Practical LDF Interpolation for Well-Behaved IBNR

Ira Robbin, Ph.D.

Abstract

Actuaries have devised numerous methods for interpolating annual evaluation loss development factors (LDF) to arrive at quarterly evaluation factors. Not all of these work as well as might be hoped. Some introduce oscillations not found in the original factors. Many lead to IBNR projections that move erratically or have blips that are hard to explain. This paper advances the approach to interpolation by taking a whole curve perspective, defining properties of well-behaved interpolates, and focusing on attributes of the resulting IBNR projections. It demonstrates a set of simple practical techniques including a backfill algorithm to compute factors at immature ages.

Keywords Loss Development Patterns, Interpolation, Equilibrium, IBNR

1. INTRODUCTION

Many practicing property casualty reserving actuaries face a recurring challenge each quarter: how to update IBNR balances for a multitude of splits by line of business, distribution channel, market segment, and geographic division. Given the lack of time and resources, doing a complete granular analysis is simply not practical. Further many of the splits do not have sufficient data to support a credible full-triangle analysis when the data is evaluated by quarter.

How do actuaries meet this challenge? One popular solution is to take the year-ending IBNR balances and use loss development factors (LDF) at quarterly evaluations to estimate the run-off. The quarterly LDF are often derived by interpolating annual LDF. To obtain the annual evaluation LDF, actuaries tend to rely on a segment's own data if it is sufficiently credible. However, when the data for a cell is too volatile even after grouping it at annual evaluation points, it is a common and accepted practice to derive default annual evaluation factors based on triangles of loss data aggregated over similar lines and segments. Both aggregation and annual evaluation increase the stability of the factors. The resulting annual evaluation default LDF are sometimes further refined by cell based on a review of industry data, claims department statistics, and other information. [1] Once the annual evaluation

[1] For example default LDF for northwest region small commercial risk division general liability (GL) losses might be derived from loss triangles for the full general liability line of business and then reduced slightly based on the actuary's belief that risks in the small commercial division have losses that develop a bit more quickly than other GL business.

development factors for a particular segment are selected, the next step is to interpolate them by quarter.[2]

Though interpolation of LDF might seem a trivial task, there are many available techniques and they can produce a range of answers. Some are vulnerable to anomalies or require too many actuarial overrides. Others induce seasonality that does not exist or an apparent trend that later turns out to be illusory. Many don't work well at early ages because they fail to distinguish *development of exposure to loss* from *development of loss on exposures that have already occurred*. Others implicitly forecast blips in expected quarterly IBNR run-off. At this time, no particular interpolation approach has been universally accepted. Actuaries want a set of interpolation techniques that are simple to implement, yet robust and free from anomalies. This aim of this paper is to provide a framework for achieving that goal.

1.1 Three Properties of Well-Behaved Interpolates

The first specific objective this paper is to propose a non-exhaustive set of properties that well-behaved interpolation algorithms should satisfy. In this paper three will be proposed.

The first is that the method should not introduce extra oscillations. The term, *inherited monotonicity*, will be used to describe this:

- **Inherited Monotonicity**: The quarterly age-to-age (ATA) LDF interpolates do not oscillate more often than the original annual ATA LDF. For example, suppose the 24-36 ATA LDF was larger than the 36-48 ATA LDF. A violation of inherited monotonicity would exist if the 36-39 month interpolate was larger than the 33-36 month factor. See Table 1 for an example of such a violation.

[22] Another option is to interpolate the default annual LDF for the aggregation and use those as default interpolates for each cell.

Table 1

Inherited Monotonicity Violation								
	Annual Evaluation Factors							
Age	24-36				36-48			
ATA LDF	1.500				1.300			
	Quarterly Interpolates				Quarterly Interpolates			
Age	24 - 27	27 - 30	30 - 33	33 - 36	36 - 39	39 - 42	42 - 45	45 - 48
ATA LDF	1.150	1.120	1.090	1.068	1.120	1.065	1.050	1.038

The second and third properties are defined by examining the resulting IBNR evolution on a hypothetical book of business produced by a growth model in equilibrium. In this growth model, it is assumed all accident years have the same actual ultimate losses and the same pattern of development: The second and third properties are *equilibrium IBNR stability* and *monotonicity of total runoff from all prior years*:

- **Equilibrium IBNR Stability**: Once equilibrium is achieved, total IBNR stays level each quarter. Each quarter the growth of IBNR from the new accident year is exactly offset by the total of IBNR runoff from all prior accident years. Table 2 shows an example of a violation of Equilibrium IBNR stability normalized so the year-ending balance is $1,000 and quarter ending "0" is the end of the first year in which equilibrium is attained.

Table 2

Equilibrium IBNR Stability Violation						
	Quarter ending IBNR balance					
Qtr	0	1	2	3	4	5
IBNR All Prior AY	1,000	800	625	450	300	225
IBNR Current AY	-	300	450	500	700	575
IBNR Total	1,000	1,100	1,075	950	1,000	800

- **Monotonically Decreasing Total Prior Year IBNR Runoff:** In equilibrium, the quarterly totals of IBNR runoff from all prior accident years form a monotonic decreasing sequence under the assumption the development pattern never goes negative (i.e. the LDF are never below unity). Table 3 has an example of this.

Table 3

Prior Year IBNR Runoff Monotonicity Violation						
	Quarter ending IBNR balance					
Qtr	0	1	2	3	4	5
Prior AY IBNR	$1,000	$700	$600	$450	$300	$225
Prior Year IBNR Runoff		$300	$100	$150	$150	$75

It might be initially surprising to realize that an arbitrary interpolation scheme will not necessarily satisfy any of these properties. Many methods introduce oscillating LDF, non-level equilibrium IBNR and a bouncy ride for the prior year IBNR run-off pattern.

Some may object that equilibrium conditions are unrealistic and of not much relevance to real-world situations. However, it is more accurate to think of it in the converse. If an interpolation routine produces IBNR fluctuations in the ideal conditions of level-growth equilibrium, then who knows what mischief may ensue in actual scenarios. In real-world scenarios problems do not jump out as clearly as they do in equilibrium. Later in this paper, it will be proved that an accident year LDF pattern will satisfy equilibrium IBNR stability if it is generated from uniform exposure to loss, the usual assumption made for a non-seasonal accident year, and a fixed underlying claim development pattern.

1.2 Three Interpolation Tools

This paper will present several practical techniques for use in the interpolation process. The first, *tail-tapering*, is not strictly an interpolation tool but rather a procedure that quickly smooths out the tail of the initial set of annual evaluation factors. However, it is essential to taper the tail before attempting to interpolate and in that sense it is the first step of the interpolation process. Tail-tapering takes the user selected percent of ultimate value at the user-selected tapering onset age and then employs a straightforward routine to smoothly taper to ultimate at the user-selected selected ultimate age.[3]

The second is *normalized cross-year increment smoothing with monotonicity adjustment.* This starts by computing level quarterly increments separately for each year. Then simple arithmetic smoothing is applied over all quarters beyond month 12. The increments by year are then normalized so as to reproduce the original annual evaluation LDF. If the provisional results violate the inherited monotonicity property, averages against the initial level increments are performed for any year in need of correction. This stage produces interpolates that are relatively smooth and which inherit monotonicity. Other methods unknowingly court difficulty when they examine each year in isolation and pay no attention to the transition from one year to the next. The averaging across years is one very simple way (not necessarily the only or the best way) to address that neglect.

The third tool is the *stability backfill* technique. This is an algorithm for determining the factors at immature ages by requiring the resulting factors to produce IBNR values satisfying the Equilibrium IBNR Stability property.

The overall method with tail tapering, cross-year smoothing, and stability backfill will be identified by the acronym, SWIMON (**S**moothing **W**ith **I**ncrements - **Mo**notonically **N**ormalized).

1.2.1 Tapering the Tail

It is best to first taper the tail of the annual evaluation LDF before performing quarterly interpolation. This assumes the initial tail goes all the way to ultimate. More sophisticated approaches are needed if this is not true and the tail factors must be extrapolated. Also it is assumed that the actuary has LDF deemed acceptable up to a certain age. They may be all-weighted year averages for example or averages ex Hi/Lo. The problem in this situation is that the tail factors may be quite erratic even if close to unity. There may be a few unity factors interspersed with occasional blips up and down that over the span of a few years might add up to point or two. Some actuaries would set the curve to unity and write-off this small amount. Others will try their hand at smoothing by eye. This tends to absorb an inordinate amount of actuarial effort, with students tapering by eye and managers and chief actuaries refining the numbers. For example, a student upon seeing annual LDF machine averages of

[33] See the Appendix for the definitions of increments, age-to-age-factors, tail decay rates and other representations of loss development.

1.008, 0.995, and 1.003, might propose a string of three factors equal to 1.002. The manager may refine that to 1.0025, 1.0020, and 1.00195. Others will try curve fitting that sometimes works well, but which is sometimes confounded by the oscillations in the tail and the need to remove outliers to arrive at a good fit. Even after fitting there may be a small tail out to infinity that the actuary would like to close out.

So how is it possible to extricate actuaries from this tedious and low value-added part of the process? The solution to be demonstrated in Chapter 2 is to taper the annual evaluation factors from a selected age onward to a selected ultimate age. The resulting tapered annual evaluation factors can then be grafted onto the body of the curve. Essentially the idea is to take the three key parameters that the actuary can readily select to define the tail and use those to construct a smooth tail. The tail-tapered curve can be interpolated by quarter as will be explained in the next section.[4]

1.2.2. Avoiding Middle Age Interpolation Disorders

Assuming relatively stable patterns of LDF in the middle and later stages of development, the problem is how to interpolate to a quarterly basis without inducing seasonal bias or producing erratic patterns going from one quarter to the next. For instance, a method might overstate the IBNR takedown for the first quarter of each prior accident year so the company more often than not sees what looks like beneficial prior year development in the first quarter of each year. Note that the IBNR runoff in a quarter is the expected development. If the IBNR runoff is overstated, then actual development will tend to come in low relative to this false benchmark. The company may conclude results are better than they truly are. By the time this gets corrected in the remaining quarters the biased figures may have led to incorrect business decisions. Another problem is that some interpolation routines yield answers prone to jumps at year-end. These routines usually generate quarterly expected development that proceeds nicely from quarter to quarter during the year and all seems fine. However, the pattern then might break sharply for the first quarter of the subsequent year (quarter 5 from the starting quarter). This can only be explained if the annual factors increase instead of decrease from one year to the next. Otherwise this would be a manifestation of a failure of

---

[4] Preliminary tail-tapering is often useful even if one is not doing quarterly interpolation. It may improve the performance of curve-fitting routines being used to smooth out factors at earlier ages. Even the step of setting factors to unity beyond a selected ultimate age is beneficial since some machine–generated averages that appear to be unity on a display are not. These can lead to small sums that make their appearance in unexpected places.

inherited monotonicity. The overall point is that faulty interpolation routines lead to blips in IBNR evolution that are difficult to explain.

The increment smoothing, normalization, and monotonicity adjustment procedure is designed to address these potential problems. It is presented in more detail in Chapter 3.

1.2.3. Exposure Growth Problems in Early Age Interpolation

Finally there is the question of what to do about the start-up period. Many methods fail to extend reasonably to early ages simply because they fail to account for the increasing exposure separately from the development of losses already incurred. The general solution as explained by Robbin [3] and Robbin and Homer [4] is to explicitly account for the dependence of loss development patterns on underlying exposure period development. Those papers describe fitting different parametric forms against data. In this paper a simpler backfill technique will be used in which the early age factors are determined so that the IBNR stays fixed each quarter in a level growth model. The approach will be demonstrated in Chapter 4. In Chapter 5, it will be shown that an accident year LDF pattern generated via the Robbin formula under reasonable uniformity assumptions will produce IBNR that automatically satisfies the backfill formula.

1.3 Existing Literature

Recent works by Boor [2] and by Bloom [1] provide useful quick methods ("hacks") for interpolating LDF. Bloom's paper shows interpolates of 12, 24, 36 … month factors at ages 15, 27, 39, ..., computed with a variety of methods including Linear, Inverse Power Curve (IVP), IVP decay, Exponential, and Exponential Decay. Her paper also has methods for extrapolating to immature ages.

Boor fits a Weibull curve form to the implicit IBNR percentages derived from the original annual evaluation factors. He then uses the Weibull curve shape to arrive at monthly interpolates between the annual factors. He extends to early ages and makes monthly exposure adjustments to convert the scaled Weibull factors to be on an accident year basis.

This paper is intended to advance actuarial interpolation tools and concepts beyond what is found in these works and other existing literature. It promotes a new "whole-curve" perspective on interpolation and highlights the need to define properties of behavior for interpolates. It also adds to the literature by stressing the importance of evaluating the qualities of interpolates by examining the resulting evolution of IBNR. The three techniques demonstrated in this paper are offered as useful if basic additions to the actuarial toolbox of practical methods.

To be clear, many actuaries do produce IBNR projections as a standard component of reserve analysis. However, documentation of this important part of the process does not appear to have previously found its way into the literature or at least not in the standard articles on interpolation of LDF.

1.4 Comparison Example

To clarify the distinction between different methods and the properties of their resulting interpolates, methods from the Bloom paper, the Boor paper, and this paper will be used to interpolate the sample annual factors from the Bloom paper. This will be done in Chapter 6.

First the IVP method in the Bloom paper will be extended to show interpolates at all intermediate quarters beyond age 12 months.(e.g. for ages 18 and 21, not just age 15). Then the "Method of 12" described in that paper will be used to fill in the early quarters.[5] Boor's Weibull fitting and splicing method will be applied to the same set of factors and summarized by quarter.[6] The three alternative sets of interpolated LDF will then be compared.

1.5 Expected Quarterly Development and Projected IBNR Run-off

A fundamental message of this paper is that the actuary should review predicted amounts of expected quarterly development by accident year over at least five projected calendar quarters. Dubious patterns of expected development indicate a poorly performing interpolation method. The actuary should be able to explain any strange blips or else go back and derive new interpolates.

The schedule of expected quarterly IBNR and IBNR run-off based on the SWIMON interpolates will be computed starting with an arbitrary hypothetical set of year-end balances. This will be done in Chapter 7. It should be noted that many reserving actuaries already produce IBNR runoff projections and study them carefully for anomalies.

1.6 Equilibrium Run-off Comparison

Equilibrium IBNR projections by quarter will be computed for the SWIMON, IVF/12, and Spliced Weibull IBNR interpolates in Chapter 8. Some may initially feel this has little

[5] Bloom presented many methods and did not recommend these over any others.
[6] Boor shows interpolates on a monthly basis.

relevance since there are few stable equilibrium scenarios in the real world. The author's perspective is that any equilibrium oscillations need to be subtracted out of real world indications. A scale that is not calibrated properly will yield incorrect results. In effect, the equilibrium analysis can indicate if a set of interpolates is appropriately balanced.

1.7 Conclusion

It is hoped the practical techniques presented in this paper will achieve acceptance as useful additions to the actuarial toolbox. The tail-tapering technique could be employed in deriving LDF patterns outside of an interpolation context. Also, there is nothing to prevent the actuary from applying the interpolation methods in this paper to interpolate Paid LDF and then project estimated Unpaid Losses instead of IBNR.

While the comparison of methods was necessary to clarify distinctions between different algorithms, the fundamental message of this paper is not that one method did or did not work better than others on a specific example. It is that actuaries should analyze the behavior and characteristics of the interpolated LDF and the resulting IBNR evolution. Indeed, many already do and in that sense this paper can be viewed as an initial attempt to codify and extend existing practice Whether actuaries accept or reject those particular interpolation techniques, a major objective of the author will have been achieved if it fosters a greater awareness of the importance of examining the behavior of the whole curve of LDF interpolates and the resulting quarterly IBNR run-off projections.

2. TAIL TAPERING AND TRUNCATION

Given that an initial percent of ultimate selection, $PCT_0(t_I)$, has been made for month t_I, which is divisible by 12, and a subsequent decay rate of unreported loss, q, has been selected, the infinitely extrapolated annual evaluation percent of ultimate series PCT*(t) for $t>t_I$ is generated inductively via:

$$PCT^*(t_I) = PCT_0(t_I) \tag{2.1}$$

$$PCT^*(t_I + k \cdot 12) = PCT^*(t_I + (k-1) \cdot 12) + q \cdot Q^*(t_I + (k-1) \cdot 12)$$

where Q= 1-PCT and k is a positive integer

For example, if PCT_0 is 90% at 120 months and q is 40%, then PCT* is 94% at 132 and

96.4% at age 144.

Now suppose the actuary selects an age, t_F, at which it is desired the development pattern will reach ultimate. Set the multiplier, M, via:

$$M = \frac{1 - PCT^*(t_I)}{PCT^*(t_F) - PCT^*(t_I)} \quad (2.2)$$

Then set annual increments, INC, between t_I and t_F, via

$$INC^*(t_I + k \cdot 12) = PCT^*(t_I + k \cdot 12) - PCT^*(t_I + (k-1) \cdot 12) \quad (2.3)$$

$$INC(t_I + k \cdot 12) = M \cdot INC^*(t_I + k \cdot 12)$$

The actuary should set initial and final ages and the value of q so that the increments appear reasonable. An example is shown in Table 4.

Table 4 Tail-tapering Example

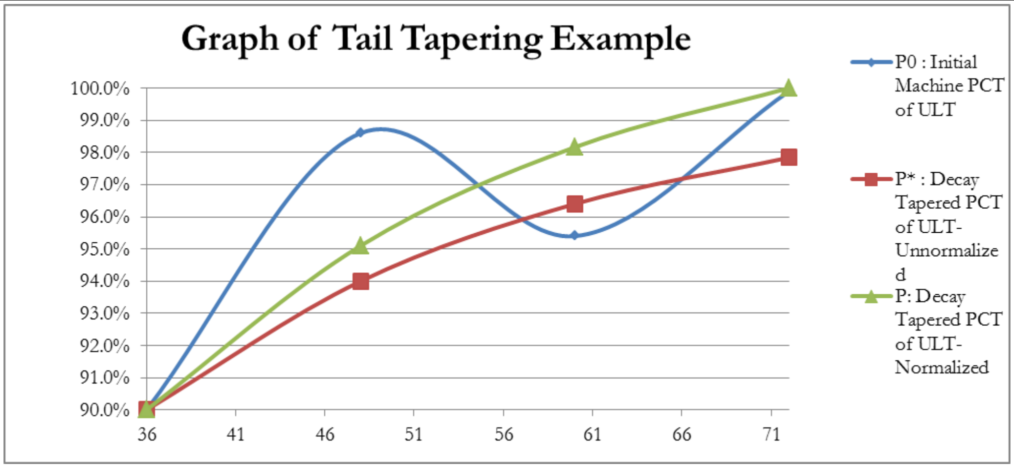
	Age	Pct Ult
Initial	36	90%
Ultimate	72	100%

Decay Rate	40.0%
Multiplier	1.276

Age (Months)	36	48	60	72
P0 : Initial Machine PCT of ULT	90.0%	98.6%	95.4%	99.9%
P* : Decay Tapered PCT of ULT-Unnormalized	90.0%	94.0%	96.4%	97.8%
P: Decay Tapered PCT of ULT-Normalized	90.0%	95.1%	98.2%	100.0%

The initial machine generated percentages of ultimate, the un-normalized, and final normalized tail-tapered curves are shown in Graph 1.

Graph 1



3. CROSS-YEAR QUARTERLY SMOOTHING, NORMALIZATION, AND MONOTONICITY FIXING

The next step in the SWIMON procedure is to obtain annual increments of development. This is done by taking differences between the percent of ultimate values. After that preparatory step, each annual increment is divided equally to get initial increments by quarter. For example if the percent of ultimate goes from 80.0% to 90.0% over months 48 to 60, then the increment for year five is 10.0% and the initial set of quarterly increments for year five is 2.5% for each quarter.

The next step is to smooth these across all quarters starting with quarter five out to ultimate. In the example shown in Exhibit 1B, three point smoothing is done twice. The initial annual evaluation LDF are taken from the example in Bloom's paper. The smoothed increments are then renormalized to preserve the annual totals.

Though the initial level increments will satisfy the inherited monotonicity property, the same cannot be guaranteed after they are smoothed and normalized. So the resulting increments are examined and if any violation is found, it can be removed by averaging the increments for the year in which the violation occurs with the initial level increments for that year. This is also shown in Exhibit 1B. Overall, this procedure tempers the jump from one year to the next and leads to quarterly increments that evolve more reasonably than the initial flat values, but which still balance to the desired annual totals.

4. IMMATURE AGE IBNR EQUILIBRIUM STABILITY BACKFILL

To extrapolate back to quarters over the first year, the SWIMON approach is to backfill so as to achieve level IBNR each quarter in the equilibrium growth phase on a level book of business. The key idea is the IBNR added from the new accident year must offset the sum of IBNR run-off for all prior accident years.

The mathematical construction is begun with some general definitions. Let IBNR%(t) be the IBNR percentage for the t^{th} month of development of an accident year as a percent of ultimate loss and let IBNRQ(w,k) be the IBNR percentage for the w^{th} prior accident year as of the kth calendar quarter after the end of year y-1. Here k runs from 1 to 4. For example IBNRQ(2,3) is the IBNR percentage as of the end of the third quarter for the second prior accident year. Let w=0 correspond to the current accident year. It follows that:

IBNR Definitions (4.1)

$$IBNRQ(0,k) = \frac{k}{4} - PCT(3k) \qquad \text{for w = 0}$$

$$IBNRQ\ (w,k) = IBNR\%\ (12w + 3k) = 1 - PCT(12w + 3k)$$

$$for\ w = 1, 2, \ldots$$

When w= 0, the "k/4" term is needed because it is the percent of ultimate exposure incurred as of the kth quarter under the usual uniformity assumptions for an accident year. For example, if k=3, and the percent of ultimate as of the end of the third quarter is 40%, then the IBNR% for the third quarter of the current accident year is 75%-40% = 35%. The "k/4' term gets replaced by unity when w = 1, 2, …. For example, the IBNR for the second prior accident year as of the third quarter after year-end is the IBNR percentage at month 33 which is 100% minus the percent of ultimate at month 33.

The quarterly IBNR run-off for the w^{th} prior AY as of the kth subsequent quarter is defined as the difference in IBNR for the k-1^{st} and k^{th} quarters and denoted as R(IBNRQ)(w,k):

IBNR Run-off (4.2)

$$R(IBNRQ)(w,k) = -\Delta IBNRQ\ (w,k) = IBNRQ(\,w,k-1) - IBNRQ(w,k)$$
$$= IBNR\%\ \big(12 * w + 3(k-1)\big) - IBNR\%\ (12w + 3k)$$

For example the third quarter IBNR Runoff percentage for the second prior accident year is the difference between the IBNR percentage at 30 (2*12+3*2) months and 33 (2*12 + 3*3) months.

The next part of the exposition is to determine formulas for IBNR in equilibrium under uniform growth assumptions. The equilibrium and level growth assumptions mean that ultimate losses are the same for all accident years and that IBNR totals can be obtained by summing the appropriate percentages. Thus, to attain stability in equilibrium, the increase in IBNR for the current accident year must equal the total runoff for the prior years:

(4.3)

$$\Delta IBNRQ(0,k) = R(IBNRQ)(All\ Prior\ AY,k) = \sum_{w=1} R(IBNRQ)(w,k)$$
$$for\ k = 1,2,3,4$$

Recall w=0 is used here to stand for the current accident year.

Knowing the change in IBNR is enough to solve for the incremental percent of ultimate, INCQ, for the current accident year. Let ETD(k) be the percentage of ultimate loss exposure earned to date as of the kth quarter. For an accident year, the ETD function is 25%, 50%, 75%, and 100% for the first four quarters and 100% thereafter. Then for k= 1, 2, 3, 4, it follows that:

(4.4)

$$INCQ(k) = PCT(3k) - PCT(3(k-1)) = \Delta ETD - \Delta IBNR(0,k)$$

For example, if total prior year IBNR runoff for the second quarter is 14.0%, then the incremental increase in percent of ultimate in the second quarter is 11.0% (25%-14%).

This method is shown in Exhibit 1C again using the example from Bloom's paper and the mature year interpolates derived in Exhibit 1B. The quarterly interpolated LDF are then grafted together to make one curve from age 3 months on to ultimate. This is shown in Exhibit 1A

As will be proved in the next section, under level growth model assumptions, the IBNR for immature periods of a uniform accident year must grow enough to offset the run-off for all prior years.

5. EQUILIBRIUM IBNR STABILITY

Many readers accept the concept of equilibrium IBNR stability because it is intuitively appealing. Others might not be entirely convinced and perhaps wonder if some non-seasonal development pattern might nonetheless give rise to IBNR oscillations in equilibrium. In this section it will be shown that under the usual uniformity assumptions and other reasonable assumptions, the IBNR must be stable in equilibrium under level growth.

To set the groundwork, it is necessary to quickly summarize the general loss development pattern representation theory of Robbin and Homer [3] and an additional accident year result from Robbin [2]. Under slightly revised notation, let T be the underlying claim settlement lag random variable defined as the time elapsed from when a claim occurs until it settles. Let A be a loss exposure bucketing random variable defined as the lag from the start of an exposure period until a loss occurs. For an accident year under the usual assumptions, A is uniform on [0, 1]. The percent of ultimate for the underlying development variable T and the exposure bucketing variable A is given by the convolution integral:

Robbin-Homer Convolution Formula for Percent of Ultimate (5.1)

$$PCT_{T|A}(t) = F_{A+T}(t) = \int_0^t ds\, f_A(s) * F_T(t-s)$$

The integral representation assumes the random variables A and T are independent. Independence can be asserted based on the general grounds that the manner in which loss exposures are bucketed for purposes of accounting and reporting should not have any impact on how the claims are settled.

For an accident year, Equation 5.1 can be expressed using formulas that include the limited expected value of T, denoted here as LEV:

Robbin Accident Year Percent of Ultimate Formula Based on LEVs (5.2)

$$PCT_{T|A}(t) = \begin{Bmatrix} t - LEV(t) & for\ t < 1 \\ 1 - (LEV(t) - LEV(t-1) & for\ t > 1 \end{Bmatrix}$$

The proof is in Robbin [2]. Equation 5.2 provides a convenient way to generate accident year loss development curves given a parametric non-negative random variable such as a Pareto or exponential that has a tractable limited expected value formula.

The new result in this paper is that Equation 5.2 implies IBNR stability in equilibrium.

AY Equilibrium IBNR Stability (5.3)

$$Let\ an\ AY\ development\ pattern, PCT^*(t), be\ given.$$

$$If\ \ there\ exists\ a\ non-negative\ random\ lag\ variable, T, with\ finite\ mean\ such\ that$$

$$PCT^*(t) = \ PCT_{T|A}(t),$$

$$then\ IBNR\ is\ constant\ in\ equililibrium\ in\ a\ growth\ model\ with\ \ level\ growth.$$

Proof: The change in IBNR for quarter k is given using 4.2 as

$$\Delta IBNRQ(k) = \sum_{w=0} \Delta IBNRQ\ (w,k) \tag{5.4}$$

Expanding each of the change in IBNR terms for an accident year, A, with a fixed development distribution T in terms of the PCTs of ultimate and then substituting in 5.2, one finds for w=0:

$$\begin{aligned} \Delta IBNRQ_{T|A}(0,k) &= \frac{1}{4} - \ \left\{PCT_{T|A}(3k) - PCT_{T|A}\big(3(k-1)\big)\right\} \\ &= \frac{1}{4} - \left\{\frac{3k}{4} - E(3k) - \left(\frac{3(k-1)}{4} - E(3(k-1))\right)\right\} \\ &= \left\{E(3k) - E\big(3(k-1)\big)\right\} \end{aligned} \tag{5.5}$$

For w =1, 2, …

$$\Delta IBNRQ_{T|A}(w,k) = \tag{5.6}$$
$$1 - PCT_{T|A}(12w+3k) - \{1 - PCT_{T|A}(12w+3(k-1))\}$$
$$= E(12(w-1)+3k) - E(12w+3k)$$
$$-\{E(12(w-1)+3(k-1)) - E(12w+3(k-1))\}$$

Plugging 5.5 and 5.6 back into 5.4, one finds that each new term in the sum offsets the residual of previous term and leaves a residual that is offset by the next term.

For example with k=2, one has after 4 terms:

$$\Delta IBNRQ(2) = \tag{5.7}$$
$$E(6) - E(3)$$
$$+E(18) - E(6) - \{E(15) - E(3)\}$$
$$+E(30) - E(18) - \{E(27) - E(15)\}$$
$$E(42) - E(30) - \{E(39) - E(27)\}$$
$$= E(42) - E(39)$$

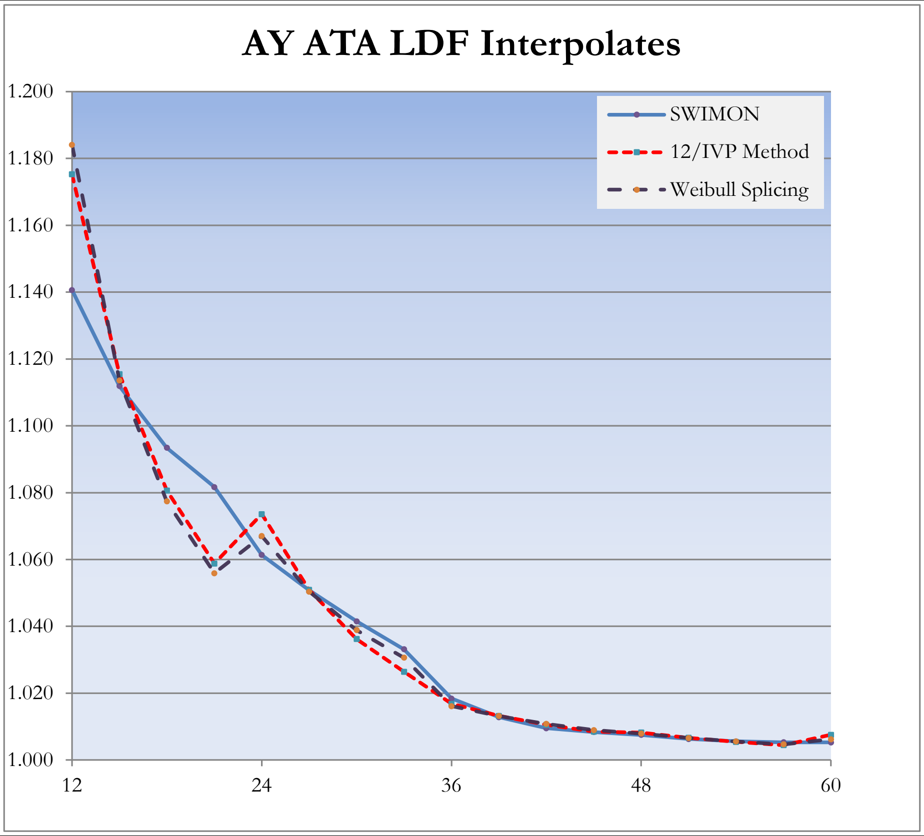
Assuming T has a finite mean, the difference in the limited expected values must go to zero. It follows the ΔIBNRQ(k)=0. Therefore total IBNR does not change by quarter in equilibrium for an accident year pattern generated by A given T.

So the entire suite of AY development curves that can be generated by Equation 5.2 are curves that will satisfy equilibrium IBNR stability.

6. COMPARISON OF LDF FOR DIFFERENT METHODS

In this chapter, different interpolation methods are compared on the specific set of annual factors in the Bloom paper [1]. Interpolations from the SWIMON procedure are derived and compared with those derived from the IVP Method and Method of 12 as shown in Bloom [1] and the fitted Weibull Spliced IBNR model presented by Boor[2]. The derivation and results are shown in Exhibits 2 and 3 respectively. Readers with questions about those methods should refer back to the Bloom and Boor papers. The resulting sets of ATA and ATU LDF are compared in Exhibit 4A and the corresponding percent of ultimate and incremental curves are shown in Exhibit 4B. Graph 2 shows the ATA LDF.

Graph 2



Since the original annual LDF are monotonically decreasing, the bounce in the 12/IVP and Weibull spliced curves indicate a violation of the inherited monotonicity property.

7. QUARTERLY INTERPOLATED LDF AND INDICATED IBNR

Any set of quarterly interpolated LDF can be used to project IBNR Runoff by quarter for each prior accident year. Starting with the year-end prior accident year IBNR balances at the end of the prior calendar year as given, this chapter will show how the LDF can be used to compute IBNR run-off percentages or equivalent IBNR decay factors.

7.1 IBNR Runoff by Accident Year

Let INCQ(w,k) be the percentage increment of development during the kth quarter after year end for the w^{th} prior AY. Let PCT(t) be the interpolated percent of ultimate pattern derived from the interpolated LDF, where t is expressed in months. Then the increments are given as:

$$INCQ(w,k) = PCT(12w + 3k) - PCT(12w + 3(k-1)) \quad (7.1)$$

The resulting IBNR run-off percentages, RUNQ(w,k), as factors against their respective year-end balances are given as:

$$RUNQ(w,k) = \frac{INCQ(w,k)}{1 - PCT(12y)} \quad (7.2)$$

The runoff can also be expressed as a series of decay ratios applied against the each prior IBNR balance.

$$DRQ(w,k) = 1 - \frac{INCQ(w,k)}{IBNRQ(w,k-1)} \quad (7.3)$$

For example, if IBNR for the second prior accident year was 48% of ultimate as of year-end and 40% of ultimate for the as of the end of the second quarter of the current calendar year and the increment during the third quarter was 4.0%, then the Run-off percentage would 8.25% (=4/48) and the Decay Ratio for the third quarter would be 90% (=1-4/40).

Exhibit 5 shows IBNR Runoff tables that result from applying the SWIMON interpolates of the Bloom annual LDF to a set of sample year-ending IBNR balances. These balances are not derived from any equilibrium condition, but are instead meant to typify a real-world situation. Nonetheless, using the SWIMON interpolates, the resulting IBNR Runoff schedule evolves in a reasonable fashion.

8. EQUILIBRIUM IBNR COMPARISON

In this section Equilibrium IBNR percentages by quarter are computed under the assumption of level growth and based on the three different methods of interpolation applied to the annual factors from Bloom's example. Formulas from Chapters 4 and 5 are used and all values are expressed as percentages of ultimate loss for an accident year. Results are shown in Exhibit 6 for the SWIMON method, in Exhibit 7 for the 12/IVP procedure, and in Exhibit 8 for the Weibull Splices approach. The "B" sections of these exhibits show the computation of the change in IBNR by quarter based on the interpolated factors. The "A" sections show the change in IBNR by accident year and quarter for five subsequent quarters. The "A" sections have prior year and current year totals and the grand totals for each quarter. A summary comparison is provided in Exhibit 9.

Exhibit 9 shows that the SWIMON method is the only one to satisfy Equilibrium IBNR stability. It also shows that the SWIMON and the Weibull Splicing methods satisfy the monotonic decreasing total prior year IBNR runoff property.

9. CONCLUSION

This paper has made the initial effort in defining some basic properties that are desirable in an LDF interpolation routine. It has gone beyond the purely mathematical aspects of general interpolation to focus on the particular qualities of LDF interpolation. It has documented the widespread actuarial practice of producing quarterly IBNR run-off schedules and highlighted the importance of examining the IBNR run-off projections out to five quarters at least.

It has demonstrated one set of simple tools for interpolating LDF. The tail-tapering is useful in its own right. The cross-year averaging of increments of development with annual normalization and monotonicity adjustment combines a series of mathematically basic steps to produce a robust result. The strategy of cross-year smoothing, of not looking at each year in isolation, is an advance over splicing. Even though the back-filling for level equilibrium IBNR is computationally straightforward, it has a stronger conceptual foundation than various numerical extension routines and it eliminates unintended, algorithmic-induced seasonality.

In conclusion, it has been argued in this paper that LDF interpolation should be done on a whole curve basis with focus on the behavior of the resulting IBNR projections. Other approaches that examine years in isolation or ignore IBNR evolution are effectively missing one of the key reasons why actuaries interpolate LDF in the first place. This paper was written to address the challenge faced by reserving actuaries in updating and projecting IBNR each quarter. Such a practical focus has led to a better understanding of the conceptual attributes of desirable interpolation routines. It is hoped others will advance this line of thinking further perhaps by proposing more sophisticated sets of properties interpolates should satisfy or by developing more sophisticated set of tools to produce even better-behaved interpolations.

Appendix A – Different Representations of Loss Development

One of the practical observations offered in this paper is that there is useful flexibility to be gained in keeping on hand several equivalent ways to describe loss development. The actuary can then adopt whatever perspective is most convenient for solving a particular problem. The different representations are:

- age-to-age factors
- age-to-ultimate factors
- percent of ultimate values
- incremental percentages = IBNR takedown schedules
- IBNR and tail decay rates

For t = 1, 2, 3, …, , let X(t) be the incremental amount of loss development in the t^th^ period for one particular exposure period and let S(t) be the cumulative development so that:

$$S(t) = X(1) + X(2) + \ldots, + X(t)$$

Define the Age-to-Age factor:

$$ATA(t) = S(t+1)/S(t).$$

Let X(t) = B*INC(t) and S(t) = B* PCT(t) where

$$INC(t) = PCT(t) - PCT(t-1).$$

Also define the Age-to-Ultimate factor

$$ATU(t) = 1/PCT(t).$$

In this construction, B is the ultimate loss, PCT is the percent of ultimate, and INC is the increment of development. Note that B, S, X, PCT, INC, ATA, and ATU are all random variables.

Define random variables, Q(1), Q(2), … ,Q(t), where $0 < Q(t) < 1$, via:

$$Q(1) = PCT(1) \qquad \text{Eq(1)}$$

$$Q(t+1) = \frac{INC(t+1)}{1 - PCT(t)}$$

The Q random variables are called the tail decay rate random variables. Q(t) is called the decay rate and is interpreted as the fraction of the loss development tail remaining after time, t-1, that will be reported during the t^{th} period. If one has a set of decay rate variables, the process can be run in reverse to generate a percent of ultimate pattern.

Eq (2)

$$PCT(t) = 1 - \prod_{s=1} (1 - Q(s)) \tag{2.1}$$

$$INC(t) = Q(t) \cdot \prod_{s=1} (1 - Q(s)) \tag{2.2}$$

For example, if Q(1) is 20% and Q(2) is 10%, then PCT(2) = 1-(.8)(.9) = 28% and INC(2) = .10*(1-.8) = 8%.

EXHIBITS

Glossary of Exhibits

1	Interpolation: SWIMON	
	1A	Full curve
	1B	Smoothing Increments for Mature AY
	1C	Early Age Equilibrium Backfill
2	12/IVP Interpolation	
3	Weibull Splicing	
4	Interpolation Methods Comparison	
	4A	ATA and ATU LDF
	4B	PCT ULT and Increments
5	IBNR Runoff under SWIMON	
6	Equilibrium IBNR: SWIMON	
7	Equilibrium IBNR: 12/IVP	
8	Equilibrium IBNR: Weibull Spliced	
9	Equilibrium IBNR Comparison	

Exhibit 1A

Quarterly LDF Interpolation
SWIMON

(1) Age	(2) Interval	(3) AY ATA LDF	(4) AY ATU LDF	(5) AY PCT ULT	(6) AY Increm by Yr	(7) AY Increm Interp by Q	(8) AY PCT ULT	(9) AY ATA LDF	(10) AY ATU LDF	(11) ITD ATU LDF
						From Ex 1C	Running sum of (7)	Ratios of consec rows of (8)	1/(9)	ITD Expos as % of AY*(10)
0				0.00%						
3	0 - 3					9.38%	9.38%	2.268	10.661	2.665
6	3 - 6					11.90%	21.28%	1.648	4.700	2.350
9	6 - 9					13.79%	35.06%	1.429	2.852	2.139
12	9 - 12	1.500	1.996	50.11%	50.11%	15.04%	50.11%	1.156	1.996	1.996
15	12 - 15					7.83%	57.94%	1.113	1.726	1.726
18	15 - 18					6.52%	64.46%	1.087	1.551	1.551
21	18 - 21					5.61%	70.07%	1.073	1.427	1.427
24	21 - 24	1.200	1.331	75.16%	25.05%	5.09%	75.16%	1.061	1.331	1.331
27	24 - 27					4.61%	79.77%	1.051	1.254	1.254
30	27 - 30					4.05%	83.82%	1.041	1.193	1.193
33	30 - 33					3.48%	87.30%	1.033	1.145	1.145
36	33 - 36	1.050	1.109	90.19%	15.03%	2.89%	90.19%	1.018	1.109	1.109
39	36 - 39					1.66%	91.85%	1.013	1.089	1.089
42	39 - 42					1.18%	93.03%	1.010	1.075	1.075
45	42 - 45					0.89%	93.91%	1.008	1.065	1.065
48	45 - 48	1.025	1.056	94.70%	4.51%	0.79%	94.70%	1.008	1.056	1.056
51	48 - 51					0.71%	95.41%	1.006	1.048	1.048
54	51 - 54					0.60%	96.02%	1.006	1.041	1.041
57	54 - 57					0.54%	96.55%	1.005	1.036	1.036
60	57 - 60	1.020	1.030	97.07%	2.37%	0.51%	97.07%	1.005	1.030	1.030
63	60 - 63					0.51%	97.58%	1.005	1.025	1.025
66	63 - 66					0.50%	98.08%	1.005	1.020	1.020
69	66 - 69					0.48%	98.56%	1.005	1.015	1.015
72	69 - 72	1.010	1.010	99.01%	1.94%	0.45%	99.01%	1.003	1.010	1.010
75	72 - 75					0.30%	99.31%	1.002	1.007	1.007
78	75 - 78					0.25%	99.55%	1.002	1.004	1.004
81	78 - 81					0.22%	99.78%	1.002	1.002	1.002
84	81 - 84	1.000	1.000	100.00%	0.99%	0.22%	100.00%	1.000	1.000	1.000

Exhibit 1B

Quarterly LDF Interpolation

Normalized Cross-Year Smoothing of AY Increments

Fixed to Inherit Monotonicity

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Age	Interval	AY ATA LDF	AY ATU LDF	AY PCT ULT	AY Increm by Yr	Initial AY Increm by Qtr	AY Increm Smooth 1	AY Increm Smooth 2	Norm AY Increm	Mono Fixed Norm AY Increm
						By year, (6)/4	*3-pt smooth of (7)*	*3-pt smooth of (8)*	*Normalize by year to match*	*(10) or Average (10) and (7)*
0				0.00%						
3	0 - 3					12.53%				
6	3 - 6					12.53%				
9	6 - 9					12.53%	*12.53%*			
12	9 - 12	1.500	1.996	50.11%	50.11%	12.53%	*10.44%*	*10.44%*		
15	12 - 15					6.26%	8.35%	8.35%	7.83%	7.05%
18	15 - 18					6.26%	6.26%	6.96%	6.52%	6.39%
21	18 - 21					6.26%	6.26%	5.98%	5.61%	5.94%
24	21 - 24	1.200	1.331	75.16%	25.05%	6.26%	5.43%	5.43%	5.09%	5.68%
27	24 - 27					3.76%	4.59%	4.59%	4.61%	4.61%
30	27 - 30					3.76%	3.76%	4.04%	4.05%	4.05%
33	30 - 33					3.76%	3.76%	3.47%	3.48%	3.48%
36	33 - 36	1.050	1.109	90.19%	15.03%	3.76%	2.88%	2.88%	2.89%	2.89%
39	36 - 39					1.13%	2.00%	2.00%	1.66%	1.66%
42	39 - 42					1.13%	1.13%	1.42%	1.18%	1.18%
45	42 - 45					1.13%	1.13%	1.07%	0.89%	0.89%
48	45 - 48	1.025	1.056	94.70%	4.51%	1.13%	0.95%	0.95%	0.79%	0.79%
51	48 - 51					0.59%	0.77%	0.77%	0.71%	0.71%
54	51 - 54					0.59%	0.59%	0.65%	0.60%	0.60%
57	54 - 57					0.59%	0.59%	0.58%	0.54%	0.54%
60	57 - 60	1.020	1.030	97.07%	2.37%	0.59%	0.56%	0.56%	0.51%	0.51%
63	60 - 63					0.49%	0.52%	0.52%	0.54%	0.51%
66	63 - 66					0.49%	0.49%	0.50%	0.51%	0.50%
69	66 - 69					0.49%	0.49%	0.46%	0.47%	0.48%
72	69 - 72	1.010	1.010	99.01%	1.94%	0.49%	0.41%	0.41%	0.42%	0.45%
75	72 - 75					0.25%	0.33%	0.33%	0.30%	0.30%
78	75 - 78					0.25%	0.25%	0.27%	0.25%	0.25%
81	78 - 81					0.25%	0.25%	0.25%	0.22%	0.22%
84	81 - 84	1.000	1.000	100.00%	0.99%	0.25%	0.25%	0.25%	0.22%	0.22%

Exhibit 1C

Quarterly LDF Interpolation
Backfill for Equilibruim IBNR Stability

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Age	Interval	AY Interp Qtrly Increm after 12 mos	Change in IBNR	Cal Q in Year y	Prior AY Total Change in Equil IBNR	AY Loss Exposure ITD	Change in Exposure	Change in Equil IBNR	AY Increm
			-(3)			*min((1),12)/ 12*	*Diff of consec rows of (7)*	*-(6)*	*(8)-(9)*
0						0.00%			
3	0 - 3			1	-15.62%	25.00%	25.00%	15.62%	9.38%
6	3 - 6			2	-13.10%	50.00%	25.00%	13.10%	11.90%
9	6 - 9			3	-11.21%	75.00%	25.00%	11.21%	13.79%
12	9 - 12			4	-9.96%	100.00%	25.00%	9.96%	15.04%
15	12 - 15	7.83%	-7.83%			100.00%	0.00%	-7.83%	7.83%
18	15 - 18	6.52%	-6.52%			100.00%	0.00%	-6.52%	6.52%
21	18 - 21	5.61%	-5.61%			100.00%	0.00%	-5.61%	5.61%
24	21 - 24	5.09%	-5.09%			100.00%	0.00%	-5.09%	5.09%
27	24 - 27	4.61%	-4.61%			100.00%	0.00%	-4.61%	4.61%
30	27 - 30	4.05%	-4.05%			100.00%	0.00%	-4.05%	4.05%
33	30 - 33	3.48%	-3.48%			100.00%	0.00%	-3.48%	3.48%
36	33 - 36	2.89%	-2.89%			100.00%	0.00%	-2.89%	2.89%
39	36 - 39	1.66%	-1.66%			100.00%	0.00%	-1.66%	1.66%
42	39 - 42	1.18%	-1.18%			100.00%	0.00%	-1.18%	1.18%
45	42 - 45	0.89%	-0.89%			100.00%	0.00%	-0.89%	0.89%
48	45 - 48	0.79%	-0.79%			100.00%	0.00%	-0.79%	0.79%
51	48 - 51	0.71%	-0.71%			100.00%	0.00%	-0.71%	0.71%
54	51 - 54	0.60%	-0.60%			100.00%	0.00%	-0.60%	0.60%
57	54 - 57	0.54%	-0.54%			100.00%	0.00%	-0.54%	0.54%
60	57 - 60	0.51%	-0.51%			100.00%	0.00%	-0.51%	0.51%
63	60 - 63	0.51%	-0.51%			100.00%	0.00%	-0.51%	0.51%
66	63 - 66	0.50%	-0.50%			100.00%	0.00%	-0.50%	0.50%
69	66 - 69	0.48%	-0.48%			100.00%	0.00%	-0.48%	0.48%
72	69 - 72	0.45%	-0.45%			100.00%	0.00%	-0.45%	0.45%
75	72 - 75	0.30%	-0.30%			100.00%	0.00%	-0.30%	0.30%
78	75 - 78	0.25%	-0.25%			100.00%	0.00%	-0.25%	0.25%
81	78 - 81	0.22%	-0.22%			100.00%	0.00%	-0.22%	0.22%
84	81 - 84	0.22%	-0.22%			100.00%	0.00%	-0.22%	0.22%

Exhibit 2

Quarterly LDF Interpolation
IVP and Method of 12

Early Age	Plus 12 Method
Mature Age	IVP Decay for each year
ln(ATU-1) = ln(a)+ b*ln(1/T)	

(1) Age	(2) AY ATA LDF	(3) AY ATU LDF	(4) AY PCT ULT	(5) Exposure (ETD) as % of AY ULT	(6) Interval	(7)	(8)	(9) AY ATU LDF	(10) AY PCT ULT	(11) AY ATA LDF
			1/(4)							
0			0.00%	0.00%	0 - 3	Plus 12 Method			0.00%	
3				25.00%	3 - 6	{[ATU12)]^(12+12+t)/12}/ETD		13.404	7.46%	2.377
6				50.00%	6 - 9			5.639	17.73%	1.783
9				75.00%	9 - 12			3.163	31.62%	1.585
12	1.500	1.996	50.11%	100.00%	12 - 15			1.996	50.11%	1.175
15				100.00%	15 - 18	a	51.9	1.698	58.89%	1.115
18				100.00%	18 - 21	b	1.6	1.522	65.69%	1.081
21				100.00%	21 - 24			1.409	70.99%	1.059
24	1.200	1.331	75.16%	100.00%	24 - 27			1.331	75.16%	1.074
27				100.00%	27 - 30	a	2008.5	1.239	80.69%	1.051
30				100.00%	30 - 33	b	2.7	1.179	84.80%	1.036
33				100.00%	33 - 36			1.138	87.87%	1.026
36	1.050	1.109	90.19%	100.00%	36 - 39			1.109	90.19%	1.017
39				100.00%	39 - 42	a	428.0	1.090	91.71%	1.013
42				100.00%	42 - 45	b	2.3	1.076	92.92%	1.011
45				100.00%	45 - 48			1.065	93.90%	1.009
48	1.025	1.056	94.70%	100.00%	48 - 51			1.056	94.70%	1.008
51				100.00%	51 - 54	a	2479.2	1.047	95.48%	1.007
54				100.00%	54 - 57	b	2.8	1.040	96.12%	1.005
57				100.00%	57 - 60			1.035	96.64%	1.004
60	1.020	1.030	97.07%	100.00%	60 - 63			1.030	97.07%	1.008
63				100.00%	63 - 66	a	1.8E+09	1.022	97.80%	1.005
66				100.00%	66 - 69	b	6.1	1.017	98.33%	1.004
69				100.00%	69 - 72			1.013	98.72%	1.003
72	1.010	1.010	99.01%	100.00%	72 - 75	Linear ATU		1.010	99.01%	1.002
75				100.00%	75 - 78	a	######	1.008	99.26%	1.002
78				100.00%	78 - 81	b	134.4	1.005	99.50%	1.002
81				100.00%	81 - 84			1.003	99.75%	1.003
84	1.000	1.000	####	100.00%	84 -			1.000	100.00%	1.000

Exhibit 3

Quarterly LDF Interpolation
Weibull Splicing

Weibull Fit age 12-60	a	(1.7469)
IBNR= exp(-c*T^b)	b	0.7517
T = Avg Maturity	C= exp(a)	0.1743

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Age	Avg Loss Maturity	AY ATA LDF	AY ATU LDF	IBNR	ln(-ln(IBNR))	ln(Maturity)	Weibull Fit IBNR Curve	Weibull IBNR Interp	PCT ULT for Maturity	ATU LDF for Maturity	AY ATU LDF	AY ATA LDF
				1-1/(4)	*ln(-ln((5)))*	*ln((2))*		*Scale (8) by year to hit (5)*	*1-(9)*	*1/(10)*	*(11)*min (1,12/(1))*	*(12)/(12) next row*
0	0.0			100.0%			100.0%	100.0%	0.0%			
3	1.5						78.9%	78.4%	21.6%	4.629	#####	3.119
6	3.0						67.2%	66.3%	33.7%	2.968	5.937	1.906
9	4.5						58.3%	57.2%	42.8%	2.336	3.115	1.561
12	6.0	1.500	1.996	49.89%	(0.363)	1.792	51.2%	49.9%	50.1%	1.996	1.996	1.184
15	9.0						40.3%	40.7%	59.3%	1.685	1.685	1.114
18	12.0						32.4%	33.9%	66.1%	1.514	1.514	1.077
21	15.0						26.3%	28.8%	71.2%	1.405	1.405	1.056
24	18.0	1.200	1.331	24.84%	0.331	2.890	21.6%	24.8%	75.2%	1.331	1.331	1.067
27	21.0						17.9%	19.8%	80.2%	1.247	1.247	1.050
30	24.0						15.0%	15.8%	84.2%	1.187	1.187	1.039
33	27.0						12.5%	12.5%	87.5%	1.143	1.143	1.031
36	30.0	1.050	1.109	9.81%	0.842	3.401	10.6%	9.8%	90.2%	1.109	1.109	1.016
39	33.0						8.9%	8.4%	91.6%	1.091	1.091	1.013
42	36.0						7.6%	7.1%	92.9%	1.077	1.077	1.011
45	39.0						6.5%	6.1%	93.9%	1.065	1.065	1.009
48	42.0	1.025	1.056	5.30%	1.078	3.738	5.5%	5.3%	94.7%	1.056	1.056	1.008
51	45.0						4.7%	4.6%	95.4%	1.048	1.048	1.007
54	48.0						4.1%	3.9%	96.1%	1.041	1.041	1.006
57	51.0						3.5%	3.4%	96.6%	1.035	1.035	1.005
60	54.0	1.020	1.030	2.93%	1.261	3.989	3.0%	2.9%	97.1%	1.030	1.030	1.006
63	57.0						2.6%	2.3%	97.7%	1.024	1.024	1.005
66	60.0						2.3%	1.8%	98.2%	1.018	1.018	1.005
69	63.0						2.0%	1.4%	98.6%	1.014	1.014	1.004
72	66.0	1.010	1.010	0.99%	1.529	4.190	1.7%	1.0%	99.0%	1.010	1.010	1.003
75	69.0						1.5%	0.7%	99.3%	1.007	1.007	1.003
78	72.0						1.3%	0.4%	99.6%	1.004	1.004	1.002
81	75.0						1.1%	0.2%	99.8%	1.002	1.002	1.002
84	78.0	1.000	1.000	0.00%			1.0%	0.0%	100.0%	1.000	1.000	1.000

Exhibit 4A

Interpolation Methods Comparison
ATU and ATA

			AY ATU LDF			AY ATA LDF		
Age	Interval	Original ATU LDF	SWIM	12-12 and IVF Method	Weibull Splicing	SWIM	12-12 and IVF Method	Weibull Splicing
0								
3	0 - 3		10.661	13.404	18.518	2.268	2.377	3.119
6	3 - 6		4.700	5.639	5.937	1.648	1.783	1.906
9	6 - 9		2.852	3.163	3.115	1.429	1.585	1.561
12	9 - 12	1.996	1.996	1.996	1.996	1.156	1.175	1.184
15	12 - 15		1.726	1.698	1.685	1.113	1.115	1.114
18	15 - 18		1.551	1.522	1.514	1.087	1.081	1.077
21	18 - 21		1.427	1.409	1.405	1.073	1.059	1.056
24	21 - 24	1.331	1.331	1.331	1.331	1.061	1.074	1.067
27	24 - 27		1.254	1.239	1.247	1.051	1.051	1.050
30	27 - 30		1.193	1.179	1.187	1.041	1.036	1.039
33	30 - 33		1.145	1.138	1.143	1.033	1.026	1.031
36	33 - 36	1.109	1.109	1.109	1.109	1.018	1.017	1.016
39	36 - 39		1.089	1.090	1.091	1.013	1.013	1.013
42	39 - 42		1.075	1.076	1.077	1.010	1.011	1.011
45	42 - 45		1.065	1.065	1.065	1.008	1.009	1.009
48	45 - 48	1.056	1.056	1.056	1.056	1.008	1.008	1.008
51	48 - 51		1.048	1.047	1.048	1.006	1.007	1.007
54	51 - 54		1.041	1.040	1.041	1.006	1.005	1.006
57	54 - 57		1.036	1.035	1.035	1.005	1.004	1.005
60	57 - 60	1.030	1.030	1.030	1.030	1.005	1.008	1.006
63	60 - 63		1.025	1.022	1.024	1.005	1.005	1.005
66	63 - 66		1.020	1.017	1.018	1.005	1.004	1.005
69	66 - 69		1.015	1.013	1.014	1.005	1.003	1.004
72	69 - 72	1.010	1.010	1.010	1.010	1.003	1.002	1.003
75	72 - 75		1.007	1.008	1.007	1.002	1.002	1.003
78	75 - 78		1.004	1.005	1.004	1.002	1.002	1.002
81	78 - 81		1.002	1.003	1.002	1.002	1.003	1.002
84	81 - 84	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Exhibit 4B

Interplation Methods Comparison
PCT ULT and Increments

			AY PCT ULT			AY Increments		
Age	Interval	Original AY PCT ULT	SWIM	12-12 and IVF Method	Weibull Splicing	SWIM	12-12 and IVF Method	Weibull Splicing
0								
3	0 - 3		9.4%	7.5%	5.4%	9.4%	7.5%	5.4%
6	3 - 6		21.3%	17.7%	16.8%	11.9%	10.3%	11.4%
9	6 - 9		35.1%	31.6%	32.1%	13.8%	13.9%	15.3%
12	9 - 12	50.1%	50.1%	50.1%	50.1%	15.0%	18.5%	18.0%
15	12 - 15		57.9%	58.9%	59.3%	7.8%	8.8%	9.2%
18	15 - 18		64.5%	65.7%	66.1%	6.5%	6.8%	6.7%
21	18 - 21		70.1%	71.0%	71.2%	5.6%	5.3%	5.1%
24	21 - 24	75.2%	75.2%	75.2%	75.2%	5.1%	4.2%	4.0%
27	24 - 27		79.8%	80.7%	80.2%	4.6%	5.5%	5.0%
30	27 - 30		83.8%	84.8%	84.2%	4.1%	4.1%	4.0%
33	30 - 33		87.3%	87.9%	87.5%	3.5%	3.1%	3.3%
36	33 - 36	90.2%	90.2%	90.2%	90.2%	2.9%	2.3%	2.7%
39	36 - 39		91.9%	91.7%	91.6%	1.7%	1.5%	1.5%
42	39 - 42		93.0%	92.9%	92.9%	1.2%	1.2%	1.2%
45	42 - 45		93.9%	93.9%	93.9%	0.9%	1.0%	1.0%
48	45 - 48	94.7%	94.7%	94.7%	94.7%	0.8%	0.8%	0.8%
51	48 - 51		95.4%	95.5%	95.4%	0.7%	0.8%	0.7%
54	51 - 54		96.0%	96.1%	96.1%	0.6%	0.6%	0.6%
57	54 - 57		96.6%	96.6%	96.6%	0.5%	0.5%	0.5%
60	57 - 60	97.1%	97.1%	97.1%	97.1%	0.5%	0.4%	0.5%
63	60 - 63		97.6%	97.8%	97.7%	0.5%	0.7%	0.6%
66	63 - 66		98.1%	98.3%	98.2%	0.5%	0.5%	0.5%
69	66 - 69		98.6%	98.7%	98.6%	0.5%	0.4%	0.4%
72	69 - 72	99.0%	99.0%	99.0%	99.0%	0.5%	0.3%	0.4%
75	72 - 75		99.3%	99.3%	99.3%	0.3%	0.2%	0.3%
78	75 - 78		99.6%	99.5%	99.6%	0.2%	0.2%	0.3%
81	78 - 81		99.8%	99.8%	99.8%	0.2%	0.2%	0.2%
84	81 - 84	100.0%	100.0%	100.0%	100.0%	0.2%	0.2%	0.2%

Exhibit 5A

AY IBNR Run-off by Q
LDF Interpolation: SWIMON

AY	Year end IBNR	Q1	Q2	Q3	Q4	Q5
y						
y-1	800	674	570	480	398	324
y-2	610	497	397	312	241	200
y-3	320	266	227	199	173	150
y-4	500	433	376	325	277	228
y-5	80	66	52	39	27	19
y-6	10	7	5	2	-	-
Total Prior AY	2,320	1,943	1,627	1,357	1,116	921

IBNR Run-off AY	Q1	Q2	Q3	Q4	Q5
y					
y-1	126	105	90	82	74
y-2	113	99	85	71	41
y-3	54	38	29	26	23
y-4	67	57	51	49	48
y-5	14	14	13	12	8
y-6	3	3	2	2	-
Total Prior AY	377	315	270	241	194

Exhibit 5B

IBNR Runoff Calculations
LDF Interpolation: SWIMON

(1) Age	(2) Interval	(3) Interp ATA LDF	(4) ATU LDF	(5) AY PCT ULT	(6) Tail of ULT Loss	(7) Increment	(8) IBNR Runoff Factor	(9) Exposure to Date (ETD)%	(10) ETD IBNR%	(11) Change in IBNR (% AY ULT)
			Running back product of (3)	*1/(4)*	*1-(5)*	*Row Diffs of (6)*	*(7)/(6)*	*AY Uniform Expos*	*(9)-(5)*	
0				0.00%	100.00%			0.00%		
3	0 - 3	2.268	10.661	9.38%	90.62%	9.38%		25.00%	15.62%	15.62%
6	3 - 6	1.648	4.700	21.28%	78.72%	11.90%		50.00%	28.72%	13.10%
9	6 - 9	1.429	2.852	35.06%	64.94%	13.79%		75.00%	39.94%	11.21%
12	9 - 12	1.156	1.996	50.11%	49.89%	15.04%		100.00%	49.89%	9.96%
15	12 - 15	1.113	1.726	57.94%	42.06%	7.83%	15.69%	100.00%	42.06%	-7.83%
18	15 - 18	1.087	1.551	64.46%	35.54%	6.52%	15.51%	100.00%	35.54%	-6.52%
21	18 - 21	1.073	1.427	70.07%	29.93%	5.61%	15.79%	100.00%	29.93%	-5.61%
24	21 - 24	1.061	1.331	75.16%	24.84%	5.09%	17.00%	100.00%	24.84%	-5.09%
27	24 - 27	1.051	1.254	79.77%	20.23%	4.61%	18.56%	100.00%	20.23%	-4.61%
30	27 - 30	1.041	1.193	83.82%	16.18%	4.05%	20.03%	100.00%	16.18%	-4.05%
33	30 - 33	1.033	1.145	87.30%	12.70%	3.48%	21.50%	100.00%	12.70%	-3.48%
36	33 - 36	1.018	1.109	90.19%	9.81%	2.89%	22.77%	100.00%	9.81%	-2.89%
39	36 - 39	1.013	1.089	91.85%	8.15%	1.66%	16.94%	100.00%	8.15%	-1.66%
42	39 - 42	1.010	1.075	93.03%	6.97%	1.18%	14.44%	100.00%	6.97%	-1.18%
45	42 - 45	1.008	1.065	93.91%	6.09%	0.89%	12.70%	100.00%	6.09%	-0.89%
48	45 - 48	1.008	1.056	94.70%	5.30%	0.79%	12.92%	100.00%	5.30%	-0.79%
51	48 - 51	1.006	1.048	95.41%	4.59%	0.71%	13.45%	100.00%	4.59%	-0.71%
54	51 - 54	1.006	1.041	96.02%	3.98%	0.60%	13.15%	100.00%	3.98%	-0.60%
57	54 - 57	1.005	1.036	96.55%	3.45%	0.54%	13.48%	100.00%	3.45%	-0.54%
60	57 - 60	1.005	1.030	97.07%	2.93%	0.51%	14.94%	100.00%	2.93%	-0.51%
63	60 - 63	1.005	1.025	97.58%	2.42%	0.51%	17.44%	100.00%	2.42%	-0.51%
66	63 - 66	1.005	1.020	98.08%	1.92%	0.50%	20.62%	100.00%	1.92%	-0.50%
69	66 - 69	1.005	1.015	98.56%	1.44%	0.48%	24.94%	100.00%	1.44%	-0.48%
72	69 - 72	1.003	1.010	99.01%	0.99%	0.45%	31.34%	100.00%	0.99%	-0.45%
75	72 - 75	1.002	1.007	99.31%	0.69%	0.30%	29.82%	100.00%	0.69%	-0.30%
78	75 - 78	1.002	1.004	99.55%	0.45%	0.25%	35.62%	100.00%	0.45%	-0.25%
81	78 - 81	1.002	1.002	99.78%	0.22%	0.22%	50.00%	100.00%	0.22%	-0.22%
84	81 - 84	1.000	1.000	100.00%	0.00%	0.22%	100.00%	100.00%	0.00%	-0.22%

Exhibit 6A

IBNR Change Projection in Equilibrium Assuming Level Growth
Interpolation: SWIMON

Change in IBNR Projected by Q					
AY	Q1	Q2	Q3	Q4	Q5
y	15.62%	13.10%	11.21%	9.96%	-7.83%
y-1	-7.83%	-6.52%	-5.61%	-5.09%	-4.61%
y-2	-4.61%	-4.05%	-3.48%	-2.89%	-1.66%
y-3	-1.66%	-1.18%	-0.89%	-0.79%	-0.71%
y-4	-0.71%	-0.60%	-0.54%	-0.51%	-0.51%
y-5	-0.51%	-0.50%	-0.48%	-0.45%	-0.30%
y-6	-0.30%	-0.25%	-0.22%	-0.22%	
AY y	15.62%	13.10%	11.21%	9.96%	-7.83%
All Prior	-15.62%	-13.10%	-11.21%	-9.96%	-7.79%
Total	0.00%	0.00%	0.00%	0.00%	-15.62%

Exhibit 6B

Calculation of IBNR Change Assuming Level Equilibrium
Interpolation: SWIMON

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Age	Interval	Interp ATA LDF	ATU LDF	AY PCT ULT	Tail of AY ULT Loss	Increm	Expos to Date (ETD) %	Change in ETD	Tail of Loss ETD as % of AY Ult	Change in IBNR
			Running back product of (3)	*1/(4)*	*1-(5)*	*Row Diffs of (6)*	*min(12,(1))/12*	*Row Diffs of (8)*	*(8)-(5)*	*(9)-(7)*
0				0.00%	100.00%		0.00%			
3	0 - 3	2.268	10.661	9.38%	90.62%	9.38%	25.00%	25.00%	15.62%	15.62%
6	3 - 6	1.648	4.700	21.28%	78.72%	11.90%	50.00%	25.00%	28.72%	13.10%
9	6 - 9	1.429	2.852	35.06%	64.94%	13.79%	75.00%	25.00%	39.94%	11.21%
12	9 - 12	1.156	1.996	50.11%	49.89%	15.04%	100.00%	25.00%	49.89%	9.96%
15	12 - 15	1.113	1.726	57.94%	42.06%	7.83%	100.00%	0.00%	42.06%	-7.83%
18	15 - 18	1.087	1.551	64.46%	35.54%	6.52%	100.00%	0.00%	35.54%	-6.52%
21	18 - 21	1.073	1.427	70.07%	29.93%	5.61%	100.00%	0.00%	29.93%	-5.61%
24	21 - 24	1.061	1.331	75.16%	24.84%	5.09%	100.00%	0.00%	24.84%	-5.09%
27	24 - 27	1.051	1.254	79.77%	20.23%	4.61%	100.00%	0.00%	20.23%	-4.61%
30	27 - 30	1.041	1.193	83.82%	16.18%	4.05%	100.00%	0.00%	16.18%	-4.05%
33	30 - 33	1.033	1.145	87.30%	12.70%	3.48%	100.00%	0.00%	12.70%	-3.48%
36	33 - 36	1.018	1.109	90.19%	9.81%	2.89%	100.00%	0.00%	9.81%	-2.89%
39	36 - 39	1.013	1.089	91.85%	8.15%	1.66%	100.00%	0.00%	8.15%	-1.66%
42	39 - 42	1.010	1.075	93.03%	6.97%	1.18%	100.00%	0.00%	6.97%	-1.18%
45	42 - 45	1.008	1.065	93.91%	6.09%	0.89%	100.00%	0.00%	6.09%	-0.89%
48	45 - 48	1.008	1.056	94.70%	5.30%	0.79%	100.00%	0.00%	5.30%	-0.79%
51	48 - 51	1.006	1.048	95.41%	4.59%	0.71%	100.00%	0.00%	4.59%	-0.71%
54	51 - 54	1.006	1.041	96.02%	3.98%	0.60%	100.00%	0.00%	3.98%	-0.60%
57	54 - 57	1.005	1.036	96.55%	3.45%	0.54%	100.00%	0.00%	3.45%	-0.54%
60	57 - 60	1.005	1.030	97.07%	2.93%	0.51%	100.00%	0.00%	2.93%	-0.51%
63	60 - 63	1.005	1.025	97.58%	2.42%	0.51%	100.00%	0.00%	2.42%	-0.51%
66	63 - 66	1.005	1.020	98.08%	1.92%	0.50%	100.00%	0.00%	1.92%	-0.50%
69	66 - 69	1.005	1.015	98.56%	1.44%	0.48%	100.00%	0.00%	1.44%	-0.48%
72	69 - 72	1.003	1.010	99.01%	0.99%	0.45%	100.00%	0.00%	0.99%	-0.45%
75	72 - 75	1.002	1.007	99.31%	0.69%	0.30%	100.00%	0.00%	0.69%	-0.30%
78	75 - 78	1.002	1.004	99.55%	0.45%	0.25%	100.00%	0.00%	0.45%	-0.25%
81	78 - 81	1.002	1.002	99.78%	0.22%	0.22%	100.00%	0.00%	0.22%	-0.22%
84	81 - 84	1.000	1.000	100.00%	0.00%	0.22%	100.00%	0.00%	0.00%	-0.22%

Exhibit 7A

IBNR Change Projection in Equilibrium Assuming Level Growth
Interpolation: 12/IVP

	Change in IBNR Projected by Q				
AY	Q1	Q2	Q3	Q4	Q5
y	17.54%	14.73%	11.12%	6.51%	-8.78%
y-1	-8.78%	-6.80%	-5.30%	-4.17%	-5.53%
y-2	-5.53%	-4.11%	-3.07%	-2.32%	-1.52%
y-3	-1.52%	-1.21%	-0.98%	-0.80%	-0.78%
y-4	-0.78%	-0.63%	-0.52%	-0.43%	-0.73%
y-5	-0.73%	-0.53%	-0.39%	-0.29%	-0.25%
y-6	-0.25%	-0.25%	-0.25%	-0.25%	
AY y	17.54%	14.73%	11.12%	6.51%	-8.78%
All Prior	-17.59%	-13.53%	-10.51%	-8.26%	-8.81%
Total	-0.05%	1.19%	0.61%	-1.75%	-17.59%

Exhibit 7B

Calculation of IBNR Change Assuming Level Equilibrium
Interpolation: 12/IVP

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Age	Interval	Interp ATA LDF	ATU LDF	AY PCT ULT	Tail of AY ULT Loss	Increm	Expos to Date (ETD) as % of AY ULT	Change in ETD	Tail of Loss ETD as % of AY Ult	Change in IBNR
			Running back product of (3)	*1/(4)*	*1-(5)*	*Row Diffs of (6)*	*min(12,(1))/12*	*Row Diffs of (8)*	*(8)-(5)*	*(9)-(7)*
0				0.00%	100.00%		0.00%			
3	0 - 3	2.377	13.404	7.46%	92.54%	7.46%	25.00%	25.00%	17.54%	17.54%
6	3 - 6	1.783	5.639	17.73%	82.27%	10.27%	50.00%	25.00%	32.27%	14.73%
9	6 - 9	1.585	3.163	31.62%	68.38%	13.88%	75.00%	25.00%	43.38%	11.12%
12	9 - 12	1.175	1.996	50.11%	49.89%	18.49%	100.00%	25.00%	49.89%	6.51%
15	12 - 15	1.115	1.698	58.89%	41.11%	8.78%	100.00%	0.00%	41.11%	-8.78%
18	15 - 18	1.081	1.522	65.69%	34.31%	6.80%	100.00%	0.00%	34.31%	-6.80%
21	18 - 21	1.059	1.409	70.99%	29.01%	5.30%	100.00%	0.00%	29.01%	-5.30%
24	21 - 24	1.074	1.331	75.16%	24.84%	4.17%	100.00%	0.00%	24.84%	-4.17%
27	24 - 27	1.051	1.239	80.69%	19.31%	5.53%	100.00%	0.00%	19.31%	-5.53%
30	27 - 30	1.036	1.179	84.80%	15.20%	4.11%	100.00%	0.00%	15.20%	-4.11%
33	30 - 33	1.026	1.138	87.87%	12.13%	3.07%	100.00%	0.00%	12.13%	-3.07%
36	33 - 36	1.017	1.109	90.19%	9.81%	2.32%	100.00%	0.00%	9.81%	-2.32%
39	36 - 39	1.013	1.090	91.71%	8.29%	1.52%	100.00%	0.00%	8.29%	-1.52%
42	39 - 42	1.011	1.076	92.92%	7.08%	1.21%	100.00%	0.00%	7.08%	-1.21%
45	42 - 45	1.009	1.065	93.90%	6.10%	0.98%	100.00%	0.00%	6.10%	-0.98%
48	45 - 48	1.008	1.056	94.70%	5.30%	0.80%	100.00%	0.00%	5.30%	-0.80%
51	48 - 51	1.007	1.047	95.48%	4.52%	0.78%	100.00%	0.00%	4.52%	-0.78%
54	51 - 54	1.005	1.040	96.12%	3.88%	0.63%	100.00%	0.00%	3.88%	-0.63%
57	54 - 57	1.004	1.035	96.64%	3.36%	0.52%	100.00%	0.00%	3.36%	-0.52%
60	57 - 60	1.008	1.030	97.07%	2.93%	0.43%	100.00%	0.00%	2.93%	-0.43%
63	60 - 63	1.005	1.022	97.80%	2.20%	0.73%	100.00%	0.00%	2.20%	-0.73%
66	63 - 66	1.004	1.017	98.33%	1.67%	0.53%	100.00%	0.00%	1.67%	-0.53%
69	66 - 69	1.003	1.013	98.72%	1.28%	0.39%	100.00%	0.00%	1.28%	-0.39%
72	69 - 72	1.002	1.010	99.01%	0.99%	0.29%	100.00%	0.00%	0.99%	-0.29%
75	72 - 75	1.002	1.008	99.26%	0.74%	0.25%	100.00%	0.00%	0.74%	-0.25%
78	75 - 78	1.002	1.005	99.50%	0.50%	0.25%	100.00%	0.00%	0.50%	-0.25%
81	78 - 81	1.003	1.003	99.75%	0.25%	0.25%	100.00%	0.00%	0.25%	-0.25%
84	81 - 84	1.000	1.000	100.00%	0.00%	0.25%	100.00%	0.00%	0.00%	-0.25%

Exhibit 8A

IBNR Change Projection in Equilibrium Assuming Level Growth
Interpolation: Weibull Splice

Change in IBNR Projected by Q					
AY	Q1	Q2	Q3	Q4	Q5
y	19.60%	13.56%	9.74%	6.99%	-9.22%
y-1	-9.22%	-6.74%	-5.11%	-3.98%	-5.04%
y-2	-5.04%	-4.04%	-3.28%	-2.68%	-1.46%
y-3	-1.46%	-1.21%	-1.01%	-0.84%	-0.75%
y-4	-0.75%	-0.63%	-0.53%	-0.45%	-0.60%
y-5	-0.60%	-0.52%	-0.44%	-0.38%	-0.30%
y-6	-0.30%	-0.26%	-0.23%	-0.20%	
AY y	19.60%	13.56%	9.74%	6.99%	-9.22%
All Prior	-17.37%	-13.39%	-10.60%	-8.53%	-8.15%
Total	2.23%	0.16%	-0.85%	-1.53%	-17.37%

Exhibit 8B

Calculation of IBNR Change Assuming Level Equilibrium
Interpolation: Weibull Splice

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Age	Interval	Interp ATA LDF	ATU LDF	AY PCT ULT	Tail of AY ULT Loss	Increm	Expos to Date (ETD) as % of AY ULT	Change in ETD	Tail of Loss ETD as % of AY Ult	Change in IBNR
			Running back product of (3)	*1/(4)*	*1-(5)*	*Row Diffs of (6)*	*min(12,(1))/12*	*Row Diffs of (8)*	*(8)-(5)*	*(9)-(7)*
0				0.00%	100.00%		0.00%			
3	0 - 3	3.119	18.518	5.40%	94.60%	5.40%	25.00%	25.00%	19.60%	19.60%
6	3 - 6	1.906	5.937	16.84%	83.16%	11.44%	50.00%	25.00%	33.16%	13.56%
9	6 - 9	1.561	3.115	32.10%	67.90%	15.26%	75.00%	25.00%	42.90%	9.74%
12	9 - 12	1.184	1.996	50.11%	49.89%	18.01%	100.00%	25.00%	49.89%	6.99%
15	12 - 15	1.114	1.685	59.33%	40.67%	9.22%	100.00%	0.00%	40.67%	-9.22%
18	15 - 18	1.077	1.514	66.07%	33.93%	6.74%	100.00%	0.00%	33.93%	-6.74%
21	18 - 21	1.056	1.405	71.18%	28.82%	5.11%	100.00%	0.00%	28.82%	-5.11%
24	21 - 24	1.067	1.331	75.16%	24.84%	3.98%	100.00%	0.00%	24.84%	-3.98%
27	24 - 27	1.050	1.247	80.20%	19.80%	5.04%	100.00%	0.00%	19.80%	-5.04%
30	27 - 30	1.039	1.187	84.24%	15.76%	4.04%	100.00%	0.00%	15.76%	-4.04%
33	30 - 33	1.031	1.143	87.51%	12.49%	3.28%	100.00%	0.00%	12.49%	-3.28%
36	33 - 36	1.016	1.109	90.19%	9.81%	2.68%	100.00%	0.00%	9.81%	-2.68%
39	36 - 39	1.013	1.091	91.65%	8.35%	1.46%	100.00%	0.00%	8.35%	-1.46%
42	39 - 42	1.011	1.077	92.85%	7.15%	1.21%	100.00%	0.00%	7.15%	-1.21%
45	42 - 45	1.009	1.065	93.86%	6.14%	1.01%	100.00%	0.00%	6.14%	-1.01%
48	45 - 48	1.008	1.056	94.70%	5.30%	0.84%	100.00%	0.00%	5.30%	-0.84%
51	48 - 51	1.007	1.048	95.45%	4.55%	0.75%	100.00%	0.00%	4.55%	-0.75%
54	51 - 54	1.006	1.041	96.08%	3.92%	0.63%	100.00%	0.00%	3.92%	-0.63%
57	54 - 57	1.005	1.035	96.61%	3.39%	0.53%	100.00%	0.00%	3.39%	-0.53%
60	57 - 60	1.006	1.030	97.07%	2.93%	0.45%	100.00%	0.00%	2.93%	-0.45%
63	60 - 63	1.005	1.024	97.67%	2.33%	0.60%	100.00%	0.00%	2.33%	-0.60%
66	63 - 66	1.005	1.018	98.19%	1.81%	0.52%	100.00%	0.00%	1.81%	-0.52%
69	66 - 69	1.004	1.014	98.63%	1.37%	0.44%	100.00%	0.00%	1.37%	-0.44%
72	69 - 72	1.003	1.010	99.01%	0.99%	0.38%	100.00%	0.00%	0.99%	-0.38%
75	72 - 75	1.003	1.007	99.31%	0.69%	0.30%	100.00%	0.00%	0.69%	-0.30%
78	75 - 78	1.002	1.004	99.58%	0.42%	0.26%	100.00%	0.00%	0.42%	-0.26%
81	78 - 81	1.002	1.002	99.80%	0.20%	0.23%	100.00%	0.00%	0.20%	-0.23%
84	81 - 84	1.000	1.000	100.00%	0.00%	0.20%	100.00%	0.00%	0.00%	-0.20%

Exhibit 9

IBNR Change Projection in Equilibrium Assuming Level Growth
Comparison of Interpolation Methods

	Change in IBNR Projected by Q								
	SWIMON			**12/IVP**			**Weibull Spliced**		
Qtr	AY y	All Prior AY	Total	AY y	All Prior AY	Total	AY y	All Prior AY	Total
Q1	15.62%	-15.62%	0.00%	17.54%	-17.59%	-0.05%	19.60%	-17.37%	2.23%
Q2	13.10%	-13.10%	0.00%	14.73%	-13.53%	1.19%	13.56%	-13.39%	0.16%
Q3	11.21%	-11.21%	0.00%	11.12%	-10.51%	0.61%	9.74%	-10.60%	-0.85%
Q4	9.96%	-9.96%	0.00%	6.51%	-8.26%	-1.75%	6.99%	-8.53%	-1.53%
Q5	-7.83%	-7.79%	-15.62%	-8.78%	-8.81%	-17.59%	-9.22%	-8.15%	-17.37%

REFERENCES

[1] Lynne Bloom, "Interpolation Hacks and their Efficacy", *CAS E-Forum,* **Fall 2015,** p.1-63.
[2] Joseph Boor, "Interpolation Along a Curve", *Variance,* **Vol 8, Issue 1,** 2016, p.9-21.
[3] Ira Robbin, "Exposure Dependent Modeling of Percent of Ultimate Curves", *CAS Forum*, **spring 2004**, p. 401-458.
[4] Ira Robbin and David Homer, "Analysis of Loss Development Patterns Using Infinitely Decomposable Percent of Ultimate Curves", *CAS Discussion Paper Program*, **May, 1988**, p501-538.
[5] Richard E. Sherman, "Extrapolating, Smoothing and Interpolating Development Factors", *PCAS*, **1984,** p122-155

Abbreviations and Notations

ATA, Age-to-Age
ATU, Age-to-Ultimate
LDF, Loss Development Factor
PCT ULT, Percent of Ultimate

Biography of the Author

Ira Robbin is currently Assistant Vice-President in Economic Capital Modeling at TransRe in New York City. Ira received a Bachelor's Degree in Math from Michigan State University and a PhD in Math from Rutgers University. He has served in a variety of research, actuarial pricing, reserving, and corporate roles over his career at companies including the Insurance Company of North America (INA), CIGNA Property and Casualty, ACE, Partner RE, Endurance, and AIG. While developing new techniques and theories, he has headed large risk property and casualty pricing units, developed pricing algorithms, produced price monitors, conducted reserve reviews, priced treaties, allocated capital, and computed ROE. He has written several Proceedings, Forum, and Study Note papers on a range of subjects, taught exam preparation classes and made numerous presentations at actuarial meetings.

Disclaimers

Opinions expressed in this paper are solely those of the author. They are not presented as the express or implied positions of the authors' current or prior employers or clients. No warranty is given that any formula or assertion is accurate. No liability is assumed whatsoever for any losses, direct or indirect, that may result from use of the methods described in this paper or reliance on any of the views expressed therein.