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E-Forum, Fall 2014-Volume 2

The CAS *E-Forum*, Fall 2014-Volume 2

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Risk Based Capital (RBC) Underwriting Risk Charges: Differences in Premium and Reserve Risk Charges by Ceded Reinsurance Usage

Report 9 of the CAS Risk Based Capital (RBC) Research Working Parties
Issued by the RBC Dependencies and Calibration Working Party (DCWP)

Abstract: The paper examines the extent to which indicated Premium Risk Factors (PRFs) and Reserve Risk Factors (RRFs) are related to ceded reinsurance usage.

Prior DCWP work found that company insurance impairment probabilities generally increase as ceded reinsurance usage increases.[1]

In this report, we show that that PRFs and RRFs are higher for companies with an elevated reinsurance usage for all-lines combined, for nearly all lines of business (LOBs) and generally across LOB-sizes than for companies with lower usage. This result allows us to connect the earlier impairment rate findings, which did not imply particular capital charges to mitigate the increased risk, to the indicated risk charges for the RBC formula.

This is one of several papers being issued by the Risk Based Capital (RBC) Dependencies and Calibration Working Party.

Keywords: Risk-Based Capital, Capital Requirements, Analyzing/Quantifying Risks, Assess/Prioritizing Risks, Integrating Risks

1. INTRODUCTION AND FINDINGS

1.1 Prior DCWP Research

DCWP Report 4, Review of Historical Insurance Company Impairments,[2] showed that relative impairment probability increased as the ceded reinsurance usage increased (Report 4 Table 7). This pattern applied to all group sizes combined and separately to each size group, except for the smallest 20%, of company-sizes (Report 4 Tables 7 - 10).

Further not-yet-published DCWP research, using a multi-factor regression model, also showed that elevated ceded reinsurance usage ("Elevated Re") generally implied higher relative impairment probabilities, after controlling for company characteristics such as size, LOB concentration, and state concentration.

1.2 Findings

In this report, we test the extent to which Elevated Re is associated with higher indicated PRFs

[1] A Review of Historical Insurance Company Impairments (**Report 4**), CAS E-Forum, Fall 2012, http://www.casact.org/pubs/forum/12fforumpt2/RBC-DCWPRpt4.pdf
[2] Ibid.

and RRFs 3

In DCWP Reports 6 and 7[4] indicated PRFs and RRFs by line of business (LOB) are determined using accident year (AY) premiums and loss ratios (Report 6) and initial reserves and reserve runoff ratios (Report 7) net of ceded reinsurance. The PRFs and RRFs by LOB indicated by the methods described in Reports 6 and 7 are determined using data from all companies, regardless of the level of ceded reinsurance usage. For this analysis, we segment the companies based on the ceded reinsurance usage.

Table 1.1 on the next page shows that PRFs and RRFs increase as reinsurance usage increases.

The horizontal axis in Table 1.1 represents the percentage of ceded reinsurance that is used to separate the data between Base ceded reinsurance usage ("Base Re") and Elevated Re. The PRF or RRF for Base Re is the PRF or RRF indicated using the data points with reinsurance usage at or below the selected Separation Point, ("Reinsurance Separation Point" or "Separation Point"). The PRF or RRF for Elevated Re is the PRF or RRF indicated using the data points with reinsurance percentage usage above the selected Separation Point.

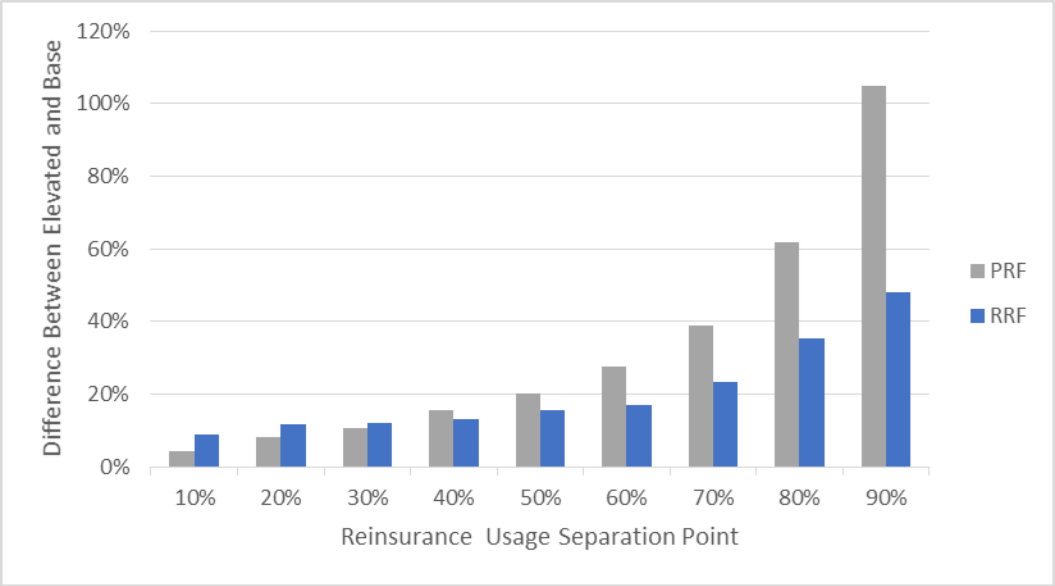
The vertical axis is the difference between PRFs or RRFs for the data points with Elevated Re compared to the PRFs or RRFs for the data points with Base Re. For example, the vertical axis value of approximately 20% at the horizontal value of 50% reinsurance Separation Point means that the indicated PRF for all data points with reinsurance usage above 50% is 20% of Net Earned Premium (NEP) higher than the indicated PRF for all data points with ceded reinsurance usage of 50% or less.

[3]A higher value of the 87.5th percentile of loss ratios or runoff ratios could be due to higher mean values, greater variability around the mean or a combination thereof.

[4] Casualty Actuarial Society E-Forum, CAS Research Working Party on Risk-Based Capital Dependencies and Calibration (DCWP), Risk Based Capital (RBC) Premium Risk Charges – Improvements to Current Calibration Method, **Report 6**, Fall 2013, and

Casualty Actuarial Society E-Forum, CAS Research Working Party on Risk-Based Capital Dependencies and Calibration (DCWP), Risk Based Capital (RBC) Reserve Risk Charges – Improvements to Current Calibration Method, **Report 7**, Winter 2014.

Table 1.1
PRF and RRF Variations by Reinsurance Usage
Differences in Indications between Elevated Re vs. Base Re
All Lines Combined



In addition to the all-lines combined analysis summarized in Table 1.1, we also examined the experience at three sub-levels: (a) by company-size for all-lines combined, (b) by LOB[4] and (c) by LOB-size[5] within each LOB.

- By all-lines company-size: We also observed higher indicated PRFs/RRFs for Elevated Re. The differences were largest for the smallest company-size group and the differences generally decreased with company size.
- By LOB: Weobserved higher indicated PRFs/RRFs for Elevated Re for most LOBs.
- By LOB-size groups: The same pattern was evident, but less consistent. The smallest LOB-size group (first 15th percentile) showed the largest yet most variable differences. As LOB-size grew above the 50th percentile size group, the differences between Base and Elevated PRFs/RRFs are typically smaller. The 75th-95th percentile LOB-size group had several LOBs where Elevated Re produced lower (rather than higher) PRFs and RRFs.

Table 1.2 below summarizes the analysis and the tables found in Section 3.

[5] For LOBs and LOB-size groups with more than a minimum number of data points (50) in Elevated Re.

Table 1.2
Summary of Findings
PRFs and RRFs for Elevated Re vs. Base Re

Table(s)	Data Group	Findings
3.1a-3.1b	All lines combined	PRF/RRF differences grow as the Separation Point between Elevated versus Base Re grows. At the 40% Separation Point, the comparison of Elevated to Base Re is: - PRF is 15.5% of NEP higher - RRF is 13.0% of initial reserves higher.
3.2a-3.2b	All lines combined By company size	PRFs/RRFs for Elevated Re are higher than for Base Re across all company-size groups. The smallest companies have the largest differences.
3.3a-3.3b	By LOB All sizes combined	For most lines, PRFs at Elevated Re are at least 12% higher than at Base Re, and RRF's are at least 10% higher. Exceptions are usually LOB's with a lower number of data points.
3.3a-3.3b, 3.4a-3.4b and Appendix A	By LOB; by LOB-size group	With Elevated Re, PRFs and RRFs are generally higher across all size groups. The smallest LOB-sizes have the largest differences. PRFs, compared to RRFs, show more consistent downward trend in difference as the LOB-size group increases.

In the remainder of this report:

- Section 2 provides more detail regarding (a) the ceded reinsurance data (b) and the risk data used to establish the PRFs and RRFs.
- Section 3 presents the results of our analyses.
- Section 4 discusses the relationship between the findings in this report and the reinsurance credit risk charge in R3.
- Section 5 provides thoughts on further research.
- Section 6 is a glossary of key terms.
- Appendix A provides further detail regarding risk factor differences between Base and Elevated Re, by LOB and LOB-size group.
- Appendix B provides a summary of an analysis using ceded reinsurance data from Schedule F, to compare to the analysis in Section 3 based on ceded reinsurance data from Schedule P.

- Appendix C provides further detail on filtering of data points used in the analysis.
- Appendix D shows that the maturity of PRF and RRF data points does not affect our findings.

1.3 Terminology, Assumed Reader Background and Disclaimer

This paper assumes the reader is generally familiar with the property/casualty RBC formula[6] and has a working knowledge of DCWP Reports 6 and 7.

In this paper, references to "we" and "our" refer to the principal authors of this paper. The "working party," and "DCWP" refer to the CAS RBC Dependencies and Calibration Working Party.

The analysis and opinions expressed in this report are solely those of the authors, and in particular are not those of the authors' employers, the Casualty Actuarial Society, or the American Academy of Actuaries.

DCWP make no recommendations to the NAIC or any other body. DCWP material is for the information of CAS members, policy makers, actuaries and others who might make recommendations regarding the future of the P&C RBC formula. In particular, we expect that the material will be used by the American Academy of Actuaries.

This paper is one of a series of articles prepared under the direction of the DCWP.

2. DATA

In our analysis we use two types of data:

- Information to assess indicated PRFs and RRFs (Risk Data).
- Information to determine ceded reinsurance usage (Reinsurance Usage Date).

The sections below discuss risk data (2.1) and reinsurance usage data (2.2).

2.1 Risk Data

The data we used to determine indicated PRFs and RRFs is as described in DCWP Reports 6 and 7.

In brief, the premium risk data consists of AYs 1988-2010 loss and loss adjustment expense ratios, net of reinsurance, at the latest available maturity from Schedule P, Part 1, in the 1997-2010 Annual Statements, by LOB and by company for individual companies and DWCP-defined group pools, as indicated. Thus, each data point is a single AY and LOB for a single company or pool

[6] For a detailed description of the formula and its basis, see Feldblum, Sholom, NAIC Property/Casualty Insurance Company Risk-Based Capital Requirements, Proceedings of the Casualty Actuarial Society, 1996 and NAIC, Risk-Based Capital Forecasting & Instructions, Property Casualty, 2010.

(LOB-Company-AY).[7]

Similarly, the reserve risk data consists of reserve runoff ratios for initial reserve dates 1988-2009. The ratios, net of reinsurance, are through the latest available maturity from Schedule P, Parts 2 and 3, in the 1997-2010 Annual Statements, by LOB and company for individual companies and DWCP-defined group pools, as indicated. Thus, each data point is the runoff ratio from a single initial reserve date and LOB for a single company or pool (LOB-Company-Initial Reserve Date).

For this analysis, we also constructed all-lines data points. For PRFs, the all-lines premium for each company-AY data point is the sum of the NEP for all LOBs in the PRF data set. The all-lines loss ratio is the all-lines average loss ratio weighted by NEP by LOB for the LOBs included in the PRF data.[8] For RRFs, the all-lines reserve for each company-initial reserve date is the sum of the initial reserves for all LOBs in the RRF data set. The reserve runoff ratio is the all-lines average reserve runoff ratio weighted by initial reserve by LOB.

2.2 Reinsurance Usage Data

For each LOB-Company-AY PRF data point and each LOB-Company-Initial Reserve Date RRF data point, the ceded reinsurance percentage is the AY Ceded Premium divided by AY Direct plus Assumed Premium by LOB by company, from the 1998-2010 Annual Statements (i.e., values in Schedule P, Part 1, column 2 divided by column 1.)

For the PRF data points this reinsurance usage ratio closely matches the premium in the data point, and the RBC charge is based on that premium. For RRF data points, the reinsurance usage ratio in the initial reserves would be the most closely related reinsurance usage ratio. However, the ceded reinsurance reserves for initial reserve dates earlier than 1996 cannot be obtained from the Annual Statements available for our research. Therefore we use the ceded premium reinsurance percentage as a proxy for the ceded reinsurance applicable to reserve runoff ratios. The effect using the premium proxy is a matter for further research. We did find the RRF results using a loss-based reinsurance usage ratio (the three-year ceded AY incurred loss, including IBNR, divided by the three-year direct plus assumed AY incurred loss) produced results that were very close to the results using the premium-based reinsurance ratio from the PRF analysis.

2.3 Filtering

We excluded data points that had negative direct plus assumed earned premium or cession percentages equal to or greater than 100%. We also used a minimum cession percentage of 0% when

[7] In constructing the PRF and RRF data sets we removed certain data points using the filtering process explained in Section 3.2.2 of DCWP Reports 6 and 7, for premium risk and reserve risk, respectively.

[8] Because the all-lines data points are constructed from the filtered LOB data points, the all-lines data only includes lines in which the LOB component of the LOB data point satisfied the Report 6 and 7 filtering tests.

the calculated ratio indicated a negative cession.

We also excluded data points in which there was a significant inconsistency in the AY NEP between the risk data and the Schedule P data. These inconsistencies are all in the "two-year" lines,[9] and the extent of the inconsistencies is summarized in Appendix C. These inconsistencies arise because the risk data is obtained from the NAIC 10 year RBC reporting of "two-year" lines while the reinsurance usage data is from the two years of data publically available in Annual Statements. We looked at the effect of including or excluding the data points with these inconsistencies and we found that the impact was negligible.

2.4 Schedule F

Schedule F provides an alternative way to assess reinsurance usage. In Appendix B, we show the effect of using Schedule F data for reinsurance usage rather than the Schedule P data. Schedule F data is less useful in that it is not available by LOB, it is not available by accident year and it is only available for 1996 and subsequent evaluation dates. On the other hand, Schedule F allows us to separate reinsurance with affiliates from other reinsurance. The Schedule F results are similar to the Schedule P results.

3. ANALYSIS

We measured the effect of ceded reinsurance usage on PRFs and RRFs at four levels of detail:

- All LOBs combined, all company-size groups combined
- By company-size, all LOBs combined
- By LOB, all LOB-size groups combined
- By LOB-size group, by LOB

The analyses are discussed in sections 3.1, 3.2 and 3.3 below.

3.1 All Lines Combined / All Company-Sizes Combined

In this section we describe the analysis of Elevated Re on PRFs and RRFs for all lines/all-company-sizes combined. In this analysis, we used the all-lines PRF and RRF risk data points constructed as described in Section 2.1. Each all-line data point is assigned a Schedule P ceded reinsurance percentage described in Section 2.2. At each Reinsurance Separation Point from 10% to 90%, in 10% increments, and we calculated the difference between:

a) the all-lines indicated PRFs and RRFs for data points with ceded reinsurance above the Separation Point, i.e., Elevated Re and

[9] Special Property, Auto Physical Damage, Fidelity and Surety, and Other

b) the all-lines indicated PRFs and RRFs for data points with ceded reinsurance at or below the Separation Point, i.e., Base Re.

Table 3.1a below shows the results. Column (1) shows the Separation Points. Column (2) shows the PRF for Base Re given the Column (1) Separation Point. Column (3) shows the PRF for Elevated Re given the Column (1) Separation Point. The difference between the two is shown in Column (4). Column (5) shows the percentage of PRF data points that had ceded percentages less than or equal to the ceded percentage in Column (1). Columns (6) through (9) show the same information as Columns (2) through (5) but for RRFs.

For example, the 30% row in Table 3.1a column (4) shows that premium risk data points with a ceded reinsurance percentage more than 30% have a PRF 10.6% of NEP higher than the PRF data points with ceded reinsurance percentages of 30% or less. Looking at the share of data points, column (5), 61.8% of PRF data points have a ceded reinsurance percentage of 30% or less.

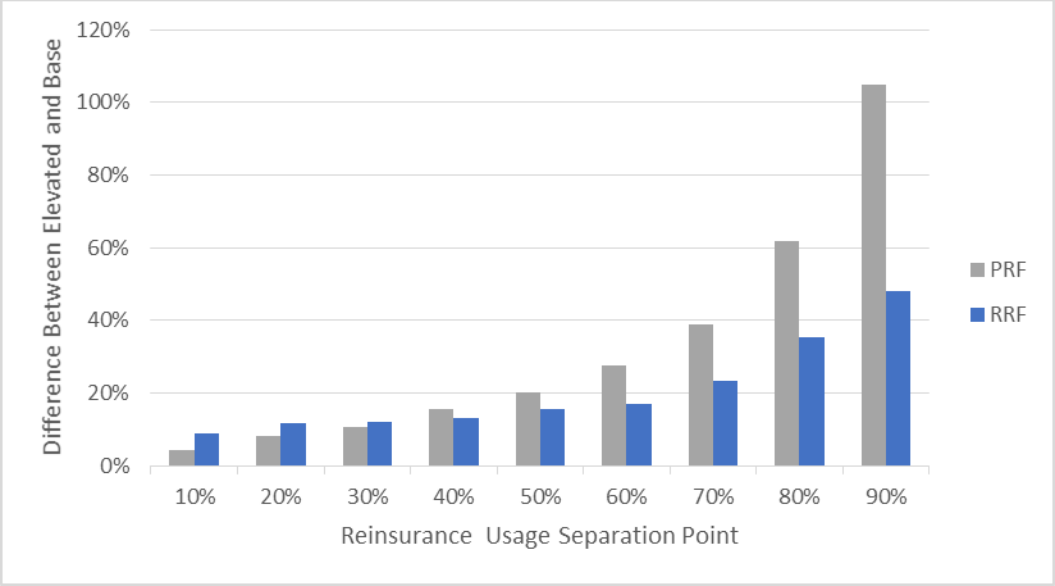
Similarly, the 30% row in Table 3.1a column (8) also shows that that reserve risk data points with ceded reinsurance percentage of higher than 30% have a RRF 11.9% of reserves higher than the RRF data points with ceded reinsurance percentage 30% or less. Looking at the share of data points, column (9), 62.5% of RRF data points have a ceded reinsurance percentage of 30% or less.

Table 3.1a
PRF and RRF Variations by Reinsurance Usage
Difference in Indications between Elevated Re vs. Base Re
All-Lines Combined

Reins. Usage Separation	--------- PRF --------			% Base	--------- RRF --------			% Base
Point	Base	Elevated	Difference	Data Pts	Base	Elevated	Difference	Data Pts
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
10%	91.3%	95.6%	4.4%	30.2%	21.6%	30.4%	8.8%	30.2%
20%	90.3%	98.4%	8.1%	48.4%	22.2%	34.0%	11.8%	48.8%
30%	90.4%	101.0%	10.6%	61.8%	23.7%	35.7%	11.9%	62.5%
40%	90.3%	105.8%	15.5%	71.6%	24.7%	37.6%	13.0%	72.4%
50%	90.7%	110.9%	20.2%	79.3%	25.5%	40.9%	15.4%	80.2%
60%	91.3%	118.7%	27.4%	85.4%	25.9%	42.8%	16.9%	86.1%
70%	91.6%	130.4%	38.8%	89.5%	26.1%	49.5%	23.4%	90.2%
80%	92.2%	154.1%	61.9%	93.3%	26.4%	61.8%	35.4%	93.8%
90%	93.0%	197.8%	104.8%	96.5%	27.1%	75.1%	48.0%	96.8%

Table 3.1b below, the same as Table 1.1 above, shows the Table 3.1 information graphically.

Table 3.1b
PRF and RRF Variations by Reinsurance Usage
Differences in Indications between Elevated Re vs. Base Re
All-Lines Combined



For PRFs, this table shows a consistent increase in PRF differences with increases in the Separation Point between Base Re and Elevated Re. The growth in the difference becomes steeper after the 30% Separation Point.

For RRFs, this table also shows a consistent increase in RRF differences with increases in the Separation Point. The difference does not grow as sharply as the PRF difference.

For purposes of our analysis of the effect of ceded reinsurance usage on PRFs and RRFs by LOB and by size, we selected a Separation Point of 40%.

With 40% ceded reinsurance percentage as the Separation Point, Table 3.1a column (4) shows that the PRF is 15.5% of NEP higher for Elevated Re and the RRF, column (8), is 13.0% of initial net reserves higher for the Elevated Re. Somewhat more than 70% of the data points are in Base Re, as shown in columns (5) and (9).

3.2 All Lines Combined / By Company-Size

In this section we describe the analysis of Elevated Re on PRFs and RRFs by company size, utilizing the 40% Separation Point between Base Re and Elevated Re.

Tables 3.2a and 3.2b below show the differences between all-lines indicated PRFs and RRFs,

respectively, for companies with Elevated Re to companies with Base Re, separately for each of four company-size quartiles based on the NEP or initial reserve associated with each data point.

For PRFs, Table 3.2a Columns (1) and (2) show the range of premium volume in each quartile. Column (3) shows the PRFs for all data points in each quartile. Column (4) shows the PRFs for Base Re and Column (5) shows the PRFs for Elevated Re. Column (6) is the difference, as a percentage of NEP, between the PRF for Elevated Re and the PRF for Base Re.

Columns (7) – (9) show the number of data points at Base Re, Elevated Re and in total. Column (10) shows the number of companies represented in that quartile. Columns (11) and (12) show the simple average ceded percentage for each premium volume group, separately for Base Re and Elevated Re.

Table3.2a – PRF
Base Re vs. Elevated Re
By Company Size/All Lines Combined

Premium Vol Grps ($000's)		------ Reinsurance Usage ------				--- Data Counts ---				% Ceded	
From	To	All	Base	Elevated	Elev Less Base	Base	Elevated	Total	Companies	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
0	2,956	112.1%	98.7%	136.3%	37.6%	4,097	3,007	7,104	1,049	13.5%	70.1%
2,956	13,357	92.2%	88.6%	99.9%	11.3%	5,047	2,055	7,102	1,058	15.1%	64.0%
13,357	63,251	90.7%	89.7%	94.7%	5.0%	5,310	1,791	7,101	885	15.4%	62.2%
63,251	& greater	89.6%	89.2%	91.0%	1.8%	5,883	1,220	7,103	573	14.3%	54.9%
All groups		94.3%	90.3%	105.8%	15.5%	20,337	8,073	28,410	1,758	14.6%	64.5%

- A company can be counted in more than one size group.
- % Ceded statistics are simple averages over all data points.

Table 3.2a column (6) shows that higher PRFs are indicated for Elevated Re, regardless of company size. The difference is largest for the smallest companies and decreases consistently with company size. Column (12) shows that for data points in the smallest size group, Elevated Re has an average ceded reinsurance of 70.1%. For data points in the largest size group, the average ceded reinsurance is 54.9% for Elevated Re.

Table 3.2b shows analogous information regarding RRFs.

Table3.2b – RRF
Base Re vs. Elevated Re
By Company Size/All Lines Combined

Reserve Vol Grps ($000's)		------ Reinsurance Usage ------				--- Data Counts ---			% Ceded		
From	To	All	Base	Elevated	Elev Less Base	Base	Elevated	Total	Companies	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
0	1,515	50.0%	42.2%	63.2%	21.0%	4,052	2,427	6,479	1,103	14.3%	67.7%
1,515	9,835	29.9%	27.7%	33.0%	5.3%	4,462	2,011	6,473	1,113	14.8%	64.7%
9,835	60,501	21.2%	20.4%	22.7%	2.4%	4,884	1,591	6,475	890	15.4%	62.5%
60,501	& greater	21.7%	19.5%	34.6%	15.1%	5,362	1,114	6,476	534	14.4%	56.4%
All groups		27.9%	24.7%	37.6%	13.0%	18,760	7,143	25,903	1,904	14.7%	63.9%

- A company can be counted in more than one size group.
- % Ceded statistics are simple averages over all data points.

As was the case for PRFs, Table 3.2b column (6) shows that higher RRFs apply for Elevated Re, regardless of company size, and the difference is largest for the smallest companies. Unlike the case for PRFs, the difference does not consistently decrease with company size. Column (12) shows that for data points in the smallest size group, Elevated Re has an average ceded reinsurance of 67.7%. For data points in the largest size group, the average ceded reinsurance is 56.4% for Elevated Re.

3.3 Analysis by LOB and LOB-size

In this section we describe the analysis of Elevated Re on PRFs and RRFs by LOB and by LOB-size within each LOB utilizing a Separation Point of 40%.

Tables 3.3a and 3.3b below show the differences between indicated PRFs and RRFs, respectively, for Elevated Re and Base Re by LOB for all LOB-sizes combined. The tables also summarize information on the indications by LOB-size group. Further details on LOB by size analysis are presented in Appendix A.

In Table 3.3a, columns (1) and (2) identify the LOB. Columns (3) and (4) show the PRFs for Base Re and Elevated Re, respectively. Column (5) shows the difference between the PRF for Elevated Re and Base Re. Column (6) shows minimum and maximum differences in indicated PRFs across LOB-size groups with more than 50 data points in Elevated Re. Column (7) shows the number of size segments with PRF differences greater than zero out of the total number of size segments with more than the minimum amount of data. Columns (8) and (9) show the number of data points within Base and Elevated Re for all LOB-size groups.

Column (5) shows that for all LOBs other than the LOB "Other" (credit and A&H business), the PRFs are larger with Elevated Re than when it is with Base Re. Column (7) shows that the PRFs by LOB-size group are almost always larger for Elevated Re.

Table3.3a – PRF
Base Re vs. Elevated Re Usage by LOB

LOB	Sch P Line	PRF Base	PRF Elevated	Difference: Elev - Base	By Size Range	Credible Size Frequency of Positive Diff.	No. of Data Points Base	No. of Data Points Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Homeowners	A	95.2%	112.2%	17.0%	-0.3% to 22.4%	7 out of 8	7,029	2,427
PPA	B	97.7%	110.2%	12.5%	-2.1% to 21.1%	6 out of 7	6,969	1,686
CAL	C	97.9%	119.9%	22.0%	-0.3% to 27.1%	6 out of 7	4,867	1,170
WC	D	102.5%	120.4%	17.9%	-4.7% to 46.0%	5 out of 7	4,868	1,115
CMP	E	85.8%	104.9%	19.1%	4.3% to 35.3%	7 out of 7	5,244	1,948
MEDMAL - OCC	F1	145.7%	217.2%	71.5%	97.1% to 97.1%	1 out of 1	843	187
MEDMAL - CM	F2	114.3%	139.9%	25.6%	-4.0% to 162.7%	2 out of 4	1,953	489
Spec Liab	G	92.6%	99.4%	6.8%	-7.8% to 47.0%	2 out of 4	696	429
Other Liab	H	94.0%	122.5%	28.5%	7.5% to 103.0%	8 out of 8	5,710	2,977
Spec Property	I	78.1%	90.7%	12.7%	-6.4% to 35.7%	6 out of 7	3,402	2,005
Auto Phys Dam	J	86.3%	95.0%	8.7%	-0.3% to 15.7%	6 out of 7	4,008	1,410
Fidelity & Surety	K	58.2%	90.4%	32.3%	-11.2% to 39.5%	1 out of 2	703	239
Other	L	96.2%	94.8%	-1.4%	-16.0% to 0.3%	1 out of 2	753	317
International	M					Sufficient data not available		
Non Prop Reins - Prop/Fin	N&P	123.5%	150.6%	27.1%			838	216
Non Prop Reins Casualty	O	129.6%	149.7%	20.1%			951	168
Product Liability	R	106.8%	178.0%	71.2%			434	222
Financial Guarantee	S					Sufficient data not available		
Warranty	T					Sufficient data not available		

The information in Table 3.3b is analogous to Table 3.3a, but for RRFs and reserves. Table 3.3b column (5) shows that for most LOBs, the RRFs are larger for Elevated Re. The exceptions are LOBs "other" (credit and A&H business), medical malpractice claims made, special liability (ocean marine, boiler and machinery, etc.), and non-proportional property reinsurance. That those are exceptions may be due to the fact that the number of data points for those lines is relatively small or because of the specialty nature of those lines.

Column (7) shows that the RRFs by LOB-size group are almost always larger for Elevated Re than for Base Re.

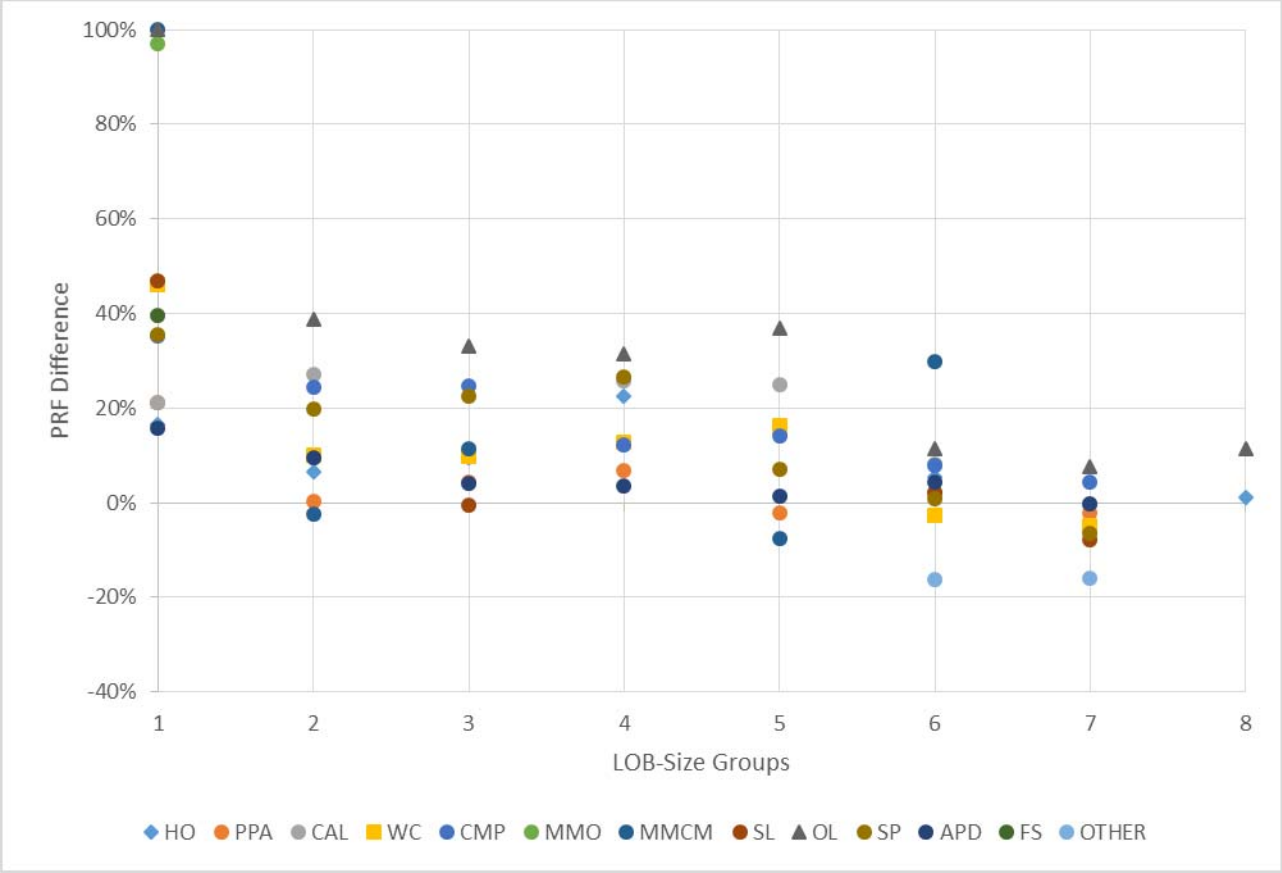
Table 3.3b – RRF
Base Re vs. Elevated Re by LOB

LOB	Sch P Line	RRF Base	RRF Elevated	Difference: Elev - Base	By Size Range	Credible Size Frequency of Positive Diff.	No. of Data Points Base	No. of Data Points Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Homeowners	A	21.6%	40.1%	18.5%	-18.1% to 32.1%	6 out of 8	6,287	2,053
PPA	B	17.9%	39.6%	21.7%	1.3% to 23.4%	7 out of 7	6,335	1,509
CAL	C	32.6%	52.1%	19.5%	0.0% to 24.7%	7 out of 7	4,480	1,157
WC	D	31.8%	47.5%	15.7%	-3.1% to 20.3%	6 out of 7	4,595	1,033
CMP	E	50.2%	65.9%	15.7%	-6.8% to 23.2%	7 out of 8	4,893	1,750
MEDMAL - OCC	F1	31.3%	75.5%	44.2%		1 out of 1	803	153
MEDMAL - CM	F2	10.1%	6.3%	-3.8%	-22.2% to 57.1%	1 out of 6	1,708	434
Spec Liab	G	46.3%	32.6%	-13.8%	-27.3% to -15.9%	0 out of 2	632	381
Other Liab	H	52.4%	54.4%	2.0%	-26.7% to 30.8%	6 out of 8	5,064	2,700
Spec Property	I	31.9%	41.7%	9.8%	-15.5% to 57.4%	5 out of 7	2,930	1,691
Auto Phys Dam	J	12.8%	38.1%	25.2%	-3.2% to 26.6%	6 out of 7	1,986	717
Fidelity & Surety	K	68.5%	78.9%	10.4%			599	172
Other	L	27.3%	23.6%	-3.6%	-21.2% to 9.3%	1 out of 2	668	295
International	M					Sufficient data not available		
Non Prop Reins - Prop/Fin	N&P	45.0%	41.4%	-3.6%		1 out of 1	760	242
Non Prop Casualty	O	66.1%	75.5%	9.4%			908	172
Product Liability	R	80.1%	122.6%	42.5%			372	197
Financial Guarantee	S					Sufficient data not available		
Warranty	T					Sufficient data not available		

As a further way to show the relationship between the risk factor differences and the LOB-size, we provide Table 3.4a and Table 3.4b, scatter plots for PRF and RRF differences respectively, by LOB-size group. Size is measured by the data point NEP for PRFs and the initial reserve for RRFs. Table 3.4a and 3.4b show differences in indicated PRFs and RRFs for each LOB-size group that had more than 50 data points in Elevated Re. There are eight size groups: 0-15th percentile, 15th-25th, 25-35th, 35th-45th, 45-55th, 55th-75th, 75-95th, and the top 5th. These are labeled 1-8 on the horizontal axes of these tables.

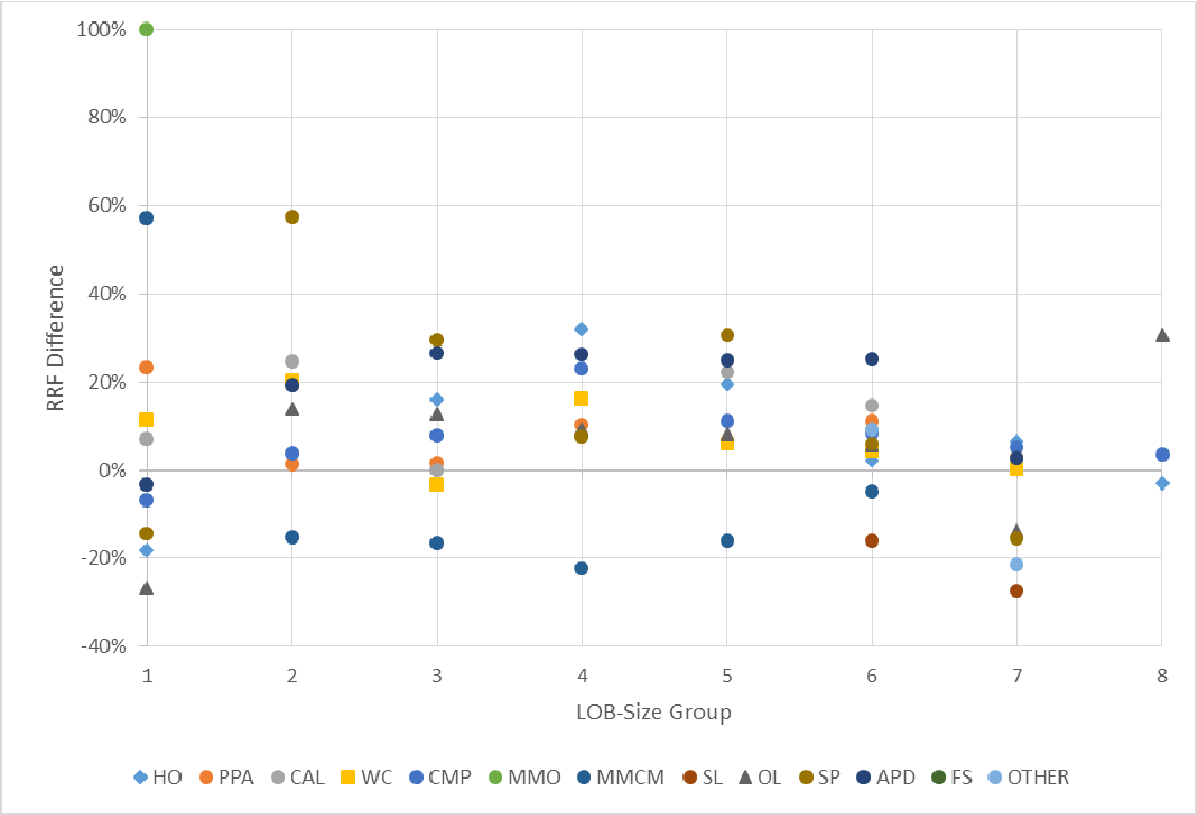
For the first size group, 0-15th percentile, the indicated PRF and RRF differences between Base and Elevated Re are larger and more variable than for the other size groups. As shown in Table 3.4a, there is a noticeable downward trend in the PRF differences (Table 3.4a) and RRF differences (Table 3.4b) as the LOB-sizes increase.

Table 3.4a
PRF Differences by Size Group within LOB



Note: the largest PRF differences (Base vs. Elevated) are limited to 100% on this table. This affects Medical Malpractice Claims Made (163%) and Other Liability (103%) in the first 15% size group.

Table 3.4b
RRF Differences by Size Group within LOB



Note: the largest RRF difference (Base vs. Elevated) is limited to 100% on this table. This affects Medical Malpractice Occurrence (165%) in the first 15% size group.

The details on the indicated PRFs and RRFs by LOB-size group are discussed in Appendix A.

4. IMPLICATIONS FOR REINSURANCE CREDIT RISK CHARGE (R3)

The scope of this paper does not include the evaluation of the implications of these findings on R3; however, as an outline of possible further research issues, we note the issues described below.

First, we note that the calibration of the risk called 'reinsurance credit risk' depends on the extent to which reinsurance-related risks are reflected in R4 and R5.[10] The R3-reinsurance credit risk charge can be viewed as the segment of the RBC formula that reflects any reinsurance-related risks not otherwise considered in the RBC formula. From that perspective, any changes in R4 or R5 based on the characteristics of ceded reinsurance should consider related changes in R3.

Second, we note that, among other factors, the increase in observed PRFs/RRFs with increasing use of reinsurance might be due (a) in part to companies with higher underwriting risk (volatility of business, concentration by state or line, new companies, etc.) purchasing more reinsurance and (b) in part be due to transfer of less risk per dollar of ceded premium (reserve) than is retained by the ceding company. This paper does not evaluate to what extent either of these factors have contributed to the observed patterns.

Third, we note that the observed increase in PRFS/RRFS might be expressed as a risk charge applied to the ceded premium/reserves rather than as an increase in risk charge applied to net premium/reserves. This would be consistent with the view that the increase in PRFS/RRFS results from reinsurance that does not transfer 100% of the risk from cedant to reinsurer.

Table 1.1 appears to show that PRFs are more strongly related to reinsurance usage than RRFs. This observation should be tempered recognizing that reserves can be multiples of one year premium. Thus, a smaller RRF increase might have a larger effect on RBC than a large increase in PRF. This paper did not explore any implications of that RBC sensitivity.

Finally, the analysis suggests reinsurance-related risk charges that might vary by line of business. This paper does not explore the extent to which that can be applied in practice.

[10] For example, American Academy of Actuaries, "Report on Reinsurance Credit Risk Charge in the NAIC Property/Casualty Risk-Based Capital System, page 10, March 29, 2013

5. FURTHER RESEARCH

Areas of further research arising from this work include the following:

- Evaluating the relationship between R3 and R4 and R5, as discussed in Section 4.
- Evaluating the extent to which the reinsurance usage effect is connected to other risk factors, e.g., high growth or specialization by state or line, as companies with those characteristics might tend to be associated with Elevated Re.
- Evaluating the relationship between PRFS/RRFS by ceded reinsurance usage at risk levels above the 87.5th percentile level evaluated in this report.
- Further review of differences between Schedule P and Schedule F methods of measuring ceded reinsurance usage.
- Alternative methods of measuring ceded reinsurance usage on reserve risk.
- Evaluating the reinsurance sensitivity of the combined effect of the premium and reserve risk factors.

6. GLOSSARY

Term	Interpretation
AY	Accident year
Base Re	Data points with reinsurance usage ratio at or below a specified Separation Point. In most analyses, this Separation Point is 40%.
Data point	Each PRF data point for Section 3.3 and Appendix A analyses is an AY-LOB for a single company or pool at the latest available maturity within the database. Each RRF data point for Section 3.3 and Appendix A is an LOB-runoff ratio, for a single company or pool, at the latest available maturity. For Sections 3.1-3.2, the data points are AY-All Lines Combined as of the latest maturity.
DCCE[11]	Defense and cost containment expenses
DCWP	Dependency and Calibration Working Party
Elevated Re	Data points with reinsurance usage ratios above a specified Separation Point. In most analyses, this Separation Point is 40%.
Initial Reserve	The loss reserve amount, net of reinsurance, for the current and all prior AYs evaluated at the initial reserve date.
Initial Reserve Date	December 31st for the year specified (i.e., December 31, 2008 is the initial reserve date for the 2008 net loss reserve which includes AY's 2008 and prior)
LOB	Schedule P Lines of Business. Note that the two parts (occurrence and claims made) have been combined for Other Liability (Line H) and Products Liability (Line R). Line N (Non-Proportional Property) and Line P (Non-Proportional Financial) have also been combined.

[11] "Defense and Cost Containment Expenses" are called "Allocated Loss Adjustment Expenses" (ALAE) in older Annual Statements. In our analysis we treat DCCE and ALAE as equivalent.

Term	Interpretation
LOB-size	Line of business size based on NEP (for PRF analyses) or Initial Reserve (for RRF analyses).
NEP	Net Earned Premium
PRF	Premium Risk Factor
RBC Formula or Formula	The 2010 NAIC Property-Casualty RBC Formula
Reinsurance Usage or Reinsurance Usage Percentage	Ceded premium divided by Direct + Assumed premium from Schedule P (base analysis) or Schedule F (Appendix B alternative analysis).
Reserves or Loss Reserves	Case, bulk and IBNR loss and defense and cost containment expense (DCCE) reserves net of reinsurance, as shown in Schedule P – Part 2 and 3.
RRF	Reserve Risk Factor
Runoff ratio or Reserve Runoff Ratio	The ratio of (a) incurred movement from the initial reserve date to the latest available evaluation date, for all constituent AYs combined to (b) the Initial Reserve.

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APPENDIX A - ANALYSIS AND COMMENTS BY LOB-SIZE GROUPS

We discuss the findings by LOB-size groups in the sections below.

Each LOB has two tables – one for PRF differences and the second for RRF differences. Column (1) shows the size group. There are eight size groups:[12] 0-15th percentile, 15th-25th, 25-35th, 35th-45th, 45th-55th, 55th-75th, 75-95th, and the top 5th. These are labeled 15%, 25% … 100% on the tables. The size is measured by the NEP associated with the data point in the case of the PRFs. For the RRFs, the measure for size is the initial reserve associated with the data point. Columns (2) and (3) are the upper and lower bounds for the size group in terms of NEP or net loss reserve ($000s). The next three columns are the PRF and RRF factors. Column (4) is the overall factor for the group; Column (5) is the factor for the Base Re (i.e., 40% or less) and Column (6) is the factor for the Elevated Re.[13]

Most of the comments for these sections pertain to column (7), the PRF or RRF difference between Elevated Re and the Base Re. Column (7) is shown in color if the value is positive, as is typically the case, or no color if it is negative. For some LOBs /size of business / reinsurance usage, there were no data points or only a limited number of data points. Column (7) is blank if there are 50 or fewer data points with Elevated Re within the LOB (perhaps as few as 3 companies). The values in column (7) are the source for the values shown in columns (6) and (7) in Tables 3.3a and 3.3b along with the values plotted in Tables 3.4a and 3.4b. Thus, if the Elevated Re data point count is 50 or fewer within a LOB-size group, the PRF or RRF differences are ignored when the establishing ranges and frequencies in columns (6) and (7) in Tables 3.3a and 3.3b or when plotting points for Tables 3.4a and 3.4b.

Columns (8) and (9) show the number of data points for each LOB-size group for Base Re (less than or equal to 40%) and Elevated Re (greater than 40%) Re, respectively.

Looking at these individual LOB tables and the summary tables in Section 3.4, we observe the following:

- The first 15th percentile (i.e., smallest) size groups had the largest and most volatile

[12] These groups consist of the smallest 15% and largest 5% LOB-sizes, sizes that DCWP typically identifies separately because the smallest and largest LOB-sizes are often different from the other 80% of LOB-sizes. The groups are summarized in 10% size bands in the smaller LOB-sizes and 20% size bands for the larger LOB-size bands, as, in some cases, the proportion of Elevated Re data points is larger for the smaller LOB-sizes than for the larger LOB-sizes. To that extent, including more companies in the large LOB-size bands better distributes the number of Elevated Re data points within the LOB-size bands. In any case, the exact banding structure does not appear to effect the conclusions.

[13] Column (4) is the 87.5th percentile of the total data, and, as such, it is not the average of the column (5) and column (6) values. Particularly when there are a small number of data points, the column (4) value might be the same as the value in column (5) or the value in column (6). Also, if 87.5 times the number of data points in not an integer, the percentile value may be an interpolation and the column (6) value can fall outside the range of the column (4) and (5) values.

differences.

- The middle groups showed more consistent but still significant differences.
- An apparent downward trend in the differences appears after the 50th percentile. This trend is more apparent with the PRF differences.
- The 75th-95th percentile did show some frequency of negative differences.

Our line-by-line comments follow.

A1. Homeowners

HO LOB Size Group			----------PRF----------				Data Points	
Perc End	Lo Prem	Hi Prem	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	730	128.7%	119.2%	135.7%	16.4%	574	853
25%	730	1,483	102.2%	99.9%	106.4%	6.5%	671	273
35%	1,483	2,758	97.8%	95.9%	105.5%	9.6%	717	232
45%	2,758	5,022	95.8%	94.4%	116.8%	22.4%	738	211
55%	5,022	16,382	94.0%	92.4%	106.8%	14.4%	753	197
75%	16,382	61,546	93.5%	92.9%	98.3%	5.4%	1,565	319
95%	61,546	252,884	93.0%	93.1%	92.8%	-0.3%	1,607	279
100%	252,884	10,820,092	94.5%	94.0%	95.1%	1.0%	404	63
		Overall	98.5%	95.2%	112.2%	17.0%	7,029	2,427

Data points above the 75th LOB-size percentile have limited PRF differences between Elevated Re and Base Re. The differences are all greater than 5% positive for the other LOB-size groups.

HO LOB Size Group			----------RRF----------				Data Points	
Perc End	Lo Res	Hi Res	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	169	86.3%	96.6%	78.6%	-18.1%	538	612
25%	169	357	43.1%	42.0%	45.8%	3.8%	518	263
35%	357	672	28.2%	26.1%	42.0%	16.0%	609	194
45%	672	1,276	26.1%	21.8%	53.9%	32.1%	638	166
55%	1,276	4,820	27.3%	24.7%	44.2%	19.5%	639	185
75%	4,820	19,776	19.8%	19.4%	21.8%	2.3%	1,397	315
95%	19,776	74,565	9.9%	8.7%	15.2%	6.5%	1,547	262
100%	74,565	27,109,142	10.2%	10.4%	7.6%	-2.8%	401	56
		Overall	24.9%	21.6%	40.1%	18.5%	6,287	2,053

The RRF differences by LOB-size are erratic, with most of the large positive differences within the mid percentile groups. The first 15% LOB-size percentile shows a fairly significant negative difference.

A2. Personal Auto Liability

PPA LOB Size Group			----------PRF----------				Data Points	
Perc End	Lo Prem	Hi Prem	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	1,596	124.3%	117.0%	138.1%	21.1%	811	490
25%	1,596	3,634	101.9%	101.7%	102.0%	0.4%	612	253
35%	3,634	6,667	100.3%	99.6%	103.8%	4.3%	660	201
45%	6,667	11,219	101.3%	100.1%	106.8%	6.8%	677	188
55%	11,219	28,352	97.1%	97.6%	95.5%	-2.1%	728	139
75%	28,352	130,201	96.8%	96.2%	98.1%	1.9%	1,469	264
95%	130,201	580,234	94.1%	94.2%	92.1%	-2.1%	1,600	129
100%	580,234	18,406,826	89.4%	89.3%	102.3%		412	22
		Overall	99.9%	97.7%	110.2%	12.5%	6,969	1,686

The PRF differences are small for most LOB-size groups after the first 15%.

PPA LOB Size Group			----------RRF----------				Data Points	
Perc End	Lo Res	Hi Res	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	813	82.9%	78.3%	101.7%	23.4%	564	351
25%	813	1,953	41.1%	40.0%	41.3%	1.3%	469	262
35%	1,953	4,006	31.5%	31.3%	32.9%	1.6%	539	216
45%	4,006	7,450	26.0%	24.7%	35.0%	10.2%	617	184
55%	7,450	20,753	18.0%	17.4%	24.3%	6.9%	689	119
75%	20,753	105,435	13.2%	10.8%	21.9%	11.0%	1,412	231
95%	105,435	543,374	7.3%	6.8%	9.8%	3.0%	1,607	134
100%	543,374	17,069,357	8.1%	7.8%	8.6%		438	12
		Overall	21.4%	17.9%	39.6%	21.7%	6,335	1,509

There are RRF positive differences amongst all LOB-size groups although no discernible relationship.

A3. Commercial Auto Liability

CAL LOB Size Group			----------PRF----------				Data Points	
Perc End	Lo Prem	Hi Prem	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	767	126.1%	117.5%	138.7%	21.2%	612	298
25%	767	1,491	107.0%	100.0%	127.1%	27.1%	466	140
35%	1,491	2,755	100.9%	98.6%	109.3%	10.7%	473	131
45%	2,755	4,639	99.5%	96.2%	121.9%	25.7%	483	123
55%	4,639	13,680	98.9%	97.0%	122.0%	24.9%	519	81
75%	13,680	53,660	97.9%	97.1%	105.2%	8.0%	992	209
95%	53,660	189,338	95.4%	95.5%	95.2%	-0.3%	1,038	169
100%	189,338	1,875,641	93.8%	93.1%	102.3%		284	19
		Overall	100.7%	97.9%	119.9%	22.0%	4,867	1,170

There are significant PRF positive differences amongst most LOB-size groups although no discernible relationship.

CAL LOB Size Group			----------RRF----------				Data Points	
Perc End	Lo Res	Hi Res	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	511	111.8%	110.3%	117.3%	7.1%	483	261
25%	511	1,238	59.4%	54.4%	79.2%	24.7%	401	128
35%	1,238	2,534	37.0%	36.9%	37.0%	0.0%	390	141
45%	2,534	4,555	36.8%	35.4%	43.3%	7.9%	433	113
55%	4,555	14,677	34.1%	32.9%	55.0%	22.1%	481	89
75%	14,677	62,677	26.8%	24.7%	39.3%	14.6%	967	192
95%	62,677	241,324	25.3%	24.6%	27.3%	2.7%	1,034	205
100%	241,324	2,785,549	13.0%	12.9%	11.6%		291	28
		Overall	35.9%	32.6%	52.1%	19.5%	4,480	1,157

There are significant RRF positive differences amongst most LOB-size groups although no discernible relationship.

A 4. Workers' Compensation

WC LOB Size Group			----------PRF----------				Data Points	
Perc End	Lo Prem	Hi Prem	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	1,756	131.7%	117.3%	163.3%	46.0%	562	353
25%	1,756	3,872	122.4%	118.5%	128.5%	10.1%	471	141
35%	3,872	6,827	110.4%	108.6%	118.5%	9.9%	480	133
45%	6,827	12,098	107.9%	105.0%	117.7%	12.8%	516	98
55%	12,098	37,341	102.0%	101.9%	118.3%	16.4%	525	88
75%	37,341	148,020	96.7%	97.1%	94.5%	-2.6%	1,076	147
95%	148,020	518,403	96.4%	96.5%	91.8%	-4.7%	1,020	117
100%	518,403	7,918,320	99.5%	98.9%	101.8%		218	38
		Overall	104.9%	102.5%	120.4%	17.9%	4,868	1,115

The smallest premium group has by far the largest positive PRF difference. The largest credible LOB-size groups have small negative differences. Overall this LOB has the most variation in differences by LOB-size.

WC LOB Size Group			----------RRF----------				Data Points	
Perc End	Lo Res	Hi Res	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	1,819	60.0%	55.4%	66.7%	11.3%	458	279
25%	1,819	3,836	40.6%	36.2%	56.5%	20.3%	389	141
35%	3,836	7,763	45.4%	45.9%	42.8%	-3.1%	425	102
45%	7,763	14,380	42.7%	40.9%	57.2%	16.3%	449	112
55%	14,380	48,220	42.5%	42.1%	48.0%	5.9%	472	94
75%	48,220	258,752	29.3%	28.4%	32.7%	4.3%	1,045	141
95%	258,752	1,262,334	22.5%	22.4%	22.8%	0.3%	1,134	124
100%	1,262,334	16,176,596	28.4%	27.3%	32.2%		223	40
		Overall	34.7%	31.8%	47.5%	15.7%	4,595	1,033

This table shows some positive significant RRF differences in 4 of the first 5 LOB-size groups, but no consistent relationship to LOB-size.

A5. Commercial Multi-Peril

CMP LOB Size Group			----------PRF----------				Data Points	
Perc End	Lo Prem	Hi Prem	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	681	109.3%	90.0%	125.3%	35.3%	575	503
25%	681	1,520	87.7%	81.2%	105.6%	24.4%	490	230
35%	1,520	2,841	87.7%	79.4%	104.0%	24.6%	468	251
45%	2,841	4,810	88.3%	85.3%	97.5%	12.3%	539	181
55%	4,810	14,256	89.9%	84.1%	98.3%	14.2%	553	166
75%	14,256	54,619	87.3%	86.2%	94.0%	7.9%	1,131	307
95%	54,619	294,101	86.9%	86.3%	90.7%	4.3%	1,178	261
100%	294,101	2,970,994	90.1%	88.9%	99.0%		310	49
		Overall	89.9%	85.8%	104.9%	19.1%	5,244	1,948

There is a consistent positive PRF difference amongst all credible LOB-size groups and a noticeable trend downward as the premium group LOB-size increases.

CMP LOB Size Group			----------RRF----------				Data Points	
Perc End	Lo Prem	Hi Prem	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	254	138.2%	141.2%	134.4%	-6.8%	572	368
25%	254	696	70.2%	69.2%	73.0%	3.8%	421	195
35%	696	1,537	58.9%	57.8%	65.7%	7.8%	432	185
45%	1,537	3,299	49.7%	46.4%	69.6%	23.2%	498	186
55%	3,299	11,589	56.8%	49.6%	60.7%	11.0%	490	158
75%	11,589	48,771	47.2%	46.1%	54.4%	8.3%	1,023	319
95%	48,771	324,375	35.6%	34.0%	39.2%	5.2%	1,144	274
100%	324,375	4,184,264	31.1%	30.0%	33.6%	3.6%	313	65
		Overall	54.8%	50.2%	65.9%	15.7%	4,893	1,750

Like many of the other lines, the majority of the RRF differences for LOB-size groups are positive. The differences are quite erratic and the first 15% LOB-size group has a negative difference greater than 5%.

A6. Medical Malpractice – Occurrence

MM Occ- LOB Size Group			----------PRF----------				Data Points	
Perc End	Lo Prem	Hi Prem	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	823	238.8%	182.7%	279.9%	97.1%	94	72
25%	823	1,595	140.2%	143.4%	129.0%		81	26
35%	1,595	2,623	118.8%	118.8%	108.1%		81	16
45%	2,623	4,087	142.2%	139.8%	214.4%		91	7
55%	4,087	11,654	123.4%	117.3%	237.7%		83	12
75%	11,654	44,393	156.5%	156.0%	158.4%		158	34
95%	44,393	152,900	141.5%	138.6%	239.4%		200	20
100%	152,900	516,498	144.8%	144.8%			55	0
		Overall	151.2%	145.7%	217.2%	71.5%	843	187

The Elevated Re data is too limited to give reliable strong indications on the sensitivity of the PRF differences by LOB-size group.

MM-Occ LOB Size Group			----------RRF----------				Data Points	
Perc End	Lo Res	Hi Res	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	1,940	170.8%	119.0%	284.2%	165.2%	105	51
25%	1,940	5,327	59.0%	49.2%	67.5%		74	30
35%	5,327	11,727	32.4%	33.9%	22.0%		92	19
45%	11,727	20,166	17.3%	12.7%	75.5%		87	8
55%	20,166	73,546	4.8%	5.3%	-8.3%		76	6
75%	73,546	248,051	44.4%	44.4%	43.9%		145	24
95%	248,051	728,864	8.7%	8.6%	19.5%		180	15
100%	728,864	3,130,491	7.8%	7.8%			44	0
		Overall	37.4%	31.3%	75.5%	44.2%	803	153

Again, there is limited data for Elevated Re. Nonetheless, the first 15% LOB-size groups and the LOB overall show PRF and RRF positive differences that are large.

A7. Medical Malpractice – Claims Made

MM CM- LOB Size Group			----------PRF----------				Data Points	
Perc End	Lo Prem	Hi Prem	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	1,422	173.1%	124.7%	287.4%	162.7%	224	139
25%	1,422	2,642	94.8%	95.8%	93.4%	-2.3%	172	70
35%	2,642	4,082	114.4%	108.4%	119.8%	11.4%	174	70
45%	4,082	6,520	106.5%	106.6%	106.0%		196	49
55%	6,520	19,211	103.7%	106.4%	98.7%	-7.6%	186	60
75%	19,211	58,551	121.7%	119.4%	149.3%	29.9%	420	65
95%	58,551	142,452	118.7%	118.5%	118.8%		465	28
100%	142,452	726,535	117.7%	122.1%	90.4%		116	8
		Overall	118.3%	114.3%	139.9%	25.6%	1,953	489

The smallest percentile LOB-size group shows largest positive PRF difference. There are a limited number of Elevated Re data points in the other LOB-size groups. Overall PRF difference is much lower than the indication for the smallest percentile.

MM-CM LOB Size Group			----------RRF----------				Data Points	
Perc End	Lo Res	Hi Res	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	1,337	106.6%	98.3%	155.5%	57.1%	185	83
25%	1,337	2,976	12.8%	16.6%	1.6%	-15.0%	160	51
35%	2,976	5,447	15.5%	22.2%	5.6%	-16.5%	160	70
45%	5,447	10,560	9.2%	17.7%	-4.5%	-22.2%	163	55
55%	10,560	35,165	6.1%	10.9%	-5.2%	-16.1%	152	66
75%	35,165	135,321	7.3%	7.8%	2.9%	-4.9%	344	70
95%	135,321	397,713	-2.9%	-3.7%	13.7%		436	29
100%	397,713	1,478,669	-0.9%	-0.5%	-2.9%		108	10
		Overall	9.5%	10.1%	6.3%	-3.8%	1,708	434

Even though smallest percentile LOB-size group has a large significant positive difference, the overall difference is negative.

A8. Special Liability

Spec Liab - LOB Size Group			----------PRF----------				Data Points	
Perc End	Lo Prem	Hi Prem	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	1,031	116.0%	85.0%	132.0%	47.0%	84	78
25%	1,031	2,069	111.4%	95.6%	122.0%		63	50
35%	2,069	3,416	94.2%	94.0%	93.5%	-0.5%	58	55
45%	3,416	6,024	100.5%	102.2%	95.6%		74	39
55%	6,024	14,995	104.1%	94.4%	105.7%		71	43
75%	14,995	66,873	89.4%	87.8%	90.0%	2.2%	154	73
95%	66,873	231,342	90.3%	93.2%	85.4%	-7.8%	142	85
100%	231,342	594,515	70.2%	70.0%	70.0%		50	6
		Overall	95.6%	92.6%	99.4%	6.8%	696	429

There is limited credible data amongst the LOB-size groups. The positive difference for the smallest percentile LOB-size group is much larger than the overall difference.

Spec Liab LOB Size Group			----------RRF----------				Data Points	
Perc End	Lo Res	Hi Res	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	352	67.8%	123.0%	57.4%		58	50
25%	352	1,053	19.7%	15.5%	27.3%		64	29
35%	1,053	2,155	35.0%	29.5%	40.0%		51	48
45%	2,155	3,591	103.7%	93.9%	102.6%		55	45
55%	3,591	12,909	18.8%	21.9%	13.5%		70	33
75%	12,909	73,690	35.4%	37.3%	21.4%	-15.9%	153	71
95%	73,690	153,328	31.3%	45.1%	17.8%	-27.3%	139	86
100%	153,328	507,687	6.0%	9.2%	-1.7%		42	19
		Overall	38.9%	46.3%	32.6%	-13.8%	632	381

There is also limited credible data within LOB-size groups for the RRF differences. The smallest percentile LOB-size group, with only 50 data points had a large negative difference. This is one of the few lines with an overall negative difference.

A9. Other Liability

Other Liab - LOB Size Group			----------PRF----------				Data Points	
Perc End	Lo Prem	Hi Prem	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	481	137.0%	97.0%	200.0%	103.0%	666	630
25%	481	1,087	102.7%	84.5%	123.2%	38.7%	511	355
35%	1,087	2,008	107.9%	92.6%	125.8%	33.2%	579	292
45%	2,008	3,584	107.3%	95.5%	126.9%	31.3%	599	270
55%	3,584	10,389	102.4%	86.5%	123.5%	37.0%	616	254
75%	10,389	49,079	99.7%	96.3%	107.8%	11.5%	1,134	606
95%	49,079	210,786	99.7%	97.1%	104.6%	7.5%	1,278	462
100%	210,786	9,366,624	98.2%	94.1%	105.5%	11.4%	327	108
		Overall	104.3%	94.0%	122.5%	28.5%	5,710	2,977

This line showed the most consistency with respect to a significant difference between Elevated Re and Base Re amongst LOB-size groups. There also appears to be a fairly consistent inverse relationship between premium LOB-size and the PRF difference.

Other Liab LOB Size Group			----------RRF----------				Data Points	
Perc End	Lo Res	Hi Res	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	391	147.3%	160.0%	133.3%	-26.7%	601	497
25%	391	1,226	68.6%	64.4%	78.3%	13.9%	442	289
35%	1,226	2,530	45.3%	39.8%	52.7%	12.9%	478	255
45%	2,530	4,699	38.0%	34.3%	43.4%	9.1%	499	260
55%	4,699	17,931	29.9%	26.7%	34.8%	8.1%	518	252
75%	17,931	97,476	34.0%	30.5%	36.1%	5.6%	1,016	598
95%	97,476	526,700	61.9%	65.8%	52.0%	-13.7%	1,209	425
100%	526,700	23,638,870	72.3%	65.8%	96.5%	30.8%	301	124
		Overall	53.0%	52.4%	54.4%	2.0%	5,064	2,700

The RRF differences are not as consistent. Large negative differences for the first 15% and the 75-95% LOB-size group help make the overall difference rather small.

A10. Special Property

Spec Prop - LOB Size Group			----------PRF----------				Data Points	
Perc End	Lo Prem	Hi Prem	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	487	103.4%	82.7%	118.4%	35.7%	489	360
25%	487	931	76.6%	71.2%	91.0%	19.8%	388	191
35%	931	1,683	77.6%	76.0%	98.4%	22.4%	427	189
45%	1,683	2,913	81.5%	75.3%	101.7%	26.5%	381	207
55%	2,913	9,021	77.0%	73.5%	80.5%	7.0%	399	250
75%	9,021	36,266	79.1%	78.6%	79.6%	1.0%	595	485
95%	36,266	144,658	81.7%	83.3%	76.9%	-6.4%	640	276
100%	144,658	2,748,838	91.9%	83.6%	94.8%		83	47
		Overall	81.5%	78.1%	90.7%	12.7%	3,402	2,005

This line had fairly large PRF differences for the premium LOB-size groups within the first 50% and limited differences in the top 50%.

Special Property LOB Size Group			----------RRF----------				Data Points	
Perc End	Lo Res	Hi Res	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	46	109.4%	119.5%	105.2%	-14.3%	508	200
25%	46	107	46.7%	32.2%	89.6%	57.4%	328	143
35%	107	216	35.2%	27.5%	57.1%	29.6%	357	173
45%	216	391	30.0%	26.4%	33.9%	7.6%	349	194
55%	391	1,598	20.6%	12.1%	42.8%	30.7%	295	213
75%	1,598	8,722	20.3%	18.4%	24.3%	5.9%	518	449
95%	8,722	44,881	21.7%	30.4%	14.9%	-15.5%	497	276
100%	44,881	2,227,919	40.1%	41.9%	21.3%		78	43
		Overall	35.1%	31.9%	41.7%	9.8%	2,930	1,691

Large RRF differences in the middle percentile LOB-size groups offset the negative differences at the smallest and largest LOB-size groups.

A11. Auto Physical Damage

APD - LOB Size Group			----------PRF----------				Data Points	
Perc End	Lo Prem	Hi Prem	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	1,133	108.0%	101.3%	117.0%	15.7%	632	379
25%	1,133	2,445	89.9%	87.1%	96.6%	9.5%	436	177
35%	2,445	4,415	87.7%	86.6%	90.5%	4.0%	461	173
45%	4,415	7,293	85.8%	85.2%	88.7%	3.5%	456	205
55%	7,293	19,194	85.4%	84.9%	86.4%	1.5%	449	195
75%	19,194	91,334	85.4%	84.0%	88.3%	4.3%	854	198
95%	91,334	343,654	82.7%	82.7%	82.5%	-0.3%	578	79
100%	343,654	12,748,056	80.1%	80.2%	54.5%		142	4
		Overall	88.3%	86.3%	95.0%	8.7%	4,008	1,410

The PRF differences are mostly positive and below 10%. The smallest LOB-size group is near 16%, but the highest credible group has a small negative difference.

APD LOB Size Group			----------RRF----------				Data Points	
Perc End	Lo Res	Hi Res	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	73	56.9%	57.5%	54.3%	-3.2%	293	161
25%	73	150	44.6%	34.5%	53.6%	19.2%	216	92
35%	150	269	19.9%	7.0%	33.6%	26.6%	230	99
45%	269	469	11.4%	9.5%	36.0%	26.4%	237	73
55%	469	1,528	21.0%	13.8%	38.7%	24.8%	224	71
75%	1,528	7,161	9.1%	0.3%	25.5%	25.2%	425	122
95%	7,161	24,872	1.2%	0.1%	2.9%	2.8%	293	87
100%	24,872	3,404,975	9.7%	8.2%	9.8%		68	12
		Overall	19.3%	12.8%	38.1%	25.2%	1,986	717

The RRF differences are much more significant and for the most part positive except for the smallest LOB-size group.

A12. Fidelity and Surety

Fid & Sur - LOB Size Group			----------PRF----------				Data Points	
Perc End	Lo Prem	Hi Prem	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	848	99.0%	75.5%	115.0%	39.5%	99	52
25%	848	1,657	72.6%	24.2%	100.2%		69	26
35%	1,657	3,168	69.2%	44.0%	85.5%		68	38
45%	3,168	5,357	91.0%	75.8%	96.4%		79	38
55%	5,357	10,817	68.0%	69.6%	40.1%		73	21
75%	10,817	30,582	55.0%	48.8%	56.5%		143	47
95%	30,582	109,891	37.5%	36.3%	37.6%		150	17
100%	109,891	974,546	35.6%	35.6%			22	0
		Overall	65.5%	58.2%	90.4%	32.3%	703	239

There is limited data here but the smallest 50% of the LOB-size group data points have a much higher difference than the largest 50%.

Fid&Sur LOB Size Group			----------RRF----------				Data Points	
Perc End	Lo Res	Hi Res	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	848	303.6%	152.6%	400.0%		89	17
25%	848	1,657	48.7%	49.9%	26.6%		58	22
35%	1,657	3,168	160.2%	56.4%	194.5%		60	15
45%	3,168	5,357	67.8%	73.6%	52.0%		64	21
55%	5,357	10,817	127.3%	169.6%	74.5%		63	19
75%	10,817	30,582	43.6%	59.4%	22.7%		99	47
95%	30,582	109,891	31.9%	25.5%	72.2%		146	31
100%	109,891	974,546	52.3%	52.3%			20	0
		Overall	71.1%	68.5%	78.9%	10.4%	599	172

There are no credible LOB-size groups for the RRF differences.

A13. Other

Other - LOB Size Group			----------PRF----------				Data Points	
Perc End	Lo Prem	Hi Prem	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	1,052	111.4%	114.2%	92.9%		129	30
25%	1,052	2,105	91.6%	82.8%	99.0%		94	25
35%	2,105	4,638	88.2%	86.4%	88.3%		83	31
45%	4,638	8,326	85.5%	82.1%	113.7%		78	34
55%	8,326	24,267	89.1%	81.2%	103.8%		82	44
75%	24,267	88,823	98.1%	99.8%	83.5%	-16.3%	142	91
95%	88,823	243,019	88.1%	90.3%	74.4%	-16.0%	125	57
100%	243,019	2,477,354	90.0%	91.0%	74.7%		20	5
		Overall	96.0%	96.2%	94.8%	-1.4%	753	317

This is the only line with an overall indication (although small) that Base Re had a higher PRF than Elevated Re. The "Other" LOB includes credit and A&H business and the unusual pattern may be the result of that.

Other LOB Size Group			----------RRF----------				Data Points	
Perc End	Lo Res	Hi Res	Overall	Base	Elevated	Difference	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
15%	0	140	60.0%	47.2%	221.9%		99	38
25%	140	325	32.3%	38.1%	24.6%		80	21
35%	325	772	38.0%	38.0%	41.2%		88	24
45%	772	1,840	12.7%	9.3%	14.6%		67	36
55%	1,840	5,131	20.9%	15.9%	28.3%		62	31
75%	5,131	22,550	15.3%	12.3%	21.6%	9.3%	134	76
95%	22,550	60,856	16.4%	20.8%	-0.4%	-21.2%	114	66
100%	60,856	1,215,858	32.5%	40.3%	1.4%		24	3
		Overall	26.6%	27.3%	23.6%	-3.6%	668	295

The RRF also had a negative overall difference but the LOB-size group differences are erratic and mostly lack credibility.

A14. Remaining Lines

Both reinsurance lines lacked credibility within Elevated Re, consistent with a view that the extent of retrocessions is usually much smaller than regular reinsurance cessions.

The international, warranty, and financial guarantee lines also did not have sufficient data points to make any notable comments on LOB-size impacts.

APPENDIX B - REINSURANCE USAGE BASED ON SCHEDULE F

In the main part of this report we used Schedule P data to measure reinsurance usage. Schedule F is an alternative source of reinsurance usage data. From Schedule F we determined reinsurance usage as the ratio of (a) ceded reinsurance premium to non-affiliates divided by (b) the direct and assumed premium from non-affiliates. The premium is on a written basis and represents the cumulative total of all available years between 1996 and 2010 inclusive. The ratio includes all lines combined, so each LOB-AY data point within the risk data set for a single company is assigned the same percent reinsurance ceded.

Comparison of Ceded Reinsurance Percentage Measures

Useful features of the Schedule F view of reinsurance data set are that (a) it provided the premium data specifically involving reinsurance from non-affiliates and (b) it was constructed as an all-year view that avoids the effect of year-to-year fluctuations in ceded reinsurance levels. The later point is potentially useful for the RRF analysis which is based on reserve runoff ratios that include multiple accident years.

Useful features of the Schedule P view are that it is by LOB, by accident year,[14] including older years, back to 1988. Because Schedule P is adjusted for pooling, affiliate reinsurance in the form of pooling is not counted in the ceded reinsurance percentage even in the Schedule P view.

The use of affiliated reinsurance, which can be significant for some companies, is arguably, more a matter of internal capital structure and less a matter of managing underwriting risk within individual companies. Comparison of the schedule P view and the Schedule F view tests the extent to which including non-pool reinsurance activity amongst affiliates differentiate risk differences in companies that have high dependence on external reinsurance.[15]

Analysis and Findings

In the subsections below we compare the Schedule F results to the Schedule P results for (a) all lines/all sizes combined, (b) by company size, (c) by LOB and (d) by LOB/LOB-size.

[14] Schedule F would allow separation of ceded reinsurance percentages by calendar year, at least for years 1996-2010, but we did not make that separation in this study.

[15] There are some companies / pools in the risk data (from Report 6) that are not present in the Schedule F data. These include state workers compensation funds and liability joint underwriting associations (JUA's). We have thus eliminated these companies from the results and accumulations for this analysis. We eliminated companies which had negative or zero direct + assumed premium within the reinsurance data set. In summary, we had to eliminate 161 or 9.1% companies from the 1,778 companies in the PRF risk data set and 481 or 23.5% companies from the 2,040 company RRF risk data set.

All Lines Combined / All Company-Sizes Combined

From the summaries below, it appears that there is no material difference in the main indications, i.e., PRFs and RRFs increase with increasing use of reinsurance no matter the Separation Point between Base Re and Elevated Re.

There are some differences in the patterns. We observe that the Schedule F RRF differences are higher than Schedule P differences, but the reverse is the case for PRFs. The differences grow as the Separation Point increases. RRF differences using Schedule F reinsurance usage show similar high growth as the PRF which was not the case with the Schedule P data. Tables B.1 and B.2 compare the Schedule F results developed in this Appendix to the Schedule P results from Table 1.1, for PRFs and RRFs, respectively.

Table B.1
PRF - Comparison of Schedule P and Schedule F Analysis
All Lines Combined

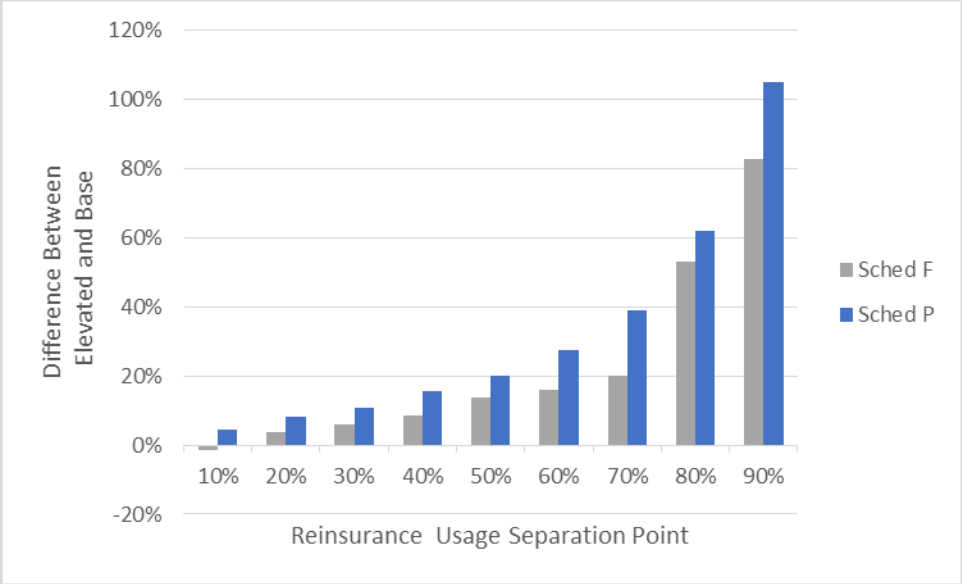
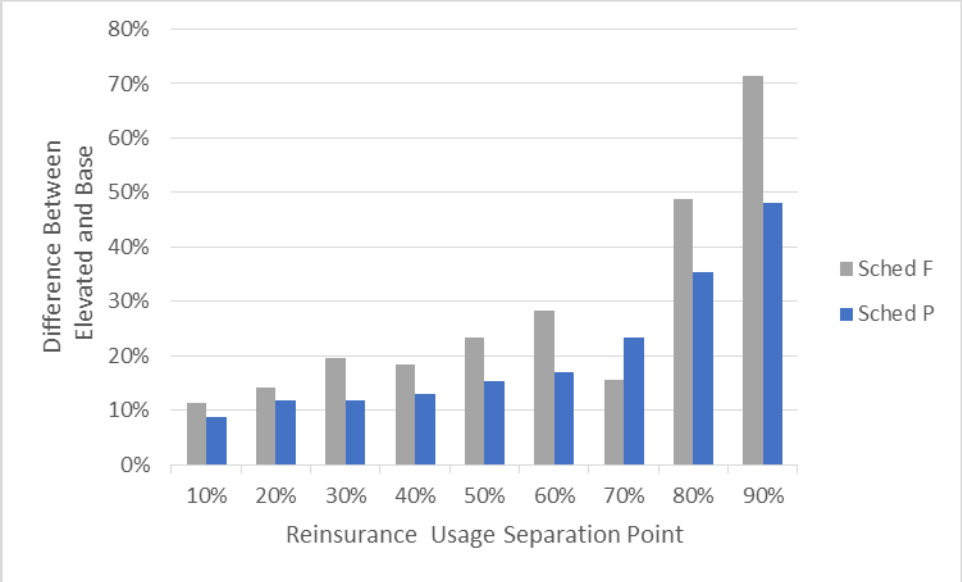


Table B.2
RRF - Comparison of Schedule P and Schedule F Analysis
All Lines Combined



All Lines Combined / By Company-Size

As in the Schedule P analysis, the Schedule F analysis shows evidence that the risk factor differences are greater for smaller companies than larger companies on an all-line basis, as shown in

Tables B.3 and B.4, corresponding to Tables 3.2a and 3.2b.

Table B.3
PRFs - Base Re versus Elevated Re
Schedule F Reinsurance Usage
By Company Size/All Lines Combined

Premium Vol Grps ($000's)		------ Reinsurance Usage ------				--- Data Counts ---				% Ceded	
From	To	All	Base	Elevated	Elev Less Base	Base	Elevated	Total	Companies	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
0	3,035	110.9%	108.7%	116.7%	8.0%	5,041	1,833	6,874	970	14.6%	89.1%
3,035	13,641	92.1%	91.2%	95.8%	4.5%	5,731	1,142	6,873	983	15.1%	56.8%
13,641	64,253	90.6%	90.6%	91.6%	1.0%	6,102	771	6,873	838	12.8%	57.9%
64,253	& greater	89.3%	89.2%	92.2%	3.0%	6,422	452	6,874	544	12.7%	88.5%
All groups		93.9%	92.8%	101.4%	8.6%	23,296	4,198	27,494	1,758	13.7%	74.7%

- A company can be counted in more than one size group.
- % Ceded statistics are simple averages over all data points.

Table B.4
RRFs - Base Re versus Elevated Re
Schedule F Reinsurance Usage
By Company Size/All Lines Combined

Reserve Vol Grps ($000's)		------ Reinsurance Usage ------				--- Data Counts ---				% Ceded	
From	To	All	Base	Elevated	Elev Less Base	Base	Elevated	Total	Companies	Base	Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
0	3,537	53.3%	48.5%	73.9%	25.4%	4,822	1,304	6,126	892	14.6%	60.5%
3,537	14,795	28.3%	26.9%	35.0%	8.1%	5,099	1,026	6,125	1,015	14.6%	61.6%
14,795	75,659	23.6%	21.9%	40.2%	18.3%	5,412	714	6,126	813	14.9%	65.8%
75,659	& greater	22.5%	21.4%	35.6%	14.3%	5,681	443	6,124	485	12.7%	75.0%
All groups		29.6%	27.1%	45.5%	18.4%	21,014	3,487	24,501	1,896	14.2%	63.8%

- A company can be counted in more than one size group.
- % Ceded statistics are simple averages over all data points.

Analysis by LOB and LOB-Size

As in the Schedule P analysis, the Schedule F analysis shows that PRFs and RRFs increase with reinsurance usage across most LOBs. This is shown in Tables B.5 and B.6 below, corresponding to Tables 3.3a and 3.3b.

Table B.5 – PRF
Schedule F Reinsurance Usage
Base Re vs. Elevated Re by LOB

LOB	Sch P Line	PRF Base	PRF Elevated	Difference: Elev - Base	By Size Range	Credible Size Frequency of Positive Diff.	No. of Data Points Base	No. of Data Points Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Homeowners	A	96.9%	106.5%	9.6%	-7.9% to 16.1%	4 out of 7	7,547	1,667
PPA	B	98.4%	112.1%	13.7%	-2.8% to 9.5%	5 out of 6	7,670	753
CAL	C	99.5%	113.4%	13.9%	-5.7% to 9.8%	5 out of 6	5,183	678
WC	D	104.1%	112.5%	8.4%	-23.2% to 12.4%	3 out of 6	5,257	538
CMP	E	88.3%	102.6%	14.3%	-1.6% to 22.3%	6 out of 7	5,983	992
MEDMAL - OCC	F1	152.3%	147.5%	-4.8%	0.0% to 0.0%	0 out of 0	960	40
MEDMAL - CM	F2	118.6%	115.4%	-3.3%	0.0% to 0.0%	0 out of 0	2,215	196
Spec Liab	G	94.3%	94.8%	0.5%	-13.3% to -13.3%	0 out of 1	875	199
Other Liab	H	100.3%	125.7%	25.4%	3.2% to 37.0%	7 out of 7	6,903	1,484
Spec Property	I	80.7%	95.5%	14.8%	0.1% to 54.9%	8 out of 8	7,127	1,515
Auto Phys Dam	J	84.5%	91.5%	7.1%	-2.5% to 8.3%	4 out of 7	8,394	947
Fidelity & Surety	K	65.8%	81.4%	15.6%	0.0% to 0.0%	0 out of 0	1,203	217
Other	L	93.8%	96.6%	2.7%	-13.2% to -13.2%	0 out of 1	1,433	214
International	M					Sufficient data not available		
Non Prop Reins - Prop/Fin	N&P	129.4%	145.8%	16.4%			889	88
Non Prop Reins Casualty	O	134.2%	119.4%	-14.9%			897	112
Product Liability	R	109.8%	190.6%	80.8%			546	99
Financial Guarantee	S					Sufficient data not available		
Warranty	T					Sufficient data not available		

Table B.6 – RRF
Schedule F Reinsurance Usage
Base Re vs. Elevated Re Usage by LOB

LOB	Sch P Line	RRF Base	RRF Elevated	Difference: Elev - Base	By Size Range	Credible Size Frequency of Positive Diff.	No. of Data Points Base	No. of Data Points Elevated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Homeowners	A	21.7%	48.1%	26.4%	-3.2% to 36.5%	5 out of 7	6,104	1,022
PPA	B	18.6%	49.3%	30.6%	-7.3% to 26.5%	6 out of 7	7,441	861
CAL	C	36.0%	70.3%	34.4%	-0.5% to 71.1%	6 out of 7	5,376	802
WC	D	34.6%	46.6%	12.0%	-23.8% to 63.5%	5 out of 7	5,873	744
CMP	E	51.6%	65.2%	13.6%	-8.1% to 26.0%	4 out of 7	5,717	867
MEDMAL - OCC	F1						1,091	26
MEDMAL - CM	F2	13.8%	12.0%	-1.9%	0.0% to 0.0%	0 out of 0	2,175	165
Spec Liab	G	49.2%	27.2%	-22.0%	-30.2% to -30.2%	0 out of 1	898	194
Other Liab	H	54.3%	66.5%	12.2%	-49.8% to 56.6%	4 out of 7	6,438	1,442
Spec Property	I	39.9%	43.4%	3.5%	-123.2% to 27.0%	3 out of 8	4,551	959
Auto Phys Dam	J	23.3%	22.0%	-1.3%	-23.7% to 4.3%	2 out of 5	3,551	478
Fidelity & Surety	K	72.4%	46.2%	-26.2%			760	158
Other	L	37.2%	59.1%	22.0%	-48.3% to -48.3%	0 out of 1	997	205
International	M					Sufficient data not available		
Non Prop Reins - Prop/Fin	N&P	44.6%	50.8%	6.3%		0 out of 0	982	57
Non Prop Casualty	O	68.8%	43.5%	-25.3%			1,038	134
Product Liability	R	71.1%	267.8%	196.7%			519	99
Financial Guarantee	S					Sufficient data not available		
Warranty	T					Sufficient data not available		

Tables B.5 and B.6 shows that the risk factor differences for LOB-size groups show a wider range of indications than with the Schedule P data. Also note that the number of Elevated Re data points for medical malpractice – occurrence in the Schedule F view is below 50.

As shown in Tables B.7 and B.8, corresponding to Tables 3.4a and 3.4b, there is also only a slight indication of a decrease in the difference as the LOB-size increases.

Table B.7 – PRF
Schedule F Reinsurance Usage
Base Re vs. Elevated Re by LOB Size Group

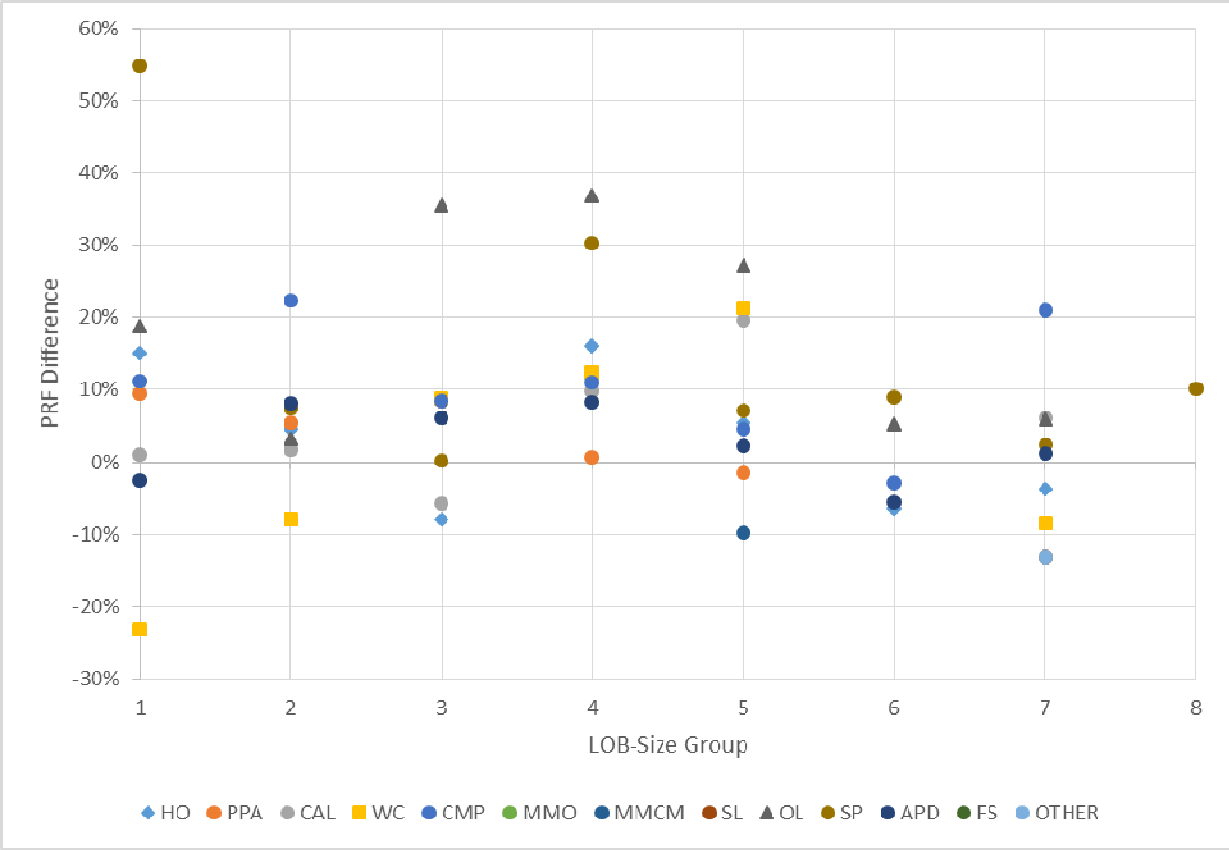
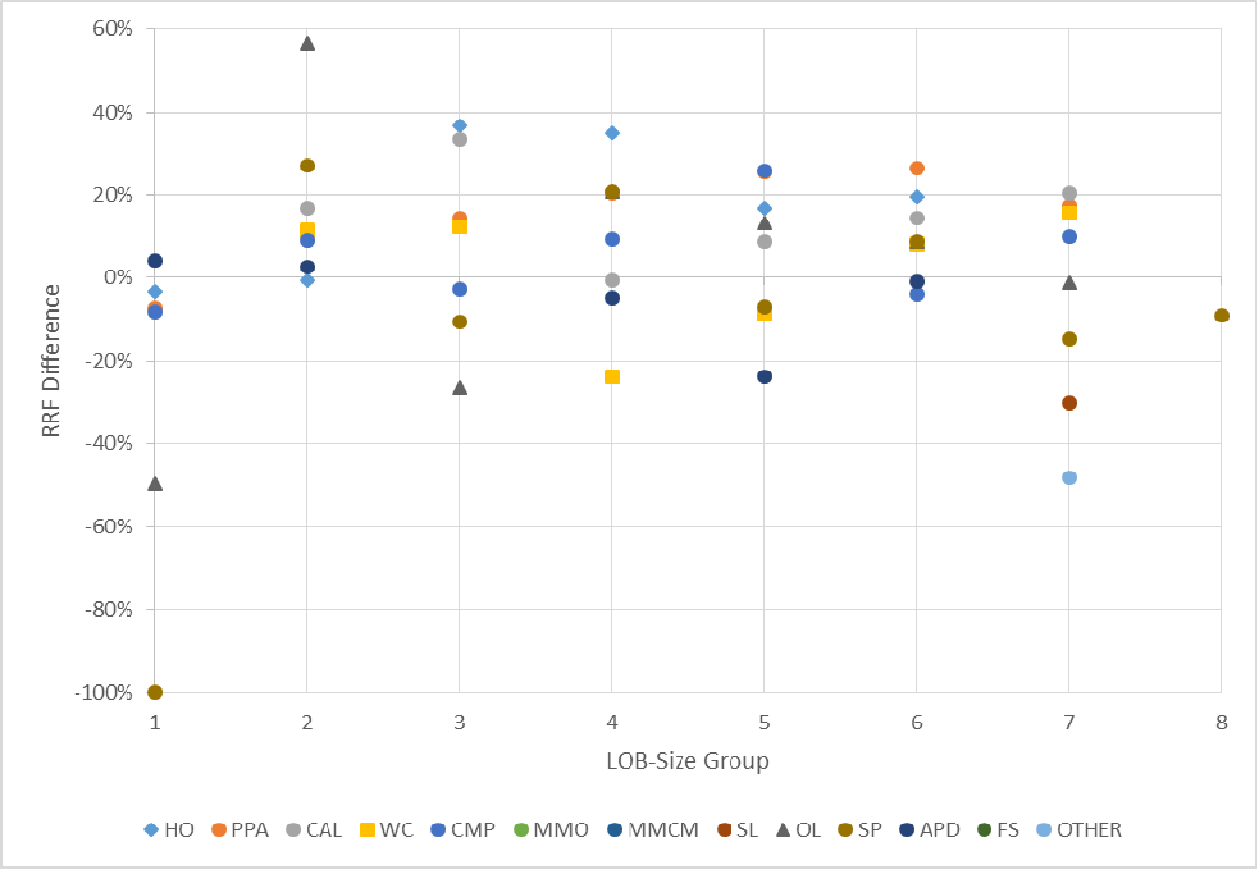


Table B.8 – RRF
Schedule F Reinsurance Usage
Base Re vs. Elevated Re by LOB Size Group



Note that the difference for Special Property at the smallest LOB-size group was capped at -100%. The actual indication was -123%.

Table B.9 below summarizes the findings from our various segments of the risk data using the Schedule F reinsurance usage indication.

Table B.9
Summary of Findings
PRF and RRF for Base Re versus Elevated Re
Schedule F Reinsurance Usage

Table(s)	Data Group	Findings
B.1-B.2	All lines combined	PRF/RRF differences grow as the Separation Point between Elevated Re versus Base Re grows. At the 40% Separation Point, the comparison of Elevated Re to Base Re is: - PRF is 8.6% of NEP higher - RRF is 18.4% of initial reserves higher
B.3-B.4	All lines combined By company-size	Elevated Re PRFs/RRFs are higher than Base Re PRFs/RRFs across all size groups. The smallest companies have the largest differences.
B.5-B.6	By LOB All sizes combined	PRFs and RRFs are generally higher with Elevated Re than Base Re. Differences are more variable across lines than from the Schedule P analysis. As Schedule F reinsurance is not by LOB, the Schedule F ceded reinsurance might not be a sufficiently close match to the risk points.
B.5-B.6 and B.7-B.8	LOB; by LOB-size group	Elevated Re PRFs/RRFs are generally higher than Base Re PRFs/RRFs across all size groups, with many variations. The Schedule F pattern is less consistent than was the case in the Schedule P analysis.

Appendix C - Risk Data Filtering

"Two-Year" Lines

We filtered out data points in which there was a significant inconsistency in the AY NEP between the risk data and the Schedule P data. These inconsistencies are all in the "two-year" lines.[16]

These inconsistencies arise because the risk data is obtained from the NAIC 10 year RBC

[16] Special Property, Auto Physical Damage, Fidelity and Surety, and Other

reporting of "two-year" lines; the reinsurance usage data is from the two years of data publically available in Annual Statements. We looked at the effect of including or excluding these points but the impact to our observations was deemed negligible.

The following Table C.1 summarizes the population of the risk data set and what was eliminated from this analysis but what was used in Reports 6 and 7:

Table C.1 – Summary of Data Point Filtering

	PRF Data Points	**Premium ($000s)**	**RRF Data Points**	**Net Reserve ($000s)**
Report 6 and 7 Starting Point	75,515	6,471,209	71,352	7,682,622
No Matching % Ceded Info	6,502	427,704	10,787	725,659
Schedule P Data Eliminated -- negative direct or net premium -- cessions at or greater than 100%	278	12,370	1,680	54,006
Inconsistent Premium	2,365	645,326	1,142	85,861
Data Points Used	66,370	5,385,809	57,743	6,817,095

The line-by-line effect is summarized in Table C.2 below.

Table C.2
Line by Line Summary of Inconsistent Data Points

LOB	PRF Inconsist. Data Point	Percent of LOB Total	Associated Net EP (000s)	Percent of LOB Total
I - Special Property	795	8.9%	182,369	54.3%
J - Auto Physical Damage	1,326	14.0%	406,393	36.3%
K - Fidelity and Surety	108	7.5%	15,290	37.9%
L - Other	136	8.0%	41,274	40.0%
Total	2,365	10.5%	645,326	38.5%

LOB	RRF Inconsist. Data Point	Percent of LOB Total	Associated Net EP (000s)	Percent of LOB Total
I - Special Property	518	7.3%	61,658	62.5%
J - Auto Physical Damage	509	11.2%	10,521	18.1%
K - Fidelity and Surety	49	4.2%	3,901	33.0%
L - Other	66	4.5%	9,781	33.5%
Total	1,142	7.3%	85,861	28.8%

Company Counts

Since we are counting data points for each line of business / accident year / company combination, a company can contribute multiple data points. Thus it is worth noting, that there are a total of 1,778 companies in our PRF data set, including 204 pooled entities, and 2,040 companies in the RRF data set, including 210 pooled entities.

For the PRF data set, we used only 1,758 of these companies for the all lines analysis in that 20 companies had no data points where the Schedule P direct or NEP was greater than 0 or the ceded percentage was less than 100%.

For the RRF data set, we used only 1,904 of these companies for the all lines analysis in that 136 companies had no data points where the Schedule P direct or NEP was greater than 0 or the ceded percentage was less than 100%.

It is also worth noting that some LOBs had too few data points within Elevated Re from which to draw credible observations. Because of this, we have removed the LOB indications resulting from international, warranty, and financial guarantee.

APPENDIX D - TEST OF IMPACT OF MATURITY IMPACT

DCWP has observed that the maturity of data points can affect the PRF and RRF indications.

In order to test the extent to which maturity age filtering might have an impact to our observations regarding risk factors and reinsurance usage, we compared the results presented in Sections 3.1 through 3.3 to the results produced when the risk data sets only contained data points that had at least four years of maturity from the start of the most recent accident year or from the initial reserve date. The differences in the Elevated Re versus Base Re risk factors are shown in the tables below.

As shown in the Tables below, there does not appear to be appreciable difference in the indications such that we would change our findings regarding the impact of reinsurance usage on the premium and reserve risk factors. The reinsurance split graphs similar to Table 3.1 show little difference when the maturity filter is applied. The all line by size segmentation similar to Tables 3.2a and 3.2b has more noticeable differences but nothing that would indicate a consistent shift in the differences. The LOB differences, as originally show in Tables 3.3a and 3.3b, did show some significant change but effectively only in the LOB's which had high differences and volatile indications (i.e., medical malpractice occurrence and product liability).

Table D.1
PRF - Comparison of All Maturity to 4 Years and Older Maturity
Schedule P Reinsurance Usage
All Lines Combined

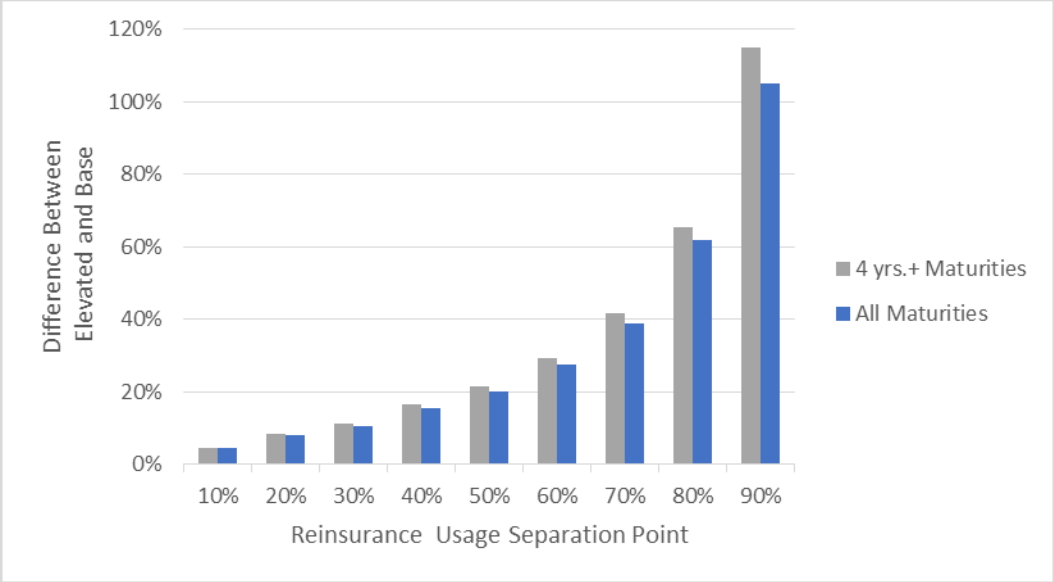


Table D.2
RRF - Comparison of All Maturity to 4 Years and Older Maturity
Schedule P Reinsurance Usage
All Lines Combined

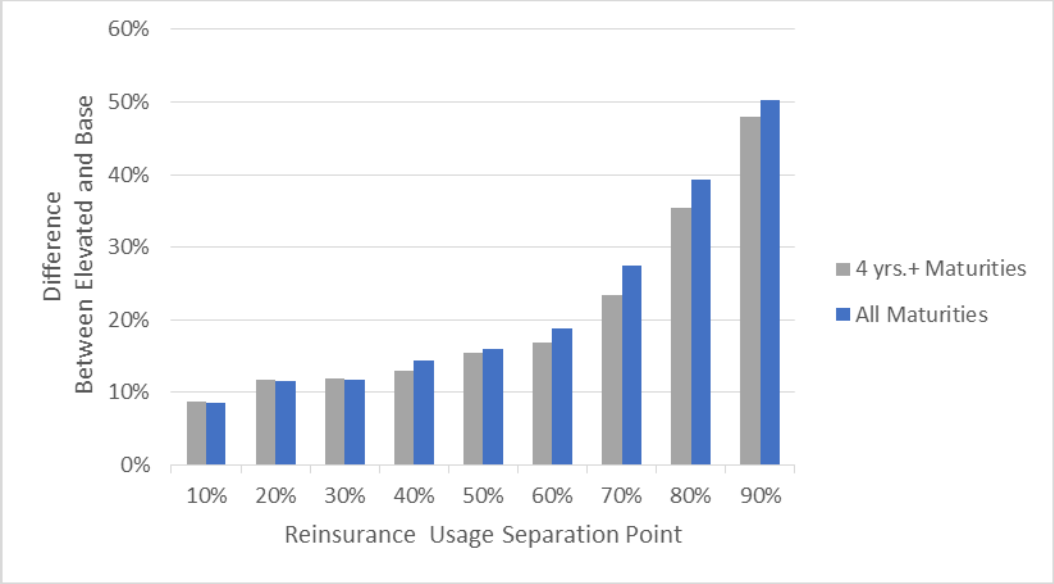


Table D.3
PRF – Comparison by Size of All Maturity to 4 Years and Older Maturity
Schedule P Reinsurance Usage
All Lines Combined

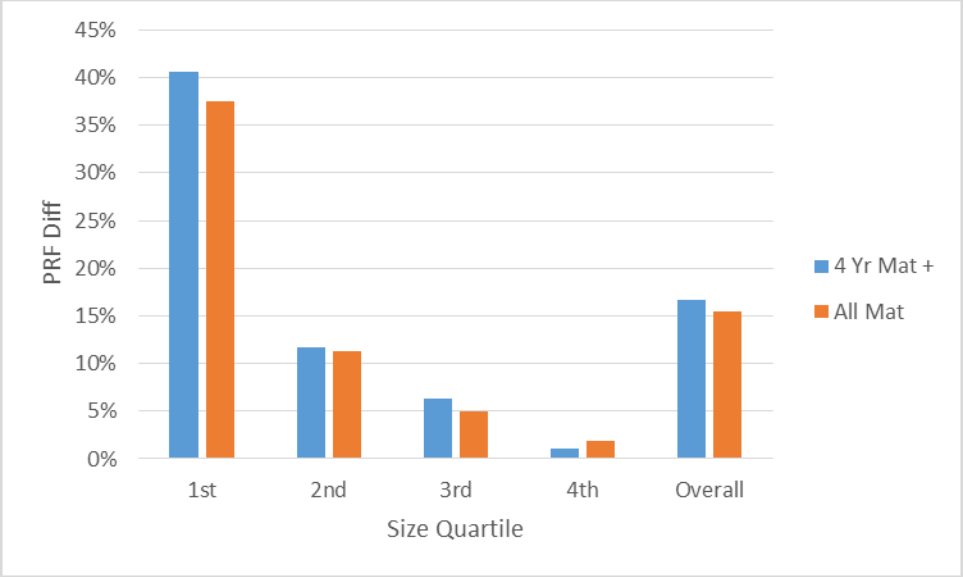


Table D.4
RRF – Comparison by Size of All Maturity to 4 Years and Older Maturity
Schedule P Reinsurance Usage
All Lines Combined

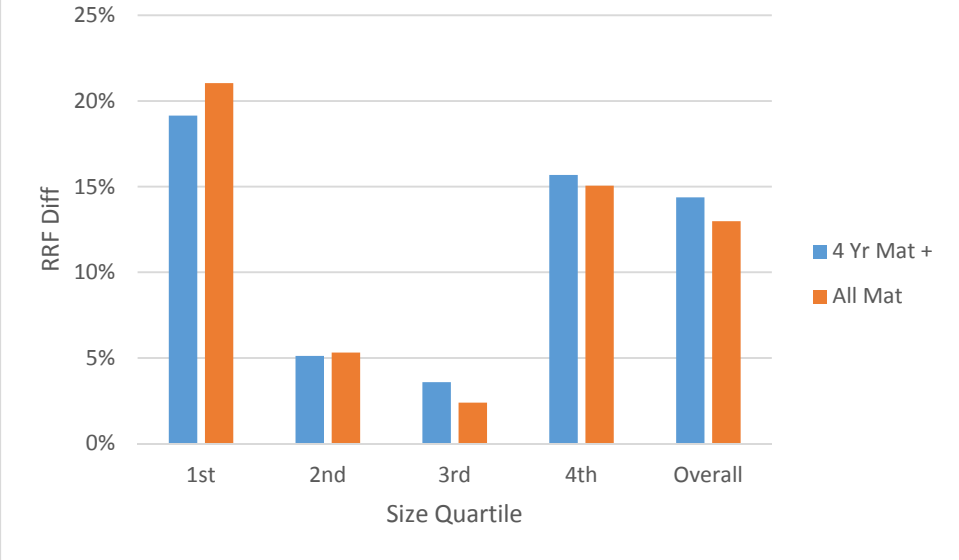


Table D.5
PRF – Comparison of All Maturity to 4 Years and Older Maturity
Schedule P Reinsurance Usage
By Line

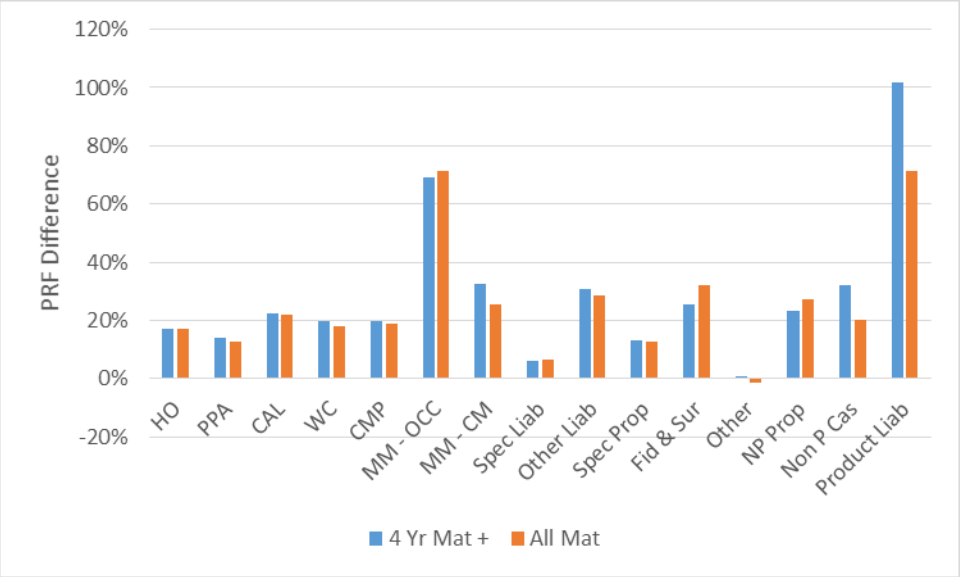
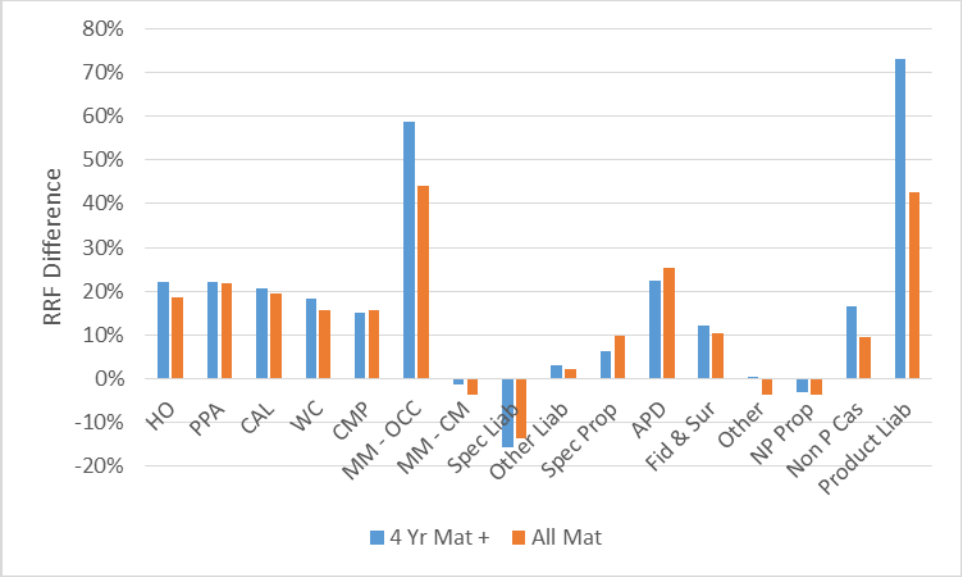


Table D.6
RRF – Comparison of All Maturity to 4 Years and Older Maturity
Schedule P Reinsurance Usage
By Line



On the Use of Stock Index Returns from Economic Scenario Generators in ERM Modeling

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Abstract

The modeling of insurance company enterprise risks requires correlated forecasts of the future values of various economic variables. These forecasts, especially as they pertain to interest rates, inflation, stock market performance and other economic variables needed for asset modeling, are typically obtained from an economic scenario generator (ESG). With respect to stock market performance, third-party ESGs generally provide forecast returns for various market indexes; the output is not tailored to reflect an insurer's own equity portfolio composition. For that reason, ESG stock return scenarios cannot be used for insurer ERM modeling without adjustment to reflect the insurer's own equity portfolio composition and idiosyncratic risk. This paper describes two methods for making the necessary adjustment to the ESG market return scenarios based on the assumption of normally-distributed 1) arithmetic returns, and 2) logarithmic returns.

Keywords. CAPM; correlated sampling; economic scenario generator; enterprise risk management; stochastic modeling.

1. INTRODUCTION

An economic scenario generator (ESG) produces a set of future scenarios for the values of economic and investment variables. The scenarios, typically stochastically generated, are intended to be mutually coherent, meaning that they reflect realistic dependencies and correlations among the modeled variables. The list of modeled variables from commercial ESGs is extensive; the key variables for risk modeling in non-life insurance include interest rate term structures, credit spreads, inflation, equity returns and other economic variables of interest to insurers. For a general overview of economic scenario generators, see Ahlgrim, D'Arcy and Gorvett [1].

The sole focus of this paper is on ESG *equity return* scenarios and how to use them appropriately in insurance company enterprise risk management (ERM) stochastic modeling. The future equity returns modeled by an ESG generally pertain to one or more stock market indexes representing the broad market, e.g., the S&P 500 in the U.S. and comparable international indexes elsewhere. ESG data sets are not typically tailored to reflect an insurer's own equity portfolio composition.

If an insurer holds a common stock portfolio that exactly matches one of the equity indexes for which an ESG provides modeled future market returns, then the ESG output can

be used without adjustment to model the risk and return of the insurer's own portfolio. However, unless the insurer invests solely in index funds, it is unlikely that the match would be exact.

In the more likely case in which the insurer's portfolio deviates from the ESG's index portfolio, the ESG returns must be adjusted to reflect the characteristics of the insurer's own equity portfolio before they can be used to model the risk in that portfolio.

2. ADJUSTING ESG EQUITY RETURNS TO REFLECT INSURER PORTFOLIO CHARACTERISTICS

One method for making the necessary adjustment to ESG equity return output is based on the Capital Asset Pricing Model (CAPM),[1] which expresses the expected return $E(r_i)$ on capital asset i (in our case the insurer's equity portfolio) as a function of the risk-free return r_f, the expected market return $E(r_M)$ and a factor β_{iM} (known as "beta"):

$$E(r_i) = r_f + \beta_{iM} \cdot (E(r_M) - r_f), \tag{2.1}$$

where

$$\beta_{iM} = \frac{\sigma_{iM}}{\sigma_M^2} = \rho_{iM} \cdot \frac{\sigma_i}{\sigma_M}, \tag{2.2}$$

and σ_{iM} and ρ_{iM} are, respectively, the covariance and correlation coefficient of the portfolio return with the market return, σ_M is the standard deviation of the market return and σ_i is the standard deviation of the portfolio return.[2]

β_{iM} is frequently described, especially in the non-academic press, as a measure of the volatility of a portfolio relative to the market. For example, according to the Bloomberg terminal guide posted on numerous university websites (and widely echoed elsewhere),

> You can think of beta as the tendency of a security's returns to respond to swings in the market. For example, if a stock's beta is 1.2, then it is theoretically 20% more volatile than the market.[3]

---

[1] While there is much criticism of the validity of CAPM, there is also widespread acceptance among and active use by financial professionals. See the CAPM discussion in Bodie, Kane and Marcus [2], particularly pp. 299-300.

[2] Strictly speaking, these should all be defined with respect to the market and portfolio returns *excess* of the risk-free rate, but they are generally expressed in terms of the total rather than excess returns. The origin of that convention is probably due to the fact that, if r_f is a constant, then there is no difference. That is not true if r_f is regarded as variable, but in most practical situations the difference is likely to be too small to matter.

[3] Excerpt from the Bloomberg Guide posted on the Brigham Young University website at the following link: http://guides.lib.byu.edu/content.php?pid=53518&sid=401576. The same language also appears on a number of other university websites, all of which appear to license the Bloomberg terminal. Similar language is widely used elsewhere. See, for example, http://www.thestreet.com/topic/46048/beta.html.

2.1 Using β_{iM} to Adjust ESG Market Returns – An Incomplete Solution

Given the conception of β_{iM} as a measure of volatility as expressed at the end of the previous section, it is tempting to model portfolio returns excess of the risk-free rate by simply multiplying the excess portion of the market return $\hat{r}_M$ obtained from the ESG by the portfolio beta β_{iM}:

$$\hat{r}_i - r_f = \beta_{iM} \cdot (\hat{r}_M - r_f), \tag{2.3}$$

where $\hat{r}_i$ represents the modeled portfolio total return, the idea being that the resulting $\hat{r}_i$ values would be used to represent the insurer's equity returns in its ERM modeling .

However, *that would be wrong*, at least if one stopped there. Despite Bloomberg's description, β_{iM} does not fully capture the volatility of the portfolio return, which is *actually* given by σ_i. What β_{iM} captures is only the portion of volatility that is correlated with the market. To see this, consider the following.

The standard deviation of the modeled portfolio excess return from Formula (2.3) is:

$$\sigma(\hat{r}_i - r_f) = |\beta_{iM}| \cdot \sigma(\hat{r}_M - r_f), \tag{2.4}$$

which, if we treat r_f as a constant, simplifies to

$$\begin{aligned} \sigma(\hat{r}_i) &= |\beta_{iM}| \cdot \sigma_M \\ &= \left|\rho_{iM} \cdot \frac{\sigma_i}{\sigma_M}\right| \cdot \sigma_M \\ &= |\rho_{iM}| \cdot \sigma_i. \end{aligned} \tag{2.5}$$

Formula (2.5) shows that using Formula (2.3) to model portfolio returns by simply multiplying the ESG market excess returns by β_{iM} and adding back the risk-free rate results in the understatement of portfolio return volatility whenever $|\rho_{iM}| < 1$.

To illustrate the consequences for ERM modeling, consider the following scenario in which $r_f = 3\%$, $E(r_M) = 7\%$, $\beta_{iM} = 0.9$, $\rho_{iM} = 0.8$, $\sigma_M = 20\%$ and $\sigma_i = 22.5\%$. In this scenario Formula (2.3) implies an expected portfolio return $E(r_i)$ of 6.6%. If equity returns

are normally distributed, the one-year portfolio value-at-risk (VaR) at the 99.5% level is given by:[4]

$$VaR(r_i)_{0.995} = -(E(r_i) + N^{-1}(0.005) \cdot \sigma_i) \tag{2.6}$$

$$= -(6.6\% - 2.576 \cdot 22.5\%)$$

$$= 51.36\%$$

In contrast, if we were to model the portfolio returns according to Formula (2.3), the value of σ_i would be replaced by $|\rho_{iM}| \cdot \sigma_i$ and the calculated $VaR(r_i)_{0.995}$ would be only 39.77%, which is 11.59 points (or nearly one-quarter) *lower than the correct amount*.

2.2 Using β_{iM} to Adjust ESG Market Returns – More Complete Solution

It is possible to repair the erroneous Formula (2.3) by adding an independent error term ϵ_i with a mean of zero that reflects the portfolio's idiosyncratic risk, i.e., the extent to which it displays additional variation not explained by market movements:

$$\hat{r}_i - r_f = \beta_{iM} \cdot (\hat{r}_M - r_f) + \epsilon_i, \tag{2.7}$$

in which case the corrected formula for the portfolio total return is then given by:

$$\hat{r}_i = r_f + \beta_{iM} \cdot (\hat{r}_M - r_f) + \epsilon_i. \tag{2.8}$$

In order for the required relationship of $\sigma(\hat{r}_i) = \sigma_i$ to hold, the standard deviation of the error term $\sigma(\epsilon_i)$ must be:

$$\sigma(\epsilon_i) = \sqrt{1 - \rho_{iM}^2} \cdot \sigma_i. \tag{2.9}$$

Under these conditions, and again treating r_f as a constant, the standard deviation of the modeled portfolio return $\sigma(\hat{r}_i)$ consistent with Formula (2.8) now matches σ_i as required:

$$\sigma(\hat{r}_i) = \sqrt{0^2 + |\rho_{iM}|^2 \cdot {\sigma_i}^2 + (1 - \rho_{iM}^2) \cdot {\sigma_i}^2}$$

$$= \sqrt{{\rho_{iM}}^2 + (1 - \rho_{iM}^2)} \cdot \sigma_i = \sigma_i.$$

[4] Value-at-risk is a downside risk measure. VaR amounts are generally shown as positive numbers even though they signify losses. For that reason, in Formula (2.6) we must change the sign on the 0.5th percentile result from the return distribution (in which losses are negative numbers). The 99.5th percentile of the resulting distribution is $VaR(r_i)_{0.995}$.

2.2.1 Normal arithmetic return assumption

Let's assume that equity returns r_M and r_i as well as the error term ϵ_i in Formula (2.8) all are normally distributed, and that r_M is modeled accordingly within the ESG.

Then, given a value of $\hat{r}_M$ drawn from the ESG, we can generate a related portfolio return $\hat{r}_i$ as follows:

$$\hat{r}_i = r_f + \beta_{iM} \cdot \left(\hat{r}_M - r_f\right) + z \cdot \sqrt{1 - \rho_{iM}^2} \cdot \sigma_i, \tag{2.10}$$

where z represents a random draw from the standard normal distribution.

To illustrate the application of Formula (2.10), let's again assume the risk-free rate $r_f = 3\%$, $\beta_{iM} = 0.9$, $\rho_{iM} = 0.8$ and $\sigma_i = 22.5\%$. Under those conditions Formula (2.10) simplifies to:

$$\hat{r}_i = 3\% + 0.9 \cdot (\hat{r}_M - 3\%) + 0.135 \cdot z. \tag{2.11}$$

Next, we obtain a market return value $\hat{r}_M$ from the ESG and randomly draw a standard normal random number z. Let's say $\hat{r}_M = 12\%$ and $z = -0.8416$. Then Formula (2.11) yields:

$$\hat{r}_i = 11.10\% + 0.135 \cdot (-0.8416)$$

$$= -0.26\%.$$

A different draw of z would, of course, result in a different value of $\hat{r}_i$. For example, if $\hat{r}_M = 12\%$ and $z = 0.4823$, Formula (2.11) yields:

$$\hat{r}_i = 11.10\% + 0.135 \cdot (0.4823)$$

$$= 17.61\%.$$

Note that Formula (2.10) requires little explicit information about the ESG's market return random variable r_M. All that is needed from the ESG is the market return observation $\hat{r}_M$. For further insight into Formula (2.10), see Section (A.1) of the Appendix.

2.2.2 Normal logarithmic return assumption

As an alternative to the assumption that arithmetic equity returns are normally distributed, market practitioners sometimes use a lognormal model in which *logarithmic* returns (or excess returns) are normally distributed. If the arithmetic market return, for

example, is $\hat{r}_M$, the corresponding logarithmic return is given by the expression $\ln(1+\hat{r}_M)$. In a lognormal return model, $\ln(1+r_M)$ is normally distributed, but r_M is not.

If we replace the arithmetic excess returns in Formula (2.10) with the corresponding logarithmic returns and the arithmetic return standard deviation σ_i with the standard deviation of the logarithmic return $\sigma_i(LN)$, and then isolate $\hat{r}_i$, we obtain:[5] [6]

$$\ln(1+\hat{r}_i-r_f)=\beta_{iM}\cdot\left(1+\hat{r}_M-r_f\right)+z\cdot\sqrt{1-\rho_{iM}^2}\cdot\sigma_i(LN), \tag{2.12}$$

$$\hat{r}_i=r_f+\exp\left(\beta_{iM}\cdot\ln\left(1+\hat{r}_M-r_f\right)+z\cdot\sqrt{1-\rho_{iM}^2}\cdot\sigma_i(LN)\right)-1 \tag{2.13}$$

For additional insight into Formula (2.13), see Section (A.2) of the Appendix.

We illustrate the application of Formula (2.13) using the same values for r_f, β_{iM}, ρ_{iM} and σ_i assumed in Section 2.2.1, which together imply a value for the lognormal sigma parameter $\sigma_i(LN)$ of 21.47%. Given the same ESG market return of $\hat{r}_M=12\%$ and the same $z=-0.8416$, the lognormal model yields a slightly different value for the portfolio return $\hat{r}_i$ of -0.04% (vs. -0.26%):[7]

$$\hat{r}_i=3\%+\exp(0.9\cdot\ln(1.09)+(-0.8416)\cdot(0.6)\cdot(21.47\%))-1$$

$$\hat{r}_i=3\%+\exp(7.755\%-10.841\%)-1$$

$$=-0.04\%.$$

For the second draw of $z=0.4823$ used in Section 2.2.1, with everything else being equal, Formula (2.12) produces the following value of $\hat{r}_i$:

$$\hat{r}_i=3\%+\exp(7.755\%+6.213\%)-1$$

$$=17.99\%.$$

[5] As a first step the r_f term is moved from the right side to the left side of the equal sign in Formula (2.10).

[6] $\sigma_i(LN)=\text{sqrt}[\ln(CV^2+1)]$, where $CV=\sigma_i/(1+E(r_i)-r_f)$ refers to the coefficient of variation. Strictly speaking, the values of β_{iM}, ρ_{iM} should also be recalculated, but the differences are typically small and thus we omit that step from our illustration.

[7] The insurer's ERM team must make a decision about whether the normal or lognormal return model is more appropriate, in light of what is known about the characteristics of the ESG's market equity return distribution.

3. SUMMARY

In this paper we have demonstrated that ESG data as it pertains to equity market index returns cannot be used without adjustment in the modeling of an insurer's enterprise risks, except in that rare instance in which the insurer only invests in equity indexes. Having identified that problem, we have described two methods for making the necessary adjustment, one based on the assumption that arithmetic equity returns are normally-distributed and the other based on the assumption that logarithmic equity returns are normally distributed. The decision about which, if any, of these two methods is most appropriate depends on the distributional assumptions underlying the ESG supplying the market return scenarios.

APPENDIX

A.1 Normal Arithmetic Return Model – Additional Information

If we assume that the excess return $r_M - r_f$ is normally distributed with mean:

$$\mu_{MX} = E(r_M) - r_f, \tag{A. 1}$$

and standard deviation σ_M, then the market return scenario $\hat{r}_M$ can be expressed as a function of the standard normal random number z_1 corresponding to that scenario:

$$\hat{r}_M = r_f + \mu_{MX} + \sigma_M \cdot z_1, \tag{A. 2}$$

and Formula (2.10) from Section 2.2.1 can be restated in terms of independent standard normal random numbers z_1 and z_2 (where z_2 is the same as the z in Formula (2.10) but now with a subscript to distinguish it better from z_1):

$$\hat{r}_i = r_f + \beta_{iM} \cdot (\mu_{MX} + z_1.\sigma_M) + z_2 \cdot \sqrt{1 - \rho_{iM}^2} \cdot \sigma_i,$$

$$= r_f + \beta_{iM} \cdot \mu_{MX} + z_1 \cdot \rho_{iM} \cdot \sigma_i + z_2 \cdot \sqrt{1 - \rho_{iM}^2} \cdot \sigma_i. \tag{A. 3}$$

We can write Formula (A.3) more succinctly as:

$$\hat{r}_i = r_f + \mu_{iX} + z_2^* \cdot \sigma_i, \tag{A. 4}$$

where

$$z_2^* = z_1 \cdot \rho_{iM} + z_2 \cdot \sqrt{1 - \rho_{iM}^2}, \tag{A. 5}$$

is a standard normal random number correlated with z_1,[8] and

$$\mu_{iX} = \beta_{iM} \cdot \mu_{MX}. \tag{A.6}$$

Circling back to the illustration in Section 2.2.1, let's assume the mean market excess return embedded in the ESG is $\mu_{MX} = 4\%$. From our other assumptions we can infer $\sigma_M = 20\%$ by using Formula (2.2). Then, given ESG market return scenario $\hat{r}_M = 12\%$, we can use Formula (A.3) to solve for the corresponding value of z_1:

$$z_1 = \frac{(\hat{r}_M - r_f) - \mu_{MX}}{\sigma_M} \tag{A.7}$$

$$= \frac{9\% - 4\%}{20\%}$$

$$= 0.25.$$

Given $z_2 = -0.8416$, we can first use Formula (A.5) to obtain the value of z_2^*=0.0864 and then Formula (A.4) to calculate the portfolio return $\hat{r}_i = -0.26\%$ (matching the result in Section 2.2.1) as follows:

$$z_2^* = 0.25 \cdot 0.8 + (-0.8416) \cdot 0.6$$

$$= -0.3050.$$

$$\hat{r}_i = 6.6\% + (-0.3050) \cdot 22.5\%$$

$$= -0.26\%.$$

As we have just illustrated, Formula (A.4), like Formula (2.10), gives us the means to generate the correlated portfolio return $\hat{r}_i$ in cases where we have already obtained the $\hat{r}_M$ scenario from the ESG. However, Formula (A.4) has the further advantage that it can address circumstances in which we do not yet have a known value of $\hat{r}_M$ as well as cases in which we simply prefer to sample from both of the market and portfolio return

[8] For a complete discussion of correlated sampling of two or more standard normal random variables, see Rubenstein [3] (pp. 65-67), which rests on the use of the Cholesky decomposition of the correlation or covariance matrix to transform a set of independent standard normal random numbers into a set of correlated normal random numbers (which might be *standard* normal or not, depending on whether the correlation or covariance matrix is used).

distributions simultaneously. By randomly selecting independent standard normal random numbers z_1 and z_2, and computing z_2^*, we can then obtain correlated random observations $\hat{r}_M$ and $\hat{r}_i$.

For example, if we draw $z_1 = 0.4309$ and $z_2 = -0.3572$, which together with $\rho_{iM} = 0.8$ imply z_2^*=0.1304, we can use Formulas (A.2) and (A.4) to obtain the following values for $\hat{r}_M$ and $\hat{r}_i$, respectively:

$$\hat{r}_M = 3\% + 4\% + 0.4309 \cdot 20\%$$

$$= 15.62\%.$$

$$\hat{r}_i = 3\% + 3.6\% + 0.1304 \cdot 22.5\%$$

$$= 9.53\%.$$

A.2 Normal Logarithmic Return Model – Additional Information

If the ESG equity returns are based on a logarithmic return model, then the market return scenario $\hat{r}_M$ can be expressed as:

$$\hat{r}_M = r_f + \exp\big(\mu_{MX}(LN) + z_1 \cdot \sigma_M(LN)\big) - 1, \qquad \text{(A. 8)}$$

where $\mu_{MX}(LN)$ and $\sigma_M(LN)$ are the mean and standard deviation, respectively, of the logarithmic excess market return $\ln(1 + \hat{r}_M - r_f)$, and z_1 is a random draw from the standard normal distribution.

The argument of the exponential function in Formula (A.8) can be expressed as the natural logarithm of the accumulated value of the market excess return:

$$\mu_{MX}(LN) + z_1 \cdot \sigma_M(LN) = \ln(1 + \hat{r}_M - r_f) \qquad \text{(A. 9)}$$

If, in Formula (2.13), we substitute the expression on the left side of Formula (A.9) for the expression $\ln(1 + \hat{r}_M - r_f)$, we see that the portfolio return $\hat{r}_i$ can then be modeled as:

$$\hat{r}_i = r_f + \exp\left(\beta_{iM} \cdot (\mu_{MX}(LN) + z_1 \cdot \sigma_M(LN)) + z_2 \cdot \sqrt{1 - \rho_{iM}^2} \cdot \sigma_i\right) - 1$$

$$= r_f + \exp\left(\mu_{iX}(LN) + z_1 \cdot \rho_{iM} \cdot \sigma_i(LN) + z_2 \cdot \sqrt{1 - \rho_{iM}^2} \cdot \sigma_i\right) - 1, \qquad \text{(A. 10)}$$

$$\hat{r}_i = r_f + \exp\left(\mu_{iX}(LN) + z_2^* \cdot \sigma_i(LN)\right) - 1, \qquad \text{(A. 11)}$$

where $\mu_{iX}(LN) = \ln\left(1 + E(r_i) - r_f\right) - 0.5 \cdot \sigma_i(LN)^2$ and z_2^* defined by Formula (A.5).

Formula (A.11), like Formula (2.13), supports correlated sampling of r_i in cases where we have already obtained the $\hat{r}_M$ scenario from the ESG. In addition, however, Formula (A.11) has the feature that it can address cases in which we do not already know $\hat{r}_M$ as well as those situations in which we prefer to sample from both of the market and portfolio return distributions simultaneously. By randomly selecting numbers z_1 and z_2, and computing z_2^* using Formula (A.5), we can then obtain correlated random observations $\hat{r}_M$ and $\hat{r}_i$ using Formulas (A.8) and (A.11).

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Abbreviations and Notations

β_{iM}	CAPM beta of portfolio *i* with respect to the market index *M* .
CAPM	Capital Asset Pricing Model.
ϵ_i	Random variable for an independent error term reflecting the idiosyncratic variability of the return on portfolio *i*.
ERM	Enterprise risk management.
ESG	Economic scenario generator.
μ_{iX}	Mean of arithmetic portfolio *i* return excess of risk-free rate: $E(r_i - r_f)$; mu parameter of normal distribution of arithmetic (excess) portfolio *i* return.
μ_{MX}	Mean of arithmetic market index *M* return excess of risk-free rate: $E(r_M - r_f)$; mu parameter of normal distribution of arithmetic (excess) market index *M* return.
$\mu_{iX}(LN)$	Mean of logarithmic portfolio *i* return excess of risk-free rate: $E(\ln(r_i - r_f))$; mu parameter of the lognormal distribution of logarithmic (excess) portfolio return.
$\mu_{MX}(LN)$	Mean of logarithmic market index *M* return excess of risk-free rate: $E(\ln(r_M - r_f))$; mu parameter of the lognormal distribution of logarithmic (excess) market return.
$N^{-1}(z)$	Inverse distribution function of the standard normal distribution evaluated at *z*, where *z* is a standard normal number.
r_f	Risk-free rate of return, treated as a constant.
r_i	Random variable representing the arithmetic return on portfolio *i.*
$\hat{r}_i$	One modeled or observed arithmetic return on portfolio *i*
r_M	Random variable representing the arithmetic return on market index *M.*

$\hat{r}_M$	One modeled or observed arithmetic return on market index *M.*
ρ_{iM}	Correlation coefficient between portfolio *i* and market index *M* returns.
σ_{iM}	Covariance between portfolio *i* and market index *M* equity returns.
σ_i	Standard deviation of r_i; sigma parameter of normal distribution of arithmetic portfolio *i* return.
$\sigma(\epsilon_i)$	Standard deviation of the idiosyncratic portion of the arithmetic return on portfolio *i*.
$\sigma_i(LN)$	Standard deviation of portfolio *i* logarithmic return; sigma parameter of the lognormal distribution of logarithmic portfolio *i* return.
σ_M	Standard deviation of r_M; sigma parameter of normal distribution of arithmetic market index *M* return.
$\sigma_M(LN)$	Standard deviation of market index *M* logarithmic return; sigma parameter of the lognormal distribution of logarithmic market index *M* return.
$VaR(r_i)_{0.995}$	Value-at-risk of portfolio *i* return at the 99.5th percentile, where negative returns are expressed as positive numbers. It corresponds to the 0.5th percentile of the unadjusted return distribution multiplied by -1.
z	A standard normal random number; sometimes given with a subscript (z_1 and z_2).
z_2^*	A standard normal random number correlated with z_1; determined from a pair of independent standard normal random numbers and a correlation coefficient.

Biography of the Author

Michael G. Wacek is Executive Vice President and Chief Risk Officer of Odyssey Re Holdings Corp., based in Stamford, CT. Over the course of more than 35 years in the insurance / reinsurance industry, including nine years in the London Market, he has seen the business from the vantage point of a primary insurer, reinsurance broker and reinsurer in actuarial, underwriting, risk management and executive management roles. He has a BA from Macalester College (Math, Economics), is a Fellow of the Casualty Actuarial Society, a Chartered Enterprise Risk Analyst and a member of the American Academy of Actuaries. He is the author of a number of actuarial papers on insurance, reinsurance and enterprise risk management issues.