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This *E-Forum* contains Report 11 of the CAS Risk-Based Capital Dependencies and Calibration Working Party (Reports 1 and 2 are posted in *E-Forum* Winter 2012-Volume 1; Reports 3 and 4 in *E-Forum* Fall 2012-Volume 2; Report 5 in *E-Forum* Summer 2012; Report 6 in *E-Forum* Fall 2013; Report 7 in *E-Forum* Fall 2013; Report 8 in *E-Forum* Spring 2014; Report 9 in *E-Forum* Fall 2014-Volume 2); and Report 10 in *E-Forum* Winter 2016.

This *E-Forum* also contains one independent research paper.

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Risk-Based Capital (RBC) Underwriting Risk Factor Safety Levels

Report 11 of the CAS Risk-Based Capital (RBC) Research Working Parties

Issued by the RBC Dependencies and Calibration Working Party (DCWP)

Abstract: The underwriting elements in the NAIC Property Casualty RBC Formula (RBC Formula) are not selected to achieve a particular total safety level. We examine the historical variability in underwriting experience and measure the achieved safety level in terms of a Value at Risk (VaR). As explained in this paper, we consider a Policyholder View for measuring safety level as opposed to a Company View. We demonstrate that the line of business (LOB) risk factors for premium and reserves, while calibrated to an 87.5th percentile safety margin with a Company View, produce a safety margin higher than 87.5% on a Policyholder View.

We show that the underwriting risk charge resulting from the combined effects of individual line of business premium and reserve risk charges, the diversification credits, and the dependency between premium and reserve risk in the 2010 RBC Formula produces a 91% safety level.

This analysis does not evaluate the effect on the safety level of other elements of the RBC Formula, i.e., the R0, R1, R2, R3 risks including R3-Reinsurance Credit Risk, the own company adjustment factors, loss sensitive contract discounts, the growth risk charge or the choice of 5% interest rate assumption in the investment income offset. The paper identifies potential biases in observed safety level due to the use of immature data in the analysis.

This is one of several papers being issued by the Risk-Based Capital (RBC) Dependencies and Calibration.

Keywords: Risk-Based Capital, Capital Requirements, Analyzing/Quantifying Risks, Assess/Prioritize Risks, Integrate Risks

1. Introduction and Findings

1.1 Background

The underwriting elements in the RBC Formula are not selected to achieve a particular total safety level. When the RRFs and PRFs, collectively underwriting (UW) risk factors, were updated by the NAIC in 2008, 2009 and 2010, those elements of the RBC Formula were selected to equal the 87.5th percentile of company-LOB data points in the ten accident years (AYs)/runoff years of data from the most recent Annual Statement, for all companies above a threshold size level, excluding anomalous data and subject to limits on fluctuations in risk factors from year to year.[1] These calibrations represent a VaR approach based on the frequency of company UW results above the VaR threshold levels used to establish the RBC Company Action Level.[2]

[1] American Academy of Actuaries, P/C Risk-Based Capital Working Group, "An Update to P/C Risk-Based Capital Underwriting Factors," September 2007, page 6.

[2] The RBC Formula is used to produce several capital values such that if company capital falls below those levels company or regulatory action is triggered. The first trigger, corresponding the highest of those capital amounts, is called the Company Action Level. For more details, see the NAIC, "Risk-Based Capital Forecasting & Instructions," Property Casualty, 2010. Our analysis of safety level is relative to Company Action Level of RBC.

We assess the effect of that LOB-calibration on the broader RBC Formula considering other facets of the RBC Formula. In addition to the LOB risk factors we consider the premium concentration factor (PCF), the loss concentration factor (LCF) and the dependency between premium risk and reserve risk.

Moreover, in addition to considering the safety level based on the number of companies with underwriting variability below the Company Action Level RBC (Company View), we examine the safety level by summing the premiums plus reserves[3] for companies with variability below that RBC level. This alternative approach gives more weight to large companies with more policyholders and claimants than does counting the number of companies. We refer to the premium + reserve basis as the Policyholder View of safety level.[4] The distinction between number of policyholders/claimants and number of companies is important because, generally, larger LOBs have lower variability and achieve higher safety levels when risk charges do not vary by LOB-size.[5,6]

We use 23 years of loss ratio experience, accident years 1988-2010, and 22 years of reserve runoffs from 1988-2009. We measure the achieved premium safety level as the percentage of Net Earned Premium or (NEP) for companies with loss and loss adjustment expense ratios (LRs) or AY underwriting results that are more favorable than the RBC UW factors. We measure the achieved reserve safety level as the percentage of reserves[3] for companies with runoff results that are more favorable than the RBC UW factors.

1.2 Findings

1.2.1 Findings - Observed Safety Levels

We test how well the current UW risk factors stand up against the observed variability of company reserve development and accident year loss ratios observed in the prior 22 and 23 years respectively of experience within our data set at three levels of detail:

By LOBs,

All-lines reserve risk and premium risk separately, and

All-lines premium and reserve risks combined.

Table 1.1 below shows that for all-lines premium and reserve risk combined, the 2010 RBC Formula produces a safety level of 91.2%[7]. By this we mean that 91.2% of NEP plus reserves[3] from

[3] Reserves here include A&O

[4] We recognize that premium and claim reserves reflect many variables in addition to the number of policyholders and claimants. Nonetheless, we believe the reference is a useful contrast to the alternative Company View.

[5] DCWP Report 6 pages 21-25 and 60-64 and Report 7 pages 25-30 and 60-66, regarding premium risk and reserve risk by LOB-size, respectively.

[6] This issue affects Solvency II calibrations as discussed in EIOPA, "Calibration of the Premium and reserve risk Factors in the Standard Formula of Solvency II, Report of the Joint Working Group on Non-Life and Health NSLT Calibration, pg. 31-33 and 57-58.

[7] Before adjustment for maturity issues discussed below.

company/year combinations that have an observed risk from NEP and reserves that is less than the risk determined from the RBC Formula for premiums and reserves[3].

Looking at reserve risk alone for all lines combined, 91.1% of reserves[3] are from company/year combinations that have an observed risk that is less than the reserve risk determined from the RBC Formula. Looking at premium risk alone, 90.5% of NEP is from company/year combinations that have an observed risk that is less than the premium risk from the RBC Formula.

Table 1.1
Reserve and Premium Safety Levels

Risk	Safety Levels[8]	Basis
Reserve	91.1%	Percentage of reserve including A&O
Premium	90.5%	Percentage of NEP
Premium & Reserve Combined	91.2%	Percentage of NEP and reserve including A&O

1.2.2 Findings - Maturity Effect

DCWP Reports 6 and 7 show that the least mature data points indicate low PRF and RRF values that develop upward at later maturities, and, therefore that the use of the least mature data might understate the risk factors.[9]

We test the potential impact of that finding on the observed safety level. To do so we repeat the analysis excluding the least mature data points. We find that removing the four least mature years from the data history for premium and reserves reduces the safety level from 91.2% to 88.6%.[10]

Table 1.2
Effect of Maturity Adjustment
Reserve and Premium Safety Levels Excluding Data with the Least 4 Mature Points[11]

Risk	Safety Level	Basis
Reserve	88.4%	Percentage of reserve including A&O
Premium	89.2%	Percentage of NEP
Premium & Reserve Combined	88.6%	Percentage of NEP and reserve including A&O

The decrease in safety level from Table 1.1 to Table 1.2 could be due, in part or in whole, to factors other than maturity. For example, our test excludes recent data that might be more favorable than the

[8] Before adjustment for maturity issues discussed below.
[9] DCWP Report 6 pages 25-30. DCWP Report 7 pages 30-34.
[10] A portion of the decline relates to the difference in years included. A portion relates the maturity of the data points. It is beyond the scope of this paper to determine the portion related to maturity.
[11] For reserves this excludes data points with maturities of 24, 36, 48 and 60 months. For premiums this excludes data points with an AY maturity of 12, 24, 36 and 48 months.

long term history. Nonetheless, the observation supports the need for further research on the effect of maturity on risk factor calibration and back-testing.

1.2.4 Remainder of Report

In the remainder of this report:

- Section 2 provides more detail regarding our approach,
- Section 3 describes the safety level analyses by LOB,
- Section 4 describes the safety level analyses for all LOBs combined, separately for premium risk and reserve risk,
- Section 5 describes the safety level analysis for all LOBs combined, for combined premium and reserve risk,
- Section 6 presents further results on the impact of excluding the least mature years,
- Section 7 lists areas for further research, and
- Appendices A, B and C contain further information about our data and a number of sensitivity tests.

1.3 Terminology, Assumed Reader Background and Disclaimer

This paper assumes the reader is generally familiar with the property/casualty RBC Formula[12] and has a working knowledge of DCWP Reports 6 and 7.

In this paper, references to "we" and "our" refer to the principal authors of this paper. The "working party," and "DCWP" refer to the RBC Dependencies and Calibration Working Party.

The analysis and opinions expressed in this report are solely those of the authors, and in particular are not those of the members' employers, the Casualty Actuarial Society, or the American Academy of Actuaries.

DCWP makes no recommendations to the NAIC or any other body. DCWP material is for the information of CAS members, policy makers, actuaries and others who might make recommendations regarding the future of the P&C RBC Formula. In particular, we expect that the material will be used by the American Academy of Actuaries.

This paper is one of a series of articles prepared under the direction of the DCWP.

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[12] For a detailed description of the formula and its basis, see Feldblum, Sholom, "NAIC Property/Casualty Insurance Company Risk-Based Capital Requirements," *Proceedings of the Casualty Actuarial Society*, 1996 and NAIC, "Risk-Based Capital Forecasting & Instructions," Property Casualty, 2010.

2. Approach

Our approach to measuring the implied safety level in the RBC Formula is as follows:

1. We obtain the observed Reserve Runoff Ratios (RRRs) and observed LRs for each LOB/company/year from 1996-2010 Annual Statements as described in DCWP Reports 6 and 7.
2. We apply the RBC Formula to each LOB/company/year NEP and loss reserve and use the Company Action Level RBC as the predicted or modeled values of expected variability (or risk) for each LOB/company/year data point.
3. We interpret the observed RRRs and observed LRs as reflecting the anticipated future distribution (or risk) of actual RRRs and LRs.
4. We count the <u>number</u> of data points where the RRR or LR, respectively, do not exceed the modeled value. The proportion of data points with RRR or LR below the modeled value can be interpreted as a "per Company View" of the safety level for LOB risk factors.
5. We total the <u>reserve</u> or <u>premium</u> for the data points where the RRR or LR, respectively, do not exceed the modeled value. Summing the premiums plus reserves gives more weight to large companies with more policyholders and claimants than does counting the number of companies. We refer to the premium + reserve basis as the Policyholder View of safety level.[13]
6. We focus on the Policyholder View, rather than the Company View, in that this gives the security level on a "per policyholder basis" or "per claimant basis".[14] The "per policyholder/per claimant" safety level will tend to be higher than the per company safety level because larger LOBs tend to have less variation in experience than smaller LOBs, and larger LOBs have a high proportion of the policyholders/claimants.
7. We apply this approach for premium and reserve risk by LOB, for premium and reserve risks with all-lines combined, and for all-lines with premium and reserve risks combined.

2.1 Important Approximations/Simplifications

Several issues that are particularly important in interpreting the results of this paper are discussed below.

First, our analysis does not include all of the elements of the RBC Formula.

Within the premium and reserve risk calculations, we do not consider the effect on safety level from the following: (1) the own-company adjustment, (2) loss sensitive business discount, (3) the growth risk

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[13] We recognize that premium and claim reserves reflect many variables in addition to the number of policyholders and claimants. We believe the reference is a useful contrast to the alternative Company View.

[14] The Solvency II "99.5%" safety level also considered both the company view and the policy view. Joint Working Group on Non-Life and Health NSLT Calibration, pg. 31-33 and 57-58.

charge (3) the choice of 5% interest rate assumption in the investment income offset, or (5) the portion of R3-Reinsurance Credit Risk associated with underwriting risk. We discuss the potential effect of these simplifications in Section 7, "Future Research".

Second, the paper identifies potential size of biases in observed safety level due to the use of some immature data in the measurement of achieved safety level, but does not attempt to fully quantify that.

Third, we also do not test the R0, R1, R2 or R3 elements of the RBC Formula.

Fourth, the reliability of these estimates of safety levels depends on the extent to which there is enough data, both in number of data points and with respect to variability in economic and insurance market conditions. For example, the other liability LOB results for reserves are influenced by continuing adverse development of asbestos liabilities. To the extent that the severity and longevity of that source of claim development is not representative of the future, the data may understate the safety level for the other liability LOB. On the other hand, the adverse medical malpractice reinsurance experience of the 1970-1985 time period is not represented in the calibration data.

3. Safety Levels by LOB

The subsections below discuss modeled risk, observed risk and observed safety level, the key elements used in this analysis of the safety levels resulting from RRFs and PRFs by LOB.

3.1 Data

3.1.1 Data for Modeled Risk

The key elements in the modeled risk are the NEP and reserves[3] used to calculate premium and reserve risk respectively. For each LOB/company/year that information is available in our risk data set.

We use the 2010 RBC PRF and RRF factors shown in Appendix A.

3.1.2 Data on Observed Risk

The observed RRRs and LRs are the LOB/Company/Year data points described in DCWP Reports 6 and 7.[15]

In brief, the RRR data consists of reserve runoff ratios for initial reserve dates 1988-2009. The ratios, net of reinsurance, are developed through the latest available maturity from Schedule P, Parts 2 and 3, in the 1997-2010 Annual Statements, by LOB and company for individual companies and DWCP-defined group pools, as indicated. Thus, each data point is the runoff ratio from a single reserve date and LOB for a single company or pool (LOB/company/year).

Similarly, the LR data consists of AYs 1988-2010 loss and loss adjustment expense ratios, net of

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[15] DCWP Report 6 pages 7-16. DCWP Report 7 pages 9-21.

reinsurance, at the latest available maturity from Schedule P, Part 1, in the 1997-2010 Annual Statements, by LOB and by company for individual companies and DWCP-defined group pools, as indicated. Thus, each data point is a single AY and LOB for a single company or pool (LOB/company/year).

3.1.3 Selection of Data Points (Matching Year "Y-1" and Year "Y")

To calculate observed and modeled premium and reserve risk, as we use those terms in this Report, we use Risk Data, as follows:

- Premium Risk:

  For modeled premium risk, following the RBC Formula, we use NEP for year Y-1 as the base for premium risk as that is the base for premium risk arising from year Y.

  For observed premium risk we use the LR[16] and NEP for year Y as those represent the incurred loss that emerges in observed year Y.

  For example, we use year 2000 NEP and PRF to predict observed 2001 NEP and LR.

- Reserve Risk:

  For modeled reserve risk, following the RBC Formula, we use reserve for year Y-1 as that is base for reserve risk in the RBC Formula for year Y-1.

  For observed reserve risk we use the RRR from year Y-1 and the initial reserve at year Y-1, as those represent the runoff in calendar year Y and subsequent on year Y-1 unpaid claims.

  For example, we use year 2000 initial reserve and RRF to predict reserve runoff in 2001 and subsequent on claims unpaid at year end 2000.

- Combined Risk:

  We use the data required for both premium risk and reserve.

Table 3-1 below summarizes the data requirements.

Table 3-1
Data Required to Evaluate Modeled Risk vs. Observe Risk for Year Y[17]

Premium Risk	Reserve Risk	Premium + Reserve Risk
NEP –Year Y-1	Initial reserve Year Y-1	NEP –Year Y-1
NEP – Year Y	RRR – Year Y-1	NEP – Year Y
LR – Year Y		LR – Year Y
		Initial reserve Year Y-1
		RRR – Year Y-1

[16] The observed incurred losses and LAE for each LOB/Year/Company data point are at the most mature evaluation date available. For example, from the 2010 Annual Statement the 2002 AY would be evaluated at 9 years maturity, 2003 AY at 8 years of maturity and so on until 2010 at 1 year of maturity.

[17] As explained below, for premium risk and combined premium and reserve risk we also use Company Expense Ratios (CERs) for Year Y-1 and Y.

In some cases we have data required for premium, but we do not have data required for reserves, or conversely. In those cases we could do either the premium analysis of the reserve analysis, but we cannot do the combined premium + reserve analysis. However, while it is more stringent than necessary, we chose to do all the analyses using only the data points for which we have the data required for the premium + reserve analysis.[18][19]

In the LOB analysis in this section we simplify the comparison. We use year Y data for both modeled and observed risk. With this simplification we remove the effect of year-to-year changes in premium levels and we thereby specifically test the model LR distribution against the actual LR distribution. This simplified approach corresponds to the way the PRFs are calibrated.

3.1.4 Approximations

In applying the portion of the RBC Formula that we analyze, we make two approximations:

1. For premium risk, the RBC Formula uses net written premium (NWP). We use NEP, which is more readily available. The effect of using NEP rather than NWP would tend to reduce the measure of actual risk, as NWP can vary more from year to year than is the case for NEP.
2. For reserve risk, the RBC Formula uses unpaid loss plus Defense and Cost Containment Expenses (DCC) plus Adjusting and Other expenses (A&O). Our observed data only includes loss plus DCC. We thus use loss + DCC reserves and apply a factor to approximate and include the A&O value. In Appendix B we discuss how we determined the A&O factors and its impact on the results.

3.2 Modeled Risk

The modeled risk in this analysis is the premium or reserve risk charge produced by the RBC Formula. We call these the Modeled Premium Risk (MPR_{LOB}) and Modeled Reserve Risk (MRR_{LOB}), respectively. These values are the 2010 PRF or RRF factors multiplied by the reserve or NEP for the LOB/company/year data points.

Specifically, the modeled risk values are calculated as follows:

$$MRR_{LOB,YEAR,COMPANY} = Reserve_{LOB,YEAR,COMPANY} * (1+A\&O\%_{LOB,YEAR,COMPANY}) * RRF_{LOB}$$

$$MPR_{LOB,YEAR,COMPANY} = NEP_{LOB,YEAR,COMPANY} * PRF_{LOB}$$

[18] Excluding the company/year combinations without a full match increases the premium safety level by 0.3% and the reserve safety level by less than 0.001%.

[19] Note that in the section 3 LOB analysis we used premium or reserve data points regardless of whether there were corresponding reserve or premium data points, respectively, and regardless of whether we had data points for both the current and subsequent years.

3.3 Observed Risk

The observed risk is the distribution of reserve runoff and incurred losses by LOB/company/year. We call these the Observed Reserve Risk ($ORR_{LOB,YEAR,COMPANY}$) and the Observed Premium Risk ($OPR_{LOB,YEAR,COMPANY}$).[20]

These calculations are as follows:

$ORR_{LOB,YEAR,COMPANY} = RRR_{LOB,YEAR,COMPANY} * Reserve_{LOB,YEAR,COMPANY} * (1+A\&O\%_{LOB,YEAR,COMPANY})$

$OPR_{LOB,YEAR,COMPANY} = LR_{LOB,YEAR,COMPANY} * NEP_{LOB,YEAR,COMPANY}$

3.4 Observed Safety Level

The observed safety level is the percentage of reserve[3] (as opposed to data point counts) for which the $ORR_{LOB,YEAR,COMPANY}$ is less than the $MRR_{LOB,YEAR,COMPANY}$ or percentage of NEP for which the $OPR_{LOB,YEAR,COMPANY}$ is less than the $MPR_{LOB,YEAR,COMPANY}$ charge for that line of business.

For each LOB,

LOB Reserve Safety Level = (Sum of all reserves including A&O in data points for which $ORR_{LOB,YEAR,COMPANY} \leq MRR_{LOB,YEAR,COMPANY}$) divided by (Sum of all reserves including A&O)

LOB Premium Safety Level = (Sum of all NEP in data points for which $OPR_{LOB,YEAR,COMPANY} \leq MPR_{LOB,YEAR,COMPANY}$) divided by (Sum of all NEP)

The results are shown in Table 3.2 below.

[20] The LRs are evaluated at the most recent evaluation within the data set for each LOB/Year/Company data point. For example, from the 2010 Annual Statement the 2002 AY would be evaluated at 9 years maturity, 2003 AY at 8 years of maturity and so on until 2010 at 1 year of maturity. Likewise the RRRs are evaluated at the most recent evaluation of incurred development for that LOB/Year/Company data point. For example, from the 2010 Annual Statement the 2002 AY evaluated at 9 years of maturity and so on until 2009 AY at 2 years of maturity.

Table 3.2
Summary of Observed Safety Levels by LOB

Line of Business	Reserve	Premium
Homeowners/Farmowners	94.9%	86.2%
Priv. Passenger auto Liability	97.0%	94.3%
Commercial Auto Liability	90.7%	90.4%
Workers Comp	91.6%	86.2%
Commercial Multi-Peril	93.8%	91.2%
Medical Mal – Occurrence	96.2%	95.0%
Medical Mal - Claims Made	94.2%	77.5%
Special Liability	86.1%	90.6%
Other Liability	81.5%	90.2%
Special Property	79.7%	92.3%
Auto Physical Damage	94.4%	91.6%
Fidelity & Surety	83.5%	91.9%
Other	83.2%	79.1%
International	77.4%	89.9%
Reinsurance A&C	89.3%	92.4%
Reinsurance B	89.5%	93.2%
Products Liability	72.4%	90.1%
Financial Guarantee	95.2%	90.6%
Warranty	84.7%	91.9%
All Lines*	91.1%	90.5.%

*Note: The "All lines" value is calculated in Section 4.

In Table 3.2, we observe that even though the risk charges are intended to be calibrated to the 87.5th percentile safety level by LOB, for most lines, the calculated safety level is higher than 87.5%. This arises largely because the 87.5th percentile LR or RRR varies significantly by LOB-size.[21]

Tables 3.2A and 3.2B below show how the 87.5th percentile RRRs vary by LOB-size[22] and how they compare to the overall 87.5th percentile for the Private Passenger Auto Liability (PPA) and Other Liability[23] (OL) LOBs. Each of the 11 horizontal bars represents a group of companies with LOB reserve-size within a size band. The height of the bar represents the 87.5th percentile RRR for LOBs within that LOB-reserve size band. The horizontal line represents the 87.5th percentile RRR for all

[21] The differences between 87.5% and the observed safety levels by LOB might also arise if the PRFs and RRFs used in the RBC Formula were not consistent with the 87.5th percentile for the data used in this back-testing. To test for that possibility we calculate the PRFs and RRFs by LOB that would be indicated based on the data used in this back-testing. In Appendix D, Table D.1, we show that, even if using those indicated PRFs and RRFs, the patterns in observed safety levels by LOB are similar to the patterns in Table 3.1.

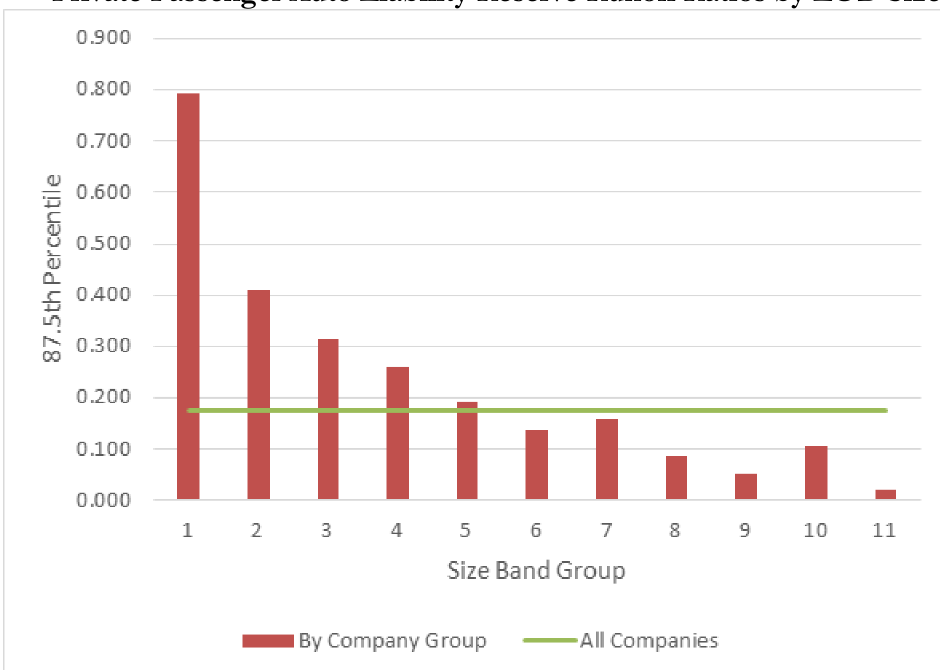
[22] LOB sizes are expressed in bands: band 1= 0-15% smallest, band 2= 15%-25%, band 3 = 25-35%, … band 9 = 85-95%, band 10 = 95%-100% less the 100 largest data points, band 11 is for the 100 largest data points, approximately 5 companies. The band percentiles are by number of companies and are not by reserve/premium size. The highest numbered band sizes contain the bulk of total reserve amounts and the bulk of total premium amounts. Size band 1 includes PPA reserves size of $0 to $0.8m. Size band 5, the median size band, covers PPA reserves sizes of $7.4m to $12.5m. Size bands 9 and 10, the largest 95% of reserve sizes covers reserves sizes of $105m to $17 billion. The band sizes are based on reserve for loss and DCC only. DCWP Report 7 page 60 and 62.

[23] Other Liability is a combination of data from the Other Liability Occurrence and Other Liability Claims Made lines

companies except those below a selected minimum size threshold.[24]

In table 3.2A we see that the 87.5th percentile RRFs for the largest LOB-sizes are the lowest, and, in particular, are lower than the combined all-size 87.5th percentile RRF. As the largest LOB-sizes contribute disproportionately to the total industry reserve dollars, the observed reserve safety level we calculated for this line is higher than the 87.5th percentile. The PPA pattern is directionally typical of many LOBs, although the magnitude of the decrease is more significant for PPA than for other LOBs.

Table 3.2A
Private Passenger Auto Liability Reserve Runoff Ratios by LOB-Size



In table 3.2B we see a different pattern that applies to some lines, including the Other Liability LOB. For Other Liability the largest LOB-sizes have RRFs that are larger than the combined all-size indicated RRF. As the largest companies contribute disproportionately to overall reserve dollars, the observed reserve safety level we calculated for this line is lower than the 87.5th percentile. DCWP Reports 6 and 7 provide further detail on variation in indicated 87.5th percentile safety levels by LOB-size for all each of the LOBs.[25]

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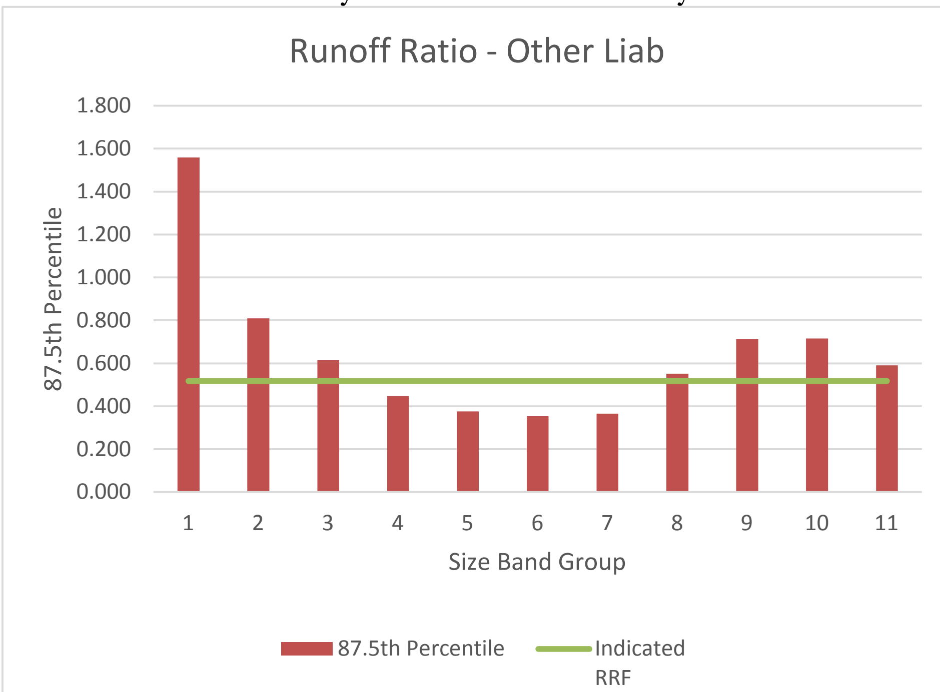
[24] $350k for PPA and $1,250 for OL. Less than 25% of LOB data points are smaller than that.
[25] DCWP Report 6 pages 21-25 and 60-64 and Report 7 pages 25-30 and 60-66, regarding premium risk and reserve risk, respectively.

Table 3.2B
Other Liability Reserve Runoff Ratios by LOB- Size



4. Reserve and Premium Safety Levels for All Lines Combined

The subsections below discuss modeled risk, observed risk and observed safety level, the key elements used in our analysis of the all-lines combined reserve and all-lines combined premium risk safety levels.

4.1 Data

The required data for this analysis includes the LOB NEP and reserve[3] amounts, and the LOB LRs and RRRs used in the previous sections.

In addition, we use investment income offsets (IIO's) by LOB, company expenses (all-lines, by company and year), and we calculate premium and reserve concentration factors (PCFs and LCFs) for each company/year combination.

4.1.1 IIO's

Although U.S. Statutory Accounting is based on loss reserves that are not discounted, the premium and reserve risk charges in the RBC Formula are reduced to the extent that future investment income on assets corresponding to unpaid claims and unearned premium is available to offset adverse outcomes. Specifically, the factor called the Investment Income Offset (IIO) in the RBC Formula serves to calculate the available investment income, and it is applied to produce a risk charge that is reduced accordingly. The IIO uses a 5% interest rate [26]. IIOs vary by line of business based on the LOB payment patterns. We represent the Premium and Reserve IIOs by LOB as IIO_P_{LOB} and IIO_R_{LOB}, respectively.

In examining individual LOBs, in Section 3, we do not reflect the IIOs, as modeled and actual risk levels have the same IIOs. In combining LOBs, as we do in this section, we use IIO's by LOB to recognize that $1 of adverse reserve development or underwriting loss in a short tail line has more impact on the financial condition of the company than $1 of adverse development or underwriting loss in a long tail line. We use the 2010 IIO_P_{LOB}'s and 2010 IIO_R_{LOB}'s that are based on a 5% interest rate regardless of the AY or initial reserve date.

4.1.2 Company Expense Ratios (CER%s)

For all lines combined, we convert PRFs to Premium Risk Charges (PRCs) using all-lines company expenses to calculate the underwriting loss that would apply if the loss ratio were at the PRF level. The PRC equals the PRF plus CER% minus 100%. This is the underwriting loss produced by a loss ratio equal to the PRF.

Appendix B provides details on how we estimate the company expenses by company/year.

4.1.3 Concentration/Diversifications Factors

In the RBC Formula the reserve and premium LOB risk charges after discounting are combined using concentration factors. The degree of concentration for each company/year combination is measured by taking the largest LOB[27] NEP or reserve[3], for premium risk and reserve risk, respectively, divided by the total NEP or reserve[3]. This ratio is 100% for mono-line companies. The ratio might be close to zero for highly diversified companies.[28] We refer to this method of measuring of concentration as the "Max Line%" approach.

The RBC Formula uses Max Line% and a maximum diversification credit of 30% to calculate PCFs and LCFs as follows:

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[26] We do <u>not</u> test the effect that interest rates changing over time on the safety margin.

[27] In measuring premium and loss (reserve) concentration, the RBC Formula combines data for Other Liability Occurrence and Other Liability Claims Made and does the same for Product Liability.

[28] 0% concentration is not achievable, but premium or reserves equally spread among 17 LOBs would produce a concentration value of 1/19 or 5.9%.

$PCF_{YEAR,COMPANY} = 0.7 + 0.3 * \text{Max Line \% (NEP)}_{YEAR,COMPANY}$

$LCF_{YEAR,COMPANY} = 0.7 + 0.3 * \text{Max Line \% (reserves)}_{YEAR,COMPANY}$

4.2 Modeled Risk

We calculate the modeled risk using the PRF and RRF values, PCF and LCF values, IIO_P and IIO_R values as discussed below.

4.2.1 Modeled Reserve Risk

For each LOB we calculate the model reserve risk ($MRR_{LOB,YEAR,COMPANY}$) using the LOB reserve, the $A\&O\%_{LOB,YEAR,COMPANY}$ RRF_{LOB}, and IIO_R_{LOB}. We combine these using the $LCF_{YEAR,COMPANY}$ as follows:

First we calculate the following for each LOB/company/year combination:

$MRR_{LOB,YEAR,COMPANY} = ((RRF_{LOB} +1) * IIO\_R_{LOB} -1.0) * Reserve_{LOB,YEAR,COMPANY} * (1+A\&O\%_{LOB,YEAR,COMPANY})$

Subject to the following condition:

If $((RRF_{LOB} + 1) * IIO\_R_{LOB} -1.0) < 0$, then $\text{Reserve Risk}_{LOB,YEAR,COMPANY} = 0.0$[29]

Then, for each company/year combination, we calculate the $MRR^{All\text{-}Lines}_{YEAR,COMPANY}$ as follows:

$MRR^{All\text{-}lines}_{YEAR,COMPANY} = [\sum \text{(over all LOBs) of } MRR_{LOB,YEAR,COMPANY}] * LCF_{YEAR,COMPANY}$

4.2.2 Modeled Premium Risk

For each LOB we calculate the model premium risk ($MPR_{LOB\ YEAR,COMPANY}$) using the PRF_{LOB}, $NEP_{LOB,YEAR,COMPANY}$, IIO_P_{LOB}, and $CER\%_{YEAR,COMPANY}$.[30] We combine these using $PCF_{YEAR,COMPANY}$.

First, we calculate the following for each LOB/company/year combination:

$MPR_{LOB,YEAR,COMPANY} = (PRF_{LOB} * IIO\_P_{LOB} + CER\%_{YEAR,COMPANY} -1.0) * NEP_{LOB,YEAR,COMPANY}$

Subject to the following conditions:

If $(PRF_{LOB} * IIO\_P_{LOB} + CER\%_{YEAR,COMPANY} -1.0) < 0$, then $\text{(Premium risk)}_{LOB,YEAR,COMPANY} = 0$,

and

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[29] This condition is always satisfied in our analysis. If the own-company or loss sensitive business discount were applied, the condition might affect the result for companies with favorable adjustments.

[30] Consistent with the RBC Formula, the all-lines company operating expense ratio is applied to each LOB. Therefore we have CER% rather than $CER\%_{LOB}$.

If $CER\%_{YEAR,COMPANY} > 400\%$ then $CER\%_{YEAR,COMPANY} = 400\%$[31]

Then, for each company/year combination we calculate $MPR^{All\text{-}Lines}_{YEAR,COMPANY}$ as follows:

$$MPR^{All\text{-}lines}_{YEAR,COMPANY} = [\sum \text{(over all LOBs) of Modeled Premium Risk}_{LOB,YEAR,COMPANY}] * PCF_{YEAR,COMPANY}$$

4.3 Observed Risk

The observed risk is the distribution of all-lines reserve runoff and AY UW losses by company/year after reserve and premium adjustments for investment income.

4.3.1 Observed Reserve Risk

We calculate the actual all lines reserve risk charge $ORR^{All\text{-}Lines}_{YEAR,COMPANY}$ for each company/year combination as follows:

$$ORR^{All\text{-}lines}_{YEAR,COMPANY} = \sum\text{(over all LOBs) of } [\{(1+ RRR_{LOB,YEAR,COMPANY}) * IIO\_R_{LOB} - 1\} * Reserve_{LOB,YEAR,COMPANY} * (1+A\&O\%_{LOB,YEAR,COMPANY})]$$

4.3.2 Observed Premium Risk

We calculate the actual premium risk charge $OPR^{All\text{-}Lines}_{YEAR,COMPANY}$ for each company/year combination as follows:

$$OPR^{All\text{-}lines}_{YEAR,COMPANY} = \sum\text{(over all LOBs) of } [LR_{LOB,YEAR,COMPANY} * IIO\_P_{LOB} + CER\%_{YEAR,COMPANY} -1.0) * NEP_{LOB,YEAR,COMPANY}]$$

Unlike the modeled premium risk, the CER% is not capped at 400% for the observed premium risk.

4.4 Observed Safety Level

We determine the observed safety level for reserve risk and premium risk as described below.

4.4.1 Reserve Safety Level

For each company/year combination, we compare the $ORR^{All\text{-}lines}_{YEAR,COMPANY}$ to the $MRR^{All\text{-}lines}_{YEAR,COMPANY}$. We calculate the industry reserve safety level as:

Reserve Safety Level = (Sum of all reserves including A&O in data points for which

[31] The 400% limit is in the RBC Formula.

$ORR^{All\text{-}lines}_{YEAR,COMPANY} \leq MRR^{All\text{-}lines}_{YEAR,COMPANY}$) divided by (Sum of all reserves including A&O)

4.4.2 Premium Safety Level

For each company/year combination, we compare the $OPR^{All\text{-}lines}_{YEAR,COMPANY}$ to the $MPR^{All\text{-}lines}_{YEAR\text{-}1,\ COMPANY}$. We calculate the industry premium safety level as:

Premium Safety Level = (Sum of all NEP in data points for which

$OPR^{All\text{-}lines}_{YEAR,COMPANY} \leq MPR^{All\text{-}lines}_{YEAR\text{-}1,\ COMPANY}$) divided by (Sum of all NEP)

4.4.3 Results

The results are shown in Table 4.1

Table 4.1
Safety Level for all-lines Premium and Reserve Risk

Risk	Safety Level	Basis
Reserve	91.1%	Percentage of reserve including A&O
Premium	90.5%	Percentage of NEP

5. Combined Premium + Reserve Safety Level

The subsections below discuss modeled risk, observed risk and observed safety level, the key elements used in the analysis of the combined premium + reserve ("Underwriting" or "UW") safety level.

5.1 Modeled Risk

We described the separate all-lines premium and all-lines reserve modeled risk charges ($MRR^{All\text{-}lines}_{YEAR\text{-}1,\ COMPANY}$ and $MPR^{All\text{-}lines}_{YEAR\text{-}1,\ COMPANY}$) in section 4 above. The charges are combined with a square root rule to give the combined modeled UW risk:

$MUWR^{All\text{-}lines}_{YEAR,COMPANY}$ = Square Root ($MRR^{All\text{-}lines}_{YEAR\text{-}1,\ COMPANY}$ ^2 + $MPR^{All\text{-}lines}_{YEAR\text{-}1,\ COMPANY}$ ^2)

Note that we use year Y-1 data to model the underwriting risk in year Y.

The square root rule reflects the RBC Formula assumption that premium risk and reserve risk are not correlated.

5.2 Observed Risk

The observed risk is the distribution of discounted all-lines reserve runoff plus the discounted all-lines accident year underwriting results by company/year, expressed as a dollar amount.

The accident year LRs and RRRs in this analysis are constructed as discussed in Section 4. They are combined as follows:

$$OUWR^{\text{All-lines}}_{\text{YEAR,COMPANY}} = ORR^{\text{All-lines}}_{\text{YEAR-1, COMPANY}} + OPR^{\text{All-lines}}_{\text{YEAR,COMPANY}}$$

In the OUWR, the reserve runoff affecting year Y is the reserve runoff from year Y-1, hence the mixture of "Y" and "Y-1" in the OUWR formula above.

5.3 Observed Safety Level

The safety level for the combined is measured as the amount of NEP and reserves[3] from the company/year combinations that have an $OUWR^{\text{All-lines}}_{\text{YEAR,COMPANY}} \leq MUWR^{\text{All-lines}}_{\text{YEAR,COMPANY}}$.

Table 5.1 shows the results for premium and reserve combined, comparing those to the separate premium and reserve results from Table 4.3.

Table 5.1
Summary of Reserve and Premium Safety Levels

Risk	Safety Level	Basis
Reserve	91.1%	Percentage of reserve including A&O
Premium	90.5%	Percentage of NEP
Premium & Reserve Combined	91.2%	Percentage of NEP and reserve including A&O

6. Impact of Maturity

DCWP Reports 6 and 7 show that the least mature data points indicate low PRF and RRF values that develop upward at later maturities, and therefore the use of the least mature data might understate the risk factors.

We test the potential impact of that observation on the achieved safety level. To do so we repeat the analysis, excluding the least mature LOB/company/year data points. As the less mature years are excluded the combined level safety decreases, as we show in Table 6.1.

Table 6.1
Impact on Safety Level of Excluding Least Mature Years

Maturities Excluded	Combined Risk as % NEP & Reserves including A&O	Premium Risk as % NEP	Reserve Risk as % Reserves Including A&O
None	91.2%	90.5%	91.1%
Least Mature Year	90.6%	90.6%	90.6%
Least Two Mature Years	90.0%	90.2%	89.9%
Least Three Mature Years	89.4%	89.9%	89.3%
Least Four Mature Years	88.6%	89.2%	88.4%

The decrease in safety level shown in Table 6.1 could be due, in part or in whole, to factors other than maturity. For example, our test excludes recent data that might be more favorable than the long term history. Nonetheless, the observed decreases in indicated safety level support the need for further research on the effect of maturity on risk factor calibration and back-testing.

7. Further Research

The observed safety level measured in our analysis would be affected if we had considered additional elements of the RBC Formula. Those areas are outlined below.

7.1 IIO

We did not test the 5% interest rate assumption used in the Investment Income Offset. As interest rates have declined over the course of the 24 year period, the safety level at current interest rates is likely lower than shown, if all else were constant. Evaluating the investment income impact is a matter for further research.

7.2 R3-Reinsurance Credit Risk

The RBC Formula includes a 10% charge on reinsurance balances receivable[32] on reinsurance ceded to non-Affiliates less any applicable reinsurance penalty. This charge is referred to as R3-Reinsurance Credit Risk. In most cases,[33] half of the 10% charge is included with R4, reserve risk and half in included in R_3, credit risk.

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[32] Reinsurance balances receivable includes any amounts due on paid and unpaid, plus unearned premiums. The 10% charge does not apply to reinsurance with U.S affiliates, State Mandated Involuntary Pools and associations or to Federal insurance programs.

[33] If R4-reserve risk (loss portion only) is less than the sum of half of the R3-Reinsurance credit risk and other Credit Risk, then R3-Reinsurance Credit Risk is included fully in the credit risk category. If it's more than, then half the R3-Reinsurance Credit Risk is included in the R4 reserve risk.

The R3-Reinsurance Credit Risk includes an (unspecified) element for the expectation that overall underwriting risk is higher for companies that use higher amounts of reinsurance.[34] That portion of R3-Reinsurance Credit Risk is realized in the observed reserve and premium risks, ORR and OPR.

Since the observed risk is part of the data, it would be reasonable to include the related portion of the modeled R3-Reinsurance Credit Risk in the analysis. If we had done so, modeled risk value, MUWR, would have been higher and the safety level would be higher than shown, if all else were constant.

We did not include the R3 consideration at this point, but should be considered in further research.

7.3 Other Elements of RBC Formula

Other elements of the RBC Formula that are not reflected in our analysis of observed safety level are:

- Own-company adjustment - This is the ratio of Company Development to Industry Development for reserves and Company Loss and Expense ratio to Industry Loss Ratios for premium, shown on Line 3 of RBC forms PR016 and PR017 for reserves and premium respectively. Directionally, the effect of including the own-company adjustment factors is uncertain. On one hand, we would expect that including the own-company adjustment factors might increase the apparent safety level, as the worse performing companies would have higher risk charges than assumed in our tests. On the other hand, larger companies, with lower indicated risk, might have favorable own-company adjustments that would lower the apparent safety level.
- Loss sensitive contract discount - The RBC Formula allows a 30% discount in risk charge for loss sensitive business written directly and a 15% discount in risk charge is allowed for business assumed. No allowance has been made for loss sensitive business in our calculation.
- Growth risk charge – The RBC Formula includes an increase in premium and risk charges for companies with three year average growth rates in excess of 10%. The magnitude of the increase in risk charges depends on the difference between the growth rate and the 10% threshold level. Directionally, including the effects of the growth rate would increase the modeled risk with no change in the observed risk. Therefore, the observed safety level would be higher than shown, if all else were constant.
- NEP and NWP - We use NEP rather than NWP in both the modeled risk calculation and the observed risk calculation because historical NEP by schedule P is available in our Risk Data and NWP is not. This simplification is applied in both modeled risk and observed risk. The simplification affects the results to the extent that the year-to-year change in NEP is different

[34] American Academy of Actuaries, P&C Risk Based Capital Committee, "Report on Reinsurance Credit Risk charge in the NAIC Property/Casualty Risk-Based Capital," March 29, 2013.

from the year-to-year change in NWP and to the extent that the reserves-to-NEP differs from reserves to NWP.

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9. GLOSSARY

Term	Interpretation
A&O	Adjusting and Other Expense; a part of LAE
A&O%	Adjusting and Other Expense % that is applied to Loss and DCC reserves
AY	Accident year
CER%	Company underwriting expense ratio as specified by company within the RBC Formula
DCC or DCCE	Defense and cost containment expenses; a part of LAE
DCWP	Dependency and Calibration Working Party
IIO	Generic for IIO_P or IIO_R
IIO_P	Premium investment income offset from 2010 RBC Formula
IIO_R	Reserve investment income offset from 2010 RBC Formula
LAE	Loss adjustment expenses
LCF	Reserve Concentration Factor as calculated in 2010 RBC Formula
LOB	Schedule P Lines of Business
LOB-size	Line of business size based on NEP or Initial Reserve, as appropriate.
LR	AY Loss and LAE ratios
MPR	Modeled Premium Risk – See sections 3 & 4 for definitions.
MRR	Modeled Reserve Risk – See sections 3 & 4 for definitions.
MUWR	Modeled Underwriting Risk – See section 5.1
NEP	Net Earned Premium
NWP	Net Written Premium
OPR	Observed Premium Risk – See sections 3 and 4 for definitions
ORR	Observed Reserve Risk – See sections 3 and 4 for definitions
OUWR	Observed Underwriting Risk – See section 5.2
PCF	Premium Concentration Factor as calculated in 2010 RBC Formula
PPA	Private Passenger Auto Liability
PRF	Premium Risk Factor from 2010 RBC Formula
R_0	Asset Risk – Insurance affiliate investment and (non-derivative) off-balance sheet risk.
R_1	Asset Risk – Fixed Income Investments
R_2	Asset Risk – Equity
R3-Reinsurance Credit Risk	The portion of R3-Credit Risk applicable to ceded reinsurance balances
RBC Formula	The 2010 NAIC RBC Formula
Initial Reserve Date or Reserve Date	Each year-end in our data set, December 31, 1987 through December 31, 2010
Reserves or Loss Reserves	Case, bulk and IBNR loss and defense and cost containment expense (DCCE) [35] reserves net of reinsurance, as shown in Schedule P – Part 2 and 3 for current AY and all prior AYs; Reserves include or exclude A&O as indicated.
RRF	Reserve Risk Factor from the 2010 RBC Formula

[35] "Defense and Cost Containment Expenses" are called "Allocated Loss Adjustment Expenses" (ALAE) in older Annual Statements. In our analysis we treat DCCE and ALAE as equivalent.

Term	Interpretation
RRR	Runoff ratio or Reserve Runoff Ratio The ratio of: (a) the incurred movement from the initial reserve date to the latest available evaluation date, for all constituent AYs combined to b) the Initial Reserve
TVaR	Tail Value at Risk
UW	Underwriting, the combination of AY results and development on prior year reserves
VaR	Value At Risk

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Appendix A

A.1 Risk and IIO Factors

LOB	2010 RBC Factors						DCWP Indicated[36]	
	Reserves			Premium			Reserves	Premiums
	RRF	IIO_R		PRF	IIO_P		RRF	PRF
Homeowners/Farmowners	0.201	0.938		0.937	0.954		0.201	0.958
Priv. Passenger Auto Liability	0.192	0.928		0.969	0.925		0.156	0.984
Commercial Auto Liability	0.230	0.911		0.988	0.890		0.320	1.001
Workers' Comp	0.324	0.830		1.033	0.839		0.336	1.053
Commercial Multi-Peril	0.465	0.876		0.921	0.896		0.462	0.897
Medical Mal – Occurrence	0.431	0.865		1.822	0.767		0.314	1.512
Medical Mal – Claims Made	0.306	0.883		1.092	0.827		0.106	1.203
Special Liability	0.257	0.890		0.904	0.898		0.449	0.963
Other Liability	0.511	0.852		1.042	0.816		0.518	1.038
Special Property	0.191	0.966		0.941	0.949		0.311	0.830
Auto Physical Damage	0.112	0.976		0.843	0.971		0.165	0.855
Fidelity & Surety	0.325	0.940		0.883	0.904		0.612	0.674
Other	0.172	0.967		0.893	0.947		0.271	0.941
International	0.327	0.874		1.169	0.905		0.490	0.832
Reinsurance Prop and Financial	0.286	0.901		1.349	0.893		0.422	1.290
Reinsurance B	0.769	0.838		1.507	0.777		0.657	1.328
Products Liability	0.643	0.841		1.214	0.774		0.894	1.196
Financial Guarantee	0.200	0.926		1.482	0.884		0.000[37]	1.553
Warranty	0.325	0.940		0.883	0.904		0.032[37]	1.238

[36] 87.5th percentile LR or RRR, as appropriate, for all data points in the risk data excluding the smallest data points. The smallest data points are defined as those having premium or reserve amounts below a threshold level that varies from $1m to $100k by LOB (threshold values listed in Reports 6 and 7).
[37] The indicated values for Financial Guarantee and Warranty are lower than would be used in practice. If higher RRFs were used in practice, the observed safety level would be higher than indicated using these values.

Appendix B
Operating Expense / Adjusting and Other Expenses
B.1 Company Expense Ratios – Premium Risk

To calculate the MPR and OPR we need all-lines company/year expense ratios. We obtain this information from industry databases for the years 1996 – 2010. The data is on an individual company basis and we combine companies into DCWP-defined pools when necessary[38]. For years 1995 and prior, we used the NWP weighted average of the calendar years 1998-1996. For company/year combinations for which no expense ratio information was available we used the industry premium weighted average for that year. In accordance with the RBC Formula, where the company CER% was greater than 400%, the MPR is based on 400%. The OPR is based on actual expenses, without limit.

B.2 Adjusting and Other Expenses (A&O) – Reserve Risk

The RBC Formula applies RRF to losses including A&O. Our data, however, is from, Schedule P Parts 2 and 3, which do not include A&O. That feature of the data would not affect our analysis if A&O were a constant percentage of loss plus DCC for all companies, all LOBs and at all stages of maturity. As that is not the case, for our back testing we include A&O to the extent possible. We consider three issues:

1. A&O% can vary as losses develop.
2. A&O% can be higher on data points with RRRs that are higher than average and lower on data points with RRRs that are lower than average.
3. A&O% varies by LOB.

The implication of these issues is discussed below.

B.2.1 - A&O% can develop

The A&O% used to calculate the MRR is not necessarily the same as the A&O% used to calculate the ORR. The A&O% used in the MRR should be the A&O% at the initial reserve date. The A&O% used in the ORR should be the A&O% for the developed reserves. To the extent that A&O develops differently than loss plus DCC, the A&O% for developed reserves will not be the same as the A&O% in the initial reserve.

As the developed A&O is not available in Schedule P, Parts 2 and 3, we assume the ratio for the developed data is the same as the ratio at the initial reserve date, i.e., A&O development is proportional to loss and DCC development and the A&O% for the initial reserve is the same as the A&O% for the ultimate reserve.[39] However, we cannot test this assumption without comparing multiple annual

[38] Pooling Adjustment - We pool participants as described in Appendix G of DWP paper 6. The NWP for the pooled entity is the sum of the premium for all pool members. The expense ratio for the pooled entity is the weighted average of the expense ratios for the individual pool members, weighted by NWP.

[39] RRFs are calibrated on the assumption that A&O develops at the same rate as loss plus DCC.

statements, and we did not do that for this study.

B.2.2 - A&O% can be higher on 'unfavorable' data points than on 'favorable' data points

Assuming that the A&O% does not develop, then for the calculation of safety levels by LOB, the A&O% equally affects the modeled and actual results by LOB and would not impact whether the actual result exceeded the modeled result and therefore would not affect the company view of safety level.

However, if A&O%s are higher for companies with unfavorable RRRs than is the case for companies with more favorable RRRs, then the portion of reserve including A&O from companies above the safety level would be higher than would appear to be the case for reserves excluding A&O. We address that by calculating an A&O% for each company.

Table B.1 below shows that there is only a small effect from using the A&O adjustment in the calculations of individual LOB safety levels. Column 2 shows the safety levels by LOB from Table 3.1. Column 3 shows the safety levels that would have been produced if we had not made the A&O adjustment.

Table B1
Reserve LOB Safety Levels

(1)	(2)	(3)
Line of Business	**Base Analysis**	**No A&O Adj**
Homeowners/Farmowners	94.9%	94.9%
Priv. Passenger Auto Liability	97.0%	96.9%
Commercial Auto Liability	90.7%	90.7%
Workers' Comp	91.6%	91.6%
Commercial Multi-Peril	93.8%	93.7%
Medical Mal – Occurrence	96.2%	96.2%
Medical Mal – Claims Made	94.2%	94.1%
Special Liability	86.1%	86.1%
Other Liability	81.5%	81.4%
Special Property	79.7%	79.7%
Auto Physical Damage	94.4%	93.5%
Fidelity & Surety	83.5%	83.6%
Other	83.2%	83.1%
International	77.4%	77.5%
Reinsurance A&C	89.3%	89.3%
Reinsurance B	89.5%	89.5%
Products Liability	72.4%	72.2%
Financial Guarantee	95.2%	95.2%
Warranty	84.7%	84.7%
All Lines	**90.1%**	**90.0%**

Column (2) From Table 3.2 except for All Lines.

All lines row is the weighted average or safety level from all-lines data points

B.2.3 - A&O varies by LOB

Third, for the calculation of safety levels for all lines combined, the A&O% affects the observed safety level to the extent that A&O% varies by LOB.

We apply the A&O adjustment by LOB.

B.2.4 - LR Data Points

We make no A&O adjustment to the LR points as the LRs we obtain from Schedule P Part 1 include A&O.

B.2.5 - Calculating the A&O%

To determine an A&O% for LOB/company/year combination we extract the following data by calendar year from industry databases for calendar years 1997-2010:

1. A&O
2. Direct and Assumed Loss and all LAE
3. Ceded Loss and all LAE

We calculate:

4. A&O% = (1) / [(2)-(3)-(1)]

The data is on an individual company basis and we combine it as necessary for DCWP-defined pools[40]. We select A&O% for years prior to 1997[41] and company/year combinations where no expense ratio was available as follows:

- For years 1996 and prior we use the reserve weighted average of the years 1997-1999.
- For LOB/company/year combinations where no data was available we use the industry average for that Year/LOB combination.

For all years we apply the following filters:

- If A&O ≤ 0, we set A&O to be $0
- If A&O % >2 * Industry A&O%, we set A&O % = Industry A&O % for that LOB.

[40] The A&O percentage for the pooled entity is the weighted average of the A&O percentage for the individual pool members, weighted by reserve.

[41] Data was available back to 1996, but we extracted data beginning with 1997.

Appendix C
Sensitivity Testing

In this section we discuss certain aspects of the Risk Data and the effect on the safety level calculations.

C.1 LOB Risk Data – All Data vis-à-vis Calibration Data

In all cases we begin with the LOB Risk Data:

- The premium file contains 157,622 LOB/company/year data records.
- The reserve file contains 128,439 LOB/company/year data records.

These are all data points other than those with anomalous values and after consolidating pools into a single data point.

In Reports 6 and 7 we discuss calibration of LOB risk factors using a subset of the data that excludes (a) new-LOBs (with NEP for less than 5 years) and (b) minor lines (LOB containing less than 5% of the company total business). That data set consists of the following:

- The premium file contains 86,861 LOB/company/year data records.
- The reserve file contains 71,352 LOB/company/year data records.

Table 3.1 showed the LOB safety levels based on the larger data set. Table C.1 below shows the LOB safety levels based on the Report 6 and 7 calibration data. Columns (3) and (5), compared to columns (2) and (4) show that the safety levels are in general higher using the calibration data than all the data. This would be expected as the filters were designed to remove the volatile points.

Table C.1
LOB Safety Levels – All Data vs. Calibration Data

	Reserve Risk		Premium Risk	
(1)	**(2)**	**(3)**	**(4)**	**(5)**
Line of Business	**All Data**	**Calibration Data**	**All Data**	**Calibration Data**
Homeowners/Farmowners	94.9%	95.5%	86.2%	86.2%
Priv. Passenger Auto Liability	96.9%	97.4%	94.3%	94.4%
Commercial Auto Liability	90.7%	92.6%	90.4%	90.6%
Workers' Comp	91.6%	91.7%	86.2%	86.0%
Commercial Multi-Peril	93.7%	94.2%	91.2%	92.1%
Medical Mal – Occurrence	96.2%	98.0%	95.0%	96.5%
Medical Mal – Claims Made	94.1%	97.1%	77.5%	79.2%
Special Liability	86.1%	87.5%	90.6%	91.0%
Other Liability	81.4%	81.4%	90.2%	90.1%
Special Property	79.7%	79.5%	92.3%	92.7%
Auto Physical Damage	93.5%	95.4%	91.6%	91.6%
Fidelity & Surety	83.6%	89.3%	91.9%	97.7%
Other	83.1%	78.3%	79.1%	80.8%
International	77.5%	92.1%	89.9%	93.7%
Reinsurance A&C	89.3%	91.5%	92.4%	93.9%
Reinsurance B	89.5%	91.1%	93.2%	94.4%
Products Liability	72.2%	90.2%	90.1%	92.5%
Financial Guarantee	95.2%	97.5%	90.6%	85.7%
Warranty	84.7%	100.0%	91.9%	93.9%
All Lines	90.0%	91.2%	90.3%	90.6%

All lines row is the weighted average or safety level from all-lines data points. The values in Table 4.1 differ from the total row in the above Table. The total values in Table C.1 above represent the weighted average of the LOB safety levels. The Table 4.1 values represent the safety level when all lines are combined into a single modeled or observed risk data point for company/year combinations.

The values in columns (2) and (3) are calculated excluding the A&O adjustment. Column (2) is from Table B1 column (3)

C.2 All Lines Risk Data:

We use the larger data set to construct all-lines risk data points for each company/year combination. Each all-lines data point is the weighted average of LOB data points.

- For reserve risk, there are 31,949 all-lines company/year combinations for calculating the all-lines company/year Actual Reserve Runoff.
- For premium risk, there are 35,750 all-lines company/year combinations for calculating the all-lines company/year Loss Ratios.
- For premium and reserve risk combined there are a total of 39,544 unique all-lines company/year combinations. This count is greater than either the number of premium all-lines data points or the number of reserve all-lines data points because some combinations have premium data points only and some combination have reserve risk data points only.

Our all-lines premium, reserve and combined premium+reserve safety level analysis uses four key elements MRR_{year-1}, MPR_{year-1}, OPR_{year} and ORR_{year-1}. We use only those where all the components exist. Of the 39,544 unique company/year combinations there are 30,292 company/year combinations where all four data points exist. The other combinations are:

- 2,644 company/year combinations where OPR_{year} exists but MRR_{year-1}, MPR_{year-1}, and ORR_{year-1} do not exist (i.e. year exists in the premium data file but the year y-1 is not in the reserve or premium data files).
- 4,961 company/year combinations where MPR_{year-1} exists but the MRR_{year-1} and ORR_{year-1} do not exist (i.e. year y-1 exists in the premium data file but the year y-1 is not in the reserve data file).
- 1,160 company/year combinations where MRR_{year-1} and ORR_{year-1} exist but the MPR_{year-1} does not exist (i.e. year y-1 exists in the reserve data file but the year y-1 is not in the premium data file).
- 497 company/year combinations where MRR_{year-1} and ORR_{year-1} exist but the OPR_{year} does not exist (i.e. year y-1 exists in the reserve data file but the year is not in the premium data file).

Table C.2 shows the differences in calculated safety level between using the 30,292 data set and the 39,544 data set and a number of intermediate sized data sets. It shows that the combined safety level is slightly higher, 91.17%, using company/years with all data points, compared to 91.01% when we include all unique company/year combinations available regardless of if all data points exist. Thus, the impact of excluding data from the combined safety level is minor, less than 0.2%.

For the premium risk alone, the difference in safety level is slightly higher, 0.34% (90.47% compared to 90.12%).

For the reserve risk alone, the difference in safety level is minor, less than 0.0001% difference.

Table C.2
Comparison of Analyses Using Different Company/Year Combinations

	Number CO/YRs	**Combined Safety Level**	**Premiums Safety Level**	**Reserves Safety Level**	**Reserve $millions**	**NEP $millions**
(A)	**30,292**	**91.17%**	**90.47%**	**91.15%**	**9,194,901**	**7,044,283**
(A) - (B)		0.163%	0.344%	0.000%	-30,182	-265,131
(B)	39,554	91.01%	90.12%	91.15%	9,225,084	7,309,414
(C)	36,910	91.12%	90.35%	91.15%	9,225,084	7,096,628
(C) - (B)	2,644	0.108%	0.222%	0.000%	0	-212,786
(D)	31,949	91.17%	90.46%	91.15%	9,225,084	7,046,155
(D) - (C)	4,961	0.053%	0.117%	0.000%	0	-50,473
(E)	30,789	91.18%	90.47%	91.15%	9,206,799	7,044,283
(E) - (D)	1,160	0.004%	0.004%	0.003%	-18,284	-1,872
(F)	30,292	91.17%	90.47%	91.15%	9,194,901	7,044,283
(F) - (E)	497	0.00%	0.000%	-0.003%	-11,898	0

Notes to Table C.2

(A)	**Only used company/year combinations where we have a full set of data - i.e. company/year (x) combination (ORR, MRR, MPR) exists in both the reserve and premium file and the company/year (x+1) exists in the premium file (OPR)**
(B)	Use all company/year combinations that exist.
(C)	Exclude 2,644 company/year combinations where year x+1 in premium (OPR) but year x not in reserve (ORR, MRR) or premium file (MPR)
(D)	Exclude 4,961 company/year combinations where year x in premium (MPR) but not in reserve (ORR, MRR)
E	Exclude 1,160 company/year combinations where year x in reserve (ORR, MRR) but not in premium (MPR)
F	Exclude 497 company/year combinations remaining where no year x+1 in premium (OPR)

C.3 Measuring Safety Level Based on Premium

We measure premium safety level based on NEP, reserves safety level based on reserves including A&O, and combined premium + reserves safety level based on combined NEP and reserves including A&O.

We considered evaluating safety level based on NEP in all cases. That would require matching each reserve data point to a corresponding premium data point, e.g., AY 1990 premium and 1990 reserves.

However, we found a premium match for only 89% of reserve entries. The unmatched entries are due to factors including:

- Some reserve data points (e.g., runoff companies) have no corresponding premium, and
- Some data was eliminated by our filtering of exceptional values.

Rather than removing about 10% of our data due to unmatched entries we decided to measure the safety levels of the reserves using reserve data.

Table C.3 below summarizes the match by line of business.

Table C.3
Matching Premium and Reserve Records

LOB	NEP (Premium File) $000	Number Premium Entries	Reserves $000	Number Reserve Entries	Matched NEP from Premium file to Reserve File $000	Number Unmatched Reserve Entries	% of Reserve Entries Unmatched
Homeowners/Farmowners	837,779,469	14,388	283,743,839	12,890	752,237,134	849	7%
Priv. Passenger Auto Liability	1,665,757,585	12,648	1,484,867,521	11,825	1,557,250,433	1,059	9%
Commercial Auto Liability	339,751,660	12,563	475,364,115	11,725	318,861,103	1,059	9%
Workers' Compensation	850,602,677	10,752	2,285,323,369	10,767	799,713,126	1,642	15%
Commercial Multi-Peril	514,883,064	12,988	606,805,258	12,011	479,399,100	1,179	10%
Medical Mal - Occurrence	45,844,899	3,463	251,165,745	3,922	40,187,542	1,191	30%
Medical Mal - Claims made	113,039,619	4,224	279,537,964	3,695	101,607,051	334	9%
Special Liability	100,242,320	5,045	93,242,192	4,559	92,159,493	804	18%
Other Liability	605,485,345	20,790	1,607,856,355	17,557	552,581,070	1,462	8%
Special Property	433,757,648	17,081	114,752,709	10,970	326,843,828	494	5%
Auto Physical Damage	1,165,687,705	15,040	63,696,563	6,759	540,233,500	311	5%
Fidelity & Surety	90,811,841	7,517	31,768,591	3,505	62,741,739	234	7%
Other	200,447,319	5,294	46,788,712	3,758	126,916,957	870	23%
International	7,409,062	659	11,818,154	785	5,883,989	316	40%
Reinsurance A&C	98,929,534	4,179	124,874,904	3,659	87,561,671	627	17%
Reinsurance B	178,061,703	4,046	713,338,940	4,537	159,822,581	1,383	30%
Products Liability	53,086,437	6,195	255,964,385	5,235	49,081,648	732	14%
Financial Guarantee	1,972,460	540	847,662	211	554,636	38	18%
Warranty	5,863,372	210	318,810	69	1,090,852	25	36%
All Lines	**7,309,413,719**	**157,622**	**8,732,075,788**	**128,439**	**6,054,727,453**	**14,609**	**11%**

APPENDIX D
Sensitivity Tests

D.1 Safety Level with 2010 Risk Factors vs. Safety Level with Indicated Risk Factors

In Table 3.2, we observed that differences between the observed safety levels by LOB and 87.5% might arise if the PRFs and RRFs used in the 2010 RBC Formula were not consistent with the 87.5th percentile for the data used in this back-testing. To test for that possibility we calculate the PRFs and RRFs by LOB that would be indicated based on the data used in this back-testing. Table D.1 shows that, even if using those indicated PRFs and RRFs, the patterns in observed safety levels by LOB are similar to the patterns in Table 3.2.

The indicated factors for this purpose are the 87.5th percentile LR and RRR by company count based on the current data using the work completed in DCWP Reports 6 and 7.[42]

Table D.1 Column (2) repeats the results from Table 3.2 and Column (4) from Table C1 Column (4) (which is Table 3.2 re-calculated with no A&O adjustment)... Columns (3) and (5), compared to columns (2) and (4), show that the safety levels are only slightly higher overall, with no consistent pattern across LOBs.

[42] Indicated risk factors are shown in Appendix A. These are the 87.5th percentile LR or RRR, as appropriate, for all data points in the risk data excluding the smallest data points. The smallest data points are defined as those having premium or reserve amounts below a threshold level that varies from $1m to $100k by LOB (threshold values listed in Reports 6 and 7).

Table D.1
Premium Equivalent Safety level of 2010 and Indicated Risk Factors[43]

	Safety Level in Premium Risk Factors		Safety Level in Reserve Risk Factors	
(1)	(2)	(3)	(4)	(5)
	2010 Factors	Indicated Factors	2010 Factors	Indicated Factors
Homeowners/Farmowners	86.2%	89.0%	94.9%	92.4%
Priv. Passenger Auto Liability	94.3%	97.1%	96.9%	96.3%
Commercial Auto Liability	90.4%	91.6%	90.7%	94.6%
Workers' Comp	86.2%	87.0%	91.6%	92.7%
Commercial Multi-Peril	91.2%	89.0%	93.7%	94.3%
Medical Mal – Occurrence	95.0%	87.9%	96.2%	95.0%
Medical Mal – Claims Made	77.5%	84.0%	94.1%	87.8%
Special Liability	90.6%	92.9%	86.1%	93.0%
Other Liability	90.2%	89.2%	81.4%	82.8%
Special Property	92.3%	86.1%	79.7%	83.2%
Auto Physical Damage	91.6%	93.9%	93.5%	93.9%
Fidelity & Surety	91.9%	85.8%	83.6%	90.8%
Other	79.1%	90.1%	83.1%	89.0%
International	89.9%	72.1%	77.5%	80.1%
Reinsurance A&C	92.4%	90.6%	89.3%	92.5%
Reinsurance B	93.2%	90.7%	89.5%	87.8%
Products Liability	90.1%	89.9%	72.2%	81.2%
Financial Guarantee	90.6%	91.5%	95.2%	67.7%
Warranty	91.9%	99.5%	84.7%	76.6%
All Lines	90.3%	91.4%	90.0%	90.7%

Column (2) from Table 3.2, Column (4) from Table C1.

All lines row is weighted average or safety level from all-lines data points

[43] Calculated with no A&O adjustment.

Dependencies in Stochastic Loss Reserve Models

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Abstract

Given a Bayesian Markov Chain Monte Carlo (MCMC) stochastic loss reserve model for two separate lines of insurance, this paper describes how to fit a bivariate stochastic model that captures the dependencies between the two lines of insurance. A Bayesian MCMC model similar to the Changing Settlement Rate (CSR) model, as described in Meyers (2015), is initially fit to each line of insurance. Then taking a sample from the posterior distribution of parameters from each line, this paper shows how to produce a sample that represents a bivariate distribution that maintains the original univariate distributions as its marginal distributions. This paper goes on to compare the predicted distribution of outcomes by this model with the actual outcomes, and a bivariate model predicted under the assumption that the lines are independent. It then applies the Watanabe-Akaike Information Criterion to compare the fits of the two models.

Key Words: Bayesian MCMC, Stochastic Loss Reserving, Correlation, Dependencies.

1. INTRODUCTION

Recent attempts to apply enterprise risk management principles to insurance have placed a high degree of importance using stochastic models to quantify the uncertainty on the various estimates. For general insurers, the most important liability is the reserve for unpaid losses. Over the years, a number of stochastic models have been developed to address this problem. Some of the more prominent nonproprietary models are those of Mack (1993, 1994), England and Verrall (2002) and Meyers (2015).

As good as these models may be, they fall short of quantifying the uncertainty in the insurer's liability as they do not address the issue of correlation (or more generally – dependencies) between lines of insurance. The failure to resolve this problem analytically has resulted in judgmental adjustments to various risk-based capital formulas. Herzog (2011) provides a summary of some current practices.

Zhang and Dukic (2013) describe what I believe to be a very good attempt at solving this problem. As this paper uses their paper as a starting point, it would be good to provide an outline of their approach[1].

[1] As this paper deals with lognormal models of claim amounts, its description of the Zhang-Dukic ideas are not as general as they put forth in their paper. Their results apply for more general copulas, where this paper deals only with the more specialized multivariate lognormal distribution.

But first, we need to set our notation. Let C_{wd}^{X} be the cumulative paid claim amount in line of insurance X for accident year, w = 1, …. , K and development year d = 1, … , K. Since this paper works with Schedule P data taken from the CAS Loss Reserve Database,[2] we can set $K = 10$. In this paper, X will be *CA* for Commercial Auto, *PA* for Personal Auto, *WC* for Workers Compensation, or *OL* for Other Liability.

Now suppose that we have models for two different lines of insurance such as

$$\begin{aligned}\log\left(C_{wd}^{X}\right) &\sim Normal\left(\mu_{wd}^{X},\sigma_{d}^{X}\right)\\ \log\left(C_{wd}^{Y}\right) &\sim Normal\left(\mu_{wd}^{Y},\sigma_{d}^{Y}\right)\end{aligned} \tag{1.1}$$

As we shall see below, the parameters μ_{wd}^{X} will be functions of w and d and the parameters σ_{d}^{X} will be subject to constraints for each line X. That feature can be ignored for now as we are setting up the problem.

As shown in Meyers (2015), it is possible to use a Bayesian MCMC model to generate a large sample, say of size 10,000, from the posterior distributions of $\left\{\left\{{}_{i}\mu_{wd}^{X}\right\},\left\{{}_{i}\sigma_{d}^{X}\right\}\right\}_{i=1}^{10000}$ for each line of insurance X.

The idea put forth by Zhang and Dukic is to fit a bivariate Bayesian MCMC model of the following form given the Bayesian MCMC models described by Equation (1.1).

$$\begin{pmatrix}\log\left(C_{wd}^{X}\right)\\ \log\left(C_{wd}^{Y}\right)\end{pmatrix} \sim \text{Multivariate Normal}\left(\begin{pmatrix}\mu_{wd}^{X}\\ \mu_{wd}^{Y}\end{pmatrix},\begin{pmatrix}\left(\sigma_{d}^{X}\right)^{2} & \sigma_{d}^{X}\cdot\rho\cdot\sigma_{d}^{Y}\\ \sigma_{d}^{X}\cdot\rho\cdot\sigma_{d}^{Y} & \left(\sigma_{d}^{Y}\right)^{2}\end{pmatrix}\right) \tag{1.2}$$

The correlation parameter, ρ, describes the dependency between Line X and Line Y.

Zhang and Dukic then use a Bayesian MCMC model to obtain a large sample from the posterior distribution:

[2] The CAS Loss Reserve Database is on the CAS website at
http://www.casact.org/research/index.cfm?fa=loss_reserves_data

$$\left\{\left\{\begin{matrix} {}_i\mu_{wd}^{X*} \\ {}_i\mu_{wd}^{Y*} \end{matrix}\right\},\left\{\begin{matrix} \left({}_i\sigma_d^{X*}\right)^2 & {}_i\sigma_d^{X*}\cdot{}_i\rho\cdot{}_i\sigma_d^{Y*} \\ {}_i\sigma_d^{X*}\cdot{}_i\rho\cdot{}_i\sigma_d^{Y*} & \left({}_i\sigma_d^{Y*}\right)^2 \end{matrix}\right\}\right\}_{i=1}^{10000}.$$

The asterisk (*) on the μ and σ parameters calls attention to the fact that the posterior distributions from the models in Equation (1.1) may, and often do, differ significantly from the corresponding marginal posterior distributions from the models in Equation (1.2). To the actuary who prepares loss reserve reports, this presents a problem. Typically actuaries analyze their reserves by individual line of insurance. With a Bayesian MCMC model, they can quantify the uncertainty of the outcomes for that line. Now suppose that there is a demand to quantify the uncertainty in the sum of losses for two or more lines of insurance using the Zhang-Dukic framework. They will need to explain, for example, why the univariate distribution for Commercial Auto produces different results than the marginal distribution for Commercial Auto when combined with Personal Auto. And the marginal distribution could be different still when combined with Workers Compensation.

Scalability is also a problem. For example, the univariate model used in this paper has 31 parameters. Using this model with the bivariate Zhang-Dukic framework yields a model with 31+31+1=63 parameters. In theory, Bayesian MCMC software can handle it, but in practice I have found that running times increase at a much faster rate than the number of parameters. I have coded models using the bivariate Zhang-Dukic framework that work well for some pairs of loss triangles, but others took several hours of running time to obtain convergence of the MCMC algorithm.

The purpose of this paper is to present a framework similar to that of Zhang and Dukic that preserves the univariate models as the marginal distributions.

Before we go there, we should note that a suboptimal model might produce artificial dependencies. To illustrate, consider Figure 1.1 below where y_1 and y_2 are independent random deviations off two parabolic functions of x. We want to fit a bivariate distribution to the ordered pair $(y_1(x), y_2(x))$ of the form:

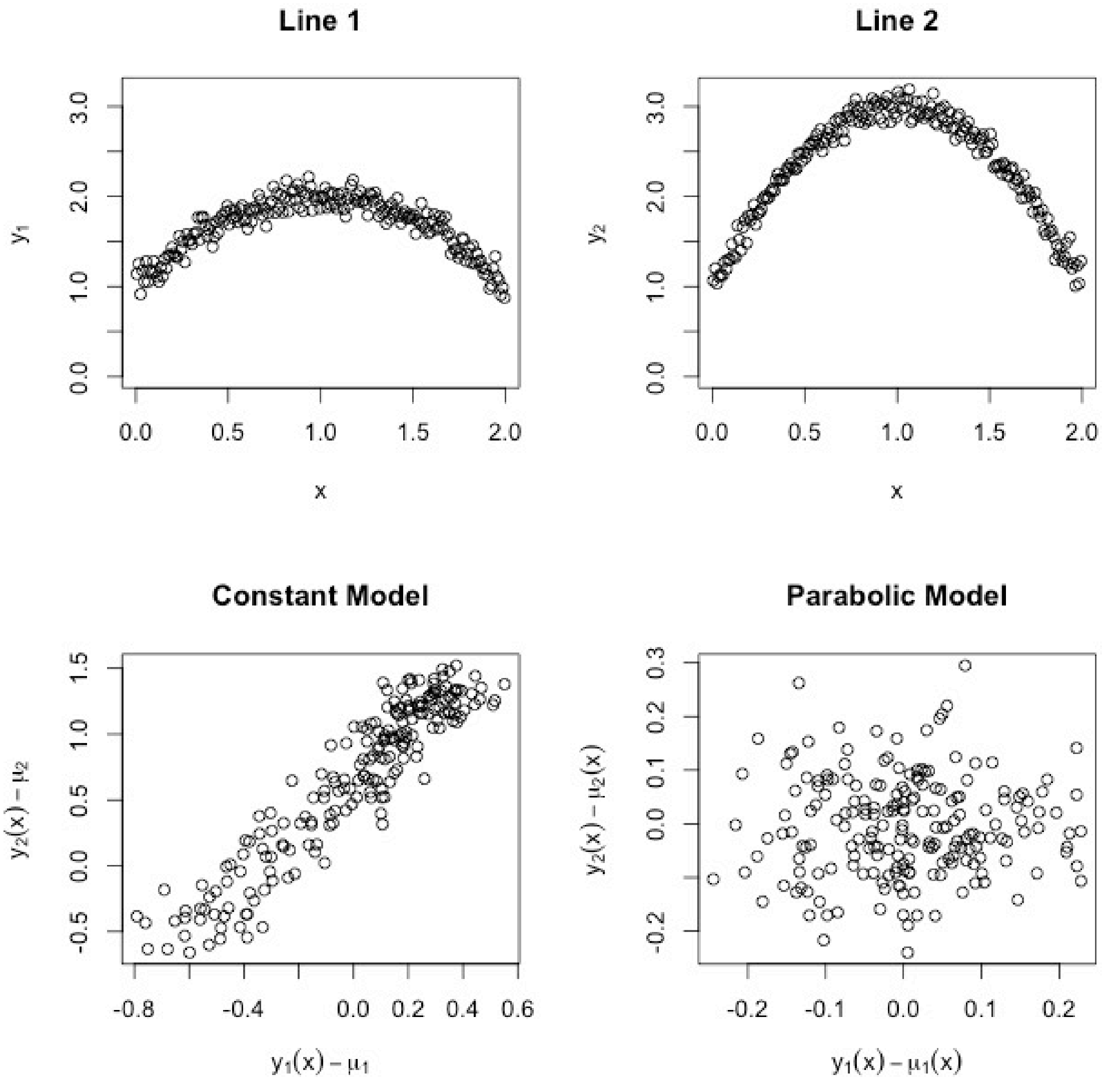
$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \sim \text{Multivariate Normal}\left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix},\begin{pmatrix} \sigma_1^2 & \sigma_1\cdot\rho\cdot\sigma_2 \\ \sigma_1\cdot\rho\cdot\sigma_2 & \sigma_2^2 \end{pmatrix}\right) \qquad (1.3)$$

The lower left plot of Figure 1.1 shows a scatter plot of $y_1(x) - \mu_1$ and $y_2(x) - \mu_2$ for the (suboptimal) model $\mu_i(x)$ is a constant. The lower right plot is a scatter plot of $y_1(x) - \mu_1(x)$ and $y_2(x) - \mu_2(x)$ for the (correct) parabolic model. This example shows how suboptimal models for the marginal distribution can cause an artificial nonzero correlation the multivariate model.

The next section will describe the data used in this paper. Section 3 will describe the univariate (marginal) models and illustrate some diagnostics to test the appropriateness of the model. Section 4 will show how to obtain a random sample from the posterior distribution of parameters subject to the constraint that the marginal distribution is the same as those obtained by the corresponding univariate models. Section 5 will describe statistical tests to test the hypothesis that the correlation parameter, ρ, in the bivariate distribution is significantly different from zero. Section 6 will address the sensitivity of the results to the choice of models, and Section 7 will discuss the conclusions.

This paper assumes that the reader is familiar with Meyers (2015).

Figure 1.1 – Illustration of Artificial Correlation



2. THE DATA

The data used in this paper comes from the CAS Loss Reserve Database[3]. The Schedule P loss triangles taken from this database are listed in Appendix A of Meyers (2015). There are 200 loss triangles, 50 each from the CA, PA, WC and OL lines of insurance. Univariate models from all 200 loss triangles will be analyzed in Section 3 and 6.

At the time of writing the monograph, Meyers (2015), I did not envision a dependency study. But it turned out that there were 102 within-group pairs of triangles (29 CA-PA, 17 CA-WC, 17 CA-OL, 14 PA-WC, 15 PA-OL and 10 WC-OL) that were suitable for studying dependency models. Preferring to use loss triangles that have already been vetted, I decided to stick with these within-group pairs of triangles.

This paper will provide detailed analyses for two illustrative insurers (Groups 620 and 1066) for the CA and PA lines of business. The complete loss triangles and outcomes are in Table 2.1 below. The upper data triangle used to fit each model is printed with the ordinary font. The lower data triangle used for retrospective testing is printed with bold and italicized font.

A complete list of the insurer groups used in this paper is included in a spreadsheet titled "Appendix." The sheets in the Appendix contain:

- The 200 groups along with the associated calculations in Section 3.
- The R scripts that produce the univariate model calculations described in Section 3 and 6.
- The 102 within-group pairs with associated calculations in Sections 4 and 5.
- The R scripts that produce the bivariate model calculations described in Sections 4, 5 and 6.

[3] http://www.casact.org/research/index.cfm?fa=loss_reserves_data

Table 2.1 – Data for Illustrative Insurers

Group 620 - Commercial Auto

AY	Premium	DY1	DY2	DY3	DY4	DY5	DY6	DY7	DY8	DY9	DY10
1	30,224	4,381	9,502	15,155	18,892	20,945	21,350	21,721	21,934	21,959	21,960
2	35,778	5,456	9,887	13,338	17,505	20,180	20,977	21,855	21,877	21,912	***21,981***
3	42,257	7,083	15,211	21,091	27,688	28,725	29,394	29,541	29,580	***29,595***	***29,705***
4	47,171	9,800	17,607	23,399	29,918	32,131	33,483	33,686	***34,702***	***34,749***	***34,764***
5	53,546	8,793	19,188	26,738	31,572	34,218	35,170	***36,154***	***36,201***	***36,256***	***36,286***
6	58,004	9,586	18,297	25,998	31,635	33,760	***34,785***	***35,653***	***35,779***	***35,837***	***35,852***
7	64,119	11,618	22,293	33,535	39,252	***42,614***	***44,385***	***44,643***	***44,771***	***45,241***	***45,549***
8	68,613	12,402	27,913	39,139	***45,057***	***47,650***	***50,274***	***50,505***	***50,554***	***50,587***	***50,587***
9	74,552	15,095	27,810	***35,521***	***44,066***	***48,308***	***50,061***	***51,337***	***51,904***	***52,016***	***53,895***
10	78,855	16,361	***28,545***	***40,940***	***50,449***	***54,212***	***56,722***	***57,658***	***57,734***	***57,883***	***57,906***

Group 620 - Personal Auto

AY	Premium	DY1	DY2	DY3	DY4	DY5	DY6	DY7	DY8	DY9	DY10
1	48,731	15,318	27,740	35,411	40,204	42,388	43,726	44,217	44,277	44,400	44,431
2	49,951	15,031	30,132	37,946	42,371	43,875	44,518	44,738	45,089	45,094	***45,146***
3	52,434	16,994	31,614	39,599	44,943	46,342	47,653	47,866	48,085	***48,097***	***48,241***
4	58,191	17,717	33,767	42,741	46,881	49,117	50,419	50,641	***50,787***	***50,942***	***50,980***
5	61,873	17,842	31,117	39,436	44,871	46,810	47,421	***48,209***	***48,724***	***48,815***	***49,133***
6	63,614	20,266	37,466	45,721	50,641	52,244	***53,241***	***53,794***	***54,093***	***54,468***	***54,471***
7	63,807	18,778	33,216	42,030	47,695	***49,252***	***50,002***	***50,546***	***50,799***	***50,887***	***50,890***
8	61,157	19,900	36,442	43,585	***49,177***	***52,052***	***53,150***	***53,420***	***53,488***	***53,649***	***53,659***
9	62,146	20,395	35,797	***43,816***	***47,687***	***50,468***	***51,085***	***51,598***	***51,754***	***51,756***	***51,914***
10	68,003	20,622	***36,466***	***44,589***	***50,539***	***52,860***	***53,886***	***54,610***	***54,796***	***55,048***	***55,080***

Group 1066 - Commercial Auto

AY	Premium	DY1	DY2	DY3	DY4	DY5	DY6	DY7	DY8	DY9	DY10
1	5,103	1,060	3,034	4,580	5,243	4,178	4,347	4,399	4,598	4,582	4,629
2	5,196	1,224	3,751	5,735	4,902	5,295	5,486	5,941	5,976	5,977	***5,977***
3	6,947	1,252	3,568	5,265	6,102	6,607	6,315	6,343	6,370	***6,445***	***6,419***
4	9,482	1,606	3,875	5,439	6,507	8,021	8,098	8,282	***8,300***	***8,328***	***8,378***
5	10,976	1,750	4,038	5,662	6,293	6,779	7,048	***7,048***	***7,047***	***7,047***	***7,047***
6	11,893	1,125	4,322	5,263	6,036	6,462	***6,617***	***6,647***	***6,649***	***6,654***	***6,654***
7	13,029	1,403	3,746	5,800	6,737	***7,078***	***7,110***	***7,225***	***7,346***	***7,366***	***7,366***
8	12,511	1,541	4,620	5,746	***6,171***	***6,462***	***6,680***	***6,714***	***6,713***	***6,728***	***6,729***
9	14,372	1,986	4,532	***4,817***	***5,653***	***5,932***	***5,988***	***6,036***	***6,038***	***6,051***	***6,043***
10	7,371	1,970	***2,730***	***3,214***	***3,376***	***3,502***	***3,605***	***3,744***	***3,750***	***3,777***	***3,780***

Group 1066 - Personal Auto

AY	Premium	DY1	DY2	DY3	DY4	DY5	DY6	DY7	DY8	DY9	DY10
1	24,988	5,135	11,980	16,368	18,163	20,189	20,462	20,715	20,749	20,720	20,813
2	26,082	5,655	15,108	19,498	23,097	23,819	24,296	24,622	24,735	24,736	***24,741***
3	29,606	6,648	17,982	23,078	25,334	26,596	26,983	27,096	27,150	***27,195***	***27,206***
4	33,802	5,722	14,677	19,356	21,906	22,497	22,732	23,149	***23,207***	***23,197***	***23,254***
5	37,261	5,906	14,864	18,305	20,075	21,779	22,277	***22,425***	***22,466***	***22,424***	***22,536***
6	35,849	6,439	15,146	19,187	21,576	22,539	***22,941***	***23,037***	***23,029***	***23,135***	***23,174***
7	35,053	6,934	15,703	19,748	21,300	***21,948***	***22,004***	***22,043***	***22,136***	***22,211***	***22,210***
8	33,254	6,194	12,183	15,282	***17,315***	***18,550***	***18,697***	***18,876***	***19,014***	***19,040***	***19,210***
9	29,101	5,314	10,915	***13,854***	***15,179***	***15,537***	***16,083***	***16,057***	***16,088***	***16,101***	***16,137***
10	29,149	4,301	***9,758***	***11,914***	***13,216***	***13,740***	***14,098***	***14,427***	***14,448***	***14,491***	***14,513***

3. THE CHANGING SETTLEMENT RATE (CSR) MODEL

The univariate model used in this paper will be a minor modification to the CSR model used in Meyers (2015). Here is the model. Let:

1. $\alpha_w \sim \text{Normal}\left(0, \sqrt{10}\right)$ for w = 2, … ,10. $\alpha_1 = 0$.
2. *logelr* ~ Uniform(-1, 0.5).
3. $\beta_d \sim$ Uniform(-5, 5) for d = 1, … ,9. $\beta_{10} = 0$.
4. $S_1 = 1$, $S_w = S_{w-1} \cdot (1 - \gamma - (w-2) \cdot \delta)$ for w = 2, … ,10. $\gamma \sim$ Normal (0, 0.05), $\delta \sim$ Normal(0, 0.01).
5. $\mu_{wd} = \log(\text{Premium}_w) + logelr + \alpha_w + \beta_d \cdot S_w$.
6. $\sigma_d^2 = \sum_{i=d}^{10} a_i$, $a_i \sim$ Uniform(0, 1).
7. $\log(C_{wd}) \sim \text{Normal}(\mu_{wd}, \sigma_d)$.

This model differs from the CSR model described in Meyers (2015) in three aspects.

1. The parameter γ, allows for a speedup (or slowdown when γ is negative) of the claim settlements. By including the δ parameter, this version of the CSR model allows the settlement rate to change over time.
2. Forcing $\alpha_1 = 0$ eliminates some overlap between the α_w parameters and the *logelr* parameter. In the Meyers (2015) version of the model, a constant addition to each α_w parameters could be offset by a subtraction in the *logelr* parameter. Correcting features of this sort tend to speed up convergence of the MCMC algorithm.
3. The MCMC software used for the calculation described in this paper is Stan. See http://mc-stan.org for installation instructions. I have found that, in general, the MCMC algorithm implemented by Stan converges faster than that of JAGS. Stan also allows one to compile a model (in C++) in advance of its use. Using a compiled model can greatly speed up the processing when one uses the same model repeatedly (as we will do below) with different inputs.

The R script that implements this version of the CSR model is available in the appendix spreadsheet. The script produces a sample from the posterior distribution of the parameters for line X,

$$\left\{\left\{{}_{i}\alpha_{w}^{X}\right\}_{w=2}^{10},\left\{{}_{i}\beta_{d}^{X}\right\}_{d=1}^{9},\left\{{}_{i}\sigma_{d}^{X}\right\}_{d=1}^{10},{}_{i}logelr^{X},{}_{i}\gamma^{X},{}_{i}\delta^{X}\right\}_{i=1}^{10000}.$$

Following Meyers (2015), the script then simulates 10,000 outcomes $\left\{{}_{i}C_{w,10}^{X}\right\}_{i=1}^{10000}$ from which we can calculate various summary statistics such as the predictive mean and standard deviation of the outcomes and the percentile of the actual outcome. Table 3.1 gives a summary of the result of these calculations for the Commercial Auto ($X=CA$) and the Personal Auto ($X=PA$) lines of business.

Figure 3.1 gives the test for uniformity of the predictive percentiles of this version of the CSR model. When compared with Meyers (2015) Figure 22, we see that allowing the claim settlement rate to change over time improves the model so that the percentiles are (within 95% statistical bounds) uniformly distributed for all four lines.

Table 3.1. CSR Models on Illustrative Insurer Data

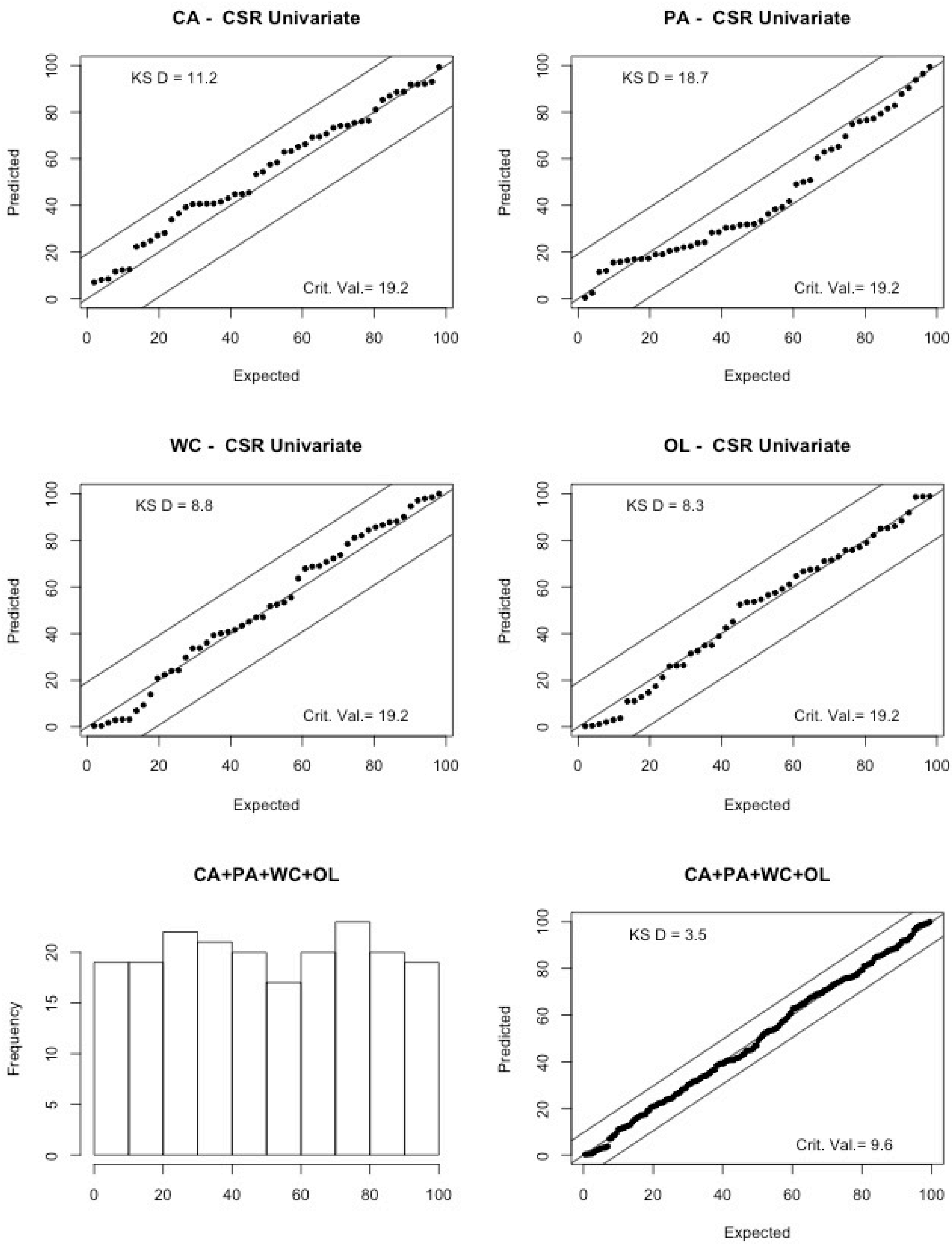
CA Insurer Group 620			Outcome Percentile = 39.24		
w	Premium	Estimate	Std. Dev.	C.V.	Outcome
1	30,224	22,119	0	0.0000	21,960
2	35,778	21,896	453	0.0207	21,981
3	42,257	30,068	685	0.0228	29,705
4	47,171	34,052	852	0.0250	34,764
5	53,546	36,638	1,106	0.0302	36,286
6	58,004	35,192	1,342	0.0381	35,852
7	64,119	45,387	2,305	0.0508	45,549
8	68,613	53,215	4,061	0.0763	50,587
9	74,552	55,166	7,439	0.1348	53,895
10	78,855	63,922	17,493	0.2737	57,906
Total	553,119	397,656	27,378	0.0688	388,485

PA Insurer Group 620			Outcome Percentile = 65.14		
w	Premium	Estimate	Std. Dev.	C.V.	Outcome
1	48,731	44,535	0	0.0000	44,431
2	49,951	45,453	366	0.0081	45,146
3	52,134	48,304	386	0.0080	48,241
4	58,191	51,003	457	0.0090	50,980
5	61,873	48,335	511	0.0106	49,133
6	63,614	54,243	712	0.0131	54,471
7	63,807	50,779	877	0.0173	50,890
8	61,157	52,674	1,351	0.0256	53,659
9	62,146	52,704	2,437	0.0462	51,914
10	68,003	52,910	5,125	0.0969	55,080
Total	589,907	500,941	8,709	0.0174	503,945

CA Insurer Group 1066			Outcome Percentile = 12.59		
w	Premium	Estimate	Std. Dev.	C.V.	Outcome
1	5,103	4,727	0	0.0000	4,629
2	5,196	6,077	363	0.0597	5,977
3	6,947	6,439	415	0.0645	6,419
4	9,482	7,855	620	0.0789	8,378
5	10,976	7,300	606	0.0830	7,047
6	11,893	6,218	659	0.1060	6,654
7	13,029	7,117	867	0.1218	7,366
8	12,511	7,260	1,160	0.1598	6,729
9	14,372	8,305	2,013	0.2424	6,043
10	7,371	9,299	4,380	0.4710	3,780
Total	96,880	70,597	7,573	0.1073	63,022

PA Insurer Group 1066			Outcome Percentile = 81.50		
w	Premium	Estimate	Std. Dev.	C.V.	Outcome
1	24,988	20,888	0	0.0000	20,813
2	26,082	24,943	290	0.0116	24,741
3	29,606	27,471	367	0.0134	27,206
4	33,802	23,274	328	0.0141	23,254
5	37,261	22,564	367	0.0163	22,536
6	35,849	22,960	466	0.0203	23,174
7	35,053	23,370	605	0.0259	22,210
8	33,254	18,117	669	0.0369	19,210
9	29,101	15,515	985	0.0635	16,137
10	29,149	11,704	1,727	0.1476	14,513
Total	314,145	210,804	3,617	0.0172	213,794

Figure 3.1. Uniformity Tests for the CSR Model



While the observation that the CSR model performs well on a large number of old triangles with outcome data is encouraging, it should not relieve the actuary from testing the assumptions underlying their model of their current data. Traditional tests, such has those provided by Barnett and Zehnwirth (2000) plot residuals (i.e. differences between observed and expected values) along accident year, development year and calendar year dimensions.

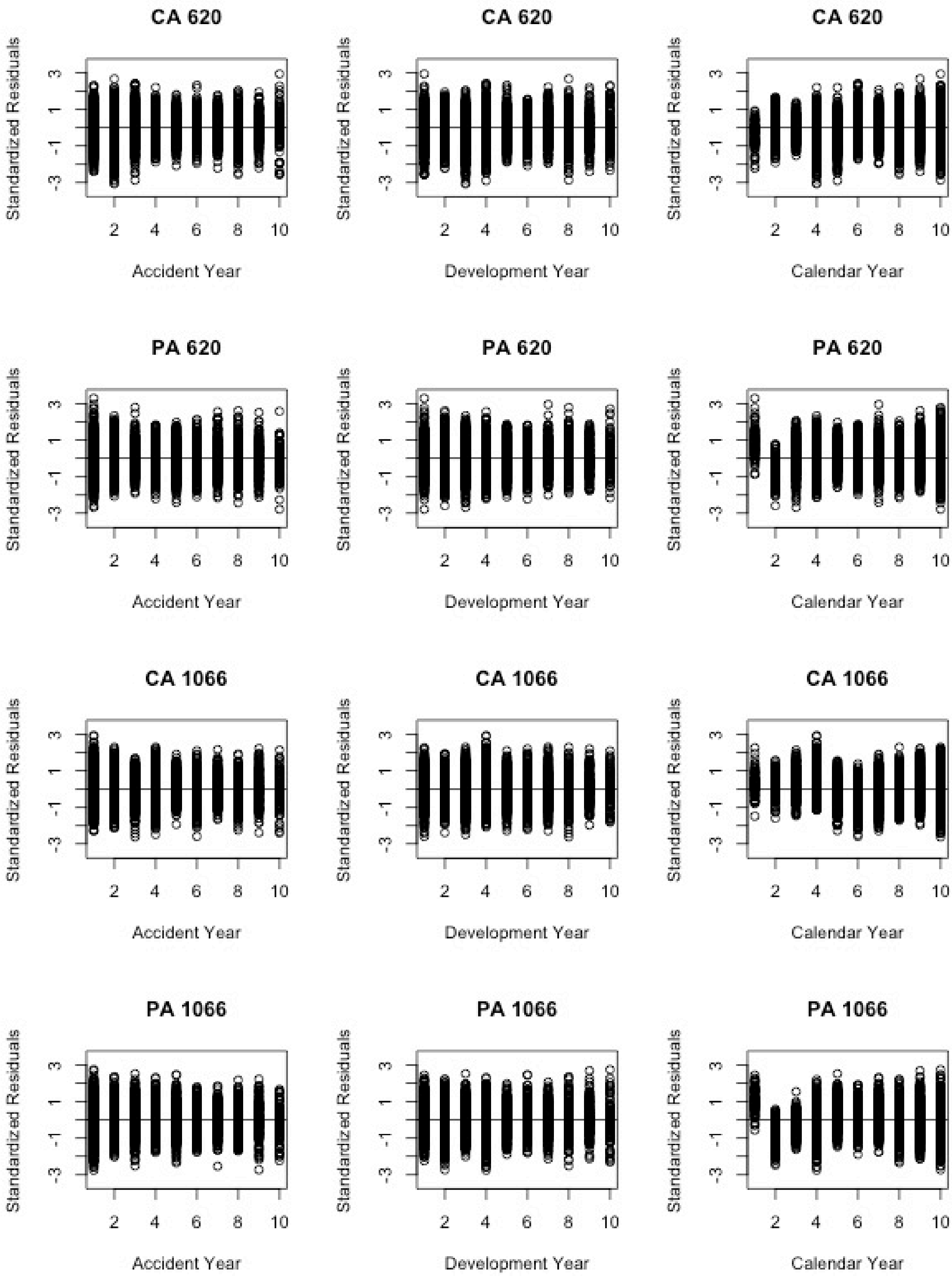
The Bayesian MCMC models in this paper provide a sample of size 10,000 from a posterior distribution of parameters. Given that we have this large sample, I consider it to be more informative if we take a subsample, *I*, of (say) size 100, then calculate the standardized residuals for each *w* and *d* in the upper loss triangle, and *i* in the subsample

$$\mathrm{R}_I^X = \left\{ \frac{\log\left(C_{wd}^X\right) - {}_i\mu_{wd}^X}{{}_i\sigma_d^X} \right\}_{i \in I} \qquad (3.1).$$

In general we should expect these residual plots to have a standard normal distribution with mean 0 and standard deviation 1. Figure 3.2 shows plots of these standardized residuals against the accident year, development year and calendar year for the illustrative insurers. I have made similar plots for other insurers as well. For accident years and development years, the plots have always behaved as expected. Deviations for the early calendar years as shown in two of the four plots are not uncommon. I have chosen to regard them as unimportant, and attach more importance to later calendar years.

If the standardized residual plots look like those of the illustrative insurers, we should not have to worry about artificial correlations.

Figure 3.2 – Standardized Residual Plots for the CSR Model



4. A TWO-STEP BIVARIATE MODEL

The last section presented a univariate model that performed well on data in the CAS Loss Reserve Database. This section shows how to construct a bivariate distribution that has the univariate distributions as marginal distributions.

To shorten the notation let

$$_i\theta^X = \left\{ \left\{ _i\alpha_w^X \right\}_{w=2}^{10}, \left\{ _i\beta_d^X \right\}_{d=1}^{9}, \left\{ _i\sigma_d^X \right\}_{d=1}^{10}, {_i\mathrm{logelr}^X}, {_i\gamma^X}, {_i\delta^X} \right\}$$

for line X and $i = 1, \ldots, 10000$.

The first step is to obtain the univariate samples, $\left\{ _i\theta^X \middle| \left\{ C_{wd}^X \right\} \right\}_{i=1}^{10,000}$ and $\left\{ _i\theta^Y \middle| \left\{ C_{wd}^Y \right\} \right\}_{i=1}^{10,000}$ where $C_{wd}^X \in$ Upper Triangle of line X and $C_{wd}^Y \in$ Upper Triangle of line Y. Then repeatedly for each i, use Bayesian MCMC to take a sample from the posterior distribution of $\left\{ \rho \middle| \left\{ C_{wd}^X \right\}, \left\{ C_{wd}^Y \right\}, {_i\theta^X}, {_i\theta^Y} \right\}$ where ρ has a $\beta(2,2)$ prior distribution translated from (0,1) to (-1,1). Next we randomly select a single $_i\rho$ from that sample and use $\left\{ _i\theta^X, {_i\theta^Y}, {_i\rho} \right\}_{i=1}^{10,000}$ to calculate the derived parameters in the bivariate distribution given by Equation (1.2). This amounts to using the two univariate distributions as the prior distribution for the second Bayesian step. From that two-step bivariate distribution, one can simulate outcomes from the "posterior" distribution of parameters and calculate any statistic of interest. Be reminded that this can be different from the usual Bayesian posterior distribution $\left\{ _i\theta^X, {_i\theta^Y}, {_i\rho} \middle| \left\{ C_{wd}^X \right\}, \left\{ C_{wd}^Y \right\} \right\}_{i=1}^{10,000}$ that comes out of the Zhang-Dukic approach.

At first glance, one might expect the run time for 10,000 Bayesian MCMC simulations to be unacceptably long. But there are a number of considerations that allow one to speed up the calculations.

1. The MCMC simulation is for a single parameter that runs much faster than a multi-parameter simulation that one normally runs with stochastic loss reserve models.
2. We have a good starting value, $\rho = 0$. The burn-in period is short and convergence is rapid.
3. Since we are repeatedly running the same model with different inputs, we need only compile the model once, which the Stan software permits.
4. Using the "parallel" package in R allows one to distribute the simulations to separate cores on a multi-core computer.

Taking these factors into account, my laptop[4] usually turns out this bivariate distribution in about 6 minutes. As I mentioned above, the R scripts that produce these calculations are made available to the reader in the Appendix.

The purpose of getting a bivariate distribution is to predict the distribution of the sum of the outcomes for the two lines of insurance. Table 4.1 gives results analogous to Table 3.1 for the sum of CA and PA lines for the two illustrative insurers. Also included are the sums of the two lines predicted under the assumption of independence. Figure 4.1 contains histograms of the two-step posterior distributions for ρ for the illustrative insurers.

[4] Apple MacBook Pro with quad-core processor – purchased in late 2013.

Table 4.1. Combined CSR Models on Illustrative Insurer Data[5]

	Insurer Group 620			Outcome Percentile = 41.34		
	w	Premium	Estimate	Std. Dev.	C.V.	Outcome
	1	78,955	66,647	0	0.0000	66,391
	2	85,729	66,928	567	0.0085	67,127
	3	94,691	78,326	844	0.0108	77,946
Two-Step Bivariate	4	105,362	85,532	1,006	0.0118	85,744
Model	5	115,419	84,311	1,189	0.0141	85,419
	6	121,618	90,935	1,516	0.0167	90,323
	7	127,926	95,970	2,452	0.0255	96,439
	8	129,770	105,814	4,259	0.0402	104,246
	9	136,698	107,501	7,770	0.0723	105,809
	10	146,858	118,650	18,581	0.1566	112,986
	Total	1,143,026	900,614	28,999	0.0322	892,430

	Insurer Group 620			Outcome Percentile = 44.05		
	w	Premium	Estimate	Std. Dev.	C.V.	Outcome
	1	78,955	66,654	0	0.0000	66,391
	2	85,729	67,349	586	0.0087	67,127
	3	94,691	78,372	789	0.0101	77,946
Indepencence	4	105,362	85,054	972	0.0114	85,744
Assumption	5	115,419	84,973	1,225	0.0144	85,419
	6	121,618	89,435	1,528	0.0171	90,323
	7	127,926	96,166	2,470	0.0257	96,439
	8	129,770	105,890	4,269	0.0403	104,246
	9	136,698	107,870	7,810	0.0724	105,809
	10	146,858	116,833	18,172	0.1555	112,986
	Total	1,143,026	898,597	28,667	0.0319	892,430

	Insurer Group 1066			Outcome Percentile = 26.62		
	w	Premium	Estimate	Std. Dev.	C.V.	Outcome
	1	30,091	25,610	0	0.0000	25,442
	2	31,278	30,680	444	0.0145	30,718
	3	36,553	33,773	580	0.0172	33,625
Two-Step Bivariate	4	43,284	31,429	710	0.0226	31,632
Model	5	48,237	29,451	693	0.0235	29,583
	6	47,742	30,056	813	0.0270	29,828
	7	48,082	30,392	1,047	0.0344	29,576
	8	45,765	25,397	1,329	0.0523	25,939
	9	43,473	23,672	2,208	0.0933	22,180
	10	36,520	21,726	5,018	0.2310	18,293
	Total	411,025	282,187	8,617	0.0305	276,816

	Insurer Group 1066			Outcome Percentile = 29.78		
	w	Premium	Estimate	Std. Dev.	C.V.	Outcome
	1	30,091	25,615	0	0.0000	25,442
	2	31,278	31,020	464	0.0150	30,718
	3	36,553	33,910	552	0.0163	33,625
Indepencence	4	43,284	31,129	705	0.0226	31,632
Assumption	5	48,237	29,864	709	0.0237	29,583
	6	47,742	29,178	810	0.0278	29,828
	7	48,082	30,487	1,056	0.0346	29,576
	8	45,765	25,376	1,333	0.0525	25,939
	9	43,473	23,819	2,236	0.0939	22,180
	10	36,520	21,003	4,733	0.2253	18,293
	Total	411,025	281,401	8,417	0.0299	276,816

[5] I attribute the differences in the "Estimate" column by insurer to simulation error. The expected values for the bivariate and independence assumptions are equal.

Figure 4.1 – Posterior Distribution of ρ for CSR Model

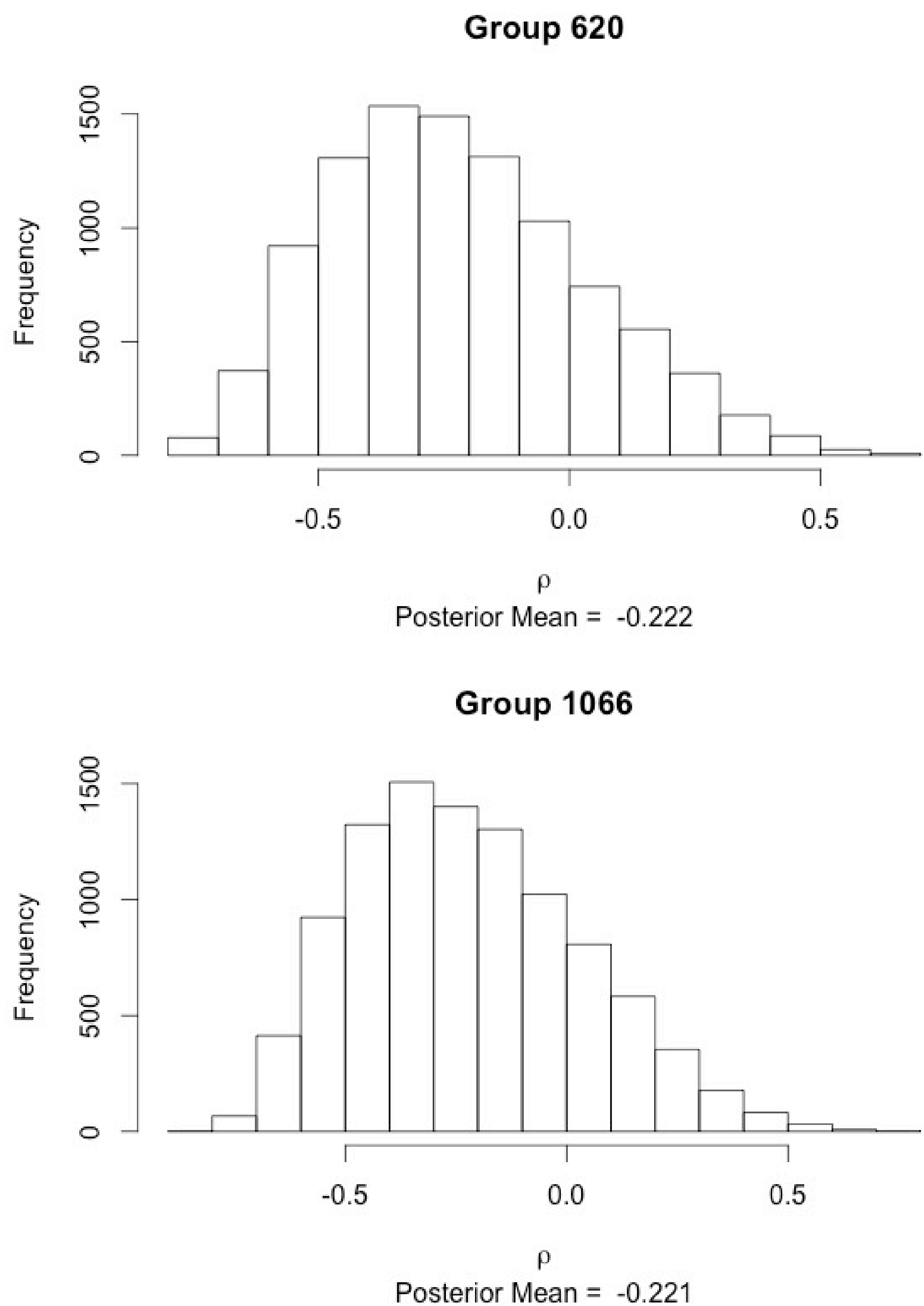
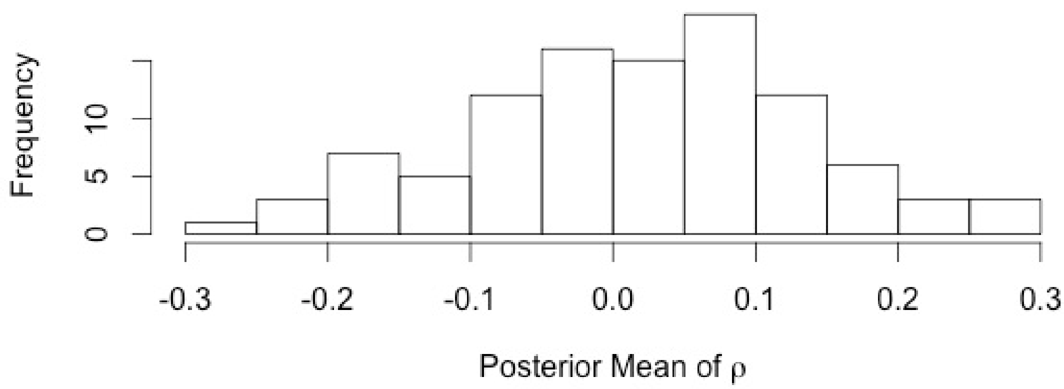


Table 4.1 and Figure 4.1 are notable in two aspects. First, the output from the bivariate model is not all that different from the output created by taking independent sums of losses from the univariate model. Second, the posterior distributions of ρ from the two-step bivariate model have a fairly wide range. The posterior distributions of ρ for both groups are predominantly negative.

Typically the posterior mean ρ over all the within-group pairs of lines is not all that different from zero. Figure 4.3 shows the frequency distribution of posterior mean ρs from the insurer group sample.

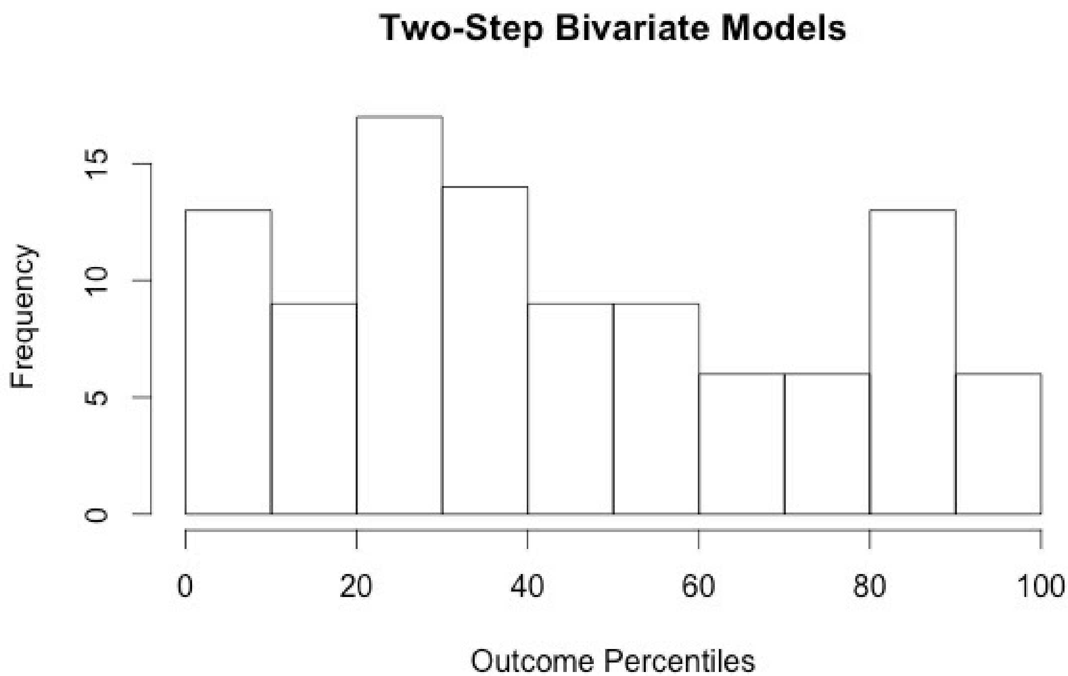
Figure 4.3 – Posterior Mean ρs from the Within-Group Pairs of Lines



This section concludes with a test of uniformity of the outcome percentiles of the within-group pairs for the sum of two lines predicted by the two-step bivariate model and the independence assumption. As Figure 3.1 shows, the univariate models pass our uniformity test, one would think that a valid bivariate model would also pass a uniformity test. Figures 4.4 and 4.5 show the results.

It turns out that both the two-step bivariate model and the independence assumption pass the uniformity test, with the independence assumption performing slightly better. This suggests that the lines of insurance are independent for many, if not all, insurers. In the next section we will examine the independence assumption for individual pairs of loss triangles.

Figure 4.4 - Uniformity Tests of Outcome Percentiles



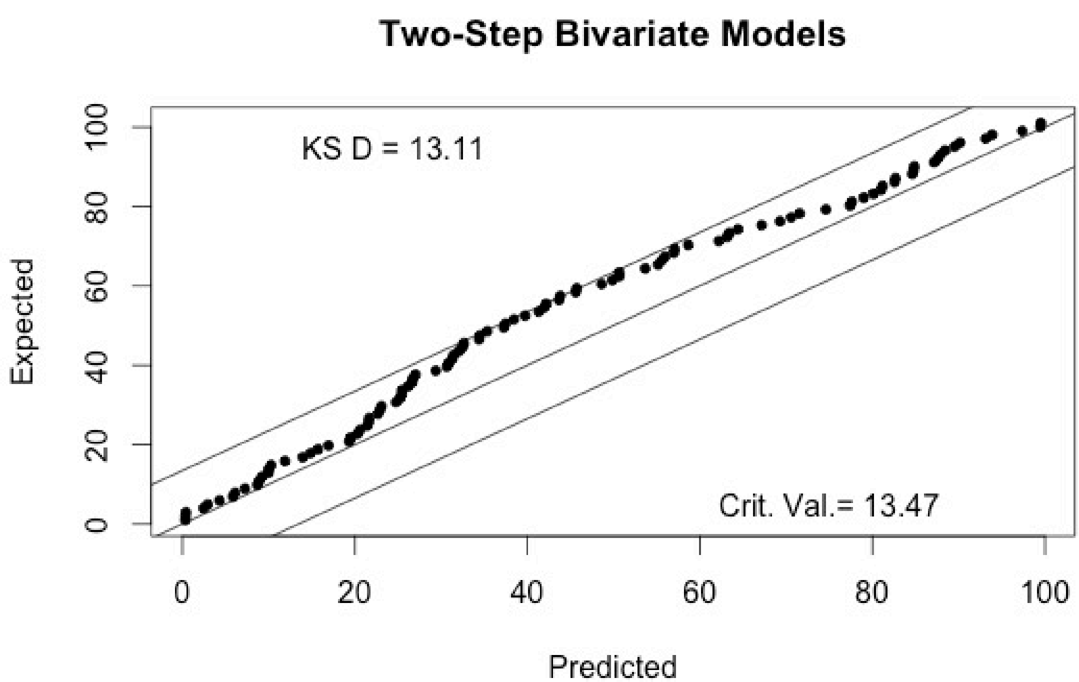
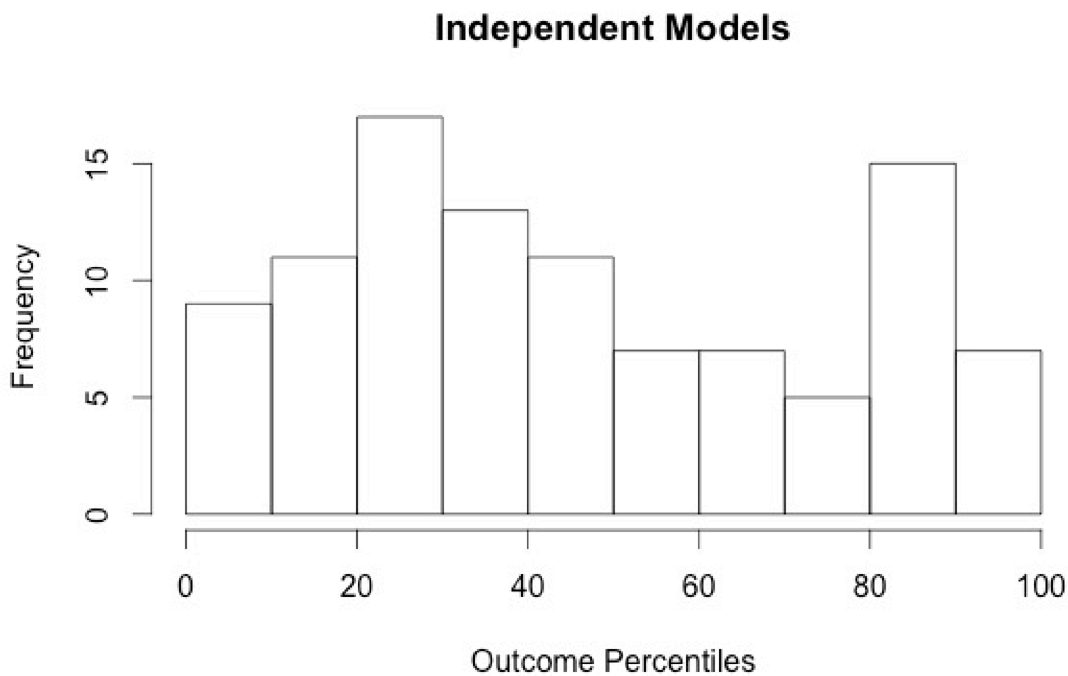
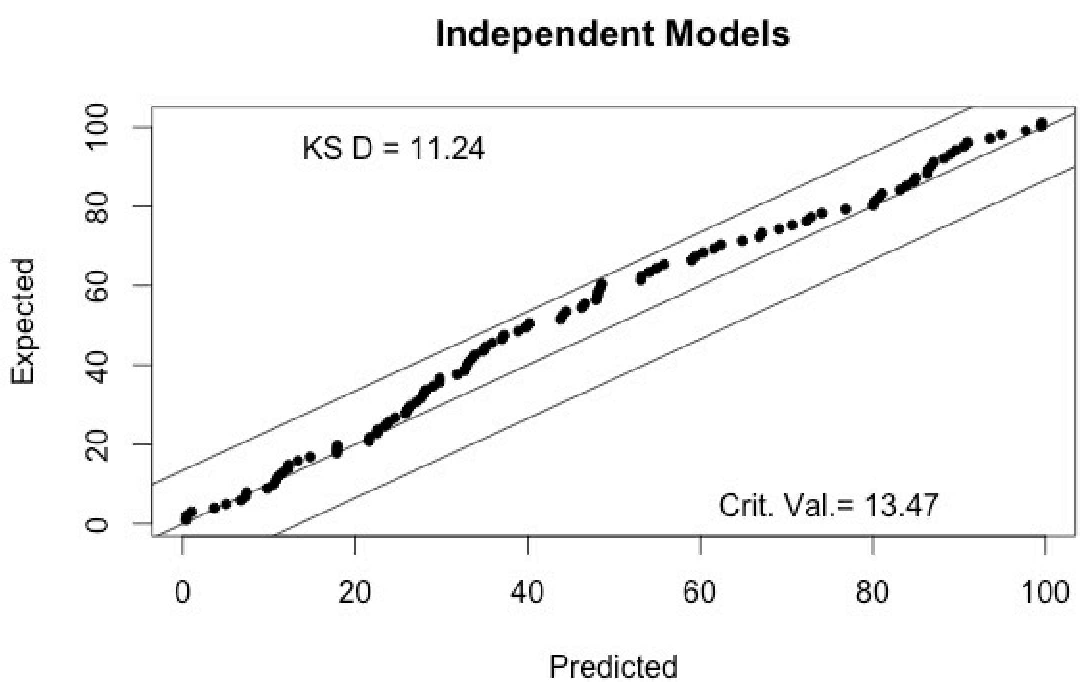


Figure 4.5 - Uniformity Tests of Outcome Percentiles





5. MODEL SELECTION[6]

Let's start the discussion with a review of the Akaike Information Criteria (AIC).

Suppose that we have a model with a data vector, $\boldsymbol{x}$, and a parameter vector θ, with p parameters. Let $\hat{\boldsymbol{\theta}}$ be the parameter value that maximizes the log-likelihood, L, of the data, $\boldsymbol{x}$. Then the AIC is defined as

$$\text{AIC} = 2 \cdot p - 2 \cdot L\left(\hat{\theta}\right). \tag{5.1}$$

Given a choice of models, the model with the lowest AIC is to be preferred. This statistic rewards a model for having a high log-likelihood, but it penalizes the model for having more parameters.

There are problems with the AIC in a Bayesian MCMC environment. Instead of a single maximum likelihood estimate of the parameter vector, there is an entire sample of parameter vectors taken from the model's posterior distribution. There is also the sense that the penalty for the number of parameters should not be as great in the presence of strong prior information.

To address these concerns, Gelman *et. al.* (2014) and Vehtari and Gelman (2014) describe a statistic, called the Watanabe-Akaike Information Criterion (WAIC) that generalizes the AIC in a way that is appropriate for Bayesian MCMC models[7].

First define the computed log pointwise predictive density (made specific for this paper) as

$$L_{\text{WAIC}} = \sum_{j=1}^{55} \log\left(\frac{1}{10000}\sum_{i=1}^{10000} \phi\left(\log\left(C^X_{w(j),d(j)}\right), \log\left(C^Y_{w(j),d(j)}\right) \Big|\, {}_i\theta^X, {}_i\theta^Y, {}_i\rho\right)\right). \tag{5.2}$$

where ϕ is a multivariate normal distribution such as that given in Equation(1.2). The L_{WAIC} statistic replaces the log-likelihood L in Equation(5.1) with an average log-likelihood taken over the sample from the posterior distribution.

Next, define the effective number of parameters p_{WAIC} as

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[6] For more information about the model selection statistics in this section, see Section 7.2 of Gelman, *et. al.*.

[7] Another popular statistic designed for Bayesian MCMC models is the Deviance Information Criterion (DIC) that is available in the MCMC software WINBUGS and JAGS. Gelman *et. al.* (2014) and Vehtari and Gelman (2014) make the case that the WAIC is a better statistic as it is based on the entire sample from the posterior distribution as opposed to a point estimate.

$$p_{WAIC} = \sum_{j=1}^{55} Var_i\left[log\left(\phi\left(log\left(C^X_{w(j),d(j)}\right), log\left(C^Y_{w(j),d(j)}\right) \Big| {}_i\theta^X, {}_i\theta^Y, {}_i\rho\right)\right)\right]. \tag{5.3}$$

p_{WAIC} has the property that it decreases with the tightness of the prior distribution. Of possible general interest is that Vehtari and Gelman discuss situations, e.g. flat priors and a large number of data points, where p_{WAIC} is equal to the nominal number of parameters, p. But none of that applies to the examples in this paper where we have only 110 observations, some non-flat priors and, in addition, have some constraints between some of the parameters.

The final expression for the WAIC is analogous to Equation (5.1) and is given by

$$\text{WAIC} = 2 \cdot p_{WAIC} - 2 \cdot L_{WAIC}. \tag{5.4}$$

The WAIC statistics for the bivariate two-step model were calculated with the posterior distribution $\left\{ {}_i\theta^X, {}_i\theta^Y, {}_i\rho \right\}_{i=1}^{10000}$. For the model that assumes independence of the univariate models, the WAIC statistics were calculated with the posterior distribution $\left\{ {}_i\theta^X, {}_i\theta^Y, 0 \right\}_{i=1}^{10000}$. Table 5.1 gives these statistics for the illustrative insurers. The lower WAIC statistic for the assumption of independence is the preferred model for both insurer groups.

Table 5.1 – WAIC Statistics for the Illustrative Insurer Groups For the CSR Model

Group	Model	p_{WAIC}	L_{WAIC}	WAIC
620	Bivariate	31.09	255.31	-448.44
620	Independent	27.23	252.92	-451.38
1066	Bivariate	30.89	180.41	-299.04
1066	Independent	27.12	178.03	-301.82

The WAIC statistics for the *all* the within-group pairs, given in the Appendix, indicate that the assumption of independence is the preferred model!

6. ILLUSTRATION OF MODEL SENSITIVITY

In discussions with my actuarial colleagues over the years, I have sensed a general consensus among most actuaries is that there is some degree of dependence between the various lines of insurance. But as pointed out in the introduction, using a suboptimal model can lead to artificial dependencies. This section takes a stochastic version of a currently popular model and demonstrates that it is suboptimal for our sample of insurers. It also shows that given this model, there are significant dependencies between the various lines of insurance suggesting that the "general consensus" is understandable given the state of the art that has existed over the years.

One of the most popular loss reserving methodologies is given by Bornhuetter-Ferguson (1972). A key input to the loss reserve formula given in that paper is the expected loss ratio, which must be judgmentally selected by the actuary. Presentations by Clark (2013) and Leong (2013) suggest that the Bornhuetter-Ferguson method that assumes a constant loss ratio provides a good fit to industry loss reserve data.

Actuaries who want to use data to select the expected loss ratio can use the "Cape Cod" model that is given by Stanard (1985). A stochastic version of the Cape Code model can be expressed as a special case of the CSR model by setting the parameters $\alpha_w = 0$ for $w = 1,\ldots, 10$, $\gamma = 0$ and $\delta = 0$. Let's call this model the Stochastic Cape Cod (SSC) model.

Figure 6.1 gives the standardized residual plots of the SCC model for the illustrative insurers that are analogous to those in Figure 3.2. Figure 6.2 gives the posterior distribution of the ρ parameters for the two-step bivariate SCC model. Noteworthy is that the mean ρ for Group 1066 is quite high compared to any of the results for the CSR model. In Table 6.1 we see that the WAIC statistic for Group 1066 is lower for the bivariate model than the independent model indicating that the bivariate model is favored. The reverse is true for Group 620.

Figure 6.1 – Standardized Residual Plots for the SSC Model

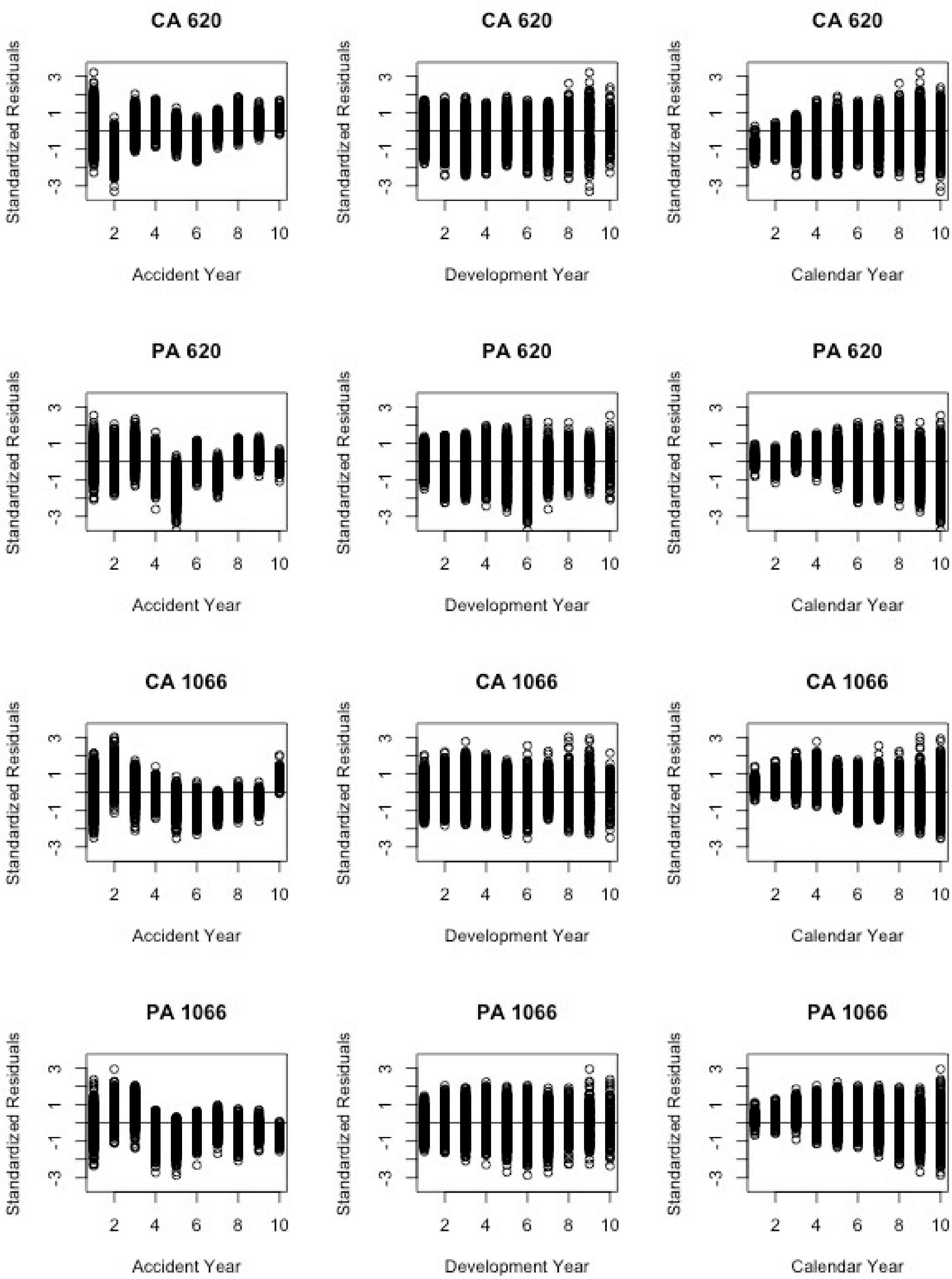


Figure 6.2 – Posterior Distribution of ρ for SCC Model

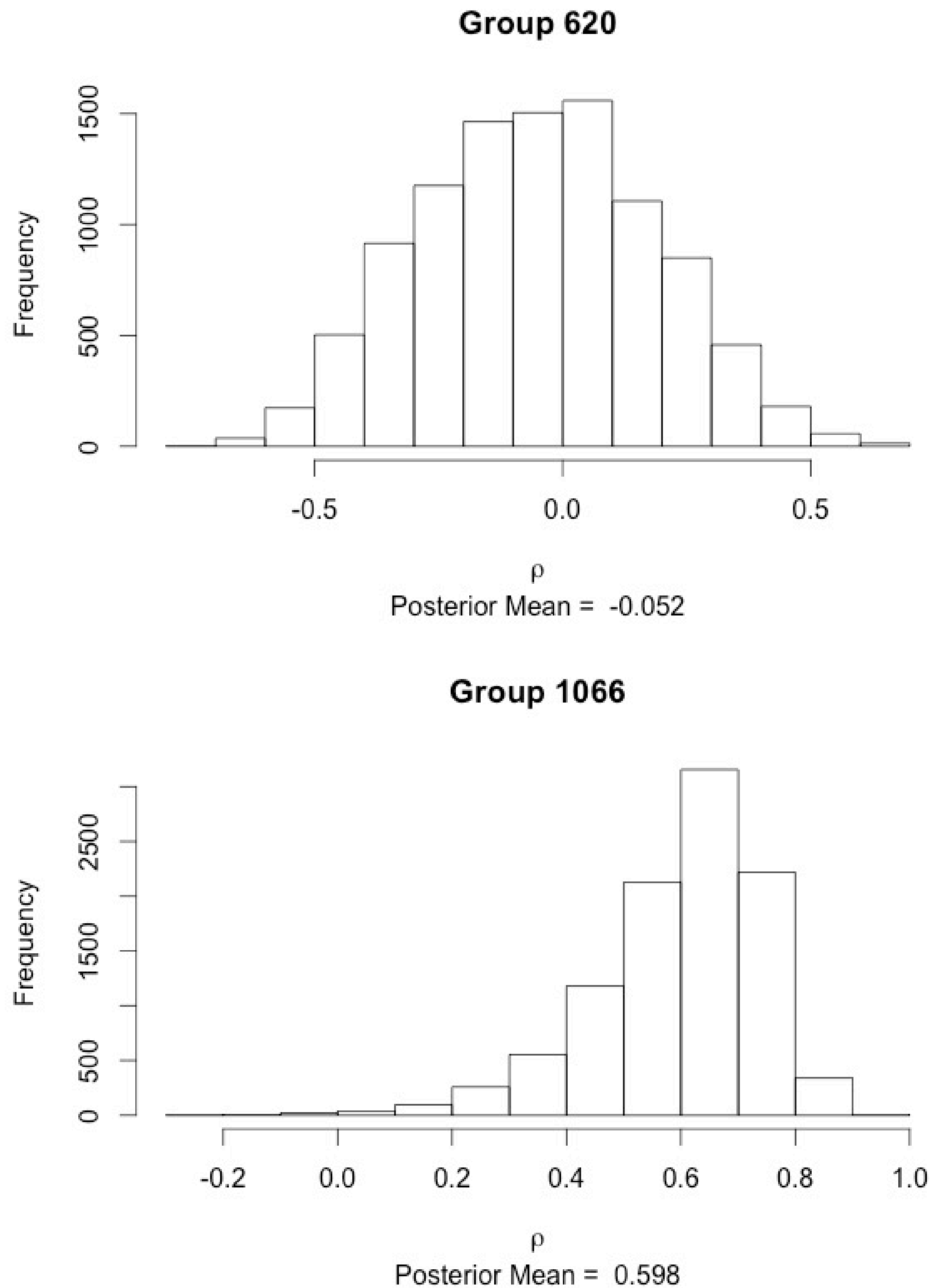


Table 6.1 – WAIC Statistics for the Illustrative Insurer Groups For the SCC Model

Group	Model	p_{WAIC}	L_{WAIC}	WAIC
620	Bivariate	17.79	122.10	-208.62
620	Independent	16.34	121.55	-210.42
1066	Bivariate	23.61	18.99	9.24
1066	Independent	14.53	6.68	15.7

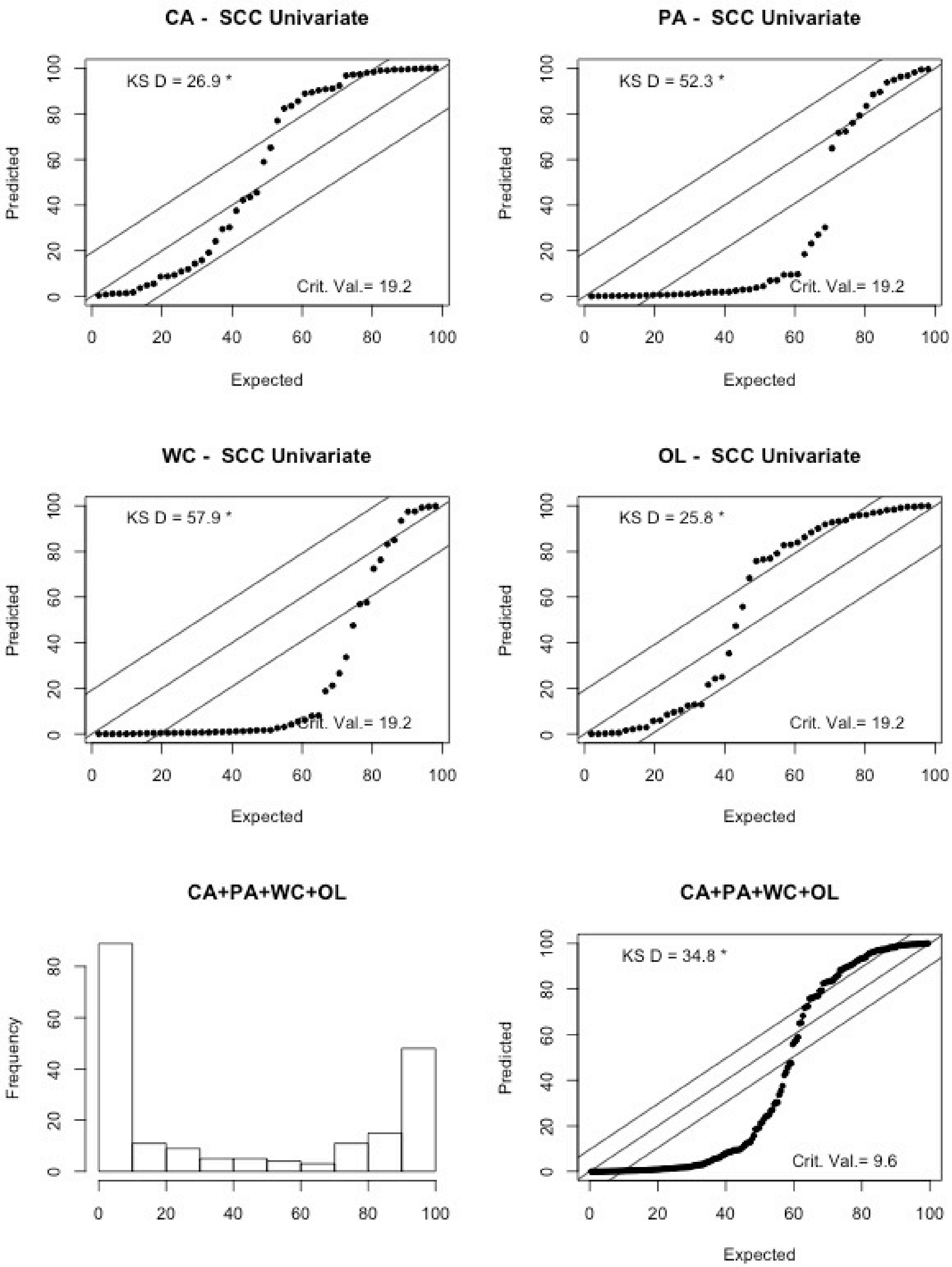
By examining the standardized residual plots in Figure 6.1 across accident years we can see a possible explanation for these results. First note that the standardized residual plots for the SCC model are not as well behaved as the similar plots for the CSR model in Figure 3.2. But the pattern of the errors in the CA and PA plots are dissimilar for Group 620, but similar for Group 1066. The similarity of the plots for Group 1066 leads to the overwhelmingly positive posterior distribution of ρ, and the indicated preference of the bivariate model over the independent model.

Over the entire sample of insurer groups, the bivariate model was the preferred model for 39 of the 102 within-group pairs of triangles.

It is also worth noting that, as shown in Figure 6.3, the SCC model fails the uniformity test that the CSR model passed, as shown in Figure 3.1.

Here we see an example where the suboptimal SCC model leads to artificial dependencies between lines, whereas the less suboptimal CSR model leads to independence between lines for our sample of insurer loss triangles.

Figure 6.3. Uniformity Tests for the SCC Model



7. SUMMARY AND CONCLUSIONS

The purpose of this paper was to illustrate how to build a model that creates a bivariate distribution given two univariate Bayesian MCMC models that preserve the original univariate distributions. While this modeling technique was applied to lognormal stochastic loss reserve models, it should not be difficult to apply this two-step approach to other Bayesian MCMC models using bivariate copulas as was done by Zhang and Dukic.

While a statistical study such as that done in this paper can never carry the weight of a mathematical proof, its conclusion was derived from the analysis of a large number of within-group pairs of loss triangles. It should be noted that these loss triangles came from NAIC Schedule Ps reported in the same year.

The conclusion that the within-group pairs of loss triangles are independent for the CSR model may come as a surprise to some. But the evidence supporting this conclusion is as follows.

1. The univariate models pass two fairly restrictive tests (i.e. the retrospective test in Figure 3.1 and the standardized residual tests in Figure 3.2) that could disqualify many suboptimal models. Thus we should not expect to see an artificial appearance of dependency due to a bad model.
2. The retrospective results of Section 4 indicate support the independence assumption for the bivariate two-step model. The range of ρs for the 102 within-group pairs contained both positive and negative values, which appear to be random in light of the tests performed in this paper.

I feel fortunate that I was able to find a model that indicated independence between lines of insurance. Before taking on this line of research, there was no guarantee that I would be able to find such a model. In fact, initially I did not believe the independence results that I was getting. The lesson learned is that if one has a model with statistically significant dependences between lines of insurance, one should search for a more optimal model.

The reason that the dependency problem is so important is that risk-based insurer solvency standards are based on the total risk to the insurance company. Ignoring a true dependency could understate the total risk faced by an insurer. On the other hand, too stringent of a solvency standard could limit the supply of insurance. If this holds, then the current practice in some jurisdictions could limit the supply of insurance.

While retrospective results can be informative, there is a need for criteria testing the independence assumption that can be applied prospectively. That was the purpose of Section 5. The prospective test consists of; (1) fitting the two-step bivariate model; (2) fitting a bivariate model that assumes independence; and (3) calculating the WAIC statistic to see which model is favored. It turned out that the WAIC statistic favored the independence assumption in *every one* of the 102 within-group pairs of triangles.

So for now, the CSR model with the independence assumption is looking pretty good. But in light of the high stakes involved, assumptions of this sort need a stringent peer review and replication with new and different data. I look forward to seeing this happen.

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