limits distribution over time should be obtained; otherwise, Solomon-like judgment must be displayed by the underwriter/actuary.

The excess claim counts can be developed by the usual methods, oı better methods if one can manage (see Section 3). Claim develop- ment statistics from the reinsurer's book of similarly exposed treaties can also be used, and credibility methods can balance tł answers. Excess claim severity development can be studied in tot by accident year, by report year, etc. using various actuarial methods, or individual claims can be analyzed judgementally.

Besides the usual development triangles, a fairly good exhibit fc examining excess development, making judgements and explaining th to oneself and to underwriters and marketing people is displayed Exhibit (5.2.18). This type of format allows you to see if your purely technical answers make sense.

Sections 1 and 2 of Exhibit (5.2.18) have no actuarial estimates all except for the adjusted on-level subject premiums. Displayed are the actual closed and known excess claim count and average si growth over the past two years.

The actuarial estimates are concentrated in Section C, columns (5 (6) and (7). The reasonableness of these estimates may be judged in comparison with the facts to date. Try various reasonable estimates on for size. The mean frequency and severity estimates derived from these are displayed in columns (12) and (10) respectively. If we use the estimates which exclude the last two accident years of data, we have a mean frequency of 1.266 per $1,000,000 of subject risk earned premium and a mean severity of $92,348 in the layer $500,000 excess of $100,000. This frequency

GP:IBNRREV
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EXHIBIT (5.2.18) COMMERCIAL AUTOMOBILE LIABILITY

SECTION A: CLOSED CLAIMS (TRENDED) $400,000 EXCESS $100,000

(1) YEAR	(2) ADJ SUBJECT EARNEDPREM (1,000's)	(3) CLAIM COUNT AT 12/31/87	(4) AVERAGE SEVERITY 12/31/87	(5) CLAIM COUNT AS OF 12/31/88	(6) AVERAGE SEVERITY 12/31/88	(7) CLAIM COUNT AS OF 6/30/89	(8) AVERAGE SEVERITY 6/30/89	(9) AGGREGATE (7)*(8) 6/30/89	(10) CLAIM FREQ ((7)/(2)) *1,000
1979	$8,165	10	$74,401	12	$79,447	13	$81,158	$1,055,059	1.592
1980	8,521	7	52,237	8	56,559	8	56,559	452,472	0.939
1981	8,852	12	87,912	13	91,472	14	100,747	1,410,461	1.582
1982	9,650	4	146,114	4	146,114	5	152,280	761,400	0.518
1983	10,423	8	91,562	10	103,700	10	103,700	1,037,004	0.959
1984	11,200	3	51,128	3	51,128	4	43,475	173,899	0.357
1985	9,523	3	78,155	4	75,386	6	89,522	537,134	0.630
1986	9,822	4	59,554	6	79,707	7	76,414	534,899	0.713
1987	10,023	1	30,000	7	50,117	7	50,117	350,822	0.698
1988	9,927	0	0	0	0	2	91,200	182,400	0.201
TOTAL	96,106	52	79,570	67	82,096	76	85,468	6,495,551	0.791
EXCLUDES LAST 2 YRS	76,156	51	80,542	60	85,827	67	88,990	5,962,329	0.880

SECTION B: REPORTED CLAIMS(TRENDED) $400,000 EXCESS $100,000

(1) YEAR	(2) ADJ SUBJECT EARNEDPREM (1,000's)	(3) CLAIM COUNT AT 12/31/87	(4) AVERAGE SEVERITY 12/31/87	(5) CLAIM COUNT AS OF 12/31/88	(6) AVERAGE SEVERITY 12/31/88	(7) CLAIM COUNT AS OF 6/30/89	(8) AVERAGE SEVERITY 6/30/89	(9) AGGREGATE (7)*(8) 6/30/89	(10) CLAIM FREQ ((7)/(2)) *1,000
1979	$8,165	13	$78,391	13	$80,259	13	$81,158	$1,055,059	1.592
1980	8,521	8	64,025	8	56,559	9	63,139	568,247	1.056
1981	8,852	15	82,561	18	89,704	17	98,851	1,680,460	1.920
1982	9,650	6	117,410	7	141,633	7	143,905	1,007,336	0.725
1983	10,423	14	89,182	12	99,143	13	100,940	1,312,225	1.247
1984	11,200	6	62,772	7	59,128	7	59,128	413,897	0.625
1985	9,523	5	70,610	10	65,614	12	82,261	987,134	1.260
1986	9,822	6	69,703	9	66,471	9	72,767	654,90[illegible]	0.916
1987	10,023	2	60,000	10	54,582	11	53,711	590,819	1.097
1988	9,927	0	0	1	60,000	3	70,800	212,400	0.302
TOTAL	96,106	75	79,875	95	79,639	101	83,985	8,482,480	1.051
EXCLUDES LAST 2 YRS	76,156	73	80,419	84	82,856	87	88,267	7,679,261	1.142

EXHIBIT (5.2.18) COMMERCIAL AUTOMOBILE LIABILITY

SECTION C: PREDICTIONS FOR CLAIMS(TRENDED) $400,000 EXCESS $100,000

(1) YEAR	(2) ADJ SUBJECT EARNEDPREM (1,000's)	(3) OPEN CLAIMS 6/30/89	(4) OPEN AVER SIZE 6/30/89	(5) OPEN AVER SIZE EST. ULT.	(6) IBNR CLAIM COUNT 6/30/89	(7) IBNR AVER SIZE 6/30/89	(8) IBNR AGGREGATE (6)*(7)	(9) TOTAL AGGREGATE	(10) TOTAL AVER SIZE	(11) LOSS COST RATE (9)/(2)	(12) TOTA CLAI FREQUE
1979	$8,165	0	$0	$100,000	0.00	$100,000	$0	$1,055,059	$81,158	12.92%	1.5
1980	8,521	1	115,775	100,000	0.09	100,000	9,091	561,563	61,772	6.59%	1.0
1981	8,852	3	90,000	100,000	0.53	100,000	52,577	1,763,038	100,597	19.92%	1.9
1982	9,650	2	122,968	100,000	0.45	100,000	44,681	1,006,081	135,102	10.43%	0.7
1983	10,423	3	91,740	100,000	1.44	100,000	144,444	1,481,448	102,562	14.21%	1.3
1984	11,200	3	79,999	100,000	1.09	100,000	109,249	583,148	72,060	5.21%	0.7
1985	9,523	6	75,000	100,000	2.69	100,000	268,788	1,405,923	95,720	14.76%	1.5
1986	9,822	2	60,001	100,000	3.11	100,000	311,306	1,046,205	86,370	10.65%	1.2
1987	10,023	4	59,999	100,000	6.63	100,000	662,821	1,413,642	80,192	14.10%	1.7
1988	9,927	1	30,000	100,000	4.09	100,000	409,220	691,620	97,518	6.97%	0.7
TOTAL	96,106	25	79,477	100,000	20.12	100,000	2,012,176	11,007,727	90,881	11.45%	1.2
									STD DEV =	4.29%	0.4
EXCLUDES LAST 2 YRS	76,156	20	85,847	100,000	9.40	100,000	940,136	8,902,465	92,348	11.69%	1.2
									STD DEV =	4.41%	0.4

translates into 8.229 claims excess of $100,000 for $6,500,000 of risk premium ($10,000,000 * .65).

The reader can verify the following exposure-rating values:

19. Prob[X > 300,000 | X > 100,000 } = 0.1479
    Prob[X + A > 300,000 | X + A > 100,000 } = 0.1552

20. Expected claim count excess $100,000 = 7.8898 if ALAE is added to each claim, or 5.9277 otherwise.

21. Expected claim severity excess $100,000 and up to $500,000 = $98,113 if ALAE is added to each claim, or $95,238 otherwise.

If the Exhiibit 5.2.18 experience includes ALAE added to claims, then the experience claim count estimate of 8.229 is incredibly close to the exposure expectation of 7.8898. Likewise, the experience claim severity estimate of $92,348 is reasonably close to the exposure expectation of $98,113. Patrik (1980), Hogg and Klugman (1984) and Patrik and John (1980) discuss methods for estimating Pareto or other model parameters directly from the excess claims severity data, and the testing of those estimates. If the various answers differ but cannot be further reconciled, final answers for excess $100,000 and excess $300,000 claim count and severity can be based upon a credibility balancing of the separate estimates. However, all these differences should not be ignored, but should indeed be included in your estimates of parameter (and model) uncertainty, thus giving rise to more realistic measures of variances, etc. and to risk.

As with the exposure rating, these credibility balanced exposure/experience rating estimates are assembled into totals for expectations and variances, and perhaps higher moments. Any other overall adjustments are made, such as for clash exposure.

STEP 5: The last step is specifying the cover terms, and explaining and negotiating. Suppose our totals come out too high with respect to (11). Suppose that the estimated 1990 expected aggregate loss is $10,000,000 with estimated standard deviation (including all of our uncertainty) of $5,000,000 (u = 2.191, s = .4724). The reader can verify that if the distribution of the aggregate excess loss is assumed to be lognormal (a simplistic assumption), then:

22. Prob[L > $10,000,000] = .41

23. Prob[L > $15,000,000] = .14

24. Prob[L > $20,000,000] = .04

Suppose that, based upon both the excess loss data and upon data for similarly exposed contracts,the reinsurer believes that a loss discount factor of .80 is reasonable and the reinsurer needs IXL = 5% for this cover. If the reinsurer believes that, based both upon the above probabilities (22) through (24) and upon general considerations for this type of exposure, he needs an RER = 15%, then the reader can verify that the reinsurer determines a flat premium of:

25. Reinsurance premium = $9,900,000 (rounded)

Suppose that the cedent believes that the excess loss potential is significantly less than our estimate. And he wants either a flat rate no higher than 8% of subject premium or a retrospectively-rated treaty with a maximum 10% rate. Its time again for the reinsurer to sharpen his pencil.

The best bet is to recommend that the attachment point be increased to $350,000 or $400,000. Attachment points should naturally increase over time in an inflationary environment. Many cedents have trouble accepting this fact, and the marketing of indexed attachment point contracts in the U.S. market in the late 1970's was an attempt to unobtrusively enforce a status quo balance between the cedent and reinsurer. However, this reasonable idea turned out to be impractical in the U.S. reinsurance market because it complicated the accounting of claims and because of anti-selection against the reinsurers who sold the idea (the best cedents found fixed attachment point excess coverage).

The ambitious reader can verify that for the exposure in Table (5.2.15):

26. Expected claim count excess $350,000 = .9231

28. Expected claim severity excess $350,000 = $142,775

29. Expected aggregate loss excess $350,000 = $131,793

compare these results to Table (5.2.16). For other exposure categories the effects will be similar. Suppose that for a cover of $650,000 excess of $350,000 the reinsurer believes that the expected aggregate loss is $7,000,000 with a standard deviation of $4,000,000.

The reader can verify that analogously to (22) through (24):

30. Prob[L > $10,000,000] = .27

31. Prob[L > $15,000,000] = .04

32. Prob[L > $20,000,000] = .01

In this case the reinsurer might offer a retrospectively-rated treaty of the following form:

33. Provisional rate = 7%

34. Rate = reinsurer's aggregate loss (including an IBNR provision) divided by subject premium + 0.5% (reinsurer's margin).

35. Minimum rate = 5%

36. Maximum rate = 10%

37. Profit and deficit carryforward (into successive coverage periods)

The reader can verify, using Formula (5.2.11):

38. If L = \$7,000,000, then AER = 21%

39. If L = \$10,000,000, then AER = 16%

40. If L = \$15,000,000, then AER = -26%

The astute reader will recognize that Formula (5.2.11) should be modified slightly for retrospective-rated contracts since, if the aggregate loss is large, the additional premium is only transferred to the reinsurer as the losses are reported and is thus not available for investment from the beginning. This could be handled by modifying the discount factor.

As with facultative covers, it is clear that much of the above can and should be programmed into an interactive PC package for the underwriters and actuaries. And it is also extremely important to monitor the results of large treaties and groups of treaties. The monitoring of the pricing experience and the monitoring of the loss development and IBNR experience on the reinsurer's books and the reconciliation of both is important.

G. Pricing higher exposed layer and clash layer excess treaties

Since losses are not "expected" for higher exposed and clash layers, historical loss data is sparse or nonexistent. And yet the layers are exposed, or else the cedent wouldn't want to buy cover. Prices for these covers are usually set by market conditions: what the traffic will bear. The actuarial price is largely determined by the risk loading, and may or may not be comparable to the market price.

Where there is policy limits exposure, an exposure rate may be determined in the same manner as for a working cover. An experience rate may be determined by experience rating the working layers below and using these working cover rates as bases for estimating the higher exposed layer rate. The higher the layer, the more significant becomes the workers compensation exposure, if any, and the clash exposure.

For pure clash layer pricing, the reinsurer should keep experience statistics on all clash covers combined to see how often and to what degree these covers are penetrated, and to see if the historical market-dictated rates have been reasonable overall. The rates for various clash layers should bear reasonable relationships to each other depending upon the underlying exposures and the distance of the attachment points from the policy limits sold. Underwriters sometimes view the market rates with regard to a notion of payback - the premium should cover one loss every m years for some selected m - but this is technically inexplicable.

H. Pricing property catastrophe treaties

The price for a windstorm catastrophe treaty should depend upon the attachment point, upon the cedent's accumulation of property exposure in storm-prone localities and upon the cedent's net position on each policy after all other reinsurance. Historical experience, large losses and exposure, should be adjusted to the level of the contemplated coverage period. Changes in the cedent's non-catastrophe net retentions may have a great effect upon the catastrophe exposure excess of a given attachment point. That is, a reinsurance program can be very tricky: a small change here can have a big effect there.

The actuary can be very useful in estimating the reinsurer's total accumulated exposure to catastrophe shock losses. For each windstorm-prone locality, reinsurance exposure from every property contract should be estimated. For each contract, this would be based upon the cedent's exposed policy limits and the reinsurance coverage. Thus the reinsurer can see where too much catastrophe potential is accumulated and can better structure his own catastrophe retrocessional program. Similarly for earthquake exposure.

Various actuaries are working on catastrophe simulation computer programs which estimate loss distributions based upon an insurer's geographic exposure distribution. It may be possible that such programs can be modified for reinsurance use.

I. Pricing aggregate excess treaties

Aggregate excess treaties are sometimes called stop loss covers. They may be used to protect the cedent's net loss ratio. For example, suppose the cedent's expected loss ratio is 65% of net earned premium. A stop loss cover might cover 50% of all net loss payments in excess of a 75% loss ratio up to a 90% loss ratio (the loss ratios are with respect to subject premium net of all other reinsurance coverage). The exposure subject to the treaty could be all or part of the cedent's net exposure.

In a sense, this is the ultimate reinsurance cover for protecting the cedent's net position if all else fails. Because of the magnitude of the risk transfer, one can guess that these covers are quite expensive, and often not available unless written on a nontraditional basis, as we shall see.

Another form of aggregate excess treaty provides coverage over an aggregate deductible on an excess layer. This is more interesting; so we will illustrate this alternative. The concepts for pricing a net stop loss are similar.

(5.2.19) EXAMPLE

Facts:

1. The facts are the same as in Example (5.2.13) except that the cedent wants to retain the first $10,000,000 of payments on the layer $700,000 excess of $300,000.

The cedent might want a \$10,000,000 deductible to avoid trading dollars with the reinsurer for fairly certain loss payments. Keeping the premium for the deductible, the cedent also keeps the investment income.

We (reinsurer) would have to perform the same analysis we did in Example (5.2.13) except we would now be much more careful in our estimation of the distribution of aggregate excess loss. As we noted before, the best way to build up this distribution is from individual excess claims and amounts in Formula (5.2.17). But now, instead of simply estimating the first two or three moments, we may want to use more exact methods such as simulation or the methods espoused by Phillip Heckman and Glenn Meyers (Heckman-Meyers, 1981) or by Harry Panjer (Panjer, 1984). Gary Venter's method (Venter, 1985) of using continuous models for aggregate loss are also useful when the expected number of excess claims is large, so that the probability of having no claims is close to zero. For most low frequency cases, there is a positive probability, $Prob[L = 0] > 0$, a cluster point at 0, that must be accounted for. The Panjer method is especially good for the low frequency case, and has the advantage of being fairly easily explainable to non-mathematicians. We would also be very careful to account for our model and parameter uncertainty, so to get a proper spread for the aggregate loss distribution.

Suppose we have done all our homework and now have a rigorous estimate of the distribution of aggregate loss in the layer $700,000 excess of $300,000 together with an analytic representative of our model and parameter uncertainty. Suppose that once again, we believe that this can be described by a lognormal with mean $10,000,000 and standard deviation $5,000,000. For the degree of accuracy we need here, this is now probably not a good model, but we use it here simply for convenience.

Remembering item 22 under Example (5.2.13), we have:

2. Prob[L > $10,000,000] = .41

So even though the deductible is set at the reinsurer's expectation, there is significant probability of loss and thus there should be a substantial expectation in excess of the deductible. For notational convenience, define the reinsurer's loss L10 in terms of the aggregate excess loss L as follows:

(5.2.20)

$$L10 = \begin{cases} 0 & \text{if } L < \$10,000,000 \\ L - \$10,000,000 & \text{otherwise} \end{cases}$$

The reader can verify the following (using Appendix B):

3. Prob[L10 > $5,000,000] = .14

   (See Example (5.2.13), #23)

4. Prob[L10 > $10,000,000] = .05

5. E[L10] = $1,867,000

6. Standard deviation of L10 = $3,686,500

For an unlimited cover, the risk transfer (if measured by (6)) overwhelms the expectation of $1,867,000, so the reinsurer must charge a substantial margin. The cedent will probably be reluctant to pay so large a premium and the reinsurer may be reluctant to assume so great a risk. To cut down the risk transfer and the price, the reinsurer would want an aggregate limit. Suppose the aggregate limit is $5,000,000. The reader can verify the following:

7. E[L10;5] = $1,251,700

   (expectation of L10 in the layer $0 to $5,000,000)

8. Standard deviation of L10;5 = $1,886,500

The aggregate of $5,000,000 may be acceptable to the cedent if he believes an aggregate excess loss of $15,000,000 is impossible. Remember that our lognormal says that the probability of exceeding $15,000,000 is about 14%.

Along with our evaluation of the incurred aggregate loss distribution, we would more carefully estimate the excess loss payout distribution. Suppose that for each value of L10;5, we estimate a discount factor DF(L10;5) based upon the expected payou pattern of L10;5, available U.S. Treasury instruments, the effect of taxes and the risk that the payout may be different than expected.

Suppose we need IXL = 10% for this limited cover, and RER = 10% on an undiscounted basis and RER = 30% on a discounted basis, where a average discount factor is E[DF(L10;5)] = .60. Remembering that C = BF = 0, the reader can use Formula (5.2.9) to verify the following rounded premiums:

9. Reinsurance premium 1 (undiscounted) = $1,545,000

10. Reinsurance premium 2 (discounted) = $1,200,000

The cedent may object to paying such a large premium for a pure risk cover - where we believe the chance of the aggregate losses reaching the reinsurer is only 41%, and the chance of a complete $5,000,000 loss is 14%. So a cover with a long-term funding mechanism and decreased risk transfer (thus lower reinsurance margin) may be more desirable. We will discuss these types of covers in the next section.

J. <u>Pricing nontraditional reinsurance covers</u>

The simplest example of a reinsurance cover which might be classified as nontraditional is a financial quota share. The Example (5.2.10) quota share could be modified in various ways to emphasize the financial aspects of the cover and diminish the risk transfer, thus diminishing the reinsurer's margin. For example:

(5.2.21) EXAMPLE

Facts: Same as Example (5.2.10) except for:

1. Commission slides 1% with each loss ratio 1%
2. Minimum commission = 20%
3. Reinsurance aggregate limit = 90% of reinsurance premium
4. Premium and loss payments quarterly
5. Penalty negative commission if canceled in a deficit position by cedent
6. Reinsurer expense and profit margin = 2% (at a 63% loss ratio)

The cedent still gets surplus relief from the commission on the ceded unearned premium reserve, and still decreases his premium-to-surplus ratio. You can see the drop in the reinsurer's margin from 10% in Example (5.2.10); the 2% here is just for illustration. The reinsurer's margin is meant to cover the surplus loan as before, a small charge for absorbing premium volume and thus indirectly utilizing further surplus (the reinsurer's surplus is thus not available to support other business), the reinsurer's expenses and a small risk charge. If casualty exposure were covered, the reinsurer would credit the cedent with most of the investment income earned on the contract's cash balance according to some specified formula. As long as the contract is in a profitable position, this would be returned to the cedent as an additional commission upon cancellation or sooner.

Assuming IXL = 2% for this case, the reader can verify the following, with the other facts remaining as in Example (5.2.10):

7. Actual economic return (AER) = 6.32%

8. AER at a 78% loss ratio = 5.89%

9. AER at an 82.81% loss ratio = 0%

10. AER at a 90% loss ratio = -8.80%

When most insurance people think of nontraditional reinsurance, they think of loss portfolio transfers. A cedent may cede all or part of the its liability as of a specified accounting date; this may be for a line of business or territory no longer written, or for other reasons. Most loss portfolio transfers in the U.S. market are written in order to give the cedent the statutory surplus benefit arising from the implicit discounting of loss reserves. That is, the reinsurance premium is essentially the present value of the transferred estimated liability, plus reinsurer's expense, surplus use and risk charges. And the cedent can take reinsurance credit for the liability ceded, thus offsetting all or part of the loss reserve previously set up.

An example may clarify this. Suppose the cedent in Example (5.2.13) has been told by its domiciliary Insurance Department that it should increase loss reserves as of December 31, 1989 for the subject exposure by 20%. With Insurance Department approval, the cedent wishes to purchase a loss portfolio cover for this additional liability. Suppose the cedent would like to minimize the statutory surplus effect as much as possible. Suppose we have the following situation:

(5.2.22) **EXAMPLE**

Facts: Cedent as in Example (5.2.13), plus:

1. Carried loss reserve 12/31/89 = $150,000,000

2. Required additional reserve = $30,000,000

Based upon a thorough analysis of the cedent's financial reports, historical exposure, historical reinsurance program, net loss development and payout distributions by line and in aggregate, we determine that the additional $30,000,000 could easily be funded by a $15,000,000 payment. To get to this point, besides evaluating the adequacy of the cedent's loss reserves, we would pay careful attention to the historical loss payout patterns and their fluctuations. Has the recent exposure changed in such a way to cause a significant change in future loss payout? Have there been major changes in the cedent's Claim Department or claims processing? A common analytical technique is to study ratios of cumulative loss payout for each accident year divided by ultimate estimates for each category of exposure.

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(5.2.23) PRIVATE PASSENGER AUTOMOBILE LIABILITY (FICTITIOUS EXAMPLE)
CUMULATIVE PAID LOSS AS A PERCENTAGE OF ULTIMATE LOSS

ACCIDENT YEAR	EST. ULT. LOSS	EVALUATION YEAR (END OF): 1	2	3	4	5
1984	$6,000,000	0.500	0.650	0.800	0.950	1.000
1985	$7,000,000	0.400	0.550	0.700	0.900	
1986	$8,000,000	0.630	0.700	0.750		
1987	$9,000,000	0.350	0.550			
1988	$10,000,000	0.400				
	1 WEIGHTED AVERAGE	0.450	0.610	0.748	0.923	1.000
	2 3-YR WTD AVERAGE	0.451	0.600	0.748	0.923	1.000
	3 MAXIMUM	0.630	0.700	0.800	0.950	1.000
	4 MINIMUM	0.350	0.550	0.700	0.900	1.000
	5 TRIMMED AVERAGE	0.433	0.600	0.750	0.923	1.000
	6 SELECTED "MEAN"	0.450	0.610	0.748	0.923	1.000
	7 SELECTED EXTREME	0.630	0.700	0.800	0.950	1.000

SCENARIO 1: MEAN PAYOUT

ACCIDENT YEAR	EST. LIABILITY AS % OF TOTAL	PERCENT OF LIABILITY TO BE PAID IN YEAR: 1989	1990	1991	1992	1993
1984	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
1985	7.7%	100.0%	0.0%	0.0%	0.0%	0.0%
1986	25.2%	69.5%	30.5%	0.0%	0.0%	0.0%
1987	39.0%	35.3%	45.0%	19.7%	0.0%	0.0%
1988	55.0%	29.1%	25.0%	31.9%	14.0%	0.0%

ACCIDENT YEAR	ESTIMATED LIABILITY	PERCENT TO BE PAID IN YEAR: 1989	1990	1991	1992	1993
1984	$0	$0	$0	$0	$0	$0
1985	538,462	538,462	0	0	0	0
1986	2,019,048	1,403,663	615,385	0	0	0
1987	3,510,000	1,238,571	1,579,121	692,308	0	0
1988	5,502,500	1,602,500	1,376,190	1,754,579	769,231	0
TOTAL	11,570,009	4,783,196	3,570,696	2,446,886	769,231	0

A simplified example is displayed in Table (5.2.23). This table also displays the "mean" dollar payout prediction by calendar year. We would also want to determine maximal extremes based upon extreme values for the ultimate incurred loss by accident year and upon faster than expected payout. The payout predictions from all the covered liabilities would be assembled. If a lower risk, lower margin treaty were contemplated, greater care would be taken with the loss discounting: the reinsurer might consider pricing the payout stream via the use of an immunizing asset portfolio. The bond maturities would be selected to allow adequate margin for the stochastic nature of the payout.

To zero-out the surplus effect on the cedent, we would look for an attachment point where the cash payment for the loss portfolio transfer would approximately match the resulting loss reserve takedown. For example, suppose a reinsurance premium of $35,000,000 is sufficient for a cover of $65,000,000 excess of $115,000,000. This transaction would not change the cedent's beginning surplus (before reserving the additional $30,000,000).

Another example of a nontraditional reinsurance treaty is a funded aggregate excess cover. Let us transform Example (5.2.19) into such a cover.

(5.2.24) EXAMPLE

Facts: Same as Example (5.2.19) except for:

1. Aggregate limit of $5,000,000

2. Cedent desires a low cost funding cover.

Recall that the reinsurer's loss expectation is E[L10;5] = $1,251,700 with a standard deviation of $1,886,500 and a (discounted) reinsurance premium of $1,200,000 ((5.2.19),#7, #8 and #10). One possible structure for a funded cover would be that the reinsurer takes an initial premium of $2,000,000, deducts an expense and profit margin of only 5%, instead of the 40% previously, and allocates 90% of the investment income to the "loss fund". The aggregate limit might be equal to the fund plus $1,000,000 up to a maximum of $5,000,000, and loss payments might be only annually, to allow the fund to grow as large as possible. As with the quota share (5.2.21), there probably would be a penalty negative commission if the cedent cancelled in a deficit position.

Recalling that Prob[L10 > 0] is estimated to be 0.41 and the likelihood of a total loss in one year is estimated to be 0.14 ((5.2.19), #2 and #3), we would expect the fund to build to $5,000,000 fairly rapidly, at which time the cedent and reinsurer could decide to increase the limit. It should be noted that the aggregate excess attachment point of $10,000,000 should be adjusted appropriately each coverage year to reflect aggregate loss inflation.

K. Conclusion of the reinsurance pricing section

We have seen some examples of how standard actuarial methods and some not-so-standard actuarial methods apply to reinsurance pricing. We must remember that there is no one right way of pricing reinsurance. But there are plenty of wrong ways. Common actuarial methods should be used only to the extent they make sense. To avoid major blunders, an underwriter/actuary must always understand as well as possible the underlying primary insurance exposure and must always be aware of the differences between the reinsurance cover contemplated and that primary exposure. The differences usually involve much less specificity of information, report and settlement timing delays and often much smaller frequency together with much larger severity, all inducing a distinctly higher risk situation. But with this goes a glorious opportunity for actuaries and other technically sophisticated people to use their theoretical mathematical and stochastic modeling abilities fully.

In the next section, we will see how reinsurance loss reserving differs from primary insurance loss reserving, and we will discuss some simple methods for dealing with these differences.

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5.3 REINSURANCE LOSS RESERVING

A. General considerations

For a reinsurance company, the loss reserve is usually the largest indeterminant number in the statement of the company's financial condition. It is also the most uncertain. To properly estimate a loss reserve, we must study the runoff of the past business of the company. As a result of this process, we should not only be able to estimate a loss reserve as of a certain point in time, but we should also be able to estimate historical loss ratios, loss reporting patterns and loss settlement patterns by year, by line and by type of business in enough detail to know whether or not a particular contract or business segment is unprofitable and when. This information should also be applicable to future pricing and decision-making. The goal is to deliver good management information regarding the company's historical contract portfolio, and also deliver some indications of where the company may be going.

Reinsurance loss reserving has many of the same problems as primary insurance loss reserving, and many of the same methods can be used. But there are also various technical problems which make reinsurance loss reserving somewhat more difficult. I will outline some of these problems and then suggest various techniques for handling them.

B. Reinsurance loss reserving problems

There seem to be seven major technical problems which make reinsurance loss reserving somewhat more difficult than loss reserving for a primary company. These technical problems are:

(5.3.1) Claim report lags to reinsurers are generally longer, especially for casualty excess losses.

The claim report lag, the time from date of accident until first report to the reinsurer, is exacerbated by the longer reporting pipeline - a claim reported to the cedent must first be perceived as being reportable to the reinsurer, then must filter through the cedent's report system to his reinsurance accounting department, then may journey through an intermediary to the reinsurer, then must be booked and finally appear upon the reinsurer's claim system. The report lag is also lengthened by the undervaluation of serious claims by the cedent - for a long time, an ultimately serious claim may be valued below the reinsurance reporting threshold. This is not an indictment of primary company claims staffs, but simply an observation that a claimsperson, faced with insufficient and possibly conflicting information about a potentially serious claim, may tend to reserve to probable "expectation". While this "expectation" may be sufficent for most claims with a certain probable fact pattern, it is those few which blow up above this average which will ultimately be covered by the reinsurer. Thus these larger claims generally are reported later to the reinsurer than are the smaller claims the cedent carries net.

Also, certain kinds of mass tort claims with really extreme discovery or reporting delays to the cedent, such as asbestosis, are reinsured heavily, so their extreme report lags have a big impact on the reinsurer's experience. Just as we saw these time delays adding greatly to the uncertainty in reinsurance pricing, they also add greatly to the uncertainty in reinsurance loss reserving.

(5.3.2) There is a persistent upward development of most claim reserves.

Economic and social inflation cause this development. It may also be caused by a tendency of claimspeople to reserve at average values, as noted in (5.3.1). Also, there seems to be a tendency to under-reserve allocated loss adjustment expenses. Thus, early on, the available information may indicate that a claim will pierce the reinsurance retention, but not yet indicate the ultimate severity.

(5.3.3) Claims reporting patterns differ greatly by reinsurance line, by type of contract and specific contract terms, by cedent and possibly by intermediary.

The exposure assumed by a reinsurance company can be extremely heterogeneous. This is a problem because most loss reserving methods require the existence of large homogeneous bodies of data. The estimation methods depend upon the working of the so-called law of large numbers; that is, future development en masse will duplicate past development because of the sheer volume of data with similar underlying exposure. Reinsurers do not have this theoretical luxury, since many reinsurance contracts are unique, and even when there exists larger aggregates of similar exposure, loss frequency may be so low and report lags so long that there is extreme fluctuation in historical loss data. Thus, normal actuarial loss development methods may not work very well.

As we discussed in Section 2, a reinsurer knows much less about the specific exposures being covered than does a primary carrier. Also, the heterogeneity of reinsurance coverages and specific contract terms creates a situation where the actuary never has enough information and finds it difficult to comprehend what is being covered and the true exposure to loss. This is especially true for a reinsurer writing small shares of brokered business.

(5.3.4) Because of the heterogeneity in (5.3.3), it is difficult to use industry statistics.

Every two years, the Reinsurance Association of America (RAA) publishes a summary of casualty reinsurance loss development statistics. These statistics give a very concrete demonstration of the long report and development lags encountered by reinsurers. However, as is noted by the RAA, the heterogeneity of the exposure and reporting differences by company make the statistics of questionable use for particular loss reserving situations.

Likewise, for any two reinsurers, Annual Statement Schedules O and P by-line exposure and loss development are essentially incomparable. Annual Statement lines of business do not provide a good breakdown of reinsurance exposure into reasonably homogeneous exposure categories useful for loss reserving; proper categorization follows the pricing categories we have already seen, and will vary by reinsurance company according to the types of business the company specializes in. This is a problem for two reasons:

1. many people who are not expert in reinsurance insist upon evaluating a reinsurer's loss reserves according to Schedule O and P statistics, and 2. for an actuary examining a reinsurer for the purpose of loss reserving, an appropriate exposure categorization for the particular company may not be as apparent or as easily accomplished as for a primary company.

(5.3.5) The reports the reinsurer receives may be lacking certain information.

Most proportional covers require only summary claims information; often the data are not even split by accident year or by coverage year. Since loss liabilities must be evaluated as of the end of an accident period, calendar year or underwriting year (akin to policy year - all premiums and losses for a contract are assigned to effective or renewal date of the contract) statistics are not sufficient; various interpretations and adjustments must be made.

Even when there is individual claims reporting such as on excess-of-loss covers, there often is insufficient information for the reinsurer's claimspeople to properly evaluate each claim without exerting great effort in pursuing information from the cedent.

This is why it is necessary to have a reasonably large, professional reinsurance claims staff even though the cedent's claims staff is handling the claims. Also, reinsurance claims-people are more accustomed to handling large claims with catastrophic injuries, thus being able to advise the cedent's staff and limit the ultimate payments (and advise in the rehabilitation of seriously injured parties).

For loss reserving, it is useful to have an exposure measure against which to compare loss estimates. One possible measure is reinsurance premium by year by Annual Statement line. On most contracts, losses may be coded correctly by Annual Statement line, but very often the reinsurance premium is assigned to line according to a percentage-breakdown estimate made at the beginning of the contract. To the degree that this estimate does not accurately reflect the reinsurer's loss exposure by Annual Statement line, any by-line comparisons of premiums and losses may be distorted. This adds to the problems noted in (5.3.4).

For most treaties, premiums and losses are reported quarterly in arrears; they may not be reported (and paid) until some time in the following quarter. Thus there is an added IBNR exposure for both premiums and losses. The actuary must remember that the latest-year premiums may be incomplete, so they may not be a good measure of latest-year exposure.

(5.3.6) Because of the heterogeneity in coverage and reporting requirements, reinsurers often have data coding and EDP systems problems.

All reinsurers have management information systems problems. The business has grown in size and complexity faster, and expectations regarding the necessary level of data detail have grown faster, than reinsurers' data systems' abilities to handle and produce the reports requested by marketing, underwriting, claims, accounting and actuarial staffs. This problem may be endemic to the insurance business, but it is even more true for reinsurance.

(5.3.7) The size of an adequate loss reserve is greater for a reinsurer.

This is not a purely technical problem; it is more a management problem, and many reinsurance companies have stumbled over it. All the above problems act to increase the size of an adequate loss reserve and also make it more uncertain. It is difficult for the actuary to overcome the disbelief on the part of management and marketing people and convince them to allocate adequate resources for loss liabilities. Eventually, claims emerging on old exposure overwhelms this disbelief, at least for those who listen. A cynic might say that many reinsurance managers change jobs often enough to stay ahead of their IBNR. Start-up operations in particular have this problem - if there is no concrete runoff experience to point to, why believe a doom-saying actuary.

These seven problems imply that uncertainty in measurement and its accompanying financial risk are large factors in reinsurance loss reserving. This has become an even more important item because of the 1986 Tax Act requiring discounting of loss reserves for income tax purposes. This discounting eliminates the implicit margin for adverse deviation which had been built into insurance loss reserves simply by not discounting. Insurers have lost this implicit risk buffer. Since this buffer now flows into profits and thus is taxed sooner, assets decrease. This clearly increases insurance companies' risk level. The effect on reinsurers is greater.

C. Components of a reinsurer's loss reserve

The general components of a reinsurer's statutory undiscounted loss reserve are as follows:

(5.3.8) COMPONENTS OF A REINSURER'S LOSS RESERVE

1. Case reserves reported by the ceding companies.

These may be individual case reports or may be reported in bulk, depending upon the loss reporting requirements of each individual contract. Most excess-of-loss contracts require individual case reports, while most proportional contracts allow summary loss reporting.

2. Reinsurer additional reserves on individual claims.

The reinsurer's claims department usually reviews individual case reserve reports and specifies additional case reserves (ACR) on individual claims as necessary. This second component of additional case reserves may vary considerably by contract by cedent.

3. Actuarial estimate of future development on (1) and (2).

The future development on known case reserves in total is sometimes known is IBNER, incurred (and reported) but not enough reserved.

4. Actuarial estimate of pure IBNR.

Most actuaries would prefer that separate estimates be made for (3) and (4) the estimate of pure IBNR, incurred but not reported. However, because of limitations in their data systems, in practice most reinsurers combine the estimates of (3) and (4). Depending upon the reinsurer's mix of business, these together may amount to more than half the total loss reserve.

Unless otherwise noted, the term IBNR henceforth shall stand for the sum of IBNER and pure IBNR.

5. Risk load.

The last component of a loss reserve should be the risk loading or adverse deviation loading necessary to keep the reserve at a suitably conservative level, so as not to allow uncertain income to flow into profits too quickly. Many loss reserving professionals prefer to build this into the reserve implicitly by employing conservative assumptions and methodology. Many actuaries would prefer to see it estimated and accounted for explicitly; however, this would violate current AICPA standards. Because of the long-tailed nature of much of their exposure and its heterogeneity and the uncertainty of their statistics, this component is theoretically more important for reinsurers.

The above items (1) through (5) refer to undiscounted statutory loss reserves. Not considered is a loss reserve component for the offset arising from future investment income. Even when we must estimate this and record it on our financial statements, most actuaries would prefer to account for it separately from the undiscounted statutory loss reserve. See the chapter on loss reserving for more discussion on this.

D. A general procedure

The steps involved in reinsurance loss reserving methodology are as follows:

(5.3.9) A GENERAL PROCEDURE

1. Partition the reinsurance portfolio into reasonably homogeneous exposure groups.

2. Analyze the historical development patterns. If possible, consider individual case reserve development and the emergence of IBNR claims separately.

3. Estimate the future development. If possible, estimate the bulk reserves for IBNER and pure IBNR separately.

4. Monitor and test the predictions, at least by calendar-quarter.

Let us now proceed to discuss the first step in some detail.

E. Exposure groups

It is obviously important to segregate the contracts and loss exposure into categories of business on the basis of loss development potential. Combining loss data from nonhomogeneous exposures into large aggregates can increase measurement error rather than decrease it.

Reasonably homogeneous exposure categories for reinsurance loss reserving have been discussed in the actuarial literature and follow closely the categories used for pricing.

Table (5.3.10) lists various important variables for partitioning a reinsurance portfolio. All affect the pattern of claim report lags to the reinsurer and the development of individual case amounts. The listing is meant to be in approximate priority order.

TABLE 5.3.10

PARTITIONING THE REINSURANCE PORTFOLIO

INTO REASONABLY HOMOGENEOUS EXPOSURE GROUPS

Important variables:

1. Type of contract (1): Facultative, Treaty

2. Line of business (1): Property, Casualty, Bonding, Ocean Marine, etc.

3. Type of contract (2): Quota share, surplus share, excess per-risk, excess per-occurrence, aggregate excess, catastrophe, loss portfolio transfer, etc.

4. Layer: primary, working, higher excess, clash.

5. Contract Terms: flat-rated, retro-rated, sunset clause, share of loss adjustment expense, claims-made or occurrence coverage etc.

6. Type of Cedent: Small, large, or E&S company.

7. Line of Business (2): Annual Statement Line.

8. Intermediary

Obviously, a partition by all seven variables would split a portfolio into numerous pieces with too little credibility. However, after partitioning by the first three variables, it may be desirable to recognise the effects of various of the other variables. For example, for Treaty Casualty Excess business, certain reinsurers have found that the type of cedent company (6) is an important indicator of report lag.

Since each reinsurer's portfolio is unique and extremely heterogeneous, in order to determine a suitable partition of exposure for reserving and results analysis, one must depend greatly upon the knowledge and expertise of the people writing and underwriting the exposures, the people examining individual claim reports and the people processing data from the cedents. Their knowledge, together with elementary data analysis (look at the loss development statistics), point the actuary toward the most important variables.

One possible first-cut partition of assumed reinsurance exposure is shown in Table (5.3.11), remembering that there is no such thing as a "typical" reinsurance company.

TABLE 5.3.11

EXAMPLE OF MAJOR EXPOSURE GROUPS FOR

A "TYPICAL" REINSURANCE COMPANY

Treaty Casualty Excess

Treaty Casualty Proportional

Treaty Property Excess

Treaty Property Proportional

Treaty Property Catastrophe

Facultative Casualty

Facultative Property

Surety Excess

Surety Proportional

Fidelity Excess

Fidelity Proportional

Ocean Marine Treaty

Ocean Marine Facultative

Nontraditional Reinsurance

Miscellaneous Special Contracts

Within these major categories, the exposure should be further refined. For example, Treaty Casualty Excess exposure may be further segregated by type of retention (per occurrence excess vs aggregate excess), by type of cedent company (small vs large vs E&S carriers), by layer of coverage (working vs higher clash layers) and by line of business(2) (automobile liability, general liability, workers compensation, medical professional liability). Each of these categories would be expected to have distinctly different lags for claims reported to the reinsurer.

Categories for Treaty Casualty Proportional business would be similar. As we have seen, many contracts classified as proportional are not shares of first dollar primary layers, but rather shares of higher excess layers; thus, whether the exposure is ground-up or excess may be an important variable.

Loss reserving categories for Facultative Casualty would certainly separate out automatic primary programs and automatic umbrella programs; the certificate exposure could be split into buffer versus umbrella, and then further by line of business(2).

Likewise for property and other exposures, the loss reserving categories should correspond closely to the pricing categories.

It is convenient to split the consideration of the historical analysis and estimation ((5.3.9), #2 and #3) according to the length of the claim report lag for different exposure categories.

F. Methodology for short-tailed exposure categories

As is generally true, the best methodologies to use are those which provide reasonable accuracy for least effort and cost. For short-tailed lines of business, such as most property coverage exposure, losses are reported and settled quickly, so loss liabilities are relatively small and run off very quickly. Thus, elaborate loss development estimation machinery is unnecessary.

Reinsurance categories of business which may usually be considered to be short-tailed (as with anything about reinsurance, be careful of exceptions) are listed in Table (5.3.12).

TABLE 5.3.12

REINSURANCE CATEGORIES WHICH ARE USUALLY SHORT-TAILED

(WITH RESPECT TO CLAIM REPORTING AND DEVELOPMENT)

CATEGORY	COMMENTS*
Treaty Property Proportional	Beware of recent catastrophes
Treaty Property Catastrophe	Beware of recent catastrophes
Treaty Property Excess	Possibly exclude high layers
Facultative Property	Exclude construction risks
Fidelity Proportional	

* Exclude all international exposure, if possible.

Many reinsurers reserve property business by setting IBNR equal to some percentage of the latest-year earned premium. A rule of thumb in the industry seems to be that 5-6% is a reasonable percentage. Since most property losses are not coded by accident year, the possible improvement in estimation by using more sophisticated methodology is probably not cost effective.

However, it is a good idea to separately consider major storms and other major catastrophes. A recent catastrophe may cause real IBNR liability to far exceed the 5-6% rule loss reserve. Hurricane losses, even on proportional covers, may not be finalized for a few years.

Another simple method used for short-tailed exposure, for new lines of business or for other situations where the reinsurer has little or no loss statistics is to reserve up to a selected loss ratio. For short-tailed exposure, as long as the selected loss ratio bears some reasonable relationship to past years' experience and as long as it is larger than that computed from already-reported losses, this may be a reasonable method.

Another useful method for short-tailed lines of business is to use the standard American Chainladder (CL) Method of age-to-age factors on cumulative aggregate incurred loss triangles. As long as accident year data exists and the report lags are small, this is sufficiently good methodology. An advantage of this method is that it correlates future development with an overall lag pattern and very definitely correlates it with reported losses for each accident year. A major disadvantage, at least for long-tailed lines, is simply that the IBNR is so heavily correlated with reported losses; so for recent, immature years, the reported, very random nose wags the extremely large tail estimate.

For some proportional treaties, summary loss reporting may assign claims by underwriting year, according to inception or renewal date of the reinsurance treaty instead of by accident year; the reinsurer's claims accounting staff has no choice but to book the claims thusly. So the loss statistics for each false "accident" year may show great development because of future occurring accidents. To get a more accurate loss development picture and estimate IBNR properly, one can assign these "accident" year losses to approximate true accident year by percentage estimates based upon the underwriting year inception date and the general report lag for the type of exposure. Summary claims reported on a calendar (accounting) year basis can likewise be assigned to accident year by percentage estimates, if necessary. For short-tailed lines reserved by a percentage of premium or reserved up to a selected loss ratio, these re-assignments are unnecessary.

G. Methodology for medium-tailed exposure categories

Let us consider any exposure for which claims are almost completely settled within five years and with average aggregate report lag of one to two years to be medium-tailed for this discussion. Reinsurance categories of business one might consider here are listed in Table (5.3.13).

TABLE 5.3.13

REINSURANCE CATEGORIES WHICH ARE USUALLY MEDIUM-TAILED

(WITH RESPECT TO CLAIM REPORTING AND DEVELOPMENT)

CATEGORY	COMMENTS*
Treaty Property Excess Higher Layers	If it is possible to separate these from working layers
Construction Risks	If it is possible to separate these from other property exposure
Surety	
Fidelity Excess	
Ocean Marine	
Any International Property Exposure	

Even if a property claim is known almost immediately, its ultimate value may not. Thus it may take longer to penetrate a higher per-risk excess attachment point. This happens more often if time element coverage is involved. The discovery period for construction risk covers may extend years beyond the contract (loss occurrence) period. So for both these exposures, claim report lags may be significantly longer than normal for property business.

For Surety exposure, it is wise to consider losses gross of salvage and, separately, salvage recoveries. The gross losses are reported fairly quickly, but the salvage recoveries have a long tail. It is instructive to consider for mature years the ratio of salvage to gross loss; this ratio is fairly stable and may help explain predictions for recent coverage years as long as the underwriters can predict how the salvage ratio may have slowly changed over time.

For medium-tailed exposure, the CL Method using aggregate reported losses, with or without ACRs, will yield reasonably accurate answers. An alternative estimation method is the so-called Bornheuter-Ferguson (BF) Method (Bornheuter, R. and Ferguson, R., 1973) which is discussed in the chapter on loss reserving. This method uses a selected loss ratio for each coverage year and an aggregate dollar report lag pattern specifying the percentage of ultimate aggregate loss expected to be reported as of any evaluation date. An advantage of this method is that it correlates future development with an exposure measure:

exposure = reinsurance premium * selected loss ratio.

A disadvantage is that the BF IBNR estimate is very dependent upon arbitrarily selected loss ratios; also, the estimate for each accident year does not reflect the particular to-date reported losses for that year, unless the selected loss ratio is chosen with this in mind. The to-date reported loss for a given accident year is strongly correlated with the place of that year in the reinsurance profitability cycle; it would seem to be desirable to use this fact in the IBNR estimate. As noted before, the reinsurance profitability cycles are more extreme than primary insurance cycles. Thus, when using the BF Method, one must select the accident year loss ratios carefully.

An estimation method which overcomes some of the problems with the CL and BF Methods was independently derived by Edward Weissner (written up in Patrik (1978) and Weissner (1981)) and Hans Buehlmann (internal Swiss Re publications). As with the CL and BF Methods, this method, let us call it the Weissner-Buehlmann (WB) Method, uses an aggregate known loss lag pattern which may be estimated via the CL Method. The key innovation is that the ultimate expected loss ratio for all years combined is estimated from the overall known loss experience, instead of being selected arbitrarily. The problem with the WB Method is that it does not tell the user how to adjust the overall loss ratio an appropriate a priori loss ratio by accident year. It is left to the user to adjust each year's premium to reflect the profit cycle on a relative basis. A simple example will explain this.

(5.3.14) EXAMPLE:

For a given exposure category with five years experience, assume that the yearly risk earned premiums (net of reinsurance commissions and brokerage fees) can be adjusted to remove any suspected rate differences by year and the internal expense ratio is constant year to year, so that a single ELR can represent the expected loss ratio for each year. In primary insurance terms, assume that the premiums have been put on-level. Let ELR represent this expected loss ratio to adjusted risk earned premium. Suppose that Table (5.3.15) displays the current experience for this category:

TABLE 5.3.15

EXAMPLE

Data as of 12/31/88 in 1,000's

(1) Cal/Acc Year	(2) Risk Earned Premium	(3) Adjusted Earned Premium	(4) Aggregate Reported Loss	(5) Aggregate Reported Loss Lag
1984	$ 6,000	$ 8,000	$ 7,000	95%
1985	7,000	7,000	5,000	85%
1986	8,000	6,000	3,000	70%
1987	9,000	7,000	2,000	50%
1988	10,000	10,000	4,000	30%
TOTAL	40,000	38,000	21,000	NA

The IBNR estimate is given by (5.3.16):

(5.3.16) WB IBNR est. = Sum (year i IBNR est.)

= Sum (ELR est. x adj. earned premium x (1 - lag))

= ELR est. x Sum (adj. earned premium x (1 - lag))

= ELR est. x ((8,000 x .05) + (7,000 x .15) + (6,000 x .3) + (7,000 x .5) + (10,000 x .7))

= ELR est. x 13,700

The ELR estimate may be written as in (5.3.17):

$$(5.3.17)\quad \text{WB ELR est.} = \frac{(\text{Total reported losses}) + (\text{Total IBNR est.})}{\text{Total adjusted earned premium}}$$

$$= (\$21{,}000 + \text{IBNR est.})/\ \$38{,}000$$

The trick is putting these two together:

$$(5.3.18)\quad \text{WB ELR est.} \times 13{,}700 = \text{WB IBNR est.}$$

$$= (\text{WB ELR est.} \times 38{,}000) - 21{,}000$$

or

$$(5.3.19)\quad \text{WB ELR est.} \times (38{,}000 - 13{,}700) = 21{,}000$$

or

$$(5.3.20)\quad \text{WB ELR est.} = 21{,}000/24{,}300 = .864$$

Table (5.3.21) compares IBNR and estimated ultimate loss ratios for CL and WB Methods; the BF and WB Methods cannot be compared, since the BF loss ratios are not estimated by formula.

TABLE 5.3.21

COMPARISON OF CHAINLADDER AND WEISSNER-BEUHLMANN METHODS

(1)	(2)	(3)	(4)	(5)	(6)
Cal/ Accident Year	Risk Earned Premium	Chainladder Estimates IBNR	Chainladder Estimates Loss Ratio	Weissner-Buehlmann Estimates IBNR	Weissner-Buehlmann Estimates Loss Ratio
1984	$6,000	$ 368	123%	$ 346	122%
1985	7,000	882	84%	907	84%
1986	8,000	1,386	54%	1,555	57%
1987	9,000	2,000	50%	3,024	56%
1988	10,000	9,333	133%	6,048	100%
TOTAL	40,000	13,969	87%	11,880	82%

As long as the rate relativity adjustments to yearly risk earned premium are reasonably accurate, the yearly and overall results are more accurate with the WB Method. It is easy to see that the above example would be more vivid if a longer-tailed example were used.

H. Methodology for long-tailed exposure categories

Just as for pricing, the real problem in loss reserving is long-tailed exposure, especially excess-of-loss casualty reinsurance. Reinsurance categories of business usually considered to be long-tailed are listed in Table (5.3.22).

TABLE 5.3.22

REINSURANCE CATEGORIES WHICH ARE USUALLY LONG-TAILED

(WITH RESPECT TO CLAIM REPORTING AND DEVELOPMENT)

CATEGORY	COMMENTS
Treaty Casualty Excess	Includes the longest lags
Treaty Casualty Proportional	Some of this exposure may possibly be medium-tailed
Facultative Casualty	Some of this exposure may possibly be medium-tailed

The first step is to separate these exposures into finer categories. This is, of course, an iterative process. Depend upon the company's marketing, underwriting, claims and accounting personnel for the first stage categorization. Further refinements will then depend upon your hypothesis testing and upon your investigation of various comments from the marketing and underwriting people as they receive the estimated IBNR by major contract or category based upon the latest categorization. It may be desirable to treat claims-made exposure separately, if possible.

It may be necessary to separate out certain types of losses for special consideration; for example, claims arising from asbestosis, environmental and other mass tort claims. Because of the catastrophic significance of these types of claims (nothing for many years, then suddenly, gigantic totals), they would terribly distort the development statistics if left in the normal loss data. Also, it is unlikely that normal actuarial loss development techniques, if used blindly, would yield reasonable answers for these types of losses. The question of which claims should be specially treated is difficult and should be discussed thoroughly with the company's claims staff.

For long-tailed exposure, current methodology is usually the CL, BF or WB Methods. However, with the extreme lags encountered here, it may pay to consider the estimation of IBNER separately from the estimation of pure IBNR. For the estimation of pure IBNR, it is appropriate to consider the estimation of IBNR counts and amounts separately. These separate estimates can be input to standard risk theoretic models for aggregate losses so that the loss runoff can be viewed as a stochastic process.

An advantage of using a claim count/claim severity model is that we can contemplate intuitively satisfying models for various lag distributions, such as the time from loss event occurrence until first report and the lag from report until settlement. We can then connect these lags with appropriate models for the dollar reserving and payments on individual claims up through settlement.

Perhaps the best way to describe a simple modeling approach is through use of a simple example.

(5.3.23) **EXAMPLE**

Facts:

1. We want to estimate gross assumed IBNER and pure IBNR for our Treaty Casualty Excess working cover business assumed from large companies; these working covers would have fairly high attachment points.

2. We have the usual development triangles for reported and paid aggregate dollars and claim counts for the last 25 accident years; the claims are separated by major line: automobile liability, general liability, workers compensation.

3. We have risk earned premiums by year for the last 15 calendar years.

4. We have talked with the marketing people, underwriters and claimspeople to see if there are any special contracts, exposures or types of claims which should be treated separately or particularly large individual claims which should be censored so as not to have an undue random impact on the estimation.

As a result of our discussions(4), we decide to separate out only asbestosis-related claims. Also, we decide to censor (limit) to a value of $5,000,000 each six large reported claims, so to smooth their impact upon the claim severity estimates; the final claim severity estimates will be adjusted to account for potential severity excess of $5,000,000.

We shall pay particular attention to the estimation of the report lag distribution, the time from claim occurrence until first report. By first report we shall mean the month in which the claim first appears in the reinsurer's claims database with a significant (nonprecautionary) dollar value. If, in addition to the summary claim count development triangles, we also have individual claims data with accident date and first report date for each claim, then for various selected probabilistic models, we can obtain parameters via maximum likelihood estimation as discussed by Weissner (1978 and 1981) and John (1982).

Alternatively, suppose we have only the summary development triangles of reported claims. Also in this case, maximum likelihood estimates of model parameters may be made on these data by treating the increments for each development interval as grouped data, exactly as discussed by Hogg and Klugman (1984) for claims severity. The reported claim counts for each accident year can be considered to be a sample from a truncated model (unknown tail). A slight practical problem here may be negative increments, but for the estimation, the time intervals for the grouping of the data need not necessarily all be one year periods, so the intervals can always be adjusted to avoid negative increments. Or negative increments can be handled separately by estimating claim dropout rates (closed no payment). To simplify this discussion, let us assume that claims closing without payment drop out of the count.

Various statistical and reasonableness tests can then help us decide which model best describes the data and which we believe will best predict future claims arrivals. This model with the fitted parameters can then be used to predict IBNR claim emergence.

Assume now that we have estimated claim report lag distributions for each line. Assume also that we don't trust the breakdown of reinsurance premiums by line, so they cannot serve as a by-line exposure base for IBNR estimation. Without a reasonable by-line exposure, the only achievable by-line IBNR estimates are via the CL

Method, hardly credible for immature accident years. An alternative is to estimate the overall report lag distribution by weighing together the lags for each line. Technically, the weights could vary by accident year. Let us assume in this case that constant weights over all accident years are reasonable.

For simplicity, let us suppose that our combined all-lines report lag is estimated to be a lognormal with mean = 6 years and coeffient of variation = 1 (u = 1.2918, s = 1). Suppose that Table (5.3.24) displays our claims situation:

TABLE 5.3.24

TREATY CASUALTY EXCESS WORKING COVER EXAMPLE - as of 12/31/88

(1) Accident Year	(2) Reported Claims	(3) Estimated Report Lag	(4) CL IBNR Est. Claims
1964	39	96.89%	1.3
1965	27	96.58%	1.0
:	:	:	:
1984	20	58.41%	14.2
1985	11	48.45%	11.7
1986	13	35.36%	23.8
1987	5	18.77%	21.6
1988	0	2.36%	0
TOTAL	473	NA	231.4

Suppose that we can adjust the annual reinsurance premiums for rate relativities and relative excess frequency year by year with some, but not total, confidence. Then we can also estimate IBNR via the WB Method and a reasonable "credibility" method in Table (5.3.25):

TABLE 5.3.25

TREATY CASUALTY EXCESS WORKING COVER EXAMPLE - as of 12/31/88

(1) Acc Year	(2) Adj.Earn Premium (1,000's)	(3) Reported Claims	(4) Report Lag	(5) CL IBNR Claims	(6) WB IBNR Claims	(7) Cred IBNR Claims	(8) Claim Freq. (3+7)/2
1964	.	39	96.89%	1.25	1.25	1.325	NA
1965	.	27	96.58%	0.96	0.96	0.96	NA
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
1984	$ 8,000	20	58.41%	14.24	13.31	13.85	4.231
1985	8,000	11	48.45%	11.70	16.50	14.17	3.146
1986	7,000	13	35.36%	23.76	18.10	20.10	4.729
1987	8,000	5	18.77%	21.64	25.99	25.17	3.771
1988	9,000	0	2.36%	0.0	35.15	34.32	3.813
'74-'88	100,000	166	6.00	205.93	234.02	231.21	3.972
TOTAL	100,000	473	Average Lag	231.42	259.54	256.66	NA

The reader can verify the following:

5. For those years with adjusted premium, the WB IBNR estimated claims in column 6 are computed with respect to a claim frequency of 4.0 per $1,000,000 (estimated via the WB Method for the most recent 15 years). The earlier years simply use the CL IBNR estimated claims.

6. The "credibility" IBNR estimated claims in column 7 is a weighing of the CL and WB estimated claims using the report lag as a weight for each CL estimate.

Exhibit (5.3.26) is a picture of claim count and claim reporting lags for accident year 1984 from an actuarial point of view (using the total credibility claim count of 33.9 and the expected report lag distribution). Suppose that the commonly used Poisson distribution, with parameter n say, is a good model for the total claim count N. Then the number of claims reported in the ith year, N(i), will also be Poisson with parameter n*p(i), where p(i) is the lag probability for the ith year; that is, p(i) is the probability that a claim will be reported between i-1 and i years after its occurrence. This Poisson assumption allows us to make interconnected probability statements about claim reports from year to year. Under these assumptions, the reader can verify the following:

7. The standard deviation of the accident year 1984 Credibility IBNR is 3.72 claims.

8. Prob[225 < Cred. IBNR total < 288] = .95 (approximate)

Please note that these estimates simplistically assume that we know the true parameter n for the Poisson. In reality, sample error on the estimate of n should be considered, perhaps, also simplistically, inducing a Negative Binomial distribution.

In addition to estimating report lag distributions and estimating IBNR claim counts, we must also estimate IBNER and the IBNR claim severities. Various techniques from the chapter on Loss Reserving may be used. Or, if you have the information, an approach similar to that displayed by Table (5.2.18) might be used.

Once the various distributions for counts and amounts are estimated, the aggregate losses reported in the ith year of run-off can then be modeled via the standard risk theoretic model under suitable assumptions for the claim sizes. We already saw this model in (5.2.17), repeated here for convenience:

(5.3.27) $L(t) = X(t,1) + X(t,2) + \ldots + X(t,N)$

where N = number of (paid) claims (or occurrences)

and $X(t,i)$ = amount of the ith claim at time t

The N, L and X's here may represent the pure IBNR for a particular accident year. Given appropriate models for N and X and suitable assumptions, we can approximate the probability distribution of L. Then we can ask various probability questions as we did in Section 2.

Given models like (5.3.27) for each part of IBNER and IBNR, the various L's can be added and we can talk about the joint distribution of the sum.

We should note here that various authors have used very different approaches to estimate the distribution of aggregate IBNER and IBNR. The reader should refer to the bibliography for advanced and/or different methodologies, especially see Taylor (1985), Eegehn, J. van (1981) and various Advanced Techniques sessions in Casualty Loss Reserve Seminar transcripts.

A problem with increasingly sophisticated methodologies is that the answers may become less intuitive and may be much more difficult for the actuary to understand and explain to management and others. Here I recommend the use of an exhibit format like (5.2.17); the few actual estimates (expected settlement average size for reported open and IBNR claims and the expected IBNR count) are cleanly separated from the known numbers, but are juxtaposed for comparison. Various probable future settlement scenarios can be displayed for comparison. These and the monitoring reports to be discussed in the next section are important for management (and actuarial) decision-making.

I. <u>Monitoring and testing predictions</u>

A loss reserve or an IBNR reserve is an hypothesis about future claims settlements for past events. In order to validate your methodology, you must test your predictions against actual future experience. Monitoring and testing quarterly claims runoff against predictions may provide early warning of problems.

For short-tailed and medium-tailed lines, this can be fairly simple. As long as current accident year claims can be reasonably separated from past accident year runoff, the runoff can be compared with the previous year-end reported open and IBNR reserves.

For long-tailed lines, slightly more sophisticated comparisons are necessary. Table (5.3.28) is one possible format:

Table 5.3.28

TREATY CASUALTY EXCESS WORKING COVER EXAMPLE

(1) ACC YEAR	(2) REPORTED CLAIMS 12/31/88	(3) CRED IBNR ESTIMATE 12/31/88	(4) PREDICTED EMERGENCE 3/31/89	(5) REPORTED CLAIMS 3/31/89	(6) ACTUAL EMERGENCE (5)-(2)	(7) ACTUAL - PREDICTED (6)-(4)	(8) REPORT LAG 3/31/89
1964	39	1.25	0.03	39	0	-0.03	96.96%
1965	27	0.96	0.02	28	1	0.98	96.66%
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
1984	20	13.85	0.70	20	0	-0.70	60.50%
1985	11	14.17	0.76	13	2	1.24	51.20%
1986	13	20.10	1.12	12	(1)	-2.12	38.97%
1987	5	25.17	1.38	9	4	2.62	23.21%
1988	0	34.32	1.18	2	2	0.82	5.71%
TOTAL	473	256.66	10.26	487	14	3.74	6.00%

TOTAL(7)/TOTAL(4) = 36%

Columns 2 and 3 are from Table (5.3.25). Column 8 is the lognormal adjusted by one quarter. The reader can verify the following:

9. Column 4 is obtained from Column 3 using the lags at 12/31/88 and 3/31/89.

10. Assuming that the emergence is Poisson with mean = Total(4), then there is approximately a 12% probability that the actual emergence is 14 or more (use a Normal approximation to the Poisson).

As a result of (10), perhaps we wouldn't alter our opinion of the magnitude of the accuracy of the report lags and the estimated IBNR. However, we may want to pay close attention to the claim emergence over the next quarter. Note the negative emergence for 1986; most likely a claim was settled for less than the excess attachment point.

In addition to monitoring and testing claim count predictions, one should also review claim severities. Besides just report emergence, one can and should monitor and test claim settlements.

J. Conclusion of the reinsurance loss reserving section

We have seen some examples of how standard actuarial methods and some not-so-standard actuarial methods apply to reinsurance loss reserving. We must remember that there is no one right way of estimating reinsurance loss reserves. But there are plenty of wrong ways. Common actuarial methods should be used only to the extent they make sense. To avoid major blunders, the actuary must always understand as well as possible the types of reinsurance exposure in his company's portfolio. The differences from primary company loss reserving mainly involve much less specificity of information, report and settlement timing delays and often much smaller claim frequency together with much larger severity, all inducing a distinctly higher risk situation. But with this goes a glorious opportunity for actuaries to use their theoretical mathematical and stochastic modeling abilities and data analytical abilities fully.

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APPENDIX A: PARETO DISTRIBUTION

1. Support: $X > 0$

2. Parameters: $b > 0,\ q > 0$

3. C.d.f.: $F(X) = 1 - (b/(b + X))^{q}$

4. P.d.f.: $f(X) = qb^{q}/(b + X)^{q+1}$

5. Moments: $E[X^{n}] = b^{n} * (n!)/((q-1)*(q-2)*...*(q-n))$

6. Censored c.d.f.: (general definition)

$$F(X;c) = \begin{cases} F(X) & \text{if } X < c \\ 1 & \text{otherwise} \end{cases}$$

7. Censored moments: If q is not an integer, then

$$E[X^{n};c] = E[X^{n}] - q*(b/(b+c))^{q} * \{ (b+c)^{n-1}/(q-n) + .$$

$$. . + (-1)^{i} * ((n!)/((i!)*(n-i)!)) * b^{i} * (b+c)^{k-i}/(q-n+i)$$

$$+ . . . + (-1) * b^{n} / q^{n} - c^{n}/q \}$$

8. Censored expectation: $E[X;c] = E[X]*\{ 1 - (b/(b+c))^{q-1} \}$

9. Conditional probability:

$$\text{Prob}[X>y|X>x] = \{(b+x)/(b+y)\}^{q}$$

10. Truncated (conditional) distribution:

Definition: $Xd = X - d$ for $X > d$

Then Xd is Pareto with parameters b+d, q:

$$F(Xd) = 1 - \{(b+d)/((b+d)+Xd)\}^{q}$$

11. Trended distribution:

Definition: $Y = t*X$

Then Y is Pareto with parameters t*b, q:

$$F(Xd) = 1 - \{(t*b)/((t*b)+Y)\}^{q}$$

APPENDIX B: LOGNORMAL DISTRIBUTION

1. Support: $X > 0$

2. Parameters: $-oo < u < oo,\ s > 0$

3. C.d.f.: $F(X) = O((\ln X - u)/s)$

4. P.d.f.: $f(X) = (1/X)*o((\ln X - u)/s)$

5. Moments: $E[X^n] = \exp(nu + n^2 s^2/2)$

6. Censored moments:

$$E[X^n;c] = E[X^n]*O((\ln(c) - u)/s - n*s)$$

$$+ c^n*(1 - O((\ln(c) - u)/s))$$

7. Truncated (conditional) distribution: $(Xd = X - d)$

   a) $E[Xd] = E[X] - E[X;d]$

   b) $E[(Xd)^2] = (E[X^2] - E[X^2;d]) - 2*d*E[Xd]$

   c) $Var[Xd] = E[(Xd)^2] - E[Xd]^2$

8. Censored truncated distribution:

   a) $E[Xd;c-d] = E[X;c] - E[X;d]$

   b) $E[(Xd)^2;c-d] = (E[X^2;c] - E[X^2;d]) - 2*d*E[Xd;c-d]$

   c) $Var[Xd;c-d] = E[(Xd)^2;c-d] - E[Xd;c-d]^2$

APPENDIX C: AGGREGATE LOSS MODEL

1. Aggregate loss $L = X(1) + X(2) + \ldots + X(N)$

   where N = rv denoting number of claims

   $X(i)$ = rv denoting the value of the <u>ith</u> claim

Assume that N and the X(i)'s are mutually independent and the X(i)'s are identically distributed with c.d.f. F (x). (Note: These are usually reasonable assumptions when the parameters for the distributions of N and X are assumed to be known.) Then the following statements are true:

2. C.d.f.: $F(x) = \text{Prob}[N=n]*F^{*n}(x)$

   where F^{*n} is the <u>nth</u> convolution

3. Moments:

$$E[L] = E[N]*E[X]$$

$$Var(L) = E[N]*E[X^2] + (Var(N) - E[N])*E[X]^2$$

$$= E[N]*(Var(X) + (Var(N)/E[N])*E[X]^2)$$

$$E[(L - E[L])^3] = E[N]*E[(X - E[X])^3]$$

$$+ E[(N - E[N])^3]*E[X]^3$$

$$+ 3*Var(N)*E[X]*Var(X)$$

LOSS RESERVING
(TEXTBOOK CHAPTER DRAFT)

Chapter 3
LOSS RESERVING

by Ronald F. Wiser, FCAS, ASA, MAAA

Introduction

Accounting Concepts

In order to understand the loss reserving process, it is essential to have a conceptual understanding of basic accounting principles.

The accounting process produces two major end products (among others): the balance sheet and the income statement, that document the financial position and performance of a firm, respectively. The accuracy of both of these statements are very dependent on loss reserves.

i.) The Balance Sheet

The balance sheet presents the status of the firm at a specific point in time. It is a report on the financial position of the firm. It reports on the levels of assets and liabilities, and the status of the shareholders equity, or surplus for the property and casualty company. The reporting follows the simple equation:

Assets = Liabilities + Owners' Equity

Assets are any economic resource that is held by the firm. This could be cash, agents' receivables, real estate, or stocks and bonds, for example.

Liabilities are claims on the resources of the firm, to satisfy obligations of the firm. These could be mortgages, bank debt, bonds issued, premiums received from clients but not yet earned, or benefits payable on behalf of clients due to contractual obligations, for example.

Owners' equity is the owners' claim on the assets of the firm. The owners' claim is always subordinate to all other liabilities of the firm. It is actually the balancing item in the equation above.

Thus, when liabilities exceed assets, the value of the owners' equity is negative, and the firm is bankrupt. A more suggestive way to write the equation of financial position is then given by:

Owners' Equity = Assets - Liabilities

Through common usage the term "loss reserve" has come to denote the property and casualty company's liability for claims by or against policyholders. Loss reserving is the process of estimating the amount of the company's liabilities for such claims ("losses") which the company has contracted to settle for its policyholders.

Loss Reserving

Although an essential part in the preparation of any insurance concern's financial statement preparation, loss reserving is the function of the actuary because it is an estimation process that involves the current financial evaluation of future contingent events. These contingencies apply to obligations that have already been assumed by the company through the insurance contract. They are:

1.) Future developments on claims already reported, and

2.) Future claims to be reported, based on events that have already occurred.

ii.) The Income Statement

The income statement is the second major product of the accounting process. It measures certain changes in Owners' Equity during a stated period of time. Owners' Equity can be subdivided into the capital contributed by the owners and any earnings of the firm from past periods retained in the firm. Thus

Owners' Equity = Contributed Capital + Retained Earnings

The income statement measures the firm's performance in the period ended as follows:

Income = Revenues - Expenses

Revenue measures the inflow of net assets from providing services. Expenses measure the outflow of net assets that are consumed in providing the firm's services.

Income may be used to either increase Owners' Equity in the firm (i.e. increase Retained Earnings) or may be distributed to owners as dividends. This can be written as

Income = Change in Retained Earnings + Dividends to Owners

This series of equations then defines the relationship between the balance sheet and the income statement. This relationship can be obtained by chaining together the basic accounting equations above.

Income = Change in Retained Earnings + Dividends to Owners

But,

Retained Earnings = Owners' Equity - Contributed Capital

and,

Owners' Equity = Assets - Liabilities.

Thus,

Retained Earnings = Assets - Liabilities - Contributed Capital

and,

Income = Change in Assets - Change in Liabilities - Change in Contributed Capital + Dividends to Owners.

iii.) The Definition of Liabilities

An obligation satisfies the accounting definition of a liability if it possesses three essential characteristics:

1.) The obligation involves a probable future sacrifice of resources at a specified or determinable date;

2.) The firm has little or no discretion to avoid the transfer; and

3.) The transaction or event giving rise to the obligation has already occurred.

A claim liability of a property and casualty insurer satisfies the second and third characteristics above. The first requirement is not generally satisfied in property and casualty claim situations. For instance, in a workers compensation pension claim the payments must be made periodically at specified times, often weekly. However, in a third party liability situation it is not possible to specify the date on which settlement will be made.

A loss reserve is a contingent liability in the sense that each specific claim under adjustment depends on some future contingent event to determine the extent of the insurer's liability. For most firms, contingent liabilities are not treated as accounting liabilities. Two tests are proposed by the accounting literature to determine if a contingent liability should be recognized on the company's balance sheet. These are

1.) Information at the time of preparation of the financial reports indicates that it is likely that a liability has been incurred, and

2.) the amount of the liability can be reasonably estimated.

Clearly, an insurer's loss reserves satisfy both these conditions.

iv.) Cash versus Accrual Basis of Accounting

The balance sheet and income statement may be prepared under different accounting bases - cash or accrual.

The cash basis recognizes revenues when they are received, and expenses are reported in the period expenditures are made. For very simple businesses such as professional services, the cash basis may be adequate. However, for most businesses the cash basis of accounting does an unsatisfactory job of matching revenues and expenses appropriately.

The accrual basis of accounting recognizes revenue as it is earned by the firm. Likewise costs are reported as expenses in the same period as the revenues giving rise to those costs are recognized. This results in an income statement that matches costs with appropriate revenues.

For the property and casualty operation this results in recognition of earned premium as revenue rather than written premiums. Earned premiums are generally calculated through the use of a liability account termed the unearned premium reserve. Thus,

Earned Premium = Written Premium + Beginning Unearned

Premium Reserve - Ending Unearned Premium Reserve.

The unearned premium reserve is that portion of the written premium that has not been exposed to loss. These are funds that have been received from the policyholders, but for which services have not yet been provided. For the property and casualty company the service is exposure to the chance of loss from the perils insured against over a period of time. Thus these are liabilities of the company to the policyholders.

Generally this is an easy calculation, since most property and casualty policies are for a fixed term, say six months or one year, and the premium revenues can be considered as earned pro-rata over the policy term. A company may calculate its unearned premium liability on a daily pro-rata basis, or monthly. For daily pro-rata calculations, a $1000 policy written on January 1, would require an unearned premium reserve of $657.53 after 125 days of coverage had elapsed on April 15.

Similarly, expenses incurred are policyholder benefits, which must be matched to the revenues earned on the policies. It would clearly be inappropriate to count only paid losses and paid loss adjustment expense as expenses. This is due to the long delays between the time period for which insurance protection is afforded under the policy, and the actual liquidation of the obligations assumed under the terms of the policies.

The expenses incurred for policy benefits for the income statement can be computed through use of the "loss reserve" liability account. The formula is given by

Incurred Losses = Paid Losses + Ending Claim Liability - Beginning Claim Liability.

Unlike most businesses, the property and casualty business is characterized by strong positive cash flows. This is a result of the fact that premiums for the policy protection are received at the beginning of the policy period. Also, there are various delays until the actual liquidation of policy benefits. The measurement and financial quantification of these delays is the result of the loss reserve estimation process.

v.) Illustration of Cash Flows and Profitability

It is possible to construct a simplified model of the cash flows and accruals of a typical property and casualty insurance mechanism. All of the concepts reviewed above can be observed in such a model. The difference between a balance sheet that appears healthy on a cash flow basis, but is insolvent on an accrual accounting basis is demonstrated below.

Because of the long delays inherent in the recognition and liquidation of claims arising under the insurance contract, the differences between cash and accrual accounting can be quite dramatic. A simplified example of the difference between cash and accrual balances can demonstrate the differences.

The lags in claim recognition can be recognized by development patterns. The development pattern may be interpreted as the probability that a dollar of loss is reported within the given development time period. Assume the following reporting pattern for losses:

Development Patterns by Year since Year of Occurrence

Year	0	1	2	3
Reported	40.0%	30.0%	20.0%	10.0%

This implies that all losses are reported within four years of the start of the year of occurrence, and 40% of incurred claim dollars are reported in the same 12 month period as the accident period.

Assume that once a claim has been reported its liquidation probabilities are given as follows:

Loss Reserving

Development Patterns by Year since Report Year

Year	0	1	2
Paid	35.0%	35.0%	30.0%

This means that 35% of dollars to be paid on claims reported in a given calendar year (the report year) will be paid out in that initial report year.

We assume the following loss ratios and expense ratios, as well as a starting position with $1 million equity invested half in cash and bonds.

	Starting Balance	Yield
Starting Cash:	$500,000	5.0%
Starting Bonds:	$500,000	7.0%

This results in a starting balance sheet as follows

Assets		Liabilities and Policyholders Surplus	
Cash	$500,000	Liabilities	$0
Bonds	$500,000		
Total	$1,000,000	Policyholders Surplus	$1,000,000
		Total	$1,000,000

Underwriting assumptions for the seven year simulation below are given by the following,

Underwriting Assumptions	
Loss Ratio	75% of Earned Premium
Expenses	25% of Written Premium
Combined Ratio	100%

At a given combined ratio the gap between net cash flow and changes in surplus varies directly with the growth rate. A high rate of growth at a high combined ratio can generate a comfortable positive cash flow position while the surplus position of the insurer can be eroding.

Growth Rate	10%
Cash Invested in Bonds	80% of new cash flow

Loss Reserving

This model assumes 80% of each year's net cash flow is invested in bonds yielding 7%. Cash balances yield 5%. Income taxes are ignored for simplicity.

Based on the assumptions above, premiums written are derived. Unearned premiums are set at 1/2 of the written premiums. The earned premiums follow from the accounting formula:

Earned Premium = Written Premium + Beginning Unearned - Ending Unearned

Ultimate losses are based on the ultimate loss ratio assumption multiplied by the earned premiums.

Losses reported are based on the lag factors assumed multiplied by the ultimate losses. Losses paid are based on the amount of loss reported multiplied by the lag factors for liquidation of reported losses.

Amounts in $1,000's

Premiums	1982	1983	1984	1985	1986	1987	1988
Written	$1,000	$1,100	$1,210	$1,331	$1,464	$1,611	$1,772
Unearned	$500	$550	$605	$666	$732	$805	$886
Earned	$500	$1,050	$1,155	$1,271	$1,398	$1,537	$1,691

Losses	1982	1983	1984	1985	1986	1987	1988
Ultimate	$375	$788	$866	$953	$1,048	$1,153	$1,268
Reported	$150	$428	$658	$836	$957	$1,053	$1,158
Paid	$53	$202	$425	$651	$825	$954	$1,061
Reserved	$323	$908	$1,349	$1,651	$1,874	$2,073	$2,280
Expenses	$250	$275	$303	$333	$366	$403	$443
Inv. Income	$60	$110	$158	$201	$237	$270	$305

Cash	1982	1983	1984	1985	1986	1987	1988
Cash In	$1,060	$1,210	$1,368	$1,532	$1,701	$1,881	$2,077
Cash Out	$303	$477	$727	$984	$1,191	$1,357	$1,504
Net Cash Flow	$758	$733	$641	$548	$510	$524	$573

Assets	1982	1983	1984	1985	1986	1987	1988
Bonds	$1,106	$1,692	$2,205	$2,643	$3,051	$3,471	$3,929
Cash	$652	$798	$926	$1,036	$1,138	$1,243	$1,357
Total	$1,758	$2,490	$3,131	$3,679	$4,189	$4,713	$5,286

Liabilities	1982	1983	1984	1985	1986	1987	1988
Reserves	$323	$908	$1,349	$1,651	$1,874	$2,073	$2,280
Unearned	$500	$550	$605	$666	$732	$805	$886
Surplus	$935	$1,032	$1,177	$1,363	$1,583	$1,835	$2,120
Total	$1,758	$2,490	$3,131	$3,679	$4,189	$4,713	$5,286

A proper accounting statement for a property casualty insurer requires a correct evaluation of reserves. The results below show that even an insurer headed toward insolvency can show a very strong net cash flow position as long as growth is maintained. The results produced by this very simple underwriting model are given in the table below for a range of differing inputs. Needed loss reserves have been accurately stated.

Some possible outcomes from this model are given in the table below.

Annual Premium Growth	Combined Ratio	Bond Yields	1988 Surplus	1988 Invested Assets	1988 Cash Flow	Annual Surplus Growth
0%	125%	7.0%	$283	$2,733	($71)	-16.5%
5%	125%	7.0%	$58	$3,173	$16	-33.4%
10%	125%	7.0%	($210)	$3,717	$144	NA
20%	125%	7.0%	($905)	$5,191	$568	NA
0%	110%	7.0%	$1,354	$3,512	$118	4.4%
5%	110%	7.0%	$1,281	$4,029	$236	3.6%
10%	110%	7.0%	$1,188	$4,658	$401	2.5%
20%	110%	7.0%	$929	$6,334	$916	-1.0%
0%	100%	7.0%	$2,068	$4,031	$244	10.9%
5%	100%	7.0%	$2,096	$4,599	$383	11.2%
10%	100%	7.0%	$2,120	$5,286	$573	11.3%
20%	100%	7.0%	$2,151	$7,096	$1,148	11.6%
0%	95%	7.0%	$2,425	$4,290	$307	13.5%
5%	95%	7.0%	$2,503	$4,885	$457	14.0%
10%	95%	7.0%	$2,586	$5,600	$659	14.5%
20%	95%	7.0%	$2,763	$7,477	$1,264	15.6%

From these examples, we can conclude that growth, even at ruinous combined ratios can often generate healthy cash flows for an insurer. However, unless the business is profitable on a combined ratio basis, the growth that enhances cash flow can actually decrease surplus. The accurate reporting of the drain on surplus will take place only if the needed reserves are correctly evaluated.

Loss Reserving Activity Model

A look at the structure of a typical loss reserve inventory will help us to better understand the processes at work when we observe loss development. For the moment, let us consider loss development as simply the change in the amount of a loss reserve inventory in a specified period of time. While in the aggregate we may only be able to observe the increase in total dollars of loss reserve (e.g., given only Annual Statement data), the change in total case reserve inventory is the result of a number of natural transition processes that can be found in any reserve inventory. An understanding of this structure will allow the loss reserve actuary to investigate reasons for observed development patterns.

Initially, losses must enter the case inventory. They will enter either as new claims (2.), reopened claims (3.), or zero reserves (4.). A reopened claim is one that has previously been closed, but requires a pending claim file, because of further adjusting activity. This must be distinguished from a closed claim that simply requires an additional payment after closure, i.e. a prematurely closed claim (included in line 15.). Such a claim is not reopened because no further adjusting effort is expected to be necessary after the single payment.

A particular type of new claim that should be distinguished from others is the zero reserve claim or precautionary reserve claim. This is used to establish a file as a means of monitoring a potential liability situation. No dollar value, or a nominal amount, is put up on the claim file because there is not yet a strong enough fact situation that a liability of the company exists. However, there is potential for liability and the situation must be closely monitored by the company. The use of these precautionary files is most often found on excess or reinsurance losses. If the primary carrier is another company, there may be very little information in the file initially established, other than the mandatory notice required by the excess policy wording.

Many companies also use a "fast track" claim category (line 14). This is simply a claim that is paid without a claim file ever being established. This procedure is often used on small property claims, such as auto physical damage or homeowners.

		Counts	Amounts
1.	Beginning Outstanding	1,015	$5,673,633
2.	New Reserves	80	$270,850
3.	Reopened Reserves	29	$84,472
4.	Zero Reserves	2	$ 0
5.	Reserve Increases	28	$163,995
6.	Subtotal: Increases (2+3+4+5)	139	$519,317
7.	Reserve Decreases	81	($ 47,433)
8.	Closed with Payment	30	($713,281)
9.	Closed without Payment	71	($147,291)
10.	Subtotal: Decreases (7+8+9)	182	($908,005)
11.	Total Reserve Change Counts (2+3+4-8-9) Amounts(6+10)	10	($388,688)
12.	Final Payments	30	$693,180
13.	Partial payments	82	$60,514
14.	Fast Track Payments	8	$29,281
15.	All Other	51	$32,943
16.	Total Payments (2+13+14+15)	171	$815,918
17.	Salvage/Subrogation	28	($3,269)
18.	Incurred loss (11+16+17)		$423,961
19.	Ending Outstanding (1+11)	1,025	$5,284,945

A new claim may be distinguished by the actuary with regard to date incurred but such a distinction is usually not of interest to the claim adjuster managing the file.

Once a claim has entered the inventory, it is managed by a claim adjuster until closure of the file, or settlement. For reserving purposes, we focus only on the financial changes. Referring to the above, we have categorized the types of financial actions we are interested in recording. Reserve increases (line 5) or decreases (line 7) are changes in open claim file valuations, and the file remains open after the change in valuation. These changes in reserve valuation may be accompanied by a loss or expense payment. Note that partial payments may be split into payments with and without incurred effect for the file. A payment may have no incurred effect if the remaining reserve is reduced by the amount of the payment. Recall the basic accounting formula for incurred losses:

Incurred Loss = Paid Loss + Ending Loss Reserve

- Beginning Loss Reserve.

Thus if the claim adjuster reduces the case reserve by the amount of payment, there is no incurred loss effect. Often, automated claim systems will reduce the case reserve by the amount of the payment. This requires the adjuster to take specific reserve action only if he intends to change his total valuation of the claim file.

One of the most important statistics to monitor for any claim inventory is the number of claim closings (lines 8 and 9). Note that we distinguished between claims closed without payment and those closed with some payment of loss or expense. In terms of simple monitoring of reserve activity, the rate of claim closings should be carefully watched.

The incurred effect of reserve closings can be calculated from the same accounting formula:

Incurred Loss = Paid Loss - Beginning Reserve.

Note that the ending reserve on a closed claim is zero, hence the formula simplifies as above. For our example, for the 101 closing payments (30 closed with payment, 71 without payment) we can calculate that:

Incurred Loss = $693,180 - $713,281 - $147,291

Incurred Loss = ($167,392).

Note that for this period, there is actually a savings on claims closed. This can often be the case, especially for lines of business that generate a high proportion of claims closed with no payment. If we only consider claims that close with payment, we calculate that

Incurred Loss = $693,101 - $713,281 = ($20,101)

In addition to claim payments associated with reserve files, we also can have payments to which no currently open files are attached. Fast Track payments have been mentioned above. In addition there are other miscellaneous payments, including payments on files already closed.

Note that almost all of the 82 partial claim payments on line 13 are associated with reserve decreases on line 7. This is a result of the automatic decrease of case reserves to offset the amount of partial payments.

Once a file is closed with regard to the insurer's obligation to the policyholder, it may still be pursued for recoveries from third parties of some part of the indemnity amount. Thus, the assumption of the obligation to defend and settle a policyholder's claim usually carries with it the assumption of the policyholder's right to recovery of the amount of damages. This right of recovery is called subrogation. An example of subrogation is the payment of a collision claim by an insurer. If a third party was responsible for the damage, the insurer making the collision coverage payment to its insured, has the right to recover the amount of damages from the responsible party.

In addition to subrogation situations, the payment of first party benefits is usually accompanied by the insurer's taking of title to the damaged property. This property can often be disposed of for a partial recovery of the amount paid to the policyholder for adjustment of the loss. This property is called salvage. An example of a common salvage situation is an automobile accident in which the insured's vehicle is a total loss. The insurer reimburses the insured for the value of the vehicle and takes title to the vehicle. The auto is then disposed of for any scrap value and the proceeds kept by the insurer.

Note that both of these activities serve to reduce the insurer's net payout. Then the total amount paid in the period is

Paid loss = \$815,918 -\$3,269 = \$812,649

An Actuarial Model of Loss Development

We have dealt with the accounting models as well as the claims model of the reserving process. Each of these models deals with aggregates over a certain time period. Further, the claim department is concerned with individual file actions. An actuarial model can be constructed that supplies a structure behind the aggregate financial descriptions of claims activity. This can serve as a starting point for the analysis of reserves from the actuary's viewpoint.

The basic mathematical form of an actuarial loss development model is outlined below. However the rest of this chapter will document existing practices of loss reserve development as opposed to a theoretical development of the modeling approach.

Let $v(x)$ be the amount of loss arising from instant x, i. e. $v(x)$ can be thought of as the loss density. Then the amount of ultimate loss in the time period (a,b) can be calculated as

$$\int_a^b v(x)\,dx.$$

All observations of loss reserve situations are observations of various aggregate amounts. The form of v(x) cannot be observed directly.

Since most observations of the loss amounts are at periods short of ultimate development, we need to recognize the development of loss statistics over time. This can be done by introducing a development function d(t), where d is a continuous function with

$$d(t) = 0, \text{ for } t < 0,$$

$$d(t) = 1, \text{ for } t > T,$$

where loss development continues for a duration of T.

Then aggregate losses from period (a,b) developed through time c are given by

$$\int_a^b \int_0^c v(x)d(x+t)\, dt\, dx.$$

The actuarial model requires that a proper form and parameters for the functions v and d be found that fit the observed aggregate calendar period loss data.

For instance if v(x) = k, a constant volume of losses, then

$$\int_a^b \int_0^c v(x)d(x+t)\, dt\, dx = k\int_a^b \int_0^c d(x+t)\, dt\, dx$$

If the aggregate data yields the following set of accident quarter loss development factors

Developed Months	87-3	87-4	88-1	88-2	88-3	88-4
3-6	1.773	1.994	1.941	1.936	2.087	1.974
6-9	1.355	1.336	1.311	1.349	1.465	
9-12	1.181	1.223	1.177	1.237		
12-15	1.111	1.087	1.099			
15-18	1.066	1.041				
18-21	1.030					

one can set up the following set of equations for third quarter 1987,

$$(1.773)\int_{0}^{.25}\int_{0}^{.25} v(x)d(x+t)\,dt\,dx = \int_{0}^{.25}\int_{0}^{.5} v(x)d(x+t)\,dt\,dx.$$

$$(1.355)\int_{0}^{.25}\int_{0}^{.5} v(x)d(x+t)\,dt\,dx = \int_{0}^{.25}\int_{0}^{.75} v(x)d(x+t)\,dt\,dx.$$

etc ...

Similarly, we have the following equations for fourth quarter 1987,

$$(1.994)\int_{.25}^{.50}\int_{0}^{.25} v(x)d(x+t)\,dt\,dx = \int_{.25}^{.50}\int_{0}^{.50} v(x)d(x+t)\,dt\,dx.$$

$$(1.336)\int_{.25}^{.50}\int_{0}^{.5} v(x)d(x+t)\,dt\,dx = \int_{.25}^{.50}\int_{0}^{.75} v(x)d(x+t)\,dt\,dx.$$

etc ...

From the form of these equations it is clear that the rate of growth $v(x)$ in the underlying incurred losses is embedded in the observed loss development factors. The impact of growth on loss development is extremely important. Generally, in a growing book of business, loss development factors will be less than in a steady state situation. Likewise, in a declining volume of losses, loss development factors will be higher than steady state factors. The benefit of the actuarial model of loss development is the capability to factor out growth effects and measure the underlying "true" loss development.

These equations can be solved for the best fit set of parameters for the data and the chosen form of the development function $d(t)$. The area of appropriate functions that represent loss development patterns is just starting to be explored by actuaries.

Loss Reserving

Loss Reserving Principles

Loss reserve estimation is approached by the actuary from a much different perspective than that of the claim adjuster. The loss reserve model reviewed above is very close to the claims functions view of the financial aggregate loss reserves. The actuary must understand the claims and accounting perspectives of the total loss reserve, but will most often deal with issues inherent in the actuarial approach to the loss reserve aggregate. The principles by which the actuary completes the reserve estimation process have been formulated by the CAS Committee on Loss Reserves.

First it is essential to define basic loss reserve terminology that can be used to standardize discussions of the loss reserve estimation process.

Loss reserve estimation procedures can only be properly applied to well defined groups. A loss reserve inventory is said to be well defined if it deals with claim files arising from a time period with an explicit beginning and ending date. The start and end dates must relate to one of the distinctive dates in the life of a claim file. This could be the date of reporting, the date of loss, the date of policy inception, or the date of claim closing. The dates specified must be unambiguous and characteristic of an important event in the life of a claim.

The reserve inventory will also often be specified in terms of claims arising from a specific geographic location, as well as specified policy coverages. This aspect is important to considerations of homogeneity and credibility.

Accounting date

A loss reserve is an estimate of the liability for unpaid claims as of a given date, called the accounting date. An accounting may be any date. However it is generally a date for which a financial statement is prepared. This is most often a month end, quarter end, or year end.

Valuation date

A loss reserve inventory as of a fixed accounting date may be evaluated at a date different than the accounting date. The valuation date of a reserve liability is the date as of which the evaluation of the reserve liability is made. Thus we need to evaluate reserve liabilities as of the close of a financial period. The valuation and accounting date would be identical. However for monitoring the accuracy of our evaluations we do a quarterly hindsight evaluation of the year end. These quarterly valuation dates would be different than the year end accounting of the inventory we are evaluating. As a further example, it is common in liability rate studies to evaluate the latest accident year loss reserves as of 15 months of age.

Loss Reserving

Since the loss reserve liability is always an estimate, and the amount of the estimate will change as of successive valuation dates, we should establish some conventional terminology to discuss the results of the loss reserve process.

The **required loss reserve** for a well defined group of losses as of a given accounting date is the amount that must be paid to settle all claim liabilities of the loss inventory. The true value of the required loss reserve can only be known when all claims in the defined inventory have been finally settled. Thus the required loss reserve as of a given accounting date is a fixed number that does not change at different valuation dates. However, the value of the required loss reserve is generally unknown for an extremely long period of time.

The **indicated loss reserve** is the result of the actuarial analysis of a reserve inventory as of a given accounting date conducted as of a certain valuation date. This indicated loss reserve is the actuaries opinion of the amount of the required loss reserve. This estimate will change with successive valuation dates and will converge to the required loss reserve as the time between valuation date and the accounting date of the inventory increases.

The **carried loss reserve** is the amount of unpaid claim liability shown on external or internal financial statements. The carried loss reserve for any subgroup of business is the result of the method of generating carried reserves used by the reporting entity for financial reporting reasons.

The **loss reserve margin** is the difference between the carried reserve and the required reserve. Since the required reserve is an unknow quantity we usually speak of an indicated margin. The indicated loss reserve margin is defined to be the carried loss reserve minus the indicated loss reserve. One should not generally expect the margin to be zero, since for any subset of an entity's business it is unlikely that th carried loss reserve will be identical to either the indicated or require loss reserve. Even further, when the loss reserve is split into componen the carried reserve for any component will most often not be identical to the indicated loss reserve.

The loss reserve can be considered to consist of two major subdiv: the reserve for known claims and the reserve for unknown claims. Each of major divisions can then be further broken into subdivisions. Known clai those claims for which the entity has actually recorded some liability on books at some point in time. Thus a known claim may have been considered at one point, but later need to be reopened for further adjustment.

The reserve for known claims may be considered to consist of case reserves, a reserve for future development on case reserves, and a reserve for reopened claims. Note that the total required reserve for known reserves is estimated by the indicated reserve for known claims. The indicated reserve for known claims is the sum of the carried case reserves for known claims, the indicated provision for future development on known claims, and the indicated provision for reopened claims.

The case reserve is defined as the sum of the values assigned to specific claims by the entity's case reserving procedure. Most often a claims file is valued by an estimate placed on the file by the claims examiner. The term adjusters' estimates is used to refer to the aggregate of the estimates made by claims personnel on individual claims, based on the facts of those particular claims. Formula reserves may be placed on reported cases. Formula reserves are reserves established by formulas for groups or classes of claims. The formulas may be based on any of a number of factors such as coverage, state, age, limits, severity of injury, or other variables.

Total reserve for unreported claims consists of a reserve for claims incurred but not recorded (IBNR). This reserve can be further subdivided into a reserve for claims incurred but not yet reported to the company, and a reserve for those claims reported to the company but not yet recorded on the company's books as specific cases. This reserve may sometimes be referred to as a pipeline reserve. This distinction takes on more importance under claims made coverages, when the pipeline reserve is the only IBNR reserve needed. Most data used for estimation measures the lags from the time a loss is incurred to the time the claim is recorded on the entity's books and records. If such data is used for estimation process then the estimated liability for both pipeline and unreported claims will result.

Development is defined as the difference, on successive valuation dates, between observed values of certain fundamental quantities that may be used in the loss reserve estimation process. These changes can be changes in paid and carried amounts. Development on reported claims as of two valuation dates consists of the additional paid on case reserves plus the change in case reserves from the beginning of the calendar period. Recall that this is also the definition of incurred loss in a calendar period. Thus, Development in the period from x to x+y on Known Cases as of x-w =

(Paid in the period from x to x+y on Known Cases as of x-w) +
(Change in Carried Reserves in the period from x to x+y on Known Cases as of x-w)

It is often the case that we speak of development on a case reserve inventory as of a certain date. Another type of development relates to IBNR claims. The development of IBNR claim counts is often referred to as **emergence of IBNR**. In reviewing the development on the prior year end reserve it is often of interest to divide the total development into its case development and IBNR emergence components.

The **loss adjustment expense reserve** for a particular exposure period is the amount required to cover all future expenses required to investigate and settle claims incurred in the exposure period. This covers claims yet to be reported as well as claims already known.

Loss adjustment reserves may be charged to specific claims or may be general claims expense not directly attributable to any one file. This distinction lead to separate consideration of allocated loss adjustment expense and unallocated loss adjustment expense.

Allocated loss adjustment expenses are those expenses such as attorneys' fees and legal expense which are incurred with and are assigned to specific claims. **Unallocated loss adjustment expenses** are all other claim adjustment expenses, such as salaries,heat, light and rent, which are associated with the claim adjustment function but are not readily assignable to specific claims.

Data Organization and Availability

The availability of proper data is essential to the task of estimating loss and loss adjustment expense reserve needs. The actuary is responsible for assuring the entity provides sufficiently detailed and quality data to allow the actuary to arrive at reliable reserve estimates. Some general considerations regarding data sources have been established by the CAS Statement of Principles.

The data must first be grouped into homogeneous categories. Homogeneity can be an issue across any of several different dimensions. The first choice of the analyst must be the dimensions along which homogeneity must be attained. These dimensions could be locations, coverages, limits or layers, classes, organizational units, among others.

Data must be presented that clearly displays development of losses by accident year, policy year, or report year, to enable the actuary to project the ultimate level of losses.

The effectiveness of the method depends very much on the organization of the data history. Suppose that the paid losses for a well defined line of business totalled $68,402,000 in 1988. This is the sort of information one might be able to obtain from simple accounting exhibits. It would be more useful to know the composition of this paid amount according to some "aging" criterion. Thus we would be interested in the information that $9,705,000 of this paid amount relates to occurrences with 1988 date of loss. Similarly, we would want to know that $16,267,000 of the 1988 payments relate to losses with 1987 dates of loss, and so forth. Thus the 1988 paid loss amount contains more information if we can split it into components by year loss. The same paid amount could also be split into components by year of reported loss. By date of loss occurrence, we would find:

Paid on 1988 losses:	$11,346 thousand
Paid on 1987 losses:	$16,567 thousand
Paid on 1986 losses:	$19,935 thousand
Paid on 1985 losses:	$11,956 thousand
Paid on 1984 losses:	$5,985 thousand
Paid on 1983 losses:	$3,211 thousand
Paid on 1982 losses:	$2,274 thousand
	---- ----
Total paid loss in 1988	$71,273 thousand

Since we now know that $11,346,000 was paid on 1988 losses during the year 1988, we would like to know the comparable amount paid on 1987 losses during 1987. We can find that a total of $73,972 was paid in 1987 on this line of business, and that $17,001,000 is for losses that occurred during 1987. Further, the full 1987 paid amount can be split into amounts paid on accidents from different years as was done for 1988 payments:

Paid on 1987 losses:	$17,001	thousand
Paid on 1986 losses:	$22,343	thousand
Paid on 1985 losses:	$13,036	thousand
Paid on 1984 losses:	$9,098	thousand
Paid on 1983 losses:	$6,235	thousand
Paid on 1982 losses:	$4,693	thousand
Paid on 1981 losses:	$1,567	thousand
Total paid loss in 1987	$73,972	thousand

Comparison of these amounts by loss year for several calendar years would quickly become awkward. This calls for a more useful method of data organization that facilitates the comparisons we want to make between the components of calendar year paid amounts.

One of the most common ways to organize such data is the loss development triangle. For a given period, say loss year, all payments on claims from that loss year are displayed in the same column. Each row indicates a subsequent year of payments on claims of that accident year.

For instance, the payments on accident years 1982 through 1988 can be displayed as follows:

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$22,603	$22,054	$20,166	$19,297	$20,555	$17,001	$11,346
24	$17,461	$21,916	$18,981	$18,058	$22,343	$16,567	
36	$14,237	$14,767	$12,172	$13,036	$19,935		
48	$9,813	$13,104	$9,098	$11,956			
60	$7,143	$6,235	$5,985				
72	$4,693	$3,211					
84	$2,274						

Now we see that the loss payments of the 1988 calendar year appear on the diagonal of the triangle. Similarly, the 1987 calendar year payments appear on the second most recent diagonal. This mode of data organization greatly facilitates comparison of the development history expected of a loss year. For instance, it is immediately apparent that payments on a loss year during the second annual period of payment seem to be roughly equivalent to the amount paid in the first 12 months of payment.

While this arrangement shows the amount paid in each 12 month period, it is often convenient to accumulate the payments on a given loss year. This would result in the following triangle of cumulative loss payments:

Developed Months	1980	1981	1982	1983	1984	1985	1988
12	$22,603	$22,054	$20,166	$19,297	$20,555	$17,001	$11,346
24	$40,064	$43,970	$39,147	$37,355	$42,898	$33,568	
36	$54,301	$58,737	$51,319	$50,391	$62,832		
48	$64,114	$71,841	$60,417	$62,347			
60	$71,257	$78,076	$66,402				
72	$75,950	$81,287					
84	$78,224						

Review of the data is conveniently done in the triangular format discussed above. This array of data follows the development of a given fixed grouping of claims until all claims reach their ultimate settlement amounts. Some of the possible data arrays that may be inspected are presented below. Berquist and Sherman give some rules for relevant data to be used for reserve analysis.

1. Data may be provided by accident year, report year, policy year, or calendar year (in descending order of preference), by development year.

2. The number of years of development must be great enough so that further developments will be negligible.

3. Allocated loss expenses must be included, or shown separately.

Reserve Estimation Strategy

The overall approach to a reserve valuation problem can be broken into four phases;

1) Review of the data to identify its key characteristics and possible anomalies. Balancing of data to other verified sources should be undertaken at this point.

2) Application of appropriate reserve estimation techniques.

3) Evaluation of the conflicting results of the various reserve methods used, with an attempt to reconcile or explain the bases for different projections. At this point the proposed reserving ultimates are evaluated in contexts outside their original frame of analysis.

4) Prepare projections of reserve development that can be monitored over the subsequent calendar periods. Deviations of actual projected developments of counts or amounts is from one of the most useful diagnostic tools in evaluating accuracy of reserve estimates.

Loss Reserving

There are several issues that govern selection of loss reserving methods.

1.) Understanding of trends and changes affecting the data base. A review of the available data before the application of the reserve methods should take place to identify any trends or abrupt changes that may be evident from the data itself. A number of possible influences could be evident;

a) underwriting policies or procedures can change over the time frame included in a reserve review. Usually some accompanying fact situations can be documented such as changes in classes written, geographic areas, or changes in key underwriting personnel.

b) claims adjusting changes can be the result of expansion or consolidation of offices,or changes in claims department management. Changes in claims office procedures or automated claim support and payment systems can change claim development history.

c) accounting changes may be apparent from the data. A very common cause is a change in a computerized accounting routines.

d) the legal/social environment can change abruptly. This is especially a possibility if the data is concentrated from one state.

2.) Subdivide or combine data as necessary to achieve the largest possible block of homogeneous data.

3.) In a block of data it is necessary to understand the key provisions of the underlying contracts. Endorsements that may affect loss development, such as reporting endorsements may require a further separation of the data. The deductible and limit profiles of the business may have changed over the time span. Both can have a dramatic effect on loss development. Likewise reinsurance contracts can impact loss development, so that the reinsurance history of a block of business needs to be documented to allow the analyst to interpret the development history.

4.) Homogeneity of data is necessary in order to draw valid conclusions about the likely future loss development outcomes. In this sense we are only interested in the requirement of homogeneity as it affects loss development. While a block of data may be considered heterogeneous in terms of locations or other descriptors, we are only interested in homogeneity of behavior from the aspect of loss development.

5.) Credibility of the data requires a volume of data that will allow a stable history of development patterns. The amount of data needed to give credibility to indicated results will often depend on the average size of loss and the variability of loss size.

6.) It is necessary to be able to identify the expected development patterns of the business. This is useful if the data is sparse, and recourse to external development patterns is necessary. Books of business with very different development patterns should not be combined for loss reserve studies. Issues of homogeneity and credibility for loss reserving should most often be thought of in terms of their impact on loss on development patterns.

7.) The existence of a block of reserves discounted for interest in a book of business adds additional complexities. The discounted case reserves should be isolated and evaluated separately. This includes discounting for mortality as well as interest rates. Hence any pension reserves, as in workers compensation, should be separately identified by data coding. The evaluation of the reserve should be made taking the discounted case reserves into consideration.

8.) Groupings of data should be made of similar frequency and severity characteristics. Clearly, a high frequency, low severity line could easily mask the development of a low frequency, high severity line. The two should be separated before the loss reserve analysis is performed.

9.) There is extremely different susceptibility to reopened claims by line of insurance. In some lines like physical damage, the potential for reopens can almost be ignored. In other cases, like workers comp, we must isolate the reopened claim effect and evaluate it explicitly. The key issue to reopened claim liabilities is that the exposure base for reopened claims is the past inventory of all closed claims, and not the current volume of business.

10.) Credibility and homogeneity of data from even the same line of business is greatly enhanced if the policy limits or layers of loss are very similar. For example, loss development data on General Liability excess reinsurance may not be very stable or useful if not grouped by underlying limits and layer widths. Similarly, history on a book of basic limits auto liability business is of little value in evaluating a new book with $1 million limits. Aggregate limits of liability or even large losses that are capped at policy limits are very important facts for the loss reserve actuary to take into account in his analysis.

11.) The potential for recoveries from salvage or subrogation, or even large deductibles is extremely important. Most often these recovery potentials should be estimated separately from the gross data, before recoveries. This requires appropriate data coding be set up.

12.) Members of pools and associations often get results reported in bulk without appropriate loss or record dates. This data must be isolated from internal company records and a separate evaluation of loss reserve liabilities made.

13.) Changes in company operations can be the source of some of most important impacts on loss development patterns. A complete history of all company actions that the loss reserve actuary feels could have had a significant impact on reserve history should be kept as part of the analysis discussion.

14.) External changes in state laws or judicial precedents must be evaluated and commented on. Occasionally, high impact changes like the introduction of no-fault auto insurance clearly changes business environment. However, even less noticeable changes, such as administrative rulings allowing cash settlements for workers compensation lifetime indemnity cases will clearly have a significant impact on the loss reserve actuary's ultimate estimated liabilities.

Exploratory Data Analysis

Before the actuary begins his attempts to project immature loss data to ultimate loss estimates, it is important to review the data. The objective of this review is to understand the data in terms of

1) rate of development,

2) smoothness of development,

3) presence of large losses,

4) volume of data.

Review of the data will allow the analyst to form conclusions about:

1) appropriate projection methodologies,

2) anomalies in the data,

3) appropriate questions to ask management concerning issues that manifest themselves in the data, that will further the analyst's understanding of the book of business that generated the data.

A review of the more common data displays that should be reviewed by the actuary follows.

Incurred losses: An incurred loss triangle contains the history of paid losses and case reserves. This example is an accident year triangle. A review of the incurred loss triangle points to a fluctuating level of losses since 1982. Note that the dip in losses reported on the 1985 accident year as of 12 months did not result in less loss reported through 48 months of development. This should alert the analyst to search for possible processing slowdowns at year end 1985, or major fluctuations in case reserve adequacy. In light of this, the analyst must consider how to interpret the low level of 1988 accident year incurred losses. Clearly, some measure of exposure is called for - whether earned exposures, earned premiums, or even policies in force. This will help determine whether a level of ultimate incurred losses proportional to the low reported 1986 incurred is reasonable. The situation with loss processing as well as case reserve adequacy needs to be probed in order to decide on the proper 1988 accident year reserve.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$58,641	$63,732	$51,779	$40,143	$55,665	$43,401	$28,800
24	$74,804	$79,512	$68,175	$67,978	$80,296	$57,547	
36	$77,323	$83,680	$69,802	$75,144	$87,961		
48	$77,890	$85,366	$69,694	$77,947			
60	$80,728	$88,152	$70,041				
72	$82,280	$87,413					
84	$82,372						

Paid Losses: A paid loss triangle contains the history of paid losses. Small variations in paid loss as of 12 months can be seen to be indicative of very large differences in ultimate accident year losses. Also the low reported incurred on 1985 accident year is also paralleled by a lower paid loss amount on the 1985 accident year. This indicates the extent of the low 1988 paid losses cannot be due to solely underreporting. Then we would look for evidence of lower 1988 exposure levels to explain the reported losses.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$22,603	$22,054	$20,166	$19,297	$20,555	$17,001	$11,346
24	$40,064	$43,970	$39,147	$37,355	$42,898	$33,568	
36	$54,301	$58,737	$51,319	$50,391	$62,832		
48	$64,114	$71,841	$60,417	$62,347			
60	$71,257	$78,076	$66,402				
72	$75,950	$81,287					
84	$78,224						

Incremental Incurred Losses: This triangle shows the incremental incurred losses in each successive 12 month period. It is useful for the analyst to gauge the reasonability of yearly aggregate loss accumulations on an accident year. Note the "speedup" of incurred losses in 12 to 24 months aging of the 1987 accident year (calendar year 1988), when incurred losses increased $27,835,000. It appears that the second annual development on the 1986 accident year of $24,632,000 is also unusually large when compared to accident years 1986 and prior. Thus the analyst must suspect that processing problems were also apparent in the organization at year end 1988. Questions to key managers in Claims and Underwriting should help the analyst gather information to confirm this suspicion. This triangle also shows directly the amount of development in the oldest developments.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$58,641	$63,732	$51,779	$40,143	$55,665	$43,401	$28,800
24	$16,163	$15,779	$16,396	$27,835	$24,632	$14,147	
36	$2,519	$4,168	$1,627	$7,166	$7,665		
48	$567	$1,686	($107)	$2,803			
60	$2,838	$2,786	$347				
72	$1,552	($739)					
84	$92						

Incremental Paid Losses: This triangle shows the incremental paid losses in each successive 12 month period. We see immediately that payments during the second annual development period on an accident year are roughly equal to the amount paid in the first annual development period. Thus we form an expectation that any development pattern to ultimate must yield $9-10 million projected paid losses during the 12 to 24 month development on 1988 accident year.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$22,603	$22,054	$20,166	$19,297	$20,555	$17,001	$11,346
24	$17,461	$21,916	$18,981	$18,058	$22,343	$16,567	
36	$14,237	$14,767	$12,172	$13,036	$19,935		
48	$9,813	$13,104	$9,098	$11,956			
60	$7,143	$6,235	$5,985				
72	$4,693	$3,211					
84	$2,274						

Paid Claims as a Percent of Incurred Claims; This triangle divides paid losses at each development by reported losses as at the same development age. This statistic tests the consistency of development of paid and reported losses. It also may give warning of case reserve inadequacies. This statistic clearly flags the 1985 accident year as being inconsistent with history. The high ratio indicates that the case reserve portion of the 1985 accident year incurred losses as of year end 1985 was much weaker than historically. One benefit of this statistic is that it appears concurrent with the analysis, and does not rely on hindsight. The crucial 1988 year looks normal with regard to case reserves.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	38.5%	34.6%	38.9%	48.1%	36.9%	39.2%	39.4%
24	53.6%	55.3%	57.4%	55.0%	53.4%	58.3%	
36	70.2%	70.2%	73.5%	67.1%	71.4%		
48	82.3%	84.2%	86.7%	80.0%			
60	88.3%	88.6%	94.8%				
72	92.3%	93.0%					
84	95.0%						

Reported Claim Counts: Claim count history is extremely important in any loss reserve analysis. This triangle simply displays all reported claims by annual development period. Essentially all claims are reported within 24 months on this line. Note both 1985 and 1986 have developed from low bases as of 12 months to almost catch up with the 1982 accident year in number of losses. The 1988 accident year does have a much lower level of incurred loss than prior accident years. A radical change in volume such as 1988 accident year displays should alert us to consider the "Simon Effect" on our loss development patterns (see Simon).

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	32,751	33,736	27,067	24,928	25,229	17,632	15,609
24	41,201	39,528	32,740	29,796	31,930	21,801	
36	41,618	39,926	33,084	30,074	32,281		
48	41,755	40,044	33,183	30,169			
60	41,773	40,072	33,209				
72	41,774	40,072					
84	41,774						

Closed Paid Claim Counts: The cumulative claims closed with payment are displayed. Note that this is not all closed claims. See claims closed with no payment below. The processing problem at the end of 1985 and 1986 again appears in this statistic.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	23,355	22,662	18,951	16,631	17,381	12,666	10,592
24	31,940	30,294	25,197	22,894	24,581	16,669	
36	33,288	31,588	26,214	23,806	25,765		
48	33,860	32,129	26,582	24,229			
60	34,091	32,323	26,777				
72	34,247	32,433					
84	34,294						

No-claim Counts: Claims may also be closed without any payment. The claims closed with no payment could easily account for over half of all claims reported for some lines such as such as medical malpractice. This display does not show any unusual patterns.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	2,646	3,142	2,752	2,343	2,238	1,749	1,246
24	6,285	6,529	5,366	4,744	4,666	3,458	
36	6,935	7,053	5,840	5,132	5,375		
48	7,240	7,308	6,050	5,400			
60	7,353	7,411	6,185				
72	7,393	7,465					
84	7,412						

Closed as Percent of Reported Claim Counts : Total closed claims can be related to claims reported. This is a monitor on closing activities. We can see a very steady performance of this ratio as of 12 and 24 months of development for accident years 1982 to 1987. Note the unusually low closing ratio on the 1988 accident year. This should be explored with the claims department. On a property line this could be the result of a catastrophe in December, but in a liability line it indicates potential processing problems.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	79.4%	76.5%	80.2%	76.1%	77.8%	81.8%	75.8%
24	92.8%	93.2%	93.4%	92.8%	91.6%	92.3%	
36	96.6%	96.8%	96.9%	96.2%	96.5%		
48	98.4%	98.5%	98.3%	98.2%			
60	99.2%	99.2%	99.3%				
72	99.7%	99.6%					
84	99.8%						

No- Claim Counts : This triangle displays open claims, i.e. claims reported less all claims closed.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	6,750	7,932	5,364	5,954	5,610	3,217	3,771
24	2,976	2,705	2,177	2,158	2,683	1,674	
36	1,395	1,285	1,030	1,136	1,141		
48	655	607	551	540			
60	329	338	247				
72	134	174					
84	68						

Average Open Claim Amount: This triangle tracks the average amount reserved on open claims. Note the 1988 average open case reserve has dropped from the prior year's level. This should be investigated with the claim department.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$5,339	$5,254	$5,894	$3,501	$6,258	$8,206	$4,629
24	$11,671	$13,137	$13,334	$14,190	$13,941	$14,324	
36	$16,499	$19,405	$17,939	$21,798	$22,026		
48	$21,029	$22,285	$16,832	$28,896			
60	$28,782	$29,820	$14,722				
72	$47,240	$35,209					
84	$60,722						

Increase in Average Open Claim; This triangle charts the annual increase in average open reserve as of each development age. This is useful for determining if case reserves are keeping up with reasonable inflationary increases. The 1985-86 increases at 12 months of development average out to a 6.1% annual increase during the two years. Since the 1982 column has no logical entry it can be used to report the multi-year average growth in case reserve.

Developed Months	Average	1983	1984	1985	1986	1987	1988
12	97.6%	98.4%	112.2%	59.4%	178.7%	131.1%	56.4%
24	104.2%	112.6%	101.5%	106.4%	98.2%	102.7%	
36	107.5%	117.6%	92.4%	121.5%	101.0%		
48	111.2%	106.0%	75.5%	171.7%			
60	71.5%	103.6%	49.4%				
72	74.5%	74.5%					
84	NA						

Average Closed Claim;

This triangle shows paid losses divided by closed with payment counts. Note that these average are very regular with no reversals across accident years.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$968	$973	$1,064	$1,160	$1,183	$1,342	$1,071
24	$1,254	$1,451	$1,554	$1,632	$1,745	$2,014	
36	$1,631	$1,859	$1,958	$2,117	$2,439		
48	$1,894	$2,236	$2,273	$2,573			
60	$2,090	$2,415	$2,480				
72	$2,218	$2,506					
84	$2,281						

Increase in Average Closed Claim; This triangle shows the annual increase in average paid claim amount as of identical development periods. The column for 1982 is also used to display the multi-year average increase in closed claim values. It is useful to compare the annual increases in open case reserves to these increases in closed claim amounts. This allows the actuary to evaluate if the claim department reserves are keeping pace with inflationary increases in settlements.

Developed Months	Average	1983	1984	1985	1986	1987	1988
12	101.7%	100.6%	109.3%	109.0%	101.9%	113.5%	79.8%
24	109.9%	115.7%	107.0%	105.0%	107.0%	115.4%	
36	110.6%	114.0%	105.3%	108.1%	115.2%		
48	110.8%	118.1%	101.6%	113.2%			
60	108.9%	115.6%	102.7%				
72	113.0%	113.0%					
84	NA						

Closed Claims as a Percent of Open Claims; The rate of claim closure is one of the most important indicators of the condition of the claim department. This statistic shows some deterioration in claims performance in calendar year 1987. Only 78% of the 1986 inventory and 60% of the 1985 inventory was closed during 1987. Some level of recovery is evident during calendar year 1988 on the 1986 and 1985 inventories. However, it appears that the closing activity on the 1987 inventory may have slipped in order to allow the catch up activity on the older inventories. These indications should be probed with questions for claims management.

Developed Months	1982	1983	1984	1985	1986	1987	1988
24	80.4%	80.3%	80.3%	80.1%	78.2%	77.3%	
36	67.1%	67.2%	68.5%	60.2%	70.6%		
48	62.8%	61.9%	56.1%	60.8%			
60	52.5%	48.9%	59.9%				
72	59.6%	48.5%					
84	49.3%						

Loss Reserve Estimation Methodologies

A. Aggregate versus Structural Reserving Methodologies

While there are many schemes for the projection of reserves to ultimate values, these methods fall into two very simple groups; aggregate methods and structural methods.

An aggregate reserve method is one which simply projects the growth behavior of a reserve inventory, without taking advantage of any particular knowledge of the structure of the processes within the inventory. Most simple and frequently used reserve methods fall in this category. An aggregate reserve method takes no advantage of the fact that we are estimating loss reserve requirements. The methods can be equally well used to project ultimate values of any growth process. Some examples might be the ultimate response rate to a direct mail campaign, or the loan defaults in a loan portfolio.

A structural reserve method is one which uses some particular aspect of the dynamics of the insurance or loss reserving process to obtain ultimate incurred loss estimates. These methods model some aspect of the loss reserve process, and generally are more complex than aggregate methods. One example is the Fisher Lange reserve method that estimates claim counts and closure patterns and values.

In actuarial practice, detailed models of loss reserve development are seldom used to actually establish reserves. Most reserve estimation work is done using aggregate reserve methods supplemented with the experience of the reserve analyst.

B. Aggregate Reserving Methodologies

1. Triangular Methods

One of the most common methods used to estimate ultimate loss levels consists of tracking the history of a group of claims with similar definitional groupings. The data for this purpose is arranged in a triangular loss format as discussed above. An undeveloped loss year is "completed" to its expected ultimate payout, based on the assumption that each loss year will be completed in some fashion "analogous" to prior years. The assumptions the actuary makes about the relationship of past patterns on future ones defines the nature of this "analogy".

For instance, suppose we have a triangular display of cumulative paid losses as described above.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$22,603	$22,054	$20,166	$19,297	$20,555	$17,001	$11,346
24	$40,064	$43,970	$39,147	$37,355	$42,898	$33,568	
36	$54,301	$58,737	$51,319	$50,391	$62,832		
48	$64,114	$71,841	$60,417	$62,347			
60	$71,257	$78,076	$66,402				
72	$75,950	$81,287					
84	$78,224						

A review of the data in the first row "as of 12 months" indicates that some rather extreme fluctuations in loss volumes seem to have taken place over the last seven years. This should be checked out by a review of the historical claim count triangles, and also earned exposure history, or earned premiums at a uniform exposure level as a proxy. These concerns are discussed above under the data exploration topics.

Our concern is that these shifts in volume of losses make it difficult to reach any conclusion about this development. Thus we would like to "normalize" the development history by removing this volume effect. This is accomplished by studying the aging effect within each accident year as follows.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	1.773	1.994	1.941	1.936	2.087	1.974	
24-36	1.355	1.336	1.311	1.349	1.465		
36-48	1.181	1.223	1.177	1.237			
48-60	1.111	1.087	1.099				
60-72	1.066	1.041					
72-84	1.030						

This triangular display represents the historical development of each accident year. For instance, the development from 12 to 24 months on accident year 1984 is an increase in incurred losses of 97.4% ($33,568 / $17,001). Now the range of variation is considerably reduced. One can see that 12-24 month development seems to vary from a low of 1.773 to 2.087. The high 1986 development of 2.087 seems to be outside a reasonable range of development factors observed in recent time periods. If we can predict the next 12-24 month development that we expect to see take place during 1987, we would be able to forecast the 1988 accident year losses at 24 months of development.

Thus our next task should consist of predicting the 1988 accident year 12-24 month incurred loss development factor. One common technique is to inspect several averages of the age-to-age factors. The averages should provide a guide in selecting the next calendar period's development on that accident year.

Accident Year	Developed Months 12-24	24-36	36-48	48-60	60-72	72-84
1982	1.773	1.355	1.181	1.111	1.066	1.030
1983	1.994	1.336	1.223	1.087	1.041	
1984	1.941	1.311	1.177	1.099		
1985	1.936	1.349	1.237			
1986	2.087	1.465				
1987	1.974					
Average	1.951	1.363	1.205	1.099	1.053	1.030
Avg Last 3	1.999	1.375	1.213	1.099	1.053	1.030
Avg Last 4	1.985	1.365	1.213	1.099	1.053	1.030
Avg Exc Hi & Lo	1.961	1.347	1.202	1.087	1.053	1.030
Weighted Average	1.948	1.364	1.205	1.099	1.053	1.030
Harmonic Mean	1.948	1.362	1.204	1.099	1.053	1.030

There is a practically unlimited number of ways to average the historical development factors. The key point to remember is that these averages are only guides to selection of the next reasonable development, based on all information the loss reserve actuary has developed from his reviews with management as well as the historical loss development.

Let's review the averages displayed above. The "average" line is simply the arithmetic average of all historical loss development factors at that stage of development. Similarly, the "Avg Last 3" and "Avg Last 4" are simple arithmetic averages of the latest 3 and latest 4 development factors at a given point of development. The "Avg Exc Hi & Lo" are the arithmetic average of the developments other than the highest and lowest. The "Weighted Average" is weighted by amount of incurred loss. The harmonic mean is the n'th root of the n historical development factors.

Notice that we are generally interested in examining the averages of the latest few development periods. Hence we calculate averages of the latest three or four factors. An alternative display of averages that allow us to inspect the behavior of the averages as more historical points are added is often useful. Like many actuarial procedures, the analyst is asked to make a judgement of the most appropriate trade-off between stability (i.e., more development periods in the average computation) and responsiveness (i.e., only include the latest few development periods in the average).

The familiar triangular format also becomes a convenient way to inspect the addition of more development points in the averages.

Developed Months	Arithmetic Average of... Latest	Last 2	Last 3	Last 4	Last 5	Last 6
12-24	1.974	2.031	1.999	1.985	1.986	1.951
24-36	1.465	1.407	1.375	1.365	1.363	
36-48	1.237	1.207	1.213	1.205		
48-60	1.099	1.093	1.099			
60-72	1.041	1.053				
72-84	1.030					

Inspection of such past historical trends allows the analyst to complete the lower triangle by filling it in with his projections of future development factors. For instance, let's assume the analyst chose the following development patterns along the right diagonal as the most likely over the next 12 months.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	1.773	1.994	1.941	1.936	2.087	1.974	2.000
24-36	1.355	1.336	1.311	1.349	1.465	1.350	
36-48	1.181	1.223	1.177	1.237	1.270		
48-60	1.111	1.087	1.099	1.085			
60-72	1.066	1.041	1.060				
72-84	1.030	1.018					
84-Ult	1.01						

The selected expected developments for the 1989 calendar period are shown below the actual historical developments for each accident year. Note that often the selected developments are not identical to any of the selected averages. For instance the 12 to 24 month expected development is 2.000. Notice that the historical 12 to 24 month developments have been trending upwards since 1982 with exception of the 1986 year. Review of this history with claims and systems manager indicates that a new claim processing system was installed in 1986 that necessitated a longer installation period than anticipated. Thus the analyst views the 1986 development as an anomaly. This is supported by the recovery of the 1987 development to a value that seems to fit in with the trend of increasing developments. It is extremely important to realize that once the suspected anomalous development pattern on 1986 accident year has been identified, further information must be sought about company operations that may have caused this anomaly. This information can in most cases not be acquired through further study of the numbers, but requires the actuary to gather additional information, often of a qualitative nature.

The search for an explanation of recent unusual historical developments is critical to the reserve estimate, because the analyst must decide whether the situation causing the abnormal loss development is still a factor that can affect future developments. In this case the new claim processing system has been in place since the end of 1986, and the 1987 year was closed with a return to more normal loss development patterns in the 1988 calendar year. This leads the analyst to discount the 1986 development from 12 to 24 months as a nonrecurring situation. In turn the analyst does not wish to use the averages for two reasons:

a) the data shows a clear trend, making averaging inappropriate,

b) the unusual 1986 development should not be included in any average.

Contrast this situation to the following. Suppose the analyst had determined that the unusual development was due to several large losses that required large case reserve increases in 1987. This is clearly a situation that could happen again in any particular year, absent any changes in policy limit profiles or reinsurance retentions, and should receive some weight in future scenarios. In this situation, the averages with the 1986 accident year development from 12 to 24 months should be used as a guide to the 1989 calendar year development on the 1988 accident year.

The choice of the next projected development of 24 to 36 months is difficult because of the unusually high development of 146% on the 1986 reported incurred losses from 24 to 36 months. At this point the analyst must regard the 1986 accident year as very unusual. A note should be made to inspect the projected ultimate on this accident year very carefully once development factors have been selected.

Investigation of events in 1988 indicates that several large case reserve increases were evident on 1986 accident year cases, but this was on classes of business no longer written by the company. Accordingly the 1986 development cannot be used again in the choice of the next calendar year's development on the 1987 accident year.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	1.773	1.994	1.941	1.936	2.087	1.974	2.000
24-36	1.355	1.336	1.311	1.349	1.465	1.350	1.350
36-48	1.181	1.223	1.177	1.237	1.270	1.290	
48-60	1.111	1.087	1.099	1.085	1.100		
60-72	1.066	1.041	1.060	1.060			
72-84	1.030	1.018	1.018				
84-Ult	1.01	1.01					

Once the development factors for calendar year 1989 have been chosen, for accident years 1983 through 1988, the analyst must forecast development for the 1990 calendar year. The 1990 calendar year developments are not necessarily the same as the 1989 calendar year forecast development factors. For instance the developments from 36 to 48 months have shown a distinct trend upward in our history. The analyst chooses to believe that this tend will continue in the developments observed in 1989 and 1990 calendar years. This is a critical assumption that will add a significant amount to the estimated loss reserve liabilities of the entity. This sort of assumption needs to be highlighted in the report of the reserve analysis. The situation causing the trending developments must be discussed with the claims function and an understanding of claim settlement practices that may be causing this trended development sought. Subsequent reserve analyses must retest this trended development assumption. The analyst must monitor whether actual 1987 and 1988 calendar year developments prove to follow the assumed pattern.

Note the 1990 development on the 1986 accident year is higher than the similar 48 to 60 month development selected for the 1985 accident year. This selection reflects the analyst's finding that the 1986 accident year contained more hazardous long development classes than the following years. These classes were assumed to account for the higher development on the 1986 accident year from 24 to 36 months. While the selected factor of 1.100 is only slightly higher than the "normal" selected development of 1.085 at this age, the analyst feels some recognition must be given to the past behavior of these more hazardous business mix. Since there are no data older than 1986 in this business, the higher selected factor on 1986 is an example of purely a judgement call on the part of the analyst. In the report on the loss reserves the analyst will disclose his assumptions that the 1986 accident year will reflect a more severe development pattern due to information he has learned about its more severe business mix.

In reviewing the assumptions, the analyst now finds that he needs to have a more consistent opinion on the 1986 accident year. That is, although the development from 36 to 48 months is trended, it should probably be even higher to reflect the business mix of the 1986 accident year. The amount of the adjustment is in question. The analyst finds that the 24 to 36 month development on 1986 at 1.465 is higher than a selected average development based on the observed developments of 1982-85 of 1.360. It is also assumed that this differential of 1.0772 (the relativity of the observed 1.465 to the "projected" 1.360) should dampen back to unity with the passage of time. The analyst decides to reflect this dampening effect by taking the square root of the differential with the passage of each year. Thus the differential for the 36 to 48 month development should be (1.0772)^0.5, or 1.038. This results in a development from 36 to 48 months of 1.318, given by 1.27*1.038.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	1.773	1.994	1.941	1.936	2.087	1.974	2.000
24-36	1.355	1.336	1.311	1.349	1.465	1.350	1.350
36-48	1.181	1.223	1.177	1.237	1.318	1.290	1.310
48-60	1.111	1.087	1.099	1.095	1.100	1.085	1.085
60-72	1.066	1.041	1.060	1.060	1.060	1.060	1.060
72-84	1.030	1.030	1.018	1.018	1.018	1.018	1.018
84-Ult	1.010	1.010	1.010	1.010	1.010	1.010	1.010
Dev to Ult	1.010	1.040	1.090	1.193	1.580	2.059	4.183

In the above triangle all development factors have been chosen for future periods. This is a forecast of the loss development to be expected on these accident years in calendar years 1987 and forward. As can be seen from the process of selection of development factors that the analyst must:

1. be able to recognize normal levels of random fluctuation in developments.
2. recognize aberrations in development patterns and be able to isolate their causes, and determine if they are ongoing or onetime changes in development.
3. recognize trends in loss development patterns.

This triangle can then be transformed into its dollar equivalents, as shown below.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$22,603	$22,054	$20,166	$19,297	$20,555	$17,001	$11,346
24	$40,064	$43,970	$39,147	$37,355	$42,898	$33,568	$22,692
36	$54,301	$58,737	$51,319	$50,391	$62,832	$45,317	$30,634
48	$64,114	$71,841	$60,417	$62,347	$82,829	$58,458	$40,131
60	$71,257	$78,076	$66,402	$68,270	$91,112	$63,427	$43,542
72	$75,950	$81,287	$70,386	$72,366	$96,579	$67,233	$46,154
84	$78,224	$83,726	$71,653	$73,669	$98,317	$68,443	$46,985
Ultimate	$79,006	$84,563	$72,369	$74,405	$99,301	$69,128	$47,455
Reserve	782	$3,276	$5,968	$12,058	$36,468	$35,560	$36,109
					Total Reserve		$130,221

This particular analysis of the paid loss data indicates that a reserve of about $130 million is necessary to provide for unpaid loss reserve liabilities from accident years 1982 through 1988.

While an estimate is available for a reserve need for this book of business, we can note at least two deficiencies in our knowledge at this point. First, we have not made any use of other available information, such as claim counts or case reserve values. Second, we have no means of evaluating prospectively the confidence we should have in this single forecast of the future. Both of these concerns can be addressed by alternative forecasts of ultimate loss reserve need using other information available to us.

A triangular development analysis can also be developed using paid losses plus case reserves. Case reserves could be either adjuster determined or set by use of average values. Assume incurred losses as presented below.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$58,641	$63,732	$51,779	$40,143	$55,665	$43,401	$28,800
24	$74,804	$79,512	$68,175	$67,978	$80,296	$57,547	
36	$77,323	$83,680	$69,802	$75,144	$87,961		
48	$77,890	$85,366	$69,694	$77,947			
60	$80,728	$88,152	$70,041				
72	$82,280	$87,413					
84	$82,372						

Age to age development factors may be calculated for this data.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	1.276	1.248	1.317	1.693	1.443	1.326	
24-36	1.034	1.052	1.024	1.105	1.095		
36-48	1.007	1.020	0.998	1.037			
48-60	1.036	1.033	1.005				
60-72	1.019	0.992					
72-84	1.001						

Use the triangular format as a convenient way to inspect the addition of more development points in the averages.

		Arithmetic Average of ...				
Developed Months	Latest	Last 2	Last 3	Last 4	Last 5	Last 6
12-24	1.326	1.384	1.487	1.445	1.405	1.384
24-36	1.095	1.100	1.075	1.069	1.062	
36-48	1.037	1.018	1.019	1.016		
48-60	1.005	1.019	1.025			
60-72	0.992	1.005				
72-84	1.001					

Assume the analyst chose the following development patterns along the right diagonal as the most likely over the next 12 months.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	1.276	1.248	1.317	1.693	1.443	1.326	1.350
24-36	1.034	1.052	1.024	1.105	1.095	1.095	
36-48	1.007	1.020	0.998	1.037	1.020		
48-60	1.036	1.033	1.005	1.020			
60-72	1.019	0.992	1.000				
72-84	1.001	1.000					
84-Ult	1.000						

Now complete the selections of all other development factors as below.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	1.276	1.248	1.317	1.693	1.443	1.326	1.350
24-36	1.034	1.052	1.024	1.105	1.095	1.095	1.095
36-48	1.007	1.020	0.998	1.037	1.020	1.020	1.020
48-60	1.036	1.033	1.005	1.020	1.020	1.020	1.020
60-72	1.019	0.992	1.000	1.000	1.000	1.000	1.000
72-84	1.001	1.000	1.000	1.000	1.000	1.000	1.000
84-Ult	1.000	1.010	1.010	1.010	1.010	1.010	1.010
Dev to Ult	1.000	1.010	1.010	1.030	1.051	1.151	1.553

Based on factors chosen above the complete projection of incurred losses by accident year may be completed.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$58,641	$63,732	$51,779	$40,143	$55,665	$43,401	$28,800
24	$74,804	$79,512	$68,175	$67,978	$80,296	$57,547	$38,880
36	$77,323	$83,680	$69,802	$75,144	$87,961	$63,014	$42,574
48	$77,890	$85,366	$69,694	$77,947	$89,720	$64,275	$43,426
60	$80,728	$88,152	$70,041	$79,506	$91,515	$65,560	$44,294
72	$82,280	$87,413	$70,041	$79,506	$91,515	$65,560	$44,294
84	$82,372	$87,413	$70,041	$79,506	$91,515	$65,560	$44,294
Ultimate	$82,372	$88,287	$70,741	$80,301	$92,430	$66,216	$44,737
Paid/Date	$78,224	$81,287	$66,402	$62,347	$62,832	$33,568	$11,346
Reserve	$4,148	$7,000	$4,339	$17,954	$29,598	$32,648	$33,391
					Total Reserve		$129,078

This particular analysis of the incurred loss data indicates that a reserve of about $129 million is necessary to provide for unpaid loss and loss adjustment expenses from accident years 1982 through 1988. This is very close to the $130 million reserve estimate obtained through a paid loss projection. Any difference in estimates clearly raises questions that need to be investigated by the analyst in an attempt to reconcile the reserve estimate using two sets of loss data.

The pattern of claim reporting should be reviewed in the same fashion.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	32,751	33,736	27,067	24,928	25,229	17,632	15,609
24	41,201	39,528	32,740	29,796	31,930	21,801	
36	41,618	39,926	33,084	30,074	32,281		
48	41,755	40,044	33,183	30,169			
60	41,773	40,072	33,209				
72	41,774	40,072					
84	41,774						

Age to age development factors may be calculated for this data.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	1.258	1.172	1.210	1.195	1.266	1.236	
24-36	1.010	1.010	1.011	1.009	1.011		
36-48	1.003	1.003	1.003	1.003			
48-60	1.000	1.001	1.001				
60-72	1.000	1.000					
72-84	1.000						

Use the triangular format as a convenient way to inspect the addition of more development points in the averages.

		Arithmetic Average of ...				
Developed Months	Latest	Last 2	Last 3	Last 4	Last 5	Last 6
12-24	1.236	1.251	1.232	1.227	1.216	1.223
24-36	1.011	1.010	1.010	1.010	1.010	
36-48	1.003	1.003	1.003	1.003		
48-60	1.001	1.001	1.001			
60-72	1.000	1.000				
72-84	1.000					

Assume the analyst chose the following development patterns along the right diagonal as the most likely over the next 12 months.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	1.258	1.172	1.210	1.195	1.266	1.236	1.200
24-36	1.010	1.010	1.011	1.009	1.011	1.012	1.012
36-48	1.003	1.003	1.003	1.003	1.003	1.003	1.003
48-60	1.000	1.001	1.001	1.001	1.001	1.001	1.001
60-72	1.000	1.000	1.000	1.000	1.000	1.000	1.000
72-84	1.000	1.000	1.000	1.000	1.000	1.000	1.000
84-Ult	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Dev to	1.000	1.000	1.000	1.001	1.004	1.016	1.219

Ult Based on factors chosen above the complete projection of incurred loss counts by accident year may be completed.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	32,751	33,736	27,067	24,928	25,229	17,632	15,609
24	41,201	39,528	32,740	29,796	31,930	21,801	18,731
36	41,618	39,926	33,084	30,074	32,281	22,063	18,956
48	41,755	40,044	33,183	30,169	32,378	22,129	19,012
60	41,773	40,072	33,209	30,199	32,410	22,151	19,031
72	41,774	40,072	33,209	30,199	32,410	22,151	19,031
84	41,774	40,072	33,209	30,199	32,410	22,151	19,031
Ultimate	41,774	40,072	33,209	30,199	32,410	22,151	19,031
Reported	41,774	40,072	33,209	30,169	32,281	21,801	15,609
Unreported	0	0	0	30	129	350	3,422
				Total Unreported			3,932

This analysis implies about 3,900 claims remain to be reported. By itself, this analysis is useful as an indicator of true IBNR reporting. However the projected ultimate claims may also be used to reduce the paid and incurred triangles to an average basis. Note that these incurred counts include those claims closed without loss payment. It is possible to project the net claim count after claims closed without payment are excluded.

Let the following triangle represent the history of claims reported that are closed with no loss payment.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	2,646	3,142	2,752	2,343	2,238	1,749	1,246
24	6,285	6,529	5,366	4,744	4,666	3,458	
36	6,935	7,053	5,840	5,132	5,375		
48	7,240	7,308	6,050	5,400			
60	7,353	7,411	6,185				
72	7,393	7,465					
84	7,412						

As a percent of total reported claims, these no-claims show the following relationships.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	8.1%	9.3%	10.2%	9.4%	8.9%	9.9%	8.0%
24	15.3%	16.5%	16.4%	15.9%	14.6%	15.9%	
36	16.7%	17.7%	17.7%	17.1%	16.7%		
48	17.3%	18.2%	18.2%	17.9%			
60	17.6%	18.5%	18.6%				
72	17.7%	18.6%					
84	17.7%						

This triangular display may be completed to obtain an ultimate estimate of the percent of reported claims closed with no indemnity payment, or alternatively, the reported claims could be reduced for the claims closed with no indemnity.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	30,105	30,594	24,315	22,585	22,991	15,883	14,363
24	34,916	32,999	27,374	25,052	27,264	18,343	
36	34,683	32,873	27,244	24,942	26,906		
48	34,515	32,736	27,133	24,769			
60	34,420	32,661	27,024				
72	34,381	32,607					
84	34,362						

Age to age development factors may be calculated for this data.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	1.160	1.079	1.126	1.109	1.186	1.155	
24-36	0.993	0.996	0.995	0.996	0.987		
36-48	0.995	0.996	0.996	0.993			
48-60	0.997	0.998	0.996				
60-72	0.999	0.998					
72-84	0.999						

Use the triangular format as a convenient way to inspect the addition of more development points in the averages.

Developed Months	Arithmetic Average of ... Latest	Last 2	Last 3	Last 4	Last 5	Last 6
12-24	1.155	1.170	1.150	1.144	1.131	1.136
24-36	0.987	0.991	0.993	0.993	0.993	
36-48	0.993	0.994	0.995	0.995		
48-60	0.996	0.997	0.997			
60-72	0.998	0.999				
72-84	0.999					

Assume the analyst chose the following development patterns along the right diagonal as the most likely over the next 12 months.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	1.160	1.079	1.126	1.109	1.186	1.155	1.120
24-36	0.993	0.996	0.995	0.996	0.987	0.996	0.996
36-48	0.995	0.996	0.996	0.993	0.994	0.994	0.994
48-60	0.997	0.998	0.996	0.996	0.996	0.996	0.996
60-72	0.999	0.998	0.998	0.998	0.998	0.998	0.998
72-84	0.999	0.999	0.999	0.999	0.999	0.999	0.999
84-Ult	0.999	0.999	0.999	0.999	0.999	0.999	0.999
Dev to Ult	0.999	0.998	0.996	0.992	0.986	0.982	1.100

Based on factors chosen above the complete projection of net incurred loss counts by accident year may be completed.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	30,105	30,594	24,315	22,585	22,991	15,883	14,363
24	34,916	32,999	27,374	25,052	27,264	18,343	16,087
36	34,683	32,873	27,244	24,942	26,906	18,270	16,022
48	34,515	32,736	27,133	24,769	26,744	18,160	15,926
60	34,420	32,661	27,024	24,670	26,637	18,087	15,862
72	34,381	32,607	26,970	24,620	26,584	18,051	15,831
84	34,362	32,574	26,943	24,596	26,558	18,033	15,815
Ultimate	34,328	32,542	26,916	24,571	26,531	18,015	15,799

These ultimate nonzero incurred claim counts can be used to restate the average paid loss amounts.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$658	$678	$749	$785	$775	$944	$718
24	$1,167	$1,351	$1,454	$1,520	$1,617	$1,863	
36	$1,582	$1,805	$1,907	$2,051	$2,368		
48	$1,868	$2,208	$2,245	$2,537			
60	$2,076	$2,399	$2,467				
72	$2,212	$2,498					
84	$2,279						

Age to age development factors may be calculated for this data.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	1.773	1.994	1.941	1.936	2.087	1.974	
24-36	1.355	1.336	1.311	1.349	1.465		
36-48	1.181	1.223	1.177	1.237			
48-60	1.111	1.087	1.099				
60-72	1.066	1.041					
72-84	1.030						

Use the triangular format as a convenient way to inspect the addition of more development points in the averages.

		Arithmetic Average of ...				
Developed Months	Latest	Last 2	Last 3	Last 4	Last 5	Last 6
12-24	1.974	2.031	1.999	1.985	1.986	1.951
24-36	1.465	1.407	1.375	1.365	1.363	
36-48	1.237	1.207	1.213	1.205		
48-60	1.099	1.093	1.099			
60-72	1.041	1.053				
72-84	1.030					

Assume the analyst chose the following development patterns along the right diagonal as the most likely over the next 12 months.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	1.773	1.994	1.941	1.936	2.087	1.974	2.000
24-36	1.355	1.336	1.311	1.349	1.465	1.450	1.450
36-48	1.181	1.223	1.177	1.237	1.235	1.235	1.235
48-60	1.111	1.087	1.099	1.085	1.085	1.085	1.085
60-72	1.066	1.041	1.060	1.060	1.060	1.060	1.060
72-84	1.030	1.015	1.015	1.015	1.015	1.015	1.015
84-Ult	1.010	1.010	1.010	1.010	1.010	1.010	1.010
Dev to Ult	1.010	1.025	1.087	1.179	1.456	2.111	4.223

Based on factors chosen above the complete projection of average paid loss by accident year may be completed.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$658	$678	$749	$785	$775	$944	$718
24	$1,167	$1,351	$1,454	$1,520	$1,617	$1,863	$1,436
36	$1,582	$1,805	$1,907	$2,051	$2,368	$2,702	$2,083
48	$1,868	$2,208	$2,245	$2,537	$2,925	$3,337	$2,572
60	$2,076	$2,399	$2,467	$2,753	$3,173	$3,620	$2,791
72	$2,212	$2,498	$2,615	$2,918	$3,364	$3,838	$2,958
84	$2,279	$2,535	$2,654	$2,962	$3,414	$3,895	$3,002
Ultimate	$2,302	$2,561	$2,681	$2,992	$3,448	$3,934	$3,033
Counts	34,328	32,542	26,916	24,571	26,531	18,015	15,799
Ultimate	$79,006	$83,331	$72,156	$73,509	$91,490	$70,873	$47,911
Paid2Date	$78,224	$81,287	$66,402	$62,347	$62,832	$33,568	$11,346
Reserve	$782	$2,044	$5,754	$11,162	$28,658	$37,305	$36,565
					Total Reserve		$122,269

The 1988 average paid claim at ultimate appears to be completely unreasonable at $3,033 per claim, compared to past values projected. A review of annual increases in the ultimate average paid amount indicates

	1982	1983	1984	1985	1986	1987	1988
Ultimate	$2,302	$2,561	$2,681	$2,992	$3,448	$3,934	$4,406
Annual Increase		11.3%	4.7%	11.6%	15.3%	14.1%	12.0%

that a 12% increase from the 1987 value might be more in line with past experience. Using this increase yields an average payment per claim closed with payment of $4,406.

Ultimate	$2,302	$2,561	$2,681	$2,992	$3,448	$3,934	$4,406
Counts	34,328	32,542	26,916	24,571	26,531	18,015	15,799
Ultimate	$79,006	$83,331	$72,156	$73,509	$91,490	$70,873	$69,613
Paid2Date	$78,224	$81,287	$66,402	$62,347	$62,832	$33,568	$11,346
Reserve	$782	$2,044	$5,754	$11,162	$28,658	$37,305	$58,267
					Total Reserve		$143,972

The resulting reserve estimate based on average paid data is now much higher than the reserve estimate produced by the incurred loss development method of $129 million.

A similar projection can be made of the average amount of loss incurred per reported claim.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$1,948	$2,083	$2,130	$1,777	$2,421	$2,733	$2,005
24	$2,142	$2,409	$2,490	$2,713	$2,945	$3,137	
36	$2,229	$2,546	$2,562	$3,013	$3,269		
48	$2,257	$2,608	$2,569	$3,147			
60	$2,345	$2,699	$2,592				
72	$2,393	$2,681					
84	$2,397						

Age to age development factors may be calculated for this data.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	1.100	1.157	1.169	1.527	1.216	1.148	
24-36	1.041	1.056	1.029	1.110	1.110		
36-48	1.012	1.024	1.003	1.045			
48-60	1.039	1.035	1.009				
60-72	1.020	0.993					
72-84	1.002						

Use the triangular format as a convenient way to inspect the addition of more development points in the averages.

		Arithmetic Average of ...				
Developed Months	Latest	Last 2	Last 3	Last 4	Last 5	Last 6
12-24	1.148	1.182	1.297	1.265	1.243	1.220
24-36	1.110	1.110	1.083	1.076	1.069	
36-48	1.045	1.024	1.024	1.021		
48-60	1.009	1.022	1.028			
60-72	0.993	1.007				
72-84	1.002					

Assume the analyst chose the following development patterns along the right diagonal as the most likely over the next 12 months.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	1.100	1.157	1.169	1.527	1.216	1.148	1.195
24-36	1.041	1.056	1.029	1.110	1.110	1.100	1.100
36-48	1.012	1.024	1.003	1.045	1.025	1.025	1.025
48-60	1.039	1.035	1.009	1.025	1.025	1.025	1.025
60-72	1.020	0.993	1.000	1.000	1.000	1.000	1.000
72-84	1.002	1.003	1.003	1.003	1.003	1.003	1.003
84-Ult	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Dev to Ult	1.000	1.003	1.003	1.028	1.054	1.159	1.385

Based on factors chosen above the complete projection of average incurred losses by accident year may be completed.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$1,948	$2,083	$2,130	$1,777	$2,421	$2,733	$2,005
24	$2,142	$2,409	$2,490	$2,713	$2,945	$3,137	$2,396
36	$2,229	$2,546	$2,562	$3,013	$3,269	$3,451	$2,636
48	$2,257	$2,608	$2,569	$3,147	$3,351	$3,537	$2,702
60	$2,345	$2,699	$2,592	$3,226	$3,435	$3,626	$2,769
72	$2,393	$2,681	$2,592	$3,226	$3,435	$3,626	$2,769
84	$2,397	$2,689	$2,600	$3,235	$3,445	$3,637	$2,778
Ultimate	$2,397	$2,689	$2,600	$3,235	$3,445	$3,637	$2,778
Counts	34,328	32,542	26,916	24,571	26,531	18,015	15,799
Ultimate	$82,290	$87,500	$69,970	$79,496	$91,400	$65,514	$43,882
Paid2Date	$78,224	$81,287	$66,402	$62,347	$62,832	$33,568	$11,346
Reserve	$4,066	$6,213	$3,568	$17,149	$28,568	$31,946	$32,536
					Total Reserve		$124,046

The ultimate reserve estimate resulting from this average incurred history indicates a reserve of $124 million is needed.

The triangular loss development factor methods applied to the four different loss statistics above yields the following different sets of ultimate accident year incurred losses.

	1982	1983	1984	1985	1986	1987	1988
Paid	$79,006	$84,563	$72,369	$74,405	$99,301	$69,128	$47,455
Incurred	$82,372	$88,287	$70,741	$80,301	$92,430	$66,216	$44,737
Avg Paid	$79,006	$83,331	$72,156	$73,509	$91,490	$70,873	$69,613
Avg Inc'd	$82,290	$87,500	$69,970	$79,496	$91,400	$65,514	$43,882

Note that there is substantial variation from method to method. The analyst must still choose his point estimate in some fashion.

2. Reserve Development Methods

The triangular methods have used either paid or incurred data, but have not made any use of historical relationships between paid amounts and case reserved amounts. The reserve development method attempts to analyze the adequacy of case reserves based on the history of payments against those case reserves. In order to be able to interpret the procedure in terms of payments on case reserves, we present the following report year paid and reserve data.

Case Loss Reserves by Report Year

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$46,770	$53,422	$41,802	$40,334	$47,500	$42,219	$30,416
24	$31,944	$36,588	$28,899	$28,266	$35,455	$27,221	
36	$18,832	$21,214	$15,798	$18,312	$22,225		
48	$9,559	$11,345	$95,600	$8,724			
60	$4,999	$8,049	$5,403				
72	$2,821	$3,701					
84	$1,693						

Incremental Paid by Report Year

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$30,001	$29,421	$26,601	$24,981	$27,595	$25,886	$15,220
24	$16,021	$18,081	$17,078	$15,251	$18,196	$17,700	
36	$14,144	$16,904	$13,169	$12,665	$17,687		
48	$8,238	$10,811	$7,522	$9,465			
60	$5,923	$4,942	$4,739				
72	$3,119	$2,930					
84	$1,145						

The fundamental idea of the reserve development method of reserve evaluation is to track the development of a case reserve amount into subsequent paid losses and remaining reserves. For instance, the $42,219 in reserves from report year 1987 cases has developed into $17,700 of paid loss during 1988, with $27,221 remaining in reserve as of the end of 1988. We are then interested in the amount we expect to be paid on the $27,221 reserve during the next 12 months. The entire liquidation pattern of the report year reserves can then be charted and used to evaluate the ultimate liquidation value of the report year case reserves.

Consider the following ratios of paid on open reserves,

Developed	1982	1983	1984	1985	1986	1987	1988
Months	0.340	0.338	0.409	0.378	0.383	0.420	
12-24	0.442	0.462	0.454	0.448	0.500		
24-36	0.438	0.514	0.473	0.519			
36-48	0.621	0.440	0.495				
48-60	0.636	0.364					
60-72	0.392						
72-84							

Likewise, we can create the array of ratios of remaining in reserve

Developed	1982	1983	1984	1985	1986	1987	1988
Months	0.683	0.685	0.691	0.701	0.746	0.645	
12-24	0.590	0.580	0.547	0.648	0.627		
24-36	0.508	0.535	6.051	0.476			
36-48	0.523	0.709	0.057				
48-60	0.564	0.460					
60-72	0.600						
72-84							

Note that the sum of these two ratios, gives a history of the amount developed on reserves.

Developed	1982	1983	1984	1985	1986	1987	1988
Months	1.023	1.023	1.100	1.079	1.130	1.065	
12-24	1.032	1.042	1.001	1.096	1.126		
24-36	0.945	1.049	6.524	0.995			
36-48	1.144	1.149	0.551				
48-60	1.200	0.824					
60-72	0.992						
72-84							

In order to complete the projection of ultimate reserve outcomes both the paid on reserve and the remaining reserve on reserve developments must be projected.

Use the triangular format as a convenient way to inspect the averages of paid on reserve ratios.

Developed Months	Latest	Arithmetic Average of ... Last 2	Last 3	Last 4	Last 5	Last 6
12-24	0.420	0.402	0.394	0.398	0.386	0.378
24-36	0.500	0.474	0.467	0.466	0.461	
36-48	0.519	0.496	0.502	0.486		
48-60	0.495	0.467	0.519			
60-72	0.364	0.500				
72-84	0.392					

Assume the analyst chose the following development patterns along the right diagonal as the most likely over the next 12 months.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	0.340	0.338	0.409	0.378	0.383	0.420	0.420
24-36	0.442	0.462	0.454	0.448	0.500	0.500	0.500
36-48	0.438	0.514	0.473	0.519	0.500	0.500	0.500
48-60	0.621	0.440	0.495	0.500	0.500	0.500	0.500
60-72	0.636	0.364	0.500	0.500	0.500	0.500	0.500
72-84	0.392	0.400	0.400	0.400	0.400	0.400	0.400
84-Ult	1.000	1.000	1.000	1.000	1.000	1.000	1.000

The same exercise must carried out for the remaining on reserve

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	0.683	0.685	0.691	0.701	0.746	0.645	
24-36	0.590	0.580	0.547	0.648	0.627		
36-48	0.508	0.535	6.051	0.476			
48-60	0.523	0.709	0.057				
60-72	0.564	0.460					
72-84	0.600						

Use the triangular format as a convenient way to inspect the addition of more development points in the averages.

Developed Months	Latest	Last 2	Last 3	Last 4	Last 5	Last 6
		Arithmetic Average of ...				
12-24	0.645	0.696	0.697	0.696	0.694	0.692
24-36	0.627	0.637	0.607	0.600	0.598	
36-48	0.476	3.264	2.354	1.893		
48-60	0.057	0.383	0.430			
60-72	0.460	0.512				
72-84	0.600					

Assume the analyst chose the following development patterns along the right diagonal as the most likely over the next 12 months.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	0.683	0.685	0.691	0.701	0.746	0.645	0.690
24-36	0.590	0.580	0.547	0.648	0.627	0.635	0.635
36-48	0.508	0.535	6.051	0.476	0.530	0.530	0.530
48-60	0.523	0.709	0.057	0.600	0.600	0.600	0.600
60-72	0.564	0.460	0.500	0.500	0.500	0.500	0.500
72-84	0.600	0.600	0.600	0.600	0.600	0.600	0.600
84-Ult	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Note that this statistic is the amount remaining on reserve so that the 84 month to ultimate development must be zero. Thus selection of tail factors is not an issue in this reserve projection methodology.

Once development factor scenarios have been constructed, it is necessary to complete the settlement projections in their dollar terms. An example using the 1986 report year is the most direct illustration of the completion technique involved.

There is $22.2 million of case reserves outstanding on the 1986 report year as of the end of 1988. The completed development factors indicate that 53% of this reserve will remain in reserve, while 50% of the reserve will be paid out, for a total adverse development of 3% on the 1986 report year cases during 1988.

We can complete the projection of remaining reserves

Case Loss Reserves by Report Year

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$46,649	$53,415	$41,793	$40,328	$47,471	$42,122	$29,490
24	$31,939	$36,568	$28,986	$28,248	$35,406	$27,097	$20,348
36	$18,827	$21,014	$15,907	$18,243	$22,200	$17,207	$12,921
48	$9,552	$11,235	$9,577	$8,698	$11,766	$9,119	$6,848
60	$4,991	$8,043	$5,376	$5,219	$7,060	$5,472	$4,109
72	$2,799	$3,656	$2,688	$2,609	$3,530	$2,736	$2,054
84	$1,685	$2,194	$1,613	$1,566	$2,118	$1,642	$1,233
Ultimate	$0	$0	$0	$0	$0	$0	$0

Based on the amounts remaining in reserve, annual paid amounts by report period can be derived simply by using the selected paid on reserve factors.

Paid Losses by Report Year

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$30,001	$29,421	$26,601	$24,981	$27,595	$25,886	$15,220
24	$16,021	$18,081	$17,078	$15,251	$18,196	$17,700	$12,386
36	$14,144	$16,904	$13,169	$12,665	$17,687	$13,549	$10,174
48	$8,238	$10,811	$7,522	$9,465	$11,100	$8,603	$6,461
60	$5,923	$4,942	$4,739	$4,349	$5,883	$4,560	$3,424
72	$3,119	$2,930	$2,688	$2,609	$3,530	$2,736	$2,054
84	$1,145	$1,462	$1,075	$1,044	$1,412	$1,094	$822
Ultimate	$1,685	$2,194	$1,613	$1,566	$2,118	$1,642	$1,233

Thus the incremental paid loss history is projected to become

Paid Losses by Report Year

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$30,001	$29,421	$26,601	$24,981	$27,595	$25,886	$15,220
24	$46,022	$47,502	$43,679	$40,232	$45,791	$43,586	$27,606
36	$60,166	$64,406	$56,848	$52,897	$63,478	$57,135	$37,780
48	$68,404	$75,217	$64,370	$62,362	$74,578	$65,738	$44,240
60	$74,327	$80,159	$69,109	$66,711	$80,461	$70,298	$47,664
72	$77,446	$83,089	$71,797	$69,320	$83,991	$73,033	$49,719
84	$78,591	$84,551	$72,872	$70,364	$85,403	$74,128	$50,541
Ultimate	$80,276	$86,745	$74,485	$71,930	$87,521	$75,769	$51,773

Reserve:							
Required	$1,685	$3,656	$5,376	$9,568	$24,043	$32,183	$36,553
Carried	$1,685	$3,656	$5,376	$8,698	$22,200	$27,097	$29,490
Supplemental Case	$0	$0	$0	$870	$1,843	$5,086	$7,063
							$14,862
Reported Case Adequacy	100.0%	100.0%	100.0%	90.9%	92.3%	84.2%	80.7%

This analysis indicates a case reserving pattern with case reserves deficient 20% in the first 12 months, 15% at age 12 to 24 months, 7-8% at age 24 to 48 months. After 48 months we expect case reserves to be adequate.

While the reserve development method is simplest to interpret on report year data, it may also be used on accident period data. In order to apply the method to accident period data one must be able to assume that IBNR claim activity is related consistently to claims already reported. This assumption is a reasonable one for most lines of business that have the bulk of their claims reported in the first accident period to serve as a stable base for IBNR projections.

Assume we take the accident year paid and incurred triangles that were presented above.

Remaining Case Reserve By Accident Year

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$36,038	$41,679	$31,613	$20,846	$35,110	$26,400	$17,454
24	$34,740	$35,542	$29,028	$30,623	$37,399	$23,980	
36	$23,022	$24,943	$18,483	$24,753	$25,129		
48	$13,776	$13,525	$9,277	$15,600			
60	$9,471	$10,076	$3,639				
72	$6,330	$6,126					
84	$4,148						

Incremental Paid by Accident Year

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$22,603	$22,054	$20,166	$19,297	$20,555	$17,001	$11,346
24	$17,461	$21,916	$18,981	$18,058	$22,343	$16,567	
36	$14,237	$14,767	$12,172	$13,036	$19,935		
48	$9,813	$13,104	$9,098	$11,956			
60	$7,143	$6,235	$5,985				
72	$4,693	$3,211					
84	$2,274						

Consider the following ratios of paid on open reserves,

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	0.485	0.526	0.600	0.866	0.636	0.628	
24-36	0.410	0.415	0.419	0.426	0.533		
36-48	0.426	0.525	0.492	0.483			
48-60	0.519	0.461	0.645				
60-72	0.495	0.319					
72-84	0.359						

Likewise, we can create the array of ratios of remaining in reserve.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	0.964	0.853	0.918	1.469	1.065	0.908	
24-36	0.663	0.702	0.637	0.808	0.672		
36-48	0.598	0.542	0.502	0.630			
48-60	0.687	0.745	0.392				
60-72	0.668	0.608					
72-84	0.655						

Note that the sum of these two ratios, gives a history of the amount developed on reserves, now including IBNR claims.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	1.449	1.379	1.519	2.335	1.702	1.536	
24-36	1.073	1.117	1.056	1.234	1.205		
36-48	1.025	1.068	0.994	1.113			
48-60	1.206	1.206	1.037				
60-72	1.164	0.927					
72-84	1.015						

In order to complete the projection ultimate reserve outcomes both the paid on reserve and the remaining reserve on reserve developments must be projected.

Use the triangular format as a convenient way to inspect the addition of more development points in the averages.

		Arithmetic Average of ...				
Developed Months	Latest	Last 2	Last 3	Last 4	Last 5	Last 6
12-24	0.628	0.632	0.710	0.683	0.651	0.623
24-36	0.533	0.479	0.459	0.448	0.441	
36-48	0.483	0.488	0.500	0.482		
48-60	0.645	0.553	0.542			
60-72	0.319	0.407				
72-84	0.359					

Assume the analyst chose the following development patterns along the right diagonal as the most likely to be paid as percent of reserve over the next 12 month periods.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	0.485	0.526	0.600	0.866	0.636	0.628	0.600
24-36	0.410	0.415	0.419	0.426	0.533	0.460	0.460
36-48	0.426	0.525	0.492	0.483	0.495	0.495	0.495
48-60	0.519	0.461	0.645	0.500	0.500	0.500	0.500
60-72	0.495	0.319	0.510	0.510	0.510	0.510	0.510
72-84	0.359	0.430	0.430	0.430	0.430	0.430	0.430
84-Ult	1.010	1.010	1.010	1.010	1.010	1.010	1.010

The same exercise must carried out for the remaining on reserve

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	0.964	0.853	0.918	1.469	1.065	0.908	
24-36	0.663	0.702	0.637	0.808	0.672		
36-48	0.598	0.542	0.502	0.630			
48-60	0.687	0.745	0.392				
60-72	0.668	0.608					
72-84	0.655						

Use the triangular format as a convenient way to inspect the addition of more development points in the averages.

	Arithmetic Average of ...					
Developed Months	Latest	Last 2	Last 3	Last 4	Last 5	Last 6
12-24	0.908	0.987	1.148	1.090	1.043	1.030
24-36	0.672	0.740	0.706	0.705	0.696	
36-48	0.630	0.566	0.558	0.568		
48-60	0.392	0.569	0.608			
60-72	0.608	0.638				
72-84	0.655					

Assume the analyst chose the following development patterns of reserves as ratio to prior accident period reserves.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	0.964	0.853	0.918	1.469	1.065	0.908	1.000
24-36	0.663	0.702	0.637	0.808	0.672	0.750	0.750
36-48	0.598	0.542	0.502	0.630	0.600	0.600	0.600
48-60	0.687	0.745	0.392	0.600	0.600	0.600	0.600
60-72	0.668	0.608	0.650	0.650	0.650	0.650	0.650
72-84	0.655	0.655	0.655	0.655	0.655	0.655	0.655
84-Ult	0.000	0.000	0.000	0.000	0.000	0.000	0.000

We can complete the projection of remaining reserves

Case Loss Reserves by Accident Year

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$36,038	$41,679	$31,613	$20,846	$35,110	$26,400	$17,454
24	$31,939	$36,568	$28,986	$28,248	$35,406	$27,097	$17,454
36	$18,827	$21,014	$15,907	$18,243	$22,200	$20,323	$13,091
48	$9,552	$11,235	$9,577	$8,698	$13,320	$12,194	$7,854
60	$4,991	$8,043	$5,376	$5,219	$7,992	$7,316	$4,713
72	$2,799	$3,656	$3,494	$3,392	$5,195	$4,756	$3,063
84	$1,685	$2,395	$2,289	$2,222	$3,403	$3,115	$2,006
Ultimate	$0	$0	$0	$0	$0	$0	$0

Based on the amounts remaining in reserve, annual paid amounts by accident period can be derived simply by using the selected paid on reserve factors.

Paid Losses by Accident Year

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$22,603	$22,054	$20,166	$19,297	$20,555	$17,001	$11,346
24	$15,879	$18,080	$17,078	$15,251	$18,196	$17,697	$10,473
36	$14,117	$16,904	$13,169	$12,665	$17,687	$12,465	$8,029
48	$8,241	$10,811	$7,522	$9,465	$10,989	$10,060	$6,480
60	$5,935	$4,942	$4,739	$4,349	$6,660	$6,097	$3,927
72	$3,172	$2,930	$2,742	$2,662	$4,076	$3,731	$2,403
84	$1,097	$1,572	$1,503	$1,459	$2,234	$2,045	$1,317
Ultimate	$1,702	$2,419	$2,312	$2,244	$3,437	$3,146	$2,026

Thus the incremental paid loss projections accumulate to become

Paid Losses by Accident Year

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$22,603	$22,054	$20,166	$19,297	$20,555	$17,001	$11,346
24	$38,482	$40,134	$37,244	$34,548	$38,751	$34,698	$21,819
36	$52,599	$57,038	$50,413	$47,213	$56,438	$47,163	$29,848
48	$60,840	$67,849	$57,935	$56,678	$67,427	$57,223	$36,327
60	$66,775	$72,791	$62,674	$61,027	$74,087	$63,319	$40,255
72	$69,947	$75,721	$65,416	$63,688	$78,163	$67,051	$42,658
84	$71,044	$77,293	$66,918	$65,147	$80,397	$69,095	$43,975
Ultimate	$72,746	$79,711	$69,230	$67,391	$83,833	$72,241	$46,002

Reserve:							
Required	$1,702	$3,991	$6,556	$10,713	$27,395	$37,543	$34,656
						Total	$122,556
Carried	$1,685	$3,656	$5,376	$8,698	$22,200	$27,097	$17,454
						Total	$86,166

3. **Bornhuetter - Ferguson Method**

There are many cases where historical case incurred amounts are reported over a long time period (10 years or longer) and very little of the incurred loss is reported in the first two to three years. Most excess lines of insurance, as well as reinsurance lines fit into this category. In addition, many lines such as contract bond surety may report faster than excess lines, but are subject to very large occasional losses. This behavior makes it inappropriate to use reported losses as the sole base for projecting ultimate losses.

The Bornhuetter-Ferguson technique smooths out the projected ultimates by relying on smoothed development factors and expected losses to project future incurred development. Again assume we have the following incurred loss data as follows, by accident year.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$58,641	$63,732	$51,779	$40,143	$55,665	$43,401	$28,800
24	$74,804	$79,512	$68,175	$67,978	$80,296	$57,547	
36	$77,323	$83,680	$69,802	$75,144	$87,961		
48	$77,890	$85,366	$69,694	$77,947			
60	$80,728	$88,152	$70,041				
72	$82,280	$87,413					
84	$82,372						

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	1.276	1.248	1.317	1.693	1.443	1.326	
24-36	1.034	1.052	1.024	1.105	1.095		
36-48	1.007	1.020	0.998	1.037			
48-60	1.036	1.033	1.005				
60-72	1.019	0.992					
72-84	1.001						

Use the triangular format as a convenient way to inspect the addition of more development points in the averages.

	Arithmetic Average of ...					
Developed Months	Latest	Last 2	Last 3	Last 4	Last 5	Last 6
12-24	1.326	1.384	1.487	1.445	1.405	1.384
24-36	1.095	1.100	1.075	1.069	1.062	
36-48	1.037	1.018	1.019	1.016		
48-60	1.005	1.019	1.025			
60-72	0.992	1.005				
72-84	1.001					

Assume the analyst chose the following development patterns of incurred loss amounts for future development periods.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	1.276	1.248	1.317	1.693	1.443	1.326	1.400
24-36	1.034	1.052	1.024	1.105	1.095	1.070	1.070
36-48	1.007	1.020	0.998	1.037	1.020	1.020	1.020
48-60	1.036	1.033	1.005	1.020	1.020	1.020	1.020
60-72	1.019	0.992	1.000	1.000	1.000	1.000	1.000
72-84	1.001	1.000	1.000	1.000	1.000	1.000	1.000
84-Ult	1.010	1.010	1.010	1.010	1.010	1.010	1.010

These development factors for one period can be chained together to produce age to ultimate factors.

Age to Ultimate Factors

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-Ult	1.419	1.385	1.366	2.000	1.660	1.491	1.574
24-Ult	1.112	1.110	1.038	1.181	1.151	1.124	1.124
36-Ult	1.076	1.055	1.013	1.069	1.051	1.051	1.051
48-Ult	1.068	1.034	1.015	1.030	1.030	1.030	1.030
60-Ult	1.031	1.002	1.010	1.010	1.010	1.010	1.010
72-Ult	1.011	1.010	1.010	1.010	1.010	1.010	1.010
84-Ult	1.010	1.010	1.010	1.010	1.010	1.010	1.010

These age to ultimate factors can be interpreted to mean that an accident year is 99% reported as of 60 months, since the development from 60 months to ultimate is given by a factor of 1.010. Likewise, as of 12 months the 1988 accident year is expected to be 63.5% reported in incurred amounts, given by 1/1.574. This implies that 36.5% of the ultimate incurred loss for 1986 remains to be reported as of the end of 1988.

Clearly, if one had an estimate for 1988 accident year ultimate losses, one should set a reserve amount equal to 36.5% of this estimate as the appropriate reserve as of 12/31/88. The Bornhuetter-Ferguson technique is the use of this estimated reserve as the appropriate reserve. The issue is then what should be used as the estimate of ultimate incurred losses. One ready candidate is the earned premium for the accident year times the expected loss ratio, or the pricing assumption ultimate incurred loss amount.

If the earned premiums and expected loss ratios for each year are given below we can apply this method to our incurred losses.

Bornhuetter - Ferguson

	1982	1983	1984	1985	1986	1987	1988
Earned Premium	$102,918	$111,212	$95,494	$92,218	$112,206	$97,140	$66,376
Reported Loss Ratio	80.0%	78.6%	73.3%	84.5%	78.4%	59.2%	43.4%
Expected Loss Ratio	78.0%	78.0%	78.0%	78.0%	78.0%	78.0%	78.0%
Ultimate	$80,276	$86,745	$74,485	$71,930	$87,521	$75,769	$51,773
Development to Ultimate	1.010	1.010	1.010	1.030	1.051	1.124	1.574
Remaining Development	1.0%	1.0%	1.0%	2.9%	4.8%	11.1%	36.5%
Remaining in Reserve	$795	$859	$737	$2,109	$4,231	$8,380	$18,883
						Total	$35,994
Developed to Date	$82,372	$87,413	$70,041	$77,947	$87,961	$57,547	$28,800
Ultimate Estimate	$83,167	$88,272	$70,778	$80,055	$92,193	$65,928	$47,683
Ultimate Loss Ratio	80.8%	79.4%	74.1%	86.8%	82.2%	67.9%	71.8%

A simple analysis shows that this reserve method is related to the incurred loss development method above. Let d(a,n) be the n to ultimate development factor for accident year a. Let U(a) be an estimate of ultimate incurred losses for period a, and l(a,n) be the amount of reported incurred losses due to period a. Then the new revised ultimate U'(a) is given by $U'(a) = l(a,n) + U(a) \times (1-1/d(a,n))$.

which can be written

$$U'(a) = (l(a,n) \times d(a,n))/d(a,n) + U(a) \times (1-1/d(a,n)).$$

But $l(a,n) \times d(a,n)$ is simply the development factor projection ultimate for period a. Hence the Bornhuetter-Ferguson ultimate is simply a weighted average of the ultimate estimate U(a) with the development factor projection for period a, where the ultimate estimate U(a) is given weight (1-1/d(a,n)). Note that at very early developments, when the development factor d(a,n) is very large, the bulk of the weight is given the initial ultimate estimate U(a). Thus, this method acts as a smoothing of very long-tailed or unstable loss development lines.

Loss Adjustment Expenses

An extremely important part of the loss reserve evaluation process is the evaluation of loss adjustment expense liabilities. One approach could be to combine loss adjustment expenses with losses and estimate the total liability. Generally, this approach will not be desirable because the two loss development patterns will be quite different. Thus combining loss adjustment expense with losses is often similar to combining two non-homogeneous lines of business. Different analyses of loss and loss adjustment expense are also necessary to allow for monitoring of actual loss and adjustment expenses versus projected developments of each separate component.

Loss adjustment expenses need always to be split into their allocated and unallocated components. While allocated loss adjustment expenses could be combined with their associated losses this is not possible for unallocated loss adjustment expenses.

The allocated loss adjustment expenses can often be further split into subcategories. The most important subcategory is legal fees. It will often be conducive to obtaining better estimates of loss adjustment expense to develop legal expense separate from all other allocated expense items.

Most often, case reserve estimates are not established for loss adjustment expenses. This means that the actuary only has paid allocated loss adjustment expenses to work with. The allocated expense reserve is established on a bulk basis by actuarial estimates, or may be spread to cases by some formula approach. In either case, allocated paid amounts are the only meaningful history available for the analysis.

A common analysis procedure is to compare the allocated expenses paid to the paid losses on the same claims, and follow the development of the relationship of paid allocated expense to paid loss over time.

Assume the same paid loss history from our previous example.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$22,603	$22,054	$20,166	$19,297	$20,555	$17,001	$11,346
24	$40,064	$43,970	$39,147	$37,355	$42,898	$33,568	
36	$54,301	$58,737	$51,319	$50,391	$62,832		
48	$64,114	$71,841	$60,417	$62,347			
60	$71,257	$78,076	$66,402				
72	$75,950	$81,287					
84	$78,224						

We also have available the history of paid allocated loss expense by accident year.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$554	$485	$446	$405	$388	$357	$216
24	$1,110	$1,244	$1,104	$953	$1,025	$843	
36	$2,118	$2,256	$1,981	$1,809	$2,161		
48	$3,231	$3,578	$2,973	$2,905			
60	$4,211	$4,567	$3,785				
72	$4,170	$5,202					
84	$5,429						

The relationship of paid allocated loss expense to paid loss is then derived as follows for this history.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	2.45%	2.20%	2.21%	2.10%	1.89%	2.10%	1.90%
24	2.77%	2.83%	2.82%	2.55%	2.39%	2.51%	
36	3.90%	3.84%	3.86%	3.59%	3.44%		
48	5.04%	4.98%	4.92%	4.66%			
60	5.91%	5.85%	5.70%				
72	5.49%	6.40%					
84	6.94%						

Age to age development factors applying to the ratios of paid allocated loss adjustment expense are given below. We are trying to estimate the ultimate ratio of loss adjustment expense to loss. Once the ultimate ratio is chosen it can be applied to the estimate of ultimate losses to obtain an estimate of ultimate loss adjustment expense. It often helps to think of these ratios of paid allocated expense to paid loss as the cost to settle $100 of loss.

Allocated cost to settle $100 of loss

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$2.45	$2.20	$2.21	$2.10	$1.89	$2.10	$1.90
24	$2.77	$2.83	$2.82	$2.55	$2.39	$2.51	
36	$3.90	$3.84	$3.86	$3.59	$3.44		
48	$5.04	$4.98	$4.92	$4.66			
60	$5.91	$5.85	$5.70				
72	$5.49	$6.40					
84	$6.94						

The development triangle of these expense amounts is given by

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	1.131	1.286	1.276	1.214	1.265	1.195	
24-36	1.408	1.357	1.369	1.408	1.439		
36-48	1.292	1.297	1.275	1.298			
48-60	1.173	1.175	1.159				
60-72	0.929	1.094					
72-84	1.264						

We can select development factors expected for future periods by inspecting the behavior of the historical developments and their averages.

		Arithmetic Average of ...				
Developed Months	Latest	Last 2	Last 3	Last 4	Last 5	Last 6
12-24	1.195	1.230	1.225	1.238	1.247	1.228
24-36	1.439	1.424	1.405	1.393	1.396	
36-48	1.298	1.286	1.290	1.290		
48-60	1.159	1.167	1.169			
60-72	1.094	1.011				
72-84	1.264					

Assume the analyst chose the following development patterns along the right diagonal as the most likely over the next 12 months.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12-24	1.131	1.286	1.276	1.214	1.265	1.195	1.300
24-36	1.408	1.357	1.369	1.408	1.439	1.400	1.400
36-48	1.292	1.297	1.275	1.298	1.295	1.295	1.295
48-60	1.173	1.175	1.159	1.160	1.160	1.160	1.160
60-72	0.929	1.094	1.025	1.025	1.025	1.025	1.025
72-84	1.264	1.010	1.010	1.010	1.010	1.010	1.010
84-Ult	1.010	1.010	1.010	1.010	1.010	1.010	1.010
Dev to Ult	1.010	1.020	1.046	1.213	1.571	2.199	2.859

Based on factors chosen above the complete projection of incurred losses by accident year may be completed.

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	2.45%	2.20%	2.21%	2.10%	1.89%	2.10%	1.90%
24	2.77%	2.83%	2.82%	2.55%	2.39%	2.51%	2.47%
36	3.90%	3.84%	3.86%	3.59%	3.44%	3.51%	3.46%
48	5.04%	4.98%	4.92%	4.66%	4.45%	4.55%	4.48%
60	5.91%	5.85%	5.70%	5.41%	5.17%	5.28%	5.19%
72	5.49%	6.40%	5.84%	5.54%	5.30%	5.41%	5.32%
84	6.94%	6.46%	5.90%	5.60%	5.35%	5.46%	5.38%
Ultimate	7.01%	6.53%	5.96%	5.65%	5.40%	5.52%	5.43%
Ultimate Loss	$78,487	$83,842	$73,849	$73,771	$97,708	$67,190	$79,071
Allocated Expenses: Ultimate							
	$5,501	$5,474	$4,401	$4,170	$5,279	$3,709	$4,295
Reserve	$73	$271	$616	$1,264	$3,118	$2,866	$4,079

This analysis indicates a reserve need of $12.4 million for allocated loss adjustment expenses.

Many variations of this approach are of course possible. In addition, it is possible to simply develop the paid allocated loss expense history in its own right to obtain ultimate estimates. This approach does have the drawback that the estimate is not related to ultimate level of losses, hence it could produce widely varying results in allocated expense paid per $100 of claim paid. The premise of the analysis above is that the relationship of allocated expense to loss dollars is usually fairly stable.

Variations on the above method include developing the additive increments to the allocated expense to loss ratios in place of the multiplicative development of these ratios. If the ratios are very small at early maturities, the additive approach seems to be more stable. In addition, the ratios of incremental allocated loss adjustment expense to incremental paid loss could be developed. Finally, it is sometimes useful to develop an average paid allocated loss expense amount per paid claim count. Clearly, the methods chosen will depend heavily on a review of the data and its characteristics.

Unallocated Loss Adjustment Expenses

In addition to those loss adjustment expense items that are directly involved in the defense of a claim, an insuring entity also has the responsibility to manage each case to conclusion. This requires the recognition of the accrued liability for the general expenses of maintaining a claims department and its attendant rent, utilities, and salary costs. The estimation of the amount of accrued liability is difficult without some detailed expense study available.

Unallocated loss adjustment expenses are recorded on the Annual Statement, however they are simply calendar period claim department expenses. If they are split to lines of business it is usually through an internal expense allocation procedure. The New York Insurance Department's Regulation 30 gives a detailed set of instructions for allocating expenses back to lines of business for the purposes of preparing the Insurance Expense Exhibit. Some combination of claim counts is probably most often used to allocate the unallocated loss adjustment expense to line of business. Some mixture of the following bases is probably most commonly used: number of claims incurred during the year, claims closed during the year, numbers of claims remaining open during the year, number of days claims are open, or number of payment or reserve transactions during the year. Ideally, standard costs could be assigned to each of these, or similar transactions, and total claim department costs allocated in accordance to the distribution of standard costs.

Once unallocated loss expense payments have been assigned to lines of business, we can begin to estimate the needs. The most common procedure is to estimate the amount of loss adjustment expense that is needed per $100 of claims payments.

Suppose the following history of unallocated loss expense payments exists for the line of business we are reviewing, by calendar year of payment,

1982	1983	1984	1985	1986	1987	1988
$12,345	$13,826	$15,486	$17,344	$19,425	$21,756	$24,367

Comparing these paid expenses to the paid losses for these same calendar years

1982	1983	1984	1985	1986	1987	1988
$91,955	$100,576	$111,530	$130,708	$145,889	$164,051	$171,397

Comparing the paid loss adjustment expenses to paid losses for each calendar year results in the following costs per $100 of paid loss

1982	1983	1984	1985	1986	1987	1988
$13.43	$13.75	$13.88	$13.27	$13.32	$13.26	$14.22
					Average	$13.59

These amounts average about $13.59 in unallocated loss adjustment expense per $100 of paid loss. However, the amounts paid per paid loss amount comes from a mixture of new and pending claims. The common approach to estimating the liability for unallocated loss adjustment expense requires an assumption concerning the amount of unpaid loss adjustment expense paid on setup of a new claim. One procedure simply assumes that 50% of the total unallocated loss adjustment expense is paid at the outset of the claim. However, it is preferable to review with the claim department the extent of unallocated loss adjustment expense incurred on the establishment of a claim. Assume that this review indicates that 40% of unallocated loss expense is paid to set up an initial claim. Then the estimated liability for unallocated loss adjustment expense is given by

$$.1359 \times \text{IBNR Reserve} + .1359 \times (1 - .40) \times \text{Case Reserve}.$$

EVALUATION OF ULTIMATE LOSS ESTIMATES

The application of any particular reserve methods to a given body of data will yield a set of estimated ultimate losses. However, each method applied will result in a different set of ultimate losses and an associated reserve estimate. The actuary must still decide on either a best estimate reserve, or a range of possible reserve estimates, or both. Of course, for financial statement purposes, a point estimate of loss reserve requirements must be supplied for the balance sheet.

While a substantial amount of judgement has been an element of the selection and application of each reserving method, the selection of a final reserve estimate is most often a subject of the actuary's experience and judgement. In this section we will present a number of practical tests that will allow one to test a set of estimated ultimate losses for reasonability.

It is important to evaluate the results of each reserving method by attempting to diagnose the reasons the methods vary. The explanation must be the result of the actuary's analysis and experience.

The analysis conducted on the data presented above yielded six different estimates of ultimate losses by accident year. These estimates are the result of different methodologies that are sensitive to different aspects of the reserve development process. This is to be expected, since each method uses only a limited amount of data about the loss development process.

Estimated Ultimate Losses by Accident Year and Method

	<---Accident Years					--->
Method	1982	1983	1984	1985	1986	1987	1988
Paid	$79,006	$84,563	$72,369	$74,405	$99,301	$69,128	$47,455
Incurred	$82,372	$88,287	$70,741	$80,301	$92,430	$66,216	$44,737
Avg Paid	$79,006	$83,331	$72,156	$73,509	$91,490	$70,873	$69,613
Avg Inc'd	$82,290	$87,500	$69,970	$79,496	$91,400	$65,514	$43,882
B-F Method	$83,167	$83,139	$81,466	$79,999	$81,555	$83,184	$77,523
Rsrv Dev	$72,746	$79,711	$69,230	$67,391	$83,833	$72,241	$46,002

Method	Required Reserve
Paid	$130,221
Incurred	$129,078
Average Paid	$143,972
Average Inc'd	$124,046
Bornhuetter-Ferguson	$174,027
Reserve Development	$95,149

Suppose the analyst initially selects the incurred loss development ultimates as candidates for his selections.

	1982	1983	1984	1985	1986	1987	1988
Selected	$82,372	$88,287	$70,741	$80,301	$92,430	$66,216	$44,737

The initial selection of these ultimate loss estimates should be tested for reasonability by comparing them to various loss development history displays for reasonability.

Comparing these selected ultimates to paid history yields the following display of paid as a percent of ultimate.

Paid Losses as % of Ultimate

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	27.4%	25.0%	28.5%	24.0%	22.2%	25.7%	25.4%
24	48.6%	49.8%	55.3%	46.5%	46.4%	50.7%	
36	65.9%	66.5%	72.5%	62.8%	68.0%		
48	77.8%	81.4%	85.4%	77.6%			
60	86.5%	88.4%	93.9%				
72	92.2%	92.1%					
84	95.0%						
Ultimate	$82,372	$88,287	$70,741	$80,301	$92,430	$66,216	$44,737

A similar review of the ultimates with respect to incurred losses is also useful.

Incurred Losses as % of Ultimate

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	71.2%	72.2%	73.2%	50.0%	60.2%	65.5%	64.4%
24	90.8%	90.1%	96.4%	84.7%	86.9%	86.9%	
36	93.9%	94.8%	98.7%	93.6%	95.2%		
48	94.6%	96.7%	98.5%	97.1%			
60	98.0%	99.8%	99.0%				
72	99.9%	99.0%					
84	100.0%						
Ultimate	$82,372	$88,287	$70,741	$80,301	$92,430	$66,216	$44,737

A similar review of the ultimates with respect to carried case reserves is also useful. This display of course is merely the difference between the previous two displays.

Case Reserves as % of Ultimate

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	43.8%	47.2%	44.7%	26.0%	38.0%	39.9%	39.0%
24	42.2%	40.3%	41.0%	38.1%	40.5%	36.2%	
36	27.9%	28.3%	26.1%	30.8%	27.2%		
48	16.7%	15.3%	13.1%	19.4%			
60	11.5%	11.4%	5.1%				
72	7.7%	6.9%					
84	5.0%						
Ultimate	$82,372	$88,287	$70,741	$80,301	$92,430	$66,216	$44,737

Based on the selected ultimates, the required reserve is the difference between the incurred losses and the ultimate. This required reserve is also a "hindsight" test of the selected ultimate, similar to that presented in Schedule P, Part 3.

Required Reserves as % of Ultimate

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	72.6%	75.0%	71.5%	76.0%	77.8%	74.3%	74.6%
24	51.4%	50.2%	44.7%	53.5%	53.6%	49.3%	
36	34.1%	33.5%	27.5%	37.2%	32.0%		
48	22.2%	18.6%	14.6%	22.4%			
60	13.5%	11.6%	6.1%				
72	7.8%	7.9%					
84	5.0%						
Ultimate	$82,372	$88,287	$70,741	$80,301	$92,430	$66,216	$44,737

It is also useful in some cases to review the ratio of the required reserve to the carried reserve, as this ratio can be very stable for some lines.

Required Reserves as % of Carried Reserves

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	165.9%	158.9%	160.0%	292.6%	204.7%	186.4%	191.3%
24	121.8%	124.7%	108.8%	140.2%	132.4%	136.1%	
36	121.9%	118.5%	105.1%	120.8%	117.8%		
48	132.5%	121.6%	111.3%	115.1%			
60	117.4%	101.3%	119.2%				
72	101.5%	114.3%					
84	100.0%						
Ultimate	$82,372	$88,287	$70,741	$80,301	$92,430	$66,216	$44,737

Review of these statistics indicates that the paid ratios on the 1987 and 1988 ultimates are somewhat high, at 50.7% and 25.4% of ultimate, respectively. However the comparison to 1985 and 1986 is difficult because of the anomalous behavior of these two years.

Once ultimates have been selected, it is extremely important for the analyst to be able to derive projections of the upcoming periods loss development aggregates. These predictions can be monitored over the next period - month, quarter, or year. If actual loss statistics, such as paid losses, case reserves, IBNR counts, and CWP's actually come in close to the forecast development amount, the analyst can have more confidence in his analysis and understanding of the reserve situation.

For example, based on the incurred loss development method estimate of ultimate losses of $44,737,000 on the 1988 accident year, we should expect to see emergence of incurred losses amounting to $10,080,000 during 1989 on the 1988 accident year. These expected loss emergence forecasts come directly from the diagonal of the incurred loss triangle used by the analyst to develop his ultimate estimates. The forecast incurred loss expected in calendar year 1989 is

1982	1983	1984	1985	1986	1987	1988
$0	$0	$0	$1,559	$1,759	$5,467	$10,080
					Total	$18,865

This indicates a total of $18,865,000 of incurred loss should emerge during 1989 based on the analyst's selection of loss development factors. The benefit of monitoring the near term forecast is clear. The accuracy of the ultimate estimate on accident year 1988 will take several more years to ascertain. The development projection for the next calendar year is available and its accuracy can be measured in only one year.

MISCELLANEOUS TOPICS

Reserve Discounting

In establishing the liabilities for losses and loss adjustment expenses, it is often necessary to recognize the time value of money. Recall that in all of the loss reserve estimation procedures reviewed above, we have not taken interest into account.

In discounting the loss reserve liability for the time value of money, we need a payout schedule for the liability amount. If the liability estimate is given by the paid loss development estimate as above, we have an undiscounted liability of $xx,xxx,000. The payout pattern can be deduced from the completed triangle established by the selection of paid loss development factors.

The paid loss projection is given as follows,

Developed Months	1982	1983	1984	1985	1986	1987	1988
12	$22,603	$22,054	$20,166	$19,297	$20,555	$17,001	$11,346
24	$40,064	$43,970	$39,147	$37,355	$42,898	$33,568	$22,692
36	$54,301	$58,737	$51,319	$50,391	$62,832	$45,317	$30,634
48	$64,114	$71,841	$60,417	$62,347	$82,829	$58,458	$40,131
60	$71,257	$78,076	$66,402	$68,270	$91,112	$63,427	$43,542
72	$75,950	$81,287	$70,386	$72,366	$96,579	$67,233	$46,154
84	$78,224	$83,726	$71,653	$73,669	$98,317	$68,443	$46,985
Ultimate	$79,006	$84,563	$72,369	$74,405	$99,301	$69,128	$47,455

which yields the following forecast of paid amounts by calendar year and accident year.

Calendar Year	Paid on Accident Year 1982	1983	1984	1985	1986	1987	1988
1989	$782	$2,439	$3,984	$5,923	$19,997	$11,749	$11,346
1990	$0	$837	$1,267	$4,096	$8,283	$13,142	$7,942
1991	$0	$0	$717	$1,303	$5,467	$4,969	$9,497
1992	$0	$0	$0	$737	$1,738	$3,806	$3,411
1993	$0	$0	$0	$0	$983	$1,210	$2,613
1994	$0	$0	$0	$0	$0	$684	$831
1995	$0	$0	$0	$0	$0	$0	$470

This results in the following payout pattern for the 12/31/88 liability for loss reserves.

Calendar Year	Payout	Discount Rate	Discounted Payout
1989	$56,220	96.7%	$54,365
1990	$35,567	90.3%	$32,117
1991	$21,951	84.4%	$18,527
1992	$9,692	78.9%	$7,647
1993	$4,806	73.8%	$3,547
1994	$1,515	68.9%	$1,044
1995	$470	64.4%	$303
	$130,221		$117,550

Discounting this payout pattern for 7% interest results in a discount of almost $13 million, or 10% of the undiscounted amount.

Reserve Estimate Ranges

Throughout our analyses, we have focused on obtaining point estimates of the loss reserve liability. However, we have also found that it is extremely difficult to obtain one single estimate of the loss reserve liability. Each method results in a different answer. Further, to the extent that we are dealing with the estimation of the mean of a stochastic process, the actual result will almost always differ from the estimate.

Clearly, a range of results and a statement of our confidence that the observed reserve liability at final development will be within the stated range is preferable for this sort of process. However, the balance sheets will continue to require a point estimate of the reserve liability.

Working with risk theoretical concepts, it is possible to develop a model of the reserve inventory, in terms of frequency and severities. This model can be used to develop confidence intervals for the development of the reserve. The development of such a risk theory model is outside of the scope of this chapter.

RISK CLASSIFICATION
(TEXTBOOK CHAPTER DRAFT)

Chapter 4 - Risk Classification

by: Robert J. Finger

I. INTRODUCTION

Risk classification involves similar concepts to both ratemaking (Chapter 1) and individual risk rating (Chapter 2). Risk classification is used primarily in ratemaking when there is not sufficient information to estimate a price for a given individual. In order to derive a price, individuals that are expected to have the same costs are grouped together. The actuary then calculates a price for the group and assumes that the price is applicable to all of the members of the group. This, in simple terms, is the substance of risk classification.[1]

Premiums should vary if the underlying costs vary. Costs may vary among groups for all of the elements of insurance cost and income: losses, expenses, investment income, and risk. For losses, as an example, groups may have varying accident frequency or average

[1]For more detail on the general problem of risk classification, see SRI International (1979) and SRI (1976).

claim costs. Expenses may also differ among groups and in some lines, such as boiler and machinery, inspection expense is a major portion of the premium. Investment income may vary among groups; for example, some insureds may be sued more quickly (emergency physicians versus obstetricians) or claims may be settled more quickly. Finally, risk, defined as variation from the expected, may vary among different types of insureds. For example, more heterogeneous groups are subject to more adverse selection and, hence, more risk. In the remainder of this chapter, the term "costs" will refer to all of the above considerations.

Risk classification is "the formulation of different premiums for the same coverage based on group characteristics". These characteristics are called rating variables. For automobile insurance, examples are geography and driver characteristics. Rating variations due to individual claim experience, as well as those due to limits of coverage and deductibles, are not considered as part of the classification problem.

This chapter first considers the interaction between classifications and other rating mechanisms, such as exposure bases, marketing, underwriting, and individual risk rating. This chapter will then review the various criteria (actuarial, operational, social, and legal) for selecting rating variables. It then turns to historical examples of classification systems. Next, measures of classification efficiency are examined. Finally, the chapter briefly reviews problems in and approaches to estimating class relativities.

II. RELATIONSHIP TO OTHER RATING MECHANISMS

The classification process must be considered within the overall context of marketing, underwriting, and rating. The overall goal is to price an insured properly for a given coverage. This may be accomplished in different ways. Risk classification is one step in a process that makes significant pricing decisions in many ways.

Exposure Base

An important consideration is the exposure base. For automobile insurance, the exposure base is an insured car-year. For workers' compensation, exposure can be total payroll, hours worked, or limited payroll (i.e., payroll up to some limit for a given time period). Manual premiums are calculated as the exposure times a rate. For example, if payroll is $1 million and the rate is $5 per $100 of payroll, manual premium is $50,000.

Exposure bases should be as closely proportional to costs as possible. For example, consider workers' compensation, which has both medical and indemnity benefits. If a worker is injured, the worker's medical costs are paid and the worker receives indemnity payments for time lost from work. Indemnity benefits typically are calculated as two-thirds of wages, subject to a maximum equal to the statewide average wage. For example, assume the maximum benefit is $400 per week. If the worker's wages are $600 or more, the worker receives $400; if the wages are $450, the worker receives $300. The most appropriate exposure base would be hours worked for medical benefits and limited payroll (limited to $600 per week per employee) for indemnity benefits. These exposure bases would be proportional to costs, assuming that all workers have the same accident frequency per hour worked and no differences other than wages in average claim size.[2]

If all employers pay the same wages (or have proportionately the same number of workers at different wage levels), total payroll is an adequate exposure base. If one employer pays higher wages than another, however, total payroll is not as accurate an exposure base as the combination of hours worked and limited payroll. Because accident frequency or severity varies among different insureds, some element of cost variance remains to be rated by a classification system or other means.

Individual Risk Rating

As mentioned above, the goal in all pricing is to properly evaluate the potential costs. When the individual insured is large enough

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[2]It may be argued that accident frequency, the duration of indemnity benefits, or the total amount of medical expense is related to wages. If so, total payroll could be more accurate than hours worked or limited payroll.

to have credible claims experience, that claims data can be used to modify the rate that would have been charged by a classification plan alone.

Schedule rating, based on underwriting judgment, is often part of individual risk rating plans. It is based on items that are not quantifiable or not includable in a classification or experience rating plan. Schedule rating has the potential for predicting costs more accurately, but it is often used to meet competitive prices.

<u>Marketing and Underwriting</u>

Insurers may use different strategies for pricing business. As will be pointed out below, many factors that are related to cost potential cannot be objectively defined and rated. Instead of pricing these factors, insurers may adjust their marketing and underwriting practices to account for them.

Two common strategies are: (1) adjust price according to individual cost potential, and (2) accept an individual only if the existing price structure is adequate.[3] It often happens that commercial lines underwriters follow the first strategy and personal lines underwriters follow the second. Part of the reason may be the size

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[3]For more detail see Glendenning & Holtom (1977) and Launie, Lee & Baglini (1976).

of the accounts; with a larger account, more expense dollars, and more meaningful claims history, an underwriter may feel more comfortable in formulating an individual price.

An alternative to the second strategy is to have several different rate structures within the same insurance group. Due to rate laws, this often requires several different insurance companies. For example, one company in a group may charge bureau rates; one charge 20% more; another charges 10% less than bureau rates; and a fourth company charges 25% less than bureau rates. Using all available information, the underwriter makes a subjective judgment about which rate level is the most appropriate. In practice, competitive rate quotes may have an influence on the underwriter's judgment.

In the above case, the underwriter is working with an existing classification plan. Each rate level, presumably, has the level of class detail that can be supported by objective rating variables. The underwriter assesses all other relevant information, including difficult to quantify data, to fix the charged rate.

In practice, a certain number of insureds will be considered to be uninsurable. This can happen when the number of potential insureds with certain characteristics is so small that cost experience will not be credible. Along the same line, the insureds within any given group may be thought to be too heterogeneous. That is, there is a great risk in writing such an insured because the cost may be much higher than the average (or measured) cost of the group. In both cases, the individual insureds are difficult to rate properly because there is not enough experience with analogous types of insureds.

Notwithstanding the above situation, insurers compete on price for larger individual risks and classes of business. An important consideration is the ability and propensity of insureds to shop for the best price. The more insureds shop, the more an insurer must refine its classification plan. Insurers also vary in their ability to select lower-cost insureds within a classification through underwriting. More successful insurers are said to be "skimming the cream".

When an insurer receives a disproportionate number of higher-cost insureds, relative to its classification plan, it is being adversely selected against. If the adverse selection continues, the insurer must increase its premiums. Such premium increases may induce the insurer's lower-cost insureds to move to another insurer, creating more adverse selection and producing a need to further premium increases. The insurer can become insolvent, unless it can adequately price its book of business.

In summary, premiums for the same amount of coverage can vary among insureds due to exposure bases, individual risk rating plans, and marketing or underwriting approaches. Classification plans are one aspect, integrated with these others, of accurately pricing individual insureds.

III. CRITERIA FOR SELECTING RATING VARIABLES

Criteria for selecting variables may be summarized into the following categories: actuarial, operational, social, and legal. Following this discussion, the section describes the ramifications of restricting the use of rating variables.

Actuarial

These criteria may also be denominated "statistical" criteria. They include accuracy, homogeneity, credibility, separation and reliability. Foremost is accuracy. Rating variables should be related to costs. If costs do not differ, the usual methods for estimating rate relativities will produce the same relativity; thus the variable adds to administrative expense, and possibly consumer confusion, but does not affect premiums. As an example, most insurers do not charge different automobile insurance premiums for drivers between the ages of 30 and 64, solely due to age. Presumably costs do not vary much by age, or cost variances are due to other identifiable factors. As a practical matter, insurers may maintain cost information on a more detailed basis than the pricing structure; data is maintained so that premium differences may be introduced if there actually are cost differences.

Accuracy is important for at least two reasons: the market mechanism and fairness. In a market economy, insurers that price their products more accurately can be more successful. Suppose, for example, that the cost (including a reasonable profit) of insuring group A is $100 and the cost of insuring group B is $200. If an insurer charges both groups $150, it is likely to be undersold in group A by another insurer. The first insurer will tend to insure more people in group B and, consequently, to lose money. Thus, to the extent that insurers accurately can identify costs, they can compete more successfully. There is thus an incentive to charge more accurate premiums. For the most part, this incentive also produces more rating variables and a more detailed classification system.

Another reason for the importance of accuracy is fairness. In the example above, it is fair for group A members to pay $100 and group B members to pay $200, because this is the cost of the goods and services provided to them. (Of course, if there are subgroups within group A whose costs are $50, $150, and $250, it would be fairer to charge those costs to those subgroups). This concept is often called "actuarial fairness" and it is based on the workings of a market economy. Of course, other concepts of fairness may appeal to some people. For example, income taxation is supposedly progressive, meaning that people pay for government services based on ability to pay rather than services received.

The second actuarial criterion is homogeneity. This means that all members of a group that receive the same rate or premium should have similar expected costs. As a practical matter, it is difficult to know if all group members do have similar costs. The reason for grouping is the lack of credibility of individual experience. Consequently, for many rating groups, subdivisions of the group may not have sufficiently more credibility than individual insureds.

The third actuarial criterion, alluded to above, is credibility. A rating group should be large enough to measure costs with sufficient accuracy. There will always be the desire to estimate costs for smaller groups or subdivisions, even down to the individual insured level. Random fluctuations in claims experience may make this difficult, however. There is an inherent trade off between theoretical accuracy (i.e., the existence of premiums for smaller and smaller groups) and practical accuracy (i.e., consistent premiums over time).

The fourth actuarial criterion is separation: different groups should have different mean costs. If two different groups have the same mean cost and are charged the same premium, it may not serve any purpose to have separate classifications.

The goals of separation and homogeneity may conflict in practice. Two subgroups with similar mean costs may have different levels of homogeneity. Separate classes may reduce adverse selection. In addition, two heterogeneous subgroups may be combined into one class, even though the mean costs are different, if cost variations within the subgroups significantly exceed differences in mean costs.

The fifth actuarial criterion is reliability or predictive stability. Based on a given set of loss data, the apparent cost of different groups may be different. The differences, however, may be due to random fluctuations (analogous to the problem discussed under credibility, above). In addition, the cost differences may change over time. For example, historical cost differences between genders may diminish or disappear as societal roles change. Technology may change relative cost differences.

In summary, actuarial classification criteria attempt to most accurately group individual insureds into groups that: (1) are relatively homogeneous (all group members have similar costs), (2) are sufficiently large to estimate relative cost differences (credibility), (3) have different mean costs (separation), and (4) maintain different mean costs over time (reliability).

<u>Operational</u>

These actuarial criteria must be tempered by practical or operational considerations. The most important is that the rating variable have an objective definition. There should be little ambiguity; class definitions should be mutually exclusive; and the opportunity for administrative error should be minimized. For example, automobile insurance underwriters often talk of "maturity" and "responsibility" as important criteria for youthful drivers. These are difficult to define objectively and apply consistently. The actual rating variables, age, gender, and marital status, may be seen as proxies for the more "fundamental" sources of cost variation. Maturity might be a more accurate variable, but it is not practical.

Another important practical consideration is administrative expense. The cost of obtaining and verifying information may exceed the value of the incremental accuracy. For example, driving mileage (or even, when and where a person drives) may be a very good indicator of cost. It is probably too expensive to obtain and verify, however. Assume that drivers driving under 7,500 miles per year cost 20% less than those who drive 7,501 to 12,000 miles, who cost 20% less than those who drive more than 12,000 miles. Assume also that the middle group costs $100 per year and that it costs $20 per driver to obtain, process, and verify annual mileage data. In a system utilizing mileage, drivers driving under 7,500 would pay $100 (their previous cost of $80 plus $20 for the additional expense), the middle group would pay $120 and the highest cost group, $145. Nobody would pay less than before! Although this example may be extreme, it demonstrates that added expense to classify may not serve insured (or insurers) any better than not classifying.

Another practical consideration, alluded to above, is verifiability. If insureds know that they can pay lower premiums by lying, some percentage of them will do so. The effect is to cause honest insureds to pay more than they should to make up for the dishonest insureds that pay less than they should. There is a practical tradeoff between verifiability, administrative expense, and accuracy. Few rating variables are free from manipulation by insureds. Indeed most insurance rating information is supplied by insureds and much of it is only verified to a limited extent. At some point, the value (in expense savings) of relying upon unverified information is outweighed by its inaccuracy. In practice, variables are added, at a tolerable cost, as long as they result in improved overall accuracy.

There are several other practical considerations in selecting rating variables. The variables should be intuitively related to costs. Age, in life insurance, is intuitively related (i.e., older people are more likely to die). Age in automobile insurance is less so. Younger operators may tend to be more reckless and older operators may tend to be less observant, but the correlation between age and these factors is less precise than with mortality. Intuitive relationships also improve acceptability, which will be discussed below.

Pertinent to the cost-verifiability issue, it is often better to use measures that are available for another purpose. If the variable is used only for insurance rating, it is more likely to be manipulated and it may be more difficult to verify. Payroll and sales records, for example, are kept for other purposes (such as taxation). These may be manipulated for those purposes, as well as insurance purposes, but there may be other ramifications of manipulation (such as criminal penalties or adverse relations with suppliers or bankers).

Still another practical consideration is the avoidance of extreme discontinuities. If group A's rate is $100 and group B's rate is $300, a group B insured may obtain a significant premium reduction by qualifying for group A rates. Thus the incentive to cheat and the expense to verify will be higher if there are fewer classes, with larger differences in premiums. It may be difficult in practice, however, to construct gradual changes in rates because there may be very small numbers of very high cost insureds. Thus, for credibility purposes, there may be fewer classes, with widely differing rates.

Social

This section has discussed the actuarial goals of classification and some of the operational difficulties. Another limitation on classification is "social acceptability" or social considerations. A number of key terms, such as "causality", "controllability", "discrimination", and "affordability" have been debated in public. This section will briefly describe some of the public concerns.

Privacy is an important concern. People may be reluctant to disclose personal information. This affects accuracy of classification, verifiability, and administrative cost. In automobile insurance, for example, a psychological or behavioral profile might be strongly correlated with claims cost. (It may also be expensive to obtain). Many people might resist this intrusiveness, however. Although insurer A might have a more accurate rating system, using a psychological profile, it might not obtain a sufficient amount of business. Insureds may choose to pay more to avoid disclosing personal information.

Discrimination is an emotionally charged term when used in racial, religious, or gender contexts. In fact, all people discriminate every day, in their choice of food, clothing, friends, etc. Life is a matter of making choices, which involves discrimination. Some types of discrimination may be morally disreputable, but discrimination itself is inevitable. Risk classification is discrimination. Different insureds are charged different amounts of premiums. Furthermore, risk classification is discrimination based on group characteristics. The insured is charged a premium based on the costs of the group, assuming that the insured belongs to that group.

What differentiates risk classification from the opprobrious types of discrimination is that it is (or should be) objective and based on prospective costs. Racial discrimination is condemned because it is not objective. Fair discrimination involves the use of relevant characteristics that have a measurable relationship to costs.

In this connection, the terms "correlation" and "causality" are often invoked. Assume there is some rating variable, X, which divides insureds into groups A, B, C, etc. The rating variable is correlated with costs if the mean costs for the various groups are significantly different. There may be other variables for which there are similar correlations. The "real" reason for the differences in costs may be some entirely different variable or combination of variables. Nevertheless, X is correlated to the cost of providing insurance. X may be a proxy for the "real" cost difference.

"Causality" implies a closer relationship to costs than correlation.[4] Mileage in automobile insurance might be considered

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[4]See, for example, Mass. Division of Insurance (1978), p.22.

a causal variable; the more miles a driver drives, the higher the cost of insurance should be (other things being equal). "Causality" is difficult to define in operational terms, but it conveys a direct relationship with costs. Loss costs, for example, can be divided into claim frequency and average claim cost. "Causal" variables then, could be considered to be directly related to claim frequency and average claim cost. Automobile mileage, presumably, is proportional to claim frequency. Proximity to fire protection, in fire insurance, may be proportional to the average claim cost.

Unfortunately, however, the categorization of variables as "causal" or "non-causal" is ambiguous. With automobile mileage, for example, when and where one drives may be more relevant to costs than mileage. Driving in a large city, with more vehicles, more intersections, and more distractions is probably more hazardous than driving in rural areas. Driving at night or when tired or drunk may be more hazardous than driving in daytime or when fully rested or sober.

Clearly, "causality" is a valuable concept. "Causal" variables are probably better at cost prediction than non-causal variables. The issue, as usually put forward by insurance industry critics, is whether mere correlation should justify the use of rating variables. In automobile insurance, for example, it is argued that age, gender, and marital status are not "causal" variables and, therefore, are not "socially acceptable." It is usually not disputed that there are correlations between costs and age, gender and marital status. These variables, by themselves, are not the true cause of the cost variances (according to critics). In the sense that some younger drivers have lower cost potential than some older drivers, this is true. There are reasons, albeit unknown, for the cost differences between younger and older drivers. Are the true reasons: immaturity, inexperience, recklessness, or something else? Some of these possible reasons can be measured and used as rating variables. For example, inexperience could be measured by the number of years licensed. (Of course, the quality of experience, such as total mileage and mileage under various driving conditions, would be difficult to assess). Most of the other plausible reasons tend to fail under the above practical considerations (e.g., objective definitions or cost) or other social considerations (e.g., privacy).

The dilemma can be summarized as follows. Certain variables will be correlated to costs, but (at least in the opinion of certain critics) not causally related. That is, the relationship between the variable is not direct enough; it may be a proxy for other, "real", causes or it may be a spurious or fleeting correlation. If non-causal variables are prohibited, insurers would have an incentive to develop causal variables, which are seen to be better, or other, less opprobrious non-causal variables. The ultimate problem, however, is that no "causal" variables may satisfy all of the other actuarial, practical, and social constraints. Thus there may be a question of using a non-causal variable or using nothing at all. From an actuarial point of view, correlated variables provide more accurate premiums and are thus more desirable in a competitive market place. Eliminating correlated non-causal variables may produce certain market corrections. Those will be discussed later.

Several other concepts of "social acceptability" have been debated by the insurance industry and its critics. These include (1) unfair discrimination, (2) controllability, and (3) affordability.

Most property-casualty rating laws prohibit "unfair discrimination." The number of reported legal decisions that have construed this term are few. From an actuarial point of view, this would normally mean premiums out of line with costs. From a social point of view, this may be analogized to legal cases involving race, ethnic, or religious discrimination. Legal considerations will be discussed below.

"Controllability" is seen as desirable by most insurance industry critics. A controllable variable is one which is under the control of the insured. If the insured moderates behavior in a certain way, premiums will be reduced. For example, by installing burglar alarms, the insured reduces claims cost potential and should receive some discount. Accident prevention can be encouraged by the use of controllable rating variables.

From a practical view point, there may not be very many useful controllable variables. The make and model of automobile in physical damage insurance is controllable. Geographical location is controllable in a broad sense, but not very useful in making day-to-day or short-term decisions. (Moving a warehouse or petroleum refinery is not practical; nevertheless, the decision to locate a structure is controllable and insurance costs may possibly be a factor in the decision). Driver training course credits for automobile insurance are also controllable.

Even though variables are controllable, they may not have much impact on the rating system. Most people take some sort of driver training, for example, so the rate differential will only apply to a small group of drivers. In addition, burglar alarms may reduce the frequency of burglaries, but some thefts will occur anyway and theft costs may be a small proportion of homeowners and commercial property premiums.

Controllable variables may increase administrative costs. If the insured has control over premium costs, the insured can manipulate the rating system and insurers may require verification. As with "causality", "controllability" is a useful concept but there is a shortage of usable rating variables that apply.

Another social consideration is "affordability". In the context of risk classification, it usually arises where classification schemes are more refined, with the attendant spreading of rates. Thus high rates are often seen as causing affordability problems (even if, for example, the high rate is generated by a youthful operator driving a Corvette in New York City). Another example is the correlation of incomes and insurance rates. In automobile insurance, rates are often highest in urban areas, where, allegedly, most poor people live. In reality, wealthy people also live in urban areas; youthful drivers that can afford any car or a high-priced car are not necessarily poor. Thus both high rates, per se, and higher rates for lower-income groups pose an affordability concern.

Another aspect of the affordability issue is the necessity of insurance. Many states require automobile (liability) insurance. Most mortgagors require property insurance. To some insurance industry critics, this implies the necessity of a subsidy for some consumers. (Of course, owning a car or a house is optional. The value of controllable variables, providing an incentive to prevent accidents or reduce costs, is ignored in this context).

Still another aspect of affordability is availability. If rates are arbitrarily leveled or reduced below cost, the market may not voluntarily provide coverage. Thus rates may be "affordable" but insurers may be very reluctant to insure people at those rates.

This section digresses at this point to consider the interaction of the arguments often posed by insurance industry critics. Causality is good and correlation (without causality) is bad. When discussing affordability problems, however, the alleged correlation between incomes and premiums is sufficient to cause concern. Further, a controllable variable, such as geographical location, is an anathema for the affordability issue, while controllability, in general, is much preferred to immutable characteristics (such as age, and gender). This is not to say that insurance industry practices are above criticism or that insurance industry critics do not have valid concerns. The point is that classification criteria are multi-faceted and risk classification is a difficult problem.

A theme that is stressed by insurance industry critics, particularly in the causality-correlation and controllability debates, is that of individual characteristics. Analogizing to racial, ethnic, and religious discrimination cases, critics essentially attack the basing of rates on group characteristics. This is seen to be unfair to individuals whose costs may differ from those of the group. A common rhetorical device is the "overlap theory", which can apply to almost any rating variable. As an example, this section will use gender in automobile insurance. Youthful females generally have lower automobile insurance costs than youthful males. Some youthful females, however, have higher costs than some youthful males, and, in a perfect rating system, should be charged more than some males. The overlap theory conclusion is that it is unfair to charge males more than females.

The overlap theory relies on the concept, however impractical, of a perfect system. The real risk classification issue is whether the male-female rating system is more accurate than a system that does not use that variable. (It could be argued that a value system based on something other than accuracy is being used. That argument is defeated by the dependency of the overlap theory on the "unfairness" of certain individuals being charged more than their costs. Notions of accuracy, as a criterion for risk classification, are central to the overlap theory.)

The overlap theory essentially ignores the nature of insurance and the practical necessity of using group characteristics for rating individuals. Costs in insurance are fortuitous. Individual insureds have a cost potential, but this potential is not directly measurable. Cost potential can be estimated using subjective probability. The actuary can use a wide range of information, including historical cost information, to make a subjective judgment about future costs. Subjective judgments include what rating variables are related to costs and whether certain individuals belong in certain groups that are used for rating purposes. Since the costs of individuals are unknown, and since group characteristics are used, there will be an overlap, by necessity. The overlap reflects the reality of the insurance situation: that costs have a fortuitous element and that group characteristics must be used for rating purposes.

The critics might contend that the use of group characteristics is unjustified; that each individual should be judged on his or her own merits. This may be appropriate in work situations or some other contexts, but it is not feasible in an insurance situation.

What is meant by using individual characteristics? Presumably these are a collection of enough different factors so that almost any two individuals would compute a different "score" or have a different combination. That is, they would be individuals because they would not be exactly alike some one else. For each of these factors, the actuary could subjectively determine the prospective impact on costs. To do this, the actuary will evaluate group experience for each of the different factors. The actuary cannot evaluate individual experience because, from the definition at the beginning of the chapter, that experience is not credible. Thus the only way the actuary can proceed is to project group costs. If costs are projected for enough different factors, the resulting rates may approach individual rates (in the sense that no two individuals have exactly the same combination of factors).

The upshot is that <u>more</u> variables should be used to rate individuals, rather than fewer. More variables and a more detailed rating plan will provide more accurate rates and more individualized rates. The conclusion that the critics draw from the overlap theory (i.e., the abolition of a rating variable) is self-contradictory.

With the exception of the affordability issue, these social issues are based on accuracy arguments. The basic limitations on accuracy are the competitiveness of the insurance industry and the credibility of the cost information. These factors are related in some cases. As long as the insurance industry is competitive, there are incentives (profitability, solvency) to accurately price individual insureds. These incentives may be minimal, however, for small groups of heterogeneous insureds. Restrictions on competition are unlikely to produce a more accurate rating system. The ramifications of restrictions will be discussed after a brief review of legal considerations.

<u>Legal</u>

This section has considered actuarial, practical, and social considerations. It now turns to the legal context of risk classifications. The following discussion is necessarily brief, but it provides an overview. The circumstances of each particular case (e.g., rating variable, line of insurance, state statutes and constitution), will determine its outcome. The following is based on general concepts and principles.

Risk classification may be affected by constitutions (state and federal), statutes, and regulations. Generally, constitutions govern statutes and statutes govern regulations. Constitutions are usually very general, statutes are more specific, and regulations may be the most specific.

Both federal and state constitutions may apply to a risk classification situation. There must, however, be a specific phrase or section that is applicable. The federal constitution is quite broad and vague. The "equal protection clause" ("EPC") might be applicable. Other clauses probably are not. State constitutions are often much more specific. Gender discrimination, for example, may be specifically mentioned.

The federal equal protection clause reads: "No state shall ...deny to any person within its jurisdiction the equal protection of the laws." This points to two requirements; (1) state action and (2) unequal treatment. "State action" generally means that the state has acted, either on its own or by officially sanctioning the conduct of private individuals. Purely private discrimination is usually not actionable under the EPC. With insurance, the requisite state action is probably the promulgation of rates; the mere approval or acquiescence in rates probably is not state action. If rates are not regulated at all, rating classifications are probably exempt from the EPC.

Unequal treatment is also a requirement under the EPC. Arguably, basing premium differences on demonstrable cost differences is not unequal treatment.

Because of the state action requirement, constitutional challenges to insurance rating classifications are unlikely to succeed. Statutes, however, can impose restrictions on insurers. In this case, it is the insurers who will try to invoke constitutional provisions to invalidate the statutes. Several federal clauses, such as "due process," "takings", and "contracts" may be applicable. As a general rule, however, courts have been very solicitous of legislatures in their regulation of businesses. Most likely, any statutory restriction on rating variables would be constitutional.

Finally, regulations issued by state insurance departments may affect classifications. Under a constitutional theory (known as the "delegation doctrine") only the legislature may promulgate substantive law; the executive branch merely carries out the will of the legislature. Although states vary considerably, broad discretionary grants of power to executive agencies may be found unconstitutional.

In summary, constitutional provisions, statutes, and insurance department regulations may all potentially affect the freedom of insurers to select and use rating variables. As this brief discussion indicates, constitutional provisions are probably not applicable; statutes are practically invulnerable; and regulations may or may not be subject to challenge by insurers.

Ramifications Of Restrictions

Legislatures may abolish the use of certain rating variables or relativities may be capped. The consequence will be similar for each, although more extreme for abolition. The discussion below deals with abolition. Insurers can react in three ways: pricing, marketing, and underwriting. In pricing, they can try to find replacement variables. As stated above, there may not be many variables that are suitable, given the above actuarial, operational, and social criteria. Insurers generally do have incentives to create better variables, and the current ones in use are considered to be the best. If no replacement variables are found, rates will be levelled and subsidies created. For example, if Group A's cost is $50 and Group B's cost is $150, but the distinction between them cannot be used in rating, both groups may pay $100. Group A would be overcharged by $50 and Group B would be subsidized by $50.

The effect of abolishing rating variables in a competitive market, is to create availability problems (unless there are suitable replacement variables). Insurers may withdraw from marketing the coverage to certain groups or refuse to insure them. This will produce, most likely, a larger residual market. (Residual markets, such as assigned risk plans in automobile insurance, exist to provide insurance to those not voluntarily insured). Abolition of variables may also affect insurer profitability and solvency. If an insurer, in the above example, has a large percentage of Group B business, it will need to raise its rates or else it will be unprofitable. When it raises its rates, it may drive more of its better business to competitors, who have lower rates; this will further increase its costs and require a further rate increase. In the long run, solvency may be threatened.

Abolition of rating variables has social consequences, as well. To some extent, abolition will create subsidies. Insurers may voluntarily subsidize certain groups. Otherwise, residual markets will expand; since most residual markets are subsidized by the voluntary market, subsidies will be created. Such subsidies, deriving from legislation, are a tax-in-kind. Certain insured pay more for insurance than they otherwise would have, while others pay less. There is a redistribution of income from the disfavored group to the favored group.

In addition to the subsidies, abolition of rating variables can reduce accident prevention incentives. That is, to the extent accurate pricing promotes accident prevention, less accurate pricing reduces it.

Thus the abolition of rating variables probably will reduce the accuracy of the rating system, which either creates subsidies or else produces availability problems. In either case, accident prevention incentives are reduced.

IV. EXAMPLES OF CLASSIFICATION SYSTEMS

So far this chapter has discussed the general principles for developing classification systems. In this section, specific systems, with particular emphasis on automobile insurance, will be discussed. To be concrete, some assumptions will be made that may not be widely accepted within either the actuarial profession or the insurance industry. The objective is not to specify all of the relevant factors and only relevant factors, but to present an approach that a knowledgeable actuary may follow. Risk classification is a difficult subject area. In theory, not enough is known about either the underlying causes of loss or the variations in costs between insureds. In practice, there is never sufficient data for formulating and testing hypotheses.

Forces Affecting Classification Systems

Classification systems vary over time. Automobile liability originally had only one classification. Prior to World War II there were three classes (adult, youthful operator, and business use). These became refined into nine classes by sub-dividing the youthful class and adding more use categories. In 1965, the National Bureau of Casualty Underwriters (a rating bureau predecessor to today's Insurance Services Office) introduced a plan which had 260 classifications. In 1970, the number of classes was reduced to 217. Most of the classifications were for combinations of age-gender-marital status and use, for youthful operators.

Many forces, chiefly those related to competition, influence classification plans. Generally, the more competitive the marketplace, the more classifications there will be. Assume one insurer charges the same rate, $100, to groups A and B, but their costs are different, $50 for A and $150 for B. Another insurer could charge group A $50 and still be profitable. Thus, to the extent insurers can actually identify cost differences, they will tend to make price differentials. Not to do so affects their profitability and, ultimately, their solvency.

Classification systems may also become more refined as coverage becomes more expensive. From the buyer's side, shopping for favorable prices is encouraged when coverage is more expensive. From the insurer's side, more expense dollars may be available to classify and underwrite; in addition, the cost of making mistakes, or of not having as refined a system, is higher when premiums are higher. For example, towing coverage may be priced the same for all automobiles, even though older cars may be more likely to break down and towing costs may be higher in rural areas; at a low premium (e.g. $10 per year), it may not be cost effective to have rate differentials.

Classification systems usually are more refined for larger markets. Considering the credibility of available cost data, more classifications can be supported by larger amounts of insured exposure.

Finally, classification systems probably have become more refined as information technology has progressed. More information can be handled more cost-effectively today than yesterday. This section now turns to automobile liability classifications.

Automobile Insurance Classifications

Automobile liability insurance classifications can be categorized into the following types of variables: (1) age-gender-marital status, (2) use, (3) geography, and (4) others. Classification plans vary significantly among insurers.[5] Certain types of factors are widely used; many factors are used by only one or a few insurers.

Age-gender-marital status primarily distinguishes among youthful operators, although most insurers have a separate class for drivers over 65. Youthful operators generally are those under 25, although most insurers separate single males under 30. Some insurers have separate classes for each age; some group ages, such as 17 to 20

[5]See SRI (1976).

and 21 to 24. Most insurers distinguish between single male principal operators (using the automobile 50% or more) and occasional operators. Many insurers do not distinguish between single and married female operators, or between principal and other operators for females and married males.

Use categories typically are: pleasure, drive to work (sometimes over or under a given number of miles, one-way, such as 10), business, and farm. Added to this may be annual driving mileage over or under a given amount (such as 7500). Use categories may vary between adult and youthful operators.

Geographical territories are commonly used in classification plans. Contiguous areas, often delineated by city or county boundaries, are the most common. Some insurers use zip codes, sometimes combining adjacent areas or using some other criteria (such as population density). Territories are the same for all age-gender-marital status classes and all use classes. Territories sometimes vary by coverage. For example, there may be fewer claims for uninsured motorist coverage, so there are fewer separate rating territories.

Several other rating variables are in common use. These include good student and driver training discounts for youthful operators; multiple-car discounts; accident and violation surcharges, and sports car surcharges.

In addition to the above variables, several other variables are used for automobile physical damage insurance. These generally consider the value of the automobile, its crashworthiness, and its age. Most insurers use the make and model of the car; various makes and models are combined into a series of different rate groups.

Cost Variation in Automobile Insurance

Above are the classification variables that are commonly used in automobile insurance. Some are "causal"-type variables; others are correlated to costs. Below, this section will discuss potential reasons for cost differences. Some of these are incorporated into rating variables, while others are used only in "underwriting" (i.e., risk selection or rejection).

Cost differences can be classified into four broad categories: (1) use of the automobile, (2) driving ability, (3) interaction with the claims mechanism, and (4) the extent of damages. In many of these areas, the available evidence is more subjective than objective. What is presented is thought to be relevant to costs, even though concrete data may be elusive.

Different uses of the automobile contribute to varying cost potential. More driving should produce more exposure to liability and collision claims. Driving conditions (time of day, traffic density, weather) are also important. Automobile theft is a significant factor for comprehensive coverage, therefore location of the car in higher crime neighborhoods is relevant for that coverage.

Mileage may be used directly in rating, although commonly the only distinctions are annual mileage over or under a given amount and mileage to-work. Indirectly, mileage may be correlated with multiple-car discounts and some age-gender-marital status classifications. For example, over 65 drivers may drive less or under more favorable conditions; females may drive less than males; married males may drive less than single males. Driving conditions are taken into account, at least indirectly, in geographical territories. The territory is usually defined by the principal garage, which may differ, of course, from where the car is usually operated. Driving conditions are considered more directly in the use variables.

Cost differences may be due to differences in driving ability, arising from familiarity with the driving conditions, experience and training, reaction time, eyesight and hearing, concentration, condition of the automobile, and driving style. Some classification variables are related indirectly to these cost differences. For example, youthful operators have less familiarity and less experience; over 65 drivers may have poorer eyesight or hearing; discounts for driver training are available. Admittedly, individual performance varies greatly within the given rating classes.

Cost differences may also arise from interactions with the claims mechanism. Some people are more claims conscious than others. This affects the physical damage, personal injury protection, and medical payments coverages for the insured. Geographical differences may be apparent for liability coverages. Some people may be more or less sympathetic to a jury. Some people may press dishonest claims. Some people may be more cooperative in submitting claims or in helping to defend claims. Most of these differences are quite subjective and difficult to quantify in a rating variable. Where cost differences can be discerned, it is more likely that insurers would refuse to insure an individual, rather than try to determine an accurate rate.

Finally, cost differences may result from the extent of damages, given that an accident has occurred. Crashworthiness of the automobile is an obvious rating variable. The same type of accident may produce $100 of damage in one car and $1000 of damage in another. The speed with which a car is driven will also affect damages. The use of safety devices, such as air bags or seat belts, will affect costs. Physical impairments may produce higher loss costs. Some of these differences may only be relevant to certain coverages.

To some extent, existing rating variables consider these differences in costs. Sports cars are often surcharged, presumably because they are driven at higher speeds, are prone to greater damage, cause greater damage, or are more prone to lawsuits.

In summary, a variety of factors have been presented that affect claims costs. Some of these are more objective and lend themselves more readily to becoming rating variables. Many factors, however, are quite subjective, very difficult to measure, and almost infeasible to use as rating variables; these tend to be used by underwriters to decline coverage or assign to a higher-rated company in the group.

To conclude this section, other lines of business are briefly reviewed. Most lines of business use geographical rating. Workers' compensation classes are mostly based on occupations or industries. There are some 600 different classes used by the National Council on Compensation Insurance in one or more states. Medical malpractice classes are based on specialties, paying particular attention to the amount of surgery performed. Boiler and machinery rates vary by type of object, because inspection costs are a significant element of premium. Products liability classes are defined by the type of product. Premises liability is defined by the character of the operation or activity. Homeowners and dwelling fire rating variables include the number of units in the structure and the age and type of the structure. Fire insurance rates are based on the type of construction, type of occupancy, protection features, and special exposure to loss.

V. MEASURES OF EFFICIENCY

The quantitative description of the accuracy of classification systems has concerned actuaries for many years. Recently, however, public debate on risk classification has encouraged new research and analysis. This section will define "efficiency" as a measure of a classification system's accuracy.

The reason for developing classification systems is the variability in costs from one insured to another. The key to measuring efficiency is understanding this variability. Costs vary because claim frequency varies and because claim sizes vary. A perfect classification system would produce the same variability as the insured population. Conversely, a classification system that has less variability than the insured population cannot be perfect, because two insureds may receive the same rate when their costs are actually different.

A complicating factor is the fortuitous nature of insurance. Costs are unknown. When measurements are made of cost variability, it is after certain events have already happened. The same events probably will not happen again. It is uncertain whether the actual events that occurred are representative of what will occur in the future. The future may have more or less variability than the past.

Most existing measures of classification efficiency use the statistical measure of variance. Other measures are possible, include average deviation, and average absolute deviation. Variance has the advantage of being widely used in many types of statistical applications (e.g., regression analysis and analysis of variance). This section will use variance concepts as an operational measure of efficiency, but other measures could be used.

Likewise, there are many possible specific formulas for efficiency. The measure most commonly used compares the variance explained by the classification system to the total variance underlying the insured population.[6] If the classification system were perfect, the efficiency would be 100%. If the classifications had no predictive value (i.e., were random with respect to potential costs), the efficiency would be 0%.

This formula requires the calculation of two items: (1) the variance of the classification system and (2) the variance of the insured population. The former is relatively easy to calculate; the latter is unknowable. Each will be discussed in turn.

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[6]See SRI (1976), Woll (1979).

To determine the variability of the class plan, one needs the class relativities and the percentage of exposures by class. It is assumed that the relativities are the expected values of actual cost differences; if not, the latter should be used instead.

Although formulated in terms of "variance", efficiency can be measured by other numerical calculations. For simplicity, this chapter uses the concept of the coefficient of variation, ("CV") which is the standard deviation divided by the mean. The square of the CV can be used to measure efficiency, as proposed above, in terms of variance. (It is assumed that both the class plan costs and the underlying population costs have the same mean; if not, adjustments can be made).

For a numerical example, see the table below.

Relativity	Percentage of Exposures	Mean (Extension)	Deviation From Mean	Deviation Squared	Variance (Extension)
.5	.10	.05	1.0	1.00	.1000
1.0	.40	.40	0.5	0.25	.1000
1.5	.30	.45	0.0	0.00	.0000
2.0	.10	.20	0.5	0.25	.0250
3.0	.05	.15	1.5	2.25	.1125
5.0	.05	.25	3.5	12.25	.6125
Total	1.00	1.50			.9500

The coefficient of variation is the standard deviation (.975) divided by the mean (1.5), or 0.65. This numerical example points out several truisms. First, high efficiencies necessarily require extreme rates. Almost two-thirds of the variance is due to the highest cost 5% of insureds. Second, the key to designing highly efficient systems is to find variables that can isolate the highest and lowest cost individuals. Many insured populations seem to have a coefficient of variation of about 1.0.[7] If this is true for the numerical example above, the efficiency would be about 42% ($.65^2$).

The basic difficulty in computing efficiency is determining the variability of the insured population. Because costs depend upon fortuitous events, the variability is unknowable. It is possible to apply concepts of risk theory, however, to develop some plausible estimates.

The basic types of variability that should be considered are:

- Inherent variability in expected accident frequency,
- Inherent variability in expected claim size,
- Variability in frequency and claim size for an individual insured over time, and
- Variability in the actual frequency and claim size, given the expected values.

The list could go on, but it already contains enough substance to challenge the mathematically sophisticated. Few practical applications have involved variability in claim sizes. Most published research includes only expected and actual claim frequency. Woll (1979) has mentioned changing individual

[7]See SRI (1976).

frequency over time. (Woll refers to this as the individual's exposure to loss, which he treats as a stochastic process.)

The underlying variability will be measured from actual claim experience. Any such measurement, of course, will only be accurate if the actual data derives from a suitable situation. It is subject to random fluctuations, since the actual data is the result of random processes.

To provide a framework for the measurement, Woll (1979) defines X as the actual number of claims; M, as the distribution of expected frequency for the individual insureds; and P, as the distribution of the individual insured's frequency over time. He derives the following formula:

$$\mathrm{Var}(X) = \mathrm{Var}(M) + E(M) + E(\mathrm{Var}(P))$$

What is required is Var(M), the underlying variability in expected claim frequency. Woll gives four formulas for calculating Var(M). These are illustrated in Exhibit I. In that Exhibit, 1000 drivers are observed over two periods. They are categorized by the number of claims in the first and second periods. The first formula was used by SRI and assumes no variation in loss costs over time. The second uses the difference in frequency between insureds with zero and one prior accidents. The third multiplies the claim-free discount by the variance of the observed frequency. The fourth is due to Woll. $\alpha(j)$ is the claim frequency for insureds with j prior claims. r_j is the percentage of insureds with j claims. Note that $E(M) = E(X)$.

$$
\begin{aligned}
(1)\quad Var(M) &= Var(X) - E(X) \\
&= .1179 - .11 \\
&= .0079 \\
CV(M) &= .808
\end{aligned}
$$

$$
\begin{aligned}
(2)\quad Var(M) &= [E(M)]^2 \; \frac{\alpha(1) - \alpha(0)}{\alpha(0)} \\
&= .11^2 \; \frac{(.1333) - (.1044)}{.1044} \\
&= .00335 \\
CV(M) &= .526
\end{aligned}
$$

$$
\begin{aligned}
(3)\quad Var(M) &= \left[1 - \frac{\alpha(0)}{E(X)}\right] Var(X) \\
&= \left[1 - \frac{.1044}{.11}\right] .1179 \\
&= .0060 \\
CV(M) &= .704
\end{aligned}
$$

$$
\begin{aligned}
(4)\quad Var(M) &= \sum_{j=0} j \; r_j \; \alpha(j) - E(X)^2 \\
&= 1(.09)\,(.1333) + 2(.01)\,(.4000) - .11^2 \\
&= .0079 \\
CV(M) &= .808
\end{aligned}
$$

Other formulas are certainly available. One clear message from this example is that empirical data may not provide a suitable estimate of efficiency.

Measures of efficiency, even if they can be calculated with accuracy and consistency, do not provide a complete answer. The cost of the classification process itself is ignored, for example. The availability of a feasible, more accurate system is unknown. Efficiency may be low in any given case, but no better system may be available at a reasonable cost.

What are the implications of efficiency measures for the design of classification systems? To produce a higher efficiency there must be higher percentage of insureds at more extreme relativities. This is necessary to produce a higher variance or CV. This process, however, runs counter to much of the current criticism. Higher rates mean less affordability. In addition, greater efficiency can be produced by any variable that can accurately refine the classification system. Thus, the preference for causal variables is irrelevant to increased efficiency; correlated variables can be just as efficient if they can distinguish cost potential. Similarly, controllable variables are useless unless they can produce greater efficiency. Indeed, controllability and causality are irrelevant; what is important to efficiency is being correlated with costs.

Risk classification efficiency can be approached from another point of view. Insurers have economic incentives to accurately classify insureds. The classification system should be as good as the market allows. In other words, if a group is too small to have credible experience or poses too great a risk (in that there is too much variability in costs within the group), the group may not be very accurately rated. If the group is large and relatively homogeneous, insurers have an incentive to properly classify and rate it.

In summary, the importance of classification efficiency may be overrated. Existing efficiency measures are a comparison to an abstract ideal, that probably has little relevance to practical situations. They do not provide useful information about what practical, cost-effective variables might be utilized. In addition, the market will probably dictate how refined classification systems will be. The more competitive the market, the more refined the classification system may be.

Number of Drivers with X_i Claims

Second Period Count	First Period Count 0	1	2	Total	Count	Frequency
0	814	79	7	900	93	.1033
1	78	10	2	90	14	.1556
2	8	1	1	10	3	.3000
Total Drivers	900	90	10	1000	110	.1100
Claim Count	94	12	4	110		
(1) Frequency α(j)	.1044	.1333	.4000	.1100 = E(X) = E(M)		
(2) Deviation	-.11	.89	1.89			
Deviation Squared	.0121	.7921	3.5721			
(3) Variance (Extension)	.0109	.0713	.0357	.1179 = Var(X)		

Notes:

(1) Count/drivers.
(2) Count - overall frequency (.11).
(3) Sum over first period counts (percentage of drivers times deviatic squared).

VI. ESTIMATING CLASS RELATIVITIES

This final section will discuss several actuarial problems involved in estimating classification relativities. These include: 1) whether relativities should be additive or multiplicative, 2) how to estimate multiple sets of relativities, 3) how to obtain more or more reliable data, and 4) how to select the appropriate credibility complement for groups with less than fully credible data. This topic is also discussed in Chapter 2.

Relativities are usually calculated for classification variables, rather than pure rates for each class, because they are used in different contexts. For example, relativities between classes are likely to be the same from state to state, even though the absolute value of the rate may be quite different. For example, state A may have double the medical malpractice costs of state B. The relativity in costs between general surgeons and general practice (with no surgery), however, may be about 3 to 1 in both states.

The two most common mathematical constructs for relativities are additive and multiplicative factors. For two-dimensional structures, there would be no practical difference in results. For three sets of factors, however, there may be significant differences. For example, in automobile liability the first dimension is combinations of age-gender-marital status. The second dimension is use, such as pleasure-farm and all other. A third dimension could be good student-driver training discounts.

Philosophically, are the third level differentials additive (i.e., a function of the base rates for a given territory) or multiplicative (i.e., a function of the specific age-gender-marital status and use differentials)? For example, is a good student discount worth 20% of the base (i.e., adult) rate (additive) or 10% of the actual rate (multiplicative)? The actual rate may be 360% of the base for a 17 year-old male principal operator (multiplicative good student discount equals 36% of base rate) or 150% for a 20 year-old female (multiplicative good student discount equals 15% of base rate). Does "good student" status reduce costs equally for all insureds (additive) or does it affect costs proportionally (multiplicative)?

Whether a variable should be additive or multiplicative is difficult to determine; the type of variable is important. Most often variables are treated as multiplicative. This makes the relativities somewhat easier to calculate and analyze.

Regardless of which form is chosen for the relativities, estimation is not necessarily straightforward. Certain subdivisions of a rating variable may have a disproportionate share of another rating variable; that is, two rating variables may be highly correlated with each other. For example, assume group A costs twice group B and group X costs twice group Y. Also assume that AX occurs 40% of the time, AY, 10%, BX 10%, and BY, 40%. See Exhibit II.

The empirical cost for X is 3.6, and for Y, 1.2. Thus the empirical relativity is 3.0, when we know the actual cost is only double. This has happened because a disproportionate amount of exposure is concentrated in higher and lower cost groups. In determining the relative cost of X and Y, one may expect half of the exposure to be in group A and half in group B. Instead, 80% of X's exposure is in high-cost group A and 80% of Y's exposure is in low-cost group B. Thus X looks relatively higher in cost than it actually is.

Various methods can be used to adjust for unequal distributions of underlying exposures. Bailey (1963) provides a method for doing this. Premiums at present rates can be calculated for each cell using current rate relativities. The comparable number of base class exposures also be calculated for each cell. For example, if A is priced at three times B, the base class, each class A exposure is multiplied by three.

Another estimation problem concerns the credibility of the data. Since competition encourages insurers to refine their classification systems, refinement will generally continue to the point where the credibility of the data becomes minimal.

In the context of classification, credibility involves the assessment of the relative meaningfulness of a group's cost versus the meaningfulness of the credibility complement's cost. Assume for example, that the task is to estimate the cost of group A. If group A has a large body of data, that experience alone may be sufficient for estimating its cost. As group A becomes smaller, at some point it will be useful to compare group A's empirical costs to the cost of some other group. This other group is the credibility complement. Group A's empirical cost may be twice the cost of the complement. Since group A has less data or less reliable data, the actuary may decide that group A's true cost is only 60% higher than the complement.

Thus two credibility related problems emerge: (1) how to obtain more data or more reliable data, and (2) what is the most appropriate credibility complement? Each of these matters can be discussed at length. The purpose here is to provide an overview.

Obtaining more or more reliable data can be done in several ways. Most obviously, more years of data or, possibly, data from several states (or countrywide) can be used. Of course, the threshold question is whether the broader base actually applies. Has there been a change over time? Do countrywide indications apply in each state?

Another method is to give more weight to more stable phenomena. For example, relativities can be based primarily on frequency (by looking only at claim counts or by limiting the size of claims), instead of pure premiums. Partial pure premiums can be calculated. For example, property damage liability costs may be more stable than bodily injury liability; workers' compensation medical costs may be more stable than deaths or permanent disabilities. In determining relativities, more emphasis (credibility) is given to the more stable phenomena.

The choice of credibility complement may be more difficult than obtaining more or more reliable data. It may not be clear which group is most nearly the same as the group in question. National or regional data may be applicable. Related industry group data may be applicable. In most of these cases, adjustments must be made because the level of costs can be quite different for the complement. Often, the percentage change in the complement is considered, rather then the actual value. As a last resort, the complement may be based on the prior year's analysis; this, in effect, takes more years of data into account.

EXHIBIT II

I. Exposure

	A	B	Total
X	40	10	50
Y	10	40	50
Total	50	50	100

II. Pure Premium

	A	B
X	4	2
Y	2	1

III. Costs

	A	B	Total	Exposures	Relativity
X	160	20	180	50	3.0
Y	20	40	60	50	1.0
Total	180	60	240	100	
Exposures	50	50			
Relativity	3.0	1.0			

Exhibit III illustrates some of the credibility issues. The problem is choosing rate relativities for a group of surgical specialties. At the current time, all specialties shown on Exhibit III are being charged 8.4 times the base. Data is grouped for various combinations of accident years (all groups ending with 1988). Relativities to the base are shown for claim frequency, severity, and pure premium. The severity relativity for all surgery classifications is about 1.25.

The frequencies seem to be different for the different groups, although groups B and C could possibly have the same frequency. The severities are much different for group C, although the number of claims is relatively small (17 for the 10-year period).

Selected relativities were based on judgment rather than a formal credibility formula. Essentially, claim frequency was given high credibility. The overall severity for surgeons (1.25) was used for groups A and B, although actual data is not much different. The severity for group C reflects a small upward adjustment to the overall surgeons' relativity (about 15% credibility). The selected pure premium relativities were rounded.

EXHIBIT III

CLASS RATING EXAMPLE

Current Relativity = 8.4

Rating Group		Years	Exposures	Relativites to Group 1 Frequency	Severity	Pure Prem
I.	Raw Data					
	A	79-88	420	4.2	1.15	4.9
		81-88	340	4.6	1.18	5.4
		84-88	193	4.6	1.10	5.1
		86-88	93	4.7	1.36	6.3
	B	79-88	846	5.1	1.16	5.9
		81-88	635	5.6	1.22	6.9
		84-88	304	5.2	1.07	5.6
		86-88	147	6.0	1.26	7.6
	C	79-88	293	5.9	1.93	11.3
		81-88	233	6.1	1.98	12.1
		84-88	133	4.8	1.72	8.3
		86-88	69	4.5	1.69	7.6
II.	Conclusions					
	A			4.8	1.25	6.0
	B			5.6	1.25	7.0
	C			6.0	1.33	8.0

SUMMARY

Risk classification involves the formulation of different premiums for the same coverage based on group characteristics. That is, the task is to price an individual insured, but the available claim data for that individual is insufficient for the purpose. The recourse is to measure group costs and assume that the individual belongs to a certain group.

Premiums should vary because underlying costs vary. Costs may vary due to different claim frequency or average claim size, different administrative expense requirements, different investment income potential, or differing assessments of risk. Risk classification proceeds by identifying variables that distinguish these costs among different insureds. In addition to classification variables, premiums can also vary due to the choice of different exposure bases, individual risk rating methods, and marketing or underwriting strategies.

Various criteria, actuarial, operational, social, and legal, have been suggested for formulating classification variables. Actuarial criteria attempt to most accurately group individual insureds into groups that, (1) are relatively homogeneous (2) are sufficiently large to estimate relative cost differences (credibility) (3) have

different mean costs (separation) and (4) maintain different mean costs over time (reliability).

Operational criteria include objective definitions, reasonable administrative expense, and verifiability. Social criteria include privacy, causality, controllability, and affordability.

A competitive market tends to produce more refined classifications and accurate premiums. Competition may be limited, however, when the premium volume for a group is small or where there is significant heterogeneity in costs within the group. Most of the social criteria are based on concepts of accuracy. The abolition of certain rating variables, which is seen as desirable by various insurance industry critics, likely will reduce rating accuracy, as well as creating subsidies or availability problems. The inadequacy in the current rating systems is primarily determined by the level of competition and the statistical difficulty of rating small groups of insureds.

The absolute efficiency of current classification systems can be estimated, but the estimates depend upon some measurement of the variability in costs among all insureds (which can never be observed directly). Knowing the absolute efficiency, however, is not particularly useful in determining whether more and better rating variables are available.

Chapter 4
<u>Risk Classification</u>

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INDIVIDUAL RISK RATING
(TEXTBOOK CHAPTER DRAFT)

Chapter 2 - Individual Risk Ratemaking
Margaret Wilkinson Tiller **September 5, 1988**

I. INTRODUCTION

Manual ratemaking determines what rates should be charged groups of entities for specified coverage and entity characteristics. Individual risk ratemaking works within the rating groups to modify the group rates in whole or in part to reflect an individual entity's experience.

If all entities in all rating groups were truly homogeneous, fluctuations in experience would be fortuitous. While this is the goal of manual ratemaking, it is not usually possible to achieve. In addition, some entities are so large that their experience is, to some extent, "credible." The combination of non-homogeneous rating groups and entities with credible experience indicates that individual risk ratemaking is appropriate.

A. Goals of Individual Risk Ratemaking

For an insurer, the primary goal of individual risk ratemaking is to more accurately price the coverage provided than if rates were based only on manual rates. Non-traditional risk financing mechanisms may also call for individual risk ratemaking. For groups of entities, such as pools or risk retention groups, the primary goals of individual risk ratemaking (sometimes referred to as cost allocation) are to more accurately allocate costs to participants and to motivate participation in risk control programs. These are also the goals of individual risk ratemaking for individual entities retaining ("self-insuring") all or part of their risks and allocating the associated costs to departments or other units. Individual entities purchasing insurance may similarly wish to allocate the insurance costs to their departments or other units. For individual entities in either situation, the units to which the costs are being allocated take the role of participants or "insureds." Some entities may participate in individual risk ratemaking systems as both allocator and allocatee.

Motivation of participation in risk control programs is a secondary goal of insurers using individual risk ratemaking. Other goals of insurers and other entities using individual risk ratemaking are to appropriately balance risk sharing and risk bearing and to provide information to design or modify risk control programs. For individual entities, the allocation of costs to units allows for more accurate pricing of products and services.

B. Attributes of Good Individual Risk Ratemaking Systems

Good individual risk ratemaking systems have the following attributes:

- serve the needs of the organization using them,
- appropriately balance risk sharing and risk bearing,
- are not subject to internal or external manipulation,
- are simple to administer,
- are easy to understand, and
- do not subject the entities subject to them to large fluctuations in costs from one year to the next due to fortuitous experience.

Some of the attributes listed above are sufficient but not necessary to insure that other attributes listed above are met. As practical considerations may override one or more of these attributes, all are listed.

Prior to designing any individual risk ratemaking system, the organization using it should determine its needs for the system. These needs may simply be the goals listed above. Or the entity may have different needs. For example, a public entity wishing to allocate the costs of its general liability insurance back to its various departments may wish to allocate half the cost to its public utility which can recover costs in its rates.

An individual risk ratemaking system should appropriately balance risk sharing and risk bearing. The costs for small entities whose experience is not at all credible should be determined solely based on risk sharing. Large entities whose experience is completely credible should have their costs solely based on risk bearing. Entities between these extremes should have their costs based on a weighting of risk sharing and risk bearing.

Individual risk ratemaking systems should not be subject to internal or external manipulation. Manipulation is internal if the entity to which costs are being allocated can influence the cost allocation. An example is when the entity to which costs are being allocated sets the case reserves used in the individual risk ratemaking calculation. Manipulation is external if some agency other than the entity to which costs are being allocated can influence the cost allocation. An example is when a marketing manager can override the pricing results of the individual risk ratemaking calculation.

As a practical consideration, individual risk ratemaking systems should be simple to administer. If one is very complicated to administer, it may not be used. A system which is simple to administer is also more likely to be easy to understand. Understanding is important particularly in those situations in which participation in risk control programs is one of the goals: the easier a system is to understand, the better will be the motivation, assuming the system is appropriately designed.

A good individual risk ratemaking system does not subject the entities subject to it to large fluctuations in costs from one year to the next due to fortuitous experience. An individual risk ratemaking system should reflect an entity's experience only to the extent that it is credible. Fortuitous experience is not credible because it is the result of chance alone. An individual risk ratemaking system that reasonably balances risk sharing and risk bearing usually has this attribute of moderating the impact of fortuitous cost fluctuations. However, a system could have this attribute without reasonably balancing risk sharing and risk bearing.

C. Overview of Individual Risk Ratemaking

There are two basic types of individual risk ratemaking systems: prospective and retrospective. Prospective systems use past experience to determine costs for the future. Retrospective systems use experience to determine the final costs for the experience (past) period.

Retrospective systems are more responsive to experience changes than prospective changes. This is an advantage when a primary goal is to motivate participation in risk control programs. This responsiveness also means that retrospective systems result in less stable costs from one time period to the next than do prospective systems.

While different systems use different formulae, all individual risk ratemaking systems weight experience and exposure. The weight assigned to the experience portion is a reflection of the credibility (degree of belief) that the entity's experience is valid.

There are practical considerations that impact individual risk ratemaking systems. These include such items as using alternative exposure measures and data to those desired if those desired are not readily available. If one of the goals is to motivate participation in risk control programs and the results of the experience rating calculation do not make a material difference to the entity to which costs are being allocated, there will probably be no such motivation.

For individual entities allocating risk financing costs to units, different tax rates and systems, the ability of units to purchase their own insurance, and whether and how unit managers get the benefits or penalties of the costs allocated to their units all impact how effectively an individual risk ratemaking system will meet its goals.

D. Terminology

The insurance industry is notorious for using words in different ways, even within the same company. It is important to understand in every situation how terms are used as different usage could produce different results. Below are discussed some basic terminology used in this chapter.

1. Claims and Occurrences

A claim is a demand by an individual or other entity to recover for loss. An occurrence is a series of incidents happening over a period of time that collectively results in personal injury or property damage. Note that one occurrence may have multiple claims associated with it. An example is an automobile accident in which several people, each of whom files a claim, are injured.

"Claim" is often used when "occurrence" is meant. Additionally, some entities count the different components of a claim as separate claims. For example, a general liability claim with both bodily injury and property damage may be counted as two claims.

"Claim" is often used also to refer to "losses." "Claim" and "occurrence" are zero/one words: they indicate either presence or absence and not amount.

Many individual risk ratemaking systems limit the losses used in the experience portion of the calculation. These limits are usually applied to each occurrence. Some formulae for the credibility used in the individual risk ratemaking calculation rely on number of occurrences.

2. Losses, ALAE, and ULAE

"Losses" refers to the amount associated with a claim. This is the amount a claim is worth, not the request for payment. For liability, losses includes bodily injury, property damage, and personal damage. For workers' compensation, losses includes medical and indemnity.

Allocated loss adjustment expenses (ALAE) are attorneys' fees, investigative fees, etc., associated with settling a particular claim. Unallocated loss adjustment expenses (ULAE) are expenses associated with settling claims which can not be allocated to settlement of a particular claim.

For an insurer, ALAE are usually the cost of outside legal counsel and investigators and ULAE are usually the costs of the claim department, including office space, salaries and benefits, supplies, etc. However, some insurers use no outside resources to settle claims (and have no ALAE) while other insurers keep time and expense records for the claim department and charge the costs to claims as ALAE (and have no ULAE). Similar situations can occur with non-traditional risk financing mechanisms.

"Losses" sometimes refers to losses only and sometimes to losses and ALAE. Different individual risk ratemaking systems treat losses, ALAE, and ULAE differently. And the same system used by different entities may produce different results if ALAE and ULAE are defined differently.

3. Time Periods

An accident period is the period during which an occurrence occurs, regardless of when any policies covering it are written or when the occurrence is reported or paid. A policy period is the period during which an occurrence occurs for policies written during a specified time, regardless of when the claim is reported or paid.

Exhibit I illustrates the accident period and policy period concepts for policies written to cover accidents occurring during the policy period (referred to as an occurrence basis policy). The accident years are represented by vertical lines; the policy years by 45 degree lines. Note that accident year 1981 contains accidents (occurrences) associated with policies written in 1980 and 1981, and policy year 1981 contains some accidents that occur in 1981 and some that occur in 1982. A policy written December 31, 1981 will have almost all accidents associated with it occurring in 1982. For any one entity with an occurrence basis policy, the policy period is the same as the accident period.

Not all policies are written to cover accidents occurring during the policy period. Two other options are "claims-made" and "claims-paid." Claims-made policies cover claims (or occurrences) reported during the policy period, regardless of when claim occur or are paid if they occur after the retroactive date. Claims-paid policies cover the claims (or occurrences) paid during the policy period, regardless of when claims occur or are reported if they occur and are reported after the appropriate retroactive dates. Retroactive dates are used to prevent duplicate coverage in converting from occurrence basis policies to claims-made or claims-paid. If an entity changes from occurrence basis to claims-made to claims-paid, two retroactive dates, one for occurrences and one for reporting, would be necessary.

There are many coverage questions that arise with claims-made and claims-paid policies because of poor coverage wording. The two main questions are:

- Is the coverage for claims or occurrences?
- How is the report (or payment) date defined?

Non-traditional risk financing mechanisms also have these time period concepts. For example, individual entities retaining risk may decide to fund during each fiscal year only for those occurrences reported during the fiscal year.

The time period concepts are important in individual risk ratemaking because the first step in designing or understanding such a system is to know what costs are involved. This depends on the coverage provided or the funding basis, which is a function of the time period under consideration.

4. Loss Components

Paid losses are losses that have been paid. Outstanding losses, or case reserves, are estimates by the claim examiner of the remaining amount required to settle particular claims based on the knowledge about those claims at a particular date. Case reserving is an art, not a science, so different examiners may set reserves on the same claim at different amounts.

Case reserves, when added to the payments on open claims, do not and are not supposed to reflect the ultimate settlement amount. Case reserves are based on knowledge at a particular point in time. In general, additional information about a claim tends to be worse, rather than better. This means that there is usually an upward development of the payments on open claims plus case reserves on a given group of claims. It is possible to have downward development, but this is very unusual. The difference between the current total of payments on open claims plus case reserves and the ultimate settlement value for a given group of claims is called "case reserve development." Note that occurrence basis and claims-made coverage will need a reserve to reflect case reserve development.

Occurrence basis coverage will also need a reserve to reflect unreported occurrences. Claims-made coverage will need a reserve to reflect unreported claims if coverage is provided for occurrences reported during a particular period since not all claims associated with an occurrence are reported at the same time. The unreported occurrences/claims reserve is the true "IBNR" (incurred but not reported) reserve.

"Reported losses" refers to the sum of payments plus case reserves. "Unreported losses" refers to the case reserve development plus unreported occurrences/claims reserve. "Incurred losses" refers to the sum of reported and unreported losses. Note that unreported losses and incurred losses contain different items for occurrence and claims-made coverage and may contain different items for different types of claims-made coverage.

"Case reserves" are sometimes used when "reported losses" are meant. Many entities refer to "reported losses" as "incurred losses" and to "unreported losses" and "IBNR." The result is confusion, with incurred losses plus incurred but not reported losses equaling incurred losses.

Exhibit 2 illustrates the loss component terminology used in this chapter. These terms also apply to ALAE. Sometimes losses and ALAE are treated together, sometimes separately, and sometimes as a mixture. An example of the last is treating paid losses and paid ALAE separately, but setting case reserves for losses and ALAE combined or for losses only.

To properly design, understand, or use an individual risk ratemaking system, loss component terminology for the system and the data available to be used by the system must be clarified.

5. Frequency and Severity

Two other terms that have different usage in different situations that arise in conjunction with individual risk ratemaking systems are "frequency" and "severity." Frequency is the number of claims (or occurrences) per exposure unit. "Frequency" is sometimes incorrectly used to refer to the number of claims or occurrences. Frequency is a relative, not an absolute, measure.

Severity is the average loss per claim (or per occurrence). Note that loss may include or exclude ALAE.

E. What is to be Allocated

The first task in designing or understanding an individual risk ratemaking system is to determine what is to be allocated. For traditional insurance, the answer often is all costs. These include losses, ALAE, ULAE, reinsurance premium, risk control costs, overhead, taxes, miscellaneous expenses, and profit associated with insurance policies of the type being written (e.g., occurrence).

Non-traditional risk financing mechanisms and individual entities allocating risk financing costs back to units also may want to allocate all costs associated with insurance policies of the type being written. Those costs may include different items, such as excess insurance premium and a risk margin (money for adverse loss and ALAE experience), and exclude others, such as taxes and profit. Non-traditional risk financing mechanisms and individual entities allocating costs back to units and even some insurers may want to allocate only some subset of costs, such as losses, ALAE, and ULAE, with other costs treated in a different manner.

Note that part of the determination of what is to be allocated involves determining the basis on which policies are written or funding occurs. This is necessary so that the various components subject to the allocation are appropriately tabulated and adjusted.

II. PROSPECTIVE SYSTEMS

There are three basic types of prospective individual risk ratemaking systems: schedule rating, experience rating, and some types of composite rating. Schedule rating takes into consideration characteristics that should impact loss and ALAE experience but that are not reflected in that experience. Experience rating uses an entity's actual experience to modify manual rates (determined by the entity's rating group). Composite rating simplifies the premium calculation for large, complex entities and, in some instances, allows the entities' experience to impact the manual rates or determine the rates regardless of rating group.

A. Schedule Rating

Schedule rating is the only individual risk ratemaking system that does not directly reflect an entity's experience: it recognizes characteristics that should impact an entity's experience but that are not actually reflected in that experience. These characteristics could result from recent changes in exposure or risk control programs.

Schedule rating systems usually take the form of percentage credits and debits. These credits and debits are sometimes applied before and sometimes after experience rating. There may be a limit to the total debit or credit that an entity can receive.

Note that schedule credits and debits apply only to those characteristics which should impact an entity's loss and ALAE experience. If a characteristic is listed which should not impact a particular entity's loss and ALAE experience, there should be no adjustment to the manual rates for that characteristic for that entity.

Also note that the application of schedule credits and debits may take considerable underwriting judgment. A schedule rating system that is based on objective criteria will result in more consistent treatment of entities subject to it than a system that relies on subjective evaluation. This is illustrated by the two examples of schedule rating that follow.

1. Insurance Services Office General Liability Schedule Rating Plan

For eligible entities, the manual rates may be modified according to the table below in addition to any experience rating modification. The maximum schedule rating modification is 25% up or down.

ISO General Liability Schedule Rating Table

A.	Location			
	(i) Exposure Inside Premises	-5%	to	+5%
	(ii) Exposure Outside Premises	-5%	to	+5%
B.	Premises - Condition, Care	-10%	to	+10%
C.	Equipment - Type, Condition, Care	-10%	to	+10%
D.	Classification Peculiarities	-10%	to	+10%
E.	Employees - Selection, Training Supervision, Experience	-6%	to	+6%
F.	Cooperation			
	(i) Medical Facilities	-2%	to	+2%
	(ii) Safety Program	-2%	to	+2%

This plan relies heavily on subjective evaluation. For example:

- What is it about the condition and care of the premises that results in a credit of 10%, 9%, etc?

- o Will different underwriters give identical schedule credits and debits in identical situations?

- o Will the same underwriter give identical schedule credits and debits in identical situations?

2. Roller Skating Rink Risk Retention Group Schedule Rating Plan

This schedule rating plan is similar to one developed for a roller skating rink risk retention group offering general liability coverage. All participating entities are eligible. There is no maximum schedule credit other than the one inherent in the plan (40%). Note that only credits are given. The manual rates assume none of the characteristics in the schedule rating plan exist.

The general credit list is as follows:

A.	Floor supervision	+10%
B.	Premises	+5%
C.	Rental Skates	+5%
D.	Management	+5%
E.	Incident Report	+10%
F.	First Aid	+5%
	Total	+40%

Details of the floor supervision credit follow:

Rink must meet or exceed industry safety standard of one floor supervisor per 200 skaters at all times.

Rink has a written policy or procedure which includes:

- o a distinctive uniform or vest for floor supervisors,
- o floor supervisors must be paid employees, owners, or family members of owners,
- o floor supervisors must be at least 18 years of age, and

- a written training program for floor supervisors.

The floor supervisor training program must include the following items at a minimum:

- Floor guards should inspect the floor for foreign objects at all times.
- During special numbers or events, floor guards should keep unqualified skaters off the floor.
- Floor guards should have written policy regarding unruly skaters.
- Floor guards should have explicit, written instructions in case of an accident including:
  - not moving the injured skater,
  - diverting skaters from the injured skater,
  - notifying management of an incident, and
  - procedure for contacting emergency medical/police/fire assistance.

Floor supervisor training must include a minimum of one safety meeting per calendar quarter.

Floor supervisor training must be recorded and verified by the employee.

ALL OF THE ABOVE MUST BE PRESENT TO EARN THE 10% CREDIT. NO PARTIAL CREDIT WILL BE GIVEN.

The other credits similarly rely on objective criteria that can be verified by audit and/or surprise inspections. All credits encourage activities which should favorably impact loss and ALAE experience. Note that credit is given for activities which a rink has just begun, regardless of its actions in the past.

B. Experience Rating

All individual risk ratemaking systems are a form of experience rating. However, the term "experience rating" has come to mean a particular type of prospective system, which is discussed in this section.

Experience rating assumes that the past, with appropriate adjustments, is predictive of the future. Actual losses, and sometimes ALAE, for a prior period are compared to expected losses (and ALAE). To have an "apples to apples" comparison, several different experience and exposure base combinations can be used, including the following:

- actual paid losses (and ALAE) at a particular date and the expected paid losses (and ALAE) at that date both for the experience period,
- reported losses (and ALAE) at a particular date and the expected reported losses (and ALAE) at that date both for the experience period,
- projected ultimate losses (and ALAE) and expected losses both for the experience period, and
- projected ultimate losses (and ALAE) for the experience period adjusted to the current exposure and dollar levels and expected losses at the current dollar and exposure levels.

Projected ultimate losses are the expected ultimate settlement value of all subject claims/occurrences. Projected ultimate ALAE are the expected ultimate ALAE costs of all subject claims/occurrences. The expected losses (and ALAE) are based on past or current exposure, as appropriate. A weighting of the actual and expected components results in the cost to the entity which is the subject of the system for the current period.

The three components of experience, exposure, and credibility (the weighting factor) and some additional considerations are discussed below.

1. Experience

The experience base should be related to the exposure base, as detailed above, and the basis on which policies are written or funding occurs. If the policy to be rated is written on an occurrence basis, any of the four options listed above for accidents occurring in the experience period could be used. If the policy to be rated is written on a claims-paid basis, the paid losses option or the projected ultimate losses option adjusted to current exposure and dollar levels, both for payments made during the experience period, are the options of choice. If the costs to be allocated include ALAE, ALAE should be included with losses in the calculation.

The length of the experience rating period usually ranges from two to five years. The shorter the period, the more responsive the plan will be to changes that truly impact loss (and ALAE) experience, such as changes in the risk control program, and the more subject to fortuitous fluctuations in loss (and ALAE) experience. Conversely, a longer period will result in less responsiveness to changes and to fortuitous occurrences.

To reduce the impact of fortuitous occurrences, many experience rating plans place per occurrence limits on the losses (and ALAE) used in the experience rating calculation. These limits sometimes apply to losses only, with ALAE unlimited or treated in a different manner, and sometimes to losses and ALAE combined. Note that if actual losses (and ALAE) are limited, the expected losses (and ALAE) must be also to maintain an "apples to apples" comparison. If losses (and ALAE) are limited, the cost of expected losses (and ALAE) above the per occurrence limit must be collected through a different part of the experience rating formula than the weighting of experience and exposure. Annual or other period aggregate limits may also be used.

If projected ultimate losses are to be used in the experience rating calculation, they can be developed in a number of ways similar to those used to develop projected ultimate losses used to determine manual rates. Projected ultimate losses are often based on paid or reported losses at a particular date.

For the last experience option listed above, projected ultimate losses are adjusted to current exposure and dollar levels. Dollar level adjustments should include both economic and social inflation. The latter category includes such items as changes in the legal atmosphere and law changes.

Exposure adjustments include both converting the experience period to the current period (e.g., dividing by three to go from a three-year experience period to a one-year current period) and adjusting for changes in the magnitude of the exposure. Both can be accomplished at once by dividing the projected ultimate losses for the experience period, adjusted to current dollar level, by the exposure for the experience period, adjusted to current dollar level if appropriate, and applying this "rate" to the exposure for the current period.

2. Exposure

The expected losses are a function of the past or current exposure base, as appropriate. The exposure base should be related to the experience base, as detailed above. For the first three options listed above, the past exposure base is used; for the last option, the current exposure base is used.

Expected losses are usually a product of an expected loss rate and the exposure. The expected loss rate can be based on the manual rates for the prior or current period, adjusted to the appropriate dollar level. For example, to develop expected loss rates for a prior period, the current expected loss rate could be adjusted to the prior period's dollar level, or the prior period's expected loss rates could be used directly. The former approach is usually better if there have been no underlying changes in the nature of the exposure because the current expected loss rate is based on more recent information than the prior period's loss rates.

The exposure measure used should reflect the underlying risk of loss and ALAE. It is not always possible to use the theoretically optimal exposure measure. In practice, insurers and non-traditional risk financing mechanisms often use whatever exposure measure insurers use in their premium calculations.

For general liability, exposure measures often used are sales, payroll, total operating expenditures, and square footage. For workers' compensation the exposure measure is usually payroll adjusted for differences in payroll type (e.g., a coal miner is expected to have more losses and ALAE per payroll dollar than a secretary, even though both are employed by the same entity). For property, exposure measures often used include actual cash value, stated amount, or replacement cost.

Non-traditional risk financing mechanisms may use different exposure measures for different costs. For example, for a public entity workers' compensation pool the exposure base for all administrative costs may be full-time-equivalent employees while the exposure base for losses and ALAE is payroll, with both full-time-equivalent employees and payroll adjusted for differences in payroll type. The use of two exposure measures may be the result of different payroll scales being used by different participants.

Individual entities allocating risk financing costs to units may also use different exposure measures for different costs. And some costs, such as the cost of a policy that applies only to one unit, may be allocated without using the experience rating plan.

3. Credibility

The actual (experience) and expected (exposure) components as of the experience rating calculation are weighted to produce the costs the entity under consideration will pay. The weight assigned to the experience component is called "credibility." The weight assigned to the exposure component is one (1.000) minus the credibility.

Credibility reflects the degree of belief that the entity's experience is valid. Credibility has three criteria that must be met:

1. Credibility should not be less than zero or greater than one.

2. Credibility should increase as the size of risk increases.

3. The percentage charge for any loss of a given size should decrease as the size of risk increases.

These criteria can also be shown as mathematical relationships:

Z - credibility

E - size of risk

1. $0 \leq Z \leq 1.$

2. $$\frac{dZ}{dE} > 0.$$

3. $$\frac{d}{dE}\left[\frac{Z}{E}\right] < 0.$$

These three criteria are met if credibility follows the curve shown in Exhibit 3. Note that size of risk is represented in the diagram by exposure. Size of risk can also be based on expected losses or expected number of claims. Chapter 6 contains a detailed discussion of credibility.

4. Other Considerations

Experience rating plans may be designed so that there is a minimum or maximum premium charge. These are often based on the prior year's premium adjusted for changes in exposure. For example, the maximum premium change from one year to the next may be the change indicated by any exposure changes plus or minus 25%. This means that if there is an increase of 15% because of an increase in exposure, the total increase possible after application of the experience rating plan is 40%.

The premium collected under experience rating plans may not equal the expected premium in total. This means that the plan is "off-balance." If this can be anticipated, the experience rating plan can include as a last step, multiplication by a factor to correct for this off-balance. Alternatively, the manual rates can include an off-balance correction.

5. Insurance Services Office General Liability Experience Rating Plan

The Insurance Services Office general liability experience rating plan is illustrated in Exhibits 4 and 5. This example is used throughout the following discussion of the plan.

This plan may be used for occurrence and claims-made general liability coverages, with a few exceptions, for those entities meeting the eligibility criteria specified in the plan. The coverage in the example is premises/operations and products/completed operations for policy period 1/1-12/31/88 written on a third-year claims-made basis.

The experience basis is projected ultimate losses and ALAE for the experience period. The exposure basis is expected losses and ALAE for the experience period. Both the projected ultimate losses and ALAE and expected losses and ALAE are limited to basic limits, which applies to losses only, and by a maximum single limit per occurrence (MSL), which applies to the basic limits losses and unlimited ALAE.

The experience period is the three policy periods completed at least six months prior to the policy effective date for the calculation being performed. If three policy periods are not available, one or two may be used. Occurrences associated with tail coverage on claims-made policies are excluded. In the example, the three policy periods are 1/1-12/31/84, 1/1-12/31/85, and 1/1-12/31/86. The older two were written on an occurrence basis; the most recent on a first-year claims-made basis. The evaluation date is 9/30/87.

The projected ultimate losses and ALAE limited by basic limits and MSL for the experience period are the sum of the reported losses and ALAE at 9/30/87 and the expected unreported losses and ALAE at 9/30/87, both limited by basic limits and the MSL. The experience component is the actual loss and ALAE ratio, the projected ultimate losses and ALAE limited by basic limits and MSL divided by the subject premium (the total basic limits premium subject to experience rating).

The exposure measure is premium. The exposure component is the adjusted expected loss ratio. The actual and expected loss ratios are compared using a credibility factor to arrive at the experience credit (percentage reduction in premium) or debit (percentage increase in premium). This plan has no minimums, maximums, or explicit off-balance correction.

Exhibit 4 shows the basic calculation. Exhibit 5 shows the calculation of the expected unreported losses and ALAE at 9/30/87 and subject premium. The expected unreported losses and ALAE at 9/30/87 are the product of the subject premium, adjusted expected loss and ALAE ratio, and expected percentage losses and ALAE unreported at 9/30/87. These three quantities reflect the impact of basic limits losses and the MSL.

Note that there is no adjustment for unreported losses and ALAE for the claims-made policies, even though there may be case reserve development. This results in an probable understatement of the actual loss and ALAE ratio and a resulting probable overstatement of any credits or understatement of any debits.

The subject premium is the product of the current basic limits premium, two policy adjustment factors, and policy period adjustment factors. The Type 1 policy adjustment factors adjust premium to an occurrence level. The Type 2 policy adjustment factors adjust for the experience period being claims-made. In 1/1-12/31/86 in the example, the third-year claims-made premium is adjusted up to an occurrence basis by the Type 1 factor and down to a first-year claims-made basis by the Type 2 factor because the experience for the 1/1-12/31/86 period is first-year claims-made. The policy period adjustment factors adjust current premium to policy period dollar levels. It is not clear if these factors also adjust for changes in coverage, such as changes in exclusions.

The calculation performed to determine the experience credit/(debit) is as follows:

AELR Adjusted Expected Loss and ALAE Ratio
ALR - Actual Loss and ALAE Ratio
Z - Credibility
CD - Credit/(Debit)

$$CD = \frac{AELR - ALR}{AELR} \times Z.$$

This can be rearranged to a more familiar form:

E - Expected Losses and ALAE Limited by Basic Limits and MSL
A - Actual Losses and ALAE Limited by Basic Limits and MSL
Z - Credibility
M - Modification Factor

$$M = \frac{(A \times Z) + (E \times (1 - Z))}{E}$$

where

E = Subject Premium x AELR

and

A = Projected Ultimate Losses and ALAE Limited by Basic Limits and MSL.

Note that

M = 1 - CD.

For the example:

CD = 4.6% from Exhibit 4,
E = 60,641,
A = 55,828,
Z = 0.580, and
M = 0.954 = 1 - 0.046.

This plan has special rules for treating non-standard expense allowances, deductibles, and experience periods with no claims.

6. Workers' Compensation Pool Experience Rating Plan

The experience rating plan of a workers' compensation pool for fire districts in one state is illustrated in Exhibits 6 through 8. This example is used throughout the following discussion of the plan.

This plan is used for occurrence workers' compensation coverage written on a guaranteed cost basis for all entities participating in the pool. Pool participation has been constant since the pool's inception and is not expected to change for 7/1/88-89, the policy period in question. All policies renew 7/1.

The costs to be allocated using a weighting of experience and exposure are the expected losses and ALAE for 7/1/88-89, discounted for anticipated investment income. The estimated discounted expected expenses other than ALAE for 7/1/88-89 are distributed to participant based on the expected full-time-equivalent (FTE) personnel for 7/1/88-89.

The experience basis is reported losses and ALAE at 6/30/87 for the experience period, adjusted for changes in FTE personnel. The exposure basis is expected full-time-equivalent employees for the 7/1/88-89 period. The reported losses and ALAE at 6/30/87 are limited to $25,000 per occurrence. The experience period is the latest three complete policy periods, i.e., 7/1/84-85, 7/1/85-86, and 7/1/86-87. Credibility is based on FTE employees for the experience period.

FTE personnel are used rather than payroll as an exposure measure, the credibility base, and to allocate estimated discounted expenses for 7/1/88-89 due to the presence in some of the districts of volunteer firefighters and pay scale discrepancies between districts. Volunteer firefighters are covered by workers' compensation law. The generally fortuitous nature of workers' compensation claims for firefighters and the pay scale discrepancies indicate that some costs and credibility are more closely related to FTE personnel than payroll.

The plan has a built-in minimum: the estimated discounted administrative expenses for 7/1/88-89, as allocated based on expected FTE personnel for 7/1/88-89. The plan also has a maximum for each participant: 25% above the prior year's contribution (for 7/1/87-88 in this example), adjusted for any increase in total recommended contribution but not for any decrease (a 30% increase in this example, from $853,000 to $1,109,000). The total increase allowable in this example is 62.5% ((1.300 x 1.250) - 1.000).

Because pool participation has been and is expected to remain constant, it is possible to calculate the exact off-balance and adjust accordingly so that the total dollars collected are the total recommended contribution for the group. The off-balance may need to be recalculated after application of minimums and maximums, depending on their impact.

Exhibit 6 shows the premium determination. Exhibit 7 shows the determination of A, the discounted expected losses and ALAE for 7/1/88-89 allocated based on experience. Exhibit 8 shows the determination of E (the discounted expected losses and ALAE for 7/1/88-89 allocated based on exposure), Z (credibility), minimum premium, and maximum premium.

The premium before adjustment for off-balance, minimums, and maximums is determined as follows:

Unadjusted Premium = Minimum Premium + ((A x Z) + ((E x (1.000 - Z))).

The unadjusted premium for the example is shown in column (7) of Exhibit 6. Column (8) of Exhibit 6 shows the premium adjusted for the off-balance. Column (9) of Exhibit 6 shows the premium adjusted for maximum premiums combined with an additional off-balance calculation. Note that in the example, no participant's premium was lower than the applicable minimum. Any amounts under minimum premiums would have to be reallocated similarly to the reallocation of the amounts over maximum premiums.

A is the discounted expected losses and ALAE for 7/1/88-89 allocated based on experience (calculated in Exhibit 7). The reported losses and ALAE at 6/30/87 for accident period 7/1/84-87 are limited to $25,000 per occurrence. The ratio of these to FTE personnel for 7/1/84-87 results in the raw annual loss and ALAE rate. The raw annual loss and ALAE rate is applied to the expected FTE personnel for 7/1/88-89 to obtain unadjusted A's. The unadjusted A's are adjusted so that the desired total of $832,000 of discounted expected losses and ALAE for 7/1/88-89 would be collected if all participants had credibilities of 1.00.

E is the discounted expected losses and ALAE for 7/1/88-89 allocated based on exposure. The E's are calculated in Exhibit 8 by distributed the $832,000 in proportion to the expected FTE personnel for 7/1/88-89. This is what would be collected if all participants had credibility of 0.00. The credibilities (Z) are based on FTE personnel for 7/1/84-87 and the formula in Exhibit 8. The minimum and maximum premiums are also calculated in Exhibit 8.

C. <u>Composite Rating</u>

Composite rating is an administrative tool to facilitate the rating of large, complex risks upon audit. Instead of rating different coverages using different exposure measures, all applicable coverages are rated using one, composite exposure measure.

The composite rate applied to the composite exposure measure is determined at the beginning of the policy period under consideration based on estimated exposures. It is used to determine the deposit premium based on the estimated composite exposure and the final premium based on the audited composite exposure. The composite rate may be based on manual rates to which the appropriate experience modification factors have been applied or on the entity's experience. The remainder of this section discusses the latter case, using the loss rating portion of the Insurance Services Office Composite Rating Plan for Automobile Physical Damage, Automobile Liability, General Liability, Glass and Theft Insurance as an example.

Exhibit 9 shows the basic formulae for the ISO Composite Rating Plan loss rated risks example. Eligibility for loss rating is based on the reported losses and ALAE at the latest evaluation date, limited to various per occurrence limits, for the same period of time as the experience period to be used in the calculation. Different eligibility requirements apply for different combinations of coverage and limits. The premium charged is based solely on the entity's experience, adjusted for differences in coverage type (occurrence or claims-made year), trends in losses and ALAE and exposure, and other factors which may impact the appropriateness of the composite rate.

The composite rate is the adjusted premium for the experience period divided by the adjusted composite exposure for the experience period. The adjusted premium for the experience period is sum of the adjusted projected ultimate losses and ALAE, converted from occurrence to claims-made basis if appropriate, divided by the expected loss and ALAE ratio, for each type of loss. The adjusted composite exposure for the experience period is the composite exposure for the experience period, adjusted by exposure trend factors, converted from occurrence to claims-made if appropriate. The projected ultimate losses and ALAE are the reported losses and ALAE at latest evaluation date developed to ultimate, converted from claims-made to occurrence if appropriate, trended to the year for which the composite rate is being calculated, and adjusted for other changes if appropriate.

The reported losses and ALAE used in the calculation are subject to various per occurrence limits. The plan has special rules for treating non-standard expense allowances, deductibles, and limits larger than those used in the composite rating calculation. The deposit premium is not subject to experience rating since it is based solely on the entity's experience up to the limits used in the calculation. The final premium may be subject to retrospective rating. Both deposit and final premiums may be subject to schedule rating.

III. RETROSPECTIVE RATING

While experience rating and some forms of composite rating assume that the past, with appropriate adjustments, is predictive of the future, retrospective rating uses the experience during the period to determine the costs for the period. This makes costs based on retrospective rating plans more responsive to changes in experience and more subject to fortuitous fluctuations in experience than experience rating or composite rating plans. However, retrospective rating is very similar to prospective experience rating in that many of the elements are the same.

As with experience rating, actual losses, and sometimes ALAE, are compared to expected losses (and ALAE), although in this case they are both for the current period. To have an "apples to apples" comparison, several different experience and exposure base combinations can be used, including the following:

- o actual paid losses (and ALAE) at a particular date and the expected paid losses (and ALAE) at that date both for the experience period,

- o reported losses (and ALAE) at a particular date and the expected reported losses (and ALAE) at that date both for the experience period, and

- o projected ultimate losses (and ALAE) and expected losses both for the experience period.

These are the same as the first three options listed for experience rating.

As with experience rating, the experience base should be related to the exposure base and the basis on which policies are written or funding occurs. If the costs to be allocated include ALAE, ALAE should be included with losses in the calculation.

The length of the retrospective rating period is usually one or three years. As with experience rating, the shorter the period, the more responsive the plan will be to changes that truly impact loss and ALAE experience, such as changes in the risk control program, and the more subject to fortuitous fluctuations in loss and ALAE experience. Conversely, a longer period will result in less responsiveness to changes and to fortuitous occurrences.

Retrospective rating plans may also limit losses (and ALAE) per occurrence and in aggregate to reduce the impact of fortuitous occurrences, as may experience rating plans.

If projected ultimate losses are to be used in the retrospective rating calculation, they can be developed in a number of ways similar to those used to develop projected ultimate losses used to determine manual rates. Projected ultimate losses are often based on paid or reported losses at a particular date.

The expected losses are a function of the current exposure base. The exposure base should be related to the experience base, as detailed above. As for experience rating, expected losses are usually a product of an expected loss rate and the exposure.

Also as for experience rating, the exposure measure used should reflect the underlying risk of loss and ALAE. It is not always possible to use the theoretically optimal exposure measure. In practice, insurers and non-traditional risk financing mechanisms often use whatever exposure measure insurers use in their premium calculations.

Credibility has the same function and is used in the same way for retrospective rating as for experience rating. Retrospective rating plans also may have minimum or maximum premium charges and need to be corrected for off-balance, as with experience rating plans.

The deposit premium for retrospective rating plans may be based on an experience rating plan. Retrospective adjustments are made periodically after the end of the experience period for a pre-determined number of adjustments or until the insurer and insured agree to end the adjustments.

Two examples of retrospective rating plans are discussed below.

A. National Council on Compensation Insurance Retrospective Rating Plan

The National Council on Compensation Insurance Retrospective Rating Plan applies to workers' compensation and employer's liability for eligible insureds. An insured must elect to participate in the plan and the insurer must agree.

The basic formulae are shown in Exhibit 10. Losses, minor ALAE for workers' compensation, and all ALAE for employer's liability are the subject of the allocation. Some aircraft-related claims are excluded and the costs of some accidents involving more than one person are limited. All other costs are collected as a function of the losses, exposure (as represented by the standard premium), or, for taxes only, the retrospective premium before taxes. All policies are written on an occurrence basis.

The deposit premium collected at the beginning of the period is the experience rated premium. Retrospective adjustments are made using claim data at 18, 30, 42, ... months after the beginning of the policy period, if it is a one-year retrospective period, until insurer and insured agree there will be no more. For a three-year retrospective period, the claim data are evaluated at 42, 54, 66, ... months after the beginning of the policy period.

For losses under any applicable limits, the experience is given credibility of 1.000. Losses over any applicable limits are given zero credibility, and money for them is collected based on exposure, as represented by standard premium.

The plan has minimum and maximum retrospective premiums. Costs above the maximum less those below the minimum are collected from/credited to the insured based on exposure, as represented by standard premium. Various minimum and maximum retrospective premium combinations are possible (including no minimum or maximum). The choice of minimum and maximum premiums impacts the basic premium. The plan has no explicit off-balance correction.

The general retrospective rating formula calculates retrospective premium as the sum of basic premium and converted losses, both multiplied by the tax multiplier. The basic premium, which is a function of the standard premium (exposure), provides for the following costs:

- insurer expenses such as acquiring and servicing the insured's account;
- risk control services, premium audit, and general administration of the insurance;
- an adjustment for limiting the retrospective premium between the minimum and maximum retrospective premiums; and
- an allowance for the insurer's possible profit or contingencies.

The converted losses are the reported limited losses at the evaluation date multiplied by the loss conversion factor. The loss conversion factor covers the ALAE not included with the losses and ULAE. The tax multiplier covers licenses, fees, assessments, and taxes which the insurer must pay on the premium it collects.

There are two additional elements the insured may elect, if the insurer agrees: a loss limitation resulting in an excess loss premium and a retrospective development premium. Both these premiums are subject to the tax multiplier. The retrospective rating formula with these elective premium elements is also shown in Exhibit 10.

If the loss limitation is accepted, the reported limited losses at any evaluation are further limited to an agreed-upon amount per accident. The cost of losses above this amount and related ALAE and ULAE are collected through the excess loss premium. It is a function of standard premium (exposure).

Because reported limited losses tend to develop over time upwards to the ultimate limited losses, the first retrospective adjustment is likely to result in the insurer returning premium to the insured. Successive retrospective adjustments will probably result in most of, if not all of or more than, this amount being returned by the insured to the insurer. To smooth out these back-and-forth payments, some insureds opt to use the retrospective development premium, which attempts to offset this process. The retrospective development premium is a function of standard premium (exposure). It is used only for the first three retrospective adjustments and decreases over time.

Note that there does not seem to be any part of the formula that recovers for the cost of the excluded aircraft-related claims and amounts above limits on accidents involving more than one person. There is also an overlap of the excess loss premium and basic premium.

The excess loss premium collects for losses and related expenses above the per accident limit; the basic premium collected for losses and related expenses above the maximum limit, some of which are the result of losses above the per accident limit.

B. Automobile Physical Damage Insurance Retrospective Allocation to Units by Single Entity

Exhibit 11 illustrates the retrospective allocation of automobile physical damage insurance premium to units by a single entity. The coverage is actual cash value, written on an occurrence basis for one year.

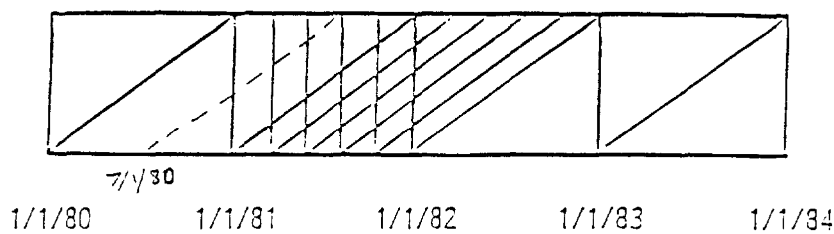
The deposit premium collected from the units at the beginning of the period is based on the expected cost of insurance, allocated to unit based on exposure as represented by the expected number of vehicles. There is no distinction for different types of vehicles. This is reasonable if each unit has the same vehicle expected cost per vehicle.

There is one retrospective adjustment, made using data at 18 months after the beginning of the policy year. Only one adjustment is made because automobile physical damage claims are reported and settled very quickly and the actual exposure is known shortly after the year ends. The actual cost of the insurance is allocated based on audited exposure (actual number of vehicles) and based on reported losses and ALAE. These two allocations are weighted using credibility. Losses and ALAE are unlimited because the cost of any one occurrence is limited by the actual cash value of the vehicle in the accident plus any ALAE, which should be small. All experience is given a credibility of 0.25 regardless of the exposure size to make the plan easier for the unit managers to understand.

The plan has no minimum and maximum retrospective premiums. The plan has no off-balance correction as none is needed because the credibility factors are the same for all units.

Accident Year/Policy Year Illustration for Occurrence Policies

Exhibit 1



ACCIDENT YEAR 1981 CONTAINS ACCIDENTS ASSOCIATED WITH POLICIES WRITTEN IN 1980 AND 1981.

POLICY YEAR 1981 CONTAINS SOME ACCIDENTS THAT OCCUR IN 1981 AND SOME THAT OCCUR IN 1982.

Loss Terminology Illustration

Exhibit 2

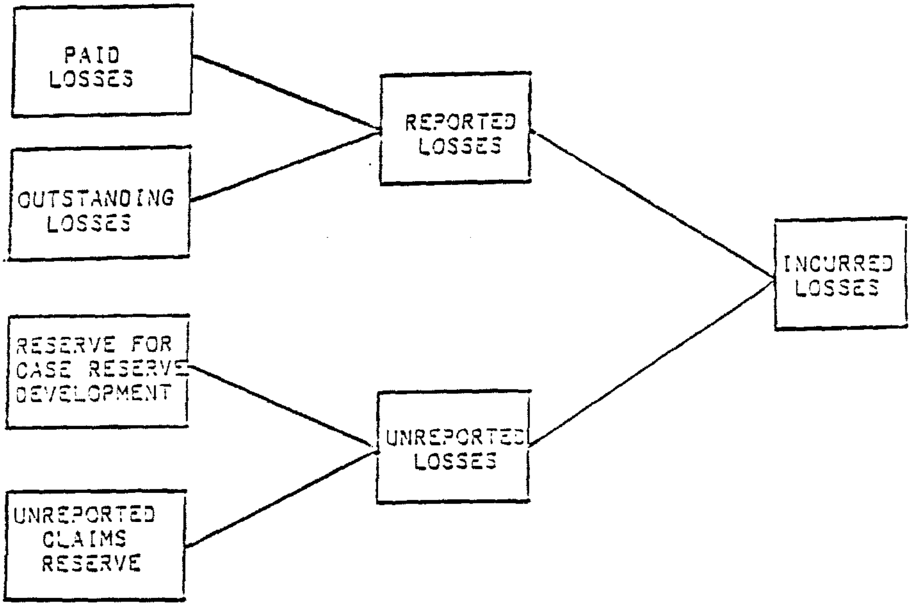


Exhibit 3

General Credibility Illustration

Credibility

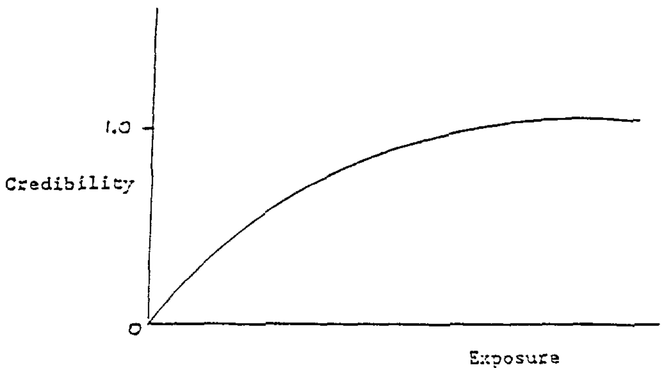


Exhibit 4

ISO Experience Rating Sample Calculation

Basic Calculation

Coverage:	Premises/Operations and Products/Completed Operations
Policy Being Rated:	1/1-12/31/88 Third-Year Claims-Made
Experience Period:	1/1-12/31/84 Occurrence 1/1-12/31/85 Occurrence 1/1-12/31/86 First-Year Claims-Made

I. Experience Component:	
A. Reported Losses and ALAE at 9/30/87 Limited by Basic Limits and MSL	44,300
B. Expected Unreported Losses and ALAE at 9/30/87 Limited by Basic Limits and MSL (See Exhibit 5)	11,528
C. Projected Ultimate Losses and ALAE Limited by Basic Limits and MSL ((A) + (B))	55,828
D. Total Basic Limits Premium Subject to Experience Rating (See Exhibit 5)	136,272
E. Actual Loss and ALAE Ratio ((C) / (D))	0.410
II. Exposure Component: Expected Loss and ALAE Ratio	0.445
III. Credibility:	0.580
IV. Experience Credit/Debit): ((((II) - (I-E)) / (II)) x (III))	4.6%

Notes: MSL is the maximum single limit per occurrence, applied to basic limits losses and unlimited ALAE. It is based on the total basic limits premium subject to experience rating (subject premium).

The adjusted expected loss ratio and credibility are supplied by ISO. Credibility is based on the total basic limits premium subject to experience rating.

ISO Experience Rating Sample Calculation

Exhibit 5

Expected Unreported Losses and ALAE at 9/30/87
and Subject Premium

Policy Period	Coverage	Current Basic Limits Premium	Policy Adjustment Factors Type 1	Policy Adjustment Factors Type 2	Policy Period Adjustment Factors	Subject Premium	Adjusted Expected Loss & ALAE Ratio	Expected Percentage B/L Losses & ALAE Unreported at 9/30/87	Expecte B/L Loss & ALAE Unreport at 9/30/
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1/1-12/31/84	Prem/Ops	47,500	1.06	1.00	0.701	35,295	0.445	11.3%	1,7
	Products	15,500	1.16	1.00	0.732	13,161	0.445	46.8%	2,7
1/1-12/31/85	Prem/Ops	47,500	1.06	1.00	0.766	38,568	0.445	19.4%	3,3
	Products	15,500	1.16	1.00	0.792	14,240	0.445	58.1%	3,6
1/1-12/31/86	Prem/Ops	47,500	1.06	0.67	0.837	28,236	0.445	0.0%	
	Products	15,500	1.16	0.44	0.856	6,772	0.445	0.0%	
Total						136,272			11,52

Notes: (3) is for the 1/1-12/31/88 third-year claims-made policy.

(4) adjusts premium up to an occurrence level.

(5) adjusts for experience period being claims-made, reflecting claims-made year.

(6) adjusts current premium to policy period dollar levels.

(7) = (3) x (4) x (5) x (6).

(9) and (10) also reflect MSL.

(10) = (7) x (8) x (9).

(4), (5), (6), (8), and (9) are supplied by ISO.

MSL is the maximum single limit per occurrence, applied to basic limits losses and unlimited ALAE. It is based on the total basic limits premium subject to experience rating (subject premium).

Workers' Compensation Pool Experience Rating Sample Calculation

Premium Determination

Fire District	Minimum Premium	Maximum Premium	A	E	Z	Premium for 7/1/88-89 Unadjusted	Adjusted #1	Adjusted #2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
A	93,384	372,825	290,914	280,491	0.59	380,075	376,698	372,825
B	1,494	8,634	821	4,487	0.00	5,980	5,927	5,958
C	18,810	93,623	15,286	56,497	0.12	70,319	69,694	70,063
D	8,409	53,402	6,163	25,257	0.00	33,666	33,367	33,544
E	28,546	171,593	172,188	85,742	0.25	136,014	134,805	135,518
F	38,615	222,414	128,716	115,985	0.33	158,778	157,368	158,200
G	166	1,599	0	499	0.00	664	659	662
H	6,805	41,251	44,007	20,439	0.00	27,243	27,001	27,144
I	13,167	72,625	10,922	39,548	0.09	50,228	49,782	50,045
J	52,999	270,257	121,658	159,188	0.39	197,500	195,745	196,780
K	4,868	28,141	37,010	14,623	0.00	19,491	19,318	19,420
L	1,715	9,593	145	5,151	0.00	6,866	6,805	6,841
M	2,987	15,670	4,105	8,973	0.00	11,960	11,854	11,917
N	5,034	24,623	65	15,121	0.00	20,156	19,977	20,082
Total	277,000	1,386,250	832,000	832,000		1,118,941	1,109,000	1,109,000

Notes:
(2) is (6) of Exhibit 8.
(3) is (8) of Exhibit 8.
(4) is (7) of Exhibit 7.
(5) is (3) of Exhibit 8.
(6) is (5) of Exhibit 8.
(7) = (2) + (((4) x (6)) + ((5) x (1.00-(6)))).
(8) = ((7) / Total (7)) x 1,109,000. 1,109,000 is the recommended contribution for 7/1/88-89.
(9) = (8), adjusted for maximum premiums with amount over maximum premiums reallocated based on (8).

Workers' Compensation Pool Experience Rating Sample Calculation

Exhibit 7

Determination of A

Fire District	Reported Limited Losses & ALAE at 6/30/87	FTE Personnel 7/1/84-87	Raw Annual Loss & ALAE Rate ((2)/(3))	FTE Personnel 7/1/88-89	A: Unadjusted ((4)x(5))	A: Adjusted
(1)	(2)	(3)	(4)	(5)	(6)	(7)
A	350,240	463.3	755.97	168.8	127,607	290,914
B	1,000	7.5	133.33	2.7	360	821
C	15,126	76.7	197.21	34.0	6,705	15,286
D	8,892	50.0	177.84	15.2	2,703	6,163
E	193,214	132.0	1463.74	51.6	75,529	172,188
F	147,365	182.3	808.39	69.8	56,460	128,716
G	0	1.6	0.00	0.3	0	0
H	56,654	36.1	1569.36	12.3	19,303	44,007
I	13,809	68.6	201.30	23.8	4,791	10,922
J	130,682	234.6	557.04	95.8	53,365	121,658
K	47,965	26.0	1844.81	8.8	16,234	37,010
L	185	9.0	20.56	3.1	64	145
M	4,768	14.3	333.43	5.4	1,801	4,105
N	63	20.0	3.15	9.1	29	65
Total/Avg.	970,463	1322.5	733.81	500.7	364,951	832,000

Note: (7) = 832,000 x ((6) / Total (6)). 832,000 is the discounted expected losses and ALAE for 7/1/88-89.

Exhibit 8

Workers' Compensation Pool Experience Rating Sample Calculation

Determination of E, Z, Minimum Premium, and Maximum Premium

Fire District	FTE Personnel 7/1/88-89	E	FTE Personnel 7/1/84-87	Credibility (Z)	Estimated Discounted Admin. Costs 7/1/88-89	Contribution 7/1/87-88	Maximum Premium 7/1/88-89
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
A	168.8	280,491	463.3	0.59	93,384	229,410	372,825
B	2.7	4,487	7.5	0.00	1,494	5,313	8,634
C	34.0	56,497	76.7	0.12	18,810	57,609	93,623
D	15.2	25,257	50.0	0.00	8,409	32,860	53,402
E	51.6	85,742	132.0	0.25	28,546	105,586	171,593
F	69.8	115,985	182.8	0.33	38,615	136,858	222,414
G	0.3	499	1.6	0.00	166	984	1,599
H	12.3	20,439	36.1	0.00	6,805	25,383	41,251
I	23.8	39,548	68.6	0.09	13,167	44,688	72,625
J	95.8	159,188	234.6	0.39	52,999	166,297	270,237
K	8.8	14,623	26.0	0.00	4,868	17,316	28,141
L	3.1	5,151	9.0	0.00	1,715	5,903	9,593
M	5.4	8,973	14.3	0.00	2,987	9,642	15,670
N	9.1	15,121	20.0	0.00	5,034	15,151	24,623
Total	500.7	832,000	1,322.5		277,000	853,000	1,386,250

Notes: (3) is distributed based on (2). 832,000 is the discounted expected losses and ALAE for 7/1/88-89.

(5) is determined based on (4) as follows:

FTE Personnel Years	Credibility
60 or less	0.00
60 - 1,199	$\left(\frac{\text{FTE personnel years} - 60}{1,140}\right)^{1/2}$
1,200 or more	1.00

(6) is distributed based on (2). This is the minimum premium.

(8) = (7) x (1,109,000 / 353,000) x 1.25. 1,109,000 is the total recommended contribution for 7/1/88-89.

ISO Composite Rating Plan Loss Rated Risks Example

Exhibit 9
Page 1 of 2

Types of Losses Covered: General Liability, Automobile Liability, Automobile Physical Damage, Glass, and Theft

Experience Period: Five years beginning between six and five and one-half years prior to the date the composfte rate is to be effective. As few as three years, beginning between four and three and one-helf years prior to the date the composite rate is to be effectitve, may used if that is all that are available.

Experience: For each type of loss, calculate by accident year and total the adjusted projected ultimate Losses and ALAE as follows:

Reported Limited Losses & ALAE at Latest Evaluation Date	X	Loss & ALAE Development Factor	X	Conversion Factor From Claims-Made to Occur.	X	Loss & ALAE Trend Factors to Current Year	X	Factors to Reflect Other Changes

Adjusted Composite Exposure for Experience Period: For the experience period, calculated the adjusted composite exposure as follows:

Composite Exposure for Exper. Period	X	Exposure Trend Factors	X	Conversion Factor From Occur. to Claims-Made

Adjusted Premium for Experience Period: For each type of loss, calculate the loss premium as following:

[Adjusted Projected Ultimate Losses & ALAE	X	Conversion Factor From Occur. to Claims-Made]	/	Expected Loss & ALAE Ratio

Total these to get the adjusted premium for the experience period.

Composite Rate: The composite rate is calculated as follows:

Adjusted Premium for Exper. Period	Adjusted Composite Exposure for Exper. Period

Exhibit 9
Page 2 of 2

Final Premium: The final premium is calculated as follows:

Audited Exposure for Policy Period X Composite Rate

Notes: Various per occurrence limits apply to reported losses and ALAE.

For automobile physical damage, exclude ALAE.

The following are provided by ISO:

- o loss and ALAE development factors,
- o conversion factors from occurrence to claims-made,
- o loss & ALAE trend factors,
- o exposure trend factors, and
- o conversion factors from claims-made to occurrence,
- o expected loss and ALAE ratios.

Exhibit 10

NCCI Retrospective Rating Plan Example

Experience Period: One or Three Years

Deposit Premium: Experience Rated Premium

Retrospective Adjustments: Uses claim data at 18, 30, 42, ... months from the beginning of a one-year policy period and claim data at 42, 54, 66, ... months from the beginning of a three-year policy period.

Retrospective Rating Formula:

$$\text{Retro. Premium} = \left[\text{Basic Premium} + \text{Converted Losses}\right] \times \text{Tax Multiplier}$$

$$\text{Basic Premium} = \text{Standard Premium} \times \text{Basic Premium Factor}$$

Standard Premium = Manual Premium modified for experience rating, loss constants, and minimum premium excluding premium discount and expense constants.

$$\text{Converted Losses} = \text{Reported Limited Losses at Eval. Date} \times \text{Loss Conversion Factor}$$

Reported limited losses include interest on judgments, expenses incurred in obtaining third party recoveries, and ALAE for employer's liability claims, exclude some aircraft-related claims, and have limits on some accidents involving more than one person.

Retrospective Rating Formula With Elective Premium Elements:

$$\text{Retro. Premium} = \left[\text{Basic Premium} + \text{Converted Losses} + \text{Loss Premium} + \text{Devel. Premium}\right] \times \text{Tax Multiplier}$$

$$\text{Excess Loss Premium} = \text{Standard Premium} \times \text{Excess Loss Premium Factor} \times \text{Loss Conversion Factor}$$

$$\text{Retro. Devel. Premium} = \text{Standard Premium} \times \text{Retro. Devel. Factor} \times \text{Loss Conversion Factor}$$

Converted losses are calculated as above, but reported limited losses now also have a per accident limit.

Minimum and Maximum Retrospective Premiums:

$$\text{Minimum Retro. Premium} = \text{Standard Premium} \times \text{Minimum Retro. Premium Factor}$$

Maximum				Maximum
Retro.	=	Standard	x	Retro.
Premium		Premium		Premium
				Factor

Note: The following are provided by NCCI:

- Basic Premium Factor,
- Excess Loss Premium Factor,
- Loss Conversion Factor,
- Maximum Retrospective Premium Factor,
- Minimum Retrospective Premium Factor,
- Retrospective Development Factor, and
- Tax Multiplier.

Exhibit 11

Automobile Physical Damage Insurance Retrospective
Allocation to Units by Single Entity Example

Deposit Premium

Unit	Expected Number of Vehicle Years	Expected Cost of Insurance Allocated Based on Exposure
(1)	(2)	(3)
A	500	50,000
B	1,000	100,000
C	750	75,000
D	500	50,000
E	2,500	250,000
Total	5,250	525,000

Note: (3) is allocated based on (2). (3) is the deposit premium.

Retrospective Premium

Unit	Actual Number of Vehicle Years	Actual Cost of Insurance Allocated Based on Exposure	Reported Losses & ALAE at 18 Months	Actual Cost of Insurance Allocated Based on Experience	Credibility	Retro. Premium
(1)	(2)	(3)	(4)	(5)	(6)	(7)
A	525	48,659	35,000	52,778	0.25	49,688
B	1,050	97,317	60,000	90,476	0.25	95,607
C	600	55,610	60,000	90,476	0.25	64,326
D	500	46,341	30,000	45,238	0.25	46,066
E	2,450	227,073	130,000	196,032	0.25	219,313
Total	5,125	475,000	315,000	475,000		475,000

Notes: (3) is allocated based on (2).
(5) is allocated based on (4).
(7) = ((3) x (1.00 - (6))) + ((5) x (6)).