Casualty Actuarial Society E-Forum, Spring 2015

The CAS *E-Forum*, Spring 2015

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This *E-Forum* contains eight reinsurance call papers and two independent research papers. The reinsurance papers were created in response to a call issued by the CAS Committee on Reinsurance. Some of the Reinsurance Call Papers will be presented at the 2015 Seminar on Reinsurance on June 1-2, 2015, in Philadelphia, PA.

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The Actuary's Role in Transfer Pricing

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Abstract
When related parties enter into cross border intercompany reinsurance, most countries require that the intercompany pricing be consistent with an "arm's-length standard". An arm's-length standard is an internationally accepted concept that the price of a transaction needs to be reasonably consistent with what would have been negotiated between unrelated parties. In the U.S., regulations governing the intercompany prices are in the Internal Revenue Code ("IRC") Section 482 and Treasury Regulations promulgated thereunder. The analysis and documentation surrounding these regulations is referred to as transfer pricing analysis. Actuaries often play a key role in creating transfer pricing documentation since it requires an in depth knowledge of reinsurance pricing and a fundamental understanding of the reinsurance market. In this paper, we will provide an overview of transfer pricing regulations and acceptable documentation. Further, we will explore and demonstrate the methods that are commonly used to support the pricing of such transactions, which include Return on Economic Capital, Market Based, Expected Profits, Rate-on-Line and Contract Comparison. We will also give practical examples and provide considerations for the actuary performing these analyses.

Keywords. Reinsurance, Transfer Pricing, Tax

1. INTRODUCTION

The property and casualty insurance industry increasingly operates on a global level. As part of an overall global business strategy, many companies utilize intercompany reinsurance to manage risk and capital more effectively while ultimately improving profitability. Accordingly, taxing authorities in many jurisdictions are focusing on and challenging more and more the pricing associated with these related party transactions (i.e., transfer pricing).

A common circumstance arises for U.S. domiciled insurance companies that have affiliates in jurisdictions such as Bermuda that have no corporate taxes. In such cases, the Internal Revenue Service (IRS), the U.S.' taxing authority, may challenge the U.S. company with regard to its pricing of reinsurance ceded to such an affiliate, with the concern that the U.S. company is paying reinsurance premiums that are greater than what might be observed between unrelated parties. Since the companies are affiliated, the IRS may take the view that the ceding company has an incentive to pay excessive premium for the risk being reinsured, because it reduces the U.S. taxable income and thus the tax obligation.

Taxing authorities in many jurisdictions have regulations that guide companies on how to appropriately develop evidence for the pricing of intercompany transactions. Typically, such guidance requires that the pricing be consistent with that which would be charged between unrelated parties; this is sometimes referred to as the "arms-length standard." While transfer pricing applies not only to reinsurance but to all intercompany transactions, demonstrating that pricing is arms-length is often more challenging for reinsurance as it does not have a listed market price. As a result, actuaries often play a key role in creating transfer pricing documentation since it requires an in-depth knowledge of reinsurance pricing and a fundamental understanding of the reinsurance market.

However, relatively few actuaries perform transfer pricing analyses or are even aware of the regulatory need for such analyses. In this paper, we will provide a high level overview of transfer pricing regulations and acceptable documentation. Further, we will explore and demonstrate the methods that are commonly used to support the pricing of such transactions, which include Return on Economic Capital, Market Based, Expected Profits, Rate-on-Line and Contract Comparison. We will also give practical examples and provide considerations for the actuary performing these analyses.

1.1 Research Context

Although no specific papers addressing the role of actuaries in transfer pricing have been published, the general concepts are covered to a certain degree by other authors, notably Rodney Kreps in "Investment Equivalent Insurance Pricing" and Lee R Steeneck in "Loss Portfolios: Financial Reinsurance."

1.2 Objective

The objective of this paper is to provide education and support to actuaries when performing transfer pricing analyses, including practical examples of several methods that are commonly used in such circumstances.

1.3 Outline

The remainder of the paper proceeds as follows:

Section 2: Background and Current Tax Regulations

Section 3: Types of Methods

Section 4: Methodology and Examples

Section 5: Potential Issues and Variations

Section 6: Conclusions

2. BACKGROUND AND CURRENT TAX REGULATIONS

Many multinational companies use intercompany reinsurance as a key component of their business strategy and often need to consider certain country-specific regulations when dealing with cross-border transactions. When related parties enter into reinsurance contracts, most countries require that pricing of these intercompany transactions be consistent with an arm's-length standard. An arm's-length standard is an internationally accepted concept requiring the price of a transaction to be reasonably consistent with the price that unrelated parties would have negotiated.

In the U.S., regulations governing intercompany reinsurance transaction pricing may be found in Internal Revenue Code ("IRC") Section 482 and Treasury Regulations promulgated thereunder, as well as a penalty provision prescribed in IRC Section 6662. However, these regulations do not prescribe a particular method for determining the pricing of such a transaction. To avoid the risk of penalties resulting from the IRS disagreeing with the intercompany reinsurance pricing and imposing an adjustment, a taxpayer must prepare and maintain documentation to substantiate its pricing of an intercompany transaction by the time it files its tax return. Section 6662 requires documentation including, but not limited to, the following:

- An overview of the taxpayer's business,
- A description of the intercompany transaction(s),
- Selection of the method used to demonstrate that the pricing is consistent with an arm's-length transaction, and
- An analysis to substantiate the intercompany pricing.

Taxing authorities in many jurisdictions outside the U.S. have similar transfer pricing requirements. Since no two reinsurance contracts are identical, demonstrating arms-length intercompany contract pricing can be challenging. Nevertheless, the documentation and judgments made therein should support the intercompany pricing because taxing authorities will heavily scrutinize the documentation, and the level of scrutiny will increase as the transaction decreases the entity's tax obligation.

2.1 Definition of "Price" for a Reinsurance Contract

For excess of loss reinsurance contracts, "price" is commonly expressed as the contract premium. In some cases it is expressed as a percentage of underlying subject premium, but, effectively, the price is still the final premium. However, on a quota share contract, the determination of price arises in effect from the ceding commission. Since premiums and losses covered under quota share percentages are determined as a contractually stated proportion of the underlying reinsured contracts, the contractual commissions are the actual determinant of the contract pricing. The higher the expected ceding commission, the lower the effective price of the contract.

3. TYPES OF METHODS

The approaches that actuaries typically use to determine the price of intercompany reinsurance contracts fall into four general categories of methods:

1. Capital Based
2. Market Based
3. Contract Comparison (including Rate on Line)
4. Expected Profits

As a starting point, for transfer pricing purposes it is helpful to evaluate a reinsurance contract in the same manner that a pricing actuary in an actuarial department would price a reinsurance contract. However, the approaches used for transfer pricing support may be different from traditional pricing approaches. The actuary is trying to determine a reasonable market price and may operate at a different level of detail than the company pricing actuaries (level of detail and breadth of methods used may be more or less). Also the transfer pricing actuary may derive a range of acceptable prices, the width of which would vary depending on the type of business and the uncertainty in the market. There are also specific company considerations that may alter the price of the reinsurance contract. For example, a company may place more value on a contract because it contains a certain class of business that balances its portfolio. In addition, some of the methods used are hybrid methods, utilizing market data, company data and specific contract data and generally do not fit squarely into one of the four approaches.

3.1 Capital Based Methods

The most commonly used and most complicated approaches are capital based methods, whereby price is determined based on economic variables and a theoretical construct, described below. The basic components that determine the price are:

1. Expected amount of covered losses, discounted to present value
2. Internal expenses
3. Cost of capital that the assuming company would maintain over time for the risk inherent in the contract.

Capital Based methods require an estimate of capital associated with the policy as well an estimate of what investors demand as a return on that capital. This class of methods is useful for both excess of loss and quota share contracts. It tends to be an especially useful method for evaluating lines of business where the pricing tends to be highly dependent on the uncertainty and duration of the cash flows. For portfolios of business that may be evaluated in a loss portfolio transfer or a commutation, variations of this method are almost exclusively used as the other methods described herein are often not applicable.

There are various approaches in which the capital required by the assuming company is estimated. Some common ways to determine capital are:

1. A solvency ratio, for example the 99.5th percentile of the loss distribution, with further consideration given to diversification within the reinsurer's portfolio of business.
2. Observed leverage ratios in the property/casualty insurance sector, comprised of premium to surplus and/or reserve to surplus ratios.
3. Based on a risk-based capital (RBC) prescribed ratio applied to premiums and estimated unpaid claims, which vary by line of business.
4. An allocation of total company capital.

The principal advantage of capital based methods is that they are generally the most consistent with common actuarial pricing approaches. Capital based methods directly consider the distribution of expected losses, expected payment pattern, cost of capital, and profitability targets to estimate price.

Nevertheless, there are potential limitations with this approach. In applying this method, there are numerous assumptions required, notably the selections of a capital requirement and an appropriate return on capital are othen be subjective, particularly if the assuming company does not provide reinsurance to unrelated parties. These assumptions may be made and the overall model may lack real market significance and may not reflect changes in cycle or market forces that drive price. As such, these methods are often not the primary methods used for coverages that cover predictable and more homogeneous exposures.

As an additional consideration, if the assuming company writes reinsurance to unrelated parties, it is helpful to demonstrate to taxing authorities that key assumptions used in the pricing (i.e., required capital, expected return, etc.) are the same between third party contracts and intercompany contracts.

3.2 Market Based Methods

Market based methods essentially amount to industry comparisons of commonly used market benchmarks, such as combined ratios from publicly available information. Combined ratios are most commonly applicable for quota share contracts, as excess of loss contracts are typically more difficult to determine market benchmarks for. Considering the impact of the time value of money can be a challenging nuance of this method.

This method is typically performed on a line of business level, such as commercial auto liability, or at times by general class of business, such as Reinsurance Type B. This may be a higher or different level of aggregation than typically used by reinsurance pricing actuaries.

The principle advantage of these types of methods is simplicity. They also reflect current market conditions and have "real world" significance. They are easy to explain to others and defendable. Additionally, they do not require assumptions regarding capital requirements and expected return on capital.

A disadvantage to these methods is that they may not reflect the nuances of a particular contract, and as such the more uncertainty and/or the longer the payout of claims, the less reasonable these methods are for transfer pricing. These methods will therefore work best for short tail contracts, where price and contract features are more homogenous in the market.

3.3 Contract Comparison Methods

In determining an arms-length price, an actuary may leverage insights gained from the pricing of contracts with unrelated parties. This may include directly comparing the pricing for similarly reinsured business, indirect comparisons, and an approach we refer to as "the rate-on-line method." Rate-on-line is defined as the price of a layer divided by the width of the layer. In application, this method leverages information regarding rates-on-line from externally placed reinsurance to estimate rates-on-line on other layers being reinsured between related parties for the same underlying business.

An important advantage to these methods is that they directly or indirectly provide evidence that the pricing is consistent with actual contracts between unrelated parties.

A disadvantage to these methods might be that they don't consider a broader market or economic perspective or unique contract features because they are focused on just a few contracts.

These methods can work equally well for both quota share and excess of loss contracts, for various levels of risk.

3.4 Expected Profit Methods

The expected profit method is used for straight quota share contracts only, and it compares the expected profit of the assuming company to the expected profit of the ceding company. All else equal, taxing authorities may expect that the ceding and assuming companies share profits consistent with their proportional share as contractually set under the contract. Oftentimes, in our experience, we have found that the ceding company retains somewhat more of its proportional share as this entity typically owns and controls the business and would tend to negotiate a somewhat greater share in the open market.

An advantage to this method is its logical appeal and simplicity. However, there are several disadvantages. One, apart from acquisition expenses, it is not clear how the assuming company's operating expenses are considered. Two, it is not clear if the equivalence of profit is performed before or after income taxes. The application of this method can yield significantly different results depending on how these assumptions are set.

4. METHODOLOGY AND EXAMPLES

This section will present individual methods within the classes listed above and give examples of the application for each method. The examples are meant to provide simple illustrations as to how the methods could be applied in practice, and in certain cases we used simplifying assumptions for ease of the illustration. In practice, the methods used to support transfer pricing of reinsurance contracts should strive to be reasonable from an actuarial perspective, yet understandable to taxing authorities. In striking this balance, the methods used are often less sophisticated than those used to price reinsurance transactions in the open market.

To illustrate these methods, we perform the methods with a sample quota share contract, a sample aggregate excess of loss contract and/or a sample property excess contract.

4.1 Contracts

4.1.1 The Quota Share Contract

Assume you have the following quota share contract:

1. Underlying Subject Premium = 100,000
2. Percent Ceded = 50%
3. Actual Ceding Commission = 25.0%
4. Lines of Business = Other Liability Occurrence
5. Acquisition costs = 25% or $25,000
6. Assuming Company expense ratio = 2%
7. Ceding Company is U.S. based with a tax rate of 35%
8. Assuming Company is domiciled in Bermuda and pays no corporate taxes.

Historical Data is given as follows:

Calendar Year	Earned Premium	Paid Loss and Expense	Caried Ultimate	Loss Ratio
2004	$ 238,416	$ 118,942	$ 132,090	55.4%
2005	233,273	120,946	132,616	56.9%
2006	246,685	109,425	126,153	51.1%
2007	201,719	110,161	133,728	66.3%
2008	140,162	67,477	91,263	65.1%
2009	97,008	44,881	75,019	77.3%
2010	86,469	30,429	67,909	78.5%
2011	72,845	25,854	61,078	83.8%
2012	66,176	6,339	44,688	67.5%
2013	53,467	2,685	45,257	84.6%
	$ 1,436,220	$ 637,139	$ 909,801	63.3%
	Coefficient of Variation of Loss Ratio			19.0%

4.1.2 The Aggregate Excess of Loss Contract

Assume that the underlying business above had an aggregate excess cover written for losses between a 72.5% and 92.5% loss ratio. Also assume the price of the contract is 6.75% of underlying subject premium. All other info between the two parties is the same.

4.1.3 The Property Excess of Loss Contract

Assume this is a property excess of loss contract covering the layer from $25 million excess of $40 million. The ceding company reinsures layers up through $40 million with third party reinsurers. The current data available is as follows:

Layer	Premium Charged 2013
10 M xs 15 M	$ 4,875,000
10 M xs 25 M	1,100,000
5 M X 35 M	300,000

All other company information is as above. The contract has been priced at $1,027,000.

4.2 Return on Economic Capital Method (ROEC Method)

The ROEC method is a common variation of a capital based method, where an estimate of the premium is made considering commissions paid to the ceding insurer, an estimate of losses that will be covered under the contract, and an estimated return on economic capital that is commensurate with the assuming company's target rate of return or opportunity cost of capital.

Economic capital is a theoretical construct representing the amount of capital an assuming company would need to dedicate to a specific block of business in order to maintain solvency a high percentage of the time. For purposes of our illustrations, we assumed that the assuming company would price these agreements based on dedicating capital that would result in 99.5th percentile of certainty that it would be sufficient to cover the uncertainties under the transaction, as this percentage is one we commonly observe being applied in practice. The assuming company must hold this amount of capital over the life of the contract and therefore will incur an opportunity cost of maintaining this capital rather than investing it in other investments. The opportunity cost, along with the total value of losses, is considered as part of the cost of assuming business.

There are several alternatives that can be used to the way we derive economic capital, and we list some of the alternatives below:

1. Using industry or target premium to surplus or reserve to surplus ratios to determine capital (this method will be illustrated below).

2. Using RBC ratios or other industry benchmark ratios to determine capital.
3. Using an allocation of total company capital.

The advantage to the way we will illustrate the capital requirement is that it is relatively easy to calculate. A transfer pricing review might offer several alternative versions of this method to illustrate the range of prices in which a reasonable arms-length price might fall.

Since the amount of premium is dependent on the overall capital charge over the life of the contract and since the overall level of capital required is dependent on how much premium is received, the determination of premium is made through an iterative process. When a reinsurance contract is written, the expected outcome is that premium will cover the losses associated with the contract. However, there is a reasonable probability that the actual losses under the contract will exceed the consideration, creating the need for required capital. However, the more adequate the premium, the less need for capital; therefore, the amount of capital required is dependent on premium charged.

We will illustrate this method for our quota share and aggregate excess of loss methods since the data and information provided lends well for pricing those contracts. It should be noted however, that industry data specific to type of business can replace many of the components in our analysis, where needed.

4.2.1 General Formula

The general premium formula employed in the ROEC method is:

Premium = Discounted value of losses plus expenses plus the discounted cost of capital over the life of the contract.

4.2.2 Considerations and Assumptions

The following components are determined to perform the ROEC method:

1. *Total Variability and Expected Distribution of Losses* - A first step in this approach is to estimate the expected losses, as well as the potential variability of such losses. The greater the variability, the more economic capital the assuming company would need to maintain and thus the greater the premium. To estimate variability, losses might be modeled using lognormal distributions and a selected coefficient of variation (CV). Using historical data above, selection might be presented as follows:

Calendar Year	Earned Premium	Paid Loss and Expense	Caried Ultimate	Loss Ratio
2004	$ 238,416	$ 118,942	$ 132,090	55.4%
2005	233,273	120,946	132,616	56.9%
2006	246,685	109,425	126,153	51.1%
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2012	66,176	6,339	44,688	67.5%
2013	53,467	2,685	45,257	84.6%
	$ 1,436,220	$ 637,139	$ 909,801	63.3%
	(a) Coefficient of Variation of Loss Ratio			19.0%
	Parameter Risk Load as a % of Variance			50.0%
			(b) Final CV	23.2%

Where (a) = Standard Deviation of the loss ratio column divided by the weighted average (63.35%) of that column and (b) = square root ((a)^2 x (1+ Parameter Risk Load)) since in our experience parameter loads are more commonly applied to variance rather than standard deviation. The parameter risk load is selected judgmentally based on industry norms. Note for simplicity this example does not include enhancement such as on-leveling of premium and loss trends. The appropriate CV is a matter of judgment and the Actuaries best judgment with regards to the appropriate data and final selection.

In addition for contracts with more than one line of business, the distribution of the aggregate business should consider correlations among the lines of business. A full review of appropriate modeling of losses is beyond the scope of this paper.

The higher the selected CV, the higher the need for capital, and therefore the higher the capital charge and the higher the premium (or lower the commission for a quota share contract).

2. *Solvency Standard* - Our example uses the 99.5th percentile of the above distribution to determine total capital needs. In another words, we assume that the reinsurer's risk appetite is such that no more than a one in 200 chance of ruin is acceptable. The selected percentile is an assumption that can be varied.

3. *Time Value of Money* - This approach considers the time value of money on the premium and economic capital. Accordingly, the U. S. Treasury security interest rates when the contract would have been priced are commonly used in practice. A good resource can be found at http://www.treasury.gov. We used the treasury yield curve to match cash flows to the appropriate risk free interest rates.

4. *Capital Charge* – In this context, the capital charge in essence reflects the amount of return expected above the risk free rate which is commensurate with the risks of writing this type of reinsurance. Note that because of the nature of the way we perform our calculations, we assume a pre-tax rate.

   This assumption may be benchmarked using the reinsurers own recent experience or using industry data. It is a very subjective assumption, and as such it may be useful to calculate premiums using a range of estimates based on a range of capital charges. The rates we have observed in the industry have varied widely, albeit more recently we have observed rates between 4% and 10%. For purposes of our illustrations, we used a charge of 5%.

5. *Expected Payout Pattern of Losses* – For our example, we used the payout pattern implied by the historical data and smoothed the tail. This pattern is important as it determines how long capital will need to be held. To the extent that losses are not yet paid, uncertainty remains and capital must be held. The longer the payout pattern, the longer the need for capital and the higher the capital charge. A higher capital charge will increase premium; however, the longer payout pattern will decrease the discounted value of the losses.

6. *Expected Loss Ratio on Underlying Business* – This estimate will be determined by available data and underwriting expectations. We judgmentally selected a loss ratio of 70% based on recent years' experience in our example.

7. *Diversification Benefit* – The actuary should also consider a diversification benefit present to the assuming company in adding the contract to its portfolio of business. In some cases, this is not relevant as the assuming company may write no other business besides a contract from its affiliate. Or conversely, the assuming company may write a highly diversified portfolio, and thus the contract may require significantly less capital due to diversification. This also tends to be an assumption that requires significant judgment.

   Suppose in the case of our other liability quota share contract, the reinsurer writes mostly property business for the rest of its book. In such a case, it would be logical to assume that the contract will not require as much additional capital. If the history is as follows, we might assign a diversification benefit around 87%

Calendar Year	Other Liability Loss Ratio	Property Loss Ratio	Weighted	Portfolio
2004	55.4%	74.0%		
2005	56.9%	64.7%		
2006	51.1%	85.0%		
2007	66.3%	84.7%		
2008	65.1%	85.1%		
2009	77.3%	83.8%		
2010	78.5%	75.3%		
2011	83.8%	73.5%		
2012	67.5%	68.0%		
2013	84.6%	70.0%		
	63.3%	76.9%		
Select Loss Ratio	70.0%	68.0%	68.8%	68.8%
CV of Loss Ratio	19.0%	10.1%		
Parameter Risk Load (% of Variance)	50.0%	50.0%		
Correlation				-13.9%
Weights	40.0%	60.0%		
Final CV	23.2%	12.3%		11.0%
99.5th Percentile	123.1%	92.6%	104.8%	90.8%
Diversifications Benefit				86.7%

The weighted column simply weights the 99.5th percentile loss ratio by the 40/60 weights which would be derived from expected losses (in the year the contract is priced for). The portfolio column uses the portfolio variance formula:

$$\sigma_P^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_Aw_B\sigma_A\sigma_B\rho_{AB}$$

In the formula above, Rho, shown in the formula as (ρ), is the correlation observed between lines. (Note this is used for simplicity and to show the effect of such a correlation. While we have observed diversification benefits on multi-line portfolios it is beyond the scope of this paper to explore the best ways to estimate correlation. The actuary should use their best judgment and for purposes of demonstrating to IRS, keep it simple and well documented.) The diversification benefit of 86.7% is derived by taking the 99.5th percentile based on the portfolio CV and dividing it by the 104.8% which is based on a weighted average and therefore would assume 100% correlation. The capital required for this contract therefore is reduced to 86.7% of original capital requirement.

4.2.3 Results – Quota Share Contract

The following table is an illustration of the application of the ROEC method performed for the quota share contract, considering the assumptions as described in this section. A more detailed version of this exhibit is presented in Exhibit 1.

Calendar Year	Paid Loss (%)	Duration Matched Rate (%)	Discount Factor To Time Zero	Disc. Percent Paid	Percent Outs.	Disc. Percent Outs.	Disc. Outs. Loss	Needed Capital	Capital Charge at 5.00%	Disc. Capital Charge
			1.000		100.00	92.00	32,200	13,321		
2014	5.93	0.10	1.000	5.93	94.07	86.11	30,140	13,009	329	329
2015	8.25	0.26	0.996	8.22	85.81	78.15	27,351	11,648	650	648
2016	28.14	0.58	0.986	27.74	57.67	50.84	17,794	6,933	582	574
2017	2.48	1.02	0.965	2.39	55.19	49.45	17,306	7,057	347	335
2018	15.02	1.51	0.935	14.04	40.17	36.02	12,608	5,154	353	330
2019	14.11	1.93	0.900	12.71	26.06	23.29	8,152	3,301	258	232
2020	8.44	2.28	0.864	7.29	17.62	15.83	5,541	2,277	165	143
2021	4.36	2.55	0.828	3.61	13.26	12.16	4,255	1,844	114	94
2022	4.46	2.75	0.794	3.54	8.80	8.21	2,874	1,300	92	73
2023	3.80	2.94	0.759	2.88	5.00	4.79	1,677	806	65	49
2024	5.00	3.07	0.728	3.64	0.00	0.00	0	0	40	29

Total Charge	2,836
Economic Premium	35,751
Nominal Premium	50,000
Implied Commission	28.50%

Note the initial level of needed capital is determined as the 99.5th percentile (including diversification benefit) of discounted outstanding loss minus the total economic premium (nominal premium less ceding commission). As time progresses, the capital becomes the 99.5th percentile of the remaining outstanding loss minus the nominal held reserves at each point in time. In our example, we assumed a proportional relationship between capital and reserves overtime. Although the actuary can model this more scientifically, we feel this is adequate for transfer pricing documentation purposes. Another simplifying assumptions is that the payout pattern at the 99.5th percentile and the expected value are the same. Although it is possible to conceive two very different patterns, we feel using one pattern is suitable for transfer pricing purposes. In essence you are providing a corroborative range around price.

The economic premium must also equal the (discounted losses plus the cost of capital)/(1-the reinsurer expense ratio of 2%). Since the amount of capital depends on economic premium, the economic premium must be calculated iteratively. Commission is then determined by comparing the economic premium with the nominal premium.

4.2.4 Aggregate Excess of Loss Contract

For the aggregate excess of loss contract over the same book of business, we simply apply that same lognormal loss distribution to the layer of the contract. For this we use the Mean Excess Value (MEV) function of the lognormal distribution:

$$e(x) = \frac{\exp\left(\mu + \frac{\sigma^2}{2}\right)\left\{1 - \Phi\left(\frac{\ln x - \mu - \sigma^2}{\sigma}\right)\right\}}{\left\{1 - \Phi\left(\frac{\ln x - \mu}{\sigma}\right)\right\}} - x$$

Therefore, expected value in layer = MEV (attachment point or 72.5% loss ratio) x probability that losses are above 72.5% loss ratio – MEV (limit or 92.5% loss ratio) x probability that losses are above 92.5% loss ratio. In this case, the expected value of the layer as a percentage of subject premium is 4.3%. The 99.5th percentile of the underlying losses cover the whole layer and therefore the 99.5th percentile of the aggregate contract is 20% of the underlying premium.

The following are the results of applying the ROEC method for this contract. More detail is displayed in Exhibit 2..

Calendar Year	Paid Loss (%)	Duration Matched Rate (%)	Discount Factor To Time Zero	Disc. Percent Paid	Percent Outs.	Disc. Percent Outs.	Disc. Outs. Loss	Needed Capital	Capital Charge at 5.00%	Disc. Capital Charge
			1.000		100.00	89.48	3,876	10,466		
2014	-	0.10	1.000	-	100.00	89.52	3,878	13,573	258	258
2015	5.93	0.26	0.996	5.91	94.07	83.89	3,634	12,703	679	676
2016	8.25	0.58	0.986	8.13	85.81	76.53	3,315	11,589	635	626
2017	28.14	1.02	0.965	27.16	57.67	50.03	2,167	7,507	579	559
2018	2.48	1.51	0.935	2.32	55.19	49.16	2,129	7,441	375	351
2019	15.02	1.93	0.900	13.52	40.17	36.02	1,560	5,464	372	335
2020	14.11	2.28	0.864	12.19	26.06	23.43	1,015	3,557	273	236
2021	8.44	2.55	0.828	6.99	17.62	16.01	693	2,438	178	147
2022	4.36	2.75	0.794	3.47	13.26	12.32	534	1,890	122	97
2023	4.46	2.94	0.759	3.39	8.80	8.43	365	1,306	95	72
2024	8.80	3.07	0.728	6.40	0.00	0.00	0	0	65	47
									Total Charge	3,405
									Economic Premium	7,430
									Nominal Premium	100,000

Note that we did not recalculate a diversification benefit for this contract. It is often more challenging to estimate correlation reliably on an aggregate excess contract versus a portfolio of relatively homogeneous first dollar claims. Nevertheless, such correlation should still be considered to the extent the actuary believes it is meaningful to the estimates.

4.3 Leverage Ratio Method (LR Method)

This method is essentially identical to the ROEC method, except that capital is determined by observing premium and reserves to surplus ratios in the property/casualty insurance industry. While less "actuarial" than the ROEC method, the LR Method has an advantage of simplicity in that the approach is essentially the same but there are fewer assumptions surrounding required capital that need to be made. Instead, required capital is estimated at the property/casualty insurance sector level considering broader industry statistics. However, as a disadvantage, in cases where the assuming entities in the industry have significant risks beyond underwriting, such as significant reserve uncertainty, using the leverage ratio method alone may not produce a reasonable estimate.

For this reason, we use this method on the quota share contract only, as industry aggregate leverage ratios may not fit an aggregate excess contract. In lieu of using ratios for the industry as a whole, we may consider ratios for companies that are similar to the assuming company or that reinsure predominantly the lines of business covered by the contract. For this illustration, we used the industry as a whole and ratios of net premium plus reserves divided by average surplus averaged to approximately 1.8 over the latest 5 calendar years.

This method also has to be solved iteratively as initial capital is determined based on initial premium and total premium must also equal discounted losses plus capital charge plus expenses. These are our results for the quota share contract:

Calendar Year	Paid Loss (%)	Duration Matched Rate (%)	Discount Factor To Time Zero	Disc. Percent Paid	Percent Outs.	Disc. Percent Outs.	Disc. Outs. Loss	Needed Capital	Capital Charge at 5.00%	Disc. Capital Charge
			1.000		100.00	92.00	32,200	20,635		
2014	5.93	0.10	1.000	5.93	94.07	86.11	30,140	18,291	510	509
2015	8.25	0.26	0.996	8.22	85.81	78.15	27,351	16,686	915	911
2016	28.14	0.58	0.986	27.74	57.67	50.84	17,794	11,214	834	822
2017	2.48	1.02	0.965	2.39	55.19	49.45	17,306	10,732	561	541
2018	15.02	1.51	0.935	14.04	40.17	36.02	12,608	7,812	537	502
2019	14.11	1.93	0.900	12.71	26.06	23.29	8,152	5,068	391	352
2020	8.44	2.28	0.864	7.29	17.62	15.83	5,541	3,427	253	219
2021	4.36	2.55	0.828	3.61	13.26	12.16	4,255	2,578	171	142
2022	4.46	2.75	0.794	3.54	8.80	8.21	2,874	1,711	129	102
2023	3.80	2.94	0.759	2.88	5.00	4.79	1,677	972	86	65
2024	5.00	3.07	0.728	3.64	0.00	0.00	0	0	49	35

Total Charge	4,201
Economic Premium	37,144
Nominal Premium	50,000
Implied Commission	25.71%

Further detail can be found in Exhibit 3.

4.4 Other Capital Based Methods

There are multiple alternatives to calculation of capital such as using RBC ratios or allocation of total company capital. Such methods would be applied in an identical manner as the ROEC, except with a different amount for required capital.

The application of these methods is from the perspective of the assuming company and how much capital the company is expected to hold against the contract at a given point in time. The cost of such capital then becomes part of our calculation of premium. An alternative method would be to calculate returns form the point of view of the investor and recreate financial statements to derive when capital has to be invested and released. The cash flows are then discounted at the investor's required rate of return and the premium can be set such that the net present value to the investor is zero. We have not illustrated this alternative approach; however, in our experience the results tend to be substantially the same as those produced by the ROEC method.

4.5 Market Combined Ratio Method

The market combined ratio method compares the expected combined ratios using industry benchmarks to that expected to be generated by the reinsurance contract being evaluated. The source of industry benchmarks is typically aggregate industry data, refined by line of business as applicable. This method is most commonly used for quota share contracts, as it is typically easier to obtain industry benchmarks.

Below are the results of an illustration using industry combined ratios. In this illustration, the industry combined ratios were lower than we observed in the data for our sample contract. Our sample contract has a loss ratio of 70% and actual ceding commission of 25%. The indicated commission for this method would be the commission that sets the combined ratio equal to that reported by the industry for the coverage type and selected group of accident years:

	Industry Other Liability Combined Ratio (%)
Lower Quartile	61.0
Median	74.3
Upper Quartile	86.8
Contract Expected Combined Ratio	95.0
Equalizing Commission	
Lower Quartile	(9.0)
Median	4.3
Upper Quartile	16.8

Based on this data, for the other liability line of business as a whole, we might conclude that our contract is providing reinsurance to risks that are less variable than the broader industry since the expected combined ratio for our sample contract is much higher. Generally, the market combined ratio method works well with shorter tail and less varied lines such as nonstandard personal auto or accident and health quota share. In such cases, using sector combined ratios provides reinsurance pricing estimates that are consistent with observed industry practice, and such estimates tend to be greater than estimates based on methods that derive rates based on perceived uncertainty.

4.6 Indirect Industry Comparison Method

Indirect comparison methods often tend to equalize the playing field depending on the amount of uncertainty present in the contract. For this approach, we measure the risk of the contract by its CV. All else equal, a reinsurer should expect to receive a greater risk margin for increased uncertainty - the greater the uncertainty, the lower the expected combined ratio. While we acknowledge that many other factors and nuances in pricing, this method merely shows that a contract is in line with industry risk / price relationships in general and can be a useful tool in demonstrating fair pricing.

The indirect industry comparison method may be applied using aggregate industry data for combined ratios over, for example, a 5 to 10 year period. For each company that is included in the comparison, we can calculate the standard deviation of those reported combined ratios and the current combined ratio. By doing this for a group of companies for lines of business related to the contract we can establish a relationship between risk and price.

Our analysis of industry data for other liability revealed the following average combined ratios (%):

CV greater than 1.0	56.7
CV greater than .5 and less than 1.0	73.2
CV greater than .25 and less than .5	86.4
CV less than .25	100.6

In our illustration, the CV for the quota share contract is 23.2% and the CV of the aggregate excess contract is 170.3%. The combined ratios at current prices are 95% and 64.2% respectively. In addition to evaluating at averages by industry band, a line could be fit to individual CV data points to provide another estimate of combined ratios. So in this case, for the aggregate excess contract, the CV of 170.3% is fitted to a combined ratio of 63.6% which compares well with the priced combined ratio of 64.2%. For the quota share contract, the fitted price of 92.7% also compares well with the expected combined ratio of 95.%. The last to columns display what the price would have been for expected combined ratio to match the fitted. This is illustrated in the table below:

	Contract CV	Fitted Combined Ratio (%)	Expected Combined Ratio (%)	Implied Comission at Fitted Combined Ratio	Implied Price at Fitted Combined Ratio
Aggregate Excess Fitted Value	170.3%	63.6	64.2		6.8
Quota Share	23.2%	92.7	95.0	22.7	

4.7 Contract Comparison Method

An analysis of other contracts that are written or entered into by either the ceding or assuming company can be relevant to preparing support in a transfer pricing analysis, and both ceded and assumed contracts are considered as long as they were entered into between unrelated parties. Tax experts often consider comparable contracts to be the strongest support when evidencing transfer pricing. Unfortunately, in most cases, the pricing in one reinsurance contract is not directly comparable to the pricing in another, particularly for excess of loss contracts.

Another area that the actuary may want to investigate is pricing practices of the reinsurer. If third parties are all priced using the same ROEC method or the same table of underwriting benchmarks, it is important that the intercompany contract follow the same set of rules.

Lastly, the Indirect Industry Comparison Method can be used on third party contracts in which the ceding company and assuming company are engaged. The following table shows what a typical comparison of existing contracts might look like:

Third Party Reinsured	Contract Type	Coefficient of Variation	Expected Combined Ratio	Margin
Company A	All Lines QS	16.5%	95.5%	4.5%
Company B	Marine QS	16.0%	96.0%	4.0%
Company C	Property Catastrophe QS	58.7%	66.8%	33.2%
Company D	General Liability & Liquor Liability QS	16.4%	94.0%	6.0%
Company E	Property QS	16.7%	100.0%	0.0%
Company F	Workers' Compensation XOL	76.9%	90.4%	9.6%
Company 5	Auto QS Retro Reinsurance	11.9%	97.0%	3.0%
Company H	Workers' Compensation XOL	26.3%	91.9%	8.1%
Company I	Medical Professional Liability Clash XOL	60.0%	73.0%	27.0%
Company J	Property Catastrophe Retrocession	125.0%	68.4%	31.6%
	Minimum	11.9%	66.8%	0.0%
	Maximum	125.0%	100.0%	33.2%

Qualitatively, we can say that our current other liability contracts are in line with existing contracts in terms of the relationship of risk to price.

4.8 Rate on Line Method (ROL Method)

In the absence of sufficient data to conduct other methods, as in the case of the sample property excess contract, it is often useful to use a ROL method. ROL is defined as the price of a reinsurance layer divided by the width of that layer. The premise of this method is that as the attachment point of the insurance layer increases, the rate on line should decrease, since the frequency of losses decreases. To the extent the ceding company has entered into contracts with unrelated parties for certain layers of coverage, the rates on line observed can be leveraged to estimate a range of rates on line for a layer of coverage written between related parties.

The following is an illustration of an application of the Rate on Line Method. There are various considerations that may impact the evaluation of the results, such as expense ratios and margin requirements – these are typically considered in developing a range.

Layer	Premium Charged 2013	Width Of Layer	Charged Rate on Line	Low Selected ROL	High Selected ROL	Low Premium	High Premium
10 M xs 15 M	$ 4,875,000	10,000,000	48.8%				
10 M xs 25 M	1,100,000	10,000,000	11.0%				
5 M xs 35 M	300,000	5,000,000	6.0%				
25 M xs 40M		25,000,000		3.0%	5.0%	$ 750,000	$ 1,250,000

4.9 Expected Profits Method

The expected profits method is applicable for traditional quota share contracts. The basic premise of this method is that all else being equal, the ceding company and the assuming company should receive their proportionate share of expected profits. Note for simplicity, we did this on a nominal basis. The following displays our analysis.

	Ceding Company	Assuming Company
Premium	50,000	50,000
Expenses	(25,000)	(1,000)
Expected Commission	12,500	(12,500)
Expected Losses	(35,000)	(35,000)
Margin	5.0%	3.0%
After Tax Margin	3.3%	3.0%
Equalizing Commission		
Before Tax	24.0%	
After Tax	24.8%	
Commission at 24.8%	12,424	(12,424)
Margin	4.8%	3.2%
After Tax Margin	3.2%	3.2%

There are several key assumptions in the analysis that require judgment. First, for both the ceding and assuming companies, the expenses applicable to performing this exercise needs to be estimated. For the reinsurer, expenses should be the nominal amount to write the contract. Taxation also needs to be considered. If the balancing of profits is performed on an after-tax basis, then in effect the ceding company is receiving a share of the tax benefit from the transaction and reducing the estimated price. In practice, we observe both to determine the fairness of a contract.

5. POTENTIAL ISSUES AND VARIATIONS

During the course of performing transfer pricing analyses, there are several challenges that may arise.

5.1 Loss Portfolio Transfers

Transfer pricing applies to loss portfolio transfers (LPT) between related parties, even though for U.S. statutory purposes company management may be tempted to book the transaction at book value to ensure the transaction is surplus neutral. This may not produce results that are consistent with transfer pricing approaches. For LPTs, the ROEC method generally works very well since it captures the risk specific to the transfer and the unique payment patterns that may arise. Other methods described herein are generally less appropriate.

The trade-off between cost of capital and discount in the losses will be important in determining if price should be greater than, equal to or less than book value. Because of the inherent uncertainty in pricing such a transaction, it is also useful to vary assumptions such as capital charge and capital requirement to derive a range in price.

5.2 Captive Reinsurance Companies

The arms-length principle and pricing approaches described herein are equally applicable for pricing business ceded to affiliated captive reinsurance companies. However, there may be additional considerations that arise with captive reinsurance companies, such as:

1. The capital held in the captive may be much less than required under an economic capital analysis.
2. Internal expenses for captives are generally much lower than other reinsurance companies.
3. Captives may be subject to different tax laws, depending on the jurisdiction.

Accordingly, when performing transfer pricing on captive reinsurance transactions, the actuary should modify the methods appropriately.

5.3 Limited Industry Data

,Because of their multi-jurisdictional nature of transactions, transfer pricing engagements may involve classes of business not typically covered by industry sources such as Best's or SNL financials, which deal with statutory lines of business.

However, the principle of the relationship between risk and price is the same as are the economic principles above. In this case, methods like the indirect industry method and contract comparison methods can be very useful. Also, the actuary can find Schedule P lines that are very similar to foreign business.

5.4 Multiple Jurisdiction and Contracts

For some cross-border reinsurance contracts, there are multiple jurisdictions that may be impacted. In these cases, each taxing authority involved in tested transactions will have an interest in the fairness of price, and in particular an interest in not unfairly losing tax revenue. Several points are important to note in this situation:

1. Each intercompany contract should be fairly priced on its own. For example, it is generally not appropriate to have an excessively priced contract be offset with an underpriced one. Taxing authorities may only focus on the excessively priced contract.
2. A jurisdiction may be a country or a state, depending on the tax laws. It is important to have a comprehensive understanding of the tax treatment for each entity. Companies that are locating in certain jurisdictions may be taxed in another region, depending on the relevant corporate and tax laws.
3. Pricing methodologies between transactions in the group should use consistent methodology. This is similar to the assertion that pricing assumptions must be consistent with the company's pricing of third party transactions.

5.5 Taxing Authority Challenges

Taxing authorities, such as the IRS in the U.S., may challenge transfer pricing documentation and assess the company for the difference between what it considers to be an appropriate price and the actual price charged multiplied by the tax rate. The IRS may challenge assumptions or a certain methodology. For example, for a given transaction between affiliates, if the IRS determines that premiums paid from the U.S. to the Bermuda affiliate were excessive, it would assess the company for additional taxes. In addition, if the company did not maintain transfer pricing documentation, there would be an additional penalty assessed. As such, documentation of the assumptions and methodologies that were used to support the transfer pricing provide "penalty protection" for the U.S. taxpayer.

6. CONCLUSIONS

Transfer pricing is of increasing importance for many companies that operate internationally. Taxing authorities are focusing to a greater extent on intercompany agreements, including related party reinsurance contracts. Many casualty actuaries have an effective blend of reinsurance pricing training and experience, as well as broader reinsurance market insights and access to industry data to support transfer pricing evaluations on these contracts.

The methods to fairly price reinsurance contracts are not limited to what is presented in this paper. However, we believe that this paper provides useful descriptions and illustrations for an actuary conducting transfer pricing work in coordination with tax professionals.

The Appendix contains a summary of our illustrations as well as a sample presentation of results.

Appendix A

Exhibits are contained in Appendix A which show further details of examples provided in this paper.

5. REFERENCES

[1] Sample Industry Data from SNL Financials 2011 – 2013 Data.

Biography(ies) of the Author(s)

Lynne Bloom, FCAS, MAAA, is a Director at PwC in Philadelphia, PA. She has a B.B.A. in Finance from the Wharton Business School at the University of Pennsylvania. She is a Fellow of the CAS and a Member of the American Academy of Actuaries. Lynne is the chairman of the CAS Research Oversight Committee and Vice President of CAMAR.

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Appendix A

Summary of Indications

	Quota Share Commission	Aggregate Excess Rate	Property Excess Premium
ROEC	28.5%	7.4%	
LR	25.7%		
Market Combined Ratio Median	4.3%		
Market Combined Ratio Upper	16.8%		
Indirect Industry	22.7%	6.8%	
Rate On Line Low			$ 750,000
Rate On Line High			$ 1,250,000
Expected profits	24.8%		
Actual	25.0%	6.8%	$ 1,027,000

Appendix A

Exhibit 1

Return on Economic Capital Method - Quota Share

(1)	(2)	(3)	(4)	(5)	(6)		(7)	(8)	(9)	(10)	(11)	(12)
Calendar Year	Paid Duration	Paid Loss (%)	Duration Matched Rate (%)	Discount Factor To Time Zero	Disc. Percent Paid	Net Premium paid in	Percent Outs.	Disc. Percent Outs.	Disc. Outs. Loss	Needed Capital	Capital Charge at 5.00%	Disc. Capital Charge
				1.000		35,751	100.00	92.00	32,200	13,321		
2014	0.500	5.93	0.10	1.000	5.93	-	94.07	86.11	30,140	13,009	329	329
2015	1.500	8.25	0.26	0.996	8.22		85.81	78.15	27,351	11,648	650	648
2016	2.500	28.14	0.58	0.986	27.74		57.67	50.84	17,794	6,933	582	574
2017	3.500	2.48	1.02	0.965	2.39		55.19	49.45	17,306	7,057	347	335
2018	4.500	15.02	1.51	0.935	14.04		40.17	36.02	12,608	5,154	353	330
2019	5.500	14.11	1.93	0.900	12.71		26.06	23.29	8,152	3,301	258	232
2020	6.500	8.44	2.28	0.864	7.29		17.62	15.83	5,541	2,277	165	143
2021	7.500	4.36	2.55	0.828	3.61		13.26	12.16	4,255	1,844	114	94
2022	8.500	4.46	2.75	0.794	3.54		8.80	8.21	2,874	1,300	92	73
2023	9.500	3.80	2.94	0.759	2.88		5.00	4.79	1,677	806	65	49
2024	10.500	5.00	3.07	0.728	3.64		0.00	0.00	0	0	40	29

(a) Total Charge	2,836
(b) Economic Premium	35,751
(c) Nominal Premium	50,000
(d) Implied Commission	28.50%

Calculations

(5) 1/(1+(4)/100)^(2)

(6) (3) x (5)

(7) 100 - Cumulative of (3)

(8) Sumproduct of future (3) and (5) divided by current (5)

(9) (8) x (c) x Expected loss Ratio of 70%

(10) Initial Value: (9) x Loss ratio of 123.1 (99.5th percentile) / Expected Loss Ratio of 70.0 x Diversification Benefit of 86.7% - (b)
(b) represents premium and therefore held unearned premium at time contract is written
Subsequent Values subtract Nominal Loss reserves held at each point in time = (7) x (c) x Expected loss Ratio of 70%

(11) Previous (10) x capital charge of 5%

(12) (11) x (5)

(a) Sum of (12)

(b) Solved iteratively such that it is equal to [(a) plus initial value of (9)]/(1-expense ratio of 2%]

(d) 1 - (b)/(c)

Appendix A

Exhibit 2

Return on Economic Capital Method - Aggregate Excess

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Calendar Year	Paid Duration	Paid Loss (%)	Duration Matched Rate (%)	Discount Factor To Time Zero	Disc. Percent Paid	Percent Outs.	Disc. Percent Outs.	Disc. Outs. Loss	Needed Capital	Capital Charge at 5.00%	Disc. Capital Charge
	-			1.000		100.00	89.48	3,876	10,466		
2014	0.500	-	0.10	1.000	-	100.00	89.52	3,878	13,573	258	258
2015	1.500	5.93	0.26	0.996	5.91	94.07	83.89	3,634	12,703	679	676
2016	2.500	8.25	0.58	0.986	8.13	85.81	76.53	3,315	11,589	635	626
2017	3.500	28.14	1.02	0.965	27.16	57.67	50.03	2,167	7,507	579	559
2018	4.500	2.48	1.51	0.935	2.32	55.19	49.16	2,129	7,441	375	351
2019	5.500	15.02	1.93	0.900	13.52	40.17	36.02	1,560	5,464	372	335
2020	6.500	14.11	2.28	0.864	12.19	26.06	23.43	1,015	3,557	273	236
2021	7.500	8.44	2.55	0.828	6.99	17.62	16.01	693	2,438	178	147
2022	8.500	4.36	2.75	0.794	3.47	13.26	12.32	534	1,890	122	97
2023	9.500	4.46	2.94	0.759	3.39	8.80	8.43	365	1,306	95	72
2024	10.500	8.80	3.07	0.728	6.40	0.00	0.00	0	0	65	47

(a) Total Charge	3,405
(b) Economic Premium	7,430
(c) Nominal Premium	100,000
(d) Rate	7.43%

Calculations

(5) 1/(1+(4)/100)^(2)

(6) (3) x (5)

(7) 100 - Cumulative of (3)

(8) Sumproduct of future (3) and (5) divided by current (5)

(9) (8) x Expected cost of Layer of 4.3% x (c)

(10) Initial Value: (9) x 20% of (c) (99.5th percentile) x initial value of (8)/100 - (b)

(b) represents premium and therefore held unearned premium at time contract is written

Subsequent Values subtract Nominal Loss reserves held at each point in time = 4.3% x (7)

(11) Previous (10) x capital charge of 5%

(12) (11) x (5)

(a) Sum of (12)

(b) Solved iteratively such that it is equal to [(a) plus initial value of (9)]/(1-expense ratio of 2%]

(d) (b)/(c)

Exhibit 3

Leverage Ratio Method - Quota Share

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Calendar Year	Paid Duration	Paid Loss (%)	Duration Matched Rate (%)	Discount Factor To Time Zero	Disc. Percent Paid	Percent Outs.	Disc. Percent Outs.	Disc. Outs. Loss	Needed Capital	Capital Charge at 5.00%	Disc. Capital Charge
	-			1.000		100.00	92.00	32,200	20,635		
2014	0.500	5.93	0.10	1.000	5.93	94.07	86.11	30,140	18,291	510	509
2015	1.500	8.25	0.26	0.996	8.22	85.81	78.15	27,351	16,686	915	911
2016	2.500	28.14	0.58	0.986	27.74	57.67	50.84	17,794	11,214	834	822
2017	3.500	2.48	1.02	0.965	2.39	55.19	49.45	17,306	10,732	561	541
2018	4.500	15.02	1.51	0.935	14.04	40.17	36.02	12,608	7,812	537	502
2019	5.500	14.11	1.93	0.900	12.71	26.06	23.29	8,152	5,068	391	352
2020	6.500	8.44	2.28	0.864	7.29	17.62	15.83	5,541	3,427	253	219
2021	7.500	4.36	2.55	0.828	3.61	13.26	12.16	4,255	2,578	171	142
2022	8.500	4.46	2.75	0.794	3.54	8.80	8.21	2,874	1,711	129	102
2023	9.500	3.80	2.94	0.759	2.88	5.00	4.79	1,677	972	86	65
2024	10.500	5.00	3.07	0.728	3.64	0.00	0.00	0	0	49	35

(a) Total Charge	4,201
(b) Economic Premium	37,144
(c) Nominal Premium	50,000
(d) Implied Commission	25.71%

Calculations

(5) 1/(1+(4)/100)^(2)

(6) (3) x (5)

(7) 100 - Cumulative of (3)

(8) Sumproduct of future (3) and (5) divided by current (5)

(9) (8) x (c) x Expected loss Ratio of 70%

(10) Initial Value: (b) / 1.8

(b) represents premium and therefore held unearned premium at time contract is written

Subsequent Values use Nominal Loss reserves held at each point in time = (7) x (c) x Expected loss Ratio of 70% / 1.8

(11) Previous (10) x capital charge of 5%

(12) (11) x (5)

(a) Sum of (12)

(b) Solved iteratively such that it is equal to [(a) plus initial value of (9)]/(1-expense ratio of 2%]

(d) 1 - (b)/(c)

Summary of Indications

	Quota Share Commission	Aggregate Excess Rate	Property Excess Premium
ROEC	28.5%	7.4%	
LR	25.7%		
Market Combined Ratio Median	4.3%		
Market Combined Ratio Upper	16.8%		
Indirect Industry	22.7%	6.8%	
Rate On Line Low			$ 750,000
Rate On Line High			$ 1,250,000
Expected profits	24.8%		
Actual	25.0%	6.8%	$ 1,027,000

Return on Economic Capital Method - Quota Share

(1)	(2)	(3)	(4)	(5)	(6)		(7)	(8)	(9)	(10)	(11)	(12)
Calendar Year	Paid Duration	Paid Loss (%)	Duration Matched Rate (%)	Discount Factor To Time Zero	Disc. Percent Paid	Net Premium paid in	Percent Outs.	Disc. Percent Outs.	Disc. Outs. Loss	Needed Capital	Capital Charge at 5.00%	Disc. Capital Charge
				1.000		35,751	100.00	92.00	32,200	13,321		
2014	0.500	5.93	0.10	1.000	5.93	-	94.07	86.11	30,140	13,009	329	329
2015	1.500	8.25	0.26	0.996	8.22		85.81	78.15	27,351	11,648	650	648
2016	2.500	28.14	0.58	0.986	27.74		57.67	50.84	17,794	6,933	582	574
2017	3.500	2.48	1.02	0.965	2.39		55.19	49.45	17,306	7,057	347	335
2018	4.500	15.02	1.51	0.935	14.04		40.17	36.02	12,608	5,154	353	330
2019	5.500	14.11	1.93	0.900	12.71		26.06	23.29	8,152	3,301	258	232
2020	6.500	8.44	2.28	0.864	7.29		17.62	15.83	5,541	2,277	165	143
2021	7.500	4.36	2.55	0.828	3.61		13.26	12.16	4,255	1,844	114	94
2022	8.500	4.46	2.75	0.794	3.54		8.80	8.21	2,874	1,300	92	73
2023	9.500	3.80	2.94	0.759	2.88		5.00	4.79	1,677	806	65	49
2024	10.500	5.00	3.07	0.728	3.64		0.00	0.00	0	0	40	29

(a) Total Charge	2,836
(b) Economic Premium	35,751
(c) Nominal Premium	50,000
(d) Implied Commission	28.50%

Calculations
(5) 1/(1+(4)/100)^(2)
(6) (3) x (5)
(7) 100 - Cumulative of (3)
(8) Sumproduct of future (3) and (5) divided by current (5)
(9) (8) x (c) x Expected loss Ratio of 70%
(10) Initial Value: (9) x Loss ratio of 123.1 (99.5th percentile) / Expected Loss Ratio of 70.0 x Diversification Benefit of 86.7% - (b)
(b) represents premium and therefore held unearned premium at time contract is written
Subsequent Values subtract Nominal Loss reserves held at each point in time = (7) x (c) x Expected loss Ratio of 70%
(11) Previous (10) x capital charge of 5%
(12) (11) x (5)
(a) Sum of (12)
(b) Solved iteratively such that it is equal to [(a) plus initial value of (9)]/(1-expense ratio of 2%]
(d) 1 - (b)/(c)

Return on Economic Capital Method - Aggregate Excess

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Calendar Year	Paid Duration	Paid Loss (%)	Duration Matched Rate (%)	Discount Factor To Time Zero	Disc. Percent Paid	Percent Outs.	Disc. Percent Outs.	Disc. Outs. Loss	Needed Capital	Capital Charge at 5.00%	Disc. Capital Charge
	-			1.000		100.00	89.48	3,876	10,466		
2014	0.500	-	0.10	1.000	-	100.00	89.52	3,878	13,573	258	258
2015	1.500	5.93	0.26	0.996	5.91	94.07	83.89	3,634	12,703	679	676
2016	2.500	8.25	0.58	0.986	8.13	85.81	76.53	3,315	11,589	635	626
2017	3.500	28.14	1.02	0.965	27.16	57.67	50.03	2,167	7,507	579	559
2018	4.500	2.48	1.51	0.935	2.32	55.19	49.16	2,129	7,441	375	351
2019	5.500	15.02	1.93	0.900	13.52	40.17	36.02	1,560	5,464	372	335
2020	6.500	14.11	2.28	0.864	12.19	26.06	23.43	1,015	3,557	273	236
2021	7.500	8.44	2.55	0.828	6.99	17.62	16.01	693	2,438	178	147
2022	8.500	4.36	2.75	0.794	3.47	13.26	12.32	534	1,890	122	97
2023	9.500	4.46	2.94	0.759	3.39	8.80	8.43	365	1,306	95	72
2024	10.500	8.80	3.07	0.728	6.40	0.00	0.00	0	0	65	47

(a) Total Charge	3,405
(b) Economic Premium	7,430
(c) Nominal Premium	100,000
(d) Rate	7.43%

Calculations

(5) 1/(1+(4)/100)^(2)

(6) (3) x (5)

(7) 100 - Cumulative of (3)

(8) Sumproduct of future (3) and (5) divided by current (5)

(9) (8) x Expected cost of Layer of 4.3% x (c)

(10) Initial Value: (9) x 20% of (c) (99.5th percentile) x initial value of (8)/100 - (b)

(b) represents premium and therefore held unearned premium at time contract is written

Subsequent Values subtract Nominal Loss reserves held at each point in time = 4.3% x (7)

(11) Previous (10) x capital charge of 5%

(12) (11) x (5)

(a) Sum of (12)

(b) Solved iteratively such that it is equal to [(a) plus initial value of (9)]/(1-expense ratio of 2%]

(d) (b)/(c)

Exhibit 3

Leverage Ratio Method - Quota Share

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Calendar Year	Paid Duration	Paid Loss (%)	Duration Matched Rate (%)	Discount Factor To Time Zero	Disc. Percent Paid	Percent Outs.	Disc. Percent Outs.	Disc. Outs. Loss	Needed Capital	Capital Charge at 5.00%	Disc. Capital Charge
	-			1.000		100.00	92.00	32,200	20,635		
2014	0.500	5.93	0.10	1.000	5.93	94.07	86.11	30,140	18,291	510	509
2015	1.500	8.25	0.26	0.996	8.22	85.81	78.15	27,351	16,686	915	911
2016	2.500	28.14	0.58	0.986	27.74	57.67	50.84	17,794	11,214	834	822
2017	3.500	2.48	1.02	0.965	2.39	55.19	49.45	17,306	10,732	561	541
2018	4.500	15.02	1.51	0.935	14.04	40.17	36.02	12,608	7,812	537	502
2019	5.500	14.11	1.93	0.900	12.71	26.06	23.29	8,152	5,068	391	352
2020	6.500	8.44	2.28	0.864	7.29	17.62	15.83	5,541	3,427	253	219
2021	7.500	4.36	2.55	0.828	3.61	13.26	12.16	4,255	2,578	171	142
2022	8.500	4.46	2.75	0.794	3.54	8.80	8.21	2,874	1,711	129	102
2023	9.500	3.80	2.94	0.759	2.88	5.00	4.79	1,677	972	86	65
2024	10.500	5.00	3.07	0.728	3.64	0.00	0.00	0	0	49	35

(a) Total Charge	4,201
(b) Economic Premium	37,144
(c) Nominal Premium	50,000
(d) Implied Commission	25.71%

Calculations

(5) 1/(1+(4)/100)^(2)

(6) (3) x (5)

(7) 100 - Cumulative of (3)

(8) Sumproduct of future (3) and (5) divided by current (5)

(9) (8) x (c) x Expected loss Ratio of 70%

(10) Initial Value: (b) / 1.8

(b) represents premium and therefore held unearned premium at time contract is written

Subsequent Values use Nominal Loss reserves held at each point in time = (7) x (c) x Expected loss Ratio of 70% / 1.8

(11) Previous (10) x capital charge of 5%

(12) (11) x (5)

(a) Sum of (12)

(b) Solved iteratively such that it is equal to [(a) plus initial value of (9)]/(1-expense ratio of 2%]

(d) 1 - (b)/(c)

The Lognormal Random Multivariate

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Abstract

For decades the lognormal random variable has been widely used by actuaries to analyze heavy-tailed insurance losses. More recently, especially since ERM and Solvency II, actuaries have had to solve problems involving the interworking of many heavy-tailed risks. Solutions to some of these problems may involve the relatively unknown extension of the lognormal into the multivariate realm. The purpose of this paper is present the basic theory of the lognormal random multivariate.

Keywords: lognormal, multivariate, moment generating function, positive-definite

1. INTRODUCTION

The lognormal random variable $Y = e^{X \sim N(\mu, \sigma)}$ is familiar to casualty actuaries, especially to those in reinsurance. It vies with the Pareto for the description of heavy-tailed and catastrophic losses. However, unlike the Pareto, all its moments are finite. Moreover, the formula for the lognormal moments is rather simple: $E[Y^n] = e^{n\mu + n^2\sigma^2/2}$. So its first two moments are $E[Y] = e^{\mu + \sigma^2/2}$ and $E[Y^2] = e^{2\mu + 2\sigma^2} = E[Y]^2 e^{\sigma^2}$. Hence, its variance is $Var[Y] = E[Y]^2 \left(e^{\sigma^2} - 1\right)$, a formula so well known that actuaries commonly refer to $e^{\sigma^2} - 1$ as the "CV squared" of the lognormal. But in recent years, with the rise of ERM and capital modeling, actuaries have needed to model many interrelated random variables. If these random variables are heavy-tailed, it may be apt to model them with the lognormal random multivariate, which we will now present.[1]

[1] The standard reference for the lognormal distribution is Klugman [1998, Appendix A.4.1.1]. On the subject of heavy-tailed distributions, see Klugman [1998, §2.7.2] and Halliwell [2013].

2. MOMENT GENERATION AND THE LOGNORMAL MULTIVARIATE

The lognormal random multivariate is $\mathbf{y} = e^{\mathbf{x}}$, where $\mathbf{x} = \begin{bmatrix} X_1 \\ \vdots \\ X_n \end{bmatrix}$ is an $n\times1$ normal multivariate with $n\times1$ mean μ and $n\times n$ variance Σ. As a realistic variance, Σ must be positive-definite, hence invertible.[2]

The probability density function of the normal random vector **x** with mean μ and variance Σ is:[3]

$$f_{\mathbf{x}}(\mathrm{x}) = \frac{1}{\sqrt{(2\pi)^n|\Sigma|}} e^{-\frac{1}{2}(\mathrm{x}-\mu)'\Sigma^{-1}(\mathrm{x}-\mu)}$$

Therefore, $\int_{\mathrm{x}\in\Re^n} f_{\mathbf{x}}(\mathrm{x})dV = 1$. The single integral over $\Re^n$ represents an *n*-multiple integral over each x_j from –∞ to +∞; $dV = dx_1 \dots dx_n$. The moment generating function of **x** is $M_{\mathbf{x}}(\mathrm{t}) = E\left[e^{\mathrm{t}'\mathbf{x}}\right] = E\left[e^{\sum_{j=1}^{n} t_j X_j}\right]$, where t is an $n\times1$ vector. Partial derivatives of the moment generating function evaluated at $\mathrm{t} = 0_{n\times1}$ equal moments of ***x***, since:

$$\left.\frac{\partial^{k_1+\dots k_n} M_{\mathbf{x}}(\mathrm{t})}{\partial^{k_1} x_1 \dots \partial^{k_n} x_n}\right|_{\mathrm{t}=0} = E\left[X_1^{k_1} \dots X_n^{k_n} e^{\mathrm{t}'\mathbf{x}}\right]\Big|_{\mathrm{t}=0} = E\left[X_1^{k_1} \dots X_n^{k_n}\right]$$

The lognormal moments come directly from the normal moment generating function. For example, if $\mathrm{t} = \mathrm{e}_j$, the j^{th} unit vector, then $M_{\mathbf{x}}(\mathrm{e}_j) = E\left[e^{\mathrm{e}_j'\mathbf{x}}\right] = E\left[e^{X_j}\right] = E\left[Y_j\right]$. Likewise,

[2] For a review of positive-definite matrices see Judge [1988, Appendix A.14].
[3] See Johnson and Wichern [1992, Chapter 4] and Judge [1988, §2.5.7].

$M_{\mathbf{x}}(\mathrm{e}_j+\mathrm{e}_k)=E\left[e^{X_j}e^{X_k}\right]=E\left[Y_jY_k\right]$. So the normal moment generating function is the key to the lognormal moments.

The moment generating function of the normal random vector **x** is:

$$\begin{aligned}M_{\mathbf{x}}(\mathrm{t})&=E\left[e^{\mathrm{t'x}}\right]\\&=\int_{\mathrm{x}\in\Re^n}\frac{1}{\sqrt{(2\pi)^n|\Sigma|}}e^{-\frac{1}{2}(\mathrm{x}-\mu)'\Sigma^{-1}(\mathrm{x}-\mu)}e^{\mathrm{t'x}}dV\\&=\int_{\mathrm{x}\in\Re^n}\frac{1}{\sqrt{(2\pi)^n|\Sigma|}}e^{-\frac{1}{2}\left\{(\mathrm{x}-\mu)'\Sigma^{-1}(\mathrm{x}-\mu)-2\mathrm{t'x}\right\}}dV\end{aligned}$$

A multivariate "completion of the square" results in the identity:

$$(\mathrm{x}-\mu)'\Sigma^{-1}(\mathrm{x}-\mu)-2\mathrm{t'x}=(\mathrm{x}-[\mu+\Sigma\mathrm{t}])'\Sigma^{-1}(\mathrm{x}-[\mu+\Sigma\mathrm{t}])-2\mathrm{t'}\mu-\mathrm{t'}\Sigma\mathrm{t}$$

We leave it for the reader to verify the identity. By substitution, we have:

$$\begin{aligned}M_{\mathbf{x}}(\mathrm{t})&=\int_{\mathrm{x}\in\Re^n}\frac{1}{\sqrt{(2\pi)^n|\Sigma|}}e^{-\frac{1}{2}\left\{(\mathrm{x}-\mu)'\Sigma^{-1}(\mathrm{x}-\mu)-2\mathrm{t'x}\right\}}dV\\&=\int_{\mathrm{x}\in\Re^n}\frac{1}{\sqrt{(2\pi)^n|\Sigma|}}e^{-\frac{1}{2}\left\{(\mathrm{x}-[\mu+\Sigma\mathrm{t}])'\Sigma^{-1}(\mathrm{x}-[\mu+\Sigma\mathrm{t}])-2\mathrm{t'}\mu-\mathrm{t'}\Sigma\mathrm{t}\right\}}dV\\&=\int_{\mathrm{x}\in\Re^n}\frac{1}{\sqrt{(2\pi)^n|\Sigma|}}e^{-\frac{1}{2}(\mathrm{x}-[\mu+\Sigma\mathrm{t}])'\Sigma^{-1}(\mathrm{x}-[\mu+\Sigma\mathrm{t}])}dV\cdot e^{\mathrm{t'}\mu+\mathrm{t'}\Sigma\mathrm{t}/2}\\&=1\cdot e^{\mathrm{t'}\mu+\mathrm{t'}\Sigma\mathrm{t}/2}\\&=e^{\mathrm{t'}\mu+\mathrm{t'}\Sigma\mathrm{t}/2}\end{aligned}$$

The reduction of the integral to unity in the second last line is due to the fact that $\frac{1}{\sqrt{(2\pi)^n|\Sigma|}}e^{-\frac{1}{2}(\mathrm{x}-[\mu+\Sigma\mathrm{t}])'\Sigma^{-1}(\mathrm{x}-[\mu+\Sigma\mathrm{t}])}$ is the probability density function of the normal random vector with mean $\mu+\Sigma\mathrm{t}$ and variance Σ.

So the moment generating function of the normal multivariate $\mathbf{x} \sim N(\mu, \Sigma)$ is $M_{\mathbf{x}}(\mathrm{t}) = e^{\mathrm{t}'\mu + \mathrm{t}'\Sigma\mathrm{t}/2}$. As a check:[4]

$$\frac{\partial M_{\mathbf{x}}(\mathrm{t})}{\partial \mathrm{t}} = (\mu + \Sigma\mathrm{t})e^{\mathrm{t}'\mu + \mathrm{t}'\Sigma\mathrm{t}/2} \Rightarrow E[\mathbf{x}] = \left.\frac{\partial M_{\mathbf{x}}(\mathrm{t})}{\partial \mathrm{t}}\right|_{\mathrm{t}=0} = \mu$$

And for the second derivative:

$$\begin{aligned}
\frac{\partial^2 M_{\mathbf{x}}(\mathrm{t})}{\partial \mathrm{t}\partial \mathrm{t}'} &= \frac{\partial(\mu + \Sigma\mathrm{t})e^{\mathrm{t}'\mu + \mathrm{t}'\Sigma\mathrm{t}/2}}{\partial \mathrm{t}'} \\
&= \left(\Sigma + (\mu + \Sigma\mathrm{t})(\mu + \Sigma\mathrm{t})'\right)e^{\mathrm{t}'\mu + \mathrm{t}'\Sigma\mathrm{t}/2} \\
&\Rightarrow E[\mathbf{xx}'] = \left.\frac{\partial^2 M_{\mathbf{x}}(\mathrm{t})}{\partial \mathrm{t}\partial \mathrm{t}'}\right|_{\mathbf{t}=0} = \Sigma + \mu\mu' \\
&\Rightarrow Var[\mathbf{x}] = E[\mathbf{xx}'] - \mu\mu' = \Sigma
\end{aligned}$$

The lognormal moments follow from the moment generating function:

$$E[Y_j] = \left[e^{X_j}\right] = E\left[e^{\mathrm{e}_j'\mathbf{x}}\right] = M_{\mathbf{x}}(\mathrm{e}_j) = e^{\mathrm{e}_j'\mu + \mathrm{e}_j'\Sigma\mathrm{e}_j/2} = e^{\mu_j + \Sigma_{jj}/2}$$

The second moments are conveniently expressed in terms of first:

$$\begin{aligned}
E[Y_jY_k] &= E\left[e^{X_j}e^{X_k}\right] = E\left[e^{(\mathrm{e}_j + \mathrm{e}_k)'\mathbf{x}}\right] \\
&= e^{(\mathrm{e}_j + \mathrm{e}_k)'\mu + (\mathrm{e}_j + \mathrm{e}_k)'\Sigma(\mathrm{e}_j + \mathrm{e}_k)/2} \\
&= e^{\mu_j + \mu_k + (\Sigma_{jj} + \Sigma_{jk} + \Sigma_{kj} + \Sigma_{kk})/2} \\
&= e^{\mu_j + \Sigma_{jj}/2 + \mu_k + \Sigma_{kk}/2 + (\Sigma_{jk} + \Sigma_{kj})/2} \\
&= e^{\mu_j + \Sigma_{jj}/2} \cdot e^{\mu_k + \Sigma_{kk}/2} \cdot e^{(\Sigma_{jk} + \Sigma_{kj})/2} \\
&= e^{\mu_j + \Sigma_{jj}/2} \cdot e^{\mu_k + \Sigma_{kk}/2} \cdot e^{(\Sigma_{jk} + \Sigma_{jk})/2} \\
&= E[Y_j]E[Y_k] \cdot e^{\Sigma_{jk}}
\end{aligned}$$

So, $Cov[Y_j, Y_k] = E[Y_jY_k] - E[Y_j]E[Y_k] = E[Y_j]E[Y_k]\left(e^{\Sigma_{jk}} - 1\right)$, which is the multivariate equivalent of the well-known scalar formula $CV^2\left[e^X\right] = e^{\sigma^2} - 1$. The whole variance matrix can be expressed as

[4] The vector formulation of partial differentiation is explained in Judge [1988, Appendix A.17].

$Var[\mathbf{y}] = \left(E[\mathbf{y}]E[\mathbf{y}]' \right) \circ \left(e^{\Sigma} - 1_{n \times n} \right)$, where '$\circ$' represents elementwise multiplication (the Hadamard product). Defining the diagonalization of a vector as $diag(\mathrm{v}_{n\times 1}) = \begin{bmatrix} v_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & v_n \end{bmatrix}$, we may express the variance in terms of the usual matrix multiplication as $Var[\mathbf{y}] = diag(E[\mathbf{y}])\left(e^{\Sigma} - 1_{n\times n}\right)diag(E[\mathbf{y}])$. Because $diag(E[\mathbf{y}])$ is diagonal in positive elements (hence, symmetric and positive-definite), $Var[\mathbf{x}]$ is positive-definite if and only if $e^{\Sigma} - 1_{n\times n}$ is positive-definite. Although beyond the scope of this paper, it can be proven[5] that if Σ is positive-definite, as stipulated above, then so too is $\mathrm{T} = e^{\Sigma} - 1_{n\times n}$.[6]

3. CONCLUSION

The mean and the variance of the lognormal multivariate are straightforward extensions of their scalar equivalents. Simulating lognormal random outcomes is nothing more than exponentiating simulated normal random multivariates. Therefore, one faced with the problem of modeling several heavy-tailed random variables in a mean-variance framework may find an acceptable solution in the lognormal random multivariate.

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[5] A proof involving Shur's Product Theorem forms part of an unpublished paper by the author, "Complex Random Variables," which he will make available for the asking.

[6] The converse is not necessarily true: there exist positive-definite T for which $\Sigma = \ln\left(1_{n\times n} + \mathrm{T}\right)$ is not positive-definite. Lognormal variance is a proper subset of (normal) variance. Hereby one can test whether variance T is realistic for interrelated random variables with heavy tails.

Stochastic Ordering of Reinsurance Structures

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Abstract

The paper offers a simple framework for ranking the common reinsurance structures in practice with the theory of stochastic orders. The basic idea is to slice the space of reinsurance structures into groups by expected loss cost to facilitate the comparisons within the group and between groups. Given the standard risk aversion assumption in economics, a spectrum of reinsurance structures with the same expected loss cost can be compared analytically with one another and sequenced based on their risk coverages under the convex order. The paper then expands the dimension of the comparison to groups of reinsurance structures with different expected loss costs, which can be ranked under the increasing convex order and the usual stochastic order. As such, the paper maps out the ordering for the entire space of reinsurance structures and presents it in a matrix format for quick reference. The implication of this stochastic ordering to reinsurance pricing is also investigated.

Keywords: Reinsurance; Usual Stochastic Order, Convex Order; Increasing Convex Order; Stochastic Dominance; Insurance Premium Principles.

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1 Introduction

Reinsurance is one of the most frequently used risk management tools by insurance companies in managing their portfolios. Insurance companies regularly evaluate and if necessary, modify the structure of their reinsurance program to adjust their overall risk exposures in an evolving business environment. For example, an enterprise risk management (ERM) analysis may compare the coverages and the efficiency between the current reinsurance program and alternative reinsurance structures. These alternative reinsurance structures may involve increasing or decreasing the retention level of an excess of loss reinsurance, adding an aggregate deductible or an aggregate limit, and adjusting the placement ratio.

To find the optimal reinsurance *contract* that maximizes an objective variable, such as the net underwriting income, the typical industry approach is to run a simulation model with as many potential reinsurance structures as possible. One of the key challenges in the ERM evaluation process is how to set the reinsurance prices for these alternative options, which to a large extent determines the efficiencies of the options. Given that the ERM modelers usually do not have the benefit of market quotes for all the options, it is important that these reinsurance structures can be properly ordered and priced in the model. The abundance of reinsurance choices together with the complexity of reinsurance pricing, however, often makes the selection process very difficult.

The goal of this paper is to provide actuaries, underwriters, and brokers a framework to compare common reinsurance structures so that unnecessary simulation may be avoided and reasonable results can be obtained quickly in an ERM analysis. We first explore the risk ranking of common reinsurance structures using the convex order from the theory of stochastic orders (e.g., Shaked and Shanthikumar (2007), Müller and Stoyan (2002), and Denuit *et al.* (2005)). We then further expand the dimension of the comparison to reinsurance structures with different expected loss costs using the usual stochastic order (equivalently, the First-order Stochastic Dominance)[1] and the increasing convex order (dual to the general Second-order Stochastic Dominance).

The convex order is dual to the concave order, which is the familiar Rothschild-Stiglitz second-order stochastic dominance (R-S SSD) with equal means as pioneered by Rothschild and Stiglitz (1970) in economics. Heyer (2001) uses the general SSD to rank reinsurance *contracts* on an empirical distribution basis through simulation. Assuming a risk-averse principal (or equivalently an increasing concave utility function), if the net underwriting income resulting from reinsurance structure A is larger in "size" and less volatile than reinsurance structure B, then A is second-order stochastic dominating B from a cedant's point of view. However, the result of the underwriting income comparison using the general SSD is often inconclusive as demonstrated in Heyer's analysis. This paper will focus on the loss distributions, rather than the underwriting income distributions, of the reinsurance structures as there exists a natural ordering for the former, but not necessarily for the latter.

[1]See Levy (1998) for a general introduction to stochastic dominance, and see Heyer (2001) for an application of stochastic dominance to reinsurance.

The convex order allows us to compare alternatives that have the same expected value, and thus eliminate the need to compare "size" or "magnitude." The focus of the comparison, instead, can then be on the "variability" or the pure risk of the reinsurance structures. We will show analytically that any risk-averse individual under the convex order can distinguish and rank basic reinsurance structures given their natural orders in "variability." In short, under the convex order, the stop-loss reinsurance is more risky than the quota share reinsurance, which in turn is more risky than the reinsurance with an aggregate limit (i.e., 100% quota share with a cap):

$$\textbf{Aggregate Limit} \preceq_{cx} \textbf{Quota Share} \preceq_{cx} \textbf{Stop-loss}$$

where $A \preceq_{cx} B$ means B dominates A under the convex order.

This line of reasoning can be extended to analyzing the aggregate loss treaties with more than one contract feature. For example, a quota share treaty with a stop-loss threshold can be compared with a quota share treaty with an aggregate limit. More parameters need to be calibrated within a treaty to make sure that the mean loss is the same across all treaties as required by the convex order. Note that these combination structures with two contract features form a continuum of options that are bounded by the three basic reinsurance structures. Outlined below are the rankings of some possible combinations.

$$\begin{aligned} &\textbf{Aggregate Limit} \\ \preceq_{cx}&\textbf{Mixture of Quota Share and Aggregate Limit} \\ \preceq_{cx}&\textbf{Quota Share} \\ \preceq_{cx}&\textbf{Mixture of Stop-loss and Quota Share} \\ \preceq_{cx}&\textbf{Stop-loss.} \end{aligned}$$

The approaches we have used in analyzing the aggregate loss reinsurance can also be applied to the excess of loss (XOL) reinsurance treaties with features such as annual aggregate deductible (AAD), higher per claim retention[2], partial placement (or equivalently cedant co-participation) and aggregate limit. Note that the convex order is closed under convolutions. That is, when the claim count distribution is independent of the severity distributions, the dominance relationship between the severity distributions at the per risk/per occurrence level can be carried over to the aggregate layer loss level. This closure property is crucial in proving the relationship between XOL with Partial Placement and XOL with Higher Retention.

We will show that under the convex order, these XOL reinsurance treaties along

[2]Here the per risk/per occurrence retention is raised, but the sum of the retention and limit is the same as that for the original layer. See Definition 5.6

with the corresponding hybrid structures can be ranked analytically as follows:

$$
\begin{aligned}
&\textbf{XOL with Aggregate Limit}\\
\preceq_{cx}&\textbf{XOL with Mixture of Partial Placement and Aggregate Limit}\\
\preceq_{cx}&\textbf{XOL with Partial Placement}\\
\preceq_{cx}&\textbf{XOL with Mixture of Higher Retention and Partial Placement}\\
\preceq_{cx}&\textbf{XOL with Higher Retention}\\
\preceq_{cx}&\textbf{XOL with Mixture of Aggregate Deductible and Higher Retention}\\
\preceq_{cx}&\textbf{XOL with Aggregate Deductible.}
\end{aligned}
$$

The next step is to expand the dimension of the comparison to reinsurance structures with different expected loss costs using the usual stochastic order (equivalently, the First-order Stochastic Dominance) and the increasing convex order (the dual to the general Second-order Stochastic Dominance). The usual stochastic order ($\preceq_{st}$) can be established between any two structures that are of the same type, but have different expected losses. If different types of structures are involved in the one-on-one comparison, we may be able to establish dominance under the weaker increasing convex order ($\preceq_{icx}$).

The use of the usual stochastic order and increasing convex order greatly expands the range of reinsurance structures that can be compared and ranked. While it appears that the number of comparison combinations may be infinite, some reinsurance treaties, however, are not comparable under any of the three stochastic orders. Particularly, the comparison is inconclusive between a quota share treaty and a treaty with both an aggregate limit and an aggregate deductible. The reason for inconclusiveness is that neither treaty has thicker tails on both ends of the density function, which is required for the dominance relationship. But we will show that the inconclusiveness follows a predictable pattern based on the types of reinsurance structure.

Section 2 of the paper defines the three stochastic orders and Section 3 compares the risk rankings of basic reinsurance structures under the convex order. The paper then extends the analysis to the reinsurance structures with different expected values in Section 4 while Section 5 applies the same methodology to excess of loss reinsurance. We then compare aggregate reinsurance structures with XOL reinsurance structures in Section 6. The implications of this risk ranking analysis to reinsurance pricing and the optimal reinsurance literature are considered in Section 7 and the concluding remarks are in Section 8. In the appendix, we analyze some XOL reinsurance structures that cannot be compared with those structures analyzed in Section 5. The implication is that we may need to divide reinsurance structures into subsets such that the members of the subsets can be compared with one another.

2 Preliminaries

Assume a standard collective risk model where $x > 0$ is a continuous ground-up loss random variable for a single occurrence or a single risk with mean $0 < E(x) < \infty$

and variance $0 < Var(x) < \infty$. Let $N \geq 0$ be an integer-based random variable for the ground-up loss frequency and independent of x. S denotes the corresponding aggregate loss and $S = \sum_{i=1}^{N} x_i$, where i is the index for N and $S = 0$ when $N = 0$. We first define the usual stochastic order and then introduce the increasing convex order and the convex order.

Definition 2.1. Usual Stochastic Order - *(Definition 1.A.1 in Shaked and Shanthikumar (2007))* Let X and Y be two random variables such that $P(X > t) \leq P(Y > t)$ for all $t \in (-\infty, \infty)$. Then X is said to be smaller than Y in the usual stochastic order or $X \preceq_{st} Y$.

In economics, the usual stochastic order is called the first-order stochastic dominance (FSD). The definition implies that at every percentile, Y has a higher value than X. It can be characterized as $X \preceq_{st} Y$ if, and only if, $E(\phi(X)) \leq E(\phi(Y))$ for all non-decreasing functions $\phi : R \to R$, provided the expectations exist. Clearly, if $X \preceq_{st} Y$, then $E(X) \leq E(Y)$ and $Var(X) \leq Var(Y)$ as both the expectation and the variance functions are non-decreasing.

Definition 2.2. Increasing Convex Order - *(Definition 4.A.1 in Shaked and Shanthikumar (2007))* Let X and Y be two random variables such that $E(\phi(X)) \leq E(\phi(Y))$ for all increasing convex functions $\phi : R \to R$, provided the expectations exist. Then X is said to be smaller than Y in the increasing convex order or $X \preceq_{icx} Y$.

The increasing convex order is a dual order to the increasing concave order or the second-order stochastic dominance (Theorem 7.3.10, Kass *et al.*(2009)), which is often used by financial economists to analyze investment decision-making under uncertainty. In other words, if a risk-averse individual prefers Y to X under the second-order stochastic dominance, he/she would equivalently also prefer $-X$ to $-Y$ under the increasing convex order. Thus it is usually a matter of convenience and intuition to use the increasing convex order rather than the increasing concave order or the second-order stochastic dominance when the objects for comparison are losses rather than assets.

Definition 2.3. Convex Order - *(Definition 3.A.1 in Shaked and Shanthikumar (2007))* Let X and Y be two random variables such that $E(\phi(X)) \leq E(\phi(Y))$ for all convex functions $\phi : R \to R$, provided the expectations exist. Then X is said to be smaller than Y in the convex order or $X \preceq_{cx} Y$.

The convex order is closely related to the increasing convex order and second-order stochastic dominance. The difference between the convex order and the increasing convex order is that the convex order requires that $E(\phi(X)) \leq E(\phi(Y))$ holds for *all* convex functions ϕ. Since $\phi(x) = x$ and $\phi(x) = -x$ are both convex, $X \preceq_{cx} Y$ implies that X and Y must have the same expected value, i.e., $E(X) = E(Y)$.

In a sense, the increasing convex order compares both the "size" and the "variability" of random variables while the convex order compares only the "variability," given that the underlying random variables must have the same expected value. Focusing on the convex order first allows us to make comparison between reinsurance structures of the same "size." This is essentially the concept of risk defined by Rothschild and Stiglitz (1970) in economics. The standard characterizations for these stochastic orders are summarized as follows:

Proposition 2.1. *(Theorems 4.A.3, 3.A.1, and 4.A.6 in Shaked and Shanthikumar (2007)) Let X and Y be two random variables. The stop-loss premium function of X is defined as $\pi_X(d) = \int_d^\infty (1 - F(x))dx$, where F is the distribution function and d is a stop-loss threshold.*

$$\begin{aligned}
&(1) X \preceq_{icx} Y \text{ if, and only if, } \pi_X(d) \leq \pi_Y(d), \forall d \geq 0\\
&(2) \text{Given } E(X) = E(Y), X \preceq_{cx} Y \text{ if, and only if, } \pi_X(d) \leq \pi_Y(d), \forall d \geq 0\\
&(3) X \preceq_{icx} Y \text{ if, and only if, there exist a random variable } Z \text{ such that}\\
&\quad X \preceq_{st} Z \preceq_{cx} Y \text{ or } X \preceq_{cx} Z \preceq_{st} Y
\end{aligned}$$

Proposition 2.1 says that having a larger stop-loss premium is a necessary and sufficient condition for both the convex order and the increasing convex order. It can be shown that it is just a necessary condition for the usual stochastic order. Thus if $X \preceq_{st} Y$, then $X \preceq_{icx} Y$. Or equivalently in economics, if $-X$ is first-order stochastic dominating $-Y$, $-X$ is also second-order stochastic dominating $-Y$. Item (3) of the proposition above is the well-known separation theorem that links the three stochastic orders and will be used in Section 4 to show the dominance relationship between reinsurance structures with different expected loss costs.

Assuming equal means, a *sufficient* condition for one random variable having larger stop-loss premium than the other random variable for every stop-loss threshold is that the distribution functions of the two random variable cross only once.

Definition 2.4. Single Crossing Condition[3] - The distribution functions F and G satisfy the single crossing condition if for some u^* in $(0, 1)$,

$$\begin{cases} F^{-1}(u) \leq G^{-1}(u) & \text{if } u \geq u^* \\ F^{-1}(u) \geq G^{-1}(u) & \text{if } u < u^*. \end{cases}$$

The following proposition shows that this single crossing property together with the equality of the means can be used to establish the convex order between two random variables.

Proposition 2.2. *(Theorem 3.3.C in Rüschendorf (2013); Property 3.4.19 in Denuit et al. (2005)) - Let X and Y be two random variables with distribution functions F and G, respectively, such that $E(X) = E(Y)$. Then $X \preceq_{cx} Y$ if for some u^* in $(0, 1)$,*

$$\begin{cases} F^{-1}(u) \leq G^{-1}(u) & \text{if } u \geq u^* \\ F^{-1}(u) \geq G^{-1}(u) & \text{if } u < u^*. \end{cases}$$

[3] Also known as the Karlin-Novikov Cut Criterion in its simplest form or the Thicker Tail condition in the actuarial literature (Denuit *et al.* (2005)).

We will see below in Section 3 that the comparison of the basic reinsurance structures can fit neatly into the framework with the single crossing condition. On the other hand, multiple crossings can happen between the distribution functions of other types of reinsurance such as excess of loss reinsurance. To establish the ranking for those reinsurance structures, we need to use the property of closure under convolutions for the three stochastic orders *(Theorems 1.A.3, 3.A.13, and 4.A.9 in Shaked and Shanthikumar (2007))* as shown in Sections 5 and 6.

3 Aggregate Loss Reinsurance

We first investigate three basic reinsurance structures - those with stop-loss, aggregate limit, or quota share. The distribution functions for all these structures are defined on the same space as the gross aggregate loss ($0 \leq S < \infty$), which is continuous and increasing. Assuming that these structures have the same means, we'll show that they can be ranked using the convex order since their distribution functions cross only once when compared in pairs.

3.1 Three Basic Reinsurance Structures

Definition 3.1. Stop-Loss - The stop-loss reinsurance S_D with a threshold $D > 0$ is

$$S_D = \begin{cases} 0 & \textit{if } 0 \leq S < D \\ S - D & \textit{if } D \leq S. \end{cases}$$

Definition 3.2. Quota Share - Let $0 < q < 1$ be a quota share percentage. The quota share reinsurance is $S_q = qS$.

Definition 3.3. Aggregate Limit (i.e., 100 % quota share with a cap)- The reinsurance S_L with an aggregate limit $L > 0$ is

$$S_L = \begin{cases} S & \textit{if } 0 \leq S < L \\ L & \textit{if } L \leq S. \end{cases}$$

To illustrate the interrelationship of these reinsurance structures, the distribution functions F_{S_D}, F_{S_q}, and F_{S_L} for the reinsurance contracts with stop-loss, quota share and aggregate limit, respectively, are graphed below in the typical Lee graph format (Lee (1988)) with the y-axis as loss amount and the x-axis as distribution percentile. The area under each curve is the expected value of the respective loss random variable, which is assumed the same for all reinsurance structures in the illustration.

In Figure 1, the blue curve is the distribution function for aggregate gross loss S while the red curve represents a stop-loss reinsurance S_D, which stays flat until the aggregate loss amount reaches the stop-loss threshold D at around 40^{th} percentile and then increases with the same incremental amounts as S. The expected retained loss amount by the cedant would be equivalent to the area between the two curves.

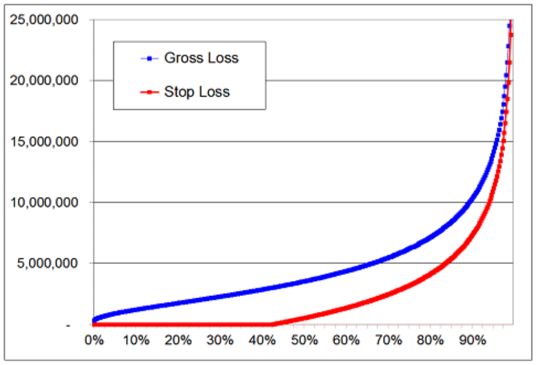

Figure 1

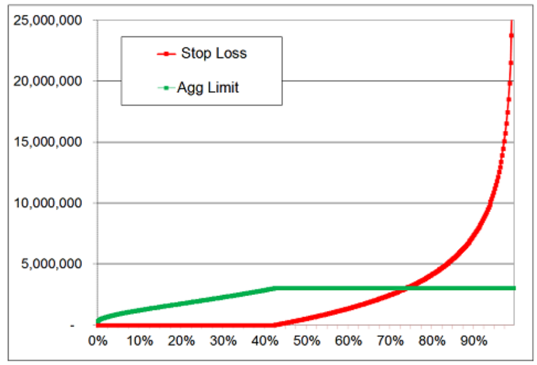

Figure 2

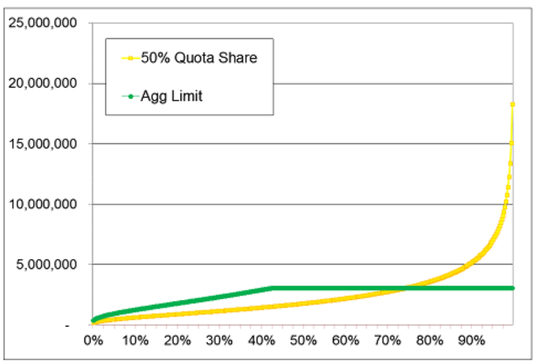

Figure 3

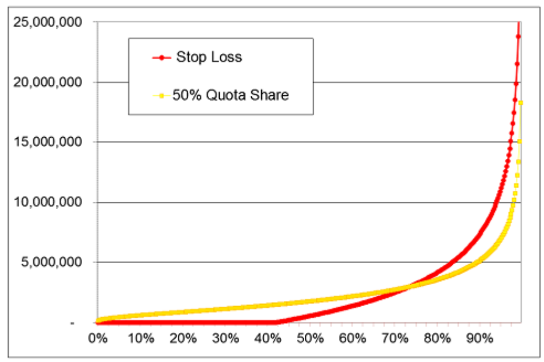

Figure 4

The green curve in Figure 2 represents the distribution function for a reinsurance with an aggregate limit (S_L) while the red curve is for a stop-loss reinsurance (S_D). The S_L curve follows the same path as the gross loss curve S and then becomes flat at the aggregate limit L. The areas between the curves before and after the intersection are the same and represent the trade-off between the two reinsurance structures. The curve for the stop-loss reinsurance is more spread out with higher weights in the upper tail.

Figure 3 compares the curves between the aggregate limit reinsurance S_L and the quota share reinsurance S_q while Figure 4 compares the latter with the stop-loss reinsurance S_D. Notice the differences between the curves in Figures 3 and 4 are less than those in Figure 2 as it will be shown later that the stop-loss reinsurance and aggregate limit reinsurance are the two extreme options in terms of riskiness.

The reason that we can conveniently graph the distribution functions of S, S_D, S_L, and S_q in the same space is that S_D, S_L, and S_q are non-decreasing functions of S and are in fact comonotone (Definition 1.9.1 in Denuit *et al.* (2005)). That is, given a specific aggregate loss S^* and its percentile u^* on the distribution function of S, the corresponding S_D^*, S_L^*, and S_q^* are all at the same percentile u^* on the distribution functions of S_D, S_L, and S_q, respectively. This makes the comparison of reinsurance structures much straightforward.

3.2 Risk Rankings of Basic Structures

The steps to show that stop-loss reinsurance dominates quota-share reinsurance follow the classical results in van Heerwaarden, Kass and Goovaerts (1998), where they show that a risk-averse cedant would prefer the stop-loss reinsurance contract to all other contracts if all contracts have the same expected loss cost.

Proposition 3.1. *Assume that the stop-loss reinsurance S_D, the aggregate limit reinsurance S_L, and the quota share reinsurance S_q for a ground-up aggregate loss S defined above have the same expected value. Under the convex order, the stop-loss reinsurance is more risky than the quota share reinsurance, which in turn is more risky than the reinsurance with an aggregate limit. That is $S_L \preceq_{cx} S_q \preceq_{cx} S_D$, or*

$$\textbf{Aggregate Limit} \preceq_{cx} \textbf{Quota Share} \ \preceq_{cx} \textbf{Stop-Loss}.$$

Proof. Based on Theorem 6.1 in van Heerwaarden, Kass and Goovaerts (1989), the CDF of the retained loss net of a reinsurance with an aggregate deductible intersects only once with the CDF of the retained loss net of any other reinsurance structure given the equality of the mean losses. This also implies that the CDF of S_D crosses only once with the CDF of any other reinsurance structures including S_q. Let S^* denote the gross loss at the intersection of S_D and S_q. That is $S^* - D = qS^*$, or $S^* = \frac{D}{1-q}$. The value of S_D and S_q at the intersection would be $S^*_D = S^*_q = \frac{qD}{1-q}$ and $F_S(S^*) = F_{S_D}(\frac{qD}{1-q}) = F_{S_q}(\frac{qD}{1-q})$. Note that when $S^* < S$, $S_q < S_D$ since S_D increases faster than S_q. Similarly, $S_D \leq S_q$ when $S \leq S^*$. Thus the CDF of S_D crosses the CDF of S_q from below (in the context of a Lee graph). That is,

$$\begin{cases} F_{S_q}(x) \leq F_{S_D}(x) & \textit{if } x \leq \frac{qD}{1-q} \\ F_{S_D}(x) \leq F_{S_q}(x) & \textit{if } \frac{qD}{1-q} \leq x. \end{cases}$$

By Proposition 2.2, $S_q \preceq_{cx} S_D$ and the stop-loss reinsurance is more risky than the quota share reinsurance under the convex order. Similarly, we can demonstrate $S_L \preceq_{cx} S_q$. By the transitivity property of the convex order, $S_L \preceq_{cx} S_q \preceq_{cx} S_D$. □

Example 1: The XYZ Insurance Company writes \$50 million general liability insurance annually at an expected loss ratio of 70%. Currently the company has a 20% quota share treaty with a 20% ceding commission and no aggregate limit. XYZ is considering replacing the quota share with a stop-loss reinsurance treaty that attaches at an 80% loss ratio with a reinsurance premium of \$5.5 million. The company estimates that the expected loss cost of the stop-loss treaty is \$3.5 million, which means the implied margin is \$2 million.

The table below shows the treaty premium, treaty expected loss and implied reinsurance margin for each structure.

Table 1 : Ranking Comparison

Option Type	Option Description	Insurance/Reins. Premium	Expected Loss Cost	Implied Margin
S	Gross	\$50M	\$35M	
S_q	20% Quota Share	\$(10-2)M=\$8M	\$7M	\$1M
S_q	10% Quota Share	\$(5-1)M=\$4M	\$3.5M	\$0.5M
S_D	Stop-Loss	\$5.5M	\$3.5M	\$2M

Based on the risk ordering analysis, the comparable treaties under the convex order are the stop-loss treaty and the 10% quota share treaty as both have the same expected loss cost and the former is more risky than the latter. This is also reflected in the extra margin charge of (2M−0.5M) = 1.5M. Note that the stop-loss threshold D is at an 80% loss ratio or \$40M. The intersection point of the 10% quota share and the stop-loss treaties is at $S = D/(1-q) = 40/0.9 = 44.44$M or $S_D = S_q = qD/(1-q) =$ \$4.44M. In other words, the 10% quota share treaty recovers more than the stop-loss treaty when the gross loss is less than \$44.44M. The extra margin is meant to cover the uncertainty of the loss beyond \$44.44M. The company should weigh their risk preference against the extra margin in selecting their reinsurance program.

3.3 Hybrid Reinsurance Structures

This line of reasoning and analysis can be extended to the aggregate loss reinsurance treaties with more than one contract feature, which include combinations of stop-loss, quota share, and aggregate limit. As more features are included in a reinsurance structure, more parameters such as stop-loss threshold and aggregate limit need to be calibrated to make sure that the mean losses are the same across all treaties as required by the convex order. We define below two additional types of reinsurance and show that they can be properly ordered under the convex order along with the three basic reinsurance structures.

Definition 3.4. Quota Share with Aggregate Limit - The reinsurance $S_{q,L}$ with an aggregate limit $L > 0$ and a quota share percentage $0 < q < 1$ is

$$S_{q,L} = \begin{cases} qS & \textit{if } 0 \le qS < L \\ L & \textit{if } L \le qS. \end{cases}$$

Definition 3.5. Quota Share with Stop-Loss - The reinsurance $S_{D,q}$ with a stop-loss threshold $D > 0$ and a quota share percentage $0 < q < 1$ is

$$S_{D,q} = \begin{cases} 0 & \textit{if } 0 \le qS < D \\ qS - D & \textit{if } D \le qS. \end{cases}$$

Proposition 3.2. *Denote q, q_1, and q_2 as quota share percentages, D and D_2 as stop-loss thresholds, and L and L_1 as aggregate limits. Let* $0 < q < q_1 < 1$, $0 < q < q_2 < 1$,

$0 < D_2 < D$, and $0 < L < L_1$ such that the reinsurance options, S_L, S_{q_1,L_1}, S_q, S_{D_2,q_2}, and S_D for a ground-up aggregate loss S have the same expected value. That is,

$$E(S_L) = E(S_{q_1,L_1}) = E(S_q) = E(S_{D_2,q_2}) = E(S_D).$$

Then the following orderings can be established:

$$S_L \preceq_{cx} S_{q_1,L_1} \preceq_{cx} S_q \preceq_{cx} S_{D_2,q_2} \preceq_{cx} S_D$$

or

Aggregate Limit
$\preceq_{cx}$**Mixture of Quota Share and Aggregate Limit**
$\preceq_{cx}$**Quota Share**
$\preceq_{cx}$**Mixture of Stop-loss and Quota Share**
$\preceq_{cx}$**Stop-loss**

Proof. $S_L \preceq_{cx} S_{q_1,L_1}$ since the distribution function of S_{q_1,L_1} intersects only once with the distribution function of S_L from below at L and the single crossing condition applies. Similarly, since $0 < q < q_1 < 1$, the distribution function of S_q intersects only once with the distribution function of S_{q_1,L_1} from below at L_1 and thus $S_{q_1,L_1} \preceq_{cx} S_q$.

Since $0 < q < q_2 < 1$, S_{q_2} represents a larger layer than S_q. Similar to the proof in Proposition 3.1, the loss distribution function from a larger layer with a stop-loss such as S_{D_2,q_2} crosses only once with the distribution function of any other reinsurance option such as S_q, given that $E(S_q) = E(S_{D_2,q_2})$. And we conclude that $S_q \preceq_{cx} S_{D_2,q_2}$. The proof of $S_{D_2,q_2} \preceq_{cx} S_D$ follows the same argument in Proposition 3.1. □

Example 2: Continuing the example in Section 3.1, the XYZ insurance company considers lowering the stop-loss threshold from 80% to 75% loss ratio, but taking a 10% co-participation in the stop-loss treaty. It also considers adding an overall aggregate limit to the quota share reinsurance. It has been determined that both the new stop-loss reinsurance and a 12% quota share reinsurance with 9.6 million aggregate limit have an expected loss cost of 3.5 million.

Based on the risk ranking analysis and the quotes received earlier in the example in Section 3.2, the reinsurance premium for the new stop-loss option should be between \$5.5M and \$4M and the premium for the 12% quota share reinsurance with a \$9.6M aggregate limit should be less than \$4M. In this case, Option S_L would be a 100% quota share reinsurance with a small overall aggregate limit such as \$5M.

The new option S_{D_2,q_2} is obviously not the only treaty that can be ranked between the stop-loss reinsurance S_D and the quota share reinsurance S_q. Contract options can be created by decreasing the stop-loss threshold from the 80% loss ratio and reducing the quota share percentage from 100% such that the combination of the stop-loss threshold and quota share percentage have the same expected loss as before. Then in theory, infinite number of options can be ordered and placed in between Options S_D and S_q. Under the convex order, the option with a higher stop-loss threshold would dominate those with lower stop-loss thresholds. Similarly, a continuum of options can fill the space between Options S_L and S_q by changing the quota share percentage and aggregate limit while keeping the expected loss cost constant.

Table 2 : Ranking Analysis

Option Type	Option Description	Quoted Reins. Premium	Expected Loss Cost	Implied Margin
S_L	100% Quota Share, $5M Aggregate Limit.		$3.5M	
S_{q_1,L_1}	12% Quota Share, $9.6M Aggregate Limit.		$3.5M	
S_q	10% Quota Share, No Aggregate Limit.	$(5-1)M=$4M	$3.5M	$0.5M
S_{D_2,q_2}	Stop-loss attaching at 75% LR, 90% Quota Share.		$3.5M	
S_D	Stop-loss attaching at 80% LR, 100% Quota Share.	$5.5M	$3.5M	$2M

4 Beyond Convex Order

We have shown in Section 3 that the basic reinsurance structures and their combinations can be compared in pairs and ranked using the convex order. The comparison is static in nature as the range of the structures is limited to those having the same expected loss cost. In this section we expand the comparison to the structures with different expected loss costs. The tools that we use are the usual stochastic order and the increasing convex order as defined in Section 2. We show in the following proposition that the dominance relationship under these two stochastic orders for structures with different expected values can be clearly mapped out. On the other hand, some reinsurance structures are not comparable even though their expected loss costs may be far apart.

Proposition 4.1. *Denote q, q_1, and q_2 as quota share percentages, D, and D_2 as stop-loss thresholds, and L, and L_1 as aggregate limits. Let $0 < q < q_1 < 1$, $0 < q < q_2 < 1$, $0 < D_2 < D$, and $0 < L < L_1$ such that the reinsurance options, S_L, S_{q_1,L_1}, S_q, S_{D_2,q_2}, and S_D for a ground-up aggregate loss S have the same expected value m. That is,*

$$E(S_L) = E(S_{q_1,L_1}) = E(S_q) = E(S_{D_2,q_2}) = E(S_D) = m.$$

Consider a similar set of reinsurance structures, $S_{L'}$, $S_{q'_1,L'_1}$, $S_{q'}$, $S_{D'_2,q'_2}$, and $S_{D'}$ for the same ground-up aggregate loss S, where $0 < q' < q'_1 < 1$, $0 < q' < q'_2 < 1$, $0 < D'_2 < D'$, and $0 < L' < L'_1$ such that

$$E(S_{L'}) = E(S_{q'_1,L'_1}) = E(S_{q'}) = E(S_{D'_2,q'_2}) = E(S_{D'}) = n > m.$$

Then the following orderings can be established:

	$S_{L'}$	$S_{q_1',L_1'}$	$S_{q'}$	$S_{D_2',q_2'}$	$S_{D'}$
S_L	$\preceq_{st}$	$\preceq_{icx}$	$\preceq_{icx}$	$\preceq_{icx}$	$\preceq_{icx}$
S_{q_1,L_1}		$\preceq_{icx}$ *if* $q_1' < q_1, L_1 < L_1'$ $\preceq_{st}$ *if* $q_1 \leq q_1', L_1 \leq L_1'$	$\preceq_{icx}$	$\preceq_{icx}$	$\preceq_{icx}$
S_q			$\preceq_{st}$	$\preceq_{icx}$	$\preceq_{icx}$
S_{D_2,q_2}				$\preceq_{icx}$ *if* $q_2 < q_2', D_2 < D_2'$ $\preceq_{st}$ *if* $q_2 \leq q_2', D_2' \leq D_2$	$\preceq_{icx}$
S_D					$\preceq_{st}$

where the table reads, from left to right, $S_L \preceq_{st} S_{L'}$, $S_L \preceq_{icx} S_{q_1',L_1'}$, $S_{q_1,L_1} \preceq_{icx} S_{q_1',L_1'}$ *if* $q_1' < q_1$ *and* $L_1 < L_1'$, *and so on.*

Proof. We first show the usual stochastic orderings ($\preceq_{st}$) on the diagonal of the table above. Since $n > m$, the following inequalities must be true : $L < L'$, $q < q'$ and $D' < D$. Then the distribution functions of $S_{L'}$, $S_{q'}$, and $S_{D'}$ are above those of S_L, S_q, and S_D, respectively, at every percentile. Thus we have $S_L \preceq_{st} S_{L'}$, $S_q \preceq_{st} S_{q'}$, and $S_D \preceq_{st} S_{D'}$. Similarly, if $q_1 \leq q_1'$ and $L_1 \leq L_1'$, the distribution function of $S_{q_1',L_1'}$ are above that of S_{q_1,L_1} at every percentile. By the same token, if $q_2 \leq q_2'$ and $D_2' \leq D_2$, the distribution function of $S_{D_2',q_2'}$ are above that of S_{D_2,q_2} at every percentile. This proves all the usual stochastic orderings ($\preceq_{st}$) on the diagonal.

Now we prove $S_{q_1,L_1} \preceq_{icx} S_{q_1',L_1'}$ if $q_1' < q_1$ and $L_1 < L_1'$. According to Proposition 3.2, we can find a $q^* < q_1$ such that $S_{q_1,L_1} \preceq_{cx} S_{q^*,L_1'}$. Since $E(S_{q_1',L_1'}) = n > m$ $= E(S_{q^*,L_1'}) = E(S_{q_1,L_1})$, then $q^* < q_1'$, and $S_{q_1,L_1} \preceq_{cx} S_{q^*,L_1'} \preceq_{st} S_{q_1',L_1'}$. By Proposition 2.1, $S_{q_1,L_1} \preceq_{icx} S_{q_1',L_1'}$. Similarly, we can show $S_{D_2,q_2} \preceq_{icx} S_{D_2',q_2'}$ if $q_2 < q_2'$ and $D_2 < D_2'$.

Note that $S_L \preceq_{st} S_{L'} \preceq_{cx} S_{q_1',L_1'}$. By Proposition 2.1, $S_L \preceq_{icx} S_{q_1',L_1'}$. All the other increasing convex ordering pairs on the upper right corner of the table follow the same argument. □

Note that when $q_1 < q_1'$, and $L_1' < L_1$, no relationship can be derived between S_{q_1,L_1} and $S_{q_1',L_1'}$ as the former has a larger left tail while the latter has a larger right tail. Similarly, no ordering can be established for S_{D_2,q_2} and $S_{D_2',q_2'}$ when $q_2' < q_2$ and $D_2' < D_2$. Figures 5 and 6 illustrate this point.

In Figure 5, the curve VV^* is the collection of reinsurance treaties with the same expected loss m, where each point on the curve represents a different combination of quota share percentage q_2 and stop-loss threshold D_2. For example, point V represents a reinsurance treaty with a \$50M stop-loss threshold and a 60% quota share. Similarly, the curve $V'V''$ is the collection of reinsurance treaties, all having the same expected loss n, where $n > m$. Proposition 4.1 says that the relationship between the points on the $V'V''$ curve and the point V is such that the treaties above point V' on the $V'V''$ curve are riskier than V under the increasing convex order and the points between V' and V'' including V' and V'' are riskier than V under the usual stochastic order. The treaties below V'', however, do not have any dominating relationship with V.

Similarly in Figure 6, the WW^* and the $W'W''$ curves represent the reinsurance structures having expected loss costs of m and n, respectively, where $n > m$. The treaties between W' and W'' are dominating W under the usual stochastic order while the treaties along the curve above W' are dominating W under the increasing convex order. No dominating relationship exists between W and those treaties below W'' on the $W'W''$ curve.

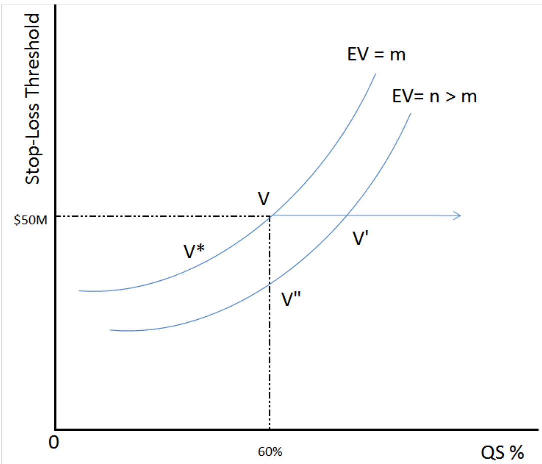

Figure 5

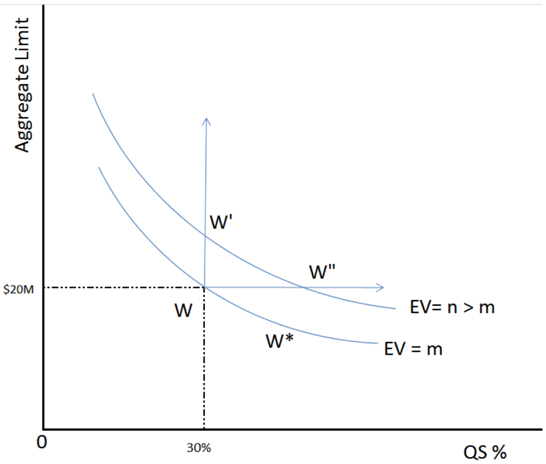

Figure 6

5 Application to Excess of Loss Reinsurance

The approach above can be applied to the excess of loss (XOL) reinsurance except that the terminologies used in XOL are slightly different. The equivalent of a stop-loss threshold in an XOL reinsurance is called an aggregate deductible while the equivalent of a quota share in XOL is called partial placement or co-participation from a cedant's point of view. We first define the various XOL options.

5.1 Basic XOL Definitions and Risk Rankings

Definition 5.1. Excess of Loss - For $l, r > 0$, the (l xs r) layer loss for a risk or an occurrence is

$$(x - r)_+ \wedge l = \begin{cases} 0 & \textit{if } 0 \leq x < r \\ x - r & \textit{if } r \leq x < r + l \\ l & \textit{if } r + l \leq x \end{cases}$$

Definition 5.2. Aggregate Layer Loss - Let $Y = \sum_{i=1}^{N}((x_i - r)_+ \wedge l)$ denote the aggregate layer loss for the (l xs r) layer where the summation is over the ground-up loss frequency random variable N with index i. $Y = 0$ when $N = 0$.

Definition 5.3. XOL with Aggregate Deductible - The XOL reinsurance Y_D with an aggregate deductible $D > 0$ is

$$Y_D = \begin{cases} 0 & \textit{if } 0 \leq Y < D \\ Y - D & \textit{if } D \leq Y \end{cases}$$

Definition 5.4. XOL with Aggregate Limit - The XOL reinsurance Y_L with an aggregate limit $L > 0$ is

$$Y_L = \begin{cases} Y & \textit{if } 0 \leq Y < L \\ L & \textit{if } L \leq Y \end{cases}$$

Definition 5.5. XOL with Partial Placement - Let $Y_q = qY$ denote the XOL reinsurance with partial placement where $0 < q < 1$ is the ratio ceded to reinsurers and $(1 - q)$ is the cedant's co-participation ratio in the reinsurance.

Proposition 5.1. *Assume that the XOL reinsurance with aggregate deductible, aggregate limit and partial placement have the same expected value. Under the convex order, the XOL reinsurance with an aggregate deductible is more risky than the XOL reinsurance with partial placement, which in turn is more risky than the XOL reinsurance with an aggregate limit. That is* $Y_L \preceq_{cx} Y_q \preceq_{cx} Y_D$, *or*

XOL with Aggregate Limit
$\preceq_{cx}$**XOL with Partial Placement**
$\preceq_{cx}$**XOL with Aggregate Deductible.**

Proof. Similar to the proof of Proposition 3.1. □

Example 3: The XYZ Insurance Company writes \$100 million of commercial auto insurance annually. The company is presented with three reinsurance options: (1) \$4M xs \$1M XOL reinsurance with unlimited free reinstatements, (2) \$4M xs \$1M XOL reinsurance with an aggregate deductible of \$3M and unlimited free reinstatements, or (3) \$4M xs \$1M XOL reinsurance with three free reinstatements. The company estimates that the expected loss costs for options 1, 2 and 3 are \$6M, \$4M, and \$5.5M, respectively, and quoted reinsurance premiums are \$8M, \$5.8M and \$7M, respectively.

The following table summarizes the estimated expected loss cost and market quotes for each of the reinsurance options:

Table 3 : Ranking Analysis

Option Type	Variation of 4x1 XOL	Quoted Reins. Premium	Expected Loss Cost	Implied Margin
Y	Free unlimited reinstatements	\$8M	\$6M	\$2M
Y_D	\$3M aggregate deductible	\$5.8M	\$4M	\$1.8M
Y_q	Free unlimited reinstatements 66.6% (=4/6) placement	\$5.33M	\$4M	\$1.33M
Y_q	Free unlimited reinstatements 91.7% (=5.5/6) placement	\$7.33M	\$5.5M	\$1.83M
Y_L	3 free reinstatements (aggregate limit =16M)	\$7M	\$5.5M	\$1.5M

According to the risk ranking analysis, a relevant comparison can be made between the 4x1 XOL reinsurance with a 66.6% placement and the 4x1 reinsurance with a \$3M aggregate deductible as the expected loss costs are the same at \$4M. The extra margin charge is \$0.47M (1.8M-1.33M) for the risky aggregate deductible option. On the other hand, the theory indicates that the 4x1 XOL reinsurance with a 91.7% placement is more risky than the 4x1 XOL reinsurance with an aggregate limit of \$16M (implied by the three reinstatements). The extra margin charge for the 4x1 XOL reinsurance with a 91.7% placement is \$0.33M (1.83M-1.5M).

5.2 Higher XOL Retention as An Option

In XOL reinsurance, insurers can consider another option, namely adjusting their per risk/per occurrence retentions. Insurers often make these adjustments in response to changes in the underlying exposure and the implication to capital requirements. In this section, we will explore how an XOL reinsurance with a higher retention is stacking up against other types of XOL reinsurance in terms of risk ranking. Again we will assume all reinsurance structures under consideration in this section have the same expected value.

Definition 5.6. XOL with Higher Retention - Given an (l xs r) layer, the (l_H xs r_H) layer is a layer with a higher retention if $r < r_H$, $l_H < l$ and $(r + l) = (r_H + l_H)$. Let $Y_H = \sum_{i=1}^{N}((x_i - r_H)_+ \wedge l_H)$ denote the aggregate layer loss for the (l_H xs r_H) layer where the summation is over the ground-up loss x with frequency N.

For example, by definition, a \$3M xs \$2M XOL layer is a higher layer than a \$4M xs \$1M XOL layer while the sums of the respective limits and retentions are identical at \$5M. Suppose the cedant co-participates in the \$4M xs \$1M layer so that the resulting \$4M xs \$1M XOL reinsurance with partial placement has the same expected value as the \$3M xs \$2M XOL reinsurance. We will show in the following proposition that the latter is more risky than the former under the convex order.

Proposition 5.2. *(Higher Retention vs. Partial Placement) Let Y_q denote the (l xs r) XOL with partial placement and Y_H denote the (l_H xs r_H) XOL where $r_H > r$, $l_H < l$, and $r + l = r_H + l_H$. Assuming $E(Y_q) = E(Y_H)$, then under the convex order, Y_H is more risky than Y_q. That is $Y_q \preceq_{cx} Y_H$, or*

Partial Placement $\preceq_{cx}$ Higher Retention

Proof. First we analyze the two per risk/occurrence severity random variables, $q[(x-r)_+ \wedge l]$ and $[(x-r_H)_+ \wedge l_H]$. The relationship between $q[(x-r)_+ \wedge l]$ and $[(x-r_H)_+ \wedge l_H]$ is similar to that of a quota share reinsurance with q as the quota share percentage and a stop-loss reinsurance with $(r_H - r)$ as the stop-loss threshold. Note that $qE[(x-r)_+ \wedge l]$ $= E[(x-r_H)_+ \wedge l_H]$. Then the single crossing condition and the equality of the means imply that on the individual severity distribution basis,

$$q[(x - r)_+ \wedge l] \preceq_{cx} [(x - r_H)_+ \wedge l_H].$$

That is, under the convex order $[(x - r_H)_+ \wedge l_H]$ is more risky than $q[(x - r)_+ \wedge l]$. Note that $Y_q = \sum_{i=1}^{N} q[(x_i - r)_+ \wedge l]$ and $Y_H = \sum_{i=1}^{N}[(x - r_H)_+ \wedge l_H]$ where N is the number of risks/occurrences and $E(Y_q) = E(Y_H)$. As the convex order is closed under convolution (Theorem 3.A.13, Shaked & Shanthikumar (2007)) and the frequency random variable N is independent, we get $Y_q \preceq_{cx} Y_H$.

□

Proposition 5.3. *(Higher Retention vs. Aggregate Deductible) Let Y_D denote the $(l \text{ xs } r)$ XOL reinsurance with aggregate deductible D and let Y_H denote the $(l_H \text{ xs } r_H)$ XOL reinsurance where $r < r_H$, $l_H < l$ and $(r + l) = (r_H + l_H)$. Assuming $E(Y_D) = E(Y_H)$, then under the convex order, Y_D is more risky than Y_H. That is $Y_H \preceq_{cx} Y_D$, or*

$$\textbf{XOL with Higher Retention} \preceq_{cx} \textbf{XOL with Aggregate Deductible}$$

Proof. Similar to the proof for Proposition 3.1, Theorem 6.1 in van Heerwaarden, Kass and Goovaerts (1989) implies that the CDF of Y_D also crosses only once with the CDF of any other XOL reinsurance structures such as Y_H given that the (l_H xs r_H) layer is a subset of the original layer. Given the equality of the means and the single crossing property, Proposition 2.2 implies that $Y_H \preceq_{cx} Y_D$.

□

Combining Propositions 5.1, 5.2, and 5.3 and using the transitivity of the convex order, we obtain the following result:

$$\begin{aligned} &\textbf{XOL with Aggregate Limit} \\ \preceq_{cx} &\textbf{XOL with Partial Placement} \\ \preceq_{cx} &\textbf{XOL with Higher Retention} \\ \preceq_{cx} &\textbf{XOL with Aggregate Deductible.} \end{aligned}$$

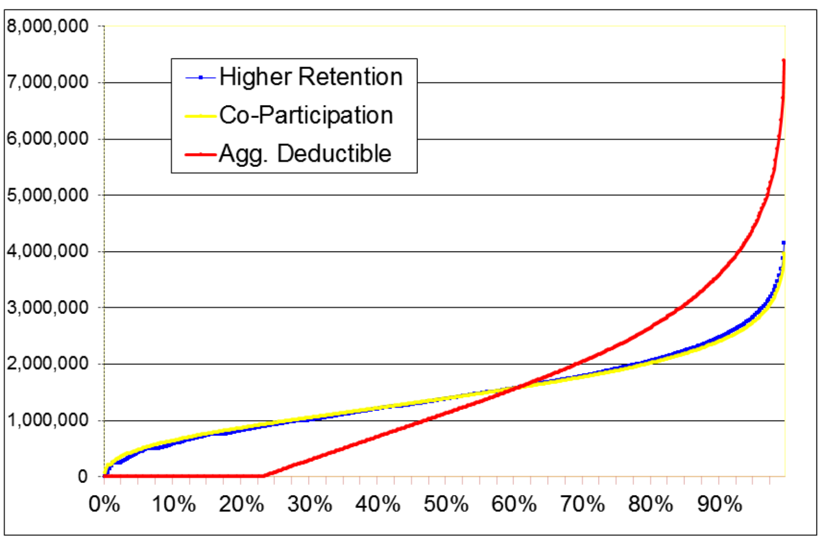

Figure 7

The red curve in Figure 7 represents the distribution function for a reinsurance with aggregate deductible Y_D, which stays flat until the aggregate layer loss amount reaches the deductible threshold D at the 25^{th} percentile and then increases with the same incremental amounts as Y.

The yellow curve represents the distribution function of an XOL reinsurance with partial placement Y_q while the blue curve is for an XOL reinsurance Y_H with a per risk/occurrence retention level higher than that for Y. Given that Y_q and Y_H have the same expected loss, the blue Y_H curve starts under the yellow Y_q curve, then the two curves intertwine over most of the percentiles, and finally the Y_H curve takes over after the last intersection at the 70^{th} percentile. Notice that the convex order does allow multiple crossings of the CDF curves as long as the stop-loss premium requirement in Proposition 2.1 is satisfied. When multiple crossings occur, it is difficult to discern convex order dominance empirically. Thus the analytical proof is an important confirmation of the dominance relationship and serves as an indication tool for reinsurance pricing.

5.3 Hybrid XOL Structures

Again this line of reasoning and analysis can be extended to the XOL treaties with more than one contract feature, which include combinations of aggregate deductibles, higher retentions, partial placement, and/or aggregate limits. For common reinsurance structures with at most two contract features, proving stochastic ordering may be straightforward. We define below three additional types of reinsurance and show that they can be properly ordered under the convex order along with the four basic XOL reinsurance structures. For these reinsurance structures with two contract features, the proof of stochastic ordering is similar to those in Propositions 5.2 and 5.3.

Definition 5.7. Mixture of Partial Placement and Aggregate Limit - The XOL reinsurance $Y_{q,L}$ with an aggregate limit $L > 0$ and a placement ratio $0 < q < 1$ is

$$Y_{q,L} = \begin{cases} qY & \text{if } 0 \le qY < L \\ L & \text{if } L \le qY. \end{cases}$$

Definition 5.8. Mixture of Higher Retention and Partial Placement - The XOL reinsurance $Y_{H,q}$ with a placement ratio $0 < q < 1$ and a higher retention as defined in Definition 5.6 is $Y_{H,q} = qY_H$.

Definition 5.9. Mixture of Aggregate Deductible and Higher Retention - The XOL reinsurance $Y_{D,q}$ with an aggregate deductible $D > 0$ and a higher retention as defined in Definition 5.6 is

$$Y_{H,D} = \begin{cases} 0 & \text{if } 0 \le Y_H < D \\ Y_H - D & \text{if } D \le Y_H. \end{cases}$$

Similar to Proposition 4.1, the following proposition uses the usual stochastic order and the increasing convex order and extends the analysis to include XOL reinsurance structures with different expected losses.

Proposition 5.4. *Denote H, H_2, and H_3 as higher retention layers, q, q_2, and q_3 as placement ratios, D and D_3 as aggregate deductibles, and L and L_1 as aggregate limits. Let $0 < q < q_1 < 1$, $0 < q < q_2 < 1$, $0 < D_3 < D$, $0 < L < L_1$, and H be a higher layer than either H_2 or H_3 such that the XOL reinsurance options, Y_L, Y_{q_1,L_1}, Y_q, Y_{H_2,q_2}, Y_H, Y_{H_3,D_3}, and Y_D for an aggregate layer loss Y have the same expected value, m. That is,*

$$E(Y_L) = E(Y_{q_1,L_1}) = E(Y_q) = E(Y_{H_2,q_2}) = E(Y_H) = E(Y_{H_3,D_3}) = E(Y_D) = m.$$

Then the following orderings can be established:

$$Y_L \preceq_{cx} Y_{q_1,L_1} \preceq_{cx} Y_q \preceq_{cx} Y_{H_2,q_2} \preceq_{cx} Y_H \preceq_{cx} Y_{H_3,D_3} \preceq_{cx} Y_D$$

or

XOL with Aggregate Limit
$\preceq_{cx}$**XOL with Mixture of Partial Placement and Aggregate Limit**
$\preceq_{cx}$**XOL with Partial Placement**
$\preceq_{cx}$**XOL with Mixture of Higher Retention and Partial Placement**
$\preceq_{cx}$**XOL with Higher Retention**
$\preceq_{cx}$**XOL with Mixture of Aggregate Deductible and Higher Retention**
$\preceq_{cx}$**XOL with Aggregate Deductible.**

Moreover, consider a similar set of reinsurance structures, $Y_{L'}$, $Y_{q'_1,L'_1}$, $Y_{q'}$, $Y_{H'_2,q'_2}$, $Y_{H'}$, $Y_{H'_3,D'_3}$ and $Y_{D'}$, on the same layer aggregate loss Y, where $0 < q' < q'_1 < 1$, $0 < q' < q'_2 < 1$, $0 < D'_3 < D'$, $0 < L' < L'_1$, and H' is a higher layer than either H'_2 or H'_3 such that

$$E(Y_{L'}) = E(Y_{q'_1,L'_1}) = E(Y_{q'}) = E(Y_{H'_2,q'_2}) = E(Y_{H'}) = E(Y_{H'_3,D'_3}) = E(Y_{D'}) = n > m.$$

Then the following orderings can be established:

	$Y_{L'}$	$Y_{q'_1,L'_1}$	$Y_{q'}$	$Y_{H'_2,q'_2}$	$Y_{H'}$	$Y_{H'_3,D'_3}$	$Y_{D'}$
Y_L	$\preceq_{st}$	$\preceq_{icx}$	$\preceq_{icx}$	$\preceq_{icx}$	$\preceq_{icx}$	$\preceq_{icx}$	$\preceq_{icx}$
Y_{q_1,L_1}		*see **	$\preceq_{icx}$	$\preceq_{icx}$	$\preceq_{icx}$	$\preceq_{icx}$	$\preceq_{icx}$
Y_q			$\preceq_{st}$	$\preceq_{icx}$	$\preceq_{icx}$	$\preceq_{icx}$	$\preceq_{icx}$
Y_{H_2,q_2}				*see ***	$\preceq_{icx}$	$\preceq_{icx}$	$\preceq_{icx}$
Y_H					$\preceq_{st}$	$\preceq_{icx}$	$\preceq_{icx}$
Y_{H_3,D_3}						*see ****	$\preceq_{icx}$
Y_D							$\preceq_{st}$

where the table reads, from left to right, $Y_L \preceq_{st} Y_{L'}$, $Y_L \preceq_{icx} Y_{q'_1,L'_1}$, and so on.

$*$	$Y_{q'_1,L'_1}$
Y_{q_1,L_1}	$\preceq_{icx}$ *if* $q'_1 < q_1, L_1 < L'_1$ $\preceq_{st}$ *if* $q_1 \leq q'_1, L_1 \leq L'_1$

$**$	$Y_{H'_2,q'_2}$	$***$	$Y_{H'_3,D'_3}$
Y_{H_2,q_2}	$\preceq_{icx}$ *if* $H_2 < H'_2, q_2 < q'_2$ $\preceq_{st}$ *if* $H'_2 \leq H_2, q_2 \leq q'_2$	Y_{H_3,D_3}	$\preceq_{icx}$ *if* $H'_3 < H_3, D_3 < D'_3$ $\preceq_{st}$ *if* $H'_3 \leq H_3, D'_3 \leq D_3$

where in general $H < H'$ means H' has a higher per risk/occurrence retention than H.

Proof. The proof of $Y_L \preceq_{cx} Y_{q_1,L_1} \preceq_{cx} Y_q$ is similar to the proof of Proposition 3.2 while the proof of $Y_q \preceq_{cx} Y_{H_2,q_2} \preceq_{cx} Y_H$ is similar to the proof of Proposition 5.2, where the convex order is established first at the per risk/occurrence level. Use the closure by convolution property to prove the ordering at the aggregate layer loss level. Similarly, use Proposition 5.3 to prove $Y_H \preceq_{cx} Y_{H_3,D_3} \preceq_{cx} Y_D$ since $D_3 < D$ and H is a higher layer than H_3, which in turn is a higher layer than the original layer for Y_D.

The usual stochastic orderings ($\preceq_{st}$) and the increasing convex orderings ($\preceq_{icx}$) in the large table above are similar to those in Proposition 4.1 except the relationship for the structures with the higher retention layers. We need to show $Y_H \preceq_{st} Y_{H'}$ and the relationships in the (**) grid and (***) grid. Since $n > m$, H is a higher layer than H'. Then the distribution function of $Y_{H'}$ must be above that of Y_H at every percentile, hence $Y_H \preceq_{st} Y_{H'}$.

If $H'_2 \leq H_2$ and $q_2 \leq q'_2$, the distribution function of $Y_{H'_2,q'_2}$ must be above that of Y_{H_2,q_2} at every percentile. By the same token, if $H'_3 \leq H_3$ and $D'_3 \leq D_3$, the distribution function of $Y_{H'_3,D'_3}$ must be above that of Y_{H_3,D_3} at every percentile. This proves all the usual stochastic ordering on the diagonal.

If $q_2 < q'_2$ and $H_2 < H'_2$, based on the first half of this proposition, we can find a q^* greater than q_2 such that $Y_{H_2,q_2} \preceq_{cx} Y_{H'_2,q^*}$. Since $E(Y_{H'_2,q'_2}) = n > m = E(Y_{H'_2,q^*}) = E(Y_{H_2,q_2})$, then q^* must be smaller than q'_2 and $Y_{H_2,q_2} \preceq_{cx} Y_{H'_2,q^*} \preceq_{st} Y_{H'_2,q'_2}$. By Proposition 2.1, $Y_{H_2,q_2} \preceq_{icx} Y_{H'_2,q'_2}$. Similarly, we can show $Y_{H_3,D_3} \preceq_{icx} Y_{H'_3,D'_3}$ if $H'_3 < H_3$ and $D_3 < D'_3$, and $Y_{q_1,L_1} \preceq_{icx} Y_{q'_1,L'_1}$ if $q'_1 < q_1$ and $L_1 < L'_1$. □

Similar to Figures 5 and 6 for aggregate loss reinsurance, Figures 8 and 9 illustrate the relationships between $Y_{H'_2,q'_2}$ and Y_{H_2,q_2} and between $Y_{H'_3,D'_3}$ and Y_{H_3,D_3}, respectively. In Figure 8, the curve VV^* represents the collection of reinsurance treaties with the same expected loss m, where each point on the curve is a different combination of placement percentage q_2 and layer retention H_2. Similarly, the curve $V'V''$ is the collection of reinsurance treaties, all having the same expected loss n, where $n > m$. Proposition 5.4 says that the relationship between the points on the $V'V''$ curve and the point V is such that the treaties above point V' on the $V'V''$ curve are riskier than V under the increasing convex order and the points between V' and V'' including V' and V'' are riskier than V under the usual stochastic order. The treaties below V'', however, do not have any dominating relationship with V. Similar interpretation can be made for Figure 9, where the WW^* and the $W'W''$ curves represent reinsurance structures with different combinations of layer retentions and aggregate deductibles and having expected loss costs of m and n, respectively ($n > m$).

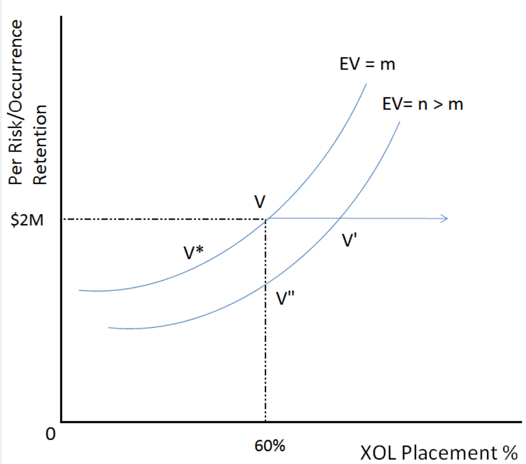

Figure 8

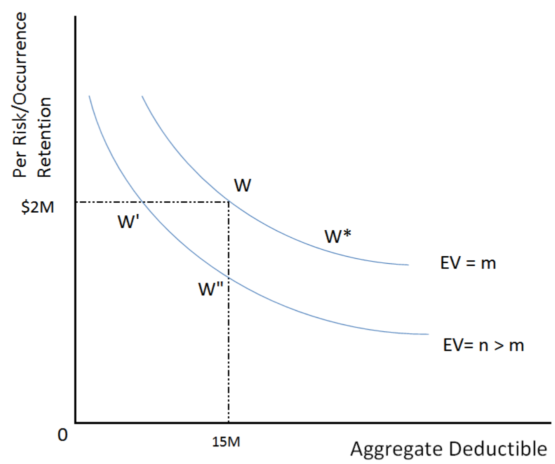

Figure 9

In general, with equal means, options with an aggregate deductible would dominate those without an aggregate deductible under the convex order. If both options have an aggregate deductible, then the one with a higher aggregate deductible would dominate the other with a lower aggregate deductible. Similarly, options without an aggregate limit would dominate those with an aggregate limit. If both options have an aggregate limit, then the one with a higher aggregate limit would dominate the other with a lower aggregate limit.

Example 4: Continuing the example in Section 5.1, the XYZ insurance company decides to explore other options by increasing the retention level of the XOL reinsurance for commercial auto liability and is willing to co-participate up to 20%. The company determines that the \$3M xs \$2M XOL reinsurance and the \$2.5M xs \$2.5M XOL reinsurance have the expected loss costs of \$5 million and \$4 million, respectively. Both options assume unlimited reinstatements. In addition, the company estimates that adding a \$1.5 million aggregate deductible to the \$3M xs \$2M XOL reinsurance can reduce the expected loss cost to \$4M. Similarly, decreasing the number of free reinstatements from three to one for the \$4M xs \$1M XOL reinsurance also reduces the expected loss cost to \$4M.

Based on the risk ranking analysis and the quotes received earlier for the \$4M xs \$1M layer, the reinsurance premiums for these new options should be less than \$5.8M and greater than \$5.33M and should be in the order as shown in Table 4.

An interesting question can be raised as to how the reinsurance premium for an XOL layer (e.g., Option Y_H) can be approximated in general. Based on the risk ranking results, one can find a premium lower bound for an XOL layer from a lower retention layer with partial placement (Option Y_{H_2,q_2}) and an upper bound from a lower retention layer with an aggregate deductible (Option Y_{H_3,D_3}). Clearly the closer the layers and the smaller the aggregate deductible, the better the approximation of the reinsurance premium.

Table 4 : Ranking Analysis

Option Type	Option Description	Quoted Reins. Premium	Expected Loss Cost	Implied Margin
Y_L	4x1 XOL, 100% placement, 1 free reinstatement.		\$4M	
Y_{q_1,L_1}	4x1 XOL, 71.7% placement, 3 free reinstatements.	\$5.09M	\$4M	\$1.09M
Y_q	4x1 XOL, 66.6% placement, free unlimited reinstatements.	\$5.33M	\$4M	\$1.33M
Y_{H_2,q_2}	3x2 XOL, 80% placement, free unlimited reinstatements.		(0.8*5) =\$4M	
Y_H	2.5x2.5 XOL, 100% placement, free unlimited reinstatements.		\$4M	
Y_{H_3,D_3}	3x2 XOL, 100% placement, free unlimited reinstatements, \$1.5M aggregate deductible.		\$4M	
Y_D	4x1 XOL, 100% placement, free unlimited reinstatements, \$3M aggregate deductible.	\$5.8M	\$4M	\$1.8M

6 A Global Comparison

In reinsurance practice, the need to compare XOL reinsurance structures with the reinsurance on aggregate losses arises constantly. The metrics used in comparison are usually the distribution moments, such as mean and standard deviation along with some tail measures. In this section, we use the stochastic ordering approach to comparing reinsurance options that are on either an aggregate loss basis or an XOL basis.

Proposition 6.1. *Denote q as an XOL placement ratio, D and D_1 as a stop-loss threshold and an XOL aggregate deductible, respectively, and L and L_1 as an aggregate limit and an XOL aggregate limit, respectively. Let $0 < q < 1$, $0 < D_1 < D$, $0 < L < L_1$ and H be a higher layer such that the reinsurance options, S_L, Y_{L_1}, Y_q, Y_H, Y_{D_1}, and S_D have the same expected loss value, m. That is,*

$$E(S_L) = E(Y_{L_1}) = E(Y_q) = E(Y_H) = E(Y_{D_1}) = E(S_D) = m.$$

Then the following orderings can be established:

$$S_L \preceq_{cx} Y_{L_1} \preceq_{cx} Y_q \preceq_{cx} Y_H \preceq_{cx} Y_{D_1} \preceq_{cx} S_D$$

or

$$\begin{aligned}
&\textbf{Aggregate Limit}\\
\preceq_{cx}&\textbf{XOL with Aggregate Limit}\\
\preceq_{cx}&\textbf{XOL with Partial Placement}\\
\preceq_{cx}&\textbf{XOL with Higher Retention}\\
\preceq_{cx}&\textbf{XOL with Aggregate Deductible}\\
\preceq_{cx}&\textbf{Stop-loss}
\end{aligned}$$

Consider a similar set of reinsurance structures, $S_{L'}$, $Y_{L'_1}$, $Y_{q'}$, $Y_{H'}$, $Y_{D'_1}$, *and* $S_{D'}$, *with regard to the same underlying loss, where* $0 < q' < 1$, $0 < D'_1 < D'$, *and* $0 < L' < L'_1$ *and* H' *is a higher per risk/occurrence layer such that*

$$E(S_{L'}) = E(Y_{L'_1}) = E(Y_{q'}) = E(Y_{H'}) = E(Y_{D'_1}) = E(S_{D'}) = n > m.$$

Then the following orderings can be established:

	$S_{L'}$	$Y_{L'_1}$	$Y_{q'}$	$Y_{H'}$	$Y_{D'_1}$	$S_{D'}$
S_L	$\preceq_{st}$	$\preceq_{icx}$	$\preceq_{icx}$	$\preceq_{icx}$	$\preceq_{icx}$	$\preceq_{icx}$
Y_{L_1}		$\preceq_{st}$	$\preceq_{icx}$	$\preceq_{icx}$	$\preceq_{icx}$	$\preceq_{icx}$
Y_q			$\preceq_{st}$	$\preceq_{icx}$	$\preceq_{icx}$	$\preceq_{icx}$
Y_H				$\preceq_{st}$	$\preceq_{icx}$	$\preceq_{icx}$
Y_{D_1}					$\preceq_{st}$	$\preceq_{icx}$
S_D						$\preceq_{st}$

where the table reads, from left to right, $S_L \preceq_{st} S_{L'}$, $S_L \preceq_{icx} Y_{L'_1}$, $Y_{L_1} \preceq_{st} Y_{L'_1}$ *and so on.*

Proof. The proof for the first half of the proposition follows the proofs in Proposition 3.1 and Proposition 5.4. The proof of the ($\preceq_{icx}$) and ($\preceq_{st}$) relationship in the grid follows the first half of this proposition and Propositions 4.1 and 5.4, where we show that if $A \preceq_{cx} B \preceq_{st} C$, then $A \preceq_{icx} C$. □

Note that the quota share reinsurance is not compatible with this comparison framework involving XOL reinsurance. The right tail of the quota share reinsurance is always thicker than that of the XOL reinsurance while the opposite is true for the left tail. Proposition 6.1 also indicates that stop-loss reinsurance and the reinsurance with an aggregate limit serve as the upper and lower boundaries for the XOL reinsurance options. To make the comparison more complete, we can add the hybrid XOL reinsurance from Section 5.3. Given the transitivity of the convex order, the ranking of those hybrid XOL reinsurances would be the same as indicated in Proposition 5.3.

7 Implications to Pricing and Optimal Reinsurance

Assuming reinsurance companies are also risk averse, it is reasonable to assume that they would adopt premium principles that observe the established ordering above. Suppose the reinsurance structures under consideration have the same expected value and reinsurance companies employ the expected loss premium principle in calculation of the reinsurance premium. The actuarial literature indicates that stop-loss reinsurance would always be preferred by the cedant as it passes more risk to the reinsurer and costs the same as all the other options. Thus the implication of the risk ranking analysis above is that if reinsurance A is found to be more risky than reinsurance B, then reinsurance A should be priced higher than reinsurance B to compensate for the higher risk. As such, these ranking results may serve as an elementary tool in identifying inconsistent market quotes.

In the optimal reinsurance literature (e.g., Cheung (2010)), the frequently used approach in finding optimal reinsurance is by maximization/minimization of an objective function over a convex constraint. The convex objective function (to be minimized) can be VaR or TVaR of the retained exposure, which is defined as total exposure minus ceded exposure plus the reinsurance premium for the ceded exposure.

Definition 7.1. VaR objective function - The VaR objective function is

$$\mathbf{MinVaR}[X - f(X) + PR(f(X))]$$

where $f(X)$ is the ceded loss and $PR(f(X))$ is the corresponding reinsurance premium.

Obviously if the premium calculation is expected value based, the optimal reinsurance would always be the stop-loss reinsurance given that a tail measure is the selection criterion. Thus it is more realistic if the premium principle is convex in the maximization/minimization process (e.g., Chi (2012), Guerra & Centeno (2010)).

The standard deviation principle and the variance principle along with the Wang principle are known to observe the second order stochastic dominance relationship. It would be interesting to evaluate the pricing differentials among the reinsurance structures using the three premium principles, which could be a subject for future research.

8 Conclusions

Reinsurance can be regarded as financial derivatives on a random loss process, which determines how reinsurers and insurers would share the loss upon its realization. The major technical difference between reinsurance and other financial derivatives such as stock options is that common reinsurance structures are comonotone with the underlying loss process. This makes the comparison of reinsurance structures intuitive and sometimes straightforward.

Following the classical results on optimal reinsurance in the actuarial literature, the paper[4] has shown that many common reinsurance structures in practice can be ranked either under the convex order if they have the same expected loss costs or under the increasing convex order and the usual stochastic order if they have different expected loss costs. Using the results of the paper, actuaries and underwriters can easily compare the riskiness of various reinsurance structures in an ERM and/or reinsurance retention analysis. The results also imply that reinsurers should price these reinsurance contracts with premium principles that recognize the risk rankings established in the paper.

[4]Loss discount and other accounting treatments that may be associated with specific reinsurance treaties are not considered here and are beyond the scope of this paper.

Appendix

A.1 Alternative Definition for Higher Layers

In this section, we expand the definition of higher layers in Section 5.2 by dropping the requirement that a higher layer must be a subset of the original layer. Thus a higher layer and the original layer can be overlapping or even disjoint as long as the retention of the higher layer is greater than that of the original layer. Then results similar to Proposition 5.2 are still valid while Proposition 5.3 may not be true for all reinsurance structures under this new definition. We redefine a higher layer as follows:

Definition A.4. XOL with Higher Retention - *Given an* $(l\ xs\ r)$ *layer, the* $(l_H\ xs\ r_H)$ *layer is a layer with a higher retention if* $r < r_H$ *and* $(r+l) \le (r_H + l_H)$. *Let* $Y_H = \sum_{i=1}^{N}((x_i - r_H)_+ \wedge l_H)$ *denote the aggregate layer loss for the* $(l_H\ xs\ r_H)$ *layer where the summation is over the ground-up loss* x *with frequency* N.

Proposition A.1. *(Higher Retention vs. Partial Placement) Let* Y_q *denote the* $(l\ xs\ r)$ *XOL with partial placement and* Y_H *denote the* $(l_H\ xs\ r_H)$ *XOL where* $r < r_H$ *and* $(r+l) \le (r_H + l_H)$. *Assuming* $E(Y_q) = E(Y_H)$, *then under the convex order,* Y_H *is more risky than* Y_q. *That is* $Y_q \preceq_{cx} Y_H$, *or*

$$\textbf{XOL with Partial Placement} \ \preceq_{cx} \textbf{XOL with Higher Retention}$$

Proof. Again we consider the two per risk/occurrence severity random variables, $q[(x-r)_+ \wedge l]$ and $[(x-r_H)_+ \wedge l_H]$. Note that $qE[(x-r)_+ \wedge l] = E[(x-r_H)_+ \wedge l_H]$. We need to consider the following three cases:

$$\begin{cases} (r+l) = (r_H + l_H) & \textit{higher layer is a subset;} \\ (r+l) \le r_H < (r_H + l_H) & \textit{two disjoint layers;} \\ r_H < (r+l) < (r_H + l_H) & \textit{two overlapping layers.} \end{cases}$$

Proposition 5.2 covers the first case, where $(r+l) = (r_H + l_H)$. If $(r+l) \le r_H < (r_H + l_H)$, the CDF of $[(x-r_H)_+ \wedge l_H]$ must cross the CDF of $q[(x-r)_+ \wedge l]$ once from below at some $x^* \ge r_H$, where $q[(x-r)_+ \wedge l]$ already reaches its maximum at ql. If $r_H < (r+l) < (r_H + l_H)$, the crossing of the two CDF's can only occur when x is between r_H and $(r_H + l_H)$ since the CDF of $[(x-r_H)_+ \wedge l_H]$ becomes flat when $(r_H + l_H) \le x$ or when $x \le r_H$. By applying Theorem 6.1 in van Heerwaarden, Kass and Goovaerts (1989) again, we can show that the two CDF's crosses only once since the CDF of the retained loss net of $[(x-r_H)_+ \wedge l_H]$ is flat between r_H and $(r_H + l_H)$.

For the latter two cases, the single crossing condition and the equality of the means imply that on the individual severity distribution basis,

$$q[(x-r)_+ \wedge l] \preceq_{cx} [(x-r_H)_+ \wedge l_H].$$

That is, under the convex order $[(x-r_H)_+ \wedge l_H]$ is more risky than $q[(x-r)_+ \wedge l]$. Note that on the aggregate basis, $Y_q = \sum_{i=1}^{N} q[(x_i - r)_+ \wedge l]$ and $Y_H = \sum_{i=1}^{N}[(x_i - r_H)_+ \wedge l_H]$, where N is the number of risks/occurrences and $E(Y_q) = E(Y_H)$. As the convex order is closed under convolution (Theorem 3.A.13, Shaked & Shanthikumar (2007)) and the frequency random variable N is independent, we get $Y_q \preceq_{cx} Y_H$. □

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The Consideration of Loss Timing for Risk Transfer Analysis

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Abstract

Motivation. An important consideration in a risk transfer analysis is the potential variability of loss timing. By excluding this variability, a risk transfer analysis could lead to materially different results, thereby causing users to draw different conclusions about risk transfer.

Method. This paper specifically illustrates the variation in payment patterns commonly found in paid loss and allocated loss adjustment expense development patterns (payment patterns) then provides an example of one method that can be used to model this payment pattern volatility. The impact of modeling this payment pattern volatility is illustrated with Expected Reinsurer Deficit (ERD) results under a hypothetical reinsurance structure. Important model considerations also reflected are correlation and discount rate assumptions. The ERD test is also used to illustrate the sensitivity of these modeled assumptions.

Results. The change in the results of a risk transfer test such as the ERD test can be material after consideration of payment pattern timing.

Conclusions. Modeling the variation of payment patterns is important for a broad spectrum of actuarial analyses. When evaluating reinsurance risk transfer test statistics it is important to keep in mind features that are sensitive to the variation of loss payment timing. The loss payment timing may have a significant impact on the present value of losses ceded to a reinsurer. At the very least the variation in timing will have an impact on the present value of losses used in the ERD test statistic, particularly with larger discount rates. Correlation of payment timing (or duration) with ultimate loss and allocated loss adjustment expense (ALAE) modeled is also an important consideration that can impact the results of the ERD test. The results below show the sensitivity of changes in correlation and discount rates combined with modeling the variation in the payment timing of ceded paid loss and ALAE.

Keywords. Risk Transfer, Timing Risk, ERD test, Correlation, Sensitivity of Assumptions.

1. INTRODUCTION

The timing of all cash flows between a primary insurer and reinsurer is an important consideration when assessing a reinsurance structure for risk transfer. This paper specifically looks at the timing of losses associated with variations in loss and ALAE development patterns. We will illustrate the potential impact loss timing variability can have on risk transfer test statistics such as the ERD.

1.1 Research Context

Reinsurance contractual features and the variability in loss and ALAE development patterns can have a material impact on the results of traditional risk transfer tests such as the ERD test statistic. Discount rate and correlation between simulated payment pattern and ultimate loss and ALAE are important assumptions to consider and can impact the results of the ERD test.

1.2 Objective

The intent of this paper is to provide high level insight into the importance of capturing loss and ALAE payment timing risk in models used to assess risk transfer. This is accomplished by providing illustrative examples of the variation in paid loss and ALAE timing, a simple model to simulate this timing, and the results of the ERD test under various assumptions.

1.3 Outline

The remainder of the paper proceeds as follows. Section 2.1 will discuss the risk transfer requirements under the guidance in the Statement of Financial Accounting Standards No. 113 (FAS 113). Section 2.2 will briefly discuss the Expected Reinsurer Deficit (ERD) test statistic for evaluation risk transfer. Section 2.3 will illustrate an example of the actual timing difference commonly found in loss development patterns. Section 2.3 will also give an example of a correlation analysis and simulated payment pattern. Section 2.4 shows the sensitivity of ERD results to payment pattern timing (i.e., variable versus fixed), correlation, and discount rates under a hypothetical reinsurance program.

2. BACKGROUND AND METHOD

2.1 Requirements for Risk Transfer

Timing of losses is a fundamental component of the "significant insurance risk" requirement under the guidance in the Statement of Financial Accounting Standards No. 113 (FAS 113). To summarize FAS 113: There are *two requirements* that must be met for a short duration contract to be considered as "*indemnifying the cedant*".

1. Reinsurer assumes significant insurance risk under the reinsured portions of insurance contracts; and
2. It is reasonably possible that the reinsurer may realize significant loss from the transaction.

Note: Contracts are exempt from risk transfer requirements when the reinsurer assumes *"substantially all"* of the insurance risk relating to the reinsured portions of the underlying insurance contracts (e.g., straight quota share contracts). It is still good practice to test this type of reinsurance deal for risk transfer and thoroughly understand the contract terms. This includes understanding the potential limitations that certain terms may have on the reinsurer's ultimate underwriting performance compared to the cedant.

To evaluate requirement (1), there must be a possibility of significant variation in the amount or timing of cash flows between assuming and ceding companies. When developing a stochastic loss model to evaluate the variation in the amount or timing of cash flows, consideration should be given to the distribution of probable loss outcomes and the timing of losses ceded to the reinsurer. To evaluate requirement (2), the present value of all cash flows between the reinsurer and the cedant under reasonably possible scenarios must be evaluated.

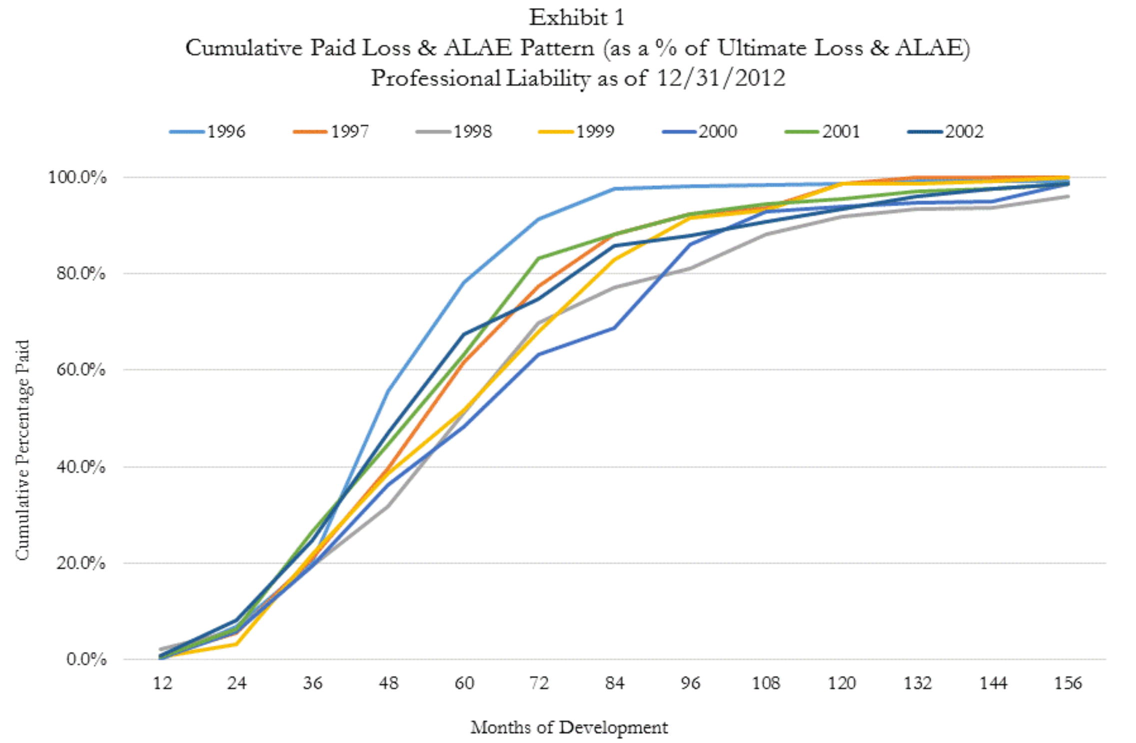
2.2 ERD Risk Transfer Test

The ERD test statistic reflects the probability of a net present value underwriting loss for the reinsurer multiplied by the net present value of the average severity of the underwriting loss. In this context, underwriting loss is the amount by which the present value of losses plus expenses exceeds the present value of premium as of the effective date of the reinsurance policy. The average severity in this context is the average underwriting loss (as a percentage of premium). A commonly accepted but not endorsed ERD threshold is 1% where an indicated ERD % greater than 1% passes risk transfer. This is consistent with the 10-10 test's 10% probability times a 10% underwriting loss (i.e., at least a 10% chance of an underwriting loss ratio of at least 110%), however the ERD test also considers severity of underwriting loss. It is important to note "ERD has not been explicitly endorsed by any professional body. However, while the CAS Working Party paper stopped short of endorsing the ERD, they prefer its use as the de facto standard over the 10-10 rule." [1] The risk transfer results illustrated Exhibit 5 below only consider the ERD test. Once one considers the timing risk associated with the potential variation in paid loss and ALAE the conclusions of risk transfer could potentially change.

2.3 Timing Differences in Historical Cumulative Loss Patterns

The sensitivity of risk transfer can be assessed by looking at risk transfer statistics such as the ERD test statistic and gross versus ceded cash flows at various probability levels. The variation in payment timing can be better understood after an investigation of historical data that has had time to develop to full maturity. Consider the following cumulative paid loss and ALAE percentages (as a percentage of ultimate loss and ALAE) for policy years 1996 through 2002 for a professional liability insurer[2]. Note this time horizon extends across only seven accident years of data, but illustrates the loss timing differences commonly found in other long tailed lines of insurance reviewed by the author.

2.3.1 Actual Example of Timing Differences in Paid Loss and ALAE Data



The above payment patterns are based on accendent year loss and allocated loss adjustment expense development triangles in a rate filing for American Casualty Company of Reading - PA.

The variation in cumulative paid loss and ALAE percentages as of 60 months of development ranges from 48% to 78% for the 7 accident years of data displayed above. The relationship of this potential variation in payment pattern timing and the variation in ultimate loss and ALAE settlements for a policy period is an important consideration when assessing a reinsurance contract for risk transfer. Consider the following section as a potential analysis in assessing this relationship.

2.3.2 Correlation of Loss & ALAE Payment Timing and Ultimate Loss and ALAE Data

Exhibit 2
Correlation of Payment Pattern Timing
and Ultimate Loss and ALAE

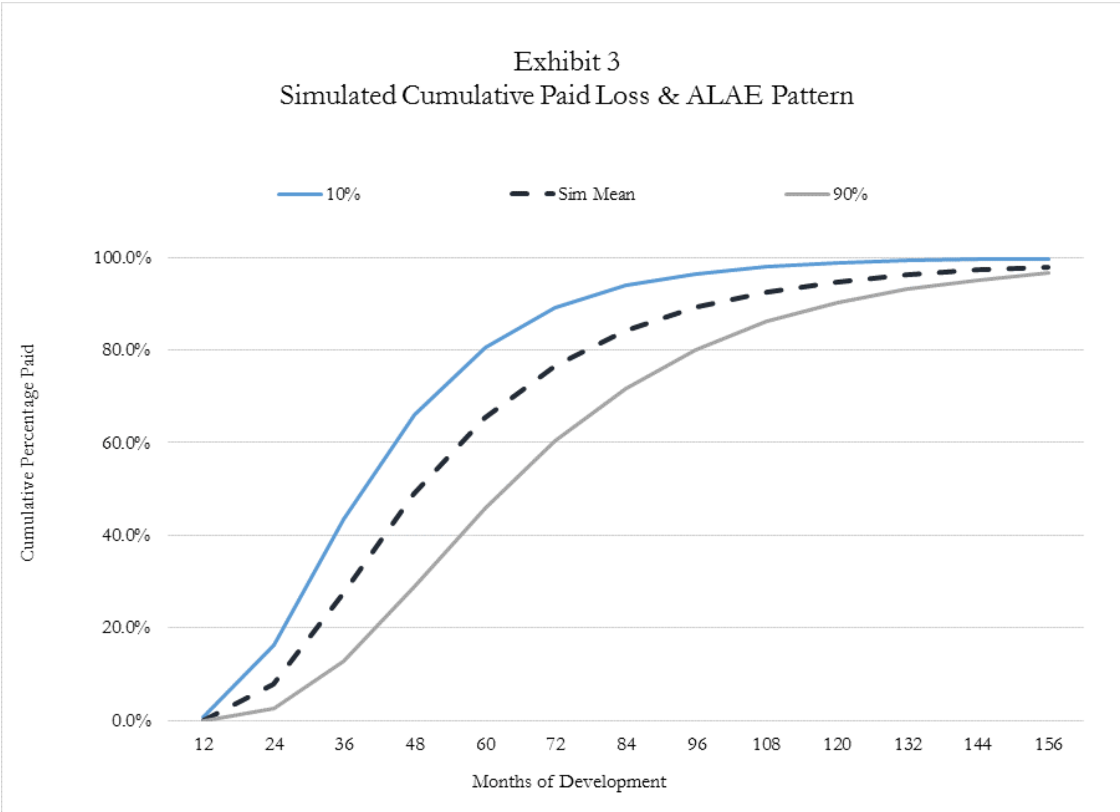
Loss Year	Selected Ultimate Loss & ALAE	Cumulative Paid Loss & ALAE at 60 Months of Dev	Cumulative Paid Loss & ALAE % at 60 Months of Dev [1]	Duration [2] of Paid Loss & ALAE (in Years)
1996	16,893	13,216	78.2%	4.0
1997	22,113	13,600	61.5%	4.6
1998	27,316	14,004	51.3%	5.3
1999	29,292	15,121	51.6%	4.9
2000	32,160	15,292	47.6%	5.3
2001	45,879	29,124	63.5%	4.5
2002	50,889	34,397	67.6%	4.6
2003	66,981	43,100	64.3%	4.7
2004	58,066	34,926	60.1%	4.9
			Correlation to Ult Loss & ALAE	Correlation to Ult Loss & ALAE
		Correlation 96'-02'	-4.6%	10.4%
		Correlation 96'-03'	7.6%	5.5%
		Correlation 96'-04'	6.1%	9.5%

(1) As a percentage of Ultimate Loss & ALAE
(2) Duration is based on a discount rate of 2% and payments occurring mid-year

The indicated correlation between selected ultimate loss & ALAE and the payment pattern timing is not highly negative or positive based on the professional liability rate filing data[2] illustrated in Exhibit 2. Note this is based on a limited number of mature loss years of data and further research based on a longer horizon of mature data may lead to different conclusions. Also, note historical exposure/policy counts underlining the selected ultimate loss & ALAE is unknown and trending ultimate loss and ALAE for changes in exposure could lead to different correlation indications. In the author's opinion, it is likely that the duration of the payment pattern generally has a small positive correlation to ultimate loss and ALAE. As such, the sensitivity of the ERD test results will be shown under several correlation scenarios. First, let us consider a model to simulate the payment pattern timing seen in Exhibit 1.

2.3.3 Fitted Payment Pattern

Exhibit 3 shows the simulated cumulative paid loss percentages by maturity at the 10th percentile, mean and 90th percentile. These percentages are fitted to the professional liability filing loss patterns illustrated in Section 2.3.1 above. As such, these simulated outcomes reflect payment pattern variation consistent with the actual variation in historical cumulative payment patterns. The author selected the lognormal distribution with a fitted mean and coefficient of variation to produce the simulated mean payment pattern shown in Exhibit 3. To produce the simulated mean pattern shown in Exhibit 3, the author selected the lognormal distribution based on the results of Excel Solver. The lognormal distribution produced the best fit (i.e., the lowest MSE) after considering several other continuous distributions such as the beta and gamma. Further, the author allowed the mean parameter to vary uniformly between a selected min and max thereby resulting in the distribution of paid loss and ALAE patterns shown below. The selections were made using best fit and judgment.



The variability in the above simulated payment patterns is consistent with actual historical paid loss and ALAE development patterns for the long tailed lines of business that the author has observed, such as professional liability, medical malpractice, workers' compensation, mortgage insurance, etc. Note the range of 10% to 90% in Exhibit 3 above represents 80% of simulated accident year events in the reinsurance risk transfer analysis. The variability in loss timing can lead to materially different ERD test results especially after considering the combined correlation and discount rate assumptions.

2.4 Illustrative Example of ERD Results

To illustrate the potential impact of timing risk under various assumptions of payment pattern timing, correlations, and discount rates, first consider the following hypothetical captive reinsurance program and set of assumptions.

- The primary insurer cedes $260,000 in premium on January 1, 2014 to the captive reinsurer with a 30% ceding commission;

- The captive reinsurance program attaches on an aggregate excess of loss basis where primary insurer loss and ALAE for policy year 2014 above $475,000 is covered by the reinsurance policy and reinsures loss and ALAE up to a limit of $225,000. This equates to a maximum underwriting loss ratio to the reinsurer of approximately 16.5% (i.e., [($225,000 + 30% x $260,000) / ($260,000)] -1;
- Coverage is provided on an occurrence basis for policy year 2014 for professional liability;
- Direct ultimate policy-year losses of the primary insurer follow a lognormal loss distribution with an expected loss of $550,000 and a coefficient of variation of 40%;
- Based on the correlation analyses in Exhibit 2 above, a 0% correlation is assumed when modeling the correlation between the duration of simulated paid loss and ALAE and ultimate paid loss and ALAE;
- The timing of paid loss and ALAE is modeled with a lognormal distribution using a fitted mean and standard deviation; and
- A discount rate of 2% is selected based on current U.S. treasury yields. Discussion of the interest rate selection is beyond the scope of this paper.

Exhibit 4 shows ERD results under the assumptions above:

Exhibit 4
ERD Test Cash Flow and Results
(Discount: 2.0%, Simulated Payment Pattern, Correlation: 0.0%)

Cumulative Probability Distribution %	Present Value Ceded Loss	Present Value Ceding Commission	Present Value Ceded Premium	Underwriting Deficit
99%	$ 210	$ 78	$ 260	10.74%
98%	$ 209	$ 78	$ 260	10.32%
95%	$ 207	$ 78	$ 260	9.46%
90%	$ 203	$ 78	$ 260	8.13%
80%	$ 192	$ 78	$ 260	3.78%
70%	$ 132	$ 78	$ 260	0.00%
60%	$ 75	$ 78	$ 260	0.00%
50%	$ 31	$ 78	$ 260	0.00%
	Average Underwriting Deficit (ERD Ratio)			1.64%

The results above are based on 10,000 Monte Carlo simulated trials using the simulation software Oracle Crystal Ball. After considering these results let us now consider the sensitivity of the ERD ratio in assuming a fixed payment pattern (i.e., not simulating the payment pattern). As shown in Exhibit 5, the ERD ratio produced by assuming a static or fixed payment pattern decreases slightly under this reinsurance structure and modeled assumptions. Exhibit 5 also shows the results of the ERD test across various combinations of correlation and discount rate assumptions.

Exhibit 5
ERD Test Results Under Various Scenarios [1]

Correlation[2]	Discount Rate	ERD % Simulated Payment Pattern	ERD % Fixed Payment Pattern
0%	2%	1.64%	1.53%
25%	2%	1.50%	1.53%
50%	2%	1.36%	1.53%
0%	4%	0.32%	0.14%
25%	4%	0.21%	0.14%
50%	4%	0.11%	0.14%

(1) The above results illustrate how the results of the ERD test are sensitive to modeled assumptions of correlation, discount rates, and variability in payment pattern timing.
(2) Reflects correlation between simulated ultimate loss and ALAE and the average duration of the simulated payment pattern. Correlation assumption does not affect the ERD results for the fixed payment pattern.

Exhibit 5 illustrates how the ERD result is sensitive to the assumptions of payment pattern timing, correlation, and discount rate. Other reinsurance structures are likely more or less sensitive to these assumptions depending on the contractual terms, economic environment, line of business reinsured, etc. The variability in the timing of losses is affected by numerous events, including but not limited to government moratoriums, economic trends, claims practice changes, changes in TPA, changes in reserving practices, and changes in the distribution of business written. Reinsurance contractual features sensitive to the timing risk component of risk transfer such as commutation options, fixed coverage periods, and working covers should also be considered.

3. CONCLUSIONS

Modeling this variation in loss timing is important for a broad spectrum of actuarial analyses. This includes pro forma analyses, risk transfer analyses, and premium deficiency reserve analyses. When evaluating reinsurance risk transfer statistics it is important to keep in mind features that are sensitive to the variation of loss payment timing, particularly when the ERD result is near a threshold where risk transfer is questionable. In addition to payment pattern timing, discount rate and correlation are assumptions that can have a material impact on the result of the modeled ERD statistic. It is important to understand the sensitivity of those assumptions as they may change under different economic environments, reinsurance structures and lines of business reinsured. The loss variation may have a significant impact on the amount of losses ceded to a reinsurer. At the very least, the variation in timing will have an impact on the present value of losses used in the ERD test statistic, particularly with larger discount rates.

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Abbreviations and notations

ALAE, allocated loss adjustment expense
Dev, development
ERD, expected reinsurer deficit
MSE, mean squared error
Ult, ultimate

Biography of the Author

Peter J. Johnson is a consulting actuary at Bartlett Actuarial Group, Ltd. in Charleston, SC. He is responsible for reserving, pricing and risk transfer analyses. This includes work in property & casualty traditional lines of insurance including workers' compensation and medical malpractice as well as the captive marketplace. He has a degree in Applied Math & Computer Science from the University of Wisconsin - Stout. He is a Fellow of the CAS and a Member of the American Academy of Actuaries. He participates on the CAS examination committee and the Committee on Reinsurance Research.

Commutation Pricing – Cedent and Reinsurer Perspectives

Brian MacMahon, FCAS, CERA

Abstract:

A commutation is an agreement between the cedent and the reinsurer. In exchange for a onetime payout to the cedent, the commutation completely releases the reinsurer from an identified set of reserves that fall under the reinsurance contract. Reinsurers and cedents agree to commute claim obligations for a variety of reasons. Foremost on this list is reinsurer or cedent insolvency. In the case of reinsurer insolvency there is rarely a use for a pricing formula as all of the reinsurer's cedents will likely get some negotiated fraction of their outstanding obligations from the reinsurer. In other cases, including cedent insolvency, pricing formulas are useful. However, even if the pricing methodology is agreed between cedent and reinsurer, the parameters used in these formulas often vary between the reinsurer and the cedent. In some cases, this will widen the gap of acceptable prices and make it harder for an agreement to be reached. In other cases it will do the opposite.

In this paper, I consider a variety of factors that would influence how a cedent and, separately, how the reinsurer would value a commutation. Examples are given to broadly illustrate how these factors could be included in a pricing formula. At the end, there is also a short discussion on more qualitative considerations that may override pricing formulae.

1. INTRODUCTION

Once a motivation has been established that brings the cedent and the reinsurer to the table to discuss a commutation, the key factors that influence the acceptable price for each party must be valued.

For example, the cedent may consider:

1. Valuation of the reserves to be reassumed, especially the worst case scenarios. In the case of significant Bodily Injury, assumptions on increases in medical utilization and medical inflation are important.
2. Tax value based on both internal effective tax rate and value of the IRS discount unwind
3. Capital required to take back the reserves, considering both internal calculation of economic capital and rating agency required capital
4. Cost of capital
5. Value of eliminating credit risk and any Sch. F penalties
6. Value of reduction in recoverables, if overloaded on the given reinsurer
7. Internal new money investment rate compared to the risk free yield
8. Value of cash in the prevailing investment environment

9. Impact on financial statements – generally an income loss at time of transaction
10. Value of avoiding costly litigation when dispute exists over coverage with the reinsurer
11. Expense savings due to elimination of future claims and processing expenses

The reinsurer is likely to consider the flip side of most of these issues. However, there are likely to be differences in how each party interprets and values the same items. For example, the valuation of the IRS discount unwind is likely to be based on different discount factors, perhaps higher discount factors if the reinsurer relies on an excess of loss table while the cedent relies on a primary line of business table. The reinsurer may have a different tax position than the cedent and the impact of tax may be more or less significant. The reinsurer is likely to have a different and perhaps higher cost of capital than the cedent, given the relatively higher probability of ruin for a reinsurance company compared to that of a primary company, all else being equal. The reinsurer may strive to attain a high rating from the rating agencies, and thus need more capital, if this impacts their ability to be on the authorized list for the various ceding companies. If the transaction results in an income gain in the financial statement, and income already meet targets for the year, the fact that the transaction generates income may not be important. Even the magnitude of the income impact may differ between the reinsurer and the cedent if they are not carrying the same reserves. The cost of potential insolvency of the cedent to the reinsurer will have a different value (mostly based on the notion that claims will not be handled as robustly as when the cedent is solvent) than the removal of the credit risk of the reinsurer has to the cedent. The reinsurer may also have a different investment strategy than the cedent and paying cash may be more or less costly to the reinsurer.

In the remainder of the paper, I will consider several scenarios that reflect some of these differences in viewpoint and illustrate a way to price for them.

2. THE EFFECT OF TAX ON COMMUTATION VALUES

Using the formula put forth by Connor and Olsen[1], we calculate the commutation price for the cedent and separately, the reinsurer, as the ambivalence point where the cost of not commuting is equal to the cost of commuting. If the commutation price for the reinsurer is larger than the commutation price for the cedent, then the commutation is feasible.

2.1 Cedent

The cost of not commuting is equal to the tax benefit that would accrue due to unwind of IRS discount on reserves. In other words, the cedent, by transferring reserves to the reinsurer has lost the tax benefit that would exist if they had kept the reserves.

[1] Connor and Olsen, "Commutation Pricing in the Post-Tax Reform Era", CAS *Proceedings* 1991

The cost of commuting is equal to present value of the reserves taken back less the commutation payment plus the tax the cedent would pay on any tax based profit from the transaction.

Putting this into equation form:

$$Cost\ of\ Not\ Commuting = Tax\ Benefit\ on\ IRS\ Discount\ Unwind_C \quad (2.1)$$

$$Cost\ of\ Commuting = NPV(Loss) - CP_C + (CP_C - Disc_{IRSC}(Loss)) * Tax\ Rate_C \quad (2.2)$$

Setting the two equations equal and solving for the commutation price, CP_C,

$$CP_C = (NPV(Loss) - Tax\ Disc\ Unwind_C - Disc_{IRSC}(Loss) * Tax\ Rate_C)/(1 - Tax\ Rate_C) \quad (2.3)$$

CP_C = *Cedent Commutation Price*

$Disc_{IRSC}$ = *Discounted Value of Unpaid Loss, using Cedent IRS discount factors*

$Tax\ Rate_C$ = *Cedent Tax Rate*

2.2 Reinsurer

The cost of not commuting is equal to the present value of the reserves less the tax benefit from the unwind of IRS discount on reserves.

The cost of commuting is equal to the commutation payment made plus the tax paid on any tax based profit from the transaction. Note that the profit on the transaction, itself, is not included here because the profit would be realized in the future in the form of investment income on reserves.

$$Cost\ of\ Not\ Commuting = NPV(Loss) - Tax\ Benefit\ on\ IRS\ Discount\ Unwind_R \quad (2.4)$$

$$Cost\ of\ Commuting = CP_R + (Disc_{IRSR}(Loss) - CPR) * Tax\ Rate_R \quad (2.5)$$

Setting the two equations equal and solving for CP_R,

$$CP_R = (NPV(Loss) - Tax\ Disc\ Unwind_R - Disc_{IRSR}(Loss) * Tax\ Rate_R)/(1 - Tax\ Rate_R) \quad (2.6)$$

CP_R = *Reinsurer Commutation Price*

$Disc_{IRSR}$ = *Discounted Value of Unpaid Loss, using Reinsurer's IRS discount factors*

$Tax\ Rate_R$ = *Reinsurer's Tax Rate*

In other words, the two equations are equal if the loss estimations and payout patterns are the same, the discount rate used to present value the losses are the same, the IRS discount factors are the same and the effective tax rates are the same.

Under what conditions would the commutation be feasible, i.e. the payment acceptable to the cedent is less than the commutation payment the reinsurer is willing to make?

If the effective tax rate of the cedent is higher than the tax rate of the reinsurer, the cedent will get more benefit from taking back the reserves than the reinsurer will lose from giving up the reserves. Thus the cedent's acceptable price will be lower than the reinsurer's.

Likewise, if the IRS discount unwind benefit is larger for the cedent than the reinsurer, the cedent's price will be lower than the reinsurer's. This means the cedent would need to be using a longer IRS payment pattern than the reinsurer. The reverse is more likely, which will make the commutation less feasible.

The example below assumes an effective tax rate of 30% for the cedent and 25% for the reinsurer. In addition, it assumes that the cedent uses the IRS discount factors for the "Other Liability" line while the reinsurer uses the factors from the "Reinsurance Non-Proportional Assumed Liability". The tax rate difference facilitates the commutation since the cedent gets more benefit from the tax aspects of the commutation than the reinsurer. On the other hand, the longer IRS discount pattern impedes the commutation since the cedent has a smaller tax discount unwind. The tax rate differential has a slightly larger impact so the commutation is feasible.

Table 1

	Cedent	Reinsurer	Govt
Tax Rate	30.0%	25.0%	35.0%
Tax Table	Other Liab	Reinsurance	
New $ Rate	2.50%	2.5%	1.00%

Cedent Discounts at their New Money Rate of 2.5%

	(1)	(2)	(3)	(4)
Calendar Years	Unpaid	Payout Pattern	Disc Factor	Disc Loss
2014	20,000,000	35.0%	0.99	6,914,107
2015	13,000,000	25.0%	0.96	4,818,193
2016	8,000,000	20.0%	0.94	3,760,541
2017	4,000,000	15.0%	0.92	2,751,615
2018	1,000,000	5.0%	0.89	894,834
2019	-	0.0%	0.87	-
		100.0%		19,139,291

IRS Discount Factor - "Other Liability - Occurrence" 2013

(5)	(6) = (1)*(5)	(7) = (1) - (6)	(8) = $(7)_i$ - $(7)_{i+1}$	(9) = (8)*(3)
IRS Disc Factor	IRS Disc Unpaid	IRS Discount	IRS Disc Unwind	NPV(IRS Discount Unwind)
93.5%	18,702,600	1,297,400		
94.4%	12,278,383	721,617	575,783	554,847
95.0%	7,597,072	402,928	318,689	299,611
95.6%	3,822,832	177,168	225,760	207,068
96.4%	964,140	35,860	141,308	126,447
96.6%	-	-	35,860	31,306
			1,297,400	1,219,279

Cedent - Cost to Not Commute

(a) = $(4)_{Total}$	(b) = $(9)_{Total}$	(c)	(d) = (b)*(c)	(e) = (d)
NPV Loss	NPV Tax Disc Unwind	Tax Rate	Tax Hit on Unwind of Discount	Cost to Not Commute
19,139,291	1,219,279	30.0%	365,784	365,784

Cedent - Cost to Commute

(f)=(a)	(g) = [(f) - (d) - $(6)_{2014}$ * (c)]/[1-(c)]	(h) = (g) - $(6)_{2014}$	(i) = (c)*(h)	(j) = (f) - (g) + (i)
NPV Loss	Commutation Payment	Profit	Tax on Profit	Cost to Commute
19,139,291	**18,803,896**	101,296	30,389	365,784

Reinsurer Discounts at their New Money Rate of 2.5%

	(1)	(2)	(3)	(4) = [$(1)_i$ - $(1)_{i+1}$]*(3)
Calendar Years	Unpaid	Remaining Payout Pattern	Disc Factor	Disc Loss
2014	20,000,000	35.0%	0.99	6,914,107
2015	13,000,000	25.0%	0.96	4,818,193
2016	8,000,000	20.0%	0.94	3,760,541
2017	4,000,000	15.0%	0.92	2,751,615
2018	1,000,000	5.0%	0.89	894,834
2019	-	0.0%	0.87	-
		100.0%		19,139,291

IRS Discount Factor - "Reinsurance Non-Proportional Assumed Liability" 2013

(5)	(6) = (1)*(5)	(7) = (1) - (6)	(8) = $(7)_i$ - $(7)_{i+1}$	(9) = (8)*(3)
IRS Disc Factor	IRS Disc Unpaid	IRS Discount	IRS Disc Unwind	NPV(IRS Discount Unwind)
92.2%	18,435,480	1,564,520		
88.0%	11,433,695	1,566,305	(1,785)	(1,720)
89.0%	7,118,872	881,128	685,177	644,159
93.6%	3,743,648	256,352	624,776	573,048
96.0%	959,635	40,365	215,987	193,273
95.3%	-	-	40,365	35,239
			1,564,520	1,443,998

Reinsurer - Cost to Not Commute

(a) = $(4)_{Total}$	(b) = $(9)_{Total}$	(c)	(d) = (b)*(c)	(e) = (a) - (d)
NPV Loss	NPV Tax Disc Unwind	Tax Rate	Tax Benefit on Unwind Disc	Cost to Not Commute
19,139,291	1,443,998	25.0%	361,000	18,778,291

Reinsurer - Cost to Commute

(f) = [(a) - (d) - $(6)_{2014}$ * (c)]/[1-(c)]	(g) = (6), 2014 row	(h) = (g) - (f)	(i) = (c)*(h)	(j) = (f) + (i)
Commutation Payment	IRS Reserves Taken Down	Profit	Tax on Profit	Cost to Commute
18,892,562	18,435,480	(457,082)	(114,270)	18,778,291

CP = (NPV(Loss) - Tax Disc Unwind - IRS Disc (Loss)*Tax Rate)/(1-Tax Rate)

CP_C = (19,139,291 - 365,784 - 18,702,600*30%)/(1-30%) = **18,803,896**

CP_R = (19,139,291 - 361,000 - 18,435,480*25%)/(1-25%) = **18,892,562**

3. INCORPORATING RISK LOAD

We are going to start with the same commutation price formula as above, except now we are going to factor in the cost of capital. Capital is required to support negative variation in reserve outcomes. When the reinsurer commutes reserves, capital supporting the reserves can be taken down, which is a benefit if you assume that there is an immediate business use for the capital. Conversely, the cedent in taking back the reserves has to put capital up, which could otherwise be used to generate profits. This is a cost to the cedent, assuming that the cedent does not have unused capital with no immediate use.

The relationship between this benefit to the reinsurer and the cost to the cedent depends on how much capital is employed and the relative costs of capital. The combination of capital and cost of capital can be viewed as a risk load. If the risk load required by the cedent is less than the risk load released by the reinsurer, then the transaction will be facilitated.

Note that risk load is an internal computation that does not impact the income statement, so there are no tax consequences.

We can use the equations from above.

3.1 Cedent

The cost of not commuting remains the same as above, no risk load is needed.

The cost of commuting has the additional cost of the risk load.

$$CP_C = (NPV(Loss) - Tax\ Disc\ Unwind_C - Disc_{IRSC}(Loss) * Tax\ Rate_C + RL_C)/(1 - Tax\ Rate_C)$$

$$RL_C = Cedent\ Risk\ Load \tag{3.1}$$

3.2 Reinsurer

The cost of commuting remains the same, no risk load is needed.

The cost of not commuting has the additional cost of the risk load.

$$CP_R = (NPV(Loss) - Tax\ Disc\ Unwind_R - Disc_{IRSR}(Loss) * Tax\ Rate_R + RL_R)/(1 - Tax\ Rate_R)$$

$$RL_R = Reinsurer's\ Risk\ Load \tag{3.2}$$

Now let's look at how the risk load for the cedent could compare to the risk load for the reinsurer. As mentioned above, there are two components to a risk load – the required capital and the target return on capital. The concepts discussed in this section will be applicable to any capital approach. There are many ways to set required capital. For example, it could be set using an economic approach such as Value at Risk (VaR) or Tail Value at Risk (TVaR); based on regulatory required capital or rating agencies requirements for a given rating; or on a more simplified formula like reserves/capital. For the purposes of illustration, we are going to use an economic approach based on the 99th percentile VaR. Nowadays, most reserving software will produce reserve distributions that can be used to estimate this. Let's assume that both the reinsurer and the cedent set capital based on the profit/loss at the 99th worst income result at the time of commutation and run off this capital as the 99th worst income diminishes with loss payments. Let's assume that the runoff of the capital can be expressed as a factor against the initial 99th worst income. Appendix A shows the derivation of this runoff factor. The risk load required is then equal to the cost of capital (think of this as "target ROE"), reduced for the after-tax investment income rate earned on the capital, multiplied by the 99th worst income multiplied by the Factor for Runoff of capital.

$$RL_C = [(Target\ ROE_C - Investment\ Rate\ after\ Tax)]*Income_{99thC}*F_{RunoffC} \quad (3.3)$$

$$RL_R = [(Target\ ROE_R - Investment\ Rate\ after\ Tax)]*Income_{99thR}*F_{RunoffR} \quad (3.4)$$

RL = Risk Load

F = Factor for Runoff of Capital

Target ROE = Companies cost of capital

As before, subscripts C and R stand for Cedent and Reinsurer, respectively.

It is unlikely that these two risk loads will be equal. For example, the reinsurance business, being inherently more volatile than primary insurance, will generally require a higher cost of capital. The view of the reinsurer on the 99th worst outcome could be better or worse than the cedent and their view on how uncertainty diminishes over the lifetime of the reserve payments will also differ. For example, information on the claims making up the reserves is asymmetric. The cedent has more information on the reserves than the reinsurer. Generally, more information will allow for less parameter risk in estimating the aggregate loss distribution. This means the 99th worst outcome will be lower for the cedent, all else being equal. With a higher cost of capital and a worse 99th income, both of which imply a higher risk load for the reinsurer, the transaction will be facilitated.

Let's now introduce a diversity factor. If the relationship between the 99th percentile of the commutation reserves and the 99th percentile of the rest of the company's reserves do not move in full unison (for example as measured by correlation or a tail copula value), there will be a reduction in the required risk load for the commutation reserves. The diversity factor is the marginal impact of the 99th worst outcome for the commutation reserves on the 99th worst outcome of the company's other reserves. For example, in property catastrophe, if the set of commutation reserves comes from one cat zone, the diversity will be more significant if the insurer has cat reserves spread over many cat zones. If all their reserves come from one cat zone (e.g. a mono-state writer), then there will be little to no diversity. For casualty, the same impact would result if the commutation reserves came from one line of business and the insurer book is spread over many lines of business.

It may be that a large multi-line primary insurer has more diversity than a reinsurer focused on higher layers. This would also facilitate the commutation since the risk load released by the reinsurer would be larger than the risk load required by the cedent. Of course, if the cedent is a less diversified insurer, such as a small mono-line writer, the opposite may be true.

In the example below the reinsurer's cost of capital is assumed to be 15%, while the cedent's is 10%. In addition, the cedent is assumed to have a diversity factor of 50%, i.e. that only half the capital that the transaction requires on a stand-alone basis is needed when it is considered part of the cedent's whole reserve portfolio. The reinsurer's diversity factor is assumed to be 75%. "Capital" as used here refers to the economic capital at the 99^{th} income percentile, as above, not capital required by a rating agency.

The factor for the runoff of capital is based on the simplified notion that the risk in the reserves diminishes in accordance with the reduction in outstanding losses. Thus, it is fully determined by the payout pattern and the discount rate. The calculation of this is shown in Appendix A. A more sophisticated reserve variability model could show that risk falls off faster than outstanding reserves.

Note, in the calculation of the risk load below, that a "premium" is calculated in order to determine the capital required, since the required capital considers the downside "profit" not just the downside losses.

One way to think of this is that when the reinsurance contract incepted, the reinsurer received funds from the cedent to pay for the risk assumed. The capital put up by the reinsurer would be based on the 99^{th} worst outcome considering both inflows and outflows. Inflows would be initial premium and the expected value of reinstatement or sliding scale premiums (if any). Outflows would be expected losses, ceding commissions (if any), reinsurer expenses, brokerage (if involving a broker market reinsurer), and any loss sensitive profit features like a profit commission, no claims bonus, sliding scale cede, etc., although the profit features are not likely to apply in the worst outcome scenarios used to set the capital.

What I'm doing in the example below is inferring the premium needed to hit the ROE target. This is purely notional because it has nothing to do with the commutation payment, other than its use in determining the risk load. One could think of it as the original target ROE premium reduced for the proportional reduction in risk since the contract incepted. The derivation of the premium is shown in Appendix B. Note that line (6), "NPV Profit (after Tax)" is the risk load.

Calculation of Risk Load:

Table 2

	Cedent	Reinsurer	Govt
Tax Rate	30.0%	25.0%	35.0%
Tax Table	Other Liab	Reinsurance	
Discount Rate	2.50%	2.50%	1.00%

		Cedent	Reinsurer
(1)	Premium	22,372,729	25,190,017
(2)	Expected Loss	20,000,000	20,000,000
(3)	Discounted Loss	19,139,291	19,139,291
(4) = 1-3	NPV Profit (before Tax)	3,233,438	6,050,726
Tax	Tax Rate	30.0%	25.0%
(6) = 4*(1-Tax)	NPV Profit (after Tax)	**2,263,407**	**4,538,044**
(7)= Disc Rate*(1-Tax)	Passive Return	1.8%	1.9%
(8) = 15	Capital	27,435,234	34,575,576
(9) = 7 + 6/8	ROE	10.0%	15.0%

Cost of Capital

	Risk Free	Risk Margin	Total
Reinsurer	1.0%	14.0%	15.0%
Cedent	1.0%	9.0%	10.0%

Capital Calculation

		Cedent	Reinsurer
(10) Agg Loss Curve	99th Downside Loss (Disc)	40,000,000	40,000,000
(11) = 1-10	99th Downside NPV Profit	(17,627,271)	(14,809,983)
(12) Selected	Diversity Factor	0.50	0.75
(13)=11*12*-1	First Year Capital	8,813,635	11,107,487
(14)=Sum NPV(O/S)	Runoff Multiplier	3.11	3.11
(15) = 13*14	All Years Capital	27,435,234	34,575,576

Calculation of Commutation Price:

Table 3

Cedent

Cost to Not Commute

(1) NPV Tax Disc Unwind	(2) Tax Rate	(3) Tax Hit on Unwind Disc			(4) = 3 Cost to Not Commute
1,219,279	30.0%	365,784			365,784

Cost to Commute

(1) NPV Loss	(2) Risk Load	(3) CP	(4) Profit	(5) Tax on Profit	(6) = 1 + 2 + 5 - 3 Cost to Commute
19,139,291	2,263,407	**22,037,334**	3,334,734	1,000,420	365,784

Reinsurer

Cost to Not Commute

(1) NPV Loss	(2) NPV Tax Disc Unwind	(3) Tax Rate	(4) Tax Benefit on Unwind Disc	(5) Risk Load	(6) = 1 + 5 - 4 Cost to Not Commute
19,139,291	1,443,998	25.0%	361,000	4,538,044	23,316,336

Cost to Commute

(1) CP	(2) Reserves Taken Down	(3) Profit on Transaction	(4) Tax on Transaction		(6) = 1 + 4 Cost to Commute
24,943,288	18,435,480	(6,507,808)	(1,626,952)		23,316,336

CP = (NPV(Loss) - Tax Disc Unwind - IRS Disc (Loss)*Tax Rate + RL)/(1-Tax Rate)

CP_C = (19,139,291 - 365,784 - 18,702,600*30% + 2,263,407)/(1-30%) = **22,037,334**

CP_R = (19,139,291 - 361,000 - 18,435,480*25% + 4,538,044)/(1-25%) = **24,943,288**

As you can see the commutation is facilitated by the fact that the risk load released by the reinsurer is larger than the risk load put up by the cedent.

Required capital could also be driven by a rating agency's required capital level to maintain a given rating. If you assume that primary insureds are less sophisticated than insurers, it would follow that a higher rating would be more valuable to a reinsurer than an insurer. It is also likely that a rating agency will require a reinsurer to hold more capital for a given level of reserves for a given rating level than an insurer. Both of these factors would increase the reinsurer's risk load relative to the cedent and facilitate the commutation.

Generally, a cedent or a reinsurer that is concerned with financial ratings may base their required capital, for pricing purposes, on the capital required by the rating agency to achieve their desired rating. One would think that this is likely to exceed the economic capital required, since rating agencies should build in a margin of error.

4. INCORPORATING REINSURER CREDIT RISK AND SCHEDULE F PENALTIES

Unlike the previous costs and benefits discussed above, we will assume that reinsurer credit risk will impact only the cedent's commutation ambivalence point. It could be argued that there is a possible benefit to the reinsurer of commuting reserves, if it has an impact on their rating agency credit rating. A commutation would generally have to be very large and the credit rating unstable for this to have any impact. We'll ignore this potential benefit to the reinsurer.

In the required capital formula for one of the larger rating agencies, there is a fixed factor applied to reinsurance recoverables of 10%. This factor can vary considerably based on the rating of the reinsurer, the dependence of the cedent upon reinsurance (leverage of recoverables plus ceded premiums to surplus), and the concentration of recoverables with the given reinsurer. It could be as low as 2% and as high as 100%. The charge for credit risk requires the cedent to put up capital to support the ceded reserves. The cost can be viewed as a risk load based on the cedent's cost of capital. Unlike the cedent risk load discussed above, which only arises when the reserves are commuted, this risk load exists when the reserves are not commuted and disappears when they are.

Commuting will eliminate this risk load which will lower the required commutation price for the cedent.

$$CP_C = (NPV(Loss) - Tax\ Unwind_C - Disc_{IRSC}(Loss) * Tax\ Rate_C + RL_{Economic} - RL_{Credit}) / (1\text{-}Tax\ Rate_C) \quad (4.1)$$

Note that this charge is an internal cost to the cedent and does not enter into the income statement, so there is no tax impact.

An alternative approach to assessing an economic impact of reinsurance credit risk, as opposed to the rating agency charge, is to use transition matrices, such as those calculated by S&P. These matrices will give the probability of default over a specified time horizon for a given starting rating value. These can then be extended as far as desired into the future and a cumulative default rate determined. To that default rate, an assumed percentage of recovery has to be applied to get a total loss amount. Note that this calculation does not reflect the impact of a slowdown in reinsurance payments such as that reflected in Schedule F.

Schedule F penalties are generally a charge against surplus. This means that the ceded reserves are assigned additional capital. So another risk load is required to support the ceded reserves. The impact is exactly identical in form to the impact of credit risk. Commutation will eliminate this cost, and the cedent will accept a lower commutation price to eliminate this risk load.

$$CP_C = (NPV(Loss) - Tax\ Unwind_C - Disc_{IRSC}(Loss) * Tax\ Rate_C + RL_{Economic} - RL_{Credit} - RL_{SchF}) / (1\text{-}Tax\ Rate_C) \quad (4.2)$$

Schedule F penalties (or equivalents for GAAP accounting) apply to financial statement capital. Presumably, rating agencies will have factored in slowdown in payments or other drivers of Schedule F penalties into the factor they select above, which means that it would be double counting to include risk loads for both components.

In the example below, both loads are included under the assumption that "credit risk" refers to the default of the reinsurer, while Schedule F penalties applies to the slowdown in payments that can occur independent of default.

The credit risk charge is based upon the assumption that the cedent has a reinsurance leverage of 100% (current recoverables plus ceded premiums is equal to surplus) and the charge is 45%. To simplify the calculation, we assume that the charge is then the capital needed to support the transaction *45%, i.e. we assume no extra diversity reduction. The risk load associated with it is 10% of capital, i.e. an ROE of 10%. The Schedule F penalty is based on the reinsurer being classified as a slow payer. The capital in this case is the reinsurance recoverable of $20m * Sch. F Penalty of 20%.

Calculation of Risk Load:

Table 4

	Cedent	Reinsurer	Govt
Tax Rate	30.0%	25.0%	35.0%
Tax Table	Other Liab	Reinsurance	
New $ Rate	2.50%	2.50%	1.00%
Reinsurance Leverage	100.0%	N/A	
Credit Charge (Rating Agency)	45%		
Schedule F Penalty	20%		

		Cedent	Reinsurer
(1)	Premium	22,372,729	25,190,017
(2)	Expected Loss	20,000,000	20,000,000
(3)	Discounted Loss	19,139,291	19,139,291
(4) = 1-3	NPV Profit (before Tax)	3,233,438	6,050,726
Tax	Tax Rate	30.0%	25.0%
(6) = 4*(1-Tax)	NPV Profit (after Tax)	2,263,407	4,538,044
(7)= Disc Rate*(1-Tax)	Passive Return	1.8%	1.9%
(8) = 17	Capital	27,435,234	34,575,576
(9) = 7 + 6/8	ROE	**10.0%**	**15.0%**

Cost of Capital

	Risk Free	Risk Margin	Total
Reinsurer	5.0%	10.0%	15.0%
Cedent	5.0%	5.0%	10.0%

Capital Calculation

		Cedent	Reinsurer
(10) Agg Loss Curve	99th Downside Loss (Disc)	40,000,000	40,000,000
(11) = 1-10	99th Downside NPV Profit	(17,627,271)	(14,809,983)
(12) Selected	Diversity Factor	0.50	0.75
(13)=11*12*-1	First Year Capital	8,813,635	11,107,487
(14)=Sum NPV(O/S)	Years Held Multiplier	3.11	3.11
(15) = 13*14*Reins Lev*Credit Chg	Credit Risk Capital	12,345,855	-
(16) = 2*Sch F Penalty	Sch F Capital	4,000,000	-
(17) = 13*14	Economic Risk Load Capital	27,435,234	34,575,576

Calculation of Commutation Price:

Table 5

Cedent

Cost to Not Commute

NPV Tax Disc Unwind	Tax Rate	Tax Hit on Unwind Disc	Credit Risk Load	Sch F Risk Load	Cost to Not Commute
1,219,279	30.0%	365,784	1,234,586	400,000	2,000,369

Cost to Commute

NPV Loss	Economic Risk Load	Commutation Payment	Profit on Transaction	Tax on Profit	Cost to Commute
19,139,291	2,263,407	**19,702,212**	999,612	299,884	2,000,369

Reinsurer

Cost to Not Commute

NPV Loss	NPV Tax Disc Unwind	Tax Rate	Tax Benefit on Unwind Disc	Risk Load	Cost to Not Commute
19,139,291	1,443,998	25.0%	361,000	4,538,044	23,316,336

Cost to Commute

Commutation Payment	Reserves Taken Down	Profit on Transaction	Tax on Transaction		Cost to Commute
24,943,288	18,435,480	(6,507,808)	(1,626,952)		23,316,336

CP_C = (NPV(Loss) - Tax Disc Unwind - IRS Disc (Loss)*Tax Rate + RL - Risk Load$_{Credit}$ - Risk Load$_{Sch\ F}$)/(1-Tax Rate)

CP_C = (19,139,291 - 365,784 - 18,702,600*30% + 2,263,407 - 1,234,586 - 400,000)/(1-30%) = **19,702,212**

CP_R = (19,139,291 - 361,000 - 18,435,480*25% + 4,538,044)/(1-25%) = **24,943,288**

Note that the commutation price is unchanged for the reinsurer.

5. INCORPORATING FUNDING OF COMMUTATION PAYMENTS

This addresses the issue of the cost to the reinsurer of converting reserves into cash for a commutation payment and the cost to the insurer of investing cash to pay for the future payments on the commuted reserves.

In a rising interest rate environment, relative to when the reserves were funded by the reinsurer, if the reinsurer is matching assets with liabilities and has to liquidate assets to fund the commutation, it is likely to realize capital losses in the process. This will increase the cost to it and lower its acceptable commutation price. The opposite is true in a falling interest rate environment.

In a very low and flat yield curve environment such as that prevailing in 2014, there is likely to be much more investment in short-term instruments since liquidity is valuable and there is less to be gained from investing long. In this case, the reinsurer is likely to have sufficient cash or short-term instruments on hand to fund the commutation payment. Thus, there may not be the need to realize capital losses to fund the commutation.

On the other hand, the cedent will often invest new cash in a variety of instruments that maximize the overall portfolio return without slavish regard to the exact matching of assets and liabilities. In this case, the new money rate of the cedent is the defacto maximum discount rate the cedent will use to value the commutation reserves, even if long term assets that match the liability duration have a higher return.

The literature often advocates the use of risk-free discount rates under the assumption that any higher discount rate involves risk for the cedent and should be separated from the pure commutation value (or another risk load added). In practice, the risk free rate is determined by the lowest risk available investments. This is often considered to be the rates offered by US Government securities. However, there remains a risk of default, even of the US Government, so the theoretical risk-free rate should be even lower.

At the present time, the rates on US Government securities have been maintained per monetary policy at extremely low rates. These rates would probably not be feasible to use in valuing a commutation. For example, if the cedent invested in US Government bonds, the current yield curve for the duration of reserves in our example would imply a yield of 0.43%. The examples above have assumed a 2.5% new money rate. If the 0.43% rate was used, how would the cedent's commutation price change? First, the NPV(Loss) would increase. Second the discounted 99th worst loss outcome would increase, which would increase the capital required. The income would go down because the discounted loss is higher and the passive return on capital would decrease. The rating agency credit charge, since it is applied against required capital (before diversity) would also increase, which increases the credit risk load. This would be slightly offset by an increase in the present value of the IRS discount unwind. The net effect would drive the cedent to a higher risk load and a higher required commutation price.

NPV(Loss) would increase from $19.1m to $19.8m

99th worst loss outcome increases from $40.0m to $41.5m

NPV(IRS Discount Unwind) from increase from $365k to $385k

Years Held Multiplier would increase from 3.11 to 3.27

Economic risk load would increase from \$2.3m to \$2.8m

Credit risk load would increase from \$1.2m to \$1.3m

Commutation price would increase from \$19.7m to \$21.4m

Commutation Price at 2.5% new money rate

CP_C = (19,139,291 - 365,784 - 18,702,600*30% + 2,263,407 - 1,234,586 - 400,000)/(1-30%) = **19,702,212**

Commutation Price at 0.43% Government Rate

CP_C = (19,846,402 - 385,021 - 18,702,600*30% + 2,794,042 - 1,296,339 - 400,000)/(1-30%) = **21,354,721**

This calculation assumes that the cedent will match the pure loss component of the commutation proceeds with the liability duration, i.e. that the current risk-free rate will be used to price the commutation. In this historically low interest rate environment, various other strategies may be employed to maximize the return on the commutation proceeds. For example, the cedent could hold all the proceeds in short-term investments and reinvest to match the remaining loss duration if and when the interest rates turn upwards. An example illustrating this approach is given in Appendix D.

If there is a cost to liquidating investments to fund the commutation, the Commutation Price formula for the reinsurer includes an additional term:

$$Cost\ of\ Commuting = CP_R + (Disc_{IRSR}(Loss) - CP_R) * Tax_{\ RateOrdinary\ Income} + Realized\ Capital\ Losses * (1 - Tax\ Rate_{Capital\ Gains}) \quad (5.1)$$

The realized investment losses are netted for capital gains tax. The commutation payment formula then includes an additional term for the realized capital loss. It is a negative term because the larger the capital loss, the lower the acceptable commutation price for the reinsurer.

$$CP_R = (NPV(Loss) - Tax\ Disc\ Unwind_R - Disc_{IRSR}(Loss) * Tax_{\ RateOrdinary\ Income} + RL_R - Realized\ Capital\ Losses * (1 - Tax\ Rate_{Capital\ Gains}))/(1 - Tax\ Rate_R) \quad (5.2)$$

Calculation of Capital Loss on Commutation:

Table 7

		Strips At Time of Purchase			Strips At Time of Commutation		
Calendar			Yield			Yield	
Year	Payments	Maturity	Curve	Price	Maturity	Curve	Price
2014	7,000,000	1	1.0%	6,930,693	1	3.00%	6,796,117
2015	5,000,000	2	1.3%	4,877,305	2	3.25%	4,690,184
2016	4,000,000	3	1.5%	3,825,268	3	3.50%	3,607,771
2017	3,000,000	4	1.8%	2,798,876	4	3.75%	2,589,219
2018	1,000,000	5	2.0%	905,731	5	4.00%	821,927
	20,000,000			19,337,873			18,505,218
	Capital Loss at Time of Commutati			**832,655**			

CP_R = (19,139,291 - 361,000 - 18,435,480*25% + 4,538,044 - **832,655*(1-20%)**)/(1-25%) = **24,055,123**

This is a decrease of $881k.

6. MEDICAL COST, MEDICAL UTILIZATION AND TORT LIABILITY TRENDS

So far, we have avoided any discussion of disagreement between the insurer and the reinsurer in the size of the ultimate reserves. For reserves that are likely to be impacted by future medical inflation, medical utilization or tort liability award trends, the cedent is likely to have a more conservative view than the reinsurer. Complicating the situation is that some cedents may not explicitly include the cost of medical inflation or medical utilization into their reserve estimates. The reinsurer may take the reserves presented by the cedent at face value without knowing whether or not such future costs are built in. However, the cedent will include this cost when negotiating the commutation potentially creating a large gap between the reserves held by the reinsurer and the ultimate values estimated by the cedent.

Workers Compensation claims involving permanent total injuries with an expectation of lifetime medical payments often have these characteristics. In order to calculate a discounted reserve value for a single such claim, the following information is required:

- Information about weekly indemnity payments, COLAs associated with them and any time limit on indemnity,
- Ongoing medical costs, anticipated future surgeries, medication costs, home care, etc. and the inflation associated with these costs
- Estimated life expectancy and the appropriate life table

Differences in inflation assumptions can have a huge impact, especially on excess of loss layers. The following example illustrates the impact of assuming a 3% medical inflation (near-term medical CPI) vs. a 6% inflation (longer-term medical CPI). In practice, different inflation assumptions would be made for each medical cost component, such as medications (including brand label becoming generic), anticipated advances in medical devices/surgeries which may have a very high initial cost, inflation in home health care, cost of prosthetics, end of life spike in costs, etc.

In the example below, under both scenarios, the full $5m limit is exhausted. However, the present value of the reserves, when the medical inflation is 6%, is $1.5m compared to $1.1m when the medical inflation is 3%.

In this case, distributions around the key cost parameters should be employed in order to arrive at a fair expected value. Improvements in life expectancy, not always captured in the latest available life table, would be one such key parameter.

Many of these considerations also play a role in serious automobile claims, product liability claims, latent injury claims, etc. that have the possibility of catastrophic bodily injury.

Table 8

Example of Medical Inflation Rate on Commutation Value

Parameters		
Date of Loss	1/1/2008	
Evaluation Date:	12/31/2014	
Rated Age:	65	
Gender	M	
Est'd Annual Indem. Pmt:	$ 20,000.00	Per State Formula
Est'd Annual Med. Pmt:	$ 50,000.00	Estimated by Cedent
Cost of Living Adjustment:	2.00%	Specified by State as 20 year COLA
Est'd Medical Cost Infl'n:	**6.00%**	
Indemnity Paid to Date:	200,000	
Medical Paid to Date:	500,000	
Reins. Attachment Point:	1,000,000	
Reins. Limit:	5,000,000	
Discount Rate:	2.50%	
100% Expected Layer Pmt, Discounted	1,508,781	

Cal Yr.	Incremental Indemnity Payment	Incremental Medical Payment	Total Payment	Cumulative Payment	Excess of Attachment	Probability of Surviving to the Pmt Yr	2.5% Discount Factor	Expected Disc't Pmt
2014	200,000	500,000	700,000	700,000	0		1.00	
2015	20,199	51,478	71,677	771,677	0	100.0%	0.99	-
2016	20,603	54,567	75,170	846,847	0	98.6%	0.96	-
2017	21,015	57,841	78,856	925,703	0	97.0%	0.94	-
2018	21,435	61,311	82,747	**1,008,450**	8,450	95.3%	0.92	7,389
2019	21,864	64,990	86,854	**1,095,304**	95,304	93.6%	0.89	72,725
2020	22,301	68,889	91,191	**1,186,494**	186,494	91.7%	0.87	72,980
2021	22,747	73,023	95,770	**1,282,264**	282,264	89.6%	0.85	73,122
2022	23,202	77,404	100,606	**1,382,871**	382,871	87.5%	0.83	73,141
2023	23,666	82,048	105,715	**1,488,586**	488,586	85.2%	0.81	73,015
2024	24,140	86,971	111,111	**1,599,696**	599,696	82.7%	0.79	72,718
2025	24,622	92,190	116,812	**1,716,508**	716,508	80.1%	0.77	72,230
2026	25,115	97,721	122,836	**1,839,344**	839,344	77.4%	0.75	71,551
2027	25,617	103,584	129,201	**1,968,546**	968,546	74.5%	0.73	70,679
2028	26,130	109,799	135,929	**2,104,474**	1,104,474	71.4%	0.72	69,589
2029	26,652	116,387	143,039	**2,247,514**	1,247,514	68.3%	0.70	68,245
2030	27,185	123,370	150,556	**2,398,069**	1,398,069	64.9%	0.68	66,618
2031	27,729	130,773	158,502	**2,556,571**	1,556,571	61.3%	0.67	64,696
2032	28,283	138,619	166,902	**2,723,473**	1,723,473	57.7%	0.65	62,491
2033	28,849	146,936	175,785	**2,899,259**	1,899,259	53.9%	0.63	60,010
2034	29,426	155,752	185,178	**3,084,437**	2,084,437	50.0%	0.62	57,259
2035	30,015	165,097	195,112	**3,279,549**	2,279,549	46.1%	0.60	54,237
2036	30,615	175,003	205,618	**3,485,167**	2,485,167	42.2%	0.59	50,974
2037	31,227	185,503	216,731	**3,701,898**	2,701,898	38.2%	0.57	47,496
2038	31,852	196,634	228,485	**3,930,383**	2,930,383	34.3%	0.56	43,842
2039	32,489	208,432	240,921	**4,171,304**	3,171,304	30.4%	0.55	40,055
2040	33,139	220,938	254,076	**4,425,380**	3,425,380	26.7%	0.53	36,187
2041	33,801	234,194	267,995	**4,693,375**	3,693,375	23.2%	0.52	32,295
2042	34,477	248,245	282,723	**4,976,098**	3,976,098	19.8%	0.51	28,442
2043	35,167	263,140	298,307	**5,274,405**	4,274,405	16.7%	0.49	24,690
2044	35,870	278,929	314,799	**5,589,204**	4,589,204	13.9%	0.48	21,100
2045	36,588	295,664	332,252	**5,921,456**	4,921,456	11.3%	0.47	17,730
2046	37,319	313,404	350,724	6,272,180	5,000,000	9.1%	0.46	3,276
2047	38,066	332,208	370,274	6,642,454	5,000,000	7.1%	0.45	-
2048	38,827	352,141	390,968	7,033,422	5,000,000	5.5%	0.44	-
2049	39,604	373,269	412,873	7,446,295	5,000,000	4.1%	0.43	-
2050	40,396	395,666	436,061	7,882,357	5,000,000	3.0%	0.42	-
2051	41,204	419,405	460,609	8,342,966	5,000,000	2.0%	0.41	-
2052	42,028	444,570	486,598	8,829,563	5,000,000	1.1%	0.40	-
2053	42,868	471,244	514,112	9,343,676	5,000,000	0.4%	0.39	-
2054	43,726	499,519	543,244	9,886,920	5,000,000	0.0%	0.38	-
2055	44,600	529,490	574,090	10,461,010	5,000,000	0.0%	0.37	-
2056	45,492	561,259	606,751	11,067,761	5,000,000	0.0%	0.36	-
Total								1,508,781

Table 9

Example of Medical Inflation Rate on Commutation Value

Parameters		
Date of Loss	1/1/2008	
Evaluation Date:	12/31/2014	
Rated Age:	65	
Gender	M	
Est'd Annual Indem. Pmt:	$ 20,000.00	Per State Formula
Est'd Annual Med. Pmt:	$ 50,000.00	Estimated by Cedent
Cost of Living Adjustment:	2.00%	Specified by State as 20 year COLA
Est'd Medical Cost Infl'n:	**3.00%**	
Indemnity Paid to Date:	200,000	
Medical Paid to Date:	500,000	
Reins. Attachment Point:	1,000,000	
Reins. Limit:	5,000,000	
Discount Rate:	2.50%	
100% Expected Layer Pmt, Discounted	1,098,766	

Cal Yr.	Incremental Indemnity Payment	Incremental Medical Payment	Total Payment	Cumulative Payment	Excess of Attachment	Probability of Surviving to the Pmt Yr	2.5% Discount Factor	Expected Disc't Pmt
2014	200,000	500,000	700,000	700,000	0		1.00	
2015	20,199	50,744	70,943	770,943	0	100.0%	0.99	-
2016	20,603	52,267	72,870	843,813	0	98.6%	0.96	-
2017	21,015	53,835	74,850	918,663	0	97.0%	0.94	-
2018	21,435	55,450	76,885	995,548	0	95.3%	0.92	-
2019	21,864	57,113	78,977	**1,074,526**	74,526	93.6%	0.89	62,402
2020	22,301	58,827	81,128	**1,155,654**	155,654	91.7%	0.87	64,927
2021	22,747	60,592	83,339	**1,238,993**	238,993	89.6%	0.85	63,631
2022	23,202	62,409	85,612	**1,324,604**	324,604	87.5%	0.83	62,240
2023	23,666	64,282	87,948	**1,412,552**	412,552	85.2%	0.81	60,743
2024	24,140	66,210	90,350	**1,502,902**	502,902	82.7%	0.79	59,130
2025	24,622	68,196	92,819	**1,595,721**	595,721	80.1%	0.77	57,394
2026	25,115	70,242	95,357	**1,691,078**	691,078	77.4%	0.75	55,545
2027	25,617	72,349	97,967	**1,789,044**	789,044	74.5%	0.73	53,592
2028	26,130	74,520	100,650	**1,889,694**	889,694	71.4%	0.72	51,528
2029	26,652	76,756	103,408	**1,993,102**	993,102	68.3%	0.70	49,337
2030	27,185	79,058	106,243	**2,099,345**	1,099,345	64.9%	0.68	47,011
2031	27,729	81,430	109,159	**2,208,504**	1,208,504	61.3%	0.67	44,556
2032	28,283	83,873	112,156	**2,320,660**	1,320,660	57.7%	0.65	41,993
2033	28,849	86,389	115,238	**2,435,899**	1,435,899	53.9%	0.63	39,340
2034	29,426	88,981	118,407	**2,554,305**	1,554,305	50.0%	0.62	36,612
2035	30,015	91,650	121,665	**2,675,970**	1,675,970	46.1%	0.60	33,820
2036	30,615	94,400	125,015	**2,800,985**	1,800,985	42.2%	0.59	30,992
2037	31,227	97,232	128,459	**2,929,444**	1,929,444	38.2%	0.57	28,152
2038	31,852	100,149	132,000	**3,061,444**	2,061,444	34.3%	0.56	25,328
2039	32,489	103,153	135,642	**3,197,086**	2,197,086	30.4%	0.55	22,551
2040	33,139	106,248	139,386	**3,336,472**	2,336,472	26.7%	0.53	19,852
2041	33,801	109,435	143,236	**3,479,709**	2,479,709	23.2%	0.52	17,261
2042	34,477	112,718	147,196	**3,626,904**	2,626,904	19.8%	0.51	14,808
2043	35,167	116,100	151,267	**3,778,171**	2,778,171	16.7%	0.49	12,520
2044	35,870	119,583	155,453	**3,933,624**	2,933,624	13.9%	0.48	10,420
2045	36,588	123,170	159,758	**4,093,382**	3,093,382	11.3%	0.47	8,525
2046	37,319	126,865	164,185	**4,257,566**	3,257,566	9.1%	0.46	6,848
2047	38,066	130,671	168,737	**4,426,303**	3,426,303	7.1%	0.45	5,392
2048	38,827	134,591	173,418	**4,599,722**	3,599,722	5.5%	0.44	4,157
2049	39,604	138,629	178,233	**4,777,955**	3,777,955	4.1%	0.43	3,132
2050	40,396	142,788	183,184	**4,961,138**	3,961,138	3.0%	0.42	2,302
2051	41,204	147,072	188,275	**5,149,413**	4,149,413	2.0%	0.41	1,544
2052	42,028	151,484	193,511	**5,342,925**	4,342,925	1.1%	0.40	859
2053	42,868	156,028	198,897	**5,541,822**	4,541,822	0.4%	0.39	323
2054	43,726	160,709	204,435	**5,746,256**	4,746,256	0.0%	0.38	(0)
2055	44,600	165,530	210,131	**5,956,387**	4,956,387	0.0%	0.37	(0)
2056	45,492	170,496	215,988	6,172,375	5,000,000	0.0%	0.36	(0)
Total								1,098,766

7. QUALITATIVE CONSIDERATIONS IN COMMUTATIONS

The following illustrate some of the considerations that may cause the cedent or the reinsurer to be motivated to commute beyond the formula dynamics described above. These examples are merely a sampling of reasons and are far from exhaustive.

1. Distressed Reinsurer or Cedent - Here, the first party to the negotiation table generally will get the best outcome (as long as the distressed company is expected to run-off without going into receivership. Otherwise any agreement may be subject to unwind due to the principle of "voidable preference"). So the solvent party may be highly motivated to settle. Settling for discounted loss values (effectively harvesting the imbedded value in the undiscounted reserves) may bolster the solvency of the distressed party. These situations will often involve global commutations and can be quite large. Usually cost considerations such as risk loads, different tax treatment, credit risk, etc. will not play into the settlement.
2. Two bombs are detonated in a large city within blocks of each other 1 hour apart. The cedent has a WC reinsurance treaty for \$5m xs \$5m on an occurrence basis. The issue at hand: is it one or two occurrences? The total loss is calculated at \$25m. If it is considered two occurrences, the insurer believes it is entitled to collect \$10m in recoveries; if one occurrence, only \$5m. This same issue is affecting many reinsurance contracts and the resolution is tied up in court proceedings that will take many years to resolve. However, the insurer has already paid the full \$25m loss. The amount of the reinsurance recovery, and the delay in determining it, will affect both the balance sheet and the income statement for several years. The cedent is motivated to commute. The insurer and reinsurer may agree to commute for a compromise payment of \$7.5m.

   Another example of dispute over the number of occurrences could arise in consecutive risks attaching property cat treaties with an interlocking clause that limits the recovery from a single event to one occurrence limit, even when both treaties are involved. If there is a question about the number of events, such as when a hurricane hits in one area, strengthens and then later hits in another area, this can tie up recoveries.

3. A cedent has a WC reinsurance treaty for the layer \$8m xs \$2m. One of the employees of an insured, 25 years old, has suffered a traumatic brain injury and will need lifetime care. The rated age of the injured worker is 75 years. The claim is valued at \$10m and the duration of the claim is expected to be 30 years. The discounted value is \$4.1m. This splits into a discounted value of \$1.4m for the insurer and \$2.6m for the reinsurer. The insurer has the opportunity to enter into a structured settlement for \$3.0m. The strict application of the reinsurance language would allow a recovery of only \$1.0m. However, if the settlement is not

entered into, both the insurer and reinsurer will pay more on a present value basis. The duration of the first $1m of payments is 12 years and the duration of the $8m xs $2m payments is 39 years.

No settlement:

Insurer: $1.4m

Reinsurer: $2.6m

Structured Settlement with no commutation:

Insurer: $2.0m

Reinsurer: $1.0m

Structured Settlement with allocation of discount to each layer:

Insurer: $1.3m

Reinsurer: $1.7m

The insurer and reinsurer agree to commute the claim for $1.7m. The reinsurer saves $0.9m and the insurer saves $0.1m and both eliminate future uncertainty.

4. A large multi-line insurer decides to exit the surety line. The book consists largely of contract surety bonds. They have an uncapped quota share treaty on a risks attaching basis. The book has produced a higher than expected combined ratio result of 90%. A lower combined ratio was anticipated because the insurer price included a profit load higher than 10%, due to the systemic catastrophe potential. Claims handling is crucial for this line of business. In particular, the extension of credit to obligees can often ameliorate potential cash-flow induced claims. The ascertaining of where this is likely to lower losses involves substantial involvement and ongoing discussions with the obligees. The reinsurer is concerned about the claims handling expertise that will be applied in the run-off of this book. The insurer believes their run-off results will be equal to or better than historical results. The reinsurer may be motivated to commute for a combined ratio of 100% on the run-off exposure and the insurer may be happy to accept this.
5. Casualty reinsurance purchased in the time period 1960 – 1980. Attachment points and limits are quite low relative to current cost levels – $4m xs $1m layers in today's dollars are equivalent to $400k xs $100k or $800k xs $200k during those time periods. Any remaining claim recoveries will be small in magnitude and few in number. Both insurer and reinsurer are motivated to commute just to eliminate future administrative costs. The commutation amounts are likely to be small as well, which will also facilitate the commutation.

6. Sidecar/Cat Bond/Hedge Fund Reinsurer – the hedge fund has provided the initial capital to fund the agreement (generally in the form of a "special purpose vehicle") and now wants to commute in order to repatriate capital to investors. The underlying business in these agreements is typically short-tail business where reserves are paid quickly and commutation values can be agreed soon after the expiration of the agreement. In these cases, usually commutation is an up-front expectation so the mechanism for calculating the commutation does not involve compromise.

   Another example of a pre-agreed commutation is when a reinsurance contract includes a mandatory commutation clause. This was often seen in worker's compensation excess of loss facultative certs where one of the reinsurers was a life company. In these cases all the parameters for calculation of the individual claim values were generally spelled out in the cert.

7. Coverage dispute on specific underlying claims. For example, Cedent A may write a layer of an insured's program. There may be a strong case for an expected and intended defense (i.e. the insured knew of the loss before the policy period). Cedent A settles the claim for a small discount on the full layer value without taking the case to trial. However, the reinsurer also covers Cedent B participating higher up on the insured's coverage tower. Cedent B more aggressively fights the claim and eventually gets a better result at a higher discount on the layer value. The reinsurer disputes the settlement value of Cedent A. Cedent A and the reinsurer may agree to commute the claim using the higher discount on the lower layer.

8. Conclusion

A cedent and a reinsurer may agree to commute individual claims or entire books of business for many reasons. When both parties are solvent, the commutation negotiation may involve many cost/benefit considerations beyond the simple discounted value of the outstanding reserves. Some of those addressed here include tax value embedded in the reserves, capital needed to support the reserves, reinsurer credit risk, funding considerations and differing viewpoints on cost inflation. When these costs/benefits are included in the commutation price, the commutation may be facilitated or hindered depending on the magnitude of the cost/benefit of these items to both parties.

There are also many other financial reasons that may drive commutation settlements that may not allow for such a detailed cost/benefit analysis. Principal among these is commutation involving a distressed counterparty. Some other reasons include disputed claims, structured settlements, cedent exit from a line of business, expense considerations on low activity treaties and prior commutation expectations, such as a mandatory commutation clause. There are, of course, many other reasons that are not enumerated here.

DISCLAIMERS

The opinions expressed are solely those of the author and are not presented as a statement of the views or practices of any past or present employer. The author assumes no liability whatsoever from any damages that may result, directly or indirectly, from use or reliance on any observation, opinion, idea or method presented in this paper.

Biography of the Author

Brian MacMahon is Chief Actuary, Reinsurance for the Liberty Mutual Reinsurance unit within Liberty Mutual Group. He is responsible for all aspects of actuarial support for reinsurance including treaty pricing, treaty reserving, financial planning and capital allocation. He has a Master's of Science in Applied Mathematics from California State University at Hayward. He is a Fellow of the CAS, a Chartered Enterprise Risk Analyst (CERA), and a Member of the American Academy of Actuaries. He participates on the CAS Reinsurance Research committee, and is a frequent presenter at industry symposia.

Appendix A – Derivation of Capital Run-off Factor (Section 3)

The capital run-off factor is the multiplier applied to initial capital in order to get at total capital. Initial capital is set at the time of commutation and is based on the selected approach. In this paper, we have used the 99th percentile VaR of income. In other words, if the final amount of reserves to be paid was known at the end of the year and it was the 99th worst outcome, we would utilize the full initial capital held to pay the losses.

However, if the payout pattern extends over several years, and the outcome remains uncertain over those years, we have to continue to hold capital until the final outcome either requires us to use part or all of the capital or release the capital for other uses. Generally, the uncertainty reduces as the outstanding losses are paid down. This means that the 99th worst outcome also reduces. So, while capital has to be held for many years, the amount of that capital reduces each year. In this paper, we have made the simplifying assumption that capital reduces proportionally as outstanding losses reduce. An argument can be made that the major risk is frontloaded in the payment pattern and capital should be taken down more rapidly than the reduction in the outstanding losses. Certainly one can construct cases where the risk is front loaded and other cases where it is back loaded.

In Table A below, the calculation of the capital runoff factor, 'F', is shown. The implication with this method is that the initial variation as a percentage of the mean, remains the same as reserves are paid out. For example, if the aggregate income curve was a lognormal, the coefficient of variation would determine the 99th percentile. This method would be equivalent to assuming that the CoV remains the same each year as the reserves are paid out, i.e. the mean would shrink but the CoV would stay the same. This is probably conservative for claims that are certain as to amount but the payment is slow. One example would be a set of high excess Fortune 100 casualty claims, all of which are large enough to exhaust the insurer's layers with certainty but payment is slow because of negotiations with all of the insurers on the full tower of coverage. On the other hand, there are claims where payments do not indicate a proportional reduction in future uncertainty. One example would be Worker's Compensation lifetime pension cases, where the biggest source of variability is the future medical inflation and utilization. As payments are made (especially indemnity payments), this variability may not decrease in proportion to the reduction in outstanding loss.

In formula terms, the runoff factor, 'F', can be expressed as:

$$Runoff\ Factor = \text{'}F\text{'} = 1 + \sum_{i=2}^{n}\sum_{j=i}^{n} NPV_0(Unpaid_{j-1} - Unpaid_j) \qquad (A.1)$$

Where,

NPV_0 = *NPV back to time of commutation*

$i = 1$ *is time of commutation*

$i = n$ *is time of last payment*

Table A

i	Year	O/S Loss	(1) O/S Reserves	(2) = (1)i-1 - (1)i Payout	(3) = NPV[Sum(2$_i$ to 2$_6$)]/(1+d)^Year$_i$ % of Intial Capital
1	0.00	20,000,000	100.0%		100.0%
2	0.50	13,000,000	65.0%	35.0%	93.4%
3	1.50	8,000,000	40.0%	25.0%	59.6%
4	2.50	4,000,000	20.0%	20.0%	36.1%
5	3.50	1,000,000	5.0%	15.0%	17.8%
6	4.50	-	0.0%	5.0%	4.4%
				Runoff Factor 'F':	3.11

Appendix B – Derivation of Premium used in Risk Load Calculation (Section 3)

The example in Table 2 of Section 3 did not include any expense. In this derivation, it will be included. The premium we are deriving here can be viewed as the premium that the reinsurer would charge if it was to assume the same reserves that are being considered for commutation. In other words, it would want the premium to cover the expected value of the reserves, all expenses associated with the transaction -both internal and external- and a profit load that yields their target after-tax return on capital. Losses, expenses and the stream of capital supporting the reserves would all be considered on a present value basis.

Premium is equal to the sum of discounted losses, discounted expenses and profit margin. Now, profit margin does not have to cover the entire target return on capital. Capital is invested while it is used to support the reserves. Let's call the after-tax investment income earned on capital, the "passive" return. The after-tax profit margin on the insurance cash flows, we'll call the "active" return. We'll assume that the discount rate used to present value the cash flows is equal to the investment income rate on the capital. The sum of the passive and active returns has to equal the after-tax target return on capital.

The after-tax profits from both the active and passive returns have to equal the target after-tax ROE times the capital. In formulaic terms:

$$\text{Premium} = \text{Loss}_{Disc} + \text{Expense}_{Disc} + \text{Active Profit} \quad \text{(B.1)}$$

$$\text{Premium} = \text{Loss}_{Disc} + \text{Expense}_{Disc} + \text{Total Profit} - \text{Passive Profit} \quad \text{(B.2)}$$

$$\text{Premium} = \text{Loss}_{Disc} + \text{Expense}_{Disc} + \text{ROE*Capital/(1-Tax Rate)} - \text{Discount Rate*Capital} \quad \text{(B.3)}$$

$$\text{Capital} = (99^{th}\text{ percentile worst loss} - \text{Premium})\text{*Diversity Factor*Runoff Multiplier} \quad \text{(B.4)}$$

Note that the profit due to the active return is just the premium less the loss and expense,

$$\text{Active Profit} = \text{ROE*Capital/(1-Tax Rate)} - \text{Discount Rate*Capital} \quad \text{(B.5)}$$

After Tax, it becomes

$$\text{After-Tax Active Profit} = \text{ROE*Capital} - \text{Discount Rate*Capital*(1-Tax Rate)} \quad \text{(B.6)}$$

While the after-tax passive profit is:

$$\text{After-Tax Passive Profit} = \text{Discount Rate*Capital*(1-Tax Rate)} \quad \text{(B.7)}$$

Finally, the sum of the active and passive profits equals:

$$\textit{After-Tax Active Profit} + \textit{After-Tax Passive Profit} = ROE*\textit{Capital} = \textit{Target Profit} \quad \text{(B.8)}$$

Replacing Capital in equation (A.3) with (A.4) and solving for Premium, gives the following expression:

$$\textit{Premium} = [L_{Disc}+E_{Disc} + (ROE/(1\text{-}T) - d)*L_{99th}*D*F]/[1+ (ROE/(1\text{-}T) - d)*D*F] \quad \text{(B.9)}$$

L_{Disc} = Present Value of Expected Loss

E_{Disc} = Present Value of Expenses

ROE = Target after-tax Return on Capital

T = Tax Rate on income

d = Discount Rate

L_{99} = Present Value of 99th percentile worst loss (not income)

D = Diversity Factor

F = Runoff multiplier

Appendix C – Sensitivity of Commutation Price to Variations in Factors

Section 2

	Tax Rate			
	20.0%	25.0%	30.0%	35.0%
IRS Table = "Other Liab"				
Discount Rate				
2.50%	18,943,644	18,878,428	18,803,896	18,717,897
5.00%	17,976,062	17,851,064	17,708,209	17,543,376
IRS Table = "Reinsurance"				
Discount Rate				
2.50%	18,954,244	18,892,562	18,822,068	18,740,729

Section 3

Tax Rate of 30% and IRS Discount Pattern = "Other Liab"

Risk Load

	ROE			
Disc Rate	5%	10%	15%	20%
2.5%	984,091	2,263,407	3,323,005	4,215,012
5.0%	428,495	1,684,923	2,728,443	3,608,946

Commutation Price

	ROE			
Disc Rate	5%	10%	15%	20%
2.5%	20,209,740	22,037,334	23,551,046	24,825,342
5.0%	18,320,344	20,115,242	21,605,985	22,863,846

Appendix D – Example of Investing Short (Section 5)

In Section 5, a component of the commutation payment is duration matched to the loss payments using the risk free rate. In today's historically low interest rate environment, one could ask if there are other better strategies to investing than duration matching. For example, what would happen if assets were kept short with the expectation of an imminent upward movement in interest rates? An upward movement in interest rates, as long as the yield curve was upward sloping, would have less impact on the short assets than the longer duration matched assets. The short assets could then be reinvested at the longer duration needed to match the remaining liabilities. It is possible that the realized loss, when interest rates rise, on the short assets would be more than offset by the gain in return on the reinvested longer assets. This strategy would provide funds in excess of the required loss payments. This is a good strategy as long as interest rates are expected to rise significantly enough and soon enough relative to the payment of the losses. There are multiple dimensions involved here – the timing of the change, the magnitude of the change, the slope of the yield curve, etc. The breakeven solution would be a bounded surface along these dimensions. There may be no solutions or a continuum of solutions. For example, take the following table:

Table 6

Breakeven Increases in Interest Rate by Year

Years	Payments	2014 Yield Curve	Discounted Payments	Interest Rate Rise in Year 2	Discounted Payments	Interest Rate Rise in Year 3	Discounted Payments	Interest Rate Rise in Year 4	Discounted Payments
0.5	7,000,000	0.125%	6,995,629	0.125%	6,995,629	0.125%	6,995,629	0.125%	6,995,629
1.5	5,000,000	0.250%	4,981,308	**1.358%**	4,960,192	0.125%	4,990,640	0.125%	4,990,640
2.5	4,000,000	0.750%	3,925,973	**1.483%**	3,907,758	**4.940%**	3,894,971	0.125%	3,987,527
3.5	3,000,000	1.250%	2,872,358	**1.983%**	2,852,722	**5.065%**	2,778,744	**32.607%**	2,595,438
4.5	1,000,000	1.500%	935,196	**2.483%**	916,593	**5.565%**	871,192	**32.732%**	651,492
Realized Loss, Discounted to Inception:					77,572		179,290		489,740
			19,710,466		19,710,466		19,710,466		19,710,466

The column labeled "2014 Yield Curve" uses the risk-free US Governmental bond yield curve effective in October, 2014. This is our base case: duration matching using the current risk-free yield curve. The three remaining interest rate columns are derived by holding assets at the 6 month rate until interest rates rise. The yield curve is assumed to move upward, using the slope of the current yield curve, at the beginning of Year 2, Year 3 or Year 4. Table 6 shows the required interest rate, highlighted in red, at 6 months and each year thereafter in order to breakeven with the current yield curve. When the 6 month interest rate rises, the assets held at the 6 month rate must be liquidated to be reinvested at the higher interest rates. When they are liquidated, there is a realized loss. This is indicated in the table above as "Realized Loss, Discounted to Inception". The sum of the discounted payments and the realized loss has to equal the original discounted payments. This table shows that large increases in interest rates are required to offset the loss in income from investing short. This is due to the short payment pattern in this example. Results would be more reasonable for a long tail liability type payment pattern.

Realized Loss:

Let A_i = Funds assumed to be the investable at the end of Year i. Year 1 is the first year after the date of commutation. A_0 = NPV(Loss) using original yield curve assumed to be starting funds.

Let P_i = Payment in Year i

Let r_i = investment rate for a duration of i – 6 months from original yield curve. r_1 is our initial short-term (6 month) rate

Let R_i = investment rate for a duration of i – 6 months after increase in yield curve. R_1 is our short-term (6 month) rate after the increase in yield curve

Let H_i = realized loss at beginning of Year i from liquidating assets at investment rate r_1

Assets investable at end of Year i, assuming that assets are kept at the 6 month rate:

$$A_i = [A_{i-1}*(1+r_1)^{0.5} - P_i]*(1+r_1)^{0.5} \quad \text{(D.1)}$$

Realized loss at beginning of Year i:

$$H_i = [A_{i-1}*(1+R_1)^{0.5} - A_{i-1}*(1+r_1)^{0.5}]/(1+R_1)^{0.5} \quad \text{(D.2)}$$

Using the example in Table 6 for the 6 month interest rate increasing in year 2,

A_0 = \$19,710,466

A_1= [\$19,710,466*(1.00125)$^{0.5}$ - \$7,000,000]*(1.00125)$^{0.5}$ = \$12,730,730

H_2 = [\$7,743,520*(1.01358)$^{0.5}$ - \$7,743,520*(1.00125)$^{0.5}$]/(1.01358)$^{0.5}$ = \$77,669

Discounting H_3 back to inception requires discounting for one years at R_1 = 1.01358

Realized loss, discounted back to inception:

\$77,669/(1.01358)^1 = \$77,572

Clearly, H_2 cannot be calculated without knowing R_1, which is calculated in the following section.

Required Interest Rate Change:

Depending on how long the assets are kept at the investment rate r_1, the increase in the discounted loss payments compared to those derived from the duration matched yield curve, must be made up by the lower discounted value of the future loss payments discounted at rate R_1, R_2, etc. In addition, the discount on the future loss payments must be enough to also offset the realized loss, calculated above.

Using the original yield curve, the formula for the NPV of losses is:

$$\sum_{i=1}^{n} P_i / (1+r_i)^{(i-0.5)} \qquad \text{(D.3)}$$

When the yield curve increases in year j+1, the formula for the NPV of losses combined with the realized loss is:

$$\sum_{i=1}^{j} P_i / (1+r_1)^{(i-0.5)} + \sum_{i=j+1}^{n} P_i / [(1+r_1)^{(j)} (1+R_{i-j})^{(i-j-0.5)}] + H_{j+1} / (1+r_1)^{(j-1)} \qquad \text{(D.4)}$$

Set (D.3) equal to (D.4) and solve for R_1. You need to use the consistent slope assumption in the following formula:

$$R_i = R_1 + (r_i - r_1) \qquad \text{(D.5)}$$

Note that the formula for H_{j+1} also includes R_1.

In the case where the 6 month investment rate changes in Year 2, R_1 = 1.1358%

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Abbreviations and notations

CP, commutation price
NPV, net present value
RL, risk load
T, tax rate
ROE, return on equity
F, runoff factor for capital
d, discount rate
D, diversity factor

An Enhanced Understanding of Using the RAA Excess Casualty Loss Development Study For Reserve Analysis

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Abstract:

This article explores the differences between the various studies published by the RAA over the years. In comparing the reporting patterns for the different lines of business in the RAA study, I attempt to determine what factors can have an effect on the reporting patterns. Based on the data I show that these factors include the underwriting cycle, data quality and data manipulation to minimize the impact of any one company. I also show how the actuary can incorporate this information in using the RAA data in his reserving analysis.

Keywords: RAA, Benchmarks, Underwriting Cycle, Reporting Patterns, Reserving, Reinsurance

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1. INTRODUCTION

The RAA publishes a bi-annual study of incurred and paid loss triangles of the reinsurance companies that are members of the RAA. The triangles that are published are comprised of four casualty lines of business: Auto Liability, General Liability excluding Asbestos and Pollution, Medical Malpractice and Workers' Compensation. Besides triangles for the entire line of business, the data is also broken out by attachment point, divided into five attachment point ranges. As a disclaimer, in the introduction to its study, the RAA cautions that for various reasons the results of one study will not necessarily match up to the results of a prior or subsequent study.

The RAA triangles are often used to help the actuary in determining the ultimate loss for the non-proportional and facultative reinsurance triangles. In the casualty lines, especially for the long-tailed, high attachment point lines, there is often not enough credible company data to determine an appropriate ultimate loss. By incorporating the RAA studies, the actuary can come to a more reasonable conclusion in selecting an ultimate loss. However, if the RAA studies do change over time, and it is in fact true that one cannot assume a later study will match up with an earlier study, then what will be the impact to a company's results when a new RAA study is published? This paper will attempt to demonstrate if differences between the studies do exist, and if they exist, several suggestions will be offered to explain these differences. Several reserving procedures that utilize the RAA data will then be shown, with an attempt to show if any of the possible explanations could have an impact on the procedures. This will help the actuary decide when to use the RAA benchmarks and what assumptions need to be made when using them.

1.1 Research Context

To the best of my knowledge, there has been no prior research done that compares the RAA studies. However, the RAA in its bi-annual study details the limitations that one should be aware of before using the study. These limitations can be helpful in understanding the potential differences between the studies. Furthermore, one area which is explored is the effect of the underwriting cycle on the different RAA studies. There has been some research published showing the impact that the underwriting cycle might have on the amount of reserves held by a company. In particular, the working party paper presented at the 2008 General Insurance Convention (Hilder), as well as the paper published by Line (et al) (Line), focus extensively on this issue.

1.2 Objectives

The primary goal of this paper is to understand what is driving the differences between the various studies published by the RAA. This is important for a couple of reasons. First of all, there might exist within a company some reserving groups where the company's historical data is sparse or volatile which will necessitate heavy reliance on benchmarks. Significant changes in these benchmarks may lead to significant changes in the reserve indications for reasons which are external to the reserve portfolio. This in turn may compromise the credibility of the actuaries in the eyes of end users of actuarial indications such as company management. Understanding why the RAA data has changed can go a long way in minimizing the concerns of management.

Secondly, from the actuarial side, an actuary might be tempted to continue using the benchmarks from a prior study even when a newer study is available. If in fact the newer study does give different results than the prior study, and the actuary does not update his projections, the reserves could wind up being either deficient or redundant. Furthermore, by understanding what differences exist, and why they exist, will help the actuary decide when it is appropriate to use the RAA benchmarks and what assumptions should be made in using them. Understanding these differences can help the reserving actuary make the necessary adjustments in the actuary's projections.

1.3 Outline

This paper will focus on the reporting patterns for the Auto Liability line of business. I will compare the reporting patterns by attachment point for the last four RAA studies. Where differences exist, I will propose some possible explanations and test the assumptions from the RAA data. Finally, based on my findings, I will make some recommendations for the reserving actuary to keep in mind when using the RAA study as a benchmark.

2 METHODOLOGY

In this paper I will use the incurred loss triangles from the last four [1] RAA studies to produce a set of loss reporting patterns for the different attachment point triangles produced by the RAA. Although patterns are available for the General Liability, Medical Malpractice and Workers' Compensation lines of business, in this paper I will just present the results for Auto Liability. A cursory review on the GL and WC lines seems to produce similar results to the Auto Liability line so for the sake of simplicity I have focused solely on the Auto Liability line. A more in-depth study would be needed for the other lines and it would be interesting to compare the results of each of the lines.

In order to eliminate any bias due to the judgmental selection of factors between the various studies, I used the same procedure for each of the triangles. The all year weighted averages were selected for each triangle, without eliminating any high or low factors. By choosing the average for all years, the hope is that the outliers, both high and low, will balance each other out. Secondly, in selecting the tail factors, if based on the experience, the cumulative reported loss percentage was at 100% in a period with at least 5 years of experience, then no curve fitting was performed. Where the reporting percentage was more than 100%, then at the period where the reporting percentage reached 100%, a factor of 1.00 was chosen for the tail. In the event that it was necessary to select a tail factor other than 1.00, I used curve fitting to project the tail. Since my intention was not to figure out what the appropriate tail is, but rather to compare the studies, I chose the same curve fit for each study. The curve fit used was the one which gave the highest R^2 for the 2012 study. This curve fit was then used for that particular triangle in each of the studies.

2.1 Results

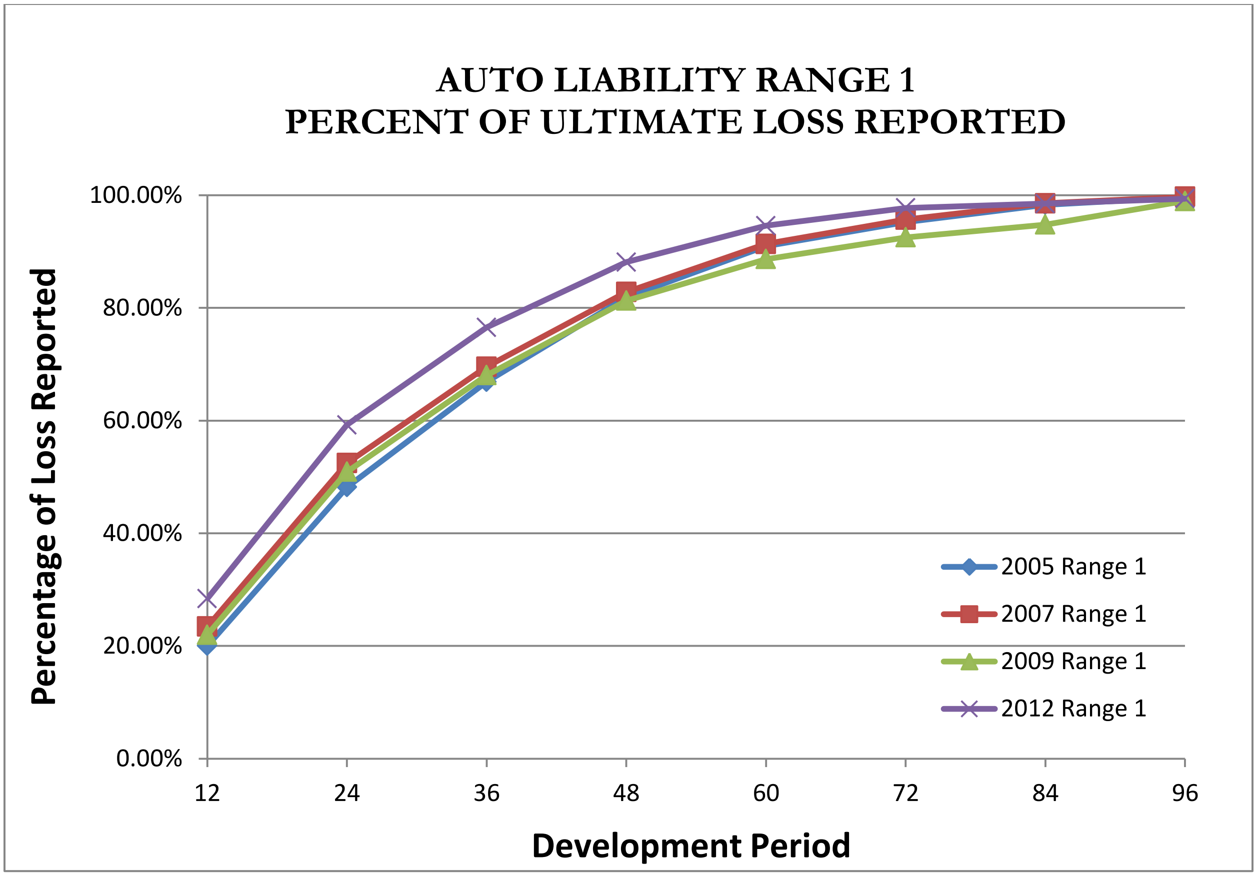
The RAA publishes triangles by various attachment points. In the exhibit below is a table detailing the five different attachment point ranges published by the RAA.

[1] This includes the 2005, 2007, 2009 and the 2012 RAA studies.

Range Name	Attachment Point Range
Range 1	1 to 210,000
Range 2	210,001 to 500,000
Range 3	500,001 to 2,050,000
Range 4	2,050,001 to 5,500,000
Range 5	5,500,001 and greater

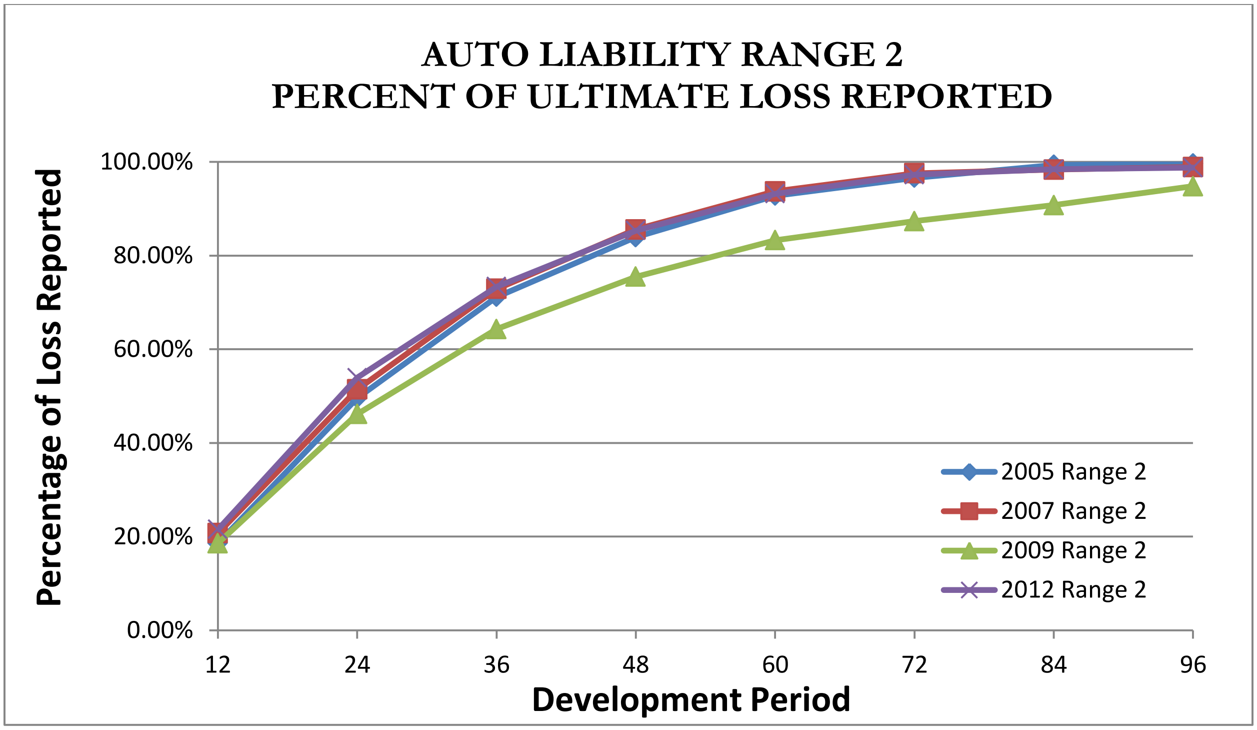
Looking at a comparison between the studies, at the various attachment points,[2] it is clear that the loss reporting pattern for the 2009 study is slower than the other studies. Even for Range 1 where the 2009 study seems to match up pretty well with the 2005 and 2007 study, it is still significantly slower than the 2012 study. There are several possible explanations for this and I will attempt to explore each of the possibilities.

[2] For Auto Liability, Range 4 data was only published in the 2005 and 2012 study. Therefore, this paper will only focus on Ranges 1, 2 and 3.



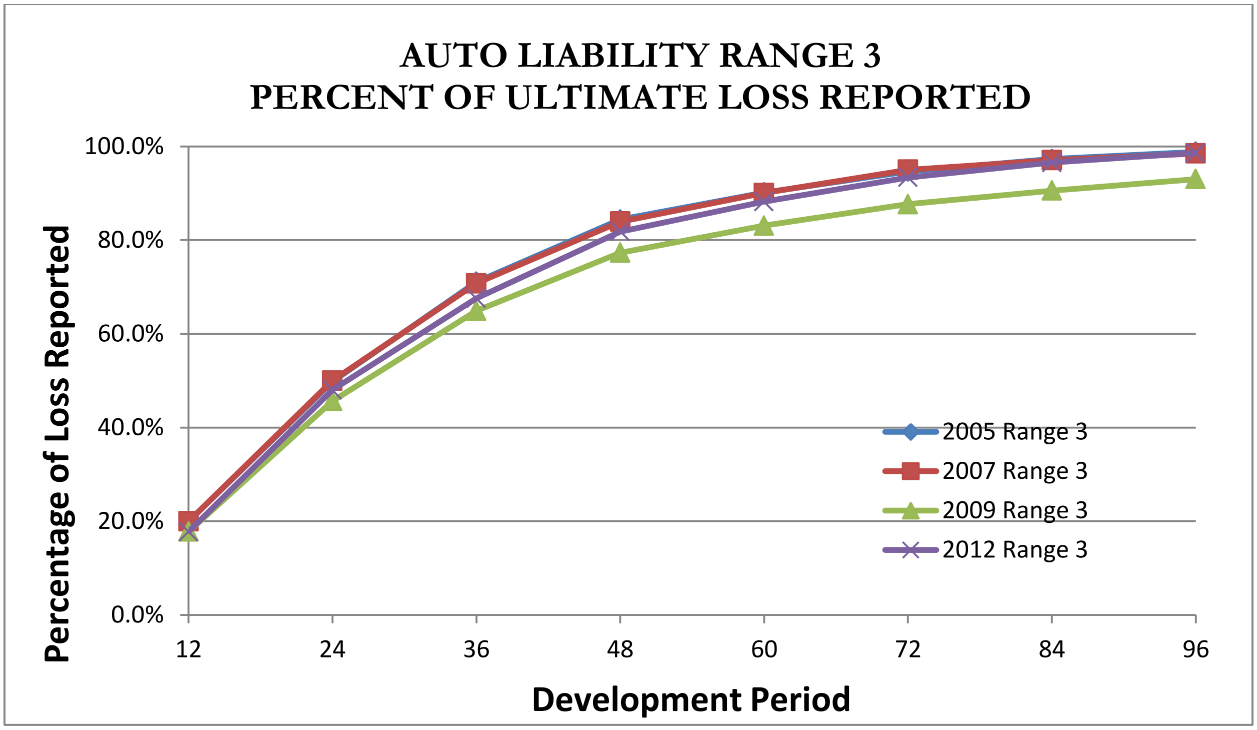
	12	24	36	48	60	72	84
2007	23.4%	52.4%	69.6%	82.8%	91.4%	95.7%	98.6%
2009	22.0%	50.9%	68.0%	81.3%	88.7%	92.5%	94.8%
% difference	-6.2%	-2.9%	-2.2%	-1.8%	-3.0%	-3.3%	-3.8%

	12	24	36	48	60	72	84
2009	22.0%	50.9%	68.0%	81.3%	88.7%	92.5%	94.8%
2012	28.4%	59.2%	76.5%	88.1%	94.6%	97.8%	98.5%
% difference	29.3%	16.3%	12.5%	8.4%	6.7%	5.6%	3.9%



	12	24	36	48	60	72	84
2007	20.7%	51.4%	72.9%	85.6%	93.7%	97.6%	98.3%
2009	18.6%	46.2%	64.3%	75.5%	83.3%	87.3%	90.8%
% difference	-10.4%	-10.2%	-11.8%	-11.8%	-11.1%	-10.5%	-7.7%

	12	24	36	48	60	72	84
2009	18.6%	46.2%	64.3%	75.5%	83.3%	87.3%	90.8%
2012	21.5%	53.9%	73.4%	85.3%	93.2%	97.3%	98.4%
% difference	16.1%	16.8%	14.1%	13.0%	11.9%	11.4%	8.4%



	12	24	36	48	60	72	84
2007	19.9%	50.0%	70.8%	83.9%	90.1%	95.0%	97.1%
2009	17.8%	45.7%	64.9%	77.3%	83.1%	87.7%	90.6%
% difference	-10.9%	-8.6%	-8.3%	-7.9%	-7.7%	-7.7%	-6.7%

	12	24	36	48	60	72	84
2009	17.8%	45.7%	64.9%	77.3%	83.1%	87.7%	90.6%
2012	17.7%	48.0%	67.6%	81.8%	88.2%	93.4%	96.6%
% difference	-0.2%	5.2%	4.1%	5.8%	6.2%	6.5%	6.6%

2.2 UW Year Cycle

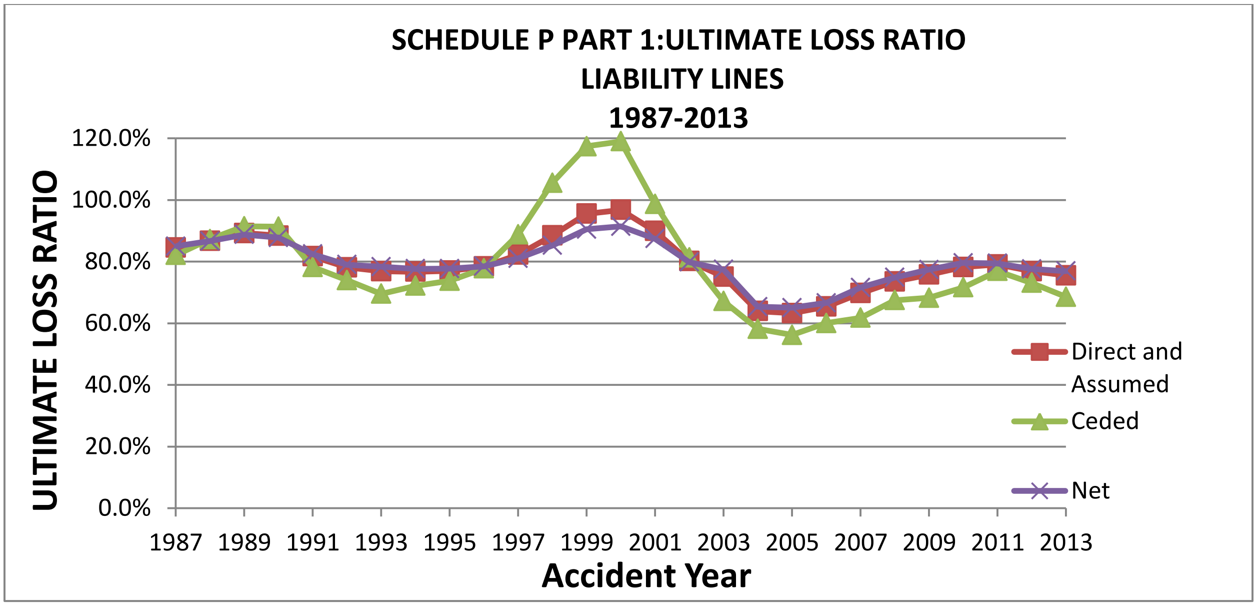
One possible explanation for the slower reporting pattern in the 2009 study can be due to the position within the underwriting cycle. An underwriting cycle is the cyclical manner in which profits within the sector tend to rise and fall over a period of time. Over the last decade, studies have been done to show that there is a relationship between the underwriting cycle and reserving cycle. A reference was made by Bob Conger (Conger), a past president of the CAS, during his keynote address to the 2002 GIRO convention. Subsequently, several papers have been published showing that there is indeed a relationship between the underwriting cycle and the reserving cycle, and that the underwriting cycle can distort development patterns. Line (et al) (Line) attempted to offer several hypotheses why this might be the case. Although the authors were not able to confirm or refute their hypotheses beyond doubt, they did point out that the soft market years appeared to develop more slowly than the hard market years.

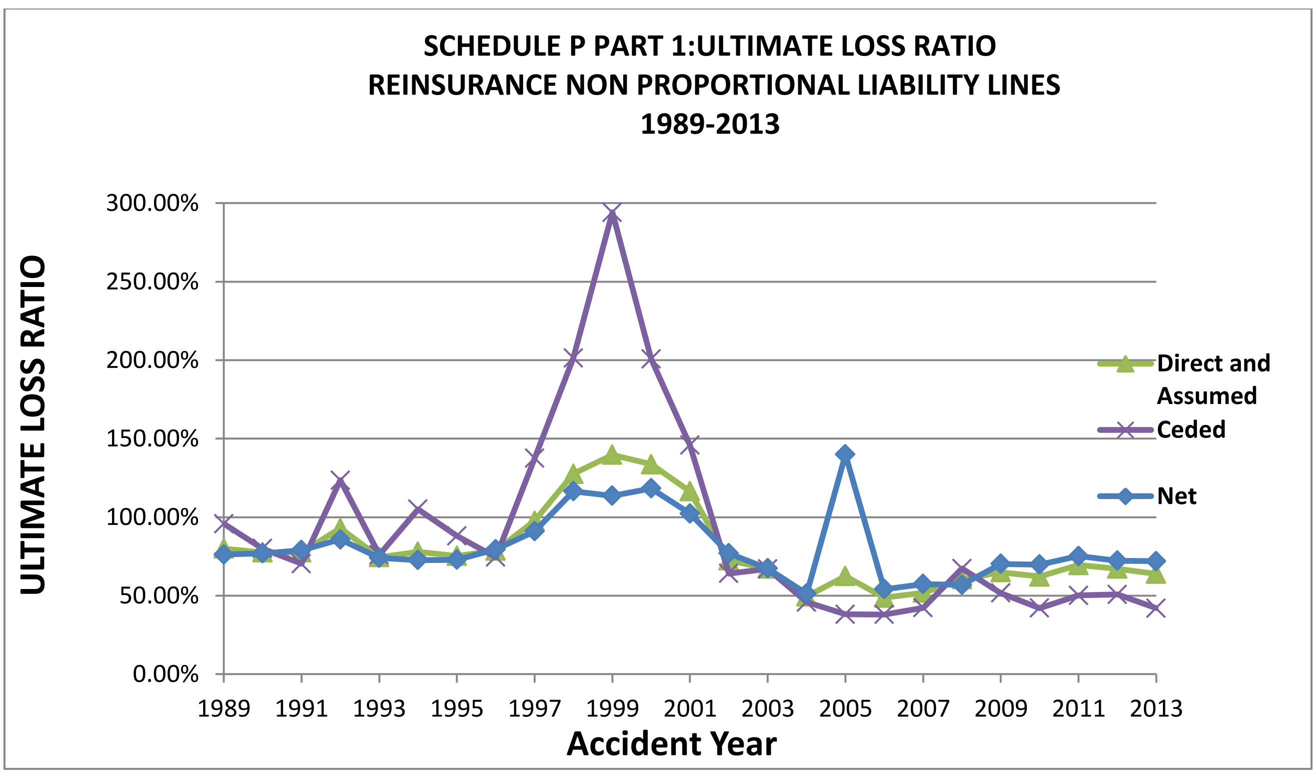
If this is indeed the case, then it is quite possible that the underwriting cycle is driving the difference in the benchmarks. The patterns selected for each study are based on the all year weighted averages for each period. It should be pointed out that the later studies will contain more accident years in the weighted averages for a particular development period compared to the earlier studies. For example, the weighted averages for the 2012 study will contain two more accident years (accident years 2009 and 2010) in the average than the 2009 study (where the latest accident year is 2008). However, even taking this into account, to the extent that a soft market year is given more weight in the average, it would stand to reason that the overall weighted average will be slower. Conversely, if the hard market years are given more weight, then the overall average for a particular period will be faster.

In order to test this theory, it is first necessary to determine which years are the hard market years and which years are the soft market years. It is widely assumed that AY 1997-2001 were the soft market years for reinsurance. In fact if one looks at Schedule P data[3] from the 2013 year-end annual statements for the years 1987-2013, one can clearly see that the reinsurance results for AY 1997-2001 were worse than other years. It appears that we can say that these years were in fact the soft market years.[4]

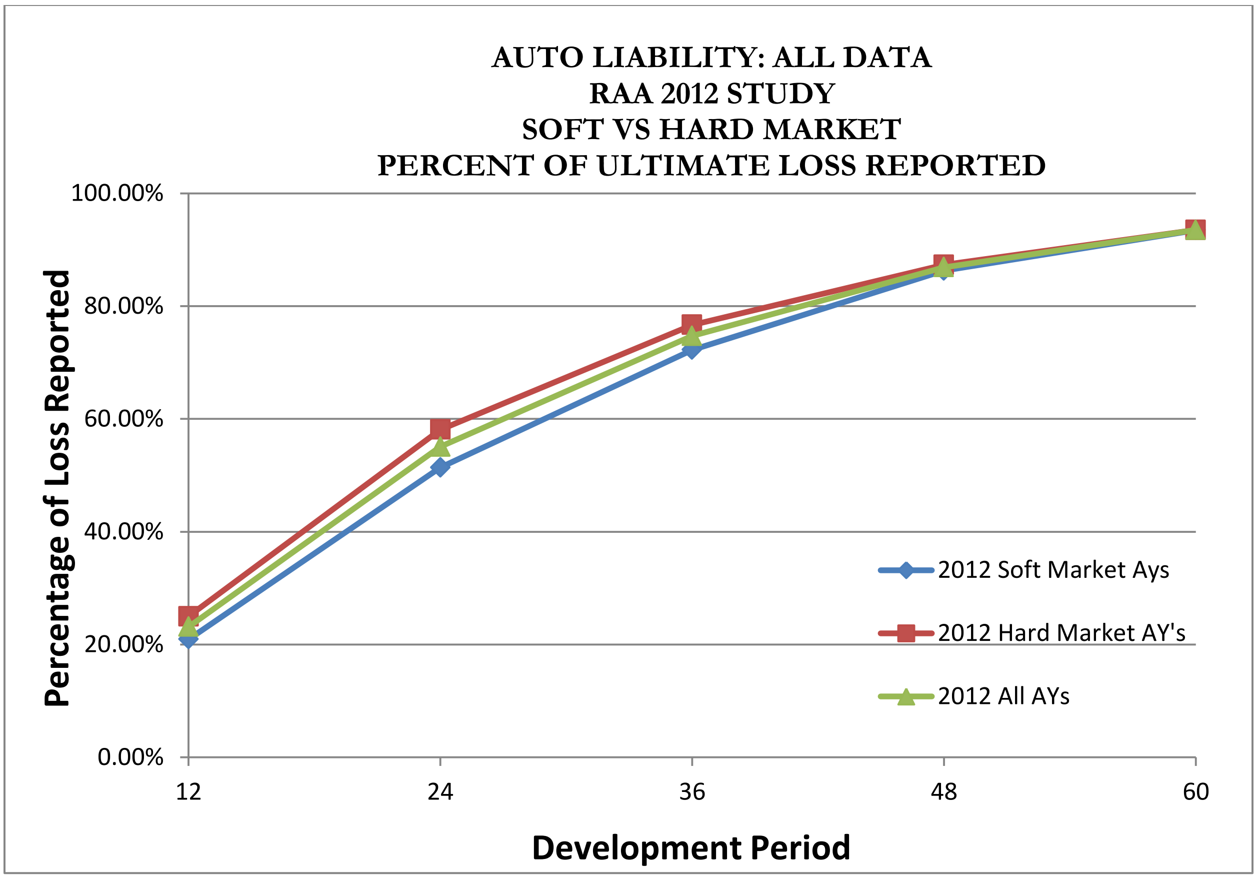
[3] Schedule P Part 1 data was taken from the 1996, 2003 and 2013 Annual Statements using data collected by SNL Financial

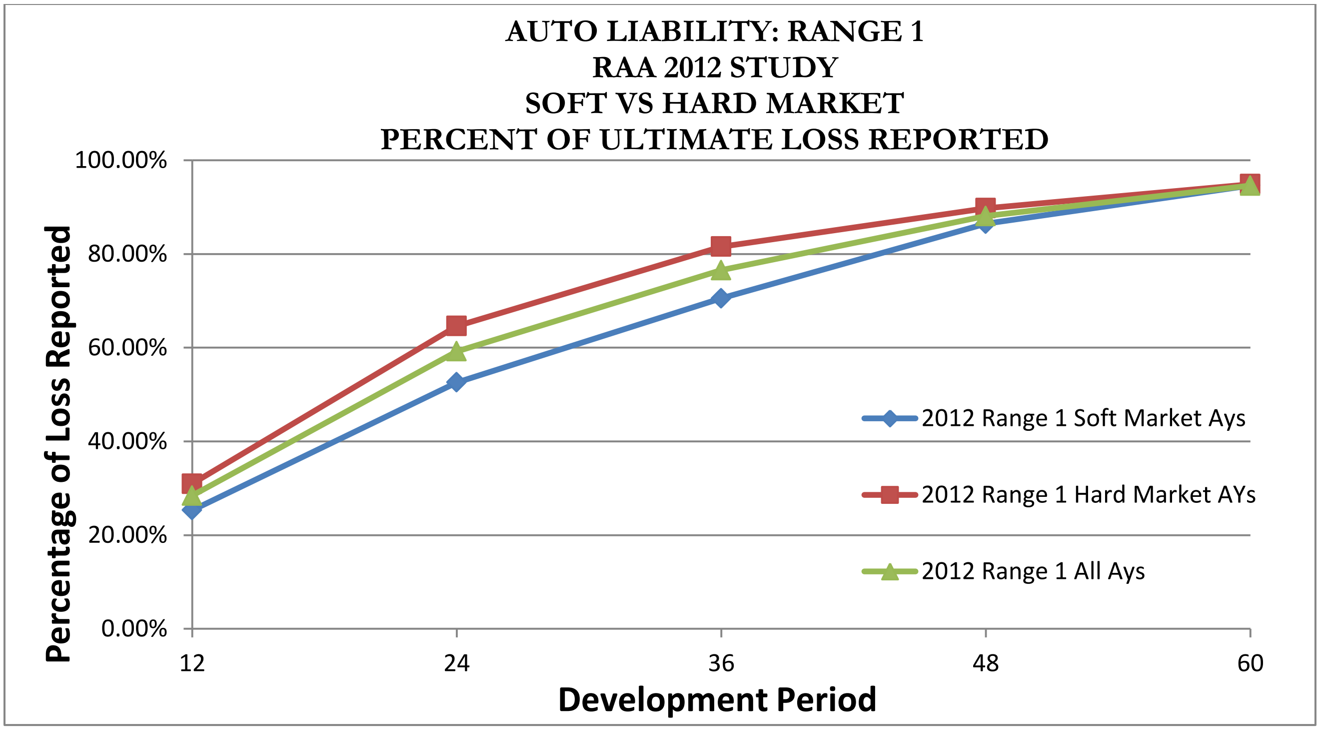
[4] The following exhibits have been adapted from a presentation given by Christopher Bozman of Towers Watson.

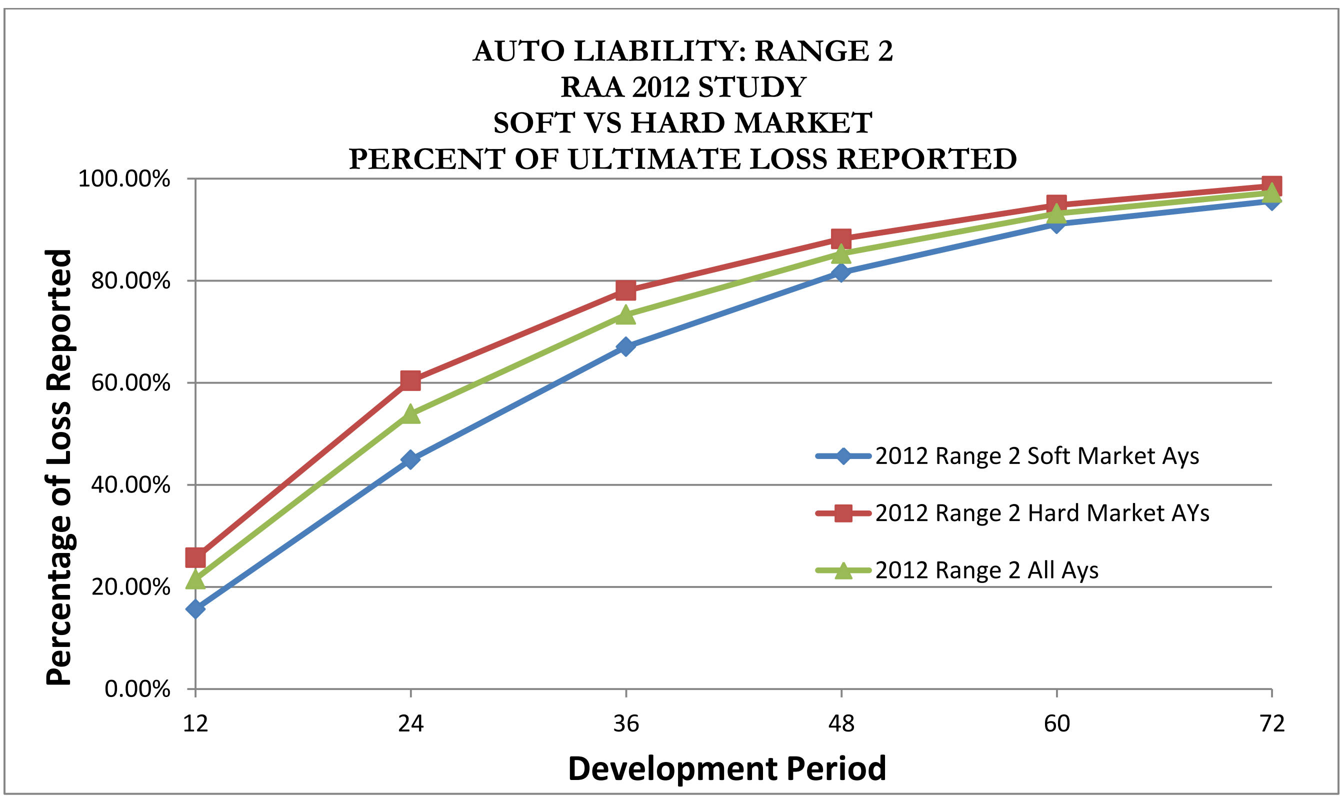

SCHEDULE P PART 1:ULTIMATE LOSS RATIO
LIABILITY LINES
1987-2013
ULTIMATE LOSS RATIO
120.0%
100.0%
80.0%
60.0%
40.0%
20.0%
0.0%
1987 1989 1991 1993 1995 1997 1999 2001 2003 2005 2007 2009 2011 2013
Accident Year
Direct and Assumed
Ceded
Net

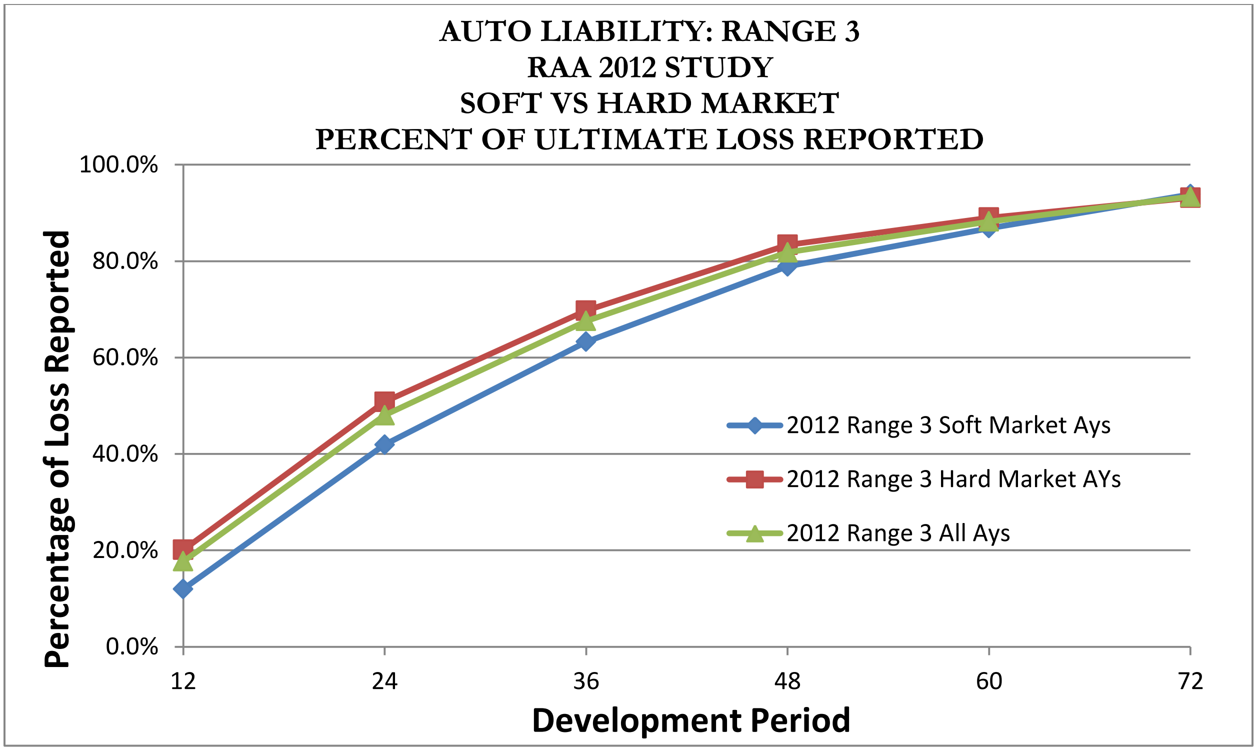

SCHEDULE P PART 1:ULTIMATE LOSS RATIO
REINSURANCE NON PROPORTIONAL LIABILITY LINES
1989-2013
ULTIMATE LOSS RATIO
300.00%
250.00%
200.00%
150.00%
100.00%
50.00%
0.00%
1989 1991 1993 1995 1997 1999 2001 2003 2005 2007 2009 2011 2013
Accident Year
Direct and Assumed
Ceded
Net

Furthermore, the following exhibits show a comparison between the soft market years and the hard market years from the most recent RAA study. It seems clear from the RAA data, that the soft market years do in fact produce slower reporting patterns than the other years.




AUTO LIABILITY: RANGE 1
RAA 2012 STUDY
SOFT VS HARD MARKET
PERCENT OF ULTIMATE LOSS REPORTED
Percentage of Loss Reported
100.00%
80.00%
60.00%
40.00%
20.00%
0.00%
12
24
36
48
60
Development Period
2012 Range 1 Soft Market Ays
2012 Range 1 Hard Market AYs
2012 Range 1 All Ays

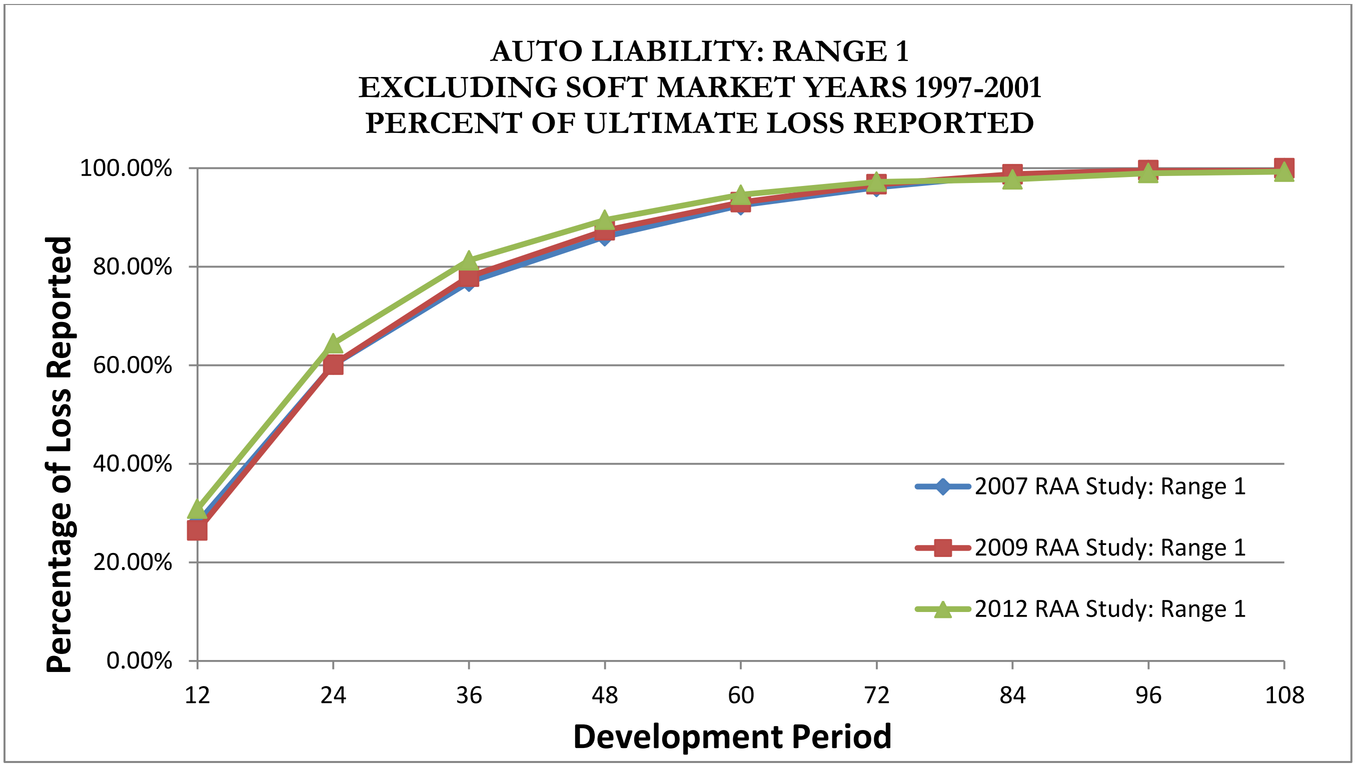

AUTO LIABILITY: RANGE 2
RAA 2012 STUDY
SOFT VS HARD MARKET
PERCENT OF ULTIMATE LOSS REPORTED
Percentage of Loss Reported
100.00%
80.00%
60.00%
40.00%
20.00%
0.00%
12
24
36
48
60
72
Development Period
2012 Range 2 Soft Market Ays
2012 Range 2 Hard Market AYs
2012 Range 2 All Ays

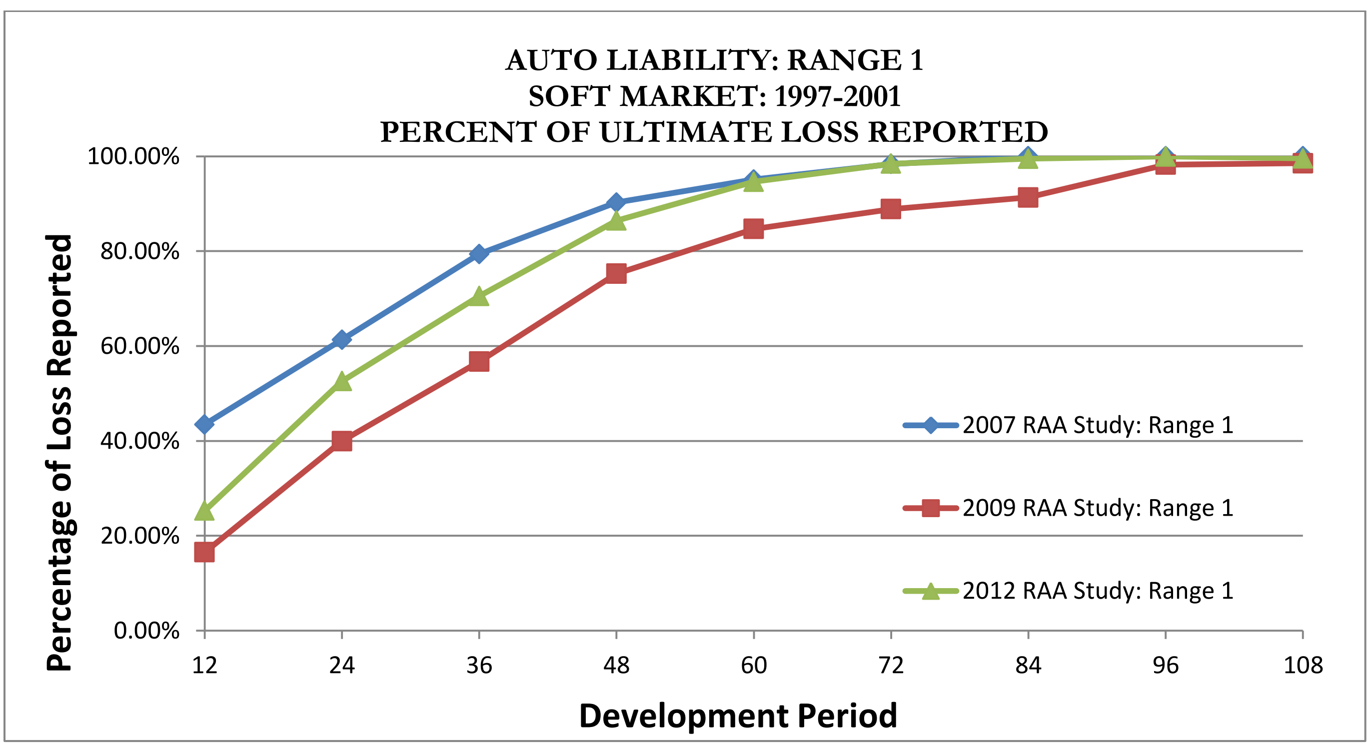


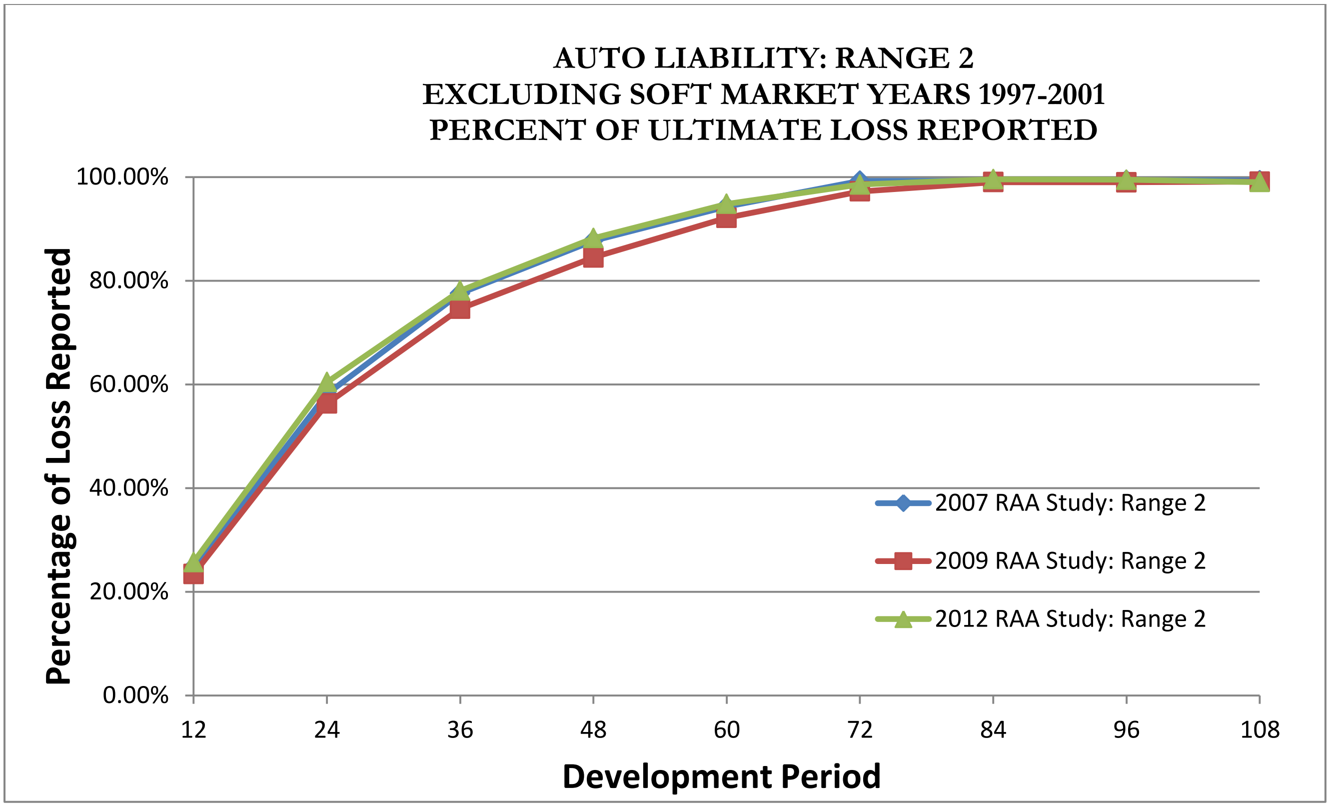
We are now left with determining if in fact the soft market years of the 2009 study are the reason why its reporting pattern is slower than the other studies. If we compare the reporting patterns of each of the studies excluding the soft market years as well as the patterns for just the soft market years,[5] we get the following results.[6]

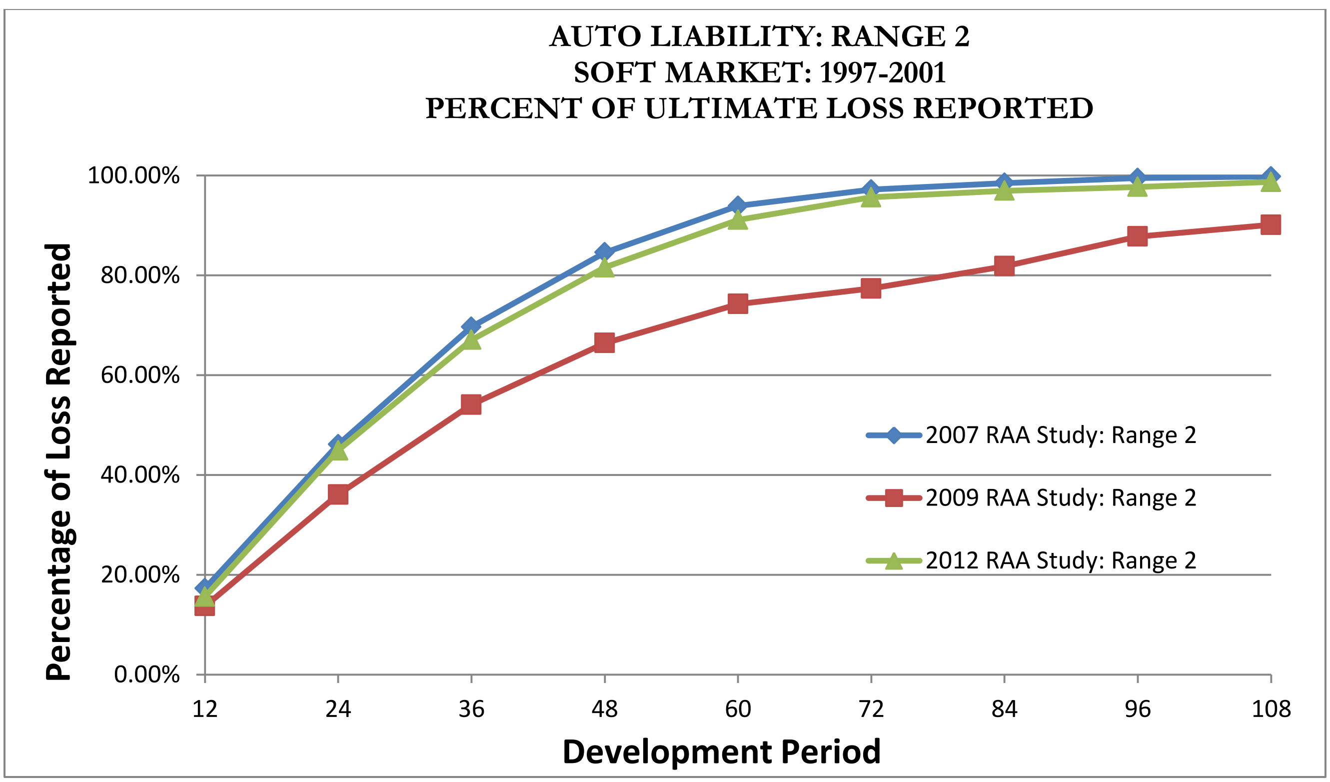
[5] As shown above, we have determined that the soft market years are the underwriting years 1997-2001

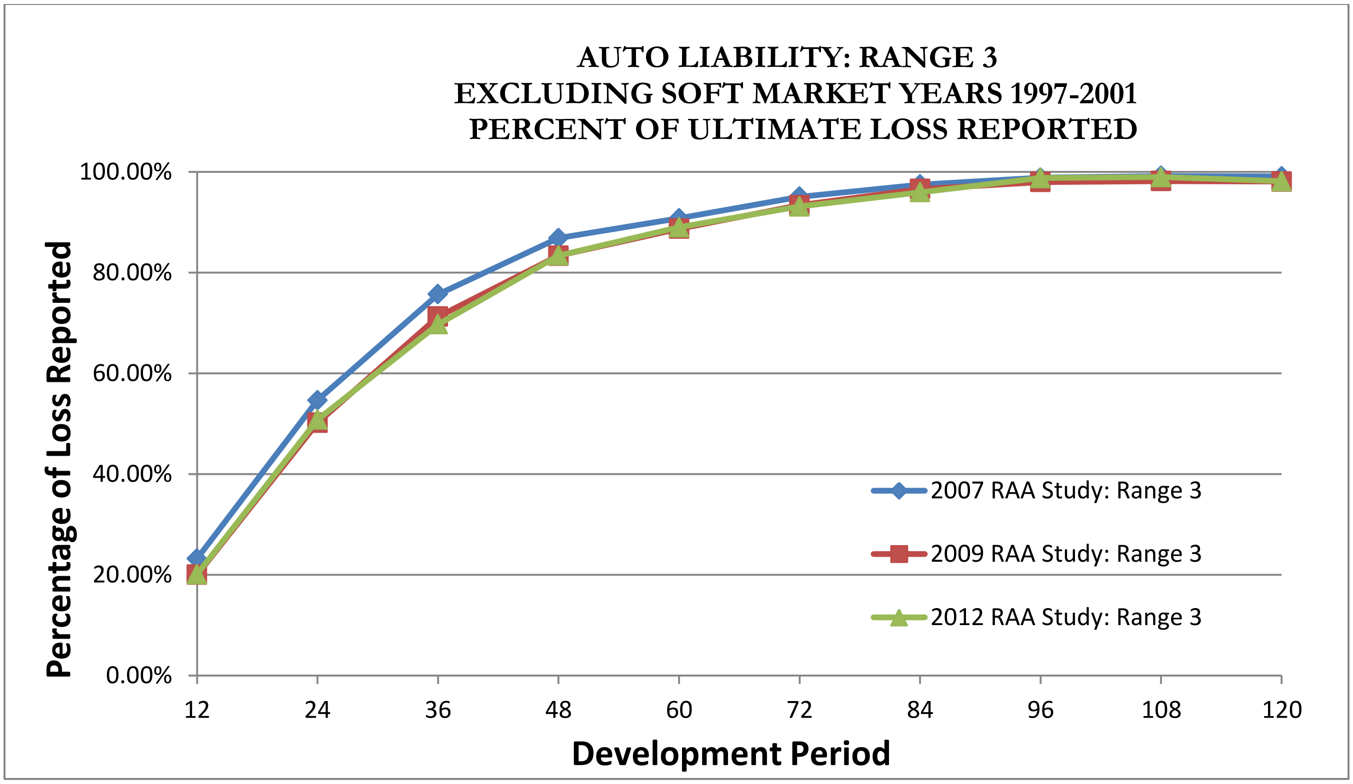
[6] For simplicity and to make the exhibits easier to read, I have left out the patterns from the 2005 study. The patterns from the 2005 study are similar to the 2007 and 2012 studies.

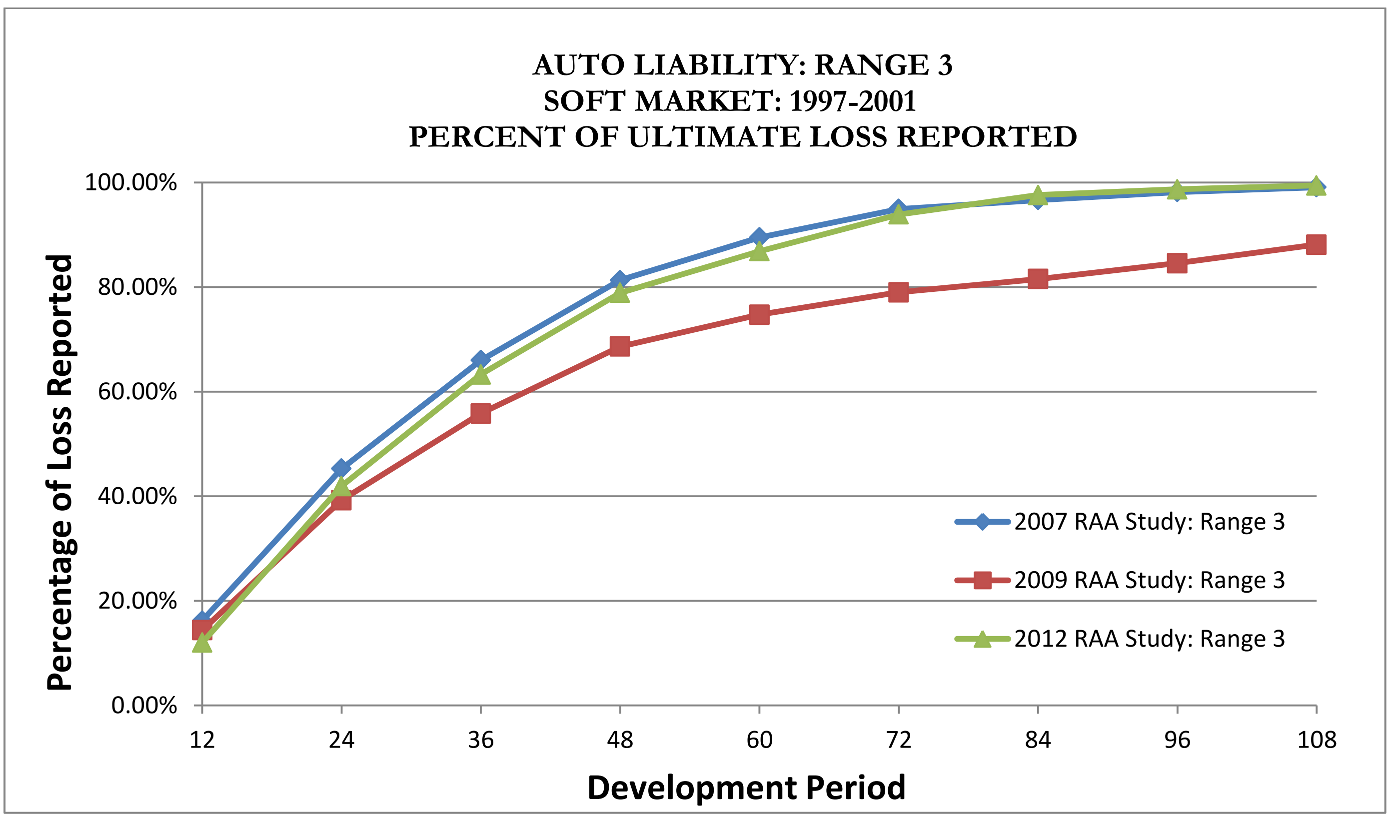

AUTO LIABILITY: RANGE 1
EXCLUDING SOFT MARKET YEARS 1997-2001
PERCENT OF ULTIMATE LOSS REPORTED
Percentage of Loss Reported
100.00%
80.00%
60.00%
40.00%
20.00%
0.00%
12
24
36
48
60
72
84
96
108
Development Period
2007 RAA Study: Range 1
2009 RAA Study: Range 1
2012 RAA Study: Range 1


AUTO LIABILITY: RANGE 1
SOFT MARKET: 1997-2001
PERCENT OF ULTIMATE LOSS REPORTED
Percentage of Loss Reported
100.00%
80.00%
60.00%
40.00%
20.00%
0.00%
12
24
36
48
60
72
84
96
108
Development Period
2007 RAA Study: Range 1
2009 RAA Study: Range 1
2012 RAA Study: Range 1


AUTO LIABILITY: RANGE 2
EXCLUDING SOFT MARKET YEARS 1997-2001
PERCENT OF ULTIMATE LOSS REPORTED
Percentage of Loss Reported
100.00%
80.00%
60.00%
40.00%
20.00%
0.00%
12
24
36
48
60
72
84
96
108
Development Period
2007 RAA Study: Range 2
2009 RAA Study: Range 2
2012 RAA Study: Range 2


AUTO LIABILITY: RANGE 2
SOFT MARKET: 1997-2001
PERCENT OF ULTIMATE LOSS REPORTED
Percentage of Loss Reported
100.00%
80.00%
60.00%
40.00%
20.00%
0.00%
12
24
36
48
60
72
84
96
108
Development Period
2007 RAA Study: Range 2
2009 RAA Study: Range 2
2012 RAA Study: Range 2


AUTO LIABILITY: RANGE 3
EXCLUDING SOFT MARKET YEARS 1997-2001
PERCENT OF ULTIMATE LOSS REPORTED
Percentage of Loss Reported
100.00%
80.00%
60.00%
40.00%
20.00%
0.00%
12
24
36
48
60
72
84
96
108
120
Development Period
2007 RAA Study: Range 3
2009 RAA Study: Range 3
2012 RAA Study: Range 3


AUTO LIABILITY: RANGE 3
SOFT MARKET: 1997-2001
PERCENT OF ULTIMATE LOSS REPORTED
Percentage of Loss Reported
100.00%
80.00%
60.00%
40.00%
20.00%
0.00%
12
24
36
48
60
72
84
96
108
Development Period
2007 RAA Study: Range 3
2009 RAA Study: Range 3
2012 RAA Study: Range 3

If we look at the reporting patterns for the non-soft market years, we see that the Range 1 and Range 2 triangles show the same reporting pattern for each of the RAA Studies. The Range 3 triangle actually shows a faster reporting pattern for the 2007 study, but this could be due to other factors as well. In comparison, the triangles for underwriting years 1997-2001, the soft market years, show a completely different result. The 2009 RAA study has a much slower reporting pattern than both the 2007 and 2012 studies. This would suggest that the soft market years have a significant impact to the overall all year weighted average reporting pattern for the 2009 study as opposed to the other studies.

We can understand that the reason the soft market years affect the 2009 study moreso than the 2005 or 2007 is because by 2009 we are further along in the development and the adverse development has more of an impact on the 2009 tail. For example, if we look at the actual triangle we can see that the additional two years of development increase the average for the development periods significantly.

RAA 2009 Study: Auto Range 2

Origin Period	12	24	36	48	60	72	84	96	108	120	132
1997	2.660	1.571	1.313	1.118	1.072	1.012	1.017	1.001	0.998	1.013	1.007
1998	3.093	1.474	1.276	1.107	1.028	1.045	0.999	1.002	0.999	1.020	
1999	2.964	1.473	1.263	1.100	1.013	1.000	1.014	1.016	1.068		
2000	2.690	1.481	1.219	1.160	1.076	1.004	1.167	1.074			
2001	2.039	1.533	1.114	1.108	1.038	1.217	1.143				

RAA 2007 Study: Auto Range 2

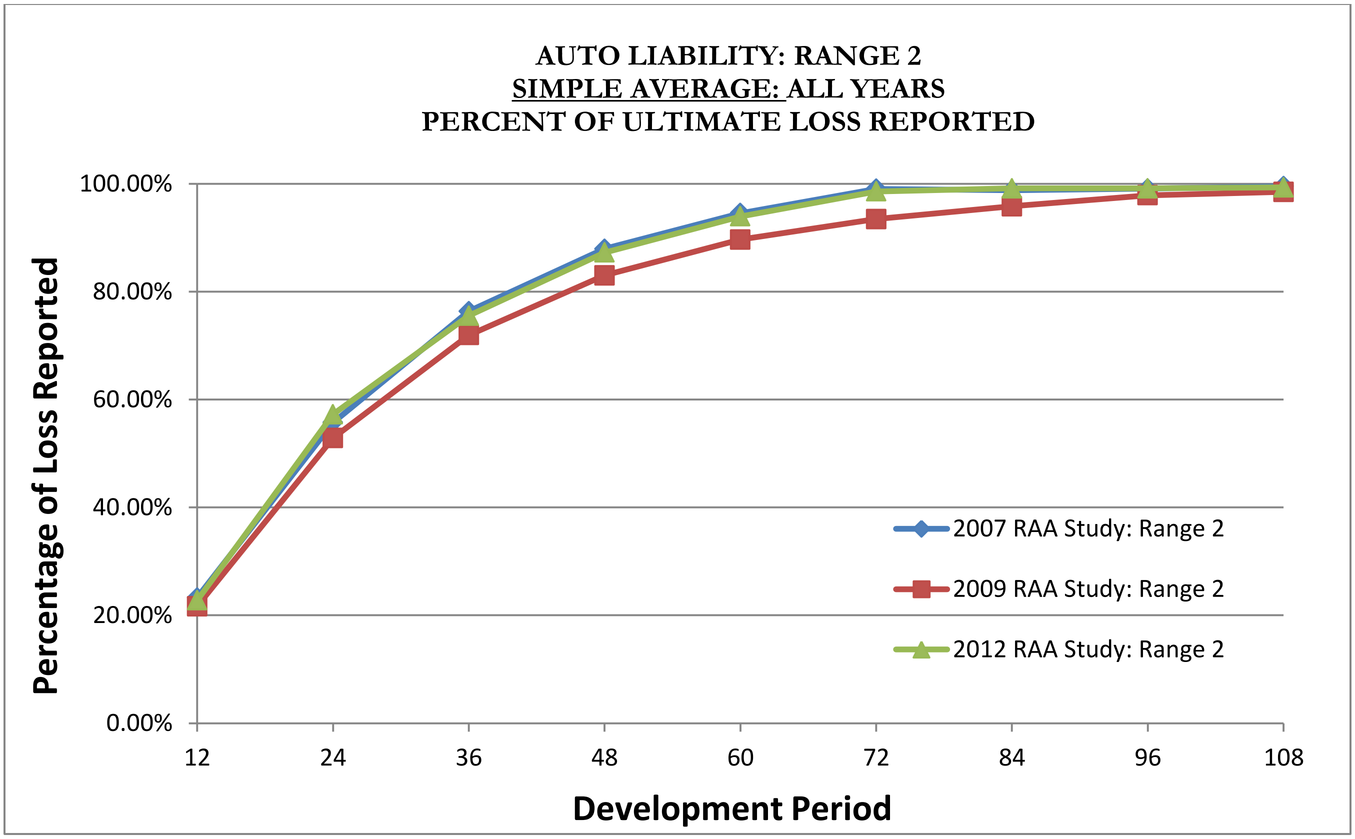
Origin Period	12	24	36	48	60	72	84	96	108	120	132
1997	2.691	1.550	1.278	1.109	1.063	1.019	1.011	0.990	0.999		
1998	3.157	1.495	1.267	1.098	1.027	1.037	1.004	0.999			
1999	3.019	1.537	1.234	1.100	1.012	0.998	1.015				
2000	2.548	1.462	1.200	1.149	1.059	1.004					
2001	2.077	1.530	1.103	1.099	1.030						

However, why do we not see a similar impact on the 2012 study?

2.2.1 Effect of Using Volume Weighted Averages

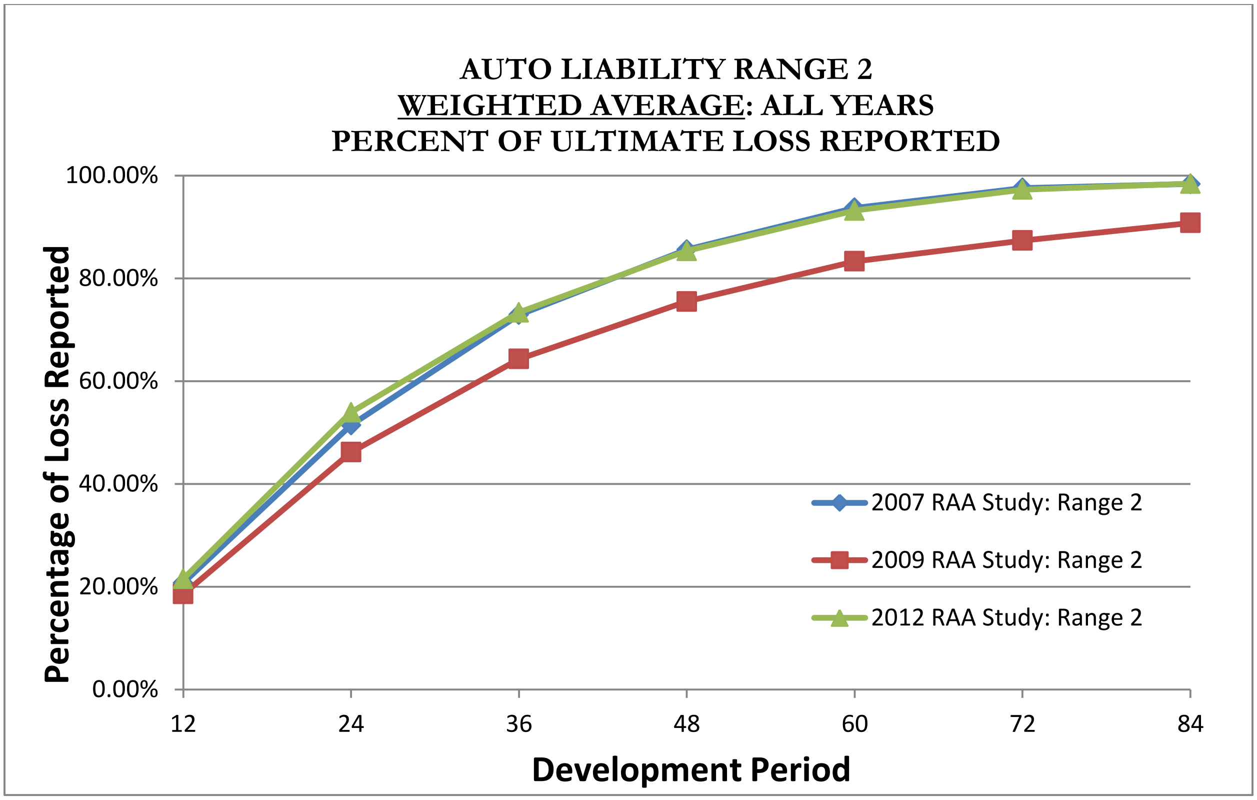
One possible explanation is that to ensure that a single company's data does not dominate the triangle in the latest study put out in 2012, the RAA scaled individual company data and adjusted the data volume by applying a certain percentage to the entire triangle. Although the magnitude of the actual development factors is not affected, the volume of losses is affected (RAA Historical Loss Development Study 2012 edition). Given that the patterns were calculated using volume weighted averages, it is quite possible that the volume of data in the 2012 study has been artificially changed, resulting in a different reporting pattern than would otherwise have been calculated.

If instead of using volume weighted averages, we use straight averages we can eliminate the distortion caused by any artificial change to the actual data. For example, if we look at the straight averages for both the Range 2 and Range 3 triangles, we see that the 2009 study is still slower than the other studies. This would indicate that the difference between the studies is not solely affected by the volume of data. However, being that the difference between the RAA studies is less when we use the simple averages, as opposed to using the weighted averages, this does lend support to the idea that the artificial change to the volume of data is affecting the comparison.



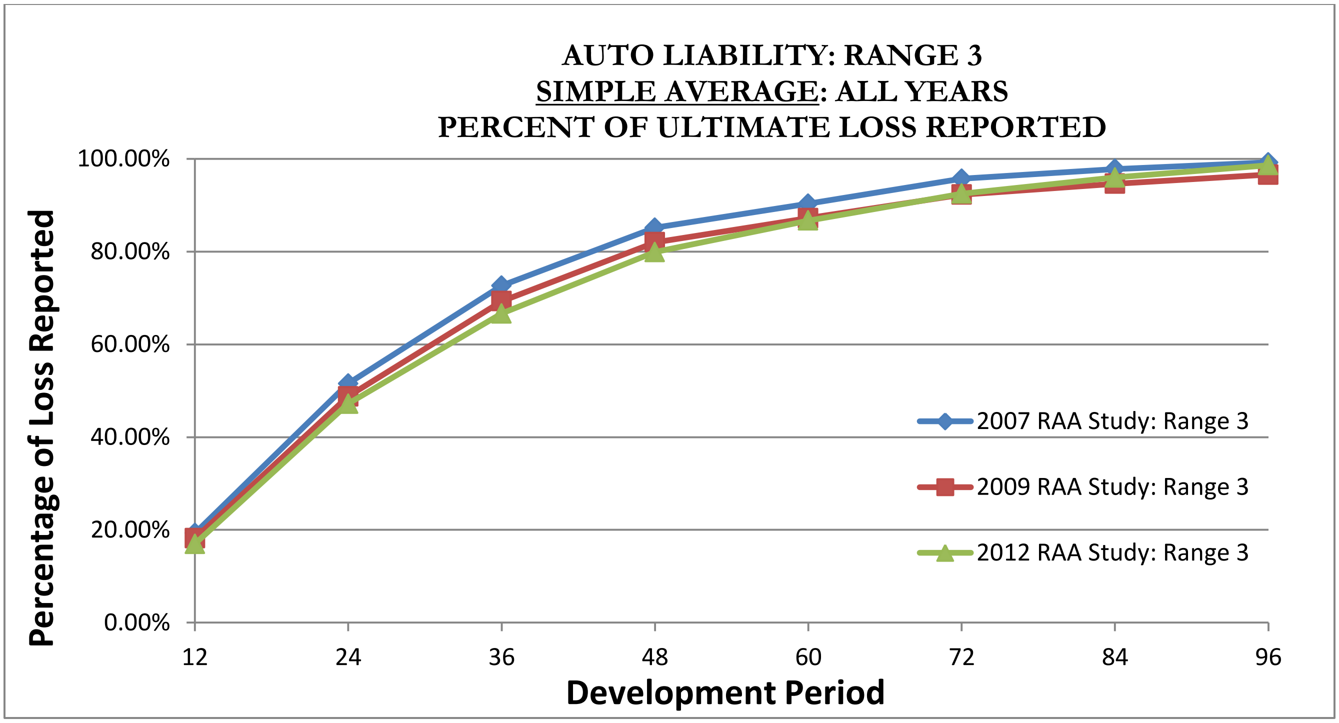
	12	24	36	48	60	72	84
2007	23.3%	55.8%	76.4%	88.0%	94.6%	99.1%	98.8%
2009	21.7%	52.9%	72.0%	83.0%	89.7%	93.5%	95.8%
% difference	-6.8%	-5.1%	-5.8%	-5.6%	-5.2%	-5.6%	-3.0%

	12	24	36	48	60	72	84
2009	21.7%	52.9%	72.0%	83.0%	89.7%	93.5%	95.8%
2012	22.8%	57.3%	75.6%	87.3%	94.0%	98.6%	99.2%
% difference	5.0%	8.3%	5.0%	5.1%	4.8%	5.5%	3.5%



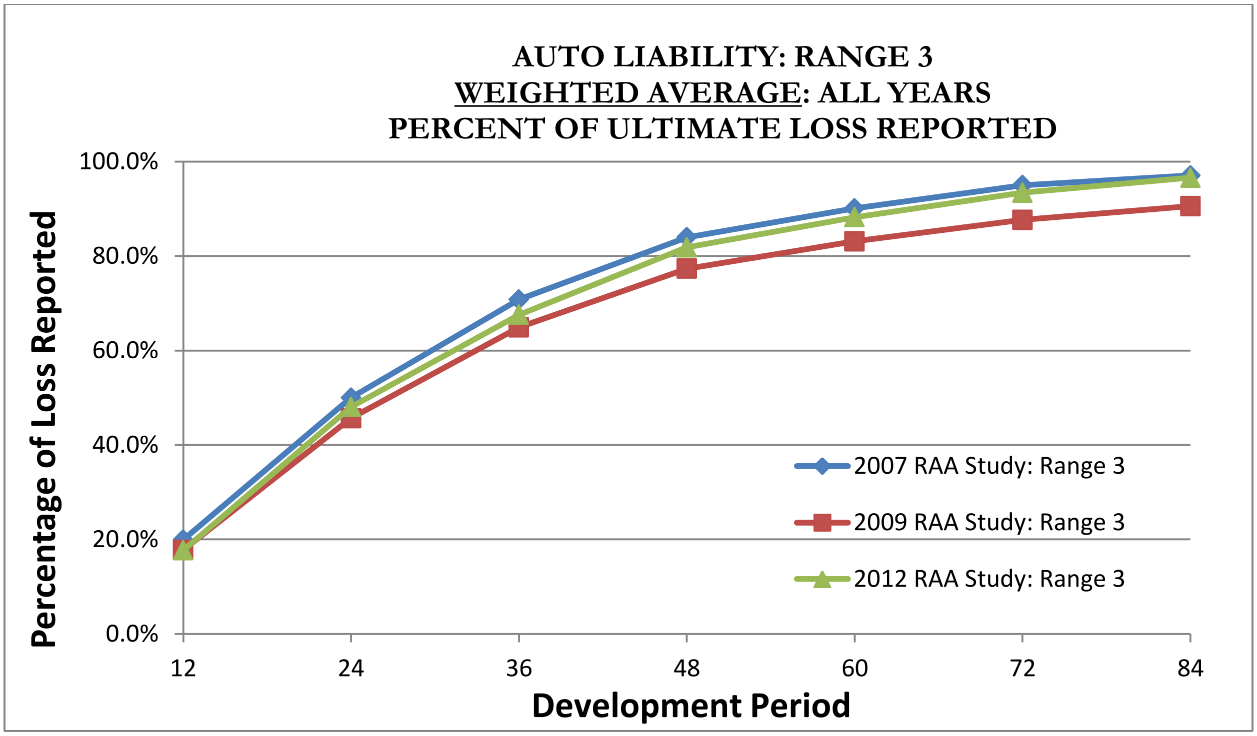
	12	24	36	48	60	72	84
2007	20.7%	51.4%	72.9%	85.6%	93.7%	97.6%	98.3%
2009	18.6%	46.2%	64.3%	75.5%	83.3%	87.3%	90.8%
% difference	-10.4%	-10.2%	-11.8%	-11.8%	-11.1%	-10.5%	-7.7%

	12	24	36	48	60	72	84
2009	18.6%	46.2%	64.3%	75.5%	83.3%	87.3%	90.8%
2012	21.5%	53.9%	73.4%	85.3%	93.2%	97.3%	98.4%
% difference	11.7%	11.4%	13.3%	13.4%	12.5%	11.7%	8.3%



	12	24	36	48	60	72	84
2007	19.3%	51.5%	72.6%	85.1%	90.3%	95.7%	97.8%
2009	18.2%	48.8%	69.3%	82.0%	87.2%	92.3%	94.6%
% difference	-5.8%	-5.3%	-4.6%	-3.7%	-3.5%	-3.6%	-3.2%

	12	24	36	48	60	72	84
2009	18.2%	48.8%	69.3%	82.0%	87.2%	92.3%	94.6%
2012	17.0%	47.2%	66.7%	79.9%	86.7%	92.5%	96.0%
% difference	-6.7%	-3.3%	-3.8%	-2.6%	-0.6%	0.2%	1.4%



	12	24	36	48	60	72	84
2007	19.9%	50.0%	70.8%	83.9%	90.1%	95.0%	97.1%
2009	17.8%	45.7%	64.9%	77.3%	83.1%	87.7%	90.6%
% difference	-10.9%	-8.6%	-8.3%	-7.9%	-7.7%	-7.7%	-6.7%

	12	24	36	48	60	72	84
2009	17.8%	45.7%	64.9%	77.3%	83.1%	87.7%	90.6%
2012	17.7%	48.0%	67.6%	81.8%	88.2%	93.4%	96.6%
% difference	-0.2%	5.2%	4.1%	5.8%	6.2%	6.5%	6.6%

2.2.2 Commutation Effect

A second possible explanation is that the RAA study is net of commutations. It is quite possible that by the time the 2012 study was done, several reinsurers took steps to commute the unprofitable business from these years.[7] Without the bad business from the soft market years in the triangle, the effect on the reporting patterns would not be as severe as it is in the 2009 study. This could explain why the reporting patterns for the 2012 study are more similar to the 2005 and 2007 study than they

[7] It is also possible that some of the unprofitable reinsurers dropped out of the RAA study.

are to the 2009 study. Although one would still see a slower reporting pattern in the 2012 study for the soft market years, the pattern would follow more closely the 2005 and 2007 study.

However, this explanation is not very likely. The soft market years were from 1997-2001, and the deteriorating results should have already been apparent to companies after a few years. This is especially true with Auto Liability, which has a shorter tail than other casualty lines. If there were any significant commutations, the impact on the triangles should have already been noticeable in the 2005 and 2007 RAA studies. Furthermore, a look at the data seems to lend support that commutations are not an adequate explanation. If we compare the actual reported losses in Range 3 for both the 2009 and 2012 studies we see the following results.

Range 3: Difference between 2012 and 2009 study (in millions)

Accident Year	1	2	3	4	5	6	7	8	9
1993	(6.3)	(12.5)	(15.1)	(13.6)	(12.8)	(10.2)	(9.0)	(9.1)	(8.2)
1994	(7.6)	(14.1)	(22.8)	(26.0)	(26.5)	(26.2)	(24.6)	(24.4)	(24.4)
1995	(7.2)	(12.9)	(16.1)	(18.8)	(14.5)	(15.0)	(15.8)	(15.2)	(14.8)
1996	(11.3)	(20.3)	(25.3)	(23.0)	(21.4)	(21.3)	(20.4)	(21.3)	(22.5)
1997	(11.4)	(14.8)	(15.2)	(20.9)	(19.9)	(18.9)	(16.2)	(15.8)	(16.3)
1998	(8.7)	(18.0)	(20.3)	(19.2)	(19.8)	(20.1)	(19.5)	(19.9)	(21.4)
1999	(5.0)	(16.8)	(20.1)	(18.3)	(19.2)	(16.7)	(15.2)	(14.7)	(21.4)
2000	(13.4)	(19.4)	(32.5)	(37.3)	(40.6)	(37.7)	(34.9)	(44.2)	(53.1)
2001	(2.9)	(0.5)	6.5	0.5	0.4	(1.8)	(9.0)	(15.7)	

Range 3: Percentage Difference between 2012 and 2009 study

Accident Year									
1993	-39.3%	-35.0%	-33.5%	-28.4%	-25.6%	-20.2%	-17.6%	-17.5%	-16.0%
1994	-39.8%	-37.1%	-44.1%	-44.6%	-43.4%	-41.9%	-38.7%	-37.3%	-37.3%
1995	-44.8%	-36.8%	-33.7%	-35.1%	-26.9%	-26.0%	-26.7%	-25.5%	-25.1%
1996	-65.9%	-54.5%	-50.4%	-43.4%	-38.1%	-37.1%	-36.4%	-37.3%	-38.6%
1997	-60.9%	-38.0%	-29.2%	-31.8%	-28.5%	-25.4%	-21.1%	-20.2%	-20.6%
1998	-51.7%	-41.5%	-33.0%	-26.0%	-24.9%	-23.2%	-22.1%	-22.4%	-23.7%
1999	-23.3%	-22.1%	-20.3%	-16.1%	-15.9%	-13.5%	-12.0%	-11.5%	-15.9%
2000	-47.7%	-33.4%	-35.9%	-32.3%	-31.4%	-28.0%	-25.7%	-30.3%	-34.2%
2001	-22.9%	-0.9%	8.4%	0.5%	0.4%	-1.5%	-7.0%	-11.6%	

The Range 3 reported losses for the years 1997-2001 in the 2012 study are significantly less than the 2009 study. However, a look at other accident years also shows a significant decrease in losses in the 2012 study as compared to the 2009 study. This would suggest that the first explanation of a data volume offset is a more probable explanation. It would be interesting to compare the future studies to the 2012 study and see if the data volume is consistent or has changed.

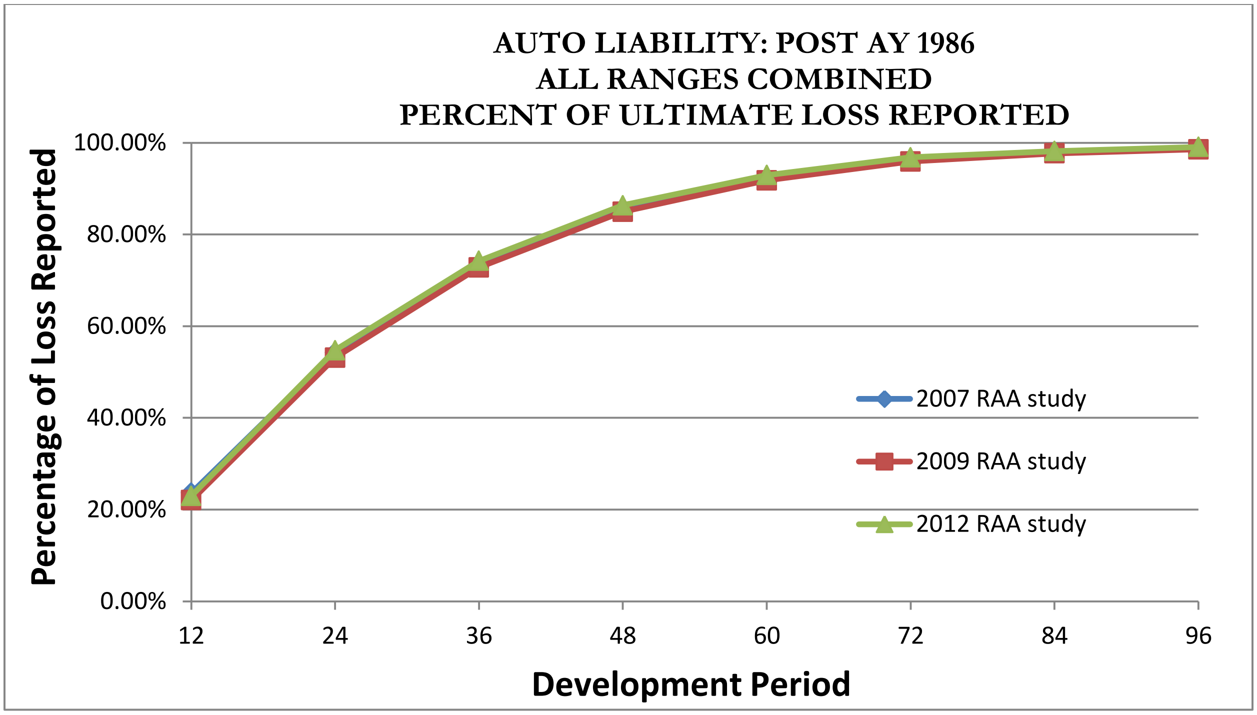
2.3 Other Explanations

2.3.1 Change in Volume of Data by Attachment Point

Another explanation for the differences is something that the RAA cautions about and that is the availability of the data by attachment point. The RAA relies on its members to not only provide the data but to also segment the data by attachment point. It is quite possible that a particular company did not have the data available by attachment point for one study, yet it was available for a prior or subsequent study. If this would be the case, then there could be a change in the data reported from one study to the next.

To check this, we can look at the data for all ranges combined to see how the patterns compare.[8]

[8] The Auto Liability triangle for the total reported losses starts with AY 1973 while the attachment point data starts with AY 1986. However, the RAA points out that the data before 1986 can be distorted due to the existence of long tailed PIP claims. Therefore, I have shown the total data starting from 1986.



In looking at the patterns, it seems that in the aggregate the reporting patterns for the various RAA studies are similar. It is only when the data is broken out by attachment point range, is there a difference. This does lend support to the hypothesis that the breakout of data by attachment point has changed from study to study. However, we previously showed that the differences in the studies are isolated to the soft market years. Therefore, it is quite possible that when looking at the total triangle, the volume of data for the non-soft market years compensate for the differences in the soft market years.

2.3.2 Number of Companies Reporting Data

It is also possible that there was a change in the volume of data being reported. As the tables below show, the number of companies reporting data changed from study to study. It is quite possible that the change in volume due to the number of companies reporting data had an impact on the reporting patterns. Furthermore, there was also a change in the number of companies reporting data for a particular attachment point. This also could have had an impact on the reporting patterns.

Number Of Companies Reporting Data

Total			
AY	2007	2009	2012
1995	17	9	10
1996	16	9	10
1997	16	9	10
1998	15	9	10
1999	15	9	10
2000	16	9	12
2001	16	11	13
2002	15	12	14
2003	15	14	16
2004	15	14	16
2005	15	14	16
2006	15	15	16

Range 1			
	2007	2009	2012
1995	6	5	5
1996	6	5	5
1997	6	4	5
1998	7	5	5
1999	7	5	5
2000	7	5	6
2001	7	5	7
2002	6	4	7
2003	7	5	7
2004	7	5	8
2005	7	6	8
2006	7	7	8

Range 2			
	2007	2009	2012
1995	7	5	5
1996	7	5	5
1997	7	5	5
1998	7	4	4
1999	7	5	5
2000	8	5	7
2001	9	6	7
2002	7	6	7
2003	8	7	8
2004	9	8	9
2005	9	7	8
2006	9	7	7

Range 3			
	2007	2009	2012
1995	5	7	5
1996	5	7	5
1997	5	7	4
1998	5	7	5
1999	5	7	5
2000	5	7	6
2001	6	8	6
2002	6	7	5
2003	7	8	5
2004	7	8	4
2005	8	8	5
2006	7	8	7

3 SOME PRACTICAL APPLICATIONS

It would be instructive to take a look at some of the explanations offered in this paper, and understand how it might affect the reserving process.

We have shown that some of the RAA data might have been manually adjusted to limit the impact of any one company and that this manual adjustment has an effect on the volume weighted averages. Therefore, it would be prudent for the actuary to keep this in mind and to realize that one's LDF selections might be distorted due to the adjustments made to the data volume. It would not be unreasonable to suggest that simple averages rather than volume weighted averages should be used in projecting RAA benchmarks.

Although the RAA triangles can be used as benchmarks in the reserving process, care must be taken when using them to make sure that the appropriate set of triangles are used. Obviously, if one uses a triangle by attachment point then one must make sure that it matches the attachment point of the experience. However, one must also be careful to determine if the RAA data is a good proxy for the company's experience. There a couple of procedures that can be used to adjust the RAA data to fit the company experience. Let us see if any of the issues mentioned above would have an impact on these procedures.

3.1 Adjusting the Triangle Using Relativities

One procedure that can be used is for a situation where the experience triangle has a different attachment point mix for different accident years. However, rather than using development factors derived from the RAA data, one might still want to project the losses based on the actual experience. There will be a concern that the historical development for a particular development period is not on a consistent basis because of the fact that the attachment point levels are not consistent across all the accident years. In the example we will use, the losses for AY 1984-1991 consists of contracts attaching at RAA Range 4, while AY 1992-2001 attach at RAA Range 3.

One can use the RAA data to bring the triangle onto the same attachment point basis through a procedure which is conceptually similar to the Berquist-Sherman Method (Berquist & Sherman). The Berquist-Sherman Method adjusts the historical paid loss data based on the current settlement rate, resulting in an adjusted paid development pattern. Similarly, in this procedure we can restate part of the triangle using a set of relativities calculated from the RAA data.

The first step is to select age-to-age development factors for both the Range 3 and Range 4 triangles. We then select which range will be restated. In our example, we will restate Range 4, AY 1984-1991 to be on a Range 3 basis so that the entire triangle is equivalent to a Range 3 attachment point triangle. We will take the selected factors from the RAA Range 3 triangle at each period and divide by the RAA Range 4 selected factors for that period. We now have relativities for each of the 12-24, 24-36 etc. periods. These relativities are then applied to each of the age-to-age factors from the portion of the triangle that contains Range 4 data. We now have an entire triangle that attaches at Range 3. When we look at the development in this adjusted triangle, we can assume that any differences one sees in one particular development period between two or more accident years are not due to the change in attachment point.

	(1)	(2)	(1)/(2)
	Range 3 Age-to-Age	Range 4 Age-to-Age	Relativity
12	2.25126	2.42411	92.9%
24	1.27361	1.26709	100.5%
36	1.24862	1.14338	109.2%
48	1.14113	1.23178	92.6%
60	1.13399	1.12219	101.1%
72	1.09131	1.04160	104.8%
84	1.07609	1.16705	92.2%
96	1.04185	1.13838	91.5%

Original Triangle: Range 4

AY	(1) 12-24	(2) 24-36	(3) 36-48	(4) 48-60	(5) 60-72
1984	2.813	2.513	2.555	2.112	1.731
1985	1.101	42.313	2.136	1.053	1.520
1986	1.417	1.512	13.592	2.128	1.013
1987	1.006	1.088	1.736	2.355	1.006
1988	1.101	3.390	5.178	1.696	1.119
1989	1.101	5.273	1.366	1.808	1.487
1990	1.149	1.124	1.115	1.506	0.864
1991	2.331	1.154	0.874	1.022	1.013
	12	24	36	48	60
Relativity Factor	92.9%	100.5%	109.2%	92.6%	101.1%

Adjusted Triangle: Range 4 * Relativity Factor

AY	(1) 12-24	(2) 24-36	(3) 36-48	(4) 48-60	(5) 60-72
1997	2.612	2.526	2.791	1.957	1.749
1998	1.022	42.531	2.333	0.976	1.536
1999	1.316	1.519	14.843	1.971	1.024
2000	0.935	1.094	1.895	2.182	1.017
2001	1.022	3.408	5.655	1.571	1.131
2002	1.022	5.300	1.492	1.675	1.503
2003	1.067	1.130	1.217	1.395	0.873
2004	2.165	1.160	0.955	0.947	1.024

In this example, the accident years we are adjusting were not from the soft market years. However, the RAA benchmarks we are using includes the slower development attributed to the soft market years. Is the underwriting year cycle effect distorting the calculated relativities? We can check this by calculating relativities from an RAA triangle that excludes the soft market years. Here are the results.

Relativity Excluding Soft Market

	(1)	(2)	(1)/(2)
	Range 3 Age-to- Age	Range 4 Age-to-Age	Relativity
12	1.85240	2.43082	76.2%
24	1.18198	1.25624	94.1%
36	1.17540	1.13989	103.1%
48	1.11454	1.22035	91.3%
60	1.10902	1.13657	97.6%
72	1.11861	0.98295	113.8%
84	1.07652	1.21306	88.7%
96	1.04258	1.17257	88.9%

Original Triangle: Range 4

AY	(1) 12-24	(2) 24-36	(3) 36-48	(4) 48-60	(5) 60-72
1984	2.813	2.513	2.555	2.112	1.731
1985	1.101	42.313	2.136	1.053	1.520
1986	1.417	1.512	13.592	2.128	1.013
1987	1.006	1.088	1.736	2.355	1.006
1988	1.101	3.390	5.178	1.696	1.119
1989	1.101	5.273	1.366	1.808	1.487
1990	1.149	1.124	1.115	1.506	0.864
1991	2.331	1.154	0.874	1.022	1.013

	12	24	36	48	60
Relativity Factor	76.2%	94.1%	103.1%	91.3%	97.6%

Adjusted Triangle: Range 4 * Relativity Factor

AY	(1) 12-24	(2) 24-36	(3) 36-48	(4) 48-60	(5) 60-72
1984	2.143	2.365	2.635	1.929	1.689
1985	0.839	39.812	2.203	0.962	1.483
1986	1.080	1.422	14.015	1.943	0.989
1987	0.767	1.024	1.790	2.151	0.982
1988	0.839	3.190	5.339	1.549	1.092
1989	0.839	4.961	1.409	1.651	1.451
1990	0.876	1.058	1.149	1.375	0.843
1991	1.776	1.086	0.901	0.934	0.988

If we compare the all year average from each adjusted triangle, we can conclude that the underwriting cycle effect can have an impact on the relativities. Therefore, if one decides to calculate relativities from the RAA study, one must keep in mind the possibility that the effects of underwriting cycle will influence the results.

1997-2014 All Year Avg. Including Soft Market

(1) 12-24	(2) 24-36	(3) 36-48	(4) 48-60	(5) 60-72
4.358	4.355	2.154	1.365	1.109

1997-2014 All Year Avg. Excluding Soft Market

(1) 12-24	(2) 24-36	(3) 36-48	(4) 48-60	(5) 60-72
4.246	3.814	1.890	1.239	1.019

3.2 Calculating the Tail

Another area in which the RAA benchmarks can be useful is in calculating the tail factor. In the long tailed casualty lines, very often there is not enough data to calculate a credible tail factor. One approach is to use the tail found in the RAA triangles. However, there are times when one is not confident that the RAA data is a perfect fit for the experience. In such a case one can use a procedure described in a paper written by the CAS Working Party on Tail Factors (The CAS Tail Factor Working Party). In this procedure, one can compare the age-to-age factors from the experience data to the benchmark age-to-age factors prior to the development of the tail. The relativities from these factors can then be used to estimate an adjustment multiplier for the benchmark tail factor. Here is an example using data from the RAA Workers' Compensation Range 2.

	(1)	(2)= (1)-1	(3)	(4)= (3)-1	(5)=(2)/(4)
Maturity	Experience Age to Age	Development Portion	Benchmark Age to Age	Development Portion	Relativity
12	3.906	2.906	3.960	2.960	98.2%
24	1.837	0.837	1.988	0.988	84.7%
36	1.325	0.325	1.408	0.408	79.6%
48	1.238	0.238	1.256	0.256	93.0%
60	1.191	0.191	1.188	0.188	101.5%
72	1.130	0.130	1.128	0.128	102.0%
84	1.081	0.081	1.064	0.064	126.1%
96	1.073	0.073	1.077	0.077	94.1%
108	1.053	0.053	1.067	0.067	80.3%
120	1.044	0.044	1.041	0.041	108.8%
132	1.029	0.029	1.033	0.033	88.1%
144	1.017	0.017	1.021	0.021	80.0%
156	1.021	0.021	1.034	0.034	63.0%
				Average (last 6 periods)	85.7%
				Tail	1.287
				Adjusted Tail	1.246

How would the results be different if we assumed that the development in our experience triangle is not affected by the soft market because these years were commuted? If we adjusted the RAA data to remove the soft market patterns, would our results change?

Maturity	Experience Age to Age	Development Portion	Benchmark Age to Age	Development Portion	Relativity
12	3.906	2.906	3.869	2.869	101.3%
24	1.837	0.837	1.731	0.731	114.6%
36	1.325	0.325	1.257	0.257	126.3%
48	1.238	0.238	1.222	0.222	107.3%
60	1.191	0.191	1.193	0.193	98.6%
72	1.130	0.130	1.132	0.132	98.3%
84	1.081	0.081	1.097	0.097	83.9%
96	1.073	0.073	1.068	0.068	105.9%
108	1.053	0.053	1.042	0.042	128.5%
120	1.044	0.044	1.048	0.048	93.0%
132	1.029	0.029	1.025	0.025	114.2%
144	1.017	0.017	1.013	0.013	130.1%
156	1.021	0.021	1.010	0.010	221.5%
				Average	132.2%
				Tail	1.203
				Adjusted Tail	1.268
				% Difference from prior exhibit	1.8%

In this scenario, it does not seem that the underwriting cycle effect impacts this procedure. Intuitively, this makes sense as we are comparing the RAA benchmark to the experience and applying the adjustment factor to the RAA tail. When we compare the two scenarios, we see that the adjustment factor for scenario 1 is 35% lower than scenario 2. However, the development portion of the tail factor for scenario 1 is 41% higher. In effect the lower adjustment factor is cancelled out by the higher tail.

4 CONCLUSIONS

We have presented evidence to show that the different RAA studies in fact do produce different results. In trying to understand the differences we have suggested several explanations. Among the explanations presented were the effects of the underwriting cycle and the manual adjustment to the volume of data. We have also shown how both of these suggestions can have an impact on how the RAA data is used in creating benchmarks to be used in a reserving analysis.

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An Integrated Approach to the Design of a Reinsurer's Data Architecture

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Abstract

A reinsurer's internal database can be a valuable source of data that has the potential of providing a competitive advantage. This data can be used to refine pricing, business steering, contract design, new product development, planning, reserving, capital utilization and much more. To maximize the value of this internal database, it is important that the data be aligned, complete, and as granular as possible. This paper presents some of the significant uses of internal data, describes some of the most common challenges and discusses elements of an ideal database. The paper ends with a detailed discussion on line of business structure and describes an ideal way of allocating data elements to a more granular unit with particular application to IBNR allocation to contract.

Keywords: Reinsurance Data, Reinsurance Information Management, IBNR allocation, Reinsurance line of business

1. INTRODUCTION

Insurers and reinsurers have long been aware of the value of data. Access to ISO and NCCI data has enabled US insurers to build sophisticated data driven models. Even more valuable than public data is proprietary data that is not available to competitors. Access to proprietary data can give an insurer a significant competitive advantage in its development of pricing parameters. The internal data can also provide an insurer with a deeper understanding of the accuracy of their pricing as well as a more detailed understanding of their own book of business. For example, a reinsurer can gain insight into the following questions:

> What were the profitability relativities of the various sub-segments of the book of business? Were large accounts or small accounts more profitable? Supported umbrella or unsupported? What were the relativities between new business and renewal business? The answers to these and many other similar questions can help a reinsurer steer its business to greater profits.
>
> How accurate were the reinsurer's estimate of the pricing components such as frequency, severity, rate level, development patterns, and so on? Even if the loss ratio estimates were accurate, a better understanding of the components can help a reinsurer fine tune its pricing and develop a more profitable book. It can also improve the reinsurer's ability to help their clients.

These and many more examples are discussed in detail in Section 2.

The proprietary data is generated during the many processes that make up the life cycle of an insurance or reinsurance contract. One normal path for the data flow of a reinsurer is that it begins with pricing where the goal is to analyze the risk and develop pricing estimates. It then continues with the underwriting and contract process where the goal is to negotiate, determine, and bind the terms of coverage. This part of the process is sometimes referred to as the administration process.

Once the contract incepts, the data flow continues with the accounting and claims functions where cedant data is entered. The data entered during this process is sometimes referred to as the booking data. Generally the data life cycle ends with finance where additional estimates are entered and the company's financials are produced.

While the main focus at each step is to satisfy the narrow requirements of the function entering the data, the value of the proprietary database is maximized when contract data can be tracked through the whole data life cycle. The technical IT capabilities exist, but the challenge is to develop a robust data architecture and implement the database protocols to support it. The impetus to accomplish this will come from those with the vision that this data can catapult internal profitability analyses to a new dimension. There is no professional in an insurance company better suited than the actuary to combine the business vision with the technical capabilities required to execute, especially in North America.

This paper explores

1) Specific ways in which a reinsurer can use their internal data to improve their competitive position
2) Some common issues that inhibit the use of a reinsurer's internal database
3) Some basic concepts underlying the ideal design of a reinsurer's database that will allow the reinsurer to maximize its ability of using it as a competitive tool.

The paper concludes with an in-depth discussion of the line of business attribute as well as an allocation approach applied to IBNR.

2. DATA USAGE

2.1 Internal Data Used to Improve Pricing and Business Steering

2.1.1 Actual versus Expected (AvE) Analysis. This is critical in the effort to validate or fine-tune pricing assumptions. The most elementary application is a comparison of actual loss ratio and pricing expected loss ratio. AvE can be expanded to validate assumptions on: frequency, severity, paid and incurred patterns, cause of loss, probability of multi claimant occurrences, primary ELR, primary rate levels, loss trends, et cetera. A sophisticated AvE analysis will separate the catastrophe from the non-catastrophe exposures and allow analysis of the contract at coverage level.

2.1.2 Development of Pricing Parameters. Assumptions on pricing parameters such as expected loss ratios, loss development patterns, frequency and severity assumptions, trend factors, size of loss distributions are essential to any sophisticated pricing analysis. Generally, most of these assumptions are developed from industry data. This data is broadly available and, therefore, may not provide any competitive advantage. Any ability to supplement the industry data with internal data creates the opportunity to gain a competitive advantage. Internal data can be even more valuable in cases where little or no industry data exists. For specially designed reinsurance covers such as clash covers, do we know their historic experience? Can we distinguish between the loss experience of separate components of clash covers such as runaway allocated loss adjustment expense (ALAE), multi-line accumulation, multi-insured accumulation, extra contractual obligations (ECO) and excess of policy limits (XPL)?

2.1.3 Business Steering. What are the profitability relativities of large clients versus small clients, new reinsurance contracts versus renewals, mature primary books of business versus new ventures, broad multi-line reinsurance contracts versus very specifically defined covers? Similarly what are the profitability relativities of single state versus multi-state WC reinsurance covers, reinsurance covers on admitted carriers versus excess and surplus lines carriers, claims made versus occurrence medical reinsurance covers, and small law firms versus large law firms liability policies? How do supported versus unsupported umbrella reinsurance covers compare? How about primary versus excess umbrella? What are the separate loss rates for the auto versus general liability exposure of umbrella covers? Can we separate the catastrophe versus non-catastrophe components when calculating profitability? These are just examples of the many questions that one can ask when deciding on a strategy of choosing what business to reinsure.

2.1.4 Contract Features. Many contract features have an economic impact on the contract profitability that is at best estimated and frequently totally ignored. One such example is the cost of covering ECO and XPL. Another example is a treaty clause that gives a cedant choosing to non-renew a treaty, the option to cancel on a run-off basis or a cut-off basis. There is little or no industry data that can be used to quantify the impact of many standard (and non-standard) contract terms. Properly coded internal data can provide the required data.

2.1.5 Renewal Analysis. At the annual renewal of a treaty, we perform the standard experience and exposure rating to arrive at a quote. If this treaty has been written for several years, we can examine how well these pricing methods have predicted ultimate treaty results in prior years. Consistent biases may indicate something is not adequately considered in the pricing. An analysis of the complete profitability of the relationship with the client is also important. This is especially true when making a difficult decision on a particular renewal.

2.1.6 New Product Development and Client Services. Detailed data on the cause and consequence of loss, industry segment, subline, et cetera can help a reinsurer develop and price profitable new products. Alternatively, these insights can be shared with clients to help them become more profitable. The ability of a reinsurer to use their own data to help clients better understand the profit drivers of their business can significantly strengthen the value added by the reinsurance relationship.

2.2 Internal Data Used to Improve Internal Processes

2.2.1 Pricing Reserving Linkage. The expected loss ratio, premium earnings pattern and expected incurred and paid loss lag patterns of the contract are important feeds from Pricing to Reserving. In addition, Pricing's most recent view of past years' results can be important input into the IBNR calculation.

2.2.2 Recalculation of the Reinsurance Layer Expected Loss Ratio (ELR) At the time of pricing, the reinsurer makes assumptions on the primary ELR, primary rate changes, underlying claim frequency, loss trend, et cetera. Many of these assumptions are known with much greater certainty a year or two after contract inception. Yet for non-proportional covers, losses are still mostly unreported. A properly designed pricing database would allow easy, automatic recalculation of an updated ELR for the reinsurance layer using the more recently known values of these pricing parameters.

2.2.3 Accumulation control. A key part of risk management is tracking a company's loss exposure to a single event. The most obvious example is tracking loss exposure to a natural catastrophe (nat cat) such as a hurricane or an earthquake. Accumulation of loss by nat cat scenario (San Francisco earthquake or Gulf Coast hurricane) from pricing models is a standard feature of most catastrophe modeling tools. Casualty lines of business are also exposed to accumulation of loss from a single event. Examples include asbestos, various pharmaceutical events, and environmental catastrophes. In the absence of detailed data by contract, liability accumulations by insured, product, industry segment, et cetera are a challenge, especially for a large multinational reinsurer.

2.2.4 Asset Liability Matching & Capital management. A key requirement of both enterprise risk management and sophisticated investment management is an understanding of the probability distribution of future cash flows. Most specifically both the mean and variance of the duration of liabilities need to be estimated. Automated data feeds from the pricing database to the reserving database and from there to the enterprise risk model support this process.

2.2.5 Planning. Each year, reinsurers develop a plan detailing the expected premium, loss ratio and profit by line of business expected in the following year. A sophisticated planning process generally starts with individual planning of all large contracts. An automated feed of premium, expected loss ratio, expected cash flows and expected profitability for in-force contracts by line of business enables efficient planning down to contract level.

2.2.6 Legal Entity Data. Legal entity data is required, at a minimum for regulatory purposes. This data may also be required for tax purposes and for rating agencies. For a large global group with many legal entities (in some cases hundreds of legal entities) it is not a trivial task to ensure accurate legal entity data. A properly constructed and maintained database can simplify this process.

3. DATA ISSUES

Some of the main difficulties encountered in the goal of building an ideal internal database are discussed below.

3.1 Data Completeness

It is important to ensure that the many valuable data elements that are calculated during the pricing process or during the claims management process are stored in a database. These include

3.1.1 Expected Loss Ratio. The individual contract pricing expected loss ratio needs to be captured and transferred to reserving to serve as an a priori loss ratio.

3.1.2 Pricing Loss Lag Patterns. Portfolio patterns are generally available from reserving. However, these are historic patterns reflecting historic business mix, attachment points, et cetera. Pricing patterns reflect changes in business mix, attachment points, et cetera At a minimum these can be aggregated to serve as a check on the reserving patterns. Ideally, the individual account patterns can be used to more accurately allocate IBNR to individual accounts and to measure profitability by account.

3.1.3 Subject Premium. The reinsurer will always record the reinsurance premium. For conducting rate level and trend analyses, the reinsurance premium alone may be insufficient. The reinsurer should strive to record the underlying exposure. For example, for personal motor it would be vehicle count. For commercial motor it might be miles driven or number of power units. For hospital liability it might be number of beds. This would enable the reinsurer to track excess loss costs and excess frequency relative to an absolute exposure base that is not affected by the insurance cycle. If that is not available, then the subject premium (or equivalently the reinsurance rate) should be recorded. This, at least removes the effect of the reinsurance cycle on the exposure base.

3.1.4 Detailed Pricing Data for Advanced Applications. This includes frequency and severity expectations as well as exposure and experience rating details. For example, tracking the expected loss estimates developed from the exposure rating and experience rating of each account and comparing each of these estimates to the actual developed ultimate loss can provide feedback on how well each of these pricing approaches is performing. Capturing the expected primary loss ratio enables a straightforward update to the pricing a priori loss ratio as primary loss ratios become known.

3.1.5 Ground up Loss. When a reinsurance agreement covers an excess contract, the ground up loss needs to be recorded. For example, a reinsurance agreement covers 80% of a $4 million xs $1 million layer on a primary policy. This primary policy is excess of a $10 million lower layer covered by a different primary insurer. There is a ground up $15 million loss. The first $10 million is covered by the first primary policy. The second primary policy records a $5 million gross loss. It keeps the first $1 million and cedes 80% of the next $4 million to the reinsurer. The ground up loss reported by the second primary insurer to the reinsurer may be defined as $5 million. However, to develop accurate size of loss distributions we would need the full $15 million loss.

3.1.6 Other Claim Data. Without belaboring the point, cause of loss, consequence of loss, ECO/XPL claims, et cetera need to be recorded in order to enable sophisticated analyses. Events with multiple claimants need to be identified and the claim data split by claimant. Claims that exceed the reinsurer's layer need to be identified and where possible, the full market loss should be recorded.

3.2 Data Alignment

3.2.1 Contract ID. It is essential to have the ability to track the contract (or contract segment) from pricing database to underwriting and contract database to accounting and claims database to Finance database. If different contract IDs are used in pricing and in the underwriting systems it can make it difficult, if not impossible, to compare actual experience versus expected.

3.2.2 Line of Business. Many reinsurance contracts provide coverage for more than one line of business. For example, a casualty excess treaty may provide coverage for general liability, umbrella, motor and workers compensation. We need to ensure that for each contract the premium and loss allocations to line of business are consistent in each of the applications. The following are some examples where an issue may arise. A homeowners quota share may be booked by accounting as 100% property or while in pricing it was split between property and liability. The property portion itself may be booked by accounting as 100% fire while in pricing it was split between fire and hurricane. Database protocols need to be established to ensure that all contracts are recorded consistently.

3.2.3 Data Corrections

When data is passed from one application to another, a misalignment may occur when data is corrected in the original application. Frequently the interface between the two applications occurs only once and the corrected data is not sent to the receiving application. Database protocols need to be established to handle such cases.

3.2.4 Other examples of data alignment challenges

Cedant must be entered identically in all applications. While this may appear trivial, it is not. Even small differences in the spelling of a cedant may make it difficult to combine data by cedant. As we discuss later, selecting the cedant from a drop down menu is an ideal way of solving this issue.

A quota share reinsurance cover on a primary excess contract can be called proportional, following the reinsurance structure, or excess based on its absolute structure. Clear database protocols need to be in place to ensure that a quota share reinsurance cover on a primary excess contract is adequately and consistently encoded in all systems.

A no-claims bonus can be considered commission (an expense) or negative premium. Again a clear set of guidelines is necessary.

A reinsurance contract may be written to provide coverage on a losses occurring basis. In this case, there is a premium portfolio transfer to the reinsurer representing the reinsurer's portion of the primary insurer's unearned premium reserve at the inception of the reinsurance contract. In addition there will be quarterly installments paid to the reinsurer representing the reinsurer's portion of the primary insurer's premium incepting during the term of the reinsurance contract. If the reinsurance contract includes a commission paid by the reinsurer to the primary insurer, practices can differ whether to book the portfolio premium net of commission or gross of commission. Clear database protocols need to be in place to guarantee clarity and consistency.

As part of pricing, a contract specific loss payment pattern may be derived from the cedant's submission. In such cases, it is then necessary that this contract specific pattern be used for the contract profitability calculation in the profitability evaluation systems.

3.3 Data Granularity

3.3.1 Granularity. It is important that data be entered in the most granular form possible. For example, premium and loss on a treaty covering medical malpractice should be entered as medical malpractice and not to the more general professional liability. This level of granularity is necessary to enable a reinsurer to monitor the profitability of its medical malpractice business.

3.3.2 Multiline Contracts. It is important that data be separately entered for each line of a multiline contract. For example, property treaty data should be entered separately for fire and for nat cat and not entered as 100% fire. An auto quota share treaty that covers both auto liability as well as auto physical damage should be split to show premium and loss separately for liability and physical damage and not entered as 100% auto liability.

These line of business issues will be discussed in greater detail in Section 6.

3.4 Allocations

Probably the most important allocation to individual contract is IBNR. Since in most cases, IBNR is calculated at the portfolio level, it is necessary to allocate IBNR to contract in order to evaluate contract profitability. A simplistic allocation methodology may cause serious data quality issues with account profitability data.

Internal expenses, capital charges and taxes all need to be allocated if full profitability data at the contract level is desired. Again, care is necessary in developing the allocation methodology.

In section 7, this paper presents an allocation methodology that will generally produce reasonable results.

3.5 Data Inconsistency

Two examples of situations where different systems may calculate the same thing in different ways are the currency conversion routine and the discounting methodology. It is important that a uniform methodology should be used in all applications.

4. BASIC PRINCIPLES

4.1 Consistent and Aligned Contract Identification and Structure

4.1.1 A Single Internal and Universally Used Contract ID. An internal contract ID should be created for each contract and used throughout the life cycle of the contract, from submission through pricing, underwriting, accounting, claims, and finance. This ID should be contained on each record of every contract. It may be necessary to have multiple contract IDs, especially when using external vendor applications. For example, a reinsurer may use externally provided software for its premium and loss accounting or for its contact administration. These may have protocols regarding contract ID that are not consistent with the internal ID. This is acceptable. These external contract IDs, however, need to be linked to the internal contract ID and at least in the internal databases, the internal contract ID should appear on every record as well.

4.1.2 A Single Contract Structure. A single consistent contract structure should be used throughout the life cycle of the contract. This includes a consistent line of business structure, type of business (proportional versus non-proportional) structure, et cetera.

4.2 Full Income Statement at Granular Level.

Assume for example that the lowest granular level of data is line of business/underwriting year/contract. Call this the contract unit. All elements of the income statement should be calculated or allocated down to the contract unit. This includes all premium, commission and loss (including IBNR). It also includes internal expenses, capital charges and taxes. This would be done on both a nominal and discounted basis.

If all income statement components are pushed down to contract unit level, it will be possible to develop full profitability analyses on any dimension. In particular the data will be available to answer the questions raised in section 2.1.3.

4.3 Data Consistency

4.3.1 Consistent (unique) Definition and Rules for Each Variable. Each variable should be clearly defined with a unique meaning and set of rules. Examples of data elements that require special attention were discussed in Section 3.

4.3.2 Consistent Protocols for Each Calculation.

The protocols for currency conversion and investment income calculation can be very complex. Should a single point conversion be used or should a dynamic conversion routine be used? When using a dynamic conversion routine what dates should be used for estimates? In either case, should the rates be daily, monthly, or quarterly? Should they be end of period or mid-point?

4.4 Industry Standards

An industry data standard would help to align the consistency of submission, contract and financial data and to enhance the data quality end to end. Many of the issues discussed throughout this paper, could be more easily resolved if there was an accepted industry data standard. An industry data standard is currently not available.

5. IMPLEMENTATION

5.1 Corporate Culture Supporting Quality Internal Data

A high quality integrated internal database requires a corporate commitment to invest the necessary funding and resources. This is especially true for a large multi-national reinsurer. Local practices that differ by region may need to be consolidated. Practices that favor a narrow departmental view may need to be replaced by practices that support the broader corporate benefit. Specific resources dedicated to data quality management and review may need to be created.

5.1.1 Communicating and Marketing the Value of Data. Employees across the company need to understand that the data they enter is crucial to the continuing success of the corporation. Senior executives should stress that the internal data is a key component of competitive advantage. Executives should periodically publish actual examples of how data was used to generate profitable business. Occasional awards to employees responsible for significant improvements in data value should be given. These types of recognition will motivate employees towards high standards of data quality. Management support of data quality initiatives is critical to validate the necessary costs.

5.1.2 Responsibility for Data Quality. If data is really viewed as a source of value then responsibility for data entry needs to be assigned with the goal of assuring a high level of data quality. If responsibility for data entry stops at a junior level, it is not likely that the highest standards of data quality will be achieved. When a reinsurance agreement is consummated and the contract is entered into the reinsurer's database, a senior member of the deal team should sign off on the coding.

5.2 Centralized Data Functions

An integrated database requires some degree of central oversight. One way of accomplishing this is a small specialized central data unit under the guidance of a data management board that represents the various corporate functions and business units. This board will make the tough decisions on tradeoff between cost and granularity.

5.2.1 Single Uniform Definition of all Data Elements. A data dictionary needs to be established that is used throughout the company. It needs to be mandatory that all systems utilize the data dictionary. This includes the definition of each data field and all allowable values. For example, the field "Type of Business" will mean the same thing and have the same allowable values in each system. If there is a need for multiple versions of a data element, separate names must be used and each version must be clearly defined. For example, the original pricing expected loss ratio for a contract may be modified to reflect information received after contract inception. This modified expected loss ratio is used by reserving as the contract a-priori loss ratio. These two expected loss ratios need to separate names and definitions. Education and training, including online easily available reference material, needs to be available.

5.2.2 Single Set of Booking Rules. Similarly, a single set of rules needs to be promulgated to define how contract data is to be recorded. Specifically, rules need to clearly define how to deal with multi-year contracts, nat cat exposure on homeowners contracts, proportional shares of excess contracts, and no claims bonuses, et cetera.

5.2.3 Data Quality Reviews. Peter Drucker famously said, "What gets measured gets improved." A common finding in the data quality area is that any field that is not used or reviewed can be expected to have very low data quality. A detailed discussion of data reviews is beyond the scope of this paper. The following describe major components of a data quality review.

> *Data Validity* – Data fields are tested to ensure that they contain only valid data. For example, a numeric field whose values should be between 0 and 1 can be checked to verify that all entered data is between 0 and 1. A field containing a code can be checked to verify that the entered code is valid. Ideally, data should be automatically verified at time of entry. A data quality review would check fields that are not automatically verified.
>
> *Data Reasonability* – Data fields are tested to ensure they contain reasonable values. For example, an expected paid loss lag pattern for a reinsurance contract is designed to display the cumulative percentage of ultimate loss that is expected to be paid at each yearend following the contract inception. Values that do not appear reasonable can be identified either by comparing them against a predetermined reasonable range or by testing for outliers. Values that fail the reasonability check are not necessarily invalid. There may be a reason why the data for a particular contract behaves differently than expected. These values are candidates for further investigation.
>
> *Data Alignment* – Data accessed from different sources that are expected to be similar can be compared. For example, the expected premium by line of business within contract can be compared to the actual accounted premium by line of business within contract. Large differences are candidates for further investigation. This example will be covered in great detail in section 6.
>
> *Data Accuracy* – Data is manually compared to source documents. This is standard data auditing.

5.3 Technical Standards

5.3.1 Header records. At the first entry of a contract into company systems (usually this will occur when the submission is received), a header record should be created. This record will contain basic information about the contract, most importantly an internal contract ID. This header record will be contained within all systems containing contract data. It will be part of any record where data is transmitted from one system to another.

5.3.2 Single internal contract ID. This single internal contract ID from the header record is critical to ensure that all contract information can be tracked and combined. Especially when some of the systems are external, a unique contract ID cannot be ensured. Some systems may require a purely numeric contact ID while others will have alphanumeric components. The header internal contract ID will always be the same and this allows each system to define, if necessary, a second contract ID according to its unique internal system requirements without compromising the ability to match contract data in different systems.

5.3.3 Drop down menus. Wherever possible, data entry should be from a drop down menu rather than entered directly. For example, a cedant company name could be directly entered. However, this will likely lead to multiple versions of the name. An ideal way to ensure that the cedant company name will always appear identically the same, is to force that data element to be selected from a drop down menu.

5.3.4 Single Data Warehouse. Ideally all contract information should be stored in a single data warehouse. This should include data from submission, pricing, underwriting and contract, accounting and claims, IBNR, and finance.

5.3.5 Golden copy. Original data is often fed into downstream systems and from there it may be fed further downstream. Each data transfer carries with it the risk of data modification. There may be criteria that restrict full data transfer. For example, non-traditional transactions or intra group retrocessions may be excluded. In other cases, data may be modified by currency conversions, line of business mappings, et cetera. Within the data warehouse, each data element should have a "golden copy." This is the original and most accurate source for that data element. For example, the pricing expected loss ratio "golden copy" is the one that comes directly from pricing.

A more ideal solution may that each data element is only stored in one place. All reporting is handled by dynamically linked tables and queries. This may be more easily accomplished in universe-based data environment.

5.3.6 Mapping matrix. In some instances, it may not be possible for the coding in two systems to be identical. This is not a desirable situation and it violates the ideals described in this paper. It may, however, not be economically viable to correct the situation. In such cases, it is important to create a mapping matrix that shows how to map from one structure to the other.

5.3.7 Data Extraction and Report Generation. Data necessary for an analysis may need to be drawn from several data sources, each with a different reporting tool. This can be a daunting task for many potential users who are not expert on each data source and reporting tool. Databases and reporting tools should be designed to make data accessible to all users. Wherever possible, screens should be standardized across reporting tools. An online facility should be available to help users find the data they need.

6. DETAILED DISCUSSION OF LINE OF BUSINESS (LOB) ISSUES

6.1 LoB Structure - Line of business is a complicated combination of different characteristics. The lines of business in the US NAIC Annual Statement include: peril (fire, earthquake), industry segment (farmowners, homeowners), coverage (occurrence versus claims made), object insured (airplane in aircraft coverage, ship in ocean marine coverage, automobile in auto physical damage coverage), et cetera. The lines of business used by many reinsurers are even more complex. For example, umbrella and clash are really coverage combinations of underlying lines.

The LoB attribute may have special importance to a reinsurer since this may be the most granular level for the accounting of a reinsurance contract. For example, a single reinsurance treaty may cover many primary segments of business. In addition to the segments mentioned above, these may include personal and commercial segments, different classes of business such as lawyers liability and accountants liability, and so on. On a reinsurer's books, the premium and loss for the treaty may only be split into lines of business.

Ideally, the LoB attribute would be split into at least these four attributes: industry segment, object insured, coverage and peril. Such a split allows for a much richer data structure. This may be difficult to implement because of cost considerations and because of culture shock. If this split Lob structure cannot be implemented, the following issues need to be considered.

6.1.1 Nat Cat Exposure on Other Lines of Business. Property nat cat is generally a subline of property. However, many other lines, including workers compensation, motor, marine, and aviation are also exposed to nat cat events. Let us take motor as an example. Unless we duplicate the nat cat structure into motor, we are faced with the choice of either coding the exposure to nat cat (in which case it will be considered property business and not motor) or to motor (in which case we will not be able to identify it as nat cat). Either way, how does a reinsurer track its nat cat experience on motor business?

One possible solution is to utilize the pricing nat cat component of the pricing expected loss ratio. This can be applied to the earned premium to obtain an estimate of the portion of the earned premium covering the nat cat exposure. The nat cat losses can be identified by the cause of loss code. This approach provides a breakdown of the premium and loss into cat and noncat. This approach requires a high quality alignment between the pricing ELR data and the premium and loss database, good data quality for the cause of loss data, and the ability to insert this data into the standard corporate profitability reports.

6.1.2 Personal versus Commercial. This is very similar to the above situation. Unless we duplicate lines of business we may not be able to distinguish between personal and commercial experience.

Examples are: nat cat on commercial property vs homeowners and auto assigned risk on commercial vs personal auto.

6.1.3 Coverage. Most frequent example is claims made vs occurrence. If a treaty covers both, how do we separately code the premium and loss?

6.2 LoB Alignment.

For a multiline treaty, a line of business structure needs to be defined and the premium and loss need to be allocated to the lines of business. In some cases, the submission data used for pricing and the accounting data are provided on a consistent basis and the coding is straightforward. In other cases the data is not provided on a consistent basis and the coding can be challenging. For the reasons discussed in section 2, it is important that the structure and allocation be identical (or at the very minimum aligned) throughout the life cycle of the contract.

The following outlines a process to achieve this goal.

6.2.1 The LoB Structure Available for Coding is Identical in All Systems. This includes pricing, underwriting, accounting and finance. While this may sound obvious, this is not always the case.

6.2.2 The LoB Structure is Set During the Pricing Analysis. This structure will be based on the submission data and the expected accounting data. The pricing premium for each LoB of a multi-line treaty with a single indivisible premium rate will be calculated in a way that expected profitability is equal among the LoBs.

6.2.3 The Pricing LoB Structure and Premium Allocation is Fed into the Underwriting Systems. The underwriter has the ability to adjust the pricing structure and allocation but they must be aligned.

6.2.4 Aligning the Pricing and Reporting LoB Structure. The case of a non-proportional treaty with a single non-divisible rate against subject premium is discussed first. Losses are individually reported with full detail.

6.2.4.1 Submission Information is More Granular than the Accounting Information. A reinsurance professional liability treaty covering lawyers liability and accountants liability, will be used to illustrate the issues. Assume the submission provided detailed experience. Separate loss models were developed for the lawyers liability business and the accountants liability business. These loss models were combined and a single rate was quoted to the cedant for their professional liability subject premium. In the pricing database, based on the individual loss models, the reinsurance premium was allocated to the two sublines in a manner that made them equally profitable. The accounting data is reported with losses separately coded to lawyers liability and accountants liability but with a single premium for professional liability.

One alternative to alignment is to separately code each of the pricing and accounting data to the maximum granularity available. In the above example, the pricing data is separately coded to accountants liability and lawyers liability while the accounting data is coded separately for the loss data but the premium data is combined. Theoretically, the pricing data can then be used to separate the aggregate professional liability premium into the sublines. This will allow profitability analyses, AvE analyses, recalculation of APLR for reserving, et cetera by subline. In practice, this approach has the following two disadvantages.

This additional step will need to be performed at the contract level for each separate analysis of lawyers liability versus accountants liability, thus creating an inefficiency.

Many standard reports will not include this extra step and will thus provide incomplete data.

The preferred approach is that at the time the individual accounting records are entered, the premium is separately coded to the two sublines according to the percentages coming from the pricing analysis. Since the individually accounted losses will have detailed coding from the cedant, the losses will be accurately recorded by subline. This data will now flow into all the standard corporate reports and allow for automated reporting of detailed profitability data by professional liability subline.

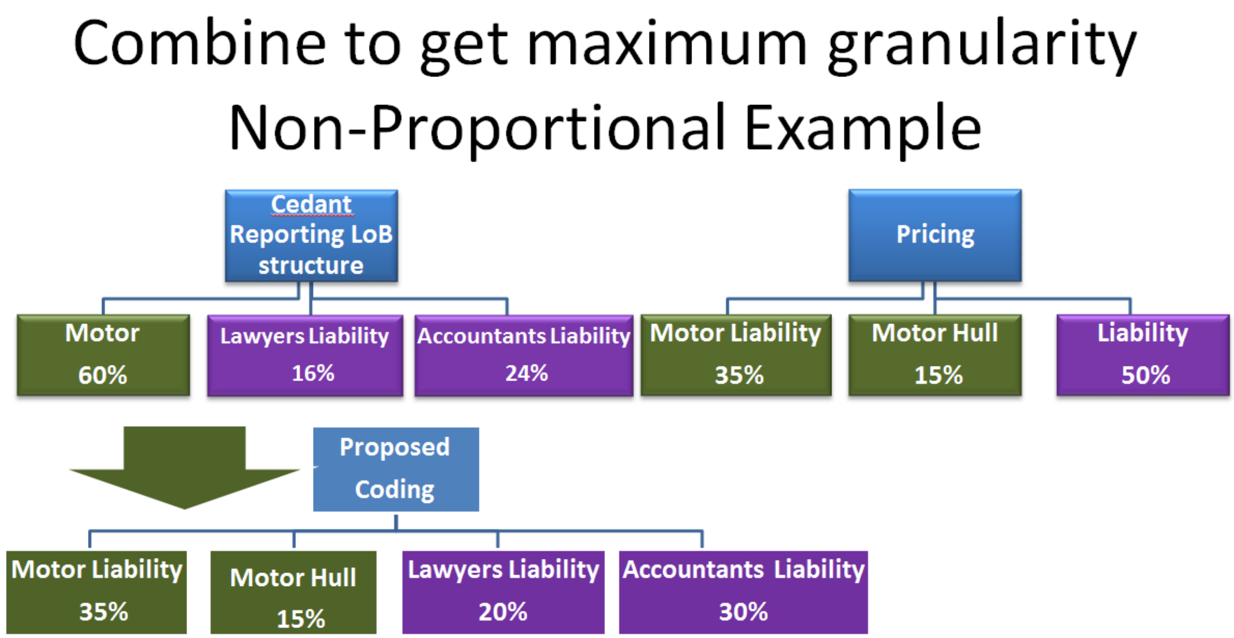
Please note that in this case even if the cedant reported a premium split between lawyers and accountants based on primary exposure that was different than the pricing percentages it is likely that the pricing percentages should be used. The reason is that the pricing allocations estimate exposure at the excess layer covered by the treaty. Primary premium distribution may not be the best indicator of how to distribute the excess premium.

6.2.4.2 More Complex Example. The following chart illustrates a more complex example. Here, the treaty covers multiple line of business. In some cases, the pricing information is more granular and in some cases the cedant reports are more granular. The proposal below, is an effort to maximize data granularity.

In this example we accept from pricing that the treaty is 50% liability and 50% motor and that the motor premium is split 70% liability and 30% hull. We accept from cedant reporting that the liability premium is split 40% lawyers and 60% accountants.

Combining all this we get the following distribution:

Reporting LoBs	**LoB %**	Calculation
motor liability	**35%**	.70 X .50
motor hull	**15%**	.30 X .50
lawyers liability	**20%**	.40 X .50
accountants liability	**30%**	.60 X .50



The mapping matrix referred to in section 5.2.7 for this example is shown below.

Pricing LoBs

Reporting LoBs	LoB %	Motor Liability 35%	Motor Hull 15%	Liability 50%
motor liability	35%	100%		
motor hull	15%		100%	
lawyers liability	20%			40%
accountants liability	30%			60%

6.2.4.3 Proportional Example. The proportional treaty case is generally treated the same way with two important differences:

If the cedant data differs in the allocation percentages from the original pricing expectation, then we will accept the cedant percentages. The reason for this difference is that in the proportional case the different allocation percentages are assumed to be caused by a shift in the underlying exposure. Please note that in the non-proportional case, we can also take into account shifts in underlying exposure. But, in order to do so we need to store deeper pricing information. In addition to the expected pricing premium by line, we need to store expected underlying cedant exposure and excess intensities by line. This would represent a nice additional sophistication.

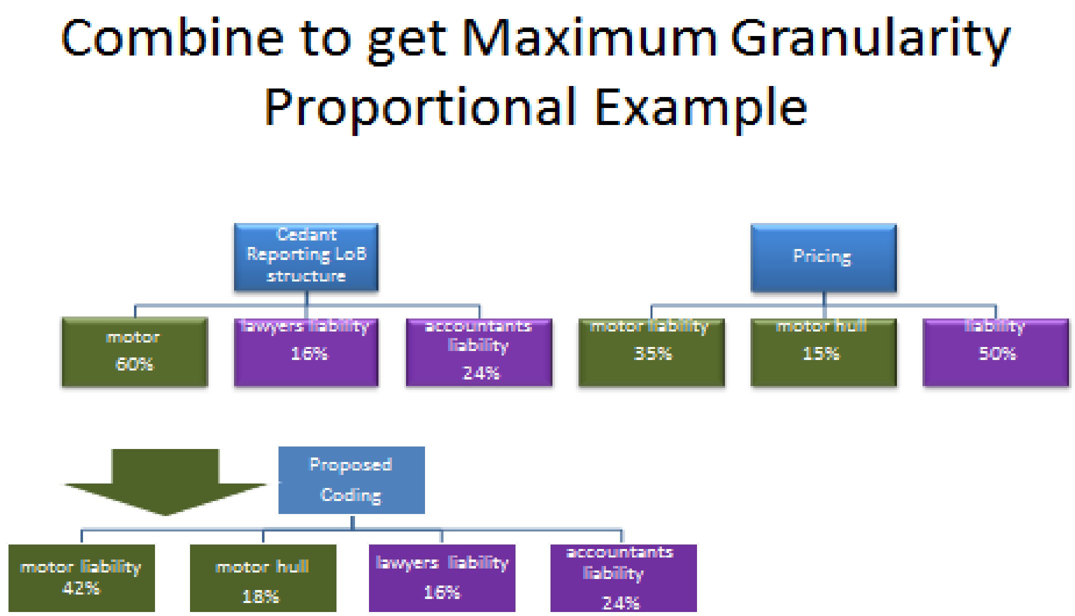
A second difference, is that since losses are generally not reported individually, then it is necessary to allocate the losses to line of business as well. The pricing percentages for loss by line of business would be used the same way they are used for premium. Please note, that the pricing percentages for loss can be different than for premium. The reason for this is that the pricing may have different expected loss ratios by line.

In this example we accept from cedant reporting that the treaty is 40% liability and 60% motor and that the liability premium is split 40% lawyers and 60% accountants. We accept from pricing that the motor premium is split 70% motor liability and 30% motor hull.

Combining all this we get the following distribution:

Reporting LoBs	LoB %	Calculation
motor liability	**42%**	.70 X .60
motor hull	**18%**	.30 X .60
lawyers liability	**16%**	.40 X .40
accountants liability	**24%**	.60 X .40

The chart for the proportional case would be as follows.



7. IBNR ALLOCATION AND OTHER ALLOCATIONS

7.1 IBNR Allocation

Many data items need to be allocated from an aggregated level to a more granular level. Examples may include: IBNR, internal expenses, capital or capital charges, taxes, et cetera. Generally speaking, the preferred approach is to calculate each of these bottom up using the individual contract features and then "truing – up" the bottom up results to match the corporate figures. In this section, this approach is applied to the allocation of IBNR from the portfolio level to the individual contract level.

IBNR is normally calculated at portfolio levels using aggregated data. But since reinsurers need to understand the profitability of historic results by client and even contract, IBNR calculated at the portfolio level is allocated to individual contract. Since the profitability of business at the client and contract level is a critical component of business decisions, care needs to be taken to allocate using the best possible estimate.

One way to improve the reasonableness of any allocation methodology is to calculate the values at the granular level using all available information and then make only relatively small adjustments to ensure that the aggregation of the bottom up numbers match the calculated numbers at the portfolio level. Ideally, the granular level calculation would take into account type and age of claim, would incorporate a methodology based on claim counts as well as claim amounts, would distinguish between paid loss and loss reserves, would separately calculate incurred but not enough reported (IBNER) and pure IBNR, et cetera. This sophisticated approach may be too complex and difficult to implement. The following simpler and more practical approach is suggested.

The following data is necessary by line of business within contract:

Earned premium (EP) (from financial systems)

Expected loss ratio (ELR) (from pricng)

Expected loss reporting pattern (LAG_t) (from pricing). This will be displayed as a cumulative percentage of expected reported loss at time t.

The initial bottom-up contract Bornhuetter-Fergusson IBNR is given by the following formula

$$\text{Initial IBNR}_t = \text{EP X ELR X } (1 - \text{LAG}_t)$$

This IBNR is aggregated over all contracts and compared to the calculated IBNR at the portfolio level. The initial contract IBNR is multiplied by an adjustment factor AF_t to ensure that the sum of the contract IBNR is equal to the portfolio IBNR. So the final contract IBNR at time t is given by

$$\text{IBNR}_t = \text{EP X ELR X } (1 - \text{LAG}_t) \text{ X AF}_t$$

The advantages of this approach are:

The contract IBNR is transparent and easily explainable. The EP is not disputed. the ELR and the lag pattern were agreed to by the deal team at the time the contract was written. The AF adjustment should hopefully be relatively small.

Assuming the AF is close to unity, the majority of the IBNR is determined by the individual contract metrics. So, it has an excellent chance of being a best estimate.

This approach automatically provides an alternate view of the portfolio IBNR. If the AF is small then the bottom up methodology supports the top down result. If the AF is large, it provides a flag to indicate which portfolios might require a more detailed analysis. This alternate approach can be particularly valuable when the portfolio is undergoing change (retentions, limits, underlying business, et cetera).

This approach clearly requires the availability of the contract/LoB ELR and Lag. This in turn requires that the pricing database captures and stores contract/LoB ELRs and Lags. In addition it requires that the pricing database and the accounting database are aligned in terms of contract ID and LoB structure. If this data is available, the approach outlines above is easy to implement and will significantly improve the credibility of the allocated IBNR as compared to an allocation based only on earned premium and incurred loss.

7.2 Expected Emerged Loss

As a byproduct of the above IBNR allocation methodology, the contract/LoB expected emerged $loss_t$ is calculated as

$$\text{expected emerged loss}_t = EP \text{ X } ELR \text{ X } LAG_t$$

A comparison of expected emerged loss and actual emerged loss can serve as an excellent metric of how a contract is performing. It is especially valuable because it is independent of any portfolio effect or impact of reserving conservatism or lack thereof. It can form the basis of both internal and external discussion without the often emotional arguments surrounding the IBNR. It can also serve as an important feedback to pricing since it uses pricing's own estimates to compare to actual.

7.3 Capital Allocation and Expense Allocation

The concept of allocating capital and expense to granular levels has been extensively discussed in the actuarial literature and a detailed discussion of these allocations is beyond the scope of this paper. However, it deserves noting that the above allocation methodology can also be effectively used for other allocations including expense allocation and capital allocation. The concept is to develop the best possible formula to calculate these items on a contract level given basic contract characteristics such as line of business, type of business, country, premium size, expected loss, number of expected claims, risk metrics such as variability and shortfall, new vs renewal, et cetera. These items are then calculated at the contract level, aggregated to the portfolio level and compared to a portfolio value that was determined previously. The individual contract values are then scaled to assure that the sum of the contract values is equal to the portfolio value.

8. CONCLUSION

Hopefully, this paper will motivate reinsurance actuaries to spearhead an increased realization of the value of a company's internal data and create the desire to develop a data architecture that will enable significantly more sophisticated data analyses. The potential benefit to those leading this effort can be very large. There is nobody better suited to be passionate about this cause than the actuarial community.

Sharpe Ratio Optimization
of an
Excess of Loss Reinsurance Contract

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Abstract

Effects of the sharing and layering of losses by means of an excess of loss reinsurance contract are examined. In particular, the covariance of loss at any given position across the layers of an excess of loss reinsurance contract as a function of the first and second moments of the frequency and individual layer loss severity distributions is derived. Based on this result, a risk management application is presented and the Sharpe Ratio based investment equivalent paradigm for measuring profitability is explored.

1 Introduction

An excess of loss reinsurance contract facilitates the layering and sharing of losses by allowing individual reinsurers to take a position on the contract consisting of a set of shares across the various layers. In order to appropriately assess the profitability of a given reinsurer at its position on a variance based risk adjusted basis, it is necessary to take into account the interdependence between those layers. That is, evaluating any such measure of profitability for each layer individually will not take into consideration the correlation of loss between the layers at the reinsurers shares. Moreover, in the case that the cedent retains a portion of the reinsured layers then the measurement of the variance of loss to the cedent net of reinsurance is complicated for the same reasons also.

In the first part of this paper, a method for determining the variance of loss at a given position on an excess of loss reinsurance contract as a function of individual layer loss metrics is presented. In the second part, Kreps' investment equivalent approach is generalized to a multi-layer framework analogous to the Markowitz Portfolio Model whereby investors providing the capital required to back the reinsurer's position on the contract are concerned with the Sharpe Ratio of their investment. However, in this case it is the various layers of the reinsurance contract that serve as the individual risky assets for the investor to take a position across. In the third part of the paper, the Sharpe Ratio optimization paradigm will be examined through numerical examples exploring the view of the reinsurer as a price taker. This is intended as a thought experiment to examine the implications of layer risk load relativities on the behavior of market participants under this classical paradigm. Finally, an example is presented to demonstrate how the variance formula can be applied to the cedent from a risk management perspective.

2 Background and Methods

2.1 Variance of Shared and Layered Losses

While many more sophisticated measures are commonly used in practice, variance still remains at the least a benchmark measure of risk. For an excess of loss contract however, the effect of sharing and layering losses complicates the estimation of the variance at given shares of each layer. In particular, while if taking the same shares of each layer then the variance of loss at that position is just the square of the share times the total variance, at unequal shares of any two layers it is necessary to explicitly take into consideration the covariance of loss between those layers. This would also be the case from the perspective of the cedent if retaining some portion of one or more of the reinsured layers while of course retaining all of the loss beneath the attachment point of the contract which can be viewed as an underlying layer. And so, to appropriately account for the variance of loss to either a participating reinsurer or the cedent, it is necessary to take into consideration the individual layer variances as well as the covariances of loss between those layers.

Collective risk loss models separate the task of modelling losses into two distinct frequency and severity components by assuming that the number of losses and the severity of those losses are independent and that the loss severity distribution is the same for each loss. Unlike first dollar losses, for excess layers the loss severity can be defined based on large losses above some chosen threshold amount up to the attachment point of the contract. Then, the frequency is based on the number of losses in excess of the threshold and the severity is based on the amount of loss at a given position across the layers given a loss in excess of the threshold. That is, the random variable representing the severity here is the amount of loss at a given position as a function of the large loss random variable, the set of shares across the layers comprising the given position, and the various layer attachments and limits.

The collective risk loss model approach has the advantage of being able to capture certain loss characteristics entirely by either the frequency or the severity distribution resulting in a more accurate and flexible loss model [1]. For example, inflation can be explained by the severity distribution alone while exposure growth can be explained by that of the frequency. For an excess of loss contract, since the allocation of loss between the layers is a function of the loss severity only, the correlation of loss between those layers is explained entirely by the severity distribution. As will be shown, by taking a collective loss model approach it is possible to express the covariance of loss between any two layers of an excess of loss reinsurance contract at given shares of each layer as a function of individual layer loss metrics.

[1] Loss Models, Klugman et al. pg. 137

Let

a_i = attachment point of the i_{th} layer
b_i = upper bound of the i_{th} layer
s_i = share of the i_{th} layer
N = random variable representing the number of losses in excess of the threshold
L = random variable representing the amount of loss given a loss in excess of the threshold
L_i = loss to the i_{th} layer given a loss in excess of the threshold
L_s = loss at a given position across the layers given a single loss in excess of the threshold
$f_L(l)$ = probability density function of L
μ_i = expected loss to the i_{th} layer given a single loss in excess of the threshold
σ_i = standard deviation of the loss to the i_{th} layer given a single loss in excess of the threshold
T = total loss at a given position across the layers, where T = 0 if N = 0
σ_T = standard deviation of the total loss at a given position across the layers

For this collective risk model, by the law of total variance we have

$$\begin{aligned}\sigma_T^2 &= E(N)Var(L_s) + E^2(L_s)Var(N)\\ &= E(N)Var(\sum_i s_iL_i) + E^2(\sum_i s_iL_i)Var(N)\\ &= E(N)[\sum_i Var(s_iL_i) + 2\sum_{j<k}\sum_k Cov(s_jL_j, s_kL_k)] + E^2(\sum_i s_iL_i)Var(N)\\ &= E(N)[\sum_i s_i^2\sigma_i^2 + 2\sum_{j<k}\sum_k s_js_kCov(L_j, L_k)] + (\sum_i s_i\mu_i)^2Var(N)\end{aligned}$$

Now to derive the covariance term, let any two layers be given and denoted by j and k for the lower and upper layers respectively. Then by definition, the covariance of L_j and L_k is given by

$$\begin{aligned}Cov(L_j, L_k) &= E(L_jL_k) - E(L_j)E(L_k)\\ &= \int_{l=-\infty}^{l=\infty} L_j(l)L_k(l)f_L(l)dl - \mu_j\mu_k\end{aligned}$$

But since L_j and L_k are both non-zero only when $L > a_k$, and since $L_j = (b_j - a_k)$ in that case, we have

$$\begin{aligned}Cov(L_j, L_k) &= \int_{l=a_k}^{l=\infty} (b_j - a_j)L_k(l)f_{L(l)}dl - \mu_j\mu_k\\ &= (b_j - a_j)\mu_k - \mu_j\mu_k \qquad (1)\end{aligned}$$

Plugging this into the equation for total variance we have

$$\sigma_T^2 = E(N)[\sum_i s_i^2\sigma_i^2 + 2\sum_{j<k}\sum_k s_js_k[(b_j - a_j)\mu_k - \mu_j\mu_k]] + (\sum_i s_i\mu_i)^2 Var(N) \tag{2}$$

And so we have the formula for the variance of the total loss at a given position across the layers of an excess of loss reinsurance contract as a function of the expected value and variance of the number of losses in excess of the threshold and the expected value and variance of the individual layer losses given a loss in excess of the threshold. Then if we estimate those individual layer metrics, the variance of the total loss at a given position across the layers can be determined formulaically.

A simplifying assumption that is often made when using a collective risk model is that the frequency is distributed Poisson. Under this assumption, and since for a Poisson distribution the mean and variance are equal, the total variance formula simplifies to

$$\begin{aligned} \sigma_T^2 &= \lambda[Var(L_s) + E^2(L_s)] \\ &= \lambda[E(L_s^2)] \end{aligned} \tag{3}$$

Where λ is the expected number of claims in excess of the given loss threshold. Now, since

$$\begin{aligned} Var(L_s) + E^2(L_s) &= \sum_i s_i^2\sigma_i^2 + 2\sum_{j<k}\sum_k s_js_k[(b_j - a_j)\mu_k - \mu_j\mu_k] + (\sum_i s_i\mu_i)^2 \\ &= \sum_i s_i^2 E(L_i^2) + 2\sum_{j<k}\sum_k s_js_k(b_j - a_j)\mu_k \end{aligned}$$

we have

$$E(L_s^2) = \sum_i s_i^2 E(L_i^2) + 2\sum_{j<k}\sum_k s_js_k(b_j - a_j)\mu_k \tag{4}$$

and

$$\sigma_T^2 = \lambda[\sum_i s_i^2 E(L_i^2) + 2\sum_{j<k}\sum_k s_js_k(b_j - a_j)\mu_k] \tag{5}$$

And so we have the formula for the variance of total loss at any given position under the Poisson assumption as a function of the expected number of losses in excess of the chosen loss threshold and the first and second moments of loss to the individual layers given a loss in excess of the threshold.

2.2 Investment Equivalent Reinsurance Pricing

2.2.1 Kreps' Paradigm

In Kreps' paper Investment Equivalent Reinsurance Pricing, he presents a paradigm for setting risk loads for layers of a single excess of loss reinsurance contract on a standalone basis by viewing the contract as an investment alternative to a given target investment. In particular, the risk load and capital allocated at contract inception by the investor to back the contract must be such that the expected return and risk as measured by variance of that return are at least as favorable as those of the target investment. Further, the amount of capital allocated to back the contract must be such that the probability of ruin not exceed some given loss safety level.

Kreps explicitly considers the reinsurance investment alternative as a combination of the reinsurance contract and a financial technique that is used to earn investment income on the total funds available to the reinsurer. Those funds consist of both the premium received and capital allocated at contract inception until losses are paid. And so, the risk and return to the investor providing the capital required to participate on the reinsurance contract are derived from both underwriting as well as investment income.

2.2.2 Dissimilarities of a Reinsurance Contract and Traditional Investments

A key difference between traditional investments such as stocks and bonds and a reinsurance contract is the determination of the amount of capital required to participate. While for stocks and bonds the capital required is the market price of the asset, for a reinsurer the amount of capital allocated at inception in order to back the contract must be determined. Further, while for stocks and bonds the greatest possible loss to the investor is the initial cost of acquiring the asset, for a reinsurer it is possible to lose more than the initial capital allocation requiring either additional capital to be provided by the investor to fund the additional losses or insolvency of the reinsurer. The greater the amount of capital initially allocated to back the contract, the less likely the investor will have to provide additional capital to cover losses and in turn the less likely that the cedent will not be reimbursed due to insolvency of the reinsurer. As such, the amount of capital allocated at inception can be viewed as a measure of underwriting conservatism [2] from the reinsurer's perspective and as a measure of security with respect to insolvency from the perspective of the cedent. Of course, credit risk to the cedent associated with collecting reinsurance recoverables still exists even in the case that the reinsurer remains solvent. Only in the extreme case where the investor collateralizes the reinsurer in the amount of the full limit provided is there no risk associated with the collectability of reinsurance recoverables for the cedent.

Another important difference between a reinsurance contract and more traditional investments is the liquidity of the investor's position. While with stocks and bonds an investor can usually liquidate their position at most any time, with a reinsurance contract the matter is more complicated. Firstly, the sale would occur in the private market and so would require the investor to find a buyer. Secondly, even if a buyer is found reinsurance contracts typically contain a special termination clause that requires the approval of the cedent in order to change control of the reinsurer. And so, in order to liquidate their position the investor would have to find an interested buyer and then get approval from the cedent in order to sell the reinsurer and liquidate their position. Assuming investors prefer liquidity, then this difference should necessitate a liquidity premium in the form of additional return to the investor.

[2] Kreps, Investment Equivalent Reinsurance Pricing

2.2.3 Sharpe Ratio of the Reinsurance Investment

While many measures for the performance of an investment are used in practice, a classical benchmark still used today is the Sharpe Ratio [3]. First introduced by William Sharpe in 1966, the ratio compares the expected excess return over the risk free rate to the standard deviation of that excess return. What this measure indicates is the risk premium provided to the investor per unit of risk taken as measured by standard deviation of the excess return, and so allows for a comparison of different investments on a risk adjusted basis. Under the assumption of unlimited borrowing and lending as is the case in the Markowitz Portfolio Selection Model,[4] by combining the risky asset with the risk free asset individual investors can take on a level of risk commensurate with their own degree of risk aversion based on individual utility and achieve a certain level of expected return. As such, when comparing two investments and given the individual risk appetite of the investor, the investment with the higher Sharpe Ratio will yield the higher expected return at that given level of risk chosen by the investor. And so, assuming that investors are risk averse, an investor will prefer the risky asset with the higher Sharpe Ratio as that will result in a higher expected return at the same level of risk.

In the case of a per risk excess of loss contract, if we assume that the investor capitalizing the reinsurer to write a single stand-alone contract has the ability to borrow or lend at the risk free rate, then a similar strategy as with the Markowitz Portfolio Selection Model can be taken. In particular, rather than target the individual expected return and variance of the target investment, the investor may instead consider the Sharpe Ratio of the return on the capital provided to capitalize the reinsurer. As is the case with the Markowitz portfolio selection model, the investor can then combine the reinsurance investment with the risk free asset in order to achieve a certain level of risk and corresponding expected return.

In order to derive the Sharpe Ratio of the reinsurance investment, the following assumptions are made:

1. The reinsurer writes a single standalone reinsurance contract
2. The term of the reinsurance contract is one year
3. There is a single loss payment at the end of the year
4. The ceded premium and allocated capital are invested at the risk free rate from contract inception until losses are paid

[3] Investments 9th Edition, Bodie, Kane, Marcus, pg. 133
[4] Investments 9th Edition, Bodie, Kane, Marcus, pg. 211

Let

$P =$ ceded premium at a given position across the layers of the reinsurance contract
$R_i =$ risk load defined as the difference between the ceded premium and the discounted expected loss for the i_{th} layer
$R =$ risk load at a given position across the layers of the reinsurance contract
$C =$ capital allocated to the reinsurer by the investor in order to take a given position on the reinsurance contract
$r =$ return on the capital provided by the investor
$\widehat{r} =$ expected return on capital
$\sigma_r =$ standard deviation of the return on capital
$r_f =$ risk free rate of return
$\mu_T =$ expected total loss at a given position across the layers of the reinsurance contract
$SR =$ Sharpe Ratio of the return on capital

Starting with the definition of the Sharpe Ratio we have

$$SR = \frac{\widehat{r} - r_f}{\sigma_r}$$

Now,

$$\begin{aligned} r &= \frac{P(1+r_f) + Cr_f - T}{C} \\ &= \frac{[R + \frac{\mu_T}{(1+r_f)}](1+r_f) + Cr_f - T}{C} \\ &= \frac{R(1+r_f) + r_f C - (T - \mu_T)}{C} \end{aligned}$$

And so,

$$\widehat{r} = \frac{R(1+r_f) + r_f C}{C} \text{ and } \sigma_r = \frac{\sigma_T}{C}$$

Which gives us

$$SR = \frac{R(1+r_f)}{\sigma_T}$$

If we assume a collective risk loss model with poisson(λ) frequency distribution, then substituting equation (3) for σ_T^2 we have

$$SR = \frac{R(1+r_f)}{[\lambda E(L_s^2)]^{\frac{1}{2}}} \tag{6}$$

$$= \frac{(\sum_i s_i R_i)(1+r_f)}{[\lambda[\sum_i s_i^2 E(L_i^2) + 2\sum_{j<k}\sum_k s_j s_k (b_j - a_j)\mu_k]]^{\frac{1}{2}}} \tag{7}$$

2.3 Reinsurance Market Dynamics

As mentioned by Kreps, reinsurance pricing is usually described as market driven. Prices are generally thought to be determined by the supply of capital available to provide reinsurance and the demand for coverage by cedents. Under the assumption that investors are concerned with the Sharpe Ratio of their investment, then the willingness of investors to allocate capital to provide coverage will depend on how the Sharpe Ratio of the reinsurance investment opportunity compares to that of the target investment. That is, taking into account the differences in the nature of the two investments, the Sharpe Ratio of the reinsurance investment opportunity must be sufficient in comparison to that of the target investment in order to entice investors to participate. The reinsurance supply curve under the current paradigm then is the aggregate of many reinsurers all concerned with the Sharpe Ratio of their investment but with differing views of the pertinent loss metrics.

2.3.1 Reinsurer View

Suppose that an investor is presented with a reinsurance investment opportunity consisting of a single two layer excess of loss reinsurance contract. Further, the investor here is a price taker in the sense that the ceded premium and contract terms are taken as given. The investor has the option to either capitalize a reinsurer to participate at shares of each layer of its own choosing such that a given internal ceded premium target is met or decline to participate entirely.

In order to assess the quality of the investment opportunity under the current paradigm, the investor must first determine the shares of each layer that will result in the highest Sharpe Ratio given the target ceded premium. For a two layer excess of loss reinsurance contract, assuming a poisson distribution with mean λ for the number of losses we have

$$SR = \frac{(s_1R_1 + s_2R_2)(1+r_f)}{[\lambda[s_1^2E(L_1^2) + s_2^2E(L_2^2) + 2s_1s_2[(b_1-a_1)\mu_2]^{\frac{1}{2}}} \tag{8}$$

In order to maximize the Sharpe Ratio of the investment, we look for points of extrema for the Sharpe Ratio formula. Let,

$$K = \text{reinsurer premium target}$$
$$P_i = \text{ceded premium for the } i_{th} \text{ layer}$$

Now, the target premium constraint given is $K = s_1 P_1 + s_2 P_2$ which reduces the optimization problem to one dimension.

If we substitute $s_2 = \frac{K - s_1 P_1}{P_2}$ and then take the derivative with respect to s_1, we get

$$SR' = \frac{(1+r_f)}{f} \frac{[R' E(L_s^2)^{\frac{1}{2}} - \frac{1}{2} R E(L_s^2)^{\frac{-1}{2}} E(L_s^2)']}{E(L_s^2)}$$

which is zero when

$$\frac{R'}{R} = \frac{1}{2} \frac{E'(L_s^2)}{E(L_s^2)}$$

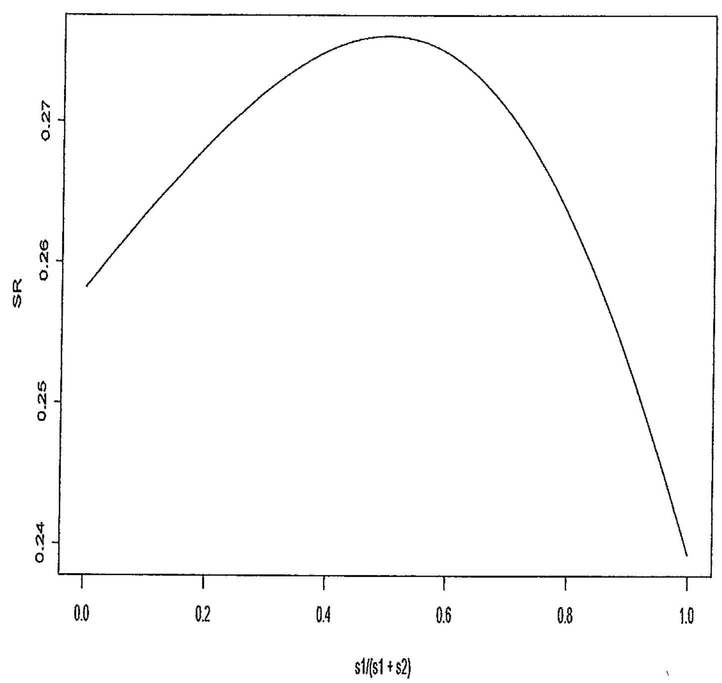
And so, positions of extrema occur when the rate of change in the percentage of the risk load equals half of the percentage change of the second moment of the loss severity distribution. However, to the author's knowledge no simple form of the solution for s_1 is available and so numerical methods will be relied upon in the following three examples that illustrate the effect of risk load sharing on the optimal position of the reinsurer.

Example 1

A reinsurer is presented with a two layer excess of loss reinsurance contract consisting of a 10M xs 10M and a 30M xs 20M layer. The reinsurer models losses to the cover using a collective risk model at a loss threshold of 5M and assumes that the severity of loss is distributed Pareto with alpha 1.8 and that the frequency is distributed Poisson with mean 2.2. The resulting individual layer loss metrics based on the loss distribution as estimated by the reinsurer along with the full coverage ceded premium amounts are summarized in the table below.

layer	a_i	b_i	μ_i	$E(L_i^2)$	P_i
1	10M	20M	1,527,951	12,143,363,968,303	4,463,585
2	20M	50M	1,071,173	23,499,405,810,277	4,087,942

Based on the reinsurers estimated individual layer loss metrics and frequency pick, using equation (8) the following relationship between the Sharpe Ratio of the reinsurance investment and the proportion of total shares in the first layer is obtained.

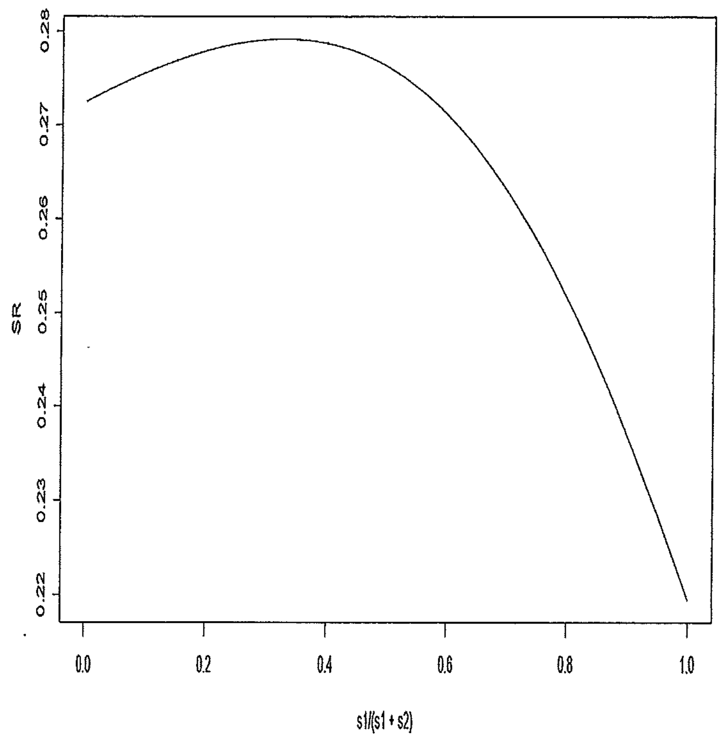


In this example, the risk load relativity between the layers is such that the optimal Sharpe Ratio occurs at equal shares of each layer and so the risk loading is balanced in the sense that the reinsurer does not have a preference to take more or less of any one of the layers. The optimal Sharpe Ratio here is 0.276 while the individual layer Sharpe Ratios are 0.239 and 0.258 for layers 1 and 2 respectively. It is interesting to note that setting the Sharpe Ratio's of the individual layers equal will not result in a balanced risk loading.

Example 2

Now, suppose instead that the total ceded premium amount of 8.6M is allocated as follows, all else the same as in the first example.

$P_1 = 4.4M, P_2 = 4.2M$

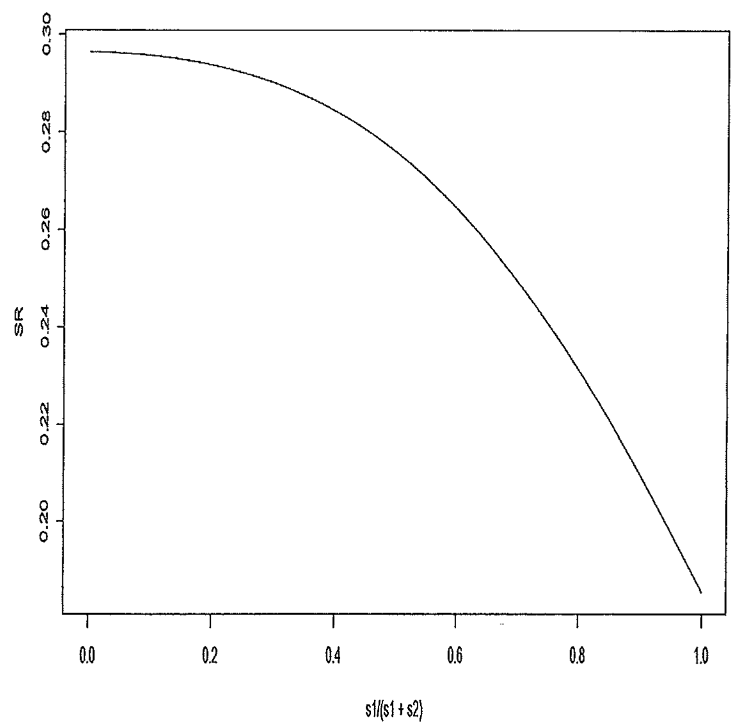


In this example, the second layer rates much better than the first layer with Sharpe Ratio's of 0.272 and 0.219 respectively. However, the Sharpe Ratio actually increases as shares are shifted from the second layer to the first and remains greater than the first layer alone until near the balance point. The optimal Sharpe Ratio is 0.279 and occurs around a two to one ratio of shares of layer 2 to layer 1.

Example 3

Again, all else the same only now let

$P_1 = 4.2M, P_2 = 4.4M$



In this example, the first layer rates significantly better than the second with Sharpe Ratio's of 0.297 and 0.185 respectively. There is no benefit to diversify in this case and the optimal position for the reinsurer is to have the entire target premium from the first layer. It is interesting to note however, that even in this case there is enough of a diversification benefit such that a fairly flat region on the curve results as shares of the second layer are added to the first over which the reinsurer does not lose too greatly by taking up a small portion of shares of the less profitable layer.

2.3.2 Cedent View

While reinsurers are concerned primarily with the profitability of the reinsurance contract, cedents have other considerations to take into account such as capital management and regulation. From the view of the cedent, reinsurance functions more as a risk management tool and less as an investment and so decisions regarding the purchase of reinsurance must take into consideration regulatory and internal risk management objectives. That said, cedents are still concerned with giving away profits and so depending on their views of the pricing of their coverage relative to that of the market, the cedent may choose to cede more or less of a given layer taking into consideration any constraints on the reinsurance purchase regulatory or otherwise.

Example 4

An insurer would like to purchase a two layer excess of loss reinsurance treaty to manage the volatility of its book of business. Company management desires that the standard deviation of loss not exceed 1/3 of its current capital base of 10.7M. The cedent actuary estimates that the number of claims follows a Poisson distribution with mean 1200 and produces the following estimates for the first and second moments of the severity of loss for each of the reinsured layers as well as the underlying retained layer.

layer	a_i	b_i	μ_i	$E(L_i^2)$
0	0M	5M	14,544	423,049,790
1	5M	10M	1,956	19,138,644
2	10M	20M	1,500	58,223,477

Based on their internal analysis, the cedent believes that the market rate to fully place the first layer is too high and as such would rather cede less of that layer. At the rate that the cedent is willing to pay for the first layer, the supply of reinsurance will be sufficient for a 50 percent placement. In such a case where the insurer cedes all of the second layer while retaining half of the first layer and all of the underlying layer, the standard deviation of loss to the insurer is given by equation (5) as 3.5M which amounts to a minimum capital requirement of 10.5M. Since the cedent's current capital base exceeds the required amount, the cedent is able to meet its profitability objective within the minimum capital requirement constraint by ceding all of the second layer and half of the first layer.

3 Discussion

By taking a collective risk model approach it is possible to express the variance of loss at any combination of shares across the layers of an excess of loss reinsurance contract as a function of the first and second moments of the frequency and the individual layer loss severity distributions. While the presentation here was specifically in the context of an excess of loss reinsurance contract, the results are applicable to any setting involving the sharing and layering of losses including primary subscription policies or the combination of an excess of loss contract coupled with an underlying quota share.

The correlation of loss between the layers is dependent on the severity distribution only to the extent of the first and second moments and does not require any assumption about the distribution otherwise. And so, while it is common to assume a certain distribution for the severity of losses when modelling an excess of loss reinsurance contract, no such assumption is required in order to estimate the variance of loss at a given position. Moreover, the covariance term here is estimable to the same extent that the pertinent individual layer loss metrics upon which it is based are. While parameter risk was not taken into consideration and is beyond the scope of this paper, the applicability of the methods presented here is limited by the estimability of the pertinent loss metrics which may be difficult especially for more remote layers.

A key assumption underlying the collective risk model is the independence of the frequency and severity components. However, the validity of this underlying assumption may be questionable in certain cases. For example, for an excess of loss treaty covering a heavily catastrophe exposed property book, a large catastrophic event may impact several risks resulting in an unusually high frequency while the severity also is unique since all of those losses are caused by the same set of underlying extreme physical circumstances. And so there may be some dependence of the severity distribution on the number of losses in certain cases. Another common assumption made in collective risk modelling is that the frequency is distributed Poisson which simplifies the model to a more compact and mathematically tractable form. However, excess of loss reinsurance contracts typically have aggregate features such as occurrence limits and limits on reinstatements that may invalidate this assumption even as a reasonable approximation. Further, the type of cover may also invalidate the Poisson assumption as would be the case for an aggregate excess of loss cover for which the frequency is binary. Thus, caution should be exercised when employing the collective risk model approach in terms of both the underlying independence of frequency and severity assumption as well as the simplifying Poisson frequency distribution assumption.

When presented with the opportunity to participate on an excess of loss reinsurance contract, a reinsurer may choose to take a position consisting of unequal shares of the layers for various reasons including its views of the relative profitability of those layers. In such a case, the collective risk loss model approach can be employed to assess the profitability of different positions on a variance based risk adjusted basis. One such measure of profitability is the Sharpe Ratio of the reinsurance investment. In deriving the formula for the Sharpe Ratio, it was assumed that the funds available to the reinsurer over the duration of the contract are invested at the risk free rate, or as referred to by Kreps the 'Swap' financial technique. Under this assumption, the resulting formula for the Sharpe Ratio is independent of the capital allocated to back the contract and so this profitability measure does not require any assumption about how capital is allocated. For other financial techniques such as the 'Put' technique presented by Kreps however, the amount of capital allocated will need to be taken into consideration.

In order to further examine the implications of taking a multi-layer view, a thought experiment was presented analogous to the Markowitz Portfolio Selection Model whereby investors are concerned with maximizing the Sharpe Ratio of their investment by choosing shares of each layer that will result in an optimal portfolio while meeting a given ceded premium target. In essence, this amounts to examining the implications of risk load relativities across the layers of an excess of loss reinsurance contract on the behavior of reinsurers under the Markowitz paradigm. Three examples were presented that varied only in the allocation of risk load across the layers of a two layer contract in order to demonstrate the impact of the risk load relativity. It was interesting to note that only in the most extreme example where the two layers were most mispriced from the view of the reinsurer assessing the deal was there no benefit to take at least some of the less profitable layer. In the second example, even though there was a large difference in the Sharpe Ratio of the individual layers, the point of optimality is actually achieved by including some of the less profitable layer. And so, while one layer may rate better than the other on an individual basis, there may still be some benefit to diversify by taking at least some shares of the less profitable layer. This result is important from a practicality standpoint as cedents often restrict the choice of shares and may require reinsurers to take at least some shares of all layers in order to participate on the contract. As was the case in the third example where the disparity in profitability between the two layers was greatest, there was a fairly flat segment of the Sharpe Ratio curve as shares were shifted from the second to the first layer with the Sharpe Ratio holding fairly well until near to a two to one ratio of shares in the more profitable layer to the less profitable layers.

4 Conclusion

Under the framework of a collective risk loss model, it is possible to derive the covariance of loss at any given position across an excess of loss reinsurance contract as a function of the first and second moments of the frequency and the individual layer loss severity distributions. The covariance term facilitates the extension of a single layer variance based approach to a multi-layer setting formulaically and allows for a more holistic view of risk as variance to be taken. One such possible application of this result is the Markowitz Portfolio Selection Model applied to the various layers of a given excess of loss reinsurance contract. Under such a framework, the relativity of the risk load between the layers of the reinsurance contract will govern the behavior of reinsurers and ultimately determine the relative demand for the layers by the market. Even in the case where one layer rates poorly relative to others, there may still be a benefit to having some shares of that layer in the portfolio. The explicit consideration of the covariance of loss between layers is also important from the perspective of the cedent for variance based risk management measures in the case that their net position varies across the layers of their book.

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A New Model for Weathering Risk: CDOs For Natural Catastrophes

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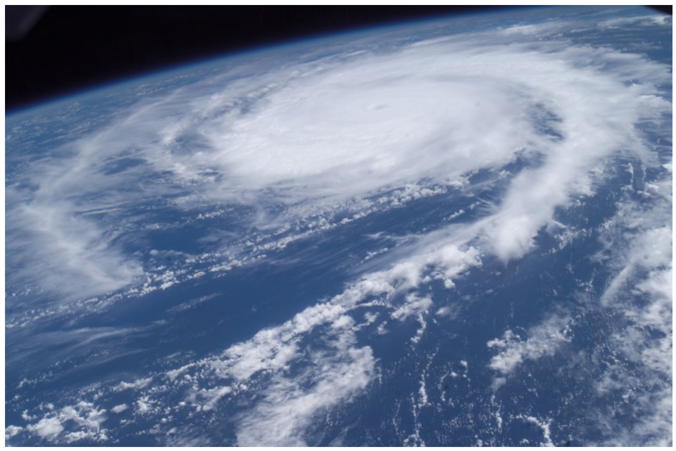

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1. INTRODUCTION

A fundamental function of reinsurance is to provide financial recovery from natural catastrophes. A well-functioning insurance market can enable rebuilding efforts that would overwhelm the resources of individual households and communities.

By its nature, catastrophe risk is often not diversifiable on a local or even regional scale. Instead, insurers usually look to the global reinsurance markets for catastrophe risk protection.

Recently, a third option has risen to prominence as hedge funds, pension funds, and other institutional investors (hereafter "alternative capital") have sought to "directly" invest in catastrophe risk. Through investments in insurance-linked securities (ILS) and collateralized reinsurance, alternative capital has increased the available supply of property catastrophe risk coverage and driven ILS prices toward all-time lows.

Yet alternative capital may be most remarkable not for its impacts to date, but for its vast untapped potential. ILS is still a niche asset class, and the existing market for catastrophe risk is dwarfed by the pool of available institutional capital. Nevertheless, price competition on existing property catastrophe risk may have already reached the point of diminishing returns. Product innovation is needed to support the growth rate of alternative capital and to produce further improvements in the availability and cost of catastrophe coverage.

In this paper, the use of pooling and tranching techniques similar to those used in collateralized debt obligations (CDOs) is proposed as a tool for expanding the market for catastrophe risk.

Given the somewhat infamous legacy of CDOs and inherent complexity of catastrophe risk, the pairing of the two may seem problematic. However, catastrophe risk is a far stronger candidate for inclusion in a CDO-type structure than economic assets such as securitized mortgages. An appropriately designed *collateralized risk obligation* (CRO) would have significantly less systemic vulnerability than the subprime mortgage-fueled CDOs at the heart of the recent financial crisis.

In fact, the concept of a CRO is not entirely novel. Limited numbers of CRO-type instruments were issued in the early to mid-2000s, only to largely disappear at the onset of the financial crisis. The market for catastrophe risk has matured in the interim, yet suffers from limitations that a CRO is well-suited to address.

CROs should improve efficiency and stimulate growth in the catastrophe risk market in two key ways.

First, CROs would *simultaneously increase the availability of investment-grade catastrophe risk and high-yielding catastrophe risk*. The vast majority of recently securitized catastrophe risk is either unrated or assigned a speculative, or "junk," rating. Large institutional investors frequently place limits on the amount of non-investment-grade risk they will hold in their portfolios. On the other end of the spectrum, falling yields on catastrophe risk have led to heightened demand for high-yielding catastrophe securities. Thus, CROs should further expand the supply of alternative capital.

Second, CROs should *encourage investment in heretofore underinsured perils and geographic locations*. In a CRO, unusual or diversifying perils provide enhanced value, which is due to their low correlation with the other assets in the portfolio. CROs should contribute to the globalization of a predominantly US and European market, and stimulate the growth of insurance in developing economies.

Guide to this paper

Building a CRO: Sections 2 and 3

Section 2 presents a brief overview of alternative capital, its recent increase in popularity, and the emerging need for product innovations such as the CRO.

Section 3 outlines a basic design framework for CROs. It also addresses a key question: How are CROs different from the CDOs that underpinned the financial crisis? This section demonstrates that the primary pitfalls of pre-crisis CDOs are largely mitigated for CROs, which is due to the nature of insurance risk and the structure of insurance markets.

Case study: Section 4

Section 4 provides a stylized example of a CRO. This “sample CRO” is used to discuss potential pricing and rating methodologies for CROs. In addition, it illustrates the differences in achievable credit enhancement (i.e., leverage) between CROs and traditional CDOs.

Practical considerations, market history, and conclusion: Sections 5 and 6

Section 5 considers several practical and historical questions surrounding the implementation of a CRO. Who are the likely sponsors? What kinds of CRO-type instruments were issued prior to the financial crisis? What lessons can be drawn from their history?

Section 6 provides conclusions.

2. THE RISE OF ALTERNATIVE CAPITAL

2.1 – Market transformation

2013 was a banner year for alternative capital investments in their various forms (see Sidebar 1). Catastrophe bonds enjoyed their second largest issuance year on record, and reached an all-time peak for the amount of total principal outstanding (approximately US$20.2 billion).[1] Collateralized reinsurance had even stronger growth, surpassing the traditional catastrophe bond market in size for the first time.[2] This significant influx of alternative capital led to rapidly falling prices, with some sources quoting a year-over-year decrease in catastrophe bond spreads of nearly 40%.[3]

Yet, this may be just the beginning of alternative capital's entry into catastrophe risk markets. While current estimates peg the amount of invested alternative capital at around US$50 billion as of early 2014, this pales in comparison to the estimated US$30 *trillion* of existing worldwide pension fund assets. Further allocations to catastrophe risk of even 1% to 2% of these assets could double or triple the capacity of the existing US$300 billion USD catastrophe risk market.[4] Many market analysts expect an explosive next five years for alternative capital, with projections ranging from $40 billion USD to $150 billion USD in *new* alternative capital entering the marketplace.[5,6,7]

However, for every risk investor there must also be a risk seller, and there are limits on alternative capital's ability to grow purely through price competition on property catastrophe risk. If supply

<u>**Sidebar 1:**</u>

Major Existing Types of Alternative Capital Investments

<u>**Catastrophe bonds:**</u> Investments in Special Purpose Vehicles (SPVs) in which a limit of catastrophe coverage is fully collateralized by outside investors, who are paid periodic risk-based coupons by the ceding (re)insurer through the SPV. After a triggering event, the investor may lose the principal for the ceding reinsurer to cover claims.

<u>**Collateralized reinsurance:**</u> Reinsurance coverage in which capital markets investors fully collateralize the reinsurance limit offered in exchange for an up-front reinsurance premium.

<u>**Industry Loss Warranties (ILWs):**</u> Dual-trigger reinsurance or derivative contracts (typically fully collateralized) in which the payout is based upon both an industry loss threshold and the ultimate net loss to the cedent.

<u>**Sidecars:**</u> Financial structures designed to allow outside investors to take on a quota-share portion of the risk written by a (re)insurer, by establishing a collateralized limit of coverage for which reinsurance premiums are paid. Generally designed to have a limited lifespan and intended to capture the increase in rates often witnessed after a major catastrophe.

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[1] Swiss Re [33]
[2] Artemis.bm [2]
[3] Plenum Insurance Linked Capital [26]
[4] Guy Carpenter [14]
[5] Artemis.bm [3]
[6] Artemis.bm [4]
[7] BNY Mellon [8]

outpaces demand, investors will at some point reach a minimum acceptable return for a given level of risk. Indeed, some reports have suggested that prices on certain risks have already begun to reach this lower boundary.[8,9] Alternative capital also faces a stiff test from traditional providers of catastrophe risk protection. Despite public promises to avoid a pricing "race to the bottom," catastrophe-focused reinsurers are unlikely to simply let profitable business walk away. Longtime client relationships and add-on services provide reinsurers an edge that in some instances may overcome the lower prices of alternative capital.

Thus, the long-term growth prospects for alternative capital depend on the ability to leverage its primary advantage over the traditional reinsurance model—a lower cost of capital—into the development of *market-expanding* and *market-completing* innovations.

2.2 – Market expansion

Market-expanding innovations introduce new exposures to the alternative capital market. To a certain extent, the globalization of the insurance industry will sow the seeds of opportunity for *market-expanding* innovation. As the epicenter of insurance growth shifts toward Asia-Pacific and similar regions,[10] opportunities for investing in new catastrophe risks will multiply. The quality of catastrophe models and data for these regions will also improve, providing potential investors with better tools for measuring risk and assessing investment opportunities.

Alternative capital investors have already demonstrated enthusiasm for the limited number of developing market catastrophe securitizations to date. Catastrophe bonds for diversifying perils—such as Mexican hurricane risk, Mexican earthquake risk, and Turkish earthquake risk—have enjoyed high investor demand and coupon spreads well below the market average. More opportunities may be on the horizon, as officials in countries such as India and the Philippines have recently voiced interest in securitizing a portion of their countries' catastrophe risks.[11,12]

Another candidate for *market-expanding* innovation is the securitization of new types of risk, including terrorism risk and catastrophic liability risk. These risks are significantly harder to model than natural catastrophe risk. For instance, any terrorism risk model must contend with terrorists' intention of avoiding predictability. Nevertheless, falling margins in property catastrophe risk may eventually push alternative capital into these harder-to-model perils: At some point, every risk must have its price.

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[8] Munich Re [24]
[9] Carrier Management [11]
[10] Munich Re [23]
[11] The Economic Times [13]
[12] Reuters [29]

2.3 – Market completion

Market-completing innovations bring new investors into the market. Despite recent success, the catastrophe risk market still lacks many of the characteristics exhibited by more mature financial markets. Certain *market-completing* innovations stand to grow the catastrophe risk market by removing existing supply-side limitations.

For example, most catastrophe bonds to date[13] have been identified as speculative or "junk"-grade risk, receiving ratings between BB- and BB+. Compared to other fixed-income alternatives such as corporate bonds, catastrophe bonds are disproportionately high-risk investments.

The low ratings of existing catastrophe risk instruments serve to limit the pool of potential alternative capital investors, as institutional investors are frequently limited in the amount of non-investment-grade risk they can hold. Similarly, financial services companies subject to risk-based capital standards (e.g., banks and insurers) are required to hold more capital for low-rated, non-investment-grade assets.

2.4 – The role of the CRO

Assume that you wish to build a new product for the property catastrophe risk market with both *market-expanding* and *market-completing* properties. These goals would appear to be at odds. Market expansion usually requires the inclusion of previously uncharted risks, entailing greater uncertainty and requiring correspondingly higher returns. Conversely, market completion through the introduction of investment-grade investment options seems to require the creation of *safer*, lower-risk catastrophe instruments.

The introduction of CROs may be able to achieve both of these objectives. CROs would produce a wide spectrum of rated catastrophe risk, opening up investment opportunities for a broader range of potential investors. It would also promote the securitization of new types of risk in order to help diversify the catastrophe risk assets collateralizing the pool.

Of course, catastrophe risk is very different from the credit risk found in traditional CDOs. As such, it is important to understand how a CRO might differ from a traditional CDO, and the impacts of these differences on the success and sustainability of the CRO.

[13] The current market also contains a number of unrated instruments offering investment opportunities generally similar to speculative rated securities.

3. DIVIDE AND CONQUER: CATASTROPHE RISK AND THE CRO

3.1 –CDOs and insurance companies

Set aside catastrophe risk for a moment and begin with the underlying structure: How does a basic CDO function? During the financial crisis, CDOs (particularly those containing high concentrations of risky subprime mortgages) became notorious for their complexity and inscrutability. However, these derivative structures bear striking structural similarities to a much older and more familiar type of financial vehicle—the insurance company. The resemblances are illustrated by two key parallels between CDOs and insurers: tranching and the law of large numbers.

A CDO is comprised of a pool of financial assets[14] carved into *tranches,* a series of ordered claims to the pool's cash flows. Investors in *senior tranches* have first claim to pool profits, and are followed in order by holders of *mezzanine* and *equity* (or *junior*) *tranches*. In exchange for bearing a larger share of the pool's default risk, equity trancheholders are compensated with the highest potential returns. At the opposite end, senior tranches appeal to risk-averse investors willing to accept lower returns in exchange for holding highly rated assets.

The appeal of a CDO is that the most senior tranches[15] can often be structured to satisfy rating agencies' requirements for an exceptionally strong (typically AAA) credit rating. Generally, the credit ratings of these tranches significantly exceed those of the underlying pool collateral assets were they to be rated individually, which is due to the security provided by the subordinate tranches.

The structure of an insurance company is fundamentally similar. As with a CDO, an insurer carves up a pool of underlying assets (in this case, the profits or losses on insurance policies) into *de facto* "tranches." The tranched structure of an insurer is illustrated by considering the priority order of the insurer's liabilities in a run-off scenario. Outstanding policyholder obligations (e.g., loss and unearned premium reserves) receive the highest priority, and are analogous to senior CDO tranches. The insurer's other debt obligations are equivalent to mezzanine tranches, and the equityholders of an insurer match to equity CDO trancheholders (see Figure 1). This tranche-based description of insurers has been examined in detail elsewhere, notably in the context of analyzing reinsurance arrangements.[16]

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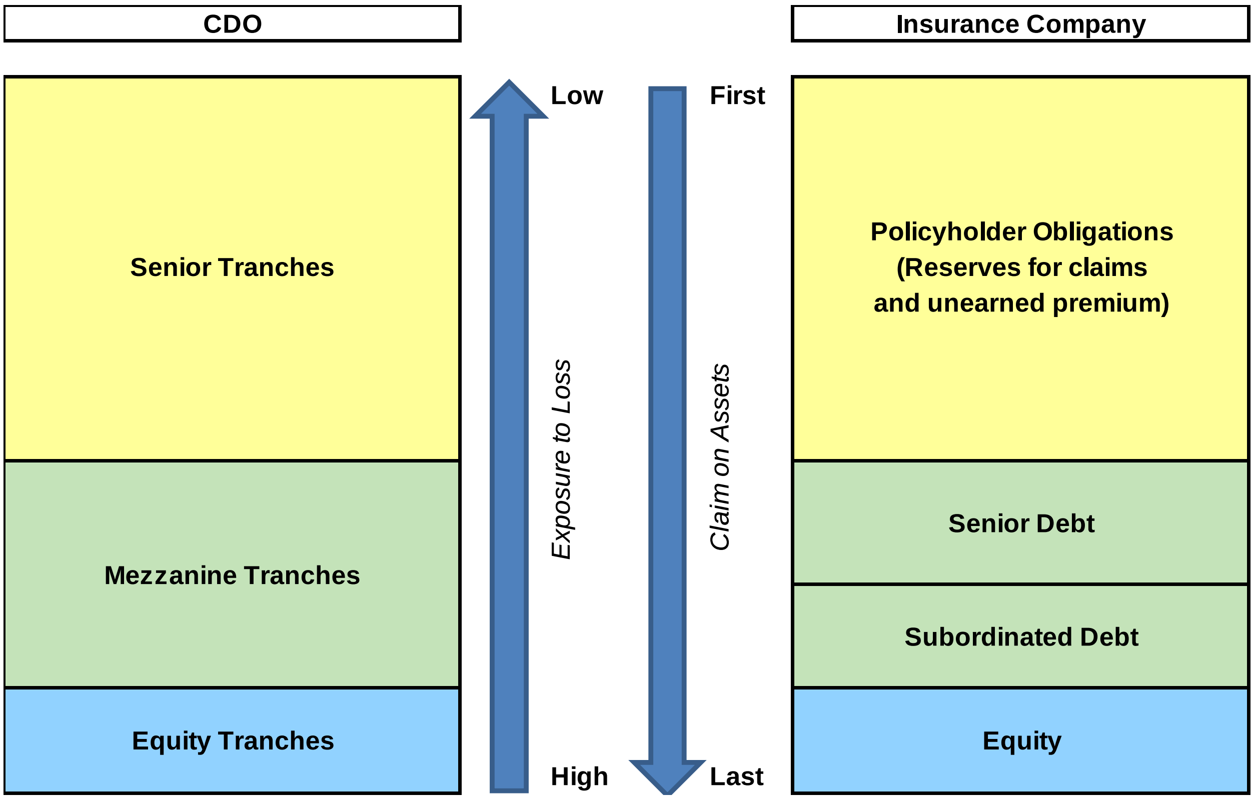
[14] These assets are often assumed to be homogenous.

[15] Complex CDO structures can have upwards of 10 (or more) tranches, with several senior tranches, a number of *mezzanine* tranches, and one or more equity tranches.

[16] Mango, D. & Bunick, C. [21]

Figure 1: Structural Comparison of a CDO and an Insurance Company



The second parallel is that CDOs, like insurers, derive their economic value from the law of large numbers. Both are vulnerable when the assumptions underlying the that law does not hold and single events can cause a large volume of highly correlated losses—Hurricane Andrew and the mortgage downturn being prominent examples.

These similarities beget an obvious question from comparing many decades of insurance industry success to the financial conflagration caused almost immediately by CDOs: Why the enormous discrepancy? Further, could a CRO avoid the pitfalls that undermined the CDOs of the mid-2000s?

The remainder of this section seeks to answer these questions. First, the composition of a CRO will be roughly outlined and compared to that of a pre-crisis CDO. Then, the CRO will be scrutinized in the context of the key structural factors contributing to the collapse of the CDO market.

3.2 – Designing a CRO

As noted above, catastrophe risk can pose a threat to the law of large numbers. By nature, catastrophes affect a large number of policies simultaneously, making catastrophe risk diversifiable only on a global scale.

There is, however, potential for diversification - of two kinds - if sufficient variety of global catastrophe risk can be collected into a single pool. First, this structure would enjoy *geographic* diversification: For instance, Asian typhoon activity is not fully correlated (and in fact may be inversely correlated) with North American hurricane activity.[17]

Second, a broadly based catastrophe risk pool benefits from *typological* diversification. Natural disasters can be geophysical (e.g., earthquake), meteorological (e.g., convective storms and tropical cyclones), or even climatological (e.g., drought) in nature.[18] Each type is driven by forces that are not fully correlated—and often, are not correlated at all. For example, the occurrence of a Japanese earthquake is unlikely to impact the likelihood of a major U.S. hurricane.

The ideal CRO will pool the broadest range of natural catastrophe risks possible to ensure ample diversification. But what form will these risks take? The most basic building block of insurance risk is the individual insurance policy, but securitized instruments are ill-suited to insuring single policies. Instead, the pooled "risks" in a CRO must be pooled *portfolios* of catastrophe risk collected by insurers.

The cost efficiency of the CRO is further enhanced if the pooled catastrophe risks are already securitized and tradable. Catastrophe bonds have a somewhat liquid secondary market, and have already undergone the initial modeling and pricing process. Fractional shares of existing catastrophe bonds likely represent strong building blocks for the CRO.

But does the market have enough existing catastrophe risk material to support CROs? Outstanding catastrophe bonds number in the dozens, while a single mortgage-based CDO pooled thousands of individual mortgages. It was this numerousness (and the law of large numbers) that enabled the credit enhancement found in CDOs.

Sidebar 2:
Impact of Asset Quantity on a CDO

High (1,000+ assets):

Benefits
- Achieves greater spread of diversifiable risk
- Creates proportionally more highly-rated securities
- Allows for greater structuring flexibility and complexity

Drawbacks
- Requires significantly stronger modeling assumptions
- Creates more of a "black box" – complexity may not fully be understood
- Requires higher ongoing management costs

Low (5-10 assets):

Benefits
- Simpler to model – may be able to fully specify relationships between each pair of assets
- Uses less resources for gathering and managing pool assets

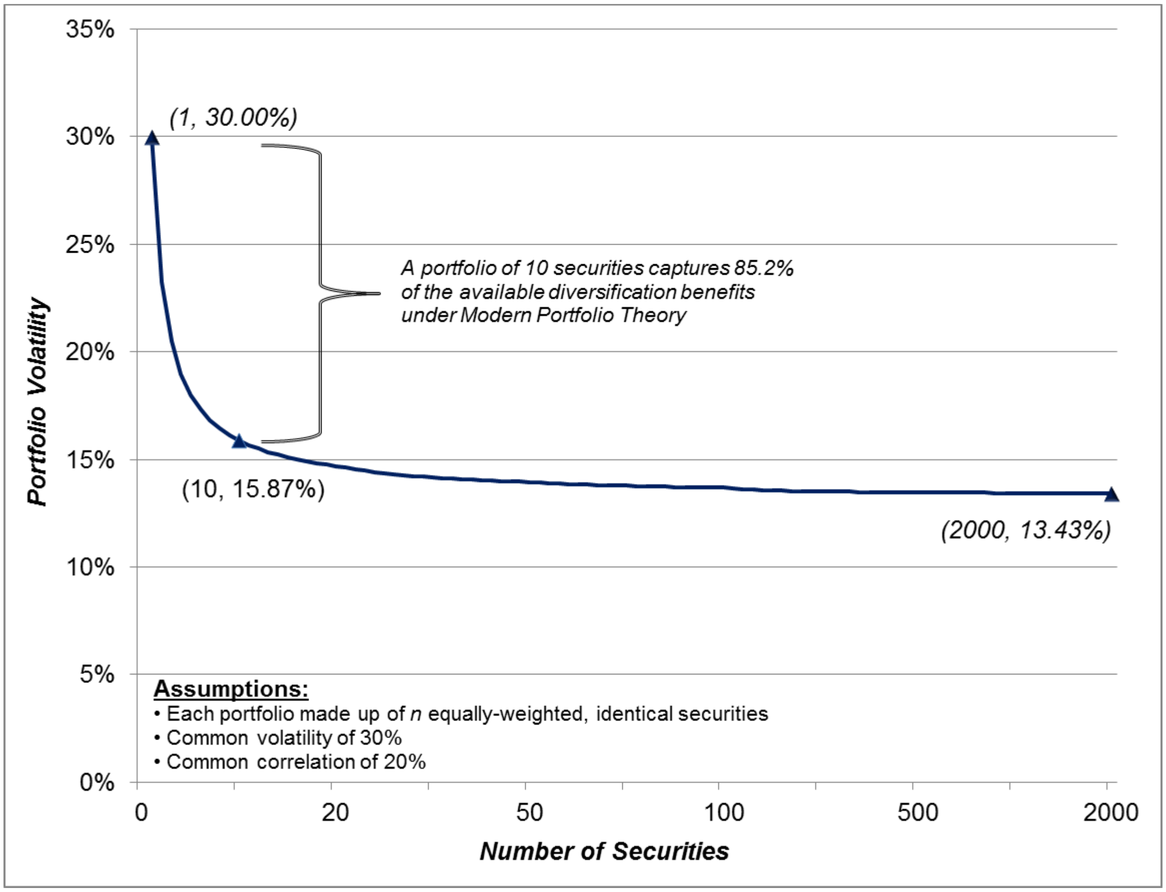
Drawbacks
- Achieves less credit enhancement
- May be harder to reach cost-efficient pool size

Nonetheless, it should not be necessary to acquire thousands of catastrophe assets to create a

[17] Maloney, E. & Hartmann, D. [20]
[18] Among other types not listed.

well-functioning CRO. The key to the feasibility of the CRO is the fact that a relatively low number of pooled assets is needed to capture the vast majority of portfolio diversification benefits. For instance, under the assumptions shown in Figure 2, approximately 85% of the available diversification benefits are captured by a pool of only 10 securities. After this point, the marginal benefit of adding further assets to the pool decreases rapidly.

Figure 2: Diversification Effects on Portfolios of Varying Size



Compared to catastrophe risk securities, individual mortgages are comparatively low in value: A large number of mortgages are required to create an economically viable pool.[19] On the other hand, individual catastrophe bonds are frequently issued with several hundred million dollars of principal at stake, each bond covering a portfolio of thousands of individual insurance policies. A fractional share of a single catastrophe bond offering can represent an investment of many millions of dollars.

It should be feasible to create a CRO with a relatively low number of underlying assets, perhaps between five and 10. The resulting structure will likely be smaller than the average pre-crisis subprime mortgage CDO (for which one source provides an average size of $829 million).[20]

[19] Ashcraft, A. and Schuermann, T. [6]
[20] Barnett-Hart, A.K. [7]

However, CROs should also have a lower size threshold for economic viability, because of the reduced expense load for ongoing management of several securities as opposed to thousands.

3.3 – Does the CRO have the same vulnerabilities as pre-crisis subprime CDOs?

The effort spent designing a CRO is wasted if the resulting structure exhibits the same weaknesses that led to the collapse of the CDO market during the financial crisis. What are the key factors that led to those losses, and how should we expect a CRO to fare in comparison?

After the onset of the crisis, many sought to diagnose the causes behind the collapse of the CDO market. Their conclusions, while wide-ranging, tended to highlight similar themes. These themes can be separated into two categories:

- **Modeling-focused observations:** Addressed *which* assumptions failed to match reality.
- **Behavioral-focused observations:** Addressed *why* assumptions failed to match reality.

The balance of this section provides a brief overview of the fall of pre-crisis CDOs through the lens of the categories above. It finds that a well-designed CRO should fare better on almost every test of systemic vulnerability, proving to be significantly more robust than its pre-crisis subprime predecessors.

3.4 – Modeling: The actuary and the Gaussian copula

In standard CDO models, two types of input parameters must be estimated for each underlying asset. The simpler is the *default profile*—the likelihood of default across time, independent of any other asset in the pool. If this information is not readily available for each asset, then simplifying homogeneity assumptions can streamline the model.

A more significant challenge is quantifying an asset's *dependency profile* with each of the other assets—in short, how its likelihood of default is affected by surrounding defaults. For a portfolio of 1,000 mortgages, modeling the interactions among each asset using a traditional linear correlation matrix requires close to a half million parameter estimates.[21] This approach is usually unwieldy—and for a long time represented the biggest barrier to CDO modeling.[22]

In 2000, actuary David X. Li proposed pricing CDOs with copula models, which were then primarily found in biostatistics and actuarial science.[23] In particular, Li's paper presented the use of a Gaussian copula constructed from a multivariate normal distribution. Unlike many other copula forms, the Gaussian has the practical advantage of being easily generalizable from the two-variable

[21] Because the correlation coefficient is assumed to be 1 along the diagonal, and is the same pairwise for analogous cells on either side of the diagonal.
[22] Mathematically, it is nearly impossible to ensure a positive semi-definite correlation matrix.
[23] Li, D. [18]

situation (find the single dependency between *x* and *y*) to the *n*-variable situation (find the various dependencies among *a*, *b*, *c*, *d*…etc.). To do so, it utilizes a crucial simplification: It assumes that the entire dependency structure for the pool of assets is driven by a common factor, which can be estimated as a single pool default correlation parameter ρ (rho).

The Gaussian copula model (and the similar models that followed) provided solutions to what had seemed an impossible mathematical problem—but in return, it required the assumption that the relationships among thousands of mortgages could be fully expressed by a solitary constant. This key parameter held enormous sway over the model output, particularly due to the significant leverage inherent in CDOs. Thus, the most senior CDO tranches were almost indestructible *assuming the models used to price and rate them were correct*—and highly susceptible to downgrade if they were not.[24,25,26,27]

The models, of course, turned out to be wildly optimistic. As the housing bubble burst, mortgage default rates skyrocketed past all recent historical benchmarks, nationwide. This caused rating agencies to reassess their models and downgrade AAA tranches at an unprecedented pace. The ensuing collateral calls and liquidity crunch kicked off the financial crisis and crystallized the public's image of the CDO: A structured finance vehicle both incomprehensible and toxic.

In comparison, CROs should be able to rely upon a more accurate and robust modeling process. Natural catastrophe models forecast physical events, while economic models forecast human behavior: The former lie more in the realm of science, the latter social science. While it is important not to downplay the amount of uncertainty in catastrophe models (which is significant), it is also true that they need not capture the additional behavioral component inherent in financial markets - which often drives tail outcomes (e.g., a "run on the bank"). At least in the short run, humans can have little impact over the occurrence or severity of any particular natural catastrophe—the very reason that catastrophe risk is desired as a *zero-beta* investment.[28]

Further, the low number of assets in a CRO allows for more transparent pricing. With accurate exposure information and access to catastrophe models, it is possible to research each CRO asset in detail.[29] In a CRO, relationships among specific assets can be identified and modeled on a case-by-case basis as opposed to relying on a single, catch-all assumption to represent the entire dependency structure. Potential CRO modeling techniques are described in Section 4.2 below.

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[24] Coval, J., Jurek, J., & Stafford, E. [12]
[25] Krahnen, J. & Wilde, C. [17]
[26] Hull, J. & White, A. [16]
[27] Heitfield, E. [15]
[28] That is, an investment showing no correlation with the performance of the equity markets as a whole.
[29] Some CDOs, to further complicate the matter, were comprised solely out of tranches of other CDOs – creating a third layer of tranching.

3.5—Behavior: A matter of incentives

While it is important to understand the weaknesses in the CDO pricing models used in practice during the mid-2000s, it is more important to understand *why* the modeling assumptions turned out to be wholly inaccurate.

At the heart of the matter was an incentives problem: Most of the key participants in the life cycle of a CDO were compensated according to the volume of completed CDO transactions. Worse, most CDO originators retained little to none of the downside risk associated with the securities:

- Mortgage writers adopted an "originate-to-distribute" model that removed their portfolios of subprime mortgages from their balance sheets and led to a loosening of loan standards.[30]
- Major banks then turned loan portfolios into securitized instruments, collecting a healthy underwriting fee while typically retaining little to no risk.[31] The lower-rated tranches of these mortgage-backed securities were then re-tranched into CDOs, providing yet another opportunity for fees.[32]

A misguided incentive structure plagued not only the formation of CDOs, but their evaluation by the major credit rating agencies. As the CDO market exploded, so too did the fees paid to rating agencies - who were paid not only on volume, but by the *arranger* of the security (and not by the ultimate investor). For the ratings agencies, taking a more pessimistic view than the competition often meant watching arrangers take their subsequent (and highly lucrative) business elsewhere.[33]

As a result, the only participants with a strong incentive to accurately assess the quality of the assets were the investors themselves. In reality, many investors were either unable or unwilling to invest the resources necessary to obtain their own view of CDO risk, instead putting their faith in the CDOs' sterling credit ratings. Only the eventual market implosion revealed what is obvious in hindsight: Real skin in the game—fundamental to appropriately motivating CDO intermediaries—was absent at nearly every stage.

Fortunately, these issues are largely absent in a CRO. Securitized insurance risk is generally written on an *excess-of-loss* basis, often with cedent co-participation in the reinsured layer. Unlike pre-crisis mortgage originators, primary insurers thus have every incentive to write good business—because they retain the vast majority of risk on their policies.

In addition, reinsurance markets are highly relationship-based. Many of the same intermediaries

[30] Purnanandam, A. [28]
[31] Ashcraft Schuermann [6]
[32] Many of the losses eventually suffered by major banks were incurred by those that weren't quite *good enough* at removing the toxic assets they were creating from their balance sheets.
[33] Barnett-Hart [7]

that serve the catastrophe bond market also provide services for a range of non-catastrophe reinsurance transactions by similar (or even the same) parties. There arguably exists a stronger reputational incentive for the actors along the catastrophe securitization structuring chain to respect the interests of the other involved parties.

Thus, a properly structured CRO may avoid both the modeling and incentives problems that plagued pre-crisis CDOs. Significant expertise is still needed to grasp the numerous sources of risk inherent in catastrophe contracts: Nevertheless, the robustness of insurance markets should allow the CRO to avoid becoming simply the latest example of a "toxic" structured finance asset.

4. A CRO PRICING EXAMPLE

4.1 – Overview

This section considers the pricing of a theoretical five-asset CRO. It is designed to highlight the key features of the structure without excessive functional detail. Analysis is presented in a simplified form, with most mathematical details left to the Appendix.

For our sample CRO, we assume that the asset pool consists of fractional shares of single-peril Rule 144(A)—that is, publicly issued—catastrophe bonds with a one-year duration. Each bond has thus been evaluated by a third-party catastrophe model vendor during the initial pricing and issuance process. We assume that we have a stand-alone exceedance probability (EP) curve for each asset representing its loss profile. We also have the following summary statistics for each bond:

- **Attachment probability:** The likelihood that a bond will suffer a nonzero loss to its principal.
- **Expected loss:** The average percent of principal that a bond is expected to lose.
- **Exhaustion probability:** The likelihood that a bond will suffer a complete loss to its principal.

Figure 3 shows the selected parameters for each of the five securities in our sample CRO, based loosely on existing market securities. The current, public Standard & Poor's rating table for catastrophe bonds was utilized to estimate a rating for each security. None of the securities listed below qualifies for an investment-grade credit rating.

Figure 3: Sample CRO Composition

	Term (Years)	*Assumed Expected Loss*	*Estimated Attachment Probability*	*Estimated Exhaustion Probability*	*S&P Implied Rating*
Florida Hurricane - FLH	1	4.00%	5.33%	2.67%	B+
New England Hurricane - NEH	1	2.00%	2.67%	1.33%	BB
US (California) Earthquake - USQ	1	3.00%	4.00%	2.00%	BB-
Japan Earthquake - JPQ	1	3.00%	4.00%	2.00%	BB-
Turkey Earthquake - TUQ	1	1.50%	2.00%	1.00%	BB+

Next, we aim to estimate similar performance metrics for each CRO tranche.[34] To do so, we must first define the tranches' attachment and detachment (or exhaustion) points.

In a CDO, tranches are designed to maximize the size of the highest-rated (usually AAA) tranches. The endpoints of each tranche are frequently calibrated to precisely meet the minimum standards of a given rating category. With only five assets, such a level of refinement is less feasible for a CRO. Instead, we will divide the sample pool into four illustrative tranches:

- **Equity tranche:** Eroded by aggregate losses of the first 0% to 20% of pool collateral.
- **Mezzanine tranche:** Eroded by aggregate losses from 20% to 40% of pool collateral.
- **Senior tranche:** Eroded by aggregate losses from 40% to 60% of pool collateral.
- **Super-Senior tranche:** Eroded by aggregate losses from 60% to 100% of pool collateral.

4.2 – CRO dependency modeling

The prices of CRO tranches, as with those of a traditional CDO, depend heavily on how the individual risks in the collateral pool relate to one another. These effects are magnified greatly in senior tranches, whose loss profiles are highly leveraged on the pool's dependency patterns.

There are at least two plausible approaches to reflecting asset dependencies in CRO pricing.

The first approach is through *event simulation*, using the same techniques that are used to price stand-alone catastrophe bonds. Each major catastrophe modeler produces a simulated "event set" of natural catastrophes. Using this event set, simulated years are generated for the portfolio of exposures—which in this case represents all of the securities within the asset pool. From these simulations, one can obtain the EP curve and summary statistics referenced above. These are then used to price the CRO tranches.

However, investors may consider the outputs of the major catastrophe modelers to be somewhat of a black box. Further, perhaps detailed exposure information is not available for each asset in the CRO, or the model becomes unwieldy and hard to analyze on a portfolio-wide basis. In any of these cases, investors may desire another approach for establishing their own "view of risk."

An alternative to event simulation is a *portfolio analysis* approach. For our sample CRO, this consists of taking each of the five individual EP curves (i.e., catastrophe model outputs) and relating them to one another—as opposed to trying to create a comprehensive portfolio EP curve out of the combined exposure sets.

[34] Note that for a CRO with sequentially stacked, non-overlapping tranches, the exhaustion probability of one tranche is equivalent to the attachment probability of the next higher tranche.

Catastrophe modeling firms already enable this type of analysis by producing pairwise linear correlation matrices for the existing universe of catastrophe bonds.[35] However, using this particular type of matrix for CRO pricing may be problematic. It best suits analysis based on the normal distribution—the use of which was one of the issues with traditional CDO pricing.

Recall that normal Gaussian copulas (and related forms) were used for CDO pricing because of the challenges in extending two-dimensional dependency modeling to higher dimensions. Fortunately, there are alternative methodologies that are likely superior for pools with relatively few assets, such as a CRO.

The use of *vine copulas* is one such alternative. Vine copulas tackle the multi-dimensional challenge by modeling pairs of copulas in two dimensions and then linking them together in "vines." Importantly, this procedure allows for the use of tail-heavy copulas that fit the tail-heavy distributions being modeled—and better, it allows for the use of *different* copulas to model each pair of assets. This eliminates the need for an overarching (and often ill-fitting) assumption about which single copula best fits the data.

4.3 – Rating the sample CRO with a vine copula approach

A vine copula model can be used to estimate the loss parameters and credit ratings of the various tranches of our sample CRO.[36] To illustrate the importance of asset interdependencies, we rate the CRO tranches under two assumptions:

- *Independence model:* The performance of each asset in the CRO is assumed to be fully independent of the performance of each of the other assets.
- *Dependencies model:* While independence is assumed for many pairs of assets, a few are assumed to have positive loss correlation captured by a *Clayton copula*:[37]
    - There is assumed to be a slight positive correlation between Florida and Northeast hurricane risk.
    - Similarly, there are assumed to be slight positive correlations among earthquake risks in the U.S., Japan, and Turkey.
    - There is assumed to be no correlation between hurricane and earthquake risk types.

Figures 4 and 5 show the results of modeling under each set of assumptions. Modeling details, mathematical derivations, and additional key assumptions are outlined in the Appendix.

[35] Risk Management Solutions, Inc. Miu Platform [30]
[36] After the key loss statistics and ratings have been estimated, the tranches can be priced through any number of theoretical approaches or by comparison to existing market securities.
[37] For the purposes of this paper, the various *taus* are selected judgmentally and for illustrative purposes only.

Figure 4: Estimated Independence Model Tranche Loss Parameters

Sample CRO Independence Model				
Tranche	Tranche Range	Default Probability	Expected Loss	Implied Rating
Junior	0-20%	16.79700%	12.86343%	CCC+
Mezzanine	20-40%	1.03720%	0.65358%	BB+
Senior	40-60%	0.02670%	0.01458%	A
Super-Senior	60-100%	0.00020%	0.00003%	AAA

Based on Monte Carlo Simulation

Based on the Independence model, we find that both the senior and super-senior tranches of the sample CRO have simulated default probabilities low enough to quality for investment-grade ratings on the S&P ratings table for structured finance instruments.[38] In addition, the super-senior tranche (representing a full 40% of the CDO collateral) meets the ratings table standards for a AAA rating.

Figure 5: Estimated Dependencies Model Tranche Loss Parameters

Sample CRO Dependencies Model				
Tranche	Tranche Range	Default Probability	Expected Loss	Implied Rating
Junior	0-20%	16.62980%	12.75911%	CCC+
Mezzanine	20-40%	1.17590%	0.74806%	BB+
Senior	40-60%	0.04010%	0.02176%	A-
Super-Senior	60-100%	0.00050%	0.00006%	AA+

Based on Monte Carlo Simulation

In contrast, the Dependencies model concentrates a higher percentage of the losses in tail outcome events—that is, in higher tranches of the CRO. As a result, both the senior and super-senior tranches of the sample CRO are rated one notch lower than the comparable tranches in the Independence model.

[38] Barnett-Hart [7]

Even though the absolute effects on default probability and expected loss may be small (e.g., the expected default probability on the super-senior tranche goes up 0.0003%), the *relative* impacts of the Dependencies model are significant, as shown in Figure 6.

Figure 6: Modeled Loss Comparisons by Tranche

Sample CRO Model Comparison				
	(1)	(2)	(3)	(4)
	Independence Model Expected Loss	Dependencies Model Expected Loss	(2) - (1) Difference	(3) / (1) Relative Percent Change
Tranche				
Junior	12.86343%	12.75911%	-0.10431%	-0.811%
Mezzanine	0.65358%	0.74806%	0.09448%	14.456%
Senior	0.01458%	0.02176%	0.00717%	49.164%
Super-Senior	0.00003%	0.00006%	0.00003%	73.380%

Based on Figures 4 and 5

The incorporation of dependencies shifts risk from the junior tranche to the other tranches, with the relative impact growing as the level of seniority increases. This finding, which matches the results of a number of financial crisis analyses, highlights the importance of accurate incorporation of dependencies into a CRO model—despite the fact that correlations among insurance risks may be low compared to those found in the financial markets.

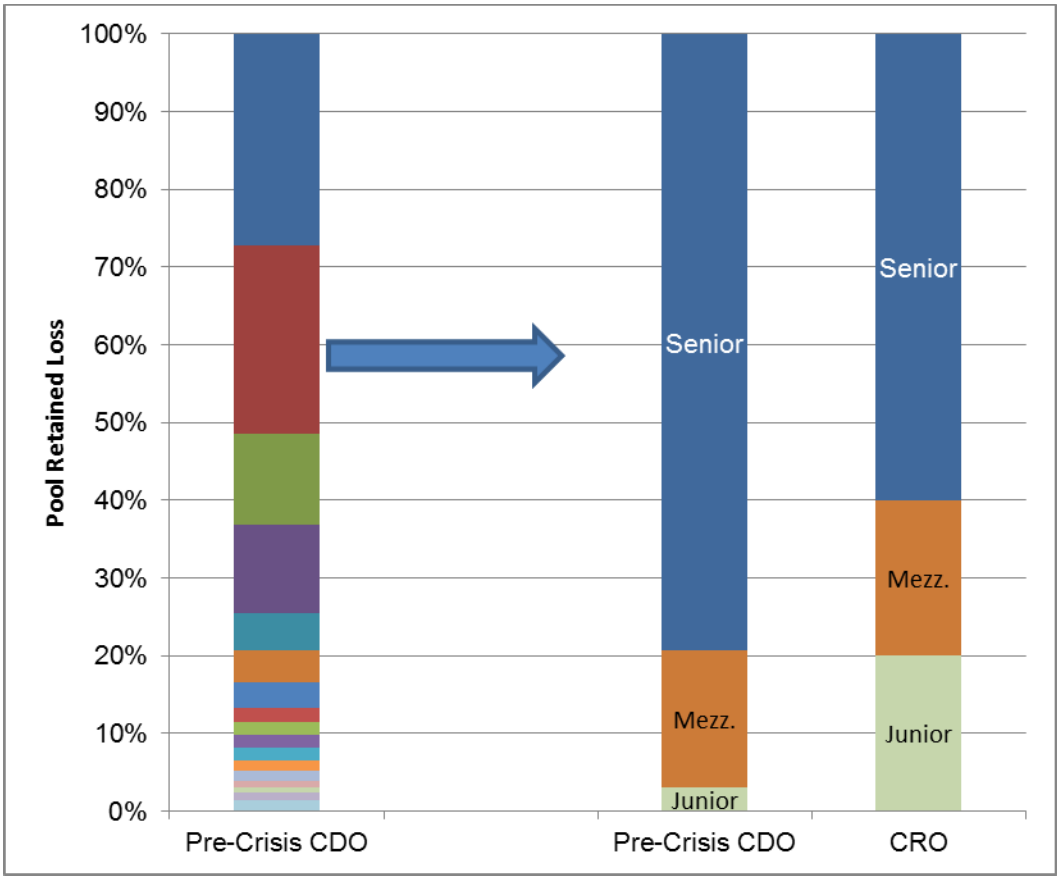
4.4 – Other observations

Despite the potential advantages a CRO may enjoy in terms of modeling accuracy and stability, it nevertheless provides less credit enhancement (e.g., fewer AAA-rated assets) than a traditional CDO. Figure 7 compares our sample CRO tranching structure to that of a pre-crisis mortgage-backed CDO—Goldman Sachs' GASMP Trust 2006-NC2 (GSAMP). GSAMP was split into the following 17 tranches:[39]

- 5 senior tranches, each rated AAA by S&P
- 9 mezzanine tranches, with investment-grade ratings ranging from AA+ to BBB-
- 3 junior tranches (including an equity tranche), with speculative-grade ratings ranging from BB+ to unrated.

[39] Ashcraft Schuermann [6]

Figure 7: Tranche Structure, Pre-crisis CDO vs. Sample CRO



GSAMP contains a significantly higher proportion of senior risk than the sample CRO. On a proportional basis, the difference is most striking between the sets of junior tranches, which make up 20% of the CRO but only 3% of GSAMP.

However, this may not be too severe of a drawback for the CRO. In some instances, the alternative capital market has shown significant appetite for ILS with a default risk similar to or higher than that of the sample CRO's junior tranche. For example, USAA's Residential Re 2013-2 (Class 1) catastrophe bond exposed investors to an attachment probability of 21.38% and an expected loss of 13.06% - and received one of the lowest pricing ratios of coupon-to-expected loss ever seen in the market.

In fact, the CRO's creation of junk junior tranches may be a significant benefit to alternative capital investors who seek a high-yielding portfolio of catastrophe risk as opposed to a well-diversified one.[40] For these investors, rapidly falling spreads on peak perils such as Florida hurricane

[40] The rationale for these investors is often that holding catastrophe risk itself (in small quantities) serves as the macro-level diversifier for the rest of their portfolios. Under this paradigm, pursuing diversification within the catastrophe portfolio can lead to an unnecessary erosion of returns.

risk have threatened their return objectives. This potentially opens the door for the use of small, targeted allocations to CRO junior tranches as part of a larger investment strategy.

5. POTENTIAL SPONSORS AND CAPITAL CONSIDERATIONS

5.1 – Market history: Multiple-event securitizations

Prior to the financial crisis, a few major insurers and reinsurers experimented with creating investment-grade securitizations of their risk. These *multiple-event securitizations* worked similarly to CROs: High credit ratings were obtained by insuring only the second, third, or further subsequent events happening in a given period across a worldwide portfolio. In essence, they were CROs that simply excluded the junior tranche.

The first catastrophe bond to have a tranche receive an “A” rating in such a manner was issued by the French reinsurer SCOR Group in December 2001.[41] Atlas Reinsurance II covered European windstorm, Japanese earthquake, and Californian earthquake risk. It had two tranches: Class B notes provided coverage for the second qualifying catastrophe in the contract period, while Class A notes provided coverage for the third.[42] While the Class B notes received a BB+ rating, Class A received the coveted “A”—with an annual expected loss of 0.05% and coupon spread above LIBOR of 2.38%.[43,44]

This rating reversed S&P’s policy of maintaining a BBB+ ceiling for catastrophe bonds, which was due to the “cliff risk” inherent in a first-event cover: No matter how unlikely the event, the owner of a first-event catastrophe security faces the risk of full default with little or no warning. Because Atlas II required an accumulation of events to be triggered, S&P was comfortable that cliff risk was sufficiently mitigated. In the event of a first triggering event, investors would have the opportunity to reassess their holdings - and offload them if they believed the risk profile no longer suited their objectives.[45]

Other major insurance players such as Swiss Re and Converium also issued multiple-event securitizations in the early to mid-2000s. Since then, the market for such products appears to have largely disappeared: The share of investment-grade catastrophe risk fell from roughly a quarter of the market in 2007 to zero in 2013.[46] Why might this have occurred?

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[41] Woo, G. [37]
[42]SCOR retains the first qualifying catastrophe without assistance from Atlas Reinsurance II.
[43] Woo [37]
[44]Artemis Deal Directory [5]
[45] Boyle C. [9]
[46] Willis Re Capital Markets [36]

5.2 – Capital adequacy: The challenge of single-cedent CROs

To illustrate, consider the *return periods* (the "1 in X" odds) for triggering each tranche of the sample CRO under the Dependencies model. By implied rating, the mezzanine and senior tranche are approximately equivalent to Atlas II's Class B and A notes, respectively.

Figure 8: Return Periods by Tranche, Sample CRO

Sample CRO Dependencies Model		
Tranche	**Default Probability**	**Return Period**
Junior	16.62980%	6 Years
Mezzanine	1.17590%	85 Years
Senior	0.04010%	2,494 Years
Super-Senior	0.00050%	200,000 Years

The senior tranche of the sample CRO has a return period of approximately 2,500 years. In comparison, the Solvency Capital Requirement (SCR) under Solvency II requires European insurance companies to hold capital to protect against a 1-in-200-year series of events. Most companies hold capital to protect against events beyond this 1-in-200-year standard: Ratings agencies will generally insist on it. However, holding capital is expensive. Companies may not care (and it could be argued, *shouldn't* care) about risk at return periods far exceeding their internal capital adequacy targets.

A similar line of argument provides a case against a single-cedent CRO. For example, a company with the risks contained in the sample CRO and a capital adequacy horizon that doesn't extend to 2,500 years could simply buy protection up to the exhaustion point of the mezzanine tranche (e.g., by securitizing only the first two events occurring on the global portfolio). This would prevent the company from paying for coverage that is not in line with its overall strategic plan.

There are still a number of reasons for a company to consider a full-fledged CRO solution. Perhaps an insurance group's regionally based companies need reinsurance cover that cannot be consumed by catastrophes in another region, or the company has capital management goals that go beyond a simple analysis of return period adequacy. Nevertheless, it is not surprising that the popularity of the single-cedent multiple-event securitization has waned over time.

5.3 – Market history: Gamut Re

An alternative vision of the CRO (and the primary one offered in this paper) combines securitized risks from a number of companies. This avoids the problems described in the prior section, as each company's risks can be securitized at a lower return period (e.g., the 25- to 100-year periods commonly found in today's catastrophe bonds). For this type of CRO, the historical precedents are more infrequent—perhaps limited to a single structure established immediately prior to the financial crisis.

In June 2007, the hedge fund Nephila Capital raised over $300 million to sponsor Gamut Re Ltd., a sidecar-type vehicle whose returns from investing in catastrophe risk were allocated across five tranches.[47,48] The catastrophe portfolio held in Gamut was actively managed by Nephila Capital, and ran through the end of 2009. Details are shown in Figure 9

Figure 9: Gamut Re Tranche Structure.

Nephila Capital - Gamut Re
Tranche Structure, Ratings, and Yields

Class	Size (in M $USD)	Coupon*	S&P Rating
A	60	1.4%	A-
B	120	3.0%	BBB+
C	60	7.0%	BB-
D	25	15.0%	NR
E	45	Equity	NR

*Represents spread over LIBOR

Source: PR Newswire

Gamut Re expired at the end of 2009 and was not renewed, with Nephila citing the increased cost of debt in the immediate post-crisis markets.[49] Since then, it appears that no similar transactions have been attempted (at least publicly).

Yet conditions have changed since 2009. Fixed-income coupon rates currently approach all-time lows and an unprecedented (and increasing) number of investors have turned their attention to the catastrophe risk markets. As the diversity and number of catastrophe securitizations continue to

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47 PR Newswire [27]

48 SIFMA: Insurance & Risk-Linked Securities Conference [31]

49 Trading Risk [34]

increase, so too does the feasibility of the CRO—and with it, the expansion of catastrophe risk markets to investors and risks that heretofore have remained on the outside looking in.

6. CONCLUSION: NEW SKIES AHEAD?

More than most other economic assets, catastrophe risk securitizations are well-suited to inclusion in tranche-based leveraging structures. As evidenced by the lessons of the recent financial crisis, such structures do not offer a panacea for maturing financial markets: Nevertheless, the CRO may serve as a powerful tool for completing and expanding the existing market for catastrophe risk.

Because of their inherent similarity to CDOs—the fuel for the financial crisis meltdown—we can expect that the concept of tranched catastrophe risk might require patient exploration. However, a well-structured CRO is likely to avoid many of the systemic modeling and incentive-based vulnerabilities that were fatal to the pre-crisis CDO market. In contrast, CROs are well-positioned to take advantage of recent advances in dependency and catastrophe modeling to provide a nuanced, powerful, and relatively transparent basis for market analysis.

Above all, the CRO is arguably the optimal tool for generating investment-grade catastrophe risk, a missing ingredient in the current market. Securitizing risk that is too far out in the tail (either on a single-event or multiple-event basis) is unlikely to appeal to many companies on a stand-alone basis. As the catastrophe risk market continues to expand, however, it becomes increasingly possible to generate investment-grade risk by combining the risk of a number of different companies—opening up new possibilities for the financial markets to spread the risk from natural catastrophes on a global basis.

APPENDIX: MODELS FOR CRO PRICING

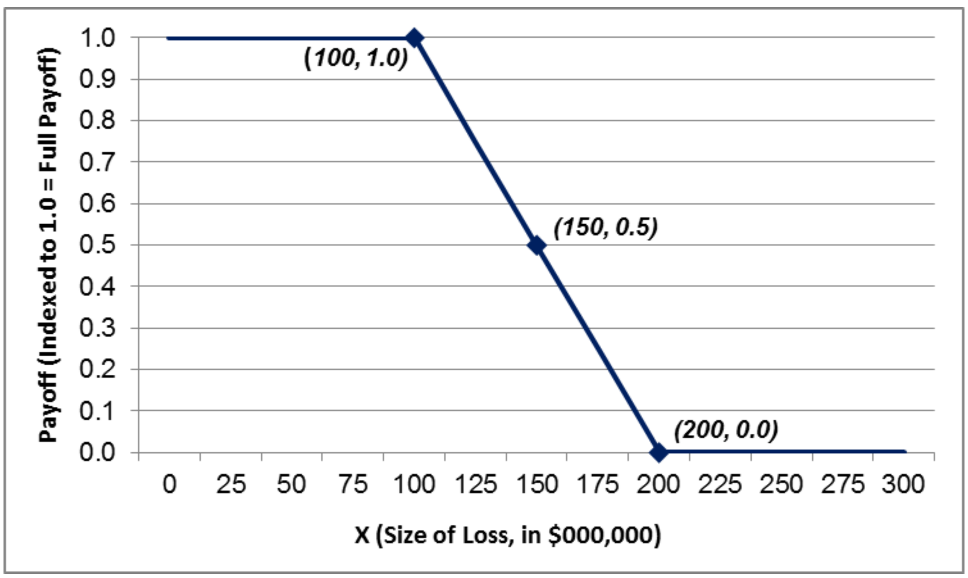
A.1 – Key modeling assumptions and background

The following assumptions and background discussion apply to each of the two models discussed in this Appendix.

Assume that the assets are equally weighted in the CRO—that is, each asset represents 20% of the overall portfolio. Further, assume that the catastrophe bonds are zero-coupon: That is, a single payment is made at the end of the payment term to the bondholder if the bond has yet to default.[50]

On an individual basis, we assume that the payoff profile for each catastrophe bond asset in the pool could be replicated by a bull put spread (assuming the appropriate options existed). For example, a catastrophe bond written to cover a 50% quota share of the layer $100 million excess of $100 million would have a payoff profile as a function of *X*, the random variable representing size of loss, as shown in Figure 10.

Figure 10: Sample Catastrophe Bond Payoffs by Size of Loss



By combining the payoff function with the exceedance probability function (that is, the survival distribution function of the size of loss), we can derive the full payoff probability function for each asset. In practice, the exceedance probability function will be given by the modeled EP curve. For the sake of this example, we assume that each of our assets has the following simplified payoff structure:

[50] This assumption likely would not be justified in an actual pricing model, but does not materially change the generalizable conclusions of this paper.

- For each asset, assume the chance of loss hitting the insured layer (the "attachment probability") is y_i, which is given by the historical data.
- Similarly, assume that the chance of a full-limits loss to the insured layer (the "detachment probability") is z_i, which is given by the historical data.
- Assume that the exceedance function is linear between the attachment and detachment points.[51] Let the expected loss (EL) for each asset be the probability-weighted expectation of the amount of payoff not received by the bondholder that is due to catastrophe loss. Given this, $EL_i = (y_i + z_i)/2$

For each of the assets in our sample CRO, we assume a term-to-expiration of one year and default parameters designed to approximate current market offerings. We determine the implied credit rating for each asset based on the most recent Standard and Poor's (S&P) rating matrix for catastrophe securities, shown in Figure 11.[52]

Figure 11: Illustrative S&P Catastrophe Risk Rating Table

Portion Of Nat-Cat Risk Factor Table

(%)	aaa	aa+	aa	aa-	a+	a	a-	bbb+	bbb	bbb-
1	0.003	0.010	0.015	0.025	0.040	0.060	0.085	0.234	0.353	0.547
2	0.027	0.048	0.074	0.106	0.150	0.200	0.264	0.514	0.825	1.279
3	0.052	0.085	0.133	0.188	0.260	0.340	0.443	0.850	1.405	2.177
4	0.076	0.123	0.191	0.269	0.370	0.480	0.621	1.246	2.073	3.213
5	0.100	0.160	0.250	0.350	0.480	0.620	0.800	1.704	2.812	4.359

	bb+	bb	bb-	b+	b	b-	ccc+	ccc	ccc-
1	1.632	2.525	3.518	4.510	5.824	8.138	23.582	45.560	66.413
2	3.211	4.946	6.915	8.885	11.751	16.674	38.104	59.145	79.233
3	4.758	7.230	10.095	12.960	17.152	24.004	46.752	64.835	82.905
4	6.276	9.380	13.037	16.694	21.921	30.025	52.288	68.078	84.581
5	7.763	11.403	15.745	20.087	26.089	34.945	56.158	70.313	85.650

A.2 – The Independence model

The following section prices the CRO tranches under the assumption that the performance of each of the underlying single-peril securities is unrelated to each other asset in the CRO.

The concept and mathematics behind the Independence model are simple. Given the assumption of independence, it becomes a straightforward three-step process to assess the risk profile for each CRO tranche:

[51] Outside of the range between the attachment and detachment points, the payoff function is constant.
[52] Standard &Poors [32]

1. Using a Monte Carlo simulation generator, simulate X number of years of performance for each of the assets using the payoff profiles and exceedance curve given above. In this paper, X = 1,000,000.
2. For each simulated year, add up the probability-weighted losses for the assets to get the total loss as a percent of pool collateral.
3. Assign the total pool losses to tranches according to the tranching algorithm outlined in Section 4.

The results are presented in Figure 4 above.

A.3 – The Dependencies model: Clayton copulas

The Dependencies model produces a more nuanced view of the risk profile for each tranche. We take a vine copula modeling approach in conjunction with a set of Clayton copulas. The Clayton copula concentrates risk into the left tail of the dependency structure, which in this case we will take to mean high-loss outcomes leading to low-payout states of the security. A major benefit of the Clayton copula is that it is solvable in closed form, leading to a relatively straightforward simulation process when one of the variables is already known.

This procedure, per Venter (2007), is as follows:[53]

- u and v represent the inverse single-variable cumulative distribution function for x and y respectively, that is:
  - $u = F_X^{-1}(x)$
  - $v = F_Y^{-1}(y)$
- a represents the *Kendall's tau* (τ) for the relationship between the two single-variable distributions. See below for description of Kendall's tau.
- $C_u(u,v)$ represents the partial first derivative with regards to the first argument

Then, the following holds for the Clayton copula (Figure 12):

[53]Venter, G. [35]

Figure 12: Derivation of Simulation Formula for Clayton Copula

$$C(u,v) = [u^{-1/a} + v^{-1/a} - 1]^{-a}, \quad a > 0$$

$$C_u(u,v) = [u^{-1/a} + v^{-1/a} - 1]^{-a-1} u^{-1-1/a}$$

$$\frac{C_u(u,v)}{u^{-1-1/a}} = [(1-u)^{-1/a} + (1-v)^{-1/a} - 1]^{-a-1}$$

$$[\frac{C_u(u,v)}{u^{-1-1/a}}]^{1/(-a-1)} + 1 - u^{-1/a} = v^{-1/a}$$

$$\left\{[\frac{C_u(u,v)}{u^{-1-1/a}}]^{1/(-a-1)} + 1 - u^{-1/a}\right\}^{-a} = v$$

Thus—with knowledge of $C_u(u,v)$, u, and constant a – we can simulate the variable v as the output variable conditioned on the independently simulated variables $C_u(u,v)$ and u.

To utilize this model, we must first estimate Kendall's tau. Although Kendall's tau differs from the standard Pearson product-moment correlation coefficient, its form is much the same—a number between -1 and 1 (inclusive) with the following meanings:

- A tau of -1 represents a pair of fully anti-correlated (negatively correlated) assets
- A tau of 0 represents a pair of uncorrelated assets
- A tau of 1 represents a pair of fully correlated assets

The impacts of Kendall's tau on the modeled outputs from the Clayton copula are shown below. Simulated relationships between variables using a Kendall's tau of 0.0 (Figure 13) and a Kendall's tau of 0.4 (Figure 14) are shown below. Note that while the joint distribution of the variables is evenly spread in Figure 13, it instead shows a concentration in the lower left-hand corner (and a smaller amount of concentration in the upper right-hand corner) in Figure 14.

Figure 13: Clayton copula simulation, Tau = 0.0

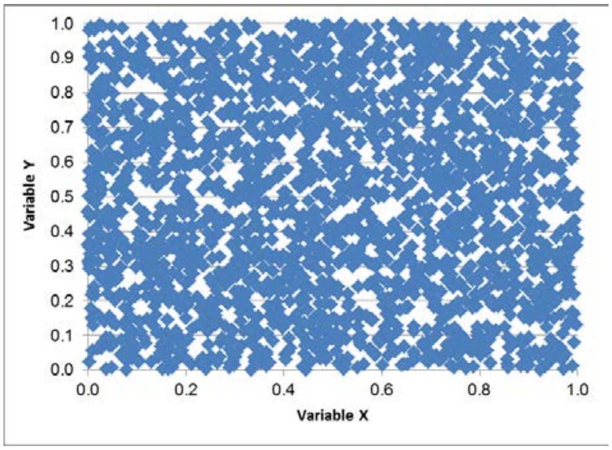
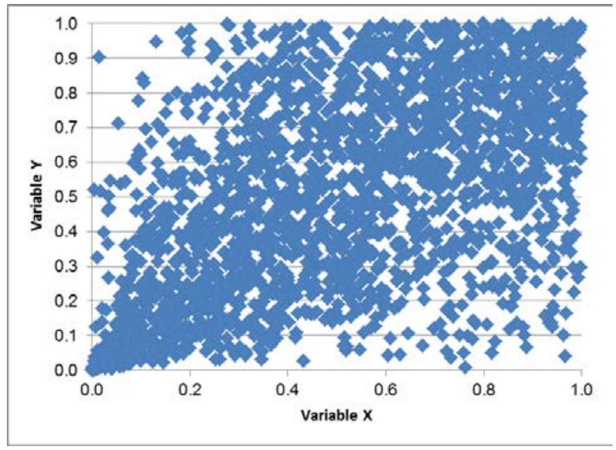


Figure 14: Clayton copula simulation, Tau = 0.4



In practice, *tau* estimates will likely be selected for each pair of assets based on a combination of historical data, existing catastrophe models, and expert judgment. For the purposes of this paper, estimates of tau are selected judgmentally and for purely academic purposes as follows:

- The relationship between Florida and New England hurricane risk is captured by a tau of 0.2
- The relationship between US earthquake and Japan earthquake risk is captured by a tau of 0.2
- The relationship between US earthquake and Turkey earthquake risk is captured by a tau of 0.2
- All other relationships (direct or conditional) are independent (captured by a tau of 0.0).

These relationships are captured in the pairwise tau matrix shown in Figure 15 below. Note that despite being captured in a similar form, these constants do *not* represent the linear Pearson's constants typically shown in correlation matrices.

Figure 15: Tau Matrix for Sample CRO

	FLH	NEH	USQ	JPQ	TUQ
FLH	**1.0**	**0.2**	**0**	**0**	**0**
NEH	*0.2*	**1.0**	**0**	**0**	**0**
USQ	*0*	*0*	**1.0**	**0.2**	**0.2**
JPQ	*0*	*0*	*0.2*	**1.0**	**0**
TUQ	*0*	*0*	*0.2*	*0*	**1.0**

A.4 – The Dependencies model: Vine copulas

V*ine copula* models enjoy a number of advantages over the higher-dimensional copulas traditionally used by financial practitioners to model CDOs:

Traditional copula models are limited to the use of a single copula to describe the entire dependency structure. In comparison, a vine copula model is highly flexible: A different copula may be selected for each relationship, reflecting its specific attributes (e.g., tail heaviness).

As a result, vine copula models require far fewer assumptions regarding the behavior of the pool, particularly regarding the homogeneity of pool assets.

The primary multivariate copula models (e.g., Gaussian, Student's t) are generally either symmetric or have only moderate tail heaviness, particularly in higher dimensions. Thus, these models may fail to capture the true tail risk contained in a CDO, particularly given that modeling asymmetries in the dependency structure (i.e., choosing the shape of the copula) can sometimes have a greater impact on results than modeling asymmetries in the marginal distributions of the individual assets themselves.[54]

The primary weakness of a vine copula model is the large number of parameter estimates needed as the size of the pool grows. Assuming a single-parameter copula is used for each dependency, a pool of n assets requires the estimation of (n)(n-1)(0.5) dependency parameters for a fully specified model. The number of estimated parameters can frequently be reduced by careful vine structuring and/or a constant parameter assumption for all conditional dependencies past a certain vine level. Nevertheless, vine copula models are likely to be far more accurate for pools containing a limited number of securities, where the additional precision of the individual dependency estimates is not overwhelmed by the increased risk from estimating many parameters.

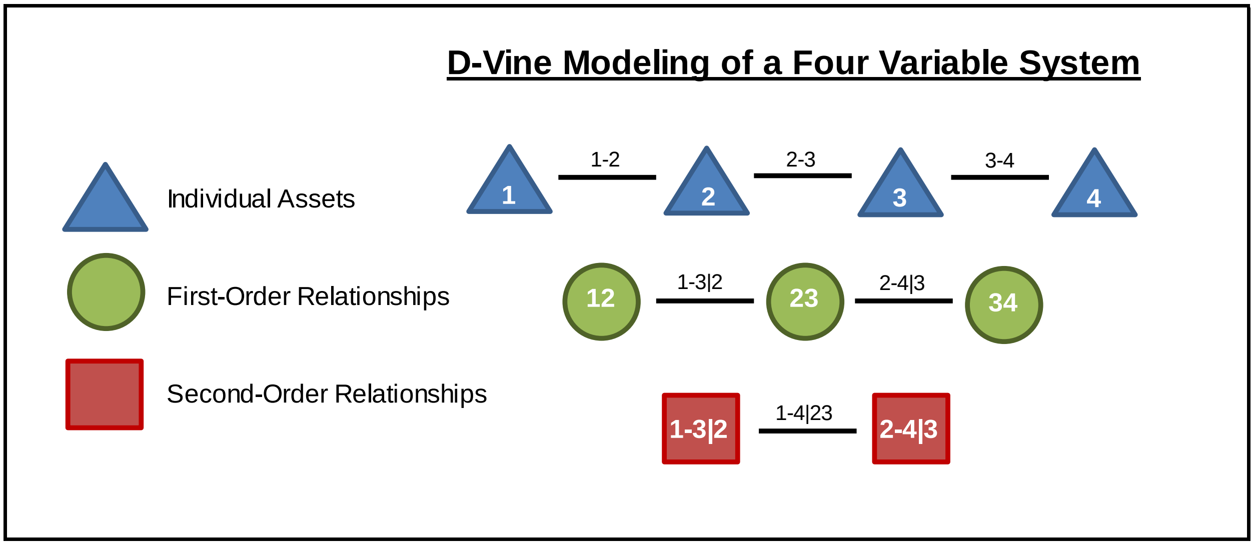
Recent empirical testing of financial return data suggests that vine copulas offer improvements over existing models for pools of up to 10 to 12 assets.[55] As a result, CROs are ideal candidates for vine copula modeling, allowing for more precise pricing specifications than previously available for a tranche-based security.

There are a number of ways to build vine copula models, and there are a number of sources offering more detailed explorations of the theory of vine copula modeling.[56,57,58] For the purposes of this paper, a *D-Vine* copula model will be used to evaluate the sample CRO under the Dependencies

54 Low, R. K. Y. et al. [19]
55 Ibid.
56 Nikoloulopoulos, A., Joe, H., & Li, H. [25]
57 Brechmann, E. & Czado, C. [10]
58 Mendes, B., Semeraro, M., & Leal, R. [22]

model. For illustration, a D-Vine structure for four assets is shown in Figure 16.[59]

Figure 16: D-Vine Copula Modeling Structure



The first level in the vine contains direct pairwise dependencies. Subsequent levels contain *conditional* dependencies based on relationships identified in higher vine levels and conditioned on the shared variables.[60] Once a pool of assets is decomposed into pairwise direct and conditional relationships, pool results are simulated recursively.[61]

With a modified version of the Monte Carlo procedure used above and the simulation methodology provided in Aas et al. (2006),[62] we obtain the revised tranche estimates of default probability and expected loss shown in Section 4.

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[59] The other prominent vine types are C-Vines and R-Vines, respectively.

[60] For instance, the combination of the 1-2 and 2-3 relationships results in the dependency between variables 1 and 3, conditioned on 2.

[61] Using the inverse of selected partial derivatives of the copula function.

[62] Aas, K. et al. [1]

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Abbreviations and notations

Collect here in alphabetical order all abbreviations and notations used in the paper

CDO, collateralized debt obligation
CRO, collateralized risk obligation
EP, exceedance probability
GSAMP, Goldman Sachs GSAMP Trust 2006-NC2
ILS, insurance-linked securities
ILW, industry loss warranty
S&P, Standard and Poor's
SPV, special purpose vehicle

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Aaron C. Koch, FCAS, MAAA is a Consulting Actuary with Milliman, Inc. Aaron works with a diverse set of clients in the property and casualty industry, including multiline insurers, reinsurers, captives, risk-retention groups, and municipalities. His experience includes reserving and ratemaking for commercial lines of insurance such as property, general liability, workers' compensation, products liability, and medical malpractice.

Aaron develops innovative solutions for the rapidly growing catastrophe risk and alternative capital markets. He works with leaders in this field—including specialist hedge funds and reinsurers—to provide independent analysis and establish best-practice standards for asset valuation, post-event loss reserving, and operational reviews. In addition, Aaron is a frequent writer and speaker on alternative financing structures for catastrophe risk.

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A Frequency-Severity Stochastic Approach to Loss Development

Uri Korn, FCAS, MAAA

Abstract

In this paper, we present a stochastic loss development approach that models all the core components of the claims process separately. The benefits of doing so are discussed, including the providing of more accurate results by increasing the data available to analyze. This also allows for finer segmentations, which is very helpful for pricing and profitability analysis.

Keywords. Loss Development, Frequency, Severity, Reserve Variability, Cox Proportional Hazards Model

1. INTRODUCTION

Over the recent past, there has been much development and discussion of new stochastic models for loss development. These models apply a more scientific approach to the old problem of estimating unpaid losses, but most still stick with the same strategy of using aggregate losses. Using aggregate losses discards much useful information that can be used to improve predictions, such as the reporting times of unpaid claims, the number of currently open claims, and separate frequency and severity information.

In many cases, working with aggregate data may be satisfactory and the extra work involved in building a more detailed model may not justify the benefit. But for some cases such as those involving low-frequency/high-severity losses, where fine segmentations are desired, or when there are relatively fewer years of data available, this pushes the limits of what aggregate data can do, even with the most sophisticated stochastic models. In this paper we present a stochastic loss development model that analyzes all of the underlying parts of the claims process separately, while still keeping the model as simple as possible.

1.1 Research Context

There have been other works as well that recommend using more detailed data to help produce more accurate results. Zhou et al. 2009 uses a Generalized Linear Model approach to loss development modeling on frequency and severity separately. Meyers 2007 does this as well, but within a Bayesian framework. And recently, Parodi 2013 handles the frequency component of pure IBNR by modeling on claim emergence times directly, one of the components of our model as well, but has more complicated formulas for handling the bias caused by data that is not at ultimate

1.2 Objective

This goal of our method is to model the underlying claims process in more detail and to improve the accuracy of predictions. The models mentioned above as well as other similar approaches do not use many useful pieces of information available in the data, such as the reporting times (and number, for some) of unpaid claims, how the likelihood of a claim being paid changes as the claim ages, and individual detail of how outstanding reserved claims have been settling. There is also no framework for handling mixes of policy retentions and limits and for dealing with changes in this mix over time. Lastly, modeling processes that are more abstracted and removed from reality like the total development process of aggregate losses makes it harder to fit simple parametric models that can be used to smooth volatility and produce more accurate fits; this will be elaborated later on as well. The approach developed in this paper was designed to use as much information as possible while not being overly complicated.

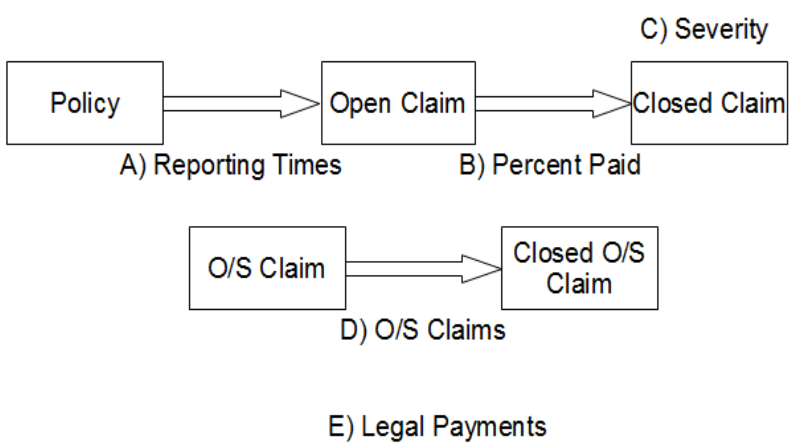
There are many benefits of individually modeling each component of the claims process separately. This can be compared to analyzing data for a trend indication. Combining frequency and severity information can often mask important patterns in the data while separating them out usually yields better predictions. This is because when there are different underlying drivers affecting the data, it becomes harder to see what the true patterns in the data are. Take, for example, two incurred triangles for two different segments, in which the first segment has a slower reporting pattern, but more severe losses than the second. More severe losses tend to be reserved for sooner and more conservatively, and so this will make the aggregate loss development pattern faster. On the other hand, the slower reporting pattern will obviously make the pattern slower than the second. When comparing these two aggregate triangles, it may be difficult to judge whether the differences are caused mostly from volatility, or whether there are in fact real differences between these two segments. In contrast, looking at each component separately will yield clearer details and results. The example we gave applied to comparing two separate triangles, but this will also create problems when attempting to select development factors for a single, unstable triangle. High volatility compounds this issue.

Second, by looking at every component separately, we increase the data available to analyze since, for example, only a fraction of reported claims end up being paid or reserved for. When looking at aggregate data, we only see the paid or incurred claims, but if we analyze the claim reporting pattern separately, we are able to utilize every single claim, even those that close without payment or reserve setup. When making predictions, we are also able to take into account the number and characteristics of claims that are currently open, which will add to the accuracy of our predictions.

Lastly, as mentioned, by separating out each piece, it becomes much easier to fit parametric models to the data that we can be confident in. It is difficult to find an appropriate curve that provides a good fit to the development patterns in aggregate data. But it is relatively easy to find very good fits for each of the individual pieces of the development process, such as the reporting and settlement times and the severity of each loss. Fitting parametric models involves estimating fewer parameters than relying on empirical data where every single duration needs to be estimated independently, and so helps lower the variance of the predictions, since prediction variance increases with the number of parameters being estimated, as is known[1]. We show an example later based on simulated data that demonstrates that the prediction volatility can be cut by more than half by using this method over standard triangle methods. Fitting parametric models to each piece will also help us control for changes in retentions and limits, as well as enable us to create segmentations in the data, as will be explained more later.

1.3 Outline

For this model, we break the claims process down into five separate pieces, as shown in the diagram below. Each piece will be discussed below in more detail.



The five parts we will analyze are as follows:

A) The reporting time of each claim

1 That is, with keeping the data the same. By separating out each piece, even though we now need to estimate separate parameters for each piece, this does not increase the variance, since we are working with more data. This is analogous to how separating out frequency and severity trend information would not increase the variance even though we now have to estimate two trend parameters instead of one.

B) The percent of reported claims that are paid, as well as the settlement times of reported claims

C) The severity of each paid claim

D) The final settlement amount of each claim that has outstanding case reserves

E) Legal payments

The next section will discuss fitting distributions when right truncation is present in the data, which will be used for some of these pieces; it will also discuss the fitting of hyper-parameters, which is not absolutely necessary to build this model, but can be used to make it more refined. Section 3 will then discuss each of these modeling steps in detail and section 4 will discuss the how to use each piece to calculate the unpaid and ultimate loss and legal estimates. Section 5 will show a numerical example of using this method on simulated data. Section 6 will discuss ways to check this model, and finally, section 7 will discuss some alternatives and other uses of this model, such as to calculate the volatility of ultimate losses.

2. TECHNICAL BACKGROUND

Before we delve into the details of each piece, we first need to explain the process of right truncation and how to build a model when it is present in the data. This will be discussed in the first two parts of this section. It will also be helpful to understand the process of fitting hyper-parameters, which will be discussed in the third part of this section.

2.1 Maximum Likelihood Estimation with Right Truncation

When modeling insurance losses, we normally have to deal with left truncation and right censoring. Left truncation is caused by retentions where we have no information regarding the number of claims below the retention. Right censoring is caused by policy limits and is different from truncation in that we know the number of claims that pierce the limit, even if we still do not know the exact dollar amounts. Reported claim counts, for example, which we will be analyzing in this paper, are right truncated, since we have no information regarding the number of claims that will occur after the evaluation date of the data.

We will be using Maximum Likelihood Estimation (MLE) to model reporting times, and MLE can handle right truncation similar to how it handles left truncation. To handle left truncation, the likelihood of each item is divided by the survival function at its truncation point; similarly, to handle right truncation, each item's likelihood should be divided by the cumulative distribution function (CDF) at its truncation point.

2.2 Reverse Kaplan-Meier Method for Right Truncation

When fitting a distribution to data, it is a good idea to compare the fitted curve to the empirical to help judge the goodness of fit. Probably the most common method actuaries use to calculate the empirical distribution when dealing with retentions and limits (i.e. left truncation and right censoring) is the Kaplan-Meier method. Here, however, we have data that is right truncated, which is not handled by this method. We propose a modification to work with right truncated data that we will refer to as the reverse-Kaplan-Meier method.

In the normal Kaplan-Meier method, we start from the left and calculate the conditional survival probabilities at each interval. For example, we may first calculate the probability of being greater than 1 conditional on being greater than 0, i.e. $s(1) / s(0)$. We may then calculate $s(2) / s(1)$, and so on. For this second interval, we would exclude any claims with retentions greater than 1, with limits less than 2, and with claims less than 1. To calculate the value of $s(2)$ for example, we would multiply these two probabilities together, that is:

$$s(2)= \frac{s(1)}{s(0)} \times \frac{s(2)}{s(1)}$$

To accommodate right truncation, we will instead start from the right and calculate the conditional CDF probabilities, e.g. $F(9) / F(10)$, followed by $F(8) / F(9)$, etc. To calculate the value of F(8) for example, we can multiply these probabilities together:

$$\frac{F(8)}{F(10)} = \frac{F(9)}{F(10)} \times \frac{F(8)}{F(9)}$$

This is the value of $F(8)$ conditional on the tail of the distribution at t=10. We can plug in this tail value from the fitted distribution and use this empirical curve to test the goodness of fit of our fitted distribution. Using this method, all points of the calculated empirical distribution depend on the tail portion, which can be very volatile because of the thinness in this portion of the data. For the comparison with the fitted distribution to be useful, the right-most point should be chosen at a point before the data gets too volatile. It may be helpful to choose a couple of different right-most points for the comparison.

2.3 Hyper-Parameters

This method can be used to help refine some pieces of the model, but is not absolutely necessary. It involves fitting a distribution to data via MLE but letting one or more of the distribution parameters vary based on some characteristic of each data point. We refer to this technique as the hyper-parameters method, since the distribution's parameters themselves have parameters, and these are known as hyper-parameters. This can be useful, for example, if we want our reporting times distribution to vary based on the retention.

To set this method up, each claim should have its own distribution parameters. These parameters are a function of some base parameters (that are common to all claims), the claim's retention, in this example, and another adjustment parameter that helps determine how fast the parameter changes with retention. These base parameters can be the distribution parameters at a zero retention or at the lowest retention. Both the base parameters and the adjustment parameters are then all solved for using MLE. If there are different segments, each segment can be given its own base parameters but share the same adjustment parameters. Either one or more of the distribution's parameters can contain hyper-parameters. It is also possible to reparameterize the distribution to help obtain the relationship we want, as will be shown in the below example.

In this example, we will assume that we are fitting a Gamma distribution, with parameters alpha and beta, to the reporting times of all claims (which will be explained more later), and that we wish the mean of this distribution to vary with the retention, with the assumption that claims at higher retentions are generally reported later. The mean of a Gamma distribution is given by alpha divided by beta, and so we need to reparameterize the distribution. We will reparameterize our distribution to have parameters for the mean (mu) and for the coefficient of deviation (CV). The original parameters can be obtained by alpha = 1 / CV^2, and beta = 1 / (mu x CV^2). Only the first parameter, mu, will vary with the retention.

The first step is to determine the shape of an appropriate curve to use for this parameter. For this, we fit the data with MLE allowing only one parameter for the CV, but having different parameters for the mean for each group of retentions. Plotting these points can help determine whether a linear of a logarithmic curve is the most appropriate. The final curve can then be plotted against these points to help judge the goodness of fit. After doing this, assume that we decided to use the equation, $\log(mu_r) = \log(mu_{base}) + \exp(theta) \times \log(r / base)$, where r is the retention of each claim, base is the retention of the lowest claim, and $\log(mu_{base})$ and theta are parameters that are fit via MLE, in addition to the CV parameter which is common across all claims. We took the exponent of theta to ensure that the mu parameter is strictly increasing with retention. Once this is done, we have a distribution that is appropriate for every retention.

3. MODELING STEPS

The modeling of each of the five parts will now be explained in detail. Using all of these pieces for the calculation of the unpaid and ultimate projections will be discussed in the following section.

Table 1 below shows the data that will be needed for each of the steps.

Table 1:

Part	Data	Fields Needed
A) Reporting Times	Claim Level, All Claims	Accident Date, Report Date
B) Percent Paid and Settlement Times	Claim Level, All Closed Claims (May also include open outstanding claims as well)	Report Date, Closed Date, Final State of Claim (Paid or Not)
C) Severity	Claim Level, All Closed Claims	Claim Amount, Retention, Policy Limit, Accident Date, Closed Date
D) Case Outstanding Claims	Claim Level, All Closed Claims That Have Had an Outstanding Reserve At Some Point	Average Outstanding Value, Ultimate Paid Amount (including zeros), Policy Limit
E) Legal Payments	Aggregate Claim Data, All Data	Paid Losses and Paid Legal Amounts by Total Duration

3.1 Part A: Reported Times

In this section, we will explain how to model the reporting lag, that is, the time from the accident date of a claim to the report date. (If report date is unavailable, the create quarter can be used instead by using the first quarter that each claim number first appears.) This will be used to help estimate the pure IBNR portion of unpaid losses later. This data is right truncated since we have no information about the number of claims that will occur after the evaluation date. The right truncation point for each claim is the evaluation date of the data minus the accident date of the claim. We will use MLE to fit a distribution to these times. The Exponential, Weibull, and Gamma distributions all appear to fit this type of data very well. (A log-logistic curve may also be appropriate in some cases with a thicker tail, although the tail of this distribution should be cut off at some point so as not to be too severe.)

After this data is fit with MLE using right truncation, the goodness of fit should be compared against the empirical curve which can be obtained using the reverse-Kaplan-Meier method, all as described in the previous section. Using this approach, as opposed to using aggregate data, makes it much easier to see if the reporting lag distribution has any significant historical changes. There is also no need to estimate a separate tail piece as this is already included in the reporting times distribution[2].

3.2 Part B: The Likelihood of a Claim Being Paid

The second component to be modeled is the percent of reported claims that will ultimately be paid. This can be done very simply by dividing the number of paid claims by the total number of closed claims, but this estimate may be biased if closed with no payment (CNP) claims tend to close faster than paid claims. If this is true and we do not take this into account, we will underestimate the percent of claims that are paid, since our snapshot of data being used will have relatively more CNP claims that would be present after all claims are settled. To give an extreme example to help illustrate this point, say there are two report years of data. All CNP claims settle in the first year, and all paid claims settle in the second year. There are 100 claims each year, and 50% of claims are paid. The evaluation date of the data is one year after the latest year. The first year will have 50 CNP claims and 50 paid claims. When looking at the second year however, we will see 50 CNP claims and no paid claims, since all of the claims that will ultimately be paid are still open (and we do not know what their final state will be). When we calculate the percent of claims paid using the available data, we will get the following:

$$\frac{50\, paid\ claims}{50\ paid\ claims + 100\, closed\ claims} = \frac{1}{3}$$

which is less than the correct value of 50%.

Instead, we will suggest an alternative approach. For the first step, we fit distributions to all paid claims and to all CNP claims separately. (If the distributions do not appear different, then the paid likelihood can be calculated simply by dividing and there is no need to go further.) There will still be many open claims in the data that we do not know what their ultimate state will be making the ultimate number of paid and CNP claims unknown, and so this data is right truncated as well. The right truncation point for each claim is equal to the reported date subtracted from the evaluation date. The Exponential, Weibul, and Gamma distributions all appear to be good candidates for this type of data as well.

The ultimate number of paid claims is equal to the following, where *F(x)* is the cumulative distribution function evaluated at x:

$$\sum_{i = All\ Paid\ Claims} 1 / F_{Paid}\,(Evaluation\, Date - Report\ Date_i)$$

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2 This tail may only be accurate if relatively small, otherwise, it is an extrapolation, which may not be accurate. The Gamma tail seems slightly better than the Weibull, but this observation is based off of limited data.

And the ultimate number of unpaid claims is equal to:

$$\sum_{i=\text{All CNP Claims}} 1/F_{CNP}(Evaluation\,Date - Report\,Date_i)$$

And so, the ultimate percent of claims that are paid is equal to:

$$\frac{Ultimate\,Paid\,Claims}{Ultimate\,Paid\,Claims + Ultimate\,CNP\,Claims}$$

Dividing each claim by the CDF at the right truncation point is similar to performing a chain ladder method. The most recent years may have high development factors and may be unstable. To address this, we can make the method more similar to a Cape Cod-like method by weighting each year appropriately according to the credibility of each year. To do this, the weight for each year can be set to the average of the calculated CDF values of each claim multiplied by the claim volume. The paid distribution or the CNP distribution can be used to calculate this CDF, or it can be taken as the average of the two. To give more recent, relevant experience slightly more weight, an exponential decay factor can be applied as well. Alternatively, the actual number of claims per year can be used instead. For this version, the ultimate claim counts for each year should be multiplied by the ratio of the actual claim count to the ultimate claim count for that year. Using this reweighting technique (that is, dividing by the CDF and then multiplying by an off-balance factor for each year) will not change the number of claims, but still addresses the bias that is caused from our data being right truncated. We will refer to this approach as right truncated reweighting. This approach will be used when building more complicated models on this type of data.

So far, we have calculated the total percentage of claims that will be paid; this will be used for the calculation of pure IBNR. We also need to determine how this percentage changes with duration to be able to apply this to currently open claims for calculation of IBNER. If paid claims have a longer duration than CNP claims, then it should be expected that the paid percentage should increase with duration, since relatively more CNP claims will have already closed earlier. So the longer a claim is open, the more chance it has of being paid. To calculate this, we can use Bayes' formula as follows:

$$P(Paid \mid t \geq x) = \frac{P(t \geq x \mid Paid) \times P(Paid)}{P(t \geq x \mid Paid) \times P(Paid) + P(t \geq x \mid CNP) \times P(CNP)}$$

$$= \frac{s_{Paid}(x) \times P(Paid)}{s_{Paid}(x) \times P(Paid) + s_{CNP}(x) \times P(CNP)} \tag{3.1}$$

where t is the time from the reported date of the claim and x is the duration for each year. It is also possible to calculate the paid likelihoods for claims closing at exactly a given duration (that is, not conditional as in the above) by using the PDFs instead of the survival functions in formula 3.1.

These values can then be used to compare against the actual paid likelihoods by duration as a sanity check. The conditional likelihoods cannot be used for this since these likelihoods represent the probability of a claim being paid given that it has been open for at least a certain number of years, but not exactly at that time.

A more detailed model that also incorporates outstanding claims can be built as well, where instead of just modeling the lags and probabilities of two states (paid and CNP), the outstanding state is modeled as well. Once claims are in the outstanding state, they can then transition to either the paid or CNP states. All of these states and transitions can be modeled using the same techniques discussed in this section. The ultimate probability of a claim being paid is then equal to the probability of a reported claim being paid (before transitioning to an outstanding state, that is) plus the product of the probabilities of transitioning to an outstanding state and of transitioning from an outstanding state to a paid state. This is a mini Markov Chain model, with bias correction caused from the right truncation of the data. If open claims are assigned different "signal" reserves that represent information about the possibility of payment for each claim, then a more detailed Markov Chain model can be built that incorporates the probability of transitioning to and from each of these "signal" states as well.

Another possible refinement is to have the paid (or other state) likelihoods vary by various factors, such as the type of claim or the reporting lag, by building a GLM on the claim data. To account for the bias caused from the data being at an incomplete state, right truncated reweighting can be used to calculate the weights for the GLM, and a weighted regression can be performed; this will account for the bias without altering the total number of observations. The settlement lag distributions can even be allowed to vary by various factors as well using the hyper-parameters approach. The resulting probabilities will be the paid (or other) likelihoods from time zero, which can be applied to new, pure IBNR claims. For currently open claims for calculation of IBNER, Bayes' formula (3.1) should be used to calculate the conditional probabilities given that a claim has been open for at least a certain amount of time. If the settlement lag distributions were allowed to vary, the appropriate distribution should be used for this calculation as well.

We should note that using right truncated reweighting for the GLM and then again adjusting the resulting probabilities is not double counting the effects of development. The former is to account for the fact that the data used for modeling is not at ultimate, while the latter is needed to reflect how the probability of a claim being paid varies over time.

It may seem odd at first that the probabilities for open claims are developed and so will always be higher than the probabilities used to apply to new, pure IBNR claims (if this is how claims develop, which it often is). If everything develops as expected, the total predicted number of paid claims will not change, as will be illustrated. Using an example similar to the above, there are 100 claims and half of these claims will be paid. All unpaid claims close in the first year and all paid claims close in the second year. The initial, unconditional probability to apply to new claims is 50%. After a year, we will assign 100% probability of being paid to all the remaining claims. Initially we predicted that half of the 100 claims will be paid, which is 50 claims. After a year, no actual claims were paid and we will predict that 100% of the 50 remaining claims will be paid, which also equals 50 claims. This estimate would be biased downwards if we did not apply this adjustment to calculate the conditional probabilities.

3.3 Part C: Severity Portion

This portion involves fitting an appropriate severity distribution to the claim data. Before doing so, all losses should be trended to a common year. We will also need to take into account that more severe claims tend to be reported and settled later. It is technically possible to have the paid settlement time distribution vary with claim size and use right truncated reweighting here as well, but this approach will likely not be accurate since only a few large claims may have settled earlier. Because this problem is also relevant to constructing Increased Limit Factors in general, we will elaborate on this in detail. There are many ways that this can be accounted for, but we will only discuss a couple.

The first way is to use the hyper-parameters approach discussed earlier. Claim severity can be a function of the reporting lag, the settlement lag, both, or the sum of the two, which is the total duration of the claim. If these lag distributions were made to vary by retention or by other factors, it may be more accurate to model on the percentile complete instead of the actual lag. To give an example of using the hyper-parameters approach, if we allowed the scale parameter of our distribution to vary with duration, this would be assuming that each claim increases by the same amount on average, no matter the size of the claim. (Note that this may be a poor assumption as it is more likely that the tail potential increases with duration, since the more severe claims tend to arrive at the later durations.) The limited expected value (LEV) at any lag can now be calculated. This LEV can be used directly if solving for ultimate losses by simulating claim arrival times. If using a closed form solution, a weighted average of the LEVs can be calculated by using the (conditional) reporting times and/or settlement times distributions. If the total duration was used, the distribution for total duration can be obtained by calculating the discrete convolution of the reporting and settlement times distributions.[3] If we wanted to calculate a single distribution that represents the expected amount of claims that will be settled in each duration, we can do the following. We will first note that if survival values are generated from a loss distribution, and these survival values are then converted into a probability density function (PDF) by taking the differences of the percentages at each interval, and then this data is refit via MLE using these PDF percentages as the weights (by multiplying each log-likelihood by its weight), the original distribution parameters will be produced. (This can be confirmed via simulation.) The values for each likelihood can either be the average of the two values for each interval, or more accurately, can be represented as a range. MLE can be performed using ranges by setting each likelihood to the difference of the CDFs at the two interval values. This can also be done by generating the PDF values from the distribution directly, but in order to be accurate, this would need to be done at very fine increments. Using this, we can generate a single distribution based on the percentages of claims expected to be settled in each duration by generating the PDF tables for each duration as mentioned, and then setting the total sum of the weights for each duration to equal the percentage of claims expected to be settled in each duration. (It is possible that this mixed distribution of durations may not be the same as the original distribution used to fit a single duration. If this is the case, parameters can be added by creating a mixed distribution of the same type as the original distribution. There is no fear of adding too many parameters and over-fitting here, since we are not fitting to actual data, but to values that have already been smoothed.) The survival percentages generated should start at and be conditional on the lowest policy retention and go up to the top of the credible region for the severity curve. This will make the mixing of the different duration curves more properly reflect the actual claim values and make the final fitted distribution more accurate.

Another way to account for the increasing severity by duration, is to use a survival regression model called the Cox Proportional Hazards Model. This model does not rely on any distribution assumptions for the underlying data, as it is semi-parametric. It can also handle retentions and limits, i.e. left truncation and right censoring. As opposed to a GLM that models on the mean, the Cox model tells how the hazard function varies with various parameters. The Cox Model is multiplicative, similar to a log-link function in a GLM. The form of the model is: $H_i(t) = H_0(t)\exp(B_{i1}X_{i1} + B_{i2}X_{i2} + \ldots)$, where $H_i(t)$ is the cumulative hazard function for a particular risk at time t, $H_0(t)$ is the baseline hazard, roughly similar to an intercept (although this is

3 A discrete convolution is calculated by first converting each of these continuous distributions to be discrete. The probabilities for each amount, x, are then calculated by multiplying the probabilities of each distribution that add up to x. For example, for x = 3, this can be achieved by a reporting lag of 0 and a settlement lag of 3, or a reporting lag of 1 and a settlement lag of 2, etc.

not returned from the model), and the B's and X's are the coefficients and the data for a particular risk, respectively. The cumulative hazard function, H(t) is equal to: *H(t) = exp[-s(t)]*, and so *s(t) = -ln[H(t)]*. It can be seen from this formula that a multiplicative factor applied to the cumulative hazard function is equivalent to taking the survival function to a power[4]. We will use this fact below. A full discussion of the Cox model is outside the scope of this paper[5].

Assuming that we are modeling on the total duration of each claim, with this approach we are assuming that the hazard function of the data changes with the duration. The hazard can be thought of very roughly as the thickness of the tail, and so we are assuming that the tail is what increases with duration.

Initially, a Cox model should be run on the individual loss data with a coefficient for each duration to help judge the shape of the curve for how the hazard changes with duration. Next, another model should be fit with a continuous coefficient either for the duration or the log of duration, or any other function of duration that is appropriate. Different segments that may be changing by year can also be controlled for with other coefficients.[6]

Assuming the log of duration was used, the pattern for how the severity curve changes with duration, d, can be obtained from the results of the Cox model, as follows:

$$Relative\ Hazard\,(d) = \exp\left(Cox\ Duration\ Coefficient \times \log(d)\right) = d^{Cox\ Duration\ Coefficient} \quad \textbf{(3.2)}$$

There are two ways that will be discussed to create severity distributions using this information. Before we explain the first method, we first need to mention that if an empirical survival curve is generated from claim data using the Kaplan-Meier method, and this survival function is then converted to a PDF and fitted with MLE, as explained, the parameters will match those that would be obtained from fitting the claim data directly with MLE. (This can be confirmed via simulation as well.) The first way involves first calculating the empirical survival curve at the base duration, where the base duration is the duration that is assigned a coefficient of zero in the Cox model. To do this, instead of using the probably more familiar Kaplan-Meier method to calculate the empirical survival function, we use the Nelson-Aalen method to calculate the empirical cumulative hazard function. As a note on the Nelson-Aalen method, calculating the cumulative hazard and then taking the negative of the natural logarithms to convert to a survival function will produce very similar values to the survival values produced from the Kaplan-Meier method. The Nelson-Aalen estimate is equal

4 Even though the Cox Model technically models on the instantaneous hazard function, since it also assumes that the hazards always differ by a constant multiplicative factor, this model can also be viewed as modeling on the cumulative hazard as well, since the ratios between the instantaneous and cumulative hazards will be the same.
5 For a longer explanation, see Fox 2002.
6 These segments should ideally be treated as separate strata in a stratified model.

to:

$$H(t)=\sum_{i\le t}\frac{d_i}{n_i}$$

Where d_i is the number of events in each interval and n_i is the number of total risks that exist at each interval. To calculate the hazard at the base duration using the coefficients from the Cox model, the following formula can be used:

$$H_0(t)=\sum_{i\le t}\frac{\sum_{Each\,Risk} 1/\exp(coefficient(d_i))}{n_i} \tag{3.3}$$

The only difference from the normal Nelson-Aalen formula is that instead of counting all events the same, as one, each event is counted as the inverse of the exponent of the sum of its coefficients.

Using this, we can calculate the survival function at the base hazard by taking the negative of the natural logarithm of the cumulative hazard. With the base survival function, we can now calculate the survival function at any duration, d, using the following formula:

$$s_d(t)=s_{Base}(t)^{Relative\,Hazard(d)} \tag{3.4}$$

The survival functions at each duration can then be converted to probability distribution functions and then fit with MLE as shown above. Doing this will produce a distribution for each duration (or duration group, if durations were combined to simplify this procedure). A single distribution representing a weighted average of the expected durations can also be obtained by combining the data from multiple durations together and weighting each according to the expected percentage of claims expected to be settled at each duration. (Note that this new distribution may not be the same type as the original distribution as mentioned above.) Alternatively, another way that does not require fitting a distribution at every duration is to only fit a distribution to the base duration. The fitted survival values can be produced at the base duration using this distribution, and the survival values at any duration can then be obtained by taking this base survival function to the appropriate power. The limited expected values can now be obtained by "integrating" the survival values at the desired duration, since:

$$LEV(Retention, Policy\,Limit)=\int_{Retention}^{Retention+Policy\,Limit} s(x)\,dx$$

Where by LEV(Retention, Policy Limit), we mean the limited expected value from the retention up to the retention plus the policy limit. To do this discretely, we can use this formula as an approximation:

$$LEV(Retention, Policy\ Limit) = (Width\ of\ s(x)\ Increments) \times \sum_{Retention}^{Retention + Policy\ Limit} s(x)$$

The thinner the increment width that the survival values are calculated at, the more accurate this will be. Putting this together, the formula to calculate the LEV at each duration *d* is as follows:

$$LEV_d(Retention, Policy\ Limit) = Width \times \sum_{Retention}^{Retention + Policy\ Limit} s(x)^{Relative\ Hazard(d)} \quad \textbf{(3.5)}$$

The second method to construct distributions for each duration is similar except that it involves adjusting the actual claim values instead of the survival or hazard functions. We can use the well known relationship for adjusting a distribution for trend, $F(x) = F'(ax)$ (Rosenberg et al. 1981), where $F(x)$ is the cumulative distribution function of the original distribution before adjusting for trend, $F'(x)$ is the same after adjusting for trend, and a is the trend adjustment factor. Similarly here, using survival functions instead of cumulative distribution functions, we can solve for the adjustment factor for every value of x that satisfies, $s(x) = s'(ax) = s(ax)^{Desired\ Adjustment}$, or equivalently, $s(x)^{1/Desired\ Adjustment} = s(ax)$, since the latter is computationally quicker to solve. The survival values can be determined from either the empirical Kaplan-Meier survival function or from a fitted survival function applied to the entire data set. This factor, a, can be determined for every claim amount and duration by backing into the value of a that satisfies the equality. Once this is done, all of the original loss data can be adjusted to the base duration, and then a loss distribution can be fit to this data. We can use this same method to adjust the claim data to any duration, or alternatively, any of the methods discussed above in this section can be performed to derive LEVs at all of the durations.

If one is using a one- or two-parameter Pareto distribution, this process becomes simpler since taking the survival function to a power is equivalent to multiplying the alpha parameter by a factor. This can be easily seen by looking the Pareto formulas, which will not be shown here. Once the distribution is fit at the base duration using one of the methods discussed, the distribution for any duration can be obtained by adjusting the alpha parameter, as follows:

$$\alpha_d = \alpha_{base} \times Relative\ Hazard(d) \quad \textbf{(3.6)}$$

Similar methods can be used if using other types of regression models as well, such as a GLM or an Accelerated Failure Time model, which will not be elaborated on here.

3.4 Part D: Outstanding Reserved Claims

This section explains the estimating of the ultimate settlement values of claims that currently have outstanding reserves. Note that this is different from open, non-reserved claims in that the reserve amounts here are significant. For example, some companies set up a reserve amount of one dollar or a similar amount to indicate that a claim is open, but that no real estimate of the claim's ultimate settlement value is available yet.

To calculate the ultimate paid amounts, we will use a logistic GLM (that is a GLM with a logit link and a binomial error term) on all closed claims that have had an outstanding reserve set up at some point in the claim's lifetime. We will model on the dollar amounts divided by the policy limits using the following regression equation:

$$\frac{Paid}{Policy\ Limit} = B_1 \frac{Average\ O/S}{Policy\ Limit} + B_2 \exp\left(\frac{Average\ O/S}{Policy\ Limit}\right) \quad \textbf{(3.7)}$$

We used the average outstanding value for each claim since the reserve amount of a claim may have changed over time[7]. Note that this ratio can also be calculated directly by dividing the sum of ultimate paid dollars by the sum of outstanding reserves, but this result may be biased since the ultimate settlement values depend on the dollar amount of reserves setup, and this amount depends on the duration. It is also not as refined as it could be. CNP claims can be included or excluded from this model. If they are excluded, a separate model will need to be built to account for. If they are included, right truncated reweighting should be performed on the claims to avoid any bias.

Formula 3.7 seems to provide a very good fit to some types of data, although sometimes logarithms or other alternatives (such as splines) are more appropriate, depending on the book of business and the company. The logistic model will ensure that the predicted value is always less than one, since the claim cannot (usually) settle for more than the limit. (Some GLM packages may give a warning when modeling on data that is not all ones and zeros, but it should still return appropriate results.) Once again, the fit should be compared to the actual. This model will capture the fact that claims reserved near the policy limit tend to settle for lower on average (since they only have one direction to move), while claims reserved for lower amounts have a tendency to develop upwards, on average. It is also possible to add coefficients for the type of claim and other factors if desired.

3.5 Part E: Legal Payments

The legal percentages should be calculated for each duration, since this percentage usually increases with duration. To address credibility issues with looking at each duration separately, a

7 Alternatively, it is also possible to include every outstanding amount in the model, weight appropriately so that all of the rows for each claim add up to one, and use a Generalized Linear Mixed Model to account for the correlation between the data points.

curve should be fit to this data. Once this is done, cumulative percentages should be calculated for each duration by taking a weighted average of the legal percentages from each duration until the last duration. The weights should be based on the expected amount of paid dollars per duration. This pattern can be obtained by looking at the aggregate data, or by using the model from this paper and simulating all years' losses from the beginning. (This will be discussed a bit more later as well). These cumulative legal percentages will be applied to the unpaid losses for each accident year.

The approach we chose to use here is not as refined as it could be. It is also possible to build a more robust model that determines the legal payments separately for each of the parts from Table 1, and takes into account the number claims as well as the limits and retentions by year, etc. We used a simpler approach here so as not to over-complicate our approach.

4. CALCULATION OF UNPAID LOSSES

Each part of the unpaid loss plus legal expenses now needs to be calculated. Table 2 below shows the data that is needed for each part that will be described in detail below. The right-most column also shows which parts of the modeling steps from Table 1 each piece depends on.

Table 2:

Part	Data	Fields Needed	Depends On
1) Pure IBNR	Grouped Policy Data	Average Expected Accident Date (Average of the Effective Date and the Earlier of the Expiration Date and the Evaluation Date), Retention, Policy Limit, Sum of Exposures or On-Level Premiums	A, B, C
2) IBNER on Non-Reserved Claims	Claim Level Detail, All Open Non-Reserved Claims	Accident Date, Report Date, Retention, Policy Limit	B, C
3) IBNER on Reserved Claims	Claim Level Detail, All Open Reserved Claims	Outstanding Amount, Policy Limit	D
4) Legal Payments	None	None	E

4.1 Part 1: Pure IBNR

For the calculation of pure IBNR, we will calculate the frequency of a claim for each policy using a Cape Cod-like method while also controlling for differences in retentions between policies. We will use the following formula to calculate the frequency per exposure unit:

$$Frequency = \frac{Total\ Reported\ Claims}{Used\ Exposure\ Units} \quad \textbf{(4.1)}$$

Where *F(x)* and *s(x)* are the CDF and survival function, respectively, calculated at *x* and Used Exposures Units is defined as:

$$Exposure\ Units \times F_{Report\ Time}(Eval\ Date - Avg\ Accident\ Date) \times s_{Severity}(Retention) \quad \textbf{(4.2)}$$

The severity distribution should be detrended to the appropriate year before calculating this value. Doing this will take care of the frequency trend component that is a result of retention erosion. If there is a non-zero ground up frequency trend as well, this should also be accounted for. If using premiums, the exposure units can be the on-level premiums divided by the LEV for the policy layer. Dividing by the LEV takes the severity component out of the premium. Similar to the Cape Cod method, we multiply the exposures by the percentage of claims that were expected to have already been reported at this point in time. We obtain this percentage by applying the CDF of reported claim times (Part A) to the right truncation point for each group of policies. So as not give too much weight to older years, decaying weights can be used here as well. To take different retentions into account, we need to consider that a policy with a retention of $100,000 may only see 50% of the ground up claims while a policy with a retention of $200,000 may only see 20%. By multiplying the exposures by the survival function at the retention, we adjust for this. (The severity distribution that should be used should not be calculated at a specific duration, but should be the overall average distribution that would be used to price accounts.)

We then calculate the expected IBNR frequency per policy using this formula:

$$Frequency \times Exposures \times s_{Report\ Time}(Eval\ Date - Avg\ Acc\ Date) \times s_{Severity}(R) \quad \textbf{(4.3)}$$

Where "Eval Date" is the evaluation data, "Avg Acc Date" is the average accident data, and "R" is the retention. The exposures times the survival function of the reported times represents the unused portion of the exposures. Once we have this, we can multiply the expected frequency per policy by the paid likelihood, obtained from Part B to get the expected number of paid IBNR claims. We then apply Part C to calculate the average severity for each paid claim by calculating the conditional severity of each paid claim above the retention, that is, *LEV(Retention, Policy Limit) / s(Retention)*. The claim distribution should be detrended to the appropriate year if it is desired to have losses on a historical basis. Otherwise, if trended losses are needed for pricing or profitability purposes, no detrending is needed. The durations, reporting lags, and/or settlement lags should be taken into account if the severity distribution was made dependent on these, by using the appropriate conditional distributions given the current reporting lag of each claim.

4.2 Part 2: IBNER on Non-Reserved Claims

For each open non-reserved claim, we need to calculate the probability of it being paid given its current duration using formula 3.1 from Part B above. Severities can be calculated taking into account each claim's reporting lag and the conditional settlement times distribution given its current settlement lag. Multiplying these two pieces together yields the expected value of IBNER for each claim. Summing up all of these values will yield the total IBNER on opened, non-reserved claims for the entire book.

4.3 Part 3: IBNER on Reserved Claims

All that is needed for this part is to apply the model from Part D to all open reserved claims to produce the expected paid ratio to policy limit for each claim, and then multiply each percentage by the policy limit to obtain the dollar amount. Subtracting the total outstanding reserves from this number will yield the IBNER for these claims. Note that this amount can be both positive and negative.

4.4 Part 4: Legal Payments

The appropriate cumulative legal percentage from Part E should then be applied to each accident year's total unpaid losses to calculate the total expected legal payments, taking into account the age of each year. This part is only needed if legal payments are paid outside of the policy limits; otherwise, they should be included in Part C, in the average severity.

4.5 IBNR and Ultimate Losses

Taking the sum of the four parts above will yield the unreported loss plus legal estimates per year. Adding this to the incurred losses will produce the ultimate indications. It is also possible to calculate the losses for a prospective year of policies with the expected makeup of retentions and policy limits from the beginning to derive an estimate of the expected ultimate losses for the prospective period. This can be done for historical periods as a check as well.

5. NUMERICAL EXAMPLE

We will now illustrate this method with an example using simulated data. To simplify, we will not include any outstanding claims or legal payments, so only Parts A (reporting times), B (percent of claims paid and settlement times), and C (claim severity) will be needed. We will also assume that the claim severity does not change with duration or year, and that all claims occur on the first day of each year. We first walk through an example using a particular simulation run chosen at random, and then discuss the results of running many simulations.

Claim reporting and settlement times were simulated from Exponential distributions, with a mean of 2 years for reporting times, and means of 4 and 3 years for the settlement times of claims that end up being paid and unpaid, respectively. Claim frequencies were simulated from a Negative Binomial distribution having a variance-to-mean ratio of 2 and a frequency per policy of 0.5 (for claims above the retention). Each claim had a probability of 20% of being paid. Claim severity was simulated from a Lognormal distribution with mu and sigma parameters of 9 and 2, respectively. All policies had a retention of half a million and a policy limit of one million. We simulated ten years of data, with 1,000 accounts each year. The two tables below show what the aggregate loss triangle looks like for this simulation run, and the respective link ratios for that run. Note the large amount of volatility in the link ratios.

Year / Duration	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
2004	$2,603	$7,733	$13,900	$18,985	$22,930	$28,700	$32,359	$33,268	$36,414	$38,731
2005	$1,565	$5,296	$14,285	$23,152	$27,106	$31,980	$34,089	$37,308	$38,502	
2006	$708	$6,249	$10,862	$16,483	$19,533	$25,779	$31,793	$35,490		
2007	$1,479	$4,321	$9,433	$14,885	$19,508	$24,071	$25,798			
2008	$1,068	$5,550	$9,263	$20,372	$26,033	$29,437				
2009	$1,350	$10,322	$19,760	$27,413	$33,388					
2010	$1,065	$3,656	$10,077	$17,731						
2011	$2,732	$7,055	$14,523							
2012	$2,356	$9,900								

Year	**1:2**	**2:3**	**3:4**	**4:5**	**5:6**	**6:7**	**7:8**	**8:9**	**9:10**
2004	2.970	1.798	1.366	1.208	1.252	1.127	1.028	1.095	1.064
2005	3.384	2.698	1.621	1.171	1.180	1.066	1.094	1.032	
2006	8.824	1.738	1.517	1.185	1.320	1.233	1.116		
2007	2.922	2.183	1.578	1.311	1.234	1.072			
2008	5.195	1.669	2.199	1.278	1.131				
2009	7.647	1.914	1.387	1.218					
2010	3.432	2.756	1.760						
2011	2.582	2.059							
2012	4.203								

We will now use the method described in this paper. Following Part A, the first step is to fit an Exponential distribution to the reporting times of all claims using MLE, taking the right truncation point of each year's claims into account. Doing this yielded a mean of 1.99, very close to the actual value of 2, which is not surprising given the large number of reported claims. Using this, we calculated the value of the CDF at the right truncation point for every policy (which is the evaluation date of the data minus the average accident data of each policy), and then multiply this by the number of exposures to produce the number of used exposures per year. Dividing the total number of claims by this number yields the excess claim frequency per policy. Normally, we would also multiply by the survival function at each claim's retention to produce the ground up frequency (as in formula 4.2); we chose to skip this step for simplicity since all policies have the same retention in this example. The results are shown in the table below. The bottom right of this table shows that the final calculated frequency per policy was 0.500, which matches the actual value used to simulate the data. Again, this accuracy is not surprising given the large number of total claims.

Year	Used Exposures	Claims	Frequency
2004	993	521	52.4
2005	989	476	48.1
2006	982	502	51.1
2007	970	499	51.4
2008	951	471	49.5
2009	918	433	47.1
2010	865	424	49.0
2011	778	399	51.3
2012	633	307	48.5
2013	394	206	52.3
TOTAL	8474	4238	50.0

We now continue with Part B, and fit distributions to all of the paid and CNP claims separately, also with taking the right truncation point of each claim into account. The fitted means of the Exponential distributions for the paid and CNP claims were 4.17 and 2.91, not far from the actual values of 4 and 3, respectively. We then develop each claim by taking the inverse of the CDF at the right truncation point, and add up all of these values to produce the ultimate number of paid and CNP claims per year as detailed in section 3.2. We can then estimate the percentage of claims that are paid each year by dividing. To be more similar to a Cape Cod-like method, as mentioned, to calculate the weights given to each year, we first calculate the average of the paid and the CNP CDF values for each claim. We then take the average of these values across all claims for each year. Using this, older, more mature years are given more weight and newer, greener years are given less. To place some more weight on the more recent experience, a yearly exponential decay factor can be applied, as mentioned above in section 3.2, but we did not do so in this example for simplicity. The results are shown in the table below. The final calculated value for the percent of claims paid was 21.2%, close to the true value of 20%.

Year	Ultimate Paid Claims	Ultimate CNP Claims	Relative Weight	Percent Paid
2004	123	409	0.88	23.1
2005	86	392	0.87	18.1
2006	100	401	0.82	19.9
2007	87	404	0.78	17.8
2008	89	349	0.71	20.3
2009	104	336	0.64	23.6
2010	99	304	0.55	24.6
2011	84	277	0.45	23.1
2012	101	231	0.32	30.4
2013	36	191	0.17	15.8
TOTAL	908	3294	NA	21.2

Note how both the results in this table (minus the latest two years) as well as the previous table that shows claim frequency were relatively stable by year, even with volatile data such as this. This is usually not the case with loss development factors, as can be seen from the triangle above.

We then use formula 3.1 shown above to solve for the conditional percent of claims paid given that a claim has been open for a certain amount of time. This percentage needs to be calculated for every open claim and depends on the evaluation date of the data and the report lag of each claim. The average percentages for each year are shown in the table below. Note how the likelihood of being paid is higher for claims from older years which have been open for longer; this was expected since the average settlement time for paid claims was longer than that of unpaid claims.

Year	Percent
2004	35.6
2005	31.9
2006	32.0
2007	29.2
2008	28.2
2009	26.4
2010	25.1
2011	24.3
2012	23.0

The final piece is Part C, where we estimate the parameters of the severity distribution. Fitting a Lognormal distribution to the data using MLE, taking the retention and limit of each claim into account produced mu and sigma parameters of 11.5 and 1.45, compared to the true parameters of 9 and 2. Using these parameters to calculate the average limited expected value for the appropriate retention and limit yields $479,726; the actual value was $469,588. (In practice, if all retentions and limits are the same and average severity does not appear to significantly change with the duration, it would be more efficient to calculate the average of the claim values directly, instead of fitting a distribution.)

We now use the results from the three steps above to estimate the unpaid losses per year. We use the formulas above to calculate the pure IBNR and the IBNER per policy. Recall that pure IBNR is calculated at the policy level by multiplying the unused exposures by the claim frequency and multiplying that by the expected percentage of claims that will be paid and the claim severity (formula 4.3). IBNER is calculated at the claim level by multiplying the likelihood that each claim will be paid given its current duration (formula 3.1) by the severity. Results are then aggregated by year. Adding paid losses yields our ultimate projections. The results are shown in the table below.

Year	Paid	Pure IBNR	IBNER	Total Unpaid	Ultimate
2004	38.7	0.3	9.6	9.9	48.6
2005	38.5	0.6	9.5	10.0	48.5
2006	35.5	0.9	13.3	14.3	49.8
2007	25.8	1.5	14.9	16.4	42.2
2008	29.4	2.5	21.2	23.7	53.2
2009	33.4	4.1	20.5	24.7	58.0
2010	17.7	6.8	24.0	30.8	48.6
2011	14.5	11.3	27.8	39.1	53.6
2012	9.9	18.7	22.5	41.2	51.1
2013	2.5	30.8	17.8	48.6	51.1

The below table shows how the results from this simulation compare to the actual.

Year	Estimated Unpaid	Actual Unpaid	Estimated Ultimate	Actual Ultimate	Unpaid Difference	Unpaid Percent Difference	Ultimate Difference	Ultimate Percent Difference
2004	9.9	12.0	48.6	47.0	-2.1	-17.5%	1.7	3.6%
2005	10.0	11.7	48.5	47.0	-1.6	-13.7%	1.6	3.4%
2006	14.3	16.6	49.8	47.0	-2.3	-13.9%	2.8	6.0%
2007	16.4	13.2	42.2	47.0	3.1	23.5%	-4.8	-10.2%
2008	23.7	24.1	53.2	47.0	-0.4	-1.7%	6.2	13.2%
2009	24.7	25.4	58.0	47.0	-0.8	-3.2%	11.1	23.6%
2010	30.8	24.3	48.6	47.0	6.5	26.8%	1.6	3.4%
2011	39.1	31.1	53.6	47.0	8.0	25.7%	6.6	14.0%
2012	41.2	37.1	51.1	47.0	4.1	11.1%	4.1	8.7%
2013	48.6	40.7	51.1	47.0	8.0	19.7%	4.2	8.9%
TOTAL / AVERAGE	258.7	236.1	50.5	47.0	22.6	9.6%	3.5	7.5%

Running many simulations confirms that this method is unbiased, even with a tail that extends for another 10 to 15 years past the evaluation date of the data. For comparison with a standard triangle method, we used the Cape Cod method with the modified Bondy method (Boor 2006) for estimating the tail, where the tail is set to the square of the latest loss development factor; this was about correct, although we did not penalize for any overall tail bias. Running 5,000 simulations showed a coefficient of variation for total unpaid losses for our method of 11.1% compared to 23.1% for the aggregate triangle method, meaning that in this example, our method cut the standard deviation down by more than half. The difference in the ultimate projections was a bit under 40%, also quite dramatic. As the data became sparser and we decreased the number of accounts per year, the benefit of our method over the triangle method became more pronounced, and it became smaller as we increased the number of years or accounts, both as expected. Any change that made the data more volatile, such as increasing the frequency variance-to-mean ratio, increasing the sigma parameter in the severity distribution, or extending the settlement times of claims all decreased the difference between the two methods, although not too significantly. At first, the direction of this change may seem surprising, but the fact is that as data becomes more volatile, there is less that can be done with it. As an extreme example, for data that is so volatile that has almost no credibility, any method used on it will perform just as poorly, since the volatility is coming all from the data and not from the predictions.

We should mention that the differences in volatility mentioned are overstated since no human input was used for selecting the best loss development factors. On the flip side though, no penalty was given for any inaccuracy of the tail estimate. But regardless, it should not be surprising that this method can lower the volatility by a very large margin; each parameter needed for predicting ultimate losses is estimated using the entire data, as opposed to the triangle method where each "parameter" only uses data from a single duration. In addition, the estimated parameters from the latter part of the triangle are often very volatile and affect the entire estimate since they feed into all the earlier age-to-ultimate factors.

6. CHECKING

The most obvious way to check this model is to compare the ultimate results to that produced from a standard triangle analysis. Results are not expected to match, but this should still give some indication as to the appropriateness of the model.

If settlement times from Part B were calculated for times of paid claims only, that is, not including outstanding reserved claims, then paid loss development factors can be produced by starting each year from the beginning and calculating the expected losses at each duration. Loss development factors can then be calculated from these expected payments by duration, and these can then be compared to the factors obtained from a triangle method as a sanity check. It is also possible to use these paid loss development factors directly as an alternative. Producing incurred loss development factors is more complicated as we would also need to take into account when reserves are set up, how they change, and when they will ultimately be paid.

7. REFINEMENTS AND ALTERNATIVE MODELS

7.1 Paid Only Model

A simplified version of this model can be used that only uses paid losses and does not consider reported or reserved claims. With this approach, Parts B (percent paid and settlement times) and D (reserved claims) can be left out of the model since we are only interested in the settlement of paid claims. Part A (reporting times) will be modified to only include paid claims and will now model the complete reporting plus settlement duration of each claim. This approach does not take advantage of all of the data that the full model does, but is much easier to implement. With this version, we also do not have to worry about dependencies between reporting and settlement times, and so this can also serve as a test for the full version of the model.

With this paid-only model, more accurate modeling by retentions can also be performed. In the full model, we modeled on the retention of each policy, so for example, a 50 million dollar claim on a policy with a one million retention would only be considered under the one million retention group. With this new model, however, a Kaplan-Meier like approach can be used and this claim can be counted under all retentions up to 51 million, since this claim would still have occurred at all of these retentions. To model this, we would use the MLE hyper-parameters method similar to the above, but claims can be counted multiple times in all of the retention groups that they could have occurred at. Normally, the Kaplan-Meier method is done at increments of every claim level, but this is clearly not possible here because of performance constraints. Instead, the method can be performed using wider intervals. This approach is not possible with the full version since the ultimate paid amounts for each claim in the model is unknown.

7.2 Segmentations using Mixed Models and Bayesian Credibility

Our model consists of a bunch of different parametric distributions and GLMs. Each distribution can be broken into finer segments and incorporate credibility by building a Bayesian model. Similarly, instead of using GLMs, Generalized Linear Mixed Models (GLMMs) can be used to incorporate credibility by segment. To produce credibility weighted estimates, it is better to run a prospective year from the beginning instead of adding the credibility weighted unpaid estimates to actual losses. If this is not done, the unpaid portion may be credibility weighted, but the actual losses that already occurred still need to take credibility into account in order to be useful for a prediction. Alternatively, initial estimates can be produced without taking credibility into account, and these estimates can then be credibility weighted. Further discussion is outside the scope of this paper.

7.3 Differences by Retention

All of the reporting and settlement time distributions can be made to vary with the retention of each claim by using the hyper-parameters approach discussed above in section 2.3. This will take into account that larger claims, and thus policies with higher retentions, may have slower reporting and settlement of claims.

7.4 Copulas

As an alternative to using the hyper-parameters and the other approaches mentioned, normal or t copulas can be used instead to take into account the dependencies of the reporting, settlement, and claim severity distributions. A further discussion is outside the scope of this paper.

7.5 Calculation of Volatility

This model can also be used to estimate the volatility in the IBNR or ultimate losses, either in closed form or via simulation. Alternatively, our framework can also be used estimate the uncertainty in the loss predictions resulting from a regular triangle method. To do this, losses will need to be simulated and triangles can be generated from these losses. Simulating a paid triangle is relatively straightforward, but building a reported triangle is more difficult since it involves simulating the changes in each claim's outstanding reserve values over time. The frequency of each claim having a reserve change per year or quarter can be calculated directly. For the average magnitude of each change, Part D above (section 3.4) can be modified to model all reserve changes, instead of just changes from the outstanding amount to the paid amount. Now, given a starting reserve (as a percentage of the limit), we can calculate the expected reserve after the change. To be able to simulate though, we need to build distributions around these expected values. To do this, a Beta distribution can be fit to the data using the hyper-parameters approach to set the mean equal to the predicted value from the GLM and allowing the volatility (that is the sum of the alpha and beta parameters) to be solved for using MLE. Once this is done, a Beta distribution will be available for each starting reserve amount that can be used to simulate the magnitude of the change. Once a triangle is simulated, LDFs can be calculated (and ideally smoothed) and a method similar to that used to calculate the actual IBNR and ultimate losses can be performed. Running many simulations will yield the distribution of the prediction errors, either on an absolute basis, or for a one year time horizon, which is needed for Solvency II.

8. CONCLUSIONS

The goal of the frequency-severity development approach presented in this paper is improved accuracy and better segmentation. This model can also produce valuable information regarding the expected frequency and severity of individual policies, provide a better framework for investigating how the reporting and settlement patterns may be changing over time, and generate volatility estimates. A large loss load can be easily calculated as well using the severity distribution. All of the benefits of this model, however, need be evaluated against the additional effort involved. For cases involving very volatile or sparse data, including low frequency-high severity books of business, aggregate triangle methods start to struggle and their predictions can even become very questionable at times. In these scenarios, the case for building a more detailed model, such as the one presented in this paper, becomes even stronger. This model also takes many factors into account that triangle methods do not, such as the settlement lag of each claim and the outstanding amounts of each reserved claim, individually and not in aggregate, and so can be used to produce more accurate, refined estimates.

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