The Actuary & Enterprise Risk Management: Integrating Reserve Variability

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Abstract

Motivation. The development of a wide variety of reserve variability models has been primarily driven by the need to quantify reserve uncertainty. This quantification can serve as the basis for satisfying a number of Solvency II requirements in Europe, can be used to enhance Own Risk Solvency Assessment (ORSA) reports, and is often used as an input to DFA or Dynamic Risk Models, to name but a few. Moving beyond quantification, the purpose of this paper is to explore other aspects of reserve variability which allow for a more complete integration of these key risk metrics into the larger Enterprise Risk Management framework.

Method. This paper will primarily use a case study to discuss and illustrate the process of integrating the output from periodic reserve and reserve variability analysis into the enterprise risk management process. Consequences of this approach include the production of valuable performance indicators and an increase in the lines of communication between the actuarial function and other insurance functional departments, both of which are valuable to management.

Results. By expanding the regular reserving process to include regular variability analysis and expanding the dialogue with management, the actuary can greatly contribute to the understanding of risks related to claim management within an enterprise.

Conclusions. The value of this process is not limited to reserving as it can logically and directly be extended into pricing, reinsurance optimization, etc.

Availability. In lieu of technical appendices, companion Excel workbooks are included that illustrate the calculations described in this paper. The companion materials are summarized in the Supplementary Materials section and are available at [CAS to fill in location].

Keywords. Reserve variability, enterprise risk management, actual versus expected, back-testing, deviations from expectation, one-year time horizon, validation, reserve distribution testing, assumption consistency, run-off analysis, key performance indicator.

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1. Introduction

Never has it been more important for actuaries to improve their understanding of reserve variability. Updated International Financial Reporting Standards (IFRS Phase II) will likely require all insurance companies to record an independently measured and updated risk margin. In Europe, Solvency II directives already require the recognition of a risk margin and validation standards require the Actuarial Function to comment on material deviations from prior expectations.

A range of reasonable estimates can be selected based on the results of deterministic methods, some scenario testing, and a few basic rules of thumb. Such a range, together with some heroic assumptions, can provide an unsophisticated aid to management in selecting a risk margin. More commonly, however, the calibration of risk margins makes use of modern stochastic modelling techniques, resulting in a distribution of possible outcomes,[1] with the outcomes providing the ability to measure statistical properties such as the mean, mode, percentiles, etc. There are a number of uses for the results of stochastic modelling techniques beyond the calibration of a risk margin, many of which can be incorporated for use within the Enterprise Risk Management ("ERM") process such that "new" information can be quickly used to assess performance. For example, key performance indicators ("KPIs") can be developed based on a range of percentiles around the expected outcomes.

Back-testing is a validation technique that enables the reserving actuary to assess the "new" information inherent in the loss triangles, relative to "known" information and future expectations inherent in the prior analysis. However, without an analysis of reserve variability, an assessment of the ***significance of deviations*** from expectations on both a granular level (individual accident periods) and an aggregate level (by reserving segment, by line of business, or by Company) is not quantifiable. Even with an analysis of reserve variability, bifurcating significant deviations as being the result of mean estimation error, variance estimation error, and/or random error is difficult.

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[1] A distribution of possible outcomes is an expression of the "full breadth" of the possibilities of the future payouts. Note that the estimation of unpaid claims involves significant uncertainties that cannot be completely estimated, so "full breadth" should be thought of as a reasonable estimate of the distribution to the extent that it can be estimated using historical data (for independent risk) and a subjective adjustment to account for variability attributable to systemic risk. Further, the available historical data may be limited such that an adjustment to account for events not in the data ("ENID") may also be necessary. For this reason, a distribution of possible outcomes may not be possible using the most sophisticated actuarial techniques available.

A systematic back-testing process as part of a comprehensive ERM system, which uses the output of prior reserve variability analyses, significantly increases the ability of the actuary to assess deviations from expectations and provides management with an early indication of the current period's performance relative to the actuary's expectations. Further, a systematic back-testing process allows for the evaluation of the universe of deviations, relative to the distributional expectations for the current period.

Within the comprehensive ERM solution, assumption consistency becomes an important consideration. When selecting a central estimate[2] for an unpaid claim estimate, the practicing actuary commonly weights the results from multiple methods. By assigning weights to multiple methods, the actuary is partially accepting or rejecting the assumptions inherent in each method that contributes to the selection of their central estimate.[3]

Therefore the future expectation for each data element (e.g., incremental paid losses) is a weighted average of the respective expected data element from each of the methods which received weight. Likewise, the inherent uncertainty in the selected estimate is more appropriately modeled as a weighted average of the expected uncertainty in the methodology which underlies each model used to estimate uncertainty as this also helps to address model risk.[4] In contrast, an approach which uses a single model (e.g., Mack or an ODP bootstrap of the paid chain ladder method alone) to estimate the uncertainty around a point estimate based on multiple methods, uses an assumption set for the variance which is at best partially rejected during the selection of the point estimate and at worst involves assumptions which are completely different from those used for the point estimate.

This paper will develop and examine a framework for reserve distribution testing and validation and demonstrate its use with real datasets within an Enterprise Risk Management framework. It will also illustrate how stochastic results based on a one-year time horizon (as specified in Solvency II) can be used in the subsequent year's process of estimating reserves

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[2] This paper uses the term "central estimate," consistent with Actuarial Standard of Practice No. 43, "Property/Casualty Unpaid Claim Estimates," promulgated by the Actuarial Standards Board [1]. With respect to Solvency II and IFRS Phase II, regulations and guidance use the term "best estimate" to mean the same thing.

[3] Accepting or rejecting assumptions is a simplification of the entire process and all considerations. For example, not giving weight to a method for a specific year is not rejection of the method or any specific assumption within the method as the method may be given some weight for another year. Thus, this description of the process of weighing methods to arrive at a central estimate should be interpreted as including all considerations an actuary uses.

[4] Weighting deterministic methods is also a way to address model risk. The entire process of weighting multiple models is outside the scope of this paper, but common issues (like consistency of variances between models) are assumed to have been resolved when selecting weights.

to get an early indication of the expected reserve changes due to the emergence of new information.

1.1 Research Context

The importance of assumption consistency should not be underestimated. Paragraph 3.6.2 of ASOP 43 [1] states that an actuary "should use assumptions that, in the actuary's professional judgment… are not internally inconsistent." Also note that Article 122.2 of the Solvency II Framework Directive [10] ("FD") states that models "used to calculate the probability distribution forecast shall… be consistent with the methods used to calculate technical provisions." Finally, section C from Technical Actuarial Standards: Modelling ("TAS-M") [11] states that assumptions should be consistent in "a model or in a suite of models." TAS-M further suggests that different assumptions (i.e., use of multiple methods that use different assumptions) are "not always inconsistent. For example, if several independent models are used in conjunction to provide better estimates than any one model could provide on its own, different assumptions might be chosen deliberately." If however, inconsistent assumptions are used, TAS-M requires a disclosure statement.

Actuarial literature includes a number of approaches to quantifying the uncertainty of reserve estimates based on the variability observed in the actual historical development of the claims under consideration. In practice, the most frequently used approaches are statistical approximations to relatively simple regression models. Such approaches have the advantages of being (relatively) straightforward to implement, interpret, and explain. They can be applied equally well to accident or underwriting period data to generate results on the same basis. Two regression models in particular tend to dominate: the Mack [18] linear regression model and the ODP bootstrap model originally developed by England & Verrall [7, 8].

In both cases, the expected values of the reserve estimate are equal to the results of the deterministic paid chain ladder method using the all-year volume-weighted average development factors, which is rarely the sole basis for the central estimate, especially for immature accident periods. Some practitioners of such models get around this limitation by "shifting" the modelled distribution such that the mean of the distribution is equal to the central estimate and the standard deviation from the model is maintained. The "shift" is usually implemented in an additive fashion by adding to each iteration the difference between the central estimate and the result of the paid chain ladder method (using the all-year volume-weighted average link ratios) by accident period. In order to get to the expected payments by development period, the "shift" will also need to be allocated to the incremental payments, which is often done in proportion to the overall expected average incremental payments before the shift.

As originally framed, the Mack [18] model (and by extension, the Merz-Wüthrich [19] model) provides a method for estimating a coefficient of variation ("CoV") for the reserve estimate. In order to convert the CoV into an estimate at a specific confidence level, however, it is necessary to select a particular parametric probability distribution whose parameters can be determined by the CoV together with the central estimate.

The ODP Bootstrap model originally developed by England & Verrall [7, 8] is often used in a similar manner to Mack [18] in the sense that the distributional output for the basic "chain ladder" model with paid data is "shifted" so the mean matches the central estimate. However, the ODP bootstrap approach can be extended to simulate any number of methods without requiring the selection of a particular parametric probability distribution as described in Shapland [27]. It is this approach which enables the actuary to maximize the assumption consistency between the central estimate of loss reserves and the calibration of reserve variability.

1.2 Objective

The objective of integrating loss reserve variability into the ERM process is to improve the estimation and management of loss reserves and reserving risk.

In order to manage reserve risk, one needs to measure it first. Integrating reserve risk into a continuously monitored ERM process ensures that assumptions are tracked and validated over time and that changes in assumptions are justified relative to the performance of prior assumptions.

Back-testing is a validation technique which can provide insight which improves a reserving process in that inevitable deviations from expectations are forced to be understood and future decision points (i.e., assumptions and expert judgement) can be based on the performance of past decision points.

2. Notation

The notation in this paper is from the CAS Working Party on Quantifying Variability in Reserve Estimates Summary Report [5] since it is intended to serve as a basis for further research.

Many models visualize loss data as a two-dimensional array, (w,d) with accident period or policy period w, and development age d (think w = "when" and d = "delay"). For this discussion, it is assumed that the loss information available is an "upper triangular" subset for rows $w = 1, 2, \ldots, n$ and for development ages $d = 1, 2, \ldots, n-w+1$. The "diagonal" for which $w+d$ equals the constant, k, represents the loss information for each accident period w as of accounting period k.[5]

For purposes of including tail factors, the development beyond the observed data for periods $d = n+1, n+2, \ldots, u$, where u is the ultimate time period for which any claim activity occurs – i.e., u is the period in which all claims are final and paid in full – must also be considered.

The paper uses the following notation for certain important loss statistics:

$c(w,d)$: cumulative loss from accident year w as of age d.[6]

$q(w,d)$: incremental loss for accident year w from d - 1 to d.

$c(w,n) = U(w)$: total loss from accident year w when claims are at ultimate values at time n, or with tail factors[7]

$c(w,u) = U(w)$: total loss from accident year w when claims are at ultimate values at time u.

$R(w)$: future development after age d for accident year w, i.e., = $U(w) - c(w,d)$.

$f(d)$: factor applied to $c(w,d)$ to estimate $q(w,d+1)$ or can be used more generally to indicate any factor relating to age d.

$F(d)$: factor applied to $c(w,d)$ to estimate $c(w,d+1)$ or $c(w,n)$ or can be

[5] For a more complete explanation of this two-dimensional view of the loss information, see the *Foundations of Casualty Actuarial Science* [12], Chapter 5, particularly pages 210-226.

[6] The use of accident year is for ease of discussion. All of the discussion and formulas that follow could also apply to underwriting year, policy year, report year, etc. Similarly, year could also be half-year, quarter or month.

[7] This would imply that claims reach their ultimate value without any tail factor. This is generalized by changing n to $n+t=u$, where t is the number of periods in the tail.

used more generally to indicate any cumulative factor relating to age d.

$G(w)$: factor relating to accident year w – capitalized to designate ultimate loss level.

$h(k)$: factor relating to the diagonal k along which $w+d$ is constant.[8]

$e(w,d)$: a random fluctuation, or error, which occurs at the w,d cell.

$E(x)$: the expectation of the random variable x.

$Var(x)$: the variance of the random variable x.

$Dist(x)$: the distribution of the random variable x.

$P_y(x)$: the y percentile of the distribution of the random variable x.

$\hat{x}$: an estimate of the parameter x.

What are called factors here could also be summands, but if factors and summands are both used, some other notation for the additive terms would be needed. The notation does not distinguish paid vs. incurred, but if this is necessary, capitalized subscripts P and I could be used.

3. Back-Testing

Back-testing is a process of comparing actual results with the expected results in order to answer the question "are the actual results better or worse than expected?" This simple question has many important nuances and ramifications, including psychological implications in the sense that people naturally tend to assume or hope for more "better than expected" back-tests than "worse than expected". While people also intuitively understand that a "worse than expected" back-test is "normal" the tendency to want more "better than expected" back-tests can creep into the initial expected results in the form of a bias for setting expectations higher than they may have otherwise been set. On the other hand, pressure to publish better financial results can push initial expectations lower.

In its simplest form a back-test can be formulated as in (3.1) for a particular incremental

[8] Some authors define $d=0,1,\ldots,n-1$ which intuitively allows $k=w$ along the diagonals, but in this case the triangle size is $n \times n-1$ which is not intuitive. With $d=1,2,\ldots,n$ defined as in this paper, the triangle size $n \times n$ is intuitive, but then $k=w+1$ along the diagonals is not as intuitive. A way to think about this which helps tie everything together is to assume the w variables are the beginning of the accident periods and the d variables are at the end of the development periods. Thus, if years are used then cell $c(n,1)$ represents accident year n evaluated at 12/31/*n*, or essentially 1/1/*n+1*.

value.

$$q(w,d) - E[\hat{q}(w,d)] \tag{3.1}$$

By subtracting the expected result from the actual result a "better than expected" result means that the actual result was less than the expected result. Somewhat counterintuitively, however, this "better than expected" result is actually a negative number.

The term "run-off" or a run-off analysis is often used interchangeably with "back-test" as the goal is to watch how actual results compare to the initial expectations. However, the run-off outcome is generally formulated as in (3.2) for a particular incremental value.

$$E[\hat{q}(w,d)] - q(w,d) \tag{3.2}$$

For the run-off test a "better than expected" result also means that the actual result was less than the expected result, but in this case the value is positive and perhaps more intuitive. Even as "back-test" and "run-off" can be used interchangeably, formulas (3.1) and (3.2) could also be interchanged between terms. For simplicity, from this point forward the paper will only refer to "back-testing" and will assume the reader can transition between terms and formulas (3.1) and (3.2) as preferred.

A back-test can be performed at either a granular or at a higher level. At a granular level, this would involve testing a single method or even a specific assumption within a method, with the goal of understanding the efficacy of that method or assumption. At a higher level the back-test will provide insight into the sum total of all methods and assumptions used to produce a final estimate. Granular level back-testing tends to be more of an academic or technical review whereas the higher level back-testing tends to focus at a management level, which is where the remainder of this paper will focus.

Within the ERM vernacular, the output of back-testing can be considered a KPI. As with other KPIs within an ERM system, information about deviations from expected outcomes provides valuable information for management.

3.1 Deterministic Back-Testing

For deterministic methods, the resulting point estimate is the sole source of the "expectation" from which to test deviations.[9] Consider, for example, the back-test results in Table 3.1. While a final back-test of the ultimate projection will be useful when all the claims

[9] For a deterministic analysis the point estimate does not contain any specific statistical meaning such as a mean, mode or median, so the term "expectation" likewise does not have any statistical connotation other than being a convenient reference to the central estimate.

are completely settled, the value of the back-test is typically drawn from the interim evaluations in order to check whether the incremental amounts are consistent with the development to date with respect to the ultimate projection.

In Table 3.1, actual accruals for accident year ("AY") 2015 are shown but expected accruals for AY 2015, and therefore differences, are not shown. This is because the 2015 calendar year ("CY") experience includes payments and case reserve changes attributable to AY 2015 and prior. The expectations, on the other hand, are based on the reserve analysis as of the prior year-end, in this case for AY 2014 and prior (i.e., as of 31 December 2014). In this paper the term "AY < CY" is used to denote the subtotal of all accident years not including the current accident year and "AY = CY" is used to denote the experience for the most recent AY which does not have a comparable expectation based on the prior reserve analysis alone.

Table 3.1 Back-Testing Example: Deterministic Actual vs. Expected

Sample Insurance Company Consolidation of All Segments Deterministic Actual vs. Expected as of December 31, 2015							
AY	Age	Actual Paid	Expected Paid	Difference	Actual Incurred	Expected Incurred	Difference
2006	120	3,069	3,701	(632)	1,863	2,158	(295)
2007	108	5,905	7,405	(1,500)	3,145	2,794	351
2008	96	8,986	10,073	(1,087)	3,553	6,142	(2,589)
2009	84	18,992	19,027	(35)	9,872	11,285	(1,413)
2010	72	51,003	47,151	3,852	25,942	26,873	(931)
2011	60	105,067	103,127	1,940	52,012	54,534	(2,522)
2012	48	202,932	194,479	8,453	106,624	106,020	604
2013	36	334,434	325,644	8,790	189,908	192,143	(2,235)
2014	24	841,484	833,793	7,691	454,217	479,073	(24,856)
2015	12	1,798,138			2,528,235		
Totals		3,370,010			3,375,371		
AY<CY		1,571,872	1,544,400	27,471	847,136	881,022	(33,886)

The "Difference" columns in Table 3.1 are calculated using formula (3.1), but like all deterministic back-tests the amounts reveal more about the direction of the outcome than the significance. Similar comparisons of actual and "expected" values are not difficult to compile for a number of other data elements (e.g., closed claims, reported claims, etc.), but while the total numbers of positive and negative deviations may be instructive it does not overcome the lack of a measure of significance. The only area where care needs to be exercised is in the calculation of the expected incremental amounts. For this, each method used should be converted into the incremental value being tested (e.g., paid claims) and then weighted together to arrive at an expectation which is consistent with the overall assumptions used to determine the selected estimate by accident period.[10] A typical short cut of multiplying the selected estimate by a selected development pattern will create a disconnection between assumptions at the macro and micro levels and should therefore be avoided.

A logical extension of this back-test is to check if the actual outcome falls within the reasonable range that was used to develop and select the central estimate. With a range, the formulation of the back-test can take the form of a percent, with a result between 0 and 100% indicating the outcome was within the range, a result greater than 100% indicating the outcome was above the range, and a result less than zero indicating the outcome was below the range.

$$\frac{q(w,d) - Min[\hat{q}(w,d)]}{Max[\hat{q}(w,d)] - Min[\hat{q}(w,d)]} \tag{3.3}$$

Continuing the example above, the back-test using a range is illustrated in Table 3.2, with the "Range Percent" columns calculated using formula (3.3).

Table 3.2 Back-Testing Example: Actual to Deterministic Range of Estimates

Sample Insurance Company
Consolidation of All Segments
Deterministic Actual vs. Method Range as of December 31, 2015

AY	Age	Actual Paid	Paid Minimum	Paid Maximum	Range Percent	Actual Incurred	Incurred Minimum	Incurred Maximum	Difference
2006	120	3,069	3,701	3,704	-21075%	1,863	2,158	2,162	-6790%
2007	108	5,905	5,827	8,983	2%	3,145	1,210	4,380	61%
2008	96	8,986	9,887	10,277	-231%	3,553	5,955	6,356	-599%
2009	84	18,992	17,726	20,381	48%	9,872	9,981	12,657	-4%
2010	72	51,003	44,889	49,487	133%	25,942	24,600	29,236	29%
2011	60	105,067	100,495	106,278	79%	52,012	51,856	57,857	3%
2012	48	202,932	191,183	198,745	155%	106,624	102,222	110,845	51%
2013	36	334,434	310,031	338,355	86%	189,908	174,120	205,898	50%
2014	24	841,484	794,706	853,821	79%	454,217	436,298	503,306	27%
2015	12	1,798,138				2,528,235			
Totals		3,370,010				3,375,371			
AY<CY		1,571,872	1,481,602	1,586,896	86%	847,136	811,568	929,564	30%

The range used for this test can vary based on preferences or testing criteria. For example, the range could include only methods given some weight by accident year (the "weighted range"), the range could include all methods given weight for any accident year (the "method range"), or the range could be expanded to include methods not given any weight or scenario testing (the "possible range").

[10] The "Results – Deterministic" sheet in the "LOB Backtest.xlsm" file illustrates the process of combining weighted estimates of the incremental values consistently with the overall unpaid estimates by accident year.

While the relationship between the actual outcome and the range is a bit more instructive than the back-test of actual to "expected", unfortunately it is still more about direction than significance.

3.2 Stochastic Back-Testing

The only way to test the significance of the deviations from expected is to start with a reserve variability analysis to estimate the distribution of possible outcomes – i.e., instead of simply reviewing whether the outcome is better or worse than expected, the question becomes "is the outcome significantly different than expected?" As with a deterministic back-test, the calculation of expected values will reflect the models employed during the analysis and requires assumption consistency with the methods contributing to the selected unpaid claim estimate. More importantly, in order to dissect the efficacy of the models and assumptions used in a stochastic analysis of unpaid claims, consistency of assumptions for both mean and variance is important. As noted in Section 1.1, using multiple methods to select a point estimate and then using a single "shifted" model approach is quite inconsistent in the sense that the assumptions for the mean and variance are completely different.

Assuming that model and assumption consistency is maintained within a reserve variability analysis, the assessment of the significance or materiality of the resulting differences is a straightforward process using a percentile function. Formula (3.4) uses the Excel PERCENTRANK.INC function, but percentile functions for other software would be similar.[11]

$$P_x[q(w,d)] = \text{PERCENTRANK.INC}\{Dist[\hat{q}(w,d)], q(w,d)\} \tag{3.4}$$

Like for the deterministic back-test, the only area where care needs to be exercised is in the development of the distributions for each incremental value. The output of stochastic models may only include the simulations for the totals by year, but most software will include the simulations of incremental amounts as an output option. Assuming the incremental simulations are available, then the only issue remaining is to insure that the incremental output has been weighted and shifted consistently with the overall model

[11] In Excel, the =PERCENTRANK.INC(*Array,X*) function has two required parameters, *Array*, which is the range of values which can be used to determine relative standing within the range and, *X*, which is the value for which you want to determine the rank. The function returns the rank of *X* within the *Array* as a percentage (0, 1, inclusive) of the range of values.

assumptions.[12]

For the examples used in this paper a reserve variability analysis was completed using four variations of the ODP bootstrap model (i.e., Paid Chain Ladder, Incurred Chain Ladder, Paid Bornhuetter-Ferguson, Incurred Bornhuetter-Ferguson), including weighting and shifting to match the assumptions and unpaid claim estimates for a deterministic analysis using the same methods in order to estimate the expected distribution of possible outcomes. The approach was used for three sample reserving segments and correlated to derive an aggregate distribution in order to illustrate the process for a whole company.[13]

Table 3.3 Back-Test Example: Stochastic Actual vs. Expected

Sample Insurance Company Aggregation of All Segments Stochastic Actual vs. Expected as of December 31, 2015							
AY	Age	Actual Paid	Expected Paid	Percentile	Actual Incurred	Expected Incurred	Percentile
2006	120	3,069	4,077	31.8%	1,863	2,115	49.8%
2007	108	5,905	6,163	47.9%	3,145	1,819	80.6%
2008	96	8,986	10,176	33.6%	3,553	6,026	20.9%
2009	84	18,992	20,033	39.0%	9,872	10,399	46.3%
2010	72	51,003	48,298	71.6%	25,942	25,562	55.3%
2011	60	105,067	104,415	54.3%	52,012	53,101	44.8%
2012	48	202,932	196,083	74.2%	106,624	104,075	61.7%
2013	36	334,434	331,701	57.1%	189,908	185,173	64.0%
2014	24	841,484	839,689	52.8%	454,217	469,822	29.3%
2015	12	1,798,138			2,528,235		
Totals		3,370,010			3,375,371		
AY<CY		1,571,872	1,560,637	61.2%	847,136	858,093	37.6%

Large (small) deviations between actual and expected values are expected when a reserve variability analysis concludes that uncertainty is high (low). The use of an expected distribution of possible outcomes for each accident period and in total (i.e. AY < CY) implies that the use of percentiles automatically adjusts for differences in uncertainty by year or segment as illustrated in Table 3.3.

Note that for simplicity the examples and case study do not include an expected distribution of possible outcomes for most recent accident period (i.e., AY = CY), as this would require modeling that is generally not included in the reserving analysis for the prior period. However, if the reserving analysis is extended to include a distribution of the next

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[12] For a useful reference see Shapland [27]. The "RawSimResults" sheets in the "LOB Backtest.xlsm" file assume that the incremental output by year and by iteration has been weighted and shifted as described in Shapland [27].

[13] While the terms can be used interchangeably, in this paper "consolidation" is used to mean a deterministic sum of the parts or segments whereas "aggregation" is used to mean the stochastic correlation of the parts or segments.

accident year (perhaps in a "pricing risk" calibration) then this could be included with the back-test. The only caveat to the inclusion of pricing risk is that it will be based on expectations of future exposures, so any back-test should first adjust the distribution for the actual exposures prior to calculation of percentiles in order to more properly compare these once future exposures to all the prior years which were based on actual exposures.

Deviations expressed as a percentile provide an indication as to the materiality. Note that deviations expressed as extreme percentiles do not necessarily indicate a problem with the methodology employed during the prior analysis, as observations at the extreme levels of a distribution of possible outcomes should occur.

3.3 Stochastic Key Performance Indicators

Reviewing a single percentile is instructive, but hardly useful. In the greater scheme of determining materiality, the single observation is more about random noise than materiality. Only with a large number of observations can the analyst start to detect material issues by observing patterns or biases in the percentiles. It is in the detection of patterns that the key performance indicators add value to the stochastic analysis. Consider for example Figure 3.1 which graphically displays pre-defined thresholds which are used to define stochastic KPI thresholds.

Figure 3.1 Pre-defined KPI thresholds

0%	**5%**	**25%**	**75%**	**95%**	**100%**

As illustrated in Figure 3.1, the case study in this paper uses thresholds at the 25th and 75th percentile, the 5th and 95th percentile, as well as the simulated minimum and maximum of the distribution of possible outcomes to denote material deviations from expected. Such deviations can be communicated visually using a table of numbers (see Tables 3.3 and 5.10), a chart of individual accident periods (see Figures 3.2a and 3.2b), or a chart of the total calendar year – i.e., all accident years combined (see Figures 3.3a and 3.3b).

Figure 3.2a Paid KPI Thresholds by Accident Year

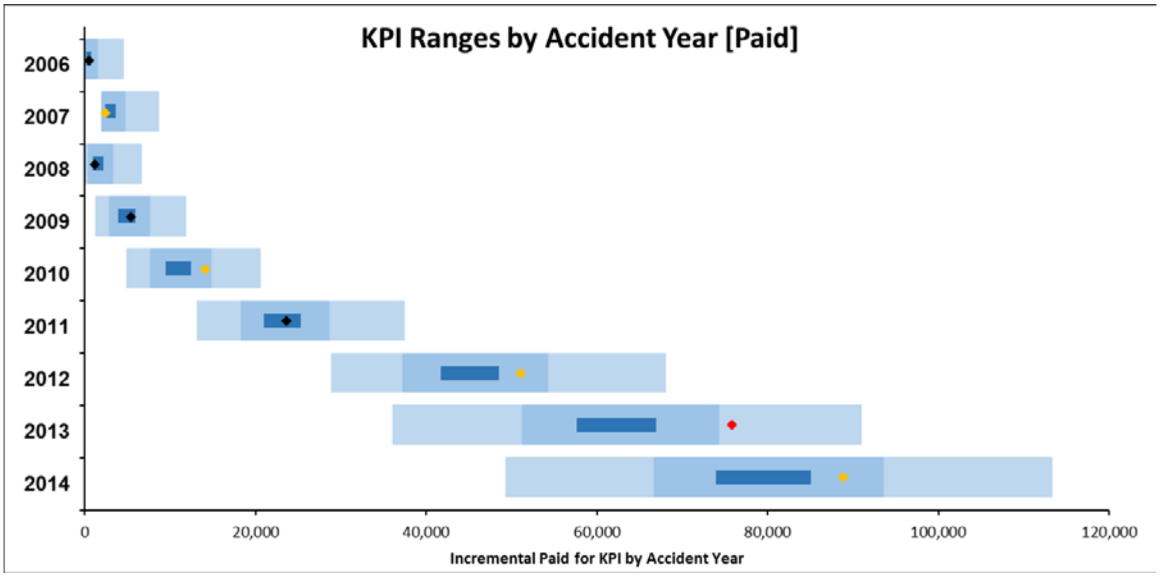
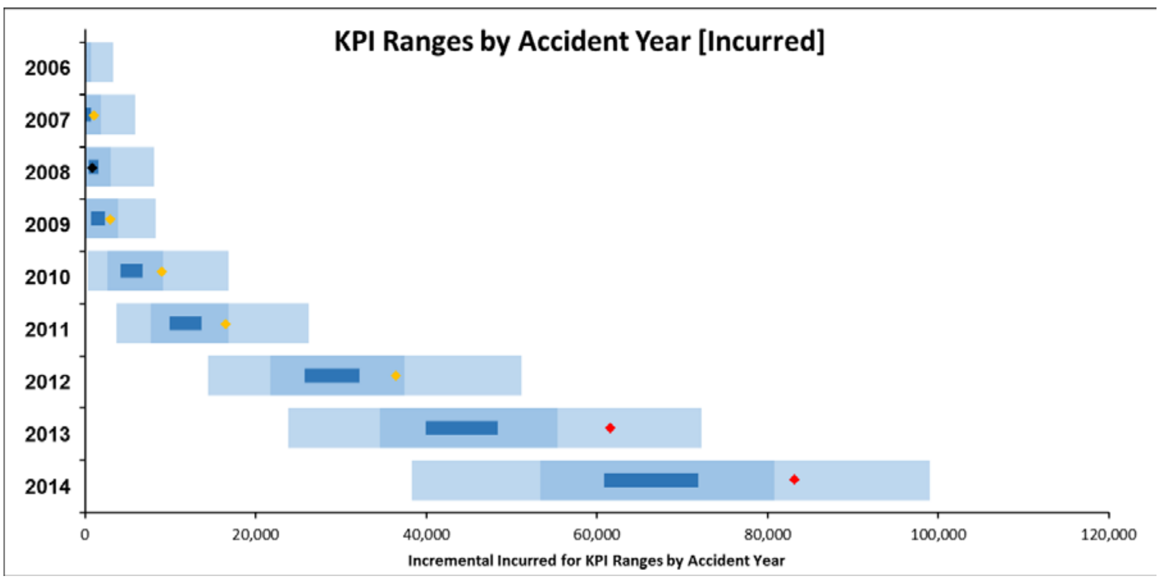


Figure 3.2b Incurred KPI Thresholds by Accident Year



Figures 3.2a and 3.2b show where the actual incremental paid and actual incremental incurred by accident year for a single reserving segment; the black, orange, and red points, fall within the thresholds of the expected distribution of possible outcomes. Note that the blue color coded areas represent the areas defined by the pre-defined thresholds as defined in Figure 3.1.

Figures 3.3a and 3.3b show where the actual incremental paid and actual incremental incurred for the calendar year (i.e., all accident years AY < CY) for a Segment; the orange and red points, fall within the expected distribution of possible outcomes. Again, the blue color coded areas represent the areas defined by the pre-defined thresholds.

Figure 3.3a Calendar Year Paid KPI

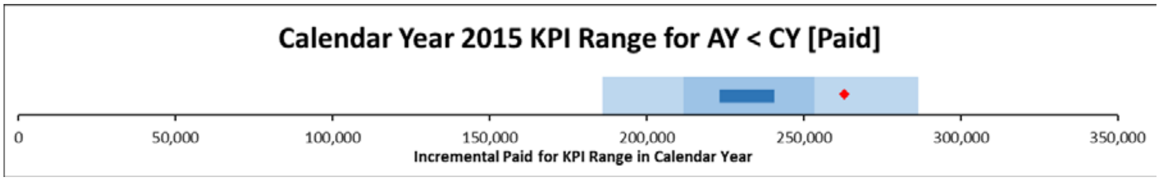
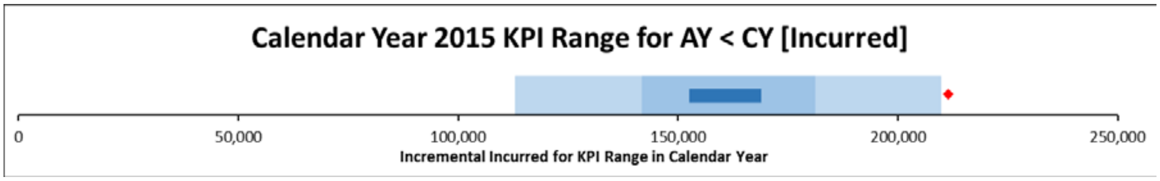


Figure 3.3b Calendar Year Incurred KPI



When using tables or charts, the materiality of the deviation can be better understood by using color coded fonts (see Tables 3.3 and 5.10) or color coded areas representing breaches of pre-defined thresholds (see Figure 3.1) within the distribution of possible outcomes.

There are caveats to this approach such as:

1. Various assumptions (each requiring validation) need to be made in order to produce a distribution of possible outcomes (distributional predictions);
2. The approach tends to work well for high frequency segments on a gross of reinsurance basis but not necessarily for low frequency segments or on a net or ceded basis; and
3. Analysis of industry performance over the past few decades show that some ODP bootstrap model variations, absent adjustment for model weaknesses, may underestimate reserve risk (i.e. the distribution of possible outcomes could be wider).

4. Reserving Within an ERM Framework

There are numerous definitions of ERM. The common themes and principles that emerge from the various definitions, as summarized by the 2016 International Actuarial Association paper [16] "Actuarial Aspects of ERM for Insurance Companies," are:

1. ERM is a continuous process;
2. ERM adopts a holistic view to risk and assesses risk from the perspective of the company's aggregate position as well as from a standalone perspective;

3. ERM is concerned with all risks, including those that are unquantifiable or difficult to quantify;
4. ERM considers uncertainty from both a positive and negative viewpoint;
5. ERM aims to achieve greater value for all stakeholders by assisting in achieving an appropriate risk-reward balance; and
6. ERM considers both the short term and the long term aspects of risk.

Key components of a company's ERM system include risk governance, risk strategy, and the steps that make up the core risk management process consisting of risk identification, risk assessment, risk measurement, risk response, risk monitoring and risk reporting.

Risk governance generally includes the assignment of roles and responsibilities, the establishment of risk policies and procedures, robust internal control systems, and risk culture. For the assignment of roles and responsibilities, many companies adopt a "three lines of defense" model. The first line is responsible for the regular operations of the business. The second line is responsible for overseeing of the operations of the first line. Finally, the third line is responsible for independent review (i.e., audit) and assurance of the operations of the first and second lines.

Once risk has been identified, analyzed and measured then management is faced with responding to the risks. Responses are often characterized as avoiding, accepting, mitigating, or sharing.

The ERM process does not change the way that an actuarial function manages loss reserves and the corresponding reserving risk. Rather, the ERM process formalizes the governance around the process and ensures a consistent and continuous approach. In the case study below, one such approach is described. With or without an ERM process, the actuarial function within an insurance entity is responsible for the reliability and adequacy of the calculation of loss reserves, including:

- Promptly reporting major deviations from expectations such that management has the relevant information necessary for the decision-making process; and
- Investigating the causes of deviations such that changes to the assumptions and methodologies can be suggested in order to improve the central estimate of loss reserves.

The ERM process adds a change control process such that unauthorized changes to the model are restricted and changes are documented.

Risk monitoring is linked to risk measurement and reporting in that the quality of measurement and reporting often determines the extent of monitoring possible. In the case study below, a high quality measurement process which increases the scope of typical monitoring of loss reserves is described, including:

- Clear assignment of risk ownership and establishment of timely automatic reporting mechanisms;
- Consistent, accurate, and auditable controlling of both the deterministic method(s) and methodology supporting the selected central estimate, and the stochastic model(s) supporting the corresponding reserve uncertainty conclusion in the form of an expected distribution of possible outcomes;
- Producing metrics than an actuarial function can use to identify deviations from prior expectations and efficiently allocate analysis resources, prior to commencing with the current analysis;
- Allowing for analysis resources to hypothesize and monitor whether deviations from expectations are the result of mean estimation error, variance estimation error, or random error;
- Producing performance indicators that management can use to anticipate the conclusions of the actuarial analyses, based on how the prior assumptions have held up; and
- Expanding the discussion to interested parties outside of the actuarial function, regarding major deviations from expectations.

Monitoring would normally be done with a frequency that is appropriate to the risk in question. Monitoring should be sufficiently frequent to allow decisions to be made and for action to be taken on an informed basis. In the case study below, a process that uses annual analyses is described, which is typical, but a more frequent basis can be similarly achieved as long as the data and processes are established accordingly.

5. Enterprise Risk Management in Action – A Case Study

With the foundation established, the rest of the paper will illustrate the advantages of integrating reserve variability into the Enterprise Risk Management system by using a case study. Summary tables and graphs for each LOB and the aggregate results are shown in Appendices C, D, E, and F, respectively.

5.1 Introduction

The case study presents the work cycle for an actuarial function within a sophisticated ERM system, including a more robust estimation process for the unpaid claim estimates (i.e., loss reserves) as of 31 December 2015. To set the stage, a general timeline of activity is established before presenting the details.

- Prior to year-end 2015: Levels of back-testing granularity are defined[14] to be Entity Total, Segment Total (where Entity Total = ΣSegment), and AY for each Segment (where Segment Total = Σ AY for each Segment).[15]
- Prior to year-end 2015: Two levels of thresholds are defined,[16] such that observations in the 5% tail areas (i.e., less than the 5th percentile and greater than the 95th percentile) and 25% tail areas initiate action.[17]
- Prior to year-end 2015: Elements included in the automatic back-testing system are defined to include paid loss and incurred loss. ***Other elements, such as reported and closed claim counts, could be included in a live system but they are excluded here for simplicity.***
- Prior to year-end 2015: Enhanced documentation standards[18] of assumptions and expert judgement are established for the analysis and validation of each reserving segment.[19]

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[14] Note that changes in the segmentation, and the ramifications to the ERM system, need to be thoroughly addressed prior to the year-end.

[15] Note that it is often more practical to exclude special Segments and very mature AYs, such that "Entity Total = ΣSegment + excluded segments" and "Segment Total = Σ AY for each Segment + prior AYs".

[16] Note that thresholds could be nominal (e.g., differences larger than $1 million), relative (e.g., differences 150% larger than the mean expected), or distributional (e.g., observations above the 95th percentile of possible future outcomes).

[17] Note that the identification of a threshold breach does not imply that an error in the prior calculation has been identified. Rather, a breach brings attention to large deviations such that the assumptions and methodology underlying the expectation can be reviewed.

[18] Note that enhanced documentation includes a list of relevant and material assumptions for each segment, the results of sensitivity testing material assumptions, segment specific diagnostics with qualitative descriptions supporting the conclusions, and justification (if available) for material expert judgement exercised.

[19] Note that enhanced documentation together with the automated back-testing ensures that a change in employee personnel does not unnecessarily render the historical assumption set and rationale less transparent or understandable (i.e., the institutional memory stays intact.)

- 4 January 2016: The accounting function closes the books such that all data elements as of the 31 December 2015 valuation date are available on an AY and CY basis.

- 5 January 2016: Granular results of automated back-testing of the current CY (i.e., CY 2015) and deviations[20] from the predictions for CY 2015 (based on the loss reserve analysis as of 31 December 2014) are available.

  - Previously identified segments (or previously identified data elements from a segment) are included in the automated back-testing procedure where a robust validation of the CY 2015 accruals can be achieved.

  - AY 2014 and prior incremental accruals (i.e., AY < CY) are compared to the expectations as of 31 December 2014, based on the final distribution of possible outcomes estimated by the actuarial function in the prior reserving analysis. ***The process can be expanded to include specific models, but that is not done here only for simplicity***.

  - AY 2015 incremental accruals (i.e., AY = CY) can be compared to the expectations for losses related to the unearned premium as of 31 December 2014, with adjustment for actual new business written during 2015. ***For simplicity, these amounts are not included in the details of the case study shown below, although it should be noted that deviations from expectations can be described as a mixture of reserve risk and premium risk.***

- 5 January 2016: The actuarial function determines an efficient allocation of analysis resources so that segments and/or AYs which exhibit a large number of significant deviations receive additional attention.

- 5 January 2016: Breaches in the 25% tail areas initiate additional hindsight analysis including hypothesis testing as to whether the breach could have been caused by an assumption error in either the deterministic or stochastic analysis, a systematic effect (e.g., an explainable change in the internal or external environment), or random variation.

- 5 January 2016: Breaches in the 5% tail areas initiate an alert system intended to collect relevant information from other departments (e.g., data quality, underwriting, claims, and reinsurance).

[20] The automated back-test identifies areas where the deviations from predictions breach a pre-defined threshold (for multiple levels of granularity and for multiple data elements.)

- 5 January 2016: Conditional reserve estimates using the 1-year time horizon analysis as of 31 December 2014 are available to management as an early indication of the reserve changes that will occur for the 31 December 2015 evaluation. (See Appendix A for an overview of the one-year time horizon.)
- 5 January 2016: Armed with a view of how each segment performed during CY 2015, relative to the expectations inherent in the actuarial methodology as of 31 December 2014, the actuarial function can commence with its valuation analysis as of 31 December 2015.
- 5-26 January 2016: During the analysis, diagnostics and statistical tools are used to review assumptions and calibrate the parameters of each of the methods and models which comprise the segment's methodology. Such diagnostics and tests are retained in a log so that they can be referenced in the actuarial report. Also interaction with interested parties outside of the actuarial function provide a critical sounding board for expert judgement exercised.
- 27 January 2016: At the conclusion of the analysis a recommendation for the loss reserve is sent to management, taking the form of an actuarial function report.
- 10 February 2016: After the dust settles, the expectations for CY 2016 are compiled by the actuarial function, based on the expectations inherent in the analysis as of 31 December 2015. Further analyses of change are completed and documented. Suggestions for the enhancement of the robust estimation process for CY 2016 (levels of granularity, thresholds, data elements, diagnostic retention and other enhanced documentation) are considered, based on the performance and the collective findings of the analysis.

5.2 Basis of Underlying Data

In producing this case study real industry data was used.[21] To ensure confidentiality, triangular data for 10 accident years was aggregated from a small number of insurance entities writing Commercial Auto ("CA"), Private Passenger Auto ("PPA"), and Homeowners ("HO"), as of consecutive year-ends. This produced a data set for a fictitious entity.

By performing a deterministic and stochastic analysis on the annual data for this fictitious

[21] The data comes from historical Schedule P triangles, as compiled by SNL Financial.

entity, an exercise which is often undertaken by actuarial departments every year-end, the case study attempts to highlight the wealth of information that is ripe for integration within an ERM framework to enhance the understanding of the underlying dynamics, including the production of KPIs for reserving risk.

The deterministic analysis was limited to four methods, namely: the paid and incurred chain ladder ("Pd CL" and "Inc CL") methods and the paid and incurred Bornhuetter-Ferguson ("Pd BF" and "Inc BF") methods. The selected ultimate loss estimates for each accident year are a weighted average of the four methods. To maximize assumption consistency, four ODP bootstrap models consistent with each of the deterministic methods were used. The selected distribution of possible outcomes for each accident year are a weighted average of the four ODP bootstrap models (using the same weights as for the deterministic methods),[22] shifted such that the mean of the distribution for each accident year is equal to the selected unpaid loss.

It is reasonable to expect that the underlying data within the fictitious entity would be available by the first Monday of the year (4 January 2016) and that the generous management of the fictitious entity allows the actuarial department to spend three weeks in completing its work. Within such tight schedules, the importance of activity before the year-end is emphasized, which calibrates the framework such that diagnostics and KPIs are produced as soon as the underlying data is available.

In the case study, the diagnostics and KPIs focus on the performance of the most recent period (i.e., the past CY). The framework and approach can just as easily focus on multiple periods, which for some reserving segments would be appropriate. The multiple period approach provides insight that could be used to reduce unnecessary adjustments in the underlying actuarial assumptions (i.e., additional volatility caused by overreaction to single period observations).

5.3 Validation of the Prior Analysis

As noted above, enhanced documentation standards of assumptions and expert judgement are established for the analysis and validation of each reserving segment. A non-

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[22] Note that weighting distributions together requires that possible outcomes mean the same thing in each model. For example, the unadjusted output for an ODP bootstrap model applied to a paid (an incurred) loss triangle would result in a distribution of possible unpaid loss (IBNR) outcomes. Prior to weighting, the incurred ODP bootstrap models implemented were adjusted such that the outputs were distributions of possible unpaid loss outcomes as described in Shapland [27].

exhaustive list of assumptions that require validation and examples of enhanced documentation could include the following:

5.3.1 Selected Loss Development Factors ("LDFs")

The Mack [18] paper introduced three assumptions which underlie the chain ladder method, the first two of which are validated as part of the enhanced documentation for the fictitious entity.

$$E[c(w,d+1) \mid c(w,1),...,c(w,d)] = c(w,d) \times F(d) \tag{5.1}$$

$$\{c(i,1),...,c(i,n)\} \ \& \ \{c(j,1),...,c(j,n)\} \text{ are independent for } i \neq j \tag{5.2}$$

$$Var[c(w,d+1) \mid c(w,1),...,c(w,d)] = c(w,d) \times \sigma_d^2 \tag{5.3}$$

Assumption (5.1) says that the all year loss weighted average ("AYLWA") multiplied by the value in the last diagonal is equivalent to the expected value of the next diagonal given the observations to date. The validation test for this assumption (shown in Figures 5.1 and 5.2) compares the LDF which is a regression through the origin (red line) relative to an alternative approach that uses an intercept term (green line).[23] If the regression with an intercept is not significantly different than the regression through the origin, then the LDF is validated.

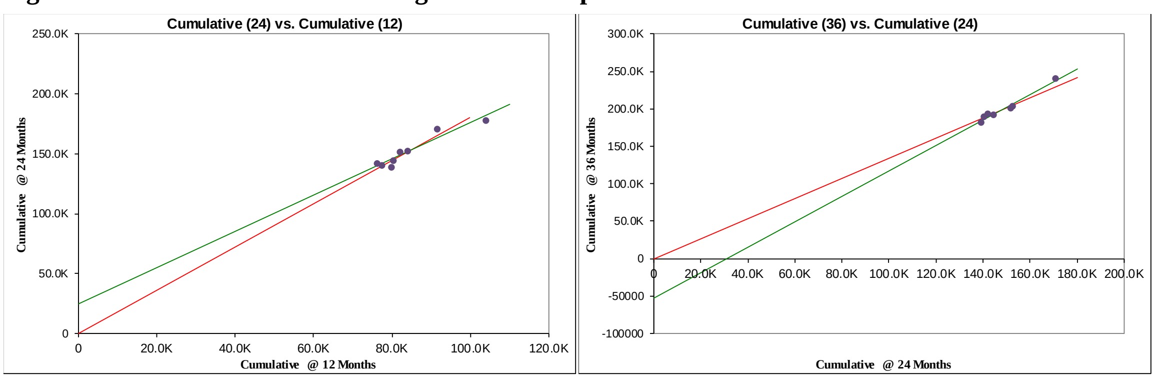
[23] A more complete exposition of tests which can be used to validate the three Mack assumptions are provided in Venter [29]. The graphs in Figures 5.1, 5.2, 5.3 and 5.4 were created using the "Bootstrap Models.xlsm" companion Excel file for Shapland [27].

Table 5.1 Commercial Auto: Chain Ladder Methods

Sample Insurance Company Commercial Auto -- Paid Data Chain Ladder Development as of December 31, 2014										
AY	12	24	36	48	60	72	84	96	108	120
2006	77,401	140,425	189,316	223,326	243,182	250,182	254,305	256,672	257,689	
2007	76,085	142,122	193,196	224,406	246,220	257,226	263,698	264,871		
2008	79,850	139,041	181,905	209,366	228,012	237,792	240,300			
2009	80,323	144,482	192,134	227,723	249,165	259,339				
2010	83,919	152,487	203,761	245,150	270,525					
2011	82,001	151,768	201,189	245,541						
2012	91,514	170,696	240,652							
2013	103,957	177,709								
2014	105,547									
	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132
ATA	1.805	1.347	1.184	1.095	1.039	1.018	1.007	1.004	1.002	1.002
CDF	3.385	1.875	1.392	1.176	1.074	1.033	1.015	1.008	1.004	1.002
Unpaid	0.705	0.467	0.282	0.149	0.069	0.032	0.015	0.008	0.004	0.002

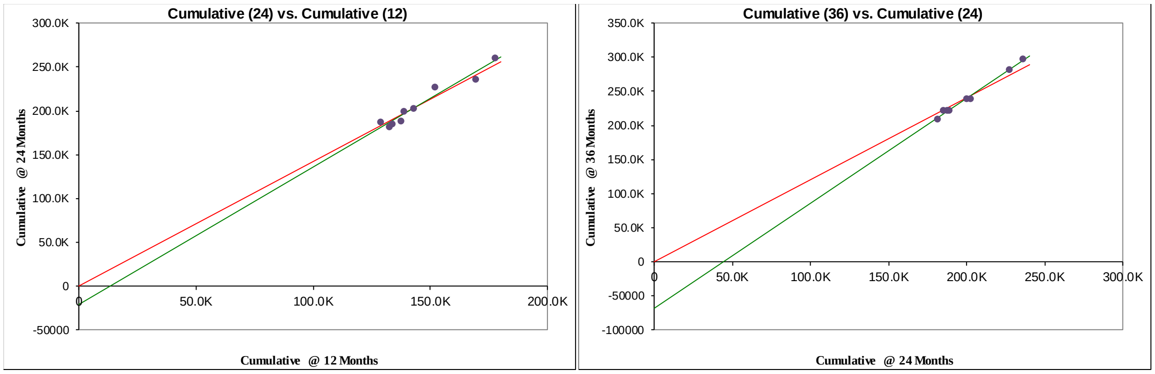
Sample Insurance Company Commercial Auto -- Incurred Data Chain Ladder Development as of December 31, 2014										
AY	12	24	36	48	60	72	84	96	108	120
2006	133,521	185,161	221,635	241,420	251,646	255,508	256,596	258,041	258,524	
2007	128,727	187,403	222,093	247,345	258,712	265,636	269,558	270,758		
2008	132,567	181,263	209,262	226,237	236,863	241,107	242,171			
2009	137,295	188,962	222,624	247,335	258,856	265,496				
2010	142,862	202,363	239,239	269,940	281,376					
2011	138,650	199,791	239,719	266,101						
2012	151,778	227,353	282,394							
2013	169,171	235,983								
2014	177,611									
	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132
ATA	1.418	1.193	1.106	1.045	1.022	1.008	1.005	1.002	1.001	1.001
CDF	2.029	1.431	1.200	1.085	1.038	1.016	1.008	1.003	1.001	1.001
Unrptd	0.507	0.301	0.166	0.078	0.037	0.016	0.008	0.003	0.001	0.001

Figure 5.1 Commercial Auto: Testing the first two paid LDFs



Corr. = 0.952 P-Value = 0.000 Int. P-Value = 0.045 Corr. = 0.985 P-Value = 0.000 Int. P-Value = 0.004

Figure 5.2 Commercial Auto: Testing the first two incurred LDFs



Corr. = 0.974 **P-Value = 0.000** **Int. P-Value = 0.005** **Corr. = 0.996** **P-Value = 0.000** **Int. P-Value = 0.000**

For the fictitious entity, the LDFs were validated, so the CL methods using the AYLWA are reasonable. Note that each ODP bootstrap model is 100% consistent with using the AYLWA for the deterministic method, so none of the residuals were removed (i.e., no outliers were selected in the calibration of the ODP bootstrap models). The a priori loss ratios and tail factors used in the ODP bootstrap models were also consistent, except that variance assumptions were also added.

Note that the implementation of a "picker approach" (to reflect observable trends) in selecting LDFs would necessitate additional validation of each "pick" and consideration of consistent treatment of the residuals in the calibration of the ODP bootstrap model, but that was not done in the case study in keeping with the theme of simplicity.

5.3.2 Accident Year Independence

Regarding assumption (5.2), the independence of the accident years can be validated using a table of the individual LDFs and color coding the LDFs which are smaller (green shading) or larger (red shading) than the median LDF for each development period, as illustrated in Figure 5.3. This color coding aids in searching for patterns in the LDFs which could indicate that they are not independent. For example, the independence assumption could be violated if there were a strong diagonal trend, or clustering, of one of the colors.

Figure 5.3 Commercial Auto: Testing independence of accident years

Test of the Independence Between Accident Years (Paid)

AY	12	24	36	48	60	72	84	96	CY Small	CY Large
2006	1.81	1.35	1.18	1.09	1.03	1.02	1.01	1.00	1	0
2007	1.87	1.36	1.16	1.10	1.04	1.03	1.00		0	2
2008	1.74	1.31	1.15	1.09	1.04	1.01			2	1
2009	1.80	1.33	1.19	1.09	1.04				4	0
2010	1.82	1.34	1.20	1.10					3	2
2011	1.85	1.33	1.22						1	3
2012	1.87	1.41							1	5
2013	1.71								4	3
Median	1.82	1.34	1.18	1.09	1.04	1.02	1.01	1.00		

Test of the Independence Between Accident Years (Incurred)

AY	12	24	36	48	60	72	84	96	CY Small	CY Large
2006	1.39	1.20	1.09	1.04	1.02	1.00	1.01	1.00	1	0
2007	1.46	1.19	1.11	1.05	1.03	1.01	1.00		0	2
2008	1.37	1.15	1.08	1.05	1.02	1.00			2	0
2009	1.38	1.18	1.11	1.05	1.03				3	1
2010	1.42	1.18	1.13	1.04					3	1
2011	1.44	1.20	1.11						2	4
2012	1.50	1.24							1	6
2013	1.39								4	2
Median	1.41	1.19	1.11	1.05	1.02	1.00	1.01	1.00		

In practice, the independence of the accident years can be distorted by certain calendar year effects like major changes in the claims handling process or in case reserve strengthening.

5.3.3 A Priori BF Loss Ratios ("IELR")

In the case study, the a priori or initial expected loss ratios ("IELR") used in the BF methods were based on published figures (i.e., selected ultimate loss amounts from Schedule P), expressed as a percentage of premium. IELRs are an important assumption and an example of expert judgement which requires additional validation.

Table 5.2 Commercial Auto: IELRs

Sample Insurance Company Commercial Auto				
AY	Paid CL ULR	Inc CL ULR	Management IELR	Selected ULR
2006	73.2%	73.2%	73.3%	73.2%
2007	76.0%	77.3%	77.4%	76.7%
2008	64.5%	64.5%	64.6%	64.5%
2009	62.8%	63.2%	63.2%	63.0%
2010	60.4%	60.7%	60.8%	60.6%
2011	53.2%	53.2%	53.4%	53.2%
2012	57.9%	58.5%	58.5%	58.2%
2013	54.5%	55.3%	54.7%	54.9%
2014	57.3%	57.7%	52.9%	54.7%

Validation, in this case, would likely take the form of sensitivity testing the important assumptions underlying the IELR. The common sources of expert judgement in this case would be renewal studies performed by the underwriting department and actuarial analyses summarizing average premium levels achieved relative to the expected premium level.

5.3.4 Weighting Scheme

No single method is perfect. For this reason, it has become best practice for actuaries estimating an insurer's unpaid claim estimate to review and assess the merits of multiple methods for each reserving segment in the actuarial analysis.

Traditional unpaid claim projection methods are generally based on averages that produce an indication of the unpaid claims reserves or a "reasonable estimate" for each accident period and in total. The results of these methods, being based on different data and assumptions, give different answers. For example, chain ladder approaches applied to aggregate paid losses and aggregate incurred losses will produce different estimates of ultimate losses for each accident period and in total.

Expert judgement supported by tangential information (e.g., expected loss ratios, severities, and frequencies from underwriting and claims experts) can be helpful in the reconciliation of the results from various methods. The reconciliation of the method results is a process where an actuary investigates and rationalizes large differences at a granular level (i.e., by reserving segment and accident period) in the results from multiple methods.

Although the reconciliation process is generally a source of significant insight, a common outcome is that a subset of implemented methods each produce different but reasonable outcomes for a given accident period. In this case, the actuary often chooses to credibility weight the results of the methods which have produced reasonable results, rather than selecting a single method for that accident period.

Estimates for immature accident periods benefit from expert judgement supported by tangential information. For these accident periods, payments are few and case reserves are based on incomplete information, which means that chain ladder methods can be easily distorted by the behavior of a few claims. As accident periods mature, the actuary tends to rely more on period-specific information as found in chain ladder methods. This is because settlement amounts are known for closed claims and future payments for open claims become more predictable as more claim specific information is collected (e.g., loss survey, repair estimates, details of injury).

Table 5.3 Commercial Auto: Weighting scheme

Sample Insurance Company
Commercial Auto
Calculation of Weighted Ultimate as of December 31, 2014

		Ultimate Values by Method				Weights by Method				Weighted
AY	Age	Paid CL	Inc CL	Paid BF	Inc BF	Paid CL	Inc CL	Paid BF	Inc BF	Ultimate
2006	108	258,835	258,835	258,837	258,836	50.0%	50.0%	0.0%	0.0%	258,835
2007	96	267,103	271,591	267,143	271,592	50.0%	50.0%	0.0%	0.0%	269,347
2008	84	243,981	244,137	243,991	244,141	50.0%	50.0%	0.0%	0.0%	244,059
2009	72	267,942	269,784	267,999	269,783	50.0%	50.0%	0.0%	0.0%	268,863
2010	60	290,475	292,079	290,608	292,092	50.0%	50.0%	0.0%	0.0%	291,277
2011	48	288,645	288,592	288,785	288,669	50.0%	50.0%	0.0%	0.0%	288,618
2012	36	335,023	338,775	335,956	338,702	25.0%	25.0%	25.0%	25.0%	337,114
2013	24	333,220	337,698	333,662	336,635	0.0%	0.0%	50.0%	50.0%	335,149
2014	12	357,305	360,286	338,097	344,953	0.0%	0.0%	50.0%	50.0%	341,525
Totals		2,642,529	2,661,779	2,625,078	2,645,402					2,634,788

As illustrated in Table 5.3, the selection of a weighting scheme is an example of exercising expert judgement, which should be adequately documented, including: the inputs on which the judgement is based; the objectives and decision criteria; the materiality of the expert judgement made; any material limitations and the steps taken to mitigate the effect of these limitations; and the validation carried out for the expert judgement. Other selections based on expert judgment should also be adequately documented.

Article 77 of the Solvency II FD states that the "value of technical provisions shall be equal to the sum of a best estimate and a risk margin." Ignoring discounting and the risk margin for the purposes of this case study, the best estimate is further defined to correspond to the "probability weighted average of future cash flows."[24] Note that Article 122.2 of the

[24] A strong interpretation of the required correspondence to a probability weighted average of future cash flows is that a "distribution of possible outcomes" needs to be modelled. Note that deriving such a distribution of possible outcomes may not be possible using even the most sophisticated actuarial techniques available. The best attempt at such, however, would require the consideration of multiple (deterministic) methods and multiple (stochastic) models in order to calibrate a distribution of possible outcomes. In addition, such a distribution would require consideration of systemic risks that may not have been adequately modelled otherwise. A weaker interpretation of the required correspondence to a probability weighted average of future cash flows is that each actuarial method produces future cash

FD ensures that models "used to calculate the probability distribution forecast shall… be consistent with the methods used to calculate technical provisions." Consistency would include elements of expert judgement exercised by the actuary during the calculation of technical provisions, including the use of shorter term average development factors, adjustment for trends, etc.

5.3.4 Other Manual Adjustments

It can happen that adjustments to the ultimate loss estimate are implemented based on (i.e., after) the weighting of multiple methods or models. In the case study, the weighting of paid and incurred chain ladder methods for accident year 2007 results in an IBNR value less than 0 for Commercial Auto. Such a scenario implies that the case reserve may be redundant. The suggested course of action is to interact directly with the claims team, if possible, to determine the likelihood of this conclusion. For purposes of the case study, a small IBNR has been added and the consequences of this decision is included in the expected values of the subsequent year's back-test as illustrated in Table 5.4. Throughout the tables in the "LOB Backtest.xlsm" file, deviations from the weighted results are highlighted in green.

Table 5.4 Commercial Auto: Manual Adjustment of Accident Year 2007

Sample Insurance Company
Commercial Auto
Total Unpaid Reconciliation as of December 31, 2014

AY	Age	Paid to Date	Incurred to Date	Weighted Ultimate	Case Reserve	IBNR	Total Unpaid	Selected Ultimate	Selected IBNR	Total Unpaid
2006	108	257,689	258,524	258,835	835	311	1,146	258,835	311	1,146
2007	96	264,871	270,758	269,347	5,887	(1,411)	4,476	271,500	742	6,629
2008	84	240,300	242,171	244,059	1,871	1,888	3,759	244,059	1,888	3,759
2009	72	259,339	265,496	268,863	6,157	3,367	9,524	268,863	3,367	9,524
2010	60	270,525	281,376	291,277	10,851	9,901	20,752	291,277	9,901	20,752
2011	48	245,541	266,101	288,618	20,560	22,517	43,077	288,618	22,517	43,077
2012	36	240,652	282,394	337,114	41,742	54,720	96,462	337,114	54,720	96,462
2013	24	177,709	235,983	335,149	58,274	99,166	157,440	335,149	99,166	157,440
2014	12	105,547	177,611	341,525	72,064	163,914	235,978	341,525	163,914	235,978
Totals		2,062,173	2,280,414	2,634,788	218,241	354,374	572,615	2,636,941	356,527	574,768

5.3.5 Coefficient of Variation of the IELR

In the case study, the uncertainty in the IELR is required as an input to the ODP bootstrap for the BF models and was calibrated to follow a lognormal distribution with a Coefficient of Variation ("CoV") of 8%. The purpose of this assumption is to include uncertainty in the IELR by simulating from a lognormal distribution a different IELR for each iteration.

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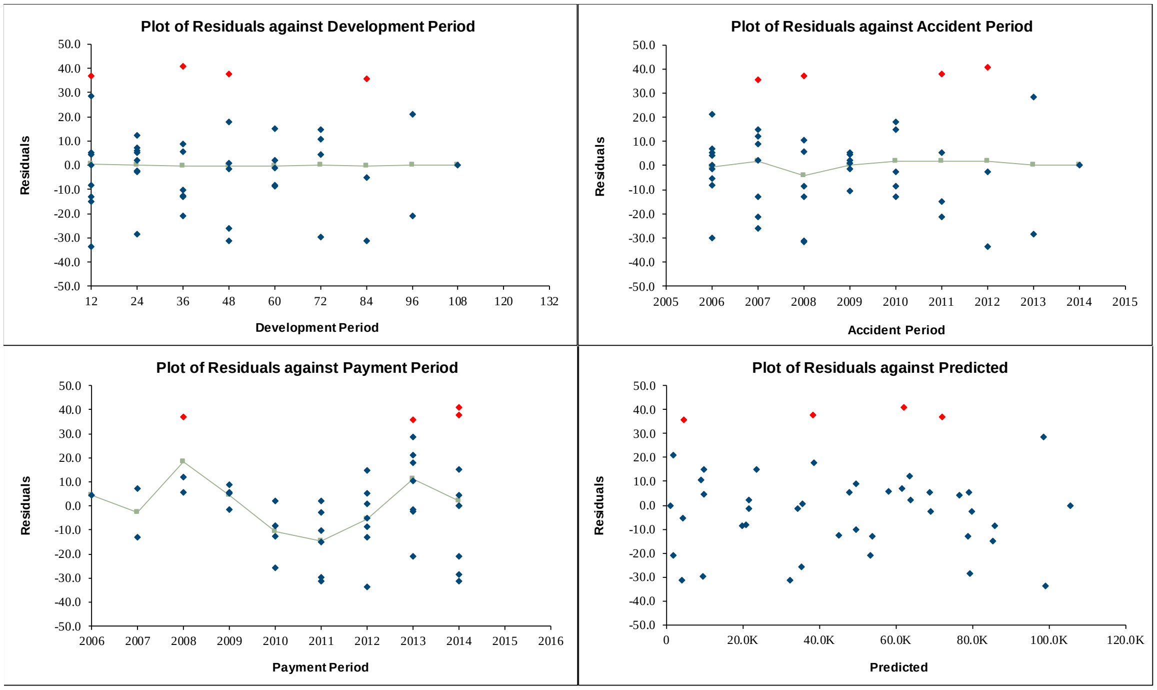
flows unique to the assumptions underlying the respective method as applied to an accident period and reserving segment. These competing cash flow projections are weighted together based on the subjective credibility assigned to each accident period of each method.

5.3.6 Heteroscedasticity

An analysis of residuals by itself is an example of a validation technique. For the case study, the residuals are analyzed to identify trends or other features in the data that may not be completely modeled by the chain ladder approach.

Figure 5.4 Commercial Auto: Plots of Residuals (Paid)



Particularly important are the identification of heteroscedasticity and outliers. In the ODP bootstrap model,[25] residuals are resampled with replacement – that is, they are taken from any location in the residual triangle, and placed in another random location to form the sample triangle. Therefore, the residuals should all be independent, identically distributed random numbers (i.e., homoscedastic). Heteroscedasticity occurs when the residuals are not identically distributed. By looking at the variability of the residuals by period (e.g., by accident year) you can visually inspect them to make sure the variability is consistent between periods. If they are not consistent, this is an indication that heteroscedasticity is present in the residuals and additional parameters may be needed to adjust for the different variances by period.[26]

The adjustment for heteroscedasticity is typically made by focusing on the Plot of

[25] The typical ODP bootstrap model is semi-parametric, but conditions could exist for the implementation of a fully parametric ODP bootstrap, which allows for the sampling of residuals from a distribution (a more robust solution).

[26] For a more complete discussion, see Shapland [27] section 4.6 and section 5.

Residuals against Development Period (see Figure 5.4) and identifying columns with similar dispersion of residuals. While it is tempting to add hetero groupings to force additional consistency of the residuals (e.g., at 60 months where the dispersion appears low), this will be done at the expense of adding more parameters to an already highly parameterized model. This is not to say that trying other hetero groups is never justified, just that the ODP bootstrap already has one parameter for every development period and one parameter for every accident period (minus one), so adding parameters for heteroscedasticity must be decided carefully.

5.3.7 Process Variance adjustment to the ODP Bootstrap

One of the last steps in the ODP bootstrap is the use of a distributional assumption in order to add process variance to the simulated future incremental values. Without this step the projected incremental values would be point estimates rather than possible outcomes. In the case study, the Gamma distribution was used as this is the most common choice. The Normal or Lognormal distributions are possible alternative distributions which could be tested to see if they produce material differences in results, but that is outside the scope of the case study.

5.3.8 Correlation Between Segments

Thus far the list of assumptions which could be tested has been focused at the segment or model level. As the case study is intended to replicate a complete ERM system, correlation to derive an aggregate distribution is also included.

In general, the aggregate distribution of unpaid claims can be materially narrower than the sum of the individual distributions, after considering correlation between the segments. This difference between the correlated aggregate and the sum of the segments would not be as material in cases where the segments are all strongly positively correlated, where there is little variability in the individual distributions, or where one segment is far larger than the rest.

For the case study, correlation was measured using a pairwise approach.[27] A more robust solution, e.g., a maximum likelihood estimation ("MLE") copula, could be used to solve for all correlations at once since it is done analyzing all of the data at once. However, the MLE copula approach can be less than ideal when data is excluded or missing for one or more

[27] The pairwise approach is used in the "Aggregation.xlsm" companion file for the Shapland [27] paper, which was used to create Tables 5.5 and 5.6.

segments.[28,29] The measurement of correlation could be done using paid residuals and/or incurred residuals, both before and after heteroscedasticity adjustments. The resulting correlation matrices for paid loss residuals before heteroscedasticity are shown in Table 5.5.

Table 5.5 Pairwise Rank Correlation of Residuals and P-values– Paid Loss

Rank Correlation of Residuals prior to Hetero Adjustment - Paid

	PPA	**CA**	**HO**
PPA	1.000	0.276	-0.142
CA	0.276	1.000	0.027
HO	-0.142	0.027	1.000

P-Values of Rank Correlation of Residuals prior to Hetero Adjustment - Paid

	PPA	**CA**	**HO**
PPA	0.000	0.066	0.352
CA	0.066	0.000	0.860
HO	0.352	0.860	0.000

In order to aggregate distributions of possible outcomes for the entity, one needs to evaluate the inherent correlation by segment. For this, the p-values can be reviewed to assess the significance of the correlation between each pair of segments. In this test, the smaller the p-value the more significant the calculated correlation and a larger p-value (e.g., greater than 0.05 is a typical threshold) indicates that the correlation is not significantly different than zero. Therefore, the p-values of 0.352 (HO x PPA) and 0.860 (HO x CA) imply that the measured correlation is not significantly different from zero, while the p-value of 0.066 implies that the measured correlation is close to the true correlation. The selected correlation in Table 5.6 reflects the consideration of the p-values.

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[28] For example, if you are only using two year average age-to-age ratios for one segment, then only the data for the last three diagonals can be used in the estimation process. The maximum likelihood copula only uses data points that are common for every segment, so it is possible to have a problematic situation where there are no common data points for all segments.

[29] It is important to note any adjustments to the ODP bootstrap model (i.e., anything less than the AYLWA for the link ratios or exclusion of outliers) will result in some of the residuals (that would otherwise be included) being excluded from the correlation matrix calculations.

Table 5.6 Selected Correlation Matrix

Assumed Correlation Matrix	PPA	CA	HO
PPA	1.000	0.276	0.000
CA	0.276	1.000	0.000
HO	0.000	0.000	1.000

The validation of correlation assumptions is a challenge. Monitoring both the measured rank correlation and corresponding p-values over time can provide some insight as to the stability of the correlation assumptions. Even so, the selected correlation assumption may also consider the impact of issues not in the measured coefficients, such as contagion or lack of prior catastrophe losses.

5.4 Implied Expected Values from Multiple Methods

Future expected incremental values (i.e., paid loss, reported claims, etc.) could be produced in a number of ways. For example, they could be independently calculated based on an independent analysis or they could be calculated based on consecutive differences of cumulative estimates which result from a curve fit. Although such practice is common, a continuous ERM process intends to improve the models and methods employed in the estimation process. Therefore, the approach used here is to estimate the future incremental values that arise from the methods (and models) which have received weight and any subsequent adjustments. The idea is that deviations can be traced back to the underlying deterministic calculations, for which validated assumptions with enhanced documentation is available and subsequent adjustments, for which documentation of decision points are available.

One challenge that immediately arises from this approach is that expected future incremental paid (and incurred) loss values must be gleaned from the expectations inherent in incurred (and paid) methods. In the extreme case where the incurred chain ladder method receives 100% of the weight for all accident years, expected incremental paid losses still need to be produced even though no paid method received weight. In order to address this challenge, the collection of methods as a whole is considered in order to rely on analogous paid methods. Continuing the example from the case study (see above for LDF validation and weighting scheme), the formulas (5.4) to (5.7) are used to derive expected cumulative amounts, for a particular method, from which incremental amounts follow.[30]

$$E[\hat{c}_P(w,d)]_{P-Method} = E[\hat{c}_P(w,d-1)]_{P-Method} \times F(d-1)_{P-Method} \tag{5.4}$$

$$E[\hat{c}_P(w,d)]_{I-Method} = E[\hat{c}_P(w,d)]_{P-Method} \times \frac{U(w)_{I-Method}}{U(w)_{P-Method}} \tag{5.5}$$

$$E[\hat{c}_I(w,d)]_{I-Method} = E[\hat{c}_I(w,d-1)]_{I-Method} \times F(d-1)_{I-Method} \tag{5.6}$$

$$E[\hat{c}_I(w,d)]_{P-Method} = E[\hat{c}_I(w,d)]_{I-Method} \times \frac{U(w)_{P-Method}}{U(w)_{I-Method}} \tag{5.7}$$

Note that a consequence of this approach is that any IBNR adjustment made subsequent to the weighting of methods will have an impact on both expected paid and incurred amounts. With cumulative paid and incurred amounts by development period so derived for each method, the weighting scheme can be applied to determine the weighted cumulative paid and incurred amounts, from which the incremental amounts can be derived. Examples of the next diagonal of incremental values (i.e., for Calendar Year 2015 during the year end 2014 analysis) are shown in Tables 5.7 and 5.8.

Table 5.7 Commercial Auto: Implied Expected Paid Losses

Sample Insurance Company Commercial Auto Expected Paid Losses during CY 2015						
AY	Paid CL	Inc CL	Paid BF	Inc BF	Weighted	Selected
2006	572	572	573	572	572	572
2007	1,049	5,518	1,068	5,497	3,284	**4,863**
2008	1,642	1,797	1,647	1,796	1,720	1,720
2009	4,560	6,375	4,590	6,348	5,468	5,468
2010	10,624	12,177	10,695	12,130	11,401	11,401
2011	23,280	23,230	23,355	23,247	23,255	23,255
2012	44,341	47,533	44,779	47,112	45,941	45,941
2013	61,648	64,865	61,823	63,957	62,890	62,890
2014	85,007	86,597	78,521	82,254	80,388	80,388
AY<CY	232,723	248,663	227,052	242,913	234,917	**236,497**

[30] Formulas (5.4) and (5.6) may seem redundant in the sense that the expected incremental development for the paid and incurred methods, respectively, are derived directly from the method itself. The formulas are included for completeness of exposition and as a link to the calculations in the "LOB Backtest.xlsm" file.

Table 5.8 Commercial Auto: Implied Expected Incurred Losses

Sample Insurance Company Commercial Auto Expected Incurred Losses during CY 2015						
AY	Paid CL	Inc CL	Paid BF	Inc BF	Weighted	Selected
2006	155	155	157	156	155	155
2007	(3,976)	507	(3,937)	507	(1,735)	**912**
2008	1,062	1,217	1,070	1,220	1,140	1,140
2009	288	2,116	345	2,115	1,202	1,202
2010	4,482	6,061	4,608	6,067	5,271	5,271
2011	11,967	11,915	12,068	11,956	11,941	11,941
2012	26,520	29,980	27,409	29,941	28,462	28,462
2013	41,780	45,513	42,556	45,037	43,797	43,797
2014	72,073	74,156	63,052	67,932	65,492	65,492
AY<CY	154,351	171,620	147,327	164,931	155,725	**158,372**

5.5 Advantages of Using the ODP Bootstrap

In the case study, the ODP bootstrap approach is relied on to model uncertainty. A main advantage of this approach is that the assumption set in the uncertainty calibration is largely consistent with the assumption set in the point estimate calibration, while areas of inconsistency (or adjustment) are identified, documented, and (to the extent possible) validated for reasonableness. Of course the uncertainty calibration required additional assumptions to be made, each of which required documentation and validation.[31]

Alternatively, the Mack [18] method could be used for the uncertainty calibration, but in doing so a number of additional challenges arise, only some of which can be overcome.

1. The variance assumptions in the Mack method would be largely inconsistent with the assumptions used to calibrate a point estimate. Recall that the selected weights imply a full rejection of the chain ladder methods for the most recent accident years.

2. The Mack method produces a variance estimate for each accident year and in total, but a distribution needs to be postulated in order to translate this variance estimate into a distribution of outcomes. The likelihood is low that such a distribution includes all possible outcomes and validation of such may not be possible.

3. The Mack formula and resulting variance estimate (on an ultimate basis) would need to be bifurcated such that variance estimates would be available for each development period between the valuation date and the date at which time the losses are fully

[31] This does not imply that the ODP bootstrap model is the only model suited for this process. In actual practice many other models can be considered with their assumptions validated, documented, etc.

developed (at ultimate).

4. The practicing actuary learns very little about the data and underlying uncertainty when using a closed form model such as Mack. This follows because such models require limited calibration to get a result and limited diagnostics regarding the underlying assumptions. Further, the uncertainty is highly dependent on the observable loss development factors, relative to the AYLWA, which in the tail area can be limited.
5. The practicing actuary has little ability to adjust the results of the Mack method in cases where the output from the closed form solution is inconsistent with expectations.

5.6 ERM Governance Elements and Automatic Alert System

The manipulation and validation of methods and models, while interesting and attractive to actuaries, is only a small part of the case study. The real benefit of a well-defined ERM process results from a governance structure that allows the actuary to actively manage resources and to escape the confines of their office to actively engage with professionals from other departments.

5.6.1 Governance

The ERM system used in the case study includes several KPIs that result from the reserving process. For each KPI, the risk owner and risk reviewer are defined. At the highest level, the KPIs for aggregate (i.e., entity-wide) paid loss and aggregate incurred loss could be defined such that the Chief Actuary is the Risk Owner and the Chief Executive Officer ("CEO") is the Risk Reviewer.

In discussing governance, KPIs, and thresholds, it is important to remember that 1 in 100 realizations is expected to fall above the 99th percentile. Stated differently, just because a deviation is large does not necessarily mean that the prior methods and models were calibrated incorrectly. On the contrary, there are three possible explanations which can be investigated:

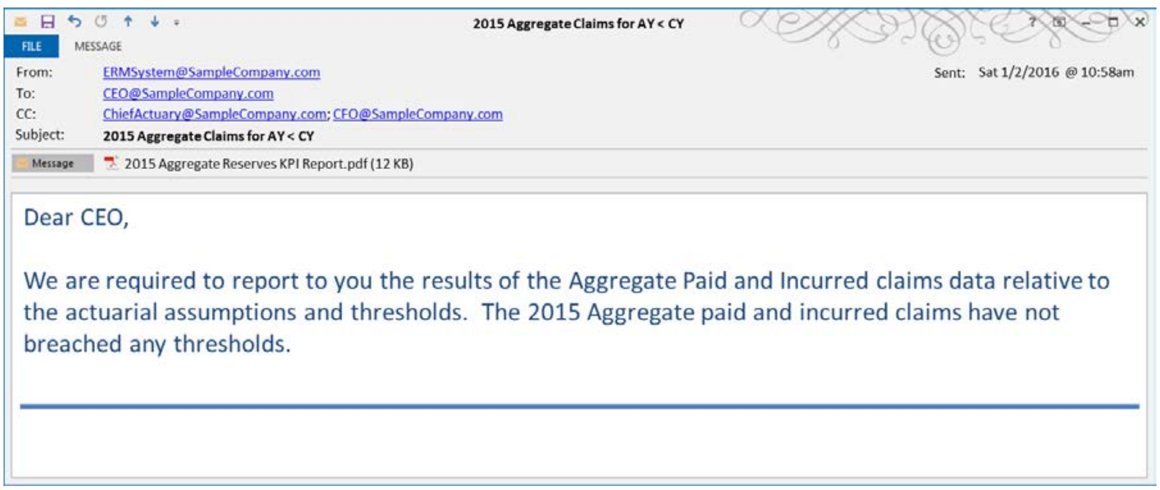
1. There could be a change in an internal process which was unknown at the time of the prior analysis contributing to the large deviation;
2. One or more of the prior modelling assumptions, with respect to the deterministic methods and stochastic models, may be causing the large deviation; or
3. A large deviation could simply be the result of a random occurrence.

5.6.2 Automatic Alert System

Further, the realized values are subject to thresholds, each with well-defined consequences in case of a breach. The case study uses thresholds at the 25th and 75th percentile, the 5th and 95th percentile, as well as the simulated minimum and simulated maximum of the distribution of possible outcomes to denote material deviations from expected, as illustrated in Figure 3.1.

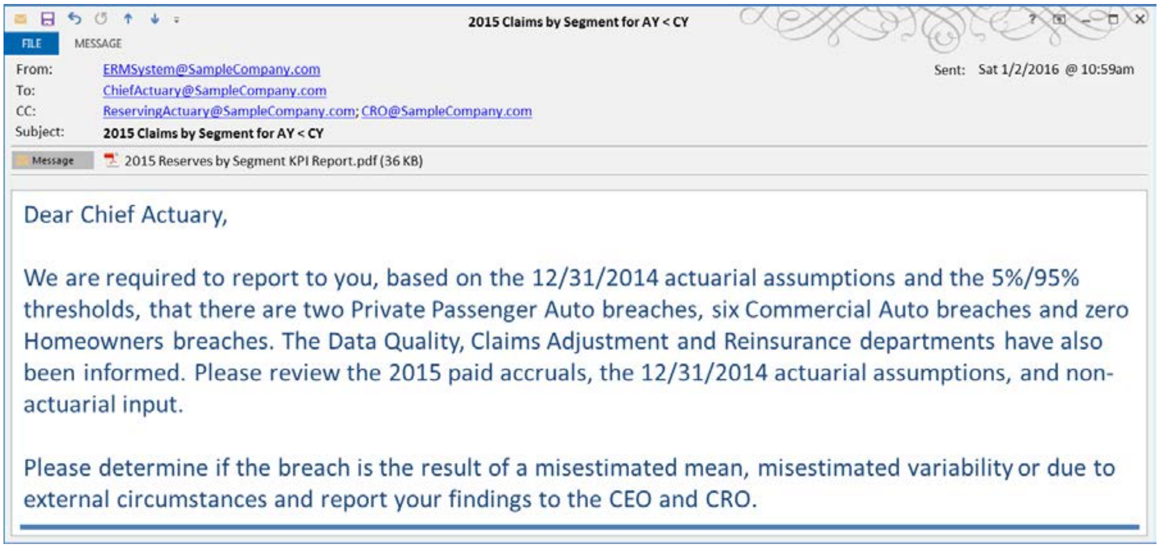
The CEO receives an immediate and automatic email from the ERM system on the first day of the analysis period confirming whether the 5% or 95% thresholds were breached by the aggregate paid loss or aggregate incurred loss.

Figure 5.5 Sample Automated E-Mail #1 to the CEO



The automatic alert system will send as many emails as needed based on the pre-defined thresholds to the appropriate Risk Owners and Risk Reviewers. For example, while the CEO is the risk reviewer and the Chief Actuary is the risk owner of the aggregate results, for the results by segment the Chief Actuary is the risk reviewer and the Reserving Actuary is the risk owner.

Figure 5.6 Sample Automated E-Mail #2 to the Chief Actuary


FILE MESSAGE

2015 Claims by Segment for AY < CY

From: ERMSystem@SampleCompany.com

Sent: Sat 1/2/2016 @ 10:59am

To: ChiefActuary@SampleCompany.com

CC: ReservingActuary@SampleCompany.com; CRO@SampleCompany.com

Subject: 2015 Claims by Segment for AY < CY

Message | 2015 Reserves by Segment KPI Report.pdf (36 KB)

Dear Chief Actuary,

We are required to report to you, based on the 12/31/2014 actuarial assumptions and the 5%/95% thresholds, that there are two Private Passenger Auto breaches, six Commercial Auto breaches and zero Homeowners breaches. The Data Quality, Claims Adjustment and Reinsurance departments have also been informed. Please review the 2015 paid accruals, the 12/31/2014 actuarial assumptions, and non-actuarial input.

Please determine if the breach is the result of a misestimated mean, misestimated variability or due to external circumstances and report your findings to the CEO and CRO.

For the emails illustrated in Figures 5.5 and 5.6 there is also a report attached which the recipients can open to review the specific results. The reports attached to the email, which also highlight any breached thresholds, are shown in Appendix B. For higher levels of management a more aggregate view will tend to be the first priority and at lower levels of management a more detailed view will be important as the automated system will reflect the responsibilities of the individuals.

5.6.3 One-Year Time Horizon as Preliminary Monitoring Tool

On the first day of the analysis, the Actuarial Function is capable of sharing even more information with the CEO & CFO, which is a valuable early warning system related to both the direction and potential magnitude of aggregate reserve changes on financial results. The value comes from estimating the one-year time horizon reserves which are conditional on the possible outcomes of the ultimate time horizon distribution. No matter whether the early warning is positive or negative, management as a whole can keep their eye on the risk management issues related to reserve changes from the beginning of the reserving exercise instead of reacting to surprises toward the end of the exercise, just prior to the publishing of financial results.

The one-year time horizon has been developed and promoted by entities subject to the Solvency II regime in Europe using both an ODP bootstrap approach and as a modification to the Mack model developed by Merz & Wüthrich [19]. Essentially, because entities are required to hold sufficient capital to be 99.5% certain of staying solvent over a one-year time horizon, actuaries have developed techniques which bifurcate measures of reserving risk into two pieces, the reserving risk over a single year and the reserving risk over all subsequent years.

The calibration of reserving risk over a one-year time horizon using the ODP bootstrap approach produces a conditional reserve at each probability level and involves a two-step process:[32]

1. Possible outcomes are simulated as usual but only the simulations of the first calendar year cash flows are retained (the one-year time horizon). These simulated diagonals are used to re-parameterize the ODP bootstrap model based on the original data plus the simulated diagonals;

2. Point estimates for the remainder of the unpaid claims subsequent to the one-year time horizon are created for each possible outcome of the original triangle plus the simulated one-year diagonal. Note that point estimates in this case have not been adjusted for process variance as they are intended to represent a reserve estimate which is conditional on the outcome of the one-year time horizon.

Table 5.9 Differences between Expected and Conditional Reserves

Sample Insurance Company
Aggregation of All Segments
Summary of Conditional Reserves as of December 31, 2015

	Private Passenger Auto			Commercial Auto			Homeowners			Total (Sum)		
AY	Conditional Reserve	Expected Reserve	Change	Conditional Reserve	Expected Reserve	Change	Conditional Reserve	Expected Reserve	Change	Conditional Reserve	Expected Reserve	Change
2006	2,680	2,991	(311)	643	603	40	-	747	(747)	3,323	4,341	(1,018)
2007	7,248	5,498	1,750	3,257	4,242	(985)	164	721	(557)	10,669	10,461	208
2008	8,654	10,061	(1,406)	1,675	2,582	(907)	1,367	1,640	(272)	11,697	14,283	(2,586)
2009	15,635	19,472	(3,836)	5,593	4,121	1,472	(1,153)	1,793	(2,946)	20,075	25,386	(5,311)
2010	31,595	38,066	(6,470)	13,946	6,632	7,313	3,722	340	3,381	49,263	45,039	4,224
2011	73,359	71,302	2,057	20,073	19,441	632	3,979	6,894	(2,915)	97,412	97,638	(227)
2012	151,670	156,061	(4,390)	57,978	45,442	12,536	12,839	9,468	3,370	222,487	210,971	11,516
2013	292,882	322,812	(29,930)	110,701	81,627	29,075	21,590	26,615	(5,024)	425,174	431,054	(5,880)
2014	581,448	574,019	7,430	170,589	147,146	23,442	59,458	80,333	(20,875)	811,496	801,499	9,997
2015												
Totals	1,165,174	1,200,281	(35,107)	384,456	311,837	72,619	101,967	128,553	(26,586)	1,651,596	1,640,671	10,926
AY<CY	1,159,897	1,200,281	(40,385)	390,213	311,837	78,376	96,676	128,553	(31,876)	1,646,786	1,640,671	6,115

By calculating the percentile of the actual calendar year paid within the distribution of expected calendar year paid using (3.4), then the conditional reserve would be the same percentile of the distribution of point estimates subsequent to the one-year time horizon using formula (5.8). The expected reserve for the new analysis is equal to the expected reserve for the prior analysis less the actual amount paid during the year as shown in (5.9). In other words, the new expected reserve is equal to the prior expected reserve if the estimate

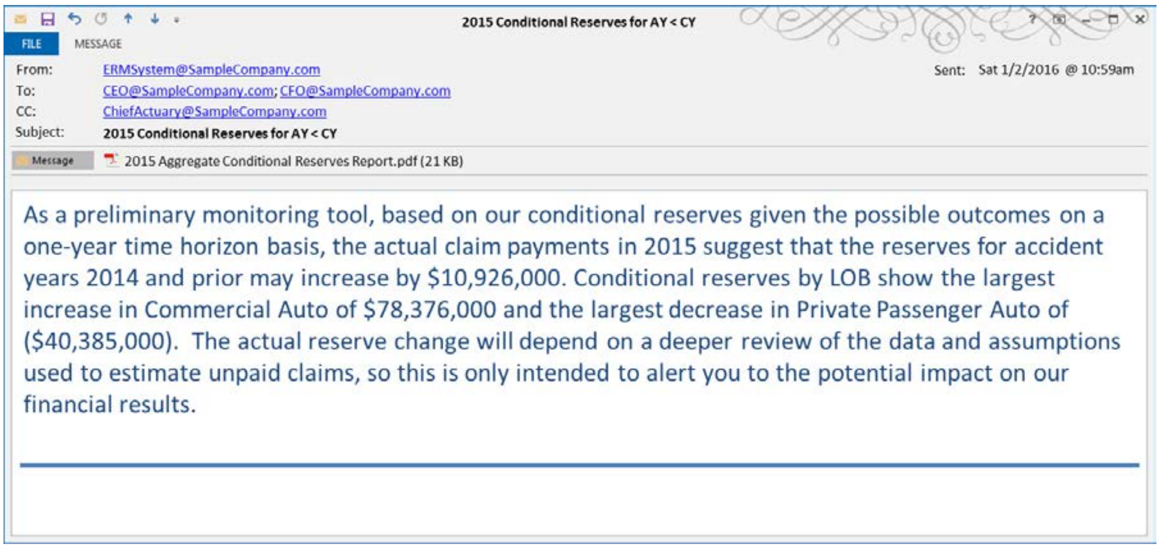
[32] See Appendix A for a graphical overview of the one-year time horizon calculations using the ODP bootstrap model.

of ultimate loss did not change at all. The estimated reserve change, therefore, is represented by the difference between conditional reserve and the expected reserve, i.e., (5.8) minus (5.9).

$$E[\hat{R}(w,d+1)\,|\,x] = PERCENTILE.INC\{Dist[\textstyle\sum_{d=t+1}^{u} \hat{q}(w,d)], P_x[q(w,d)]\} \tag{5.8}$$

$$E[\hat{R}(w,d+1)] = E[\hat{R}(w,d)] - q(w,d) \tag{5.9}$$

Figure 5.7 Automated E-Mail #3 to the CEO and CFO


2015 Conditional Reserves for AY < CY

FILE MESSAGE

From: ERMSystem@SampleCompany.com Sent: Sat 1/2/2016 @ 10:59am

To: CEO@SampleCompany.com; CFO@SampleCompany.com

CC: ChiefActuary@SampleCompany.com

Subject: 2015 Conditional Reserves for AY < CY

Message 2015 Aggregate Conditional Reserves Report.pdf (21 KB)

As a preliminary monitoring tool, based on our conditional reserves given the possible outcomes on a one-year time horizon basis, the actual claim payments in 2015 suggest that the reserves for accident years 2014 and prior may increase by $10,926,000. Conditional reserves by LOB show the largest increase in Commercial Auto of $78,376,000 and the largest decrease in Private Passenger Auto of ($40,385,000). The actual reserve change will depend on a deeper review of the data and assumptions used to estimate unpaid claims, so this is only intended to alert you to the potential impact on our financial results.

The CEO and CFO receive an immediate and automatic email from the ERM system on the first day of the analysis period stating a preliminary estimate for the change in reserves, based on the conditional reserves given the possible outcomes under a one-year time horizon and the actual paid loss observed during the most recent calendar year. The report attached to the email is shown in Appendix B. Based on the conditional reserves, the aggregate increase of $10.9 million may not be of immediate concern, but the Commercial Auto increase of $78.4 million will certainly draw attention.

5.6.4 Allocating Resources

In addition to the conditional reserves by segment, it is possible to quantify and rank the deviation from expected for each of the outcomes. For the case study, 80 outcomes include 10 paid observations and 10 incurred observations, calculated as 9 AYs and Segment Total (i.e. AY < CY), for 3 Segments and the Aggregate (i.e., after correlation).

A ranked list of deviations allows for an alternative approach to managing actuarial resources. Actuarial managements often use an approach that assigns individuals to segments. An advantage of this approach is that an individual develops an area of expertise and relationships with corresponding claims and underwriting professionals. A disadvantage of this approach is that the methodology and corresponding documentation may receive less external challenge, increasing the risk that business will be disrupted in case the current expert needs to be replaced.

An alternative approach, using the ranked list of deviations, includes the allocation of resources based on the quantitative deviation from expected. This alternative approach envisions assigning resources based on need. If the methods and models are producing large deviations from expected, assignment of a resource with a proven ability to "put out fires" may be advantageous. This approach pre-supposes that the department manager has a strong sense of the strengths and weaknesses of their team.

5.6.5 Additional Indicators of Performance

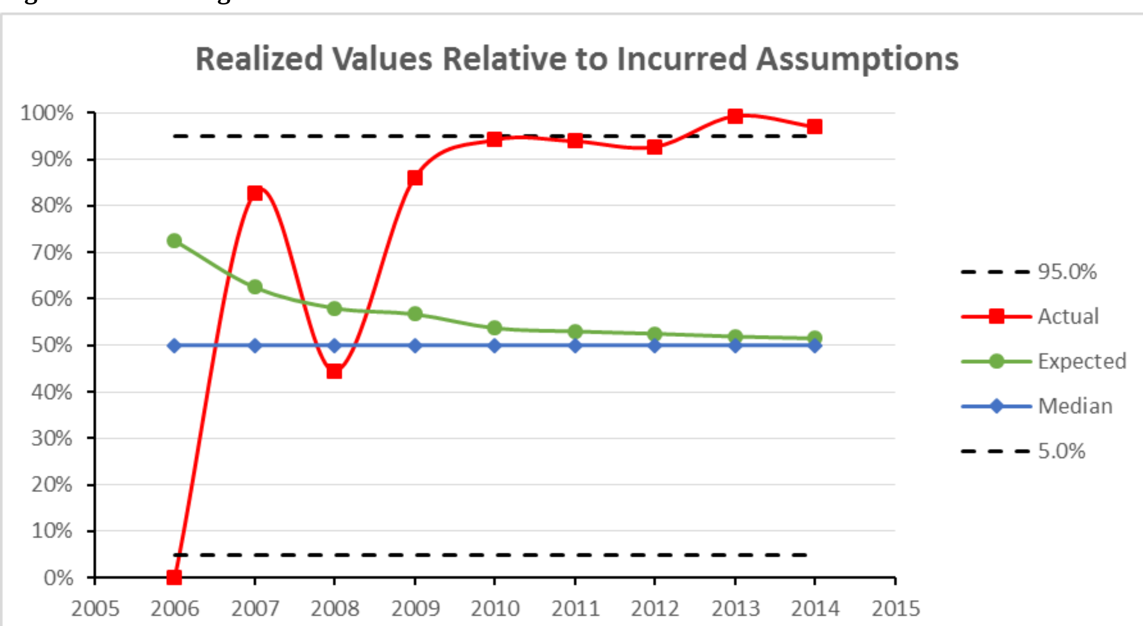
In the case of the Commercial Auto segment, the experience observed on day one of the analysis is quite poor so immediately digging into the drivers will be important. As shown in Table 5.10, two of the incurred observations (highlighted with grey shading) have breached the minimum and maximums defined by the prior models. A further two incurred and two paid observations have breached the 5%/95% threshold (highlighted with red font); and 5 incurred and 4 paid observations have breached the 25%/75% threshold (highlighted with orange font). Only 5 observations sit comfortably in the core 50%, from 25% to 75% of the distribution of possible outcomes. Absent changes in the methodology and modelling, the one-year time horizon exercise implies a deterioration of more than 13% (equal to 78,376 / [262,931 + 311,837], referring to values found in Tables 5.9 and 5.10).

Table 5.10 Assessing the 20 Observations for Commercial Auto

Sample Insurance Company Commercial Auto Stochastic Actual vs. Expected as of December 31, 2015							
AY	Age	Actual Paid	Expected Paid	Percentile	Actual Incurred	Expected Incurred	Percentile
2006	120	543	571	57.9%	(47)	154	0.0%
2007	108	2,387	3,131	21.8%	1,040	448	82.8%
2008	96	1,177	1,665	33.5%	851	1,167	44.5%
2009	84	5,403	5,044	63.1%	2,954	1,669	86.1%
2010	72	14,120	11,061	91.1%	9,035	5,606	94.2%
2011	60	23,636	23,276	56.1%	16,524	11,960	93.9%
2012	48	51,020	45,272	86.7%	36,454	29,103	92.7%
2013	36	75,813	62,481	96.5%	61,541	44,392	99.3%
2014	24	88,832	79,698	86.1%	83,154	66,555	97.0%
2015	12	99,123			178,539		
Totals		362,054			390,045		
AY<CY		262,931	232,199	98.9%	211,506	161,054	100.0%

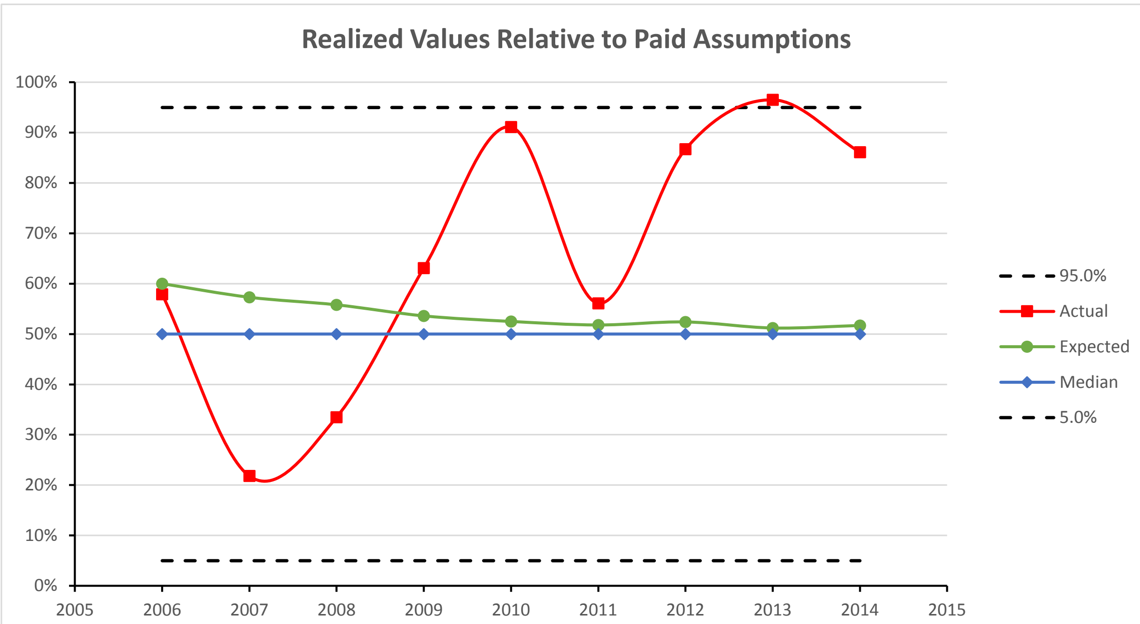
Looking closer at the incurred observations in Table 5.10 and Figure 5.8, notice that immature AYs appear to have been significantly underestimated. Though not conclusive, the realized values imply there may have been a problem with the deterministic methods underlying the prior analysis. Although the minimum and maximum have been breached, the prior uncertainty estimates may have been too narrow or the mean was too low or a combination of both, as 8 of the 10 realizations are above the 75th percentile of the distribution.

Figure 5.8 Assessing the Incurred AY Observations for Commercial Auto



Looking closer at the paid observations in Table 5.10 and Figure 5.9, notice that immature AYs appear to have again been significantly underestimated. Though not conclusive, the realized values imply again that there may have been a problem with the deterministic methods underlying the prior analysis. Again the prior uncertainty estimates may have been too narrow or the means too low or both (but to a lesser extent than observed in the incurred KPIs).

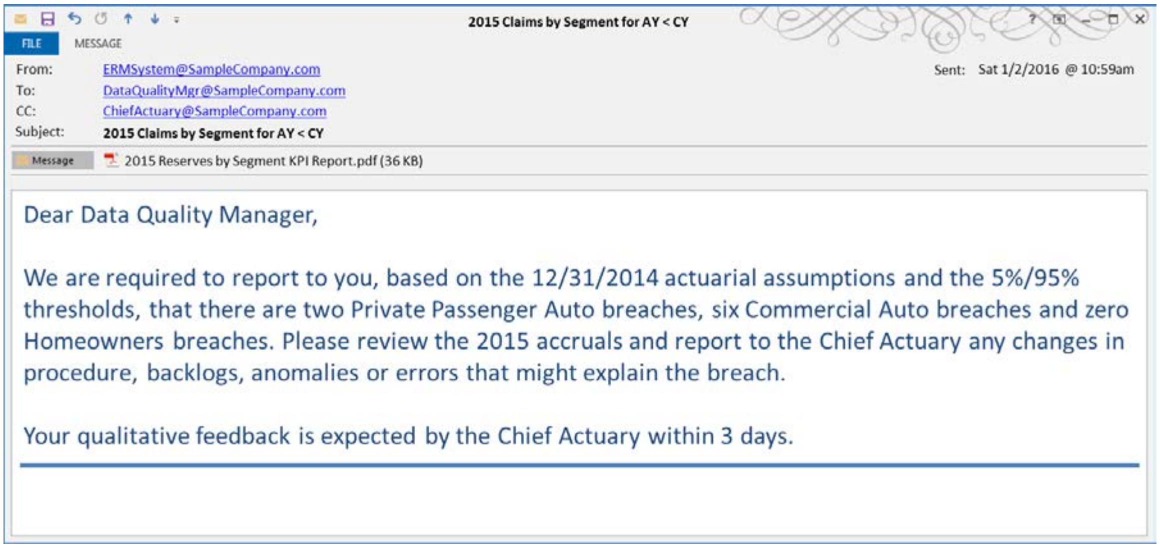
Figure 5.9 Assessing the Paid AY Observations for Commercial Auto



Note the skewness across AYs in the models underlying both the incurred and paid expectations by observing the differences between the expected values or means (the green line) and median values (the blue line) in the Figures 5.8 and 5.9.

An ERM system also has pre-defined actions, which are conditional on the breaching of the 95th percentile threshold. For Commercial Auto, these actions include immediate and automatic emails from the ERM system to the Data Quality Manager, Claims Manager, and Reinsurance Manager, among others; as illustrated in Figures 5.10 to 5.12. This presupposes some training of non-actuarial professionals so that they understand that 5 of the 100 observations should breach the 95th percentile and that a breach does not necessarily indicate that the methods and models were calibrated incorrectly. However, as part of the risk management collaboration that is being cultivated, these emails move all concerned to action.

Figure 5.10 Automated E-Mail #4 to the Data Quality Manager



2015 Claims by Segment for AY < CY

FILE MESSAGE

From: ERMSystem@SampleCompany.com

Sent: Sat 1/2/2016 @ 10:59am

To: DataQualityMgr@SampleCompany.com

CC: ChiefActuary@SampleCompany.com

Subject: 2015 Claims by Segment for AY < CY

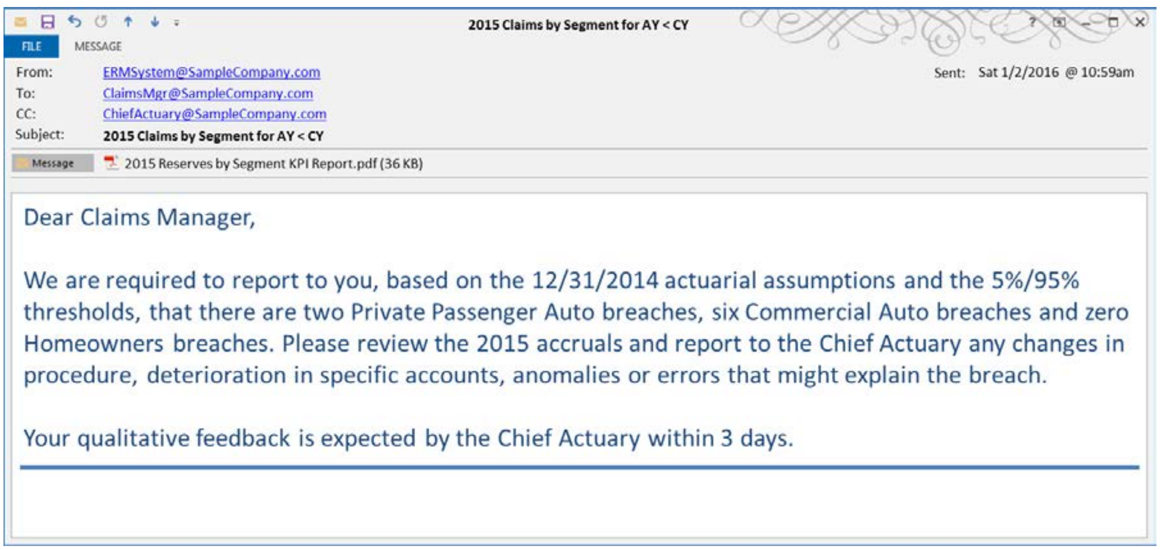
Message 2015 Reserves by Segment KPI Report.pdf (36 KB)

Dear Data Quality Manager,

We are required to report to you, based on the 12/31/2014 actuarial assumptions and the 5%/95% thresholds, that there are two Private Passenger Auto breaches, six Commercial Auto breaches and zero Homeowners breaches. Please review the 2015 accruals and report to the Chief Actuary any changes in procedure, backlogs, anomalies or errors that might explain the breach.

Your qualitative feedback is expected by the Chief Actuary within 3 days.

Figure 5.11 Automated E-Mail #5 to the Claims Manager



2015 Claims by Segment for AY < CY

FILE MESSAGE

From: ERMSystem@SampleCompany.com

Sent: Sat 1/2/2016 @ 10:59am

To: ClaimsMgr@SampleCompany.com

CC: ChiefActuary@SampleCompany.com

Subject: 2015 Claims by Segment for AY < CY

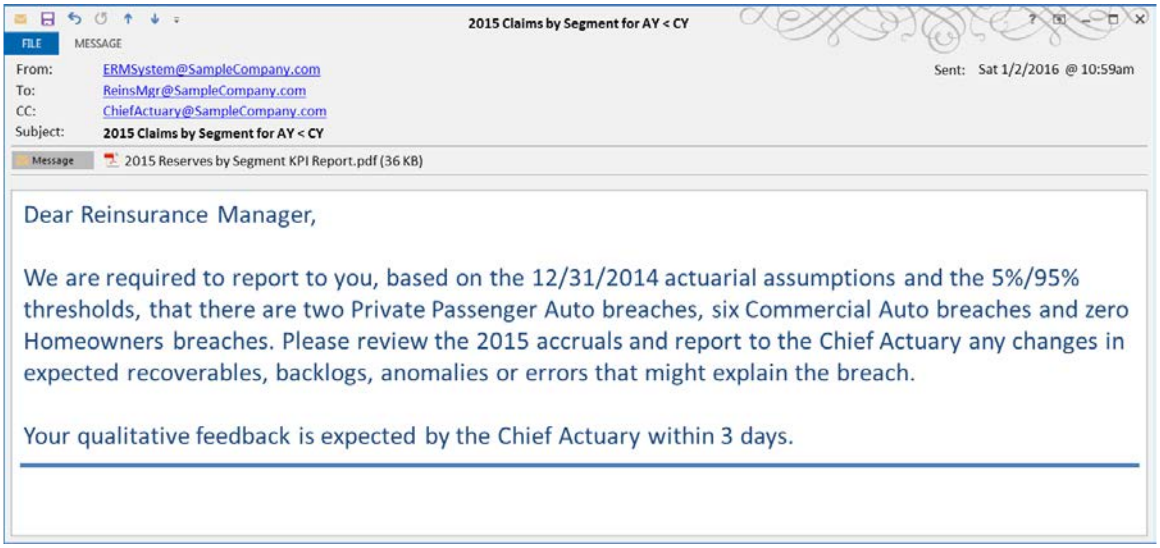
Message 2015 Reserves by Segment KPI Report.pdf (36 KB)

Dear Claims Manager,

We are required to report to you, based on the 12/31/2014 actuarial assumptions and the 5%/95% thresholds, that there are two Private Passenger Auto breaches, six Commercial Auto breaches and zero Homeowners breaches. Please review the 2015 accruals and report to the Chief Actuary any changes in procedure, deterioration in specific accounts, anomalies or errors that might explain the breach.

Your qualitative feedback is expected by the Chief Actuary within 3 days.

Figure 5.12 Automated E-Mail #6 to the Reinsurance Manager



For the emails illustrated in Figures 5.10, 5.11, and 5.12 there is also a report attached which the recipients can open to review the specific results. The reports attached to the email, which also highlight any breached thresholds, are shown in Appendix B.

5.7 Using Back-testing Diagnostics to Assess Uncertainty

As noted above, a single observation has limited value related to assessing the overall quality of the variability estimates. However, it can be a value added exercise to review a large number of observed percentiles relative to the expectations. For the example in Table 5.11, 50% of the observations are expected to manifest within the 25th to 75th percentile. Likewise, 90% of the observations are expected to manifest within the 5th to 95th percentile and 10% of the observations are expected to manifest either below the 5th or above the 95th percentiles.

Table 5.11 Assessing Uncertainty in the 80 Observations

Sample Insurance Company Summary of Theshold Activity by Segment as of December 31, 2015												
	Number						Percentage					
	25% < X < 75%		5% < X < 95%		5% > X < 95%		25% < X < 75%		5% < X < 95%		5% > X < 95%	
	Expected	Actual	Expected	Actual	Expected	Actual	Expected	Actual	Expected	Actual	Expected	Actual
PPA	10	14	18	18	2	2	50.0%	70.0%	90.0%	90.0%	10.0%	10.0%
CA	10	**5**	18	**14**	2	**6**	50.0%	**25.0%**	90.0%	**70.0%**	10.0%	**30.0%**
HO	10	12	18	20	2	0	50.0%	60.0%	90.0%	100.0%	10.0%	0.0%
AGG	10	18	18	20	2	0	50.0%	90.0%	90.0%	100.0%	10.0%	0.0%
Total	40	49	72	72	8	8	50.0%	61.3%	90.0%	90.0%	10.0%	10.0%

Based solely on the 80 observations, the Commercial Auto line of business appears to need attention (which is consistent with the conditional reserves). Further, the Homeowners and Private Passenger Auto lines of business appear to be behaving with less uncertainty than expected. While not definitive, this process provides clues as to where the ODP bootstrap models may have been underestimating or overestimating the inherent uncertainty.

While it is tempting to draw conclusions, restraint is required as random noise can easily have a larger or smaller number of extreme observations than witnessed in Table 5.11. Nevertheless, evidence is mounting that Commercial Auto deserves the most attention.

5.8 The Feedback Loop

A critical and common part of reserving and ERM is the feedback loop. Reviewing and re-evaluating models and assumptions is a healthy part of any reserve analysis and an open discussion of risks within the ERM framework naturally leads back to the original assumptions. In the case study, all assumptions discussed in Section 5.3 were systematically reviewed and alternative assumptions tested to determine if there was a material difference in the back-test with the benefit of hindsight.

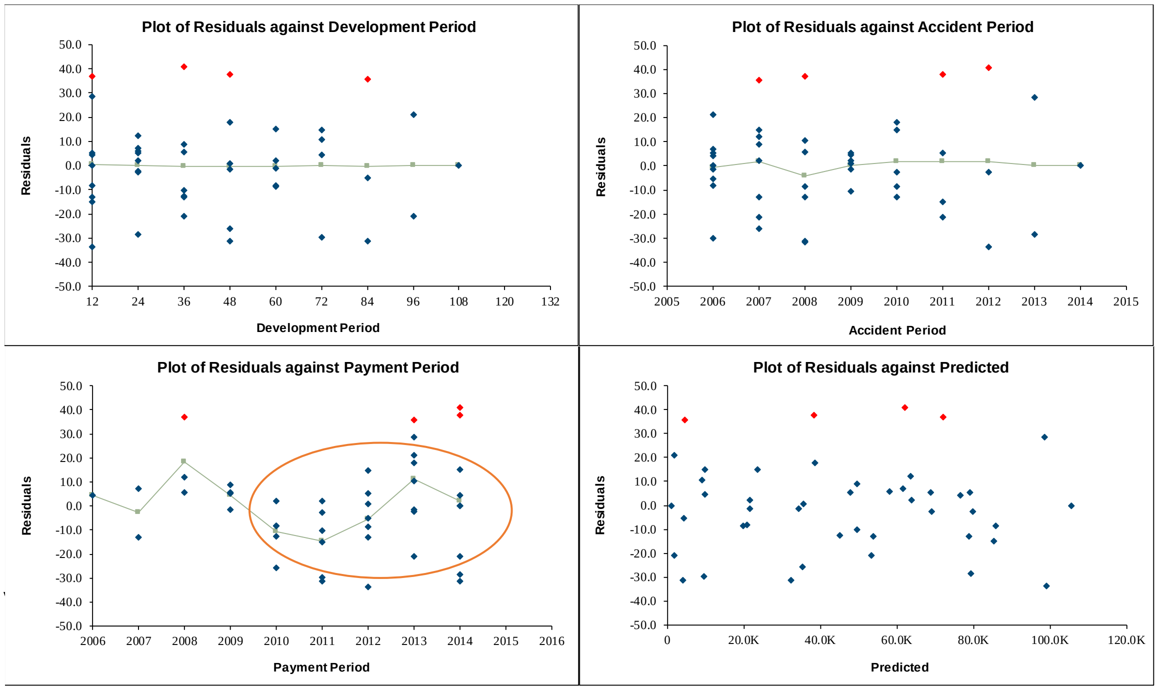
The only assumption that proved to have more than an insignificant impact on the back-test was the a priori loss ratio assumption for the Bornhuetter-Ferguson models. As shown in Table 5.2, the management IELR of 52.9% for 2014 is a bit low compared to the projected loss ratios from the Pd CL and Inc CL models, so for the back-test the 2014 IELR was changed to 57.5%. Comparing Table 5.12 with Table 5.10, the back-test of this assumption has a significant impact on the paid results for 2014, but the incurred results for 2014 are not as significant and the impact on the AY < CY results were insignificant.

Table 5.12 Revised Observations for Commercial Auto after A Priori Adjustment for 2014

Sample Insurance Company Commercial Auto Stochastic Actual vs. Expected as of December 31, 2015							
AY	Age	Actual Paid	Expected Paid	Percentile	Actual Incurred	Expected Incurred	Percentile
2006	120	543	571	57.9%	(47)	154	0.0%
2007	108	2,387	3,131	21.8%	1,040	448	82.8%
2008	96	1,177	1,665	33.5%	851	1,167	44.5%
2009	84	5,403	5,044	63.1%	2,954	1,669	86.1%
2010	72	14,120	11,061	91.1%	9,035	5,606	94.2%
2011	60	23,636	23,276	56.1%	16,524	11,960	93.9%
2012	48	51,020	45,272	86.7%	36,454	29,103	92.7%
2013	36	75,813	62,481	96.5%	61,541	44,392	99.3%
2014	24	88,832	85,603	65.4%	83,154	73,782	85.3%
2015	12	99,123			178,539		
Totals		362,054			390,045		
AY<CY		262,931	238,104	96.7%	211,506	168,281	99.9%

While the assumed loss ratios over the past few years have been decreasing, in the light of the back-testing it seems more likely that the loss ratios have remained constant at best or have been increasing.

Figure 5.13 Commercial Auto: Plots of Residuals (Paid)

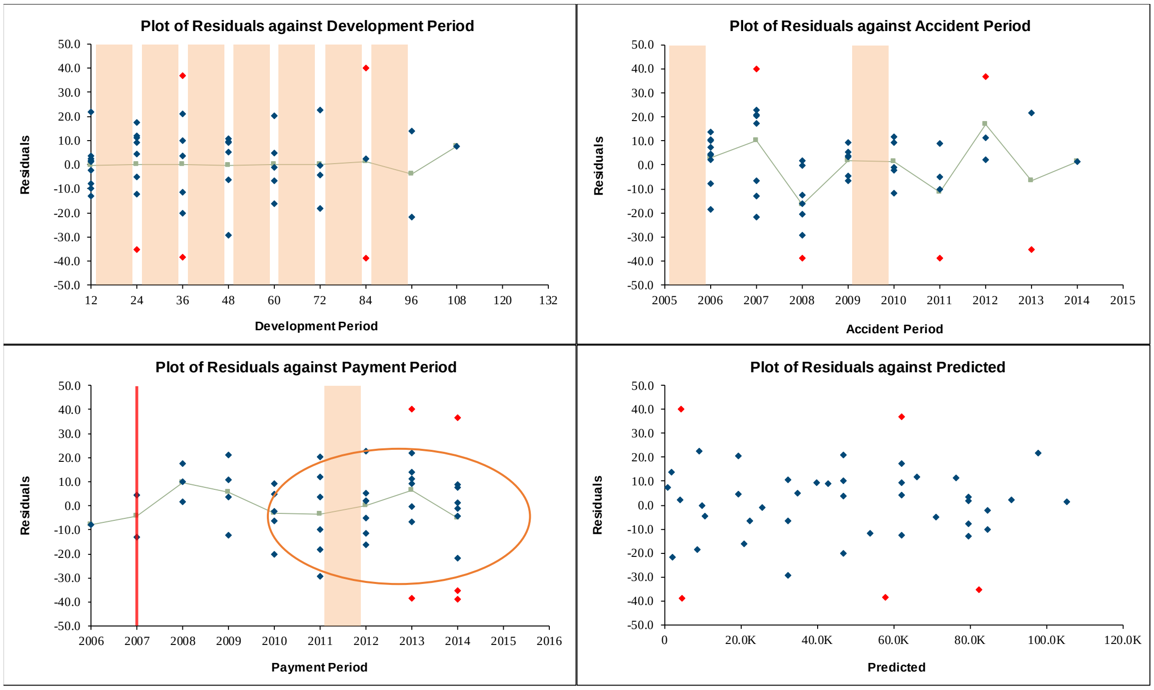


The benefit of hindsight led to an observation that a calendar year trend was evident yet overlooked (see bottom left graph in Figure 5.13). It is important here to pause and contemplate how frequently such trends are observed and disregarded (or considered immaterial). The point here is that the enhanced documentation provides an evidence trail that confirms that the trend was not addressed. With the benefit of hindsight, however, more attention is given to such diagnostics as a material driver of performance.

After identification of this possible explanation, a new model as of the previous valuation date can be calibrated. In this case, the relationship between the ODP bootstrap model and the GLM it is based on became useful. The ODP bootstrap model uses one parameter for every development year and one parameter for every accident year (minus one). Therefore the ODP bootstrap model is unable to add parameters to account for calendar year effects without removing corresponding accident year or development year parameters.

New GLM Bootstrap models based on paid and incurred data were calibrated with calendar year parameters, which was able to model the calendar year effect (see Figure 5.14, where shading refers to the parameters being used). The underlying calendar year trends inherent in the new GLM Bootstrap models imply no trend from 2006 until 2011, but an annual trend of 7.3% for years 2011 and subsequent using the paid data and a trend of 6.4% using the incurred data.

Figure 5.14 Commercial Auto: Plots of Residuals (Paid) for GLM Bootstrap Model



The new GLM Bootstrap models based on paid and incurred data performed better than the prior selected models, as seen in Table 5.13, and many of the model statistics are better.

At first glance Table 5.13 does not appear to be significantly better than Table 5.10. However, a review of Figures 5.15 and 5.16 (for the GLM Bootstrap) reveals that adding the calendar year trend to the models counteracts the upward trend in Figures 5.8 and 5.9 (prior to GLM Bootstrap) to a significant degree (more for paid than incurred) which provides a rationale (or evidence) for the increasing loss ratios over the last few years. This corroborates the earlier back-test of the Bornhuetter-Ferguson a priori loss ratios. The resulting variations in Figures 5.15 and 5.16 also indicates that the variability of the potential outcomes may still be too narrow (e.g., Bornhuetter-Ferguson a priori variance could be larger), but this is just a preliminary review.

Table 5.13 Assessing the Commercial Auto Observations for the GLM Bootstrap Models

Sample Insurance Company Commercial Auto Stochastic Actual vs. Expected as of December 31, 2015							
AY	Age	Actual Paid	Expected Paid	Percentile	Actual Incurred	Expected Incurred	Percentile
2006	120	543	432	69.4%	(47)	228	2.0%
2007	108	2,387	942	96.6%	1,040	516	86.8%
2008	96	1,177	2,117	14.0%	851	1,181	37.9%
2009	84	5,403	5,001	64.1%	2,954	2,665	64.7%
2010	72	14,120	12,100	82.3%	9,035	6,659	89.8%
2011	60	23,636	27,514	11.8%	16,524	13,869	84.2%
2012	48	51,020	46,010	87.6%	36,454	31,896	87.7%
2013	36	75,813	66,910	94.6%	61,541	50,020	98.5%
2014	24	88,832	88,362	54.1%	83,154	78,184	77.8%
2015	12	99,123			178,539		
Totals		362,054			390,045		
AY<CY		262,931	249,388	86.0%	211,506	185,218	98.7%

The ERM process has provided the information to identify the problem segment and the enhanced documentation has allowed quick testing of the prior assumptions to provide an alternative model which can be considered and implemented by the actuarial resources for the current valuation. Additionally, the GLM approach has both identified when the positive calendar year trend begins (i.e., the break point) and quantified the trend rates, which allows the actuary to engage more directly with the claims department, where deeper knowledge may exist to improve the modeling process.

Figure 5.15 Assessing the Incurred AY Observations for Commercial Auto (GLM Bootstrap Model)

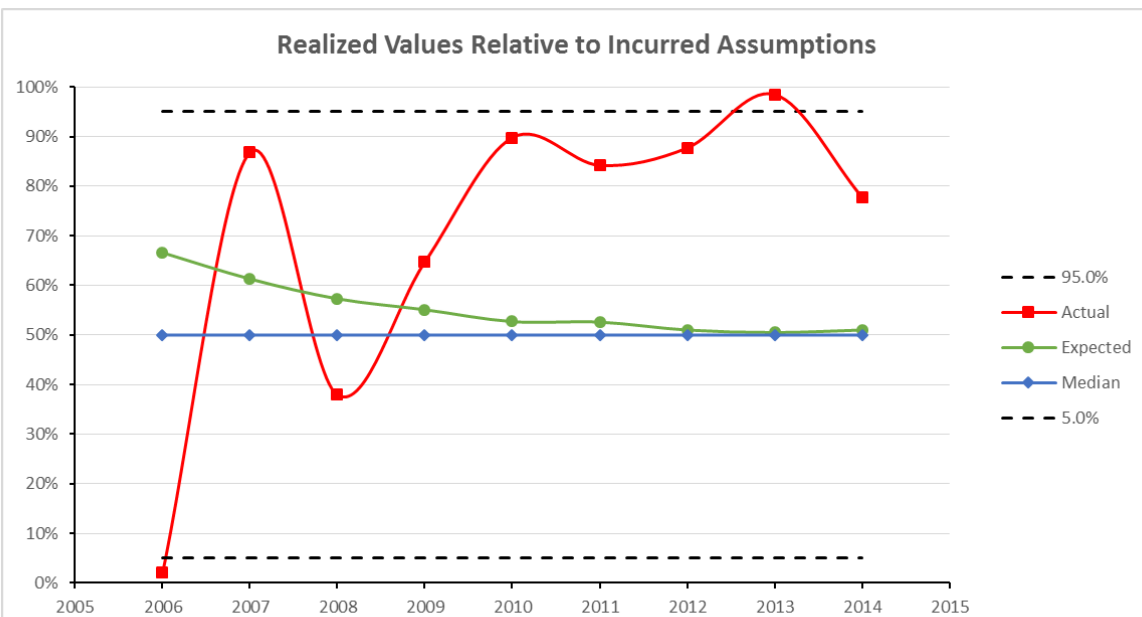
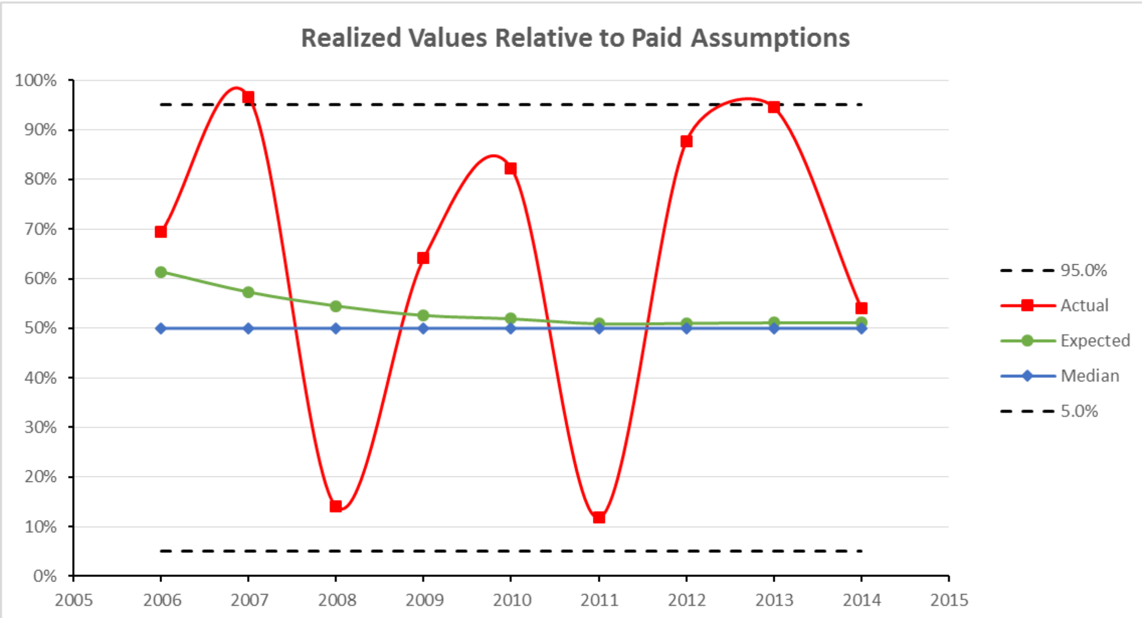
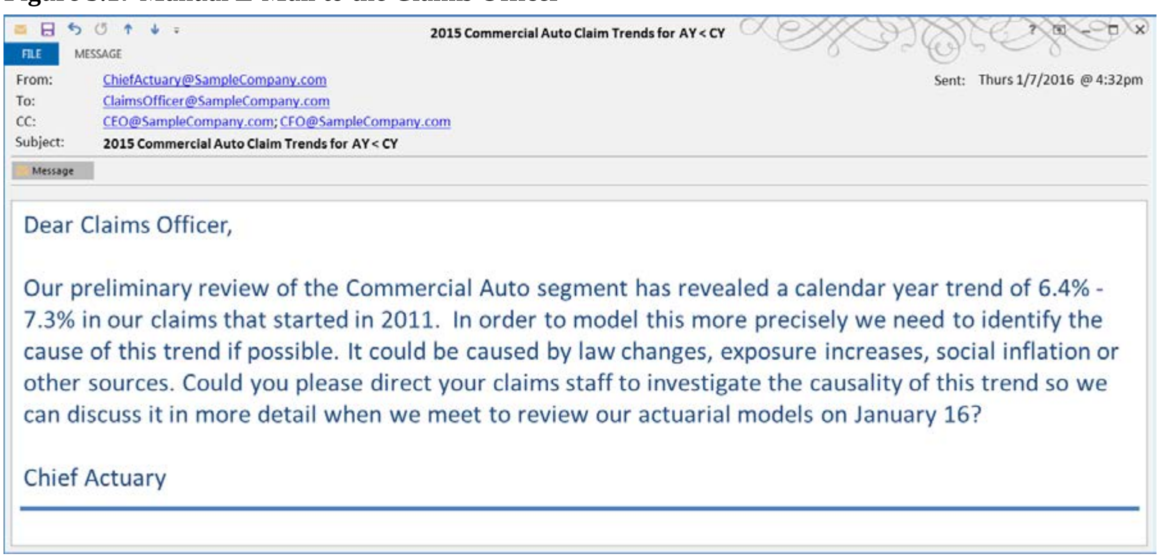


Figure 5.16 Assessing the Paid AY Observations for Commercial Auto (GLM Bootstrap Model)



A direct email from the Chief Actuary to the relevant Claims Officer, as illustrated in Figure 5.17, is the logical next step in the process so that communication around this issue can begin. Note that the process allows the actuary to speak to the claims officer in the language the claims officer understands: no mention of triangles, IBNR, accident years, or any other actuarial concepts that may be unfamiliar.

Figure 5.17 Manual E-Mail to the Claims Officer



2015 Commercial Auto Claim Trends for AY < CY

FILE MESSAGE

From: ChiefActuary@SampleCompany.com Sent: Thurs 1/7/2016 @ 4:32pm
To: ClaimsOfficer@SampleCompany.com
CC: CEO@SampleCompany.com; CFO@SampleCompany.com
Subject: 2015 Commercial Auto Claim Trends for AY < CY

Message

Dear Claims Officer,

Our preliminary review of the Commercial Auto segment has revealed a calendar year trend of 6.4% - 7.3% in our claims that started in 2011. In order to model this more precisely we need to identify the cause of this trend if possible. It could be caused by law changes, exposure increases, social inflation or other sources. Could you please direct your claims staff to investigate the causality of this trend so we can discuss it in more detail when we meet to review our actuarial models on January 16?

Chief Actuary

The value of this active feedback loop on reserving risk within the ERM process can't be overestimated. Not only does it naturally expand the actuarial conversation regarding risk drivers to the entire firm, but it also flows into other risks such as claims management and pricing risk. Indeed, consider the impact that identifying this trend will have on future pricing discussions for Commercial Auto.

6. Conclusions

While the value of including reserve variability estimates as part of the "normal" reserving cycle processes is questioned by some, and perhaps feared by others, the purpose of this paper is to show how making reserve variability estimates a routine part of the analysis can greatly benefit the risk management process. Keeping these estimates in the "back room" or "hidden until needed" does not benefit anyone. If casualty actuaries are going to truly embrace Enterprise Risk Management, then deep discussions of reserving risk must become part of the actuarial lexicon.

Acknowledgments

The authors gratefully acknowledge the many authors listed in the References (and others not listed) that contributed to the foundation of stochastic reserving and Enterprise Risk Management, without which this research would not have been possible. The authors are also grateful to Wayne Blackburn for his thorough review and insightful comments. The authors are also grateful to the participants in various seminars and sessions at GIRO, the CLRS, and the European Actuarial Academy stochastic modeling seminars where the concepts in the paper were first presented and discussed. Finally, the authors are grateful to the CAS Committee on Reserves for their comments which also greatly improved the quality of the paper.

Supplementary Material

There are companion files designed to give the reader a deeper understanding of the concepts discussed in the paper. The files are all in the "Actuary & ERM.zip" file. The files are:

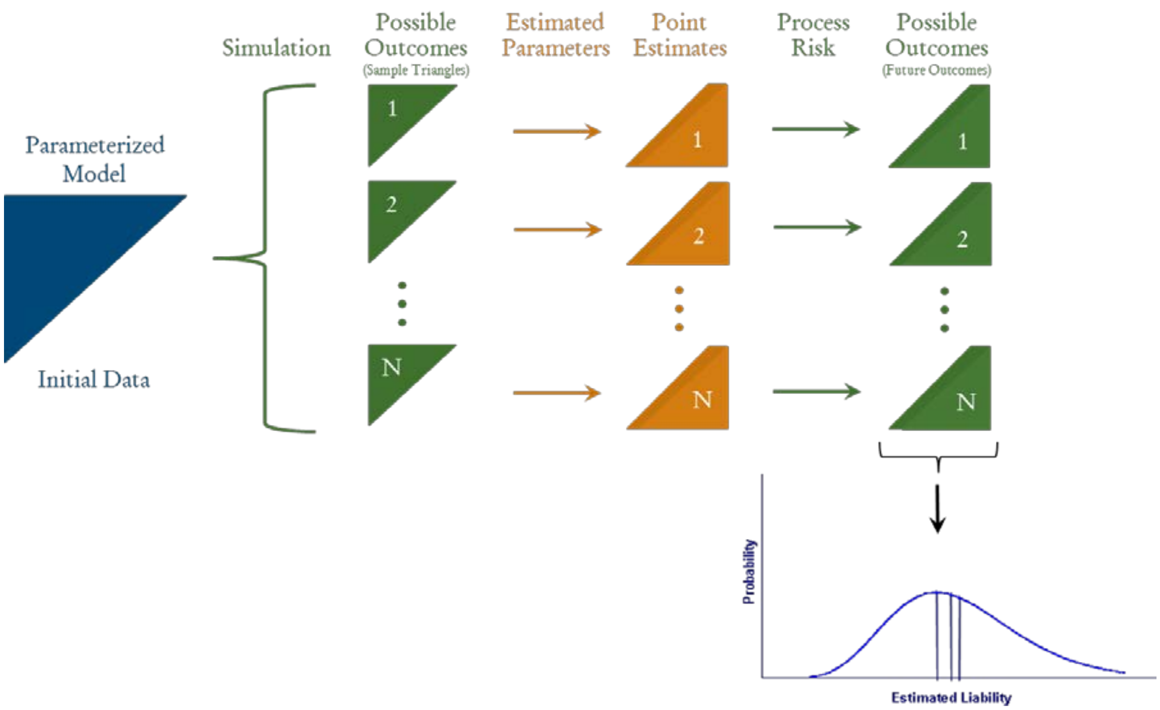
LOB Backtest.xlsm – this file contains the detailed calculations described in this paper for a single segment or line of business. Data can be entered and simulation output can be added for calculating both expected and actual outcomes, along with various statistical measures and results. Deterministic calculations and results are also included for comparison to stochastic results.

AGG Backtest.xlsm – this file can be used to summarize the deterministic and stochastic results from the LOB Backtest.xlsm file (selected results need to be copied to this file) for three lines of business. Aggregate simulation output can be added for calculating both expected and actual outcomes, along with various statistical measures and results.

APPENDICIES

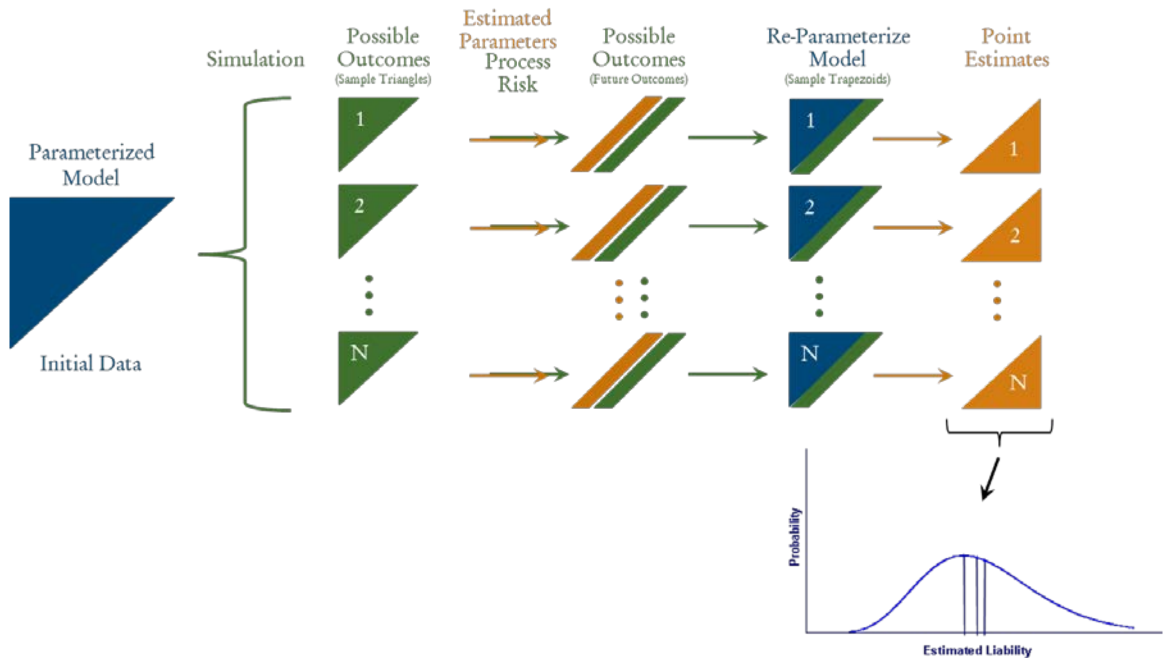
Appendix A – Overview of One-Year Time Horizon

A "standard" ODP bootstrap model can be represented graphically as follows:



- The "standard" model is based on paid data, but incurred data can also be used to reflect information in case reserves and converted to a random payment stream.
- The "standard" model is based on the chain ladder methodology, but other methods such as Bornhuetter-Ferguson and Cape Cod can also be included.
- Multiple models can also be "weighted" and "shifted" to reconcile with the deterministic "best estimate".
- Aggregation of the segment results can be done to derive a consolidated corporate result, even though these graphs are for one segment.

By using the first diagonal of the possible future outcomes and then calculating a point estimate for the remaining unpaid claims, the one-year time horizon can be represented graphically as follows:



- The "one-year" model is based on paid data, but incurred data can also be used to reflect information in case reserves and converted to a random payment stream for the first diagonal and expected payments for the remaining diagonals.
- The "one-year" model is based on the chain ladder methodology, but other methods such as Bornhuetter-Ferguson and Cape Cod can also be included. For internal consistency, all of the assumptions for the "standard" model should apply unchanged for the "one-year" model.
- Multiple models can also be "weighted" and "shifted" to reconcile with the deterministic "best estimate". The weights should be the same as for the "standard" model and "shifting" should be consistent with "standard" model so that the first diagonal after shifting is identical.
- Distributions of conditional point estimates can also be created for each accident year even though the total of all accident years combined is shown in the graphs.
- Aggregation of the segment results can be done to derive a consolidated corporate result, even though these graphs are for one segment.

Appendix B – Reports Attached to Emails

Figure B.1 – Report on 2015 Aggregate Exposures

Stochastic Model Results

2015 Aggregation of All Segments Exposure

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Stochastic Model Detail

Edit Delete Clone

Model Name	2015 Aggregation of All Segments Exposure	Assumption Owner	Chief Actuary
Description	Expected Aggregation of All Segments claim payments during 2015 for exposure periods prior to 2015 based on data generated by claims system as of 12/31/2015 relative to the 12/31/2014 actuarial assumptions.	Reports To	Chief Executive Officer
Assumption Value	Expected Value	Assumption Value Date	12/31/2014
Assumption Minimum	5.0%	Next Update Due	12/31/2015
Assumption Maximum	95.0%		

▼ Realized Value

Paid Actual	1,571,872	Incurred Actual	847,136
Paid Expected	1,560,637	Incurred Expected	858,093
Paid Percentile	61.2%	Incurred Percentile	37.6%

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Stochastic Values New Value Help

Action	Number	Exposure Period	Age	Paid Actual	Paid Expected	Paid Percentile	Incurred Actual	Incurred Expected	Incurred Percentile
Edit | Del	0001	12/31/2006	120	3,069	4,077	31.8%	1,863	2,115	49.8%
Edit | Del	0002	12/31/2007	108	5,905	6,163	47.9%	3,145	1,819	80.6%
Edit | Del	0003	12/31/2008	96	8,986	10,176	33.6%	3,553	6,026	20.9%
Edit | Del	0004	12/31/2009	84	18,992	20,033	39.0%	9,872	10,399	46.3%
Edit | Del	0005	12/31/2010	72	51,003	48,298	71.6%	25,942	25,562	55.3%
Edit | Del	0006	12/31/2011	60	105,067	104,415	54.3%	52,012	53,101	44.8%
Edit | Del	0007	12/31/2012	48	202,932	196,083	74.2%	106,624	104,075	61.7%
Edit | Del	0008	12/31/2013	36	334,434	331,701	57.1%	189,908	185,173	64.0%
Edit | Del	0009	12/31/2014	24	841,484	839,689	52.8%	454,217	469,822	29.3%
Edit | Del	0010	12/31/2015	12	1,798,138	0		2,528,235	0	

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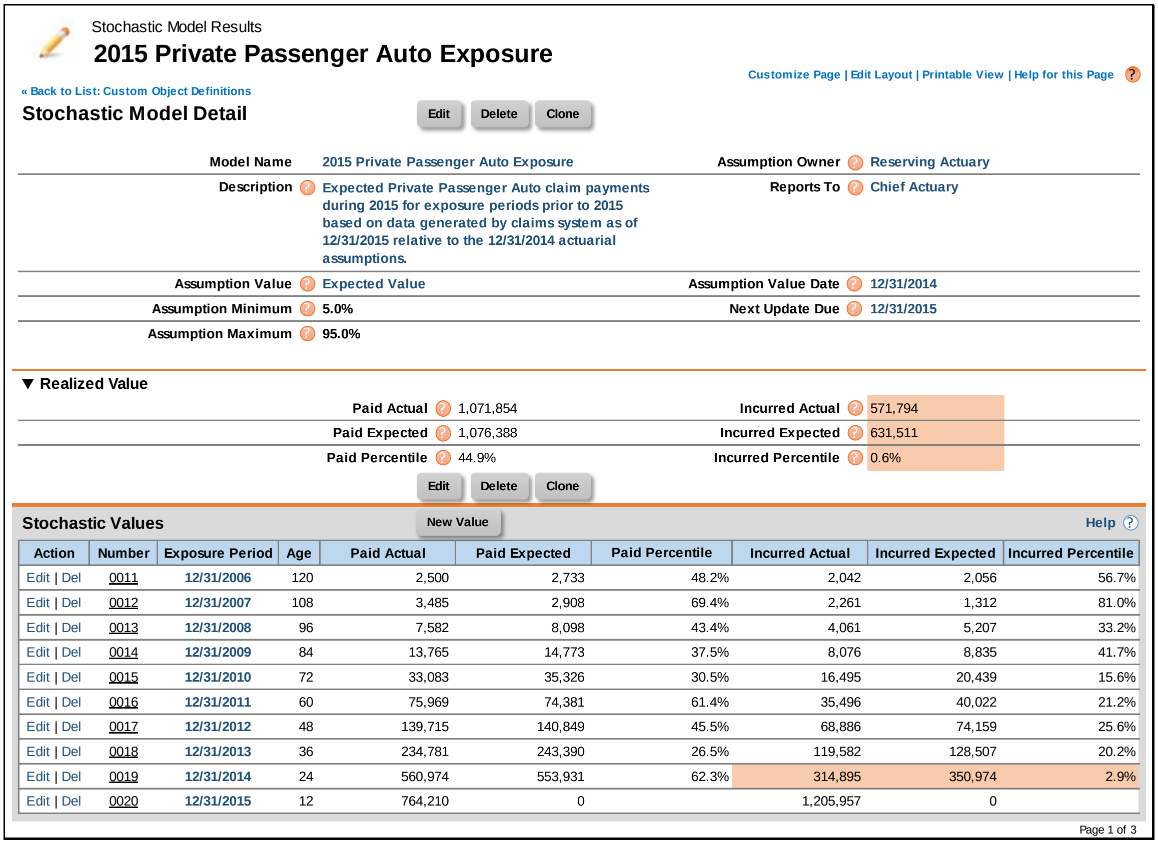
Figure B.2 – Report on 2015 Private Passenger Auto Exposures



Stochastic Model Results

2015 Private Passenger Auto Exposure

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Stochastic Model Detail

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Model Name	2015 Private Passenger Auto Exposure	Assumption Owner	Reserving Actuary
Description	Expected Private Passenger Auto claim payments during 2015 for exposure periods prior to 2015 based on data generated by claims system as of 12/31/2015 relative to the 12/31/2014 actuarial assumptions.	Reports To	Chief Actuary
Assumption Value	Expected Value	Assumption Value Date	12/31/2014
Assumption Minimum	5.0%	Next Update Due	12/31/2015
Assumption Maximum	95.0%		

▼ Realized Value

Paid Actual	1,071,854	Incurred Actual	571,794
Paid Expected	1,076,388	Incurred Expected	631,511
Paid Percentile	44.9%	Incurred Percentile	0.6%

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Stochastic Values

New Value

Help

Action	Number	Exposure Period	Age	Paid Actual	Paid Expected	Paid Percentile	Incurred Actual	Incurred Expected	Incurred Percentile
Edit | Del	0011	12/31/2006	120	2,500	2,733	48.2%	2,042	2,056	56.7%
Edit | Del	0012	12/31/2007	108	3,485	2,908	69.4%	2,261	1,312	81.0%
Edit | Del	0013	12/31/2008	96	7,582	8,098	43.4%	4,061	5,207	33.2%
Edit | Del	0014	12/31/2009	84	13,765	14,773	37.5%	8,076	8,835	41.7%
Edit | Del	0015	12/31/2010	72	33,083	35,326	30.5%	16,495	20,439	15.6%
Edit | Del	0016	12/31/2011	60	75,969	74,381	61.4%	35,496	40,022	21.2%
Edit | Del	0017	12/31/2012	48	139,715	140,849	45.5%	68,886	74,159	25.6%
Edit | Del	0018	12/31/2013	36	234,781	243,390	26.5%	119,582	128,507	20.2%
Edit | Del	0019	12/31/2014	24	560,974	553,931	62.3%	314,895	350,974	2.9%
Edit | Del	0020	12/31/2015	12	764,210	0		1,205,957	0	

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Figure B.3 – Report on 2015 Commercial Auto Exposures

Stochastic Model Results

2015 Commercial Auto Exposure

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Stochastic Model Detail

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Model Name	2015 Commercial Auto Exposure	Assumption Owner	Reserving Actuary
Description	Expected Commercial Auto claim payments during 2015 for exposure periods prior to 2015 based on data generated by claims system as of 12/31/2015 relative to the 12/31/2014 actuarial assumptions.	Reports To	Chief Actuary
Assumption Value	Expected Value	Assumption Value Date	12/31/2014
Assumption Minimum	5.0%	Next Update Due	12/31/2015
Assumption Maximum	95.0%		

▼ Realized Value

Paid Actual	262,931	Incurred Actual	211,506
Paid Expected	232,199	Incurred Expected	161,054
Paid Percentile	98.9%	Incurred Percentile	100.0%

Edit | Delete | Clone

Stochastic Values

New Value

Help

Action	Number	Exposure Period	Age	Paid Actual	Paid Expected	Paid Percentile	Incurred Actual	Incurred Expected	Incurred Percentile
Edit | Del	0021	12/31/2006	120	543	571	57.9%	(47)	154	0.0%
Edit | Del	0022	12/31/2007	108	2,387	3,131	21.8%	1,040	448	82.8%
Edit | Del	0023	12/31/2008	96	1,177	1,665	33.5%	851	1,167	44.5%
Edit | Del	0024	12/31/2009	84	5,403	5,044	63.1%	2,954	1,669	86.1%
Edit | Del	0025	12/31/2010	72	14,120	11,061	91.1%	9,035	5,606	94.2%
Edit | Del	0026	12/31/2011	60	23,636	23,276	56.1%	16,524	11,960	93.9%
Edit | Del	0027	12/31/2012	48	51,020	45,272	86.7%	36,454	29,103	92.7%
Edit | Del	0028	12/31/2013	36	75,813	62,481	96.5%	61,541	44,392	99.3%
Edit | Del	0029	12/31/2014	24	88,832	79,698	86.1%	83,154	66,555	97.0%
Edit | Del	0030	12/31/2015	12	99,123	0		178,539	0	

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Figure B.4 – Report on 2015 Homeowners Exposures

Stochastic Model Results

2015 Homeowners Exposure

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Stochastic Model Detail Edit Delete Clone

Model Name	2015 Homeowners Exposure	Assumption Owner	Reserving Actuary
Description	Expected Homeowners claim payments during 2015 for exposure periods prior to 2015 based on data generated by claims system as of 12/31/2015 relative to the 12/31/2014 actuarial assumptions.	Reports To	Chief Actuary
Assumption Value	Expected Value	Assumption Value Date	12/31/2014
Assumption Minimum	5.0%	Next Update Due	12/31/2015
Assumption Maximum	95.0%		

▼ Realized Value

Paid Actual	237,087	Incurred Actual	63,836
Paid Expected	252,049	Incurred Expected	65,528
Paid Percentile	28.4%	Incurred Percentile	50.2%

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Stochastic Values New Value Help

Action	Number	Exposure Period	Age	Paid Actual	Paid Expected	Paid Percentile	Incurred Actual	Incurred Expected	Incurred Percentile
Edit | Del	0031	12/31/2006	120	26	773	13.9%	(132)	(95)	83.5%
Edit | Del	0032	12/31/2007	108	33	125	61.9%	(156)	59	31.4%
Edit | Del	0033	12/31/2008	96	227	414	57.2%	(1,359)	(349)	23.5%
Edit | Del	0034	12/31/2009	84	(176)	217	14.1%	(1,158)	(105)	18.5%
Edit | Del	0035	12/31/2010	72	3,800	1,911	85.6%	412	(482)	67.2%
Edit | Del	0036	12/31/2011	60	5,462	6,758	37.5%	(8)	1,119	12.2%
Edit | Del	0037	12/31/2012	48	12,197	9,961	74.9%	1,284	813	81.4%
Edit | Del	0038	12/31/2013	36	23,840	25,830	40.5%	8,785	12,274	37.9%
Edit | Del	0039	12/31/2014	24	191,678	206,060	28.0%	56,168	52,293	62.7%
Edit | Del	0040	12/31/2015	12	934,805	0		1,143,739	0	

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Figure B.5 – Report on 2015 Conditional Reserves

Stochastic Model Results

2015 Conditional Reserves by Segment

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Stochastic Model Detail Edit Delete Clone

Model Name	2015 Conditional Reserves by Segment	**Assumption Owner**	Chief Actuary
Description	Expected conditional reserves as of 12/31/2015 for exposure periods prior to 2015 based on data generated by claims system during CY 2015 relative to the 12/31/2014 actuarial assumptions.	**Reports To**	Chief Executive Officer
Assumption Value	Percentile of One-Year Horizon	**Assumption Value Date**	12/31/2014
Output Value	One-Year Reserve Estimate	**Next Update Due**	12/31/2015

▼ Realized Value

Sum of Yrs	(2,154)	**Sum of Yrs**	10,926	**Sum of Yrs**	72,619	**Sum of Yrs**	(35,107)
CY 2015	(2,086)	**CY 2015**	6,115	**CY 2015**	78,376	**CY 2015**	(40,385)
Overall Change:	Aggregation of All Segm	**Overall Change:**	Sum of Segments	**Largest Increase:**	CA	**Largest Decrease:**	PPA

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Stochastic Values New Value Help

Aggregation of All Segments

Action	Number	Exposure Period	Age	Original	Actual Paid	Current	Paid Percentile	Conditional	Change
Edit | Del	0001	12/31/2006	120	7,410	3,069	4,341	31.8%	2,539	(1,802)
Edit | Del	0002	12/31/2007	108	16,366	5,905	10,461	47.9%	11,349	888
Edit | Del	0003	12/31/2008	96	23,269	8,986	14,283	33.6%	10,961	(3,322)
Edit | Del	0004	12/31/2009	84	44,378	18,992	25,386	39.0%	21,615	(3,771)
Edit | Del	0005	12/31/2010	72	96,042	51,003	45,039	71.6%	49,308	4,269
Edit | Del	0006	12/31/2011	60	202,705	105,067	97,638	54.3%	97,157	(481)
Edit | Del	0007	12/31/2012	48	413,903	202,932	210,971	74.2%	222,250	11,279
Edit | Del	0008	12/31/2013	36	765,488	334,434	431,054	57.1%	427,667	(3,387)
Edit | Del	0009	12/31/2014	24	1,642,982	841,484	801,499	52.8%	795,671	(5,828)
Edit | Del	0010	SUM OF YRS		3,212,543	1,571,872	1,640,671		1,638,516	(2,154)
Edit | Del	0011	CY 2015		3,212,543	1,571,872	1,640,671	61.2%	1,638,584	(2,086)

Sum of All Segments

Action	Number	Exposure Period	Age	Original	Actual Paid	Current	Paid Percentile	Conditional	Change
Edit | Del	0012	12/31/2006	120	7,410	3,069	4,341	N/A	3,323	(1,018)
Edit | Del	0013	12/31/2007	108	16,366	5,905	10,461	N/A	10,669	208
Edit | Del	0014	12/31/2008	96	23,269	8,986	14,283	N/A	11,697	(2,586)
Edit | Del	0015	12/31/2009	84	44,378	18,992	25,386	N/A	20,075	(5,311)
Edit | Del	0016	12/31/2010	72	96,042	51,003	45,039	N/A	49,263	4,224
Edit | Del	0017	12/31/2011	60	202,705	105,067	97,638	N/A	97,412	(227)
Edit | Del	0018	12/31/2012	48	413,903	202,932	210,971	N/A	222,487	11,516
Edit | Del	0019	12/31/2013	36	765,488	334,434	431,054	N/A	425,174	(5,880)
Edit | Del	0020	12/31/2014	24	1,642,982	841,484	801,499	N/A	811,496	9,997
Edit | Del	0021	SUM OF YRS		3,212,543	1,571,872	1,640,671		1,651,596	10,926
Edit | Del	0022	CY 2015		3,212,543	1,571,872	1,640,671	N/A	1,646,786	6,115

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Figure B.5 – Report on 2015 Conditional Reserves (Cont.)

Stochastic Model Results

2015 Conditional Reserves by Segment

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Stochastic Model Detail

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Model Name	2015 Conditional Reserves by Segment	Assumption Owner	Chief Actuary
Description	Expected conditional reserves as of 12/31/2015 for exposure periods prior to 2015 based on data generated by claims system during CY 2015 relative to the 12/31/2014 actuarial assumptions.	Reports To	Chief Executive Officer
Assumption Value	Percentile of One-Year Horizon	Assumption Value Date	12/31/2014
Output Value	One-Year Reserve Estimate	Next Update Due	12/31/2015

▼ **Realized Value**

Edit Delete Clone

Stochastic Values New Value Help

Private Passenger Auto (PPA)

Action	Number	Exposure Period	Age	Original	Actual Paid	Current	Paid Percentile	Conditional	Change
Edit | Del	0023	12/31/2006	120	5,491	2,500	2,991	48.2%	2,680	(311)
Edit | Del	0024	12/31/2007	108	8,983	3,485	5,498	69.4%	7,248	1,750
Edit | Del	0025	12/31/2008	96	17,643	7,582	10,061	43.4%	8,654	(1,406)
Edit | Del	0026	12/31/2009	84	33,237	13,765	19,472	37.5%	15,635	(3,836)
Edit | Del	0027	12/31/2010	72	71,149	33,083	38,066	30.5%	31,595	(6,470)
Edit | Del	0028	12/31/2011	60	147,271	75,969	71,302	61.4%	73,359	2,057
Edit | Del	0029	12/31/2012	48	295,776	139,715	156,061	45.5%	151,670	(4,390)
Edit | Del	0030	12/31/2013	36	557,593	234,781	322,812	26.5%	292,882	(29,930)
Edit | Del	0031	12/31/2014	24	1,134,993	560,974	574,019	62.3%	581,448	7,430
Edit | Del	0032	SUM OF YRS		2,272,135	1,071,854	1,200,281		1,165,174	(35,107)
Edit | Del	0033	CY 2015		2,272,135	1,071,854	1,200,281	44.9%	1,159,897	(40,385)

Commercial Auto (CA)

Action	Number	Exposure Period	Age	Original	Actual Paid	Current	Paid Percentile	Conditional	Change
Edit | Del	0034	12/31/2006	120	1,146	543	603	57.9%	643	40
Edit | Del	0035	12/31/2007	108	6,629	2,387	4,242	21.8%	3,257	(985)
Edit | Del	0036	12/31/2008	96	3,759	1,177	2,582	33.5%	1,675	(907)
Edit | Del	0037	12/31/2009	84	9,524	5,403	4,121	63.1%	5,593	1,472
Edit | Del	0038	12/31/2010	72	20,752	14,120	6,632	91.1%	13,946	7,313
Edit | Del	0039	12/31/2011	60	43,077	23,636	19,441	56.1%	20,073	632
Edit | Del	0040	12/31/2012	48	96,462	51,020	45,442	86.7%	57,978	12,536
Edit | Del	0041	12/31/2013	36	157,440	75,813	81,627	96.5%	110,701	29,075
Edit | Del	0042	12/31/2014	24	235,978	88,832	147,146	86.1%	170,589	23,442
Edit | Del	0043	SUM OF YRS		574,768	262,931	311,837		384,456	72,619
Edit | Del	0044	CY 2015		574,768	262,931	311,837	98.9%	390,213	78,376

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Figure B.5 – Report on 2015 Conditional Reserves (Cont.)

Stochastic Model Results

2015 Conditional Reserves by Segment

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Stochastic Model Detail

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Model Name	2015 Conditional Reserves by Segment	Assumption Owner	Chief Actuary
Description	Expected conditional reserves as of 12/31/2015 for exposure periods prior to 2015 based on data generated by claims system during CY 2015 relative to the 12/31/2014 actuarial assumptions.	Reports To	Chief Executive Officer
Assumption Value	Percentile of One-Year Horizon	Assumption Value Date	12/31/2014
Output Value	One-Year Reserve Estimate	Next Update Due	12/31/2015

▼ Realized Value

Edit | Delete | Clone

Stochastic Values

New Value

Help

Homeowners (HO)									
Action	Number	Exposure Period	Age	Original	Actual Paid	Current	Paid Percentile	Conditional	Change
Edit | Del	0045	12/31/2006	120	773	26	747	13.9%	0	(747)
Edit | Del	0046	12/31/2007	108	754	33	721	61.9%	164	(557)
Edit | Del	0047	12/31/2008	96	1,867	227	1,640	57.2%	1,367	(272)
Edit | Del	0048	12/31/2009	84	1,617	(176)	1,793	14.1%	(1,153)	(2,946)
Edit | Del	0049	12/31/2010	72	4,140	3,800	340	85.6%	3,722	3,381
Edit | Del	0050	12/31/2011	60	12,356	5,462	6,894	37.5%	3,979	(2,915)
Edit | Del	0051	12/31/2012	48	21,665	12,197	9,468	74.9%	12,839	3,370
Edit | Del	0052	12/31/2013	36	50,455	23,840	26,615	40.5%	21,590	(5,024)
Edit | Del	0053	12/31/2014	24	272,011	191,678	80,333	28.0%	59,458	(20,875)
Edit | Del	0054	SUM OF YRS		365,640	237,087	128,553		101,967	(26,586)
Edit | Del	0055	CY 2015		365,640	237,087	128,553	28.4%	96,676	(31,876)

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Appendix C – Back-Testing Results for Private Passenger Auto

Table C.1 – Calculation of Weighted Ultimate (Deterministic)

Sample Insurance Company
Private Passenger Auto
Calculation of Weighted Ultimate as of December 31, 2014

		Ultimate Values by Method				Weights by Method				Weighted
AY	Age	Paid CL	Inc CL	Paid BF	Inc BF	Paid CL	Inc CL	Paid BF	Inc BF	Ultimate
2006	108	1,218,574	1,218,574	1,218,578	1,218,577	50.0%	50.0%	0.0%	0.0%	1,218,574
2007	96	1,376,278	1,375,860	1,376,284	1,375,866	50.0%	50.0%	0.0%	0.0%	1,376,069
2008	84	1,439,598	1,439,241	1,439,624	1,439,261	50.0%	50.0%	0.0%	0.0%	1,439,420
2009	72	1,561,673	1,558,592	1,561,726	1,558,664	50.0%	50.0%	0.0%	0.0%	1,560,133
2010	60	1,649,696	1,645,907	1,649,700	1,646,004	50.0%	50.0%	0.0%	0.0%	1,647,802
2011	48	1,669,252	1,665,339	1,670,112	1,665,994	50.0%	50.0%	0.0%	0.0%	1,667,295
2012	36	1,746,970	1,739,396	1,750,509	1,741,935	25.0%	25.0%	25.0%	25.0%	1,744,703
2013	24	1,841,516	1,816,296	1,855,755	1,827,462	0.0%	0.0%	50.0%	50.0%	1,841,608
2014	12	1,897,487	1,829,829	1,944,009	1,877,128	0.0%	0.0%	50.0%	50.0%	1,910,569
Totals		14,401,045	14,289,034	14,466,298	14,350,890					14,406,172

Table C.2 – Reconciliation of Total Unpaid (Deterministic)

Sample Insurance Company
Private Passenger Auto
Total Unpaid Reconciliation as of December 31, 2014

AY	Age	Paid to Date	Incurred to Date	Weighted Ultimate	Case Reserve	IBNR	Total Unpaid	Selected Ultimate	Selected IBNR	Total Unpaid
2006	108	1,213,083	1,214,471	1,218,574	1,388	4,103	5,491	1,218,574	4,103	5,491
2007	96	1,367,086	1,369,955	1,376,069	2,869	6,114	8,983	1,376,069	6,114	8,983
2008	84	1,421,777	1,427,920	1,439,420	6,143	11,500	17,643	1,439,420	11,500	17,643
2009	72	1,526,896	1,538,117	1,560,133	11,221	22,016	33,237	1,560,133	22,016	33,237
2010	60	1,576,653	1,604,722	1,647,802	28,069	43,080	71,149	1,647,802	43,080	71,149
2011	48	1,520,024	1,584,626	1,667,295	64,602	82,669	147,271	1,667,295	82,669	147,271
2012	36	1,448,927	1,583,503	1,744,703	134,576	161,200	295,776	1,744,703	161,200	295,776
2013	24	1,284,015	1,535,603	1,841,608	251,588	306,005	557,593	1,841,608	306,005	557,593
2014	12	775,576	1,238,406	1,910,569	462,830	672,163	1,134,993	1,910,569	672,163	1,134,993
Totals		12,134,037	13,097,323	14,406,172	963,286	1,308,849	2,272,135	14,406,172	1,308,849	2,272,135

Table C.3 – Expected Incremental Development – Paid (Deterministic)

Sample Insurance Company
Private Passenger Auto -- Paid Data
Expected Incremental Future Development as of December 31, 2014

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2006										2,742	2,749	5,491
2007									2,783	3,097	3,104	8,983
2008								8,029	3,128	3,239	3,247	17,643
2009							13,923	8,893	3,390	3,511	3,519	33,237
2010						34,453	16,297	9,393	3,581	3,708	3,717	71,149
2011					73,449	36,693	16,490	9,504	3,623	3,752	3,761	147,271
2012				139,035	79,111	38,585	17,340	9,994	3,810	3,946	3,955	295,776
2013			237,853	152,195	84,565	41,245	18,536	10,683	4,073	4,218	4,227	557,593
2014		547,018	256,629	157,719	87,634	42,742	19,208	11,071	4,220	4,371	4,381	1,134,993

Table C.4 – Expected Incremental Development – Incurred (Deterministic)

Sample Insurance Company
Private Passenger Auto -- Incurred Data
Expected Incremental Future Development as of December 31, 2014

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2006										2,050	2,053	4,103
2007									1,481	2,315	2,319	6,114
2008								5,322	1,331	2,421	2,425	11,500
2009							9,743	5,576	1,443	2,624	2,629	22,016
2010						21,433	8,685	5,890	1,524	2,772	2,776	43,080
2011					40,949	19,818	8,788	5,959	1,542	2,805	2,809	82,669
2012				76,014	41,204	20,892	9,264	6,282	1,626	2,957	2,962	161,200
2013			135,434	78,332	44,616	22,622	10,031	6,802	1,760	3,201	3,207	306,005
2014		361,322	130,571	82,786	47,153	23,908	10,601	7,189	1,860	3,383	3,389	672,163

Table C.5 – Actual vs. Expected Back-test (Deterministic)

Sample Insurance Company Private Passenger Auto Deterministic Actual vs. Expected as of December 31, 2015							
AY	Age	Actual Paid	Expected Paid	Difference	Actual Incurred	Expected Incurred	Difference
2006	120	2,500	2,742	(242)	2,042	2,050	(8)
2007	108	3,485	2,783	702	2,261	1,481	780
2008	96	7,582	8,029	(447)	4,061	5,322	(1,261)
2009	84	13,765	13,923	(158)	8,076	9,743	(1,667)
2010	72	33,083	34,453	(1,370)	16,495	21,433	(4,938)
2011	60	75,969	73,449	2,520	35,496	40,949	(5,453)
2012	48	139,715	139,035	680	68,886	76,014	(7,128)
2013	36	234,781	237,853	(3,072)	119,582	135,434	(15,852)
2014	24	560,974	547,018	13,956	314,895	361,322	(46,427)
2015	12	764,210			1,205,957		
Totals		1,836,064			1,777,751		
AY<CY		1,071,854	1,059,284	12,569	571,794	653,748	(81,954)

Table C.6 – Actual to Range of Estimates Back-test (Deterministic)

Sample Insurance Company Private Passenger Auto Deterministic Actual vs. Method Range as of December 31, 2015									
AY	Age	Actual Paid	Paid Minimum	Paid Maximum	Range Percent	Actual Incurred	Incurred Minimum	Incurred Maximum	Difference
2006	120	2,500	2,742	2,744	-12977.0%	2,042	2,050	2,052	-332.1%
2007	108	3,485	2,574	2,993	217.7%	2,261	1,272	1,691	236.3%
2008	96	7,582	7,851	8,218	-73.5%	4,061	5,144	5,515	-291.9%
2009	84	13,765	12,402	15,469	44.5%	8,076	8,215	11,282	-4.5%
2010	72	33,083	32,601	36,307	13.0%	16,495	19,564	23,302	-82.1%
2011	60	75,969	71,579	75,753	105.2%	35,496	39,041	43,372	-81.8%
2012	48	139,715	134,970	143,551	55.3%	68,886	71,591	80,910	-29.0%
2013	36	234,781	222,411	249,543	45.6%	119,582	117,907	148,270	5.5%
2014	24	560,974	500,290	570,167	86.8%	314,895	308,639	389,322	7.8%
2015	12	764,210				1,205,957			
Totals		1,836,064				1,777,751			
AY<CY		1,071,854	987,421	1,104,745	72.0%	571,794	573,423	705,671	-1.2%

Table C.7 – Estimated Unpaid Claims by Accident Year (Stochastic)

Sample Insurance Company Private Passenger Auto Stochastic Estimates as of December 31, 2014 Estimated Unpaid Claims by Accident Year											
AY	Mean	Std Dev	CoV	Min	Max	5%	25%	Median	Mode	75%	95%
2006	5,491	2,751	50.1%	19	16,929	1,188	3,538	5,318	19	7,256	10,281
2007	8,983	3,423	38.1%	(395)	27,201	3,633	6,557	8,844	13,467	11,195	14,917
2008	17,643	4,155	23.6%	5,353	34,375	11,018	14,771	17,448	14,798	20,330	24,790
2009	33,237	5,245	15.8%	15,269	60,704	24,910	29,619	33,085	32,036	36,639	42,225
2010	71,149	6,902	9.7%	48,314	99,369	60,123	66,324	71,033	72,699	75,783	82,763
2011	147,271	9,088	6.2%	114,275	187,688	132,806	141,043	147,027	142,651	153,290	162,219
2012	295,776	14,568	4.9%	244,570	348,069	272,495	285,945	295,225	281,357	305,146	320,628
2013	557,593	25,394	4.6%	457,369	651,838	516,980	540,414	556,720	552,490	574,475	599,860
2014	1,134,993	46,822	4.1%	973,312	1,337,053	1,062,388	1,102,616	1,132,386	1,181,722	1,165,441	1,216,110
Total	2,272,135	59,102	2.6%	2,064,755	2,479,344	2,177,063	2,231,575	2,270,627	2,295,340	2,311,669	2,371,532

Table C.8 – Estimated Claims Paid by Calendar Year (Stochastic)

Sample Insurance Company
Private Passenger Auto
Stochastic Estimates as of December 31, 2014
Estimated Paid Claims by Calendar Year

CY	Mean	Std Dev	CoV	Min	Max	5%	25%	Median	Mode	75%	95%
2015	1,076,388	31,344	2.9%	949,483	1,213,672	1,025,966	1,054,657	1,075,871	1,048,875	1,096,712	1,129,462
2016	551,046	19,390	3.5%	479,596	631,486	519,806	537,516	550,967	553,695	564,102	582,949
2017	311,957	13,916	4.5%	259,341	367,185	289,477	302,543	311,686	316,778	321,297	335,118
2018	163,631	9,937	6.1%	130,776	200,970	147,538	156,774	163,477	162,064	170,225	180,340
2019	80,988	7,270	9.0%	52,760	116,518	69,328	76,043	80,859	84,649	85,870	93,146
2020	40,653	5,645	13.9%	20,217	62,342	31,712	36,714	40,478	39,787	44,381	50,138
2021	22,548	4,548	20.2%	7,784	40,869	15,431	19,416	22,362	21,178	25,499	30,348
2022	12,196	3,877	31.8%	(166)	29,026	6,142	9,531	12,012	8,133	14,672	18,808
2023	8,412	3,700	44.0%	(121)	27,344	2,614	5,876	8,238	(121)	10,742	14,779
2024	4,316	2,311	53.6%	(50)	15,575	764	2,652	4,155	(50)	5,756	8,407
Total	2,272,135	59,102	2.6%	2,064,755	2,479,344	2,177,063	2,231,575	2,270,627	2,295,340	2,311,669	2,371,532

Table C.9 – Mean Future Incremental – Paid (Stochastic)

Sample Insurance Company
Private Passenger Auto - Paid
Mean Future Incremental as of December 31, 2014

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2006										2,733	2,758	5,491
2007									2,908	3,022	3,053	8,983
2008								8,098	3,080	3,226	3,239	17,643
2009							14,773	8,493	3,216	3,363	3,392	33,237
2010						35,326	15,895	9,164	3,479	3,614	3,670	71,149
2011					74,381	36,251	16,246	9,369	3,594	3,713	3,719	147,271
2012				140,849	78,253	38,124	17,114	9,886	3,733	3,891	3,925	295,776
2013			243,390	149,664	83,084	40,493	18,186	10,534	3,985	4,107	4,150	557,593
2014		553,931	253,630	155,843	86,574	42,317	19,004	10,953	4,164	4,262	4,316	1,134,993

Table C.10 – Standard Deviation of Future Incremental – Paid (Stochastic)

Sample Insurance Company
Private Passenger Auto - Paid
Standard Deviation Future Incremental as of December 31, 2014

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2006										1,534	1,543	2,751
2007									1,496	1,721	1,722	3,423
2008								2,135	1,567	1,785	1,763	4,155
2009							2,748	2,262	1,679	1,864	1,895	5,245
2010						4,154	2,887	2,321	1,745	1,952	1,988	6,902
2011					5,827	4,105	2,892	2,358	1,770	1,987	2,013	9,088
2012				8,864	6,479	4,403	3,076	2,516	1,860	2,084	2,091	14,568
2013			13,598	9,804	6,879	4,728	3,270	2,652	1,990	2,215	2,225	25,394
2014		25,362	14,095	10,125	7,121	4,866	3,297	2,703	2,032	2,275	2,311	46,822

Table C.11 – Coefficient of Variation of Future Incremental – Paid (Stochastic)

Sample Insurance Company
Private Passenger Auto - Paid
CoV Future Incremental as of December 31, 2014

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2006										56.1%	55.9%	50.1%
2007									51.4%	57.0%	56.4%	38.1%
2008								26.4%	50.9%	55.3%	54.4%	23.6%
2009							18.6%	26.6%	52.2%	55.4%	55.9%	15.8%
2010						11.8%	18.2%	25.3%	50.2%	54.0%	54.2%	9.7%
2011					7.8%	11.3%	17.8%	25.2%	49.3%	53.5%	54.1%	6.2%
2012				6.3%	8.3%	11.5%	18.0%	25.5%	49.8%	53.5%	53.3%	4.9%
2013			5.6%	6.6%	8.3%	11.7%	18.0%	25.2%	49.9%	53.9%	53.6%	4.6%
2014		4.6%	5.6%	6.5%	8.2%	11.5%	17.3%	24.7%	48.8%	53.4%	53.6%	4.1%

Table C.12 – Estimated Unpaid Claims by Accident Year in 2015 (Stochastic)

Sample Insurance Company Private Passenger Auto - Paid Stochastic Estimates as of December 31, 2014 Estimated Unpaid Claims by Accident Year, Calendar Year 2015 Only											
AY	Mean	Std Dev	CoV	Min	Max	5%	25%	Median	Mode	75%	95%
2006	2,733	1,534	56.1%	9	9,689	444	1,629	2,563	9	3,697	5,509
2007	2,908	1,496	51.4%	(269)	10,441	750	1,873	2,750	(252)	3,766	5,640
2008	8,098	2,135	26.4%	1,608	20,022	4,867	6,616	7,934	8,649	9,413	11,850
2009	14,773	2,748	18.6%	6,175	26,858	10,506	12,878	14,607	13,421	16,523	19,567
2010	35,326	4,154	11.8%	19,713	52,817	28,828	32,396	35,169	36,788	38,033	42,514
2011	74,381	5,827	7.8%	52,662	98,238	65,082	70,380	74,239	70,540	78,233	84,209
2012	140,849	8,864	6.3%	105,135	178,702	126,665	134,837	140,706	140,360	146,614	155,792
2013	243,390	13,598	5.6%	189,263	302,308	221,056	234,122	243,174	238,506	252,536	266,186
2014	553,931	25,362	4.6%	462,086	667,072	513,991	536,419	553,004	547,742	570,306	597,839
Total	1,076,388	31,344	2.9%	949,483	1,213,672	1,025,966	1,054,657	1,075,871	1,048,875	1,096,712	1,129,462

Table C.13 – Actual vs. Expected Back-test & Conditional Reserve (Stochastic)

Sample Insurance Company Private Passenger Auto Stochastic Actual vs. Expected as of December 31, 2015										
AY	Age	Actual Paid	Expected Paid	Percentile	Actual Incurred	Expected Incurred	Percentile	Conditional Reserve	Expected Reserve	Change
2006	120	2,500	2,733	48.2%	2,042	2,056	56.7%	2,680	2,991	(311)
2007	108	3,485	2,908	69.4%	2,261	1,312	81.0%	7,248	5,498	1,750
2008	96	7,582	8,098	43.4%	4,061	5,207	33.2%	8,654	10,061	(1,406)
2009	84	13,765	14,773	37.5%	8,076	8,835	41.7%	15,635	19,472	(3,836)
2010	72	33,083	35,326	30.5%	16,495	20,439	15.6%	31,595	38,066	(6,470)
2011	60	75,969	74,381	61.4%	35,496	40,022	21.2%	73,359	71,302	2,057
2012	48	139,715	140,849	45.5%	68,886	74,159	25.6%	151,670	156,061	(4,390)
2013	36	234,781	243,390	26.5%	119,582	128,507	20.2%	292,882	322,812	(29,930)
2014	24	560,974	553,931	62.3%	314,895	350,974	2.9%	581,448	574,019	7,430
2015	12	764,210			1,205,957					
Totals		1,836,064			1,777,751			1,165,174	1,200,281	(35,107)
AY<CY		1,071,854	1,076,388	44.9%	571,794	631,511	0.6%	1,159,897	1,200,281	(40,385)

Figure C.1 – Graph of KPI Thresholds by Accident Year – Paid (Stochastic)

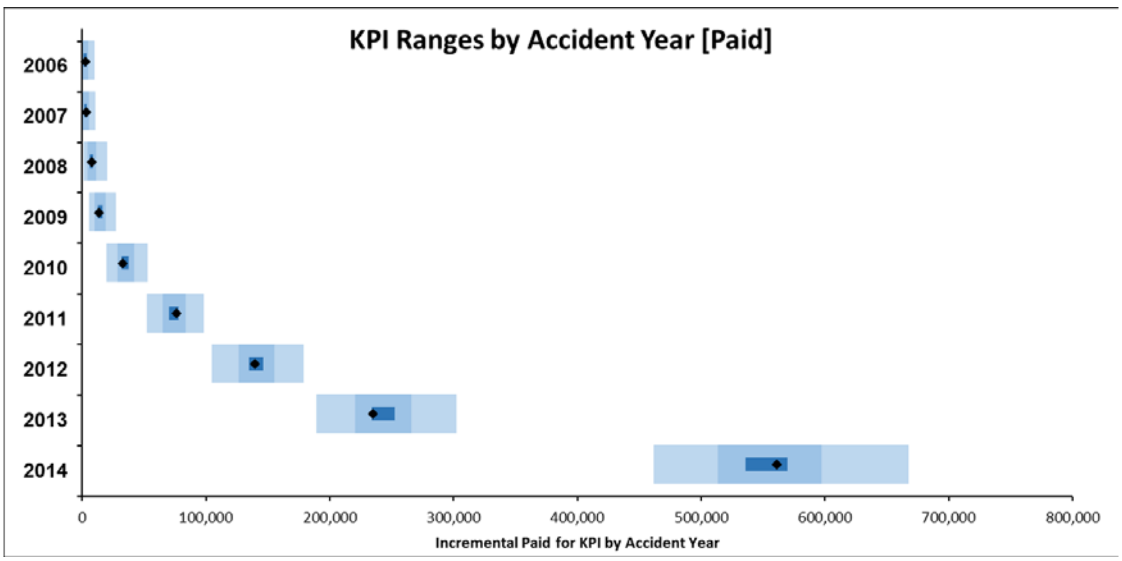


Figure C.2 – Graph of KPI Thresholds by Calendar Year – Paid (Stochastic)

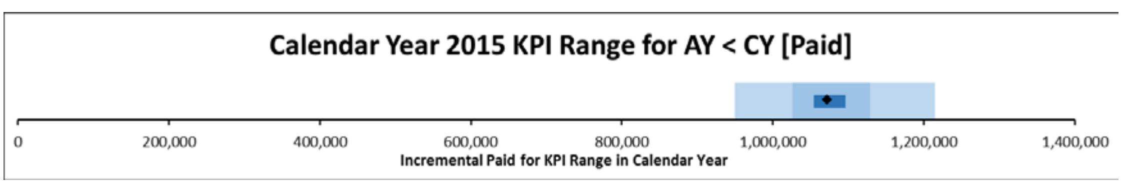


Figure C.3 – Graph of KPI Thresholds by Accident Year – Incurred (Stochastic)

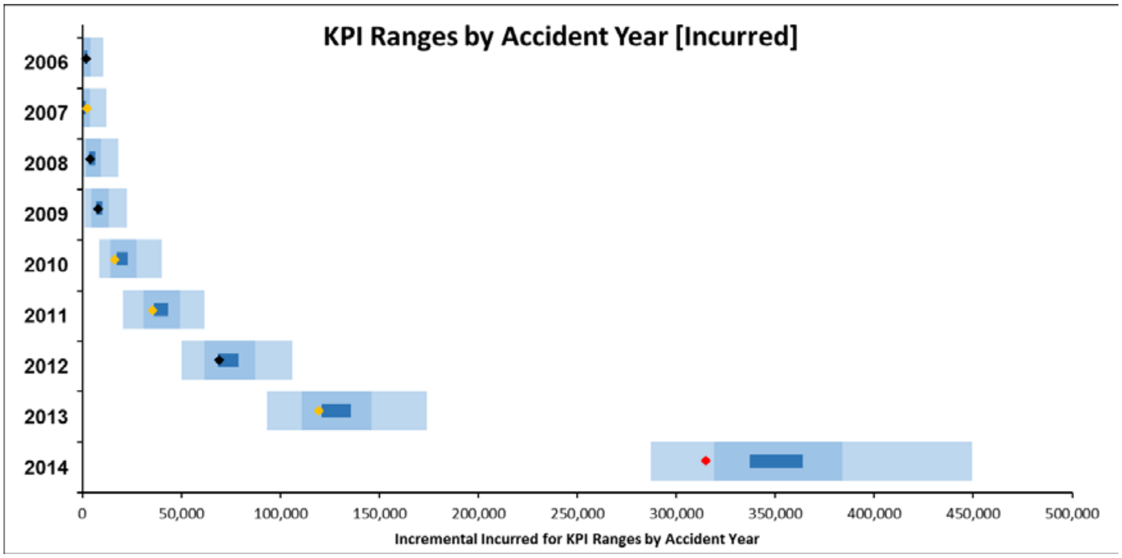


Figure C.4 – Graph of KPI Thresholds by Calendar Year – Incurred (Stochastic)

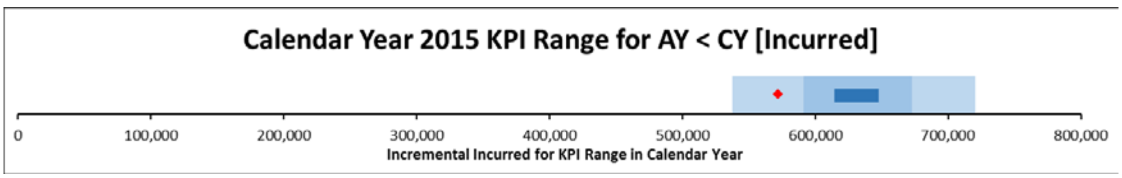


Figure C.5 – Graph of Realized Values vs. Assumptions – Paid (Stochastic)

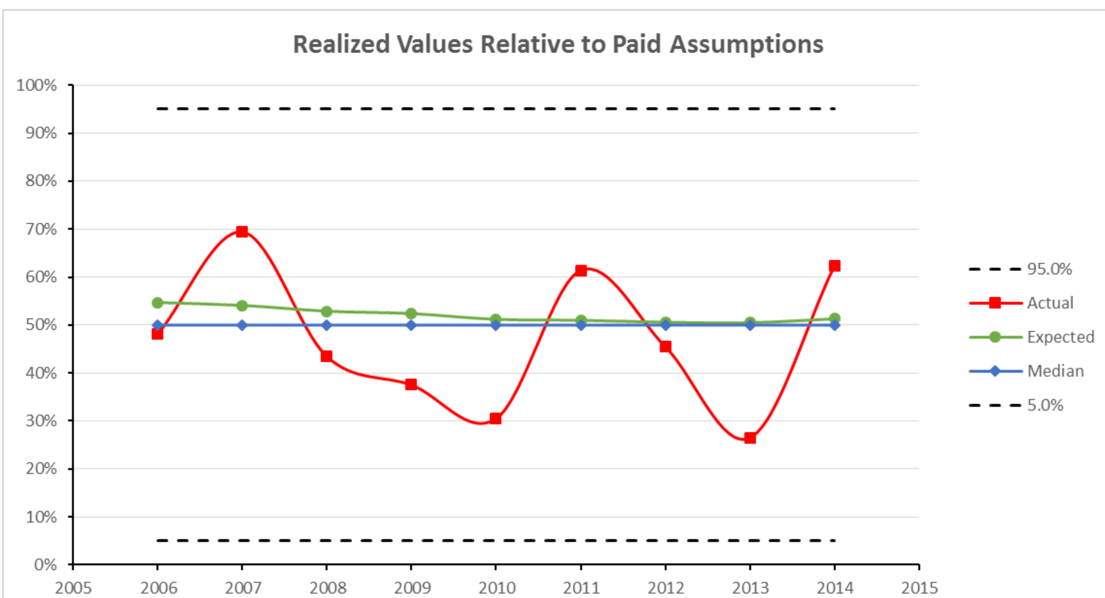
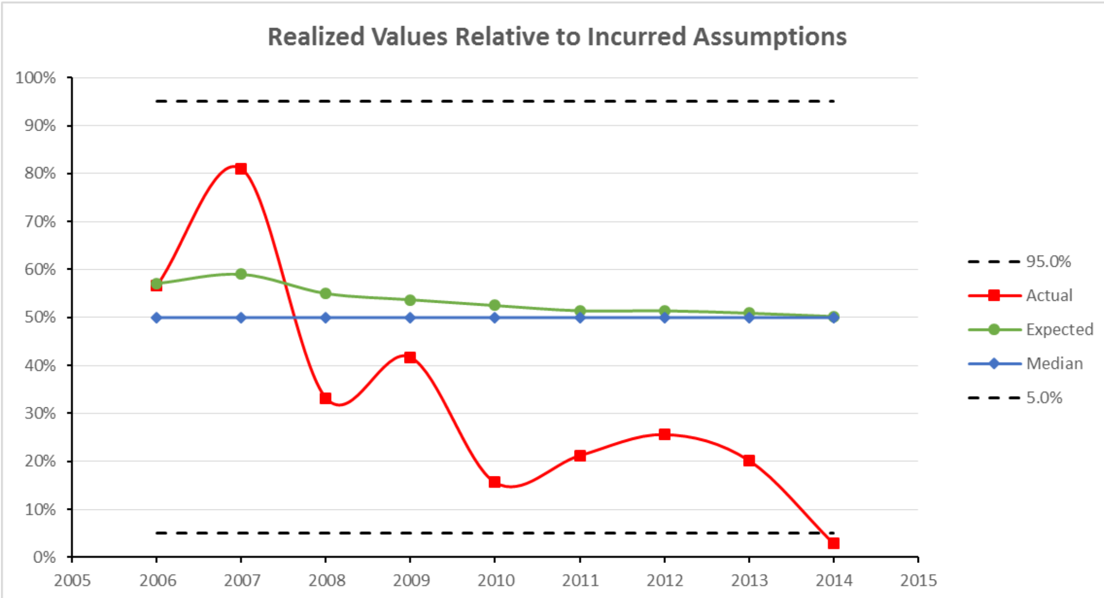


Figure C.6 – Graph of Realized Values vs. Assumptions – Incurred (Stochastic)



Realized Values Relative to Incurred Assumptions
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
95.0%
Actual
Expected
Median
5.0%

Appendix D – Back-Testing Results for Commercial Auto

Table D.1 – Calculation of Weighted Ultimate (Deterministic)

Sample Insurance Company
Commercial Auto
Calculation of Weighted Ultimate as of December 31, 2014

		Ultimate Values by Method				Weights by Method				Weighted
AY	Age	Paid CL	Inc CL	Paid BF	Inc BF	Paid CL	Inc CL	Paid BF	Inc BF	Ultimate
2006	108	258,835	258,835	258,837	258,836	50.0%	50.0%	0.0%	0.0%	258,835
2007	96	267,103	271,591	267,143	271,592	50.0%	50.0%	0.0%	0.0%	269,347
2008	84	243,981	244,137	243,991	244,141	50.0%	50.0%	0.0%	0.0%	244,059
2009	72	267,942	269,784	267,999	269,783	50.0%	50.0%	0.0%	0.0%	268,863
2010	60	290,475	292,079	290,608	292,092	50.0%	50.0%	0.0%	0.0%	291,277
2011	48	288,645	288,592	288,785	288,669	50.0%	50.0%	0.0%	0.0%	288,618
2012	36	335,023	338,775	335,956	338,702	25.0%	25.0%	25.0%	25.0%	337,114
2013	24	333,220	337,698	333,662	336,635	0.0%	0.0%	50.0%	50.0%	335,149
2014	12	357,305	360,286	338,097	344,953	0.0%	0.0%	50.0%	50.0%	341,525
Totals		2,642,529	2,661,779	2,625,078	2,645,402					2,634,788

Table D.2 – Reconciliation of Total Unpaid (Deterministic)

Sample Insurance Company
Commercial Auto
Total Unpaid Reconciliation as of December 31, 2014

AY	Age	Paid to Date	Incurred to Date	Weighted Ultimate	Case Reserve	IBNR	Total Unpaid	Selected Ultimate	Selected IBNR	Total Unpaid
2006	108	257,689	258,524	258,835	835	311	1,146	258,835	311	1,146
2007	96	264,871	270,758	269,347	5,887	(1,411)	4,476	271,500	742	6,629
2008	84	240,300	242,171	244,059	1,871	1,888	3,759	244,059	1,888	3,759
2009	72	259,339	265,496	268,863	6,157	3,367	9,524	268,863	3,367	9,524
2010	60	270,525	281,376	291,277	10,851	9,901	20,752	291,277	9,901	20,752
2011	48	245,541	266,101	288,618	20,560	22,517	43,077	288,618	22,517	43,077
2012	36	240,652	282,394	337,114	41,742	54,720	96,462	337,114	54,720	96,462
2013	24	177,709	235,983	335,149	58,274	99,166	157,440	335,149	99,166	157,440
2014	12	105,547	177,611	341,525	72,064	163,914	235,978	341,525	163,914	235,978
Totals		2,062,173	2,280,414	2,634,788	218,241	354,374	572,615	2,636,941	356,527	574,768

Table D.3 – Expected Incremental Development – Paid (Deterministic)

Sample Insurance Company
Commercial Auto -- Paid Data
Expected Incremental Future Development as of December 31, 2014

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2006										572	574	1,146
2007									4,863	882	884	6,629
2008								1,720	959	540	541	3,759
2009							5,468	1,810	1,056	595	596	9,524
2010						11,401	4,957	1,961	1,144	644	646	20,752
2011					23,255	10,556	4,912	1,943	1,134	638	640	43,077
2012				45,941	27,285	12,374	5,758	2,277	1,329	748	750	96,462
2013			62,890	44,425	27,071	12,277	5,712	2,259	1,319	742	744	157,440
2014		80,388	61,679	44,125	26,889	12,194	5,674	2,244	1,310	737	739	235,978

Table D.4 – Expected Incremental Development – Incurred (Deterministic)

Sample Insurance Company
Commercial Auto -- Incurred Data
Expected Incremental Future Development as of December 31, 2014

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2006										155	156	311
2007									912	(85)	(85)	742
2008								1,140	455	147	147	1,888
2009							1,202	1,341	502	161	162	3,367
2010						5,271	2,284	1,452	544	175	175	9,901
2011					11,941	5,989	2,263	1,439	539	173	173	22,517
2012				28,462	13,911	6,991	2,642	1,680	629	202	202	54,720
2013			43,797	29,442	13,736	6,903	2,609	1,659	621	200	200	99,166
2014		65,492	44,040	28,917	13,491	6,780	2,562	1,629	610	196	196	163,914

Table D.5 – Actual vs. Expected Back-test (Deterministic)

Sample Insurance Company Commercial Auto Deterministic Actual vs. Expected as of December 31, 2015							
AY	Age	Actual Paid	Expected Paid	Difference	Actual Incurred	Expected Incurred	Difference
2006	120	543	572	(29)	(47)	155	(202)
2007	108	2,387	4,863	(2,476)	1,040	912	128
2008	96	1,177	1,720	(543)	851	1,140	(289)
2009	84	5,403	5,468	(65)	2,954	1,202	1,752
2010	72	14,120	11,401	2,719	9,035	5,271	3,764
2011	60	23,636	23,255	381	16,524	11,941	4,583
2012	48	51,020	45,941	5,079	36,454	28,462	7,992
2013	36	75,813	62,890	12,923	61,541	43,797	17,744
2014	24	88,832	80,388	8,444	83,154	65,492	17,662
2015	12	99,123			178,539		
Totals		362,054			390,045		
AY<CY		262,931	236,497	26,434	211,506	158,372	53,134

Table D.6 – Actual to Range of Estimates Back-test (Deterministic)

Sample Insurance Company Commercial Auto Deterministic Actual vs. Method Range as of December 31, 2015									
AY	Age	Actual Paid	Paid Minimum	Paid Maximum	Range Percent	Actual Incurred	Incurred Minimum	Incurred Maximum	Difference
2006	120	543	572	573	-1947.6%	(47)	155	157	-11482.4%
2007	108	2,387	2,629	7,097	-5.4%	1,040	(1,329)	3,154	52.8%
2008	96	1,177	1,642	1,797	-300.2%	851	1,062	1,220	-133.1%
2009	84	5,403	4,560	6,375	46.4%	2,954	288	2,116	145.9%
2010	72	14,120	10,624	12,177	225.1%	9,035	4,482	6,067	287.2%
2011	60	23,636	23,230	23,355	323.6%	16,524	11,915	12,068	3013.1%
2012	48	51,020	44,341	47,533	209.3%	36,454	26,520	29,980	287.1%
2013	36	75,813	61,648	64,865	440.3%	61,541	41,780	45,513	529.3%
2014	24	88,832	78,521	86,597	127.7%	83,154	63,052	74,156	181.0%
2015	12	99,123				178,539			
Totals		362,054				390,045			
AY<CY		262,931	228,631	250,242	158.7%	211,506	149,974	174,267	253.3%

Table D.7 – Estimated Unpaid Claims by Accident Year (Stochastic)

Sample Insurance Company Commercial Auto Stochastic Estimates as of December 31, 2014 Estimated Unpaid Claims by Accident Year											
AY	Mean	Std Dev	CoV	Min	Max	5%	25%	Median	Mode	75%	95%
2006	1,146	814	71.0%	(10)	5,794	78	535	1,001	(10)	1,614	2,674
2007	6,629	1,224	18.5%	4,226	12,888	4,900	5,718	6,480	5,217	7,369	8,901
2008	3,759	1,453	38.6%	301	11,438	1,635	2,703	3,633	2,931	4,649	6,345
2009	9,524	2,142	22.5%	3,182	20,485	6,275	8,015	9,377	10,379	10,869	13,349
2010	20,752	3,200	15.4%	10,281	35,184	15,708	18,540	20,585	18,785	22,831	26,235
2011	43,077	4,575	10.6%	26,937	64,990	35,935	39,920	42,912	45,008	46,064	50,902
2012	96,462	8,635	9.0%	64,159	131,809	82,929	90,631	96,052	94,959	101,869	111,214
2013	157,440	14,252	9.1%	106,918	218,146	134,900	147,693	157,063	161,109	166,699	181,556
2014	235,978	20,115	8.5%	165,204	320,049	204,296	222,059	235,235	228,038	249,252	269,810
Total	574,768	27,218	4.7%	472,897	687,879	530,792	556,111	574,426	558,264	592,649	620,040

Table D.8 – Estimated Claims Paid by Calendar Year (Stochastic)

Sample Insurance Company Commercial Auto Stochastic Estimates as of December 31, 2014 Estimated Paid Claims by Calendar Year											
CY	Mean	Std Dev	CoV	Min	Max	5%	25%	Median	Mode	75%	95%
2015	232,199	12,743	5.5%	186,133	286,448	211,733	223,345	231,854	239,707	240,793	253,653
2016	155,214	10,078	6.5%	123,220	202,461	138,975	148,466	154,950	152,408	161,829	172,239
2017	94,488	7,627	8.1%	67,914	124,583	82,240	89,213	94,253	97,115	99,485	107,381
2018	49,452	5,311	10.7%	33,520	73,129	40,823	45,820	49,320	49,423	52,929	58,355
2019	22,776	3,557	15.6%	10,658	37,548	17,087	20,273	22,624	21,106	25,137	28,853
2020	10,624	2,554	24.0%	2,401	21,272	6,697	8,827	10,460	11,167	12,231	15,060
2021	4,974	1,804	36.3%	522	13,768	2,328	3,680	4,783	5,419	6,057	8,218
2022	2,823	1,412	50.0%	(123)	11,759	872	1,773	2,649	2,360	3,651	5,416
2023	1,476	950	64.4%	8	7,844	222	771	1,325	8	2,002	3,244
2024	741	621	83.8%	4	4,737	28	275	596	4	1,045	1,956
Total	574,768	27,218	4.7%	472,897	687,879	530,792	556,111	574,426	558,264	592,649	620,040

Table D.9 – Mean Future Incremental – Paid (Stochastic)

Sample Insurance Company Commercial Auto - Paid Mean Future Incremental as of December 31, 2014												
AY	12	24	36	48	60	72	84	96	108	120	132	Total
2006										571	575	1,146
2007									3,131	1,735	1,763	6,629
2008								1,665	983	557	555	3,759
2009							5,044	1,988	1,170	657	666	9,524
2010						11,061	5,146	2,028	1,189	658	672	20,752
2011					23,276	10,564	4,895	1,925	1,135	636	646	43,077
2012				45,272	27,668	12,508	5,837	2,304	1,348	757	768	96,462
2013			62,481	44,600	27,194	12,354	5,746	2,265	1,308	744	746	157,440
2014		79,698	61,955	44,373	26,936	12,267	5,703	2,264	1,311	730	741	235,978

Table D.10 – Standard Deviation of Future Incremental – Paid (Stochastic)

Sample Insurance Company Commercial Auto - Paid Standard Deviation Future Incremental as of December 31, 2014												
AY	12	24	36	48	60	72	84	96	108	120	132	Total
2006										515	519	814
2007									881	534	538	1,224
2008								908	826	500	500	1,453
2009							1,465	990	879	523	533	2,142
2010						2,208	1,565	1,042	912	547	559	3,200
2011					3,189	2,197	1,559	1,027	908	563	556	4,575
2012				5,203	3,869	2,573	1,795	1,181	1,062	626	625	8,635
2013			7,006	5,566	4,081	2,625	1,792	1,197	1,056	629	634	14,252
2014		8,276	6,947	5,516	4,013	2,599	1,783	1,182	1,064	623	621	20,115

Table D.11 – Coefficient of Variation of Future Incremental – Paid (Stochastic)

Sample Insurance Company Commercial Auto - Paid CoV Future Incremental as of December 31, 2014												
AY	12	24	36	48	60	72	84	96	108	120	132	Total
2006										90.1%	90.2%	71.0%
2007									28.2%	30.8%	30.5%	18.5%
2008								54.6%	84.0%	89.8%	90.1%	38.6%
2009							29.0%	49.8%	75.2%	79.6%	80.0%	22.5%
2010						20.0%	30.4%	51.4%	76.7%	83.2%	83.2%	15.4%
2011					13.7%	20.8%	31.8%	53.4%	80.0%	88.5%	86.1%	10.6%
2012				11.5%	14.0%	20.6%	30.7%	51.3%	78.8%	82.7%	81.3%	9.0%
2013			11.2%	12.5%	15.0%	21.2%	31.2%	52.8%	80.8%	84.5%	84.9%	9.1%
2014		10.4%	11.2%	12.4%	14.9%	21.2%	31.3%	52.2%	81.2%	85.4%	83.8%	8.5%

Table D.12 – Estimated Unpaid Claims by Accident Year in 2015 (Stochastic)

Sample Insurance Company
Commercial Auto - Paid
Stochastic Estimates as of December 31, 2014
Estimated Unpaid Claims by Accident Year, Calendar Year 2015 Only

AY	Mean	Std Dev	CoV	Min	Max	5%	25%	Median	Mode	75%	95%
2006	571	515	90.1%	(5)	4,550	7	182	441	(5)	813	1,573
2007	3,131	881	28.2%	1,923	8,619	2,052	2,457	2,966	2,052	3,634	4,804
2008	1,665	908	54.6%	47	6,639	440	990	1,522	1,421	2,191	3,355
2009	5,044	1,465	29.0%	1,265	11,797	2,893	3,975	4,902	5,069	5,945	7,666
2010	11,061	2,208	20.0%	4,960	20,538	7,667	9,509	10,915	10,312	12,486	14,886
2011	23,276	3,189	13.7%	13,209	37,472	18,316	21,040	23,131	21,086	25,331	28,725
2012	45,272	5,203	11.5%	28,879	68,025	37,212	41,731	44,991	42,206	48,538	54,277
2013	62,481	7,006	11.2%	36,066	90,980	51,265	57,668	62,265	61,583	67,022	74,418
2014	79,698	8,276	10.4%	49,321	113,281	66,688	74,012	79,329	73,977	85,090	93,641
Total	232,199	12,743	5.5%	186,133	286,448	211,733	223,345	231,854	239,707	240,793	253,653

Table D.13 – Actual vs. Expected Back-test & Conditional Reserve (Stochastic)

Sample Insurance Company
Commercial Auto
Stochastic Actual vs. Expected as of December 31, 2015

AY	Age	Actual Paid	Expected Paid	Percentile	Actual Incurred	Expected Incurred	Percentile	Conditional Reserve	Expected Reserve	Change
2006	120	543	571	57.9%	(47)	154	0.0%	643	603	40
2007	108	2,387	3,131	21.8%	1,040	448	82.8%	3,257	4,242	(985)
2008	96	1,177	1,665	33.5%	851	1,167	44.5%	1,675	2,582	(907)
2009	84	5,403	5,044	63.1%	2,954	1,669	86.1%	5,593	4,121	1,472
2010	72	14,120	11,061	91.1%	9,035	5,606	94.2%	13,946	6,632	7,313
2011	60	23,636	23,276	56.1%	16,524	11,960	93.9%	20,073	19,441	632
2012	48	51,020	45,272	86.7%	36,454	29,103	92.7%	57,978	45,442	12,536
2013	36	75,813	62,481	96.5%	61,541	44,392	99.3%	110,701	81,627	29,075
2014	24	88,832	79,698	86.1%	83,154	66,555	97.0%	170,589	147,146	23,442
2015	12	99,123			178,539					
Totals		362,054			390,045			384,456	311,837	72,619
AY<CY		262,931	232,199	98.9%	211,506	161,054	100.0%	390,213	311,837	78,376

Figure D.1 – Graph of KPI Thresholds by Accident Year – Paid (Stochastic)

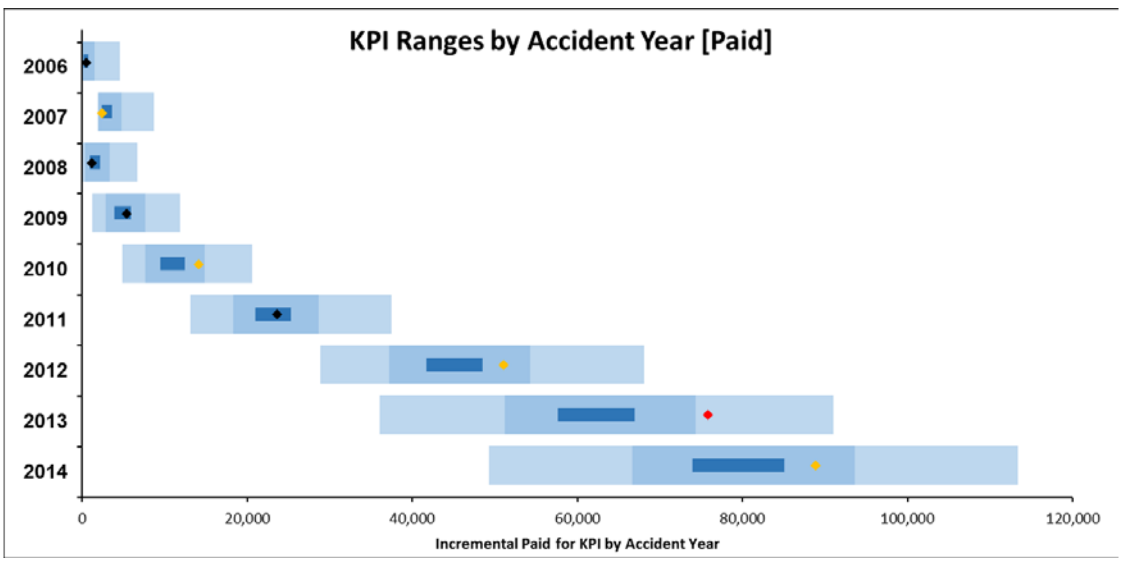


Figure D.2 – Graph of KPI Thresholds by Calendar Year – Paid (Stochastic)

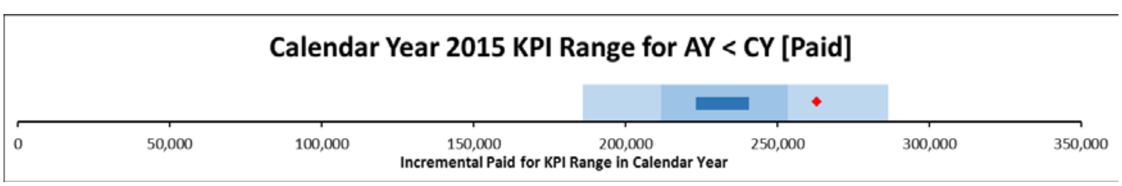


Figure D.3 – Graph of KPI Thresholds by Accident Year – Incurred (Stochastic)

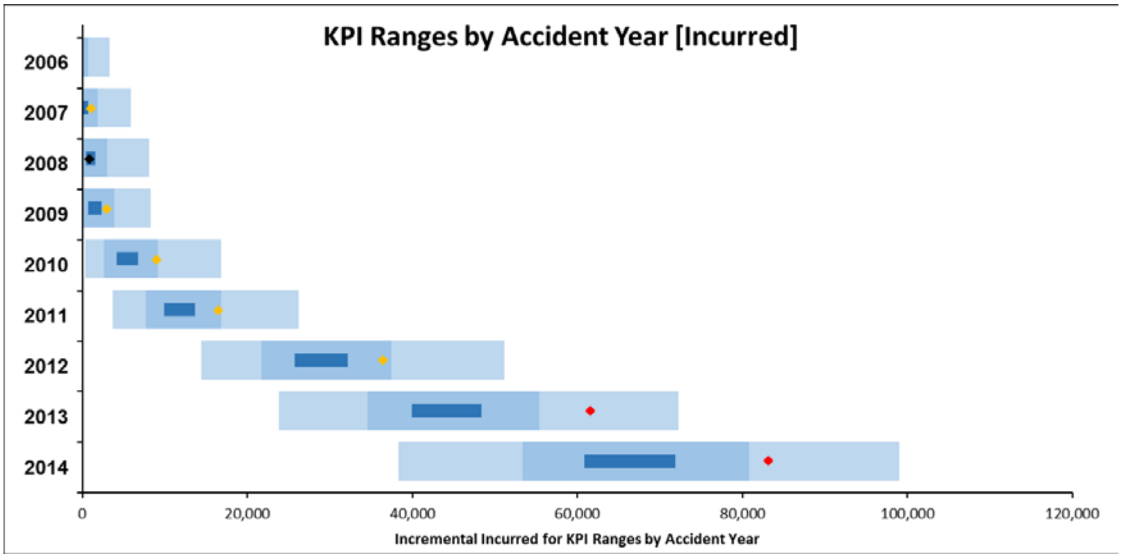


Figure D.4 – Graph of KPI Thresholds by Calendar Year – Incurred (Stochastic)

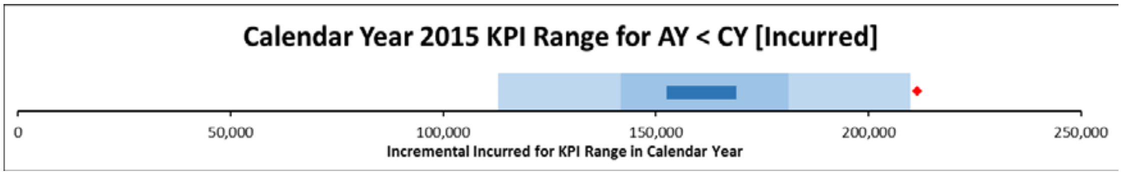


Figure D.5 – Graph of Realized Values vs. Assumptions – Paid (Stochastic)

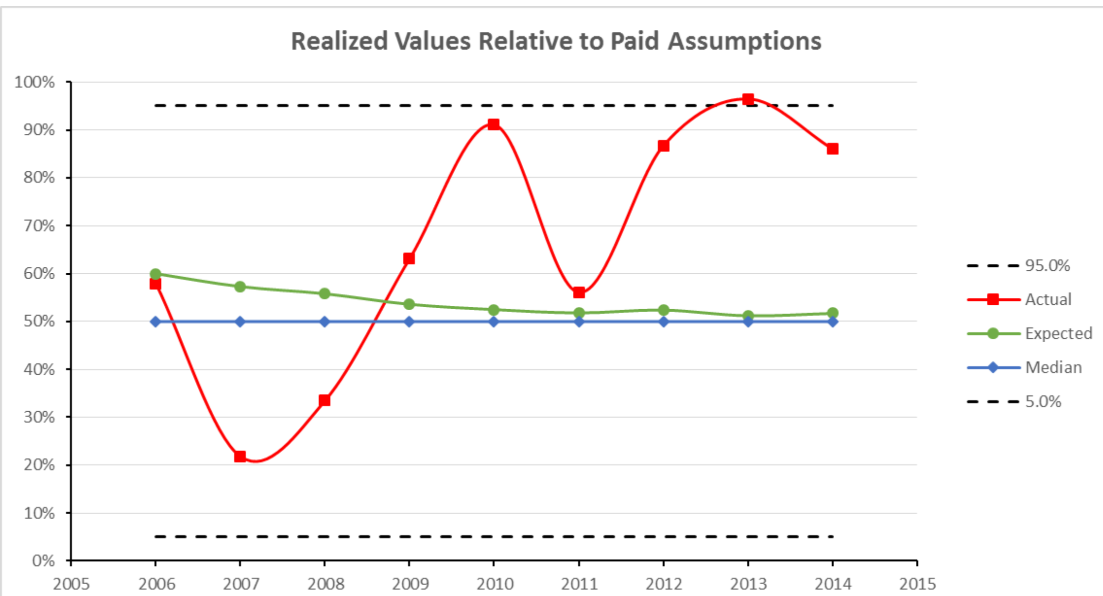
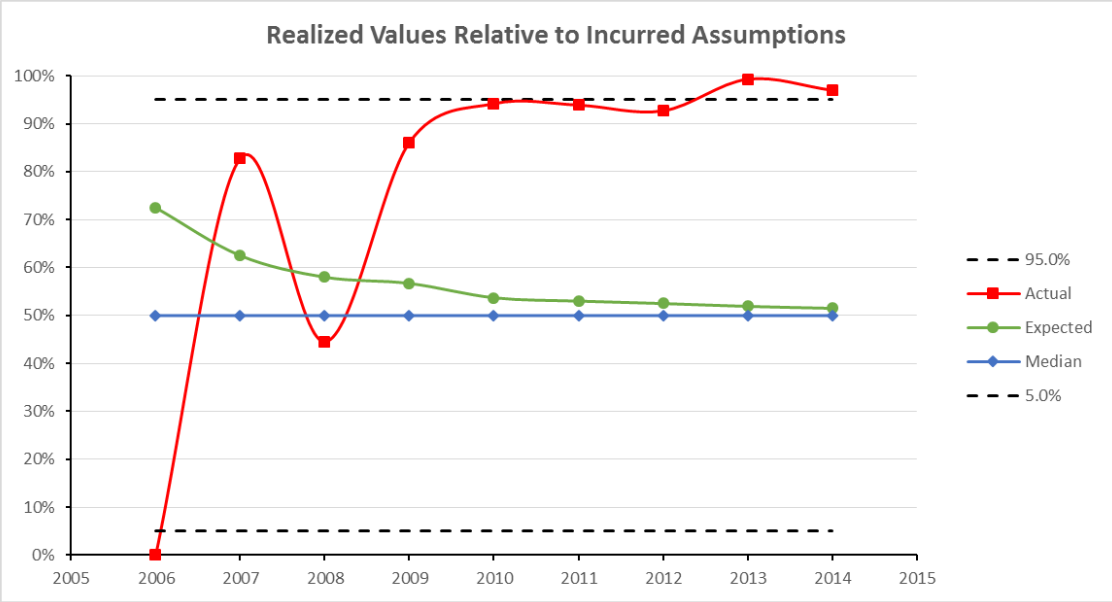


Figure D.2 – Graph of Realized Values vs. Assumptions – Incurred (Stochastic)



Appendix E – Back-Testing Results for Homeowners

Table E.1 – Calculation of Weighted Ultimate (Deterministic)

Sample Insurance Company
Homeowners
Calculation of Weighted Ultimate as of December 31, 2014

		Ultimate Values by Method				Weights by Method				Weighted
AY	Age	Paid CL	Inc CL	Paid BF	Inc BF	Paid CL	Inc CL	Paid BF	Inc BF	Ultimate
2006	108	328,806	328,806	328,806	328,806	50.0%	50.0%	0.0%	0.0%	328,806
2007	96	423,382	422,484	423,380	422,484	50.0%	50.0%	0.0%	0.0%	422,933
2008	84	542,749	542,575	542,751	542,574	50.0%	50.0%	0.0%	0.0%	542,662
2009	72	551,124	549,747	551,123	549,745	50.0%	50.0%	0.0%	0.0%	550,435
2010	60	680,803	678,422	680,808	678,412	50.0%	50.0%	0.0%	0.0%	679,612
2011	48	758,487	757,002	758,506	756,997	50.0%	50.0%	0.0%	0.0%	757,744
2012	36	702,481	700,796	702,653	700,788	25.0%	25.0%	25.0%	25.0%	701,679
2013	24	801,498	797,111	801,473	797,161	0.0%	0.0%	50.0%	50.0%	799,317
2014	12	992,257	996,379	993,794	996,481	0.0%	0.0%	50.0%	50.0%	995,137
Totals		5,781,585	5,773,322	5,783,294	5,773,446					5,778,327

Table E.2 – Reconciliation of Total Unpaid (Deterministic)

Sample Insurance Company
Homeowners
Total Unpaid Reconciliation as of December 31, 2014

AY	Age	Paid to Date	Incurred to Date	Weighted Ultimate	Case Reserve	IBNR	Total Unpaid	Selected Ultimate	Selected IBNR	Total Unpaid
2006	108	328,033	328,901	328,806	868	(95)	773	328,806	(95)	773
2007	96	422,179	422,654	422,933	475	279	754	422,933	279	754
2008	84	540,795	543,199	542,662	2,404	(537)	1,867	542,662	(537)	1,867
2009	72	548,818	550,729	550,435	1,911	(294)	1,617	550,435	(294)	1,617
2010	60	675,472	680,658	679,612	5,186	(1,046)	4,140	679,612	(1,046)	4,140
2011	48	745,388	758,597	757,744	13,209	(853)	12,356	757,744	(853)	12,356
2012	36	680,014	701,622	701,679	21,608	57	21,665	701,679	57	21,665
2013	24	748,862	787,351	799,317	38,489	11,966	50,455	799,317	11,966	50,455
2014	12	723,126	930,676	995,137	207,550	64,461	272,011	995,137	64,461	272,011
Totals		5,412,687	5,704,387	5,778,327	291,700	73,940	365,640	5,778,327	73,940	365,640

Table E.3 – Expected Incremental Development – Paid (Deterministic)

Sample Insurance Company
Homeowners -- Paid Data
Expected Incremental Future Development as of December 31, 2014

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2006										386	387	773
2007									(240)	497	497	754
2008								325	266	638	638	1,867
2009							(364)	418	270	647	647	1,617
2010						1,297	397	516	333	798	799	4,140
2011					6,423	2,763	443	575	371	890	891	12,356
2012				9,503	6,648	2,568	412	535	345	827	828	21,665
2013			24,902	11,755	7,541	2,913	467	607	391	939	940	50,455
2014		206,388	33,665	14,702	9,432	3,643	584	759	489	1,174	1,175	272,011

Table E.4 – Expected Incremental Development – Incurred (Deterministic)

Sample Insurance Company
Homeowners -- Incurred Data
Expected Incremental Future Development as of December 31, 2014

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2006										(48)	(47)	(95)
2007									401	(61)	(61)	279
2008								(319)	(61)	(78)	(78)	(537)
2009							340	(412)	(62)	(80)	(80)	(294)
2010						169	(432)	(509)	(76)	(98)	(98)	(1,046)
2011					1,645	(1,143)	(482)	(568)	(85)	(109)	(109)	(853)
2012				1,543	839	(1,064)	(449)	(528)	(79)	(102)	(102)	57
2013			12,913	745	955	(1,212)	(511)	(602)	(90)	(116)	(116)	11,966
2014		52,259	13,378	925	1,185	(1,504)	(634)	(747)	(112)	(144)	(144)	64,461

Table E.5 – Actual vs. Expected Back-test (Deterministic)

Sample Insurance Company
Homeowners
Deterministic Actual vs. Expected as of December 31, 2015

AY	Age	Actual Paid	Expected Paid	Difference	Actual Incurred	Expected Incurred	Difference
2006	120	26	386	(360)	(132)	(48)	(84)
2007	108	33	(240)	273	(156)	401	(557)
2008	96	227	325	(98)	(1,359)	(319)	(1,040)
2009	84	(176)	(364)	188	(1,158)	340	(1,498)
2010	72	3,800	1,297	2,503	412	169	243
2011	60	5,462	6,423	(961)	(8)	1,645	(1,653)
2012	48	12,197	9,503	2,694	1,284	1,543	(259)
2013	36	23,840	24,902	(1,062)	8,785	12,913	(4,128)
2014	24	191,678	206,388	(14,710)	56,168	52,259	3,909
2015	12	934,805			1,143,739		
Totals		1,171,892			1,207,575		
AY<CY		237,087	248,619	(11,532)	63,836	68,902	(5,066)

Table E.6 – Actual to Range of Estimates Back-test (Deterministic)

Sample Insurance Company
Homeowners
Deterministic Actual vs. Method Range as of December 31, 2015

AY	Age	Actual Paid	Paid Minimum	Paid Maximum	Range Percent	Actual Incurred	Incurred Minimum	Incurred Maximum	Difference
2006	120	26	386	386	-143771.0%	(132)	(48)	(47)	-33682.3%
2007	108	33	(688)	207	80.5%	(156)	(48)	850	-12.1%
2008	96	227	235	413	-4.6%	(1,359)	(407)	(229)	-534.5%
2009	84	(176)	(1,051)	322	63.7%	(1,158)	(350)	1,030	-58.5%
2010	72	3,800	99	2,485	155.1%	412	(1,028)	1,372	60.0%
2011	60	5,462	5,673	7,170	-14.1%	(8)	900	2,417	-59.9%
2012	48	12,197	8,582	10,415	197.2%	1,284	650	2,526	33.8%
2013	36	23,840	22,756	27,002	25.5%	8,785	10,700	15,091	-43.6%
2014	24	191,678	203,968	207,819	-319.1%	56,168	49,431	53,586	162.1%
2015	12	934,805				1,143,739			
Totals		1,171,892				1,207,575			
AY<CY		237,087	243,694	253,519	-67.2%	63,836	63,878	73,919	-0.4%

Table E.7 – Estimated Unpaid Claims by Accident Year (Stochastic)

Sample Insurance Company
Homeowners
Stochastic Estimates as of December 31, 2014
Estimated Unpaid Claims by Accident Year

AY	Mean	Std Dev	CoV	Min	Max	5%	25%	Median	Mode	75%	95%
2006	773	920	119.1%	(18)	7,510	(16)	121	459	(18)	1,101	2,668
2007	754	1,334	176.9%	(2,345)	11,715	(831)	(164)	445	(446)	1,359	3,384
2008	1,867	1,847	98.9%	(2,791)	15,138	(541)	573	1,534	1,422	2,847	5,402
2009	1,617	1,975	122.1%	(4,363)	14,310	(989)	206	1,315	921	2,700	5,238
2010	4,140	2,932	70.8%	(4,812)	24,814	9	2,020	3,791	1,561	5,885	9,480
2011	12,356	4,435	35.9%	404	35,123	5,775	9,158	11,996	12,056	15,160	20,191
2012	21,665	5,686	26.2%	5,673	46,724	13,069	17,642	21,254	23,445	25,267	31,717
2013	50,455	9,708	19.2%	23,208	98,051	35,582	43,515	49,808	41,265	56,737	67,307
2014	272,011	30,285	11.1%	176,947	402,593	224,048	250,890	271,241	293,093	291,855	323,755
Total	365,640	33,369	9.1%	247,985	505,728	312,138	342,419	364,523	360,985	387,991	421,695

Table E.8 – Estimated Claims Paid by Calendar Year (Stochastic)

Sample Insurance Company
Homeowners
Stochastic Estimates as of December 31, 2014
Estimated Paid Claims by Calendar Year

CY	Mean	Std Dev	CoV	Min	Max	5%	25%	Median	Mode	75%	95%
2015	252,049	25,430	10.1%	171,900	348,486	211,598	234,404	251,252	261,859	269,070	294,959
2016	55,570	9,158	16.5%	29,368	103,028	41,386	49,232	55,076	52,236	61,369	71,445
2017	26,772	6,387	23.9%	7,593	56,696	17,092	22,144	26,470	27,827	30,888	37,890
2018	14,401	4,923	34.2%	333	38,744	7,102	10,932	13,965	13,221	17,409	23,173
2019	6,241	3,422	54.8%	(2,952)	24,140	1,334	3,813	5,881	5,630	8,306	12,436
2020	3,212	2,583	80.4%	(4,367)	18,449	(318)	1,383	2,867	2,281	4,693	7,986
2021	2,735	2,471	90.3%	(5,722)	17,438	(656)	1,006	2,423	770	4,070	7,339
2022	2,318	2,271	98.0%	(3,834)	15,984	(819)	769	1,965	1,163	3,562	6,552
2023	2,340	1,852	79.1%	0	18,642	155	940	1,938	-	3,281	5,981
Total	365,640	33,369	9.1%	247,985	505,728	312,138	342,419	364,523	360,985	387,991	421,695

Table E.9 – Mean Future Incremental – Paid (Stochastic)

Sample Insurance Company
Homeowners - Paid
Mean Future Incremental as of December 31, 2014

AY	12	24	36	48	60	72	84	96	108	120	Total
2006										773	773
2007									125	629	754
2008								414	237	1,215	1,867
2009							217	293	205	903	1,617
2010						1,911	319	403	259	1,248	4,140
2011					6,758	2,604	416	545	348	1,685	12,356
2012				9,961	6,391	2,487	402	503	333	1,588	21,665
2013			25,830	11,299	7,304	2,814	459	585	373	1,792	50,455
2014		206,060	33,797	14,743	9,478	3,682	608	775	527	2,340	272,011

Table E.10 – Standard Deviation of Future Incremental – Paid (Stochastic)

Sample Insurance Company
Homeowners - Paid
Standard Deviation Future Incremental as of December 31, 2014

AY	12	24	36	48	60	72	84	96	108	120	Total
2006										920	920
2007									831	1,054	1,334
2008								952	995	1,243	1,847
2009							704	934	1,030	1,236	1,975
2010						1,805	844	1,062	1,187	1,397	2,932
2011					3,045	1,966	892	1,170	1,287	1,508	4,435
2012				3,658	2,927	1,919	867	1,092	1,236	1,419	5,686
2013			6,340	4,080	3,298	2,086	951	1,234	1,378	1,574	9,708
2014		24,137	7,203	4,746	3,852	2,459	1,138	1,508	1,636	1,852	30,285

Table E.11 – Coefficient of Variation of Future Incremental – Paid (Stochastic)

Sample Insurance Company
Homeowners - Paid
CoV Future Incremental as of December 31, 2014

AY	12	24	36	48	60	72	84	96	108	120	Total
2006										119.1%	119.1%
2007									665.2%	167.5%	176.9%
2008								229.9%	419.4%	102.3%	98.9%
2009							324.5%	318.6%	503.5%	136.9%	122.1%
2010						94.4%	264.4%	263.5%	458.1%	112.0%	70.8%
2011					45.1%	75.5%	214.7%	214.7%	369.8%	89.5%	35.9%
2012				36.7%	45.8%	77.2%	215.6%	217.1%	370.6%	89.4%	26.2%
2013			24.5%	36.1%	45.2%	74.1%	207.1%	210.9%	370.0%	87.9%	19.2%
2014		11.7%	21.3%	32.2%	40.6%	66.8%	187.1%	194.6%	310.6%	79.1%	11.1%

Table E.12 – Estimated Unpaid Claims by Accident Year in 2015 (Stochastic)

Sample Insurance Company
Homeowners - Paid
Stochastic Estimates as of December 31, 2014
Estimated Unpaid Claims by Accident Year, Calendar Year 2015 Only

AY	Mean	Std Dev	CoV	Min	Max	5%	25%	Median	Mode	75%	95%
2006	773	920	119.1%	(18)	7,510	(16)	121	459	(18)	1,101	2,668
2007	125	831	665.2%	(1,973)	6,958	(1,083)	(157)	(63)	(74)	294	1,701
2008	414	952	229.9%	(2,175)	9,496	(742)	(26)	118	(26)	693	2,285
2009	217	704	324.5%	(1,892)	9,688	(523)	(96)	(27)	(96)	360	1,645
2010	1,911	1,805	94.4%	(2,885)	14,491	(317)	565	1,550	(564)	2,884	5,331
2011	6,758	3,045	45.1%	47	22,789	2,482	4,544	6,378	4,282	8,579	12,327
2012	9,961	3,658	36.7%	1,207	28,737	4,701	7,304	9,587	9,740	12,199	16,585
2013	25,830	6,340	24.5%	8,694	52,980	16,319	21,257	25,371	19,688	29,857	37,189
2014	206,060	24,137	11.7%	132,533	295,967	167,429	189,609	205,307	200,574	221,714	247,353
Total	252,049	25,430	10.1%	171,900	348,486	211,598	234,404	251,252	261,859	269,070	294,959

Table E.13 – Actual vs. Expected Back-test & Conditional Reserve (Stochastic)

Sample Insurance Company
Homeowners
Stochastic Actual vs. Expected as of December 31, 2015

AY	Age	Actual Paid	Expected Paid	Percentile	Actual Incurred	Expected Incurred	Percentile	Conditional Reserve	Expected Reserve	Change
2006	120	26	773	13.9%	(132)	(95)	83.5%	-	747	(747)
2007	108	33	125	61.9%	(156)	59	31.4%	164	721	(557)
2008	96	227	414	57.2%	(1,359)	(349)	23.5%	1,367	1,640	(272)
2009	84	(176)	217	14.1%	(1,158)	(105)	18.5%	(1,153)	1,793	(2,946)
2010	72	3,800	1,911	85.6%	412	(482)	67.2%	3,722	340	3,381
2011	60	5,462	6,758	37.5%	(8)	1,119	12.2%	3,979	6,894	(2,915)
2012	48	12,197	9,961	74.9%	1,284	813	81.4%	12,839	9,468	3,370
2013	36	23,840	25,830	40.5%	8,785	12,274	37.9%	21,590	26,615	(5,024)
2014	24	191,678	206,060	28.0%	56,168	52,293	62.7%	59,458	80,333	(20,875)
2015	12	934,805			1,143,739					
Totals		1,171,892			1,207,575			101,967	128,553	(26,586)
AY<CY		237,087	252,049	28.4%	63,836	65,528	50.2%	96,676	128,553	(31,876)

Figure E.1 – Graph of KPI Thresholds by Accident Year – Paid (Stochastic)

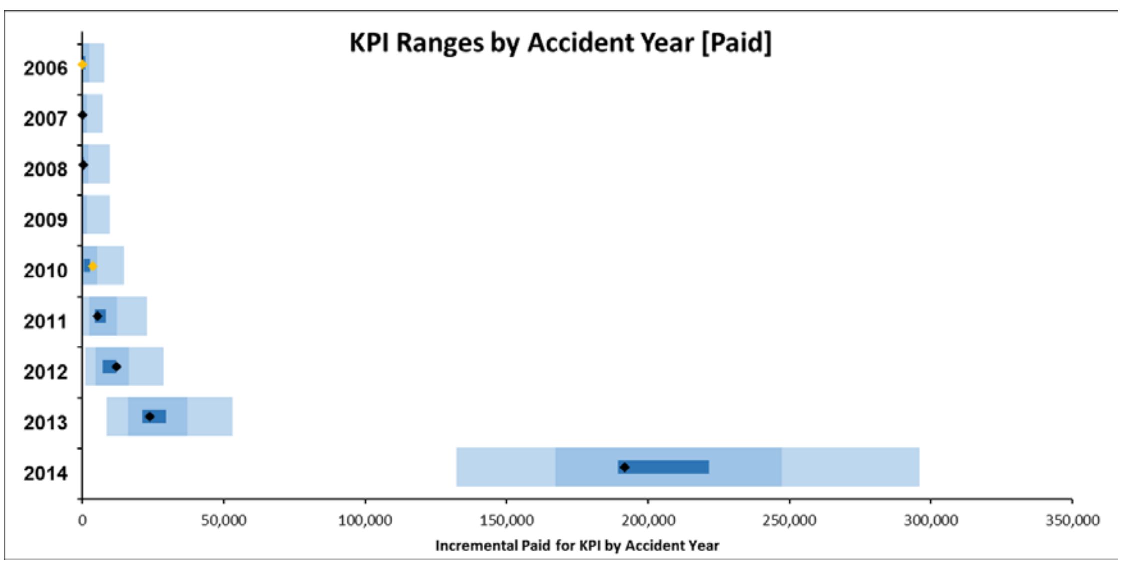


Figure E.2 – Graph of KPI Thresholds by Calendar Year – Paid (Stochastic)

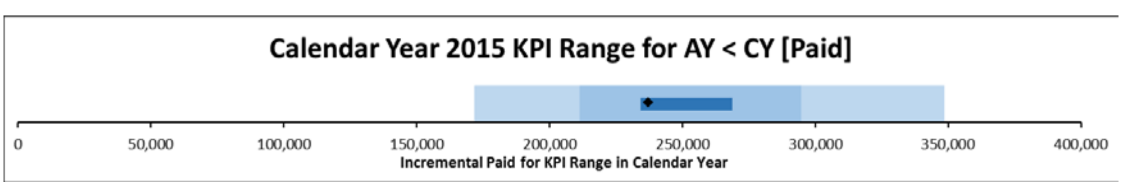


Figure E.3 – Graph of KPI Thresholds by Accident Year – Incurred (Stochastic)

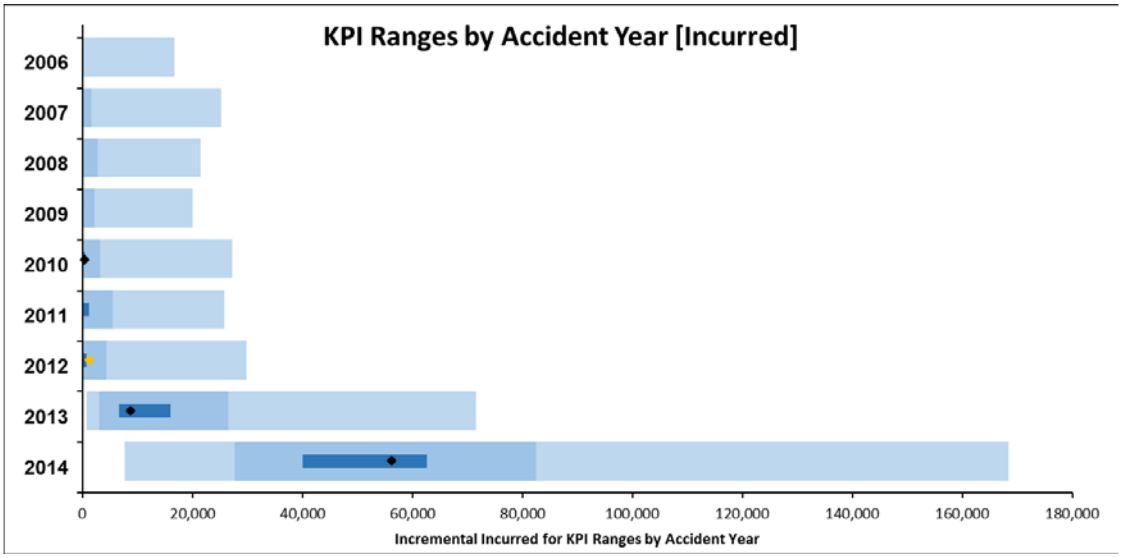


Figure E.4 – Graph of KPI Thresholds by Calendar Year – Incurred (Stochastic)

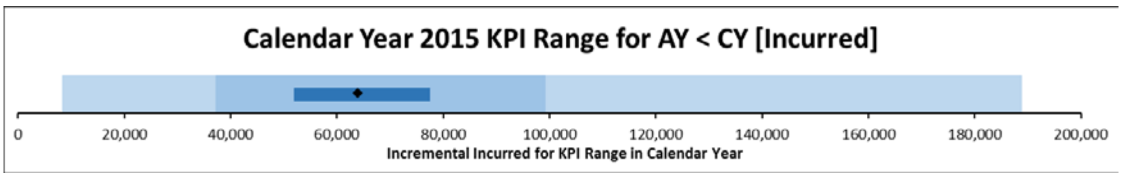


Figure E.5 – Graph of Realized Values vs. Assumptions – Paid (Stochastic)

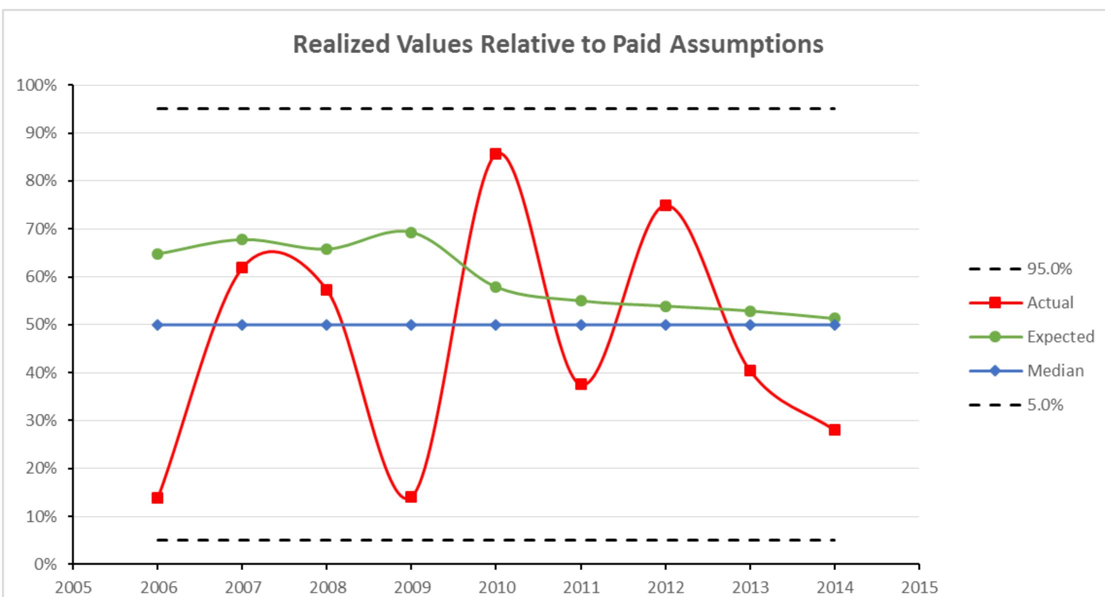
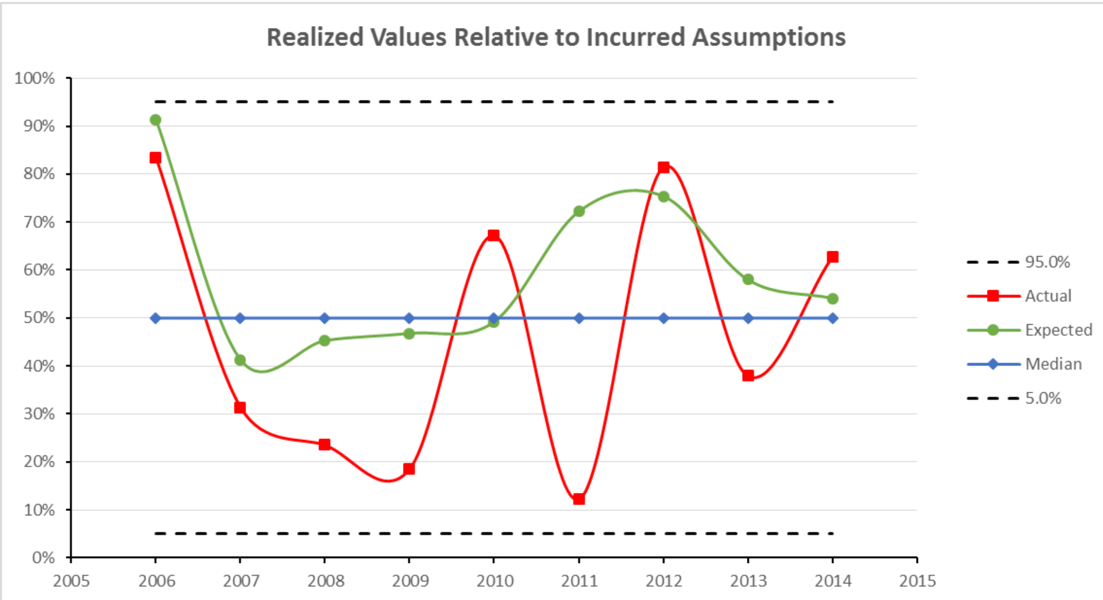


Figure E.6 – Graph of Realized Values vs. Assumptions – Incurred (Stochastic)



Appendix F – Back-Testing Aggregate Results

Table F.1 – Reconciliation of Total Unpaid (Deterministic)

Sample Insurance Company
Consolidation of All Segments
Total Unpaid Reconciliation as of December 31, 2014

AY	Age	Paid to Date	Incurred to Date	Weighted Ultimate	Case Reserve	IBNR	Total Unpaid	Selected Ultimate	Selected IBNR	Total Unpaid
2006	108	1,798,805	1,801,896	1,806,215	3,091	4,319	7,410	1,806,215	4,319	7,410
2007	96	2,054,136	2,063,367	2,068,349	9,231	4,982	14,213	2,070,502	7,135	16,366
2008	84	2,202,872	2,213,290	2,226,141	10,418	12,851	23,269	2,226,141	12,851	23,269
2009	72	2,335,053	2,354,342	2,379,431	19,289	25,089	44,378	2,379,431	25,089	44,378
2010	60	2,522,650	2,566,756	2,618,692	44,106	51,936	96,042	2,618,692	51,936	96,042
2011	48	2,510,953	2,609,324	2,713,658	98,371	104,334	202,705	2,713,658	104,334	202,705
2012	36	2,369,593	2,567,519	2,783,496	197,926	215,977	413,903	2,783,496	215,977	413,903
2013	24	2,210,586	2,558,937	2,976,074	348,351	417,137	765,488	2,976,074	417,137	765,488
2014	12	1,604,249	2,346,693	3,247,231	742,444	900,538	1,642,982	3,247,231	900,538	1,642,982
Totals		19,608,897	21,082,124	22,819,287	1,473,227	1,737,163	3,210,390	22,821,440	1,739,316	3,212,543

Table F.2 – Expected Incremental Development – Paid (Deterministic)

Sample Insurance Company
Consolidation of All Segments -- Paid Data
Expected Incremental Future Development as of December 31, 2014

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2006										3,701	3,709	7,410
2007									7,405	4,476	4,485	16,366
2008								10,073	4,353	4,417	4,426	23,269
2009							19,027	11,120	4,716	4,752	4,762	44,378
2010						47,151	21,651	11,869	5,058	5,151	5,162	96,042
2011					103,127	50,012	21,845	12,022	5,128	5,281	5,292	202,705
2012				194,479	113,044	53,527	23,509	12,806	5,484	5,521	5,533	413,903
2013			325,644	208,375	119,178	56,435	24,715	13,549	5,783	5,899	5,911	765,488
2014		833,793	351,973	216,546	123,955	58,580	25,466	14,073	6,020	6,282	6,295	1,642,982

Table F.3 – Expected Incremental Development – Incurred (Deterministic)

Sample Insurance Company
Consolidation of All Segments -- Incurred Data
Expected Incremental Future Development as of December 31, 2014

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2006										2,158	2,161	4,319
2007									2,794	2,169	2,172	7,135
2008								6,142	1,726	2,489	2,494	12,851
2009							11,285	6,504	1,883	2,706	2,711	25,089
2010						26,873	10,537	6,833	1,991	2,849	2,853	51,936
2011					54,534	24,663	10,569	6,831	1,995	2,868	2,873	104,334
2012				106,020	55,954	26,819	11,457	7,434	2,175	3,057	3,062	215,977
2013			192,143	108,519	59,307	28,313	12,129	7,859	2,291	3,285	3,291	417,137
2014		479,073	187,988	112,628	61,829	29,184	12,530	8,072	2,358	3,436	3,441	900,538

Table F.4 – Actual vs. Expected Back-test (Deterministic)

Sample Insurance Company
Consolidation of All Segments
Deterministic Actual vs. Expected as of December 31, 2015

AY	Age	Actual Paid	Expected Paid	Difference	Actual Incurred	Expected Incurred	Difference
2006	120	3,069	3,701	(632)	1,863	2,158	(295)
2007	108	5,905	7,405	(1,500)	3,145	2,794	351
2008	96	8,986	10,073	(1,087)	3,553	6,142	(2,589)
2009	84	18,992	19,027	(35)	9,872	11,285	(1,413)
2010	72	51,003	47,151	3,852	25,942	26,873	(931)
2011	60	105,067	103,127	1,940	52,012	54,534	(2,522)
2012	48	202,932	194,479	8,453	106,624	106,020	604
2013	36	334,434	325,644	8,790	189,908	192,143	(2,235)
2014	24	841,484	833,793	7,691	454,217	479,073	(24,856)
2015	12	1,798,138			2,528,235		
Totals		3,370,010			3,375,371		
AY<CY		1,571,872	1,544,400	27,471	847,136	881,022	(33,886)

Table F.5 – Actual to Range of Estimates Back-test (Deterministic)

Sample Insurance Company Consolidation of All Segments Deterministic Actual vs. Method Range as of December 31, 2015									
AY	Age	Actual Paid	Paid Minimum	Paid Maximum	Range Percent	Actual Incurred	Incurred Minimum	Incurred Maximum	Difference
2006	120	3,069	3,701	3,704	-21075.4%	1,863	2,158	2,162	-6790.5%
2007	108	5,905	5,827	8,983	2.5%	3,145	1,210	4,380	61.0%
2008	96	8,986	9,887	10,277	-230.8%	3,553	5,955	6,356	-599.0%
2009	84	18,992	17,726	20,381	47.7%	9,872	9,981	12,657	-4.1%
2010	72	51,003	44,889	49,487	133.0%	25,942	24,600	29,236	28.9%
2011	60	105,067	100,495	106,278	79.1%	52,012	51,856	57,857	2.6%
2012	48	202,932	191,183	198,745	155.4%	106,624	102,222	110,845	51.1%
2013	36	334,434	310,031	338,355	86.2%	189,908	174,120	205,898	49.7%
2014	24	841,484	794,706	853,821	79.1%	454,217	436,298	503,306	26.7%
2015	12	1,798,138				2,528,235			
Totals		3,370,010				3,375,371			
AY<CY		1,571,872	1,481,602	1,586,896	85.7%	847,136	811,568	929,564	30.1%

Table F.6 – Estimated Unpaid Claims by Accident Year (Stochastic)

Sample Insurance Company Aggregation of All Segments Stochastic Estimates as of December 31, 2014 Estimated Unpaid Claims by Accident Year											
AY	Mean	Std Dev	CoV	Min	Max	5%	25%	Median	Mode	75%	95%
2006	7,410	3,000	40.5%	209	20,930	2,762	5,258	7,230	7,126	9,376	12,584
2007	16,366	3,857	23.6%	4,326	35,971	10,293	13,681	16,160	13,955	18,874	23,025
2008	23,269	4,798	20.6%	7,340	41,630	15,697	19,961	23,038	24,448	26,387	31,552
2009	44,378	6,012	13.5%	23,290	73,490	34,774	40,249	44,172	43,645	48,324	54,552
2010	96,042	8,137	8.5%	68,354	129,130	82,986	90,380	95,868	97,281	101,523	109,899
2011	202,705	11,141	5.5%	162,433	245,913	184,872	195,065	202,429	213,672	210,093	221,392
2012	413,903	18,019	4.4%	348,396	495,863	385,145	401,826	413,324	431,386	425,535	444,597
2013	765,488	31,256	4.1%	643,540	893,747	714,958	744,538	764,726	758,282	786,020	818,610
2014	1,642,982	62,139	3.8%	1,378,415	1,972,517	1,544,716	1,602,194	1,641,001	1,633,958	1,682,508	1,746,787
Total	3,212,543	79,355	2.5%	2,811,937	3,596,084	3,084,602	3,161,789	3,211,505	3,295,980	3,261,725	3,343,252

Table F.7 – Estimated Claims Paid by Calendar Year (Stochastic)

Sample Insurance Company Aggregation of All Segments Stochastic Estimates as of December 31, 2014 Estimated Unpaid Claims by Calendar Year											
CY	Mean	Std Dev	CoV	Min	Max	5%	25%	Median	Mode	75%	95%
2015	1,560,637	43,888	2.8%	1,326,487	1,761,442	1,490,151	1,531,594	1,560,068	1,569,675	1,589,323	1,634,164
2016	761,830	24,692	3.2%	671,495	861,974	721,379	745,435	761,974	778,026	778,144	802,553
2017	433,217	17,767	4.1%	368,636	499,640	404,462	420,952	433,003	430,492	445,020	463,153
2018	227,484	12,686	5.6%	180,708	277,701	206,908	218,837	227,342	231,979	235,833	248,870
2019	110,005	8,936	8.1%	81,148	145,658	95,506	104,003	109,870	108,106	115,810	124,858
2020	54,489	6,783	12.4%	30,217	81,348	43,677	49,928	54,233	53,345	58,990	65,976
2021	30,258	5,508	18.2%	11,536	54,292	21,555	26,490	30,113	31,602	33,792	39,599
2022	17,338	4,694	27.1%	1,748	38,761	9,925	14,127	17,132	15,736	20,273	25,447
2023	12,228	4,234	34.6%	351	31,873	5,612	9,261	12,025	15,750	14,892	19,631
2024	5,057	2,388	47.2%	(46)	15,791	1,427	3,333	4,900	4,363	6,546	9,313
Total	3,212,543	79,355	2.5%	2,811,937	3,596,084	3,084,602	3,161,789	3,211,505	3,295,980	3,261,725	3,343,252

Table F.8 – Mean Future Incremental – Paid (Stochastic)

Sample Insurance Company Aggregation of All Segments - Paid Mean Future Incremental as of December 31, 2014												
AY	12	24	36	48	60	72	84	96	108	120	132	Total
2006										4,077	3,333	7,410
2007									6,163	5,387	4,816	16,366
2008								10,176	4,300	4,998	3,794	23,269
2009							20,033	10,774	4,591	4,922	4,058	44,378
2010						48,298	21,360	11,595	4,927	5,520	4,342	96,042
2011					104,415	49,419	21,556	11,839	5,077	6,033	4,365	202,705
2012				196,083	112,311	53,119	23,353	12,692	5,415	6,236	4,693	413,903
2013			331,701	205,564	117,582	55,662	24,391	13,384	5,665	6,643	4,896	765,488
2014		839,689	349,382	214,959	122,988	58,266	25,315	13,992	6,001	7,332	5,057	1,642,982

Table F.9 – Standard Deviation of Future Incremental – Paid (Stochastic)

Sample Insurance Company
Aggregation of All Segments - Paid
Standard Deviation Future Incremental as of December 31, 2014

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2006										1,851	1,623	3,000
2007									1,927	2,080	1,809	3,857
2008								2,494	2,030	2,244	1,833	4,798
2009							3,202	2,660	2,162	2,280	1,974	6,012
2010						5,017	3,331	2,742	2,331	2,477	2,065	8,137
2011					7,305	5,065	3,417	2,795	2,369	2,568	2,101	11,141
2012				10,921	8,101	5,518	3,644	3,008	2,443	2,580	2,185	18,019
2013			16,733	12,067	8,683	5,833	3,853	3,164	2,615	2,786	2,312	31,256
2014		36,658	17,799	12,858	9,241	6,087	3,943	3,330	2,814	2,992	2,388	62,139

Table F.10 – Coefficient of Variation of Future Incremental – Paid (Stochastic)

Sample Insurance Company
Aggregation of All Segments - Paid
CoV Future Incremental as of December 31, 2014

AY	12	24	36	48	60	72	84	96	108	120	132	Total
2006										45.4%	48.7%	40.5%
2007									31.3%	38.6%	37.6%	23.6%
2008								24.5%	47.2%	44.9%	48.3%	20.6%
2009							16.0%	24.7%	47.1%	46.3%	48.6%	13.5%
2010						10.4%	15.6%	23.6%	47.3%	44.9%	47.6%	8.5%
2011					7.0%	10.2%	15.8%	23.6%	46.7%	42.6%	48.1%	5.5%
2012				5.6%	7.2%	10.4%	15.6%	23.7%	45.1%	41.4%	46.6%	4.4%
2013			5.0%	5.9%	7.4%	10.5%	15.8%	23.6%	46.2%	41.9%	47.2%	4.1%
2014		4.4%	5.1%	6.0%	7.5%	10.4%	15.6%	23.8%	46.9%	40.8%	47.2%	3.8%

Table F.11 – Estimated Unpaid Claims by Accident Year in 2015 (Stochastic)

Sample Insurance Company
Aggregation of All Segments - Paid
Stochastic Estimates as of December 31, 2014
Estimated Unpaid Claims by Accident Year, Calendar Year 2015 Only

AY	Mean	Std Dev	CoV	Min	Max	5%	25%	Median	Mode	75%	95%
2006	4,077	1,851	45.4%	4	12,459	1,386	2,758	3,891	3,545	5,211	7,424
2007	6,163	1,927	31.3%	92	14,962	3,317	4,823	5,994	6,136	7,317	9,584
2008	10,176	2,494	24.5%	2,955	24,018	6,391	8,444	9,987	8,710	11,747	14,546
2009	20,033	3,202	16.0%	9,752	35,160	15,071	17,795	19,882	19,530	22,094	25,607
2010	48,298	5,017	10.4%	27,691	69,353	40,292	44,825	48,117	49,900	51,560	56,893
2011	104,415	7,305	7.0%	76,379	135,132	92,822	99,305	104,299	105,433	109,283	116,607
2012	196,083	10,921	5.6%	157,181	242,812	178,556	188,588	195,828	193,134	203,222	214,311
2013	331,701	16,733	5.0%	257,765	396,823	304,516	320,387	331,465	315,168	342,845	359,464
2014	839,689	36,658	4.4%	679,077	1,011,508	781,489	815,305	839,033	862,142	862,844	900,811
Total	1,560,637	43,888	2.8%	1,326,487	1,761,442	1,490,151	1,531,594	1,560,068	1,569,675	1,589,323	1,634,164

Table F.12 – Actual vs. Expected Back-test & Conditional Reserve (Stochastic)

Sample Insurance Company
Aggregation of All Segments
Stochastic Actual vs. Expected as of December 31, 2015

AY	Age	Actual Paid	Expected Paid	Percentile	Actual Incurred	Expected Incurred	Percentile	Conditional Reserve	Expected Reserve	Change
2006	120	3,069	4,077	31.8%	1,863	2,115	49.8%	2,539	4,341	(1,802)
2007	108	5,905	6,163	47.9%	3,145	1,819	80.6%	11,349	10,461	888
2008	96	8,986	10,176	33.6%	3,553	6,026	20.9%	10,961	14,283	(3,322)
2009	84	18,992	20,033	39.0%	9,872	10,399	46.3%	21,615	25,386	(3,771)
2010	72	51,003	48,298	71.6%	25,942	25,562	55.3%	49,308	45,039	4,269
2011	60	105,067	104,415	54.3%	52,012	53,101	44.8%	97,157	97,638	(481)
2012	48	202,932	196,083	74.2%	106,624	104,075	61.7%	222,250	210,971	11,279
2013	36	334,434	331,701	57.1%	189,908	185,173	64.0%	427,667	431,054	(3,387)
2014	24	841,484	839,689	52.8%	454,217	469,822	29.3%	795,671	801,499	(5,828)
2015	12	1,798,138			2,528,235					
Totals		3,370,010			3,375,371			1,638,516	1,640,671	(2,154)
AY<CY		1,571,872	1,560,637	61.2%	847,136	858,093	37.6%	1,638,584	1,640,671	(2,086)

Figure F.1 – Graph of KPI Thresholds by Accident Year – Paid (Stochastic)

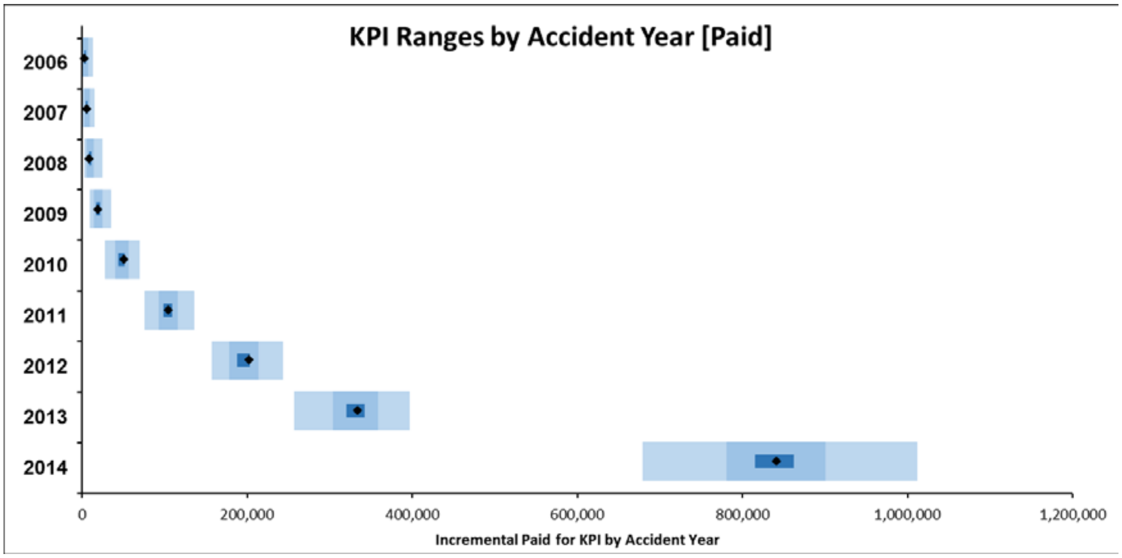


Figure F.2 – Graph of KPI Thresholds by Calendar Year – Paid (Stochastic)

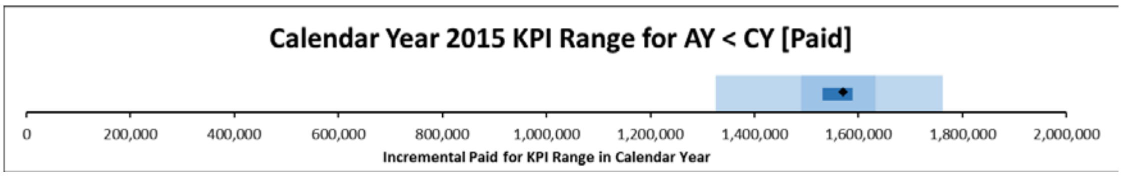


Figure F.3 – Graph of KPI Thresholds by Accident Year – Incurred (Stochastic)

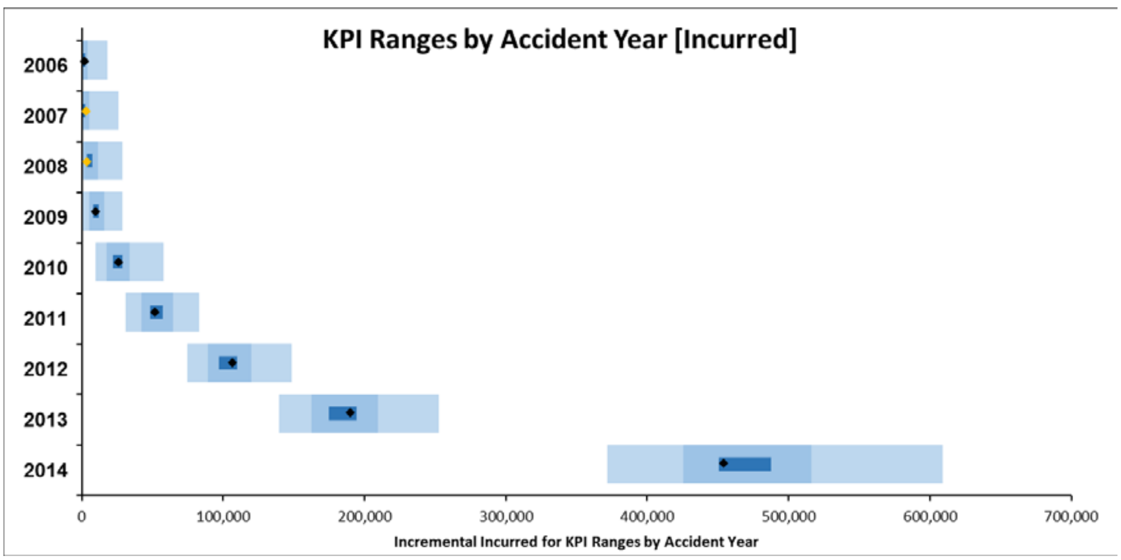


Figure F.4 – Graph of KPI Thresholds by Calendar Year – Incurred (Stochastic)

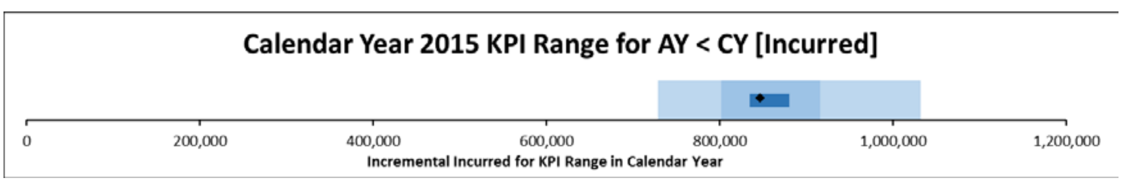


Figure F.5 – Graph of Realized Values vs. Assumptions – Paid (Stochastic)

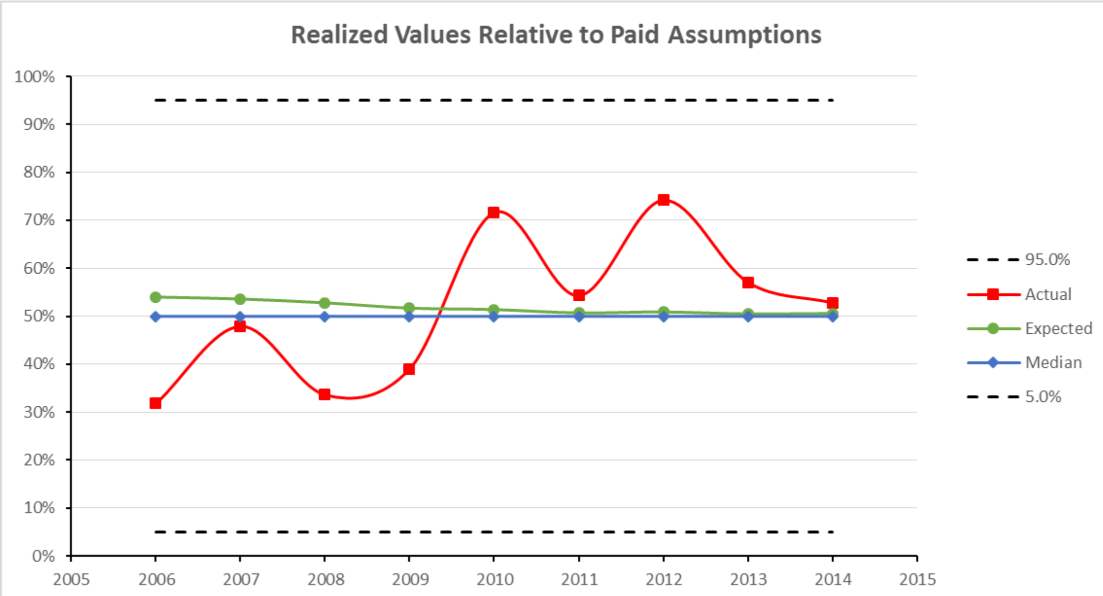


Figure F.6 – Graph of Realized Values vs. Assumptions – Paid (Stochastic)

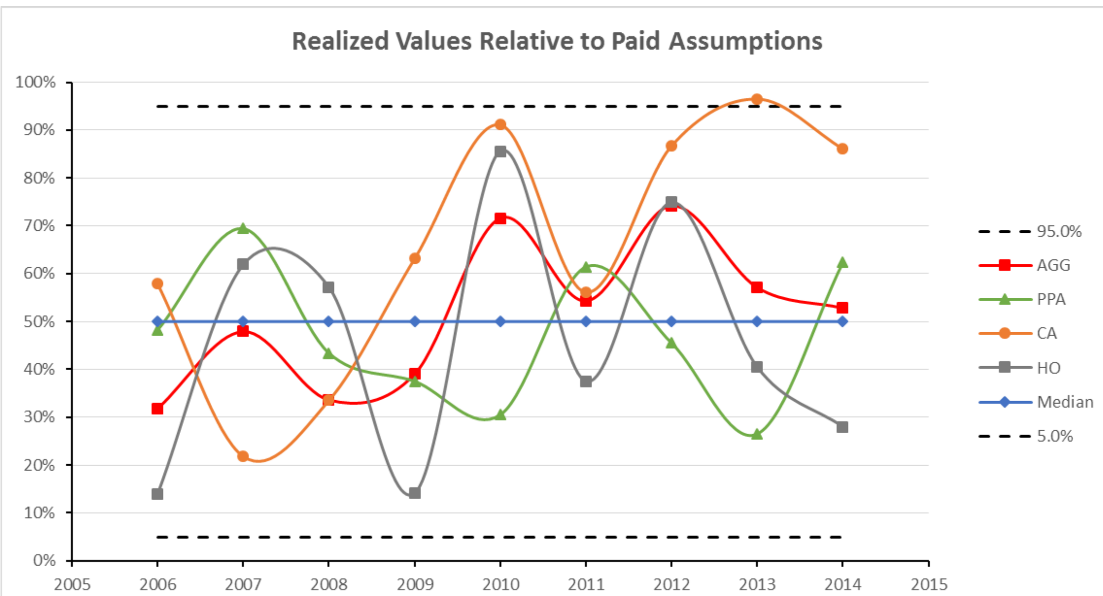


Figure F.7 – Graph of Realized Values vs. Assumptions – Incurred (Stochastic)

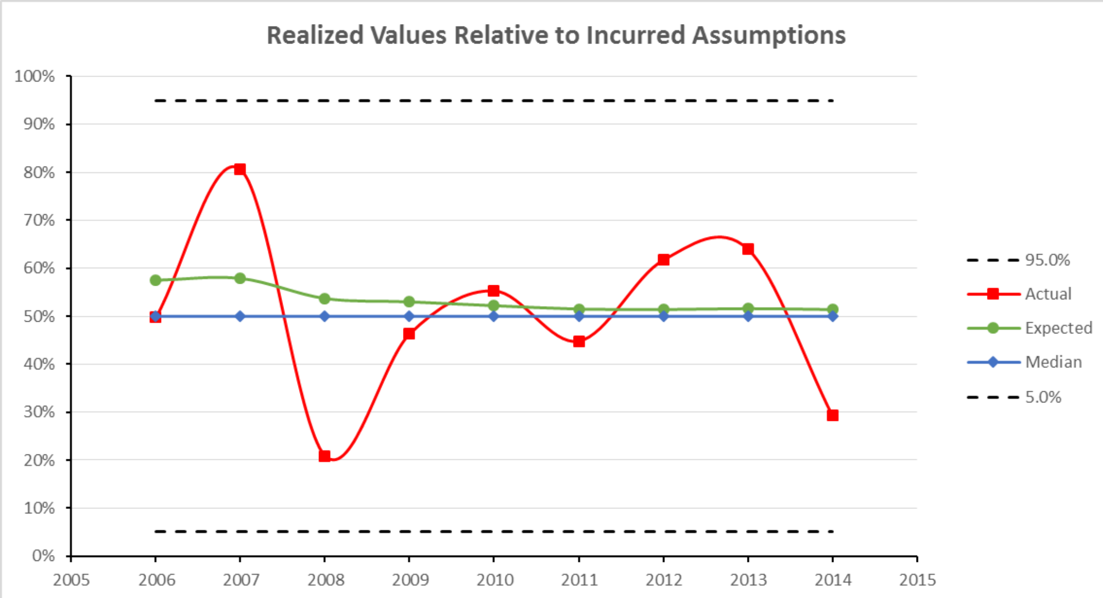
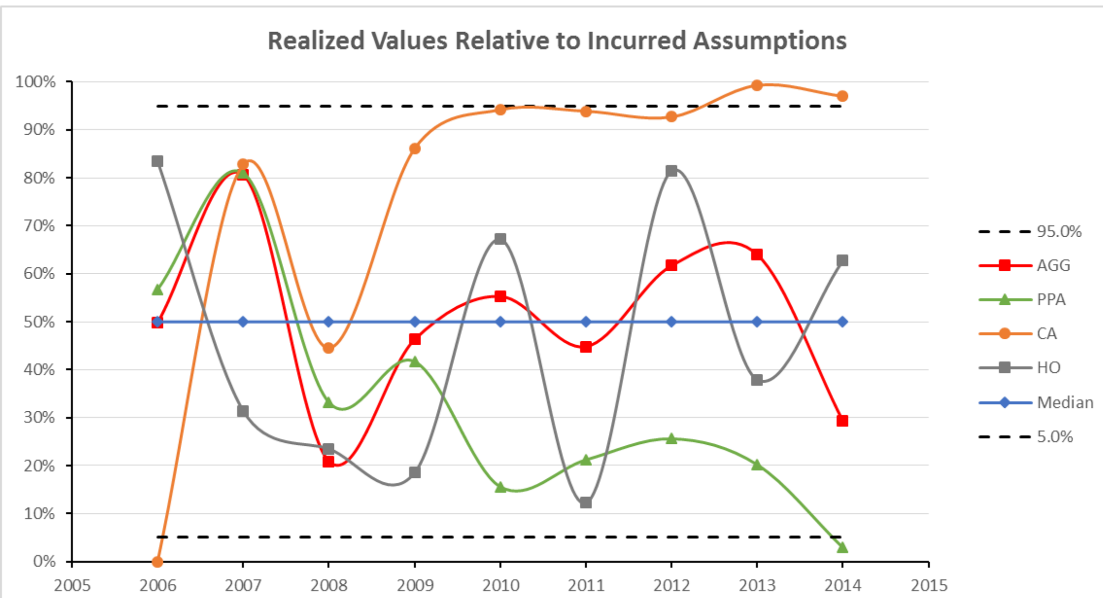


Figure F.8 – Graph of Realized Values vs. Assumptions – Incurred (Stochastic)



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Abbreviations and notations

The following abbreviations and notations are used in the paper.

AY, Accident Year
AY = CY, the latest AY for which there is no comparable expectation based on the prior annual reserve analysis

AYLWA, All Year Loss Weighted Average
BF, Bornhuetter-Ferguson
CA, Commercial Automobile
CEO, Chief Executive Officer
CL, Chain Ladder
CoV, Coefficient of Variation
ENID, Events Not In the Data
ERM, Enterprise Risk Management
FD, Framework Directive
GLM, Generalized Linear Models
HO, Homeowners
CY, Calendar Year
AY < CY, all AYs except the latest AY for which there is a comparable expectation based on the prior annual reserve analysis
IELR, Initial Expected Loss Ratio
Inc BF, Incurred Bornhuetter-Ferguson Method
Inc CL, Incurred Chain Ladder Method
KPI, Key Performance Indicator
LDF, Loss Development Factor
MLE, Maximum Likelihood Estimation
ODP, over-dispersed Poisson
Pd BF, Paid Bornhuetter-Ferguson Method
Pd CL, Paid Chain Ladder Method
PPA, Private Passenger Automobile
TAS-M, Technical Actuarial Standard: Modelling

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