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Relative Unpaid Claims Loss Reserving

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"Let no one say that I have said nothing new; the arrangement of the subject is new."

—Blaise Pascal

Abstract: This paper derives an elementary Relative Unpaid Claims Loss Reserving Model using only: (i) accident year incremental losses that were paid during the same calendar year as the accounting date; (ii) relativities of successive accident year unpaid losses as of the accounting date; and (iii) unpaid losses for the oldest included accident year as of the accounting date. Methods to apply the Model are presented along with considerations and techniques to improve accuracy. Several methods derived from the Model are applied to the CAS loss reserve data base historical experience and the resulting unpaid claim estimates are compared to indications using traditional loss reserving methods. Performance accuracy of competing methods is evaluated using a retrospective hindsight test of subsequent emergence. Advantages of the Relative Unpaid Claims Loss Reserving Model include that it requires less data and fewer assumptions than traditional chain ladder methods and results in unpaid claim estimates that empirically appear at least as accurate as estimates derived from comparable generally accepted actuarial loss reserving methods.

Keywords: loss reserve; reserving; unpaid claims; IBNR; recursive model; relative

1. INTRODUCTION

1.1 Historical Background

As expounded upon by Friedland [4], basic loss reserving methods are fundamentally rooted in loss development triangles and associated loss development factors. After appropriate investigation, traditional loss reserve analyses typically proceed with compilations of historically based accident year[1] loss development triangles intended to be representative of expected future development. Loss development factors derived from these historical development patterns are applied to accident year experience as of the valuation date to extrapolate historical development into the future and, thereby, estimate ultimate accident year losses. Unpaid loss estimates as of a particular accounting date are indirectly calculated by subtracting cumulative loss payments through the accounting date from estimated accident year ultimate losses.

Even where "expected loss" is introduced to improve the accuracy of estimated ultimate losses, commonly applied methods (e.g., Bornhuetter-Ferguson, Cape Cod) also require loss development factor selections. Basic frequency-severity (counts & averages) methods are similarly organized into development triangles and require selections for some combination of loss development factors,

[1] Accident year is referenced throughout this paper since it is the most common categorization of historical data. Techniques described in this paper are also applicable to data categorized by other time intervals including policy year, underwriting year, report year and fiscal year. Similarly, the techniques are applicable to monthly, quarterly and biannual data.

frequency trend, severity trend, and disposal rates.

1.2 Objective

This paper presents a straightforward and robust Relative Unpaid Claims Loss Reserving model conceived from a different perspective than traditional chain ladder loss development models. Methods to apply the relative unpaid claims model use estimated ratios of unpaid claims[2] as of the accounting date for successive accident years and an estimate of unpaid claims for the oldest accident year to directly estimate unpaid losses for each accident year. These methods are relatively easy to apply and, optimally, improve the accuracy of unpaid claim estimates while requiring less data and fewer assumptions than traditional chain ladder loss development triangle methods.

1.3 Outline

The remainder of this paper presents a framework and describes techniques to estimate unpaid claims from relationships derived in Section 2:

- Section 2 presents Relative Unpaid Claims Loss Reserving basics;
- Section 3 provides an illustrative example;
- Section 4 discusses measures of relative unpaid losses at common maturities;
- Section 5 addresses unpaid losses for the oldest included accident year;
- Section 6 explores empirical evidence using the CAS loss reserve data base to compare results of methods that apply the Relative Unpaid Claims Loss Reserving model to the results of several generally accepted actuarial loss reserving methodologies;
- Section 7 summarizes relevant results; and
- Section 8 presents the main conclusions and areas for future research.

2. RELATIVE UNPAID CLAIMS LOSS RESERVING BASICS

We derive a relative unpaid claims model from definitions.

2.1 Definitions

For consecutive accident years m through n ($n>m$), define:

[2] The techniques presented are applicable to loss dollars, claim counts, ALAE (DCCE), and loss & ALAE (DCCE) combined.

$U_{i,j}$ = accident year *i* unpaid losses as of year-end *j*, where $j \geq i$,

$p_{i,j}$ = accident year *i* payments during calendar year *j*, where $j \geq i$.

As of accounting date year-end *d*, define the ratio of unpaid loss at common maturities:

$r_i = \dfrac{U_{i,d}}{U_{i-1,d-1}}$, where $m + 1 \leq i \leq n$, $d \geq n$.

r_i equals the **relativity** of accident year *i* unpaid losses as of accounting date year-end *d* to accident year *i*-1 unpaid losses as of accounting date year-end *d*-1.

2.2 Relative Unpaid Claims Loss Reserving Model ("Model")

Beginning with initial value $U_{m,d}$, each $U_{i,d}$ ($m + 1 \leq i \leq n$, $d \geq n$) may be computed using the recursive algorithm:

$$U_{m+1,d} = r_{m+1}\left[U_{m,d} + p_{m,d}\right]$$
$$U_{m+2,d} = r_{m+2}[U_{m+1,d} + p_{m+1,d}]$$
$$\ldots$$
$$U_{n,d} = r_n[U_{n-1,d} + p_{n-1,d}]$$

Proof:

The proof follows directly from definitions.

It is self-evident that:

$$U_{i-1,d-1} = U_{i-1,d} + p_{i-1,d} \tag{2.1}$$

From the definition of r_i:

$$U_{i,d} = r_i U_{i-1,d-1} \tag{2.2}$$

Substituting the expression for $\mathrm{U}_{i-1,d-1}$ of (2.1) into (2.2) gives us

$$U_{i,d} = r_i\left[U_{i-1,d} + p_{i-1,d}\right], \text{ where } m + 1 \leq i \leq n,\ d \geq n. \tag{2.3}$$

Given a base value for $U_{m,d}$, recursive application of (2.3) commencing with i=m+1 and ending with i=n results in the Model algorithm. Q.E.D.

A closed-form expression for each $U_{i,d}$ is presented in Appendix A.

The Model demonstrates that, in order to determine accident year *m* through *n* unpaid losses as of accounting date year-end *d*, it is sufficient to know: (i) $p_{i,d}$, the incremental paid losses during calendar year *d* for each accident year *i*=*m* through *i*=*n*-1; (ii) r_i, the ratio of accident year *i* unpaid losses as of accounting date year-end *d* to accident year *i*-1 unpaid losses as of accounting date year-end *d*-1 for each *i*=*m*+1 through *i*=*n*; and (iii) $U_{m,d}$, the unpaid losses of accident year *m* as of accounting date year-end *d*.

The Model specifies an unpaid claims algorithm that provides an exact representation of unpaid losses (i.e., perfectly accurate Model parameters result in perfectly accurate unpaid losses for each accident year; whereas, in a traditional chain ladder model setting, the most accurate loss development factor selections are not expected to result in perfectly accurate unpaid loss estimates for each accident year). Generally, model risk is the risk that the methods are not appropriate to the circumstances or the models are not representative of the specified phenomenon. Since the Model provides an exact representation of unpaid losses, the second aspect of model risk is eliminated. For application of the Model, we refer to items (i) - (iii) above: (i) $p_{i,d}$ will typically be known as part of the historical data base for the vast majority of loss reserve analyses; (ii) r_i will typically be unknown and estimated; and (iii) $U_{m,d}$ will typically be unknown and estimated using methods analogous to tail factor development methods. Various methods to derive unpaid claims estimates using the Model will be explored in greater depth. However, we immediately proceed to a simple Relative Unpaid Claims Loss Reserving illustrative example.

3. ILLUSTRATIVE EXAMPLE METHOD

This section presents an example to illustrate use of the Model's algorithm to estimate unpaid losses from a large business segment of actual Other Liability – Occurrence experience[3]. Though the term 'loss' is used for convenience, examples presented in this paper are actually comprised of combined loss & ALAE (DCCE) experience. All loss dollar data presented throughout this paper are displayed in thousands of dollars (i.e., $000 omitted).

3.1 Rudimentary Assumptions

For this example: (i) $p_{i,d}$, the incremental paid losses during calendar year *d*=1997 for each accident year *i*=*m*=1988 through *i*=*n*-1=1996, are known; (ii) r_i, the ratio of accident year *i* unpaid losses as of accounting date year-end *d*=1997 to accident year *i*-1 unpaid losses as of accounting date year-end

[3] CAS Loss Reserve Data Base [7]: Other Liability Data Set; NAIC Company Code 1767

d-1=1996 for each i=m+1=1989 through i=n=1997 are assumed to equal the ratio of corresponding case reserves; and (iii) $U_{m,d}$, unpaid losses of accident year m=1988 as of accounting date year-end d=1997 is assumed to equal the corresponding company filed loss reserves (including IBNR).

By utilizing the ratio of case reserves as of the latest common maturities, (ii) assumes that this ratio is an accurate proxy for the relativity of all (including IBNR) unpaid losses as of the most recent common maturities. By accepting the company filed loss reserves (including IBNR) as of year-end d=1997 for the oldest included accident year m=1988, (iii) assumes that these filed loss reserves accurately provide for the corresponding unpaid claims.

Table 3.1 displays these assumptions. Table 3.1, Column (4) derives each estimated r_i as specified in (ii) above. Table 3.1, Column (5) displays the company filed loss reserves (including IBNR) as of 12/31/97 for oldest accident year 1988 as in (iii) above.

TABLE 3.1

ASSUMPTIONS SELECTION

(1)	(2)	(3)	(4)= (3)/[Prior(2)]	(5)
Accident Year	Case Reserves as of 12/31/96	Case Reserves as of 12/31/97	Selected Ratio Unpaid Loss	Selected Unpaid Loss of Oldest Accident Year as of 12/31/97
i			Selected r_i	Selected $U_{1988,1997}$
1988	1,588	116		1,048
1989	2,838	1,419	0.8935768	
1990	4,883	1,436	0.5059901	
1991	7,016	3,282	0.6721278	
1992	23,466	11,991	1.7090935	
1993	31,248	15,482	0.6597631	
1994	56,994	46,505	1.4882552	
1995	66,826	55,399	0.9720146	
1996	54,941	70,761	1.0588843	
1997		61,839	1.1255529	

(1) m=1988; n=1997
(2), (3) CAS Loss Reserve Data Base [7]
(5) CAS Loss Reserve Data Base [7] = company filed loss reserves (including IBNR) as of 12/31/97

3.2 Derive Unpaid Claims Estimate

Table 3.2, Column (4) uses Table 3.1 assumptions to apply the Model and derive estimated unpaid losses as of accounting date 12/31/97[4] for each accident year 1988 through 1997. The Table 3.2, Column (4) accounting date 12/31/97 indicated total accident year 1988 through 1997 unpaid losses equals $853,442.

TABLE 3.2

INDICATED UNPAID LOSSES

(1)	(2)	(3)	(4)	(5)
Accident Year	Case Reserve Ratio Unpaid	Incremental Paid Loss During 1997	Indicated Unpaid Loss as of 12/31/97	Actual Emergence
i	Selected r_i	$p_{i,1997}$	Indicated $U_{i,1997}$	
1988		2,064	1,048	1,048
1989	0.8935768	5,085	2,781	2,229
1990	0.5059901	3,432	3,980	4,875
1991	0.6721278	13,032	4,982	8,939
1992	1.7090935	17,241	30,787	27,175
1993	0.6597631	23,924	31,687	38,236
1994	1.4882552	56,447	82,764	75,947
1995	0.9720146	77,480	135,315	130,558
1996	1.0588843	72,104	225,325	216,789
1997	1.1255529	21,098	334,772	309,458
Total			**853,442**	815,254

(1) m=1988; n=1997
(2) Table 3.1, Column (4)
(3) CAS Loss Reserve Data Base [7]
(4) d=1997
For $i = m = 1988$: Table 3.1, Column (5)
For $1989 \le i \le 1997$: (2)x[Prior (3)+Prior(4)]
(5) Computed from CAS Loss Reserve Data Base [7]
Actual Emergence = cumulative losses paid subsequent to 12/31/97 through nine years subsequent to accident year + company filed loss reserves (including IBNR) nine years subsequent to accident year

3.3 Retrospective Testing

For the purposes of examples throughout this paper, the term "actual emergence" is defined as

[4] All examples in this paper as of accounting date 12/31/97 estimate unpaid losses as of valuation date 12/31/97.

cumulative losses paid subsequent to 12/31/97 through nine[5] years subsequent to the accident year added to company filed loss reserves (including IBNR) as of nine years subsequent to the accident year. The Table 3.2, Column (5) total actual emergence equals $815,254. Comparison of the Table 3.2, Column (4) indicated unpaid losses with the Table 3.2, Column (5) actual emergence provides a retrospective test of indicated unpaid claim estimate accuracy. This retrospective test demonstrates that the method results in accounting date 12/31/97 indicated total unpaid losses within 5% of the total actual emergence.

3.4 Initial Observations

Implementation of the Model using the method described in this illustrative example has several advantages over traditional chain ladder loss development reserving methods:

- The method is more efficient to apply;
- The method only requires experience from the most recent calendar year. As such, this method requires less data and information than chain ladder loss development methods since there is no need to produce loss development triangles and no need to select loss development factors;
- It is not necessary to understand or analyze how possible changes in claim payment patterns, case reserve adequacy or other potential distortions have wended their way through an entire historical loss development triangle. As such, it is unnecessary to attempt to adjust for these changes over an entire historical loss development triangle; and
- Given the most recent calendar year payments by accident year, all that is required to effectively employ this method is: case reserves for accident year i at accounting date year-end d divided by case reserves for accident year i-1 at accounting date year-end d-1 reasonably estimate the corresponding ratio of total unpaid losses; and a reasonable estimate of unpaid losses for the oldest included accident year m as of accounting date year-end d.

[5] Nine years subsequent to the accident year is the maximum number of development years available from the CAS loss reserve data base.

4. ESTIMATING RELATIVITY OF UNPAID LOSSES r_i

Selection of appropriate r_i is critical for successful application the Model. It can be useful to conceptualize appropriate r_i as reasonable measures of relative exposure to unpaid losses. For example, the case reserve ratio assumption (ii) in Section 3 is tantamount to the assumption that case reserve ratios measure the corresponding relative exposure to total (including IBNR) unpaid losses.

While the Section 3 illustrative example uses the ratio of case reserves to estimate r_i, several issues may cause case reserve ratios, or other measures, to be a distorted measure of relative unpaid losses. Distortions may result from three general areas: internal (e.g., shifts in mix of business, changes in claim settlement procedures, changes in case reserve adequacy); external (e.g., law changes, inflation, social influences); and credibility (i.e., randomness or sparseness of data renders it unrepresentative of the future). Potential distortions may occur in isolation or concurrently. In Section 2 of their paper "Accident Year/Development Year Interactions" [2], Clark and Rangelova discuss internal and external considerations in the context of loss development patterns. Generally, these considerations are also pertinent to estimating r_i. Credibility distortions arise when potential r_i measures do not have sufficient predictive power to reasonably measure the relativity of unpaid losses.

The following subsections discuss r_i characteristics and potential r_i measures or proxies.

4.1 Reproduction of Actual Emerged Losses

Pursuant to the Model, incremental calendar year *d* payments for each accident year together with foreknowledge of the actual ratios of unpaid losses r_i and foreknowledge of unpaid losses for the oldest included accident year $U_{m,d}$ determine unpaid losses for all accident years as of year-end *d*. It is instructive to derive unpaid losses for the Section 3 example based upon the r_i and $U_{m,d}$ implicit in actual emergence. Table 4.1.1 uses foreknowledge of the actual emergence to solve for r_i and $U_{m,d}$.

TABLE 4.1.1

ASSUMPTIONS SELECTION

(1) Accident Year i	(2) Unpaid Loss as of 12/31/97 Actual Emergence	(3) Incremental Paid Loss During 1997	(4)= (2)+(3) Unpaid Loss as of 12/31/96 Emergence	(5)= (2)/[Prior (4)] Selected Ratio Unpaid Loss Selected r_i	(6) Selected Unpaid Loss of Oldest Accident Year as of 12/31/97 Selected $U_{1988,1997}$
1988	1,048	2,064	3,112		1,048
1989	2,229	5,085	7,314	0.7162596	
1990	4,875	3,432	8,307	0.6665299	
1991	8,939	13,032	21,971	1.0760804	
1992	27,175	17,241	44,416	1.2368577	
1993	38,236	23,924	62,160	0.8608610	
1994	75,947	56,447	132,394	1.2217986	
1995	130,558	77,480	208,038	0.9861323	
1996	216,789	72,104	288,893	1.0420644	
1997	309,458	21,098	330,556	1.0711855	

(1) m=1988; n=1997

(2) Table 3.2, Column (5)

Actual Emergence = cumulative losses paid subsequent to 12/31/97 through nine years subsequent to accident year + company filed loss reserves (including IBNR) nine years subsequent to accident year

(3) Table 3.2, Column (3)

(4) cumulative losses paid subsequent to 12/31/96 through nine years subsequent to accident year + company filed unpaid losses (including IBNR) nine years subsequent to accident year

(6) CAS Loss Reserve Data Base [7] = company filed loss reserves (including IBNR) as of 12/31/97

Table 4.1.2 inputs the resulting r_i and $U_{m,d}$ into the Model and, as we would expect, demonstrates that actual emergence is indeed reproduced for each accident year.

TABLE 4.1.2

INDICATED UNPAID LOSSES

(1)	(2)	(3)	(4)	(5)
Accident Year	Selected Ratio Unpaid	Incremental Paid Loss During 1997	Indicated Unpaid Loss as of 12/31/97	Actual Emergence
i	Selected r_i	$p_{i,1997}$	Indicated $U_{i,1997}$	
1988		2,064	1,048	1,048
1989	0.7162596	5,085	2,229	2,229
1990	0.6665299	3,432	4,875	4,875
1991	1.0760804	13,032	8,939	8,939
1992	1.2368577	17,241	27,175	27,175
1993	0.8608610	23,924	38,236	38,236
1994	1.2217986	56,447	75,947	75,947
1995	0.9861323	77,480	130,558	130,558
1996	1.0420644	72,104	216,789	216,789
1997	1.0711855	21,098	309,458	309,458
Total			815,254	815,254

(1) m=1988; n=1997
(2) Table 4.1.1, Column (5)
(3) Table 4.1.1, Column (3)
(4) d=1997
For i = m = 1988: Table 4.1.1, Column (6)
For 1989 ≤ i ≤ 1997: (2)x[Prior (3)+Prior(4)]
(5) CAS Loss Reserve Data Base [7]
Actual Emergence = cumulative losses paid subsequent to 12/31/97 through nine years subsequent to accident year
+ company filed loss reserves (including IBNR) nine years subsequent to accident year

4.2 Case Reserves

Section 3 uses the ratio of case reserves as a rudimentary measure of r_i under the assumption that case reserve ratios are a reasonable estimate of relative total (including IBNR) unpaid losses (or, alternatively, relative total unpaid loss exposure). The following provides several advantages and potential distortions in the use of case reserve ratios as proxies for r_i:

Advantages -

- Case reserves are typically readily available.
- Case reserves reflect actual loss experience.
- The ratio of case reserves at common maturities measures the implicit aggregate relative case reserves established by claims personnel acting on behalf of the insuring entity. Accordingly, if claims personnel have behaved consistently, the ratio of case reserves at common maturities as a measure of the ratio of all unpaid losses (including IBNR) is intuitively appealing.

Potential Distortions -

- Non-homogenous mix of business.
- Case reserves may have established at different levels of adequacy. This might occur due to changing conditions (e.g., claims personnel practices) or external conditions (e.g., inflation).
- Although case reserves may be evaluated at a common time maturity, such common time maturity may correspond to different stages of development and, thereby, distort case reserve ratios as an appropriate measure of relative total unpaid losses.
- The relativity of IBNR losses may be different than the corresponding case reserve ratio.
- Sparse case reserve experience may reduce the credibility of the case reserve ratio as a reasonable measure of relative unpaid exposure. This may be especially true for: relatively small books of business with relatively low volume; older accident years which are more fully developed and have relatively few remaining case reserves; and recent accident years for slow developing lines of business where only relatively few (or no) case reserves have yet been established.

It may be possible to partition, aggregate or adjust data to eliminate or mitigate distortions in the use of case reserve ratios as a measure of relative total unpaid losses. As discussed by Gross [5], actuaries may use claims level predictive analytics to build their own models of unbiased case reserves based upon detailed objective information about claims and exposure. In general, when considering r_i candidates (case reserve ratios or otherwise), it is prudent to weigh strengths and weaknesses of competing measures.

4.3 Calendar Year *d* Reported Emergence

While the ratio of case reserves at common maturities is an obvious candidate to estimate r_i, we may not have taken full advantage of all available information. To estimate each r_i, we have not yet made use of reported emergence during calendar year *d*.

Appendix B, Sheet 1 displays historical incremental paid losses and case reserves for the Section 3 business segment. Appendix B, Sheet 2 displays the case reserves to left of the corresponding one year reported[6] losses emerged along with the resultant underlined one-year loss development factor[7] displayed underneath. The one-year loss development factors are case reserve development factors that develop case reserves as of year-end to subsequent one year reported emergence (i.e., to payments during the next calendar year plus case reserves as of the next calendar year-end).

As a result of reversion to the mean, if the one-year loss development factors as of year 2 are samples from the same random variable, then an average of the sample loss development factors is generally a more accurate estimate of the future one-year loss development factor as of year 2 than simply repeating the most recent value. The same is true for one-year loss development factors as of years 3, 4, . . . The final underlined row of Appendix B, Sheet 2 displays the (up to) three most recent years dollar weighted average of one-year loss development factors.

The dollar weighted average one-year loss development factors from the final underlined row of Appendix B, Sheet 2 are selected to derive Table 4.3.1, Column (5) estimates of unpaid loss as of 12/31/97 reported as of 12/31/98 for the numerator of r_i. Table 4.3.1, Column (6) displays estimated r_i that incorporate 12/31/96 unpaid losses reported emergence during calendar year 1997.

[6] One year reported losses for accident year *i* as of year *x* is defined as: accident year *i* incremental losses paid during maturity year *x* plus accident year *i* case reserves as of maturity year-end *x*.

[7] One-year loss development factor for accident year *i* as of *x* is defined as: one year reported losses for accident year *i* as of year *x* divided by accident year *i* case reserves as of maturity year-end *x*-1.

TABLE 4.3.1

ASSUMPTIONS SELECTION

(1) Accident Year	(2) Unpaid Loss as of 12/31/96 Reported as of 12/31/97	(3) Case Reserves as of 12/31/97	(4) Selected One Year Development Factor	(5)= (3)x(4) Estimated Unpaid Loss as of 12/31/97 Reported as of 12/31/98	(6)= (5)/[Prior (2)] Selected Ratio Unpaid Loss	(7) Selected Unpaid Loss of Oldest Accident Year as of 12/31/97
i					Selected r_i	Selected $U_{1988,1997}$
1988	2,180					1,048
1989	6,504	1,419	1.3727960	1,948	0.8935768	
1990	4,868	1,436	1.6909393	2,428	0.3733378	
1991	16,314	3,282	1.3999528	4,595	0.9438465	
1992	29,232	11,991	1.7282284	20,723	1.2702701	
1993	39,406	15,482	1.2571046	19,462	0.6657941	
1994	102,952	46,505	1.4460186	67,247	1.7065192	
1995	132,879	55,399	1.6082550	89,096	0.8654103	
1996	142,865	70,761	1.8627350	131,809	0.9919475	
1997		61,839	2.7249017	168,505	1.1794715	

(1) m=1988; n=1997
(2) Appendix B, Sheet 2; One Year Reported final diagonal
(3) Appendix B, Sheet 2; final diagonal
(4) Appendix B, Sheet 2; Wt'd Avg. Dev. Factor
(7) CAS Loss Reserve Data Base [7] = company filed loss reserves (including IBNR) as of 12/31/97

Table 4.3.2 inputs Table 4.3.1 assumptions into the Model to derive estimated unpaid losses as of 12/31/97. The Table 4.3.2, Column (4) total unpaid loss estimate of $799,986 is closer to the Column (5) actual emergence of $815,254 than the Table 3.2, Column (4) total unpaid loss estimate of $853,442. Indeed, the Table 4.3.2, Column (4) total unpaid claim estimate has narrowed the retrospective test accuracy from within 5% to within 2% of the actual emergence. Nonetheless, one should not generally presume that incorporating one year reported emergence during calendar year *d* will necessarily yield more accurate unpaid claim estimates than use of more rudimentary assumptions such as (ii) from Section 3.1.

TABLE 4.3.2

INDICATED UNPAID LOSSES

(1)	(2)	(3)	(4)	(5)
Accident Year	Selected Ratio Unpaid Loss	Incremental Paid Loss During 1997	Indicated Unpaid Loss as of 12/31/97	Actual Emergence
i	Selected r_i	$p_{i,1997}$	Indicated $U_{i,1997}$	
1988		2,064	1,048	1,048
1989	0.8935768	5,085	2,781	2,229
1990	0.3733378	3,432	2,937	4,875
1991	0.9438465	13,032	6,011	8,939
1992	1.2702701	17,241	24,190	27,175
1993	0.6657941	23,924	27,584	38,236
1994	1.7065192	56,447	87,900	75,947
1995	0.8654103	77,480	124,919	130,558
1996	0.9919475	72,104	200,770	216,789
1997	1.1794715	21,098	321,847	309,458
Total			**799,986**	815,254

(1) m=1988; n=1997
(2) Table 4.3.1, Column (6)
(3) Table 3.2, Column (3)
(4) d=1997
For i = m = 1988: Table 4.3.1, Column (7)
For 1989 ≤ i ≤ 1997: (2)x[Prior (3)+Prior(4)]
(5) Computed from CAS Loss Reserve Data Base [7]
Actual Emergence = cumulative losses paid subsequent to 12/31/97 through nine years subsequent to accident year
+ company filed loss reserves (including IBNR) nine years subsequent to accident year

While obviously not the complete foreknowledge of Section 4.1, incorporating actual calendar year *d* one year reported emergence into estimated r_i includes additional loss experience available as of the valuation date that reflects more mature emergence toward the actual value of r_i than merely estimating r_i as case reserve ratios of Section 3.1. As such, the credibility of resulting r_i may be increased. It should also be noted that this procedure reintroduces a form of the loss development factor approach, albeit, for only one development year.

The foregoing procedure employs calendar year *d* reported emergence in the context of an incurred development method framework. More generally, this approach is applicable in the context of any loss reserving methodology that implicitly estimates accident year age-to-age development by calendar year.

4.4 Steady State Value for r_i = 1 + trend rate

A steady state system is defined herein as the same real (i.e., without consideration of frequency or severity trend) unpaid claim exposure as of common maturities for each accident year. In a steady state system, r_i = 1 + (net impact of frequency and severity trend between $U_{i,d}$ and $U_{i-1,d-1}$). Consequently, if there were no unpaid frequency trend and no unpaid severity trend, steady state r_i would equal 1 for each *i*. These are important benchmark properties of r_i to bear in mind while considering appropriate r_i. To the extent that an indicated r_i moves further away from 1 (or, more precisely, 1 + unpaid expected trend rate), it is worthwhile to confirm that such r_i are reasonable and that the accident year-over-year indicated change in unpaid loss exposure is warranted. r_i would be expected to deviate from steady state values if there were a significant change in the expected unpaid loss volume between successive accident years at common maturities. Note that the actual r_i derived from the actual emergence of the Section 3 example fall within a range from .66 to 1.24 as displayed in Table 4.1.1, Column (5).

Similar steady state properties are absent from chain ladder development methods since there is no universal steady state CDF value. Steady state r_i properties remain valid regardless of development period length. On the other hand, the greater the expected development from a particular maturity, the higher the corresponding indicated CDF will be as of that maturity. CDFs from early maturities for slow developing lines of business are typically significantly greater than 1. Under near steady state conditions, indicated CDFs for long tailed lines may also be highly leveraged. While the r_i implicit in actual emergence from the Section 3 other liability example cluster near unity, the corresponding actual emergence accident year 1997 one year-to-ultimate incurred development CDF[8] equals 3.986 and the corresponding actual emergence one year-to-ultimate payment development CDF[9] equals 15.668.

4.5 Earned Premium

As a result of the relatively high volume in the numerator and denominator, the ratio of successive accident year earned premium may provide stability and credibility to corresponding r_i indications. Initially, it is preferable to set all earned premium to a common rate adequacy level before estimating r_i using earned premium as this would normally be expected to provide a more accurate measure of relative exposure than unadjusted earned premium[10]. In addition to inconsistent premium adequacy, potential weaknesses of r_i based upon earned premium ratios are: they measure relative total accident year exposure rather than relative unpaid loss exposure; actual loss experience is not directly reflected; and expected unpaid losses are not directly considered. While the relative high volume of earned

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[8] Computed as (21,098 + 309,458)/(21,098 + 61,839) = 3.986 derived from Table 3.1 and Table 4.1.2

[9] Computed as (21,098 + 309,458)/21,098 = 15.668 derived from Table 4.1.2

[10] Pure Premium, the provision in the premium for loss & DCCE, would typically be an even more accurate basis for estimating r_i.

premium may add stability and credibility, a countervailing consideration is that resulting indicated r_i might suffer from reduced credibility as a result of potential earned premium ratio weaknesses. Table 4.5 uses earned premium from the Section 3 business segment to derive indicated r_i. For the final selection of r_i, it would typically be appropriate to complement earned premium ratio indications with other r_i measures since estimated r_i based solely upon earned premium would ignore the impact of recent loss experience through the valuation date.

TABLE 4.5

(1)	(2)	(3)= (2)/[Prior (2)]
Accident Year	Earned Premium	Indicated Ratio Unpaid Loss
i		Indicated r_i
1988	138,743	
1989	163,183	1.1761530
1990	162,184	0.9938780
1991	177,393	1.0937762
1992	197,770	1.1148692
1993	225,434	1.1398797
1994	267,578	1.1869461
1995	318,426	1.1900306
1996	363,402	1.1412447
1997	400,300	1.1015349

(2) CAS Loss Reserve Data Base [7]

4.6 Unpaid Claim Counts and Severity Indices

Where claim counts are available, their use may result in more accurate r_i estimates than other basic measures. When considering the use of claim counts, it is important that the definition and treatment of claim counts has been consistent. Potential claim count inconsistencies include, but are not limited to, changes in claim processing systems; treatment of incident claims; proportion of claims closed without payment; method of recording number of claims versus number of claimants; and time to establish claims. It may be possible to adjust raw claim counts to a more consistent basis in order to mitigate or eliminate potential inconsistencies. It may also be possible to employ inconsistent claim counts in a manner that would minimize the impact of potential distortions.

Unpaid claim counts may be estimated by various actuarial claim count methods including application of the Model. Unpaid claim counts together with an unpaid severity trend are used to estimate r_i as:

$$\text{Estimated } r_i = \frac{\text{Estimated } C_{i,d}}{\text{Estimated } C_{i-1,d-1}} \text{ x} \frac{\text{Estimated } S_{i,d}}{\text{Estimated } S_{i-1,d-1}},$$

Where $C_{i,j}$ = accident year i number of claims unpaid as of year-end j, where $j \geq i$,

$S_{i,j}$ = accident year i unpaid severity as of year-end j, where $j \geq i$.

The entire quantity $\frac{\text{Estimated } S_{i,d}}{\text{Estimated } S_{i-1,d-1}}$ may be estimated as the estimated unpaid severity percent increase of accident year i losses as of accounting date year-end d over estimated unpaid severity for accident year i-1 losses as of accounting date year-end d-1. For example, unpaid severity of accident year i losses as of accounting date year-end d estimated to be 3% greater than estimated unpaid severity of accident year i-1 losses as of accounting date year-end d-1 corresponds to $\frac{\text{Estimated } S_{i,d}}{Estimated\ S_{i-1,d-1}}$ equals 1.03.

Where data are organized by report year, claim counts are generally known by report year end. As such, the relative ratio of unpaid claims are known. Consequently, only an estimate of unpaid severity trend is required in order to estimate r_i in such a report year setting.

4.7 Other Measures and Adjustments

Depending upon the line of business, it may be worthwhile to investigate exposure measures not previously discussed. These include payroll, number of vehicles, miles driven, operating expenditures, square footage, average occupied beds, outpatient visits, and number of employees. Accident year-over-year comparisons of these types of measures may provide additional insight into appropriate estimated r_i.

It may be appropriate to adjust exposure measures for features that may not otherwise be captured. Adjustments may be appropriate for items such as policy limits and deductibles, reinsurance provisions, law changes, and tabular reserves. Littmann [6] and Struzzieri and Hussian [8] explore exposure adjustment concepts in greater detail. For the purposes of applying the Model, the key question of whether to adjust relative exposure candidate(s) is: Does the proposed adjustment(s) improve the accuracy of estimated r_i?

4.8 Optimal Estimated Relative Unpaid Losses r_i

As evidenced by the foregoing discussion, an optimal measure of r_i estimates cannot be universally prescribed to cover all circumstances. Further investigation may be warranted when competing initial r_i candidates result in divergent r_i indications. Additional insight may also be gained by exploring the sensitivity of unpaid claim estimates to several reasonable r_i indications. Within a business segment, it may be plausible that appropriate exposure measures may vary by accident year. It may also be reasonable to use a weighted average of different potential r_i measures as an appropriate r_i measure. The key principle is that optimal estimated r_i is the relative exposure measure (or combination of exposure measures) that most accurately estimates the ratio of exposure to unpaid losses for accident year *i* as of accounting date year-end *d* relative to unpaid losses for accident year *i*-1 as of accounting date year-end *d*-1.

Where indicated r_i have low credibility, it may be advisable to restrict the number of successive accident years included in the application of the Model. For example, relatively small remaining unpaid claim exposure for the oldest several accident years may result in volatile low credibility r_i indications for these accident years. It may be prudent to exclude these accident years, especially to the extent that low credibility r_i would have a leveraged effect on the unpaid loss indications for subsequent accident years. An extreme example would be a relatively old accident year with no remaining unpaid claims liability that results in an undefined or indeterminate (division by zero) r_i indication. This issue is discussed further in Section 5.

5. OLDEST ACCIDENT YEAR UNPAID LOSSES $U_{m,d}$

Successful implementation of the Model requires a reasonable estimate of unpaid losses (including IBNR) for the oldest included accident year $U_{m,d}$. Examples in this paper have accepted the company filed loss reserves (including IBNR) for the oldest accident year as the corresponding unpaid losses. Estimating unpaid losses for the oldest accident year is akin to estimating the tail in traditional loss development methods. The CAS Committee on Reserves [1] has compiled an extensive set of techniques to estimate tail factors. Many of these techniques may be readily adapted to estimate unpaid losses for the oldest included accident year.

Each application of the Model requires one to consider the oldest accident year *m* to include in the calculation. Under optimal circumstances: *m* is set at the oldest accident year with unpaid claim exposure as of accounting date year-end *d*; $U_{m,d}$ and each r_i are credible; and relatively small changes in $U_{m,d}$ and r_i result in relatively small changes in the resulting unpaid claims estimate. Where these conditions are not met, it may be more appropriate to set *m* equal to a later year than the oldest accident year in order to more closely approximate optimal Model conditions. Unpaid losses for accident years prior to *m* would normally be expected to be relatively small and may be estimated by methods other than applying the Model.

6. EMPIRICAL EVIDENCE

The CAS loss reserve data base [7][11] can be used to empirically compare the relative accuracy of commonly used loss reserving methods versus methods derived from the Model. Although the goal of the CAS data base is to "prepare a clean and nice data set of loss triangles that could be used for claims reserving studies," several issues preclude the use of every included company business segment for unbiased comparison (e.g., data abnormalities, sparseness). Consequently, each business segment is pre-screened for inclusion in the comparisons. For the 46 business segments that meet qualifying criteria, Table 6 uses actual emergence as a retrospective test to compare accuracy of 12/31/97 unpaid loss estimates for (a) the Payment Development Method, (b) the Incurred Development Method, (c) the Bornhuetter-Ferguson Method, and four (4) relative unpaid claims methods (*d*)-(*g*) derived from application of the Model.

TABLE 6

RETROSPECTIVE ACCURACY TEST OF 12/31/97 UNPAID CLAIM ESTIMATES:
46 Qualifying CAS Loss Reserve Data Base U.S. Property/Casualty Business Segments

(1) Loss Reserving Method	(2) Number of Business Segments where Estimate Falls Within 20% of Actual Emergence	(3) Number of Business Segments where Estimate Falls Within 10% of Actual Emergence
Payment Development (a)	19	13
Incurred Development (b)	26	17
Bornhuetter-Ferguson (c)	32	21
Relative Unpaid Claims 1 (d)	30	16
Relative Unpaid Claims 2 (e)	27	18
Relative Unpaid Claims 3 (f)	38	21
Relative Unpaid Claims 4 (g)	33	23

(2) Number of 46 Business Segments where $1/1.2 \leq$ (Estimated Unpaid Loss)/(Actual Emergence) ≤ 1.2
(3) Number of 46 Business Segments where $1/1.1 \leq$ (Estimated Unpaid Loss)/(Actual Emergence) ≤ 1.1

6.1 Criteria for Inclusion

Business segments were pre-selected from the CAS data base for consistency, credibility and

[11] The CAS data base is "a data set that contains [net of reinsurance] run-off triangles of six lines of business [private passenger auto liability/medical; commercial auto/truck liability/medical; workers' compensation; medical malpractice – claims made; other liability – occurrence; and products liability] for all U.S. property casualty insurers. The triangle data correspond to claims of accident year 1988 -1997 with 10 years of development lag. Both upper and lower triangles are included so that one could use the data to develop a model and then test its performance retrospectively". The Section 3 example uses data from a large business segment (Company Code 1767) of other liability experience drawn from the CAS data base.

compatibility with each of the seven (7) methods under consideration. Recalling that all dollar figures presented throughout this paper are displayed with thousands of dollars omitted, each selected business segment must meet the following criteria:

- Actual emergence of at least $25,000;
- Positive earned premium for each calendar year 1988 through 1997;
- Non-negative calendar year 1997 loss payments for each accident year 1988 through 1997;
- Each accident year 1988 through 1996 case reserve as of 12/31/96 at least equal to $25 and each accident year 1989 through 1997 case reserve as of 12/31/97 at least equal to $25; and
- No division by zero in working through any of the seven methods.

This filtering results in 46 business segments for comparison testing including the Section 3 example business segment.

6.2 Seven Unpaid Claim Methods

Ordinarily, sound actuarial practice would not blindly rely upon mechanical 'cookbook' procedures. Nevertheless, in order to objectively analyze and compare method performance, it is necessary to make standardized assumptions. If the only information available were the CAS loss reserve data base experience as of accounting date 12/31/97, we attempt to standardize how a practicing actuary might typically implement three commonly applied loss reserving methods – Payment Development Method, Incurred Development Method, and Bornhuetter-Ferguson Method. Four methods derived from the Model are also standardized.

All seven methods accept accident year 1988 company filed loss reserves (including IBNR) as of accounting date 12/31/97 as the estimate for the corresponding unpaid losses. For the calculation of CDFs, it follows that this filed loss reserve plus accident year 1988 cumulative paid losses through 12/31/97 are assumed to be accident year 1988 ultimate losses. The 10 year-ultimate tail payment (or reported) development CDF is, therefore, assumed to equal these accident year 1988 ultimate losses divided by accident year 1988 cumulative loss payments (or reported losses) through 12/31/97.

The seven standardized methods used to estimate 12/31/97 accounting date unpaid losses are discussed below:

<u>Payment Development Method[12] (a)</u>- For each development period, select each LDF equal to the (up to) three most recent dollar weighted average payment LDFs as of accounting date 12/31/97.

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12 Friedland [4], Chapter 7

Incurred Development Method[13] (b)- For each development period, select each LDF equal to the (up to) three most recent dollar weighted average reported LDFs as of accounting date 12/31/97.

Bornhuetter-Ferguson Method[14] (c)- Select Expected Loss Ratio equal to combined accident years 1988 through 1990 Incurred Development Method estimated ultimate loss ratio[15]. For accident years where Incurred Development method CDF>1.000, select these CDFs for use in the Bornhuetter-Ferguson Method. For accident years where Incurred Development method CDF≤1.000, select accident year Incurred Development Method estimated ultimate losses.

Relative Unpaid Claims Method 1 (*d*)- Assume r_i equals case reserve ratios as implemented in Section 3.

Relative Unpaid Claims Method 2 (e)- Assume r_i equals estimated one year reported emergence ratios as implemented in Section 4.3.

Relative Unpaid Claims Method 3 (f)- Assume r_i equals 0.75 x (r_i of Relative Unpaid Claims Method 1) + 0.25 x (earned premium ratios as in Table 4.5, Column (3)). Assigning 75% weight to case reserve ratios and 25% weight to earned premium ratios is one approach to estimating r_i by blending a loss experience-based estimate with an *a priori* earned premium based estimate.

Relative Unpaid Claims Method 4 (*g*)- Assume r_i equals 0.75 x (r_i of Relative Unpaid Claims Method 2) + 0.25 x (earned premium ratios as in Table 4.5, Column (3)).

Since the CAS data base does not capture claim count experience, it does not permit us to also explore and compare unpaid claim estimates using reserving methods that rely upon claim counts.

6.3 Accuracy Measure

Unpaid loss estimates are calculated using all seven Section 6.2 methods for each of the 46 qualifying business segments. Table 6 is a retrospective accuracy test that displays the number of business segments where the 12/31/97 estimated unpaid claim estimate fall within 20% and 10%[16] of actual emergence. Notwithstanding randomness, methods where more of the 46 business segments

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13 Ibid

14 Ibid, Chapter 9, Incurred Bornhuetter-Ferguson version

15 Selection of an Expected Loss Ratio is, perhaps, the most challenging assumption to standardize. Other possibilities were considered such as: choosing a different number of years than the oldest three accident years- however, three years seems to strike a reasonable balance between capturing loss ratio information and not simply reiterating Incurred Loss Development method indications; using the company incurred losses (including IBNR) for more than the oldest accident year in the numerator of the expected loss ratio calculation instead of ultimate losses indications from the Incurred Loss Development method- however, this would incorporate company knowledge absent from the other six methods; choosing a fixed expected loss ratio (e.g., 60%, 65%) across all accident years for all business segments- however, this would ignore the loss ratio tendencies of the particular business segment; and choosing expected loss ratios in conjunction with the historical industry underwriting cycle- however, this would use information external to the CAS loss reserve data base unlike any of the other six methods.

16 Since the distribution of liability unpaid losses is typically right skewed:
Actual emergence within 20% of estimate is defined as 1/1.2 ≤ (estimated unpaid loss)/(actual emergence) ≤ 1.2;
Actual emergence within 10% of estimate is defined as 1/1.1 ≤ (estimated unpaid loss)/(actual emergence) ≤ 1.1.

have unpaid claim estimates that fall within a specified range are empirically more accurate than those methods where fewer fall within that range.

6.4 Discussion of Results

Based upon review of Table 6, we observe the empirical comparative accuracy of the seven loss reserving methods tested.

The relatively poor performance of the Payment Development Method is consistent with Forray's [3] observation that this method should not generally receive the weight it often does. The Incurred Development Method is best compared with Relative Unpaid Claims Method 1 and Relative Unpaid Claims Method 2 since these all only rely upon payments and case reserves (or estimated one year reported emergence) without reference to earned premium exposure. Although requiring much less historical experience, Relative Unpaid Claims Method 1 performs similarly to the Incurred Development Method. Relative Unpaid Claims Method 2 slightly outperforms the Incurred Development Method.

The Bornhuetter-Ferguson Method outperforms the other two traditional reserving methods. This is also consistent with Forray's [3] inference that the incurred Bornhuetter-Ferguson Method is the best performing method in common use[17]. The Bornhuetter-Ferguson Method is most comparable to Relative Unpaid Claims Methods 3 and 4 since these all consider earned premium exposure. Unlike the Bornhuetter-Ferguson Method, Relative Unpaid Claims Methods 3 and 4 have the significant advantage that selection of expected loss ratios is not required. By assigning one-quarter weight to earned premium ratios, we are attempting to bring stability and additional credibility to estimated r_i. Relative Unpaid Claims Methods 3 and 4 perform at least as well as the Bornhuetter-Ferguson Method. The best performing method for the 20% range is Relative Unpaid Claims Method 3 and the best performing method for the 10% range is Relative Unpaid Claims Method 4.

While Relative Unpaid Claims Methods 3 and 4 use one particular weighting scheme (75% weight to case reserve, or estimated one year reported emergence, ratios; 25% weight to earned premium ratios) to estimate r_i, many other weightings between case reserve (or estimated one year reported emergence) ratios and earned premium ratios may also be reasonable. One possibility is to formulate a credibility weighting scheme between case reserve (or estimated one year reported emergence) ratios and earned premium ratios. Another avenue for exploration is to incorporate r_i steady state properties into a credibility weighting procedure. Investigation of suitable credibility weightings is a fertile area for future research.

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17 Forray measures comparative performance via relative "Method Skill". The expected loss ratios used in Forray's incurred Bornhuetter-Ferguson formulation are industry-based.

No attempt is made to apply rigorous statistical tests of significance to our observations regarding unpaid claims estimates derived from the Model compared with traditional actuarial loss reserving methods. However, our heuristic approach generally suggests that unpaid claim estimates derived from applications of the Model are at least as accurate as comparable unpaid loss estimates derived from commonly applied actuarial loss reserving methods. In any case, perceived overall improved accuracy over a specific historical data set would not guarantee improved accuracy for any particular future instance where the Model may be applied.

7. SUMMARY RESULTS AND DISCUSSION

This paper presents a straightforward Relative Unpaid Claims Loss Reserving Model. Examples are presented to highlight practical applications of the Model and considerations are explored to offer guidance in the selection of appropriate parameters for methods that apply the Model. In general, methods that apply the Model require less data and information and fewer assumptions than traditional chain ladder loss development methods. Empirical testing suggests that unpaid claim estimates derived from applications of the Model are generally as accurate, if not more accurate, than comparable unpaid claim estimates derived from commonly applied actuarial loss reserving methods. In consideration of the above, the loss reserving paradigm set forth in this paper provides a very practical and powerful tool for the estimation of unpaid claims.

8. CONCLUSION AND FUTURE RESEARCH

With its focus on appropriate parameters that measure prospective emergence, the Relative Unpaid Claims Loss Reserving Model provides actuaries the opportunity and flexibility to tailor methods to the circumstances of business segments under review and to directly estimate unpaid losses. While the paper explores many Model parameter options, additional research is encouraged to study techniques to further improve parameter accuracy and, thereby, increase the accuracy of resultant unpaid claims estimates. Additional research topics include: rigorous statistical tests comparing the accuracy of Relative Unpaid Claims Loss Reserving versus basic loss reserving methods; special considerations for small books of business and low credibility data; and appropriate treatment of negative loss payments.

Although this paper introduces Relative Unpaid Claims Loss Reserving and has concentrated on unpaid claims point estimates, it also paves the way toward future work that would cast the Relative Unpaid Claims Loss Reserving Model in a stochastic framework.

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Abbreviations and notations

ALAE, allocated loss adjustment expenses
CAS, Casualty Actuarial Society
CDF, cumulative loss development factor
DCCE, defense and cost containment expenses
IBNR, incurred but not reported loss (i.e., all unreported development beyond case reserves)
LDF, age-to-age loss development factor
NAIC, National Association of Insurance Commissioners

$ dollars are displayed with thousands of dollars omitted throughout this paper.

"Actual emergence" is defined throughout this paper as cumulative losses paid subsequent to 12/31/97 through nine years subsequent to the accident year plus company filed loss reserves (including IBNR) as of nine years subsequent to the accident year.

Biography of Author

Bertram A. Horowitz is President of Bertram Horowitz, Inc. Actuarial and Risk Consultants which provides property/casualty and title insurance actuarial and risk assessment services. He has a B.S. degree in Applied Mathematics from the State University of New York at Stony Brook and a M.S. in Mathematics from Brown University. He is a Fellow of the CAS and a Member of the American Academy of Actuaries. Mr. Horowitz is the former Special Deputy Superintendent and Financial Actuary of the New York State Insurance Department (now the New York State Department of Financial Services). He has served on the CAS Committee on Reserves and has been an active participant in the development of actuarial research, principles and standards.

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APPENDIX A

Closed-Form Model Representation:

$$U_{i,d} = U_{m,d} \prod_{m+1 \le j \le i} r_j + \sum_{m \le k \le i-1} [p_{k,d}] \prod_{k+1 \le j \le i} r_j \text{, where } m+1 \le i \le n$$

Proof:

$$U_{m+2,d} = r_{m+2}[r_{m+1}(U_{m,d} + p_{m,d}) + p_{m+1,d}$$

$$U_{m+3,d} = r_{m+3}\{r_{m+2}[r_{m+1}(U_{m,d} + p_{m,d}) + p_{m+1,d}] + p_{m+2,d}\}$$

$$= r_{m+3}r_{m+2}r_{m+1}(U_{m,d} + p_{m,d}) + r_{m+3}r_{m+2}p_{m+1,d} + r_{m+3}p_{m+2,d}$$

$$U_{i,d} = U_{m,d} \prod_{m+1 \le j \le i} r_j + \sum_{m \le k \le i-1} [p_{k,d}] \prod_{k+1 \le j \le i} r_j$$

Q.E.D.

APPENDIX B, SHEET 1

SECTION 3 EXAMPLE BUSINESS SEGMENT
HISTORICAL INCREMENTAL PAID LOSSES AND CASE RESERVES

Accident Year	**Paid During Year 1**	*Case Reserves as of Year 1*	**Paid During Year 2**	*Case Reserves as of Year 2*	**Paid During Year 3**	*Case Reserves as of Year 3*	**Paid During Year 4**	*Case Reserves as of Year 4*	**Paid During Year 5**	*Case Reserves as of Year 5*	**Paid During Year 6**	*Case Reserves as of Year 6*	**Paid During Year 7**	*Case Reserves as of Year 7*	**Paid During Year 8**	*Case Reserves as of Year 8*	**Paid During Year 9**	*Case Reserves as of Year 9*	**Paid During Years 10**	*Case Reserves as of Year 10*
1988	3,962	*18,455*	22,325	*32,519*	26,713	*24,536*	39,637	*10,366*	13,699	*6,640*	10,341	*3,393*	3,939	*4,025*	3,161	*3,177*	2,079	*1,588*	2,064	*116*
1989	6,066	*18,674*	22,231	*27,084*	31,838	*16,408*	23,890	*13,583*	19,061	*10,691*	14,446	*6,809*	4,848	*3,791*	3,734	*2,838*	5,085	*1,419*		
1990	3,751	*15,681*	27,752	*32,388*	36,613	*26,127*	32,308	*19,254*	12,364	*12,150*	10,289	*6,913*	6,004	*4,883*	3,432	*1,436*				
1991	3,336	*22,485*	42,852	*38,265*	49,612	*40,475*	39,363	*24,041*	17,983	*16,674*	12,284	*7,016*	13,032	*3,282*						
1992	6,647	*31,730*	42,672	*48,726*	45,541	*43,345*	34,446	*25,248*	18,929	*23,466*	17,241	*11,991*								
1993	8,056	*44,945*	73,031	*69,391*	66,645	*48,541*	45,543	*31,248*	23,924	*15,482*										
1994	9,720	*41,128*	55,619	*62,428*	64,964	*56,994*	56,447	*46,505*												
1995	7,171	*51,969*	75,651	*66,826*	77,480	*55,399*														
1996	16,696	*54,941*	72,104	*70,761*																
1997	21,098	*61,839*																		

CAS Loss Reserve Data Base [7]

APPENDIX B, SHEET 2

SECTION 3 EXAMPLE BUSINESS SEGMENT
HISTORICAL ONE YEAR REPORTED EMERGENCE

Accident Year		*Case Reserves as of Year 1*	**One Year Reported as of Year 2**	*Case Reserves as of Year 2*	**One Year Reported as of Year 3**	*Case Reserves as of Year 3*	**One Year Reported as of Year 4**	*Case Reserves as of Year 4*	**One Year Reported as of Year 5**	*Case Reserves as of Year 5*	**One Year Reported as of Year 6**	*Case Reserves as of Year 6*	**One Year Reported as of Year 7**	*Case Reserves as of Year 7*	**One Year Reported as of Year 8**	*Case Reserves as of Year 8*	**One Year Reported as of Year 9**	*Case Reserves as of Year 9*	**One Year Reported as of Year 10**
1988		18,455	54,844	32,519	51,249	24,536	50,003	10,366	20,339	6,640	13,734	3,393	7,964	4,025	6,338	3,177	3,667	1,588	2,180
	1 Year LDF		2.972		1.576		2.038		1.962		2.068		2.347		1.575		1.154		1.373
1989		18,674	49,315	27,084	48,246	16,408	37,473	13,583	29,752	10,691	21,255	6,809	8,639	3,791	6,572	2,838	6,504	1,419	
	1 Year LDF		2.641		1.781		2.284		2.190		1.988		1.269		1.734		2.292		
1990		15,681	60,140	32,388	62,740	26,127	51,562	19,254	24,514	12,150	17,202	6,913	10,887	4,883	4,868	1,436			
	1 Year LDF		3.835		1.937		1.974		1.273		1.416		1.575		0.997				
1991		22,485	81,117	38,265	90,087	40,475	63,404	24,041	34,657	16,674	19,300	7,016	16,314	3,282					
	1 Year LDF		3.608		2.354		1.566		1.442		1.157		2.325						
1992		31,730	91,398	48,726	88,886	43,345	59,694	25,248	42,395	23,466	29,232	11,991							
	1 Year LDF		2.880		1.824		1.377		1.679		1.246								
1993		44,945	142,422	69,391	115,186	48,541	76,791	31,248	39,406	15,482									
	1 Year LDF		3.169		1.660		1.582		1.261										
1994		41,128	118,047	62,428	121,958	56,994	102,952	46,505											
	1 Year LDF		2.870		1.954		1.806												
1995		51,969	142,477	66,826	132,879	55,399													
	1 Year LDF		2.742		1.988														
1996		54,941	142,865	70,761															
	1 Year LDF		2.600																
1997		61,839 `																	
W'td Avg. Dev. Factor			2.7249017		1.8627350		1.6082550		1.4460186		1.2571046		1.7282284		1.3999528		1.6909393		1.3727960

One Year Reported as of Year x = Appendix B, Sheet 1: Paid During Year x + Case Reserves as of Year x
Wt'd Avg. Dev. Factor equals dollar weighted average of (up to 3) most recent years underlined 1 Year LDFs

The Role of the Reserving Actuary in the Closing Process

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Abstract: The paper discusses the processes that the reserving actuary needs to be aware of and the contributions that the actuary can make during the annual or quarterly company financial close process ("the Closing Process"). In describing the actuary's role, I attempt to show that it is not enough to be concerned with just the bottom line and determining the "right" number. Rather I try to formalize and illustrate analysis and communications that need to take place during the closing process. I provide some tips and guidance on anomalies to look for in the data and results and provide options for dealing with them and calculating a reasonable amount of IBNR. I also show how having a robust closing process can help the actuary stay informed about the business environment in order to make the actuarial projections more robust. Finally, I discuss the use of reserve ranges and consideration of professional standards of practice during the closing process.

Keywords: Reserving, IBNR, Pricing, Standards of Practice, Reserve Ranges

1. INTRODUCTION

One of the most critical processes that a company executes either on a monthly or quarterly basis is the closing of its accounting books. It is during this window of time that a company reconciles its balance sheet and finalizes the profit or loss for the period under consideration. There has not been much formal discussion in the actuarial literature, describing the role that an actuary can play in this process. It could be that such a discussion has not been deemed necessary as the process is viewed as simple and straightforward. However, like most things in life, I believe that the complexity of the process becomes apparent once one is actively involved in such a process. Based on my own experience, I believe there may be many actuaries who are not very familiar with the process and may not appreciate what is involved. Furthermore, even those who are familiar with process may overlook some critical details. The paper attempts to discuss on a formal basis several aspects of the closing process. The first section of the paper explains the role and responsibilities of the reserving actuary during the closing process, a brief overview of the timeline of the closing process and how this is linked to the company's reserve analysis. The second and the third sections presents suggestions on how to estimate the IBNR during the closing process. The paper then presents an approach on how to review the results and explain the types of communication the actuary should have with business partners in the company. The next section explains the role that reserve ranges play in the closing process. Finally, I conclude with a discussion on the importance of the application of the actuarial standards of practice in the closing process.

2. THE CLOSING PROCESS

2.1 Defining the Process

Before we begin to explain the role of the actuary, we first must define what is meant by the closing process.

The accounting closing process, also called closing the books, is the steps required to prepare accounts for financial statement preparation and the start of the next accounting period. The closing process consists of steps to transfer temporary account balances to permanent accounts and make the general ledger ready for the next accounting period. For an insurance company this means making sure that all the premiums and losses are recorded on the balance sheet. Both the unearned premium reserves and loss reserves from the prior close are reversed out and rebooked for the current close and the appropriate commissions are booked.

Insurance Companies have different approaches as to how frequently they close their books. Whereas primary companies close monthly, reinsurance companies tend to follow a quarterly closing process. I have tried to be as generic as possible in recognition of the differences between primary and reinsurance companies. Where possible, I have used the term "closing period" as opposed to a "monthly" or "quarterly" period. I have also used the term "year" rather than specify whether it is an accident year or underwriting year. However, where necessary for the sake of clarity, the exhibits produced are from the perspective of a reinsurer that closes their books every quarter. Although the terminology will be reinsurance specific, the ideas and thoughts can be utilized by primary companies as well.

2.2 The Responsibilities of the Actuary

The reserving actuary has several roles to play during the closing process. The most obvious one is finalizing and updating the IBNR for the closing period. However, besides calculating the IBNR, there are several other crucial areas for which the actuary is responsible. For example, there are various calculations like audit premium, reinstatement premiums and profit commissions that are often dependent on loss and IBNR information. Although these numbers can be automatically generated by a company's information system, very often it is the job of the actuary to make sure that these calculations are being performed correctly. The actuary can also be responsible for calculating the ULAE or the appropriate reserve discount in cases where the company discounts its reserves. Finally, the actuary should also be prepared to verify the paid loss, case reserves and premiums that go into the IBNR calculation. The actuary must make sure that the correct losses and premium information is accurately being reflected in what is being booked during the closing process. This last step is especially crucial because if the premium and losses are not correct, the applicable IBNR will also not be correct.

2.3 Timeline of the Closing Process

Most companies will have an official start to the closing process. Once the closing process starts, the actuary is responsible for performing all the calculations mentioned above. Throughout the closing process, the actuary will be constantly changing and tweaking the results as new information becomes available. At some point, the closing process will come to an end ("pencils down") and all the calculations will need to be finalized. These final calculations will be what is booked on the company's official balance sheet.

Although the general approach is the same for all companies, the timing and the length of the closing process can vary from company to company. For example, when it comes to the frequency of the closing process, there can be differences between a primary and reinsurance company. A primary company, where losses are reported more frequently, might close its books every month. A reinsurer on the other hand, which might only receive losses quarterly, will only close its books quarterly. Regarding the length of the process, some companies might opt for a quick turn around and allow only a short period of time from the start of the close until the end, while other companies will allow for a longer period.

The length of time allotted for the closing period, will have a major impact on how the actuary goes about calculating the IBNR. If the closing period is long enough, then the actuary will have the opportunity to perform a full reserve analysis and book the IBNR based on the full analysis. However, for those companies with a short turn around, a full reserve analysis is impractical. Instead, a decision will have to be made whether to perform a full analysis before the close or after the close. There are several factors that can determine if the reserve review is done before or after the close. For example, there just might not be enough time due to other actuarial responsibilities to perform an adequate analysis pre-close. Another factor to consider is the impact the new information received during the close will have on the IBNR. If the IBNR needs to be adjusted significantly for the new closing data, then it might just make sense to wait until after the close to perform a reserve analysis. In any event, if the analysis is done before the close, then the IBNR booked during the close will be based on pre close numbers but it will need to be updated for the results of the analysis and any new information that becomes available. If the analysis is done after the close, then the IBNR booked during the close will be calculated by taking the pre-close IBNR and making adjustments based on the new information that comes in during the close.

3. RECALCULATING THE IBNR

As previously mentioned, one of the responsibilities of the reserving actuary is to finalize and update the IBNR for the closing period. In the event that a full reserve review is impractical during the closing period, the question arises, what is the best way to determine the IBNR?

3.1 Rebook Prior IBNR

There are many different approaches and there is no one size fits all approach. For example, one option might be to look at what is driving the need to adjust the IBNR. It is possible that there was a large loss like a catastrophe that caused the IBNR to drop more than expected. It is also possible that the company instead of loading the IBNR in dollars, chooses to load loss ratios which calculate the IBNR. If this were the case then maybe the premium was corrected downward by accounting, and this caused the IBNR to drop as well. In both these cases, one can correct the IBNR by simply rebooking the IBNR from the prior close after subtracting the non-cat losses that were reported in the most recent period and adding IBNR for any new premium earned in the period. An example is shown below:

TABLE 1

Closing Period Results (in thousands)				
(1)	(2)	(3)	(4)	(5)
Additional Earned Premium	Expected Loss Ratio	Old IBNR	New Losses (incl. catastrophe losses)	Non-Catastrophic Losses
1,000	75%	6,000	5,000	450

(6) = (3) - (4)	(7) = (1) * (2)	(8) = (3) - (5)	(9) = (7) + (8)
IBNR before adjustment	IBNR for new Earned Premium	IBNR adj for losses	New IBNR
1,000	750	5,550	6,300

However, this approach has its limitations. First of all, it is only useful for those situations where the IBNR has changed due to unique loss events or accounting corrections. For the cases where IBNR is affected by your typical premium and loss activity, this approach would not work. Furthermore, one must be mindful and careful to include any IBNR for new earned premium and to only exclude the unique loss events from the IBNR calculation. Determining which are the loss events that should have no impact on the IBNR is not necessarily straight forward. Do you exclude only losses related to catastrophes or do you use a dollar threshold? How one answers these questions will have an impact on the final IBNR selected for the period. Finally, although the calculations in this approach are simple, sorting through all the data and deciding which years require adjustment can be tedious and time consuming.

3.2 Statistical Method

A second possible approach is to use a statistical distribution around your reserve range. Within the actuarial literature there are numerous papers describing various stochastic models that can be used to simulate a reserve range. Although a discussion of the various models that are utilized by actuaries is beyond the scope of this paper, the CAS literature has many papers devoted to the subject. One useful source is the report published by The CAS Working Party on Quantifying Variability in Reserve Estimates ((The CAS Working Party on Quantifying Variability in Reserve Estimates, 2005) which summarizes some of these methods. As it relates to the closing process, one could decide on an appropriate IBNR estimate based on a probability distribution and during the closing process ensure that the booked IBNR falls within this range. Of course, this approach will only work for those companies who incorporate probability distributions in their regular reserving analysis. Another complication with this method is that it is not always so clear how to incorporate the distribution of IBNR into one final number. The CAS working party admits as much in their final conclusion in Section 7.8 on page 125. They conclude "What to do with the estimate of variability is beyond the scope of this paper. … Assuming a reasonable distribution can be estimated, what to book becomes an issue for various professional organizations concerned with the financial statements such as the AAA, AICPA SEC, IRS". This is especially true for the closing process, where one might want to utilize this approach to tweak the IBNR by line of business and/or by year. One would need to figure out the best way to ensure the appropriate IBNR level for each line of business and year using the probability distribution.

3.3 Roll-Forward Approach

A third option for adjusting IBNR based on the most recent data, is a roll forward approach that adjusts the parameters underlying the reserve study analysis and updates the IBNR calculation based on the updated data. There are several ways that this can be updated depending on the underlying methods in the reserve study. For examples for the chain ladder method this would involve interpolating the loss development factors from the prior analysis to the current quarter. The interpolated link ratios are then applied to the most recent data, either by using the chain ladder method or the Bornhuetter-Fergusin method, to recalculate the IBNR.

The advatntage to this approach is that it is a quick way to recalculate your IBNR using the assumptions from your prior reserve analysis. The exhibit should be easy to produce and any changes in IBNR can be easily explained. This method might not be appropriate for those companies which might rely on frequency-severity methods or stochastic methods to produce their IBNR. However, for those companies who tend to rely on the chain-ladder method and Bornhuetter-Ferguson method, this approach works quite well.

As an example, let us assume that the most recent analysis was done at year end 2018, and we are now closing the books for the third quarter in 2019. We would first interpolate the link ratios, (or the percent reported) so that the 12-ult, 24-ult, 36-ult etc. link ratios would be interpolated to 9-ult, 21-ult, 33-ult etc. The interpolation method can vary, ranging from a simple linear interpolation to an interpolation based on curve fitting, like a cubic-spline approach. It should be noted, that different interpolation methods can produce different results, so care should be taken in choosing an interpolation method. Using the new premiums and losses for the quarter, one can simply recalculate the chain ladder and Bornhuetter-Ferguson methods to come up with updated IBNR amounts. The exhibit below shows an example of this approach. The percent reported in the table below represent the interpolated percentages derived from the most recent analysis. Also, in this table there are four options to choose from; an updated chain ladder method, an updated B-F method and a loss ratio option using either the a-priori or the best estimate from the most recent analysis. Of course if there are other methods that you want to roll forward, you can include them as well.

TABLE 2

Roll Forward Exhibit (in thousands)													
UY	Earned Premium	Ultimate Premium	Incurred Loss	% Reported	A-Priori Loss ratio	Roll-Forward Chain Ladder Ultimate Loss Ratio	Roll-Forward Chain Ladder IBNR	Roll-Forward B-F Ultimate Loss Ratio	Roll-Forward B-F IBNR	Latest Best Ultimate Loss Ratio	Selected Ultimate Loss Ratio	Selected IBNR	Current IBNR
2010	1,034	1,034	579	100%		56%	0			54%	56%	0	15
2011	857	857	1,446	99%		171%	15			40%	171%	15	54
2012	1,387	1,387	945	97%		70%	29			40%	70%	29	76
2013	1,267	1,267	1,000	93%		85%	78			84%	85%	78	123
2014	1,387	1,387	1,312	88%	75%	107%	178	104%	124	78%	107%	178	234
2015	1,243	1,243	922	81%	75%	92%	218	88%	178	73%	92%	218	323
2016	1,389	1,389	939	71%	60%	96%	393	85%	246	79%	85%	246	250
2017	1,803	1,803	757	57%	60%	74%	579	68%	469	71%	68%	469	400
2018	1,655	1,839	649	40%	60%	88%	809	71%	531	92%	60%	345	345
2019	766	1,179	230	23%	60%	85%	418	66%	273	0%	60%	230	230
TOTAL	**12,789**	**13,385**	**8,778**									**1,807**	**2,049**

There are actually several advantages in using this approach. First of all, it is a simple method that is easy to calculate in a spreadsheet. The results can be easily understood and explained to management and it mirrors the approach taken for the full reserve analysis. Also, if IBNR adjustments need to be made for a particular line or year, all you need to do is look at the results of the roll-forward to get the proposed IBNR. There are no additional calculations that are necessary. Finally, the current IBNR can easily be compared to the roll forward IBNR in order to determine which areas are carrying too much or too little IBNR.

It should be pointed out that just because the roll forward method indicates that the IBNR is deficient or a surplus, does not mean that one should automatically book the new IBNR. One needs to understand that if a particular line is relatively volatile, then although for this closing period, the roll-forward shows a surplus, the next period might show a deficiency. This is especially true for the longer tail lines like casualty or workers comp. Unless the booked IBNR is significantly different than the roll-forward indications, it may be prudent not to touch the IBNR during the closing period and revisit the results during the reserve analysis. However, for the short tail property lines, it is probably worthwhile to look at the roll forward each quarter. Another area of caution is to be careful about cherry picking your results. One might see that for some years the roll forward IBNR indicates a surplus and for other years it indicates a deficiency. For example, in the above exhibit, 2015 indicates a surplus (the roll forward IBNR is lower than booked) while for 2017 the results indicate a deficiency (the roll forward IBNR is higher than booked). It might not be appropriate to just select the roll forward IBNR for 2015 while ignoring the 2017 year. Unless one has a good reason, you should either change both years or leave them both as is.

3.4 Actual vs Expected

Another useful tool for the actuary to consider would be an Actual vs Expected projection. This is where one looks at the results that have come in over a period of time as compared to what was expected to come in. One could look at periods of a month, a quarter or a year. The time frame might depend on how frequently (monthly or quarterly) the company closes its books. Also, depending on the credibility and volume of the data one might decide that looking at a monthly or quarterly Actual vs Expected might not be as meaningful as an Actual vs Expected over the entire year. This approach is not necessarily used to project the amount of IBNR needed, but rather it is useful as a diagnostic tool to aid in assessing the appropriate level of IBNR. It can be used to provide justification for adjusting IBNR in the current closing period. Finally, it can also be used as an early warning system to identify the lines of business and years in which the currently booked IBNR may not hold.

For example, as part of the AvE calculation, one can look at ratios of actual over expected and highlight those areas in which the ratio lies outside a predetermined range. One would then have an idea about which lines and years need to be investigated in greater detail. Based on the results of the investigation, a determination could be made about whether the current IBNR levels are adequate. One could also set up a graph as a visual aid to help show the difference between the actual vs expected results. This would be extremely useful when sharing the results with management or the underwriters. Finally, one can create a graph to identify any trends, like reporting frequency, that might exist in the data.

There are several different options in how to set up an AvE calculation. For example, one can look at the AvE based on paid losses or an AvE based on reported losses. What follows is an example of an AvE calculation using reported losses and projecting the expected losses for the first quarter 2019 with some short explanations.

TABLE 3

Actual vs Expected (in thousands)								
	(1)	(2)	(3)	(4)	(5)	(6) = (1) * [(4) - (3)]	(7) = (6) - (5)	(8) = (7) / (2)
UWY	Ultimate Losses 2018 analysis	IBNR 2018 analysis	Cumulative Percent Reported Development as of Q4 2018	Cumulative Percent Reported Development as of Q1 2019	Actual Reported Losses in the Quarter	Expected Losses to be Reported in the Quarter	Actual vs Expected	Actual vs Expected (as % of IBNR)
2005	8,214	83	97%	100%	0	210	(210)	-253%
2006	4,526	66	96%	97%	0	22	(22)	-33%
2007	7,862	167	96%	96%	0	38	(38)	-22%
2008	14,025	432	95%	95%	0	66	(66)	-15%
2009	5,124	230	94%	94%	0	24	(24)	-10%
2010	5,234	367	93%	93%	(0)	24	(25)	-7%
2011	7,564	702	92%	92%	(52)	35	(87)	-12%
2012	9,875	1,616	91%	91%	69	45	24	1%
2013	10,254	2,716	89%	90%	70	152	(82)	-3%
2014	13,268	6,952	80%	83%	607	335	272	4%
2015	14,523	6,925	66%	70%	2,467	588	1,879	27%
2016	19,823	11,817	52%	55%	(9)	731	(741)	-6%
2017	25,000	14,204	36%	40%	3,130	1,056	2,074	15%
2018	21,986	20,734	15%	20%	568	1,052	(484)	-2%

i. Column 1 is the projected ultimate loss from the most recent analysis. These losses will be used to calculate the expected losses for the quarter.

ii. Column 3 is the percent of losses expected to be reported at the prior quarter and Column 4 are the percentages for the current quarter. The difference between the two is the incremental percentage or the percentage of losses that we expect to see this quarter.

iii. Column 6 multiplies the incremental percentage by the projected ultimate loss to calculate the expected loss to quarter.

iv. Colum 8 is the ratio of AvE to initial IBNR.

Rather than taking the actual ratio of Actual/Expected, we are taking the Actual- Expected as a ratio to IBNR from the prior analysis. The advantage of calculating it this way is that it allows you to see the impact the Actual vs Expected has on your IBNR. A high positive ratio indicates that more losses came in than expected and it will eat up more of your IBNR. If the ratio is positive but low, we can conclude that although more losses came in than expected, the impact to your IBNR is minimal. A high negative ratio tells us that reported losses were less than expected and maybe our

IBNR is too high.

In the exhibit above, the actual losses for 2015 are significantly higher than what was expected. This should serve as a warning that we would want to investigate this year to understand why the losses were higher than expected. Looking at the ratio of AvE to IBNR for this year, it seems that the greater than expected adverse development will use up 27% of the booked IBNR. Depending on what threshold we have determined is significant, this could indicate that we need to increase the IBNR for this year.

It is possible that the reason your actual losses were significantly different than your expectations is because your initial assumptions are no longer valid. It is possible that your ultimate loss projection or the percent reported from the prior reserve analysis are too low. Had your assumptions been correct then the actual losses would have matched your expectations. However, for the most part, we are assuming that the assumptions from the prior reserve analysis are still valid and do not need to be tweaked for each closing period.

4. IBNR ADJUSTMENTS

The reserving actuary's main role during the closing period is to help determine the amount of IBNR that the company needs. I would like to suggest some recommendations of how and when adjustments to IBNR should be made.

Within the framework of the closing process, there are several IBNR adjustments that should automatically be made, even before determining the appropriate level of IBNR. Although these adjustments are intuitive and might seem obvious, for those who are not familiar with the closing process it might be helpful to provide a brief description of these adjustments.

4.1 Negative IBNR

The first area of concern during the closing process is to look at the negative IBNR being generated. With the exception of certain lines like surety and auto physical damage, companies generally do not carry negative IBNR. Negative IBNR is a result of the reported loss ratio being higher than the ultimate loss ratio that was booked at the end of the prior period. For example, let us say that at the end of the 1st quarter in 2019, Actuarially Accurate Reinsurance Company (AA Re for short) has determined that the expected ultimate loss ratio for its Non-Proportional Casualty book in underwriting year 2019 is 75%. Furthermore, let's assume that at the end of the 1st quarter the earned premium was $10 M and the reported losses were $5 M. This would mean that the ultimate loss for underwriting year 2019 was projected to be $7.5 M which would have required AA Re to carry $2.5 M in IBNR (7.5% * 10 M – 5 M). If during the second quarter, an additional $3 M of losses came in then the reported loss ratio is now 80%, ($8 M / $10 M) while the ultimate loss ratio is still only 75% and the IBNR being carried is now -$ .5 M [(75% - 80%) * $10 M]. The company's results will show no loss for the quarter when in actuality there will be a small loss once the Ultimate Loss Ratio and IBNR is adjusted upward. The following exhibit shows this more clearly.

TABLE 4.1

Keeping Ultimate Loss Ratio @ 75% (in thousands)				
Quarter	Earned Premium	Inception-to-date Reported Losses	IBNR	Ultimate Losses
1st quarter	10,000	5,000	2,500	7,500
2nd quarter	10,000	8,000	-500	7,500

Activity for the Quarter (in thousands)				
	Earned Premium	Reported Losses	IBNR	Profit/(Loss)
Quarter-to-date results	0	3,000	3,000	0

TABLE 4.2

Adjusting Ultimate Loss Ratio to 85% (in thousands)				
Quarter	Earned Premium	Inception-to-date Reported Losses	IBNR	Ultimate Losses
1st quarter	10,000	5,000	2,500	7,500
2nd quarter	10,000	8,000	-500	8,500

Activity for the Quarter (in thousands)				
	Earned Premium	Reported Losses	IBNR	Profit/(Loss)
Quarter-to-date results	0	3,000	-2,000	-1,000

As you can see from the exhibits, keeping the 2nd quarter loss ratio steady at 75% results in no loss for the quarter, while adjusting the loss ratio upward to account for the new losses that came in results in a small loss for the quarter. Although by making these adjustments, the loss for the quarter will increase, one should not be concerned about this. First of all, the job of an actuary is to make sure the IBNR is reasonable and if the negative IBNR is not fixed, the IBNR will not be correct . Furthermore, these corrections are necessary in order to accurately represent the results of the closing period.

4.2 Minimizing the impact of catastrophe losses

Another type of adjustment that can be automatic is a situation where a particular year previously had large catastrophe losses but is still earning premium. Because of these catastrophe losses, the ultimate loss ratio for this book of business could be unusually high. This will cause a problem when premium is still being earned and the new premium is being hit with a loss ratio that is too high. An example will clarify the situation.

In 2018 AA Re, experienced a $28 M loss in its property segment related to a major earthquake in California. At the end of the 1st quarter in 2019, the earned premium is currently $10 M and the reported losses including the losses from the earthquake is $31.5 M. AA Re is also carrying $500 K in IBNR which brings the ultimate loss ratio to 320%. However, during the 2nd quarter an additional $1.5 M of premium is earned, and $500 K of non- cat losses are reported. Although relatively speaking this is not a lot of premium, because the booked loss ratio is 320%, an additional IBNR amount of $4.3 M will automatically be generated. If the expected loss ratio for additional earned premium was 65%, the IBNR being generated should have only been $475 K, (IBNR on new earned premium minus new losses for the period). Unless the loss ratio is adjusted downward, AA Re will be carrying an additional IBNR of $3.825 M and more importantly, rather than showing a $525 K profit for the quarter, they will be showing a technical loss (excluding the effect of commissions) of $3.3 M.

The exhibit below will highlight this.

TABLE 5.1

Inception-to-date Results including catastrophe losses (in thousands)					
Quarter	Earned Premium	Inception-to-date Reported Losses	IBNR	Ultimate Losses	Ultimate Loss Ratio
1st quarter	10,000	31,500	500	32,000	320 %
2nd quarter	11,500	32,000	4,800	36,800	320 %

TABLE 5.2

Impact of prior year catastrophe on Technical Results for the Quarter (in thousands)				
	(1)	(2)	(3)	(4) = (1) – (2) – (3)
	Earned Premium	Reported Losses	IBNR	Profit/(Loss)
Quarter-to-date results	1,500	500	4,300	(3,300)

TABLE 5.3

Technical Results for the Quarter excluding impact of prior year catastrophe (in thousands)					
	(1)	(2)	(3)	(4) = (1) * (2) – (3)	(4) = (2) – (3) – (4)
	Loss Ratio on New Earned Premium	Earned Premium	Reported Losses	IBNR	Profit/(Loss)
Quarter-to-date results	65%	1,500	500	475	525

There are several ways to fix this problem, each of which could be perfectly legitimate. For example, one could utilize the roll forward method and recalculate the IBNR. If this becomes too difficult either because the reporting patterns are not readily available or because you don't have accurate updated information on the catastrophe losses, there is another method that can be used. The expected loss ratio can be applied to the new premium earned this quarter, and then you can subtract out the non-catastrophe reported losses for the quarter and add this amount to the prior quarter IBNR. The IBNR and Ultimate Loss Ratio will then be adjusted downward, and the correct technical results will be shown for the quarter. Below is an example of this second approach.

TABLE 5.4

Additional IBNR needed based on Quarter Results (in thousands)				
	(1)	(2)	(3)	(4) = (1) * (2) – (3)
	Loss Ratio on Quarter-to-date Earned Premium	Earned Premium	Reported Losses	Additional IBNR to Book
Quarter-to-date results	65%	1,500	500	475

TABLE 5.5

Recalculating loss ratio for prior year catastrophe losses					
Quarter	Earned Premium	Reported Losses	IBNR	Ultimate	Ultimate Loss Ratio
1st quarter	10,000	31,500	500	32,000	320%
Quarter-to-date results	1,500	500	475	525	
Adj 2nd quarter	11,500	32,000	975	32,975	287%

4.3 Negative Reported Losses

A third area of concern is if there has been a decrease in losses for a given year, this could, depending on how the IBNR is recalculated, cause the IBNR to automatically be adjusted upward. This would happen in a scenario where the company books an ultimate loss ratio as opposed to an IBNR amount. If the booked ultimate loss ratio has not been changed, and no additional premium has been earned, then any increase or decrease in reported losses will automatically lead to a decrease or increase in IBNR.

Take for example the following two scenarios.

TABLE 6.1 — Scenario 1:

Increase of losses leads to decrease of IBNR (in thousands)				
Quarter	Earned Premium	Total Reported Losses	IBNR	Ultimate Loss
1st quarter	50	20	15	35
2nd quarter	50	30	5	35

	Quarter-to-date Earned Premium	Quarter-to-date Reported Losses	Quarter-to-date IBNR	Profit/(Loss)
Quarter-to-date results	0	10	(10)	0

TABLE 6.2 — Scenario 2:

Decrease of losses leads to increase of IBNR (in thousands)				
Quarter	Earned Premium	Total Reported Losses	IBNR	Ultimate Loss
1st quarter	50	30	5	35
2nd quarter	50	20	15	35

	Quarter-to-date Earned Premium	Quarter-to-date Reported Losses	Quarter-to-date IBNR	Profit/(Loss)
Quarter-to-date results	0	(10)	10	0

The first example is a typical scenario. The ultimate losses have been held steady, and an increase in losses leads to a decrease in IBNR. This is perfectly normal, and no adjustments need to be made. However, the second scenario needs an adjustment. In this case, the ultimate losses have also been held steady. However, rather than there being an increase in reported losses, the reported losses have actually gone down. Mechanically, this results in the IBNR going up. However, this could be counter intuitive. If our reported loss has decreased, one could argue that this should lead to a decrease in ultimate losses as well. Consequently, rather than the IBNR increasing, either the IBNR should be kept at the same level as the prior quarter or be taken down based on any new projections.

In this scenario, there could be a difference in the adjustments made depending on the line of business. If we are talking about a casualty or workers compensation line, the results for these lines can be volatile and take a while to develop. It might not make sense to decrease the IBNR just because the reported losses decreased in the quarter. Given the volatility inherent in these lines, a decrease one quarter might be offset by an increase next quarter. However, for a shorter tail line like property, these adjustments would be appropriate.

4.4 Roll Forward Adjustments

Even after the adjustments mentioned above are made, we are still interested in making sure that all reasonable adjustments have been made. This would include increasing and decreasing the IBNR as necessary. The challenge is how does one modify IBNR while ensuring it remains at the same reserve adequacy level. This is where the roll-forward method mentioned above can be utilized. It is a perfect tool to use in discovering which areas of IBNR to be adjusted. This is especially true for the short tail property lines. For longer tailed lines updating the patterns for the current quarter will probably not make a major difference to the quarter results. Furthermore, given the volatility inherent in these lines it doesn't make sense to constantly change the IBNR every quarter. Although the overall ultimate loss for the long tail lines might be stable, the IBNR level from period to period can fluctuate. However, for short tail lines like property, where the tail can end after 24 or 36 months, using a roll forward method makes sense and the savings can be significant.

5. UNDERSTANDING THE RESULTS

As previously mentioned, having a robust closing process not only leads to more accurate results, but also will help facilitate better communication between the actuary and the other departments. Therefore, one of the main goals of the closing process should not only be to finalize the quarterly profit or loss but also to understand what lines of business in particular contributed to the overall results of the company.

The challenge is that even in a small company, the amount of information can seem to be overwhelming. The short timeframe of the closing process also requires a method that is efficient and focused. How is one supposed to calculate and update the IBNR as well as investigate what is driving the profit/loss for the quarter?

One possible option is to take a top down approach as described in the exhibits below. The exhibits below show the closing results for Actuarially Accurate Reinsurance Company, a reinsurance company that closes it's books every quarter.

Here are the quarterly results for AA Re by market segment.

This first exhibit shows the technical results that occurred during the quarter, sorted by magnitude of loss, for each market segment. The numbers below reflect the changes that have occurred in the quarter. Column 1 represents the amount of premium earned during the quarter while columns 2 and 3 represent the changes during the quarer to the reported loss and IBNR respectively. Column 4 shows the amount of commissions that were paid out during the quarter. The technical result for the quarter is Earned Premium-Reported Loss-IBNR-Commission.

For example, ProportionalAuto earned $3.8 M in premium during the quarter, the losses increased by $6.9 M and the IBNR for this line also increased by $2.5 M. The total commssions paid out were $.6 M which means that the overall result for the line during the quarter was a loss of $6.2 M.

TABLE 7.1

AA Re Quarterly Results (in thousands)					
	(1)	(2)	(3)	(4)	(5) = (1) - (2) - (3) - (4)
Market Segment	**Earned Premium**	**Reported Losses**	**IBNR**	**Earned Commissions**	**Technical Result**
Proportional Auto	3,829	6,905	2,535	604	-6,215
Fac Casualty	6,194	1,830	5,974	1,006	-2,616
Non-Proportional Casualty	1,057	504	159	82	312
Non-Proportional Auto	4,452	3,920	-657	606	583
Proportional Casualty	3,202	-93	1,445	-123	1,973
Total	18,734	13,066	9,456	2,175	-5,963

In this example, Proportional Auto shows a loss to the quarter of $6.2 M, while Proportional Casualty has a quarterly profit of $1.9 M. Since the Proportional Auto segment shows the biggest loss for the quarter, it makes sense to start with that segment.

TABLE 7.2

Proportional Auto Quarterly Results (in thousands)					
UY	**Earned Premium**	**Reported Losses**	**IBNR**	**Earned Commissions**	**Technical Result**
2009	0	0	0	0	0
2010	0	0	0	0	0
2011	0	-200	0	0	200
2012	0	0	0	0	0
2013	0	0	0	0	0
2014	0	0	0	0	0
2015	0	-500	500	0	0
2016	0	1,500	-1,500	0	0
2017	699	941	450	250	-942
2018	1,530	2,164	1,870	145	-2,649
2019	1,600	3,000	1,215	209	-2,825
Total	**3,829**	**6,905**	**2,535**	**604**	**-6,215**

The next step would be to drill down and look at the individual Underwriting Years. In our example, you can easily see that the years which contribute the most to the results are the last two years, 2018-2019. However, in addition to looking into 2018 and 2019, we still would want to investigate 2016. This is because although the technical loss for 2016 is $0, this is only because the $1,500 loss mechanically led to a decrease in the IBNR. However, we still want to know what caused the loss for 2016. Another outlier is 2011. Although the loss is small, the fact that there was a loss seems to stick out and it might be worth investigating.

The third and final step would be to drill down for each of the years and look at which cedents and treaties contributed to the loss. In the event that claim detail is available, it might also be helpful to look at the underlying claims.

Once you have this framework in place, it should be relatively easy to go through each market segment and pick out the areas that need investigating. Even if you don't have the answers of what happened, you are in a better position to explain to management what happened during the period and the rationale for any of your IBNR adjustments.. You are also well equipped to share your concerns with accounting and claims as will be explained below.

6. COMMUNICATION

Another important area to focus on is discussions with the various departments. These discussions can take place both during and after the closing. Given the tight closing schedule that some companies operate under, it might not be feasible to have all these discussion during the close. Although some discussions should be held during the closing process, other conversations like explaining the results to the underwriters can take place after the close. Besides the obvious benefits of building relationships, there are other benefits in that it helps create a deeper and more substantial understanding of the business the company writes and enables better reserving decisions. This will also have a positive impact on the reserving analysis that is performed every year and allow the reserving actuary to make better informed choices in calculating the IBNR. Below are some suggestions on how to communicate with the various departments as well as what areas to focus on.

6.1 Different Viewpoints

When analyzing the results, there are several different views that will needed, depending on the purpose of the discussion and the stakeholders involved. For example, one can either look at the overall P&L or focus on the results of the balance sheet. Each of these views are necessary and help tell a different story. Looking at the results from the perspective of the balance sheet will show the profitability of the company for the current period or year. Furthermore, looking at the balance sheet results can help the accounting department determine if the accounts are being booked properly. The P&L results on the other hand will help the company understand if its overall business is profitable and if any changes to its overall business plan is necessary. Another reason for these different views is that different stakeholders might be interested in different views. The finance and accounting department might be more interested in the balance sheet results, while an underwriter might prefer to focus on the P&L.

Another consideration is in looking at the results for that particular period as well as for the entire calendar year. Looking at results on a monthly or quarterly basis might be too short of a time to make any decisions. Having a view of the entire calendar year can give a broader perspective. This is especially true for a line where the premiums might all be earned in the early part of the year but losses do not start coming in until the end of the year. Focusing on the first half of the year will show high profits while the balance sheet for the end of the year period will show a loss. By looking at the book of business for the entire year in total, one can determine the true profitability for the year.

One can also look at the results for the prior years separately from the current year. For the prior years, most of the premium has been earned and no new business is being written. Any changes to the ultimate loss indications are a reflection on the performance of the book of business. An increase in the ultimate loss projection could indicate the book is not performing well while a might indicate the book is performing better than expected. However, when it comes to the current year, the closing results will also reflect the new business being written throughout the year. As an example, at the start of the year there are five accounts written with an overall projedcted ultimate loss ratio of 60%. Midway through the year, 5 more accoutts are written with a projected loss ratio of 75%. The projected loss ratio for the entire book will have gone up, but this is not a reflection on the books poor performance but rather on the growth of the portfolio.

One way to highlight whether the change to ultimate loss is coming from new premium being earned or from adverse loss conditions is by using the following two formulas.

a. (Change in EP) * (Current Ultimate LR).
b. (Prior EP) * (Change in Ultimate LR)

The first formula represents the change in ultimate loss due to new premium earned during the period. The second formula represents the change in ultimate loss due to a change in ultimate loss ratio. Both formulas together represent the total change to ultimate loss that occurred during the closing period.

6.2 Accounting

One advantage to maintaining a robust closing process and using the exhibits above, is the ability to easily spot accounting issues that need to be addressed. One simple example will highlight this very clearly.

TABLE 8.1

Closing Period Results (in thousands)						
Year	Earned Premium	Paid Losses	Case Reserves	IBNR	Commissions	Profit/Loss
2015	5	-250	+100	150	-2,000	-1,995

The exhibit above shows the closing results for one market segment, in thousands, for the 2015 year. As can be seen in the exhibit, some of the numbers stand out. For example, it seems that only $5,000 in premium has been earned. Yet, despite almost no premium in that year, the exhibit still shows $2 M in commissions being paid out for the quarter. Now there could be a perfectly legitimate reason for this to happen. Maybe these are our profit commissions being booked this quarter which are not dependent on earned premiums. However, it could also be an accounting error and there should be no commissions paid out in this year. By pointing out this anomaly to the accounting department, they will be able to investigate and make the appropriate changes. In this particular case, if the commissions need to be fixed, the profit/loss will go from showing a quarterly loss to potentially showing a profit. This is an outcome that will be greatly appreciated by management.

6.3 Claims

The Claims department is another area with whom there should be a discussion and again the robust closing process described in the paper can beneficial in this regard as well.

During the close, one could just focus on maintaining the IBNR and leave any data investigations for the reserve review. The downside is that by the time the review comes around you potentially could be are dealing with large volume of loss development. The amount of data you need to investigate can be overwhelming and there might not be enough time to investigate everything. It is quite possible that some claims will slip through the cracks and you will miss important information that affects your ultimate loss projections.

The following example will show how one can use the closing to avoid this problem.

TABLE 8.2

Closing Period Results (in thousands)						
Year	Earned Premium	Paid Losses	Case Reserves	IBNR	Commissions	Profit/Loss
2018	3,000	-3,000	-15,000	4,000	-900	-11,900

In this example, the reserves for this line in Year 2018 stand out as being unusually high. By reaching out to the claims department, you might discover that these reserves are from a large catastrophic event that occurred in the quarter. Armed with this knowledge you will be able to ensure that whatever roll forward method is being used takes this information into account. This would especially be material if you were using the chain ladder method for this particular year. By not removing these losses, you will be unnecessarily developing the catastrophe losses, which would lead to a higher ultimate loss than is warranted. Furthermore, when it comes time for the reserve review, you will have a better idea about what losses to include or exclude from your reserve triangles. Alternatively, the large loss might just reflect a claim that has recently started developing. Knowing about this claim during the closing process will allow you to keep track of the claim as the year develops. If the claim keeps developing, you will already be aware of it by the time you get around to doing a full reserve analysis. Knowing about the claim in advance will help you make better decisions during the reserve review. Although, you might have picked up on this claim during the reserve review, it is also possible that it would have just gotten lost in the sheer volume of data that you need to look at. Finally, maybe the unusual loss reflects a change in the way the claims department pays losses or sets up reserves. Finding this information out during the closing process, will allow you the time and flexibility to schedule deeper discussions with the claims department and make the appropriate adjustments to your full analysis.

6.4 Underwriting

Underwriters are chiefly concerned with their individual accounts. However, most reserving analysis are done by line of business or specific market segments, and it is quite possible for the individual treaties to get lost in the process. More often than not, the actuary is mostly concerned with the aggregate patterns and there isn't always time to focus on the individual accounts. Furthermore, communicating with the underwriters is an important part of managing the reserves. Having an open and ongoing dialogue with them can give you an understanding of why the book is behaving as it does. However, due to time constraints and various deadlines these conversations can slip by the wayside. By incorporating these discussions as part of the closing process, it ensures that these discussions take place and become part of the company business practices. Also, the underwriters will feel that it is not just them giving information, but they will be also be getting important feedback on their book of business. The closing process is a perfect opportunity to do an individual account analysis and provide feedback to the underwriters.

Taking a top down approach, you can share with the underwriters the exhibits produced above. After showing how each line did in total, you can then drill down to highlight which years had the worse results. Finally, a similar exhibit can be produced for each year, showing the results by treaty or program.

Some of the issues that you might want to discuss are as follows:

- How the segment performed this quarter and was this better or worse than expected?
- Besides the quarterly results how are the results overall since inception?
- What is driving the loss/profit for the quarter. Is it one or two specific treaties or is it due to a frequency of losses that occurred?
- Were there any changes in the market which might be contributing to the results?
- How do the commissions look relative to what was expected? Are they being booked properly?

6.5 Pricing

Besides having these discussions with the underwriters, it can also be beneficial to speak to the pricing actuaries. The same types of exhibits and questions mentioned above can be shared with the pricing team as well. However, there are a couple of additional questions and information that might be more relevant for the pricing team. Specifically, this relates to the expected pricing loss ratios and some of the actuarial reserving assumptions that went into calculating the IBNR. Although some of these questions should be asked as part of the reserve review, incorporating these questions in the closing process can help facilitate the discussion.

The following questions can be asked of the pricing team.

- Are the a-priori loss ratios being used in line with the expected pricing loss ratios?
- How does the inception to date loss ratios compare with the pricing loss ratios, both for individual treaties and for the market segment in total?
- How do the reporting patterns used in the roll-forward compare with the pricing patterns?
- For those segments which have Catastrophe losses, is there a way to break out the cat load to calculate the attritional loss ratio?

The advantage of understanding the cat load is beneficial for the more recent years which tend to rely more heavily on the expected loss ratio. As an example, let us take a property segment where the 2019 pricing loss ratio is 65%, with a 45% attritional loss ratio and a 20% load for Earthquake and Hurricane exposure. Since it is a new year and there is very little loss experience, we will want to book the IBNR based on the pricing loss ratio. If we were to calculate the IBNR using the 65% loss ratio, we would be adding 20 points of IBNR for a Hurricane or Earthquake event that hasn't even occurred. For every $1 M of earned premium, we would be adding an extra $200 K of IBNR. If the premium volume is large, this can quickly add up. Furthermore, even in the event that a Hurricane has happened, the claims department might have a very good idea of the potential losses and they will already have set up the appropriate case reserves. Even for the claims which have not yet been reported, the claims department might have a good idea if there is a need to set up a bulk case reserve for these claims. Any IBNR booked for this event could potentially be redundant. In this instance, the appropriate approach would be to remove the cat load and book the attritional loss ratio of 45%. Understanding what the cat load should be can go a long way in ensuring a better estimate of the results.

7. RESERVE RANGES

Another important area that needs to be given consideration is the setting of a reserve range and the impact on the closing process. The IBNR that will be booked during the closing process is a point estimate ad a specific dollar amount. However, this does not mean that this is the only appropriate number that should be booked.

ASOP 43 is the Actuarial Standard of Practice that is there to "provide guidance to actuaries when performing professional services relating to the estimation of loss and loss adjustment expense for unpaid claims for property/casualty coverages.". The ASOP states in section 3.7.3:

> 3.7.3 Presentation—The actuary may present the unpaid claim estimate in a variety of ways, such as a point estimate, a range of estimates, a point estimate with a margin for adverse deviation, or a probability distribution of the unpaid claim amount. The actuary should consider the intended purpose or use of the unpaid claim estimate when deciding how to present the unpaid claim estimate.

In other words, the Actuarial Standard of Practice recognizes that there is not just one single appropriate IBNR amount. Calculating a reserve range is as equally valid as calculating a point estimate. At the same time, for purposes of the balance sheet and financial statements, the company must eventually decide on one number to book. During the closing process, the decision of what number is most appropriate can be the subject of much internal discussion and back and forth. The number decided on by management, i.e. Management's best Estimate, might not be the same number as the Actuarial Central Estimate. As part of the management team, the actuary has a responsibility to consider the management estimate as well. It is important to realize that there is not necessarily one correct number and different perspectives can lead to different results[1]. However, at the same time it is important to be aware of the appropriate reserve range. Booking a point estimate too high or too low in the range can lead to a situation down the road where due to adverse or positive development the company finds itself carrying an IBNR amount outside the range. If the IBNR is not adjusted, this can raise flags with regulators and auditors who will want to know why the IBNR is seemingly redundant or deficient even by the company's projections. In their paper Applications of Reserve Ranges and Variability in Practice , (Walker & Littman, 2013) , the authors point out that the SEC has been active in questioning insurers in light of large reserve redundancies/deficiencies being posted in their financial statements. The SEC focused on understanding how the "best estimate" was developed and it required discussion to help the investors understand the risks involved. Furthermore, as per ASOP 36, the Standards of Practice providing guidance to the Statement of Actuarial Opinion, Section 4.2 requires the actuary to disclose if the reserve amount is deficient or redundant.

Another important consideration is the possibility that there is a risk of material adverse deviation in the reserves. Both Section 3.6 and Section 4.2.e of ASOP 36 state that an actuary must evaluate the risk of material deviation and disclose any materiality. If the IBNR amount booked during the closing is towards the low end of the range and there is a good chance that results can get worse, the actuary might be forced to comment on this in the Statement of Actuarial Opinion.

The conclusion from all this is that the actuary must be prepared to communicate with management and explain what the reserve range represents. Furthermore, the actuary must be able

[1] In his paper "Reserving Styles-Are Actuaries In-Sync with their Stakeholders", Mark Littman discusses the different perspectives between actuaries and other stake holders that can lead to different results. (Littman, 2015)

to clearly make the arguments, pro and con, of booking an amount that falls towards the high or low end of the range.

There are a lot of variables that management must consider in finalizing the closing numbers, and it is the responsibility of the actuary that the actuarial concerns are given their full attention.

8. PROFESSIONALISM AND ACTUARIAL STANDARDS OF PRACTICE

The financial close process is an area in which professionalism is very important. There are a couple of Actuarial Standards of Practice that are relevant to our discussion and as one goes through the closing process it would be prudent to keep them in mind. Interpreting an ASOP and its relevancy to a topic can be subjective and debatable. Therefore, I will just highlight a couple of the ASOP's that I feel are material and I will leave it to the reader to determine the practical applications.

ASOP 23

The purpose of ASOP 23 as described in the Standards of Practice is "to provide guidance to the actuary when performing actuarial services involving data". Although the ASOP is quite long, I believe the section that relates to review of the data is relevant to our discussion.

The ASOP defines "Review of Data" as follows:

> A review of data may not always reveal defects. Nevertheless, the actuary should perform a review, unless, in the actuary's professional judgment, such review is not necessary or not practical. In exercising such professional judgment, the actuary should consider the purpose and nature of the assignment, any relevant constraints, and the extent of any known checking, verification, or audit of the data that has already been performed.

The ASOP then explains further:

> If the actuary performs a review, the actuary should … make a reasonable effort to identify data values that are questionable or relationships that are significantly inconsistent. If the actuary believes questionable or inconsistent data values could have a significant effect on the analysis, the actuary should consider taking further steps, when practical, to improve the quality of the data.

Clearly, what seems to be required by this ASOP is a good faith attempt to ensure that the data being used is correct and accurate. Although the Standards of Practice allow for using inferior data, nevertheless it seems it is incumbent on the actuary to try to validate the data as much as possible.

ASOP 43

A second ASOP that seems relevant to the closing process is ASOP 43, which is the standard of practice as it relates to Property/Casualty Unpaid Claim Estimates.

As described in the Standards of Practice the purpose of ASOP 43 as is "provide guidance to actuaries when performing professional services relating to the estimation of loss and loss adjustment expense for unpaid claims for property/casualty coverages."

The ASOP states in section 3.6.1 that the actuary should consider methods or models for estimating unpaid claims that, in the actuary's professional judgment, are appropriate.

Furthermore, the actuary is instructed to:

> …consider the reasonableness of the assumptions underlying each method or model used. Assumptions generally involve significant professional judgment as to the appropriateness of the methods and models used and the parameters underlying the application of such methods and models. … The actuary should use assumptions that, in the actuary's professional judgment, have no known significant bias to underestimation or overestimation of the identified intended measure and are not internally inconsistent. Note that bias with regard to an expected value estimate would not necessarily be bias with regard to a measure intended to be higher or lower than an expected value estimate.

One of the challenges is how to make the appropriate IBNR adjustments during the short closing period, and still ensure that the results are reasonable and unbiased.

Furthermore, section 3.7.3, the section on reserve ranges is also relevant to our discussion, as explained above.

9. CONCLUSION

This paper was written with the hope of generating discussion on the closing process. I have attempted to provide some insight into some of the methodology and thought that goes into setting the reserves every closing period. Open discussion and dialogue with the various departments as well as management is a crucial part of the process. The approach to the closing process can have far reaching implications for the growth and profitability of the company.

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Reserving for the Cost of Credit Security

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Abstract: **This paper is a discussion on the estimation of a reserve related to a common form of credit security, typically in the form of a bond or standby letter of credit, that stems from loss and loss adjustment expense reserves when there is credit risk (e.g., large deductible and captive insurance company fronting arrangements) in certain circumstances. For purposes of this paper, the author focuses on the policyholder perspective, as is easily defined as a liability under Generally Accepted Accounting Principles. Applications under Statutory Accounting Principles are not discussed, as the concepts and estimation methods should be comparable where applicable. The primary basis for this reserve derives from Generally Accepted Accounting Principles Statement of Financial Accounting Standard number five regarding recognition of contingent liabilities in circumstances when there are recurring future costs to purchase security for loss reserves. This can be a material reserve that requires actuarial analysis to estimate. In this paper the author discusses applicable accounting practices, an estimation methodology, the process for selecting certain assumptions, and thoughts on classifying the security reserve liability. Included is a detailed example of the estimate for this reserve using workers' compensation data on a nominal and discounted basis.**

1. INTRODUCTION

This paper is a discussion on the estimation of a reserve related to a common form of security, typically in the form of a bond or standby letter of credit ("LOC"), that stems from loss and loss adjustment expense reserves ("loss reserves") when there is credit risk (e.g., large deductibles, captive insurance fronting arrangements, and reinsurance collateral agreements) in certain circumstances. For purposes of this paper, the author focuses on the policyholder financial statement perspective, as that is easily defined as a liability under Generally Accepted Accounting Principles ("GAAP"), is a commonly encountered situation for actuaries, is readily understood, and can be extended to other comparable forms of credit security by analogy.

When a policyholder purchases insurance for a coverage with a long period of loss reserve runoff[1] (e.g., workers' compensation) and a large deductible, the insurer usually requires that a form of security be posted to "guarantee" the policyholder will pay for the deductible portion of any claims, typically as reimbursement to the insurer. Given the nature of a LOC, this creates a long term liability. Statement of Financial Accounting Standard number five ("SFAS

[1] Runoff in this context means the time period for all the claims that have occurred, as of the financial reporting date, to fully pay out and final close. At the end of this period, loss reserves are expected to be zero. This runoff period can arise from active, expired and terminated policies. The runoff period can be shortened by commutation.

5") from the Financial Accounting Standards Board, regarding recognition of contingent liabilities, provides GAAP accounting guidance for establishing this liability, which is explained in more detail below. The liability arises in such circumstances because there are recurring future costs to purchase the credit security during the period of loss reserve runoff. The cumulative cost of purchasing this security for future periods is a long term liability (hereafter referred to as the "security reserve").

After reviewing this paper, practicing actuaries should be able to recognize the liability and estimate the security reserve for a company and discuss with accountants the basis for booking the reserve under GAAP. Potentially, this is an area where actuarial services can be added, likely in addition to estimating loss reserves.

2. BACKGROUND AND METHODOLOGY

2.1 How the Security Reserve Arises

The security reserve arises in circumstances where there are recurring future costs to purchase security for loss reserves. For a LOC, this is the annual purchase cost. The following example demonstrates how the security reserve arises in a common business insurance transaction:

> A corporation purchases a large deductible policy covering its workers' compensation exposures. Based on estimates by the corporation's actuary, the ultimate loss incurred for the policy deductible (excluding premiums and other costs) is approximately $5 million for the year with the runoff period to pay all claims expected under the policy deductible to be approximately 35 years[2]. The corporation is contractually obligated by the insurance policy governing the deductible to post collateral to secure the loss reserves owed the insurance company under the deductible until all claims are paid. Assuming the insurer agrees that the ultimate is $5 million, the security requirement would be $5 million at policy inception and the corporation would need to post

[2] As a point of reference, 35 years is commonly used as a claim runoff period by the National Council on Compensation Insurance in its' workers' compensation cash flow modeling. Actuarial literature documents longer periods of runoff for this coverage in certain states. Many factors can affect the length of the runoff period, discussion of which is outside the scope of this paper.

collateral to cover this obligation. The corporation purchases a LOC to cover the security requirement. Based on market prices for this corporation at the time of purchase, the LOC cost is estimated to be 1.5 percent of the LOC amount. The first year LOC cost is $75,000 (.015 x 5,000,000) and decreases gradually to zero at 35 years for this policy. Assuming no commutation occurs, when projected and summed over the 35 year runoff, the corporations' LOC cost adds up to a significant amount that will be paid by the corporation in future years. The corporation over time renews this policy, adding additional security requirements for each policy added, each of which is in various stages of runoff. As these future LOC costs are contractual obligations and can be reasonably estimated by an actuary, they meet the requirements of SFAS 5 and should be recognized as a liability.

2.2 The Standby Letter of Credit[3]

Corporate policyholders commonly, but not always, use LOC's to provide security for loss reserves under a large deductible because it is often cheaper and more convenient than posting marketable securities or purchasing specialty bonds, particularly for large companies with good credit ratings. LOC's are a promise by a lender (e.g., bank or brokerage) to provide credit up to the amount of the LOC to a third party on behalf of the purchaser. These are typically made in two parts – one agreement between the lender and the insurer and another agreement between the lender and the policyholder. The policy is a third agreement between the insurer and policyholder. The right to draw from the LOC is specified in the agreements between the insured, insurer and lender based on certain contingencies. A few examples of such contingencies that might be considered in a LOC used for large deductible security include:

- The policyholder fails to make a reimbursement payment for claims paid by the insurer under a large deductible within a certain time period.
- Breach of contract by the policyholder.
- Failure of the policyholder to renew the LOC as agreed.

[3] This section discusses common aspects of the standby letter of credit. It does not cover all variations and/or all nuances associated with these credit instruments.

- Failure to increase the LOC as requested by the insurer.
- The policyholder's credit rating and/or financial condition deteriorates.

LOC's are often required to have "evergreen" clauses, which automatically extend the LOC beyond the expiration date (e.g., for an additional year) without modification. This clause provides extra time for an insurer to review the policy in the event of policy termination, non-renewal, policyholder financial condition changes, and for other matters that may affect credit security. LOC's are also often required to be irrevocable.

When drawn, the LOC converts to a loan by the lender to the policyholder with the proceeds paid directly to the insurer, effectively transferring security credit risk from the insurer to the lender.

LOC's have an annual cost. Factors that may affect the cost include credit market conditions and the financial condition of the purchaser. There are various other factors that may affect LOC costs, which vary by lender, similar in some respects to factors considered for loans. For purposes of estimating the security reserve, the author assumes all such factors are reflected in the annual cost of the LOC. In recent years, the author has reviewed data indicating LOC costs in the range of approximately 0.5 percent to 3.5 percent for corporations with good credit ratings. The actual range for LOC costs can be considerably wider (e.g., up to 10 percent) based on specific policyholder circumstances, market conditions, and lender requirements.

A significant risk of relying on LOC's is variability of the policyholders' credit rating. For example, as credit worthiness of the policyholder declines, the LOC cost may increase and if this decline in credit worthiness is sufficiently large, the LOC will not be available for purchase from any lender.

Credit market conditions are another significant risk of relying on LOC's and these conditions vary over time due to many factors. For example, when credit is readily available, the market cost of LOC's, all else equal, is likely to be lower. When credit market conditions harden, as they did during the 2008 financial crisis, LOC's may be harder to obtain and cost more than expected.

2.4 SFAS 5 Guidance[4]

Typically, a corporation with a large deductible will have multiple policy periods with a significant amount of loss reserves running off as of the financial reporting date. As the LOC is paid for annually over the life of the loss reserve runoff, it creates a stream of future cash outflows. Because loss reserves decline over time during the runoff period, the associated cost of the LOC, which is proportional to the loss reserve, will also decline over time (all else equal). When the runoff period is sufficiently long, as it is in many casualty lines of business (e.g., workers compensation, general liability, products, and certain professional liability coverages), the cumulative future cost for the LOC's can be material.

SFAS 5 states in summary, that an estimated loss from a loss contingency shall be accrued if information available prior to issuance of the financial statements indicates that it is probable that a loss has been incurred as of the date of the financial statements and the amount of loss can be reasonably estimated. Since the security requirement is a condition of the insurance contracts entered into prior to the financial reporting date, since the security requirement is expected to be maintained over the period of loss reserve runoff irrespective of future policy renewals, and since the security reserve can be estimated (most likely by an actuary), these conditions appear to be met. There could be exceptions, but in most cases regarding a large deductible secured by a LOC, the author believes the security reserve falls within the requirement to recognize a liability per SFAS 5.

2.4 Estimating LOC Cost Relative to Loss Reserves

The LOC cost can be estimated from the policyholder's own data, credibility considerations permitting. This information may be in the form of actual LOC contract details for the large deductible or it may be accounting data showing the amounts paid and total LOC's purchased. When estimating the cost of the LOC for purposes of the security reserve calculation shown in this paper, it is desirable to convert the cost of the LOC to a percentage of loss reserves

[4] The discussion in this section is from the perspective of an actuary practicing in a field that regularly involves the application of SFAS 5 and does not constitute an accounting opinion regarding the applicability of any accounting standard. Opinions expressed by CPA's may differ.

("LOC cost ratio"), if the data is not already in that form. For example, one could use actuarial estimates of loss reserves as the denominator, rather than carried amounts or actual security reserve requirements.

When using aggregate accounting period data, care should be taken to approximately match LOC payments to the corresponding loss reserves historically, as corporations often purchase LOC's for various reasons beyond securing large deductible loss reserves.

Consideration should also be given to the use of long term versus short term averages of historical LOC cost to reserve ratios.

An example of the estimate of the LOC cost ratio for a hypothetical policyholder with a large deductible is shown in Table 1 below:

Table 1
Workers' Compensation LOC Historical Cost Ratio
($ Amounts in 000)

Calendar Year	(1) LOC Amount	(2) LOC Cost Paid	(3) Actuarial Indicated Loss Reserve	(4) Ratio of LOC Cost Paid to Indicated Reserve	(5) Ratio of LOC Cost Paid to LOC Amount	(6) Moody's AAA Bond Yield 12-month Average
2005	$ 50,000	$ 900	$ 27,000	3.3%	1.8%	5.2%
2006	50,000	700	26,000	2.7%	1.4%	5.6%
2007	55,000	700	25,000	2.8%	1.3%	5.6%
2008	53,000	600	35,000	1.7%	1.1%	5.6%
2009	53,000	550	35,000	1.6%	1.0%	5.3%
2010	60,000	590	35,000	1.7%	1.0%	4.9%
2011	60,000	580	34,000	1.7%	1.0%	4.6%
2012	50,000	490	33,000	1.5%	1.0%	3.7%
2013	40,000	250	32,000	0.8%	0.6%	4.2%
2014	38,000	250	27,000	0.9%	0.7%	4.2%
2015	36,000	250	26,000	1.0%	0.7%	3.9%
2016	34,000	260	27,000	1.0%	0.8%	3.7%
2017	32,000	270	28,000	1.0%	0.8%	3.7%
2018	30,000	300	27,000	1.1%	1.0%	3.9%
Total	$ 641,000	$ 6,690	$ 417,000	1.6%	1.0%	
			Selected	**1.5%**		

(4)=(2)/(3)
(5)=(2)/(1)
(6) Source: FRED Economic Data, Federal Reserve Bank of St. Louis

Data for this example was compiled by calendar year. Data for columns (1) and (2) were provided by the policyholder and data for column (3) came from the policyholders' actuarial analysis of large deductible loss reserves. The LOC amount in column (1) is the estimated face value of the LOC's purchased during the year to secure the workers' compensation large deductible. Column (1) is typically based on the insurers' estimate of the amount needed to cover both existing loss reserves at policy renewal plus the ultimate in the current policy year less expected runoff of loss reserves during the subsequent year (a new LOC amount would typically be estimated at each policy renewal). Cost paid in column (2) is shown in the year in which the LOC was purchased and was purchased at policy renewal. Loss reserves in column (3) are the indicated amounts at the financial reporting date. Column (4) shows the LOC cost paid as a percentage of the loss reserve ("LOC %") and is used in the estimate of the security reserve shown in Exhibit 1 (discussed below). Columns (5) and (6) were included for consideration in selecting the value to be used from column (4).

During the period for which the data was available, this hypothetical policyholder experienced varying amounts of loss reserves, falling LOC costs for a time and then increasing LOC costs more recently. Based on review of the history with the policyholder, the cost variations were the result of credit market conditions, with policyholder credit ratings being relatively stable. The LOC cost data shown in the example fell during the financial crisis of 2008 along with interest rates, but the timing and magnitude of those changes did not necessarily correlate closely with changes in market interest rates (based on Moody's AAA bond yields).

The required LOC purchase amounts in column (1) of this example were different than the policyholder's financial reporting estimates of loss reserves for the large deductible in column (3) for several reasons including:

- Timing differences: The security requirements of the insurer were based on data typically older than what the policyholder's actuary used as they were calculated using data available several months prior to the policy inception date and the inception date was several months prior to the financial reporting date. Policy periods did not get a security requirement update until the payroll audits were completed (audits occurred 18 months after inception). The required LOC amount included the insurers' estimate of expected security requirements for the renewal policy period, a portion of which may not yet be reflected in the loss reserves as of the financial reporting date.

- Policy provisions: The insurer's loss development factors used for future security adjustments were specified in the policy at inception and were not modified for future security reviews unless requested by the policyholder, which the policyholder did not consistently do.
- Actuarial methods and assumptions: The policyholders' actuary used other methods in addition to the loss development method. The insurer relied on the insurer's own loss development factors rather than the policyholder's development data.

For purposes of this example, the author selected a LOC % of 1.5 percent. In selecting the LOC %, which is used in Exhibit 1 (discussed below), the following considerations are highlighted:

- The continued stability of the policyholders' credit ratings in the future. Given a stable history of credit ratings, continued stability was assumed.
- The credit market conditions during the experience period in Table 1 compared to what might arise in the future. Longer term averages of the LOC % were considered to be more appropriate than short term averages, as the security reserve is projected over many future years when credit market conditions will likely vary.
- The differences between column (1) and column (3) discussed above affect the historical values in column (4). For example, the insurer's estimate of LOC requirements reflected in column (1) may include a provision for the new policy period when renewed, a portion of which may be unexpired as of the financial reporting date[5].

While additional LOC cost data would be desirable, the data shown in Table 1 was all that was available. Comparable to the loss reserve estimate, the security reserve estimate should

[5] To the extent this is considered material to the policyholders' balance sheet, various methods could be used to address this including adding the unexpired portion of ultimate to the loss reserves in column (3) and/or giving some weight to the values in column (5).

probably be updated periodically to reflect new data.

2.4 Estimating the Security Reserve

An example of the estimate of the security reserve for a hypothetical policyholder with a large deductible is shown in Exhibit 1 (attached). Cost estimates were based on the average costs for LOC's as a percentage of loss reserves from Table 1. The estimated loss payout pattern shown in row (1) was derived from 20 years of paid loss development data fitted to an inverse power curve. Remaining payout at the tail of the calculation was assumed to occur in the last projection year to simplify this example - such simplification may not be appropriate in all cases.

The exhibit is shown on an accident year basis, as a single policyholder year is equivalent to an accident year. The estimate is for a financial reporting period that ends approximately one year after the latest policy period incepts (i.e., most recent policy period has already paid out one year). Historical accident years are shown on the left as rows, with future payout years shown as column headings at the top.

The inverted triangle in the central part of the exhibit are payout patterns for each of the loss reserve amounts shown to the right and represent the expected payout in future periods for those reserves. For example, the oldest accident year is assumed to fully payout in the next period, and as the accident years move closer to present, the number of future payout periods increases. These are derived from the complete payout pattern shown in row (1).

Footnotes in Exhibit 1 provide additional information regarding the calculations used.

Both nominal and discounted security reserves are estimated in this exhibit. An interest rate of 3 percent was assumed for the discounted estimate. The totals for rows (9) and (11) show the estimated security reserve on a nominal ($2.5 million) and discounted ($2.1 million) basis, respectively. As a percentage of the indicated loss reserve of $27 million, the security reserve is approximately 9 percent on a nominal basis[6].

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[6] Because the security reserve was assumed to be a constant percentage of the loss reserve during the reserve runoff period in this example, the discounted security reserve as a percentage of the discounted loss reserve, which the author did not include in this example, should also be approximately 9 percent.

This estimate is sensitive to the LOC %, the loss reserve amounts (including how loss reserves are distributed by accident year), and the payout patterns.

2.5 Additional Considerations

Where should this reserve be carried on the balance sheet – as loss adjustment expense or elsewhere? One argument for calling it loss adjustment expense is that the security reserve would not be required if not for the insurance contract. It is not loss, adjusting, or defense. It might be considered a form of cost containment, as large deductibles are used by policyholders to reduce insurance costs and insurers believe such policies encourage policyholders to pay more attention to cost controls. It is also a derivative cost of loss reserves and could be considered part of "other" loss adjustment expense. For these reasons and simplicity, the author believes the security reserve can be classified as loss adjustment expense. Other classifications, including credit expenses (comparable to interest expenses) or financial guaranty/surety bond expenses might be used to classify the security reserve. Experts in financial reporting (e.g., accountants, finance managers, regulators, other actuaries) should be consulted to determine the most appropriate classification for this reserve.

Does the security reserve qualify for discounting? This may depend on how it is classified and how GAAP or other accounting standards apply. The author shows estimates on both a nominal and discounted basis. In cases where the security reserve is considered a loss adjustment expense, if the loss adjustment expense reserves are discounted, it seems reasonable that this reserve should also be discounted.

How does treating the LOC expenses as a security reserve compare to other practices? Based on the authors' experience, many policyholders book the costs of the LOC as annual costs of operation rather than a reserve for future costs. Such costs are often included with the overall cost of credit and never reviewed as a SFAS 5 liability related to loss reserves. It is the author's hope that information contained in this paper will help to change this practice to one more consistent with SFAS 5 and related accounting standards.

3 SUMMARY AND CONCLUSIONS

This paper demonstrates how SFAS 5 can be applied to cases where the cost to provide credit security (e.g., in the form of a LOC to secure loss reserves under a large deductible) is an annual expense that continues beyond the expiration of the policy. Examples of how to estimate this expense and estimate the security reserve that results from the future cash outflows related to this expense are included. Readers are encouraged to look for comparable situations in their work environment where these actuarial methods can and likely should be used to estimate the security reserve, where applicable.

Examples shown in this paper were based on the author's experience with estimating security reserves and were created for instructional purposes only. Data shown in these examples should be treated as hypothetical in nature.

About the Author

Bruce Ollodart is Managing Member and Consulting Actuary at American Actuarial LLC, a property and casualty consulting firm. He has degrees in applied mathematics and physics from the George Washington University. He is a Fellow of the CAS and a Member of the American Academy of Actuaries. He has been working in the actuarial field since the early 1980's and has authored previous papers for the CAS on topics including uncollectible reinsurance and mass tort liabilities.

Workers' Compensation
Estimated Secuirty Reserve
($ Amounts in 000)

Exhibit I

(1) Loss Payout Pattern for Complete Accident Year:

Cumulative Paid	24.8%	52.7%	65.4%	72.7%	77.5%	80.8%	83.3%	85.2%	86.7%	88.0%	89.0%	89.8%	90.6%	91.2%	91.8%	92.3%	92.7%	93.1%	93.4%	93.7%	94.0%	94.3%	94.5%	100.0%
(2) Incremental Paid	24.8%	27.9%	12.7%	7.3%	4.8%	3.3%	2.5%	1.9%	1.5%	1.2%	1.0%	0.9%	0.7%	0.6%	0.6%	0.5%	0.4%	0.4%	0.3%	0.3%	0.3%	0.3%	0.2%	5.5%

Accident	Payout Year																									(4)
Year																										Loss
Ending	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032	2033	2034	2035	2036	2037	2038	2039	2040	2041	2042	Total	Reserve
(3) Adjusted Payout Pattern (percentage of loss reserve to be paid in period):																										
1996	100.0%																								100.0%	540
1997	4.2%	95.8%																							100.0%	162
1998	4.4%	4.0%	91.6%																						100.0%	162
1999	4.6%	4.2%	3.8%	87.4%																					100.0%	162
2000	4.8%	4.4%	4.0%	3.6%	83.2%																				100.0%	216
2001	5.0%	4.6%	4.1%	3.8%	3.5%	79.0%																			100.0%	270
2002	5.3%	4.8%	4.3%	3.9%	3.6%	3.3%	74.8%																		100.0%	324
2003	5.6%	5.0%	4.5%	4.1%	3.7%	3.4%	3.1%	70.6%																	100.0%	378
2004	5.9%	5.3%	4.7%	4.2%	3.8%	3.5%	3.2%	2.9%	66.4%																100.0%	432
2005	6.3%	5.6%	4.9%	4.4%	4.0%	3.6%	3.3%	3.0%	2.7%	62.2%															100.0%	486
2006	6.8%	5.9%	5.2%	4.6%	4.1%	3.7%	3.3%	3.0%	2.8%	2.5%	58.0%														100.0%	540
2007	7.2%	6.3%	5.5%	4.8%	4.3%	3.8%	3.4%	3.1%	2.8%	2.6%	2.4%	53.8%													100.0%	648
2008	7.8%	6.7%	5.8%	5.0%	4.4%	3.9%	3.5%	3.2%	2.9%	2.6%	2.4%	2.2%	49.6%												100.0%	648
2009	8.5%	7.2%	6.1%	5.3%	4.6%	4.1%	3.6%	3.2%	2.9%	2.6%	2.4%	2.2%	2.0%	45.4%											100.0%	702
2010	9.3%	7.7%	6.5%	5.5%	4.8%	4.2%	3.7%	3.3%	2.9%	2.6%	2.4%	2.2%	2.0%	1.8%	41.2%										100.0%	756
2011	10.2%	8.3%	6.9%	5.8%	5.0%	4.3%	3.8%	3.3%	2.9%	2.6%	2.4%	2.1%	1.9%	1.8%	1.6%	37.0%									100.0%	864
2012	11.4%	9.1%	7.4%	6.1%	5.2%	4.4%	3.8%	3.3%	2.9%	2.6%	2.3%	2.1%	1.9%	1.7%	1.6%	1.4%	32.8%								100.0%	1,188
2013	12.9%	9.9%	7.9%	6.4%	5.3%	4.5%	3.8%	3.3%	2.9%	2.6%	2.3%	2.0%	1.8%	1.6%	1.5%	1.4%	1.2%	28.5%							100.0%	1,350
2014	14.8%	11.0%	8.5%	6.7%	5.5%	4.5%	3.8%	3.3%	2.8%	2.5%	2.2%	1.9%	1.7%	1.5%	1.4%	1.3%	1.2%	1.1%	24.3%						100.0%	1,620
2015	17.4%	12.2%	9.1%	7.0%	5.6%	4.5%	3.7%	3.2%	2.7%	2.3%	2.0%	1.8%	1.6%	1.4%	1.3%	1.2%	1.0%	1.0%	0.9%	20.1%					100.0%	2,160
2016	21.2%	13.7%	9.7%	7.2%	5.5%	4.4%	3.6%	3.0%	2.5%	2.1%	1.8%	1.6%	1.4%	1.3%	1.1%	1.0%	0.9%	0.8%	0.8%	0.7%	15.8%				100.0%	3,240
2017	26.9%	15.5%	10.0%	7.1%	5.2%	4.0%	3.2%	2.6%	2.2%	1.8%	1.6%	1.3%	1.2%	1.0%	0.9%	0.8%	0.7%	0.7%	0.6%	0.6%	0.5%	11.6%			100.0%	4,320
2018	37.0%	17.0%	9.7%	6.3%	4.4%	3.3%	2.5%	2.0%	1.6%	1.4%	1.1%	1.0%	0.8%	0.7%	0.7%	0.6%	0.5%	0.5%	0.4%	0.4%	0.3%	0.3%	7.3%		100.0%	5,832
																									Total	27,000
(5) Amount Paid	5,969	3,358	2,345	1,771	1,446	1,229	1,074	958	868	793	730	702	622	574	528	502	539	506	488	502	554	518	424			
(6) Cumulative Pd	5,969	9,327	11,671	13,442	14,888	16,117	17,191	18,149	19,017	19,810	20,540	21,242	21,865	22,439	22,968	23,470	24,009	24,514	25,002	25,504	26,058	26,576	27,000			
(7) Reserve Balance	21,031	17,673	15,329	13,558	12,112	10,883	9,809	8,851	7,983	7,190	6,460	5,758	5,135	4,561	4,032	3,530	2,991	2,486	1,998	1,496	942	424			164,233	
(8) LOC %	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%				
(9) LOC Cost	315	265	230	203	182	163	147	133	120	108	97	86	77	68	60	53	45	37	30	22	14	6			**2,463**	
(10) Discount Factor	0.985	0.957	0.929	0.902	0.875	0.850	0.825	0.801	0.778	0.755	0.733	0.712	0.691	0.671	0.651	0.632	0.614	0.596	0.579	0.562	0.546	0.530				
(11) Discounted LOC Cost	311	254	214	183	159	139	121	106	93	81	71	61	53	46	39	33	28	22	17	13	8	3			**2,057**	

Notes (correspond to line numbers shown above):

(1) Payout pattern derived from paid loss development analysis
(3) = (2) adjusted to eliminate payout for portions already paid and scaled to sum to 100%
(4) Hypothetical reserve amounts totaling $27 million
(5) Sum of (3)*(4) for each payout year

(6) Cumulative of (5)
(7) Total of (4) less (6)
(8) Assumed LOC Cost Per $/Reserve based on Table 1: 1.5%
(9) = (8)*(7)

(10) Discount factor with interest rate of: 3.0%
Calculated assuming mid-year payment dates.
(11) = (10)*(9)

A Case Study
Using Bayesian MCMC Stochastic Loss Reserve Models

Glenn Meyers

June 30, 2019

Abstract

At the 2018 annual meeting of the Casualty Actuarial Society, Bob Wolf and Mary Frances Miller presented a loss reserve analysis[1] on real data (scaled to maintain anonymity). These data consisted of 16 x 16 paid and incurred loss triangles. Features of the data included.

- Rapid premium growth
- Change in claims philosophy?
- Underestimates of outstanding liability in previous years

Mr. Wolf provided me with an electronic copy of those data. The paper analyzes those data using Bayesian MCMC starting with models described in Meyers (2019). It ends up by making changes to these models suggested by various diagnostics.

[1] Session C-24 - Learning Lounge Case Study: Material Adverse Reserve Development? When is it just that stuff happens?

Editorial Notes

- I call this document a "paper" even though it is written in a presentation format. I proposed this format to the CAS editorial staff and they agreed to it as an experiment.
- I chose this format because much of what people, including myself, read these days is on a screen. I want to make it easy to navigate between text, tables and graphics.[2]
  - There are section titles on the sidebar. Clicking on a section title will take you directly to that section. There is also direct access the plots.
  - Advancing the pages with consecutive plots will make it easier to compare the plots.
- The discussion of the models will be at a fairly high level. To fill in the details, one will have to look at my monograph, Meyers (2019).

[2] Printing this paper is permitted.

Supplemental Data Files

- The following data files are included with this paper. If they are placed in the same directory as the file for this pdf document, you will be able to see them by clicking on the link.
- The loss triangles
  - LL_Paid_Triangle.csv
  - LL_Incurred_Triangle.csv
- Summary statistics for the posterior distributions
  - Posterior_Stats.xls

R/Stan Scripts for the Models in This Paper

- I use RStudio to run my scripts. When I click on one of the links below, the script comes up in RStudio ready to run.
- To run these scripts on your computer you will have to:
  1. Change the R "setwd" function to the same directory as this pdf.
  2. Install the "rstan", "loo" and "data.table" R packages.
- These R/Stan Scripts should be in the same directory as this pdf.
  - LL_CRC.R
  - LL_CSR_w.R
  - LL_CSR_c.R
  - LL_CSR_vc.R
  - LL_POS_vc.R

The Question to be Addressed

- As the next two pages show, taken from the "Learning Lounge" presentation, the opining actuary, underestimated the liability in the three prior years.
- The question posed by the presentation was — Is this simply a case of "bad stuff" that sometimes happens?
- In this paper I pose the question as — "Is there a loss reserve model that does a better job of predicting the "bad stuff?"
- To properly answer this question:
  - We need a model with features that allow us to predict the "bad stuff."
  - If such features cannot be identified, we need a stochastic model that provide a range of possible outcomes.

Page 4 from "Learning Lounge" Presentation

Run-off of Net Carried Loss and DCC (aka ALAE Reserves) ($000s)
Adequacy of Net reserves in Hindight at Prior-Year Ends
Source- Derivations from using December 31, 2017 Schedule P Data

		1 Year Later	2 years later	3 years later
		(Paid + Remaining Reserves)		
Carried Reserves as of December 31, 2014	$ 145,170	$ 158,865 9.4%	$ 166,370 14.6%	$ 182,100 25.4%
		$ -	$ -	$ -
Annual Change		$ 13,695	$ 7,505	$ 15,730
Cumulative Change		$ 13,695	$ 21,200	$ 36,930
		1 Year Later	2 years later	3 years later
		(Paid + Remaining Reserves)		
Carried Reserves as of December 31, 2015	$ 207,945	$ 208,500 0.3%	$ 229,905 10.6%	
		$ -	$ -	
Annual Change		$ 555	$ 21,405	
Cumulative Change		$ 555	$ 21,960	
		1 Year Later	2 years later	3 years later
		(Paid + Remaining Reserves)		
Carried Reserves as of December 31, 2016	$ 244,470	$ 272,095 11.3%		
		$ -		
Annual Change		$ 27,625		
Cumulative Change		$ 27,625		
		1 Year Later	2 years later	3 years later
		(Paid + Remaining Reserves)		
Carried Reserves as of December 31, 2017	$ 306,365			
Annual Change				
Cumulative Change				

Page 5 from “Learning Lounge” Presentation

Actuarial Opinion- History

	Appointed Actuary	Carried Reserves	Appointed Actuary Central Estimate	Actuarial Opinion Summary Range			RMAD
2/31/2014	A	145,170	145,170	141,396	to	174,494	
		Reasonable	0.0%	-2.6%		20.2%	NO
2/31/2015	A	207,945	207,945	188,190	to	245,583	
		Reasonable	0.0%	-9.5%		18.1%	NO
2/31/2016	A	244,470	253,675	229,504	to	299,465	
		Reasonable	3.8%	-6.1%		22.5%	NO
2/31/2017	B	306,365	306,365	290,434	to	321,377	
		Reasonable	0.0%	-5.2%		4.9%	NO

Stochastic Loss Reserve Models

- Start with the model framework in Meyers (2019).

$$C_{wd} \sim \text{lognormal}(\mu_{wd}, \sigma_d)$$

- where:
  - w =Accident Year (AY), $w = 1, \ldots, W$
  - d =Development Year (DY), $d = 1, \ldots, D$
  - Also, let c =Calendar Year (CY), $c = w + d - 1$
- This paper will initially examine models where:

$$\mu_{wd} = \log(Premium_w) + logelr + \alpha_w + \beta_d \cdot Sp(t)$$

- The $Sp(t)$, i.e. the "Speedup", function specifies how the "development factors" change over the time, t, where t could be measured by accident year, or calendar year.
- This paper explores alternative $Sp(t)$ functions in an effort to find a model that makes better predictions of the ultimate losses.

Interpreting the Model Parameters

- To prevent overdetermining the model, set:

$$\alpha_1 \equiv 0 \text{ and } \beta_D \equiv 0$$

- Thus the expected ultimate loss, U_w for accident year w, is the mean of a lognormal distribution, i.e.

$$U_w \equiv Premium_w \cdot \exp(logelr + \alpha_w + \sigma_D^2/2) \qquad (1)$$

- If the reported losses are near ultimate, the parameter σ_D will be very small. Thus for $w = 1$ the ultimate loss is approximately equal to $Premium_1$ times the expected loss ratio, $\exp(logelr)$. The α_w parameters account for accident year differences in the loss ratio.

Interpreting the Model Parameters — Continued

- Note that since $\beta_D = 0$ the $Sp(t)$ does not *directly* affect the projected ultimate loss.
- However, the $Sp(t)$ indirectly affects the ultimate loss parameters, *logelr* and α_w, through the Bayesian MCMC fitting algorithm.
- Recall

$$C_{wd} \sim \text{lognormal}(\mu_{wd}, \sigma_d)$$

$$\mu_{wd} = \log(Premium_w) + logelr + \alpha_w + \beta_d \cdot Sp(t)$$

- Heuristically speaking, it is the entire sum of the terms in the expression for μ_{wd} that is "attracted" to C_{wd}. The values of $\beta_d \cdot Sp(t)$ will influence values of *logelr* and α_w.

Interpreting the Speedup Function, $Sp(t)$

- What will distinguish the models in this paper is the choice of the $Sp(t)$ function. Let's discuss its meaning.
- Recall that $\beta_D = 0$. If $Sp(1) > Sp(2) > \cdots$, then the product $\beta_d \cdot Sp(t)$ is moving closer to 0 as t increases.
- For paid losses, this means losses are being settled more quickly over time.
- For incurred losses, this means that losses are being recognized more quickly over time.
- The reverse is true if $Sp(1) < Sp(2) < \cdots$. That is, paid losses are being settled more slowly over time, and incurred losses are being recognized more slowly over time.

Models Considered in This Paper

- The CRC Model — $Sp(w) \equiv 1$
  - This model most closely resembles the standard actuarial models that do not allow the development patterns to change over time.
- The CSR-w Model — $Sp(w) = (1-\gamma)^{w-1}$
  - $\gamma > 0$ gives us a decreasing $Sp(w)$ as the accident year, w increases from 1 to W. $\gamma < 0$ gives us an increasing $Sp(w)$.
- The CSR-c Model — $Sp(c) = (1+\gamma)^{C-c}$
  - $\gamma < 0$ gives us a increasing $Sp(c)$ as the calendar year, c, increases from 1 to $C-1$. $\gamma > 0$ gives us a decreasing $Sp(c)$
- We refer to the γ parameter as the speedup rate. We call a negative speedup rate a slowdown.

Models Considered in This Paper - Continued

- The CSR-vc Model —

$$
\begin{aligned}
Sp(C) &= 1 \\
Sp(C-i) &= Sp(C-i+1)\cdot(1+\gamma_{C-i}) \\
&\quad \text{for } i = 1, \cdots, C-1
\end{aligned}
$$

  - This model allows the speedup rate to vary by calendar year.
- The first two models are described in Meyers (2019). The next two were developed during the research that led to this paper. As we shall see, analyses of the shortcomings of these models point to another model, the POS-vc model that I will describe below.

The Run ID

- The various model runs in this paper will be fit on a given set of calendar years of either the paid or incurred loss triangle.
- Each model run will have an identifier with three components.
  1. The model name
  2. The loss triangle used — either “P” or “I”
  3. The calendar year range.
- For example, the run id “CSR-vc P-7:16” means that the CSR-vc model was fit to the paid loss triangle using data from the calendar years from 7 to 16.

Invoking Bayesian MCMC

- As described in Meyers (2019), the Bayesian MCMC fitting algorithm produces 10,000 equally likely parameter sets[3] $\{logelr\}$, $\{\alpha_w\}_{w=1}^{W}$, $\{\beta_d\}_{d=1}^{D}$, $\{\gamma\}$ and $\{\sigma_d\}_{d=1}^{D}$.
- The R/Stan scripts for the five models are included with this paper.
  - LL_CRC.R
  - LL_CSR_w.R
  - LL_CSR_c.R
  - LL_CSR_vc.R
  - LL_POS_vc.R
- The scripts allow for the user to select which triangle (paid or incurred) , and the calendar years within each triangle to use in fitting the model.

[3] The presence of brackets $\{\cdot\}$ around a parameter will indicate that it is a sample of 10,000 values from the posterior distribution.

Parameter Summary Statistics

- The spreadsheet titled "Posterior_Stats.xls" contains the means and standard deviations of the 10,000 parameter sets for the models run in this paper.
- Some observations:
  - While somewhat volatile, values taken from the γ parameter sample are generally negative — indicating a slowdown in claim settlements.
  - The ranges of the $\{\alpha_w\}$ parameter samples are small in the earlier accident years where the reported losses are well known. But the ranges grow wider in the later accident years.
  - One would expect the ranges of the $\{\alpha_w\}$ parameter samples for the P-1:16 models to be smaller that those for the P-7:16 models, since there are more observations in the P-1:16 dataset. Instead the opposite is true.

Statistics of Interest

- With a sample of 10,000 parameter sets, one can use Equation 1 to to obtain a sample of 10,000 expected ultimate losses, $\{U_w\}$
- Define $\{U_{Tot}\} = \sum_{w=1}^{16}\{U_w\}$.
- Also of interest is a sample of 10,000 possible unpaid losses (ultimate loss less current paid loss), $\{R_c\}$, at calendar year c where:

$$R_c = \sum_{w=1}^{c} U_w - \sum_{d=1}^{c} C_{c+1-d,d} \tag{2}$$

Statistics of Interest — Continued

- From the samples $\{R_c\}$ and $\{U_{Tot}\}$, we can calculate statistics of interest, such as:
  - Ultimate Loss $=$ mean$\{U_{Tot}\}$
  - Ultimate Standard Error $=$ standard deviation$\{U_{Tot}\}$
  - Reserve Low $=$ 2.5th percentile of $\{R_{16}\}$
  - Reserve $=$ mean$\{R_{16}\}$
  - Reserve High $=$ 97.5th percentile of $\{R_{16}\}$

Running the MCMC Models on P-1:16 Data

- The table below shows the calculations for the above statistics for each of the four models fit with the P-1:16 data.
- Note that the results vary significantly by model. To resolve these differences we need some model diagnostics — the $\widehat{elpd}_{loo}$ and the Standardized Residual Boxplots that we now turn to describing.

Run ID	Ult Loss	Ult SE	Res Low	Reserve	Res High	$\widehat{elpd}_{loo}$
CRC P-1:16	1,147,142	38,750	118,780	187,214	272,284	222.13
CSR-w P-1:16	1,284,563	63,262	213,704	324,635	460,932	229.51
CSR-c P-1:16	1,328,029	69,501	248,047	368,101	519,552	230.03
CSR-vc P-1:16	1,251,066	80,223	159,958	291,137	473,496	238.60

The Expected Log Predictive Density ($\widehat{elpd}_{loo}$)[4]

- For each observation, C_{wd} in the loss triangle:
  1. Remove that observation from the data.
  2. Fit the selected model to the data in the triangle that remains and obtain the parameter sets $\{\theta(-wd)\}$ (consisting of all the $\{\alpha_w\}$s, $\{\beta_d\}$s, etc.)
  3. Calculate the average likelihood, $p(C_{wd}|\{\theta(-wd)\})$ over all 10,000 parameter sets.
- Then

$$\widehat{elpd}_{loo} = \sum_{w,d} log(p(C_{wd}|\{\theta(-wd)\}))$$

- The "loo" term refers to the "leave one out" feature in bullet #1 above.

[4] More details about this statistic are in Section 6 of Meyers (2019).

$\widehat{elpd}_{loo}$ — Continued

- A higher $\widehat{elpd}_{loo}$ indicates a better fit. By this measure, the CSR-vc model fits the P-1:16 data the best.
- Since the likelihoods are calculated on holdout data, there is no penalty for fitting models with a large number of parameters.

Standardized Residual Boxplots

- The models in this paper all assume a lognormal distribution with the parameters μ_{wd} and σ_d. Thus we expect that

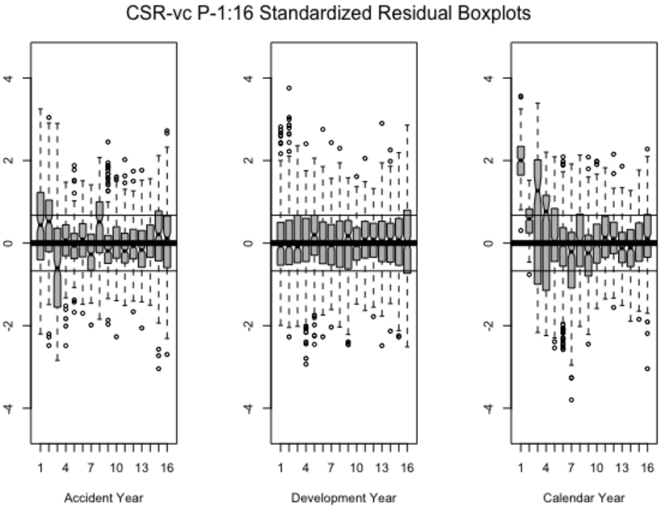
$$\frac{log(C_{wd}) - \{\mu_{wd}\}}{\{\sigma_d\}}$$

will have a normal(0,1) distribution.
- To test this graphically we split the residuals, in turn by accident year, development year and calendar year and plot a sample of size 200 in each "year" with the R "boxplot" function.

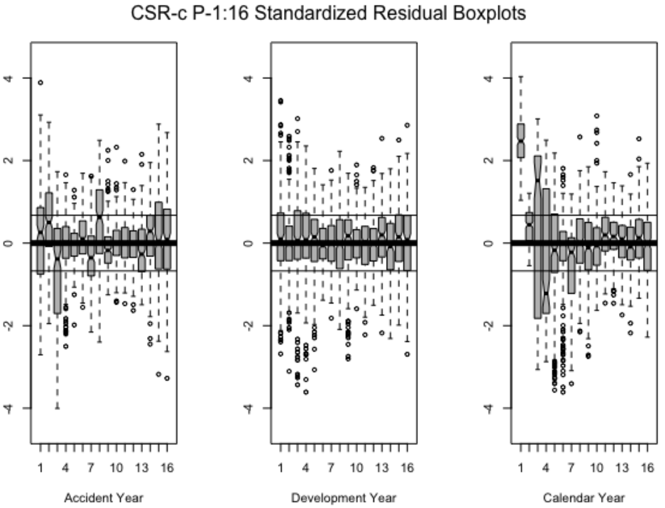
Expected Results with the R "boxplot" Function

- The following four pages contain the standardized residual Boxplots for the four models on the P-1:16 data.
- The gray bars correspond to the interquartile range. Ideally the bars should be centered on 0. The endpoints of those bars should be touching the black lines representing the interquartile range of the standard normal distribution.
- Most of the remaining residuals should be between $\pm$ 2. A few could be in the (-3,-2) or the (2,3) ranges. Very few should be outside the $\pm$ 3 range.
- Now flip through the next four pages to see how close the Boxplots are to the "ideal" Boxplot. I will give my take on the other side.

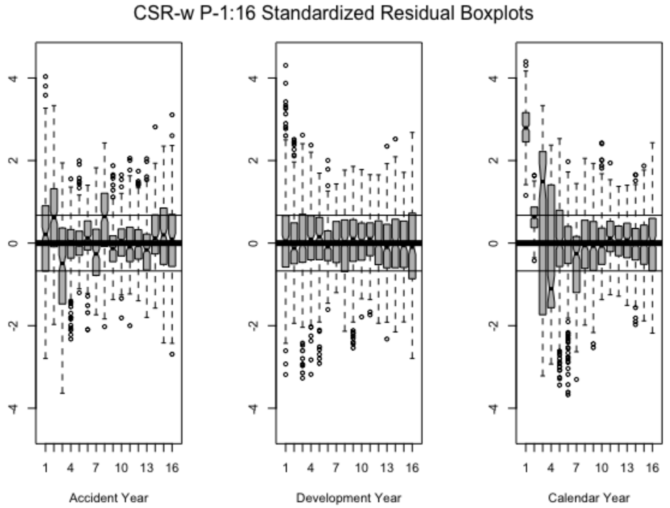
Boxplots P-1:16



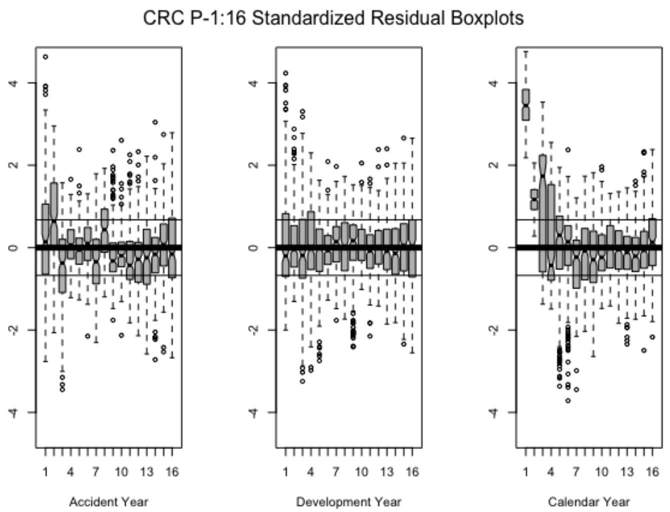
Boxplots P-1:16



Boxplots P-1:16



Boxplots P-1:16



P-1:16 Discussion

- The Boxplots of all four models:
  - Accident year — Not ideal
  - Development year — Pretty good
  - Calendar year — Bad for the early calendar years
- AY, DY and CY Boxplots get worse as you flip from CSR-vc ⇒ CSR-c ⇒ CSR-w ⇒ CRC.
- CSR-vc had the best fit according to the $\widehat{elpd}_{loo}$ statistic.
- The CSR-vc appears to be the best model for P-1:16.
- The problem appears most prominently in the calendar year Boxplots. It appears that the calendar year changes in speedup rate are more complicated than assumed by the speedup function.

Choosing a Subset of the P-1:16 Data

- When we have a portion of the data that does not fit our current model we have two options.
  1. Find a modification to your model that fits all the data.
  2. Dropping that portion of the data that does not fit our current model.
- Lacking access to the claim handlers for these data, I elected to use the most recent 10 calendar years.
- Dropping older calendar years while fitting loss reserve models is a fairly common practice among actuaries

Running the MCMC Models on P-7:16 Data

Run ID	Ult Loss	Ult SE	Res Low	Reserve	Res High	$\widehat{elpd}_{loo}$
CRC P-7:16	1,186,501	26,857	175,749	226,573	283,168	234.94
CSR-w P-7:16	1,233,188	46,782	185,870	273,260	369,290	235.05
CSR-c P-7:16	1,280,649	53,024	227,348	320,721	439,200	236.06
CSR-vc P-7:16	1,256,436	73,076	179,454	296,510	460,811	240.85

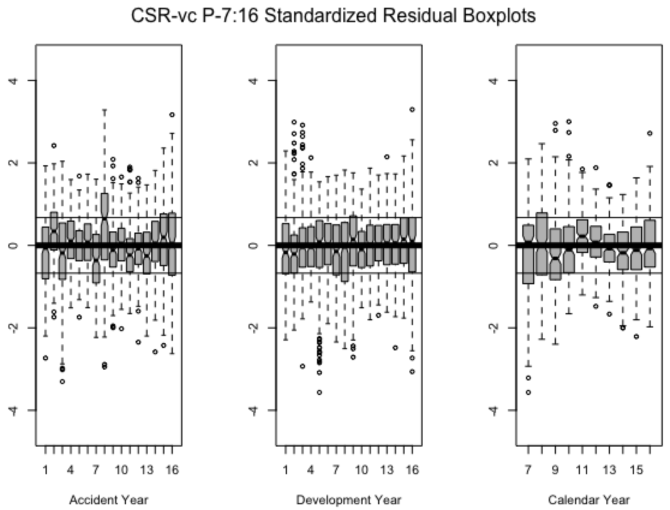
Some observations

- The CSR-vc model had the highest $\widehat{elpd}_{loo}$ statistic.
- The mean reserve estimates vary significantly by model.
- Look at the "Gamma" tab in "Posterior_Stats.xls".
  - The speedup parameter is -0.0156 for the CSR-w model, -0.0291 for the CSR-c model. For the CSR-vc model it starts as 0.0131 and moves down to fluctuate between the -0.012 to -0.035 range for the later calendar years.
  - A negative speedup parameter means a slowdown in claim settlements, and hence a higher predicted ultimate loss.

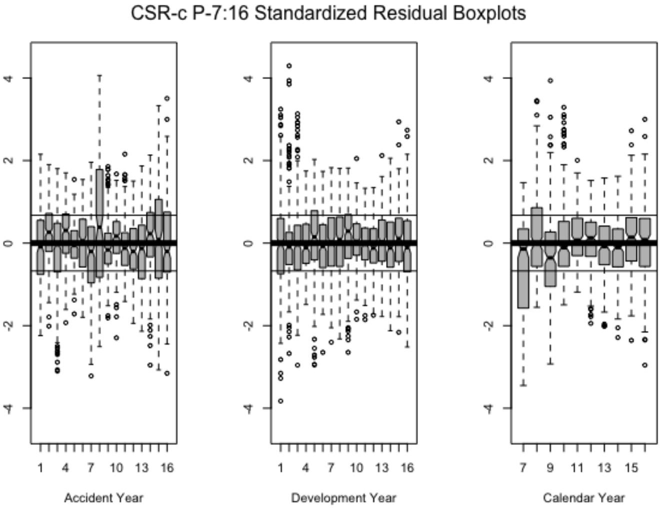
Observation on the P-1:16 and P-7:16 Datasets

- Looking at the CSR-vc model on each dataset:
  - The ultimate loss estimates were fairly close.
  - The range of the loss estimates are narrower for the P-7:16 data.
  - As the P-7:16 data has fewer observations, one should expect the reverse to be true.
- I attribute this reversal to model error with the P-1:16 data.
- Now scroll through the Boxplots for these models.

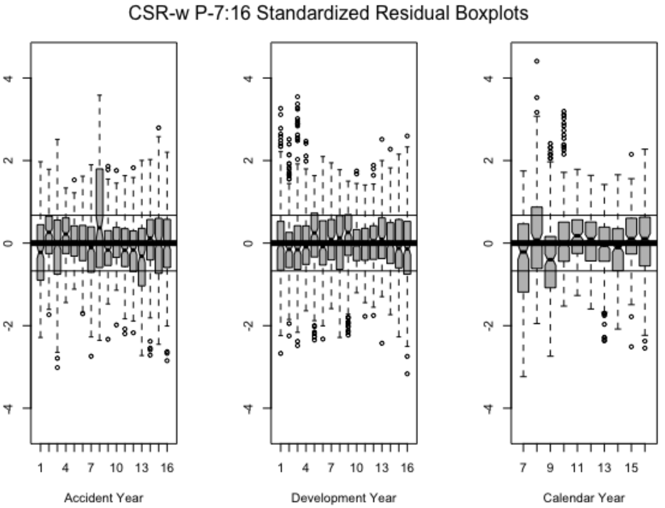
Boxplots P-7:16



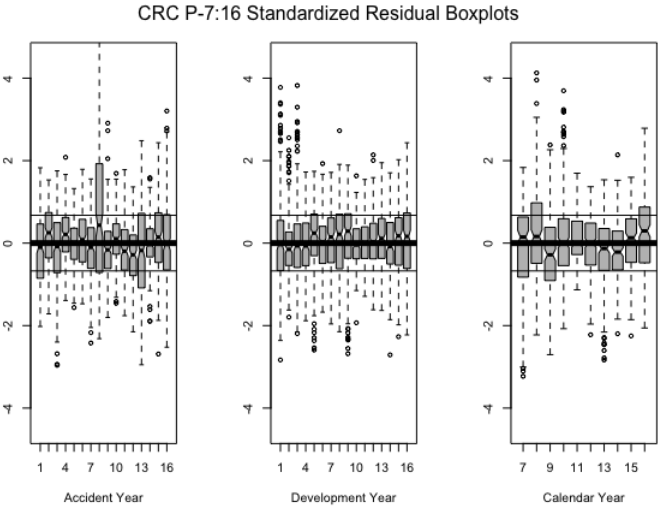
Boxplots P-7:16



Boxplots P-7:16



Boxplots P-7:16



P-7:16 Discussion

- I judge the CSR-vc model to have the best Boxplots.
  - The interquartile ranges are about the same and all pretty good.
  - The CSR-vc model has noticeably fewer outliers in the Boxplots, i.e. outside the ± 2 range.
- This combined with its having the highest $\widehat{elpd}_{loo}$ statistic make it the model of choice for the paid data.

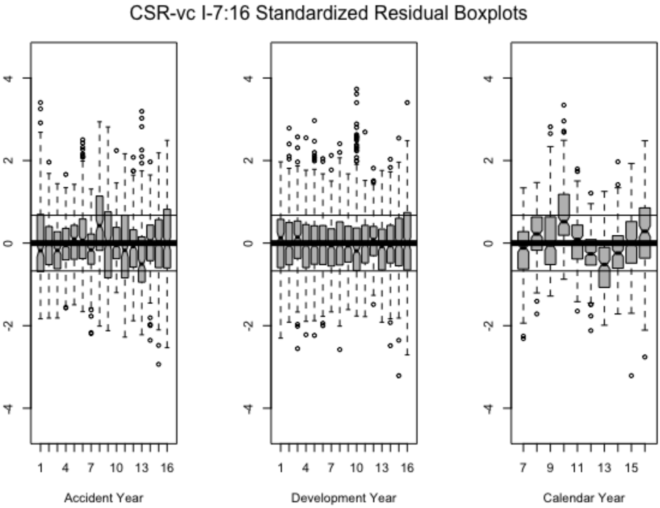
Running the MCMC Models on I-7:16 Data

Run ID	Ult Loss	Ult SE	Res Low	Reserve	Res High	$\widehat{elpd}_{loo}$
CRC I-7:16	1,230,151	29,800	214,940	270,223	333,281	235.64
CSR-w I-7:16	1,193,518	37,085	167,331	233,590	313,557	232.50
CSR-c I-7:16	1,317,128	75,412	243,610	357,200	531,381	235.19
CSR-vc I-7:16	1,262,187	58,618	201,157	302,261	430,947	241.27

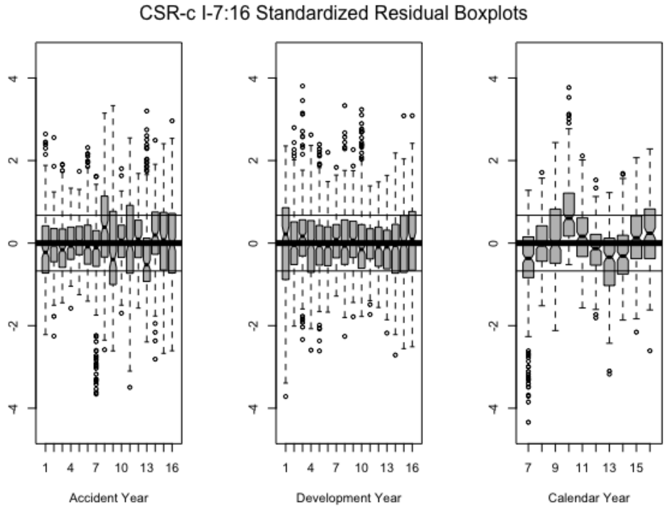
Some observations

- The CSR-vc model had the highest $\widehat{elpd}_{loo}$ statistic.
- The mean reserve varies significantly by model.
- Look at the "Gamma" tab in "Posterior_Stats.xls".
  - The speedup parameter is a *positive* 0.0375 for the CSR-w model, a *negative* 0.0675 for the CSR-c model. For the CSR-vc model it starts close to zero and moves up around the -0.02 to -0.04 range for the later calendar years.
  - A negative speedup parameter for incurred losses can also indicate a decreasing recognition of outstanding losses, and hence a higher predicted ultimate loss.
- Now scroll through the Boxplots for these models.

Plots I-7:16

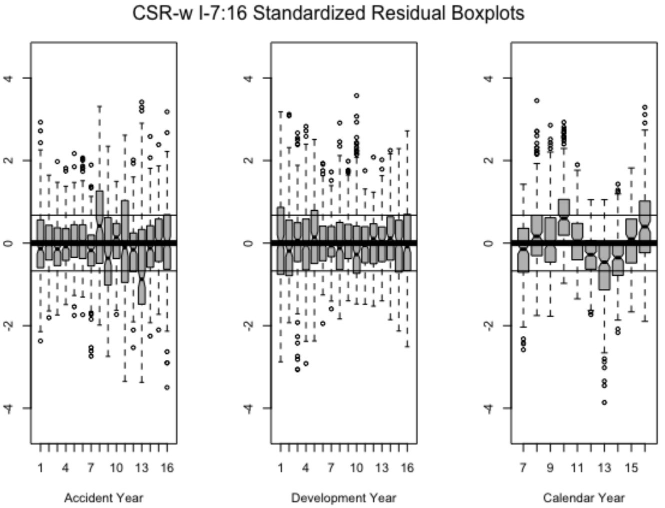

CSR-vc I-7:16 Standardized Residual Boxplots
4
2
0
-2
-4
1 4 7 10 13 16
Accident Year
1 4 7 10 13 16
Development Year
7 9 11 13 15
Calendar Year

Plots I-7:16

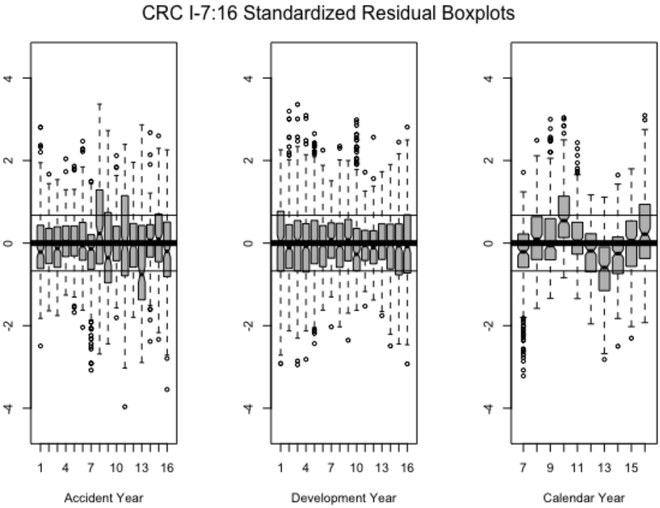


CSR-c I-7:16 Standardized Residual Boxplots
4
2
0
-2
-4
1 4 7 10 13 16
Accident Year
Development Year
7 9 11 13 15
Calendar Year

Plots I-7:16


CSR-w I-7:16 Standardized Residual Boxplots
4
2
0
-2
-4
1 4 7 10 13 16
Accident Year
Development Year
7 9 11 13 15
Calendar Year

Plots I-7:16

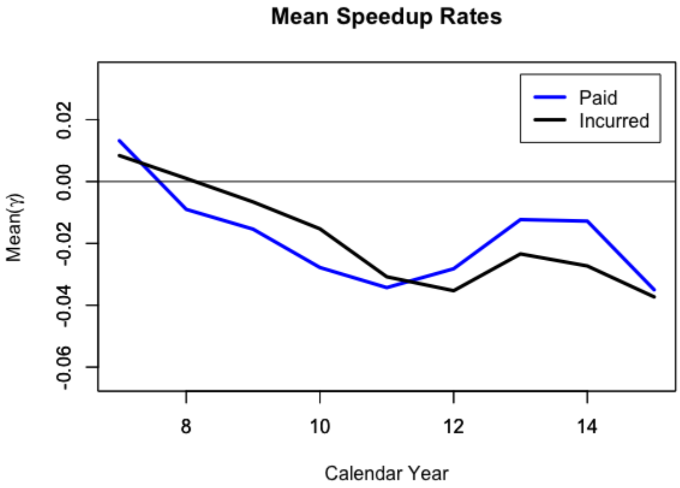


CRC I-7:16 Standardized Residual Boxplots
4
2
0
-2
-4
1 4 7 10 13 16
Accident Year
Development Year
7 9 11 13 15
Calendar Year

I-7:16 Discussion

- I judge that the CSR-vc model has the best Boxplots.
  - Slightly better by accident year and development year.
- The Boxplots by calendar year suggests that there as been a change in case reserving practices.
- The next page shows plots of the mean speedup rates, i.e. γ parameters, for the paid and the incurred models. One would expect to see the plots track closely with each other as a substantial portion of the incurred losses are already paid.
- But — As we can see from these plots, there is a noticeable difference between the plots. And moreover, they cross.
- This suggests that there should be separate $\{\gamma\}$ parameters for paid and outstanding losses.

Mean Speedup Rates for the CSR-vc P-7:16 and the CSR-vc I-7:16 Models



Integrated Paid and Outstanding (POS) Models

- This section proposes a model that simultaneously fits both paid and incurred losses.[5].
- This model has lognormal distributions for each of the paid and incurred losses.
  - The μ_{wd} parameter of the distribution for paid losses is the same as above.
  - The μ_{wd} of the incurred losses are equal to the sum of the μ_{wd} for the paid losses, plus a separate factor representing outstanding losses.
- More details on the next page.

[5] A more detailed discussion of fitting models simultaneously to paid and incurred is discussed in Section 9 of Meyers (2019)

The POS-vc Model

The prefixes P, I and OS denote "Paid", "Incurred" and "Outstanding" respectively.

$$
\begin{aligned}
{}_P\mu_{wd} &= \log(Premium_w) + logelr + \alpha_w + {}_P\beta_d \cdot {}_PSp(c) \\
{}_I\mu_{wd} &= {}_P\mu_{wd} + {}_{OS}\beta_d \cdot {}_{OS}Sp(c)
\end{aligned}
$$

$$
{}_P\beta_D \equiv 0, \text{ and } {}_{OS}\beta_D \not\equiv 0
$$

Where

$$
\begin{aligned}
{}_XSp(C) &= 1 \\
{}_XSp(C-i) &= Sp(C-i+1) \cdot (1 + {}_X\gamma_{C-i}) \\
&\quad \text{for } i = 1, \cdots, C-1 \text{ and } X = P \text{ or } OS
\end{aligned}
$$

Then

$$
\begin{aligned}
{}_PC_{wd} &\sim \text{lognormal}({}_P\mu_{wd}, {}_P\sigma_d) \\
{}_IC_{wd} &\sim \text{lognormal}({}_I\mu_{wd}, {}_I\sigma_d)
\end{aligned}
$$

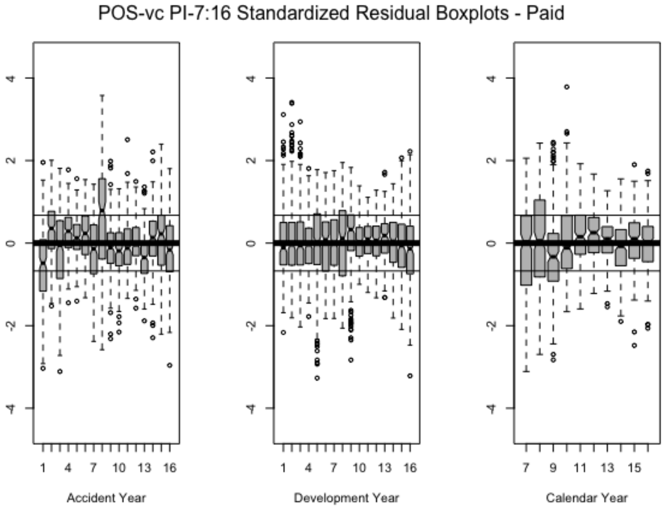
Running the POS Model on PI-7:16 Data

- Results for the comparable CSR model runs are also given.
- The script for this model is in LL_POS_vc.R

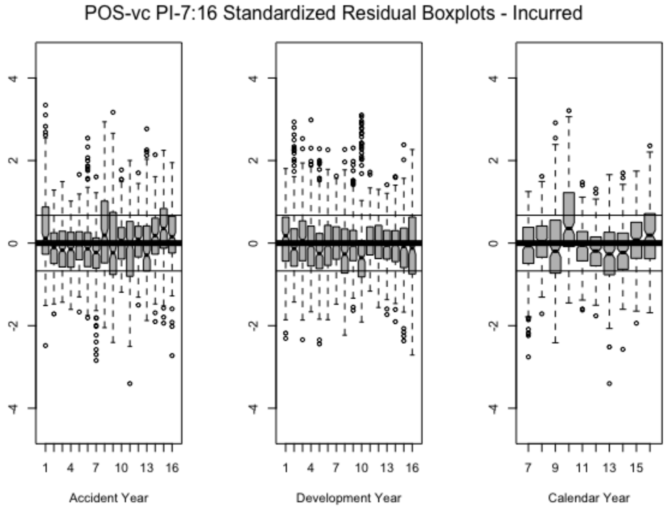
Run ID	Ult Loss	Ult SE	Res Low	Reserve	Res High	$\widehat{elpd}_{loo}$
CSR-vc P-7:16	1,256,436	73,076	179,454	296,510	460,811	240.85
CSR-vc I-7:16	1,262,187	58,618	201,157	302,261	430,947	241.27
POS-vcp PI-7:16	1,262,897	58,400	205,683	302,969	432,516	251.85
POS-vci PI-7:16	1,262,897	58,400	205,691	302,969	432,529	255.56

- The $\widehat{elpd}_{loo}$ statistics are calculated separately on the paid and incurred data in the POS model. These statistics are significantly better for the POS-vc model than they are for the corresponding CSR-vc models.
- The standardized residual Boxplots are on the following three pages. Compared with the corresponding CSR-vc Boxplots:
  - The POS-vc plots look a bit worse for the paid losses.
  - They look a bit better for the incurred losses.
  - They look pretty good for the combined losses.

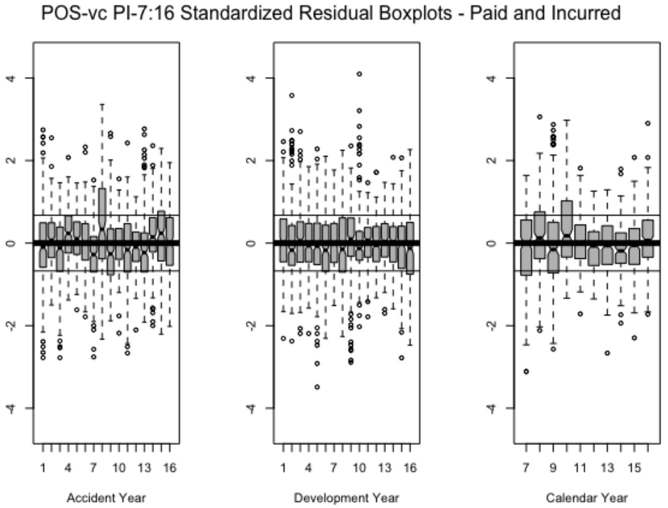
Boxplots for POS-vc Model



Boxplots for POS-vc Model



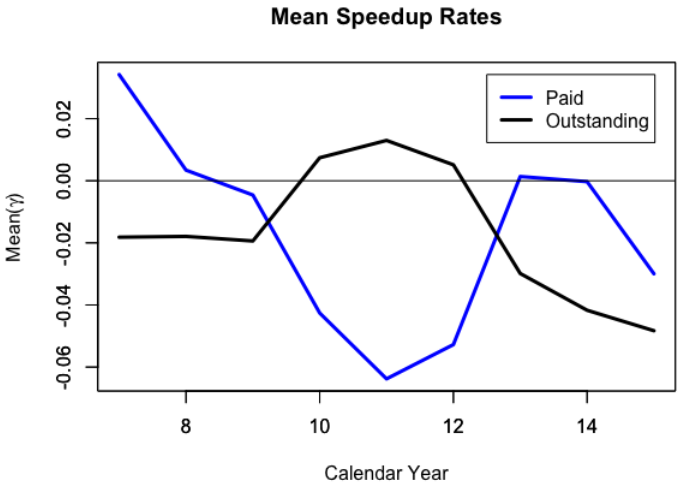
Boxplots for POS-vc Model



Claims Department Practices

- The following page has plots of the the mean paid claim speedup rate, mean$\{_P\gamma\}$, and the mean outstanding claim speedup rate, mean$\{_{OS}\gamma\}$.
- The claims department appears to be slowing down the paid claim settlements, while speeding up the recognition of outstanding claims, and vice versa.
- This observation should be discussed with the claims department.

Mean Speedup Rates for the POS-vc Model



Estimating Ultimate Losses

- Recall from Equation 1 that the ultimate expected loss for accident year w is equal to the expected value

$$Premium_w \cdot \mathrm{E}\{\exp(logelr + \alpha_w + {}_X\sigma_D^2/2)\}$$

where X can refer to either paid, P, or incurred, I, losses.

- For the POS-vc model the expected ultimate incurred loss is slightly more complicated.

$$Premium_w \cdot \mathrm{E}\{\exp(logelr + \alpha_w + {}_{OS}\beta_D + {}_I\,\sigma_D^2/2)\}$$

- After 16 years of development, the values of ${}_{OS}\beta_D$ and ${}_X\sigma_D$ are close to zero. So the paid and incurred loss estimates are very close to each other.
- The following three pages give the ultimate loss estimates by accident year for the CSR-vc and POS-vc models.

Accident Year Exhibit for CSR-vc P-7:16

AY	Premium	Estimate	SE	CV
2002	13,750	7,035	36	0.0051
2003	28,052	11,172	89	0.0080
2004	44,853	27,882	237	0.0085
2005	70,507	42,229	397	0.0094
2006	80,285	45,451	459	0.0101
2007	96,286	58,149	659	0.0113
2008	130,481	66,126	817	0.0124
2009	142,059	49,960	715	0.0143
2010	131,024	70,952	1,150	0.0162
2011	131,870	89,695	1,702	0.0190
2012	122,125	83,745	2,025	0.0242
2013	125,456	88,474	2,794	0.0316
2014	201,129	105,300	4,505	0.0428
2015	271,351	148,458	10,143	0.0683
2016	297,237	180,482	22,320	0.1237
2017	292,035	181,328	51,519	0.2841
Total	2,178,500	1,256,436	73,076	0.0582

Accident Year Exhibit for CSR-vc I-7:16

AY	Premium	Estimate	SE	CV
2002	13,750	7,037	35	0.0050
2003	28,052	11,056	84	0.0076
2004	44,853	27,643	231	0.0084
2005	70,507	41,556	376	0.0090
2006	80,285	44,764	439	0.0098
2007	96,286	57,033	619	0.0109
2008	130,481	65,057	765	0.0118
2009	142,059	49,100	658	0.0134
2010	131,024	70,228	1,078	0.0154
2011	131,870	89,487	1,542	0.0172
2012	122,125	82,593	1,863	0.0226
2013	125,456	87,515	2,526	0.0289
2014	201,129	105,847	4,581	0.0433
2015	271,351	148,245	9,905	0.0668
2016	297,237	184,843	21,850	0.1182
2017	292,035	190,185	46,733	0.2457
Total	2,178,500	1,262,187	58,618	0.0464

Accident Year Exhibit for POS-vc PI-7:16

AY	Premium	Estimate(p)	Estimate(i)	SE(p)	SE(i)	CV(p)	CV(i)
2002	13,750	7,038	7,038	34	34	0.0048	0.0048
2003	28,052	11,117	11,117	68	68	0.0061	0.0061
2004	44,853	27,779	27,779	184	184	0.0066	0.0066
2005	70,507	41,907	41,907	299	299	0.0071	0.0071
2006	80,285	45,135	45,135	347	347	0.0077	0.0077
2007	96,286	57,636	57,636	489	489	0.0085	0.0085
2008	130,481	65,630	65,630	603	603	0.0092	0.0092
2009	142,059	49,658	49,658	513	513	0.0103	0.0103
2010	131,024	70,692	70,692	792	792	0.0112	0.0112
2011	131,870	89,930	89,930	1,175	1,175	0.0131	0.0131
2012	122,125	83,456	83,456	1,373	1,373	0.0165	0.0165
2013	125,456	88,619	88,619	1,909	1,909	0.0215	0.0215
2014	201,129	106,701	106,701	3,372	3,372	0.0316	0.0316
2015	271,351	149,816	149,816	7,690	7,690	0.0513	0.0513
2016	297,237	183,299	183,299	17,254	17,254	0.0941	0.0941
2017	292,035	184,484	184,484	37,469	37,469	0.2031	0.2031
Total	2,178,500	1,262,897	1,262,897	58,400	58,400	0.0462	0.0462

Predictive Distribution of Loss Reserve Liability

- Following Equation 2, a sample of the predictive distribution of the outstanding losses is given by:

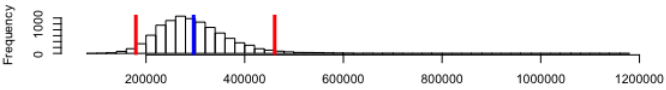
$$\{_X R_C\} = \sum_{w=1}^{C} \{_X U_w\} - \sum_{d=1}^{C} C_{c+1-d,d}$$

where $X =$CSR-vc P-7:16, CSR-vc I-7:16 or POS-vc 7:16.

- Histograms of the predictive distributions for these models are given in the next page.
- Note that the POS-vc model reduces the range of ultimate estimates, by a lot for paid losses, and by a little for incurred losses.

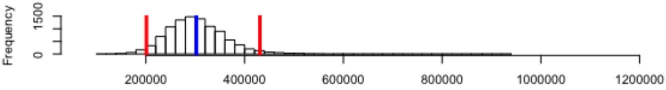
Predictive Distribution of Loss Reserve Liability

CSR-vc P-7:16 Predictive Distribution of the Loss Reserve



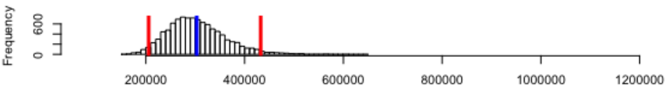
Mean = 296510
Low End (2.5%) = 179454 High End (97.5%) = 460811

CSR-vc I-7:16 Predictive Distribution of the Loss Reserve



Mean = 302261
Low End (2.5%) = 201157 High End (97.5%) = 430947

POS-vci PI-7:16 Predictive Distribution of the Loss Reserve



Mean = 302969
Low End (2.5%) = 205691 High End (97.5%) = 432529

Hindsight Reserves

- A “Hindsight Reserve” is a reserve for an earlier calendar year calculated with parameters derived from current data.
- In the notation of this paper, let’s define:

$$\text{Reserve}(CY, CY_{Data}) \equiv \text{Mean}[\{R_{CY}\}]$$

where the parameters $\{logelr\}$, $\{\alpha_w\}$ and $\{\sigma_D\}$ were calculated from a loss triangle compiled in the year CY_{Data}.

- If $CY < CY_{Data}$, Reserve(CY, CY_{Data}) is called a hindsight reserve.
- If $CY = CY_{Data}$, Reserve(CY, CY_{Data}) is the original posted reserve.

Hindsight Reserves — Continued

- The next three pages compare hindsight reserves with the originally posted reserves using the following models.
  - POS-vc on P-7:c The results for I-7:c are very similar.
  - CSR-vc on P-7:c as the best model for paid data.
  - CSR-vc on I-7:c to demonstrate the effect of the poor fit along the calendar year dimension.
  - CRC on P-7:c is the model that ignores any changes in the claim speedup rate.
- Notation for the following tables
  - The original posted reserves are in **bold**.
  - Note — The relationship between the nominal calendar year, CY, and the calendar year index, c is $CY = c + 2001$. CY_{Data} for the dataset P-7:c is equal to $c + 2001$.

Hindsight Reserves for POS-vc P-7:c Models

	CY_{Data}			
CY	2017	2016	2015	2014
2014	164,503	156,036	143,788	**122,639**
2015	209,566	206,311	**201,284**	
2016	255,896	**284,712**		
2017	**299,989**			

Changes from Original to Latest Hindsight

2014 $+34.1\%$

2015 $+4.1\%$

2016 -10.1%

Hindsight Reserves for CSR-vc P-7:c Models

	CY_{Data}			
CY	2017	2016	2015	2014
2014	165,380	163,306	161,387	**157,006**
2015	209,387	215,340	**223,681**	
2016	253,559	**297,939**		
2017	**296,510**			

Changes from Original to Latest Hindsight

2014 +5.3%

2015 −6.4%

2016 −14.9%

Hindsight Reserves for CSR-vc I-7:c Models

	CY_{Data}			
CY	2017	2016	2015	2014
2014	158,126	140,872	123,894	**110,558**
2015	201,920	182,147	**176,653**	
2016	250,453	**247,591**		
2017	**302,261**			

Changes from Original to Latest Hindsight

2014 +43.0%

2015 +14.2%

2016 +1.2%

Hindsight Reserves for CRC P-7:c Models

	CY_{Data}			
CY	2017	2016	2015	2014
2014	156,230	144,382	130,750	**123,818**
2015	191,356	177,752	**158,059**	
2016	220,532	**212,310**		
2017	**226,575**			

Changes from Original to Latest Hindsight
2014 +26.2%
2015 +21.2%
2016 +3.9%

Discussion of Hindsight Results[6]

- Let's examine the difference between the original and hindsight reserves more closely. What affects future hindsight reserves?
- It follows from our lognormal assumption that for a given calendar year c:

$$\log(C_{wd}) = \log(Premium_w) + \mathrm{E}[\{logelr + \alpha_w + \beta_d \cdot Sp(c)\}]$$

- Suppose for calendar year $\mathscr{C} > C$ we obtain a new set of parameters for the model and obtain for a w and d in calendar year $\mathscr{C}$:

$$\log(C_{wd}) = \log(Premium_w) + \mathrm{E}[\{logelr' + \alpha'_w + \beta'_d\}]$$

[6] This analysis applies to the CSR-vc model. For the POS-vc model, the derivation has more terms, but follows the same logic.

Explaining Hindsight Changes - The Math

- Since $\log(C_{wd})$ should be the same for models fit in each calendar year c and $\mathscr{C}$ we can combine the two equations on the last page to get:

$$\mathrm{E}[\{logelr + \alpha_w - logelr' - \alpha'_w\}] = \mathrm{E}[\{\beta'_d - \beta_d \cdot Sp(c)\}]$$

- If

$$\mathrm{E}[\{\beta_d \cdot Sp(c)\}] < \mathrm{E}[\{\beta'_d\}]$$

then we expect

$$\mathrm{E}[\{logelr + \alpha_w\}] > \mathrm{E}[\{logelr' + \alpha'_w\}]$$

which (according to Equation 1), will nudge the estimate of the hindsight reserve higher for accident year w.

Projecting Next Year's Hindsight Reserve

- Suppose, after a conversation with the claims department, we expect next year's speedup rate to be equal to γ'_C.
- Given current data, how would this change our reserve estimate?
- Keeping the same structure of the model:

$$\begin{aligned} \{\beta_d\} &= \{\beta'_d\} \cdot (1 + \gamma_C) \\ \{Sp'(c)\} &= \{Sp(c)\} \cdot (1 + \gamma_C) \end{aligned}$$

Since $\mathrm{E}[C_{wd}]$ is unchanged, this implies that

$$\begin{aligned} \{\beta_d \cdot Sp(c)\} &= \{\beta'_d \cdot Sp'(c)\} \text{ and} \\ \mathrm{E}[\{logelr + \alpha_w\}] &= \mathrm{E}[\{logelr' + \alpha'_w]\}) \end{aligned}$$

Projecting Next Year's Hindsight Reserve

- Thus this projection does not change the reserve estimate!
- If there is a difference between the hindsight reserve and the current estimated reserve, it is due to the new data that changes the parameters in ways *unanticipated* by the model.
- For the paid data, the current and the hindsight reserve estimates are relatively close. This encourages confidence in the model for paid data.
- But for the incurred data, the current and hindsight estimates for the CSR and POS models are noticeably different.
- The POS model helps the incurred data somewhat, but I still think there is something different in the incurred data.

The Question Addressed by This Paper

- In prior years the original opining actuary underestimated the loss reserve liability.
- Was this a case of "bad stuff" that sometimes happens?
- Or was it the case that there is a loss reserve model that does a better job of predicting the "bad stuff?"
- The Learning Lounge presentation mentioned a number of red flags, e.g. declining paid to current ultimate and declining incurred to current ultimate ratios, and slowdown in claim settlement due to rapid premium growth.
- In looking at the "Actuarial Opinion - History" slide in the introduction, it appears that the opining actuary and the Learning Lounge presenters recognized by 2016 and 2017, that earlier reserve estimates were understated because of the slowdown in the claim settlements. To my way of thinking, this means that they needed a better model.

A Proposal for the “Bad Stuff” i

- This paper proposes the “vc” models that explicitly recognize changes in the claim speedup rate by calendar year.
- The CRS-vc model works well with paid losses, but not very well with incurred losses.
- POS-vc model obtains a better fit with the incurred losses, there were still shortcomings identified in the hindsight analyses.

Comparison with More Traditional Approaches

- In the right hands, traditional actuarial judgment can be pretty good.
- The next page compares the ultimate estimates by accident year obtained by Mary Frances, the company actuary and the CSR-vc P-7:16 model. The CSR-vc model estimates are close, but in general, are a bit lower than the estimates in the Learning Lounge presentation.

Page 62 from "Learning Lounge" Presentation

A Case Study

Glenn Meyers

Introduction

Models

MCMC

P-1:16 Data

$\widehat{elpd}_{loo}$

Boxplots

P-7:16 Data

Boxplots

I-7:16 Data

Boxplots

Speedup Rates

POS Model

Boxplots

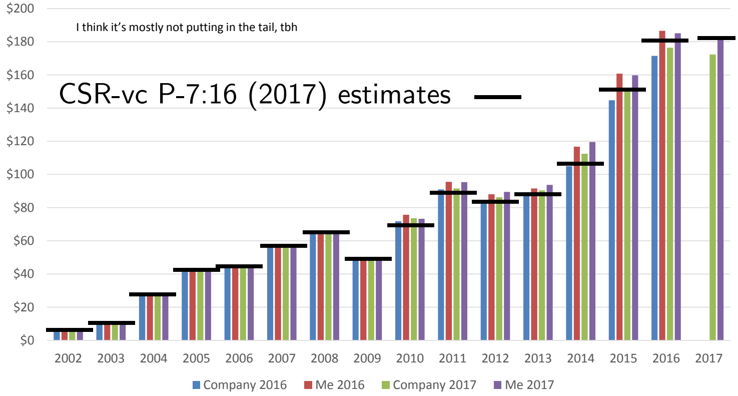
Speedup Rates

Hindsight

Discussion

Bad Stuff?

Commentary



My Loss Reserving Philosophy

- I view loss reserving as a dialogue between an actuarial department and its corresponding claims department. One way this dialogue might work is as follows.
  1. In talking with the claims department, the actuaries try to find out how the claims adjustment process works.
  2. They then formulate a model that describe the claims adjustment process. Then test the model thoroughly.
  3. If testing reveals unexpected differences between the model and the data, repeat Steps 1-2 above as necessary.
- Advantages of using Bayesian MCMC for model building
  1. Flexibility in model building — If you can code the likelihood function, you can run the model.
  2. Bayesian models are transparent and reproducible. Your judgments are made explicit in your choice of models and prior distributions.
  3. Bayesian models provide output that can be used for calculating risk margins. See Section 11 of Meyers (2019).

Final Comments

- The flexibility of Bayesian MCMC models was very helpful in this exercise. It enabled me to to easily explore beyond my existing collection of models.
- Over time, I expect that I, and others, will add to our collection of such models in the future.
- I want to thank Bob and Mary Frances for making these data available to the public. It was interesting to see how well the estimates derived from a Bayesian MCMC model tracked with the estimates from experienced reserving actuaries. I was glad for my model and for the actuarial profession, to see that the estimates were reasonably close.
- Generally speaking, I am willing to try fitting a Bayesian MCMC model to any real loss triangle that is, or can be made, publicly available.

References

- This paper refers extensively to Meyers (2019). This also applies to the references in that monograph.
- I want to make a call out to Ben Zehnwirth, who for years has been insisting on a calendar year model for loss reserving. See, for example, Barnett and Zehnwirth (2000).
- This paper introduces a calendar year effect to my collection of Bayesian MCMC models. My reason for not doing this before is that until now, I had not figured out how to make sense out of a calendar year model applied to cumulative loss data. My attempts to come up with a satisfactory model for incremental loss data have failed.