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The CAS *E-Forum*, Spring 2019

The Spring 2019 edition of the CAS *E-Forum* is a cooperative effort between the CAS *E-Forum* Committee and various CAS committees, task forces, working parties and special interest sections. This *E-Forum* contains three reports from two CAS Research Working Parties: Reports 14 and 15 of the CAS Risk-Based Capital Dependencies and Calibration Working Party and a report of the Predictive Analytics in Capital Modeling Working Party. (For Reports 1-13 of the CAS Risk-Based Capital Dependencies and Calibration Working Party, visit https://www.casact.org/pubs/forum/.) This Spring 2019 E-Forum also contains two independent research papers.

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CAS *E-Forum*, Spring 2019

Table of Contents

CAS Research Working Parties Reports

Risk Based Capital — Calibration of LOB Diversification in Underwriting Risk Charges
Report 14 of the CAS Risk Based Capital (RBC) Research Working Parties Issued by
The RBC Dependencies and Calibration Working Party (DCWP) 1-72

Risk Based Capital — Calibration of Investment Income Offset
Report 15 of the CAS Risk-Based Capital (RBC) Research Working Parties Issued by
The RBC Dependencies and Calibration Working Party (DCWP) 1-44

Upgrading an Existing Capital Model — A Common Risk Driver Application
A Report of the CAS Predictive Analytics in Capital Modeling Working Party 1-18
Excel supplement: **03_Risk_Driver_Mathematics_v11.xls**

Independent Research

Another Pioneering Use of DFA: New Zealand Earthquake Commission
Ian McLean 1-13

A Note on Euler Allocation for Performance Measurement
Shayan Sen, ACAS, Ph.D. 1-12

For information on submitting a paper to the *E-Forum*, visit http://www.casact.org/pubs/forum/.

Risk-Based Capital — Calibration of LOB Diversification in Underwriting Risk Charges

Report 14 of the CAS Risk Based Capital (RBC) Research Working Parties Issued by the RBC Dependencies and Calibration Working Party (DCWP)

Abstract: In this paper we analyze the Line of Business (LOB) diversification elements of the RBC Formula.

We compare the diversification credit produced by the NAIC Property/Casualty RBC Formula to the indicated diversification credit, i.e., the observed reduction in risk[1] with increasing diversification. For the larger/more diversified companies, with the bulk of the premium/reserves and receiving the bulk of the diversification credit, we find that:

- The data supports the approach in the RBC Formula, i.e., the data supports a diversification credit that is linear with respect to 100% minus the percentage of reserves/premium in the largest line of business, by company.
- The indicated maximum diversification credit is at least at least 50%, for premium risk and reserves risk, rather than the 30% maximum credit in the 2010 RBC Formula.

Three natural alternatives to the diversification approach in the RBC Formula are the correlation[2] matrix approach, the Herfendahl-Hirschman Index (HHI) approach, and the RBC approach applied to risk amounts rather than reserves/premium volume. We apply some simple tests of the extent to which each of these approaches fits the data. With our tests, the correlation approach is better than the approach in the RBC Formula for reserves, but the reverse is the case for premium. More interestingly, the RBC approach applied to risk amounts rather than reserves/premium volume is better than the approach in the RBC Formula for both premium and reserves.

This is one of several papers being issued by the CAS RBC Dependencies and Calibration Working Party (DCWP).

Keywords: Risk-Based Capital, Capital Requirements, Analyzing/Quantifying Risks, Assess/Prioritizing Risks, Integrating Risks, Diversification, Correlation

1. INTRODUCTON

The NAIC Property/Casualty RBC Formula (RBC Formula) has six main risk categories, R_0 – R_5. Underwriting risk is represented in two of these categories, R_4[3] and R_5, reserve risk and premium risk, respectively. The all-lines R_4 and R_5 values include a credit for diversification. The diversification credit in R_4 is based on the ratio of reserves for the LOB with the largest reserves to the total reserves. Similarly, the diversification credit in R_5 is based on the ratio of premium for the LOB with the largest premium to the total premium. We refer to this method of measuring diversification as the Company Maximum Line Percentage of Business or the CoMaxLine% Approach. We refer to the ratios as the $\text{CoMaxLine\%}_{\text{PREMIUM}}$ and the $\text{CoMaxLine\%}_{\text{RESERVES}}$, or CoMaxLine% generically, for either.

In this paper we evaluate the RBC Formula 30% Maximum Diversification Credit (MDC) and the assumption that diversification is proportional to 100%-CoMaxLine%.

We also evaluate alternatives to the diversification approach in the RBC Formula, e.g., the

[1] Risk, in our analysis, is 87.5th percentile RRR, for reserve risk, and the 87.5th percentile accident year ultimate operating loss (AUYL), for premium risk.

[2] We use the term correlation to describe a factor-based method for combining individual risks to produce risk measures for the combination of several risks. The source of the factor might be linear correlation, copulas or other techniques. In using this term, we do not intend to imply that the assumptions related to linear correlation are appropriate.

[3] When applied in the RBC Formula, the pure reserve risk component is combined with a portion of the reinsurance credit risk component. This paper deals with the pure reserve risk component of R_4.

correlation[4] matrix approach, the Herfendahl-Hirschman Index (HHI) approach, and RBC approach applied to risk amounts rather than reserves/premium volume (CoMaxLine%-Risk).

In Section 2 we describe the nature of our risk data. In section 3 we evaluate the CoMaxLine% Approach. In section 4 we compare the performance of the CoMaxLine% Approach to the performance of the alternative approaches.

1.1 Terminology, Assumed Reader Background and Disclaimer

This paper assumes the reader is generally familiar with the property/casualty RBC Formula[5] and has a working knowledge of risk data and line of business risk factor calibration approach described in DCWP Reports 6 and 7.

In this paper we use the term diversification, rather than its complement,[6] concentration unless the context makes the alternative clearer.

Although the term multi-line insurance company is commonly used to refer to an insurer that is well-diversified across LOBs, in this paper we will use the term more broadly to refer to any company for which the diversification credit is greater than zero.

References to we and our mean the principal authors of this paper. The working party, and DCWP refer to the CAS RBC Dependencies and Calibration Working Party.

The analysis and opinions expressed in this report are solely those of the authors, and are not those of the authors' employers, the Casualty Actuarial Society, or the American Academy of Actuaries.

DCWP makes no recommendations to the NAIC or any other body. DCWP material is for the information of CAS members, policy makers, actuaries and others who might make recommendations regarding the future of the RBC Formula. We expect that the material will be used by the American Academy of Actuaries.

This paper is one of a series of articles prepared under the direction of the DCWP.

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[4] We use the term correlation to describe a factor-based method for combining individual risks to produce risk measures for the combination of several risks. The source of the factor might be linear correlation, copulas or other techniques. In using this term, we do not intend to imply that the assumptions related to linear correlation are appropriate.

[5] For a detailed description of the formula and its basis, see Feldblum, Sholom, NAIC Property/Casualty Insurance Company Risk-Based Capital Requirements, Proceedings of the Casualty Actuarial Society, 1996 and NAIC, Risk-Based Capital Forecasting & Instructions, Property Casualty, 2010.

[6] A company with a concentration ratio of 80% can equivalently be described as a having a diversification ratio of 20%, 100%-80%.

2. RISK DATA

We describe our risk data in DCWP Reports 6[7] and 7,[8] and we summarize the characteristics of that data below.

For each year-end (Initial Reserve Date), the reserve risk data consists of the reserve amount (Initial Reserve[9]) and reserve development data. We summarize the reserve development data into a Reserve Runoff Ratio (RRR). The RRR is the ratio of (a) movement in incurred loss and defense and cost containment expense (DCCE) from the Initial Reserve date to the most mature valuation date available to (b) the Initial Reserve for loss and DCCE. The ratios in that RRR calculation are net of reinsurance, from Schedule P, Parts 2 and 3, in the 1998-2010 Annual Statements, by LOB and by company for individual companies and DWCP-defined pools, as indicated.[10] Thus, each reserve data point is the Initial Reserve and RRR from a single Initial Reserve Date and LOB for a single company or DCWP-defined pool (LOB-Company-Initial Reserve Date). We have data for Initial Reserve dates 1987-2009.[11]

Similarly, the premium risk data consists of net earned premium (NEP) and accident year (AY) loss and loss adjustment expense ratios (LRs) for AYs 1988-2010, net of reinsurance, at the latest available maturity from Schedule P, Part 1, in the 1998-2010 Annual Statements, by LOB and by company or DCWP-defined pool, as indicated (LRs). Thus, each premium data point consists of the NEP and LR for a single AY and LOB for a single company or DCWP-defined pool (LOB-Company-AY).[12]

For this analysis of diversification, we also construct all-lines data points. For reserve risk, the all-lines Initial Reserve for each Company-Initial Reserve Date is the sum of the Initial Reserves for each

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[7] http://www.casact.org/pubs/forum/13fforum/01-Report-6-RBC.pdf

[8] http://www.casact.org/pubs/forum/14wforum/Report-7-RBC.pdf

[9] Reserve for loss and defense and containment expenses, but not including adjusting and other expenses.

[10] The Risk Data points are filtered as we describe in DCWP Report 6 (on PRFs) and Report 7 (on RRFs). In brief, the main filters are that we exclude anomalous values; treat pool company data on a combined basis (DCWP-defined group pools); exclude Minor Lines data points (see Glossary); exclude the smallest LOBs data points, defined as those in smallest 15th percentile of LOB-size, by AY; exclude companies with less than 5 AYs of NEP; use values at the latest available maturity; and include companies regardless of whether they filed a 2010 Annual Statement (Survivorship Adjustment).

The runoff ratio includes movement related to "all prior year" element of Schedule P.

Those filters are largely the same as the filters used in the 2016 American Academy of Actuaries calibration report 2016 Update to Property and Casualty Risk-Based Capital Underwriting Factors http://www.actuary.org/files/publications/PC_RBC_UWFactors_10282016.pdf

[11] The most recent RRRs in our data are from the runoff on Initial Reserve Date December 2009, which represents one year of reserve development, from December 2009 to December 2010. There is one fewer year of reserve development than there are of AYs in that for the latest year, 2010, we have AY LRs, but no runoff on the 2010 Initial Reserve.

[12] In the remainder of the text when we refer to 'company' or 'companies' we mean companies or DCWP-defined pools, as appropriate.

of the company LOBs in the risk data. The all-lines RRR is the all-lines average RRR weighted by Initial Reserves by LOB. [13] For premium risk, the all-lines NEP for each Company-AY data point is the sum of the NEP for each of the company LOBs in the risk data. The all-lines LR is the all-lines average LR weighted by NEP by LOB.

There are 30,000 all-lines Company-Initial Reserve Date reserve risk data points and 29,000 all-lines Company-AY premium risk data points in the resulting all-lines data set. We categorize each of these points into size and diversification bands, as we describe below.

Company size bands

We measure company size based on all-lines Initial Reserve or all-lines NEP, for reserves and premium, respectively. We classify each company as being in one of five company size bands, selected so that 20% of the company data points are in each company size band. We label these company size bands A (smallest) through E (largest).

Company diversification bands

We determine the degree of diversification for each all-lines data point using the CoMaxLine%, correlation matrix, HHI or CoMaxLine%-Risk approaches, as appropriate for the analysis.[14] We use 6 diversification bands. Diversification band 0 is for monoline companies. [15] We select the other diversification bands so that 20% of the multi-line company data points are in each diversification band. We call those diversification bands 1 (least diversified, multi-line companies) through 5 (most diversified).

2.1 Company Size and Diversification Characteristics of Risk Data

In this section we describe the characteristics of the data by company size and company diversification.

Number of Company-Year Data Points

[13] Because the all-lines data points are constructed from the filtered LOB data points, the all-lines data does not include lines in which the LOB component of the LOB data point do not satisfied the Report 6 and 7 filtering tests, most importantly excluding Minor Lines data points and data points with less than four years of net earned premium.

[14] The diversification index for CoMaxLine% Approach is 100% - CoMaxLine%. The diversification index for HHI Approach is 100% – HHI value%. The diversification index for CoMaxLine%-Risk Approach is 100% - CoMaxLine%-Risk. The diversification index for correlation matrix approach is 100% – risk value after diversification/sum of LOB risk charge%s without diversification, as a percentage.

With different diversification metrics, e.g., correlation or HHI, the diversification band might differ. In practice, we find that the diversification metrics produce ranking of companies by diversification level. That is consistent with the findings in DCWP Report 14, showing that the RBC UW Risk Values are similar across a variety of diversification metrics.

[15] For our purpose, "monoline" means zero diversification credit in the Risk Data. This includes companies with one "major line" and, possibly, several Minor Lines, each of which has less than 5% of all-lines NEP. When we apply the correlation matrix approach, monoline incudes a company with two lines that are 100% correlated.

Tables 2-1A and 2-1B show the number of company-year data points for reserve risk and premium risk, respectively, in each of the thirty company size/diversification cells (using CoMaxLine% Approach to measuring diversification). The cells highlighted in yellow/bold are the largest and most diversified companies.

Table 2-1A
Number of Reserve Data Points by Size and Diversification

Number of Data Points						
Div Band	Size Band					
	A	B	C	D	E	Total
0	3,870	2,801	2,388	1,824	1,005	11,888
1	539	815	812	764	720	3,650
2	536	718	718	769	909	3,650
3	532	659	**763**	**811**	**885**	3,650
4	452	645	**793**	**925**	**835**	3,650
5	101	387	**553**	**934**	**1,674**	3,649
Total	6,030	6,025	6,027	6,027	6,028	30,137

Table 2-1B
Number of Premium Data Points by Size and Diversification

Number of Data Points						
Div Band	Size Band					
	A	B	C	D	E	Total
0	3,442	2,449	1,798	1,291	688	9,668
1	825	843	909	801	462	3,840
2	529	765	969	885	691	3,839
3	549	806	**813**	**904**	**767**	3,839
4	340	665	**778**	**870**	**1,186**	3,839
5	88	244	**506**	**1,022**	**1,979**	3,839
Total	5,773	5,772	5,773	5,773	5,773	28,864

There are approximately 30,000 data points for each of the premium and reserve data sets (30,137 for reserves and 28,864 for premium). Over 1/3 of companies are monoline entities with zero diversification (11,888 for reserves and 9,668 for premium). That might be viewed as more monoline companies than anticipated, but the observation is consistent with two features of the data. First, our data records are individual companies, but not company-groups.[16] Second, our data records exclude

[16] We consolidate data across groups only if the data is affected by pooling, as described in Reports 6 and 7.

Minor Line[17] data points by LOB. Some of the monoline companies have other lines, but none of those LOBs has more than 5% of the total premium in that company.

In both tables, looking at the diagonal of data records from the left top (Size A/Div 0) to the bottom right (Size E/Div 5), we see that, monoline companies tend to be smaller and the most diversified companies tend to be larger. Nonetheless, large companies (size band E) are represented in all diversification bands. Almost all cells have at least 500 data points.[18]

We see that the largest companies, size band E, tend to be highly diversified (diversification band 5), although, interestingly, for reserves, the second highest number of companies in size band E is in diversification band 0, monoline.

Amount of Reserves/Premium

Tables 2-2A and 2-2B below show the Initial Reserve and NEP, respectively, in each of the thirty company size/diversification cells (using CoMaxLine% Approach to measuring diversification).

Table 2-2A
Total Reserves Amount by Size and Diversification Band (In million)

Div Band	Size Band					
	A	B	C	D	E	Total
0	954	6,569	22,267	73,472	794,126	897,388
1	199	1,888	7,620	32,420	651,723	693,850
2	190	1,709	7,168	31,488	790,745	831,300
3	195	1,537	**7,552**	**31,715**	**1,195,729**	1,236,729
4	173	1,490	**7,829**	**36,229**	**875,078**	920,800
5	40	964	**5,507**	**41,119**	**3,054,924**	3,102,554
Total	1,751	14,159	57,943	246,444	7,362,325	7,682,622

[17] A Minor Line data point is a LOB data point for which the LOB premium or initial reserve is 5% or less of the total all-lines premium and initial reserve.
[18] We imply no significance to the value of 500.

Table 2-2B
Total Premium Amount by Size and Diversification Band (In million)

NEP (millions)						
Div Band	Size Band					
	A	B	C	D	E	Total
0	2,695	10,553	24,752	61,318	277,165	376,482
1	760	3,638	12,783	38,439	273,032	328,652
2	507	3,381	14,147	44,073	393,702	455,810
3	527	3,420	**12,069**	**45,378**	**1,175,892**	1,237,285
4	386	2,843	**11,237**	**44,369**	**1,656,501**	1,715,337
5	114	1,115	**7,405**	**55,777**	**2,293,232**	2,357,643
Total	4,989	24,950	82,393	289,355	6,069,523	6,471,209

Most of the reserves and premium come from size band E that has $7.4 trillion[19] of reserves, representing 96% of the total reserves, and $6.1 trillion of premium, representing 94% of total premium. Within this company size band, diversification band 5 has the most reserves ($3.1 trillion) and premium ($2.3 trillion), over 35% of total reserve for reserves and premium.

The yellow/bold cells mark the larger/more diversified companies. Table 2-2A shows these represent $5.3 trillion, representing 68% of all reserves. Looking back at Table 2-1A, we see that the yellow/bold cells have 8,173 data points. This is about 27% of all companies, and slightly over 50% of multiline companies (diversification band >0) with size greater than the smallest 20% (size bands B-E).

The yellow/bold cells in Table 2.2B include $5.3 trillion of premium, representing 82% of all premiums. Looking back at Table2-1B, we can see that the yellow/bold cells have 8,825 data points, about 31% of the total and slightly over 50% of multiline companies (diversification bands 1-5) with size greater than the smallest 20% (size bands B-E).

Average Reserve/Premium

Tables 2-3A and 2-3B below show the average reserve and average premium amounts by size and diversification band. The average reserve amount in Table 2-3A is the reserve amount in Table 2-2A divided by the number of data points in Table 2-1A. The average premium amount in Table 2-3B is the value in Table 2-2B divided by the number of data points in Table 2-1B.

As expected, size band E has the largest average reserve or premium size and size A has the lowest. The size range between companies is large. For example, the ratio of the average size for the largest size band divided by the average size for the smallest size band is a factor of over 4,000 for reserves

[19] The amounts seem large because they represent the sum of reserve amounts at year for each of 22 years of reserve data. The reserve at December 2009 alone was $492 Billion.

($0.3 million to $1.2 billion) and over 1,000 for premium.[20]

Table 2-3A
Average Reserves Amount by Size and Diversification Band (In million)

Average Reserve Volume by NAIC Band (millions)						
Div Band	Size Band					
	A	B	C	D	E	Total
0	0.2	2.3	9.3	40.3	790.2	75.5
1	0.4	2.3	9.4	42.4	905.2	190.1
2	0.4	2.4	10.0	40.9	869.9	227.8
3	0.4	2.3	**9.9**	**39.1**	**1,351.1**	338.8
4	0.4	2.3	**9.9**	**39.2**	**1,048.0**	252.3
5	0.4	2.5	**10.0**	**44.0**	**1,824.9**	850.2
Total	0.3	2.3	9.6	40.9	1,221.4	254.9

Table 2-3B
Average Premium Amount by Size and Diversification Band (In million)

Average Premium Volume by NAIC Band (millions)						
Div Band	Size Band					
	A	B	C	D	E	Total
0	0.8	4.3	13.8	47.5	402.9	38.9
1	0.9	4.3	14.1	48.0	591.0	85.6
2	1.0	4.4	14.6	49.8	569.8	118.7
3	1.0	4.2	14.8	50.2	1,533.1	322.3
4	1.1	4.3	14.4	51.0	1,396.7	446.8
5	1.3	4.6	14.6	54.6	1,158.8	614.1
Total	0.9	4.3	14.3	50.1	1,051.4	224.2

Amount of Diversification Credit

Tables 2-4A and 2-4B below show the dollar amount of diversification credit by company size and diversification band. The dollar amount of diversification credit is the difference between the all-lines risk charge with no diversification credit and the all-lines risk charge after diversification credit, based on the 2010 risk factors and the diversification formula in the 2010 RBC Formula.

Following the RBC Formula, there is zero diversification credit for companies in diversification band 0. The amount of diversification credit is small for the smaller companies, size bands A and B. That is partly because the companies in those size bands are somewhat less diversified. It is more so the case because the amount the amount of reserve risk/premium risk, and therefore diversification

[20] Some of the companies in the data set may be small enough that state regulations might exempt them from making RBC filings. We do not adjust our analysis to reflect that situation.

amount, is small, regardless of degree of diversification.

The companies in the yellow/bold cells contain about 94% of the total dollar amount of diversification credit for both reserves and premium.

Table 2-4A
Total Reserve Diversification by Company Size and Diversification Band (In million)

Dollar of Diversification Credit - 2010 Reserve Risk Factors						
Div Band	Size Band					
	A	B	C	D	E	Total
0	-	-	-	-	-	-
1	1	9	35	173	3,491	3,709
2	3	26	116	538	16,132	16,815
3	5	43	**220**	**965**	**49,376**	50,609
4	7	58	**346**	**1,647**	**48,019**	50,077
5	2	54	**320**	**2,434**	**204,658**	207,469
Total	18	189	1,038	5,757	321,676	328,679

1Table 2-4B
Total Premium Diversification by Company Size and Diversification Band (In million)

Diversification Credit - 2010 Premium Risk Factors						
Div Band	Size Band					
	A	B	C	D	E	Total
0	-	-	-	-	-	-
1	9	50	176	613	3,757	4,606
2	14	97	395	1,301	11,118	12,925
3	20	137	470	1,858	39,438	41,923
4	18	139	536	2,181	74,966	77,838
5	7	66	426	3,320	147,419	151,237
Total	68	488	2,003	9,272	276,699	288,530

3. ANALYSIS – COMAXLINE% APPROACH

3.1 RBC Formula - Diversification Rule

The RBC Formula instructions present the details of the R_4 and R_5 calculations.[21] The components

[21] Also, for a detailed description of the operation of the RBC Formula, see Odomirok, et al, Chapter 19, Risk Based Capital https://www.casact.org/library/studynotes/Odomirok-etal_Financial-Reportingv4.pdf

of those calculations and the simplifications we use in our diversification analysis are as follows:

Reserve Risk (R_4)

For each company, for each of the 19 LOBs[22] used in the RBC Formula, the reserve risk value depends on the following, which vary by LOB: the loss and loss adjustment expense reserve net of reinsurance (Initial Reserve) at the valuation date (Initial Reserve Date), the Reserve Risk Factor (RRF) applied to all companies, an adjustment for the difference between company reserve development experience and industry reserve development experience (own-company adjustment), an adjustment for investment income, and a credit for loss sensitive business. The sum of the LOB results is reduced by a diversification credit based on the Loss Concentration Factor (LCF), increased for larger than normal growth and increased by a portion of reinsurance credit risk.

We refer to the ratio of the reserve risk value to the Initial Reserve as the reserve risk charge percentage (RRC%).

Premium Risk (R_5)

For each company, for each of the 19 LOBs[23] used in the RBC Formula, the premium risk value depends on the following, which vary by LOB: the written premium for the latest year net of reinsurance (NWP), the Premium Risk Factor (PRF) applied to all companies, the own-company adjustment, an adjustment for investment income, and a credit for loss sensitive business. The total is combined with the company all lines expenses, reduced by a diversification credit based on the Premium Concentration Factor (PCF), and increased for larger than normal growth.

We refer to the ratio of the premium risk value to the net written premium as the premium risk charge percentage (PRC%).

Simplifications

Our calculations include certain simplifications.

For both reserve risk and premium risk, we do not include the own-company adjustment factor, the loss sensitive business adjustment factor or the growth charge. This is as if the own-company

For an older description of the Formula and its original basis, see Feldblum, Sholom, NAIC Property/Casualty Insurance Company Risk-Based Capital Requirements, Proceedings of the Casualty Actuarial Society, 1996. http://www.casact.org/pubs/proceed/proceed96/96297.pdf.

For the actual Formula, see NAIC, Risk-Based Capital Forecasting & Instructions, Property Casualty, 2010.

[22] RBC UW risk values are determined using data in the Annual Statement Schedule P, which shows 22 LOBs. RBC calculations treat occurrence and claims made LOBs for other liability and products liability on a combined basis and treat non-proportional property and non-proportional financial on a combined basis, leaving a net of 19 LOBs.

[23] RBC UW risk values are determined using data in the Annual Statement Schedule P, which shows 22 LOBs. RBC calculations treat occurrence and claims made LOBs for other liability and products liability on a combined basis and treat non-proportional property and non-proportional financial on a combined basis, leaving a net of 19 LOBs.

adjustment and loss sensitive factors were 1.0 and as if the growth risk charge was 0%. We do not include the investment income offset, assuming that the diversification effect is the same before or after the investment income effects.

For premium risk, we use Net Earned Premium (NEP) rather than net written premium. For company expenses in the premium risk calculation, we use the average of the 2010 industry average expense ratio by LOB, weighted by the company specific premium by LOB.[24]

For reserve risk, reserve amounts do not include reserves for adjusting and other expenses. We also do not include the R_3-reinsurance credit risk component for R_4.

In this work, we assume our simplifications do not materially affect our findings.[25]

Determine the Diversification Credit

R_4 and R_5 are first calculated by line of business (LOB). The all-lines R_4, the reserve risk charge, is the sum of the R_4 risk charges by LOB, multiplied by a Loss Concentration Factor (LCF). The all-lines R_5, the premium risk charge, is the sum of the R_5 risk charges by LOB, multiplied by a Premium Concentration Factor (PCF).[26] Using the CoMaxLine% Approach, for each company, the PCF and LCF are determined as follows:

CoMaxLine% for reserves = Initial reserve for the LOB with the largest Initial Reserve divided by the total all-lines Initial Reserve.

CoMaxLine% for premium = NEP[27] for the LOB with the largest premium divided by the total all-lines NEP.

$LCF_{COMPANY} = 0.7 + 0.3 * (CoMaxLine\% \text{ (reserves)}_{COMPANY})$

$PCF_{COMPANY} = 0.7 + 0.3 * (CoMaxLine\% \text{ (premium)}_{COMPANY})$

These can also be written as:

$LCF_{COMPANY} = 100\% - 0.3 * (100\% - CoMaxLine\%_{reserve})$

$PCF_{COMPANY} = 100\% - 0.3 * (100\% - CoMaxLine\%_{premium})$

Therefore, the diversification credit equals 30% times (100%-CoMaxLine%) where the diversification index is (100%-CoMaxLine%)

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[24] We make this simplification because expenses by LOB for all years in our data set were not readily available to us.

[25] Further research will be necessary to verify that assumption.

[26] The LCF and PCF are applied to the sum of the LOB RBC amounts, where those RBC amounts reflect the investment income offset, the own-company experience adjustment, and the loss sensitive business adjustment.

[27] NWP in the RBC Formula. NEP in our simplified calculation.

<u>LOB risk factors</u>

The observed diversification relationship might depend on the selection of LOB risk factors. Therefore, in our analysis, we do not use the LOB PRFs and RRFs in the 2010 RBC Formula. Instead, we use the LOB PRFs and RRFs indicated by the reserve and premium risk data that we use in this diversification analysis. By using these indicated risk factors, we avoid possible distortions resulting from use of LOB risk factors that are not consistent with the data we use for the diversification analysis. In Appendix 1/Exhibit 1, we show the 2010 LOB risk factors and the LOB risk factors that we use in this analysis.

3.2 Analysis Method

In our analysis, we examine the data by size band and diversification band. For each of the size/diversification cells, we calculate the following:

1. Observed Risk – For reserves, this is the 87.5th percentile[28] all-lines RRR. For premium, this is the 87.5th percentile all-lines AY Underwriting Gain/Loss percentage (AYUL in dollars and AYUL%, as a percentage of premium).

   The AYUL% by company equals the company all lines average loss ratio plus the all lines company expense ratio[29] minus 100%.
2. Expected Risk – This is the average RBC Formula result, including or excluding the diversification credit, as needed, for premium and reserves separately, averaged across companies.

   We express the expected risk as a ratio to reserves, for reserve risk, and as a ratio to premium, for premium risk. We refer to those ratios as the expected reserve risk charge% and expected premium risk charge%, respectively, and expected risk charge% generically.

   In using the RBC Formula to measured expected risk, we treat the RBC Formula as the model that predicts the RRR or AYUL% at the 87.5th percentile risk level.

   In Appendix 1/Exhibits 2-3 we show an example of how we use the risk data to calculate the all-lines expected risk charge%, the diversification band and size band for a sample company/year risk data point, for reserve risk and premium risk, respectively.
3. We vary the MDC (30% in the RBC Formula) to improve the 'fit' between the observed risk and the expected risk based on the RBC Formula.

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[28] We use the 87.5th percentile because that is the safety level last used (2016) in the calibration of LOB risk factors. The diversification relationship might be different if the safety level were the 90th percentile or some other value. Evaluating the variation in indicated diversification credit with changing safety level is a matter for future research.

[29] As noted in the "Simplifications" subsection above, for company expense we use industry expenses by LOB, weighted by the company NEP by LOB.

In our analysis we examine the data in three levels of detail, as follows:

- A 2 x 2 split of monoline vs. multi-line and smallest size band vs. all other size bands combined.
- A 2 x 6 split treating each of six diversification bands separately and considering two size bands, smallest size band vs. all other size bands combined.
- A 5 x 6 split treating each diversification/size band separately.

With the 2x2 analysis we test the 30% MDC. With the 2x6 analysis we evaluate the extent to which the indicated diversification credit varies linearly with the diversification index, 100%-CoMaxLine%, as well as testing the 30% MDC. The 5x6 analysis adds more insight into the extent to which differences in experience among company sizes B, C, D and E affect the observed pattern for sizes B-E combined, used in the 2x6 analysis.

3.3 Diversification– 2x2 Analysis

In this section, we examine the data in 4 company size/diversification cells:

- By company size band– split the companies by size into the smallest 20% of companies and the other 80%, and
- By company diversification band - split the companies into two diversification bands: monoline companies and multiline companies.

3.3.1 Observed vs. Expected Effect of Diversification

<u>Expected Risk Charge%s</u>

Table 3-1, below, shows the all-lines expected reserve and expected premium risk charge%s based on the CoMaxLine% Approach, with the 30% MDC, for each of the cells in the 2x2 array by company size and company diversification.

Table 3-1
Expected Risk Charge%

Div Band	Reserves		Premium	
	Size Band		Size Band	
	<20%	>=20%	<20%	>=20%
0	34.1%	32.7%	27.8%	29.3%
>0	28.7%	**30.7%**	22.4%	**21.8%**

Note: Expected risk charge% is from application of the RBC Formula Value, with the 30% MDC. Appendix 1/Exhibits 2 and 3 show how one company-year of data enters the calculation in Table 3-1, for reserve risk and premium risk respectively.

The expected risk charge% in each cell of Table 3-1 is the unweighted average of the company-year risk charge%s from the RBC Formula for companies in that cell, i.e., the risk data points are equally weight, regardless of company reserves/premium amount.

The risk charge%s in the row >=0 would be lower than the risk charge%s in row 0 due to the diversification credit applicable to data points in the >= row, unless that credit was offset by differences in the LOB distribution between monoline and multiline companies.

The only reasons for variation between the reserve values or between the premium values in the columns <20% or >=20%, within each diversification band, in Table 3-1 are:

- Variation in distribution of reserves/premium by LOB, between smaller companies (<20%) and all other companies (>=20%), and/or,
- For diversification band >0 only,[30] variation in average LCF/PCF, by company size, for the smallest 20% of company sizes vs. all other company sizes.

Examining Table 3-1, we see that:

1. Rows: Across the rows, comparing the <20% company size column to the remaining 80% column, called >=20%, the differences are due to LOBs with higher/lower risk charge%s in larger companies.

   For example, for premium, we see that:

   a. Smaller monoline companies include more lower risk LOBs than large monoline companies, 27.8% versus 29.3%;

   b. Smaller multiline companies have higher risk LOBs lines and/or less diversification credit than bigger multiline companies, 22.4% versus 21.8%

2. Columns: Down the column, the expected risk charge%s from the 0 row to the > 0 row decrease, as the RBC Formula includes a reduction in risk charges, and that decrease is, apparently, not offset by differences, if any, in the distribution of reserves/premium by LOB for monoline versus multiline companies.

   For example, for reserves, in the <20% column, the reserve risk charge% decreases from 34.1% in the diversification row 0 to 28.7% in the diversification row labeled >0.

   Observed Risk

Table 3-2, below, shows the 87.5th percentile RRR and the 87.5th percentile AYUL%, for all company-years in the size/diversification cell. These are the indicated all-lines reserve and all-lines premium risk charge%s corresponding to the expected risk charge%s in Table 3-1.

[30] This applies for diversification band > 0 because, in the zero-diversification row, the concentration factor is 1.0, and the diversification is zero, for all companies.

Table 3-2
Indicated Risk Charge

Div Band	Reserves		Premium	
	Size Band		Size Band	
	<20%	>=20%	<20%	>=20%
0	63.0%	26.5%	56.2%	28.7%
>0	54.7%	**27.2%**	43.9%	**17.8%**

Appendix 1/Exhibits 2 and 3 show how one company-year of data enters the calculation in Table 3-2, for reserve risk and premium risk respectively.

Examining Table 3-2, we see that:

Rows: In each row, the indicated risk charge%s in the column showing company size <20% is higher than the indicated risk charge% for the remaining 80% of companies.

Thus, for example, for premium:

a. Smaller monoline companies have higher indicated LOB risk charge%s than larger monoline companies, 56.2% versus 28.7%;

b. Smaller multiline companies have higher indicated LOB risk charge%s than larger multiline companies, 43.9% versus 17.8%

DCWP Reports 6 and 7 noted this pattern of higher risk charge%s indicated for smaller LOB-sizes that would predominate in smaller companies.

Columns: We expect that indicated risk charge%s will decrease with increasing diversification, to the extent that the effect is not offset by differences in the distribution of reserves/premium by LOB between monoline and multi-line companies, and to the extent that indicated risk charge%s by LOB are the same for monoline and multi-line companies. The expected pattern of decrease is apparent for premium, but not for reserves. We discuss the other factors affecting the observed pattern later in this report.[31]

<u>Comments on comparison of expected to observed risk charges/Tables 3-1 and 3-2</u>

The Indicated MDC is Greater than 30%

If the CoMaxLine% Approach were perfect, then each value in the array of expected values, Table 3-1, would equal the corresponding value in the array of observed values, Table 3-2. That, however, is not the case. Except for the smallest multi-line companies, the observed risk charge%s are lower than the expected risk charges, so a MDC greater than 30% is indicated.[32]

For example, for reserves, in the yellow/bold cell, the expected risk charge% is 30.7%. The

[31] See Section 4 and Appendix 2 for further discussion of the extent to which LOB indicated risk charge%s vary by company level of diversification.

[32] Given the structure of the RBC Formula, the only parameter that can be adjusted is the MDC.

indicated risk charge% is 27.2%. As 27.2% is less than 30.7%, the data indicates that the 30% MDC is not giving enough diversification credit for reserve risk, for multi-line companies larger than the smallest 20%.

Similarly, for premium, in the yellow/bold cell, the expected risk charge% is 21.8%. The indicated risk charge% is 17.8%. As 17.8% is less than 21.8%, the data indicates that the 30% MDC is not giving enough diversification credit for premium risk, for multi-line companies larger than the smallest 20%.

Focus on Multi-Line Companies/Company size Excluding Smallest 20% of Companies

In comparing observed risk charge%s to expected risk charges, we focus on the yellow/bold cells because:

- Diversification band 0, monoline companies, provides no information about the benefit of diversification, as there is none, and
- The small company data in column <20% is not useful in a diversification calibration, as the risk charge%s for LOBs at that size are not consistent with the risk charge%s for the bulk of the companies that have larger sizes.[33]

3.3.2 Indicated MDC

To determine the indicated MDC, we use Tables 3-1 and 3-2, above, and Tables 3-3 through 3-5 below.

Table 3-3, below, shows the all-lines expected risk charge% based on the RBC Formula with no diversification credit. As required by the operation of the RBC Formula, the values in Table 3-3 equal the values in Table 3-1 for the 0 diversification band, and the values in Table 3-3 are higher than the values in Table 3-1 for the >0 diversification band.

[33] For similar reasons, our calibration of indicated risk charge%s by LOB in DCWP Reports 6 and 7 uses data excluding the smallest 15% of LOB data points. In these reports we observe that the indicated risk charge%s for small LOB-sizes are much higher than the risk charge%s for larger LOB-sizes that constitute the bulk of the number of companies and premium and reserve amounts. As the RBC Formula does not allow different rick charges % by LOB-size. Reports 6 and 7, and the American Academy of Actuaries analysis of risk changes, exclude experience of the smallest companies in determined risk charge%s. As small LOB-sizes will predominate in smaller companies, excluding the smallest companies from the dependency analysis is the all-lines analogue of the LOB-size strategy with respect to LOB risk charge% caligba5tion.

Table 3-3
Expected Risk Charge% Before Diversification

Div Band	Reserves Size Band		Premium Size Band	
	<20%	>=20%	<20%	>=20%
0	34.1%	32.7%	27.8%	29.3%
>0	31.2%	**34.2%**	24.8%	**25.0%**

Note: Expected risk charge% before diversification is the RBC Formula Value before applying LCF/PCF.

Table 3-4, below, shows current average diversification credit, i.e., the value based on the CoMaxLine% Approach and the 30% MDC for reserve and premium risk values.[34]

Table 3-4
Current Average Diversification Credit with RBC Formula and 30% MDC

Div Band	Reserves Size Band		Premium Size Band	
	<20%	>=20%	<20%	>=20%
0	0.0%	0.0%	0.0%	0.0%
>0	7.7%	**9.9%**	9.8%	**13.3%**

As required by the operation of the RBC Formula, the values in Table 3-4 equal zero for the diversification band 0. The value 9.9% for reserves, diversification >0 and size >=20% is the average diversification credit for companies in that size/diversification cell, and the corresponding average CoMaxLine% for those companies is 67.1%.[35]

Based on Tables 3-1 to 3-4, above, we calculate the indicated MDC in Table 3-5, below. The calculation uses the data for multiline companies, excluding the smallest 20% of companies, i.e., yellow/bold cells in Tables 3-1 to 3-4, for the reasons described in Section 3.3.1 above.

[34] This is the unweighted average of the company-year diversification credits for companies in that cell, i.e., the risk data points are equally weight, regardless of company reserves/premium amount.
[35] LCF = 1- diversification credit = 90.1%. 90.1% = 0.7 + .3 * .671.

Table 3-5
Overall Indicated MDC (2x2 Analysis)

	(1)	(2)	(3)
#	Item	Reserves	Premium
1	Observed Risk - 87.5th RRR/AYUL (Table 3-2)	27.2%	17.8%
2	Expected Risk – Apply RBC Formula before diversification (Table 3-3)	34.2%	25.0%
3	Indicated Diversification Credit 1.0-(1)/(2)%	20.6%	28.8%
4	Average Diversification Credit (Current Formula) (Table 3-4)	9.9%	13.3%
5	Indicated Maximum Credit [(3)/(4) * 30%]	62%	65%

The elements of the calculation in Table 3-5 are as follows:

- Row 1 - The <u>observed risk,</u> 87.5th percentile all-lines AYUL% and RRR. This is 27.2% for reserve risk, and 17.8% for premium risk (From Table 3-2).
- Row 2 – The <u>expected risk,</u> the all-lines reserve and premium risk charge%s calculated with from the RBC Formula, before considering the diversification adjustment. This is the average, all companies equally weighted, of the LOB premium or reserves risk charge%s, before diversification credits (From Table 3-3).
- Row 3 –The <u>indicated average diversification credit</u>, 1.0- (1)/ (2), expressed as a percentage. This is the diversification credit that, if applied on average, all companies equally weighted, would result in expected reserve and premium risk charge%s equal to observed risk reserve and premium risk charges.
- Row 4 - The <u>current average diversification credit</u>, the unweighted average, i.e., all companies equally weighted, of the value "30% * (100%-CoMaxLine%)," across all company-years in this analysis. (From Table 3-4)

  The Row 3 value is more than the Row 4 value showing that the indicated credit diversification is greater than the credit produced by the RBC Formula.
- Row 5 – The <u>indicated MDC</u>, Row (5) = Row (3)/Row (4) * 30%. The indicated MDC is 65% for premium and 62% for reserves.[36]

Thus, Table 3-5 shows that, based on 2x2 analysis, the indicated diversification formulas are:

LCF = 38% plus 62% * CoMaxLine%

PCF = 35% plus 65% * CoMaxLine%

The values 65% and 62% are more than twice the current value of 30%, driven by the fact that the

[36] Given the structure of the RBC Formula, the only parameter that can be adjusted is the MDC.

indicated diversification (20.6% and 28.8%, line 3, for reserves and premiums, respectively) are more than twice the current average diversification (9.9% and 13.3%, line 4, for reserves and premiums, respectively).

This indicated MDC reflects risk theory diversification effects and the extent to which indicated LOB risk charge%s vary by degree of diversification. We describe the latter effect in Section 4 and in Appendix 2. Regardless of the causes, Row 5 is an estimate of the MDC that is indicated by the risk data, using the selected PRFs/RRFs, given the structure of the RBC Formula.

3.4 Diversification - 2x6 Analysis (Two Size Bands/Six Diversification Bands)

In this section, we examine the data in 12 cells,

- By company size – split the companies by size into the smallest 20% and the other 80%, and
- By company diversification band - split the companies by diversification into one monoline band and five multiline bands.

In this 2x6 analysis we can test both the MDC and the extent to which the diversification credit is linear with CoMaxLine%. In Section 3.3, above, with less diversification segmentation, we only tested the value of the MDC. Our analysis, in sections 3.4.1 and 3.4.2 below, follows the approach described in sections 3.3.1 and 3.3.2 for the 2x2 analysis.

3.4.1 Observed vs. Expected Effect of Diversification Experience

Table 3-6, below, shows the all-lines expected reserve and premium risk charge%s based on the CoMaxLine% Approach with the 30% MDC, for each of the cells in the 2x6 array by company size and company diversification. Table 3-6 is a more detailed segmentation of Table 3-1.

As we noted with respect to the 2x2 analysis in section 3.3.1, Table 3-6 shows the expected risk charge%s using the RBC Formula. The only reasons for variation among these twelve reserve values or among the twelve premium values are the following:

- The variation in distribution of LOB reserves/premium by company size and company diversification, and/or
- Variation in LCF/PCF by company size, for multi-line companies, i.e. diversification bands greater than 0.

Table 3-6
Expected Risk Charge%

Div Band	Reserves		Premium	
	Size Band		Size Band	
	<20%	>=20%	<20%	>=20%
0	34.1%	32.7%	27.8%	29.3%
1	27.4%	30.0%	25.3%	28.0%
2	28.9%	29.6%	23.4%	22.3%
3	28.6%	**31.3%**	20.0%	**20.9%**
4	29.6%	**32.0%**	18.9%	**19.9%**
5	29.8%	**30.5%**	19.1%	**18.9%**
all x 0	28.7%	30.7%	22.4%	21.8%

Note: Expected risk charge% is the RBC Formula Value, including 30% MDC.

Examining Table 3-6, we see that:

1. Rows: Across the rows, the values in the >=20% column are generally larger than the values in the <20%, as would be the case to the extent that larger companies write LOBs with higher risk charges, not offset by increasing diversification credit for larger companies, within each diversification band.
2. Columns: Down the columns, the expected risk charge%s for companies with greater diversification would be lower than the values for companies with lower diversification, except to the extent that more/less diversified companies write the LOBs with higher/lower risk charge%s.

   For premium, the expected risk charge%s decrease down the column, indicating that diversification is more important than change in distribution of reserves/premium by LOB or other factors, if any.

   For reserves, the expected risk charge%s are more consistent down the column, indicating that the effects of (a) diversification and (b) distribution of reserves/premium by LOB are somewhat offsetting.

Table 3-7, below, shows the 87.5th percentile RRR and the 87.5th percentile AYUL%. These are the indicated all-lines reserve and premium risk charge%s corresponding to the expected risk charge%s in Table 3-6. Table 3-7 is a more detailed segmentation of Table 3-2. The rows 0 and all x 0 in Table 3-7 have the same values as the corresponding rows, 0 and >0 in Table 3-2.

Table 3-7
Indicated Risk Charge

Div Band	Reserves		Premium	
	Size Band		Size Band	
	<20%	>=20%	<20%	>=20%
0	63.0%	26.5%	56.2%	28.7%
1	53.4%	26.7%	44.7%	24.4%
2	54.0%	26.9%	42.1%	16.5%
3	74.6%	**28.2%**	44.1%	**18.0%**
4	44.9%	**28.5%**	32.8%	**16.7%**
5	36.5%	**25.6%**	55.9%	**16.0%**
all x 0	54.7%	27.2%	43.9%	17.8%

Examining Table 3-7, we see that:

Rows: Across the rows, the values in the <20% column are always higher than the values in the >=20% column. This is consistent with Table 3-2 and with the LOB analysis in DCWP Report 6 and 7, which show that risk charge%s are highest for the smallest LOBs that would predominate in the smallest company sizes.

Columns: Down the columns, the indicated risk charge%s for diversification band 5, the most diversified companies, is lower than the risk charge%s for less diversified companies. This is consistent with the effect we expect based on increasing diversification.

As already noted with respect to Table 3-2, the indicated risk charge%s do not decrease down the column, with increasing diversification, as uniformly as we would expect if the changes down the column were driven by diversification only. We discuss other factors affecting the observed pattern later in this report.[37]

<u>The Indicated MDC is Greater than 30% - Table 3-6 compared to Table 3-7</u>

If the CoMaxLine% Approach were perfect, then each value in the array of expected values, Table 3-6, would equal the corresponding value in the array of observed values, Table 3-7. That, however, is not the case. Except for the smallest multi-line companies, the observed risk charge%s are lower than the expected risk charges, so a MDC greater than 30% is indicated.[38]

3.4.2 Indicated MDC

To determine the indicated diversification credit with this 2x6 data segmentation, we use Tables 3-6 and 3-7, above, plus the information in Tables 3-8 to 3-11 below. The analysis is analogous to the

[37] See Section 4 and Appendix 2 for further discussion of the extent to which LOB indicated risk charge%s vary by company level of diversification.

[38] Given the structure of the RBC Formula, the only parameter that can be adjusted is the MDC.

Table 3-5 calculation in section 3.3 for the 2x2 array of data:

- Table 3-8 - Expected Risk Charge% Before Diversification Credit (analogous to Table 3-3)
- Table 3-9 - Indicated Average Diversification Credit (analogous to Table 3-5 line 3, but not shown as separate Table in section 3.3).
  These values equal 100% - Table 3-7/Table 3-8.
- Table 3-10 - Current Average Diversification Credit (analogous to Table 3-4)
- Table 3-11 - Indicated MDC (analogous to Table 3-5)
  These values equal 30% * Table 3-9/Table 3-10.

Table 3-8
Expected Risk Charge% Before Diversification

Div Band	Reserves Size Band		Premium Size Band	
	<20%	>=20%	<20%	>=20%
0	34.1%	32.7%	27.8%	29.3%
1	27.9%	30.5%	26.3%	29.2%
2	30.6%	31.3%	25.7%	24.7%
3	31.6%	**34.6%**	23.0%	**24.1%**
4	34.2%	**36.9%**	22.5%	**23.9%**
5	36.0%	**37.2%**	23.9%	**23.8%**
all x 0	31.2%	34.2%	24.8%	25.0%

Note: Expected risk charge% Before Diversification is the RBC Formula Value before applying the LCF/PCF.

Table 3-9
Indicated Average Diversification Credit

Div Band	Reserves Size Band		Premium Size Band	
	<20%	>=20%	<20%	>=20%
0	-84.7%	18.8%	-102.0%	1.9%
1	-91.3%	12.4%	-69.5%	16.5%
2	-76.5%	14.2%	-63.4%	33.1%
3	-135.8%	**18.4%**	-91.8%	**25.3%**
4	-31.3%	**22.7%**	-45.8%	**30.1%**
5	-1.6%	**31.2%**	-133.5%	**33.0%**
all x 0	-75.3%	20.6%	-77.3%	28.8%

Table 3-10
Current Average Diversification Credit with RBC Formula and 30% MDC

Div Band	Reserves		Premium	
	Size Band		Size Band	
	<20%	>=20%	<20%	>=20%
0	0.0%	0.0%	0.0%	0.0%
1	1.8%	1.7%	4.1%	4.3%
2	5.5%	5.4%	9.4%	9.5%
3	9.4%	**9.5%**	13.2%	**13.3%**
4	13.4%	**13.4%**	16.2%	**16.5%**
5	17.2%	**18.1%**	20.0%	**20.8%**
all x 0	7.7%	9.9%	9.8%	13.3%

Table 3-11
Indicated MDC

Div Band	Reserves		Premium	
	Size Band		Size Band	
	<20%	>=20%	<20%	>=20%
0	NA	NA	NA	NA
1	-1524.0%	211.9%	-513.5%	114.1%
2	-417.5%	78.2%	-203.2%	104.0%
3	-431.7%	**58.4%**	-208.9%	**57.3%**
4	-70.3%	**51.0%**	-84.6%	**54.7%**
5	-2.7%	**51.7%**	-200.3%	**47.6%**
all x 0	-291.9%	62.5%	-236.8%	64.9%

For calibration, we focus on the cells in yellow/bold because:

- Diversification band 0, monoline companies, provide no information about the benefit of diversification, as there is none.
- The small company data in column <20% is not useful in diversification calibration of as the risk charge%s for LOBs at that size are not consistent with the risk charge%s for the bulk of the companies that have reserve/premium larger sizes and the bulk of the diversification credit. [39]
- Those cells represent the overwhelming proportion of diversification credit, as shown in Table 2-4A and 2-4B.
- Moreover, the diversification bands "1" and "2" show high values for the indicated MDC,

[39] See footnote 33.

compared to the indicated MDC for diversification bands 3-5.

In Appendix 2 we show that, for diversification bands 1 and 2, the indicated LOB risk factors are different from the indicated LOB risk factors for diversification bands 3-5. Thus, the high indications for diversification levels 1 and 2 are not relevant for calibrating diversification for the companies in diversification bands 3-5 that constitute the bulk of premium and reserves amounts and the overwhelming proportion of industry total diversification credit.

For these yellow/bold cells, Table 3-11 shows that the indicated MDC is almost always more than 50%.[40]

3.4.3 Testing Linear Relationship between CoMaxLine% and Indicated Diversification Credit

Next, we use regression through the origin to test the validity of the linear relationship between indicated diversification credit and 100%-CoMaxLine% and to further test the indicated diversification credit. The dependent variable is the indicated average diversification credit (Table 3-9). The independent variable is the diversification index, "100% - CoMaxLine%," (Table 3-10 divided by 30%).[41] We exclude the smallest 20% of companies from this analysis, for the reasons discussed above.

Table 3-12, below, presents the regression results, which produce a reasonably high R-square value.[42]

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[40] Note that the typical indicated MDC in the yellow/bold cells of Table 3-10 is 50%. This is lower than the 60+% indicated MDC from Table 3-5. Looking at Table 3-11, we see that the highest indicated values for the indicated MDC are in diversification bands 1 and 2 with indicated MDC values from 75% to over 200%. Thus, the 2x6 analysis enables us to calibrate the diversification credit using the experience of companies in diversification bands 3-5, that represent the bulk of reserves, premiums and diversification credit, with no distortion from the indications for bands 1 and 2.

[41] We graph the values divided, by 30%, rather than the Table 3-10 values, so that the slope of graph is the indicated MDC.

[42] The R-squared statistic is calculated by Excel regression in Excel data pack. The Excel formula for R-squared for regression through the origin is not the same as the R-squared formula used for OLS regression. Joseph G Eisenhauer (2003), Teaching Statistics, 25(3), 76-80.

Table 3-12
Regression Analysis of Diversification Formula

Div Band	Reserves			Premium		
	1	2	3	4	5	6
	Average Div Index	Indicated Div Credit	Fitted Div Credit	Average Div Index	Indicated Div Credit	Fitted Div Credit
0	0.0%	18.8%	0.0%	0.0%	1.9%	0.0%
1	5.8%	12.4%	3.2%	14.5%	16.5%	8.3%
2	18.1%	14.2%	9.9%	31.8%	33.1%	18.4%
3	31.5%	18.4%	17.2%	44.2%	25.3%	25.5%
4	44.5%	22.7%	24.2%	55.0%	30.1%	31.7%
5	60.5%	31.2%	32.9%	69.4%	33.0%	40.1%
	Slope		54%	Slope		58%
	R-square		82%	R-square		92%

Columns 1 and 4 equal Table 3-10 divided by .30. We use the diversification index rather than the average diversification credit, for simplicity, so that the slope equals the indicated MDC.
Columns 2 and 5 from Table 3-9.
Data excludes company size band A, the 20% smallest companies.
The regression includes data from diversification band 0. If we exclude diversification band 0 and recalculate the regression, the slope is not affected but the R-squared values are 95% and 92% for reserve and premium respectively.

Table 3-13 shows the regression results graphically. Table 3-13 shows that the linear relationship through the origin is particularly close for the three data points representing the largest/most diversified companies.

Based on those results, the indicated diversification formulas are:

LCF = 46% plus 54% * CoMaxLine%

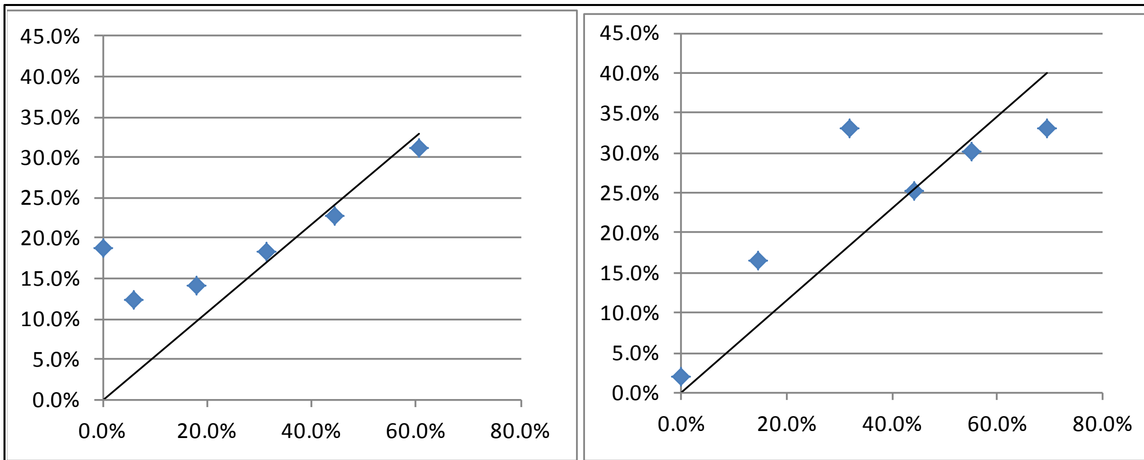
PCF = 42% plus 58% * CoMaxLine%

The regression lines show that, for reserves, every 100-basis point increase in the diversification index will result in a 54-basis point increase in the indicated diversification credit. For premium, every 100-basis point increase in the diversification index will result in a 58 basis point increases in the indicated diversification credit.

These formulas provide larger diversification credits than the current 30% MDC, over 50%, but less than the parameters from the 2x2 analysis.

Table 3-13
Regression Results

Reserves | **Premium**



	Reserves		Premium
Slope	54%	Slope	58%
R-Square	82%	R-Square	92%

X-Axis shows 100% - CoMaxLine% that equals Average Diversification Credit /0.3.
Y-Axis shows indicated diversification credit.

3.5 Diversification - 5x6 Analysis (Five Size Bands /Six Diversification Bands

In this section, we examine the data in 30 cells,

- By company size – split the companies into 5 size bands, and
- By company diversification - split the companies into 6 diversification bands

We follow the same approach as in the 2x2 and 2x6 analyses in Sections 3.3 and 3.4 respectively. We show that the findings from section 3.4, the 2x6 analysis, remain valid.

3.5.1 Observed vs. Expected Effect of Diversification Experience

Table 3-14, below, shows the all-lines expected reserve and premium risk charge%s based on the CoMaxLine% Approach with the 30% MDC, for each cell in the 5x6 array by company size and company diversification.[43] This analysis is analogous to the analysis shown in Tables 3-1 and 3-6.

As we noted with respect to the 2x2 and 2x6 analyses, in sections 3.3.1 and 3.4.1, respectively, the only reasons for variation among these thirty premium values or among the thirty reserve values are the following:

[43] Table 3-14 is a more detailed segmentation of Table 3-1 and Table 3-6.

- Variation in distribution of LOB reserves/premium by company size and/or company diversification, and/or
- Variation in LCF/PCF by company size, for multi-line companies.

Table 3-14
Expected Risk Charge%

Diversif. Band Quintiles	Reserves					Premium				
	Size Band (Quintiles)					Size Band (Quintiles)				
	A	B	C	D	E	A	B	C	D	E
0	34.1%	33.9%	33.0%	31.1%	31.3%	27.8%	28.5%	28.9%	31.1%	30.0%
1	27.4%	28.0%	30.9%	32.4%	28.6%	25.3%	26.4%	26.4%	30.3%	30.2%
2	28.9%	29.2%	29.6%	30.4%	29.2%	23.4%	22.9%	21.6%	21.8%	23.5%
3	28.6%	29.2%	**30.4%**	**30.2%**	**34.7%**	20.0%	21.2%	**20.2%**	**20.5%**	**22.0%**
4	29.6%	28.7%	**31.6%**	**31.8%**	**34.9%**	18.9%	20.0%	**19.4%**	**20.0%**	**20.1%**
5	29.8%	29.4%	**30.0%**	**29.7%**	**31.3%**	19.1%	18.8%	**18.2%**	**18.4%**	**19.3%**
All Ex 0	28.7%	28.8%	30.6%	30.9%	31.8%	22.4%	22.5%	21.6%	21.9%	21.4%

Note: Expected risk charge% is the RBC Formula Value, including 30% MDC.

Table 3-15, below, shows the 87.5th percentile RRR and the 87.5th percentile AYUL%. These are the indicated all-lines reserve and premium risk charge%s corresponding to expected risk charge%s in Table 3-13.[44] This analysis is analogous to the analysis shown in Tables 3-2 and 3-7.

Table 3-15
Indicated Risk charge%

Diversif. Band Quintiles	Reserves					Premium				
	Size Band (Quintiles)					Size Band (Quintiles)				
	A	B	C	D	E	A	B	C	D	E
0	63.0%	38.2%	25.1%	21.2%	18.2%	56.2%	29.0%	25.9%	27.2%	36.6%
1	53.4%	33.6%	27.2%	29.9%	15.1%	44.7%	20.8%	25.1%	21.8%	38.5%
2	54.0%	34.7%	29.7%	28.7%	17.0%	42.1%	19.4%	15.2%	16.5%	15.0%
3	74.6%	39.4%	**27.0%**	**22.2%**	**25.2%**	44.1%	20.7%	**17.2%**	**17.9%**	**16.6%**
4	44.9%	36.3%	**31.9%**	**22.5%**	**28.8%**	32.8%	13.7%	**18.1%**	**18.2%**	**15.7%**
5	36.5%	30.5%	**24.1%**	**23.6%**	**25.6%**	55.9%	22.0%	**15.4%**	**16.4%**	**15.3%**
All Ex 0	54.7%	35.2%	27.9%	25.1%	23.7%	43.9%	19.3%	18.2%	17.8%	16.8%

The patterns observed in Table 3-14 and 3-15 are consistent with the patterns observed in the less detailed 2x2 and 2x6 segmentations, in Tables 3-1, 3-2, 3-6 and 3-7. For example, the indicated risk charge%s for size band A are the highest. Also, the indicated risk charges, in size bands C-E are lower than the expected risk charge%s in the corresponding size and diversification bands. That indicates, as noted before, that the current diversification credits are lower than the indicated diversification credits.

[44] Table 3-15 is a more detailed segmentation of Table 3-2 and Table 3-7.

3.5.2 Indicated MDC

To examine the indicated diversification credit, we use Table 3-14 and 3-15, above, and the information in Tables 3-16 to 3-19 below. The analysis is analogous to that used in section 3.3.2, for the 2x2 analysis, and section 3.4.2, for the 2x6 analysis:

- Table 3-16 - Expected risk charge% before diversification credit (analogous to Tables 3-8 and 3-3)
- Table 3-17 - Indicated Average Diversification Credit (analogous to Tables 3-9 and 3-5 line 3). These are 100% - Table 3-15/Table 3-14
- Table 3-18 - Current Average Diversification Credit (analogous to Tables 3-10 and 3-4)
- Table 3-19 - Indicated MDC (analogous to Tables 3-11 and 3-5)
  This is 30% times Table 3-17 / Table 3-18.

Table 3-16
Expected Risk Charge% Before Diversification

Diversif. Band Quintiles	Reserves Size Band (Quintiles) A	B	C	D	E	Premium Size Band (Quintiles) A	B	C	D	E
0	34.1%	33.9%	33.0%	31.1%	31.3%	27.8%	28.5%	28.9%	31.1%	30.0%
1	27.9%	28.5%	31.4%	32.9%	29.1%	26.3%	27.6%	27.6%	31.6%	31.5%
2	30.6%	30.9%	31.3%	32.1%	30.9%	25.7%	25.3%	23.9%	24.1%	26.0%
3	31.6%	32.2%	**33.6%**	**33.4%**	**38.4%**	23.0%	24.4%	**23.3%**	**23.6%**	**25.4%**
4	34.2%	33.2%	**36.5%**	**36.7%**	**40.3%**	22.5%	23.9%	**23.2%**	**24.0%**	**24.1%**
5	36.0%	35.6%	**36.5%**	**36.2%**	**38.4%**	23.9%	23.5%	**22.9%**	**23.2%**	**24.5%**
All Ex 0	31.2%	31.6%	33.7%	34.4%	36.0%	24.8%	25.3%	24.3%	25.1%	25.4%

Note: Expected risk charge% Before Diversification is the RBC Formula Value before applying the LCF/PCF.

Table 3-17
Indicated Average Diversification Credit

Diversif. Band Quintiles	Reserves Size Band (Quintiles) A	B	C	D	E	Premium Size Band (Quintiles) A	B	C	D	E
0	-84.7%	-12.6%	24.0%	31.8%	41.8%	-102.0%	-1.7%	10.4%	12.5%	-22.0%
1	-91.3%	-18.0%	13.5%	9.1%	48.2%	-69.5%	24.5%	9.0%	31.0%	-22.3%
2	-76.5%	-12.5%	5.0%	10.7%	45.0%	-63.4%	23.2%	36.6%	31.6%	42.1%
3	-135.8%	-22.4%	**19.5%**	**33.7%**	**34.2%**	-91.8%	15.4%	**26.1%**	**24.2%**	**34.7%**
4	-31.3%	-9.3%	**12.8%**	**38.8%**	**28.5%**	-45.8%	42.8%	**21.9%**	**24.1%**	**35.0%**
5	-1.6%	14.4%	**33.9%**	**34.8%**	**33.5%**	-133.5%	6.4%	**32.7%**	**29.3%**	**37.4%**
All Ex 0	-75.3%	-11.5%	17.4%	27.0%	34.3%	-77.3%	23.6%	25.4%	29.2%	33.7%

Table 3-18
Current Average Diversification Credit with RBC Formula and 30% MDC

Diversif. Band Quintiles	Reserves					`				
	Size Band (Quintiles)					Size Band (Quintiles)				
	A	B	C	D	E	A	B	C	D	E
0	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
1	1.8%	1.7%	1.6%	1.7%	2.0%	4.1%	4.4%	4.4%	4.3%	4.2%
2	5.5%	5.4%	5.5%	5.4%	5.4%	9.4%	9.5%	9.5%	9.6%	9.6%
3	9.4%	9.4%	**9.4%**	**9.6%**	**9.4%**	13.2%	13.2%	**13.3%**	**13.2%**	**13.3%**
4	13.4%	13.3%	**13.3%**	**13.4%**	**13.4%**	16.2%	16.3%	**16.5%**	**16.6%**	**16.6%**
5	17.2%	17.4%	**17.8%**	**18.0%**	**18.5%**	20.0%	20.2%	**20.2%**	**20.5%**	**21.2%**
All Ex 0	7.7%	8.3%	9.0%	10.1%	11.3%	9.8%	11.3%	11.8%	13.2%	15.8%

Table 3-19
Indicated MDC

Diversif. Band Quintiles	Reserves					Premium				
	Size Band (Quintiles)					Size Band (Quintiles)				
	A	B	C	D	E	A	B	C	D	E
0	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
1	-1524.0%	-310.2%	247.1%	165.0%	731.8%	-513.5%	167.1%	61.4%	218.5%	-159.0%
2	-417.5%	-69.6%	27.3%	59.1%	248.8%	-203.2%	73.4%	115.6%	98.9%	131.1%
3	-431.7%	-71.6%	**61.9%**	**105.7%**	**108.8%**	-208.9%	35.0%	**58.9%**	**54.9%**	**78.1%**
4	-70.3%	-20.9%	**28.7%**	**87.0%**	**64.0%**	-84.6%	78.8%	**39.9%**	**43.7%**	**63.3%**
5	-2.7%	24.8%	**57.2%**	**58.0%**	**54.3%**	-200.3%	9.6%	**48.5%**	**42.8%**	**52.9%**
All Ex 0	-291.9%	-41.5%	57.7%	80.3%	91.0%	-236.8%	62.9%	64.3%	66.3%	63.9%

We focus on data cells highlighted in yellow/bold, for the reasons we discuss in Section 3.4.2. Those yellow/bold cells in Table 3-19 show indicated MDCs that average over 50% for reserve and premium risk charges. This is consistent with the findings from Table 3-11, the 2x6 analysis.

3.5.3 Testing Linear Relationship between CoMaxLine% and Indicated Diversification Credit

Next, we use regression through the origin to further test both the indicated MDC and to test the validity of the linear relationship between 100%-CoMaxLine% and the indicated diversification credit. The dependent variable is the indicated average diversification credit (Table 3-17). The independent variable is 100% - CoMaxLine% (Table 3-18 divided by 30%).

Table 3-20A, below, presents the regression results showing that the indicated MDC, the value of the slope, is approximately 50%, although with lower R-square[45] values than in the 2x6 analysis. For reserves, for every 100-basis point increase in the diversification index will result in 48 basis point

[45] The R-squared statistic is calculated by Excel regression in Excel data pack. The Excel formula for R-squared for regression through the origin is not the same as the R-squared formula used for OLS regression. Joseph G Eisenhauer (2003), Teaching Statistics, 25(3), 76-80.

increases in the diversification credit. For premium, for every 100-basis point increase in the diversification index will result in 54 basis point increases in the diversification credit.

Table 3-20A
Regression Analysis of Diversification Formula
Excluding Smallest Companies and Monoline Companies

Div Band	Size Band	Reserves			Premium		
		1	2	3	4	5	6
		Average Div Level	Indicated Div Credit	Fitted Div Credit	Average Div Level	Indicated Div Credit	Fitted Div Credit
1	B	5.8%	-18.0%	2.8%	14.7%	24.5%	7.9%
1	C	5.5%	13.5%	2.6%	14.7%	9.0%	7.9%
1	D	5.5%	9.1%	2.6%	14.2%	31.0%	7.7%
1	E	6.6%	48.2%	3.1%	14.0%	-22.3%	7.6%
2	B	18.0%	-12.5%	8.6%	31.6%	23.2%	17.0%
2	C	18.2%	5.0%	8.7%	31.6%	36.6%	17.0%
2	D	18.2%	10.7%	8.7%	32.0%	31.6%	17.2%
2	E	18.1%	45.0%	8.6%	32.1%	42.1%	17.3%
3	B	31.3%	-22.4%	14.9%	44.1%	15.4%	23.8%
3	C	31.5%	19.5%	15.0%	44.2%	26.1%	23.8%
3	D	31.9%	33.7%	15.2%	44.0%	24.2%	23.7%
3	E	31.4%	34.2%	15.0%	44.5%	34.7%	24.0%
4	B	44.5%	-9.3%	21.2%	54.3%	42.8%	29.3%
4	C	44.5%	12.8%	21.2%	54.9%	21.9%	29.6%
4	D	44.6%	38.8%	21.3%	55.2%	24.1%	29.7%
4	E	44.6%	28.5%	21.3%	55.3%	35.0%	29.8%
5	B	57.9%	14.4%	27.6%	67.2%	6.4%	36.2%
5	C	59.3%	33.9%	28.3%	67.5%	32.7%	36.3%
5	D	59.9%	34.8%	28.6%	68.4%	29.3%	36.9%
5	E	61.7%	33.5%	29.4%	70.7%	37.4%	38.1%
		Slope		48%	Slope		54%
		R-square		40%	R-square		72%

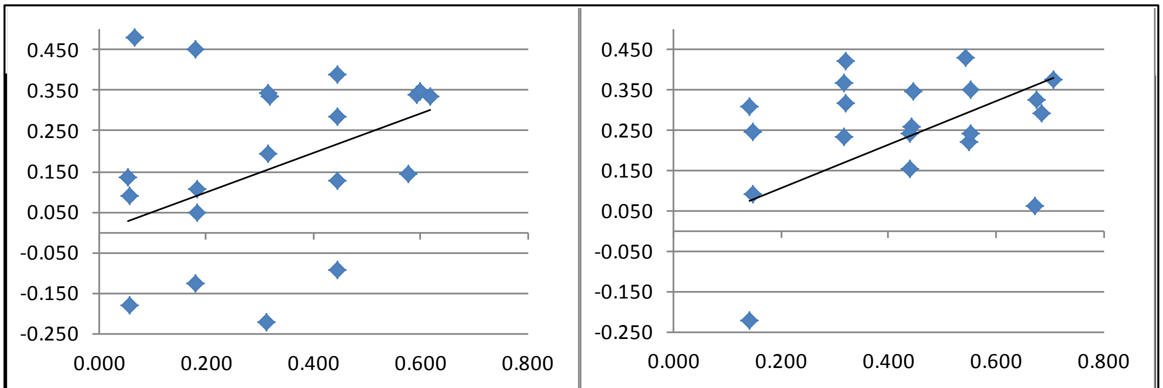
Columns 1 and 4 equal the values in Table 3-18/30%.
Columns 2 and 5 from Table 3-17.
Column 3 is based on regression through the origin.
The R-squared values based on regression through the origin.[46]

Table 3-20B shows the regression results graphically.

[46] The R-squared statistic is calculated by Excel regression in Excel data pack. The Excel formula for R-squared for regression through the origin is not the same as the R-squared formula used for OLS regression. Joseph G Eisenhauer (2003), Teaching Statistics, 25(3), 76-80.

Table 3-20B
Table 3-20A Graphically

Reserves	Premium



Slope	48%	Slope	54%
R-Square	40%	R-Square	72%

X-Axis shows 100% - CoMaxLine% (Average Diversification Credit /0.3).
Y-Axis shows indicated diversification factor.

Tables 3-21A and 3-21B, below, show the same information as 3-20A and 3-20B, above, for the nine data points, C3 to E5, which represent the largest and most diversified companies that constitute the bulk of the reserve, premium and diversification credit amounts. The nine-point regressions in Tables 3-21A and 3-21B have a much higher R-square value than the 20-point regressions in Tables 3-20A and 3-20B. Based on the 9-point regression, for reserves, every 100-basis point increase in the diversification index will result in a 63 basis point increases in the diversification credit. For premium, every 100-basis point increase in diversification index will result in a 52 basis point increases in the diversification credit.

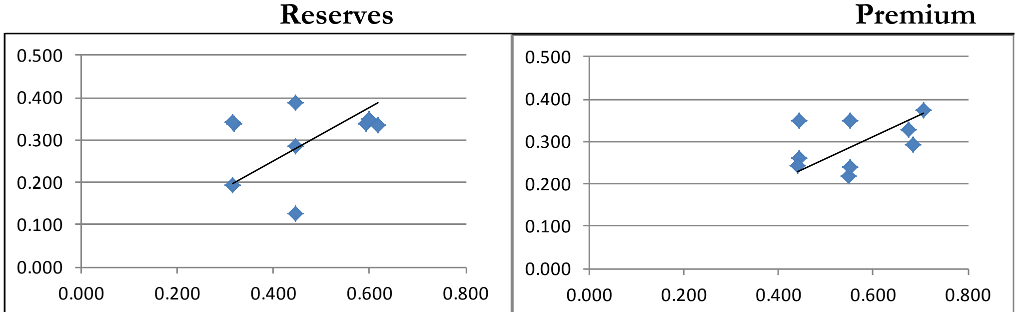
Table 3-21A
Regression Analysis of Diversification Formula All (Large and Diversified Only)
Size Band B-E/Diversification Bands 3-5

Div Band	Size Band	Reserves			Premium		
		1	2	3	4	5	6
		Average Div Level	Indicated Div Credit	Fitted Div Credit	Average Div Level	Indicated Div Credit	Fitted Div Credit
C	3	31.5%	19.5%	19.8%	44.2%	26.1%	22.8%
D	3	31.9%	33.7%	20.1%	44.0%	24.2%	22.7%
E	3	31.4%	34.2%	19.8%	44.5%	34.7%	23.0%
C	4	44.5%	12.8%	28.0%	54.9%	21.9%	28.3%
D	4	44.6%	38.8%	28.0%	55.2%	24.1%	28.5%
E	4	44.6%	28.5%	28.0%	55.3%	35.0%	28.5%
C	5	59.3%	33.9%	37.3%	67.5%	32.7%	34.8%
D	5	59.9%	34.8%	37.7%	68.4%	29.3%	35.3%
E	5	61.7%	33.5%	38.8%	70.7%	37.4%	36.5%
		Slope		63%	Slope		52%
		R-square		91%	R-square		96%

Columns 1-6 from selected rows of Table 3-20A
The R-squared values based on regression through the origin.[47]

Table 3-21B
Table 3-21A Graphically

Reserves **Premium**



Slope	63%	Slope	52%
R-Square	91%	R-Square	96%

X-Axis shows 100% - CoMaxLine%, or, equivalently Average Diversification Credit /0.3.
Y-Axis shows indicated diversification factor.

[47] The R-squared statistic is calculated by Excel regression in Excel data pack. The Excel formula for R-squared for regression through the origin is not the same as the R-squared formula used for OLS regression. Joseph G Eisenhauer (2003), Teaching Statistics, 25(3), 76-80.

Based on those results, the indicated diversification formulas are:

LCF = 37% plus 63% * CoMaxLine%

PCF = 48% plus 52% * CoMaxLine%

4. ALTERNATIVE DIVERSIFICATION APPROACHES

In this section we test alternatives to the CoMaxLine% Approach.

4.1 Alternatives to CoMaxLine%

From the risk theory perspective, the natural approach to diversification is to combine risk charges by LOB using correlation factors between each pair of LOBs. Individual company capital models often use this pairwise correlation approach. The Solvency II Standard Formula uses the pairwise correlation approach. The correlation approach, if applied in the RBC Formula, uses 171 parameters.[48] In contrast to the correlation approach, the RBC Formula CoMaxLine% Approach might be described as 'simple," perhaps too simple, and ad hoc.

One difference between the CoMaxLine% Approach and the correlation matrix approach, as normally applied, is that the degree of diversification in the correlation matrix approach is based on risk by LOB while the degree of diversification in the CoMaxLine% Approach is based on volume (premium amount or reserve amount) by LOB. Therefore, another alternative to CoMaxLine% and correlation matrix approaches, is the CoMaxLine%-Risk Approach, in which we apply the CoMaxLine% Approach to LOB risk rather than LOB volume, when calculating the LCF and PCF for a company.

Another alternative to the CoMaxLine% and the correlation matrix approach is the HHI approach, used by economists to measure concentration. HHI considers the relative proportions of all LOBs, the largest, second largest, third largest, and so on.[49] This is simpler than correlation approach, but it is more complex than the CoMaxLine% Approach in that the HHI approach recognizes the extent of

[48] One parameter for each pair of LOBs, i.e., 19 LOBs each need to be paired with the 18 other LOBs, thus 19x18 = 342, divided by 2 because the relationship be LOB "X" and LOB "Y" is the same as the relationship between LOB "X" and LOB "Y". Therefore, in theory that requires 171 parameters. In practice Solvency II uses 2 parameters, 25% and 50%, and judgement to decide whether each of 171 LOB pairs is lower correlation (25%) or higher correlation (50%).

[49] HHI equals the sum of the squares of the LOB shares of total. For example, if there is only one LOB, HHI is 1.0, as is the case for CoMaxLine%. With two lines split 25% and 75% HHI is 0.25 ^2 plus 0.75^2 or 0.625 compared the CoMaxLine% of 0.750, i.e., it shows less concentration/more diversification. With three lines split 50%, 25% and 25% HHI is 0.50^2 plus 0.25^2 plus 0.25^2 or 0.375, less concentration/more diversification than the CoMaxLine% of 0.5.

diversification for the 2nd, 3rd, 4th, etc. largest LOBs.[50]

Any of these approaches to diversification is an approximation. The theoretical requirements for risk theory diversification approach do not fully apply to standard formulas, at least as evidenced by our risk data, for reasons that include the following:

1. LOB charges vary not only by LOB, but within LOBs based on the degree of specialization of the insurer, extent of reinsurance usage, etc.

   For example, with our risk data, the indicated personal automobile risk charge% for a monoline, or near monoline, company is not the same as the indicated risk charge% for personal lines automobile for multi-line companies.[51] Appendix 2 shows our analysis of variation in LOB risk charge% by variation in company diversification.
2. The LOB risk charge%s and, possibly, diversification parameters, that might vary by LOB-size. The differences by LOB-size are not part of the either RBC or the Solvency II Standard Formula. As such, the LOB risk charges and the correlations relationships are, at best, correct for a particular set of LOB-sizes and/or on average across all LOB-sizes.
3. For the most plausible LOB-size distributions, the "normal-family" assumption underlying the covariance formula might not be satisfied.[52]

In addition to those three issues, which affect the theoretical framework, as a practical matter there may not be enough data for all the potential parameters. For the correlation matrix approach, even the DCWP database, with 30,000 company/year/all-line data points (for each of the premium and reserve data sets),[53] may not be adequate to support a data-driven calibration of the 171 required diversification parameters, especially if differences in the diversification relationship by company size are reflected.

4.2 Analysis of Alternatives

To apply the correlation approach for our testing, we first construct a set of pairwise correlation factors, called a correlation matrix. Following the Solvency II approach, we construct the correlation

[50] The HHI is sometimes applied to only the n-th largest segments, e.g., the degree of diversification among the top ten LOBs. The HHI index applied to the single largest segment would be very similar to the CoMaxLine%. HHI can be written as p_1^2 + p_2^2+p_3^2…p_n^2. The truncated HHI limited to one element would be p_1^2. CoMaxLine% is p_1.

[51] This feature of the data implies that a key assumption in the risk theory diversification framework not valid. In mathematical terms, the risk distribution by LOB f(x) should be the same regardless of the proportion of business from line of business y. We find that f(x|no other business) ≠ f(x| there is some other business); f(x|(company has enough y to be at Diversification level 1), ≠ f(x|(company has enough y to be at Diversification level 2), ≠ f(x|(company has enough y to be at Diversification level 2), etc.

[52] This issue might be addressed using copulas, but that requires further parameterization.

[53] To our knowledge, this database is larger than any other database used for Standard Formula calibrations.

matrix using values of 25% or 50%[54] for most of the 171 LOB-pairs. For several LOB-pairs that we consider very highly correlated we select correlation factors of 75% or 100%.[55] Appendix 3/Exhibit 1 shows the Solvency II correlation matrix for the 12 Solvency II non-life LOBs. Appendix 3/Exhibit 2 shows the correlation matrix that we use.

Then, for each of the four diversification approaches, i.e., the CoMaxLine% Approach, the correlation matrix approach, the CoMaxLine%-risk approach and the HHI approach, we compare the indicated risk charge%s to the formula risk charge%s for each of the thirty company-size/diversification band cells, separately for premium risk and reserve risk. Appendix 4 shows the calculations of indicated risk charge%s and differences between the indicated risk charge%s and the risk charge%s from the RBC Formula with the CoMaxLine% and correlation matrix dependency formulas.[56]

In Table 4-1, below, we summarize the 30 indicated versus formula results, for CoMaxLine% Approach and correlation matrix approach, from Part 5 of Appendix 4. We use three measures of indicated versus formula differences. We refer to those as 'error statistics' for each method. These error statistics are as follows:

- Standard deviation,
- Average error, and
- Average absolute error

We calculate the error statistics for each of the following three sets of points by company size/diversification band, separately for reserves and premium:

- All Points – All, excluding monoline companies (25 size/diversification segments)
- Exclude the smallest – All, other than the smallest company sizes and monoline companies, i.e. across company size/diversification bands B1-E5 (20 size/diversification segments).
- Include only the largest/most diversified - The largest, most diversified companies that constitute the bulk of the premium/reserves and diversification credit, i.e., company size/diversification bands C3-E5 (9 size/diversification segments).

[54] "Advice for Band 2 Implementing Measures on Solvency II: SCR Standard Formula Article 111(d) Correlations," (former Consultation Paper 74), January 2010, pp 39-44. See Appendix 3 of this paper for further discussion of the origin of the Solvency II correlation matrix.

[55] We select pairwise correlations of 100% for claims made and occurrence medical malpractice and for general liability, special liability and products liability. We select pairwise correlations of 75% between special property and homeowners, between private passenger automobile liability and automobile physical damage and between commercial automobile liability and automobile physical damage.

[56] The analysis for the HHI and CoMaxLine%-Risk are analogous to those in Appendix 4, for CoMaxLine% and correlation matrix. We do not present the HHI or CoMaxLine%-Risk details in this Report.

In Table 4-1, below, shows that, for reserves, the correlation approach has somewhat lower error statistics. For example, the correlation matrix approach has the lowest error statistic for 8 of the 8 tests[57], and the lowest error statistic for the 9-point test that represents the bulk of the reserves, premium and diversification credit. For premium, Table 4-1 shows that the CoMaxLine% Approach (labeled NAIC) often has somewhat lower error statistics. For example, the CoMaxLine% Approach has the lowest error statistic for 7 of the 8 tests, and the lowest error statistic for the 9-point test that represents the bulk of the reserves, premium and diversification credit.

Overall, we conclude that the correlation approach does not better represent the data than the CoMaxLine% Approach.

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[57] There are eight tests, rather than nine. The value for "Include only largest (9 points)" for Average Error is always zero because we select the best fitting risk charge%s to achieve that result.
By a "lower error score" we mean the absolute value of the difference between indicated and expected has a smaller absolute value.

Table 4-1
Error Statistics – CoMaxLine% (NAIC) vs. Correlation Matrix (Correlation) Approaches
Error Measured as % of Reserves/Premium
Multi-Line Companies Only
[Green Highlight indicates the lower value within each pair of models]

Standard Deviations				
	Reserves		Premium	
Points Included	**NAIC**	**Correlation**	**NAIC**	**Correlation**
All Points (25 points)	0.13	0.11	0.11	0.12
Exclude Smallest (20 points)	0.07	0.06	0.040	0.038
Include only Largest (9 points)	0.03	0.02	0.01	0.02

Average Error				
	Reserves		Premium	
Points Included	**NAIC**	**Correlation**	**NAIC**	**Correlation**
All Points (25 points)	6.5%	4.7%	4.4%	4.3%
Exclude Smallest (20 points)	1.2%	0.7%	-0.7%	-1.2%
Include only Largest (9 points)	0%	0.0%	0%	0%

Absolute Average Error				
	Reserves		Premium	
Points Included	**NAIC**	**Correlation**	**NAIC**	**Correlation**
All Points (25 points)	9.7%	8.0%	7.4%	7.7%
Exclude Smallest (20 points)	5.3%	4.9%	3.0%	3.1%
Include only Largest (9 points)	2.9%	1.9%	1.1%	1.5%

Green highlight indicates whether NAIC (CoMaxLine%) or Correlation Matrix approaches provide the lower error within each group of cells. Data rounded to show differences.
Note – For "Average Error" section, the overall level is set so that the average error equals zero for the largest 9 points.
We express the error statistics as a percentage of reserves/premium. Risk charge%s are approximately 20% of reserves/premium, so a 1% error premium is a 5% error in the risk charge. Thus 1% is a small, but not negligible proportion of the irks charge.

Table 4-2, below, shows the same error statistics but for all four of the methods for reserve risk and premium risk.[58]

[58] The analysis for the HHI and CoMaxLine%-Risk are analogous to those in Appendix 4, for CoMaxLine% and correlation matrix. We do not present the HHI or CoMaxLine%-Risk details in this Report.

Table 4-2
Error Statistics – CoMaxLine% (NAIC) vs. CoMaxLine%-Risk Approach
Error Measured as % of Reserves/Premium
[Green Highlight indicates the lowest value among the four models]

A. Standard Deviations								
	Reserves				Premium			
Points Included	NAIC	Correlation	HHI	CoMaxLine % - Risk	NAIC	Correlation	HHI	CoMaxLine % - Risk
All Points (25 points)	0.133	0.120	0.168	0.126	0.114	0.128	0.125	0.105
Exclude Smallest (20 points)	0.067	0.063	0.050	0.066	0.040	0.038	0.037	0.031
Include only Largest (9 points)	0.035	0.023	0.026	0.028	0.014	0.021	0.014	0.010

B. Average Error								
	Reserves				Premium			
Points Included	NAIC	Correlation	HHI	CoMaxLine % - Risk	NAIC	Correlation	HHI	CoMaxLine % - Risk
All Points (25 points)	6.5%	5.6%	9.6%	5.7%	4.37%	4.43%	5.8%	3.5%
Exclude Smallest (20 points)	1.2%	0.8%	3.3%	1.1%	-0.7%	-1.2%	0.2%	-1.4%
Include only Largest (9 points)	0%	0%	0%	0%	0.0%	0.0%	0%	0.0%

C. Absolute Average Error								
	Reserves				Premium			
Points Included	NAIC	Correlation	HHI	CoMaxLine % - Risk	NAIC	Correlation	HHI	CoMaxLine % - Risk
All Points (25 points)	9.7%	8.9%	10.5%	9.3%	7.4%	7.8%	7.7%	6.7%
Exclude Smallest (20 points)	5.3%	4.9%	4.4%	5.2%	3.0%	3.1%	2.5%	2.6%
Include only Largest (9 points)	2.9%	1.9%	2.1%	2.3%	1.1%	1.6%	1.1%	0.9%

Green highlight indicates whether NAIC (CoMaxLine%), correlation matrix, HHI or CoMaxLine%-Risk approaches provides the lower error within each group of cells. Data rounded to show differences.
Note – For "Average Error" section, the overall level is set so that the average error equals zero for the largest 9 points.
We express the error statistics as a percentage of reserves/premium. Risk charge%s are approximately 20% of reserves/premium, so a 1% error premium is a 5% error in the risk charge. Thus 1% is a small, but not negligible proportion of the irks charge.

In this 4-way comparison, we see that:

- The RBC CoMaxLine% Approach does not have the lowest error statistics for any size group for either premium or reserves; however,
- As we saw in Table 4-1, comparing CoMaxLine% and correlation matrix approaches, CoMaxLine% has lower error statistics premium while correlation matrix approach has lower error statistics for reserves.
- CoMaxLine%-Risk has lower error statistics than CoMaxLine% for both premium and reserves (8 of 8 for reserves and 7 of 8 for premium and, in particular, for the two 9-point tests). For premium, CoMaxLine%-Risk has the lowest error statistics across the four approaches.
- The difference between the RBC Approach and the method with the lowest error statistics is always less than 1.7% of reserves/premium (therefore less than about 10% of average

UW risk RBC).

LOB Risk Factors that vary by LOB-size

In Appendix 5, we address the extent to which our findings regarding diversification with CoMaxLine% Approach would be affected if the RBC Formula used risk factors that vary by LOB-size.

This question is motivated, in part, because we observe that LOB-size, company-size and diversification level are inter-related. For example, we observe that larger LOB-sizes indicate risk charge%s that are lower than the risk charges%s indicated by smaller LOB-sizes. Therefore, it could be the case higher indicated diversification credits are a proxy for lower LOB risk charge%s for larger companies.

We test that hypothesis by applying LOB risk charge%s that vary by LOB-size. We find that the indicated MDC would be different if the risk factors were determined by LOB size, we find that the indicated MDC% is greater than 30% and our conclusion regarding CoMaxLine% versus correlation matrix remains the same.[59]

5. OVERALL FINDINGS

Thus, we find that:

- The linear relationship between diversification discount and 100%-CoMaxLine%, in the CoMaxLine% Approach is not perfect, but it is a reasonable approximation, especially close for the most diversified companies.
- A MDC of at least 50% is better supported by the data than the current 30% MDC.
- The CoMaxLine%-Risk Approach may be better than the CoMaxLine% Approach.
- Neither the correlation matrix approach nor the HHI approach represents the data significantly better than the diversification approach in the RBC Formula for both reserve risk and premium risk.

6. FUTURE RESEARCH

Our analysis uses certain simplifications. The expected risk charge%s in our analysis do not include the effect of Investment Income Offset (IIO), loss sensitive business, own-company adjustment or growth risk in the expected risk charges. To convert premium risk factors to AYUL and AYUL% we use industry-total expense by LOB, adjusted to the company LOB distribution, rather than company-

[59] We did not test the comparison for HHI or CoMaxLine% risk.

by-company expenses. Our analysis uses risk data that satisfies the LOB filtering tests, describing in DCWP Reports 6 and 7, and therefore does include Minor Lines data points or other data points removed for LOB risk factor analysis. We do not include the R_3-Reinsurance Credit Risk Element of R_4. Future research could test the extent to which, if at all, those simplifications affect the indicated MDC or the conclusion that there is a linear relationship between diversification and CoMaxLine%.

We did not evaluate the HHI-Risk approach, analogous to CoMaxLine%-Risk, in which HHI is applied to amount of risk rather than amount of reserve/premium. Also, the RBC formula might consider both diversification by LOB and diversification among types of multi-line companies, e.g., personal vs. standard commercial vs. specialty. Future research could test the extent to which those approaches better reflect observed diversification patterns.

Future research could evaluate the extent to which there might be improvements to the error statistics we used to compare the alternative diversification formulas.

Our analysis is based on a target safety level of 87.5%. Future research could examine the extent to the conclusions vary if a different safety level were selected.

7. GLOSSARY

Annual Statement	US NAIC Annual Statement
CoMaxLine%	The NAIC measure of concentration, the percentage of a company's total premium or reserves from its single largest LOB.
CoMaxLine% Approach	The NAIC method of determining diversification credit. The diversification credit is (1.0 – CoMaxLine%) times 30%.
CoMaxLine%-Risk Approach	CoMaxLine% Approach based on risk charge value by LOB rather than premium or reserve volume by LOB.
Correlation approach	We use that term to characterize methods of combining LOB risk charges to produce an all-lines risk charge using 'correlation factors.' Our use of the term does not imply that the assumptions underlying individual and joint distributions of the parameters are satisfied.
Correlation Factor	A factor used to express the relationship between individual risks to produce the risk parameter of interest for the combined risk. Our use of the term does not imply that the assumptions underlying individual and joint distributions of the parameters are satisfied.
Correlation Matrix	A matrix array of correlation factors, with one factor for each pair of LOBs.
DCWP	Risk Based Capital Dependency and Calibration Working Party of the Casualty Actuarial Society
Initial Reserve	The reserve amount at the Initial Reserve Date for all accident years prior to the Initial Reserve Date.
Initial Reserve Date	December 31st for the year specified (i.e., December 31, 2010 is the Initial Reserve Date for the 2010 net loss reserve which includes AY's 2010 and prior)
LCF	Loss (Reserve) Concentration Factor as calculated in 2010 RBC Formula. Based on CoMaxLine% Approach.
LOB	Schedule P Lines of Business used in the RBC Formula. Note that three pairs of Schedule P LOBs are combined; occurrence and claims Other Liability (Line H), occurrence and claims made Products Liability (Line R), and Reinsurance: nonproportional property and Reinsurance: nonproportional financial (Lines P and N, respectively).
Loss sensitive business adjustment	An element of the RBC Formula that reduces the risk charge if unfavorable experience can be offset by increases in revenue on loss sensitive business.
MDC	Maximum Diversification Credit, 30% in the 2010 RBC Formula
NAIC	National Association of Insurance Commissioners
Own-company adjustment, or 50/50 rule	RBC premium and reserve factors are based 50% on factors calibrated based on industry data and 50% based on the industry data adjusted by the ratio of company experience to industry experience. (Subject to certain exceptions.)

PCF	Premium Concentration Factor as calculated in 2010 RBC Formula. Based on CoMaxLine% Approach.
R_0	Insurance affiliate investment and (non-derivative) off-balance sheet risk.
R_1	Asset Risk – Fixed Income Investments
R_2	Asset Risk – Equity
R_3	Credit risk (non-reinsurance plus one half of Reinsurance Credit Risk)[56]
R_3-Reinsurance Credit Risk	See Reinsurance Credit Risk
R_4	Reserve risk plus one half of R_3-reinsurance credit risk.[60] This paper uses R_4 without R_3-reinsurance credit risk.
R_5	Premium risk.
RBC	Risk-Based Capital
RBC Formula or Formula	The 2010 NAIC Property-Casualty RBC Formula
RBC UW Risk Value	The Company Action Level amount calculated for the UW risk components of the RBC Formula for a company or DCWP defined group of companies.
Reinsurance Credit Risk	An element of R_3, representing both credit risks related to reinsurance counterparty and the difference in premium and reserve risk of between companies with varying levels of ceded reinsurance.
Reserves or Loss Reserves	Case, bulk and IBNR loss and defense and cost containment expense[61] reserves net of reinsurance, as shown in Schedule P – Part 2 and 3.
Schedule P	A set of exhibits in the Annual Statement that provide most of the risk data used in our analysis.
Solvency II	EU regulation and related implementing measures
Standard Formula	A formula determining capital requirements under Solvency II, RBC or other regulatory capital systems
UW	Underwriting
UW risk	Underwriting risk – the combination of premium risk and reserve risk

[60] The 'transfer' from credit risk to reserve risk applies only if the reserve risk without the reinsurance credit risk component is larger than the reinsurance credit risk, as is most often the case.

[61] "Defense and Cost Containment Expenses" are called "Allocated Loss Adjustment Expenses" in older Annual Statements. In our analysis we treat defense and cost containment expense and allocated loss adjustment expenses as equivalent.

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APPENDIX 1- INDICATED RISK FACTORS AND SAMPLE CALCULATIONS

Appendix 1/Exhibit 1
Indicated PRC% and RRC% by LOB

Line of Business	2010 PRFs/RRFs		Indicated PRFs/RRFs for Dependency Analysis			
	(1)	(2)	(3)	(4)	(5)	(6)
	PRF	RRF	PRF	CER	PRC%	RRF
A- Homeowners/Farmowners	0.937	0.201	0.956	0.301	0.257	0.225
B- Private Passenger Auto	0.969	0.192	0.969	0.252	0.221	0.179
C- Commercial Auto	0.988	0.230	0.988	0.308	0.296	0.352
D - Workers Compensation	1.033	0.324	1.039	0.268	0.307	0.333
E - Commercial Multi-Peril	0.921	0.465	0.879	0.355	0.234	0.488
F1 - Med Prof Liab-Occ	1.822	0.431	1.458	0.280	0.738	0.306
F2 - Med Prof Liab-CM	1.092	0.306	1.146	0.280	0.426	0.106
G - Special Liability	0.904	0.257	0.947	0.344	0.291	0.455
H - Other Liability	1.042	0.511	1.015	0.303	0.318	0.525
I - Special Property	0.941	0.191	0.817	0.326	0.143	0.331
J - Auto Physical Damage	0.843	0.112	0.828	0.252	0.080	0.194
K - Fidelity/Surety	0.883	0.325	0.644	0.454	0.098	0.560
L - Other	0.893	0.172	0.923	0.358	0.281	0.274
M - International	1.169	0.327	0.899	0.400	0.299	0.508
N&P - Reinsurance-Prop/Fin	1.349	0.286	1.288	0.247	0.535	0.422
O - Reinsurance-Liabiity	1.507	0.769	1.302	0.247	0.549	0.650
R - Products Liability	1.214	0.643	1.184	0.311	0.495	0.883
S - Financial/Mort Guarantee	1.482	0.200	0.725	0.285	0.010	0.560
T - Warranty	0.883	0.325	0.879	0.359	0.238	0.488

CER = Company Expense Ratio. Equals 2010 industry average underwriting expense ratio by LOB.
F1 and F2 – same expense ratio;
H is average of H1 and H2; R is average of R1 and R2
Same expense ratio for N&P and O

Risk Data Selection

As described in DCWP Reports 6 and 7, the risk data we use in our calculation of the RRFs/PRFs shown above excludes anomalous values; treats pool company data on a combined basis; excludes Minor Lines data points; and, for premium risk data, excludes companies with less than 5 AYs of NEP. We also exclude the LOB data points for the smallest LOBs, defined as those in the smallest 15th percentile of all LOB-company-year data points, with the 15th percentile determined separately for each AY/Initial Reserve Date.

For premium risk, the data points do not include data for 2001-2010 AYs for companies that did not file a 2010 Annual Statement. For reserve risk, the data points include 2001-2000 Initial Reserve Dates, to the extent such information is in any Annual Statement.

The risk data values are the values at the latest available maturity.

To convert premium risk factors to premium risk charge%s we use 2010 industry-total expense by LOB.

Appendix 1/Exhibit 2
Example of Data Underlying Expected Risk Charge% and Indicated Risk charge% Calculation for a Sample Company
Reserve Risk Data

(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Modeled Reserve Risk			Observed Reserve Experience		
Line	**Initial Reserve**	**Modeled Risk Charge %**	**Modeled Risk Charge $**	**Observed Reserve Runoff $**	**Observed Reserve Runoff %**	**Reserve % by LOB**
A	6,458	22.5%	1,453	(1,733)	-26.8%	5.2%
B	-	17.9%	-	-	-	
C	25,334	35.2%	8,918	(4,111)	-16.2%	20.4%
D	28,643	33.3%	9,538	1,524	5.3%	23.0%
E	18,091	48.8%	8,828	(4,623)	-25.6%	14.6%
F1	-	30.6%	-	-	-	
F2	-	10.6%	-	-	-	
G	-	45.5%	-	-	-	
H	35,596	52.5%	18,688	(9,834)	-27.6%	28.6%
I	-	33.1%		-	-	
J	-	19.4%	-	-	-	
K	-	56.0%	-	-	-	
L	-	27.4%	-	-	-	
M	-	50.8%	-	-	-	
N_P	-	42.2%	-	-	-	
O	-	65.0%	-	-	-	
R	10,203	88.3%	9,009	4,098	40.2%	8.2%
S	-	56.0%	-	-	-	
T	-	48.8%	-	-	-	
Total/Avg	124,325	45.4%	56,434	(14,679)	-11.8%	100.0%

Diversification Approach	**Diversification Index**
8.CoMaxLine%	71.4%
9.CoMaxLine%-Risk	66.9%
10. HHI	79.3%
11. Correlation Matrix	76.7%

These calculations are described below, in Notes to Appendix 1/Exhibit 2.

Notes to Appendix 1/Exhibit 2

Col/ Row	Notes
Col 1	Line of Business
Col 2	Data – loss and LAE reserve for the sample company-year-line of business
Col 3	Indicated Reserve Risk Factor shown in Appendix 1/Exhibit 1/Column 6
Col 4	(2) x (3)
Col 5	Data – company-year-LOB reserve runoff from Initial Reserve Date through the latest available maturity. Negative values indicate favorable runoff.
Col 6	(5)/ (1) – reserve runoff as a percentage of Initial Reserve;
Col 7	LOB Initial Reserve / all line total Initial Reserve (2)/ All line total (2)
Row 8	100% - Maximum LOB % from column (7)
Row 9	100% - Maximum value in Column 4/Total of Column 4
Row 10	HHI calculation 100% - Sum of squares of percentages in column 7
Row 11	Calculated from correlation matrix in Appendix 3/Exhibit 1 applied to expected risk amounts column 4.

The all-lines risk information in the Total/Avg row provides a single company-year data point used to calculate expected risk and indicated risk. We use the data in Rows 8-11 to categorize each company by diversification band.

Appendix 1/Exhibit 3
Example of Data Underlying Expected Risk Charge% and Indicated Risk charge% Calculation for a Sample Company
Premium Risk Data

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Expected Premium Risk			**Observed Premium Experience**			
Line	Premium	Expected Risk Charge %	Expected Risk Charge $	Observed Loss Ratio	Industry Expense Ratio	Observed AYUL%	Premium % by LOB
A	14,903	25.7%	3,833	80.5%	30.1%	10.6%	6.9%
B	13,679	22.1%	3,018	89.2%	25.2%	14.4%	6.3%
C	18,591	29.6%	5,512	85.1%	30.8%	15.9%	8.6%
D	22,324	30.7%	6,863	72.9%	26.8%	-0.3%	10.3%
E	20,541	23.4%	4,808	101.7%	35.5%	37.2%	9.5%
F1	-	73.8%	-	-	28.0%	-	-
F2	-	42.6%	-	-	28.0%	-	-
G	-	29.1%	-	-	34.4%	-	-
H	24,492	31.8%	7,800	43.1%	30.3%	-26.6%	11.3%
I	34,772	14.3%	4,960	51.5%	32.6%	-15.9%	16.1%
J	20,933	8.0%	1,684	84.4%	25.2%	9.6%	9.7%
K	16,893	9.8%	1,660	11.8%	45.4%	-42.8%	7.8%
L	-	28.1%	-	-	35.8%	-	-
M	-	29.9%	-	-	40.0%	-	-
N_P	28,979	53.5%	15,504	75.7%	24.7%	0.4%	13.4%
O	-	54.9%	-	-	24.7%	-	-
R	-	49.5%	-	-	31.1%	-	-
S	-	1.0%	-	-	28.5%	-	-
T	-	23.8%	-	-	35.9%	-	-
Total/Avg	216,107	25.7%	55,641	68.1%	30.4%	-1.4%	100.0%

Diversification Approach	Diversification Index
9. CoMaxLine%	83.9%
10.CoMaxLine%-Risk	72.1%
11. HHI	89.2%
12. Correlation Matrix Diversification	64.8%

These calculations are described below, in Notes to Appendix 1/Exhibit 3.

Notes to Appendix 1/Exhibit 3

Col/ Row	Notes
Col 1	Line of Business
Col 2	Data – Net earned premium for the sample company-year-line of business
Col 3	Indicated Premium Risk Charge shown in Appendix 1/Exhibit 1/Column 5
Col 4	(2) x (3)
Col 5	Data – Loss and LAE ratio at the latest available maturity
Col 6	Data – 2010 industry expense ratio. Used as a proxy for company expense ratios as these are not readily available for each year in the experience period.
Col 7	(5)+(6)-100%
Col 8	Line of Business Premium/ all line total Premium; (2)/ All line total (2)
Row 9	100% - Maximum LOB % from column 8
Row 10	100% - Maximum value in Column 4/ Total of column 4
Row 11	HHI calculation 100% - Sum of squares of percentages in column 8
Row 12	Calculated from correlation matrix in Appendix 3/Exhibit 1 applied to expected risk amounts in column 4.

The all-lines risk information in the Total/Avg row provides a single company-year data point used to calculate expected risk and indicated risk. We use the data in Rows 9-12 to categorize each company by diversification band.

APPENDIX 2 - LOB RISK CHARGE%S VARY WITH DEGREE OF DIVERSIFICATION OF THE COMPANY.

In individual company capital modeling, diversification credit arises because the risk[62] associated with the combined LOB (1 + 2) business is generally less than the sum of LOB 1 risk and LOB 2 risk. The magnitude reduction depends on the extent to which the two LOBs risk characteristics are correlated. Using the correlation relationship (and some statistical assumptions) allows the determination of the LOB (1+2) risk from the separate LOB1 and LOB2 risk. This framework requires that the LOB risk charge%s are independent of the degree of diversification of the company.

In calibrating a Standard Formula, on the other hand, the LOB1 risk charge is based on data for all levels of company diversification combined, i.e., {LOB1|all diversification levels}. This {LOB1|all diversification levels} may not have the same as risk as {LOB1|monoline company} or {LOB1|given that the company writes some of LOB2 and perhaps other LOBs}. Similarly, {LOB2|all diversification levels} may not have the same risk as {LOB2|monoline Company} and {LOB2|given the companies writes some LOB1 and perhaps other LOBs}.

Therefore, the risk for LOB (1+2) (at specific diversification levels) would not necessarily follow from {LOB1|all diversification levels} and {LOB2|all diversification levels}. In fact, our review of the Risk Data we find that there are variations in LOB risk charge%s with the degree of diversification of the company. For some LOBs, for example, for the personal automobile liability LOB, monoline companies[63] have higher PPA LOB risk charge%s than diversified companies. That might follow from reduced geographic risk diversification in monoline companies, or other features of those companies. For other LOBs, e.g., monoline MPL, monoline companies have lower LOB risk charge%s than diversified companies. That might follow from benefits of specialization, the type of policies, e.g., primary vs. excess or physicians vs. hospitals, or other factors.

Regardless of the underlying causes, Appendix 2/Exhibits 1A and 1B, below, show that LOB risk charge%s vary with diversification level of the company. For more than half of the 32 LOBs (16 for each of premium and reserve risk), the indicated PRF/RRF at zero diversification is either the highest of the six values for that LOB or the lowest of the six values for that LOB. If the distribution of risk charge%s by diversification level were random, we would expect that the zero-diversification band would be the highest or lowest, on average, for about 1/3 of the LOBs. To have that be the case for

[62] As in earlier sections of this paper, we use the term "risk" to mean the 87.5th percentile of the observed distribution. The analysis in this section applies regardless of the percentile safety level and for alternative risk metrics other than VaR
[63] In our diversification Risk Data, 'monoline' includes companies with a small proportion of business (less than 5% of premium) in other LOBs, e.g., Minor Line data points that we exclude from the Risk Data.

19 or more of the 32 LOBs has a probability of less than 1%. This effect is much stronger for reserves than form premium.[64]

Appendix 2/Exhibit 1A
Indicated RRFs - Variation in LOB Risk Charge% with Variation in Company Diversification

Indicated RRF by Diversification Band								
	Diversification Band							
LOB	0	1	2	3	4	5	all	0 vs. rest
A	0.35	0.28	0.25	0.19	0.18	0.19	0.22	Highest
B	0.30	0.15	0.16	0.14	0.22	0.13	0.18	Highest
C	0.57	0.34	0.37	0.48	0.40	0.28	0.35	Highest
D	0.32	0.23	0.34	0.40	0.40	0.31	0.33	
E	0.54	0.60	0.67	0.50	0.49	0.43	0.49	
F1	0.09	0.35	0.34	0.22	0.40	0.87	0.31	Lowest
F2	0.04	0.11	0.16	0.26	0.13	0.37	0.11	Lowest
G	0.39	0.08	0.38	0.50	0.31	0.63	0.45	
H	0.29	0.85	0.56	0.57	0.55	0.54	0.53	Lowest
I	0.12	0.59	0.43	0.34	0.31	0.30	0.33	Lowest
J	0.00	0.16	0.17	0.06	0.25	0.29	0.19	Lowest
K	0.34	0.39	0.74	1.28	0.64	0.50	0.56	Lowest
L	0.11	0.26	0.47	0.73	0.21	0.34	0.27	Lowest
N&P	0.17	0.41	0.40	0.44	0.51	0.48	0.42	Lowest
O	0.66	0.43	0.58	0.59	0.68	0.76	0.65	
R	0.56	1.48	0.49	1.05	0.67	0.82	0.88	
Average	0.35	0.39	0.39	0.40	0.41	0.38	0.37	Lowest

[64] Looked at for reserves and premium, separately, the situation is less clear. The probability of 12 of 16 for reserves is well under 1%, but the probability of the observed seven or more for premium is 26%, hence not unusual by itself.

Appendix 2/Exhibit 1B
Indicated PRFs - Variation in LOB Risk Charge% with Variation in Company Diversification

Indicated PRF by Diversification Band								
	Diversification Band							
LOB	**0**	**1**	**2**	**3**	**4**	**5**	**all**	**0 vs. rest**
A	1.04	0.90	0.89	0.97	0.97	0.97	0.96	Highest
B	1.01	0.95	0.95	0.98	0.97	0.97	0.97	Highest
C	0.97	0.99	0.98	1.01	1.02	0.97	0.99	
D	1.04	0.98	1.10	1.10	1.11	1.00	1.04	
E	0.87	0.95	0.84	0.88	0.87	0.88	0.88	
F1	1.37	1.49	1.37	1.45	1.39	1.19	1.46	
F2	1.07	1.19	1.22	1.26	1.36	1.24	1.15	Lowest
G	0.99	0.81	0.92	1.03	0.92	0.94	0.95	
H	1.02	1.01	0.97	1.05	1.03	1.00	1.02	
I	0.82	0.81	0.79	0.81	0.80	0.84	0.82	
J	0.82	0.78	0.84	0.85	0.83	0.82	0.83	
K	0.41	0.69	0.78	0.75	0.86	0.70	0.64	Lowest
L	0.85	0.93	0.86	0.92	0.93	0.98	0.92	Lowest
N&P	1.14	1.16	1.37	1.14	1.36	1.25	1.29	Lowest
O	0.96	1.50	1.19	1.34	1.15	1.33	1.30	Lowest
R	1.93	1.56	1.41	1.05	1.14	1.11	1.18	
Average	0.96	0.93	0.94	0.98	0.97	0.95	0.96	

To further test the statistical significance of the pattern by LOB, including the extent to which zero diversification indicated risk factors are the highest or lowest, we construct standardized differences[65] between each value and mean for the LOB across all diversification bands. Appendix 2- Exhibits 2A, 2B, and 3, below, show those standardized differences.

Appendix 2/Exhibit 2A
Indicated RRFs – Standardized Variation in LOB Risk Charge% with Variation in Company-diversification

Standard Normal Difference LOB RRF by Diversification Band vs. LOB RRF for all Div Bands						
	Diversification Band					
LOB	**0**	**1**	**2**	**3**	**4**	**5**
A	2.0	1.0	0.5	-0.6	-0.7	-0.5
B	2.0	-0.5	-0.3	-0.6	0.8	-0.8
C	2.3	-0.1	0.2	1.3	0.5	-0.8
D	-0.2	-1.8	0.2	1.1	1.2	-0.4
E	0.7	1.4	2.3	0.1	0.0	-0.7
F1	-0.9	0.2	0.2	-0.4	0.4	2.3
F2	-0.6	0.0	0.5	1.4	0.2	2.4
G	-0.4	-2.2	-0.5	0.3	-0.9	1.0
H	-1.5	2.0	0.2	0.3	0.2	0.1
I	-1.5	1.8	0.7	0.0	-0.1	-0.2
J	-1.9	-0.3	-0.3	-1.3	0.5	1.0
K	-0.7	-0.5	0.6	2.3	0.3	-0.2
L	-0.8	-0.1	1.0	2.3	-0.3	0.3
N&P	-2.3	-0.1	-0.2	0.2	0.8	0.5
O	0.1	-2.1	-0.6	-0.5	0.3	1.1
R	-0.9	1.8	-1.2	0.5	-0.6	-0.2
Average	-1.2	1.2	1.0	1.4	2.0	0.5
Avg Absolute value	1.2	1.0	0.6	0.8	0.5	0.8

[65] For each LOB, we calculate the PRF/RRF for each diversification level, minus the PRF/RRF for all diversification levels combined, divided the standard deviation across diversification levels for the LOB.

Appendix 2/Exhibit 2B
Indicated PRFs - Standardized Variation in LOB Risk Charge% with Variation in Company-diversification

Standard Normal Difference LOB PRF by Diversification Band vs. LOB RRF for all Div Bands						
	Diversification Band					
LOB	0	1	2	3	4	5
A	1.7	-1.2	-1.3	0.3	0.2	0.2
B	2.1	-0.9	-0.9	0.6	0.2	-0.1
C	-1.0	0.0	-0.6	1.1	1.5	-1.0
D	0.0	-1.1	1.2	1.1	1.4	-0.8
E	-0.2	2.2	-1.1	0.0	-0.3	0.0
F1	-1.0	0.4	-1.0	-0.1	-0.7	-2.8
F2	-0.9	0.5	0.9	1.3	2.5	1.1
G	0.6	-2.0	-0.4	1.2	-0.4	-0.1
H	0.2	-0.1	-1.9	1.3	0.6	-0.4
I	0.0	-0.4	-2.0	-0.2	-0.9	1.3
J	-0.5	-2.2	0.5	0.9	0.0	-0.4
K	-1.7	0.3	1.0	0.8	1.5	0.4
L	-1.5	0.2	-1.5	-0.1	0.2	1.3
N&P	-1.5	-1.3	0.8	-1.5	0.7	-0.4
O	-2.0	1.1	-0.7	0.2	-0.9	0.1
R	2.4	1.2	0.7	-0.4	-0.1	-0.2
Total	0.5	-1.4	-0.9	1.4	0.9	-0.4
Avg Absolute value	1.1	1.0	1.0	0.7	0.8	0.7

Appendix 2/Exhibit 3, below, shows the premium/reserve weighted averages of the absolute values of the standardized differences between each level of diversification and the all-diversification risk charges. At diversification band 0, the PRFs/RRFs, on average, are 1.1 or 1.2 standard deviations, respectively, either above or below the mean. At diversification band 5 the PRFs/RRFs are closer to the mean, 0.7 or 0.8 standard deviations, respectively. Thus, there appears to be trends towards different LOB risk charge%s in companies with different levels of diversification.

The patterns in Appendix 2/Exhibit 3 might be the result of random effects, of course. Nonetheless, the data contributing to that pattern contribute to the observations that the indicated diversification credit does not increase smoothly with higher diversification, particularly at the lower levels of diversification (bands 0-2)

Appendix 2/Exhibit 3
Variation in Indicated LOB Risk Charge% with Variation in Company-diversification

Standardized Normal Difference Average of Absolute Values		
Diversification Band	Premium	Reserves
0	1.1	1.2
1	1.0	1.0
2	1.0	0.6
3	0.7	0.8
4	0.8	0.5
5	0.7	0.8

.

APPENDIX 3- CONSTRUCTION OF CORRELATION MATRIX FOR DIVERSIFICATION TESTING

To apply the correlation approach, we construct a set of pairwise correlation factors, called a correlation matrix. In Solvency II correlation matrix, the factors were not calibrated from analysis of data. Rather, the factors represent an expert judgment on whether the LOB pairwise correlation is lower (0.25) or higher (0.50).

In the Solvency II 4th Quantitative Impact Analysis (QIS4) analysis, the factors were sensitivity tested with additional analysis assuming a minus or plus 25 percentage points adjustment to each "non-diagonal" value. These changes resulted in capital requirements that were 25% lower and 21% higher (respectively) than the proposed QIS4 factors. After this sensitivity analysis was completed, the selected factors were maintained at the QIS3 level, *"translating the broad support there is around these parameters and the lack of more evidence for changing the correlations".*[66] Thus, the overall level represents an expert judgment much like the 30% MDC in the RBC Formula.

Appendix 3/Exhibit 1 shows the Solvency II correlation matrix for the 12 Solvency non-life LOBs.[67] Appendix 3/Exhibit 2 provides the LOB definitions.

Following the Solvency II approach,[68] we construct the correlation matrix using values of 25% or 50% for most of the 171 LOB-pairs. For a few LOB-pairs that we consider very highly correlated we select correlation factors of 75% or 100%.[69] Appendix 3/Exhibit 2 shows the correlation matrix that we use to test the diversification relationship.

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66 "CEIOPS-DOC-70/10" (Page 44, paragraph B.31)

67 (See next line) http://www.lloyds.com/~/media/files/the%20market/operating%20at%20lloyds/solvency%20ii/2016%20guidance/2015_yesf_synd_v62.xlsx., "Non-Life and Health UW Section," Tab "Premium and Reserve Risk Params"

68 "Advice for Band 2 Implementing Measures on Solvency II: SCR Standard Formula Article 111(d) Correlations," (former Consultation Paper 74), January 2010, pp 39-44.

69 We select pairwise correlations of 100% for claims made and occurrence medical malpractice and for general liability, special liability and products liability. We select pairwise correlations of 75% between special property and homeowners, between private passenger automobile liability and automobile physical damage and between commercial automobile liability and automobile physical damage.

Appendix 3/Exhibit 1
Solvency II Standard Formula Correlation Matrix for Premium and Reserves

LOB/LOB	1	2	3	4	5	6	7	8	9	10	11	12
1	100%	50%	50%	25%	50%	25%	50%	25%	50%	25%	25%	25%
2	50%	100%	25%	25%	25%	25%	50%	50%	50%	25%	25%	25%
3	50%	25%	100%	25%	25%	25%	25%	50%	50%	25%	50%	25%
4	25%	25%	25%	100%	25%	25%	25%	50%	50%	25%	50%	50%
5	50%	25%	25%	25%	100%	50%	50%	25%	50%	50%	25%	25%
6	25%	25%	25%	25%	50%	100%	50%	25%	50%	50%	25%	25%
7	50%	50%	25%	25%	50%	50%	100%	25%	50%	50%	25%	25%
8	25%	50%	50%	50%	25%	25%	25%	100%	50%	25%	25%	50%
9	50%	50%	50%	50%	50%	50%	50%	50%	100%	25%	50%	25%
10	25%	25%	25%	25%	50%	50%	50%	25%	25%	100%	25%	25%
11	25%	25%	50%	50%	25%	25%	25%	25%	50%	25%	100%	25%
12	25%	25%	25%	50%	25%	25%	25%	50%	25%	25%	25%	100%

Solvency II LOBs[70]

1	Motor vehicle liability	7	Legal expenses
2	Other motor	8	Assistance
3	Marine, aviation and transport	9	Miscellaneous financial loss
4	Fire and other damage to property	10	NP casualty reinsurance
5	General liability	11	NP marine, aviation and transport reinsurance
6	Credit and suretyship	12	NP property reinsurance

Direct LOBs include proportional reinsurance of the same type.
NP = Non-proportional

70 http://www.lloyds.com/~/media/files/the%20market/operating%20at%20lloyds/solvency%20ii/2016%20guidance/2015_yesf_synd_v62.xlsx. "Non-Life and Health UW Section," Tab "Premium and Reserve Risk Params"

DCWP Report 14: RBC - Calibration of LOB Diversification in UW Risk Charges

Appendix 3/Exhibit 2

Selected DCWP Correlation Matrix – Applied by the DCWP to US NAIC LOBs for this Study

LOB	HO	PPA	CA	WC	CMP	M-Occ	M-CM	SL	OL	SP	Ohy	Fid	Other	Intl	Re Prop	Re Liab	Prod	FG	Warrnty
HO	100%	25%	25%	25%	50%	25%	25%	25%	25%	**75%**	50%	25%	25%	25%	25%	25%	25%	25%	25%
PPA	25%	100%	50%	25%	25%	25%	25%	25%	25%	25%	**75%**	25%	25%	25%	25%	25%	25%	25%	25%
CA	25%	50%	100%	50%	50%	25%	25%	50%	50%	25%	**75%**	25%	25%	25%	25%	25%	50%	25%	25%
WC	25%	25%	50%	100%	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%
CMP	50%	25%	50%	25%	100%	25%	25%	50%	50%	50%	25%	25%	25%	25%	25%	25%	50%	25%	25%
M-Occ	25%	25%	25%	25%	25%	100%	**100%**	50%	50%	25%	25%	25%	25%	25%	25%	25%	50%	25%	25%
M-CM	25%	25%	25%	25%	25%	**100%**	100%	50%	50%	25%	25%	25%	25%	25%	25%	25%	50%	25%	25%
SL	25%	25%	50%	25%	50%	50%	50%	100%	**75%**	25%	25%	25%	25%	25%	25%	50%	**100%**	25%	25%
OL	25%	25%	50%	25%	50%	50%	50%	**75%**	100%	25%	50%	50%	25%	50%	25%	50%	**100%**	25%	25%
SP	**75%**	25%	25%	25%	50%	25%	25%	25%	25%	100%	25%	25%	25%	25%	50%	25%	25%	25%	25%
Phy	50%	**75%**	**75%**	25%	25%	25%	25%	25%	50%	25%	100%	25%	25%	25%	25%	25%	25%	25%	25%
Fid	25%	25%	25%	25%	25%	25%	25%	25%	50%	25%	25%	100%	25%	25%	25%	50%	25%	25%	25%
Other	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	100%	25%	25%	25%	25%	25%	25%
Intl	25%	25%	25%	25%	25%	25%	25%	25%	50%	25%	25%	25%	25%	100%	25%	25%	25%	25%	25%
Re Prop	25%	25%	25%	25%	25%	25%	25%	25%	25%	50%	25%	25%	25%	25%	100%	25%	25%	25%	25%
Re Liab	25%	25%	25%	25%	25%	25%	25%	50%	50%	25%	25%	50%	25%	25%	25%	100%	50%	25%	25%
Prod	25%	25%	50%	25%	50%	50%	50%	**100%**	**100%**	25%	25%	25%	25%	25%	25%	50%	100%	25%	25%
FG	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	100%	25%
Warrnty	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	25%	100%

Note: Off diagonal values other than 25%, 50% are in bold.

LOB Definitions

LOB	Abbreviation	LOB	Abbreviation	LOB	Abbreviation
Homeowners/Farmowners	HO	Special Liab	SL	International	Intl
Priv. Passenger Auto	PPA	Other Liab-Occ and CM	OL	Reinsurance-Fin and Prop	Re Prop
Commercial Auto	CA	Spec Property	SP	Reinsurance-Liab	Re Liab
Workers Compensation	WC	Auto Physical Damage	Phy	Products Liability-Occ and CM	Prod
Commercial Multi-peril	CMP	Fidelity & Surety	Fid	Financial/Mort Guarantee	FG
Medical Prof Liab - Occ	P-Occ	Other	Other	Warranty	Warrnty
Medical Prof Liab - CM	M-CM				

APPENDIX 4 - DIVERSIFICATION BASED ON CORRELATION MATRIX APPROACH

In Appendix 4/Exhibits 1 and 2, we compare how well diversification formulas for CoMaxLine% and correlation matrix approach fit the experience by company size and diversification level, for reserves and premium respectively.

Part 1 of these exhibits shows the expected risk charge%s, before diversification. These are the unweighted averages of the expected risk charge%s, for each company-year in the size/diversification bands, before application of diversification. For the CoMaxLine% section the values are the same as the values in Table 3-16. For the correlation matrix approach, the values are very similar to the values in Table 3-16. This should be the case, as the values are calculated before any diversification effect. Therefore, the values differ only to the extent that the diversification band under CoMaxLine% Approach is different from the diversification band under the correlation matrix approach.

Part 2 of these exhibits shows the indicated risk charge%s. These values are the 87.5th percentile RRR and the 87.5th percentile AYUL% for all company-years in the size/diversification cell. For the CoMaxLine% column, the values are the same as the values in Table 3-15. For the correlation matrix approach, the values are very similar to the values in Table 3-15. This is the case because the values differ only to the extent that the diversification band under CoMaxLine% Approach is different from the diversification band under the correlation matrix approach.

Part 3 of these exhibits shows the current average diversification credit.

Using Parts 1, 2 and 3, we calculate the factor that, when applied to the current average diversification credit, minimizes the difference between actual experience (Part 2) and expected experience [Part 1*(1-Part 3)] for company size/diversification bands C3.E5. We determine that factor through an iterative process. We manually "goal seek' to produce the adjustment to the Part 3 diversification credit that minimizes the sum of the differences between (a) Part 2 values and (b) the values of [Part 1*(1-Part 3) * test adjustment to the average diversification credit], for the cells in section C3.E5. In the first line below Part 2, we show the increase/decrease in diversification credit that is necessary to achieve the target diversification credit, e.g., +120% for CoMaxLine%, or an MDC of 66%, (1+1.2) x 30%.[71,72]

Part 4 equals Part 1 times the adjusted average diversification credit.

Part 5 shows the differences between indicated risk charge%s (Part 2) and expected risk charge%s at the target diversification level (Part 4).

[71] For the correlation matrix approach, the percentage is the effect that would need to be achieved by changes in pairwise correlation values.
[72] Immediately below that value, we show the remaining difference between Part 2 values and Part 5. Part 5 values are the differences between indicated and formula risk charge%s after applying adjustment factor.

Appendix 4/Exhibit 1 – Reserves
Diversification Analysis by LOB-size/Diversification (5x6 analysis)
Calculation of Normalized Variability with Array by Method

CoMaxLine%/Single Factor Risk Charge

Diversif. Band Quintiles	Expected Risk - No diversification Credit-Part 1				
	Size Band (Quinitiles)				
	A	B	C	D	E
0	34%	34%	33%	31%	31%
1	28%	28%	31%	33%	29%
2	31%	31%	31%	32%	31%
3	32%	32%	34%	33%	38%
4	34%	33%	37%	37%	40%
5	36%	36%	37%	36%	38%
All Ex 0	31%	32%	34%	34%	36%

Diversif. Band Quintiles	Indicated Risk - Part 2				
	Size Band (Quinitiles)				
	A	B	C	D	E
0	63%	38%	25%	21%	18%
1	53%	34%	27%	30%	15%
2	54%	35%	30%	29%	17%
3	75%	39%	27%	22%	25%
4	45%	36%	32%	22%	29%
5	37%	30%	24%	24%	26%
All Ex 0	55%	35%	28%	25%	24%

Calibration to Target Diversification Level 120.0%

0.004

Diversif. Band Quintiles	Current Average Diversification- Part 3				
	Size Band (Quinitiles)				
	A	B	C	D	E
0	0%	0%	0%	0%	0%
1	2%	2%	2%	2%	2%
2	5%	5%	5%	5%	5%
3	9%	9%	9%	10%	9%
4	13%	13%	13%	13%	13%
5	17%	17%	18%	18%	19%
All Ex 0	8%	8%	9%	10%	11%

Diversif. Band Quintiles	Expected Risk With Target Div Level- Part 4				
	Size Band (Quinitiles)				
	A	B	C	D	E
0	34%	34%	33%	31%	31%
1	27%	27%	30%	32%	28%
2	27%	27%	28%	28%	27%
3	25%	26%	27%	26%	30%
4	24%	23%	26%	26%	28%
5	22%	22%	22%	22%	23%
All Ex 0	26%	26%	27%	27%	27%

Diversif. Band Quintiles	Actual vs. Expected - Part 5				
	Size Band (Quinitiles)				
	A	B	C	D	E
0	29%	4%	-8%	-10%	-13%
1	27%	6%	-3%	-2%	-13%
2	27%	8%	2%	0%	-10%
3	50%	14%	0%	-4%	-5%
4	21%	13%	6%	-3%	0%
5	14%	8%	2%	2%	3%
All Ex 0	29%	9%	1%	-2%	-3%

Correlation/Single Factor Risk Charge

Diversif. Band Quintiles	Expected Risk - No diversification Credit-Part 1				
	Size Band (Quinitiles)				
	A	B	C	D	E
0	34%	34%	33%	30%	29%
1	29%	31%	34%	39%	39%
2	32%	33%	35%	36%	36%
3	32%	33%	35%	35%	38%
4	32%	31%	34%	35%	38%
5	34%	30%	32%	32%	36%
All Ex 0	31%	32%	34%	35%	37%

Diversif. Band Quintiles	Indicated Risk - Part 2				
	Size Band (Quinitiles)				
	A	B	C	D	E
0	63%	39%	26%	22%	16%
1	50%	34%	28%	31%	24%
2	65%	39%	36%	32%	22%
3	62%	30%	26%	29%	30%
4	35%	34%	25%	22%	26%
5	38%	30%	24%	16%	23%
All Ex 0	54%	35%	28%	25%	25%

Calibration to Target Diversification Level 50.0%

0.004

Diversif. Band Quintiles	Current Average Diversification- Part 3				
	Size Band (Quinitiles)				
	A	B	C	D	E
0	0%	0%	0%	0%	0%
1	3%	2%	2%	2%	2%
2	8%	8%	8%	8%	8%
3	14%	14%	14%	14%	14%
4	20%	20%	20%	19%	20%
5	25%	25%	25%	26%	26%
All Ex 0	10%	11%	13%	15%	17%

Diversif. Band Quintiles	Expected Risk With Target Div Level- Part 4				
	Size Band (Quinitiles)				
	A	B	C	D	E
0	34%	34%	33%	30%	29%
1	28%	30%	33%	38%	38%
2	28%	29%	31%	31%	31%
3	26%	26%	27%	27%	30%
4	23%	22%	24%	25%	27%
5	21%	19%	20%	19%	22%
All Ex 0	26%	26%	27%	27%	28%

Diversif. Band Quintiles	Actual vs. Expected - Part 5				
	Size Band (Quinitiles)				
	A	B	C	D	E
0	29%	6%	-7%	-9%	-13%
1	23%	4%	-4%	-7%	-13%
2	37%	10%	6%	0%	-9%
3	36%	4%	-1%	2%	0%
4	12%	12%	1%	-3%	-1%
5	16%	11%	4%	-4%	1%
All Ex 0	27%	8%	0%	-2%	-3%

Appendix 4/Exhibit 2 – Premium
Diversification Analysis by LOB-size/Diversification (5x6 analysis)
Calculation of Normalized Variability with Array by Method

CoMaxLine%/Single Factor Risk Charge

Diversif. Band Quintiles	Expected Risk - No diversification Credit-Part 1 Size Band (Quinitiles)				
	A	B	C	D	E
0	28%	29%	29%	31%	30%
1	26%	28%	28%	32%	31%
2	26%	25%	24%	24%	26%
3	23%	24%	23%	24%	25%
4	23%	24%	23%	24%	24%
5	24%	24%	23%	23%	24%
All Ex 0	25%	25%	24%	25%	25%

Diversif. Band Quintiles	Indicated Risk Charge - Part 2 Size Band (Quinitiles)				
	A	B	C	D	E
0	56%	29%	26%	27%	37%
1	45%	21%	25%	22%	39%
2	42%	19%	15%	16%	15%
3	44%	21%	17%	18%	17%
4	33%	14%	18%	18%	16%
5	56%	22%	15%	16%	15%
All Ex 0	44%	19%	18%	18%	17%

Calibration to Target Diversification Level 75.0%
(0.004)

Diversif. Band Quintiles	Current Average Diversification-Part 3 Size Band (Quinitiles)				
	A	B	C	D	E
0	0%	0%	0%	0%	0%
1	4%	4%	4%	4%	4%
2	9%	9%	9%	10%	10%
3	13%	13%	13%	13%	13%
4	16%	16%	16%	17%	17%
5	20%	20%	20%	21%	21%
All Ex 0	10%	11%	12%	13%	16%

Diversif. Band Quintiles	Modeled Risk With Target Div Level - Part 4 Size Band (Quinitiles)				
	A	B	C	D	E
0	28%	29%	29%	31%	30%
1	24%	25%	25%	29%	29%
2	22%	21%	20%	20%	22%
3	18%	19%	18%	18%	19%
4	16%	17%	17%	17%	17%
5	16%	15%	15%	15%	15%
All Ex 0	21%	20%	19%	19%	18%

Diversif. Band Quintiles	Actual vs. Expected - Part 5 Size Band (Quinitiles)				
	A	B	C	D	E
0	28%	0%	-3%	-4%	7%
1	20%	-5%	0%	-7%	9%
2	21%	-2%	-5%	-4%	-7%
3	26%	2%	-1%	0%	-3%
4	17%	-3%	2%	1%	-1%
5	40%	7%	1%	2%	0%
All Ex 0	23%	-1%	-1%	-2%	-2%

Correlation/Single Factor Risk Charge

Diversif. Band Quintiles	Expected Risk - No diversification Credit-Part 1 Size Band (Quinitiles)				
	A	B	C	D	E
0	28%	30%	30%	34%	35%
1	23%	23%	22%	23%	21%
2	24%	24%	24%	25%	25%
3	25%	25%	23%	24%	24%
4	25%	25%	23%	23%	24%
5	23%	24%	24%	24%	25%
All Ex 0	24%	24%	23%	24%	24%

Diversif. Band Quintiles	Indicated Risk Charge - Part 2 Size Band (Quinitiles)				
	A	B	C	D	E
0	57%	30%	28%	29%	46%
1	62%	17%	18%	17%	13%
2	35%	18%	18%	15%	18%
3	33%	18%	18%	18%	14%
4	51%	18%	15%	17%	16%
5	48%	25%	18%	17%	15%
All Ex 0	43%	19%	17%	17%	15%

Calibration to Target Diversification Level 45.0%
0.004

Diversif. Band Quintiles	Current Average Diversification-Part 3 Size Band (Quinitiles)				
	A	B	C	D	E
0	0%	0%	0%	0%	0%
1	3%	3%	3%	3%	3%
2	9%	9%	9%	9%	9%
3	15%	15%	16%	16%	16%
4	21%	21%	21%	22%	22%
5	28%	27%	28%	28%	29%
All Ex 0	10%	13%	14%	17%	21%

Diversif. Band Quintiles	Modeled Risk With Target Div Level - Part 4 Size Band (Quinitiles)				
	A	B	C	D	E
0	28%	30%	30%	34%	35%
1	22%	22%	21%	22%	20%
2	21%	21%	21%	21%	21%
3	20%	19%	18%	19%	18%
4	18%	17%	16%	16%	16%
5	14%	14%	14%	14%	15%
All Ex 0	20%	20%	19%	18%	17%

Diversif. Band Quintiles	Actual vs. Expected - Part 5 Size Band (Quinitiles)				
	A	B	C	D	E
0	29%	0%	-3%	-5%	11%
1	40%	-5%	-4%	-5%	-6%
2	14%	-2%	-3%	-6%	-3%
3	13%	-2%	-1%	0%	-4%
4	33%	1%	-1%	1%	-1%
5	34%	11%	4%	2%	0%
All Ex 0	23%	-1%	-1%	-1%	-2%

APPENDIX 5- DIVERSIFICATION ANALYSIS – RESULTS USING RISK FACTORS BY LOB-SIZE

In this section, we address the extent to which our findings regarding diversification with CoMaxLine% Approach would be affected if the RBC Formula used risk factors that vary by LOB-size.

This question is motivated, in part, because we observe that LOB-size, company-size and diversification level are inter-related. For example, we observe that larger LOB-sizes indicate risk charge%s that are lower than the risk charges%s indicated by smaller LOB-sizes. Therefore, it could be the case higher indicated diversification credits are a proxy for lower LOB risk charge%s for larger companies.

To analyze that question, we first use the risk data to construct reserve and premium risk factors that vary by LOB-size.[73] Appendix 5/Exhibit 1, below, shows those risk factors.

[73] We develop these risk factors by LOB-size using our calibration approach, described in DCWP Reports 6 and 7, applied separately to each LOB-size band. For this purpose, we measure "LOB-size" for each company/LOB/year as the percentile of reserve/premium amount relative to reserve/premium for all Company/LOBs in that year.

Appendix 5/Exhibit 1
Indicated PRC% and RRC% by LOB-size

Premium Risk Charge = PRF + CER - 100% by LOB-Size										
Line of Business	0%-15%	15%-25%	25%-35%	35%-45%	45%-55%	55%-65%	65%-75%	75%-85%	85%-95%	95%-100%
A- Homeowners/Farmowner	58.8%	32.4%	28.6%	26.5%	24.1%	21.5%	25.9%	24.0%	22.9%	24.8%
B- Private Passenger Auto	49.6%	27.1%	25.5%	26.5%	22.3%	22.3%	21.4%	21.2%	17.1%	14.7%
C- Commercial Auto	56.9%	37.9%	31.7%	30.3%	29.7%	28.1%	29.7%	28.1%	25.2%	24.6%
D - Workers Compensation	58.3%	49.0%	37.2%	34.8%	28.8%	24.5%	22.2%	22.4%	28.5%	37.9%
E - Commercial Multi-Peril	44.8%	23.1%	23.1%	23.7%	25.4%	24.2%	22.2%	21.0%	23.5%	25.5%
F1 - Med Prof Liab-Occ	171.5%	84.1%	54.5%	72.1%	54.1%	71.1%	97.6%	71.0%	66.0%	71.7%
F2 - Med Prof Liab-CM	104.0%	28.5%	43.5%	34.2%	31.7%	45.6%	52.1%	37.9%	49.5%	45.6%
G - Special Liability	57.6%	45.8%	28.9%	34.5%	38.5%	21.4%	30.9%	28.7%	19.2%	4.4%
H - Other Liability	68.6%	32.7%	38.2%	37.6%	32.8%	31.7%	28.8%	32.6%	26.5%	28.4%
I - Special Property	32.6%	9.5%	9.6%	12.5%	9.9%	15.4%	14.6%	18.4%	16.1%	18.2%
J - Auto Physical Damage	29.1%	13.1%	9.7%	9.4%	8.7%	7.2%	10.0%	6.6%	4.4%	4.2%
K - Fidelity/Surety	43.1%	13.7%	8.8%	21.1%	11.9%	1.3%	7.6%	1.0%	10.2%	1.0%
L - Other	44.9%	27.0%	23.6%	19.1%	27.6%	31.0%	29.6%	15.3%	33.3%	27.1%
M - International	46.8%	25.1%	25.1%	25.7%	27.4%	26.2%	24.2%	23.0%	25.5%	27.5%
N&P - Reinsurance-Prop/Fin	109.6%	53.0%	85.1%	55.3%	40.0%	65.9%	43.8%	47.0%	32.7%	26.4%
O - Reinsurance-Liabiity	95.7%	68.4%	42.2%	53.5%	51.9%	58.3%	54.1%	42.1%	61.8%	28.6%
R - Products Liability	72.7%	14.1%	80.8%	23.9%	81.8%	46.9%	131.9%	45.5%	39.8%	34.4%
S - Financial/Mort Guarante	43.1%	13.7%	8.8%	21.1%	11.9%	1.3%	7.6%	1.0%	10.2%	1.0%
T - Warranty	44.8%	23.1%	23.1%	23.7%	25.4%	24.2%	22.2%	21.0%	23.5%	25.5%

Reserve Risk Charge = RRF by LOB-Size										
Line of Business	0%-15%	15%-25%	25%-35%	35%-45%	45%-55%	55%-65%	65%-75%	75%-85%	85%-95%	95%-100%
A- Homeowners/Farmowner	83.3%	41.1%	33.6%	28.8%	27.7%	27.5%	14.1%	8.3%	12.2%	10.4%
B- Private Passenger Auto	79.4%	41.0%	31.3%	26.0%	19.3%	13.5%	15.7%	8.7%	5.2%	8.0%
C- Commercial Auto	126.5%	69.8%	44.9%	39.4%	35.3%	32.4%	26.0%	34.0%	23.1%	13.1%
D - Workers Compensation	69.5%	36.4%	49.1%	41.7%	44.6%	29.1%	30.7%	24.0%	22.8%	27.3%
E - Commercial Multi-Peril	134.9%	76.4%	57.1%	52.4%	58.1%	54.2%	41.1%	32.9%	41.2%	31.5%
F1 - Med Prof Liab-Occ	195.2%	67.8%	32.8%	31.4%	17.4%	58.4%	40.2%	12.2%	7.6%	7.1%
F2 - Med Prof Liab-CM	67.2%	20.1%	21.0%	14.8%	12.6%	12.0%	10.6%	1.0%	1.0%	1.0%
G - Special Liability	172.6%	18.4%	78.9%	119.6%	39.4%	36.0%	35.3%	31.8%	29.5%	6.0%
H - Other Liability	155.8%	81.0%	61.5%	44.8%	37.6%	35.4%	36.6%	55.2%	71.3%	67.2%
I - Special Property	120.0%	45.3%	35.2%	29.1%	27.0%	25.9%	26.1%	34.1%	36.4%	43.4%
J - Auto Physical Damage	62.8%	44.6%	19.4%	15.8%	27.1%	15.0%	9.4%	10.3%	24.9%	9.2%
K - Fidelity/Surety	188.9%	43.7%	103.7%	71.4%	127.3%	112.4%	33.5%	42.4%	26.2%	30.8%
L - Other	118.6%	38.7%	37.9%	12.9%	19.1%	11.9%	22.7%	91.3%	19.1%	27.9%
M - International	136.9%	78.4%	59.1%	54.4%	60.1%	56.2%	43.1%	34.9%	43.2%	33.5%
N&P - Reinsurance-Prop/Fin	74.1%	39.7%	51.3%	34.5%	72.4%	53.1%	40.0%	42.4%	31.3%	6.5%
O - Reinsurance-Liabiity	114.1%	55.2%	78.7%	58.3%	94.0%	43.8%	46.4%	68.8%	66.4%	104.2%
R - Products Liability	138.9%	68.7%	73.0%	137.1%	70.0%	28.2%	180.3%	74.6%	22.8%	1.0%
S - Financial/Mort Guarante	188.9%	43.7%	103.7%	71.4%	127.3%	112.4%	33.5%	42.4%	26.2%	30.8%
T - Warranty	134.9%	76.4%	57.1%	52.4%	58.1%	54.2%	41.1%	32.9%	41.2%	31.5%

Minimum of 1% PRC% and PRF% applied as needed

2x2 Analysis – Risk Factors by LOB-size

Table 3-5 shows the indicated MDC based on all multiline companies and all company sizes larger than the smallest 20%. We found that the indicated MDC was 62% and 65% for reserve risk and premium risk respectively. Appendix 5/Exhibit 2, below, shows that if the RBC Formula used LOB risk factors based on LOB-size, the indicated MDC would be higher, 76% and 85% for reserves and premium, respectively (column C/line 5).

Appendix 5/Exhibit 2
Indicated MDC – 2x2 Analysis

	Reserves	Single RRF	RRF by LOB-size
	(A)	(B)	(C)
#	Item	Premium	Premium
1	Observed Risk - 87.5th RRR/AYUL	27.2%	27.2%
2	Expected Risk - 87.5th RRR/AYUL before diversification	34.2%	36.2%
3	Indicated Diversification Credit - 100%-(1)/(2)%	20.6%	24.9%
4	Current Average Diversification Credit	9.9%	9.9%
5	Indicated Maximum Credit (3)/(4) * 30%	62.5%	75.7%

	Premium	Single PRF	PRF by LOB-size
	(A)	(B)	(C)
#	Item	Reserves	Reserves
1	Observed Risk - 87.5th RRR/AYUL	17.8%	17.8%
2	Expected Risk - 87.5th RRR/AYUL before diversification AYULedit	25.0%	28.7%
3	Indicated Diversification Credit - 100%-(1)/(2)%	28.8%	37.8%
4	Current Average Diversification Credit	13.3%	13.3%
5	Indicated Maximum Credit (3)/(4) * 30%	64.9%	85.4%

The column "Single PRF/RRF" is the same as Table3-5

Notes:
The values in column B are the same as the values in Table 3-5.
Row 1– Observed Risk – This is based on LRs and RRRs and is not affected by the expected risk calculation. Hence columns B and C have the same values.
Row 2 – Expected risk calculated using the single risk factor or risk factor by LOB-size, hence columns B and C are not the same.
Row 3 – Calculated as shown.
Row 4 – Current average diversification credit. It is not affected by the risk factors; hence column B and C are the same values.
Row 5 – Calculated as shown.

5x6 Analysis – Risk Factors by LOB-size

Table 3-19, in which risk factors by LOB do not vary by LOB-size, shows that the indicated MDC is generally greater than 50% for both reserve risk and premium risk, for company size/diversification bands C3 through E5. We repeat Table 3-19 below, labeled Appendix 5/Exhibit 3.

Appendix 5/Exhibit 4, below shows the corresponding indicated MDC values when the LOB-risk factors vary by LOB-size. Table 3-19 shows unexpected negative indicated MDC values for the company size bands A and B, the smallest sizes. These negative values do not appear in Appendix 5/Exhibit 4, where the LOB risk factors vary by LOB-size. The observation that the negative indicated risk factors are eliminated is evidence that the negative values in Table 3-19 are due to the variation in LOB-risk factors by IOB-size.

Looking at the indicated MDC in each of yellow/bold cells, in Appendix 5/Exhibit 4, we see that values often exceed 50%, and average over 50%.

Appendix 5/Exhibit 3
Indicated MDC - Single risk factor by LOB for all LOB-sizes
Copy of Table 3-19

Diversif. Band Quintiles	Reserves					Premium				
	Size Band (Quintiles)					Size Band (Quintiles)				
	A	B	C	D	E	A	B	C	D	E
0	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
1	-1524.0%	-310.2%	247.1%	165.0%	731.8%	-513.5%	167.1%	61.4%	218.5%	-159.0%
2	-417.5%	-69.6%	27.3%	59.1%	248.8%	-203.2%	73.4%	115.6%	98.9%	131.1%
3	-431.7%	-71.6%	**61.9%**	**105.7%**	**108.8%**	-208.9%	35.0%	**58.9%**	**54.9%**	**78.1%**
4	-70.3%	-20.9%	**28.7%**	**87.0%**	**64.0%**	-84.6%	78.8%	**39.9%**	**43.7%**	**63.3%**
5	-2.7%	24.8%	**57.2%**	**58.0%**	**54.3%**	-200.3%	9.6%	**48.5%**	**42.8%**	**52.9%**
All Ex 0	-291.9%	-41.5%	57.7%	80.3%	91.0%	-236.8%	62.9%	64.3%	66.3%	63.9%

Appendix 5/Exhibit 4
Indicated MDC - LOB-risk factors by LOB-size

Diversif. Band Quintiles	Reserves					Premium				
	Size Band (Quintiles)					Size Band (Quintiles)				
	A	B	C	D	E	A	B	C	D	E
0	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
1	449.6%	383.1%	296.2%	4.2%	475.6%	139.4%	295.0%	93.8%	234.6%	-203.4%
2	172.3%	147.9%	55.1%	5.4%	165.1%	77.1%	159.3%	138.5%	102.2%	121.1%
3	39.6%	69.5%	**87.5%**	**80.0%**	**81.2%**	31.0%	112.0%	**82.2%**	**59.3%**	**69.1%**
4	111.2%	66.3%	**50.0%**	**79.3%**	**46.5%**	60.8%	123.9%	**69.1%**	**47.4%**	**55.1%**
5	109.3%	87.9%	**75.6%**	**52.1%**	**37.6%**	-15.0%	75.3%	**75.9%**	**48.7%**	**47.8%**
All Ex 0	129.2%	104.2%	83.0%	62.0%	60.3%	53.2%	137.6%	91.8%	71.5%	56.4%

Appendix 5/Exhibit 5 below compares the error statistics for CoMaxLine% approach and correlation matrix approach with risk factors that vary (by LOB-size) and risk factors that are the same for all LOB-sizes (as in RBC Formula).

Appendix 5/Exhibit 5
Error Statistics - Diversification Models/Size Bands
Error Measured as % of Reserves/Premium
[Green Highlight indicates the lower value within each pair of models]
Standard Deviations – Part A

Points Included	Reserves			
	NAIC	Correlation	NAIC	Correlation
	Single LOB Risk Factor		LOB Risk Factor Varies	
All Points (25 points)	0.13	0.11	0.08	0.12
Exclude Smallest (20 points)	0.07	0.06	0.04	0.05
Include only Largest (9 points)	0.03	0.02	0.029	0.032

Points Included	Premium			
	NAIC	Correlation	NAIC	Correlation
	Single LOB Risk Factor		LOB Risk Factor Varies	
All Points (25 points)	0.11	0.12	0.09	0.08
Exclude Smallest (20 points)	0.040	0.038	0.07	0.05
Include only Largest (9 points)	0.01	0.02	0.021	0.022

Average Error - Part B

Points Included	Reserves			
	NAIC	Correlation	NAIC	Correlation
	Single LOB Risk Factor		LOB Risk Factor Varies	
All Points (25 points)	6.5%	4.7%	-4.3%	-3.5%
Exclude Smallest (20 points)	1.2%	0.7%	-1.8%	-2.3%
Include only Largest (9 points)	0%	0%	0%	0%

Points Included	Premium			
	NAIC	Correlation	NAIC	Correlation
	Single LOB Risk Factor		LOB Risk Factor Varies	
All Points (25 points)	4.4%	4.3%	-0.2%	-2.2%
Exclude Smallest (20 points)	-0.7%	-1.2%	-1.7%	-4.0%
Include only Largest (9 points)	0%	0%	0%	0%

Average Absolute Error - Part C

Points Included	Reserves			
	NAIC	Correlation	NAIC	Correlation
	Single LOB Risk Factor		LOB Risk Factor Varies	
All Points (25 points)	9.7%	8.0%	6.2%	8.0%
Exclude Smallest (20 points)	5.3%	4.9%	3.5%	4.2%
Include only Largest (9 points)	2.9%	1.9%	2.7%	2.9%

Points Included	Premium			
	NAIC	Correlation	NAIC	Correlation
	Single LOB Risk Factor		LOB Risk Factor Varies	
All Points (25 points)	7.4%	7.7%	5.8%	6.0%
Exclude Smallest (20 points)	3.0%	3.1%	5.1%	4.9%
Include only Largest (9 points)	1.1%	1.5%	1.8%	2.0%

The type of information in Appendix 5/Exhibit 5 is the same as Table 4-1. The values in the columns labeled "single risk factor" are the same as the values in Table 4-1.

For risk factors that vary by LOB-size, the CoMaxLine% approach (labeled NAIC) has lower error statistics in more tests than the correlation matrix approach (7 of 8 tests for reserves and 5 of 8 tests for premium). Hence, evening using risk charges by LOB-size, it does not appear that the correlation matrix fits the data better than CoMaxLine% Approach.

Risk-Based Capital — Calibration of Investment Income Offset

Report 15 of the CAS Risk-Based Capital (RBC) Research Working Parties
Issued by the RBC Dependencies and Calibration Working Party (DCWP)

Abstract: In this paper we describe a method of calibrating the Investment Income Offset element of the RBC Formula. Our key calibration decisions are the following:
1. We select the Present Value Approach rather than the Nominal Value Approach
2. We convert the current combination of interest rate safety margins and UW risk safety targets to an equivalent UW risk safety target with no interest rate safety margin.
In our calibration, for simplicity, we apply a single interest rate approach to all LOBs. In an actual calibration interest rates might vary by LOB, for example, longer duration interest rates for LOBs with longer payment patterns.
This is one of several papers being issued by the Risk-Based Capital (RBC) Dependencies and Calibration Working Party.
Keywords: Risk-Based Capital, Capital Requirements, Analyzing/Quantifying Risks

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1. INTRODUCTON & TERMINOLOGY

The NAIC Property & Casualty RBC Formula ("RBC Formula") has six main risk categories, R_0 – R_5. Underwriting (UW) risk is represented in R_4 and R_5, reserve risk and premium risk, respectively. Appendix 1 describes all the elements of R_4 and R_5.[1]

The purpose of this report is to develop an approach to calibrating the Adjustment for Investment Income ("Investment Income Offset," "IIO," IIO_R for reserves, and IIO_P, for premium) element of R_4 and R_5, in response to changing interest rates.

Terminology and Simplifications

For our analysis we use the terms Premium Risk Factor (PRF), Reserve Risk Factor (RRF), Company UW Expense Ratio (CER), IIO_P and IIO_R which we define in our discussion of below.

Reserve Risk Charge – R_4

For each of the 19 RBC Lines of Business (LOB),[2] reserve risk R_4 is determined using an "Industry Loss and Expense RBC %" in RBC Formula, Form PR016, Line 4, a value

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[1] For a detailed description of the operation of the RBC Formula, Odomirok, et al, Chapter 19, Risk Based Capital https://www.casact.org/library/studynotes/Odomirok-etal_Financial-Reportingv4.pdf
For an older description of the formula and its original basis, see Feldblum, Sholom, NAIC Property/Casualty Insurance Company Risk-Based Capital Requirements, Proceedings of the Casualty Actuarial Society, 1996. For the actual formula, see NAIC, Risk-Based Capital Forecasting & Instructions, Property Casualty, 2010.

[2] The 19 RBC LOBs are the 22 Schedule P LOBs, with occurrence and claims made LOBs for other liability and products liability on a combined basis and non-proportional property and non-proportional financial on a combined basis, leaving a net of 19 LOBs.

applicable to all companies. We refer to this as the Reserve Risk Factor (RRF). The RRF is applied to the loss and loss adjustment expense reserve in the latest annual statement. For each company, we use a simplified version of the Reserve Risk Charge (RRC) by LOB defined as follows:[3,4]

$RRC_{LOB,COMPANY} = \{(\text{Initial Reserve}_{LOB,COMPANY})*[(1.0+RRF_{LOB})* IIO\_R_{LOB} -1.0]\}$

$RRC\%_{LOB,COMPANY} = RRC/ \text{Initial Reserve}_{LOB,COMPANY}$

Premium Risk Charge – R_5

Similarly, for each RBC LOB, premium risk R_5 is determined using an "Industry RBC Loss and Expense Ratio", in RBC Formula, Form PR017, line 4, a value applicable to all companies. We refer to this as the premium risk factor (PRF). The PRF is applied to net written premium (NWP) for the most recent year in the latest annual statement. For each company, we define a simplified version of the Premium Risk Charge (PRC) by LOB as follows:[5]

$PRC_{LOB,COMPANY} = \{\text{Net Earned Premium}_{LOB,COMPANY}* [PRF_{LOB} *IIO\_P_{LOB}+ CER_{LOB,Industry\ Avg} -100\%]\}$

$PRC\%_{LOB,COMPANY} = PRC/(\text{Net Earned Premium}_{LOB,COMPANY})$

Factors in 2010 RBC Formula

Appendix 1/Exhibit 3 shows the PRF, RRF, IIO_P, and IIO_R values, by LOB, in the 2010 RBC Formula.

Investment Income Offsets

The stated purpose of the IIO element of the RBC Formula is to reflect the investment income that can be expected to be earned on assets corresponding to the unpaid claim reserves (reserve risk) and on assets arising from premium collected for new policies (premium risk) over the period that the related premium and reserve obligations are settled. In Statutory Accounting, this investment income is available to offset adverse reserve development or adverse underwriting results.[6] Mathematically, the IIOs are the premium and reserve discount

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[3] As if the own-company adjustment is 1.0, the loss sensitive contract adjustment is zero and there is no growth risk charge.
Also, for our analysis we use the company loss and defense and cost containment expense (DCCE) reserve rather than the reserve for loss and all loss adjustment expense.
We do not include the portion of R_3- Reinsurance Credit Risk that is included in R_4 in the RBC Formula.

[4] These simplifications are the way that the LOB risk factors have been calibrated in American Academy of Actuaries reports dealing with the calibration of UW risk factors.

[5] The simplifications in footnote 3 apply. In addition, for premium risk, the RBC Formula uses the all-lines CER by company. For our analysis, we use the industry average expense by LOB instead. Also, we use net earned premium (NEP) rather than net written premium (NWP).

[6] Feldblum, 1996, page 329 says, *Statutory accounting requires that loss reserves be reported at undiscounted values. The "implicit interest margin," or the difference between the discounted value of the reserves and the undiscounted value of the reserves, serves as an*

factors by LOB.

The effect of the IIO on the total RBC UW Risk Values in the 2010 RBC Formula is significant. In total, across all companies, the effect of the IIO is to reduce the premium risk charge by 34%, to reduce the reserve risk charge by 47%, and to reduce the total RBC UW Risk Values by 44%.[7,8] Thus, calibration of the IIO is significant for the RBC Formula results.

Current Calibration of IIOs

The IIOs in the 2010 RBC Formula were calibrated using claim payment patterns by LOB and an interest rate of 5%. The NAIC periodically adjusts the IIOs for more recent payment pattern data, but it has not changed the 5% interest rates assumption since the RBC Formula was developed in the early-1990s.

Table 1, below, shows interest rates for the period 1962-2013. From a calibration perspective, the notable features of this table include the following:

- The current interest rates are well below the interest rates in the 1990's when the IIO's were calibrated and when the NAIC selected the 5% interest rate, and
- The trend in interest rates since 1981 has been downward

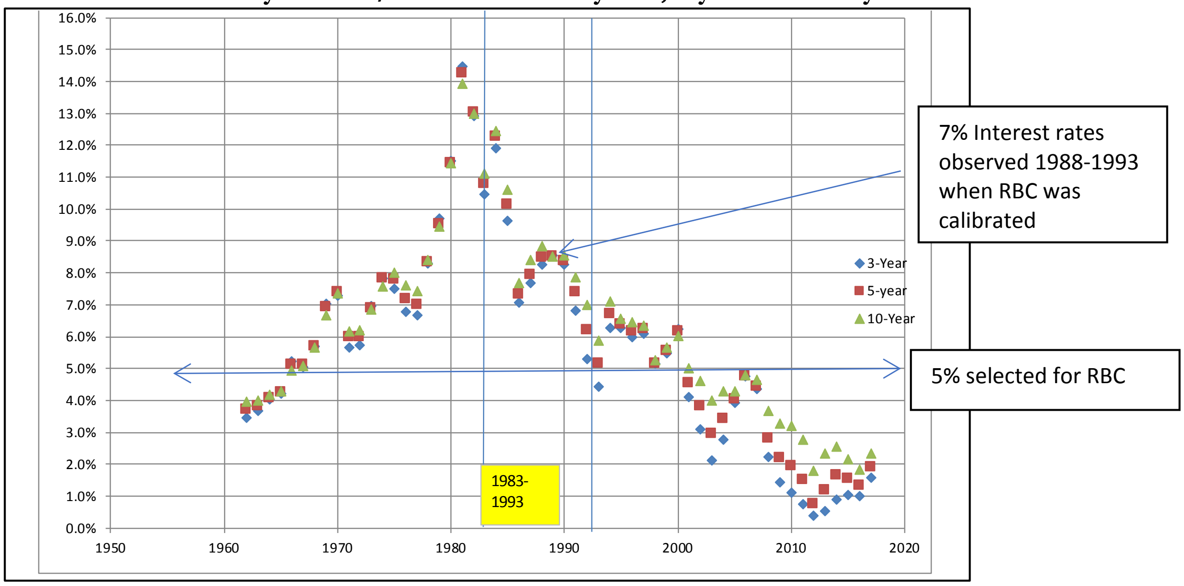
In this paper we present an approach to updating the IIO calibration to reflect the change in interest rates over time.

implicit "cushion" for solvency. Not taking this implicit "cushion" into account would double-count the required capital: an explicit capital requirement held as surplus and an implicit capital cushion held as reserves.

[7] For premium, the RBC risk value decreases from an average of 26.3% of premium, before the IIO adjustment, to 17.3% of premium after the IIO adjustment. For reserves, the RBC risk value decreases from an average of 36.9% of reserves, before the IIO adjustment, to 19.7% of reserves, after the IIO adjustment. See Appendix 1/Exhibit 3.

[8] These averages are based on 2010 industry total LOB net earned premium and net loss and loss expense reserve amounts, 2010 industry average expenses, with no adjustment for concentration/diversification, own-company adjustment, growth charge and loss sensitive adjustments. This calculation assumes the effect of the later three adjustments is proportional across LOBs and between premium risk and reserve risk. Combined premium and reserve UW risk value is based on premium and reserve risk values and the square root rule of the RBC Formula.

Table 1
Annual Interest Rates by Year- 1962-2013
US Treasury Bonds / Durations of 3 years, 5 years and 10 years[9]



See data in Table 2, Section 2.2.2 below.

1.1 Assumed Reader Background and Disclaimer

This paper assumes the reader is generally familiar with the RBC Formula.

References to "we" and "our" mean the principal authors of this paper.

The "working party" and "DCWP" refer to the CAS RBC Dependencies and Calibration Working Party.

The analysis and opinions expressed in this report are solely those of the principal authors, and are not those of the authors' employers, the Casualty Actuarial Society, or the American Academy of Actuaries.

Nether the authors nor DCWP make recommendations to the NAIC or any other body. This material is for the information of CAS members, policy makers, actuaries and others who might make recommendations regarding the RBC formula. In particular, we expect that the material will be used by the American Academy of Actuaries.

This paper is one of a series of articles prepared under the direction of the DCWP.

[9] Federal Reserve website: http://www.federalreserve.gov/releases/h15/data.htm
Market yield on U.S. Treasury securities at 3-year, 5-year and 10-year constant maturity, annual rate by day, averaged over each year.

2. CALIBRATION APPROACH

2.1 Calibration Alternatives

A key aspect in the current calibration method is described in the 1993 Report on Reserve and Underwriting Risk Factors by the American Academy of Actuaries Property/Casualty Risk-Based Capital Task Force, page 17, as follows:

> *The current factors are based on nominal reserve development and nominal loss ratios. Separately, credit for the time value of money is given using a constant 5% interest rate. This approach overlooks the correlation between the level of interest rates and industry underwriting experience. Intuitively, it makes sense that during periods of high interest rates loss ratios will be higher, because market conditions force the companies to set their prices in anticipation of investment income. Since high interest rates often occur during high inflation periods, it also makes sense that reserve development will be worse during periods of high interest rates. Interest experience over the last ten years generally supports both of those hypotheses.*[10]

The 1993 American Academy of Actuaries Task Force proposed that premium and reserve risk factors be calibrated from discounted loss and loss adjustment expense ratios (LRs) and discounted reserve runoff experience. We refer to this as the Present Value Approach (PVA), in contrast the Nominal Value Approach (NVA) used in the calibration of the IIO and underwriting risk factors in the current[11] RBC Formula.

Thus, a key decision in the calibration is to determine whether to apply the NVA or the PVA.[12] In Section 2.2, below, we describe the data we use to test the NVA and PVA approaches. In Section 2.3 we test the NVA and PVA.

2.2 Data

To examine the two calibration approaches, we use claim payment patterns, interest rates, and risk data that we describe in the sub-sections below.

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[10] American Academy of Actuaries Property/Casualty Risk-Based Capital Task Force, Report on Reserve and Underwriting Risk Factors, May 1993, https://www.casact.org/pubs/forum/93sforum/93sf105.pdf

[11] As of the 2018 RBC Formula.0

[12] Still another approach to addressing investment income potential is to report reserves on a discounted basis, increasing reported capital rather than as an offset to risk. Solvency II uses the discounted reserve approach. Statutory Accounting and RBC do not use the discounted reserve approach.

The NVA and the discounted reserve approach are similar in treating risk levels and interest rates as independent variables. The PVA differs from both, in treating interest rates risk levels as being as related.

2.2.1 Payment Patterns

The NAIC uses the 1986 IRS payment pattern methodology in its IIO updates, the last[13] of which was completed in 2010 using 2007 data. We use the same IRS methodology, but we apply it using 2010 data. In Appendix 2, we show the resulting payment patterns, we show examples of how we use the patterns to determine IIOs, and we compare the IIOs at 5% interest with our indicated payment patterns to the current IIOs. Appendix 2/Exhibit 3 shows that our all-lines average LOB IIO's at a 5% interest rate are very similar to the IIO's in the 2010 RBC Formula. Hence, our updated payment patterns are very similar to the payment patterns underlying the IIOs in the current[14] RBC Formula.

There are more refined methods of calculating payment patterns, but we did not apply those in our work. Analysis of possible refinements is a matter of future research.

2.2.2 Selecting Interest Rates

To calculate discounted LRs the American Academy of Actuaries 1993 Report used 5-year duration US Treasury interest rates, less 2%,[15] for the year in which premium was earned. To calculate discounted reserve runoff values, the in the 1993 Academy RBC Task Force used interest rates for each accident year (AY) component of reserves for reserve risk equal to the interest rate in that AY LR.[16]

We consider using the interest rates selected in the following ways:

<u>Duration</u>

- 3-year, 5-year and 10-year duration US Treasury interest rates[17]

<u>Time Period</u>

- The average interest rate[18] during the year that the premium was earned and the year of the initial reserve date, e.g., the 1989 average interest rate for 1989 AY LRs and for 1989 initial reserve dates (we refer to this period as "earned year" or "current year"),
- The average interest rate during the year prior to the year the premium was earned year, and the year prior to the initial reserve date e.g., 1988 for 1989 LRs and 1989 initial reserve dates (we refer to this as "prior year"). From the premium perspective,

[13] Last update as of 2018.
[14] As of the 2018 RBC Formula.0
[15] The interest rate in the years that were considered in that 1993 analysis ranged from 7% to 14%. In recent years, interest rates are lower. Since 2008, interest rates for some durations in some years have been less than 2%. Subtracting 2% would produce a negative interest rate, which, while possibly appropriate, should be noted.
[16] https://www.casact.org/pubs/forum/93sforum/93sf105.pdf, page 17-18.
[17] From daily per annum interest rates at constant maturity.
[18] Arithmetic average of daily per annum rates for current year, prior year and December interest rates.

premium earned in 1989 would have been written partially in 1988 and rate decisions would have been made in 1988 or prior, and

- For reserve risk, in addition to current year and prior year interest rates, we considered the interest rate for the month of December of the initial reserve year, e.g., average for December 1989 for initial reserve year 1989.

Table 2, below, shows the US treasury interest rates at durations of 3-years, 5-years and 10-years, average by calendar year or by month of December within the calendar year. Table 1 is a graphical display of values from Table 2, columns 2-4.

Table 2
Annual Interest Rates by Duration[19]

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Year	Calendar Year Average			Average for Month of Dec.		
	Duration			Duration		
	3 Year	5 Year	10 Year	3 Year	5 Year	10 Year
1978	8.3%	8.3%	8.4%	9.3%	9.1%	9.0%
1979	9.7%	9.5%	9.4%	10.7%	10.4%	10.4%
1980	11.5%	11.4%	11.4%	13.6%	13.2%	12.8%
1981	14.5%	14.2%	13.9%	13.7%	13.6%	13.7%
1982	12.9%	13.0%	13.0%	9.9%	10.2%	10.5%
1983	10.4%	10.8%	11.1%	11.1%	11.5%	11.8%
1984	11.9%	12.3%	12.5%	10.6%	11.1%	11.5%
1985	9.6%	10.1%	10.6%	8.4%	8.7%	9.3%
1986	7.1%	7.3%	7.7%	6.4%	6.7%	7.1%
1987	7.7%	7.9%	8.4%	8.1%	8.5%	9.0%
1988	8.3%	8.5%	8.8%	9.1%	9.1%	9.1%
1989	8.6%	8.5%	8.5%	7.8%	7.7%	7.8%
1990	8.3%	8.4%	8.6%	7.5%	7.7%	8.1%
1991	6.8%	7.4%	7.9%	5.4%	6.2%	7.1%
1992	5.3%	6.2%	7.0%	5.2%	6.1%	6.8%
1993	4.4%	5.1%	5.9%	4.5%	5.1%	5.8%
1994	6.3%	6.7%	7.1%	7.7%	7.8%	7.8%
1995	6.3%	6.4%	6.6%	5.4%	5.5%	5.7%
1996	6.0%	6.2%	6.4%	5.9%	6.1%	6.3%
1997	6.1%	6.2%	6.4%	5.7%	5.8%	5.8%
1998	5.1%	5.2%	5.3%	4.5%	4.5%	4.6%
1999	5.5%	5.6%	5.6%	6.1%	6.2%	6.3%
2000	6.2%	6.2%	6.0%	5.3%	5.2%	5.2%
2001	4.1%	4.6%	5.0%	3.6%	4.4%	5.1%
2002	3.1%	3.8%	4.6%	2.2%	3.0%	4.0%
2003	2.1%	3.0%	4.0%	2.4%	3.3%	4.3%
2004	2.8%	3.4%	4.3%	3.2%	3.6%	4.2%
2005	3.9%	4.0%	4.3%	4.4%	4.4%	4.5%
2006	4.8%	4.7%	4.8%	4.6%	4.5%	4.6%
2007	4.3%	4.4%	4.6%	3.1%	3.5%	4.1%
2008	2.2%	2.8%	3.7%	1.1%	1.5%	2.4%
2009	1.4%	2.2%	3.3%	1.4%	2.3%	3.6%
2010	1.1%	1.9%	3.2%	1.0%	1.9%	3.3%
2011	0.7%	1.5%	2.8%	0.4%	0.9%	2.0%
2012	0.4%	0.8%	1.8%	0.4%	0.7%	1.7%
2013	0.5%	1.2%	2.4%	0.7%	1.6%	2.9%
2014	0.9%	1.6%	2.5%	1.1%	1.6%	2.2%
2015	1.0%	1.5%	2.1%	1.3%	1.7%	2.2%
2016	1.0%	1.3%	1.8%	1.5%	2.0%	2.5%
2017	1.6%	1.9%	2.3%	2.0%	2.2%	2.4%

Market yield on U.S. Treasury securities at 3-year, 5-year and 10-year constant maturity, per annum rate by day, arithmetic average of daily rates per over each year for columns 2-4 and month of December for columns 5-7.

<u>Selecting Duration and Time Period for Our Calibration</u>

We use 3-year US Treasury interest rates by year by year in the PVA/NVA analysis in

[19] Federal Reserve website: http://www.federalreserve.gov/releases/h15/data.htm

Section 3. For LRs we use the interest rates for the year that proceeded the year that the premium was earned. For the reserve runoff values, we use the interest rates for the year-ending 12 months before the initial reserve date. That is, for example, we used 1988 average interest rates to discount AY 1989 LRs and December 31, 1989 reserve runoff values. We refer to those interest rates as the "actual" interest rates by year.

In Appendix 3 we test the effect of alternative interest rates. Based on the material in Appendix 3, we believe that using the 3-year duration/prior-year average interest rates is reasonable overall, for purposes of this Report, although other choices would also be reasonable. Further research, as part of an actual calibration, could assess of the extent to which the selected interest rates should vary by LOB and/or vary between premium risk and reserve risk. The research could also examine the effect of using 1993 American Academy of Actuaries approach of discounting the reserve using separate interest rates for each AY component of the initial reserve amount.

2.2.3 Risk Data

We describe our risk data in DCWP Reports 6[20] and 7,[21] and we summarize the characteristics of that data below.

Our premium risk data consists of net earned premium (NEP) and LRs for AYs 1988-2010, net of reinsurance, at the latest available maturity from Schedule P, Part 1, in the 1997-2010 Annual Statements, by LOB and by company for individual companies and DWCP-defined group pools, as indicated.[22] Each LOB data point is the NEP and LR for a single AY and LOB for a single company or pool (LOB-Company-AY).

Similarly, the reserve risk data consists of the Initial Reserve amount (Initial Reserve[23]) reserve runoff ratios (RRRs) for initial reserve dates 1988-2009. The RRR for each initial reserve date is the ratio of (a) movement in incurred loss and defense and cost containment

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[20] http://www.casact.org/pubs/forum/13fforum/01-Report-6-RBC.pdf

[21] http://www.casact.org/pubs/forum/14wforum/Report-7-RBC.pdf

[22] The Risk Data points are filtered as we describe in DCWP Report 6 (on PRFs) and Report 7 (on RRFs). In brief, the main filters are that we exclude anomalous values; treat pool company data on a combined basis (DCWP-defined group pools); exclude Minor Lines data points (see Glossary); exclude the smallest LOBs data points, defined as those in smallest 15th percentile of LOB-size, by AY; exclude companies with less than 5 AYs of NEP; use values at the latest available maturity; and include companies regardless of whether they filed a 2010 Annual Statement (Survivorship Adjustment).

Those filters are largely the same as the filters used in the 2016 American Academy of Actuaries calibration report 2016 Update to Property and Casualty Risk-Based Capital Underwriting Factors http://www.actuary.org/files/publications/PC_RBC_UWFactors_10282016.pdf

[23] Reserve for loss and defense and containment expenses, but not including adjusting and other expenses.

expense (DCCE) from the initial reserve date to the most mature valuation date available, to (b) the Initial Reserve for loss and DCCE. The ratios in that RRR calculation are net of reinsurance, from Schedule P, Parts 2 and 3, in the 1997-2010 Annual Statements, by LOB and by company for individual companies and DWCP-defined group pools, as indicated.[24] Each LOB data point is the Initial Reserve amount and RRR from a single initial reserve date and LOB for a single company or pool (LOB-Company-Initial Reserve Date).

As we describe in DCWP Reports 6 and 7, using that data, the indicated PRFs by LOB are the 87.5th percentile value of LRs by LOB. The indicated RRFs by LOB are the 87.5th percentile of RRRs. We calculate all-year indicated risk factors using data from all years combined. We calculate year-by-year indicated risk factors using the risk data separately for each AY or reserve runoff year.

From indicated PRFs and RRFs we determine indicated PRC%s and RRC%s percentages, by LOB. The indicated PRC% equals the indicated PRF plus industry expense ratios, by LOB minus, 100%. The indicated RRC% equals the indicated RRF.

2.2.4 Discounted Risk Data

Using the payment patterns developed as described in 2.2.1, the 3-year US treasury interest rates, and alternative interest rates for sensitivity testing, we construct discount factors for LRs and RRRs, by LOB, by year. Appendix 2/Exhibits 1 to 2 show how we determine the discount factors for LRs and RRRs, respectively, using the NAIC 5% interest rate and using interest rates for sample years.

The discounted $LR_{COMPANY, LOB, YEAR}$ equal the $LR_{COMPANY, LOB, YEAR}$ multiplied by the discount factor. The discounted $RRR_{COMPANY, LOB, YEAR}$ equals $(1+RRR_{COMPANY, LOB, YEAR})$ multiplied by discount factor – 1.0. The indicated discounted PRF_{LOB} and RRF_{LOB} are the 87.5th percentile value of discounted $LR_{COMPANY, LOB, YEAR}$ or discounted $RRR_{COMPANY, LOB, YEAR}$.

From discounted PRFs and RRFs we determine indicated discounted premium risk charge

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[24] The Risk Data points are filtered as we describe in DCWP Report 6 (on PRFs) and Report 7 (on RRFs). In brief, the main filters are that we exclude anomalous values; treat pool company data on a combined basis (DCWP-defined group pools); exclude Minor Lines data points (see Glossary); exclude the smallest LOBs data points, defined as those in smallest 15th percentile of LOB-size, by AY; exclude companies with less than 5 AYs of NEP; use values at the latest available maturity; and include companies regardless of whether they filed a 2010 Annual Statement (Survivorship Adjustment).

The runoff ratio includes movement related to "all prior year" element of Schedule P.

Those filters are largely the same as the filters used in the 2016 American Academy of Actuaries calibration report 2016 Update to Property and Casualty Risk-Based Capital Underwriting Factors http://www.actuary.org/files/publications/PC_RBC_UWFactors_10282016.pdf percentages (PRC%) and indicated discounted reserve risk charge (RRC%) percentages. Discounted RRC% equals the discounted RRF as a percentage of reserves. Discounted PRC% equals the discounted PRF as a percentage of premium, plus industry expenses by LOB, minus 100%.

2.2.5 Important Caution Regarding Data Adequacy

Our interest rate data and our risk data include a single long period of declining interest rates. Therefore, any observed relationship between risk and interest rates may be due to long term trends in factors other than interest rates. We address the implication of this caution later in our analysis.

3. INDICATED RISK CHARGE VARIATION WITH INTEREST RATES

NVA and PVA imply different relationships between risk charges and interest rates, as follows:

- The NVA is more applicable for calibration if indicated undiscounted risk charges, year-by-year, are independent of the interest rates.

  If this were the case, NVA indicated risk charges would vary only randomly when interest rates change.
- The PVA is more applicable for calibration if indicated risk charges vary with interest rates, higher when interest rates are higher and lower when interest rates are lower; such that indicated present value risk charges, year-by-year, are independent of interest rates.

  If this were the case, PVA indicated risk charges would vary only randomly when interest rates change.

Based on this understanding, we test which approach is better represented by the data. We first examine the private passenger automobile liability (PPA) and workers compensation (WC) LOBs, as examples. We then examine the extent to which the patterns from those two LOBs apply to other LOBs.

3.1 PPA and WC Examples

For PPA, Table 3 shows the actual interest rate by year, column 2, and the indicated nominal and discounted PRFs (column 3 and 4), corresponding PRC%s (column 5 and 6), the indicated nominal and discounted RRFs (column 7 and 8). and the corresponding RRC%

values (also columns 7 and 8).

Table 3
PPA – Indicated Nominal and Discounted Risk Factors and Risk Charges by Year

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Year	Interest Rate	PRF		PRC%		RRF=RRC%	
		Nominal	Discounted	Nominal	Discounted	Nominal	Discounted
1988	7.7%	1.048	0.933	30.1%	18.5%	24.6%	11.5%
1989	8.3%	1.074	0.948	32.6%	20.0%	26.0%	11.9%
1990	8.6%	1.069	0.939	32.1%	19.2%	23.9%	9.6%
1991	8.3%	0.971	0.857	22.3%	10.9%	15.4%	2.5%
1992	6.8%	0.968	0.871	22.0%	12.4%	13.7%	2.9%
1993	5.3%	0.958	0.882	21.0%	13.4%	11.6%	3.1%
1994	4.4%	0.942	0.878	19.4%	13.0%	13.3%	6.0%
1995	6.3%	0.948	0.860	20.0%	11.3%	17.5%	7.1%
1996	6.3%	0.929	0.844	18.2%	9.6%	19.6%	9.1%
1997	6.0%	0.930	0.847	18.2%	9.9%	19.5%	9.4%
1998	6.1%	0.897	0.816	15.0%	6.8%	14.0%	4.2%
1999	5.1%	1.009	0.931	26.1%	18.3%	22.6%	13.5%
2000	5.5%	1.103	1.013	35.6%	26.5%	33.5%	23.0%
2001	6.2%	1.067	0.969	31.9%	22.1%	32.1%	20.6%
2002	4.1%	1.035	0.970	28.7%	22.2%	23.3%	16.0%
2003	3.1%	0.904	0.860	15.6%	11.2%	18.7%	13.2%
2004	2.1%	0.855	0.827	10.8%	7.9%	8.8%	5.3%
2005	2.8%	0.850	0.812	10.2%	6.5%	10.9%	6.3%
2006	3.9%	0.858	0.806	11.1%	5.9%	13.9%	7.4%
2007	4.8%	0.914	0.848	16.6%	10.0%	9.4%	1.9%
2008	4.3%	0.931	0.869	18.3%	12.1%	11.2%	4.1%
2009	2.2%	0.978	0.944	23.1%	19.6%	11.9%	8.1%
2010	1.4%	0.973	0.950	22.5%	20.2%	NA	NA

Column 2 interest rate = 3-year US Treasury constant maturity interest rates average for year prior to year shown. For example, 1988 year shows 7.7% is interest rate. This is interest rate average for 1987 applicable to AY 1988 and initial reserve date 1988.
Appendix 1/Exhibit 3 shows the of nominal and discounted PRC, PRC% and RRC% with 2010 risk factors.

We show the Table 3 information graphically in Tables 4A and 4B, arrayed by interest rate rather than year, for premium risk and reserve risk, respectively. In each of Tables 4A and 4B the horizontal axis represents the annual interest rates; the vertical axis represents the year-by-year indicated PRC%/RRC%; and, each diamond-shaped point on the chart shows the indicated PRC%/RRC% compared to its annual interest rate, for premium risk and reserve risk respectively.

The graph on the left shows how undiscounted, i.e., NVA indicated risk charges vary with interest rates. The graph on the right shows how indicated discounted, i.e., PVA, indicated risk charges vary with interest rates.

For example, as marked on Table 4A, for AY 1990 the interest rate we use is 8.6%.[25] The

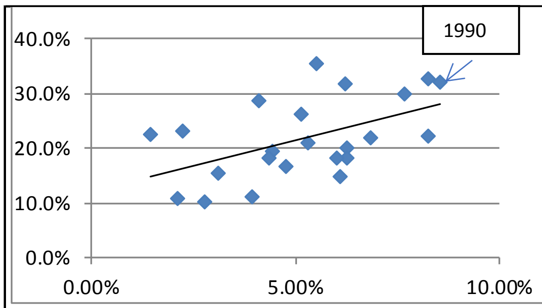
[25] Reminder, for 1990 we use the average interest rate for 1989 (the year prior to 1990) for 3-year duration US Treasury securities.

indicated PRC% with 1990 data is 32.1% on an NVA basis and 19.2% on a PVA basis. Thus, the NVA graph, on the left, includes the data point x=8.66% and y=32.1%, (8.6%, 32.1%) in coordinate notation. The present value graph, on the right, includes the data point x=8.6% and y=19.2%, (8.6%, 19.2%), in coordinate notation.

In the NVA indicated risk charge exhibit on the left of each Table, the line slopes upward showing that indicated PRC%s and RRC%s (Tables 4A and 4B, respectively) tend to increase as interest rates increases. In the discounted value indicated risk charge exhibit on the right side of each graph, the slope and R-squared values are much lower than for the NVA indicated PRC%s/RRC%s.

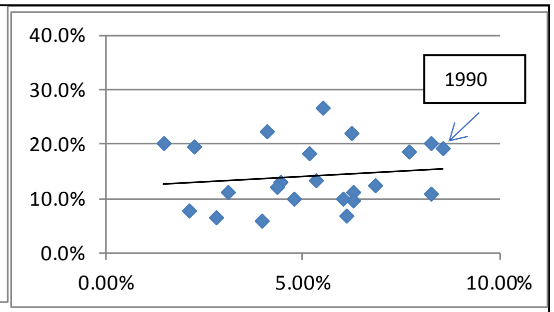
Table 4A - - PPA- Premium Risk
Indicated PRC%s by interest rate

Nominal Value



Slope	1.9
R-Squred	27%

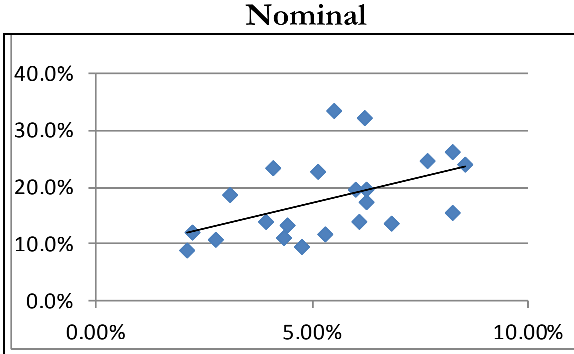
Present Value



Slope	0.4
R-Squred	2%

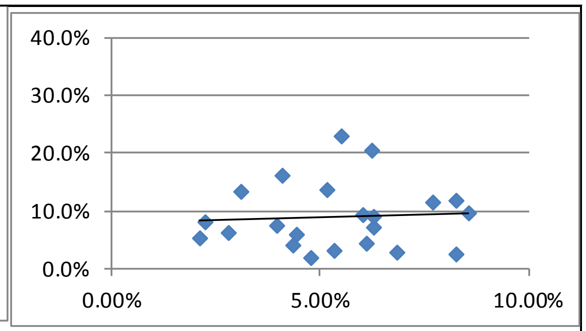
Table 4B - PPA-Reserve Risk
Indicated RRC%s by interest rate

Nominal



Slope	1.8
R-Squred	24%

Present Value

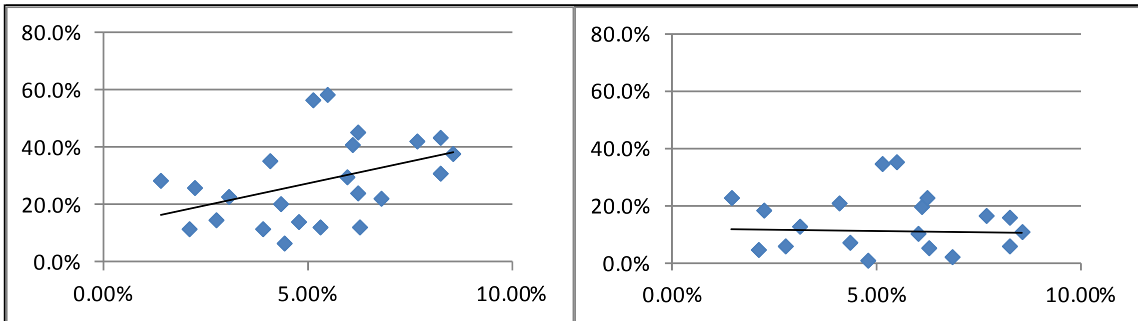


Slope	0.2
R-Squred	0%

Tables 5A and 5B, below, show the same information for the workers compensation LOB. The workers compensation patterns are similar to the PPA patterns.

Table 5A - Workers Compensation- Premium Risk
Indicated PRC%s by interest rate

Nominal | **Present Value**

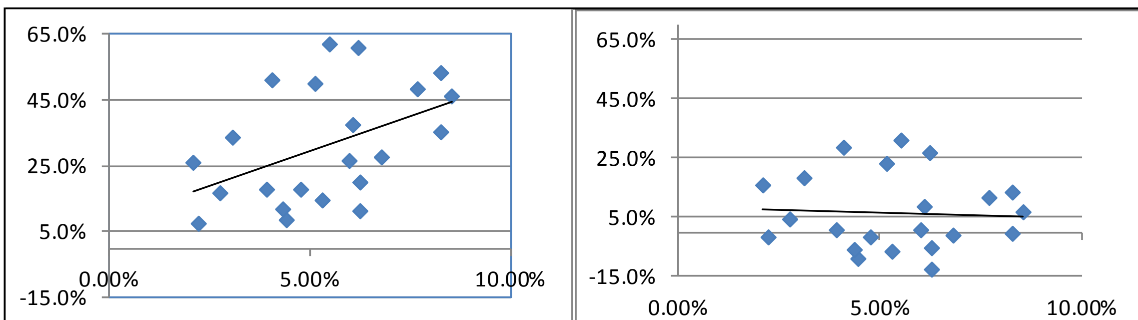


Nominal	
Slope	3.0
R-Squred	17%

Present Value	
Slope	-0.2
R-Squred	0%

Table 5B - Workers Compensation – Reserve Risk
Indicated RRC%s by interest rate

Nominal | **Present Value**



Nominal	
Slope	4.2
R-Squred	21%

Present Value	
Slope	-0.3
R-Squred	0%

Thus, for PPA and WC, the data is more consistent with PVA than with NVA.

The upward slope in the NVA indications is an important consideration when calibrating risk factors for a low interest rate environment, as is currently the case. An NVA calibration based in multiple years of data would be satisfactory when the current interest rate is close to the interest rate typical during the risk data experience period, in our case 1988-2010. The indication would not be correct for a year when interest rates were higher than typical in the experience period. In that case the NVA indications, based on a 'typical' interest rate would be too low. For a year when interest rates were lower than typical in the experience period, the NVA indications, based on typical interest rates, would be too high.

3.2 Line-by-Line by Results

Tables 6A and 6B, below, for premium and reserves, respectively, show that the patterns demonstrated above for PPA and WC apply to most LOBs. Columns (1) and (4) show the slope of NVA and PVA indicated risk charges. Columns (2) and (5) show the R-squared value for the regression of risk charge compared to interest rates, respectively. Columns (3) and (6) show the slope compared to the standard error of the slope, a measure of the statistical significance of the slope, respectively.

On the NVA basis, the slope is upward for nearly all LOBs. On the PVA basis the slope is more randomly up or down. Thus, the data overall, is more consistent with PVA than with NVA.

Table 6
Summary of Relationship of Indicated PRC% to interest rate
Table 6A-Premium

Lines of Business	Nominal			Discounted			2010 NEP %
	(1) Slope	(2) R-Sqr	(3) Slope/ Std Err	(4) Slope	(5) R-Sqr	(6) Slope/ Std Err	
A- Homeowners/Farmowners	0.0	0%	0.0	(0.7)	3%	(0.8)	15%
B- Private Passenger Auto	1.9	27%	2.8	0.4	2%	0.7	23%
C- Commercial Auto	3.1	25%	2.7	1.0	4%	0.9	4%
D - Workers Compensation	3.0	17%	2.1	(0.2)	0%	(0.2)	8%
E - Commercial Multi-Peril	1.3	5%	1.1	(0.2)	0%	(0.2)	7%
F1 - Med Prof Liab-Occ	1.9	1%	0.6	(3.7)	9%	(1.5)	1%
F2 - Med Prof Liab-CM	2.0	2%	0.7	(1.1)	1%	(0.5)	2%
G - Special Liability	4.6	32%	3.1	2.5	15%	1.9	1%
H - Other Liability	1.3	3%	0.7	(1.8)	8%	(1.3)	9%
I - Special Property	0.4	1%	0.4	(0.4)	1%	(0.5)	8%
J - Auto Physical Damage	0.4	1%	0.6	0.0	0%	0.0	16%
K - Fidelity/Surety	(2.0)	8%	(1.4)	(2.8)	18%	(2.2)	1%
L - Other	(0.6)	3%	(0.8)	(1.5)	15%	(1.9)	2%
M - International	8.1	7%	1.3	5.0	3%	0.9	0%
N&P - Reinsurance-Prop/Fin	6.0	7%	1.3	3.1	2%	0.7	2%
O - Reinsurance-Liabiity	6.0	12%	1.7	1.0	1%	0.4	1%
R - Products Liability	1.9	1%	0.5	(2.1)	2%	(0.7)	1%
S - Financial/Mort Guarantee	NA	NA	NA	NA	NA	NA	NA
T - Warranty	NA	NA	NA	NA	NA	NA	NA
Wtd Avg	1.36	10%	1.25	(0.22)	3%	(0.16)	1.00

Table 6B-Reserves

Lines of Business	Nominal			Discounted			% 2010 Rsv
	(1) Slope	(2) R-Sqr	(3) Slope/ Std Err	(4) Slope	(5) R-Sqr	(6) Slope/ Std Err	
A- Homeowners/Farmowners	(0.0)	0%	(0.0)	(1.4)	13%	(1.7)	4%
B- Private Passenger Auto	1.8	24%	2.5	0.2	0%	0.2	17%
C- Commercial Auto	1.7	11%	1.6	(0.5)	1%	(0.5)	4%
D - Workers Compensation	4.2	21%	2.3	(0.3)	0%	(0.2)	21%
E - Commercial Multi-Peril	6.2	33%	3.1	2.7	11%	1.5	7%
F1 - Med Prof Liab-Occ	(2.6)	6%	(1.2)	(5.2)	26%	(2.7)	2%
F2 - Med Prof Liab-CM	(0.3)	0%	(0.2)	(2.3)	14%	(1.8)	3%
G - Special Liability	7.2	30%	2.9	4.0	14%	1.8	1%
H - Other Liability	5.1	20%	2.2	0.7	1%	0.3	24%
I - Special Property	0.7	2%	0.7	(0.1)	0%	(0.1)	2%
J - Auto Physical Damage	1.6	4%	0.9	1.0	2%	0.6	1%
K - Fidelity/Surety	2.8	2%	0.6	1.0	0%	0.2	1%
L - Other	3.4	9%	1.4	2.5	6%	1.1	1%
M - International	9.4	26%	2.2	6.3	17%	1.7	0%
N&P - Reinsurance-Prop/Fin	10.7	44%	4.0	7.2	31%	3.0	1%
O - Reinsurance-Liabiity	6.6	10%	1.5	0.7	0%	0.2	7%
R - Products Liability	8.4	16%	1.9	2.8	3%	0.8	3%
S - Financial/Mort Guarantee	NA	NA	NA	NA	NA	NA	NA
T - Warranty	NA	NA	NA	NA	NA	NA	NA
Wtd Avg	3.86	18%	1.98	0.37	3%	0.09	100.00%

Table 6C – Premium Plus Reserves

Wtd Average	2.78	15%	1.67	0.11	3%	(0.01)

LOBs S and T show NA and are not included in the averages as there is insufficient year-by-year data.

Because essentially no companies have an industry distribution by LOB, the weighted average is a summary, but does not represent a typical company.

3.3 Analysis and Summary – NVA or PVA

Considerations Favoring NVA

From a statistical perspective, the data for slope, R-squared and slope/standard error statistics, by LOB, and on average, from Table 6A and 6B, are not, by themselves, strong statistical proof that the interest rate is a significant explanatory variable.

Also, our data consists of a single long period of declining interest rates. Therefore, the pattern we observe may be due to factors other than interest rates that affected risk during the past 30 years. For example, if, during that period, there were improved company business controls on pricing and reserving and/or improved regulatory oversight, those improvements might have produced effects that would appear in the statistical tests as being related to interest rates.

Moreover, the effects we observe may be due to the downward trend in interest rates, rather than the absolute level of the interest rates.

Considerations Favoring PVA

However, patterns for some LOBs are notable. The R-squared value is over 15% for many of the larger LOBs, suggesting that the interest rate alone explains as much as 15% of the variability from year to year. Average value for slope/standard error of 2.0 at levels of statistical significance for one-tail (slopes greater than zero) hypothesis testing, for some LOBs. Moreover, the upward slopes for nominal risk charges are much greater than the slopes for discounted risk charges, consistently across LOBs.

Thus, most of the statistical evidence is more consistent with PVA rather than NVA.

Also, as explained in the 1993 American Academy of Actuaries Report, it is not surprising that the risk factors vary with interest rates. For premium risk, the relationship between interest rates and the 87.5^{th} percentile LR might reflect the extent that pricing reflects investment earning potential. The relationship might reflect that rate adequacy is harder to achieve in high inflation rate environments that often occur in high interest rate environments. Similarly, for reserve risk, the relationship between interest rates and 87.5^{th} percentile runoff ratios might reflect several factors. For example, it might reflect difficulties in reserve estimates in higher inflation environment that typically arise in higher interest rate environments; and it might reflect that reserve adequacy could follow the premium adequacy, in that reserves for the least mature AYs are often based on LRs.

Our Calibration

For our calibration we use the PVA for the following reasons:

- The PVA perspective is a plausible interpretation of risk experience.
- The statistics are more consistent with PVA than with NVA.
- Even if improvements in business controls and regulatory oversight were the drivers for the apparent relationship between interest rates and; risk, reflecting the business controls and regulatory oversight using change in interest rates as proxies is more accurate than ignoring the business and regulatory changes, as would be the case by using the NVA.
- As we show in the next section, NVA indications are for larger increases in risk charges than are PVA indications. Therefore, implementing indications on the PVA would be a step towards implementing the NVA, if the latter were, ultimately, proved more appropriate.

Implementing and Updating Risk Factors with PVA

Calibration based on the PVA has the following implications with respect to the RBC Formula and future risk factor updates:

- Formula Impact of PVA - With the PVA, separate IIOs and risk factors are not required. A single risk factor could be used to represent the combined effect. However, in implementing the RBC Formula, the existing IIO structure could be retained, by splitting the indicated discounted risk factor into an IIO component and an undiscounted risk factors component, by LOB.
- Calibration Implications of PVA - Regardless of how the PVA were implemented, risk factors and payment patterns in the RBC Formula should be reviewed periodically. However, with PVA there is be no need to review the interest rate component of the formula, as the interest rates are reflected in the discounted risk data used in the calibration.

4. SELECTING THE TARGET SAFETY LEVEL

When the NAIC selected a 5% interest rate for the RBC Formula, the actual interest rate was higher than 5%. For example, interest rates averaged 7%[26] for the period 1988-1993. As a result, the safety margin effectively included in the UW risk charge in the original RBC Formula had two components:

[26] The basis for the 5% selection is not documented. The average interest rate for 1988-1993 was 6.9% and 7.3% for 3-year and 5-year duration US treasury securities, respectively. In the period 1980-1996 interest rates averaged over 9%, for all bond durations from 1 year to 10 years. (See Table 2)

- One component is the selection of the PRF/RRF safety level. We call that the UW margin, i.e., the 87.5th percentile, as we discussed in Reports 6 and 7 and in American Academy of Actuaries risk factor calibration reports, most recently in 2016.[27]
- The second component arises from the selection of the interest rate for the IIO, i.e., 5% selected versus 7% observed at the time.

In selecting interest rates for IIO calibrations, for either the NVA and the PVA, there are three regulatory choices for UW risk safety margins and interest rate safety margins:

Option	**UW Risk Safety Margin**	**Interest Rate Safety Margin**[28]
1	87.5th percentile	No interest rate safety margin
2	87.5th percentile	Interest rate safety margin>0
3	Greater than 87.5th percentile	No interest rate safety margin

Among these options, Option 1 produces the lowest risk charges and the lowest target safety level. The target safety level would be lower than the safety level achieved by the original RBC calibration, with respect to UW and interest rate combined. On the other hand, there is no documentation to demonstrate that the interest rate difference was intended as a 'safety margin.' Perhaps the 5% was intended as a long-term value,[29] around which safety levels might vary from year to year. If so, the implicit safety effect may not have been intended, and, arguably, there is no reason to continue it.

Options 2 and 3 could be designed to produce equal overall safety levels[30] if we select the UW risk safety level in Option 3 to offset the interest rate risk margin in Option 2. Option 3 is less complex, in that it has fewer discretionary safety level parameters, i.e., an underwriting safety level rather than both an underwriting safety level and an interest rate safety margin. Also, Option 3 is more transparent in that safety levels are generally[31] described in terms of percentiles of outcomes, not as interest rate safety margins or a mixture of percentiles of outcomes and interest rate safety margins.

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27 AAA 2016 report
http://www.actuary.org/files/publications/PC_RBC_UWFactors_10282016.pdf

28 If there is an interest rate safety margin, its effect applies to both the NVA and PVA. In the NVA, the interest rate safety margin is reflected in the IIO value. In the PVA, the interest rate safety margin in reflected in the discount factors applied to calibration risk data.

29 A current assessment of a long-term interest rate might be lower than the 5% selected about 25 years ago.

30 Although the change in safety level would vary between LOBs.

31 For example, in individual company capital models, in solvency II and regulatory capital formulas following solvency II.

7.1 Effect of Alternative Target Safety Levels-Options 1, 2 and 3

In Tables 7A and 7B, below, we compare PVA indicated risk charges by LOB, based on options 1, 2 and 3, with various interest rate safety margins and UW safety levels, for premium risk and reserve, respectively.

- We consider interest rates margins of 0%, 1%, 1.5% and 2%.[32] 2% being our estimate of the implicit interest rate safety margin when the underwriting risk factors and IIOs were first calibrated.
- We consider underwriting safety levels of 87.5%, 89% and 90%.

Tables 7A and 7B show the following:

- Regarding Option 1: Column 2 shows the indicated risk charges by LOB with an underwriting safety margin of 87.5% and actual interest rates by year with zero interest rate safety margin using the PVA.
- Regarding Option 2: Columns 3-5 show the indicated risk charges by LOB with an underwriting safety margin of 87.5% and actual interest rates by year minus interest rate safety margins of 1%, 1.5% and 2%, respectively, using the PVA.
- Regarding Option 3: Columns 6-9 show the indicated risk charges by LOB with zero interest rate safety margin with underwriting safety margins of 87.5% to 90%, as shown, using the PVA.

We observe the following:

- Column 2 values, "current years" rate less 0%," have the same values as the 87.5th percentile results in column 6, i.e., a 16.5% premium risk charge, as column 2 and column 6 are the same calculation.
- Columns 3 to 5 show that the indicated PRC%s and RRC%s (for example, the all-lines values of 18.6%, 18.9%, and 19.8% for premium risk) increase as we reduce the annual interest rate by increasing the interest rate safety margins from 1%, to 1.5% to 2% and UW safety margin of 87.5%.
- Columns 6 to 9 show that the indicated PRC%s and RRC%s (for example, the all-lines values of 16.5%, 17.3%, 19.0% and 20.7% for premium risk) increase as we increase the safety level from 87.5%, 88%, 89% and 90%, with zero interest rate safety margins.
- For all lines combined, the indicated PRC% and RRC% with interest rate safety

[32] US Treasury rates since 2008 have often been lower than these safety margin. As such, the interest rate used in the present value calculation would sometimes be a negative number. That is not necessarily incorrect, but it is worth noting.

margins of 1.5% or 2% (the circled values in columns 4 and 5), are close to at the indicated PRC% and RRC% with 89%-90% safety level (the circled values in column 8 and 9).

Thus, the PVA calibration with a safety level of 89%-90% can be viewed as producing the same overall the target safety level in the original NAIC calibration considering both the UW margin and the interest rate safety margin.[33]

Table 7 Re-Calibration of UW Risk Charges
Indicated Risk Charges at Various Safety Level Combinations
Table 7A – Premium

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Lines of Business	PV Indicated at 87.5th %-ile 3 Yr/Current Year Rates Less:				PV Indicated at 3 Yr/Current Year Rates: At %-ile			
	0%	1%	1.5%	2%	87.5%	88.0%	89.0%	90.0%
A- Homeowners/Farmowners	22.1%	23.3%	23.4%	23.9%	22.1%	22.7%	24.2%	26.0%
B- Private Passenger Auto	15.3%	17.0%	17.3%	18.0%	15.3%	15.8%	16.8%	17.8%
C- Commercial Auto	20.1%	22.4%	23.1%	24.2%	20.1%	20.6%	22.1%	23.9%
D - Workers Compensation	14.1%	18.1%	18.5%	20.2%	14.1%	14.6%	16.2%	17.9%
E - Commercial Multi-Peril	15.6%	17.6%	18.0%	18.8%	15.6%	16.2%	17.5%	19.3%
F1 - Med Malp - Occ	46.7%	55.9%	56.6%	60.1%	46.7%	48.6%	51.4%	53.9%
F2 - Med Malp - CM	27.6%	32.7%	33.2%	35.1%	27.6%	28.7%	31.0%	33.2%
G - Special Liability	21.6%	24.0%	24.2%	25.1%	21.6%	22.2%	24.3%	27.3%
H - Other Liability	14.3%	18.3%	19.3%	21.2%	14.3%	15.2%	17.5%	20.0%
I - Special Property	11.0%	12.2%	12.1%	12.5%	11.0%	11.9%	13.4%	15.2%
J - Auto Physical Damage	6.5%	7.1%	7.2%	7.4%	6.5%	7.0%	8.1%	9.3%
K - Fidelity/Surety	6.1%	7.5%	7.8%	8.4%	6.1%	7.0%	8.8%	11.6%
L - Other	24.2%	25.4%	25.6%	26.1%	24.2%	24.7%	26.3%	27.7%
M - International	18.0%	21.6%	20.5%	21.4%	18.0%	18.8%	20.2%	21.7%
N&P - Reinsurance Prop/Fin	36.2%	39.5%	40.0%	41.3%	36.2%	37.1%	39.7%	43.4%
O - Reinsurance - Liabiity	23.2%	29.9%	30.4%	32.9%	23.2%	25.0%	27.3%	31.2%
R - Products Liability	26.8%	32.4%	34.5%	37.3%	26.8%	27.9%	29.2%	32.4%
S - Financial/Mort Guarantee	60.0%	63.0%	64.8%	66.5%	60.0%	68.3%	83.5%	88.1%
T - Warranty	73.4%	73.3%	74.7%	75.2%	73.4%	73.7%	74.4%	75.0%
Average	16.5%	18.6%	18.9%	19.8%	16.5%	17.3%	19.0%	20.7%

Note: Because essentially no companies have an industry distribution by LOB, the weighted average is a summary, but does not represent a typical company.

[33] As the weighted average indicated risk factor is not necessarily the risk factor for the typical company, in an actual calibration this 'equivalence' test might be applied by type of company to examine whether there are types of company that are affected in an unexpected way.

Table 7B – Reserve

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Lines of Business	PV Indicated at 87.5th %-ile 3 Yr/Current Year Rates Less:				PV Indicated at 3 Yr/Current Year Rates: At %-ile			
	0%	1%	1.5%	2%	87.5%	88.0%	89.0%	90.0%
A- Homeowners/Farmowners	13.9%	15.3%	16.0%	16.8%	13.9%	15.1%	17.1%	19.2%
B- Private Passenger Auto	9.1%	10.7%	11.5%	12.4%	9.1%	9.8%	11.5%	12.9%
C- Commercial Auto	22.5%	24.6%	25.7%	26.8%	22.5%	23.4%	25.6%	29.1%
D - Workers Compensation	8.3%	12.3%	14.4%	16.5%	8.3%	9.2%	11.0%	13.8%
E - Commercial Multi-Peril	30.6%	33.4%	34.9%	36.4%	30.6%	31.9%	34.6%	37.8%
F1 - Med Malp - Occ	11.6%	14.8%	16.4%	18.1%	11.6%	13.5%	17.3%	20.2%
F2 - Med Malp - CM	-1.5%	0.9%	2.1%	3.4%	-1.5%	-0.7%	0.5%	3.6%
G - Special Liability	25.4%	27.9%	29.1%	30.4%	25.4%	28.3%	35.6%	40.7%
H - Other Liability	29.6%	33.4%	35.3%	37.4%	29.6%	31.3%	34.7%	39.4%
I - Special Property	24.1%	24.9%	25.3%	25.8%	24.1%	25.7%	29.6%	32.9%
J - Auto Physical Damage	9.7%	10.3%	10.5%	10.8%	9.7%	11.8%	15.2%	18.9%
K - Fidelity/Surety	39.0%	40.8%	41.7%	42.6%	39.0%	41.7%	50.2%	56.3%
L - Other	22.2%	23.1%	23.5%	24.0%	22.2%	23.2%	28.5%	31.6%
M - International	23.4%	25.4%	26.5%	27.5%	23.4%	24.7%	28.3%	32.3%
N&P - Reinsurance Prop/Fin	27.4%	30.0%	31.4%	32.7%	27.4%	28.5%	31.1%	32.8%
O - Reinsurance - Liabiity	33.4%	38.7%	41.7%	44.6%	33.4%	35.8%	40.4%	43.2%
R - Products Liability	52.9%	57.8%	60.2%	62.8%	52.9%	55.2%	58.1%	64.6%
S - Financial/Mort Guarantee	-10.6%	-9.4%	-8.8%	-8.2%	-10.6%	-9.3%	-6.8%	-4.3%
T - Warranty	-16.8%	-16.4%	-16.2%	-15.9%	-16.8%	-16.5%	-16.0%	-15.5%
Average	18.3%	21.3%	22.8%	24.5%	18.3%	19.6%	22.3%	25.5%

Note: Because essentially no companies have an industry distribution by LOB, the weighted average is a summary, but does not represent a typical company.

4.2 Impact on RBC of alternative approaches to IIO calibration – Summary

Table 8, below, compares PVA indications for all lines combined from Table 7A and 7B, with 2010 factors and two NVA indications.

Rows 1-3 shows the combined premium and reserve risk factors using the 2010 RBC Formula values and two NVA indications, as follows:

Row 1: 2010 risk factors and IIOs

Row 2: Indicated risk factors and IIOs at 5% interest rate

Row 3: Indicated risk factors and IIOs at 0% interest rate

Rows 4-7 show risk charges based on PVA with UW safety margins of 87.5% to 90 %, and interest rate interest rates selected with interest rate margins of 0% or 2%.[34]

In each row, column 6 shows the total premium and reserve risk charge as a percentage of premium, derived from columns 4 and 5, premium risk and reserve risk, respectively. In each row, column 9 shows the change in total risk charge compared to the 'starting point' in row 1, 2010 risk factors and IIOs. Row 1, column 9 is 0.0% because that is the base for these

[34] Table 8 Row 4-7 values are from Table 7A and 7B, columns 2, 5, 8 and 9, respectively.

comparisons. Row 2, column 9 equals -0.5% (1.319/1.320-1.0 as a percentage), is small, on average across LOBs, showing that using the new risk data and our selected payment patterns does not produce much overall change in risk charges.

Row 3, column 9, shows the result of an NVA calculation with a zero percent interest rate, based, say, on 2% interest rate and a 2% safety margin. Those assumptions imply an 85.8% increase from current risk charges. The row 3 assumptions are consistent with the original calibration of the RBC Formula, but, for the reasons described above, we do not believe that the NVA provides an appropriate measure of risk. The NVA indications will be too high when interest rates are low, as is currently the case, and too low when interest rates are high.

Table 8

Impact of Changing Interest Rates on All-Line Average Indicated PRC%s and RRC%s Before Diversification

Row	Option	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		Calibration Approach			**Average Risk Charges**			**% Increase in RBC over Current**		
		NAV/PVA	**UW %**	**Interest Rate Safety Margin**	**PRC%**	**RRC%**	**Total%**	**PRC%**	**RRC%**	**Total%**
A. Before Considering Changing Interest Rates										
1	a	2010 Factors	87.5th	5% IIO	17.3%	19.7%	32.0%	0.0%	0.0%	0.0%
2	b	NVA indicated	87.5th	5% IIO	17.1%	19.7%	31.9%	-1.4%	-0.2%	-0.5%
B. After Considering Changing Interest Rates										
3	c	NVA indicated	87.5th	0% IIO	23.1%	40.1%	59.5%	33.5%	103.5%	85.8%
4	1	PVA Indicated	87.5th	0% safety margin	16.5%	18.3%	30.0%	-4.4%	-7.2%	-6.4%
5	2	PVA Indicated	87.5th	2% safety margin	19.8%	24.5%	38.9%	14.4%	24.3%	21.5%
6	3	PVA Indicated	89th	0% safety margin	19.0%	22.3%	36.0%	9.6%	13.3%	12.2%
7	3	PVA Indicated	90th	0% safety margin	20.7%	25.5%	40.6%	19.4%	29.5%	26.6%

For each row, column 7= (1.0+ col 4) divided by (1.0+ column 74for row 1) – 1.0 as a percentage. Similarly, for columns 8 and 9, based on columns 5 and 6, respectively.

Total% in column (6) equals the square root of $(\text{column } 4)^2 + (1.37 \text{ x column } 5)^2$, assuming reserves = 1.37 * premium based on 2010 industry totals.

Because essentially no companies have an industry distribution by LOB, the weighted average is a summary, but does not represent a typical company.

Column 9, Rows 4-7 show PVA calculations that produce changes in UW risk charges as follows:

- Row 4: A 6.4% decrease, using the PVA, with no interest rate safety margin.
- Row 5: 21.5% increase based on the PVA, an interest rate safety margin of 2% and an UW safety margin of 87.5%.
- Row 6: 12.2% increase based on the PVA, actual interest rates and an UW margin of 89%; and
- Row 7: 26.6% increase based on the PVA, actual interest rates and an UW margin of 90%.

Depending on regulatory selection of safety levels, the indicated increase to reflect current interest rates, using the option 3 method of expressing safety margins, is an increase of 12.2% to 26.6%.

5. FUTURE RESEARCH

For simplicity in our PVA calibration, for each AY, we use a single interest for all LOBs, and we for each initial reserve date we use a single interest rate for all LOBs. We used the same "prior year/3-year duration interest rate" approach for both premium risk and reserve risk. Further research, could assess of the extent to which the selected interest rates should vary by LOB and/or vary between premium risk and reserve risk. The research could also examine the effect of using the 1993 American Academy of Actuaries approach of discounting the reserve using separate interest rates for each AY component of the Initial Reserve amount.

In our work, we used the IRS payment pattern methodology. Analysis of more refined methods would be useful.

The interest rate history includes years with a wide range of interest rates, which is helpful for our analysis. However, the risk data is from 1988 to 2010, a portion of a single period of falling interest rates. That means there is less certainty that our results would apply than if our calibration were based on multiple periods of rising/falling interest rates. An analysis of experience from earlier time periods, when interest rates were rising would be useful in testing the extent to which our calibration has been affected by the interest environment we used. In particular, it could re-test the NVA and PVA assumptions in a wider range of interest rate environments.

Glossary

Term	Interpretation
AY	Accident year
Data point	Each PRF data point is the LR for an AY, LOB and a single company or pool at the latest available maturity within the database. Each RRF data point is the RRR for an initial reserve date, LOB and a single company or pool, at the latest available maturity.
DCCE	Defense and cost containment expenses DCCE is called "Allocated Loss Adjustment Expenses" in older Annual Statements. In our analysis we treat DCCE and ALAE as equivalent.
DCWP	Dependency and Calibration Working Party
Formula RBC Formula	The 2010 RBC Formula
IIO, IIO_P, IIO_R	Investment Income Offset, called Adjustment for Investment Income in the RBC Formula. IIO_P for premium risk. IIO_R for reserve risk.
Initial Reserve	The loss and loss adjustment expense reserve amount, net of reinsurance, for the current and all prior AYs evaluated at the initial reserve date.
Initial Reserve Date	December 31st for the year specified (i.e., December 31, 2008 is the initial reserve date for the 2008 net loss reserve which includes AY's 2008 and prior)
LOB RBC LOB	RBC uses 19 LOBs from the Annual Statement Schedule P. Schedule P shows 22 LOBs. RBC calculations treat occurrence and claims made LOBs for other liability (H1 and H2) and products liability (R1 and R2) on a combined basis and treats non-proportional property and non-proportional financial (N and O) on a combined basis, leaving a net of 19 LOBs.
LR	Ratio of net loss and all loss adjustment expense to net earned premium by AY
Minor lines	LOB whose data points are excluded due to LOB-size versus total company size
NAIC	National Association of Insurance Commissioners
PRC	Premium Risk Charge
PRC%	Premium Risk Charge as a percentage of premium
PRF	Premium Risk Factor
RBC	Risk-Based Capital
RBC Formula or Formula	The 2010 NAIC Property-Casualty RBC Formula

Term	Interpretation
RBC UW Risk Value	The Company Action Level amount calculated for the UW risk components of the RBC Formula for a company or group of companies.
RBC Value	The Company Action Level amount calculated from the RBC Formula for a company or group of companies.
Reserves or Loss Reserves	Case, bulk and IBNR loss and DCCE reserves net of reinsurance, as shown in Schedule P – Part 2 and 3, in our analysis. In the RBC Formula the Reserves include Adjusting and Other Expense.
RRC	Reserve Risk Charge
RRC%	Reserve Risk Charge as a percentage of initial reserve amount
RRF	Reserve Risk Factor
RRR Reserve Runoff Ratio Runoff ratio	The ratio of (a) incurred movement from the initial reserve date to the latest available evaluation date, for all constituent AYs combined, including "all prior year" reserve development to (b) the Initial Reserve.
UW	Underwriting
UW risk	Underwriting risk – the combination of premium risk and reserve risk.

6. REFERENCES

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APPENDICES

Appendix 1 - UW Risk - RBC Components R_4 & R_5

This Appendix describes the calculation of R_4 and R_5 risk values and the simplifications we use in our IIO analysis. The NAIC 2010 RBC Forecasting & Instructions publication and Odomirok, et al, 2014, provide further details on the RBC calculations.

R_4 – Reserves

The RBC charge for reserve risk, R_4, measures the susceptibility of loss reserves to adverse development.[35,36]

We describe, the calculation of the R_4 risk charge amount as follows: For each of 19 RBC LOBs,[37] we multiply the net carried loss and loss adjustment expense (LAE) reserve by the industry-RRF adjusted for the difference between company and industry experience; we reduce that result to recognize future investment income; and, we reduce it further with credits for the presence of loss-sensitive business[38] within a company's portfolio. We calculate the sum of the reserve risk RBC by LOB; we reduce that total to reflect diversification across the 19 LOBs; and we increase that value by the growth charge, if applicable. The result is further increased by a portion of the R_3-Reinsurance Credit Risk charge.

For each RBC LOB, Appendix 1/Exhibit 1 below shows the key items used in the calculation of R_4:

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[35] Feldblum, Sholom, "NAIC Property/Casualty Risk-Based Capital Requirements," Proceedings of the Casualty Actuarial Society, 1996, www.casact.org/pubs/proceed/proceed96/96297.pdf, page 324

[36] The reserve risk charge does not, measure the adequacy of reported reserves for the company. The assessment of reserve adequacy is addressed outside the RBC framework through measures including financial examinations, regulatory examinations, and the Statement of Actuarial Opinion provided by each individual company's appointed actuary.

[37] RBC UW risk values are determined using data in the Annual Statement Schedule P, which shows 22 LOBs. RBC calculations treat occurrence and claims made LOBs for other liability and products liability on a combined basis and treat non-proportional property and non-proportional financial on a combined basis, leaving a net of 19 LOBs.

[38] Loss sensitive business in that for which additional income can be collected by the insurer if experience is worse than bands specified in the contract. Detailed definitions are contained in the NAIC Annual Statement Instructions for Schedule P Part 7. Also, it is described in Feldblum, 1996 pages 344-349.

Appendix 1/Exhibit 1
Components of Reserve Risk (R_4)
Line references are to RBC Form PR0016 in the 2010 RBC Formula

Item	Risk Element By LOB:	Description
1	Net Unpaid Loss and Loss Adjustment Expense (Lines 6 and 7)	The risk charge varies with the volume of the company's carried reserve, net of reinsurance, gross of all interest discount other than workers compensation tabular discount on indemnity claims.
2	Reserve Risk Factor (RRF) "Industry Loss and Expense RBC %", (Line 4)	A factor applied to all companies, selected by NAIC to represent the reserve risk at the desired safety level.
3	Factor to adjust for company reserve development experience that is better or worse than the industry (own-company adjustment), (Lines 1-3)	Compares the company reserve development over the most recent nine years with the reserve development for the average company.[39]
4	Adjustment for Investment Income, (also called Investment Income Offset for reserves, or IIO_R), (Line 8)	A factor applied to all companies, selected by the NAIC to represent the potential for investment income to offset adverse loss development. Based on 5% per annum investment return.
5	Adjustment for loss sensitive business (Lines 10, 11)	Reduces the risk charge if unfavorable experience can be offset by increases in premium on loss sensitive business.
	For all LOBs combined:	
6	Loss Concentration Factor (LCF), (Line 14)	A discount for the extent to which company has a diversified distribution of reserves by line.
7	Growth charge (RBC Form PR0015)	For companies with premium growth in excess of 10% per year over the past three years.
8	Portion of reinsurance credit risk from R_3 (RBC Form PR0011)	50% of R_3-reinsurance credit risk, if R_3-reinsurance credit risk is lower than the otherwise determined reserve risk.[40] Zero otherwise.

[39] Annually the NAIC determines the reserve development factor to be used by each company eligible to apply the own-company adjustment. The reserve development for the average company is the unweighted average reserve development of those companies, using the prior year's data.

[40] The 50% 'transfer' from credit risk to reserve risk applies only if the reserve risk without the Reinsurance Credit Risk component is larger than 50% of the reinsurance credit risk plus credit risk other than reinsurance credit risk, as is most often the case. Otherwise the entire R_3-Reinsurance Credit Risk charge is added to the R_3-Other Credit Risk charge.

In formula terms, the calculation is as follows:

R_4 = {∑ [Over all RBC LOBs] (Initial $\text{Reserve}_{LOB,COMPANY}$)* [((1.0+$RRF_{LOB}$*$(3)_{LOB,COMPANY}$) *$IIO\_R_{LOB}$ -1.0)- $(5)_{LOB,COMPANY\ LOB}$]}*$LCF_{Company}$*(7)+0.5 *(8),

where items (3), (5) and (7) are defined in Appendix 1/Exhibit 1.

As we describe in Section 1, in this paper we make certain simplifications. We treat the own-company adjustment (Appendix 1/Exhibit 1 Line 3) as 1.0. We treat the reduction for loss sensitive business (Appendix 1/Exhibit 1 Line 5) and the growth charge (Appendix 1/Exhibit 1 Line 7) and as 0%. We do not include any of =the R_3-Reinsurance Credit Risk Charge (Appendix 1/Exhibit 1 Line 8). We use the reserve for loss and DCCE and rather than the reserve for loss and all loss adjustment expense. We do not use the LCF (Appendix 1/Exhibit 1 Line 6) because are analyzing risk charges by LOB.

We refer to the simplified portion of R_4 as RRC_{LOB}, defined as follows:

$RRC_{LOB,COMPANY}$={(Initial $\text{Reserve}_{LOB,COMPANY}$)*[((1.0+$RRF_{LOB}$)*$IIO\_R_{LOB}$-1.0)]}

$RRC\%_{LOB,COMPANY}$ = $RRR_{LOB,COMPANY}$/ Initial $\text{Reserve}_{LOB,COMPANY}$

R_5 – Premium

The RBC charge for premium risk, R_5, measures the potential that the company's future business will be unprofitable. Because the volume of business that will be written during the year is unknown, the volume of the most recent calendar year is used as a proxy for future premium writings.[41]

[41] Feldblum, "Risk Based Capital Requirements," 1996, pages 334-335.

We describe, the calculation of the R_5 risk charge amount as follows: For each of 19 RBC LOBs,[42] we multiply the NWP by the industry-PRF adjusted for the difference between company and industry experience; we reduce that result to recognize future investment income; and, we reduce it further with credits for the presence of loss-sensitive business[43] within a company's portfolio. The LOB $_{risk}$ charge is that result, plus the company all-lines operating expense ratio, less 100%. We calculate the sum of the reserve risk RBC by LOB; we reduce that total to reflect diversification across the 19 LOBs; and we increase that value by the growth charge, if applicable.

For each LOB, Appendix 1/Exhibit 2 below shows the key items used in the calculation of R_5.

[42] RBC UW risk values are determined using data in the Annual Statement Schedule P, which shows 22 LOBs. RBC calculations treat occurrence and claims made LOBs for other liability and products liability on a combined basis and treats non-proportional property and non-proportional financial on a combined basis, leaving a net of 19 LOBs.

[43] Loss sensitive business in that for which additional income can be collected by the insurer if experience is worse than bands specified in the contract. Detailed definitions are contained in the NAIC Annual Statement Instructions for Schedule P Part 7. Also, it is described in Feldblum, 1996 pages 344-349.

Appendix 1/Exhibit 2
Components of Premium Risk (R_5)
Line references are to RBC Form PR0016 in the 2010 RBC Formula

Item	Risk Element	Description
	By LOB:	
1	Net written premium (NWP) (Line 8)	The risk charge varies with the volume of the company's net written premium in the latest year.
2	Premium Risk Factor (PRF) "Industry RBC Loss and Expense Ratio" (Line 4)	A factor applied to all companies, selected by NAIC to represent the premium risk at the desired safety level.
3	Factor to adjust for company LR experience that is better or worse than the industry (own-company adjustment), (Lines 1-3)	Compares the company LR over the most recent ten years with the LR for the average company.[44]
4	Company Expense Ratio (CER)	Other UW expenses for all LOBs combined.
5	Adjustment for Investment Income, (also called Investment Income Offset, or IIO_P) (Line 7)	A factor applied to all companies, selected by NAIC to represent the potential for investment income to offset unfavorable LRs. Based on 5% per annum investment return.
6	Adjustment for loss sensitive business (Lines 10 and 11)	Reduces the risk charge if unfavorable experience can be offset by increases in income on loss sensitive business.
	For all LOBs combined:	
7	Premium Concentration Factor (PCF), (Line 14)	A discount for the extent to which company has a diversified distribution of premium by line.
8	Growth charge (PR00015)	For companies with premium growth in excess of 10% per year over the past three years.

In formula terms, the calculation is as follows:

$$R_5 = \{\sum \text{[Over all RBC LOBs]} (NWP_{LOB,\ COMPANY})*[PRF_{LOB}*(3)_{LOB,COMPANY}*IIO\_P_{LOB}+ CER_{COMPANY} -100\%-(6)]\}*PCF_{LOB}*(8)_{LOB,COMPANY}$$

Where items (3), (6) and (8) are defined in Appendix 1/Exhibit 2.

As we describe in Section 1, in this paper we make certain simplifications. We treat the

[44] Annually the NAIC determines the LR for each company eligible to apply the own-company adjustment. The LR for the average company is the unweighted average LR of those companies, using the prior year's data.

own-company adjustment (Appendix 1/Exhibit 1 Line 3) as 1.0. We treat the reduction for loss sensitive business (Appendix 1/Exhibit 1 Line 5) and the growth charge (Appendix 1/Exhibit 1 Line 7) and as 0%. We do not use the PCF (Appendix 1/Exhibit 2 Line 7) because are analyzing risk charges by LOB. We also use NEP rather than NWP.

We refer to the simplified portion of R_5 by LOB as the PRC_{LOB}, defined as follows:

$PRC_{LOB, COMPANY}=\{(NEP_{LOB,COMPANY})* [(PRF_{LOB} *IIO\_P_{LOB}+ CER_{Industry, LOB} -100\%)]\}$

$PRC\%_{LOB, COMPANY}= PRC_{LOB, COMPANY}/NEP$

Appendix 1/Exhibit 3, below, shows the PRF, RRF, IIO_P, IIO_R. RC%s, PRC%s and RRC% values using the 2010 RBC Formula with our simplifications

Appendix 1/Exhibit 3
2010 RBC Formula UW Risk Parameters

Line of Business	2010 RBC Factors							
	Premium					Reserves		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	PRF	IIO_P	2010 Avg Co Exp	PRC% before IIO	PRC% (Discounted)	RRF/ RRC% before IIO	IIO_R	RRC% (Discounted)
A- Homeowners/Farmowners	0.937	0.954	30.10%	23.8%	19.5%	0.201	0.938	12.7%
B- Private Passenger Auto	0.969	0.925	25.20%	22.1%	14.8%	0.192	0.928	10.6%
C- Commercial Auto	0.988	0.890	30.80%	29.6%	18.7%	0.230	0.911	12.1%
D - Workers Compensation	1.033	0.839	26.80%	30.1%	13.5%	0.324	0.830	9.9%
E - Commercial Multi-Peril	0.921	0.896	35.50%	27.6%	18.0%	0.465	0.876	28.3%
F1 - Med Malp - Occ	1.822	0.767	28.00%	110.2%	67.7%	0.431	0.865	23.8%
F2 - Med Malp - CM	1.092	0.827	28.00%	37.2%	18.3%	0.306	0.883	15.3%
G - Special Liability	0.904	0.898	34.40%	24.8%	15.6%	0.257	0.890	11.9%
H - Other Liability	1.042	0.816	30.30%	34.5%	15.3%	0.511	0.852	28.7%
I - Special Property	0.941	0.949	32.60%	26.7%	21.9%	0.191	0.966	15.1%
J - Auto Physical Damage	0.843	0.971	25.20%	9.5%	7.1%	0.112	0.976	8.5%
K - Fidelity/Surety	0.883	0.904	45.40%	33.7%	25.2%	0.325	0.940	24.6%
L - Other	0.893	0.947	35.80%	25.1%	20.4%	0.172	0.967	13.3%
M - International	1.169	0.905	40.00%	56.9%	45.8%	0.327	0.874	16.0%
N&P - Reinsurance Prop/Fin	1.349	0.893	24.70%	59.6%	45.2%	0.286	0.901	15.9%
O - Reinsurance - Liabiity	1.507	0.777	24.70%	75.4%	41.8%	0.769	0.838	48.2%
R - Products Liability	1.214	0.774	31.10%	52.5%	25.1%	0.643	0.841	38.2%
S - Financial/Mort Guarantee	1.482	0.884	28.50%	76.7%	59.5%	0.200	0.926	11.1%
T - Warranty	0.883	0.904	35.90%	24.2%	15.7%	0.325	0.940	24.6%
Weighted Avg	0.976	0.912	28.75%	26.3%	17.3%	0.369	0.919	19.7%
Average effect of IIO					-34%			-47%

(4) -PRC% before IIO = (1)+(3)-100%
(5) – PRC% Discounted = (1)*(2)+(3)-100%
(8) – RRC% Discounted = (1+(6))*(7)-100%

Appendix 2 – Calculating IIOs with Selected Interest Rates

The IIOs are premium and reserve discount factors by LOB, which are calculated with payment patterns and interest rates. In this Appendix we show our payment patterns, we illustrate how we calculate the IIOs, and we compare our indicated IIOs to the IIOs in the

2010 RBC Formula.

Payment Patterns and Discount Factors – LRs used in PRF Calculation

We develop AY payment patterns using a methodology like the 1986 IRS approach, as was done in the current calibration approach.[45,46] Specifically, for each LOB, we calculate the AY payment pattern as follows:

- Use the 2010 industry total Schedule P Part 1, for each AY, to determine the ratio of (a) cumulative loss and LAE paid through December 31, 2010 to (b) ultimate loss and LAE, evaluated at December 31, 2010.

  We use these ratios as the cumulative payment percentage for the paid development age of each component the AY.
- Determine the difference between those cumulative payment ratios.

  We use these differences as the incremental AY payment percentages.
- Next, consistent with past RBC payment pattern calibrations, we extend the payment patterns for LOBs with payment patterns that appear longer than 10 years (for 10-year lines) or longer than 2 years (for 2-year LOBs).

[45] NAIC RBC 2010 Formula, page 20 says:
Line 08 (for reserves) – Adjustment for Investment Income - This discount factor assumes a 5 percent interest rate. For lines of business other than workers compensation and the excess reinsurance lines, the payment pattern is determined using an IRS type methodology applies to industry-wide Schedule P data by line of business; otherwise a curve has been fit to the data to estimate the average payout over time. The discount factor for workers' compensation is adjusted to reflect the tabular portion of the reserves that is already discounted. The factors are provided by the NAIC and are shown on the Underwriting RBC Summary by line of business. The 2010 Formula provides similar explanation for premium risk, Line 07.

[46] There are more precise ways to measure payment patterns, but we did not apply those in our work. We believe using the "IRS-type" methodology is sufficient for the analysis described in this Paper. Use of more refined methods is a matter for future research.

Appendix 2/Exhibit 1 shows the resulting payment patterns by LOB and illustrates the calculation of the LR discount factors.

Payment Patterns and Discount Factors – Loss Reserves used in RRF Calculation

For each LOB, we calculate the loss reserve payment pattern as follows:

- From the 2010 industry total Schedule P Part 1, we determine the percentage of the total reserve attributable to each AY component, evaluated at December 31, 2010.
- For each accident year component, we calculate the expected payments in future calendar years, using the appropriate 'tail' portion of the selected AY payment pattern.
- We assume payments are made in the middle of each calendar year.
- We calculate the discounted value for each AY component of the reserve.
- We calculate the total of the discounted AY portions of the total reserve.
- The discount factor is the ratio of the discounted total reserve to the undiscounted total reserve.

We show the AY discount factors at interest rates of 5% (current rate for IIOs), 8.3% (the 3-year duration US Treasure rate for 1988 that we use for AY 1989) and 1.4% (the 3-year duration US Treasure rate for 2009 that we use for AY 2010).

Appendix 2/Exhibit 2 illustrates the calculation of the reserve date discount factors.

Appendix 2/Exhibit 3 compares (a) the 2010 IIOs, based on 5% per annum interest rates to (b) the IIOs we calculate using our selected payment patterns with a 5% per annum interest rate. Our new IIOs differ somewhat from the current IIOs, as the payment patterns are not identical, but are close for most lines and close overall.

Appendix 2/Exhibit 1
Selected Payment Patterns and Discount Factor at Various Annual Interest Rate

Year	A	B	C	D	E	F1	F2	G	H	I
0	70.7%	43.0%	25.7%	23.4%	42.2%	1.2%	6.5%	36.6%	9.4%	55.9%
1	22.0%	29.0%	22.6%	22.4%	21.3%	3.1%	16.9%	25.0%	15.7%	33.8%
2	3.7%	12.8%	19.6%	12.6%	12.6%	7.7%	18.9%	13.4%	15.2%	10.2%
3	1.3%	7.5%	14.2%	7.6%	4.7%	13.5%	14.7%	8.7%	13.8%	
4	1.0%	3.9%	8.4%	5.4%	5.9%	17.2%	14.4%	6.6%	12.2%	
5	0.1%	1.7%	4.2%	5.4%	5.8%	14.7%	11.0%	4.5%	9.1%	
6	0.6%	0.8%	2.4%	2.4%	2.0%	11.9%	4.3%	0.2%	5.7%	
7	0.3%	0.4%	1.2%	2.3%	0.4%	13.5%	4.6%	1.4%	4.6%	
8	0.1%	0.2%	0.4%	2.6%	0.9%	4.2%	3.6%	0.6%	3.0%	
9	0.1%	0.2%	0.2%	2.6%	1.4%	4.8%	2.6%	-1.6%	0.5%	
10	0.2%	0.4%	1.1%	2.6%	2.7%	8.3%	2.5%	4.7%	5.5%	
11				2.6%					2.8%	
12				2.6%					1.4%	
13				2.6%					0.7%	
14				2.6%					0.3%	
15									0.2%	
PV factors by LOB @ sample interest rates										
@ 5%	0.955	0.924	0.890	0.837	0.902	0.760	0.830	0.902	0.814	0.951
@8.3%	0.929	0.882	0.830	0.760	0.850	0.645	0.743	0.849	0.724	0.922
@1.4%	0.986	0.977	0.966	0.946	0.969	0.922	0.946	0.969	0.940	0.985

Year	J	K	L	M	O	N&P	R	S	T
0	90.3%	22.9%	54.7%	14.7%	3.5%	19.5%	6.7%	6.2%	85.4%
1	9.5%	32.9%	29.6%	48.6%	19.7%	38.1%	10.1%	33.5%	14.1%
2	0.3%	22.1%	15.7%	17.9%	20.5%	14.2%	14.8%	20.1%	0.5%
3		11.0%		0.0%	-4.8%	6.5%	8.3%	20.1%	
4		11.0%		8.0%	9.1%	7.8%	13.3%	20.1%	
5				4.9%	32.0%	8.0%	8.8%		
6				1.1%	-3.5%	1.8%	6.8%		
7				3.1%	1.7%	1.3%	6.8%		
8				1.6%	1.9%	-0.1%	4.6%		
9					-0.2%	-0.1%	4.4%		
10					4.3%	1.5%	7.9%		
11					3.9%	0.8%	3.9%		
12					3.5%	0.4%	2.0%		
13					3.1%	0.2%	1.0%		
14					2.8%	0.1%	0.5%		
15					2.5%	0.0%	0.2%		
PV factors by LOB @ sample interest rates									
@ 5%	0.971	0.907	0.948	0.896	0.782	0.887	0.777	0.881	0.969
@8.3%	0.954	0.854	0.917	0.839	0.682	0.827	0.673	0.815	0.950
@1.4%	0.992	0.972	0.984	0.968	0.927	0.965	0.927	0.963	0.991

Notes on the following page.

Appendix 2/Exhibit 1- Notes
Selected Payment Patterns and Discount Factor at 5% Annual Interest Rate
Notes

- For most 10-year LOBs we use the 10-year AY payment pattern derived in that fashion from Schedule P Part 1.
- For workers compensation, other liability, non-proportional reinsurance (property and lability) and product liability we extend the payment pattern to 15 years assuming uniform payment or exponentially decaying payments, by year for years 10-15.
- For the 2-year LOBs we use payment patterns of 3 years or 5 years.
- For lines H and R, other liability and products liability, we use the combined H1 (occurrence) and H2 (claims made) and the combined R1 (occurrence) and R2 (claims made), respectively.

  For LOB O the data method has some time periods with negative incremental payments. We found that the results are not particularly sensitive to that feature of the payment pattern, and we did not over-ride those negative values.

Appendix 2/Exhibit 2
Sample Calculation of Reserve Discount Factor
5% Annual Interest Rate
HO/FO LOB=A

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Row #	Year	Pattern Pattern	Discount Factor @5%	Reserve for Year at the maturity (month):									
1	**Remaining Reserve:**			14,274,039	3,346,587	1,853,884	860,646	454,076	459,418	213,429	112,488	75,384	65,191
2	**Maturity (yrs):**			1	2	3	4	5	6	7	8	9	10
3.1	1	70.7%											
3.2	2	22.0%	0.976	10,704,251	1,667,502	647,943	368,767	48,706	214,013	90,855	33,075	17,260	65,191
3.3	3	3.7%	0.929	1,778,717	586,850	516,718	52,761	188,836	104,467	36,040	18,182	58,124	
3.4	4	1.3%	0.885	625,990	467,998	73,929	204,557	92,177	41,440	19,813	61,231		
3.5	5	1.0%	0.843	499,211	66,958	286,625	99,851	36,565	22,781	66,721			
3.6	6	0.1%	0.803	71,424	259,600	139,911	39,609	20,101	76,717				
3.7	7	0.6%	0.765	276,914	126,719	55,500	21,775	67,692					
3.8	8	0.3%	0.728	135,171	50,267	30,511	73,327						
3.9	9	0.1%	0.694	53,619	27,634	102,747							
3.10	10	0.1%	0.661	29,477	93,059								
3.11	11	0.2%	0.629	99,265									
4	Total	100.0%		13,561,046	3,066,016	1,667,899	776,011	403,357	423,430	195,947	103,377	70,866	63,620
5	**Total Undiscounted**			21,715,142									
6	**Total Discounted**			20,331,569									
7	**Discount Factor**			0.936									

Notes on following page

Appendix 2/Exhibit 2 - NOTES
Sample Calculation of Reserve Discount Factor

Column 1-4:

Column 1: Row #
Column 2: Year in which reserve will be paid, where 1= first year; 2= next year, etc.
Column 3: Percentage of AY claims that would be paid, calendar year-by-calendar year
Column 4: Discount factor for payments in 0.5 years, in 1.5 years, etc., measured from the present, using 5% per annum interest rate.

Columns 5-14 – There is one column for each maturity year. Each row is as follows:

Row 1: The AY components of the 2010 HO reserve at December 2010, e.g., AY 2010 unpaid at 2010 is age 1, $14.3 million, … AY 2000 unpaid at 2010 is age 10, $65 thousand.
Row 2: The number of years of payment already made for the AY component in Row 1.
Rows 3.1-3.11 for Maturity Years 1 – 11 show the expected future payments for each of the AY components on row 1, by future year of payment, based on the payment pattern shown in column 2, which was derived as described in Appendix 2/Exhibit 1. We calculate the values in rows 3.1 to 3.11, using column 3. For example, in column 14 the entire amount unpaid at 2010 of from claims arising in AY 2000 (age 10) will be paid in the subsequent year; hence, the value in row 3.2 equals the value in row 1. For example, in column 13, the amount unpaid at 2010 from claims arising in AY 2001 (age 9) will paid in the remaining two years. The payment pattern in row 4, column 3.10 and 3.11, shows that 0.2% of the full AY payments (0.2037% before rounding) are expected to be paid in year 11 and 0.1% of the full AY payments (0.0605% before rounding) are expected to be paid in year 10. The total is 0.2542% of the full accident year, 23% (.0605/.2542) in year 1 and 77% (.2037/.2542) in year 2, producing $17k and $58k shown.
Row 4: The sum of the row 3 values times the column 4 discount factors for all years.
Row 5: The total undiscounted, i.e., the sum of the payments in calendar years 2011 and beyond, equals the sum of values in row 2.
Row 6: The total discounted, i.e., the sum of the discounted payments in calendar years 2011 and beyond, the sum of values in row 4.
Row 7: The reserve discount factor, 0.936, is the ratio of row 6, the total of discounted future payments, to row 5, the total of undiscounted future payments.

Appendix 2/Exhibit 3
2010 RBC IIO's vs. Calculated IIOs at 5% annual interest rate

Lines of Business	Premium		Reserves	
	2010 IIO_P	Indicated IIO_P	2010 IIO_R	Indicated IIO_R
A- Homeowners/Farmowners	0.954	0.955	0.938	0.936
B- Private Passenger Auto	0.925	0.924	0.928	0.928
C- Commercial Auto	0.890	0.890	0.911	0.913
D - Workers Compensation	0.839	0.837	0.830	0.822
E - Commercial Multi-Peril	0.896	0.902	0.876	0.888
F1 - Med Malp - Occ	0.767	0.760	0.865	0.862
F2 - Med Malp - CM	0.827	0.830	0.883	0.883
G - Special Liability	0.898	0.902	0.890	0.895
H - Other Liability	0.816	0.814	0.852	0.856
I - Special Property	0.949	0.951	0.966	0.967
J - Auto Physical Damage	0.971	0.971	0.976	0.975
K - Fidelity/Surety	0.904	0.907	0.940	0.935
L - Other	0.947	0.948	0.967	0.964
M - International	0.905	0.896	0.874	0.922
N&P - Reinsurance Prop/Fin	0.893	0.887	0.901	0.899
O - Reinsurance - Liabiity	0.777	0.782	0.838	0.817
P - Products Liability	0.774	0.777	0.841	0.840
S - Financial/Mort Guarantee	0.884	0.881	0.926	0.941
T - Warranty	0.904	0.969	0.940	0.980
Total	0.912	0.913	0.878	0.877

Calibration IIO's use the payment patterns by LOB from Exhibit 1, applied to LRs and reserves as described in Appendix 2/Exhibits 1 and 2.

APPENDIX 3 – SELECTION OF INTEREST RATES

Our calibration in Section 3 uses the interest rates from US Treasury securities with 3-year durations. We also analyze the effect of using interest rates based on 5-year and 10-year duration interest rates.

Also, we consider the various ways to attribute interest rates to LRs and RRRs by year. Our calibration in Section 3 uses the average interest rate for the year that proceeded the year that the premium was earned and for LRs and the year-ending 12 months prior initial reserve date for RRRs. That is, for example, we use 1988 average interest rates to discount the AY 1989 LRs and the December 31, 1989 RRR.

We consider two alternatives. We analyze data with LRs and RRRs discounted based on the interest rate in the year the year earned, i.e., using 1988 average interest rates for AY 1988 LRs and December 31, 1988 RRRs. Also, we analyze data with RRRs discounted based on average December interest rates, i.e., average interest rate for December 1988 for the 1988 RRR.

PVA vs. NVA Indicated Risk Charges with Alternative Interest Rate Selections

The values in Appendix 3/Exhibits 1A and 1B, below, are comparable to the total rows of Tables 6A and 6B, for premium and reserve risk, respectively, with various alternative interest rate approaches. This exhibit shows the PVA and NVA statistics for the weighted average of all LOBs. The first row of these exhibits, "3Year/Prior Year," is the interest rate we use in Section 3. The values in that row equal the weighted average lines from Exhibits 6A and 6B. The values in the other rows represent the comparable weighted average values based on the alternative interest rate selections, as described in the Interest Rate column.

Consistent with our observations regarding the 3-year interest rates we used in Section 3, we observe that for all interest rates selections the PVA shows lower values for the slope, R-squared values and ratios of slope to standard error. This indicates that that the variation of risk with interest rate is more random with PVA than NVA, regardless of the interest rate selection.

Appendix 3/Exhibit 1A - Premium
Indicated PRC% vs. Interest Rate Graph – Slope, R-squared and Slope/Standard Error
PVA vs. NVA
All-Lines Weighted Averages

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Interest Rate		Slope		R-sqr		Slope/std err	
		NVA	PVA	NVA	PVA	NVA	PVA
3Year	Prior Year	1.36	(0.22)	10%	2.7%	1.25	(0.16)
3Year	Current Yr	0.87	(0.70)	8%	5.0%	0.78	(0.69)
5Year	Prior Year	1.33	(0.22)	9%	2.8%	1.12	(0.14)
5Year	Current Yr	0.93	(0.61)	8%	4.2%	0.78	(0.53)
10Year	Prior Year	1.15	(0.35)	8%	3.3%	0.88	(0.22)
10Year	Current Yr	0.90	(0.59)	8%	4.3%	0.71	(0.41)

Appendix 3/Exhibit 1B - Reserves
Indicated RRC% vs. Interest Rate Graph – Slope, R-squared and Slope/Standard Error
PVA vs. NVA
All-Lines Weighted Averages

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Interest Rate		Slope		R-sqr		Slope/std err	
		NVA	PVA	NVA	PVA	NVA	PVA
3Year	Avg Dec	3.04	(0.40)	13%	4.5%	1.60	(0.44)
3Year	Prior Year	3.86	0.37	18%	3.3%	1.98	0.09
3Year	Current Yr	3.38	(0.09)	16%	3.4%	1.78	(0.20)
5Year	Avg Dec	3.48	0.04	14%	3.9%	1.66	(0.18)
5Year	Prior Year	4.01	0.56	17%	3.6%	1.85	0.16
5Year	Current Yr	3.76	0.29	16%	3.4%	1.79	0.01
10Year	Avg Dec	3.94	0.47	14%	3.8%	1.64	0.04
10Year	Prior Year	3.87	0.48	14%	3.6%	1.57	0.07
10Year	Current Yr	3.99	0.53	15%	3.6%	1.66	0.11

Selecting Interest Rate Approach

In Appendix 3/Exhibits 1A and 1B, above, the values shaded green are the lowest (most consistent with PVA assumptions) among the interest rates selections. The values shaded yellow are the second lowest (second most consistent with PVA assumptions) among the interest rates selections.

The 3-year duration, prior year average, interest is the most often "green" selection. Hence, our selection of that rate for Section 3. In implementation, both 3-year and 5- year duration are reasonable choices, and the choice might vary by LOB and between reserves and premium.

Appendix 3/Exhibit 2 shows the indicated PRC% and RRC%, by LOB, for each of the alternative interest rates. On average, across LOBs, differences in all line average indicated risk charges due to interest rate duration and time period is less significant for premium (PRC%) than for reserves (RRC%).

- PRC% - The difference in PRC% between 3, 5 and 10-year duration interest rates is approximately a 50 basis points[47] (comparing columns 2, 4 and 6 for current year average or columns 3, 5 and 7 for prior year average, in premium section of Appendix 3/Exhibit 2), respectively.

  RRC% - The difference for RRC% is more than 100 basis points[48] (comparing columns 2, 5 and 8 for current year, 3, 6 and 9 for prior year, or 4, 7 and 10 for "December, from reserve section of Appendix 3/Exhibit 2), respectively.

[47] 50 basis points is about 2% of the risk charge, using the 90th percentile indicated premium risk charge with 'actual' interest rates

[48] 100 basis points is about 4% of the risk charge, using the 90th percentile indicated reserve risk with 'actual' interest rates

- PRC% - The difference in PRC% between current year average and prior year average is 50 basis points (comparing columns 2 and 3 for 3-year duration, columns 4 and 5 for 5-year duration or columns 6 and 7 for 10-year duration), respectively.
- RRC% - The difference in RRC% between current year average and prior year average is over 100 basis points (comparing columns 2 and 3 for 3-year duration, columns 5 and 6 for 5-year duration or columns 8 and 9 for 10-year duration interest rates), respectively.

Appendix 3/Exhibit 2
Indicated PRC% and RRC% - PVA – Various Bases for Interest Rate Selection
87.5th Percentile UW Safety Level/Zero Risk Margin

PRC% - Present Value Model - Various Bases for Interest Rate Selection

Lines of Business	(1) With 5% IIO	(2) Current Year 3Yr	(3) Prior Year 3Yr	(4) Current Year 5Yr	(5) Prior Year 5Yr	(6) Current Year 10YR	(7) Prior Year 10YR	(8) % 2010 NEP
A- Homeowners/Farmowners	22.3%	22.1%	22.5%	21.9%	22.2%	21.5%	21.7%	15%
B- Private Passenger Auto	15.8%	15.3%	15.6%	14.9%	15.1%	14.4%	14.6%	23%
C- Commercial Auto	20.9%	20.1%	20.4%	19.4%	19.7%	18.9%	19.1%	4%
D - Workers Compensation	14.5%	14.1%	15.2%	13.0%	14.0%	11.8%	12.6%	7%
E - Commercial Multi-Peril	16.1%	15.6%	16.1%	15.2%	15.5%	14.5%	14.8%	7%
F1 - Med Prof Liab-Occ	44.9%	46.7%	49.4%	45.2%	47.5%	43.2%	45.7%	1%
F2 - Med Prof Liab-CM	28.0%	27.6%	29.0%	26.7%	27.8%	25.4%	25.9%	2%
G - Special Liability	22.5%	21.6%	22.1%	20.7%	21.6%	20.0%	21.1%	1%
H - Other Liability	14.5%	14.3%	15.0%	12.8%	13.7%	11.5%	12.2%	9%
I - Special Property	11.3%	11.0%	11.4%	10.9%	11.1%	10.5%	10.7%	7%
J - Auto Physical Damage	6.6%	6.5%	6.6%	6.4%	6.5%	6.3%	6.4%	16%
K - Fidelity/Surety	6.3%	6.1%	6.3%	5.7%	5.7%	5.3%	5.3%	1%
L - Other	24.5%	24.2%	24.4%	24.0%	24.2%	23.8%	23.9%	2%
M - International	16.9%	18.0%	19.9%	17.3%	18.7%	16.7%	17.5%	0%
N&P - Reinsurance-Prop/Fin	39.0%	36.2%	37.0%	35.0%	36.5%	34.6%	34.8%	1%
O - Reinsurance-Liabiity	27.7%	23.2%	25.3%	22.6%	23.9%	22.1%	21.8%	1%
R - Products Liability	23.5%	26.8%	27.9%	24.5%	26.1%	21.5%	23.2%	1%
S - Financial/Mort Guarantee	67.2%	60.0%	59.7%	59.0%	59.5%	57.5%	59.0%	2%
T - Warranty	71.5%	73.4%	72.4%	72.9%	72.3%	72.1%	72.1%	0%
Weighted Average	17.1%	16.5%	17.0%	16.0%	16.4%	15.4%	15.8%	100%

RRC% - Present Value Model - Various Bases for Interest Rate Selection

Lines of Business	(1) With 5% IIO	(2) Current Year 3Yr	(3) Prior Year 3Yr	(4) Dec Avg 3Yr	(5) Current Year 5Yr	(6) Prior Year 5Yr	(7) Dec Avg 5Yr	(8) Current Year 10Yr	(9) Prior Year 10Yr	(10) Dec Avg 10Yr	(11) % 2010 Rsv
A- Homeowners/Farmowner	14.3%	14.7%	13.9%	14.4%	14.2%	13.5%	13.8%	13.5%	13.0%	13.3%	4%
B- Private Passenger Auto	9.5%	10.0%	9.1%	9.6%	9.5%	8.6%	9.1%	8.9%	8.1%	8.6%	16%
C- Commercial Auto	23.5%	24.3%	22.5%	23.0%	23.5%	22.0%	22.4%	22.4%	21.4%	21.8%	4%
D - Workers Compensation	9.6%	11.8%	8.3%	10.1%	9.9%	6.8%	8.2%	7.8%	5.4%	6.7%	20%
E - Commercial Multi-Peril	32.3%	32.2%	30.6%	31.4%	31.5%	29.7%	30.5%	30.1%	28.7%	29.6%	6%
F1 - Med Prof Liab-Occ	13.4%	14.5%	11.6%	13.4%	13.6%	10.5%	12.7%	11.7%	9.7%	10.8%	2%
F2 - Med Prof Liab-CM	-2.3%	0.2%	-1.5%	-1.3%	-0.8%	-2.6%	-2.0%	-2.3%	-3.7%	-3.1%	3%
G - Special Liability	30.3%	27.4%	25.4%	26.3%	26.5%	25.0%	25.3%	23.9%	24.2%	24.5%	1%
H - Other Liability	30.3%	31.3%	29.6%	30.0%	30.2%	28.4%	29.0%	28.6%	27.3%	27.5%	22%
I - Special Property	24.4%	24.6%	24.1%	24.5%	24.4%	24.0%	24.2%	24.2%	23.9%	23.9%	2%
J - Auto Physical Damage	10.3%	10.2%	9.7%	10.0%	10.0%	9.7%	9.9%	9.8%	9.6%	9.6%	1%
K - Fidelity/Surety	40.2%	41.3%	39.0%	40.3%	40.9%	38.1%	39.9%	40.2%	37.8%	39.5%	1%
L - Other	22.2%	22.8%	22.2%	23.0%	22.5%	22.1%	22.5%	22.4%	22.1%	21.9%	1%
M - International	29.2%	25.6%	23.4%	25.5%	24.7%	22.2%	23.9%	23.1%	21.6%	22.4%	0%
N&P - Reinsurance-Prop/Fin	27.8%	27.4%	27.4%	26.8%	27.0%	26.2%	26.4%	26.3%	25.6%	26.2%	1%
O - Reinsurance-Liabiity	34.6%	35.7%	33.4%	35.6%	34.2%	31.5%	33.4%	32.2%	30.3%	31.0%	6%
R - Products Liability	56.5%	55.0%	52.9%	51.8%	52.5%	50.6%	52.1%	49.9%	48.2%	50.7%	3%
S - Financial/Mort Guarantee	-1.2%	-11.1%	-10.6%	-11.2%	-11.1%	-10.8%	-11.3%	-11.3%	-11.2%	-11.5%	5%
T - Warranty	4.0%	-15.6%	-16.8%	-16.1%	-15.8%	-16.8%	-16.3%	-16.1%	-16.9%	-16.6%	0%
Weighted Average	19.7%	20.1%	18.3%	19.0%	19.0%	17.2%	18.0%	17.6%	16.3%	16.9%	100%

Note: Because essentially no companies have an industry distribution by LOB, the weighted average is a summary, but does not represent a typical company.

Upgrading an Existing Capital Model — A Common Risk Driver Application

Predictive Analytics in Capital Modeling Working Party

Abstract: In this paper we apply a simple regression model to link performance of a D&O insurance line of business to the S&P 500 economic variable from an economic scenario generator (ESG). The regression structure is incorporated into an existing economic capital model. The distribution of the error term is constrained so that the final distribution of the D&O line is equivalent to the distribution previously used. We explore the impact this model change has on the existing correlation structures.

Keywords: ERM, regression, correlation, risk drivers

INTRODUCTION

The literature describes the benefits of using common risk drivers compared to the use of correlation matrices for inducing dependency relationships among risks in economic capital models. However, there is little guidance on how to calibrate the risk drivers, and still less guidance on how to introduce such linkage into an existing Economic Capital Model (ECM).

In this paper we develop a mathematical relationship between an economic variable and a line of business (LOB) in a company capital model. We then show a method of implementing that relationship, with a process that minimizes the impact on the existing LOB distribution while inducing correlation, as desired, between economic risk and the insurance risk.

The motivation for this paper is twofold. First, this is an interesting application of common risk drivers. Second, the process we demonstrate can be used to incorporate informative external variables or other sources into an existing economic capital model, with minimum disruption to the company's existing ECM.

We provide a linked illustrative Excel workbook that shows our calculations and the Tables and Figures we present in this paper.

EXISTING CAPITAL MODEL FRAMEWORK

We address the following situation:

- The company has an ECM in place. Quarterly updates and reporting are established practices.
- Individual insurance LOBs are represented by defined risk distributions in the ECM.
- Investment risk is modeled using an economic scenario generator (ESG).
- A correlation matrix represents relationships between LOBs.
- There is no explicit correlation in the model between economic risk and insurance risk variables.

Historical results, industry research and management judgement all indicate that the risk distribution for the D&O LOB is influenced by economic conditions; in this case we represent this with a broad stock index.

The company would like to introduce an explicit relationship between the D&O LOB risk distribution and economic variables *without* changing the overall D&O LOB risk distribution[1]. To illustrate our approach, we use the S&P index as the economic variable.[2]

Expert Judgment Framework

The company experts believe, with support in recent historical data and anecdotal evidence, that D&O results are influenced by economic conditions. These economic conditions include stock market movements. The company has two prior expectations:

- Prior Expectation #1 (PE1): If the stock market performs worse than expected over the projected period, then underwriting (UW) results will be worse than planned.
- Prior Expectation #2 (PE2): If the stock market performs better than expected over the projected period, then, to a lesser degree than is the case in Prior

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[1] We note that while the D&O LOB risk distribution does not change, the total company risk distribution will change due to the introduction of correlation between investments and insurance risk. Moreover, changes to the D&O LOB risk distribution will change the effect of correlations between D&O and other LOBs, absent offsetting changes.

[2] In Appendix 1 we discuss some of the alternatives we considered.

Expectation #1, the UW results will also be worse than planned.[3]

PE1 can be implemented with a linear relationship between the S&P error distribution and the loss ratio error distribution.

Combining PE1 and PE2 requires a more complicated relationship between the S&P error and the loss ratio error. That relationship exhibits a "turning point" from which both positive and negative deviations of the S&P index from expected produce increases in the LR above plan. In our example we use a quadratic relationship as a reasonably simple form meeting that requirement.

Statistical Framework

The table below shows notation we use in this paper. Variables with double dots, e.g. $\ddot{x}$, refer to historical data. Unmodified variable names, e.g. x, refer to values from distributions.

Examples of Notation used in this document

x_{err}	The distribution around an error variable one year in the future.
$x_{err,i}$	A simulated observation from x_{err}
$\ddot{x}_{err}$	The observed distribution of the historical prediction errors of X
$\ddot{x}_t$	A historical observation of the variable used to calculate historical errors.
$\ddot{x}_{err,t}$	A historical observation of the error around the predicted historical variable.
k	Number of observations from historical dataset. k = Max(year) – Min(year) + 1
N	Number of simulations run in model
E(X)	Expected value of X
SD(X)	Standard deviation of X
$F_X(x)$	Cumulative distribution function of X; i.e. Probability that X is less than x.

Definition of variables

We apply that framework to the variables of interest in our work as follows:

x-based variables refer to the explanatory variable – in this paper, the S&P index.

y-based variables refer to the predicted variable – in this paper, the loss ratio.

[3] PE1 has the obvious interpretation. PE2 is related to increased M&A activity or increased risk-taking activity, including M&A activity. Combined, the two Prior Expectations imply that predictable economic conditions produce the best UW results. Note also that 'higher' and 'lower' economic conditions are not the same as 'up/down', but rather are whether the trends in the market continue in the manner that are predicted when the planned LR is selected.

S&P Variables

The x-based variables we use in the paper are as follows:

$\ddot{x}_t$ = S&P Index at the end of year t

$$\ddot{x}_{err,t} = \frac{\text{S\&P Index at the end of year } t}{E(\text{S\&P Index at the end of year } t)}$$

For example, if at 12/31/2012 the expected value of the S&P index one year in the future, 12/31/2013, was 1,400, while the actual index value was 1,479, then

$\ddot{x}_{err,2013}$= 1,479/1,400 = 1.056.

In a company setting, the expected value of the S&P index would be obtained from an ESG. In this report we use the following simplified forecasting approach:

$$E(\text{S\&P Index at the end of year } t) = \left(\ddot{x}_{t-1} + d_{t-1}\right) * (1 + r_{t-1}),$$

where,

d_t = dividend in year t

r_t = 1yr Treasury yield at end of year t

and,

$\ddot{x}_{err}$=Observed distribution of historical S&P prediction errors

x_{err} is the distribution of the error around the predicted level of the S&P index.
Error in this paper is defined as the ratio of actual to expected.

$$x_{err,i} = \frac{\text{S\&P Index simulation i}}{Avg(\text{S\&P Index simulations})}$$

Loss Ratio Variables

The y-based loss ratio variables we use in the paper have analogous definitions, as follows:

$\ddot{y}_t$ represents the observations of historical loss ratios.

$$\ddot{y}_{err,t} = \frac{\text{Historical Loss Ratio for year } t}{\text{Planned Loss Ratio for year } t}$$

The historical loss ratio observations we use in this analysis are Schedule P industry accident year loss ratios for the Other Liability – Claims Made statutory line of business at the latest available maturity, up to 120 months developed for the most mature data points.

The planned loss ratio is the accident year loss ratio at 12 months; used as a proxy for the planned loss ratio.

$\ddot{y}_{err}$=Observed distribution of historical Loss Ratio prediction errors

y_{err} is the distribution of the error around the predicted ultimate loss ratio for the line of business, that is, the ultimate loss ratio for accident year 2017.

Error in this paper is defined as the ratio of actual to expected, so

$$y_{err,i} = \frac{\text{Loss Ratio simulation i}}{Avg(\text{Loss Ratio simulations})}$$

Original and Revised Loss Ratio Error Distribution

At this point we must introduce some notation to distinguish between our original loss ratio error distribution and the revised one we are producing with this alternative model. The goal is for the two to be as close as possible.

y_{err}^{Orig} has a lognormal distribution LN(1, sigma) and implicitly contains variability related to economic conditions.

In our alternative model:

$y_{err}^{Rev} = yP_{err}^{Rev} * yI_{err}^{Rev}$, where

yP_{err}^{Rev} is the distribution of the loss ratio predicted, based on the S&P index, versus expected loss ratio, which will be defined later as a regression on x_{err}.

yI_{err}^{Rev} is the variability in the loss ratio error that is independent of the S&P Index volatility, or rather, x_{err}. The distribution of yI_{err}^{Rev} reflects the residual (multiplicative basis) in yP_{err}^{Rev} versus y_{err}^{Rev}.

Our goal is for y_{err}^{Rev} to be as close as possible to y_{err}^{Orig}.

If y_{err}^{Orig} and yI_{err}^{Rev} and yP_{err}^{Rev} were all lognormally distributed, then we could determine a closed-form solution for yI_{err}^{Rev}. However, we want more flexibility in the choice of underlying distributions of y_{err}^{Orig} and yP_{err}^{Rev}. Therefore, we take an approach that allows us to select any appropriate regression model to represent the risk of the LOB explained by economic variables and then define the distribution of the error term using a beta distribution for its flexibility.

Initial status

In the current capital model structure, the insurance risk distribution is derived from historical observations of ultimate loss ratios by accident year. There is no explicit assumed relationship between the D&O LOB and the S&P index.

In Table 1, below, the observed S&P Index in column 1 is from public sources. In practice, the predicted S&P Index in column 2 would be the average following year-end S&P index from the Economic Scenario Generator used in the company economic capital model. In this example, the S&P prediction is equal to the sum of the prior year's S&P value and dividend inflated at the 1-year US Treasury rate (see formula above, in S&P Variables subsection).

Table 1
Historical Data

AY	(1)	(2)	(3)	(4)	(5)	(6)
	S&P Index		Loss Ratio (LR)		Observed Error	
	Predicted	**Observed**	**Planned**	**Observed**	$\ddot{x}_{err}$	$\ddot{y}_{err}$
1990	359.7	328.8	60.3	62.4	0.914	1.035
1991	364.1	388.5	70.0	59.1	1.067	0.844
1992	417.2	435.6	71.7	65.7	1.044	0.917
1993	464.2	466.0	73.3	54.5	1.004	0.744
1994	495.9	455.2	76.4	60.1	0.918	0.787
1995	502.1	614.6	76.0	58.5	1.224	0.769
1996	660.9	743.3	73.9	58.0	1.125	0.784
1997	799.9	962.4	72.0	68.1	1.203	0.945
1998	1031.8	1190.1	66.7	86.8	1.153	1.302
1999	1260.9	1428.7	68.4	108.9	1.133	1.592
2000	1531.8	1330.9	71.1	110.2	0.869	1.549
2001	1418.9	1144.9	74.2	105.6	0.807	1.423
2002	1185.9	899.2	68.9	93.7	0.758	1.359
2003	927.3	1080.6	66.1	65.4	1.165	0.989
2004	1111.9	1199.2	65.9	45.4	1.079	0.689
2005	1252.2	1262.1	63.0	45.1	1.008	0.716
2006	1340.5	1416.4	64.3	49.0	1.057	0.763
2007	1513.4	1479.2	67.0	56.2	0.977	0.839
2008	1557.3	877.6	72.6	81.9	0.564	1.128
2009	909.3	1110.4	70.6	77.2	1.221	1.095
2010	1138.1	1241.5	70.3	73.5	1.091	1.045
2011	1267.9	1243.3	71.1	74.1	0.981	1.044
2012	1271.3	1422.3	69.5	74.2	1.119	1.068
2013	1455.9	1807.8	65.9	68.4	1.242	1.038

Mean	1.030	1.019
Std Dev	0.164	0.262

The loss ratio information in columns 3 and 4 is data from industry Schedule P data for the Other Liability (Claims Made) LOB for reporting years 1999-2016. Column 3 is the industry loss ratio developed to 120 months. Column 4 is the Schedule P accident year loss ratio at 12 months; used as a proxy for the planned loss ratio.

Columns 5 and 6 show actual versus expected results as ratios, column 5 = column 2/column 1 and column 6=column 4/column 3.

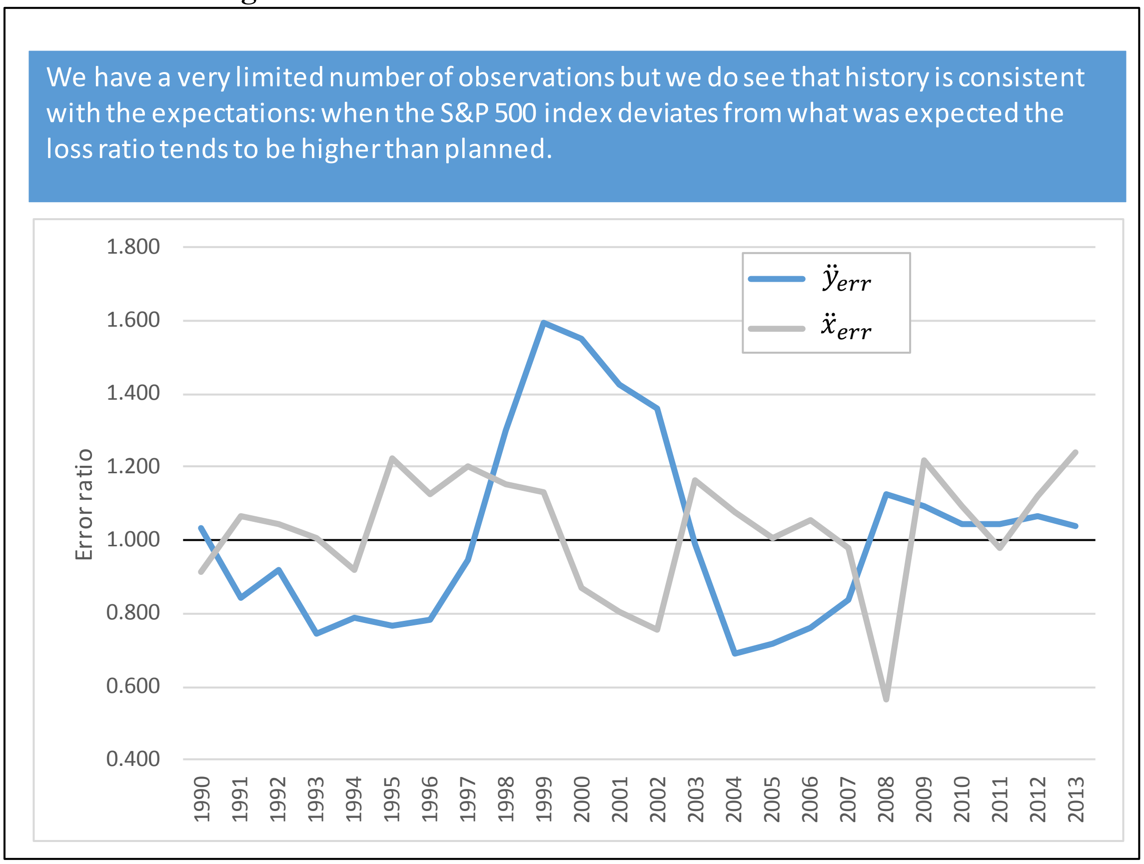
Original Loss Ratio Error Distribution

The historical data for the Loss Ratio error, Table 1 column 6, $\ddot{y}_{err}$, is seen to have a mean of 1.019 and standard deviation 0.262. We used this information to parameterize y_{err}^{Orig}, assuming a lognormal distribution with mean of 1 and standard deviation of 0.262, implies mu and sigma of 0.258 and -0.033.

Predicted Loss Ratio Error Distribution

In Figure 1 below, we compare the observed Loss Ratio error, Table 1 column 6, and the S&P index error, Table 1, column 5. In this Figure, we see that for 2000 and 2001 the S&P error indices ($\ddot{x}$ values) are lower than expected, i.e., below the "1.0" line, and Loss Ratios errors ($\ddot{y}$ values) are higher than expected, i.e., above the "1.0" line. This is consistent with Expectation #1. In general, we see LR errors and S&P index errors are on opposite sides of the "1.0" line, as expected.

Figure 1 – Loss Ratio Error vs S&P Index Error timeline



We examine Expectation #2 in Figure 2, the scatter plot on the following page.

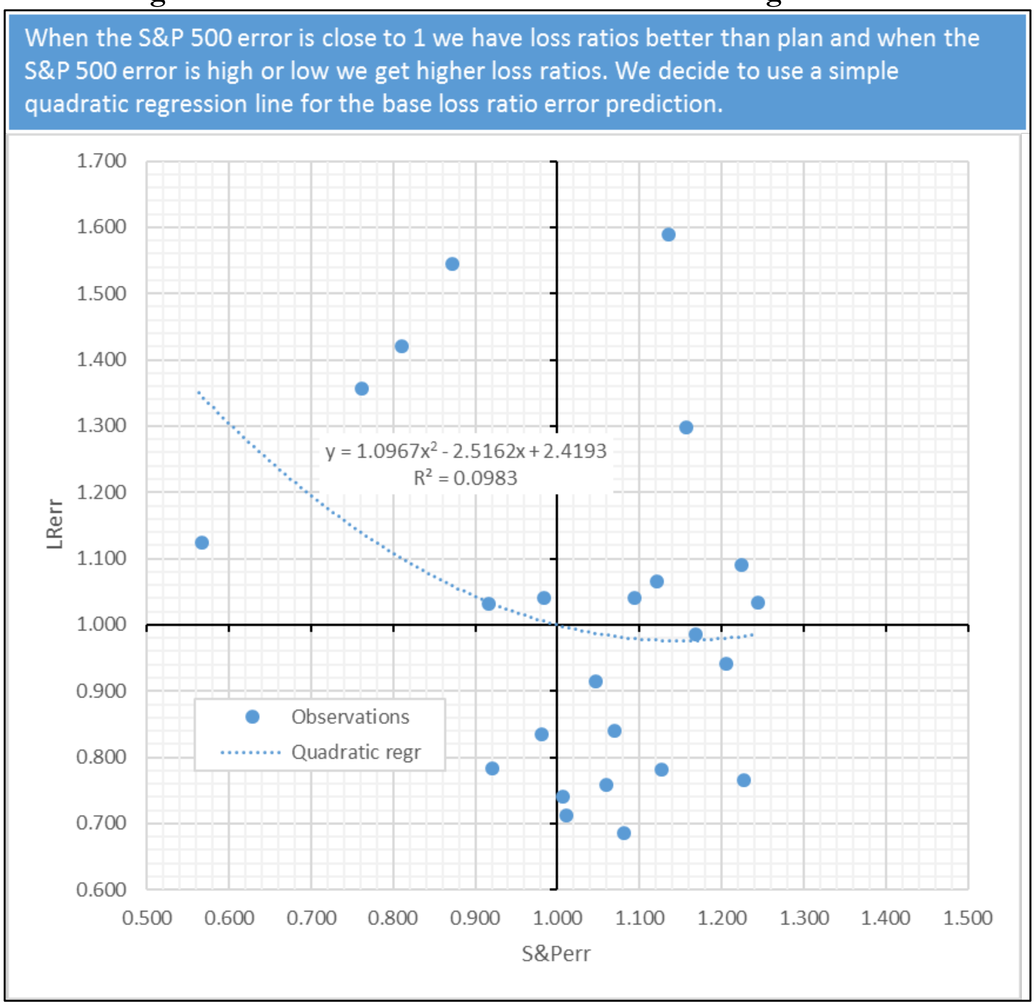
We consider four regions of S&P Error, i.e, S&P compared to expected S&P, as follows: 10% worse than expected ("<0.9); between 10% worse than expected and expected, i.e., 'bad' but not too bad (≥0.9; ≤1.0); between expected and 10% better than expected, i.e., good but not too good (≥1.0; ≤1.1)' and more than 10% better than expected (>1.1). Table 2 below shows the LR error performance in each of those regions

Table 2
LR Error Values by S&P Region

(1)	(2)	(3)	(4)	(5)
S&P Error Range	LR Error			
	<1.0	≥1.0	Total	% ≥1.0
<0.9	0	4	4	0%
≥.9; ≤1	2	2	4	50%
≥1;≤1.1	6	1	7	86%
>1.1	4	5	9	44%

Consistent with Expectation #1, the LR errors become increasingly favorable (0% over 1.0 to 86% over 1.0) in the first three regions. Consistent with Expectation #2, the LR errors become less favorable as the S&P error increases into the fourth region.

Figure 2 - Loss Ratio Error vs S&P Index Error regression



With the concession that historical data is limited, we find that it is consistent with our underwriting Expectation #2. To apply our assumptions of Expectation #1 and Expectation #2, in Figure 2, we fit the data to a quadratic curve[4]:

$$yP_{err}^{Rev} = b_2\left(x_{err}\right)^2 + b_1 x_{err} + b_0 \quad \text{(Regression coefficients found in Figure 2)}$$

[4] LINEST(LRerr,SPerr^{1,2}).

This fitted curve implies that the most favorable LR variance from expected arises when the S&P index error is about 15%. The expected LR variance becomes less favorable as the S&P index variation becomes more favorable beyond that level.

Fitting Error Distributions

At this stage we have specified yP_{err}^{Rev}, and we can leave this historical sample set to fit the distribution of the error term using simulated data consistent with our capital model. Remember, the goal of the error fit is to produce a final loss ratio error distribution, y_{err}^{Rev}, that very closely matches that of the original, y_{err}^{Orig}.

Table 3 column 2, shows the first 10 of 1,000 simulations of the S&P Index one year from the model valuation date, using the company Economic Scenario Generator. The mean value of the 1,000 simulations for the forecast year, Average (2), is 1,845.2. This is the S&P prediction in our model. Column 3 = Column 2 / Average (2). Then we calculate column 4, yP_{err}^{Rev}, using the quadratic relationship determined in the previous step:

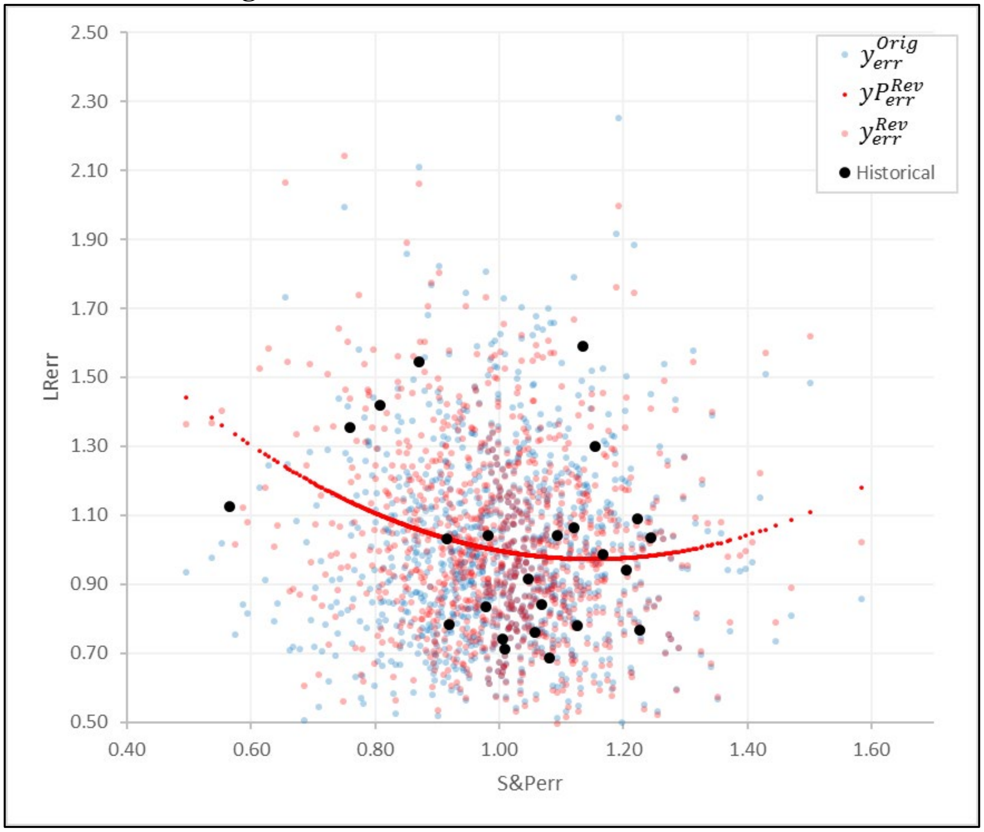
Column (4) = b_0 + b_1*(Column 3) + b_2*(Column 3)^2

Table 3
S&P Value from Company ESG and yP_{err}^{Rev} from Quadratic Model
(First 10 of 1000 simulations)

Simulated S&P Index and Error			
(1)	(2)	(3)	(4)
Sim	**S&P 500 Simulation**	x_{err}	yP_{err}^{Rev}
1	1291.6	0.70	1.20
2	1893.0	1.03	0.99
3	1638.8	0.89	1.05
4	2041.1	1.11	0.98
5	1652.2	0.90	1.05
6	1934.4	1.05	0.99
7	2022.5	1.10	0.98
8	1812.5	0.98	1.01
9	1819.4	0.99	1.00
10	1946.3	1.05	0.99

Figure 3, below, shows all the 1,000 simulated data points and the fitted quadratic relationship.

Figure 3 – Loss Ratio error vs S&P error



The next step is to calibrate yI^{Rev}_{err}.

As noted in the statistical framework, we assume that yI^{Rev}_{err} has a beta distribution. We select the beta distribution parameters to minimize the difference between y^{Rev}_{err} and y^{Orig}_{err} at selected percentiles. We do this using Excel Solver. [5]

Table 4 shows the fitted parameters for the beta distribution and the solver constraints used in fitting those parameters. Column 7 contains the solver constraints for the parameters in Column 6. The mean of the beta distribution is constrained to 1 so that the mean of y^{Rev}_{err} equals the mean of y^{Orig}_{err}.

[5] An analytical method to determine the parameters of the beta distribution requires numerical analysis. For simplicity's sake we used the excel solver.

Table 4
Beta Distribution Parameters and Solver Constraints

Final Values for Beta Distribution		
(5)	(6)	(7)
yI_{err}^{Rev} Beta parameters		Solver constraints
minY	0.21	0.0500
maxY	8.48	10.0000
alpha	8.80	0.1000
beta	83.32	0.1000
E[Impact]	1.000	1.0000
SD[Impact]	0.252	

The solver iteration that produces the values in Table 4 column 6 uses the values from Tables 4 and 5, column 8-14 as follows:

- Column (8) shows random values from a uniform distribution.
- Column (9) shows the observations $yI_{err,i}^{Rev}$, generated from a beta distribution with the parameters in (6) and the random variable values in (8).
- Column (10) = column (9) * $yP_{err,i}^{Rev}$, for i=1 to 1000, from Table 2, column 4. Column 10 is the new modeled y_{err}^{Rev} distribution.

Table 5
(First 10 of 1000 simulations)

Revised y_{err} distribution		
(8)	(9)	(10)
U	yI_{err}^{Rev}	y_{err}^{Rev}
0.1276	0.72	0.86
0.2136	0.79	0.79
0.7166	1.13	1.18
0.6149	1.05	1.03
0.0737	0.66	0.70
0.7246	1.13	1.12
0.7151	1.13	1.10
0.3371	0.88	0.88
0.5454	1.00	1.01
0.6134	1.05	1.03

We continue the calculation as follows:

- Column 11 shows the selected cumulative distribution function percentiles at which we compare y_{err}^{Rev} and the newly calculated y_{err}^{Orig} distribution.
- Column 12 is the value of y_{err}^{Orig} at the cumulative distribution probability level in column 11. In this example, the original model is lognormal, so these are the inverse cumulative lognormal values for the CDF levels in column 11. The method, however, does not require a parametric distribution for column 12.
- Column 13 shows the new $y_{err}^{Rev} = yP_{err}^{Rev} * yI_{err}^{Rev}$.
- Column 14 is the difference between column 12 and column 13, squared. The objective function is the sum of column 14. We determine the beta parameters in column 6 using Excel Solver to minimize the sum of column 14. You should be aware that there are an infinite set of beta parameters that will result in y_{err}^{Rev} fitting our needs and rerunning the solver multiple times will return a different set of parameters. That is, the beta parameters are unstable, but that does not affect the utility of the outcome as, in our tests, the distribution of yI_{err}^{Rev} is stable.

Table 6
Minimizing Differences Between y_{err}^{Orig} and y_{err}^{Rev}

Fitting Beta Parameters			
(11)	(12)	(13)	(14)
Selected CDF levels	y_{err}^{Orig}	y_{err}^{Rev}	error
0.001	0.436	0.436	0.000
0.01	0.531	0.538	0.000
0.05	0.633	0.640	0.000
0.1	0.695	0.711	0.000
0.2	0.778	0.797	0.000
0.3	0.845	0.868	0.001
0.4	0.906	0.930	0.001
0.5	0.967	0.997	0.001
0.6	1.033	1.057	0.001
0.7	1.107	1.137	0.001
0.8	1.202	1.228	0.001
0.9	1.346	1.373	0.001
0.95	1.479	1.499	0.000
0.99	1.763	1.743	0.000
0.995	1.880	1.893	0.000
0.999	2.147	2.145	0.000
			Obj: 0.007

The quality of the fit between y_{err}^{Orig} and the newly constructed y_{err}^{Rev} is good, as evidenced by a comparison of columns 12 and 13.

CONCLUSIONS

We believe this is an interesting application of common risk drivers. Moreover, it is a demonstration of a process that can be used to incorporate informative external variables or other sources into an existing economic capital model with minimum disruption to the company's existing ECM.

SUPPLEMENTARY MATERIAL

We provide a linked illustrative Excel workbook that shows our calculations and the Tables and Figures we present in this paper.

AUTHORS

Principal Authors: Mario E. DiCaro and Allan M. Kaufman.

Allan M. Kaufman is a Managing Director at FTI Consulting. He also serves as Chairman of the Board of a Lloyd's managing agency. He is a Fellow of the CAS and a Member of the American Academy of Actuaries. He currently participates on the American Academy of Actuaries Property& Casualty Risk Based Capital Committee and recently led the CAS Risk Based Capital Dependency and Calibration Research Working Party. Allan has served in a wide variety of other CAS and American Academy of Actuaries roles.

Mario E. DiCaro is a Vice President at Tokio Marine HCC and is responsible for the capital modeling work done at the group level. He is a Fellow of the CAS and a Chartered Enterprise Risk Actuary (CERA). He currently serves on the committee of the Joint Risk Management Section, a collaboration of the CAS, CIA, and SOA.

Assistance from Shira Jacobson, FCAS

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APPENDIX 1

In this Appendix we discuss three further issues:

1. Correlations to other LOBs
2. Alternative Models
3. Sensitivity Testing Extreme Values

Correlation to other LOBs

In the existing capital model, after separating the risk into components, yI_{err}^{Rev}, the component *not* related to the economic variable, is still part of the existing correlation matrix, but yP_{err}^{Rev}, the economic component, is not. Absent other adjustments, the separation reduces the correlation to other LOBs of business in ways that are not desirable.

There are two approaches to retaining the desired correlation between the D&O LOB and other LOBs. The complex approach is to update all LOBs with risk drivers. That may not be practical.

Instead, in practice, we increase the correlation factors in the copula between the D&O LOB, yI_{err}^{Rev}, and the other LOBs. That is, we increase the row/column in the matrix until the measured output correlations were consistent with what they were prior to the model changes.

In that way, we retain the pre-existing LOB correlation but we add explicit correlation between D&O loss ratios and investment risk.

Alternative Models

In this paper we used a quadratic regression model to relate LRs to the S&P Index.

We considered alternative structures and alternative variables that we outline here.

For structure, we chose quadratic rather than linear for the reasons described in the paper, even though, as noted, the data to support that decision in limited. We also considered a kernel function that uses nearest neighbors to estimate values at various simulated points. The results were interesting but given the limited historical data for calibration we chose the quadratic model.

For variables we considered the following:

- Unemployment
- Interest rates

- Changes in corporate bond spreads and yields,
- Change in average CEO salaries
- The number of securities class action lawsuits

We chose to consider only one variable because the historical data for calibration is limited.

For practical reasons, we also choose to consider only variables for which we have simulation forecast values from the ESG. The last two variables are not available in the ESG.

Unemployment, interest rates, corporate spreads and yields, and the S&P 500 index *are* variables simulated by the ESG. Plotting the relationships between the historical loss ratios and the index movements quickly revealed that many different relationships could have been used. None were clearly better than the S&P Index, particularly for use in an illustration such as the one presented in this paper. With more historical experience, it might have been clear that a different model was superior.

Sensitivity Testing Extreme Values

In this paper we did not demonstrate any tests for model validity at extreme S&P error points. That is, what would the predicted loss ratio be if the ESG produces an S&P error of 2.5, or 0.2? In practice these sorts of outcomes need to be tested for or your model may produce embarrassingly impossible outcomes in those extremes.

Another Pioneering Use of DFA: New Zealand Earthquake Commission

Ian McLean

Abstract: The New Zealand Earthquake Commission (EQC) started using DFA (Dynamic Financial Analysis)[1] in 1994 and has used DFA commercially ever since. EQC was one of the pioneers in the application of DFA to the insurance industry. Other pioneering users at the same time are described in four papers in the Casualty Actuarial Society Forum, Spring, 1996. The development of models for EQC has not previously been fully described in the literature.[2] This paper describes the development of DFA models for EQC from the viewpoint of the user.

DFA BACKGROUND

A major theoretical basis for DFA was published in English in 1969: *Risk Theory - the Stochastic Basis of Insurance* by R.E. Beard et al.[3] It provided the theory and methodology for measuring total risk and return of insurance businesses.

Some years passed after this book was published before its methods were applied. Computers at that time were far too slow to carry out the number of simulations required, particularly when dealing with catastrophe risks where distributions typically have a long tail. The same problem also arose with the simulation of asset risk, where again and again firms have discovered to their chagrin that the probability distribution of their asset values has an unexpectedly fat tail.

Stochastic modelling was clearly needed to model total risk and return:

- Relationships between the various risks affecting insurance businesses are complex and the relationship between total risk and return cannot in general be calculated through solvable algorithms.
- Over a defined period of years many different sequences of events can occur. Assessing the outcome at the end of the multi-year period requires that all likely sequences be taken into account, according to the probability of each sequence. No method of summing the possible pathways exists, apart from simulation.

Monte Carlo modelling provided the solution. By using a large number of simulations, 100,000 or even more, it was possible to sufficiently reduce the confidence limits so as to make the results reliable. And it was possible to run these large numbers of simulations within a reasonable time - hours rather than days.

DFA using large numbers of simulations was starting to be used commercially about 1994. A second important book: *Practical Risk Theory for Actuaries* by Daykin, Pentikainen, and Pesonen was

1 The term "Dynamic Financial Analysis" has been largely superseded by the term "Capital Modelling." The earlier term is used here because it was current during the time of the events described in this article.

2 David Middleton provides an outline of the development amongst other modelling in Middleton (2002)

3 Beard (1969)

published by Chapman and Hall at the end of 1993.[4] This covered much the same material as the earlier Beard work of 1969 in a slightly more user-friendly, albeit less elegant, form.

In 1994 several firms were pioneering the use of DFA for commercial purposes. This innovation was described in several papers in the Casualty Actuarial Society *Forum*, Spring, 1996:[5]

- within Liberty Mutual; authors Douglas M Hodes et al.[6];
- by Tillinghast – Towers - Perrin for RenaissanceRe; authors Stephen P. Lowe and James N. Stanard[7];
- the *MIDAS model*, client and modellers unstated; authors Steven Thoede and Janet Haby[8];
- by INSTRAT for Insurance Corporation of British Columbia (ICBC); authors Rodney E. Kreps and Michael M. Steel.[9]

In all these four documented cases, the work was novel in that it modelled the overall financial statements of insurance businesses to forecast risk and return over multi-year time horizons, and it did so fast enough to be of commercial use.

At about the same time as that modelling was being developed by these firms, the Subcommittee on Dynamic Financial Models of the Casualty Actuarial Society was working on DFA. Its purpose was to:

…to discuss and provide guidance on the important issues and considerations that confront actuaries when designing, building or selecting dynamic financial models of property-casualty risks.[10]

In addition to providing sound advice for modellers and users of DFA, the Sub-Committee report in September 1995 contained a useful bibliography of the prior literature.

THE COMMISSION —- EQWD BECOMING EQC

In 1991, on becoming Chair of the Earthquake and War Damage Commission (EQWD), I was given the task of leading a reform of the Commission.

The reform was initiated by the new Minister in Charge of the Commission (Hon Doug Kidd, now Sir Douglas) and the Treasury. The basic structure of the Commission was retained, but with major change in the cover provided:

- only domestic property was covered, with commercial property phased-out;
- the cover was changed from an indemnity basis to repair or replacement;

4 Daykin (1993).

5 https://www.casact.org/pubs/forum/96spforum/

6 Hodes (1996)

7 Lowe (1996)

8 Thoede (1996)

9 Kreps (1996)

10 Van Slyke (1995) p1

- caps were imposed on cover at $100,000[11] for buildings and $20,000 for contents, both exclusive of GST[12]
- tsunami was added to the hazards covered
- war damage was no longer covered, and consequentially the name of the institution changed to the Earthquake Commission (EQC).

The changes were driven by a very competent board, including the Deputy Chair Trevor Roberts. David Middleton came in as General Manager and brought strong insurance expertise.[13]

Of significance in the development of DFA modelling were the Commission's reinsurance and investment policies.

Its reinsurance programme was in 1991 believed to be the largest catastrophe program in the world with NZD 1 billion cover (in excess of NZD 1 billion). The cover was placed by a consortium of three of the world's leading reinsurance brokers, and led by Lloyd's underwriters together with Swiss Re.

Government policy required EQC to follow an archaic investment policy. Apart from cash, all funds were invested in New Zealand Government stock. This policy was a relic of the quite recent time when almost all the funds held by the Crown[14] and Crown agencies were centralised and pooled. Similarly, the management of risk, to the extent that it was managed at all, was mostly centralised.

Because nearly all the Commission's assets were invested in New Zealand government stock, the Crown had effectively retained much of the risk brought to it by EQC's cover of catastrophes. The economic effect of EQC realising government stock in order to pay claims would be essentially the same as government issuing new stock. This risk was managed only to the extent that reinsurance was purchased.

Moreover, by law the Crown guarantees payment of "the liabilities of the Commission."[15] The Crown thus carried the risk of liabilities exceeding assets. This risk was open-ended and unmanaged.

A change in the external environment also had an impact on the Commission and was causing great concern. The turmoil in the Lloyd's insurance market, as the LMX (London market excess) spiral of the late 1980s collapsed, led to a reduction in the reinsurance capacity available to the Commission.

The Board of the Commission sought strategies to deal with its new situation. It commissioned Frank Russell Company Pty to report on investment policy, and specifically on asset allocation strategy. The report was received in December 1992. It was written by Prof Craig Ansley, then

11 All $$$ are NZD, worth 66 US cents in 20 Aug 2018

12 GST is a value-added tax

13 Mrs Demetra Kennedy as Acting General Manager had held the organisation together over a difficult time of uncertainty.

14 In NZ as in the UK, the Central Government is commonly referred to as "the Crown" when it acts financially or legally.

15 Earthquake Commission Act (1993), s16.

NZI Professor of Finance at Auckland University.

The report was wide reaching. It was based on a model of liabilities developed by Craig Ansley.[16] He pointed out that with expected losses at $71 million exceeding premiums at $69 million, we could not expect funds to accumulate.

In a letter to the Commission dated 24 February 1993, he elaborated on this statement. He pointed out that every elementary textbook on the Theory of Risk shows that if premiums net of expenses is equal to expected losses, and no income is earned on investments, '… (eventual insolvency) is *certain'* [his emphasis]. He calculated that under the existing regime with dividends and fees of $150 million being paid to the Crown, and a premium rate of $0.050 per $100, the approximate probability of ruin was 92%.

This struck terror into our hearts.

In parallel with our consideration of investment strategy and premium rates, we were also investigating what changes should be made in reinsurance strategy in the light of the reduced capacity available in the market.

Naively, I asked the question: how do we know we are getting the best bang for our buck from reinsurance? The answers from our advisers in the industry were not measures of value. Most were qualitative in nature: 'based on your objectives', 'protection providing comfort', 'sleep easy', etc. The only quantitative responses proposed meeting PML's at minimum cost, with PML's based on modelling of scenarios.

Craig Ansley advised that it had quite recently become practical to quantify total risk and return through modelling. The theory had been around for some time[17] but faster computers now enabled modelling to be done in reasonable time. Thus, this modelling had become commercially practical.

The basis was stochastic modelling using the Monte Carlo method. The tools were the same as he had already used in building the loss model for his previous report. But these tools had hardly as yet been used in the insurance industry.

At about the same time, the Finance Manager for EQC, Paul Martin, visited a technical agency of one of our reinsurance brokers: INSTRAT, then owned by Sedgwick Payne, and located in Seattle. Several key people were involved: Mr Donald Paterson who had a deep understanding of the nature of risk and how it could be managed, Dr Rodney Kreps who had published seminal papers on the pricing of reinsurance, and Mr Michael Steel who was an accomplished modeller.

Paul Martin reported back to us that INSTRAT was able to give us measures of what insurance programme would be optimal for our situation. The management and board of EQC were

16 Ansley (1993)

17 As expounded in Beard (1969).

sceptical, but Paul was quite insistent. Eventually we opened dialogue with INSTRAT and they too offered us modelling of our total risk and return.

Both Frank Russell and INSTRAT told us that they knew of nobody in the world currently using these techniques, apart from the Insurance Corporation of British Columbia for whom INSTRAT was building a model.

EQC decided to engage the firms to each build a model of EQC total risk and return. Using Monte Carlo modelling, each model was to some extent a "black box" in that their internal logical steps could not be sufficiently analysed to determine whether the results were trustworthy. The only way to effectively check on the results of the model was to have parallel models built independently but drawing on the same data. If the two models produced similar results, one could rely upon them. If the results differed, there would be an opportunity to explore within the models the reasons for the difference. We did of course also test the models by setting various parameters, to 0,1, or perhaps 1000 (according to the limits of the range of the parameter), and examining whether the results were plausible.

PROCESS OF DEVELOPING MODELS

The two models being developed were similar in some respects:

- Both used stochastic modelling based on the Theory of Risk.
- Both were to be based on a model of the Commission's earthquake losses already developed by Prof. Craig Ansley.
- Both used the same formula for reinsurance pricing as published by Dr. Rodney Kreps.[18]
- Both modelled the same financial flows and stocks of the Commission and used the same financial structure.
- Both had as outputs the risk and return of different strategies.

But in other respects, the models were different:

- The Frank Russell model was written in Visual Basic and was hardcoded, so that any changes in parameters had to be made by the programmer.
- The INSTRAT model was written in C^{++}. It had an interface which allowed EQC users to vary some of the parameters.
- The Frank Russell model was able to explore different financial structures, and the INSTRAT model allowed more reinsurance options to be evaluated.

Despite their differences, each model was capable of representing the total risk and return of EQC, and had the ability to optimise strategies by varying key parameters.

Modelling was mostly carried out over time-horizons of 5 or 20 years. The primary measure of risk was probability of ruin: i.e. EQC exhausting its capital and hence calling upon the Crown to support payment of claims. Other risk measures were also used as confidence in the modelling developed.

18 See Kreps (1990).

At first there were considerable differences in the output of the two models. These arose partly because of different understandings of the rules inherent in the Commission's structure. The Commission's business structure was prescribed by an Act of Parliament and differed significantly in many respects from the structure of commercial insurers.

In order to determine the reasons for differences in the outputs from the two initial models, a workshop meeting was held in Seattle with both sets of modellers and me participating. We followed through the way that each model depicted EQC structure and the basic logic of the models. We examined the differences in results and resolved differences in interpretation.

The models when revised produced reasonably consistent results and were then again fine-tuned in to further improve consistency.

Output from the models

The first major output was a report from Frank Russell based on their model. It dealt with:

- solvency and probability,
- the Crown underwriting fee, dividend and taxation,
- factors affecting the probability of ruin,
- the solvency limit, and
- the financial outlook.

The model tested variations such as the size of the *Fund* (essentially the capital or the surplus of the Commission), the fees charged by the Crown, asset allocation, and economic growth. The effect of these variations on the probability of ruin were indicated.

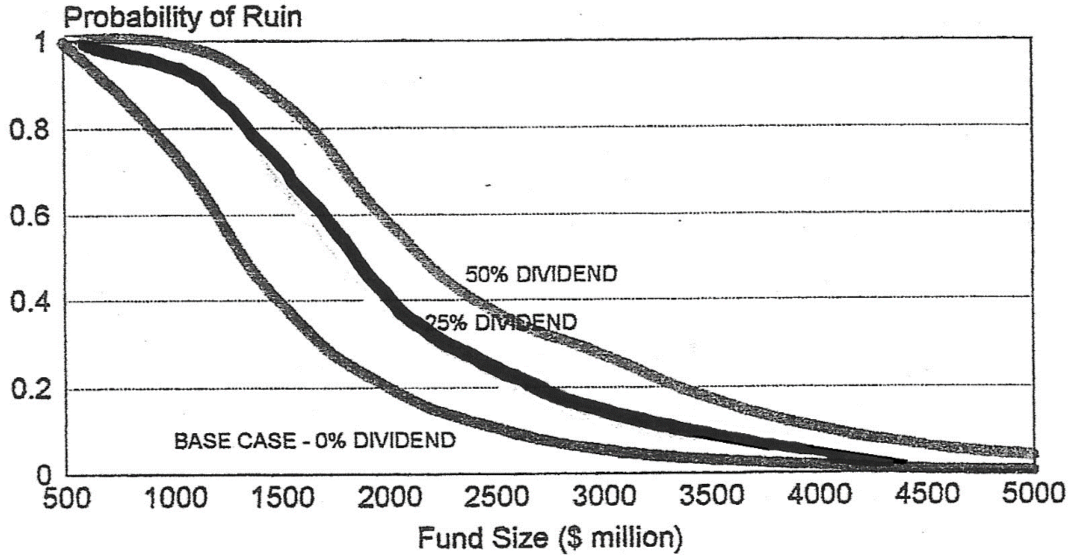
The report[19] included simple but powerful charts of which one example was Figure 14:

19 Ansley (1994)

Figure 14

PROBABILITY OF RUIN

Solvency Limit = Initial Fund Size



In this report, and in the earlier loss distribution report,[20] Craig Ansley gave several warnings which proved prescient in the light of the Christchurch earthquakes (these are discussed below).

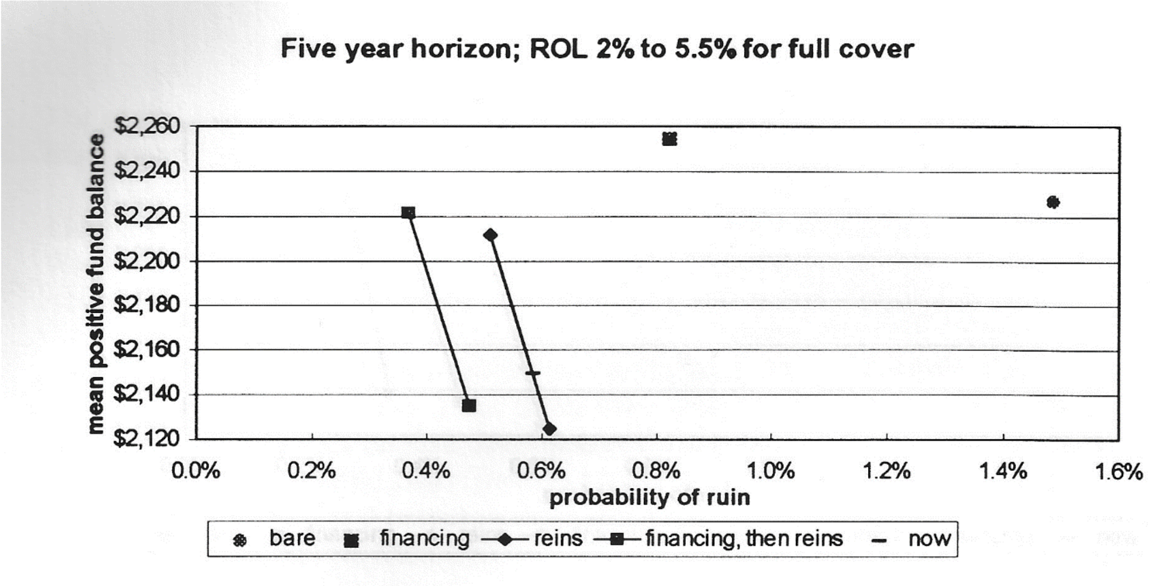
The INSTRAT model was used initially to optimise the existing conventional reinsurance program. The Commission was using all the capacity it could obtain from creditworthy reinsurers across the world at a reasonable price. The structure of this program was somewhat constrained. While moving the attachment point up or down by changing the deductible was a possibility, at high levels a minimum rate-on-line applied, despite the reduced risk. At low levels the rate-on-line increased quite sharply. Modelling indicated the optimal structure of the program.

One of the conclusions from the modelling was that multi-year covers would provide better value than single year covers at comparable rates. The advice was that such covers were difficult to obtain in the market at that time at a reasonable price. In subsequent years EQC used multiple year covers extensively once underwriters were prepared to write them.

The available capacity of traditional reinsurance was limited. It was less than the commission had purchased in the past. New products were at the time being developed in the finance and reinsurance markets in response to the shortage of traditional reinsurance capacity. We did not consider that these had reached sufficient maturity to be available for consideration by EQC. The INSTRAT model was used to explore other options of protection — especially post–event financing. This was the most significant form of alternative protection available at that time.

20 Ansley (1993)

One example is shown below in a chart provided in an INSTRAT report.



The explanation read: "Category 4 uses a combination of financing and a reinsurance with the financing paying first. This is a very interesting result because for each reinsurance ROL the fund size is slightly greater than option 2, and the probability of ruin is about 20% smaller. For this reason, we believe it is reasonable to take the view that option 4 dominates option 2 and should therefore be preferred."

In the event, EQC did not use post-event financing. The Treasury considered that any borrowing post-event should be done by the Crown itself, and that no prior arrangements were necessary for such borrowing. However, the analysis was of great value in demonstrating to the Treasury the level of risk to which it was exposed and the need to reduce the fees the Crown charged EQC (see below).

So, we explored the modelling of financing and reinsurance, and on the journey learnt about statistical dominance.

Use of the models

It took until well into 1994 before the results of the models were reasonably consistent. In the meantime, however, the Commission gained a much deeper understanding of its financial and risk structure, and the drivers of its business.

As soon as the models were developed, they were used by EQC in two specific ways:

- as a basis for negotiations with the Treasury and ministers, especially regarding investment policy and fees.
- In determining reinsurance strategy and purchases.

The Frank Russell model was used by EQC mainly as the basis of advice on its investment policy. It gave a strong quantitative basis for the Commission's discussions with The Treasury, who took some time to take any interest in the modelling, or show appreciation of its value.

The INSTRAT model was used in devising reinsurance strategy and tactics. From the first runs it gave a clear indication of the risk and return of various protection options.

Because the Frank Russell model was hardcoded, it was operated entirely by Craig Ansley and his staff. The output to EQC was in written reports.

The INSTRAT model was different in that it was designed to run on EQC laptops as well by INSTRAT itself. Considerable work was necessary to make the model user-friendly. I have copies of emails between Rodney Kreps and me discussing how version 13 of the INSTRAT model could be improved. Later, when a Windows interface was developed, the model became much easier to use.

The Commission also received reports from INSTRAT on protection strategies based on model results.

In 1994 I was able to say in the EQC Annual Report to Parliament that: "we have now developed a corporate financial model which enables changes in policy to be tested and their effect on EQC survival to be measured."[21]

Since then, DFA modelling has been continually used by EQC, sourced directly as well as through its reinsurance brokers.

VALUE OF THE MODELS

The models gave EQC board and management a much clearer appreciation of the risk and return of different strategies.

Prior to having model results, strategies were based on philosophical principles rather than hard numbers. Through the modelling, the risk brought to the Crown by EQC became more visible. This contributed to the decision by the government in 1995 not to require EQC to pay tax or dividends.[22]

In reinsurance purchases, modelling soon became routine. Initially it was done using the model constructed for EQC by INSTRAT, and later through modelling carried out by EQC's brokers as the basis of their advice. ReMetrica was used as a matter of course.

In 1999 EQC went to tender in for disaster risk and financial modelling. The tender was won by Aon who subsequently provided a model 'Minerva' to EQC.

The people involved

Craig Ansley continued to provide advice to EQC from Frank Russell after this modelling work was done.

21 See Annual Report of the Earthquake Commission 1993 – 1994, p8,

22 See Annual Report of the Earthquake Commission 1996-1997, p29

Two of the people involved in developing the INSTRAT model moved in 1996 to join Greig Fester, reinsurance brokers in London. Donald Paterson and Michael Steel developed there the modelling platform called ReMetrica. This took into account their earlier work, but had much greater functionality, and was much more user-friendly.

Greig Fester merged with Benfield Ellinger, and later with Aon. For many years this firm and its successors were lead broker or sole broker for the EQC reinsurance program. In this role they used ReMetrica extensively in providing advice on reinsurance structures and purchases.

Don Paterson and Michael Steel also led the use of DFA modelling by insurers and reinsurers worldwide. I retired from EQC in 1995, and later became associated with Greig Fester. My role was assisting with the application of ReMetrica to particular projects and demonstrating to insurers and reinsurers how DFA modelling could enable them to make better decisions.

CHRISTCHURCH EARTHQUAKE[23]

EQC was severely challenged by the Canterbury series of earthquakes commencing on 4th September 2010. It is natural to ask whether the losses experienced in these earthquakes were consistent with modelled results.

In one sense these earthquakes were not a test of the models. The model did not seek to measure the effect of single events or a series of events over a short period of time. The models dealt with periods of time of five years, 20 years or longer. Single events were modelled as they had been in the past, by the use of scenarios.

Nevertheless, the magnitude of the losses experienced in Christchurch was substantially more than the catastrophe models underlying initial DFA models had envisaged. This was so even after taking account of increases in building costs, higher building standards adding costs, and the increase in the number of dwellings.

The most important reason for the difference was that the Canterbury sequence of earthquakes was clearly a "black swan" occurrence.[24] The Greendale Fault event of September 2010 that initiated the Canterbury Sequence may have a recurrence interval of around 5300 years,[25] or it may not have moved for 20 to 30,000 years prior to 2010.[26] In either case, the recurrence interval is far beyond the time horizon over which insurers measure risk (and beyond the planning horizon for society itself!).

23 Referred to more formally as the Canterbury Earthquakes

24 As a point of interest, all the wild swans in New Zealand are black - Cygnus atratus.

25 Villamorl p21

26 Hornblow (2016), abstract. See also Van Dissen (2015) and Guiang (2014)

A BRANZ bulletin has summarised other unusual features thus[27]:

"The [Canterbury] series was unique in New Zealand and the world because:

- there were several major events in a short timeframe.
- the quakes were centred close to each other.
- there were high vertical accelerations.
- there was widespread liquefaction."

In its work about the time of the development of the models, EQC had explored the issue of multiple events, especially "after-shocks." The context was primarily the "hours clause" in reinsurance contracts. Perhaps we were lulled into false complacency by a belief that because the magnitude of after-shocks should reduce according to the Gutenberg–Richter law, the intensity of shaking and the damage caused by after-shocks would reduce similarly.

While the magnitude of the events subsequent to 4th September 2010 reduced generally as the Gutenberg–Richter law would predict, their peak ground accelerations did not. The extraordinarily high vertical accelerations of the 21 February event were an unexpected phenomenon and were due largely to the local geological structure.

The likelihood of liquefaction was appreciated, but the damage and loss it caused were not built into the initial models. The losses to EQC were also increased because it covered the loss of land under and around dwellings. Furthermore because of liquefaction and tectonic changes in the altitude of some residential areas, some land became worthless as building sites.[28]

EQC was warned by Craig Ansley about the limitations of modelling. Amongst other issues, he warned that:

- The statistical models for earthquake hazard were based on a very short data record…, and
- Damage ratio estimates were based on one set of data from one event which may not be representative of the likely level of damage for other locations….[29]

The event from which the damage ratios were derived was the Edgecumbe earthquake of 1987. Despite the village of Edgecumbe and surrounding farms being mostly built on structurally weak alluvial soils, liquefaction was limited in extent and area.[30] Thus, liquefaction after the Edgecumbe event did not cause significant damage. Hence the damage ratios based on that event were essentially derived from shaking-damage rather than liquefaction or any other hazards.

The major conclusion for modelling from the Canterbury earthquakes is that Black Swan events will still occur. Events beyond the time horizon of the model can and do occur. That means that insurers need an element of conservatism in the application of model results. Also, secondary hazards need to be incorporated into the modelled risks.

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27 BRANZ Bulletin 551.

28 For map see Te Ara Encyclopaedia of New Zealand; https://teara.govt.nz/en/zoomify/46379/eastern-suburbs-red-zone

29 Ansley (1993), pp 2-3.

30 I personally observed that limited liquefaction occurred after the Edgecumbe earthquake; also see Bastin (2017).

CONCLUSION: WHAT EQC LEARNT IN DEVELOPING DFA MODELS

Financial modelling has greatly developed in the decades since the pioneering EQC modelling described in this paper. Sophisticated models are now routine in the insurance industry.

However, the EQC experience is still relevant to innovation as well as the application of modelling:

1. **Insurers need to ask the "idiot questions."** Some leading figures in the insurance industry were disparaging and suggested that the EQC modelling was a waste of time and money. Without EQC asking questions, we may never have developed the models.
2. **Use the best people and firms that one can afford to buy**. It was quite fortuitous that Frank Russell in New Zealand and INSTRAT in Seattle had the vision and technical capacity to develop DFA models. But it was not fortuitous that EQC was using these firms, because they were world leaders in their everyday business.
3. **Be conservative in the application of modelled results**. The practice of major reinsurers (and insurers) of using multiple catastrophe models adds certainty. But models are limited in accuracy by the accuracy of the science supporting them.

AUTHOR BIOGRAPHY

In a varied career, author Ian McLean has worked as an agricultural-economist, led a United Nations Development Programme/Food and Agriculture Organization project in Tanzania, and was a member of the New Zealand Task Force on Economic and Social Planning. In his 12 years as a New Zealand Member of Parliament, he was Chair of the Parliamentary Expenditure Committee for one term. He was Chair of the New Zealand Earthquake Commission (EQC) from 1991 to 1995. He assisted in establishing the government catastrophe insurance schemes in Turkey and Romania under World Bank projects. He led the Review of the Civil Defence Emergency Management Response to the 22 February Christchurch Earthquake. McLean led the LakesWater Quality Society, which successfully stimulated the restoration program for the Rotorua Lakes. His publications include: *The Future for NZ Agriculture: Economic Strategies for the 1980s* and "Community Action and Science Help Restore New Zealand Lakes" in *Solutions Journal.* He has a BA from the University of New Zealand (Auckland) and is a Companion of the Queen's Service Order (QSO).

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A Note on Euler Allocation for Performance Measurement

Shayan Sen, ACAS, Ph.D.

Abstract

It is a well-known result that when an Euler allocation is used to allocate capital by line the overall expected return on capital can be increased by writing more business in lines where the expected return on allocated capital is greater than the overall companywide expected return. If the cost of equity capital varies by line, however, writing more business in these lines may not be the best choice for the company. In this paper we give a simple example that highlights why using an Euler allocation to allocate capital by line for the purpose of performance measurement is not always appropriate if the cost of equity capital varies by line.

Keywords. Capital allocation; Euler allocation; performance measurement; cost of equity capital.

1. INTRODUCTION

If a property and casualty insurer would like to allocate its capital for the purpose of performance measurement, the literature suggests numerous ways to do so.

One particularly appealing option is the Euler allocation. This method is the only one that guarantees that if a company writes more business in lines where the expected return on allocated capital is greater (less) than the overall company expected return on capital, then the overall company expected return on capital will increase (decrease). In the standard application of this approach, there is an underlying assumption that the risk of a line is reflected solely by the amount of capital allocated to it. In particular, the cost of equity capital is assumed to be the same for all lines of business.

There is, however, empirical evidence that the cost of equity capital may vary by line of business. In this case, we show that using an Euler allocation to allocate capital by line for performance measurement is not always appropriate.

1.1 Research Context

Venter [5] surveys capital allocation methods in the literature.

Tasche [4] defines the Euler allocation method for allocating capital and discusses some important properties of this method. Tasche [3] derives the key result regarding the use of Euler allocation for performance measurement. In Tasche's approach, the risk of a line is reflected by the amount of capital allocated to it. There is an implicit assumption that the cost of equity capital does not vary by line of business.

Cummins and Phillips [2] provide empirical evidence that the cost of equity capital may vary by line of business. Their analysis suggests "significant differences in the cost of equity capital across lines".

1.2 Objective

We give a simple example that highlights why using an Euler allocation to allocate capital for the purpose of performance measurement is not always appropriate if the cost of equity capital varies by line. In particular, our example shows how writing more business in a line that (based on an Euler allocation) is performing worse than average can actually improve the results of the company overall.

1.3 Outline

The remainder of the paper proceeds as follows. Section 2 will discuss Tasche's result regarding the use of Euler allocation for performance measurement. In Sections 3 we consider the case when the cost of equity capital varies by line of business and give a simple example that illustrates why an Euler allocation is not always appropriate for performance measurement in this context. Section 4 continues the discussion from Section 3. Section 5 concludes.

2. BACKGROUND: EULER ALLOCATION AND RORAC

Insurance companies charge premiums to cover the cost of expected claims. Actual claims costs may end up being much higher than expected, for example due to a catastrophic event, and so an insurance company must have additional funds – economic capital – available to ensure it can meet its obligations in this case. The amount of capital a firm will hold is typically determined by (or at least guided by) a risk measure such a Value at Risk, Tail Value at Risk or Standard Deviation as well as by regulatory requirements. Once the total amount of capital has been determined, the firm might want to allocate this capital by line of business and/or geographic region. This may be helpful for reasons such as risk management - for example to understand which lines/regions are driving the need to hold capital - and performance measurement (which will be discussed in more detail below). An Euler allocation is a particular method of allocating capital which is often considered very suitable for performance management purposes. If the risk measure being used to determine the overall capital requirement satisfies certain properties, we will see later in Section 2 that an Euler allocation will exist for that risk measure.

The notation, definitions and wording in the remainder of Section 2 are largely based on Tasche [3] and Tasche [4]. The wording has been modified slightly to reflect a focus on an insurance (rather than investment) context. The main result in Section 2 (Proposition 2.2) is based on Theorem 4.4 in Tasche [3] and Proposition 2.1 in Tasche [4].

Suppose that real-valued variables $X_1,\ldots,X_n$ are given and represent the profits and losses of the various lines of business written by an insurance company. Let X denote the companywide portfolio profit/loss, ie

$$X=\sum_{i=1}^{n} X_i.$$

It is useful to allow for some dynamics in this model by introducing variables $u=(u_1,\ldots u_n)$:

$$X(u)=X(u_1,\ldots u_n)=\sum_{i=1}^{n} u_i X_i.$$

Then we have obviously X=X(1,…1). For the purposes of this paper we assume that the probability distribution of the random variable $(X_1,\ldots,X_n)$ is fixed and that the u_i only take on values close to 1 (ie the company's current mix of business will not be changing drastically).

Definition 2.1

- A non-empty set U in R^n is homogeneous if for each u in U and t>0, t*u is in U.
- A function h:U → R is homogeneous if U is homogeneous and for each u in U and t>0, h(t*u)=t*h(u).

Proposition 2.1 tells us that differentiable homogeneous functions can be represented as a weighted sum of their derivatives in a canonical manner. This result is stated (in a more general form) in Tasche [3] and follows from Euler's theorem on homogeneous functions.

Proposition 2.1 Let U be a non-empty open set in R^n and h:U → R be a real-valued function. If h is totally differentiable, then it is homogeneous if and only if for all u in U,

$$h(u)=\sum_{i=1}^{n} u_i\left(\frac{\partial h}{\partial u_i}\right)(u).$$

We define a risk measure ϱ to be a function from U to R. We assume that the economic capital (EC) required by the company (ie capital as a buffer against high losses) is determined by a homogeneous and totally differentiable risk measure ϱ, ie:

$$EC(X(u)) = \varrho(X(u)).$$

Proposition 2.1 tells us that

$$\varrho(X(u)) = \sum_{i=1}^{n} u_i \left(\frac{\partial \varrho}{\partial u_i}\right)(X(u)) .$$

Definition 2.2 Let $\varrho(X_i|X)$ be the capital allocated to line i. Then

- The total portfolio Return on Risk Adjusted Capital is defined by

$$RORAC(X(u)) = E[X(u)]/\varrho(X(u)).$$

- The portfolio-related RORAC of the i-th line is defined by

$$RORAC((X_i|X)(u)) = (u_i E[X_i])/\varrho((X_i|X)(u)).$$

Definition 2.3 Let X denote the portfolio-wide profit/loss.

- A capital allocation $\varrho(X_1|X),\ldots,\varrho(X_n|X)$ of the total economic capital $\varrho(X)$ satisfies the full allocation property if

$$\sum_{i=1}^{n} \varrho(X_i|X) = \varrho(X).$$

- A capital allocation $\varrho(X_1|X),\ldots,\varrho(X_n|X)$ is RORAC compatible if there exist $\varepsilon_i>0$ such that

$$RORAC(X_i|X) > RORAC(X) \Rightarrow RORAC(X+hX_i) > RORAC(X)$$

for all $0 < h < \varepsilon_i$.

Proposition 2.2 Let ϱ be a risk measure. Assume that ϱ is homogeneous and totally differentiable. If there is a capital allocation $\varrho(X_1|X),\ldots,\varrho(X_n|X)$ that is RORAC compatible in the sense of Definition 2.3 for arbitrary expected values $m_1,\ldots,m_n$ of $X_1,\ldots X_n$, then $\varrho(X_i|X)$ is uniquely determined as

$$\varrho((X_i|X)(u)) = \varrho_{Euler}((X_i|X)(u)) = u_i\left(\frac{\partial \varrho}{\partial u_i}\right)(X(u)).$$

In this case, there also exist $\varepsilon_i>0$ such that

$$RORAC(X_i|X) < RORAC(X) \Rightarrow RORAC(X+hX_i) < RORAC(X)$$

for all $0 < h < \varepsilon_i$.

Proposition 2.2 is the key result regarding the use of Euler allocation for performance measurement. It tells us that if an Euler allocation exists, then it is the unique RORAC compatible allocation that satisfies the full allocation property. In particular, with an Euler allocation if a company writes more business in lines where the expected return on allocated capital is greater (less) than the overall company expected return on capital, then the overall company expected return on capital will increase (decrease).

3. MAIN RESULT: EULER ALLOCATION AND EXCESS RORAC

Proposition 2.2 underlines the suitability of using an Euler allocation for performance measurement when RORAC is the performance metric. However if the cost of equity capital varies by line a RORAC compatible capital allocation may not be the best choice for the company. Cummins and Phillips [2] suggest that the variation in the cost of equity capital by line may be quite significant.

The overall company cost of equity capital is a weighted average of the by line costs of equity capital, weighted by the capital allocated to each line. If the cost of equity capital is the same for all lines of business, then clearly the overall company cost of capital will not change if there are small changes in the volume of business written in each line. In this case, an increase in the overall company RORAC due to these small changes is always beneficial to the company.

If the cost of equity capital varies by line of business, however, it may not always be the case that an increase in the company RORAC due to small changes in the volume of business written in each line is beneficial to the company. In this case, we must also consider any possible impact to the overall company cost of equity capital due to these small changes. This observation motivates Definition 3.1.

Definition 3.1 Let $\varrho(X_1|X),\ldots,\varrho(X_n|X)$ be an allocation of the total economic capital $\varrho(X)$ that satisfies the full allocation property. Let t_i be the cost of equity capital for line i, and t be the overall cost of equity capital for the company. Then

- The total portfolio Excess Return on Risk Adjusted Capital is defined by

$$\text{Excess RORAC}(X) = \text{RORAC}(X) - t.$$

- The portfolio-related Excess RORAC of the i-th line is defined by

$$\text{Excess RORAC}(X_i|X) = \text{RORAC}(X_i|X) - t_i.$$

The definition of Excess RORAC for a line i is basically the same as the definition of Economic Value Added on Capital (EVAOC) for a line i in Cummins [1] except that we require the capital allocation to satisfy the full allocation property and we do not specify any constraints on how to define the profit/loss of a line.

For the remainder of Section 3, we consider an increase in the overall company Excess RORAC ("XS RORAC") as being beneficial to the company. In other words. Excess RORAC (rather than RORAC) is our performance metric.

This means that, for example, we consider a situation where the RORAC is 22% and the cost of equity capital is 20% to be preferable to a situation where the RORAC is 11.5% and the cost of equity capital is 10% since an Excess RORAC of 2% is considered preferable to an Excess RORAC of 1.5%.

Note that if the cost of equity capital is the same for all lines of business, then small changes in the volume of business written in each line will result in an increase in the overall company Excess RORAC if and only if it will result in an increase in the overall company RORAC.

We now consider whether using an Euler allocation to allocate capital by line for the purpose of performance measurement is appropriate in this context. In particular, for all i, does there exist $\varepsilon_i>0$ such that

$$\text{XS RORAC}(X_i|X) > \text{XS RORAC}(X) \Rightarrow \text{XS RORAC}(X+hX_i) > \text{XS RORAC}(X) \text{ and}$$

$$\text{XS RORAC}(X_i|X) < \text{XS RORAC}(X) \Rightarrow \text{XS RORAC}(X+hX_i) < \text{XS RORAC}(X)$$

for all $0 < h < \varepsilon_i$?

The following result suggests that this is not the case in general.

Proposition 3.1 Let ϱ be a risk measure. Assume that ϱ is homogeneous and totally differentiable. Let $\varrho(X_i|X) = \varrho_{\text{Euler}}(X_i|X)$ be the Euler allocation of the total economic capital $\varrho(X)$, and R=XS RORAC(X) be the total portfolio Excess RORAC. Then for each i,

$$\frac{\partial R}{\partial u_i} = \frac{\left[\rho m_i - E[X]\left(\frac{\partial \rho}{\partial u_i}\right)\right]}{\rho^2} - \partial t/\partial u_i$$

Proof: Let R= XS RORAC(X) = E[X] - t. Note that $E[X] = \sum_{i=1}^{n} m_i u_i$. The result follows from the Quotient Rule.

We assume for the two notes below that n=2 and XS RORAC(X_1|X) > XS RORAC(X).

Note that:

- If $t_1=t_2$ (ie the cost of equity capital doesn't vary by line), then $t=t_1=t_2$ (ie t is a constant and does not depend on u_1 or u_2) so that $\frac{\partial t}{\partial u_1} = 0$, and so our assumption that XS RORAC(X_1|X) > XS RORAC(X) implies that $\frac{\partial R}{\partial u_1} > 0$, ie R is an increasing function of u_1. This implies that there exists some ε_i>0 such that XS RORAC(X+hX1) > XS RORAC(X) for all $0 < h < \varepsilon_i$, so that an Euler allocation is appropriate for the purpose of performance measurement.

- In general, to ensure that $\frac{\partial R}{\partial u_1} > 0$, we need $\frac{\partial(\text{RAROC(X)})}{\partial u_1} > \frac{\partial t}{\partial u_1}$ which is not always the case.

The following simple example highlights why an Euler allocation is not always appropriate for the purpose of performance measurement if the cost of equity capital varies by line and Excess RORAC is the performance metric.

Suppose a company writes two lines of business and capital is allocated to the two lines based on an Euler allocation.

We assume that $\rho(X) = 2\sqrt{\text{Var (X)}}$ and that the overall capital required by the company is 100.

We further assume that X_1 and X_2 are independent so that Cov(X1,X2)=0, and also assume that $\text{Var}(X_1)=\text{Var}(X_2)=1250=\text{Var}(X)/2$ and that $u_1=u_2=1$.

Note that ϱ(X) is homogeneous and totally differentiable and that

$$\frac{\partial \rho}{\partial u_i} = \frac{2\, u_i \text{Var}(X_i)}{\sqrt{\text{Var}(X)}}.$$

Note also that

$$t = \sum_{i=1}^{2} \frac{t_i u_i^2 \text{Var}(X_i)}{\text{Var}(Z)}$$

and so

$$\frac{\partial t}{\partial u_i} = \frac{\text{Var}(Z)\left(2u_i t_i \text{Var}(X_1)) - (\sum_{j=1}^{2} t_j u_j^2 \text{Var}(X_j)\right)(2u_i \text{Var}(X_1))}{(\text{Var Z})^2}$$

The expected profit, allocated capital, RORAC, cost of equity capital and Excess RORAC for each line and for the company overall are shown in Table 1.

Table 1: Simple Example

	Profit	Capital	RORAC	Cost	XS RORAC
Line 1	3	50	6.0%	6.0%	0.0%
Line 2	4	50	8.0%	7.5%	0.5%
Total	7	100	7.0%	6.75%	0.25%

Using the approach of Proposition 2.2, Line 1 would be considered the worse performing line since its RORAC is lower than that of the company overall (6% vs 7%). Proposition 2.2 tells us that writing more business in Line 1 would lower the overall company RORAC.

The approach of Section 3 would also suggest that Line 1 is the worse performing line since its Excess RORAC is lower than that of the company overall (0.0% vs 0.25%). This means that for the Euler allocation to be suitable for performance measurement, writing more business in Line 1 should result in a lower overall company Excess RORAC.

However, applying the Quotient Rule, and using the fact that $\mathrm{Var}(X) = 2\mathrm{Var}(X_1)$, $\mathrm{Var}(X_2) = \mathrm{Var}(X_1)$ and $u_1=u_2=1$, we have:

$$\frac{\partial R}{\partial u_1} = \sqrt{2\,\mathrm{Var}(X_1)}\,\frac{(2m_1 - (m_1 + m_2))}{8\,\mathrm{Var}(X_1)} - \frac{2t_1 - (t_1 + t_2)}{2} = -0.005 + 0.0075 > 0$$

This means that the overall company Excess RORAC will increase if we increase the volume in Line 1. So the Euler allocation is not suitable for performance measurement in this case where the cost of equity capital is not the same for the two lines of business and Excess RORAC is the performance metric. Note that if the cost of equity capital was the same for both lines, we would have $t_1 = t_2$, and so $\frac{\partial R}{\partial u_1} < 0$, which would be consistent with a capital allocation that is suitable for performance measurement (which is what we would expect due to Proposition 2.2).

4. ADDITIONAL RESULT: EULER ALLOCATION AND RELATIVE RORAC

As well as the Excess RORAC, a company may also be interested in the Relative RORAC, ie the RORAC divided by the cost of equity capital. For example, if a company has a choice between two options with the same expected Excess RORAC (say, 2%) but different costs of equity capital (say, 10% and 20%), it may prefer the option with the lower cost of equity capital as this may be considered less risky. This option will also have a higher Relative RORAC (since, for example, 12%/10% = 1.2 is greater than 22%/20% = 1.1).

With this in mind, we give the following definitions:

Definition 4.1 Let $\varrho(X_1|X),\ldots,\varrho(X_n|X)$ be an allocation of the total economic capital $\varrho(X)$ that satisfies the full allocation property. Let t_i be the cost of equity capital for line i, and t be the overall cost of equity capital for the company. Then

- The total portfolio Relative Return on Risk Adjusted Capital is defined by

  Relative RORAC(X) = RORAC(X)/t.

- The portfolio-related Relative RORAC of the i-th line is defined by

$$\text{Relative RORAC}(X_i|X) = \text{RORAC}(X_i|X)/t_i.$$

Although the Relative RORAC will likely be of less interest to the company than the Excess RORAC, in this section we will examine it in much the same way that we examined the Excess RORAC in the previous section.

So in Section 4, Relative RORAC ("Rel RORAC") is our performance metric.

This means that, for example, we consider a situation where the RORAC is 11.5% and the cost of equity capital is 10% to be preferable to a situation where the RORAC is 22% and the cost of equity capital is 20% since a Relative RORAC of 1.15 is considered preferable to a Relative RORAC of 1.1.

Note that if the cost of equity capital is the same for all lines of business, then small changes in the volume of business written in each line will result in an increase in the overall company Relative RORAC if and only if it will result in an increase in the overall company RORAC.

We now consider whether using an Euler allocation to allocate capital by line for the purpose of performance measurement is appropriate in this context. In particular, for all i, does there exist $\varepsilon_i>0$ such that

$$\text{Rel RORAC}(X_i|X) > \text{Rel RORAC}(X) \Rightarrow \text{Rel RORAC}(X+hX_i) > \text{Rel RORAC}(X) \text{ and}$$

$$\text{Rel RORAC}(X_i|X) < \text{Rel RORAC}(X) \Rightarrow \text{Rel RORAC}(X+hX_i) < \text{Rel RORAC}(X)$$

for all $0 < h < \varepsilon_i$?

The following result suggests that this is not the case in general.

Proposition 4.1 Let ϱ be a risk measure. Assume that ϱ is homogeneous and totally differentiable. Let $\varrho(X_i|X) = \varrho_{Euler}(X_i|X)$ be the Euler allocation of the total economic capital $\varrho(X)$, and R=Rel RORAC(X) be the total portfolio Relative RORAC. Then for each i,

$$\frac{\partial R}{\partial u_i} = \frac{\left[(t\,\rho)m_i - E[X]\left(\frac{\partial(t\,\rho)}{\partial u_i}\right)\right]}{(t\,\rho)^2}$$

Proof: Let R= Rel RORAC(X) = E[X]/(t ϱ(X)). Note that $E[X] = \sum_{i=1}^{n} m_i u_i$. The result follows from the Quotient Rule.

We assume for the two notes below that n=2 and Rel RORAC(X_1|X) > Rel RORAC(X).

Note that:

- If $t_1=t_2$ (ie the cost of equity capital doesn't vary by line), then $t=t_1=t_2$ (ie t is a constant and does not depend on u_1 or u_2), and so our assumption that Rel RORAC(X_1|X) > Rel RORAC(X) implies that $\frac{\partial R}{\partial u_1} > 0$, ie R is an increasing function of u_1. This implies that there exists some ε_i>0 such that Rel RORAC(X+hX1) > Rel RORAC(X) for all 0 < h < ε_i, so that an Euler allocation is appropriate for the purpose of performance measurement.

- In general, to ensure that $\frac{\partial R}{\partial u_1} > 0$, we need $\frac{m_1}{\frac{\partial(t\,\rho)}{\partial u_1}} > \frac{E[X]}{t\,\rho}$ which is not always the case.

We now return to the simple example from Section 3 but with Relative RORAC (instead of Excess RORAC) as the performance metric.

Suppose a company writes two lines of business and capital is allocated to the two lines based on an Euler allocation.

We assume that $\rho(X) = 2\sqrt{\mathrm{Var}\ (X)}$ and that the overall capital required by the company is 100.

We further assume that X_1 and X_2 are independent so that Cov(X1,X2)=0, and also assume that $\mathrm{Var}(X_1)=\mathrm{Var}(X_2)=1250=\mathrm{Var}(X)/2$ and that $u_1=u_2=1$.

Note that $\varrho(X)$ is homogeneous and totally differentiable and that

$$\frac{\partial\rho}{\partial u_i} = \frac{2\ u_i \mathrm{Var}(X_i)}{\sqrt{\mathrm{Var}(X)}}.$$

Note also that

$$t\,\rho = \frac{t_1 u_1\big(2\ u_1 \mathrm{Var}(X_1)\big)}{\sqrt{\mathrm{Var}(X)}} + \frac{t_2 u_2\big(2\ u_2 \mathrm{Var}(X_2)\big)}{\sqrt{\mathrm{Var}(X)}}$$

and so

$$\frac{\partial(t\,\rho)}{\partial u_1} = \frac{4t_1 u_1 \mathrm{Var}(X_1)}{\sqrt{\mathrm{Var}(X)}} - 2\frac{u_1 \mathrm{Var}(X_1)\big(t_1 u_1^2\ \mathrm{Var}(X_1) + t_2 u_2^2 \mathrm{Var}(X_2)\big)}{\mathrm{Var}(X)^{\frac{3}{2}}}$$

The expected profit, allocated capital, RORAC, cost of equity capital and Relative RORAC for each line and for the company overall are shown in Table 2.

Table 2: Simple Example - Continued

	Profit	Capital	RORAC	Cost	Rel RORAC
Line 1	3	50	6.0%	6.0%	1.00
Line 2	4	50	8.0%	7.5%	1.07
Total	7	100	7.0%	6.75%	1.04

Using the approach of Proposition 2.2, Line 1 would be considered the worse performing line since its RORAC is lower than that of the company overall (6% vs 7%). Proposition 2.2 tells us that writing more business in Line 1 would lower the overall company RORAC.

The approach of Section 4 would also suggest that Line 1 is the worse performing line since its Relative RORAC is lower than that of the company overall (1.00 vs 1.04). This means that for the Euler allocation to be suitable for performance measurement, writing more business in Line 1 should result in a lower overall company Relative RORAC.

However, applying the Quotient Rule, and using the fact that $Var(X) = 2Var(X_1)$, $Var(X_2) = Var(X_1)$ and $u_1=u_2=1$, we have:

$$\frac{\partial R}{\partial u_1} = \sqrt{2\,Var(X_1)}\,\frac{6.5t_2 - 7.5t_1}{(t\,\rho)^2} = \frac{50(0.0375)}{(0.0675 \text{ x } 100)^2} = \frac{1.875}{(6.75)^2} > 0$$

This means that the overall company Relative RORAC will increase if we increase the volume in Line 1. So the Euler allocation is not suitable for performance measurement in this case where the cost of equity capital is not the same for the two lines of business and Relative RORAC is the performance metric. Note that if the cost of equity capital was the same for both lines, we would have $t_1 = t_2$, and so $\frac{\partial R}{\partial u_1} < 0$, which would be consistent with a capital allocation that is suitable for performance measurement (which is what we would expect due to Proposition 2.2).

5. CONCLUSIONS

Performance measurement is an important application of capital allocation and Proposition 2.2 highlights why an Euler allocation is particularly appropriate for this purpose when the cost of equity capital does not vary by line. In particular, with an Euler allocation if a company writes more business in lines where the expected return on allocated capital is greater (less) than the overall company expected return on capital, then the overall company expected return on capital will increase (decrease).

The example presented in this paper illustrates that the equivalent result in not always true if the cost of equity capital varies by line of business.

So although an Euler allocation (if it exists) will always be suitable for performance measurement when RORAC is the performance metric, it may not be suitable for performance measurement if the cost of equity capital varies by line and Excess RORAC or Relative RORAC is the performance metric. This means that care must be taken when using an Euler allocation to allocate capital by line for the purpose of performance measurement when the cost of equity capital varies by line.

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Biography of the Author

Shayan Sen is Manager, Actuarial Pricing at Wawanesa Mutual Insurance Company in Toronto. He has over 10 years of actuarial experience mostly in personal lines pricing and is an Associate of the Casualty Actuarial Society. He has a BA in Mathematics from the University of Dublin and a PhD in Mathematics from Cornell University.