Casualty Actuarial Society E-Forum, Fall 2013

The CAS *E-Forum*, Fall 2013

The Fall 2013 Edition of the CAS *E-Forum* is a cooperative effort between the CAS *E-Forum* Committee and various other CAS committees, task forces, or working parties. This *E-Forum* contains two CAS Working Party reports: report 6 of the CAS Risk-Based Capital Dependencies and Calibration Working Party (Reports 1 and 2 are posted in *E-Forum* Winter 2012-Volume 1, reports 3 and 4 in *E-Forum* Fall 2012-Volume 2 and report 5 in E-Forum Summer 2012) and the report of the Tail Factors Working Party. This *E-Forum* also includes 16 papers submitted in response to a call for non-technical reserve papers, conducted by the CAS Committee on Reserves. Several of these papers will be presented at the 2013 CAS Annual Meeting, November 3-6, in Minneapolis, MN.

Risk-Based Capital Dependencies and Calibration Research Working Party

Tail Factors Working Party

Committee on Reserves

CAS *E-Forum,* Fall 2013

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Risk-Based Capital (RBC) Premium Risk Charges – Improvements to Current Calibration Method

Report 6 of the CAS Risk-Based Capital (RBC) Research Working Parties

Issued by the RBC Dependencies and Calibration Working Party (DCWP)

Abstract: The purpose of this paper is to describe the results of research on methods to improve the Current Calibration Method (CCM) for premium risk charges for use in the NAIC RBC Formula. The paper shows how it is possible to construct risk charges that might be both more reflective of underlying risk and more stable over time than the CCM.

This paper shows the extent to which calibration of premium risk charges is affected by issues identified, but not measured, in prior research – premium size by line of business (**LOB-size), pooling, and movement over time**. The paper also identifies and measures the extent to which risk charges are affected by the following additional issues: (a) the **"minor line"** effect, which appears to distort risk charges for specialty lines of business (LOBs), (b) the effect of data **maturity,** and (c) the effect of **'survivorship',** companies that stop filing annual statements.

This is one of several papers being issued by the Risk-Based Capital (RBC) Dependencies and Calibration Working Party.

Keywords. Risk-Based Capital, Capital Requirements, underwriting risk, reserve risk, premium risk, Analyzing/Quantifying Risks, Assess/Prioritizing Risks, Integrating Risks.

1. Introduction

1.1 Background and Purpose

The NAIC RBC Formula ("Formula") has six main risk categories, R0 – R5. The underwriting risk is expressed in two of the categories, reserve risk and written premium risk, R4 and R5 respectively. This paper relates to R5, written premium risk.

For each Schedule P line of business (LOB), R5 is determined using an "Industry RBC Loss and Expense Ratio," used in PR017 line 4, a value applicable to all companies. We refer to this as the premium risk factor (PRF).

For each LOB the Premium Risk Charge (PRC) is produced using the PRF, LOB net written premium (NWP), and adjustments for investment income, differences between the company loss ratios and the industry loss ratios, the company proportion of loss sensitive contracts, and the company all-lines expense ratio.[1] For purposes of this paper we refer to the PRC divided by the NWP as the PRC%.

This paper provides a framework for deriving the PRFs by LOB.

1.2 Terminology, Assumed Reader Background, and Disclaimer

This paper assumes the reader is generally familiar with the property/casualty RBC formula.[2]

In this paper, references to "we" and "our" refer to the principal authors of this paper. "The working party" and "DCWP" refer to the CAS RBC Dependencies and Calibration Working Party.

The analysis and opinions expressed in this report are solely those of the authors, the Working Party members, and in particular are not those of the members' employers, the Casualty Actuarial Society, or the American Academy of Actuaries.

DCWP makes no recommendations to the NAIC or any other body. DCWP material is for the information of CAS members, policy makers, actuaries, and others who might make recommendations regarding the future of the property/casualty RBC formula. In particular, we expect that the material will be used by the American Academy of Actuaries RBC Committee.

In Section 3 we define a "baseline filtering" approach to selecting data for use in our analysis. The purpose of the baseline is to simplify comparison among a number of analyses; it is not presented as a recommendation.

This paper is one of a series of articles prepared under the direction of the CAS RBC Dependency and Calibration Working Party.

Special terms and acronyms are described in the Glossary.

[1] For expenses other than loss adjustment expenses. Net of reinsurance.

[2] For a more detailed description of the formula and its initial basis, see Feldblum, Sholom, NAIC Property/Casualty Insurance Company Risk-Based Capital Requirements, Proceedings of the Casualty Actuarial Society, 1996 and NAIC, Risk-Based Capital Forecasting & Instructions, Property Casualty, 2010.

1.3 Prior Research

The PRFs in the Formula were first set in 1993.[3]. Research reports on the PRFs and comparable reserve risk charges were most recently prepared by the American Academy of Actuaries (Academy) in 2007[4] with updates in 2009[5] and 2010,[6] and by the Underwriting Risk Working Party (URWP) of the Casualty Actuarial society (CAS) in 2012.[7] In this paper we refer to the method described in the 2007 Academy Report as the "Current Calibration Method" (CCM).

This paper describes new research addressing a number of the issues raised by those prior papers, particularly those identified by URWP, as follows:

1. The current data sources—confidential company RBC filings and the most recently available Schedule P—yield too few observations for stable estimates of RBC factors from one calibration cycle to the next. Additional data sources should be investigated.

2. Filtering eliminates a significant amount of company experience from the Current Calibration Method. For many lines of business the majority of the companies in the industry are eliminated; for two lines, all companies are eliminated. New ways to filter out questionable data should be investigated. Possible alternatives are discussed in the report. [8]

[URWP] … identified potential improvements to the Current Calibration Method that could be researched within the framework of the current RBC formula (including the following):

<u>Data</u>

1. Filtering strategies.

2. Additional or extended (number of years) data sources.

3. Treatment of data from pooled companies.

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[3] Academy (2007)
[4] Academy 2007
[5] Academy (2009)
[6] Academy (2010)
[7] CAS E-Forum, URWP report, Winter 2012
[8] CAS E-Forum, URWP report, Winter 2012– page 2

4. Analysis of the extent to which alternative filtering is affected by run-off and startup companies, and including procedures to mitigate that effect, if any. [9]

DCWP also reviewed Solvency II approaches to underwriting risk charge calibration and the results of that work will be described in a different paper.

1.4 Working Party Approach

To address the opportunities for improvements identified by that prior research, DCWP proceeded as follows:

1. Using information provided by the NAIC we compiled Schedule P information from 14 Annual Statements (1997-2010) from all individual companies and DCWP-defined pools,[10] for each LOB. This provides data for up to 23 accident years (AYs), many of them developed to 10 years maturity. By comparison, CCM uses only one Annual Statement with a maximum of 10 AYs and only one AY at 10 years maturity.
2. We applied less restrictive approaches to filtering data, and thereby retained more data for analysis.

In this DCWP research we continued to apply the CCM framework of measuring the PRF as the 87.5th percentile of observed loss ratios across companies and AYs.

1.5 Findings

The main findings from this research are the following, organized by section in this paper:

1. Section 2 – PRFs calibrated based on the CCM (using 10 AYs from a single Annual Statement) vary, often widely, from to Annual Statement to Annual Statement. This variation seems to be driven by the underwriting cycle, catastrophes, and other industry-wide effects. Longer-term data appears necessary to achieve more stable indicated PRFs.

[9] CAS E-Forum, URWP report, Winter 2012, page 26.
[10] Details in Appendix G.

2. Section 3 – We identified certain data points as "minor lines" data points if the Net Earned Premium (NEP) for the LOB and AY represents less than 5% of the company's all-line total premium for that LOB and AY. For certain specialty LOBs the indicated PRFs excluding the "minor lines" data points are significantly lower, and more relevant, than the PRFs based on all data points. For those LOBs, failure to exclude the minor lines data points appears to result in PRFs that are not representative of risk for companies writing the bulk of the industry LOB premium.

3. Section 3 – Pooling can distort the PRFs. The distortion can be at least partially removed.

4. Section 3 – We define a baseline filtering approach to selecting data for use in our analysis. This baseline is not a recommendation. Rather, it is a practical way to evaluate a variety of alternatives. This baseline is the starting point for the analyses described in Sections 4-8.

5. Section 4 – Looking at all 23 available years and the 'even-year/odd-year' test suggests that the 23-year data set will produce PRFs that are more stable than the CCM across calibrations from year-to-year.

6. Section 5 – We demonstrate that indicated PRFs vary with LOB-size; i.e., NEP by LOB.[11] To the extent that the RBC formula is not intended to have PRFs that vary by LOB-size, we identify two approaches to treating that issue in the context of the RBC Formula: PRFs based on the median LOB-size and PRFs based on LOB-size above a threshold. There may be other suitable approaches.

7. Section 6 – PRFs are affected by the maturity of the data to an extent that varies by LOB.

8. Section 7 – For most LOBs, PRFs are lowest for data points from companies with the longest experience period, 20 or more AYs of NEP > 0.

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[11] We use the term LOB-size to clearly distinguish between the premium size of the company and the premium size for the LOB.

9. Section 8 – PRFs are somewhat affected by "survivorship." Companies with 2010 Annual Statements have somewhat lower PRFs than companies whose last filed Annual Statements were prior to 2010.

10. While maturity and survivorship adjustments are not included in the baseline that we used for comparative purposes, it would be reasonable to include them in a final RBC calibration.

2. PRFs Based on CCM

In 2011, the URWP observed that the CCM-indicated PRFs, based on data from a single Annual Statement, vary widely from Annual Statement to Annual Statement, and URWP recommended that more data be used in determining the PRFs. In this section we provide a more detailed illustration of the year-to-year variability exhibited by the PRFs indicated by the CCM.

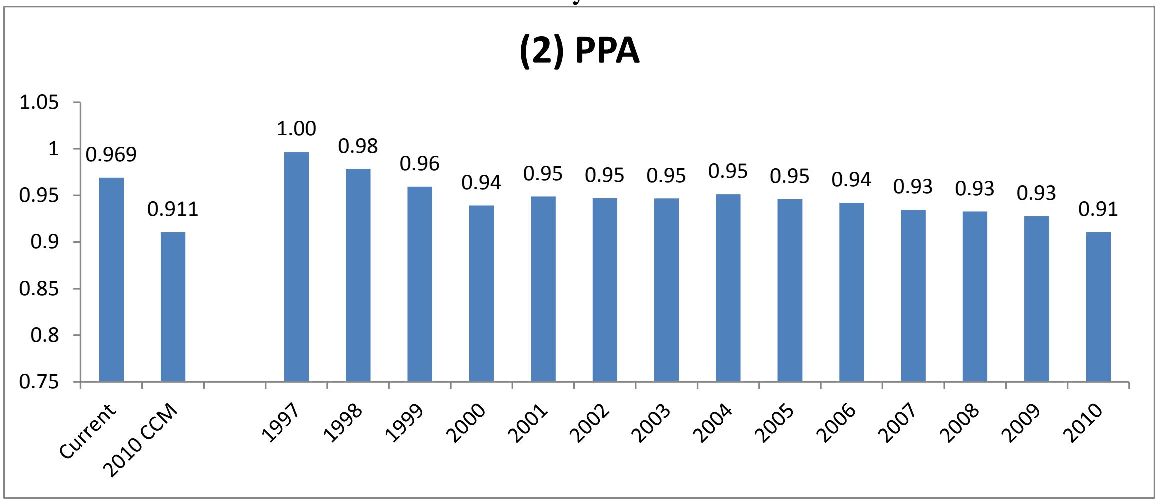
The PRFs indicated by the CCM are based on the empirical 87.5th percentile of 10 years of loss ratio data from all companies at a single Annual Statement date, with filtering described below in section 3.2.1.

Table 2.1 shows these values, as would be determined from successive Annual Statements from 1997 to 2010, for the Private Passenger Auto (PPA) LOB.

Note that for this chart, as with most charts in this paper, the vertical scale starts at 0.75, so that the height of the displayed bar can be considered representative of the PRC%, based on an illustrative underwriting expense ratio of 0.25 and before considering the investment income offset and other factors that affect the final PRC%.

Table 2.1

PPA – CCM PRFs by Annual Statement Year



For this LOB, the PRF varies from 1.00 to 0.91 over the 14 years shown: a swing of nine percentage points in PRF, a large portion of the PRC% for this LOB.

For comparative purposes, the current PRF, 0.969, is shown at the left side of the table. This is the "industry loss and expense ratio" appearing in Line 04 of the 2010 RBC report PR017. The PRF indicated using the CCM and 2010 Annual Statement data, 0.911, is also shown on the left part of the chart. The actual RBC factors were updated over the 2008-2010 period, based on the CCM but subject to limitations ("caps") in year-over-year movements. The caps were ±15% in each of 2008 and 2009, and ±5% in 2010.[12]

Table 2.2 shows the indicated PRFs for workers compensation. Here we see a swing of 11 percentage points of PRF, from 0.94 related to experience in based on year 2010 Annual Statements to 1.05 based on year 2003 Annual Statements. The values also show a pattern over time typical of the underwriting cycle.

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[12] URWP – page 5.

Table 2.2
WC – CCM PRFs by Annual Statement Year

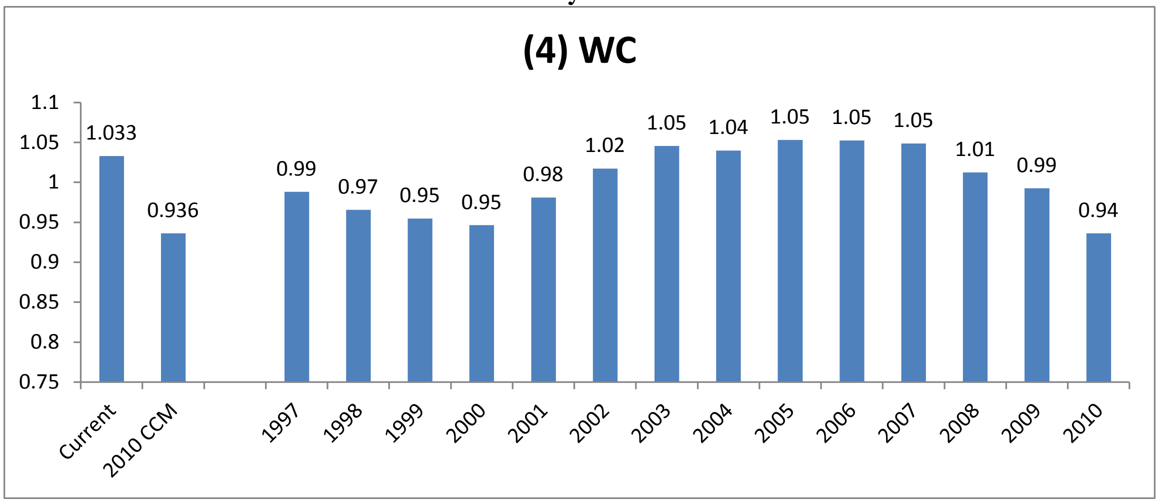
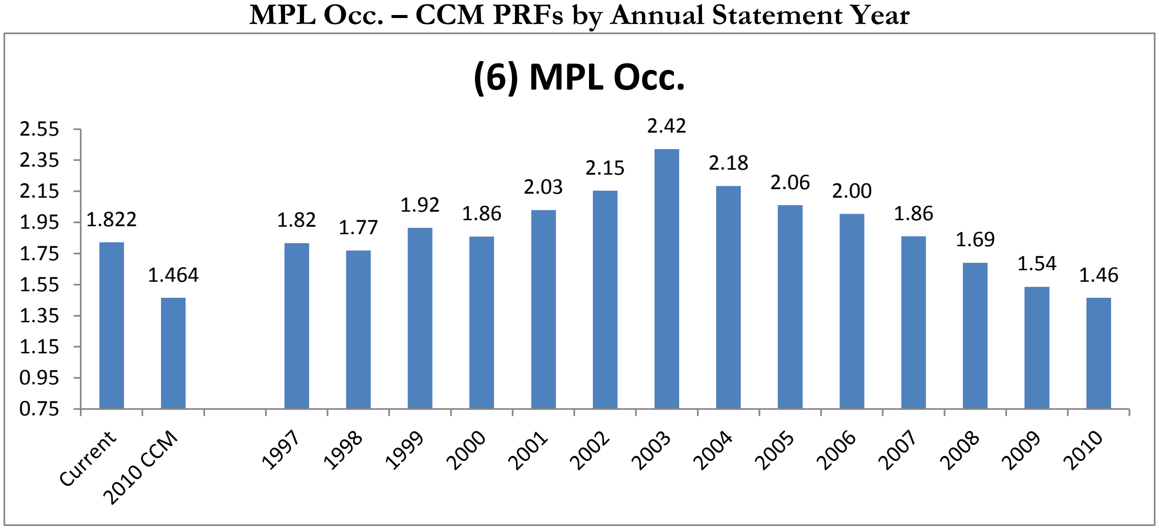


Table 2.3 shows the PRFs for the Medical Professional Liability (MPL) – Occurrence LOB. Here the PRF swing is from 1.46 to 2.42, 96 percentage points of PRF swing from Annual Statement year 2003 to Annual Statement year 2010.

Table 2.3
MPL Occ. – CCM PRFs by Annual Statement Year



Similar year-by-year PRF graphs for all LOBs are shown in Appendix A.

It seems clear that the CCM approach of using the most recent Annual Statement will not produce stable PRF indications.

3. Data and Filtering

3.1 Data

Using information provided by the NAIC, we compiled Schedule P information from 14 Annual Statements (1997-2010) from all individual companies and DWCP-defined group pools (pools). That provides over 200,000 data points, covering 23 AYs, many of them developed to 10 years maturity. The CCM uses only one Annual Statement with a maximum of 10 AYs and only one AY at 10 years maturity.

Each data point is an AY-LOB, for a single company or pool, at the latest available maturity. For each data point we have the following information:

1. net earned premium (NEP)
2. the loss and all loss adjustment expense ratio to premium
3. maturity of the AY (1 year, 2 years,… 10 years)
4. the percentage of premium for the data point LOB compared to the premium for the all LOBs for the same company (pool) for the same year, to identify 'minor lines' described under section 3.2.2.

3.2 Filtering Methodologies

We use the term "filtering" to describe the manner in which we treat data features that might affect the indicated PRFs, such as data errors, LOB-size, maturity of loss experience, etc. In the sections below we discuss the CCM filtering and DCWP filtering approaches.

3.2.1 CCM Filtering

CCM uses data from only one Annual Statement for the calibration. In the CCM <u>all</u> data associated with a LOB for a company is removed if, for the 10 years of data included in the latest Annual Statement:

1. Average AY earned premium < $500,000
2. Any AY loss ratio < 0
3. Fewer than 10 years of earned premium
4. Fewer than 8 AYs with net earned premium greater than 20% of average earned premium for all AYs (company growing or shrinking too rapidly)

For remaining data points, loss ratios are capped at 300%.[13]

CCM filtering eliminates about half of the data points and about 10% of premium dollars from the data set.[14]

3.2.2 Alternative Filtering Methods

In this analysis, the DCWP used a less restrictive filtering process.

A data point (i.e., company/LOB/AY combination) is excluded if earned premium ≤ 0 or incurred loss ≤ 0. By excluding data points rather than excluding the entire company's data, more data is retained for analysis. This filter eliminates about 11% of data points but almost 0% of premium dollars.

In the rest of this section we test the sensitivity of indicated PRFs to three other data filtering methods: pooling, minor lines, and LOB-size:

Pooling – For companies with intergroup pooling arrangements the Schedule P loss ratio for each LOB-AY would be the same for each pool member; the common loss ratio would be the weighted average net loss ratio for that LOB-AY for the entire pool rather than the individual pool member loss ratio before pooling.

That feature of the data would distort the results of our analysis in that:

1. The same loss ratio value would appear multiple times, reducing the apparent variability in the loss ratios across companies; and
2. Companies that appear small based on their pooling percentages would show the lower year-to-year variability associated with the larger size of the overall pool rather than the higher year-to-year variability associated with a company of its apparent lower size.

To mitigate these effects, we would like to combine the separate pool participants into a single date point for each LOB-AY. If that were done, the data would reflect the correct variability between companies and the proper data point LOB-size.

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13 The 300% cap would affect PRF only if the indicated PRF were above 300%. That situation does not arise.
14 URWP – page 8.

We use information in the Annual Statements to identify individual companies that appear to be part of a larger pooled entity. There are 3,730 NAIC legal entities in the initial data set. Of these, 2,695 are not part of any pool and 1,035 entities are mapped into 206 DCWP-constructed pooled entities. Thus, the total data set includes 3,730 – 1,035 + 206 = 2,901 entities in total.[15]. Our approach to identifying relevant pools is discussed in Appendix G.[16].

LOB-Size – Indicated PRFs vary by LOB-size, and in Section 5 we evaluate PRFs by LOB-size. In the subsection below, we test the effect of excluding a data point if the LOB NEP is below a threshold which varies by LOB. The selected thresholds are listed in Appendix B.

Minor Line Filtering – We defined "minor lines" data points as those where the company/LOB/year NEP was less than 5% of the all-lines NEP for that company/year. We compare the indicated PRFs using data including minor line data points and data excluding minor line data points.

3.3 Sensitivity Testing

In this section we describe how we tested the extent to which pooling, minor lines, and LOB-size affect the indicated PRFs.

Table 3.1 shows the results of our filtering sensitivity analysis for the PPA LOB.

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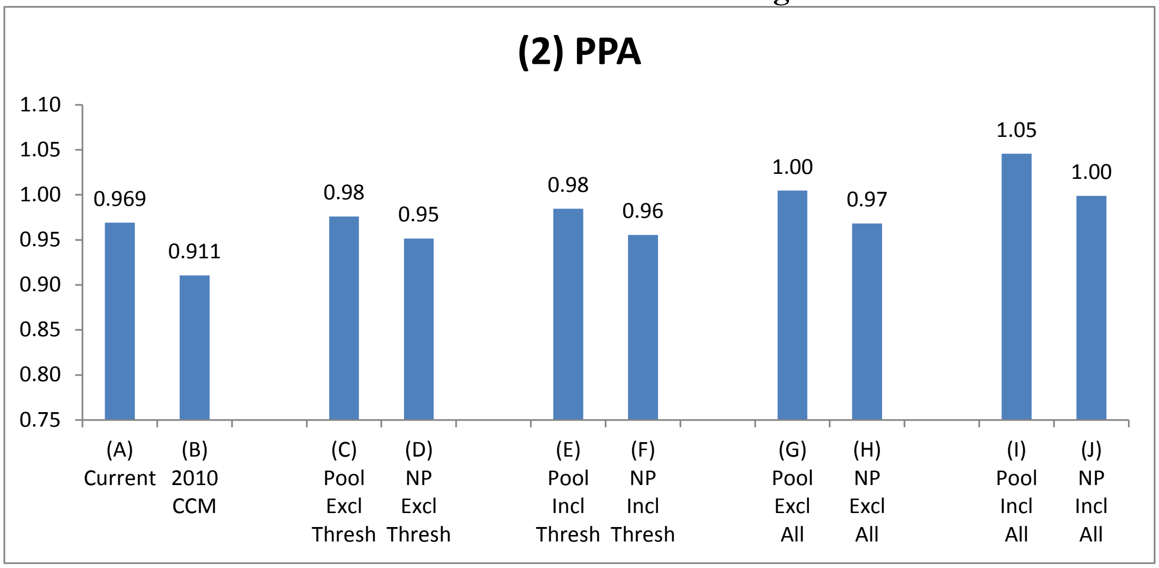
15 For each LOB, the number of entities is smaller, as not all companies have written business in each LOB.
16 As described in Appendix G, our approach is approximate, as it does not necessarily identify all pools and it may combine some LOB/companies that are not actually pooled.

Table 3.1
PPA – Effects of Alternative Filtering Methods



The "Current" and "2010 CCM" values shown in columns A and B at the left of the graph are unchanged from Section 2. We now focus on the pairs of values from right to left.

A comparison of the values in columns I and J at the far right shows the effect on indicated PRFs of pooling; the "Pool" and "NP" labels designate "Pooling" and "No Pooling" respectively, with no other filtering. Comparing columns I and J, we see an increase in the indicated PRF using pooled data, from 1.00 to 1.05.

The values in columns G and H show the indicated PRFs excluding minor lines filtering; the "Excl" label indicates that minor lines data points are excluded, and the "Incl" label indicates that minor lines data points are included. Comparing columns G to I and H to J, we observe a decrease in the indicated PRFs from 1.05 to 1.00 for pooled data and a decrease from 1.00 to 0.97 for unpooled data, when minor lines data points are excluded.

The values in columns E and F show the indicated PRFs with LOB-size filtering; the "Thresh" label indicates that the data points with LOB-size below the threshold size are excluded. The label "All" indicates that data points of all LOB-sizes are included. Comparing columns E to I and F to J, we see the effect on the calibration of removing the data points with LOB-sizes below the threshold. The size threshold for PPA is $1 million. The effect is a decrease in the indicated PRF, from 1.05 to 0.98 and from 1.00 to 0.96 for pooled and unpooled data respectively.

We note that the decrease in indicated PRF is larger based on LOB-size threshold than the decrease based on exclusion of minor lines data points. We characterize this as "LOB-size filter is more significant than minor lines filter" for PPA. This general pattern, "LOB-size filter is more significant than minor lines filter" appears to be the case for many LOBs.

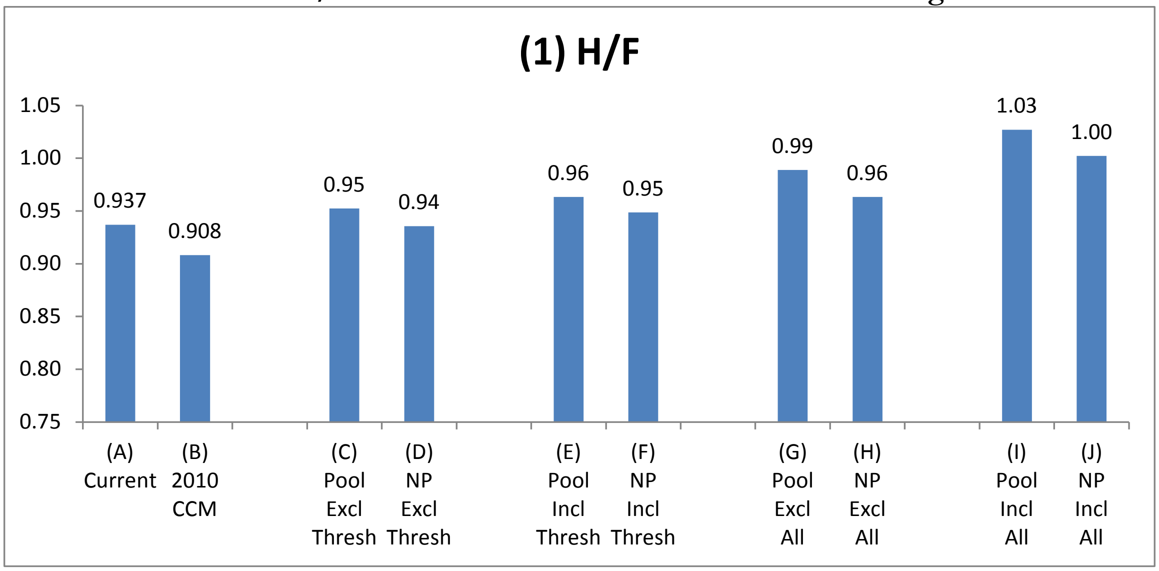
Finally, the values in columns C and D show the indicated PRFs with LOB-size and minor line filters combined. Comparing columns C and D against the other pooled/not-pooled pairs, there is a further decrease in indicated PRF by applying both the size threshold and the minor lines filters.

Table 3.2 displays the filtering sensitivity results for the Homeowners/Farmowners LOB. As with PPA:

1. The PRF based on pooled data is lower than the PRF based on unpooled data (Columns I vs. J, G vs. H, E vs. F, and C vs. D).
2. The PRF excluding minor lines data points is lower than the PRF including minor lines data points (Columns G vs. I, and H vs. J).
3. The PRF excluding LOB-size below the premium threshold[17] is lower than the PRF across all LOB-sizes (Column E vs. I and F vs. J).
4. The LOB-size filter is more significant than minor lines filter (Columns E vs. G and F vs. H).

[17] The LOB-size filter for the Homeowners/Farmowners LOB is $1 million.

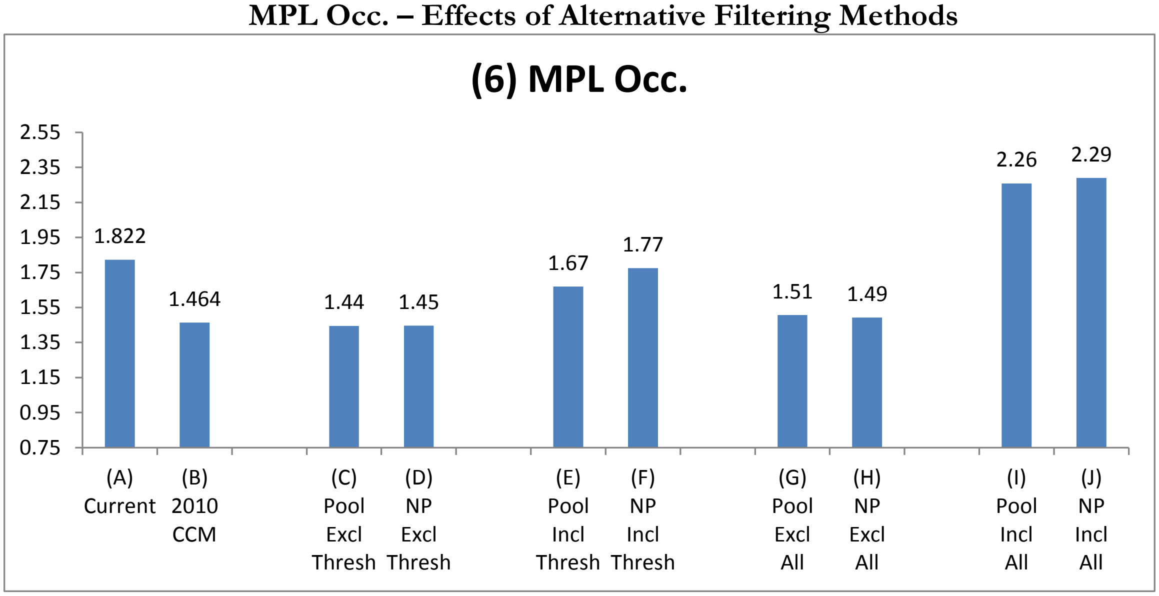
Table 3.2
Homeowners/Farmowners – Effects of Alternative Filtering Methods



For certain other LOBs, minor lines filtering is more significant than LOB-size filtering.

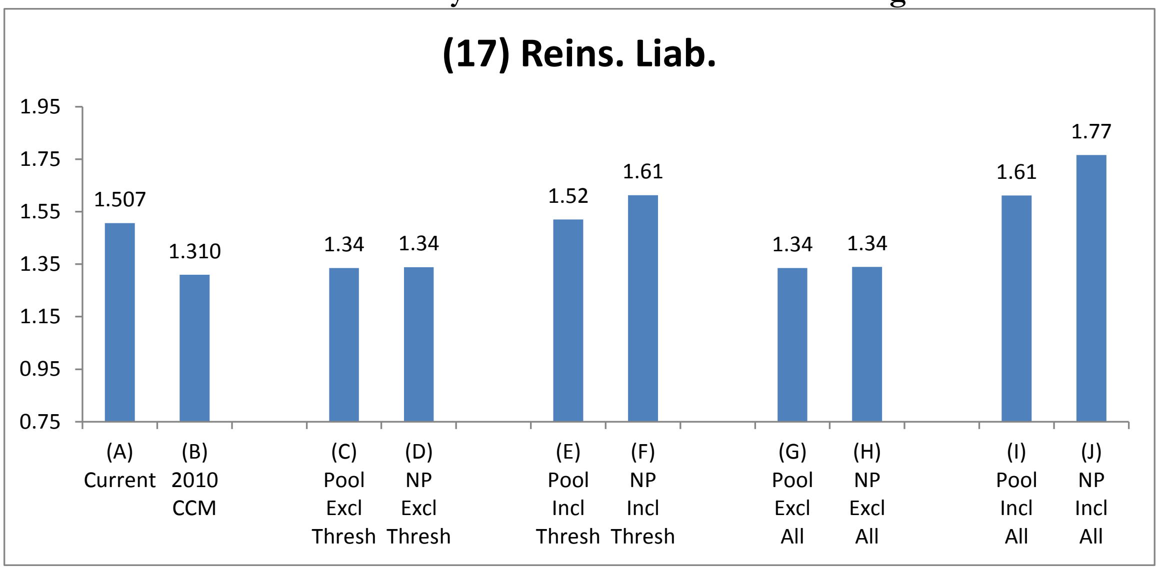
Table 3.3 shows indicated PRFs for the MPL – Occurrence LOB with the various filter combinations. In many respects the pattern is the same as for PPA and Homeowners/Farmowners.

Table 3.3
MPL Occ. – Effects of Alternative Filtering Methods



However, the pair of columns G and H is lower than the columns E and F, showing that the minor lines filter has a larger effect than the LOB-size filter.[18] This result demonstrates what might be called a "specialist effect," i.e., PRFs are larger for many insurers who write some MPL-Occurrence but for whom MPL-Occurrence is a small part of the overall business. We see a similar effect in the Reinsurance-Liability LOB in Table 3.4.

Table 3.4
Reinsurance-Liability – Effects of Alternative Filtering Methods



For Reinsurance-Liability the minor line effect is so significant that the minor lines filter alone produces the same effect as minor lines and LOB-size filters[19] combined; compare columns G and H to columns C and D.

Corresponding graphs for all LOBs are shown in Appendix B. The premium thresholds by LOB are shown at the end of Appendix B in Appendix B Table 1.

In the following sections, unless otherwise indicated we use data

- on a pooled basis,
- excluding minor lines data points, and
- excluding data points with LOB-size below the threshold.

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[18] The LOB-size filter for the MPL – Occurrence LOB is $ 800,000.
[19] The LOB-size filter for the Reinsurance – Liability LOB is $200,000.

In addition, to avoid the use of data points from immature LOBs, we exclude data points from companies with less than five years of positive NEP by LOB. We refer to the combination of these filters as the "baseline filtering."

Table 3.5shows the all-lines number of data remaining after the effects of pooling, the size threshold, minor lines filtering, and too few years of positive NEP.

Table 3.5

Number of data points and amount of premium after each step of the baseline filtering (all LOBs /all Years Combined)		
Filtering	**Premium (millions)**	**Data Points**
Un-Pooled	7,047	216,513
Pooled	7,061	121,622
Excluding Minor LOBs	6,508	79,025
Remove data points from companies with less than 5 years of positive NEP by LOB	6,471	75,515
Size above threshold (after applying minor lines and 5 Year NEP filters)	6,469	68,264

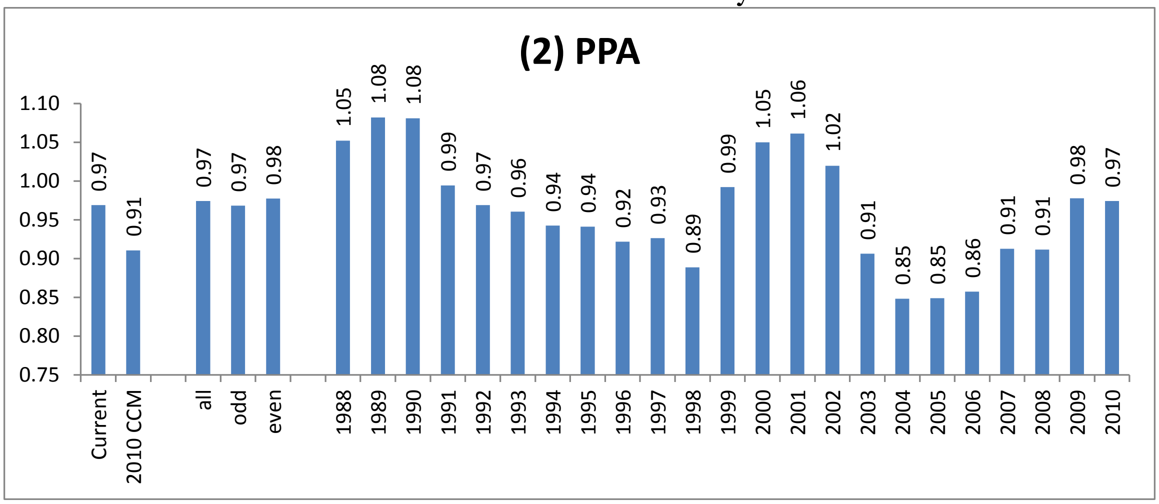
4. Indicated PRF by AY

In this section we review indicated PRFs by AY using the baseline filtering. The indicated PRF for an AY is the 87.5th percentile loss ratio for data points after baseline filtering within the LOB and AY.

Table 4.1shows the indicated year-by-year PRFs for the PPA LOB.

Table 4.1

PPA – Indicated PRFs by AY



In Table 4.1 the "Current" and "2010 CCM" values on the left side of the chart are the same as in the corresponding graph in Sections 2 and 3. The column "All" on the left shows the indicated PRF using all 23 AYs of available data, again with baseline filtering.[20] The "Odd" and "Even" values represent the results using odd and even AYs,[21] and give one perspective on whether the results will change significantly if additional years were added to the data set.

Not surprisingly, the individual year-to-year results exhibit more variability than the 10-year-rolling average CCM values shown in Section 2. The comparison of the "Odd" and "Even" results, 0.97 and 0.98, to the "All" result, 0.97, suggests that the random variation from year-to-year is significantly smoothed if spread over twelve years reflecting sufficient underwriting cycles and other systemic effects.[22]

We also tested variability across every fourth data point (sets of 4 or 5 data points). This is a smaller set, and we expect that the correlation across four years is much less than the correlation between adjacent years. The results of that test, presented at the end of

[20] The "all year" indicated PRF is not the average of the year-by-year PRFs. The all-year PRF is the 87.5th percentile loss ratio among all loss ratios, after baseline filtering, regardless of AY.

[21] The even-year PRF is the 87.5th -percentile loss ratio among all loss ratios from even numbered AYs. The odd-year PRF is the 87.5th -percentile loss ratio among all loss ratios from odd numbered AYs.

[22] It is beyond the scope of this paper to assess the extent to which the 23 AYs of experience in this data set does or does not sufficiently reflect the extent of systemic and cyclical variability in all lines of business.

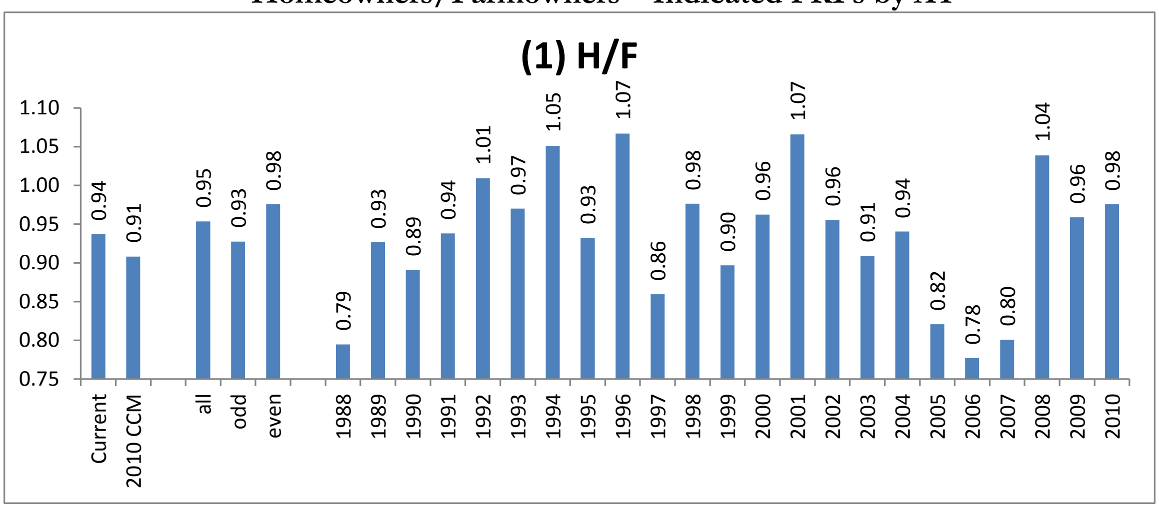
Appendix C, show more variability than the even/odd test, but still much less than the year-to-year variation in the CCM.

In examining year-by-year data, note that the oldest AYs shown are 10 years mature, and the more recent years are between one and nine years mature. In Section 6 we observe that for AYs 1997-2000, PRFs increased with increasing maturity. To the extent that recent year PRFs change with increasing maturity, then the more recent accident PRFs should be used with caution.[23]

Also note that as PRFs are the 87.5th percentile of loss ratios in each year, they will vary (a) as average loss ratio varies and (b) to the extent that variability (e.g., as measured by standard deviation) changes from year to year. We have not studied the components separately.

Table 4.2 shows the indicated PRFs for the Homeowners/Farmowners LOB.

Table 4.2
Homeowners/Farmowners – Indicated PRFs by AY



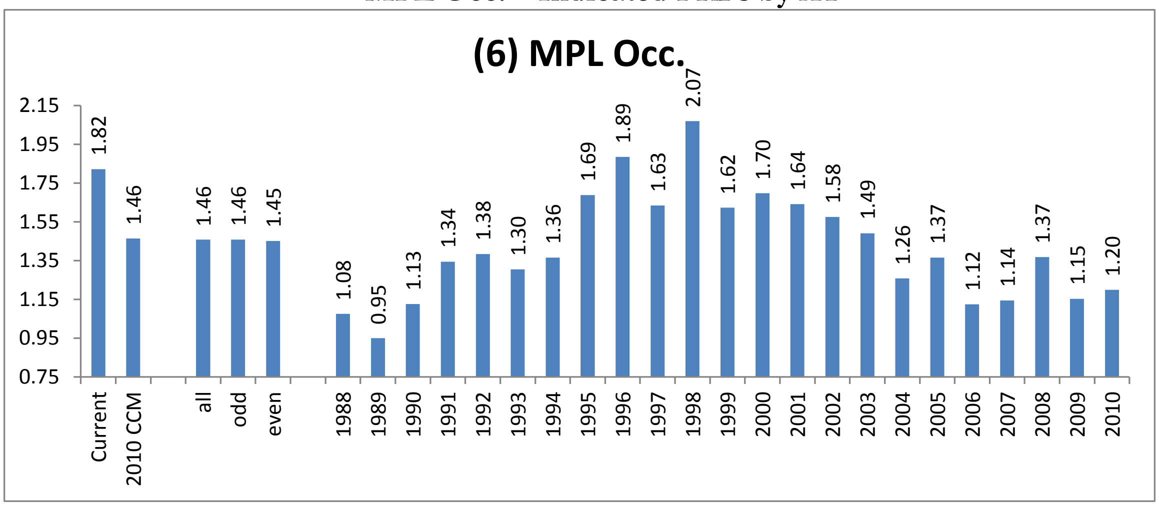
In this case the "Odd" and "Even" values are not as stable for as for PPA, a difference of 0.05 from 0.93 to 0.98. We also note that the highest years may indicate 'headline"

[23] This maturity pattern may not apply for all AYs. For example 1997-2000 might have been affected by the adverse side of the underwriting cycle for a LOB like reinsurance. AYs on the favorable side cycle might (possibly) develop less unfavorably or even develop favorably. The working party did not test these hypotheses.

catastrophe events, e.g., 1992 (Andrew) and 1994 (Northridge). For other years, the high values may be combinations of smaller natural events and adverse underwriting cycles. The slightly higher number of even-year 'high points' contributes to the difference between even-year and odd-year PRFs. If, as currently intended by the NAIC, catastrophe risk were reflected separately in the RBC formula, then the residual non-catastrophe PRF would be lower overall and more similar from year-to-year.

Table 4.3 shows the indicated PRFs for the MPL – Occurrence LOB.

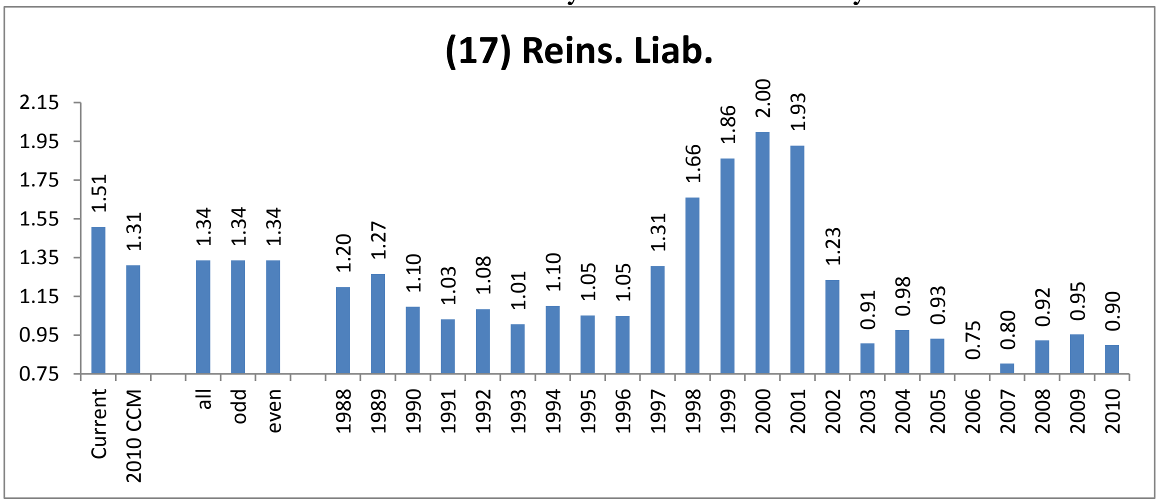
Table 4.3
MPL Occ. – Indicated PRFs by AY



Notwithstanding the large variability from year to year, the odd-year and even-year indicated MPL – Occurrence PRFs are stable at 1.45-1.46.

Finally, Table 4.4 shows the indicated PRFs for the Reinsurance – Liability LOB.

Table 4.4
Reinsurance-Liability – Indicated PRFs by AY



Again, although the year-to-year variability is large, the odd/even test again indicates the stability resulting from use of additional years of data.

Corresponding graphs for all LOBs are shown in Appendix C.

5. Analysis of LOB-size

In this section we examine the effect of LOB-size on indicated PRF.

To do this, we grouped LOB results into percentile LOB-size bands, and calculated PRFs and corresponding PRC%s for the data in each band. LOB-size bands refer to the LOB-size, regardless of the company size.

Table 5.1 displays the results for the PPA LOB. In column A, the row labels refer to upper-size end of the LOB-size band, so the first row, labeled 15%, refers to data points[24] with premium in percentiles 0%-15%. The second LOB-size band covers the next 10% of data points, up to the 25th percentile in premium LOB-size. In the final two rows of the table we show the largest 5% of data points, split between the "95% to largest 100" data points[25] (penultimate row) and the largest 100 data points (final row).

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[24] As a single company can have as many as 23 data points, one for each AY, the top 100 data points might represent only 5 or 6 companies.

[25] For some LOBs, the largest 5% of data points constitutes less than 200 data points. For those LOBs, the "largest l00" means the top 2.5% of data points, even though that is less than 100 data points.

Columns B and C show the lower and upper annual LOB-sizes corresponding to the percentile levels. Column D shows the number of data points included in each row.

Column E shows the PRF based on data within the LOB-size band. As expected, we observe in column E that the indicated PRFs are highest in the smallest LOB-size band, and generally decrease in value as we progress through the larger LOB-size bands.

Column F shows the PRF based on all LOB-size bands at or above the LOB-size for that row. For example, the first row in Column F is the PRF for all data points, regardless of LOB-size. The second row in Column F is the indicated PRF for all data points in the top 85% of LOB-sizes; the third row is the indicated PRF for data points in the top 75% of LOB-sizes, and so on. The row called "100%" shows the PRF for the largest 100 data points alone. In this row column E = column F.

Table 5.1
PPA – PRF and PRC% by LOB-size

(2) PPA							
(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)
Size Band	Premium ($000s)			87.5th Percentile LR		Risk Charge	
Endpoint Percentile	from	to	Data Points	all points in band	all points > "from"	all points in band	all points > "from"
15%	0	1,596	1,304	1.243	0.999	43%	18%
25%	1,596	3,634	869	1.019	**0.969**	20%	**15%**
35%	3,634	6,667	868	1.003	0.965	19%	15%
45%	6,667	11,219	869	1.013	0.958	20%	14%
55%	11,219	16,368	869	**0.971**	0.950	**16%**	14%
65%	16,368	28,352	869	0.971	0.945	16%	13%
75%	28,352	54,053	869	0.962	0.939	15%	12%
85%	54,053	130,201	868	0.959	0.929	14%	11%
95%	130,201	580,234	869	0.920	0.908	11%	9%
largest 100	580,234	3,936,971	334	0.895	0.894	8%	8%
100%	3,936,971	18,406,826	100	0.892	0.892	8%	8%
	Current Risk Charge Loss Ratio (PR017 Line 4)				0.969		
	Underwriting Expense Ratio in Risk Charge					19%	

Column G shows the PRC%s, by LOB-size band, calculated from the indicated PRF in Column E using an underwriting expense ratio that would produce a break-even combined ratio for all data points. For example, in Table 5.1, the average loss ratio for all data points is 0.815; this implies a break-even expense ratio of 0.185, shown as 19% in the final row of Table 5.1.

Column H, analogously to Column F, shows the PRC%s, for all LOB-size bands at or above the row, based on the cumulative PRFs in Column F.

There are various ways we might use this information to select the PRF for an RBC formula. One approach is to use the PRF indicated using data points with LOB-size above a threshold that varies by LOB (threshold approach). The threshold might be selected based on judgment, to maximize the number of data points used while minimizing distortions in the indicated PRF. This is in the baseline approach, described in Section 3. The PPA threshold in the baseline is $1 million. The LOB-size thresholds for all LOBs are shown in Appendix B Table 1.

Alternatively, the threshold might be based on a particular percentile of data points; e.g., excluding the smallest 15% of LOB-size data points. The items marked in bold and underline in Columns F and H of the row labeled "25%" (i.e., the 15%-25% row) are the PRF and PRC% obtained by setting the threshold to exclude the smallest 15% LOB data points. Here we note that the PRF based on data points above a 15th percentile threshold happens to coincide with the factor in the 2010 RBC Formula for this LOB.

A second approach is to select the PRF associated with the median LOB-size, or range of data points around the median LOB-size (median approach). The items marked in bold and underline in columns E and G of the "55%" row (i.e., values included between the 45th and 55th LOB-size percentiles) are the indicated median values. In Table 5.1, we note that the 87.5th percentile loss ratio for the median LOB-sizes, 0.971, is quite close to the 0.969 value used in the current RBC calculation for this LOB. This is not the case for all LOBs.

Another approach is to have PRFs vary by LOB-size. Currently, none of the standard formulas vary PRFs in this way; however, Table 5.1 shows that the indicated PRC% for the largest data points (8%) is only about half as large as the PRC% indicated by the median or threshold approaches (15% or 16%). Thus, using the median or threshold approach to setting the PRF and PRC% means that the safety margin for the larger companies, and therefore for most policyholders, is higher, perhaps much higher, than the 87.5th percentile.

Table 5.2 displays the results for the Homeowners/Farmowners LOB; the pattern of variation by LOB-size is similar to that of the PPA LOB. The PRFs based on median and threshold approaches are similar, but not as close to each other as they were for

PPA. The decrease in PRC% from the median to the largest data points, from 18% to 15%, is not as significant as it was for PPA. We do not have the data to test this, but one reason may be that catastrophes affect the PRF and PRC% significantly for all LOB-size's.

Table 5.2
Homeowners/Farmowners – PRF and PRC% by LOB-Size

(1) H/F							
(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)
Size Band	Premium ($000s)			87.5th Percentile LR		Risk Charge	
Endpoint Percentile	from	to	Data Points	all points in band	all points > "from"	all points in band	all points > "from"
15%	0	730	1,429	1.287	0.989	53%	23%
25%	730	1,483	951	1.023	**0.956**	27%	**20%**
35%	1,483	2,758	951	0.985	0.948	23%	19%
45%	2,758	5,022	952	0.964	0.941	21%	18%
55%	5,022	8,866	952	**0.941**	0.938	**18%**	18%
65%	8,866	16,382	952	0.914	0.938	16%	18%
75%	16,382	31,572	951	0.959	0.945	20%	19%
85%	31,572	61,546	952	0.940	0.937	18%	18%
95%	61,546	252,884	952	0.929	0.935	17%	18%
largest 100	252,884	1,499,819	375	0.951	0.947	19%	19%
100%	1,499,819	10,820,092	100	0.912	0.912	15%	15%
	Current Risk Charge Loss Ratio (PR017 Line 4)				0.937		
	Underwriting Expense Ratio in Risk Charge					24%	

Table 5.3 displays the results for the MPL – Occurrence LOB. The PRFs by LOB-size are more erratic for this line than for the two lines discussed above. The indicated PRFs appear to be smallest near the median LOB-size level and larger for both smaller LOB-sizes and larger LOB-sizes. This atypical behavior may be due to the smaller number of data points, or differences in types of business (primary vs. excess or institutions vs. individual health care providers) among the smaller, medium, and larger LOB-sizes.

The PRFs for the median and threshold approaches in Table 5.3, 1.261 and 1.458 respectively, are both lower than the current PRF, 1.822. One factor contributing to this difference is the years of data used. As shown in Table 4.3, the PRFs for MPL – Occurrence vary by year. The current charges may reflect the effects of the adverse 1995, 1996, and 1998 years. Also, the current RBC PRFs, based on less recent data, do not reflect the effects of the more favorable 2009 and 2010 years included in Table 4.3.

Another factor contributing to the difference between current PRFs and Table 5.3 indicated PRFs may be that data in Table 5.3 excludes minor lines data point experience, while data underlying the current PRFs were not adjusted in that way. Table 3.3 showed that excluding minor lines has a significant effect on the indicated PRF for this LOB.

Table 5.3
MPL Occ. – PRF and PRC% by LOB-Size

(6) MPL Occ.							
(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)
Size Band	Premium ($000s)			87.5th Percentile LR		Risk Charge	
Endpoint Percentile	from	to	Data Points	all points in band	all points > "from"	all points in band	all points > "from"
15%	0	823	168	2.434	1.521	147%	56%
25%	823	1,595	111	1.566	**1.458**	60%	**50%**
35%	1,595	2,623	111	1.265	1.447	30%	49%
45%	2,623	4,087	112	1.440	1.459	48%	50%
55%	4,087	6,672	111	**1.261**	1.464	**30%**	50%
65%	6,672	11,654	112	1.426	1.486	46%	52%
75%	11,654	24,496	111	1.696	1.521	73%	56%
85%	24,496	44,393	111	1.431	1.425	47%	46%
95%	44,393	152,900	112	1.380	1.422	42%	46%
largest 28	152,900	204,129	27	1.339	1.448	38%	49%
100%	204,129	516,498	28	1.545	1.545	58%	58%
	Current Risk Charge Loss Ratio (PR017 Line 4)				1.822		
	Underwriting Expense Ratio in Risk Charge					4%	

Table 5.4 displays the results for the Reinsurance – Liability LOB.

Table 5.4
Reinsurance-liability – PRF and PRC% by LOB-Size

(17) Reins. Liab.							
(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)
Size Band	Premium ($000s)			87.5th Percentile LR		Risk Charge	
Endpoint Percentile	from	to	Data Points	all points in band	all points > "from"	all points in band	all points > "from"
15%	0	2,339	169	1.700	1.335	86%	49%
25%	2,339	5,258	112	1.436	**1.302**	59%	**46%**
35%	5,258	9,036	112	1.175	1.278	33%	44%
45%	9,036	18,520	112	1.288	1.290	45%	45%
55%	18,520	33,620	112	**1.272**	1.290	**43%**	45%
65%	33,620	54,532	112	1.335	1.290	49%	45%
75%	54,532	105,154	112	1.293	1.265	45%	42%
85%	105,154	223,643	112	1.174	1.227	33%	39%
95%	223,643	760,588	112	1.387	1.262	55%	42%
largest 28	760,588	1,098,101	27	0.980	0.972	14%	13%
100%	1,098,101	4,178,508	28	0.931	0.931	9%	9%
	Current Risk Charge Loss Ratio (PR017 Line 4)				1.507		
	Underwriting Expense Ratio in Risk Charge					16%	

As with MPL, we also observe that the Table 5.4 indicated PRFs for threshold or median approaches, 1.302 or 1.272, respectively, are lower than the current PRF, 1.507.

One factor contributing to this difference is the years used. As shown in Table 4.4, the PRFs for Reinsurance – Liability vary widely by year; the current PRFs may have the effects of the adverse 1998-2001 years. Also, the current charges, based on less recent data, do not reflect the effects of the more favorable 2009 and 2010 years included in Table 5.5.

Another factor contributing to this difference may be that data in this analysis excludes minor lines data points, while data underlying the current PRFs did not make that adjustment. Table 3.4 showed that excluding minor lines data points has a significant effect on the indicated PRF.

Corresponding tables for all LOBs are shown in Appendix D. The tables in Appendix D also include average loss ratio, loss ratio standard deviation, and loss ratio coefficient of variation statistics.

6. Maturity

The DCWP data set includes data points of varying development maturities. The most recent AY (2010) reflects one year of payments and management reserve estimate (case+

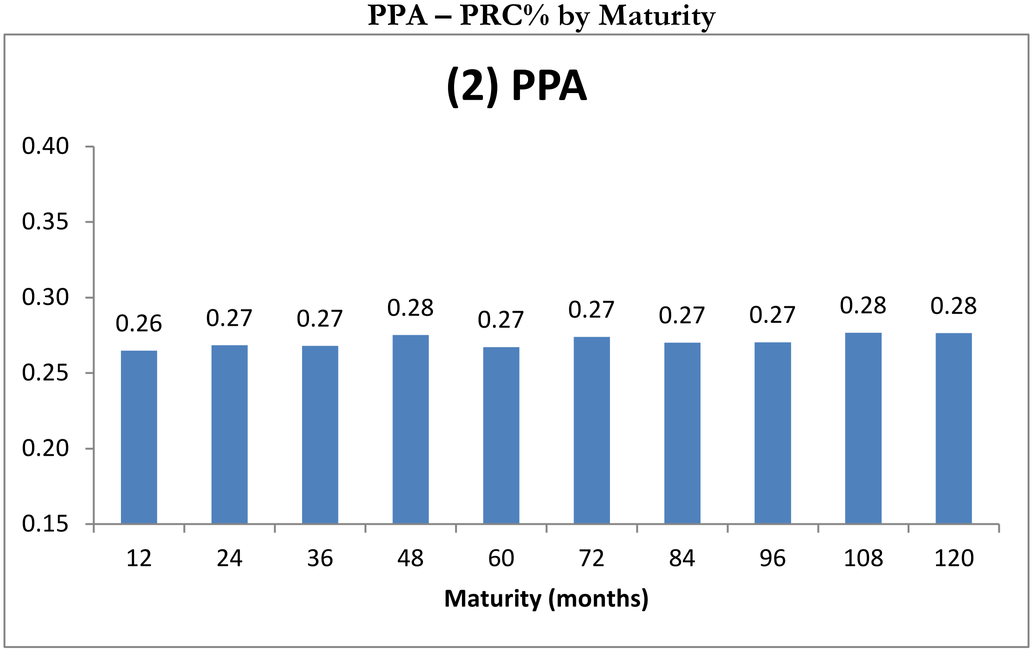
bulk + IBNR) at 12 months. AY 2009 reflects two years of payments and management reserve estimate at 24 months, etc. AYs 1997-2000 are the most mature, and reflect payments through 10 years and management reserve estimate at 120 months. The CCM and the baseline filtering in this paper treat all data points as equivalent, regardless of the maturity of the data.

In this section we test whether such equivalent treatment is appropriate. To do so, we examined data from AY 1997-2000. These are the AYs for which we have data points at every maturity from age 12 months to age 120 months. We use the same AYs for each maturity level to avoid bias that might arise from differences in PRC% by AY shown in Section 4 above.

We calculated PRC%s using data points for each maturity level separately using the baseline filtering. The results are discussed below.

Table 6.1 shows the PRC%s, for the PPA LOB, for AYs 1997-2000 combined, separately for each maturity level.

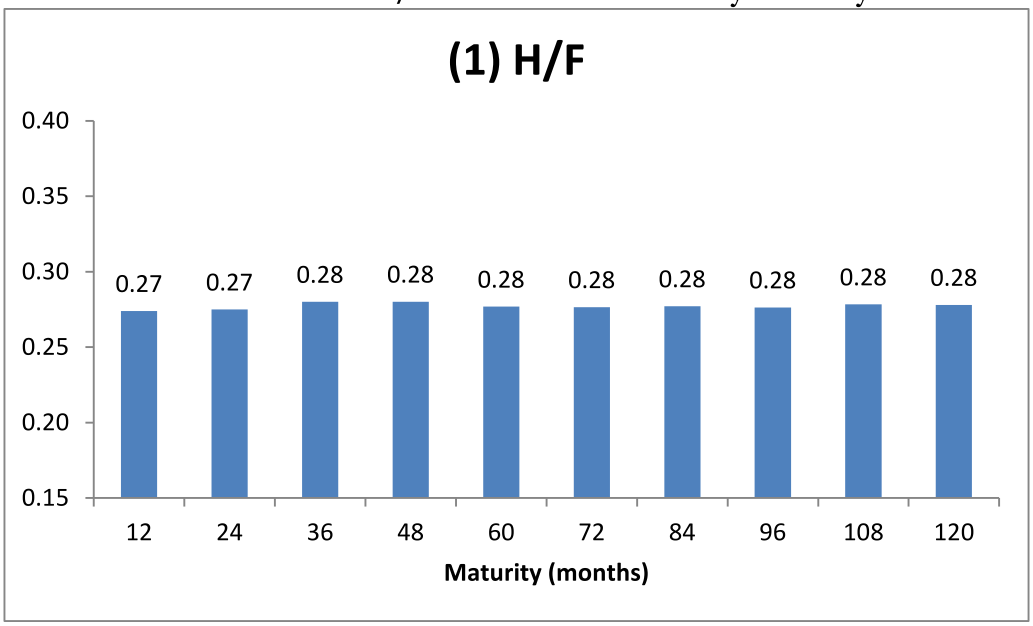
Table 6.1
PPA – PRC% by Maturity



Here we see that the PRC% reaches a stable value at 12 or 24 months of development.

Table 6.2 shows the corresponding PRC%s for the Homeowners/Farmowners LOB; the "fast development" pattern for Homeowners/Farmowners is similar to the PPA LOB pattern.

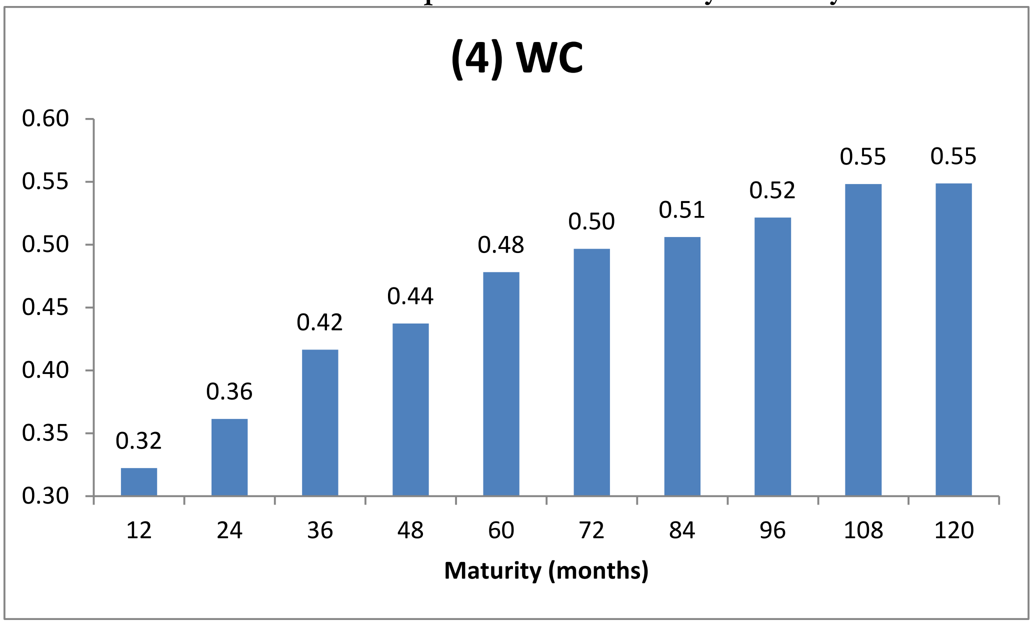
Table 6.2
Homeowners/Farmowners – PRC% by Maturity



The results shown so far are consistent with expectations for shorter-tailed liability LOBs.

Table 6.3 shows the PRC%s grouped by maturity for the workers compensation LOB.

Table 6.3
Workers Compensation – PRC% by Maturity



For the workers compensation LOB, the time required for PRC%s to reach a stable value, i.e., the "development period," is much longer than for the PPA and Homeowners/Farmowners LOBs illustrated in Tables 6.1 and 6.2.

Some of the development in workers compensation PRF might be due to emergence of tabular reserve.[26] This working party did not analyze that effect.

Table 6.4 shows the development period for the MPL – Occurrence LOB, shorter than workers compensation but longer than PPA or Homeowners/Farmowners. Nontabular reserve, which might appear for MPL lines, does not affect the PRFs and PRC%s because the Schedule P loss ratios used in our analysis are gross of nontabular discount.

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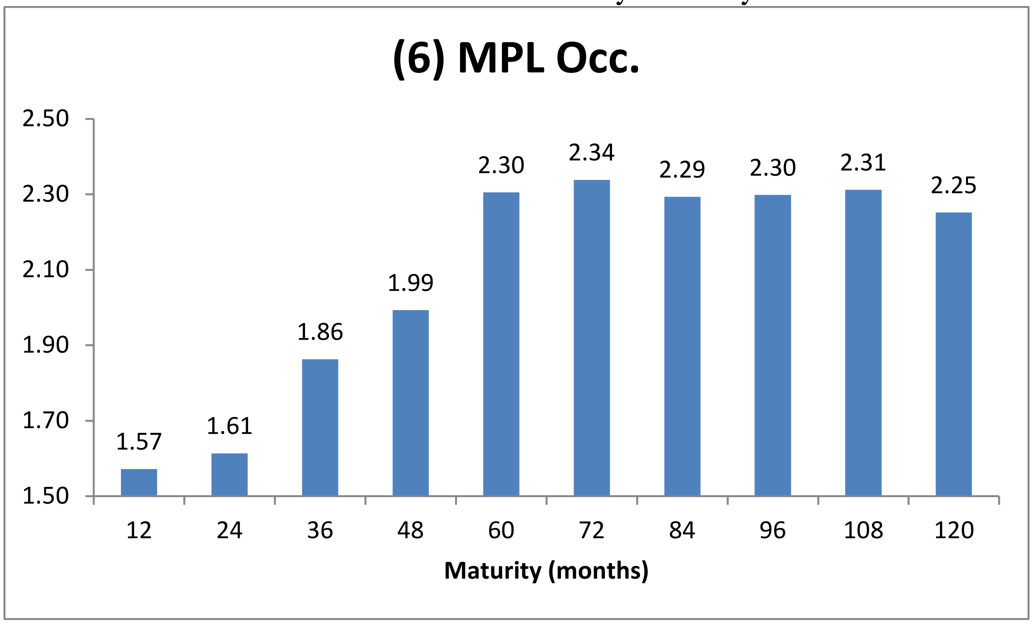
[26] The PRF should be designed with data gross of all interest discount, to the extent possible, in that Investment Income Offset in the RBC formula separately reflects the value of investment income for risk-based capital adequacy purposes.

Table 6.4
MPL Occ. – PRC% by Maturity



Corresponding tables for all LOBs are shown in Appendix E.

Table 6.5 displays the number of years of maturity required for the PRF to be within three percentage points[27] of the mature PRF for the 1997-2000 AY experience period.

It is possible that the 1997-2000 time period reflected in Table 6.5 is not typical, at least for some lines, and further research is warranted to examine that. Even given that uncertainty, the simplest way to reflect the maturity issue in calibration of PRFs would be to discard data points that are not sufficiently mature.

A more complex method would be to adjust the PRFs for expected development and use the adjusted data in an all-year PRF calculation. That would require more analysis of the extent to which the PRF "development" for AYs 1997-2000 is typical.

The working party has not tested the effect of either maturity adjustment.

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[27] 3% is an arbitrary, but we think reasonable, target for "mature".

Table 6.5
Development Years Needed to Reach Maturity[28]
AYs 1997-2000

LOB	Years to Reach Maturity
(1) H/F	1
(2) PPA	2
(3) CA	4
(4) WC	9
(5) CMP	5
(6) MPL Occ.	5
(7) MPL C-M	5
(8) SL	3
(9) OL	5
(11) Spec. Prop.	1
(12) APD	2
(10) Fidelity / Surety	9
(13) Other	8
(16) Reins. Prop. / Fin.	2
(17) Reins. Liab.	8
(18) PL	10

7. Years of NEP >0

The baseline filtering excludes data points from LOBs where the company has had less than five years of positive NEP in that LOB. The five-year trigger was selected given that some minimum seemed appropriate, and we wanted to test a criterion that was less strict than the 10-year requirement in the CCM.

To evaluate the extent to which PRFs vary by years of NEP, we grouped the data points based on the number of years of positive NEP for the LOB-company/pool and calculated the PRFs for each data group.

Table 7.1 shows the premium and number of data points in each of the NEP>0 year groupings. We see that the 20 and over group is a significant proportion of the total:

[28] For Auto Physical Damage and Special Property LOBs, the PRC%s at 12 months are slightly higher, rather than lower, than the mature PRC%s.

approximately 90% of the premium and approximately 59% of the data points. There is relatively little data in the category 0-4 years of NEP>0.[29]

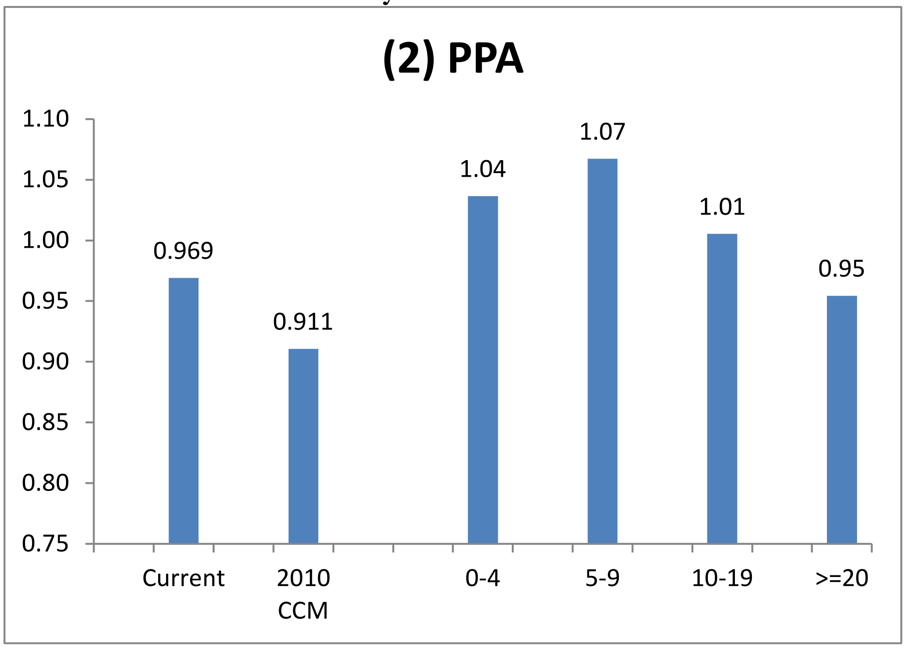
Table 7.1
Premium and Data Points by Number of Years NEP>0

	All-Year Premium ($millionss)						All-Year Data Points				
LOB	0-4	5-9	10-19	>=20	Total		0-4	5-9	10-19	>=20	Total
(1) H/F	2,305	10,256	75,576	713,396	801,534		317	920	1,862	6,735	9,834
(2) PPA	2,207	12,449	80,093	1,514,014	1,608,763		344	894	2,237	5,557	9,032
(3) CA	1,106	4,574	17,069	241,078	263,828		249	614	1,625	3,822	6,310
(4) WC	3,131	25,866	65,965	662,871	757,832		349	754	1,812	3,568	6,483
(5) CMP	1,283	5,561	32,078	408,948	447,870		240	577	1,734	4,882	7,433
(6) MPL Occ.	662	1,230	1,376	24,941	28,208		72	222	248	644	1,186
(7) MPL C-M	909	6,795	3,759	63,415	74,879		213	818	495	1,171	2,697
(8) SL	128	1,169	2,378	35,911	39,585		71	190	320	625	1,206
(9) OL	1,319	6,105	32,586	429,753	469,763		407	1,029	2,095	5,580	9,111
(11) Spec. Prop.	2,049	5,967	27,412	262,821	298,248		330	815	2,696	5,382	9,223
(12) APD	1,494	5,243	65,548	909,953	982,239		395	863	2,942	5,680	9,880
(10) Fidelity / Surety	123	699	3,505	101,102	105,429		89	213	396	836	1,534
(13) Other	454	4,358	24,704	49,338	78,854		114	325	773	609	1,821
(15) International	4,687	2,482	21,763	9,044	37,976		20	19	38	21	98
(16) Reins. Prop. / Fin.	123	1,151	5,725	45,707	52,706		70	184	311	559	1,124
(17) Reins. Liab.	238	2,851	9,510	107,832	120,431		93	227	273	620	1,213
(18) PL	288	764	3,446	26,906	31,404		55	48	225	383	711
Total	22,507	97,520	472,494	5,607,029	6,199,550		3,428	8,712	20,082	46,674	78,896
	0%	2%	8%	90%	100%		4%	11%	25%	59%	100%

Table 7.2 shows the PRFs grouped in bands by "number of years" for the PPA LOB.

[29] Some of the data points in the NEP<5 category have already been removed from the data set by the minor lines or the size threshold filters.

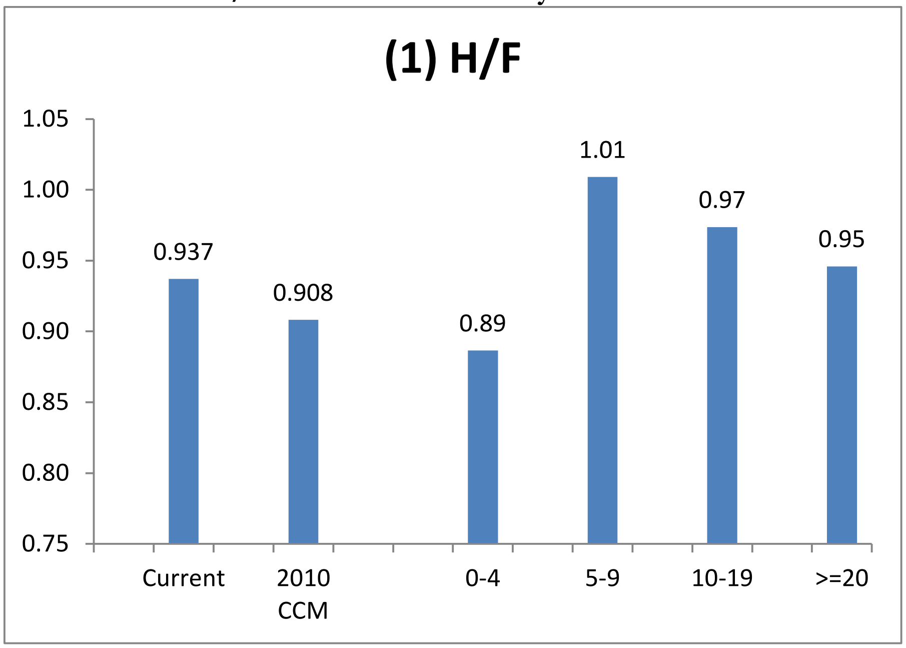
Table 7.2
PPA – PRF by Number of Years NEP>0



For the groups with more than four years of NEP>0, we see a decrease in the PRF as years of NEP>0 increase, as might be expected if variability is lower the longer a company is in business for a LOB.

Table 7.3 shows the PRFs grouped by number of years NEP>0 for the Homeowners/Farmowners LOB.

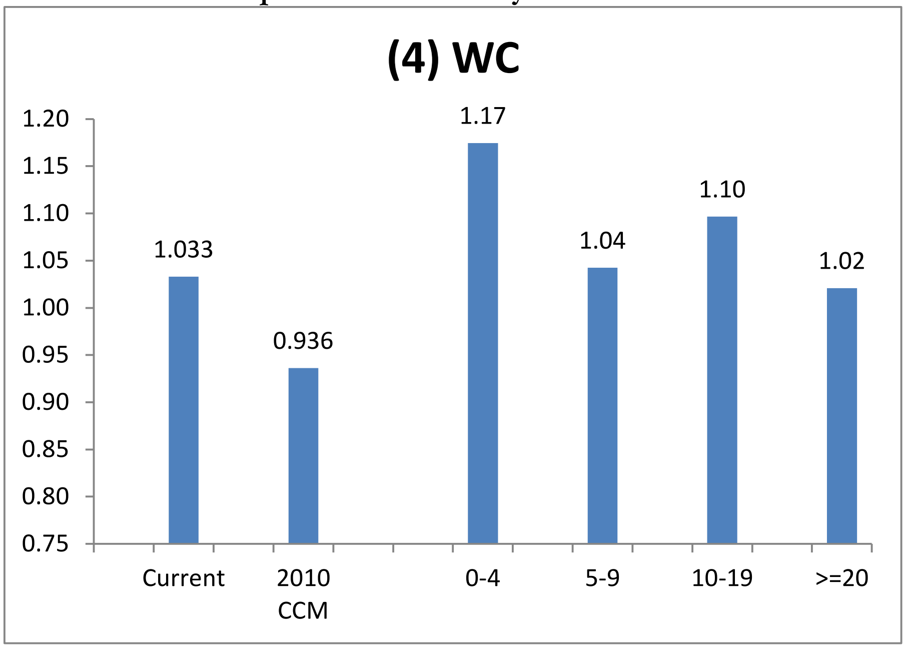
Table 7.3
Homeowners/Farmowners – PRF by Number of Years NEP>0



Here again we see a decreasing pattern for the groups with more than four years NEP>0.

Table 7.4 shows the PRFs grouped by number of years NEP>0 for the workers compensation LOB.

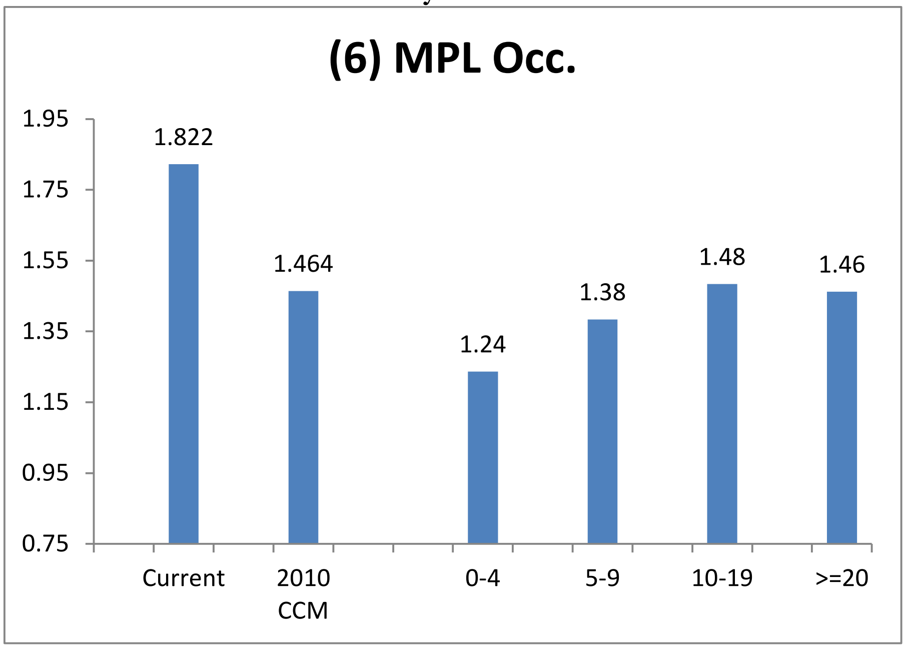
Table 7.4
Workers Compensation – PRF by Number of Years NEP>0



Unlike the case for the other LOBs, the values for groups with more than four years NEP>0 do not exhibit a monotonically-decreasing pattern.

Table 7.5 shows the PRFs grouped by number of years NEP>0 for the MPL – Occurrence LOB. As with workers compensation, there is no pattern to the PRFs based on years of NEP>0.

Table 7.5
MPL Occ. – PRF by Number of Years NEP>0



Corresponding tables for all LOBs are shown in Appendix F.

8. Survivorship

In compiling the baseline data set, we assumed that companies had a 2010 Annual Statement. Consequently, we obtained loss ratios for AYs 2001-2010 from the 2010 Annual Statement. We then used the 2009 Annual Statement to obtain AY 2000 loss ratios, the 2008 Annual Statement to obtain AY 1999 loss ratios, etc.

However, later research revealed companies which had no 2010 Annual Statement but did have data for AYs 2001-2009. To test the effect of including this additional data, we adjusted our data set to use the 10 AYs from the latest available Annual Statement, even if the latest available Annual Statement was not the 2010 Annual Statement.

This revised process added approximately 9,100 data points, an increase of about 13%. Table 8.1 below summarizes the comparison of data points and indicated PRC%s using the two data sets.

Table 8.1
Effect of "Survivorship Bias" on Indicated PRC%

	Baseline		Revised		Difference	
LOB	Data Points Used	Indicated PRC%	Data Points Used	Indicated PRC%	Data Points Used	Indicated PRC%
(1) H/F	7,720	19.6%	8,372	20.1%	652	0.5%
(2) PPA	7,828	15.9%	8,663	17.0%	835	1.0%
(3) CA	4,923	24.1%	5,580	26.0%	657	1.9%
(4) WC	5,750	25.6%	6,844	26.8%	1,094	1.2%
(5) CMP	6,640	23.0%	7,467	24.2%	827	1.2%
(6) MPL Occ.	951	49.6%	1,083	55.1%	132	5.4%
(7) MPL C-M	2,325	38.0%	2,686	43.9%	361	5.8%
(8) SL	967	29.0%	1,079	30.6%	112	1.7%
(9) OL	7,719	35.4%	8,679	37.1%	960	1.7%
(11) Spec. Prop.	8,385	25.1%	9,431	26.2%	1,046	1.1%
(12) APD	9,174	17.2%	10,402	18.5%	1,228	1.3%
(10) Fidelity / Surety	1,394	31.4%	1,655	33.8%	261	2.4%
(13) Other	1,652	31.0%	2,119	32.1%	467	1.1%
(15) International	77	23.9%	91	23.0%	14	-0.9%
(16) Reins. Prop. / Fin.	1,000	50.6%	1,143	50.1%	143	-0.6%
(17) Reins. Liab.	1,061	49.4%	1,251	48.7%	190	-0.7%
(18) PL	637	43.8%	760	46.2%	123	2.4%
(14) Financial / Mortgage	18	153.1%	70	74.1%	52	-79.1%
(19) Warranty	29	55.3%	30	52.6%	1	-2.7%
Total	68,250		77,405		9,155	

In general, the more inclusive data set produces higher PRFs and PRC%s. For most LOBs, the adjusted PRC%s are about one percent higher. The effect is larger for MPL. The effect is slightly beneficial for Reinsurance.

We refer to this adjustment as "survivorship" because it corrects for the apparent bias introduced when companies drop out of the data set.

9. Further Research

DCWP is conducting research in the following areas, and reports will be published in due course.

1. Variation in PRFs and PRC% by type of company; e.g., personal lines, professional reinsurer, etc.
2. Variation in PRFs and PRC% based on data including expenses; i.e., combined ratio rather than loss ratio.
3. Solvency II modeling approach vs. the "empirical approach" used in the research.

There are a number of other interesting issues, but DCWP is not now conducting research on those areas. These include the following:

4. Issues identified in the report:
    a. Effect of maturity for experience periods other than 1997-2000.
    b. Effect of workers compensation tabular reserve on observed maturity effect.
    c. The extent to which the 23 AYs of experience in this data set does or does not sufficiently reflect the extent of systemic and cyclical variability in all lines of business.
5. Interactions between PRF calibration and own-company adjustment and other aspects of the filtering used in final calibration. It seems logical that industry average loss ratios used in the own company adjustment process should be based on industry average from companies that satisfy the filtering used to calibrate the PRFs; e.g., excluding minor lines and LOB-size above the size threshold. This report does not examine the impact of that issue.
6. Investment Income offset – The investment income offset might best be determined considering the years used to calibrate the PRF, as higher interest rates would produce higher loss ratios and higher PRFs in the past.
7. Risk metrics
    a. Higher confidence levels, e.g., 90%, 95%,… vs. 87.5%
    b. TVaR vs. VaR vs. Butsic (risk-adjusted VaR, DCWP Report 5)

8. Risk metric – Currently it is based on a percentile over all data points all years. Alternatives include percentiles determined:

    a. within years, or

    b. within companies.

10. Authors

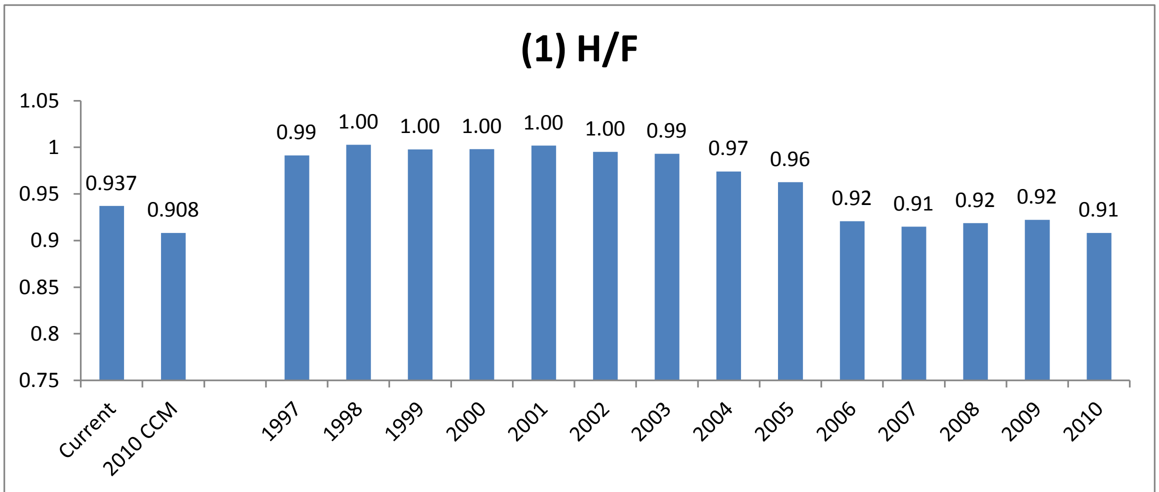
Principal Authors:
Analysis: Jennifer Wu, Allan Kaufman
Text: Dennis Franciskovich, Allan Kaufman
Other work stream members:
Karen H. Adams, Daniel M. Murphy, Timothy Sweetser, Giuseppe (Franco) LePera

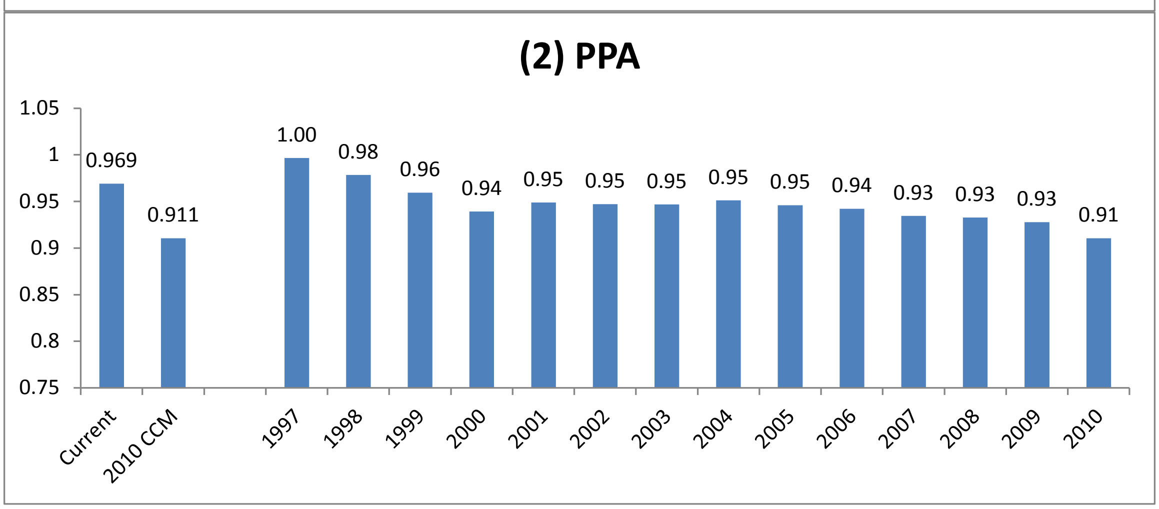
Work was supported by the DCWP working party with membership as follows:

Allan M. Kaufman, Chair
Karen H. Adams
Emmanuel Theodore Bardis
Jess B. Broussard
Robert P Butsic
Pablo Castets
Joseph F. Cofield
Jose R. Couret
Orla Donnelly
Chris Dougherty
Brian A. Fannin
Sholom Feldblum
Kendra Felisky
Dennis A. Franciskovich
Spencer Gluck
James C. Guszcza
Jed Nathaniel Isaman
Shira L. Jacobson
Shiwen Jiang
Alex Krutov
Terry T. Kuruvilla
Apundeep Singh Lamba
Giuseppe F. LePera
Zhe Robin Li
Lily (Manjuan) Liang
Thomas Toong-Chiang Loy
Eduardo P. Marchena
Mark McCluskey
James P. McNichols
Glen G. Meyers
Daniel M. Murphy
Douglas Robert Nation
G. Chris Nyce
Jeffrey J. Pfluger
Yi Pu
Ashely Arlene Reller
David A. Rosenzweig
David L. Ruhm
Andrew Jon Staudt
Timothy Delmar Sweetser
Anna Marie Wetterhus
Jennifer X. Wu
Jianwei Xie
Ji Jyo
Linda Zhang
Christina Tieyan Zhou

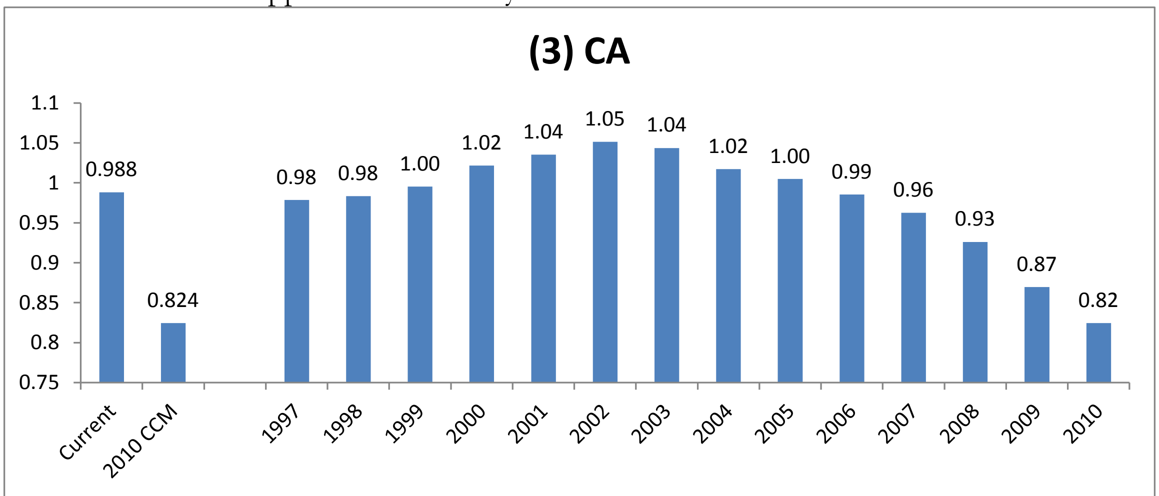
CAS Staff – Karen Sonnet
Actuarial Students – Damon Chom, Francis Guo

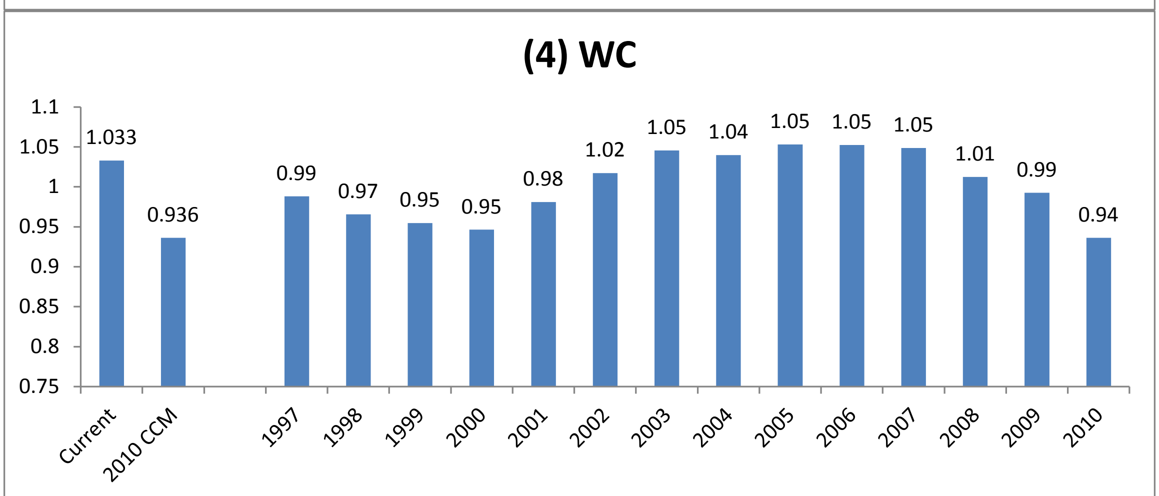
Appendix A –PRF by Statement Year Based on CCM

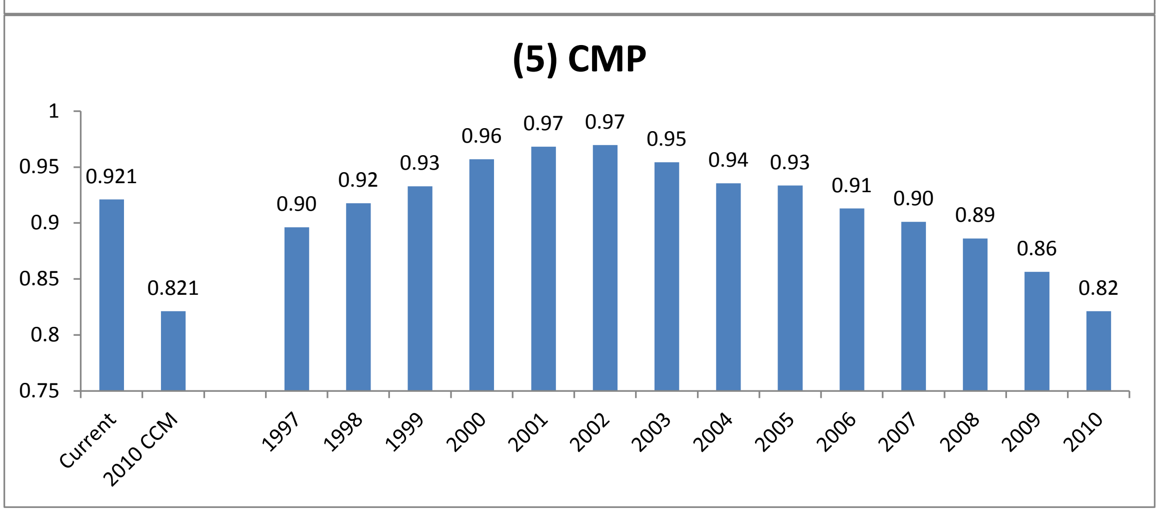




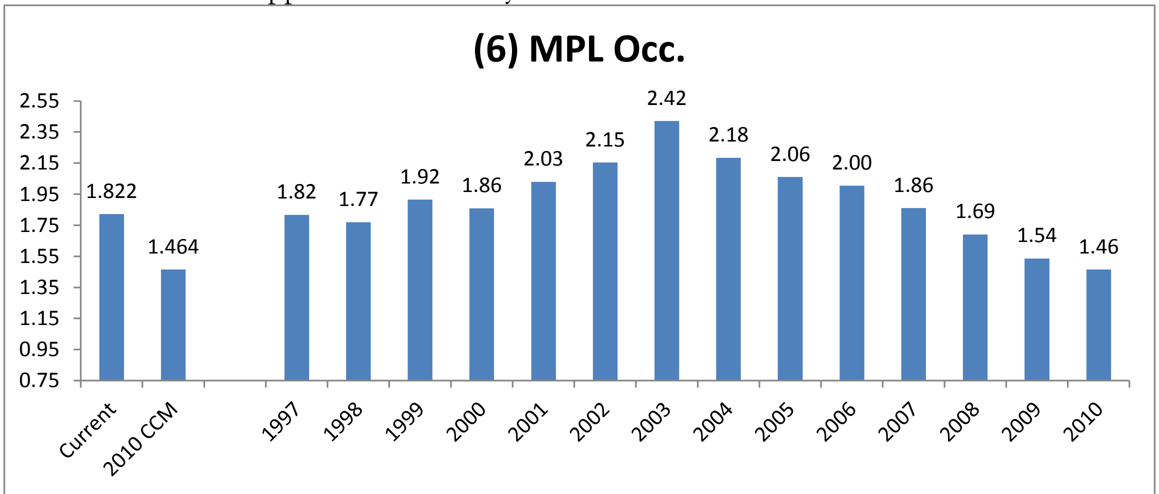
Appendix A - PRF by Statement Year Based on CCM

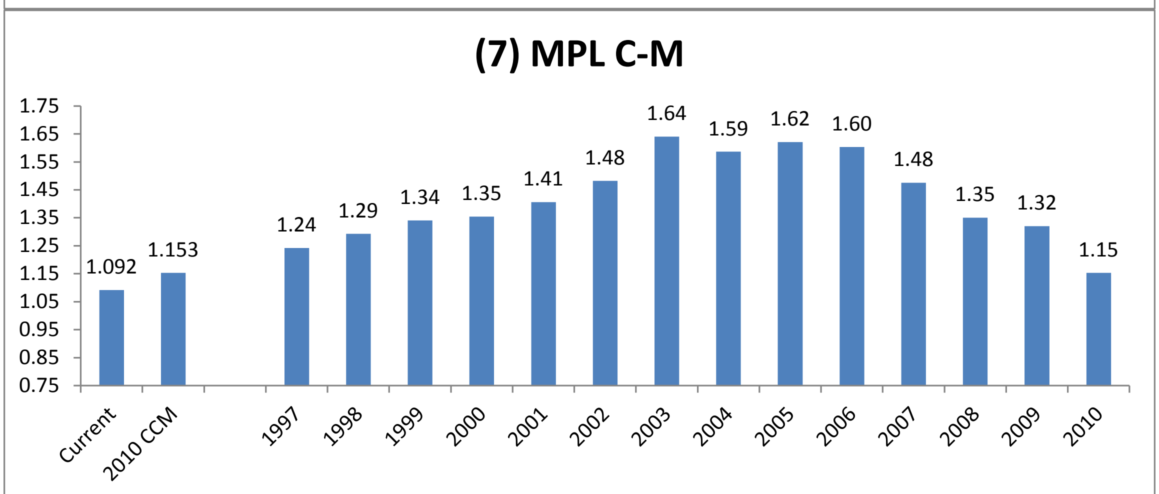


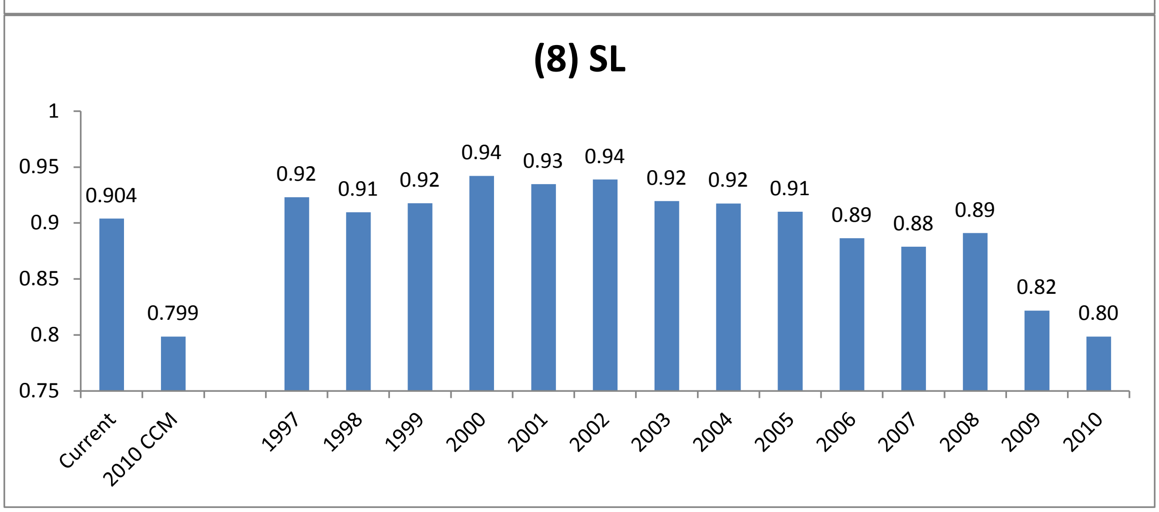




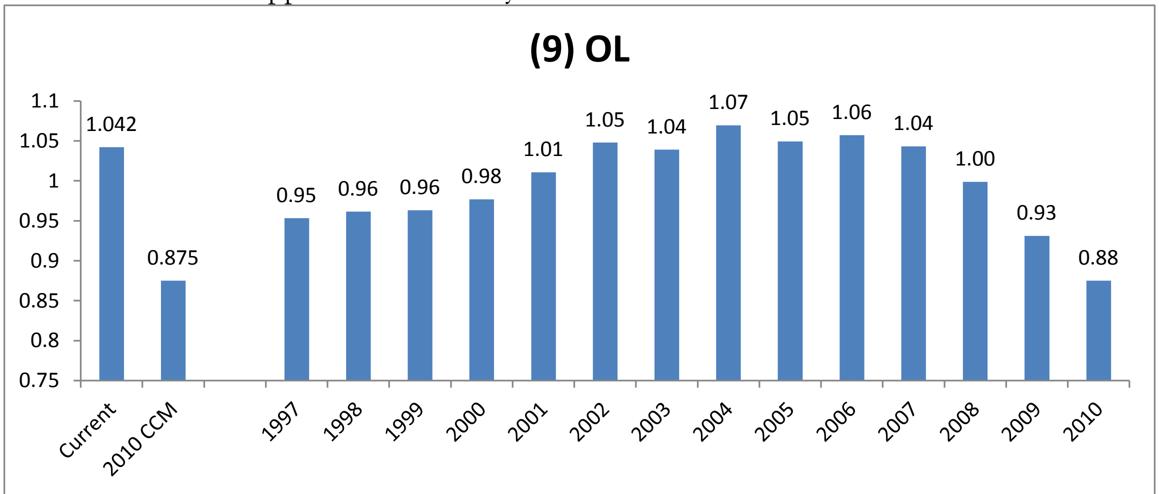
Appendix A - PRF by Statement Year Based on CCM

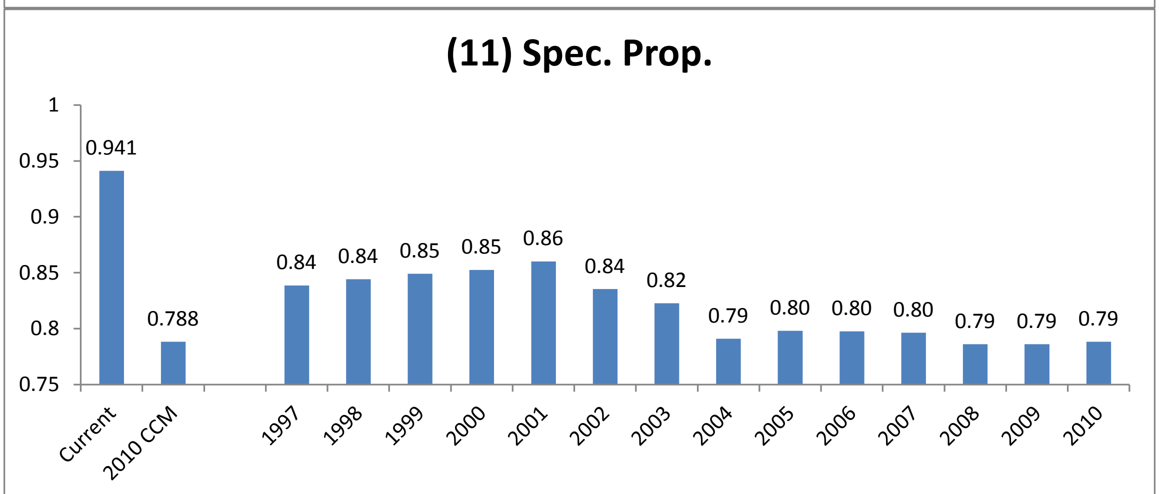


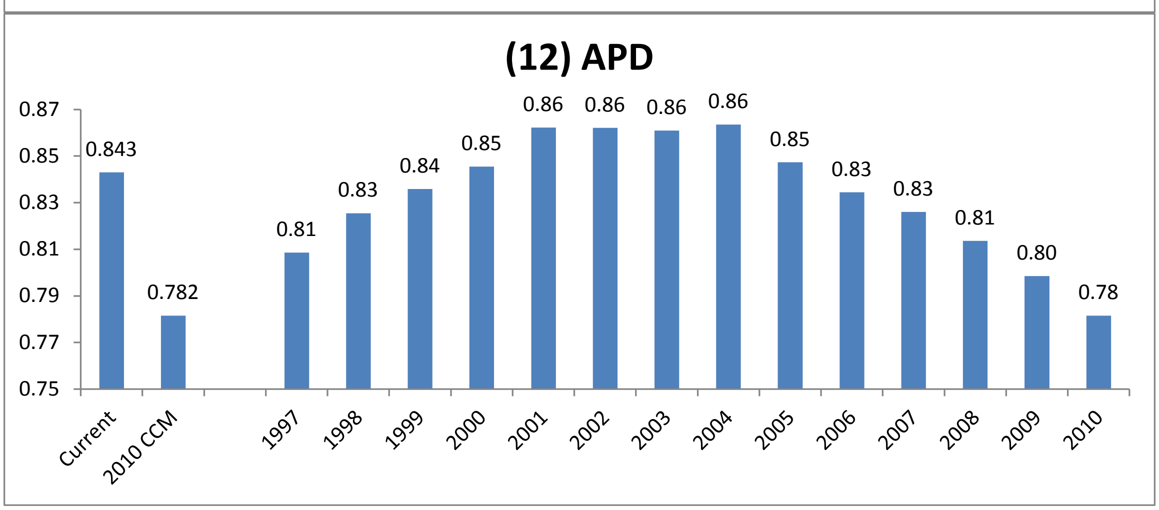




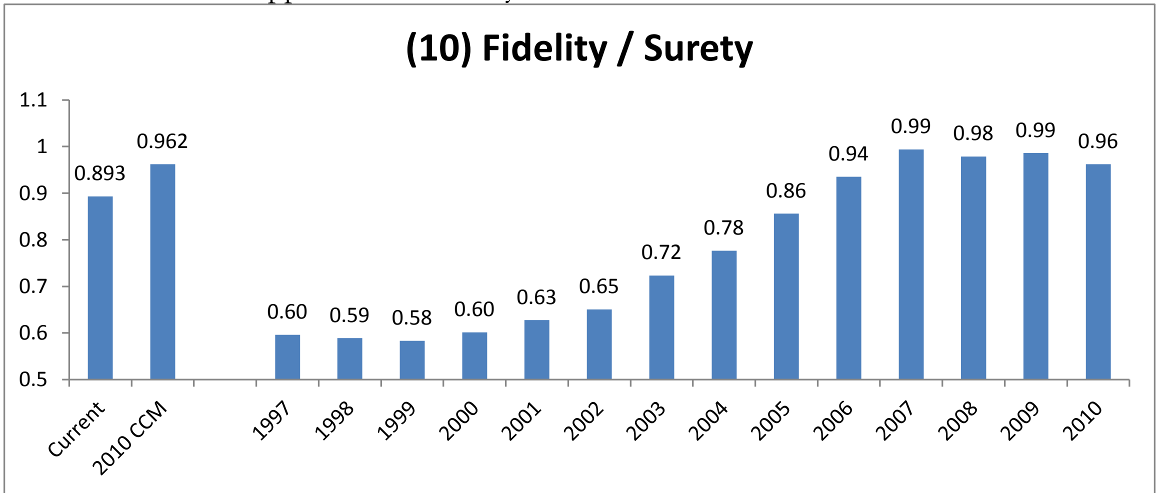
Appendix A - PRF by Statement Year Based on CCM

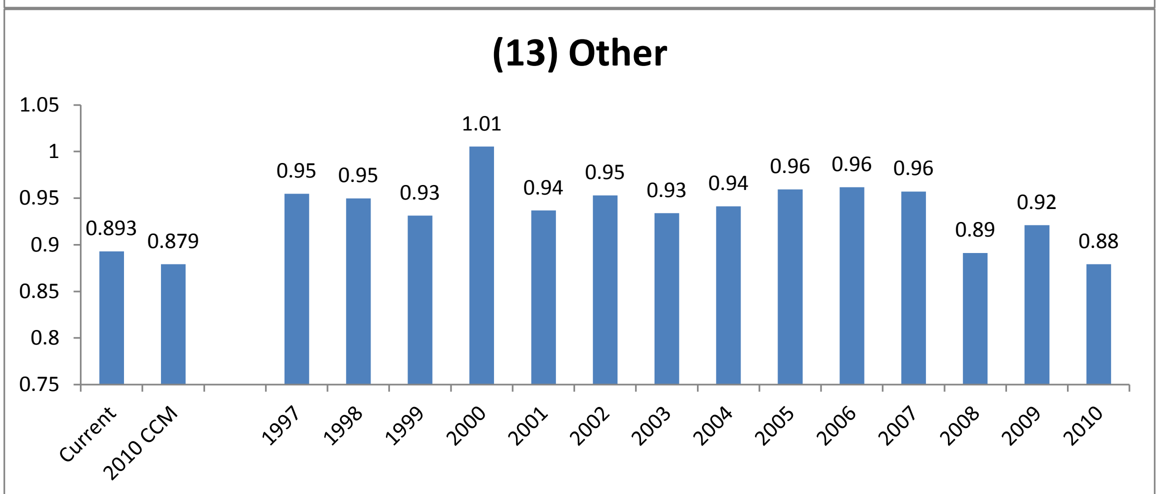


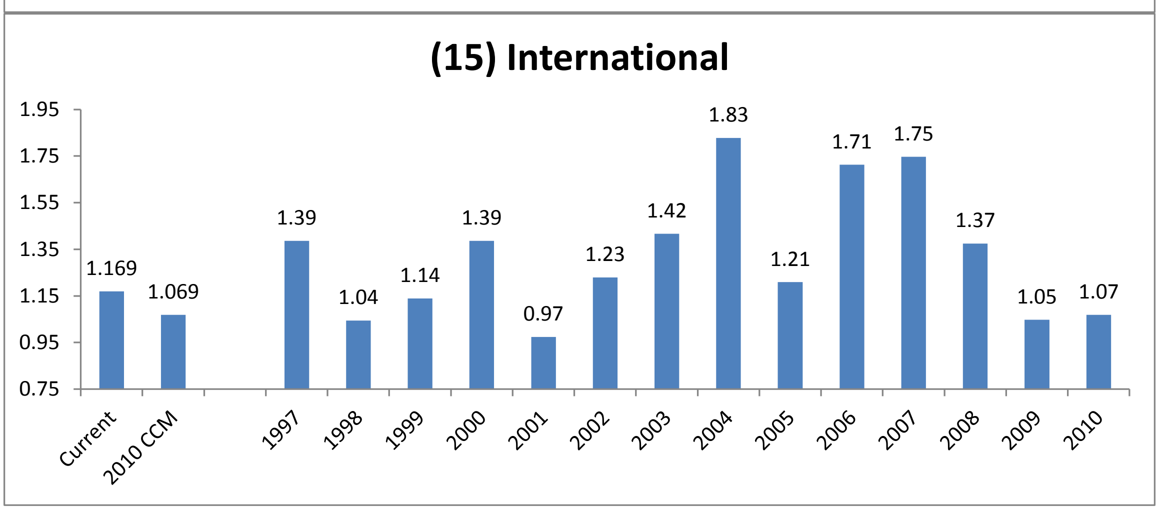




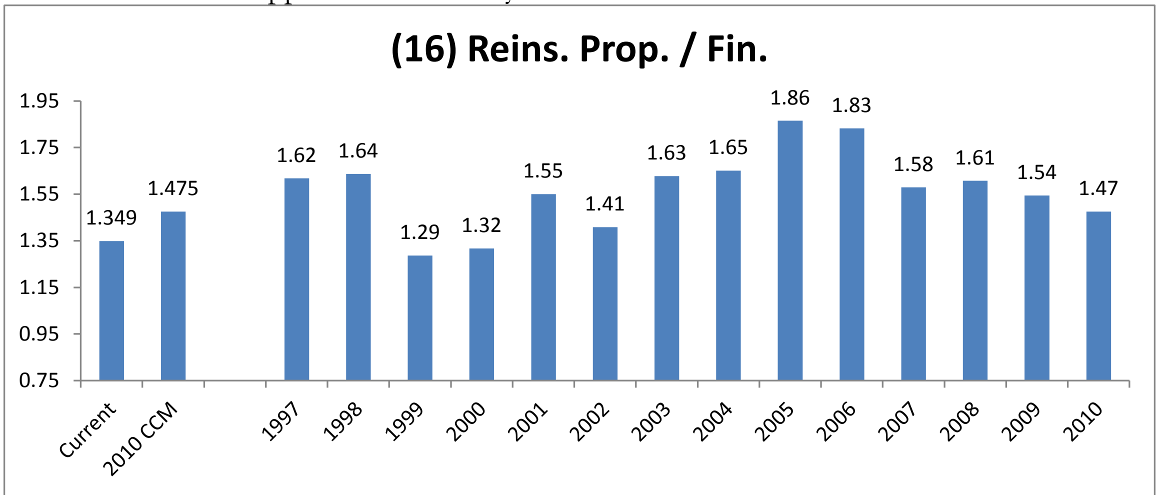
Appendix A - PRF by Statement Year Based on CCM

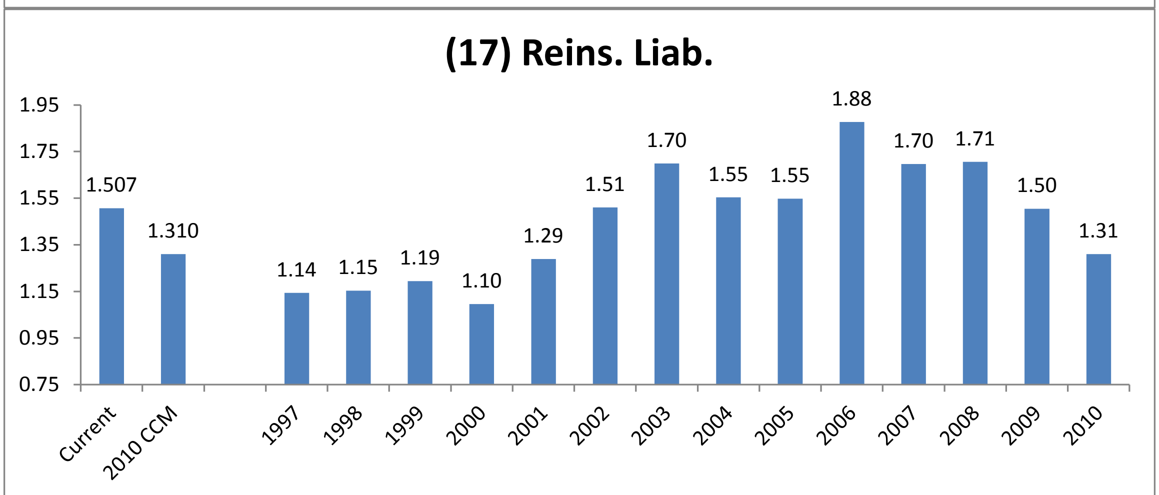


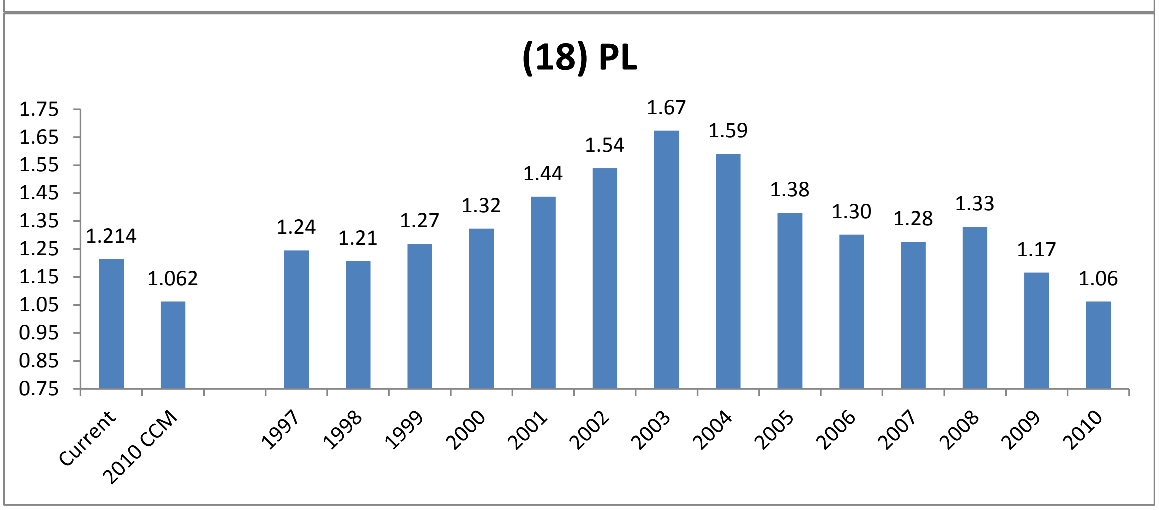




Appendix A - PRF by Statement Year Based on CCM

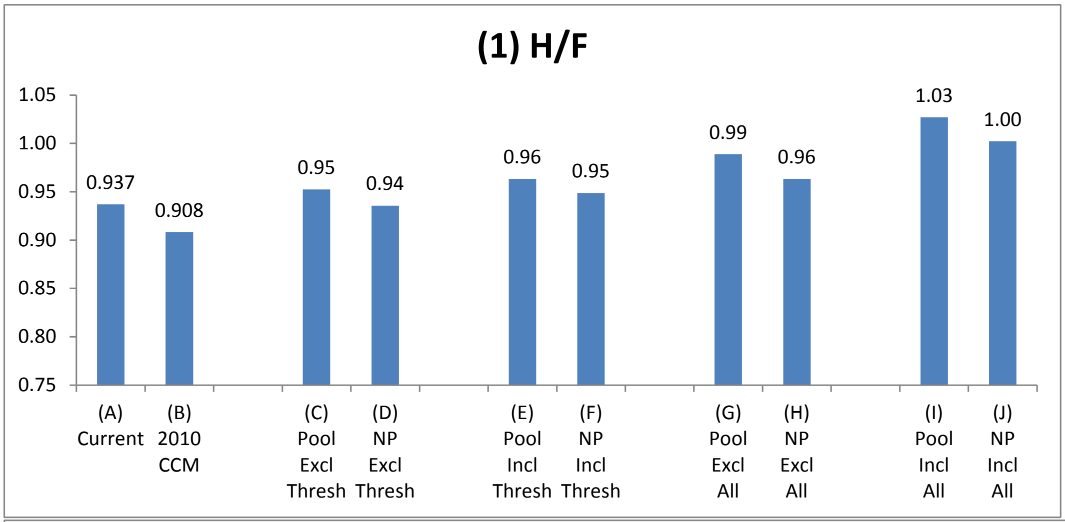


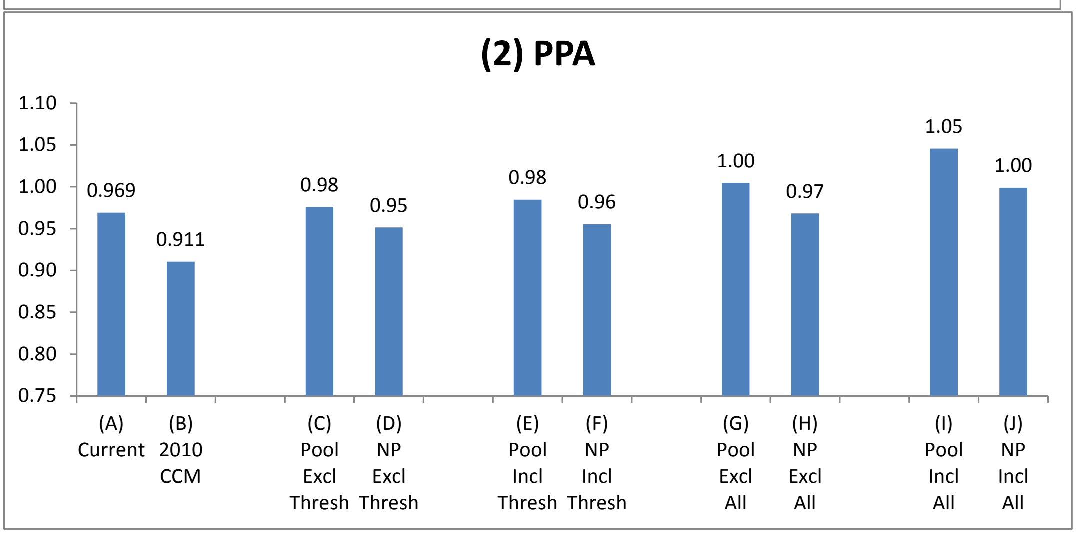


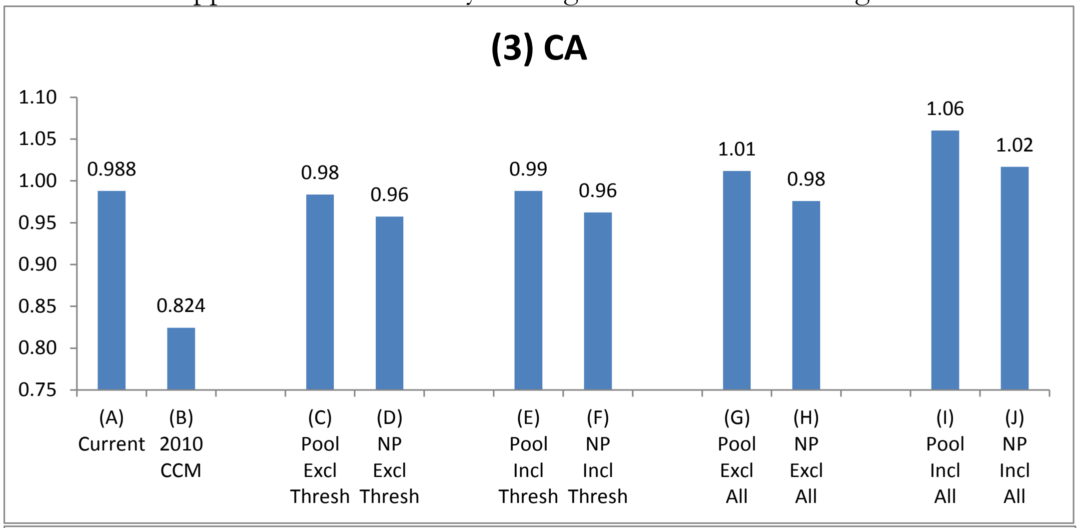


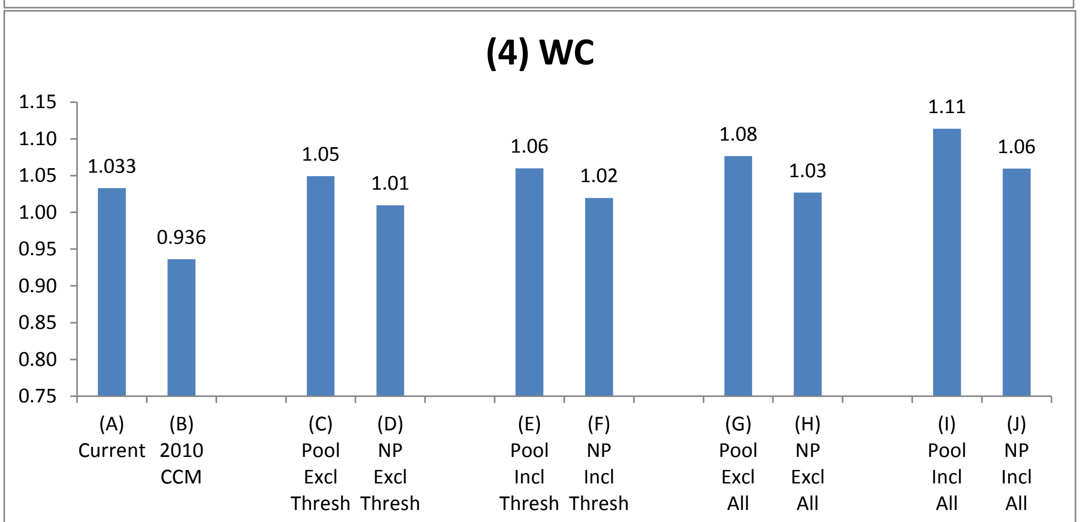
Note: (14) Financial/Mortgage and (19) Warranty LOBs are not shown as data for those lines is so new and sparse that charts are not meaningful.

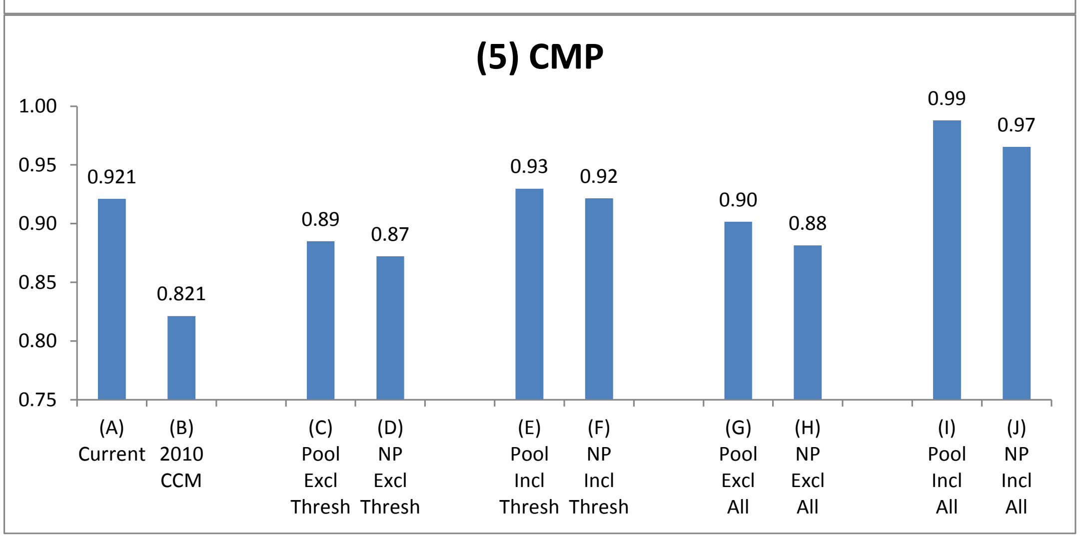
Appendix B – Sensitivity Testing of Alternative Filtering Methods

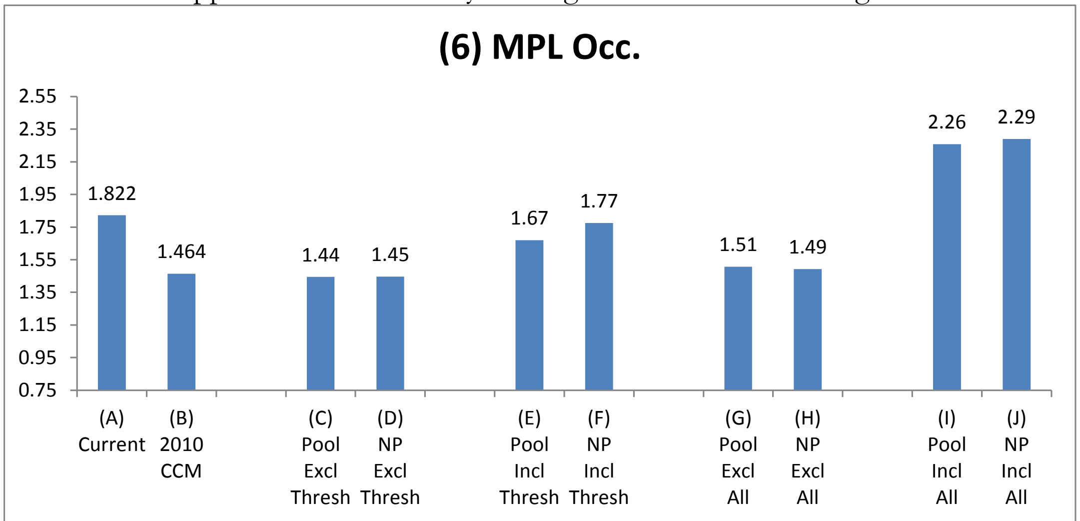


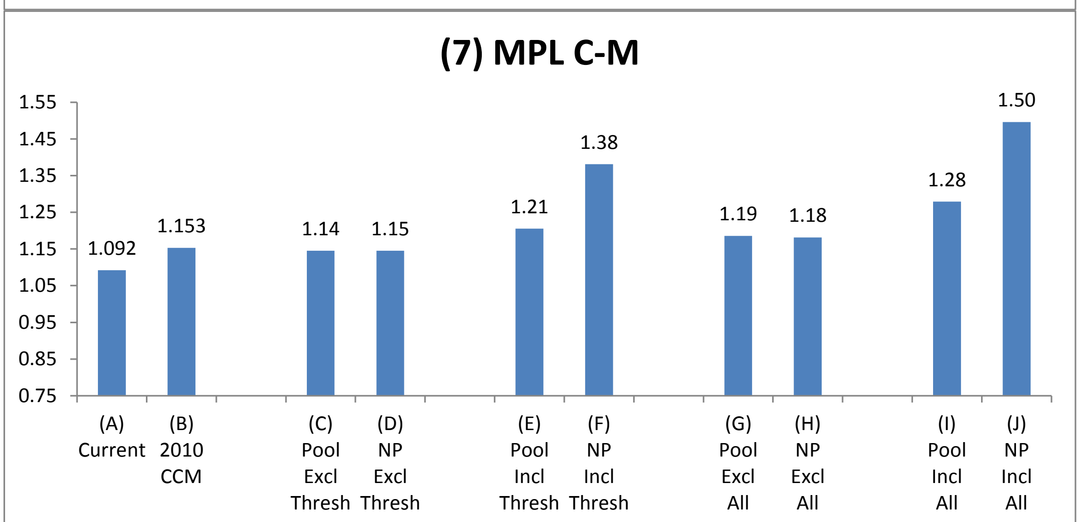


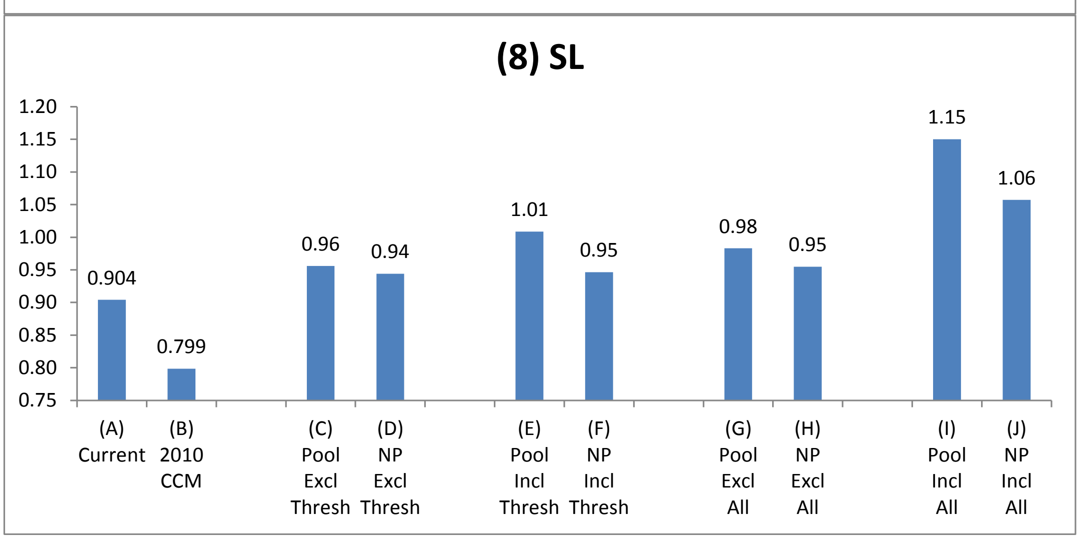

(3) CA
1.10
1.05
1.00
0.95
0.90
0.85
0.80
0.75
0.988
0.824
0.98
0.96
0.99
0.96
1.01
0.98
1.06
1.02
(A) Current
(B) 2010 CCM
(C) Pool Excl Thresh
(D) NP Excl Thresh
(E) Pool Incl Thresh
(F) NP Incl Thresh
(G) Pool Excl All
(H) NP Excl All
(I) Pool Incl All
(J) NP Incl All

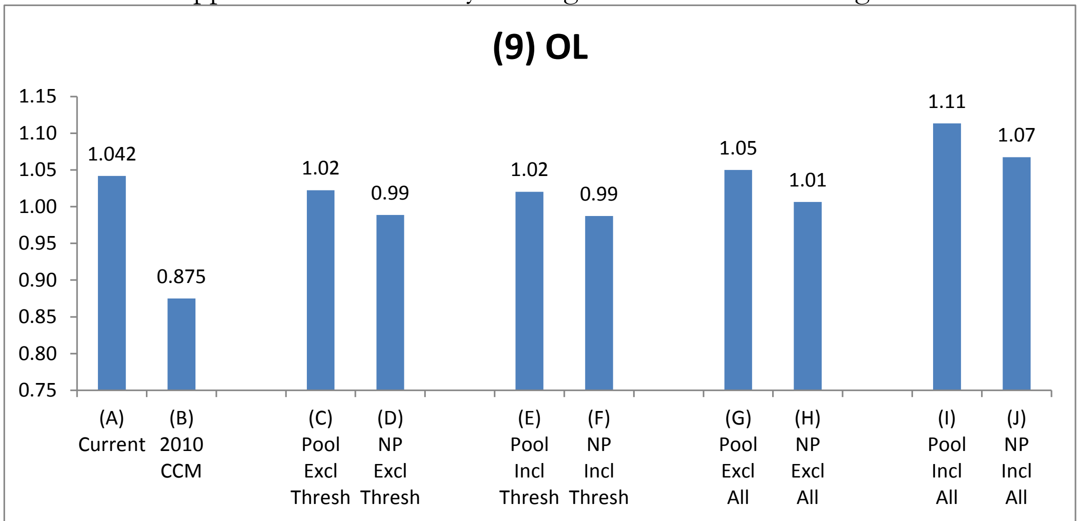

(4) WC
1.15
1.10
1.05
1.00
0.95
0.90
0.85
0.80
0.75
1.033
0.936
1.05
1.01
1.06
1.02
1.08
1.03
1.11
1.06
(A) Current
(B) 2010 CCM
(C) Pool Excl Thresh
(D) NP Excl Thresh
(E) Pool Incl Thresh
(F) NP Incl Thresh
(G) Pool Excl All
(H) NP Excl All
(I) Pool Incl All
(J) NP Incl All

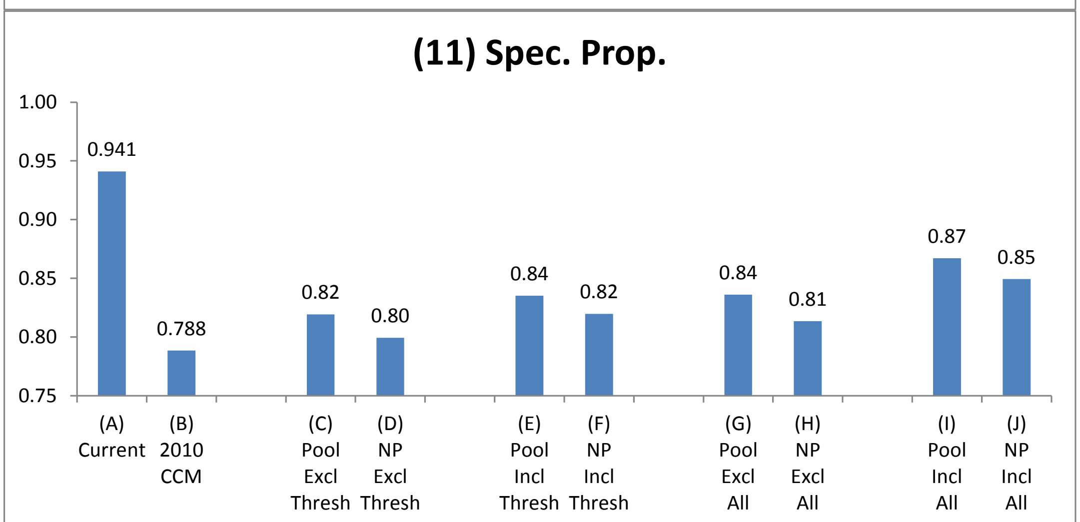

(5) CMP
1.00
0.95
0.90
0.85
0.80
0.75
0.921
0.821
0.89
0.87
0.93
0.92
0.90
0.88
0.99
0.97
(A) Current
(B) 2010 CCM
(C) Pool Excl Thresh
(D) NP Excl Thresh
(E) Pool Incl Thresh
(F) NP Incl Thresh
(G) Pool Excl All
(H) NP Excl All
(I) Pool Incl All
(J) NP Incl All

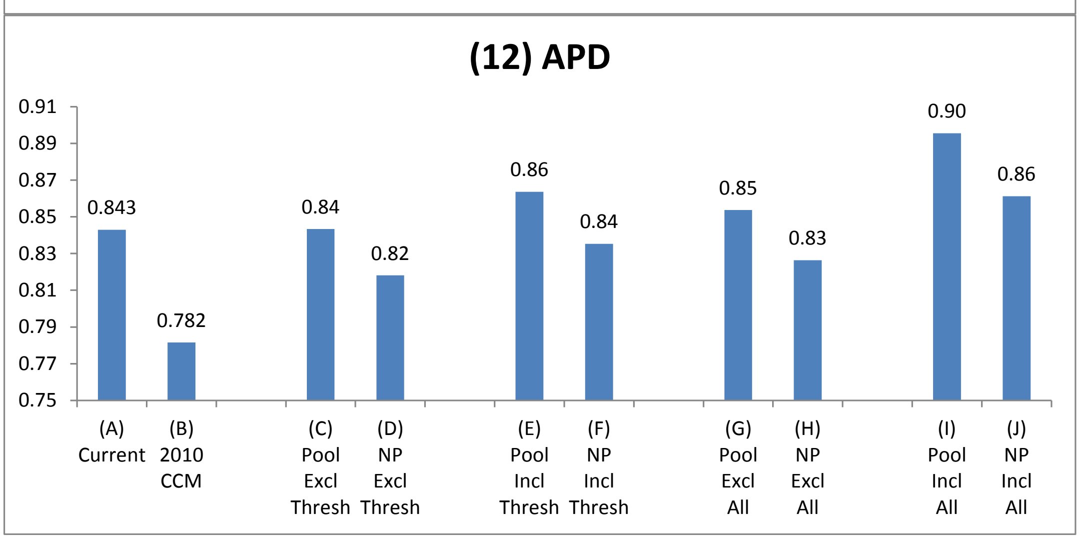

(6) MPL Occ.
2.55
2.35
2.15
1.95
1.75
1.55
1.35
1.15
0.95
0.75
1.822
1.464
1.44
1.45
1.67
1.77
1.51
1.49
2.26
2.29
(A) Current
(B) 2010 CCM
(C) Pool Excl Thresh
(D) NP Excl Thresh
(E) Pool Incl Thresh
(F) NP Incl Thresh
(G) Pool Excl All
(H) NP Excl All
(I) Pool Incl All
(J) NP Incl All

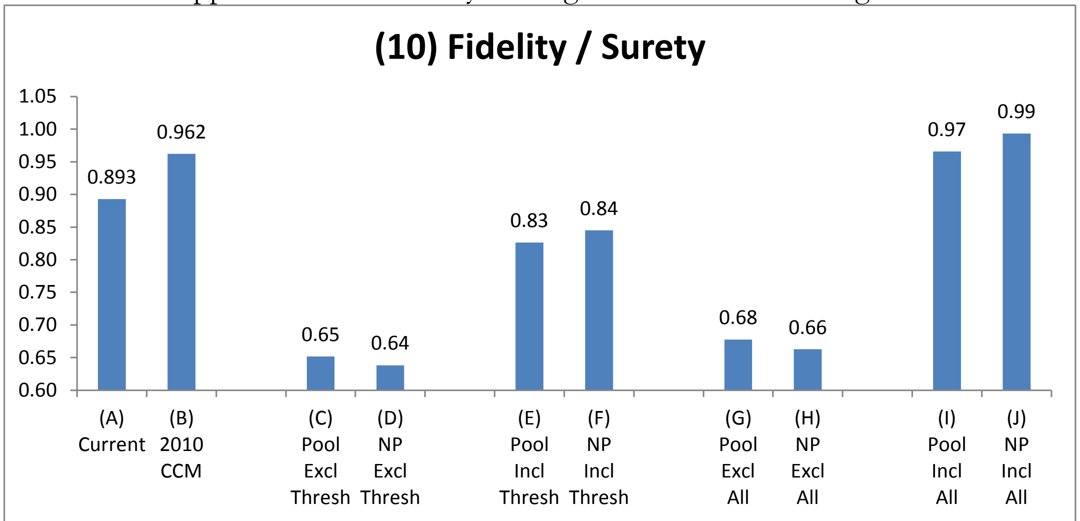

(7) MPL C-M
1.55
1.45
1.35
1.25
1.15
1.05
0.95
0.85
0.75
1.092
1.153
1.14
1.15
1.21
1.38
1.19
1.18
1.28
1.50
(A) Current
(B) 2010 CCM
(C) Pool Excl Thresh
(D) NP Excl Thresh
(E) Pool Incl Thresh
(F) NP Incl Thresh
(G) Pool Excl All
(H) NP Excl All
(I) Pool Incl All
(J) NP Incl All

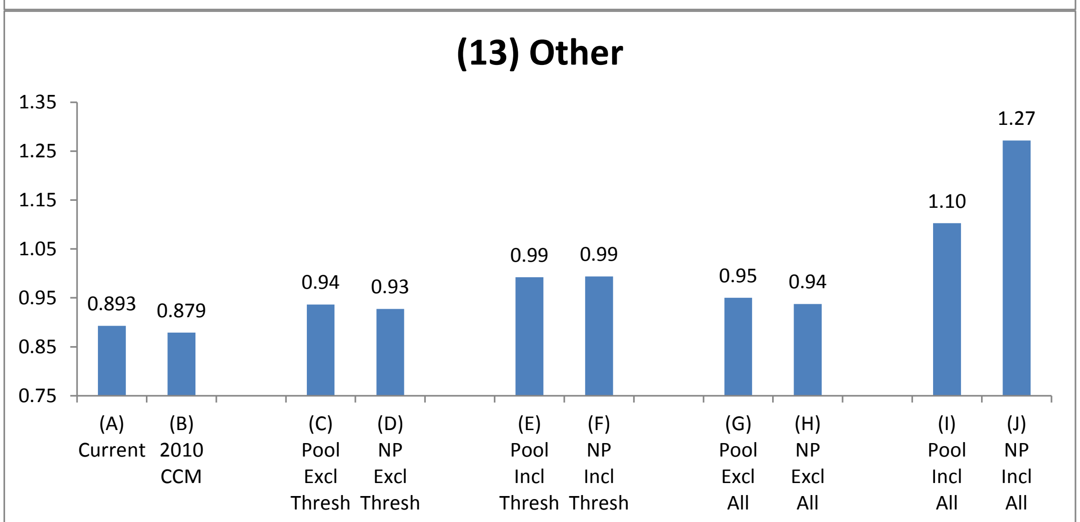

(8) SL
1.20
1.15
1.10
1.05
1.00
0.95
0.90
0.85
0.80
0.75
0.904
0.799
0.96
0.94
1.01
0.95
0.98
0.95
1.15
1.06
(A) Current
(B) 2010 CCM
(C) Pool Excl Thresh
(D) NP Excl Thresh
(E) Pool Incl Thresh
(F) NP Incl Thresh
(G) Pool Excl All
(H) NP Excl All
(I) Pool Incl All
(J) NP Incl All

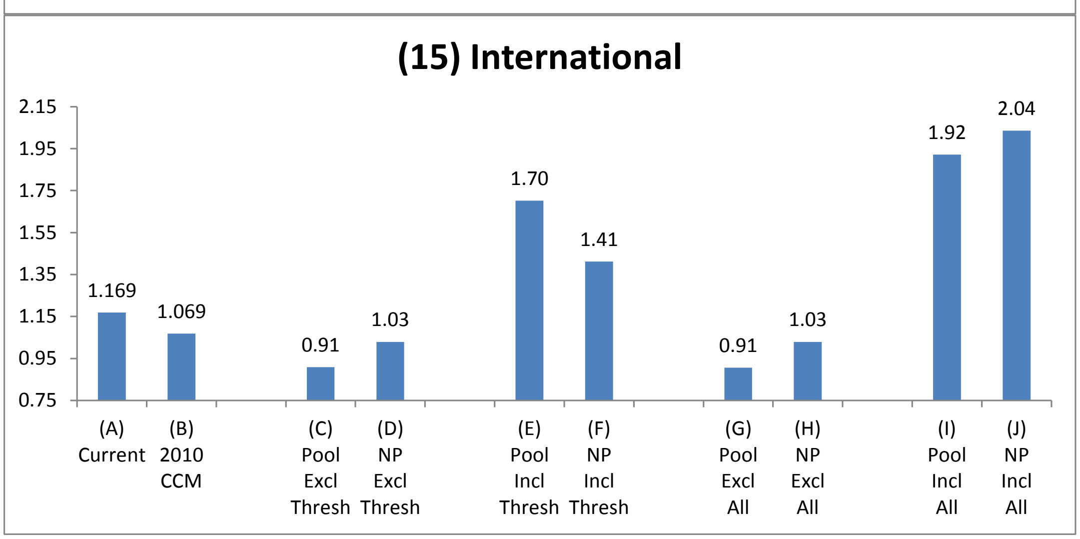

(9) OL
1.042
0.875
1.02
0.99
1.02
0.99
1.05
1.01
1.11
1.07
(A) Current
(B) 2010 CCM
(C) Pool Excl Thresh
(D) NP Excl Thresh
(E) Pool Incl Thresh
(F) NP Incl Thresh
(G) Pool Excl All
(H) NP Excl All
(I) Pool Incl All
(J) NP Incl All

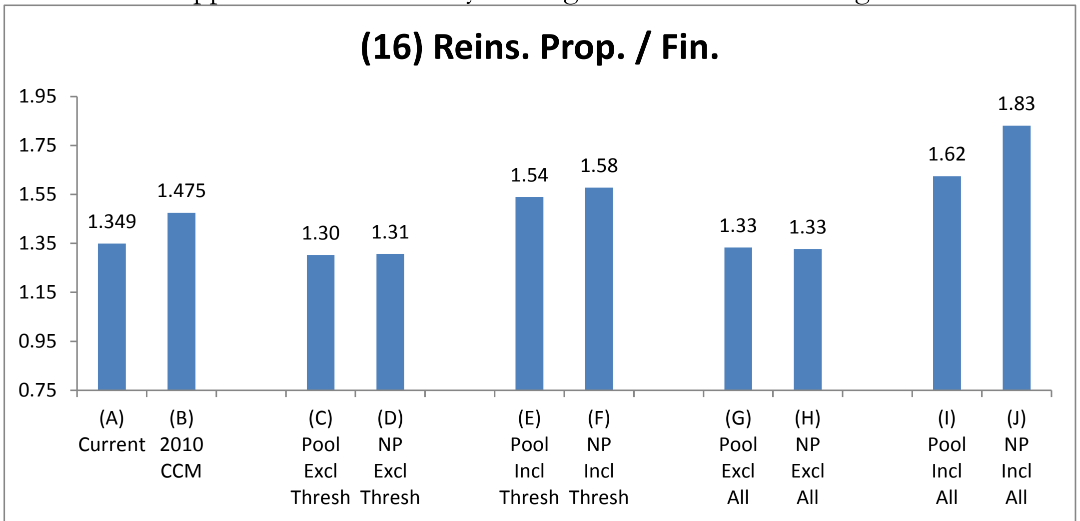

(11) Spec. Prop.
0.941
0.788
0.82
0.80
0.84
0.82
0.84
0.81
0.87
0.85
(A) Current
(B) 2010 CCM
(C) Pool Excl Thresh
(D) NP Excl Thresh
(E) Pool Incl Thresh
(F) NP Incl Thresh
(G) Pool Excl All
(H) NP Excl All
(I) Pool Incl All
(J) NP Incl All

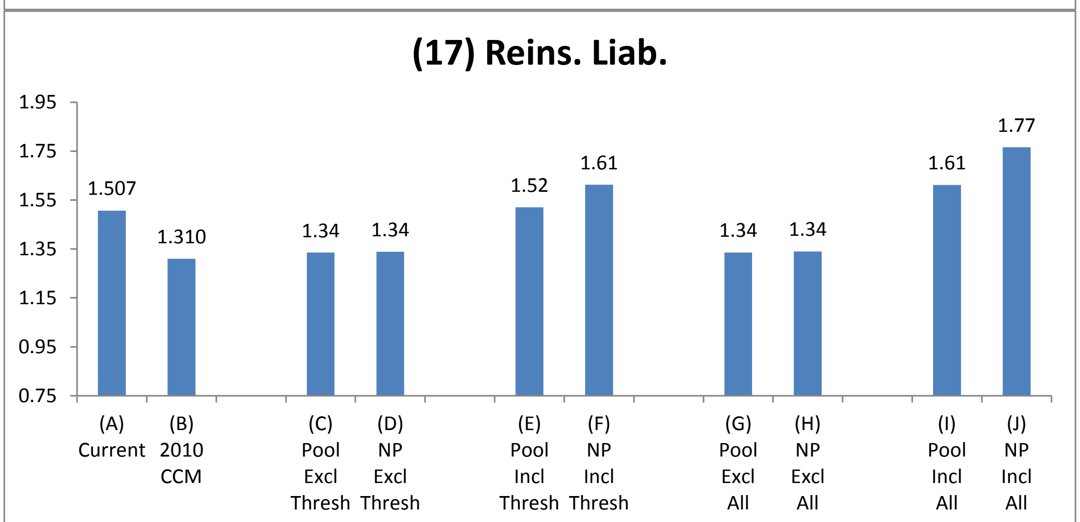

(12) APD
0.843
0.782
0.84
0.82
0.86
0.84
0.85
0.83
0.90
0.86
(A) Current
(B) 2010 CCM
(C) Pool Excl Thresh
(D) NP Excl Thresh
(E) Pool Incl Thresh
(F) NP Incl Thresh
(G) Pool Excl All
(H) NP Excl All
(I) Pool Incl All
(J) NP Incl All

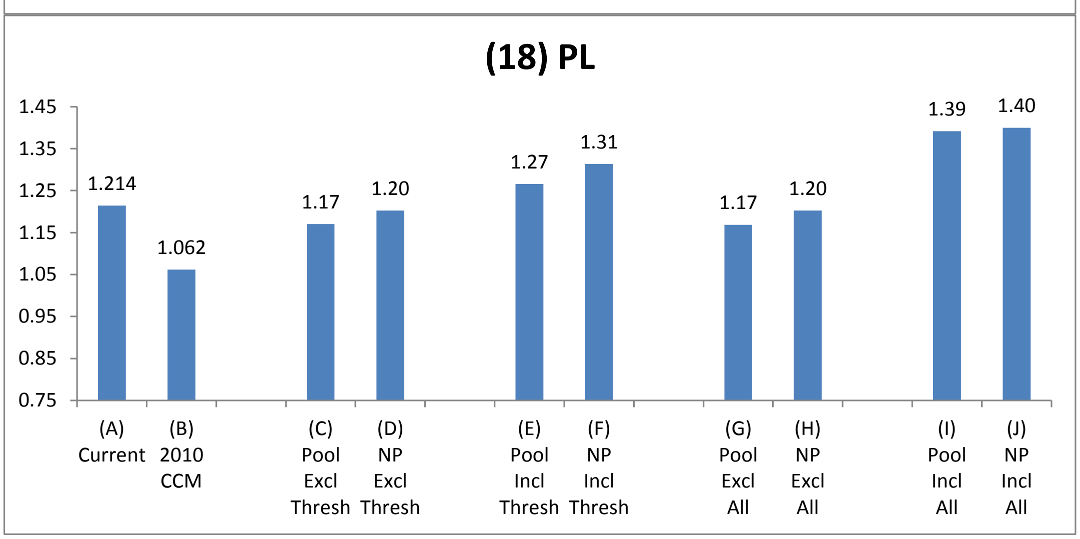

(10) Fidelity / Surety
1.05
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
0.893
0.962
0.65
0.64
0.83
0.84
0.68
0.66
0.97
0.99
(A) Current
(B) 2010 CCM
(C) Pool Excl Thresh
(D) NP Excl Thresh
(E) Pool Incl Thresh
(F) NP Incl Thresh
(G) Pool Excl All
(H) NP Excl All
(I) Pool Incl All
(J) NP Incl All


(13) Other
1.35
1.25
1.15
1.05
0.95
0.85
0.75
0.893
0.879
0.94
0.93
0.99
0.99
0.95
0.94
1.10
1.27
(A) Current
(B) 2010 CCM
(C) Pool Excl Thresh
(D) NP Excl Thresh
(E) Pool Incl Thresh
(F) NP Incl Thresh
(G) Pool Excl All
(H) NP Excl All
(I) Pool Incl All
(J) NP Incl All


(15) International
2.15
1.95
1.75
1.55
1.35
1.15
0.95
0.75
1.169
1.069
0.91
1.03
1.70
1.41
0.91
1.03
1.92
2.04
(A) Current
(B) 2010 CCM
(C) Pool Excl Thresh
(D) NP Excl Thresh
(E) Pool Incl Thresh
(F) NP Incl Thresh
(G) Pool Excl All
(H) NP Excl All
(I) Pool Incl All
(J) NP Incl All


(16) Reins. Prop. / Fin.
1.95
1.75
1.55
1.35
1.15
0.95
0.75
1.349
1.475
1.30
1.31
1.54
1.58
1.33
1.33
1.62
1.83
(A) Current
(B) 2010 CCM
(C) Pool Excl Thresh
(D) NP Excl Thresh
(E) Pool Incl Thresh
(F) NP Incl Thresh
(G) Pool Excl All
(H) NP Excl All
(I) Pool Incl All
(J) NP Incl All


(17) Reins. Liab.
1.95
1.75
1.55
1.35
1.15
0.95
0.75
1.507
1.310
1.34
1.34
1.52
1.61
1.34
1.34
1.61
1.77
(A) Current
(B) 2010 CCM
(C) Pool Excl Thresh
(D) NP Excl Thresh
(E) Pool Incl Thresh
(F) NP Incl Thresh
(G) Pool Excl All
(H) NP Excl All
(I) Pool Incl All
(J) NP Incl All

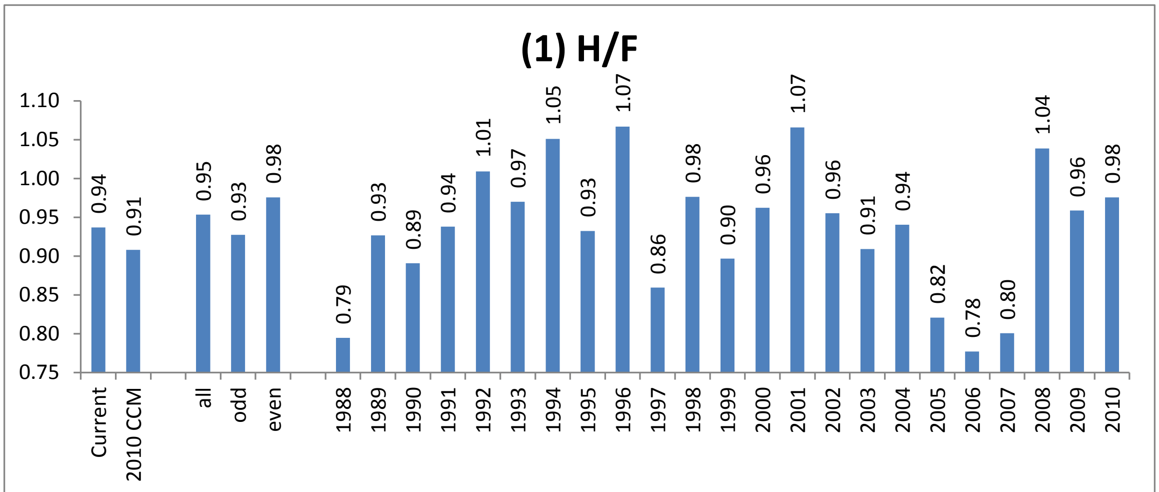

(18) PL
1.45
1.35
1.25
1.15
1.05
0.95
0.85
0.75
1.214
1.062
1.17
1.20
1.27
1.31
1.17
1.20
1.39
1.40
(A) Current
(B) 2010 CCM
(C) Pool Excl Thresh
(D) NP Excl Thresh
(E) Pool Incl Thresh
(F) NP Incl Thresh
(G) Pool Excl All
(H) NP Excl All
(I) Pool Incl All
(J) NP Incl All

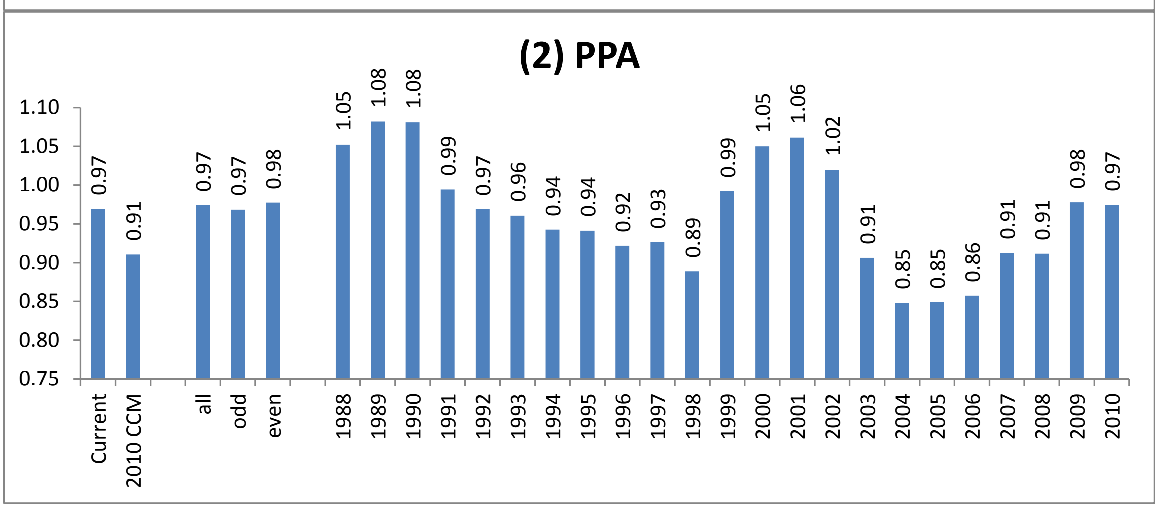
The baseline filter used the judgmentally selected thresholds by line of business shown in Table B1.

Appendix B - Table 1
Selected Baseline LOB-size Thresholds

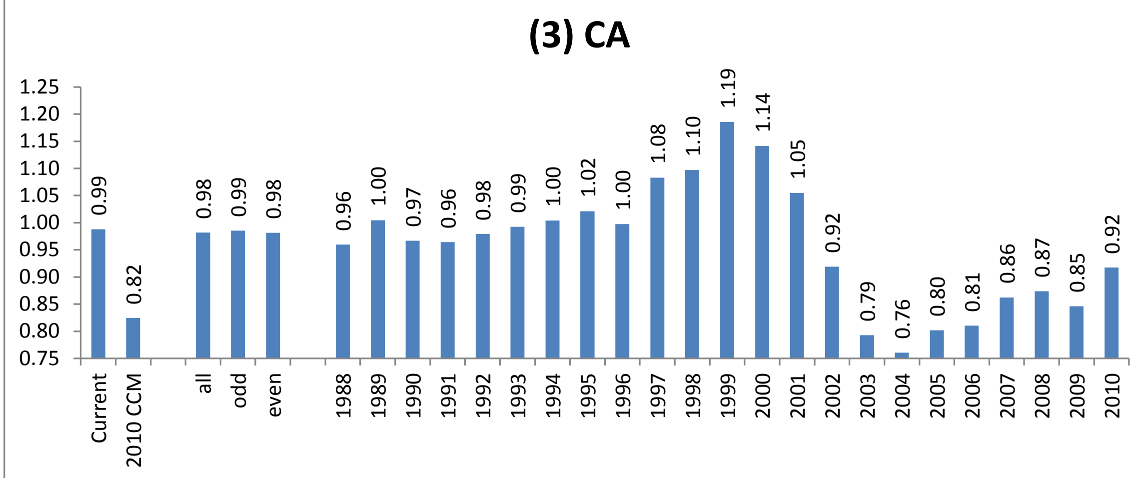
Line of Business	Premium Threshold (000's)
A Homeowners/Farmowners	1,000
B Priv. Passenger Auto Liability	1,000
C Commercial Auto Liability.	1,000
D Workers Compensation	600
E Commercial Multiperil	300
F1 Medical Malpractice – Occurrence	800
F2 Medical Malpractice - Claims made	600
G Special Liability	1,000
H Other Liability	300
I Special Property	200
J Auto Physical Damage	200
K Fidelity & Surety	200
L Other	200
M International	200
N&P Reinsurance A &C (property and financial)	200
O Reinsurance B (liability)	300
R Products Liability	200
S Financial Guarantee	100
T Warranty	0

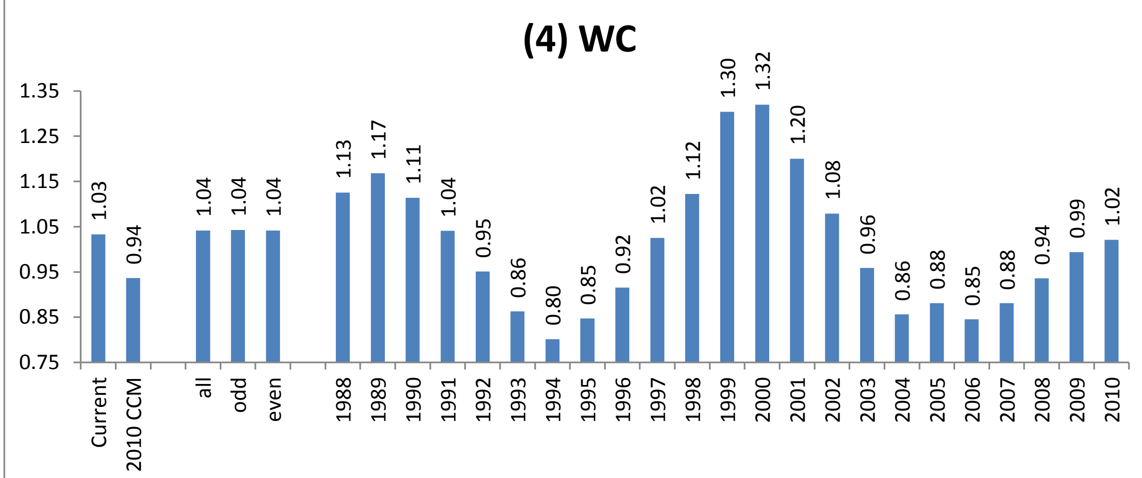
Appendix C –PRF by AY (with Baseline Filtering)

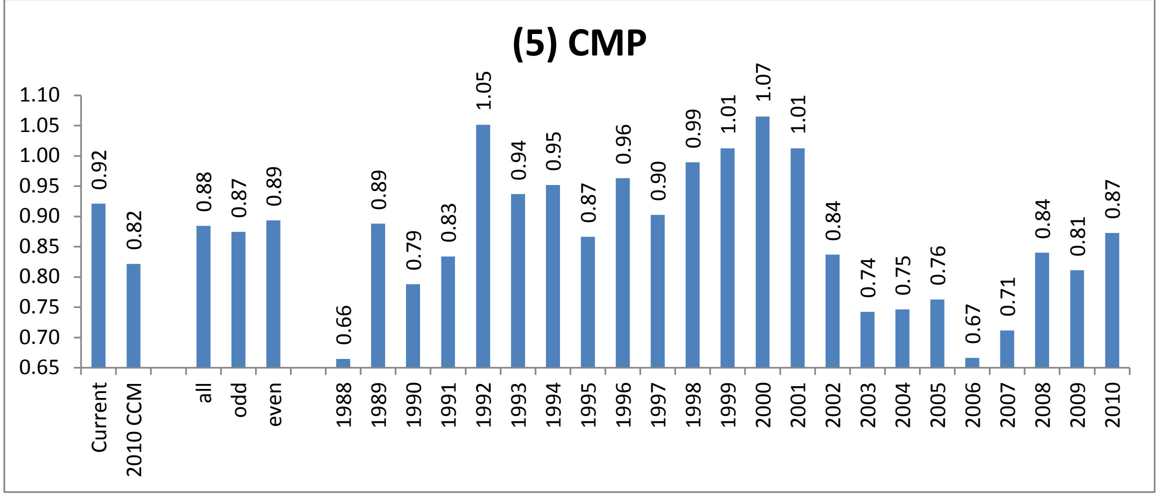




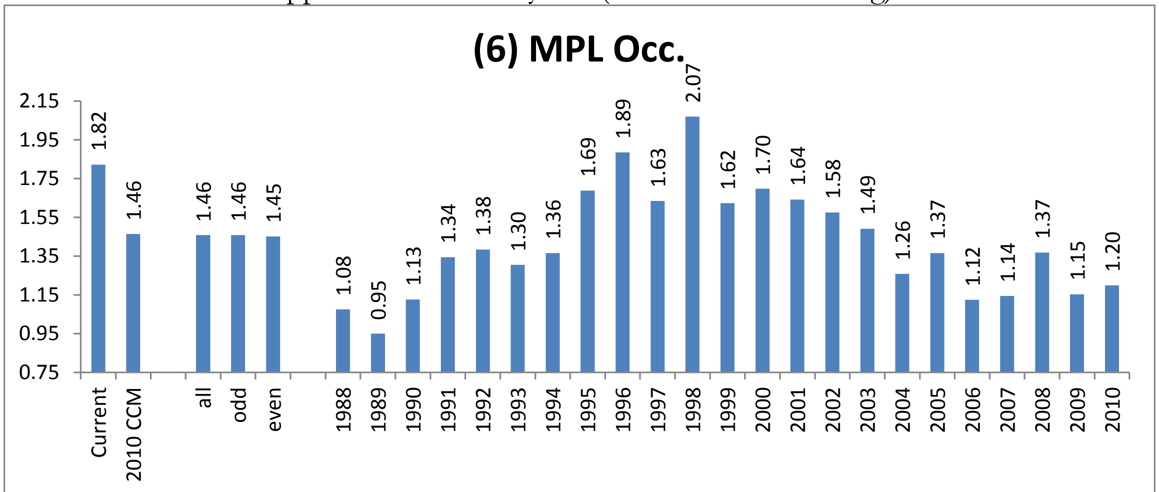
Appendix C – PRF by AY (with Baseline Filtering)

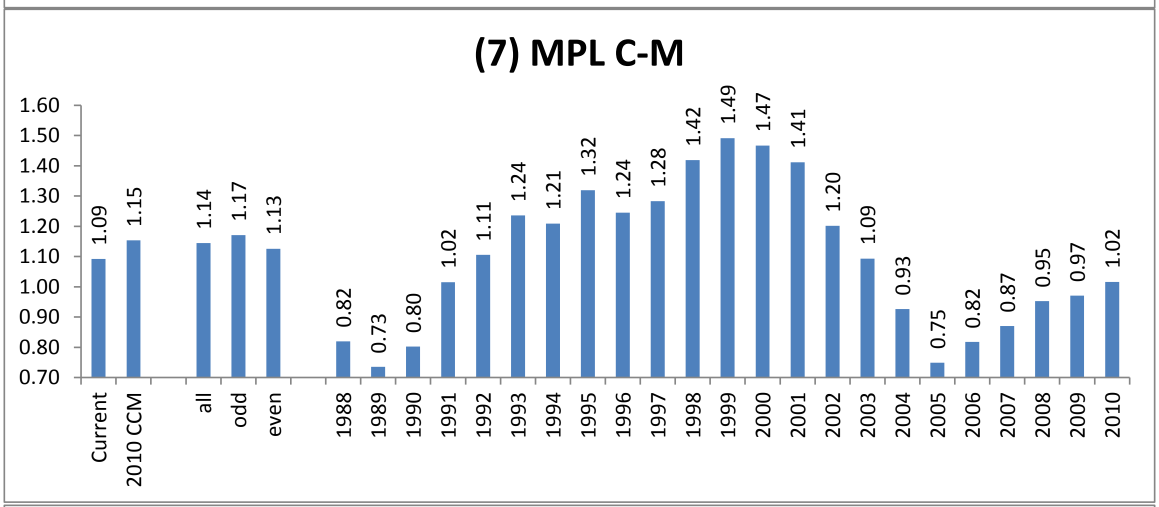


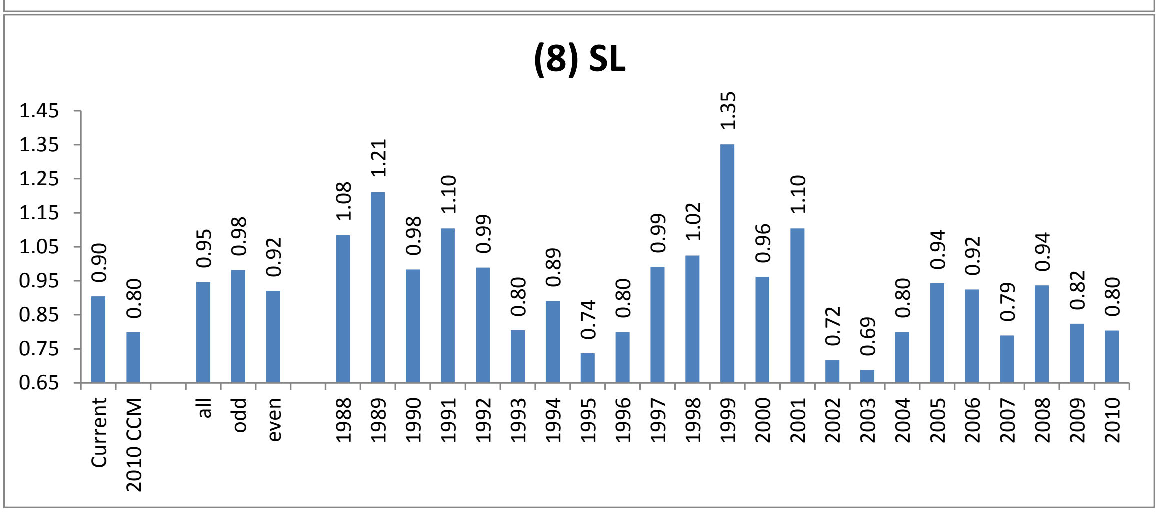




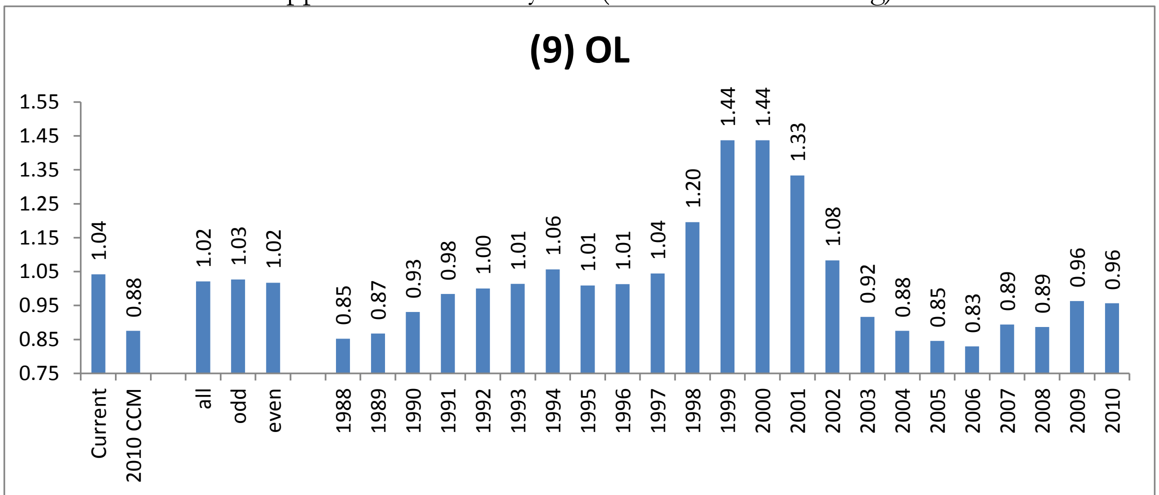
Appendix C – PRF by AY (with Baseline Filtering)

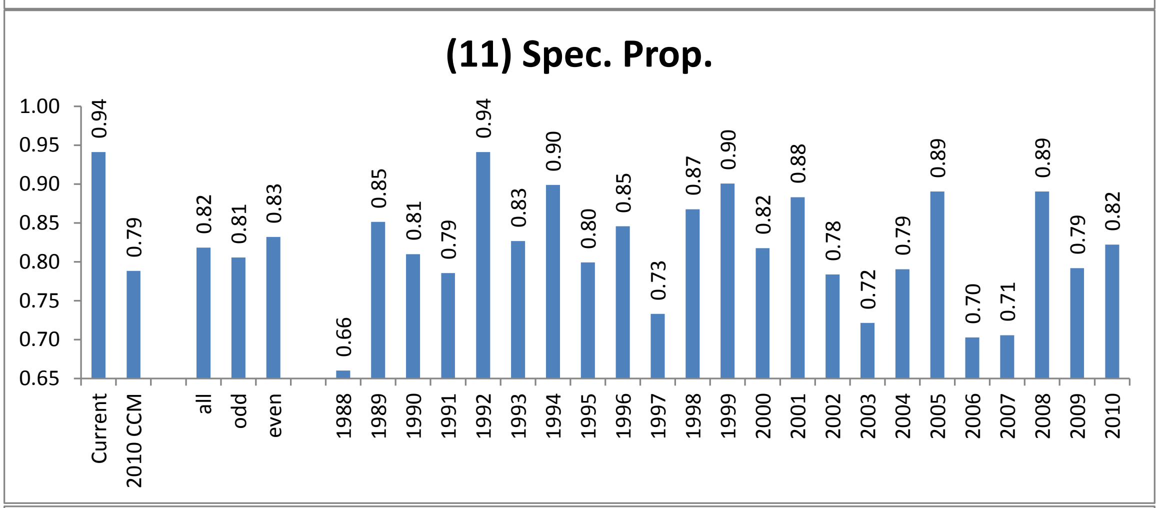


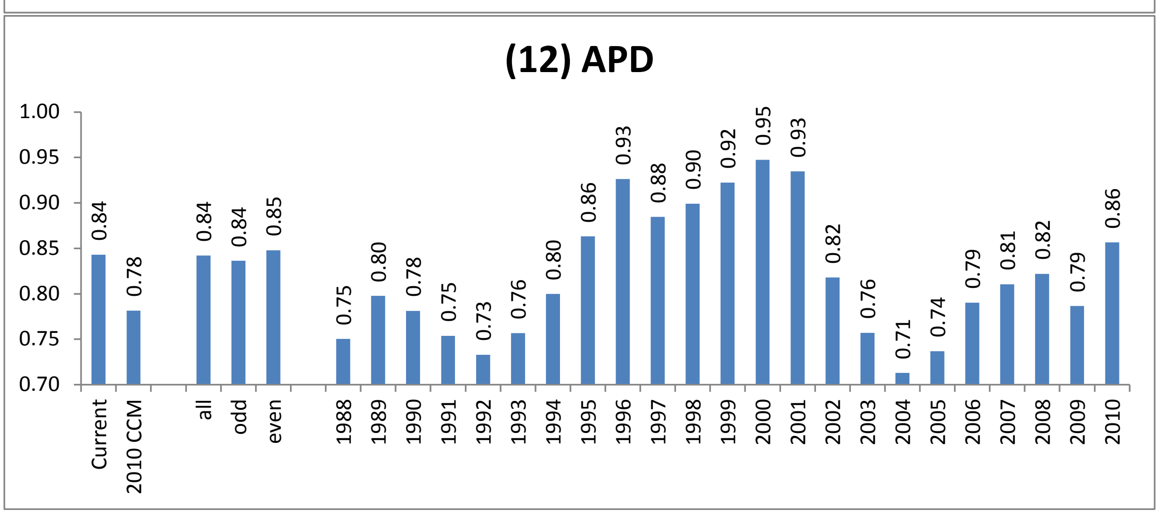


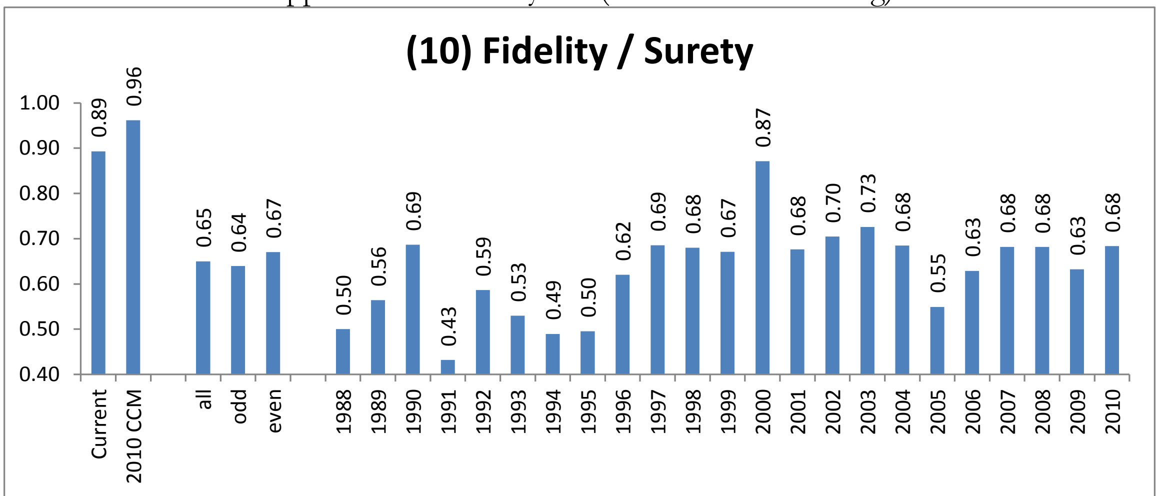


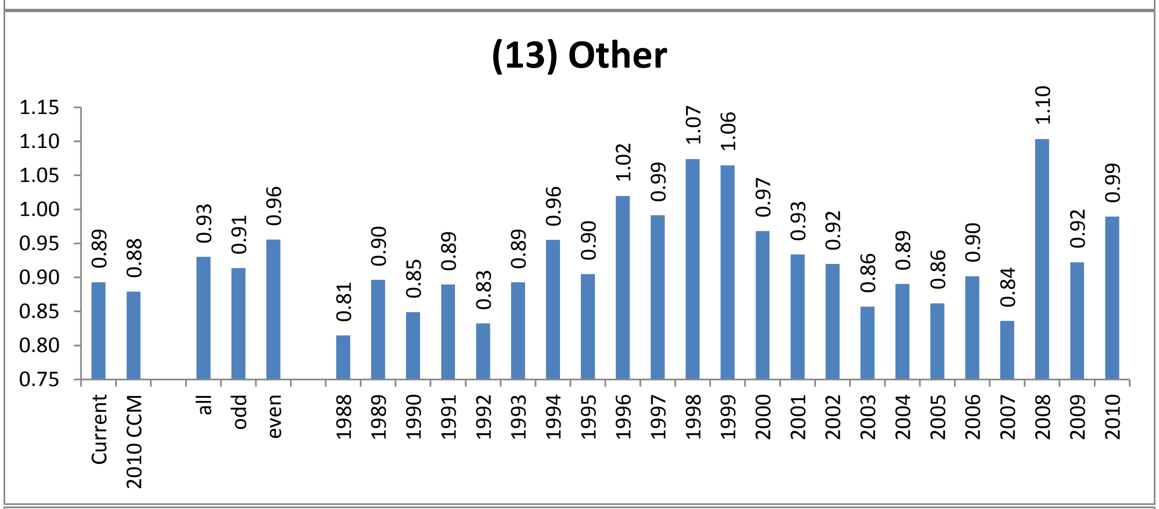
Appendix C – PRF by AY (with Baseline Filtering)

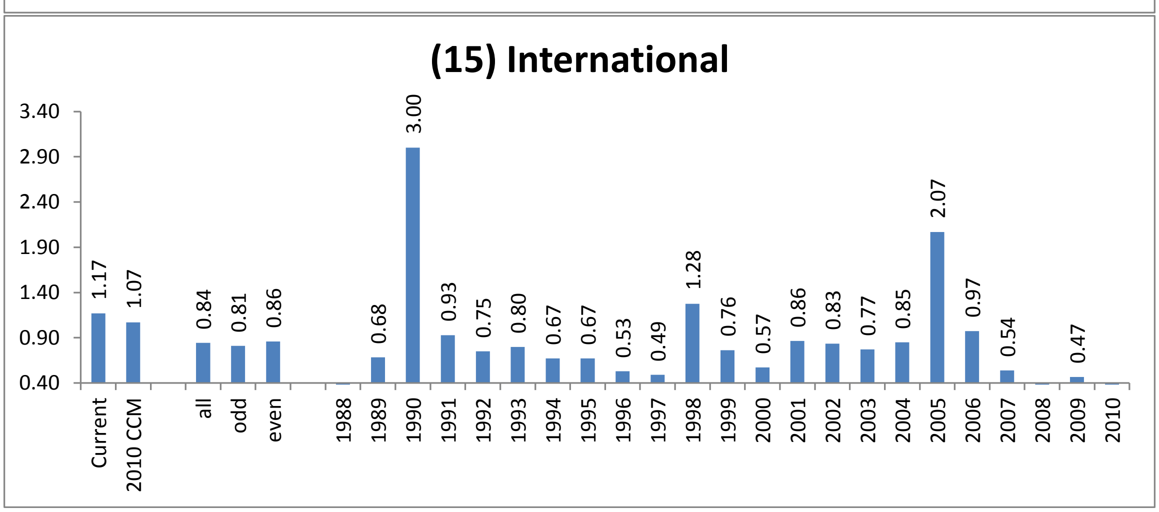




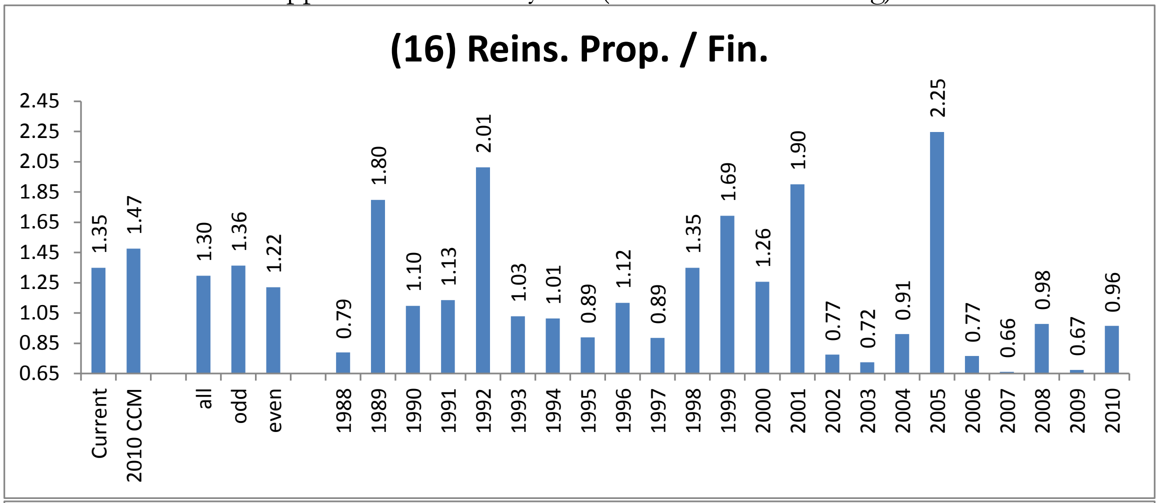


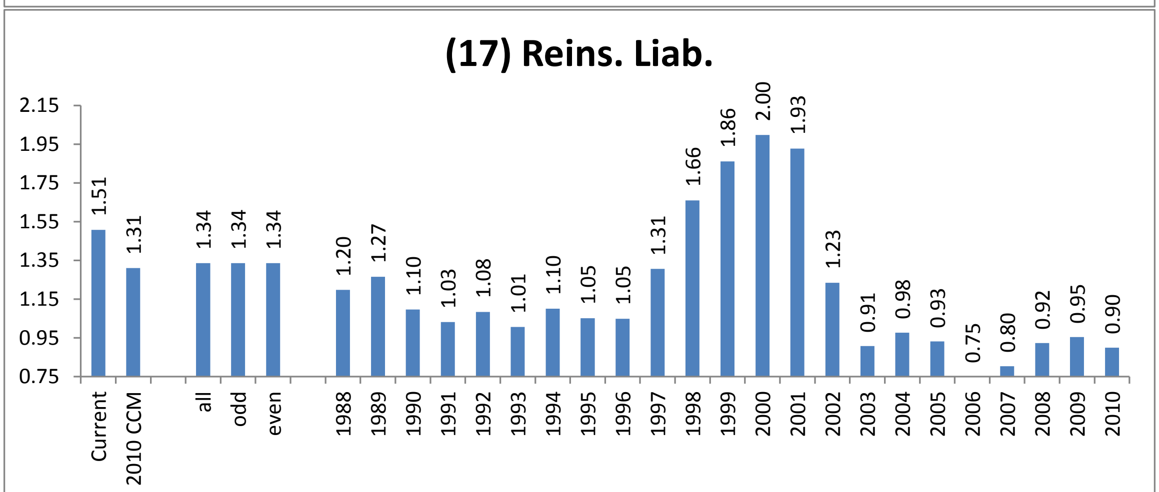

(10) Fidelity / Surety
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.89 0.96 0.65 0.64 0.67 0.50 0.56 0.69 0.43 0.59 0.53 0.49 0.50 0.62 0.69 0.68 0.67 0.87 0.68 0.70 0.73 0.68 0.55 0.63 0.68 0.68 0.63 0.68
Current
2010 CCM
all
odd
even
1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010

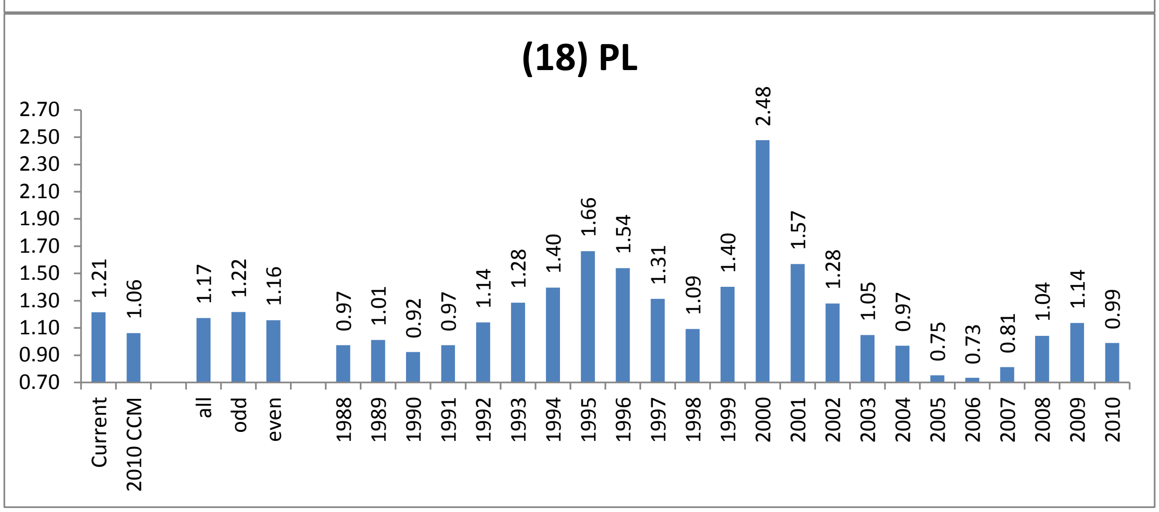

(13) Other
1.15
1.10
1.05
1.00
0.95
0.90
0.85
0.80
0.75
0.89 0.88 0.93 0.91 0.96 0.81 0.90 0.85 0.89 0.83 0.89 0.96 0.90 1.02 0.99 1.07 1.06 0.97 0.93 0.92 0.86 0.89 0.86 0.90 0.84 1.10 0.92 0.99
Current
2010 CCM
all
odd
even
1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010


(15) International
3.40
2.90
2.40
1.90
1.40
0.90
0.40
1.17 1.07 0.84 0.81 0.86 0.68 3.00 0.93 0.75 0.80 0.67 0.67 0.53 0.49 1.28 0.76 0.57 0.86 0.83 0.77 0.85 2.07 0.97 0.54 0.47
Current
2010 CCM
all
odd
even
1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010

Appendix C – PRF by AY (with Baseline Filtering)







Appendix C Appendix C – PRF by AY (with Baseline Filtering)
Even/Odd and Every-Fourth-Year Tests

Premium Risk - LLAE Ratio
Baseline Filtering

			Segment		Segment (in fourths)				Differences - Segment minus All					
Accident Year Percentile		all	odd	even	0mod4	1mod4	2mod4	3mod4	odd	even	0mod4	1mod4	2mod4	3mod4
(1) H/F	A	0.953	0.928	0.976	0.998	0.954	0.955	0.903	-0.026	0.022	**0.044**	0.001	0.002	**-0.050**
(2) PPA	B	0.974	0.968	0.977	0.981	0.985	0.975	0.947	-0.006	0.003	0.007	0.011	0.001	-0.027
(3) CA	C	0.982	0.985	0.981	0.983	0.982	0.979	0.986	0.003	-0.001	0.001	0.000	-0.003	0.004
(4) WC	D	1.042	1.042	1.041	1.057	1.053	1.021	1.029	0.001	0.000	0.015	0.012	-0.020	-0.013
(5) CMP	E	0.885	0.874	0.894	0.917	0.889	0.870	0.861	-0.010	0.009	0.032	0.005	-0.014	-0.023
(6) MM Occurrence	F1	1.458	1.459	1.451	1.448	1.422	1.448	1.494	0.001	-0.006	-0.009	-0.036	-0.009	0.036
(7) MM CM	F2	1.145	1.171	1.125	1.110	1.150	1.135	1.194	0.026	-0.019	-0.035	0.005	-0.009	**0.049**
(8) SL	G	0.946	0.982	0.920	0.917	0.996	0.920	0.920	0.036	-0.026	-0.029	0.050	-0.026	-0.027
(9) OL	H	1.021	1.027	1.017	1.011	1.015	1.020	1.046	0.006	-0.003	-0.010	-0.006	-0.001	0.025
(11) Spec Prop	I	0.818	0.806	0.832	0.836	0.829	0.828	0.777	-0.013	0.014	0.017	0.011	0.010	**-0.041**
(12) Auto Phys Damage	J	0.842	0.836	0.848	0.863	0.837	0.832	0.835	-0.006	0.006	0.021	-0.005	-0.010	-0.007
(10) Fidelity & Surety	K	0.650	0.639	0.671	0.672	0.638	0.655	0.639	-0.011	0.021	0.022	-0.012	0.005	-0.011
(13) Other	L	0.930	0.914	0.955	0.942	0.914	0.961	0.905	-0.017	0.025	0.012	-0.017	0.031	-0.026
(15) International	M	0.844	0.809	0.858	0.739	0.884	1.102	0.767	-0.035	0.014	**-0.104**	**0.040**	**0.258**	**-0.076**
(16) Rein Property & Financial	N&P	1.295	1.364	1.219	1.343	1.544	1.123	1.147	**0.068**	**-0.076**	**0.047**	**0.248**	**-0.172**	**-0.149**
(17) Reinsurance Liab	O	1.335	1.336	1.335	1.331	1.345	1.343	1.331	0.001	0.000	-0.004	0.010	0.007	-0.005
(18) Products Liability	R	1.173	1.216	1.156	1.201	1.273	1.075	1.090	**0.043**	-0.016	0.028	**0.100**	**-0.098**	**-0.083**
(14) Fin & Mort	S	2.410	2.796	1.542	1.903	1.219	1.210	2.942	**0.386**	**-0.868**	**-0.507**	**-1.191**	**-1.200**	**0.532**
(19) Warranty	T	1.270	1.226	1.178	0.924	1.139	1.438	1.143	**-0.045**	**-0.092**	**-0.346**	**-0.132**	**0.168**	**-0.127**

Differences over .040 shown bolded and underlined

Appendix D – PRF and PRC% by LOB-Size

(1) H/F

(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)	(M)	(N)
Size Band	Premium ($000s)			87.5th Percentile LR		Risk Charge		Average Loss Ratio		LR Std. Dev.		LR Coeff. Var.	
Endpoint			Data	all points	all points	all points	all points	all points	all points	all points	all points	all points	all points
Percentile	from	to	Points	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"
15%	0	730	1,429	1.287	0.989	53%	23%	0.844	0.757	0.570	0.336	0.675	0.444
25%	730	1,483	951	1.023	**0.956**	27%	**20%**	0.745	0.742	0.305	0.272	0.410	0.366
35%	1,483	2,758	951	0.985	0.948	23%	19%	0.733	0.741	0.325	0.267	0.443	0.360
45%	2,758	5,022	952	0.964	0.941	21%	18%	0.740	0.743	0.294	0.257	0.398	0.346
55%	5,022	8,866	952	**0.941**	0.938	**18%**	18%	0.737	0.743	0.293	0.250	0.397	0.336
65%	8,866	16,382	952	0.914	0.938	16%	18%	0.721	0.745	0.279	0.239	0.387	0.321
75%	16,382	31,572	951	0.959	0.945	20%	19%	0.746	0.751	0.220	0.225	0.295	0.300
85%	31,572	61,546	952	0.940	0.937	18%	18%	0.747	0.754	0.241	0.227	0.322	0.302
95%	61,546	252,884	952	0.929	0.935	17%	18%	0.752	0.758	0.209	0.218	0.278	0.287
largest 100	252,884	1,499,819	375	0.951	0.947	19%	19%	0.770	0.769	0.244	0.234	0.317	0.305
100%	1,499,819	10,820,092	100	0.912	0.912	15%	15%	0.763	0.764	0.193	0.193	0.252	0.252
	Current Risk Charge Loss Ratio (PR017 Line 4)				0.937								
	Underwriting Expense Ratio in Risk Charge					24%							

(2) PPA

(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)	(M)	(N)
Size Band	Premium ($000s)			87.5th Percentile LR		Risk Charge		Average Loss Ratio		LR Std. Dev.		LR Coeff. Var.	
Endpoint			Data	all points	all points	all points	all points	all points	all points	all points	all points	all points	all points
Percentile	from	to	Points	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"
15%	0	1,596	1,304	1.243	0.999	43%	18%	0.878	0.815	0.458	0.240	0.522	0.295
25%	1,596	3,634	869	1.019	**0.969**	20%	**15%**	0.798	0.803	0.223	0.174	0.279	0.216
35%	3,634	6,667	868	1.003	0.965	19%	15%	0.796	0.804	0.220	0.166	0.277	0.206
45%	6,667	11,219	869	1.013	0.958	20%	14%	0.809	0.805	0.204	0.156	0.253	0.194
55%	11,219	16,368	869	**0.971**	0.950	**16%**	14%	0.789	0.805	0.186	0.145	0.235	0.181
65%	16,368	28,352	869	0.971	0.945	16%	13%	0.804	0.808	0.168	0.135	0.209	0.166
75%	28,352	54,053	869	0.962	0.939	15%	12%	0.814	0.810	0.144	0.123	0.177	0.152
85%	54,053	130,201	868	0.959	0.929	14%	11%	0.822	0.808	0.130	0.114	0.158	0.141
95%	130,201	580,234	869	0.920	0.908	11%	9%	0.799	0.798	0.107	0.101	0.134	0.126
largest 100	580,234	3,936,971	334	0.895	0.894	8%	8%	0.796	0.796	0.090	0.087	0.113	0.109
100%	3,936,971	18,406,826	100	0.892	0.892	8%	8%	0.797	0.797	0.078	0.078	0.098	0.098
	Current Risk Charge Loss Ratio (PR017 Line 4)				0.969								
	Underwriting Expense Ratio in Risk Charge					19%							

(3) CA

(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)	(M)	(N)
Size Band	Premium ($000s)			87.5th Percentile LR		Risk Charge		Average Loss Ratio		LR Std. Dev.		LR Coeff. Var.	
Endpoint			Data	all points	all points	all points	all points	all points	all points	all points	all points	all points	all points
Percentile	from	to	Points	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"
15%	0	767	911	1.260	1.006	52%	26%	0.711	0.741	0.555	0.324	0.781	0.437
25%	767	1,491	606	1.070	**0.988**	33%	**25%**	0.695	0.746	0.367	0.262	0.528	0.352
35%	1,491	2,755	605	1.009	0.979	27%	24%	0.727	0.753	0.301	0.244	0.414	0.324
45%	2,755	4,639	606	0.995	0.975	25%	23%	0.739	0.757	0.284	0.234	0.384	0.309
55%	4,639	8,038	606	**0.989**	0.971	**25%**	23%	0.739	0.760	0.256	0.224	0.346	0.294
65%	8,038	13,680	606	0.973	0.965	23%	22%	0.752	0.765	0.262	0.215	0.349	0.282
75%	13,680	23,821	606	0.989	0.964	25%	22%	0.769	0.769	0.232	0.200	0.301	0.260
85%	23,821	53,660	606	0.973	0.952	23%	21%	0.780	0.768	0.224	0.186	0.287	0.241
95%	53,660	189,338	606	0.944	0.944	20%	20%	0.759	0.761	0.161	0.154	0.212	0.203
largest 100	189,338	526,117	203	0.916	0.938	17%	20%	0.763	0.766	0.136	0.140	0.179	0.182
100%	526,117	1,875,641	100	0.974	0.974	23%	23%	0.772	0.773	0.146	0.146	0.189	0.189
	Current Risk Charge Loss Ratio (PR017 Line 4)				0.988								
	Underwriting Expense Ratio in Risk Charge					26%							

Appendix D – PRF and PRC% by LOB-Size

(4) WC

(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)	(M)	(N)
Size Band	Premium ($000s)			87.5th Percentile LR		Risk Charge		Average Loss Ratio		LR Std. Dev.		LR Coeff. Var.	
Endpoint			Data	all points	all points	all points	all points	all points	all points	all points	all points	all points	all points
Percentile	from	to	Points	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"
15%	0	1,756	921	1.315	1.062	53%	28%	0.821	0.786	0.590	0.334	0.719	0.425
25%	1,756	3,872	613	1.223	**1.039**	44%	**25%**	0.824	0.780	0.373	0.264	0.452	0.339
35%	3,872	6,827	613	1.104	1.018	32%	23%	0.777	0.774	0.304	0.245	0.390	0.317
45%	6,827	12,098	614	1.079	1.008	29%	22%	0.783	0.773	0.284	0.235	0.363	0.304
55%	12,098	21,267	613	**1.020**	0.994	**23%**	21%	0.755	0.771	0.269	0.225	0.356	0.291
65%	21,267	37,341	614	0.977	0.990	19%	20%	0.753	0.775	0.234	0.214	0.310	0.276
75%	37,341	70,403	613	0.954	0.993	17%	21%	0.744	0.781	0.214	0.207	0.287	0.265
85%	70,403	148,020	613	0.956	1.006	17%	22%	0.768	0.796	0.190	0.203	0.248	0.255
95%	148,020	518,403	614	1.017	1.047	23%	26%	0.799	0.815	0.193	0.209	0.241	0.256
largest 100	518,403	1,521,266	206	1.121	1.107	34%	32%	0.845	0.846	0.247	0.235	0.292	0.277
100%	1,521,266	7,918,320	100	1.074	1.074	29%	29%	0.846	0.846	0.206	0.206	0.244	0.244

Current Risk Charge Loss Ratio (PR017 Line 4) 1.033

Underwriting Expense Ratio in Risk Charge 21%

(5) CMP

(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)	(M)	(N)
Size Band	Premium ($000s)			87.5th Percentile LR		Risk Charge		Average Loss Ratio		LR Std. Dev.		LR Coeff. Var.	
Endpoint			Data	all points	all points	all points	all points	all points	all points	all points	all points	all points	all points
Percentile	from	to	Points	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"
15%	0	681	1,079	1.093	0.899	44%	24%	0.667	0.655	0.569	0.332	0.853	0.507
25%	681	1,520	720	0.877	**0.879**	22%	**22%**	0.612	0.653	0.327	0.269	0.534	0.412
35%	1,520	2,841	719	0.877	0.879	22%	22%	0.612	0.658	0.313	0.260	0.512	0.395
45%	2,841	4,810	720	0.883	0.880	23%	22%	0.625	0.666	0.302	0.250	0.483	0.376
55%	4,810	7,866	719	**0.899**	0.879	**24%**	22%	0.637	0.673	0.261	0.239	0.410	0.355
65%	7,866	14,256	719	0.887	0.875	23%	22%	0.679	0.681	0.280	0.233	0.413	0.342
75%	14,256	25,346	719	0.868	0.875	21%	22%	0.668	0.682	0.205	0.217	0.307	0.318
85%	25,346	54,619	720	0.855	0.876	20%	22%	0.669	0.687	0.213	0.221	0.318	0.322
95%	54,619	294,101	719	0.881	0.890	23%	23%	0.687	0.699	0.249	0.226	0.363	0.324
largest 100	294,101	1,063,131	259	0.924	0.901	27%	25%	0.739	0.723	0.174	0.168	0.236	0.233
100%	1,063,131	2,970,994	100	0.855	0.855	20%	20%	0.678	0.680	0.141	0.141	0.209	0.208

Current Risk Charge Loss Ratio (PR017 Line 4) 0.921

Underwriting Expense Ratio in Risk Charge 34%

(6) MPL Occ.

(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)	(M)	(N)
Size Band	Premium ($000s)			87.5th Percentile LR		Risk Charge		Average Loss Ratio		LR Std. Dev.		LR Coeff. Var.	
Endpoint			Data	all points	all points	all points	all points	all points	all points	all points	all points	all points	all points
Percentile	from	to	Points	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"
15%	0	823	168	2.434	1.521	147%	56%	1.086	0.961	0.890	0.588	0.819	0.612
25%	823	1,595	111	1.566	**1.458**	60%	**50%**	0.897	0.939	0.645	0.513	0.719	0.547
35%	1,595	2,623	111	1.265	1.447	30%	49%	0.796	0.945	0.475	0.493	0.597	0.522
45%	2,623	4,087	112	1.440	1.459	48%	50%	0.897	0.968	0.608	0.492	0.678	0.508
55%	4,087	6,672	111	**1.261**	1.464	**30%**	50%	0.874	0.981	0.537	0.466	0.615	0.475
65%	6,672	11,654	112	1.426	1.486	46%	52%	0.953	1.004	0.403	0.445	0.423	0.443
75%	11,654	24,496	111	1.696	1.521	73%	56%	1.113	1.019	0.509	0.456	0.457	0.447
85%	24,496	44,393	111	1.431	1.425	47%	46%	1.002	0.982	0.447	0.427	0.446	0.435
95%	44,393	152,900	112	1.380	1.422	42%	46%	0.927	0.969	0.423	0.413	0.457	0.426
largest 28	152,900	204,129	27	1.339	1.448	38%	49%	0.993	1.054	0.303	0.376	0.306	0.357
100%	204,129	516,498	28	1.545	1.545	58%	58%	1.118	1.110	0.426	0.426	0.381	0.383

Current Risk Charge Loss Ratio (PR017 Line 4) 1.822

Underwriting Expense Ratio in Risk Charge 4%

(7) MPL C-M

(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)	(M)	(N)
Size Band	Premium ($000s)			87.5th Percentile LR		Risk Charge		Average Loss Ratio		LR Std. Dev.		LR Coeff. Var.	
Endpoint			Data	all points	all points	all points	all points	all points	all points	all points	all points	all points	all points
Percentile	from	to	Points	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"
15%	0	1,422	373	1.760	1.185	100%	42%	0.826	0.764	0.781	0.451	0.946	0.591
25%	1,422	2,642	249	1.004	**1.146**	24%	**38%**	0.631	0.753	0.465	0.362	0.737	0.481
35%	2,642	4,082	248	1.159	1.160	39%	40%	0.686	0.770	0.410	0.343	0.598	0.445
45%	4,082	6,520	248	1.062	1.160	30%	40%	0.717	0.782	0.300	0.329	0.418	0.421
55%	6,520	11,635	249	**1.037**	1.184	**27%**	42%	0.690	0.794	0.332	0.333	0.481	0.419
65%	11,635	19,211	248	1.206	1.198	44%	43%	0.824	0.817	0.357	0.328	0.433	0.402
75%	19,211	32,649	249	1.239	1.198	47%	43%	0.829	0.816	0.373	0.320	0.450	0.392
85%	32,649	58,551	248	1.099	1.184	33%	42%	0.772	0.810	0.277	0.296	0.359	0.365
95%	58,551	142,452	248	1.215	1.212	45%	45%	0.845	0.836	0.309	0.305	0.366	0.365
largest 62	142,452	214,411	62	1.274	1.177	51%	41%	0.843	0.818	0.300	0.294	0.355	0.360
100%	214,411	726,535	62	1.081	1.081	32%	32%	0.792	0.794	0.287	0.287	0.362	0.361

Current Risk Charge Loss Ratio (PR017 Line 4) 1.092

Underwriting Expense Ratio in Risk Charge 24%

(8) SL

(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)	(M)	(N)
Size Band	Premium ($000s)			87.5th Percentile LR		Risk Charge		Average Loss Ratio		LR Std. Dev.		LR Coeff. Var.	
Endpoint			Data	all points	all points	all points	all points	all points	all points	all points	all points	all points	all points
Percentile	from	to	Points	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"
15%	0	1,031	171	1.233	0.964	58%	31%	0.631	0.656	0.616	0.396	0.977	0.603
25%	1,031	2,069	113	1.114	**0.947**	46%	**29%**	0.685	0.661	0.547	0.342	0.798	0.517
35%	2,069	3,416	114	0.945	0.931	29%	27%	0.669	0.658	0.444	0.304	0.664	0.462
45%	3,416	6,024	113	1.005	0.925	35%	27%	0.657	0.656	0.328	0.276	0.499	0.421
55%	6,024	9,096	114	**1.041**	0.912	**38%**	26%	0.682	0.656	0.373	0.265	0.547	0.405
65%	9,096	14,995	113	0.871	0.880	21%	22%	0.654	0.650	0.194	0.234	0.297	0.361
75%	14,995	31,064	114	0.964	0.886	31%	23%	0.660	0.649	0.286	0.245	0.434	0.377
85%	31,064	66,873	113	0.944	0.859	29%	20%	0.686	0.645	0.213	0.226	0.311	0.350
95%	66,873	231,342	114	0.849	0.825	19%	17%	0.666	0.618	0.204	0.229	0.306	0.371
largest 28	231,342	323,270	28	0.699	0.702	4%	5%	0.541	0.520	0.145	0.244	0.268	0.469
100%	323,270	594,515	28	0.711	0.711	5%	5%	0.487	0.500	0.311	0.311	0.638	0.621

Current Risk Charge Loss Ratio (PR017 Line 4) 0.904

Underwriting Expense Ratio in Risk Charge 34%

(9) OL

(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)	(M)	(N)
Size Band	Premium ($000s)			87.5th Percentile LR		Risk Charge		Average Loss Ratio		LR Std. Dev.		LR Coeff. Var.	
Endpoint			Data	all points	all points	all points	all points	all points	all points	all points	all points	all points	all points
Percentile	from	to	Points	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"
15%	0	481	1,306	1.383	1.043	72%	38%	0.681	0.667	0.753	0.464	1.106	0.696
25%	481	1,087	871	1.024	**1.015**	36%	**35%**	0.585	0.664	0.473	0.391	0.809	0.589
35%	1,087	2,008	871	1.079	1.014	41%	35%	0.644	0.675	0.482	0.378	0.748	0.560
45%	2,008	3,584	870	1.073	1.010	41%	34%	0.623	0.680	0.405	0.359	0.650	0.528
55%	3,584	6,057	870	**1.024**	1.000	**36%**	33%	0.653	0.690	0.344	0.349	0.527	0.506
65%	6,057	10,389	870	1.014	0.996	35%	33%	0.676	0.698	0.392	0.349	0.580	0.500
75%	10,389	19,960	871	0.984	0.991	32%	32%	0.693	0.704	0.372	0.336	0.537	0.477
85%	19,960	49,079	870	1.023	0.993	36%	33%	0.722	0.708	0.368	0.320	0.510	0.452
95%	49,079	210,786	870	0.962	0.969	30%	30%	0.686	0.700	0.301	0.283	0.439	0.405
largest 100	210,786	1,059,392	335	0.939	0.982	27%	32%	0.707	0.727	0.246	0.241	0.348	0.332
100%	1,059,392	9,366,624	100	1.042	1.042	38%	38%	0.792	0.792	0.213	0.213	0.269	0.269

Current Risk Charge Loss Ratio (PR017 Line 4) 1.042

Underwriting Expense Ratio in Risk Charge 33%

(11) Spec. Prop.

(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)	(M)	(N)
Size Band	Premium ($000s)			87.5th Percentile LR		Risk Charge		Average Loss Ratio		LR Std. Dev.		LR Coeff. Var.	
Endpoint			Data	all points	all points	all points	all points	all points	all points	all points	all points	all points	all points
Percentile	from	to	Points	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"
15%	0	487	1,337	1.000	0.834	43%	27%	0.617	0.568	0.591	0.370	0.958	0.651
25%	487	931	888	0.769	**0.817**	20%	**25%**	0.527	0.559	0.337	0.314	0.640	0.562
35%	931	1,683	888	0.770	0.820	20%	25%	0.540	0.563	0.339	0.310	0.628	0.551
45%	1,683	2,913	890	0.799	0.828	23%	26%	0.563	0.567	0.341	0.306	0.606	0.539
55%	2,913	4,933	889	**0.772**	0.832	**20%**	26%	0.521	0.568	0.311	0.299	0.597	0.526
65%	4,933	9,021	889	0.827	0.838	26%	27%	0.560	0.578	0.333	0.295	0.595	0.510
75%	9,021	16,814	889	0.820	0.842	25%	27%	0.547	0.583	0.301	0.283	0.551	0.485
85%	16,814	36,266	890	0.857	0.851	29%	28%	0.594	0.598	0.297	0.274	0.500	0.458
95%	36,266	144,658	889	0.835	0.846	27%	28%	0.590	0.601	0.250	0.257	0.423	0.428
largest 100	144,658	644,456	344	0.907	0.855	34%	29%	0.628	0.623	0.280	0.270	0.446	0.434
100%	644,456	2,748,838	100	0.810	0.810	24%	24%	0.605	0.607	0.234	0.234	0.386	0.385

Current Risk Charge Loss Ratio (PR017 Line 4) 0.941

Underwriting Expense Ratio in Risk Charge 43%

Appendix D – PRF and PRC% by LOB-Size

(12) APD

(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)	(M)	(N)
Size Band	Premium ($000s)			87.5th Percentile LR		Risk Charge		Average Loss Ratio		LR Std. Dev.		LR Coeff. Var.	
Endpoint			Data	all points	all points	all points	all points	all points	all points	all points	all points	all points	all points
Percentile	from	to	Points	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"
15%	0	1,133	1,423	1.039	0.850	37%	18%	0.716	0.670	0.427	0.238	0.597	0.355
25%	1,133	2,445	949	0.880	**0.828**	21%	**16%**	0.653	0.662	0.255	0.184	0.390	0.279
35%	2,445	4,415	948	0.845	0.822	17%	15%	0.645	0.663	0.192	0.173	0.297	0.261
45%	4,415	7,293	949	0.842	0.819	17%	15%	0.641	0.666	0.187	0.170	0.291	0.255
55%	7,293	11,829	948	**0.835**	0.817	**16%**	15%	0.665	0.671	0.217	0.166	0.326	0.248
65%	11,829	19,194	949	0.820	0.814	15%	14%	0.663	0.672	0.202	0.152	0.305	0.227
75%	19,194	38,239	948	0.849	0.812	18%	14%	0.677	0.675	0.159	0.135	0.235	0.200
85%	38,239	91,334	949	0.814	0.798	14%	13%	0.671	0.674	0.140	0.124	0.209	0.184
95%	91,334	343,654	948	0.792	0.792	12%	12%	0.667	0.676	0.116	0.112	0.174	0.165
largest 100	343,654	2,115,343	374	0.786	0.790	12%	12%	0.686	0.692	0.104	0.100	0.151	0.144
100%	2,115,343	12,748,056	100	0.804	0.804	13%	13%	0.712	0.714	0.082	0.082	0.115	0.115

Current Risk Charge Loss Ratio (PR017 Line 4) 0.843

Underwriting Expense Ratio in Risk Charge 33%

(10) Fidelity / Surety

(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)	(M)	(N)
Size Band	Premium ($000s)			87.5th Percentile LR		Risk Charge		Average Loss Ratio		LR Std. Dev.		LR Coeff. Var.	
Endpoint			Data	all points	all points	all points	all points	all points	all points	all points	all points	all points	all points
Percentile	from	to	Points	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"
15%	0	848	217	0.977	0.665	64%	33%	0.462	0.336	0.697	0.414	1.508	1.231
25%	848	1,657	146	0.684	**0.644**	35%	**31%**	0.292	0.314	0.441	0.336	1.510	1.069
35%	1,657	3,168	143	0.635	0.641	30%	30%	0.280	0.317	0.373	0.319	1.331	1.005
45%	3,168	5,357	145	0.762	0.641	43%	30%	0.408	0.323	0.419	0.309	1.028	0.958
55%	5,357	7,721	144	**0.653**	0.603	**32%**	27%	0.323	0.307	0.275	0.282	0.853	0.917
65%	7,721	10,817	145	0.560	0.599	22%	26%	0.283	0.304	0.255	0.283	0.898	0.932
75%	10,817	17,464	144	0.625	0.600	29%	26%	0.314	0.310	0.409	0.290	1.303	0.938
85%	17,464	30,582	145	0.520	0.600	18%	26%	0.260	0.308	0.248	0.226	0.953	0.735
95%	30,582	109,891	144	0.648	0.614	31%	28%	0.325	0.340	0.224	0.205	0.691	0.602
largest 36	109,891	208,596	36	0.610	0.538	27%	20%	0.422	0.369	0.173	0.155	0.410	0.419
100%	208,596	974,546	36	0.408	0.408	7%	7%	0.302	0.318	0.104	0.104	0.343	0.327

Current Risk Charge Loss Ratio (PR017 Line 4) 0.883

Underwriting Expense Ratio in Risk Charge 66%

(13) Other

(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)	(M)	(N)
Size Band	Premium ($000s)			87.5th Percentile LR		Risk Charge		Average Loss Ratio		LR Std. Dev.		LR Coeff. Var.	
Endpoint			Data	all points	all points	all points	all points	all points	all points	all points	all points	all points	all points
Percentile	from	to	Points	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"
15%	0	1,052	257	1.091	0.938	47%	32%	0.646	0.620	0.644	0.415	0.997	0.669
25%	1,052	2,105	170	0.902	**0.922**	28%	**30%**	0.582	0.616	0.509	0.360	0.875	0.584
35%	2,105	4,638	171	0.877	0.924	26%	30%	0.551	0.620	0.337	0.335	0.610	0.539
45%	4,638	8,326	171	0.831	0.925	21%	30%	0.556	0.631	0.314	0.333	0.565	0.528
55%	8,326	14,318	170	**0.917**	0.936	**30%**	32%	0.611	0.645	0.327	0.334	0.534	0.519
65%	14,318	24,267	171	0.951	0.938	33%	32%	0.605	0.652	0.301	0.336	0.498	0.515
75%	24,267	46,152	171	0.937	0.927	32%	31%	0.653	0.665	0.347	0.344	0.532	0.517
85%	46,152	88,823	170	0.795	0.923	17%	30%	0.597	0.670	0.306	0.342	0.513	0.511
95%	88,823	243,019	171	0.974	0.957	35%	34%	0.698	0.719	0.411	0.356	0.588	0.495
largest 43	243,019	360,682	42	0.958	0.913	34%	29%	0.803	0.760	0.223	0.201	0.278	0.264
100%	360,682	2,477,354	43	0.889	0.889	27%	27%	0.716	0.720	0.165	0.165	0.231	0.229

Current Risk Charge Loss Ratio (PR017 Line 4) 0.893

Underwriting Expense Ratio in Risk Charge 38%

(15) International

(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)	(M)	(N)
Size Band	Premium ($000s)			87.5th Percentile LR		Risk Charge		Average Loss Ratio		LR Std. Dev.		LR Coeff. Var.	
Endpoint			Data	all points	all points	all points	all points	all points	all points	all points	all points	all points	all points
Percentile	from	to	Points	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"	in band	> "from"
15%	0	5,074	12	0.731	0.842	13%	24%	0.395	0.605	0.277	0.447	0.701	0.738
25%	5,074	8,286	8	2.212	**0.849**	161%	**24%**	1.177	0.643	0.854	0.461	0.726	0.717
35%	8,286	11,464	8	0.937	0.815	33%	21%	0.628	0.570	0.375	0.310	0.598	0.543
45%	11,464	14,601	8	0.973	0.791	37%	19%	0.757	0.561	0.235	0.297	0.310	0.529
55%	14,601	17,653	7	**0.640**	0.701	**3%**	10%	0.397	0.525	0.217	0.292	0.546	0.555
65%	17,653	21,188	8	0.712	0.701	11%	10%	0.551	0.550	0.160	0.299	0.289	0.543
75%	21,188	30,030	8	0.586	0.694	-2%	9%	0.504	0.549	0.129	0.329	0.255	0.598
85%	30,030	48,593	8	0.685	0.694	8%	9%	0.550	0.567	0.133	0.382	0.241	0.673
95%	48,593	83,510	8	0.674	0.692	7%	9%	0.467	0.579	0.186	0.489	0.399	0.845
largest 2	83,510	88,539	1	0.190	1.528	-42%	92%	0.190	0.804	0.000	0.844	0.000	1.051
100%	88,539	105,750	2	1.749	1.749	114%	114%	1.074	1.008	0.899	0.899	0.837	0.892

Current Risk Charge Loss Ratio (PR017 Line 4) 1.169

Underwriting Expense Ratio in Risk Charge 39%

(16) Reins. Prop. / Fin.

(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)	(M)	(N)
Size Band	Premium ($000s)			87.5th Percentile LR		Risk Charge		Average Loss Ratio		LR Std. Dev.		LR Coeff. Var.	
Endpoint Percentile	from	to	Data Points	all points in band	all points > "from"	all points in band	all points > "from"	all points in band	all points > "from"	all points in band	all points > "from"	all points in band	all points > "from"
15%	0	1,624	159	1.833	1.315	104%	53%	0.860	0.789	0.756	0.532	0.880	0.674
25%	1,624	3,429	105	1.288	**1.288**	50%	**50%**	0.813	0.777	0.542	0.481	0.666	0.619
35%	3,429	7,180	105	1.604	1.286	81%	50%	0.835	0.772	0.584	0.472	0.700	0.611
45%	7,180	11,004	106	1.305	1.236	52%	45%	0.816	0.762	0.483	0.451	0.592	0.592
55%	11,004	16,375	105	**1.156**	1.222	**37%**	43%	0.778	0.752	0.494	0.444	0.635	0.591
65%	16,375	27,959	106	1.409	1.230	62%	44%	0.863	0.746	0.493	0.432	0.572	0.579
75%	27,959	50,634	105	1.191	1.178	40%	39%	0.746	0.713	0.402	0.407	0.539	0.571
85%	50,634	104,700	105	1.224	1.168	44%	38%	0.748	0.700	0.461	0.408	0.616	0.583
95%	104,700	349,120	106	1.079	1.079	29%	29%	0.699	0.667	0.352	0.365	0.504	0.546
largest 26	349,120	477,622	26	1.197	1.030	41%	24%	0.648	0.605	0.453	0.380	0.699	0.628
100%	477,622	2,472,954	26	0.804	0.804	1%	1%	0.552	0.564	0.281	0.281	0.509	0.498

Current Risk Charge Loss Ratio (PR017 Line 4) 1.349

Underwriting Expense Ratio in Risk Charge 21%

(17) Reins. Liab.

(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)	(M)	(N)
Size Band	Premium ($000s)			87.5th Percentile LR		Risk Charge		Average Loss Ratio		LR Std. Dev.		LR Coeff. Var.	
Endpoint Percentile	from	to	Data Points	all points in band	all points > "from"	all points in band	all points > "from"	all points in band	all points > "from"	all points in band	all points > "from"	all points in band	all points > "from"
15%	0	2,339	169	1.700	1.335	86%	49%	0.892	0.842	0.788	0.506	0.883	0.601
25%	2,339	5,258	112	1.436	**1.302**	59%	**46%**	0.882	0.833	0.606	0.437	0.688	0.524
35%	5,258	9,036	112	1.175	1.278	33%	44%	0.822	0.826	0.385	0.408	0.468	0.494
45%	9,036	18,520	112	1.288	1.290	45%	45%	0.748	0.827	0.431	0.412	0.576	0.498
55%	18,520	33,620	112	**1.272**	1.290	**43%**	45%	0.829	0.841	0.437	0.406	0.527	0.483
65%	33,620	54,532	112	1.335	1.290	49%	45%	0.901	0.844	0.464	0.399	0.515	0.473
75%	54,532	105,154	112	1.293	1.265	45%	42%	0.837	0.827	0.327	0.377	0.390	0.456
85%	105,154	223,643	112	1.174	1.227	33%	39%	0.862	0.823	0.413	0.395	0.479	0.480
95%	223,643	760,588	112	1.387	1.262	55%	42%	0.857	0.797	0.402	0.381	0.469	0.478
largest 28	760,588	1,098,101	27	0.980	0.972	14%	13%	0.679	0.679	0.348	0.300	0.512	0.442
100%	1,098,101	4,178,508	28	0.931	0.931	9%	9%	0.671	0.678	0.246	0.246	0.366	0.362

Current Risk Charge Loss Ratio (PR017 Line 4) 1.507

Underwriting Expense Ratio in Risk Charge 16%

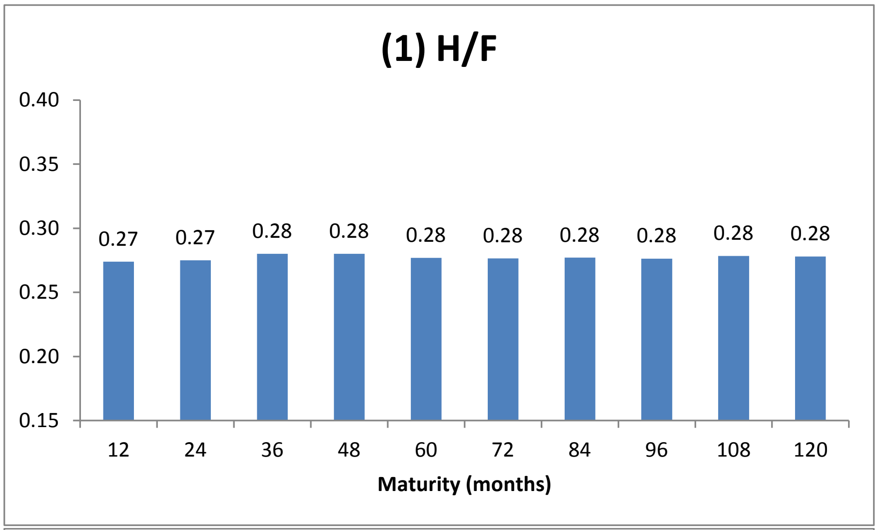
(18) PL

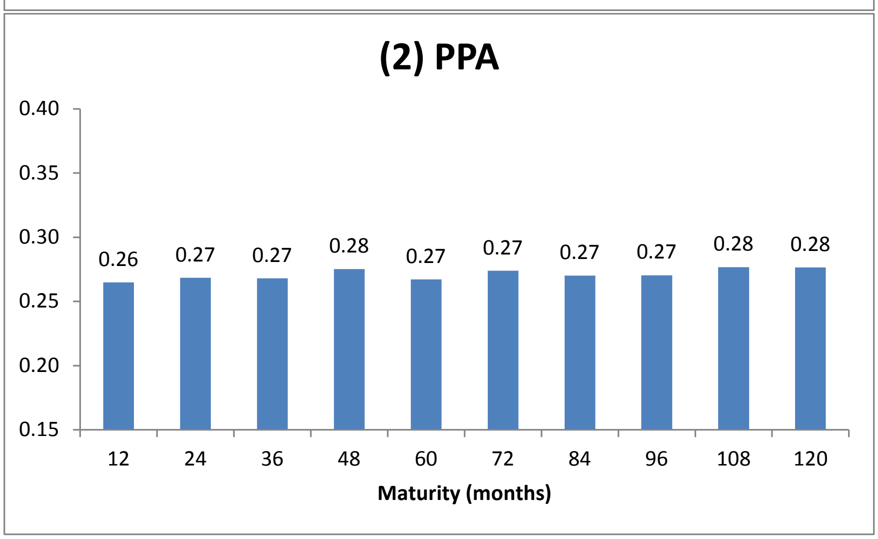
(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)	(K)	(L)	(M)	(N)
Size Band	Premium ($000s)			87.5th Percentile LR		Risk Charge		Average Loss Ratio		LR Std. Dev.		LR Coeff. Var.	
Endpoint Percentile	from	to	Data Points	all points in band	all points > "from"	all points in band	all points > "from"	all points in band	all points > "from"	all points in band	all points > "from"	all points in band	all points > "from"
15%	0	792	99	1.416	1.225	68%	49%	0.806	0.734	0.752	0.620	0.933	0.845
25%	792	1,209	66	0.826	**1.184**	9%	**45%**	0.412	0.722	0.329	0.593	0.798	0.822
35%	1,209	1,999	65	1.510	1.280	78%	55%	0.762	0.763	0.656	0.608	0.861	0.797
45%	1,999	3,430	66	0.926	1.175	19%	44%	0.525	0.763	0.429	0.600	0.817	0.787
55%	3,430	6,400	65	**1.519**	1.250	**78%**	52%	0.862	0.807	0.806	0.617	0.935	0.764
65%	6,400	10,699	66	1.157	1.171	42%	44%	0.826	0.795	0.682	0.566	0.827	0.712
75%	10,699	18,112	66	2.008	1.173	127%	44%	0.892	0.786	0.731	0.527	0.819	0.671
85%	18,112	34,768	65	1.144	1.096	41%	36%	0.750	0.743	0.377	0.410	0.503	0.552
95%	34,768	77,989	66	1.087	1.085	35%	35%	0.794	0.739	0.467	0.430	0.588	0.583
largest 16	77,989	100,642	16	0.952	1.038	22%	30%	0.561	0.627	0.279	0.311	0.496	0.495
100%	100,642	216,048	16	1.105	1.105	37%	37%	0.678	0.689	0.329	0.329	0.486	0.478

Current Risk Charge Loss Ratio (PR017 Line 4) 1.214

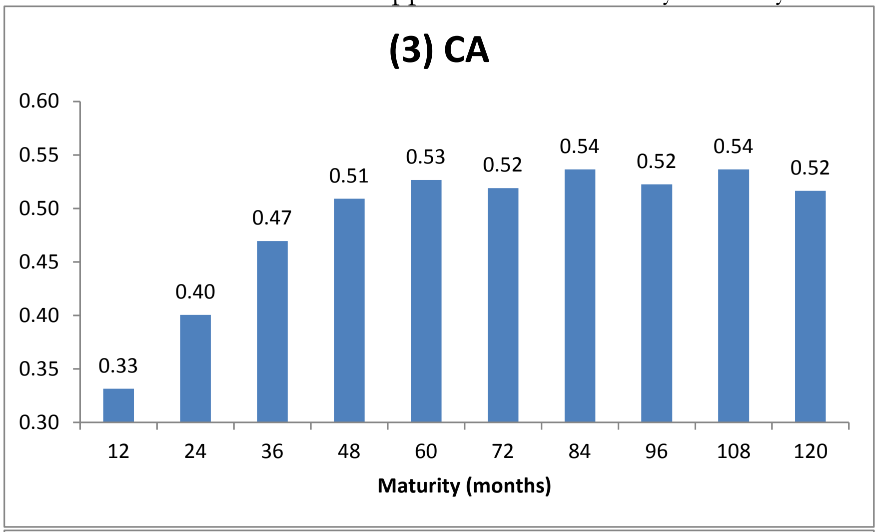
Underwriting Expense Ratio in Risk Charge 27%

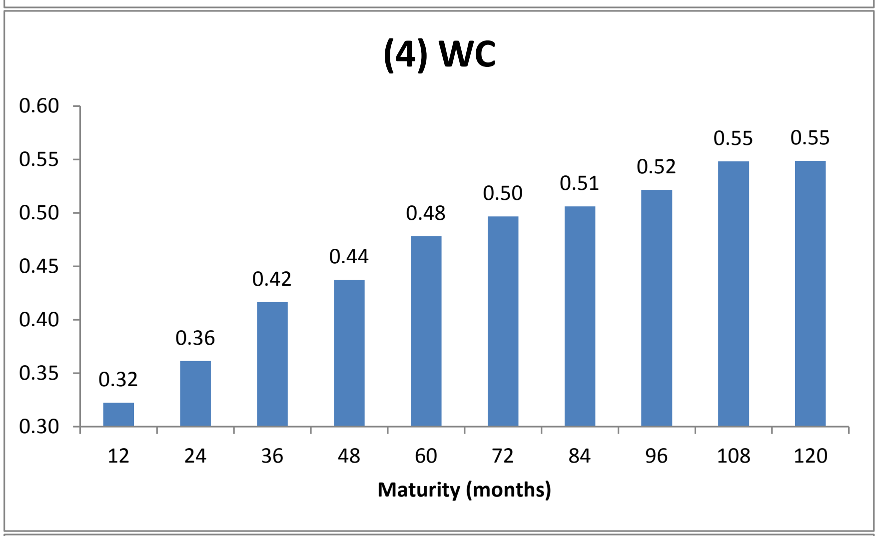
Appendix E – PRC% by Maturity

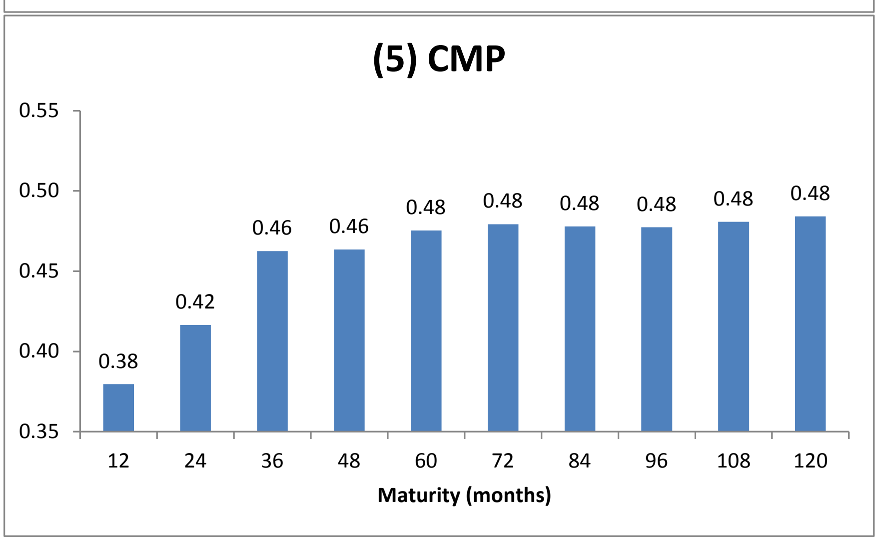




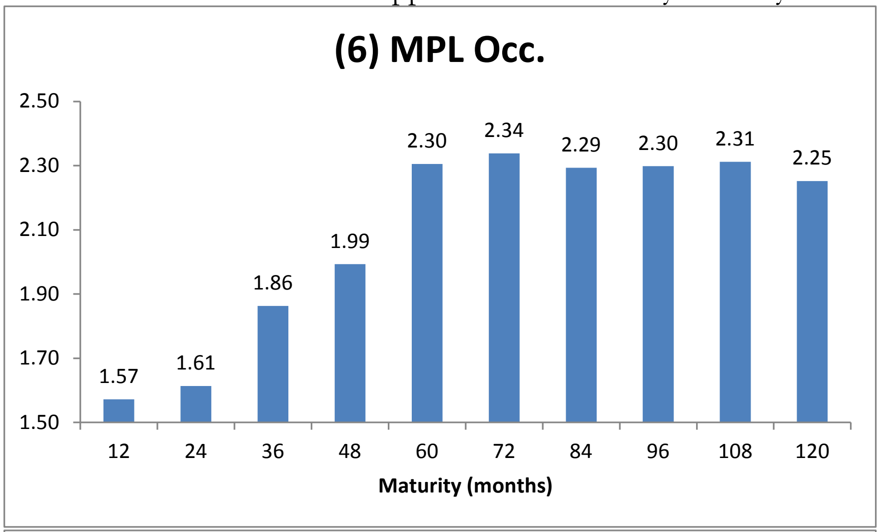
Appendix E – PRC% by Maturity

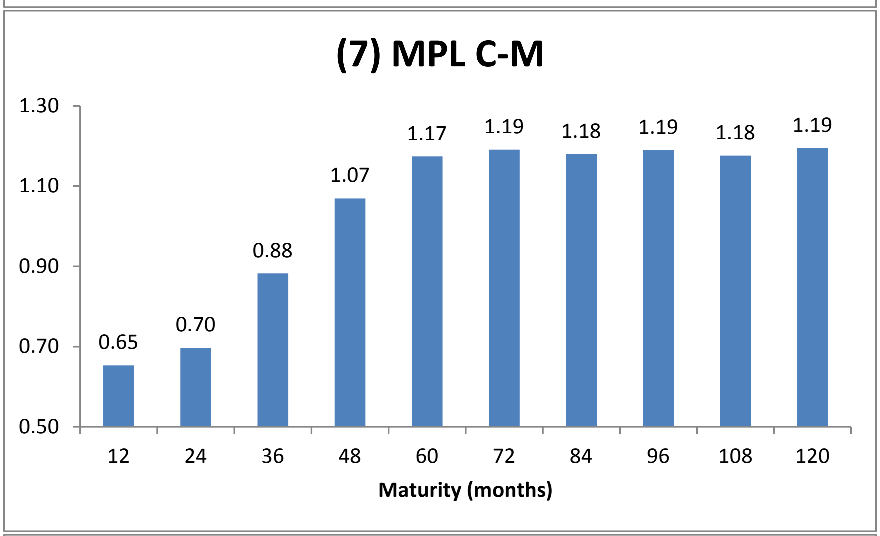


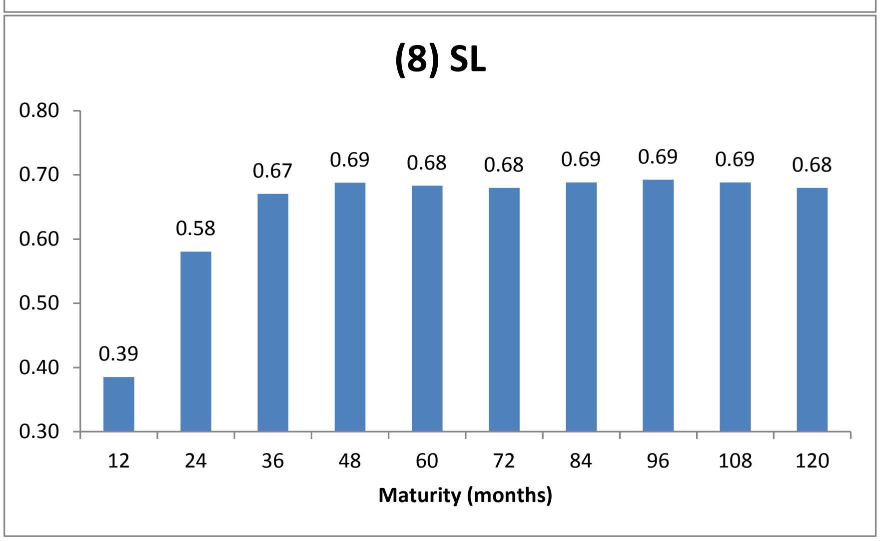




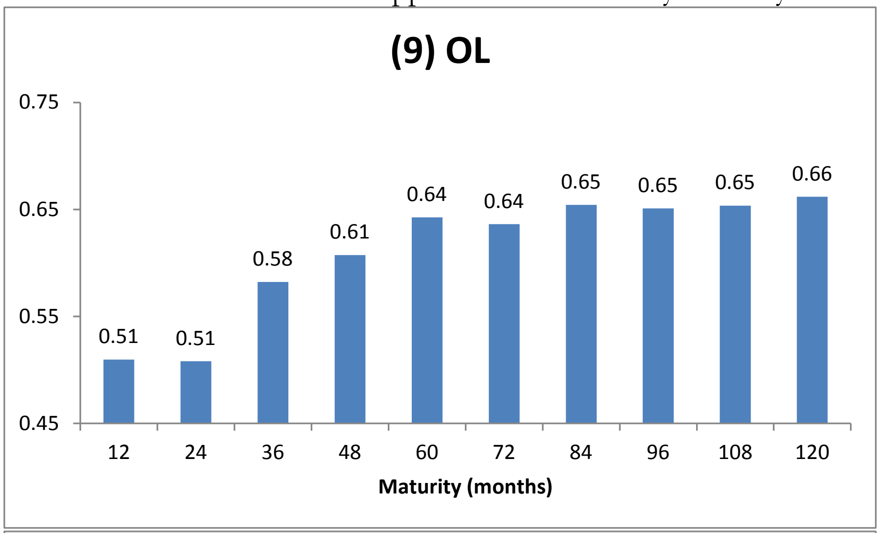
Appendix E – PRC% by Maturity

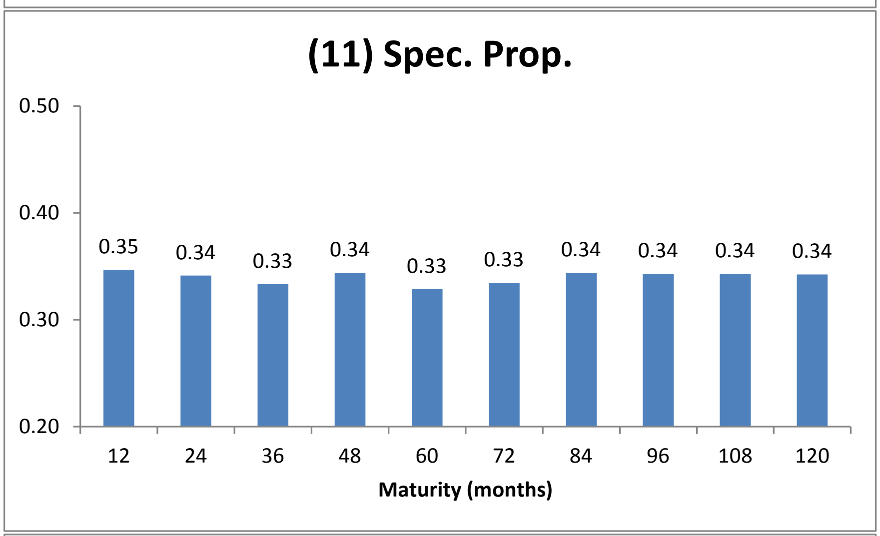


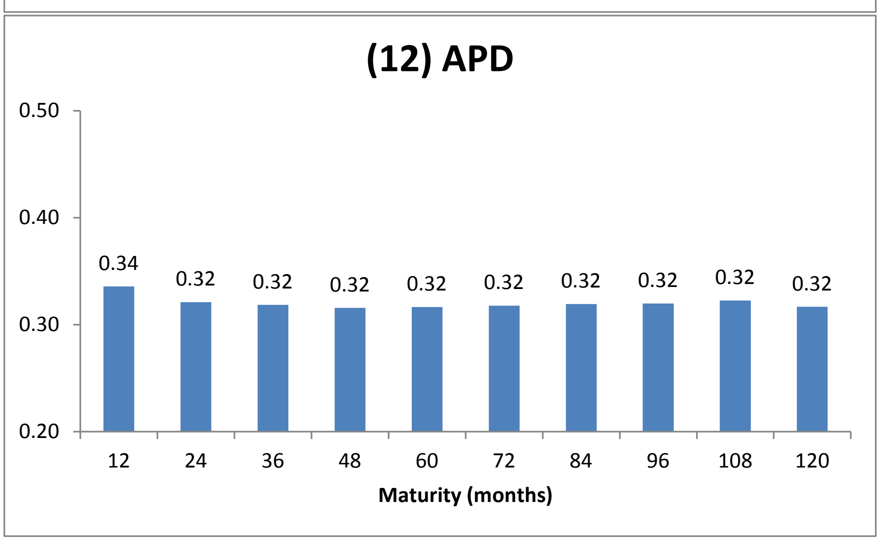


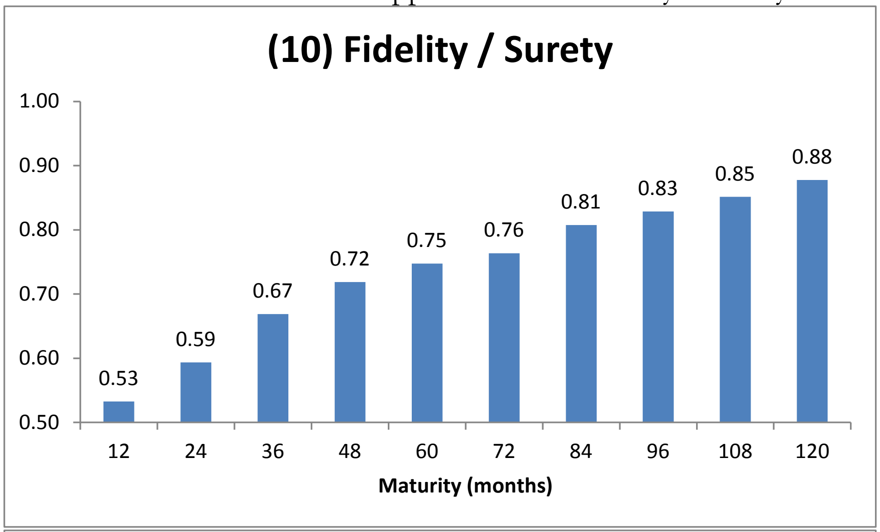


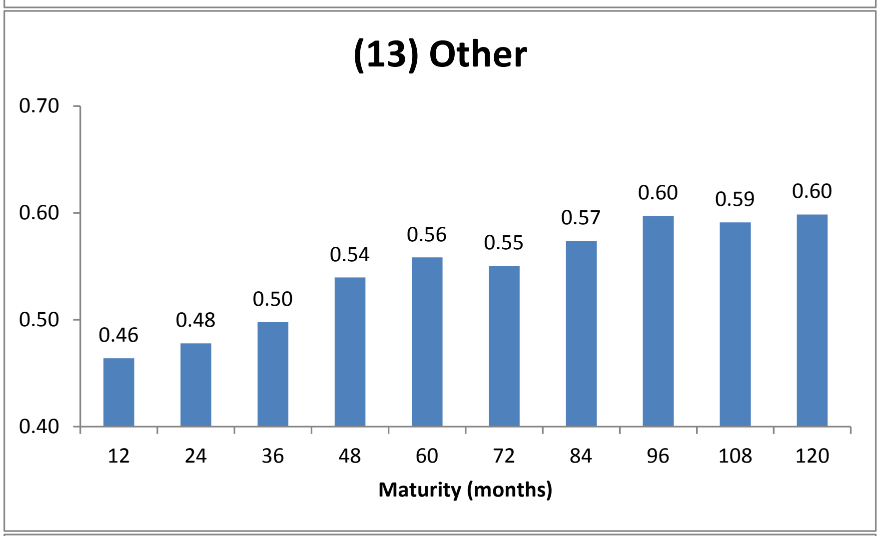
Appendix E – PRC% by Maturity

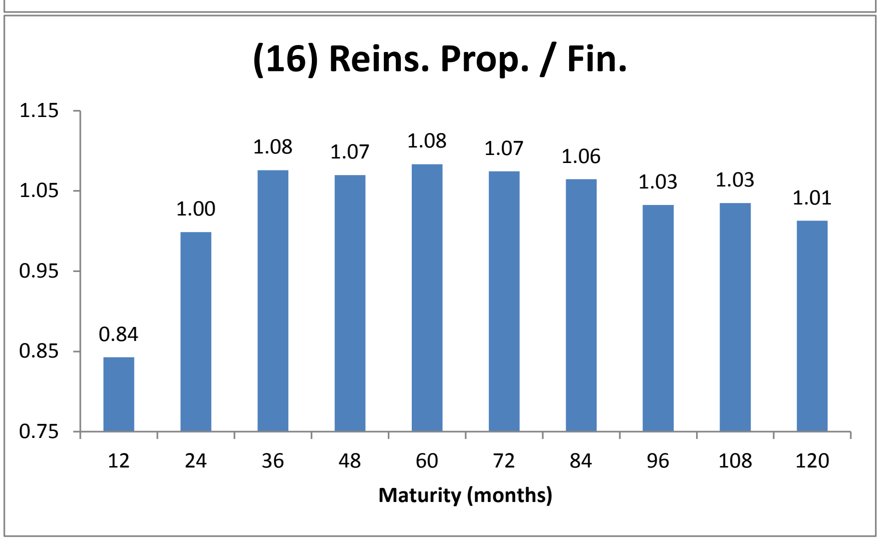




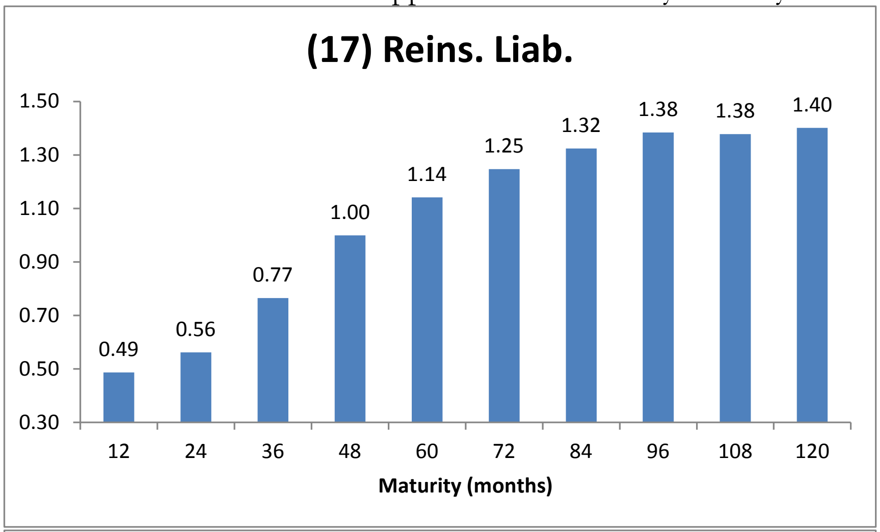


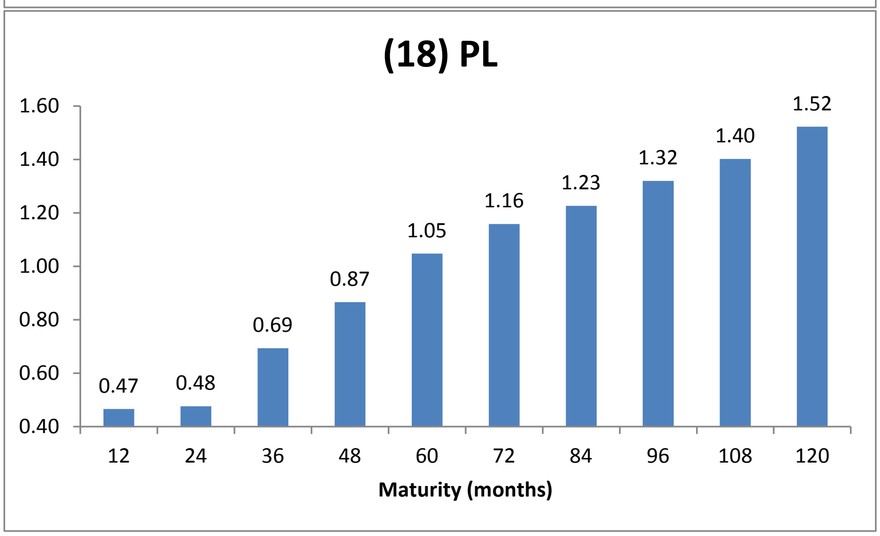

(10) Fidelity / Surety
1.00
0.90
0.80
0.70
0.60
0.50
0.53
0.59
0.67
0.72
0.75
0.76
0.81
0.83
0.85
0.88
12
24
36
48
60
72
84
96
108
120
Maturity (months)


(13) Other
0.70
0.60
0.50
0.40
0.46
0.48
0.50
0.54
0.56
0.55
0.57
0.60
0.59
0.60
12
24
36
48
60
72
84
96
108
120
Maturity (months)


(16) Reins. Prop. / Fin.
1.15
1.05
0.95
0.85
0.75
0.84
1.00
1.08
1.07
1.08
1.07
1.06
1.03
1.03
1.01
12
24
36
48
60
72
84
96
108
120
Maturity (months)

Appendix E – PRC% by Maturity





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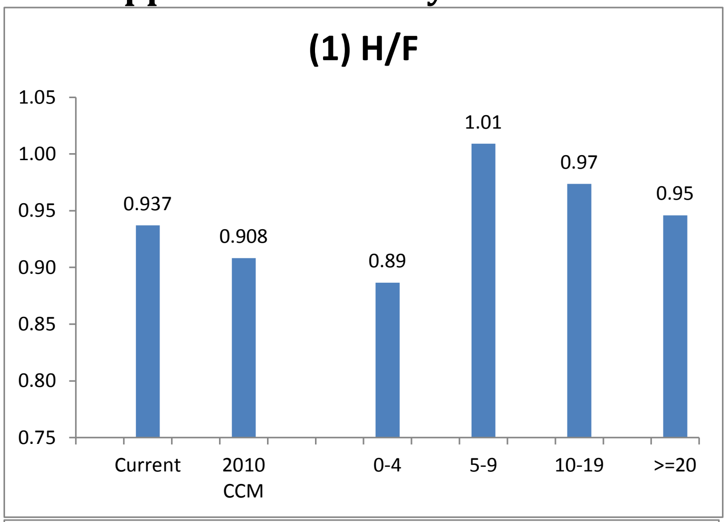
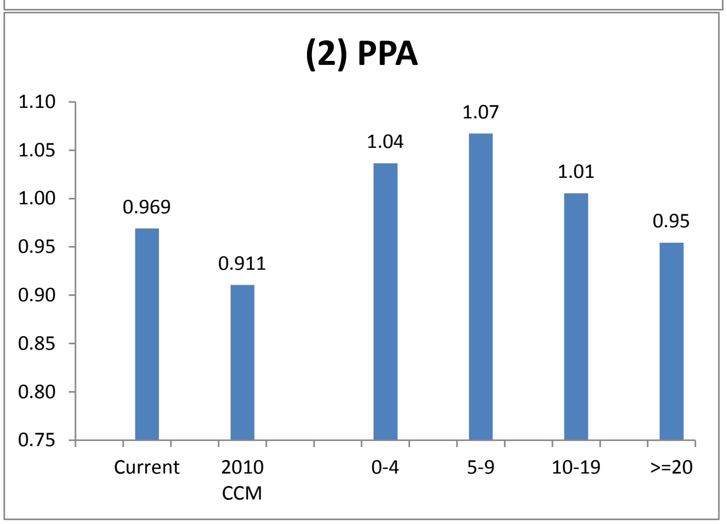
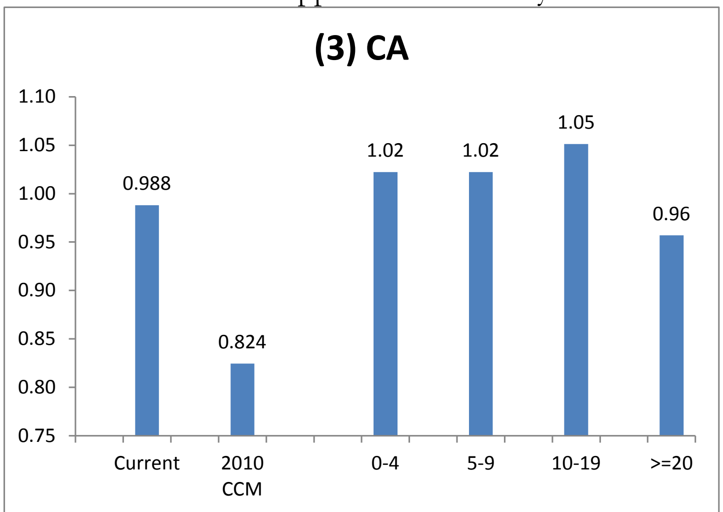
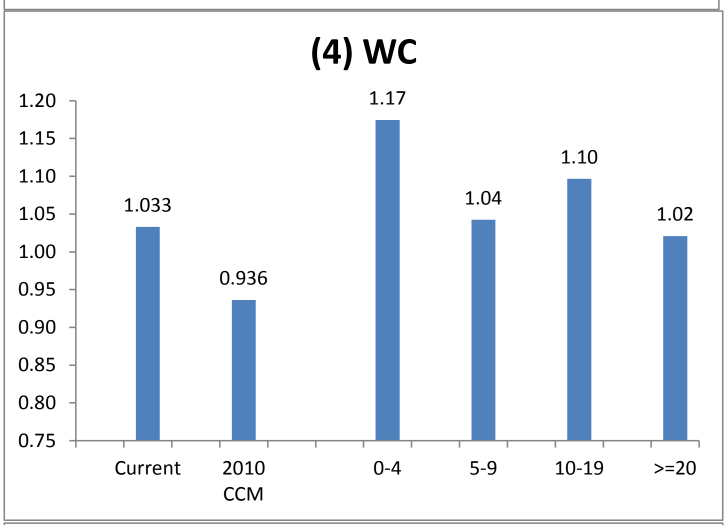
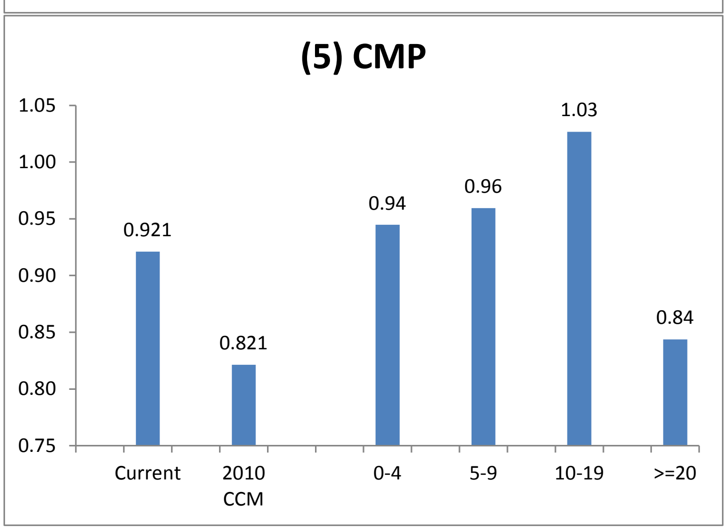




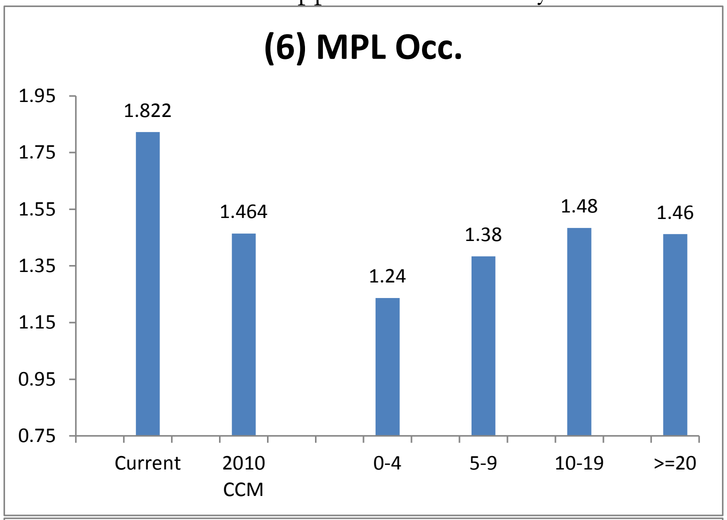
Appendix F – PRF by Number of Years NEP>0

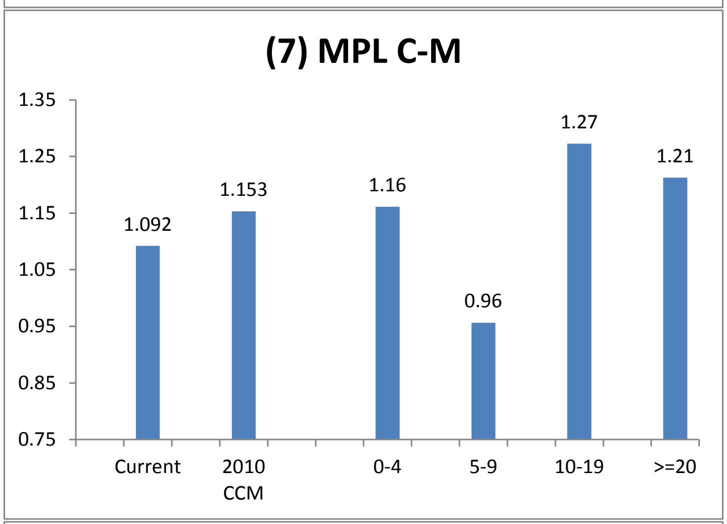


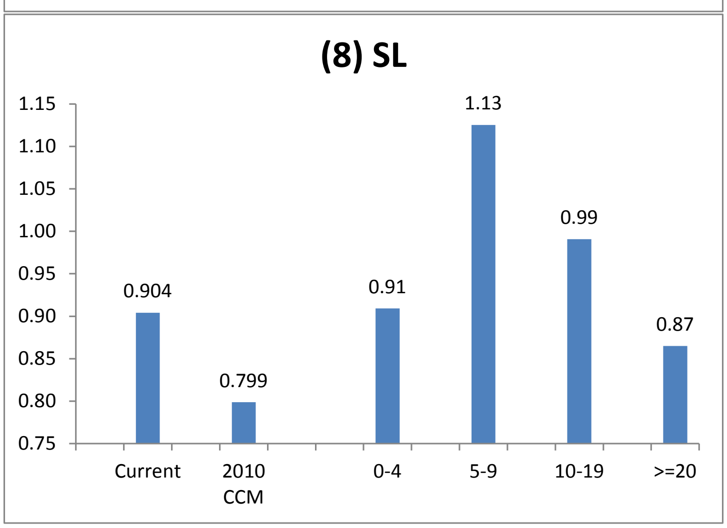




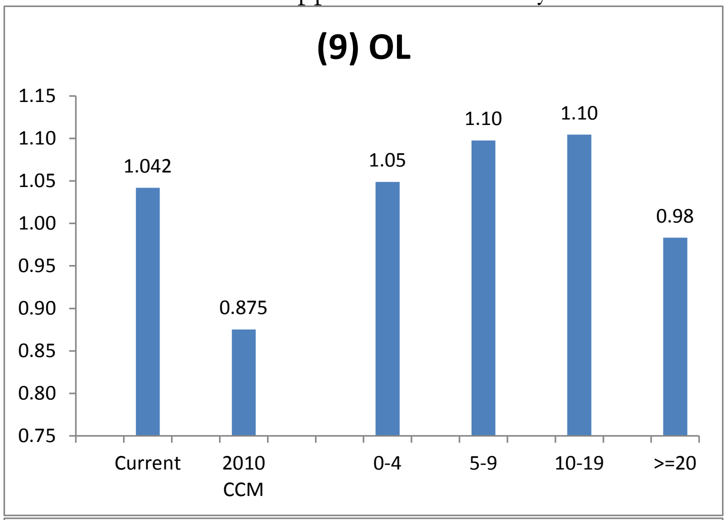
Appendix F – PRF by Number of Years NEP>0

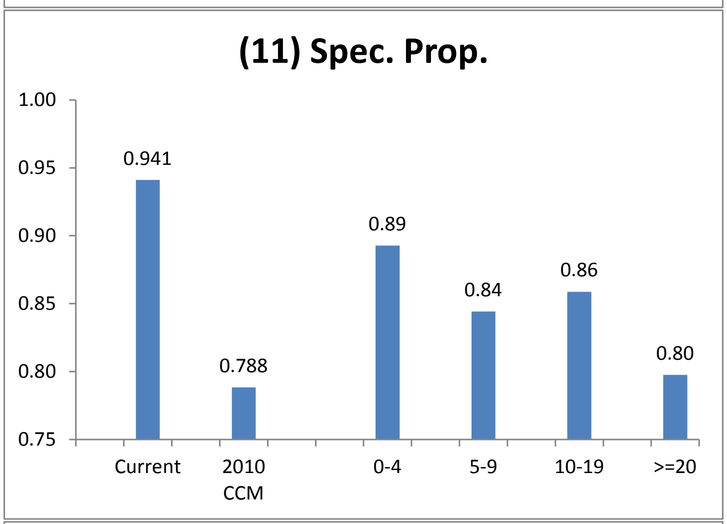


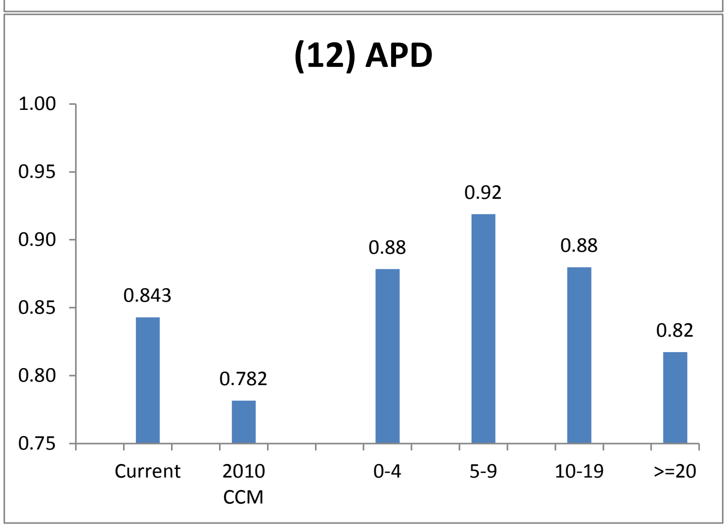




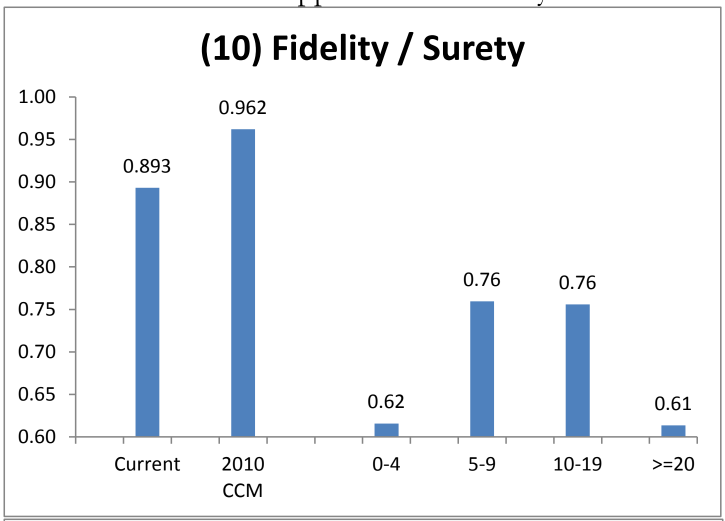
Appendix F – PRF by Number of Years NEP>0

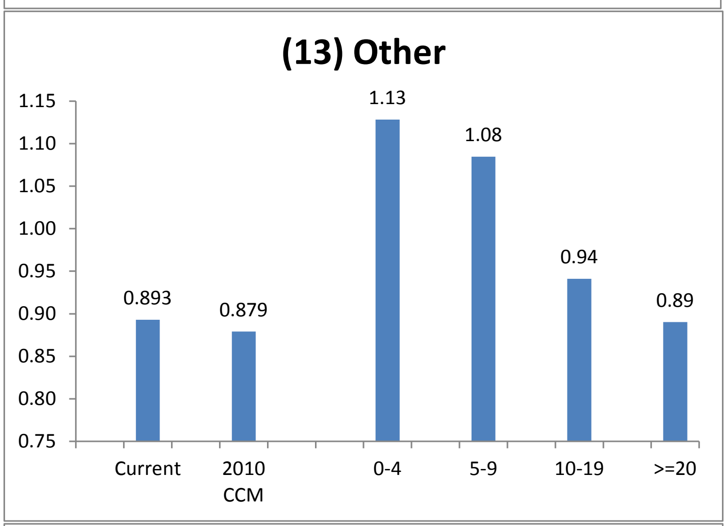


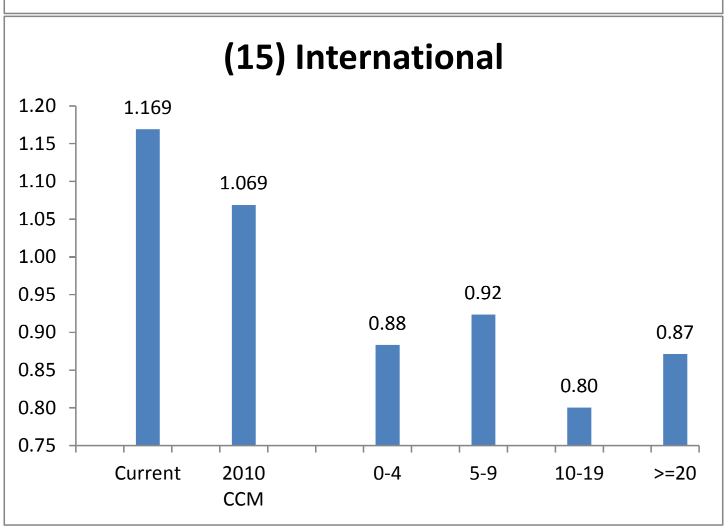




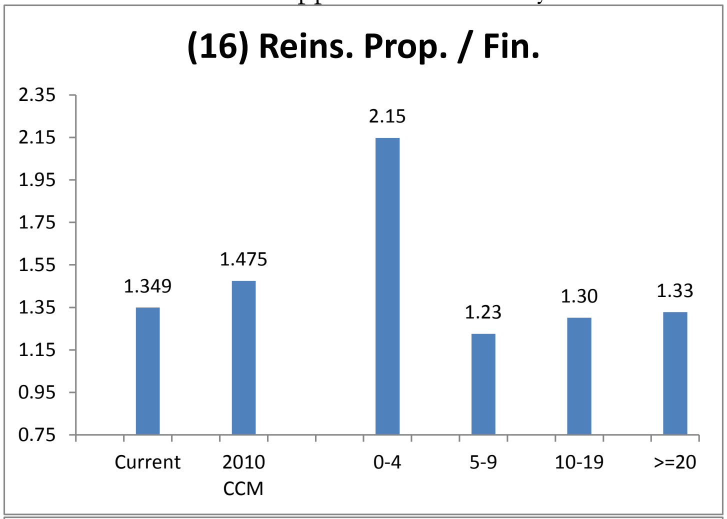
Appendix F – PRF by Number of Years NEP>0

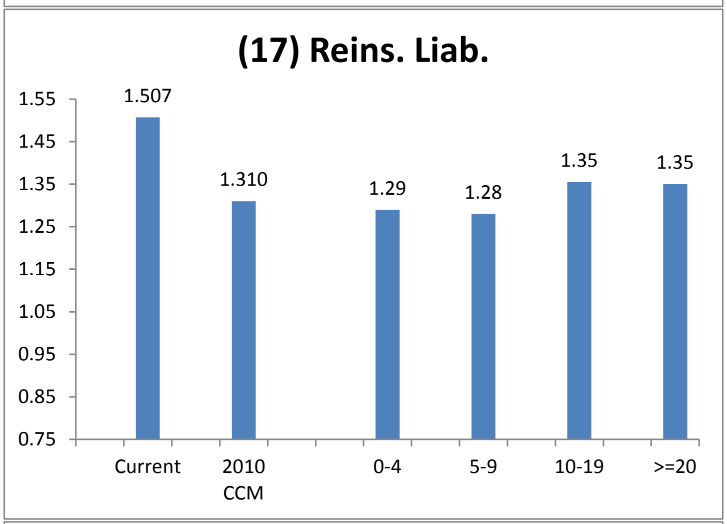


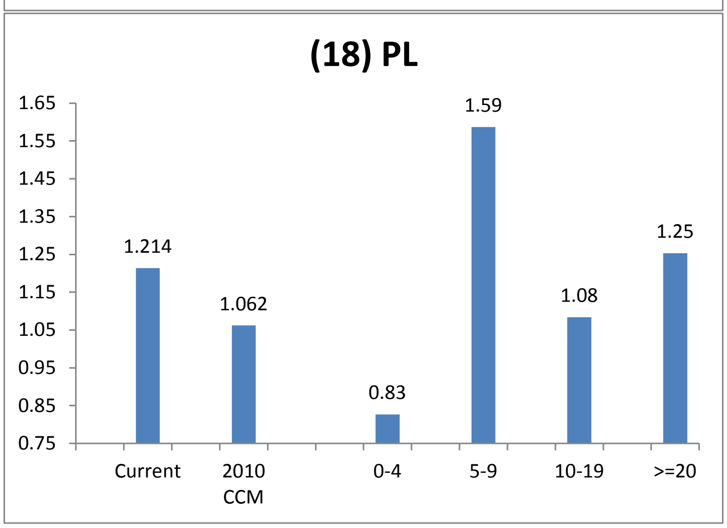




Appendix F – PRF by Number of Years NEP>0







Appendix G – Pooling

As described by Feldblum and Blanchard, CAS Study Note for NAIC Annual Statement, October 2010:

> Many property/casualty insurance groups in the US have intercompany pooling arrangements (pools or pooling) among at least some of their group members. These arrangements typically take the form of a quota-share reinsurance treaty with no expiration date. The companies generally cede 100% of the business to the lead company in the pool, and then assume back a fixed percentage of the pooled results from the lead company.
>
> …
>
> Schedule P requires members of an intercompany pool to ignore the separate cessions to the lead company and assumptions from the lead company. Instead these pool members are required to first determine the Schedule P for the pool as a whole, and then apply their pool percentage to the pool's Schedule P. They are then required to report this scaled-down version of the total Schedule P, instead of reflecting the individual cessions and assumptions between pool members. [30]

This aspect of US P&C business affects the Annual Statement Schedule P data used for the DCWP research. In particular for each LOB-AY, the Schedule P loss ratio would be the same for each pool member; the common loss ratio would be the average net loss ratio for that LOB-AY for the entire pool rather than the individual pool member loss ratio before pooling.

That feature of the data would distort the results of our analysis in that:

1. The same loss ratio value would appear multiple times, reducing the apparent variability in the loss ratios across companies;
2. Companies that appear small based on their pooling percentages would show the lower year-to-year variability from year associated the larger size of the overall pool

[30] Feldblum, Sholom and Ralph Blanchard, CAS Study Note for NAIC Annual Statement, October 2010

rather than the higher year-to-year variability associated with a company of its apparent size.

To mitigate these effects, we would like to combine the separate pool participants into a single data point for each LOB-AY. If that were done, the data would reflect the correct variability among companies and the proper data point LOB-size.

Approach

Data

There are four sources of information on the extent to which Schedule P data reflects pooling:

- NAIC group code
- NAIC "consolidated company code"
- Pooling percentage data in Schedule P
- Schedule F reserves for "Affiliates – U. S. Intercompany Pooling".

Each source provides some information and none is perfect for this purpose of this research.

Methodology

For each current NAIC group, we identified the member companies that had either non-zero Schedule P "pooling percentages" or non-zero Schedule F reserves for "Affiliates – U.S. Intercompany Pooling" for seven or more of the fourteen Annual Statement years, for all LOBs combined. Within each group, we treated all such member companies as "pooled" and created a single "pooled entity."

The premium for the pooled entity is the sum of the premium for all pool members. The loss ratio for the pooled entity is the weighted average of the loss ratios for the individual pool members. The multiple individual pool member AY-LOB data points are removed from our data set. The pooled entity AY-LOB data point is added to the data set; in effect, the newly created data point replaces the multiple pool member data points.

Discussion of data considerations

We use seven years rather than 14 years because pooling arrangements change over time, but not so frequently that it seemed necessary to track pools by year. The seven-year rule might include some data points that are not pooled and not include some points that are pooled.

We used the current group structure to identify possible pool members. To the extent that group structures change over time, this approach might group some currently unrelated companies and might fail to group some historically, but not currently, related companies.

Those aspects of our approach might cause some of the issues noted below.

- The Schedule P and Schedule F information might be expected to identify the same pools, but we found pools identified in Schedule P that were not identified by Schedule F and vice versa.
- The pooling data appears reasonable in that the total pooling percentages for a LOB for group within a year typically was a round 100% (or sometimes 200% or 300%, as would happen if there was two or three pools within the group.) This was not universally the case. For some groups the companies showed pooling that did not total an even 100%, 200%, or 300%.
- 20 companies with pooling percentages were not part of a current group.

Effect on company counts in the data set

There are 3,730 NAIC legal entities in the initial data set.

The DCWP approach results in 2,901 entities. 2,695 are individual companies not affected by pooling approach. 206 are pooled entities formed from 1,035 consolidated entities. (2,901 = 2,695 + 206).

The pooled entities we used in this analysis are not the same as "NAIC Groups" or "NAIC Consolidated Companies". Our pooled entity approach retains more entities (and therefore more data points) than would be the case if we had based relied on either "NAIC Groups" or "NAIC Consolidated Companies".

- If we had combined all companies within an NAIC Group into a "group entity" there would be only 1,884 entities (vs. 2,901 entities used): 1,362 stand-alone companies and 522 groups with more than one member.

G Appendix G - Pooling

- If we had combined all NAIC consolidated entities into a "Consolidated Company entities," there would be only 2,387 consolidated entities (vs. 2,901 entities used), 1,359[31] stand-alone companies not part of a group or consolidated company, 698 individual companies that are part of an NAIC Group but not part of a consolidated company statement, and 330 consolidated companies.

Final Comment

Our approach does not necessarily identify all pools and it may combine some LOB/companies that are not actually pooled. Therefore, some pooling effects remains in the data used for this analysis.

However, we are confident this adjustment, while not perfect, is an improvement over using all companies as if there were no pooling.

Future research might refine this work by identifying data points as pooled by company-by-year rather than more simply by company as we did for this research.

[31] The 1,359 entities is 1,362 stand-alone companies not in group minus three companies in consolidated statements in which there is only one member company (probably a consolidation that included more than one member in the past).

GLOSSARY

Term	Interpretation
AY	Accident year
Baseline filtering	As defined in Section 3.4
CCM	Current Calibration Method
Data point	Each data point is an AY-LOB, for a single company or pool, at the latest available maturity (for most analyses) or at successive annual evaluation dates (in the maturity analysis in Section 6)
DCWP	CAS RBC Dependency and Calibration Working Party
Formula RBC Formula	The 2010 NAIC Property-Casualty RBC Formula
LOB	Line of Business
LOB-size	Line of business size, expressed as NEP
Loss ratio	Loss and all loss adjustment expenses net of reinsurance divided by earned premium net of reinsurance", as shown in Schedule P – Part 1, column 31.
Minor lines	A company (pool) LOB-AY for which the NEP represents less than 5% of all-lines total NEP by AY
MPL or MM	Medical Professional Liability/ Medical Malpractice
NEP	Net Earned Premium
NWP	Net Written Premium
PPA	Private Passenger Automobile liability
PRC	Premium Risk Charge
PRC%	PRC divided by NWP
PRF	Premium Risk Factor
RBC	Risk Based Capital
Survivorship	The extent to which PRFs are affected by included companies that did not file 2010 Annual Statements

REFERENCES

[1] American Academy of Actuaries' P/C Risk-Based Capital Committee, 2009 Update to P/C Risk-Based Capital Underwriting Factors presented to National Association of Insurance commissioners' P/C Risk-Based Capital Working Group, December 2008

http://www.actuary.org/files/publications/CPC_P-C_RBC_Committee_Update_to_Underwriting_Risk_Factors_to_NAIC_Property_RBC_Working_Group_120908.pdf

[2] American Academy of Actuaries' P/C Risk-Based Capital Committee, 2010 Update to P/C Risk-Based Capital Underwriting Factors presented to National Association of Insurance commissioners' P/C Risk-Based Capital Working Group, March 2010

http://actuary.org/pdf/casualty/rbc_update_mar10.pdf

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THE ESTIMATION OF LOSS DEVELOPMENT TAIL FACTORS: A SUMMARY REPORT

CAS Tail Factor Working Party

Steven C. Herman, co-chair
Mark R. Shapland, co-chair
Mohammed Q. Ashab
Joseph A. Boor
Anthony R. Bustillo
David A. Clark
Robert J. Foskey
Aaron Halpert
Sejal Haria
Bertram A. Horowitz
Gloria A. Huberman
Richard Kollmar
Joshua Merck
Rasa V. McKean
Michael R. Murray
Bernard A. Pelletier
Susan R. Pino
Anthony J. Pipia
F. Douglas Ryan
Scott G. Sobel

Reviewed by: The CAS Committee on Reserves

ABSTRACT

Motivation. Tail factors are used by actuaries to estimate the additional development that will occur after the eldest maturity in a given loss development triangle, or after the eldest credible link ratio. Over the years, many valuable contributions have been made to the CAS literature that describes various methods for calculating tail factors. The CAS Tail Factor Working Party prepared this paper on the methods currently used by actuaries to estimate loss development 'tail' or 'completion' factors. Standard terminology for discussing aspects of link ratios and tail development is communicated within the paper. Descriptions of the advantages and disadvantages of each method are included as well general indications of what types of entities (companies, rating bureaus, or consulting firms) typically use each method.

Method. An extensive survey of existing CAS literature was performed, along with surveys of methods currently in use by various rating bureaus, insurers, and consulting organizations. The methods identified by the Working Party are grouped into six basic categories: (1) "Bondy Methods"; (2) algebraic methods that focus on relationships between paid and incurred loss; (3) methods based on use of benchmark data; (4) curve-fitting methods; (5) methods based on remaining open counts; (6) methods based on peculiarities of the remaining open claims; and (7) the remaining unclassified methods.

Results. Comparisons of the results of several key tail factor methodologies to the actual post-ten year development for a number of long-tail lines using multiple realistic data sets are included, along with the advantages and vulnerabilities of each method.

Availability. A copy of the Working Party's paper and companion Excel template can be found on the CAS website at http://www.casact.org/pubs/forum/13fforumpt/.

Keywords. Tail Factors; Completion Factors; Link Ratios; Age-to-Age Factors; Development Factors; Loss Reserving; Curve Fitting; Bondy Method; Benckmark; Loss Development.

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1. INTRODUCTION

1.1 Importance of Loss Development Tail Factors

The loss development tail factors (sometimes referred to as completion factors) are an important part of any reserve analysis. They have a highly leveraged impact since they form a portion of the loss development applied to each of the accident years being analyzed. However, the discussions of tail factor estimation methods used, when they are contained in the CAS literature at all, are generally just as adjuncts to the main topics of papers. Further, some methods are used in practice that are not described in the CAS literature at all. Therefore, the CAS Committee on Reserving sponsored a Tail Factor Working Party to undertake an exhaustive survey of the tail factor estimation methods in use and describe and comment on each method.

1.2 Research Context

As stated above, tail factors have a highly leveraged impact on loss development since they form a portion of the loss development of all accident years analyzed. Further, tail loss development reflects development occurring after the last development period in the reserving data triangle and is therefore somewhat more difficult to estimate than the various link ratios developed from the data triangle. For both those reasons, the Tail Factor Working Party believes it is helpful to provide information concerning tail factor estimation methods to practitioners.

1.3 Objective

This paper is designed to be as exhaustive a listing of methods used to estimate tail loss development as is reasonably possible at the time of its writing. The Tail Factor Working Party hopes this will expose the various approaches to a wider audience, and help actuaries choose the best method for each reserving circumstance from a larger toolkit. Further, this paper lists at least some of the advantages and disadvantages of each method, which could help the practitioner decide which method to use in a given circumstance.

1.4 Disclaimer

While this paper is the product of a CAS Working Party, its findings do not represent the official view of the Casualty Actuarial Society. Moreover, while we believe the approaches we describe are very good examples of how to estimate tail development in reserving, ratemaking and selecting the best method for a given circumstance, we do not claim they are the only acceptable ones or that we have ultimately addressed all of the issues that must be considered in selecting a tail factor or tail factor methodology.

1.5 Section References to Methods

The classes of methods presented are discussed in the next sections. Within each class of method, an introduction to the class of method, a summary of the methods, any particular findings, and conclusions are presented.

1.6 Alternate Grouping of Methods Included in the Paper

While organizing this paper, working party members noted that the groupings of methods were not inherently absolute and that the methods could be grouped in alternate ways. The commentary and listing in Appendix A represents an alternate but still logical view of how the various methods relate to each other.

1.7 Notation

This paper describes many tail factor methods identified in the actuarial literature and elsewhere. For the sake of uniform notation, where appropriate we have adopted (and expanded) the notation used by the CAS Working Party on Quantifying Variability in Reserve Estimates. In the paper produced by that Working Party, some models visualize loss statistics as a two-dimensional triangle array. In the notation, the row dimension is the period[1] by which the loss information is subtotaled, most commonly an accident period.[2] For each accident period w, development age d the (w,d) element of the array is the total of the loss information as of development age d.[3]

For this discussion, we assume that the loss information available is an upper left triangular subset of the two-dimensional array for rows $w = 1,2,\ldots,n$. For each row w, the information is available for development ages 1 through $n-w+1$. If we think of period n as the latest accounting period for which loss information is available, the triangle represents the loss information as of accounting dates 1 through n. The diagonal for which $w+d$ equals a constant k represents the loss information for each accident period w as of accounting

---

1 Most commonly the periods are annual (years), but as most methods can accommodate periods other than annual we will use the more generic term "period" to represent year, half-year, quarter, month, etc. unless noted otherwise.

2 Other exposure period types, such as policy period and report period, also utilize tail factor methods. For ease of description, we will use the generic term "accident" period to mean all types of exposure periods, unless otherwise noted.

3 Depending on the context, the (w,d) cell can represent the cumulative loss statistic as of development age d or the incremental amount occurring during the d^{th} development period.

period k .[4]

In general, the two-dimensional array will also extend to columns $d = 1, 2, \ldots, n$. For purposes of calculating tail factors, we are interested in understanding the development beyond the observed data for periods $d = n+1, n+2, \ldots, u$, where u is the ultimate time period for which any claim activity occurs – i.e., u is the period in which all claims are final and paid in full.

The paper uses the following notation for certain important loss statistics:

$c(w,d)$: cumulative paid or incurred loss from accident period w as of development ages d. (w and d may be thought of as representing "when" and "delay," respectively.) In the context of this and other notation, $c_{Paid}(w,d)$ denotes cumulative paid loss and $c_{Inc}(w,d)$ denotes cumulative case incurred loss.

$q(w,d)$: incremental paid or incurred loss on accident period w during the development age from $d-1$ to d. Also denoted as $q_{Paid}(w,d)$ or $q_{Inc}(w,d)$.

$s(w,d)$: case reserves at end of development age d for accident period w.

$c(w,u) = U(w)$: total loss from accident period w when at the end of ultimate development.

$R(w)$: future development after age $d = n-w+1$ for accident period w, i.e., = $U(w) - c(w, n-w+1)$.

$S(d)$: estimated ratio of unpaid costs to case reserves at the end of the triangle data d.

S: estimated ratio of unpaid costs to case reserves as of the end of the triangle data.

$f(d) = 1 + v(d)$: factor applied to $c(w,d)$ to estimate $c(w,d+1)$ or more generally any factor relating to age d. This is commonly referred to as a link ratio. $v(d)$ is referred to as the 'development portion' of the link ratio, which is used to estimate $q(w,d+1)$. The other portion, the number one, is referred to

[4] For a more complete explanation of this two-dimensional view of the loss information see the *Foundations of Casualty Actuarial Science* [5], Chapter 5, particularly pages 210-226.

as the 'unity portion' of the link ratio.

$\hat{f}(d) = 1 + \hat{v}(d)$: an estimate of the link ratio for development age to development age $d+1$.

$F(d) = 1 + V(d)$: ultimate development factor relating to development age d. The factor applied to $c(w,d)$ to estimate $c(w,u)$ or more generally any cumulative development factor relating to development age d. The capital indicates that the factor produces the ultimate loss level. As with link ratios, $V(d)$ denotes the 'development portion' of the loss development factor, the number one is the 'unity portion' of the loss development factor. $G(d)$ is used interchangeably with $F(d)$ and by convention, G may also be used to denote the ultimate loss development factor needed for period w when written as $G(w)$.

$T = T(n)$: tail factor at end of triangle data.

$\hat{T}$: estimate of the tail factor.

$h(w+d)$: factor relating to the diagonal k along which $w+d$ is constant.

$e(w,d)$: a mean zero random fluctuation that occurs at the w,d cell.

$r(k)$: annual rate of loss cost inflation, in this case related to payment period, although in cases where r is either constant or estimated as a constant, r is the cumulative impact over k years $(1+r)^k$.

$\hat{r}$: an estimate of the rate of annual loss cost inflation.

m: development or delay time in months.

$D(m)$: rate of loss cost inflation per month, when D is constant over m, the impact over m months is $(1+D)^m$.

$\hat{D}$: an estimate of the rate of monthly loss cost inflation.

l: lag until payouts start. Used in McClenahan and Sherman methods.

$B(d) = 1 + b(d)$: notation for a benchmark link ratio and the 'development portion' of the benchmark. Note that $B_T = 1 + b_T$ represents the benchmark tail factor.

i: a specific accident month, similar to w.

p_i: the month-to-month decay rate of the pre-inflation loss payouts for a given accident month, also used as a constant over all months, p.

$q_i = 1 - p_i$: the complement of p, also used as a constant over all months, q.

$A(i)$: constant of proportionality reflecting total expected pre-inflation losses in a given accident month i.

$H(w)$: a constant of proportionality used in curve-fitting. Often, for global curve-fitting across an entire triangle, simply used as H.

a and b: constant terms representing the multiplier and exponent of an inverse power curve, respectively.

RE: the reinsurance retention applying to a given triangle. $RE(w)$ refers to the retention of a specific period w.

$E(x)$: the expectation of the random variable x.

$Var(x)$: the variance of the random variable x.

$U(w)$: ultimate loss amount in accident period w = c(w,u).

Also, for some methods, additional or slightly different notation is used.

2. BONDY-TYPE METHODS

2.1 Introduction and Description of Bondy-Type Methods

This class of methods is discussed first due to its simplicity. Martin Bondy suggested this method of just repeating the last observed link ratio for use as the tail factor. Note, that at the time Bondy developed his method in the 1960s, most lines of insurance were believed to be "short-tailed" in nature compared to assumptions assumed for many casualty lines of insurance today. Bondy's Original Method (see section 2.2) may seriously understate the needed tail factor for "long-tail" lines or for any case where substantial development occurs in the tail. Several alternate versions of the Bondy approach have been developed in an attempt to mitigate the original method's shortcomings.

The formulas for the Bondy-Type methods are described in the sub-sections below. Starting with the original method, we move through modifications that lead to a fully generalized method.

2.2 Bondy's Original Method

Bondy's Original Method used the link ratio $f(n-1)$ at the last observed development age, n, to develop losses to ultimate; that is

$$F(n) = f(n-1). \qquad (2.1)$$

The assumption for age-to-age development factors in the tail is that

$$f(d) = \sqrt{f(d-1)}\,. \tag{2.2}$$

2.3 Modified Bondy Method

In these revisions of Bondy's Original Method, some recognition is given to more extended development patterns; the first approach is multiplicative, the second additive.

The first approach consists of simply squaring the last link ratio, rather than just repeating it:

$$F(n) = f(n-1)^2\,. \tag{2.3}$$

The second approach, utilized by some practitioners, is to merely double the development portion of the last link ratio:

$$F(n) = 1 + [2 \times v(n-1)]\,. \tag{2.4}$$

2.4 Generalized Bondy Method

Subsequently, Weller [16] suggested a generalization by setting $f(n) = f(n-1)^B$, where B is a number between 0 and 1. We call B the Bondy exponent. It follows that

$$F(n) = f(n-1)^B f(n-1)^{B^2} \cdots = f(n-1)^{B/(1-B)}\,. \tag{2.5}$$

Thus, if $B = \dfrac{1}{2}$, we recover the original Bondy method.

Let $f(d)$ be the development ratio chosen for age $d-1$ to age d. In his paper, Weller used the average of the latest three observed development ratios for $f(d)$. (Fewer or more observations could be utilized.) Set $l_d = \log f(d)$, $\hat{B}$ the estimated Bondy parameter, $\hat{f}(i)$ the estimated development ratio for the earliest development period used to estimate the parameters, and $\hat{l}_i = \log \hat{f}(i)$. The parameters, $\hat{f}(i)$ and $\hat{B}$, are chosen to minimize

$$\sum_{d=i}^{n} \left(l_d - \hat{l}_i \hat{B}^{d-1} \right)^2 . \tag{2.6}$$

The parameters, $\hat{f}(i)$ and $\hat{B}$, can be calculated easily using a readily available spreadsheet optimization function such as the "Solver" function in Microsoft® Excel.

2.5 Fully Generalized Bondy Method

Gile [6] devised a further generalization by letting the estimated development ratios vary by accident period, while using the same estimated Bondy parameter for each accident period. Two parameters, as well as the development ratios, are chosen for each accident period by minimizing the sum of squared differences using more than one development period for each accident period.

2.6 Examples

See Appendix B, Section B.2.

2.7 Advantages and Disadvantages of the Bondy Methods

The method is easily implemented using standard spreadsheet functions. It only uses the data in cumulative paid or incurred loss triangles. Finally, loss development is described in terms of only one factor, the Bondy exponent.

The fully generalized Bondy method is not always useful for incurred loss data because it may produce Bondy exponents not in the range from 0 to 1. For this same reason, the method fails to give meaningful answers when the pattern of development factors is increasing. Since the Bondy method describes loss development in terms of only one parameter, the method may also fail if the development pattern is complicated in some other way.

2.8 Users

The Bondy-type methods (including the specific forms discussed above) are widely accepted and used in current practice.

2.9 Summary

Bondy methods give a simple solution to the problem of determining tail factors. They are easy to explain and to implement. However, they describe loss development in terms of only one parameter so that complicated development patterns may not be accurately projected.

3. ALGEBRAIC METHODS

3.1 Introduction to Algebraic Methods

Algebraic methods are methods that focus on the relationships between the paid and incurred loss triangles. They are based on relatively simple calculations in the sense that complex mathematical formulae and curve fitting, etc. is not required. Additionally, ancillary information beyond readily available paid and incurred data is not required for any of these methods.

3.2 Equalizing Paid and Incurred Development Ultimate Losses

This method is one of the oldest tail factor methods used and also has perhaps the broadest usage of all the methods. It was designed to provide an easy methodology for determining a paid loss tail factor when the incurred loss tail factor is available.

3.2.1 Description[5]

This method is most useful when incurred loss development essentially stops after a certain stage (i.e., the link ratios are near to unity or are equal to unity). Then, due to the absence of continuing development, the current case incurred (e.g., case incurred as of end of most recent accounting period, sometimes called reported) losses are a good predictor of the ultimate losses for the older or oldest years without the need for additional tail factor development. A tail factor suitable for paid loss development can then be computed as the ratio of the case incurred for the oldest accident period in the triangle divided by the paid losses to date for the same accident period. This results in a paid to ultimate development factor estimate which when multiplied by the cumulative paid equals the ultimate (which are also the current) incurred losses for that oldest accident year.

This method relies on one axiomatic (meaning plainly true rather than an assumption as such) assumption and two true assumptions. The axiomatic assumption is that the paid loss and incurred loss development estimates are estimating the same quantity, therefore the ultimate loss estimates they produce should be equal. The second assumption (the first true assumption) is that the incurred loss estimate of the ultimate losses for the oldest accident period is accurate. The last assumption is that the other periods will show the same development in the tail as the oldest period. An appropriate way to test this assumption is to estimate the paid loss tail based on several accident periods.

This method may also be generalized to the case where the current case incurred is still showing development near the tail. In this situation, the implied paid loss tail factor is

$$\frac{\text{ultimate incurred loss development estimate for the oldest accident period}}{\text{paid losses to date for the oldest accident period}}, \text{ or}$$

$$\frac{c_{Inc}(1,u)}{c_{Paid}(1,n)}. \tag{3.1}$$

5 Section 3.2.1 is reproduced from [1] with permission. Minor edits have been made for consistency with the rest of this Report.

In this instance, the incurred loss development estimate for the oldest accident period is usually the current case incurred losses for the oldest period multiplied by an incurred loss tail factor developed using other methods.

3.2.2 Example

We are given the following selected incurred loss development factors:

12-24 months	2.000
24-36	1.500
36-48	1.250
48-60	1.125
60-72	1.063
72-84	1.031
84-96	1.016
96-108	1.008
108-120	1.004

Incurred losses for the oldest year in the triangle as of 120 months is $50,000,000 and the corresponding paid loss is $40,000,000. The incurred estimated ultimate using the 1.004 tail factor is $50,200,000. The paid loss tail factor to equalize the paid estimated ultimate to the incurred estimated ultimate would be $50,200,000 divided by $40,000,000 or 1.255.

3.2.3 Advantages and Disadvantages

This method has a substantial advantage in that it is based solely on the information in the triangle itself. One of its weaknesses is that a reliable estimate of the ultimate loss for the oldest year is needed before it can be used. In addition, if the ultimate incurred loss development of the oldest accident year is estimated using a tail factor estimate, then this method also relies on the incurred loss tail factor. Lastly, there is an assumption that the ratio of the case incurred loss to the paid loss will be the same for less mature years once they reach the level of maturity used initially to calculate the paid tail. This assumption can be tested by looking at the stability of the paid to incurred ratio.

3.2.4 Users

This method is such a basic part of most loss development analyses that it is probably under-reported on surveys. For example, most users will attempt to at least compare the estimated ultimate paid and estimated ultimate incurred loss for the oldest years.

3.2.5 Summary

This method is both simple and widely used. However, a major limitation is that unless development of the oldest accident period is complete at least one tail factor (incurred or paid) must be calculated by other means before this approach can be used.

3.3 Sherman-Boor Method

This method was developed by Sherman in [13], and later by Joseph Boor in the course of analyzing very long tail workers compensation data during the 1987-1989 periods. Although it was originally published some time ago as an adjunct to other tail factor methods, it has only recently received much attention. Thus, a comparatively small percentage of practicing actuaries are aware of it. It was developed largely to provide an alternative to the use of fitted curves and their heavy reliance on theoretical assumptions.

3.3.1 Description

This method relies solely on the triangles themselves and does not require a pre-existing ultimate loss estimate, involve curve-fitting assumptions, or require external data. For data triangles with high statistical reliability as predictors, this can represent a powerful and reliable predictor of tail development.

This method involves simply determining the ratio of case reserves to paid loss for the oldest period in the triangle, then adjusting the case reserves by an estimate of the ratio of the unpaid loss to carried case reserves. In essence, the case reserves of the oldest accident period are 'grossed up' to estimate the true unpaid loss using a factor. The estimate of the (true unpaid loss)/(case reserves) factor is based on how many dollars of payments are required to 'eliminate' a dollar of case reserve.

The mathematical formula requires computing a triangle containing incremental rather than cumulative paid losses. The formula for incremental paid losses for accident period w, from development age $d-1$ to d is:

$$q_{Paid}(w,d) = c_{Paid}(w,d) - c_{Paid}(w,d-1). \tag{3.2}$$

The next step begins with a triangle of case reserves. The incremental case reserve disposed of in a development period is calculated as the beginning case reserve of that period minus the ending case reserve of that period. The formula for case reserves disposed of is essentially a decrement-type process (process of reduction rather than process of increase), so it is stated in negative terms as:

$$-q_{Case}(w,d) = s(w,d) - s(w,d-1). \tag{3.3}$$

Alternately, it may be stated positively as:

$$q_{Case}(w,d) = s(w,d-1) - s(w,d)\,, \tag{3.3.1}$$

where $s(w,d)$ represents case reserves at the end of development age d for accident period w. Next the ratios of incremental paid to reserve disposed for each element in the triangle~~s~~ is computed. Noting that the case decrement at the first column (which may be either $d=0$ or $d=1$ in context) is essentially undefined, we get a triangle relating the costs of disposing of case reserves to the amount of case reserves that are disposed of

$$\text{Relative Disposal Costs}\,(w,d) = q_{paid}(w,d)\,/\,q_{Case}(w,d) \tag{3.4}$$

Reviewing the above matrix (triangle) of relative disposal costs, a final adjustment ratio for ending case reserves, S is selected.[6] The final step involves multiplying that selected S ratio times the ratio of the remaining case reserves of the oldest accident period (which provides an estimate of remaining payments) and dividing by the cumulative paid loss of the oldest accident period. The result is an estimate of the development portion of the paid loss tail factor. The tail factor formula is:

$$\hat{T}_{Paid} = 1.0 + S \times \frac{s(1,n)}{c_{inc}(1,n)} \tag{3.5}$$

For the incurred tail factor, it must be recognized that the unity (1.0) portion of the case is already accrued in the incurred loss. So, the incurred tail factor formula is:

$$\hat{T}_{Inc} = 1.0 + (S-1) \times \frac{s(1,n)}{c_{inc}(1,n)} \tag{3.6}$$

3.3.2 Example

See Appendix B, Section B.3.1.

3.3.2.1 Considerations

It is important to consider the primary activity within each development stage.

When using multiple periods to estimate a tail factor, it is relatively important that the periods reflect the same general type of claims department activity as that which takes place in the tail. For example, in the early 12 to 24 month stage of workers compensation, the primary development activity is the initial reporting of claims and the settlement and closure of small claims. The primary factors influencing development are how quickly the claims are reported and entered into the system, and the average reserves (assuming the claims department

[6] However, it is important to focus the review on the period in the triangle where the same 'type' of activity is occurring, as will be discussed later.

initially just sets a 'formula reserve', or a fixed reserve amount for each claim of a given type such as medical or lost time) used when claims are first reported.

In the 24 to 36-48 month period, claims department activity is focused on ascertaining the true value of long-term claims and settling claims. After 48-60 months most of the activity centers on long-term claims. So, the 12-24 link ratio has relatively little relevance for the tail, as the driver behind the link ratio is reporting and the size of initial formula reserves rather than the handling of long-term cases. Similarly, if the last credible link ratio in the triangle is the 24 to 36 or 36 to 48 link ratio, that triangle may be a poor predictor of the required tail factor.

Another consideration that could improve this method is using multiple years to estimate the tail factor. This method assumes that the current ratio of case incurred loss to paid loss that exists in the oldest year will apply to the other years when they reach that same level of maturity. For a large, high dollar volume triangle with relatively low underlying policy limits that may be a reasonable assumption, but for many reserving applications the 120-month ratio of case incurred to paid loss may depend on whether a few large, complex claims remain open or not. Therefore, it may be wise to supplement the tail factor derived from the oldest available accident period with that implied by the following accident period or even the second following accident period. This method is particularly useful when the later development portion of the triangle has some credibility, but the individual link ratio estimates from the development triangle are not fully credible.

The process is fairly straightforward: compute the tail factor for each succeeding accident period by the method above, and divide each such tail factor by remaining link ratios in the triangle.

An example using the data in Appendix B may help clarify matters. The 2000 accident period at development age 108 has $7,934 of paid loss and $584 of case reserves. Assume that the best estimate of the 108-120 paid loss link ratio is (using 2000 accident period data) 1.024. Assuming S is 3.073, then the 108-month paid loss tail would be 1.0+ (3.073*584/7,934) = 1.226. Then, dividing out the 108-120 link ratio of 1.024 would give a 108-month paid tail factor of 1.226/1.024=1.197. By comparison, the analysis in the Appendix using 2000 instead of 2001 gives a 120-month tail factor estimate of 1.149. Both indicate tail factors in the 1.15-1.20 range and averaging the estimates would be reasonable. The use of averaging greatly limits the impact of any unusually low or high case reserves that may be present in the oldest year in the triangle.

Note also, that the improvement above involved computing an alternate tail factor using the accident period with one year less development age than the oldest accident period. A similar analysis could also be performed on the next oldest year, except that two paid development link ratios plus the tail factor are needed to estimate the paid loss tail factor.

3.3.3 Advantages and Disadvantages

The significant strengths of this method are that it requires only the data already in the triangles. The weakness is that it can be distorted if the adequacy of the ending case reserve has changed significantly over time.

3.3.4 Users

At present this method has not been published and as such is not widely known or used.

3.3.5 Summary

This method can be a reasonable approach in predicting tail factors without reliance on extensive assumptions, but it needs to be focused on data mature enough so that the overwhelming majority of claims have been reported.

3.4 NCCI Method

This section describes the methodology used by the National Council on Compensation Insurance (NCCI) to derive an indicated 19th-to-ultimate tail factor for use in aggregate ratemaking specifically for workers compensation. NCCI applies this method in most states where it provides ratemaking services.

3.4.1 Introduction

NCCI uses the Accident Year Call for Experience (Call 5) submitted by its affiliates for the calculation of the accident year incurred 19^{th}-to-ultimate tail factor used for ratemaking. The loss data collected on Call 5 includes cumulative paid losses, case loss reserves, bulk reserves, and IBNR for the most recent 20 accident years individually, and in total for years prior to the 20^{th} accident year. [7] Throughout the examples in this section, the notation $c(w, d)$ will be used to denote cumulative incurred losses including paid, case, bulk and IBNR reserves for accident year w and development period d. Similarly, $q(w, d)$ will be used to denote incremental incurred (paid plus change in case, bulk and IBNR) losses for accident year w during the period from $d-1$ to d.

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7 Beginning with data valued as of December 31, 2007, NCCI began the process of expanding Call 5 by adding an additional accident year each reporting year until 30 accident years are reported individually, with years prior to the 30th accident year reported in total. However, as of the time of this writing, NCCI continues to calculate a 19th-to-ultimate tail factor as described in this section.

3.4.2 Calculation of the Accident Year Incurred 19th-to-Ultimate Tail Factor

An estimate of all future incurred development beyond 19th report for a given accident year is estimated as the sum of i) reported incurred development from 19th to 20th report on the given accident year and ii) adjusted reported incurred development during the same calendar year for all prior accident years.[8] The incurred development on prior accident years is adjusted by a "growth factor" to reflect the difference in overall loss levels between those years and the given accident year.

The incurred 19th-to-ultimate tail factor for a given accident year is then obtained by adding unity to the ratio of a) estimated future incurred development beyond 19th report to b) incurred losses at 19th report for the given accident year:

$$\text{AY incurred } 19^{th}\text{-to-ultimate tail factor} = 1 + \frac{\text{Estimated AY incurred development beyond } 19^{th}}{\text{AY incurred losses at } 19^{th}}$$

Where:

$$\begin{array}{c} \text{Estimated AY} \\ \text{incurred} \\ \text{development} \\ \text{beyond } 19^{th} \end{array} = \begin{array}{c} \text{Incurred} \\ \text{development on} \\ \text{given AY from} \\ 19^{th} \text{ to } 20^{th} \end{array} + \frac{\text{Nominal CY incurred development on all prior AYs}}{\text{Growth factor}}$$

$$= \textbf{(a)} + \frac{\textbf{(b)}}{\textbf{(c)}}.$$

OR:

$$F(19) = 1 + \frac{R(w)}{c(w,19)}. \tag{3.7}$$

Where:

$$R(w) = q(w,20) + \frac{\sum_{d=21}^{n} q(n-d+1,d)}{g}. \tag{3.8}$$

This is best illustrated by an example. Displayed below is a historical incurred loss triangle through 2010 evaluated at 12/31/2010. Note that values to the right of the jagged line (for

[8] The development on all prior accident years during a calendar year, i.e., calendar year development, is a reasonable approximation of the future development on the given accident year assuming development patterns and exposure levels are constant.

development periods beyond 20th) are not available individually, but are shown for the purpose of this example.

Cumulative Incurred Loss Triangle $c(w, d)$

	1	…	19	20	21	22	23	…
1986	$c(1986,1)$	…	$c(1986,19)$	$c(1986,20)$	$c(1986,21)$	$c(1986,22)$	$c(1986,23)$	…
1987	$c(1987,1)$	…	$c(1987,19)$	$c(1987,20)$	$c(1987,21)$	$c(1987,22)$	$c(1987,23)$	…
1988	$c(1988,1)$	…	$c(1988,19)$	$c(1988,20)$	$c(1988,21)$	$c(1988,22)$	$c(1988,23)$	
1989	$c(1989,1)$	…	$c(1989,19)$	$c(1989,20)$	$c(1989,21)$	$c(1989,22)$		
1990	$c(1990,1)$	…	$c(1990,19)$	$c(1990,20)$	$c(1990,21)$			
1991	$c(1991,1)$	…	$c(1991,19)$	$c(1991,20)$				
1992	$c(1992,1)$	…	$c(1992,19)$					
$\vdots$	$\vdots$							
2010	$c(2010,1)$							

The values below are shown for illustrative purposes and are not intended to reflect realistic incurred loss development patterns.

Cumulative Incurred Loss Triangle $c(w,d)$

	1	…	19	20	21	22	23	…
⋮		…						
1986	6,000	…	30,000	30,600	30,906	31,061	31,154	…
1987	8,000	…	40,000	40,800	41,208	41,414	41,538	…
1988	10,000	…	50,000	51,000	51,510	51,768	51,923	
1989	12,000	…	60,000	61,200	61,812	62,121		
1990	14,000	…	70,000	71,400	72,114			
1991	16,000	…	80,000	81,600				
1992	18,000	…	90,000					
⋮	⋮							
2010	50,000							

Note that in this example, accident year 1991 is the most recent accident year for which data is available at 20th report. The 19th-to-ultimate tail factor for this accident year is calculated below. Since the underlying data is evaluated as of 12/31/2010, the formula uses incurred loss development on all prior accident years that occurred during calendar year 2010. The components of formula (3.8) are calculated as follows:

(a) Incurred development on given AY from 19th to 20th report

$$\begin{aligned}&= c(1991,20) - c(1991,19)\\&= 81{,}600 - 80{,}000\\&= 1{,}600.\end{aligned}$$

(b) Incurred development on all prior AYs

$$\begin{aligned}&= \sum_{d=21}^{n} q\left(2010 - d + 1, d\right)\\&= q\left(1990,21\right) + q\left(1989,22\right) + q\left(1988,23\right) + \cdots\\&= \left[c(1990,21) - c(1990,20)\right] + \left[c(1989,22) - c(1989,21)\right] + \left[c(1988,23) - c(1988,22)\right] + \cdot\\&= \left(72{,}114 - 71{,}400\right) + \left(62{,}121 - 61{,}812\right) + \left(51{,}923 - 51{,}768\right) + \cdots\\&= 714 + 309 + 155 + \cdots\\&= 3{,}000 \quad \text{(datapoints not shown).}\end{aligned}$$

(c) Growth factor, g (the rationale for the selection of the elements used to calculate g is discussed below.)

$$= \frac{\left(\frac{1}{5}\right)\times\left[c(1986,19)+c(1987,19)+c(1988,19)+c(1989,19)+c(1990,19)\right]}{c(1991,19)}$$

$$= \frac{\left(\frac{1}{5}\right)\times(30{,}000+40{,}000+50{,}000+60{,}000+70{,}000)}{80{,}000}$$

$$= 0.625.$$

Substituting,

(3.8)	Estimated AY incurred development beyond 19th	=	(a)	+	$\frac{(b)}{(c)}$
		=	1,600	+	$\frac{3{,}000}{0.625}$
		=	6,400.		

(3.7)	AY incurred 19th-to-ultimate tail factor	= 1 +	$\frac{\text{Estimated AY incurred development beyond } 19^{\text{th}}}{\text{AY incurred losses at } 19^{\text{th}}}$
		= 1 +	$\frac{6{,}400}{80{,}000}$
		= 1.08.	

3.4.3 Derivation of the Formula

Assuming that all claims are closed and all losses paid out at n^{th} report, the actual incurred development on accident year 1991 from 19th report to ultimate is:

$$F(1991,19) = \frac{c(1991,\ n)}{c(1991,\ 19)}$$

$$= \frac{c(1991,19)+q(1991,20)+\sum_{d=21}^{n} q(1991,d)}{c(1991,19)}$$

$$= 1+\frac{q(1991,20)}{c(1991,19)}+\frac{q(1991,21)+q(1991,22)+\ldots}{c(1991,19)}$$

$$= 1+\frac{q(1991,20)}{c(1991,19)}+\frac{\left[q(1990,21)\times\frac{q(1991,21)}{q(1990,21)}\right]+\left[q(1989,22)\times\frac{q(1991,22)}{q(1989,22)}\right]+\ldots}{c(1991,19)}$$

$$= 1+\frac{q(1991,20)}{c(1991,19)}+\frac{\sum_{d=21}^{n}\left[q(2010-d+1,d)\times\frac{q(1991,d)}{q(2010-d+1,d)}\right]}{c(1991,19)}$$

$$= 1 + \frac{q(1991,20)}{c(1991,19)} + \frac{\sum_{d=21}^{n} q(2010-d+1,d)}{c(1991,19)} \times h, \tag{3.9}$$

where $$h = \frac{\sum_{d=21}^{n}\left[q(2010-d+1,d) \times \frac{q(1991,d)}{q(2010-d+1,d)}\right]}{\sum_{d=21}^{n} q(2010-d+1,d)}, \tag{3.10}$$

which can be described as a weighted average of the terms

$$\frac{q(1991,d)}{q(2010-d+1,d)},$$

using as weights

$$\frac{q(2010-d+1,d)}{\sum_{d=21}^{n} q(2010-d+1,d)}.$$

Each of the terms in this series is a ratio of incremental incurred losses for accident year 1991 relative to an earlier accident year. However, in each term the numerator is unknown (because this development has yet to occur), and the denominator is not available (because only 20 development years of data are reported individually). Therefore, NCCI approximates these terms by measuring accident year 1991 incurred losses against each of the earlier accident years at an earlier, known report level. Because the incremental incurred losses for one report can vary widely, cumulative losses are compared in each of the terms, as follows:

$$\frac{c(1991,19)}{c(1990,19)}, \frac{c(1991,19)}{c(1989,19)}, \frac{c(1991,19)}{c(1988,19)}, \ldots$$

Substituting into formula (3.10):

$$h \approx \frac{\sum_{d=21}^{n}\left[q(2010-d+1,d) \times \frac{c(1991,19)}{c(2010-d+1,19)}\right]}{\sum_{d=21}^{n} q(2010-d+1,d)}. \tag{3.11}$$

For a given term (which measures accident year 1991 against a given accident year) in the weighted average described by formula (3.11), the weight applied to that term is the given accident year's proportion of calendar year incurred development on all accident years prior to 1991. Since the calendar year incurred development on accident years prior to 1991 is only available in total and not by accident year, NCCI approximates the weighted average on the right-hand side of equation (3.11) with a simple average of a subset of the first k terms. With this approximation, equation (3.11) simplifies to:

$$h \approx \frac{c(1991,19)}{\frac{1}{k} \times \sum_{d=21}^{21+(k-1)} c(2010-d+1,19)}.$$

Currently, NCCI uses a simple average of the first five terms ($k=5$) to approximate this "growth factor." This selection is discussed in further detail below. With $k=5$, we have:

$$h \approx \frac{1}{g} = \frac{c(1991,19)}{\frac{1}{5} \times \sum_{d=21}^{25} c(2010-d+1,19)}. \tag{3.12}$$

Substituting into formula (3.9):

$$\begin{aligned} F(1991,19) \quad & \approx 1 + \frac{q(1991,20)}{c(1991,19)} + \frac{\sum_{d=21}^{n} q(2010-d+1,d)}{c(1991,19)} \times \frac{1}{g} \\ & \approx 1 + \frac{q(1991,20) + \left[\frac{1}{g} \times \sum_{d=21}^{n} q(2010-d+1,d) \right]}{c(1991,19)}. \end{aligned} \tag{3.13}$$

Formula (3.13) is the form used by NCCI. [9]

3.4.3.1 Growth Factor

The tail factor method used by NCCI has evolved since its initial implementation. While the derivation of the formula above accurately describes the rationale underlying the current approach, the method originated from a simpler form that initially did not incorporate the growth factor adjustment. Using the current formula (3.13), removing the growth adjustment would be equivalent to setting $g=1$. In an environment of increasing exposure (loss volume), failure to incorporate a growth adjustment would result in an understated tail factor. Conversely, the tail factor would be overstated if exposure is decreasing and $g=1$.

Since it is not possible to calculate the growth adjustments shown in formulas (3.10) or (3.11) with the data collected on financial calls, NCCI approximates the growth adjustment using formula (3.12). This approximation compares the cumulative incurred losses at 19th report for the most recent accident year to the average cumulative incurred losses at 19th report for the five prior accident years.[10] The five-year average was selected (as opposed to

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9 The discussion above illustrates the calculation of the incurred 19th-to-ultimate tail factor for a single accident year using the most recent data. In NCCI filings, the final tail factor is selected based on a review of at least the most recent five accident year tail factors.

10 For tail factors using data valued prior to December 31, 2008, NCCI used 8th report losses in the calculation

shorter- or longer-term averages) judgmentally with consideration given to the following items:

1. **Incurred loss development pattern beyond 19th report –** Workers compensation is a long-tailed line of insurance in which the ultimate cost of claims incurred during a given accident year may not be known for several decades. When using a simple average of a fixed number of accident years for the growth factor adjustment, a longer tail would suggest using more years in the average. Conversely, a shorter tail would suggest using fewer years in the average.
2. **Exposure growth rates –** Exposure (loss volume) can increase or decrease over time due to a number of factors (e.g., inflation, benefit changes). Given constant incurred loss development beyond 19th report for all accident years, a higher rate of exposure growth would suggest using a fewer number of years in the average for the growth factor adjustment.
3. **Impact of the growth factor adjustment –** In some states, incurred loss development beyond 19th report may be minimal (especially for indemnity benefits, which are typically limited in duration by statute). In these cases, the growth factor has little to no impact on the calculated tail factor, making the number of years used in calculating the growth factor an immaterial selection.
4. **Data constraints –** The number of years used in the average for the growth adjustment is limited on the upper end by data constraints. Specifically, the oldest accident year for which data was reported individually at 19th report is 1979.

3.4.3.2 Conversion Ratios

In determining 1st-to-19th loss development factors, NCCI organizes loss data in a variety of ways (policy year or accident year, on a paid or paid + case basis). Therefore, a "conversion ratio" is required to convert the accident year incurred 19th-to-ultimate tail factor to the corresponding 1st-to-19th loss development basis. For instance, in a state where link ratios from 1st-to-19th report are based on accident year paid + case losses, a paid + case-to-incurred

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of the growth factor. When the growth adjustment was introduced to the formula in the late 1980s, data reported to NCCI included only eight individual accident years. Over time, the financial calls were expanded to include 20 individual accident years of data—adding one additional accident year at each subsequent reporting date. Growth factors could not be calculated using data at a 19th report until there were six valuations of data that each included 20 individual accident years of loss experience.

conversion ratio at 19^{th} report is divided into the accident year incurred 19^{th}-to-ultimate tail factor to calculate an accident year paid + case 19th-to-ultimate tail factor.

For 1st-to-19th development on a policy year basis, the 18th-to-19th policy year link ratio is first raised to the two-thirds power to approximate accident year experience at 19th report.[11]

The various conversions are illustrated in the following table:

1^{st}-to-19^{th} Loss Development Basis	**18^{th}-to-19^{th} Link Ratio**		**Tail Factor Conversion Formula**
AY Paid+Case:	AY Paid+Case 18^{th}-to-19^{th} Link Ratio	x	$\dfrac{\text{Incurred } 19^{th}\text{-to-Ult Tail}}{\text{AY Paid+Case-to-Inc Conv Ratio @ } 19^{th}}$
AY Paid:	AY Paid 18^{th}-to-19^{th} Link Ratio	x	$\dfrac{\text{Incurred } 19^{th}\text{-to-Ult Tail}}{\text{AY Paid-to-Inc Conv Ratio @ } 19^{th}}$
PY Paid+Case:	(PY Paid+Case 18^{th}-to-19^{th} Link Ratio)$^{2/3}$	x	$\dfrac{\text{Incurred } 19^{th}\text{-to-Ult Tail}}{\text{AY Paid+Case-to-Inc Conv Ratio @ } 19^{th}}$
PY Paid:	(PY Paid 18^{th}-to-19^{th} Link Ratio)$^{2/3}$	x	$\dfrac{\text{Incurred } 19^{th}\text{-to-Ult Tail}}{\text{AY Paid-to-Inc Conv Ratio @ } 19^{th}}$

As part of ongoing efforts to improve its ratemaking methodologies, NCCI continues to research alternative methods to address tail development. As of the time of this writing, NCCI is currently considering the following potential changes to the method described above:

1. Elimination of bulk and IBNR reserves from the calculation[12]
2. Change to the number of years used in the growth factor
3. Algebraic revision to the growth factor formula

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[11] The justification of the two-thirds power adjustment to bring the maturity level of the policy year experience more in line with the maturity level of the accident year experience is beyond the scope of this paper.

[12] Calculating the tail factor using paid + case losses would eliminate the need for the paid + case-to-incurred conversion ratios. In addition, without the need for IBNR data (only reported on an accident year basis), a policy year 19th-to-ultimate tail factor could be calculated directly, eliminating the need for the "two-thirds power" adjustment.

3.4.3.3 Adjustment for Capped Methodology

In 2004, NCCI enhanced its aggregate ratemaking methodology to mitigate the possible distortions that catastrophic events and extremely large individual claims can create in state premium level indications.[13] NCCI uses this large loss ratemaking procedure in most of the states where it provides ratemaking services. Essentially, the methodology derives ultimate losses using reported losses capped at a given dollar threshold per claim and later adds a provision for expected losses in excess of that threshold.

In order to develop capped losses to ultimate, loss development factors on a capped basis are needed. From 1st to 19th report, NCCI caps individual claims prior to calculating loss development factors. However, individual claim detail for large claims is only reported for claims with accident dates on or after January 1, 1984. Therefore, to calculate the capped 19th-to-ultimate tail factor, NCCI derives a factor to adjust the selected uncapped paid + case tail factor to a capped basis.

In general terms, the tail adjustment factor is the ratio of capped (for a given threshold) to uncapped paid + case loss development beyond 19th report on a countrywide basis. NCCI uses excess ratios and excess loss development factors to calculate the adjustment factor by threshold and then applies the factor as follows: [14]

$$\begin{array}{c}\text{Capped 19}^{\text{th}}\text{-to-}\\ \text{ultimate paid + case}\\ \text{tail factor}\end{array} = 1 + \left[\begin{array}{c}\text{Tail}\\ \text{adjustment}\\ \text{factor}\end{array} \times \left(\begin{array}{c}\text{Uncapped 19}^{\text{th}}\text{-}\\ \text{to-ultimate paid}\\ \text{+ case tail factor}\end{array} - 1\right)\right]$$

3.4.4 Advantages and Disadvantages

One strong advantage of this method is that it uses the total for all prior accident years (the 'prior' row) available in the financial call data submitted to NCCI. Further, although this calculation may appear relatively complex, the core approach of the method (looking at one year's runoff of all prior years during the current calendar year) is actually fairly simple. A disadvantage is that the growth factor used by NCCI is an approximation, and the number of years of data used in the calculation is selected judgmentally. Also of note, this method requires that a sufficient history of accident years and volume of loss activity exists in the

13 In this paper, discussion of NCCI's large loss methodology is restricted to that portion affecting the tail factor calculation. For a more thorough treatment of the procedure used by NCCI, see "Catastrophes and Workers Compensation Ratemaking," by Tom Daley, CAS *Forum*, Winter 2007.

14 If the selected uncapped 19th-to-ultimate paid + case tail factor is less than 1.0, the tail adjustment factor is set equal to 1 so that the capped tail factor equals the uncapped tail factor.

'prior' row.

3.4.5 Users

This method is used most by its developers, NCCI, but it is sometimes used by consulting firms as well.

3.4.6 Summary

At its core, this method was designed by a rating bureau for their specific situation. However, it has evolved from a fairly simple and understandable concept. Therefore, as long as there is an adequate volume in the runoff from prior years and an appropriate and reliable growth correction can be made, it can be a very useful method.

3.5 Summary of Algebraic Methods

The algebraic methods key off basic and very reasonable assumptions about the relationship of development in the tail to quantities which are relatively simple to compute from basic reserving data. As such, they are very useful reserving tools.

4. BENCHMARK-BASED METHODS

4.1 Introduction to Benchmark-Based Methods

If a suitable benchmark can be found, the use of benchmark data from a larger pool of losses, typically those that contain development detail at greater maturity than the data being developed, can supplement the data being developed. This can feature advantages due to a higher credibility of the link ratios near the tail, or may have more years of development than a start-up type program.

4.2 Directly Using Tail Factors from Benchmark Data

4.2.1 Description

Many actuaries review benchmark data when selecting a tail factor. Benchmark data can be used in place of or as a supplement to more company-specific data when selecting the tail factor. In some cases, the benchmark is comprised of industry data triangles and the tail must be derived; in other cases the tail factor and development pattern have been selected by the organization producing the benchmark data. At its simplest, the benchmark method involves copying the benchmark age to ultimate development factor at the maturity desired for the tail factor. If the tail factor needed is a different age than available, it will be necessary to interpolate (assuming the age is in between two ages available in the benchmark) or extrapolate (if the age needed is outside the range of ages available in the benchmark data).

For extrapolation, it may be possible to use one of the other methods described in this paper. If the source does not directly compute a tail factor, it will be necessary to derive a tail factor.

4.2.2 Data Sources

Perhaps the most common benchmark data triangles are those that can be developed from Best's Aggregates and Averages for each of the Schedule P lines. This source presents summarized development triangles on an industry basis out to 120 months. Triangles are available for Paid, IBNR and Total Incurred (paid loss + case reserves + IBNR) to 120 months for the last 10 accident years. An incurred loss triangle excluding IBNR can be derived by subtracting the IBNR triangle from the Total Incurred triangle. Aggregates and Averages do not generate a tail factor or development pattern directly; a tail factor must be calculated. This can be done using one of the other methods described in this paper (on what should be a very credible set of data) or a tail factor can be inferred based on the IBNR booked by the industry. For example, if one needed a paid tail factor from 96 months to ultimate for a particular period, you could compute the ratio of the ultimate losses of the accident period at 96 months to paid loss at 96 months to determine the tail factor. Alternatively, you could use the ratio of ultimate loss for all accident periods older than 96 months to the sum of paid loss at 96 months for those same accident periods.

The two larger rating bureaus, the National Council on Compensation Insurance (NCCI) and Insurance Services Office (ISO), as well as the Reinsurance Association of America (RAA), all publish benchmark loss development data. Benchmarks are also available from the state workers compensation rating bureaus. The rating bureaus will generally select a development pattern and tail factor based on the statistical data reported to them by insurance companies and other writers in the case of workers compensation coverage.

Another source of benchmark data is the annual statements of individual insurance companies. This data is basically in the same form as Aggregates and Averages. The annual statements can be found at each state's insurance department. Tail factors can be derived as described above, but this method is more heavily dependent upon the adequacy of the reserve estimates for a single company, and would be less credible. On the other hand, this data would more specifically capture the reserving practices of the company used. Also, the annual statement of a company known to be writing business on risks similar to those of the company under review may be of particular interest.

4.2.3 Usage

This method is very commonly used by consulting actuaries and actuaries at smaller companies where data either are inadequate or do not exist.

4.2.4 Advantages and Disadvantages

One key advantage of using tail factors from benchmark data is that benchmark data is easily available through common industry sources. In addition, benchmark tail factors are typically based on a high volume of data, which can help reduce process variance that is often inherent in smaller data sets.

The primary disadvantage of this method is that the benchmark tail development may not be representative of the book of business being analyzed. Considerations such as differences in the way claims are adjusted or reserved, differences in the types or mix of types of claims (medical vs. indemnity), differences in the potential for long-developing high-value claims, differences in the initial reporting pattern of claims (claims-made vs. occurrence, whether or not there is an innately long discovery period, etc.), and differences in the adjudication process of litigated claims can all cause differences in development patterns. It is important to consider those factors along with the statistical reliability of the benchmark triangle when selecting the most appropriate benchmark tail factor.

4.2.5 Summary

This is the most basic and most common of the benchmark-based methods. It is dependent on the benchmark data being a 'good match' to the data in question. However, for low-credibility data, where it is most often used, any mismatch in data must be measured against the unreliability of the data in the triangle being analyzed.

4.3 Use of Benchmark Tail Factors Adjusted to Match Pre-Tail Link Ratios

4.3.1 Description

One way to address differences between the benchmark development pattern and the development pattern of a given book of business is to try to adjust the benchmark data to take into account differences in the subject book of business. One common practice is to compare the age-to-age link ratios from the subject data to the benchmark age-to-age link ratios prior to the tail development stage. The relativities from those stages are used to estimate an adjustment multiplier for the benchmark tail factor. Of note, generally just the development portions of the link ratios ($v(d)$ of $1+v(d)$) are compared.

4.3.2 An Example

An example will help to illustrate how the process works. Consider the following two patterns where we simply compute the ratio of the development portion of our triangle-based link ratios to the development portion of the matching benchmark link ratios:

(1)	(2)	(3)	(4)	(5)	(6)
	Selected Link Ratio		Benchmark Link Ratio		Selected to Benchmark Ratio =
Maturity	Estimated by Triangle $f(d)=1+v(d)$	Development portion $v(d)$	Ratio	Development portion $v(d)$	(3)/ (5)
12	2.000	1.000	2.000	1.000	100%
24	1.450	.450	1.350	.350	129%
36	1.200	.200	1.150	.150	133%
48	1.150	.150	1.100	.100	150%
60	1.100	.100	1.050	.050	200%
72	1.080	.080	1.030	.030	267%
84	1.050	.050	1.025	.025	200%
96	1.035	.035	1.020	.020	175%
108	1.010	.010	1.010	.010	100%
Tail			1.050		
Chosen Ratio		200%			

$T(n)$ = 1+.050*200% =1 + .100 = 1.100

In the example above, 200% is chosen as the ratio of subject development portions of the age-to-age factors to the benchmark based on the 60- through 108-month relativities.

The underlying assumption of this adjustment is the underlying processes in our subject data that are causing the (in this case) higher development than seen in the benchmark data will continue throughout the life of the claim. This may or may not be the case. From a practical standpoint, it is generally not possible to examine all aspects of claims handling to the degree necessary to make this determination. The example above is representative of a reasonable adjustment one might make based on the data, but it is a qualitative adjustment, not a statistically based adjustment.

4.3.3 Usage of This Method

This method is very commonly used by consulting actuaries and actuaries at smaller companies where data either are inadequate or do not exist.

4.3.4 Advantages and Disadvantages

The main advantages of this method are (1) it is easy to apply and (2) it presents a very broad representation of the potential outcomes of the subject data. Industry-wide benchmark data represents an industry-wide view of the possible outcomes of the claims adjustment process. Even a complete set of data for a smaller company may not adequately represent the potential for very long-term claims. This broad perspective can also be one of the major weaknesses of this method. The benchmark may be too broad, as it is often difficult to find a perfect match in terms of all the factors (claims handling, case reserving, potential for large claims, etc.) that affect loss development.

Another issue with benchmarks is the availability of data beyond 120 months. Most available benchmark data does not extend beyond 108 or 120 months. Deriving a tail factor for an age beyond this time frame would require some form of extrapolation.

4.3.5 Summary

This method is a relatively simple way to improve the tail predictions generated by benchmark data. It is used a little less commonly than the 'straight benchmark,' though, there are many different ways to adjust benchmark data. Presumably, adjustment can improve the effectiveness of benchmark data significantly.

4.4 Benchmark Average Ultimate Severity Method

4.4.1 Description

This method relies on a benchmark average severity and the reported average severity near the tail to derive a tail factor. It requires two key assumptions. Specifically, one must first assume that the average ultimate severity of the oldest accident period being analyzed is equal to or similar to some benchmark ultimate severity. Second, one must assume that the number of reported claims is equal to the total ultimate number of claims (or, equivalently, one must be able to derive a highly reliable estimate of the total ultimate number of claims generated by the oldest period). The method then involves the simple act of using the ratio of the benchmark average ultimate severity to the reported severity as the tail factor. If, for an accident period, the estimate of the ultimate number of claims is higher than the oldest period's number of reported claims, then the ratio of benchmark ultimate severity to the reported average severity must be multiplied by the reported claims count tail factor to derive the tail factor.

In mathematical terms, the first case may be stated as:

$$T(n) = Average\,Severity(u) \,/\, \frac{c_{inc}(1,n)}{c_{reported\,count}(1,n)}\,. \tag{4.1}$$

The second case may be stated as:

$$T'(n) = T(n) \times T_{reported\,count}(n)\,, \tag{4.2}$$

where we recall that n represents the last development age of a given accident period as well as the most recent accounting period for which data is available. $T_{reported\ count}(n)$ is the development factor to ultimate for reported claim counts for the oldest maturity in the triangle n.

When using this method it is absolutely imperative that the benchmark severity be appropriate for the eldest period. Note that when the triangle data has low or medium credibility, the true average severity may be strongly affected by the vicissitudes of fortune with respect to large, late settling claims. If more than the average number of large claims are present in the data, this test may improperly suggest negative development in the tail. If the oldest period contains fewer large claims than average (which is more common), then this method will suggest more development than actually occurs. On the other hand, in the rare cases where a large number of large claims emerge and balance out the average number of large claims, the development will be relatively greater than this method implies. Of note, if only a limited amount of development is available in the triangle (say four to five periods in long tail lines), and the larger claims all occur after the oldest maturity n, then (as long as the benchmark is appropriate) this test may have higher relative reliability. That is because this test can comfortably assume an average number of large losses without the data being distorted by variance in the number of large losses already reported. Further, note that the class of business must be such that a reliable benchmark that matches the type of data in the triangle is available. For example, if one has a large volume of private passenger auto data all from standard classes, for which benchmark data is readily available, this method may prove to be useful.

4.4.3 Example

Consider a triangle going out 10 periods (120 months) containing private passenger auto data. Suppose that all the claims are clearly reported by 120 months but some remain unsettled. Specifically, suppose that the total incurred loss for the oldest period is

$c_{inc}(1,10)$ = \$120 million.

Further, suppose the corresponding reported counts are

$c_{reported\ count}(1,10)$ = 6,000.

So, the reported severity at 120 months is \$20,000. If the benchmark average ultimate severity is \$20,200, then the implied tail factor would be

$T(10)$ = 20,200/20,000 = 1.01.

4.4.3.1 A Second Example

The other utility mentioned for this approach involves long-tail data that requires a tail for a medium term triangle. Say, for example, that a workers compensation triangle is available, but it only has five 12-month periods of data and hence stops at 60 months. Suppose you know that the average severity benchmark data, for the hazard group mix contained in your data, at ultimate is $50,000 per claim, counting both initial claims closed with any type of payment and reopened claims closed with any type of payment separately in the denominator. Further, suppose that this larger benchmark workers compensation data says that the reported claims count tail factor at 60 months is 1.02.

Then all you need from your data are the reported counts and incurred losses for the oldest period. (Again, all reported count figures only include those with payment, and count reopened claims as claims in themselves in this example). Suppose they are:

$c_{inc}(1,5)$ = $4 million, and

$c_{reported\,count}(1,5)$ = 100 claims.

Then the current reported severity of the oldest period would be $4 million/100 = $40,000 per claim;

and the implied tail factor would be

1.02 ×$50,000/40,000 = 1.02 ×1.25 = 1.275.

4.4.4 Advantages and Disadvantages

As mentioned above this method is only suitable when a reliable benchmark average severity is available and when the presence or absence of a few large losses are not factors in the eldest period's data. Due to the relative rarity of those situations, this method is not widely used.

4.4.5 Users

This method does not currently have widespread usage. A few actuaries in consulting and primary company actuaries have been observed to use this method.

4.4.6 Summary

This method involves applying an average severity from benchmark data to correct the severity shown in the case incurred data. Because of the difficulty in finding reliable benchmark severities, its utility and use in practice is somewhat limited.

4.5 Use of Industry-Booked Tail Factors

This method is also referred to as the "industry-booked" method, as it relies on the adequacy of booked industry IBNR in older accident years to determine the tail factor. While it can be argued that this factor should represent the "best" estimate of the industry actuaries of the additional reserve need, history has shown that this figure has often been inadequate. This would suggest that tail factors based on this method would be understated.

4.5.1 Description

The general practice while using this method is to simply look at the (direct or net) IBNR booked by the industry (per Best's Aggregates and Averages, or other sources) for the oldest year in schedule P, then divide that by the (direct or net) case incurred loss for that year. The result forms the industry booked incurred tail. Similarly, dividing the (direct or net) case reserves + IBNR for the oldest year by the (direct or net) paid loss for that year yields the industry-booked paid loss tail.

4.5.2 Example

Assume that the industry Schedule P for the year 2007 shows the following values for accident year 1998 (the oldest year in that Schedule P):

A. Direct Paid Loss	5,000,000
B. Direct Loss Case Reserves	2,500,000
C. Direct IBNR	2,500,000

Then, we first compute some intermediate values (the total incurred and total reserve):

D. Total Case Incurred Loss (= A+B)	7,500,000
E. Total Reserves (Case + IBNR) (= B+C)	5,000,000

We then can compute the development portions of the tail factors as described above, and the tail factors themselves.

F. Development Portion of Incurred Tail Factor (= C/D)	0.33
G. Incurred Loss Tail Factor (= 1.0 + F)	1.33
H. Development Portion of Paid Tail Factor (= E/A)	1.00
I. Paid Loss Tail Factor (= 1.0 + H)	2.00

4.5.3 Usage of this Method

In spite of the potential problems with industry reserve inadequacy, this method is in broad usage in consulting firms and by actuaries at small- to medium-sized insurance companies. Generally, larger companies tend to have better alternatives. There is a smaller group of large firms that prefer to benchmark relative to their peers that may use this approach.

4.5.4 Advantages and Disadvantages of this Method

The pros and cons of this method revolve around two main points: first, the data is easy to obtain and the method itself is easy to perform; but, second, for many lines it may be unrealistic to expect industry booked IBNR to be adequate. Another important concern would be whether or not the industry would be a suitable benchmark for the book of business being analyzed.

4.5.5 Summary

It must be noted that this method is based on what may be an incorrect assumption (that industry IBNR is adequate). Nonetheless, many actuaries use this method. That is perhaps a tribute to its simplicity.

4.6 Benchmark Tail Factors Adjusted for Company-Specific Case Reserving

The use of benchmark data, as discussed earlier, is often necessary due to lack of credibility in triangles with low data volume near the tail. However, this can be problematic when the entity handling claims for the subject book of business uses different case reserving standards than the industry at large. In such cases, it is common to include a correction to the benchmark tail factors to reflect the specific case reserve adequacy of the subject book of business.

4.6.1 Description

This method is very similar to the use of benchmark tail factors, excepting that a secondary factor is included that adjusts the case reserves near the tail to industry level. Most commonly, the adjustment will be generated by a claims audit. That will involve sending a highly experienced claims person, preferably one specializing in claims audits, to the claims handling office for a formal audit. Typically, such a claims auditor will review a sample of the claim files and, based on what is in the file and his or her claims expertise, estimate what case reserve should be carried on the file at industry standard case reserve levels. Such efforts may be focused on the tail by sampling solely from the most mature years in the triangle. That is because the case adequacy may be different near the tail than it is an early and intermediate maturity. Using the results of the audit, one can compute a case reserve adjustment factor to industry reserving levels as the ratio of the total case reserves suggested by the claims auditor divided by the carried case reserves on the claims in the sample.

As the final step in producing the corresponding tail factor, one need only multiply the benchmark tail factor by a factor to adjust the business' total case incurred losses to industry levels. To compute that factor to adjust the business' total case incurred to industry levels we sum up the ratio of the eldest years' cumulative paid losses to its case incurred losses plus the claims auditor's case reserve adjustment factor times the eldest years' ratio of case reserves to case incurred losses. So, in total we have the following equation.

$$\hat{T} = T_{Benchmark} \times \{(c_{Paid} / c_{Incurred}) + [Adj\ Factor \times (s / c_{Incurred})]\} \quad (4.3)$$

4.6.2 Example

Suppose the benchmark tail factor is 1.2. Further, suppose the cumulative paid loss for the eldest year is 85% of the case incurred, so the case reserves ('*s*' above) are 15% of the case incurred at the tail. Then suppose a claims audit says that in order to bring case reserves in line with industry reserve adequacy, the case reserves should be twice what they are. Then, the adjusted benchmark tail is:

1.2 × (.85 + 2 × .15) = 1.2 × 1.15 = 1.38.

4.6.3 Advantages and Disadvantages

This method offers a significant opportunity to improve the accuracy of benchmark tail factors. However, claims audits can impose a significant cost and, more importantly, require the use of highly trained claims auditors. These resources are not available to every actuary. Further, the auditors need not only to be highly trained but also have to be extremely objective, or else the results will be misleading. Perhaps another approach would be to ask an objective auditor for 'industry best practices,' which might be different from the 'industry average.' In order to recognize that difference, claims adjusted using industry best practices could be developed using a benchmark that is more mature than the data. Certainly the more experience an actuary has in working with an auditor and watching the tails develop, the more trust he or she can place in this method. Also the fact that history has shown benchmark IBNR data is often inadequate must be considered.

4.6.4 Users

This method is used primarily by large commercial and reinsurance carriers that must reserve data from a multitude of different claims handlers. Some actuarial firms that work with data from many different claims entities use it as well.

4.6.5 Summary

In summary, this method can be a useful adjunct to the use of benchmark tail factors, but does require an extensive set of resources. Further, it requires a great deal of vetting of not just the case reserves but the claims auditor as well.

4.7 Summary of Benchmark-Based Methods

Benchmark data can serve a useful function, especially in the small-to-medium credibility situations. One must be careful, though, to make sure that either the benchmark data is a good match for the book of business being analyzed, or that appropriate adjustments are applied either to the benchmark data or the book of business being analyzed.

CURVE-FITTING METHODS

5.1 Introduction to Curve-Fitting Methods

One strategy for developing tail factors is to posit some relationship between the link ratios at various development ages (or, some similar quantity such as incremental paid by development age), and use that relationship as an assumption to fit a curve to the link ratios. Projected link ratios in the development ages covered by the tail factor can then be generated. All those projected link ratios can then be multiplied together to provide an estimate of the tail factor. The methods below represent only those methods where curve-fitting is the primary source of the tail factor. There are several methods (e.g., Mueller's method, which is discussed later) that involve curve-fitting but are not solely curve-fitting type methods.

The topic of modeling loss development for various purposes such as projecting ultimate losses or estimating variability in development factors has been discussed in various actuarial articles and papers such as McClenahan [9], Finger [4] and Hayne [8]. A common characteristic of probability distributions selected for modeling is that they indicate that incremental losses emerge or are paid out at a monotonically decreasing rate (decay function). The exponential distribution is one of many probability distributions used in practice for modeling a decay process.

5.2 Exponential Decay Method

5.2.1 Description

The method utilizes link ratios, $f(d_i)$, as opposed to cumulative or incremental paid loss. Define the function $v(d_i)$, the development portion of the link ratio, as follows: $f(d_i) = 1 + v(d_i)$. In contrast to the McClenahan method (see section 5.3) and Skurnick method (see section 5.4), this method assumes that the $v(d_i)$'s decay at a constant rate, r, i.e., $v(d_{i+1}) = v(d_i) \times r$.

The process consists of first fitting an exponential curve to $v(d_i)$'s. This can be accomplished by using a regression to the natural logarithms (natural log) of $v(d_i)$'s. Next, the decay constant r can be estimated as the inverse natural log of the slope of the fitted curve. The remaining development, from a given development age d , can be estimated as:

$$T(d) = \prod_{m=1}^{\infty} = (1 + v(d) \times r^m) . \tag{5.1}$$

For small $v(d)$, remaining development can be approximated by:

$$T(d) \approx 1 + v(d) \times \sum_{m=1}^{\infty} r^m = 1 + v(0) \times r^{(d)} / (1 - r) . \tag{5.2}$$

5.2.2 Example

Appendix B, Section 5.2, shows a contrived example of fitting the following link ratios:

Age in Months	Period	Link Ratio
12	1	1.5
24	2	1.25
36	3	1.125
48	4	1.0625
60	5	1.03125
72	6	1.015625
84	7	1.007813

The outputs from the curve fit and actual and approximated tail calculations are shown below:

From curve fit to column (5) in the Appendix	
$\ln(r)$ = -0.6931	r = 0.5000
$\ln[v(0)]$ = 0.0000	$v(0)$ = 1.0000

Product of Age 8 to Age 22 Link Ratios	1.007830	$= T(8)$
Approximation formula	1.007813	$= 1 + v(0) \times r^8 / (1 - r)$

5.2.2.1 Another Example (Appendix B, Section 4.1)

The above example was contrived for purposes of demonstrating the method. A more realistic data pattern helps highlight certain issues that can arise when using this method.

In the Appendix example the "error in fit" (actual minus fitted) suggests a possible poor fit of the curve to the data. In the next section, a method of addressing the issue of less than optimal fit is presented.

5.2.2.2 Adjustment to Exact Fitting

In this enhancement, the development portion of the derived tail factor is adjusted by the "actual to fitted ratio" from the last stage. For example, suppose that the development portion of the 108-month tail factor is 0.03 and the ratio of the actual-to-fitted link ratio is 1.7. The adjusted tail factor is now 1 + (0.03) x (1.7) = 1.051. This 'adjustment' increases the tail factor, but this resulting value is considerably different from the tail factor produced by the method. Without further knowledge of the underlying data, such as what is the line of business, what are the claims department's reserving/payment practices, etc., there remains uncertainty as to which result is the better estimate or whether either estimate is appropriate.

5.2.2.3 Fitting Curve to Mature Periods Only

Since the focus of the curve-fitting is in estimating the development in the more mature ages, one possible enhancement to the methodology is to only fit the curve to the latter development periods.

5.2.3 Advantages and Disadvantages of Exponential Decay Method

This method is fairly straightforward to construct, intuitive in nature and there exists a closed-form approximation, which can be applied in most situations. The assumptions underlying the method are: (1) loss development from period to period decays in a constantly decreasing pattern, (2) the exponential decay rate is constant throughout the entire loss development pattern.

Exponential decay can produce relatively fast development compared to the development resulting from other distributional models. In certain circumstances (for instance high excess lines or long tail liability lines) other models might produce a more appropriate development result. In addition, in the case where paid losses do not continue to decay at a constant rate such as workers compensation indemnity, an alternative approach might be more appropriate. This method is not generally applicable to incurred losses for such reasons as (1) changing reserve patterns and (2) negative development, which would refute the decay constant assumption and can produce erroneous results from the fitting, if any at all.

5.2.4 Users

This method is used to a varying extent by consulting actuaries and actuaries at smaller companies where data either are inadequate or do not exist, or when development experience to date for a newly underwritten line of business does not reflect patterns from alternative sources such as industry aggregated data. Since a key assumption of this technique is a constant decay rate this might generally run contrary to other assumptions underlying the development patterns assumed in a reinsurance application.

5.2.5 Summary of Exponential Decay Method

The exponential decay method is based on a few assumptions concerning the rate of decay in incremental loss paid. From these assumptions, a curve can be fit to the development portion of age-to-age factors, which are calculated from observed paid loss data, and a resulting tail factor can be developed from the slope of the fitted curve. This method will produce suboptimal results for lines of business for which the decay rate "stalls out" or varies by development period, but adjustments such as fitting the curve to the most mature development periods will sometimes improve results.

5.3 McClenahan's Method

This method is derived from Charles McClenahan's loss model [9], which assumes incremental paid losses decay at a constant monthly rate after an initial few months lag in which no claims are paid.

5.3.1 Description

Let the monthly decay rate, p, be defined as the ratio of {accident month m incremental losses paid during month d to $d+1$} to {accident month m incremental losses paid during month $d-1$ to d},

$$p = q^*_{Paid}(m, d+1) / q^*_{Paid}(m, d) \tag{5.3}$$

for all accident months m and accident maturities (in months) $d \geq a$, where a is the average lag time (in months) until a claim begins to be paid. Since total loss from accident month m can be expressed as the sum of all monthly payments made on these claims over time, we have

$$U^*(m) = \sum_{d=a}^{\infty} q^*_{Paid}(m, d). \tag{5.4}$$

Since we assume a constant monthly decay rate, for some constant A, the incremental losses paid in month a can be expressed as $q_{Paid}(m, a) = A \times (1-p)$. Using the theorem $\sum_{n=0}^{\infty} p^n = 1/(1-p)$, it can be shown the constant A is in fact the ultimate or total loss incurred in accident month m.

$$U^*(m) = \sum_{d=0}^{\infty} A \times (1-p) \times p^d = A. \tag{5.5}$$

Under this assumption, additional payments are theoretically determined once the parameters p and a are estimated. The monthly decay rate is constant, so the annual decay rate, r, for annual periods after the initial lag period in which no claims are paid is also a constant, $r = p^{12}$. Given an average annual decay rate, the monthly decay rate p can be estimated as the 12th root of the average annual decay rate. McClenahan suggests estimation of a can be derived from the average report lag (average date of report – average date of occurrence). In any event, the final selection of the parameter a should consider the overall fit of the decay curve to the selected link ratios.

For any accident period at d months development, the tail factor is just unity divided by the percentage of total losses paid at d months, or

$$\mathrm{T}(d/12) = 1 \;/\; (1 - \text{percentage unpaid at } d \text{ months}). \tag{5.6}$$

In his paper, McClenahan presents several closed-form formulas for various loss statistics.[15]

Assuming $U^*(m)$ is constant for all m, and letting $q = 1 - p$, we can derive the following expressions for the total loss for year w, and future development of accident year w respectively,

$$U(w) = \sum_{m=0}^{11} U^*(m) = (12 \times A \times q)/(1-p)\,, \tag{5.7}$$

$$R(w, m/12) = \left\{U(w) \times q \times p^{m-a-10} \times (1-p^{12})\right\}/(1-p)^2\,. \tag{5.8}$$

Substituting (5.7) and (5.8) into equation (5.6) produces the closed-form expression for a tail factor at m months in terms of a, m, and p

$$T(m/12) = \left\{12 \times (1-p)\right\} / \left\{12 \times (1-p) - p^{m-a-10} \times (1-p^{12})\right\} \tag{5.9}$$

R is used here with the same meaning as in McClenahan's work, rather than as defined in Section 1.7.

5.3.2 Example (An additional example is in Appendix B, Section 4.2)

Reviewing an example may help the reader follow the application of the model discussed above. Even though this method is presented as applicable to incremental paid loss, with actual loss data, it would be highly unlikely that paid incremental losses for different accident periods will be the same, therefore we begin with the selection of age-to-age factors from an eight year (96-month) triangle:

Selected Age-to-age Factors

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15 For the purpose of this exercise, the variables McClenahan incorporates in his model for trend in severity, frequency, etc. can be collapsed into the decay rate and total loss for the accident period, hence there can be certain simplifications utilized in applying McClenahan's formulas for $U(w)$ and $R(w, m/12)$.

12-24	24-36	36-48	48-60	60-72	72-84	84-96
5.7720	1.5290	1.1870	1.0851	1.0424	1.0220	1.0116

Next we convert these to cumulative paid loss amounts by selecting a base amount for the first development period paid loss, for simplicity sake we use $100 in our example. To determine incremental paid losses by period we subtract successive cumulative loss amounts, and then we have the following table:

DEVELOPMENT DATA

(1) Development Age	(2) Selected Age-to-age Factors	(3) Age in Months	(4) Cumulative Paid	(5) Incremental Paid
		12	100.00	100.00
12-24	5.7720	24	577.20	477.20
24-36	1.5290	36	882.54	305.34
36-48	1.1870	48	1,047.57	165.03
48-60	1.0851	60	1,136.72	89.15
60-72	1.0424	72	1,184.92	48.20
72-84	1.0220	84	1,210.99	26.07
84-96	1.0116	96	1,225.04	14.05

Taking successive ratios of incremental paid amounts for the accident periods produces estimates of the annual decay constant r. This example was contrived to produce an estimate of r, but in practice any of a variety of curve-fitting techniques using the incremental paid loss regressed on age can be employed to develop an estimate of r from Column 5.

In order to avoid distortions in the "true" annual decay rate caused by the payment lag, for our example we will next fit the curve to incremental accident period losses starting with the third annual development period.

By definition $p = r^{1/12}$, and for the sake of the example, we will assume a lag constant of $a = 7$ months (see above discussion on estimating a). Once the value of r is calculated, with the value of p estimated, we can develop an estimate of $T(8)$ using equation (5.7) above.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Selected					Fitted
	Age-to-age	Age in	Cumulative	Incremental		Incremental
Age	Factors	Months	Paid	Paid	Age-to-age	Paid
		12	100.00	100.00		
12-24	5.7720	24	577.20	477.20	4.7720	
24-36	1.5290	36	882.54	305.34	0.6399	306.02
36-48	1.1870	48	1047.57	165.03	0.5405	165.35
48-60	1.0851	60	1136.72	89.15	0.5402	89.35
60-72	1.0424	72	1184.92	48.20	0.5407	48.28
72-84	1.0220	84	1210.99	26.07	0.5409	26.09
84-96	1.0116	96	1225.04	14.05	0.5389	14.09

r =0.5403 From Curve Fit to column (5)
p = 0.9500
a = 7
m = 96

$$T(8) = 1.0135 = \{12 \times q\} / \{12 \times q - p^{m-a-10} \times (1 - p^{12})\}$$

5.3.2.1 Exact Fitting to the Oldest Period

Curve fitting commonly has the problem of producing parameters that result in a less than desired fit in the tail of the curve, relative to actual results observed for these older periods. This can be due to a variety of factors relating to the assumptions underlying the structure or parameters of the fitted curve or random fluctuations within the actual data in earlier development periods. By comparing actual incremental paid loss to fitted results at the latest stage of development, we can usually improve the quality of the tail prediction.

In the above example, assume the actual link ratio for the development stage 84-96 was 1.0175 producing an incremental paid amount significantly greater than the overall annual decay rate r which is still expected to be 0.5403. In this case, the actual decay rate is less in the older development periods, hence the incremental paid loss in these latter development stages maybe expected to be higher than is implied by the model.

One approach would be to adjust the development portion of the initial estimate of the tail factor $\{T(m/12)-1\}$ by the ratio of the actual to the fitted incremental paid loss,

$$T(m/12_A) = 1 + (q_{Paid}^{Actual} / q_{Paid}^{Fitted}) \times \{T(m/12)-1\}. \tag{5.10}$$

Applying this adjustment to the example above, we have the following table:

(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Selected					Fitted
	Age-to-age	Age in	Cumulative	Incremental		Incremental
Age	Factors	Months	Paid	Paid	Age-to-age	Paid
		12	100.00	100.00		
12-24	5.7720	24	577.20	477.20	4.7720	
24-36	1.5290	36	882.54	305.34	0.6399	306.00
36-48	1.1870	48	1047.57	165.03	0.5405	165.33
48-60	1.0851	60	1136.72	89.15	0.5402	89.33
60-72	1.0424	72	1184.92	48.20	0.5407	48.27
72-84	1.0220	84	1210.99	26.07	0.5409	26.08
84-96	1.0175	96	1232.18	21.19	0.8128	14.09

$T(8) = 1.0135$ From initial Example
$Act / Fit = 150\% = 21.19 / 14.09$
$T_A(8) = 1.0203 = 1 + (Act / Fit) \times [T(8)-1]$

5.3.2.2 Using Multiple Periods to Estimate the Tail

This enhancement is similar to exact fitting to the oldest period adjustment, but provides an alternative in situations when the "tail" of the triangle is believed to possess some credibility, but individual link ratios are less than fully credible.

For example, assume as above, the actual (selected) link ratio for the 84-96 development period is 1.0175. In addition, assume the actual link ratio for the 72-84 period is 1.0440 instead of 1.0220. As in the prior example, the ratio of actual to fitted incremental paid loss for the 72-84 development period is now significantly different than 1.000. It should be noted the change in the link ratio for the 72-84 development period also has an effect on the paid incremental loss for the 84-96 development period, hence changes the adjustment ratio (actual to fitted) for this development period as well. These two adjustment ratios can be credibility-weighted to reflect the predictive accuracy of each factor. For the purpose of the example, each factor is assigned 50% weight. This results in an estimate of the tail factor as outlined in the following table:

(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Selected					Fitted
	Age-to-age	Age in	Cumulative	Incremental		Incremental
Age	Factors	Months	Paid	Paid	Age-to-age	Paid
		12	100.00	100.00		
12-24	5.7720	24	577.20	477.20	4.7720	
24-36	1.5290	36	882.54	305.34	0.6399	306.00
36-48	1.1870	48	1047.57	165.03	0.5405	165.33
48-60	1.0851	60	1136.72	89.15	0.5402	89.33
60-72	1.0424	72	1184.92	48.20	0.5407	48.27
72-84	1.0440	84	1237.06	52.14	1.0817	26.08
84-96	1.0175	96	1258.71	21.65	0.4152	14.09

$T(8) = 1.0135$ From initial Example

$$(Act/Fit)_2 = 200\% = 52.14/26.08$$

$$(Act/Fit)_1 = 154\% = 21.65/14.09$$

$$(Act/Fit)_{Avg} = 177\%$$

$$T_{A2}(8) = 1.0239 = 1 + (Act/Fit)_{Avg} \times [T(8) - 1]$$

5.3.3 Advantages and Disadvantages

This method is relatively easy to apply and produces a closed-form solution. The assumptions underlying the method are: (1) for a given accident period, losses decay at a constant decreasing pattern after an initial payment lag; (2) the reduction in paid incremental losses is proportional to the most current payout; and (3) the exponential decay rate is constant throughout the entire payout pattern (all accident periods, all development periods). If little is known about the "true" development pattern for the data, these assumptions appear to be minimal and reasonable, but care should be taken to assure that these assumptions do apply to the situation in which the method is being applied.

This method is subject to many of the same disadvantages as the exponential decay method such as (1) not being applicable to incurred loss or lines with potential for negative development between evaluation periods, (2) exponential decay at an indicated rate developed from the observed data that can produce a relatively faster development than other models for certain long tail liability lines, and (3) a suboptimal fit would be obtained for lines with variable decay rates across evaluation periods such as workers compensation or if the decay rate varies by accident period.

5.3.4 Users

This method is a variation of the exponential decay method utilizing incremental paid loss in place of the development portion of the link ratio. Usage of this method or similar variants of the exponential decay method (for example see Skurnick's method in section 5.4) are used to varying extents by consulting actuaries and actuaries at smaller companies where data either is inadequate or does not exist or when development experience to date for a newly underwritten line of business does not reflect patterns from alternative sources (i.e., industry aggregated data). Usage by reinsurance actuaries is assumed to be infrequent due to the constant rate of decay assumption.

5.3.5 Summary

Based on most of the assumptions underlying McClenahan's loss model along with the rate of decay estimated from the incremental paid experience data, a closed form equation for the tail factor can be developed. The results of the method can be adjusted in cases where the fit using all periods is less than optimal (different decay rate at later maturities) or credibility in the older development periods is less than fully credible.

5.4 Skurnick's Method

This method is derived from the loss model developed by David Skurnick [15] in his discussion of Charles McClenahan's loss model [9].

5.4.1 Description

This method is based on the same underlying loss model as McClenahan's method discussed in section 5.3 with a few simplifying assumptions. First, the model is developed on annual incremental payments and an annual decay rate. Second, no average delay constant is assumed (i.e., no delay between accident occurrence and accident payment). Third, we assume the annual rate of decay can vary by accident period (this assumption is not necessarily a simplifying one).

More formally stated, the annual decay rate, r_w, is defined as ratio of {accident year w incremental losses paid during development period d to $d+1$} to {accident year w incremental losses paid during development period $d-1$ to d}, i.e., $q_{Paid}(w,d+1)/q_{Paid}(w,d)$. Since total loss from accident period w can be expressed as the sum of all annual payments made on these claims over time we have,

$$U(w) = \sum_{d=0}^{\infty} q_{Paid}(w,d). \tag{5.11}$$

Given a constant rate of decay, the incremental losses paid in period 0 can be expressed in terms of some constant A and the decay rate, $q_{Paid}(w,0) = A \times (1-r)$. Using the theorem

$$\sum_{d=0}^{\infty} r_w^d = 1/(1-r_w), \tag{5.12}$$

we can show the constant A is the total loss for accident period w,

$$U(w) = \sum_{d=0}^{\infty} q_{Paid}(w,d) = \sum_{d=0}^{\infty} A \times (1-r_w) \times r_w^d = A. \tag{5.13}$$

For any accident year w at D period's development, by definition, the tail factor times the sum of the incremental loss paid to date will produce an estimate of the ultimate loss for accident year w. In equation format this can be expressed as:

$$T(D) * \sum_{d=0}^{D} U(w) \times (1-r_w) \times r_w^d = U(w). \tag{5.14}$$

Based on the following theorem for finite summations:

$$\sum_{d=0}^{D} ar^i = \left[ar^{D+1} - a\right]/(r-1) \ \text{ if } r \neq 1 \tag{5.15}$$

we can develop a closed form solution for the tail factor as:

$$T(D) = 1/(1-r_w^{D+1}). \tag{5.16}$$

5.4.2 Example

Assume the following incremental loss payouts for an accident period:

Age in Months	Period	Incremental Paid
12	0	4,000
24	1	2,000
36	2	1,000
48	3	500
60	4	250
72	5	125
84	6	62.5
96	7	31.25

Fitting a line to the natural logarithms of the incremental paid losses in each development period, we can develop the estimates for $\ln(r_w)$ and $\ln\left[U(w) \times (1-r_w)\right]$ by using the identity derived above:

$$q_{Paid}(w,0) = U(w) \times (1-r) \times r_w^d, \ \text{ hence}$$

$$\ln\left[q_{Paid}(w,0)\right]=\ln\left[U(w)\times(1-r)\right]+d\times\ln\left[r_w\right].$$

(1)	(2)	(3)	(4)	(5)	(6)
Age in Months	Period	Incremental Paid	Log of (3)	Fitted Incremental Loss	Fit Error
12	0	4000	8.294050	4000	0
24	1	2000	7.600902	2000	0
36	2	1000	6.907755	1000	0
48	3	500	6.214608	500	0
60	4	250	5.521461	250	0
72	5	125	4.828314	125	0
84	6	62.5	4.135167	62.5	0
96	7	31.25	3.442019	31.25	0

$\ln(r)=$ -0.6931 $\quad\quad r=$ 0.5000

From Curve Fit to column (4)

$\ln[U(w)\times(1-r)]=$ 8.294 $\quad\quad U(w)\times(1-r)=$ 4000

$T_w(6)=$ 1.0079

$T_w(7)=$ 1.0039

Taking the exponential of the estimate of the natural log of r produces estimates of the annual decay constant, from which we can estimate the tail factor at given development stages for this accident period. This example was contrived to produce an estimate of r with no error term (column (6) = column (3) minus column (5)). The next example demonstrates the effect of an increase in incremental loss in an early development period, followed by a return to a constant decay pattern.

(1) Age in Months	(2) Period	(3) Incremental Paid	(4) Log of (3)	(5) Fitted Incremental Loss	(6) Fit Error
12	0	1000	6.907755	2,245	-1,245
24	1	2000	7.600902	1,260	740
36	2	1000	6.907755	707	293
48	3	500	6.214608	397	103
60	4	250	5.521461	223	27
72	5	125	4.828314	125	0
84	6	62.5	4.135167	70	-8
96	7	31.25	3.442019	39	-8

$\ln(r) =$ -0.5776 $\qquad r =$ 0.5612

From Curve Fit to column (4)

$\ln[U(w)\times(1-r)] =$ 7.7164 $\qquad U(w)\times(1-r) =$ 2245

$T_w(6) =$ 1.0178 $\qquad =1/[1-r^{D+1}]$

$T_w(7) =$ 1.0099

The resulting curve does not fit the incremental losses as well in the earlier development periods. The tail factor produced by the estimated decay constant, r, is much larger than in the previous example, though the observed decay rate in the incremental losses in the later development periods are the same for both examples.

5.4.2.1 Limit Curve Fitting to the More Mature Development Periods

Increases in incremental paid losses from period to period, especially in early stages of development, are a common phenomenon. As demonstrated in the second example above, this can lead to less than an optimal curve fit, and possible distortions in the estimated tail factor. Putting more emphasis on the behavior of losses in the latter stages of development, at a point where a strictly monotonic decease in incremental paid losses is observed, is one approach that can provide a more optimal fit. An example of fitting a curve to actual incremental paid loss, only in the latter stages of development, is shown below:

(1)	(2)	(3)	(4)	(5)	(6)
Age in Months	Period	Incremental Paid	Log of (3)	Fitted Incremental Loss	Fit Error
60	4	250	5.521461	250.00	0
72	5	125	4.828314	125.00	0
84	6	62.5	4.135167	62.50	0
96	7	31.25	3.442019	31.25	0

$\ln(r) =$ -0.6931 $\qquad r =$ 0.5000

From Curve Fit to column (4)

$\ln[U(w)\times(1-r)] =$ 8.294 $\qquad U(w)\times(1-r) =$ 4000

$T_w(6) =$ 1.0079 $\qquad =1/[1-r^{D+1}]$

$T_w(7) =$ 1.0039

In this example, the tail factors produced for the 84- and 96-month development periods are the same as those produced in the original, contrived example.

5.4.2.2 Excluding the Latest Development Periods to Estimate the Tail

This enhancement can be used when the last development period incremental data is believed to be less than credible. The procedure is to (1) fit the curve to all periods but the last development period incremental paid loss, (2) compute the corresponding tail factor for the next to last stage of development, and (3) divide this result by the last observed link ratio.

5.4.2.3 Adjustment to Exact Fitting

In this enhancement, the development portion of the derived tail factor is adjusted by the "actual-to-fitted ratio" from the last stage. Using the second example above, the development portion of the 96-month tail factor is 0.0099 and the actual to fitted ratio is 31.25/39 = 0.795. The adjusted tail factor is now 1 + (.0099) * (0.795) = 1.0079. Given the observed data utilized in these examples, this 'correction' appears to move the factor in the right direction. This is an example of a situation in which the type of curve fitted to the data is not appropriate, based on the pattern of the data.

5.4.3 Advantages and Disadvantages

This method is simpler in construction than the McClenahan model, and produces a closed-form solution. The assumptions underlying the method are: (1) for a given accident period losses decay at a constant decreasing pattern; (2) the reduction in paid incremental losses is proportional to the most current payout; (3) the exponential decay rate, though constant over evaluation periods for a given accident period, may be a different rate for other accident periods; and (4) there is no lag between accident occurrence and accident payment.

Some of the draw backs to this method include: (1) Exponential decay assumes a monotonically decreasing function, therefore this method does not accommodate increases in incremental paid losses from one period to the next (hump-shaped patterns) very well; (2) The method breaks down when, in a given accident period, there are periods of no payments or negative payments; (3) This method is not applicable to incurred losses since they are often subject to negative development or changes in reserving practices (refutes constant decay rate); (4) For less mature accident periods with few valuations, the regression line fit could be less than optimal; and (5) This method is subject to most of the potential pitfalls of the McClenahan method such as the fitted exponential decay rate might be faster than is appropriate for the line of business, or the decay rate might vary by development period for lines such as workers compensation.

5.4.4 Users

This method is a variation of the exponential decay method utilizing incremental paid loss in place of the development portion of the link ratio. Usage of this method or similar variants of the exponential decay method (for example see McClenahan's method in section 5.3) are used to varying extents by consulting actuaries and actuaries at smaller companies where data either is inadequate, does not exist or when development experience to date for a newly underwritten line of business does not reflect patterns from alternative sources (i.e., industry-aggregated data). Usage by reinsurance actuaries is assumed to be infrequent due to the constant rate of decay assumption.

5.4.5 Summary

This method is similar in many respects to the McClenahan method (see section 5.3). Differences of this method from the McClenahan method include that (1) simplifications that reduce the calculations required in the closed-form solution, (2) the ability to vary the decay rate by accident periods, and (3) there is no need for payment lag in the calculation.

5.5 Sherman's Method

This method, first articulated by Richard Sherman [14], relies on fitting "inverse power" curves to link ratios.

5.5.1 Description

5.5.1.1 Sherman's Original Method

In this method, we fit "inverse power" curves of the form $1+ad^b$ (d representing development age) to the link ratios. The identity below enables us to base the fitted curve on a simple regression.

$$\ln(\mathrm{f(d)}-1)=\ln(\mathrm{v(d)})\approx\ln(1+ad^{b}-1)=\ln(ad^{b})=\ln(a)\ +b[\ln(d)]\ . \tag{5.17}$$

Unfortunately, there does not appear to exist a simple closed-form approximation to the tail this curve generates. The tail factor must then be estimated by multiplying together successive link ratios after the tail begins, until the impact of additional link ratios is negligible.

5.5.1.2 Sherman's Revised Method

In his study of the inverse power curve, Sherman [14] noted that the fit could sometimes be improved by adding a lag parameter to the curve. He used the formula

$$\mathrm{f(d)}=1+\mathrm{v(d)}\approx 1+a(d-c)^{b}\ . \tag{5.18}$$

In this case, the mechanics of fitting the curve are somewhat more complex.

5.5.2 Example (See Appendix B, Section B.4.3 for additional example)

The following illustrative data will be used in the appendix to illustrate Sherman's Methods.

First determine the development portion, $v(d)$, of each link ratio. The natural logarithms of $v(d)$ and the age d then represent the dependent and independent variables in our regression, respectively.

(1)	(2)	(3)	(4)	(5)
Development Age d	Link Ratio $f(d)=1+v(d)$	Development Portion $v(d)$	'X' $\ln[d]$	'Y' $\ln[v(d)]$
1	2.034	1.034	0.000	0.034
2	1.560	0.560	0.693	(0.580)
3	1.321	0.321	1.099	(1.137)
4	1.184	0.184	1.386	(1.692)
5	1.106	0.106	1.609	(2.240)
6	1.074	0.074	1.792	(2.601)
7	1.047	0.047	1.946	(3.065)
8	1.032	0.032	2.079	(3.438)
9	1.024	0.024	2.197	(3.731)

The fitted parameters of the dependent and independent variables of the fitted curve then are:

Fitted-Curve Parameters	
Slope = b	(2.386)
Intercept	4.806
$a=e^{Intercept}$	1.137

The tail factor (T) is then estimated as the product of link ratios for development ages 10 through d , where d is sufficiently large that the fitted age-to-age is close 1.00.

Several possible alternatives to Sherman's method exist. For example, in determining the appropriate curve, we could rely on link ratios of only the first 5 or 10 development ages or we could rely on the link ratios of only "mature" development ages. In addition, as discussed above, Sherman's revised formula introduces a lag parameter to the curve.

5.5.3 Advantages and Disadvantages

As with all curve-fitting methods, Sherman's method of fitting inverse-power curves to link ratios has advantages and disadvantages. The primary advantage is its relative simplicity and flexibility in evaluating multiple variations, once established in spreadsheet form. The primary disadvantage, on the other hand, is that it makes specific mathematical assumptions about the link ratio pattern when there is no compelling reason for the link ratios to follow any pattern whatsoever.

Sherman's revised formula has an added level of complexity. The modeler must evaluate whether the resulting degree of accuracy warrants the added level of complexity and work.

5.5.4 Users

This method enjoys fairly broad acceptance both with consulting firms and within insurance companies. It is not used quite as often as some of the other methods (e.g., industry booked tail), but is perhaps the most common medium complexity method in use.

5.5.5 Summary

Per Sherman's analysis in the paper describing this method, this method does appear to fit link ratio data better than the various exponential approaches (exponential decay of development, McClenahan's method, and Skurnick's method). The calculations, though they are readily doable by most actuaries, involve a little more mathematics than most audiences are prepared for. Nevertheless, this generates a very useful estimator of the tail factor.

5.6 Pipia's Method

This method determines the tail factor that best fits selected age-to-age factors by fitting a Weibul curve to the historical age-to-age data. The best fitting curve is determined by minimizing the squared ratio of the difference between the fitted age-to-age factors derived from the curve and the historical age-to-age factors. The curve represents the age to ultimate factor. The indicated age-to-age factor from the curve is found by dividing the value of the curve at time d by the value at time $d+1$.

5.6.1 Description

Age-to-age factors are selected from historical data or from an industry source; age to ultimate factors are calculated from this data. A tail factor is selected that minimizes the squared differences between selected age-to-age factors and the age-to-age factors implied by the curve representing the age to ultimate factors. For workers compensation, the Weibul distribution, $1-e^{-\lambda(d+c)^t}$, has been found to provide a good fit to age-to-ultimate factors. The age to ultimate factor at time d equals $1/1-e^{-\lambda(d+c)^t}$ where c is a shift parameter.

5.6.2 Pipia's Example (See Appendix B, Section B.4.4)

5.6.3 Advantages and Disadvantages

This method is relatively easy to apply and produces a tail factor consistent with the underlying historical observations. It is also easily adaptable to alternative selections of the distribution to be used for other lines of business. A good starting point may be the underlying loss distribution for the line of business since development is often related to the claim size distribution. This method, although it does not produce development factors less than 1.000, does not fail when actual factors below unity are in the historical data being fitted. Another advantage is that the historical data need not be complete or have consistent evaluation dates for each accident year. It provides a means to calculate development factors for a risk that only has scattered loss reports at different and inconsistent evaluation dates. This model can also be used to calculate development factors at intermediate points as well as points prior to or after the historical data. This last item is useful when one is using some benchmark data such as the NCCI Annual Statistical Bulletin, which provides incremental development factors at annual evaluations through 96 months.

This method is subject to many of the same disadvantages as all loss development methods such as changes in case reserving, payout pattern, statutory changes that affect loss development and the appropriateness of the selected distribution for a line of business.

5.6.4 Users

It is understood that the developer has used this method to provide another estimate of the tail factor in conjunction with other methods, and that he has also used it when using benchmark data such as Schedule P data from the annual statement. However, due to its limited distribution to date, the specific Weibull-curve method is only used by a few actuaries.

5.6.5 Summary

The method fits expected incremental development factors to the actual historical factors to generate an age to ultimate curve. The curve provides the age to ultimate for the average age of an accident year. The average age input can be outside the historical data range as well as at an intermediate point within the historical data period. It provides an alternative estimate of development factors as well as a tail factor. This should be used as one of several alternative methods in making a tail factor selection.

5.7 England-Verrall Method

For an excellent introduction to this method, see Section 8, "Discussion And Conclusions," of the research paper itself. Sections of the paper are quoted in their entirety below, although not in the same order in which they appear in the research paper. For consistency, the notation in the following subsections differs slightly from the notation of Section 1.7. The notation of this section follows that of mathematical probability while the notation of Section 1.7 is that of loss development in actuarial science. Table 5.7.2.1 below retains the notation of Section 1.7.

5.7.1 Description

Currently, given a triangle of data, a simple reserving exercise might proceed by fitting a chain ladder model (usually a 3, 4, or 5 period volume-weighted average chain ladder) and looking at the resultant development factors. It would then be common to smooth the factors and consider the necessity of a tail factor for projecting beyond the range of data observed. A number of methods, including judgment might be used to smooth the factors with the aim of smoothing out random variations, particularly in the later stages of development, while leaving the systematic trend intact. A tail factor might be chosen, by a variety of methods.

To construct a flexible framework for stochastic claims reserving, within which several of the models can be regarded as special cases, for incremental paid claims $c(w,d)$ define

$$E\left[c(w,d)\right]=m_{w,d}\ , \tag{5.19}$$

$$Var\left[c(w,d)\right]=\phi m_{w,d}^{\rho} \tag{5.20}$$

and

$$\ln(m_{w,d})=\eta_{w,d}=\mu_{w,d}+\delta k+c+s\theta_w(w)+s\theta_d(d)+s\theta_d(\ln(d))\ . \tag{5.21}$$

Equations (5.19), (5.20), and (5.21), which correspond to Equations (3.3), (3.4), and (3.5) on page 16 of the original research paper, specify a generalized additive model with power variance function and constant scale parameter. The power ρ dictates the choice of error distribution, with normal, Poisson, gamma and inverse Gaussian specified by ρ = 0, 1, 2, and

3, respectively. The predictor is linked to the expected value of the response through the logarithmic link function. The offsets $\mu_{w,d}$ and inflation term δk are optional (where $k = w + d$), and may be suggested by a particular context. The function $s(w)$ represents a smooth of accident period w, obtained using a smoothing spline with smoothing parameter θ_w. Similarly, the functions $s(d)$ and $s(\ln(d))$ represent smoothing splines specifying the shape of the runoff pattern, with smoothing parameter θ_d chosen (for simplicity) to be the same for both functions. In practice, it may not be necessary to include smoothers in both d and $\ln(d)$. It should be noted that both accident period w and development age d are considered as continuous covariates.

When θ_w is zero, there is no smoothing and the model is forced to pass through each value of w, which treats accident period w as though it is a factor. The same is true of θ_d; when θ_d is zero, the model is forced to pass through each value of d, and development time is treated as though it is a factor. When θ_d tends to infinity, the part of the model relating to development time is linear in d and $\ln(d)$, giving the Hoerl curve. It is also necessary to choose the power function ρ to complete the model specification.

Having chosen the model specification, the model can be fitted using maximum quasi likelihood to obtain parameter estimates (and their approximate standard errors). At this point the authors make use of standard statistical software packages which have the facility to fit generalized additive models. Currently the choice is limited, although greater choice is likely in the future as the popularity of generalized additive models increases. The authors used S-PLUS for the example.

Having fitted the model, reserve estimates are obtained by summing the appropriate predicted values in the southeast region of the claims rectangle. All that remains is the estimation of variability in the reserve estimates.

One of the principal advantages of stochastic reserving models is the availability of estimates of precision. Commonly used in prediction problems is the standard error of prediction, also known as the prediction error, or root mean square error of prediction. For claim payments in development period d for accident period w (yet to be observed), the mean square error of prediction is given by

$$E\left[\left\{c(w,d)-\hat{c}(w,d)\right\}^2\right] \approx Var\left[c(w,d)\right]+Var\left[\hat{c}(w,d)\right]. \tag{5.22}$$

Note that the mean square error of prediction can be considered as the sum of two components: variability in the data (process variance) and variability due to estimation (estimation variance).

For the general model defined above, the process variance is given by Equation (5.20). For the estimation variance, note that

$$\hat{c}(w,d) = \hat{m}_{w,d} = e^{\hat{\eta}_{w,d}} . \tag{5.23}$$

$$E\left[\left\{c(w,d) - \hat{c}(w,d)\right\}^2\right] \approx \phi \hat{m}_{w,d}^p + \hat{m}_{w,d}^p Var\left[\hat{\eta}_{w,d}\right]. \tag{5.24}$$

The final component of Equation (5.24), the variance of the (linear) predictor, is usually available directly from statistical software packages, enabling the mean square error to be calculated without difficulty. The standard error of prediction is the square root of the mean square error of prediction.

The mean square error of prediction of the origin period reserve, the total reserve estimate, and the mean square error of prediction of the total reserve are found in the original research paper as Equations (4.3) and (4.4).

Although Equations 4.3 and 4.4 of the original research paper look fairly complex, they are relatively easy to calculate by summing the appropriate elements. The only components not readily available from statistical software packages are the covariance terms. Provided the design matrix and variance-covariance matrix of the parameter estimates can be extracted from the statistical software package used, a full matrix of the covariance terms can be calculated without difficulty for any specification of the predictor η. Indeed, the variances of the (linear) predictors are simply the diagonal of such a matrix. Although natural in stochastic claims reserving, it is unusual to focus on the shape of the decay of incremental claims using traditional actuarial methods, in which it is common to focus on the relative increase in cumulative claims through development factors, the traditional "parameters" in a standard chain ladder exercise. After fitting a stochastic claims reserving model, it is straightforward to obtain equivalent development factors by applying the standard chain ladder model to the fitted values of the stochastic model. If the model is fully parametric, it may be possible to obtain a relationship between the model parameters and the chain ladder development factors.

Incremental paid losses from an aggregation of classes of business are shown in Table 6.1 on page 23 of the paper, and are used to illustrate the methodology. The incremental claims fall fairly rapidly, but are not completely run-off by the end of the tenth development period, implying the necessity for a tail factor greater than 1.0 when using the traditional chain ladder model.

5.7.2 Example (See Appendix B, Section B.4.5)

5.7.3 Advantages and Disadvantages

Advantages of this procedure are that it is extremely flexible, and it forces the actuary to look at the data. Disadvantages are that it is time-consuming and statistically inefficient.

The main strength of the method presented in this paper is that both the smoothing and extrapolating can be performed at the same time in the same model. The actuary simply has to choose one parameter for smoothing across the whole range of development time, choose an error distribution, and choose how far to extrapolate (an additional parameter is necessary if smoothing over accident years). Further advantages are that it is also possible to obtain measures of precision of the reserve estimates, and investigate where the data deviate from the fitted model by viewing residual plots. Choosing smoothing parameters at the extremes is a useful additional feature since at one extreme the model may be considered over-parameterized, and at the other the structure may be too rigid.

Incremental data are used for the method put forward in this paper: This is both an advantage and a disadvantage. It is advantageous since the method can be used when the data history is incomplete. If incremental data were recorded by accident year only after a certain date, accident years prior to that date will have incomplete runoff information, and a section of the claims triangle in the northwest corner will be missing (this is a reasonably common occurrence). This presents difficulties using standard deterministic techniques that rely on cumulative data, but is not a problem for stochastic techniques, which treat the unobserved data as "missing" and estimate the data as part of the fitting procedure. The disadvantage is that negative incremental values sometimes occur in data based on paid losses, and frequently occur in data based on incurred losses where case estimates are often set on a conservative basis and overestimated. The method proposed is robust to a small number of negative incremental claims (as in the example), but will always produce positive fitted values (due to the use of the logarithmic link function) and hence will always produce development factors greater than one. For this reason, the techniques are often not suitable for use with incurred data, which often include a series of negative incremental losses in the later stages of development requiring development factors less than one.

5.7.4 Users

As a newly developed method, there were no known users identified in our survey at this time.

5.7.5 Summary

Stochastic models have been constructed with the aim of producing exactly the same reserve estimates as the traditional deterministic chain ladder model. Advantages are that measures of precision are readily available, and the assumptions underlying the chain ladder model are clarified. More importantly, the models provide a bridge between traditional methods and stochastic methods, which is useful for the practitioner who is familiar with traditional methods and needs a starting point for exploring stochastic methods.

The aim of the England-Verrall paper is to present a flexible framework for stochastic claims reserving which allows the practitioner to choose whether to use the basic chain ladder model, or to apply some smoothing, or to use a parametric curve for the runoff. Several of the models proposed to date fit within this framework, and further extensions are possible that have not yet been tried.

5.8 Summary of Curve-Fitting Methods

Several curve-fitting methods were presented, three that involve some sort of exponential decay process, and one that involves alternate assumptions about the decay of the development portion of the link ratios. It must be recognized that, by their very nature, the exponential decay methods will all tend to produce similar answers. So, the addition of the Sherman method is a welcome improvement. However, it must be recognized that all curve-fitting methods make some very significant assumptions as to how development factors will decay. In using curve-fitting methods, it is a good idea to compare the results of several different curve-fitting techniques, considering the potential for bias in (1) the choice of the function, (2) actual points used in the fit and, (3) estimation of parameters. So, the user is cautioned to not just use them blindly.

6. METHODS BASED ON REMAINING OPEN COUNTS

6.1 Introduction to Open-Count Based Methods

There is a class of methods that involve first estimating an average cost per open count for each calendar period and multiplying by the projected number of claims remaining open in that period. Summing together the results for all calendar periods in the tail gives the unpaid loss at the tail period. Dividing that by the paid loss up to the tail produces a paid loss tail factor. As it happens, the two methods presented use mortality to project the claims remaining open, although other approaches are possible.

6.2. Static Mortality Method

The static mortality method is also known as the incremental paid to prior open method. It separately treats changes in workers compensation incremental severities (due to annual rates of medical cost escalation) and the slow decline in the number of open claims (due to mortality). It is an adaptation of the (classic) structural methods of Fisher/Lange and Adler/Kline.

6.2.1 Description

Incremental payments for every development year are estimated by taking the product of the number of open claims at the end of the prior development year and an estimated claim severity. For mature development years, future incremental payments are essentially a function of how many claims are still open and the average size of incremental payments per open claim. Changes in the number of open claims can be estimated beyond years in the triangle via mortality rates and inclusion of the small number of newly reported claims and net closures for other reasons. Analogous incurred loss development patterns can be estimated if one defines total case reserves as the product of the latest year's incremental payments times the average annuity factor for all living permanent disability (PD) claimants.

6.2.2 Example

Section 3 of the Sherman-Diss paper includes a detailed example of this method.

While the static mortality method is of limited value for early development periods, its merit relative to other reserving methods is substantial in estimating reserves for future MPD payments (the medical component of permanent disability claims) for more mature development periods. For such mature development periods, future incremental payments are essentially a function of how many claims are still open and the average size of incremental payments per open claim. In contrast, future incremental MPD payments have almost no causal linkage to payments for rapidly settled claims during early development periods.

The specific steps to be taken in applying the incremental paid per prior open claim method are:

(1) Incremental paid losses and open counts are compiled by accident year and development period.

(2) Historical averages of incremental paid per prior open are compiled in triangle format starting at 24 months, computed using the above incremental paid and open count data.

(3) Each historical average is trended to the expected severity level for the first calendar year after the evaluation date.

(4) Development factors of open counts at successive period-ends are computed.

(5) The selected ratios from (4) by development period are used to project the number of open claims for each future development period of each accident year, thereby completing the triangle of open counts.

(6) Future values of incremental paid per prior open are selected for each development period based on the trended data in (3) above.

(7) Projections of incremental paid losses for future development periods for each accident year are determined as the product of the projected open counts from the completed triangle and the projected values of incremental paid per prior open selected in (6).

The percentage declines in prior open counts reflect the composite effects of three factors affecting the number of open claims: (1) increases due to newly reported claims, (2) decreases due to the death of a few claimants, and (3) net decreases due to other reasons (including increases due to reopened claims). After 20 periods of development, newly reported claims and net claim closures (1 and 3 above) become negligible. Thus, after 20 periods of development, virtually all claim closures are attributable to the death of claimants. Consequently, changes in the number of open claims at the end of each development period beyond 20 periods can be predicted almost entirely on the basis of mortality rates. And changes in the number of open claims can be estimated beyond 15 periods via mortality rates and inclusion of the small number of newly reported claims and net closures for other reasons. This is subject to fine-tuning due to the possibility that the mortality rates of disabled claimants might be higher than those of the general populace, although recent improvements in medical technology have reduced the influence of medical impairment on mortality rates.

If the historical database includes only the total number of closed claims, the number of claimant deaths may be estimated based on mortality tables and any additional claim closures are presumed to be for other reasons. In the Sherman-Diss model of Section 8.5, the breakdown is derived by estimating the number of claim closures due to death from the 2000 Social Security Administration (SSA) mortality tables.

Just as the authors have modeled the expected paid loss development factor (PLDF) patterns for MPD losses, analogous incurred loss development factor (ILDF) patterns can be estimated by defining total case reserves as the product of the latest period's incremental payments times the average annuity factor for all living PD claimants.

6.2.3 Advantages and Disadvantages

While this method is of limited value for early development years, its merit relative to other reserving methods is substantial in estimating reserves for future MPD (medical permanent disability) payments for more mature development periods. The method is subject to fine-tuning due to the possibility that the mortality rates of disabled claimants might be higher than those of the general populace. In other words, a substandard mortality table may be required. It is important to note that the applicability of the method is not only dependent on the open claim count retention level, but also (1) the presence (or absence) of PD claimants with ongoing medical costs, and (2) the specific provisions of state workers compensation laws. However, the Sherman-Diss paper focuses primarily on MPD claims, which generally do not vary significantly between states.

6.2.4 Users

The method is utilized by the SAIF Corporation and the Oregon State Fund.

6.2.5 Summary

After 20 years of development, virtually all workers compensation claim closures are attributable to the death of claimants. Consequently, changes in the number of open claims at the end of each development year beyond 20 years can be predicted almost entirely on the basis of mortality rates. Medical cost escalation rates and the force of mortality are the key drivers of MPD tail factors. The paid loss development method is not designed to treat these two influences separately. This method (incremental paid per prior open) provides for the separate, explicit treatment of the effects of these two drivers. The above method can be applied satisfactorily to workers compensation total medical loss experience for development years 20 and higher.

6.3 Trended Mortality Method

The trended mortality method is an adaptation of the (classic) structural methods of Fisher/Lange and Adler/Kline. The model explicitly accounts for the compounding effects of downward trends in future mortality rates and persistently high rates of future medical cost escalation.

6.3.1 Description

The method is similar to the static mortality method of Section 6.2. The key difference is that the change in the number of open claims for every future development period of every accident year is determined by applying mortality tables forecasted by the SSA for the appropriate future development year. The rest of the method is essentially unchanged. The use of *forecasted* mortality is the distinctive feature of the trended mortality method.

6.3.2 Example

An example of the method is given in Section 6.2, the static mortality method. A few comments should be made, which refer specifically to the trended mortality method.

Small improvements in the annual survival rate of remaining claimants result in major differences in the number of claims still open at higher development periods. Given that the greatest differences occur during development periods in the distant future, when the effects of medical inflation have had an opportunity to compound over decades, the total reserve indicated by the trended mortality method is decidedly greater than that indicated by the static mortality method.

Paid loss development factors for earlier (as well as middle) development periods will not hold constant over successive accident periods. However, it is also evident that the rate of increase over the short to middle term in these paid development factors on account of mortality is small. It is small enough that it would not be detectable to an experienced actuary reviewing historical PLDFs (paid loss development factors).

Even though it is true that past declines in mortality rates are implicitly embedded in historical PLDFs, it would be incorrect to assume that the selection of historical factors as estimates of future PLDFs would implicitly incorporate the effects of future declines in mortality rates. With respect to mortality, the past experience of the data under review may very well not be a good indication of future mortality. What would be more appropriate would be to select representative PLDFs for each development period based on recent historical factors and then to trend these upward in a manner parallel to the PLDFs indicated by a realistic model such as mortality tables forecasted by SSA.

6.3.3 Advantages and Disadvantages

Advantages and disadvantages are similar to those for the static mortality method of Section 6.2.

6.3.4 Users

The method is utilized by the SAIF Corporation and the Oregon State Fund.

6.3.5 Summary

The Trended Mortality Method is similar to the Static Mortality Method described above but additionally, incorporates the compounding effects of the drivers. The above method can be applied satisfactorily to workers compensation total medical loss experience for development years 20 and higher.

6.4 Summary of Future Remaining Open Claims Methods

Two methods were presented, both of which rely on mortality to estimate open claim counts. These methods are correct to point out that workers' compensation link ratios can actually increase at certain durations due to medical inflation and the slow rate of withdrawals/deaths from the system.

7. METHODS BASED ON PECULIARITIES OF THE REMAINING OPEN CLAIMS

7.1 Introduction

Although tail factors are generally intended to cover 'average' development beyond the data triangle, the actual development of the oldest year may be heavily driven by whether some particularly difficult claims are left in the oldest year. So, while these methods do not generally result in a tail factor applicable to the less mature years (that may or may not have a similar open claim portfolio when they become the oldest year in the triangle), it can be very useful for analyzing the oldest year and other years near the top of the triangle.

7.2 The Maximum Possible Loss Method

7.2.1 Description

This method is a variant of the unclosed count method. However, it does not create a tail factor per se but establishes a maximum tail for the older years. The core idea of this method is that, given that the maximum net liability of an insurer is some net retention R, the liability for all the open claims should not be more than the sum of R minus paid to date across all the open claims. For simplicity we assume the coverage period of the pertinent reinsurance agreement coincides to an accident period. To use this method, given that an accident year is sufficiently mature that no IBNR claims are reasonably possible, the remaining amounts to reach the retention (R - paid-to-date) are summed across all remaining open claims in the accident year to produce the liability of open claims.

The result is an upper bound on tail development for that specific year. So, if application of the tail factor to a given year suggests more development than is 'possible' per the remaining amounts to reach the retention in the accident year, the ultimate unpaid loss for that accident year might be capped at the amounts remaining to reach the retention.

In the (fairly unusual) event that there are enough claims left open for this to be a statistically valid predictor of the development of the more recent years, it could be used in estimating the tail factor for all the accident years. But, one would have to be certain that this finding was statistically consistent with the initial tail factor analysis. For example, if the initial tail factor came from a curve fitting, it might be statistically reasonable that the curve fitting was simply using the wrong curve. However, if the initial tail factor came from a 'paid overdisposed' method that also used the actual data in the triangle itself, the tail findings would suggest the data is internally inconsistent. In that case, greater care must be taken to understand which method is most accurate for the tail factor to be applied to the more recent years.

7.2.2 Example

Consider the following list of claims remaining open for the oldest year in a triangle (assumed to be 1991)

Claims Remaining Open in Oldest Year (1991)

Claim Number	Retention	Paid at Year-End
1	300,000	150,000
2	300,000	200,000
3	300,000	250,000
4	300,000	275,000
Total		875,000

Note that retention is the same for all claims as it is presumed that one reinsurance program was in place throughout all of accident year 1991. Then, we compute the total amount unpaid up to the retention, on each individual claim.

Claim Number	Retention	Retention- Paid at Year-End
1	300,000	150,000
2	300,000	100,000
3	300,000	50,000
4	300,000	25,000
Total		325,000

In the event that no closed claims reopen, the total of the remainders to hit the retention is the maximum possible unpaid loss. Continuing in that vein, we divide the total possible maximum loss by the paid-to-date on all 1991 claims, and get a corresponding maximum possible tail factor.

Paid-to-Date (All Claims) for Oldest Year (1991)
2,000,000
Cap on Development Portion (Total Max Unpaid/Paid All Claims)
0.16
Maximum Possible Tail Factor for 1991 (1+Cap)
1.16

A similar process can be used to compute maximum IBNR, using case-incurred loss instead of paid losses.

7.2.3 Advantages and Disadvantages

This method improves on the average unpaid loss method by dint of the fact that the amount to reach the retention need not be estimated. Rather, it is fact. However, it only produces an upper bound, not an actual best estimate.

Like the average unpaid loss method, there are often statistical reliability issues when making inferences about the tail factors of the more recent years. But, one cannot readily dispute the results as an upper bound for the older years on which the method is applied, at least as long as one is certain the prospect of additional IBNR claims is immaterial. So, like the average unpaid loss method, one must be very careful to make sure the proper assumptions hold when using it. But, unlike the average unpaid loss method, it has far more certainty surrounding the loss sizes.

7.2.4 Users

This method is used by some consulting firms and some insurance companies.

7.2.5 Summary

As stated, this method may be a powerful tool for setting an upper bound on development on the oldest year or years. Yet, it does not generalize well to the more recent years. So, it does not lend itself to a tail factor that can be applied to all the years.

7.3 Judgment Estimate Method

7.3.1 Description

A method to derive the tail for the oldest claims is to examine the particular fact pattern of each reported outstanding claim and rely upon claims evaluation expertise to estimate the remaining settlement value for each claim. The sum of the estimated outstanding reported remaining settlement values by accident period is added to the cumulative payments by accident period to derive estimated ultimate settlement values by accident period. The estimated ultimate settlement value divided by the reported (or cumulative paid) losses to date by accident period results in the incurred (or paid) tail factors implied by this method. As this method is essentially a claims audit for the oldest claims, the method should probably not be strictly classified as an actuarial method.

The method is intended to be applied only to the oldest periods where there is no reasonable expectation that additional claims will be reported. Of course, the resulting estimate of the tail will only be as useful as the quality of the claims expertise used to evaluate remaining claim settlement values.

7.3.1.2 Example

Consider the following cumulative paid loss triangle:

Cumulative Paid Loss Triangle $c_{Paid}(w,d)$

	12	24	36	48	60	72
1991	1,000	2,000	2,500	2,800	2,950	3,100
1992	1,100	2,400	3,000	3,500	3,900	
1993	1,300	2,500	3,000	3,400		
1994	1,200	2,300	3,100			
1995	1,400	2,800				
1996	1,490					

By 72 months of development, it is believed that all claims have been reported for the oldest accident year—1991. There are six (6) claims outstanding for accident year 1991 as of 72 months of development. A professional claims examiner is engaged to evaluate the fact pattern of each of the six claims in order to derive an estimate of the remaining settlement value for each outstanding claim.. The claims examiner estimates are as follows: claim #1- 100; claim #2- 300 (the policy limit); claim #3- 0 (i.e., expected to be closed without payment); claim #4- 300; claim #5- 250; and claim #6- 250. The actuary reviews the claims examiner estimates for possible additional adjustments. Although claim #2 is expected to settle at the policy limit, the actuary believes there will be some loss adjustment expense to settle the case and, as such, adds 50 to the estimate for this claim. Similarly, the actuary adds 50 to the claim #3 estimate to reflect future allocated loss adjustment expenses. Claim #6 is expected to be settled in several years and the actuary believes the claims examiner has not fully reflected severity inflation through time of settlement. The actuary adds 50 to this claim in order to account for additional severity inflation beyond which has been reflected by the claims examiner. After actuarial adjustment, the individual claim estimates are as follows: claim #1- 100; claim #2- 350; claim #3- 50 claim #4- 300; claim #5- 250; and claim #6- 300. These claim estimates total 1,350. Accordingly, the 72-ultimate payment tail development factor is derived as

$$(3{,}100+1{,}350)/3{,}100=1.435$$

The actuary further notes that the payment tail factor is only based upon an evaluation of six (6) claims and, as such, may not have full credibility.

7.3.2 Advantages and Disadvantages

Strengths of this method are:

(1) The tail estimate is based upon the particulars of actual reported outstanding claims without reliance on theoretical models.

(2) The tail estimates may be improved by better claims settlement evaluation expertise.

(3) The method is readily understood by nontechnical users of the resulting actuarial

work product.

(4) The method may provide insight into the plausible upper and lower bounds for the tail by period. A lower bound may be derived by assuming all reported remaining outstanding claims are closed without payment. An upper bound may be obtained by assuming all reported remaining outstanding claims are settled at the retention or policy limits. However, the use of these upper and lower bounds has its limitations, as discussed below.

Weaknesses of this method are:

(1) The method is only applicable: where there is access to individual claim information; when individual claim evaluation expertise is available; and for periods where there is no reasonable expectation that additional claims will be reported.

(2) The results of the method are highly subject to the expertise and judgment of the examiner/auditor performing the claim evaluation. There is typically no fitting or testing of historical experience and no statistical support for the assumptions used in the claim evaluation.

(3) Claims that are subject to worsening of claimant condition, such as long-term workers compensation (or short-term benefits that are escalated to long-term), or liability claims where adverse facts may have yet to emerge, are difficult or impossible to quantify. A claims examiner/auditor estimate may have a tendency to underestimate the liability for such claims as the emergence of adverse facts might be difficult for a claims examiner/auditor to justify for any particular claim. Additional actuarial adjustments would be required to the extent that the examiner/auditor has omitted consideration of the potential for future adverse facts.

(4) Claims examiners/auditors may have a tendency to perform their evaluation on the basis of the estimated current value to dispose of the claim. Claims estimated on this basis tend to be underestimated since severity inflation through the time of final settlement is not considered. Additional actuarial adjustments would be required to the extent that the claims examiner/auditor has omitted consideration of severity inflation through final settlement.

(5) Even where there is reasonable expectation that all claims have been reported, there may be risk that additional claims may emerge due to unexpected new claims; reopened claims (e.g., for workers compensation); changes or broadening in interpretation of coverage; changes in classification of claims by period; or other unforeseen circumstances. Additional actuarial adjustments would be required to the extent that the examiner/auditor has omitted these considerations.

(6) Even where there is reasonable expectation that all claims have been reported, there is risk that the remaining settlement value of outstanding claims may be effectively less than

zero because of changes in classification of claims by period; salvage and subrogation recoveries; other recoveries on prior claims; or other unforeseen circumstances.

(7) Even where there is reasonable expectation that all claims have been reported, there is risk that the remaining settlement value of outstanding claims may be greater than the sum of the remaining policy limit amounts for each claim by period as a result of the emergence of additional claims; ALAE costs (if included in the reserve provision); changes or broadening in interpretation of coverage; bad faith claims; punitive damage awards; or other unforeseen circumstances.

7.3.3 Users

One of the key hurdles to overcome in using this approach is the need for experienced claims auditors. So, this method tends to be used the most often by those with access to claims auditors, which includes, insurance companies that work with multiple third-party administrators, consulting firms, and, occasionally, state insurance solvency regulators.

7.3.4 Summary

This method has the advantage of reflecting only the claims left open, even if the judgment estimate may sometimes be biased. It can certainly be used, though, in conjunction with tail factors developed from industry benchmark data. It is perhaps better thought of as a method for developing older years, than as a method for developing greener years that may have a different open claims pattern near the tail. It has its disadvantages in terms of the limits of what a claims auditor can reasonably ascertain. But, it is also fairly easy to explain to lay people.

7.4 Summary of Methods Based on Peculiarities of the Remaining Open Claims

These methods can produce significant improvements in estimates of the total costs of the oldest years, especially when only a few claims remain open in those years. But, the user is cautioned to avoid assuming that similar tail factors will be accurate for the less mature years.

8. OTHER METHODS

8.1 Introduction

There are several other methods discussed below that do not fall into any of the previous classes.

8.2 Restate Historical Experience Method

8.2.1 Description

When the historical reported losses are inconsistent (e.g., there has been a substantive change in the claim counting or case reserving philosophy) and/or incompatible with industry benchmark experience (e.g., the most recent case reserves are substantially lower than comparable industry case reserves), it may be useful to attempt to restate the historical experience using concepts from the judgment estimate method .

One possibility is to restate the entire reported loss history using claims evaluation expertise to estimate the case reserves of the outstanding reported losses as of each stage of development. After restatement of historical reported losses in this manner, the tail factor may be estimated using many of the methods described in this summary report. Indeed, once the historical reported losses have been restated on a consistent basis, all development factors may be estimated using traditional actuarial methods. This method shares several of the strengths and weaknesses of the judgment estimate method. However, this method has several serious additional weaknesses: (1) it is ordinarily extremely difficult to reconstruct the contemporaneous claim file information as of each previous historical development period; (2) in order to properly implement this method, the claims auditor must ignore claim developments that are known or knowable subsequent to each development period; and (3) in order to properly implement this method, the claims auditor must evaluate each previous open claim as if the evaluation were performed at a prior historical date corresponding to the development period.

Generally, a more practical approach is to use claims evaluation expertise to estimate the current value of all open claims only as discussed in the judgment estimate method and apply comparable industry tail development factors. If the current open claims are estimated at industry standard levels and the industry development factors are truly comparable, then this method is applicable for all periods rather than only the oldest periods where there is no reasonable expectation that additional claims will be reported.

8.2.2 Example

Consider the following cumulative paid loss triangle:

Cumulative Paid Loss Triangle $c_{Paid}(w,d)$

	12	24	36	48	60	72
1991	1,000	2,000	2,500	2,800	2,950	3,100
1992	1,100	2,400	3,000	3,500	3,900	
1993	1,300	2,500	3,000	3,400		
1994	1,200	2,300	3,100			
1995	1,400	2,800				
1996	1,490					

Remaining claims open as of 72 months for accident year 1991 and remaining claims open as of 60 months for accident year 1992 are evaluated at industry standard levels. A claims examiner estimates that the industry standard value of the six (6) accident year 1991 outstanding claims as 1,000 and the eleven (11) accident year 1992 outstanding claims as 1,400. An appropriate source of compatible industry-incurred development factors indicates that the 72-ultimate comparable industry incurred development factor is 1.100 and the 60-ultimate comparable industry incurred development factor is 1.150. Accordingly, the indicated accident year 1991 72–ultimate payment tail development factor is:

[(3,100 + 1,000)/(3,100)] x 1.100= 1.455.

Similarly, the indicated accident year 1992 60-ultimate payment tail development factor is

[(3,900 + 1,400)/(3,900)] x 1.150= 1.563.

A similar procedure could be adopted for each accident year.

The actuary considers whether the industry is truly reserving up to the levels of the claims examiner industry standard. If the actuary believes that the industry is not reserving up to the level of the claims examiner industry standard, then the actuary would increase the indicated tail development factors to reflect additional expected development.

8.2.2 Advantages and Disadvantages

Strengths of this method are:

(1) Estimates of ultimate losses may be improved by better claims settlement evaluation expertise at the industry standard.

(2) The method of adjustment is more readily understood by non-technical users of the resulting actuarial work product than highly theoretical models.

(3) The method relies upon industry development factors which are often compiled and

may be readily available.

Weaknesses of this method are:

(1) The method is only applicable where there is access to individual claim information and when individual claim evaluation expertise is available.

(2) The results of the method are highly subject to the expertise and judgment of the examiner/auditor performing the claim evaluation. Evaluation of claims at the industry standard is subjective. There is often no fitting or testing of historical experience and no statistical support for the assumptions used in the claim evaluation.

(3) Appropriate industry development factors may not be readily available. Selection of appropriate industry development factors is not always clear in consideration of policy limits, mix of business, reinsurance, deductibles, etc. There is often considerable judgment required to select appropriate industry development factors. It may be appropriate to use a weighted average of several industry development factors in order to improve the comparability of the development factors with the restated historical experience. The appropriate weighting scheme of industry development factors itself may also be subject to a high degree of judgment.

(4) Industry standard may be a higher value than the industry actuarial reserves. An adjustment (i.e., increase) to industry development factors may be required to reflect that the industry may actually reserve at values lower than industry standard levels.

8.2.3 Users

As with the judgment estimate method, one of the key hurdles to overcome in using this approach is the need for experienced claims auditors. So, this method tends to be used the most often by those with access to claims auditors, which includes insurance companies that work with multiple third-party administrators, consulting firms, and, occasionally, state insurance solvency regulators.

8.2.4 Summary

This subsection is a brief summary of the method and its utility. This method has the advantage of reflecting only the claims left open, even if the judgment estimate may sometimes be biased. Successful application of the method requires that the claims auditor accurately tracks the industry standard and that the industry development factors selected are appropriate for the line of business under consideration. Its disadvantages are the limits of the claims auditor's ability to ascertain industry standard and the uncertainty in the appropriate industry development factor to apply to develop the auditor's recast incurred loss to ultimate.

As with most methods, the uncertainty is greatest for the least mature years. On the other hand, the method is relatively easy to explain to non-technical users.

8.3 Mueller Incremental Tail Method

Named in recognition of the work done by Conrad Mueller, ACAS, the Mueller Incremental Tail (MIT) method was developed by Mueller internally at the SAIF Corporation.

8.3.1 Description

The MIT method is used in the Sherman-Diss model to calculate empirical 37 to 65 tail factors using paid incremental data on old accident years. See Section 8.5 of this paper for a synopsis of the Sherman-Diss paper. The method involves three stages:

1. Incremental age-to-age factors

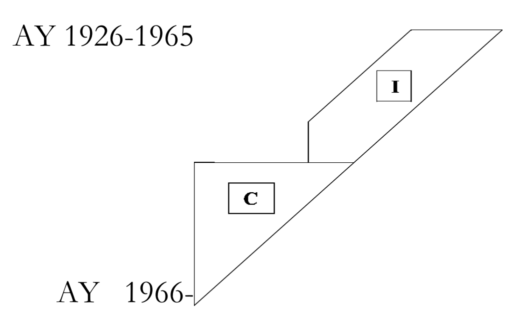
2. Anchored decay factors

3. Tail factors

8.3.2 Example

In the following example, table and figure numbers shown in parentheses refer to the original research paper by Sherman and Diss. Figure 8.3.2.1 provides a graphic summary of the portions of the incremental medical component of permanent disability claims (MPD) payments experience of the SAIF Corporation that are available. A complete triangle of MPD payments exists for AYs 1966-2002. This region is the triangle labeled "C" to designate that cumulative paid losses are available for all of these AYs. In addition, since calendar period 1985, incremental MPD payments have been captured for AYs 1926-1965 for development years 29 and higher. This region is the diagonally shaped area labeled "I" to designate that only incremental payments are available.

Figure 8.3.2.1 (Figure 2.1) Configuration of SAIF's MPD Paid Loss Data



Since paid MPD for AYs 1926-1965 has only been available for calendar periods since 1985, it was necessary to construct an actuarial method of estimating the tail factor based on decay ratios of incremental payments. This method is called the MIT method.

The MIT method was used to calculate empirical 37 to ultimate tail factors using the incremental data on old accident periods. The empirical data ended at 65 years of development, which, for purposes of this section, will be considered to be ultimate. The method is described in three stages mentioned earlier:

(1) Incremental age-to-age decay ratios
(2) Anchored decay factors
(3) Tail factors

$$f(d) = c_{\mathrm{Paid}}(w,d)/\mathrm{c}_{\mathrm{Paid}}(w,d\text{-}1) = [c_{\mathrm{Paid}}(w,d\text{-}1) + q_{\mathrm{Paid}}(w,d)]/c_{\mathrm{Paid}}(w,d\text{-}1) = 1 + q_{\mathrm{Paid}}(w,d)/\mathrm{c}_{\mathrm{Paid}}(w,d\text{-}1)$$

Then, $f(d)-1 = q_{Paid}(w,d)/c_{Paid}(w,d-1)$, which is equal to $v(d)$.

1. **Incremental age-to-age decay ratios.** The first step is to calculate incremental age to age decay ratios:

$$q_{Paid}(w,d+1)/q_{Paid}(w,d),\ q_{Paid}(w,d+2)/q_{Paid}(w,d+1),\ q_{Paid}(w,d+3)/q_{Paid}(w,d+2),$$

etc.

With the SAIF data, Sherman and Diss were are able to calculate ratios of incremental paid at age $d+1$ to incremental paid at age d, for d ranging from 29 to 65 years, using 20-year-weighted averages. Because of the sparseness of claims of this age, the empirical development ratios needed to be smoothed before they could be used. The smoothing was done using five-year centered moving averages.

2. **Anchored decay factors.** After calculating incremental age-to-age decay ratios, the factors are anchored to a base year and thereafter termed anchored age-to-age factors. In the illustration that follows, development year d is the anchor year.

$d_d = q_{Paid}(w,d)/q_{Paid}(w,d) = 1$, $d_{d+1} = q_{Paid}(w,d+1)/q_{Paid}(w,d)$,
$d_{d+2} = q_{Paid}(w,d+2)/q_{Paid}(w,d)$, … all relative to $q_{Paid}(w,d)$.

In general

$$q_{Paid}(w,d+r)/q_{Paid}(w,d) = q_{Paid}(w,d+1)/q_{Paid}(w,d) \times q_{Paid}(w,d+2)/q_{Paid}(w,d+1) \times \ldots \times q_{Paid}(w,d+r)/q_{Paid}(w,d+r-1).$$

The anchored decay factors are cumulative products of the age-to-age decay ratios and represent payments made in year $d+r$ relative to payments made in the anchor year d.

Table 8.3.2.2 shows the anchored decay factors for payments made in accident years of age

40, 45, 50, and 55 relative to payments made in an accident years of age 37 (our anchor year).

Table 8.3.2.2 (Table 2.3)

Indicated Decay Factors Relative to Anchor Year 37 Incremental Payments

Year of Development	Decay Factors
55	.962
50	1.880
45	1.724
40	1.211
Anchor Year 37	1.000

For example, payments made in development year 50 are, on average, almost double (88.0% greater) the payments made in development year 37.

Payments made in ages 38 to 65 relative to payments made in year 37 are obtained by summing the anchored decay factors from 38 to ultimate. The authors refer to these as anchored cumulative decay factors, D_d s, where

$D_{d+1} = q_{Paid}(w,d+1)/q_{Paid}(w,d) + q_{Paid}(w,d+2)/q_{Paid}(w,d) + ... = \sum d_i$ for $i = d+1$ to 65.

The sums of the decay factors are similar to tail factors, but instead of being relative to cumulative payments they are relative to the incremental payments made in the anchor year.

The process can be repeated using a different anchor year. In addition to anchor year 37, the calculations were also performed using anchor years 36, 35, 34 and 33. In each case, the payments from 38 to ultimate were compared to the payments made in the selected anchor year. Table 8.3.2.3 shows the cumulative decay factors for each of these anchor years:

Table 8.3.2.3 (Table 2.4)

Cumulative Decay Factors Relative to Incremental Payments During Different Anchor Years

Anchor Year	Cumulative Decay Factors
37	30.071
36	30.115
35	29.508
34	28.280
33	26.961

The cumulative decay factors can be interpreted as follows: Payments made from ages 38 to ultimate are 30.071 times the payments made in age 37. Similarly, payments made in ages 38 to ultimate are 30.115 times the payments made in age 36, etc.

3. **Tail Factors.** To convert these cumulative decay factors into tail factors, the authors make use of the selected cumulative loss development factors from the customary cumulative paid loss development triangle.

The Tail Factor from d to ultimate $=\left\{c_{Paid}(w,d)+\left[\sum_{d=1}^{65} q_{Paid}(w,d)\right]\right\}/c_{Paid}(w,d)$,

$$=1+\left[\sum_{d=1}^{65} q_{Paid}(w,d)\right]/c_{Paid}(w,d)$$

$$=1+q_{Paid}(w,d+1)/c_{Paid}(w,d)+q_{Paid}(w,d+2)/c_{Paid}(w,d)+\ldots$$

$$=1+\left[q_{Paid}(w,d)/c_{Paid}(w,d)\right]\times\left[q_{Paid}(w,d+1)/q_{Paid}(w,d)+q_{Paid}(w,d+2)/q_{Paid}(w,d)+\ldots\right]$$

.

But $q_{Paid}(w,d)/c_{Paid}(w,d)=\left[q_{Paid}(w,d)/c_{Paid}(w,d-1)\right]/\left[c_{Paid}(w,d)/c_{Paid}(w,d-1)\right]=$ $(f(d)-1)/f(d)$.

So the tail factor is $1+[(f(d)-1)/f(d)]\times D_{d+1}$ where $f(d)$ is the paid loss development factor for the *d*th year of development, and D_{d+1} is the cumulative decay factor for payments made during years ($d+1$) to ultimate relative to payments made in anchor year d.

In a similar way, an age-to-age loss development factor (less 1.0) extending beyond the cumulative triangle is

$$\left[d(d+1)-1\right]=[(f(d)-1)]\times d_{n+1}/f(d),$$

where d_{n+1} is the decay factor for payments made in year ($n+1$) relative to payments made in anchor year n.

This method is sensitive to f_n, the 37:36 paid loss development factor less 1. For this reason the analysis can be repeated using the 36, 35, 34 or 33 anchor years. Table 8.3.2.4 shows the 37 to 65 tail factor calculated using each of these anchor years.

Table 8.3.2.4 (Table 2.5)

37 to Ultimate MPD Tail Factors Based on Different Anchor Years

AnchorYear	37 to Ultimate MPD Tail Factors
37	1.964
36	1.808
35	1.496
34	1.439
33	1.369

Selected	1.581*

* Average excluding the high and low.

The empirically calculated 37 to ultimate MPD tail factors range from a low of 1.369 to a high of 1.964. The value is sensitive to relatively small changes either in incremental age-to-age factors in the tail or in the cumulative age-to-age factors at the end of the cumulative triangle.

8.3.3 Advantages and Disadvantages

The Mueller Incremental Tail method can be applied satisfactorily to workers compensation total medical loss experience for development years 20 and higher since virtually all medical payments are MPD payments at such maturities. A disadvantage is that it may be sensitive to the anchor year. However, the process may be repeated with various anchor years to reduce the high volatility of the tail data. This method may not be predictive if the payment patterns are changing over time but this is a disadvantage of any tail factor methodology.

8.3.4 Users

The method is utilized by the SAIF Corporation, Oregon's State Fund.

8.3.5 Summary

Workers compensation tail data is often difficult to obtain and may be of dubious quality. The Mueller method is based on decay ratios of incremental paid data and may be used to anchor a tail factor at 20 to 35 years of maturity.

8.4 Corro's Method

Daniel R. Corro published this method in his 2003 research paper titled "Annuity Densities with Application to Tail Development."

8.4.1 Description

The paper considers the task of modeling "pension" claims whose durations may vary, but whose payment pattern is uniform and flat. The aggregate payout pattern is derived from the duration density and can be applied to calculating tail development factors.

For consistency, the notation in the following subsections differs slightly from the notation of Section 1.7. The tail factor notation is the same as in Corro's original research paper.

The following assumptions are made. All payments on all claims are of the same amount. Payments are made periodically at a common uniform time interval immediately following a common time of loss, $t=0$, to claim closure. For every claim of duration x, the model assumes a continuous and constant payment rate of $1 until the claim closes. For pension claims, as described here, the entire payment schedule of a claim is completely determined by the claim duration. With the assumption that for any time t, $0<t<b$, all claims with duration t have the same predetermined and differentiable payment pattern.

Let $S(t)$ denote a survival function on the time interval $(0,b)$. Regard $S(t)$ as a distribution of closure times and let $F(t)=1-S(t)$ be the corresponding cumulative distribution function [CDF]. In effect, all claims are assumed to close on or before time *b*.

We are interested in a related CDF, denoted by $\tilde{F}(t)$ to emphasize its relation with $F(t)$, which models the paid loss development as a function of time. More precisely, $\tilde{F}(t)$ is the proportion of total loss paid by time t, i.e., the proportion paid out during $(0,t)$ (without any discount adjustment). $\tilde{F}(t)$ is the reciprocal of the paid to ultimate loss development factor and $\tilde{F}(t)$ is referred to as the paid loss development divisor [PLDD].

Consider the case when aggregate paid losses are followed over a series of N time units with $N<b$. The usual paid loss development patterns built from these N evaluations will not account for the "tail paid loss development" beyond the final evaluation at time $t=N$. With this notation, observe that this tail development factor is just $\lambda=\tilde{F}(N)^{-1}$.

It is reasonable to assume that workers compensation payments beyond some valuation, say after 10 periods, will be primarily made on pension-like claims. A model suited to such pension claims may be helpful in projecting the full payout pattern beyond 10 periods. Suppose you have a collection of PLDDs that covers the portion of the loss "portfolio" that is expected to develop beyond 10 periods. That is, for each type of claim you have a PLDD that is appropriate, at least over the time frame beyond 10 periods. The paper illustrates how to translate the mix of claims in the loss portfolio into a mixed distribution of those PLDDs. That mixed distribution then provides an estimated tail factor.

In the workers compensation work that motivated this paper, the author seeks to find a 19th to ultimate paid loss development factor. Consider a weighted sum (mixture) of PLDDs of the form

$$w\tilde{F}_{\alpha}(b_1;t)+(1-w)\tilde{F}_{\beta}(b_2;t) \text{ for } 0<w<1. \tag{8.1}$$

The assumption here is that all claims close after $Max(b_1,b_2)$ periods; one part of the loss portfolio closes by time $t=b_1$ and the complement by $t=b_2$.

Empirical loss development factor data is used to fit a non-linear model in which the mixing weight variable w is a parameter. When these simple functions are used with b_1, b_2 as selected constants, it is straightforward to set up the calculation so as to assure a closed form solution for the value of w that gives the best fit to the data.

8.4.2 Example

See appendix.

8.4.3 Advantages and Disadvantages

Advantages of the method include that it is a nonsubjective, nonlinear "fit" of the tail data, which has a closed-form solution. The subjectivity of curve fitting is removed, at least to some extent, since the same mathematical assumptions are made for any tail data to which the method is applied. Tail factors calculated empirically are often significantly greater than those derived from extrapolation techniques. The greater weight given to tail data in this method reduces the likelihood of underestimation of reserves. The added complexity of the nonlinear fit involves no added work on the part of the user. The sum of squared difference minimization is easily calculated and is a well-known procedure. Another advantage is that the procedure addresses the nature of workers compensation tail data, comprised largely of permanent disability claims.

A disadvantage of the method is that the mathematical notation may not be readily understood.

8.4.4 Users

As a newly developed method, there may be few users of the method at this time.

8.4.5 Summary

This paper considers the task of modeling "pension" claims whose durations may vary, but whose payment pattern is uniform and flat. The authors derive the aggregate payout pattern from the duration density, provide examples to show how this idea can be applied to calculating tail development factors and discuss the process.

8.5 Sherman-Diss Method

The workers compensation tail largely consists of the medical component of permanent disability claims (MPD). Yet the nature of MPD payments is not widely understood and is counter to that presumed in common actuarial models. In the Sherman-Diss paper, it is shown that common actuarial methods tend to underestimate the true MPD loss reserve. This is a serious concern because MPD loss reserves make up the bulk of total workers compensation loss reserves for all but the most recent accident periods. The authors state that the need to develop and apply new methods that directly reflect the characteristics of MPD payments is substantial.

8.5.1 Description

The Sherman-Diss paper presents an analysis of medical payments based on 160,000 permanently disabled claimants for accident periods 1926-2002, and a method utilizing incremental payment data prior to the standard triangle to extend development factors beyond the end of the triangle.

Presented is an analysis of the extensive paid loss development database of the SAIF Corporation, Oregon's state fund, extending out to 77 periods of development, separately for medical and indemnity, and separately by injury type. Medical paid loss development factors compiled by the California Workers Compensation Insurance Rating Bureau (WCIRB) and the medical paid loss history of the Washington Department of Labor and Industries (WA LNI) are presented as additional support.

Ordinarily, it would be expected that paid loss development factors for subsequent development periods would slowly decline below the last factor as a continuation of the pattern of slowly decreasing factors exhibited, for example, during development periods 10 through 15. Since common actuarial methods assume that the pattern of declining factors for these development periods will continue in the future, the projected paid loss development factors fall increasingly below the actual historical factors. This pattern of divergence continues during development periods 27 through 37, as shown in Table 8.5.1.1. Table and figure numbers shown in parentheses throughout this section refer to the original research paper.

Table 8.5.1.1 (Table 1.3) A Comparison of Historical MPD Paid Loss Development Factors with Projections Based on Development Periods 10 through 15

	Development Period										
	27	28	29	30	31	32	33	34	35	37	38
Historical	1.020	1.023	1.027	1.026	1.022	1.018	1.015	1.017	1.018	1.029	1.033
Projections Based on Development Periods 10 – 15											
Linear Decay	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Exp. Decay	1.004	1.004	1.003	1.003	1.003	1.003	1.003	1.002	1.002	1.002	1.002
Inverse Power	1.006	1.005	1.005	1.005	1.005	1.004	1.004	1.004	1.004	1.004	1.003

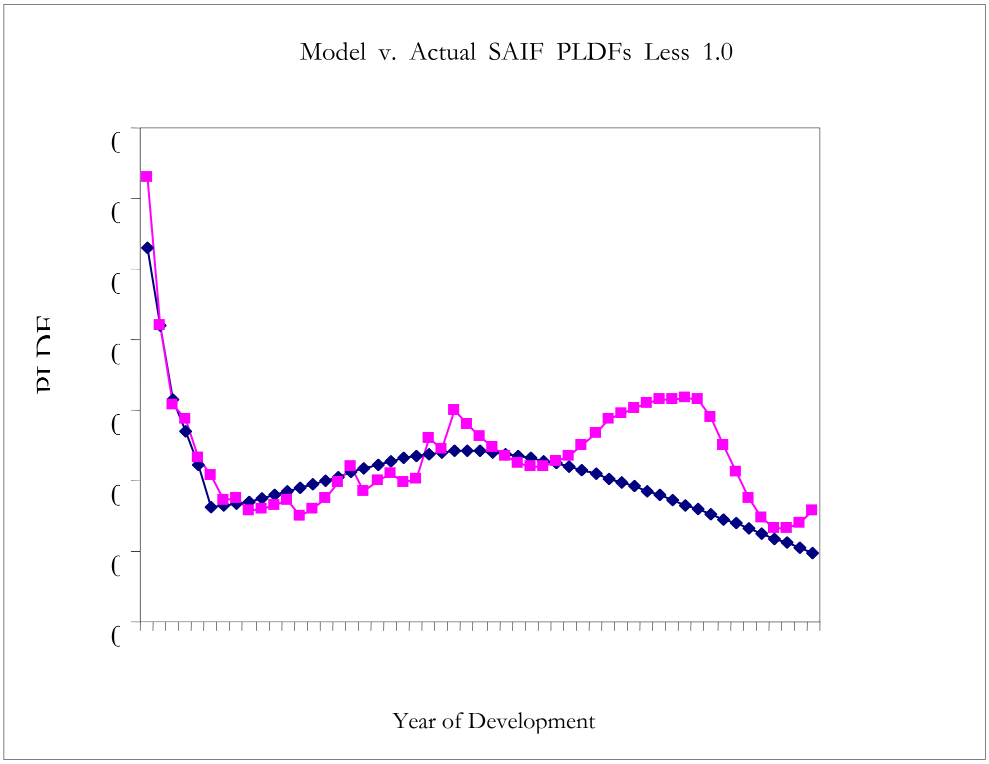
Paid loss development factors for MPD are not monotonically decreasing. Because of this seemingly anomalous behavior, estimates of the MPD tail by common actuarial methods could be seriously understated. This potentially surprising behavior is due to the fact that medical inflation rates are expected to be greater than the rate of closure of permanent disability claims due to death during these periods of development. For the most mature periods of development, the increasing force of mortality overtakes the effects of medical inflation and causes a slow reduction in incremental payments. That rate of reduction is surprisingly small.

This paper presents a reserving model that largely explains the seemingly anomalous behavior of increasing paid loss development factors at "mature" development years. The Sherman-Diss model explicitly accounts for the separate effects of inflation and mortality on paid MPD during all periods of development. This is done by directly incorporating recent mortality rates into an incremental paid per prior open loss reserving method. It will be referred to as the static mortality model.

A second reserving model is presented that explicitly accounts for the compounding effects of downward trends in future mortality rates and persistently high rates of future medical inflation. It will be referred to as the trended mortality model.

In Figure 8.5.1.1, the paid loss development factors indicated by the static mortality model are compared with SAIF's empirical paid loss development factors.

Figure 8.5.1.1 (Figure 1.1)



8.5.2

Organization of the Sherman-Diss Paper

This paper is divided into ten sections:

1. Summary and Introduction
2. Using Prior Incremental Paid Data to Extend the PLDF Triangle
3. Incorporating the Static Mortality Model into the Incremental Paid to Prior Open Method
4. Mortality Improvement
5. The Trended Mortality Model
6. A Comparison of Indicated Tail Factors
7. Sensitivity Considerations
8. Estimating the Expected Value of MPD Reserves
9. Estimating the Variability of the MPD Reserve with a Markov Chain Simulation
10. Concluding Remarks

The paper also includes five appendices:

A. The Mueller Incremental Tail (MIT) Method
B. Historical PLDFs for All Other workers compensation
C. Incorporating the Static Mortality Model into the Incremental Paid to Prior Open

Method

D. Incorporating the Trended Mortality Model into the Incremental Paid to Prior Open Method

E. Quantifying the Elder Care Cost Bulge

8.5.3 Example

The authors of the research paper believe that the most appropriate approach to estimating gross workers compensation loss reserves is to separately evaluate MPD loss reserves by one (or more) of the methods presented in their paper. Lacking separate MPD loss experience, the Static Mortality and Trended Mortality models, and the Mueller Incremental Tail method can be applied satisfactorily to total medical loss experience for DYs 20 and higher since virtually all medical payments are MPD payments at such maturities. Examples of the above three methods are given in Section 6.2, Section 6.3, and Section 8.3, of this paper, respectively.

8.5.3.1 A Comparison of Indicated Tail Factors

Table 8.5.2.1 provides a comparison of the MPD tails indicated by SAIF's own loss experience with those indicated by the static and trended mortality methods.

Table 8.5.2.1 (Table 6.1) A Comparison of Indicated MPD Tail Factors

Maturity (Years)	Based on SAIF's Experience	Based on Static Mortality Model	Based on Trended Mortality Model
10	2.469	2.684	3.025
15	2.328	2.469	2.783
25	2.054	2.019	2.271
35	1.680	1.594	1.776

8.5.3.2 Estimating the Expected Value of MPD Reserves

Consider a hypothetical permanent disability male claimant injured at age 35.9, and expected to live another 40 periods. Two different methods of estimating the medical case reserve for this claimant at the end of the first period of development are common. They are:

1. **First Method:** *Zero Inflation Case Reserve Based on Projected Payments Through Expected Period of Death*. Estimated annual medical expenses of $5,000 per period (during the first full period of development) are multiplied by the life expectancy of 40 periods to obtain a case reserve of $200,000.

2. **Second Method:** *9% Inflation Case Reserve Based on Projected Payments Through Expected Period of Death*. Escalating medical expenses are cumulated up through age 75, yielding a total incurred of $1,689,000. (Other rates of inflation may be considered appropriate.)

Two additional methods may also be applied. Each of these produces much higher, and more accurate, estimates of the expected value of the case reserve:

3. **Third Method:** *Expected Total Payout Weighted by Probability of Occurrence Over Scenarios of All Possible Periods of Death*. This method yields an expected reserve of $2,879,000.

4. **Fourth Method:** *Expected Value of Trials from a Markov Chain Simulation*. This method yields an expected reserve of $2,854,000.

8.5.3.3 Estimating the Variability of the MPD Reserve with a Markov Chain Simulation

The size of loss distribution for the medical component of a single permanent disability claim is far more skewed to the right than can be modeled by distributions commonly used by actuaries. In attempting to find a distribution to produce a reasonable fit, the authors found it necessary to first transform the ultimate cost amounts by taking the natural log of the natural log of the natural log and then taking the nth root—before a common distribution could be found. Taking the fifth root of the triple natural log appears to produce a distribution of ultimate costs that conforms well with an extreme value distribution. The fact that such intense transformations were needed suggests that a totally different approach than fitting commonly used distributions should be used.

Simulating the variability of the MPD reserve for unreported claims is naturally more complicated. First, the total number of IBNR claims should be represented by a Poisson (or similar) distribution. Then census data of the age at injury of recent claimants can be used to randomly generate these ages for unreported claimants. Then, future payments for each unreported claimant can be simulated. The degree of variability of the MPD reserve for unreported claimants is exceptionally high—because some of those claimants may have been quite young when injured, and the total expected future payment for workers injured at a young age is dramatically higher than for those injured at an older age. Estimates also vary dramatically according to the gender and age of each claimant at the time of the analysis. This suggests that the variability of the total MPD reserve can best be modeled by simulating the variability of the future payout for each claim separately.

8.5.4 Advantages and Disadvantages

The methods presented in the Sherman-Diss paper were tested against actual historical data and provide a reasonable estimate of future loss development extending out to 85 years of development. Such development is possible; a worker could be injured at age 16 and live to be over 100. No other method in the actuarial literature has been successful in doing so. One disadvantage is that total medical loss experience for development years 20 and higher is needed to successfully implement the methods. Such data may be difficult for a user to obtain. Another disadvantage is that medical and mortality rates may be difficult to obtain or estimate. A sub-standard mortality table may be necessary.

8.5.5 Users

The method is currently utilized by the SAIF Corporation, Oregon's State Fund.

8.5.6 Summary

The Sherman-Diss paper presents an analysis of medical payments based on 160,000 permanently disabled claimants—for accident years 1926-2002, and a method utilizing incremental payment data prior to the standard triangle to extend development factors beyond the end of the triangle, up to 85 years of development.

9. COMPARISON OF SELECTED RESULTS

9.1 Discussion

The working party obtained data from a number of different sources with the goal of applying the methods presented in order to (1) provide a comparison of results, and (2) enhance the discussion of each the method's value and validity under various circumstances. To the extent possible, we used a common data set to illustrate the various methods and also used this data in the companion Excel workbook (which illustrates, where possible, many of the examples shown in the appendix). One exception to this approach involves methods previously detailed in CAS papers. In these cases, we generally used the data as originally presented in the paper.

In general, the methods discussed may require different types of data – such as different historical periods, differing granularity of data (i.e., separate medical versus indemnity losses), incremental versus cumulative, absence of incurred data and completeness of data, as examples. As a result, it was not always meaningful to use the same data set for each method.

Even using the same data set, different methods produce a range of results. In addition to differences caused by the dynamics of the methods themselves, individual judgments and selections may also contribute to differences in results. For example, methods that require an assumption of link ratios or ratios of incurred loss to paid loss for each evaluation point may require actuarial judgment of the most appropriate "average" to differ between methods. To the extent possible, we have held actuarial judgment and assumptions consistent among the various methods for testing and comparison purposes. The actuary should be aware that differences in indicated tail factors can vary both as a result of the method used as well as due to the underlying assumptions which rely on actuarial judgment or selection.

The table below shows the indicated ten-period to ultimate (120 months) paid loss development tail for the methods using the common 10 year loss history shown in the appendix.

Method	Indicated Paid Tail
Generalized Bondy Method	1.025
Fully Generalized Bondy Method	1.043
Sherman-Boor	1.096
Exponential Fit	
Using all Points	1.032
Using last 6 Points	1.044
McClenahan's Method	1.055
McClenahan's Adjusted Method	1.040
Sherman's Method	1.137
Sherman's Method with Lag Adjustment	1.135
Pipia's Method (Weibull Fit) Using all Factors	
Using all historical factors	1.098
Using selected development factors	1.049

As is evidenced by the range of indicated tail factors, it is important for the actuary to understand the underlying exposure being evaluated and to use judgment in determining the most appropriate method(s) for each situation. The coverage being evaluated, the layer (i.e., excess versus primary), and claims handling practices are examples of items that should be considered in selecting the appropriate methodology for calculating a tail factor. As discussed in the sections above, each method has its own specific advantages and disadvantages and therefore, some advice was provided on whether each specific method is optimum in the reserving context of specific situations; this is intended to be helpful when selecting a method to estimate tail loss development (i.e., a method to compute a tail factor).

9.2 Future Research

The Working Party believes that this is an area of future research using simulated data wherein the ultimate values of the simulated data are also known. Thus testing of the various methods would provide a clearer sense of which methods work best based on the different types of data aberrations built into the simulations. One key point is to create as many varying simulations as possible to properly test all methods.

10. CONCLUSIONS

The Tail Factor Working Party undertook an exhaustive study of all the methods for computing tail factors that are believed to be available to actuaries. While it is possible that some methods in use were not identified by the working party, this document is believed to present the vast majority of the available methods. As the document shows, each method has its own specific advantages and disadvantages.

It should also be noted that many methods were identified that had only a handful of current users. Therefore this document can serve an important function by introducing these new approaches to a broader actuarial audience.

Again, as this document is primarily a survey paper, listing and describing all or most of the methods in existence, it is difficult to draw conclusions on tail factor methods in general. The most appropriate approach for a given analysis is likely to depend on the circumstances of the analysis. As stated above, it is certainly reasonable to conclude that there are more methods available to actuaries than are in general use. Hopefully this document will act to expand the repertoire of tail factor methods in the resources of the typical actuary.

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Supplementary Material

A companion Excel file with a sample of each of the methods in this paper is on the CAS website at http://www.casact.org/pubs/forum/13fforumpt/.

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Biographies of Working Party Members

Steven C. Herman, co-chair, is owner and consulting actuary of Asset Discovery Associates LLC. His specialty is commutation analysis including creditor filing support for foreign schemes of arrangement. Prior to the establishment of Asset Discovery Associates LLC, Steve was Executive Vice President & Actuary for Global Resource Managers, a business unit of CNA, where he was a member of the senior team managing the run-off of numerous discontinued operations. He has a B.S. degree in Computer Science from the City College of New York School of Engineering. He is a Fellow of the Casualty Actuarial Society and a Member of the American Academy of Actuaries.

Mark R. Shapland, co-chair, is a Consulting Actuary in Milliman, Inc.'s Atlanta office. He is responsible for various reserving projects including reserve variability modeling and modeling of Asbestos Liabilities. He has a B.S. degree in Actuarial Science from the University of Nebraska-Lincoln. He is a Fellow of the Casualty Actuarial Society, an Associate of the Society of Actuaries and a Member of the American Academy of Actuaries. He is a former Chair of the Dynamic Risk Modeling Committee of the CAS and a former chair of the CAS Committee on Reserves. He was also a member of the CAS Working Party on Quantifying Variability in Reserve Estimates and the CAS Working Party on the Dynamic Risk Modeling Handbook.

Mohammed ("Mo") Q. Ashab is a Vice President of Signal Mutual Indemnity Association where he presents his actuarial analyses and risk management reports to the Board of Directors. Previous to Signal, Mo was a Director at Standard & Poor's where he was responsible for the credit ratings of several large multi-line global P&C insurance companies. Previous to Standard & Poor's, Mo was a vice president and risk manager at GE Capital where he had deal responsibility for the structuring and execution of securitization transactions. Mo began his career as a practicing actuary and held positions of increasing responsibility at several large firms including Guy Carpenter, Metropolitan Property & Casualty, Milliman USA and John Hancock Mutual Life. Mo graduated from the UMass Boston with a B.S. in Pure and Applied Mathematics and Computer Science. Mo is an Associate of the Casualty Actuarial Society and Society of Actuaries and a member of the American Academy of Actuaries.

Joseph A. Boor is an Actuary in the State of Florida Office of Insurance Regulation. His responsibilities include review of rate filings for medical malpractice and rate filings related to insurance products by dealers, and preparation of the Office's annual medical malpractice report. He has been a Fellow of the CAS since 1988 and has contributed to the *Proceedings of the Casualty Actuarial Society*, the CAS *Forum* and the CAS Examination Syllabus, including a relevant paper on tail factors. He also speaks regularly at CAS meetings. He has a Bachelor's degree in Mathematics from Southern Illinois University at Carbondale, a Masters degree in Financial Mathematics from Florida State University and a Doctoral degree in Mathematics (specializing in Actuarial Science) from Florida State University.

Anthony R. Bustillo is a Manager in KPMG's Property/Casualty Actuarial Services Practice. He has provided actuarial support for a number of different clients related to their property and casualty insurance programs, including insurance and reinsurance companies, captives, risk retention groups, individual and group self-insureds, regulatory agencies, and public entities. He is a Fellow of the Casualty Actuarial Society and Member of the American Academy of Actuaries. He is an active member of various actuarial society committees and has created and participated in numerous insurance and trade related seminars. Mr. Bustillo graduated from the University of Michigan with a Bachelor of Science degree in Mathematics.

David A. Clark is an Actuarial Research Manager with Esurance in San Francisco. He received his FCAS in 2004. Previously, he was an Associate Professor of Mathematics at Brigham Young University.

Robert J. Foskey is Senior Vice President & Chief Actuary of The Oil Group of Companies in Bermuda. He is responsible for overseeing all property and casualty actuarial functions for each of The Oil Group's two subsidiaries. Rob graduated from Florida State University with a Bachelors of Science degree in Mathematics - Actuarial Science, is a Fellow of the Casualty Actuarial Society and Member of the American Academy of Actuaries. He is an active volunteer in the Casualty Actuarial Society including serving on the Exam 9 Syllabus Committee as Senior Part Specialist and as a member of the Exam 9 Examination Committee

Aaron Halpert is an independent consulting actuary after recently retiring from KPMG. He joined the firm in 1984, and was a Principal in KPMG's U.S. Property/Casualty Actuarial Services practice. During his tenure with the firm, he Chaired KPMG's Global Actuarial Group. He has over 35 years of professional experience in the insurance industry, eight with a major insurance service organization, and 27+ as a consulting actuary. Aaron serves on the Casualty Actuarial Society (CAS) Strategic Planning Committee, and Chairs the CAS Risk Management Committee.

Sejal Haria is a Senior Consultant in the actuarial and insurance management solutions team at PricewaterhouseCoopers. She has advised a wide range of clients ranging from worldwide reinsurers to composites writing commercial and personal lines business. Major assignments include the due diligence for a Lloyds' motor player and the development of pricing parameters for U.K. liability business. She is a member of the Institute of Actuaries and sits on the working party "Application of Strategic Theory to the General Insurance Markets." She is currently heading up an internal project looking at reserve variability and is a product champion for the reinsurance and alternative risk transfer solutions team at PricewaterhouseCoopers. She holds an MA in Economics from Cambridge University.

Bertram A. Horowitz is President of Bertram Horowitz, Inc. Actuarial and Risk Consultants which provides property/casualty and title insurance actuarial services to regulators, insurers and other clients. He is a Fellow of the Casualty Actuarial Society and a Member of the American Academy of Actuaries. He has been active in the development of actuarial principles and standards, has served on the CAS Committee on Reserves and is currently a member of the CAS Working Party on Bornhuetter-Ferguson-Initial Expected Losses. Mr. Horowitz is the former Special Deputy Superintendent and Financial Actuary of the New York State Insurance Department (now the New York State Department of Financial Services). He has a B.S. in Applied Mathematics from the State University of New York at Stony Brook and a M.S. in Mathematics from Brown University.

Gloria A. Huberman is the Supervising Casualty Actuary in the Property Bureau - Financial Division of the New York State Insurance Department. She is responsible for assessing the reserve adequacy of New York licensed companies. She has been an Associate of the CAS since 1995. She has a B.S. degree in Accounting and Psychology from Brooklyn College and an M.S. degree in Quantitative Analysis from New York University.

Richard Kollmar is an Alternative Collections Director for CNA, located in Cranbury, NJ. He is responsible for reserving and modeling projects in support of the running off of various discontinued operations. He has a B.A. degree in Mathematics from Clark University. He is an Associate of the Casualty Actuarial Society and a Member of the American Academy of Actuaries.

Joshua Merck is a Manager with Deloitte Consulting's Human Capital practice in McLean, Virginia. His experience includes loss reserve analysis for various insurance coverages, such as workers compensation, medical malpractice, general liability, auto liability, property liability, and asbestos liability as well as several specialized projects while with Deloitte. He has a B.S. degree in Mathematics from Marymount University.

Rasa V. McKean is the Manager of Actuarial Services at the New York Compensation Insurance Rating Board in New York City. She has a Masters degree in Mathematics from the Courant Institute of Mathematical Sciences, New York University, a Masters degree in Quantitative Finance (Financial Engineering) from CIMS, NYU, and a Bachelors degree in Chemistry from the University of Chicago. She is an Associate of the CAS and a Member of the American Academy of Actuaries. She currently serves as the Chair of the CAS Valuations, Finance, and Investments Committee.

Michael R. Murray is Assistant Vice President for Financial Analysis at ISO. He is also a current member of the advisory board of the Howe School Alliance for Technology Management, as well as a former member of the board and past president of the Society of Insurance Research, having earned a Masters degree in Economics from the University of Chicago. His responsibilities at ISO include analyzing industry loss reserve adequacy, econometric modeling of claim frequency and severity, and generating quarterly reports on the industry's financial performance. His publications include numerous ISO special studies, journal articles, and Catastrophe Insurance: Consumer Demand, Markets and Regulation (Kluwer Academic Publishers, 2003).

Bernard A. Pelletier is an FCAS working in the National Accounts Department of St. Paul Travelers. He establishes prices and reserves for this book of business. The large amount of excess Workers Compensation and General Liability have afforded him ample opportunity to work on the problem of tail estimation.

Susan R. Pino is a Director in Deloitte's Human Capital practice in McLean, Virginia. She is an Associate of the Casualty Actuarial Society, a member of the American Academy of Actuaries and an Associate in Risk Management and is designated as Firm National Insurance Industry Specialist. Ms. Pino leads teams delivering strategic financial and technical projects for both federal and commercial clients. Her experience includes developing and designing pricing models, forecasting, valuation, program analysis and strategic planning. Ms.

Pino has served on several external professional committees including the Joint Casualty Loss Reserve Seminar Planning Committee and the CAS Professional Education Policy Committee. She is currently a member of the CAS Committee on Reserves.

Anthony J. Pipia, ACAS, MAAA, FCA, is the owner and consulting actuary with Actuaries, Inc. His company provides consulting services for commercial and personal lines of business related to pricing, development of customized individual risk rating plans, and loss reserving for small- and medium-sized companies. Actuaries, Inc. also provides services to regulators related to Financial Examinations of insurance companies, captive insurers, and self-insurance funds. Mr. Pipia has an extensive background in all aspects of workers compensation insurance developed in part through his tenure at NCCI and subsequent experience with consulting firms that serviced workers compensation self-insurance funds.

F. Douglas Ryan is Chief Actuary for Key Risk Insurance Company, a W.R Berkley Company, in Greensboro, NC. Key Risk underwrites Workers Compensation insurance and provides TPA services for clients in the southeast U.S. Doug earned a BA in Mathematics from the University of North Carolina at Asheville and a MS in Applied Mathematics from North Carolina State University. Doug is currently serving on the CAS Committee on the Theory of Risk, the Open Source Software Committee and was the former chair of the Joint Committee on Academic Relations.

Scott G. Sobel joined EagleEye Analytics in October 2007. Prior to joining EagleEye, Scott worked at the National Council on Compensation Insurance, Bankers Insurance Group, and the Florida Farm Bureau. Scott obtained his Fellowship in the Casualty Actuarial Society in 2003, is a Member of the American Academy of Actuaries, and has 23 years of experience in industry research, ratemaking, and predictive modeling for personal and commercial lines including workers compensation. He is currently participating in the CAS R Working Party (2010), and has participated in the CAS Ratemaking and Product Management Seminar Planning Committee (2008-2009), CAS Research Paper Classifier Working Party (2005), CAS Tail Factors Working Party (2004), and CAS Executive Level Decision Making Using Dynamic Risk Modeling Working Party (2004). He currently serves as a CAS University Liaisons Committee (1999–present), and served as President of the Casualty Actuaries of the Southeast (2009) and VP-Admin (2005-2007). He holds a BS in Statistics with Minors in Actuarial Science, Mathematics, and Business from the University of Florida.

APPENDIX A – Alternative Organization of the Methods

Many methods in this report could conceivably be placed in different categories than the ones the working party assigned them in this document. We have listed some alternate groupings below (along with each method's reference section within this document).

A.1 Bondy-Type & Decay Methods

- Bondy methods (2.1.1-2.1.4)
- Exponential decay method (5.2)
- McClenahan's loss model (5.3)
- Skurnick method (5.4)
- Mueller Incremental tail method (8.3)

In reviewing the relationships between these distinct, but related methods, the Working Party has the following comments about the underlying decay concept and how it weaves through the methods. Of note, other methods use similar decay concepts, but may not show an explicit year-to-year decay.

Bondy methods decay the last estimated development factor over time. This is in accordance with a half-life type function where the rate of decay is assumed to be constant over time. The most common form of this method assumes a decay rate of 0.5 (i.e., each successive factor is the square root of the previous), which generates a result where the tail factor is equal to the last estimated development factor.

The physical interpretation is that the claims are being settled at a rate proportional to the current outstanding claims. This is probably not an accurate model of how claims departments work in practice but does have the benefit of generating a smooth function.

Other variations of this method include decaying with a constant number between 0 and 1. Certain lines of business are expected to exhibit thicker tails.

The exponential decay method is a way of obtaining an appropriate factor using curve-fitting techniques. It is described more fully in section 5.1.1.

The McClenahan and Skurnick methods are variations on this basic concept. With the Skurnick method the decay rate is allowed to vary by accident period.

Similar drawbacks apply, to a varying extent, to all these methods. These include:

- They are generally not applicable to lines with negative development between evaluation periods without additional adjustment – i.e., they will generally fail on incurred data.

- Exponential decay assumes a monotonically decreasing function, therefore these methods do not accommodate increases in incremental losses from one period to the next ("hump" shaped patterns).
- Exponential decay at an indicated rate developed from the observed data can produce a relatively faster development than other models for certain long tail liability lines.
- A sub-optimal fit will be obtained for lines with variable decay rates across evaluation periods such as workers compensation.

The Mueller Incremental Decay Method and the Generalized Bondy Method tackle some of the first couple of points above by considering a variety of decay factors based on differing anchor periods and estimating tail factors. It is to be noted however that this method is relatively sensitive to the choice of anchor periods and small changes in the incremental age-to-age factors.

A.2 Algebraic Methods that Focus on Relationships between Paid and Incurred

- Equalizing Paid and Incurred Development (3.2)
- Sherman-Boor Method (3.3)
- NCCI Method (3.4)
- Static Mortality Method (6.2)
- Trended Mortality Method (6.3)
- Judgment Estimate Method (7.3)

This batch of methods considers the information available from the case handlers estimates of outstanding claim reserves in estimating a tail factor for the paid claims data.

Assumptions:

- The case reserves for the final year are a true reflection of the reserves required.
- Settlement and reporting patterns are unchanged over time and claims department reserving is similar over time.
- No future pure IBNR claims will materialize for the benchmark year.

The static and trended mortality methods examine the incremental paid per prior open to estimate the paid tail going forward. The number of open claims in any period is determined using mortality tables. These two methods have been applied in practice only to the medical component of permanent disability claims. Other algebraic methods may use alternate projection techniques to estimate the number of open claims.

By their nature, algebraic methods that focus on the paid to incurred loss amounts cannot be used to estimate an incurred tail. However, once a payment stream is calculated by means of the static or trended mortality methods, expected values of case reserves may be estimated for the same payment stream. The Sherman-Diss model of Section 8.5 describes the procedure.

A.3 Methods Based on Benchmark Data

- Benchmark Data Based Methods (4.2-4.6)
- Restate historical experience (8.2)

Benchmark methods are used when the data/experience of the book is not robust. This could be due to a number of reasons including data scarcity, change in the mix of business over time or where the historical development has been distorted by changes in settlement/reporting or claims estimation practices.

In addition these methods are often used as fall-back to test the reasonability of other approaches.

The major disadvantage of this approach is that appropriate industry development factors are not always available. In addition the performance of the book may be faster/slower than the industry average; for this reason it is often instructive to compare the actual historical development to that indicated by the benchmark data and adjust as required.

A.4 Stochastic and Curve-Fitting Methods

- Exponential decay method (5.2)
- McClenahan's loss model (5.3)
- Skurnick's method (5.4)
- Sherman's method (5.5)
- Pipia's method (5.6)
- England and Verrall (5.7)

This selection of methods aims to fit curves to the data and extrapolate an appropriate tail factor. The process is similar and involves four stages:

(a) specification of the functional form (this is normally defined by the method)

(b) optimizing function and assessment of goodness of fit

(c) estimation of parameters using curve-fitting techniques

(d) reading off the curve to develop an implied tail factor.

Most of the curves that tend to be used are exponential/logs based and are generally monotonically decreasing. As such they do not allow for "humps" or negative developments in the data. Specific features like these, or even structural breaks in the development, are smoothed out as part of the fitting process; these curves do not capture these phenomena even if they are a consequence of a true underlying process rather than just as a result of random data volatility.

The Sherman-Diss Method of Section 8.5 allows for breaks in structural development. In fact, the static and trended mortality methods of the Sherman-Diss model bear much resemblance to the classic structural methods developed by Fisher/Lange and Adler/Kline.

The England-Verrall Method allows for humps and negative development by the stochastic nature of the method although the development may also be judgmentally smoothed. Stochastic methods are an enhancement of traditional methods in this respect. The England-Verrall Method simulates paid claim amounts by stochastic means. Traditional chain ladder reserving techniques may then be applied to the triangle of simulated claims payments.

The curve-fitting methods do have the advantage that they tend to consider the entire loss development, rather than focusing on the northwest corner of the triangle, where arguably there is the most volatility.

A.5 Methods Based on Future Remaining Open Counts

- Static mortality method: incremental paid per open count (6.2)
- Trended mortality method (6.3)

The static and trended mortality methods examine the incremental paid per prior open to estimate the paid tail going forward. The number of open claims in any period is determined using mortality tables. These two methods have been applied in practice only to the medical component of permanent disability claims. Other algebraic methods may use alternate projection techniques to estimate the number of open claims. Once a payment stream is calculated by means of the static or trended mortality methods, expected values of case reserves may be estimated for the same payment stream. The Sherman-Diss Method of Section 8.5 describes the procedure.

A.6 Methods Based on the Peculiarities of the Remaining Open Claims

- Maximum Possible Loss Method (7.2)
- Judgment Estimate Method (7.3)

While these methods do not generally result in a tail factor for the less mature years (that may or may not have a similar open claim portfolio when they become the oldest year in the triangle), they can be very useful for analyzing the oldest year and other years near the top of the triangle.

A.7 Other Methods

- Restate Historical Experience Method (8.2)
- Mueller Incremental Tail Method (8.3)
- Corro's Method (8.4)
- Sherman-Diss Method (8.5)

Corro's technique can be used to estimate tail factors for claims, which are duration dependent but whose payment period is flat and uniform (e.g., credit insurance claims). A "mixing weight parameter" is calculated to allocate probabilities to two specified durations.

APPENDIX B – Examples

B.1 Introduction

This appendix will show additional details and illustrations of specific methods discussed in the main body of the paper. To the extent possible, the examples shown in this appendix reference a single data set, which is shown below. This data is also included in the accompanying Excel file.

B.1.1 Paid Loss

Cumulative Paid Loss Data										
Accident Year	12	24	36	48	60	72	84	96	108	120
2000	1,202	2,685	4,132	5,323	6,059	6,406	6,812	7,208	7,440	7,618
2001	1,297	2,712	4,232	5,314	6,062	6,786	7,375	7,687	7,934	
2002	1,342	2,566	4,058	5,388	6,480	7,141	7,801	8,109		
2003	1,293	2,716	4,228	5,587	6,661	7,626	8,040			
2004	1,387	2,555	4,017	5,460	6,743	7,479				
2005	1,487	2,738	4,125	5,683	6,793					
2006	1,499	2,920	4,781	6,285						
2007	1,587	3,287	5,006							
2008	1,221	2,775								
2009	1,321									
Paid Loss Development Triangle										
Accident Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	
2000	2.234	1.539	1.288	1.138	1.057	1.063	1.058	1.032	1.024	
2001	2.092	1.560	1.256	1.141	1.119	1.087	1.042	1.032		
2002	1.911	1.582	1.328	1.203	1.102	1.092	1.039			
2003	2.100	1.557	1.321	1.192	1.145	1.054				
2004	1.842	1.572	1.359	1.235	1.109					
2005	1.841	1.507	1.378	1.195						
2006	1.948	1.637	1.314							
2007	2.071	1.523								
2008	2.272									
Straight Average	2.034	1.560	1.321	1.184	1.106	1.074	1.047	1.032	1.024	
Volume Weighted Average	2.026	1.559	1.320	1.185	1.107	1.074	1.046	1.032	1.024	
5 Year Volume Weighted	1.988	1.559	1.339	1.193	1.107	1.074	1.046	1.032	1.024	
3 Year Volume Weighted	2.085	1.555	1.349	1.207	1.119	1.077	1.046	1.032	1.024	
Selected LDF	2.034	1.560	1.321	1.184	1.106	1.074	1.047	1.032	1.024	

B.1.2 Incurred Loss

Cumulative Incurred Loss Data										
Accident Year	12	24	36	48	60	72	84	96	108	120
2000	2,539	4,479	5,650	6,639	7,224	7,224	7,464	7,778	7,892	7,987
2001	2,672	4,667	6,049	6,988	7,355	7,819	8,171	8,296	8,518	
2002	2,808	4,676	6,207	7,064	7,601	7,984	8,390	8,628		
2003	3,073	5,099	6,292	7,237	7,749	8,386	8,604			
2004	3,070	4,527	5,915	6,986	7,780	8,197				
2005	2,932	4,750	6,041	7,144	7,771					
2006	3,095	5,104	6,770	7,821						
2007	3,228	5,526	7,204							
2008	2,877	5,122								
2009	2,890									
Incurred Loss Development Triangle										
Accident Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	
2000	1.764	1.261	1.175	1.088	1.000	1.033	1.042	1.015	1.012	
2001	1.747	1.296	1.155	1.053	1.063	1.045	1.015	1.027		
2002	1.665	1.327	1.138	1.076	1.050	1.051	1.028			
2003	1.659	1.234	1.150	1.071	1.082	1.026				
2004	1.474	1.307	1.181	1.114	1.054					
2005	1.620	1.272	1.183	1.088						
2006	1.649	1.326	1.155							
2007	1.712	1.304								
2008	1.781									
Straight Average	1.675	1.291	1.162	1.081	1.050	1.039	1.029	1.021	1.012	
Volume Weighted Average	1.671	1.291	1.162	1.081	1.050	1.039	1.028	1.021	1.012	
5 Year Volume Weighted	1.646	1.289	1.161	1.080	1.050	1.074	1.046	1.032	1.024	
3 Year Volume Weighted	1.712	1.301	1.172	1.090	1.062	1.040	1.028	1.032	1.024	
Selected	1.675	1.291	1.162	1.081	1.050	1.039	1.029	1.021	1.012	

B.1.3 Case Reserves

Case Reserve										
Accident Year	12	24	36	48	60	72	84	96	108	120
2000	1,337	1,795	1,518	1,316	1,164	817	652	570	452	369
2001	1,376	1,955	1,816	1,674	1,293	1,033	796	609	584	
2002	1,466	2,111	2,149	1,677	1,121	843	589	520		
2003	1,780	2,384	2,063	1,650	1,087	760	565			
2004	1,684	1,972	1,899	1,526	1,036	718				
2005	1,445	2,012	1,916	1,460	978					
2006	1,596	2,184	1,988	1,537						
2007	1,640	2,239	2,199							
2008	1,656	2,348								
2009	1,569									

B.2 Bondy-Type Methods

Using the paid loss data shown above and specifically the selected paid loss development pattern, we can estimate the tail factor based on the various Bondy methods. A summary of those results and the cumulated development pattern is shown below.

Selected Paid Loss Development Factor											
	12	24	36	48	60	72	84	96	108	Tail	
Selected	2.034	1.560	1.321	1.184	1.106	1.074	1.047	1.032	1.024		
Bondy Estimated Age-Ultimate											
Original	6.680	3.283	2.105	1.594	1.346	1.217	1.133	1.082	1.049	**1.024**	**= Prior A-A**
Modified 1	6.840	3.362	2.156	1.632	1.379	1.246	1.160	1.108	1.074	**1.049**	**= (Prior A-A)^2**
Modified 2	6.837	3.360	2.154	1.631	1.378	1.245	1.159	1.108	1.073	**1.048**	**= (Prior A-A) * 2**
Generalized	6.632	3.260	2.092	1.586	1.334	1.197	1.119	1.073	1.045	**1.028**	
Fully Generalized	8.119	3.574	2.264	1.695	1.413	1.243	1.153	1.102	1.067	**1.043**	

The calculation of the generalized Bondy method is shown in the table below. Column 4 in this table uses formula 2.6 from the main body of the report and the sum of column 4 is minimized using the Excel "solver" function.

Generalized Bondy: Parameters and Development Factors							
(1)	(2)	(3)	(4)	(5)	(6)	(7)	
Period	LDF	Ln (2)	Formula 2.6	Fitted A-A	Fitted A-U	Index i	
12-24	2.034	0.710	0.000000	2.034	6.632	1	
24-36	1.560	0.444	0.000001	1.558	3.260	2	
36-48	1.321	0.278	0.000001	1.319	2.092	3	
48-60	1.184	0.169	0.000017	1.189	1.586	4	
60-72	1.106	0.101	0.000048	1.114	1.334	5	
72-84	1.074	0.072	0.000016	1.070	1.197	6	
84-96	1.047	0.046	0.000012	1.043	1.119	7	
96-108	1.032	0.032	0.000028	1.027	1.073	8	
108-120	1.024	0.024	0.000052	1.017	1.045	9	
					1.028	***= [Last AA ^ (B / 1 - B)]***	
		Total	0.000175	*Note: Must use "Solver" to minimize least squares [Sum of (4)]*			
Bondy Parameter		0.625	*= B*				
Estimated Ratio for 12-24		2.034	*= d^*				

The fully generalized Bondy method allows the estimated development ratios (*d*) to vary by accident period, while using the same Bondy parameter (B). In example shown below, the formula 2.6 is calculated for each of the last three accident periods at every maturity and the sum of the entire triangle is minimized using Excel.

Fully Generalized Bondy:Parameters, Development Factors, and Squared Error											
Accident	Parameter	**=[Ln(Actual AA) - Ln(d)*B^(Index - 1)]^2**									
Year	Estimate (d)	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	
2000	2.090							0.000	0.000	0.000	
2001	1.969						0.000	0.000	0.000		
2002	1.835					0.000	0.000	0.000			
2003	1.932				0.000	0.000	0.000				
2004	2.070			0.000	0.000	0.001					
2005	1.954		0.001	0.002	0.000						
2006	1.994	0.001	0.002	0.000							
2007	2.023	0.001	0.001								
2008	2.272	0.000									
Bondy:	0.648	*= B*									
	Index	1	2	3	4	5	6	7	8	9	
Minimum Least Squares		0.009	*Note: Must use "Solver" to minimize least squares [Sum of triangle]*								
	Period	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	Tail
	Fitted LDF	2.272	1.579	1.336	1.200	1.137	1.078	1.046	1.033	1.023	
	Factor to Ultimate	8.119	3.574	2.264	1.695	1.413	1.243	1.153	1.102	1.067	1.043

B.3 Algebraic Methods

B.3.1 Sherman-Boor Method

This method requires two triangles, one of paid loss and one of case reserves. Using the triangle shown in the introduction and the formulas from the main body of the paper, we can then calculate triangles of the incremental paid loss and incremental disposed case reserves. Specifically, the incremental paid loss triangle is computed as: given a cell in the cumulative paid loss triangle, then we subtract the previous cell in the same row of the cumulative paid loss triangle. Subtracting the current cell from the previous cell in the case reserve triangle to obtain the triangle of case reserves disposed of. The incremental triangles are shown below:

Accident	Incremental Paid Loss (Formula 3.2)									
Year	12	24	36	48	60	72	84	96	108	120
2000	1,202	1,483	1,448	1,191	736	347	406	396	232	178
2001	1,297	1,416	1,520	1,082	748	723	589	312	247	
2002	1,342	1,223	1,493	1,329	1,093	661	659	308		
2003	1,293	1,422	1,513	1,359	1,074	965	413			
2004	1,387	1,168	1,462	1,443	1,283	736				
2005	1,487	1,251	1,387	1,559	1,109					
2006	1,499	1,421	1,861	1,503						
2007	1,587	1,700	1,719							
2008	1,221	1,553								
2009	1,321									

Accident	Incremental Case Reserves Disposed Of (Formula 3.3)									
Year	12	24	36	48	60	72	84	96	108	120
2000		(457)	277	202	151	347	166	82	118	83
2001		(579)	139	142	381	260	237	187	25	
2002		(644)	(38)	472	556	278	254	70		
2003		(604)	320	414	562	327	196			
2004		(289)	74	373	489	318				
2005		(567)	96	456	482					
2006		(588)	195	452						
2007		(598)	40							
2008		(692)								
2009										

Then divide the incremental paid loss by the case reserves eliminated. These ratios will be used to calculate estimators of 'S'.

Accident	Relative Disposal Costs (Formula 3.4 = Ipaid(w,d) / Icase(w,d))									
Year	12	24	36	48	60	72	84	96	108	120
2000		(3.241)	5.232	5.885	4.862	1.000	2.446	4.846	1.965	2.151
2001		(2.444)	10.950	7.612	1.964	2.781	2.486	1.670	9.972	
2002		(1.898)	(39.244)	2.815	1.965	2.378	2.601	4.427		
2003		(2.355)	4.725	3.285	1.910	2.949	2.113			
2004		(4.047)	19.857	3.871	2.622	2.314				
2005		(2.206)	14.469	3.419	2.302					
2006		(2.416)	9.522	3.326						
2007		(2.841)	42.912							
2008		(2.244)								
					Adjustment Factor	3.073	='S', which is selected here as			
							as average of last 5 columns			
					Oldest Period, Current Case Reserve	369				
					Older Period, Cumulative Paid	7,618				
					Paid Tail Factor	1.149	*Formula 3.5*			
					Incurred Tail Factor	1.096	*Formula 3.6*			

Because the early development involves not just elimination of case reserves through payments, but also substantial emergence of IBNR claims, the early maturities could be potentially distorted. Looking at the various ratios at the 'mature' development stage it would appear that they average around 3.0, so we will use that as our adjustment factor 'S' for the case reserves.

Utilizing \$369 of case left on the 2000 accident period at 120 months development, and the cumulative paid on 2000 accident period of \$7,618, the development portion of the paid loss tail factor would be (\$369/\$7,618)×3.079 = .149 So, the paid loss tail factor would be 1.149.

For the incurred loss tail factor, first note that only the 'development portion' of the S =3.073, or $S-1$=2.073, need be applied (the remaining case is already contained in the incurred). Second, a ratio of the case reserves to incurred loss is needed (which is $c(1,n)/c_{Incurred}(1,n)=c(2000,120)/c_{Incurred}(2000,120)=$ \$369/\$7,987 = .046. Multiplying the two numbers creates an estimate of the development portion of the tail at 2.073 ×.046 = 0.096. So, the incurred loss tail factor estimate would be 1.096.

B.4 Curve-Fitting Methods

B.4.1 Exponential Method

The main body of the report illustrates an exponential fit using data provided by Joe Boor. Below, the exponential fit is applied to the same data used in other sections of this appendix to illustrate the fit two different ways. Specifically, the table below develops the fit using all of the selected development factors (result in column 6) and a fit using only the 6 most mature periods (with result in column 8).

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
					Curve Fit Results			
	Development Period	Selected	v(d)	ln [v(d)]	Using All Periods		Using Last 6 Periods	
		LDF	= (2) - 1	= ln(3)	Fitted A-A	Fit Error	Fitted A-A	Fit Error
1	12-24	2.034	1.034	0.034	1.855	0.180		
2	24-36	1.560	0.560	-0.580	1.532	0.027		
3	36-48	1.321	0.321	-1.137	1.332	-0.011		
4	48-60	1.184	0.184	-1.692	1.207	-0.023	1.169	0.015
5	60-72	1.106	0.106	-2.240	1.129	-0.022	1.113	-0.006
6	72-84	1.074	0.074	-2.601	1.080	-0.006	1.075	-0.001
7	84-96	1.047	0.047	-3.065	1.050	-0.003	1.050	-0.003
8	96-108	1.032	0.032	-3.438	1.031	0.001	1.033	-0.001
9	108-120	1.024	0.024	-3.731	1.019	0.005	1.022	0.002
10					1.012		1.015	
11					1.008		1.010	
12					1.005		1.007	
13					1.003		1.004	
14					1.002		1.003	
15					1.001		1.002	
16					1.001		1.001	
17					1.000		1.001	
18					1.000		1.001	
19					1.000		1.000	
20					1.000		1.000	

Fitting a line to the natural logarithms of the development portion of the link ratios (column 6), we estimate the slope and intercept of the fitted line. The inverse natural logarithm of the slope parameter becomes the decay constant, r. The complete fitted parameters are shown below. Note that for this data set and truncating the age-to-age factors through period 20, the tail factor based on the approximate formula and the cumulative of the age-to-age factors is very similar.

Curve Fit Parameters	Decay Rate	Coefficient	Tail Factor At Period 10 Truncated	Tail Factor At Period 10 Approximate
Using All Points	0.623	1.372	1.032	1.032
Using Last 6 Points	0.666	0.863	1.044	1.044

Decay = e^[slope of the linear fit of (1) and (5)]
Coefficient = intercept of linear fit of (1) and (5)
Truncated Tail = Product of remaining fitted A-A
Approximate = 1 + Coefficient x Decay ^ [Period / (1-Decay)]
From Formula 5.2

B.4.2 McClenahan's Method

Here we have replicated the McClenahan method discussed in the body of the report using the same data shown here in the Appendix. Specifically, we are using selected paid loss development factors and again converting these to cumulative paid loss amounts by selecting a base amount for the first development period paid loss, for simplicity sake we use $100. To determine incremental paid losses by period we subtract successive cumulative loss amounts, and then we have the following:

(1)	(2)	(3)	(4)	(5)	(6)
Development Period		Age	Selected A-A Factor	Cumulative Paid	Incremental Paid
1		12		100	100
2	12-24	24	2.034	203	103
3	24-36	36	1.560	317	114
4	36-48	48	1.321	419	102
5	48-60	60	1.184	496	77
6	60-72	72	1.106	549	53
7	72-84	84	1.074	590	41
8	84-96	96	1.047	617	28
9	96-108	108	1.032	637	20
10	108-120	120	1.024	652	15

We can continue this table by taking successive ratios of incremental paid amounts for the accident periods to produces estimates of the annual decay constant r. In practice any of a variety of curve-fitting techniques using the incremental paid loss regressed on age can be employed to develop an estimate of r from Column 8, in this example we have used a linear fit of the natural log of the r's.

(7)	(8)	(9)	(10)
		Curve Fit Results	
Incremental A-A	Log Incremental Ln (6)	Fitted Incremental	Actual / Fitted Ratio
		377	0.265
1.034	4.639	273	0.379
1.101	4.735	197	0.577
0.893	4.622	143	0.713
0.758	4.346	103	0.747
0.685	3.967	75	0.707
0.771	3.707	54	0.753
0.675	3.314	39	0.703
0.721	2.988	28	0.701
0.770	2.726	20	0.745
		Average	0.724
Curve Fit on Boxed Points			
Monthy Decay	0.973		
Annual Decay	0.724		

The decay rates shown are the result of a linear fit of the boxed values in column 8. The monthly decay uses monthly maturities in column 3 and the annual decay uses in period number in column 1. Note that .973^12 = .724 (when accounting for decimals rounded off), however, McLenahan's formula uses the monthly decay rate to calculate the tail (where p is the monthly decay rate and $= r(1/12)$).

For the sake of the example, we will assume a lag constant of $a = 6$. Once the value of p is calculated, we can develop an estimate of the tail at 120 months or $T(10)$ using equation 5.7. We can also estimate an adjusted tail using the actual to fitted ratio from column 10.

	Months	
Initial Lag in Report	6	
Tail at 120	1.055	From Formula 5.7
Adjusted Tail at 120	1.040	Calculated Tail, Adjusted for Actual / Fitted Ratio

Formula 5.7: $T(m/12) = \{12 \times (1 - p)\} / \{12 \times (1 - p) - p^{m-a-10} \times (1 - p^{12})\}$

B.4.3 Sherman's Method

Given the selected paid loss ink ratios, we first determine the development portion, $v(d)$, of each link ratio. The natural logarithms of the age d and $v(d)$ then represent the dependent and independent variables in our regression, respectively.

	Curve Fit Using An Inverse Power Function				
	Development Period	Selected LDF	Development Portion	Log of Development Age	Log of Development Portion
1	12-24	2.034	1.034	0.000	0.034
2	24-36	1.560	0.560	0.693	(0.580)
3	36-48	1.321	0.321	1.099	(1.137)
4	48-60	1.184	0.184	1.386	(1.692)
5	60-72	1.106	0.106	1.609	(2.240)
6	72-84	1.074	0.074	1.792	(2.601)
7	84-96	1.047	0.047	1.946	(3.065)
8	96-108	1.032	0.032	2.079	(3.438)
9	108-120	1.024	0.024	2.197	(3.731)
10					
11					
12		**I. Curve Fit With No Lag Parameter**			
13					
14		Exponent = slope	(2.386)		
15		Coefficient = e ^ Intercept	4.806		
16		Tail Factor	1.137		
17					
18					
19		**II. Curve Fit With Optimal Lag Parameter**			
20					
21		Lag Parameter	(0.076)		
22		Minimal Squared Error	0.000		
23		Tail Factor	1.135		
24					
25	Note: Must use "Solver" to minimize squared error each time new ratios are selected				

The fitted parameters of the curve are based on a linear regression of the boxed factors. The tail factor is the determined by cumulating the estimated age-to-age factors for each future period, where the factor $f(d) = 1 +$ coefficient * age slope.

Several possible alternatives to the above example exist. For example, we might have chosen a to rely on link ratios of only the first 5 or 8 development ages, we could rely on the link ratios of only "mature" development ages, etc.

To estimate the optimum lag, you can use a bisection process, specifically following the process above using different potential lags; finding the lowest value of the squared error across a group of values; and progressively narrowing the range. Alternatively, you can also use the 'solver' Excel function.

B.4.4 Pipia's Method

The following example is based the cumulative paid loss; the method can also be applied to incurred losses. In addition, other choices for the dimensions of the triangle can easily be substituted.

The parameter being minimized is the square of the ratio of the difference between the actual and fitted incremental development to the expected incremental development. As shown in the triangle below, the difference is taken for each of the age-to-age factors and the total difference for the triangle is minimized using Excel.

Squared Difference									
Accident Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
2000	0.100	0.001	0.004	0.031	0.240	0.037	0.001	0.048	0.047
2001	0.156	0.006	0.003	0.026	0.004	0.011	0.062	0.049	
2002	0.245	0.014	0.043	0.044	0.008	0.032	0.090		
2003	0.153	0.005	0.034	0.021	0.085	0.095			
2004	0.285	0.010	0.105	0.161	0.001				
2005	0.286	0.001	0.154	0.027					
2006	0.226	0.050	0.025						
2007	0.166	0.000							
2008	0.088								

Total Squared Difference 3.2776

Curve Parameters	
λ	-0.231
c	0.000
t	1.044
Implied Tail	1.098

Note: Must use "Solver" to minimize least squares each time new ratios are selected

Curve Fit Using All Historical Development Factors										
Age	12	24	36	48	60	72	84	96	108	120
Average Age of Claim	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5
Fitted Age to Age	2.806	1.521	1.271	1.168	1.112	1.078	1.056	1.041	1.031	
Fitted Age to Ultimate	9.454	3.369	2.215	1.742	1.492	1.342	1.244	1.178	1.131	1.098

The above estimated tail is only one way to minimize the squared difference. The estimated tail shown below was determined after minimizing the difference between the fitted and selected factors for only the 24 to 108 age-to-age factors, rather than the entire triangle.

Curve Fit Using Selected Factors Only										
Selected		**1.560**	**1.321**	**1.184**	**1.106**	**1.074**	**1.047**	**1.032**	**1.024**	
Fitted Age to Age	3.219	1.597	1.298	1.176	1.112	1.073	1.049	1.033	1.023	
Fitted Age to Ultimate	10.889	3.383	2.119	1.632	1.388	1.249	1.163	1.109	1.073	1.049

Squared Difference								
12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
	0.0039	0.0058	0.0022	0.0021	0.0001	0.0030	0.0015	0.0027

Total Squared Difference 0.0212

Curve Parameters	
λ	-0.218
c	0.000
t	1.175
Implied Tail	1.049

B.4.5 England-Verrall

Sections 6 and 7 of the England-Verrall paper present examples to illustrate the methodology. A comparison of predictor structures is included in Section 6. The Tables shown below include original table numbers in parentheses. Three models are fitted utilizing an over-dispersed Poisson model ($\rho = 1$ in Equation (5.20)) with a logarithmic link function. For all three models:

$$E\left[c(w,d)\right] = m_{w,d}\ , \tag{5.19}$$

$$Var\left[c(w,d)\right] = \phi m_{w,d}^{\rho} \tag{5.20}$$

$$\ln(m_{w,d}) = \eta_{w,d} \tag{5.21}$$

The models differ only in the choice of the predictor, θ_w and θ_d .

Model 1: $\eta_{w,d} = c + \alpha_w + \beta_d$ (5.21.1)

Model 2: $\eta_{w,d} = u_d + c + \alpha_w + \beta \ln(d) + \gamma_d$ (5.21.2)

Model 3: $\eta_{w,d} = u_d + c + \alpha_w + s\theta_d \ln(d)$ (5.21.3)

Models 1 and 2, shown in Table 5.7.2.1, can be fitted in any statistical software package that fits generalized linear models. Model 3 can only be fitted in statistical software packages that fit generalized additive models. Equivalent development factors are shown in Table5.7.2.2, together with the actual development factors obtained by applying the standard chain ladder model to the data in Table5.7.2.1. The reserve estimates implied by Models 1, 2 and 3 are shown in Table 5.7.2.4, together with their prediction errors (as a percentage of the reserves).

TABLE 5.7.2.1 (TABLE 6.1)

Incremental Paid Losses Formed by Aggregating Across Different Classes

	1	2	3	4	5	6	7	8	9	10
1	45,630	23,350	2,924	1,798	2,007	1,204	1,298	563	777	621
2	53,025	26,466	2,829	1,748	732	1,424	399	537	340	
3	67,318	42,333	1,854	3,178	3,045	3,281	2,909	2,613		
4	93,489	37,473	7,431	6,648	4,207	5,762	1,890			
5	80,517	33,061	6,863	4,328	4,003	2,350				
6	68,690	33,931	5,645	6,178	3,479					
7	63,091	32,198	8,938	6,879						
8	64,430	32,491	8,414							
9	68,548	35,366								
10	76,013									

TABLE 5.7.2.2 (TABLE 6.2)

Equivalent Development Factors: Overdispersed-Poisson Model

Delay Year	Standard Chain Ladder	Model 1 Stochastic Chain Ladder	Model 2 Hoerl Curve	Model 3 GAM (dof = 5)
2	1.4906	1.4906	1.4496	1.489 1
3	1.0516	1.0516	1.0796	1.0537
4	1.0419	1.0419	1.0372	1.0395
5	1.0268	1.0268	1.0238	1.0292
6	1.0254	1.0254	1.0180	1.0224
7	1.0149	1.0149	1.0150	1.0163
8	1.0130	1.0130	1.0135	1.0120
9	1.0067	1.0067	1.0127	1.0091
10	1.0078	1.0078	1.0124	1.0071
11			1.0125	1.0057
12			1.0129	1.0047
13			1.0135	1.0039
14			1.0144	1.0033
15			1.0156	1.0029
16			1.0171	1.0025

A comparison of error structures is included in Section 7 of the original paper. The same three model predictors are used, but with a Gamma error structure ($\rho = 2$) giving:

$$E\left[c(w,d)\right] = m_{w,d}\,, \tag{5.19}$$

$$Var\left[c(w,d)\right] = \phi m_{w,d}^2 \tag{5.20}$$

$$\ln(m_{w,d}) = \eta_{w,d} \tag{5.21}$$

and

Model 4: $\eta_{w,d} = c + \alpha_w + \beta_d$ (5.21.4)

Model 5: $\eta_{w,d} = u_d + c + \alpha_w + \beta \ln(d) + \gamma_d$ (5.21.5)

Model 6: $\eta_{w,d} = u_d + c + \alpha_w + s\theta_d \ln(d)$ (5.21.6)

Equivalent development factors are shown in Table 5.7.2.3

TABLE5.7.2.3 (TABLE7.1)

Equivalent Development Factors: Gamma Model

Delay Period	Standard Chain Ladder	Model 4 Stochastic Chain Ladder	Model 5 Hoerl Curve	Model 6 GAM (dof = 5)
2	1.4906	1.4969	1.4515	1.4771
3	1.0516	1.0470	1.0799	1.0512
4	1.0419	1.0381	1.0372	1.0357
5	1.0268	1.0259	1.0237	1.0280
6	1.0254	1.0251	1.0178	1.0221
7	1.0149	1.0154	1.0148	1.0165
8	1.0130	1.0131	1.0131	1.0125
9	1.0067	1.0084	1.0123	1.0098
10	1.0078	1.0086	1.0119	1.0079
11			1.0119	1.0066
12			1.0122	1.0055
13			1.0127	1.0048
14			1.0135	1.0041
15			1.0145	1.0036
16			1.0157	1.0032

Reserve estimates and prediction errors are shown in Table 5.7.2.5

TABLE 5.7.2.4 (TABLE 6.3)

Reserve Estimates and Prediction Errors: Overdispersed-Poisson Model

	Reserve Estimates			Prediction Error		
Accident Period	Model 1 Stochastic Chain Ladder	Model 2 Hoerl Curve	Model 3 GAM (dof = 5)	Model 1 Stochastic Chain Ladder	Model 2 Hoerl Curve	Model 3 GAM (dof = 5)
1	0	0	0	—	—	—
2	683	1,085	622	159%	95%	110%
3	1,792	3,101	1,998	100%	61%	62%
4	4,363	6,129	4,470	63%	46%	43%
5	5,657	7,173	5,940	50%	43%	38%
6	8,209	8,689	8,106	40%	39%	33%
7	10,914	11,031	11,106	34%	34%	29%
8	15,199	14,765	15,112	28%	30%	25%
9	21,135	24,002	21,293	24%	23%	22%
10	60,335	59,625	60,377	17%	17%	16%
Total	128,286	135,600	129,024	15%	15%	12%

TABLE 5.7.2.5 (TABLE 7.2)

Reserve Estimates And Prediction Errors: Gamma Model

	Reserve Estimates			Prediction Error		
Accident Period	Model 4 Stochastic Chain Ladder	Model 5 Hoerl Curve	Model 6 GAM (dof = 5)	Model 4 Stochastic Chain Ladder	Model 5 Hoerl Curve	Model 6 GAM (dof = 5)
1	0	0	0	—	—	—
2	488	675	450	62%	46%	43%
3	2,086	3,296	2,205	43%	36%	33%
4	5,240	6,818	5,300	36%	32%	29%
5	6,169	7,061	6,313	32%	30%	28%
6	9,750	9,305	9,427	31%	29%	28%
7	15,080	13,029	15,097	31%	29%	29%
8	18,498	15,069	17,671	32%	30%	31%
9	20,470	24,400	20,896	36%	35%	35%
10	60,043	59,576	58,519	52%	48%	48%
Total	137,824	139,229	135,878	25%	23%	24%

Chain Ladder Reserving Methods for Liabilities with Per Occurrence Limits

Karen H. S. Adams,[1] ACAS, MAAA

Abstract

Motivation: As an insurance regulator, I regularly see instances where maximum limit losses are removed from incurred and/or paid losses prior to application of the development factors. In some of these instances, the triangles and LDFs are created with limited losses, as opposed to unlimited losses.

Method: This paper simulates loss development triangles that include maximum limit losses. It compares exclusion vs. inclusion of maximum limit losses to show how each option affects the accuracy of the results. This paper provides simulated empirical probabilities obtained by randomly dispersing large losses throughout a triangle, then calculating the ultimate limited losses by two different methods.

Conclusion: If limited LDFs are calculated using triangles that include truncated maximum limit losses, then excluding maximum limit losses prior to application of the LDF produces an understated ultimate and reserve.

Availability. Calculations were performed using @RISK Standard version 5.0, from Palisade Corporation, Ithaca, NY, U.S.A. The commercial software package @Risk was used to simulate loss triangles and to create graphs of empirical loss distributions. The Excel/@Risk spreadsheets used for calculating triangles with randomly disbursed large losses are available through the author.

Keywords. Loss development; reserving, data organization, net reserves, gross reserves, ceded reserves, reserving methods, aggregate excess/stop loss; simulation

1. INTRODUCTION

It is not uncommon to see a reserve analysis in which the actuary has removed full limit losses from paid or incurred data prior to application of loss development factors. (The full limit losses are added back in after application of the LDFs) This paper provides examples showing that if used improperly, this commonly used technique understates reserves. If the LDFs are estimated using all losses, including truncated losses, and the LDFs are applied only to the losses below the limits, then the reserve is under-estimated. This is due to the fact that losses reaching the limits no longer develop over time and hence the LDFs estimated using all losses are smaller than the LDFs estimated using only the losses below the limits.

1.1 Research Context

The focus area addressed is reserving methods applicable to data limited to a certain per occurrence limit.

[1] Jennifer Wu, an actuary at the Texas Department of Insurance went above and beyond the call of duty as a reviewer of this paper.

There are several papers that discuss issues tangentially related to the one discussed here. For example, Daley [3] and Klemmt [5] discuss potential increases in accuracy gained by applying methods differently to large losses vs. small losses. Several papers such as Brown [1], Halliwell [4] and Pinto [6] discuss using and or calculating percentages of losses within various layers. However, I was unable to find any papers focusing on the issue addressed by this particular paper i.e. removal of large losses prior to application of the LDF, but where the LDFs were calculated with the truncated losses included. It is possible that no one has written such a paper because the conclusion appeared to be obvious. Nevertheless, the technique is used[2], so consequently I am writing this paper.

1.2 Objective

The objective of this paper is to increase awareness within the actuarial community that application of a commonly used technique is actuarially unsound.

1.3 Outline

The remainder of the paper will provide an example and some simulation results showing that it is more accurate to apply the limited LDFs to all the losses rather than to only the losses that are below the limit. The paper will provide some discussion about why intuitively these results make sense.

2. BACKGROUND AND METHODS

2.1 Background – Applying the LDF to the Losses

Suppose that you are given the following information

Total Case Inc. Limited Losses:	$3M
Insured Limit:	$500K
Losses exceeding 100K:	120K, 450K, 500K
Applicable Incurred LDF:	1.2

Note that I did not explain how the incurred LDF was calculated. This is an important piece of information. However, for now, let us suppose that you do not know how the LDF was calculated.

[2] One reviewer of the proposal exclaimed, "Make them stop!"

With the information at hand there are a couple of different ways to proceed.

2.1.1 Method-A

We could multiply $3M by 1.2 to obtain $3.6M as the ultimate loss, and 600K as the IBNR.

2.1.2 Method-X

We could reason that one loss has already reached the limit, and the other one, when multiplied by the LDF will exceed the limit. We remove the two largest losses from the incurred amount and limit their development to the limit. The ultimate values of the $500K & 450K losses will be assumed to be $500K

We would calculate the ultimate loss as follows:
($3M – 450K – 500K) *1.2 + 500K + 500K =
($2.05M)*1.2 + $1M =
$2.46M + $1M =
$3.46M

2.1.3 Method-A vs. Method-X

Method-A gave us an ultimate of $3.6 million whereas Method-X gave us an ultimate of $3.46 million. Consequently, the IBNR from Method-X is $140K lower[3].

Notice that the result from Method-X will always be less than or equal to the result from Method-A. The two will be equal if there are no large losses in the accident year. Sometimes there is pressure for an actuary to produce a lower value of IBNR, and so the second method may be attractive. Nevertheless, as actuaries, we must be careful to use methods that are actuarially sound.

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[3] 140K/600K is about 23%, a significant difference in IBNR.

2.2 Background – Notation

Papers written for the CAS are required to use notation consistent with that used in ***The Analysis and Estimation of Loss & ALAE Variability: A Summary Report written*** by the CAS Working Party on Quantifying Variability in Reserve Estimates. This paper uses standard actuarial triangles such as those referred to in the above mentioned report. Some notation follows.

w: The accident year

d: The age of the losses. If the accident year is 2010, then *d*=1 at 12/31/2010 and *d*=2 at 12/31/2011

f(*d*) : Incremental LDF. f(*d*) is applied to a value at age *d* to estimate the value at age *d*+1

F(*d*) : Cumulative LDF. F(*d*) is applied to a value at age *d* to estimate the value at age n. In our examples, n=10, and there is no development after age 9, so F(*d*) estimates the ultimate value of the developing quantity.

$f^T(d)$: true value of f(*d*) for unlimited losses.

$F^T(d)$: true value of F(*d*) for unlimited losses.

Throughout this paper, losses are expressed in thousands (000), or "K" and the retention/limit is $500K.

2.3 Background – Different Sources of LDFs

Suppose that you are given the four triangles below, which are all created with the same underlying data. Triangle “A” contains the aggregated unlimited losses by accident year. Triangle “B” is the same as Triangle “A” except that any occurrences of 500K[4] or more have been limited to 500K (the retention). Triangle “C” is composed only of losses less than 450K (90% of the retention) at the most recent valuation. Triangle “D” is composed only of losses greater than or equal to 450K at the last evaluation, and each of the losses has been limited to 500K. Note that triangle “B” =

Table 1

A) Unlimited Triangle

	1	2	3	4	5
2009	415	853	1,258	1,654	2,051
2010	180	370	546	717	-
2011	580	1,192	1,758	-	
2012	180	370	-	-	
2013	415	-	-	-	

	1	2	3	4	5
f(d)	2.06	1.48	1.32	1.24	1.00
F(d)	4.94	2.41	1.63	1.24	1.00

B) Limited Triangle 500K per Occ

	1	2	3	4	5
2009	415	839	1,000	1,158	1,316
2010	180	370	546	717	-
2011	580	1,178	1,500	-	
2012	180	370	-	-	
2013	415	-	-	-	

	1	2	3	4	5
f(d)	2.03	1.28	1.21	1.14	1.00
F(d)	3.58	1.76	1.38	1.14	1.00

C) Small Only - Only Losses <= 450K

	1	2	3	4	5
2009	165	339	500	658	816
2010	180	370	546	717	-
2011	180	370	546	-	
2012	180	370	-	-	
2013	415	-	-	-	

	1	2	3	4	5
f(d)	2.06	1.48	1.32	1.24	1.00
F(d)	4.94	2.41	1.63	1.24	1.00

D) Large Only Limited to 500K per Occ

	1	2	3	4	5
2009	250	500	500	500	500
2010	-	-	-	-	-
2011	400	808	955	-	
2012	-	-	-	-	
2013	-	-	-	-	

	1	2	3	4	5
f(d)	2.01	1.11	1.00	1.00	1.00
F(d)	2.24	1.11	1.00	1.00	1.00

“C”+”D”.

The LDFs calculated by the actuary will depend on the triangle used. In some consulting situations, the actuary may only be provided with triangle “B”. When this paper refers to “true LDFs” or “true unlimited LDFs” it is referring to LDFs calculated using the unlimited losses, as in triangle “A”. When this paper refers to limited LDFs, it is referring to LDFs calculated from a

[4] Actually any occurrences at last evaluation that are 90% of 500K =450K have been limited to 500K. the assumption is that if a loss is 450K at the most recent evaluation, then it will develop to a loss greater than or equal to 500K.

triangle such as "B". A third method, illustrated in the Appendices uses the LDFs from triangles "C" and "D". Within this paper, it is assumed that the "true LDFs" are known and deterministic. In this paper, the universe of examples is created by the author, and in order to simplify the picture of what is happening, the author (me) has assumed[5] that the value of all *unlimited* incurred losses at year 2 is equal to the [unlimited value at year 1] x 2.055 and that the incurred unlimited value at the end of year 3 is equal to the [unlimited value at year 2] x 1.475 etc. etc. This is a very simple model that allows for easy comparison of accuracy of two methods. I do not believe that introducing random fluctuations in the losses would change the result, but it would make the reasoning harder to follow. See the Appendices for some sensitivity testing with regard to changes in LDFs and the ratio of small to large losses. Another author is welcome to explore the effects of random fluctuations in the incurred losses, but in this paper it is assumed that unlimited incurred losses follow the deterministic path described by the LDFs below. The superscript "T" is used to indicate "true unlimited LDFs"

Table 2-True Unlimited LDFs

d	1	2	3	4	5	6	7	8	9	10
$f^T(d)$	2.055	1.475	1.315	1.240	1.200	1.175	1.145	1.125	1.110	1.000
$F^T(d)$	9.964	4.849	3.287	2.500	2.016	1.680	1.430	1.249	1.110	1.000

For every simulated *unlimited* triangle in this paper[6], the calculated LDFs will be f^T and F^T. Note, however, that if random large losses are added to the triangles, and the losses are *limited* to 500K per occurrence, then the limited LDFs will be different for every triangle and dependent on the number, size and accident year of the random large losses. Another way to say this is that changing the large losses in triangle "D" above will change the LDFs calculated from triangle "B" = "C"+ "D".

[5] In the appendices different assumptions are explored.

[6] In Appendix E, the effects of using different values for f^T and F^T are examined, but in the main part of the paper, only values in Table 2 are used.

2.4 Creating a Simulated Triangle

2.4.1 An Accident Year of Unlimited Occurrences

Suppose we have 14 losses in accident year 2006. One of them has an initial value of 250K, a second has an initial value of 150K and the rest begin at 15K each. The losses would develop as follows. The unlimited development follows Table 2.

Table 3

Year	*d*=1	*d* =2	*d* =3	*d* =4	*d* =5	*d* =6	*d* =7	*d* =8
2006	250	514	758	996	1,236	1,483	1,742	1,995
2006	150	308	455	598	741	890	1,045	1,197
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
Total	580	1,194	1,753	2,314	2,865	3,441	4,047	4,632

The row in the unlimited triangle would look as follows. If you calculate the incremental LDFs you will see that they match those in Table 2.

Table 4 – Row in an Unlimited Triangle

Year	*d* =1	*d* =2	*d* =3	*d* =4	*d* =5	*d* =6	*d* =7	*d* =8
2006	580	1,194	1,753	2,314	2,865	3,441	4,047	4,632

2.4.2 A Row in a Limited Triangle

The per occurrence values are the same except for the two losses exceeding 500K.

Table 5

Year	*d* =1	*d* =2	*d* =3	*d* =4	*d* =5	*d* =6	*d* =7	*d* =8
2006	250	500	500	500	500	500	500	500
2006	150	308	455	500	500	500	500	500
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
Total	580	1,180	1,495	1,720	1,888	2,068	2,260	2,440

The row in the *limited* triangle would look as follows. If you calculate the incremental LDFs you will see that they are lower than those in Table 2. [7]An examples of one iteration of *limited* occurrences is given in Appendix A[8]

Table 6-Row in a Limited Triangle

Year	*d* =1	*d* =2	*d* =3	*d* =4	*d* =5	*d* =6	*d* =7	*d* =8
2006	580	1180	1495	1720	1888	2068	2260	2440

2.4.3 Actual Values of Ultimate Losses

Since unlimited loss development is known exactly, we can calculate the exact ultimate values of each loss on both a limited and unlimited basis.

Table 7 – Distribution of Random Losses

Value of Occurrence at *d* =1	$F^T(1)$	Ultimate Unlimited Loss	Ultimate Limited Loss
15	9.964	149.46	149.46
150	9.964	1494.60	500.00
250	9.964	2491.00	500.00

[7] The row below in Table 6 is similar to the 2006 row in Table 9. The only reason for differences is rounding.

[8] Applicable mathematical formulae are included in Appendix C.

The fact that the model is set up so that the "true" answer is known means that we can evaluate different methods to see which method is closer to the true answer. The occurrences are aggregated to get results by accident year.

2.5 Methods

First, initial occurrence values are selected for each accident year, and unlimited occurrences are developed using $f^T(d)$, i.e. the "true" incremental LDFs. Then, the occurrences are limited to 500K, and a triangle is created by aggregating the occurrences by accident year.

1) Limited LDFs, $f(d)$ & $F(d)$, are calculated from the limited triangle using an all-year weighted average.
2) The loss development factors from 1) are applied to
   a. All the limited losses. This will be referred to as "Method-A".
   b. Incurred losses excluding the losses within 90% of the limit. After application of the LDF, the large losses are added back in at full limits. This will be referred to as "Method-X"
3) The methods above are investigated for accuracy, bias and adequacy.

The number of occurrences in each accident year stays the same from trial to trial. Also in each accident year, most occurrences are static at 15K, but there are two random losses. The probability distribution of the two random occurrences is given below. In each accident year, there is a possibility that zero, one or two occurrences will have ultimate values greater than or equal to the retention. The incurred value of each individual claim at age 1 is chosen from the values of \$15K, \$150K and \$250K. Values of \$15K at 1 year do not reach the retention limit at maturity. Initial values of 150K and 250K both exceed the 500K limit after some development. Table 5 shows one accident year of simulated losses. A full set of simulated losses from one iteration is shown in Appendix A.

Table 8 – Random Losses for Each Accident Year

Size of Random Occurrences at d=1	Probability
15K & 15K	64%
250K & 15K	16%
150K & 15K	16%
250K&150K	2%
250K & 250K	1%
150K & 150K	1%

During each iteration, a limited triangle is simulated and set of limited LDFs is derived from this triangle. Below is one simulated triangle with losses limited to 500K and the associated all-year weighted incurred LDFs. Note that the LDFs calculated from the limited incurred triangle are smaller than f^T and F^T. Since we know the true development factors, we can calculate the *actual* ultimate losses, and can compare methods for accuracy. We will first look at a single iteration.

2.5.1 Limited Triangle from One Iteration

Table 9

	1	2	3	4	5	6	7	8	9	10
2004	180	370	546	717	890	1,068	1,254	1,436	1,616	1,794
2005	180	370	546	717	890	1,068	1,254	1,436	1,616	-
2006	580	1,178	1,500	1,717	1,890	2,068	2,254	2,436	-	
2007	180	370	546	717	890	1,068	1,254	-		
2008	415	839	1,000	1,158	1,316	1,479	-			
2009	180	370	546	717	890	-	-			
2010	225	462	682	897	-	-	-			
2011	400	808	955	-						
2012	180	370	-	-						
2013	285	-	-	-						

2.5.2 Limited LDFs from One Iteration

Table 10

	1	2	3	4	5	6	7	8	9	10
f(d)	2.04	1.33	1.24	1.18	1.15	1.14	1.11	1.13	1.11	1.00
F(d)	7.19	3.53	2.66	2.15	1.83	1.59	1.39	1.25	1.11	1.00

2.5.3 Results from the Application of Method-A

Table 11

Accident Year	Age - d	Incurred $(000)	F(d)	Method A Estimate $(000)	True Ultimate $(000)	Method A IBNR $(000)	True IBNR $(000)	Error $(000)	Error as a % of True IBNR
		(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
		modeled	modeled	=(a)*(b)		=(c) -(a)	=(d)-(a)	=(f)-(e)	=(g)/(f)
2004	10	1,794	1.00	1,794	1,794	-	-	-	-
2005	9	1,616	1.11	1,794	1,794	178	178	-	0%
2006	8	2,436	1.25	3,042	2,794	606	357	248.7	70%
2007	7	1,254	1.39	1,746	1,794	491	539	(47.7)	-9%
2008	6	1,479	1.59	2,350	2,144	871	666	205.6	31%
2009	5	890	1.83	1,624	1,794	735	904	(169.2)	-19%
2010	4	897	2.15	1,928	2,242	1,031	1,345	(314.1)	-23%
2011	3	955	2.66	2,541	1,995	1,586	1,040	546.0	53%
2012	2	370	3.53	1,305	1,794	935	1,424	(488.7)	-34%
2013	1	285	7.19	2,050	1,845	1,765	1,560	204.4	13%
Total		**11,975**		**20,173**	**19,988**	**8,198**	**8,013**	**185.1**	**2%**
2004-2011		**11,320**		**16,818**	**16,349**	**5,498**	**5,029**	**469.3**	**9%**

2.5.4 Results from the Application of Method-X

Table 12- Application of Method-X

Accident Year	Age - d	F(d)	Incurred $(000)	Large Losses	Inc X Known Large Losses	Method X Estimate $(000)	True Ultimate $(000)	Method X IBNR $(000)	True IBNR $(000)	Error $(000)	Error as a % of True IBNR
		(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
		modeled	modeled	modeled	=(b)-(c)	=(a)*(d) + (c)		=(e) -(b)	=(f)-(b)	=(g)-(h)	=(i)/(h)
2004	10	1.00	1,794	-	1,794	1,794	1,794	-	-	-	$ -
2005	9	1.11	1,616	-	1,616	1,794	1,794	178	178	-	0%
2006	8	1.25	2,436	1,000	1,436	2,794	2,794	357	357	(0)	0%
2007	7	1.39	1,254	-	1,254	1,746	1,794	491	539	(48)	-9%
2008	6	1.59	1,479	500	979	2,055	2,144	577	666	(89)	-13%
2009	5	1.83	890	-	890	1,624	1,794	735	904	(169)	-19%
2010	4	2.15	897	-	897	1,928	2,242	1,031	1,345	(314)	-23%
2011	3	2.66	955	500	455	1,710	1,995	755	1,040	(285)	-27%
2012	2	3.53	370	-	370	1,305	1,794	935	1,424	(489)	-34%
2013	1	7.19	285	-	285	2,050	1,845	1,765	1,560	204	13%
Total			11,975	2,000		**18,799**	**19,988**	**6,824**	**8,013**	**(1,189)**	**-15%**
2004-2011			**11,320**	**2,000**		**15,444**	**16,349**	**4,124**	**5,029**	**(905)**	**-18%**

2.5.5 Comments on Results

Note that the results from Method-A (i.e. applying the limited LDF to all losses) is more accurate. Note also that the result of Method-A is conservative and the result from Method-X is deficient.

2.6 Simulation Results 10,000 Trials

A model was created that simulates 10 years of loss data. The same techniques used in the prior sub-sections were applied. Large losses are randomly allocated to the accident years. For each year, there is a possibility of between zero and two large losses. Table 8 provides the probability of the incurred losses by accident year and severity at *d*=1. The probability of large losses in any one year is independent of the number and size of losses in any other year.

Method-A :(All Losses) The LDF was applied to all limited losses regardless of size.

Method-X :(All losses excluding max limit losses.) All losses within 90% of the retention were removed from the incurred losses prior to application of the LDF. The LDF was then applied to all remaining losses. After application of the LDF, the large losses were added back in at the max retention.

In each case, the percentage error between the true ultimate losses and the calculated ultimate losses was found.

2.6.1 Comparison of Methods: Mean, Bias, Adequacy

The results of 10,000 simulations are shown below. A negative error indicates an aggressive (low) estimate, whereas a positive error indicates a conservative (high) estimate.

Table 13

	Method	10th Percentile	25th Percentile	Mean Error	75th Percentile	90th Percentile	Std Dev	Mean
		Error as a Percentage of IBNR						Error as % of Ult
All Years	A	-19%	**-10%**	**3.4%**	15%	28%	19%	1.35%
2004-2011	A	-20%	**-11%**	**1.9%**	14%	25%	17%	0.54%
All Years	X	-34%	**-27%**	**-20%**	-12%	-5%	11%	-7.90%
2004-2011	X	-32%	-25%	-18%	-9%	-3%	11%	-5.54%

Note that both methods are biased, but Method-X more so. Method-X is so biased that 2004-2011 estimates are less than or equal to the true value in 99% of the simulations.

2.6.2 Comparison of Methods: Distance from the True Ultimate

The Table below calculates the absolute value of the error from Method-X minus the absolute value of the error from Method-A. The fact that the mean is positive indicates that the result from Method-A is expected to be closer than that from Method-X. Note that the difference is more pronounced if the methods exclude the two most recent years.

Table 14 - Difference Between Absolute Errors
Abs (Error Method-X) – Abs (Error Method A)

	10^{th} Percentile	Mean	90^{th} Percentile	Std Dev
All Years	-19%	1.7%	18%	15%
2004-2011	-18%	4.0%	20%	15%

2.6.3 Comparison of Methods: A Subjective Measure

In many instances an overestimate of reserves is preferable to an underestimate. An underestimate could lead to underpricing, (negative income) or unfavorable reserve adjustments in later years. If "conservative" error is preferable to "aggressive" error, the percentages in the above table are understated. For example, if conservative error is determined to be only 70% as "wrong" as a low estimate, then a new error term "Subjective Error" could be defined where

EA = (total error for 1 iteration / True IBNR for 1 iteration) for Method-A

EX = (total error for 1 iteration / True IBNR for 1 iteration) for Method-X

Subjective_Error-A = MAX [70%(EA), -EA]

Subjective_Error-X = MAX [70%(EX), -EX], and the subjective superiority of Method-A could be defined by: (Subjective_ Error-X – Subjective_ Error-A)

Note that based on the above definitions, the difference between the two methods is positive if Method-A is closer to the true answer and negative if Method-X is closer to the true answer, where the "closeness" is adjusted for conservative estimates. If reserve deficiency is less desirable than reserve redundancy by a ratio of 10-to-7 then the following results occur.

Table 15 Difference Between Subjective Errors
(Subjective_Error-X) – (Subjective_Error-A)

	10th Percentile	Mean	90th Percentile	Std Dev
All Years	-13%	4%	19%	12%
2004-2011	-10%	6%	21%	12%

The results above show that if conservative error is favored then Method-A on average is more accurate by about 6% for years 2004 through 2011 and about 4% for all years combined. Additional simulation results are shown in Appendix B.

3. RESULTS AND DISCUSSION

Table 16 – Sample Incurred Losses

Year	Unlimited Incurred (000)	Limited Incurred (000)	#Losses 500K or More	Sum of Max Limit Losses (000)
2004	897	897	-	0
2005	936	936	-	0
2006	1,922	922	2	1000
2007	1,701	1,701	-	0
2008	3,296	2,296	2	1000
2009	769	769	-	0
2010	1,603	1,103	1	500
2011	3,346	2,400	2	954
2012	244	244	-	0
2013	257	257	-	0

Let's suppose that you are the consulting actuary for a company with a self-insured retention of \$500K per claim. You have been given the above incurred data as of 12/31/2013. Also, the company has provided you with an incurred triangle with losses limited to 500K.

3.1 The Only Triangle Available is Limited to 500K

From the triangle with limited losses you obtain the following factors.

Table 17

d	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
f(*d*)	4.03	1.23	1.32	1.10	1.05	1.05	1.05	1.03	1.00
F(*d*)	8.61	2.13	1.74	1.31	1.19	1.14	1.08	1.03	1.00

You note that there 7 maximum limit losses that can not develop beyond 500K

What should you do? Based on the results of this paper, the most accurate method is to apply the LDFs obtained from the limited triangle to *all* losses, regardless of whether or not each individual loss has reached the maximum limit.

Intuitively this makes sense. If a factor is developed using all the truncated losses, then the factor should be applied to all the truncated losses so that the factor is consistent with the underlying data. It doesn't make sense to apply a factor developed with one type of data to dissimilar data where the differences are known and avoidable.

3.2 If Detailed Data is Available

If you are able to obtain detailed data, and create a triangle "S" that contains only the losses less then 90% of the retention, then the factors from "S" could be applied to the smaller losses. You could create two triangles and sets of LDFs: one for small losses and the other for large losses. It is not the intent of this paper to prove that separation of large and small losses is preferable. However, for the examples and simulations in this paper, separation of large and small losses is more accurate on average. See Appendix D for an example.

4. CONCLUSIONS

It is not actuarially sound to remove truncated/limited losses from incurred and/or paid results if limited loss development factors are applied. The loss development factors should be consistent with the losses to which they are applied to the extent possible.

Acknowledgment

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Appendix A

A single iteration of the simulated occurrences is listed below.

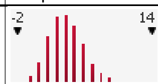
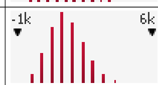
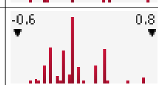
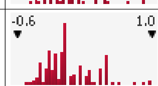
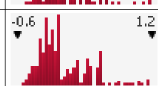
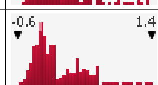
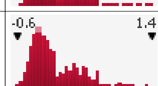
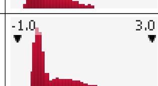
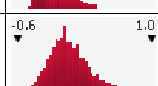
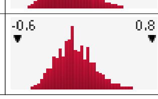
Year	1	2	3	4	5	6	7	8	9	10	True Ult	Large Amt
2004	15	31	45	60	74	89	105	120	135	149	149	0
2004	15	31	45	60	74	89	105	120	135	149	149	0
2004	15	31	45	60	74	89	105	120	135	149	149	0
2004	15	31	45	60	74	89	105	120	135	149	149	0
2004	15	31	45	60	74	89	105	120	135	149	149	0
2004	15	31	45	60	74	89	105	120	135	149	149	0
2004	15	31	45	60	74	89	105	120	135	149	149	0
2004	15	31	45	60	74	89	105	120	135	149	149	0
2004	15	31	45	60	74	89	105	120	135	149	149	0
2004	15	31	45	60	74	89	105	120	135	149	149	0
2004	15	31	45	60	74	89	105	120	135	149	149	0
2004	15	31	45	60	74	89	105	120	135	149	149	0
2005	15	31	45	60	74	89	105	120	135	-	149	0
2005	15	31	45	60	74	89	105	120	135		149	0
2005	15	31	45	60	74	89	105	120	135		149	0
2005	15	31	45	60	74	89	105	120	135		149	0
2005	15	31	45	60	74	89	105	120	135		149	0
2005	15	31	45	60	74	89	105	120	135		149	0
2005	15	31	45	60	74	89	105	120	135		149	0
2005	15	31	45	60	74	89	105	120	135		149	0
2005	15	31	45	60	74	89	105	120	135		149	0
2005	15	31	45	60	74	89	105	120	135		149	0
2005	15	31	45	60	74	89	105	120	135		149	0
2005	15	31	45	60	74	89	105	120	135		149	0
2006	250	500	500	500	500	500	500	500			500	500
2006	150	308	455	500	500	500	500	500			500	500
2006	15	31	45	60	74	89	105	120			149	0
2006	15	31	45	60	74	89	105	120			149	0
2006	15	31	45	60	74	89	105	120			149	0
2006	15	31	45	60	74	89	105	120			149	0
2006	15	31	45	60	74	89	105	120			149	0
2006	15	31	45	60	74	89	105	120			149	0
2006	15	31	45	60	74	89	105	120			149	0
2006	15	31	45	60	74	89	105	120			149	0
2006	15	31	45	60	74	89	105	120			149	0
2006	15	31	45	60	74	89	105	120			149	0
2006	15	31	45	60	74	89	105	120			149	0
2006	15	31	45	60	74	89	105	120			149	0
2007	15	31	45	60	74	89	105	-			149	0
2007	15	31	45	60	74	89	105	-			149	0
2007	15	31	45	60	74	89	105				149	0
2007	15	31	45	60	74	89	105				149	0
2007	15	31	45	60	74	89	105				149	0
2007	15	31	45	60	74	89	105				149	0
2007	15	31	45	60	74	89	105				149	0
2007	15	31	45	60	74	89	105				149	0
2007	15	31	45	60	74	89	105				149	0
2007	15	31	45	60	74	89	105				149	0
2007	15	31	45	60	74	89	105				149	0
2007	15	31	45	60	74	89	105				149	0
2008	250	500	500	500	500	500	-				500	500
2008	15	31	45	60	74	89	-				149	0
2008	15	31	45	60	74	89					149	0
2008	15	31	45	60	74	89					149	0
2008	15	31	45	60	74	89					149	0
2008	15	31	45	60	74	89					149	0
2008	15	31	45	60	74	89					149	0
2008	15	31	45	60	74	89					149	0
2008	15	31	45	60	74	89					149	0
2008	15	31	45	60	74	89					149	0
2008	15	31	45	60	74	89					149	0
2008	15	31	45	60	74	89					149	0

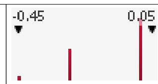
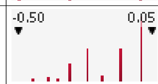
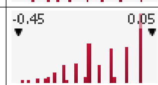
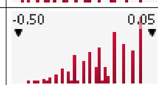
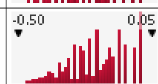
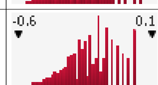
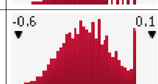
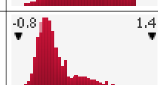
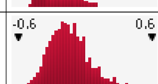
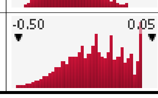
Year	1	2	3	4	5	6	7	8	9	10	True	Large
2009	15	31	45	60	74	-					149	0
2009	15	31	45	60	74	-					149	0
2009	15	31	45	60	74						149	0
2009	15	31	45	60	74						149	0
2009	15	31	45	60	74						149	0
2009	15	31	45	60	74						149	0
2009	15	31	45	60	74						149	0
2009	15	31	45	60	74						149	0
2009	15	31	45	60	74						149	0
2009	15	31	45	60	74						149	0
2009	15	31	45	60	74						149	0
2009	15	31	45	60	74						149	0
2010	15	31	45	60	-						149	0
2010	15	31	45	60	-						149	0
2010	15	31	45	60							149	0
2010	15	31	45	60							149	0
2010	15	31	45	60							149	0
2010	15	31	45	60							149	0
2010	15	31	45	60							149	0
2010	15	31	45	60							149	0
2010	15	31	45	60							149	0
2010	15	31	45	60							149	0
2010	15	31	45	60							149	0
2010	15	31	45	60							149	0
2010	15	31	45	60							149	0
2010	15	31	45	60							149	0
2010	15	31	45	60							149	0
2011	250	500	500	-							500	500
2011	15	31	45	-							149	0
2011	15	31	45	-							149	0
2011	15	31	45	-							149	0
2011	15	31	45	-							149	0
2011	15	31	45	-							149	0
2011	15	31	45	-							149	0
2011	15	31	45	-							149	0
2011	15	31	45	-							149	0
2011	15	31	45								149	0
2011	15	31	45								149	0
2012	15	31	-								149	0
2012	15	31	-								149	0
2012	15	31	-								149	0
2012	15	31	-								149	0
2012	15	31	-								149	0
2012	15	31	-								149	0
2012	15	31	-								149	0
2012	15	31	-								149	0
2012	15	31	-								149	0
2012	15	31	-								149	0
2012	15	31	-								149	0
2012	15	31	-								149	0
2013	150	-									500	0
2013	15	-									149	0
2013	15	-									149	0
2013	15	-									149	0
2013	15	-									149	0
2013	15	-									149	0
2013	15	-									149	0
2013	15	-									149	0
2013	15	-									149	0
2013	15	-									149	0

Appendix B

Selected results of a simulation are shown below. Rows labeled Meth_X refer to the results of applying Method-X, Rows labeled Meth_A refer to the results of applying Method-A. Rows labeled "Total" refer to all accident years combined, and rows labeled"Total_04_11" refer to results from combining all accident years except for the two most recent.

Table 18

Name	Graph	Min	Mean	Max	5%	95%
Number of Large Losses		-	4	13	1	7
Amount of Losses 450K or more on the evaluation date.		-	1,691	5,455	500	3,000
Meth A-2005-error		-43%	3%	74%	-25%	34%
Meth A-2006-error		-46%	0%	70%	-28%	32%
Meth A-2007-error		-44%	3%	96%	-30%	43%
Meth A-2008-error		-46%	3%	112%	-31%	51%
Meth A-2009-error		-48%	4%	135%	-32%	61%
Meth A-2010-error		-53%	-1%	129%	-37%	60%
Meth A-2011-error		-57%	8%	244%	-40%	93%
Meth A-2012-error		-61%	5%	324%	-43%	112%
Meth A-2013-error		-54%	6%	253%	-39%	104%
Meth A-Total-error		-44%	3%	86%	-25%	37%
Meth A-2004-2011-error		-43%	2%	76%	-25%	33%

Meth X-2005-error		-43%	-10%	0%	-25%	0%
Meth X-2006-error		-46%	-11%	0%	-33%	0%
Meth X-2007-error		-44%	-13%	0%	-32%	0%
Meth X-2008-error		-46%	-14%	0%	-33%	0%
Meth X-2009-error		-48%	-17%	0%	-36%	0%
Meth X-2010-error		-53%	-20%	0%	-39%	0%
Meth X-2011-error		-57%	-23%	0%	-42%	-2%
Meth X-2012-error		-61%	-16%	122%	-44%	39%
Meth X-2013-error		-54%	6%	253%	-39%	104%
Meth X-Total-error		-52%	-13%	49%	-35%	14%
Meth X-2004-2011-error		-50%	-18%	0%	-36%	-1%

Appendix C– Mathematical Formulae

Much of the following is taken from ***The Analysis and Estimation of Loss & ALAE Variability: A Summary Report*** written by the CAS Working Party on Quantifying Variability in Reserve Estimates.

The row dimension is the annual period by which the loss information is subtotaled, most commonly an accident year or policy year. For each accident period, w, the (w, d) element of the array is the total of the loss information as of development age d. Here the development age is expressed as the number of time periods after the accident or policy year. For example, the loss statistic for accident year 2 as of the end of calendar year 4 has development age 3 years.

For this discussion, we assume that the loss information available is an "upper triangular" subset of the two-dimensional array for rows $w = 1,2,\ldots,n$. For each row, w, the information is available for development ages 1 through $n - w+1$. If we think of year as the latest accounting year for which loss information is available, the triangle represents the loss information as of accounting dates 1 through n. The "diagonal" for which $w + d = k$, a constant, represents the loss information for each accident period w as of accounting year k.

The creation of simulated losses within this paper assumes that unlimited loss development is known exactly. An initial loss is chosen at d=1 year. Losses at subsequent ages are found by multiplying by the incremental development factors. If the loss exceeds the retention at any age then, then the limited loss is set at the retention. The following table provides the mathematical formulae for calculating simulated limited losses.

$$c(w, i, d+1) = \mathrm{MIN}[c(w, i, d)*f^{T}(d), \text{retention}] \qquad \text{(E.1)}$$

$$U(w, i) = \mathrm{MIN}[c(w, i, 1)*F^{T}(1), \text{retention}] \qquad \text{(E.2)}$$

w: The accident year

d: The age of the losses. If the accident year is 1/1/2010 to 12/31/2010, then d=1 at 12/31/2010 and d=2 at 12/31/2011

i: denotes an occurrence within an accident year.

$c(w, d)$: cumulative loss from accident (or policy) year w as of age d.

$c(w, i, d)$: cumulative loss from the i^{th} occurrence in accident (or policy) year w as of age d .

$c(w, d) = \sum_{i} c(w, i , d)$.

$c\,(w, i, \infty) = U\,(w, i)$**:** ultimate loss from the i^{th} occurrence in accident year w.

f(*d*): factor applied to c(*w*, *i*, *d*) or *c*(*w*, *d*) to estimate c(*w*, *i*, *d*+1) or c(*w*, *d* +1) respectively.

F(*d*) : factor applied to c(*w*, *i*, *d*) or *c*(*w*, *d*) to estimate c(*w*, *i*, *n*) or c(*w*, *n*) respectively.

$f^T(d)$: true value of f(*d*) for unlimited losses.

$F^T(d)$: true value of F(*d*) for unlimited losses.

U(*w*, *i*)**:** ultimate loss for the i^{th} occurrence in accident year *w*.

$U(w) = \sum_i U(w, i)$: The ultimate loss for accident year *w*.

Throughout this paper, losses are expressed in thousands (000), or "K", the retention = $500K. Also, *w* is the accident year, *d* is the age of the accident year, and *i* refers to a particular occurrence within the accident year. The values of F(*d*) and f(*d*) are cumulative and incremental LDFs respectively, and will vary depending on the triangle and methods used in their calculation. The true LDFs, $f^T(d)$ and $F^T(d)$ are defined by the following table (rounded). For more information on notation, please see Appendix C.

Table 19

d	1	2	3	4	5	6	7	8	9	10
$f^T(d)$	2.055	1.475	1.315	1.240	1.200	1.175	1.145	1.125	1.110	1.000
$F^T(d)$	9.964	4.849	3.287	2.500	2.016	1.680	1.430	1.249	1.110	1.000

Here are some examples:

c(2010, 3, 1) = 15 means that the 3rd occurrence in accident year 2010 has a value of 15K at an age of 1.

c(2010, 3, 2) = MIN[c(2010, 3, 1) * f^T (1), 500] =15*2.055 = 30.825 is the incurred value at age 2 for the 3rd occurrence

c(2010, 3, 3) = MIN[c(2010, 3, 2)*f^T (2), 500] = 30.825*1.475 ≈ 45.467 , the incurred value at age 3.

U(2010, 3) = c(2010, 3, 1)*F^T (1) = 15*9.964≈ 148.466. This is the ultimat e value of the third incurred loss in accident year 2010. Note that none of the incurred values for the 3rd occurrence in accident year 2010 will ever exceed the retention.

c(2011, 1, 1) = 250 means that the first occurrence in accident year 2011 has a value of 250K at age *d*=1.

c(2011, 1, 2) = MIN[c(2011, 1, 1)*f^T (1), 500] = MIN[250*2.055, 500] = 500.

Furthermore, f(2011, 1, 3)=500 and U(2011, 1) = 500 because once an incurred loss hits the retention it stays there for subsequent ages.

Examples of these calculations are included implicitly in Appendix A[9]

c(*w*,*d*) is the value of the incurred triangle in the row corresponding to accident year *w*, and column *d* where $c(w,d) = \sum_i c(w,i,d)$ = the sum of all occurrences in accident year *w* at age *d*. See Appendix A for some actual simulated liabilities.

[9] The mathematical formulae make the process look more complicated than it really is. Imagine a natural method for developing losses from unlimited age-to-age factors, then limiting the losses by occurrence limits. This is what the formulae are doing.

Appendix D – Separate LDFs for large and Small Losses

The following triangle and associated development factors result from only including losses less than 450K (90% of 500K) at the time of evaluation.

Table 20- Triangle of Small Losses

Year	1	2	3	4	5	6	7	8	9	10
2004	180	370	546	717	890	1,068	1,254	1,436	1,616	1,794
2005	180	370	546	717	890	1,068	1,254	1,436	1,616	-
2006	180	370	546	717	890	1,068	1,254	1,436	-	-
2007	180	370	546	717	890	1,068	1,254	-	-	-
2008	165	339	500	658	816	979	-		-	-
2009	180	370	546	717	890	-			-	-
2010	225	462	682	897	-				-	-
2011	150	308	455	-	-	-	-	-	-	-
2012	180	370	-							
2013	285	-								

Table 21-LDFs for Small Losses

d	1	2	3	4	5	6	7	8	9
f(*d*)	2.06	1.48	1.32	1.24	1.20	1.18	1.15	1.13	1.11
F(*d*)	9.96	4.85	3.29	2.50	2.02	1.68	1.43	1.25	1.11

Note that the removal of the large losses allowed for calculation of the true LDFs as shown in Table 2, and that these LDFs are significantly higher than those calculated with truncated losses.

The next triangle and associated development factors result from only including losses less than 450K *or more* at the time of evaluation.

Table 22 - Triangle of Large Losses

Year	1	2	3	4	5	6	7	8	9	10
2004									-	
2005	-	-	-	-	-	-	-	-	-	-
2006	400	808	955	1000	1000	1000	1000	1000	-	-
2007	-	-	-	-	-	-	-	-	-	-
2008	250	500	500	500	500	500	-	-	-	-
2009						-	-	-	-	-
2010	-	-	-	-	-	-	-	-	-	-
2011	250	500	500	-	-	-	-	-	-	-
2012	-	-	-	-	-	-	-	-	-	
2013	-	-	-	-	-	-	-	-	-	

Table 23- LDFs for Large Losses

d	1	2	3	4	5	6	7
f(*d*)	2.01	1.08	1.03	1.00	1.00	1.00	1.00
F(*d*)	2.24	1.11	1.03	1.00	1.00	1.00	1.00

We will now apply these LDFs separately to large and small losses.

Table 24-Chain Ladder for Large and Small Losses Separately

Accident Year	Age - *d*	Small Losses	F(*d*)	Ultimate Part-1		Large Losses	F(*d*)	Ultimate Part-2
		(a)	(b)	'(c)		(d)	(e)	(f)
		Table 17	Table 18	=(a)*(b)		Table 19	Table 20	=(d)*(e)
2004	10	1,794	1.00	1,794		-	1.00	-
2005	9	1,616	1.11	1,794		-	1.00	-
2006	8	1,436	1.25	1,794		1,000	1.00	1,000
2007	7	1,254	1.43	1,794		-	1.00	-
2008	6	979	1.68	1,644		500	1.00	500
2009	5	890	2.02	1,794		-	1.00	-
2010	4	897	2.50	2,242		-	1.00	-
2011	3	455	3.29	1,495		500	1.03	516
2012	2	370	4.85	1,794		-	1.11	-
2013	1	285	9.96	2,840		-	2.24	-
Total		**9,975**		**18,982**		**2,000**	**11**	**2,016**
2004-2011		**9,320**		**14,349**		**2,000**	**8**	**2,016**

Table 25-Error as a Percentage of IBNR

Accident Year	Small+Large Ult	True Ult	Difference	Error as % of IBNR
	(g)	(h)	(i)	(c)
	(c)+(f)	calc		(i)/[(h)-(a)-(d)]
2004	1,794	1,794	-	0%
2005	1,794	1,794	-	0%
2006	2,794	2,794	-	0%
2007	1,794	1,794	-	0%
2008	2,144	2,144	-	0%
2009	1,794	1,794	-	0%
2010	2,242	2,242	-	0%
2011	2,010	1,995	16	1%
2012	1,794	1,794	-	0%
2013	2,840	1,845	995	64%
Total	20,998	19,988	1,010	13%
2004-2011		**16,349**	**16**	**0%**

Note that the result is nearly perfect. The only year that is off significantly is 2013. That year is "off" because there is a loss in 2013 that will reach over 500K, but has not yet been detected as a large loss. This paper is not designed to explore methods of separating large and small losses, or to prove that separating the two is always better.

Appendix E– Sensitivity

This appendix explores the effects of altering the LDFs, and altering the percentage of large (limited) losses in relationship to ultimate losses. For the tables below only 5000 simulations were used in each of the nine scenarios. In this appendix, a slightly different method is used for identifying large losses. If the value at time *d* multiplied by F(*d*) is larger than 500K, then in Method-X, the loss for that occurrence is limited to 500K. The ratio of large to small large losses was changed by altering the value of the static small losses. It can be seen that the magnitude of the errors changes, but the fact that Method-X is biased toward low estimates is unchanged.

Table 26- All Years Combined – Sensitivity of Mean Error to LDF and Percentage of Large Losses

		Highest LDF		**High LDF**		**Moderate LDF**	
Ratio of Large Losses to Total Losses - Ultimate Limited Basis	**Method**	**Mean Error as % of IBNR**	**Mean Error as % of Ultimate**	**Mean Error as % of IBNR**	**Mean Error as % of Ultimate**	**Mean Error as % of IBNR**	**Mean Error as % of Ultimate**
15%	A	7%	3%	4%	1%	2%	0%
	X	-33%	-12%	-23%	-6%	-16%	-3%
10%	A	4%	1%	2%	1%	1%	0%
	X	-25%	-10%	-16%	-5%	-12%	-2%
5%	A	1%	1%	1%	0%	0%	0%
	X	-14%	-6%	-9%	-3%	-6%	-1%

Table 27- 2004-2011 – Sensitivity of Mean Error to LDF and Percentage of Large Losses

		Highest LDF		**High LDF**		**Moderate LDF**	
Ratio of Large Losses to Total Losses - Ultimate Limited Basis	**Method**	Mean Error as % of IBNR	Mean Error as % of Ultimate	Mean Error as % of IBNR	Mean Error as % of Ultimate	Mean Error as % of IBNR	Mean Error as % of Ultimate
15%	**A**	5%	1%	3%	1%	2%	0%
	X	-26%	-7%	-20%	-4%	-18%	-2%
10%	**A**	2%	1%	1%	0%	1%	0%
	X	-18%	-6%	-14%	-3%	-12%	-2%
5%	**A**	1%	0%	0%	0%	0%	0%
	X	-10%	-3%	-8%	-2%	-6%	-1%

The LDFs used in the sensitivity analysis are as follows.

Table 28 – Highest LDFs

d	1	2	3	4	5	6	7	8	9	10
f(*d*)	2.055	1.475	1.315	1.240	1.200	1.175	1.145	1.125	1.110	1.000
F(*d*)	9.964	4.849	3.287	2.500	2.016	1.680	1.430	1.249	1.110	1.000

Table 29 – High LDFs

d	1	2	3	4	5	6	7	8	9	10
f(*d*)	1.541	1.263	1.179	1.138	1.116	1.102	1.085	1.073	1.065	1.000
F(*d*)	3.973	2.579	2.042	1.733	1.523	1.365	1.239	1.143	1.065	1.000

Table 30 – Moderate LDFs

d	1	2	3	4	5	6	7	8	9	10
f(*d*)	1.296	1.150	1.104	1.081	1.068	1.060	1.050	1.043	1.038	1.000
F(*d*)	2.289	1.766	1.536	1.391	1.287	1.205	1.137	1.083	1.038	1.000

5. REFERENCES

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[2] CAS Working Party on Quantifying Variability in Reserve Estimates, "The Analysis and Estimation of Loss & ALAE Variability: A Summary Report", *Casualty Actuarial Society Forum, Fall,* **2005**

[3] Daley, Tom, "Catastrophes in Workers Compensation Ratemaking," *Casualty Actuarial Forum Winter* , **2007,** 1-42

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[5] Klemmt, Heinz-Jurgen, "Separation of Claims Development Triangles into Basic Losses and Large Losses," *Blatter DGVFM Deutsche Gesellschaft fur versicherungs und finanzmathematik e.V. – Germany,* **2005***: Band XXVII, Heft 1,* 49-58.

[6] Pinto, Emanuel, Gogol, Daniel F. "An analysis of Excess Loss Development," *PCAS*, **1987**, *Vol. LXXIV, Part2, No. 142*, 227-255.

Calculations performed using @RISK Standard version 5.0, from Palisade Corporation, Ithaca, NY, U.S.A.

Abbreviations and notations

K, one thousand
LDF, loss development factor
w, accident year
F(d) cumulative LDF applicable to accident year at age *d*
(000) number in thousands
CL, Chain Ladder Method
d, delay or age of accident year
f(d) incremental LDF applicable to accident year at age *d*

Biography of the Author

Karen Adams is the P&C Actuary at the Arizona Department of Insurance. She is responsible for reserving, capital modeling, pricing, and review of applications. She has a BS in Mathematics from the Virginia Tech, and an MS in Mathematics from the University of Houston. She is an Associate of the CAS and a Member in good standing of the American Academy of Actuaries.

KAdams@AZInsurance.gov, 602.364.3249.

Structured Tools to Help Organize One's Thinking When Performing or Reviewing a Reserve Analysis

Jennifer Cheslawski Balester, FCAS, MAAA

Gerald S. Kirschner, FCAS, MAAA

Abstract

Actuaries regularly update the results of prior analyses that leverage more current information. Actuaries will often apply similar methodologies and thought processes from the prior analysis to the current one. In doing so, actuaries are employing techniques that help them to evaluate the reasonability of prior assumptions as compared to the most recent data emergence and using judgment to update assumptions ranging from selection of loss development factors to initial expected loss rates to point estimates or ranges of ultimate losses. However, actuarial literature to date provides little guidance on the questions that one can ask during each of these steps and the calculations that can be done to help bring a repeatable rigor to the analysis being done.

This paper will identify three distinct series of exercises that can be performed to help bring just such a repeatable rigor to the analysis. Along the way, the exercises will help the actuary frame answers to the following questions:

1. How did losses emerge between the prior review and the current review in relation to what was expected to emerge?
2. Are the selected loss development factors (LDFs) generally in line with the patterns in the underlying data triangles?
3. What is driving the change in ultimate loss estimates from the prior to the current analysis? Is it data (i.e., loss emergence), change of assumptions (i.e., loss development factors or initial expected loss rates), or change in judgment (i.e., the manner in which a point ultimate is chosen relative to a paid or incurred ultimate loss projection for a given accident period)?

By giving actuaries a structured and repeatable methodology to apply in search of answers to these questions, we are providing actuaries with a framework that will bring them a structure to their analyses and help them to identify areas in their analyses that might benefit from further investigation and study.

Keywords: reserving, suitability testing, data diagnostics

1. INTRODUCTION

The approach described in this paper was developed over several years of working as actuarial consultants and training actuarial students in how to perform an actuarial analysis in a way that considers the work that was done before. It was very easy to hand an actuarial student a client project that they had not worked on before and ask that they update the study, only to have the student do so in a very mechanical way that did not engender the student asking insightful questions about where and why things might have changed from the prior study to the current one.

To remedy this gap, we developed a series of three structured processes that were intended to stimulate critical thinking about the data and the current analysis in a way that would lead the student to identify potential data issues, pattern changes, or other things that would benefit from deeper investigation before concluding the current analysis.

The three structured processes are:

1. Review of Actual versus Expected loss emergence
2. Review of selected loss development factors relative to the factors indicated by the data without an overlay of actuarial judgment
3. A calculation of the source of change between prior and current ultimate loss selections, broken down into three subsets: data, assumptions, and judgment

2. REVIEW OF ACTUAL VERSUS EXPECTED LOSS EMERGENCE

We assume that actuaries are doing an "actual versus expected" study as part of updating an actuarial study. We also assume, unless specifically stated, that there have been no changes to claims handling or case reserving practices. Our methodology employs two actual versus expected calculations and asks questions about the results. Our thought process going into this structured process is to enable us to comment on the following questions:

1. How have the assumptions and conclusions reached in the prior reserve analyses held up when compared to the most recent claims emergence?
2. Are there any significant differences between the actual versus expected results for incurred versus paid claims emergence?
3. Are there any significant differences between the actual versus expected results for direct versus indirect expected claims projections?
4. If the current claims activity is in line with the prior projection, we might reasonably expect current assumptions and ultimate losses to be close to prior assumptions and ultimate losses. Are they?

The two calculation methods we employ in this structured process are "direct" emergence and "indirect" or "percent of reserves" emergence.

2.1 Direct Emergence Method

The formula for calculating expected cumulative incurred losses[1] at time t for Accident Year X using the Direct Emergence Method between time t-1 and time t is as follows:

$$\begin{pmatrix}\text{Cumulative Incurred Losses} \\ \text{@ time } t\text{ - }1 \text{ for Accident Year X}\end{pmatrix} \times \left(\frac{\begin{matrix}\text{Cumulative Development Factor} \\ \text{@ time } t\text{ - }1 \text{ for Accident Year X}\end{matrix}}{\begin{matrix}\text{Cumulative Development Factor} \\ \text{@ time } t \text{ for Accident Year X}\end{matrix}} \right) \tag{1.1}$$

The following table, Table 1, provides an example of the calculation of Direct Emergence Expected Cumulative Losses. The example assumes time t-1 was 12/31/2011 and time t is 12/31/2012.

[1] The methodology for calculating expected emergence of cumulative paid losses is identical to what is shown in the cumulative incurred loss expected emergence formula, except that cumulative paid losses and paid loss development factors are used in place of cumulative incurred losses and incurred loss development factors.

Table 1: Example of Direct Emergence Expected Cumulative Loss Calculation

Accident Year	Current Age	Prior Age	Cumulative Incurred Losses at 12/31/2011	Cumulative Development Factor (CDF) from Prior Actuarial Study	CDF Interpolated to Current Claim Age[2]	Expected Cumulative Incurred Losses at 12/31/2012
			(1)	(2)	(3)	(4) = (1) * (2) / (3)
2004	108	96	621	1.025	1.012	629
2005	96	84	1,468	1.046	1.025	1,498
2006	84	72	1,283	1.072	1.046	1,315
2007	72	60	1,064	1.104	1.072	1,096
2008	60	48	1,510	1.181	1.104	1,615
2009	48	36	857	1.264	1.181	917
2010	36	24	847	1.706	1.264	1,143
2011	24	12	108	22.182	1.706	1,404
TOTAL			**7,758**			**9,618**

2.2 Indirect (Percent of Reserves) Emergence Method

The formula for calculating expected cumulative incurred losses at time *t* for Accident Year X

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[2] The CDF for the oldest loss year cannot be interpolated from the CDFs calculated in the prior study. Instead, the CDF must be extrapolated from the decay pattern in the CDFs in the prior study. The methodology used to derive the 1.012 value was to (a) calculate the rate of change in the three oldest CDFs in Column (2); (b) fit an exponential curve to the resulting rates of change using Excel's "Growth" function; (c) extrapolate the fitted exponential curve one time period into the future; and (d) apply the extrapolated value to the 1.025 value from column (2). The mathematics of this process were as follows:

Accident Year	Current Age	Prior Age	CDF from Prior Actuarial Study	CDF from Prior Actuarial Study - 1	Rate of Change in Column (2) Values	Fitted Rate of Change using Current Age as "X" Value	Extrapolated CDF
			(1)	(2)	(3)	(4)	(5)
2004	108	96	1.025	0.025		0.497	(0.025 * 0.497) + 1 = 1.012
2005	96	84	1.046	0.046	0.025 / 0.046 = 0.551	0.557	
2006	84	72	1.072	0.072	0.046 / 0.072 = 0.636	0.623	
2007	72	60	1.104	0.104	0.072 / 0.104 = 0.691	0.698	

using the Indirect Emergence Method between time *t*-1 and time *t* is as follows:

$$\left\{ \begin{pmatrix} \text{Selected IBNR @} \\ \text{time } t\text{ - }1 \text{ for} \\ \text{Accident Year X} \end{pmatrix} \times \left[\frac{\begin{pmatrix} \text{Percent Incurred} \\ \text{@ time } t \text{ for} \\ \text{Accident} \\ \text{Year X} \end{pmatrix} - \begin{pmatrix} \text{Percent Incurred} \\ \text{@ time } t\text{ - }1 \text{ for} \\ \text{Accident} \\ \text{Year X} \end{pmatrix}}{\begin{matrix} 1 - \text{Percent Incurred @} \\ \text{time } t\text{ - }1 \\ \text{for Accident Year X} \end{matrix}} \right] \right\} + \begin{pmatrix} \text{Cumulative} \\ \text{Incurred} \\ \text{Losses} \\ \text{@ time } t\text{ - }1 \\ \text{for Accident} \\ \text{Year X} \end{pmatrix} \qquad (1.2)$$

Note: A CDF is converted into a percent incurred factor by taking the reciprocal of the CDF, i.e., the percent incurred at a given loss year age equals (1 / CDF).

Note 2: When applying this formula to paid losses, instead of using IBNR as shown in formula (1.2), use unpaid losses.

The following table provides an example of the calculation of Indirect Emergence Expected Cumulative Losses.

Table 2: Example of Indirect Emergence Expected Cumulative Loss Calculation

Accident Year	Current Age	Prior Age	Cumulative Incurred Losses at 12/31/11	Selected IBNR at 12/31/11	Percent Incurred at Prior Age	Percent Incurred at Current Age	Expected Cumulative Incurred Losses at 12/31/2012
			(1)	(2)	(3) *	(4) **	(5) ***
2004	108	96	621	0	97.6%	98.8%	621
2005	96	84	1,468	50	95.6%	97.6%	1,490
2006	84	72	1,283	67	93.3%	95.6%	1,306
2007	72	60	1,064	86	90.6%	93.3%	1,089
2008	60	48	1,510	240	84.7%	90.6%	1,602
2009	48	36	857	443	79.1%	84.7%	975
2010	36	24	847	703	58.6%	79.1%	1,195
2011	24	12	108	1,417	4.5%	58.6%	911
TOTAL			**7,758**	**3,006**			**9,190**

* Values in column (3) equal 1 / value in Table 1, Column (2).
** Values in column (4) equal 1 / value in Table 1, Column (3).
*** Values in column (5) equal $\left\{(2)\times\left[\frac{(4)-(3)}{1-(3)}\right]\right\}+(1)$

2.3 Comparing Direct and Indirect Expected Results Using a Simplified Example

If ultimate losses are selected to be exactly equal to the direct development ultimate loss value, there will be no difference in actual versus expected results under either method. Differences only arise when selected ultimate losses are different than the direct development ultimate loss value.

This can be demonstrated with the following simplified data and example:

Table 3: A Priori Expected Loss Emergence Pattern

Development Age	0 – 1	1 – 2	2 – 3	Total
Incremental Loss Emergence	1,000	500	250	
Cumulative Loss Emergence	1,000	1,500	1,750	1,750

Table 4: Selected Loss Development Pattern Based on A Priori Expected Loss Emergence Pattern

Development Age	0 – 1	1 – 2	2 – 3
Incremental LDF	n/a	1.500	1.167
Cumulative LDF	n/a	1.750	1.167
Percent Incurred	57.1%	85.7%	100.0%

Table 5: Direct versus Indirect Expected Loss Emergence @ Time 2 – assumes that incurred losses at Time 1 = 1,000 and selected ultimate losses = 1,750

Development Age	Selected Ultimate Loss	Actual Cumulative Incurred Losses @ Time 1	Expected Cumulative Incurred Losses @ Time 2
Direct Expected Loss Emergence	1,750	1,000	1000 * (1.750 / 1.167) = 1,500
Indirect Expected Loss Emergence	1,750	1,000	$750 * \frac{(0.857 - 0.571)}{1 - 0.571} + 1000 = 1{,}500$

Table 5 demonstrates that when selected ultimate losses exactly equal the direct development projection of ultimate losses, the direct and indirect expected loss emergence calculations produce equivalent results. Table 6 shows what happens when selected ultimate losses are not exactly equal to the direct development projection of ultimate losses. For Table 6, we change the example as follows:

Incurred losses at Time 1 = 1,400

Direct Development Ultimate Loss Projection = 1,400 * 1.750 = 2,450

Selected Ultimate Losses = 2,000

Table 6: Direct versus Indirect Expected Loss Emergence @ Time 2 – assumes that incurred losses at Time 1 = 1,400 and selected ultimate losses = 2,000

Development Age	Selected Ultimate Loss	Actual Cumulative Incurred Losses @ Time 1	Expected Cumulative Incurred Losses @ Time 2
Direct Expected Loss Emergence	2,000	1,400	1400 * (1.750 / 1.167) = 2,100
Indirect Expected Loss Emergence	2,000	1,400	$600 * \frac{(0.857 - 0.571)}{1 - 0.571} + 1400 = 1{,}800$

As can be seen in Table 6, when selected ultimate losses do not equal the direct development ultimate loss projection, the direct and indirect expected loss calculation produce different expected loss amounts in the projected time period.

2.4 Interpreting Actual versus Expected Results from Simplified Example

Continuing with the simplified example from Table 6, suppose the actual incurred loss amount at Time 2 was 2,000. As shown in Table 7 below, our direct development actual versus expected result shows actual losses are $100 below expected and our indirect development actual versus expected result shows actual losses to be $200 above expected.

Table 7: Actual versus Direct and Indirect Expected Loss Emergence @ Time 2 – assumes that incurred losses at Time 1 = 1,400 and selected ultimate losses = 2,000

Development Age	Expected Cumulative Incurred Losses @ Time 2	Actual Cumulative Incurred Losses @ Time 2	Actual minus Expected Losses
Direct Expected Loss Emergence	1400 * (1.750 / 1.167) = 2,100	2,000	(100)
Indirect Expected Loss Emergence	$600 * \frac{(0.857 - 0.571)}{1 - 0.571} + 1400 = 1{,}800$	2,000	200

Focusing first on the direct development result, we can interpret the result to mean that actual losses have not emerged as quickly as expected. This might reasonably lead us to conclude that any subsequent development will also be lower than what we would have expected from the selected loss development pattern underlying the direct development calculation. This would lead us to consider a new selected ultimate loss that is something less than the direct development ultimate loss projection based on actual loss emergence through Time 2 (= $2,000 * 1.167 = $2,333). Alternatively, in this example, we might argue that we saw higher than expected loss emergence during Time 1 and lower than expected loss emergence during Time 2 and going forward, we will return to a loss emergence pattern that is more consistent with the historical expectations for Time 3 and beyond than what we have seen for Times 1 and 2. This counter-argument would be a reason to select $2,333 as our new ultimate loss indication.

Turning next to the indirect development result, we can interpret the result to mean that actual losses have emerged more quickly than our selected ultimate loss pick would have led us to expect. This might reasonably suggest that our selected ultimate loss pick was too low and, given what we now know, should be increased. When taking this information in conjunction with the observation that the direct development expectation for Time 2 was higher than the actual loss emergence in Time 2, we might consider selecting a new ultimate loss estimate that is higher than the $2,000 that we chose in the prior actuarial analysis but is not as high as is indicated by the current direct development ultimate loss projection ($2,000 * 1.167 = $2,333) because we think the remaining loss emergence will follow the Time 2 pattern where actual losses emerge lower than the direct development expectation.

We can summarize our analysis methodology from this section as providing actuaries with tools to critically consider how well the ultimate loss picks from prior years' reviews are holding up when compared to actual loss emergence in the most recent time period and give guidance for the direction and magnitude by which we might want to adjust ultimate loss selections.

2.5 Actual vs. Expected Results for Original Example

We can now return to the original example from Tables 1 and 2 and compare actual loss emergence to the direct and indirect expected emergence.

Table 8: Actual vs. Expected Loss Emergence for Original Example

	Expected Loss	Actual Loss	Actual - Expected
	(1)	(2)	(2) – (1)
Direct Method	9,618	9,458	(160)
Indirect Method	9,190	9,458	268

We can see in Table 8 that actual losses have emerged $160 below expectations on a direct basis but $268 above expectation on an indirect basis. The lower than expected emergence on a direct basis implies that the selected loss development factors may be too high, as losses projected to emerge in the period were higher than losses that actually emerged. However, the higher than expected emergence on an indirect basis implies that the selected ultimates might be low. The direct method is independent of the prior selected ultimate losses and uses only the cumulative incurred losses and selected loss development pattern while the indirect method uses the cumulative incurred losses, selected loss development pattern, and the prior selected ultimate losses. Understanding this, the actuary might want to consider decreasing loss development factors but increasing initial expected losses or selecting ultimate losses based on higher methods.

2.6 Considerations When Assessing the Direct and Indirect Expected Loss Emergence Results

As was noted in Section 1.3, when ultimate losses are selected to be exactly equal to the direct development ultimate loss value, there will be no difference in actual versus expected results under either method. Differences only arise when selected ultimate losses are different than the direct development ultimate loss value. With this understanding of the driver of differences in Direct versus Indirect results, we can better evaluate the meaning of the results being produced by the two methods.

1. The Indirect method incorporates a judgmental element that the direct method does not, namely the selected ultimate loss value from the prior analysis. The Indirect Actual versus Expected result provides us with a quantitative way of assessing the consistency of the selected ultimate loss value from the prior analysis with the most recent actual loss emergence.

2. The Direct method provides us with a quantitative way of assessing the extent to which the most recent actual loss emergence is or is not consistent with the emergence pattern we believe should exist (as quantified through the emergence pattern implicit in our LDF pattern). If we know that Accident Year X losses through time t-1 were lower than (higher than) what we were expecting to see at time t-1, but we do not see the actual emergence during time t coming in higher than (lower than) the Direct method expectation, we may need to dig deeper to understand why Accident Year X's losses are coming in below (above) our a priori ultimate loss expectation. For example, is claim frequency in Accident Year X different from other accident years? Or is claim severity distorting results, as might occur if there are fewer than (more than) the expected number of large losses reported to date?

To summarize, when the Direct and Indirect methods produce results that either differ in magnitude or, as in the examples shown previously, direction, the actuary has an opportunity to think about his or her a priori ultimate loss expectations as compared to the actual data reported to date. If the actual data is deviating from the a priori expectations, is this because something structural is changing in the data, such as a change in claim frequency? Or is it because the data is inherently volatile and the differences are due to random events that do not require the actuary to change his or her long term expectations?

For example, when we see large divergences between actual and expected results when the two methods are applied to what had been blocks of business with historically stable emergence patterns, we have good reason to call into question the reliability of the actual results. In this situation, we might want to ask ourselves questions along the lines of:

- Might there be something wrong with the data we are seeing?
- Has there been a change in claims handling practices that we were not aware of that would lead to an acceleration or deceleration in claim reporting?
- Has there been a change in the way case reserves are being set?

The thought process is more complex when we are looking at blocks of business that are more volatile because we have more randomness in the data with which to contend. This does not mean we should not ask the same questions as we would ask when looking at a more stable block of business. It just means that we may need to accept more volatility in the actual versus expected results. It also means we may need to dig deeper to understand if there really is a structural change occurring as opposed to just random noise in the data. Digging deeper may mean we need to look at:

- Claim counts instead of total losses;
- Data stratifications by claim size to assess if the differences are coming from changes in the mix of large versus small losses;
- Capped versus excess losses[3]
- Historical levels of volatility in less versus more mature accident periods to evaluate if the magnitude of the differences being observed have been seen before or not.
- Adjusting the data to remove calendar year inflationary trends, if the trend rates have fluctuated significantly over the time period being used to derive the expected loss development patterns

Neither method is inherently "better" than the other. We believe maximum value is achieved when they are used in conjunction so that differences between the two methods can be identified, analyzed and understood. Additionally, we have no hard and fast rule for when ultimate loss selections should be adjusted in response to actual versus expected emergence differences. Our objective with these methods is not to provide a formulaic way to get to the "right" answer, but rather to describe tools that we have found effective at helping us identify the right questions to be asked during our analysis.

[3] When performing an actual versus expected analysis using capped losses or excess losses, an additional layer of quantitative rigor needs to be incorporated into the application of formulas 1.1 and 1.2. The expected losses being calculated need to align with the capping or claim attachment points being applied to the actual data. For example, if there are large losses in the actual data at time t-1 and the application of the Direct Development method loss development factor would cause one or more claims to exceed the selected loss cap, this excess amount needs to be removed from the Direct Development expectation.

3. REVIEW OF SELECTED LOSS DEVELOPMENT FACTORS VERSUS "LETTING THE DATA SPEAK"

Once loss development factors have been selected, a reviewer must assess the overall reasonability of the LDF selections. We split this exercise into two subsets:

1. Age to age factors for which there is historical data
2. Tail factors, where there are no (or no reliable) observable data points

3.1 Review of Age to Age Factors

The review of age to age factors is done by performing a series of sensitivity tests on the underlying data while keeping the selected tail factor (and possibly the oldest age to age factors for which there is limited data) unchanged. The objective of these sensitivity tests is to assess the extent to which the selected LDFs are in line with the patterns in the data. However, having the selected LDFs in line with the patterns in the data does not necessarily mean the selected LDFs are reasonable. There could be numerous reasons that the selected LDFs should not be in line with the patterns in the data. For example, the data might contain large loss distortions that should be ignored or smoothed when selecting LDFs. Another example is changes in business mix in the historical data that is driving a change in the loss emergence pattern. In this case, the history might not be reasonably reflective of the current book of business.

These questions serve to highlight that the true importance of this test is not if the selected LDFs align or do not align with the historical patterns; rather it is so the reviewer can think about what the selected LDFs ought to look like as compared to the historical patterns and assess if the selections are consistent with his/her expectations.

The sensitivity testing compares the selected LDFs to the LDFs that would be indicated by different calculated averages. The calculated averages to use for sensitivity comparison should include different time periods (e.g., 3 year average, 5 year average) and different weighting schemes (e.g., 5 ex hi/lo, highest or 2nd highest (lowest) of the last five, weighted versus straight averages). This range of weighting schemes will include some combinations that can reasonably be expected to be biased high (such as the highest of the last five observations) and others that we expect will be biased low (such as the lowest of the last five or the 5 ex hi/lo average[4]). By including weighting schemes that will tend to be biased in one direction or the other, the selected LDFs can be compared to a sufficiently wide range of LDF alternatives to facilitate a comparison of the selected LDFs to the unadjusted patterns present in the data alone.

Additionally, we can tie our Section 1 analysis into this analysis. Where the most recent diagonal of claim emergence differs from what was expected to emerge, this difference will be reflected the most recent diagonal of LDFs. By including this most recent diagonal in the various LDF averages being calculated for comparison against the selected LDFs, we are implicitly factoring into this section's analysis our Direct Development actual versus expected results so that we can further assess how we might want to adjust our new LDF picks in response to the actual versus expected results.

For this example, we will use the following table of incurred losses:

Table 9: Incurred Loss Data Triangle (Dollars in Thousands)

Accident	Development Age								
Year	12	24	36	48	60	72	84	96	108
2004	49	402	504	570	569	624	652	621	621
2005	37	1,297	1,529	1,448	1,384	1,423	1,468	1,452	
2006	122	777	988	1,086	1,300	1,283	1,232		
2007	137	804	935	888	1,064	1,131			
2008	57	751	1,407	1,510	1,759				
2009	56	830	857	850					
2010	38	847	1,122						
2011	108	1,291							
2012	114								

[4] For discussion of the downward bias in the 5 ex hi/lo average, see "Downward Bias of Using High-Low Averages for Loss Development Factors" by Cheng-Sheng Peter Wu, Casualty Actuarial Society Summer 1997 Forum, Volume 1, pages 197-240 and 1999 Proceedings of the Casualty Actuarial Society, Volume LXXXVI, pages 699 – 735.

The corresponding loss development factors from this data triangle are as follows:

Table 10: Incurred Loss Development Factors and Loss Development Factors Averages[5]

	Development Period								
Accident Year	**12 - 24**	**24 - 36**	**36 - 48**	**48 - 60**	**60 - 72**	**72 - 84**	**84 - 96**	**96 - 108**	**108 - Ult**
2004	8.204	1.254	1.131	0.998	1.097	1.045	0.952	1.000	
2005	35.054	1.179	0.947	0.956	1.028	1.032	0.989		
2006	6.369	1.272	1.099	1.197	0.987	0.960			
2007	5.869	1.163	0.950	1.198	1.063				
2008	13.175	1.874	1.073	1.165					
2009	14.821	1.033	0.992						
2010	22.289	1.325							
2011	11.954								
2012									
3 point average	16.355	1.410	1.005	1.187	1.026	1.012	0.971	1.000	
5 point average	13.622	1.333	1.012	1.103	1.044	1.012	0.971	1.000	
7 point average	15.647	1.300	1.032	1.103	1.044	1.012	0.971	1.000	
3 point wtd avg	14.693	1.395	1.015	1.183	1.024	1.007	0.978	1.000	
5 point wtd avg	11.422	1.324	1.012	1.104	1.033	1.007	0.978	1.000	
7 point wtd avg	11.886	1.286	1.021	1.104	1.033	1.007	0.978	1.000	
5 point ex hi/lo	13.317	1.253	1.005	1.120	1.044	1.012			
Largest LDF	35.296	1.873	1.132	1.198	1.096	1.045	0.989	1.000	
2nd largest LDF	22.131	1.324	1.099	1.197	1.063	1.032	0.953		
2nd smallest LDF	6.363	1.162	0.950	0.997	1.028	1.032	0.989		
Smallest LDF	5.876	1.032	0.947	0.955	0.987	0.960	0.953	1.000	
Selected LDF	13.000	1.400	1.070	1.070	1.030	1.020	1.015	1.007	1.005

For the remainder of this example, we replace the calculated averages for ages 84 and beyond with the selected LDF for ages 84 and beyond. Doing this provides stability to the different averages where the data is very sparse.

[5] When fewer data points are available than are needed to calculate a particular average or weighted average loss development factor, the averaging formula is adjusted to use the number of data points that are available. For example, the age 27-39 "7 point average" value is an average of the six available age 27-39 LDFs and the age 39-51 "7 point average" value is an average of the five available age 39-51 LDFs.

Table 11: Loss Development Factors Averages Being Used for Sensitivity Testing

	Development Period								
	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108	108 - Ult
3 point average	16.355	1.410	1.005	1.187	1.026	1.012	1.015	1.007	1.005
5 point average	13.622	1.333	1.012	1.103	1.044	1.012	1.015	1.007	1.005
7 point average	15.647	1.300	1.032	1.103	1.044	1.012	1.015	1.007	1.005
3 point wtd avg	14.693	1.395	1.015	1.183	1.024	1.007	1.015	1.007	1.005
5 point wtd avg	11.422	1.324	1.012	1.104	1.033	1.007	1.015	1.007	1.005
7 point wtd avg	11.886	1.286	1.021	1.104	1.033	1.007	1.015	1.007	1.005
5 point ex hi/lo	13.317	1.253	1.005	1.120	1.044	1.012	1.015	1.007	1.005
Largest LDF	35.296	1.873	1.132	1.198	1.096	1.045	1.015	1.007	1.005
2nd largest LDF	22.131	1.324	1.099	1.197	1.063	1.032	1.015	1.007	1.005
2nd smallest LDF	6.363	1.162	0.950	0.997	1.028	1.032	1.015	1.007	1.005
Smallest LDF	5.876	1.032	0.947	0.955	0.987	0.960	1.015	1.007	1.005
Selected LDF	13.000	1.400	1.070	1.070	1.030	1.020	1.015	1.007	1.005

Table 12: Cumulative Loss Development Factors Averages Being Used for Sensitivity Testing

	12 - Ult	24 - Ult	36 - Ult	48 - Ult	60 - Ult	72 - Ult	84 - Ult	96 - Ult	108 - Ult
3 point average	29.344	1.794	1.272	1.266	1.067	1.040	1.027	1.012	1.005
5 point average	21.997	1.615	1.211	1.197	1.085	1.040	1.027	1.012	1.005
7 point average	25.118	1.605	1.235	1.197	1.085	1.040	1.027	1.012	1.005
3 point wtd avg	26.062	1.774	1.272	1.253	1.059	1.034	1.027	1.012	1.005
5 point wtd avg	18.054	1.581	1.194	1.180	1.068	1.034	1.027	1.012	1.005
7 point wtd avg	18.424	1.550	1.205	1.180	1.068	1.034	1.027	1.012	1.005
5 point ex hi/lo	20.383	1.531	1.222	1.216	1.085	1.040	1.027	1.012	1.005
Largest LDF	105.476	2.988	1.595	1.409	1.176	1.073	1.027	1.012	1.005
2nd largest LDF	43.437	1.963	1.482	1.349	1.127	1.060	1.027	1.012	1.005
2nd smallest LDF	7.632	1.199	1.032	1.086	1.090	1.060	1.027	1.012	1.005
Smallest LDF	5.338	0.908	0.880	0.930	0.973	0.986	1.027	1.012	1.005
Selected LDF	22.487	1.730	1.236	1.155	1.079	1.048	1.027	1.012	1.005

To calculate ultimate losses using the different CDF averages, we take the cumulative incurred losses for each accident year at time *t* and multiply by the CDFs in Table 12.

Table 13: Projected Ultimate Losses Using Different CDF Averages from Table 12

Accident Year	2012	2011	2010	2009	2008	2007	2006	2005	2004
Incurred Loss	114	1,291	1,122	850	1,759	1,131	1,232	1,452	621
3 point average	3,345	2,316	1,428	1,076	1,877	1,176	1,266	1,469	624
5 point average	2,508	2,085	1,359	1,017	1,909	1,176	1,266	1,469	624
7 point average	2,863	2,072	1,386	1,017	1,909	1,176	1,266	1,469	624
3 point wtd avg	2,971	2,290	1,427	1,065	1,862	1,169	1,266	1,469	624
5 point wtd avg	2,058	2,041	1,339	1,003	1,879	1,169	1,266	1,469	624
7 point wtd avg	2,100	2,001	1,352	1,003	1,879	1,169	1,266	1,469	624
5 point ex hi/lo	2,324	1,976	1,371	1,033	1,909	1,176	1,266	1,469	624
Largest LDF	12,024	3,858	1,790	1,198	2,069	1,214	1,266	1,469	624
2nd largest LDF	4,952	2,534	1,663	1,147	1,982	1,199	1,266	1,469	624
2nd smallest LDF	870	1,548	1,158	924	1,917	1,199	1,266	1,469	624
Smallest LDF	609	1,173	988	790	1,712	1,115	1,266	1,469	624
Selected LDF	2,564	2,233	1,386	982	1,898	1,185	1,266	1,469	624

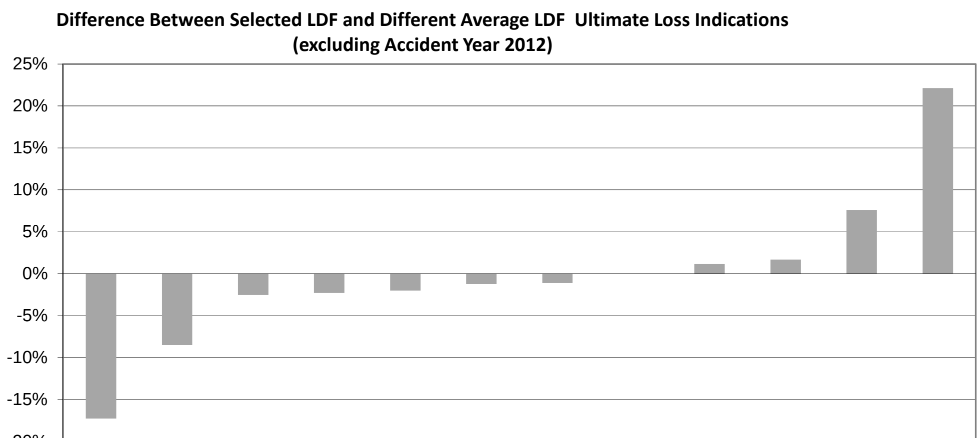
A judgmental decision is required relating to the inclusion or exclusion of the least mature accident years in the LDF comparison. Because the loss development factors being applied to the least mature accident years can contain a high degree of volatility from average to average, the analysis might benefit from excluding one or more years from consideration. In Table 14, we exclude the 2012 year for just this reason.

Table 14: Comparison of Ultimate Loss Indications Between Selected and Different Average LDF Calculations

	Total Ultimate Losses	Dollar Variance with Selected Total	Percentage Variance with Selected Total	Total Ultimate Losses ex. AY 2012	Dollar Variance with Selected Total ex. AY 2012	Percentage Variance with Selected Total ex. AY 2012
Incurred Loss	9,572			9,458		
3 point average	14,577	970	7%	11,232	188	2%
5 point average	13,413	-194	-1%	10,905	-138	-1%
7 point average	13,783	176	1%	10,919	-124	-1%
3 point weighted average	14,143	536	4%	11,172	129	1%
5 point weighted average	12,849	-758	-6%	10,791	-253	-2%
7 point weighted average	12,864	-743	-5%	10,764	-279	-3%
5 point ex hi/lo	13,147	-460	-3%	10,824	-220	-2%
Largest LDF	25,513	11,906	87%	13,489	2,445	22%
2nd largest LDF	16,836	3,229	24%	11,884	840	8%
2nd smallest LDF	10,975	-2,632	-19%	10,105	-938	-8%
Smallest LDF	9,745	-3,862	-28%	9,137	-1,906	-17%
Selected LDF	13,607			11,043		

We can also look at the results of Table 14 graphically, as follows:

Table 15: Comparison of Ultimate Loss Indications Between Selected and Different Average LDF Calculations



	Smallest LDF	2nd smallest LDF	7 point weighted average	5 point weighted average	5 point ex hi/lo	5 point average	7 point average	Selected LDF	3 point weighted average	3 point average	2nd largest LDF	Largest LDF
Variance to Selected	-17%	-8%	-3%	-2%	-2%	-1%	-1%	0%	1%	2%	8%	22%

From the results in Tables 14 and 15, we observe that if we consider just the 3, 5 and 7 year averages for accident years excluding 2012, the ultimate indications are all within 2 to 3% of the ultimate indications using the selected LDFs. While the consistency of the different averages should give us comfort that the selected LDFs are a reasonable representation of the historical data pattern, we might also observe that the shorter the average, the higher the indicated ultimates. This observation should cause us to (a) examine the historical data more closely for indications LDFs are increasing and (b) assess if our selected LDFs might be aligned more closely to the 3 year averages than being somewhere between the 3 and 5 year averages.

3.2 Review of Tail Factors

As the tail factor selection impacts the ultimate loss indication for every accident period not yet at ultimate, the value being selected can have a considerable impact on the overall reserve indication. We do not ignore the importance of this actuarial assumption; however it is one that has been written about in several other papers. Rather than reiterate what was discussed in those other papers, we refer the reader to a 2006 paper by Joseph A. Boor ("Estimating Tail Development Factors: What to Do When the Triangle Runs Out", by Joseph A. Boor, Casualty Actuarial Society Winter 2006 Forum, pages 345-390) for further guidance in selecting appropriate tail factors and assessing the reasonability of the tail factors being selected.

4. "SOURCE OF CHANGE" CALCULATION

Often the first thing we will look at when reviewing an analysis is the change in ultimate losses. Our methodology breaks down the drivers of this change into three categories: data, assumptions, and judgment. This enables us to comment on the following questions:

1. What is the impact on ultimate loss estimates of data emerging in a different pattern than expected?
2. What impact will changing an assumption have on the ultimate loss estimates?
3. Do any changes in assumptions make sense in relation to what is happening in the data?
4. Are ultimates selected in a consistent manner relative to the method results? And if not, is this inconsistency reasonable and explainable?

In order to measure the sources of change, we must first calculate three Bornhuetter-Ferguson (BF)[6] method values[7].

A. BF method with prior data and prior assumptions: this is the BF method from the prior analysis that uses data as of time t-1 and assumptions underlying the analysis as of time t-1

B. BF method with current data and prior assumptions: this is an interim value that uses updated data as of time t but assumptions underlying the analysis as of time t-1. Note that LDFs must be interpolated to the proper ages as of time t.

C. BF method with current data and current assumptions: this is the BF method from the current analysis that uses updated data as of time t and updated assumptions underlying the analysis as of time t.

These method results will be referred to as Method A, Method B, and Method C throughout the remainder of this section.

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[6] "The Actuary and IBNR" by Ronald L, Bornhuetter and Ronald E. Ferguson, 1972 Proceedings of the Casualty Actuarial Society, Volume LIX, pages 181-195.

[7] If exposures are not available, the same process can be followed using the Loss Development Method as a base. However, our experience in using this methodology is that it works best with paid and incurred BF method results because the BF methods tend to stabilize potential swings in the indicated ultimate losses as compared with direct development methods.

The following table, Table 16, provides an example of the calculation of Methods A, B, and C. The example assumes time *t*-1 was 12/31/2011 and time *t* is 12/31/2012.

Table 16: Example of BF Method Recalculation

Accident Year	Prior Incurred Loss	Prior Initial Expected Loss	Prior Percent Incurred at time *t*-1	Method A
	(1)	(2)	(3)	(4) *
2004	621	682	97.6%	638
2005	1,468	1,470	95.6%	1,533
2006	1,283	1,405	93.3%	1,377
2007	1,064	1,045	90.6%	1,162
2008	1,510	1,600	84.7%	1,755
2009	857	1,574	79.1%	1,186
2010	847	1,539	58.6%	1,484
2011	108	1,539	4.5%	1,578
TOTAL	**7,758**	**10,854**		**10,713**

Accident Year	Current Incurred Loss	Prior Percent Incurred at time *t*	Method B
	(5)	(6)	(7) **
2004	621	98.8%	629
2005	1,452	97.6%	1,488
2006	1,232	95.6%	1,294
2007	1,131	93.3%	1,201
2008	1,759	90.6%	1,910
2009	850	84.7%	1,091
2010	1,122	79.1%	1,443
2011	1,291	58.6%	1,928
TOTAL	**9,458**		**10,984**

Accident Year	Current Initial Expected Loss	Current Percent Incurred at time *t*	Method C
	(8)	(9)	(10) ***
2004	621	99.5%	624
2005	1,475	98.8%	1,470
2006	1,350	97.4%	1,268
2007	1,150	95.4%	1,183
2008	1,750	92.7%	1,887
2009	1,300	86.6%	1,024
2010	1,442	80.9%	1,397
2011	1,875	57.8%	2,082
TOTAL	**10,963**		**10,935**

* Values in column (4) equal (2) * [100% - (3)] + (1)
** Values in column (7) equal (2) * [100% - (6)] + (5)
** Values in column (10) equal (8) * [100% - (9)] + (5)

4.1 Change Due to Data

The first source of change considered is the change driven by the actual data. Unless losses have emerged exactly as expected, updating the loss experience in the analysis will change the resulting method values. We can quantify the difference driven by the data by recalculating the BF test using current data but keeping the assumptions the same as the prior analysis (interpolated to the current ages) and comparing this to the BF test in the prior analysis. This is Method B minus Method A.

Continuing the example from above, Table 17 shows the change due to data

Table 17: Example of Ultimate Loss Change Due to Data

Method B	Method A	Data Difference
(1)	(2)	(1) – (2)
10,984	10,713	272

The results should be similar to the indirect actual vs. expected results. However, rather than just telling us how much actual loss emergence differed from expected within the period, the change due to data extrapolates that difference to tell us how much the change in data impacts the ultimate loss estimates.

An increase in method results due to the change in data implies that either the assumptions underlying the prior analysis projected too little development in the period or that the ultimate losses from the prior analysis should be increased or some combination of the two.

4.2 Change Due to Changes in Assumptions

The next source of change considered is the change due to changing assumptions, such as loss development factors or initial expected losses. The additional insight that comes from having an additional year of data may lead us to change our assumptions. We can isolate this change by comparing the BF tests calculated with the same data where the only difference is changing the prior assumptions to the current assumptions. This is Method C minus Method B.

Continuing the example from above, Table 18 shows the change due to changes in assumptions. [8]

Table 18: Example of Ultimate Loss Change Due to Change in Assumptions

Method C	Method B	Assumptions Difference
(1)	(2)	(1) – (2)
10,935	10,984	-49

The results show us that the actuary has lowered assumptions from the prior analysis to the current analysis.

4.3 Change Due to Judgment

The remaining change in ultimate loss is attributable to the often elusive concept of "actuarial

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[8] For methods with multiple assumptions, we can break out the change into assumptions into each individual assumption change, if desired. To accomplish this, calculate successive method values changing one assumption at a time and calculating the difference between each successive step. As an example, we look at the BF method and isolate the change in age to age factors, tail factor, and initial expected loss. Calculate the following:
Method B1: BF method using current data and all prior assumptions (interpolated to the current age)
Method B2: BF method using current data, current age to age factors, prior tail factor (interpolated to the current age) and prior initial expected loss
Method B3: BF method using current data, current age to age factors, current tail factor, and prior initial expected loss
Method C: BF method using current data and all current assumptions

It then follows that:
Method B2 – Method B1 = change due to change in age to age factors
Method B3 – Method B2 = change due to change in tail factor
Method C – Method B3 = change due to change in initial expected loss

judgment". However, we can actually calculate this judgment component from the selected ultimate losses and the calculated method values.

If we define judgment as the amount that the selected ultimate loss differs from the method values, we can then calculate the change in judgment in successive actuarial analyses. The key is that the base method used for comparison (whether a single method or some combination of methods) must be the same base method used in the change in assumptions analysis and the same method must be used as a base for both the prior and current analysis. In this example, our base method is the incurred BF method.

Table 19 calculates the judgment built into ultimate loss selections in both the current and prior analyses in our example.

Table 19: Calculation of Judgment

Accident Year	Prior BF Method (Method A)	Prior Selected Ultimate Loss	Judgment in Prior Analysis	Current BF Method (Method C)	Current Selected Ultimate Loss	Judgment in Current Analysis
	(1)	(2)	(3) = (2) – (1)	(4)	(5)	(6) = (5) – (4)
2004	638	621	-17	624	621	-3
2005	1,533	1,475	-58	1,470	1,425	-45
2006	1,377	1,350	-27	1,268	1,250	-18
2007	1,162	1,150	-12	1,183	1,168	-15
2008	1,755	1,750	-5	1,887	1,788	-99
2009	1,186	1,300	114	1,024	1,038	14
2010	1,484	1,550	66	1,397	1,450	53
2011	1,578	1,525	-53	2,082	1,900	-182
TOTAL	**10,713**	**10,721**	**8**	**10,935**	**10,640**	**-295**

The change due to judgment is thus Column (6) minus Column (3). This can also be written out as Change due to Judgment = [Current Selected Ultimate Loss – Current BF Method] – [Prior Selected Ultimate Loss – Prior BF Method].

Table 20: Example of Ultimate Loss Change Due to Change in Judgment

Judgment in Current Analysis	Judgment in Prior Analysis	Judgment Difference
(1)	(2)	(1) – (2)
-295	8	-304

We can also demonstrate that this is simply the remaining difference in selected ultimate losses after defining the difference due to data and the difference due to assumptions.

Table 21: Remaining Difference in Ultimate Loss Change

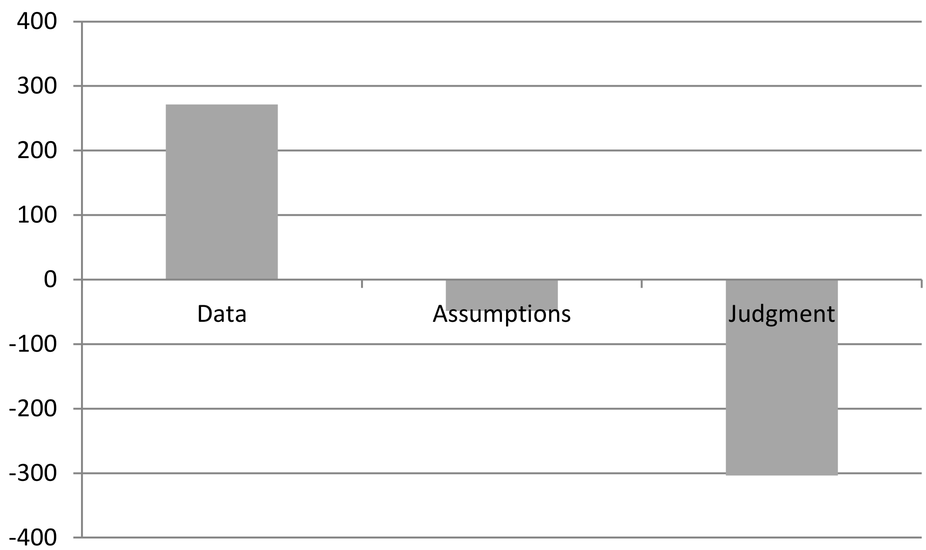
Prior Selected Ultimate Loss	Current Selected Ultimate Loss	Change in Selected Ultimate Loss	Data Difference	Assumption Difference	Remaining Difference (Judgment Difference)
(1)	(2)	(3) = (2) – (1)	(4)	(5)	(6) = (3) – (4) – (5)
10,721	10,640	-81	272	-49	-304

4.4 Interpreting Source of Change Results

Examining the sources of change allows us to ask and answer many questions about the analysis. We now know how much of the change in ultimate losses is due to the actual loss data emerging differently than expected as opposed to changes that the actuary is making in either assumptions or judgment.

We have also often found it beneficial to present this information graphically, such as the following.

Table 22: Sources of Change



In our example we see that the impact of data is an increase of $272 which is offset by a decrease in assumptions of $49 and a decrease in judgment of $304. At this point, we may ask why the actuary is lowering assumptions and judgment when the data is indicating an increase. There may be valid reasons for this, such as if the increase in data is driven by a single large loss or adverse loss emergence in a single year.

If the results do not make sense at first glance, it is often helpful to break the changes down into smaller steps. One can break out the assumptions into individual changes as discussed in footnote 5, or look at the change for each component for each individual accident year. Often there is a single year that skews overall results and if we look at results excluding that year, the picture becomes clearer. Continuing our example from above, we look at the changes for 2011 alone and all years excluding 2011.

Table 23: Sources of Change for Accident Year 2011

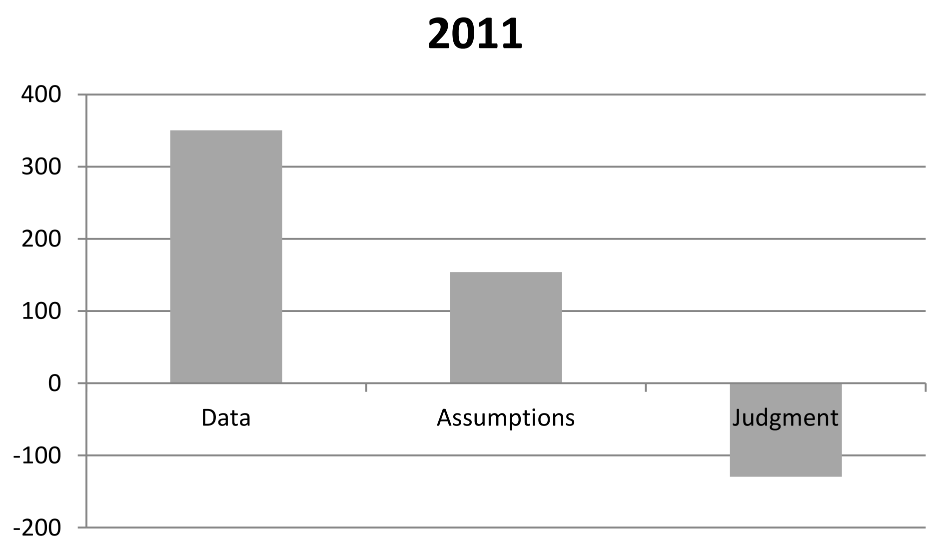
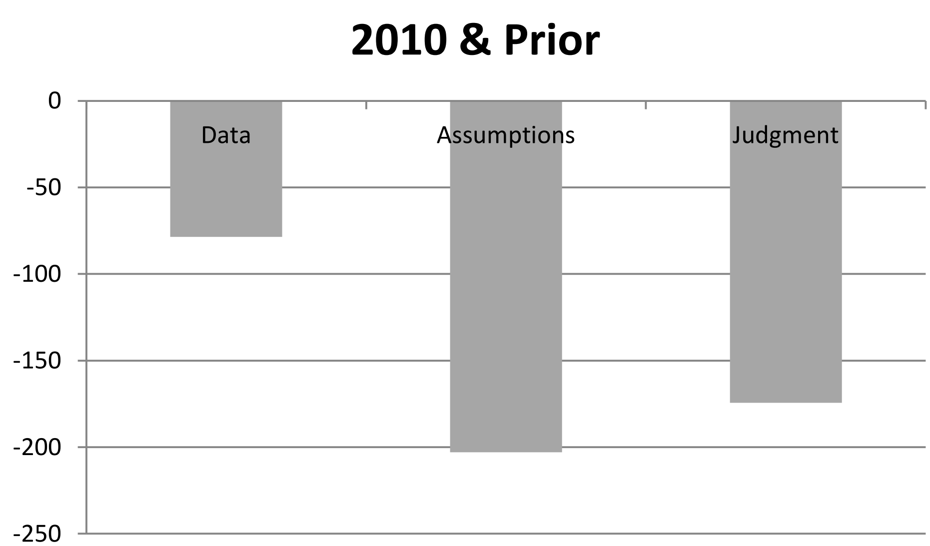


Table 24: Sources of Change for Accident Years 2010 and Prior



We can see that in our example, the increase in data is driven primarily by the 2011 year. When Accident Year 2011 is excluded, the data implies a decrease of $79. Given that data is the source of a decrease, it now makes sense that the actuary might lower assumptions so that if the current loss emergence patterns continue the data emergence in the next period might line up with lower expectations.

4.5 Common Questions

The following questions are common questions that may come up in the course of the analysis.

Do I worry if the change due to data is inconsistent with the actual vs. expected results?

In our application of the Source of Change methodology, we will often average the paid and incurred loss results at each step of the Source of Change process. This can lead to situations in which the average change due to data is greater (less) than zero, while one of the actual versus expected results is less (greater) than zero. In this case, the perceived inconsistency is not really an inconsistency at all, but rather a distortion that comes from averaging the paid and incurred data in the Source of Change but not averaging the paid and incurred data in the Actual versus Expected calculations.

Another way in which a potential inconsistency might arise is if the prior analysis Initial Expected Loss (IEL) is very different than the prior analysis direct development ultimate loss indication or the prior analysis selected ultimate loss. The Source of Change calculation uses the prior analysis IEL value to calculate the effect of data changes, whereas the Actual versus Expected calculation are based on either actual losses without regard to the prior analysis IEL or the prior analysis selected ultimate loss. A sizable difference between the values entering each of these calculations can result in one calculation showing actual loss emergence to be greater (less) than expected while another shows actual loss emergence to be less (greater) than expected.

Either of these apparent inconsistencies can be explained by an examination of the data and the calculations being done, thereby eliminating the perception of an inconsistency between the Source of Change and Actual versus Expected results.

Do I worry if I see different directional changes in my LDF picks and my IELR?

This is the type of result that should lead to some follow-up questions about the conclusions being drawn. We can imagine an example of when such an outcome might be reasonable as follows: we see that actual versus expected experience is showing different results for older versus immature accident periods. We have seen just such results when older accident years do not wind down as quickly as we had previously expected, resulting in higher LDFs for these older development ages. At the same time, we see immature accident periods showing accelerated claim closure rates that can be attributed to greater emphasis being placed on resolving claims early. In these situations we do not necessarily believe the higher LDFs that we have selected for the older accident periods are going to be needed for the immature accident periods. However, we only have one set of LDFs for the entire loss triangle, so the CDFs being calculated for the immature accident periods are now overstated because they include the accumulation of the older period LDFs. In order to counteract this LDF overstatement, we might lower our IELR pick from what was previously selected.

Do I worry if I see a large judgment impact?

There are various reasons why the judgment change may be significant.

- If ultimate losses are not selected based on the method(s) used for the baseline in the source of change, the judgment change could be large even though selection methodology is consistent from one analysis to the next.
- Consider the case where the baseline method is the average of the paid and incurred BF methods, and suppose there is a large loss in the prior analysis where very little has been paid. The incurred method will give higher results than the paid method, and the actuary would likely select closer to the incurred method since the paid method is skewed low by virtue of not including the large loss. "Judgment" in the prior analysis would appear to be a large positive number as the incurred method is above the average of the two methods. In the current analysis, a portion of the large loss is paid, bringing the paid method in line with the incurred method. Since the method results are now similar, the selected ultimate loss will now be close to the average and the "judgment" in the current analysis will be minimal. This would manifest in the source of change as a large decrease in judgment. However, there is not really a change in judgment, but is rather driven by the fact that the prior paid BF method was skewed by the large loss.
- The actuary may have a valid reason for changing the methods relied upon when selecting ultimate losses (e.g., change in case reserving practices leads the actuary to rely more on paid methods, or discovery that exposure estimates are not reliable leading the actuary to rely more on non-exposure based estimates).

5. CONCLUSION

This methodology is not designed to provide answers, but rather provide a structured framework through which to examine a reserve analysis. The results of each step in the analysis lead the actuary to ask questions that lead to a better understanding of the results of the actuarial analysis. We have used this methodology with great success as a way of teaching less experienced actuarial practitioners the types of critical thinking questions that should be asked when doing an analysis. We have also used multiple years' worth of Source of Change results to evaluate the trends in our analysis over time. For example, because the Source of Change methodology provides a consistent structural format for dissecting movement in ultimate losses into component parts, we are able to understand if we are lowering LDFs in one analysis, only to raise them in the subsequent analysis, or if we are steadily increasing (decreasing) them from analysis to analysis. Lastly, we have found the visual depiction of the Source of Change results shown in Tables 22, 23, and 24 to be very effective when communicating results to a non-actuarial audience.

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Abbreviations and notations

BF: Bornhuetter Ferguson
LDF: loss development factor
CDF: cumulative loss development factor

Biographies of the Author(s)

Jennifer Cheslawski Balester is a Senior Manager at Deloitte Consulting LLP where her work focuses on reserve analyses for insurance companies and self-insured entities as well as insurance process design and improvment. She has a degree in Mathematics and History from New York University. She is a Fellow of the CAS and a Member of the American Academy of Actuaries. She participates on the CAS examination committee.

Gerry Kirschner is Deputy Chief Risk Officer for the U.S at AIG. In this position, Gerry works closely with AIG U.S. senior management to identify and assess all sources of risk and to ensure that AIG manages the risk profile of the U.S. property casualty business to optimize returns and support sustainable business growth. In addition, Gerry is part of a team leading the effort to institute appropriate governance in AIG's U.S. operations to ensure proper controls of all risk exposures.

A Methodology for Avoiding the Pitfalls of Excess Loss Development

Lynne Bloom, FCAS, MAAA and Lela Patrik, FCAS, MAAA

Abstract
Given the long-tailed nature of certain lines of business, such as workers' compensation, and the impact of inflation on claim costs, determination of development factors, particularly in the tail, can be challenging. Reliance on excess loss development triangles can present challenges from both a credibility and volatility perspective. Furthermore, the application of excess development factors selected directly from excess loss triangles does not fully account for the impact of claim cost inflation, which has a greater impact on excess claims than on claims limited to a retention. Therefore many actuaries fall back on industry development patterns that are not necessarily indicative of the individual company's development and may be impacted by other distortions (e.g., non consistent interpretation of limits or retentions across companies in the compilation of data). We will discuss these distortions and the limitations of reliance on excess data and then present an alternative approach that relies on more stable ground-up data and can adjust for changing retention levels by year via calculation of excess development factors using excess loss factors (ELFs). We will discuss the theory behind the formula and its own benefits and limitations.

Keywords. Workers Compensation, Excess Loss Development, Reserving, Deductible

1. INTRODUCTION

One of the fundamental insurance coverages in the Unites States is statutorily required workers' compensation. Typically workers' compensation policies include defined loss retentions whereby, insureds retain a specified deductible on each claim; these deductibles - particularly for large insureds – can be quite large (several hundred thousand dollars or more).

Not only is there variety in the coverage based on the defined statutory requirements and various deductible options, but also in the variety of underlying exposures that range from workers at desk jobs to those working in industrial plants. The type of exposure not only impacts the potential frequency and severity of claims, but also the longevity of the claims. It may take several years for an injury to emerge (e.g., back injuries) and payments may continue for years into the future. Furthermore, exposure is impacted by changes in technology (e.g., carpal tunnel syndrome in desk workers) and economic conditions (e.g., a recession may result in workers collecting benefits for longer as employment options are limited).

While the indemnity portion of the coverage grows at a pace similar to the general inflation rate, the medical portion is impacted by a medical inflation rate that captures the rising medical costs and, particularly in recent years, has been greater than general inflation[1].

In addition, large deductible policies came into existence after the workers' compensation market crisis in the late 1980s and early 1990s. By 2002, the NCCI reported that 31.4% of manual-equivalent premium was written using a deductible of $100,000 or greater.

Given the wide range of risk contained by the workers' compensation coverage and its susceptibility to economic and technological factors, there is inherent volatility in the associated claim frequency and severity. The volatility of these long-tailed losses is further exacerbated when the insurance coverage offered is for an excess layer. The impact of direct mix of business changes and inherent changes in risk controls can take years to emerge. Furthermore, the underlying retentions of the insurance policies are not typically linked to an inflation index, but instead increase in stair step intervals.

For these compounding reasons, the estimation of workers' compensation losses in the excess layer is inherently volatile and is subject to large errors in the projection of ultimate losses.

In particular, we will focus on examining the magnitude of the error in the estimated ultimate, as compared to "true" ultimate, which results when utilizing a standard loss development method for which the development expectations are determined based on the inspection of loss data in excess of an average retention. While a Cape Cod or Bornhuetter-Ferguson approach may be utilized to reduce the error in the estimated ultimate, due to the lag between exposure changes and pricing adjustments, the error may only be slightly dampened.

We then will demonstrate the reduction in error when excess loss development factors are determined using loss information contained within the limited and unlimited data triangles, as opposed to reliance on loss information in the excess layer. This approach not only gleans information from the primary layer of data, but also directly considers the mix of retentions instead of relying on an average. We will refer to this approach as the Alternative Method.

1.1 Research Context

The Alternative Method has been described by Emanuel Pinto and Daniel Gogol[2] and further discussed by George Levine[3], who identified the basic formula that underlies the Alternative Method. We have expanded the use of this Alternative Method into practical solutions used in our audit work and further explored the potential benefits of such methodology.

1.2 Objective

The objective of this paper is to provide valuable insights to the practicing actuary on the nuances of excess development such that reserving decisions can be made with improved comprehension of the factors that drive the error in the estimation approaches commonly utilized. We will also provide a layout of the Alternative Method and accompanying methodology that can be used by actuaries under various circumstances.

1.3 Outline

The remainder of the paper proceeds as follows:

Section 2: A Typical Approach

Section 3: A Simple Example: Illustration of the Problem

Section 4: An Alternative Method

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2. A TYPICAL APPROACH

In practice, working at an audit firm, we see a large quantity and variety of loss development analyses that not only span different lines of business and layers of coverage, but also present a variety of methodologies and assumptions utilized in determining the estimate of ultimate loss. For assessments of the ultimate loss expected on an excess layer of coverage, it is not unusual to see an actuary utilize loss development triangles that consider claims history in excess of a specific attachment or high deductible (note we will used the words deductible, retention and attachment interchangeably for purposes of this paper). To supplement the historical average loss development factor (LDF) indications from these excess loss triangles, an actuary may consider industry data particularly in selecting the development pattern's tail factor (an often utilized source is data published biannually by the Reinsurance Association of America, or RAA). Alternatively, the actuary may fit a curve to the factors selected based on the development triangle in order to estimate the development pattern's tail. As we will demonstrate, both of these approaches typically results in significant errors in the projection of ultimate loss when these resulting loss development patterns are utilized in a standard loss development approach.

In addition, the excess loss triangles utilized by the actuary are often in excess of a fixed retention or in excess of a mix of varying retentions. Although actuaries are aware of the potential effects of inflation on the triangles in theory, the implication of the leveraging impact of inflation on the estimate of ultimate is often ignored in practice. In an attempt to recognize this resulting volatility, actuaries may provide a wide range of estimates, or rely on Bornhuetter-Ferguson or Cape Cod methods. Due to length of the tail associated with excess workers' compensation exposures, use of exposure-based methods result in significant reliance being placed on pricing loss ratios; to the extent that these initial expectations of loss later prove to be inadequate/excessive, this reliance may further exacerbate the error in the estimate of ultimate.

In our experience, reliance on excess loss development triangles in the selection of development patterns for excess layer workers' compensation is generally accepted and is oftentimes referred to as "the best we can do".

3. A SIMPLE EXAMPLE – ILLUSTRATION OF THE PROBLEM

The following simple example follows the typical approach of considering excess loss development history in the selection of development patterns, as described above, and demonstrates the error that results.

Consider a typical accident year consisted of only four claims of the following ground up unlimited amounts as of 12 months for accident Year One:

Claim 1	100
Claim 2	375
Claim 3	250
Claim 4	500
Total as of 12 months	1,225

Assume that these claims will develop to ultimate by age 36 as follows:

12:24 Unlimited Age to Age Factor	3.60
24:36 Unlimited Age to Age Factor	1.25

Therefore the ultimate value of these claims will be 5,513 in total. Also assume that there is a fixed inflation rate of 5% such that the ultimate losses for Year Two will be 5,788 and the ultimate losses for Year Three will be 6,078. The development will emerge as follows:

Unlimited Development	12	24	36
Year One	1,225	4,410	5,513
Year Two (5 % Inflation)	1,286	4,631	5,788
Year Three (5 % Inflation)	1,351	4,862	6,078

Unlimited Age to Age Factors	12:24	24:36	12:Ult
Year One	3.60	1.25	4.50
Year Two (5 % Inflation)	3.60	1.25	4.50
Year Three (5 % Inflation)	3.60	1.25	4.50

Now suppose that the insured has a fixed deductible of 350 per claim. The insurer's excess loss development triangle would look as follows:

Excess of 350 Development	12	24	36
Year One	175	3,010	4,113
Year Two (5 % Inflation)	219	3,231	4,388
Year Three (5 % Inflation)	265	3,462	4,678

Excess Age to Age Factors	12:24	24:36	12:Ult
Year One	17.20	1.37	23.50
Year Two (5 % Inflation)	14.77	1.36	20.06
Year Three (5 % Inflation)	13.08	1.35	17.67

The example above demonstrates that when inflation impacts the losses, the unlimited loss history continues to provide undistorted development factors. However, when considering loss development in excess of a fixed deductible, the resulting loss development factors are distorted by the impact of inflation, such that reliance on Year One development factors to project the estimated ultimate on future accident years would result in an overstatement of ultimate losses. Note that although inflation has a greater impact on the loss amounts in higher excess layers, the impact here results in the reduction of the excess loss development factors. This is driven by the relationship between the losses, the amount of inflation, and the underlying deductible.

If the actuary were to rely on the excess loss development triangle to estimate the ultimate loss for Year Three at 12-month maturity, the application of the weighted average development factors based on this history would produce the following estimate of ultimate:

Excess of 350 Development	12	24	36	Projection
Year One	175	3,010	4,113	4,113
Year Two (5 % Inflation)	219	3,231		4,414
Year Three (5 % Inflation)	265			5,732

Excess Age to Age Factors	12:24	24:36	12:Ult
Year One	17.20	1.37	
Year Two (5 % Inflation)	14.77		
Year Three (5 % Inflation)			
Weighted Average	15.85	1.37	21.65

Compared to the "true" ultimate loss of 4,678, as noted above, this estimate of the Year Three ultimate loss of 5,732 is overstated by 22.5%.

4. AN ALTERNATIVE METHOD

Given the volatility demonstrated above which results from the use of excess loss development patterns and consideration of the significant uncertainty that is associated with the available methods to estimate the tail, we believe it is prudent to examine an alternate approach. The fundamental relationship that is explored to determine this alternate approach is that an excess development factor is simply excess ultimate losses divided by the excess losses reported as of a given maturity. To estimate this relationship indirectly without reliance on an excess loss development triangle, the actuary can use a limited loss development triangle, along with an Excess Loss Factor (ELF), both of which are available in the data we have already considered above. Based on our review of the errors that result when the Alternative Method is followed, the limited data should provide more stability than the excess triangle and be less sensitive to exogenous factors such as inflation.

Consider the related loss development triangle of claims limited to 350 and the associated development factors:

Limited to 350 Development	12	24	36
Year One	1,050	1,400	1,400
Year Two (5 % Inflation)	1,068	1,400	1,400
Year Three (5 % Inflation)	1,086	1,400	1,400

Limited Age to Age Factors	12:24	24:36	12:Ult
Year One	1.33	1.00	1.33
Year Two (5 % Inflation)	1.31	1.00	1.31
Year Three (5 % Inflation)	1.29	1.00	1.29

If the actuary relies on the limited loss development triangle to estimate the limited ultimate loss for Year Three at 12-month maturity, the application of the weighted average development factors based on this history produces the following estimate of ultimate:

Limited to 350 Development	12	24	36	Projection
Year One	1,050	1,400	1,400	1,400
Year Two (5 % Inflation)	1,068	1,400		1,400
Year Three (5 % Inflation)	1,086			1,436

Limited Age to Age Factors	12:24	24:36	12:Ult
Year One	1.33	1.00	
Year Two (5 % Inflation)	1.31		
Year Three (5 % Inflation)			
Weighted Average	1.32	1.00	1.32

Compared to the "true" ultimate loss of 1,400, as noted above, this estimate of the Year Three ultimate loss of 1,436 is overstated by 2.5 %. Remember that estimation of unlimited ultimate losses is not distorted by inflation. However, per our example above, the excess loss development approach was distorted by an error of 22.5%. In order to combine the less distorted limited loss estimate and the undistorted unlimited loss estimate to produce an excess loss estimate, ELFs must be utilized. If the actuary considers the ultimate loss estimates that result from consideration of unlimited and limited loss development history, ELFs can be developed by accident year directly from the underlying data:

	Unlimited	Limited	ELF
Year One	5,513	1,400	0.746
Year Two	5,788	1,400	0.758
Year Three	6,078	1,436	0.764

We can then develop excess cumulative LDFs (CDFs) as follows (shown for 12 to Ultimate):

Excess CDF = Excess Ultimate Loss / Excess Reported Loss to Date

Assuming that unlimited losses are scaled to 1.00, we get:

Excess CDF = [ELF] / [Unlimited Reported Loss – Limited Reported Loss]

Excess CDF = [ELF] / [(1/Unlimited CDF) x (1.00) – (1/Limited CDF) x (1-ELF)]

Utilizing the information from the simple example above, the following excess CDF for Year Three is as follows:

Excess CDF (Year Three) = (.764) / [(1 / 4.50) – (1 / 1.32) x (1-.764)] = 17.54

Using this approach, the ultimate losses are as follows:

	Excess CDF	Excess Ult	Actual	Error
Year One	1.00	4,113	4,113	0.00%
Year Two	1.36	4,388	4,388	0.00%
Year Three	17.54	4,642	4,678	-0.77%

There are two important things to note about the use of this method:

1) In this simple example the resulting excess ultimate loss estimate from the Alternative Method is equal to the estimate obtained by subtracting the projected limited ultimate loss from projected unlimited ultimate loss. However, this would not be true if the historical data that was considered in selecting our development patterns and ELFs did not match the data being developed. Oftentimes, triangle data is utilized to develop patterns only and the ELFs are based on industry benchmarks. In a later section, the approach of utilizing industry benchmark LDFs and ELFs in situations where appropriate historical data is not available will be discussed.

2) This simple example is based on the assumption that all claims develop to ultimate by the same development pattern, which is not a realistic assumption. However, it does highlight the resulting distortion that can be caused by the leveraging impact of inflation on excess layer development patterns. In a later section, we will summarize and discuss the multitude of scenarios

demonstrating the potential sources of distortion.

5. TESTING OF THE APPROACH - ASSUMPTIONS

We tested the impact of various factors on excess loss development projections by running scenarios based on simulated data. The following are the assumptions underlying the model, including descriptions of how some of these baseline assumptions were developed:

1. Average claim value is $65,000; this is based on consideration of California claim size data[4].
2. Claim severity is lognormally distributed.
3. Using the above parameters, we derived a table of ELFs and compared it to California industry data for an average hazard group.
4. The standard deviation of the distribution was set such that the error between the generated ELFs and California ELFs for the determined hazard group was minimized.
5. Claim frequency was modeled using a Poisson distribution and a Monte Carlo simulation method was used to determine total losses by accident year.
6. Inflation applies evenly on an accident year basis.
7. Unlimited loss development was simulated based on the NCCI loss development pattern for California.
8. Eight accident years of data - both reported to date and the associated "true" ultimate loss values - were simulated. (Note that the following references to ultimate losses and error pertain to the aggregate of all eight accident years.)
9. For the base scenarios, the limited loss development pattern was determined based on the California ELFs, the NCCI unlimited loss development pattern for California, and the excess loss development pattern based on the lowest RAA attachment point of $400,000. We utilized the RAA data and associated patterns to provide baseline assumptions that mimic realistic limited loss development.

   Limited CDF = (1-ELF)/[(1/% Reported Unlimited) - (1/% Reported Excess) x (ELF)]
10. Limited loss development patterns were determined for varying limits using the same mathematical relationship and by interpolating between excess loss development patterns for subsequent RAA attachment points or extrapolating development patterns for limits greater

than provided by the RAA by using an exponential fit on existing development patterns.

11. Limited development factors vary by accident year due to the impact of inflation. If the retention is held constant, the limited pattern will react as if the retention is decreasing.
12. The limited loss development pattern varies according to a lognormal distribution around the development factors and Coefficients of Variation (CVs) that decrease with maturity, ranging from 1.00 to 0.10, for our base scenario. (In one of the alternative scenarios, the development factor CVs do not decrease with maturity after 36 months.)

The following loss development methods were utilized and the associated ultimate loss indications were compared to the "true" (simulated) ultimate loss to examine the resulting error: a standard loss development method utilizing LDFs based on historical averages from the development triangle, a standard loss development method utilizing the LDFs from the latest diagonal in the development triangle, and a standard loss development method using the Alternative Method to determine the excess LDFs. The first two methods were applied to unlimited, limited, and excess loss data to evaluate the resulting errors for each layer; the third method was applicable to calculations of excess ultimate loss only. In addition, we measured the error implicit in the selection of the age-to-age factors versus the error associated with the tail factor. We note that the error is quantified based on comparison of the resulting projection method to the "true" ultimate loss, in the aggregate for the eight accident years examined.

Based on the above assumptions and techniques, 216 scenarios were populated in the base case set based on combining the following assumption permutations:

- Pattern can be constant or varied;
- Frequency trend can be increasing by 1%, decreasing by 1%, or non-existent;
- Frequency can be constant or varied;
- Inflation can be 0, 3%, or 10%;
- Severity randomization can be based on the same variability for all years (the "1 Year" scenario) or different random seed for each of the 8 years (the "8 Year" scenario);
- Retention can be constant, move exactly with inflation, or increase by round number increments that mimic inflation; and
- Basic Retention is $400,000.

It is assumed that the actuary performing the method applies industry tail factors to the unlimited, limited, and excess loss development patterns, interpolating between available limits to get to the applicable retention or average retention (when changing).

6. TESTING OF THE APPROACH – RESULTS

6.1 The Base Case

Starting with the most basic scenario - no inflation, no variation simulated in the underlying development factors, constant frequency, constant retention, and the 1 Year random seed for severities (such that all accident years have the same expected number of losses that exceed the retention) - we noted that the difference between the projection based on the loss development approach and the "true" ultimate (i.e., the "error") was zero for the limited and unlimited losses; for the excess layer there was an error isolated to the tail. The tail error results from using industry data to derive the tail that relates to the limited and unlimited patterns, but which was derived using a different ELF than is implied by the actual data. Using the Alternative Method outlined above, this error is removed by the use of an ELF derived from the underlying data. We recognize that the industry tail on limited and unlimited data could in theory also be misstated, whereas our simulation assumes them to be correct. We display this error since we believe it may be easier to obtain more stable industry data on unlimited factors and reliable tail selections for limited data than for excess data.

As noted above, the ELFs were not judgmentally selected, but instead based on the underlying data. In a later section, we will address how to estimate ELFs when data is not available at each deductible and the impact of the associated error resulting from that estimation.

When the severity distribution underlying the simulated data is changed such that the random number seed is different for each accident year (the 8 Year approach), the resulting ELFs differ by accident year. While the presence of this changing severity distribution will introduce error into the standard development method based on excess loss patterns, this error can be removed by using the Alternative Method to determine the excess LDFs.

The introduction of a positive inflation factor causes the limited results to be overstated, particularly when the retention does not increase at the rate of inflation. The development on the excess loss layer, excluding the tail error, is erratic. The tail is again overstated, yielding overall overstated results. In most cases, use of the Alternative Method reduces error, often substantially. The following is a table of the error results described above. An example of how the results are calculated is provided in Appendix 1; Scenarios 10 and 18 only are shown for illustrative purposes.

Scenario	Inflation	Severity Randomization	Retention	Wtd Average Limited	Wtd Average Excess	Non Tail Wtd Avg Excess	Alternative Method Excess
1	0.0%	1 year	Constant	0.00%	8.37%	0.00%	0.00%
2	0.0%	1 year	Rounded	0.00%	8.37%	0.00%	0.00%
3	0.0%	1 year	Exact	0.00%	8.37%	0.00%	0.00%
4	0.0%	8 year	Constant	0.00%	1.01%	-6.79%	0.00%
5	0.0%	8 year	Rounded	0.00%	1.01%	-6.79%	0.00%
6	0.0%	8 year	Exact	0.00%	1.01%	-6.79%	0.00%
7	3.0%	1 year	Constant	1.00%	9.52%	1.06%	-1.08%
8	3.0%	1 year	Rounded	0.03%	9.93%	-0.18%	0.24%
9	3.0%	1 year	Exact	0.18%	9.85%	-0.30%	-0.30%
10	3.0%	8 year	Constant	0.99%	3.46%	-4.53%	-1.22%
11	3.0%	8 year	Rounded	0.04%	1.97%	-7.40%	0.46%
12	3.0%	8 year	Exact	0.18%	2.01%	-7.41%	-0.35%
13	10.0%	1 year	Constant	3.51%	14.59%	5.74%	-3.20%
14	10.0%	1 year	Rounded	0.61%	13.28%	-1.20%	-1.28%
15	10.0%	1 year	Exact	0.60%	13.25%	-0.96%	-0.96%
16	10.0%	8 year	Constant	3.49%	10.09%	1.59%	-3.56%
17	10.0%	8 year	Rounded	0.61%	4.04%	-9.26%	-1.70%
18	10.0%	8 year	Exact	0.60%	4.18%	-8.90%	-1.15%

A full table of scenario results is included in Appendix 2; results of the "Latest Diagonal Method" only are shown. Note that the "Weighted Average Method" refers to the approach in which the actuary has selected the loss development pattern based on the weighted average development history; the "Latest Diagonal Method" refers to the approach in which the actuary has utilized the LDFs along the latest diagonal as the selected loss development pattern.

The following subsections identify some of the different factors that impact the distortion in the projection of ultimate loss and summarize the impact to the error.

6.2 Variation of Pattern

When the development is simulated with the inclusion of variation in the loss development factors across the accident years, by maturity, the following changes in errors result:

	Wtd Average Unlimited	Wtd Average Limited	Wtd Average Excess	Non Tail Wtd Avg Excess	Alternative Method Excess
Constant	0.00%	0.66%	7.46%	-2.42%	-0.78%
Varied	4.31%	4.13%	15.93%	4.91%	0.98%

Introducing variation in the loss development pattern by accident year increases the errors in the limited and excess ultimate loss projections. Overall, the Alternative Method performs very well.

6.3 Frequency Trend and Variation of Frequency

Inclusion of frequency trend and variation in the frequency of claims by accident year does not have a significant impact on results, as shown below:

	Wtd Average Unlimited	Wtd Average Limited	Wtd Average Excess	Non Tail Wtd Avg Excess	Alternative Method Excess
Frequency Trend					
None	2.14%	2.40%	11.11%	0.72%	0.09%
1.0%	2.18%	2.42%	11.60%	1.16%	0.10%
-1.0%	2.14%	2.37%	12.38%	1.86%	0.11%
Frequency					
Constant	2.16%	2.40%	11.83%	1.40%	0.11%
Varied	2.15%	2.39%	11.56%	1.09%	0.10%

6.4 Severity Randomization

When severity randomization is introduced (i.e., the random number seed is varied across accident years) the excess results are stabilized somewhat due to offsetting random fluctuations and the law of large numbers. The Alternative Method reduces the error to nearly zero.

	Wtd Average Unlimited	Wtd Average Limited	Wtd Average Excess	Non Tail Wtd Avg Excess	Alternative Method Excess
1 year	2.20%	2.45%	14.41%	3.70%	0.16%
8 year	2.11%	2.34%	8.98%	-1.21%	0.04%

6.5 Inflation and Retention

Since inflation impacts loss size and retention defines the amount of loss in the excess layer, it makes the most sense to inspect these two variables together. As we expect, when retention does not move with inflation, errors in the excess triangle grow large. Much of this error is still concentrated in the tail, but becomes more significant in the rest of the triangle when inflation rate increases. The Alternative Method reduces the overall magnitude of the error substantially. The following table summarizes the average error across scenarios:

If we assume the loss development pattern will vary by accident year (rather than simulating data under the assumption that the loss development pattern is constant for all accident years), as is more realistic, our results are as follows:

Inflation	Retention	Wtd Average Unlimited	Wtd Average Limited	Wtd Average Excess	Non Tail Wtd Avg Excess	Alternative Method Excess
0.00%	Constant	2.05%	1.68%	9.08%	0.45%	0.79%
0.00%	Exact	2.05%	1.68%	9.08%	0.45%	0.79%
0.00%	Rounded	2.05%	1.68%	9.08%	0.45%	0.79%
3.00%	Constant	*2.13%*	*2.69%*	*11.22%*	*2.42%*	*-0.22%*
3.00%	Exact	2.13%	1.93%	10.61%	0.19%	0.51%
3.00%	Rounded	2.13%	1.80%	10.54%	0.18%	1.12%
10.00%	Constant	*2.28%*	*5.15%*	*17.53%*	*8.23%*	*-2.26%*
10.00%	Exact	2.28%	2.47%	14.05%	-0.46%	-0.12%
10.00%	Rounded	2.28%	2.47%	14.07%	-0.71%	-0.48%

As seen above, the introduction of variation in the pattern when the retention is fixed can cause the non-tail error to become quite large.

6.6 Multiple Retentions

We ran the same scenarios described above on data with basic retentions of $250,000 and

$550,000 with nearly identical errors in all scenarios (see Appendix 2). We then aggregated the three excess development triangles (i.e., in excess of $250,000, $400,000, and $550,000) to examine the resulting errors when dealing with a high deductible triangle composed of losses from underlying policies with varying retentions. Based on our experience, we note that it is common industry practice for the actuary to estimate the excess ultimate loss based on projection methods that consider that average retention of the underlying policies. Therefore, examination of the impact of this average approach on the error in the resulting projection is included below with an illustrative example.

Assume an insurer writes policies in excess of the following deductibles:

Policy Count		Expected Value of Excess	Deductible
	15	907,140	100,000
	5	185,400	400,000
	5	118,224	1,000,000
Total		1,210,764	
Average Deductible			233,818

The actuary utilizes the following unlimited and limited loss development patterns and ELFs, based on either actual loss history or industry information, as follows:

Policy Count	Expected Value of Excess	Deductible	12 Month Limited CDF	Industry ELF	Expected Value of Limited	Limited Reported at 12 months
15	907,140	100,000	2.299	0.552	1,117,727	486,207
5	185,400	400,000	2.565	0.310	83,296	32,469
5	118,224	1,000,000	2.758	0.213	31,997	11,600
Total	1,210,764		2.283	0.495	1,233,019	530,276
Average Deductible		233,818	2.459	0.355		

In the above table, the expected limited losses are derived from the expected value of excess losses and the ELF; the limited reported losses are derived from the expected value of limited losses divided by the 12 month CDF. For the limited loss layer, we observe that the weighted CDF for all policies of 2.283 deviates from the average deductible CDF of 2.459. This deviation between the weighted CDF and the average deductible CDF becomes even more significant when we examine the excess loss development pattern:

Policy Count	Expected Value of Excess	Deductible	12 Month Limited CDF	Industry ELF	Expected Value of Limited	Limited Reported at 12 months	Unlimited CDF	Excess CDF	Excess Reported at 12 Months
15	907,140	100,000	2.299	0.552	1,117,727	486,207	3.274	4.993	181,689
5	185,400	400,000	2.565	0.310	83,296	32,469	3.274	8.500	21,812
5	118,224	1,000,000	2.758	0.213	31,997	11,600	3.274	10.590	11,164
Total	1,210,764		2.283	0.495	1,233,019	530,276		5.640	214,664
Average Deductible		233,818	2.459	0.355			3.274	8.244	

For the excess layer the weighted CDF of 5.640 deviates from the average deductible CDF of 8.244 by a greater error than on the limited loss layer. This example demonstrates the importance of considering the loss development by underlying retention when determining the excess loss development pattern for a book of business that has policies written with varying retentions.

6.7 Use of Alternate Methods

In addition to examining the errors that result when standard loss development methods are applied, we inspected the results using a Cape Cod approach (assuming a constant exposure base across all accident years). We assumed the actuary would use the trend in losses as of 12 months in the method. We found distortions in results tended to be amplified with the average error increasing from 11.69% to 14.13%.

6.8 Increase in Variability

In the base case scenario, we had assumed that the CV of each development factor decreased with maturity. We also created an alternative scenario, where after 36 months the CV does not decrease at all; we believe that the triangle variability in this scenario looks more realistic.

The adjustment in this scenario to consider development factors with constant CV across maturities impacted the non-tail error of the excess methods the most. Note also, that the Alternative Method error increases significantly. This is due to the variability making it difficult to correctly predict the ELF using unlimited and limited data. The highlights of the results are as follows:

Inflation	Retention	Wtd Average Unlimited	Wtd Average Limited	Wtd Average Excess	Non Tail Wtd Avg Excess	Alternative Method Excess
0.00%	Constant	7.07%	4.55%	21.40%	9.41%	6.67%
0.00%	Exact	7.07%	4.55%	21.40%	9.41%	6.67%
0.00%	Rounded	7.07%	4.55%	21.40%	9.41%	6.67%
3.00%	Constant	*7.22%*	*5.44%*	*23.60%*	*11.39%*	*5.76%*
3.00%	Exact	7.22%	4.87%	23.32%	9.36%	6.43%
3.00%	Rounded	7.22%	4.79%	23.11%	9.23%	6.93%
10.00%	Constant	*7.50%*	*7.55%*	*29.51%*	*16.73%*	*3.81%*
10.00%	Exact	7.50%	5.52%	27.55%	9.08%	5.88%
10.00%	Rounded	7.50%	5.51%	27.74%	8.95%	5.62%
	All Varied Scenarios	7.26%	5.26%	24.34%	10.33%	6.05%
	All Varied Scenarios Lower CV as above	4.31%	4.13%	15.93%	4.91%	0.98%

6.9 Application of the Methodology Using Actual Data

We also applied the methodology to actual data, to assess the impact of the method in a real world scenario. Although there was no way to observe the error versus the "true" ultimate, given that it is still unknown, we were able to observe significant difference in results between using an excess development triangle and using our methodology with ground up data.

We observed that for the $500,000 and $1,000,000 deductible triangles (and even for the unlimited triangle), the data indicated that as of 372 months, it would be reasonable to select a tail factor of 1.00. On the other hand, the development behavior of the excess triangle was so erratic that the actuary might conclude that it is necessary to select a development factor significantly greater than 1.00.

If we assume the actuary would pick a tail factor based on consideration of the RAA industry patterns, then the excess method would produce an answer 44.18% higher for the $500,000 deductible and 117.43% higher for the $1,000,000 deductible. If we assume that a tail factor of 1.00 is selected for excess, the ultimate chosen from otherwise consistent development pattern selection (weighted average throughout) would be 25.29% and 29.47% higher than the methodology for the $500,000 and $1,000,000 deductibles respectively. We believe this shows that methods commonly employed when working with excess development triangles may overstate liabilities significantly.

The triangles and results of this are shown in detail in Appendix 3.

7. SUPPLEMENTING THE DATA

The Alternative Method discussed in this paper is applicable in situations where data is available at varying retentions and in magnitudes that provide for credible statistics. However, it may not be easy or even possible to obtain data limited to each retention that exists in the underlying book of business.

In these situations, the available limited loss development patterns can be interpolated and extrapolated between various retentions and up to the unlimited patterns (using a very high assumed limit). To do this, the following equation can be utilized to determine CDFs at the desired retention at each maturity (where A is a fitted constant and B is a fitted scalar):

(CDF -1) = A x [Retention ^B]

ln (CDF - 1) = ln A + B x ln (Retention)

To derive excess patterns at each retention, ELFs at each retention are required. We can use the ultimate losses as shown in Appendix 1, but to derive ELFs between retentions is not as simple. We found that interpolated values did not always make sense. A good way to understand the pattern of ELFs by retention is to look at the implied rate on line between them. In our example from above this would look as follows:

	Policy Count	Expected Value of Excess	Deductible	Industry ELF	ROL per Million
	15	907,140	100,000	0.552	0.807
	5	185,400	400,000	0.310	0.162
	5	118,224	1,000,000	0.213	
Total		1,210,764		0.495	

Any selected ELFs should reflect a decreasing rate on line as the deductible or retention increases. The Rate on Line (ROL) is equal to the [ELF for the lower deductible – the ELF for the next highest deductible] / [Difference in retentions] x 1,000,000.

As an example to demonstrate how this might be done, consider that the insurer above also writes policies with underlying deductibles of $350,000 and $200,000. Assume that the insurer only has credible data to compose limited loss development history and estimate ELFs for the $100,000 and $1,000,000 deductible.

Limited CDFs can be estimated for the $350,000 and $200,000 deductibles as follows:

	Deductible	Limited 12:Ult	Future Development	Ln (Future Development) (y)	ln (Deductible) (x)
Known	100,000	2.299	1.299	0.261	11.513
Known	1,000,000	2.758	1.758	0.564	13.816
Fitted	200,000	2.423	1.423	0.353	12.206
Fitted	350,000	2.532	1.532	0.426	12.766

Fitted (y) = Trend function for known y's and known x's on ln(deductible) for fitted deductible
Future Development Fitted = e^y
12:Ult Fitted = e^y +1

Also assume that for the $100,000 and $1,000,000 deductibles, we have estimated ELFs based on ultimate loss projections. We can select ELFs and the resulting excess patterns based on our formula as follows:

Excess CDF = [ELF] / [(1/Unlimited CDF) x (1.00) – (1/Limited CDF) x (1-ELF)]

Deductible	Limited 12:Ult	Unlimited 12:Ult	ELF from Data	ROL per Million	Excess 12:Ult
100,000	2.299	3.274	0.450	0.600	6.799
200,000	2.423	3.274	*0.390*	0.400	7.267
350,000	2.532	3.274	*0.330*	0.154	8.093
1,000,000	2.758	3.274	0.230		8.753
ROL 100k - 1000K			0.244		

Note that we have to judgmentally select the ELFs for the desired deductible levels (as defined in the box) such that the ROL decreases for each subsequent deductible. The resulting excess CDFs should also increase as the deductible increases. The actuary can use a mixture of judgment and these rules of thumb to estimate ELFs at additional deductibles.

The tool we developed to accompany this paper (available at the CAS website) demonstrates this further.

It should be noted, however, that the use of selected or industry ELF's introduces a new element of error into the methodology, which may potentially surpass the error in the original excess triangles. For example, using simulated data we observed that a 5% understatement in ELF each year, led to approximately a 5% overstatement in ultimate, whereas a 25% understatement in ELF, could lead to up to a 50% overstatement in ultimate. Overstatement of ELF by 50% only leads to about a 25% understatement of ultimate. Using actual data, we noticed similar results, with much less sensitivity to overstatement than understatement. Given the restrictions and judgments above it would be difficult to misstate the ELF by a large amount. It should also be noted, that this would be a potential parameter selection error (inherent in all actuarial processes), whereas the errors noted from using the excess triangle development are errors in the methodology itself.

8. CONCLUSIONS

We have demonstrated how the Alternative Method originally proposed by Pinto and Gogol creates in many scenarios a more accurate answer. Obviously, there will be situations where its use is limited because a large volume of data is unavailable. We hope that the results and considerations in this paper will give actuaries the tools to make more informed decisions. The uncertainty in predicting excess workers' compensation losses creates a quandary for reserve estimation and, ultimately, financial reporting. Given the nature of the tail liabilities, actual results will take significant time to emerge. It is also important for pricing decisions to have a more accurate handle on these books of business.

The Alternate Method and results here are not specific to workers' compensation. They can be used on any line of business, or indemnity and medical separately (which would increase inflation effect). The method merely demonstrates the effects of volatility and inflation on leveraged triangles and resultant methods.

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Supplementary Material

The Appendix to this paper and a practical tool are available electronically at the CAS website at http://www.casact.org/pubs/forum/13fforum/. The practical tool demonstrates interpolation, ELF selection, the methods used and the simulation.

9. REFERENCES

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Abbreviations and notations

CV, coefficient of variation
ELF, Excess Loss Factor
LDF, loss development factor
CDF, cumulative loss development factor

Biographies of the Authors

Lynne Bloom is a Director at PwC in Philadelphia, PA. She has a B.B.A. in Finance from the Wharton Business School at the University of Pennsylvania. She is a Fellow of the CAS and a Member of the American Academy of Actuaries. Lynne is the chairman of the CAS committee on reserves and the Bornhuetter-Ferguson Initial Expected Loss Ratio Working Party.

Lela Patrik is a Manager at PwC in Philadelphia, PA. She has a B.A. in Mathematics and Anthropology from Swarthmore College. She is a Fellow of the CAS and a Member of the American Academy of Actuaries. Lela is a member of the CAS Exam Committee.

Sample Error Calculations

Appendex 1 - Scenario 10, Page 1

Scenario Number	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention
10	Constant	None	Constant	3%	8 year	Constant

Incurred Losses Limited to Retention (400k Base)

Accident Year	12	24	36	48	60	72	84	96
2005	23,241,215	39,292,815	47,148,265	50,907,573	52,938,318	54,224,754	54,969,488	55,644,587
2006	26,167,877	44,327,361	53,156,399	57,373,508	59,623,428	61,017,692	61,816,479	
2007	25,132,102	42,656,111	51,120,611	55,155,784	57,281,502	58,568,568		
2008	22,842,812	38,846,428	46,526,134	50,180,059	52,080,158			
2009	29,693,892	50,596,177	60,561,231	65,293,217				
2010	28,001,716	47,806,207	57,186,350					
2011	26,163,306	44,754,977						
2012	27,779,342							

Accident Year	12:24	24:36	35:48	48:60	60:72	72:84	84:96	96:ult
2005	1.6907	1.1999	1.0797	1.0399	1.0243	1.0137	1.0123	
2006	1.6940	1.1992	1.0793	1.0392	1.0234	1.0131		
2007	1.6973	1.1984	1.0789	1.0385	1.0225			
2008	1.7006	1.1977	1.0785	1.0379				
2009	1.7039	1.1970	1.0781					
2010	1.7073	1.1962						
2011	1.7106							
All Year Weighted Average	1.7009	1.1980	1.0789	1.0389	1.0234	1.0134	1.0123	1.0715
Cumulative	2.5691	1.5104	1.2608	1.1686	1.1249	1.0992	1.0846	1.0715
Latest Diagonal	1.7106	1.1962	1.0781	1.0379	1.0225	1.0131	1.0123	1.0715
Cumulative	2.5725	1.5039	1.2572	1.1661	1.1235	1.0988	1.0846	1.0715

Sample Error Calculations

Appendex 1 - Scenario 10, Page 2

Scenario Number	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention
10	Constant	None	Constant	3%	8 year	Constant

Incurred Losses Unlimited

Accident Year	12	24	36	48	60	72	84	96
2005	29,536,519	48,864,358	60,101,035	66,875,909	71,525,566	75,667,109	78,748,017	81,743,504
2006	30,949,214	51,201,478	62,975,591	70,074,499	74,946,543	79,286,170	82,514,435	
2007	28,592,125	47,301,979	58,179,376	64,737,632	69,238,621	73,247,743		
2008	28,195,214	46,645,341	57,371,740	63,838,955	68,277,463			
2009	39,373,264	65,137,982	80,116,884	89,148,040				
2010	34,869,826	57,687,626	70,953,270					
2011	31,928,829	52,822,126						
2012	31,658,236							

Accident Year	12:24	24:36	35:48	48:60	60:72	72:84	84:96	96:ult
2005	1.6544	1.2300	1.1127	1.0695	1.0579	1.0407	1.0380	
2006	1.6544	1.2300	1.1127	1.0695	1.0579	1.0407		
2007	1.6544	1.2300	1.1127	1.0695	1.0579			
2008	1.6544	1.2300	1.1127	1.0695				
2009	1.6544	1.2300	1.1127					
2010	1.6544	1.2300						
2011	1.6544							
All Year Weighted								
Average	1.6544	1.2300	1.1127	1.0695	1.0579	1.0407	1.0380	1.1830
Cumulative	3.2740	1.9790	1.6090	1.4460	1.3520	1.2780	1.2280	1.1830
Latest Diagonal	1.6544	1.2300	1.1127	1.0695	1.0579	1.0407	1.0380	1.1830
Cumulative	3.2740	1.9790	1.6090	1.4460	1.3520	1.2780	1.2280	1.1830

Sample Error Calculations

Appendex 1 - Scenario 10, Page 3

Scenario Number	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention
10	Constant	None	Constant	3%	8 year	Constant

Incurred Losses Excess (400k Base)

Accident Year	12	24	36	48	60	72	84	96
2005	6,295,305	9,571,543	12,952,770	15,968,336	18,587,248	21,442,355	23,778,529	26,098,917
2006	4,781,336	6,874,117	9,819,192	12,700,991	15,323,114	18,268,479	20,697,956	
2007	3,460,023	4,645,867	7,058,765	9,581,847	11,957,119	14,679,175		
2008	5,352,401	7,798,913	10,845,606	13,658,896	16,197,304			
2009	9,679,372	14,541,805	19,555,654	23,854,824				
2010	6,868,111	9,881,419	13,766,920					
2011	5,765,523	8,067,149						
2012	3,878,894							

Accident Year	12:24	24:36	35:48	48:60	60:72	72:84	84:96	96:ult
2005	1.5204	1.3533	1.2328	1.1640	1.1536	1.1090	1.0976	
2006	1.4377	1.4284	1.2935	1.2065	1.1922	1.1330		
2007	1.3427	1.5194	1.3574	1.2479	1.2277			
2008	1.4571	1.3907	1.2594	1.1858				
2009	1.5024	1.3448	1.2198					
2010	1.4387	1.3932						
2011	1.3992							
All Year Weighted								
Average	1.4545	1.3880	1.2579	1.1956	1.1858	1.1200	1.0976	1.5397
Cumulative	6.8142	4.6851	3.3754	2.6834	2.2444	1.8927	1.6899	1.5397
Latest Diagonal	1.3992	1.3932	1.2198	1.1858	1.2277	1.1330	1.0976	1.5397
Cumulative	6.6281	4.7371	3.4001	2.7873	2.3505	1.9146	1.6899	1.5397
Method	8.2438	4.6920	3.0922	2.2184	2.0902	1.9939	1.6561	1.4208

Sample Error Calculations

Appendex 1 - Scenario 10, Page 4

Scenario Number	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention
10	Constant	None	Constant	3%	8 year	Constant

Estimated Ultimates Limited to Retention (400k Base)

Accident Year	Incurred Losses	Weighted CDF	Latest Diagonal CDF
2005	55,644,587	1.071	1.071
2006	61,816,479	1.085	1.085
2007	58,568,568	1.099	1.099
2008	52,080,158	1.125	1.124
2009	65,293,217	1.169	1.166
2010	57,186,350	1.261	1.257
2011	44,754,977	1.510	1.504
2012	27,779,342	2.569	2.573
	423,123,678		

Accident Year	Weighted LDM	Latest Diagonal LDM	Trend in 12 Month Incurred	Cape Cod with Weighted	Actual	Implied ELF from Wtd LDM's
2005	59,622,506	59,622,506	1.1953	59,692,626	59,622,506	0.3834
2006	67,049,077	67,049,077	1.0616	67,209,452	66,973,683	0.3383
2007	64,377,089	64,357,865	1.1053	64,549,172	64,172,473	0.3123
2008	58,582,518	58,513,965	1.2161	58,669,371	58,190,732	0.3654
2009	76,301,180	76,137,116	0.9355	76,265,652	75,466,751	0.4081
2010	72,100,469	71,894,242	0.9921	71,919,474	70,999,854	0.3684
2011	67,598,686	67,305,594	1.0618	67,197,712	66,183,498	0.3533
2012	71,367,961	71,462,877	1.0000	71,348,332	70,107,315	0.3114
	536,999,486	536,343,243		536,851,792	531,716,814	
Error	0.99%	0.87%		0.97%		

Sample Error Calculations

Appendex 1 - Scenario 10, Page 5

Scenario Number	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention
10	Constant	None	Constant	3%	8 year	Constant

Estimated Ultimates Unlimited

Accident Year	Incurred Losses	Weighted CDF	Latest Diagonal CDF
2005	81,743,504	1.183	1.183
2006	82,514,435	1.228	1.228
2007	73,247,743	1.278	1.278
2008	68,277,463	1.352	1.352
2009	89,148,040	1.446	1.446
2010	70,953,270	1.609	1.609
2011	52,822,126	1.979	1.979
2012	31,658,236	3.274	3.274
	550,364,817		

Accident Year	Weighted LDM	Latest Diagonal LDM	Trend in 12 Month Incurred	Cape Cod with Weighted	Actual
2005	96,702,565	96,702,565	1.0718	96,702,565	96,702,565
2006	101,327,726	101,327,726	1.0229	101,327,726	101,327,726
2007	93,610,616	93,610,616	1.1072	93,610,616	93,610,616
2008	92,311,130	92,311,130	1.1228	92,311,130	92,311,130
2009	128,908,067	128,908,067	0.8041	128,908,067	128,908,067
2010	114,163,812	114,163,812	0.9079	114,163,812	114,163,812
2011	104,534,988	104,534,988	0.9915	104,534,988	104,534,988
2012	103,649,066	103,649,066	1.0000	103,649,066	103,649,066
	835,207,967	835,207,967		835,207,967	835,207,967
Error	0.00%	0.00%		0.00%	

Sample Error Calculations

Appendex 1 - Scenario 10, Page 6

Scenario Number	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention
10	Constant	None	Constant	3%	8 year	Constant

Estimated Ultimates Excess (400k Base)

Accident Year	Incurred Losses	Weighted CDF	Latest Diagonal CDF	Method CDF	Weighted with Method Tail	Latest Diag. with Method Tail	Weighted LDM	Latest Diagonal LDM	Method LDM	Trend in 12 Month Incurred	Cape Cod with Weighted	Actual
2005	26,098,917	1.540	1.540	1.421	1.421	1.421	40,183,240	40,183,240	37,080,058	0.6162	43,128,924	37,080,058
2006	20,697,956	1.690	1.690	1.656	1.559	1.559	34,977,391	34,977,391	34,278,649	0.8113	32,756,778	34,354,043
2007	14,679,175	1.893	1.915	1.994	1.747	1.767	27,783,252	28,105,201	29,268,408	1.1211	23,704,501	29,438,142
2008	16,197,304	2.244	2.350	2.090	2.071	2.169	36,352,844	38,071,754	33,856,147	0.7247	36,669,124	34,120,397
2009	23,854,824	2.683	2.787	2.218	2.476	2.572	64,012,605	66,491,096	52,919,005	0.4007	66,313,058	53,441,315
2010	13,766,920	3.375	3.400	3.092	3.115	3.138	46,469,386	46,808,848	42,570,586	0.5648	47,053,200	43,163,958
2011	8,067,149	4.685	4.737	4.692	4.323	4.371	37,795,229	38,214,548	37,850,728	0.6728	39,499,408	38,351,489
2012	3,878,894	6.814	6.628	8.244	6.288	6.116	26,431,614	25,709,754	31,976,938	1.0000	26,574,174	33,541,750
	127,241,139						314,005,560	318,561,832	299,800,519		315,699,168	303,491,153

	Weighted LDM	Latest Diagonal LDM	Method LDM		Cape Cod with Weighted
Error	3.46%	4.97%	-1.22%		4.02%
Ultimate with Method Tail	289,756,241	293,960,652			
Non-Tail Error	-4.53%	-3.14%			

Sample Error Calculations

Appendex 1 - Scenario 18, Page 1

Scenario Number	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention
18	Constant	None	Constant	10%	8 year	Exact

Incurred Losses Limited to Retention (400k Base)

Accident Year	12	24	36	48	60	72	84	96
2005	23,241,215	39,292,815	47,148,265	50,907,573	52,938,318	54,224,754	54,969,488	55,644,587
2006	28,136,412	47,568,892	57,078,901	61,630,017	64,088,489	65,645,881	66,547,475	
2007	28,989,912	49,011,863	58,810,352	63,499,522	66,032,571	67,637,206		
2008	28,364,768	47,954,962	57,542,155	62,130,207	64,608,632			
2009	39,751,367	67,205,743	80,641,567	87,071,422				
2010	40,103,575	67,801,204	81,356,073					
2011	40,251,620	68,051,496						
2012	45,787,824							

Accident Year	12:24	24:36	35:48	48:60	60:72	72:84	84:96	96:ult
2005	1.6907	1.1999	1.0797	1.0399	1.0243	1.0137	1.0123	
2006	1.6907	1.1999	1.0797	1.0399	1.0243	1.0137		
2007	1.6907	1.1999	1.0797	1.0399	1.0243			
2008	1.6907	1.1999	1.0797	1.0399				
2009	1.6907	1.1999	1.0797					
2010	1.6907	1.1999						
2011	1.6907							
All Year Weighted								
Average	1.6907	1.1999	1.0797	1.0399	1.0243	1.0137	1.0123	1.0779
Cumulative	2.5807	1.5264	1.2721	1.1782	1.1330	1.1061	1.0911	1.0779
Latest Diagonal	1.6907	1.1999	1.0797	1.0399	1.0243	1.0137	1.0123	1.0779
Cumulative	2.5807	1.5264	1.2721	1.1782	1.1330	1.1061	1.0911	1.0779

Sample Error Calculations

Appendex 1 - Scenario 18, Page 2

Scenario Number	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention
18	Constant	None	Constant	10%	8 year	Exact

Incurred Losses Unlimited

Accident Year	12	24	36	48	60	72	84	96
2005	29,536,519	48,864,358	60,101,035	66,875,909	71,525,566	75,667,109	78,748,017	81,743,504
2006	33,052,558	54,681,190	67,255,485	74,836,844	80,039,997	84,674,551	88,122,212	
2007	32,610,492	53,949,848	66,355,966	73,835,927	78,969,490	83,542,058		
2008	34,343,280	56,816,523	69,881,851	77,759,266	83,165,606			
2009	51,218,076	84,733,694	104,218,758	115,966,792				
2010	48,442,560	80,141,962	98,571,126					
2011	47,371,350	78,369,782						
2012	50,162,011							

Accident Year	12:24	24:36	35:48	48:60	60:72	72:84	84:96	96:ult
2005	1.6544	1.2300	1.1127	1.0695	1.0579	1.0407	1.0380	
2006	1.6544	1.2300	1.1127	1.0695	1.0579	1.0407		
2007	1.6544	1.2300	1.1127	1.0695	1.0579			
2008	1.6544	1.2300	1.1127	1.0695				
2009	1.6544	1.2300	1.1127					
2010	1.6544	1.2300						
2011	1.6544							
All Year Weighted								
Average	1.6544	1.2300	1.1127	1.0695	1.0579	1.0407	1.0380	1.1830
Cumulative	3.2740	1.9790	1.6090	1.4460	1.3520	1.2780	1.2280	1.1830
Latest Diagonal	1.6544	1.2300	1.1127	1.0695	1.0579	1.0407	1.0380	1.1830
Cumulative	3.2740	1.9790	1.6090	1.4460	1.3520	1.2780	1.2280	1.1830

Sample Error Calculations

Appendex 1 - Scenario 18, Page 3

Scenario Number	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention
18	Constant	None	Constant	10%	8 year	Exact

Incurred Losses Excess (400k Base)

Accident Year	12	24	36	48	60	72	84	96
2005	6,295,305	9,571,543	12,952,770	15,968,336	18,587,248	21,442,355	23,778,529	26,098,917
2006	4,916,146	7,112,299	10,176,584	13,206,827	15,951,508	19,028,670	21,574,736	
2007	3,620,580	4,937,986	7,545,614	10,336,404	12,936,919	15,904,852		
2008	5,978,512	8,861,561	12,339,697	15,629,059	18,556,974			
2009	11,466,709	17,527,951	23,577,191	28,895,370				
2010	8,338,985	12,340,757	17,215,053					
2011	7,119,730	10,318,286						
2012	4,374,188							

Accident Year	12:24	24:36	35:48	48:60	60:72	72:84	84:96	96:ult
2005	1.5204	1.3533	1.2328	1.1640	1.1536	1.1090	1.0976	
2006	1.4467	1.4308	1.2978	1.2078	1.1929	1.1338		
2007	1.3639	1.5281	1.3699	1.2516	1.2294			
2008	1.4822	1.3925	1.2666	1.1873				
2009	1.5286	1.3451	1.2256					
2010	1.4799	1.3950						
2011	1.4493							
All Year Weighted								
Average	1.4804	1.3886	1.2620	1.1975	1.1875	1.1206	1.0976	1.6090
Cumulative	7.3012	4.9318	3.5515	2.8143	2.3501	1.9791	1.7660	1.6090
Latest Diagonal	1.4493	1.3950	1.2256	1.1873	1.2294	1.1338	1.0976	1.6090
Cumulative	7.2419	4.9970	3.5821	2.9229	2.4617	2.0023	1.7660	1.6090
Method	10.5314	4.9637	3.2010	2.2530	2.1145	2.0090	1.6502	1.4071

Sample Error Calculations

Appendex 1 - Scenario 18, Page 4

Scenario Number	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention
18	Constant	None	Constant	10%	8 year	Exact

Estimated Ultimates Limited to Retention (400k Base)

Accident Year	Incurred Losses	Weighted CDF	Latest Diagonal CDF
2005	55,644,587	1.078	1.078
2006	66,547,475	1.091	1.091
2007	67,637,206	1.106	1.106
2008	64,608,632	1.133	1.133
2009	87,071,422	1.178	1.178
2010	81,356,073	1.272	1.272
2011	68,051,496	1.526	1.526
2012	45,787,824	2.581	2.581
	536,704,714		

Accident Year	Weighted LDM	Latest Diagonal LDM	Trend in 12 Month Incurred	Cape Cod with Weighted	Actual	Implied ELF from Wtd LDM's
2005	59,978,392	59,978,392	1.9701	59,978,392	59,622,506	0.3798
2006	72,611,384	72,611,384	1.6274	72,611,384	72,180,539	0.3290
2007	74,814,002	74,814,002	1.5794	74,814,002	74,370,088	0.2993
2008	73,200,699	73,200,699	1.6142	73,200,699	72,766,358	0.3490
2009	102,585,993	102,585,993	1.1519	102,585,993	101,977,292	0.3882
2010	103,494,932	103,494,932	1.1417	103,494,932	102,880,838	0.3475
2011	103,876,989	103,876,989	1.1375	103,876,989	103,260,629	0.3302
2012	118,164,221	118,164,221	1.0000	118,164,221	117,463,086	0.2805
	708,726,610	708,726,610		708,726,610	704,521,336	
Error	0.60%	0.60%		0.60%		

Sample Error Calculations

Scenario Number	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention
18	Constant	None	Constant	10%	8 year	Exact

Estimated Ultimates Unlimited

Accident Year	Incurred Losses	Weighted CDF	Latest Diagonal CDF
2005	81,743,504	1.183	1.183
2006	88,122,212	1.228	1.228
2007	83,542,058	1.278	1.278
2008	83,165,606	1.352	1.352
2009	115,966,792	1.446	1.446
2010	98,571,126	1.609	1.609
2011	78,369,782	1.979	1.979
2012	50,162,011	3.274	3.274
	679,643,090		

Accident Year	Weighted LDM	Latest Diagonal LDM	Trend in 12 Month Incurred	Cape Cod with Weighted	Actual
2005	96,702,565	96,702,565	1.6983	96,702,565	96,702,565
2006	108,214,076	108,214,076	1.5176	108,214,076	108,214,076
2007	106,766,750	106,766,750	1.5382	106,766,750	106,766,750
2008	112,439,899	112,439,899	1.4606	112,439,899	112,439,899
2009	167,687,981	167,687,981	0.9794	167,687,981	167,687,981
2010	158,600,942	158,600,942	1.0355	158,600,942	158,600,942
2011	155,093,799	155,093,799	1.0589	155,093,799	155,093,799
2012	164,230,424	164,230,424	1.0000	164,230,424	164,230,424
	1,069,736,436	1,069,736,436		1,069,736,436	1,069,736,436
Error	0.00%	0.00%		0.00%	

Sample Error Calculations

Appendex 1 - Scenario 18, Page 6

Scenario Number	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention
18	Constant	None	Constant	10%	8 year	Exact

Estimated Ultimates Excess (400k Base)

Accident Year	Incurred Losses	Weighted CDF	Latest Diagonal CDF	Method CDF	Weighted with Method Tail	Latest Diag. with Method Tail	Weighted LDM	Latest Diagonal LDM	Method LDM	Trend in 12 Month Incurred	Cape Cod with Weighted	Actual
2005	26,098,917	1.609	1.609	1.407	1.407	1.407	41,993,460	41,993,460	36,724,173	0.6948	46,063,317	37,080,058
2006	21,574,736	1.766	1.766	1.650	1.544	1.544	38,101,508	38,101,508	35,602,692	0.8898	35,971,888	36,033,537
2007	15,904,852	1.979	2.002	2.009	1.731	1.751	31,476,807	31,846,622	31,952,748	1.2081	26,492,114	32,396,662
2008	18,556,974	2.350	2.462	2.115	2.055	2.153	43,610,428	45,681,429	39,239,200	0.7317	43,745,319	39,673,541
2009	28,895,370	2.814	2.923	2.253	2.461	2.556	81,320,206	84,456,891	65,101,989	0.3815	83,902,951	65,710,689
2010	17,215,053	3.552	3.582	3.201	3.106	3.133	61,139,632	61,666,799	55,106,010	0.5245	61,017,113	55,720,104
2011	10,318,286	4.932	4.997	4.964	4.313	4.370	50,887,308	51,560,504	51,216,810	0.6144	52,095,713	51,833,171
2012	4,374,188	7.301	7.242	10.531	6.385	6.333	31,936,772	31,677,508	46,066,203	1.0000	32,006,327	46,767,338
	142,938,376						380,466,122	386,984,722	361,009,826		381,294,743	365,215,100

	Weighted LDM	Latest Diagonal LDM	Method LDM	Cape Cod with Weighted
Error	4.18%	5.96%	-1.15%	4.40%
Ultimate with Method Tail	332,725,707	338,426,362		
Non-Tail Error	-8.90%	-7.34%		

Base Case Scenario Results
Total Error in Methods

Appendix 2 - Base Case, Page 1

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess
1	Constant	None	Constant	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	8.37%	8.37%	0.00%	0.00%	0.00%
2	Constant	None	Constant	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	8.37%	8.37%	0.00%	0.00%	0.00%
3	Constant	None	Constant	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	8.37%	8.37%	0.00%	0.00%	0.00%
4	Constant	None	Constant	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	1.01%	2.87%	-6.79%	-5.07%	0.00%
5	Constant	None	Constant	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	1.01%	2.87%	-6.79%	-5.07%	0.00%
6	Constant	None	Constant	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	1.01%	2.87%	-6.79%	-5.07%	0.00%
7	Constant	None	Constant	3.0%	1 year	Constant	0.00%	0.00%	1.00%	0.88%	9.52%	9.40%	1.06%	0.95%	-1.08%
8	Constant	None	Constant	3.0%	1 year	Rounded	0.00%	0.00%	0.03%	-0.05%	9.93%	10.13%	-0.18%	0.01%	0.24%
9	Constant	None	Constant	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	9.85%	9.85%	-0.30%	-0.30%	-0.30%
10	Constant	None	Constant	3.0%	8 year	Constant	0.00%	0.00%	0.99%	0.87%	3.46%	4.97%	-4.53%	-3.14%	-1.22%
11	Constant	None	Constant	3.0%	8 year	Rounded	0.00%	0.00%	0.04%	-0.04%	1.97%	3.81%	-7.40%	-5.73%	0.46%
12	Constant	None	Constant	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.18%	2.01%	3.86%	-7.41%	-5.74%	-0.35%
13	Constant	None	Constant	10.0%	1 year	Constant	0.00%	0.00%	3.51%	3.09%	14.59%	13.88%	5.74%	5.08%	-3.20%
14	Constant	None	Constant	10.0%	1 year	Rounded	0.00%	0.00%	0.61%	0.69%	13.28%	13.13%	-1.20%	-1.33%	-1.28%
15	Constant	None	Constant	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	13.25%	13.25%	-0.96%	-0.96%	-0.96%
16	Constant	None	Constant	10.0%	8 year	Constant	0.00%	0.00%	3.49%	3.07%	10.09%	10.39%	1.59%	1.87%	-3.56%
17	Constant	None	Constant	10.0%	8 year	Rounded	0.00%	0.00%	0.61%	0.69%	4.04%	5.82%	-9.26%	-7.70%	-1.70%
18	Constant	None	Constant	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	4.18%	5.96%	-8.90%	-7.34%	-1.15%
19	Constant	1.0%	Constant	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	5.79%	6.25%	-2.38%	-1.96%	0.00%
20	Constant	1.0%	Constant	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	5.79%	6.25%	-2.38%	-1.96%	0.00%
21	Constant	1.0%	Constant	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	5.79%	6.25%	-2.38%	-1.96%	0.00%
22	Constant	1.0%	Constant	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	4.45%	5.86%	-3.61%	-2.31%	0.00%
23	Constant	1.0%	Constant	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	4.45%	5.86%	-3.61%	-2.31%	0.00%
24	Constant	1.0%	Constant	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	4.45%	5.86%	-3.61%	-2.31%	0.00%
25	Constant	1.0%	Constant	3.0%	1 year	Constant	0.00%	0.00%	1.02%	0.89%	7.11%	7.43%	-1.16%	-0.87%	-1.12%
26	Constant	1.0%	Constant	3.0%	1 year	Rounded	0.00%	0.00%	0.02%	-0.06%	7.17%	7.85%	-2.68%	-2.06%	0.28%
27	Constant	1.0%	Constant	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	7.10%	7.58%	-2.79%	-2.36%	-0.31%
28	Constant	1.0%	Constant	3.0%	8 year	Constant	0.00%	0.00%	1.01%	0.89%	6.95%	7.87%	-1.31%	-0.46%	-1.18%
29	Constant	1.0%	Constant	3.0%	8 year	Rounded	0.00%	0.00%	0.03%	-0.04%	5.68%	7.10%	-4.03%	-2.74%	0.34%
30	Constant	1.0%	Constant	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.18%	5.73%	7.09%	-4.03%	-2.80%	-0.34%
31	Constant	1.0%	Constant	10.0%	1 year	Constant	0.00%	0.00%	3.56%	3.14%	12.92%	12.58%	4.20%	3.89%	-3.32%
32	Constant	1.0%	Constant	10.0%	1 year	Rounded	0.00%	0.00%	0.61%	0.70%	10.10%	10.47%	-3.97%	-3.65%	-1.36%
33	Constant	1.0%	Constant	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	10.10%	10.62%	-3.71%	-3.26%	-1.00%
34	Constant	1.0%	Constant	10.0%	8 year	Constant	0.00%	0.00%	3.55%	3.12%	13.30%	12.87%	4.55%	4.15%	-3.47%
35	Constant	1.0%	Constant	10.0%	8 year	Rounded	0.00%	0.00%	0.61%	0.69%	8.49%	9.67%	-5.38%	-4.35%	-1.55%
36	Constant	1.0%	Constant	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	8.58%	9.79%	-5.05%	-3.99%	-1.10%
37	Constant	-1.0%	Constant	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	10.17%	9.82%	1.66%	1.34%	0.00%
38	Constant	-1.0%	Constant	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	10.17%	9.82%	1.66%	1.34%	0.00%
39	Constant	-1.0%	Constant	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	10.17%	9.82%	1.66%	1.34%	0.00%
40	Constant	-1.0%	Constant	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	2.05%	3.63%	-5.83%	-4.37%	0.00%
41	Constant	-1.0%	Constant	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	2.05%	3.63%	-5.83%	-4.37%	0.00%
42	Constant	-1.0%	Constant	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	2.05%	3.63%	-5.83%	-4.37%	0.00%
43	Constant	-1.0%	Constant	3.0%	1 year	Constant	0.00%	0.00%	0.99%	0.86%	11.48%	11.00%	2.87%	2.43%	-1.05%
44	Constant	-1.0%	Constant	3.0%	1 year	Rounded	0.00%	0.00%	0.03%	-0.04%	11.83%	11.66%	1.55%	1.40%	0.21%
45	Constant	-1.0%	Constant	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	11.77%	11.40%	1.44%	1.11%	-0.29%

Base Case Scenario Results
Total Error in Methods

Appendix 2 - Base Case, Page 2

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess
46	Constant	-1.0%	Constant	3.0%	8 year	Constant	0.00%	0.00%	0.98%	0.85%	4.43%	5.62%	-3.64%	-2.54%	-1.16%
47	Constant	-1.0%	Constant	3.0%	8 year	Rounded	0.00%	0.00%	0.05%	-0.03%	3.15%	4.76%	-6.33%	-4.87%	0.35%
48	Constant	-1.0%	Constant	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.18%	3.18%	4.74%	-6.35%	-4.94%	-0.35%
49	Constant	-1.0%	Constant	10.0%	1 year	Constant	0.00%	0.00%	3.46%	3.04%	16.58%	15.52%	7.58%	6.59%	-3.08%
50	Constant	-1.0%	Constant	10.0%	1 year	Rounded	0.00%	0.00%	0.61%	0.69%	15.48%	14.93%	0.72%	0.24%	-1.23%
51	Constant	-1.0%	Constant	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	15.44%	15.04%	0.95%	0.60%	-0.93%
52	Constant	-1.0%	Constant	10.0%	8 year	Constant	0.00%	0.00%	3.44%	3.02%	10.77%	10.73%	2.21%	2.18%	-3.43%
53	Constant	-1.0%	Constant	10.0%	8 year	Rounded	0.00%	0.00%	0.61%	0.69%	5.57%	7.01%	-7.93%	-6.66%	-1.61%
54	Constant	-1.0%	Constant	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	5.69%	7.16%	-7.58%	-6.29%	-1.13%
55	Varied	None	Constant	0.0%	1 year	Constant	4.20%	0.70%	3.47%	0.04%	15.85%	11.58%	6.54%	2.61%	1.64%
56	Varied	None	Constant	0.0%	1 year	Rounded	4.20%	0.70%	3.47%	0.04%	15.85%	11.58%	6.54%	2.61%	1.64%
57	Varied	None	Constant	0.0%	1 year	Exact	4.20%	0.70%	3.47%	0.04%	15.85%	11.58%	6.54%	2.61%	1.64%
58	Varied	None	Constant	0.0%	8 year	Constant	4.01%	0.78%	3.28%	0.06%	9.13%	8.98%	0.36%	0.22%	1.46%
59	Varied	None	Constant	0.0%	8 year	Rounded	4.01%	0.78%	3.28%	0.06%	9.13%	8.98%	0.36%	0.22%	1.46%
60	Varied	None	Constant	0.0%	8 year	Exact	4.01%	0.78%	3.28%	0.06%	9.13%	8.98%	0.36%	0.22%	1.46%
61	Varied	None	Constant	3.0%	1 year	Constant	4.36%	0.71%	4.49%	0.79%	17.51%	12.92%	8.06%	3.84%	0.75%
62	Varied	None	Constant	3.0%	1 year	Rounded	4.36%	0.71%	3.68%	-0.01%	17.68%	13.46%	6.51%	2.68%	1.94%
63	Varied	None	Constant	3.0%	1 year	Exact	4.36%	0.71%	3.79%	0.20%	17.75%	13.26%	6.52%	2.46%	1.43%
64	Varied	None	Constant	3.0%	8 year	Constant	4.16%	0.80%	4.30%	0.79%	12.13%	11.41%	3.11%	2.45%	0.62%
65	Varied	None	Constant	3.0%	8 year	Rounded	4.16%	0.80%	3.49%	0.02%	10.40%	10.26%	-0.08%	-0.20%	1.87%
66	Varied	None	Constant	3.0%	8 year	Exact	4.16%	0.80%	3.59%	0.22%	10.63%	10.42%	0.08%	-0.12%	1.21%
67	Varied	None	Constant	10.0%	1 year	Constant	4.67%	0.73%	6.93%	2.63%	23.26%	17.82%	13.35%	8.35%	-1.07%
68	Varied	None	Constant	10.0%	1 year	Rounded	4.67%	0.73%	4.45%	0.63%	22.17%	17.07%	6.21%	1.77%	0.68%
69	Varied	None	Constant	10.0%	1 year	Exact	4.67%	0.73%	4.45%	0.55%	22.04%	17.11%	6.38%	2.08%	0.94%
70	Varied	None	Constant	10.0%	8 year	Constant	4.45%	0.83%	6.73%	2.64%	19.55%	17.39%	9.94%	7.95%	-1.24%
71	Varied	None	Constant	10.0%	8 year	Rounded	4.45%	0.83%	4.23%	0.65%	13.93%	13.56%	-0.95%	-1.28%	0.23%
72	Varied	None	Constant	10.0%	8 year	Exact	4.45%	0.83%	4.24%	0.57%	13.91%	13.57%	-0.70%	-1.00%	0.64%
73	Varied	1.0%	Constant	0.0%	1 year	Constant	4.25%	0.70%	3.53%	0.03%	13.52%	9.74%	4.39%	0.92%	1.64%
74	Varied	1.0%	Constant	0.0%	1 year	Rounded	4.25%	0.70%	3.53%	0.03%	13.52%	9.74%	4.39%	0.92%	1.64%
75	Varied	1.0%	Constant	0.0%	1 year	Exact	4.25%	0.70%	3.53%	0.03%	13.52%	9.74%	4.39%	0.92%	1.64%
76	Varied	1.0%	Constant	0.0%	8 year	Constant	4.11%	0.78%	3.32%	0.05%	12.83%	11.80%	3.76%	2.81%	1.60%
77	Varied	1.0%	Constant	0.0%	8 year	Rounded	4.11%	0.78%	3.32%	0.05%	12.83%	11.80%	3.76%	2.81%	1.60%
78	Varied	1.0%	Constant	0.0%	8 year	Exact	4.11%	0.78%	3.32%	0.05%	12.83%	11.80%	3.76%	2.81%	1.60%
79	Varied	1.0%	Constant	3.0%	1 year	Constant	4.41%	0.71%	4.56%	0.79%	15.38%	11.23%	6.11%	2.29%	0.72%
80	Varied	1.0%	Constant	3.0%	1 year	Rounded	4.41%	0.71%	3.73%	-0.03%	15.20%	11.48%	4.26%	0.90%	1.98%
81	Varied	1.0%	Constant	3.0%	1 year	Exact	4.41%	0.71%	3.84%	0.19%	15.27%	11.30%	4.27%	0.68%	1.42%
82	Varied	1.0%	Constant	3.0%	8 year	Constant	4.26%	0.80%	4.34%	0.80%	15.79%	14.19%	6.48%	5.01%	0.69%
83	Varied	1.0%	Constant	3.0%	8 year	Rounded	4.26%	0.80%	3.53%	0.00%	14.32%	13.33%	3.46%	2.57%	1.97%
84	Varied	1.0%	Constant	3.0%	8 year	Exact	4.26%	0.80%	3.63%	0.21%	14.57%	13.44%	3.64%	2.62%	1.36%
85	Varied	1.0%	Constant	10.0%	1 year	Constant	4.70%	0.73%	7.01%	2.66%	21.86%	16.73%	12.07%	7.35%	-1.16%
86	Varied	1.0%	Constant	10.0%	1 year	Rounded	4.70%	0.73%	4.49%	0.62%	19.33%	14.78%	3.74%	-0.21%	0.62%
87	Varied	1.0%	Constant	10.0%	1 year	Exact	4.70%	0.73%	4.49%	0.53%	19.23%	14.85%	3.93%	0.11%	0.91%
88	Varied	1.0%	Constant	10.0%	8 year	Constant	4.53%	0.81%	6.80%	2.67%	22.80%	19.84%	12.93%	10.21%	-1.21%
89	Varied	1.0%	Constant	10.0%	8 year	Rounded	4.53%	0.81%	4.26%	0.64%	18.48%	17.08%	3.00%	1.79%	0.45%
90	Varied	1.0%	Constant	10.0%	8 year	Exact	4.53%	0.81%	4.27%	0.56%	18.42%	17.08%	3.22%	2.05%	0.80%

Base Case Scenario Results
Total Error in Methods

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess
91	Varied	-1.0%	Constant	0.0%	1 year	Constant	4.17%	0.69%	3.43%	0.05%	17.48%	12.88%	8.04%	3.80%	1.63%
92	Varied	-1.0%	Constant	0.0%	1 year	Rounded	4.17%	0.69%	3.43%	0.05%	17.48%	12.88%	8.04%	3.80%	1.63%
93	Varied	-1.0%	Constant	0.0%	1 year	Exact	4.17%	0.69%	3.43%	0.05%	17.48%	12.88%	8.04%	3.80%	1.63%
94	Varied	-1.0%	Constant	0.0%	8 year	Constant	3.98%	0.78%	3.22%	0.08%	10.19%	9.33%	1.33%	0.54%	1.50%
95	Varied	-1.0%	Constant	0.0%	8 year	Rounded	3.98%	0.78%	3.22%	0.08%	10.19%	9.33%	1.33%	0.54%	1.50%
96	Varied	-1.0%	Constant	0.0%	8 year	Exact	3.98%	0.78%	3.22%	0.08%	10.19%	9.33%	1.33%	0.54%	1.50%
97	Varied	-1.0%	Constant	3.0%	1 year	Constant	4.33%	0.71%	4.44%	0.78%	19.27%	14.34%	9.68%	5.15%	0.76%
98	Varied	-1.0%	Constant	3.0%	1 year	Rounded	4.33%	0.71%	3.65%	-0.01%	19.40%	14.82%	8.07%	3.92%	1.90%
99	Varied	-1.0%	Constant	3.0%	1 year	Exact	4.33%	0.71%	3.76%	0.21%	19.49%	14.65%	8.08%	3.71%	1.42%
100	Varied	-1.0%	Constant	3.0%	8 year	Constant	4.13%	0.80%	4.22%	0.80%	13.11%	11.69%	4.02%	2.71%	0.65%
101	Varied	-1.0%	Constant	3.0%	8 year	Rounded	4.13%	0.80%	3.45%	0.04%	11.59%	10.77%	1.00%	0.26%	1.86%
102	Varied	-1.0%	Constant	3.0%	8 year	Exact	4.13%	0.80%	3.54%	0.24%	11.82%	10.86%	1.15%	0.28%	1.25%
103	Varied	-1.0%	Constant	10.0%	1 year	Constant	4.64%	0.72%	6.85%	2.60%	25.00%	19.24%	14.95%	9.66%	-1.00%
104	Varied	-1.0%	Constant	10.0%	1 year	Rounded	4.64%	0.72%	4.42%	0.63%	24.14%	18.66%	7.92%	3.16%	0.70%
105	Varied	-1.0%	Constant	10.0%	1 year	Exact	4.64%	0.72%	4.43%	0.55%	24.00%	18.69%	8.09%	3.46%	0.95%
106	Varied	-1.0%	Constant	10.0%	8 year	Constant	4.43%	0.82%	6.63%	2.61%	20.22%	17.48%	10.56%	8.03%	-1.18%
107	Varied	-1.0%	Constant	10.0%	8 year	Rounded	4.43%	0.82%	4.18%	0.67%	15.45%	14.22%	0.36%	-0.70%	0.32%
108	Varied	-1.0%	Constant	10.0%	8 year	Exact	4.43%	0.82%	4.19%	0.59%	15.41%	14.25%	0.60%	-0.41%	0.69%
109	Constant	None	Varied	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	8.25%	8.37%	-0.17%	-0.06%	0.00%
110	Constant	None	Varied	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	8.25%	8.37%	-0.17%	-0.06%	0.00%
111	Constant	None	Varied	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	8.25%	8.37%	-0.17%	-0.06%	0.00%
112	Constant	None	Varied	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	1.12%	2.52%	-6.74%	-5.45%	0.00%
113	Constant	None	Varied	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	1.12%	2.52%	-6.74%	-5.45%	0.00%
114	Constant	None	Varied	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	1.12%	2.52%	-6.74%	-5.45%	0.00%
115	Constant	None	Varied	3.0%	1 year	Constant	0.00%	0.00%	0.99%	0.87%	9.47%	9.45%	0.96%	0.94%	-1.09%
116	Constant	None	Varied	3.0%	1 year	Rounded	0.00%	0.00%	0.03%	-0.05%	9.84%	10.16%	-0.31%	-0.02%	0.25%
117	Constant	None	Varied	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	9.77%	9.88%	-0.43%	-0.32%	-0.30%
118	Constant	None	Varied	3.0%	8 year	Constant	0.00%	0.00%	0.99%	0.87%	3.57%	4.56%	-4.48%	-3.57%	-1.22%
119	Constant	None	Varied	3.0%	8 year	Rounded	0.00%	0.00%	0.04%	-0.04%	2.08%	3.44%	-7.35%	-6.12%	0.47%
120	Constant	None	Varied	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.18%	2.12%	3.46%	-7.37%	-6.14%	-0.35%
121	Constant	None	Varied	10.0%	1 year	Constant	0.00%	0.00%	3.48%	3.07%	14.71%	14.04%	5.79%	5.18%	-3.22%
122	Constant	None	Varied	10.0%	1 year	Rounded	0.00%	0.00%	0.61%	0.69%	13.29%	13.23%	-1.24%	-1.29%	-1.29%
123	Constant	None	Varied	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	13.26%	13.35%	-1.00%	-0.92%	-0.97%
124	Constant	None	Varied	10.0%	8 year	Constant	0.00%	0.00%	3.48%	3.07%	10.22%	9.97%	1.65%	1.42%	-3.57%
125	Constant	None	Varied	10.0%	8 year	Rounded	0.00%	0.00%	0.61%	0.69%	4.13%	5.34%	-9.23%	-8.17%	-1.70%
126	Constant	None	Varied	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	4.26%	5.48%	-8.87%	-7.81%	-1.15%
127	Constant	1.0%	Varied	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	5.97%	6.17%	-2.27%	-2.08%	0.00%
128	Constant	1.0%	Varied	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	5.97%	6.17%	-2.27%	-2.08%	0.00%
129	Constant	1.0%	Varied	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	5.97%	6.17%	-2.27%	-2.08%	0.00%
130	Constant	1.0%	Varied	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	3.43%	4.78%	-4.61%	-3.37%	0.00%
131	Constant	1.0%	Varied	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	3.43%	4.78%	-4.61%	-3.37%	0.00%
132	Constant	1.0%	Varied	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	3.43%	4.78%	-4.61%	-3.37%	0.00%
133	Constant	1.0%	Varied	3.0%	1 year	Constant	0.00%	0.00%	1.01%	0.89%	7.37%	7.40%	-0.98%	-0.95%	-1.13%
134	Constant	1.0%	Varied	3.0%	1 year	Rounded	0.00%	0.00%	0.02%	-0.06%	7.36%	7.76%	-2.56%	-2.19%	0.28%
135	Constant	1.0%	Varied	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	7.30%	7.50%	-2.67%	-2.49%	-0.31%

Base Case Scenario Results
Total Error in Methods

Appendix 2 - Base Case, Page 4

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess
136	Constant	1.0%	Varied	3.0%	8 year	Constant	0.00%	0.00%	1.00%	0.88%	6.07%	6.89%	-2.17%	-1.41%	-1.20%
137	Constant	1.0%	Varied	3.0%	8 year	Rounded	0.00%	0.00%	0.03%	-0.04%	4.64%	5.99%	-5.02%	-3.81%	0.35%
138	Constant	1.0%	Varied	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.18%	4.71%	5.99%	-5.01%	-3.85%	-0.35%
139	Constant	1.0%	Varied	10.0%	1 year	Constant	0.00%	0.00%	3.53%	3.12%	13.28%	12.64%	4.47%	3.89%	-3.33%
140	Constant	1.0%	Varied	10.0%	1 year	Rounded	0.00%	0.00%	0.61%	0.69%	10.33%	10.37%	-3.82%	-3.78%	-1.37%
141	Constant	1.0%	Varied	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	10.33%	10.52%	-3.56%	-3.40%	-1.01%
142	Constant	1.0%	Varied	10.0%	8 year	Constant	0.00%	0.00%	3.52%	3.11%	12.79%	12.20%	4.02%	3.48%	-3.51%
143	Constant	1.0%	Varied	10.0%	8 year	Rounded	0.00%	0.00%	0.61%	0.69%	7.47%	8.54%	-6.31%	-5.39%	-1.59%
144	Constant	1.0%	Varied	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	7.57%	8.67%	-5.98%	-5.01%	-1.13%
145	Constant	-1.0%	Varied	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	9.90%	9.74%	1.36%	1.21%	0.00%
146	Constant	-1.0%	Varied	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	9.90%	9.74%	1.36%	1.21%	0.00%
147	Constant	-1.0%	Varied	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	9.90%	9.74%	1.36%	1.21%	0.00%
148	Constant	-1.0%	Varied	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	1.29%	3.19%	-6.59%	-4.83%	0.00%
149	Constant	-1.0%	Varied	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	1.29%	3.19%	-6.59%	-4.83%	0.00%
150	Constant	-1.0%	Varied	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	1.29%	3.19%	-6.59%	-4.83%	0.00%
151	Constant	-1.0%	Varied	3.0%	1 year	Constant	0.00%	0.00%	0.98%	0.86%	11.25%	10.96%	2.60%	2.33%	-1.05%
152	Constant	-1.0%	Varied	3.0%	1 year	Rounded	0.00%	0.00%	0.03%	-0.05%	11.58%	11.60%	1.27%	1.29%	0.22%
153	Constant	-1.0%	Varied	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	11.52%	11.34%	1.16%	1.00%	-0.29%
154	Constant	-1.0%	Varied	3.0%	8 year	Constant	0.00%	0.00%	0.97%	0.85%	3.74%	5.23%	-4.32%	-2.95%	-1.18%
155	Constant	-1.0%	Varied	3.0%	8 year	Rounded	0.00%	0.00%	0.04%	-0.03%	2.38%	4.30%	-7.08%	-5.34%	0.38%
156	Constant	-1.0%	Varied	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.18%	2.41%	4.29%	-7.10%	-5.39%	-0.35%
157	Constant	-1.0%	Varied	10.0%	1 year	Constant	0.00%	0.00%	3.45%	3.03%	16.43%	15.54%	7.38%	6.56%	-3.11%
158	Constant	-1.0%	Varied	10.0%	1 year	Rounded	0.00%	0.00%	0.61%	0.69%	15.28%	14.91%	0.49%	0.17%	-1.24%
159	Constant	-1.0%	Varied	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	15.24%	15.02%	0.72%	0.54%	-0.94%
160	Constant	-1.0%	Varied	10.0%	8 year	Constant	0.00%	0.00%	3.43%	3.01%	10.29%	10.48%	1.71%	1.89%	-3.48%
161	Constant	-1.0%	Varied	10.0%	8 year	Rounded	0.00%	0.00%	0.61%	0.69%	4.77%	6.55%	-8.67%	-7.11%	-1.64%
162	Constant	-1.0%	Varied	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	4.89%	6.70%	-8.32%	-6.73%	-1.15%
163	Varied	None	Varied	0.0%	1 year	Constant	4.17%	0.69%	3.44%	0.04%	15.70%	11.57%	6.35%	2.55%	1.67%
164	Varied	None	Varied	0.0%	1 year	Rounded	4.17%	0.69%	3.44%	0.04%	15.70%	11.57%	6.35%	2.55%	1.67%
165	Varied	None	Varied	0.0%	1 year	Exact	4.17%	0.69%	3.44%	0.04%	15.70%	11.57%	6.35%	2.55%	1.67%
166	Varied	None	Varied	0.0%	8 year	Constant	3.95%	0.77%	3.25%	0.05%	9.11%	8.67%	0.29%	-0.12%	1.47%
167	Varied	None	Varied	0.0%	8 year	Rounded	3.95%	0.77%	3.25%	0.05%	9.11%	8.67%	0.29%	-0.12%	1.47%
168	Varied	None	Varied	0.0%	8 year	Exact	3.95%	0.77%	3.25%	0.05%	9.11%	8.67%	0.29%	-0.12%	1.47%
169	Varied	None	Varied	3.0%	1 year	Constant	4.33%	0.70%	4.46%	0.78%	17.43%	12.97%	7.93%	3.84%	0.76%
170	Varied	None	Varied	3.0%	1 year	Rounded	4.33%	0.70%	3.66%	-0.02%	17.57%	13.48%	6.35%	2.66%	1.97%
171	Varied	None	Varied	3.0%	1 year	Exact	4.33%	0.70%	3.77%	0.20%	17.64%	13.29%	6.36%	2.42%	1.45%
172	Varied	None	Varied	3.0%	8 year	Constant	4.11%	0.79%	4.27%	0.79%	12.10%	11.07%	3.04%	2.08%	0.63%
173	Varied	None	Varied	3.0%	8 year	Rounded	4.11%	0.79%	3.47%	0.01%	10.39%	9.96%	-0.15%	-0.54%	1.89%
174	Varied	None	Varied	3.0%	8 year	Exact	4.11%	0.79%	3.57%	0.22%	10.61%	10.09%	0.00%	-0.47%	1.22%
175	Varied	None	Varied	10.0%	1 year	Constant	4.64%	0.72%	6.90%	2.62%	23.33%	18.00%	13.36%	8.46%	-1.08%
176	Varied	None	Varied	10.0%	1 year	Rounded	4.64%	0.72%	4.43%	0.62%	22.14%	17.17%	6.13%	1.81%	0.69%
177	Varied	None	Varied	10.0%	1 year	Exact	4.64%	0.72%	4.44%	0.54%	22.01%	17.21%	6.30%	2.12%	0.96%
178	Varied	None	Varied	10.0%	8 year	Constant	4.41%	0.82%	6.70%	2.63%	19.58%	17.05%	9.91%	7.58%	-1.23%
179	Varied	None	Varied	10.0%	8 year	Rounded	4.41%	0.82%	4.21%	0.64%	13.92%	13.17%	-1.02%	-1.67%	0.25%
180	Varied	None	Varied	10.0%	8 year	Exact	4.41%	0.82%	4.22%	0.56%	13.89%	13.18%	-0.78%	-1.39%	0.66%

Base Case Scenario Results
Total Error in Methods

Appendix 2 - Base Case, Page 5

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess
181	Varied	1.0%	Varied	0.0%	1 year	Constant	4.18%	0.69%	3.48%	0.03%	13.63%	9.79%	4.44%	0.91%	1.63%
182	Varied	1.0%	Varied	0.0%	1 year	Rounded	4.18%	0.69%	3.48%	0.03%	13.63%	9.79%	4.44%	0.91%	1.63%
183	Varied	1.0%	Varied	0.0%	1 year	Exact	4.18%	0.69%	3.48%	0.03%	13.63%	9.79%	4.44%	0.91%	1.63%
184	Varied	1.0%	Varied	0.0%	8 year	Constant	4.08%	0.76%	3.28%	0.05%	11.92%	10.81%	2.86%	1.85%	1.56%
185	Varied	1.0%	Varied	0.0%	8 year	Rounded	4.08%	0.76%	3.28%	0.05%	11.92%	10.81%	2.86%	1.85%	1.56%
186	Varied	1.0%	Varied	0.0%	8 year	Exact	4.08%	0.76%	3.28%	0.05%	11.92%	10.81%	2.86%	1.85%	1.56%
187	Varied	1.0%	Varied	3.0%	1 year	Constant	4.34%	0.70%	4.50%	0.78%	15.57%	11.33%	6.22%	2.33%	0.69%
188	Varied	1.0%	Varied	3.0%	1 year	Rounded	4.34%	0.70%	3.68%	-0.04%	15.33%	11.53%	4.32%	0.89%	1.96%
189	Varied	1.0%	Varied	3.0%	1 year	Exact	4.34%	0.70%	3.79%	0.19%	15.41%	11.35%	4.35%	0.67%	1.40%
190	Varied	1.0%	Varied	3.0%	8 year	Constant	4.23%	0.78%	4.30%	0.79%	15.03%	13.32%	5.72%	4.16%	0.64%
191	Varied	1.0%	Varied	3.0%	8 year	Rounded	4.23%	0.78%	3.49%	0.00%	13.41%	12.31%	2.59%	1.59%	1.93%
192	Varied	1.0%	Varied	3.0%	8 year	Exact	4.23%	0.78%	3.59%	0.21%	13.68%	12.44%	2.77%	1.66%	1.31%
193	Varied	1.0%	Varied	10.0%	1 year	Constant	4.65%	0.71%	6.95%	2.64%	22.17%	16.93%	12.29%	7.47%	-1.19%
194	Varied	1.0%	Varied	10.0%	1 year	Rounded	4.65%	0.71%	4.45%	0.61%	19.52%	14.84%	3.85%	-0.21%	0.59%
195	Varied	1.0%	Varied	10.0%	1 year	Exact	4.65%	0.71%	4.45%	0.53%	19.42%	14.91%	4.04%	0.11%	0.88%
196	Varied	1.0%	Varied	10.0%	8 year	Constant	4.51%	0.80%	6.75%	2.66%	22.40%	19.29%	12.50%	9.64%	-1.28%
197	Varied	1.0%	Varied	10.0%	8 year	Rounded	4.51%	0.80%	4.23%	0.63%	17.64%	16.08%	2.22%	0.86%	0.38%
198	Varied	1.0%	Varied	10.0%	8 year	Exact	4.51%	0.80%	4.24%	0.56%	17.57%	16.08%	2.43%	1.13%	0.74%
199	Varied	-1.0%	Varied	0.0%	1 year	Constant	4.15%	0.69%	3.42%	0.05%	17.22%	12.77%	7.74%	3.65%	1.65%
200	Varied	-1.0%	Varied	0.0%	1 year	Rounded	4.15%	0.69%	3.42%	0.05%	17.22%	12.77%	7.74%	3.65%	1.65%
201	Varied	-1.0%	Varied	0.0%	1 year	Exact	4.15%	0.69%	3.42%	0.05%	17.22%	12.77%	7.74%	3.65%	1.65%
202	Varied	-1.0%	Varied	0.0%	8 year	Constant	3.96%	0.76%	3.21%	0.07%	9.48%	8.96%	0.62%	0.15%	1.47%
203	Varied	-1.0%	Varied	0.0%	8 year	Rounded	3.96%	0.76%	3.21%	0.07%	9.48%	8.96%	0.62%	0.15%	1.47%
204	Varied	-1.0%	Varied	0.0%	8 year	Exact	3.96%	0.76%	3.21%	0.07%	9.48%	8.96%	0.62%	0.15%	1.47%
205	Varied	-1.0%	Varied	3.0%	1 year	Constant	4.32%	0.70%	4.43%	0.78%	19.04%	14.27%	9.41%	5.03%	0.77%
206	Varied	-1.0%	Varied	3.0%	1 year	Rounded	4.32%	0.70%	3.64%	-0.01%	19.16%	14.74%	7.79%	3.79%	1.93%
207	Varied	-1.0%	Varied	3.0%	1 year	Exact	4.32%	0.70%	3.74%	0.21%	19.24%	14.56%	7.80%	3.58%	1.44%
208	Varied	-1.0%	Varied	3.0%	8 year	Constant	4.12%	0.78%	4.21%	0.79%	12.47%	11.37%	3.37%	2.37%	0.62%
209	Varied	-1.0%	Varied	3.0%	8 year	Rounded	4.12%	0.78%	3.43%	0.03%	10.88%	10.38%	0.30%	-0.15%	1.85%
210	Varied	-1.0%	Varied	3.0%	8 year	Exact	4.12%	0.78%	3.53%	0.23%	11.10%	10.49%	0.45%	-0.11%	1.22%
211	Varied	-1.0%	Varied	10.0%	1 year	Constant	4.64%	0.72%	6.83%	2.60%	24.85%	19.26%	14.75%	9.61%	-1.01%
212	Varied	-1.0%	Varied	10.0%	1 year	Rounded	4.64%	0.72%	4.41%	0.63%	23.93%	18.61%	7.69%	3.06%	0.71%
213	Varied	-1.0%	Varied	10.0%	1 year	Exact	4.64%	0.72%	4.42%	0.55%	23.80%	18.65%	7.85%	3.37%	0.96%
214	Varied	-1.0%	Varied	10.0%	8 year	Constant	4.43%	0.81%	6.62%	2.61%	19.81%	17.29%	10.12%	7.80%	-1.23%
215	Varied	-1.0%	Varied	10.0%	8 year	Rounded	4.43%	0.81%	4.18%	0.66%	14.73%	13.86%	-0.31%	-1.07%	0.28%
216	Varied	-1.0%	Varied	10.0%	8 year	Exact	4.43%	0.81%	4.19%	0.59%	14.70%	13.89%	-0.07%	-0.78%	0.65%

$250,000 Retention Scenario Results
Total Error in Methods

Appendix 2 - $250,000 Retention, Page 1

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess
1	Constant	None	Constant	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	6.40%	6.40%	0.00%	0.00%	0.00%
2	Constant	None	Constant	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	6.40%	6.40%	0.00%	0.00%	0.00%
3	Constant	None	Constant	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	6.40%	6.40%	0.00%	0.00%	0.00%
4	Constant	None	Constant	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	2.16%	2.95%	-3.99%	-3.24%	0.00%
5	Constant	None	Constant	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	2.16%	2.95%	-3.99%	-3.24%	0.00%
6	Constant	None	Constant	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	2.16%	2.95%	-3.99%	-3.24%	0.00%
7	Constant	None	Constant	3.0%	1 year	Constant	0.00%	0.00%	1.00%	0.87%	8.11%	7.87%	1.60%	1.37%	-0.77%
8	Constant	None	Constant	3.0%	1 year	Rounded	0.00%	0.00%	0.03%	-0.04%	7.71%	7.81%	-0.17%	-0.08%	0.14%
9	Constant	None	Constant	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	7.86%	7.86%	-0.21%	-0.21%	-0.21%
10	Constant	None	Constant	3.0%	8 year	Constant	0.00%	0.00%	1.00%	0.87%	4.13%	4.49%	-2.13%	-1.80%	-0.83%
11	Constant	None	Constant	3.0%	8 year	Rounded	0.00%	0.00%	0.01%	-0.07%	3.03%	3.70%	-4.51%	-3.88%	0.21%
12	Constant	None	Constant	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.18%	3.30%	4.05%	-4.43%	-3.73%	-0.24%
13	Constant	None	Constant	10.0%	1 year	Constant	0.00%	0.00%	3.12%	2.25%	13.36%	13.31%	6.54%	6.49%	-1.12%
14	Constant	None	Constant	10.0%	1 year	Rounded	0.00%	0.00%	0.73%	0.70%	11.33%	11.26%	-0.79%	-0.86%	-0.76%
15	Constant	None	Constant	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	11.20%	11.20%	-0.69%	-0.69%	-0.69%
16	Constant	None	Constant	10.0%	8 year	Constant	0.00%	0.00%	3.14%	2.25%	10.32%	10.73%	3.68%	4.07%	-1.09%
17	Constant	None	Constant	10.0%	8 year	Rounded	0.00%	0.00%	0.75%	0.71%	6.05%	6.67%	-5.49%	-4.94%	-0.84%
18	Constant	None	Constant	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	5.81%	6.48%	-5.50%	-4.90%	-0.79%
19	Constant	1.0%	Constant	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	4.88%	5.15%	-1.43%	-1.18%	0.00%
20	Constant	1.0%	Constant	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	4.88%	5.15%	-1.43%	-1.18%	0.00%
21	Constant	1.0%	Constant	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	4.88%	5.15%	-1.43%	-1.18%	0.00%
22	Constant	1.0%	Constant	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	4.35%	4.73%	-1.93%	-1.57%	0.00%
23	Constant	1.0%	Constant	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	4.35%	4.73%	-1.93%	-1.57%	0.00%
24	Constant	1.0%	Constant	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	4.35%	4.73%	-1.93%	-1.57%	0.00%
25	Constant	1.0%	Constant	3.0%	1 year	Constant	0.00%	0.00%	1.01%	0.89%	6.82%	6.82%	0.39%	0.39%	-0.79%
26	Constant	1.0%	Constant	3.0%	1 year	Rounded	0.00%	0.00%	0.03%	-0.05%	6.09%	6.47%	-1.67%	-1.32%	0.15%
27	Constant	1.0%	Constant	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	6.24%	6.52%	-1.71%	-1.46%	-0.22%
28	Constant	1.0%	Constant	3.0%	8 year	Constant	0.00%	0.00%	1.01%	0.89%	6.32%	6.21%	-0.08%	-0.18%	-0.82%
29	Constant	1.0%	Constant	3.0%	8 year	Rounded	0.00%	0.00%	0.01%	-0.07%	5.37%	5.58%	-2.34%	-2.15%	0.20%
30	Constant	1.0%	Constant	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.18%	5.66%	5.97%	-2.25%	-1.96%	-0.23%
31	Constant	1.0%	Constant	10.0%	1 year	Constant	0.00%	0.00%	3.17%	2.29%	12.56%	12.70%	5.78%	5.92%	-1.15%
32	Constant	1.0%	Constant	10.0%	1 year	Rounded	0.00%	0.00%	0.73%	0.70%	9.45%	9.67%	-2.47%	-2.27%	-0.78%
33	Constant	1.0%	Constant	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	9.35%	9.65%	-2.34%	-2.07%	-0.71%
34	Constant	1.0%	Constant	10.0%	8 year	Constant	0.00%	0.00%	3.19%	2.29%	12.46%	12.48%	5.70%	5.71%	-1.10%
35	Constant	1.0%	Constant	10.0%	8 year	Rounded	0.00%	0.00%	0.75%	0.71%	8.87%	8.99%	-2.98%	-2.88%	-0.82%
36	Constant	1.0%	Constant	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	8.59%	8.73%	-3.02%	-2.89%	-0.76%
37	Constant	-1.0%	Constant	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	7.96%	7.69%	1.46%	1.21%	0.00%
38	Constant	-1.0%	Constant	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	7.96%	7.69%	1.46%	1.21%	0.00%
39	Constant	-1.0%	Constant	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	7.96%	7.69%	1.46%	1.21%	0.00%
40	Constant	-1.0%	Constant	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	2.68%	3.35%	-3.50%	-2.87%	0.00%
41	Constant	-1.0%	Constant	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	2.68%	3.35%	-3.50%	-2.87%	0.00%
42	Constant	-1.0%	Constant	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	2.68%	3.35%	-3.50%	-2.87%	0.00%
43	Constant	-1.0%	Constant	3.0%	1 year	Constant	0.00%	0.00%	0.98%	0.86%	9.68%	9.17%	3.08%	2.60%	-0.74%
44	Constant	-1.0%	Constant	3.0%	1 year	Rounded	0.00%	0.00%	0.04%	-0.04%	9.36%	9.17%	1.36%	1.18%	0.12%
45	Constant	-1.0%	Constant	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	9.52%	9.23%	1.32%	1.06%	-0.21%

$250,000 Retention Scenario Results
Total Error in Methods

Appendix 2 - $250,000 Retention, Page 2

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess
46	Constant	-1.0%	Constant	3.0%	8 year	Constant	0.00%	0.00%	0.98%	0.85%	4.59%	4.82%	-1.71%	-1.49%	-0.80%
47	Constant	-1.0%	Constant	3.0%	8 year	Rounded	0.00%	0.00%	0.02%	-0.06%	3.62%	4.19%	-3.96%	-3.43%	0.20%
48	Constant	-1.0%	Constant	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.18%	3.89%	4.52%	-3.89%	-3.30%	-0.24%
49	Constant	-1.0%	Constant	10.0%	1 year	Constant	0.00%	0.00%	3.07%	2.21%	14.89%	14.57%	7.98%	7.67%	-1.10%
50	Constant	-1.0%	Constant	10.0%	1 year	Rounded	0.00%	0.00%	0.73%	0.70%	13.24%	12.86%	0.91%	0.57%	-0.74%
51	Constant	-1.0%	Constant	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	13.09%	12.78%	1.01%	0.73%	-0.67%
52	Constant	-1.0%	Constant	10.0%	8 year	Constant	0.00%	0.00%	3.09%	2.20%	10.61%	10.90%	3.95%	4.23%	-1.04%
53	Constant	-1.0%	Constant	10.0%	8 year	Rounded	0.00%	0.00%	0.75%	0.71%	6.86%	7.33%	-4.78%	-4.36%	-0.85%
54	Constant	-1.0%	Constant	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	6.60%	7.12%	-4.80%	-4.33%	-0.78%
55	Varied	None	Constant	0.0%	1 year	Constant	4.20%	0.70%	3.29%	-0.20%	13.51%	9.58%	6.44%	2.75%	1.73%
56	Varied	None	Constant	0.0%	1 year	Rounded	4.20%	0.70%	3.29%	-0.20%	13.51%	9.58%	6.44%	2.75%	1.73%
57	Varied	None	Constant	0.0%	1 year	Exact	4.20%	0.70%	3.29%	-0.20%	13.51%	9.58%	6.44%	2.75%	1.73%
58	Varied	None	Constant	0.0%	8 year	Constant	4.01%	0.78%	3.15%	-0.19%	9.44%	7.96%	2.62%	1.23%	1.82%
59	Varied	None	Constant	0.0%	8 year	Rounded	4.01%	0.78%	3.15%	-0.19%	9.44%	7.96%	2.62%	1.23%	1.82%
60	Varied	None	Constant	0.0%	8 year	Exact	4.01%	0.78%	3.15%	-0.19%	9.44%	7.96%	2.62%	1.23%	1.82%
61	Varied	None	Constant	3.0%	1 year	Constant	4.36%	0.71%	4.26%	0.50%	15.65%	11.27%	8.44%	4.33%	1.13%
62	Varied	None	Constant	3.0%	1 year	Rounded	4.36%	0.71%	3.49%	-0.26%	15.13%	11.08%	6.46%	2.72%	1.94%
63	Varied	None	Constant	3.0%	1 year	Exact	4.36%	0.71%	3.60%	-0.06%	15.38%	11.24%	6.50%	2.68%	1.61%
64	Varied	None	Constant	3.0%	8 year	Constant	4.16%	0.80%	4.11%	0.50%	11.92%	9.82%	4.95%	2.97%	1.21%
65	Varied	None	Constant	3.0%	8 year	Rounded	4.16%	0.80%	3.32%	-0.28%	10.63%	8.91%	2.31%	0.72%	2.13%
66	Varied	None	Constant	3.0%	8 year	Exact	4.16%	0.80%	3.45%	-0.05%	11.02%	9.43%	2.48%	1.01%	1.69%
67	Varied	None	Constant	10.0%	1 year	Constant	4.67%	0.73%	6.00%	1.29%	21.94%	17.36%	14.34%	10.05%	1.02%
68	Varied	None	Constant	10.0%	1 year	Rounded	4.67%	0.73%	4.34%	0.34%	19.81%	15.18%	6.53%	2.41%	1.22%
69	Varied	None	Constant	10.0%	1 year	Exact	4.67%	0.73%	4.23%	0.25%	19.57%	15.03%	6.56%	2.51%	1.32%
70	Varied	None	Constant	10.0%	8 year	Constant	4.45%	0.83%	5.86%	1.31%	19.35%	16.92%	11.91%	9.63%	1.24%
71	Varied	None	Constant	10.0%	8 year	Rounded	4.45%	0.83%	4.19%	0.37%	14.90%	13.00%	2.16%	0.47%	1.27%
72	Varied	None	Constant	10.0%	8 year	Exact	4.45%	0.83%	4.07%	0.26%	14.52%	12.70%	2.05%	0.43%	1.39%
73	Varied	1.0%	Constant	0.0%	1 year	Constant	4.25%	0.70%	3.34%	-0.22%	12.19%	8.52%	5.19%	1.75%	1.76%
74	Varied	1.0%	Constant	0.0%	1 year	Rounded	4.25%	0.70%	3.34%	-0.22%	12.19%	8.52%	5.19%	1.75%	1.76%
75	Varied	1.0%	Constant	0.0%	1 year	Exact	4.25%	0.70%	3.34%	-0.22%	12.19%	8.52%	5.19%	1.75%	1.76%
76	Varied	1.0%	Constant	0.0%	8 year	Constant	4.11%	0.78%	3.17%	-0.20%	11.88%	9.78%	4.90%	2.94%	1.84%
77	Varied	1.0%	Constant	0.0%	8 year	Rounded	4.11%	0.78%	3.17%	-0.20%	11.88%	9.78%	4.90%	2.94%	1.84%
78	Varied	1.0%	Constant	0.0%	8 year	Exact	4.11%	0.78%	3.17%	-0.20%	11.88%	9.78%	4.90%	2.94%	1.84%
79	Varied	1.0%	Constant	3.0%	1 year	Constant	4.41%	0.71%	4.31%	0.49%	14.57%	10.40%	7.42%	3.51%	1.13%
80	Varied	1.0%	Constant	3.0%	1 year	Rounded	4.41%	0.71%	3.52%	-0.28%	13.72%	9.94%	5.16%	1.67%	1.98%
81	Varied	1.0%	Constant	3.0%	1 year	Exact	4.41%	0.71%	3.64%	-0.08%	13.97%	10.11%	5.20%	1.64%	1.64%
82	Varied	1.0%	Constant	3.0%	8 year	Constant	4.26%	0.80%	4.15%	0.51%	14.30%	11.60%	7.18%	4.64%	1.21%
83	Varied	1.0%	Constant	3.0%	8 year	Rounded	4.26%	0.80%	3.33%	-0.29%	13.20%	10.83%	4.68%	2.49%	2.12%
84	Varied	1.0%	Constant	3.0%	8 year	Exact	4.26%	0.80%	3.47%	-0.06%	13.61%	11.38%	4.87%	2.81%	1.71%
85	Varied	1.0%	Constant	10.0%	1 year	Constant	4.70%	0.73%	6.06%	1.30%	21.33%	16.90%	13.77%	9.61%	1.02%
86	Varied	1.0%	Constant	10.0%	1 year	Rounded	4.70%	0.73%	4.37%	0.32%	18.18%	13.85%	5.08%	1.23%	1.23%
87	Varied	1.0%	Constant	10.0%	1 year	Exact	4.70%	0.73%	4.26%	0.23%	17.97%	13.73%	5.13%	1.35%	1.34%
88	Varied	1.0%	Constant	10.0%	8 year	Constant	4.53%	0.81%	5.90%	1.32%	21.56%	18.68%	13.98%	11.28%	1.18%
89	Varied	1.0%	Constant	10.0%	8 year	Rounded	4.53%	0.81%	4.21%	0.36%	17.88%	15.29%	4.81%	2.51%	1.28%
90	Varied	1.0%	Constant	10.0%	8 year	Exact	4.53%	0.81%	4.08%	0.25%	17.46%	14.93%	4.67%	2.42%	1.40%

$250,000 Retention Scenario Results
Total Error in Methods

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess
91	Varied	-1.0%	Constant	0.0%	1 year	Constant	4.17%	0.69%	3.25%	-0.19%	14.91%	10.69%	7.74%	3.79%	1.71%
92	Varied	-1.0%	Constant	0.0%	1 year	Rounded	4.17%	0.69%	3.25%	-0.19%	14.91%	10.69%	7.74%	3.79%	1.71%
93	Varied	-1.0%	Constant	0.0%	1 year	Exact	4.17%	0.69%	3.25%	-0.19%	14.91%	10.69%	7.74%	3.79%	1.71%
94	Varied	-1.0%	Constant	0.0%	8 year	Constant	3.98%	0.78%	3.09%	-0.17%	9.98%	8.17%	3.12%	1.43%	1.78%
95	Varied	-1.0%	Constant	0.0%	8 year	Rounded	3.98%	0.78%	3.09%	-0.17%	9.98%	8.17%	3.12%	1.43%	1.78%
96	Varied	-1.0%	Constant	0.0%	8 year	Exact	3.98%	0.78%	3.09%	-0.17%	9.98%	8.17%	3.12%	1.43%	1.78%
97	Varied	-1.0%	Constant	3.0%	1 year	Constant	4.33%	0.71%	4.21%	0.49%	17.04%	12.39%	9.74%	5.38%	1.12%
98	Varied	-1.0%	Constant	3.0%	1 year	Rounded	4.33%	0.71%	3.46%	-0.25%	16.60%	12.27%	7.83%	3.82%	1.90%
99	Varied	-1.0%	Constant	3.0%	1 year	Exact	4.33%	0.71%	3.56%	-0.05%	16.86%	12.43%	7.87%	3.78%	1.59%
100	Varied	-1.0%	Constant	3.0%	8 year	Constant	4.13%	0.80%	4.04%	0.52%	12.39%	10.00%	5.39%	3.14%	1.19%
101	Varied	-1.0%	Constant	3.0%	8 year	Rounded	4.13%	0.80%	3.26%	-0.24%	11.25%	9.22%	2.88%	1.00%	2.07%
102	Varied	-1.0%	Constant	3.0%	8 year	Exact	4.13%	0.80%	3.39%	-0.02%	11.64%	9.70%	3.05%	1.26%	1.66%
103	Varied	-1.0%	Constant	10.0%	1 year	Constant	4.64%	0.72%	5.93%	1.27%	23.22%	18.39%	15.54%	11.01%	0.99%
104	Varied	-1.0%	Constant	10.0%	1 year	Rounded	4.64%	0.72%	4.32%	0.35%	21.51%	16.56%	8.04%	3.64%	1.21%
105	Varied	-1.0%	Constant	10.0%	1 year	Exact	4.64%	0.72%	4.21%	0.26%	21.26%	16.40%	8.06%	3.73%	1.31%
106	Varied	-1.0%	Constant	10.0%	8 year	Constant	4.43%	0.82%	5.78%	1.30%	19.61%	16.99%	12.15%	9.69%	1.23%
107	Varied	-1.0%	Constant	10.0%	8 year	Rounded	4.43%	0.82%	4.14%	0.40%	15.73%	13.43%	2.90%	0.86%	1.23%
108	Varied	-1.0%	Constant	10.0%	8 year	Exact	4.43%	0.82%	4.02%	0.29%	15.33%	13.12%	2.77%	0.81%	1.36%
109	Constant	None	Varied	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	6.31%	6.39%	-0.14%	-0.06%	0.00%
110	Constant	None	Varied	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	6.31%	6.39%	-0.14%	-0.06%	0.00%
111	Constant	None	Varied	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	6.31%	6.39%	-0.14%	-0.06%	0.00%
112	Constant	None	Varied	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	2.28%	2.75%	-3.93%	-3.48%	0.00%
113	Constant	None	Varied	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	2.28%	2.75%	-3.93%	-3.48%	0.00%
114	Constant	None	Varied	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	2.28%	2.75%	-3.93%	-3.48%	0.00%
115	Constant	None	Varied	3.0%	1 year	Constant	0.00%	0.00%	0.99%	0.87%	8.08%	7.90%	1.53%	1.36%	-0.77%
116	Constant	None	Varied	3.0%	1 year	Rounded	0.00%	0.00%	0.04%	-0.03%	7.64%	7.78%	-0.28%	-0.15%	0.12%
117	Constant	None	Varied	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	7.78%	7.86%	-0.33%	-0.26%	-0.21%
118	Constant	None	Varied	3.0%	8 year	Constant	0.00%	0.00%	0.99%	0.87%	4.25%	4.28%	-2.07%	-2.04%	-0.83%
119	Constant	None	Varied	3.0%	8 year	Rounded	0.00%	0.00%	0.02%	-0.06%	3.16%	3.53%	-4.43%	-4.09%	0.19%
120	Constant	None	Varied	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.18%	3.41%	3.83%	-4.37%	-3.99%	-0.24%
121	Constant	None	Varied	10.0%	1 year	Constant	0.00%	0.00%	3.09%	2.24%	13.48%	13.49%	6.60%	6.61%	-1.14%
122	Constant	None	Varied	10.0%	1 year	Rounded	0.00%	0.00%	0.73%	0.69%	11.31%	11.33%	-0.85%	-0.84%	-0.75%
123	Constant	None	Varied	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	11.17%	11.23%	-0.76%	-0.70%	-0.69%
124	Constant	None	Varied	10.0%	8 year	Constant	0.00%	0.00%	3.12%	2.24%	10.45%	10.46%	3.75%	3.76%	-1.10%
125	Constant	None	Varied	10.0%	8 year	Rounded	0.00%	0.00%	0.75%	0.71%	6.15%	6.37%	-5.45%	-5.26%	-0.83%
126	Constant	None	Varied	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	5.92%	6.20%	-5.44%	-5.20%	-0.79%
127	Constant	1.0%	Varied	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	5.01%	5.13%	-1.35%	-1.24%	0.00%
128	Constant	1.0%	Varied	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	5.01%	5.13%	-1.35%	-1.24%	0.00%
129	Constant	1.0%	Varied	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	5.01%	5.13%	-1.35%	-1.24%	0.00%
130	Constant	1.0%	Varied	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	3.62%	3.98%	-2.66%	-2.33%	0.00%
131	Constant	1.0%	Varied	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	3.62%	3.98%	-2.66%	-2.33%	0.00%
132	Constant	1.0%	Varied	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	3.62%	3.98%	-2.66%	-2.33%	0.00%
133	Constant	1.0%	Varied	3.0%	1 year	Constant	0.00%	0.00%	1.01%	0.88%	6.96%	6.81%	0.48%	0.34%	-0.80%
134	Constant	1.0%	Varied	3.0%	1 year	Rounded	0.00%	0.00%	0.04%	-0.04%	6.23%	6.44%	-1.59%	-1.39%	0.13%
135	Constant	1.0%	Varied	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	6.38%	6.50%	-1.63%	-1.52%	-0.22%

Appendix 2 - $250,000 Retention, Page 4

$250,000 Retention Scenario Results
Total Error in Methods

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess
136	Constant	1.0%	Varied	3.0%	8 year	Constant	0.00%	0.00%	1.01%	0.88%	5.67%	5.53%	-0.73%	-0.86%	-0.83%
137	Constant	1.0%	Varied	3.0%	8 year	Rounded	0.00%	0.00%	0.01%	-0.06%	4.63%	4.86%	-3.07%	-2.85%	0.19%
138	Constant	1.0%	Varied	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.18%	4.92%	5.20%	-2.98%	-2.72%	-0.24%
139	Constant	1.0%	Varied	10.0%	1 year	Constant	0.00%	0.00%	3.14%	2.28%	12.73%	12.72%	5.90%	5.89%	-1.16%
140	Constant	1.0%	Varied	10.0%	1 year	Rounded	0.00%	0.00%	0.73%	0.69%	9.62%	9.66%	-2.36%	-2.33%	-0.76%
141	Constant	1.0%	Varied	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	9.50%	9.61%	-2.25%	-2.15%	-0.71%
142	Constant	1.0%	Varied	10.0%	8 year	Constant	0.00%	0.00%	3.16%	2.27%	12.03%	11.93%	5.24%	5.14%	-1.11%
143	Constant	1.0%	Varied	10.0%	8 year	Rounded	0.00%	0.00%	0.75%	0.71%	8.12%	8.15%	-3.69%	-3.67%	-0.82%
144	Constant	1.0%	Varied	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	7.84%	7.93%	-3.73%	-3.65%	-0.77%
145	Constant	-1.0%	Varied	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	7.75%	7.63%	1.22%	1.11%	0.00%
146	Constant	-1.0%	Varied	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	7.75%	7.63%	1.22%	1.11%	0.00%
147	Constant	-1.0%	Varied	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	7.75%	7.63%	1.22%	1.11%	0.00%
148	Constant	-1.0%	Varied	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	2.15%	3.00%	-4.04%	-3.25%	0.00%
149	Constant	-1.0%	Varied	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	2.15%	3.00%	-4.04%	-3.25%	0.00%
150	Constant	-1.0%	Varied	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	2.15%	3.00%	-4.04%	-3.25%	0.00%
151	Constant	-1.0%	Varied	3.0%	1 year	Constant	0.00%	0.00%	0.98%	0.86%	9.50%	9.13%	2.86%	2.51%	-0.74%
152	Constant	-1.0%	Varied	3.0%	1 year	Rounded	0.00%	0.00%	0.05%	-0.03%	9.16%	9.10%	1.13%	1.07%	0.11%
153	Constant	-1.0%	Varied	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	9.32%	9.19%	1.09%	0.97%	-0.21%
154	Constant	-1.0%	Varied	3.0%	8 year	Constant	0.00%	0.00%	0.98%	0.85%	4.10%	4.49%	-2.21%	-1.85%	-0.81%
155	Constant	-1.0%	Varied	3.0%	8 year	Rounded	0.00%	0.00%	0.02%	-0.05%	3.11%	3.85%	-4.48%	-3.79%	0.19%
156	Constant	-1.0%	Varied	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.18%	3.35%	4.15%	-4.43%	-3.68%	-0.24%
157	Constant	-1.0%	Varied	10.0%	1 year	Constant	0.00%	0.00%	3.05%	2.20%	14.81%	14.64%	7.85%	7.69%	-1.11%
158	Constant	-1.0%	Varied	10.0%	1 year	Rounded	0.00%	0.00%	0.72%	0.69%	13.07%	12.86%	0.71%	0.53%	-0.74%
159	Constant	-1.0%	Varied	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	12.92%	12.76%	0.81%	0.66%	-0.67%
160	Constant	-1.0%	Varied	10.0%	8 year	Constant	0.00%	0.00%	3.07%	2.20%	10.25%	10.63%	3.56%	3.92%	-1.05%
161	Constant	-1.0%	Varied	10.0%	8 year	Rounded	0.00%	0.00%	0.74%	0.71%	6.28%	6.92%	-5.34%	-4.77%	-0.84%
162	Constant	-1.0%	Varied	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	6.04%	6.74%	-5.34%	-4.72%	-0.79%
163	Varied	None	Varied	0.0%	1 year	Constant	4.17%	0.69%	3.28%	-0.20%	13.37%	9.57%	6.25%	2.69%	1.75%
164	Varied	None	Varied	0.0%	1 year	Rounded	4.17%	0.69%	3.28%	-0.20%	13.37%	9.57%	6.25%	2.69%	1.75%
165	Varied	None	Varied	0.0%	1 year	Exact	4.17%	0.69%	3.28%	-0.20%	13.37%	9.57%	6.25%	2.69%	1.75%
166	Varied	None	Varied	0.0%	8 year	Constant	3.95%	0.77%	3.13%	-0.19%	9.43%	7.80%	2.56%	1.03%	1.83%
167	Varied	None	Varied	0.0%	8 year	Rounded	3.95%	0.77%	3.13%	-0.19%	9.43%	7.80%	2.56%	1.03%	1.83%
168	Varied	None	Varied	0.0%	8 year	Exact	3.95%	0.77%	3.13%	-0.19%	9.43%	7.80%	2.56%	1.03%	1.83%
169	Varied	None	Varied	3.0%	1 year	Constant	4.33%	0.70%	4.24%	0.49%	15.57%	11.31%	8.31%	4.32%	1.13%
170	Varied	None	Varied	3.0%	1 year	Rounded	4.33%	0.70%	3.49%	-0.25%	15.00%	11.07%	6.30%	2.67%	1.94%
171	Varied	None	Varied	3.0%	1 year	Exact	4.33%	0.70%	3.59%	-0.06%	15.25%	11.25%	6.33%	2.65%	1.62%
172	Varied	None	Varied	3.0%	8 year	Constant	4.11%	0.79%	4.09%	0.50%	11.93%	9.67%	4.90%	2.78%	1.21%
173	Varied	None	Varied	3.0%	8 year	Rounded	4.11%	0.79%	3.31%	-0.27%	10.64%	8.80%	2.27%	0.56%	2.11%
174	Varied	None	Varied	3.0%	8 year	Exact	4.11%	0.79%	3.43%	-0.05%	11.01%	9.26%	2.42%	0.81%	1.70%
175	Varied	None	Varied	10.0%	1 year	Constant	4.64%	0.72%	5.98%	1.28%	22.00%	17.55%	14.34%	10.17%	1.01%
176	Varied	None	Varied	10.0%	1 year	Rounded	4.64%	0.72%	4.34%	0.34%	19.75%	15.26%	6.43%	2.44%	1.24%
177	Varied	None	Varied	10.0%	1 year	Exact	4.64%	0.72%	4.23%	0.25%	19.49%	15.08%	6.44%	2.51%	1.33%
178	Varied	None	Varied	10.0%	8 year	Constant	4.41%	0.82%	5.83%	1.30%	19.38%	16.72%	11.88%	9.40%	1.22%
179	Varied	None	Varied	10.0%	8 year	Rounded	4.41%	0.82%	4.18%	0.36%	14.88%	12.78%	2.10%	0.24%	1.30%
180	Varied	None	Varied	10.0%	8 year	Exact	4.41%	0.82%	4.06%	0.26%	14.52%	12.50%	2.01%	0.21%	1.40%

$250,000 Retention Scenario Results
Total Error in Methods

Appendix 2 - $250,000 Retention, Page 5

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess
181	Varied	1.0%	Varied	0.0%	1 year	Constant	4.18%	0.69%	3.31%	-0.22%	12.19%	8.56%	5.15%	1.75%	1.75%
182	Varied	1.0%	Varied	0.0%	1 year	Rounded	4.18%	0.69%	3.31%	-0.22%	12.19%	8.56%	5.15%	1.75%	1.75%
183	Varied	1.0%	Varied	0.0%	1 year	Exact	4.18%	0.69%	3.31%	-0.22%	12.19%	8.56%	5.15%	1.75%	1.75%
184	Varied	1.0%	Varied	0.0%	8 year	Constant	4.08%	0.76%	3.15%	-0.20%	11.16%	9.06%	4.18%	2.21%	1.82%
185	Varied	1.0%	Varied	0.0%	8 year	Rounded	4.08%	0.76%	3.15%	-0.20%	11.16%	9.06%	4.18%	2.21%	1.82%
186	Varied	1.0%	Varied	0.0%	8 year	Exact	4.08%	0.76%	3.15%	-0.20%	11.16%	9.06%	4.18%	2.21%	1.82%
187	Varied	1.0%	Varied	3.0%	1 year	Constant	4.34%	0.70%	4.28%	0.49%	14.59%	10.45%	7.39%	3.52%	1.12%
188	Varied	1.0%	Varied	3.0%	1 year	Rounded	4.34%	0.70%	3.51%	-0.27%	13.73%	9.98%	5.13%	1.66%	1.96%
189	Varied	1.0%	Varied	3.0%	1 year	Exact	4.34%	0.70%	3.61%	-0.08%	13.99%	10.15%	5.17%	1.63%	1.63%
190	Varied	1.0%	Varied	3.0%	8 year	Constant	4.23%	0.78%	4.12%	0.50%	13.68%	10.95%	6.54%	3.99%	1.18%
191	Varied	1.0%	Varied	3.0%	8 year	Rounded	4.23%	0.78%	3.32%	-0.28%	12.48%	10.13%	3.97%	1.80%	2.09%
192	Varied	1.0%	Varied	3.0%	8 year	Exact	4.23%	0.78%	3.45%	-0.06%	12.89%	10.64%	4.16%	2.08%	1.69%
193	Varied	1.0%	Varied	10.0%	1 year	Constant	4.65%	0.71%	6.03%	1.29%	21.41%	16.99%	13.79%	9.64%	1.01%
194	Varied	1.0%	Varied	10.0%	1 year	Rounded	4.65%	0.71%	4.35%	0.31%	18.25%	13.92%	5.10%	1.25%	1.24%
195	Varied	1.0%	Varied	10.0%	1 year	Exact	4.65%	0.71%	4.25%	0.23%	18.02%	13.78%	5.13%	1.35%	1.33%
196	Varied	1.0%	Varied	10.0%	8 year	Constant	4.51%	0.80%	5.87%	1.31%	21.16%	18.17%	13.55%	10.76%	1.16%
197	Varied	1.0%	Varied	10.0%	8 year	Rounded	4.51%	0.80%	4.19%	0.35%	17.19%	14.52%	4.15%	1.78%	1.27%
198	Varied	1.0%	Varied	10.0%	8 year	Exact	4.51%	0.80%	4.07%	0.25%	16.76%	14.18%	4.01%	1.71%	1.38%
199	Varied	-1.0%	Varied	0.0%	1 year	Constant	4.15%	0.69%	3.25%	-0.19%	14.69%	10.63%	7.49%	3.69%	1.72%
200	Varied	-1.0%	Varied	0.0%	1 year	Rounded	4.15%	0.69%	3.25%	-0.19%	14.69%	10.63%	7.49%	3.69%	1.72%
201	Varied	-1.0%	Varied	0.0%	1 year	Exact	4.15%	0.69%	3.25%	-0.19%	14.69%	10.63%	7.49%	3.69%	1.72%
202	Varied	-1.0%	Varied	0.0%	8 year	Constant	3.96%	0.76%	3.08%	-0.17%	9.46%	7.85%	2.58%	1.07%	1.77%
203	Varied	-1.0%	Varied	0.0%	8 year	Rounded	3.96%	0.76%	3.08%	-0.17%	9.46%	7.85%	2.58%	1.07%	1.77%
204	Varied	-1.0%	Varied	0.0%	8 year	Exact	3.96%	0.76%	3.08%	-0.17%	9.46%	7.85%	2.58%	1.07%	1.77%
205	Varied	-1.0%	Varied	3.0%	1 year	Constant	4.32%	0.70%	4.20%	0.50%	16.85%	12.34%	9.52%	5.29%	1.12%
206	Varied	-1.0%	Varied	3.0%	1 year	Rounded	4.32%	0.70%	3.46%	-0.24%	16.39%	12.20%	7.59%	3.71%	1.90%
207	Varied	-1.0%	Varied	3.0%	1 year	Exact	4.32%	0.70%	3.56%	-0.05%	16.65%	12.38%	7.63%	3.69%	1.60%
208	Varied	-1.0%	Varied	3.0%	8 year	Constant	4.12%	0.78%	4.03%	0.51%	11.92%	9.69%	4.90%	2.81%	1.17%
209	Varied	-1.0%	Varied	3.0%	8 year	Rounded	4.12%	0.78%	3.27%	-0.24%	10.74%	8.90%	2.36%	0.66%	2.05%
210	Varied	-1.0%	Varied	3.0%	8 year	Exact	4.12%	0.78%	3.39%	-0.03%	11.11%	9.37%	2.51%	0.91%	1.64%
211	Varied	-1.0%	Varied	10.0%	1 year	Constant	4.64%	0.72%	5.91%	1.27%	23.14%	18.46%	15.41%	11.03%	0.99%
212	Varied	-1.0%	Varied	10.0%	1 year	Rounded	4.64%	0.72%	4.31%	0.35%	21.33%	16.55%	7.84%	3.59%	1.22%
213	Varied	-1.0%	Varied	10.0%	1 year	Exact	4.64%	0.72%	4.21%	0.26%	21.08%	16.38%	7.85%	3.67%	1.31%
214	Varied	-1.0%	Varied	10.0%	8 year	Constant	4.43%	0.81%	5.76%	1.30%	19.29%	16.75%	11.80%	9.42%	1.20%
215	Varied	-1.0%	Varied	10.0%	8 year	Rounded	4.43%	0.81%	4.14%	0.39%	15.18%	13.07%	2.36%	0.49%	1.22%
216	Varied	-1.0%	Varied	10.0%	8 year	Exact	4.43%	0.81%	4.02%	0.29%	14.80%	12.78%	2.26%	0.46%	1.34%

$550,000 Retention Scenario Results
Total Error in Methods

Appendix 2 - $550,000 Retention, Page 1

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess
1	Constant	None	Constant	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	9.30%	9.30%	0.00%	0.00%	0.00%
2	Constant	None	Constant	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	9.30%	9.30%	0.00%	0.00%	0.00%
3	Constant	None	Constant	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	9.30%	9.30%	0.00%	0.00%	0.00%
4	Constant	None	Constant	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	-2.22%	0.72%	-10.54%	-7.85%	0.00%
5	Constant	None	Constant	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	-2.22%	0.72%	-10.54%	-7.85%	0.00%
6	Constant	None	Constant	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	-2.22%	0.72%	-10.54%	-7.85%	0.00%
7	Constant	None	Constant	3.0%	1 year	Constant	0.00%	0.00%	1.00%	0.88%	10.32%	10.18%	0.93%	0.80%	-1.35%
8	Constant	None	Constant	3.0%	1 year	Rounded	0.00%	0.00%	0.15%	0.19%	10.65%	10.50%	-0.52%	-0.65%	-0.46%
9	Constant	None	Constant	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	10.80%	10.80%	-0.37%	-0.37%	-0.37%
10	Constant	None	Constant	3.0%	8 year	Constant	0.00%	0.00%	0.99%	0.87%	0.04%	2.66%	-8.47%	-6.08%	-1.61%
11	Constant	None	Constant	3.0%	8 year	Rounded	0.00%	0.00%	0.15%	0.19%	-1.52%	1.40%	-11.46%	-8.83%	-0.80%
12	Constant	None	Constant	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.18%	-1.40%	1.56%	-11.33%	-8.67%	-0.46%
13	Constant	None	Constant	10.0%	1 year	Constant	0.00%	0.00%	3.54%	3.11%	13.42%	13.19%	3.76%	3.55%	-4.04%
14	Constant	None	Constant	10.0%	1 year	Rounded	0.00%	0.00%	0.62%	0.65%	14.32%	14.16%	-1.02%	-1.15%	-1.37%
15	Constant	None	Constant	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	14.23%	14.23%	-1.19%	-1.19%	-1.19%
16	Constant	None	Constant	10.0%	8 year	Constant	0.00%	0.00%	3.50%	3.09%	7.49%	9.17%	-1.65%	-0.13%	-4.62%
17	Constant	None	Constant	10.0%	8 year	Rounded	0.00%	0.00%	0.61%	0.65%	0.44%	3.23%	-13.03%	-10.62%	-2.03%
18	Constant	None	Constant	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	0.35%	3.29%	-13.19%	-10.65%	-1.50%
19	Constant	1.0%	Constant	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	6.51%	7.02%	-2.55%	-2.09%	0.00%
20	Constant	1.0%	Constant	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	6.51%	7.02%	-2.55%	-2.09%	0.00%
21	Constant	1.0%	Constant	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	6.51%	7.02%	-2.55%	-2.09%	0.00%
22	Constant	1.0%	Constant	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	2.31%	4.90%	-6.40%	-4.03%	0.00%
23	Constant	1.0%	Constant	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	2.31%	4.90%	-6.40%	-4.03%	0.00%
24	Constant	1.0%	Constant	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	2.31%	4.90%	-6.40%	-4.03%	0.00%
25	Constant	1.0%	Constant	3.0%	1 year	Constant	0.00%	0.00%	1.02%	0.89%	7.33%	7.68%	-1.81%	-1.48%	-1.41%
26	Constant	1.0%	Constant	3.0%	1 year	Rounded	0.00%	0.00%	0.15%	0.18%	7.67%	8.05%	-3.19%	-2.86%	-0.49%
27	Constant	1.0%	Constant	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	7.82%	8.35%	-3.04%	-2.57%	-0.38%
28	Constant	1.0%	Constant	3.0%	8 year	Constant	0.00%	0.00%	1.01%	0.89%	4.68%	6.71%	-4.23%	-2.37%	-1.54%
29	Constant	1.0%	Constant	3.0%	8 year	Rounded	0.00%	0.00%	0.15%	0.19%	3.28%	5.81%	-7.14%	-4.87%	-0.65%
30	Constant	1.0%	Constant	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.18%	3.51%	6.09%	-6.92%	-4.60%	-0.44%
31	Constant	1.0%	Constant	10.0%	1 year	Constant	0.00%	0.00%	3.59%	3.17%	10.87%	11.12%	1.43%	1.66%	-4.21%
32	Constant	1.0%	Constant	10.0%	1 year	Rounded	0.00%	0.00%	0.62%	0.65%	10.90%	11.31%	-3.98%	-3.62%	-1.45%
33	Constant	1.0%	Constant	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	10.82%	11.40%	-4.14%	-3.63%	-1.24%
34	Constant	1.0%	Constant	10.0%	8 year	Constant	0.00%	0.00%	3.56%	3.14%	11.83%	12.67%	2.31%	3.08%	-4.47%
35	Constant	1.0%	Constant	10.0%	8 year	Rounded	0.00%	0.00%	0.61%	0.65%	6.24%	8.54%	-8.01%	-6.02%	-1.78%
36	Constant	1.0%	Constant	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	6.15%	8.67%	-8.17%	-5.99%	-1.43%
37	Constant	-1.0%	Constant	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	11.07%	10.72%	1.61%	1.30%	0.00%
38	Constant	-1.0%	Constant	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	11.07%	10.72%	1.61%	1.30%	0.00%
39	Constant	-1.0%	Constant	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	11.07%	10.72%	1.61%	1.30%	0.00%
40	Constant	-1.0%	Constant	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	-0.70%	1.92%	-9.15%	-6.75%	0.00%
41	Constant	-1.0%	Constant	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	-0.70%	1.92%	-9.15%	-6.75%	0.00%
42	Constant	-1.0%	Constant	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	-0.70%	1.92%	-9.15%	-6.75%	0.00%
43	Constant	-1.0%	Constant	3.0%	1 year	Constant	0.00%	0.00%	0.99%	0.86%	12.26%	11.76%	2.70%	2.25%	-1.31%
44	Constant	-1.0%	Constant	3.0%	1 year	Rounded	0.00%	0.00%	0.15%	0.19%	12.53%	12.03%	1.18%	0.72%	-0.44%
45	Constant	-1.0%	Constant	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	12.68%	12.31%	1.32%	1.00%	-0.36%

$550,000 Retention Scenario Results
Total Error in Methods

Appendix 2 - $550,000 Retention, Page 2

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess
46	Constant	-1.0%	Constant	3.0%	8 year	Constant	0.00%	0.00%	0.98%	0.85%	1.52%	3.74%	-7.12%	-5.09%	-1.52%
47	Constant	-1.0%	Constant	3.0%	8 year	Rounded	0.00%	0.00%	0.15%	0.19%	0.15%	2.71%	-9.96%	-7.66%	-0.71%
48	Constant	-1.0%	Constant	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.18%	0.31%	2.93%	-9.80%	-7.44%	-0.45%
49	Constant	-1.0%	Constant	10.0%	1 year	Constant	0.00%	0.00%	3.49%	3.07%	15.68%	15.08%	5.83%	5.28%	-3.90%
50	Constant	-1.0%	Constant	10.0%	1 year	Rounded	0.00%	0.00%	0.61%	0.65%	16.47%	15.91%	0.84%	0.36%	-1.32%
51	Constant	-1.0%	Constant	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	16.37%	15.97%	0.67%	0.32%	-1.15%
52	Constant	-1.0%	Constant	10.0%	8 year	Constant	0.00%	0.00%	3.45%	3.03%	8.68%	9.86%	-0.57%	0.51%	-4.43%
53	Constant	-1.0%	Constant	10.0%	8 year	Rounded	0.00%	0.00%	0.61%	0.64%	2.60%	5.01%	-11.16%	-9.08%	-1.86%
54	Constant	-1.0%	Constant	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	2.51%	5.11%	-11.32%	-9.08%	-1.48%
55	Varied	None	Constant	0.0%	1 year	Constant	4.20%	0.70%	3.59%	0.21%	16.78%	12.21%	6.37%	2.21%	1.44%
56	Varied	None	Constant	0.0%	1 year	Rounded	4.20%	0.70%	3.59%	0.21%	16.78%	12.21%	6.37%	2.21%	1.44%
57	Varied	None	Constant	0.0%	1 year	Exact	4.20%	0.70%	3.59%	0.21%	16.78%	12.21%	6.37%	2.21%	1.44%
58	Varied	None	Constant	0.0%	8 year	Constant	4.01%	0.78%	3.35%	0.24%	6.49%	7.40%	-3.00%	-2.18%	0.69%
59	Varied	None	Constant	0.0%	8 year	Rounded	4.01%	0.78%	3.35%	0.24%	6.49%	7.40%	-3.00%	-2.18%	0.69%
60	Varied	None	Constant	0.0%	8 year	Exact	4.01%	0.78%	3.35%	0.24%	6.49%	7.40%	-3.00%	-2.18%	0.69%
61	Varied	None	Constant	3.0%	1 year	Constant	4.36%	0.71%	4.63%	0.96%	18.34%	13.50%	7.79%	3.38%	0.32%
62	Varied	None	Constant	3.0%	1 year	Rounded	4.36%	0.71%	3.90%	0.38%	18.49%	13.68%	6.07%	1.76%	1.09%
63	Varied	None	Constant	3.0%	1 year	Exact	4.36%	0.71%	3.92%	0.38%	18.68%	13.88%	6.26%	1.96%	1.14%
64	Varied	None	Constant	3.0%	8 year	Constant	4.16%	0.80%	4.39%	0.98%	9.40%	9.80%	-0.35%	0.02%	-0.24%
65	Varied	None	Constant	3.0%	8 year	Rounded	4.16%	0.80%	3.65%	0.41%	7.64%	8.68%	-3.65%	-2.72%	0.10%
66	Varied	None	Constant	3.0%	8 year	Exact	4.16%	0.80%	3.67%	0.41%	7.86%	8.74%	-3.43%	-2.64%	0.29%
67	Varied	None	Constant	10.0%	1 year	Constant	4.67%	0.73%	7.14%	2.87%	22.37%	17.12%	11.46%	6.68%	-1.98%
68	Varied	None	Constant	10.0%	1 year	Rounded	4.67%	0.73%	4.61%	0.77%	23.08%	17.70%	6.13%	1.48%	0.42%
69	Varied	None	Constant	10.0%	1 year	Exact	4.67%	0.73%	4.60%	0.75%	22.97%	17.69%	5.93%	1.38%	0.46%
70	Varied	None	Constant	10.0%	8 year	Constant	4.45%	0.83%	6.89%	2.86%	17.73%	16.97%	7.24%	6.54%	-2.44%
71	Varied	None	Constant	10.0%	8 year	Rounded	4.45%	0.83%	4.34%	0.80%	10.95%	11.56%	-4.33%	-3.80%	-0.65%
72	Varied	None	Constant	10.0%	8 year	Exact	4.45%	0.83%	4.34%	0.78%	10.82%	11.64%	-4.54%	-3.83%	-0.60%
73	Varied	1.0%	Constant	0.0%	1 year	Constant	4.25%	0.70%	3.65%	0.21%	14.22%	10.19%	4.04%	0.37%	1.41%
74	Varied	1.0%	Constant	0.0%	1 year	Rounded	4.25%	0.70%	3.65%	0.21%	14.22%	10.19%	4.04%	0.37%	1.41%
75	Varied	1.0%	Constant	0.0%	1 year	Exact	4.25%	0.70%	3.65%	0.21%	14.22%	10.19%	4.04%	0.37%	1.41%
76	Varied	1.0%	Constant	0.0%	8 year	Constant	4.11%	0.78%	3.41%	0.23%	11.24%	11.11%	1.33%	1.21%	1.10%
77	Varied	1.0%	Constant	0.0%	8 year	Rounded	4.11%	0.78%	3.41%	0.23%	11.24%	11.11%	1.33%	1.21%	1.10%
78	Varied	1.0%	Constant	0.0%	8 year	Exact	4.11%	0.78%	3.41%	0.23%	11.24%	11.11%	1.33%	1.21%	1.10%
79	Varied	1.0%	Constant	3.0%	1 year	Constant	4.41%	0.71%	4.70%	0.97%	15.63%	11.32%	5.32%	1.40%	0.25%
80	Varied	1.0%	Constant	3.0%	1 year	Rounded	4.41%	0.71%	3.95%	0.37%	15.77%	11.52%	3.63%	-0.17%	1.04%
81	Varied	1.0%	Constant	3.0%	1 year	Exact	4.41%	0.71%	3.97%	0.38%	15.96%	11.72%	3.82%	0.03%	1.10%
82	Varied	1.0%	Constant	3.0%	8 year	Constant	4.26%	0.80%	4.45%	0.99%	14.17%	13.49%	3.99%	3.37%	0.00%
83	Varied	1.0%	Constant	3.0%	8 year	Rounded	4.26%	0.80%	3.70%	0.40%	12.60%	12.57%	0.80%	0.77%	0.60%
84	Varied	1.0%	Constant	3.0%	8 year	Exact	4.26%	0.80%	3.72%	0.40%	12.92%	12.72%	1.11%	0.93%	0.73%
85	Varied	1.0%	Constant	10.0%	1 year	Constant	4.70%	0.73%	7.23%	2.91%	20.16%	15.33%	9.45%	5.05%	-2.14%
86	Varied	1.0%	Constant	10.0%	1 year	Rounded	4.70%	0.73%	4.65%	0.76%	19.99%	15.20%	3.46%	-0.67%	0.34%
87	Varied	1.0%	Constant	10.0%	1 year	Exact	4.70%	0.73%	4.64%	0.74%	19.88%	15.21%	3.27%	-0.76%	0.39%
88	Varied	1.0%	Constant	10.0%	8 year	Constant	4.53%	0.81%	6.96%	2.90%	22.03%	20.28%	11.15%	9.56%	-2.34%
89	Varied	1.0%	Constant	10.0%	8 year	Rounded	4.53%	0.81%	4.39%	0.79%	16.73%	16.16%	0.65%	0.16%	-0.15%
90	Varied	1.0%	Constant	10.0%	8 year	Exact	4.53%	0.81%	4.38%	0.77%	16.61%	16.27%	0.45%	0.16%	-0.10%

$550,000 Retention Scenario Results
Total Error in Methods

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess
91	Varied	-1.0%	Constant	0.0%	1 year	Constant	4.17%	0.69%	3.55%	0.21%	18.40%	13.50%	7.84%	3.39%	1.44%
92	Varied	-1.0%	Constant	0.0%	1 year	Rounded	4.17%	0.69%	3.55%	0.21%	18.40%	13.50%	7.84%	3.39%	1.44%
93	Varied	-1.0%	Constant	0.0%	1 year	Exact	4.17%	0.69%	3.55%	0.21%	18.40%	13.50%	7.84%	3.39%	1.44%
94	Varied	-1.0%	Constant	0.0%	8 year	Constant	3.98%	0.78%	3.31%	0.25%	8.00%	7.98%	-1.63%	-1.65%	0.91%
95	Varied	-1.0%	Constant	0.0%	8 year	Rounded	3.98%	0.78%	3.31%	0.25%	8.00%	7.98%	-1.63%	-1.65%	0.91%
96	Varied	-1.0%	Constant	0.0%	8 year	Exact	3.98%	0.78%	3.31%	0.25%	8.00%	7.98%	-1.63%	-1.65%	0.91%
97	Varied	-1.0%	Constant	3.0%	1 year	Constant	4.33%	0.71%	4.58%	0.95%	20.10%	14.91%	9.39%	4.67%	0.36%
98	Varied	-1.0%	Constant	3.0%	1 year	Rounded	4.33%	0.71%	3.87%	0.38%	20.21%	15.06%	7.60%	3.00%	1.11%
99	Varied	-1.0%	Constant	3.0%	1 year	Exact	4.33%	0.71%	3.88%	0.38%	20.40%	15.26%	7.80%	3.20%	1.15%
100	Varied	-1.0%	Constant	3.0%	8 year	Constant	4.13%	0.80%	4.32%	0.98%	10.86%	10.31%	0.98%	0.47%	-0.09%
101	Varied	-1.0%	Constant	3.0%	8 year	Rounded	4.13%	0.80%	3.61%	0.42%	9.27%	9.32%	-2.19%	-2.14%	0.37%
102	Varied	-1.0%	Constant	3.0%	8 year	Exact	4.13%	0.80%	3.63%	0.42%	9.53%	9.44%	-1.93%	-2.02%	0.53%
103	Varied	-1.0%	Constant	10.0%	1 year	Constant	4.64%	0.72%	7.06%	2.83%	24.38%	18.78%	13.29%	8.19%	-1.87%
104	Varied	-1.0%	Constant	10.0%	1 year	Rounded	4.64%	0.72%	4.58%	0.77%	25.03%	19.28%	7.80%	2.84%	0.46%
105	Varied	-1.0%	Constant	10.0%	1 year	Exact	4.64%	0.72%	4.57%	0.75%	24.91%	19.26%	7.61%	2.74%	0.50%
106	Varied	-1.0%	Constant	10.0%	8 year	Constant	4.43%	0.82%	6.78%	2.84%	18.88%	17.22%	8.29%	6.77%	-2.33%
107	Varied	-1.0%	Constant	10.0%	8 year	Rounded	4.43%	0.82%	4.31%	0.81%	13.00%	12.56%	-2.56%	-2.94%	-0.36%
108	Varied	-1.0%	Constant	10.0%	8 year	Exact	4.43%	0.82%	4.30%	0.80%	12.89%	12.65%	-2.75%	-2.96%	-0.32%
109	Constant	None	Varied	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	9.14%	9.21%	-0.21%	-0.15%	0.00%
110	Constant	None	Varied	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	9.14%	9.21%	-0.21%	-0.15%	0.00%
111	Constant	None	Varied	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	9.14%	9.21%	-0.21%	-0.15%	0.00%
112	Constant	None	Varied	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	-2.06%	0.39%	-10.45%	-8.21%	0.00%
113	Constant	None	Varied	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	-2.06%	0.39%	-10.45%	-8.21%	0.00%
114	Constant	None	Varied	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	-2.06%	0.39%	-10.45%	-8.21%	0.00%
115	Constant	None	Varied	3.0%	1 year	Constant	0.00%	0.00%	0.99%	0.87%	10.23%	10.16%	0.79%	0.72%	-1.36%
116	Constant	None	Varied	3.0%	1 year	Rounded	0.00%	0.00%	0.15%	0.18%	10.53%	10.45%	-0.68%	-0.76%	-0.47%
117	Constant	None	Varied	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	10.67%	10.74%	-0.54%	-0.48%	-0.37%
118	Constant	None	Varied	3.0%	8 year	Constant	0.00%	0.00%	0.99%	0.87%	0.16%	2.23%	-8.42%	-6.53%	-1.61%
119	Constant	None	Varied	3.0%	8 year	Rounded	0.00%	0.00%	0.15%	0.19%	-1.37%	1.00%	-11.38%	-9.25%	-0.80%
120	Constant	None	Varied	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.18%	-1.24%	1.19%	-11.24%	-9.06%	-0.46%
121	Constant	None	Varied	10.0%	1 year	Constant	0.00%	0.00%	3.51%	3.10%	13.51%	13.33%	3.78%	3.62%	-4.08%
122	Constant	None	Varied	10.0%	1 year	Rounded	0.00%	0.00%	0.61%	0.64%	14.30%	14.17%	-1.09%	-1.20%	-1.38%
123	Constant	None	Varied	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	14.20%	14.24%	-1.26%	-1.23%	-1.20%
124	Constant	None	Varied	10.0%	8 year	Constant	0.00%	0.00%	3.49%	3.08%	7.58%	8.57%	-1.63%	-0.73%	-4.63%
125	Constant	None	Varied	10.0%	8 year	Rounded	0.00%	0.00%	0.61%	0.64%	0.56%	2.76%	-12.97%	-11.07%	-2.04%
126	Constant	None	Varied	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	0.49%	2.84%	-13.12%	-11.09%	-1.50%
127	Constant	1.0%	Varied	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	6.62%	6.85%	-2.51%	-2.30%	0.00%
128	Constant	1.0%	Varied	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	6.62%	6.85%	-2.51%	-2.30%	0.00%
129	Constant	1.0%	Varied	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	6.62%	6.85%	-2.51%	-2.30%	0.00%
130	Constant	1.0%	Varied	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	0.94%	3.50%	-7.70%	-5.37%	0.00%
131	Constant	1.0%	Varied	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	0.94%	3.50%	-7.70%	-5.37%	0.00%
132	Constant	1.0%	Varied	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	0.94%	3.50%	-7.70%	-5.37%	0.00%
133	Constant	1.0%	Varied	3.0%	1 year	Constant	0.00%	0.00%	1.01%	0.89%	7.46%	7.52%	-1.74%	-1.69%	-1.42%
134	Constant	1.0%	Varied	3.0%	1 year	Rounded	0.00%	0.00%	0.14%	0.18%	7.81%	7.89%	-3.13%	-3.06%	-0.50%
135	Constant	1.0%	Varied	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	7.94%	8.18%	-2.99%	-2.78%	-0.38%

$550,000 Retention Scenario Results
Total Error in Methods

Appendix 2 - $550,000 Retention, Page 4

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess
136	Constant	1.0%	Varied	3.0%	8 year	Constant	0.00%	0.00%	1.00%	0.88%	3.51%	5.46%	-5.36%	-3.58%	-1.56%
137	Constant	1.0%	Varied	3.0%	8 year	Rounded	0.00%	0.00%	0.15%	0.19%	1.91%	4.36%	-8.43%	-6.23%	-0.68%
138	Constant	1.0%	Varied	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.18%	2.15%	4.68%	-8.20%	-5.93%	-0.45%
139	Constant	1.0%	Varied	10.0%	1 year	Constant	0.00%	0.00%	3.57%	3.15%	11.18%	11.07%	1.66%	1.55%	-4.23%
140	Constant	1.0%	Varied	10.0%	1 year	Rounded	0.00%	0.00%	0.62%	0.65%	11.05%	11.11%	-3.90%	-3.85%	-1.46%
141	Constant	1.0%	Varied	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	10.96%	11.20%	-4.07%	-3.86%	-1.25%
142	Constant	1.0%	Varied	10.0%	8 year	Constant	0.00%	0.00%	3.54%	3.12%	11.16%	11.77%	1.63%	2.20%	-4.54%
143	Constant	1.0%	Varied	10.0%	8 year	Rounded	0.00%	0.00%	0.61%	0.65%	4.93%	7.11%	-9.20%	-7.31%	-1.82%
144	Constant	1.0%	Varied	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	4.83%	7.26%	-9.37%	-7.27%	-1.47%
145	Constant	-1.0%	Varied	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	10.79%	10.58%	1.30%	1.11%	0.00%
146	Constant	-1.0%	Varied	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	10.79%	10.58%	1.30%	1.11%	0.00%
147	Constant	-1.0%	Varied	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	10.79%	10.58%	1.30%	1.11%	0.00%
148	Constant	-1.0%	Varied	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	-1.66%	1.30%	-10.08%	-7.38%	0.00%
149	Constant	-1.0%	Varied	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	-1.66%	1.30%	-10.08%	-7.38%	0.00%
150	Constant	-1.0%	Varied	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	-1.66%	1.30%	-10.08%	-7.38%	0.00%
151	Constant	-1.0%	Varied	3.0%	1 year	Constant	0.00%	0.00%	0.99%	0.86%	12.01%	11.66%	2.41%	2.09%	-1.32%
152	Constant	-1.0%	Varied	3.0%	1 year	Rounded	0.00%	0.00%	0.15%	0.18%	12.27%	11.90%	0.88%	0.55%	-0.45%
153	Constant	-1.0%	Varied	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	12.42%	12.19%	1.03%	0.82%	-0.36%
154	Constant	-1.0%	Varied	3.0%	8 year	Constant	0.00%	0.00%	0.97%	0.85%	0.65%	3.21%	-7.97%	-5.63%	-1.54%
155	Constant	-1.0%	Varied	3.0%	8 year	Rounded	0.00%	0.00%	0.15%	0.19%	-0.86%	2.04%	-10.92%	-8.31%	-0.72%
156	Constant	-1.0%	Varied	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.18%	-0.67%	2.29%	-10.73%	-8.07%	-0.46%
157	Constant	-1.0%	Varied	10.0%	1 year	Constant	0.00%	0.00%	3.48%	3.06%	15.52%	15.08%	5.62%	5.22%	-3.93%
158	Constant	-1.0%	Varied	10.0%	1 year	Rounded	0.00%	0.00%	0.61%	0.64%	16.26%	15.82%	0.60%	0.23%	-1.33%
159	Constant	-1.0%	Varied	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	16.16%	15.88%	0.43%	0.18%	-1.16%
160	Constant	-1.0%	Varied	10.0%	8 year	Constant	0.00%	0.00%	3.44%	3.03%	8.08%	9.56%	-1.18%	0.18%	-4.50%
161	Constant	-1.0%	Varied	10.0%	8 year	Rounded	0.00%	0.00%	0.61%	0.64%	1.57%	4.33%	-12.11%	-9.72%	-1.90%
162	Constant	-1.0%	Varied	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	1.49%	4.43%	-12.26%	-9.71%	-1.50%
163	Varied	None	Varied	0.0%	1 year	Constant	4.17%	0.69%	3.56%	0.20%	16.60%	12.12%	6.15%	2.06%	1.46%
164	Varied	None	Varied	0.0%	1 year	Rounded	4.17%	0.69%	3.56%	0.20%	16.60%	12.12%	6.15%	2.06%	1.46%
165	Varied	None	Varied	0.0%	1 year	Exact	4.17%	0.69%	3.56%	0.20%	16.60%	12.12%	6.15%	2.06%	1.46%
166	Varied	None	Varied	0.0%	8 year	Constant	3.95%	0.77%	3.32%	0.23%	6.52%	7.09%	-3.03%	-2.52%	0.70%
167	Varied	None	Varied	0.0%	8 year	Rounded	3.95%	0.77%	3.32%	0.23%	6.52%	7.09%	-3.03%	-2.52%	0.70%
168	Varied	None	Varied	0.0%	8 year	Exact	3.95%	0.77%	3.32%	0.23%	6.52%	7.09%	-3.03%	-2.52%	0.70%
169	Varied	None	Varied	3.0%	1 year	Constant	4.33%	0.70%	4.59%	0.95%	18.23%	13.48%	7.63%	3.30%	0.33%
170	Varied	None	Varied	3.0%	1 year	Rounded	4.33%	0.70%	3.87%	0.37%	18.35%	13.62%	5.88%	1.65%	1.10%
171	Varied	None	Varied	3.0%	1 year	Exact	4.33%	0.70%	3.89%	0.38%	18.54%	13.82%	6.07%	1.84%	1.16%
172	Varied	None	Varied	3.0%	8 year	Constant	4.11%	0.79%	4.36%	0.97%	9.40%	9.43%	-0.41%	-0.39%	-0.23%
173	Varied	None	Varied	3.0%	8 year	Rounded	4.11%	0.79%	3.62%	0.40%	7.67%	8.31%	-3.68%	-3.10%	0.11%
174	Varied	None	Varied	3.0%	8 year	Exact	4.11%	0.79%	3.64%	0.40%	7.90%	8.41%	-3.45%	-2.99%	0.31%
175	Varied	None	Varied	10.0%	1 year	Constant	4.64%	0.72%	7.10%	2.85%	22.42%	17.27%	11.44%	6.76%	-2.00%
176	Varied	None	Varied	10.0%	1 year	Rounded	4.64%	0.72%	4.58%	0.76%	23.04%	17.71%	6.03%	1.44%	0.44%
177	Varied	None	Varied	10.0%	1 year	Exact	4.64%	0.72%	4.58%	0.75%	22.92%	17.70%	5.83%	1.33%	0.47%
178	Varied	None	Varied	10.0%	8 year	Constant	4.41%	0.82%	6.85%	2.85%	17.72%	16.48%	7.16%	6.03%	-2.44%
179	Varied	None	Varied	10.0%	8 year	Rounded	4.41%	0.82%	4.32%	0.79%	10.97%	11.17%	-4.37%	-4.20%	-0.62%
180	Varied	None	Varied	10.0%	8 year	Exact	4.41%	0.82%	4.31%	0.78%	10.85%	11.26%	-4.56%	-4.21%	-0.57%

$550,000 Retention Scenario Results
Total Error in Methods

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Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess
181	Varied	1.0%	Varied	0.0%	1 year	Constant	4.18%	0.69%	3.59%	0.20%	14.29%	10.16%	4.04%	0.28%	1.38%
182	Varied	1.0%	Varied	0.0%	1 year	Rounded	4.18%	0.69%	3.59%	0.20%	14.29%	10.16%	4.04%	0.28%	1.38%
183	Varied	1.0%	Varied	0.0%	1 year	Exact	4.18%	0.69%	3.59%	0.20%	14.29%	10.16%	4.04%	0.28%	1.38%
184	Varied	1.0%	Varied	0.0%	8 year	Constant	4.08%	0.76%	3.36%	0.22%	10.07%	9.81%	0.20%	-0.04%	1.02%
185	Varied	1.0%	Varied	0.0%	8 year	Rounded	4.08%	0.76%	3.36%	0.22%	10.07%	9.81%	0.20%	-0.04%	1.02%
186	Varied	1.0%	Varied	0.0%	8 year	Exact	4.08%	0.76%	3.36%	0.22%	10.07%	9.81%	0.20%	-0.04%	1.02%
187	Varied	1.0%	Varied	3.0%	1 year	Constant	4.34%	0.70%	4.64%	0.96%	15.74%	11.30%	5.36%	1.32%	0.22%
188	Varied	1.0%	Varied	3.0%	1 year	Rounded	4.34%	0.70%	3.89%	0.36%	15.87%	11.50%	3.67%	-0.25%	1.00%
189	Varied	1.0%	Varied	3.0%	1 year	Exact	4.34%	0.70%	3.91%	0.37%	16.05%	11.69%	3.85%	-0.06%	1.06%
190	Varied	1.0%	Varied	3.0%	8 year	Constant	4.23%	0.78%	4.40%	0.98%	13.20%	12.35%	3.05%	2.27%	-0.09%
191	Varied	1.0%	Varied	3.0%	8 year	Rounded	4.23%	0.78%	3.66%	0.39%	11.44%	11.23%	-0.30%	-0.49%	0.50%
192	Varied	1.0%	Varied	3.0%	8 year	Exact	4.23%	0.78%	3.68%	0.39%	11.78%	11.42%	0.02%	-0.30%	0.65%
193	Varied	1.0%	Varied	10.0%	1 year	Constant	4.65%	0.71%	7.16%	2.88%	20.48%	15.45%	9.67%	5.10%	-2.18%
194	Varied	1.0%	Varied	10.0%	1 year	Rounded	4.65%	0.71%	4.60%	0.75%	20.14%	15.18%	3.53%	-0.75%	0.30%
195	Varied	1.0%	Varied	10.0%	1 year	Exact	4.65%	0.71%	4.59%	0.73%	20.02%	15.18%	3.33%	-0.84%	0.35%
196	Varied	1.0%	Varied	10.0%	8 year	Constant	4.51%	0.80%	6.91%	2.88%	21.55%	19.54%	10.65%	8.82%	-2.44%
197	Varied	1.0%	Varied	10.0%	8 year	Rounded	4.51%	0.80%	4.35%	0.78%	15.68%	14.87%	-0.31%	-1.01%	-0.25%
198	Varied	1.0%	Varied	10.0%	8 year	Exact	4.51%	0.80%	4.34%	0.77%	15.54%	14.98%	-0.52%	-1.01%	-0.20%
199	Varied	-1.0%	Varied	0.0%	1 year	Constant	4.15%	0.69%	3.54%	0.21%	18.12%	13.34%	7.53%	3.18%	1.46%
200	Varied	-1.0%	Varied	0.0%	1 year	Rounded	4.15%	0.69%	3.54%	0.21%	18.12%	13.34%	7.53%	3.18%	1.46%
201	Varied	-1.0%	Varied	0.0%	1 year	Exact	4.15%	0.69%	3.54%	0.21%	18.12%	13.34%	7.53%	3.18%	1.46%
202	Varied	-1.0%	Varied	0.0%	8 year	Constant	3.96%	0.76%	3.29%	0.24%	7.12%	7.43%	-2.49%	-2.20%	0.85%
203	Varied	-1.0%	Varied	0.0%	8 year	Rounded	3.96%	0.76%	3.29%	0.24%	7.12%	7.43%	-2.49%	-2.20%	0.85%
204	Varied	-1.0%	Varied	0.0%	8 year	Exact	3.96%	0.76%	3.29%	0.24%	7.12%	7.43%	-2.49%	-2.20%	0.85%
205	Varied	-1.0%	Varied	3.0%	1 year	Constant	4.32%	0.70%	4.56%	0.95%	19.85%	14.79%	9.10%	4.50%	0.36%
206	Varied	-1.0%	Varied	3.0%	1 year	Rounded	4.32%	0.70%	3.85%	0.37%	19.95%	14.91%	7.31%	2.81%	1.12%
207	Varied	-1.0%	Varied	3.0%	1 year	Exact	4.32%	0.70%	3.87%	0.38%	20.14%	15.11%	7.51%	3.00%	1.17%
208	Varied	-1.0%	Varied	3.0%	8 year	Constant	4.12%	0.78%	4.31%	0.97%	10.08%	9.85%	0.21%	0.00%	-0.15%
209	Varied	-1.0%	Varied	3.0%	8 year	Rounded	4.12%	0.78%	3.59%	0.41%	8.36%	8.75%	-3.06%	-2.71%	0.30%
210	Varied	-1.0%	Varied	3.0%	8 year	Exact	4.12%	0.78%	3.61%	0.41%	8.64%	8.89%	-2.79%	-2.57%	0.46%
211	Varied	-1.0%	Varied	10.0%	1 year	Constant	4.64%	0.72%	7.05%	2.83%	24.21%	18.77%	13.07%	8.11%	-1.88%
212	Varied	-1.0%	Varied	10.0%	1 year	Rounded	4.64%	0.72%	4.57%	0.77%	24.82%	19.17%	7.56%	2.69%	0.48%
213	Varied	-1.0%	Varied	10.0%	1 year	Exact	4.64%	0.72%	4.56%	0.75%	24.70%	19.15%	7.36%	2.58%	0.50%
214	Varied	-1.0%	Varied	10.0%	8 year	Constant	4.43%	0.81%	6.77%	2.82%	18.37%	17.01%	7.75%	6.51%	-2.40%
215	Varied	-1.0%	Varied	10.0%	8 year	Rounded	4.43%	0.81%	4.30%	0.80%	12.11%	11.99%	-3.39%	-3.49%	-0.44%
216	Varied	-1.0%	Varied	10.0%	8 year	Exact	4.43%	0.81%	4.29%	0.79%	11.99%	12.09%	-3.58%	-3.49%	-0.41%

Multiple Retention Scenario Results
Total Error in Methods

Appendix 2 - Multiple Retention, Page 1

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess	Sum of Ret. Method Excess
1	Constant	None	Constant	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	7.86%	7.86%	0.00%	0.00%	0.00%	0.00%
2	Constant	None	Constant	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	7.86%	7.86%	0.00%	0.00%	0.00%	0.00%
3	Constant	None	Constant	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	7.86%	7.86%	0.00%	0.00%	0.00%	0.00%
4	Constant	None	Constant	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	1.01%	2.64%	-6.35%	-4.84%	0.00%	0.00%
5	Constant	None	Constant	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	1.01%	2.64%	-6.35%	-4.84%	0.00%	0.00%
6	Constant	None	Constant	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	1.01%	2.64%	-6.35%	-4.84%	0.00%	0.00%
7	Constant	None	Constant	3.0%	1 year	Constant	0.00%	0.00%	0.99%	0.86%	9.19%	9.01%	1.23%	1.07%	-1.23%	-1.03%
8	Constant	None	Constant	3.0%	1 year	Rounded	0.00%	0.00%	0.08%	0.04%	9.29%	9.37%	-0.25%	-0.17%	-0.07%	0.00%
9	Constant	None	Constant	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	9.34%	9.34%	-0.28%	-0.28%	-0.28%	-0.28%
10	Constant	None	Constant	3.0%	8 year	Constant	0.00%	0.00%	0.98%	0.86%	3.14%	4.36%	-4.38%	-3.24%	-1.44%	-1.17%
11	Constant	None	Constant	3.0%	8 year	Rounded	0.00%	0.00%	0.07%	0.05%	1.92%	3.46%	-6.97%	-5.56%	-0.09%	0.02%
12	Constant	None	Constant	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.19%	2.05%	3.65%	-6.93%	-5.47%	-0.34%	-0.34%
13	Constant	None	Constant	10.0%	1 year	Constant	0.00%	0.00%	3.35%	2.80%	13.74%	13.44%	5.45%	5.18%	-3.30%	-2.64%
14	Constant	None	Constant	10.0%	1 year	Rounded	0.00%	0.00%	0.65%	0.68%	12.78%	12.67%	-1.01%	-1.11%	-1.05%	-1.10%
15	Constant	None	Constant	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	12.72%	12.72%	-0.92%	-0.92%	-0.92%	-0.92%
16	Constant	None	Constant	10.0%	8 year	Constant	0.00%	0.00%	3.35%	2.80%	9.62%	10.25%	1.63%	2.21%	-3.81%	-2.87%
17	Constant	None	Constant	10.0%	8 year	Rounded	0.00%	0.00%	0.65%	0.68%	4.35%	5.84%	-8.41%	-7.11%	-1.26%	-1.44%
18	Constant	None	Constant	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	4.31%	5.86%	-8.31%	-6.95%	-1.11%	-1.10%
19	Constant	1.0%	Constant	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	5.69%	6.07%	-2.01%	-1.66%	0.00%	0.00%
20	Constant	1.0%	Constant	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	5.69%	6.07%	-2.01%	-1.66%	0.00%	0.00%
21	Constant	1.0%	Constant	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	5.69%	6.07%	-2.01%	-1.66%	0.00%	0.00%
22	Constant	1.0%	Constant	0.0%	8 year	Constant	0.00%	0.00%	-0.01%	0.00%	4.11%	5.31%	-3.48%	-2.36%	0.00%	0.00%
23	Constant	1.0%	Constant	0.0%	8 year	Rounded	0.00%	0.00%	-0.01%	0.00%	4.11%	5.31%	-3.48%	-2.36%	0.00%	0.00%
24	Constant	1.0%	Constant	0.0%	8 year	Exact	0.00%	0.00%	-0.01%	0.00%	4.11%	5.31%	-3.48%	-2.36%	0.00%	0.00%
25	Constant	1.0%	Constant	3.0%	1 year	Constant	0.00%	0.00%	1.00%	0.88%	7.12%	7.31%	-0.68%	-0.51%	-1.29%	-1.08%
26	Constant	1.0%	Constant	3.0%	1 year	Rounded	0.00%	0.00%	0.07%	0.04%	6.98%	7.45%	-2.35%	-1.92%	-0.06%	0.01%
27	Constant	1.0%	Constant	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	7.02%	7.42%	-2.39%	-2.03%	-0.29%	-0.29%
28	Constant	1.0%	Constant	3.0%	8 year	Constant	0.00%	0.00%	0.99%	0.88%	6.28%	6.97%	-1.47%	-0.83%	-1.41%	-1.13%
29	Constant	1.0%	Constant	3.0%	8 year	Rounded	0.00%	0.00%	0.06%	0.04%	5.23%	6.32%	-3.95%	-2.95%	-0.07%	0.01%
30	Constant	1.0%	Constant	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.19%	5.40%	6.55%	-3.88%	-2.83%	-0.33%	-0.32%
31	Constant	1.0%	Constant	10.0%	1 year	Constant	0.00%	0.00%	3.39%	2.84%	12.22%	12.25%	4.04%	4.07%	-3.44%	-2.73%
32	Constant	1.0%	Constant	10.0%	1 year	Rounded	0.00%	0.00%	0.65%	0.68%	10.11%	10.42%	-3.36%	-3.08%	-1.09%	-1.16%
33	Constant	1.0%	Constant	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	10.08%	10.51%	-3.24%	-2.86%	-0.95%	-0.95%
34	Constant	1.0%	Constant	10.0%	8 year	Constant	0.00%	0.00%	3.40%	2.84%	12.60%	12.64%	4.40%	4.43%	-3.71%	-2.81%
35	Constant	1.0%	Constant	10.0%	8 year	Rounded	0.00%	0.00%	0.65%	0.68%	8.34%	9.28%	-4.91%	-4.08%	-1.21%	-1.32%
36	Constant	1.0%	Constant	10.0%	8 year	Exact	0.00%	0.00%	0.59%	0.60%	8.26%	9.27%	-4.84%	-3.95%	-1.06%	-1.05%
37	Constant	-1.0%	Constant	0.0%	1 year	Constant	0.00%	0.00%	-0.01%	-0.01%	9.55%	9.23%	1.56%	1.27%	0.01%	0.00%
38	Constant	-1.0%	Constant	0.0%	1 year	Rounded	0.00%	0.00%	-0.01%	-0.01%	9.55%	9.23%	1.56%	1.27%	0.01%	0.00%
39	Constant	-1.0%	Constant	0.0%	1 year	Exact	0.00%	0.00%	-0.01%	-0.01%	9.55%	9.23%	1.56%	1.27%	0.01%	0.00%
40	Constant	-1.0%	Constant	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	1.90%	3.33%	-5.52%	-4.20%	0.00%	0.00%
41	Constant	-1.0%	Constant	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	1.90%	3.33%	-5.52%	-4.20%	0.00%	0.00%
42	Constant	-1.0%	Constant	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	1.90%	3.33%	-5.52%	-4.20%	0.00%	0.00%
43	Constant	-1.0%	Constant	3.0%	1 year	Constant	0.00%	0.00%	0.97%	0.84%	10.98%	10.48%	2.89%	2.43%	-1.17%	-1.00%
44	Constant	-1.0%	Constant	3.0%	1 year	Rounded	0.00%	0.00%	0.07%	0.04%	11.07%	10.82%	1.38%	1.15%	-0.06%	-0.01%
45	Constant	-1.0%	Constant	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	11.13%	10.80%	1.36%	1.06%	-0.27%	-0.28%

Multiple Retention Scenario Results
Total Error in Methods

Appendix 2 - Multiple Retention, Page 2

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess	Sum of Ret. Method Excess
46	Constant	-1.0%	Constant	3.0%	8 year	Constant	0.00%	0.00%	0.96%	0.85%	3.97%	4.97%	-3.61%	-2.68%	-1.40%	-1.11%
47	Constant	-1.0%	Constant	3.0%	8 year	Rounded	0.00%	0.00%	0.07%	0.05%	2.92%	4.28%	-6.06%	-4.81%	-0.09%	0.01%
48	Constant	-1.0%	Constant	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.19%	3.05%	4.45%	-6.01%	-4.74%	-0.34%	-0.33%
49	Constant	-1.0%	Constant	10.0%	1 year	Constant	0.00%	0.00%	3.29%	2.75%	15.61%	14.99%	7.18%	6.61%	-3.15%	-2.55%
50	Constant	-1.0%	Constant	10.0%	1 year	Rounded	0.00%	0.00%	0.64%	0.67%	14.85%	14.37%	0.80%	0.39%	-1.00%	-1.06%
51	Constant	-1.0%	Constant	10.0%	1 year	Exact	0.00%	0.00%	0.59%	0.59%	14.77%	14.41%	0.89%	0.57%	-0.88%	-0.89%
52	Constant	-1.0%	Constant	10.0%	8 year	Constant	0.00%	0.00%	3.30%	2.75%	10.22%	10.60%	2.19%	2.54%	-3.72%	-2.76%
53	Constant	-1.0%	Constant	10.0%	8 year	Rounded	0.00%	0.00%	0.64%	0.68%	5.67%	6.92%	-7.26%	-6.16%	-1.24%	-1.37%
54	Constant	-1.0%	Constant	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	5.61%	6.93%	-7.17%	-6.01%	-1.09%	-1.08%
55	Varied	None	Constant	0.0%	1 year	Constant	4.20%	0.70%	3.46%	0.03%	15.20%	10.99%	6.45%	2.56%	1.73%	1.62%
56	Varied	None	Constant	0.0%	1 year	Rounded	4.20%	0.70%	3.46%	0.03%	15.20%	10.99%	6.45%	2.56%	1.73%	1.62%
57	Varied	None	Constant	0.0%	1 year	Exact	4.20%	0.70%	3.46%	0.03%	15.20%	10.99%	6.45%	2.56%	1.73%	1.62%
58	Varied	None	Constant	0.0%	8 year	Constant	4.01%	0.78%	3.26%	0.04%	8.91%	8.32%	0.64%	0.10%	2.12%	1.40%
59	Varied	None	Constant	0.0%	8 year	Rounded	4.01%	0.78%	3.26%	0.04%	8.91%	8.32%	0.64%	0.10%	2.12%	1.40%
60	Varied	None	Constant	0.0%	8 year	Exact	4.01%	0.78%	3.26%	0.04%	8.91%	8.32%	0.64%	0.10%	2.12%	1.40%
61	Varied	None	Constant	3.0%	1 year	Constant	4.36%	0.71%	4.45%	0.75%	17.02%	12.45%	8.13%	3.91%	0.66%	0.77%
62	Varied	None	Constant	3.0%	1 year	Rounded	4.36%	0.71%	3.70%	0.05%	16.94%	12.64%	6.39%	2.48%	1.72%	1.70%
63	Varied	None	Constant	3.0%	1 year	Exact	4.36%	0.71%	3.78%	0.18%	17.09%	12.66%	6.44%	2.42%	1.52%	1.42%
64	Varied	None	Constant	3.0%	8 year	Constant	4.16%	0.80%	4.26%	0.76%	11.60%	10.43%	3.13%	2.05%	0.86%	0.63%
65	Varied	None	Constant	3.0%	8 year	Rounded	4.16%	0.80%	3.49%	0.08%	10.16%	9.47%	0.23%	-0.41%	2.11%	1.50%
66	Varied	None	Constant	3.0%	8 year	Exact	4.16%	0.80%	3.57%	0.20%	10.43%	9.76%	0.39%	-0.22%	1.88%	1.16%
67	Varied	None	Constant	10.0%	1 year	Constant	4.67%	0.73%	6.67%	2.26%	22.47%	17.45%	13.18%	8.53%	-1.09%	-0.52%
68	Varied	None	Constant	10.0%	1 year	Rounded	4.67%	0.73%	4.47%	0.59%	21.49%	16.50%	6.30%	1.94%	0.94%	0.82%
69	Varied	None	Constant	10.0%	1 year	Exact	4.67%	0.73%	4.43%	0.52%	21.34%	16.48%	6.33%	2.07%	1.03%	0.95%
70	Varied	None	Constant	10.0%	8 year	Constant	4.45%	0.83%	6.48%	2.28%	19.10%	17.07%	10.06%	8.18%	-1.16%	-0.59%
71	Varied	None	Constant	10.0%	8 year	Rounded	4.45%	0.83%	4.25%	0.61%	13.93%	13.01%	-0.32%	-1.12%	1.22%	0.42%
72	Varied	None	Constant	10.0%	8 year	Exact	4.45%	0.83%	4.22%	0.55%	13.76%	12.93%	-0.32%	-1.05%	1.33%	0.61%
73	Varied	1.0%	Constant	0.0%	1 year	Constant	4.25%	0.70%	3.51%	0.02%	13.25%	9.44%	4.65%	1.13%	1.79%	1.62%
74	Varied	1.0%	Constant	0.0%	1 year	Rounded	4.25%	0.70%	3.51%	0.02%	13.25%	9.44%	4.65%	1.13%	1.79%	1.62%
75	Varied	1.0%	Constant	0.0%	1 year	Exact	4.25%	0.70%	3.51%	0.02%	13.25%	9.44%	4.65%	1.13%	1.79%	1.62%
76	Varied	1.0%	Constant	0.0%	8 year	Constant	4.11%	0.78%	3.30%	0.04%	12.26%	10.89%	3.74%	2.47%	2.09%	1.56%
77	Varied	1.0%	Constant	0.0%	8 year	Rounded	4.11%	0.78%	3.30%	0.04%	12.26%	10.89%	3.74%	2.47%	2.09%	1.56%
78	Varied	1.0%	Constant	0.0%	8 year	Exact	4.11%	0.78%	3.30%	0.04%	12.26%	10.89%	3.74%	2.47%	2.09%	1.56%
79	Varied	1.0%	Constant	3.0%	1 year	Constant	4.41%	0.71%	4.52%	0.75%	15.20%	10.99%	6.45%	2.56%	0.66%	0.75%
80	Varied	1.0%	Constant	3.0%	1 year	Rounded	4.41%	0.71%	3.74%	0.04%	14.87%	10.98%	4.51%	0.97%	1.78%	1.72%
81	Varied	1.0%	Constant	3.0%	1 year	Exact	4.41%	0.71%	3.82%	0.17%	15.01%	11.01%	4.55%	0.91%	1.57%	1.42%
82	Varied	1.0%	Constant	3.0%	8 year	Constant	4.26%	0.80%	4.30%	0.77%	14.92%	12.97%	6.20%	4.39%	0.84%	0.71%
83	Varied	1.0%	Constant	3.0%	8 year	Rounded	4.26%	0.80%	3.53%	0.06%	13.68%	12.19%	3.43%	2.07%	2.08%	1.66%
84	Varied	1.0%	Constant	3.0%	8 year	Exact	4.26%	0.80%	3.61%	0.20%	13.99%	12.51%	3.63%	2.28%	1.85%	1.33%
85	Varied	1.0%	Constant	10.0%	1 year	Constant	4.70%	0.73%	6.74%	2.29%	21.21%	16.45%	12.01%	7.61%	-1.16%	-0.59%
86	Varied	1.0%	Constant	10.0%	1 year	Rounded	4.70%	0.73%	4.50%	0.57%	19.10%	14.58%	4.21%	0.25%	0.97%	0.78%
87	Varied	1.0%	Constant	10.0%	1 year	Exact	4.70%	0.73%	4.47%	0.51%	18.99%	14.58%	4.26%	0.40%	1.07%	0.93%
88	Varied	1.0%	Constant	10.0%	8 year	Constant	4.53%	0.81%	6.54%	2.31%	22.13%	19.42%	12.86%	10.35%	-1.14%	-0.58%
89	Varied	1.0%	Constant	10.0%	8 year	Rounded	4.53%	0.81%	4.27%	0.60%	18.02%	16.21%	3.27%	1.68%	1.19%	0.63%
90	Varied	1.0%	Constant	10.0%	8 year	Exact	4.53%	0.81%	4.24%	0.54%	17.82%	16.10%	3.24%	1.73%	1.30%	0.80%

Multiple Retention Scenario Results
Total Error in Methods

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess	Sum of Ret. Method Excess
91	Varied	-1.0%	Constant	0.0%	1 year	Constant	4.17%	0.69%	3.41%	0.02%	16.73%	12.21%	7.87%	3.69%	1.69%	1.61%
92	Varied	-1.0%	Constant	0.0%	1 year	Rounded	4.17%	0.69%	3.41%	0.02%	16.73%	12.21%	7.87%	3.69%	1.69%	1.61%
93	Varied	-1.0%	Constant	0.0%	1 year	Exact	4.17%	0.69%	3.41%	0.02%	16.73%	12.21%	7.87%	3.69%	1.69%	1.61%
94	Varied	-1.0%	Constant	0.0%	8 year	Constant	3.98%	0.78%	3.20%	0.06%	9.82%	8.68%	1.48%	0.43%	2.06%	1.46%
95	Varied	-1.0%	Constant	0.0%	8 year	Rounded	3.98%	0.78%	3.20%	0.06%	9.82%	8.68%	1.48%	0.43%	2.06%	1.46%
96	Varied	-1.0%	Constant	0.0%	8 year	Exact	3.98%	0.78%	3.20%	0.06%	9.82%	8.68%	1.48%	0.43%	2.06%	1.46%
97	Varied	-1.0%	Constant	3.0%	1 year	Constant	4.33%	0.71%	4.40%	0.73%	18.62%	13.74%	9.61%	5.10%	0.66%	0.78%
98	Varied	-1.0%	Constant	3.0%	1 year	Rounded	4.33%	0.71%	3.66%	0.06%	18.55%	13.93%	7.86%	3.66%	1.67%	1.68%
99	Varied	-1.0%	Constant	3.0%	1 year	Exact	4.33%	0.71%	3.73%	0.18%	18.71%	13.96%	7.92%	3.60%	1.49%	1.41%
100	Varied	-1.0%	Constant	3.0%	8 year	Constant	4.13%	0.80%	4.18%	0.77%	12.44%	10.74%	3.91%	2.33%	0.84%	0.66%
101	Varied	-1.0%	Constant	3.0%	8 year	Rounded	4.13%	0.80%	3.44%	0.10%	11.17%	9.94%	1.15%	0.02%	2.04%	1.55%
102	Varied	-1.0%	Constant	3.0%	8 year	Exact	4.13%	0.80%	3.52%	0.22%	11.45%	10.20%	1.31%	0.18%	1.82%	1.22%
103	Varied	-1.0%	Constant	10.0%	1 year	Constant	4.64%	0.72%	6.59%	2.23%	24.09%	18.76%	14.67%	9.74%	-1.01%	-0.48%
104	Varied	-1.0%	Constant	10.0%	1 year	Rounded	4.64%	0.72%	4.43%	0.58%	23.33%	18.00%	7.91%	3.25%	0.93%	0.83%
105	Varied	-1.0%	Constant	10.0%	1 year	Exact	4.64%	0.72%	4.40%	0.52%	23.18%	17.97%	7.94%	3.37%	1.01%	0.96%
106	Varied	-1.0%	Constant	10.0%	8 year	Constant	4.43%	0.82%	6.39%	2.26%	19.69%	17.20%	10.60%	8.30%	-1.12%	-0.55%
107	Varied	-1.0%	Constant	10.0%	8 year	Rounded	4.43%	0.82%	4.20%	0.64%	15.23%	13.66%	0.82%	-0.55%	1.17%	0.51%
108	Varied	-1.0%	Constant	10.0%	8 year	Exact	4.43%	0.82%	4.17%	0.58%	15.05%	13.58%	0.82%	-0.48%	1.28%	0.69%
109	Constant	None	Varied	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	7.74%	7.84%	-0.17%	-0.08%	0.00%	0.00%
110	Constant	None	Varied	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	7.74%	7.84%	-0.17%	-0.08%	0.00%	0.00%
111	Constant	None	Varied	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	7.74%	7.84%	-0.17%	-0.08%	0.00%	0.00%
112	Constant	None	Varied	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	1.14%	2.36%	-6.29%	-5.15%	0.00%	0.00%
113	Constant	None	Varied	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	1.14%	2.36%	-6.29%	-5.15%	0.00%	0.00%
114	Constant	None	Varied	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	1.14%	2.36%	-6.29%	-5.15%	0.00%	0.00%
115	Constant	None	Varied	3.0%	1 year	Constant	0.00%	0.00%	0.98%	0.86%	9.14%	9.05%	1.13%	1.04%	-1.24%	-1.04%
116	Constant	None	Varied	3.0%	1 year	Rounded	0.00%	0.00%	0.08%	0.05%	9.20%	9.35%	-0.38%	-0.24%	-0.07%	0.00%
117	Constant	None	Varied	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.19%	9.25%	9.33%	-0.42%	-0.34%	-0.29%	-0.29%
118	Constant	None	Varied	3.0%	8 year	Constant	0.00%	0.00%	0.97%	0.86%	3.26%	4.05%	-4.32%	-3.59%	-1.44%	-1.17%
119	Constant	None	Varied	3.0%	8 year	Rounded	0.00%	0.00%	0.07%	0.05%	2.05%	3.19%	-6.90%	-5.87%	-0.09%	0.01%
120	Constant	None	Varied	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.19%	2.17%	3.35%	-6.87%	-5.80%	-0.34%	-0.34%
121	Constant	None	Varied	10.0%	1 year	Constant	0.00%	0.00%	3.32%	2.78%	13.86%	13.62%	5.50%	5.28%	-3.32%	-2.66%
122	Constant	None	Varied	10.0%	1 year	Rounded	0.00%	0.00%	0.65%	0.67%	12.77%	12.74%	-1.07%	-1.10%	-1.05%	-1.10%
123	Constant	None	Varied	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	12.71%	12.77%	-0.98%	-0.92%	-0.93%	-0.93%
124	Constant	None	Varied	10.0%	8 year	Constant	0.00%	0.00%	3.33%	2.79%	9.74%	9.86%	1.69%	1.80%	-3.80%	-2.88%
125	Constant	None	Varied	10.0%	8 year	Rounded	0.00%	0.00%	0.64%	0.68%	4.45%	5.45%	-8.37%	-7.50%	-1.26%	-1.44%
126	Constant	None	Varied	10.0%	8 year	Exact	0.00%	0.00%	0.59%	0.60%	4.42%	5.47%	-8.26%	-7.33%	-1.11%	-1.10%
127	Constant	1.0%	Varied	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	5.84%	6.01%	-1.93%	-1.77%	0.00%	0.00%
128	Constant	1.0%	Varied	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	5.84%	6.01%	-1.93%	-1.77%	0.00%	0.00%
129	Constant	1.0%	Varied	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	5.84%	6.01%	-1.93%	-1.77%	0.00%	0.00%
130	Constant	1.0%	Varied	0.0%	8 year	Constant	0.00%	0.00%	-0.01%	0.00%	3.14%	4.30%	-4.43%	-3.35%	0.00%	0.00%
131	Constant	1.0%	Varied	0.0%	8 year	Rounded	0.00%	0.00%	-0.01%	0.00%	3.14%	4.30%	-4.43%	-3.35%	0.00%	0.00%
132	Constant	1.0%	Varied	0.0%	8 year	Exact	0.00%	0.00%	-0.01%	0.00%	3.14%	4.30%	-4.43%	-3.35%	0.00%	0.00%
133	Constant	1.0%	Varied	3.0%	1 year	Constant	0.00%	0.00%	0.99%	0.87%	7.31%	7.27%	-0.57%	-0.60%	-1.29%	-1.08%
134	Constant	1.0%	Varied	3.0%	1 year	Rounded	0.00%	0.00%	0.07%	0.04%	7.14%	7.38%	-2.26%	-2.04%	-0.07%	0.00%
135	Constant	1.0%	Varied	3.0%	1 year	Exact	0.00%	0.00%	0.19%	0.18%	7.18%	7.35%	-2.30%	-2.15%	-0.30%	-0.30%

Multiple Retention Scenario Results
Total Error in Methods

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess	Sum of Ret. Method Excess
136	Constant	1.0%	Varied	3.0%	8 year	Constant	0.00%	0.00%	0.99%	0.87%	5.43%	6.05%	-2.31%	-1.73%	-1.44%	-1.15%
137	Constant	1.0%	Varied	3.0%	8 year	Rounded	0.00%	0.00%	0.07%	0.04%	4.24%	5.31%	-4.90%	-3.92%	-0.08%	0.01%
138	Constant	1.0%	Varied	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.19%	4.42%	5.52%	-4.82%	-3.82%	-0.33%	-0.33%
139	Constant	1.0%	Varied	10.0%	1 year	Constant	0.00%	0.00%	3.37%	2.83%	12.50%	12.28%	4.25%	4.04%	-3.45%	-2.74%
140	Constant	1.0%	Varied	10.0%	1 year	Rounded	0.00%	0.00%	0.65%	0.68%	10.30%	10.34%	-3.24%	-3.20%	-1.09%	-1.15%
141	Constant	1.0%	Varied	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	10.26%	10.42%	-3.13%	-2.99%	-0.96%	-0.96%
142	Constant	1.0%	Varied	10.0%	8 year	Constant	0.00%	0.00%	3.37%	2.83%	12.08%	11.94%	3.85%	3.73%	-3.77%	-2.85%
143	Constant	1.0%	Varied	10.0%	8 year	Rounded	0.00%	0.00%	0.64%	0.68%	7.36%	8.21%	-5.81%	-5.07%	-1.23%	-1.34%
144	Constant	1.0%	Varied	10.0%	8 year	Exact	0.00%	0.00%	0.59%	0.60%	7.28%	8.22%	-5.75%	-4.92%	-1.08%	-1.07%
145	Constant	-1.0%	Varied	0.0%	1 year	Constant	0.00%	0.00%	-0.01%	0.00%	9.30%	9.15%	1.28%	1.14%	0.01%	0.00%
146	Constant	-1.0%	Varied	0.0%	1 year	Rounded	0.00%	0.00%	-0.01%	0.00%	9.30%	9.15%	1.28%	1.14%	0.01%	0.00%
147	Constant	-1.0%	Varied	0.0%	1 year	Exact	0.00%	0.00%	-0.01%	0.00%	9.30%	9.15%	1.28%	1.14%	0.01%	0.00%
148	Constant	-1.0%	Varied	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	1.21%	2.89%	-6.22%	-4.66%	-0.01%	0.00%
149	Constant	-1.0%	Varied	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	1.21%	2.89%	-6.22%	-4.66%	-0.01%	0.00%
150	Constant	-1.0%	Varied	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	1.21%	2.89%	-6.22%	-4.66%	-0.01%	0.00%
151	Constant	-1.0%	Varied	3.0%	1 year	Constant	0.00%	0.00%	0.96%	0.84%	10.76%	10.43%	2.63%	2.33%	-1.18%	-1.01%
152	Constant	-1.0%	Varied	3.0%	1 year	Rounded	0.00%	0.00%	0.07%	0.04%	10.84%	10.74%	1.12%	1.03%	-0.06%	-0.01%
153	Constant	-1.0%	Varied	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	10.90%	10.73%	1.09%	0.94%	-0.27%	-0.28%
154	Constant	-1.0%	Varied	3.0%	8 year	Constant	0.00%	0.00%	0.96%	0.85%	3.33%	4.57%	-4.25%	-3.10%	-1.42%	-1.13%
155	Constant	-1.0%	Varied	3.0%	8 year	Rounded	0.00%	0.00%	0.08%	0.05%	2.21%	3.84%	-6.75%	-5.27%	-0.10%	0.00%
156	Constant	-1.0%	Varied	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.19%	2.34%	4.00%	-6.71%	-5.20%	-0.34%	-0.34%
157	Constant	-1.0%	Varied	10.0%	1 year	Constant	0.00%	0.00%	3.28%	2.74%	15.49%	15.03%	7.01%	6.59%	-3.17%	-2.57%
158	Constant	-1.0%	Varied	10.0%	1 year	Rounded	0.00%	0.00%	0.64%	0.67%	14.65%	14.34%	0.58%	0.31%	-1.01%	-1.07%
159	Constant	-1.0%	Varied	10.0%	1 year	Exact	0.00%	0.00%	0.59%	0.59%	14.58%	14.37%	0.66%	0.49%	-0.89%	-0.90%
160	Constant	-1.0%	Varied	10.0%	8 year	Constant	0.00%	0.00%	3.28%	2.74%	9.76%	10.33%	1.71%	2.23%	-3.76%	-2.80%
161	Constant	-1.0%	Varied	10.0%	8 year	Rounded	0.00%	0.00%	0.64%	0.68%	4.91%	6.43%	-7.96%	-6.63%	-1.26%	-1.38%
162	Constant	-1.0%	Varied	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	4.87%	6.47%	-7.86%	-6.46%	-1.11%	-1.10%
163	Varied	None	Varied	0.0%	1 year	Constant	4.17%	0.69%	3.43%	0.02%	15.05%	10.96%	6.26%	2.48%	1.72%	1.64%
164	Varied	None	Varied	0.0%	1 year	Rounded	4.17%	0.69%	3.43%	0.02%	15.05%	10.96%	6.26%	2.48%	1.72%	1.64%
165	Varied	None	Varied	0.0%	1 year	Exact	4.17%	0.69%	3.43%	0.02%	15.05%	10.96%	6.26%	2.48%	1.72%	1.64%
166	Varied	None	Varied	0.0%	8 year	Constant	3.95%	0.77%	3.23%	0.04%	8.91%	8.09%	0.58%	-0.17%	2.11%	1.41%
167	Varied	None	Varied	0.0%	8 year	Rounded	3.95%	0.77%	3.23%	0.04%	8.91%	8.09%	0.58%	-0.17%	2.11%	1.41%
168	Varied	None	Varied	0.0%	8 year	Exact	3.95%	0.77%	3.23%	0.04%	8.91%	8.09%	0.58%	-0.17%	2.11%	1.41%
169	Varied	None	Varied	3.0%	1 year	Constant	4.33%	0.70%	4.43%	0.74%	16.93%	12.48%	8.00%	3.88%	0.65%	0.78%
170	Varied	None	Varied	3.0%	1 year	Rounded	4.33%	0.70%	3.68%	0.05%	16.81%	12.63%	6.22%	2.42%	1.71%	1.71%
171	Varied	None	Varied	3.0%	1 year	Exact	4.33%	0.70%	3.75%	0.18%	16.96%	12.66%	6.27%	2.37%	1.51%	1.44%
172	Varied	None	Varied	3.0%	8 year	Constant	4.11%	0.79%	4.23%	0.76%	11.60%	10.18%	3.07%	1.76%	0.86%	0.63%
173	Varied	None	Varied	3.0%	8 year	Rounded	4.11%	0.79%	3.47%	0.07%	10.17%	9.25%	0.18%	-0.65%	2.10%	1.51%
174	Varied	None	Varied	3.0%	8 year	Exact	4.11%	0.79%	3.55%	0.20%	10.43%	9.51%	0.33%	-0.50%	1.87%	1.17%
175	Varied	None	Varied	10.0%	1 year	Constant	4.64%	0.72%	6.64%	2.25%	22.54%	17.63%	13.18%	8.64%	-1.11%	-0.53%
176	Varied	None	Varied	10.0%	1 year	Rounded	4.64%	0.72%	4.45%	0.58%	21.44%	16.58%	6.20%	1.95%	0.93%	0.84%
177	Varied	None	Varied	10.0%	1 year	Exact	4.64%	0.72%	4.42%	0.52%	21.29%	16.54%	6.23%	2.07%	1.02%	0.97%
178	Varied	None	Varied	10.0%	8 year	Constant	4.41%	0.82%	6.45%	2.27%	19.12%	16.77%	10.02%	7.85%	-1.16%	-0.59%
179	Varied	None	Varied	10.0%	8 year	Rounded	4.41%	0.82%	4.23%	0.61%	13.91%	12.71%	-0.38%	-1.43%	1.22%	0.44%
180	Varied	None	Varied	10.0%	8 year	Exact	4.41%	0.82%	4.20%	0.55%	13.76%	12.64%	-0.37%	-1.35%	1.32%	0.63%

Multiple Retention Scenario Results
Total Error in Methods

Appendix 2 - Multiple Retention, Page 5

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess	Sum of Ret. Method Excess
181	Varied	1.0%	Varied	0.0%	1 year	Constant	4.18%	0.69%	3.46%	0.01%	13.31%	9.47%	4.65%	1.11%	1.77%	1.61%
182	Varied	1.0%	Varied	0.0%	1 year	Rounded	4.18%	0.69%	3.46%	0.01%	13.31%	9.47%	4.65%	1.11%	1.77%	1.61%
183	Varied	1.0%	Varied	0.0%	1 year	Exact	4.18%	0.69%	3.46%	0.01%	13.31%	9.47%	4.65%	1.11%	1.77%	1.61%
184	Varied	1.0%	Varied	0.0%	8 year	Constant	4.08%	0.76%	3.26%	0.03%	11.36%	9.93%	2.85%	1.53%	2.07%	1.52%
185	Varied	1.0%	Varied	0.0%	8 year	Rounded	4.08%	0.76%	3.26%	0.03%	11.36%	9.93%	2.85%	1.53%	2.07%	1.52%
186	Varied	1.0%	Varied	0.0%	8 year	Exact	4.08%	0.76%	3.26%	0.03%	11.36%	9.93%	2.85%	1.53%	2.07%	1.52%
187	Varied	1.0%	Varied	3.0%	1 year	Constant	4.34%	0.70%	4.46%	0.74%	15.30%	11.05%	6.49%	2.56%	0.64%	0.73%
188	Varied	1.0%	Varied	3.0%	1 year	Rounded	4.34%	0.70%	3.70%	0.04%	14.95%	11.02%	4.53%	0.95%	1.76%	1.69%
189	Varied	1.0%	Varied	3.0%	1 year	Exact	4.34%	0.70%	3.78%	0.17%	15.10%	11.04%	4.58%	0.89%	1.55%	1.40%
190	Varied	1.0%	Varied	3.0%	8 year	Constant	4.23%	0.78%	4.26%	0.76%	14.16%	12.12%	5.44%	3.55%	0.80%	0.66%
191	Varied	1.0%	Varied	3.0%	8 year	Rounded	4.23%	0.78%	3.49%	0.06%	12.80%	11.24%	2.57%	1.15%	2.05%	1.61%
192	Varied	1.0%	Varied	3.0%	8 year	Exact	4.23%	0.78%	3.57%	0.19%	13.11%	11.54%	2.77%	1.35%	1.83%	1.28%
193	Varied	1.0%	Varied	10.0%	1 year	Constant	4.65%	0.71%	6.69%	2.27%	21.43%	16.59%	12.15%	7.69%	-1.18%	-0.61%
194	Varied	1.0%	Varied	10.0%	1 year	Rounded	4.65%	0.71%	4.47%	0.57%	19.24%	14.63%	4.28%	0.25%	0.95%	0.77%
195	Varied	1.0%	Varied	10.0%	1 year	Exact	4.65%	0.71%	4.44%	0.51%	19.11%	14.62%	4.32%	0.38%	1.05%	0.91%
196	Varied	1.0%	Varied	10.0%	8 year	Constant	4.51%	0.80%	6.50%	2.30%	21.70%	18.81%	12.40%	9.73%	-1.20%	-0.64%
197	Varied	1.0%	Varied	10.0%	8 year	Rounded	4.51%	0.80%	4.25%	0.60%	17.19%	15.23%	2.49%	0.78%	1.16%	0.57%
198	Varied	1.0%	Varied	10.0%	8 year	Exact	4.51%	0.80%	4.22%	0.54%	16.98%	15.13%	2.46%	0.84%	1.26%	0.74%
199	Varied	-1.0%	Varied	0.0%	1 year	Constant	4.15%	0.69%	3.40%	0.02%	16.48%	12.11%	7.58%	3.54%	1.69%	1.62%
200	Varied	-1.0%	Varied	0.0%	1 year	Rounded	4.15%	0.69%	3.40%	0.02%	16.48%	12.11%	7.58%	3.54%	1.69%	1.62%
201	Varied	-1.0%	Varied	0.0%	1 year	Exact	4.15%	0.69%	3.40%	0.02%	16.48%	12.11%	7.58%	3.54%	1.69%	1.62%
202	Varied	-1.0%	Varied	0.0%	8 year	Constant	3.96%	0.76%	3.19%	0.06%	9.16%	8.28%	0.81%	0.01%	2.05%	1.43%
203	Varied	-1.0%	Varied	0.0%	8 year	Rounded	3.96%	0.76%	3.19%	0.06%	9.16%	8.28%	0.81%	0.01%	2.05%	1.43%
204	Varied	-1.0%	Varied	0.0%	8 year	Exact	3.96%	0.76%	3.19%	0.06%	9.16%	8.28%	0.81%	0.01%	2.05%	1.43%
205	Varied	-1.0%	Varied	3.0%	1 year	Constant	4.32%	0.70%	4.39%	0.73%	18.40%	13.67%	9.36%	4.98%	0.66%	0.79%
206	Varied	-1.0%	Varied	3.0%	1 year	Rounded	4.32%	0.70%	3.65%	0.06%	18.32%	13.84%	7.59%	3.52%	1.67%	1.69%
207	Varied	-1.0%	Varied	3.0%	1 year	Exact	4.32%	0.70%	3.72%	0.18%	18.48%	13.88%	7.65%	3.47%	1.49%	1.42%
208	Varied	-1.0%	Varied	3.0%	8 year	Constant	4.12%	0.78%	4.17%	0.76%	11.85%	10.39%	3.30%	1.95%	0.82%	0.63%
209	Varied	-1.0%	Varied	3.0%	8 year	Rounded	4.12%	0.78%	3.44%	0.09%	10.51%	9.53%	0.49%	-0.40%	2.03%	1.52%
210	Varied	-1.0%	Varied	3.0%	8 year	Exact	4.12%	0.78%	3.51%	0.22%	10.78%	9.80%	0.65%	-0.24%	1.81%	1.19%
211	Varied	-1.0%	Varied	10.0%	1 year	Constant	4.64%	0.72%	6.58%	2.23%	23.96%	18.80%	14.49%	9.72%	-1.02%	-0.49%
212	Varied	-1.0%	Varied	10.0%	1 year	Rounded	4.64%	0.72%	4.43%	0.58%	23.14%	17.96%	7.69%	3.16%	0.93%	0.85%
213	Varied	-1.0%	Varied	10.0%	1 year	Exact	4.64%	0.72%	4.40%	0.52%	22.98%	17.92%	7.71%	3.28%	1.01%	0.97%
214	Varied	-1.0%	Varied	10.0%	8 year	Constant	4.43%	0.81%	6.37%	2.26%	19.29%	16.98%	10.18%	8.04%	-1.17%	-0.59%
215	Varied	-1.0%	Varied	10.0%	8 year	Rounded	4.43%	0.81%	4.20%	0.63%	14.55%	13.24%	0.18%	-0.97%	1.16%	0.47%
216	Varied	-1.0%	Varied	10.0%	8 year	Exact	4.43%	0.81%	4.17%	0.57%	14.38%	13.18%	0.18%	-0.88%	1.26%	0.65%

Cape Cod Scenario Results
Total Error in Methods

Appendix 2 - Cape Cod, Page 1

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Cape Cod Unlimited	Cape Cod Limited $400,000	Cape Cod Excess $400,000	Cape cod unlim	Wtd Avg Excess $400,000	Cape Cod Limited $250,000	Cape Cod Excess $250,000	Cape cod unlim	Wtd Avg Excess $250,000	Cape Cod Limited $550,000	Cape Cod Excess $550,000	Wtd Avg Excess $550,000
1	Constant	None	Constant	0.0%	1 year	Constant	0.00%	0.00%	8.37%	0.00%	8.37%	0.00%	6.40%	0.00%	6.40%	0.00%	9.30%	9.30%
2	Constant	None	Constant	0.0%	1 year	Rounded	0.00%	0.00%	8.37%	0.00%	8.37%	0.00%	6.40%	0.00%	6.40%	0.00%	9.30%	9.30%
3	Constant	None	Constant	0.0%	1 year	Exact	0.00%	0.00%	8.37%	0.00%	8.37%	0.00%	6.40%	0.00%	6.40%	0.00%	9.30%	9.30%
4	Constant	None	Constant	0.0%	8 year	Constant	0.00%	0.00%	1.10%	0.00%	1.01%	0.00%	2.31%	0.00%	2.16%	0.00%	-2.27%	-2.22%
5	Constant	None	Constant	0.0%	8 year	Rounded	0.00%	0.00%	1.10%	0.00%	1.01%	0.00%	2.31%	0.00%	2.16%	0.00%	-2.27%	-2.22%
6	Constant	None	Constant	0.0%	8 year	Exact	0.00%	0.00%	1.10%	0.00%	1.01%	0.00%	2.31%	0.00%	2.16%	0.00%	-2.27%	-2.22%
7	Constant	None	Constant	3.0%	1 year	Constant	0.00%	0.97%	9.82%	0.00%	9.52%	0.97%	8.23%	0.00%	8.11%	0.97%	10.78%	10.32%
8	Constant	None	Constant	3.0%	1 year	Rounded	0.00%	0.04%	9.82%	0.00%	9.93%	0.04%	7.66%	0.00%	7.71%	0.15%	10.66%	10.65%
9	Constant	None	Constant	3.0%	1 year	Exact	0.00%	0.18%	9.85%	0.00%	9.85%	0.18%	7.86%	0.00%	7.86%	0.18%	10.80%	10.80%
10	Constant	None	Constant	3.0%	8 year	Constant	0.00%	0.97%	4.02%	0.00%	3.46%	0.97%	4.49%	0.00%	4.13%	0.96%	0.86%	0.04%
11	Constant	None	Constant	3.0%	8 year	Rounded	0.00%	0.05%	1.97%	0.00%	1.97%	0.02%	3.15%	0.00%	3.03%	0.15%	-1.51%	-1.52%
12	Constant	None	Constant	3.0%	8 year	Exact	0.00%	0.18%	2.14%	0.00%	2.01%	0.18%	3.47%	0.00%	3.30%	0.18%	-1.39%	-1.40%
13	Constant	None	Constant	10.0%	1 year	Constant	0.00%	3.41%	15.27%	0.00%	14.59%	3.04%	13.58%	0.00%	13.36%	3.43%	14.53%	13.42%
14	Constant	None	Constant	10.0%	1 year	Rounded	0.00%	0.60%	13.34%	0.00%	13.28%	0.73%	11.37%	0.00%	11.33%	0.61%	14.34%	14.32%
15	Constant	None	Constant	10.0%	1 year	Exact	0.00%	0.60%	13.25%	0.00%	13.25%	0.60%	11.20%	0.00%	11.20%	0.60%	14.23%	14.23%
16	Constant	None	Constant	10.0%	8 year	Constant	0.00%	3.40%	11.28%	0.00%	10.09%	3.06%	10.86%	0.00%	10.32%	3.40%	9.43%	7.49%
17	Constant	None	Constant	10.0%	8 year	Rounded	0.00%	0.60%	4.33%	0.00%	4.04%	0.75%	6.36%	0.00%	6.05%	0.61%	0.59%	0.44%
18	Constant	None	Constant	10.0%	8 year	Exact	0.00%	0.60%	4.40%	0.00%	4.18%	0.60%	6.06%	0.00%	5.81%	0.60%	0.47%	0.35%
19	Constant	1.0%	Constant	0.0%	1 year	Constant	0.00%	0.00%	5.77%	0.00%	5.79%	0.00%	4.92%	0.00%	4.88%	0.00%	6.39%	6.51%
20	Constant	1.0%	Constant	0.0%	1 year	Rounded	0.00%	0.00%	5.77%	0.00%	5.79%	0.00%	4.92%	0.00%	4.88%	0.00%	6.39%	6.51%
21	Constant	1.0%	Constant	0.0%	1 year	Exact	0.00%	0.00%	5.77%	0.00%	5.79%	0.00%	4.92%	0.00%	4.88%	0.00%	6.39%	6.51%
22	Constant	1.0%	Constant	0.0%	8 year	Constant	0.00%	0.00%	4.71%	0.00%	4.45%	0.00%	4.56%	0.00%	4.35%	0.00%	2.61%	2.31%
23	Constant	1.0%	Constant	0.0%	8 year	Rounded	0.00%	0.00%	4.71%	0.00%	4.45%	0.00%	4.56%	0.00%	4.35%	0.00%	2.61%	2.31%
24	Constant	1.0%	Constant	0.0%	8 year	Exact	0.00%	0.00%	4.71%	0.00%	4.45%	0.00%	4.56%	0.00%	4.35%	0.00%	2.61%	2.31%
25	Constant	1.0%	Constant	3.0%	1 year	Constant	0.00%	0.99%	7.43%	0.00%	7.11%	0.99%	6.98%	0.00%	6.82%	0.99%	7.73%	7.33%
26	Constant	1.0%	Constant	3.0%	1 year	Rounded	0.00%	0.04%	7.05%	0.00%	7.17%	0.04%	6.09%	0.00%	6.09%	0.15%	7.56%	7.67%
27	Constant	1.0%	Constant	3.0%	1 year	Exact	0.00%	0.18%	7.08%	0.00%	7.10%	0.18%	6.28%	0.00%	6.24%	0.18%	7.70%	7.82%
28	Constant	1.0%	Constant	3.0%	8 year	Constant	0.00%	0.98%	7.64%	0.00%	6.95%	0.99%	6.72%	0.00%	6.32%	0.98%	5.76%	4.68%
29	Constant	1.0%	Constant	3.0%	8 year	Rounded	0.00%	0.04%	5.84%	0.00%	5.68%	0.01%	5.55%	0.00%	5.37%	0.15%	3.64%	3.28%
30	Constant	1.0%	Constant	3.0%	8 year	Exact	0.00%	0.18%	6.04%	0.00%	5.73%	0.18%	5.90%	0.00%	5.66%	0.18%	3.87%	3.51%
31	Constant	1.0%	Constant	10.0%	1 year	Constant	0.00%	3.46%	13.64%	0.00%	12.92%	3.09%	12.80%	0.00%	12.56%	3.49%	12.01%	10.87%
32	Constant	1.0%	Constant	10.0%	1 year	Rounded	0.00%	0.60%	10.15%	0.00%	10.10%	0.73%	9.54%	0.00%	9.45%	0.61%	10.82%	10.90%
33	Constant	1.0%	Constant	10.0%	1 year	Exact	0.00%	0.60%	10.09%	0.00%	10.10%	0.60%	9.40%	0.00%	9.35%	0.60%	10.70%	10.82%
34	Constant	1.0%	Constant	10.0%	8 year	Constant	0.00%	3.45%	14.58%	0.00%	13.30%	3.11%	13.03%	0.00%	12.46%	3.46%	13.89%	11.83%
35	Constant	1.0%	Constant	10.0%	8 year	Rounded	0.00%	0.60%	8.96%	0.00%	8.49%	0.75%	9.25%	0.00%	8.87%	0.61%	6.74%	6.24%
36	Constant	1.0%	Constant	10.0%	8 year	Exact	0.00%	0.60%	8.98%	0.00%	8.58%	0.60%	8.91%	0.00%	8.59%	0.60%	6.63%	6.15%
37	Constant	-1.0%	Constant	0.0%	1 year	Constant	0.00%	0.00%	10.18%	0.00%	10.17%	0.00%	7.92%	0.00%	7.96%	0.00%	11.15%	11.07%
38	Constant	-1.0%	Constant	0.0%	1 year	Rounded	0.00%	0.00%	10.18%	0.00%	10.17%	0.00%	7.92%	0.00%	7.96%	0.00%	11.15%	11.07%
39	Constant	-1.0%	Constant	0.0%	1 year	Exact	0.00%	0.00%	10.18%	0.00%	10.17%	0.00%	7.92%	0.00%	7.96%	0.00%	11.15%	11.07%
40	Constant	-1.0%	Constant	0.0%	8 year	Constant	0.00%	0.00%	2.11%	0.00%	2.05%	0.00%	2.82%	0.00%	2.68%	0.00%	-0.80%	-0.70%
41	Constant	-1.0%	Constant	0.0%	8 year	Rounded	0.00%	0.00%	2.11%	0.00%	2.05%	0.00%	2.82%	0.00%	2.68%	0.00%	-0.80%	-0.70%
42	Constant	-1.0%	Constant	0.0%	8 year	Exact	0.00%	0.00%	2.11%	0.00%	2.05%	0.00%	2.82%	0.00%	2.68%	0.00%	-0.80%	-0.70%
43	Constant	-1.0%	Constant	3.0%	1 year	Constant	0.00%	0.96%	11.77%	0.00%	11.48%	0.96%	9.75%	0.00%	9.68%	0.96%	12.76%	12.26%
44	Constant	-1.0%	Constant	3.0%	1 year	Rounded	0.00%	0.05%	11.74%	0.00%	11.83%	0.05%	9.27%	0.00%	9.36%	0.15%	12.62%	12.53%
45	Constant	-1.0%	Constant	3.0%	1 year	Exact	0.00%	0.18%	11.78%	0.00%	11.77%	0.18%	9.48%	0.00%	9.52%	0.18%	12.76%	12.68%
46	Constant	-1.0%	Constant	3.0%	8 year	Constant	0.00%	0.95%	4.96%	0.00%	4.43%	0.96%	4.93%	0.00%	4.59%	0.95%	2.28%	1.52%
47	Constant	-1.0%	Constant	3.0%	8 year	Rounded	0.00%	0.06%	3.12%	0.00%	3.15%	0.02%	3.73%	0.00%	3.62%	0.15%	0.10%	0.15%
48	Constant	-1.0%	Constant	3.0%	8 year	Exact	0.00%	0.18%	3.29%	0.00%	3.18%	0.18%	4.06%	0.00%	3.89%	0.18%	0.27%	0.31%
49	Constant	-1.0%	Constant	10.0%	1 year	Constant	0.00%	3.36%	17.22%	0.00%	16.58%	2.99%	15.05%	0.00%	14.89%	3.38%	16.77%	15.68%
50	Constant	-1.0%	Constant	10.0%	1 year	Rounded	0.00%	0.60%	15.55%	0.00%	15.48%	0.73%	13.24%	0.00%	13.24%	0.61%	16.57%	16.47%
51	Constant	-1.0%	Constant	10.0%	1 year	Exact	0.00%	0.60%	15.45%	0.00%	15.44%	0.60%	13.05%	0.00%	13.09%	0.60%	16.45%	16.37%

Cape Cod Scenario Results
Total Error in Methods

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Cape Cod Unlimited	Cape Cod Limited $400,000	Cape Cod Excess $400,000	Cape cod unlim	Wtd Avg Excess $400,000	Cape Cod Limited $250,000	Cape Cod Excess $250,000	Cape cod unlim	Wtd Avg Excess $250,000	Cape Cod Limited $550,000	Cape Cod Excess $550,000	Wtd Avg Excess $550,000
52	Constant	-1.0%	Constant	10.0%	8 year	Constant	0.00%	3.35%	11.96%	0.00%	10.77%	3.02%	11.14%	0.00%	10.61%	3.35%	10.59%	8.68%
53	Constant	-1.0%	Constant	10.0%	8 year	Rounded	0.00%	0.60%	5.82%	0.00%	5.57%	0.75%	7.15%	0.00%	6.86%	0.61%	2.69%	2.60%
54	Constant	-1.0%	Constant	10.0%	8 year	Exact	0.00%	0.60%	5.88%	0.00%	5.69%	0.60%	6.84%	0.00%	6.60%	0.60%	2.59%	2.51%
55	Varied	None	Constant	0.0%	1 year	Constant	5.81%	4.37%	19.93%	5.81%	15.85%	3.97%	17.17%	5.81%	13.51%	4.64%	21.06%	16.78%
56	Varied	None	Constant	0.0%	1 year	Rounded	5.81%	4.37%	19.93%	5.81%	15.85%	3.97%	17.17%	5.81%	13.51%	4.64%	21.06%	16.78%
57	Varied	None	Constant	0.0%	1 year	Exact	5.81%	4.37%	19.93%	5.81%	15.85%	3.97%	17.17%	5.81%	13.51%	4.64%	21.06%	16.78%
58	Varied	None	Constant	0.0%	8 year	Constant	5.64%	4.14%	14.64%	5.64%	9.13%	3.79%	13.83%	5.64%	9.44%	4.35%	13.07%	6.49%
59	Varied	None	Constant	0.0%	8 year	Rounded	5.64%	4.14%	14.64%	5.64%	9.13%	3.79%	13.83%	5.64%	9.44%	4.35%	13.07%	6.49%
60	Varied	None	Constant	0.0%	8 year	Exact	5.64%	4.14%	14.64%	5.64%	9.13%	3.79%	13.83%	5.64%	9.44%	4.35%	13.07%	6.49%
61	Varied	None	Constant	3.0%	1 year	Constant	5.92%	5.36%	21.57%	5.92%	17.51%	4.91%	19.20%	5.92%	15.65%	5.64%	22.67%	18.34%
62	Varied	None	Constant	3.0%	1 year	Rounded	5.92%	4.57%	21.55%	5.92%	17.68%	4.16%	18.58%	5.92%	15.13%	4.92%	22.66%	18.49%
63	Varied	None	Constant	3.0%	1 year	Exact	5.92%	4.68%	21.73%	5.92%	17.75%	4.27%	18.94%	5.92%	15.38%	4.94%	22.87%	18.68%
64	Varied	None	Constant	3.0%	8 year	Constant	5.74%	5.12%	17.63%	5.74%	12.13%	4.74%	16.20%	5.74%	11.92%	5.34%	16.09%	9.40%
65	Varied	None	Constant	3.0%	8 year	Rounded	5.74%	4.34%	15.61%	5.74%	10.40%	3.96%	14.77%	5.74%	10.63%	4.62%	14.07%	7.64%
66	Varied	None	Constant	3.0%	8 year	Exact	5.74%	4.43%	16.05%	5.74%	10.63%	4.09%	15.32%	5.74%	11.02%	4.65%	14.35%	7.86%
67	Varied	None	Constant	10.0%	1 year	Constant	6.10%	7.71%	27.02%	6.10%	23.26%	6.58%	25.10%	6.10%	21.94%	8.04%	26.51%	22.37%
68	Varied	None	Constant	10.0%	1 year	Rounded	6.10%	5.28%	25.98%	6.10%	22.17%	5.01%	23.19%	6.10%	19.81%	5.57%	27.00%	23.08%
69	Varied	None	Constant	10.0%	1 year	Exact	6.10%	5.29%	25.76%	6.10%	22.04%	4.89%	22.90%	6.10%	19.57%	5.56%	26.89%	22.97%
70	Varied	None	Constant	10.0%	8 year	Constant	5.91%	7.48%	24.59%	5.91%	19.55%	6.43%	23.18%	5.91%	19.35%	7.74%	23.98%	17.73%
71	Varied	None	Constant	10.0%	8 year	Rounded	5.91%	5.02%	19.25%	5.91%	13.93%	4.83%	19.07%	5.91%	14.90%	5.26%	17.10%	10.95%
72	Varied	None	Constant	10.0%	8 year	Exact	5.91%	5.04%	19.06%	5.91%	13.91%	4.70%	18.59%	5.91%	14.52%	5.25%	17.01%	10.82%
73	Varied	1.0%	Constant	0.0%	1 year	Constant	5.85%	4.42%	17.56%	5.85%	13.52%	4.01%	15.84%	5.85%	12.19%	4.69%	18.42%	14.22%
74	Varied	1.0%	Constant	0.0%	1 year	Rounded	5.85%	4.42%	17.56%	5.85%	13.52%	4.01%	15.84%	5.85%	12.19%	4.69%	18.42%	14.22%
75	Varied	1.0%	Constant	0.0%	1 year	Exact	5.85%	4.42%	17.56%	5.85%	13.52%	4.01%	15.84%	5.85%	12.19%	4.69%	18.42%	14.22%
76	Varied	1.0%	Constant	0.0%	8 year	Constant	5.72%	4.17%	18.33%	5.72%	12.83%	3.82%	16.21%	5.72%	11.88%	4.40%	17.88%	11.24%
77	Varied	1.0%	Constant	0.0%	8 year	Rounded	5.72%	4.17%	18.33%	5.72%	12.83%	3.82%	16.21%	5.72%	11.88%	4.40%	17.88%	11.24%
78	Varied	1.0%	Constant	0.0%	8 year	Exact	5.72%	4.17%	18.33%	5.72%	12.83%	3.82%	16.21%	5.72%	11.88%	4.40%	17.88%	11.24%
79	Varied	1.0%	Constant	3.0%	1 year	Constant	5.95%	5.41%	19.44%	5.95%	15.38%	4.96%	18.12%	5.95%	14.57%	5.70%	19.92%	15.63%
80	Varied	1.0%	Constant	3.0%	1 year	Rounded	5.95%	4.61%	19.03%	5.95%	15.20%	4.19%	17.17%	5.95%	13.72%	4.96%	19.87%	15.77%
81	Varied	1.0%	Constant	3.0%	1 year	Exact	5.95%	4.72%	19.23%	5.95%	15.27%	4.31%	17.53%	5.95%	13.97%	4.99%	20.07%	15.96%
82	Varied	1.0%	Constant	3.0%	8 year	Constant	5.81%	5.16%	21.22%	5.81%	15.79%	4.77%	18.50%	5.81%	14.30%	5.40%	20.79%	14.17%
83	Varied	1.0%	Constant	3.0%	8 year	Rounded	5.81%	4.37%	19.49%	5.81%	14.32%	3.97%	17.28%	5.81%	13.20%	4.67%	19.06%	12.60%
84	Varied	1.0%	Constant	3.0%	8 year	Exact	5.81%	4.46%	19.96%	5.81%	14.57%	4.11%	17.85%	5.81%	13.61%	4.69%	19.43%	12.92%
85	Varied	1.0%	Constant	10.0%	1 year	Constant	6.12%	7.78%	25.64%	6.12%	21.86%	6.64%	24.49%	6.12%	21.33%	8.13%	24.33%	20.16%
86	Varied	1.0%	Constant	10.0%	1 year	Rounded	6.12%	5.31%	23.12%	6.12%	19.33%	5.04%	21.56%	6.12%	18.18%	5.59%	23.84%	19.99%
87	Varied	1.0%	Constant	10.0%	1 year	Exact	6.12%	5.32%	22.93%	6.12%	19.23%	4.92%	21.30%	6.12%	17.97%	5.59%	23.74%	19.88%
88	Varied	1.0%	Constant	10.0%	8 year	Constant	5.95%	7.54%	27.68%	5.95%	22.80%	6.46%	25.27%	5.95%	21.56%	7.81%	28.06%	22.03%
89	Varied	1.0%	Constant	10.0%	8 year	Rounded	5.95%	5.05%	23.71%	5.95%	18.48%	4.84%	21.95%	5.95%	17.88%	5.28%	22.82%	16.73%
90	Varied	1.0%	Constant	10.0%	8 year	Exact	5.95%	5.06%	23.48%	5.95%	18.42%	4.70%	21.44%	5.95%	17.46%	5.28%	22.73%	16.61%
91	Varied	-1.0%	Constant	0.0%	1 year	Constant	5.78%	4.33%	21.59%	5.78%	17.48%	3.93%	18.56%	5.78%	14.91%	4.60%	22.74%	18.40%
92	Varied	-1.0%	Constant	0.0%	1 year	Rounded	5.78%	4.33%	21.59%	5.78%	17.48%	3.93%	18.56%	5.78%	14.91%	4.60%	22.74%	18.40%
93	Varied	-1.0%	Constant	0.0%	1 year	Exact	5.78%	4.33%	21.59%	5.78%	17.48%	3.93%	18.56%	5.78%	14.91%	4.60%	22.74%	18.40%
94	Varied	-1.0%	Constant	0.0%	8 year	Constant	5.64%	4.09%	15.72%	5.64%	10.19%	3.74%	14.42%	5.64%	9.98%	4.32%	14.59%	8.00%
95	Varied	-1.0%	Constant	0.0%	8 year	Rounded	5.64%	4.09%	15.72%	5.64%	10.19%	3.74%	14.42%	5.64%	9.98%	4.32%	14.59%	8.00%
96	Varied	-1.0%	Constant	0.0%	8 year	Exact	5.64%	4.09%	15.72%	5.64%	10.19%	3.74%	14.42%	5.64%	9.98%	4.32%	14.59%	8.00%
97	Varied	-1.0%	Constant	3.0%	1 year	Constant	5.90%	5.30%	23.35%	5.90%	19.27%	4.85%	20.58%	5.90%	17.04%	5.59%	24.47%	20.10%
98	Varied	-1.0%	Constant	3.0%	1 year	Rounded	5.90%	4.54%	23.30%	5.90%	19.40%	4.13%	20.05%	5.90%	16.60%	4.89%	24.44%	20.21%
99	Varied	-1.0%	Constant	3.0%	1 year	Exact	5.90%	4.64%	23.50%	5.90%	19.49%	4.23%	20.41%	5.90%	16.86%	4.91%	24.65%	20.40%
100	Varied	-1.0%	Constant	3.0%	8 year	Constant	5.74%	5.06%	18.65%	5.74%	13.11%	4.67%	16.73%	5.74%	12.39%	5.29%	17.55%	10.86%
101	Varied	-1.0%	Constant	3.0%	8 year	Rounded	5.74%	4.31%	16.83%	5.74%	11.59%	3.91%	15.44%	5.74%	11.25%	4.59%	15.72%	9.27%
102	Varied	-1.0%	Constant	3.0%	8 year	Exact	5.74%	4.39%	17.26%	5.74%	11.82%	4.04%	16.00%	5.74%	11.64%	4.62%	16.03%	9.53%

Cape Cod Scenario Results
Total Error in Methods

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Cape Cod Unlimited	Cape Cod Limited $400,000	Cape Cod Excess $400,000	Cape cod unlim	Wtd Avg Excess $400,000	Cape Cod Limited $250,000	Cape Cod Excess $250,000	Cape cod unlim	Wtd Avg Excess $250,000	Cape Cod Limited $550,000	Cape Cod Excess $550,000	Wtd Avg Excess $550,000
103	Varied	-1.0%	Constant	10.0%	1 year	Constant	6.08%	7.63%	28.75%	6.08%	25.00%	6.51%	26.36%	6.08%	23.22%	7.97%	28.51%	24.38%
104	Varied	-1.0%	Constant	10.0%	1 year	Rounded	6.08%	5.26%	27.97%	6.08%	24.14%	4.98%	24.88%	6.08%	21.51%	5.54%	29.00%	25.03%
105	Varied	-1.0%	Constant	10.0%	1 year	Exact	6.08%	5.27%	27.75%	6.08%	24.00%	4.87%	24.57%	6.08%	21.26%	5.54%	28.89%	24.91%
106	Varied	-1.0%	Constant	10.0%	8 year	Constant	5.91%	7.39%	25.31%	5.91%	20.22%	6.35%	23.51%	5.91%	19.61%	7.66%	25.16%	18.88%
107	Varied	-1.0%	Constant	10.0%	8 year	Rounded	5.91%	4.99%	20.79%	5.91%	15.45%	4.79%	19.96%	5.91%	15.73%	5.23%	19.17%	13.00%
108	Varied	-1.0%	Constant	10.0%	8 year	Exact	5.91%	5.01%	20.59%	5.91%	15.41%	4.65%	19.46%	5.91%	15.33%	5.23%	19.09%	12.89%
109	Constant	None	Varied	0.0%	1 year	Constant	0.00%	0.00%	8.20%	0.00%	8.25%	0.00%	6.28%	0.00%	6.31%	0.00%	9.11%	9.14%
110	Constant	None	Varied	0.0%	1 year	Rounded	0.00%	0.00%	8.20%	0.00%	8.25%	0.00%	6.28%	0.00%	6.31%	0.00%	9.11%	9.14%
111	Constant	None	Varied	0.0%	1 year	Exact	0.00%	0.00%	8.20%	0.00%	8.25%	0.00%	6.28%	0.00%	6.31%	0.00%	9.11%	9.14%
112	Constant	None	Varied	0.0%	8 year	Constant	0.00%	0.00%	1.25%	0.00%	1.12%	0.00%	2.43%	0.00%	2.28%	0.00%	-2.07%	-2.06%
113	Constant	None	Varied	0.0%	8 year	Rounded	0.00%	0.00%	1.25%	0.00%	1.12%	0.00%	2.43%	0.00%	2.28%	0.00%	-2.07%	-2.06%
114	Constant	None	Varied	0.0%	8 year	Exact	0.00%	0.00%	1.25%	0.00%	1.12%	0.00%	2.43%	0.00%	2.28%	0.00%	-2.07%	-2.06%
115	Constant	None	Varied	3.0%	1 year	Constant	0.00%	0.96%	9.74%	0.00%	9.47%	0.97%	8.18%	0.00%	8.08%	0.96%	10.68%	10.23%
116	Constant	None	Varied	3.0%	1 year	Rounded	0.00%	0.04%	9.69%	0.00%	9.84%	0.05%	7.57%	0.00%	7.64%	0.15%	10.52%	10.53%
117	Constant	None	Varied	3.0%	1 year	Exact	0.00%	0.18%	9.72%	0.00%	9.77%	0.18%	7.76%	0.00%	7.78%	0.18%	10.65%	10.67%
118	Constant	None	Varied	3.0%	8 year	Constant	0.00%	0.96%	4.15%	0.00%	3.57%	0.97%	4.61%	0.00%	4.25%	0.96%	1.02%	0.16%
119	Constant	None	Varied	3.0%	8 year	Rounded	0.00%	0.05%	2.11%	0.00%	2.08%	0.03%	3.29%	0.00%	3.16%	0.15%	-1.32%	-1.37%
120	Constant	None	Varied	3.0%	8 year	Exact	0.00%	0.18%	2.28%	0.00%	2.12%	0.18%	3.60%	0.00%	3.41%	0.18%	-1.19%	-1.24%
121	Constant	None	Varied	10.0%	1 year	Constant	0.00%	3.39%	15.38%	0.00%	14.71%	3.01%	13.68%	0.00%	13.48%	3.41%	14.63%	13.51%
122	Constant	None	Varied	10.0%	1 year	Rounded	0.00%	0.60%	13.32%	0.00%	13.29%	0.73%	11.33%	0.00%	11.31%	0.61%	14.32%	14.30%
123	Constant	None	Varied	10.0%	1 year	Exact	0.00%	0.60%	13.23%	0.00%	13.26%	0.60%	11.14%	0.00%	11.17%	0.60%	14.20%	14.20%
124	Constant	None	Varied	10.0%	8 year	Constant	0.00%	3.38%	11.42%	0.00%	10.22%	3.04%	10.98%	0.00%	10.45%	3.39%	9.54%	7.58%
125	Constant	None	Varied	10.0%	8 year	Rounded	0.00%	0.60%	4.46%	0.00%	4.13%	0.75%	6.46%	0.00%	6.15%	0.61%	0.76%	0.56%
126	Constant	None	Varied	10.0%	8 year	Exact	0.00%	0.60%	4.52%	0.00%	4.26%	0.60%	6.18%	0.00%	5.92%	0.60%	0.66%	0.49%
127	Constant	1.0%	Varied	0.0%	1 year	Constant	0.00%	0.00%	6.00%	0.00%	5.97%	0.00%	5.06%	0.00%	5.01%	0.00%	6.58%	6.62%
128	Constant	1.0%	Varied	0.0%	1 year	Rounded	0.00%	0.00%	6.00%	0.00%	5.97%	0.00%	5.06%	0.00%	5.01%	0.00%	6.58%	6.62%
129	Constant	1.0%	Varied	0.0%	1 year	Exact	0.00%	0.00%	6.00%	0.00%	5.97%	0.00%	5.06%	0.00%	5.01%	0.00%	6.58%	6.62%
130	Constant	1.0%	Varied	0.0%	8 year	Constant	0.00%	0.00%	3.59%	0.00%	3.43%	0.00%	3.79%	0.00%	3.62%	0.00%	1.05%	0.94%
131	Constant	1.0%	Varied	0.0%	8 year	Rounded	0.00%	0.00%	3.59%	0.00%	3.43%	0.00%	3.79%	0.00%	3.62%	0.00%	1.05%	0.94%
132	Constant	1.0%	Varied	0.0%	8 year	Exact	0.00%	0.00%	3.59%	0.00%	3.43%	0.00%	3.79%	0.00%	3.62%	0.00%	1.05%	0.94%
133	Constant	1.0%	Varied	3.0%	1 year	Constant	0.00%	0.98%	7.73%	0.00%	7.37%	0.98%	7.13%	0.00%	6.96%	0.98%	7.95%	7.46%
134	Constant	1.0%	Varied	3.0%	1 year	Rounded	0.00%	0.04%	7.29%	0.00%	7.36%	0.04%	6.24%	0.00%	6.23%	0.14%	7.78%	7.81%
135	Constant	1.0%	Varied	3.0%	1 year	Exact	0.00%	0.18%	7.33%	0.00%	7.30%	0.18%	6.43%	0.00%	6.38%	0.18%	7.90%	7.94%
136	Constant	1.0%	Varied	3.0%	8 year	Constant	0.00%	0.98%	6.71%	0.00%	6.07%	0.98%	6.04%	0.00%	5.67%	0.97%	4.49%	3.51%
137	Constant	1.0%	Varied	3.0%	8 year	Rounded	0.00%	0.04%	4.71%	0.00%	4.64%	0.02%	4.77%	0.00%	4.63%	0.15%	2.08%	1.91%
138	Constant	1.0%	Varied	3.0%	8 year	Exact	0.00%	0.18%	4.92%	0.00%	4.71%	0.18%	5.12%	0.00%	4.92%	0.18%	2.32%	2.15%
139	Constant	1.0%	Varied	10.0%	1 year	Constant	0.00%	3.44%	14.05%	0.00%	13.28%	3.06%	12.98%	0.00%	12.73%	3.46%	12.42%	11.18%
140	Constant	1.0%	Varied	10.0%	1 year	Rounded	0.00%	0.60%	10.44%	0.00%	10.33%	0.73%	9.73%	0.00%	9.62%	0.61%	11.05%	11.05%
141	Constant	1.0%	Varied	10.0%	1 year	Exact	0.00%	0.60%	10.38%	0.00%	10.33%	0.60%	9.56%	0.00%	9.50%	0.60%	10.93%	10.96%
142	Constant	1.0%	Varied	10.0%	8 year	Constant	0.00%	3.43%	14.05%	0.00%	12.79%	3.08%	12.57%	0.00%	12.03%	3.43%	13.23%	11.16%
143	Constant	1.0%	Varied	10.0%	8 year	Rounded	0.00%	0.60%	7.85%	0.00%	7.47%	0.75%	8.46%	0.00%	8.12%	0.61%	5.25%	4.93%
144	Constant	1.0%	Varied	10.0%	8 year	Exact	0.00%	0.60%	7.87%	0.00%	7.57%	0.60%	8.12%	0.00%	7.84%	0.60%	5.12%	4.83%
145	Constant	-1.0%	Varied	0.0%	1 year	Constant	0.00%	0.00%	9.87%	0.00%	9.90%	0.00%	7.68%	0.00%	7.75%	0.00%	10.83%	10.79%
146	Constant	-1.0%	Varied	0.0%	1 year	Rounded	0.00%	0.00%	9.87%	0.00%	9.90%	0.00%	7.68%	0.00%	7.75%	0.00%	10.83%	10.79%
147	Constant	-1.0%	Varied	0.0%	1 year	Exact	0.00%	0.00%	9.87%	0.00%	9.90%	0.00%	7.68%	0.00%	7.75%	0.00%	10.83%	10.79%
148	Constant	-1.0%	Varied	0.0%	8 year	Constant	0.00%	0.00%	1.26%	0.00%	1.29%	0.00%	2.25%	0.00%	2.15%	0.00%	-1.90%	-1.66%
149	Constant	-1.0%	Varied	0.0%	8 year	Rounded	0.00%	0.00%	1.26%	0.00%	1.29%	0.00%	2.25%	0.00%	2.15%	0.00%	-1.90%	-1.66%
150	Constant	-1.0%	Varied	0.0%	8 year	Exact	0.00%	0.00%	1.26%	0.00%	1.29%	0.00%	2.25%	0.00%	2.15%	0.00%	-1.90%	-1.66%
151	Constant	-1.0%	Varied	3.0%	1 year	Constant	0.00%	0.95%	11.50%	0.00%	11.25%	0.95%	9.55%	0.00%	9.50%	0.95%	12.48%	12.01%
152	Constant	-1.0%	Varied	3.0%	1 year	Rounded	0.00%	0.05%	11.45%	0.00%	11.58%	0.05%	9.05%	0.00%	9.16%	0.15%	12.32%	12.27%
153	Constant	-1.0%	Varied	3.0%	1 year	Exact	0.00%	0.18%	11.49%	0.00%	11.52%	0.18%	9.25%	0.00%	9.32%	0.18%	12.46%	12.42%

Cape Cod Scenario Results
Total Error in Methods

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Cape Cod Unlimited	Cape Cod Limited $400,000	Cape Cod Excess $400,000	Cape cod unlim	Wtd Avg Excess $400,000	Cape Cod Limited $250,000	Cape Cod Excess $250,000	Cape cod unlim	Wtd Avg Excess $250,000	Cape Cod Limited $550,000	Cape Cod Excess $550,000	Wtd Avg Excess $550,000
154	Constant	-1.0%	Varied	3.0%	8 year	Constant	0.00%	0.95%	4.20%	0.00%	3.74%	0.95%	4.42%	0.00%	4.10%	0.94%	1.31%	0.65%
155	Constant	-1.0%	Varied	3.0%	8 year	Rounded	0.00%	0.06%	2.26%	0.00%	2.38%	0.03%	3.19%	0.00%	3.11%	0.15%	-1.05%	-0.86%
156	Constant	-1.0%	Varied	3.0%	8 year	Exact	0.00%	0.18%	2.42%	0.00%	2.41%	0.18%	3.48%	0.00%	3.35%	0.18%	-0.86%	-0.67%
157	Constant	-1.0%	Varied	10.0%	1 year	Constant	0.00%	3.35%	17.05%	0.00%	16.43%	2.97%	14.95%	0.00%	14.81%	3.37%	16.59%	15.52%
158	Constant	-1.0%	Varied	10.0%	1 year	Rounded	0.00%	0.60%	15.31%	0.00%	15.28%	0.72%	13.04%	0.00%	13.07%	0.61%	16.33%	16.26%
159	Constant	-1.0%	Varied	10.0%	1 year	Exact	0.00%	0.60%	15.21%	0.00%	15.24%	0.60%	12.85%	0.00%	12.92%	0.60%	16.21%	16.16%
160	Constant	-1.0%	Varied	10.0%	8 year	Constant	0.00%	3.33%	11.43%	0.00%	10.29%	2.99%	10.75%	0.00%	10.25%	3.34%	9.95%	8.08%
161	Constant	-1.0%	Varied	10.0%	8 year	Rounded	0.00%	0.60%	4.93%	0.00%	4.77%	0.74%	6.53%	0.00%	6.28%	0.60%	1.53%	1.57%
162	Constant	-1.0%	Varied	10.0%	8 year	Exact	0.00%	0.60%	5.00%	0.00%	4.89%	0.60%	6.24%	0.00%	6.04%	0.60%	1.43%	1.49%
163	Varied	None	Varied	0.0%	1 year	Constant	5.77%	4.34%	19.80%	5.77%	15.70%	3.95%	17.01%	5.77%	13.37%	4.60%	20.94%	16.60%
164	Varied	None	Varied	0.0%	1 year	Rounded	5.77%	4.34%	19.80%	5.77%	15.70%	3.95%	17.01%	5.77%	13.37%	4.60%	20.94%	16.60%
165	Varied	None	Varied	0.0%	1 year	Exact	5.77%	4.34%	19.80%	5.77%	15.70%	3.95%	17.01%	5.77%	13.37%	4.60%	20.94%	16.60%
166	Varied	None	Varied	0.0%	8 year	Constant	5.58%	4.10%	14.59%	5.58%	9.11%	3.77%	13.77%	5.58%	9.43%	4.31%	13.08%	6.52%
167	Varied	None	Varied	0.0%	8 year	Rounded	5.58%	4.10%	14.59%	5.58%	9.11%	3.77%	13.77%	5.58%	9.43%	4.31%	13.08%	6.52%
168	Varied	None	Varied	0.0%	8 year	Exact	5.58%	4.10%	14.59%	5.58%	9.11%	3.77%	13.77%	5.58%	9.43%	4.31%	13.08%	6.52%
169	Varied	None	Varied	3.0%	1 year	Constant	5.88%	5.32%	21.51%	5.88%	17.43%	4.89%	19.11%	5.88%	15.57%	5.59%	22.62%	18.23%
170	Varied	None	Varied	3.0%	1 year	Rounded	5.88%	4.54%	21.45%	5.88%	17.57%	4.16%	18.44%	5.88%	15.00%	4.88%	22.58%	18.35%
171	Varied	None	Varied	3.0%	1 year	Exact	5.88%	4.65%	21.64%	5.88%	17.64%	4.26%	18.80%	5.88%	15.25%	4.91%	22.78%	18.54%
172	Varied	None	Varied	3.0%	8 year	Constant	5.68%	5.08%	17.57%	5.68%	12.10%	4.71%	16.17%	5.68%	11.93%	5.30%	16.05%	9.40%
173	Varied	None	Varied	3.0%	8 year	Rounded	5.68%	4.31%	15.56%	5.68%	10.39%	3.95%	14.74%	5.68%	10.64%	4.58%	14.08%	7.67%
174	Varied	None	Varied	3.0%	8 year	Exact	5.68%	4.40%	16.00%	5.68%	10.61%	4.07%	15.28%	5.68%	11.01%	4.61%	14.35%	7.90%
175	Varied	None	Varied	10.0%	1 year	Constant	6.07%	7.68%	27.11%	6.07%	23.33%	6.56%	25.16%	6.07%	22.00%	8.01%	26.60%	22.42%
176	Varied	None	Varied	10.0%	1 year	Rounded	6.07%	5.27%	25.98%	6.07%	22.14%	5.01%	23.13%	6.07%	19.75%	5.54%	27.01%	23.04%
177	Varied	None	Varied	10.0%	1 year	Exact	6.07%	5.28%	25.75%	6.07%	22.01%	4.89%	22.81%	6.07%	19.49%	5.54%	26.90%	22.92%
178	Varied	None	Varied	10.0%	8 year	Constant	5.85%	7.44%	24.58%	5.85%	19.58%	6.39%	23.18%	5.85%	19.38%	7.70%	23.93%	17.72%
179	Varied	None	Varied	10.0%	8 year	Rounded	5.85%	5.01%	19.20%	5.85%	13.92%	4.82%	19.03%	5.85%	14.88%	5.23%	17.09%	10.97%
180	Varied	None	Varied	10.0%	8 year	Exact	5.85%	5.02%	19.00%	5.85%	13.89%	4.69%	18.56%	5.85%	14.52%	5.22%	17.01%	10.85%
181	Varied	1.0%	Varied	0.0%	1 year	Constant	5.76%	4.36%	17.74%	5.76%	13.63%	3.98%	15.84%	5.76%	12.19%	4.62%	18.58%	14.29%
182	Varied	1.0%	Varied	0.0%	1 year	Rounded	5.76%	4.36%	17.74%	5.76%	13.63%	3.98%	15.84%	5.76%	12.19%	4.62%	18.58%	14.29%
183	Varied	1.0%	Varied	0.0%	1 year	Exact	5.76%	4.36%	17.74%	5.76%	13.63%	3.98%	15.84%	5.76%	12.19%	4.62%	18.58%	14.29%
184	Varied	1.0%	Varied	0.0%	8 year	Constant	5.68%	4.13%	17.50%	5.68%	11.92%	3.80%	15.51%	5.68%	11.16%	4.35%	16.88%	10.07%
185	Varied	1.0%	Varied	0.0%	8 year	Rounded	5.68%	4.13%	17.50%	5.68%	11.92%	3.80%	15.51%	5.68%	11.16%	4.35%	16.88%	10.07%
186	Varied	1.0%	Varied	0.0%	8 year	Exact	5.68%	4.13%	17.50%	5.68%	11.92%	3.80%	15.51%	5.68%	11.16%	4.35%	16.88%	10.07%
187	Varied	1.0%	Varied	3.0%	1 year	Constant	5.88%	5.35%	19.69%	5.88%	15.57%	4.93%	18.14%	5.88%	14.59%	5.63%	20.13%	15.74%
188	Varied	1.0%	Varied	3.0%	1 year	Rounded	5.88%	4.56%	19.22%	5.88%	15.33%	4.18%	17.18%	5.88%	13.73%	4.89%	20.06%	15.87%
189	Varied	1.0%	Varied	3.0%	1 year	Exact	5.88%	4.67%	19.43%	5.88%	15.41%	4.28%	17.55%	5.88%	13.99%	4.92%	20.25%	16.05%
190	Varied	1.0%	Varied	3.0%	8 year	Constant	5.78%	5.12%	20.57%	5.78%	15.03%	4.75%	17.91%	5.78%	13.68%	5.34%	20.04%	13.20%
191	Varied	1.0%	Varied	3.0%	8 year	Rounded	5.78%	4.33%	18.67%	5.78%	13.41%	3.96%	16.59%	5.78%	12.48%	4.62%	18.08%	11.44%
192	Varied	1.0%	Varied	3.0%	8 year	Exact	5.78%	4.43%	19.16%	5.78%	13.68%	4.09%	17.16%	5.78%	12.89%	4.64%	18.48%	11.78%
193	Varied	1.0%	Varied	10.0%	1 year	Constant	6.06%	7.72%	26.01%	6.06%	22.17%	6.61%	24.57%	6.06%	21.41%	8.05%	24.74%	20.48%
194	Varied	1.0%	Varied	10.0%	1 year	Rounded	6.06%	5.28%	23.39%	6.06%	19.52%	5.02%	21.65%	6.06%	18.25%	5.54%	24.09%	20.14%
195	Varied	1.0%	Varied	10.0%	1 year	Exact	6.06%	5.28%	23.19%	6.06%	19.42%	4.90%	21.36%	6.06%	18.02%	5.54%	23.97%	20.02%
196	Varied	1.0%	Varied	10.0%	8 year	Constant	5.94%	7.49%	27.41%	5.94%	22.40%	6.43%	24.92%	5.94%	21.16%	7.75%	27.82%	21.55%
197	Varied	1.0%	Varied	10.0%	8 year	Rounded	5.94%	5.02%	23.01%	5.94%	17.64%	4.83%	21.32%	5.94%	17.19%	5.25%	22.00%	15.68%
198	Varied	1.0%	Varied	10.0%	8 year	Exact	5.94%	5.03%	22.76%	5.94%	17.57%	4.70%	20.79%	5.94%	16.76%	5.25%	21.90%	15.54%
199	Varied	-1.0%	Varied	0.0%	1 year	Constant	5.76%	4.31%	21.32%	5.76%	17.22%	3.92%	18.33%	5.76%	14.69%	4.58%	22.47%	18.12%
200	Varied	-1.0%	Varied	0.0%	1 year	Rounded	5.76%	4.31%	21.32%	5.76%	17.22%	3.92%	18.33%	5.76%	14.69%	4.58%	22.47%	18.12%
201	Varied	-1.0%	Varied	0.0%	1 year	Exact	5.76%	4.31%	21.32%	5.76%	17.22%	3.92%	18.33%	5.76%	14.69%	4.58%	22.47%	18.12%
202	Varied	-1.0%	Varied	0.0%	8 year	Constant	5.61%	4.07%	15.03%	5.61%	9.48%	3.73%	13.89%	5.61%	9.46%	4.29%	13.73%	7.12%
203	Varied	-1.0%	Varied	0.0%	8 year	Rounded	5.61%	4.07%	15.03%	5.61%	9.48%	3.73%	13.89%	5.61%	9.46%	4.29%	13.73%	7.12%
204	Varied	-1.0%	Varied	0.0%	8 year	Exact	5.61%	4.07%	15.03%	5.61%	9.48%	3.73%	13.89%	5.61%	9.46%	4.29%	13.73%	7.12%

Cape Cod Scenario Results
Total Error in Methods

Appendix 2 - Cape Cod, Page 5

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Cape Cod Unlimited	Cape Cod Limited $400,000	Cape Cod Excess $400,000	Cape cod unlim	Wtd Avg Excess $400,000	Cape Cod Limited $250,000	Cape Cod Excess $250,000	Cape cod unlim	Wtd Avg Excess $250,000	Cape Cod Limited $550,000	Cape Cod Excess $550,000	Wtd Avg Excess $550,000
205	Varied	-1.0%	Varied	3.0%	1 year	Constant	5.88%	5.29%	23.11%	5.88%	19.04%	4.85%	20.38%	5.88%	16.85%	5.57%	24.23%	19.85%
206	Varied	-1.0%	Varied	3.0%	1 year	Rounded	5.88%	4.53%	23.05%	5.88%	19.16%	4.13%	19.83%	5.88%	16.39%	4.87%	24.19%	19.95%
207	Varied	-1.0%	Varied	3.0%	1 year	Exact	5.88%	4.63%	23.24%	5.88%	19.24%	4.23%	20.20%	5.88%	16.65%	4.89%	24.39%	20.14%
208	Varied	-1.0%	Varied	3.0%	8 year	Constant	5.72%	5.04%	18.04%	5.72%	12.47%	4.66%	16.27%	5.72%	11.92%	5.27%	16.83%	10.08%
209	Varied	-1.0%	Varied	3.0%	8 year	Rounded	5.72%	4.29%	16.13%	5.72%	10.88%	3.92%	14.93%	5.72%	10.74%	4.57%	14.85%	8.36%
210	Varied	-1.0%	Varied	3.0%	8 year	Exact	5.72%	4.38%	16.58%	5.72%	11.10%	4.03%	15.47%	5.72%	11.11%	4.59%	15.18%	8.64%
211	Varied	-1.0%	Varied	10.0%	1 year	Constant	6.07%	7.61%	28.59%	6.07%	24.85%	6.50%	26.27%	6.07%	23.14%	7.95%	28.35%	24.21%
212	Varied	-1.0%	Varied	10.0%	1 year	Rounded	6.07%	5.25%	27.76%	6.07%	23.93%	4.98%	24.69%	6.07%	21.33%	5.53%	28.80%	24.82%
213	Varied	-1.0%	Varied	10.0%	1 year	Exact	6.07%	5.27%	27.54%	6.07%	23.80%	4.87%	24.38%	6.07%	21.08%	5.53%	28.68%	24.70%
214	Varied	-1.0%	Varied	10.0%	8 year	Constant	5.91%	7.37%	24.94%	5.91%	19.81%	6.33%	23.20%	5.91%	19.29%	7.64%	24.75%	18.37%
215	Varied	-1.0%	Varied	10.0%	8 year	Rounded	5.91%	4.99%	20.13%	5.91%	14.73%	4.79%	19.43%	5.91%	15.18%	5.22%	18.35%	12.11%
216	Varied	-1.0%	Varied	10.0%	8 year	Exact	5.91%	5.00%	19.93%	5.91%	14.70%	4.66%	18.94%	5.91%	14.80%	5.22%	18.27%	11.99%

Increased Variation Scenario Results
Total Error in Methods

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess
1	Constant	None	Constant	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	8.37%	8.37%	0.00%	0.00%	0.00%
2	Constant	None	Constant	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	8.37%	8.37%	0.00%	0.00%	0.00%
3	Constant	None	Constant	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	8.37%	8.37%	0.00%	0.00%	0.00%
4	Constant	None	Constant	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	1.01%	2.87%	-6.79%	-5.07%	0.00%
5	Constant	None	Constant	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	1.01%	2.87%	-6.79%	-5.07%	0.00%
6	Constant	None	Constant	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	1.01%	2.87%	-6.79%	-5.07%	0.00%
7	Constant	None	Constant	3.0%	1 year	Constant	0.00%	0.00%	1.00%	0.88%	9.52%	9.40%	1.06%	0.95%	-1.08%
8	Constant	None	Constant	3.0%	1 year	Rounded	0.00%	0.00%	0.03%	-0.05%	9.93%	10.13%	-0.18%	0.01%	0.24%
9	Constant	None	Constant	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	9.85%	9.85%	-0.30%	-0.30%	-0.30%
10	Constant	None	Constant	3.0%	8 year	Constant	0.00%	0.00%	0.99%	0.87%	3.46%	4.97%	-4.53%	-3.14%	-1.22%
11	Constant	None	Constant	3.0%	8 year	Rounded	0.00%	0.00%	0.04%	-0.04%	1.97%	3.81%	-7.40%	-5.73%	0.46%
12	Constant	None	Constant	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.18%	2.01%	3.86%	-7.41%	-5.74%	-0.35%
13	Constant	None	Constant	10.0%	1 year	Constant	0.00%	0.00%	3.51%	3.09%	14.59%	13.88%	5.74%	5.08%	-3.20%
14	Constant	None	Constant	10.0%	1 year	Rounded	0.00%	0.00%	0.61%	0.69%	13.28%	13.13%	-1.20%	-1.33%	-1.28%
15	Constant	None	Constant	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	13.25%	13.25%	-0.96%	-0.96%	-0.96%
16	Constant	None	Constant	10.0%	8 year	Constant	0.00%	0.00%	3.49%	3.07%	10.09%	10.39%	1.59%	1.87%	-3.56%
17	Constant	None	Constant	10.0%	8 year	Rounded	0.00%	0.00%	0.61%	0.69%	4.04%	5.82%	-9.26%	-7.70%	-1.70%
18	Constant	None	Constant	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	4.18%	5.96%	-8.90%	-7.34%	-1.15%
19	Constant	1.0%	Constant	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	5.79%	6.25%	-2.38%	-1.96%	0.00%
20	Constant	1.0%	Constant	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	5.79%	6.25%	-2.38%	-1.96%	0.00%
21	Constant	1.0%	Constant	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	5.79%	6.25%	-2.38%	-1.96%	0.00%
22	Constant	1.0%	Constant	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	4.45%	5.86%	-3.61%	-2.31%	0.00%
23	Constant	1.0%	Constant	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	4.45%	5.86%	-3.61%	-2.31%	0.00%
24	Constant	1.0%	Constant	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	4.45%	5.86%	-3.61%	-2.31%	0.00%
25	Constant	1.0%	Constant	3.0%	1 year	Constant	0.00%	0.00%	1.02%	0.89%	7.11%	7.43%	-1.16%	-0.87%	-1.12%
26	Constant	1.0%	Constant	3.0%	1 year	Rounded	0.00%	0.00%	0.02%	-0.06%	7.17%	7.85%	-2.68%	-2.06%	0.28%
27	Constant	1.0%	Constant	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	7.10%	7.58%	-2.79%	-2.36%	-0.31%
28	Constant	1.0%	Constant	3.0%	8 year	Constant	0.00%	0.00%	1.01%	0.89%	6.95%	7.87%	-1.31%	-0.46%	-1.18%
29	Constant	1.0%	Constant	3.0%	8 year	Rounded	0.00%	0.00%	0.03%	-0.04%	5.68%	7.10%	-4.03%	-2.74%	0.34%
30	Constant	1.0%	Constant	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.18%	5.73%	7.09%	-4.03%	-2.80%	-0.34%
31	Constant	1.0%	Constant	10.0%	1 year	Constant	0.00%	0.00%	3.56%	3.14%	12.92%	12.58%	4.20%	3.89%	-3.32%
32	Constant	1.0%	Constant	10.0%	1 year	Rounded	0.00%	0.00%	0.61%	0.70%	10.10%	10.47%	-3.97%	-3.65%	-1.36%
33	Constant	1.0%	Constant	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	10.10%	10.62%	-3.71%	-3.26%	-1.00%
34	Constant	1.0%	Constant	10.0%	8 year	Constant	0.00%	0.00%	3.55%	3.12%	13.30%	12.87%	4.55%	4.15%	-3.47%
35	Constant	1.0%	Constant	10.0%	8 year	Rounded	0.00%	0.00%	0.61%	0.69%	8.49%	9.67%	-5.38%	-4.35%	-1.55%
36	Constant	1.0%	Constant	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	8.58%	9.79%	-5.05%	-3.99%	-1.10%
37	Constant	-1.0%	Constant	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	10.17%	9.82%	1.66%	1.34%	0.00%
38	Constant	-1.0%	Constant	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	10.17%	9.82%	1.66%	1.34%	0.00%
39	Constant	-1.0%	Constant	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	10.17%	9.82%	1.66%	1.34%	0.00%
40	Constant	-1.0%	Constant	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	2.05%	3.63%	-5.83%	-4.37%	0.00%
41	Constant	-1.0%	Constant	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	2.05%	3.63%	-5.83%	-4.37%	0.00%
42	Constant	-1.0%	Constant	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	2.05%	3.63%	-5.83%	-4.37%	0.00%
43	Constant	-1.0%	Constant	3.0%	1 year	Constant	0.00%	0.00%	0.99%	0.86%	11.48%	11.00%	2.87%	2.43%	-1.05%
44	Constant	-1.0%	Constant	3.0%	1 year	Rounded	0.00%	0.00%	0.03%	-0.04%	11.83%	11.66%	1.55%	1.40%	0.21%
45	Constant	-1.0%	Constant	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	11.77%	11.40%	1.44%	1.11%	-0.29%

Increased Variation Scenario Results
Total Error in Methods

Appendix 2 - Increased Variation Page 2

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess
46	Constant	-1.0%	Constant	3.0%	8 year	Constant	0.00%	0.00%	0.98%	0.85%	4.43%	5.62%	-3.64%	-2.54%	-1.16%
47	Constant	-1.0%	Constant	3.0%	8 year	Rounded	0.00%	0.00%	0.05%	-0.03%	3.15%	4.76%	-6.33%	-4.87%	0.35%
48	Constant	-1.0%	Constant	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.18%	3.18%	4.74%	-6.35%	-4.94%	-0.35%
49	Constant	-1.0%	Constant	10.0%	1 year	Constant	0.00%	0.00%	3.46%	3.04%	16.58%	15.52%	7.58%	6.59%	-3.08%
50	Constant	-1.0%	Constant	10.0%	1 year	Rounded	0.00%	0.00%	0.61%	0.69%	15.48%	14.93%	0.72%	0.24%	-1.23%
51	Constant	-1.0%	Constant	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	15.44%	15.04%	0.95%	0.60%	-0.93%
52	Constant	-1.0%	Constant	10.0%	8 year	Constant	0.00%	0.00%	3.44%	3.02%	10.77%	10.73%	2.21%	2.18%	-3.43%
53	Constant	-1.0%	Constant	10.0%	8 year	Rounded	0.00%	0.00%	0.61%	0.69%	5.57%	7.01%	-7.93%	-6.66%	-1.61%
54	Constant	-1.0%	Constant	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	5.69%	7.16%	-7.58%	-6.29%	-1.13%
55	Varied	None	Constant	0.0%	1 year	Constant	7.15%	3.32%	4.69%	1.14%	23.39%	18.80%	11.23%	7.10%	6.45%
56	Varied	None	Constant	0.0%	1 year	Rounded	7.15%	3.32%	4.69%	1.14%	23.39%	18.80%	11.23%	7.10%	6.45%
57	Varied	None	Constant	0.0%	1 year	Exact	7.15%	3.32%	4.69%	1.14%	23.39%	18.80%	11.23%	7.10%	6.45%
58	Varied	None	Constant	0.0%	8 year	Constant	7.00%	3.45%	4.45%	1.12%	18.51%	18.62%	6.83%	6.93%	6.82%
59	Varied	None	Constant	0.0%	8 year	Rounded	7.00%	3.45%	4.45%	1.12%	18.51%	18.62%	6.83%	6.93%	6.82%
60	Varied	None	Constant	0.0%	8 year	Exact	7.00%	3.45%	4.45%	1.12%	18.51%	18.62%	6.83%	6.93%	6.82%
61	Varied	None	Constant	3.0%	1 year	Constant	7.31%	3.33%	5.58%	1.76%	24.96%	20.08%	12.65%	8.25%	5.55%
62	Varied	None	Constant	3.0%	1 year	Rounded	7.31%	3.33%	4.92%	1.11%	25.28%	20.72%	11.18%	7.14%	6.68%
63	Varied	None	Constant	3.0%	1 year	Exact	7.31%	3.33%	5.00%	1.30%	25.40%	20.60%	11.23%	6.97%	6.23%
64	Varied	None	Constant	3.0%	8 year	Constant	7.15%	3.47%	5.34%	1.74%	21.22%	20.83%	9.28%	8.93%	5.96%
65	Varied	None	Constant	3.0%	8 year	Rounded	7.15%	3.47%	4.69%	1.10%	19.96%	20.05%	6.46%	6.54%	7.15%
66	Varied	None	Constant	3.0%	8 year	Exact	7.15%	3.47%	4.76%	1.28%	20.23%	20.31%	6.65%	6.72%	6.58%
67	Varied	None	Constant	10.0%	1 year	Constant	7.58%	3.32%	7.69%	3.30%	30.11%	24.48%	17.29%	12.21%	3.65%
68	Varied	None	Constant	10.0%	1 year	Rounded	7.58%	3.32%	5.64%	1.71%	30.10%	24.69%	10.98%	6.37%	5.48%
69	Varied	None	Constant	10.0%	1 year	Exact	7.58%	3.32%	5.65%	1.64%	29.88%	24.65%	11.09%	6.61%	5.70%
70	Varied	None	Constant	10.0%	8 year	Constant	7.43%	3.49%	7.46%	3.27%	27.78%	26.10%	15.19%	13.67%	4.00%
71	Varied	None	Constant	10.0%	8 year	Rounded	7.43%	3.49%	5.39%	1.71%	24.15%	24.15%	5.91%	5.91%	5.69%
72	Varied	None	Constant	10.0%	8 year	Exact	7.43%	3.49%	5.40%	1.63%	23.99%	24.02%	6.05%	6.07%	6.02%
73	Varied	1.0%	Constant	0.0%	1 year	Constant	7.20%	3.32%	4.74%	1.13%	21.45%	17.32%	9.49%	5.76%	6.52%
74	Varied	1.0%	Constant	0.0%	1 year	Rounded	7.20%	3.32%	4.74%	1.13%	21.45%	17.32%	9.49%	5.76%	6.52%
75	Varied	1.0%	Constant	0.0%	1 year	Exact	7.20%	3.32%	4.74%	1.13%	21.45%	17.32%	9.49%	5.76%	6.52%
76	Varied	1.0%	Constant	0.0%	8 year	Constant	7.10%	3.43%	4.49%	1.12%	21.59%	20.84%	9.61%	8.93%	6.86%
77	Varied	1.0%	Constant	0.0%	8 year	Rounded	7.10%	3.43%	4.49%	1.12%	21.59%	20.84%	9.61%	8.93%	6.86%
78	Varied	1.0%	Constant	0.0%	8 year	Exact	7.10%	3.43%	4.49%	1.12%	21.59%	20.84%	9.61%	8.93%	6.86%
79	Varied	1.0%	Constant	3.0%	1 year	Constant	7.35%	3.33%	5.64%	1.77%	23.20%	18.73%	11.07%	7.03%	5.59%
80	Varied	1.0%	Constant	3.0%	1 year	Rounded	7.35%	3.33%	4.96%	1.09%	23.21%	19.13%	9.35%	5.72%	6.78%
81	Varied	1.0%	Constant	3.0%	1 year	Exact	7.35%	3.33%	5.05%	1.29%	23.34%	19.01%	9.40%	5.56%	6.29%
82	Varied	1.0%	Constant	3.0%	8 year	Constant	7.24%	3.45%	5.39%	1.75%	24.24%	23.01%	12.00%	10.89%	5.91%
83	Varied	1.0%	Constant	3.0%	8 year	Rounded	7.24%	3.45%	4.72%	1.09%	23.20%	22.45%	9.33%	8.67%	7.14%
84	Varied	1.0%	Constant	3.0%	8 year	Exact	7.24%	3.45%	4.80%	1.28%	23.49%	22.67%	9.53%	8.81%	6.61%
85	Varied	1.0%	Constant	10.0%	1 year	Constant	7.61%	3.32%	7.77%	3.32%	28.98%	23.63%	16.27%	11.45%	3.61%
86	Varied	1.0%	Constant	10.0%	1 year	Rounded	7.61%	3.32%	5.68%	1.70%	27.73%	22.84%	8.96%	4.79%	5.51%
87	Varied	1.0%	Constant	10.0%	1 year	Exact	7.61%	3.32%	5.69%	1.62%	27.54%	22.81%	9.08%	5.04%	5.75%
88	Varied	1.0%	Constant	10.0%	8 year	Constant	7.48%	3.45%	7.52%	3.30%	30.40%	27.99%	17.56%	15.38%	3.88%
89	Varied	1.0%	Constant	10.0%	8 year	Rounded	7.48%	3.45%	5.42%	1.70%	27.84%	26.85%	9.05%	8.22%	5.75%
90	Varied	1.0%	Constant	10.0%	8 year	Exact	7.48%	3.45%	5.43%	1.62%	27.64%	26.71%	9.18%	8.38%	6.03%

Increased Variation Scenario Results
Total Error in Methods

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess
91	Varied	-1.0%	Constant	0.0%	1 year	Constant	7.11%	3.30%	4.64%	1.14%	24.74%	19.84%	12.45%	8.04%	6.38%
92	Varied	-1.0%	Constant	0.0%	1 year	Rounded	7.11%	3.30%	4.64%	1.14%	24.74%	19.84%	12.45%	8.04%	6.38%
93	Varied	-1.0%	Constant	0.0%	1 year	Exact	7.11%	3.30%	4.64%	1.14%	24.74%	19.84%	12.45%	8.04%	6.38%
94	Varied	-1.0%	Constant	0.0%	8 year	Constant	6.98%	3.45%	4.40%	1.15%	19.39%	18.75%	7.63%	7.06%	6.86%
95	Varied	-1.0%	Constant	0.0%	8 year	Rounded	6.98%	3.45%	4.40%	1.15%	19.39%	18.75%	7.63%	7.06%	6.86%
96	Varied	-1.0%	Constant	0.0%	8 year	Exact	6.98%	3.45%	4.40%	1.15%	19.39%	18.75%	7.63%	7.06%	6.86%
97	Varied	-1.0%	Constant	3.0%	1 year	Constant	7.27%	3.31%	5.53%	1.75%	26.41%	21.22%	13.96%	9.27%	5.50%
98	Varied	-1.0%	Constant	3.0%	1 year	Rounded	7.27%	3.31%	4.88%	1.11%	26.70%	21.83%	12.44%	8.12%	6.58%
99	Varied	-1.0%	Constant	3.0%	1 year	Exact	7.27%	3.31%	4.96%	1.30%	26.83%	21.71%	12.50%	7.96%	6.16%
100	Varied	-1.0%	Constant	3.0%	8 year	Constant	7.14%	3.47%	5.28%	1.75%	22.04%	20.92%	10.02%	9.01%	5.98%
101	Varied	-1.0%	Constant	3.0%	8 year	Rounded	7.14%	3.47%	4.65%	1.14%	20.95%	20.32%	7.34%	6.78%	7.14%
102	Varied	-1.0%	Constant	3.0%	8 year	Exact	7.14%	3.47%	4.72%	1.31%	21.22%	20.52%	7.52%	6.90%	6.62%
103	Varied	-1.0%	Constant	10.0%	1 year	Constant	7.55%	3.31%	7.62%	3.27%	31.53%	25.61%	18.57%	13.23%	3.65%
104	Varied	-1.0%	Constant	10.0%	1 year	Rounded	7.55%	3.31%	5.61%	1.71%	31.73%	25.98%	12.37%	7.47%	5.44%
105	Varied	-1.0%	Constant	10.0%	1 year	Exact	7.55%	3.31%	5.62%	1.64%	31.51%	25.93%	12.48%	7.71%	5.64%
106	Varied	-1.0%	Constant	10.0%	8 year	Constant	7.42%	3.50%	7.37%	3.26%	28.35%	26.06%	15.71%	13.64%	4.03%
107	Varied	-1.0%	Constant	10.0%	8 year	Rounded	7.42%	3.50%	5.35%	1.74%	25.41%	24.53%	6.99%	6.24%	5.77%
108	Varied	-1.0%	Constant	10.0%	8 year	Exact	7.42%	3.50%	5.37%	1.67%	25.24%	24.41%	7.12%	6.41%	6.06%
109	Constant	None	Varied	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	8.25%	8.37%	-0.17%	-0.06%	0.00%
110	Constant	None	Varied	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	8.25%	8.37%	-0.17%	-0.06%	0.00%
111	Constant	None	Varied	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	8.25%	8.37%	-0.17%	-0.06%	0.00%
112	Constant	None	Varied	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	1.12%	2.52%	-6.74%	-5.45%	0.00%
113	Constant	None	Varied	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	1.12%	2.52%	-6.74%	-5.45%	0.00%
114	Constant	None	Varied	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	1.12%	2.52%	-6.74%	-5.45%	0.00%
115	Constant	None	Varied	3.0%	1 year	Constant	0.00%	0.00%	0.99%	0.87%	9.47%	9.45%	0.96%	0.94%	-1.09%
116	Constant	None	Varied	3.0%	1 year	Rounded	0.00%	0.00%	0.03%	-0.05%	9.84%	10.16%	-0.31%	-0.02%	0.25%
117	Constant	None	Varied	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	9.77%	9.88%	-0.43%	-0.32%	-0.30%
118	Constant	None	Varied	3.0%	8 year	Constant	0.00%	0.00%	0.99%	0.87%	3.57%	4.56%	-4.48%	-3.57%	-1.22%
119	Constant	None	Varied	3.0%	8 year	Rounded	0.00%	0.00%	0.04%	-0.04%	2.08%	3.44%	-7.35%	-6.12%	0.47%
120	Constant	None	Varied	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.18%	2.12%	3.46%	-7.37%	-6.14%	-0.35%
121	Constant	None	Varied	10.0%	1 year	Constant	0.00%	0.00%	3.48%	3.07%	14.71%	14.04%	5.79%	5.18%	-3.22%
122	Constant	None	Varied	10.0%	1 year	Rounded	0.00%	0.00%	0.61%	0.69%	13.29%	13.23%	-1.24%	-1.29%	-1.29%
123	Constant	None	Varied	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	13.26%	13.35%	-1.00%	-0.92%	-0.97%
124	Constant	None	Varied	10.0%	8 year	Constant	0.00%	0.00%	3.48%	3.07%	10.22%	9.97%	1.65%	1.42%	-3.57%
125	Constant	None	Varied	10.0%	8 year	Rounded	0.00%	0.00%	0.61%	0.69%	4.13%	5.34%	-9.23%	-8.17%	-1.70%
126	Constant	None	Varied	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	4.26%	5.48%	-8.87%	-7.81%	-1.15%
127	Constant	1.0%	Varied	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	5.97%	6.17%	-2.27%	-2.08%	0.00%
128	Constant	1.0%	Varied	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	5.97%	6.17%	-2.27%	-2.08%	0.00%
129	Constant	1.0%	Varied	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	5.97%	6.17%	-2.27%	-2.08%	0.00%
130	Constant	1.0%	Varied	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	3.43%	4.78%	-4.61%	-3.37%	0.00%
131	Constant	1.0%	Varied	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	3.43%	4.78%	-4.61%	-3.37%	0.00%
132	Constant	1.0%	Varied	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	3.43%	4.78%	-4.61%	-3.37%	0.00%
133	Constant	1.0%	Varied	3.0%	1 year	Constant	0.00%	0.00%	1.01%	0.89%	7.37%	7.40%	-0.98%	-0.95%	-1.13%
134	Constant	1.0%	Varied	3.0%	1 year	Rounded	0.00%	0.00%	0.02%	-0.06%	7.36%	7.76%	-2.56%	-2.19%	0.28%
135	Constant	1.0%	Varied	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	7.30%	7.50%	-2.67%	-2.49%	-0.31%

Increased Variation Scenario Results
Total Error in Methods

Appendix 2 - Increased Variation Page 4

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess
136	Constant	1.0%	Varied	3.0%	8 year	Constant	0.00%	0.00%	1.00%	0.88%	6.07%	6.89%	-2.17%	-1.41%	-1.20%
137	Constant	1.0%	Varied	3.0%	8 year	Rounded	0.00%	0.00%	0.03%	-0.04%	4.64%	5.99%	-5.02%	-3.81%	0.35%
138	Constant	1.0%	Varied	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.18%	4.71%	5.99%	-5.01%	-3.85%	-0.35%
139	Constant	1.0%	Varied	10.0%	1 year	Constant	0.00%	0.00%	3.53%	3.12%	13.28%	12.64%	4.47%	3.89%	-3.33%
140	Constant	1.0%	Varied	10.0%	1 year	Rounded	0.00%	0.00%	0.61%	0.69%	10.33%	10.37%	-3.82%	-3.78%	-1.37%
141	Constant	1.0%	Varied	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	10.33%	10.52%	-3.56%	-3.40%	-1.01%
142	Constant	1.0%	Varied	10.0%	8 year	Constant	0.00%	0.00%	3.52%	3.11%	12.79%	12.20%	4.02%	3.48%	-3.51%
143	Constant	1.0%	Varied	10.0%	8 year	Rounded	0.00%	0.00%	0.61%	0.69%	7.47%	8.54%	-6.31%	-5.39%	-1.59%
144	Constant	1.0%	Varied	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	7.57%	8.67%	-5.98%	-5.01%	-1.13%
145	Constant	-1.0%	Varied	0.0%	1 year	Constant	0.00%	0.00%	0.00%	0.00%	9.90%	9.74%	1.36%	1.21%	0.00%
146	Constant	-1.0%	Varied	0.0%	1 year	Rounded	0.00%	0.00%	0.00%	0.00%	9.90%	9.74%	1.36%	1.21%	0.00%
147	Constant	-1.0%	Varied	0.0%	1 year	Exact	0.00%	0.00%	0.00%	0.00%	9.90%	9.74%	1.36%	1.21%	0.00%
148	Constant	-1.0%	Varied	0.0%	8 year	Constant	0.00%	0.00%	0.00%	0.00%	1.29%	3.19%	-6.59%	-4.83%	0.00%
149	Constant	-1.0%	Varied	0.0%	8 year	Rounded	0.00%	0.00%	0.00%	0.00%	1.29%	3.19%	-6.59%	-4.83%	0.00%
150	Constant	-1.0%	Varied	0.0%	8 year	Exact	0.00%	0.00%	0.00%	0.00%	1.29%	3.19%	-6.59%	-4.83%	0.00%
151	Constant	-1.0%	Varied	3.0%	1 year	Constant	0.00%	0.00%	0.98%	0.86%	11.25%	10.96%	2.60%	2.33%	-1.05%
152	Constant	-1.0%	Varied	3.0%	1 year	Rounded	0.00%	0.00%	0.03%	-0.05%	11.58%	11.60%	1.27%	1.29%	0.22%
153	Constant	-1.0%	Varied	3.0%	1 year	Exact	0.00%	0.00%	0.18%	0.18%	11.52%	11.34%	1.16%	1.00%	-0.29%
154	Constant	-1.0%	Varied	3.0%	8 year	Constant	0.00%	0.00%	0.97%	0.85%	3.74%	5.23%	-4.32%	-2.95%	-1.18%
155	Constant	-1.0%	Varied	3.0%	8 year	Rounded	0.00%	0.00%	0.04%	-0.03%	2.38%	4.30%	-7.08%	-5.34%	0.38%
156	Constant	-1.0%	Varied	3.0%	8 year	Exact	0.00%	0.00%	0.18%	0.18%	2.41%	4.29%	-7.10%	-5.39%	-0.35%
157	Constant	-1.0%	Varied	10.0%	1 year	Constant	0.00%	0.00%	3.45%	3.03%	16.43%	15.54%	7.38%	6.56%	-3.11%
158	Constant	-1.0%	Varied	10.0%	1 year	Rounded	0.00%	0.00%	0.61%	0.69%	15.28%	14.91%	0.49%	0.17%	-1.24%
159	Constant	-1.0%	Varied	10.0%	1 year	Exact	0.00%	0.00%	0.60%	0.60%	15.24%	15.02%	0.72%	0.54%	-0.94%
160	Constant	-1.0%	Varied	10.0%	8 year	Constant	0.00%	0.00%	3.43%	3.01%	10.29%	10.48%	1.71%	1.89%	-3.48%
161	Constant	-1.0%	Varied	10.0%	8 year	Rounded	0.00%	0.00%	0.61%	0.69%	4.77%	6.55%	-8.67%	-7.11%	-1.64%
162	Constant	-1.0%	Varied	10.0%	8 year	Exact	0.00%	0.00%	0.60%	0.60%	4.89%	6.70%	-8.32%	-6.73%	-1.15%
163	Varied	None	Varied	0.0%	1 year	Constant	7.10%	3.31%	4.65%	1.14%	23.31%	18.82%	11.11%	7.06%	6.50%
164	Varied	None	Varied	0.0%	1 year	Rounded	7.10%	3.31%	4.65%	1.14%	23.31%	18.82%	11.11%	7.06%	6.50%
165	Varied	None	Varied	0.0%	1 year	Exact	7.10%	3.31%	4.65%	1.14%	23.31%	18.82%	11.11%	7.06%	6.50%
166	Varied	None	Varied	0.0%	8 year	Constant	6.95%	3.45%	4.42%	1.12%	18.54%	18.45%	6.82%	6.73%	6.87%
167	Varied	None	Varied	0.0%	8 year	Rounded	6.95%	3.45%	4.42%	1.12%	18.54%	18.45%	6.82%	6.73%	6.87%
168	Varied	None	Varied	0.0%	8 year	Exact	6.95%	3.45%	4.42%	1.12%	18.54%	18.45%	6.82%	6.73%	6.87%
169	Varied	None	Varied	3.0%	1 year	Constant	7.26%	3.32%	5.55%	1.76%	24.93%	20.15%	12.57%	8.26%	5.58%
170	Varied	None	Varied	3.0%	1 year	Rounded	7.26%	3.32%	4.89%	1.10%	25.23%	20.77%	11.09%	7.13%	6.73%
171	Varied	None	Varied	3.0%	1 year	Exact	7.26%	3.32%	4.97%	1.30%	25.35%	20.64%	11.14%	6.96%	6.27%
172	Varied	None	Varied	3.0%	8 year	Constant	7.11%	3.48%	5.31%	1.73%	21.25%	20.63%	9.26%	8.70%	6.01%
173	Varied	None	Varied	3.0%	8 year	Rounded	7.11%	3.48%	4.66%	1.09%	20.01%	19.89%	6.45%	6.35%	7.21%
174	Varied	None	Varied	3.0%	8 year	Exact	7.11%	3.48%	4.73%	1.28%	20.28%	20.13%	6.64%	6.51%	6.63%
175	Varied	None	Varied	10.0%	1 year	Constant	7.55%	3.31%	7.66%	3.29%	30.21%	24.66%	17.33%	12.33%	3.65%
176	Varied	None	Varied	10.0%	1 year	Rounded	7.55%	3.31%	5.63%	1.71%	30.13%	24.80%	10.96%	6.42%	5.52%
177	Varied	None	Varied	10.0%	1 year	Exact	7.55%	3.31%	5.64%	1.64%	29.91%	24.76%	11.07%	6.66%	5.74%
178	Varied	None	Varied	10.0%	8 year	Constant	7.39%	3.51%	7.43%	3.26%	27.85%	25.90%	15.21%	13.45%	4.04%
179	Varied	None	Varied	10.0%	8 year	Rounded	7.39%	3.51%	5.37%	1.70%	24.21%	23.94%	5.92%	5.69%	5.76%
180	Varied	None	Varied	10.0%	8 year	Exact	7.39%	3.51%	5.38%	1.63%	24.05%	23.80%	6.05%	5.85%	6.08%

Increased Variation Scenario Results
Total Error in Methods

Scenario	Pattern	Frequency Trend	Frequency	Inflation	Severity Randomization	Retention	Wtd Average Unlimited	Latest Diag Unlimited	Wtd Average Limited	Latest Diag Limited	Wtd Average Excess	Latest Diag Excess	Non Tail Wtd Avg Excess	Non Tail Latest Diag excess	Alternative Method Excess
181	Varied	1.0%	Varied	0.0%	1 year	Constant	7.10%	3.29%	4.67%	1.12%	21.56%	17.47%	9.53%	5.85%	6.52%
182	Varied	1.0%	Varied	0.0%	1 year	Rounded	7.10%	3.29%	4.67%	1.12%	21.56%	17.47%	9.53%	5.85%	6.52%
183	Varied	1.0%	Varied	0.0%	1 year	Exact	7.10%	3.29%	4.67%	1.12%	21.56%	17.47%	9.53%	5.85%	6.52%
184	Varied	1.0%	Varied	0.0%	8 year	Constant	7.04%	3.41%	4.44%	1.11%	20.88%	20.27%	8.93%	8.37%	6.88%
185	Varied	1.0%	Varied	0.0%	8 year	Rounded	7.04%	3.41%	4.44%	1.11%	20.88%	20.27%	8.93%	8.37%	6.88%
186	Varied	1.0%	Varied	0.0%	8 year	Exact	7.04%	3.41%	4.44%	1.11%	20.88%	20.27%	8.93%	8.37%	6.88%
187	Varied	1.0%	Varied	3.0%	1 year	Constant	7.26%	3.30%	5.57%	1.75%	23.38%	18.93%	11.17%	7.17%	5.58%
188	Varied	1.0%	Varied	3.0%	1 year	Rounded	7.26%	3.30%	4.89%	1.07%	23.34%	19.28%	9.41%	5.81%	6.78%
189	Varied	1.0%	Varied	3.0%	1 year	Exact	7.26%	3.30%	4.99%	1.27%	23.47%	19.17%	9.47%	5.66%	6.29%
190	Varied	1.0%	Varied	3.0%	8 year	Constant	7.19%	3.43%	5.34%	1.73%	23.67%	22.54%	11.44%	10.42%	5.92%
191	Varied	1.0%	Varied	3.0%	8 year	Rounded	7.19%	3.43%	4.67%	1.08%	22.51%	21.86%	8.68%	8.10%	7.17%
192	Varied	1.0%	Varied	3.0%	8 year	Exact	7.19%	3.43%	4.75%	1.27%	22.81%	22.12%	8.89%	8.27%	6.63%
193	Varied	1.0%	Varied	10.0%	1 year	Constant	7.54%	3.30%	7.70%	3.30%	29.26%	23.92%	16.47%	11.66%	3.60%
194	Varied	1.0%	Varied	10.0%	1 year	Rounded	7.54%	3.30%	5.63%	1.69%	27.92%	23.02%	9.08%	4.91%	5.50%
195	Varied	1.0%	Varied	10.0%	1 year	Exact	7.54%	3.30%	5.64%	1.61%	27.73%	23.00%	9.20%	5.16%	5.74%
196	Varied	1.0%	Varied	10.0%	8 year	Constant	7.45%	3.44%	7.46%	3.28%	30.15%	27.78%	17.27%	15.14%	3.86%
197	Varied	1.0%	Varied	10.0%	8 year	Rounded	7.45%	3.44%	5.38%	1.69%	27.23%	26.34%	8.49%	7.73%	5.76%
198	Varied	1.0%	Varied	10.0%	8 year	Exact	7.45%	3.44%	5.39%	1.62%	27.04%	26.19%	8.61%	7.89%	6.05%
199	Varied	-1.0%	Varied	0.0%	1 year	Constant	7.10%	3.31%	4.63%	1.15%	24.56%	19.78%	12.24%	7.93%	6.43%
200	Varied	-1.0%	Varied	0.0%	1 year	Rounded	7.10%	3.31%	4.63%	1.15%	24.56%	19.78%	12.24%	7.93%	6.43%
201	Varied	-1.0%	Varied	0.0%	1 year	Exact	7.10%	3.31%	4.63%	1.15%	24.56%	19.78%	12.24%	7.93%	6.43%
202	Varied	-1.0%	Varied	0.0%	8 year	Constant	6.97%	3.45%	4.39%	1.15%	18.86%	18.61%	7.10%	6.88%	6.90%
203	Varied	-1.0%	Varied	0.0%	8 year	Rounded	6.97%	3.45%	4.39%	1.15%	18.86%	18.61%	7.10%	6.88%	6.90%
204	Varied	-1.0%	Varied	0.0%	8 year	Exact	6.97%	3.45%	4.39%	1.15%	18.86%	18.61%	7.10%	6.88%	6.90%
205	Varied	-1.0%	Varied	3.0%	1 year	Constant	7.26%	3.32%	5.52%	1.76%	26.26%	21.18%	13.77%	9.19%	5.54%
206	Varied	-1.0%	Varied	3.0%	1 year	Rounded	7.26%	3.32%	4.87%	1.11%	26.54%	21.78%	12.25%	8.02%	6.64%
207	Varied	-1.0%	Varied	3.0%	1 year	Exact	7.26%	3.32%	4.96%	1.31%	26.67%	21.66%	12.31%	7.87%	6.21%
208	Varied	-1.0%	Varied	3.0%	8 year	Constant	7.13%	3.47%	5.27%	1.75%	21.57%	20.82%	9.54%	8.87%	6.01%
209	Varied	-1.0%	Varied	3.0%	8 year	Rounded	7.13%	3.47%	4.64%	1.13%	20.42%	20.17%	6.82%	6.60%	7.19%
210	Varied	-1.0%	Varied	3.0%	8 year	Exact	7.13%	3.47%	4.71%	1.31%	20.69%	20.38%	7.00%	6.73%	6.65%
211	Varied	-1.0%	Varied	10.0%	1 year	Constant	7.55%	3.32%	7.61%	3.28%	31.44%	25.64%	18.44%	13.21%	3.67%
212	Varied	-1.0%	Varied	10.0%	1 year	Rounded	7.55%	3.32%	5.61%	1.72%	31.61%	25.96%	12.22%	7.41%	5.49%
213	Varied	-1.0%	Varied	10.0%	1 year	Exact	7.55%	3.32%	5.62%	1.65%	31.39%	25.91%	12.33%	7.65%	5.68%
214	Varied	-1.0%	Varied	10.0%	8 year	Constant	7.42%	3.50%	7.36%	3.26%	28.06%	26.05%	15.39%	13.58%	4.03%
215	Varied	-1.0%	Varied	10.0%	8 year	Rounded	7.42%	3.50%	5.35%	1.74%	24.89%	24.42%	6.50%	6.10%	5.79%
216	Varied	-1.0%	Varied	10.0%	8 year	Exact	7.42%	3.50%	5.37%	1.67%	24.72%	24.30%	6.63%	6.27%	6.08%

Unlimited

Accident Yr	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288
1981	103,747	138,893	155,864	164,517	171,942	175,493	179,000	181,996	184,490	187,870	189,288	191,809	194,378	195,035	194,962	195,964	196,574	197,308	197,447	198,676	199,147	201,480	202,700	203,285
1982	109,739	148,608	169,329	182,043	190,100	196,998	206,271	209,429	213,065	214,032	214,305	215,366	217,225	217,523	218,238	219,363	219,068	221,346	221,305	221,533	224,194	224,854	225,233	225,605
1983	107,258	150,321	177,463	195,212	205,328	214,791	220,157	228,729	231,755	232,202	230,470	231,856	232,072	231,919	234,230	235,709	236,066	236,625	236,211	236,405	238,128	239,247	239,920	239,568
1984	117,819	168,762	201,589	224,872	237,361	245,317	250,435	254,357	253,652	257,419	260,066	260,325	261,080	262,987	263,795	264,223	264,547	265,869	265,726	267,083	268,368	268,485	268,563	269,422
1985	108,566	159,254	192,000	210,607	221,498	231,217	235,526	235,756	240,226	239,671	240,990	242,344	243,673	244,368	245,441	244,818	245,471	246,522	247,442	247,888	248,256	248,729	248,951	253,482
1986	115,327	180,032	211,836	231,996	245,769	255,894	259,512	265,950	267,150	269,080	270,319	274,643	273,778	279,138	278,423	278,243	278,617	281,689	282,305	283,951	284,704	286,757	286,590	288,814
1987	153,086	230,042	266,806	290,140	304,279	315,007	320,207	322,382	325,391	325,266	327,258	329,692	330,328	331,770	333,778	336,329	338,224	341,581	342,553	342,790	343,375	346,600	346,943	347,538
1988	172,300	250,520	294,394	320,109	327,584	340,144	342,740	346,781	347,166	351,116	353,733	356,577	359,182	359,667	361,915	361,969	364,380	364,988	365,017	364,833	365,678	367,212	368,234	368,798
1989	206,549	301,734	354,369	380,862	399,472	409,246	414,389	416,767	425,713	426,934	427,302	428,850	434,942	437,220	443,279	444,264	444,437	445,716	447,148	448,450	449,494	450,679	449,638	
1990	259,719	399,206	464,548	500,012	518,761	533,554	538,490	545,881	550,085	554,355	555,281	558,829	562,229	566,397	571,016	573,574	575,026	576,753	579,161	581,634	581,192	582,272		
1991	273,827	394,533	454,242	494,252	518,771	530,341	538,952	547,295	554,573	560,478	565,240	570,791	574,642	581,946	584,135	585,209	586,124	591,353	595,581	597,489	598,684			
1992	267,683	380,406	434,716	472,469	489,208	503,129	516,789	524,840	526,775	534,430	541,630	544,276	546,974	550,185	550,698	551,221	554,647	560,008	560,774	561,978				
1993	251,840	354,810	408,472	437,338	456,766	481,423	492,096	494,554	500,205	508,885	513,348	517,574	520,864	522,123	523,951	526,293	529,245	528,796	531,985					
1994	205,281	284,928	325,984	351,904	363,686	375,656	383,380	390,480	394,253	404,341	413,176	415,890	413,937	418,001	420,955	424,337	424,068	427,663						
1995	169,032	246,400	293,409	315,122	329,911	333,729	347,292	355,028	363,438	372,294	374,845	376,749	383,738	397,219	401,774	404,235	403,885							
1996	165,360	250,278	292,433	316,704	330,128	341,800	353,291	358,609	363,494	366,833	371,540	375,997	384,124	388,734	388,808	393,386								
1997	165,752	254,299	295,345	318,803	337,098	346,877	352,681	363,104	370,080	374,314	376,534	380,872	385,418	386,028	388,703									
1998	160,614	247,459	287,834	315,100	333,349	347,520	354,717	357,559	362,039	364,771	367,673	372,355	372,845	377,522										
1999	167,190	255,539	307,870	339,875	360,079	375,252	385,217	392,272	401,466	408,245	417,782	421,991	423,222											
2000	194,999	328,045	386,296	425,294	452,850	469,359	477,308	486,919	502,059	513,813	515,859	518,794												
2001	193,399	338,983	404,093	440,697	462,380	473,701	490,623	500,238	512,669	516,964	523,016													
2002	224,214	386,518	466,827	510,594	532,235	551,593	570,648	589,292	596,885	602,848														
2003	249,463	431,515	513,406	557,541	593,191	612,742	635,454	644,289	651,341															
2004	288,781	473,041	552,844	606,548	637,317	669,479	684,292	697,768																
2005	284,446	452,941	540,039	592,984	633,277	657,280	678,405																	
2006	261,146	411,415	494,786	552,003	582,127	610,760																		
2007	265,473	432,928	527,202	572,465	601,938																			
2008	260,865	450,470	534,994	585,132																				
2009	249,953	414,847	495,269																					
2010	244,205	405,703																						
2011	222,883																							

Age-to-Age Factors

Accident Yr	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108	108 - 120	120 - 132	132 - 144	144 - 156	156 - 168	168 - 180	180 - 192	192 - 204	204 - 216	216 - 228	228 - 240	240 - 252	252 - 264	264 - 276	276 - 288	288 - 300
1981	1.339	1.122	1.056	1.045	1.021	1.020	1.017	1.014	1.018	1.008	1.013	1.013	1.003	1.000	1.005	1.003	1.004	1.001	1.006	1.002	1.012	1.006	1.003	1.003
1982	1.354	1.139	1.075	1.044	1.036	1.047	1.015	1.017	1.005	1.001	1.005	1.009	1.001	1.003	1.005	0.999	1.010	1.000	1.001	1.012	1.003	1.002	1.002	1.000
1983	1.401	1.181	1.100	1.052	1.046	1.025	1.039	1.013	1.002	0.993	1.006	1.001	0.999	1.010	1.006	1.002	1.002	0.998	1.001	1.007	1.005	1.003	0.999	1.000
1984	1.432	1.195	1.115	1.056	1.034	1.021	1.016	0.997	1.015	1.010	1.001	1.003	1.007	1.003	1.002	1.001	1.005	0.999	1.005	1.005	1.000	1.000	1.003	1.004
1985	1.467	1.206	1.097	1.052	1.044	1.019	1.001	1.019	0.998	1.006	1.006	1.005	1.003	1.004	0.997	1.003	1.004	1.004	1.002	1.001	1.002	1.001	1.018	1.002
1986	1.561	1.177	1.095	1.059	1.041	1.014	1.025	1.005	1.007	1.005	1.016	0.997	1.020	0.997	0.999	1.001	1.011	1.002	1.006	1.003	1.007	0.999	1.008	1.010
1987	1.503	1.160	1.087	1.049	1.035	1.017	1.007	1.009	1.000	1.006	1.007	1.002	1.004	1.006	1.008	1.006	1.010	1.003	1.001	1.002	1.009	1.001	1.002	1.002
1988	1.454	1.175	1.087	1.023	1.038	1.008	1.012	1.001	1.011	1.007	1.008	1.007	1.001	1.006	1.000	1.007	1.002	1.000	0.999	1.002	1.004	1.003	1.002	
1989	1.461	1.174	1.075	1.049	1.024	1.013	1.006	1.021	1.003	1.001	1.004	1.014	1.005	1.014	1.002	1.000	1.003	1.003	1.003	1.002	1.003	0.998		
1990	1.537	1.164	1.076	1.037	1.029	1.009	1.014	1.008	1.008	1.002	1.006	1.006	1.007	1.008	1.004	1.003	1.003	1.004	1.004	0.999	1.002			
1991	1.441	1.151	1.088	1.050	1.022	1.016	1.015	1.013	1.011	1.008	1.010	1.007	1.013	1.004	1.002	1.002	1.009	1.007	1.003	1.002				
1992	1.421	1.143	1.087	1.035	1.028	1.027	1.016	1.004	1.015	1.013	1.005	1.005	1.006	1.001	1.001	1.006	1.010	1.001	1.002					
1993	1.409	1.151	1.071	1.044	1.054	1.022	1.005	1.011	1.017	1.009	1.008	1.006	1.002	1.004	1.004	1.006	0.999	1.006						
1994	1.388	1.144	1.080	1.033	1.033	1.021	1.019	1.010	1.026	1.022	1.007	0.995	1.010	1.007	1.008	0.999	1.008							
1995	1.458	1.191	1.074	1.047	1.012	1.041	1.022	1.024	1.024	1.007	1.005	1.019	1.035	1.011	1.006	0.999								
1996	1.514	1.168	1.083	1.042	1.035	1.034	1.015	1.014	1.009	1.013	1.012	1.022	1.012	1.000	1.012									
1997	1.534	1.161	1.079	1.057	1.029	1.017	1.030	1.019	1.011	1.006	1.012	1.012	1.002	1.007										
1998	1.541	1.163	1.095	1.058	1.043	1.021	1.008	1.013	1.008	1.008	1.013	1.001	1.013											
1999	1.528	1.205	1.104	1.059	1.042	1.027	1.018	1.023	1.017	1.023	1.010	1.003												
2000	1.682	1.178	1.101	1.065	1.036	1.017	1.020	1.031	1.023	1.004	1.006													
2001	1.753	1.192	1.091	1.049	1.024	1.036	1.020	1.025	1.008	1.012														
2002	1.724	1.208	1.094	1.042	1.036	1.035	1.033	1.013	1.010															
2003	1.730	1.190	1.086	1.064	1.033	1.037	1.014	1.011																
2004	1.638	1.169	1.097	1.051	1.050	1.022	1.020																	
2005	1.592	1.192	1.098	1.068	1.038	1.032																		
2006	1.575	1.203	1.116	1.055	1.049																			
2007	1.631	1.218	1.086	1.051																				
2008	1.727	1.188	1.094																					
2009	1.660	1.194																						
2010	1.661																							
2011																								

Averages

	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108	108 - 120	120 - 132	132 - 144	144 - 156	156 - 168	168 - 180	180 - 192	192 - 204	204 - 216	216 - 228	228 - 240	240 - 252	252 - 264	264 - 276	276 - 288	288 - 300
NCCI Countrywide	1.346	1.103	1.052	1.028	1.020	1.012	1.011	1.085																
Volume Wtd All	1.554	1.178	1.090	1.050	1.036	1.024	1.017	1.014	1.012	1.008	1.008	1.007	1.008	1.005	1.004	1.003	1.006	1.003	1.003	1.003	1.004	1.001	1.004	1.003
Volume Wtd All	2.507	1.613	1.369	1.257	1.197	1.156	1.128	1.110	1.095	1.082	1.073	1.065	1.057	1.049	1.043	1.039	1.036	1.030	1.027	1.025	1.022	1.017	1.016	1.012

Unlimited

Accident Yr	300	312	324	336	348	360	372	Ultimate
1981	203,840	203,356	203,487	204,442	204,478	205,052	205,317	205,317
1982	225,510	225,364	225,963	224,367	224,835	224,889		225,180
1983	239,603	240,882	240,967	241,369	241,193			241,858
1984	270,385	271,243	271,733	272,839				273,725
1985	253,958	254,571	255,902					256,970
1986	291,814	293,358						295,231
1987	348,350							351,439
1988								373,238
1989								457,003
1990								592,422
1991								611,742
1992								575,824
1993								546,604
1994								440,691
1995								418,541
1996								408,717
1997								405,446
1998								395,875
1999								447,534
2000								552,312
2001								561,167
2002								652,198
2003								712,925
2004								774,374
2005								765,563
2006								705,812
2007								720,347
2008								735,269
2009								678,074
2010								654,340
2011								558,756

Age-to-Age Factors

Accident Yr	300 - 312	312 - 324	324 - 336	336 - 348	348 - 360	360 - 372	To Ult
1981	0.998	1.001	1.005	1.000	1.003	1.001	1.000
1982	0.999	1.003	0.993	1.002	1.000		
1983	1.005	1.000	1.002	0.999			
1984	1.003	1.002	1.004				
1985	1.002	1.005					
1986	1.005						
1987							
1988							
1989							
1990							
1991							
1992							
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2005							
2006							
2007							
2008							
2009							
2010							
2011							

Averages

	300 - 312	312 - 324	324 - 336	336 - 348	348 - 360	360 - 372	To Ult
NCCI Countrywide							
Volume Wtd All	1.002	1.002	1.001	1.000	1.001	1.001	1.000
Volume Wtd All	1.009	1.006	1.004	1.003	1.003	1.001	1.000

Limited to 500k

Accident Yr	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288
1981	100,483	134,956	151,969	162,027	168,761	172,607	176,875	180,181	182,025	184,314	185,559	187,030	188,348	188,386	188,189	188,923	189,188	189,586	189,530	189,262	189,517	190,297	190,389	191,119
1982	105,756	144,004	164,424	176,988	184,511	189,940	197,973	200,031	201,957	204,031	204,463	205,006	205,999	206,135	206,258	206,773	206,943	207,111	207,110	207,065	207,278	208,133	208,945	209,607
1983	106,050	148,003	174,030	189,931	199,596	209,323	214,338	220,025	222,313	222,440	223,653	224,361	224,093	224,109	224,392	224,731	224,842	224,746	224,675	225,067	225,496	226,129	226,225	226,416
1984	116,374	165,464	197,204	218,269	230,372	237,809	241,601	245,373	244,724	247,381	248,488	248,739	249,306	250,220	250,790	250,849	251,219	251,304	251,532	252,306	253,040	252,900	253,000	253,305
1985	107,052	156,743	189,149	207,061	218,065	225,683	229,396	230,103	232,123	233,090	234,573	235,013	235,228	235,144	235,415	235,658	236,095	236,723	237,339	237,410	237,587	237,482	237,838	238,837
1986	114,452	176,759	207,255	226,087	237,900	245,275	249,335	252,820	253,854	255,359	256,163	257,162	257,675	258,754	258,590	259,256	259,513	260,300	260,804	261,906	262,222	262,785	262,802	263,633
1987	152,426	227,836	261,284	284,065	297,714	307,024	311,977	313,244	316,101	316,406	318,027	319,727	320,376	321,224	322,419	323,838	325,173	327,263	327,994	327,917	327,327	327,970	328,622	328,713
1988	171,114	247,449	286,975	311,346	322,138	331,803	334,850	337,822	338,873	342,059	344,221	345,461	347,151	347,720	349,241	349,626	351,442	351,866	351,808	351,434	351,931	352,740	353,334	353,748
1989	202,169	293,053	343,596	370,101	386,304	394,791	399,617	402,444	405,655	408,401	409,083	410,645	414,414	416,335	419,549	420,757	420,574	421,389	420,972	421,838	422,240	422,366	421,763	
1990	254,078	390,418	455,265	489,131	506,711	521,129	525,149	531,397	535,408	539,294	540,770	543,163	545,756	548,198	551,418	553,363	553,944	554,444	555,739	557,070	556,720	557,104		
1991	267,444	386,947	444,267	480,190	502,884	514,901	523,503	530,997	535,185	539,619	542,520	547,821	551,193	555,340	556,231	557,009	557,913	559,787	561,374	561,577	562,324			
1992	264,342	375,842	429,728	465,971	481,967	494,625	506,624	512,500	515,088	519,479	524,565	527,219	529,580	531,632	532,246	532,837	534,286	536,844	537,453	538,201				
1993	243,657	342,907	393,051	421,341	441,436	456,537	465,537	470,680	475,477	481,903	483,534	487,171	489,022	489,686	490,073	490,551	492,156	492,258	493,683					
1994	198,596	277,041	318,536	341,800	353,961	365,598	372,992	378,331	381,925	386,376	390,757	392,287	392,346	393,899	395,023	397,193	396,856	398,040						
1995	162,351	232,536	270,168	292,073	306,245	314,760	322,909	330,217	335,451	340,916	343,503	345,000	346,802	348,391	349,734	350,475	351,035							
1996	163,065	244,348	284,536	308,261	320,981	332,212	342,837	348,136	352,707	355,432	357,409	360,095	362,766	365,844	366,494	367,867								
1997	163,531	242,590	282,637	305,347	322,579	334,109	341,554	349,555	354,743	357,688	358,719	360,659	363,844	363,392	364,548									
1998	157,814	238,276	277,683	300,376	315,898	325,775	334,488	339,454	342,005	342,910	344,974	348,088	348,147	350,398										
1999	161,241	246,750	297,343	330,183	347,754	360,966	368,221	373,129	377,470	380,718	386,371	387,003	387,708											
2000	187,581	310,162	373,244	406,690	431,555	444,467	450,405	457,948	463,124	470,334	471,548	473,832												
2001	191,548	333,068	394,077	429,877	450,924	461,807	472,930	480,429	486,067	488,457	491,231													
2002	221,741	378,731	454,259	492,125	512,558	528,898	541,630	552,410	556,834	559,837														
2003	248,572	427,534	506,723	546,245	574,755	590,902	604,520	611,238	614,640															
2004	283,417	464,978	536,604	585,867	612,331	635,546	645,499	655,687																
2005	280,924	445,208	522,040	563,225	594,714	614,207	628,355																	
2006	252,859	399,255	467,752	516,286	540,471	562,903																		
2007	261,350	421,622	506,571	547,307	574,351																			
2008	258,099	439,446	514,658	560,623																				
2009	242,843	397,542	474,603																					
2010	240,789	395,168																						
2011	219,814																							

Age-to-Age Factors

Accident Yr	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108	108 - 120	120 - 132	132 - 144	144 - 156	156 - 168	168 - 180	180 - 192	192 - 204	204 - 216	216 - 228	228 - 240	240 - 252	252 - 264	264 - 276	276 - 288	288 - 300
1981	1.343	1.126	1.066	1.042	1.023	1.025	1.019	1.010	1.013	1.007	1.008	1.007	1.000	0.999	1.004	1.001	1.002	1.000	0.999	1.001	1.004	1.000	1.004	1.001
1982	1.362	1.142	1.076	1.043	1.029	1.042	1.010	1.010	1.010	1.002	1.003	1.005	1.001	1.001	1.002	1.001	1.001	1.000	1.000	1.001	1.004	1.004	1.003	1.000
1983	1.396	1.176	1.091	1.051	1.049	1.024	1.027	1.010	1.001	1.005	1.003	0.999	1.000	1.001	1.002	1.000	1.000	1.000	1.002	1.002	1.003	1.000	1.001	0.999
1984	1.422	1.192	1.107	1.055	1.032	1.016	1.016	0.997	1.011	1.004	1.001	1.002	1.004	1.002	1.000	1.001	1.000	1.001	1.003	1.003	0.999	1.000	1.001	1.001
1985	1.464	1.207	1.095	1.053	1.035	1.016	1.003	1.009	1.004	1.006	1.002	1.001	1.000	1.001	1.001	1.002	1.003	1.003	1.000	1.001	1.000	1.002	1.004	1.001
1986	1.544	1.173	1.091	1.052	1.031	1.017	1.014	1.004	1.006	1.003	1.004	1.002	1.004	0.999	1.003	1.001	1.003	1.002	1.004	1.001	1.002	1.000	1.003	1.001
1987	1.495	1.147	1.087	1.048	1.031	1.016	1.004	1.009	1.001	1.005	1.005	1.002	1.003	1.004	1.004	1.004	1.006	1.002	1.000	0.998	1.002	1.002	1.000	1.001
1988	1.446	1.160	1.085	1.035	1.030	1.009	1.009	1.003	1.009	1.006	1.004	1.005	1.002	1.004	1.001	1.005	1.001	1.000	0.999	1.001	1.002	1.002	1.001	
1989	1.450	1.172	1.077	1.044	1.022	1.012	1.007	1.008	1.007	1.002	1.004	1.009	1.005	1.008	1.003	1.000	1.002	0.999	1.002	1.001	1.000	0.999		
1990	1.537	1.166	1.074	1.036	1.028	1.008	1.012	1.008	1.007	1.003	1.004	1.005	1.004	1.006	1.004	1.001	1.001	1.002	1.002	0.999	1.001			
1991	1.447	1.148	1.081	1.047	1.024	1.017	1.014	1.008	1.008	1.005	1.010	1.006	1.008	1.002	1.001	1.002	1.003	1.003	1.000	1.001				
1992	1.422	1.143	1.084	1.034	1.026	1.024	1.012	1.005	1.009	1.010	1.005	1.004	1.004	1.001	1.001	1.003	1.005	1.001	1.001					
1993	1.407	1.146	1.072	1.048	1.034	1.020	1.011	1.010	1.014	1.003	1.008	1.004	1.001	1.001	1.001	1.003	1.000	1.003						
1994	1.395	1.150	1.073	1.036	1.033	1.020	1.014	1.009	1.012	1.011	1.004	1.000	1.004	1.003	1.005	0.999	1.003							
1995	1.432	1.162	1.081	1.049	1.028	1.026	1.023	1.016	1.016	1.008	1.004	1.005	1.005	1.004	1.002	1.002								
1996	1.498	1.164	1.083	1.041	1.035	1.032	1.015	1.013	1.008	1.006	1.008	1.007	1.008	1.002	1.004									
1997	1.483	1.165	1.080	1.056	1.036	1.022	1.023	1.015	1.008	1.003	1.005	1.009	0.999	1.003										
1998	1.510	1.165	1.082	1.052	1.031	1.027	1.015	1.008	1.003	1.006	1.009	1.000	1.006											
1999	1.530	1.205	1.110	1.053	1.038	1.020	1.013	1.012	1.009	1.015	1.002	1.002												
2000	1.653	1.203	1.090	1.061	1.030	1.013	1.017	1.011	1.016	1.003	1.005													
2001	1.739	1.183	1.091	1.049	1.024	1.024	1.016	1.012	1.005	1.006														
2002	1.708	1.199	1.083	1.042	1.032	1.024	1.020	1.008	1.005															
2003	1.720	1.185	1.078	1.052	1.028	1.023	1.011	1.006																
2004	1.641	1.154	1.092	1.045	1.038	1.016	1.016																	
2005	1.585	1.173	1.079	1.056	1.033	1.023																		
2006	1.579	1.172	1.104	1.047	1.042																			
2007	1.613	1.201	1.080	1.049																				
2008	1.703	1.171	1.089																					
2009	1.637	1.194																						
2010	1.641																							
2011																								

Averages

	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108	108 - 120	120 - 132	132 - 144	144 - 156	156 - 168	168 - 180	180 - 192	192 - 204	204 - 216	216 - 228	228 - 240	240 - 252	252 - 264	264 - 276	276 - 288	288 - 300
Volume Wtd All	1.545	1.171	1.085	1.047	1.031	1.020	1.014	1.009	1.008	1.006	1.005	1.004	1.004	1.003	1.002	1.002	1.002	1.001	1.001	1.001	1.001	1.001	1.002	1.001
Volume Wtd All	2.328	1.507	1.286	1.186	1.132	1.098	1.076	1.061	1.052	1.044	1.038	1.032	1.028	1.024	1.022	1.019	1.017	1.015	1.014	1.013	1.012	1.010	1.009	1.007

Limited to 500k

Accident Yr	300	312	324	336	348	360	372	Ultimate	ELF
1981	191,327	191,347	191,410	191,662	191,871	191,723	191,725	191,725	0.066
1982	209,516	209,310	209,630	209,708	209,720	209,842		209,844	0.068
1983	226,101	226,890	227,458	227,846	228,636			228,624	0.055
1984	253,587	254,494	254,794	255,486				255,883	0.065
1985	239,177	239,313	239,995					240,751	0.063
1986	263,838	264,650						265,941	0.099
1987	329,125							331,320	0.057
1988								356,323	0.045
1989								425,704	0.068
1990								562,789	0.050
1991								568,914	0.070
1992								544,937	0.054
1993								500,434	0.084
1994								404,042	0.083
1995								357,149	0.147
1996								374,928	0.083
1997								372,436	0.081
1998								358,940	0.093
1999								398,578	0.109
2000								489,150	0.114
2001								509,698	0.092
2002								584,203	0.104
2003								646,718	0.093
2004								695,912	0.101
2005								676,308	0.117
2006								617,992	0.124
2007								650,412	0.097
2008								664,753	0.096
2009								610,523	0.100
2010								595,514	0.090
2011								511,645	0.084

Age-to-Age Factors

Accident Yr	300 - 312	312 - 324	324 - 336	336 - 348	348 - 360	360 - 372	To Ult
1981	1.000	1.000	1.001	1.001	0.999	1.000	
1982	0.999	1.002	1.000	1.000	1.001		
1983	1.003	1.003	1.002	1.003			
1984	1.004	1.001	1.003				
1985	1.001	1.003					
1986	1.003						
1987							
1988							
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2007							
2008							
2009							
2010							
2011							

Averages

	300 - 312	312 - 324	324 - 336	336 - 348	348 - 360	360 - 372	To Ult
Volume Wtd All	1.002	1.002	1.002	1.002	1.000	1.000	1.000
Volume Wtd All	1.007	1.005	1.003	1.002	1.000	1.000	1.000

Limited to 1000k

Accident Yr	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288
1981	102,640	137,592	154,978	164,448	171,786	175,253	178,671	181,667	184,082	187,112	188,379	190,006	191,836	191,822	191,632	192,600	193,089	193,669	193,677	193,787	194,092	195,158	195,508	196,694
1982	108,489	147,230	167,826	180,970	188,700	194,886	203,886	205,921	207,211	210,079	210,386	211,277	212,927	213,211	213,281	214,038	213,726	214,380	214,183	214,222	214,504	215,223	216,267	216,613
1983	106,889	149,867	176,501	192,774	202,614	212,134	217,503	223,632	226,867	227,251	228,333	229,736	229,861	229,695	230,664	231,384	231,824	232,332	231,944	232,463	233,025	234,168	234,528	234,177
1984	117,804	167,741	200,483	222,408	234,604	242,483	247,314	251,524	250,750	254,424	256,272	256,368	257,094	258,701	259,400	259,698	259,760	260,111	259,845	261,005	262,058	262,084	262,222	262,574
1985	108,118	158,806	191,479	210,318	221,207	230,450	234,709	234,843	238,444	238,588	239,888	241,256	241,453	240,908	241,402	241,854	242,519	243,495	244,377	244,787	245,103	245,071	245,483	246,992
1986	115,312	179,101	210,039	229,789	242,755	251,508	255,107	259,868	261,073	263,165	264,370	266,897	267,387	269,903	268,946	269,699	269,935	270,920	271,459	272,839	273,326	274,417	274,694	276,147
1987	153,071	229,809	265,403	288,714	303,083	313,386	318,916	320,773	323,933	323,788	325,703	328,054	328,712	329,931	331,954	334,364	336,135	339,509	340,557	340,387	339,966	341,255	341,956	342,322
1988	172,285	250,329	291,201	316,810	326,992	339,034	341,737	345,491	346,211	350,056	352,832	354,777	357,077	357,541	359,698	359,704	361,925	362,470	362,641	362,196	362,843	364,078	364,923	365,475
1989	205,070	296,584	348,143	375,466	393,343	402,773	408,201	410,652	414,899	416,216	416,528	418,060	423,129	425,431	429,160	430,691	430,817	431,879	431,965	432,826	434,090	434,214	433,218	
1990	258,099	395,125	461,272	496,594	515,335	530,041	534,800	541,208	545,445	549,466	550,694	553,286	556,442	558,949	563,259	565,693	566,947	568,675	571,194	573,079	572,419	573,151		
1991	272,256	392,653	450,684	489,310	513,641	524,828	532,447	540,290	545,890	552,194	556,256	562,831	566,388	573,109	575,198	576,157	577,557	580,868	583,095	583,517	584,172			
1992	267,109	379,514	433,677	470,549	487,858	501,490	514,740	521,979	524,357	530,675	537,403	540,104	543,736	546,854	547,509	548,221	551,248	555,922	556,771	557,741				
1993	248,825	350,528	402,007	430,971	451,008	467,779	477,243	482,312	488,679	495,928	497,956	502,129	505,586	506,815	507,888	509,185	511,592	511,734	514,434					
1994	203,252	282,904	323,971	348,442	359,893	372,048	380,416	386,565	390,456	396,179	403,838	406,218	406,137	408,444	409,954	412,657	412,608	415,009						
1995	166,635	238,120	276,730	298,749	312,707	321,102	330,764	338,618	345,250	353,216	355,804	357,883	361,289	365,019	367,674	368,974	370,240							
1996	164,572	247,277	288,690	312,847	325,988	337,591	349,357	354,920	360,528	364,162	367,846	371,960	377,295	381,735	383,045	385,757								
1997	165,013	246,269	286,469	309,655	328,281	340,547	348,444	358,696	364,932	369,173	371,127	374,810	379,110	378,862	381,371									
1998	159,920	242,264	282,545	305,350	321,863	333,434	342,643	348,377	352,987	354,252	356,890	361,645	361,926	365,311										
1999	163,200	249,407	300,628	333,573	353,553	368,480	377,893	384,832	392,837	399,236	407,979	410,088	411,068											
2000	190,961	315,991	380,206	416,422	441,616	457,329	465,389	474,134	484,678	494,961	497,637	500,804												
2001	192,674	336,615	400,244	437,170	458,923	470,592	484,591	494,702	504,645	509,758	514,233													
2002	223,946	382,870	460,340	501,359	523,480	541,461	559,748	576,621	581,722	586,697														
2003	249,448	430,618	511,358	555,402	587,784	606,541	626,538	634,355	639,746															
2004	287,421	471,359	544,407	596,895	625,177	655,340	668,587	681,559																
2005	284,053	450,965	532,285	576,256	611,879	634,333	652,997																	
2006	259,204	407,272	479,310	531,283	558,470	584,658																		
2007	265,317	429,055	518,971	563,493	593,976																			
2008	260,688	447,775	525,973	575,568																				
2009	246,970	403,817	483,574																					
2010	243,380	400,930																						
2011	222,158																							

Age-to-Age Factors

Accident Yr	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108	108 - 120	120 - 132	132 - 144	144 - 156	156 - 168	168 - 180	180 - 192	192 - 204	204 - 216	216 - 228	228 - 240	240 - 252	252 - 264	264 - 276	276 - 288	288 - 300
1981	1.341	1.126	1.061	1.045	1.020	1.019	1.017	1.013	1.016	1.007	1.009	1.010	1.000	0.999	1.005	1.003	1.003	1.000	1.001	1.002	1.005	1.002	1.006	1.002
1982	1.357	1.140	1.078	1.043	1.033	1.046	1.010	1.006	1.014	1.001	1.004	1.008	1.001	1.000	1.004	0.999	1.003	0.999	1.000	1.001	1.003	1.005	1.002	1.000
1983	1.402	1.178	1.092	1.051	1.047	1.025	1.028	1.014	1.002	1.005	1.006	1.001	0.999	1.004	1.003	1.002	1.002	0.998	1.002	1.002	1.005	1.002	0.999	1.000
1984	1.424	1.195	1.109	1.055	1.034	1.020	1.017	0.997	1.015	1.007	1.000	1.003	1.006	1.003	1.001	1.000	1.001	0.999	1.004	1.004	1.000	1.001	1.001	1.004
1985	1.469	1.206	1.098	1.052	1.042	1.018	1.001	1.015	1.001	1.005	1.006	1.001	0.998	1.002	1.002	1.003	1.004	1.004	1.002	1.001	1.000	1.002	1.006	1.001
1986	1.553	1.173	1.094	1.056	1.036	1.014	1.019	1.005	1.008	1.005	1.010	1.002	1.009	0.996	1.003	1.001	1.004	1.002	1.005	1.002	1.004	1.001	1.005	1.002
1987	1.501	1.155	1.088	1.050	1.034	1.018	1.006	1.010	1.000	1.006	1.007	1.002	1.004	1.006	1.007	1.005	1.010	1.003	0.999	0.999	1.004	1.002	1.001	1.001
1988	1.453	1.163	1.088	1.032	1.037	1.008	1.011	1.002	1.011	1.008	1.006	1.006	1.001	1.006	1.000	1.006	1.002	1.000	0.999	1.002	1.003	1.002	1.002	
1989	1.446	1.174	1.078	1.048	1.024	1.013	1.006	1.010	1.003	1.001	1.004	1.012	1.005	1.009	1.004	1.000	1.002	1.000	1.002	1.003	1.000	0.998		
1990	1.531	1.167	1.077	1.038	1.029	1.009	1.012	1.008	1.007	1.002	1.005	1.006	1.005	1.008	1.004	1.002	1.003	1.004	1.003	0.999	1.001			
1991	1.442	1.148	1.086	1.050	1.022	1.015	1.015	1.010	1.012	1.007	1.012	1.006	1.012	1.004	1.002	1.002	1.006	1.004	1.001	1.001				
1992	1.421	1.143	1.085	1.037	1.028	1.026	1.014	1.005	1.012	1.013	1.005	1.007	1.006	1.001	1.001	1.006	1.008	1.002	1.002					
1993	1.409	1.147	1.072	1.046	1.037	1.020	1.011	1.013	1.015	1.004	1.008	1.007	1.002	1.002	1.003	1.005	1.000	1.005						
1994	1.392	1.145	1.076	1.033	1.034	1.022	1.016	1.010	1.015	1.019	1.006	1.000	1.006	1.004	1.007	1.000	1.006							
1995	1.429	1.162	1.080	1.047	1.027	1.030	1.024	1.020	1.023	1.007	1.006	1.010	1.010	1.007	1.004	1.003								
1996	1.503	1.167	1.084	1.042	1.036	1.035	1.016	1.016	1.010	1.010	1.011	1.014	1.012	1.003	1.007									
1997	1.492	1.163	1.081	1.060	1.037	1.023	1.029	1.017	1.012	1.005	1.010	1.011	0.999	1.007										
1998	1.515	1.166	1.081	1.054	1.036	1.028	1.017	1.013	1.004	1.007	1.013	1.001	1.009											
1999	1.528	1.205	1.110	1.060	1.042	1.026	1.018	1.021	1.016	1.022	1.005	1.002												
2000	1.655	1.203	1.095	1.061	1.036	1.018	1.019	1.022	1.021	1.005	1.006													
2001	1.747	1.189	1.092	1.050	1.025	1.030	1.021	1.020	1.010	1.009														
2002	1.710	1.202	1.089	1.044	1.034	1.034	1.030	1.009	1.009															
2003	1.726	1.187	1.086	1.058	1.032	1.033	1.012	1.008																
2004	1.640	1.155	1.096	1.047	1.048	1.020	1.019																	
2005	1.588	1.180	1.083	1.062	1.037	1.029																		
2006	1.571	1.177	1.108	1.051	1.047																			
2007	1.617	1.210	1.086	1.054																				
2008	1.718	1.175	1.094																					
2009	1.635	1.198																						
2010	1.647																							
2011																								

Averages

	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108	108 - 120	120 - 132	132 - 144	144 - 156	156 - 168	168 - 180	180 - 192	192 - 204	204 - 216	216 - 228	228 - 240	240 - 252	252 - 264	264 - 276	276 - 288	288 - 300
Volume Wtd All	1.546	1.174	1.088	1.049	1.034	1.023	1.016	1.012	1.011	1.008	1.007	1.006	1.005	1.004	1.003	1.003	1.004	1.002	1.002	1.001	1.002	1.001	1.003	1.002
Volume Wtd All	2.423	1.567	1.335	1.227	1.170	1.131	1.105	1.087	1.075	1.063	1.055	1.048	1.041	1.036	1.032	1.028	1.025	1.021	1.019	1.017	1.016	1.014	1.013	1.010

Limited to 1000k

Accident Yr	300	312	324	336	348	360	372	Ultimate	ELF
1981	197,151	197,203	197,163	197,622	197,935	198,074	198,053	198,053	0.035
1982	216,610	216,354	216,664	216,827	217,123	217,264		217,241	0.035
1983	234,169	235,154	235,405	235,883	236,535			236,670	0.021
1984	263,542	264,496	264,917	265,797				266,464	0.027
1985	247,311	247,823	248,679					249,842	0.028
1986	276,779	277,656						279,387	0.054
1987	342,700							345,587	0.017
1988								369,124	0.011
1989								438,652	0.040
1990								581,049	0.019
1991								593,619	0.030
1992								567,447	0.015
1993								524,263	0.041
1994								423,842	0.038
1995								379,680	0.093
1996								396,669	0.029
1997								393,494	0.029
1998								378,440	0.044
1999								428,102	0.043
2000								524,644	0.050
2001								542,509	0.033
2002								623,733	0.044
2003								687,531	0.036
2004								741,024	0.043
2005								721,593	0.057
2006								661,056	0.063
2007								694,748	0.036
2008								706,393	0.039
2009								645,570	0.048
2010								628,234	0.040
2011								538,185	0.037

Age-to-Age Factors

Accident Yr	300 - 312	312 - 324	324 - 336	336 - 348	348 - 360	360 - 372	To Ult
1981	1.000	1.000	1.002	1.002	1.001	1.000	
1982	0.999	1.001	1.001	1.001	1.001		
1983	1.004	1.001	1.002	1.003			
1984	1.004	1.002	1.003				
1985	1.002	1.003					
1986	1.003						
1987							
1988							
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2007							
2008							
2009							
2010							
2011							

Averages

	300 - 312	312 - 324	324 - 336	336 - 348	348 - 360	360 - 372	To Ult
Volume Wtd All	1.002	1.002	1.002	1.002	1.001	1.000	1.000
Volume Wtd All	1.008	1.006	1.005	1.003	1.001	1.000	1.000

Excess of 500K

Accident Yr	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288
1981	3,264	3,937	3,896	2,490	3,181	2,887	2,125	1,815	2,465	3,556	3,729	4,779	6,030	6,649	6,773	7,041	7,385	7,722	7,917	9,414	9,630	11,183	12,311	12,166
1982	3,984	4,605	4,905	5,054	5,589	7,057	8,298	9,397	11,109	10,001	9,841	10,360	11,226	11,389	11,980	12,591	12,125	14,235	14,196	14,468	16,915	16,721	16,288	15,998
1983	1,208	2,318	3,433	5,281	5,732	5,468	5,819	8,704	9,442	9,762	6,817	7,495	7,980	7,810	9,838	10,977	11,223	11,879	11,536	11,337	12,632	13,118	13,695	13,151
1984	1,445	3,299	4,385	6,603	6,990	7,508	8,834	8,984	8,928	10,038	11,578	11,586	11,774	12,767	13,005	13,374	13,328	14,566	14,194	14,778	15,328	15,585	15,563	16,118
1985	1,514	2,511	2,851	3,546	3,433	5,534	6,129	5,652	8,104	6,580	6,417	7,331	8,445	9,223	10,026	9,160	9,376	9,799	10,103	10,479	10,668	11,248	11,113	14,645
1986	874	3,273	4,581	5,909	7,869	10,619	10,177	13,130	13,296	13,720	14,156	17,482	16,103	20,385	19,833	18,988	19,104	21,389	21,501	22,045	22,483	23,972	23,789	25,181
1987	660	2,206	5,521	6,075	6,565	7,983	8,231	9,138	9,290	8,860	9,230	9,965	9,952	10,546	11,359	12,491	13,052	14,318	14,559	14,873	16,048	18,630	18,321	18,825
1988	1,186	3,071	7,419	8,763	5,445	8,341	7,890	8,959	8,293	9,057	9,513	11,117	12,030	11,948	12,674	12,343	12,938	13,122	13,209	13,399	13,746	14,472	14,900	15,049
1989	4,380	8,682	10,773	10,761	13,169	14,455	14,772	14,323	20,057	18,533	18,220	18,204	20,528	20,885	23,730	23,506	23,863	24,327	26,176	26,612	27,254	28,313	27,875	
1990	5,641	8,788	9,283	10,881	12,050	12,424	13,341	14,485	14,678	15,061	14,511	15,666	16,473	18,200	19,598	20,211	21,082	22,309	23,423	24,564	24,472	25,168		
1991	6,383	7,587	9,975	14,062	15,887	15,440	15,448	16,298	19,388	20,859	22,720	22,970	23,449	26,606	27,904	28,200	28,210	31,566	34,207	35,912	36,360			
1992	3,341	4,564	4,988	6,498	7,241	8,504	10,165	12,340	11,686	14,952	17,065	17,057	17,393	18,552	18,451	18,384	20,361	23,165	23,320	23,777				
1993	8,183	11,903	15,421	15,996	15,330	24,886	26,559	23,874	24,729	26,982	29,814	30,403	31,842	32,437	33,878	35,741	37,089	36,538	38,303					
1994	6,685	7,887	7,448	10,104	9,726	10,058	10,388	12,149	12,328	17,965	22,419	23,603	21,591	24,102	25,932	27,144	27,212	29,623						
1995	6,681	13,864	23,241	23,049	23,667	18,969	24,383	24,810	27,987	31,378	31,343	31,749	36,936	48,828	52,040	53,761	52,850							
1996	2,295	5,930	7,897	8,443	9,147	9,588	10,455	10,473	10,787	11,401	14,131	15,902	21,358	22,890	22,313	25,519								
1997	2,222	11,709	12,708	13,456	14,518	12,768	11,127	13,549	15,337	16,626	17,815	20,213	21,574	22,636	24,155									
1998	2,800	9,183	10,151	14,724	17,451	21,745	20,229	18,106	20,033	21,861	22,700	24,267	24,699	27,123										
1999	5,949	8,789	10,527	9,692	12,325	14,286	16,996	19,143	23,995	27,528	31,411	34,989	35,513											
2000	7,418	17,883	13,052	18,605	21,295	24,892	26,903	28,972	38,935	43,479	44,310	44,962												
2001	1,852	5,915	10,016	10,821	11,455	11,895	17,693	19,810	26,603	28,508	31,785													
2002	2,473	7,786	12,568	18,470	19,677	22,695	29,018	36,882	40,051	43,011														
2003	892	3,981	6,682	11,296	18,437	21,841	30,935	33,051	36,701															
2004	5,364	8,063	16,240	20,681	24,987	33,933	38,793	42,081																
2005	3,521	7,733	17,999	29,760	38,563	43,072	50,050																	
2006	8,287	12,160	27,034	35,717	41,656	47,857																		
2007	4,123	11,307	20,631	25,158	27,588																			
2008	2,765	11,024	20,335	24,509																				
2009	7,110	17,304	20,666																					
2010	3,416	10,534																						
2011	3,069																							

Age-to-Age Factors

Accident Yr	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108	108 - 120	120 - 132	132 - 144	144 - 156	156 - 168	168 - 180	180 - 192	192 - 204	204 - 216	216 - 228	228 - 240	240 - 252	252 - 264	264 - 276	276 - 288	288 - 300
1981	1.206	0.990	0.639	1.277	0.907	0.736	0.854	1.359	1.442	1.049	1.282	1.262	1.103	1.019	1.040	1.049	1.046	1.025	1.189	1.023	1.161	1.101	0.988	1.028
1982	1.156	1.065	1.030	1.106	1.263	1.176	1.133	1.182	0.900	0.984	1.053	1.084	1.014	1.052	1.051	0.963	1.174	0.997	1.019	1.169	0.989	0.974	0.982	1.000
1983	1.919	1.481	1.538	1.085	0.954	1.064	1.496	1.085	1.034	0.698	1.099	1.065	0.979	1.260	1.116	1.022	1.058	0.971	0.983	1.114	1.038	1.044	0.960	1.027
1984	2.283	1.329	1.506	1.059	1.074	1.177	1.017	0.994	1.124	1.153	1.001	1.016	1.084	1.019	1.028	0.997	1.093	0.974	1.041	1.037	1.017	0.999	1.036	1.042
1985	1.659	1.135	1.244	0.968	1.612	1.108	0.922	1.434	0.812	0.975	1.142	1.152	1.092	1.087	0.914	1.024	1.045	1.031	1.037	1.018	1.054	0.988	1.318	1.009
1986	3.744	1.399	1.290	1.332	1.349	0.958	1.290	1.013	1.032	1.032	1.235	0.921	1.266	0.973	0.957	1.006	1.120	1.005	1.025	1.020	1.066	0.992	1.059	1.111
1987	3.343	2.503	1.100	1.081	1.216	1.031	1.110	1.017	0.954	1.042	1.080	0.999	1.060	1.077	1.100	1.045	1.097	1.017	1.022	1.079	1.161	0.983	1.027	1.021
1988	2.590	2.416	1.181	0.621	1.532	0.946	1.135	0.926	1.092	1.050	1.169	1.082	0.993	1.061	0.974	1.048	1.014	1.007	1.014	1.026	1.053	1.030	1.010	
1989	1.982	1.241	0.999	1.224	1.098	1.022	0.970	1.400	0.924	0.983	0.999	1.128	1.017	1.136	0.991	1.015	1.019	1.076	1.017	1.024	1.039	0.985		
1990	1.558	1.056	1.172	1.107	1.031	1.074	1.086	1.013	1.026	0.964	1.080	1.052	1.105	1.077	1.031	1.043	1.058	1.050	1.049	0.996	1.028			
1991	1.189	1.315	1.410	1.130	0.972	1.001	1.055	1.190	1.076	1.089	1.011	1.021	1.135	1.049	1.011	1.000	1.119	1.084	1.050	1.012				
1992	1.366	1.093	1.303	1.114	1.174	1.195	1.214	0.947	1.279	1.141	1.000	1.020	1.067	0.995	0.996	1.108	1.138	1.007	1.020					
1993	1.455	1.296	1.037	0.958	1.623	1.067	0.899	1.036	1.091	1.105	1.020	1.047	1.019	1.044	1.055	1.038	0.985	1.048						
1994	1.180	0.944	1.357	0.963	1.034	1.033	1.169	1.015	1.457	1.248	1.053	0.915	1.116	1.076	1.047	1.003	1.089							
1995	2.075	1.676	0.992	1.027	0.802	1.285	1.018	1.128	1.121	0.999	1.013	1.163	1.322	1.066	1.033	0.983								
1996	2.584	1.332	1.069	1.083	1.048	1.090	1.002	1.030	1.057	1.239	1.125	1.343	1.072	0.975	1.144									
1997	5.270	1.085	1.059	1.079	0.879	0.871	1.218	1.132	1.084	1.072	1.135	1.067	1.049	1.067										
1998	3.279	1.105	1.451	1.185	1.246	0.930	0.895	1.106	1.091	1.038	1.069	1.018	1.098											
1999	1.478	1.198	0.921	1.272	1.159	1.190	1.126	1.253	1.147	1.141	1.114	1.015												
2000	2.411	0.730	1.425	1.145	1.169	1.081	1.077	1.344	1.117	1.019	1.015													
2001	3.195	1.693	1.080	1.059	1.038	1.487	1.120	1.343	1.072	1.115														
2002	3.148	1.614	1.470	1.065	1.153	1.279	1.271	1.086	1.074															
2003	4.464	1.679	1.690	1.632	1.185	1.416	1.068	1.110																
2004	1.503	2.014	1.273	1.208	1.358	1.143	1.085																	
2005	2.196	2.327	1.653	1.296	1.117	1.162																		
2006	1.467	2.223	1.321	1.166	1.149																			
2007	2.742	1.825	1.219	1.097																				
2008	3.986	1.845	1.205																					
2009	2.434	1.194																						
2010	3.084																							
2011																								

Averages

	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108	108 - 120	120 - 132	132 - 144	144 - 156	156 - 168	168 - 180	180 - 192	192 - 204	204 - 216	216 - 228	228 - 240	240 - 252	252 - 264	264 - 276	276 - 288	288 - 300
RAA Tail (Range 2)																								
Volume Wtd All	2.051	1.428	1.238	1.134	1.144	1.127	1.084	1.138	1.085	1.062	1.063	1.059	1.105	1.054	1.032	1.017	1.071	1.031	1.034	1.039	1.055	1.004	1.041	1.041
Volume Wtd All	16.872	8.225	5.758	4.650	4.101	3.586	3.182	2.936	2.580	2.377	2.238	2.106	1.989	1.800	1.707	1.655	1.627	1.519	1.472	1.424	1.371	1.300	1.295	1.244
Method	15.350	5.584	3.269	2.877	2.535	1.835	1.783	1.865	1.804	1.581	1.619	1.405	1.379	1.362	1.367	1.324	1.162	1.237	1.205	1.299	1.178	1.177	1.123	1.124

Excess of 500K

Accident Yr	300	312	324	336	348	360	372
1981	12,513	12,009	12,077	12,779	12,608	13,329	13,592
1982	15,994	16,055	16,333	14,658	15,115	15,047	
1983	13,502	13,992	13,509	13,523	12,558		
1984	16,798	16,749	16,939	17,352			
1985	14,781	15,259	15,908				
1986	27,976	28,709					
1987	19,224						
1988							
1989							
1990							
1991							
1992							
1993							
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Ultimate

Accident Yr	Factors	Difference	No Tail	Method
1981	15,641	13,592	13,592	13,592
1982	17,657	15,335	15,344	15,335
1983	15,083	13,234	13,107	13,234
1984	20,496	17,842	17,811	17,842
1985	18,616	16,219	16,177	16,219
1986	33,914	29,290	29,471	29,290
1987	22,980	20,120	19,969	20,120
1988	18,718	16,915	16,266	16,915
1989	36,090	31,300	31,362	31,300
1990	32,715	29,634	28,429	29,634
1991	49,844	42,828	43,313	42,828
1992	33,855	30,887	29,420	30,887
1993	56,399	46,170	49,010	46,170
1994	44,992	36,648	39,097	36,648
1995	85,971	61,393	74,707	61,393
1996	42,234	33,789	36,700	33,789
1997	41,240	33,010	35,837	33,010
1998	48,815	36,935	42,419	36,935
1999	70,637	48,956	61,382	48,956
2000	94,702	63,162	82,294	63,162
2001	71,133	51,469	61,813	51,469
2002	102,245	67,994	88,848	67,994
2003	94,685	66,207	82,280	66,207
2004	123,531	78,462	107,346	78,462
2005	159,244	89,255	138,379	89,255
2006	171,622	87,820	149,136	87,820
2007	113,138	69,935	98,315	69,935
2008	113,957	70,516	99,026	70,516
2009	118,988	67,551	103,398	67,551
2010	86,643	58,826	75,291	58,826
2011	51,785	47,111	45,000	47,111
	2,007,571	1,392,406	1,744,535	1,392,406
Above Diff	44.18%		25.29%	0.00%

Age-to-Age Factors

Accident Yr	300 - 312	312 - 324	324 - 336	336 - 348	348 - 360	360 - 372	To Ult
1981	0.960	1.006	1.058	0.987	1.057	1.020	
1982	1.004	1.017	0.897	1.031	0.995		
1983	1.036	0.965	1.001	0.929			
1984	0.997	1.011	1.024				
1985	1.032	1.043					
1986	1.026						
1987							
1988							
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2011							

Averages

	300 - 312	312 - 324	324 - 336	336 - 348	348 - 360	360 - 372	To Ult
RAA Tail (Range 2)							1.151
Volume Wtd All	1.012	1.009	0.991	0.983	1.024	1.020	1.151
Volume Wtd All	1.195	1.181	1.170	1.181	1.201	1.173	1.151
Method	1.047	1.020	1.020	1.028	1.054	1.019	1.000

Excess of 1000K

Accident Yr	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288
1981	1,108	1,300	886	69	156	240	329	329	408	758	908	1,803	2,542	3,213	3,330	3,365	3,484	3,639	3,770	4,889	5,055	6,322	7,192	6,591
1982	1,250	1,378	1,502	1,072	1,400	2,111	2,385	3,507	5,854	3,953	3,919	4,089	4,298	4,312	4,956	5,325	5,342	6,967	7,122	7,310	9,690	9,631	8,967	8,992
1983	368	454	962	2,438	2,714	2,656	2,654	5,097	4,887	4,951	2,138	2,120	2,211	2,224	3,566	4,324	4,242	4,292	4,266	3,942	5,103	5,079	5,392	5,390
1984	15	1,021	1,106	2,464	2,758	2,834	3,120	2,833	2,902	2,995	3,793	3,958	3,986	4,286	4,395	4,525	4,787	5,759	5,881	6,078	6,310	6,401	6,341	6,848
1985	447	448	521	289	291	768	817	912	1,783	1,082	1,102	1,088	2,220	3,459	4,040	2,964	2,953	3,027	3,065	3,102	3,152	3,658	3,468	6,490
1986	15	931	1,797	2,206	3,014	4,386	4,405	6,082	6,076	5,915	5,949	7,747	6,391	9,235	9,477	8,544	8,682	10,769	10,846	11,112	11,379	12,341	11,896	12,668
1987	15	233	1,403	1,425	1,195	1,621	1,291	1,609	1,457	1,479	1,554	1,638	1,616	1,838	1,824	1,966	2,089	2,072	1,995	2,403	3,408	5,345	4,987	5,216
1988	15	192	3,194	3,299	592	1,110	1,003	1,290	954	1,060	901	1,801	2,105	2,127	2,217	2,265	2,455	2,518	2,377	2,636	2,835	3,134	3,311	3,323
1989	1,479	5,151	6,226	5,395	6,129	6,473	6,189	6,114	10,813	10,717	10,775	10,789	11,813	11,789	14,118	13,573	13,620	13,837	15,183	15,624	15,405	16,465	16,420	
1990	1,620	4,081	3,276	3,418	3,425	3,513	3,690	4,673	4,641	4,889	4,587	5,542	5,788	7,449	7,757	7,881	8,078	8,079	7,967	8,555	8,772	9,121		
1991	1,571	1,881	3,558	4,942	5,129	5,513	6,504	7,005	8,683	8,284	8,983	7,961	8,255	8,837	8,937	9,052	8,566	10,485	12,486	13,972	14,513			
1992	574	892	1,039	1,920	1,350	1,639	2,049	2,861	2,418	3,755	4,227	4,172	3,238	3,330	3,188	3,000	3,398	4,087	4,002	4,237				
1993	3,015	4,282	6,465	6,366	5,759	13,644	14,852	12,242	11,527	12,957	15,392	15,445	15,278	15,308	16,064	17,108	17,654	17,062	17,551					
1994	2,029	2,024	2,013	3,462	3,794	3,608	2,964	3,915	3,797	8,161	9,338	9,672	7,800	9,557	11,001	11,680	11,460	12,654						
1995	2,397	8,280	16,679	16,373	17,204	12,627	16,528	16,409	18,188	19,078	19,041	18,866	22,449	32,200	34,099	35,261	33,645							
1996	788	3,002	3,743	3,857	4,140	4,210	3,934	3,689	2,966	2,671	3,694	4,037	6,829	6,999	5,763	7,629								
1997	739	8,029	8,875	9,148	8,816	6,330	4,236	4,407	5,147	5,141	5,406	6,062	6,308	7,167	7,331									
1998	695	5,195	5,289	9,750	11,486	14,086	12,074	9,182	9,052	10,519	10,783	10,709	10,919	12,211										
1999	3,989	6,132	7,242	6,302	6,526	6,773	7,324	7,440	8,628	9,009	9,803	11,903	12,154											
2000	4,038	12,054	6,090	8,873	11,234	12,030	11,919	12,786	17,381	18,853	18,222	17,990												
2001	726	2,368	3,850	3,528	3,457	3,109	6,031	5,536	8,024	7,206	8,783													
2002	268	3,648	6,488	9,236	8,755	10,132	10,900	12,671	15,163	16,151														
2003	15	897	2,047	2,139	5,407	6,201	8,916	9,934	11,595															
2004	1,361	1,682	8,437	9,653	12,140	14,139	15,705	16,209																
2005	392	1,976	7,754	16,728	21,398	22,947	25,408																	
2006	1,942	4,143	15,476	20,720	23,657	26,102																		
2007	156	3,874	8,231	8,972	7,962																			
2008	177	2,695	9,020	9,564																				
2009	2,983	11,030	11,695																					
2010	825	4,772																						
2011	725																							

Age-to-Age Factors

Accident Yr	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108	108 - 120	120 - 132	132 - 144	144 - 156	156 - 168	168 - 180	180 - 192	192 - 204	204 - 216	216 - 228	228 - 240	240 - 252	252 - 264	264 - 276	276 - 288	288 - 300
1981	1.174	0.681	0.078	2.262	1.535	1.372	1.000	1.239	1.859	1.198	1.985	1.410	1.264	1.036	1.010	1.036	1.044	1.036	1.297	1.034	1.251	1.138	0.916	1.015
1982	1.102	1.090	0.714	1.306	1.508	1.130	1.471	1.669	0.675	0.991	1.043	1.051	1.003	1.149	1.075	1.003	1.304	1.022	1.026	1.325	0.994	0.931	1.003	0.990
1983	1.234	2.118	2.535	1.113	0.979	0.999	1.920	0.959	1.013	0.432	0.992	1.043	1.006	1.603	1.213	0.981	1.012	0.994	0.924	1.294	0.995	1.062	1.000	1.008
1984	68.047	1.083	2.229	1.119	1.028	1.101	0.908	1.024	1.032	1.267	1.043	1.007	1.075	1.025	1.030	1.058	1.203	1.021	1.033	1.038	1.014	0.991	1.080	0.999
1985	1.001	1.162	0.555	1.005	2.641	1.064	1.117	1.954	0.607	1.019	0.986	2.041	1.558	1.168	0.734	0.996	1.025	1.013	1.012	1.016	1.160	0.948	1.871	1.024
1986	62.077	1.930	1.228	1.366	1.455	1.004	1.381	0.999	0.973	1.006	1.302	0.825	1.445	1.026	0.902	1.016	1.240	1.007	1.025	1.024	1.085	0.964	1.065	1.187
1987	15.529	6.022	1.016	0.839	1.356	0.796	1.246	0.906	1.015	1.051	1.054	0.986	1.138	0.992	1.078	1.063	0.992	0.963	1.204	1.418	1.568	0.933	1.046	1.083
1988	12.773	16.668	1.033	0.179	1.876	0.903	1.287	0.740	1.110	0.850	1.998	1.169	1.010	1.042	1.022	1.084	1.025	0.944	1.109	1.075	1.106	1.056	1.004	
1989	3.482	1.209	0.867	1.136	1.056	0.956	0.988	1.768	0.991	1.005	1.001	1.095	0.998	1.198	0.961	1.003	1.016	1.097	1.029	0.986	1.069	0.997		
1990	2.519	0.803	1.043	1.002	1.026	1.051	1.266	0.993	1.054	0.938	1.208	1.044	1.287	1.041	1.016	1.025	1.000	0.986	1.074	1.025	1.040			
1991	1.197	1.892	1.389	1.038	1.075	1.180	1.077	1.240	0.954	1.084	0.886	1.037	1.071	1.011	1.013	0.946	1.224	1.191	1.119	1.039				
1992	1.553	1.165	1.849	0.703	1.214	1.250	1.397	0.845	1.553	1.126	0.987	0.776	1.029	0.957	0.941	1.133	1.203	0.979	1.059					
1993	1.420	1.510	0.985	0.905	2.369	1.089	0.824	0.942	1.124	1.188	1.003	0.989	1.002	1.049	1.065	1.032	0.967	1.029						
1994	0.998	0.994	1.720	1.096	0.951	0.821	1.321	0.970	2.149	1.144	1.036	0.806	1.225	1.151	1.062	0.981	1.104							
1995	3.454	2.014	0.982	1.051	0.734	1.309	0.993	1.108	1.049	0.998	0.991	1.190	1.434	1.059	1.034	0.954								
1996	3.808	1.247	1.030	1.073	1.017	0.934	0.938	0.804	0.900	1.383	1.093	1.692	1.025	0.823	1.324									
1997	10.860	1.105	1.031	0.964	0.718	0.669	1.040	1.168	0.999	1.052	1.121	1.041	1.136	1.023										
1998	7.478	1.018	1.844	1.178	1.226	0.857	0.761	0.986	1.162	1.025	0.993	1.020	1.118											
1999	1.537	1.181	0.870	1.036	1.038	1.081	1.016	1.160	1.044	1.088	1.214	1.021												
2000	2.985	0.505	1.457	1.266	1.071	0.991	1.073	1.359	1.085	0.967	0.987													
2001	3.264	1.626	0.916	0.980	0.899	1.940	0.918	1.449	0.898	1.219														
2002	13.619	1.779	1.424	0.948	1.157	1.076	1.162	1.197	1.065															
2003	59.804	2.282	1.045	2.528	1.147	1.438	1.114	1.167																
2004	1.236	5.015	1.144	1.258	1.165	1.111	1.032																	
2005	5.037	3.923	2.157	1.279	1.072	1.107																		
2006	2.134	3.735	1.339	1.142	1.103																			
2007	24.830	2.125	1.090	0.887																				
2008	15.214	3.347	1.060																					
2009	3.698	1.060																						
2010	5.785																							
2011																								

Averages

	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108	108 - 120	120 - 132	132 - 144	144 - 156	156 - 168	168 - 180	180 - 192	192 - 204	204 - 216	216 - 228	228 - 240	240 - 252	252 - 264	264 - 276	276 - 288	288 - 300
RAA Tail (Range 3)																								
Volume Wtd All	2.972	1.560	1.213	1.097	1.098	1.077	1.046	1.155	1.059	1.041	1.049	1.053	1.173	1.066	1.028	0.997	1.087	1.042	1.062	1.075	1.090	0.994	1.077	1.058
Volume Wtd All	37.214	12.523	8.028	6.620	6.037	5.497	5.104	4.879	4.223	3.990	3.833	3.654	3.472	2.959	2.777	2.702	2.710	2.493	2.392	2.252	2.094	1.921	1.933	1.795
Method	28.366	5.470	2.779	3.019	3.215	1.715	1.731	2.058	2.190	1.762	2.124	1.538	1.599	1.428	1.630	1.579	1.155	1.331	1.273	1.977	1.249	1.247	1.118	1.238

Excess of 1000K

Accident Yr	300	312	324	336	348	360	372		Ultimate Factors	Ultimate Difference	Ultimate No Tail	Ultimate Method
1981	6,689	6,154	6,324	6,819	6,544	6,978	7,264		12,200	7,264	7,264	7,264
1982	8,900	9,011	9,299	7,540	7,712	7,625			13,331	7,939	7,938	7,939
1983	5,434	5,727	5,562	5,486	4,658				8,341	5,188	4,967	5,188
1984	6,843	6,748	6,815	7,042					12,019	7,261	7,157	7,261
1985	6,647	6,748	7,224						11,839	7,127	7,050	7,127
1986	15,035	15,702							26,362	15,844	15,697	15,844
1987	5,650								9,588	5,852	5,709	5,852
1988									5,964	4,114	3,551	4,114
1989									31,737	18,351	18,897	18,351
1990									17,525	11,374	10,435	11,374
1991									30,391	18,123	18,096	18,123
1992									9,542	8,377	5,681	8,377
1993									41,975	22,341	24,993	22,341
1994									31,544	16,849	18,783	16,849
1995									91,179	38,861	54,291	38,861
1996									20,615	12,049	12,275	12,049
1997									20,360	11,952	12,123	11,952
1998									36,132	17,435	21,514	17,435
1999									42,193	19,432	25,123	19,432
2000									65,735	27,668	39,141	27,668
2001									33,662	18,658	20,044	18,658
2002									64,434	28,465	38,366	28,465
2003									48,967	25,394	29,157	25,394
2004									79,085	33,350	47,090	33,350
2005									129,689	43,970	77,221	43,970
2006									143,490	44,757	85,439	44,757
2007									48,068	25,599	28,621	25,599
2008									63,311	28,875	37,697	28,875
2009									93,885	32,504	55,902	32,504
2010									59,761	26,106	35,584	26,106
2011									26,987	20,570	16,069	20,570
									1,329,909	611,650	791,877	611,650
								Above Diff	117.43%		29.47%	0.00%

Age-to-Age Factors

Accident Yr	300 - 312	312 - 324	324 - 336	336 - 348	348 - 360	360 - 372	To Ult
1981	0.920	1.028	1.078	0.960	1.066	1.041	
1982	1.012	1.032	0.811	1.023	0.989		
1983	1.054	0.971	0.986	0.849			
1984	0.986	1.010	1.033				
1985	1.015	1.070					
1986	1.044						
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Averages

	300 - 312	312 - 324	324 - 336	336 - 348	348 - 360	360 - 372	To Ult
RAA Tail (Range 3)							1.679
Volume Wtd All	1.011	1.024	0.960	0.953	1.024	1.041	1.679
Volume Wtd All	1.697	1.679	1.639	1.707	1.791	1.748	1.679
Method	1.036	1.009	0.987	1.031	1.114	1.041	1.000

Justification for, and Implications of, Regulators Suggesting Particular Reserving Techniques

William J. Collins, ACAS

Abstract

Motivation. Prior to 30th June 2013, Kenya's Insurance Regulatory Authority mandated that minimum IBNR reserves for a particular class of insurance be set as a percentage of a company's calendar year net written premium for that class. While this method may provide a proxy that is easy to use by those who do not have actuarial training, it is uncertain how accurate the mandated IBNR percentage is for individual insurers. This could lead to a situation of reserves that are acceptable in the eyes of the regulator but actually deficient. Some stakeholders may not be aware of the consequences of such a method. Most importantly, any suggestion to use one particular reserving method may lead to inaccurate reserves for many insurers.

Method. This paper uses several actuarial methods to calculate the IBNR of one Kenyan general insurance company's private motor insurance line. These estimates are compared to the minimum IBNR required of the regulator.

Results. The minimum IBNR mandated by the regulator understates IBNR of the company according to the alternative methods used.

Conclusions. In Kenya, an industry-wide study could be undertaken to make sure that mandated IBNR percentages do not lead to inaccurate reserves on average. Also, the regulatory authority could determine a more reliable proxy. Individual companies should be encouraged, as they are under new Kenyan guidelines, to calculate IBNR by other methods. Both regulators and insurers should know that using a formulaic proxy to set IBNR is not a fail-safe method. Furthermore, insurers should consider multiple methods when calculating reserves.

Keywords. Reserving Methods, IBNR, Solvency, Data Quality, Data Collection and Statistical Reporting

1. INTRODUCTION

Prior to 30th June 2013, Kenya's Insurance Regulatory Authority (IRA) mandated minimum IBNR reserves. Each company was required to set IBNR for a particular class of business as a percentage[1] of the company's calendar year net written premium[2] for that class. While this method may provide a proxy that is far easier to use than more sophisticated actuarial reserving methodologies (in an environment that often does not have the technical capacity to explore such other methods), it is uncertain how accurate the mandated IBNR percentages are for the industry as a whole. Furthermore, it is uncertain to what extent the experience of individual companies varies from the industry average. This could lead to a situation of reserves that are acceptable in the eyes of

[1] The percentages set by the Kenyan regulator are detailed in Appendix A. New percentages, as a part of new reserving guidelines (excerpted in Appendix C) that became effective 30th June 2013, are detailed in Appendix D. The majority of this paper will discuss issues that existed prior to those new guidelines becoming effective, as well as general considerations regarding regulators suggesting particular reserving methods. The new guidelines set by IRA address some of the concerns discussed in this paper. Where relevant, references to the new guidelines will be made.

[2] The proxy varies in other markets. Some markets use net earned premium, for example.

the regulator but actually deficient. If such a method is to be used, there are also additional concerns: How does the regulator set such percentages? How *should* the regulator set the percentages? Clearly, a mere average IBNR could result in deficient reserves for some and redundant reserves for others. Also, what is to keep the insurer from purposely setting case outstanding reserves lower than is reasonable in order to compensate for having to set IBNR at a higher level than the company desires? In mandating minimum IBNR, the regulator does not necessarily ensure adequate reserves overall. Insurers can game the system by setting case outstanding reserves lower.

This paper uses several actuarial methods to calculate the IBNR of a small Kenyan company's private motor insurance line. These estimates are compared to the minimum IBNR required of the regulator. The analysis shows that the minimum IBNR mandated by the regulator is only 36% to 83% of what it should be for the example company according to the alternative methods used. These results are then considered in the context of the Kenyan environment, which largely lacks actuarial expertise. It is acknowledged that this simplified method might have advantages in an environment with few qualified actuaries. However, precautions must be taken.

For instance, an industry-wide study could be undertaken to make sure that mandated IBNR percentages are not deficient on average.[3] Also, the regulatory authority could determine a more reliable proxy, which would likely require collecting more detailed claims data from insurers. Individual companies could be encouraged to calculate IBNR by other methods. Both regulators and insurers should know that using a formulaic proxy to set IBNR is not a fail-safe method. Furthermore, insurers should consider multiple methods when calculating reserves.

1.1 Research Context

"Actuarial Activity in General Insurance in the Northern Countries of Europe," from 1958, by L. Wilhelmsen, discusses issues that are still relevant in many developing insurance markets. In Kenya, regulators are just starting to require the use of actuaries in non-life (general)[4] insurance.

Most actuarial literature focuses on what should be done by actuaries. The literature does not address approaches that should be taken in insurance markets that lack actuaries.

The main science discussed here is that of the reserving methods used in such environments. The

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[3] Of course, there could be issues here. By the time the analysis is completed for long-tailed lines of business, companies may have been underestimating IBNR for multiple years, which could have led to a cycle of causing premiums to be set too low, thus booking IBNR even lower.

[4] This paper will use the terms "non-life" and "general" interchangeably. In Kenya, general insurance includes everything other than life insurance, annuities, and pensions.

implications that the reserving methods have on solvency are also addressed.

1.2 Objective

Current reserving literature does not address how reserves are, or should be, set in developing markets that often lack qualified actuaries. This paper attempts to bring awareness to the fact that this lack of technical expertise can present issues to insurers, regulators, and the public. Furthermore, it makes broad suggestions on precautions that should be taken by regulators and insurers in such environments. Specifically, using IBNR as an example, it addresses the justifications for, and implications of, the regulator offering simplified formulaic proxies in lieu of requiring more detailed, company-specific actuarial analysis. This paper also discusses the implications of using any prescribed method for setting reserves.

1.3 Outline

The remainder of the paper proceeds as follows. Section 2 will discuss the methods used to test the regulator's minimum IBNR proxy. Section 3 discusses the results of the analysis and places them in the context of the market, providing practical advice for insurers and regulators in such markets. Finally, Section 4 concludes the paper by reiterating the key messages learned regarding regulators prescribing specific reserving techniques.

2. BACKGROUND AND METHODS

An anecdotal example using one Kenyan general insurance company's internal motor insurance claims data is used to test the accuracy of the minimum IBNR proxy. The results of several reserving methods are compared to the results of the proxy. The pros and cons of the proxy method are analyzed. Discussions with actuarial analysts of the regulatory authority were undertaken to understand the history of the proxy method, as well as to understand the recently updated guidelines for setting reserves in Kenya.

2.1 Reserving Analysis Using Company Internal Data

As an anecdotal example,[5] data from Company ABCD was used to test the appropriateness of the minimum proxy that the regulator mandated for motor insurance. The IBNR shown in the year-end financial statement is to be set as 5% of net written premium for the year. The results of several

[5] Aged accident year data was only available for the author's employer. Aggregated industry data is not collected in triangular accident year format. This has implications that will be discussed in more depth later in the paper.

reserving techniques[6] will be shown in comparison to the 5% proxy. The comparison is made, for each method, by first calculating the IBNR for each accident year. Then, the total IBNR for all accident years is summed and divided by the sum of net written premium for the corresponding calendar year.

2.2 Discussions with Actuarial Department of Insurance Regulatory Authority

In order to better understand the justifications for the minimum proxy method, discussions with actuarial analysts of the regulatory authority were undertaken regarding the history of the method. The proxies were set in 1984 and had not been updated until new guidelines were issued in June of 2013. It is uncertain what method was used to set the original percentages. The actuarial department knows that the percentages may no longer be accurate, and they are interested in updating the percentages or exploring alternative methods. New guidelines effective 30th June 2013 include an optional reserving method that uses a similar proxy, with some refinements and potential improvements. Still, these percentages are not based on typical reserving analysis that would require the regulatory authority to collect aged claims data.

3. RESULTS AND DISCUSSION

In our example, when compared to other reserving methods, the 5% assumption underestimates IBNR by anywhere from 17% to 64%. This misestimation of IBNR could have negative consequences for regulators, insurers, and other stakeholders of the insurance industry. There may, however, be justifications for using such a technique, and there are precautions that can be taken to mitigate the inaccuracies in calculating IBNR and to ultimately avoid the possibility of insolvency.

3.1 Summary of Reserving Method Results

The results of several common reserving techniques using ABCD's internal data are summarized in Appendix B. The results show IBNR ranging from 6% to 14% of net written premium.

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[6] A summary of results by technique is listed in Appendix B. Considering that these are common reserving techniques that have been adequately explored in previous research, they will not be further discussed here. Also, it should be noted that the author did not perform extremely detailed analysis within each technique. For instance, there was no attention given to using tail factors, adjusting for changes in case strengthening or weakening, or adjusting for changes in claim payment rates. Rather, several methods are used in a simple, straightforward way in order to illustrate a range of estimates that might be reasonable utilizing different techniques. Not adjusting the data at all might indeed misstate IBNR; however, using the methods in this fashion might point to additional problems arising from the regulator's new guidelines, which call for use of techniques such as those used here, while some insurers may not currently have the capacity to adequately use those methods.

3.2 Accuracy of Minimum IBNR Technique

In this example, the 5% minimum assumption mandated by the regulator is not appropriate. In fact, it may be drastically understating IBNR. Clearly a one-size-fits-all approach will not give an accurate estimate of every company's IBNR reserves. Differences in claims processing, management style, the underlying book of business, growth, and IBNER (just to name a few), will cause different companies to require different percentages of net written premium as their IBNR reserves. Still, the magnitude of the differences in percentage of net written premium in this example are significant and might be beyond an acceptable margin of error.[7] Several Kenyan companies used a variety of techniques when setting reserves, even prior to the new reserving guidelines that took effect on 30th June 2013. Still, some companies relied heavily on the minimum percentages supplied. Furthermore, offering a minimum without requiring the insurer to justify its use takes away the incentive for insurers to build a culture around reserving analytics. The new IRA guidelines should help to instill such a culture.

3.3 Appropriateness of Proxy

Net written premium is a simple proxy that every company surely calculates, particularly in a market that is highly driven by sales volume. Assuming that an up-to-date and accurate reserving analysis could be performed on industry data to develop reasonable estimates for a company's IBNR liabilities, then what would be a reasonable and accurate proxy? Indeed, for short-tailed lines, the current year's premium might be closely related to the amount of IBNR. But net earned premium would more closely correlate with IBNR than net written premium.

Another issue is that if IBNR liabilities exist for policies written in previous years (as would be the case for longer-tailed lines) the current year's premium would only continue to provide a reasonable estimate if the company were growing at the same rate at which the industry was growing when the analysis to calculate the IBNR percentages was performed. IRA's new reserving guidelines effective 30th June 2013 address this last concern to some extent. In the new guidelines, companies with fewer than three years of internal experience data can set IBNR using a simplified proxy method similar to the approach previously used for minimum IBNR. However, there is more refinement, as the IBNR for liability, motor commercial, and workmen's compensation classes of business is set as a distinct percentage of net written premium for each of the three preceding

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[7] This acceptable margin of error might be considered in the context of the actuary's range of reasonable estimates if there were an actuary analyzing the data. In lieu of an actuary, what is likely important is what effect the misestimation could have on earnings, capital, and solvency.

calendar years, rather than only considering the most recent calendar year. Still, premium adjustments, such as audit premiums, retrospective premium adjustments, and dividends, will likely show up after the books are closed at the end of the financial period. In this case, the reserves would be set based on incomplete premium information.

Finally, there are general issues around using premium, whether it is written or earned, whether it is net or gross of reinsurance, whether it is the total premium including adjustments, and whether or not the premium from multiple years is considered. Premium levels do not necessarily accurately reflect risk. One source of this inaccuracy is inaccurate pricing. Insurers who set inadequate premiums will also be setting reserves too low. Other companies may charge lower premiums for justifiable reasons. For instance, a company having more efficient operations that lead to a lower expense ratio would lead to that company holding lower reserves than a company with the same claims ratio but a higher expense ratio.

Another problem with using premium as a proxy for IBNR arises because of the hard and soft markets of the underwriting cycle, which might cause an insurer to charge different rates at different times for the same underlying risk. The likely lower premiums of soft markets would cause many insurers to hold lower levels of reserves than they would hold in hard markets, relatively understating the amount of risk. Soft markets can also cause companies to loosen underwriting standards or expand into new lines of business. Loosening underwriting standards might lead insurers to accept riskier business. Expanding into new lines for which the insurer does not have proper expertise or hold adequate capital might cause the risk of this new business to be understated when compared to an existing book of business.

3.4 How IBNR Affects Overall Reserve Levels and Solvency Implications

Once we consider other possible inaccuracies, such as misestimation of case reserves, it is not unthinkable that strictly following both the minimum IBNR and solvency capital requirements (solvency capital also being calculated with premium as a proxy) mandated by the Kenyan regulator could lead to complete depletion of capital for one or more firms.

By the end of 2013, the Kenya Insurance Regulatory Authority is expected to release a risk-based capital framework that should improve on the basic formula that is currently used for solvency capital. This should help insurance firms to better align their capital with the risks that they face. Still, the combination of inaccurate IBNR and case reserves could lead to insolvency or at least to significant variations in earnings once all claims are finally reported and fully paid.

Of the 25 general insurance companies in Kenya, 18 showed growth in 2011. In fact, 13 of the 25 companies showed growth in excess of 10%. Many of these companies show growth year after year.[8] This persistent growth in net written premium can surely mask under-reserving for IBNR claims because there is a mismatch between the fewer exposures associated with IBNR of previous years and the increased exposures associated with net written premium of the current year. An eventual year of slowed growth would result in IBNR claims that are a comparatively larger proportion of net written premium. This could result in large variations in earnings.

3.5 Considerations When Establishing a Regulatory Approach

There is much to gain from discussing the *accuracy* of the reserving method suggested by the regulator. However, further discussion is needed regarding the *practicality* of different methods for setting reserves. There are often restrictions on the amount or accuracy of data available for analytics. Reinsurers sometimes only receive censored claims data. Legacy IT systems may be more trouble than they are worth in terms of collecting old data for analysis. Developing markets, in particular, often lack abundance of internal data, accuracy of data, access to data, and personnel (actuaries or otherwise) who have the proper skills and training to analyze the data. As an example, the following is a list, in the opinion the author, of issues affecting the Kenyan insurance market and the data used for this paper:

- There are fewer than 10 qualified actuaries living and practicing in Kenya, and most of them are trained in life insurance, while there are 25 companies licensed to sell non-life insurance.
- Actuarial consultants are seen as too expensive for many Kenyan insurers because insurers see high consulting fees and often do not fully understand the value provided by an actuary.
- The Kenyan insurance market may have less of a "need" (due to, for instance, competition being mostly price driven) for highly technical pricing, capital modeling, and reserving (when compared to more developed insurance markets), and thus have less of a need for the skills of an actuary.

[8] Indeed, in emerging and developing markets, many firms see large growth because they are not merely fighting for customers in a saturated or near-saturated market. Instead they are finding new customers and selling new policies to existing customers, as they cater products to the needs of lower-income customers who have never before been insured, or as they benefit from high population growth rates and the emergence and growth of the middle class.

- Some companies do not have abundant (credible) internal experience data.
- Companies that do indeed have some credibility in their internal data might not have adequate access to the data needed for sophisticated analysis.
- Individual company and aggregated industry data as published by IRA only show calendar year incurred losses. This makes performing certain reserving techniques on aggregated industry data impossible.
- Without appropriate internal and external claims data, one might argue that the usefulness of an actuary is diminished, aside from the value of the actuary's extensive use of judgment.[9]
- Indeed, the data used in the ABCD example are known to have some inaccuracies. Those inaccuracies that could easily be handled were corrected. However, there are, for instance, some data duplicates that cannot easily be adjusted for without an extreme amount of manual work or a non-trivial computer algorithm.[10] Other companies may face similar issues. They may have an adequate ERP system; however, that ERP system might be suited for record keeping but not be best suited for extracting data to be used for analytics.

Given these constraints, using a simple formula based on a readily available proxy might be more accurate than requiring Kenyan insurers to calculate reserves using common actuarial techniques. Then what is an appropriate method for making sure that adequate reserves are booked in an environment lacking actuarial expertise or adequate data? This will certainly depend on the context of the market. The regulatory authority should conduct a cost-benefit analysis. This analysis should not understate the high costs of insolvency, in terms of the policyholders, employees, and investors of the insolvent company, as well as the broader consequences to other insurers who pay directly and indirectly for the insolvency.

The author suggests that regulators take into account the following general considerations:

- What is the actuarial or otherwise technical capacity of the market?
- If a proxy is to be used to calculate reserves, how is that proxy to be determined?
- Does the regulator have the data needed to calculate an accurate proxy? (For instance, is

[9] Granted, one might conversely argue that the judgment of an actuary is more important when data is limited.
[10] Because of these data issues, ABCD is embarking on a project to clean up their data.

aged accident year data available?) What would be the costs of collecting additional data?

- Forcing or allowing insurers to use one particular method for setting reserves will surely lead to inaccurate reserves for some of the insurers. It is prudent to consider multiple methods when setting reserves.[11] Additionally, providing a simple formula does not incentivize insurance organizations to adopt a culture of reserve analytics.
- Do insurers hold adequate capital that would buffer against inaccurate reserves?
- What affect do the reserves have on solvency and general market stability?
- The regulator can help insurers who lack credible experience data by combining industry data that can be used by all members. The more robust the requirements of the "data call," the more useful this data will be in helping insurers set reserves.

Furthermore, insurers can work toward ensuring adequate reserves, thus protecting themselves and their stakeholders, by taking into account these considerations:

- No formulaic method for setting reserves will be accurate for every insurer.
- The inaccuracy of (IBNR) reserves, even when using formulas supplied by the regulator (and especially when coupled with other inaccuracies), can lead to insolvency.
- A sound actuarial analysis of reserves will likely produce a more accurate estimate of unpaid liabilities than can be achieved by using a one-size-fits-all formulaic proxy.

4. CONCLUSIONS

Regulations play an important role in the business of insurance. Setting standards for reserves and solvency capital can help to ensure a functional insurance market that protects the financial stability of insurers and consumers. It is important that these regulations consider characteristics of the jurisdiction, such as the risk appetite of stakeholders, the technical knowledge of insurance personnel and consumers, and functional issues such as availability and quality of data. In the example presented here, regulators had been requiring minimum IBNR reserves based on a particular, simple reserving technique because of the low technical expertise present in the market,

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[11] In particular, U.S. standards of practice suggest that "The actuary should consider the use of multiple methods or models appropriate to the purpose, nature and scope of the assignment and the characteristics of the claims unless, in the actuary's professional judgment, reliance upon a single method or model is reasonable given the circumstances." See an excerpt of ASOP 43 in Appendix E.

while updated guidelines call for the use of techniques that many insurers do not have experience with. In more developed markets, mandated techniques might leave actuaries feeling like their hands are tied; they wish to use their expertise in particular areas, but they are forced to use only the methods that are prescribed by regulators.

In the specific case of Kenya, an industry-wide study of the Kenyan insurance market could be undertaken to make sure that mandated IBNR percentages are accurate (and especially not deficient) on average. The regulatory authority could more accurately calculate the IBNR percentages by collecting accident year data at different ages and performing (or hiring consultants to perform) an in-depth reserve analysis.[12] In addition, individual companies could be encouraged, as they are in the new guidelines effective 30th June 2013, to calculate IBNR by other methods to make sure that they are reserving adequately and ultimately avoiding insolvency. Furthermore, Kenya's regulatory authority, actuarial bodies, and others could encourage the development or transplanting of more actuaries. This would help with the issue of technical expertise, and the actuaries could also play an important role in collecting data and making it usable for analytics.

Using a proxy to set minimum IBNR reserves may offer benefits in an environment lacking actuarial expertise. However, there should be careful consideration in developing the proxy. And, above all, both regulators and insurers should know that using a formulaic proxy to set IBNR reserves is not a fail-safe method. Using any one reserving method, regardless of data quality or the level of actuarial expertise present in the market, will generally not lead to accurate reserves for most companies.

Acknowledgment

The author acknowledges John Xu and Jim Kahn for their guidance and thorough review of this paper; employees of Kenya's Insurance Regulatory Authority for their generous sharing of information and data, as well as for their commitment to make use of this paper; and ABCD for use of their data.

[12] This reserve analysis should include a variety of methods, such as those used in this paper, as well as stochastic methods that allow the regulatory authority to set reserves based on their comfort level with expected rates of insolvency.

Appendix A: IBNR Percentages by Class of General Insurance Mandated by Kenya's Insurance Regulatory Authority Prior to 30th June 2013 (C.N. Gituai, personal communication, 7 December 2012)

Class of General Insurance Business	Percentage of Net Written Premium
Aviation	0%
Engineering	5%
Fire Domestic	1%
Fire Industrial	1%
Liability	5%
Marine	3%
Motor Private	5%
Motor Commercial	5%
Personal Accident	5%
Theft	5%
Workman's Compensation	5%
Medical	5%
Miscellaneous	5%

Appendix B: Reserving Method Results Using ABCD's Experience Data

Reserving Method	Type	Details	IBNR as % of NWP
1	Incurred Chain Ladder	Straight Average, latest 5 quarters	8%
2	Incurred Chain Ladder	Weighted Average, all (18) quarters	8%
3	Incurred Chain Ladder	Weighted Average, latest 5 quarters	7%
4	Bornhuetter-Ferguson	60% expected claims ratio,[13] percent unreported from incurred chain ladder (weighted average, latest 5 quarters)	6%
5	Bornhuetter-Ferguson	60% expected claims ratio, percent unpaid from paid chain ladder (weighted average, latest 5 quarters)	14%

[13] 60% is near the industry average claims ratio for this line of business.

Appendix C: Excerpt of "Guidelines on Valuation of Technical Liabilities for General Insurers" Published by IRA, Effective 30th June 2013

4.2 Claim Reserves

4.2.1 Reserves in respect of outstanding claims incurred and reported shall be determined prudently by using Case Estimate Method, Average Cost per Claim Method or other methods recognized by the Authority.

4.2.2 Reserves in respect of incurred but not reported claims shall be valued and determined prudently by using at least two of the following methods in accordance with the risk nature, risk distribution and experiential data of the insurance lines:

i. Chain-Ladder Method;

ii. Average Cost Per Claim Method

iii. Bornhuetter-Ferguson Method

iv. Standard Development Method[14]

4.2.3 An insurer that has been in existence for not more than three years can use the Standard Development Method.

4.2.4 The percentage of net premiums written during the year should be applied when using Standard Development Method as provided in the appendix[15] to this guideline.

4.2.5 The methods to be adopted for the valuation of the Claim reserves shall depend on:

i. The particular characteristics of the class of business

ii. The reliability and volume of the available data

iii. Past experience of the insurer and the industry

iv. The robustness of the valuations models

v. Considerations of materiality

4.2.6 The value of the Claim Reserves shall include an amount in respect of the anticipated Claim adjustment expenses

4.2.7 When determining claims reserves, an insurance company shall conduct a test on the adequacy of the reserves. Where the claims reserves are inadequate, claims deficiency reserves margin shall be determined.

4.2.8 The insurer shall determine and disclose a value for its Claims Reserves for each class of business.

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[14] The Standard Development Method in the new guidelines is very similar to the method prescribed in the old guidelines, wherein IBNR is set as a percentage of net written premium.

[15] The appendix of the new guidelines, which shows the percentages to be used in the Standard Development Method, is included in this paper as Appendix D.

Appendix D: IBNR Percentages by Class of General Insurance to be Used Under the "Standard Development Method" (effective 30th June 2013)

This table is from the appendix of "Guidelines on Valuation of Technical Liabilities for General Insurers," Published by IRA, effective 30th June 2013. It shows IBNR Percentages by class of general insurance business to be used by an insurer that has been in existence for not more than three years. The previous guideline recommended a similar approach for determining minimum IBNR, but it did not dictate further analysis like that required in the new guidelines (shown in Appendix C) for those companies with more than three years of data.

No	Class of Insurance Business	Percentage of Net Premium Written
1	Aviation	2%
2	Engineering	5%
3	Fire Domestic	1%
4	Fire Industrial	1%
5	Liability	5% - Current Year
		3% - One year preceding the current year
		1% - Two years preceding the current year
6	Marine	2.50%
7	Motor Private	5%
8	Motor Commercial	5% - Current Year
		3% - One year preceding the current year
		1% - Two years preceding the current year
9	Motor Commercial (PSV)	20% - Current Year
		12.5% - One year preceding the current year
		5% - Two years preceding the current year
10	Personal Accident Insurance	5%
11	Theft	5%
12	Workmen's Compensation	5% - Current Year
		3% - One year preceding the current year
		1% - Two years preceding the current year
13	Medical	3%
14	Micro insurance	4%
15	Miscellaneous	5%

Appendix E: Excerpt of ASOP 43

The actuary should consider the following items when performing the unpaid claim estimate analysis:

3.6.1 Methods and Models—The actuary should consider methods or models for estimating unpaid claims that, in the actuary's professional judgment, are appropriate. The actuary should select specific methods or models, modify such methods or models, or develop new methods or models based on relevant factors including, but not limited to, the following:

a. the nature of the claims and underlying exposures;

b. the development characteristics associated with these claims;

c. the characteristics of the available data;

d. the applicability of various methods or models to the available data; and

e. the reasonableness of the assumptions underlying each method or model.

The actuary should consider whether a particular method or model is appropriate in light of the purpose, constraints, and scope of the assignment. For example, an unpaid claim estimate produced by a simple methodology may be appropriate for an immediate internal use. The same methodology may be inappropriate for external financial reporting purposes.

The actuary should consider whether, in the actuary's professional judgment, different methods or models should be used for different components of the unpaid claim estimate. For example, different coverages within a line of business may require different methods.

The actuary should consider the use of multiple methods or models appropriate to the purpose, nature and scope of the assignment and the characteristics of the claims unless, in the actuary's professional judgment, reliance upon a single method or model is reasonable given the circumstances. If for any material component of the unpaid claim estimate the actuary does not use multiple methods or models, the actuary should disclose and discuss the rationale for this decision in the actuarial communication.

In the case when the unpaid claim estimate is an update to a previous estimate, the actuary may choose to use the same methods or models as were used in the prior unpaid claim estimate analysis, different methods or models, or a combination of both. The actuary should consider the appropriateness of the chosen methods or models, even when the decision is made not to change from the previously applied methods or models.

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Abbreviations and notations

IRA, (Kenya) Insurance Regulatory Authority
IBNER, incurred but not enough reported
PSV, passenger service vehicle (e.g., a taxi)
ERP, enterprise resource planning
IBNR, incurred but not reported
NWP, net written premium

Biography of the Author

William (Bill) Collins recently completed a one-year fellowship with the International Labor Organization's Microinsurance Innovation Facility. He worked with a Kenyan insurance company to develop insurance programs catered to the needs of Kenya's working poor. Before working in Kenya, Bill worked on life, automobile, and crop insurance in the U.S. He has a bachelors of science in mathematics with honors from the University of Kansas. Bill is a member of the Casualty Actuarial Society. He has guest lectured on actuarial science at Strathmore University (Nairobi) and serves on the IAA Task Force on Microinsurance.

Testing the Assumptions of Assumptions Testing

Keith Curley, FCAS, MAAA

Abstract

Motivation. The growing availability and advocacy of stochastic reserving methods is outpacing their critical evaluation, testing, and indeed acceptance. I believe there has not yet been sufficient critical attention given to claims made in favor of stochastic models, and I'll focus here on the particular claims that assumptions can be tested and the models validated.

Method. We'll review some of the statistical background, especially hypothesis testing, needed to understand the issues and see how it applies to reserve modeling with aggregate loss triangles. We'll make use of the concept of statistical power, associated with Type II error, which has been previously absent from reserve modeling discussions. This concept can be used to question the reliability of modeling results and certain common modeling recommendations. A few simplified reserving models and results of simulations that help illuminate the issues are described and reported.

Results. We'll see that significance tests, and testing more generally, might have little power and recommendations based on these tests can be unwise. We'll also see the benefits of a deeper understanding of the claims process and the dangers of relying on statistical methods without that understanding.

Conclusions. This particular argument for stochastic modeling in reserving with aggregate triangles is almost certainly unsound. If this were the only reason to resort to modeling, there are more productive uses of an actuary's time. With or without modeling, better approaches probably rely on simpler methods, hard work, a skeptical and inquisitive attitude, and a deeper knowledge and understanding of the claims generation, reserving and settlement processes.

Keywords Reserving methods; reserve variability; statistical models and methods.

1. INTRODUCTION

Recent technical actuarial literature has been dominated by *advocates* of advanced quantitative techniques. This is perhaps unavoidable, but it has led to a one-sided discussion of stochastic reserve modeling.

Two particularly exaggerated claims are that modeling allows for "assumptions to be tested and models validated." I will show that this is true in only a very limited sense which is often without much practical consequence.

There is a risk that by focusing too narrowly on this one argument, which is perhaps one of the worst arguments made on behalf of modeling, I will fall into the opposite error of the advocates and be unfairly branding the whole stochastic program because of some poorly thought-out claims from its advocates: that I'll be impeding the progress of science by throwing the proverbial baby out with the bath-water while stochastic reserving is still in its infancy. All I can say is that I wish to bring greater clarity to the discussion of the merits of modeling versus traditional actuarial methods.

A fuller discussion would touch on a number of issues of which reasonable people can and probably always will disagree. All that I ask of a reader is that he or she critically evaluate my arguments and evidence and do the same whenever claims in favor of modeling are encountered.

1.1 Research Context

It is a common, though not universal, modeling practice to screen variables for inclusion within a model by means of significance testing. A number of papers in the actuarial literature have also advocated this practice when selecting variables to model yearly aggregate loss triangles. Though not a necessarily exhaustive list, in this context the usual variables considered are: yearly exposure measures, accident year trends, calendar year trends, and development year trends, which are also called loss development factors when they are a multiplicative factor of the prior development year's losses.

For illustrative purposes and because of limitations of space, we will focus in this paper on loss development factors, but I hope it will be clear that many of the concepts explored here apply equally to significance testing for any variable.

Some authors have reported that the loss development factor is often not found to be significant in their modeling experience. According to some of the authors, in this case, such a factor should then be dropped from a final loss reserve model.

In addition to significance testing, it is common practice, especially among the more thorough modelers, to run a series of additional tests and diagnostics to check that the model assumptions are probabilistically consistent with the data. The possibility of doing such tests is often offered as a distinct advantage of the modeling framework which traditional actuarial methods do not provide. Many modeling advocates make recommendations for how to develop statistical models and methods to project reserves and study reserve variability that lean very heavily on testing results.

Sometimes even bolder claims are made on behalf of the possibility, the advisability, and the effectiveness of assumption and model testing and some have tried to draw implications for which traditional reserving methods to use because of the results of this testing.

1.2 Objective

The CAS Working Party on Quantifying Variability in Reserve Estimates ("CAS Working Party"), "The Analysis and Estimation of Loss & ALAE Variability: A Summary Report," [1] correctly state under the topic of "Model or Specification Uncertainty" (page 35): "In nearly every stochastic model, the modeling begins by making the assumption that the underlying process follows the model."

Inspired by some of the bold claims in Section 1.1, let me elaborate on the comment immediately above and add a few bold claims of my own:

1. Not only do statistical models rely on various assumptions, it is important to always keep in mind that they are in fact *theories* about the world. These theories have implications for how the world must actually be in order for the assumptions to hold true. If the theories are true, they allow one to predict the future, at least probabilistically. If they are false, they might be a waste of time or even seriously misleading.

2. Although there exist various tests and diagnostics of the assumptions, it's unlikely that they will be *effective*--meaning that they would allow one, with a high probability of success, to correctly draw any conclusion about loss development or to pursue any action, such as using one loss development method rather than another.

3. The strongest and least plausible of modeling assumptions in insurance is that insurance data are observations of random variables that are *independent* and *identically* distributed. This is the main statistical assumption with modeling and if false all of the modeling results are compromised.

Although my claims are predominantly negative, we will also see along the way that any real information, which an actuary can discover about losses and how they are generated or develop, can be highly useful in the reserving process. I believe that traditional actuarial methods, in addition to having adequate statistical properties, bring the actuary in closer contact with the data, without the possibly distorting effects of false assumptions and without time spent on unnecessary tests.

I stake no particular claims to originality here. One can find scattered throughout the actuarial literature[1], comments from actuaries questioning whether standard statistical assumptions apply to insurance data. But in those papers it is usually a caveat which receives little attention while here it's the centerpiece of the paper. So I believe the emphasis and arrangement of ideas in this paper is somewhat unique.

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[1] See, for instance, the discussion of "i.i.d." by David Clark in "LDF Curve-fitting and Stochastic Reserving: A Maximum Likelihood Approach" [2] page 56.

The particular technical issue of statistical power which I'll discuss and which severely qualifies any claims for testing and validation effectiveness, although part of the Actuarial Exam Syllabus[2], has not been as far as I can tell applied to this issue before.

I owe debts to many previous actuaries, that are too numerous to name or reference here. But I owe a particular debt to the thinking of the late UC Berkeley Statistician David Freedman. The two textbooks, which I reference in this paper, and the numerous papers he has written over the years, are *models* of clear and careful statistical reasoning and how it can be applied to answer real world questions.[3]

1.3 Outline

This discussion will require a review of some basic concepts from statistics, but they are all ideas to which every actuary will have been exposed at one time or another. In particular we will review the meanings of *methods* and *models*, and the assumptions the latter usually rely on.

Unfortunately, we will have to go into some detail about statistical hypothesis testing, and this material is routinely misunderstood by both students and professionals throughout the social sciences. Apparently it is difficult for many to understand. It's possible that I might not do a great job of explaining it either. Here we discuss the importance of the *statistical power* of a test, and the consequential costs, or *loss*, associated with following the results of a test.

All I can ask is that you bear with me and be willing to think a bit. If this material is unfamiliar, I think you'll find that it's well worth learning it, and you may never look at another statistical analysis the same way again.

We will then apply these ideas to insurance reserving to see whether any claims for diagnostic effectiveness are likely to be true.

It's common in the technical actuarial literature to briefly present a model and then to elaborate at length the mathematical implications of that model. I'm not so interested in the math, but in the validity of the very first step. So we will largely travel in the other direction, and starting with models

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[2] See Stuart Klugman's Study Note *Estimation, Evaluation, and Selection of Actuarial Models* [8]

[3] James M. Robins, professor of epidemiology at the Harvard School of Public Health, once wrote about David, that he was "one of the world's leading mathematical statisticians, but he has also assumed the mantle as the skeptical conscience of statistics as it is applied to important scientific, policy and legal issues." See the obituary at: http://berkeley.edu/news/media/releases/2008/10/20_freedman.shtml

and modeling assumptions we're going to interpret them as assertions about the world and study whether those assertions are true.

We might not be able to reach any absolutely definitive conclusions, but regardless there should be some value in trying to think clearly about the relation of models to our world and the role of those models in our work.

2. BACKGROUND AND METHODS

2.1 Statistical Models and Methods

The Working Party draws a distinction between a "Method" and a "Model." Methods are (page 38) "algorithms or series of steps followed to determine an estimate," with some examples being the "chain-ladder (development factors) method or the Bornhuetter-Ferguson method." They then add that Methods "do not involve the use of any statistical assumptions that could be used to validate reasonableness or to calculate standard error."

On the other hand, a "Model" (page 67) "specifies statistical assumptions about the loss process, usually leaving some parameters to be estimated." They also add: "There are various methods that could be used for estimating the parameters, such as maximum likelihood and various robust estimators, but unless otherwise noted, 'methods' here will refer to algorithms for calculating loss future payments, not methods for estimating model parameters."

In this paper, I will not follow that convention about *methods* only applying to reserving algorithms and not parameter estimation. In the statistical context, both are functions of random variables and hence *estimators*—random variables themselves, which one hopes will take on values close to what one is estimating. Reserving methods might not explicitly rely on stochastic assumptions, but once those assumptions are introduced into the discussion, those methods become estimators.

2.1.1 An Illuminating Example of a Method and a Model

Because there is often much confusion on this score and because it will be useful throughout the paper, we should compare the ordinary least squares (OLS) *method* to the ordinary least squares (OLS) regression *model*.

The OLS *method* is merely a way of solving a system of linear equations where there are more equations than unknowns, say *n* equations in *p* unknowns with *n*>*p*. In this situation, such a system is usually inconsistent and has no solution. But by taking weighted averages of all the equations with weights given by the unknowns' coefficients in the equations, one reduces the number of equations to just *p* equations in *p* unknowns, and this usually has a unique solution. The *method* is just linear algebra.[4]

The OLS *model* is a set of statistical assumptions, for which the OLS *method* becomes a well-suited estimator of the unknowns. We will only need two unknowns in our discussion, in which case the OLS *model* assumes that (adapted from David Freedman's, *Statistical Models: Theory and Practice* [5])[5]:

1. There are two *observable* random variables $\boldsymbol{X}$ and $\boldsymbol{Y}$; they are $n \times 1$ random vectors; there is also an $n \times 1$ random vector $\boldsymbol{\varepsilon}$ that is *not* observed and is called the *random error* or *disturbance* term; $\boldsymbol{Y}$ is a linear function of $\boldsymbol{\varepsilon}$ and $\boldsymbol{X}$ via unknowns *a* and *b*, which usually have to be estimated from the data.

2. The vector relationship above unpacks into *n* ordinary equations:
$$Y_i = a + bX_i + \varepsilon_i \tag{2.1}$$

3. A fundamental assumption is that "the data on Y_i are observed values of $a + bX_i + \varepsilon_i$."
As David Freedman points out "[w]e have observed values for $\boldsymbol{X}$ and $\boldsymbol{Y}$, not the random variables themselves. We do not know [*a* and *b*] and do not observe $\boldsymbol{\varepsilon}$." Recall: *random variables* have distributions, means, standard deviations[6], etc.; *observed values* of random variables, aka *data*, are just numbers.

4. "The ε_i are independent and identically distributed, with mean 0 and variance σ^2."

5. "If $\boldsymbol{X}$ is random, we assume that $\boldsymbol{\varepsilon}$ is independent of $\boldsymbol{X}$."

   Warning: Many applications of regression and many of the standard theorems assume that $\boldsymbol{X}$ is fixed, as it could be, for instance, in an experiment where the experimenter is able to control the value of $\boldsymbol{X}$. For us, since we will be using $\boldsymbol{X}$ to represent losses during exposure

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[4] See, for instance, Gilbert Strang, *Linear Algebra and Its Applications,* 3rd Edition.
[5] Since we're focusing on only two unknowns we don't present the whole matrix formulation; and we leave out mention of certain niceties like the rank of our system of equations being at least 2 which will almost always be the case for us.
[6] Theoretically not all random variables have moments; but any which appear in insurance probably will.

> periods, $\boldsymbol{X}$ is random, not fixed, and many of the standard theorems do not apply or apply only "conditionally on $\boldsymbol{X}$ being given."[7]

As mentioned previously the OLS *method* with 2 unknowns will reduce a system of *n* equations to just 2 equations, by taking a weighted average of all the equations with weights equal to the coefficients of the unknowns in the model. So let's suppose that we have *n* pairs of observed values $(y_1,x_1), (y_2,x_2),\ldots, (y_n,x_n)$.

This gives us *n* linear equations:

$$\begin{gathered} y_1 = a + bx_1 \\ y_2 = a + bx_2 \\ \ldots \\ y_n = a + bx_n \end{gathered} \tag{2.2}$$

The coefficient of the first unknown *a* is always 1, so the first reduced equation will simply be an average of all *n* equations, where we now also employ the hat "^" over *a* and *b*, to represent that they are not the same *a* and *b* as above (in fact they cannot be the same because the above is inconsistent):

$$\bar{y} = \hat{a} + \hat{b}\bar{x}, \tag{2.3}$$

Barred variables, for instance, $\bar{y}$ and $\bar{x}$, will just indicate the averages of the data series.

We get our second equation by multiplying through on both sides of the *i*th equation by x_i and averaging those equations:

$$\begin{gathered} x_1y_1 = ax_1 + bx_1x_1 \\ x_2y_2 = ax_2 + bx_2x_2 \\ \ldots \\ x_ny_n = ax_n + bx_nx_n \end{gathered} \tag{2.4}$$

$$\overline{xy} = \hat{a}\bar{x} + \hat{b}\overline{x^2} \tag{2.5}$$

Now (2.3) and (2.5) gives us two equations in two unknowns, $\hat{a}$ and $\hat{b}$, and those equations almost always have unique solutions:

$$\hat{a} = \frac{\overline{x^2}\bar{y} - \bar{x}\overline{xy}}{\overline{x^2} - \bar{x}^2} \tag{2.6}$$

$$\hat{b} = \frac{\overline{xy} - \bar{x}\bar{y}}{\overline{x^2} - \bar{x}^2} \tag{2.7}$$

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[7] The parameter estimators tend to be *unconditionally* unbiased as well, but not necessarily the standard error estimators.

If we're going to use our estimate to, for instance, project the *n+1*-th observation, a little creative algebra and we can write:

$$y_{n+1} = \hat{a} + \hat{b}x_{n+1} \tag{2.8}$$

As:

$$y_{n+1} = (1 - Z)\bar{y} + Z\frac{\bar{y}}{\bar{x}}x_{n+1}, \tag{2.9}$$

$$\text{where } Z = \frac{\bar{x}}{\bar{y}}\frac{\overline{xy} - \bar{x}\bar{y}}{\overline{x^2} - \bar{x}^2},$$

which can also be rewritten as

$$Z = \rho\frac{CV_y}{CV_x},$$

where ρ is the correlation between the two data series x and y, and the CV's are their coefficients of variation, i.e., their standard deviations over their means.

Or, in words, as many actuaries before have noticed: if x_i represents losses from the *i*th exposure period, say accident year, at some evaluation age, and y_i represents the incremental losses at the next evaluation age, then the OLS projection y_{n+1} for the losses from the latest diagonal x_{n+1}, is just a weighted average of the standard chain ladder estimate, and the overall mean $\bar{y}$. One might replace incremental loss with cumulative loss or ultimate loss. But, regardless, actuarial methods which are now standard, such as Bornhuetter-Ferguson, Stanard-Bühlmann, and Benktander, can be viewed as methods for estimating the various parameters such as Z and $\bar{y}$ which appear in equation 2.9.

Now, in order to connect this OLS *method* with the OLS *model,* we have to invoke assumption 3, which was that "the data on Y_i are observed values of $a + bX_i + \varepsilon_i$." If we make this connection, replacing our equations in $\boldsymbol{x}$ and $\boldsymbol{y}$ above with equations in $\boldsymbol{X}$ and $\boldsymbol{Y}$, then $\hat{a}$ and $\hat{b}$ are now *estimators*, i.e. functions of random variables, and hence random variables themselves with their own distributions.

If all the other assumptions on page 6 are true as well, then we are allowed to conclude that $\hat{a}$ and $\hat{b}$ are conditionally *unbiased* estimators, meaning their expected values, conditional on a given $\boldsymbol{X}$, are equal to the unknown parameters *a* and *b*. We can also calculate their variances and correlations with each other, conditional on a given $\boldsymbol{X}$, if we know the variance of the error term σ^2; and if that's unknown we have a conditionally unbiased estimator available for it as well. Finally, conditional on

$\boldsymbol{X}$, we can show that the squared difference between $\boldsymbol{Y}$ and the linear combination of $\boldsymbol{X}$ is minimized when $a = \hat{a}$ and $b = \hat{b}$, which is of course the origin of the *ordinary least squares* method and model's name.

I apologize if all of the above is old "hat," but I think it's important that we keep in mind the difference between a method which can be applied regardless of whether any statistical assumptions are true, and a model for which that method might have some nice properties when regarded as an estimator.

For instance, if in the equations (2.2) we had dropped the *a*'s and only considered *y* as a function of *x*, i.e., taken only:

$$y_i = bx_i \tag{2.10}$$

Then there are many ways we might estimate *b*. We can simply average all the equations and divide, which would give us the standard chain-ladder over all the years, or we can take the last *m* equations for any $1 \le m$ and average them, as we might do in practice if those were more representative years.[8] More generally, we could apply any weighted average whatsoever to the *n* equations to reduce them to 1 equation in order to estimate *b*.

And referring back to the model assumptions, where now there is no *a*:

$$Y_i = bX_i + \varepsilon_i \tag{2.11}$$

As long as the ε_i have mean 0 and the weights in combination with the X_i are independent of ε_i then the resulting estimator will be an unbiased estimator of *b*.[9]

2.2 Models and Their Assumptions

If one reviews any application of statistical models, such as the example we give above, one sees that model assumptions come in five flavors[10]:

1. There is always the assumption, though usually implicit, connecting the mathematical formulation to the world: that data are *observed values* of random variables.

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[8] See Stigler, *History of Statistics* [13]. Before the advent of least squares, the Method of Averages was to use simple un-weighted averages of subsets of equations to reduce a system to solvable form.

[9] One minor caveat is to avoid dividing by anything with a non-zero probability of being zero.

[10] There is some regrettable but inconsequential overlap in my classification system: with for instance functional relationships creating dependency relationships. Also, it can be very hard conceptually to separate the two i's in i.i.d..

2. There are *functional* assumptions such as $\boldsymbol{Y}$ is a linear function of $\boldsymbol{X}$.
3. There are *parameter* assumptions, such as that the ε_i's have means of 0 and variances of σ^2, or that there are non-zero *a* and *b*, even if they usually have to be estimated from the data.
4. There are *independence (*and *identically distributed)* assumptions, such as that the ε_i are *independent (*and *identically distributed)* and that if $\boldsymbol{X}$ is random, $\boldsymbol{\varepsilon}$ is independent of $\boldsymbol{X}$.
5. Though *not* an OLS model assumption, in practice there are usually more specific *distributional* assumptions as well. For instance, in OLS one often assumes that the ε_i are *normally* distributed; this allows one to calculate t-tests, p-values, F tests, etc. and draw *inferences* about how well the model fits the data. Full distributional assumptions allow one to discuss a reserve distribution as well.

I fear that, through over-use, most of us have become deadened to the true force and meaning of *assumptions*.

Assumptions are *theories* about the world.

In *pure* mathematics, the fundamental concepts are *undefined*, since one has to start the chain of definitions somewhere; and all concepts are *un-interpreted*, meaning no particular meaning in the real world is ascribed to them.[11] One merely studies how statements (*axioms*) postulated about these concepts in terms of each other imply other statements (*theorems*) made up of them or of concepts freshly defined in terms of them.[12]

In *applied* mathematics, on the other hand, one must first *interpret* some of these undefined and un-interpreted terms, so that they refer to something in the world. The hardest term to interpret in our context is *probability* itself, and I will not attempt to do so here.[13]

But once *some* interpretation is given for that term, we can connect the assumptions and results of our statistical modeling to the real world. Assumptions, which were just conditions of theorems in pure mathematics, become in this way declarative statements about the world and how it functions. That is, they are *theories.*

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[11] This is not to say that most mathematicians don't have some interpretation in mind, merely that there is no *official* interpretation for a fundamental undefined concept such as *set membership.*

[12] This is the source of mathematical logician (later philosopher) Bertrand Russell's popular definition of mathematics as: "the science in which we do not know what we are talking about, and do not care whether what we say about it is true."

[13] See Don Gillies, *Philosophical Interpretations of Probability* [7], for a highly readable and sympathetic account of the main interpretations of probability, from a philosopher who has also done applied work in statistics.

In addition to *probability* itself, the hardest modeling assumption to comprehend is the assumption of *independent* and *identically distributed* (i.i.d.) random variables. We will not make much use of this, but I think an extremely useful tool in trying to understand the real meaning of *i.i.d.* is given by a *conceptual* model of the loss generation process as a *box* model and presented, for instance, in the textbook *Statistics,* by Freedman, Pisani, Purves ([6] page 389.) The idea is that the situation to be described by a statistical model must generate data like draws of lottery tickets from a box with fixed numbers of different tickets, where each ticket is equally likely to be drawn. There can be multiple levels of boxes and selections required, and there need not literally be a *box*, of course, but one has to be able to conceptualize the process in such a manner.

In insurance we have a ready-made box model for us in the form of Collective Risk Theory. Recall, in this model, there is a box for the claim count during a period, and a box for the claim severities. For a single period, one selects a ticket from the claim count box. Then, based on the number shown, one selects that many tickets from the severity box, making sure to replace each severity ticket after recording its value, and shaking the box thoroughly before selecting the next severity ticket. Finally, after one has drawn the requisite number of severity tickets for that period, one vigorously shakes the claim count box, selects from it for the next year, and draws again from the severity box.

We will return to the assumption of i.i.d. draws in the results section and examine a little more how well it fits to insurance. For now note that rather than establishing by means of facts, theory, or argument that their data is really from i.i.d. random variables, most modelers merely *assume* it, for among other things the enormous computational convenience it provides. Then, if they have any doubts about these assumptions and the other modeling assumptions, they rely on tests and diagnostics to indicate if they might possibly be in error. So, we had better discuss now, tests and diagnostics.

2.3 Statistical Hypothesis Testing

Tests, *diagnostics*, *validation*, *reasonability checks, goodness-of-fit*—they all mean roughly the same and can all be treated in the same general framework, which is *Statistical Hypothesis Testing.*[14]

[14] A fairly clear and concise treatment of the elements of hypothesis testing is available in Klugman's exam study note [8]. A classic graduate-level text is Lehmann and Romano's *Testing Statistical Hypotheses* [12]. As of this writing in 2013, the Wikipedia article for *Statistical Hypothesis Testing* is an informative introduction.

The CAS Working Party says (page 47) "[b]y overall model reasonability checks, we mean 'what measures can we use to judge the overall quality of the model?'" and then on page 49 "[b]y goodness-of-fit and prediction error evaluation, we mean 'what measures can we use to judge whether a model is capturing the statistical features in the data?'" They go on to list various tests, such as, "Coefficient of Variation by Year," "Validity of Link Ratios," and specific "Goodness-of-Fit Measures"...in all over a dozen criteria.

In all of these tests, the same abstract framework pertains: a *measure* is a *function* of the data being modeled and the model specification (all the various assumptions and specific parameter values,) and the measure is used to reach some *decision* about whether the current model specification is adequate or not. Sometimes it won't be the result of a single test, but a combination of tests will be examined, but in this case as well the final outcome is usually the same: a *yes* or *no* decision is reached about the whole model or some features in the model.[15]

So, we can represent our measure (or combination of measures) as a function φ which takes on the value 1, whenever we would reach a *yes* decision and the value 0, whenever we would reach a *no* decision.

Two essential concepts in hypothesis testing are: *power* and *loss*. *Power* gives us the probability that $\varphi=1$, that a *yes* decision is made, as a function of the models under consideration. *Loss* is a function of both the *decision* we make and the *models* under consideration, and measures the consequences to us of making some decision when a particular model is true.

This is all very abstract, so we will look in detail at the example which most concerns us, which is significance testing of a variable. In this type of testing we assume that, except for the *parameter* assumptions, *all* other assumptions are known and fixed: the *functional* assumptions (such as linearity,) the *distributional* assumptions (such as normality,) the *independence* assumptions (such as i.i.d. errors.)

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[15] I said "usually" because occasionally one will stop with just a probability, such as a *p-value*, and draw no particular decision or action as a result. That need not concern us. Note also: Standard Bayesian methods do not usually rely much on classical hypothesis testing for parameters, but rather rely on prior knowledge encapsulated within a prior distribution(s) which is updated with data. Nonetheless, if a Bayesian modeling exercise ever needs to reach a *yes/no* decision about variable inclusion or whether any assumptions are *true* or *not*, all of the comments in this section apply; in a Bayesian analysis one has to of course include the prior distribution(s) in all such calculations.

2.3.1 A Fully Worked Example

Consider the following simple model:

$$Y_i = bX_i + \varepsilon_i \tag{2.12}$$

Where,
X_i are *i.i.d.* lognormal with mean 1 and standard deviation of 1
ε_i are *i.i.d.* normal with mean of 0 and standard deviation of 1
All X_i's and ε_i's are independent of each other
$b = .5$

X_i might represent the cumulative losses for accident year *i* at a certain development age; *b* is an incremental multiplicative loss development factor that applies between that development age and the next; ε_i is random variation in the development; and Y_i is the resulting incremental losses for accident year *i* at that next development age.

Say we had 9 full observations of X_i and Y_i, and 1 more of just X_{10}; i.e., assume X_{10} is on the latest diagonal of a triangle.

In order to test our model specification we consider whether the model isn't really

$$Y_i = a + bX_i + \varepsilon_i, \text{ where } a \neq 0 \tag{2.13}$$

To test this we calculate the OLS parameter estimates for that model from our data.

What we are testing in particular is called a simple *null hypothesis* that $a=0$. We also state an *alternative hypothesis* which is that $a \neq 0$. In a significance test, if we find that our estimate of *a* is "significantly different" from 0, then we have some possible evidence that our null hypothesis, which corresponds to model 2.12, is incorrect. And this is possibly some evidence in favor of the alternative which corresponds to 2.13.

The significance test itself is often a *t-test* via a *t-stat*[16] which is the ratio of the parameter estimate for *a* divided by the estimate for the standard error of our estimator for *a*. Recall $\hat{a}$ is an *estimator* for *a* and a random variable, so it has a mean, standard deviation etc. When we have data we are modeling, we end up calculating one particular *estimate* which we assume is an observed value for that estimator. We also have an estimator for the standard deviation (aka standard error) of that random variable, which when applied to our data gives us an *estimate* of $\hat{a}'s$ standard error.

[16] So called because when $\boldsymbol{X}$ is fixed and $\boldsymbol{\varepsilon}$ normally distributed, the *t-stat* will follow Student's *t* distribution. Because our $\boldsymbol{X}$ is random and not fixed, and lognormal besides, we chose to simulate the results here rather than try to find any closed-form solutions.

In order to use the t-stat for any inferences, we must decide a *critical region* of values which we will regard as *significant*. Any cut-off is arbitrary, but many modelers suggest using the value of 2 for the t-stat to judge significance as an easy rule-of-thumb. If another critical region were chosen, our numbers below would of course change, but the issues would remain the same.

So, now we have our full test function φ: it will equal 1 whenever the t-stat has an absolute value of 2 or more, which corresponds to finding the parameter estimate *significant*, and otherwise it will equal 0. Our *power* will give us the probability of φ equaling 1, which is also the expected value of φ, which we can calculate because we have specified our model in (2.12.)

Estimator	Mean	Mean Standard Error	Power
$\hat{a}$	(0.00)	0.58	0.08
$\hat{b}$	0.50	0.54	0.28

With respect to $\hat{a}$, our significance test worked *exactly* the way it's supposed to: our estimate for *a* was only found to be "significantly different from 0" 8% of the time, which is good, because it is *actually 0*. But keep in mind that this is a probabilistic result: in 8% of the cases, one would conclude that *a* was *not 0* when in fact it was. This is the un-memorably named *Type I Error* rate.

But notice also that for $\hat{b}$, it was only found to be significant, 28% of the time. This despite the fact that *b* is not 0, it is .5. So, if anyone were to try to conclude whether *b* was 0, because of its significance test, he or she would be right only 28% of the time and wrong 72% of the time. The latter is the un-memorably named *Type II Error* rate. So, before one can really judge if a non-significant finding is very good evidence that a parameter is 0, one should attend to the power of the test to judge whether the test can even tell with much probability whether the parameter is "significantly different from 0."

But all the tests in the world are for naught unless one takes some action as a result of the test. The authors cited at the start of the paper suggested that one should drop a variable if its parameter estimate is found insignificant. In order to see whether this is a wise decision, we must specify some *loss* function that will measure the costs to us of taking an action: our action will be choosing one estimator over another and our loss will be a reduction in accuracy. There is infinite freedom available in choosing loss functions, but we will simply take a common measure of predictive accuracy for our loss. The one we will take is the squared difference between an estimator's prediction and the variable it is trying to predict. We will also look at just the (un-squared) difference between the two.[17] The expected value of the former is the *mean square error of prediction* (MSEP) and of the latter is the *bias*. We take the square *root* of the latter, which we call the *root mean square error of prediction* (RMSEP), to have the same units as the variable we are trying to predict.

What we are trying to predict is the incremental loss Y_{10} based on the previous 9 accident years and X_{10}.

As mentioned in section 2.1 we have infinite flexibility about what estimators and resulting predictors to use. But let's look at just the following:

1. The **Full OLS**[18] estimator, where we estimate both a and b by OLS and predict Y_{10} as $\hat{a} + \hat{b}X_{10}$.
2. The **Average** of the Y_i 's over the 9 accident years.
3. The OLS estimator for b only, with only a single variable in the regression; the **LDF Only** predictor, $\hat{b}X_{10}$.
4. The **Modeler** recommendation that we use predictor 2, the **Average**, unless the estimate of b is significant, in which case predictor 3, the **LDF Only**, is used. One should note that this is in defiance of standard actuarial practice, which would be to down-weight any unusually high LDFs.
5. So we can look at the **Anti-Modeler** (or **Actuary**), which does the exact opposite of the **Modeler** estimator, and uses predictor 3, the **LDF Only**, unless it's found to be significant, in which case it uses 2, the **Average**.

	Estimators				
Loss Criteria	**Full OLS**	**Average**	**LDF Only**	**Modeler**	**Anti-Modeler**
Bias	(0.00)	0.00	(0.00)	(0.01)	0.01
RMSEP	1.25	1.18	1.10	1.15	1.13

[17] In the standard theory, loss functions are non-negative; so un-squared difference would not usually be considered a loss function.

[18] A reviewer of parts of an earlier draft of this paper asked whether it wouldn't be more appropriate to use *weighted* regressions in some contexts. It might be, but would introduce many technical complications which would take us too far afield to address. See note 9 for instance. Suffice to say, certain numbers would change in that case, but the conceptual issues would remain the same.

The negative signs in the Bias row mean the estimator is biased high; positive that it's biased low. It's not too difficult to understand this table. The **Full OLS**, **Average**, and **LDF Only** are all unbiased estimators of the mean of Y_{10}. The **Modeler** predictor *randomly* switches between the **Average** and the **LDF Only**, based on the significance test of *b*. Since *b* is positive, including it in the model only when it's high relative to its standard error, means that the expected value of $\hat{b}$, conditional on finding it significant, will be biased high. So, the **Modeler** predictor is biased high. The **Anti-Modeler** predictor does the opposite, so is biased low.

The RMSEP of **LDF Only** is the lowest, which is nice because that reflects the true model of the data. The MSEP (before the square root) of the **Modeler** is basically the *power*-weighted MSEPs of the **LDF Only** and the **Average**. The MSEP of the **Anti-Modeler** has just the converse weights. Since *b* is found significant only 28% of the time the **Modeler** estimator spends only 28% of its time as the **LDF Only,** which has the lower RMSEP**,** and spends 72% of its time as the **Average,** which has the higher RMSEP.

Since the biases are the same, but the RMSEP of the **Anti-Modeler** is lower, in this case we would say that sometimes it is better to do the *exact opposite* of what the modelers recommend.

To generalize this example, if one *knows* the correct model form, one should design an estimator and predictor for it; significance testing has nothing to do with it. If one does *not* know the correct model form, it's at least possible, as this example shows, that significance testing will lead one to make a suboptimal choice. If *b* had been a little larger, its estimate would have been found to be significant more often; but including the variable in the model only then, would still bias the result high. As *b* grows larger, the bias due to truncating the estimator via significance testing would go to 0 as the probability of finding significance goes to 100%. But we'll see in the results section that this special case is unlikely to occur.

I bring this up as an illustration of the dangers of relying blindly on significance testing, but I will leave it as an area of future research to delineate precisely the situations where one should or shouldn't run significance tests and what actions should be taken as a result. I have seen no such delineation in any of the actuarial literature which takes into account the above issues.

2.3.2 Estimator Selection

The issue of variable selection for an estimator, as far as I can tell, is far more complicated than simply running significance tests, and may have nothing really to do with it.[19] An optimal estimator, by *whatever* loss criteria, is likely to depend on the precise distributional and parameter assumptions underlying the *true* model; an estimator might be optimal in some regions of the parameter space and less optimal in others, optimal for some distributions and not for others. Since the true model in any real application is more or less unknown, it's unclear what relevance such results would have unless the estimators are very robust to a wide range of potential models and parameter combinations. Many of the theorems available to help find estimators are restricted to *unbiased* estimators, and when *biased* estimators are included, the problem becomes so much more extensive. Finally, many of those theorems are restricted to *fixed* variables, and in insurance our potential variables are mostly *random* variables.[20]

2.3.3 Statistical Power More Generally

We mentioned previously that *power* is a *function*, but then only gave the single values of power for our regression estimators. We now complete the discussion.

So, consider a simple test function φ. φ will take on the value of 1 when the criteria we're testing for is met (or conversely not met,) or 0 if it is not (or the converse.) Given all the other model assumptions as fixed, let's let only the parameter assumptions vary. We can describe our model of the data as some joint distribution P_θ of the random variables, while θ is a possible parameter assumption.

The *power* function[21], often denoted $\beta(\Theta)$, is then a function of θ as θ varies over its possible values, and equals the probability that φ=1, calculated on the assumption that P_θ is the true model for the data. One can write this

$$\theta \to \beta(\Theta) = P_\Theta(\varphi(M) = 1), \tag{2.14}$$

where M potentially encompasses all of the random variables of the model.

[19] See Lehmann, *Theory of Point Estimation* [11], for a graduate-level treatment of estimator theory.

[20] For those who wish to pursue the topic further, the correct topic heading here appears to be *errors-in-variables* models and also *latent variable* models. Please note the early pioneering work that James Stanard did on estimator properties for certain types of loss development, and I understand Hans Bühlmann and others have continued some of that work as well.

[21] This definition is adapted from Lucien LeCam's useful comparison of frequentist and subjectivist approaches to statistics [10]. According to Wikipedia, he "was the major figure during the period 1950 – 1990 in the development of abstract general asymptotic theory in mathematical statistics."

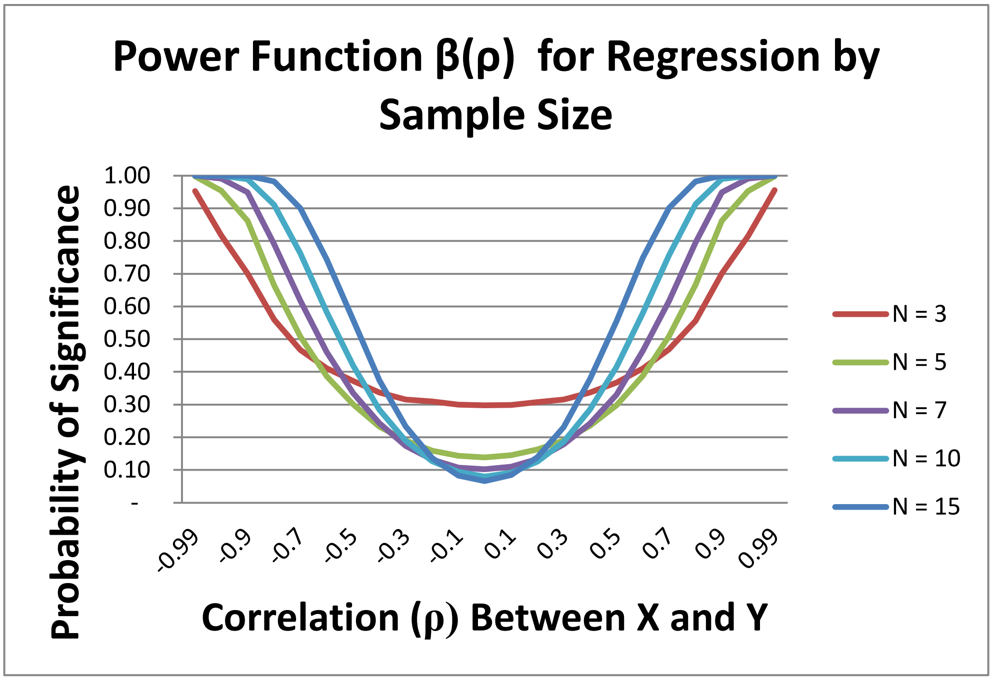
This is all a little abstract again, but the general formulation is useful to keep in mind, whenever one comes across any statistical analysis whatsoever:

1. One should ask what the model assumptions are and why; in other words what is P_θ the joint probability distribution of the random variables which correspond to the data; and then why: a useful heuristic might be, rather than contemplating "reasonableness" of assumptions, to start from the premise that the assumptions are false and see what reason there is to believe otherwise;

2. Even granting the model assumptions, if drawing any conclusion from the analysis, one should ask what the power of the analysis was to come to an opposite conclusion. If the test had low power to detect an alternative, there might be little reason to believe the results.

Returning to the specific test we're most interested in, significance, we can easily find the power function for any OLS regression as a function of the correlation ρ of two variables X and Y, where they are both normally distributed with means of 0 and SDs of 1. One can then generalize this result by adding constants for their means and scaling by different standard deviations. (Recall from formula 2.9 that the OLS coefficient in front of x_{n+1} is the product of the chain ladder estimate, the ratio of the y's mean to the x's, and the correlation and CV's, $\rho \frac{CV_y}{CV_x}$.)

Using the t-test, with the critical value of 2 discussed in the earlier example, the power function of the significance test for small sample sizes which might be relevant for yearly reserving triangles is the following:



Even with a sample of size 10, *real* correlations between X and Y that are between about (-55% and 55%) have a 50-50 chance or less of being detected. Whether in a particular modeling exercise a relationship of this size between two variables (such as the cumulative losses in one development age and the incremental losses in the next) will be declared significant is just a coin-flip. For a sample of size 5, a correlation of more than +/-70% is needed to have a better than even chance of detecting it.

Another way of looking at the same issue is to consider the number of years needed in order to have, say, a 50-50 chance of detecting a correlation of a given size. This could be an involved simulation exercise, so we will make the simplifying assumption that the sample correlation coefficient $\hat{\rho}$ is symmetric around its mean value ρ. This is probably ok unless ρ is close to +/-1, but then power isn't much of an issue anyway. There is a standard formula available for this[22]:

[22] See the Wikipedia article for *Statistical Hypothesis Testing*, for instance.

$$2 = \sqrt{\frac{\rho^2(n-2)}{(1-\rho^2)}} \tag{2.15}$$

where the first 2 is our critical value for the significance test,

the second 2 is for the number of variables, and n is the number of years.

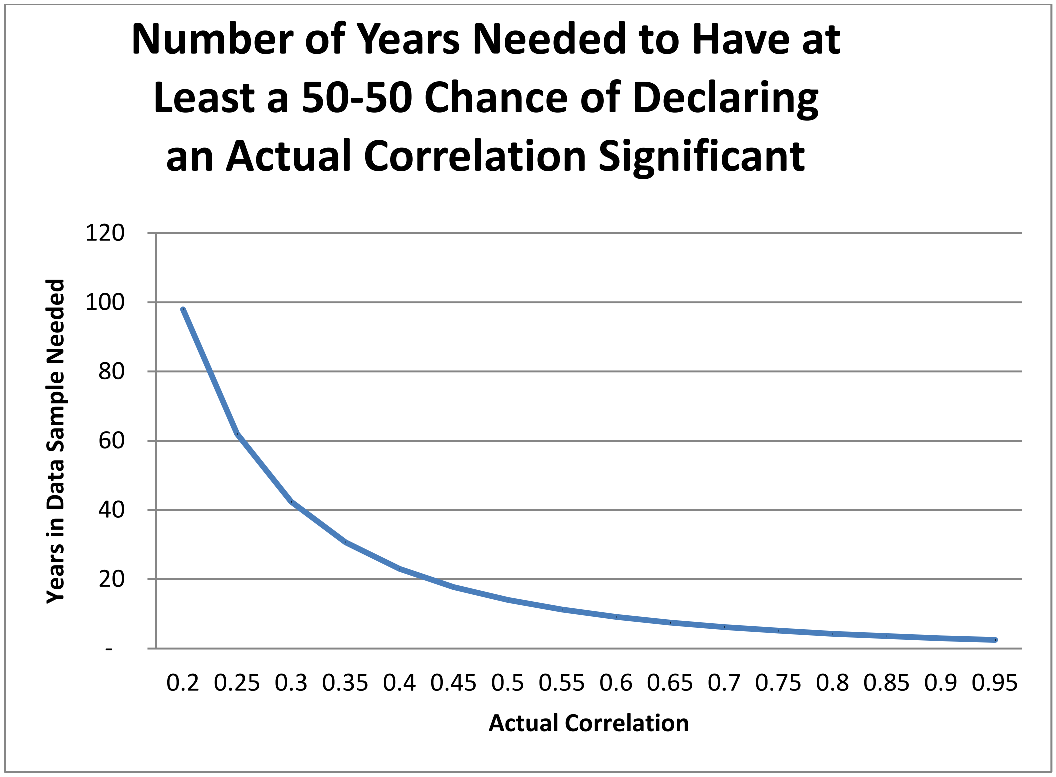
We easily rearrange this to express *n* as a function of ρ.

$$n = \frac{4}{\rho^2} - 2 \tag{2.16}$$

See Graph 2.1 on next page. I don't show the number of years needed for correlations of size .15, .10, or .05 because those are 176, 398, and 1598 respectively.

Keep in mind also that a finding of significance for an effect in a real-world modeling application is a function of: the model specification, the size of the effect, the size of the sample, and random chance. One can't in practice conclude from a significant or an insignificant finding alone which of those causes are responsible for the finding.

The most natural question to ask at this point is, what correlations would we expect to find in insurance? And we will address that issue in the next section, but first it might be worthwhile to discuss an amazing paper by David Freedman and generalize our discussion a little to other tests besides significance.



Graph 2.1

2.3.4 Diagnostic Power or the Lack Thereof

Statistical diagnostics, checks, tests, etc. usually address themselves to limited breakdowns of model assumptions. As we saw with significance, one fixes all of the distributional, independence, and functional assumptions and looks at whether or not the data is consistent with some parameters being 0. That's all the t-test is used for.

So, diagnostics are not usually direct tests of *all* of the assumptions of a model, but continue to assume some parts of that model. One should always keep in mind that a diagnostic outcome might be the result of some other model breakdown than what one is explicitly testing; this is too often forgotten in rushes to find some result, such as significance for instance.

All diagnostics have probabilistic results and all the issues with chance occurrence and power highlighted above apply. Even with non-parametric tests or robust inference procedures, once one has a probability model to work with, one can calculate the probabilities of passing or failing any test and the issue of power comes into play.

It's a common practice among modelers to also "teach to the test," meaning if they know that they'll be running some test, they will include additional features in their model, so that it will pass that particular test. A common such addition is to include some sort of time-series model in order to pass an autocorrelation test of the residuals. The *residuals* are the differences between the observed data and model estimates for that data. An autocorrelation (or serial correlation) test looks at the sample correlation between residuals in near-by years.

In some sense this is perfectly legitimate: if the residuals would otherwise show autocorrelation, there is trend within the data, and the model should perhaps be adjusted. It will definitely improve the model fit to the data. But the question is whether it will improve the model's predictions as well. The big--usually unasked--question is whether the particular model that is used to adjust for this trend is really true and why. Including such a sub-model within a model makes it harder to analyze the whole model, and may just be sweeping model misspecification issues under the rug.

If one opens up one's imagination to more general statistical assumptions than usually contemplated, then there is even more reason to question modeling results. David Freedman in a paper entitled "Diagnostics Can Not Have Much Power Against General Alternatives" [3] took one-by-one the standard assumptions underlying most statistical models, and showed that given any test, diagnostic or combination of them, which the standard assumptions would pass with a certain probability, there are alternative assumptions that are very different from the standard ones, but which would pass with the same or greater probability.

Although Freedman advises that all diagnostics should be viewed with a healthy dose of skepticism, he does *not* conclude that diagnostics should be ignored, quite the opposite, and he recommends they be employed and published more often. This is because diagnostics can still occasionally detect gross violations of model assumptions. But his results also clearly imply that one cannot simply rely on diagnostics to determine whether a model is true. One needs prior theory and experience to convincingly narrow down the model possibilities before diagnostics can be of much use.

Even then, as we saw above, with small samples or volatile processes, we need to be realistic about what we can and cannot accomplish from statistical analyses and only the data at hand.

3. RESULTS AND DISCUSSION

3.1 Some Examples of Loss Development Correlation in Insurance

3.1.1 Claim Count Development on Claims-Made Business

Consider a claims-made book of business. Suppose that X_i are the claims still open at the end of the first development period for accident year i and Y_i are the incremental changes in open claims during the next development period. Let's assume, as is often done, that X_i is Poisson distributed with mean μ. Now suppose that for each of the X_i claims, independently of the other claims, there is a certain probability p that it will close during the development period. Then Y_i is -1 times a Binomial random variable with parameters X_i and p.

Then it is possible to show that the correlation between X and Y is:

$$-\sqrt{p} \tag{3.1}$$

So, for instance, referring to Table 2.1 for a quick approximation, for a $|p|$ below .25 ($\sqrt{p}$<.5), we are unlikely to find the correlation significant with less than about 14 years. For p's over .5 however, we need only about 6 years or less.

3.1.2 Completely *Dependent* Development on Claims-Made Business

The above is a special case of a more general situation where the development on each claim during the next period is a multiple of the claim itself (in the above case, either -1 or 0.) We can generalize this example, by means of Collective Risk Theory, to include reported claims simply, rather than just open claims, reported severity distributions in the first period and then a distribution of incremental reported development factors which will be multiplied to each claim severity for the next. Let's call the development random variable λ and say it has a coefficient of variation (its standard deviation over its mean) of CV_λ.

Then the correlation is (where the -1 is if there is expected negative development):

$$\frac{\pm 1}{(1+(CV_\lambda)^2)^{1/2}} \tag{3.2}$$

Intuitively, if λ were a constant, $\boldsymbol{X}$ and $\boldsymbol{Y}$ would be perfectly correlated; but there is random variation in the development (CV_λ) which is clouding the relationship. If the expected development were 0 the CV would be infinite and there would be no correlation, though there would still be dependence via the claim count.[23]

Now, the question is what magnitudes of CV's should we expect in insurance? Well, that's going to vary by line-of-business, the specifics of a company's reserving practice and the details of the claims.

On one extreme, one could imagine that the claims-adjusters were nearly always right, at least on average, about the ultimate values of the claims they adjusted during the first period. In this case, the mean of λ would be close to 0, in which case, because the mean is in the denominator, CV_λ could be enormous. One would then have very low correlation and find it almost impossible to detect it, even though the development was still dependent on the losses from the prior period.

On another extreme, one could imagine that the claims department is stair-stepping their reserves, using claims signals for instance, or some other reserving practice which doesn't match averages, and the average development might be quite large in the next period. Then it would depend on the spread of that development, which might still be quite wide if there's a diverse set of claims.

I certainly don't know what ranges this parameter might take for different books, but the one book I did look at, which was a not too volatile professional liability account, had CV's in the second period of about 15, which means a correlation around 7%, and very little chance whatsoever of detecting that in a significance test (one would need 800 years.)

3.1.3 Completely *Independent* Development on Claims-Made Business

At another extreme, we can continue with our model for reported losses, but this time we assume that the development in the next period is completely *independent* of the severities in the prior period: they are just additive amounts that emerge for each claim independently of what the severity on the claim was previously. Because the claim count is common to both periods, the losses are still correlated however. Let's still call the development random variable λ and with a CV_λ. But now we must include the reported severity in the first period, which we'll assume has CV_S.

Then the correlation is:

[23] Zero correlation does not imply independence except with normally distributed variables for instance.

$$\frac{\pm 1}{(1+(CV_S)^2)^{1/2}(1+(CV_\lambda)^2)^{1/2}} \tag{3.3}$$

Intuitively, $\boldsymbol{X}$ and $\boldsymbol{Y}$ are correlated only by having the same claim count in common, but there is random variation in the development (CV_λ) and random variation in the severity (CV_S) which do nothing but weaken the relationship.

Again, any CV's will depend upon the particulars of a book of business, but for my one account above, I found CV's of about 3 for the reported severity in the first period and about 8 for the incremental development in the second. (Note this is the CV of λ viewed as an additive amount rather than the multiplicative amount from before.) I find a correlation here of about 4%, again undetectable for all practical purposes.

So far, I've only considered the first two development periods. As we move along the triangle, taking $\boldsymbol{X}$ as the cumulative development and $\boldsymbol{Y}$ as the incremental development, I might expect the CV's associated with **X** to grow as more information became available to precisely determine claim values. We might also expect the CV_λ to perhaps explode while the prior estimates are getting more and more accurate so the incremental development averages are close to 0, while whatever development there is might be highly volatile, and perhaps volatile enough to overcome the large number of claims undergoing no more development. But this is just speculation, and there could be all sorts of patterns of CV development. It might not even make much sense to think of these CV's as immutable parameters that could be meaningfully estimated, though the one account I looked at had much more stable CV's by report and development year than I would have expected.

3.1.4 Adding Independent IBNR

If we generalize to occurrence business and add pure IBNR claims going into the second period, which are completely independent of the claims and losses from the first period, then one can show that the correlation gets scaled down by a factor which includes the ratio of the additional variance of the new losses to the original variance.

3.1.5 Adding *Dependent* IBNR

If one adds pure IBNR claims going into the second period, which are dependent in any way on the claims and losses from the first period, as they would be if they were the result of common exposure for instance, then one can show that once again the correlation gets scaled down by the additional variance of the new losses, but a new additive term enters into the equation as well for correlation as a result of the common exposure.

3.1.6 Discussion

So, in all of the above cases, except where there is 0 expected development, there is non-zero correlation between the aggregate losses in one period and the incremental losses in the next. So if modelers have failed to find the loss development factor significant, it might very likely be due to the lack of statistical power of their analyses.

3.2 So What Development Methods Should We Use?

As I mentioned even with the simple example of 2.3.1, that is actually a very involved question. For what it's worth, based on a few tests with even tamer parameters than the ones I found for the account discussed above, I could not find much practical difference between using an average incremental development, chain-ladder, an average of the two, or the modeler or anti-modeler estimators. OLS seemed to perform slightly worse than those, but a 5-10% difference in RMSEP hardly seems to matter much.

There is a certain amount of *irreducible* uncertainty[24] to development that cannot be decreased by any estimator no matter how clever. I suspect almost any standard estimators or actuarial methods would be about as good (or bad) as any other, and as long as a number of methods are applied, there is just no practical benefit from worrying about estimator optimality.

What *does* have a real practical benefit is if an actuary can determine the parameters themselves independently of this data. Or, barring a definitive determination, a Bayesian method, as long as the prior concentrated close to the true answer, could make a practical difference as well.

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[24] There is an unfortunate ambiguity is many uses of the term *uncertainty*. Sometimes it refers to a psychological state, something akin to doubt. And it sometimes refers to random, or apparently random, variation in the world which is merely one potential *cause* of that psychological state.

3.2.1 Example of Meaningful Estimation Improvement

The real advantage of traditional actuarial methods is not their optimality properties, but that they bring the actuary quickly into close contact with the data. If there is any information the actuary can discover which will *reduce* uncertainty, then *that* could have a large effect on estimation accuracy.

Let's suppose we're at a primary medical malpractice writer. Fortunately we only write claims-made policies so except for the occasional DD&R policy, which we can always reserve separately, there's no pure IBNR, and all loss development is from claims reported in the first period. We buy reinsurance to put a cap on our maximum loss as well.

For our model let's assume that we have 10 years of data and we're trying to project the 11th. For each report year,

1. There's a fixed number of "nuisance" claims, let's say 10, of negligible value (like clearly illegitimate claims.)
2. One in every 10 years there a "catastrophic" claim (like a fetal brain injury with negligence from the OB) that will hit the reinsurance. This is a Bernoulli variable with p=0.1
3. Otherwise there are "regular" claims each year that are Poisson distributed with a mean of 10.
4. During the first development period the claims department assigns them all 1 unit (think $100,000 maybe) until it can complete an initial investigation which won't finish till at least the next year.
5. During the next incremental development period, they will discover that the 10 nuisance claims were just that, and drop all their reserves to 0, for an incremental change of -10.
6. Any catastrophic claim will be discovered and its reserve increased by 10 ($1M.)
7. Of the remaining regular claims, there's a 30% chance that each can have its reserve increased by 2 ($200K.) The others will remain as is.

Since we've specified the model we can calculate explicitly anything we can imagine.

For instance, from the above the expected losses in the first year are 20.1: 10 nuisance claims of 1, .1 catastrophic claims, and 10 regular claims.

The expected incremental losses reported in the next year are: 10 decline by 1 each, so -10; .1 increase by 10, for 1 on average; and 30% of 10, or 3, increase by 2, for 6. So the expected incremental development is -3.[25]

We can explicitly derive a number of other values such as conditional expectations based on various levels of knowledge and detail known or knowable to an actuary. But for now, let's simulate our model with 10 report years of data and apply our methods, starting with OLS regression.

		Mean		
	Mean	**Standard**	**Mean**	**Power**
	Value	**Error**	**+/-2 SE**	**(β)**
$\hat{a}$	(16.56)	8.93	(-34.43,1.31)	0.51
$\hat{b}$	0.67	0.44	(-0.21,1.55)	0.39

So, despite the effect of the number of claims from the prior period going into the next, and even $\hat{b}$'s relatively high expected value, it's only found significant 39% of the time.

One of the first things we should perhaps have done as actuaries is of course look at the age-to-age (ATA) factors in a triangle. Here is a single iteration, which represents for us something like the position we're actually in when trying to reserve: we don't have the possibility of simulating 200,000 separate draws, nor do we know the full distributions of potential outcomes with certainty. Here's one iteration where the LDF was found to be insignificant:

Report Year	Cumulative Reported Loss By Development Age 1	2	ATA LDF
1	21	15	0.71
2	22	16	0.73

[25] Negative development is not uncommon in medical malpractice.

3	21	33	1.57
4	22	26	1.18
5	22	14	0.64
6	29	29	1.00
7	18	14	0.78
8	21	21	1.00
9	16	12	0.75
10	24	30	1.25
Average:	21.60	21.00	0.96
	Incremental Development:	(0.60)	

It's a fairly typical sample year: the average of the losses at Age 1 is close to the mean of 20.1, and the incremental development is a little low compared to the mean, but nothing shocking. The one ATA LDF which stands out is the 1.57 for Report Year 3, maybe we should investigate? It turns out that is the one year that had a catastrophic claim. Maybe we should reserve that separately?

But we do not stop there. Our job is to project the 11th year from the data available. But the *real* data available is not just the sheet of numbers above; it's all of our experience, and the experience of those we can learn from in the claims department and elsewhere. Based on our level of curiosity, knowledge, and energy we might assume that we:

1. Take the lazy method and simply add an average. Let's call this **Average**.
2. Run a two variable OLS regression, call it **Full OLS**. Recall this is an average of an LDF estimator and the **Average**.
3. Apply the **Modeler** routine: test for significance first and assume a constant **Average** amount of development in the next year unless the LDF is found significant.
4. Having been around awhile and figured out that there's always 10 nuisance claims, and about 30% of claims increase by 2, while 1 in 10 years have the catastrophic claim, we parameterize a conditional expected value that tells us the expected number of claims based on the number of aggregate claims in development year 0 and apply that to report year 10. Let's call this the **Parameterized.**
5. Finally, let's suppose we are energetic and experienced enough to actually determine to which class each one of the claims belong and then apply the parameters from 4. Let's call this **Energetic**, and basically the only variability left in the reserve forecast comes

from the 30% probability of a regular claim developing upward.

Running our simulation with these loss development methods we find, with % reduction in RMSEP measured relative to the **Average** method:

Improvement in Prediction Error From Different Approaches to Reserving			
Approach	**Bias**	**RMSEP**	**% Reduction**
Average	0.00	4.82	
Full OLS	0.00	4.54	-6%
Modeler	(0.00)	4.73	-2%
Parameterized	(0.01)	4.04	-16%
Energetic	(0.00)	2.90	-40%

Please note that only the first three are standard estimators, and the latter are estimators where additional knowledge has been brought to bear on the parameters as described in 4 and 5 above.

Once again, the **Modeler** recommendations are not the best and do worse than just OLS regression, and show those recommendations can be unwise; but the practical difference in RMSEP is hardly important. But look on the other hand at how it could pay to understand and know something about the data.

The **Energetic** does best because he or she is able to *reduce* the uncertainty around the claim type, even though the uncertainty around the development of regular claims is still irreducible.

Of course if one has 20,000 claims a year rather than 20, it's not practical to personally read every claim file to determine its underlying allegations and what its claim type is. But even as claims databases get bigger it's not at all a given that the most effective computer algorithms would not be doing essentially the same that a person would do if equipped with the same patience and computational ability as a machine. I suspect every situation must be examined on a case-by-case basis to determine the most appropriate approaches. I also strongly suspect that in many cases where some advanced quantitative technique can be shown to be effective, there are simpler and more direct methods which are just as effective; but I'll have to leave the exploration of this topic as an area for future research.

The point of this example is not that modeling could not do what a traditional actuarial method could do nor vice versa. The point was merely that following a traditional actuarial method, the actuary was alerted quickly to an issue that when investigated further yielded a very large pay-off in terms of accuracy. Once an actuary *knows* that, for instance, certain claim types are an important variable, there are always ways to design programs to capture claim type or to model for it. The question is *how* is the actuary going to *first* discover this? And here I would think that the simpler the exploratory method the better.

In reality, we'll never know the parameters underlying the *true* model, though some believe that Bayesian estimation might get us close. There's a risk as well that we will be *fooled by randomness* to use Nassim Taleb's felicitous phrase, and by digging into the data and "learning" more we are just fooling ourselves that something is more predictable than it really is. There's no way to tell ahead of time, but if one doesn't look, one doesn't find.

It's also highly judgmental how much time one should spend looking for information, especially given the risk of self-deception, rather than just making a selection and moving on. I doubt any hard and fast rules can ever be given, and different actuaries will choose to spend their time differently.

Nonetheless, I believe we often face a situation in which there *is* reducible uncertainty, and with enough hard work and looking beyond the mere numbers in an aggregate loss triangle we might discover it.

Finally, in this example, the large losses came from truly horrific claims: fetal brain injuries. An insurer has a social obligation to learn as much as it can about the circumstances and possible prevention of such events, given the insurer's other social responsibilities. If as a result of improved risk management the probability of such claims dropped from .1 to .05, it might invalidate the model assumptions, but so be it. There are more important things than model validity.

3.3 The Miraculous Assumption of I.I.D.[26]

I.i.d. is the workhorse of statistical assumptions and without it little gets done.

By definition, random variables are *independent* if the probability that any one takes on a value (or set of values) remains the same no matter what values the others take on. They're *identically* distributed if they have the same distributions.

Mathematically, independence means that the *joint* distribution of the variables factors into the product of the *marginal* distributions of each variable. Identically distributed implies that all means, variances, and higher moments are the same (though not conversely.)

Statistically, if one has a series of i.i.d. random variables, the sample mean is often an excellent estimator of the random variables' means and the sample variance is unbiased and allows one to even calculate the error in one's mean estimate. In short, the knowledge of the values of any subset of the series of random variables will allow one to predict all of the others, at least probabilistically.

If one's data series comes in the form of, for instance, a loss triangle where the lower right triangle is still to come, assuming i.i.d. is no less than **assuming one can predict the future**.

If random variables are *not* i.i.d., the sample mean need not be a very good estimator of the random variables' means, the standard errors in significance tests may be wrong, and the sample variance may be very biased. In the last case, one can even think one has a much better estimate of the mean than one really has.

The Collective Risk Theory assumes that the claim counts in every year are i.i.d., the severities are independent of the claim counts, and they are i.i.d. within a particular year and across the years. [27]

The easiest way to appreciate the implausibility of the independence assumption is to recognize that any common cause that is neither certain to happen nor to not happen and that could effect, say, the means of two variables, even with different effects, would give them a non-zero correlation, and hence they'd no longer be independent.

So, any underwriting changes, marketing changes, settlement changes, inflationary changes,

---

[26] See William Kruskal's highly regarded American Statistical Association presidential address: "Statistics and Miracles: The Casual Assumption of Independence" [9] for a discussion of the importance of the independence assumption in the evaluation of testimony for miracles, among other things.
Many of the remarks in this section apply equally to certain similar concepts such as *exchangeability* in Bayesian analyses.

[27] One often adjusts for some theorized trend and portfolio changes (like deductible, etc.) first, but *then* the variables are assumed i.i.d..

weather changes, social changes, etc….anything which *could* serve to change the occurrences of claims or their settlement amounts will either act to change the distributions or to create dependence or to do both. The assumption of i.i.d. would fail.

In classical applications of statistics that are widely regarded as successful, such as for example, casino games, medical testing, population surveying, and general experimentation, i.i.d. is not simply *assumed*, it requires hard work to achieve. And even then, it's usually not perfect.

Now is it possible that all of the different dependency effects will somehow negate each other? Or that we will somehow be able to adjust accurately for all of them? Sure it's possible, but it would be little short of a miracle.

If someone is simply presenting a theorem in pure mathematics, then one can assume whatever one likes. But if one is presenting any real world conclusion, one should pay attention to the validity of one's assumptions. Since the assumers of i.i.d. are arrogating to themselves the ability to predict the future, the onus should be on them to establish that the assumption is true or at least cannot be very far from the truth. I would think that they should at least study and present the sensitivity of all of their conclusions to this assumption, but this appears to be a still largely unexplored area of actuarial research.

4. CONCLUSIONS

We saw immediately above that the fundamental assumption of statistical analysis is almost certainly false when applied to insurance.

We saw earlier that even making this assumption, one is unlikely, because of limitations to statistical power, to be able to discover by statistical means anything very useful about the claims generation process or which reserving method to use.

We saw for a few examples that some common modeling recommendations can be unwise, though we did not show that they would *always* be unwise.

We only focused on modeling recommendations which have been applied to aggregate loss triangles; the recommendations might make more sense for individual claims modeling. I believe that many of the results in this paper generalize to contexts outside of modeling just yearly aggregate loss triangles, but that has to remain an area of future research for now.

I believe that we've *practically* refuted two particular claims made on behalf of stochastic reserving of yearly aggregate triangles: the claims that assumptions can be tested and models validated *effectively*. But we've only showed this for a few examples and provided the conceptual tools to help study the issue. Others might want to research the situation and determine a precise delineation of when significance testing might lead to an optimal model and when the model diagnostics are most effective.

We saw that one might be able to make greater progress in estimating reserves by focusing on fact-finding rather than model-checking.

Outside of the aggregate triangle modeling we've considered, as datasets become larger, there are computer algorithms (such as text searches, clustering, etc.) that might prove extremely useful in the data exploratory process. Many of these methods don't rely on statistical assumptions at all, though no doubt they have their own issues. Regardless, my criticism in this paper was leveled at some careless applications of statistical assumptions and modeling, and not at all "advanced" techniques whatsoever.

There are also other arguments made on behalf of stochastic modeling. Some of these are more plausible, and, regardless, some of them are persuasive in certain situations.

Generally, I believe that actuaries need to become much more skeptical and critical of the claims made on behalf of statistical modeling. For many, the technology and the imagined power of statistical analyses are just too seductive. The result can be a lot of wasted effort and misleading models.

David Freedman in "As Others See Us: A Case Study in Path Analysis" [4], which carefully analyzes an application of path models in social science, notes about social scientists in general who apply advanced quantitative techniques that: "nobody pays much attention to the assumptions, and the technology tends to overwhelm common sense. "

Freedman also cites in that paper studies showing that major econometric forecasting models do very poorly unless frequently revised and unless some of their parameters are re-estimated *subjectively* by modelers; and even then they do no better than forecasters without models. (page 123)

The solution is simple, in fact probably too simple for many to accept. Freedman:

"My opinion is that investigators need to think more about the underlying process, and look more closely at the data, without the distorting prism of conventional (and largely irrelevant) stochastic models. Estimating nonexistent parameters cannot be very fruitful. And it must be equally a waste of time to test theories on the basis of statistical hypotheses that are rooted neither in prior theory nor in fact, even if the algorithms are recited in every statistics text without caveat."

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The opinions expressed here are not necessarily those of any but the author. And anything good about the paper is due to the above reviewers, while any errors or obnoxious opinions that remain are entirely those of the author.

Supplementary Material

Most of the articles referred to in notes of this paper are available on the internet free, for purchase, or through a library. Note in particular that many of the articles by David Freedman, including ones now published in *Statistical Models and Causal Inference* are freely available at http://www.stat.berkeley.edu/~freedman/

Klugman's *Estimation, Evaluation, and Selection of Actuarial Models* is available here: www.casact.org/library/studynotes/**klugman**4.pdf

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Abbreviations and notations

CV, coefficient of variation
DD&R, death disability and retirement
MSEP, mean square error of prediction
OLS, ordinary least squares
OB, obstetrician
RMSEP, root mean square error of prediction

Biography of the Author

Keith Curley has done reserving for Deloitte & Touche, predictive modeling for Farmers Insurance, and casualty reinsurance pricing and underwriting for Swiss Reinsurance Company. He is a Fellow of the CAS and a Member of the American Academy of Actuaries. In his work, he's been responsible for more bad models being built than anyone he knows.

The only statistics course he ever took was fortunately for him taught by David Freedman. Keith didn't appreciate it nearly as much as he should have at the time.

He can be reached at: keith_curley@hotmail.com

Runoff Collateral Requirements

James Ely

Abstract
Motivation: To provide a simple method of estimating collateral for captive reinsurance contracts that are in runoff with respect to the issuing carrier.

Method: The paper demonstrates a simplified application of individual claim development.

Results: For small open claim counts, the parameter risk and distribution risk of the estimated collateral requirement is reduced by the presence of the loss limit.

Conclusions: Individual claim development addresses the two problems of reserving subject to a loss limit and dealing with a small claim count.

1. INTRODUCTION

The purpose of this paper is to present a method of estimating collateral for captive reinsurance contracts in runoff. Runoff situations are fairly common in captive management, where each ceding company and captive pair must be evaluated separately. The problem is to estimate the reserve distribution for a small open claim count subject to the presence of a loss limit. The study demonstrates a simple example of individual claim development. It was found that the collateral requirement for a primary layer is largely independent of the form of the model distribution of development factors and only modestly impacted by variations in its parameters.

1.1 Research Context

The paper will focus on a problem common to the reserving of workers compensation for captive insurance companies or large deductibles. It will be presented in the context of captive management because that area of practice is less well documented in the actuarial literature. In runoff situations we face the challenge of estimating the workers compensation tail for a small number of claims subject to a loss limitation. I have chosen to apply the method of individual claim development. This method is discussed in the actuarial literature in the calculation of excess loss factors and in excess of loss reinsurance pricing and reserving. The most recent discussion of the NCCI's excess loss factors was provided by Dan Corro and Greg Engl [1], which builds on the work of José Couret [2]. On the reinsurance side, William Gillam and Gary Venter [3] described the method in use in 1986. Stephen Philbrick and Keith Holler [4] considered a weakness of the method in 1996. More recently, John Mahon [5] described a version with a sophisticated development process.

1.2 Objective

This paper will adapt a method common in excess of loss reinsurance to a primary insurance problem. It will consider simplifying assumptions appropriate for the new setting.

2 BACKGROUND AND METHODS

This section will provide background for the problem and the proposed solution.

2.1 Background

Group captives insure the risk of their members, while agency captives insure risk that is written by the agency owner. Both types of captives are common, many of which were formed to provide stable markets for workers compensation. Captives are prohibited from writing workers compensation directly, out of solvency concerns. Therefore, they contract with admitted carriers to write the direct coverage and cede it to the captive. This type of arrangement is referred to as "fronting" and the admitted carrier as the "fronting carrier". In a fronting arrangement the fronting carrier cedes only the primary layer to the captive, while often retaining the excess layer for its own portfolio. The fronting carrier provides the captive excess coverage, infrastructure, and credit enhancement. Our focus here is on the credit aspect. The fronting carrier is responsible for the payment of claims regardless of the captive's ability to pay. As a consequence, fronting carriers require collateral for their losses ceded to captives, most of which is provided by the premiums written.

Fronting for captives is a competitive business. Companies compete on the basis of fees, excess insurance costs, direct premium rates, underwriting appetite, and other terms and conditions. Captives occasionally change fronting carriers to obtain better pricing and/or terms, while fronting carriers occasionally non-renew unprofitable or excessively risky programs. Both situations result in runoff situations that require periodic collateral adjustments.

Reinsurance contracts are often vague with respect to the method to be used to calculate collateral requirements. It is often left up to the fronting company to determine the appropriate collateral, which typically includes a safety margin. In runoff situations the fronting company has little incentive to release collateral to the former client. Meanwhile, the owners of agency and group captives are not professional risk takers. Wide gaps in expectations regarding collateral between fronting companies and captives frequently occur, which occasionally lead to disputes and arbitration. It is hoped that better methodology will help reduce these disputes.

2.2 Selection of Method

Consider the various reserving methods used along the timeline of a fixed block of claims. Initially we have no loss information, we rely on exposure rating and expected loss ratio methods. As we begin to receive loss data we blend actual loss data with an exposure based projection in the Bornhuetter-Ferguson method. When the loss data becomes sufficient we tend to prefer link ratio projections that rely on loss data only. As the number of open claims becomes small we reach a point where it no longer makes sense to estimate a reserve that is primarily IBNER (Incurred But Not Enough Reported) from the aggregate loss data.

For ongoing programs we typically ignore this problem or make ad hoc adjustments since the mature years contribute only a small fraction of the total reserve. In runoff situations these mature years assume significance in their own right. Most workers compensation captives would be considered small portfolios relative to more typical reserving situations. It is not unusual for a captive to have fewer than 20 open claims assumed from a former fronting company five years after the end of the last exposure period.

While the number of open claims is getting small, they are also revealing their severity potential. The more serious claims will have reached or begun to approach the retention. Our projection of future development should take into account the proximity of individual claims to the retention.

Individual claim development provides a method that allows us to estimate IBNER from the open claims. It is responsive to the open claim count and it allows us to explicitly consider the presence of the specific limit. The method fits the form of the data for runoff calculations.

Individual claim development treats loss development as a stochastic process, which adds a level of complexity compared with deterministic methods. But collateral calculations require a distributional estimate of unpaid losses, so there is no net cost in effort or complexity using individual claim development in this setting.

2.3 Description of Method

The basic features of individual claim development are:

- Development factors are applied to open reserves.
- The development factors are considered as distributions.
- Unlimited development is computed claim by claim, with the loss limit applied to the resulting unlimited developed claim.

With this approach the limited expected value of each claim can be calculated explicitly, or the distribution of all open claims can be simulated. The limited expected value is useful for reserving questions, but the later findings regarding distribution risk and parameter risk would not apply. Collateral estimation requires simulation of the full distribution to protect the ceding carrier against credit losses in the event of adverse losses development.

2.4 Reserve Development Factors

We would like to develop the total reserve from the open claim data. We can do this by relating total reserves to case reserves. This makes sense because workers compensation development is dominated by reserve development after the first couple of years. IBNER is a function of the open claims.

The first step is to derive development factors for loss reserves that would produce a result consistent with the paid and incurred loss development methods. The reserve development factors can be calculated using algebra from incurred and paid loss development factors.
An example of the calculation of reserve development factors is shown below using the five-year average Arizona factors published in the NCCI 2012 Annual Statistical Bulletin[1].

Arizona

Accident Year	Development Factors Paid	Development Factors Incurred
2008	1.527	1.298
2007	1.480	1.257
2006	1.441	1.229
2005	1.409	1.210

We take the reciprocal of each factor to obtain the proportion of losses expected to be paid or incurred:

Accident	Development Factors		Proportion Emerged	
Year	Paid	Incurred	Paid	Incurred
2008	1.527	1.298	0.655	0.770
2007	1.480	1.257	0.676	0.796
2006	1.441	1.229	0.694	0.814
2005	1.409	1.21	0.710	0.826

We can then obtain the expected proportion of losses in reserve status by taking the complement of the proportion emerged. The complement of the proportion paid is the total reserve expressed as a proportion of ultimate loss. The complement of the proportion incurred is the IBNR. The proportion of ultimate losses in case reserve status is obtained by subtracting IBNR from the total reserve.

[1] Used with the permission of the NCCI.

Accident Year	Proportion Emerged Paid	Incurred	Reserve Total	IBNR	Case
2008	0.655	0.770	0.345	0.230	0.116
2007	0.676	0.796	0.324	0.204	0.120
2006	0.694	0.814	0.306	0.186	0.120
2005	0.710	0.826	0.290	0.174	0.117

Finally, the reserve development factor is the ratio of total reserves to case reserves:

Accident Year	Expected Case Reserve	Expected Total Reserve	Reserve Development Factor
2008	0.116	0.345	2.987
2007	0.120	0.324	2.706
2006	0.120	0.306	2.557
2005	0.117	0.290	2.487

Appendix A displays the reserve development factors for each NCCI state, based on the calculation described above. The reserve development factors vary greatly between states, but within each state they tend to remain fairly stable from year to year.

Individual state factors range from about 1.5 to 5. At the top of the range we find two states with escalating benefits (CT, NH) and two states that have very low tail factors (NM, SC). We should expect jurisdictional differences such as benefit laws to be reflected in reserve development factors. On the other hand, the presence of two states with small tail factors highlights the potential instability in a method that relies on a ratio of two small numbers. South Carolina presents a particularly interesting example in that the calculated reserve development factors are wildly unstable from year to year.

Excess reinsurance applications of individual claim development utilize more detailed development schemes than statewide average factors. They may include additional factors such as injury type and claim size. The excess reinsurance problem is far more complex because reinsurers are attempting to estimate potential excess losses at early development periods. In contrast, our problem demands simplicity considering the small reserves involved. Fortunately, we will see that runoff collateral calculations tend to be quite forgiving of simplification.

2.5 Development Factors Considered as a Distribution

When we consider the development of individual claims, we can be certain that they will not all develop in an identical fashion. Therefore, it makes sense to consider individual claim development as a random process. We make this random process conform with observed data by requiring that its mean is given by the reserve development factor.

I have chosen to model the reserve development factors with a lognormal distribution. The lognormal distribution model is easy to work with; it is convenient that its moment distributions are also lognormal and it is available as a built-in function in Excel. Other possible distributional forms will be considered.

2.6 Model Parameters

The parameters of the lognormal distribution of reserve development factors are completely determined by the mean and coefficient of variation. We have seen that the mean of the reserve development factor distribution can be estimated from the loss development factors, so to parameterize the model we need only fix the coefficient of variation.

I set the coefficient of variation at 0.5, as reported by Corro and Engl. On an intuitive level this seems low, but the authors discuss this point in detail, as it is one of the points of departure from the previous work on the topic.

The shape parameter σ^2 can be derived from the formula:

$$CV^2 = \exp(\sigma^2) - 1 \tag{2.1}$$

For a CV of 0.5 this yields $\sigma^2 = .223$.

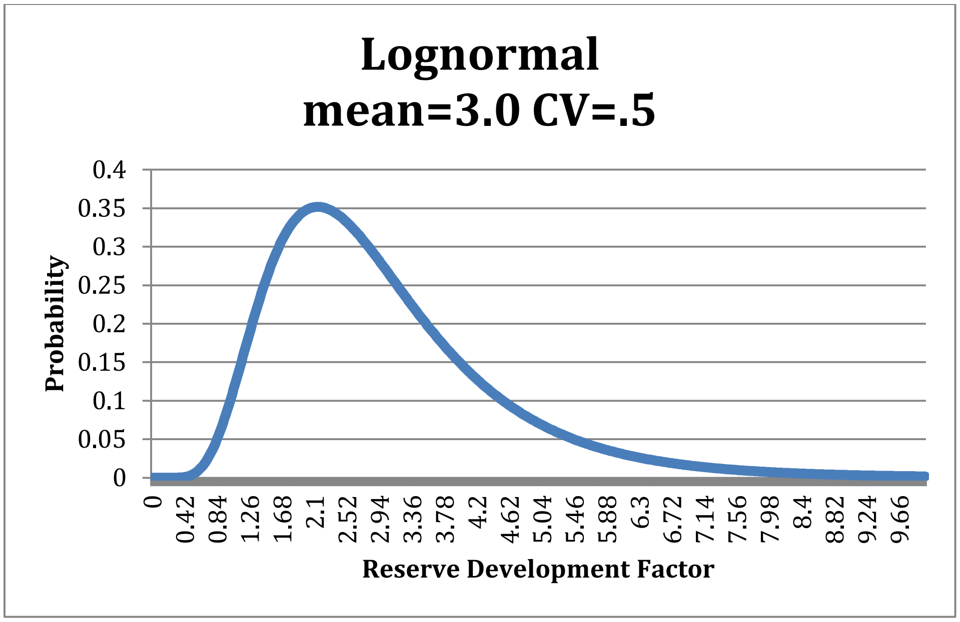
I selected a mean reserve development factor of 3.0 for the model. It is slightly higher than the indicated NCCI Countrywide factors and well within the range of factors shown in Appendix A. We can now find the location parameter μ from the well-known formula:

$$\text{mean} = \exp(\mu + \sigma^2/2) \tag{2.2}$$

$$\mu = \ln(\text{mean}) - \sigma^2/2 \tag{2.3}$$

Solving for a mean of 3.0 and CV of 0.5 yields $\mu = .987$.

A graph of the model distribution of reserve development factors is shown at the top of the next page.



In the model, there is a 98% likelihood that case reserves will develop upwards. The most likely result is that any individual reserve will double, but a few reserves will develop by a large amount.

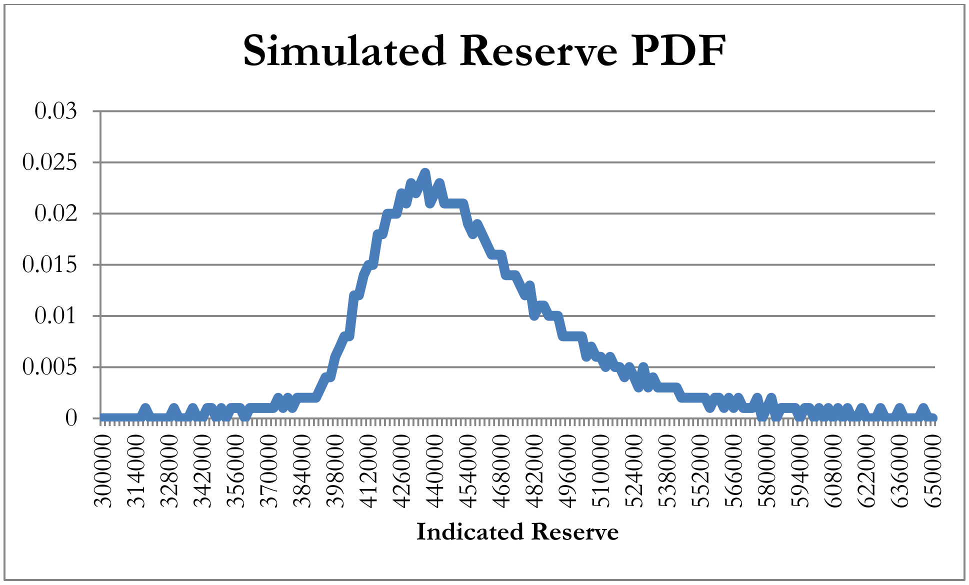
2.7 Collateral Requirement

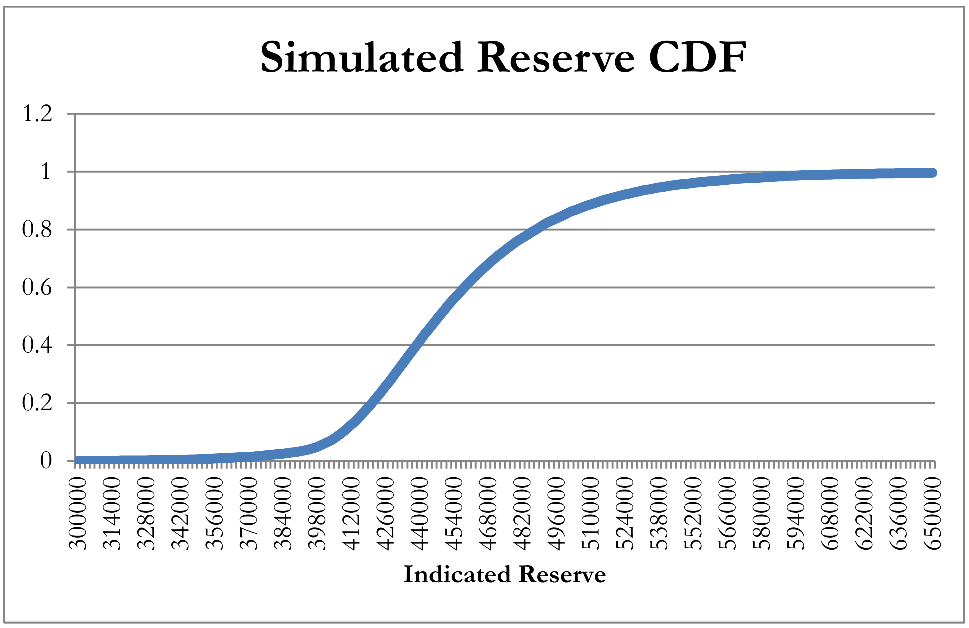
The collateral requirement for the program should consider potential for adverse development beyond the expected ultimate loss. To this end, I will estimate the distribution of limited reserve outcomes by simulation.

I will demonstrate the simulation outcomes for an actual example. The captive and fronting company relationship ran from 2004 to 2008. The captive remains in operation, but utilizes a different fronting company. The limited losses paid totaled $5.1 million through 12/31/2012, at which time there were 4 open claims with total limited reserves of $333,247. One of the claims has reached the retention of $400,000. The current values of the four open claims are as follows:

Claim	Limited Paid	Limited Case Reserve	Limited Incurred
1	217,909	182,091	400,000
2	221,190	117,844	339,034
3	-	29,500	29,500
4	16,922	3,812	20,734

For each trial in the simulation, I randomly generated 4 lognormal reserve development factors. The product of these factors and the case reserves generated the unlimited reserves, to which I added the payments and applied the retention limit.
The following graphs display the results of a simulation with 50,000 trials:





Key elements of the simulated distribution of the limited reserve are shown in the following table:

Percentile	Limited Reserve
50%	448,000
75%	478,000
90%	514,000
95%	540,000
98%	578,000
99%	604,000

3 RESULTS AND DISCUSSION

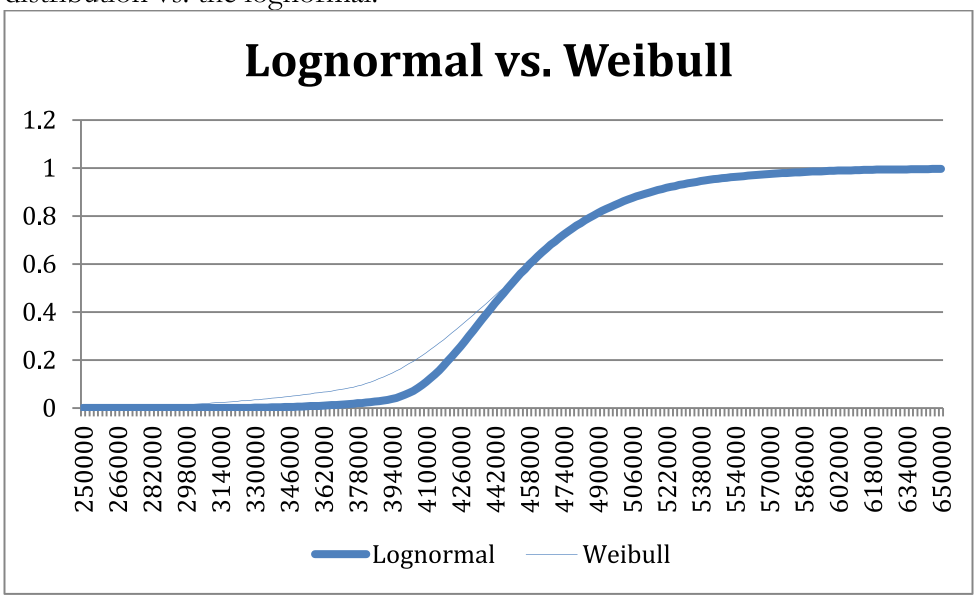
In this section we will explore the impact of varying the form of the development factor distribution and its parameters. This will be followed by a discussion of other factors not previously considered.

3.1 Alternative Distributional Forms

The Weibull distribution and the inverse translated gamma distribution will be considered as alternatives to the lognormal for the distribution of reserve development factors. We will compare the simulation results for each alternative distribution to the lognormal simulation results while holding the mean and coefficient of variation fixed.

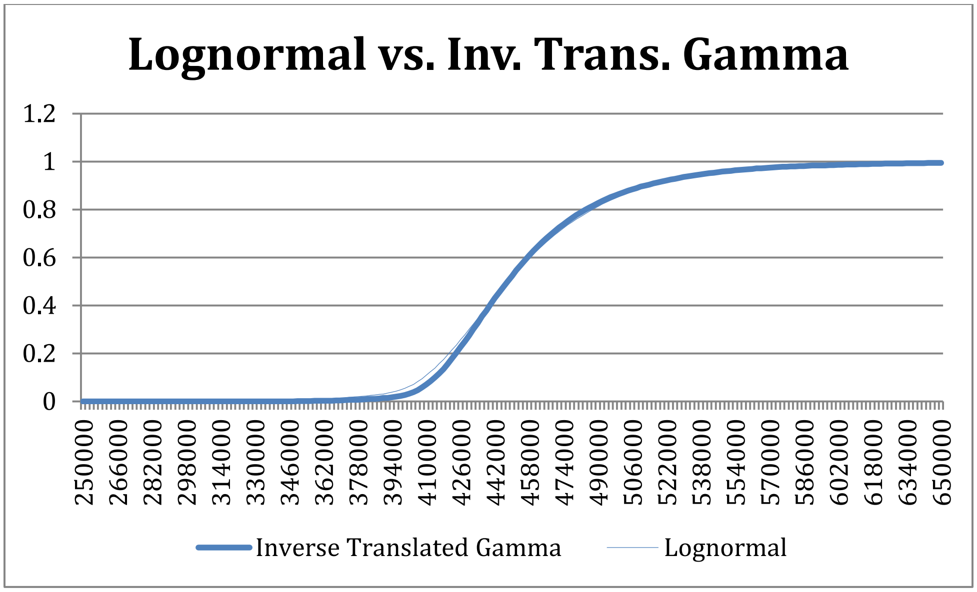
The Weibull distribution is appealing for use in simulation because it has closed form inverse. The inverse translated gamma is less simple, but was used by Corro and Engl in their update of the excess loss factors. The inverse translated gamma has three parameters, but the authors gave two of these as α = 8.7775 and τ = 0.8. This gives a coefficient of variation of 0.5 and allows a free parameter to scale the mean of the distribution.

The following graph shows a comparison of the CDF of the simulation results for the Weibull distribution vs. the lognormal:

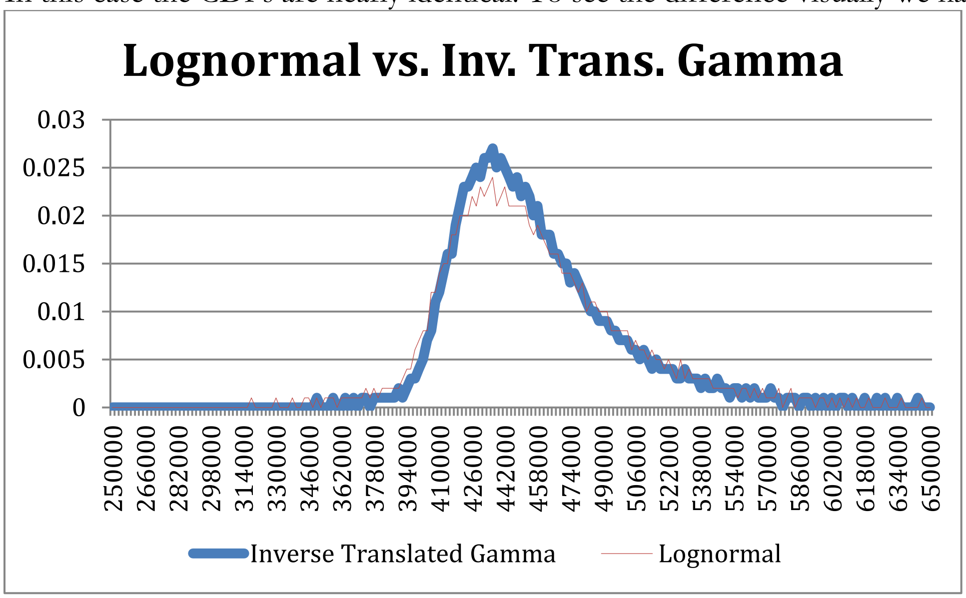


Weibull distributed development factors are more likely to produce a low simulated limited ultimate than the lognormal, but the two distributions are extremely close for higher loss amounts. We are only interested in the higher percentiles of the loss distribution when setting collateral, so it makes little difference which distribution we use.

We now consider a similar comparison of the CDF of simulation results for inverse translated gamma development factors vs. the lognormal:



In this case the CDFs are nearly identical. To see the difference visually we have to go the PDF:

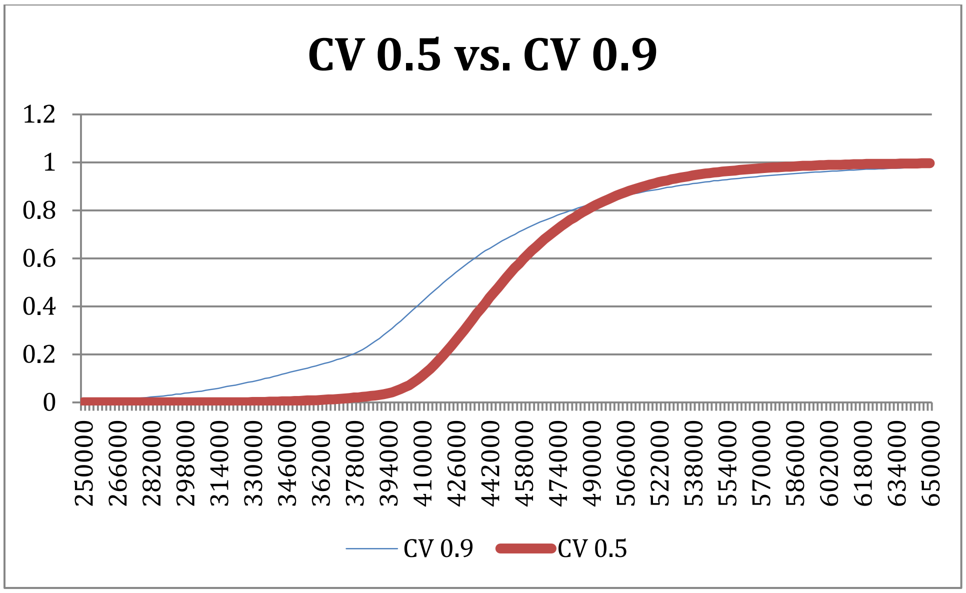


The lognormal rises a little sooner, but the inverse translated gamma has a higher peak. As we get to the higher loss amounts the two distributions are quite close.

Based on these comparisons, the form of the distribution of development factors seems to have little effect on the collateral estimate for a runoff book of business subject to a specific loss limit.

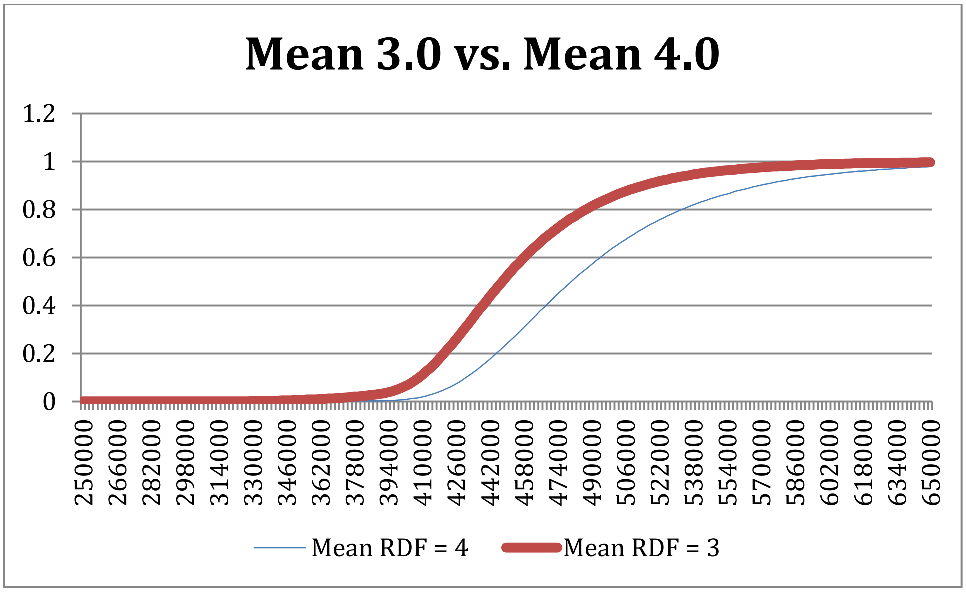
3.2 Alternative Parameters

In the first case, we will increase the coefficient of variation of the development factor and compare the resulting distribution to our original model. In the second case, we will increase the mean of the development factor distribution, while holding the CV constant. For the first case, I set the alternative CV at 0.9, which corresponds with the assumptions underlying the 1997 Excess Loss Factors by Gillam and Couret.



When we alter the coefficient of variation there is a wide gap between the simulation results for lower loss amounts, but the two curves come closer together as we go to higher loss amounts. In the absence of a single loss limit, the increase in the CV of the development factors would spread out the high end of the distribution as it has spread out the low end. However, the presence of the single loss limit causes much of the adverse development on the first two claims to fall into the excess layer.

We will next test the impact of moving the mean of the development factor distribution. Increasing the mean of the development factor distribution moves the entire limited loss distribution, but the increase at the 99th percentile of the limited ultimate loss distribution is only a small fraction of the increase in gross losses. Once again, adverse results tend to be pushed into the excess layer. This is shown in the graph at the top of the next page.



The following table provides a comparison of the upper percentile values of the simulated loss distribution under the scenarios graphed above. Increasing the mean of the loss ratio distribution from 3.0 to 4.0 increases expected gross losses by $333,000 in total, or $180,000 if the claim at the retention is excluded. Meanwhile, the 99th percentile of the simulated loss distribution increases by only $46,000.

Form	Lognormal	Lognormal	Lognormal
Mean	3	3	4
CV	0.5	0.9	0.5
90%	514,000	528,000	568,000
95%	540,000	580,000	604,000
99%	604,000	650,000	650,000

It is the nature of collateral estimates that our interest is focused on the adverse tail of the loss distribution. Given a fixed number of claims, in the presence of the single loss limit, adverse results tend to fall into the excess layer. The relative insensitivity of collateral estimates to model assumptions occurs because the higher percentiles of the reserve distribution are reached only when all of the large claims reach the retention.

3.3 Reopened Claims

It is well known that the weakness of open claim development is its treatment of IBNR and/or reopened claims. Stephen Philbrick and Keith Holler correctly stated that "To increase open reserves for anticipated development of open claims is plausible, but to increase individual claim amounts to account for newly reported counts seems unreasonable." Incurred but not yet reported (IBNYR) is not a big issue in runoff situations due to the elapsed time since the last exposure, but reopened claims present the same problem.

The direct solution is to add a reopened reserve estimated from the aggregate losses. I don't have data to estimate the reopened reserve, but I can offer an ad hoc approach. One could simply assign some fraction of the remaining aggregate tail (e.g. ¼) or some a fixed percentage of ultimate (e.g. 2%) as a provision for the reopened claims. Algebraically one could offset the reopened provision in the reserve development factor. One could ask if an offset is necessary, because as we have seen, the overstatement of the IBNER reserve will largely fall into the excess layer.

As for the distribution of the reopened reserve, the simplest approximation would seem to be to give it the same distribution as the IBNER. This approach, while not precise, has the effect of bringing the implicit provision for reopened back into the retained layer.

3.4 Other Considerations

It is reasonable to ask whether one should be making projections from open case reserves at all. Case reserves for an individual company or TPA can vary greatly from industry averages. Also, the application of large factors to a small base magnifies the variability of the outcome.

The simplified method presented here seems reasonable when it is used to cap the development of large claims. The uncertainty in the projection of the reserves for the large claims will tend to fall in the excess layer. Other methods should be considered for cases in which all of the remaining open claims are small. A lack of large claims may be an indication that case reserves are inadequate.

Open claim development is the only method of individual claim development that is available to a consultant without access to a large database. I would like to be able to experiment with incurred or paid development on open claims, but the aggregate loss data for open claims is not publicly available.

4 CONCLUSIONS

Individual claim development offers a method for estimating collateral requirements for captive fronting arrangements or workers compensation large deductible that are in runoff with respect to the issuing carrier.

The collateral question demands estimates of high percentiles of the loss distribution. This tends to reduce the parameter risk and distribution risk in the construction of the model, as those risks tend to fall into the excess layer. This effect is greatest when the model is used to limit large claims, but less so when large claims are not present.

Users of the individual claim development method should consider including a separate provision for reopened claims based on aggregate losses.

Acknowledgement

I would like to thank the committee members Susan Forray and John Alltop, whose comments helped guide me to the most interesting conclusions.

APPENDIX A

Reserve Development Factors by State

	60-ult	72-ult	84-ult	96-ult
NCCI States	2.023	2.037	2.075	2.076
AL	2.646	2.461	2.345	2.294
AK	3.139	3.393	3.015	2.885
AZ	2.987	2.706	2.557	2.487
AR	1.910	2.005	2.002	2.017
CO	1.370	1.337	1.296	1.313
CT	3.144	3.205	3.352	3.298
DC	2.265	2.269	2.251	2.209
FL	2.080	2.060	2.071	2.030
GA	2.201	2.034	2.026	2.264
HI	2.891	3.481	3.412	3.373
ID	2.662	2.333	2.510	2.519
IL	1.301	1.314	1.374	1.467
IA	2.136	2.170	2.370	2.515
KS	2.883	3.245	3.462	2.769
KY	1.854	1.826	1.780	1.710
LA	1.575	1.597	1.570	1.520
ME	1.866	1.774	1.835	1.897
MD	3.136	3.029	2.966	2.880
MS	1.897	1.912	1.910	1.798
MO	1.384	1.410	1.455	1.502
MT	3.942	3.576	3.215	3.155
NE	1.690	1.638	1.635	1.637
NH	4.599	4.984	5.001	4.183
NM	3.516	4.237	5.049	4.744
OK	2.660	2.967	2.742	2.413
OR	3.024	2.957	2.814	2.731
RI	1.117	1.289	1.346	1.366
SC	1.968	2.473	3.458	5.607
SD	1.864	1.783	1.826	1.882
TN	3.580	3.414	3.425	3.376
UT	3.441	3.296	3.302	3.223
VT	2.567	2.237	2.275	2.188
VA	2.198	2.357	2.362	2.354

Source: Derived from NCCI Annual Statistical Bulletin 2012 edition. Used with permission of the NCCI.

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Seeing the Forest with the Stems-and-Leaves

Kirk G. Fleming, FCAS, MAAA

Abstract: A picture is worth a thousand words. This paper shows how to use pictures called stem-and-leaf diagrams to display important loss patterns that might otherwise remain hidden in development triangles. These diagrams have the added benefit of appealing to the "big picture" folks in your audience. So that important patterns are always observed in these diagrams, this paper also presents some good practice suggestions that are used to review and evaluate another type of diagram, electrocardiograms (ECG) tracings.

Keywords. Reserving, development triangles, stem-and-leaf, ECG, EKG

"You can't see the forest for the trees" is an idiomatic expression that has been around for at least 500 years. It means that someone gets so engrossed in the details that they can't see the big picture. And the reverse expression is also used, albeit less frequently, "You can't see the trees for the forest." Here the expression is highlighting the danger of drawing broad general conclusions about something while missing important information that might be contained in the details.

When dealing with actuarial data, we usually move between two extremes of data detail. At one end we are looking at detailed claims data where we have information on each and every claim evaluated at periodic intervals. At the other extreme we group the individual claims data into development triangles and we look for, measure, and project broad claim trends. But anyone who has been at this for a while knows that the individual claims data is sometimes too much information and the triangles might hide important trends.

In this paper, I would like to offer an alternative way to look at claims data based on the idea of stem-and-leaf displays. Stem-and-leaf displays offer a compromise between the individual claims data and aggregate claim triangle data that presents a chance to observe additional important trends that might otherwise be lost.

So what is a stem-and-leaf display? A stem-and-leaf display is a statistical technique for presenting data where each numerical value is divided into two parts. The leading digit becomes part of the stem and the trailing digit becomes the leaf. The stems are located along the main vertical axis, and the leaves are each observation along the horizontal axis. [1]

As an example, suppose we had the following seven observations between 90 and 100: 96, 95, 93, 96, 97, 98 and 99. The stem for these seven observations would be the 9 and the leaves would be the trailing digits. Setting these up in a stem-and-leaf display we would have the following:

9|3 5 6 6 7 8 9

You just organize the trailing digits from highest to lowest.

The diagram below was created with more data and I am sure you need no further explanation in how it was constructed.

8| 8 9

9| 3 4 6 6 7 8 9

10| 3 3 4 6 7 8

11| 1 2 2 3 3 7 7 8 9

12| 0 0 4 5 5 5 7 7

13| 2 4 5 6 8 9 9

14| 2 3 8

15| 5 5 6

You can see why the diagram is called stem-and-leaf. It looks like a histogram on its side but you have kept all the information about the individual numbers in your collection.

My suggestion is that in addition to using aggregate claims data triangles you should produce stem-and-leaf displays with the emphasis on the word "display." I am going to suggest that you actually deemphasize the numbers for this exercise and just produce pictures that show the additional information that underlies your aggregate claims data.

This organization of your data will allow you to read and tell the story in your data. This presentation will appeal to the "big picture" folks in your organization. You will be able to get your important points across clearly and in a short amount of time.

So let's not beat around the bush and just get right to an example. We can go through the process with some simulated claims data. I simulated five years' worth of claims data between $100,000 and $5,000,000 for this paper. I had a particular story that I wanted to create with this example data. One year has a problem with an increase in the frequency of all sizes of claims. Then things settled down again although we will have a year where some unusually large claims popped up. These changes are the result of changes in claim frequency. The underlying exposure stays the same each year. For this example, I am only going to be using data from the first evaluation column of a development triangle but you could apply this concept to any evaluation column or columns.

The first evaluation column of the simulated data is shown below:

2008	19,406,000
2009	21,704,000
2010	41,567,000
2011	36,096,000
2012	23,557,000

Rather than break the data into equal buckets as is done with the pure stem-and-leaf display, you should decide how to create the horizontal breaks in your data based on your needs. You might want to create horizontal breakpoints that reflect your definitions of basic and excess limits data. Or you might want to break the data into layers that match provisions in your reinsurance programs. The data should be separated into manageable chunks so that you do not have hundreds of data points on one line and two or three data points on another line. Finally, the splits do not have to be based on the numerical values of the claims but could be based on any type of claim feature.

I am going to use the following stem definitions for my horizontal break points because they fit my data the best and will allow me to highlight some points:

100,000 up to 200,000

200,000 up to 300,000

300,000 up to 500,000

500,000 up to 1,000,000

1,000,000 up to 2,000,000

2,000,000 up to 3,000,000

3,000,000 up to 4,000,000

4,000,000 up to 5,000,000

Enter your data into a spreadsheet using the stem breaks that you selected. If your original selections for breakpoints do not explain your point, you can always go back to the drawing board. I copied a sample of my spreadsheet in Figure 1 on the next page.

	A	B	C	D	E	F	G	H
1								
2		2008	100	101	108	110	113	114
3			200	201	209	215	219	225
4			300	304	307	309	309	332
5			500	516	605	618	665	671
6			1,000	1,008	1,046	1,332	1,731	
7			2,000					
8			3,000					
9			4,000					
10								
11		2009	100	100	103	104	112	115
12			200	205	220	234	235	243
13			300	314	320	343	351	366
14			500	514	535	537	555	635
15			1,000	1,054	1,180	1,189	1,561	1,739
16			2,000					
17			3,000					
18			4,000					
19								

Figure 1

Once you have entered all your data, shrink the page so that you have a much smaller view of the page. In Figure 2, I shrunk my page down to 50%. We cannot read the numbers in the diagram and that is on purpose. We do not care about the numbers at this point. We are looking at the picture.

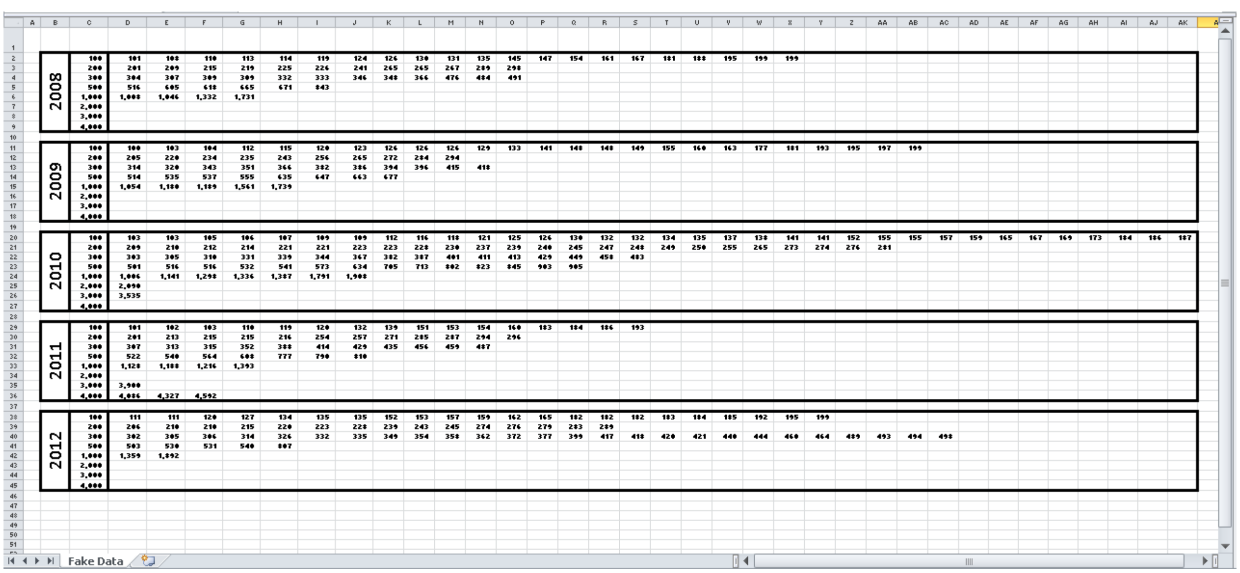


Figure 2

Before I take you through the story in the diagram in Figure 2, I will offer four good practice suggestions adopted from people who are trained to read another type of diagram, an electrocardiogram (ECG). ECG's (or EKG's if you are old school) are diagrams of the electrical activity of the heart captured by putting electrical wires around a person's heart. The heart sends electrical impulses down special internal pathways so that it contracts in a highly coordinate fashion. The typical ECG that is done in the hospital or your doctor's office shows 12 different views of the electrical activity. By looking at variations from the norm in those twelve different pictures, a doctor can diagnose electrical or physical changes in the heart that are causing variations in the pictures.

- The first suggestion is to know what the normal year's picture looks like. Once you know what is normal, then you can spot what is not normal. You might not know what is causing a year to be different but you will know that something deserves more study.
- The second suggestion is that you should adopt a systematic approach to looking at each diagram. Always follow the same steps because otherwise you run the risk of missing something critical.
- The third bit of advice is to look at the surrounding years when you find something unusual. A year will be explained by the "company it keeps." If only one year follows a pattern then it is a fluke. But if the surrounding years have the same pattern, then you have found a trend.
- The final suggestion is evaluate what you see in these diagrams with what else you know about the changes in the company's operation. As an example, if your company is moving into or out of an area of exposure, do you see the expected changes from your previous normal pattern? [2]

Let's begin looking at the diagram.

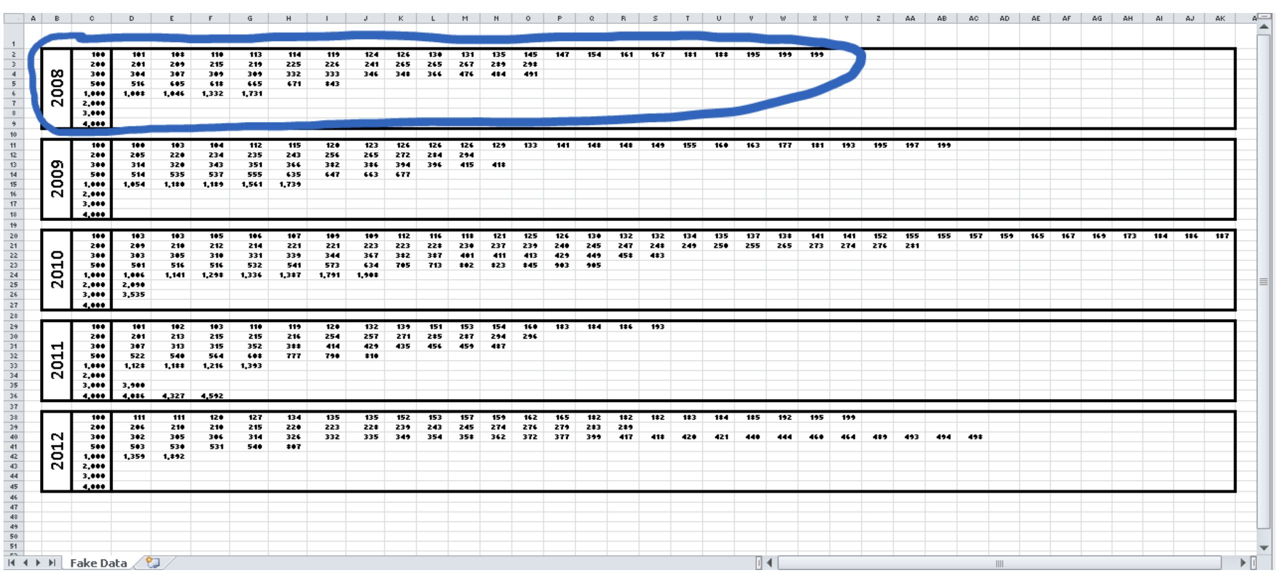


Figure 3

I will make believe that after years of looking at my fake data, I have come to recognize that 2008, the circled group in Figure 3, is a normal pattern for a year. It has a certain amount of claims in the lower layers and as we go up to higher layers, we see fewer and fewer claims until we get to the upper layers where we have no claims. If we look at 2009, the grouping right below the circled data, it is basically the same pattern. There are some random variations between the two years but basically we have the same pattern.

As far as a systematic approach to looking at the data, I am going to suggest looking at horizontal variations in the aggregate groupings and then focusing on individual years. If we look at all the years in the stem-and-leaf diagram, the first thing that catches our eyes in Figure 4 is that 2010 appears to be a bad year. You would already know that 2010 was a bad year from looking at the traditional development triangle but this diagram shows the additional insight that losses were coming in all layers as opposed to several large losses.

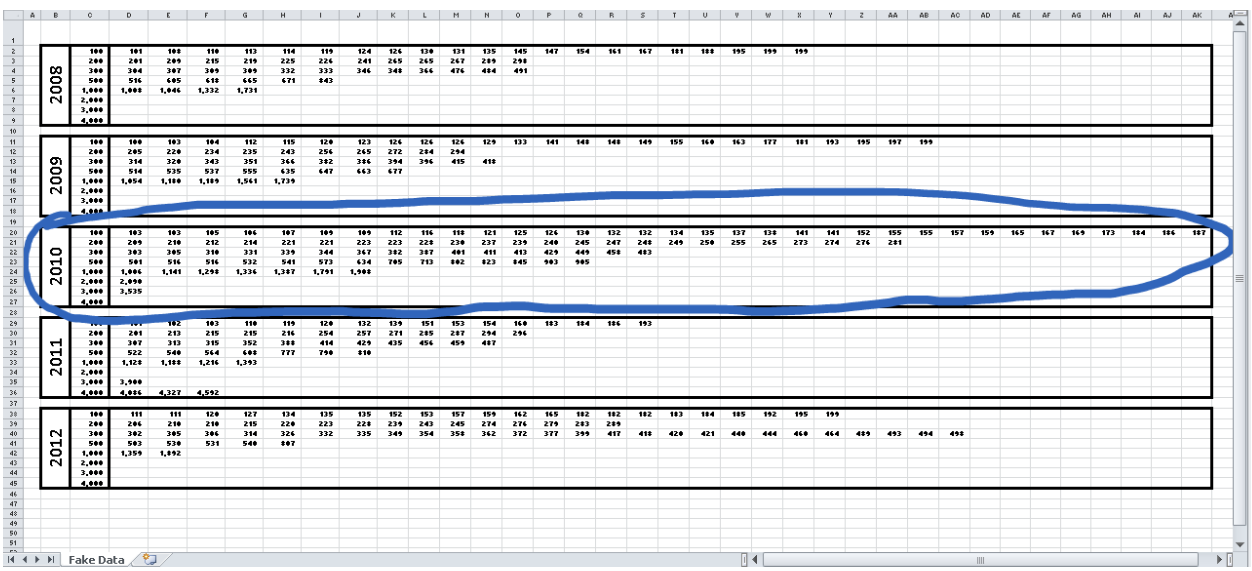


Figure 4

The 2010 stem has a lot of leaves on them. When looking at those lower layers, the leaves extend well beyond our normal year for all the layers. If you were trying to paint a picture of what was going on with a particular year, a diagram like this would help explain things. If we compare this year to all the other years, it looks unique. So this year was not part of a longer term trend.

Now let's look at some of the individual layers for the individual years. The year 2011, the circled year in Figure 5 on the next page, was the second worst year of them all. However, in this case a number of large losses are causing the problem as opposed to a frequency of losses. We can see an unusual number of large losses showing up in this year. In fact, you can quickly see that "this would have been the best year we ever had if it hadn't been for those large losses." I am going to go out on a limb and say that explanation probably does not appeal to you but at least you can see that it is true by looking at the diagram. The rest of the year looks better than our normal year. And this occurrence of large losses seems isolated to that one year. We do not have any evidence that we are moving to a new normal.

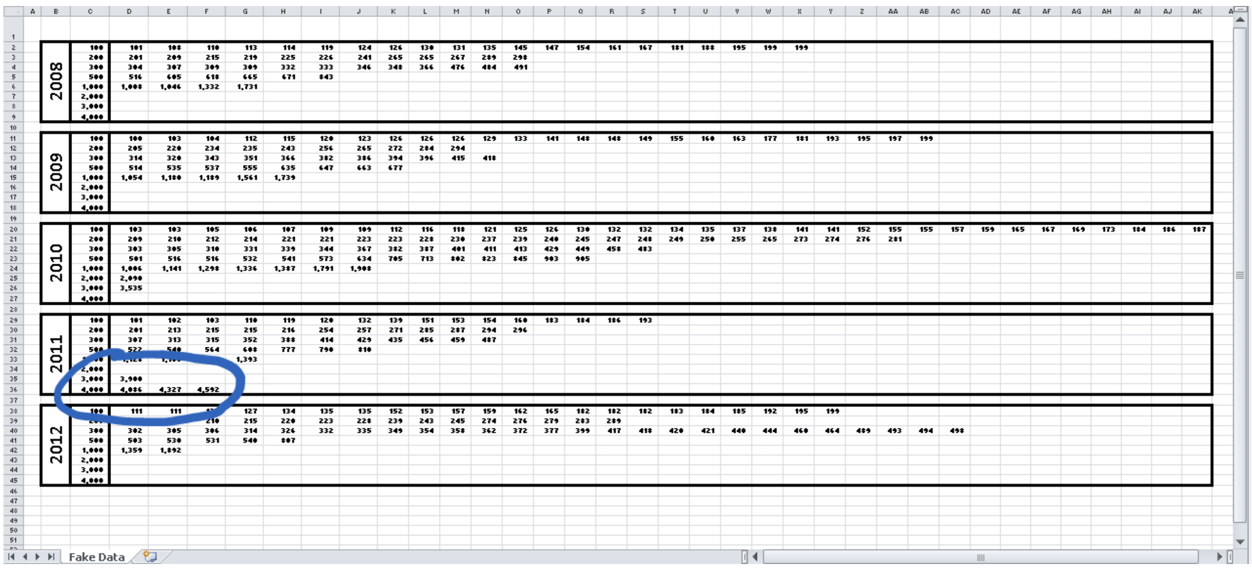


Figure 5

I will leave 2012 for you to look at and think about. How does it compare to a normal year? Using a systematic approach, do you see any unusual patterns as compared to our selected normal year? How does it compare to the prior year? Finally, you would want to ask how this year fits in with what you know about changes in the company's book of business. If the underlying exposure or type of business was changing, those changes would be part of your explanation.

Stem-and-leaf diagrams will help you quickly and clearly get your point across to the "big picture" people who are interested in your company's results. These diagrams may be a way to open up discussions with other interested parties. Just remember people may have alternative explanations for the observed changes than the explanations you offer. There will be different explanations for changes in the displays and that is what makes actuarial work and reading ECG's both an art and a science. And even though one of you might be barking up the wrong tree, hopefully these discussions will lead you in the right direction so that you all can get on the same page.

I started the paper with a 500 year old idiom. I will finish here with a relatively new one that may or may not be around in 500 years. Ladies and gentlemen, Elvis has left the building.

REFERENCES

[1] Mason, Robert D., Lind, Douglas A., Marchal, William G., *Statistical Techniques in Business and Economics*, 10th Edition, pages 31-33.
[2] Garcia, Tomas, B., Holtz, Neil, E., *Introduction to 12-Lead ECG, The Art of Interpretation*.

Biography

Kirk G. Fleming has worked as an actuary for over 30 years and has given many presentations to big picture folks. In addition, he has experience working in hospitals doing 12-lead EKG's.

Peaks and Troughs: Reserving Through the Market Cycle

Susan J. Forray, FCAS, MAAA

Zachary A. Ballweg, FCAS, MAAA

Abstract It is well-known that the carried reserve adequacy of the property & casualty industry as a whole varies across the market cycle. We examine the extent to which this variation results from actuarial methods themselves, concluding that about half of the industry's historical deficiencies and redundancies have resulted from actuarial methods. The deficiencies and redundancies that result from actuarial methods appear to be highly correlated with the economic cycle. At the same time, there is also a strong relationship between the underwriting cycle and carried reserve adequacy. Implications for uncertainty in the industry's aggregate reserve adequacy as well as for individual companies are considered.

Keywords. Reserving, reserve variability, reserving methods.

1. INTRODUCTION

It is well-known that the carried reserve adequacy of the property & casualty industry, as a whole, varies significantly across the market cycle.[1] Much less understood is the extent to which this may stem, in part, from actuarial reserving methods. If a material relation exists, any cyclicality in actuarial reserving methods could lead to over-estimated or under-estimated reserves, thus exacerbating the market cycle.

Within this paper we will assess the potentially cyclical behavior of various actuarial reserving methods. These include the paid and incurred (i.e., paid plus case) chain ladder, Berquist-Sherman, and Munich Chain Ladder methods. A complete list of methods analyzed can be found in Appendix A. For purposes of discussion, we will focus on the most commonly used of these methods, noting that the general pattern of results is consistent across all methods considered. Data has been obtained at an industry aggregate level from SNL Financial for statement years 1996 and subsequent. Data for all prior statement years was obtained from AM Best's Aggregates & Averages.

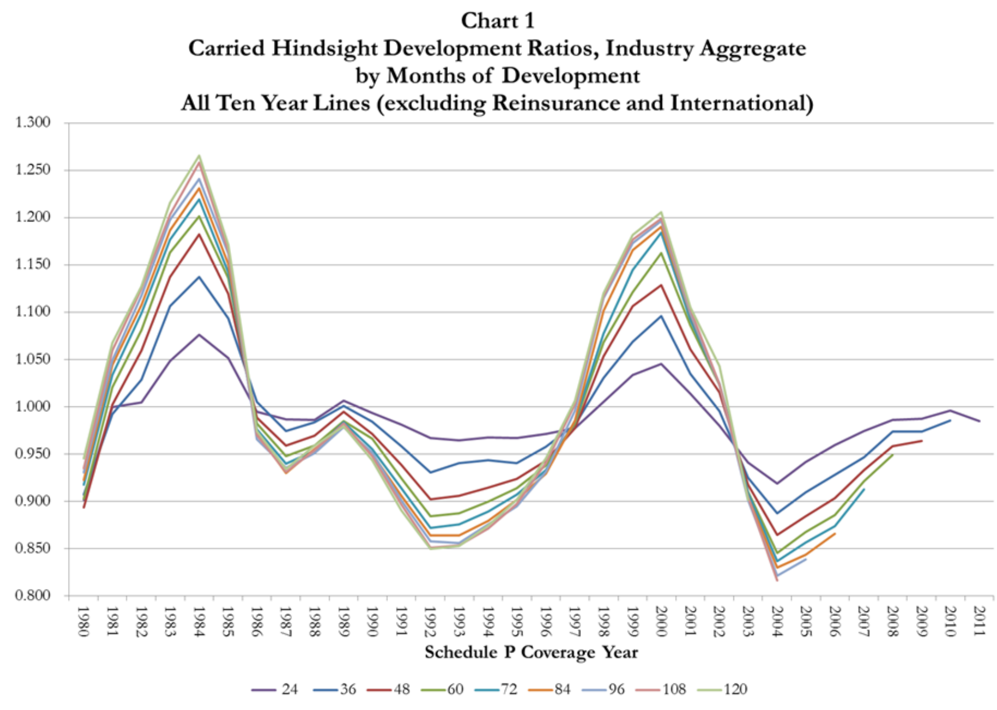
The remainder of the paper proceeds as follows. Section 2 discusses the property & casualty industry's historical carried reserve development, while Section 3 provides a summary of the actuarial research that has been performed to date in this area. Section 4 discusses the development that would have resulted from applying standard actuarial methods to data at an industry aggregate

[1] See, for example, [5], [13], and pages 13 and 14 of [8].

level and compares these results to the carried reserve development first discussed in Section 2. Section 5 discusses the relationship of the reserving cycle to the underwriting cycle and economic cycle. Lastly, Section 6 discusses certain limitations of the analysis, while Section 7 offers some conclusions.

2. DEVELOPMENT OF THE CARRIED RESERVE

Carried reserve adequacy for the property & casualty industry has varied significantly over time. This can be seen by reviewing the development of the carried loss and DCCE[2] reserve by accident year at successive evaluations. Chart 1 shows the proportional development of the industry's carried loss and DCCE by Schedule P coverage year[3] from the initial carried reserve (at twelve months of development) to the final carried amount, as measured by ratios to the initial carried loss and DCCE reserve:



[2] We will refer in this monograph to DCCE, although this term should be taken to refer to ALAE for those historical evaluations at which ALAE was provided as a subset of LAE within the Annual Statement, as opposed to DCCE. The change from ALAE to DCCE within the Annual Statement (beginning with the 1998 Annual Statement) would have a small effect on our analysis, although it is our perception that this change in Statutory accounting practice is immaterial to our results.

[3] i.e., report year for claims-made lines of business and accident year for all other lines of business.

Chart 1 provides ratios of the carried hindsight unpaid[4] loss and DCCE by accident year at various months of development (in the numerator) to the initial carried loss and DCCE reserve at twelve months of development (in the denominator). We will refer to these as hindsight development ratios. In particular, as these are based on the carried loss and DCCE reserves, we will refer to them as the carried hindsight development ratios.

Thus on an accident year basis we see that carried reserves developed adversely in the early 1980s (with the exception of coverage year 1980 when carried reserves developed favorably). During the following decade, reserve development was favorable. At the tail end of the 1990s until 2002, development was again adverse. Subsequently the industry has demonstrated favorable development, again measured on an accident year basis.

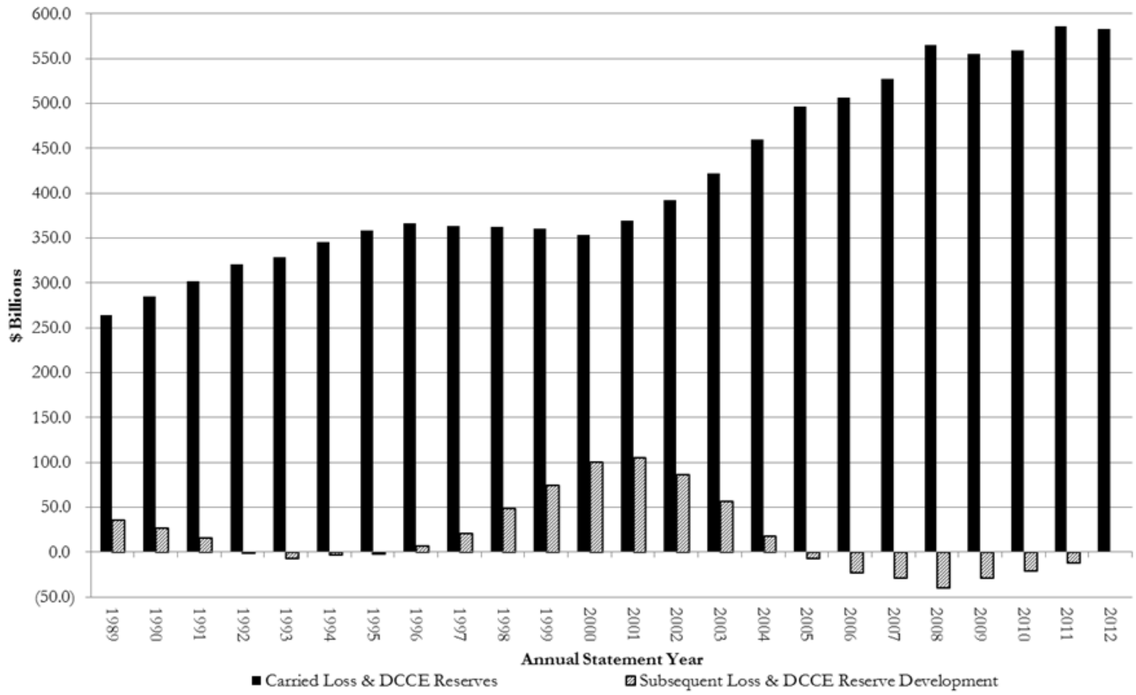
Analogous charts by line of business are provided in Appendix B. In general these charts show the same pattern of development as in Chart 1. However the degree of favorable or adverse developments is seen to be typically greater for the longer-tailed lines of business (e.g., medical professional liability and workers' compensation) and to be typically less for the shorter-tailed lines of business (e.g., auto liability and homeowners/farmowners).[5]

Thus the industry as a whole has clearly demonstrated a cyclical reserving pattern. At times the impact of this cycle on reserve adequacy has been quite significant. Chart 2 aggregates the industry's development on a statement year basis and compares it to the industry's carried reserves at the given evaluation:

[4] The hindsight unpaid is the amount unpaid as of the prior evaluation (in this case, as of twelve months of development) based on estimated liabilities at a subsequent (i.e., "hindsight") evaluation. Mathematically, the hindsight unpaid loss and DCCE can be calculated as the ultimate loss and DCCE evaluated as of a subsequent evaluation less the paid loss and DCCE as of the prior evaluation. Equivalently, the hindsight unpaid loss and DCCE is the estimated unpaid loss and DCCE as of the earlier evaluation plus any change in the estimated ultimate loss and DCCE between the initial and hindsight evaluations.

[5] Lines of business have been combined within Appendix B into those lines in place during the 1980s (e.g., the occurrence and claims-made segments of medical professional liability have been combined into a single line of business, as have personal and commercial auto liability).

Chart 2
Property & Casualty Industry Total -- All Lines of Business
Carried Loss & DCCE Reserves and Subsequent Development



Taking statement year 2000 as an example, the above chart shows that the property & casualty industry carried $353.6 billion in net loss and DCCE reserves as of December 31, 2000. Aggregating data by calendar year shows $100.8 billion in adverse development since this accounting date. In other words, with the benefit of hindsight, the industry's net carried loss and DCCE reserves as of December 31, 2000 were deficient by at least $100 billion, or 28% of the carried reserve.[6] Thus clearly the issue of reserve adequacy is significant for the property & casualty industry.

3. SUMMARY OF PRIOR RESEARCH

Surprisingly, very little research has been done to date on the source of cyclicality in carried reserve estimates, and in particular on the relationship between actuarial methods and carried reserves. We are aware of one published paper to date on this topic by a US actuary. In this paper, the author compares the booked ultimate loss and DCCE ratios for Commercial Auto Liability on an industry aggregate basis to the loss and DCCE ratios that would have been indicated by applying

[6] Given the ten-year structure of the Schedule P triangles, this estimate excludes all development subsequent to December 31, 2009, which has been consistently adverse for "prior" accident years. Offsetting this additional unknown amount, some amount of adverse development would be due to the unwinding of the discount in cases where discounting was permitted.

standard actuarial methods to the data available within Schedule P. The author's approach is similar to our own, although applied only to one line of business and only to accident years 1995 through 2001.

The author observes that, although the pattern exhibited by the carried ultimate loss and DCCE ratios by accident year has been directionally similar to the results of the actuarial indications, the carried loss and DCCE ratios have been consistently lower than the actuarial indications and have also exhibited greater error when evaluated in hindsight (i.e., when compared to the final carried amounts). He concludes that "either the booked ultimate loss ratios were based on other methods that are inferior to the chain ladder and Bornhuetter-Ferguson or judgmental adjustments were made to the indicated ultimate loss ratios that reduced the quality of the final selections."[7] The author acknowledges that "further research would be required to determine whether this is a general loss reserving phenomenon or one confined to Commercial Auto Liability during the time period studied." [8]

More research has been done on this topic by UK actuaries, as documented in [13]. In particular, a GIRO[9] working party concluded the following in 2003: [10]

a) A reserving cycle exists in the UK.

b) Standard actuarial reserving methods are probably a contributory cause of the reserving cycle.

c) There is some (inconclusive) evidence that development patterns vary with the underwriting cycle, tending to be longer-tailed when premium rates are low.

d) There is clear evidence that Lloyd's premium rate indices had tended to understate the true magnitude of the underwriting cycle.

However the GIRO working party does not appear to have considered the relationship between the reserving cycle and the economic cycle. In this paper, we will assess the extent to which the above observations hold for the US property and casualty industry, as well as the relationship between the reserving cycle and the economic cycle. To the best of our knowledge, the relationship between the economic cycle and actuarial reserving methods has not been considered previously.

[7] Page 1 of [8].
[8] Ibid.
[9] General Insurance Research Organizing Committee of the Institute and Faculty of Actuaries.
[10] As cited on page 401 of [13], from which these conclusions are paraphrased.

4. DEVELOPMENT OF THE ACTUARIALLY INDICATED UNPAID AMOUNT

A similar analysis to that shown in Section 2 above can be performed based on the results of standard actuarial methods applied to the data given within Schedule P. As an example, consider the results of the paid development method applied based on all-years weighted average development factors to each ten-year line of business within Schedule P, aggregated here:

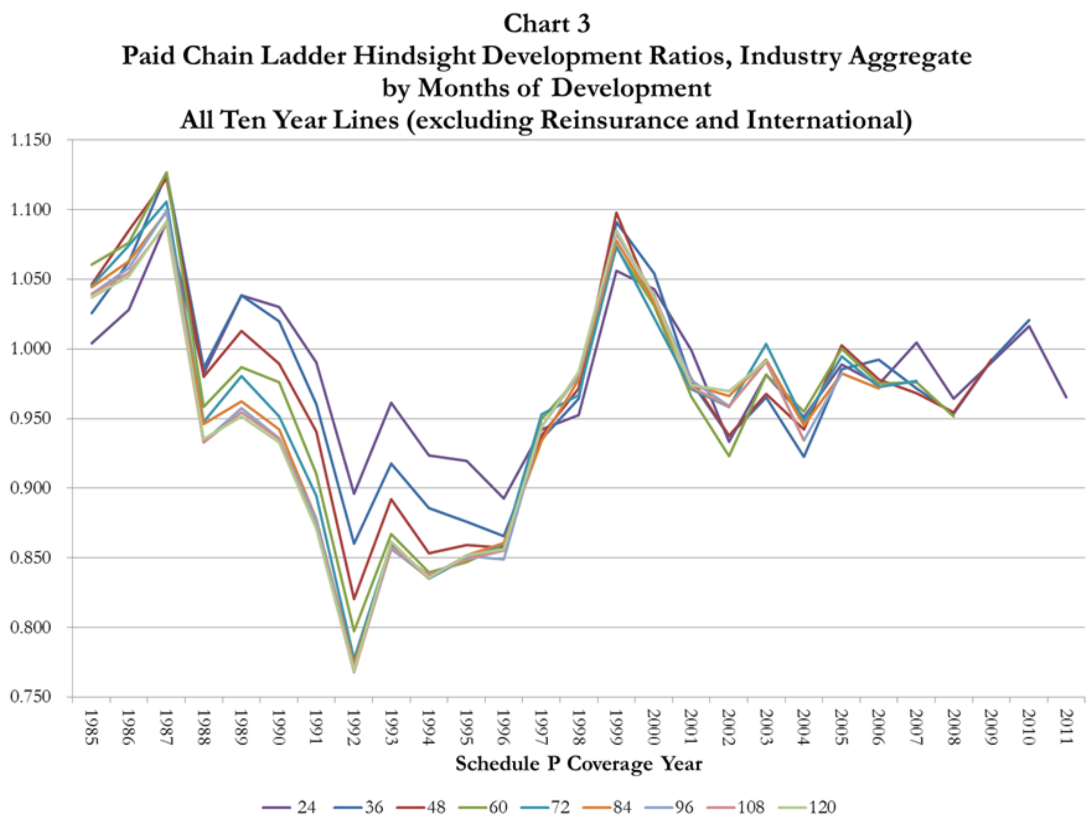
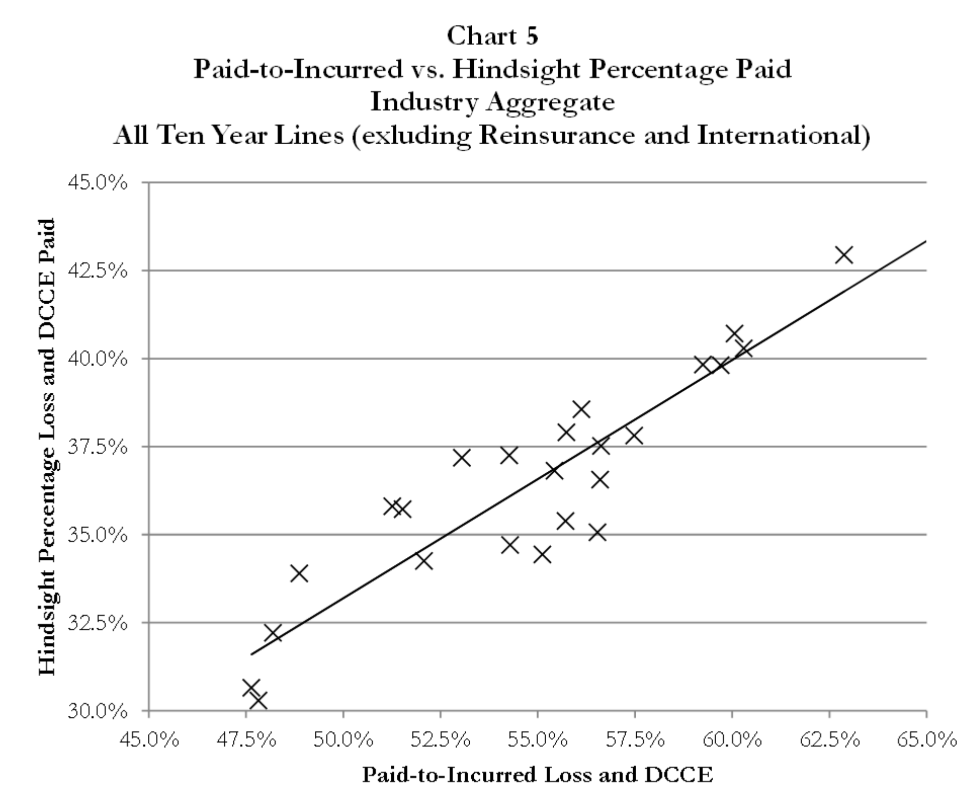


Chart 3 shows that the paid development method would have resulted in adverse development in the mid-1980s and favorable development between 1988 and 1998. Development would have been adverse for accident years 1999 and 2000 and generally favorable subsequently. It is interesting both that the paid development method evidences cyclicality in its results and also that this cycle follows the same general pattern of the carried reserves. Chart 4 compares development for the carried reserves to development of the industry aggregate paid and incurred (i.e., paid plus case) chain ladder methods, focusing on development from 12 months of development to the most recent available evaluation:

Chart 4
Hindsight Development Ratios, Industry Aggregate
All Ten Year Lines (excluding Reinsurance and International)

Thus the industry's pattern of carried reserve development generally shows a similar pattern, although at times more pronounced, as that exhibited by the paid and incurred chain ladder methods. It should be noted that based on a review of industry aggregate case reserve averages by line of business, case reserve adequacy appears to have declined at 12 months of development for accident years 1999 and 2000. This likely explains (at least in part) the greater degree of adverse development exhibited by the incurred chain ladder method for these accident years.

It is reasonable to ask whether the deficiencies or redundancies that would have resulted from the use of the paid and incurred chain ladder methods at these times could have been obviated or even eliminated. Perhaps this could result from the use of more recent (i.e., shorter-term) development factors or by adjustments stemming from diagnostic information available at the time, such as claim closure rates or paid-to-incurred ratios. There is some evidence for the predictive value of these diagnostics. As an example, consider Chart 5, which compares the ratios of paid-to-incurred loss and DCCE as of twelve months of development on the *x*-axis with the ratios of paid loss and DCCE as of twelve months of development to the ultimate loss and DCCE as of the most recent evaluation (i.e., the hindsight percentage of loss and DCCE paid, on the *y*-axis):

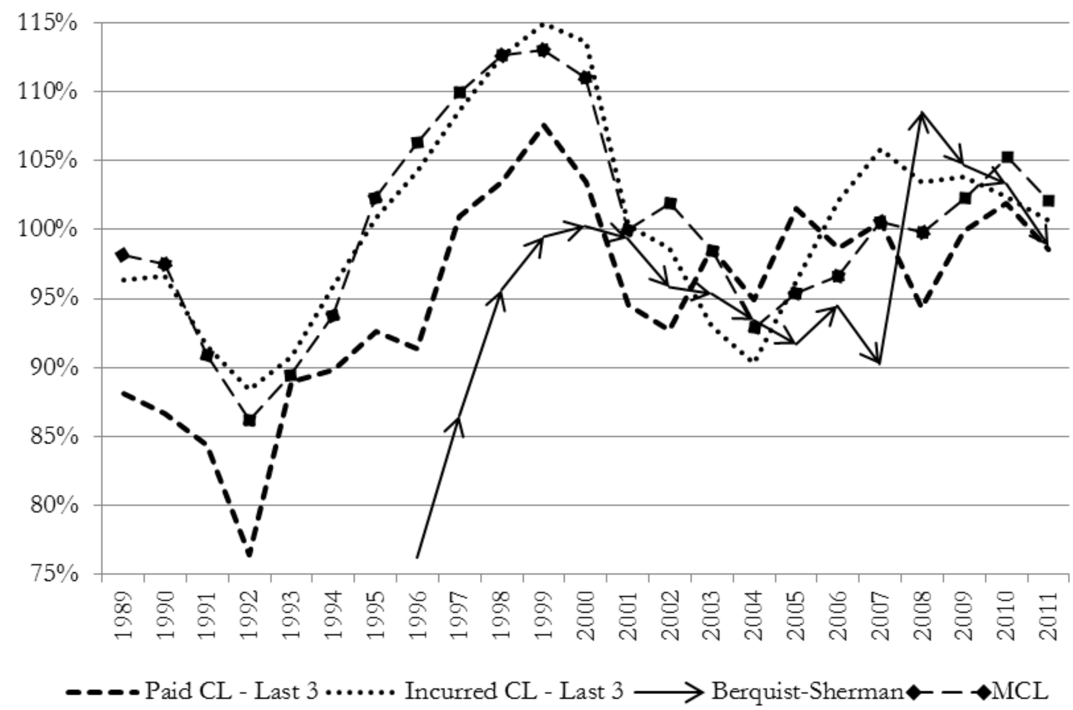


Note that the inverse of the hindsight percentage paid would be the hindsight cumulative paid development factor. Thus the paid-to-incurred ratio is clearly indicative of a paid development factor on an industry aggregate basis (note that the *R*-squared for the above linear fit is 83%). At the same time, the deviation of these points from the fitted line is demonstrative of the uncertainty that will always be present in any estimation of the future payments. While Chart 5 reflects data for all lines combined, analogous data by line of business, shown in Appendix C, demonstrates similar results. [11]

The results shown on Chart 5 strongly suggest the use of methods that would adjust for changes in payment patterns or case reserve adequacy over time. These methods would include the Berquist-Sherman and Munich Chain Ladder methods, as well as versions of the paid and incurred chain ladder methods in which more recent development factors are relied upon as the prospective selections. Chart 6, below, compares the results of these methods:

[11] The *R*-squared values by line of business range from 54% to 91%, with the exception of workers' compensation, for which the *R*-squared is 1%. This may stem from the statutory nature of workers' compensation payments, which might make fluctuations in case reserves more suggestive of changes in case reserve adequacy than changes in future payments. However, this statement may be false if triangles longer than ten years are examined, as increases in case reserves could suggest longer life expectancy, for example, which would typically not result in increased payments during a time period short enough to be reflected in a ten-year triangle.

Chart 6
Hindsight Development Ratios, Industry Aggregate
All Ten Year Lines (excluding Reinsurance and International)



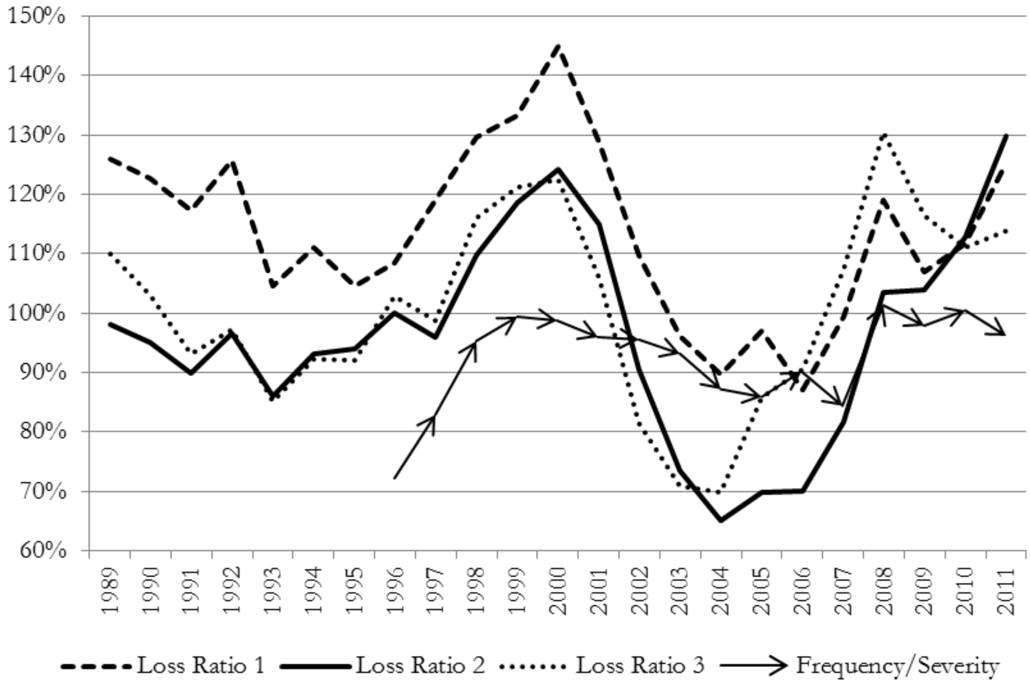
In general the Munich Chain Ladder method shows similar results to the Incurred Chain Ladder method. The Berquist-Sherman method appears to outperform the Incurred Chain Ladder method at this aggregate level at only some evaluations, and in particular the method underperformed during the first years for which claim counts were required in the Annual Statement (beginning in 1996). Presumably this is due to irregularities in the claim count data at that time. In addition, at the 2007 evaluation there appears to be an overstatement (relative to surrounding evaluations) in the number of open personal auto liability claims, which causes the Berquist-Sherman method to overstate unpaid loss and DCCE at this evaluation.

Consider the paid chain ladder method in which a weighted average of the last three development factors is assumed as the prospective selection (also included on Chart 6). For purposes of this analysis, we observe that this method appears to outperform the other methods considered, beginning in the late 1990s. For this reason, we have treated the results of this method at the most recent evaluation available as the "true" ultimate loss and DCCE, where such an ultimate was needed (i.e., the 2004 accident year and subsequent, where the incurred loss and DCCE as of 120 months of development would not yet be available). This method was used as such above in Chart 5, for example.

All methods listed in Appendix A were reviewed in an analogous fashion to the methods discussed here. In general the other methods performed similarly or in some cases underperformed the methods we have discussed in this section. Thus any solution to the cyclical behavior of actuarial reserving methods appears to be non-trivial. Appendix D provides information analogous to Chart 6 by line of business. In general results are consistent across lines of business.

As a note, the cyclical behavior we have observed also holds for methods such as the frequency/severity and loss ratio methods. Given that the loss ratios of the property & casualty industry themselves exhibit cyclicality (as a result of the underwriting cycle) it is not surprising that the loss ratio method would exhibit cyclicality in its hindsight development ratios as well. It is more interesting that the cyclicality holds even after contemporaneous attempts to adjust for the underwriting cycle. Chart 7 shows the hindsight development ratios of these methods aggregated across all ten-year lines of business:

Chart 7
Hindsight Development Ratios, Industry Aggregate
All Ten Year Lines (excluding Reinsurance and International)



Recall that definitions of the above methods are available in Appendix A. Clearly, deviation of the hindsight development ratios of the loss ratio methods from unity must result from variation in the property and casualty industry's loss and DCCE ratios over time. For example, business in the 1999 through 2001 coverage years was underpriced relative to prior coverage years, so we would naturally expect indications based on these prior coverage years to be deficient (as the lines above for Loss Ratio 2 and Loss Ratio 3 show them to be).

The contemporaneous loss and DCCE ratio estimates are given by the method Loss Ratio 1. These estimates underperform in earlier years but have improved in their performance since 2003. Note this is consistent with the conclusions of the 2003 GIRO working party report, mentioned previously, which noted that the Lloyd's premium rate indices tended to understate the magnitude of the underwriting cycle. Thus contemporaneous estimates of both the US and UK industries have historically underestimated the effect of the underwriting cycle.

By way of summarizing the above discussion, we provide the following table of correlations and *R*-squared values between the hindsight development ratios of the carried loss and DCCE and those of the actuarial indications:

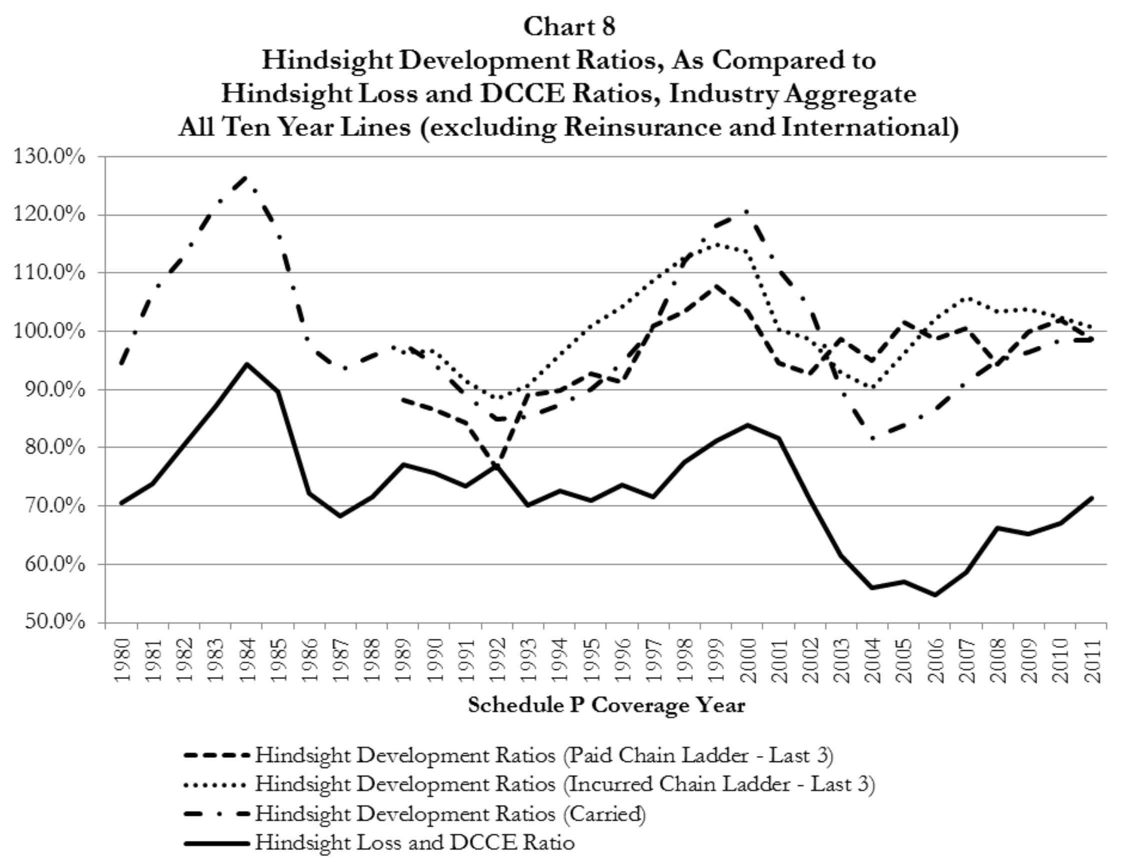
Table 1 Correlations and *R*-Squared Values of Hindsight Development Ratios of Carried Unpaid Loss and DCCE and Actuarially Indicated Unpaid Loss and DCCE		
Actuarial Method	Correlation	*R*-Squared
Paid Chain Ladder – All Years Weighted Average Development Factors	63%	40%
Paid Chain Ladder – Three Years Weighted Average Development Factors	52%	27%
Incurred Chain Ladder – All Years Weighted Average Development Factors	94%	89%
Incurred Chain Ladder – Three Years Weighted Average Development Factors	78%	61%
Berquist-Sherman	27%	7%
Munich Chain Ladder	81%	66%

For example, the 63% correlation in the first row of the above table represents the correlation between the "carried" and "paid chain ladder" lines on Chart 4 for accident years 1989 and subsequent. The 40% *R*-squared value represents the *R*-squared between these lines, where the "carried" is treated as the dependent variable and the "paid chain ladder" as the independent variable. In other words, given the variation in carried reserve adequacy at first evaluations by accident year, 40% is estimated to be due to underlying variation that is also present in the paid chain ladder method. Appendix E provides results analogous to the above table by line of business.

Given the range in the above table, we can conclude that perhaps about half of the historical variability in carried reserve adequacy can be attributed to an underlying cyclicality that is present in actuarial methods. It would greatly benefit the actuarial profession to investigate possible new methods that mitigate this cyclicality. While mitigating the cyclicality may be possible, it seems unlikely that the cyclicality can be eliminated. Some amount – likely a large amount – of uncertainty in industry reserve adequacy will always be present due to the uncertainty in future payments. Even if the cyclicality can be addressed and managed, significant uncertainty in results – even at an industry aggregate level – will continue to exist.

5. RELATIONSHIP TO THE UNDERWRITING AND ECONOMIC CYCLES

It is natural to ask whether there is a relationship between the reserving cycle and other known cycles, such as the underwriting cycle and the economic cycle. Considering first the underwriting cycle, Chart 8 demonstrates a possible relationship between reserve development and the pricing of property and casualty business:

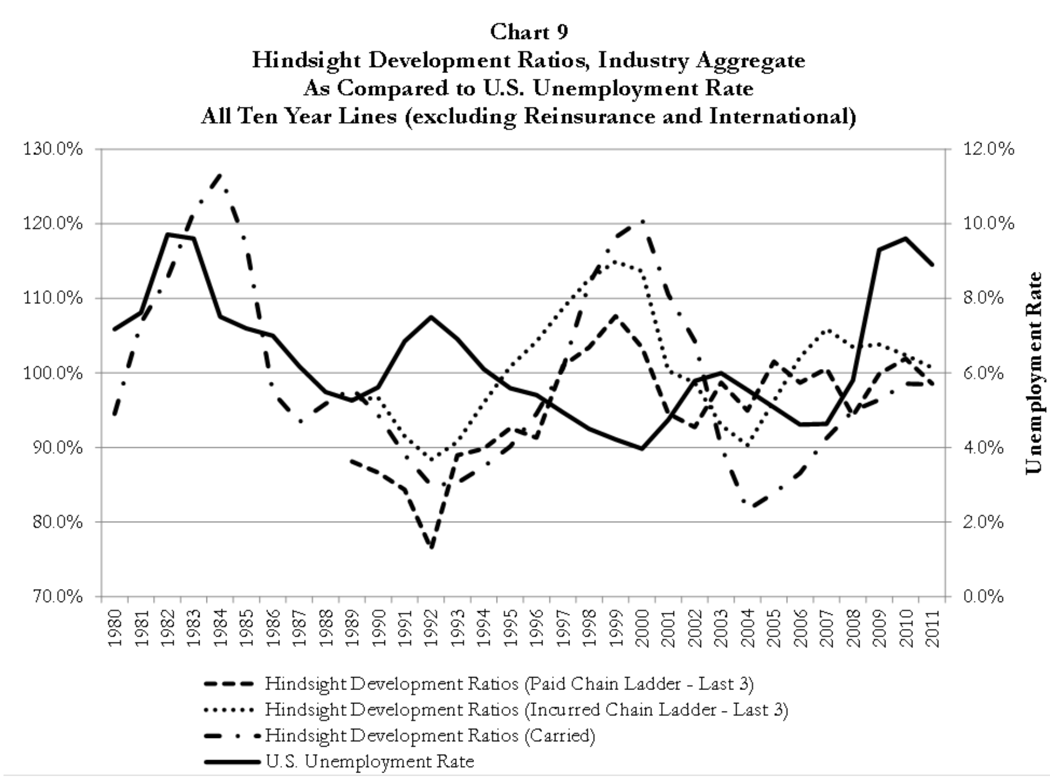


Here the underwriting cycle is represented by the hindsight (i.e., actual) loss and DCCE ratio by coverage year. Reserve development is represented by the hindsight development ratios. Chart 8 suggests a strong relationship between carried reserve adequacy and the underwriting cycle. It is interesting that, at the same time, there is essentially no correlation between the hindsight development ratios of the paid chain ladder method and the underwriting cycle. Table 2 provides the correlations between the above hindsight development ratios and the hindsight loss and DCCE ratios:

Table 2 Correlations of Hindsight Development Ratios with The Hindsight Loss and DCCE Ratios		
Indication	Correlation	Shifted Correlation[12]
Paid Chain Ladder – Three Years Weighted Average Development Factors	-18%	2%
Incurred Chain Ladder – Three Years Weighted Average Development Factors	31%	51%
Carried Reserves	82%	73%

Chart 9 is similar to Chart 8, but compares the hindsight development ratios to the economic cycle, as represented by the unemployment rate:

[12] Shifted correlation in this context refers to the correlation of the hindsight loss and DCCE ratios with the prior coverage year's hindsight development ratio. These indications suggest there may be a lagged relationship between the reserving cycle and the underwriting cycle. This may be due to an underlying relationship between these two cycles and the economic cycle (to be discussed further below). It is possible that the underwriting cycle is essentially a lagged result of the economic cycle, as has been discussed elsewhere by other authors.



The correlations between the hindsight development ratios and the unemployment rate are given in Table 3:

Table 3
Correlations of Hindsight Development Ratios with
The Unemployment Rate[13]

Indication	Correlation Measured 1989 – 2003[14]	Correlation Measured 1989 – 2008
Paid Chain Ladder – Three Years Weighted Average Development Factors	-85%	-85%
Incurred Chain Ladder – Three Years Weighted Average Development Factors	-91%	-83%
Carried Reserves	-90%	-65%

Thus Table 3 suggests a very strong relationship between the hindsight development of actuarial methods and the underlying economic cycle. More specifically, when the unemployment rate is low, subsequent reserve development for the corresponding coverage year is adverse. Conversely,

[13] Annual unemployment rate data was obtained from the Bureau of Labor Statistics at www.bls.gov.
[14] We have considered the correlations measured using data through 2003 and also through 2008. Both sets of indications are important. Data through 2003 (which we evaluated using hindsight data as of December 31, 2012) would be at an "ultimate" evaluation. Subsequent coverage years are not fully developed and consequently may fail to demonstrate a relationship between the reserve development that has been exhibited to date and the economic cycle. However a large portion of reserve development for coverage years through 2008 has been exhibited by December 31, 2012, and consequently these additional coverage years are useful to the analysis.

reserve development is favorable for coverage years with high unemployment.

One possible reason for this observation is the relationship between inflation and the economic cycle. Inflation would have a calendar year impact on payments and would presumably impact payment patterns over time. The economic cycle also likely influences underlying factors such as the propensity to report smaller claims. Even the underlying composition of claims would likely change, perhaps significantly, due to economic factors. Nonetheless, reasons for the impact of the economic cycle on the development of actuarial methods are far from understood.

For carried reserves the results are less conclusive. As noted in the headings of Table 3, we have focused on the years beginning in 1989 to measure the relevant correlations. That is because the actuarial indications are available at twelve months of development beginning with this year. The carried hindsight development ratios are available back to 1980. These show an essentially inverted relationship between the reserving cycle and the economic cycle, relative to subsequent years. This may be due in part to the high inflation of the early 1980s, which has not been observed subsequently. Perhaps more impactful would be asbestos and environmental losses stemming from these coverage years, recognized in the late 1980s or early 1990s.

Even when the years prior to 1989 are not considered, the correlations suggest a stronger relationship between the development of actuarial indications and the economic cycle than between carried reserve development and the economic cycle. There are likely factors influencing carried reserves that are not apparent in the actuarial indications. For example, carried reserves are influenced by the loss and DCCE ratios of recent coverage years.

When the loss and DCCE ratios are changing, the degree of change can be very difficult to estimate, and there may be a certain "anchoring" effect in the setting of carried reserves, whereby in setting reserves for a given coverage year a psychological difficulty is encountered in deviating from the results of prior coverage years. This might explain the strong relationship between the development of carried reserves and the underwriting cycle. Since the underwriting cycle appears to lag the economic cycle, this in turn may explain why the relationship between carried reserve development and the economic cycle is not as strong.

6. LIMITATIONS OF THE ANALYSIS

There are several limitations on any conclusions of the current analysis. In particular, we must recognize that the analysis has been performed on an industry aggregate basis. Hence, although patterns such as payment rates are more stable than when considered on an individual company basis, we are inherently limited in our ability to understand changes in these patterns when they occur.

Perhaps most importantly, we should not conclude that methods that appear to perform well on an industry aggregate basis would necessarily be the best methods to use in a company setting. For example, due to a limited amount of data for many companies, methods such as the frequency/severity and loss ratio methods can be integral to an actuarial analysis at early evaluations. On an industry aggregate basis, where a sufficient amount of loss data is available, we have observed that the frequency/severity and loss ratio methods underperform other methods considered (this would also be due in part to a lack of information on rate changes and, at times, inconsistency in claim counts within the Annual Statement from one evaluation to the next).

As another example, consider that case reserve adequacy appears to have changed significantly over time on an aggregate basis. As a result, the paid chain ladder method outperforms the incurred chain ladder method in the more recent years. However, changes in case reserve adequacy are not present for all companies. For any company for which case reserve adequacy has been stable, methods that reflect case reserves can be expected to outperform methods that are based on paid amounts alone.[15]

Lastly, we have assumed throughout the discussion that the results of actuarial methods applied to data at an industry aggregate level would be substantively similar to the aggregation of the results of actuarial methods applied to individual companies or books of business. It is possible that the results of our analysis would differ materially if performed on an individual company basis. However, is seems highly likely that the cyclicality we have observed is a phenomenon affecting all companies. This cyclicality would be difficult to observe for the vast majority of companies based on their individual data alone and may also be masked by the volatility of year-over-year results at this level. While the magnitude of our observations might differ if the analysis had been performed in a different manner, we believe the substance of the conclusions would remain the same.

[15] This is particularly true for smaller companies. See [6].

7. CONCLUSIONS

The most important conclusion of the analysis is the demonstration that actuarial methods bear some attribution for deficiencies and redundancies that have been present within the carried reserves of the property and casualty industry over time. This should not be taken to assign fault to individual actuaries or to actuaries as a whole. Given the state of actuarial science at the time, actuaries were using standard and accepted – in fact, the most accepted – actuarial methods.

However, many actuaries have previously characterized the material portion of the industry's deficiency or redundancy as being the result of management decisions or unpredictable volatility (see the discussion on prior research in Section 3). Certainly there have been cases where this observation holds. However the current analysis suggests that actuarial methods may have been as great a contributor historically to the deficiency or redundancy in carried reserve levels.

Examining the results of the analysis strongly suggests that we consider whether we may be able to improve upon our most accepted methods. However, it is unclear whether such improvements are only possible for the largest insurance companies, which generally exhibit less volatility in results (not to be confused with cyclicality), or whether such methods might be helpful for smaller companies as well. A level of prediction is possible on an industry aggregate basis that is likely not present for the smaller companies within the industry.

It is also significant that even with the use of our best methods, some degree of uncertainty will exist for the industry's reserve levels, even on an aggregate basis. Historical results show that the industry's reserves may develop – favorably or adversely – by 5% to 10% of initial indicated amounts. Such development may be unpredictable based on the current state of actuarial science. Given the inevitable uncertainty in any indication of future payments, significant improvement in indications of unpaid loss and DCCE may not be possible.

These conclusions should be considered magnified for individual insurance companies. For almost all insurance companies, development in excess of the industry aggregate benchmark of 5% to 10%, mentioned above, should be considered reasonably possible. We might characterize such benchmark development as the result of "systemic risk." For small to medium-sized insurance companies in particular, development well in excess of this benchmark is a significant possibility.

Acknowledgment

We are very grateful for the work of Ryan Skaggs, Nicholas Blaubach, Edem Togbey, Max Krueger, and Drew Groth, who created electronic data files for statement years prior to 1996, based on editions of A.M. Best Company's Aggregates & Averages dating back to 1984. This time-intensive task was instrumental to our analysis. In addition, we would also like to thank Andrew Chandler and Xi Wu for their helpful comments on an early draft of this paper.

Appendix A – Loss Reserving Methods

The following provides a list of the methods considered in the analysis, including the abbreviation used to refer to each method (note that for methods for which there are paid and incurred versions, multiple abbreviations are given). Also included is any relevant information as to how the method is applied within the current analysis, given the data limitations of Schedule P. As a result of these data limitations, the methods outlined below develop indications of loss at a 10th report (i.e., the last evaluation included within the Schedule P triangles) rather than indications of loss at ultimate.

1. **Backward Recursive Case Development (BRC)**

   This method is discussed by Marker and Mohl in [10]. The paid-on-prior-case and case-on-prior-case factors selected for our analysis are each the weighted average of the columns of these factors as given by the triangles, where the weights are proportional to the prior case. At a 10th report, we have assumed a paid-on-prior-case factor of 1.00 and a case-on-prior-case factor of 0.00.

2. **Benktander (BT)**

   The Benktander method, discussed in [9], is often referred to as the "iterated Bornhuetter-Ferguson method." In the BT method, a priori loss is equal to the indication from the BF method (in our case, BF1-I for the incurred method, and BF1-P for the paid method). The calculation of indicated loss then proceeds as described for the BF method, with calculations of the percent unpaid for the BT-P method and the percent IBNR for the BT-I method.

3. **Berquist-Sherman Case Adjustment (BS)**

   The BS method is the first of the two methods given in [2], in which an adjustment is made to the incurred loss in the prior diagonals of a given triangle for assumed changes in case reserve adequacy. This adjustment is made by de-trending the average case reserve along the most recent diagonal of the triangle (at rates that vary by line of business and evaluation date). The result is multiplied by the number of open claims within prior diagonals in order to obtain an indication of case reserves from prior diagonals at the approximate level of case reserve adequacy as the most recent diagonal. Incurred loss development factors are then developed and applied to loss along the most recent diagonal as for the LDF-I method.

4. **Bornhuetter-Ferguson 1 (BF1)**

   The first of the BF methods included in the analysis uses the indicated loss from the first loss ratio method (LR1), described below, as the a priori indicated loss. The percent unpaid and percent IBNR are then calculated as described in [3], producing both paid (BF1-P) and incurred (BF1-I) versions of this method.

5. **Bornhuetter-Ferguson 2 (BF2)**

   The second of the BF methods is an iterative procedure in which the a priori indicated loss is based on the weighted average loss ratios of preceding accident years, as based on the BF2 method indications for these years. The oldest accident year in the triangle, as well as any other accident year for which loss ratios of older accident years are not available, relies on the same a priori loss ratio as the BF1 method. Both paid (BF2-P) and incurred (BF2-I) versions of this method are calculated.

6. **Brosius Least Squares (BLS)**

   The BLS method considers that there may be both additive and multiplicative aspects of loss development. Thus the method iteratively develops both a multiplicative loss development factor, to be applied to losses paid or incurred to date, and an additive factor, to be included subsequent to the multiplication. The factors are based on a least squares regression, where the incurred loss ratio at a 10^{th} report is the dependent variable and the paid or incurred loss ratio at the given evaluation is the independent variable. The use of loss ratios rather than loss is a difference from the methodology as presented in [4], and was done so as to normalize for changes in exposure across accident years. Both paid (BLS-P) and incurred (BLS-I) versions are included.

7. **Brosius Least Squares – Weighted (BLSW)**

   Having observed certain indications produced by the BLS method, we sought to enhance the reliability of this method by giving more credibility in the regression process to years with greater premium, and presumably greater exposure. The Weighted Brosius Least Squares method that resulted uses a regression process weighted by premium, in contrast to the unweighted regression used in the BLS method itself.

8. **Cape Cod (CC)**

   The Cape Cod method is very similar to the BF method, but develops a priori loss under the assumption that in total across accident years it should be equal to the CC method indication. For the CC method as included in this analysis, we have assumed the same loss ratio for each accident year (i.e., unlike certain of the loss ratio methods discussed below, there is no a priori difference assumed by year). Both paid (CC-P) and incurred (CC-I) versions of the method are included.

9. **Case Development Factor (CDF)**
   The CDF method is based on the loss development factors from the LDF method, discussed below. In the CDF method an indicated unpaid-to-case ratio is derived from the relationship between unpaid loss and case loss implicit in the selected paid and incurred loss development factors. This factor is then applied to the case reserve to derive an indication of unpaid loss, which is added to paid loss to date for an indication of loss incurred through the 10th report.

10. **Frequency/Severity (FS)**
    The FS method is based on a projection of reported claims at a 10th report and a severity applied to these claims. Reported claims are based on the company's triangular reported claims data (i.e., Section 3 of Part 5 of Schedule P for the given line of business) developed to a 10th report using weighted average reported claim development factors. Given the relatively favorable performance of the LDF-I method as well as its general acceptance within actuarial practice, we took the LDF-I method to be the "preliminary" selected method for use in selecting severities.

    Thus the severity for each accident year is calculated as the incurred loss at a 10th report indicated by the LDF-I method divided by the indicated reported claims at a 10th report. For a given accident year, a severity is selected based on the weighted average severities of all prior accident years, where the weights are proportional to the projected reported claims. In this process, the severities are trended to the accident year in question at rates that vary by line of business and evaluation date.

11. **Hindsight Outstanding/IBNR (HS)**
    The HS method is similar to the FS method in that it relies on an equivalent projection of reported claims as well as a preliminary selected loss method (also the LDF-I method). However within the HS method, the projection of reported claims is used to calculate a triangle of "hindsight outstanding" claims, which are the difference between the projection of reported claims at a 10th report and closed claims to date. Similarly, the preliminary selected loss method is used to calculate a triangle of hindsight outstanding loss, which is the difference between the preliminary method loss projections and the paid or incurred loss to date. Thus the difference represents unpaid loss for the HS-P method and IBNR loss for the HS-I method.

    The ratios of the values within the hindsight outstanding loss triangle to the corresponding values within the hindsight outstanding claims triangle produces a triangle of hindsight outstanding severities (unpaid severities for the HS-P method and IBNR severities for the HS-I method). For a given accident year, severities from the preceding years are trended at set rates that vary by line of business and evaluation date. A weighted average of these severities, where the weights are proportional to hindsight outstanding claims, is selected.

    The weighted average hindsight severity is then applied to the number of projected outstanding claims for the given accident year to produce indications of unpaid loss for the HS-P method and IBNR loss for the HS-I method. These are then added to paid loss or incurred loss, respectively, to derive indications of incurred loss at a 10th report. This method is also referred to as the "ultimate unclosed claim severity technique" within [7].

12. Incremental Additive (IA)

In this method, incremental (i.e., calendar year) changes in paid or incurred loss are observed by accident year and compared to the premium for that year. A weighted average ratio of incremental loss to premium is selected, where the weights are proportional to the premium. These ratios are accumulated to derive an IBNR-to-premium or unpaid-to-premium ratio at the given evaluation. The ratios are applied to premium to derive IBNR or unpaid loss itself, then added to incurred loss or paid loss, respectively, for the IA-I and IA-P methods. So that the IA-P method will produce an indication of incurred loss at a 10^{th} report, the unpaid-to-premium ratio at a 10^{th} report is set equal to the case-to-premium ratio at a 10^{th} report of the earliest year in the triangle.

13. Incremental Claims Closure (ICC)

The incremental claims closure method is described by Adler and Kline in [1]. In this method, reported claims at a 10^{th} report are projected based on the reported claims triangle and weighted average reported claims development factors selected from this triangle (as above for the FS and HS methods). A closing pattern is then selected based on historical weighted average incremental closed-on-prior-open factors, where the weights are proportional to the number of claims open. These factors are then applied iteratively to project incremental closed claims, with the difference between the projected reported claims at the 10^{th} report and the projected closed claims at the 10^{th} report being the number of claims projected to close after the 10^{th} report.

As the next step, historical incremental paid loss is compared to incremental closed claims to derive incremental paid loss per closed claim by time period. These amounts are then trended at rates that vary by line of business and evaluation date to the relevant time period and a weighted average of the indications selected (where the weights are proportional to the number of closed claims). Prospective incremental paid loss by accident year is then projected as the product of the projected incremental closed claims and the projected paid loss per closed claim, each for the same time period. Ultimate loss is then the sum of these projections with paid loss to date. Within the current analysis, claims that are projected to close after the 10^{th} report are assumed to have a severity equal to that of the claims that close between the 9^{th} and 10^{th} reports, but trended one additional year.

14. Incremental Multiplicative (IM)

The incremental multiplicative method is similar to the incremental additive method in that both methods consider incremental loss triangles. However, the IM method calculates development factors that are ratios of incremental loss in one time period to the incremental loss in the preceding time period. Weighted averages of these development factors are calculated, where the weights are proportional to the incremental loss in the preceding time period.

The development factors are then applied iteratively to project incremental loss in subsequent time periods. Projections of unpaid loss and IBNR loss are derived for the IM-P and IM-I methods, respectively, by accumulating the indications of incremental paid and incremental incurred loss by time period. These projections of unpaid loss and IBNR loss are added to paid loss to date and incurred loss to date, respectively, to derive distinct indications of ultimate loss.

Within the IM-P method, a tail factor from paid loss at a 10th report to a level reflecting incurred loss at a 10th report is selected based on the oldest accident year in the triangle and the assumption that the case loss within this accident year will be paid as is. In other words, the tail factor is the case loss for this year divided by the incremental paid loss for this year in the time period preceding the 10th report. If incremental paid loss for this time period is zero, then such a ratio is undefined and assumed to be zero for purposes of our analysis.

15. **Loss Development Factor (LDF)**
The LDF methods are based on the calculation of historical loss development factors from the paid and incurred triangles. The weighted average loss development factor from all available years within the triangle is applied to loss at the given evaluation date to derive indicated loss at a 10th report. Both paid (LDF-P) and incurred (LDF-I) versions of this method are included within the analysis. For the paid method, a tail factor to develop the losses from paid at a 10th report to incurred at a 10th report is equal to the incurred-to-paid ratio at a 10th report for the earliest year in the triangle.

16. **Loss Ratio – Based on A Priori Assumption (LR1)**
Three versions of the loss ratio method are included within our analysis. Each relies on net earned premium by calendar year, consistent with the use of net paid and incurred loss within the triangles. The first of these (LR1) is based on a priori industry indications of the loss ratio for the given coverage year. These loss ratios were derived from historical A.M. Best Review & Preview reports.

17. **Loss Ratio – Based on Preliminary Selected for Prior Years (LR2)**
The remaining two loss ratio methods are each based on the use of preliminary selected incurred loss at a 10th report, which for both is set equal to the results of the LDF-I method, consistent with the preliminary selected loss in the FS and HS methods. For the LR2 method, the loss ratio for a given accident year is set equal to the weighted average of the loss ratios produced by the preliminary selected method within the preceding accident years of the triangle, where the weights are proportional to net earned premium. This loss ratio is then multiplied by net earned premium for the given calendar year to derive indicated incurred loss at a 10th report for the LR2 method.

18. **Loss Ratio – Based on Preliminary Selected for Most Recent Three Prior Years (LR3)**
The LR3 method is very similar to the LR2 method, but rather than relying on all preceding accident years within the triangle, relies on at most the preceding three accident years. Thus this method is more responsive to recent loss ratio experience, but potentially more volatile.

19. **Munich Chain Ladder (MCL)**
The MCL method is described by Quarg and Mack in [11]. Similar to the LDF method, discussed above, there are paid (MCL-P) and incurred (MCL-I) versions of the MCL method. In practice, these indications often converge on each other, although the indications are rarely equal. Due to the convergence of the two methods, no adjustment factor is included in the calculation of the MCL-P method, which is distinct from the LDF-P method.

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Abbreviations and notations

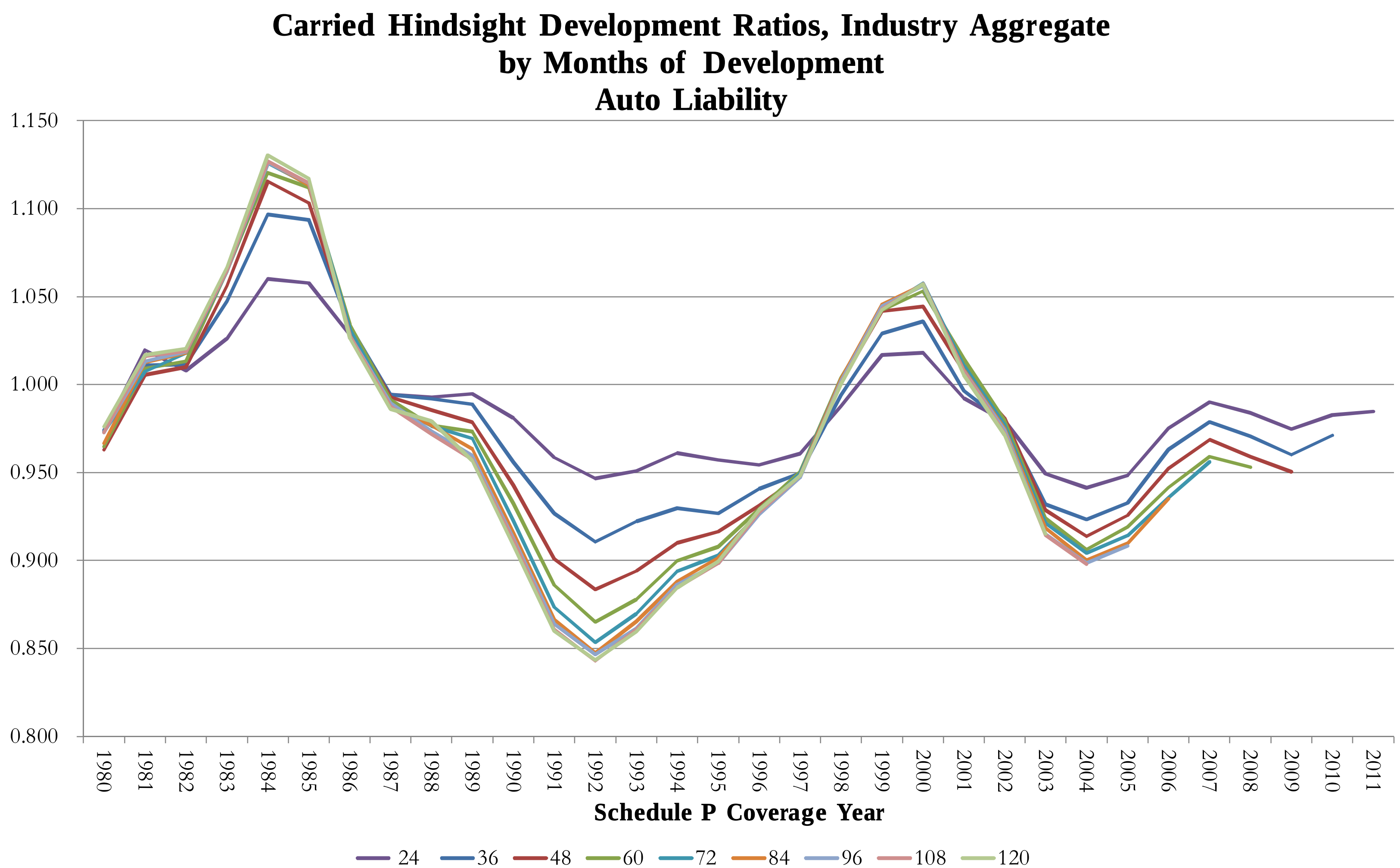
ALAE, allocated loss adjustment expense
BRC, backward recursive case development
BT, Benktander
BS, Berquist-Sherman case development
BF, Bornhuetter-Ferguson
BLS, Brosius least squares
BLSW, Brosius least squares – weighted
CAS, Casualty Actuarial Society
CC, Cape Cod
CDF, case development factor
DCCE, defense and cost containment expense
FS, frequency/severity
GIRO, general insurance research organizing committee
HS, hindsight outstanding
I, incurred
IA, incremental additive
IBNR, incurred but not reported
ICC, incremental claims closure
IM, incremental multiplicative
LAE, loss adjustment expense
LDF, loss development factor
LR, loss ratio
MCL, Munich chain ladder
P, paid
ULAE, unallocated loss adjustment expense

Biographies of the Authors

Susan J. Forray is a Principal and Consulting Actuary in the Milwaukee office of Milliman. She is a Fellow of the Casualty Actuarial Society and a Member of the American Academy of Actuaries. Susan has been active in the Casualty

Actuarial Society, having served on multiple committees. She is a frequent presenter at industry symposia, and her work has been published in the CAS *Forum*, *Best's Review*, *The Physician Insurer*, and *Contingencies*, among other venues. She can be reached at susan.forray@milliman.com.

Zachary A. Ballweg is a Consulting Actuary in the Milwaukee office of Milliman. He is a Fellow of the Casualty Actuarial Society and a Member of the American Academy of Actuaries. Zach has provided actuarial assistance to a spectrum of risk-bearing entities in both the private and public sectors, ranging from large multi-line commercial insurance companies to self-insured programs. He can be reached at zachary.ballweg@milliman.com.


Carried Hindsight Development Ratios, Industry Aggregate
by Months of Development
Auto Liability
1.150
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0.800
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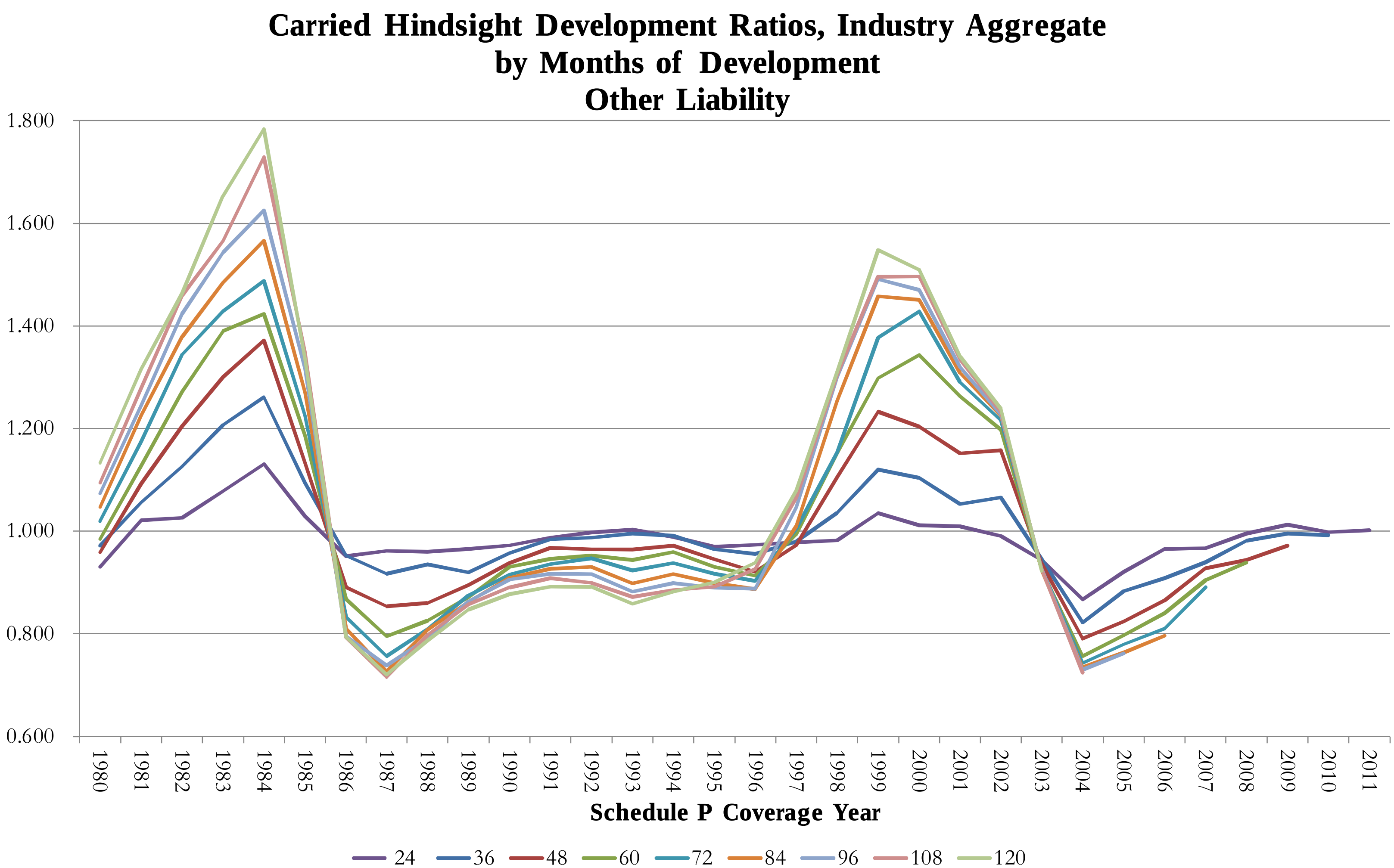
Carried Hindsight Development Ratios, Industry Aggregate
by Months of Development
Other Liability
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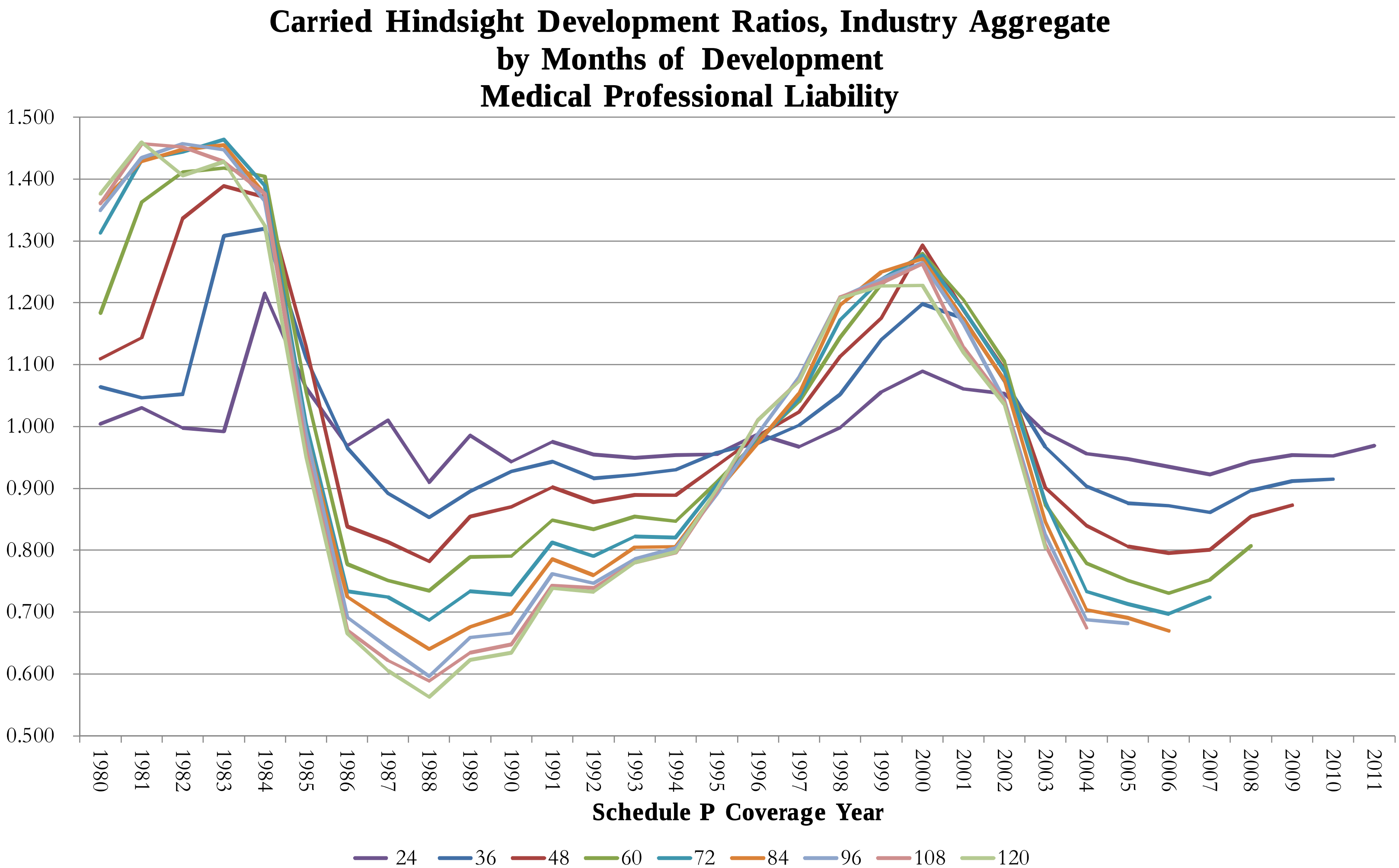
Carried Hindsight Development Ratios, Industry Aggregate
by Months of Development
Medical Professional Liability
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0.600
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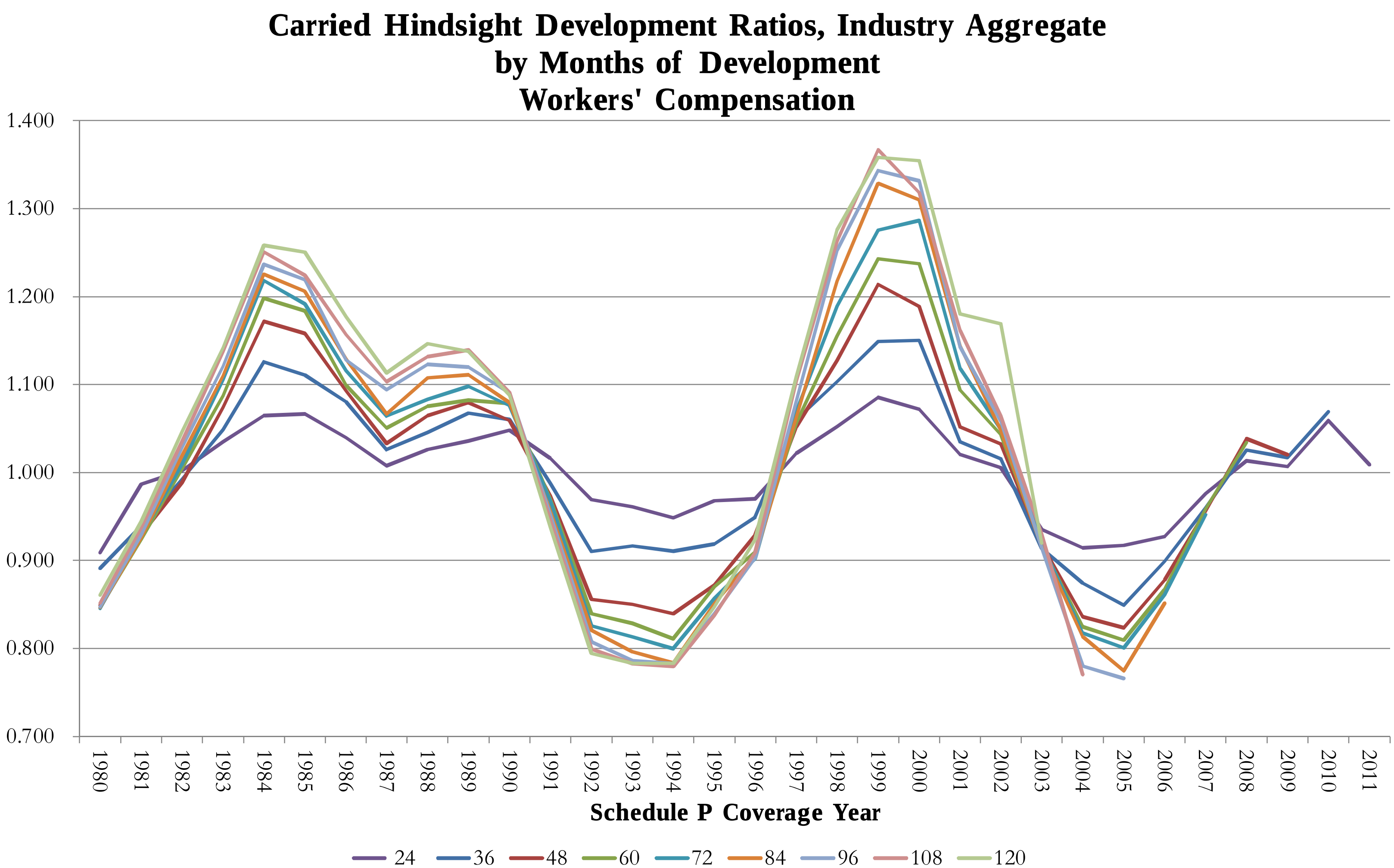
Schedule P Coverage Year
24 36 48 60 72 84 96 108 120


Carried Hindsight Development Ratios, Industry Aggregate
by Months of Development
Workers' Compensation
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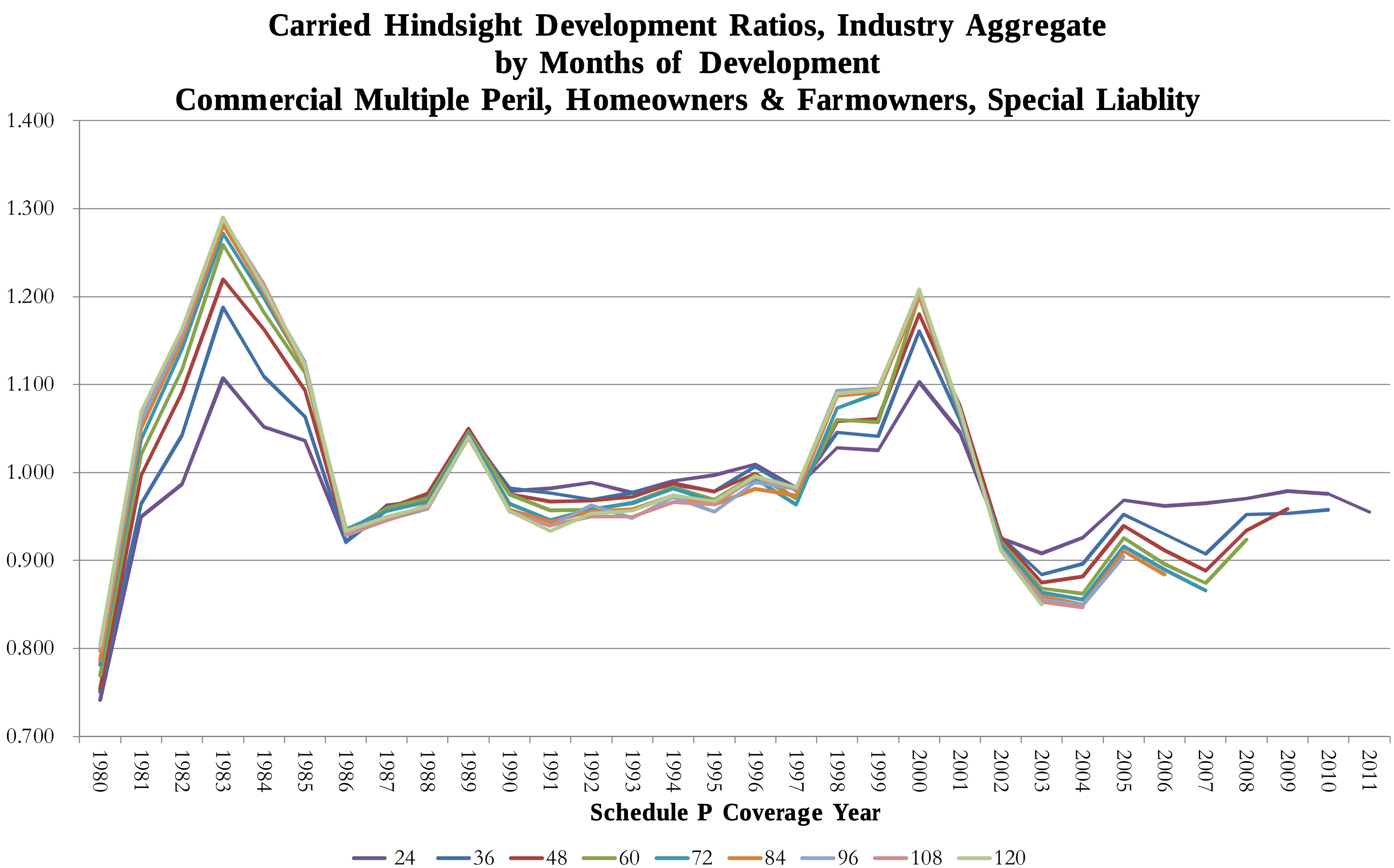
Carried Hindsight Development Ratios, Industry Aggregate
by Months of Development
Commercial Multiple Peril, Homeowners & Farmowners, Special Liablity
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1.200
1.100
1.000
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0.800
0.700
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2004
2005
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Schedule P Coverage Year
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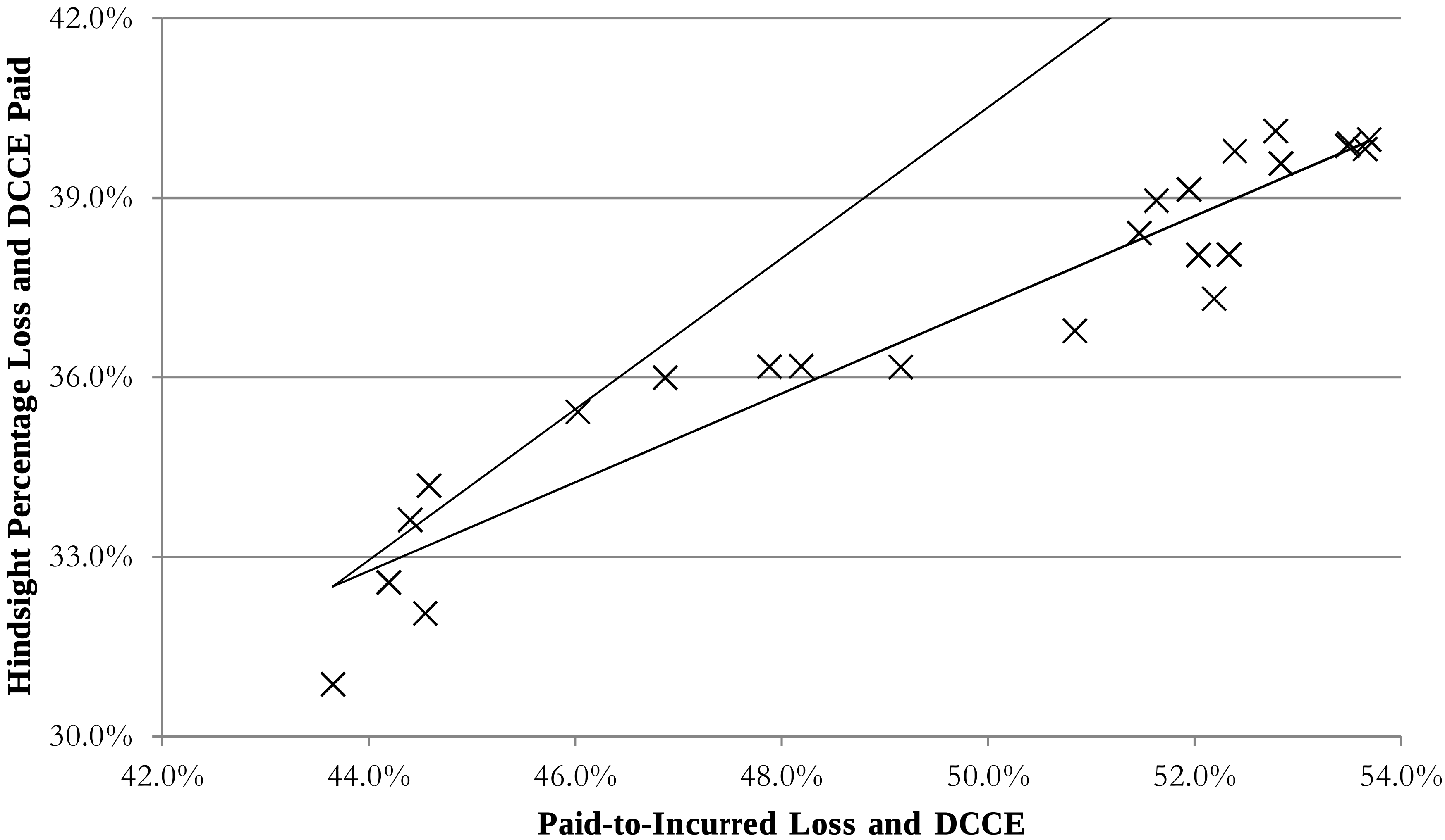
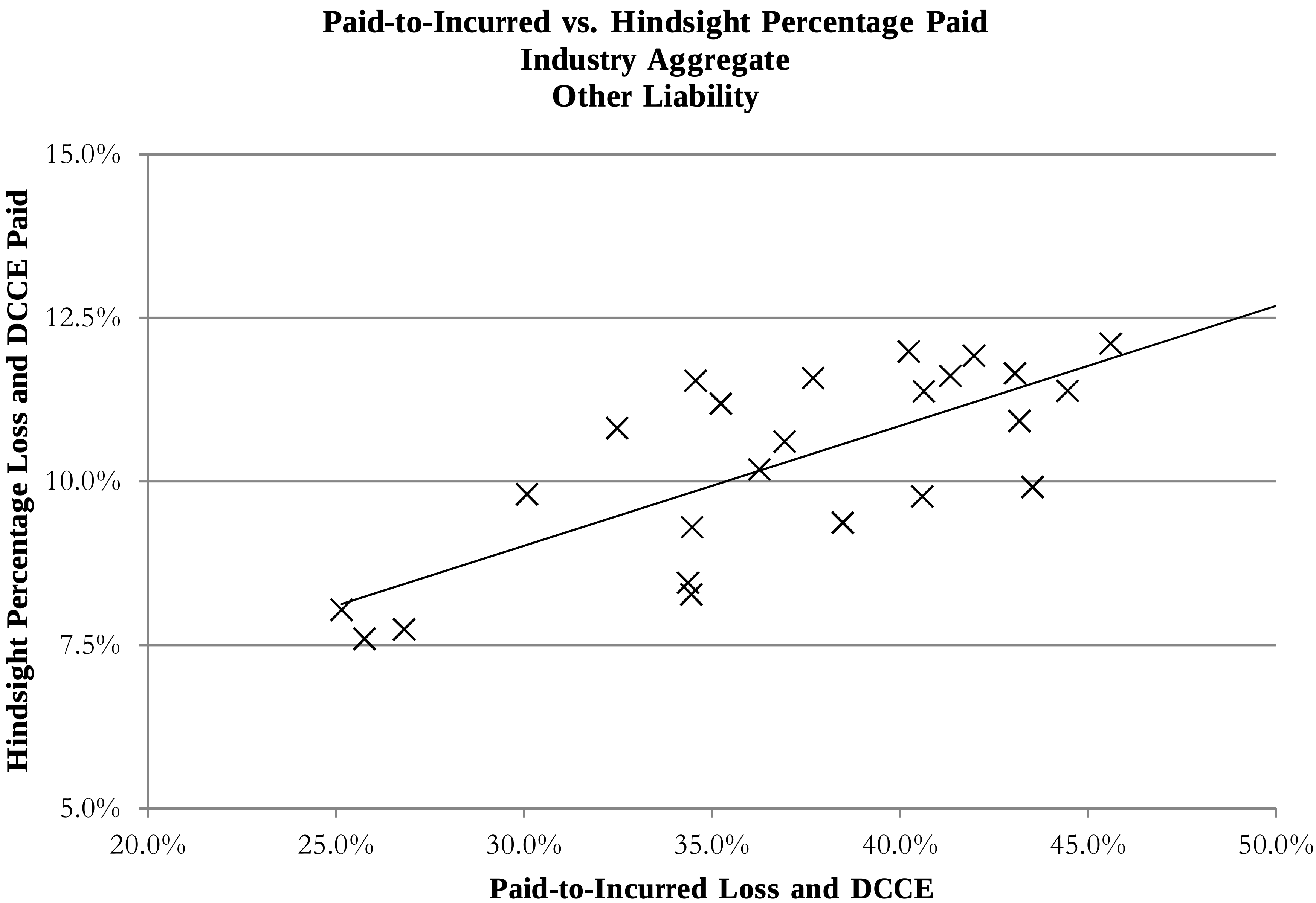
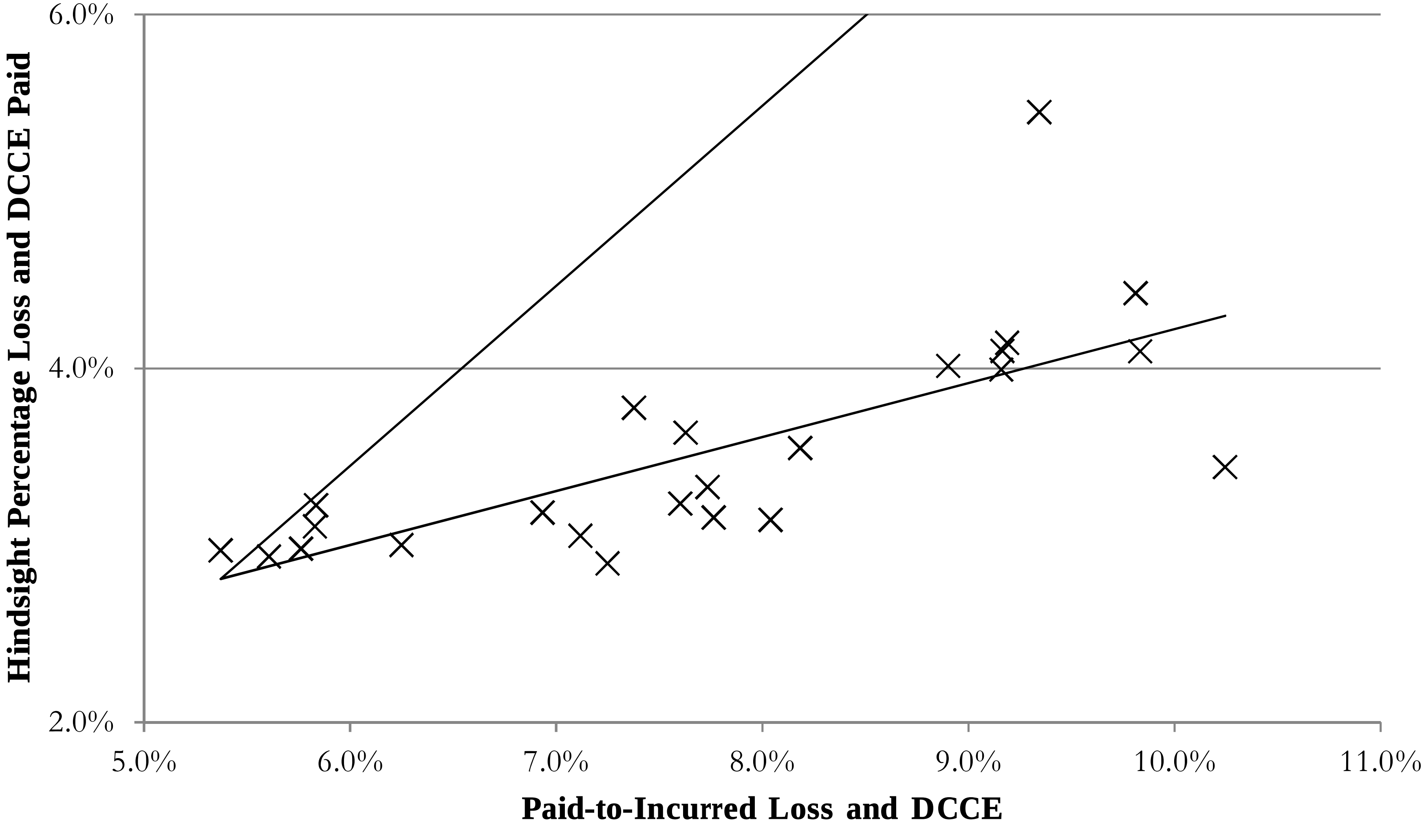
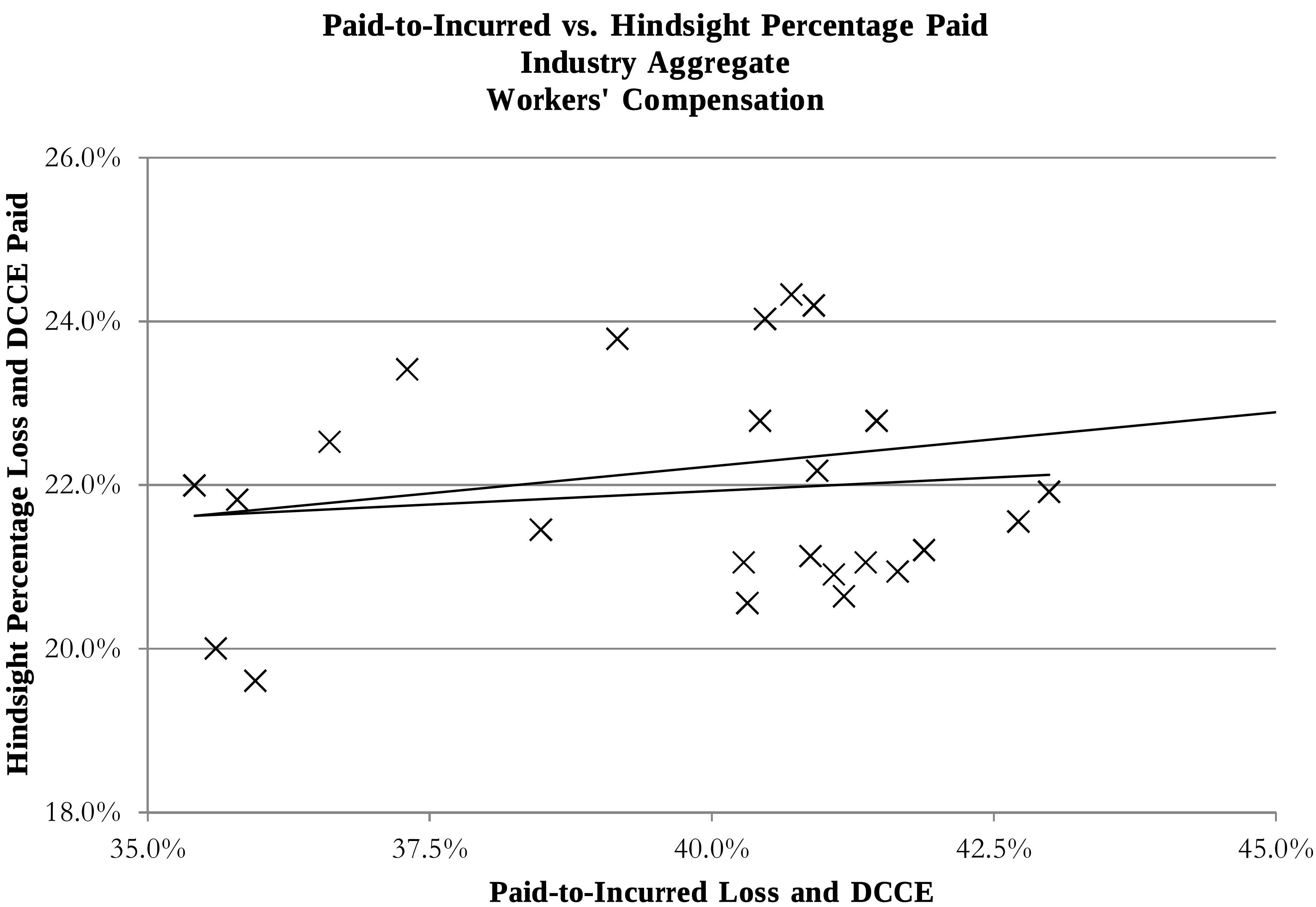
Paid-to-Incurred vs. Hindsight Percentage Paid
Industry Aggregate
Auto Liability



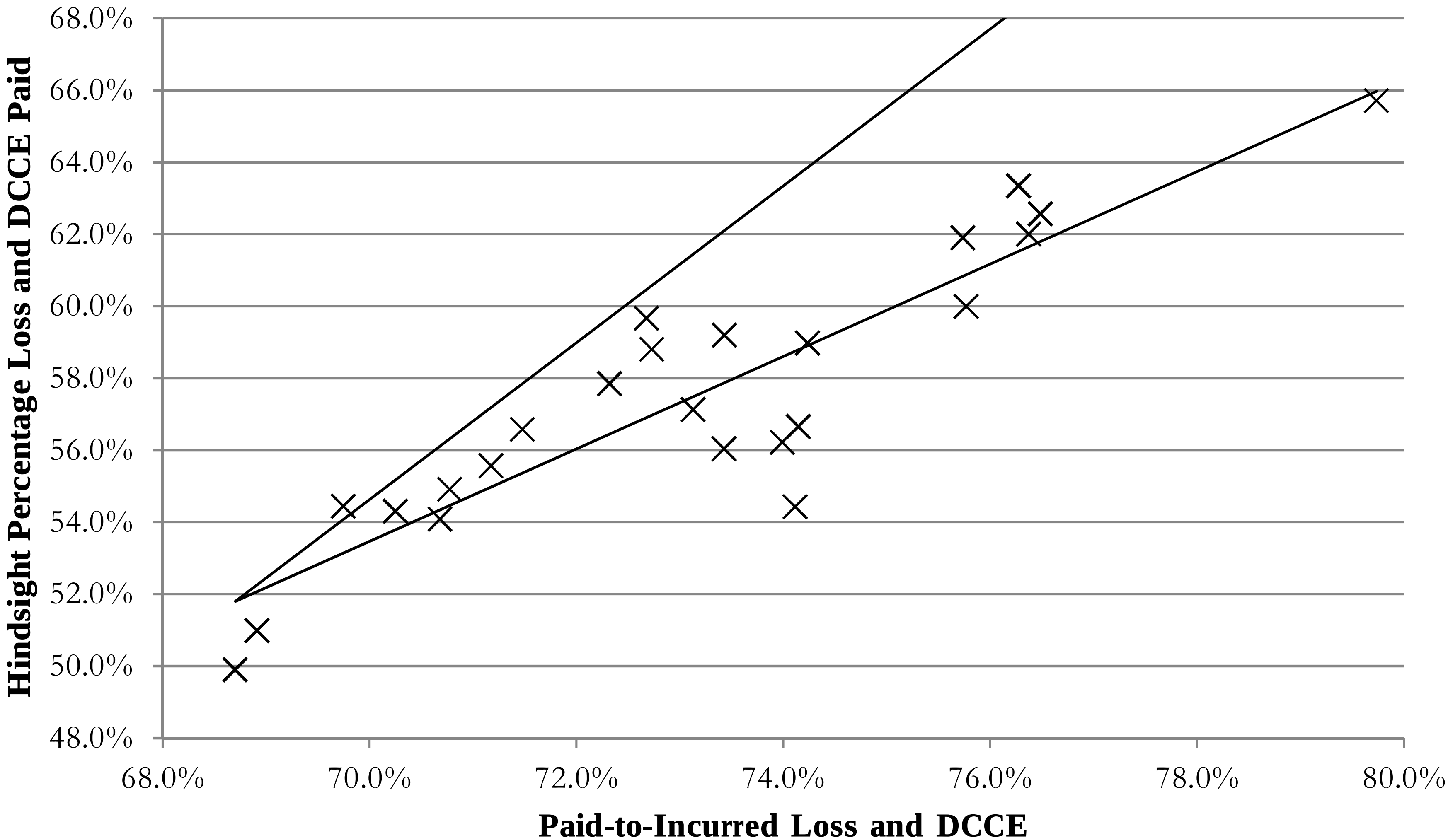

Paid-to-Incurred vs. Hindsight Percentage Paid
Industry Aggregate
Other Liability
15.0%
12.5%
10.0%
7.5%
5.0%
Hindsight Percentage Loss and DCCE Paid
20.0%
25.0%
30.0%
35.0%
40.0%
45.0%
50.0%
Paid-to-Incurred Loss and DCCE

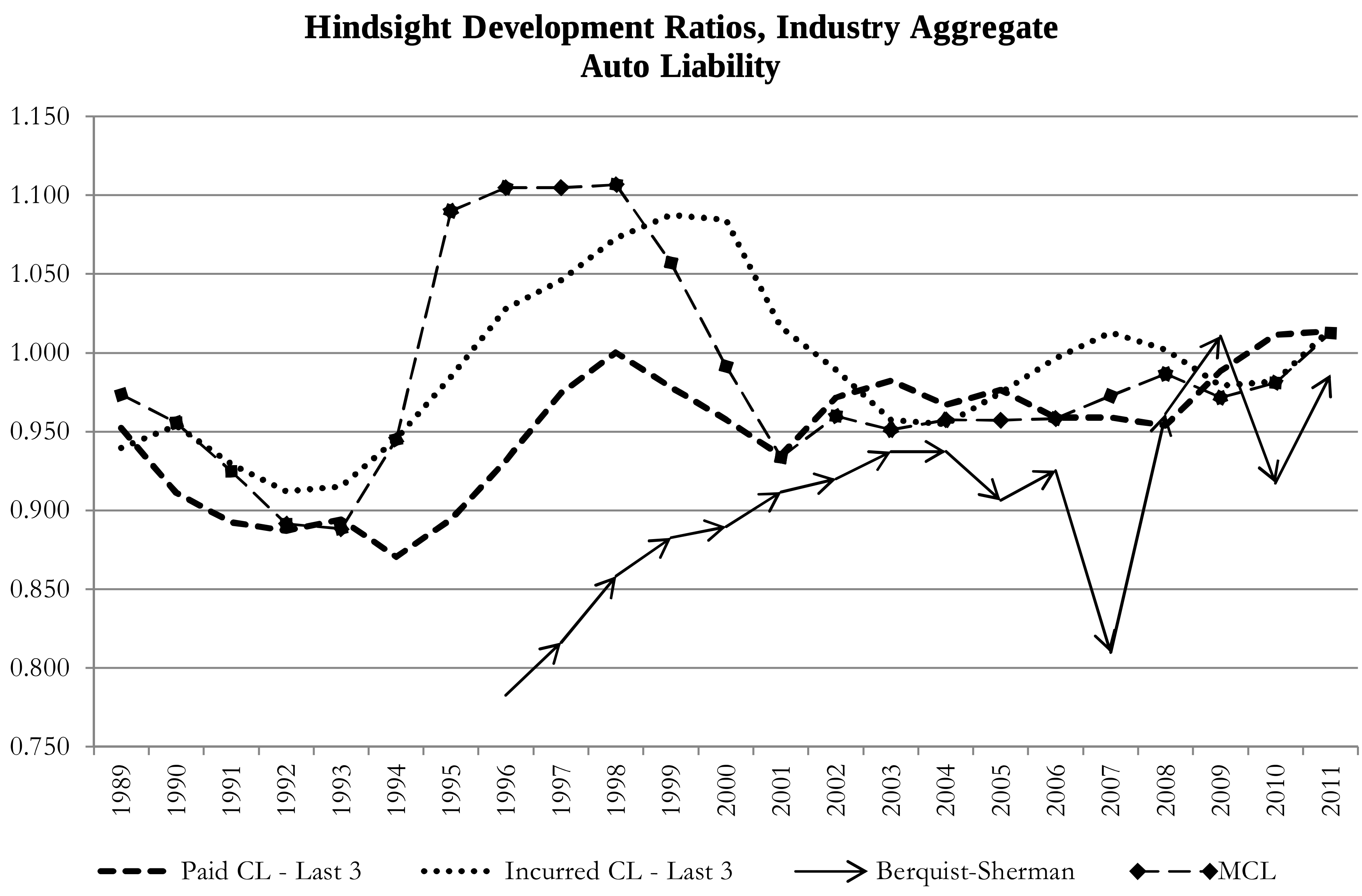
Paid-to-Incurred vs. Hindsight Percentage Paid
Industry Aggregate
Medical Professional Liability




Paid-to-Incurred vs. Hindsight Percentage Paid
Industry Aggregate
Workers' Compensation
Hindsight Percentage Loss and DCCE Paid
26.0%
24.0%
22.0%
20.0%
18.0%
35.0%
37.5%
40.0%
42.5%
45.0%
Paid-to-Incurred Loss and DCCE

Paid-to-Incurred vs. Hindsight Percentage Paid
Industry Aggregate
Commercial Multiple Peril, Homeowners & Farmowners, Special Liability




Hindsight Development Ratios, Industry Aggregate
Auto Liability
1.150
1.100
1.050
1.000
0.950
0.900
0.850
0.800
0.750
1989
1990
1991
1992
1993
1994
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2004
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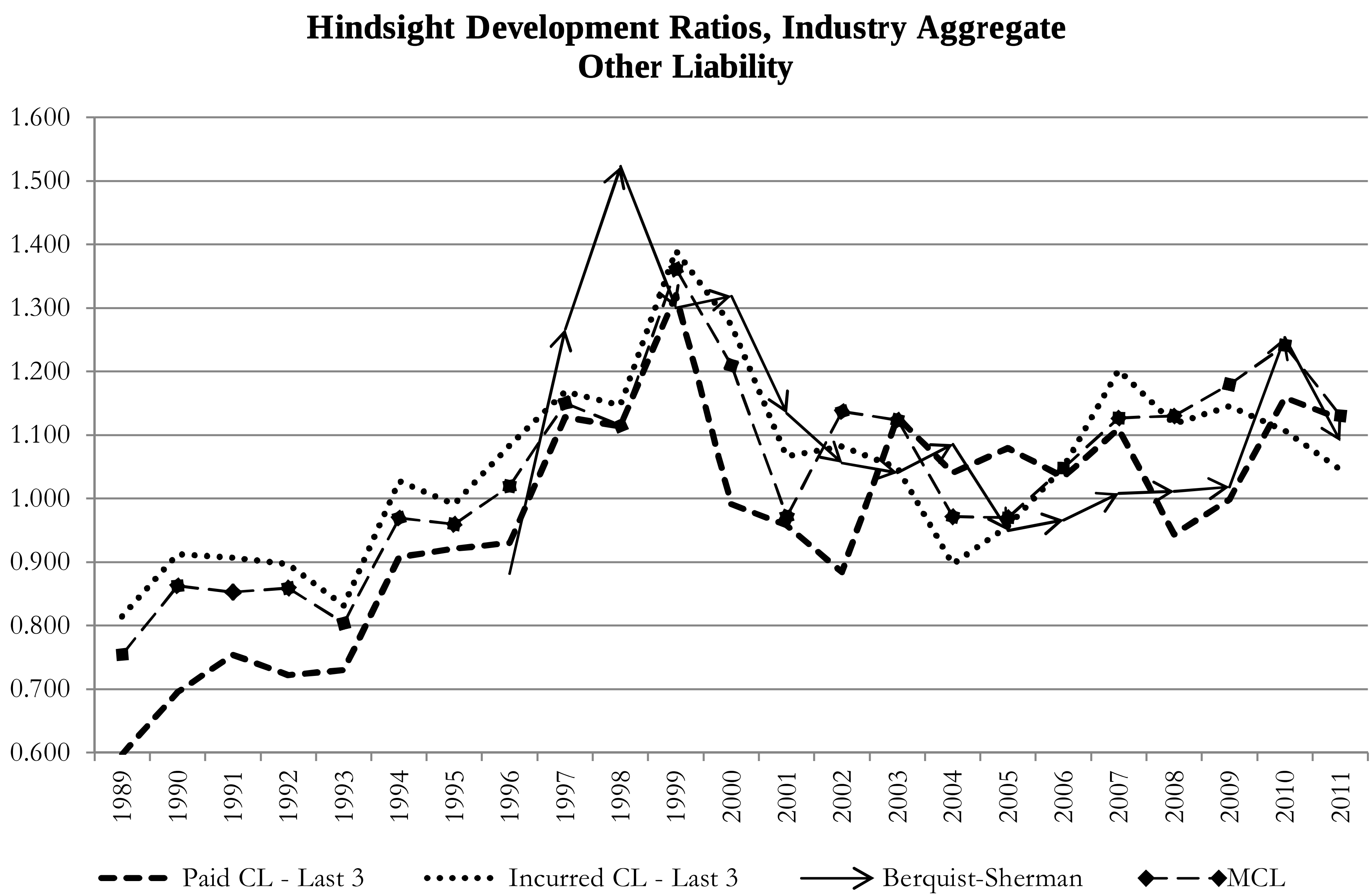
2006
2007
2008
2009
2010
2011
Paid CL - Last 3
Incurred CL - Last 3
Berquist-Sherman
MCL


Hindsight Development Ratios, Industry Aggregate
Other Liability
1.600
1.500
1.400
1.300
1.200
1.100
1.000
0.900
0.800
0.700
0.600
1989
1990
1991
1992
1993
1994
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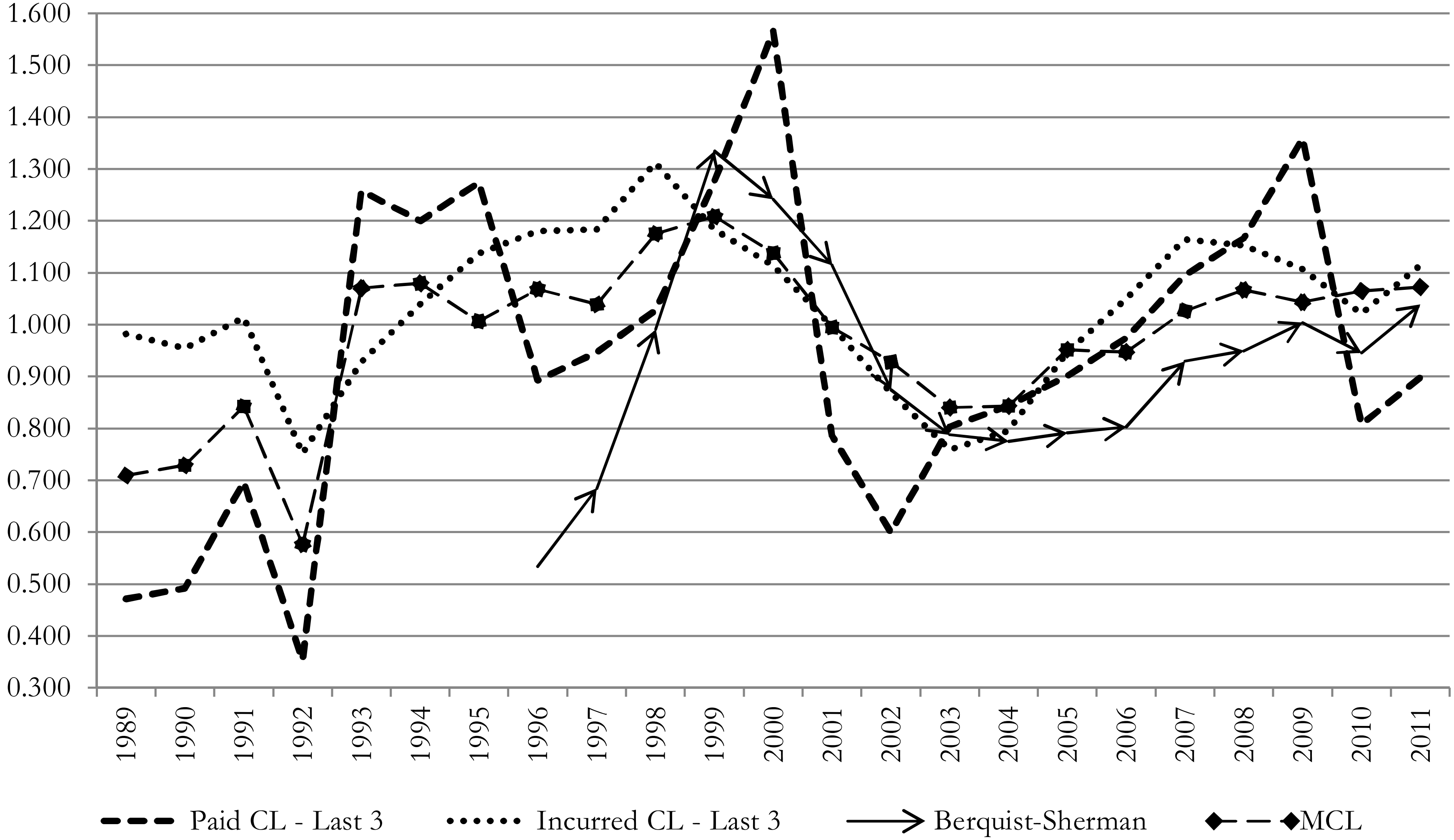
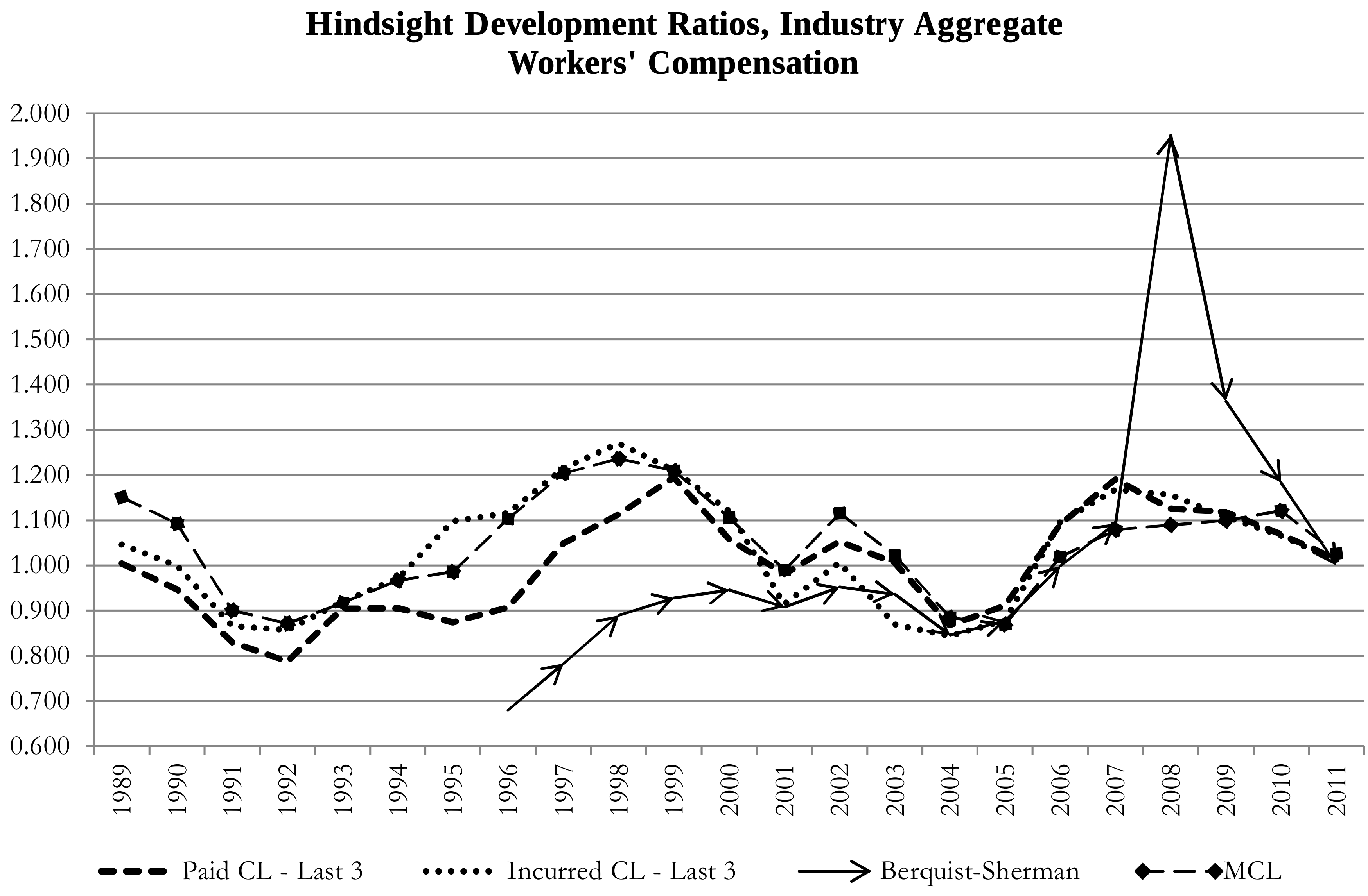
2011
Paid CL - Last 3
Incurred CL - Last 3
Berquist-Sherman
MCL

Hindsight Development Ratios, Industry Aggregate
Medical Professional Liability




Hindsight Development Ratios, Industry Aggregate
Workers' Compensation
2.000
1.900
1.800
1.700
1.600
1.500
1.400
1.300
1.200
1.100
1.000
0.900
0.800
0.700
0.600
1989
1990
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1992
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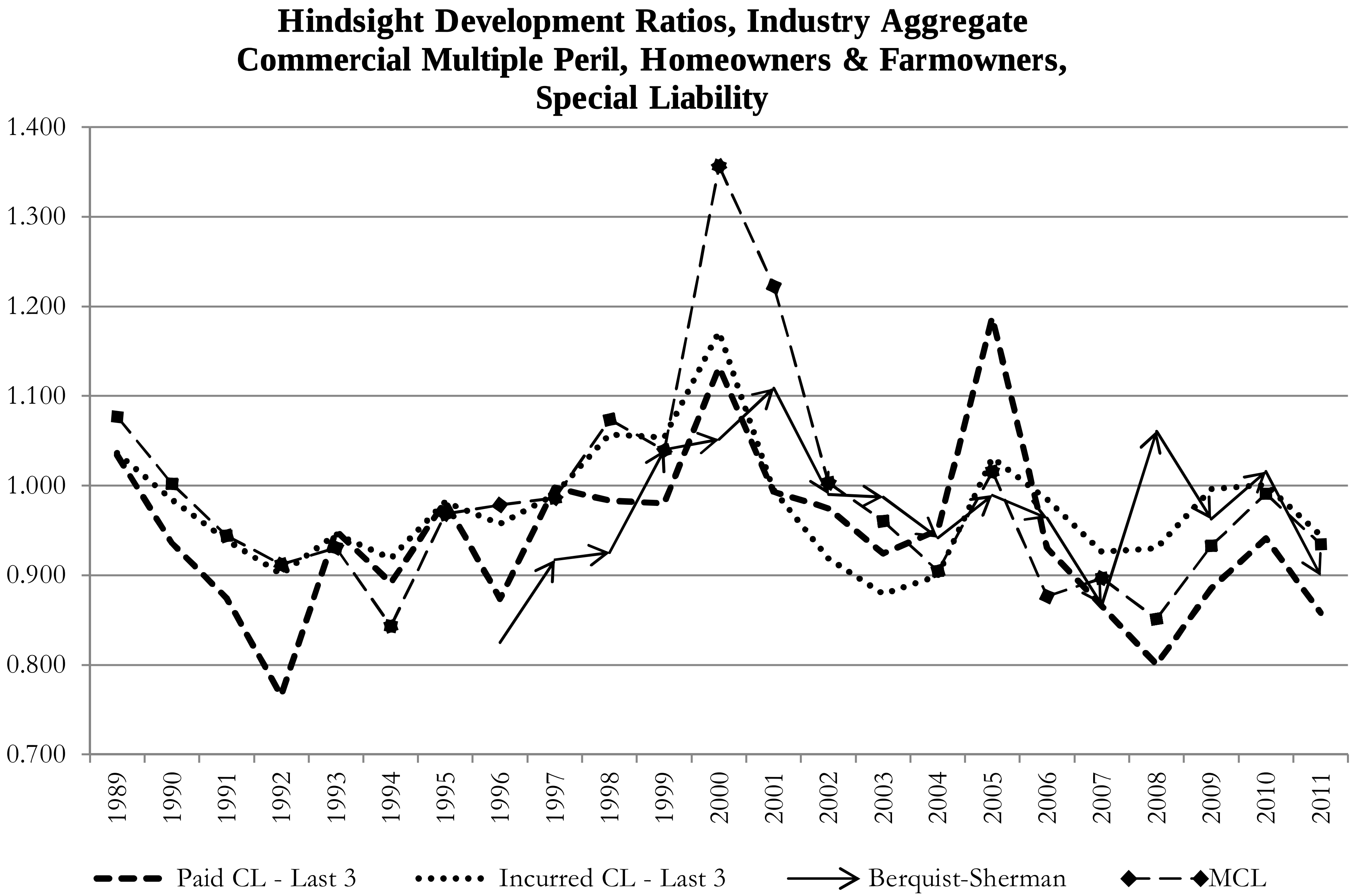
2007
2008
2009
2010
2011
Paid CL - Last 3
Incurred CL - Last 3
Berquist-Sherman
MCL


Hindsight Development Ratios, Industry Aggregate
Commercial Multiple Peril, Homeowners & Farmowners,
Special Liability
1.400
1.300
1.200
1.100
1.000
0.900
0.800
0.700
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
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2004
2005
2006
2007
2008
2009
2010
2011
Paid CL - Last 3
Incurred CL - Last 3
Berquist-Sherman
MCL

Correlations and R-Squared Values of Hindsight Development Ratios of
Carried Unpaid Loss and DCCE and
Actuarially Indicated Unpaid Loss and DCCE
Auto Liability

Actuarial Method	Correlation	R-Squared
Paid Chain Ladder *	41%	17%
Paid Chain Ladder **	67%	44%
Incurred Chain Ladder *	93%	86%
Incurred Chain Ladder **	85%	72%
Berquist-Sherman	1%	0%
Munich Chain Ladder	44%	19%

*All Years Weighted Average Development Factors
** Three Years Weighted Average Development Factors

Correlations and R-Squared Values of Hindsight Development Ratios of Carried Unpaid Loss and DCCE and Actuarially Indicated Unpaid Loss and DCCE *Other Liability*		
Actuarial Method	Correlation	R-Squared
Paid Chain Ladder *	57%	32%
Paid Chain Ladder **	38%	15%
Incurred Chain Ladder *	81%	66%
Incurred Chain Ladder **	73%	54%
Berquist-Sherman	69%	48%
Munich Chain Ladder	60%	36%

*All Years Weighted Average Development Factors
** Three Years Weighted Average Development Factors

Correlations and R-Squared Values of Hindsight Development Ratios of Carried Unpaid Loss and DCCE and Actuarially Indicated Unpaid Loss and DCCE ***Medical Professional Liability***		
Actuarial Method	Correlation	R-Squared
Paid Chain Ladder *	70%	49%
Paid Chain Ladder **	42%	18%
Incurred Chain Ladder *	75%	56%
Incurred Chain Ladder **	52%	27%
Berquist-Sherman	52%	27%
Munich Chain Ladder	68%	46%

*All Years Weighted Average Development Factors
** Three Years Weighted Average Development Factors

Correlations and R-Squared Values of Hindsight Development Ratios of Carried Unpaid Loss and DCCE and Actuarially Indicated Unpaid Loss and DCCE ***Workers' Compensation***		
Actuarial Method	Correlation	R-Squared
Paid Chain Ladder *	69%	47%
Paid Chain Ladder **	63%	40%
Incurred Chain Ladder *	93%	86%
Incurred Chain Ladder **	59%	35%
Berquist-Sherman	0%	0%
Munich Chain Ladder	80%	64%

*All Years Weighted Average Development Factors
** Three Years Weighted Average Development Factors

Correlations and R-Squared Values of Hindsight Development Ratios of Carried Unpaid Loss and DCCE and Actuarially Indicated Unpaid Loss and DCCE ***Commercial Multiple Peril, Homeowners & Farmowners, Special Liability***		
Actuarial Method	Correlation	R-Squared
Paid Chain Ladder *	26%	7%
Paid Chain Ladder **	40%	16%
Incurred Chain Ladder *	75%	56%
Incurred Chain Ladder **	82%	67%
Berquist-Sherman	34%	12%
Munich Chain Ladder	80%	63%

*All Years Weighted Average Development Factors
** Three Years Weighted Average Development Factors

Aggregate Loss Reserve Analysis by Accounting Date

Bertram A. Horowitz, F.C.A.S., M.A.A.A.

Abstract

This paper introduces new systematic procedures to estimate aggregate unpaid claims as of the current accounting date. Through the use of examples that introduce concepts in a natural progression, emphasis is placed on the reasonability and practicality of an accounting date reserving framework and its appeal to loss reserving practitioners. The accounting date framework provides a fresh perspective which differs from traditional actuarial reserving methods that typically derive unpaid claim estimates using individual accident year experience. Current accounting date aggregate unpaid claims are directly estimated from the emergence of aggregate claim experience which had been unpaid as of prior accounting dates. Exploration of this accounting date framework leads to techniques that may be understood as accounting date analogues of commonly used accident year reserving methods including the incurred development and Bornhuetter-Ferguson methods.

In addition to revealing visibly apparent aggregate unpaid claim estimates, the structure of appropriate accounting date reserving applications suggests improved accuracy over corresponding accident year development methods.

Keywords: loss reserve; reserving; unpaid claim estimate; IBNR; Bornhuetter-Ferguson; accounting date

1. INTRODUCTION

Basic loss reserving methods typically begin with individual accident year claim[1] experience and develop each accident year to an estimated ultimate value. These estimated ultimate values are reduced by cumulative claim payments as of the current accounting date resulting in an unpaid claim estimate for each accident year as of that date. In applying this procedure, the sum of the individual accident year unpaid claim estimates is understood to be an aggregate unpaid claim estimate as of the current accounting date.

Traditional accident year development methods have several important potential drawbacks:

- They are indirect. Indirectly solving for unpaid claims by estimating ultimate costs and then reducing this estimate by cumulative claim payments to date provides no immediate visible sense of the order-of-magnitude of a reasonable aggregate unpaid claim estimate.
- The aggregate unpaid claim estimate may be unduly volatile. The focus is to obtain unpaid claim estimates by individual accident year rather than directly target an aggregate unpaid claim estimate.

[1] Accident year claim (or loss) is used throughout this paper since it is the most common organization of historical data. Techniques described in this paper are also applicable to data organized in other time intervals including policy year, underwriting year, report year and fiscal year. Similarly, the techniques are applicable to monthly, quarterly and biannual data. Finally, the techniques presented are applicable to dollars, claim counts, ALAE (DCCE), and loss & ALAE combined.

- They are often highly leveraged, especially for long-tailed lines of business. Small changes in historical experience or development factor selection may lead to large changes in unpaid claim estimates. Even when exposures are directly incorporated into accident year development methods (e.g., Cape Cod), the focus remains on estimating individual accident year unpaid claims rather than an aggregate unpaid claim estimate.

This paper approaches reserving techniques from a different perspective by asking the direct question:

How might we estimate aggregate unpaid claims as of the current accounting date from the historical aggregate emergence of claims that were unpaid as of prior accounting dates?

This is addressed by examining properties of the emergence of aggregate unpaid claims under certain common and reasonable actuarial assumptions. We then endeavor to capitalize on these properties to derive estimates of aggregate unpaid claims as of the current accounting date. Exploration of the accounting date framework leads to techniques that may be understood as accounting date analogues of commonly used accident year reserving methods including the incurred development and Bornhuetter-Ferguson methods.

The accounting date techniques presented are relatively easy to apply and allow for direct estimation of aggregate unpaid claims. Since historical loss data is recast such that certain experience for all accident years is combined and the aggregate unpaid claims are estimated from this combined data, statistical volatility is expected to decrease while credibility is expected to increase as compared with traditional accident year development methods. The accounting date representation further provides an observable order-of-magnitude indication of reasonable unpaid claim estimates. Recent research suggests that certain accounting date reserving techniques are particularly consistent with the type of actuarial methodologies that tend to produce relatively accurate unpaid claim estimates in comparison with reserving methods in common use. Section 11 discusses these concepts further.

1.1 Research Context

Other than by separating historical experience into individual accident year components, surprisingly little actuarial literature exists on the subject of directly estimating aggregate unpaid claims as of an accounting date. Saltzmann [16] sought to find an appropriate "yardstick" to measure aggregate loss reserve adequacy. Khury [12] introduces the idea of using "reserve ratios" (i.e., IBNR to premium, IBNR to reported loss, IBNR to paid loss, total reserve to premium, and total reserve to paid loss) as tools for testing the reasonableness of loss reserves. The current NAIC IRIS Ratio 13 Estimated Current Reserve Deficiency to Policyholder's Surplus [15 p. 204] includes an estimate of current accounting date aggregate unpaid loss & DCCE based upon the average of developed loss & DCCE reserves to earned premium for the two prior accounting years which is then applied to current accounting date earned premium. However, all these measures are only benchmark tests and are not intended for use in actually setting loss reserves.

1.2 Objective

The purpose of this paper is to set forth a framework and systematic procedures to estimate aggregate unpaid claims as of the current accounting date. Through the use of examples that introduce concepts in a natural progression, emphasis is placed on the reasonability and practicality of this accounting date reserving paradigm and its appeal to loss reserving practitioners. Appropriate use of these accounting date concepts may increase the accuracy of aggregate unpaid claim estimates as well as bring visual clarity to the unpaid claim estimation process.

1.3 Outline

The remainder of this paper presents a framework and describes techniques by which aggregate unpaid claims may be estimated as of the current accounting date:

- Section 2 discusses actuarial assumptions relied upon to apply accounting date techniques.
- Section 3 introduces payment development by accounting date.
- Section 4 discusses incurred development by accounting date.
- Section 5 describes expected unpaid losses.
- Section 6 presents a Bornhuetter-Ferguson method by accounting date.
- Section 7 describes a Cape Cod method by accounting date.
- Section 8 explores the use of alternative exposure measures.
- Section 9 explains the broad applicability of the accounting date framework.
- Section 10 addresses certain implementation challenges.
- Section 11 discusses the major results of this paper.
- Section 12 summarizes the main conclusions of the paper.

2. ACTUARIAL ASSUMPTIONS

A loss reserve analysis usually commences with information gathering and exploration of any trends and changes that may affect the historical database. This guides the loss reserve practitioner in: consideration of the predictive power of applicable actuarial methods; choice of appropriate loss reserving techniques, and; interpretation of results.

As indicated by Berquist and Sherman [2], [10 p. 81], unpaid claim estimation cannot be reduced to a "cookbook" of rules and methods; actuarial judgment is required at many critical junctures to assure that unpaid claim estimates are neither distorted nor biased. Berquist and Sherman identify certain areas where actuarial judgment is required:

— Determining the optimal combination of the kinds of claims data to be used in the estimation of unpaid claims
— Assessing the effect of changes in an insurer's operations on the claims data that is used in estimating unpaid claims
— Adjusting the claims data for the influences of known and quantifiable events
— Evaluating the strengths and weaknesses of various estimation techniques
— Making the final selection of the unpaid claim estimate(s)

Mindful of the above, accounting date reserving techniques rely upon the following actuarial assumptions:

A1: The **requisite claim and exposure experience is available**. Techniques presented herein reorganize traditional accident year loss reserving claim and exposure experience into a new framework. Under certain conditions, less common exposure measures may be incorporated into the accounting date reserving paradigm.

A2: Except for noise (i.e., randomness in historical experience), **accident year payments subsequent to the first year of development follow the same payment pattern**.

A3: When case reserves are used as loss experience then, except for noise, **there has been no change in the adequacy of case reserves**.

A4: **The exposure metric as of each stage of development provides a reasonable measure of the relative accident year exposure to remaining development**. The exposure metric should reflect exposure volume including trend. Measurement of absolute exposure is not necessary.

A5: The **historical experience is statistically credible**.

A6: The **historical experience is homogeneous**.

A7: The **presence or absence of large claims does not distort the historical experience**.

While the valuation date and accounting date may not necessarily be equal, the current valuation date is assumed to equal the current accounting date for the purposes of this paper. Actuarial assumptions are denoted throughout this text by the shorthand references (e.g., A4) above.

When actual historical experience does not substantially satisfy certain actuarial assumptions relied upon by a particular technique (e.g., there has been a change in the claims environment), it is often possible to: restate historical experience on another basis; use alternative or supplementary data; or redefine the data to more completely satisfy actuarial assumptions. This is discussed further in Section 10.

The actuary should consider the use of multiple methods or models appropriate to the purpose, nature and scope of the assignment and the characteristics of the claims unless, in the actuary's professional judgment, reliance upon a single method or model is reasonable given the circumstances.[2] The relative strengths and weaknesses of appropriate actuarial techniques are evaluated in consideration of assignment objectives, the degree to which relevant actuarial assumptions are satisfied and the reasonableness of results.

As with all basic actuarial reserving methods, the methods presented herein provide deterministic single point estimates. Except in the most trivial situations and despite best efforts to satisfy actuarial assumptions, the actual future emergence of current accounting date unpaid claims is inherently uncertain.

3. PAYMENT DEVELOPMENT BY ACCOUNTING DATE

We introduce two payment development examples satisfying A1-A2 and A4-A7. A3 is not relevant since case reserves are not used as loss experience in payment development methods.

3.1 Static Example: No Noise

This first example contains no noise in the historical experience.

[2] Actuarial Standard of Practice No. 43 "Property/Casualty Unpaid Claim Estimates", Section 3.6.1

3.1.1 Traditional actuarial triangle chain-ladder accident year representation

Exhibit 1, Table 1 displays payment development data in the familiar CL format. Typically, selected age-to-age LDFs are derived as some average of historical LDFs. For each stage of development, the appropriate product of selected LDFs is the selected CDF. In this static example, since LDFs are identical within each age-to-age interval, simple average LDFs and volume weighted LDFs are identical within each development interval. Similarly, simple average CDFs and volume weighted CDFs are equal as of each stage of development.

Exhibit 1, Table 2 displays case reserves by accident year. Since there is no noise in this example, the ratio of case reserves to cumulative loss payments is the same for all accident years as of each stage of development.

3.1.2 Traditional payment development approach

Exhibit 1, Table 3 displays the traditional payment development method used to derive unpaid loss estimates from cumulative loss payments. The product of cumulative loss payments as of the current accounting date and their corresponding CDFs produce Column (4) estimated ultimate losses by accident year. These estimated ultimate losses are then reduced by cumulative loss payments as of the current accounting date resulting in an unpaid loss estimate for each accident year as of the current accounting date. Estimated unpaid losses by accident year are added to produce a total estimate of unpaid losses as of the current accounting date. The sum of individual accident year Column (5) unpaid loss estimates equals the total unpaid loss estimate of $434,721 as of 12/31/12.[3]

3.1.3 Accounting date representation

This paper presents an alternative approach that organizes the historical experience into an accounting date representation. Exhibit 1, Table 4 displays cumulative loss payment emergence by year-end accounting date and may be derived by the appropriate accumulation of cumulative loss payments from Exhibit 1, Table 1.

For example, year-end accounting date 2009 cumulative loss payments as of 12/31/12 (i.e., as of 3 years of emerged loss payments) of $205,714 are defined as loss payments subsequent to 12/31/09 on losses incurred during accident years 2009 & prior or, equivalently, as payments during calendar years 2010 through 2012 on accident years 2009 & prior. This may be derived from Exhibit 1, Table 1 as the

[3] Unless otherwise specified, tables in the text are displayed in rounded thousands of dollars (i.e., $000 Omitted).

sum of the appropriate accident year contributions:

85,700 - 82,993 =	2,707	accident year 2001 contribution
+ 88,350 - 81,375 =	6,975	accident year 2002 contribution
+ 95,000 - 82,000 =	13,000	accident year 2003 contribution
+ 93,840 - 76,500 =	17,340	accident year 2004 contribution
+ 86,573 - 66,290 =	20,283	accident year 2005 contribution
+ 85,999 - 59,780 =	26,219	accident year 2006 contribution
+ 79,444 - 46,607 =	32,837	accident year 2007 contribution
+ 63,163 - 28,282 =	34,881	accident year 2008 contribution
+69,857 - 18,383 =	51,474	accident year 2009 contribution
	205,714	Total[4]

The developed payments of Exhibit 1, Table 4 represent the historical emergence of aggregate losses that were incurred and unpaid as of each year-end accounting date. This representation provides useful information as it tracks the historical loss payment emergence of accounting date unpaid losses as opposed to tracking individual accident year loss payment development from accident year inception. Hence, the goal is to estimate the ultimate value of year-end accounting date 2012 (i.e., the value that corresponds to the bold rectangle in the lower right-hand corner of Exhibit 1, Table 4). How might we estimate aggregate unpaid claims as of the current accounting date from the historical aggregate emergence of claims that were unpaid as of prior accounting dates? Despite the absence of noise in this first example, the non-constant LDFs between each development interval resulting from different accident year exposure levels signifies that that an estimate of the bold rectangle value is not readily apparent directly from Exhibit 1, Table 4.

3.1.4 Accounting date representation recast at current accounting date exposure level

Exhibit 1, Table 4 year-end accounting date emergence may be recast into a form that is especially useful for estimating unpaid claims as of the current accounting date. The emerged loss payments of Exhibit 1, Table 4 are recast on Exhibit 1, Table 5 at the year-end accounting date 2012 exposure level where the case reserves of Exhibit 1, Table 2 are used as an A4 measure of the relative accident year exposure to remaining payments as of each stage of development.[5] Accordingly, Exhibit 1, Table 5

[4] Totals may not add precisely due to rounding

[5] While case reserves may not be a commonly used exposure base for traditional reserving methods that estimate individual accident year ultimate losses, case reserves can be a reasonable A4 accounting date reserving exposure metric. Exceptions would include (a) where zero case reserves at later stages of development do not signify negligible remaining exposure and (b) very long-tailed lines where few claims are reported in the early stages of development. Otherwise, when A1-A7 are satisfied, case reserves would be expected to be a reliable A4 measure of relative accident year exposure to remaining payments at each stage of development. Such case reserves would reflect the relative volume of remaining development exposure between accident years including trend. While A3 should be satisfied to accept case reserves as an

displays the Exhibit 1, Table 4 emergence recast as if each year-end accounting date had emerged at the current year-end 2012 accounting date exposure level. For example, using the loss payments of Exhibit 1, Table 1, the Exhibit 1, Table 4 year-end 2009 accounting date cumulative emerged loss payments as of 12/31/12 (i.e., after 3 years) of $205,714 is recast as:

(Year-End 2012 Accounting Date Exposure)/ (Year-End 2009 Accounting Date Exposure)			
(2,040/ 1,804) x (85,700 - 82,993)	=	3,061	accident year 2001 contribution
+ (3,958/ 3,720) x (88,350 - 81,375)	=	7,421	accident year 2002 contribution
+ (6,293/ 6,000) x (95,000 - 82,000)	=	13,635	accident year 2003 contribution
+ (9,533/ 9,180) x (93,840 - 76,500)	=	18,007	accident year 2004 contribution
+ (10,370/10,883) x (86,573 - 66,290)	=	19,327	accident year 2005 contribution
+ (15,932/13,634) x (85,999 - 59,780)	=	30,638	accident year 2006 contribution
+ (25,418/18,007) x (79,444 - 46,607)	=	46,351	accident year 2007 contribution
+ (31,399/18,855) x (63,163 - 28,282)	=	58,087	accident year 2008 contribution
+ (43,173/30,639) x (69,857 - 18,383)	=	72,531	accident year 2009 contribution
		269,056	Total

This year-end 2009 accounting date emerged loss payments as of 3 years, recast at the year-end 2012 accounting date exposure level total of $269,056, is displayed in its corresponding position on Exhibit 1, Table 5. Appendix A provides a formula to recast accounting date cumulative loss payment emergence at the current accounting date exposure level.

In order for recast year-end accounting date experience to be useful, we must be able to consistently recast each year-end accounting date through the same stage of development. Ideally, this would be though ultimate development (10 years of accident year development in this example). Section 10 discusses approaches under less than ideal circumstances.

The recast Exhibit 1, Table 5 loss payments emerged by year-end accounting date at the year-end 2012 accounting date exposure level visibly clarifies an appropriate aggregate year-end 2012 accounting date unpaid loss estimate. The recast unpaid claims for each year-end accounting date are seen to inevitably emerge towards an ultimate of $434,721. This is the same figure derived from the traditional payment development method on Exhibit 1, Table 3.

We now make several important observations:

- In contrast to traditional estimates which require an estimated ultimate for each accident year,

---

A4 exposure metric, A3 is unnecessary to perform payment development accounting date reserving. It is important to recognize that A4 exposure metrics other than case reserves may be appropriate as discussed in Sections 8 and 9.

the central goal under an accounting date representation is to directly target only one quantity, i.e., estimated aggregate unpaid claims incurred as of the current accounting date.

- Where there is no noise in the data and despite variable accident year exposure, development factors remain constant within development interval under the recast accounting date representation.
- In contrast to traditional indirect accident year estimated ultimate approaches, a reasonable unpaid claim estimate is visibly apparent under a year-end accounting date representation appropriately recast at the current accounting date exposure level.
- Where there is no noise, the recast accounting date representation results in the same unpaid claim estimate as traditional development methods.
- Tail factors converge to unity faster under accounting date representations than for corresponding traditional accident year representations.
- Accident year payments during the first calendar year are not reflected in accounting date representations.
- The final diagonal of accounting date representations contains all calendar year activity through the current accounting date on losses incurred as of each prior year-end accounting date that remained unpaid as of each year-end accounting date.
- Especially for longer tailed lines of business, the data volume for accounting date representations tends to grow faster than under corresponding traditional accident year representations.[6]

3.1.5 Estimation of aggregate unpaid loss

While we may visually observe $434,721 as an obvious unpaid claim estimate as of 12/31/12 for our 'no noise' example, this may be formalized mathematically. We can apply development procedures to the emergence of loss payments by accounting year recast at the current accounting date exposure level. The lower portion of Exhibit 1, Table 5 displays LDFs and corresponding CDFs for the recast accounting date loss payments. In this static example, since LDFs are identical within each development interval, simple average LDFs and volume weighted LDFs are identical within each development

[6] Long-tailed lines of business may exhibit little activity for recent accident years as of the current accounting date (e.g., accident year 2011 cumulative loss activity as of 12/31/12 equals 0), but would be expected to exhibit considerably more activity for recent year-end accounting dates as of the current accounting date. Accordingly, especially for long-tailed lines of business, statistical reliability and credibility (A5) would be expected to be enhanced under the recast accounting date representation since accident year activity is aggregated.

interval. As a result, simple average CDFs and volume weighted CDFs are identical at each development stage.

Exhibit 1, Table 6, Column (4) displays the indicated total emergence of unpaid year-end accounting date losses at year-end 2012 exposure levels using the recast accounting date payment development technique. As expected in this example without noise, the indicated unpaid loss for each prior year-end accounting date at the year-end 2012 accounting date exposure level equals $434,721.

3.1.6 Allocation of aggregate unpaid loss estimate to accident year

Rather than explicitly computing individual accident year unpaid claims as in the traditional payment development method, the accounting date reserving paradigm may be used to allocate the aggregate unpaid loss estimate to accident year by use of a top-down iterative approach that unwinds the exposure adjustment.

Exhibit 1, Table 6, Column (5) displays the indicated unpaid loss as of 12/31/12 at the 2012 year-end accounting date exposure level for each year-end accounting date. Beginning with accident year 2004, the oldest accident year with any remaining unpaid claim liability as of 12/31/12, we know that accident year 2004 is expected to have only one more year of loss payments beyond 12/31/12 (i.e., payments to be made during calendar year 2013). Recasting loss payments emerged at the 2012 year-end accounting date exposure level implies the following equation for accident year 2004:

$5,181= (43,173/25,500)x(acc. yr. 2004 estimated payments during yr. 10)

Solving this equation yields:

acc. yr. 2004 estimated payments during yr. 10 = (25,500/43,173)x$5,181 =
acc. yr. 2004 est. unpaid loss as of 12/31/12 = $3,060

Similarly, we have the following equation for accident year 2005:

$17,662 = (31,399/20,400)x(acc. yr. 2004 estimated payments during yr. 10)
+(43,173/24,735)x(acc. yr. 2005 estimated payments during yrs. 9,10)

Using $3,060 as the acc. yr. 2004 estimated payments during yr. 10 and solving this equation results in:

acc. yr. 2005 est. payments during years 9,10 = (24,735/43,173)x[$17,662- (31,399/20,400)x($3,060)]
acc. yr. 2005 est. unpaid loss as of 12/31/12 = $7,421

This process is continued iteratively to derive unpaid losses as of 12/31/12 for each accident year as displayed on Exhibit 1, Table 6, Column (7). Appendix C provides a formula to allocate the current accounting date aggregate unpaid loss estimate to accident year.

The total of all accident year unpaid claim estimates of the current year end accounting date equals the aggregate unpaid claims estimate. As expected in this 'no noise' example, the individual accident year unpaid losses derived in this manner equal the accident year unpaid loss estimates derived on Exhibit 1, Table 3 by using the traditional payment development method.

3.2 Payment Development with Noise

While the previous example without noise is illustrative of concepts, actual historical experience typically presents with significant noise in the historical experience. This section adds noise to the example introduced in Section 3.1.

3.2.1 Traditional actuarial triangle accident year representation

Exhibit 2, Table 1 displays loss payment experience in CL format. Since noise has been introduced, LDFs no longer remain constant within each development interval. Since interval LDFs are not constant, volume weighted average CDFs are not necessarily equal to unweighted simple average CDFs. Exhibit 2, Table 2 displays case reserves by accident year with noise added.

3.2.2 Accounting date representation

Exhibit 2, Table 3 displays the cumulative emergence of loss payments by year-end accounting date and may be derived by the appropriate accumulation of cumulative loss payments from Exhibit 2, Table 1 as described in Section 3.1.3. This tracks the historical emergence of accounting date unpaid losses and the goal is, once again, to estimate the ultimate value of year-end accounting date 2012 (i.e., the value that corresponds to the bold rectangle in the lower right-hand corner of Exhibit 2, Table 3).

3.2.3 Accounting date representation recast at current accounting date exposure level

Following procedures described in Section 3.1.4, the emerged loss payments of Exhibit 2, Table 3 are recast on Exhibit 2, Table 4 at the year-end accounting date 2012 exposure level where case reserves of Exhibit 2, Table 2 are used as an A4 measure of the relative accident year exposure to remaining payments as of each stage of development. By recasting all loss payment emergence at the 2012 year-end accounting date exposure level, LDFs within each development interval are now on a comparable basis. Weighted LDFs are weighted on the pre-recast actual loss experience of Exhibit 2, Table 3 to preserve the weighting of actual experience.[7]

Recasting the loss payments emerged as displayed on Exhibit 2, Table 4 provides an observable order-of-magnitude aggregate year-end 2012 current accounting date unpaid claim estimate. The recast unpaid claims for each recast year-end accounting date are observed to be emerging towards an ultimate somewhere in the low-to-mid four-hundred million dollar range.

3.2.4 Estimation of aggregate unpaid loss

While we may observe an order-of-magnitude unpaid claim estimate as of 12/31/12, we can apply our formal development procedure to the emergence of loss payments by accounting year recast at the current accounting date exposure level.[8]

Exhibit 2, Table 5, Column (4) displays the indicated total emergence of unpaid year-end accounting date losses at the current 2012 year-end accounting date exposure level. While each figure in Column (4) provides an estimate of unpaid losses as of 12/31/12,[9] the most recent estimate of $433,929 is accepted as the payment development accounting date unpaid loss estimate as of 12/31/12.

3.2.5 Allocation of aggregate unpaid loss estimate to accident year

Exhibit 2, Table 5, Column (7) allocates the $433,929 aggregate estimated unpaid loss as of 12/31/12 to accident year using the iterative procedure described in Section 3.1.6.

4. INCURRED (REPORTED) DEVELOPMENT BY ACCOUNTING DATE

This section presents the incurred (reported[10]) loss counterpart to the payment development discussion presented in the Section 3. We introduce two incurred development examples satisfying A1-A7.

4.1 Static Example: No Noise

4.1.1 Traditional actuarial triangle chain-ladder accident year representation

Exhibit 3, Table 1 displays reported losses in the familiar CL format. In this static example, since

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[7] Friedland's [10] Chapter 7 – Development Technique "Mechanics of the Development Technique" discussion beginning p. 85 is written in a traditional accident year development context. Her discussion may be adapted to accounting date development techniques.

[8] Friedland's [10] Chapter 7 – Development Technique 'When the Development Technique Works and When it Does Not' discussion beginning p. 95 is written in a traditional accident year development context. Her discussion may be adapted to accounting date development techniques.

[9] Section 7 revisits this important point.

[10] Reported losses equal cumulative loss payments plus case reserves.

LDFs are identical within each age-to-age interval, volume weighted LDFs are identical to simple average LDFs within each development interval and volume weighted CDFs are identical to simple average CDFs as of each stage of development.

4.1.2 Traditional incurred development approach

Exhibit 3, Table 2 displays the traditional accident year incurred development method used to derive unpaid loss estimates from reported losses. The product of reported losses as of the current accounting date and their corresponding CDFs produce Column (4) estimated ultimate losses by accident year. Estimated ultimate losses are then reduced by reported losses as of the current accounting date resulting in Column (5) IBNR estimates for each accident year as of the current accounting date. These accident year IBNR estimates are added to Column (6) current accounting date case reserves resulting in a Column (7) unpaid loss estimate for each accident year.[11] Estimated unpaid losses by accident year are added to produce a total estimate of unpaid losses as of the current accounting date. The sum of individual accident year Column (7) unpaid loss estimates equals the total unpaid loss estimate of \$434,721 as of 12/31/12. Since there is no noise, this total unpaid loss estimate is identical to the traditional payment development estimate derived in Section 3.1.2.

4.1.3 Accounting date representation

As with cumulative payments, our alternative approach organizes reported loss experience into an accounting date representation. Exhibit 3, Table 3 displays the cumulative reported losses emerged by year-end accounting date and may be derived as the sum of cumulative loss payments emerged by year-end accounting date of Exhibit 1, Table 4 and the appropriate accumulation of case reserves from Exhibit 1, Table 2.

For example, the year-end accounting date 2009 reported losses as of 12/31/12 (i.e., as of 3 years of reported loss emergence) of \$253,840 are defined as loss payments subsequent to 12/31/09 on losses incurred during accident years 2009 & prior plus case reserves as of 12/31/12 on accident years 2009 & prior. Equivalently, this may be defined as loss payments during calendar years 2010 through 2012 on accident years 2009 & prior plus case reserves as of 12/31/12 on accident years 2009 & prior. The loss payments subsequent to 12/31/09 on losses incurred during accident years 2009 & prior equal \$205,714 from Exhibit 1, Table 4. Case reserves as of 12/31/12 on accident years 2009 & prior of \$48,126 equal the sum of appropriate accident year contributions from Exhibit 1, Table 2:

[11] This derivation of unpaid loss estimates by accident year is equivalent to solving for unpaid loss estimates as Column (4) accident year estimated ultimate losses less cumulative loss payments as of the current accounting date.

2,040	accident year 2004 contribution
+ 3,958	accident year 2005 contribution
+ 6,293	accident year 2006 contribution
+ 9,533	accident year 2007 contribution
+ 10,370	accident year 2008 contribution
+ 15,932	accident year 2009 contribution
48,126	Total

The sum of these two components, \$205,714 + \$48,126, equals the \$253,840 year-end accounting date 2009 reported losses emerged as of 12/31/12.

Exhibit 3, Table 3 tracks historical reported loss emergence of accounting date unpaid losses as opposed to tracking individual accident year reported loss development from accident year inception. It is important to observe that exhibits displaying cumulative reported losses emerged by accounting date display one additional diagonal (as of 0 years) for each accounting date compared with exhibits that display the corresponding cumulative loss payments emerged.[12] In particular, the 2012 current accounting date contains an entry as of 0 years (i.e., as of 12/31/12) that equals the aggregate case reserves as of the current year-end accounting date. Our goal is to estimate the ultimate value of unpaid losses as of year-end accounting date 2012 (i.e., the value that corresponds to the bold rectangle in the lower right-hand corner of Exhibit 3, Table 3). As with Exhibit 1, Table 4, an estimate of the bold rectangle value is not readily apparent directly from Exhibit 3, Table 3.

4.1.4 Accounting date representation recast at current accounting date exposure level

Exhibit 3, Table 3 accounting year reported loss emergence may be recast into a form that is especially useful for unpaid claim estimation. Exhibit 3, Table 4 displays the recast cumulative reported losses emerged by year-end accounting date at the current accounting date exposure level and may be derived as the sum of recast cumulative loss payments emerged by year-end accounting date of Exhibit 1, Table 5 and the appropriate accumulation of recast case reserves of Exhibit 1, Table 2.

The emerged reported losses of Exhibit 3, Table 3 are recast on Exhibit 3, Table 4 at the 2012 year-end accounting date exposure level where the case reserves of Exhibit 1, Table 2 are used as an A4 measure of the relative accident year exposure to remaining reported losses (IBNR) as of each stage of development.

For example, the emerged reported losses of accounting year-end 2009 as of 12/31/12 (i.e., after 3

[12] Since there can be no emerged payments as of 0 years, reported emerged as of 0 years = case reserves as of 0 years.

years) from Exhibit 3, Table 3 of $253,840 is recast on Exhibit 3, Table 4 as $335,474. This is derived as the recast loss payments subsequent to 12/31/09 on losses incurred during accident years 2009 & prior equal to $269,056 from Exhibit 1, Table 5 plus recast case reserves as of 12/31/12 on accident years 2009 & prior of $66,418. The $66,418 of recast case reserves equals the sum of appropriate recast accident year contributions from Exhibit 1, Table 2, computed as:

(Year-End 2012 Accounting Date Exposure) / (Year-End 2009 Accounting Date Exposure)			
(9,533 / 9,180) x	2,040 =	2,118	accident year 2004 contribution
+ (10,370/10,883) x	3,958 =	3,771	accident year 2005 contribution
+ (15,932/13,634) x	6,293 =	7,354	accident year 2006 contribution
+ (25,418/18,007) x	9,533 =	13,456	accident year 2007 contribution
+ (31,399/18,855) x	10,370 =	17,269	accident year 2008 contribution
+ (43,173/30,639) x	15,932 =	22,450	accident year 2009 contribution
		66,418	Total

The year-end 2009 accounting date emerged reported losses as of 3 years, recast at the year-end 2012 accounting date exposure level of $335,474, is displayed in its corresponding position on Exhibit 3, Table 4. Appendix B provides a formula to recast accounting date reported loss emergence at the current accounting date exposure level.

It is important to observe that the recast year-end accounting date 2012 emerged reported losses of $148,116 displayed on Exhibit 3, Table 4 equals the pre-recast amount displayed on Exhibit 3, Table 3. This must always be true because the aggregate year-end accounting date 2012 case reserves recast at the 2012 year-end exposure level, by definition, equals the pre-recast aggregate year-end 2012 case reserves.

The recast Exhibit 3, Table 4 reported losses emerged by year-end accounting date at the year-end 2012 accounting date exposure level visibly clarifies an appropriate aggregate year-end 2012 accounting date unpaid loss estimate. The recast unpaid claims for each year-end accounting date are seen to inevitably emerge towards an ultimate of $434,721. This is the same figure derived from the traditional incurred development method of Exhibit 3, Table 2 as well as the 'no noise' payment development indication of Exhibit 1, Table 3 and the recast accounting date payment indication of Exhibit 1, Table 5.

The bullet point observations at the conclusion of Section 3.1.4 also apply to accounting date emerged reported loss representations. There are two additional observations for emerged reported losses under an accounting date representation:

- Accounting date reported emergence of unpaid claims converges to ultimate faster than accounting date payment emergence.
- Exhibits displaying cumulative reported losses emerged by year-end accounting date display one additional diagonal (as of 0 years) for each accounting date as compared with exhibits displaying the corresponding cumulative emerged loss payments. In particular, the current recast year-end accounting date contains an entry as of 0 years that equals total current year-end accounting date case reserves.

4.1.5 Estimation of aggregate unpaid loss

While we may observe $434,721 as an obvious unpaid claim estimate as of 12/31/12 for our 'no noise' example, this can be formalized using development factors.

Exhibit 3, Table 5, Column (4) displays the indicated total reported emergence of unpaid year-end accounting date losses at the 2012 year-end accounting date exposure level. As expected in this example with no noise, the indicated unpaid loss for each year-end accounting date at the 2012 year-end accounting date exposure level equals $434,721.

4.1.6 Allocation of aggregate unpaid loss estimate to accident year

As with loss payments, the emerged reported loss accounting date paradigm may be used to allocate the aggregate unpaid loss estimate to accident year by use of a top-down iterative approach that unwinds the exposure adjustment.

Exhibit 3, Table 5, Column (5) displays the indicated IBNR as of 12/31/12 at the 2012 year-end accounting date exposure level for each year-end accounting date. Beginning with accident year 2004, the oldest accident year with any remaining unreported losses as of 12/31/12, we know that accident year 2004 is expected to have only one more year of loss reportings beyond 12/31/12 (i.e., reportings to be made during calendar year 2013). Recasting reported losses emerged at the 2012 year-end accounting date exposure level implies the following equation for accident year 2004:

$1,727 = (43,173/25,500)x(est. acc. yr. 2004 estimated reportings during yr. 10)

Solving this equation yields:
acc. yr. 2004 estimated reportings during yr. 10 = (25,500/43,173)x$1,727 =
acc. yr. 2004 estimated IBNR as of 12/31/12 = $1,020

Similarly, we have the following equation for accident year 2005:

$7,614 = (31,399/20,400)x(acc. yr. 2004 estimated reportings during yr. 10)
+(43,173/24,735)x(acc. yr. 2005 estimated reportings during yrs. 9,10)

Using $1,020 as the acc. yr. 2004 estimated reportings during yr. 10 and solving this equation results in:

acc. yr. 2005 est. reportings during years 9,10 = (24,735/43,173)x[$7,614 –(31,399/20,400)x($1,020)]
acc. yr. 2005 estimated IBNR as of 12/31/12 = $3,463

This process is continued iteratively to derive IBNR estimates as of 12/31/12 for each accident year as displayed on Exhibit 3, Table 5, Column (7). These IBNR estimates are added to the Column (8) case reserves as of 12/31/12 resulting in the Column (9) accident year unpaid loss estimates as of 12/31/12. Appendix C provides a formula to allocate the current accounting date aggregate IBNR estimate to accident year.

The total of all accident year unpaid claim estimates of the current year end accounting date equals the aggregate unpaid claims estimate. As expected in this 'no noise' example, the individual accident year unpaid losses derived in this manner equal the accident year unpaid loss estimates derived on Exhibit 3, Table 2 by using the traditional incurred development method.

4.2 Incurred Development with Noise

This section adds noise to the example introduced in Section 4.1.

4.2.1 Traditional actuarial triangle accident year representation

Exhibit 4, Table 1 displays reported losses in the traditional CL format derived as the sum of Exhibit 2, Table 1 and Exhibit 2, Table 2. Since noise has been introduced, LDFs no longer remain constant within each development interval. Since interval LDFs are not constant, volume weighted average CDFs do not necessarily equal the unweighted simple average CDFs.

4.2.2 Accounting date representation

Exhibit 4, Table 2 displays the cumulative reported losses emerged by year-end accounting date and may be derived as the sum of cumulative loss payments emerged by year-end accounting date of Exhibit 2, Table 3 and the appropriate accumulation of case reserves from Exhibit 2, Table 2 as described in Section 4.1.3. This tracks the historical reported emergence of accounting date unpaid losses and our goal is, once again, to estimate the ultimate value of unpaid losses as of year-end accounting date 2012 (i.e., the value that corresponds to the bold rectangle in the lower right-hand corner of Exhibit 4, Table 2).

4.2.3 Accounting date representation recast at current accounting date exposure level

Following procedures described in Section 4.1.4, reported losses emerged of Exhibit 4, Table 2 are recast on Exhibit 4, Table 3 at the year-end 2012 accounting date exposure level where case reserves of Exhibit 2, Table 2 are used as an A4 measure of the relative accident year exposure to remaining reported losses (IBNR) as of each stage of development. By recasting all reported loss emergence at the 2012 year-end accounting date exposure level, LDFs within each development interval are now on a comparable basis. Weighted LDFs are weighted on the pre-recast actual loss experience of Exhibit 4, Table 2 to preserve the weighting of actual experience.[13]

It is again important to observe that recast year-end accounting date 2012 emerged reported losses of $148,006 displayed on Exhibit 4, Table 3 equals the pre-recast amount displayed on Exhibit 4, Table 2. While this relationship must be true, the fact that each prior recast year-end accounting date emerged reported loss at 0 years also equals $148,006 is only true, in this instance, because accident year case reserves are used as the A4 exposure metric. Examples using different exposure metrics, presented in subsequent sections, help clarify this point.

Recasting the reported losses emerged as on Exhibit 4, Table 3 provides an observable order-of-magnitude aggregate year-end 2012 current accounting date unpaid claim estimate. It is visually apparent that the recast unpaid claims for each year-end accounting date are emerging towards an ultimate somewhere in the low-to-mid four-hundred million dollar range.

4.2.4 Estimation of aggregate unpaid loss

While we may observe an order-of-magnitude unpaid claim estimate as of 12/31/12, we can apply our formal development treatment to the emergence of reported losses by accounting year recast at the current accounting date exposure level.[14]

The recast accounting date representation results in a CDF which is appropriate to develop the current accounting date total case reserves to ultimate. Exhibit 4, Table 4, Column (4) displays the indicated total emergence of unpaid year-end accounting date losses at the current 2012 year-end accounting date exposure level. While each figure in Column (4) provides an estimate of unpaid losses as of 12/31/12, the most recent estimate is the only one that incorporates the entire actual available 2012 year-end accounting date experience (i.e., the aggregate case reserves as of 12/31/12). As such, the most

[13] Footnote 7 applies.
[14] Footnote 8 applies.

recent estimate of $437,699 (= 148,006 x 2.957307) is accepted as the incurred development accounting date unpaid claim estimate.

While accident year case outstanding reserving methods appear in the actuarial literature [1], [8], [13], [20], the procedure described above is seen to reduce the current accounting date incurred development unpaid claim estimate to a particularly parsimonious formulation:

Aggregate Unpaid Claim Estimate = Aggregate Case Reserves x CDF

4.2.5 Allocation of aggregate unpaid loss estimate to accident year

Exhibit 4, Table 4, Column (9) allocates the $437,699 aggregate estimated unpaid loss as of 12/31/12 to accident year using the iterative Column (7) IBNR procedure described in Section 4.1.6.

5. EXPECTED UNPAID LOSSES

The key assumption of the traditional accident year expected loss technique is that the actuary can better estimate total unpaid claims based on an *a priori* (or initial) estimate than from claims experience observed to date. In certain circumstances, claims experience reported to date may provide little information about ultimate claims (e.g., assumptions A1-A7 are not generally well satisfied) especially when compared to the *a priori* estimate.[15]

To be compatible with our accounting date paradigm, expected loss by accident year is reframed as aggregate expected unpaid loss as of the current accounting date.[16] Continuing with our Section 4.2 example, comparable industry experience[17] is used to derive expected unpaid losses as of the current year-end accounting date. The critical assumption in this calculation is that the industry loss reserve to earned premium ratio by accident year as of the current accounting date is appropriate for the particular insurer under review. Exhibit 5 displays an example of this calculation which results in a Column (6) expected unpaid loss of $432,407 as of 12/31/12.

6. BORNHUETTER-FERGUSON BY ACCOUNTING DATE

The traditional Bornhuetter-Ferguson [4] method is essentially a blend of development and expected loss techniques by accident year. The Exhibit 5, Column (9) aggregate unpaid loss estimate of $434,197

[15] Adapted from Friedland's [10] Chapter 8 – Expected Claims Technique p. 131.

[16] Friedland's [10] Chapter 8 – Expected Claims Technique is written in a traditional accident year ultimate context. Her Chapter 8 discussion of expected claims may be generally adapted to accounting date expected unpaid claims.

[17] This 'comparable industry experience' is artificially constructed for illustrative purposes only and does not represent actual industry experience.

[= 148,006 + (1 - 1/2.957307)x(432,407)] as of accounting date 12/31/12 is the result of an accounting date analogue to the traditional Bornhuetter-Ferguson method.[18] As a hybrid of development and expected unpaid losses, the Bornhuetter-Ferguson by accounting date technique may be particularly suitable when assumptions A1-A7 are partially satisfied.

The accounting date analogue of the traditional Bornhuetter-Ferguson method is seen to reduce to a concise formulation:

Aggregate Unpaid Claim Estimate = Aggregate Case Reserves
+ (1-1/CDF)x(Aggregate Expected Unpaid Losses)

Column (12) displays an accident year allocation of the aggregate $434,197 unpaid claim estimate.

Application of the Bornhuetter-Ferguson method by accounting date is ill-advised where the Column (8) CDF is below unity. Caution is advised if any Column (10) implied IBNR is negative.

7. CAPE COD BY ACCOUNTING DATE

The traditional Cape Cod method is a Bornhuetter-Ferguson accident year ultimate calculation where expected losses are obtained from reported loss experience instead of an independent, and often judgmental, selection.[19] While we have previously observed relative consistency in the emergence of each recast accounting date at the current accounting date exposure level, the Cape Cod by accounting date technique explicitly reflects this important feature. Exhibit 6 displays a Cape Cod by accounting date technique applied to our example resulting in a Column (7) aggregate unpaid loss estimate of $437,867 as of accounting date 12/31/12.[20] Column (12) displays an accident year allocation of the aggregate $437,867 unpaid loss estimate.

Application of the Cape Cod method by accounting date is ill-advised when the Column (3) CDF for the current year-end accounting date is below unity. Caution is advised if any Column (10) IBNR is negative.

8. EXPOSURE MEASURES

As indicated in Section 2, the exposure metric as of each stage of development is intended to

[18] Friedland's [10] Chapter 9 – Bornhuetter-Ferguson Technique is written in a traditional accident year ultimate context. Her Chapter 9 discussion may be adapted to the Bornhuetter-Ferguson method by accounting date.

[19] Adapted from Friedland [10] Chapter 10 – Cape Cod Technique p. 174.

[20] Friedland's [10] Chapter 10 – Cape Cod Technique is written in a traditional accident year ultimate context. Her Chapter 10 discussion may be adapted to the Cape Cod method by accounting date.

provide a reasonable measure of the relative accident year exposure to remaining development. In order to properly apply the accounting date paradigm, it is important that the exposure metric reflects volume and total frequency and severity trend or, if necessary, be adjusted to reflect volume and total trend. Several alternative exposure metrics may be reasonable, as follows:

8.1 Case Reserves

Case reserves have been used as the exposure metric for examples presented in previous sections. Footnote 5 outlines situations under which case reserves may serve as a reasonable exposure measure.

8.2. Earned Premium

Earned premium is a commonly used exposure metric. Ideally, earned premium (or more precisely, the pure premium portion of earned premium) would be brought to the same premium adequacy level[21] to more accurately measure relative exposure. Exhibit 7, Table 1 displays an example of (independently derived) earned premium at the same adequacy level for each accident year. As indicated by this exhibit, earned premium is insensitive to actual emerged experience since it is remains unchanged at each stage of development.

Using earned premium at the same adequacy level as the A4 exposure metric, Exhibit 7, Table 2 and Exhibit 7, Table 3 display techniques described in Sections 3.2.3-3.2.5 to derive unpaid claim estimates based upon loss payments emerged by year-end accounting date.

Using earned premium at the same adequacy level as the A4 exposure measure, Exhibit 8, Table 1 and Exhibit 8, Table 2 display techniques described in Sections 4.2.3-4.2.5 to derive unpaid claim estimates based upon reported losses emerged by year-end accounting date. Note that, unlike Exhibit 4, Table 3 where case reserves are used as the A4 exposure measure, only the Exhibit 8, Table 1 recast year-end accounting date 2012 reported losses as of 0 years equals actual aggregate case reserves as of 12/31/12.

8.3 Claim Counts; Averages and Counts (Frequency/Severity)

Claim counts are a rich source of exposure metrics. Use of claim counts as an exposure metric allows the practitioner to incorporate and estimate average cost per claim. Claim count exposures provide a means to derive an accounting date analogue to traditional averages and counts

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[21] An example of the 'same premium adequacy level' would be where all earned premium is 7% inadequate. Under the assumption that all earned premium is at the same premium adequacy level, it would be appropriate to use actual (unadjusted) earned premium as the exposure measure. Used here, 'same premium adequacy level' is <u>not</u> to be interpreted as actual earned premium for each accident year should be brought to a common (e.g., current) rate level.

(frequency/severity) methods. While claim counts already reflect frequency trend, they need to be adjusted to additionally reflect any severity trend. As an example, Exhibit 9, Table 1 displays (independently derived) projected remaining claim counts to be closed with payment where we are confident these are reasonable estimates. These exposures are sensitive to actual emerged experience but need to be adjusted to reflect severity trend. Although a suitable severity trend index would be appropriate, Exhibit 9, Table 2 restates the Table 1 claim count exposure assuming a constant 5% annual severity trend.

Using the trend adjusted claim count exposure metric, Exhibit 9, Table 3 and Exhibit 9, Table 4 display techniques described in Sections 3.2.3-3.2.5 to derive unpaid claim estimates based upon loss payments emerged by year-end accounting date. Exhibit 9, Table 4, Column (9) displays estimated unpaid average cost per claim projected to be closed with payment.

Using the trend adjusted claim count exposure metric, Exhibit 10, Table 1 and Exhibit 10, Table 2 display techniques described in Sections 4.2.3-4.2.5 to derive unpaid claim estimates based upon reported losses emerged by year-end accounting date. Exhibit 10, Table 2, Column (11) displays estimated unpaid average cost per claim projected to be closed with payment.[22]

8.4 Other Exposure Measures

Freidland [10 p. 35, 132] extends the list of potential exposure measures to include: payroll, number of vehicles, etc. for particular coverages. The Struzzieri and Hussian [19] 'Best Exposure Base' section adds base class equivalent exposures and contains other valuable exposure discussion. Several of these other exposure measures may require trend adjustments.

Section 9 expands the meaning of "exposures" in different contexts to include exposure metrics beyond those discussed in this section.

9. BROAD APPLICABILITY

We have narrowly referred to the quantity being estimated by development methods as "losses" (or "claims") and the exposure base as "exposures". However, the accounting date paradigm has much broader application. Accounting date techniques described herein are useful any time we make a development-based projection where the ratio of remaining accident year "losses" to "exposures" is

[22] Friedland's [10] Chapter 11 – Frequency-Severity Techniques is written in a traditional accident year ultimate context. Her Chapter 11 discussion of frequency/severity techniques may be generally adapted to accounting date averages and counts methods.

expected to be equal at each stage of development. For example, if we are estimating unpaid DCCE where we expect a constant ratio of accident year unpaid DCCE to unpaid loss at each stage of development, then unpaid "losses" are unpaid DCCE and "exposures" could be estimated unpaid losses when we are confident we have reasonable estimates of unpaid losses.[23]

10. ACCOUNTING DATE IMPLEMENTATION CHALLENGES

As previously indicated, factors to consider in an unpaid claim analysis require professional actuarial judgment.[24] This section briefly addresses several accounting date implementation challenges requiring actuarial judgment.

10.1 Data Availability

For all but relatively fast developing lines of business, it is optimal to have accident year experience available for older accident years as well as several years of calendar year activity (e.g., Exhibit 1, Table 1 upper right corner experience and 10 calendar year diagonals). If this experience were not readily available, one could: (1) obtain compatible supplementary (e.g., industry, prior insurer, competitor) experience where the exposure measure is consistent with available experience; (2) perform the accounting date representation though a common (though incomplete) stage of development and estimate tail development factors; and/or (3) create pseudo-data based upon available experience. These three approaches may also be useful in situations where some available experience is relatively old and deemed unrepresentative of future development.

10.2 Supplementary Experience

As indicated in Section 10.1, supplementary experience may permit completion of accounting date representations through a further stage of development than would otherwise be possible. Supplementary data may also be used to increase the A5 credibility of experience. The use of supplementary experience should be carefully weighed and balanced with the consideration of the use of tail development factors and pseudo-data.

10.3 Tail Development Factors

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[23] This entire Section is derived from Gluck [11] p. 505-6 who also provides additional examples where we may apply this general principle.

[24] These factors are outlined in Actuarial Standard of Practice No. 43 "Property/Casualty Unpaid Claim Estimates", especially Section 3.6.

At comparable late stages of development, recast accounting date CDFs typically converge to unity more quickly than for traditional accident year reserving methods. However, additional historical data is often necessary to attain this quicker convergence. The actuary should consider the trade-offs and interplay between faster convergence, reliance on supplementary experience and the use of pseudo-data. When we perform accounting date representations through a late (but incomplete) common stage of development, we may capitalize on faster convergence and estimate tail development by adapting accident year tail factor procedures discussed in the actuarial literature.[25] When A1-A7 are satisfied, all other things being equal, faster CDF convergence implies accounting date tail development factors with less leverage and less uncertainty than for traditional accident year reserving methods.

10.4 Pseudo-Data

In addition to increasing A5 credibility, pseudo-data may also permit completion of accounting date representations through a further stage of development than would otherwise be possible. For example, if accident year 2002 & prior experience were unavailable on Exhibit 2, Table 1, then we would be unable to create Exhibit 2, Table 3 with as many year-end accounting dates and through 9 years of development. However, we could create pseudo-data to substitute for the missing experience. On the theory that accident year 2003 is the most recent fully developed accident year, a simple approach would be to use accident year 2003 experience to serve as the missing experience. A more nuanced approach would consider all accident year 2003 & subsequent experience in the creation of pseudo-data. As with previously discussed data availability tools, the actuary should consider the impact of pseudo-data and its interaction with supplementary data and tail development factors.

10.5 Actuarial Consistency Assumptions Initially Unsatisfied

Assumptions A1-A7 should be satisfied to make optimal use of accounting date reserving methods. When assumptions A1-A7 are satisfied, the noise that remains is expected to be reduced and credibility increased by aggregating all accident years.[26] When assumptions A1-A7 are not initially satisfied, it may be appropriate to pre-process the data using approaches described by Berquist and Sherman [2] that address situations where an insurer's historical experience has been inconsistent as a result of changes in

[25] Friedland's [10] Chapter 7- Development Technique "Step 5 - Select Tail Factor" is written in a traditional accident year context. Her discussion may be adapted to an accounting date framework.

[26] As a consequence of The Law of Large Numbers

operations and procedures.[27]

11. SUMMARY RESULTS AND DISCUSSION

This paper introduces the accounting date reserving paradigm. The general principle is always the same: recast the aggregate emergence of unpaid claims of prior year-end accounting dates at the current accounting date exposure level; use this recast emergence as basis to estimate the current accounting date aggregate unpaid claims; and, if necessary, allocate the aggregate unpaid claim estimate to accident year using an iterative top-down procedure.

11.1 Accounting Date Analogues to Basic Reserving Methods

The new reserving techniques presented are seen to be accounting date analogues to basic reserving methods including:

- Payment Development
- Incurred Development
- Bornhuetter-Ferguson
- Cape Cod
- Averages & Counts (Frequency/Severity)

11.2 Characteristics of Accounting Date Reserving Paradigm

As discussed, highlights of the accounting date paradigm are:

- In contrast to traditional estimates which require an estimated ultimate for each accident year, the central goal under the accounting date representation is to directly target only one quantity, i.e., the aggregate estimate of unpaid claims incurred as of the current accounting date.
- In contrast to traditional indirect accident year estimated ultimate approaches, a reasonable unpaid claim estimate is visibly apparent under a year-end accounting date representation appropriately recast at the current accounting date exposure level.
- Tail factors converge to unity faster in the accounting date representation than in the traditional accident year representation.
- Accident year payments during first year calendar year are not reflected in the accounting date representation.

[27] Friedland's [10] Chapter 13 - Berquist-Sherman Techniques provides a summary. Fleming and Mayer [7] also address an aspect of this issue.

- The final diagonal of the accounting date representation contains all calendar year activity through the current accounting date on losses incurred as of the year-end accounting date that had remained unpaid as of that accounting date.
- Especially for longer tailed lines of business, the data volume for the accounting date representation tends to grow faster than under the traditional accident year representation.
- Accounting date reported emergence of unpaid claims converges to ultimate faster than accounting date payment emergence.
- Exhibits displaying cumulative reported losses emerged by year-end accounting date display one additional diagonal (as of 0 years) for each accounting date as compared with exhibits displaying the corresponding cumulative emerged loss payments. In particular, the current recast year-end accounting date contains an entry as of 0 years that equals total current year-end accounting date case reserves.
- When appropriate assumptions are satisfied, the accounting date reserving paradigm is associated with improved accuracy over traditional accident year reserving methods as further discussed below.

11.3 Accounting Date Paradigm Consistent with Improved Accuracy

When assumptions A1-A7 are satisfied, two powerful forces imply improved accuracy of the accounting date reserving paradigm over traditional accident year reserving methods: **forward-looking** and **aggregation.**[28]

11.3.1 Forward-looking

The recent Forray [8], [9] empirical studies "…suggest that there are many more valuable methods for reserve analysis beyond the [accident year] incurred- and paid-chain-ladder methods and that the paid chain ladder, in particular, should not receive the weight it often does."[29] Forray's analysis found that the best-performing reserving methods "…were observed to satisfy the following two criteria: 1. each relies at least in part on case reserves ("Criteria 1")"; and "2. amounts paid to date do not directly influence the indicated unpaid loss ("Criteria 2")." Despite the inclination to place more reliance on paid loss triangle experience ("real money changing hands, less vulnerable to changes in case reserving

[28] When assumptions are insufficiently satisfied, these forces may serve to further mask distinctive individual accident year attributes and distort the unpaid claim estimate.

[29] Forray goes on to note: "Of course, this is a general observation, and a particular company's circumstances always should be considered in selecting methods for any reserving analysis."

practices, etc."), Meyers [14] has also recently observed instances of superior empirical results using reported loss experience.

While all accounting date reserving methods incorporate forward-looking A4 exposure measures, accounting date incurred methods also rely upon forward-looking A3 case reserves.

11.3.2 Aggregation

When assumptions A1-A7 are satisfied, the noise that remains is expected to be reduced and credibility increased as a result of aggregating accident years.

11.3.3 Excellent candidates for improved accuracy – accounting date incurred methods

The Section 4 accounting date incurred development method (i.e., aggregate case reserves x CDF): essentially relies on forward-looking case reserves (Criteria 1) in conjunction with a forward-looking exposure adjusted CDF; and uses limited amounts of paid to date (to estimate CDF) which do not directly influence the indicated unpaid loss (Criteria 2). Furthermore, when assumptions A1-A7 are satisfied, the accounting date incurred development method capitalizes on the aggregation of accident years which would be expected to result in reduced volatility and commensurate increased credibility. As such, all accounting date incurred methods[30] are excellent candidates to be relatively more accurate performing methods as compared with reserving methods in common use.

11.4 Areas for Future Research

Future areas of research include:

1. Compare accounting date reserving methods with traditional actuarial reserving methods using relative "method skill" measures [8], [9] as well as other performance analytics. Empirically test the hypothesis that incurred development accounting date methods produce relatively more accurate aggregate unpaid claim estimates than analogous accident year methods.
2. Explore the impact of changing environments (e.g., changes in payment pattern, changes in case reserve adequacy, changes in calendar year inflation trend) on accounting date reserving methods. As described by Boles and Staudt [3], compare the performance of accounting date reserving techniques to other reserving methods under changing environments.
3. Investigate techniques to organize or modify historical experience such that actuarial

[30] This includes: incurred development; (incurred) Bornhuetter-Ferguson; (incurred) Cape Cod; and (reported) averages & counts.

assumptions A1-A7 are well satisfied for application to accounting date reserving methods.

4. Consider optimal weighting scheme(s) to employ when relatively low credibility accident year experience is used to recast accounting date representations.
5. Adapt tail development factor and expected unpaid loss procedures to apply to the accounting date paradigm.
6. Analyze impacts, trade-offs, interactions and sensitivities associated with the use of various combinations of supplementary data, tail factors and pseudo-data. Consider the appropriate balance of stability and responsiveness.
7. Generalize Appendix A, B and C formulas to incorporate all situations including where no actual accident year experience has reached maturity as well as for run-off business.
8. Experiment with the most effective exposure measures to use under different circumstances. Is it advisable to use different exposures for payments versus case reserves? Would a hybrid exposure metric be more effective than any one particular exposure measure?
9. Conceive of the recast accounting date representation as sample emergence from the aggregate distribution of unpaid future payments which have been incurred and unpaid as of the current accounting date. From this perspective, consider use of the recast accounting date representation as a basis to address the stochastic analysis and estimation of loss variability [4].

12. CONCLUSION

As actuarial science has evolved, the continued widespread practice of estimating unpaid claims on an individual accident year basis may have been motivated by several considerations including: conception of the total unpaid claim estimate as the sum of individual accident year ultimate estimates reduced by cumulative payments to date; the link to ratemaking, which requires cost estimates for an individual future policy year and is often derived by trending forward individual accident year estimated ultimate loss costs; statutory annual statement Schedule P reporting requirements by individual accident year; and the natural tendency to apply familiar methods. Actuarial reserving methods that develop individual accident years to estimated ultimate values have become ingrained into common actuarial practice. However, as we have seen, this familiar paradigm may not take full advantage of reasonable actuarial assumptions.

This paper introduces a new accounting date paradigm that provides practical and powerful additions to the loss reserving methodologies available to actuaries. In addition to revealing visibly apparent aggregate unpaid claim estimates, the structure of appropriate accounting date reserving applications suggests improved accuracy over corresponding accident year development methods.

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Appendix A, B and C formulas pertain to specific exhibits presented in this paper and may not necessarily be more generally applicable.

Appendix A

Where required data for appropriate application is available, compute cumulative emerged loss payments $a_{i,j}$ as of year-end accounting date i, at year-end valuation date j, recast at current year-end accounting date c exposure level as:

$$a_{i,j} = \sum_{k=0}^{k=n-2} \left(e_{c-k}^{c} / e_{i-k}^{i}\right)\left(p_{i-k}^{j} - p_{i-k}^{i}\right)$$

where, $i < j$

n = number of years until accident year payments reach ultimate
e_m^s = exposure to remaining payments for accident year m as of year-end s
p_m^s = cumulative loss payment for accident year m through year-end s

Appendix B

Where required data for appropriate application is available, compute cumulative emerged reported losses $b_{i,j}$ for year-end accounting date i, at year-end valuation date j, recast at the current year-end accounting date c exposure level as:

$$b_{i,j} = a_{i,j} + \sum_{k=0}^{k=n-2-j+i} \left(e_{c-k}^{c} / e_{i-k}^{i}\right)\left(r_{i-k}^{j}\right)$$

where, $i \leq j$

$a_{i,j}$ = computed via Appendix A and equals 0 when $i=j$
n = number of years until accident year payments reach ultimate
e_m^s = exposure to unreported loss (IBNR) for accident year m as of year-end s
r_m^s = case reserves of accident year m as of year-end s

Appendix C

Where required data for appropriate application is available, compute the unpaid claim [or IBNR] estimate u_i iteratively for accident year i associated with the aggregate unpaid claim [or IBNR] estimate d_c at current year-end accounting date c as:

$$u_i = (e_i^i / e_c^c)\left[d_i - \sum_{k=c-n+1}^{k=i-1} \left(e_{c+k-i}^{c} / e_k^i\right) u_k\right]$$

where, $i \leq c$

n = number of years until accident year payments reach ultimate
e_m^s = remaining exposure for accident year m as of year-end s
d_i = estimated aggregate remaining unpaid [or IBNR] at year-end accounting date i at year-end accounting date c exposure level

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Abbreviations and notations

ALAE, allocated loss adjustment expenses
BF, Bornhuetter-Ferguson
CAS, Casualty Actuarial Society
CDF, cumulative age-ultimate development factor
CL, chain-ladder
DCCE, defense and cost containment expenses
IBNR, incurred but not reported loss (i.e., all unreported development beyond case reserves)
IRIS, Insurance Regulatory Information System
LDF, age-to-age loss development factor
NAIC, National Association of Insurance Commissioners
PCAS, Proceedings of the Casualty Actuarial Society

Biography of Author

Bertram A. Horowitz is President of Bertram Horowitz, Inc. Actuarial and Risk Consultants which provides property/casualty and title insurance actuarial and risk assessment services. He has a B.S. degree in Applied Mathematics from the State University of New York at Stony Brook and a M.S. in Mathematics from Brown University. He is a Fellow of the CAS and a Member of the American Academy of Actuaries. Mr. Horowitz is the former Special Deputy Superintendent and Financial Actuary of the New York State Insurance Department (now the New York State Department of Financial Services). He has served on the CAS Committee on Reserves and has been an active participant in the development of actuarial research, principles and standards.

Mr. Horowitz may be contacted at:
bert@bertramhorowitz.com

Bertram Horowitz, Inc.

Exhibit 1
Table 1

NO NOISE IN PAYMENT PATTERN

CUMULATIVE LOSS PAYMENTS BY ACCIDENT YEAR
($000 Omitted)

Accident Year	As of 1 Year		As of 2 Years		As of 3 Years		As of 4 Years		As of 5 Years		As of 6 Years		As of 7 Years		As of 8 Years		As of 9 Years		As of 10 Years
1995																	57,014	1.032609	58,873
1996															63,795	1.051429	67,075	1.032609	69,263
1997													55,873	1.067073	59,621	1.051429	62,687	1.032609	64,731
1998											51,620	1.093333	56,438	1.067073	60,223	1.051429	63,320	1.032609	65,385
1999									45,210	1.119403	50,608	1.093333	55,331	1.067073	59,042	1.051429	62,079	1.032609	64,103
2000							43,707	1.175439	51,375	1.119403	57,509	1.093333	62,876	1.067073	67,094	1.051429	70,544	1.032609	72,845
2001					39,692	1.295455	51,420	1.175439	60,441	1.119403	67,658	1.093333	73,972	1.067073	78,934	1.051429	82,993	1.032609	85,700
2002			27,900	1.466667	40,920	1.295455	53,010	1.175439	62,310	1.119403	69,750	1.093333	76,260	1.067073	81,375	1.051429	85,560	1.032609	88,350
2003	15,000	2.000000	30,000	1.466667	44,000	1.295455	57,000	1.175439	67,000	1.119403	75,000	1.093333	82,000	1.067073	87,500	1.051429	92,000	1.032609	95,000
2004	15,300	2.000000	30,600	1.466667	44,880	1.295455	58,140	1.175439	68,340	1.119403	76,500	1.093333	83,640	1.067073	89,250	1.051429	93,840		
2005	14,841	2.000000	29,682	1.466667	43,534	1.295455	56,396	1.175439	66,290	1.119403	74,205	1.093333	81,131	1.067073	86,573				
2006	15,731	2.000000	31,463	1.466667	46,146	1.295455	59,780	1.175439	70,267	1.119403	78,657	1.093333	85,999						
2007	15,889	2.000000	31,778	1.466667	46,607	1.295455	60,377	1.175439	70,970	1.119403	79,444								
2008	14,141	2.000000	28,282	1.466667	41,480	1.295455	53,736	1.175439	63,163										
2009	18,383	2.000000	36,767	1.466667	53,924	1.295455	69,857												
2010	22,428	2.000000	44,855	1.466667	65,788														
2011	23,549	2.000000	47,098																
2012	25,904																		

	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
Average LDF	2.000000	1.466667	1.295455	1.175439	1.119403	1.093333	1.067073	1.051429	1.032609
Average CDF	6.333333	3.166667	2.159091	1.666667	1.417910	1.266667	1.158537	1.085714	1.032609
Weighted LDF	2.000000	1.466667	1.295455	1.175439	1.119403	1.093333	1.067073	1.051429	1.032609
Weighted CDF	6.333333	3.166667	2.159091	1.666667	1.417910	1.266667	1.158537	1.085714	1.032609

Exhibit 1
Table 2

NO NOISE IN CASE RESERVES

CASE RESERVES BY ACCIDENT YEAR
($000 Omitted)

Accident Year	As of 1 Year	As of 2 Years	As of 3 Years	As of 4 Years	As of 5 Years	As of 6 Years	As of 7 Years	As of 8 Years	As of 9 Years	As of 10 Years
1995									1,239	0
1996								2,916	1,458	0
1997							4,088	2,726	1,363	0
1998						6,194	4,130	2,753	1,377	0
1999					7,422	6,073	4,049	2,699	1,350	0
2000				9,968	8,435	6,901	4,601	3,067	1,534	0
2001			15,336	11,727	9,923	8,119	5,413	3,608	1,804	0
2002		18,600	15,810	12,090	10,230	8,370	5,580	3,720	1,860	0
2003	25,000	20,000	17,000	13,000	11,000	9,000	6,000	4,000	2,000	0
2004	25,500	20,400	17,340	13,260	11,220	9,180	6,120	4,080	2,040	
2005	24,735	19,788	16,820	12,862	10,883	8,905	5,936	3,958		
2006	26,219	20,975	17,829	13,634	11,536	9,439	6,293			
2007	26,481	21,185	18,007	13,770	11,652	9,533				
2008	23,568	18,855	16,026	12,256	10,370					
2009	30,639	24,511	20,834	15,932						
2010	37,379	29,904	25,418							
2011	39,248	31,399								
2012	43,173									

Exhibit 1
Table 3

NO NOISE IN PAYMENT PATTERN

TRADITIONAL PAYMENT DEVELOPMENT METHOD BY ACCIDENT YEAR
($000 Omitted)

(1) Accident Year	(2) Cumulative Loss Payments as of 12/31/12	(3) Cumulative Loss Development Factor to Ultimate	(4)= (2)x(3) Payment Development Method Estimated Ultimate Losses	(5)= (4)-(2) Unpaid Loss Estimate as of 12/31/12
2003	95,000	1.000000	95,000	0
2004	93,840	1.032609	96,900	3,060
2005	86,573	1.085714	93,993	7,421
2006	85,999	1.158537	99,633	13,634
2007	79,444	1.266667	100,629	21,185
2008	63,163	1.417910	89,560	26,397
2009	69,857	1.666667	116,428	46,571
2010	65,788	2.159091	142,042	76,254
2011	47,098	3.166667	149,144	102,046
2012	25,904	6.333333	164,058	138,154
Total	712,665		1,147,386	$\boxed{\textbf{434,721}}$

(2) Exhibit 1, Table 1 final diagonal
(3) Exhibit 1, Table 1 corresponding CDF; payments completed as of 10 years

Exhibit 1
Table 4

NO NOISE IN PAYMENT PATTERN

CUMULATIVE LOSS PAYMENTS EMERGED BY YEAR-END ACCOUNTING DATE
($000 Omitted)

Cumulative Emerged Payments of Losses which were Unpaid as of Year-End Accounting Date
Derived by appropriate accumulation of Cumulative Loss Payments of Exhibit 1, Table 1

Year-End Accounting Date	As of 1 Year		As of 2 Years		As of 3 Years		As of 4 Years		As of 5 Years		As of 6 Years		As of 7 Years		As of 8 Years		As of 9 Years
2003	66,519	1.826948	121,526	1.359896	165,263	1.200125	198,337	1.125778	223,283	1.082742	241,758	1.051255	254,149	1.028684	261,439	1.011475	264,439
2004	70,308	1.825186	128,324	1.361063	174,658	1.201230	209,804	1.126951	236,439	1.082606	255,970	1.050396	268,870	1.028229	276,460		
2005	72,858	1.826059	133,043	1.360851	181,051	1.201759	217,580	1.126144	245,027	1.080913	264,852	1.049204	277,884				
2006	75,916	1.825794	138,608	1.361905	188,770	1.200954	226,705	1.124461	254,920	1.079919	275,293						
2007	78,580	1.827083	143,572	1.360128	195,277	1.198736	234,085	1.123233	262,932								
2008	79,133	1.820169	144,036	1.354522	195,100	1.196178	233,374										
2009	83,286	1.819126	151,508	1.357781	205,714												
2010	90,649	1.828898	165,788														
2011	98,688																
2012																	$\boxed{?}$

Exhibit 1
Table 5

NO NOISE IN PAYMENT PATTERN OR CASE RESERVES

LOSS PAYMENTS EMERGED BY YEAR-END ACCOUNTING DATE RECAST AT 2012 YEAR-END ACCOUNTING DATE EXPOSURE LEVEL
USING CASE RESERVES AS EXPOSURE MEASURE
($000 Omitted)

Cumulative Emerged Payments of Losses which were Unpaid as of Year-End Accounting Date
Derived by appropriate accumulation of Cumulative Loss Payments of Exhibit 1, Table 1 Exposure Adjusted to 2012 Year-End Accounting Date Exposure Level

Year-End Accounting Date	As of 1 Year		As of 2 Years		As of 3 Years		As of 4 Years		As of 5 Years		As of 6 Years		As of 7 Years		As of 8 Years		As of 9 Years
2003	107,813	1.829724	197,268	1.363909	269,056	1.202395	323,511	1.127883	364,883	1.085310	396,011	1.053151	417,059	1.029926	429,540	1.012061	434,721
2004	107,813	1.829724	197,268	1.363909	269,056	1.202395	323,511	1.127883	364,883	1.085310	396,011	1.053151	417,059	1.029926	429,540		
2005	107,813	1.829724	197,268	1.363909	269,056	1.202395	323,511	1.127883	364,883	1.085310	396,011	1.053151	417,059				
2006	107,813	1.829724	197,268	1.363909	269,056	1.202395	323,511	1.127883	364,883	1.085310	396,011						
2007	107,813	1.829724	197,268	1.363909	269,056	1.202395	323,511	1.127883	364,883								
2008	107,813	1.829724	197,268	1.363909	269,056	1.202395	323,511										
2009	107,813	1.829724	197,268	1.363909	269,056												
2010	107,813	1.829724	197,268														
2011	107,813																
2012																	434,721
Average LDF		1.829724		1.363909		1.202395		1.127883		1.085310		1.053151		1.029926		1.012061	
Average CDF		4.032178		2.203708		1.615729		1.343759		1.191399		1.097750		1.042348		1.012061	
Weighted LDF		1.829724		1.363909		1.202395		1.127883		1.085310		1.053151		1.029926		1.012061	
Weighted CDF		4.032178		2.203708		1.615729		1.343759		1.191399		1.097750		1.042348		1.012061	

Exhibit 1
Table 6

NO NOISE IN PAYMENT PATTERN OR CASE RESERVES

ACCOUNTING DATE PAYMENT DEVELOPMENT INDICATED AGGREGATE UNPAID LOSS AS OF 12/31/12;
ALLOCATION OF TOTAL UNPAID CLAIM ESTIMATE TO ACCIDENT YEAR
($000 Omitted)

(1) Year-End Accounting Date	(2) Recast Cumulative Loss Payments As of 12/31/12 at 2012 Year-End Accounting Date Exposure Level	(3) Weighted Cumulative Development Factor	(4)= (2)x(3) Indicated Total Emergence at 2012 Year-End Accounting Date Exposure Level	(5)= (4)-(2) Payment Development Indicated Unpaid Loss as of 12/31/12 at 2012 Year-End Accounting Date Exposure Level	(6) Accident Year	(7) Accident Year Allocation of Aggregate Accounting Date Payment Development Indicated Unpaid Loss as of 12/31/12
2003	434,721	1.000000	434,721		2003	
2004	429,540	1.012061	434,721	5,181	2004	3,060
2005	417,059	1.042348	434,721	17,662	2005	7,421
2006	396,011	1.097750	434,721	38,710	2006	13,634
2007	364,883	1.191399	434,721	69,838	2007	21,185
2008	323,511	1.343759	434,721	111,210	2008	26,397
2009	269,056	1.615729	434,721	165,665	2009	46,571
2010	197,268	2.203708	434,721	237,453	2010	76,254
2011	107,813	4.032178	434,721	326,908	2011	102,046
2012			$\boxed{434,721}$ *	434,721	2012	138,154
					Total	**434,721**

(2) Exhibit 1, Table 5 final diagonal
(3) Exhibit 1, Table 5 corresponding CDF
(7) Iterative Formula

* Accept most recent indication

Exhibit 2
Table 1

NOISE IN PAYMENT PATTERN

CUMULATIVE LOSS PAYMENTS BY ACCIDENT YEAR
($000 Omitted)

Accident Year	As of 1 Year		As of 2 Years		As of 3 Years		As of 4 Years		As of 5 Years		As of 6 Years		As of 7 Years		As of 8 Years		As of 9 Years		As of 10 Years
1995																	56,900	1.036922	59,001
1996															64,432	1.051174	67,730	1.031325	69,851
1997													56,432	1.069198	60,337	1.057425	63,802	1.030117	65,724
1998											52,136	1.093333	57,002	1.068401	60,901	1.044753	63,627	1.036021	65,919
1999									44,848	1.121570	50,300	1.096721	55,165	1.068755	58,958	1.042747	61,478	1.033915	63,563
2000							44,668	1.181962	52,796	1.119674	59,115	1.092251	64,568	1.065708	68,811	1.061177	73,020	1.031818	75,344
2001					39,534	1.284775	50,792	1.152742	58,550	1.125847	65,918	1.108249	73,054	1.064928	77,797	1.064181	82,790	1.031708	85,415
2002			27,370	1.532790	41,952	1.291066	54,163	1.151099	62,347	1.115752	69,564	1.094144	76,113	1.066665	81,187	1.053042	85,494	1.033287	88,339
2003	15,480	1.962209	30,375	1.445695	43,913	1.292191	56,744	1.182574	67,104	1.110873	74,544	1.093904	81,544	1.066504	86,967	1.046052	90,972	1.032186	93,900
2004	15,973	1.881226	30,049	1.470468	44,186	1.342105	59,303	1.176127	69,748	1.129863	78,805	1.090784	85,959	1.080274	92,860	1.047848	97,303		
2005	14,514	2.025562	29,400	1.433451	42,143	1.347928	56,806	1.182879	67,195	1.133108	76,139	1.089507	82,954	1.062975	88,178				
2006	15,574	2.020202	31,463	1.476000	46,439	1.301513	60,441	1.175426	71,044	1.120459	79,602	1.088167	86,621						
2007	16,365	1.955340	32,000	1.469911	47,037	1.283971	60,394	1.171531	70,754	1.120965	79,313								
2008	13,547	2.078288	28,155	1.469714	41,379	1.289659	53,365	1.175773	62,746										
2009	18,494	2.019881	37,355	1.444160	53,946	1.300354	70,149												
2010	21,082	2.055319	43,330	1.473913	63,865														
2011	24,138	1.954146	47,169																
2012	25,567																		

	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
Average LDF	1.994686	1.468456	1.303729	1.172235	1.122012	1.094118	1.068156	1.052044	1.033033
Average CDF	6.379437	3.198216	2.177945	1.670550	1.425099	1.270128	1.160869	1.086797	1.033033
Weighted LDF	1.993299	1.467589	1.303666	1.172358	1.122104	1.093804	1.068275	1.052074	1.032894
Weighted CDF	6.370319	3.195867	2.177631	1.670390	1.424813	1.269769	1.160874	1.086680	1.032894

Exhibit 2
Table 2

NOISE IN CASE RESERVES

CASE RESERVES BY ACCIDENT YEAR
($000 Omitted)

Accident Year	As of 1 Year	As of 2 Years	As of 3 Years	As of 4 Years	As of 5 Years	As of 6 Years	After 7 Years	As of 8 Years	As of 9 Years	As of 10 Years
1995									1,273	0
1996								2,779	1,516	0
1997							4,096	2,671	1,357	0
1998						6,207	4,183	3,056	1,374	0
1999					7,393	6,152	4,109	2,669	1,511	0
2000				10,297	8,409	7,005	4,615	3,156	1,580	0
2001			15,504	11,704	10,052	8,143	5,174	3,500	1,824	0
2002		18,321	15,494	11,981	10,373	8,002	5,591	3,638	1,823	0
2003	25,550	20,520	17,170	14,820	10,780	9,018	6,108	4,012	1,958	0
2004	25,245	20,318	17,721	13,432	11,523	9,345	6,120	4,015	1,973	
2005	24,191	19,748	17,324	12,978	10,949	8,905	5,925	4,068		
2006	26,062	20,535	17,686	13,457	11,502	9,420	6,255			
2007	26,428	21,397	17,953	14,004	11,069	9,476				
2008	22,885	18,987	15,946	12,170	10,391					
2009	31,313	24,732	21,064	16,315						
2010	37,903	28,588	24,910							
2011	39,680	31,618								
2012	43,001									

Exhibit 2
Table 3

NOISE IN PAYMENT PATTERN

CUMULATIVE LOSS PAYMENTS EMERGED BY YEAR-END ACCOUNTING DATE
($000 Omitted)

Cumulative Emerged Payments of Losses which were Unpaid as of Year-End Accounting Date
Derived by appropriate accumulation of Cumulative Loss Payments of Exhibit 2, Table 1

Year-End Accounting Date	As of 1 Year		As of 2 Years		As of 3 Years		As of 4 Years		As of 5 Years		As of 6 Years		As of 7 Years		As of 8 Years		As of 9 Years
2003	68,485	1.791063	122,661	1.344663	164,937	1.204728	198,705	1.125951	223,732	1.086669	243,122	1.050816	255,477	1.026816	262,327	1.011162	265,255
2004	68,252	1.826554	124,666	1.392118	173,549	1.204390	209,021	1.136102	237,469	1.082153	256,978	1.053511	270,729	1.027227	278,100		
2005	71,299	1.864343	132,926	1.377161	183,061	1.212152	221,898	1.128225	250,351	1.082149	270,917	1.046491	283,512				
2006	77,516	1.839971	142,627	1.370469	195,466	1.199810	234,522	1.124185	263,646	1.074393	283,259						
2007	80,746	1.840616	148,622	1.352661	201,035	1.196401	240,518	1.117131	268,690								
2008	82,484	1.795766	148,121	1.347481	199,591	1.188147	237,143										
2009	84,499	1.805464	152,560	1.352356	206,315												
2010	90,309	1.822621	164,599														
2011	97,321																
2012																	$\boxed{?}$

Exhibit 2
Table 4

NOISE IN PAYMENT PATTERN AND CASE RESERVES

LOSS PAYMENTS EMERGED BY YEAR-END ACCOUNTING DATE RECAST AT 2012 YEAR-END ACCOUNTING DATE EXPOSURE LEVEL
USING CASE RESERVES AS EXPOSURE MEASURE
($000 Omitted)

Cumulative Emerged Payments of Losses which were Unpaid as of Year-End Accounting Date
Derived by appropriate accumulation of Cumulative Loss Payments of Exhibit 2, Table 1 Exposure Adjusted to 2012 Year-End Accounting Date Exposure Level

Year-End Accounting Date	As of 1 Year		As of 2 Years		As of 3 Years		As of 4 Years		As of 5 Years		As of 6 Years		As of 7 Years		As of 8 Years		As of 9 Years
2003	110,337	1.793203	197,857	1.346831	266,480	1.206838	321,598	1.127627	362,643	1.088906	394,884	1.052614	415,660	1.028032	427,312	1.011532	432,240
2004	104,450	1.831614	191,313	1.396273	267,125	1.205181	321,934	1.137304	366,137	1.085009	397,262	1.056637	419,762	1.028778	431,841		
2005	105,407	1.863660	196,442	1.385869	272,243	1.212790	330,174	1.131679	373,651	1.087079	406,188	1.050341	426,636				
2006	107,687	1.854530	199,709	1.371618	273,924	1.201549	329,133	1.127578	371,123	1.079253	400,536						
2007	111,076	1.844031	204,827	1.356930	277,936	1.199481	333,378	1.121370	373,841								
2008	112,354	1.808011	203,136	1.354758	275,201	1.195522	329,009										
2009	108,263	1.815070	196,504	1.359876	267,222												
2010	106,421	1.827327	194,466														
2011	107,469																
2012																	?
Average LDF		1.829681		1.367451		1.203560		1.129112		1.085062		1.053198		1.028405		1.011532	
Average CDF		4.042031		2.209145		1.615521		1.342285		1.188798		1.095604		1.040264		1.011532	
Weighted LDF		1.829531		1.366944		1.203286		1.128899		1.084878		1.053156		1.028416		1.011532	
Weighted CDF		4.037726		2.206973		1.614531		1.341768		1.188563		1.095573		1.040275		1.011532	

Exhibit 2
Table 5

NOISE IN PAYMENT PATTERN AND CASE RESERVES

ACCOUNTING DATE PAYMENT DEVELOPMENT INDICATED AGGREGATE UNPAID LOSS AS OF 12/31/12;
ALLOCATION OF TOTAL UNPAID CLAIM ESTIMATE TO ACCIDENT YEAR
($000 Omitted)

(1) Year-End Accounting Date	(2) Recast Cumulative Loss Payments As of 12/31/12 at 2012 Year-End Accounting Date Exposure Level	(3) Weighted Cumulative Development Factor	(4)= (2)x(3) Indicated Total Emergence at 2012 Year-End Accounting Date Exposure Level	(5)= (4)-(2) Payment Development Indicated Unpaid Loss as of 12/31/12 at 2012 Year-End Accounting Date Exposure Level	(6) Accident Year	(7) Accident Year Allocation of Aggregate Accounting Date Payment Development Indicated Unpaid Loss as of 12/31/12
2003	432,240	1.000000	432,240		2003	
2004	431,841	1.011532	436,821	4,980	2004	2,924
2005	426,636	1.040275	443,819	17,183	2005	7,107
2006	400,536	1.095573	438,816	38,280	2006	13,814
2007	373,841	1.188563	444,333	70,492	2007	21,790
2008	329,009	1.341768	441,453	112,444	2008	26,195
2009	267,222	1.614531	431,437	164,216	2009	46,535
2010	194,466	2.206973	429,180	234,715	2010	75,706
2011	107,469	4.037726	433,929	326,460	2011	99,442
2012			$\boxed{433,929}$ *	433,929	2012	140,416
					Total	**433,929**

(2) Exhibit 2, Table 4 final diagonal
(3) Exhibit 2, Table 4 corresponding Weighted CDF
(7) Iterative Formula

* Accept most recent indication

Exhibit 3
Table 1

NO NOISE IN PAYMENT PATTERN OR CASE RESERVES

REPORTED LOSSES BY ACCIDENT YEAR: Exhibit 1, Table 1 + Exhibit 1, Table 2
($000 Omitted)

Accident Year	As of 1 Year		As of 2 Years		As of 3 Years		As of 4 Years		As of 5 Years		As of 6 Years		As of 7 Years		As of 8 Years		As of 9 Years		As of 10 Years
1995																	58,254	1.010638	58,873
1996															66,711	1.027322	68,534	1.010638	69,263
1997													59,962	1.039773	62,347	1.027322	64,050	1.010638	64,731
1998											57,814	1.047619	60,567	1.039773	62,976	1.027322	64,697	1.010638	65,385
1999									52,632	1.076923	56,681	1.047619	59,380	1.039773	61,742	1.027322	63,428	1.010638	64,103
2000							53,675	1.114286	59,809	1.076923	64,410	1.047619	67,477	1.039773	70,161	1.027322	72,078	1.010638	72,845
2001					55,028	1.147541	63,147	1.114286	70,364	1.076923	75,776	1.047619	79,385	1.039773	82,542	1.027322	84,797	1.010638	85,700
2002			46,500	1.220000	56,730	1.147541	65,100	1.114286	72,540	1.076923	78,120	1.047619	81,840	1.039773	85,095	1.027322	87,420	1.010638	88,350
2003	40,000	1.250000	50,000	1.220000	61,000	1.147541	70,000	1.114286	78,000	1.076923	84,000	1.047619	88,000	1.039773	91,500	1.027322	94,000	1.010638	95,000
2004	40,800	1.250000	51,000	1.220000	62,220	1.147541	71,400	1.114286	79,560	1.076923	85,680	1.047619	89,760	1.039773	93,330	1.027322	95,880		
2005	39,576	1.250000	49,470	1.220000	60,353	1.147541	69,258	1.114286	77,173	1.076923	83,110	1.047619	87,067	1.039773	90,530				
2006	41,951	1.250000	52,438	1.220000	63,975	1.147541	73,413	1.114286	81,804	1.076923	88,096	1.047619	92,291						
2007	42,370	1.250000	52,963	1.220000	64,614	1.147541	74,148	1.114286	82,622	1.076923	88,977								
2008	37,709	1.250000	47,137	1.220000	57,507	1.147541	65,991	1.114286	73,533										
2009	49,022	1.250000	61,278	1.220000	74,759	1.147541	85,789												
2010	59,807	1.250000	74,759	1.220000	91,206														
2011	62,797	1.250000	78,497																
2012	69,077																		

	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
Average LDF	1.250000	1.220000	1.147541	1.114286	1.076923	1.047619	1.039773	1.027322	1.010638
Average CDF	2.375000	1.900000	1.557377	1.357143	1.217949	1.130952	1.079545	1.038251	1.010638
Weighted LDF	1.250000	1.220000	1.147541	1.114286	1.076923	1.047619	1.039773	1.027322	1.010638
Weighted CDF	2.375000	1.900000	1.557377	1.357143	1.217949	1.130952	1.079545	1.038251	1.010638

Exhibit 3
Table 2

NO NOISE IN PAYMENT PATTERN OR CASE RESERVES

TRADITIONAL INCURRED DEVELOPMENT METHOD BY ACCIDENT YEAR
($000 Omitted)

(1) Accident Year	(2) Reported Losses as of 12/31/12	(3) Cumulative Loss Development Factor to Ultimate	(4)= (2)x(3) Incurred Development Method Estimated Ultimate Losses	(5)= (4) - (2) Incurred Development IBNR Estimate as of 12/31/12	(6) Case Reserves as of 12/31/12	(7)= (5) + (6) Unpaid Loss Estimate as of 12/31/12
2003	95,000	1.000000	95,000	0	0	0
2004	95,880	1.010638	96,900	1,020	2,040	3,060
2005	90,530	1.038251	93,993	3,463	3,958	7,420
2006	92,291	1.079545	99,633	7,341	6,293	13,634
2007	88,977	1.130952	100,629	11,652	9,533	21,185
2008	73,533	1.217949	89,560	16,026	10,370	26,397
2009	85,789	1.357143	116,428	30,639	15,932	46,571
2010	91,206	1.557377	142,042	50,836	25,418	76,254
2011	78,497	1.900000	149,144	70,647	31,399	102,046
2012	69,077	2.375000	164,058	94,981	43,173	138,154
Total	860,780		1,147,386	286,605	148,116	**434,721**

(2) Exhibit 3, Table 1 final diagonal
(3) Exhibit 3, Table 1 corresponding CDF; reportings completed as of 10 years
(6) Exhibit 1, Table 2 final diagonal

Exhibit 3
Table 3

NO NOISE IN PAYMENT PATTERN OR CASE RESERVES

CUMULATIVE REPORTED LOSSES EMERGED BY YEAR-END ACCOUNTING DATE
($000 Omitted)

Cumulative Emerged Reported Losses which were Unpaid as of Year-End Accounting Date
Derived as Exhibit 1, Table 4 plus appropriate accumulation of Case Reserves of Exhibit 1, Table 2

Year-End Accounting Date	After 0 Years		After 1 Year		After 2 Years		After 3 Years		After 4 Years		After 5 Years		After 6 Years		After 7 Years		After 8 Years		After 9 Years
2003	90,765	1.508040	136,877	1.282941	175,605	1.169036	205,289	1.108371	227,536	1.067985	243,005	1.042291	253,282	1.026560	260,009	1.013192	263,439	1.003796	264,439
2004	95,858	1.510421	144,786	1.282510	185,690	1.169247	217,117	1.108831	240,746	1.068109	257,143	1.042028	267,950	1.026124	274,950	1.012911	278,500		
2005	99,214	1.512001	150,011	1.282051	192,322	1.169163	224,856	1.108124	249,168	1.067198	265,911	1.041208	276,869	1.025329	283,882				
2006	103,372	1.510759	156,171	1.282192	200,241	1.168553	233,992	1.107412	259,126	1.066571	276,376	1.040553	287,584						
2007	106,736	1.512131	161,398	1.281310	206,801	1.167634	241,468	1.106533	267,192	1.065733	284,756								
2008	106,387	1.515387	161,217	1.279357	206,254	1.165858	240,463	1.104404	265,568										
2009	112,722	1.508262	170,015	1.280504	217,705	1.165983	253,840												
2010	124,108	1.504734	186,750	1.281565	239,332														
2011	135,349	1.504486	203,631																
2012	148,116																		$\boxed{?}$

Exhibit 3
Table 4

NO NOISE IN PAYMENT PATTERN OR CASE RESERVES

CUMULATIVE REPORTED LOSSES EMERGED BY YEAR-END ACCOUNTING DATE RECAST AT 2012 YEAR-END ACCOUNTING DATE EXPOSURE LEVEL
USING CASE RESERVES AS EXPOSURE MEASURE
($000 Omitted)

Cumulative Emerged Reported Losses which were Unpaid as of Year-End Accounting Date
Derived as Exhibit 1, Table 5 plus Case Reserves of Exhibit 1, Table 2 Adjusted to 2012 Year-End Accounting Date Exposure Level

Year-End Accounting Date	After 0 Years		After 1 Year		After 2 Years		After 3 Years		After 4 Years		After 5 Years		After 6 Years		After 7 Years		After 8 Years		After 9 Years
2003	148,116	1.505397	222,973	1.284923	286,503	1.170929	335,474	1.110057	372,396	1.069499	398,277	1.043603	415,643	1.027582	427,107	1.013784	432,994	1.003988	434,721
2004	148,116	1.505397	222,973	1.284923	286,503	1.170929	335,474	1.110057	372,396	1.069499	398,277	1.043603	415,643	1.027582	427,107	1.013784	432,994		
2005	148,116	1.505397	222,973	1.284923	286,503	1.170929	335,474	1.110057	372,396	1.069499	398,277	1.043603	415,643	1.027582	427,107				
2006	148,116	1.505397	222,973	1.284923	286,503	1.170929	335,474	1.110057	372,396	1.069499	398,277	1.043603	415,643						
2007	148,116	1.505397	222,973	1.284923	286,503	1.170929	335,474	1.110057	372,396	1.069499	398,277								
2008	148,116	1.505397	222,973	1.284923	286,503	1.170929	335,474	1.110057	372,396										
2009	148,116	1.505397	222,973	1.284923	286,503	1.170929	335,474												
2010	148,116	1.505397	222,973	1.284923	286,503														
2011	148,116	1.505397	222,973																
2012	148,116																		$\boxed{434,721}$
Average LDF		1.505397		1.284923		1.170929		1.110057		1.069499		1.043603		1.027582		1.013784		1.003988	
Average CDF		2.935012		1.949660		1.517336		1.295840		1.167364		1.091505		1.045901		1.017827		1.003988	
Weighted LDF		1.505397		1.284923		1.170929		1.110057		1.069499		1.043603		1.027582		1.013784		1.003988	
Weighted CDF		2.935012		1.949660		1.517336		1.295840		1.167364		1.091505		1.045901		1.017827		1.003988	

Exhibit 3
Table 5

NO NOISE IN PAYMENT PATTERN OR CASE RESERVES

ACCOUNTING DATE INCURRED DEVELOPMENT INDICATED AGGREGATE UNPAID LOSS AS OF 12/31/12;
ALLOCATION OF TOTAL UNPAID CLAIM ESTIMATE TO ACCIDENT YEAR
($000 Omitted)

(1) Year-End Accounting Date	(2) Recast Reported Losses As of 12/31/12 at 2012 Year-End Accounting Date Exposure Level	(3) Weighted Cumulative Development Factor	(4)= (2)x(3) Indicated Total Emergence at 2012 Year-End Accounting Date Exposure Level	(5)= (4)-(2) Indicated IBNR as of 12/31/12 at 2012 Year-End Accounting Date Exposure Level	(6) Accident Year	(7) Accident Year Allocation of Aggregate Accounting Date Incurred Development Indicated IBNR as of 12/31/12	(8) Case Reserves as of 12/31/12	(9)= (7)+(8) Accident Year Allocation of Aggregate Incurred Development Aggregate Unpaid Loss as of 12/31/12
2003	434,721	1.000000	434,721		2003			
2004	432,994	1.003988	434,721	1,727	2004	1,020	2,040	3,060
2005	427,107	1.017827	434,721	7,614	2005	3,463	3,958	7,421
2006	415,643	1.045901	434,721	19,078	2006	7,341	6,293	13,634
2007	398,277	1.091505	434,721	36,444	2007	11,652	9,533	21,185
2008	372,396	1.167364	434,721	62,325	2008	16,026	10,370	26,397
2009	335,474	1.295840	434,721	99,247	2009	30,639	15,932	46,571
2010	286,503	1.517336	434,721	148,218	2010	50,836	25,418	76,254
2011	222,973	1.949660	434,721	211,748	2011	70,647	31,399	102,046
2012	148,116	2.935012	$\boxed{434,721}$	286,605	2012	94,981	43,173	138,154
					Total	286,605	148,116	**434,721**

(2) Exhibit 3, Table 4 final diagonal
(3) Exhibit 3, Table 4 corresponding CDF
(7) Iterative Formula
(8) Exhibit 1, Table 2 final diagonal

Exhibit 4
Table 1

NOISE IN PAYMENT PATTERN AND CASE RESERVES

REPORTED LOSSES BY ACCIDENT YEAR: Exhibit 2, Table 1 + Exhibit 2, Table 2
($000 Omitted)

Accident Year	As of 1 Year		As of 2 Years		As of 3 Years		As of 4 Years		As of 5 Years		As of 6 Years		As of 7 Years		As of 8 Years		As of 9 Years		As of 10 Years
1995																	58,173	1.014232	59,001
1996															67,212	1.030270	69,246	1.008739	69,851
1997													60,529	1.040965	63,008	1.034140	65,159	1.008659	65,724
1998											58,343	1.048721	61,185	1.045299	63,957	1.016314	65,001	1.014125	65,919
1999									52,241	1.080612	56,452	1.049999	59,275	1.039695	61,627	1.022106	62,990	1.009106	63,563
2000							54,965	1.113528	61,206	1.080280	66,119	1.046330	69,182	1.040244	71,967	1.036588	74,600	1.009970	75,344
2001					55,038	1.135500	62,496	1.097708	68,602	1.079582	74,062	1.056261	78,228	1.039231	81,297	1.040801	84,614	1.009467	85,415
2002			45,691	1.257277	57,446	1.151417	66,144	1.099419	72,720	1.066629	77,566	1.053355	81,704	1.038200	84,825	1.029366	87,316	1.011716	88,339
2003	41,030	1.240434	50,895	1.200177	61,083	1.171586	71,564	1.088313	77,884	1.072903	83,562	1.048946	87,652	1.037957	90,979	1.021445	92,930	1.010438	93,900
2004	41,218	1.221975	50,368	1.229121	61,908	1.174894	72,735	1.117348	81,271	1.084654	88,150	1.044572	92,079	1.052075	96,875	1.024785	99,276		
2005	38,705	1.269811	49,148	1.209965	59,468	1.173480	69,784	1.119790	78,144	1.088300	85,044	1.045093	88,879	1.037893	92,247				
2006	41,636	1.248866	51,998	1.233239	64,126	1.152394	73,898	1.117028	82,546	1.078455	89,022	1.043283	92,875						
2007	42,794	1.247772	53,397	1.217120	64,990	1.144764	74,399	1.099791	81,823	1.085131	88,789								
2008	36,432	1.293958	47,141	1.216038	57,326	1.143205	65,535	1.115988	73,136										
2009	49,807	1.246555	62,087	1.208152	75,010	1.152700	86,464												
2010	58,985	1.219265	71,918	1.234387	88,775														
2011	63,818	1.234563	78,787																
2012	68,568																		

	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
Average LDF	1.247022	1.222831	1.155549	1.107657	1.079616	1.048507	1.041284	1.028424	1.010717
Average CDF	2.391364	1.917660	1.568214	1.357116	1.225213	1.134860	1.082358	1.039446	1.010717
Weighted LDF	1.244471	1.222709	1.155689	1.107618	1.079717	1.048268	1.041355	1.028434	1.010646
Weighted CDF	2.386138	1.917392	1.568151	1.356896	1.225058	1.134610	1.082366	1.039383	1.010646

Exhibit 4
Table 2

NOISE IN PAYMENT PATTERN AND CASE RESERVES

CUMULATIVE REPORTED LOSSES EMERGED BY YEAR-END ACCOUNTING DATE
($000 Omitted)

Cumulative Emerged Reported Losses which were Unpaid as of Year-End Accounting Date
Derived as Exhibit 2, Table 3 plus appropriate accumulation of Case Reserves of Exhibit 2, Table 2

Year-End Accounting Date	After 0 Years		After 1 Year		After 2 Years		After 3 Years		After 4 Years		After 5 Years		After 6 Years		After 7 Years		After 8 Years		After 9 Years
2003	91,421	1.521914	139,134	1.274962	177,391	1.166527	206,932	1.098568	227,328	1.070788	243,420	1.046306	254,692	1.025988	261,311	1.011381	264,285	1.003670	265,255
2004	95,895	1.494355	143,301	1.286674	184,381	1.169345	215,605	1.114225	240,233	1.075559	258,385	1.040823	268,933	1.028888	276,702	1.012183	280,073		
2005	99,240	1.519185	150,763	1.275555	192,307	1.181709	227,251	1.116661	253,762	1.068758	271,210	1.042786	282,814	1.023828	289,553				
2006	105,526	1.491873	157,431	1.299000	204,503	1.177423	240,787	1.108379	266,883	1.067745	284,963	1.037170	295,555						
2007	106,344	1.542345	164,019	1.291900	211,896	1.167557	247,400	1.103089	272,905	1.064337	290,462								
2008	106,158	1.551881	164,744	1.277332	210,433	1.160211	244,147	1.103049	269,306										
2009	113,574	1.510409	171,543	1.271869	218,180	1.167812	254,793												
2010	124,946	1.476768	184,517	1.289782	237,986														
2011	133,888	1.511168	202,327																
2012	148,006																		?

Exhibit 4
Table 3

NOISE IN PAYMENT PATTERN AND CASE RESERVES

CUMULATIVE REPORTED LOSSES EMERGED BY YEAR-END ACCOUNTING DATE RECAST AT 2012 YEAR-END ACCOUNTING DATE EXPOSURE LEVEL
USING CASE RESERVES AS EXPOSURE MEASURE
($000 Omitted)

Cumulative Emerged Reported Losses which were Unpaid as of Year-End Accounting Date
Derived as Exhibit 2, Table 4 plus Case Reserves of Exhibit 2, Table 2 Adjusted to 2012 Year-End Accounting Date Exposure Level

Year-End Accounting Date	After 0 Years		After 1 Year		After 2 Years		After 3 Years		After 4 Years		After 5 Years		After 6 Years		After 7 Years		After 8 Years		After 9 Years
2003	148,006	1.520684	225,070	1.276014	287,193	1.168440	335,568	1.099595	368,988	1.072108	395,595	1.047466	414,373	1.026993	425,558	1.011864	430,607	1.003791	432,240
2004	148,006	1.489210	220,412	1.289541	284,230	1.171306	332,921	1.115211	371,277	1.076982	399,858	1.042366	416,799	1.030754	429,617	1.012999	435,201		
2005	148,006	1.515247	224,266	1.277923	286,594	1.185282	339,695	1.118295	379,879	1.072197	407,305	1.045425	425,807	1.026139	436,938				
2006	148,006	1.490966	220,672	1.303327	287,608	1.179673	339,283	1.111792	377,212	1.070882	403,950	1.040086	420,142						
2007	148,006	1.536145	227,359	1.294217	294,252	1.171426	344,694	1.105837	381,175	1.068106	407,135								
2008	148,006	1.542578	228,311	1.283167	292,961	1.163355	340,817	1.109798	378,238										
2009	148,006	1.506422	222,960	1.274553	284,174	1.173655	333,522												
2010	148,006	1.476909	218,591	1.293966	282,850														
2011	148,006	1.513372	223,988																
2012	148,006																		?
Average LDF		1.510170		1.286589		1.173305		1.110088		1.072055		1.043836		1.027962		1.012432		1.003791	
Average CDF		2.958485		1.959041		1.522663		1.297755		1.169056		1.090482		1.044687		1.016270		1.003791	
Weighted LDF		1.509636		1.286796		1.173275		1.110150		1.071929		1.043710		1.027948		1.012448		1.003791	
Weighted CDF		2.957307		1.958953		1.522349		1.297521		1.168780		1.090352		1.044689		1.016286		1.003791	

Exhibit 4
Table 4

NOISE IN PAYMENT PATTERN AND CASE RESERVES

ACCOUNTING DATE INCURRED DEVELOPMENT INDICATED AGGREGATE UNPAID LOSS AS OF 12/31/12;
ALLOCATION OF TOTAL UNPAID CLAIM ESTIMATE TO ACCIDENT YEAR
($000 Omitted)

(1)	(2)	(3)	(4)= (2)x(3)	(5)= (4)-(2)	(6)	(7)	(8)	(9)= (7)+(8)
Year-End Accounting Date	Recast Reported Losses As of 12/31/12 at 2012 Year-End Accounting Date Exposure Level	Weighted Cumulative Development Factor	Indicated Total Emergence at 2012 Year-End Accounting Date Exposure Level	Indicated IBNR as of 12/31/12 at 2012 Year-End Accounting Date Exposure Level	Accident Year	Accident Year Allocation of Aggregate Accounting Date Incurred Development Indicated IBNR as of 12/31/12	Case Reserves as of 12/31/12	Accident Year Allocation of Aggregate Incurred Development Aggregate Unpaid Loss as of 12/31/12
2003	432,240	1.000000	432,240		2003			
2004	435,201	1.003791	436,851	1,650	2004	969	1,973	2,941
2005	436,938	1.016286	444,054	7,116	2005	3,155	4,068	7,224
2006	420,142	1.044689	438,918	18,776	2006	7,493	6,255	13,748
2007	407,135	1.090352	443,921	36,786	2007	12,007	9,476	21,483
2008	378,238	1.168780	442,078	63,839	2008	16,341	10,391	26,731
2009	333,522	1.297521	432,752	99,230	2009	30,801	16,315	47,116
2010	282,850	1.522349	430,597	147,747	2010	50,893	24,910	75,803
2011	223,988	1.958953	438,782	214,794	2011	71,103	31,618	102,721
2012	148,006	2.957307	$\boxed{437{,}699}$	289,693	2012	96,931	43,001	139,932
					Total	289,693	148,006	**437,699**

(2) Exhibit 4, Table 3 final diagonal
(3) Exhibit 4, Table 3 corresponding Weighted CDF
(7) Iterative Formula
(8) Exhibit 2, Table 2 final diagonal

Exhibit 5

NOISE IN PAYMENT PATTERN AND CASE RESERVES

ACCOUNTING DATE EXPECTED UNPAID LOSSES AS OF 12/31/12;
ACCOUNTING DATE BORNHUETTER-FERGUSON UNPAID LOSSES AS OF 12/31/12;
ALLOCATION OF TOTAL UNPAID CLAIM ESTIMATE TO ACCIDENT YEAR
($000 Omitted)

(1) Accident Year	(2) Earned Premium	(3) Industry Earned Premium	(4) Industry Loss Reserve as of 12/31/12	(5)=(4)/(3) Industry Loss Reserve to Earned Premium Ratio as of 12/31/12	(6)=(2)x(5) Expected Unpaid Loss as of 12/31/12	(7) Case Reserves as of 12/31/12	(8) Weighted Cumulative Development Factor	(9)=(7)+[1-1/(8)]x(6) BF Indicated Loss Unpaid as of 12/31/12	(10)=(6)-(7) Implied IBNR as of 12/31/12	(11) BF Indicated IBNR as of 12/31/12	(12)=(7)+(11) Accident Year Allocation of Aggregate Unpaid Loss as of 12/31/12
2003	123,500	3,723,521									
2004	122,191	3,861,662	98,800	0.025584945	3,126	1,973			1,154	1,161	3,134
2005	124,635	4,123,678	245,997	0.059654726	7,435	4,068			3,367	3,388	7,456
2006	129,911	4,446,857	463,898	0.104320506	13,552	6,255			7,298	7,343	13,598
2007	136,312	4,672,778	691,376	0.147958279	20,168	9,476			10,692	10,760	20,236
2008	116,893	4,801,223	1,105,797	0.230315732	26,922	10,391			16,532	16,636	27,026
2009	148,026	5,113,441	1,672,912	0.327159761	48,428	16,315			32,114	32,316	48,630
2010	185,947	5,117,821	2,077,899	0.406012529	75,497	24,910			50,587	50,905	75,815
2011	197,765	5,433,211	2,715,561	0.499807766	98,844	31,618			67,226	67,649	99,267
2012	210,930	5,642,668	3,703,297	0.656302564	138,434	43,001			95,433	96,034	139,034
Year-End Accounting Date 2012 Total					**432,407**	148,006	2.957307	**434,197**	284,401	286,191	**434,197**

(3), (4) figures are used here to illustrate methodology and do not represent actual Industry figures
(7) Exhibit 2, Table 2 final diagonal
(8) Exhibit 4, Table 4, Column (3) Year-End Accounting Date 2012
(11) Total = Total (9) - Total (7); otherwise (10)x[Total (11)/Total (10)]

Exhibit 6

NOISE IN PAYMENT PATTERN AND CASE RESERVES

ACCOUNTING DATE CAPE COD AGGREGATE UNPAID LOSS ESTIMATE AS OF 12/31/12;
ALLOCATION OF TOTAL UNPAID CLAIM ESTIMATE TO ACCIDENT YEAR
($000 Omitted)

(1) Year-End Accounting Date	(2) Recast Reported Losses Through 12/31/12 at 2012 Year-End Accounting Date Exposure Level	(3) Weighted Cumulative Development Factor	(4)= (2)x(3) Indicated Total Emergence at 2012 Year-End Accounting Date Exposure Level	(5)=1/(3) Development Factor Weight	(6) Volume Weight	(7) Cape Cod Indicated Total Emergence at 2012 Year-End Accounting Date Exposure Level	(8)=(7)-(2) Indicated IBNR as of 12/31/12 at 2012 Year-End Accounting Date Exposure Level	(9) Accident Year	(10) Accident Year Allocation of Aggregate Cape Cod IBNR as of 12/31/12	(11) Case Reserves as of 12/31/12	(12)=(10)+(11) Accident Year Allocation of Aggregate Cape Cod Unpaid Loss as of 12/31/12
2003	432,240	1.000000	432,240	1.000000	0.613677			2003			
2004	435,201	1.003791	436,851	0.996223	0.643548	436,855	1,654	2004	971	1,973	2,944
2005	436,938	1.016286	444,054	0.983975	0.662688	443,956	7,018	2005	3,098	4,068	7,167
2006	420,142	1.044689	438,918	0.957222	0.703464	438,877	18,735	2006	7,521	6,255	13,776
2007	407,135	1.090352	443,921	0.917135	0.713430	443,426	36,291	2007	11,724	9,476	21,201
2008	378,238	1.168780	442,078	0.855593	0.712001	441,482	63,244	2008	16,261	10,391	26,652
2009	333,522	1.297521	432,752	0.770700	0.763947	433,944	100,423	2009	32,064	16,315	48,378
2010	282,850	1.522349	430,597	0.656879	0.841387	433,121	150,271	2010	52,124	24,910	77,033
2011	223,988	1.958953	438,782	0.510477	0.903292	438,377	214,388	2011	68,465	31,618	100,083
2012	148,006	2.957307	437,699	0.338146	1.000000	**437,867**	289,861	2012	97,633	43,001	140,633
			437,953	= Expected Unpaid Loss as of 12/31/12					289,861	148,006	**437,867**

(2) Exhibit 4, Table 3 final diagonal
(3) Exhibit 4, Table 3 corresponding Weighted CDF
(4) Expected Unpaid Loss at 12/31/12 equals weighted average of Column (4), weighted on Columns (5) and (6)
(6) [Exhibit 4, Table 2 final diagonal]/[corresponding Exhibit 4, Table 3 final diagonal]
(7) (2)+[1-1/(3)]x(Expected Unpaid Loss as of 12/31/12)
(10) Iterative Formula
(11) Exhibit 2, Table 2 final diagonal

Exhibit 7
Table 1

NOISE IN EARNED PREMIUM

EARNED PREMIUM AT SAME ADEQUACY LEVEL
($000 Omitted)

Accident Year	As of 1 Year	As of 2 Years	As of 3 Years	As of 4 Years	As of 5 Years	As of 6 Years	As of 7 Years	As of 8 Years	As of 9 Years	As of 10 Years
1995									94,405	94,405
1996								110,403	110,403	110,403
1997							104,433	104,433	104,433	104,433
1998						105,912	105,912	105,912	105,912	105,912
1999					102,909	102,909	102,909	102,909	102,909	102,909
2000				115,327	115,327	115,327	115,327	115,327	115,327	115,327
2001			136,361	136,361	136,361	136,361	136,361	136,361	136,361	136,361
2002		137,150	137,150	137,150	137,150	137,150	137,150	137,150	137,150	137,150
2003	149,264	149,264	149,264	149,264	149,264	149,264	149,264	149,264	149,264	149,264
2004	149,204	149,204	149,204	149,204	149,204	149,204	149,204	149,204	149,204	
2005	145,307	145,307	145,307	145,307	145,307	145,307	145,307	145,307		
2006	152,793	152,793	152,793	152,793	152,793	152,793	152,793			
2007	158,179	158,179	158,179	158,179	158,179	158,179				
2008	143,032	143,032	143,032	143,032	143,032					
2009	184,454	184,454	184,454	184,454						
2010	220,083	220,083	220,083							
2011	226,928	226,928								
2012	252,616									

Exhibit 7
Table 2

NOISE IN PAYMENT PATTERN AND CASE RESERVES

LOSS PAYMENTS EMERGED BY YEAR-END ACCOUNTING DATE RECAST AT 2012 YEAR-END ACCOUNTING DATE EXPOSURE LEVEL
USING EARNED PREMIUM AT SAME ADEQUACY LEVEL AS EXPOSURE MEASURE
($000 Omitted)

Cumulative Emerged Payments of Losses which were Unpaid as of Year-End Accounting Date
Derived by appropriate accumulation of Cumulative Loss Payments of Exhibit 2, Table 1 Exposure Adjusted to 2012 Accounting Date Exposure Level

Year-End Accounting Date	As of 1 Year		As of 2 Years		As of 3 Years		As of 4 Years		As of 5 Years		As of 6 Years		As of 7 Years		As of 8 Years		As of 9 Years
2003	108,725	1.792728	194,915	1.348996	262,939	1.208042	317,642	1.128176	358,356	1.089345	390,373	1.052617	410,913	1.027954	422,400	1.011731	427,355
2004	103,130	1.831391	188,872	1.397195	263,891	1.205942	318,237	1.137493	361,992	1.085137	392,811	1.056868	415,149	1.028843	427,123		
2005	104,630	1.863201	194,947	1.384720	269,947	1.212657	327,353	1.131270	370,324	1.086614	402,400	1.050092	422,556				
2006	109,483	1.850491	202,597	1.372892	278,143	1.202374	334,432	1.128239	377,319	1.079334	407,254						
2007	110,657	1.838851	203,482	1.356668	276,057	1.200056	331,284	1.121657	371,587								
2008	110,808	1.806309	200,154	1.354917	271,192	1.195243	324,141										
2009	107,572	1.814544	195,195	1.360172	265,498												
2010	105,963	1.826470	193,538														
2011	105,993																
2012																	$\boxed{?}$
Average LDF		1.827998		1.367937		1.204052		1.129367		1.085107		1.053192		1.028399		1.011731	
Average CDF		4.043237		2.211839		1.616916		1.342895		1.189069		1.095807		1.040463		1.011731	
Weighted LDF		1.827809		1.367396		1.203758		1.129155		1.084912		1.053146		1.028411		1.011731	
Weighted CDF		4.038622		2.209543		1.615877		1.342360		1.188818		1.095774		1.040476		1.011731	

Exhibit 7
Table 3

NOISE IN PAYMENT PATTERN

ACCOUNTING DATE PAYMENT DEVELOPMENT INDICATED AGGREGATE UNPAID LOSS AS OF 12/31/12
USING EARNED PREMIUM AT SAME ADEQUACY LEVEL AS EXPOSURE MEASURE;
ALLOCATION OF TOTAL UNPAID CLAIM ESTIMATE TO ACCIDENT YEAR
($000 Omitted)

(1) Year-End Accounting Date	(2) Recast Cumulative Loss Payments As of 12/31/12 at 2012 Year-End Accounting Date Exposure Level	(3) Weighted Cumulative Development Factor	(4)= (2)x(3) Indicated Total Emergence at 2012 Year-End Accounting Date Exposure Level	(5)= (4)-(2) Payment Development Indicated Unpaid Loss as of 12/31/12 at 2012 Year-End Accounting Date Exposure Level	(6) Accident Year	(7) Accident Year Allocation of Aggregate Accounting Date Payment Development Indicated Unpaid Loss as of 12/31/12
2003	427,355	1.000000	427,355		2003	
2004	427,123	1.011731	432,134	5,011	2004	2,960
2005	422,556	1.040476	439,660	17,103	2005	7,249
2006	407,254	1.095774	446,258	39,004	2006	14,104
2007	371,587	1.188818	441,749	70,162	2007	21,651
2008	324,141	1.342360	435,113	110,973	2008	26,927
2009	265,498	1.615877	429,012	163,514	2009	46,271
2010	193,538	2.209543	427,631	234,093	2010	75,240
2011	105,993	4.038622	428,065	322,072	2011	98,702
2012			$\boxed{428,065}$ *	428,065	2012	134,962
					Total	**428,065**

(2) Exhibit 7, Table 2 final diagonal
(3) Exhibit 7, Table 2 corresponding Weighted CDF
(7) Iterative Formula

* Accept most recent indication

Exhibit 8
Table 1

NOISE IN PAYMENT PATTERN AND CASE RESERVES

CUMULATIVE REPORTED LOSSES EMERGED BY YEAR-END ACCOUNTING DATE RECAST AT 2012 YEAR-END ACCOUNTING DATE EXPOSURE LEVEL
USING EARNED PREMIUM AT SAME ADEQUACY LEVEL AS EXPOSURE MEASURE
($000 Omitted)

Cumulative Emerged Reported Losses which were Unpaid as of Year-End Accounting Date
Derived as Exhibit 7, Table 2 plus Case Reserves of Exhibit 2, Table 2 Adjusted to 2012 Year-End Accounting Date Exposure Level

Year-End Accounting Date	As of 0 Years		As of 1 Year		As of 2 Years		As of 3 Years		As of 4 Years		As of 5 Years		As of 6 Years		As of 7 Years		As of 8 Years		As of 9 Years
2003	146,256	1.518783	222,131	1.276210	283,486	1.169264	331,470	1.100027	364,626	1.072455	391,045	1.047640	409,674	1.026961	420,719	1.011871	425,714	1.003856	427,355
2004	146,320	1.488243	217,759	1.289847	280,876	1.171689	329,099	1.115537	367,122	1.077043	395,406	1.042466	412,197	1.030874	424,923	1.013038	430,463		
2005	146,773	1.515521	222,437	1.277448	284,152	1.184890	336,688	1.117985	376,413	1.071909	403,480	1.045138	421,692	1.025936	432,630				
2006	150,466	1.490612	224,287	1.302933	292,230	1.179661	344,733	1.111704	383,241	1.071096	410,488	1.039881	426,858						
2007	147,075	1.536631	226,000	1.293665	292,368	1.171483	342,504	1.105869	378,765	1.068237	404,611								
2008	145,669	1.543271	224,808	1.283003	288,429	1.163472	335,579	1.108985	372,152										
2009	147,019	1.505954	221,404	1.274660	282,215	1.173426	331,159												
2010	147,416	1.476158	217,609	1.293807	281,544														
2011	146,333	1.510143	220,984																
2012	148,006																		?
Average LDF		1.509480		1.286446		1.173412		1.110018		1.072148		1.043781		1.027924		1.012454		1.003856	
Average CDF		2.957138		1.959045		1.522834		1.297783		1.169155		1.090479		1.044739		1.016359		1.003856	
Weighted LDF		1.508888		1.286646		1.173365		1.110062		1.072019		1.043645		1.027907		1.012471		1.003856	
Weighted CDF		2.955693		1.958855		1.522451		1.297508		1.168861		1.090337		1.044739		1.016375		1.003856	

Exhibit 8
Table 2

NOISE IN PAYMENT PATTERN AND CASE RESERVES

ACCOUNTING DATE INCURRED DEVELOPMENT INDICATED AGGREGATE UNPAID LOSS AS OF 12/31/12
USING EARNED PREMIUM AT SAME ADEQUACY LEVEL AS EXPOSURE MEASURE;
ALLOCATION OF TOTAL UNPAID CLAIM ESTIMATE TO ACCIDENT YEAR
($000 Omitted)

(1) Year-End Accounting Date	(2) Recast Reported Losses As of 12/31/12 at 2012 Year-End Accounting Date Exposure Level	(3) Weighted Cumulative Development Factor	(4)= (2)x(3) Indicated Total Emergence at 2012 Year-End Accounting Date Exposure Level	(5)= (4)-(2) Indicated IBNR as of 12/31/12 at 2012 Year-End Accounting Date Exposure Level	(6) Accident Year	(7) Accident Year Allocation of Aggregate Accounting Date Incurred Development Indicated IBNR as of 12/31/12	(8) Case Reserves as of 12/31/12	(9)=(7)+(8) Accident Year Allocation of Aggregate Incurred Development Aggregate Unpaid Loss as of 12/31/12
2003	427,355	1.000000	427,355		2003			
2004	430,463	1.003856	432,123	1,660	2004	980	2,040	3,020
2005	432,630	1.016375	439,714	7,085	2005	3,217	3,958	7,175
2006	426,858	1.044739	445,955	19,097	2006	7,637	6,293	13,930
2007	404,611	1.090337	441,162	36,551	2007	11,975	9,533	21,508
2008	372,152	1.168861	434,994	62,842	2008	16,782	10,370	27,152
2009	331,159	1.297508	429,681	98,522	2009	30,529	15,932	46,461
2010	281,544	1.522451	428,637	147,093	2010	50,611	25,418	76,029
2011	220,984	1.958855	432,876	211,892	2011	70,577	31,399	101,976
2012	148,006	2.955693	$\boxed{437,460}$	289,454	2012	97,146	43,173	140,319
					Total	289,454	148,116	**437,570**

(2) Exhibit 8, Table 1 final diagonal
(3) Exhibit 8, Table 1 corresponding Weighted CDF
(7) Iterative Formula
(8) Exhibit 2, Table 2 final diagonal

Exhibit 9
Table 1

NOISE IN CLAIM COUNTS

PROJECTED REMAINING CLAIM COUNTS TO BE CLOSED WITH PAYMENT

Accident Year	As of 1 Year	As of 2 Years	As of 3 Years	As of 4 Years	As of 5 Years	As of 6 Years	As of 7 Years	As of 8 Years	As of 9 Years	As of 10 Years
1995									11	0
1996								25	10	0
1997							48	26	10	0
1998						72	39	21	6	0
1999					124	74	31	18	10	0
2000				228	131	75	41	24	8	0
2001			397	248	163	94	49	28	9	0
2002		624	391	233	144	79	47	23	8	0
2003	912	617	404	248	139	82	44	21	10	0
2004	904	630	432	253	157	87	51	27	10	
2005	847	579	399	233	134	72	42	23		
2006	847	580	378	224	129	72	43			
2007	801	540	350	207	127	75				
2008	690	459	304	185	110					
2009	841	563	375	227						
2010	977	652	432							
2011	976	670								
2012	1,023									

Exhibit 9
Table 2

NOISE IN CLAIM COUNTS

SEVERITY ADJUSTED PROJECTED REMAINING CLAIM COUNTS TO BE CLOSED WITH PAYMENT
Exhibit 9, Table 1 Accident Year 2003 inflated/deflated annually by 5%

Accident Year	As of 1 Year	As of 2 Years	As of 3 Years	As of 4 Years	As of 5 Years	As of 6 Years	As of 7 Years	As of 8 Years	As of 9 Years	As of 10 Years
1995									7.445	0.000
1996								17.767	7.107	0.000
1997							35.818	19.402	7.462	0.000
1998						56.414	30.558	16.454	4.701	0.000
1999					102.015	60.880	25.504	14.809	8.227	0.000
2000				196.955	113.163	64.788	35.417	20.732	6.911	0.000
2001			360.091	224.943	147.846	85.261	44.444	25.397	8.163	0.000
2002		594.286	372.381	221.905	137.143	75.238	44.762	21.905	7.619	0.000
2003	912.000	617.000	404.000	248.000	139.000	82.000	44.000	21.000	10.000	0.000
2004	949.200	661.500	453.600	265.650	164.850	91.350	53.550	28.350	10.500	
2005	933.818	638.348	439.898	256.883	147.735	79.380	46.305	25.358		
2006	980.508	671.423	437.582	259.308	149.334	83.349	49.778			
2007	973.621	656.373	425.427	251.610	154.369	91.163				
2008	880.634	585.813	387.990	236.112	140.391					
2009	1,127.020	754.474	502.536	304.202						
2010	1,374.737	917.429	607.867							
2011	1,441.997	989.895								
2012	1,587.009									

Exhibit 9
Table 3

NOISE IN CLAIM COUNTS AND PAYMENT PATTERN

LOSS PAYMENTS EMERGED BY YEAR-END ACCOUNTING DATE RECAST AT 2012 YEAR-END ACCOUNTING DATE EXPOSURE LEVEL
USING SEVERITY ADJUSTED REMAINING CLAIM COUNTS AS EXPOSURE MEASURE
($000 Omitted)

Cumulative Emerged Payments of Losses which were Unpaid as of Year-End Accounting Date
Derived by appropriate accumulation of Cumulative Loss Payments of Exhibit 2, Table 1 Exposure Adjusted to 2012 Accounting Date Exposure Level

Year-End Accounting Date	As of 1 Year		As of 2 Years		As of 3 Years		As of 4 Years		As of 5 Years		As of 6 Years		As of 7 Years		As of 8 Years		As of 9 Years
2003	110,230	1.794628	197,822	1.349383	266,937	1.209110	322,757	1.127755	363,990	1.089701	396,641	1.053049	417,682	1.028034	429,391	1.011866	434,486
2004	104,817	1.829471	191,759	1.396832	267,856	1.204772	322,705	1.137252	366,997	1.085128	398,239	1.056770	420,847	1.028814	432,973		
2005	105,955	1.858437	196,911	1.381993	272,130	1.208387	328,839	1.129128	371,301	1.085739	403,136	1.049444	423,068				
2006	108,858	1.839290	200,222	1.366073	273,518	1.199205	328,004	1.126500	369,496	1.078502	398,503						
2007	108,359	1.844729	199,893	1.358151	271,485	1.200068	325,800	1.121431	365,362								
2008	110,410	1.804541	199,239	1.354728	269,915	1.195568	322,702										
2009	110,566	1.810792	200,212	1.358820	272,052												
2010	109,800	1.822147	200,071														
2011	107,313																
2012																	?
Average LDF		1.825504		1.366568		1.202852		1.128413		1.084768		1.053087		1.028424		1.011866	
Average CDF		4.025231		2.204996		1.613528		1.341419		1.188766		1.095872		1.040627		1.011866	
Weighted LDF		1.825263		1.366010		1.202541		1.128184		1.084543		1.053022		1.028435		1.011866	
Weighted CDF		4.020162		2.202511		1.612368		1.340801		1.188459		1.095816		1.040639		1.011866	

Exhibit 9
Table 4

NOISE IN CLAIM COUNTS AND PAYMENT PATTERN

ACCOUNTING DATE PAYMENT DEVELOPMENT INDICATED AGGREGATE UNPAID LOSS AS OF 12/31/12
USING SEVERITY ADJUSTED REMAINING CLAIM COUNTS AS EXPOSURE MEASURE;
ALLOCATION OF TOTAL UNPAID CLAIM ESTIMATE TO ACCIDENT YEAR
($000 Omitted)

(1)	(2)	(3)	(4)= (2)x(3)	(5)= (4)-(2)	(6)	(7)	(8)	(9)=(7)x1,000/(8)
Year-End Accounting Date	Recast Cumulative Loss Payments As of 12/31/12 at 2012 Year-End Accounting Date Exposure Level	Weighted Cumulative Development Factor	Indicated Total Emergence at 2012 Year-End Accounting Date Exposure Level	Payment Development Indicated Unpaid Loss as of 12/31/12 at 2012 Year-End Accounting Date Exposure Level	Accident Year	Accident Year Allocation of Aggregate Accounting Date Payment Development Indicated Unpaid Loss as of 12/31/12	Number of Remaining Claims Projected to be Closed with Payment as of 12/31/12	Projected Average per Remaining Claims to be Closed with Payment as of 12/31/12 ($000 Included)
2003	434,486	1.000000	434,486		2003			
2004	432,973	1.011866	438,111	5,138	2004	3,073	10	307,285
2005	423,068	1.040639	440,261	17,193	2005	7,411	23	322,212
2006	398,503	1.095816	436,686	38,183	2006	13,946	43	324,331
2007	365,362	1.188459	434,218	68,856	2007	21,187	75	282,494
2008	322,702	1.340801	432,679	109,977	2008	26,224	110	238,396
2009	272,052	1.612368	438,648	166,596	2009	46,544	227	205,042
2010	200,071	2.202511	440,659	240,588	2010	76,525	432	177,141
2011	107,313	4.020162	431,414	324,102	2011	96,504	670	144,036
2012			$\boxed{431,414}$ *	431,414	2012	140,000	1,023	136,853
					Total	**431,414**	2,613	165,103

(2) Exhibit 9, Table 3 final diagonal
(3) Exhibit 9, Table 3 corresponding Weighted CDF
(7) Iterative Formula
(8) Exhibit 9, Table 1 final diagonal

* Accept most recent indication

Exhibit 10
Table 1

NOISE IN CLAIM COUNTS, PAYMENT PATTERN AND CASE RESERVES

REPORTED LOSSES EMERGED BY YEAR-END ACCOUNTING DATE RECAST AT 2012 YEAR-END ACCOUNTING DATE EXPOSURE LEVEL
USING SEVERITY ADJUSTED REMAINING CLAIM COUNTS AS EXPOSURE MEASURE
($000 Omitted)

Cumulative Emerged Payments of Losses which were Unpaid as of Year-End Accounting Date
Derived as Exhibit 9, Table 3 plus Case Reserves of Exhibit 2, Table 2 Adjusted to 2012 Year-End Accounting Date Exposure Level

Year-End Accounting Date	After 0 Years		After 1 Year		After 2 Years		After 3 Years		After 4 Years		After 5 Years		After 6 Years		After 7 Years		After 8 Years		After 9 Years
2003	148,713	1.516970	225,594	1.276297	287,924	1.169655	336,772	1.100249	370,533	1.072358	397,344	1.047979	416,409	1.027115	427,699	1.011922	432,798	1.003900	434,486
2004	148,440	1.489371	221,082	1.289433	285,071	1.171103	333,847	1.115058	372,259	1.076742	400,827	1.042553	417,883	1.030671	430,700	1.012934	436,271		
2005	147,818	1.516116	224,108	1.275938	285,948	1.182763	338,209	1.115700	377,340	1.070990	404,128	1.044616	422,158	1.025527	432,935				
2006	148,184	1.490925	220,932	1.299782	287,163	1.176916	337,967	1.110810	375,417	1.070042	401,712	1.039498	417,579						
2007	144,843	1.533907	222,175	1.293788	287,447	1.171842	336,843	1.105835	372,492	1.068238	397,910								
2008	145,399	1.540427	223,977	1.282804	287,319	1.163299	334,238	1.109705	370,905										
2009	150,814	1.505562	227,059	1.273481	289,156	1.173403	339,296												
2010	152,204	1.475937	224,643	1.292515	290,355														
2011	148,060	1.510921	223,707																
2012	148,006																		?
Average LDF		1.508904		1.285505		1.172711		1.109560		1.071674		1.043662		1.027771		1.012428		1.003900	
Average CDF		2.948836		1.954290		1.520251		1.296355		1.168351		1.090211		1.044602		1.016377		1.003900	
Weighted LDF		1.508351		1.285687		1.172668		1.109610		1.071536		1.043507		1.027744		1.012442		1.003900	
Weighted CDF		2.947347		1.954019		1.519826		1.296040		1.168014		1.090037		1.044590		1.016391		1.003900	

Exhibit 10
Table 2

NOISE IN CLAIM COUNTS, PAYMENT PATTERN AND CASE RESERVES

ACCOUNTING DATE INCURRED DEVELOPMENT INDICATED AGGREGATE UNPAID LOSS AS OF 12/31/12
USING SEVERITY ADJUSTED REMAINING CLAIM COUNTS AS EXPOSURE MEASURE;
ALLOCATION OF TOTAL UNPAID CLAIM ESTIMATE TO ACCIDENT YEAR
($000 Omitted)

(1)	(2)	(3)	(4)= (2)x(3)	(5)= (4)-(2)	(6)	(7)	(8)	(9)=(7)+(8)	(10)	(11)=(9)x1,000/(10)
Year-End Accounting Date	Recast Reported Losses As of 12/31/12 at 2012 Year-End Accounting Date Exposure Level	Weighted Cumulative Development Factor	Indicated Total Emergence at 2012 Year-End Accounting Date Exposure Level	Indicated IBNR as of 12/31/12 at 2012 Year-End Accounting Date Exposure Level	Accident Year	Accident Year Allocation of Aggregate Accounting Date Incurred Development Indicated IBNR as of 12/31/12	Case Reserves as of 12/31/12	Accident Year Allocation of Aggregate Incurred Development Aggregate Unpaid Loss as of 12/31/12	Number of Remaining Claims Projected to be Closed with Payment as of 12/31/12	Projected Average per Remaining Claims to be Closed with Payment as of 12/31/12 ($000 Included)
2003	434,486	1.000000	434,486		2003					
2004	436,271	1.003900	437,973	1,701	2004	1,018	1,973	2,990	10	299,035
2005	432,935	1.016391	440,031	7,096	2005	3,279	4,068	7,348	23	319,472
2006	417,579	1.044590	436,199	18,620	2006	7,519	6,255	13,774	43	320,330
2007	397,910	1.090037	433,737	35,827	2007	11,683	9,476	21,159	75	282,124
2008	370,905	1.168014	433,223	62,317	2008	16,371	10,391	26,761	110	243,286
2009	339,296	1.296040	439,741	100,445	2009	30,633	16,315	46,948	227	206,818
2010	290,355	1.519826	441,289	150,934	2010	51,271	24,910	76,180	432	176,344
2011	223,707	1.954019	437,127	213,420	2011	69,664	31,618	101,282	670	151,167
2012	148,006	2.947347	$\boxed{436,225}$	288,219	2012	96,781	43,001	139,782	1,023	136,639
					Total	288,219	148,006	**436,225**	2,613	166,944

(2) Exhibit 10, Table 1 final diagonal
(3) Exhibit 10, Table 1 corresponding Weighted CDF
(7) Iterative Formula
(8) Exhibit 2, Table 2 final diagonal
(10) Exhibit 9, Table 1 final diagonal

A Mortality-Based Approach to Reserving for Lifetime Workers' Compensation Claims

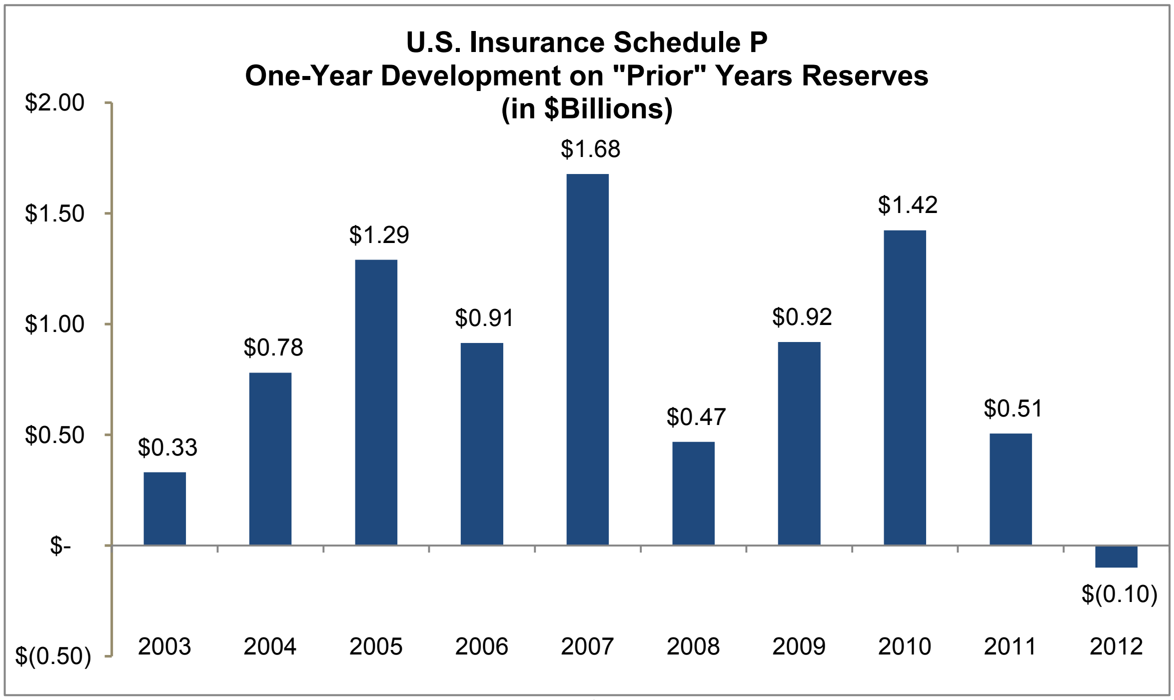
Brian A. Jones, FCAS, MAAA; Craig J. Scukas, FCAS, MAAA; Kathryn S. Frerman; Melissa S. Holt, FCAS, MAAA; Vicki A. Fendley, FCAS, ASA, MAAA

Abstract: Adverse reserve development in older accident years (i.e., related to injuries occurring more than 10 years ago) is a continuing issue in the workers' compensation industry. The use of informed judgment or the application of advanced modeling techniques for projecting this runoff (such as curve fitting) in traditional loss development methods often misstate projections. A mortality-based approach, similar to pension and life/disability insurance models, may produce more meaningful liability estimates when applied to older workers' compensation claims. This paper provides the basic framework of a mortality-based approach, including important considerations regarding the underlying assumptions and model design.

Keywords: mortality; reserving; medical trend; life expectancy/contingency; workers' compensation, pension.

1. INTRODUCTION

Adverse reserve development in older accident years is a persistent problem in the workers' compensation industry. In fact, the one-year workers' compensation reserve development of "prior years" (case and incurred but not reported or "IBNR" reserves for claims at least 10 years old) has been adverse in nine of the last ten years. The following chart shows a history of this development over the past decade.



Note: Based on Schedule P annual statement data for workers' compensation. Data excludes insolvencies.

Predicting the final cost of workers' compensation claims is particularly difficult due to the long period of time over which claimants receive statutory indemnity and medical benefit payments. Even with judgmental modifications and/or the use of advanced modeling techniques (such as curve fitting), traditional, aggregate actuarial methods typically used to project "bulk" incurred-but-not-reported (IBNR) reserves often fall short. Misestimation of reserves for these claims can result in financial reporting errors, claim settlement inequities, loss of reinsurance protection due to late reporting of large claims (through "sunset" clauses) as well as a drag on current earnings. The misestimation of reserves for lifetime workers' compensation cases can stem from many issues including:

- *Insufficient historical loss development data.* Some serious lifetime injury claims can stay open for several decades, but only limited historical loss experience may be available for analysis (e.g., 10 to 20 years).
- *Significant impact of inflation on future costs.* Generally, claims adjusters establish case reserves based on today's costs without consideration of future indemnity benefit escalation and medical inflation. Compounding this issue is the relatively high workers' compensation medical escalation rate (though tempered somewhat in very recent years) compared to general or medical consumer price indices (CPIs).

- *Increases in medical utilization over time*. Case reserves often do not anticipate future intermittent medical costs such as surgeries, prosthetic replacements, and the high cost of end-of-life care. Other significant costs, such as those resulting from technology improvements, new treatments and greater use of expensive prescription narcotics also can contribute to inadequate case reserves.
- *Implicit discounting for large, excess claims*. Current accounting guidance for insurance companies generally does not permit reserving that reflects the time value of money. A reluctance by some companies to recognize large nominal claim values today likely results in some implicit case reserve discounting.
- *Use of outdated or static life tables*. Even if case reserves reflect mortality considerations for lifetime claims, often the mortality assumptions do not reflect future improvements in life expectancy. Also, the averaging nature of a simplistic life expectancy approach generally underestimates gross claim costs in an inflationary environment (i.e., the impact on costs of claimants dying before and after the life expectancy is not offsetting) and changes the distribution of losses in various layers.
- *Industry case reserving practices*. Industry case reserving philosophies and practices vary widely and can lead to different incurred development patterns by company. For example, some organizations may only case reserve for a fixed number of years of payments (e.g., 5 years) or to a "settlement" value instead of an "ultimate value," leading to continual case reserve increases or "stair stepping."

A mortality-based approach can help address each of these issues, making it a valuable alternative or supplement to traditional actuarial methods. The prospective nature of the model, which produces a projection of future cash flows, alleviates the need for extensive loss development history of both open and closed claims. Because it is a payment-based approach, the model does not explicitly require case reserve values (although these amounts can provide a comparison for reasonableness testing). It can directly address the impact of significant changes in the environment (e.g., laws/benefits, regulation, etc.) on only outstanding cases. The approach also is amenable to identification and testing of key assumptions, including trend, discount and mortality, which can provide additional insight to management related to claim cost drivers, claim settlement options, and target areas for cost savings opportunities.

While many of the concepts introduced in this paper are not entirely new, the application of mortality-based models for estimating lifetime workers' compensation claims has gained popularity in recent years, likely due to:

- Recent high loss ratios in workers' compensation;
- Persistent adverse movement in older years' reserve values;
- Higher interest in reinsurance commutations (e.g., from the large number of workers' compensation insurers that went into runoff or insolvency in the late 1990s and early 2000s);
- The increase in limits retained by primary companies;
- Recent low investment returns turning management's focus to underwriting profitability and a better understanding of the drivers of claim cost inflation; and,
- The availability of more sophisticated technology to run (and re-run) the detailed mortality calculations.

1.1 Research Context

Previous research includes foundational discussions around the need to consider mortality in workers' compensation reserving, with later papers providing deeper analyses of other key assumptions and more detailed instructions on how to build a mortality model. In 1971, Ferguson [3] points out the necessity of considering mortality in long-term pension-type workers' compensation awards. He notes the understatement of ceded reserves when employing a simplified approach that subtracts the lower layer of loss from the expected gross reserve. In his paper, Ferguson provides mortality-based calculations which illustrate this point. Steeneck [11] provides an update to Ferguson's paper, incorporating escalation of indemnity benefits and medical inflation in mortality-based forecasts. Snader [10] expands on the use of life contingency concepts in establishing reserves for claimants requiring lifetime medical care using a three phase approach -- claim evaluation, medical evaluation and actuarial evaluation. His paper provides a comprehensive discussion of mortality modeling, including considerations for selecting key assumptions such as inflation, life expectancy, discounting and medical.

Other authors discuss specific assumptions impacting a mortality-based model. For example, Blumsohn [2] examines the errors resulting from using a deterministic approach to model parameters other than mortality, such as medical usage, medical inflation, cost of living adjustments (COLAs), and investment income. He recommends using a stochastic approach to model these parameters and demonstrates that the deterministic method produces biased estimates which understate losses in higher, excess layers. Gillam [4] focuses on mortality assumptions in his discussion of the NCCI Special Call for Injured Worker Mortality Data in 1987 and 1988 and the ensuing analysis of that data. He concluded that differences in mortality, while significant, did not, at that time, imply significant redundancy or inadequacy of the tabular reserves.

In his discussion of "ultimate" loss reserves (i.e., case plus IBNR reserves estimated on an individual claim basis) in the context of runoff operations, Kahn [5] comments on a number of important considerations, including medical escalation, longevity of claimants, and inuring reinsurance, that may impact model scenarios. Sherman and Diss [9] comment on medical cost severities, escalation rates, and mortality rates used to estimate a workers' compensation tail for the medical component of permanent disability claims. In this paper, the authors demonstrate that case reserves estimated based on the expected year of death (i.e., life expectancy approach) are significantly less than the expected value of such reserves using a life contingency cash flow approach.

1.2 Objective

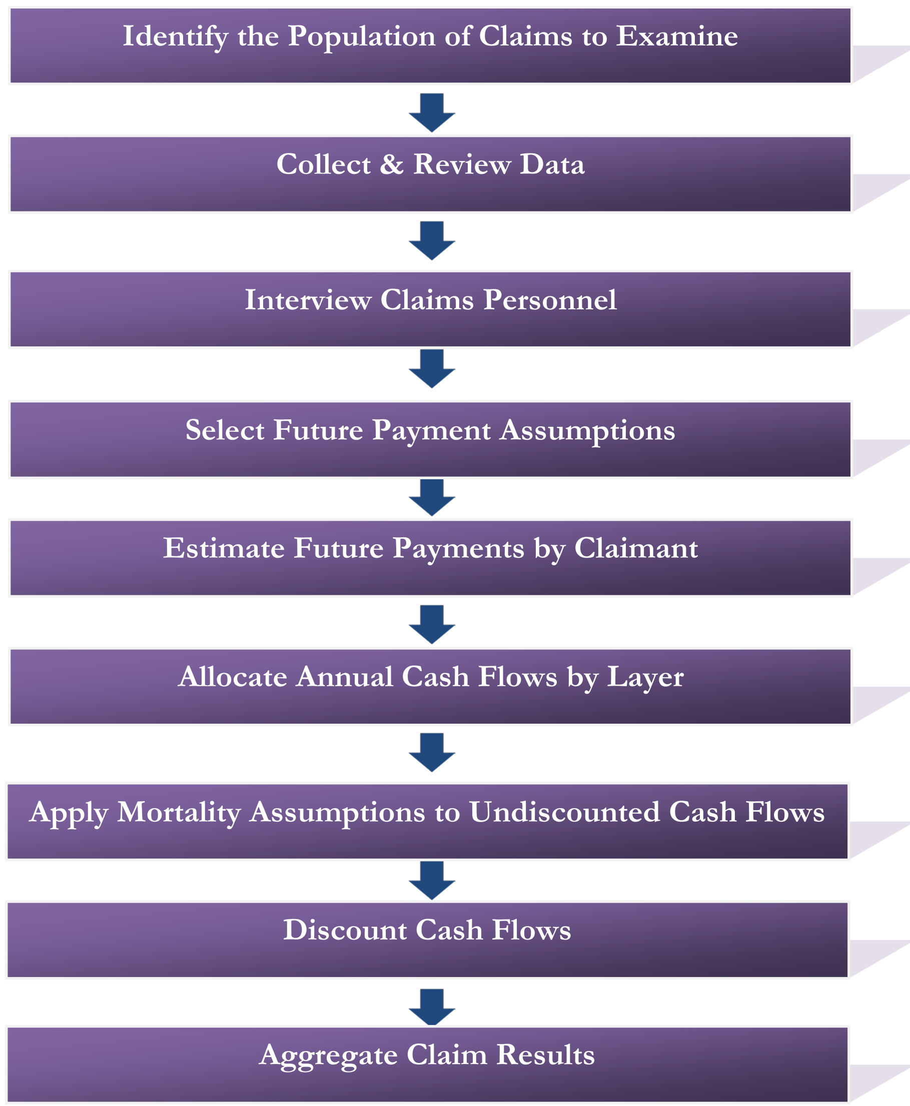
The previously noted research focused on specific assumptions, components or applications of claim-specific models. The purpose of this paper is to reintroduce and synthesize the major concepts, update certain trends and resources, and provide a practical framework to construct a mortality-based approach to model lifetime workers' compensation claims.

1.3 Outline

The remainder of the paper proceeds as follows. Section 2 includes nine steps to construct a mortality-based model for lifetime workers' compensation claims, including a detailed discussion of key model assumptions. Section 3 presents the strengths and weaknesses of the model, and Section 4 summarizes the benefits of considering a mortality-based approach as an alternative or supplement to traditional actuarial methods.

2. BUILDING A MORTALITY-BASED MODEL

The major steps in building a mortality-based model are shown below.



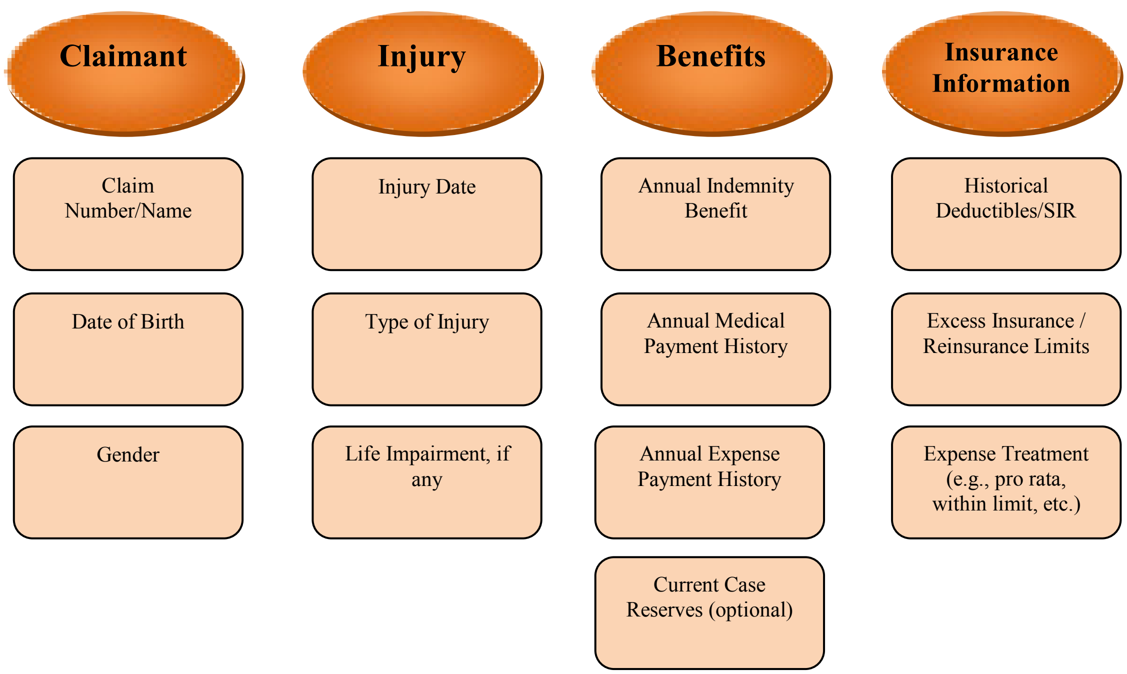
The chronological order of these steps is critical to ensuring that the model appropriately allocates losses to primary and excess layers. Specifically, applying mortality and discounting to the entire loss before layering will understate losses in the excess layer and overstate the primary layer as discussed in Step 6.

2.1 Step 1 – Identify the Population of Claims to Examine

A mortality model is appropriate for lifetime claims or claims that have reached a steady state or maintenance mode such that annual payments are normalized and reflect scheduled disability payments and/or regular, ongoing medical expenses. In cases where the focus is on estimating excess layers of loss, the model should evaluate claims well below the attachment point with the potential to develop into the excess layer due to the nature of the injury (e.g., brain, paralysis) and duration of inflationary impacts.

2.2 Step 2 – Collect and Review Data

Mortality-based models require a considerable amount of detailed claimant and injury information which claims personnel and/or a third-party administrator (TPA) typically can provide. For example, important data elements for a mortality-based model include:



The first three data categories – claimant, injury and benefits – are essential to model the ground-up losses for each claim. The fourth data category -- insurance information -- increases the complexity of model calculations; however, the model requires these elements when estimating losses by layer and, as such, these elements are critical in estimating various stakeholders' liabilities.

2.3 Step 3 – Interview Claims Personnel

Discussions with claims personnel often provide important information regarding the nature of individual claims and the general health status of claimants, including the types of treatment a claimant receives, upcoming surgical procedures and the existence of co-morbidities (i.e., diabetes, cancer, heart disease, etc.), which may impact the claimant's mortality as discussed further in Step 7. This "soft" information is useful particularly when selecting model assumptions.

2.4 Step 4 – Select Future Payment Assumptions

Lifetime workers' compensation claim payments consist of three components – statutory indemnity benefits, unlimited medical benefits and loss adjustment expenses. When selecting future payment assumptions, the modeler could review several recent years of payments, separately for indemnity, medical and expense, for each claim, including the impact of trend on the historical payments (i.e., trend adjusted or "on-level" payments). Alternatively, the model could utilize future payment projections used to determine the case reserves for each claim, which the claims department can provide. In either case, the medical payment assumption should consider expected costs for upcoming surgical procedures, prosthetic device replacements or other intermittent costs.

The selection of future payment assumptions is an important step in the estimation process due to the leveraged impact over payout periods that could extend 60 to 70 years or more into the future (although the impact is less if the model discounts these cash flows). In addition, the cumulative effect of trend over many years of future claim payments can be significant, particularly for severe cases. A variety of social and economic factors, including changes in statutory benefit levels, medical utilization and inflation, drive these trends. Since these factors influence the indemnity, medical and expense payments in different ways, a mortality-based model should project these components separately for each claim.

2.4.1 Indemnity

Projecting future indemnity payments generally is the easiest task due to state and federal workers' compensation statutes that prescribe periodic indemnity/wage replacement benefits. Under many workers' compensation statutes, indemnity benefits remain fixed once the claim is awarded, and no trending is necessary. Some state and Federal disability benefits, however, are subject to automatic escalation (e.g., COLAs), historically at 2-3% based on historical wage inflation levels. In addition, Social Security or other programs may cap or offset indemnity benefits in certain states, which increases the complexity of the model. If indemnity costs include vocational rehabilitation benefits, the model should consider the size and duration of these benefits separately.

2.4.2 Medical

Projecting future medical costs typically is one of the most controversial issues in a mortality-based model because workers' compensation medical benefits are unlimited, consist of both recurring and non-recurring costs, and may extend far into the future. For serious injuries, substantial medical payments may occur early in the life of a claim as a result of initial hospitalizations, surgeries, and treatments. These payments tend to level-off or decrease after a few years as claimants reach maximum medical improvement. Spikes in future costs still may occur for follow-up surgeries, replacement of equipment/devices, or end-of-life care which often results in additional custodial/hospital expenses, particularly for serious, permanent impairments such as brain injuries or paralysis. Alternatives to explicitly projecting such specific, non-recurring costs are building an average provision (i.e., load) into recurring costs, increasing the medical inflation rate, or employing stochastic modeling.

Selecting an appropriate medical trend assumption is another challenge in using a mortality-based approach. Numerous influences – "inflators" and "deflators" -- drive changes in workers' compensation medical costs and ultimately result in partially offsetting increases and decreases in costs over time.

INFLATORS

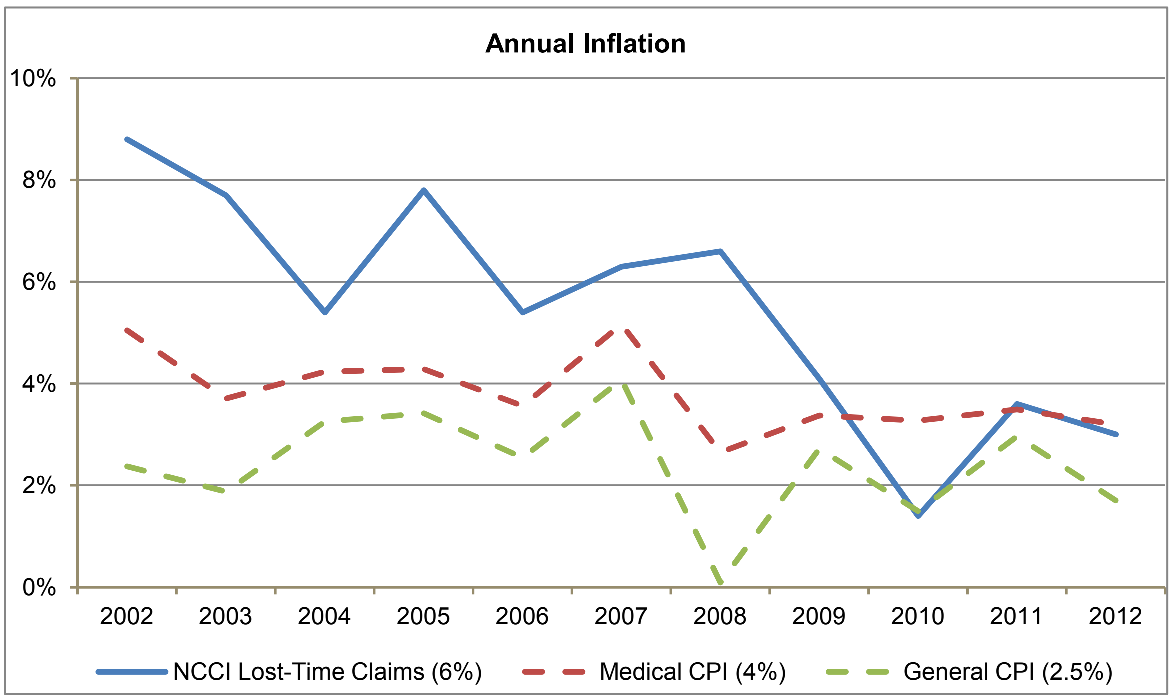
- Aging population
- Consolidated of healthcare providers
- Cost of services
- Increasing utilization
- Medical and technological advances
- Mix of services toward more expensive care alternatives
- More expensive medical devices
- Potential cost shifting from healthcare related reforms/Medicare (MSAs)
- Use of more expensive, patented drugs

DEFLATORS

- Increasing price transparency leads to competition and cost reductions for non-emergency services
- Potential cost reductions related to healthcare reform
- Recent medical supply equipment abatement with hospital consolidation, physician employment with hospitals, and insurer pressure
- Recent pharmaceutical "patent cliff" fostering the use of cost-saving generics

For some of these factors, the effect on medical costs will be short-term, while other factors may continue to influence medical costs indefinitely. Estimating changes in medical costs resulting from each factor as well as the duration of its influence is difficult and requires informed judgment, particularly in light of the significant cumulative effect of these assumptions.

Publicly available trend benchmarks include the general CPI and medical CPI on a calendar year basis and accident year changes in medical severity for NCCI lost-time claims. As the graph below shows, over the last 20 years, workers' compensation medical trends, as reported by the NCCI, have outpaced both the general CPI and the medical CPI.



Source: NCCI's State of the Line presentation for NCCI lost-time claims and the Bureau of Labor Statistics for CPI information.
Note: The average trend over the period from 2002 to 2012 is 6% for NCCI lost-time claims, 4% for the medical CPI, and 2.5% for the general CPI.

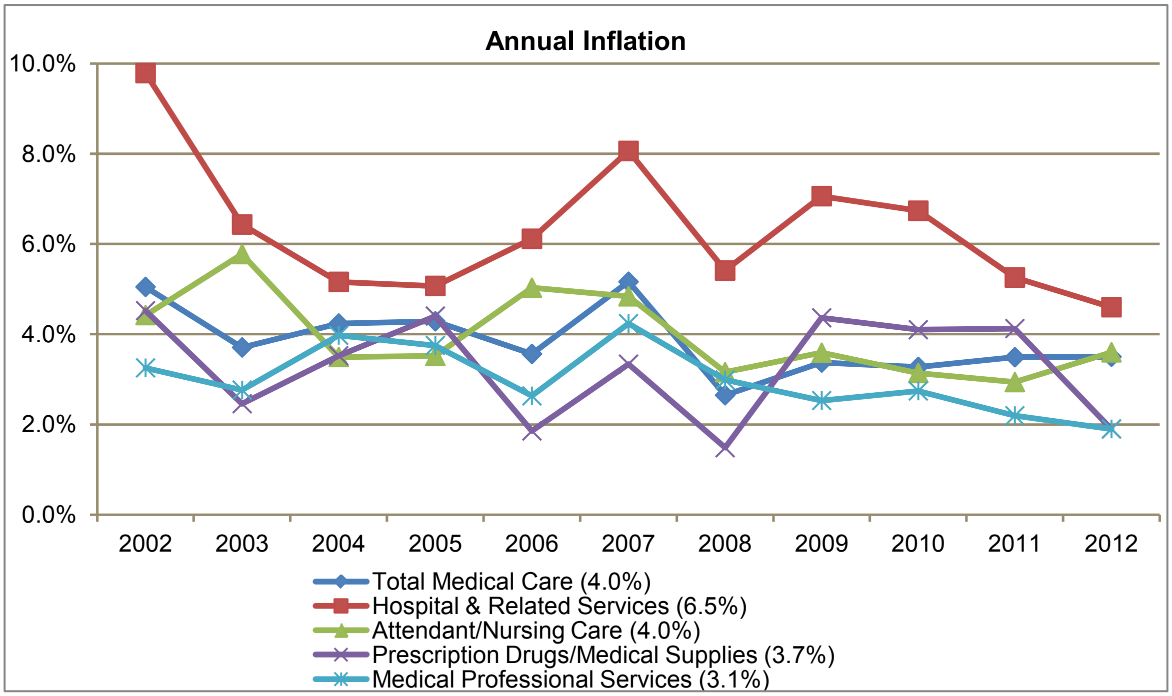
Historically, the medical CPI, which captures the trend in prices for a fixed "basket" of medical goods and services, has been about 200 basis points higher than the general CPI. Medical and technological advances, use of higher cost patented drugs, mix of services toward more expensive care alternatives, and costly medical devices are the primary drivers of this differential. All of the inflators and deflators listed above affect workers' compensation medical costs; however, not all of these factors are captured in the changes for the "basket" of care tracked by the medical CPI. As such, the trend is higher for workers' compensation claims than the medical CPI.

Similarly, changes in service costs and utilization impact workers' compensation medical costs differently than medical costs in health insurance; specifically, mandated benefits and coverage options, such as deductibles, influence health insurance utilization, but do not affect workers' compensation. Decreasing costs due to competitive pressures resulting from greater price transparency of medical services would also benefit workers' compensation costs, although perhaps not to the same degree. Further, the deflator impact of the pharmaceutical name brand "patent cliff" resulting in greater availability of generic drugs is offset by the growing use of biologics and other specialty drugs. The Patient Protection and Affordable Care Act (PPACA; i.e., healthcare reform) also may impact costs by providing incentives for healthcare providers to control costs through high performance networks and hospital readmission penalties; however, the PPACA encourages hospital consolidation, which may increase costs as the acquiring entities typically charge higher prices for services. The following table compares cost inflators and deflators impacting workers' compensation and health care with those captured by the medical CPI.

	Medical CPI	Health Insurance	Workers' Comp
Inflators			
Aging population		X	X
Consolidation of healthcare providers	X	X	X
Cost of services	X	X	X
Mandated benefits/healthcare reform		X	
Mix of claims/diagnosis		X	X *
Utilization-more expensive drugs (specialty drugs/biologics), devices, procedures		X	X
Utilization-more procedures per claimant		X	X
Deflators			
Change in care method (retail clinics, virtual access, etc.)		X	
Greater price transparency and consumer price sharing	X	X	X **
High performance health care networks providing lower-priced care	X	X	
Medical supply and equipment abatement	X	X	X
New hospital readmission penalties		X	X
Pharmaceutical "patent cliff"	X	X	X

Source: PwC's Health Research Institute's "Medical Cost Trend: Behind the Numbers 2014".
* Impacts new claims
** May not have as strong an impact

A more robust mortality model may segment or consider the mix of medical services – hospitals, physicians, drugs, attendant care, equipment, etc. – in a defined population of claims since each component may be subject to different trends over time as shown in the calendar year trends in the chart below.



Source: Bureau of Labor Statistics.

Note: The average trend over the period from 2002 to 2012 is 4.0% for total medical care, 6.5% for hospital and related services, 4.0% for attendant/nursing care, 3.7% for prescription drugs/medical supplies, and 3.1% for medical professional services.

The proportionate cost of these medical services changes over the lifetime of a workers' compensation claim. According to the NCCI Research Brief, *Medical Services by Size of Claim - 2011 Update* [6], "the medical services profile for workers with serious injuries is quite different in the later years of their treatment from the mix of services required early on." More specifically, physical therapy, hospital services, and surgery/anesthesia drive medical costs in the first six years of a claim, whereas prescription drug costs tend to represent a substantially larger proportion of total medical costs paid after the sixth year. Since the proportion of these components is different for more mature claims, the trend rate may be different than the overall medical CPI or workers' compensation medical cost trend.

The NCCI conducted a detailed study of changes in workers' compensation costs over different periods in a Research Brief titled, *The Relationship Between Medical Utilization and Indemnity Claim Severity - Comparing the Factors Driving Medical and Indemnity Severity*, [8]. The results showed a large divergence in trends for accident years 1996/97 through 2000/01, contrasted with much smaller deviations for accident years 2001/02 through 2005/06. This 2011 study presented the following observations:

- Price for workers' compensation medical moved consistently with medical inflation, its leading indicator.
- While utilization (measured as treatments per claim) was a major driver of severity increases in the first period, utilization decreased in absolute terms in the second period.
- The impact of changes in diagnosis mix was significant in the first period and eased off in the second period.

2.4.3 Expenses

A mortality model also may build in expense provisions based on separate allocated and unallocated annual expense payment and trend assumptions. While general CPI or payroll trends may align closely with the trend in unallocated costs, which consist largely of claims adjuster payroll expense, these benchmarks may not apply to allocated costs comprised of attorney and investigation fees.

An alternative method for projecting expenses is to apply a percentage loading to the model's estimated future indemnity and medical expense payments. This abbreviated practice often is deemed sufficient since these expenses typically represent a small proportion of the total claim payments for mature workers' compensation claims that have reached a steady state of annual payments and require minimal file maintenance. When using this method, however, the modeler should consider the following:

- The procedure implicitly assumes that the underlying indemnity and medical trends also are appropriate for expenses.
- The percentage of expense relative to loss for older workers' compensation claims may be lower compared to less mature claims.
- The relationship between expense and loss may change in different layers of loss, so a ground-up ratio may not be appropriate.

2.5 Step 5 – Estimate Future Payments by Claimant

A mortality-based model applies trend assumptions to the selected periodic payments, separately for indemnity benefits, medical benefits and expenses, to project future payments for each claimant. As previously mentioned, a mortality model could further segment these components into finer categories, such as vocational rehabilitation, type of medical services, or legal versus other expense. This step results in cash flows by payment type for each claim.

2.6 Step 6 – Allocate Annual Cash Flows by Layer

After estimating cash flows for each claim, the next step is to allocate the cash flows to primary and excess layers. Since the model separately estimates indemnity, medical and expense cash flows for each claim, it can accommodate varying treatments of expense (e.g., included with loss in limit, excluded from limit, pro-rata, etc.) for each layer. As noted previously, the allocation of annual cash flows by insurance layer must precede the application of mortality and discounting assumptions; otherwise, the benefit of these assumptions will inure to the highest layers of loss (i.e., the model will underestimate the excess layers and overestimate the primary/lower layers). Steeneck [11] observed that the application of mortality "impacts layering in oftentimes non-intuitive ways, especially that lower layers need not fill up fully before a higher layer becomes liable." For example, a claimant may die earlier than assumed in a reserve calculation, allowing some probability that a claim may not actually pierce the excess layer as projected. The following example shows the estimated payments by layer when the model applies mortality assumptions before allocating payments to deductible and excess layers versus after the allocation.

Male – Age 50
Estimated Annual Benefit Payments = $10,000
Assumed Benefit Trend = 4% per year
Deductible = $250,000; 1st Excess = $250,000 xs $250,000; 2nd Excess = xs $500,000
In Thousands

	2013 to 2022	2023 to 2032	2033 to 2042	2043 to 2052	2053 to 2062	2063+	Total
(1) Trended annual payments							
	$ 120.1	$ 177.7	$ 263.1	$ 389.4	$ 576.4	$ 2,272.0	$ 3,798.6
(2) Unadjusted Cash Flows Allocated by Layer							
Deductible	$120.1	$ 129.9					$ 250.0
XS Layer 1		$ 47.8	$202.2				$ 250.0
XS Layer 2			$ 60.8	$ 389.4	$ 576.4	$ 2,272.0	$ 3,298.6
(3) Probability of survival							
	98.2%	90.7%	70.7%	34.6%	5.4%	0.1%	
(4) Mortality adjusted annual payments = (1) x (3)							
	$ 117.9	$ 161.1	$ 186.0	$ 134.7	$ 31.3	$ 1.2	$ 632.3
(5) Mortality Applied Prior to Allocating Cash Flows by Layer							
Deductible	$ 117.9	$ 132.1					$ 250.0
XS Layer 1		$ 29.0	$ 186.0	$ 34.9			$ 250.0
XS Layer 2				$ 99.8	$ 31.3	$ 1.2	$ 132.3
(6) Mortality Applied After Allocating Cash Flows by Layer							
Deductible	$ 117.9	$ 120.0					$ 237.9
XS Layer 1		$ 41.1	$ 149.7				$ 190.9
XS Layer 2			$ 36.3	$ 134.7	$ 31.3	$ 1.2	$ 203.5

Note: Numbers may not add due to rounding. See Appendix A for complete cash flow calculations.

When the model applies mortality prior to allocating cash flows by layer (item 5 in the table), excess layer 2 includes $132 thousand of estimated payments versus $204 thousand when the model applies mortality to the cash flows after allocating by layer.

2.7 Step 7 – Apply Mortality Assumptions to Future Years

After allocating cash flows by layer, a life contingency model should apply mortality assumptions to estimate the undiscounted expected cash flows for each claim. A life contingency model will yield results that differ from a life expectancy approach, which is commonly used to establish case reserves. In a life expectancy approach, the claimant's future life expectancy serves as a proxy for the number of future years that a claimant will receive benefits; however, this approach underestimates the reserve as illustrated in the following example:

Male – Age 50
Life Expectancy = 30 years
Estimated Annual Benefit Payments = $10,000
Assumed Benefit Trend = 4% per Year
In Thousands

	2013	2014	2015	…	2042	2043	…	2060	…	Total
(1) Trended annual payments										
	$ 10.00	$ 10.40	$ 10.82	…	$ 31.19	$ 32.43	…	$ 63.18	…	$ 3,798.62
(2) Probability that claimant survives through year										
	99.8%	99.5%	99.3%	…	57.9%	54.2%	…	2.0%	…	
(3) Expected future annual payments = (1) x (2)										
	$ 9.98	$10.35	$ 10.74	…	$ 18.05	$ 17.57	…	$ 1.23	…	$ 632.25
(4) Life expectancy approach without trend										
	$ 10.00	$ 10.00	$ 10.00	…	$ 10.00	n/a	n/a	n/a	n/a	$ 300.00
(5) Life expectancy approach including trend										
	$ 10.00	$ 10.40	$ 10.82	…	$ 31.19	n/a	n/a	n/a	n/a	$ 560.85

Note: Numbers may not add due to rounding. See Appendix A for complete cash flow calculations.

The life expectancy approach underestimates the future liability. With trend, the reserve estimate is $561 thousand using a life expectancy approach, which compares to $632 thousand using a life contingency method. Claims professionals often do not consider trend in establishing case reserves. In this example, the estimate without trend is $300 thousand.

Blumsohn [2] further developed the comparison of a deterministic approach (using average life expectancy) versus the stochastic approach (using mortality probabilities). His paper discusses the application of a stochastic approach to medical utilization, medical inflation, COLAs, and investment income.

> "Just as it is wrong to assume a claimant's life-span is fixed, so it is wrong to assume that medical usage and inflation are fixed. Assuming a deterministic life-span leads to inaccurate calculations. Likewise, assuming deterministic medical care and inflation will lead to inaccurate calculations. A deterministic life span implies that high layers of reinsurance will not be hit, when they do, in fact, have a chance of getting hit if the claimant lives long enough. Likewise, deterministic medical care and deterministic inflation understate the costs to the highest reinsurance layers."

When selecting a mortality table for a workers' compensation claim model, the modeler should consider the applicability of the base population to the claimant population, the impact of disability on mortality, and adjustments for improvements in mortality over time as described in the following sections.

2.7.1 Applicability of the Base Population to the Claimant Population

In selecting a mortality table for use in modeling workers' compensation claims, the modeler should understand the purpose for which the mortality table was constructed and differences in the underlying population used to derive the various mortality tables. For example, the base population for a life insurance mortality table typically would include wealthier, better educated and married populations which, on average, exhibit lower levels of mortality and higher levels of improvement in mortality compared with general population mortality tables, such as tables produced by the Centers for Disease Control (CDC) from the U.S. census and Medicare data. General population mortality also is higher (i.e., higher probability of death) than pensioner (or worker) experience since some individuals in the general population are unable to work due to health conditions (e.g., mental disorders, as well as diseases of the nervous system, circulatory system, endocrine system, and respiratory system).

The most commonly used mortality tables for U.S. pension plan valuations are the RP-2000 mortality tables based on a study of over 11 million pensioner life-years from 1990 through 1994 (projected to 2000). These tables were developed separately by gender (male versus female) and health condition (healthy versus disabled) and also contain adjustments for worker type (white vs. blue collar). Since the base population used in the construction of the RP-2000 tables is "pensioners," who presumably earned wages and pensions while working, these tables also may be useful for workers' compensation claims. We note that these tables likely will be replaced by a new set of retirement plan mortality tables, which the Society of Actuaries (SOA) anticipates publishing in 2014.

2.7.2 Impact of Disability on Mortality

Separate mortality tables are available for healthy and disabled individuals, with mortality being much higher for the latter; however, the applicability of a healthy or disabled mortality table for injured workers is surprisingly debatable in workers' compensation. Although some serious injuries (e.g., brain trauma, paralysis) would likely diminish life expectancy, many lifetime cases related to other injuries (e.g., back, knee) would have little or no impact. In fact, there is some speculation that many injured worker life expectancies may even improve due to less risky work environments and better medical care. In his 1991 paper analyzing the NCCI Special Call for Injured Worker Mortality Data, Gillam found that "the mortality rate for injured workers is slightly higher than standard at ages less than 60, but very slightly lower for ages 61 to 72" and "the average life pensions on injured workers should be 1.6% lower than on standard." When answering a question regarding how injured worker's mortality could be so near standard, Gillam points to the cohort for the study, saying "an injured worker has been healthy enough to have worked in the first place. Such a person has demonstrated an ability to survive an accident long enough to be put on a pension".

A model may use a variety of approaches to address potential life impairment issues including:

- A "rated age" approach (the most common) using an estimate of the future life expectancy of claimants based upon individual facts and circumstances. The model could utilize a healthy mortality table with an adjustment to an injured worker's age (or "set forward"). Using this simple technique, a 10-year set-forward would define the probability of death for a 52-year-old male equal to that of a 62-year old male. While such an individual approach would seem optimal, it requires considerable judgment and is difficult to collect and maintain for a large population of claims.
- Use of a disabled table for only certain serious injuries and application of a healthy table for all other cases.
- A blending (e.g., 90%/10%) of healthy/disabled mortality factors applied to the entire claim population determined based on perceived impairment in the claims population. Application of a scaling factor or multiplier to healthy mortality rates based on a review of actual to expected historical death experience.

2.7.3 Adjustments for Improvements in Mortality over Time

RP-2000 Combined Healthy Employees & Annuitants - Male
Base Year = 2000
Mortality Improvement = 1.0% per year

Probability of Mortality by Year

Age (in base year)	2000	2001	2002	…	2020
40	0.108%	0.113%	0.119%	…	**0.552%**
…	…	…	…		…
58	0.527%	0.589%	**0.661%**	…	4.263%
59	0.595%	**0.668%**	0.752%	…	4.738%
60	**0.675%**	0.760%	0.858%	…	5.265%

The probability of mortality at age 60 is 0.675% in the base year. With a 1% per year improvement in mortality, the probability of mortality at age 60 for an individual who is age 59 in the base year is 0.668% (= 0.675% x [1 - 1.0%]). The probability of mortality at age 60 for an individual who is age 40 in the base year is 0.552% (= 0.675% × [1-1.0%]^{20}).

Mortality tables may be static (aka "period") life tables or generational (aka "cohort") life tables. Static life tables, such as the CDC tables, are based on the mortality experience of a population over a relatively short period of time and do not include adjustments for potential improvements in mortality. As such, adjustments to these tables, such as a scale adjustment, may be necessary to reflect the actual mortality of the claim population more accurately. For example, the RP-2000 tables are static tables that reflect mortality improvements through the table creation date (2000). The application of a related mortality improvement scale (e.g., Scale AA) for a fixed number of years easily modifies the tables. The adjustment period will depend on the purpose of the calculation or the financial reporting context of the estimates and must be updated with each valuation. When the SOA publishes its new mortality tables, it will also provide new mortality improvement rates (i.e., Scale BB).

A generational table is a more robust, and often preferred, mortality assumption as it is a series of static tables combined to reflect mortality improvements with each year of survival. To construct a generational table, an improvement scale is applied to the base table to yield a static table for each future year. The resulting series of static tables combine to form a generational table.

The following table shows a comparison of the life expectancies for males and females at various ages using four commonly cited mortality tables – GAM-83, UP-94, CDC 2007 and RP-2000. Also included are life expectancies based on the RP-2000 tables with Scale AA and generational adjustments to reflect mortality improvements.

Age	GAM-83	UP-94	CDC 2007	RP-2000 Disabled	RP-2000 Combined Healthy	RP-2000 Scaled to 2013	RP-2000 Generational
Male Life Expectancy							
30	46.5	48.5	47.1	26.9	49.5	50.8	54.6
40	36.9	38.9	37.8	22.7	39.8	41.1	44.0
50	27.7	29.5	29.0	17.7	30.3	31.6	33.6
60	19.3	20.7	20.9	13.3	21.2	22.4	23.6
70	11.9	13.3	13.7	9.3	13.4	14.3	14.8
Female Life Expectancy							
30	52.8	53.1	51.5	39.5	52.5	53.2	55.6
40	43.1	43.3	41.9	32.2	42.7	43.4	45.2
50	33.5	33.7	32.7	24.6	33.1	33.8	35.0
60	24.3	24.5	23.9	18.1	23.9	24.5	25.3
70	15.9	16.3	16.0	12.4	15.7	16.3	16.7

The RP-2000 generational table includes the largest adjustment for mortality improvements and results in the highest life expectancies. The impact of generational mortality or the reflection of mortality improvements has a greater impact on males and individuals at lower ages.

2.8 Step 8 – Discount Cash Flows (If Appropriate)

To estimate the present value of reserves by layer, discounting should be the final assumption applied to the cash flows. Discounting by layer will reflect the greater discount for the longer duration cash flows of excess layers and lesser discount for the shorter duration cash flows of deductible, or primary, layers. Similar to the application of mortality assumptions, a model should apply discounting to cash flows allocated by layer to avoid underestimating excess layers and overestimating primary layer(s). The following example shows the difference in estimated payments by layer when a model applies discounting before allocating payments to deductible and excess layers versus after allocation by layer.

Estimated Annual Benefit Payments = $10,000
Payout Period = 10 years
Assumed Benefit Trend = 4% per year
Assumed Discount Rate = 3%
Deductible = $50,000; 1st Excess = $50,000 xs $50,000; 2nd Excess = xs $100,000
In Thousands

	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	Total
(1) Trended annual payments											
	$ 10.0	$ 10.4	$ 10.8	$ 11.2	$ 11.7	$12.2	$ 12.7	$ 13.2	$ 13.7	$ 14.2	$ 120.1
(2) Undiscounted Cash Flows Allocated by Layer											
Deductible	$ 10.0	$10.4	10.8	$11.2	$ 7.5						$ 50.0
XS Layer 1					$ 4.2	$ 12.2	$ 12.7	$ 13.2	$ 7.9		$ 50.0
XS Layer 2									$ 5.8	$ 14.2	$ 20.1
(3) Discount Factor											
	0.97	0.94	0.92	0.89	0.86	0.84	0.81	0.79	0.77	0.74	
(4) Discounted annual payments = (1) x (3)											
	$ 9.7	$ 9.8	$9.9	$ 10.0	$ 10.1	$ 10.2	$ 10.3	$ 10.4	$ 10.5	$ 10.6	$ 101.4
(5) Cash Flows by Layer with Discounting Applied Prior to Allocation											
Deductible	$ 9.7	$ 9.8	$ 9.9	$ 10.0	$ 10.1	$ 0.5					$ 50.0
XS Layer 1						$ 9.7	$ 10.3	$ 10.4	$ 10.5	$ 9.2	$ 50.0
XS Layer 2										$ 1.4	$ 1.4
(6) Cash Flows by Layer with Discounting Applied After Allocation = (2) x (3)											
Deductible	$ 9.7	$ 9.8	$9.9	$ 10.0	$ 6.5						$ 45.9
XS Layer 1					$ 3.6	$ 10.2	$ 10.3	$ 10.4	$ 6.0		$ 40.5
XS Layer 2									$ 4.5	$ 10.6	$ 15.1

Note: Numbers may not add due to rounding.

When the model applies discounting prior to allocating cash flows by layer (item 5 in the table), excess layer 2 includes $1,441 of the estimated payments compared with $15,058 when the model correctly applies discounting to the cash flows after allocating by layer.

Since the average duration of lifetime payments can be quite long, e.g., 20+ years, discounting has a significant impact on claim values. The following considerations are relevant in the selection of a discount rate(s):

- The purpose of the calculation (e.g., claim settlement, commutation, etc.)
- The financial reporting context (prohibited/limited/prescribed discounting)
- The time period for future payments (average duration of the liabilities)

Under U.S. statutory accounting rules, most states allow discounting for tabular indemnity reserves, but few states allow discounting of other workers' compensation reserves. U.S. GAAP guidance generally does not allow discounting unless claim payments are fixed and reliably determinable; however, if statutory guidance allows discounting (e.g., tabular indemnity reserves), U.S. GAAP may allow an exception. Emerging IFRS guidance takes a more economic approach and allows some form of discounting, although discounting may be coupled with risk margin considerations.

One of the advantages of a mortality model is that it produces a series of future cash flows which may be discounted, using either a single blended rate or multiple rates based on a yield curve. The selection of the type of rate (e.g., risk free, high grade bond, portfolio, risk-adjusted, etc.) is dependent upon the context and purpose. From a true economic rather than accounting perspective, the gap between the inflation and discount rates also should be considered as some correlation likely exists.

2.9 Step 9 – Aggregate Claim Results

In the final step of the mortality-based approach, the model combines the indicated reserves for all claims to yield the total reserve estimate. The actuary should review the reasonability of the results and the underlying assumptions by comparing the projected payments (both by claim and in the aggregate) to the historical payments and current case reserves. Such a validation exercise may require additional discussions with claims personnel when significant differences exist between the projected future payments and current case reserves or when the model produces counter-intuitive results.

3. RESULTS AND DISCUSSION

In our exploration of mortality-based models and prior research, we have found many benefits compared to traditional casualty actuarial techniques. A mortality approach is appealing intuitively because it incorporates individual claim characteristics without requiring a long and complete history of open and closed claims experience or even case reserve values. It accelerates development to an individual claim basis (versus bulk IBNR) which allows for examination of specific facts and circumstances. In contrast to classic triangulation methods, a mortality approach can better address significant changes in factors such as benefit levels, regulation, legislation, policy limits and retentions that may impact outstanding cases. Finally, by its very nature, a mortality-based model easily allows for scenario testing of the sensitivity of important cost drivers (e.g., trend, mortality, discount, etc.)

Like any method, however, mortality-based models possess limitations and are not a panacea for achieving more "accurate" results. A mortality approach may not be applicable to all claims, but rather lifetime and other claims that have reached a relatively steady cost state (e.g., runoff books of business). This approach also requires detailed information on open claims and the modeler must apply judgment to select appropriate assumptions for future costs. In addition, the practitioner will require the requisite statistical/programming skills and software to model the liabilities, and while the initial model design can be time consuming, updates will take considerably less time.

In comparing reserve indications from a mortality approach with traditional development models, settlement activity could create differences between the liabilities under the two methods. Without adjustment, a mortality model generally assumes that payments end only with a claimant's death. However, some claimants accept lifetime settlements (prior to death), and development models incorporate these settlements, which often reflect significant discounts, at least for the time value of money. Although such information often is not available, a mortality model could include an adjustment based on a review of the incidence of such settlements and the magnitude of the difference between the two approaches.

Prior research on the use of mortality models for workers' compensation was not particularly divisive, although the authors presented some varying viewpoints, particularly in the areas of mortality and trends. The prior research does reveal many possible applications for such a model for those with the requisite skills, knowledge and capabilities.

4. CONCLUSIONS

A mortality-based approach is a valuable alternative to traditional property/casualty methods for estimating the future liability for mature claims with stable future annual payments, such as lifetime workers' compensation claims. Actuaries can use such an approach to estimate liabilities directly or to enhance traditional reserving for mature, stable, lifetime claims by corroborating tail factors used in loss development methods. Either way, consideration of a mortality calculation can enhance reserve projections, which is particularly important in the context of negotiating claim settlements, commutations and loss portfolio transfers, reserving for run-off books of business, and reinsurance reporting, as well as the collection or allocation of funds for insolvent companies, state guaranty funds and the run-off of state second injury funds. Since the mortality-based approach requires significant communication with claims personnel, including a detailed review of claim-specific information, actuaries, claims adjusters and management alike can develop a better understanding of the case reserving process and the associated liabilities. Finally, the important assumptions underlying a mortality-based model can lead to better identification of the primary drivers of claim costs over a claimant's lifetime and, therefore, potential avenues for future cost saving opportunities.

With unprecedented changes affecting workers' compensation, particularly with regard to medical and mortality trends, the estimation of workers' compensation liabilities is increasingly difficult. The use of a mortality-based approach will provide valuable insights into the variability of the liabilities through sensitivity testing of the key assumptions and provide information that may be used to better manage costs.

Appendix A

The attached appendices include complete cash flow calculations underlying the charts used throughout this paper.

5. REFERENCES

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Abbreviations and notations

ALAE, allocated loss adjustment expense
aka, also known as
CDC, Centers for Disease Control
COLA, cost of living adjustment
CPI, consumer price index
GAAP, Generally Accepted Accounting Principles
GAM, group annuity mortality
IBNR, incurred but not reported
IFRS, International Financial Reporting Standards
MSA, Medicare set-aside
NCCI, National Council on Compensation Insurance
RP, retirement plan
SCIF, State Compensation Insurance Fund
SIR, self-insured retention
SOA, Society of Actuaries
TPA, third-party administrator
ULAE, unallocated loss adjustment expense
UP, uninsured pensioner
U.S. United States

Biographies of the Authors

Brian A. Jones is a Principal with PricewaterhouseCoopers LLP. He is a Fellow of the CAS and a Member of the American Academy of Actuaries. He can be reached at brian.a.jones@us.pwc.com.

Craig J. Scukas is a Director with PricewaterhouseCoopers LLP. He is a Fellow of the CAS and a Member of the American Academy of Actuaries. He can be reached at craig.j.scukas@us.pwc.com.

Kathryn S. Frerman is a Senior Associate with PricewaterhouseCoopers LLP. She can be reached at kathryn.frerman@us.pwc.com.

Melissa S. Holt is a Manager with PricewaterhouseCoopers LLP. She is a Fellow of the CAS and a Member of the American Academy of Actuaries. She can be reached at melissa.s.holt@us.pwc.com.

Vicki A. Fendley is a Director with PricewaterhouseCoopers LLP. She is a Fellow of the CAS, an Associate of the SOA and a Member of the American Academy of Actuaries. She can be reached at vicki.a.fendley@us.pwc.com.

Male - Age 50 — Appendix A

Estimated Annual Benefit Payments = $10,000

Assumed Benefit Trend = 4% per year

Deductible = $250,000; 1st Excess = $250,000 xs $250,000; 2nd Excess = xs $500,000

								Deductible = $250,000				1st Excess = $250,000 xs $250,000				2nd Excess = xs $500,000			
Age	Year	Probability of Survival (1)	4.0% Trend Factor (2)	Trended Annual Payment (3)	Cumulative Payments Sum of (3) (4)	Expected Annual Payment (1)x(3) (5)	Expected Cumulative Payments Sum of (5) (6)	Cumulative Payments (7)	Annual Payment (8)	Expected Annual Payment (1)x(8) (9)	Expected Cumulative Payments Sum of (9) (10)	Cumulative Payments (11)	Annual Payment (12)	Expected Annual Payment (1)x(12) (13)	Expected Cumulative Payments Sum of (13) (14)	Cumulative Payments (15)	Annual Payment (16)	Expected Annual Payment (1)x(16) (17)	Expected Cumulative Payments Sum of (17) (18)
50	2013	99.8%	1.000	10,000	10,000	9,979	9,979	10,000	10,000	9,979	9,979	-	-	-	-	-	-	-	-
51	2014	99.5%	1.040	10,400	20,400	10,352	20,331	20,400	10,400	10,352	20,331	-	-	-	-	-	-	-	-
52	2015	99.3%	1.082	10,816	31,216	10,738	31,069	31,216	10,816	10,738	31,069	-	-	-	-	-	-	-	-
53	2016	99.0%	1.125	11,249	42,465	11,135	42,203	42,465	11,249	11,135	42,203	-	-	-	-	-	-	-	-
54	2017	98.7%	1.170	11,699	54,163	11,543	53,746	54,163	11,699	11,543	53,746	-	-	-	-	-	-	-	-
55	2018	98.3%	1.217	12,167	66,330	11,961	65,708	66,330	12,167	11,961	65,708	-	-	-	-	-	-	-	-
56	2019	97.9%	1.265	12,653	78,983	12,387	78,095	78,983	12,653	12,387	78,095	-	-	-	-	-	-	-	-
57	2020	97.4%	1.316	13,159	92,142	12,823	90,918	92,142	13,159	12,823	90,918	-	-	-	-	-	-	-	-
58	2021	96.9%	1.369	13,686	105,828	13,265	104,183	105,828	13,686	13,265	104,183	-	-	-	-	-	-	-	-
59	2022	96.4%	1.423	14,233	120,061	13,714	117,896	120,061	14,233	13,714	117,896	-	-	-	-	-	-	-	-
60	2023	95.7%	1.480	14,802	134,864	14,166	132,062	134,864	14,802	14,166	132,062	-	-	-	-	-	-	-	-
61	2024	95.0%	1.539	15,395	150,258	14,620	146,682	150,258	15,395	14,620	146,682	-	-	-	-	-	-	-	-
62	2025	94.1%	1.601	16,010	166,268	15,071	161,753	166,268	16,010	15,071	161,753	-	-	-	-	-	-	-	-
63	2026	93.2%	1.665	16,651	182,919	15,517	177,270	182,919	16,651	15,517	177,270	-	-	-	-	-	-	-	-
64	2027	92.1%	1.732	17,317	200,236	15,956	193,226	200,236	17,317	15,956	193,226	-	-	-	-	-	-	-	-
65	2028	91.0%	1.801	18,009	218,245	16,383	209,609	218,245	18,009	16,383	209,609	-	-	-	-	-	-	-	-
66	2029	89.7%	1.873	18,730	236,975	16,792	226,401	236,975	18,730	16,792	226,401	-	-	-	-	-	-	-	-
67	2030	88.2%	1.948	19,479	256,454	17,183	243,585	250,000	13,025	11,490	237,891	6,454	6,454	5,694	5,694	-	-	-	-
68	2031	86.6%	2.026	20,258	276,712	17,551	261,136	250,000	-	-	237,891	26,712	20,258	17,551	23,245	-	-	-	-
69	2032	84.9%	2.107	21,068	297,781	17,892	279,028	250,000	-	-	237,891	47,781	21,068	17,892	41,137	-	-	-	-
70	2033	83.0%	2.191	21,911	319,692	18,194	297,222	250,000	-	-	237,891	69,692	21,911	18,194	59,331	-	-	-	-
71	2034	81.0%	2.279	22,788	342,480	18,457	315,680	250,000	-	-	237,891	92,480	22,788	18,457	77,789	-	-	-	-
72	2035	78.8%	2.370	23,699	366,179	18,672	334,352	250,000	-	-	237,891	116,179	23,699	18,672	96,461	-	-	-	-
73	2036	76.4%	2.465	24,647	390,826	18,829	353,180	250,000	-	-	237,891	140,826	24,647	18,829	115,289	-	-	-	-
74	2037	73.8%	2.563	25,633	416,459	18,918	372,098	250,000	-	-	237,891	166,459	25,633	18,918	134,207	-	-	-	-
75	2038	71.0%	2.666	26,658	443,117	18,930	391,029	250,000	-	-	237,891	193,117	26,658	18,930	153,138	-	-	-	-
76	2039	68.0%	2.772	27,725	470,842	18,857	409,886	250,000	-	-	237,891	220,842	27,725	18,857	171,995	-	-	-	-
77	2040	64.8%	2.883	28,834	499,676	18,692	428,578	250,000	-	-	237,891	249,676	28,834	18,692	190,687	-	-	-	-
78	2041	61.4%	2.999	29,987	529,663	18,426	447,004	250,000	-	-	237,891	250,000	324	199	190,886	29,663	29,663	18,227	18,227
79	2042	57.9%	3.119	31,187	560,849	18,053	465,057	250,000	-	-	237,891	250,000	-	-	190,886	60,849	31,187	18,053	36,280
80	2043	54.2%	3.243	32,434	593,283	17,567	482,624	250,000	-	-	237,891	250,000	-	-	190,886	93,283	32,434	17,567	53,847
81	2044	50.3%	3.373	33,731	627,015	16,953	499,577	250,000	-	-	237,891	250,000	-	-	190,886	127,015	33,731	16,953	70,800
82	2045	46.2%	3.508	35,081	662,095	16,212	515,790	250,000	-	-	237,891	250,000	-	-	190,886	162,095	35,081	16,212	87,013
83	2046	42.1%	3.648	36,484	698,579	15,348	531,138	250,000	-	-	237,891	250,000	-	-	190,886	198,579	36,484	15,348	102,361
84	2047	37.9%	3.794	37,943	736,522	14,369	545,507	250,000	-	-	237,891	250,000	-	-	190,886	236,522	37,943	14,369	116,730
85	2048	33.7%	3.946	39,461	775,983	13,289	558,796	250,000	-	-	237,891	250,000	-	-	190,886	275,983	39,461	13,289	130,019
86	2049	29.5%	4.104	41,039	817,022	12,123	570,920	250,000	-	-	237,891	250,000	-	-	190,886	317,022	41,039	12,123	142,143
87	2050	25.5%	4.268	42,681	859,703	10,893	581,813	250,000	-	-	237,891	250,000	-	-	190,886	359,703	42,681	10,893	153,036
88	2051	21.7%	4.439	44,388	904,091	9,623	591,436	250,000	-	-	237,891	250,000	-	-	190,886	404,091	44,388	9,623	162,659
89	2052	18.1%	4.616	46,164	950,255	8,342	599,778	250,000	-	-	237,891	250,000	-	-	190,886	450,255	46,164	8,342	171,001
90	2053	14.8%	4.801	48,010	998,265	7,085	606,863	250,000	-	-	237,891	250,000	-	-	190,886	498,265	48,010	7,085	178,085
91	2054	11.8%	4.993	49,931	1,048,196	5,896	612,759	250,000	-	-	237,891	250,000	-	-	190,886	548,196	49,931	5,896	183,982
92	2055	9.3%	5.193	51,928	1,100,124	4,804	617,563	250,000	-	-	237,891	250,000	-	-	190,886	600,124	51,928	4,804	188,785
93	2056	7.1%	5.400	54,005	1,154,129	3,829	621,391	250,000	-	-	237,891	250,000	-	-	190,886	654,129	54,005	3,829	192,614
94	2057	5.3%	5.617	56,165	1,210,294	2,984	624,375	250,000	-	-	237,891	250,000	-	-	190,886	710,294	56,165	2,984	195,597
95	2058	3.9%	5.841	58,412	1,268,706	2,273	626,648	250,000	-	-	237,891	250,000	-	-	190,886	768,706	58,412	2,273	197,870
96	2059	2.8%	6.075	60,748	1,329,454	1,693	628,340	250,000	-	-	237,891	250,000	-	-	190,886	829,454	60,748	1,693	199,563

Male - Age 50
Estimated Annual Benefit Payments = $10,000
Assumed Benefit Trend = 4% per year
Deductible = $250,000; 1st Excess = $250,000 xs $250,000; 2nd Excess = xs $500,000

Appendix A

								Deductible = $250,000				1st Excess = $250,000 xs $250,000				2nd Excess = xs $500,000			
Age	Year	Probability of Survival	4.0% Trend Factor	Trended Annual Payment	Cumulative Payments Sum of (3)	Expected Annual Payment (1)x(3)	Expected Cumulative Payments Sum of (5)	Cumulative Payments	Annual Payment	Expected Annual Payment (1)x(8)	Expected Cumulative Payments Sum of (9)	Cumulative Payments	Annual Payment	Expected Annual Payment (1)x(12)	Expected Cumulative Payments Sum of (13)	Cumulative Payments	Annual Payment	Expected Annual Payment (1)x(16)	Expected Cumulative Payments Sum of (17)
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
97	2060	2.0%	6.318	63,178	1,392,632	1,233	629,573	250,000	-	-	237,891	250,000	-	-	190,886	892,632	63,178	1,233	200,796
98	2061	1.3%	6.571	65,705	1,458,337	878	630,450	250,000	-	-	237,891	250,000	-	-	190,886	958,337	65,705	878	201,673
99	2062	0.9%	6.833	68,333	1,526,671	611	631,062	250,000	-	-	237,891	250,000	-	-	190,886	1,026,671	68,333	611	202,285
100	2063	0.6%	7.107	71,067	1,597,738	417	631,479	250,000	-	-	237,891	250,000	-	-	190,886	1,097,738	71,067	417	202,701
101	2064	0.4%	7.391	73,910	1,671,647	278	631,757	250,000	-	-	237,891	250,000	-	-	190,886	1,171,647	73,910	278	202,979
102	2065	0.2%	7.687	76,866	1,748,513	182	631,938	250,000	-	-	237,891	250,000	-	-	190,886	1,248,513	76,866	182	203,161
103	2066	0.1%	7.994	79,941	1,828,454	117	632,055	250,000	-	-	237,891	250,000	-	-	190,886	1,328,454	79,941	117	203,278
104	2067	0.1%	8.314	83,138	1,911,592	74	632,128	250,000	-	-	237,891	250,000	-	-	190,886	1,411,592	83,138	74	203,351
105	2068	0.1%	8.646	86,464	1,998,055	46	632,175	250,000	-	-	237,891	250,000	-	-	190,886	1,498,055	86,464	46	203,397
106	2069	0.0%	8.992	89,922	2,087,978	29	632,203	250,000	-	-	237,891	250,000	-	-	190,886	1,587,978	89,922	29	203,426
107	2070	0.0%	9.352	93,519	2,181,497	18	632,221	250,000	-	-	237,891	250,000	-	-	190,886	1,681,497	93,519	18	203,444
108	2071	0.0%	9.726	97,260	2,278,757	11	632,233	250,000	-	-	237,891	250,000	-	-	190,886	1,778,757	97,260	11	203,455
109	2072	0.0%	10.115	101,150	2,379,907	7	632,240	250,000	-	-	237,891	250,000	-	-	190,886	1,879,907	101,150	7	203,462
110	2073	0.0%	10.520	105,196	2,485,103	4	632,244	250,000	-	-	237,891	250,000	-	-	190,886	1,985,103	105,196	4	203,467
111	2074	0.0%	10.940	109,404	2,594,507	3	632,247	250,000	-	-	237,891	250,000	-	-	190,886	2,094,507	109,404	3	203,470
112	2075	0.0%	11.378	113,780	2,708,288	2	632,248	250,000	-	-	237,891	250,000	-	-	190,886	2,208,288	113,780	2	203,471
113	2076	0.0%	11.833	118,332	2,826,619	1	632,249	250,000	-	-	237,891	250,000	-	-	190,886	2,326,619	118,332	1	203,472
114	2077	0.0%	12.306	123,065	2,949,684	1	632,250	250,000	-	-	237,891	250,000	-	-	190,886	2,449,684	123,065	1	203,473
115	2078	0.0%	12.799	127,987	3,077,671	0	632,251	250,000	-	-	237,891	250,000	-	-	190,886	2,577,671	127,987	0	203,473
116	2079	0.0%	13.311	133,107	3,210,778	0	632,251	250,000	-	-	237,891	250,000	-	-	190,886	2,710,778	133,107	0	203,474
117	2080	0.0%	13.843	138,431	3,349,209	0	632,251	250,000	-	-	237,891	250,000	-	-	190,886	2,849,209	138,431	0	203,474
118	2081	0.0%	14.397	143,968	3,493,177	0	632,251	250,000	-	-	237,891	250,000	-	-	190,886	2,993,177	143,968	0	203,474
119	2082	0.0%	14.973	149,727	3,642,905	0	632,251	250,000	-	-	237,891	250,000	-	-	190,886	3,142,905	149,727	0	203,474
120	2083	0.0%	15.572	155,716	3,798,621	-	632,251	250,000	-	-	237,891	250,000	-	-	190,886	3,298,621	155,716	-	203,474
Total				3,798,621		632,251			250,000	237,891			250,000	190,886			3,298,621	203,474	
Subtotals by Age Band																			
2013 to 2022		98.2%		120,061		117,896			120,061	117,896			-	-			-	-	
2023 to 2032		90.7%		177,720		161,131			129,939	119,995			47,781	41,137			-	-	
2033 to 2042		70.7%		263,069		186,029			-	-			202,219	149,749			60,849	36,280	
2043 to 2052		34.6%		389,406		134,721			-	-			-	-			389,406	134,721	
2053 to 2062		5.4%		576,416		31,284			-	-			-	-			576,416	31,284	
2063+		0.1%		2,271,950		1,189			-	-			-	-			2,271,950	1,189	

(1) Annuity factor based on RP-2000 Combined Healthy Employees & Annuitants. For example, at age 60, the probability of survival is represented by the single life annuity factor with a starting age of 60 and ending age of 61 for a male currently aged 50. For subtotals by age band, column (1) = column (5) / column (3).
(3) Trended annual payment = $10,000 x column (2).
(7) Cumulative payments = column (4) subject to $250,000 deductible.
(8) Annual payments = column (7) minus prior column (7).
(11) Cumulative payments = column (4) within $250,000 excess $250,000 layer.
(12) Annual payments = column (11) minus prior column (11).
(15) Cumulative payments = column (4) excess of $500,000.
(16) Annual payments = column (15) minus prior column (15).

Estimating Unpaid Claim Liabilities for Mortgage Insurance

David Kaye, FCAS, MAAA

Abstract
This paper will provide practical guidance for the actuary estimating loss reserves for mortgage insurance exposures. It includes a brief background on the mortgage insurance product, the accounting considerations for mortgage insurance, and introduces a practical deterministic approach for estimating unpaid claim liabilities for mortgage insurance.

Keywords. Mortgage insurance; reserving.

1. INTRODUCTION

At the depths of the housing market downturn and the recent "Great Recession," mortgage insurance (MI) companies suffered elevated incurred losses and a sustained period of unprofitability. The nearly simultaneous deterioration of several macroeconomic factors – declining home values, increasing unemployment levels, the tightening availability of credit, and a significant backlog of properties awaiting foreclosure – resulted in diminished usefulness of traditional actuarial chain-ladder techniques for estimating unpaid claim liabilities for mortgage insurers.

The development of alternatives to the traditional chain-ladder framework is critical in estimating unpaid claim liabilities for MI. While some actuarial literature address the topic, the methods presented often utilize stochastic (e.g., regression) modeling; these stochastic models can be difficult to understand without having a basic framework to understand the MI loss process.

1.1 Objective

The objective of this paper is to provide the practicing actuary with:

- Sufficient background on the MI product to understand the special requirements for estimating unpaid claim liabilities for MI;
- An overview of the accounting considerations for mortgage insurers to understand the motivation for specialized approaches to estimating MI unpaid claim liabilities; and
- A practical deterministic framework for estimating MI unpaid claim liabilities.

1.2 Outline

The remainder of the paper proceeds as follows: Section 2 provides a primer on mortgage insurance exposure and a brief introduction to the MI accounting framework; Section 3 provides a deterministic framework for estimating MI unpaid claim liabilities.

2. BACKGROUND

2.1 Development of the MI Industry

The mortgage insurance industry developed as a mechanism to spread mortgage default risk from a mortgage lending institution to a separate, unrelated party (the mortgage insurer) as part of a broader initiative to promote home ownership and provide stability to the real estate and banking industry. The industry's roots can be traced to the Great Depression and the National Housing Act of 1934, which aimed to stabilize the banking system through the creation of the Federal Housing Administration (the FHA). The FHA "provides mortgage insurance on loans made by FHA-approved lenders throughout the United States and its territories." [1]

In the 1950s, the first privately owned enterprise to compete directly with the FHA was formed when Wisconsin passed legislation that paved the way for the formation of Mortgage Guaranty Insurance Corporation. Private mortgage insurance began significant growth in the 1970s with the passage of federal legislation allowing the Government Sponsored Entities (GSEs) Fannie Mae and Freddie Mac to buy and securitize loans where the loan value divided by the home value (loan to value, or LTV) exceeded 80% provided that those loans were covered by mortgage insurance. The interplay of mortgage lenders, GSEs and private mortgage insurers that developed during the 1970s has continued into the current day, with mortgage insurers playing a critical role in collateralizing loans to the point that they comply with GSE purchasing and securitization guidelines.

2.2 MI Product Background

Several key features of MI policies include:

- MI policies are issued at the time that the mortgage is issued and can either be paid by the borrower (most common) or lender (less common).
- Premiums are paid on either a monthly (most common) or single up-front (less common) basis. The premium associated with monthly pay policies is typically paid along with the monthly mortgage payment.
- The collected monthly premiums are generally recognized as income in the period in which they are collected (that is, the monthly premiums are written and earned at the same time) meaning that there is typically a very small (or no) unearned premium reserve

associated with monthly paid MI policies. There is an unearned premium reserve associated with upfront premium policies, which is amortized over the life of the MI contract as losses associated with the contract are expected to emerge.

- MI coverage is typically expressed as a percentage of a loan's unpaid principal balance ("UPB"). These coverage percentages vary from loan to loan, but a typical average coverage percentage is around 25%[1].
- MI policies provide lenders coverage for a portion of the UPB stipulated in the contract. In addition, the MI policy generally reimburses the coverage beneficiary for loss interest payments and certain foreclosure-related expenses.
- Unlike typical Property and Casualty insurance policies – generally in force for one year and have defined termination dates – MI policies often generate premiums and losses for a number of years and there is uncertainty with regard to how long the policies will remain in force. The MI policy holder may exit the insured population for a number of reasons, including defaulting on the mortgage (i.e., becoming a claim), refinancing the loan, or paying down the principal on the loan to the point that the loan no longer requires MI[2].
- MI losses are highly correlated with macroeconomic factors such as home price inflation and unemployment. As was highly evident in 2007-2011, MI company results were adversely affected by a steep drop in home prices followed by rising levels of unemployment. Not surprisingly, the states with the sharpest decreases in home prices – CA, FL and NV – were significant drivers of adverse loss experience for the MI industry.
- As explained further below, MI loss reserves are recorded at the time when a borrower is "delinquent" in paying their mortgage; this results in an unusual accounting construct where premium earning and loss accrual are not matched. In other words, premium revenue generated for MI policies is recognized (i.e., earned) prior to the associated losses

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[1] The coverage percentage is a function of LTV and often of GSE purchasing / securitization guidelines. If a borrower puts a 10% down payment on a home, leaving a 90% LTV, and the GSE requires a 68% LTV – a typical Freddie Mac requirement – to purchase the loan, then the MI provides coverage for 24.4% = 1 - .68 / .90 ≈ 25%.

[2] Typically, private MI policies are cancelled when borrowers pay enough principal such that the LTV ratio drops below 78%.

being recognized[3].

- For a cohort of MI policies issued during a year, the premium revenue generated by the policies is the greatest during the first year and then decreases over the next ten years as policies are cancelled. The delinquencies that give rise to the recording of MI loss reserves tend to rise through fourth or fifth year after loan origination; after peaking, losses tend to decrease as policies are cancelled.

2.3 Accounting for Mortgage Insurance Losses

As described above, the accounting framework for MI results in a departure of one of the common objectives of accounting: revenue and expense matching. For typical, single-year P&C insurance products, both revenue (premium) and expense (claim costs) are recognized uniformly through the year the policy is effective[4]. For monthly-pay MI policies, premium is generally earned on a monthly basis, while losses on MI policies are not recognized until the borrower stops paying his monthly mortgage payment and the lender or loan servicer notifies the MI company that the borrower is delinquent[5].

Since MI loss reserves are not established until the MI company is made aware that a borrower is delinquent in paying their loan, MI Companies typically do not establish a provision for "pure" IBNR, i.e., IBNR for claims that have occurred but that haven't yet been reported to the Company. IBNR is typically only established to the extent that on a monthly basis, there are lags in information reported from the lender to the MI company[6].

Because information reported from the lender to the MI company is usually provided on a timely basis, the IBNR provision is a small portion of the overall loss reserve balance. The majority of the loss reserve is made up of unpaid claim estimates for loans where the borrower has been identified as being delinquent in his loan payment. This paper will focus on preparing unpaid claim estimates

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[3] Contrast this with a normal P&C insurance contract, such as an auto insurance policy. For an auto insurance policy, premium revenue is recognized uniformly over the one year contract period and losses generally occur and are recognized evenly over the life of the contract. For auto insurance policies, there is a matching of premium (revenue) and loss (expense).

[4] This description is generally accurate, although there are exceptions such as property catastrophe cover where premium and loss might not be recognized uniformly.

[5] Although not included in the scope of this paper, MI Companies must also examine whether a premium deficiency exists. A premium deficiency reserve should be established by a MI company when the sum of future incurred losses and policy expenses exceed future premium revenue.

[6] Recently, some MI Companies have added an additional component to the IBNR provision accounting for the potential reinstatement of claim denials or policy rescissions, although those items are beyond the scope of this paper.

for known delinquent loans.

2.4 Terminology and Organization of Data

Before providing a framework for estimating MI unpaid claim liabilities, it is important to introduce several additional terms as well as to lay out the key characteristics used to organize the data.

2.4.1 Terminology

Although the terminology below is not necessarily universal, it is used throughout the remainder of this paper and practitioners familiar with MI will understand it.

- Outstanding delinquency: A loan reported to the MI company when the borrower has fallen two mortgage payments behind (note, there is some variance about when a loan is identified as delinquent in the MI industry, here we are assuming the MI company has set the definition as be borrower being behind two or more payments).
- Delinquency report quarter: The quarter in which a MI is first notified that a borrower is two or more payments behind. (As noted in 2.4.2 below, a loan can become delinquent multiple times over its life; therefore, a single loan can appear in several delinquency report quarters.)
- Cured delinquency: A previously delinquent loan where the borrower has made previous missed payments and is no longer considered delinquent.
- Submitted claim: A delinquent loan where the borrower has not made mortgage payments, the lending institution has foreclosed on the subject property, and a claim has been submitted to the MI company.
- Risk in force ("RIF"): This is the exposure to loss faced by MI Companies. The RIF is calculated by multiplying the MI's coverage percentage by the loan's UPB. As noted above, in addition to the coverage percentage multiplied by the UPB, the MI company may also be required to pay lost interest and certain foreclosure expenses; for this reason, the ratio of claim payments to RIF may be greater than 100%.

2.4.2 Data organization

As in any actuarial analysis, data organization is a critical component of the analysis and must be carefully considered prior to the actuary preparing the reserve analysis. For purposes of the method described in Section 3, the data is organized by delinquency report quarter, with quarterly evaluations of the data (actuaries will be familiar with the "triangular" arrangement of the data used in the analysis)[7].

The actuary must consider how best to segment the data for use in the methods described in Section 3. Although the factors that drive MI claim behavior differ from those that P&C actuaries typically encounter when determining the optimal data segmentation for use in preparing unpaid claim estimates, the primary goal of the actuary remains the same: select the data segmentation that gives the optimal balance of homogeneity and credibility. The actuary may consider the following items (among others) when selecting appropriate data segmentation[8]:

- Foreclosure laws: Each state has its own set of laws governing the foreclosure process. These state-specific laws can generate significant differences in the length of time between delinquency notification and the foreclosure and eventual MI claims.
- Unemployment: The level of unemployment can have a significant impact on the likelihood that delinquent borrowers transition to foreclosure and ultimately become MI claims. A severe downturn in employment in a single area may have a dramatic impact on claims experience and should be considered by the actuary in developing MI unpaid claim estimates.
- Creditworthiness of borrowers: The creditworthiness of borrowers can be a significant predictor in determining borrower behavior. FICO score[9] or distinguishing between Prime and Subprime loans in developing estimates can result in better data stratification.
- Home price appreciation or depreciation: As evidenced by the housing market bubble and subsequent home price deflation of 2005-2008, borrower behavior can be significantly

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[7] As described previously, some loans may become delinquent and cure numerous times before rolling to a claim. In the MI framework described in this paper, each new delinquency notification is treated as a separate event. Therefore, a single loan could appear in our reported delinquent loan population in several report quarters.

[8] For a more thorough discussion of data segmentation, please see reference [2] at the end of this paper.

[9] FICO is a common credit scoring mechanism developed originally by the Fair Isaac Corporation. The FICO score is a numerical representation of the credit worthiness of a borrower based on the evaluation of five key pieces of information [3].

impacted by home price appreciation or depreciation. During the height of the housing market bubble around 2005, MI claim experience was very favorable and relatively few reported delinquencies resulted in claims. After the bubble began deflating in 2006 and 2007, cure rates dropped significantly as "underwater" borrowers (i.e., those who owed more principal on their loan than the house's market value) were often unable to sell their home if they were unable to pay their mortgage, and in some cases made a conscious decision to walk away from their mortgage and home. The significant downturn in home prices and decreases in cure rates resulted in elevated MI claims.

During the recent housing market downturn, MI Companies observed elevated MI claim submissions from states that had significant increases in home prices prior to the housing market downturn followed by significantly elevated unemployment levels resulting from the subsequent recession. For this reason, during the last market downturn, it was beneficial to separately analyze California, Florida, and Nevada; these states were particularly hard hit by the combination of a significant housing market collapse and elevated unemployment and displayed similar claim characteristics.

3. DETERMINISTIC FRAMEWORK FOR MI

As described in the previous section, MI claim payments arise from loans that the loan servicer reports as delinquent. Because the MI accounting framework described in this paper has very little "pure" IBNR, we do not need to estimate unreported claims, rather we need to project the probability that a delinquent loan will become a claim (or conversely, that the delinquent loan will cure). The MI claims process and accounting framework gives rise to the fundamental relationship we will utilize to develop our deterministic framework:

Estimated Unpaid Claims = N x F x S, where

- N is the number of reported delinquent loans;
- F is the probability that a delinquent loan will ultimately result in a foreclosure, triggering a claim (also referred to as the "claim rate"); and
- S is the (average) amount or severity of each claim.

In the formula above, N is an amount that is known with certainty. Methods for estimating F and S are described in the sections below and the methodology is outlined in the Appendix file, which is available on the CAS website.

3.1 Estimating the Claim Rate

To estimate the claim rate, we utilize triangular claim development methods that will be familiar to actuaries. The key to utilizing claim development methods is to recognize several important aspects of the MI claims process:

- As described above, the data utilized will be organized by the quarter in which the delinquent loan first becomes an outstanding delinquency; organizing data in this way means that we do not need to include a provision for IBNR claims in our analysis beyond the potential for delinquencies in transit. The number of loans reported in a quarter will be a certain, fixed number.
- The number of remaining outstanding delinquent loans in subsequent evaluation quarters will decrease to zero as loans resolve (cure or become a claim) over time. Further, at some future date, all reported delinquent loans must either cure or become a claim.

3.1.1 Claim Rate Methodology

Developing the claim rate using the methodology described in this paper is a three step process:

1. First, we evaluate the decline in outstanding delinquencies over time as these resolve by either curing or becoming claims by reviewing an outstanding delinquency decay pattern ("decay pattern"). The decay pattern is developed by calculating ratios of delinquent loans at each evaluation period, i+1, divided by the delinquent loans at the preceding evaluation period, i. The triangle of outstanding delinquent loans compiled based on the delinquent loan data is completed by selecting a decay factor for each evaluation period and then applying the selected decay factor at each period to the outstanding delinquent loans observed (or projected) at the end of the prior evaluation period. Performing these calculations allows the actuary to estimate the number of delinquent loans that remain open at each future period and also to estimate the number of delinquent loans that resolve during each future period.

   As an example, on Exhibit 3-4 of the Appendix file, there are 1,037 loans reported delinquent during the third quarter of 2012 that remain outstanding at the end of the third quarter of 2012 (note, some loans that are reported delinquent during the third quarter will cure or become claims by then end of the quarter). One quarter later, 249 of these loans have either cured or become claims leaving 788 delinquent loans at the end of the fourth quarter of 2012. We calculate the delinquent loan decay rate as 788 / 1,037 = 76%. We can use this calculated decay ratio to project that 823 loans will remain outstanding as of March 31, 2013 from the cohort of 1,083 loans delinquent loans reported during the fourth quarter of 2012 (823 = 1,083 * 76%). Performing similar calculations across all evaluation ages and delinquency periods allows us to project future outstanding delinquent loans and develop the item labeled "RESULT 3-4".

It is important to note that, in the provided exhibits, the selected decay rate is based on the latest period observation to make the discussion in this text easier to follow. In practice, it may be more appropriate to select longer (perhaps 4) period averages to smooth out seasonality that is often present in mortgage insurance data.

2. The second step of the procedure involves projecting period to period claims and cured delinquencies given the projected number of resolved claims calculated in item (1). In order to prepare these estimates, we review the historical delinquent loan data set to determine the number of the delinquent loans that resulted in a claim or cured delinquency each quarter from the total population of delinquent loans that resolved during the quarter.

   Continuing the example above, for the delinquent loans reported during the fourth quarter of 2012, we have projected that 260 loans will resolve (260 = 1,083 – 823). The next step of the procedure requires that we estimate the portion of the 260 resolved loans that cure and the portion that become claims.

   Again, we can utilize the data from prior delinquency report periods to guide our selection of the conditional claim and cure probabilities (the condition being that the delinquent loans have resolved during the evaluation period). For delinquent loans reported during the third quarter of 2012, we note that 249 loans resolved between September 30, 2012 and December 31, 2012; the 249 resolution consisted of 225 cured delinquencies (= 483 – 258) and 24 claims (= 25 – 1). We can utilize this data to calculate the number of projected cured delinquencies and the number of claims for the cohort of delinquent loans reported during the fourth quarter of 2012 as follows:

   - Projected cured delinquencies = 260 * 225 / 249 = 235 projected incremental cures;

- Projected claims = 260 * 24 / 249 = 25 projected incremental claims.

Adding the number of previous cures and claims yields the cumulative number of cures and claims at subsequent evaluation dates. Performing similar calculations across the entire triangle allows us to project the ultimate number of cured delinquent loans and claims. Result 3-3 and Result 3-2 of the corresponding exhibits of the Appendix outline these calculations.

3. Although the example in the Appendix does not require it, the actuary should perform a check to ensure that the sum of the projected cured delinquent loans plus the projected claims is equal to the number of initial reported delinquent loans (again, all loans must either cure or become a claim). Exhibit 3 – 1 shows an example of formulas that can be used to rebalance the projected cures and claims in order to match the initial reported delinquent loans.

Exhibit 3 – 1 presents the results of our claim count analysis. Over the projection period, we estimate that approximately 35% of all reported delinquencies will result in claims and 65% of delinquent loans will cure.

It is important to point out that the data set in this example is simplified in several ways: the resolution process occurs relatively quickly (over a period of 8 quarters); there is a relatively smooth relationship over time – the likelihood that a loan will become a claim increases over the data observation period; and the decay and conditional claim rates are relatively stable over time. With real data, the relationships may not be as obvious and the projections would not be as straightforward.

3.1.2 Benefits and limitations of the claim rate methodology

The claim rate methodology should have an appeal for actuaries since the triangular arrangement of the data is familiar to all actuaries and the mechanics of the model are intuitive and straightforward. The methodology is also appealing because in many ways it is easier to describe and demonstrate to a non-actuarial audience than methods that require an understanding of statistical concepts (e.g., regression). Such statistical based methods are often referred to as "black box" methods because the inputs and outputs of the model are easy to describe, but the actual model mechanics are difficult to describe and demonstrate; the model described above does not have this limitation.

The key limitations of the deterministic framework are:

- Using aggregate data does not allow the actuary to explicitly determine the factors that are most correlated with claim behavior. Using regression methods may allow the actuary to determine that unemployment is the most important factor in determining delinquent loan behavior and other factors (e.g., borrower credit score) are less important. Although the actuary can visually inspect the data to see which factors appear to be most critical, statistical data analytics and regression models can allow the actuary to more quickly determine the parameters most closely associated with claim behavior.
- The model does not allow for explicit sensitivity testing of the results to changes in key macroeconomic factors. For example, if the MI company is concerned about the effect of an increase in unemployment on the Company's estimates, the effect cannot be explicitly incorporated into the framework. Conversely, regression models can be developed that utilize unemployment as an explanatory variable, which allows the actuary to quickly develop alternative estimates assuming different future unemployment paths.

3.1.3 Additional observations regarding the delinquency count data

Although the data presented in the example is simulated, it does share many similarities with real mortgage insurance data. Several of the more critical caveats regarding this sample data set are outlined below:

- Delinquencies are much more likely to cure at early maturities than at late maturities. Conversely, the likelihood that a delinquent loan will become claims increases the longer the loan stays delinquent.

  Very few delinquent loans become claims at early maturities limiting the usefulness of claim data at early maturities. For this reason, it is important to monitor the cured delinquency data in addition to the claim data. A significant decrease in the number of cured delinquencies at early maturities can be a signal that results are deteriorating. For example, at the beginning of the housing market collapse of the last decade, not only did the number of delinquent loans increase dramatically, but the number of cured delinquencies at early maturities fell substantially, which was an early indicator that there were significant issues emerging in the housing sector. Monitoring the behavior of claims and cured delinquencies (particularly at early maturities) may give the actuary performing the analysis an early indication that results are deteriorating significantly.

- A limitation of organizing the data in the way this paper describes is that the data does not explicitly capture the number of payments borrowers may miss. Suppose Borrower A misses a mortgage payment one month but then begins paying his mortgage again the following month; Borrower B loses his job, misses a mortgage payment and, due to economic hardship, is unlikely to ever make a mortgage payment again. While the second borrower is certainly more likely to result in an MI claim than the first, both borrowers will appear in the same evaluation points of the outstanding delinquency triangle.

  Given this limitation, it is important that the actuary recognize that the way this paper organizes data may mask some important underlying dynamics – some borrowers fall behind and will never catch up, and some borrowers fall behind for a short time but will continue making loan payments. The actuary should consider examining the data in more detail in order to understand not just how long loans have been delinquent but also how many payments the borrowers have missed and in particular, whether any shifts in the historical data set have occurred.

- It is possible that the actuary examining real data would face a situation where the delinquent loan population was not entirely resolved at the end of the projection period; when facing a lack of data regarding behavior of older loans, the actuary might consider aggregating the data for "late stage" delinquent loans, observing their claim behavior, and selecting a tail claim rate for all loans classified as late stage delinquencies.

- Sometimes after foreclosure and claim submission, the insurance policy giving rise to the claim may be reviewed by the MI company's claims adjusters to confirm that the original loan conformed to the Company's underwriting guidelines and that the proper documentation supporting the claim submission was provided. If it is determined that the original policy did not meet the MI company's underwriting standards, then the original MI policy might be rescinded (cancelled) with a return of the collected policy premium[10]. If a claim is submitted without the proper documentation supporting the submitted claims, then the claim may be denied. Policy rescissions and claim denials need to be considered in the evaluation process either through frequency or severity based on

[10] Generally, MI policies are not underwritten by the MI company at the time the policy is issued. Rather, MI Companies give lending institutions underwriting guidelines that they are required to follow. At the time of claim submission, MI Companies generally have the right to review the original loan documentation in order to ensure the loan conformed to the MI company underwriting guidelines.

historical pattern. Historically, rescissions and denials were not significant, although they have been significant through the housing market downturn.

3.2 Estimating Claim Severity

Estimating severity for MI claims is generally more straightforward and often more predictable than estimating claim behavior. The exposure to loss from individual claim submissions is simply a combination of

a) RIF;

b) a provision for lost interest payment and certain foreclosure expenses; and

c) "Subrogation" in cases where the foreclosed property is sold for a profit, which benefits the MI company.

Item (a) can be derived directly from the outstanding delinquent loan population since the MI company has information regarding the risk associated with delinquent loans. Items (b) and (c) can be estimated by examining the ratio of claim payments to the RIF on submitted claims. For example, if the actuary examines the loss data and determines that on average, $1 of RIF translates into $1.05 of paid loss, then the actuary can estimate the size of future claim payments by multiplying the average RIF on outstanding delinquencies by 1.05.

This method for estimating severity has a key advantage in its simplicity; however it can also have limitations to the extent that the underlying claim dynamics are shifting over time. For example, if the actuary chooses to organize the data used in the analysis by credit score, the data segmentation might mask the fact that a larger portion of recent claim emergence is arising from geographies with higher average cost. More importantly, if claims are more likely to arise from loans with higher than average RIF, then using average RIF on outstanding delinquent loans might understate future claim severity. The actuary should investigate whether the average paid claim has increased or decreased over the observation period and whether the geographic distribution of paid claims has shifted over time.

Exhibit 2 of the Appendix outlines an example of the severity method above. Exhibit 2-2 displays the ratio of observed paid losses to RIF on submitted claims arranged by calendar quarter. Based on the historical data in Exhibit 2-2, we can estimate that for every $1 of RIF, the MI company has paid $1.049 of claims. Exhibit 2-1 shows the calculation of the selected severity, which is developed by multiplying the average RIF on outstanding delinquencies by 1.049. Exhibit 2-2 also provides a calculation of the average severity on ultimate claims by weighting together the estimated claim severity on outstanding delinquencies with the observed average paid claims on previously submitted claims.

3.3 Unpaid Claim Estimate

The unpaid claim estimate is calculated by multiplying the outstanding claim estimates described in section 3.1 by the severity estimates described in section 3.2. The unpaid claim estimate prepared using our sample data set is shown in Exhibit 1 of the Appendix.

4. CONCLUSIONS

The actuary who prepares unpaid claims estimates for MI must understand the unique accounting for MI losses, given that there is typically very little provision for "pure" IBNR claims and that losses are recorded only when the MI company receives notification of a loan delinquency. Although the accounting for MI differs from traditional P&C insurance products, deterministic triangular methods commonly used to develop estimates for P&C products can help actuaries project delinquent loan behavior. After the actuary has a strong grasp of MI data, accounting model and claim behavior, more complex regression or generalized linear model procedure (for example, see reference [4] below) can be utilized to further refine and enhance MI unpaid claim estimates.

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Supplementary Material

The Appendix to this paper is available electronically at the CAS website at http://www.casact.org/pubs/forum/13fforum/. The dataset provided within the Appendix was simulated using constraints generally consistent with the author's knowledge of MI claim behavior.

5. REFERENCES

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[3] http://credit.about.com/od/df/g/ficoscore.htm

[4] Taylor, Greg and Peter Mulquiney, "Modeling Mortgage Insurance as a Multistate Process", Variance 1:1, 2007, pp. 81-102.

Abbreviations and notations

GSE, government sponsored entity
IBNR, incurred but not reported
LTV, loan to value ratio
MI, mortgage insurance
RIF, risk in force
UPB, unpaid principal balance

Biography of the Author

David Kaye is a Director at PwC in Philadelphia, PA. He has a B.S. in Mathematics and a B.S. in Statistics from the Pennsylvania State University. He is a Fellow of the CAS and a Member of the American Academy of Actuaries. David participates on the CAS examination committee and the Committee on Professionalism Education.

Estimated Ultimate Loss and Unpaid Claim Estimate

Exhibit 1

Delq Report Date Year-Qtr	(1) Submitted Claims (A)	(2) Actual Paid Losses (A)	(3) Reported Severity (2)/(1)	(4) Ultimate Claims (B)	(5) Outstanding Claims (4)-(1)	(6) Indicated Ultimate Severity (C)	(7) Severity on Outstanding Claims (9)/(5)	(8) Ultimate Loss (4)*(6)	(9) Indicated Unpaid Claim Estimate (8)-(2)
2011-1	476	20,390,362	42,837	476	0	42,837	0	20,390,362	0
2011-2	454	18,888,106	41,604	462	8	41,609	41,933	19,223,571	335,466
2011-3	389	16,533,738	42,503	412	23	42,493	42,314	17,500,913	967,175
2011-4	381	16,079,727	42,204	467	86	42,150	41,911	19,669,762	3,590,034
2012-1	200	8,445,125	42,226	440	240	43,232	44,072	19,005,409	10,560,284
2012-2	89	3,729,843	41,908	417	328	42,699	42,914	17,801,705	14,071,862
2012-3	25	1,059,885	42,395	424	399	44,140	44,249	18,711,208	17,651,323
2012-4	1	42,932	42,932	443	442	43,045	43,045	19,054,559	19,011,627
Total	2,015	85,169,718	42,268	3,540	1,525	42,761	43,413	151,357,488	66,187,770

Notes
(A) Data provided by the MI Company
(B) From Exhibit 3, Page 1
(C) From Exhibit 2, Page 1

Selected Ultimate Severity

Exhibit 2
Sheet 1

Delq Report Date Year-Qtr	(1) Open Delqs (A)	(2) RIF on Delqs (A)	(3) Average RIF on Open Delqs (2)/(1)*1000	(4) Paid to RIF Ratio (C)	(5) Projected Avg. Paid on Open Delqs (3)*(4)	(6) Outstanding Claims (B)	(7) Estimated Paid on Future Claims (000s) (5)*(6)/1000	(8) Actual Paid Loss (000s) (A)	(9) Indicated Ult. Loss (000s) (7)+(8)	(10) Ultimate Claims (B)	(11) Indicated Ultimate Severity (9)/(10)*1000
2011-1	0	0	0	104.9%	0	0	0	20,390,362	20,390,362	476	42,837
2011-2	8	319,864	39,983,000	104.9%	41,933,201	8	335,466	18,888,106	19,223,571	462	41,609
2011-3	25	1,008,650	40,346,000	104.9%	42,313,907	23	967,175	16,533,738	17,500,913	412	42,493
2011-4	103	4,116,086	39,962,000	104.9%	41,911,177	86	3,590,034	16,079,727	19,669,762	467	42,150
2012-1	328	13,783,216	42,022,000	104.9%	44,071,655	240	10,560,284	8,445,125	19,005,409	440	43,232
2012-2	536	21,932,048	40,918,000	104.9%	42,913,807	328	14,071,862	3,729,843	17,801,705	417	42,699
2012-3	788	33,246,508	42,191,000	104.9%	44,248,898	399	17,651,323	1,059,885	18,711,208	424	44,140
2012-4	1,083	44,449,569	41,043,000	104.9%	43,044,904	442	19,011,627	42,932	19,054,559	443	43,045
Total	2,871	118,855,941	41,398,795			1,525	66,187,770	85,169,718	151,357,488	3,540	42,761

Notes
(A) Data provided by the MI Company
(B) From Exhibit 3, Summary
(C) From Exhibit 2, Page 2

Selected Paid to RIF Ratio

Exhibit 2
Sheet 2

Delq Report Date Year-Qtr	(1) Paid Loss (000s) (A)	(2) RIF on Submitted Claims (000s) (A)	(3) Paid Loss per RIF on Submitted Claims (1)/(2)	(4) 4 Qtr. Weighted Average
2011-1	20,390,362	19,577,880	104.2%	
2011-2	18,888,106	17,942,534	105.3%	
2011-3	16,533,738	15,580,228	106.1%	
2011-4	16,079,727	15,372,588	104.6%	104.99%
2012-1	8,445,125	8,125,000	103.9%	105.13%
2012-2	3,729,843	3,556,974	104.9%	105.05%
2012-3	1,059,885	1,012,500	104.7%	104.44%
2012-4	42,932	40,997	104.7%	104.26%
Total	85,169,718	81,208,701	104.9%	
		Selected Future Paid to RIF Ratio	104.9%	

Notes
(A) Data provided by the MI Company

Claim Summary

Exhibit 3
Summary

Delq Report Date Year-Qtr	(1) Reported Delqs (A)	(2) Reported Cured Delqs (A)	(3) Submitted Claims (A)	(4) Outstanding Delqs (A)	(5) Ultimate Claims (B)	(6) Outstanding Claims (5)-(3)	(7) Outstanding Claims to O/s Delqs (6)/(4)	(8) Estimated Claims to Reptd Delqs (5)/(1)
2011-1	1,335	859	476	0	476	0		35.66%
2011-2	1,309	847	454	8	462	8	100.00%	35.29%
2011-3	1,222	808	389	25	412	23	91.43%	33.70%
2011-4	1,357	873	381	103	467	86	83.16%	34.39%
2012-1	1,213	685	200	328	440	240	73.05%	36.24%
2012-2	1,216	591	89	536	417	328	61.18%	34.29%
2012-3	1,296	483	25	788	424	399	50.62%	32.71%
2012-4	1,337	253	1	1,083	443	442	40.78%	33.11%
Total	10,285	5,399	2,015	2,871	3,540	1,525	53.10%	34.42%

Notes
(A) Data provided by the MI Company
(B) From Exhibit 3, Sheet 1

Projected Claims
Data organized by delinquency report date and evaluation quarter

Exhibit 3
Sheet 1

Delq Report Date Year-Qtr	(1) Indicated Cured Delqs (A)	(2) Indicated Claims (B)	(3) Indicated Reported Delqs (1)+(2)	(4) Initial Projected Claims to Reported (2)/(3)	(5) Actual Reported Delqs (C)	(6) Additional Delqs (5)-(3)	(7) Additional Cured Delqs [1-(4)]*(6)	(8) Additional Claims (4)*(6)	(9) Indicated Cured Delqs (1)+(7)	(10) Indicated Ultimate Claims (2)+(8)
2011-1	859	476	1,335	35.66%	1,335	0	0	0	859	476
2011-2	847	462	1,309	35.29%	1,309	0	0	0	847	462
2011-3	810	412	1,222	33.70%	1,222	0	0	0	810	412
2011-4	890	467	1,357	34.39%	1,357	0	0	0	890	467
2012-1	773	440	1,213	36.24%	1,213	0	0	0	773	440
2012-2	799	417	1,216	34.29%	1,216	0	0	0	799	417
2012-3	872	424	1,296	32.71%	1,296	0	0	0	872	424
2012-4	894	443	1,337	33.11%	1,337	0	0	0	894	443
Total	6,745	3,540	10,285	34.42%	10,285	0	0	0	6,745	3,540

Notes
(A) From Exhibit 3, Sheet 3
(B) From Exhibit 3, Sheet 2
(C) Data provided by the MI Company

Projected Claims
Data organized by delinquency report date and evaluation quarter

Exhibit 3
Sheet 2

Submitted Claims

	1	2	3	4	5	6	7	8
2011-1	1	20	93	230	366	439	467	476
2011-2	0	20	83	218	353	430	454	
2011-3	0	18	89	218	342	389		
2011-4	1	25	95	238	381			
2012-1	0	16	75	200				
2012-2	0	18	89					
2012-3	1	25						
2012-4	1							

Incremental Claim Rate (Incremental Submitted Claims / Prior O/s Delinquent Loans)

	2/1	3/2	4/3	5/4	6/5	7/6	8/7
2011-1	1.78%	8.33%	21.71%	41.46%	55.73%	68.29%	100.00%
2011-2	1.91%	7.52%	22.09%	41.67%	59.23%	68.57%	
2011-3	1.87%	8.98%	23.63%	47.33%	55.95%		
2011-4	2.18%	8.06%	22.88%	45.69%			
2012-1	1.67%	7.42%	20.97%				
2012-2	1.87%	9.01%					
2012-3	2.31%						
2012-4							
4 Qtr Avg.	2.01%	8.37%	22.39%	44.04%	N/a	N/a	N/a
Latest point	2.31%	9.01%	20.97%	45.69%	55.95%	68.57%	100.00%
Selected	2.31%	9.01%	20.97%	45.69%	55.95%	68.57%	100.00%

RESULT 3-2: Actual and Projected Claims (Bold is Projected)

	1	2	3	4	5	6	7	8	Ultimate
2011-1	1	20	93	230	366	439	467	476	**476**
2011-2	0	20	83	218	353	430	454	**462**	**462**
2011-3	0	18	89	218	342	389	**406**	**412**	**412**
2011-4	1	25	95	238	381	**439**	**460**	**467**	**467**
2012-1	0	16	75	200	**350**	**410**	**432**	**440**	**440**
2012-2	0	18	89	**201**	**336**	**390**	**410**	**417**	**417**
2012-3	1	25	**96**	**208**	**343**	**397**	**417**	**424**	**424**
2012-4	1	**26**	**100**	**218**	**358**	**415**	**436**	**443**	**443**

Projected Cured Delinquent Loans
Data organized by delinquency report date and evaluation quarter

Exhibit 3
Sheet 3

Cured delinquent loans

	1	2	3	4	5	6	7	8
2011-1	266	439	611	777	838	855	859	859
2011-2	262	451	615	767	826	844	847	
2011-3	257	413	587	742	796	808		
2011-4	257	464	637	806	873			
2012-1	255	402	542	685				
2012-2	255	410	591					
2012-3	258	483						
2012-4	253							

Incremental Cured Rate (Incremental Cured delinquent loans / Prior O/s delinquent loans)

	2/1	3/2	4/3	5/4	6/5	7/6	8/7
2011-1	16.20%	19.63%	26.31%	18.60%	12.98%	9.76%	0.00%
2011-2	18.05%	19.57%	24.88%	18.21%	13.85%	8.57%	
2011-3	16.17%	22.00%	28.39%	20.61%	14.29%		
2011-4	18.84%	19.93%	27.04%	21.41%			
2012-1	15.34%	17.61%	23.99%				
2012-2	16.13%	22.97%					
2012-3	21.70%						
2012-4							
4 Qtr Avg.	18.00%	20.63%	26.07%	19.71%	N/a	N/a	N/a
Latest point	21.70%	22.97%	23.99%	21.41%	14.29%	8.57%	0.00%
Selected	21.70%	22.97%	23.99%	21.41%	14.29%	8.57%	0.00%

RESULT 3-3: Actual and Projected Cured delinquent loans (Bold is Projected)

	1	2	3	4	5	6	7	8	Ultimate
2011-1	266	439	611	777	838	855	859	859	**859**
2011-2	262	451	615	767	826	844	847	**847**	**847**
2011-3	257	413	587	742	796	808	**810**	**810**	**810**
2011-4	257	464	637	806	873	**888**	**890**	**890**	**890**
2012-1	255	402	542	685	**755**	**771**	**773**	**773**	**773**
2012-2	255	410	591	**720**	**783**	**797**	**799**	**799**	**799**
2012-3	258	483	**664**	**793**	**856**	**870**	**872**	**872**	**872**
2012-4	253	**488**	**677**	**811**	**877**	**892**	**894**	**894**	**894**

Projected Outstanding Delinquent Loans — Exhibit 3
Data organized by delinquency report date and evaluation quarter — Sheet 4

Outstanding Delinquent Loans

	1	2	3	4	5	6	7	8
2011-1	1,068	876	631	328	131	41	9	0
2011-2	1,047	838	611	324	130	35	8	
2011-3	965	791	546	262	84	25		
2011-4	1,099	868	625	313	103			
2012-1	958	795	596	328				
2012-2	961	788	536					
2012-3	1,037	788						
2012-4	1,083							

Outstanding Delinquent Loan Decay Rate

	2/1	3/2	4/3	5/4	6/5	7/6	8/7
2011-1	82.02%	72.03%	51.98%	39.94%	31.30%	21.95%	0.00%
2011-2	80.04%	72.91%	53.03%	40.12%	26.92%	22.86%	
2011-3	81.97%	69.03%	47.99%	32.06%	29.76%		
2011-4	78.98%	72.00%	50.08%	32.91%			
2012-1	82.99%	74.97%	55.03%				
2012-2	82.00%	68.02%					
2012-3	75.99%						
2012-4							
4 Qtr Avg.	79.99%	71.01%	51.53%	36.26%	N/a	N/a	N/a
Latest point	75.99%	68.02%	55.03%	32.91%	29.76%	22.86%	0.00%
Selected	75.99%	68.02%	55.03%	32.91%	29.76%	22.86%	0.00%

<u>RESULT 3-4</u>: Actual and Projected Outstanding Delinquent Loans

	1	2	3	4	5	6	7	8
2011-1	1,068	876	631	328	131	41	9	0
2011-2	1,047	838	611	324	130	35	8	0
2011-3	965	791	546	262	84	25	6	0
2011-4	1,099	868	625	313	103	31	7	0
2012-1	958	795	596	328	108	32	7	0
2012-2	961	788	536	295	97	29	7	0
2012-3	1,037	788	536	295	97	29	7	0
2012-4	1,083	823	560	308	101	30	7	0

O/s RIF

	2011-1	2011-2	2011-3	2011-4	2012-1	2012-2	2012-3	2012-4	Latest Diagonal
2011-1	42,524,556	34,043,112	25,307,517	13,366,000	5,321,875	1,634,055	361,746	0	0
2011-2	43,013,901	32,878,930	23,910,874	12,961,296	5,178,160	1,412,110	319,864		319,864
2011-3	40,010,830	31,768,142	21,973,770	10,886,362	3,430,392	1,008,650			1,008,650
2011-4	43,743,497	35,023,800	26,166,875	12,471,485	4,116,086				4,116,086
2012-1	39,893,994	33,002,835	24,679,168	13,783,216					13,783,216
2012-2	39,293,368	31,551,520	21,932,048						21,932,048
2012-3	43,442,004	33,246,508							33,246,508
2012-4	44,449,569								44,449,569

RIF on Claims

	2011-1	2011-2	2011-3	2011-4	2012-1	2012-2	2012-3	2012-4	Latest Diagonal
2011-1	39,327	776,400	3,711,444	9,113,290	14,832,150	17,109,586	18,306,867	19,577,880	19,577,880
2011-2	0	823,920	3,311,202	8,649,368	14,271,084	16,868,900	17,942,534		17,942,534
2011-3	0	724,230	3,499,035	8,835,104	13,882,122	15,580,228			15,580,228
2011-4	39,365	1,034,275	3,763,995	9,797,508	15,372,588				15,372,588
2012-1	0	636,288	3,143,025	8,125,000					8,125,000
2012-2	0	759,060	3,556,974						3,556,974
2012-3	41,013	1,012,500							1,012,500
2012-4	40,997								40,997

Paid

	2011-1	2011-2	2011-3	2011-4	2012-1	2012-2	2012-3	2012-4	Latest Diagonal
2011-1	40,900	807,223	3,857,304	9,633,659	15,677,583	17,852,142	19,192,919	20,390,362	20,390,362
2011-2	0	860,832	3,479,080	8,987,558	14,910,429	17,661,738	18,888,106		18,888,106
2011-3	0	753,706	3,647,044	9,179,673	14,544,299	16,533,738			16,533,738
2011-4	41,176	1,081,438	3,981,930	10,404,953	16,079,727				16,079,727
2012-1	0	673,638	3,279,118	8,445,125					8,445,125
2012-2	0	792,231	3,729,843						3,729,843
2012-3	42,621	1,059,885							1,059,885
2012-4	42,932								42,932

Outstanding Delinquent Loans

	2011-1	2011-2	2011-3	2011-4	2012-1	2012-2	2012-3	2012-4	Latest Diagonal
2011-1	1,068	876	631	328	131	41	9	0	0
2011-2	1,047	838	611	324	130	35	8		8
2011-3	965	791	546	262	84	25			25
2011-4	1,099	868	625	313	103				103
2012-1	958	795	596	328					328
2012-2	961	788	536						536
2012-3	1,037	788							788
2012-4	1,083								1,083

Submitted Claims

	2011-1	2011-2	2011-3	2011-4	2012-1	2012-2	2012-3	2012-4	Latest Diagonal
2011-1	1	20	93	230	366	439	467	476	476
2011-2	0	20	83	218	353	430	454		454
2011-3	0	18	89	218	342	389			389
2011-4	1	25	95	238	381				381
2012-1	0	16	75	200					200
2012-2	0	18	89						89
2012-3	1	25							25
2012-4	1								1

Reported Delinquent Loans

	2011-1	2011-2	2011-3	2011-4	2012-1	2012-2	2012-3	2012-4
2011-1	**1,335**	1,335	1,335	1,335	1,335	1,335	1,335	1,335
2011-2	**1,309**	1,309	1,309	1,309	1,309	1,309	1,309	1,309
2011-3	**1,222**	1,222	1,222	1,222	1,222	1,222	1,222	1,222
2011-4	**1,357**	1,357	1,357	1,357	1,357	1,357	1,357	1,357
2012-1	**1,213**	1,213	1,213	1,213	1,213	1,213	1,213	1,213
2012-2	**1,216**	1,216	1,216	1,216	1,216	1,216	1,216	1,216
2012-3	**1,296**	1,296	1,296	1,296	1,296	1,296	1,296	1,296
2012-4	**1,337**	1,337	1,337	1,337	1,337	1,337	1,337	1,337

Cured Delinquent Loans

	2011-1	2011-2	2011-3	2011-4	2012-1	2012-2	2012-3	2012-4	Latest Diagonal
2011-1	266	439	611	777	838	855	859	859	859
2011-2	262	451	615	767	826	844	847		847
2011-3	257	413	587	742	796	808			808
2011-4	257	464	637	806	873				873
2012-1	255	402	542	685					685
2012-2	255	410	591						591
2012-3	258	483							483
2012-4	253								253

An Enhanced On-Level Approach to Calculating Expected Loss Costs

Marc B. Pearl, FCAS, MAAA

Jeremy Smith, FCAS, MAAA, CERA, CPCU

Abstract.

Virtually every loss reserve analysis where loss and exposure or premium data is available includes an estimate of an expected loss cost to be used in the calculation. Most estimates are either calculated by trending forward historical loss costs or are judgmentally selected. Occasionally, a change in the underlying exposure is reflected, usually in the form of a judgmentally selected factor. The methodology we present is a simple approach to using individual risk experience in generating a series of on-level factors that can reflect changes in mix by year in the development of expected loss costs. In addition, possible enhancements to the method to reflect the incorporation of new exposures over time are included. The result is a series of expected loss costs that better reflect the composition of business in a given accident or policy year, while still including the stability gained by utilizing multiple years of experience.

Keywords. Loss costs, initial expected loss cost, exposure, on-level, trend, Bornhuetter-Ferguson method

1. INTRODUCTION

This paper presents a technique for estimating an initial expected loss cost (IELC) in situations where the underlying mix of business is shifting. The IELC is commonly used to estimate an initial expected loss for use in a Bornhuetter-Ferguson (B-F) method, and the examples in this paper are presented in that context. Of course, an IELC can be used for various other purposes such as prospective funding estimates.

1.1 Research Context

A common approach to developing an IELC is to trend losses and exposures (if applicable) to current levels and use the trended losses to calculate "on-level" loss costs for each policy period. The actuary selects the IELC after examining the historical on-level loss costs.

This traditional approach can be distorted by changes in the mix of business. For example, if a commercial auto carrier elects to non-renew a particular account consisting of a large fleet of autos with particularly high historical loss costs, then the historical loss costs could potentially be misleading when selecting the IELC.

While various approaches to selecting an expected loss cost are referenced in the actuarial literature (e.g., see Chapter 8 of [1]), we are not aware of any that address continual changes in the underlying exposures over time, a situation that occurs quite frequently. This paper will rely on knowledge of basic reserving methodologies and an understanding of the assumptions used in those methodologies, but it does not rely on any specific papers currently in the actuarial literature.

1.2 Objective

This paper presents a method for selecting the IELC that explicitly adjusts for changes in the mix of business over multiple years. It takes advantage of detailed information available now to insurers through data warehouses and advanced information technology that can enable the actuary to remove much of the judgment typically associated with mix adjustments over multiple years. In developing this approach, trend and new business growth will also be considered. This approach extends the conventional on-leveling approach, so that mix changes as well as trend are contemplated in the on-level calculation.

1.3 Outline

The remainder of the paper proceeds as follows:

Section 2 will briefly review the traditional approach to selecting an IELC. In Section 2.1, we illustrate how the traditional approach can be distorted by changes in the mix of business. In the remainder of Section 2, we will present our method to adjust for the change in mix. Specifically, in Section 2.2 we will illustrate how to adjust for non-renewed business, and in Section 2.3 we will illustrate how to adjust for the addition of new business.

Section 3 will discuss the results of the analysis and possible future enhancements while Section 4 presents the conclusions and main findings of this paper. Finally, references are provided in Section 5.

2. BACKGROUND AND METHODS

Suppose we have the data displayed in Table 1. The "Exposure" in column 1 might be car-years, payroll, or various other exposure bases. The Ultimate Loss amounts and implied loss costs in columns 2 and 3 were previously estimated (e.g. by a chain ladder method) and we want to estimate an IELC for use in a B-F method. In this example we use data organized by policy year, because changes to mix of business are most easily analyzed on a policy year basis.

Table 1: Data for Company XYZ

	(1)	(2)	(3)
Policy Year	**Exposure**	**Ultimate Loss**	**Loss Cost**
	[data]	[derived]	[(2)/(1)]
2005	14,000	56,000	4.00
2006	14,000	57,680	4.12
2007	14,000	59,410	4.24
2008	10,000	48,080	4.81
2009	14,000	54,502	3.89
2010	14,000	56,137	4.01
2011	14,000	38,603	2.76
2012	14,000	39,761	2.84

A typical approach is displayed in Table 2. Using a trend of 3% (based on judgment, analysis, or some external information), we trend all the loss costs to a 2012 level[1]. These trended loss costs, called "on-level" loss costs, are displayed in column 5, and the 2012 IELC is typically selected after considering various averages of the factors.

[1] The trend used here should be understood to be a composite of both the loss and exposure trend and would include benefit level changes if applicable. These trends are often estimated and displayed separately, but we have combined them here for simplicity of discussion.

Table 2: IELC calculation

	(1)	(2)	(3)	(4)	(5)	(6)
Policy Year	**Exposure**	**Ultimate Loss**	**Loss Cost**	**Trend**	**On-Level Loss Cost**	**B-F IELC**
	[Table 1 Col (1)]	[Table 1 Col (2)]	[Table 1 Col (3)]	[1.03^n]	[(3)*(4)]	[Selected (5)/(4)]
2005	14,000	56,000	4.00	1.23	4.92	2.85
2006	14,000	57,680	4.12	1.19	4.92	2.93
2007	14,000	59,410	4.24	1.16	4.92	3.02
2008	10,000	48,080	4.81	1.13	5.41	3.11
2009	14,000	54,502	3.89	1.09	4.25	3.20
2010	14,000	56,137	4.01	1.06	4.25	3.30
2011	14,000	38,603	2.76	1.03	2.84	3.40
2012	14,000	39,761	2.84	1.00	2.84	3.50
				Selected	3.50	

In this example we have selected an IELC of 3.50. Other selections would certainly be possible; for example we might select 4.50 (the average of policy years 2005-2011). Typically, actuaries consider various indications (3 year average, 4 year average, etc.) prior to making a selection. We arrive at the B-F IELC's in column (6) by "de-trending" this selection back to each policy year, using the trend factors displayed in column (4).

These factors might appear reasonable. True, the IELCs for 2011 and 2012 are substantially different from what the On-Level Loss Cost would indicate. But these years are immature and we might distrust the Ultimate Loss estimate in column (2) for various reasons. Thus disparity between the On-Level Loss Cost and the B-F IELC for these two years might actually be viewed as an advantage; it gives us more options when making our final selection of Ultimate Loss. Similarly, the B-F IELC is substantially lower than the On-Level Loss Costs for the older policy years; we might accept this because we are unlikely to select the B-F method for these older, relatively mature years.

2.1 Distortions Caused by Change in Mix

We investigate further by analyzing the loss experience at the individual account level. This analysis reveals the history displayed in Table 3. The shaded cells in the table indicate that the account was either non-renewed or new business that had not been written yet.

Table 3: Account-Level Data

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Account A			Account B			Account C		
Policy Year	Exposure	Ultimate Loss	Loss Cost	Exposure	Ultimate Loss	Loss Cost	Exposure	Ultimate Loss	Loss Cost
	[data]	[derived]	[(2)/(1)]	[data]	[derived]	[(5)/(4)]	[data]	[derived]	[(8)/(7)]
2005	2,000	$4,000	2.00	4,000	$12,000	3.00			
2006	2,000	$4,120	2.06	4,000	$12,360	3.09			
2007	2,000	$4,244	2.12	4,000	$12,731	3.18			
2008	2,000	$4,371	2.19						
2009	2,000	$4,502	2.25				4,000	$30,000	7.50
2010	2,000	$4,637	2.32				4,000	$30,900	7.73
2011	2,000	$4,776	2.39				4,000	$31,827	7.96
2012	2,000	$4,919	2.46				4,000	$32,782	8.20

Table 3 (Continued)

	(10)	**(11)**	**(12)**	**(13)**	**(14)**	**(15)**	**(16)**	**(17)**	**(18)**
	Account D			**Account E**			**Account F**		
Policy Year	**Exposure**	**Ultimate Loss**	**Loss Cost**	**Exposure**	**Ultimate Loss**	**Loss Cost**	**Exposure**	**Ultimate Loss**	**Loss Cost**
	[data]	[derived]	[(11)/(10)]	[data]	[derived]	[(14)/(13)]	[data]	[derived]	[(17)/(16)]
2005	8,000	$40,000	5.00						
2006	8,000	$41,200	5.15						
2007	8,000	$42,436	5.30						
2008	8,000	$43,709	5.46						
2009				8,000	$20,000	2.50			
2010				8,000	$20,600	2.58			
2011							8,000	$2,000	0.25
2012							8,000	$2,060	0.26

For simplicity, we've assumed each account has a constant loss cost, affected only by a 3% annual trend. However, when examining the loss costs for all accounts combined, the simplicity of this assumption has been hidden by the changing mix of business.

In the subsequent section, we will describe a method to adjust for the change in mix. Specifically, we will show how to calculate "Mix of Business" factors as displayed in columns (5) and (6) of Table 4. The factors in column (5) can be thought of as incremental adjustments for the change in mix within a given policy year. The factors in column (6) are simply the cumulative product of the factors in column (5), beginning at the bottom of the column.

Table 4: IELC Calculation Using Mix of Business Adjustment Factors

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Policy Year	Exposure	Ultimate Loss	Loss Cost	Trend	Mix Of Business Factor	Mix Of Business Factor (Cum.)	On-Level Loss Cost	B-F IELC
	[Table 1 Col (1)]	[Table 1 Col (2)]	[Table 1 Col (3)]	[1.03^n]	[derived] (see subsequent discussion)	Cumulative Product of (5)	[(3)*(4)*(6)]	[(Selected (7)/ ((6)*(4))]
2005	14,000	$56,000	4.00	1.23	1.000	0.577	2.84	4.00
2006	14,000	$57,680	4.12	1.19	1.000	0.577	2.84	4.12
2007	14,000	$59,410	4.24	1.16	**1.100**	0.577	2.84	4.24
2008	10,000	$48,080	4.81	1.13	**0.786**	0.525	2.84	4.81
2009	14,000	$54,502	3.89	1.09	1.000	0.668	2.84	3.89
2010	14,000	$56,137	4.01	1.06	**0.668**	0.668	2.84	4.01
2011	14,000	$38,603	2.76	1.03	1.000	1.000	2.84	2.76
2012	14,000	$39,761	2.84	1.00	1.000	1.000	2.84	2.84
						Selected	2.84	

We see that this method produces an On-Level Loss Cost of 2.84 for each year. This makes sense since after adjusting for trend and the change in mix, we should be left only with Accounts A, C, and F in 2012 which, combined, has a loss cost of 2.84. Moreover, the B-F IELCs in column (8) turn out to be exactly equal to the loss costs in column (3) because of the assumptions underlying our simplified example. In practice, the factors in column (7) will not be identical and the selected loss cost (2.84 in Table 4) will need to be estimated in the normal way by considering various averages. As a result, in practice column (8) will not be identical to column (3).

2.2 Adjusting for Non-Renewed Accounts

To show how these Mix of Business on-level factors are developed we begin with the first policy year affected by a mix change, 2008. Table 5 demonstrates how to calculate the Mix of Business factor that would be applied to the 2007 results to adjust for the non-renewal of Account B in 2008. To calculate the Mix of Business factor, first estimate an on-level loss cost for the total book as shown in columns (1) through (5) in Table 5. In this case, we have chosen 4.37 based on the results appearing in column (5). In practice, the values in column (5) will vary and our selection (indicated by the letter (A)) will be an estimate, perhaps a 3-year average.

In columns (6)-(10) we carry out the same procedure, except that Account B is excluded from the history. This yields an on-level loss cost of 4.81.

The Mix of Business factor is equal to (B)/(A), i.e. the percentage change in loss cost resulting from the non-renewal of account B. Thus, by non-renewing Account B, the underlying experience would have worsened resulting in an increase in the average loss cost of about 10%.

Table 5: Mix of Business Factor for Policy Year 2007

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(10)
	Total Company XYZ					XYZ Excluding Account B			
Policy Year	Exp.	Ultimate Loss	Loss Cost	Trend	On-Level Loss Cost	Exp.	Ultimate Loss	Loss Cost	On-Level Loss Cost
	[Table 1 Col (1)]	[Table 1 Col (2)]	[Table 1 Col (3)]	[1.03^n]	[(3)*(4)]	[(1)-Table 3 Col (4)]	[(2)-Table 3 Col (5)]	[(7)/(6)]	[(8)*(4)]
2005	14,000	56,000	4.00	1.093	4.37	10,000	44,000	4.40	4.81
2006	14,000	57,680	4.12	1.061	4.37	10,000	45,320	4.53	4.81
2007	14,000	59,410	4.24	1.030	4.37	10,000	46,680	4.67	4.81
	Selected 2008 Loss Cost (A)				4.37			**(B)**	4.81
						(B)/(A) Mix of Business Factor			1.100

The calculated value of 1.100 is included in Table 4, column (5) as the mix of business factor for 2007. As we will see, because the mix of business factor is cumulatively applied it affects not only Policy Year 2007 but also all prior Policy Years. One may wonder why a change in mix occurring in 2008 should lead to an adjustment factor for 2007. This is because the Mix of Business Factor is used to adjust Policy Year 2007 loss costs for the change in mix occurring in 2008.

The calculation shown in Table 5 is not unusual and is typically performed when a factor causing an underlying shift in loss experience occurs. In reality, changes like this occur in almost all years, not just one year. Also, usually one must contend with both the non-renewal of accounts and the writing of new accounts.

2.3 Adjusting for New Accounts

In the example given in Table 3, three things happen in policy year 2009: Accounts C and E are added and Account D is non-renewed. To adjust for the new account, we must somehow develop an estimate of its expected loss cost. If historical data is available, the actuary can analyze this information and use it to develop prospective estimates. This scenario is illustrated in Table 6, where the historical data (including exposure and ultimate loss) has been added for accounts C, E and F. The data is displayed in the shaded area of the table, in italic font as a reminder that company XYZ did not actually write the business during those policy years. The exposure and ultimate loss amounts are also not included in the company totals for the older policy years. This assumption could be trued up as experience for that account becomes more credible.

Table 6: Account-Level Data*

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Account A			**Account B**			**Account C**		
Policy Year	**Exposure**	**Ultimate Loss**	**Loss Cost**	**Exposure**	**Ultimate Loss**	**Loss Cost**	**Exposure**	**Ultimate Loss**	**Loss Cost**
	[data]	[derived]	[(2)/(1)]	[data]	[derived]	[(5)/(4)]	[data]	[derived]	[(8)/(7)]
2005	2,000	$4,000	2.00	4,000	$12,000	3.00	*4,000*	*$26,655*	*6.66*
2006	2,000	$4,120	2.06	4,000	$12,360	3.09	*4,000*	*$27,454*	*6.86*
2007	2,000	$4,244	2.12	4,000	$12,731	3.18	*4,000*	*$28,278*	*7.07*
2008	2,000	$4,371	2.19				*4,000*	*$29,126*	*7.28*
2009	2,000	$4,502	2.25				4,000	$30,000	7.50
2010	2,000	$4,637	2.32				4,000	$30,900	7.73
2011	2,000	$4,776	2.39				4,000	$31,827	7.96
2012	2,000	$4,919	2.46				4,000	$32,782	8.20

	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
	Account D			**Account E**			**Account F**		
Policy Year	**Exposure**	**Ultimate Loss**	**Loss Cost**	**Exposure**	**Ultimate Loss**	**Loss Cost**	**Exposure**	**Ultimate Loss**	**Loss Cost**
	[data]	[derived]	[(11)/(10)]	[data]	[derived]	[(14)/(13)]	[data]	[derived]	[(17)/(16)]
2005	8,000	$40,000	5.00	*8,000*	*$17,770*	*2.22*	*8,000*	*$1,675*	*0.21*
2006	8,000	$41,200	5.15	*8,000*	*$18,303*	*2.29*	*8,000*	*$1,725*	*0.22*
2007	8,000	$42,436	5.30	*8,000*	*$18,852*	*2.36*	*8,000*	*$1,777*	*0.22*
2008	8,000	$43,709	5.46	*8,000*	*$19,417*	*2.43*	*8,000*	*$1,830*	*0.23*
2009				8,000	$20,000	2.50	*8,000*	*$1,885*	*0.24*
2010				8,000	$20,600	2.58	*8,000*	*$1,942*	*0.24*
2011							8,000	$2,000	0.25
2012							8,000	$2,060	0.26

* Shaded areas represent historical estimates received from the account.

Table 7 illustrates how to compute the Mix of Business factor for policy year 2008. At the beginning of the year, the entire book of business is written except for Account B which was non renewed that year (note that "the entire book of business except for Account B" is equivalent to saying "Accounts A and D"). In policy year 2009, Account D is non-renewed while Accounts C and

E are newly written. Thus, in columns (1)-(5) we consider the total book excluding B, and in columns (6)-(10) we consider the "ongoing business", i.e. Accounts A, C and E. The rest of the procedure is identical to Table 5.

Table 7: Mix of Business Factor for Policy Year 2008

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(10)
	XYZ Excl. Acc't B					Accounts A, C and E			
Policy Year	Exp.	Ultimate Loss	Loss Cost	Trend	On-Level Loss Cost	Exp.	Ultimate Loss	Loss Cost	On-Level Loss Cost
	[Table 5 Col (6)]	[Table 5 Col (7)]	[Table 5 Col (8)]	[1.03^n]	[(3)*(4)]	[Table 6 Cols (1)+(7)+(13)]	Table 6 Cols (2)+(8)+(14)]	[(7)/(6)]	[(8)*(4)]
2005	10,000	44,000	4.40	1.126	4.95	14,000	48,424	3.46	3.89
2006	10,000	45,320	4.53	1.093	4.95	14,000	49,877	3.56	3.89
2007	10,000	46,680	4.67	1.061	4.95	14,000	51,373	3.67	3.89
2008	10,000	48,080	4.81	1.030	4.95	14,000	52,915	3.78	3.89
		Selected 2009 Loss Cost (A)			4.95			**(B)**	3.89
							(B)/(A) Mix of Business Factor		0.786

Continuing with our example, there is no change to the mix of business in policy year 2010, so the Mix of Business Factor for 2009 is 1.000. However, in 2011 Account E is non-renewed and Account F is added. Therefore, we follow the procedure illustrated in Table 8.

Table 8: Mix of Business Factor for Policy Year 2010

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(10)
	Ongoing Business in 2010: Accounts A, C & E					**Ongoing Business in 2011: Accounts A, C & F**			
Policy Year	**Exp.**	**Ultimate Loss**	**Loss Cost**	**Trend**	**On-Level Loss Cost**	**Exp.**	**Ultimate Loss**	**Loss Cost**	**On-Level Loss Cost**
	[Table 6 Cols (1)+(7)+(13)]	Table 6 Cols (2)+(8)+(14)]	[(2)/(1)]	[1.03^n]	[(3)*(4)]	[Table 6 Cols (1)+(7)+(16)]	Table 6 Cols (2)+(8)+(17)]	[(7)/(6)]	[(8)*(4)]
2005	14,000	48,424	3.46	1.194	4.13	14,000	32,330	2.31	2.76
2006	14,000	49,877	3.56	1.159	4.13	14,000	33,299	2.38	2.76
2007	14,000	51,373	3.67	1.126	4.13	14,000	34,298	2.45	2.76
2008	14,000	52,915	3.78	1.093	4.13	14,000	35,327	2.52	2.76
2009	14,000	54,502	3.89	1.061	4.13	14,000	36,387	2.60	2.76
2010	14,000	56,137	4.01	1.030	4.13	14,000	37,479	2.68	2.76

Selected 2011 Loss Cost (A) 4.13 **(B)** 2.76

(B)/(A) Mix of Business Factor 0.668

Using the Mix of Business Factors calculated above, we can calculate columns (5) and (6) of Table 4. Column (5) simply assigns the Mix of Business factors to the appropriate year. A factor 1.000 is used for years in which there is no mix change in the next subsequent year. The factors Column (6) are cumulative Mix of Business Factors and adjust for mix changes in all subsequent years.

3. RESULTS AND DISCUSSION

The above procedure is conceptually simple and easy to implement. Of course, if all the information and ultimate losses by account were determined separately, the exercise above could be conducted without the need to calculate on-level factors. In fact, if we are able to estimate credible loss costs for each account (say, by using benchmark loss costs available from an industry source), then the above procedure can essentially be bypassed. The account-level loss costs can be applied to the exposure for each account to develop ultimate losses by account, and these ultimate losses can be added together to produce an ultimate loss indication for the entire book. This indication could then be used as an initial expected loss for the B-F method.

In practice, benchmark loss costs are not always available or not reflective of the business the actuary is reviewing. Also it's possible that recalculating an ultimate loss cost, every year for every account making up the book may be impractical. In that instance, carrying out the exercises shown in Tables 5, 7 and 8 only once for each, in the year the change actually takes place might be an acceptable solution. Effectively, each mix of business factor, once calculated, would be "fixed" (similar to the way benefit level adjustments are treated for workers compensation), and an adjustment for the change in mix in the latest year would be the only one necessary. Of course, this would not reflect the fact that the implicit Mix of Business factors may change over time as the ultimate loss estimates by account change in subsequent valuations, but, unless subsequent information provides persuasive evidence that an adjustment is warranted it is not unreasonable to treat the adjustment as fixed.

In essence, the calculation of the Mix of Business factor requires the calculation of two prospective loss costs: one which includes the business prior to the change in mix and one which includes the subsequent business. The use of historical account-level data is one way to develop these estimates. However, if historical data is not available (particularly for new accounts, as displayed in the shaded cells of Table 6) one could develop the prospective estimates using other techniques. For example, one might initially assume that the new account will experience an ultimate loss cost equal to that of another account with similar characteristics, or that new business loss ratios are a multiple of that of the existing business.

Finally, we note that whereas the above examples were carried out on a policy year basis, many reserving analyses are conducted on an accident year basis. In most cases, organizing the data by policy year will prove to be the most natural approach when the mix of business is changing, since these changes will typically occur at policy expiration. If the results must be presented on an accident year basis (e.g., in the statutory annual statement), one may convert the results from policy year to accident year. Ultimately, the approach taken will depend upon the granularity of the available data, the specific details of the book of business under consideration, and the actuary's judgment.

4. CONCLUSIONS

We believe the above approach is a natural and intuitive way to adjust the traditional IELC calculation to reflect a changing mix of business. With the availability of detailed historical data at policy level, this approach enables the actuary to take advantage of this accessible information to better reflect normal changes that impact loss experience over time. This produces more accurate expected loss costs over time, and eliminates much of the "judgment" the actuary typically applies to reflect these underlying changes.

5. REFERENCES

[1] Friedland, Jacqueline "Estimating Unpaid Claims Using Basic Techniques", *Casualty Actuarial Society* 2010
[2] Bornhuetter, R.L., and R.E. Ferguson, "The Actuary and IBNR" *PCAS* **1972**, Vol. LIX, 181-195.

Abbreviations and notations
B-F, Bornhuetter-Ferguson
IELC, Initial Expected Loss Cost

Biographies of the Authors

Marc Pearl is a Director within the Actuarial, Risk and Advanced Analytics practice of Deloitte Consulting LLP in the New York City office. In that role he has performed consulting work for reinsurers, self insured entities, regulators and insurance companies. Marc is a Fellow of the Casualty Actuarial Society (CAS) and a member of the American Academy of Actuaries. He has served on various CAS and American Academy Committees, and currently serves on the CAS Committee on Reserves and Casualty Committee of the Actuarial Standards Board. Prior to joining Deloitte, Marc was employed by Continental Insurance Company and Royal Insurance.

Jeremy Smith is a Manager within the Actuarial, Risk and Advanced Analytics practice of Deloitte Consulting, LLP. He is a Fellow of the CAS and a Member of the American Academy of Actuaries. He also holds the CERA and CPCU designations. He participates on the CAS Examination and Syllabus Committees.

Reserving in Two Steps: Total IBNR = Pure IBNR + IBNER

Daniel David Schlemmer, ACAS, MAAA

Tracey Tarkowski

Abstract

Motivation. To estimate IBNR, with separate amounts estimated for pure IBNR and for IBNER or development on known claims, using methods similar to traditional triangle methods.

Results. We applied several methods to our sample data set to calculate estimates that ultimately proved to be more accurate than traditional triangle methods.

Conclusions. Estimating the pure IBNR separately from development on known claims is more cumbersome and requires additional data extraction work, but provides additional information needed in order to make optimal business decisions.

Keywords. Pure IBNR, IBNER, development on known claims

1. INTRODUCTION

Standard actuarial methods for reserving generally apply development factors to losses paid-to-date and reported-to-date to calculate an estimate of ultimate losses, which then result in an estimate of total IBNR. In this paper, we look at separately developing estimates for: 1) pure IBNR, and 2) IBNER, (sometimes called "development on known claims").

The separate estimate of the two amounts can be addressed in a number of ways, for example as a part of a claim simulation model (see, for example, Sahasrabuddhe[1]); we chose to apply a method that looks and functions much like traditional loss development methods, based on triangles, but with adjustments to allow for the separate estimates to be calculated.

We define "pure IBNR" to mean the estimate of ultimate losses for claims not yet reported; "IBNER" or "development on known claims" to mean the estimate of ultimate losses for known claims, less currently reported amounts; and "total IBNR" to mean the total of these two amounts.

1.1 Motivation and Rationale

Standard actuarial methods (development factor, Bornhuetter-Ferguson, etc.) are commonly used across a wide variety of circumstances to provide for estimates of total reserves or IBNR. Various methods are commonly used to adjust for changes in development patterns, for example, methods based on the Berquist-Sherman[2] paper.

The methods commonly used combine into one factor/projection the results of two separate processes:

- The development, to their ultimate value, of claims that have already been reported; and
- The emergence of claims that were not previously reported (which then subsequently develop to their ultimate value).

In this paper, we look at various ways to analyze these processes separately rather than a single, combined projection. Our motivation in analyzing reserves this way was twofold:

1. Accuracy of reserve estimate: The method allowed us to observe separately two distinct changes that were occurring in the book being analyzed:
   - A law change affected the late reporting of claims; the result is that patterns were distorted for pure IBNR with no impact on IBNER.
   - A court ruling affected development on the current inventory of claims; a large number of claims across various accident years saw late development that would not reasonably be expected to be repeated in the future.
2. Communication regarding our reserve estimate: Management had their own views on the expected future of development on known claims, and wanted to compare to the assumptions underlying the actuarial analysis; in order to do this, we required separate estimates of the two sources of "total IBNR."

Generally, we see the value in using these methods for situations where there is a change in the claims handling process, which could be due to legal issues (as in our case) or other reasons (for example, changes in claims handling personnel), where the change would be expected to only affect one of the two components of total IBNR, either pure IBNR or IBNER.

Difficulties in using these methods include the following:

- They require a volume of data that is often not available or hasn't previously been extracted.
- They require significantly more data manipulation work than standard methods.

- While we found that nonactuaries might understand <u>results</u> from this model as well as or better than they understood results from standard methods, they struggled even more with the underlying assumptions.

The data shown in this paper is actual data for a single coverage from a single company in a single state. Some scaling and other adjustments were made to obscure the actual data for competitive reasons, but we believe the patterns shown are representative of actual development and therefore representative of a realistic experience set. The data, being short-tailed, was treated as fully developed at 10 years. Obviously the use of these methods for longer-tailed lines (i.e., workers comp) would require significantly different assumptions (i.e., a judgmental tail factor or a larger triangle).

1.2 Outline

Section 2 of this paper covers methods for developing known claims only. Section 3 covers methods for developing Pure IBNR. Section 4 briefly addresses other issues and considerations. The Appendix summarizes and demonstrates some associated calculations.

2. METHODS FOR DEVELOPING KNOWN CLAIMS ONLY

Below we use several different methods to estimate losses associated with known claims only. Note that the methods below use paid losses to arrive at an estimate of ultimate losses on known claims; we could easily use reported losses and similar methods to arrive at a similar result.

2.1 Loss Development Factors in Three Dimensions

While the standard development methods use triangles in two dimensions (normally accident years in rows and evaluation ages in columns; or in the case of Schedule P of the Statutory Annual Statement, evaluation dates in columns), we introduce a third dimension (accident reporting date) to fit the needs of our analysis.

The following can be used for a simple analysis of our data using standard triangle techniques (following the form seen in many texts, for example, Chapter 7 of Friedland[3]):

Accident Year	12	24	36	48	60	72	84	96	108	120
2002	12,562,376	25,862,513	28,467,263	29,575,021	30,091,230	30,073,026	30,104,946	30,166,426	30,186,326	30,189,073
2003	14,619,720	25,402,485	27,155,996	27,946,880	28,224,494	28,481,593	28,418,174	28,441,572	28,501,185	
2004	9,959,858	15,436,434	16,398,604	16,682,005	16,808,181	16,817,113	16,830,511	16,833,318		
2005	6,633,610	11,007,035	11,407,596	11,591,573	11,695,406	11,757,884	11,850,579			
2006	6,290,293	9,478,911	10,085,187	10,406,254	10,685,927	10,907,900				
2007	7,336,768	11,828,200	12,709,085	13,100,700	13,429,193					
2008	7,585,085	12,634,480	13,500,587	14,841,451						
2009	10,823,234	20,222,524	23,270,335							
2010	17,829,334	33,345,851								
2011	13,138,447									

Ex hibit 1

For accidents that occurred in 2002, for example, $12,562,376 was paid out in the first 12 months; $25,862,513 was paid in the first 24 months, and so on. Dividing $25,862,513 by $12,562,376 results in a loss development factor (LDF) of 2.059. Cumulative losses at the end of 2003 are 2.059 times the value of those losses at the end of 2002. Applying this same procedure to the rest of the triangle, we get a triangle of LDFs:

Accident Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
2002	2.059	1.101	1.039	1.017	0.999	1.001	1.002	1.001	1.000
2003	1.738	1.069	1.029	1.010	1.009	0.998	1.001	1.002	
2004	1.550	1.062	1.017	1.008	1.001	1.001	1.000		
2005	1.659	1.036	1.016	1.009	1.005	1.008			
2006	1.507	1.064	1.032	1.027	1.021				
2007	1.612	1.074	1.031	1.025					
2008	1.666	1.069	1.099						
2009	1.868	1.151							
2010	1.870								
Simple Average	1.725	1.078	1.038	1.016	1.007	1.002	1.001	1.001	1.000
ATU	1.984	1.150	1.066	1.028	1.011	1.004	1.002	1.001	1.000

Exhibit 2

The last lines give an average LDF for each period and the age to ultimate (ATU), which gives a factor which can be applied to predict ultimate losses for each development period. We use the notation LDF_{12-24} to denote the development for the period of 12-24 months (here, 1.725).

In order to develop known claims only, significantly more data is required and more data organization needs to occur. Using standard methods allows LDFs to be developed for all historical accident years from a single triangle. To develop only known claims, a separate triangle is needed to develop a factor for each of the historical accident ages.

The following triangle shows development for claims known (reported) during the first 12 months of the accident year:

Accident Year	12	24	36	48	60	72	84	96	108	120
2002	12,562,376	25,118,811	27,554,751	28,588,661	29,100,263	29,085,791	29,116,019	29,165,603	29,182,707	29,185,106
2003	14,619,720	24,788,513	26,468,578	27,179,855	27,438,716	27,693,922	27,641,313	27,657,101	27,691,518	
2004	9,959,858	15,034,728	15,951,729	16,230,914	16,354,359	16,365,200	16,378,598	16,381,406		
2005	6,633,610	10,638,603	11,017,732	11,130,327	11,219,686	11,278,566	11,367,675			
2006	6,290,293	9,240,966	9,818,560	10,102,109	10,388,013	10,609,986				
2007	7,336,768	11,446,700	12,291,777	12,632,077	12,953,176					
2008	7,585,085	12,329,181	13,146,004	14,438,614						
2009	10,823,234	19,605,018	22,511,712							
2010	17,829,334	32,564,625								
2011	13,138,447									

Exhibit 3

This triangle leads to a familiar-looking triangle of LDFs:

Accident Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
2002	2.000	1.097	1.038	1.018	1.000	1.001	1.002	1.001	1.000
2003	1.696	1.068	1.027	1.010	1.009	0.998	1.001	1.001	
2004	1.510	1.061	1.018	1.008	1.001	1.001	1.000		
2005	1.604	1.036	1.010	1.008	1.005	1.008			
2006	1.469	1.063	1.029	1.028	1.021				
2007	1.560	1.074	1.028	1.025					
2008	1.625	1.066	1.098						
2009	1.811	1.148							
2010	1.826								
Average	1.678	1.077	1.035	1.016	1.007	1.002	1.001	1.001	1.000
ATU	1.921	1.145	1.064	1.027	1.011	1.004	1.002	1.001	1.000

Exhibit 4

This triangle would then show a factor of 1.678 for what we label as $LDF_{12\text{-}24}(12)$; that is, the development for the period of 12-24 months on claims known as of 12 months.

So, $LDF_{12\text{-}24}(12) < LDF_{12\text{-}24}$; the difference in the two factors being due to $LDF_{12\text{-}24} = 1.725$ including a provision for late reporting claims, while unreported claims are excluded from $LDF_{12\text{-}24}(12) = 1.678$. Exhibit 4 would then provide a development factor applicable only to known claims at 12 months of age (losses for accident year 2011, for claims known as of 12/31/2011, in the data being analyzed here).

A separate triangle would then be created to include all claims that are reported during the first 24 months of the accident year, as shown in exhibit 5.

Accident Year	12	24	36	48	60	72	84	96	108	120
2002	12,562,376	25,862,513	28,462,819	29,564,219	30,080,258	30,062,055	30,093,974	30,155,455	30,175,354	30,178,101
2003	14,619,720	25,402,485	27,151,706	27,942,495	28,220,109	28,477,208	28,413,789	28,437,186	28,496,799	
2004	9,959,858	15,436,434	16,398,604	16,682,005	16,805,451	16,814,383	16,827,781	16,830,588		
2005	6,633,610	11,007,035	11,407,596	11,591,573	11,695,406	11,757,884	11,850,579			
2006	6,290,293	9,478,911	10,082,599	10,397,520	10,677,194	10,899,167				
2007	7,336,768	11,828,200	12,693,907	13,070,724	13,392,298					
2008	7,585,085	12,634,480	13,499,537	14,839,324						
2009	10,823,234	20,222,524	23,268,025							
2010	17,829,334	33,345,851								
2011	13,138,447									

Exhibit 5

Exhibit 6 shows development factors calculated.

Accident Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
2002	2.059	1.101	1.039	1.017	0.999	1.001	1.002	1.001	1.000
2003	1.738	1.069	1.029	1.010	1.009	0.998	1.001	1.002	
2004	1.550	1.062	1.017	1.007	1.001	1.001	1.000		
2005	1.659	1.036	1.016	1.009	1.005	1.008			
2006	1.507	1.064	1.031	1.027	1.021				
2007	1.612	1.073	1.030	1.025					
2008	1.666	1.068	1.099						
2009	1.868	1.151							
2010	1.870								
Average	1.725	1.078	1.037	1.016	1.007	1.002	1.001	1.001	1.000
ATU	1.983	1.149	1.066	1.027	1.011	1.004	1.002	1.001	1.000

Exhibit 6

Exhibits 5 and 6 would then provide a set of factors: $LDF_{24\text{-}36}(24)$, $LDF_{36\text{-}48}(24)$, $LDF_{48\text{-}60}(24)$, etc.; multiplying these together would then result in $LDF_{24\text{-}ult}(24)$: the factor to develop losses as of 24 months to their ultimate value. Similar to the above, these factors would only apply to a single accident year (accident year 2010 in the analysis this data was pulled from).

Constructing the remaining triangles gives us the rest of the age-to-age LDFs:

Known up to	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
12	1.678	1.077	1.035	1.016	1.007	1.002	1.001	1.001	1.000
24		1.078	1.037	1.016	1.007	1.002	1.001	1.001	1.000
36			1.037	1.016	1.007	1.002	1.001	1.001	1.000
48				1.016	1.007	1.002	1.001	1.001	1.000
60					1.007	1.002	1.001	1.001	1.000
72						1.002	1.001	1.001	1.000
84							1.001	1.001	1.000
96								1.001	1.000
108									1.000
Simple Average	1.678	1.077	1.037	1.016	1.007	1.002	1.001	1.001	1.000

Exhibit 7

As previously stated, we assumed that losses are closed at 10 years (i.e., $LDF_{120\text{-ult}} = 1.000 = LDF_{120\text{-ult}}(x)$ for all values of x), giving the table below for age-to-ultimate LDFs:

Known up to	12-ult	24-ult	36-ult	48-ult	60-ult	72-ult	84-ult	96-ult	108-ult
12	1.921	1.145	1.064	1.027	1.011	1.004	1.002	1.001	1.000
24		1.149	1.066	1.027	1.011	1.004	1.002	1.001	1.000
36			1.066	1.028	1.011	1.004	1.002	1.001	1.000
48				1.028	1.011	1.004	1.002	1.001	1.000
60					1.011	1.004	1.002	1.001	1.000
72						1.004	1.002	1.001	1.000
84							1.002	1.001	1.000
96								1.001	1.000
108									1.000

Exhibit 8

These factors can be compared to the factors obtained from the standard loss triangle:

Accident Year	12	24	36	48	60	72	84	96	108
ATA	1.725	1.078	1.038	1.016	1.007	1.002	1.001	1.001	1.000
ATU	1.984	1.150	1.066	1.028	1.011	1.004	1.002	1.001	1.000

Exhibit 9

Contrasting Exhibits 8 and 9 shows that factors after 24 months are not significantly different for the data we used; this is a result of the fast-reporting data for the line we analyzed and wouldn't be expected to be true for many commercial lines, which would have significantly more late reporting.

As stated in the introduction, this analysis is much more cumbersome than a traditional analysis, and the previous exhibits show the issue: A traditional analysis has one loss triangle and $(n - 1)$ LDFs, where n is the number of years of development (in this case, 10). A three-dimensional analysis has $(n\text{-}1)$ triangles and $n^*(n - 1)/2$ LDFs (in this case, 45), created by constructing separate triangles for each stage of development. LDFs are chosen as normal, using simple or weighted averages similar to those used for standard methods, and actuarial judgment comes into play if those averages are not judged to be representative of future expected development.

This process is repeated for claims reported within the first 12 months, 24 months, 36 months, and so on. The result is a triangle of LDFs which gives a separate LDF for each accident year and stage of development.

There are significant limitations with using this method:

- Adding segmentations to the data can result in lower credibility.
- The large number of LDFs that must be selected increases the amount of judgment

necessary.

- Overall, the method requires significantly more data and data organization effort than standard methods.

2.2 Derivative Methods

The above LDF method is very analogous to the traditional development method, simply adjusted to allow analysis of known claims only. Other common methods could be derived using the same data that was used above and would be analogous to other methods commonly used by actuaries.

We constructed, based on Exhibit 3, a triangle of incremental losses for claims reported within the first 12 months of the accident year:

Accident Year	Exposures	12	24	36	48	60	72	84	96	108	120
2002	50,645	12,562,376	12,556,435	2,435,940	1,033,909	511,602	(14,472)	30,228	49,584	17,104	2,399
2003	68,274	14,619,720	10,168,792	1,680,065	711,278	258,861	255,206	(52,609)	15,788	34,416	
2004	55,783	9,959,858	5,074,869	917,001	279,185	123,445	10,842	13,398	2,808		
2005	44,724	6,633,610	4,004,993	379,129	112,595	89,358	58,881	89,109			
2006	42,487	6,290,293	2,950,673	577,594	283,549	285,903	221,973				
2007	44,220	7,336,768	4,109,932	845,077	340,300	321,099					
2008	47,790	7,585,085	4,744,096	816,823	1,292,610						
2009	45,849	10,823,234	8,781,784	2,906,694							
2010	44,112	17,829,334	14,735,291								
2011	29,189	13,138,447									

Exhibit 10

Then we computed incremental paid loss per exposure for each year:

Accident Year	Exposures	12	24	36	48	60	72	84	96	108	120
2002	50,645	248.05	247.93	48.10	20.41	10.10	(0.29)	0.60	0.98	0.34	0.05
2003	68,274	214.13	148.94	24.61	10.42	3.79	3.74	(0.77)	0.23	0.50	
2004	55,783	178.54	90.97	16.44	5.00	2.21	0.19	0.24	0.05		
2005	44,724	148.32	89.55	8.48	2.52	2.00	1.32	1.99			
2006	42,487	148.05	69.45	13.59	6.67	6.73	5.22				
2007	44,220	165.92	92.94	19.11	7.70	7.26					
2008	47,790	158.72	99.27	17.09	27.05						
2009	45,849	236.06	191.54	63.40							
2010	44,112	404.19	334.05								
2011	29,189	450.12									
Simple Average		235.21	151.63	26.35	11.40	5.35	2.04	0.51	0.42	0.42	0.05

Exhibit 11

The factors from Exhibit 11 can then be applied to the exposures for the 2011 accident year (12 months of age) to calculate a provision for the 2011 development. We would estimate 2011 development to be

$$29{,}189*(151.63 + 26.35 + 11.40 + 5.35 + 2.04 + 0.51 + 0.42 + 0.05) = \$5{,}784{,}189.$$

Adding development to the known loss of $13,138,447 gives a total of $18,922,635 ultimate loss on known claims in 2011.

Like in the prior calculation, the triangle above would only be useful for an accident year developed to 12 months (2011 in our example), and separate triangles would be necessary for each accident year (i.e., for each age).

2.3 Using a Mathematical Function Representing Development

In addition to these methods which rely on triangles, we used a "development curve" as another method to calculate the development on known claims. The general idea was to calculate a function that could be applied to losses at any stage of development to estimate an ultimate value for those claims. We calculated a development factor $f(a\text{-}b,b)$ by taking the ratio of (X) / (Y) where:

X = the total amount paid for all claims within "b" months of occurrence

Y = the total amount paid for all claims within "a" months of occurrence

X and Y are taken from all claims <u>which are reported by age "a" and which reach at least age "b"</u> before the evaluation date (12/31/2011 in our data). Put another way, the LDF f(22-23,23) is a function that calculates development on claims that occurred more than 23 months before the evaluation date, by comparing their value as of 22 months after occurrence with their value as of 23 months after occurrence; using only claims that are reported within 22 months of their occurrence.

The key point that is underlined above is that each LDF is calculated using a different set of claims. Therefore, X and Y will change each time the month being evaluated changes. This concept is illustrated in Exhibit 12. X1 and Y1 are calculated using all claims which are at least 23 months old and are known within 22 months of occurrence. X2 and Y2 are calculated using all claims which are at least 24 months old and are known within 23 months of occurrence. Thus, there are two values for "Paid within 23 months", depending on what factor is being calculated and the claims that the factor will be applied to.

Known at 23 months	Total Paid Loss		Known at 24 months	Total Paid Loss
Paid within 23 months (X_1)	139,341,717		Paid within 24 months (X_2)	138,213,757
Paid within 22 months (Y_1)	138,180,265		Paid within 23 months (Y_2)	137,320,270
LDF:	1.008		LDF:	1.007

Exhibit 12

We calculated the development during each month in order to derive a factor for that month of development. For example, for development during month 23, the factor would be calculated using all of the claims in the database that are at least 23 months old and are known within 22 months. Paid loss through 23 months ($139,341,717) is divided by paid loss through 22 months ($138,180,265) to get:

$f(22\text{-}23, 23) = 1.008$

In practice, it might make sense to use only claims from the most recent years or to weight the claims in order to give more credibility to the most recent years. We used all of the claims in our (10-year) database equally.

These factors could be applied to accident year using the average accident date (actual or assumed), or even to individual claims in order to develop ultimate claim amounts. In the appendix, we used an assumed average accident date of June 30, and applied the appropriate factor to each accident year.

3. METHODS FOR DEVELOPING PURE IBNR

Having developed an estimate of development on known claims, we turn to methods to calculate the remaining reserve, pure IBNR.

3.1 Exposure-Based Method

We again construct a set of historical triangles and compare the results to the associated exposures to come to a reserve estimate, in a manner similar to what we did in Section 2.2. In this case, the triangle represents the pure IBNR losses, for losses reported after a particular accident year age.

Exhibit 13 shows the development to ultimate of losses that are unreported at accident year age

12 months. The 743,702 in Exhibit 13 can be calculated by subtracting the corresponding amounts in Exhibit 1 and Exhibit 3: 743,702 = 25,862,531 – 25,118,811. Other numbers in the exhibit are calculated similarly.

Accident Year	Exposures	12	24	36	48	60	72	84	96	108	120
2002	50,645	-	743,702	912,511	986,360	990,967	987,236	988,927	1,000,823	1,003,619	1,003,967
2003	68,274	-	613,972	687,418	767,025	785,778	787,671	776,861	784,471	809,667	
2004	55,783	-	401,706	446,875	451,092	453,822	451,913	451,913	451,913		
2005	44,724	-	368,431	389,864	461,246	475,721	479,317	482,903			
2006	42,487	-	237,945	266,627	304,144	297,915	297,915				
2007	44,220	-	381,500	417,308	468,623	476,017					
2008	47,790	-	305,299	354,583	402,836						
2009	45,849	-	617,506	758,623							
2010	44,112	-	781,226								
2011	29,289	-									

Exhibit 13

We then create an incremental triangle from that in Exhibit 14. The sum of the entirety of the Accident Year 2002 row in Exhibit 14 is equal to the last value in the same row of Exhibit 13.

Accident Year	Exposures	12	24	36	48	60	72	84	96	108	120
2002	50,645	-	743,702	168,810	73,849	4,607	(3,731)	1,691	11,896	2,795	349
2003	68,274	-	613,972	73,446	79,607	18,753	1,893	(10,811)	7,610	25,196	
2004	55,783	-	401,706	45,169	4,216	2,730	(1,909)	-	-		
2005	44,724	-	368,431	21,433	71,382	14,475	3,596	3,586			
2006	42,487	-	237,945	28,682	37,517	(6,230)	-				
2007	44,220	-	381,500	35,807	51,315	7,394					
2008	47,790	-	305,299	49,284	48,253						
2009	45,849	-	617,506	141,116							
2010	44,112	-	781,226								
2011	29,189	-									

Exhibit 14

We then calculate the ratio of the values in the triangle of Exhibit 14 to the exposures, to create the triangle in Exhibit 15. The 14.68 in Exhibit 15 would represent $14.68, per exposure, of pure IBNR paid during the period from 12 months to 24 months, for claims unreported as of 12 months of age, for accident year 2002. The 3.33 would represent $3.33, per exposure, of pure IBNR paid during the period from 24 months to 36 months, for claims unreported as of 12 months of age, for accident year 2002. We would call attention to the fact that both of those numbers, and the entire triangle, are for claims unreported as of 12 months of accident year developement.

Accident Year	Exposures	12	24	36	48	60	72	84	96	108	120
2002	50,645	-	14.68	3.33	1.46	0.09	(0.07)	0.03	0.23	0.06	0.01
2003	68,274	-	8.99	1.08	1.17	0.27	0.03	(0.16)	0.11	0.37	
2004	55,783	-	7.20	0.81	0.08	0.05	(0.03)	-	-		
2005	44,724	-	8.24	0.48	1.60	0.32	0.08	0.08			
2006	42,487	-	5.60	0.68	0.88	(0.15)	-				
2007	44,220	-	8.63	0.81	1.16	0.17					
2008	47,790	-	6.39	1.03	1.01						
2009	45,849	-	13.47	3.08							
2010	44,112	-	17.71								
2011	29,189	-									
Simple Average		-	10.10	1.41	1.05	0.13	0.00	(0.01)	0.12	0.21	0.01

Exhibit 15

This is, to some extent, the complement to the method outlined in section 2.2. The two methods work functionally much the same and use the same exposure base.

Using this triangle, we would estimate

$$29{,}189*(10.10+1.41+1.05+0.13+0.00-0.01+0.12+0.21+0.01) = \$379{,}815$$

of pure IBNR for accident year 2011. Similar to the methods outlined earlier, this triangle is only appropriate for calculating the pure IBNR for the accident year age 12 months (2011). Separate triangles would again be necessary for each accident year.

3.2 Frequency / Severity

Exposures can also be used to estimate the number of unreported claims. Exhibit 16 shows a triangle of claim counts for claims that were not reported in the first 12 months. Exhibit 16 is constructed the same as Exhibit 14, simply substituting reported claim counts for paid losses.

Accident Year	Exposures	12	24	36	48	60	72	84	96	108	120
2002	50,645		252	7	3	0	1	0	0	0	0
2003	68,274		215	10	1	0	0	0	0	0	
2004	55,783		117	3	0	6	0	0	2		
2005	44,724		100	1	0	0	0	1			
2006	42,487		78	3	4	1	0				
2007	44,220		106	3	2	2					
2008	47,790		96	5	1						
2009	45,849		137	6							
2010	44,112		145								
2011	29,189										

Exhibit 16

These claims counts are then divided by the exposures and those factors are used to estimate the ultimate number of unreported claims for any year.

Accident Year	Exposures	12	24	36	48	60	72	84	96	108	120
2002	50,645		0.00498	0.00014	0.00006	-	0.00002	-	-	-	-
2003	68,274		0.00425	0.00020	0.00002	-	-	-	-	-	
2004	55,783		0.00231	0.00006	-	0.00012	-	-	0.00004		
2005	44,724		0.00197	0.00002	-	-	-	0.00002			
2006	42,487		0.00154	0.00006	0.00008	0.00002	-				
2007	44,220		0.00209	0.00006	0.00004	0.00004					
2008	47,790		0.00190	0.00010	0.00002						
2009	45,849		0.00271	0.00012							
2010	44,112		0.00286								
2011	29,189										
Simple Average			0.00273	0.00009	0.00003	0.00003	0.00000	0.00000	0.00001	-	-

Exhibit 17

We estimate 29,189*(0.00273+0.00009+0.00003+0.00003+0.00001) = 85 additional claims to be reported for accident year 2011.

To estimate an ultimate value for the additional claims, we need to multiply by the average ultimate paid per claim. We looked at a variety of claim statistics to arrive at an estimate of $5,790. Overall, this value was mostly based on the average paid per claim for older (closed or nearly closed) accident years, plus application of appropriate trend. We also considered the recent closed claim statistics and applied some actuarial judgment.

It's worth noting that looking at closed claim statistics and closed accident years was deemed appropriate based on our knowledge of our line of business that we were reviewing, and would not necessarily be applicable to all lines. We specifically considered that the claims we are estimating are claims that have a significant lag between occurrence and report date, and therefore might have different characteristics or be a different mix from those taken by looking at an entire accident year (i.e., when we look at older accident years to arrive at an average ultimate paid per claim). The specific characteristics of our line (limits, historical experience, low-severity/high-frequency) made us comfortable that this would not significantly skew our results. For other lines, it could be worth looking at the difference in severity between early-reported claims and late-reported claims and considering whether an adjustment should be made to the average ultimate paid per claim.

The results of 85 additional claims and average severity of $5,790 gives an expected pure IBNR of $491,818 for accident year 2011.

4. OTHER ISSUES AND CONSIDERATIONS

An additional benefit of our work is that the methods can be more precise and intuitive for interpolating full-year development patterns into quarterly or monthly results. In particular, the comparison to the results shown in Part 3 of the Statutory Quarterly Statement (which splits the development during the quarter into newly reported claims versus development on known) can be accomplished using the assumptions in our model.

Our methods also allow for additional explanation to management of the underlying reasons for development – for example, while unexpected development on known claims might point to an issue of claims handling/reserving, a deviation in the number of previously unreported claims from the expected number would be less likely to be the result of claims handling practices (in the specific case of the state/line of business used in our analysis, it was the result of a specific law change).

Tail factors can be added, and would generally use the same actuarial judgment as traditional methods. In our case, the development past ten years was so small as to be meaningless, but this would not be the case for many lines.

The creation of the triangles necessary for our method was heavily dependent on the use of report date, which adds an additional level of data validation – whether the report date is recorded correctly and consistently in the data systems.

Claim simulation models (for example, as used by Sahasrabuddhe) can also be used to calculate pure IBNR separate from development on known, and could be incorporated as part of the analysis, for example, comparing the results from Section 2 above to the results from a simulation model and making a judgmental selection; and similarly for Section 3. However, we found that the triangle methods we used were sufficient for our needs and did not see a reason to add a simulation.

5. CONCLUSIONS

The methods presented rely largely on the same techniques that are part of the basic actuarial toolkit, but extend them in a direction that can allow the separate review of the two components of total IBNR. As such, they allowed us to differentiate the two for reporting to management in a way that would be directly correlated with the way management was looking at the line of business. Further, the underlying assumptions developed can be used to project future development of the two components of IBNR, which can aid in understanding the source of future development and the reason for deviation from expectations.

Appendix – Supporting Calculation

Using standard methods, the projected ultimate loss for all accident years is $216,433,377. This loss includes development on known claims and IBNR.

Accident Year	Paid Loss as of 12/31/2011	Paid LDF	Standard Method Projected Ultimate
2002	30,189,073	1.000	30,189,073
2003	28,501,185	1.000	28,503,779
2004	16,833,318	1.001	16,858,046
2005	11,850,579	1.002	11,879,982
2006	10,907,900	1.004	10,955,509
2007	13,429,193	1.011	13,582,626
2008	14,841,451	1.028	15,250,843
2009	23,270,335	1.066	24,812,067
2010	33,345,851	1.150	38,338,040
2011	13,138,447	1.984	26,063,411
Total	196,307,331		216,433,377

Exhibit A

Section 2 explored methods that only develop on known claims. Using methods from section 2.1, the projected ultimate loss is $215,488,589. Using exposure methods from section 2.2, the projected ultimate loss is $206,032,673. Using the function method obtained in section 2.3, projected ultimate loss is $211,934,390.

	Development on Known			
Accident Year	Paid Loss as of 12/31/2011	3-D Projection	Exposure Projection	Function Projection
2002	30,189,073	30,189,073	30,189,073	30,189,073
2003	28,501,185	28,503,751	28,504,888	28,501,185
2004	16,833,318	16,857,055	16,871,657	16,842,083
2005	11,850,579	11,878,854	11,905,273	11,868,251
2006	10,907,900	10,954,452	10,981,252	10,928,589
2007	13,429,193	13,581,572	13,595,637	13,560,877
2008	14,841,451	15,249,371	15,282,623	15,201,215
2009	23,270,335	24,786,385	24,261,461	24,752,435
2010	33,345,851	38,247,339	35,518,174	38,565,430
2011	13,138,447	25,240,739	18,922,635	21,525,254
Total	196,307,331	215,488,589	206,032,673	211,934,390

Exhibit B

Pure IBNR was calculated the using two methods developed in section 3. The exposure method of section 3.1 projects pure IBNR of $626,372. The claim count method of section 3.2 projects a total pure IBNR of $593,139. This method uses the estimated value of $5,790 paid per claim, which was derived separately.

		Pure IBNR	
Accident Year	Paid Loss as of 12/31/2011	Exposure IBNR	Claim Count IBNR
2002	30,189,073		
2003	28,501,185		
2004	16,833,318		
2005	11,850,579		3,095
2006	10,907,900		4,315
2007	13,429,193		5,502
2008	14,841,451	6,198	14,078
2009	23,270,335	35,056	22,387
2010	33,345,851	205,303	44,997
2011	13,138,447	379,815	498,765
Total	196,307,331	626,372	593,139

Exhibit C

Combining estimates for development on known claims and pure IBNR should result in a number close to the standard projection. The following chart shows all projections, along with a minimum and maximum total projection. The range of ($206,259,851, $216,689,071) contains the point estimate that was obtained by the standard method ($216,433,377), but suggests amounts substantially less than that are likely. As the purpose of this paper is to be illustrative rather than to calculate an actual reserve to recommend to management, we used simplifications like taking straight averages rather than exercising actuarial judgment in selecting LDFs; however, the fact that these exhibits suggest an amount lower than the traditional methods was consistent with our final results and recommendation to management.

		Development on Known			Pure IBNR		Total	
Accident Year	Paid Loss as of 12/31/2011	3-D Projection	Exposure Projection	Function Projection	Exposure IBNR	Claim Count IBNR	Minimum Projection	Maximum Projection
2002	30,189,073	30,189,073	30,189,073	30,189,073			30,189,073	30,189,073
2003	28,501,185	28,503,751	28,504,888	28,501,185			28,501,185	28,504,888
2004	16,833,318	16,857,055	16,871,657	16,842,083			16,842,083	16,871,657
2005	11,850,579	11,878,854	11,905,273	11,868,251		3,095	11,871,345	11,908,368
2006	10,907,900	10,954,452	10,981,252	10,928,589		4,315	10,932,904	10,985,567
2007	13,429,193	13,581,572	13,595,637	13,560,877		5,502	13,566,379	13,601,139
2008	14,841,451	15,249,371	15,282,623	15,201,215	6,198	14,078	15,207,413	15,296,701
2009	23,270,335	24,786,385	24,261,461	24,752,435	35,056	22,387	24,283,849	24,821,441
2010	33,345,851	38,247,339	35,518,174	38,565,430	205,303	44,997	35,563,171	38,770,733
2011	13,138,447	25,240,739	18,922,635	21,525,254	379,815	498,765	19,302,450	25,739,504
Total	196,307,331	215,488,589	206,032,673	211,934,390	626,372	593,139	206,259,851	216,689,071

Exhibit D

Postscript: Writing this in August 2013, with 18 months of hindsight, it's interesting to note that the current actuary's comparable estimate as of June 30, 2013 for accident year 2011 is $20.4 million, well below the estimate from traditional methods and within the range (albeit at the low end) of results from our proposed methods. While this isn't necessarily a solid validation of our results, it provides some basis to believe that the methods did, in this case, increase our accuracy.

5. REFERENCES

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Biographies of the Author(s)

Dan Schlemmer is Director of Corporate Development and Chief Actuary at 1347 Capital, a subsidiary of Kingsway Financial which focuses on opportunities in the insurance industry as advisor, investor, and financier. He has a BS from Purdue University in Math, and an MBA from the University of Chicago. He is an Associate of the CAS and a Member of the American Academy of Actuaries.

Tracey Tarkowski is an Actuarial Analyst at Allstate and former Actuarial Analyst at 1347 Capital and Kingsway. She has a BS in Math from Carnegie Mellon and MS in Statistics from DePaul University.

Using Life Expectancy to Inform the Estimate of Tail Factors for Workers Compensation Liabilities

Michael Shane, FCAS, MAAA and Dawn Morelli, ACAS, MAAA

Abstract

Traditional accident year paid and/or reported loss development methods are often used to estimate liabilities for workers compensation claims by selecting age-to-age loss development factors which are then fitted to a curve.

This short paper shares a practical reserving technique that can inform traditional loss development methods to more accurately estimate the liabilities associated with a body of claims that has claimant mortality as the main driver of the length of the tail.

Keywords: Workers Compensation, Reserving, Reserving Methods.

1. INTRODUCTION

In his paper "Overcoming Claims Inadequacies: A Mortality-Based Approach to Reserving for Old Workers' Compensation Claims," Brian Jones [1] provides a survey of how mortality can be used in workers compensation reserving. He specifically mentions its use in tail factor estimation before exploring the means to build a ground-up, mortality-based claims model. Our paper illustrates a practical technique for the aforementioned use of mortality in tail factor estimation.

1.1 Research Context

Richard Sherman and Gordon Diss [2] note in their award-winning paper, "Estimating the Workers' Compensation Tail," that the workers compensation tail largely consists of the medical component of permanent disability claims. Their paper then presents a fairly complex method for utilizing incremental payment data prior to the standard triangle to extend development factors beyond the end of the triangle. Frank Schmid [3] further analyzed aggregate workers compensation loss triangles to explain the drivers of tail development in another technical contribution to the literature, "The Workers Compensation Tails."

In practice, however, we've found the above-referenced works difficult to put into use. These approaches require data that's often unavailable and assumptions that can result in a highly-parameterized model that may not lend itself to easy explanation.

1.2 Objective

We'd like to share a practical reserving technique that we've implemented and used to help more accurately reserve run-off books of workers compensation claims but that can be applied to any body of claims that has claimant mortality as the main driver of the length of the tail (e.g., unlimited PIP).

We believe it's a relatively simple and readily understandable extension of the traditional loss development techniques that many reserve practitioners use and can be scaled up or down in complexity based on the quality and availability of the underlying data.

Our method starts with traditional accident year paid and/or reported loss development triangles. Age-to-age factors are selected as far as the data reasonably allows. These selected factors are then fitted to a curve. This, we believe, is where many reserve practitioners stop or experience difficulty. Since most fitting techniques will allow development to go on indefinitely, the length of the tail is often selected based simply on actuarial judgment. Our method provides an actuary with a way to inform the length of the tail based on the underlying claim data.

The technique we use to inform the length of the tail begins with the determination and review of claimant life expectancy percentiles for all open claims. In its simplest application all one needs is an accident date and a date of birth for each claimant. We then try to answer the question, "If claimants in a given accident year or cohort group of accident years survive to some percentile of life expectancy, how do we expect to see their related losses develop to that point in time?"

1.3 Outline

The remainder of the paper proceeds as follows:

Section 2 will discuss data considerations.

Section 3 will present loss development and curve fitting.

Section 4 will examine mortality and life expectancy.

Section 5 will describe the adjustment of tail factors for life expectancy.

2. DATA CONSIDERATIONS

Ideally one would want to use this method for the medical component of paid losses only, as the indemnity component may be more heavily influenced by factors other than claimant mortality, such as statutory requirements or the payment of survivor benefits. However, we believe the approach has predictive power for almost any aggregation of workers compensation losses because claimant mortality is the main driver of the length of the tail regardless of the mix of indemnity versus medical or loss versus expense components, though a split of first-dollar exposures versus homogeneous groupings of excess coverages is important.

Similarly one would ideally want to match mortality tables as closely as possible to the characteristics of the underlying claimant population. In our experience though there is a declining return from increased precision unless the volume of underlying data is sufficiently credible.

3. LOSS DEVELOPMENT AND CURVE FITTING

Age-to-age loss development factors are selected as one would normally for traditional loss development methods. These selected factors are then fitted to a closed-form inverse power curve as described in Richard Sherman's "Extrapolating, Smoothing, and Interpolating Development Factors" [4].

In our example we evaluate a hypothetical book of run-off workers compensation business. Given a triangle of cumulative paid loss and expense (combined) we select age-to-age factors, including the judgmental selection of a tail factor, yielding cumulative age-to-ultimate factors. The results of this analysis are as follows:

Workers Compensation as of 12/31/2012 **($000s)**

Accident	ITD	Selected Development Factors	
Year	Paid	Age-to-Age	Cumulative
1993	62,574	1.034	1.034
1994	92,671	1.002	1.036
1995	103,027	1.003	1.039
1996	119,457	1.003	1.043
1997	169,521	1.003	1.046
1998	165,049	1.003	1.049
1999	206,325	1.004	1.053
2000	260,194	1.005	1.058
2001	279,992	1.005	1.063
2002	312,353	1.006	1.070
2003	362,792	1.007	1.078
2004	375,976	1.009	1.088
2005	294,499	1.013	1.102
2006	237,595	1.022	1.127
2007	168,798	1.031	1.162
2008	135,238	1.051	1.222
2009	125,394	1.089	1.330
2010	94,536	1.174	1.562
2011	67,674	1.378	2.151
2012	16,920	2.340	5.034

We then fit these selected age-to-age development factors, excluding the tail factor, to the inverse power curve.

$$\text{Inverse Power Curve} = f(t) = 1+a*t\hat{}b \tag{3.1}$$

where t = the age, b = the slope, $\ln(a)$ = the intercept, and $\ln(f(t)-1) = \ln(a)+b*\ln(1/t)$

Fit of Selected Development Factors to Inverse Power Curve

Accident Year	Age (t)	Selected LDF	$x = \ln(1/t)$	$y = \ln(\text{LDF-1})$
2012	1	2.340	0.000	0.293
2011	2	1.378	(0.693)	(0.974)
2010	3	1.174	(1.099)	(1.748)
2009	4	1.089	(1.386)	(2.420)
2008	5	1.051	(1.609)	(2.978)
2007	6	1.031	(1.792)	(3.466)
2006	7	1.022	(1.946)	(3.799)
2005	8	1.013	(2.079)	(4.321)
2004	9	1.009	(2.197)	(4.681)
2003	10	1.007	(2.303)	(4.893)
2002	11	1.006	(2.398)	(5.078)
2001	12	1.005	(2.485)	(5.290)
2000	13	1.005	(2.565)	(5.358)
1999	14	1.004	(2.639)	(5.603)
1998	15	1.003	(2.708)	(5.783)
1997	16	1.003	(2.773)	(5.702)
1996	17	1.003	(2.833)	(5.659)
1995	18	1.003	(2.890)	(5.963)
1994	19	1.002	(2.944)	(6.107)

The Excel functions for SLOPE and INTERCEPT are then populated with the array of x and y values from the above table yielding b = 2.28223156047852 and ln(a) = 0.539573651269289 as the inputs into the inverse power curve function. The fitted values are then calculated as shown below:

Fitted Development Factors

Accident	Development Factors		
Year	Age-to-Age	Fitted	Cumulative
1994	1.002	1.002	1.028
1995	1.003	1.002	1.030
1996	1.003	1.003	1.033
1997	1.003	1.003	1.036
1998	1.003	1.004	1.040
1999	1.004	1.004	1.044
2000	1.005	1.005	1.049
2001	1.005	1.006	1.056
2002	1.006	1.007	1.063
2003	1.007	1.009	1.073
2004	1.009	1.011	1.085
2005	1.013	1.015	1.101
2006	1.022	1.020	1.123
2007	1.031	1.029	1.156
2008	1.051	1.044	1.206
2009	1.089	1.072	1.293
2010	1.174	1.140	1.474
2011	1.378	1.353	1.994
2012	2.340	2.715	5.414

However, since this fit generates loss development factors indefinitely out into time, using the calculated cumulative loss development factors directly would likely overstate development in the tail. Due to this we review projected life expectancies for all open claimants in the underlying data and then use this information to adjust the length of the tail.

4. MORTALITY AND LIFE EXPECTANCY

As discussed by Elizabeth Arias in "United States Life Tables," 2004 [5] there are two types of mortality tables, the cohort life table and the period life table. A cohort life table presents the mortality experience of all persons born in a particular year. A period life table, which is what we use here, presents what would happen to a hypothetical cohort if it experienced throughout its entire life the mortality conditions of a particular period in time.

There are many opinions regarding which life table would be the most appropriate to use in a loss reserving context, but this paper does not attempt to answer that question. The choice of the appropriate life table to use is left to the practitioner to determine.

For this example we use the CDC's 2004 U.S. period life table for males to determine our life expectancies. We then calculate "life expectancy percentiles" for each age at various intervals from 60% to 90% which represent the percentage of lives that have left the population from a given point in time up to another point in the future. The life expectancy for a given percentile is the number of years until the remaining lives drops by the given percentage. The determination of the appropriate percentiles to calculate requires judgment and may depend on the number of underlying claimants in the data. However, the selections should also consider how much the population will need to shrink before future development is no longer likely to occur.

In addition, in certain instances where it has a material impact, we have weighted the statistics from male and female life tables together based on the gender distribution of the claimants in a given set of data. For example, if we wanted a 75% male | 25% female mix, we'd calculate the number of lives, $L(x)$, as a weighted average = $0.75\ L_m(x) + 0.25\ L_f(x)$ and then determine life expectancy percentiles using this weighted $L(x)$.

To find the p-percentile of mortality for age (x), we find the first age (a) at which: (4.1)

$$L(a) \leq (1\text{-}p) * L(x)$$

Where $L(x)$ = lives remaining at age x, and

the life expectancy at that percentile is then $(a\text{-}x)$.

For example, to find the 75th percentile of life expectancy for a 40-year old male (x = 40, p = 0.75) we start by going to the period life table and determining that L(40) = 95,527. We then calculate that (1 – 0.75) * 95,527 = 23,882. Another review of the table shows that L(87) = 24,413 and L(88) = 21,447. Therefore, a = 88 is selected and the related life expectancy is 88 – 40 = 48 years.

Returning to our hypothetical book of run-off workers compensation business, we then determine the life expectancy for claimants by accident year at each year-end from 1993 to 2012 by starting with a table (partially displayed below) of calculated life expectancies at various percentiles.

Calculated Male Life Expectancies by Age at Various Percentiles

Age	Number Surviving to Age x	Expectation of Life at Age x	Life Expectancy Percentiles		
	l(x)	e(x)	60%	75%	90%
40	95,527	37.6	43	48	53
41	95,294	36.7	42	47	52
42	95,043	35.8	41	46	51
43	94,772	34.9	40	45	50
44	94,477	34.0	39	44	49
45	94,154	33.1	38	43	48
46	93,803	32.3	37	42	47
47	93,421	31.4	36	41	46
48	93,007	30.5	35	40	45
49	92,560	29.7	34	39	44
50	92,078	28.8	34	38	43
51	91,558	28.0	33	37	43
52	90,998	27.2	32	36	42
53	90,398	26.3	31	35	41
54	89,761	25.5	30	34	40
55	89,089	24.7	29	33	39
56	88,381	23.9	28	32	38
57	87,633	23.1	27	31	37
58	86,839	22.3	26	30	36
59	85,987	21.5	25	29	35
60	85,067	20.8	24	29	34

Life expectancy percentiles for an accident year or cohort of accident years is subsequently calculated based on some weighting (e.g., the past three years of paid losses and/or open case reserves) of individual claimants:

Selected 75th Percentile Life Expectancy by Accident Year Cohort

Accident Years	Open Claims	3-Year Avg Paid	Average Case Reserve	Paid Weighted LE	Case Weighted LE	Selected LE
1993-1997	62	39,074	121,657	24.2	25.8	25.0
1998-2002	164	23,831	93,113	29.4	27.6	28.0
2003-2007	334	27,552	125,519	32.4	33.1	33.0
2008-2012	564	34,162	165,989	35.0	36.2	36.0

5. ADJUSTMENT OF TAIL FACTORS FOR LIFE EXPECTANCY

Now that we have the projected life expectancy for the claimant population the tail factor of the fitted age-to-ultimate development factors can be adjusted.

This is done by dividing the cumulative development factor (CDF) at the accident year's current age as of the evaluation period by the CDF at the accident year's current age plus the selected life expectancy percentile, or the accident year's selected terminal age. As mentioned previously, because each accident-year cohort of claims is made up of claimants with different ages, the weighted life expectancy of the cohort is used for the selected percentile.

For example: Assume the selected remaining life expectancy at the 75th percentile for Accident Year 2000 was determined to be 28 years. In addition, at the time of the analysis, Accident Year 2000 was 13 years old. In this example the, fitted cumulative loss development factor at time 13 is 1.049 and, based on the remaining life expectancy of 28 years, development is expected to end at time 41. Moving along the fitted values, the CDF at time 41 is 1.008, so the age-to-ultimate factor informed by the underlying life expectancy assumption is 1.049/1.008 = 1.042.

The results for all years in our example are displayed in the following table:

Fitted Paid Age-to-Ultimate Development Factors Adjusted for Life Expectancy

Accident Year	Evaluation Age	Selected 75th Life Expectancy Percentile	CDF at Accident Year's Current Age	CDF at Accident Year's Terminal Age	CDF Adjusted for Life Expectancy
1993	20	25	1.026	1.006	1.020
1994	19	25	1.028	1.006	1.021
1995	18	25	1.030	1.007	1.023
1996	17	25	1.033	1.007	1.026
1997	16	25	1.036	1.008	1.029
1998	15	28	1.040	1.007	1.033
1999	14	28	1.044	1.007	1.037
2000	13	28	1.049	1.008	1.042
2001	12	28	1.056	1.008	1.047
2002	11	28	1.063	1.008	1.054
2003	10	33	1.073	1.007	1.065
2004	9	33	1.085	1.007	1.077
2005	8	33	1.101	1.008	1.093
2006	7	33	1.123	1.008	1.115
2007	6	33	1.156	1.008	1.146
2008	5	36	1.206	1.008	1.197
2009	4	36	1.293	1.008	1.283
2010	3	36	1.474	1.008	1.462
2011	2	36	1.994	1.009	1.977
2012	1	36	5.414	1.009	5.365

6. RESULTS AND DISCUSSION

We typically use the calculated life expectancies directly to modify the length of the fitted tail for paid development as described in this paper. However as we generally expect reported development to end sooner than paid development, our calculated life expectancies are often judgmentally adjusted (e.g., shortened by 10 years) to reflect, on average, how long before final payment accurate case reserves are expected to be recorded for an accident year or cohort of accident years. This judgment can be informed either through discussion with the claims adjusting staff or based on a hindsight review of case reserve development for closed claims.

7. CONCLUSIONS

The practicing actuary often relies upon judgment when selecting the length of the tail to be used in estimating liabilities for workers compensation claims with traditional accident year paid and/or reported loss development methods.

This paper shares a practical reserving technique that can inform traditional loss development methods as to the length of the tail using claimant mortality. In the example presented above, and in more detail in the accompanying tool in Excel, the impact of the tail assumption is material and materially different when the life expectancy of the underlying claimant population is considered:

Comparison of Selected Gross Paid Loss and Expense Reserves ($000s)

		Selected CDFs			Selected Total Reserves		
Accident Year	ITD Paid	Traditional	Fitted	Fitted w/ LE Adj	Traditional	Fitted	Fitted w/ LE Adj
1993	62,574	1.034	1.026	1.020	2,123	1,616	1,251
1994	92,671	1.036	1.028	1.021	3,357	2,590	1,946
1995	103,027	1.039	1.030	1.023	4,006	3,128	2,370
1996	119,457	1.043	1.033	1.026	5,078	3,955	3,106
1997	169,521	1.046	1.036	1.029	7,796	6,149	4,916
1998	165,049	1.049	1.040	1.033	8,122	6,594	5,447
1999	206,325	1.053	1.044	1.037	10,952	9,135	7,634
2000	260,194	1.058	1.049	1.042	15,102	12,857	10,928
2001	279,992	1.063	1.056	1.047	17,744	15,571	13,160
2002	312,353	1.070	1.063	1.054	21,865	19,746	16,867
2003	362,792	1.078	1.073	1.065	28,307	26,389	23,581
2004	375,976	1.088	1.085	1.077	33,092	31,942	28,950
2005	294,499	1.102	1.101	1.093	30,180	29,782	27,388
2006	237,595	1.127	1.123	1.115	30,212	29,315	27,323
2007	168,798	1.162	1.156	1.146	27,409	26,276	24,645
2008	135,238	1.222	1.206	1.197	29,961	27,860	26,642
2009	125,394	1.330	1.293	1.283	41,403	36,795	35,487
2010	94,536	1.562	1.474	1.462	53,103	44,831	43,676
2011	67,674	2.151	1.994	1.977	77,913	67,273	66,117
2012	16,920	5.034	5.414	5.365	68,253	74,693	73,856
Total	3,650,585				515,978	476,496	445,290

We hope the technique described in this paper proves useful to the traditional actuarial reserving practitioner and provides the foundation for further work in this area.

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Abbreviations and Notations

CDC, Centers for Disease Control and Prevention
CDF, Cumulative Development Factor (Age-to-Ultimate)
ITD, Inception to Date
LDF, Loss Development Factor (Age-to-Age)
LE, Life Expectancy
PIP, Personal Injury Protection

Biographies of the Authors

Michael Shane, FCAS, MAAA is the Chief Actuary of The RiverStone Group, a global insurance run-off manager wholly owned by Fairfax Financial Holdings Limited. He has an MBA from Boston University and a BA in History from Rutgers College.

Dawn Morelli, ACAS, MAAA is an Actuarial Manager at The RiverStone Group responsible for the actuarial reserving and reporting function. She has a BA in Mathematics from Smith College.

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Applications of Reserve Ranges and Variability in Practice

Christopher Walker, FCAS, MAAA

Mark Littmann, FCAS, MAAA

Abstract

Motivation. Actuaries are faced with increased questions on reserve variability and "reasonable ranges." As consultants Mr. Littmann and Mr. Walker confront these questions frequently. We believe a survey of current uses of ranges, how actuaries may adequately address "reasonableness" in a way that is approachable to users, and how these methods may be bridged with more theoretical methods would be beneficial to actuaries and other interested parties.

Method. We present examples of sensitivity testing and how, using a consistent data set, illustrative conclusions on how range estimates may be derived. We also extend the examples with the application of the Mack technique for evaluating a distribution of possible outcomes and investigate the potential relationship between a range of reasonable estimates with distributions of possible outcomes.

Results. Our "results" are primarily illustrations that give the practitioner easy to develop ranges and may also provide a framework for application to a company's aggregate reserve position and "reasonable range."

Conclusions. "Actuarial judgment" will not satisfy the questions being asked of actuaries on ranges and variability. Actuaries need to have a structure in place to provide evidence/illustrations of variability.

Keywords. Reserve Variability; Reserve Ranges; Thomas Mack, sensitivity testing, distribution of possible outcomes.

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1. Introduction

There has been significant time and effort spent by our colleagues in the CAS and elsewhere in the development of models and approaches for quantifying variability in loss reserve estimates. Many of these models, whether parametric or non-parametric, are quite complex and identify important theoretical issues, such as correlation between coverage lines, etc. However, practitioners are often constrained by data, time, and budget to apply these models. Furthermore, the contexts in which the reserve estimates are applied, such as financial reporting requirements of "best estimates," often limit or eliminate the usefulness of variability model outputs.

The goal of this paper is to emphasize the increasing importance of not only recognizing variability in reserve estimates, primarily through the assessment of a "reasonable range" around an actuarial central estimate, but also in providing understandable support to the variability, or range, assessment. We believe that the role of "actuarial judgment" in the construction of reserve ranges without specific support has been significantly diminished, as users seek more quantitative evidence in support of the asserted range.

This paper begins with a discussion of areas where the concept of reasonable variability and reasonableness is commonly encountered or may be emerging. We will also provide a brief discussion of two theoretical approaches to reserve variability presented in the literature, and also the concept of sensitivity testing that is not cited in the literature but is commonly used. Next, approaches commonly used by practitioners are discussed, along with advantages and disadvantages. We then provide some simple illustrations of the application of sensitivity testing to form a view on a range of reasonable estimates, and also apply a stochastic model to evaluate the potential relationship among reasonable ranges and distributions of outcomes. We close with observations on simple techniques to address variability when aggregations of reserve segments are considered.

2. Business Applications of Variability Concepts

For those familiar with property/casualty insurance, the inherent uncertainty in the ultimate settlement value of unpaid claim liabilities is self-evident. The uncertainty arises from the fact that all events and conditions affecting the ultimate settlement value of claims have not yet occurred, and for various coverages there may be a significant portion of unasserted claims at a particular point in time. Other key sources of uncertainty include future inflation rates on the costs of goods and services, future attitudes of claimants and juries to potential valuations of damages, and future law and legal rulings that may be retrospective in nature.

Thus, actuaries evaluate historical data to assess reporting and payment patterns in order to make projections of ultimate losses and the associated unpaid claim liabilities. The actuaries may generate a point-estimate, a range of reasonable estimates, or a distribution of possible outcomes, depending on the purpose and intended use of the actuarial analysis.

For financial reporting purposes, businesses do not report "ranges" of income or "ranges" of balance sheets. Loss reserves presented on a traditional Generally Accepted Accounting Standards (GAAP) basis or a Statutory Accounting Principles (SAP) basis are a unique number, not a range of estimates. However, users of such reports, such as the SEC and investors, increasingly require not just disclosure of the carried amounts but also the relative uncertainty in these estimates. This trend leads to the discussion of reserve variability and reserve ranges.

Examples of additional disclosures include the following:

2.1 Statements of Actuarial Opinion and Actuarial Opinion Summary

State regulators require a discussion of the business and its qualities that may introduce variability into the carried loss reserves being opined upon in the Statement of Actuarial Opinion (SAO). Regulators also require that an assessment be made of the risk of material adverse deviation (RMAD) in the recorded loss reserves. The variability issue also flows to the Actuarial Opinion Summary (AOS) where the Opining actuary has the option of listing a point estimate of the reserves or a range of reserves. In determining this range, actuaries are guided by Actuarial Standard of Practice (ASOP) No. 36, *Statements of Actuarial Opinion Regarding Property/Casualty Loss and Loss Adjustment Expense Reserves*, which states that an

> "actuary should consider a reserve to be reasonable if it is within a range of estimates that could be produced by an unpaid claim estimate analysis that is, in the actuary's professional judgment, consistent with both ASOP No. 43, *Property/Casualty Unpaid Claim Estimates*, and the identified stated basis of reserve presentation."

In turn, ASOP No. 43 describes considerations to be taken into account by the actuary in the analysis, aligned with the intended purpose. In this context, this "range of reserves" concept has no set definition, but it is generally approached from the standpoint of a range of reasonable estimates as opposed to a distribution of possible outcomes.

2.2 Securities and Exchange Commission filings

A fairly common disclosure in 10-K reporting is a discussion of the analysis that developed the carried reserve and the variability inherent in that estimate. For many years, the filings have included a table that shows the development of reserve estimates recorded at prior reporting dates, updated for the subsequent valuations that have occurred up to the latest reporting date. (This disclosure is comparable to the development of accident year ultimate loss and Defense & Cost Containment (DCC) estimates presented in Schedule P, Part 2, of the statutory basis Annual Statement.) The Securities and Exchange Commission (SEC) was particularly active in questioning insurers several years ago in light of filers posting large reserve redundancies/deficiencies related to prior accident years in their financial statements. Not only did the SEC focus on management's development of its "best estimate" but it also required discussion that would help investors better understand the risks and uncertainties that are inherent in that estimate and in the business as a whole. In response, the registrants have expanded their disclosures, both qualitatively and quantitatively. For example, one company with which we are familiar dedicates over twelve pages (out of 110 in total) of its 10-K filing to loss reserves and includes specific discussions on "Significant Risk Factors," "Determination of Best Estimate," and "Reserve Sensitivities." Within these categories, and from other companies filings, we have observed a variety of quantitative disclosures on variability such as confidence levels associated with low, reasonable, and high values, assessments of the impact of changes in key assumptions such as tail factors, frequency/severity, and/or inflation, and the prospective performance of reinsurance programs.

2.3 Financial Audits

Even when regulatory reporting is not a top consideration, such as in the financial reporting of privately held non-insurance companies, there is considerable focus by auditors on estimates, including loss reserves. A typical situation could be illustrated as a privately held manufacturer that may choose to self-insure its Product Liability exposure and is required to recognize a reserve for the unpaid losses incurred as of the accounting date. The auditors are faced with assessing the reasonableness of the reserve established. In performing such assessments, the auditors may be faced with a wide range of circumstances and assumptions that may or may not make sense for the situation. Auditors recognize that there may be differences in their point of view and those taken by the company; however, they are frequently faced with the dilemma of "how much difference is too much" and have generally applied formulaic materiality thresholds. Such thresholds, such as a -5%/+5% or -10%/+10% range around an independent reserve estimate, may or may not have a theoretical basis.

Auditors often rely on other measures, such as a balance's materiality to the financial statements as a whole, and this fact may render differences between the auditor's unpaid claim estimates and carried values moot; however, for insurance companies, loss and LAE reserves are usually the largest and most material liability on the balance sheet and, therefore, small differences between estimates and carried reserves may be highly material to the audit as a whole.

2.4 Mergers and Acquisitions

In both insurance and non-insurance transactions, there may be significant unpaid liabilities involved, some highly uncertain. For example, many insurers and manufacturers have legacy asbestos liabilities that continue to be a drain on current earnings. In these cases, an understanding and quantification of the potential for continued development may be a prime consideration as there may be a "true up" (evaluation several years after the close of the deal) involved or the purchase of third-party insurance/reinsurance as a condition of completing the transaction. In these cases, the idea of a reserve confidence interval or a range of possible outcomes, as opposed to a range of reasonable estimates, may be highly relevant metrics for pricing reinsurance and establishing horizons for "true ups."

2.5 Internal Revenue Service Considerations

The Internal Revenue Service (IRS) disallows explicit loss reserve margins for the calculation of insurance company federal income tax. The IRS may review various sources of documentation, such as the report of the appointed actuary or findings from the external auditor, in its audit to evaluate whether the recorded reserves included a margin, either explicit or implicit. The IRS also appears to be performing more detailed independent reviews of loss reserves even in the absence of explicit margins to support its audits. While we are not aware of definitive guidance, and as of this writing there are various on-going legal challenges, the quality of an analysis of a "reasonable range of estimates" could ultimately factor into IRS positions on company reserve redundancies.

3. Variability Concepts in the Literature

The literature of the CAS includes a broad range of papers and presentations on the topics of variability and distribution of reserve estimates. However, these generally do not offer any guidance for which portion of the distribution would constitute a reasonable range of estimates.

3.1 Thomas Mack Method

This approach is a "distribution free" technique to measure the variability of reserve estimates generated by a traditional application of the loss development (aka, chain ladder) method to a typical loss development triangle. The technique is relatively easy to apply, including being available in an Excel spreadsheet template that is publicly available on the CAS website or in commercially-available software. This approach yields an estimate for the estimated standard error (ESE) of a distribution of unpaid claim outcomes.

3.2 Boot Strapping

A basic premise of this approach is that the available data (typically a loss development triangle) is essentially "one" observation from a distribution of possibilities. Thus, the technique assumes there is a singular loss development pattern which is indicative of the "true" pattern, and views the data as random observations from this true pattern. Thus, the user prepares an estimate of the "true" pattern, often based on an average of the observed link ratios in the historical data, and then evaluates the residuals, being the differences between the actual observations and the observations that would be consistent with the true pattern. In this way, Boot Strapping is a method of re-sampling that allows the user to make inferences on the variability of mean values and distribution of possible outcomes. It is essentially a simulation process that requires many iterations (say, 1000) and the output is a distribution of possible outcomes. Similar to the Mack approach, there is no guidance as to how "a range of reasonable estimates" may compare to the derived distribution of possible outcomes produced via Boot Strapping.

3.3 Sensitivity Testing

While not a frequent subject described in the actuarial reserving literature, we believe that sensitivity testing[1] is one of the most prevalent approaches used to establish ranges of reasonable reserve estimates. This approach is not technically advanced, nor do we consider it to be a distinct method. Sensitivity testing essentially means that an actuary tests the effects of alternate judgments for the key parameters of the chosen method(s) in order to evaluate alternate low and high estimates of the unpaid claim liabilities. Thus, the relative ease by which the approach can be explained is a distinct advantage.

ASOP No. 36 and ASOP No. 43 implicitly acknowledge the concepts of sensitivity testing. ASOP No. 36 recognizes that an actuary may consider a reserve to be reasonable if it is within a range of estimates derived from appropriate methods and reasonable assumptions. ASOP No. 43 recognizes that an "actuarial central estimate" is an "expected value over a range of reasonably possible outcomes."

Considering that alternate methods may be considered appropriate for a particular unpaid claims analysis, and that alternate assumptions for the key parameters of the methods may be considered reasonable, sensitivity-testing is a natural indicator of reserve variability determined by replicating the collection of methods applied to various data sets and substituting high and low selections for the key parameters of the actuarial analysis. The most commonly-applied actuarial methods are the loss development (chain ladder) method and the Bornhuetter-Ferguson method, which is a blending of the loss development and expected loss method. These methods' key parameters are loss development link ratios, tail factors, and *a priori* expected loss ratios. These parameters can be modified to simulate the underlying drivers of reserve variability. For example, loss development factors/tail factors can be increased/decreased to represent inflation higher/lower than that represented in the underlying data. As another example, *a priori* loss ratios can be adjusted to reflect actual rate

[1] The International Actuarial Association recently published a paper on Stress Testing and Scenario Analysis by the Insurance Regulation Committee. While the paper highlights the role of stress testing and scenario analysis to enhance the risk culture of an organization, the framework may be useful for specific consideration of the variability of estimates of unpaid claims.

and schedule rating changes that may have proven to be different that contemplated in pricing or planning projections.

As additional considerations, if the analysis includes a frequency/severity approach whereby component estimates for the number of claims and the average claim value are used to project ultimate claims costs, then varying the assumptions as to the trend rate in claim frequency and/or claim severity reflecting uncertainty in underlying loss cost drivers may be appropriate. In the case of a reserve analysis segment containing minimal historical data, incorporating different external benchmark parameters may serve as reasonable examples of sensitivity. We also consider the actuary's judgment to form a final point-estimate from among multiple preliminary projections to be a key consideration.

In essence, the application of sensitivity testing may require the actuary to perform an analysis three times reflecting low, central, and high estimates, and there may be many ways to reach each estimate. This labor-intensive feature of the approach may be considered a disadvantage. On the other hand, the approach is simple to apply, easy to understand, and, perhaps more importantly, is easily communicated to a third-party in light of central estimates.

4. Approaches in Practice

We observe that the range of approaches that are commonly used in the P&C industry to evaluate potential distributions of outcomes or to evaluate ranges of reasonable estimates is narrower than the range of approaches described in the literature. Quite simply, some of the methods in the literature, while being theoretically and conceptually sound, are difficult to apply in practice and perhaps even more difficult to explain, particularly in a financial reporting context, to the various stakeholders possessing varying degrees of analytical sophistication. We observe that the more technically-robust algorithms, such as development of specific loss distributions, are commonly applied to provide the inputs required for other applications, such as economic capital models.

Stochastic methods such as Boot Strapping or the Thomas Mack (Mack) technique may be used to evaluate distributions of possible outcomes, but we rarely observe these being used to describe ranges of reasonable estimates. These methods evaluate the variability of the historical data in the context of the chosen method for projecting ultimate claim values, while a range of reasonable estimates is more akin to a range of actuarial central estimates, or a range of expected means of the distributions given various parameter assumptions. Neither the Boot Strapping nor Mack approach can, by itself, respond to the question at the heart of financial reporting faced by practitioners: "To what portion of the distribution of outcomes does the range of reasonable estimates align?"

4.1 Judgment

In the context of reasonable ranges, judgment, or "support by experience," is often cited as the basis for an actuary's central estimate and a range of reasonable estimates. In some cases, actuaries or others may resort to "rules of thumb" or "arbitrary" judgments, such as plus or minus 5%, or plus or minus 10%. These judgments reflect merely an assumption as to the variability of the reserve estimates; in some circumstances, such as a financial reporting context, they may also reflect other metrics such as a certain proportion to shareholder equity (policyholder surplus) or net income.

We observe that ranges based on judgment alone are coming under increased scrutiny by external auditors as well as state and federal regulators. The use of "judgment" alone, without substantive analytical or qualitative evidence, is often considered a fallacious appeal to authority.

4.2 Sensitivity-Testing

This method can be used to derive ranges of reasonable estimates, though there is no common "standard" for performing sensitivity tests. However, we have observed some commonalities. For example, workers' compensation variability is often illustrated by adjusting tail factors to represent changing mortality, and property variability may be illustrated by adjusting claim severity to represent inflationary effects.

For situations where the substitution of alternate parameters in traditional actuarial methods may not be appropriate, we also see that illustrations of high and low estimates may reflect the inclusion/exclusion of high-valued events, such as policy limits Products Liability claims, in immature policy years.

5. Illustrations of Sensitivity Testing and Mack-Based Calculations

5.1 Sensitivity Testing

There are several levels at which sensitivity-testing within the framework of a typical analysis of unpaid claims estimates can be applied:

- Evaluate the dispersion of indications from one or more methods applied to one or more types of data. An actuary might elect to evaluate the dispersion of indications for all accident years combined, or for each accident year.
- Evaluate the effect of alternate judgments for the key elements of the methods as applied to the various sets of data, and generally keep the judgment about relative preferences among the methods the same.

Although we include illustrations of both approaches below, we would consider the second approach to be preferred.

The illustrations below are based on a data set that we consider indicative of personal automobile liability development and variability, but not associated with any actual company. The data consists of historical development of paid and reported losses by accident year at annual valuations. The loss development and Bornhuetter-Ferguson methods are applied to both types of data, generating four preliminary estimates of ultimate losses for each accident year. For simplicity, we keep the examples confined to the latest ten accident years, recognizing that actual company data may extend beyond ten years.

Consider the following illustrative preliminary ultimate loss projections shown in:

Table 1

	Projections of Ultimate Losses			
Accident Year (AY)	Loss Development on Paid	Loss Development on Reported	Bornhuetter-Ferguson on Paid	Bornhuetter-Ferguson on Reported
2003	1,127	1,157	1,127	1,157
2004	1,179	1,193	1,179	1,193
2005	1,089	1,119	1,090	1,119
2006	1,128	1,169	1,129	1,169
2007	1,608	1,634	1,603	1,634
2008	1,418	1,466	1,416	1,465
2009	1,430	1,463	1,430	1,463
2010	1,440	1,473	1,456	1,476
2011	1,800	1,782	1,693	1,739
2012	1,597	1,565	1,574	1,564

Using the minimum and maximum of the projections for each accident year for evaluating a potential range of reasonable estimates, the results are shown in Table 2:

Table 2

AY	Minimum	Mean	Maximum
	Projections of Ultimate Losses		
2003	1,127	1,142	1,157
2004	1,179	1,186	1,193
2005	1,089	1,104	1,119
2006	1,128	1,149	1,169
2007	1,603	1,620	1,634
2008	1,416	1,441	1,466
2009	1,430	1,447	1,463
2010	1,440	1,461	1,476
2011	1,693	1,753	1,800
2012	1,564	1,575	1,597
Sum	13,669	13,878	14,074
Inception-to-date Paid	11,690	11,690	11,690
Unpaid Claim Estimate	1,979	2,188	2,385
Difference to Mean	(209)		196
Difference as % Mean	-10%		9%

As shown in Table 2, if the actuary deems each of the projections to be reliable and is indifferent as to their relative merits, and the actuary considers that each year's estimate is independent of the next, the high and low projections are used to evaluate the end-points of a range of reasonable estimates. This approach yields a range of unpaid claim estimates that extends from 10% less than to 9% greater than the mean of the projections.

On the other hand, the actuary might choose to evaluate the dispersion of the projections on an all-years basis for the four projections, as shown in Table 3:

Table 3

	Ultimate Loss Projections			
	Loss Development on Paid	Loss Development on Reported	Bornhuetter-Ferguson on Paid	Bornhuetter-Ferguson on Reported
AY's 2003 - 2012	13,816	14,021	13,698	13,978

	Minimum	Mean	Maximum
Ultimate Loss Projection	13,698	13,878	14,021
Inception-to-date Paid	11,690	11,690	11,690
Unpaid Claims Estimate	2,008	2,188	2,331
Difference to Mean	(181)		143
Difference as % Mean	-8%		7%

The indicated range of unpaid claim estimates based on the all-years approach extends from 8% less than the mean estimate to 7% greater than the mean estimate. Due to the feature that no method consistently generated the highest or lowest of the four projections for each accident year, the range is narrower than on an "each accident year" basis.

These two variations of evaluating a range of reasonable estimates do not, however, reflect the actuary's judgment for the relative reliability and/or predictive value of the various methods and data-types. For example, for the Products Liability line of business, use of the paid loss development method for relatively immature accident years may be inappropriate and subject to extreme variation over time.

Thus, we suggest that a deliberate analysis of low and high estimates using alternate yet reasonable assumptions and judgments is preferable to a rote derivation based on maximums or minimums, whether on an "each year" basis or "all-years" basis.

In the numerical examples that follow, we utilize the illustrative matrix of weights for each projection by accident year shown in Table 4, in order to form a blended point-estimate:

Table 4

AY	Weights to the Alternate Projections Loss Development on Paid	Loss Development on Reported	Bornhuetter-Ferguson on Paid	Bornhuetter-Ferguson on Reported
2003	33%	67%	0%	0%
2004	33%	67%	0%	0%
2005	33%	67%	0%	0%
2006	33%	67%	0%	0%
2007	33%	67%	0%	0%
2008	32%	67%	0%	1%
2009	31%	66%	1%	2%
2010	29%	66%	2%	4%
2011	24%	62%	4%	10%
2012	14%	50%	8%	28%

The matrix reflects judgments that reported loss data provides more predictive reliability than the paid loss data, with the loss development projections assigned more weight than the BF projections, generally in proportion to the expected reported loss emergence pattern.

Applying the matrix of weights shown in Table 4 to the set of preliminary projections shown in Table 1 yields an ultimate loss estimate of $13,940 and a corresponding unpaid claims estimate of $2,250.

We extend the illustration with assumptions that will form a high-but-reasonable estimate. For simplicity's sake, we assume that the low-but-reasonable estimate is less than the point estimate by the same dollar amount as the high estimate is greater than the point estimate. In other words, we assume that the range of reasonable estimates would be symmetrical around the point estimate. (However, we do recognize that asymmetrical reasonable ranges are very often reported in practice.) We evaluate the effects on the estimate from alternate judgments for the key parameters of the loss development and Bornhuetter-Ferguson methods, namely the loss development factors (LDF's) and expected loss ratios (ELR's).

- Loss development factors: We considered the dispersion of various averages of the historical development factors as indicative of the potential variation of judgments that could be deemed reasonable. In simple terms, an actuary may deem the 5-year average link ratio to be indicative for ultimate loss projections. Another actuary may deem the 3-year or 7-year average to be indicative and reasonable. Alternate judgments may reflect assumptions for future inflation to be higher or lower than the levels embedded in the historical data, or for claim payment or reporting to be faster or slower than during the experience period. For reported losses, the baseline and alternate (high) link ratios and development factors to ultimate are shown in Table 5:

Table 5

Reported Loss Development – Link Ratios and Development Factors to Ultimate

	12 -24 Months	24 -36 Months	36 -48 Months	48 -60 Months	60 -72 Months	72 -84 Months	84 -96 Months	96 -108 Months	108 -120 Months	120 Months to Ultimate
Link Ratios										
Baseline	1.350	1.099	1.031	1.017	1.010	1.001	1.000	1.000	1.000	1.000
Alternate (High)	1.380	1.109	1.036	1.022	1.013	1.003	1.000	1.000	1.000	1.000

	12 Months to Ultimate	24 Months to Ultimate	36 Months to Ultimate	48 Months to Ultimate	60 Months to Ultimate	72 Months to Ultimate	84 Months to Ultimate	96 Months to Ultimate	108 Months to Ultimate	120 Months to Ultimate
Development Factors to Ultimate										
Baseline	1.573	1.165	1.060	1.028	1.011	1.001	1.000	1.000	1.000	1.000
Alternate (High)	1.646	1.193	1.076	1.038	1.016	1.003	1.000	1.000	1.000	1.000

Likewise, judgments were made for the baseline and alternate (high) link ratios for paid losses.

- Expected loss ratios. Different actuaries may have different judgments for ELR's for the Bornhuetter-Ferguson method, considering alternate sources of information. These sources include expected or target loss ratios from a company's pricing or business planning process, or peer company or industry external benchmark information. Alternatively, an actuary might adjust historical projected loss ratios for mature accident periods to current levels for loss trend and changes in pricing levels. A company's history of failing to achieve intended price changes may lead the actuary to select a higher ELR assumption as an alternative scenario. For our illustration, we considered the dispersion of alternate projections based on paid and reported development to be an indicator for alternate ELR judgments, as shown in Table 6:

Table 6

	2012	2011	2010	2009	2008	2007	2006	2005	2004	2003
Weighted LR from Loss Development Projections	68%	76%	62%	63%	63%	71%	52%	53%	59%	65%
Baseline ELR	67%	63%	65%	62%	59%	55%	59%	62%	65%	65%
Alternate (high) ELR	70%	65%	66%	62%	59%	55%	59%	62%	65%	65%

A comparison of the baseline and alternate (high) estimates based on the alternate judgments for LDF's and ELR's are shown in Table 7:

Table 7

	Estimated Ultimate		Unpaid Claims Estimate	
AY	Baseline	Alternate (High)	Baseline	Alternate (High)
2003	1,147	1,147	20	20
2004	1,188	1,188	11	11
2005	1,109	1,109	23	23
2006	1,155	1,155	35	35
2007	1,626	1,628	41	44
2008	1,451	1,457	92	99
2009	1,453	1,467	162	176
2010	1,464	1,487	286	309
2011	1,778	1,824	580	626
2012	1,570	1,646	1,000	1,076
Sum	13,940	14,108	2,250	2,418
			Difference	168
	Difference as % Baseline Unpaid Claims Estimate			7%

Thus, based on this example, the high-but-reasonable unpaid claims estimate is $2,418, or $168 (7%) greater than the baseline estimate. In the context of this example, we consider this to be indicative of the high-end of a reasonable range of unpaid claims estimates. With our assumption of symmetry of the high and low estimates relative to the central estimate, the low estimate is $2,082.

5.2 Mack-based Calculations

Continuing with the same sample data set, we supplemented the sensitivity-testing by applying the Mack approach for evaluating a measure of variation in the projections. Table 8 shows the estimated standard error (ESE) of the ultimate loss projection for each accident year and all years combined based on applying the Mack technique to the historical paid and reported loss development data with the same baseline loss development factors as used in the sensitivity testing above. The amount of the ESE of the ultimate loss projection is the same as the ESE of the unpaid claim estimate since the difference (the amount of the known inception-to-date claim payments) is a constant. We observe that the ESE calculated by the Mack approach does not incorporate the variability of any tail development beyond the oldest maturity of the historical data.

Table 8

AY	Estimated Standard Error: Paid Data	Estimated Standard Error: Reported Data
2003		
2004	1	1
2005	1	1
2006	1	2
2007	2	2
2008	21	23
2009	30	33
2010	31	33
2011	109	76
2012	166	132
All Years	219	175

We observe that the ESE based on the paid loss development data is greater than the ESE based on reported loss development data. This is a feature of the sample data set and not necessarily indicative that ESE's based on paid development data are always greater than the ESE's based on reported loss development data.

In order to generate a distribution of possible outcomes for the unpaid claims amounts, we chose an ESE of $197, based on the average of the two indicated ESE's. The chosen ESE was equivalent to 9% of the baseline unpaid claim estimate of $2,250.

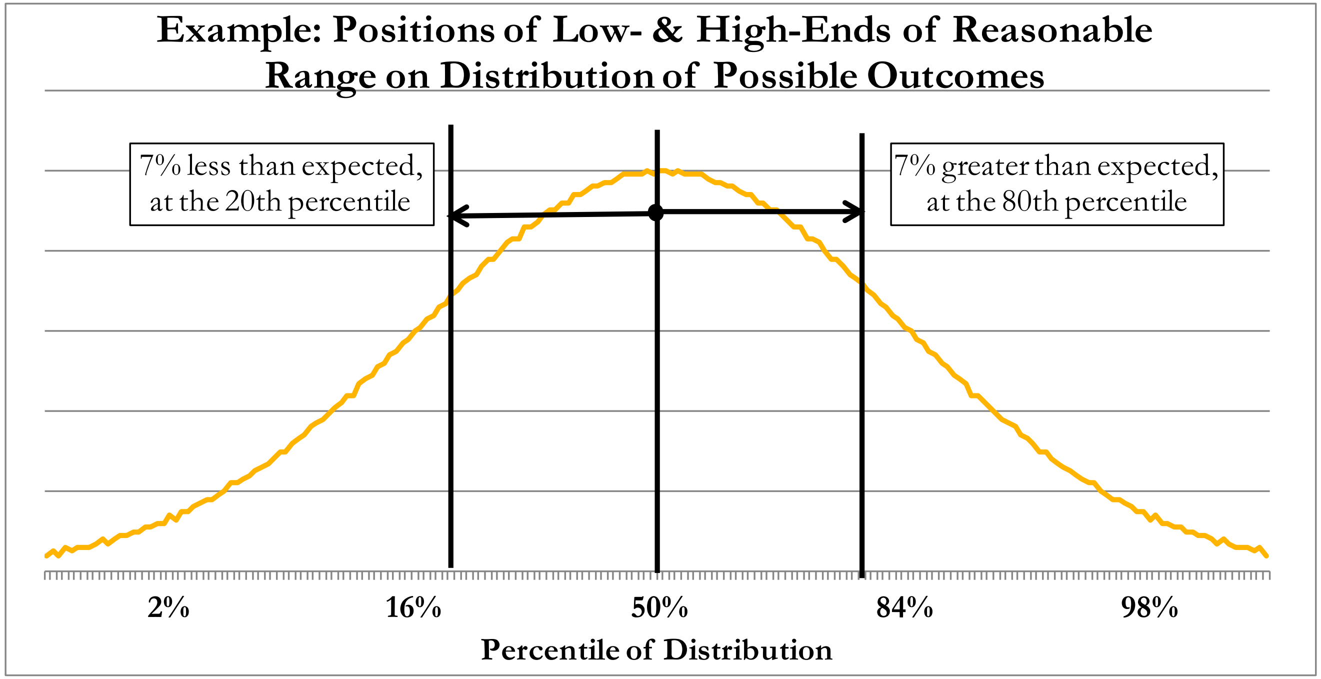
In keeping with the spirit of the non-technical nature of this review, we elected to assume a normal distribution to characterize the dispersion of possible outcomes of unpaid claim amounts. One of our goals with this paper was to describe a framework for connecting information about a reasonable range of estimates based on sensitivity testing to information about a distribution of possible outcomes based on a stochastic approach such as the Mack technique. To that end, we evaluated the end-points of a confidence interval around the mean from a normal distribution with a standard deviation based on the selected ESE from the Mack technique, where the confidence interval would align with the range of estimates generated by the sensitivity testing. The results are shown in Table 9:

Table 9

Percentiles of Distribution		Unpaid Claim Estimate	
Low	High	Low	High
20%	**80%**	2,082	2,418

In this case, the range based on the sensitivity testing extended from 7% less than to 7% greater than the baseline estimate. The 20th and 80rd percentiles of the distribution of outcomes based on our analysis with the Mack technique aligned with this range. This relationship is illustrated in Chart 1:

Chart 1



The chart illustrates the normal distribution, by the familiar bell-shaped curve, with x-axis markers at the 2nd, 16th, 50th, 84th, and 98th percentiles of the distribution.

2nd percentile	amount that is 2 standard deviations less than the mean
16th percentile	amount that is 1 standard deviation less than the mean
50th percentile	the mean amount
84th percentile	amount that is 1 standard deviation greater than the mean
98th percentile	amount that is 2 standard deviations greater than the mean

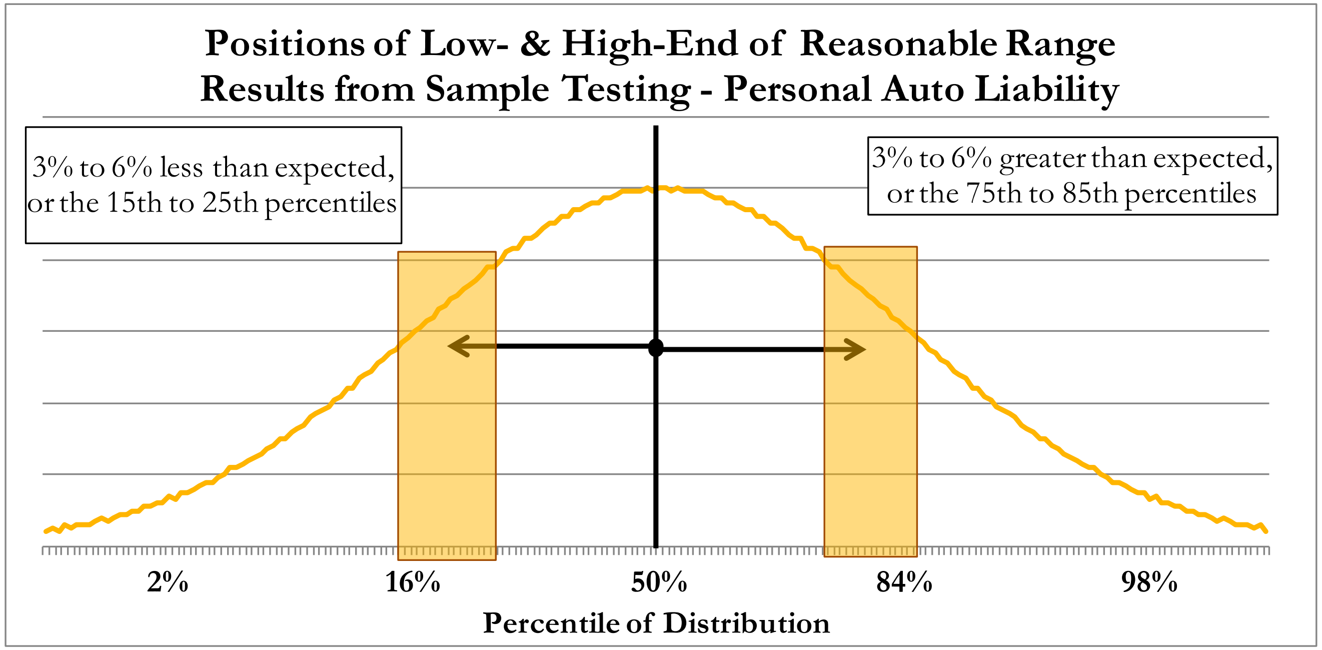
The three vertical lines correspond with the low, central, and high estimates from the example. Stated another way, the 7% differential from the baseline mean unpaid claim estimate to the high estimate based on sensitivity testing was equivalent to 0.85 of the ESE (also known as standard deviation) from the Mack-based distribution analysis.

5.3 Exploring a Potential Relationship between Sensitivity-based Ranges and Mack-based Distributions

We applied the approaches described above (supporting Tables 4 to 9) to a set of publicly-available data for Personal Auto Liability, Homeowners, and General Liability – Occurrence coverage data for 10 insurance companies. The findings shared herein are intended to be indicative of the application of the framework for integrating metrics from the sensitivity approach and a stochastic approach in order to help establish a potential connection between a range of reasonable estimates and a distribution of possible outcomes. These are not intended to be construed as "the definitive statement" on the relationship between the two approaches.

The results from our sample testing for Personal Auto Liability are summarized in Chart 2:

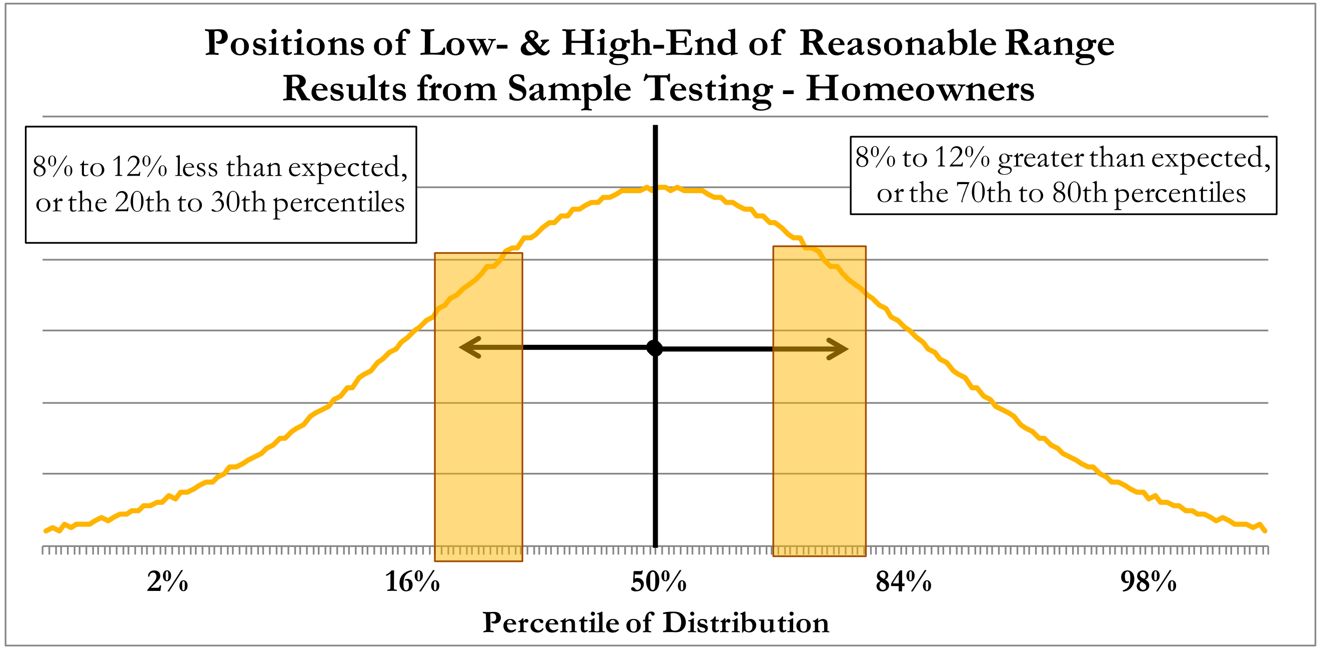
Chart 2



Based on the sample testing for Personal Auto Liability, we observe that high-ends of the reasonable estimate reserve ranges were generally 3% to 6% above the baseline estimate, and these high-ends tended to correspond with distribution percentiles at the high-end of confidence intervals that were generally in the 75% to 85% interval. With our assumption of a symmetrical range of reasonable estimates and distribution of outcomes, the low-ends of the reasonable range tended to correspond with distribution percentiles at the low-end of confidence intervals that were generally in the 15% to 25% interval.

The results from our sample testing on Homeowners data is shown in Chart 3:

Chart 3



Based on the sample testing for Homeowners multi-peril coverage, the reasonable reserve range high-ends were generally 8% to 12% above the baseline estimate, corresponding with high-end percentiles that were in the 70% to 80% range.

We also performed our testing on a sample of company data for General Liability – Occurrence coverage. The results are shown in Chart 4.

Chart 4

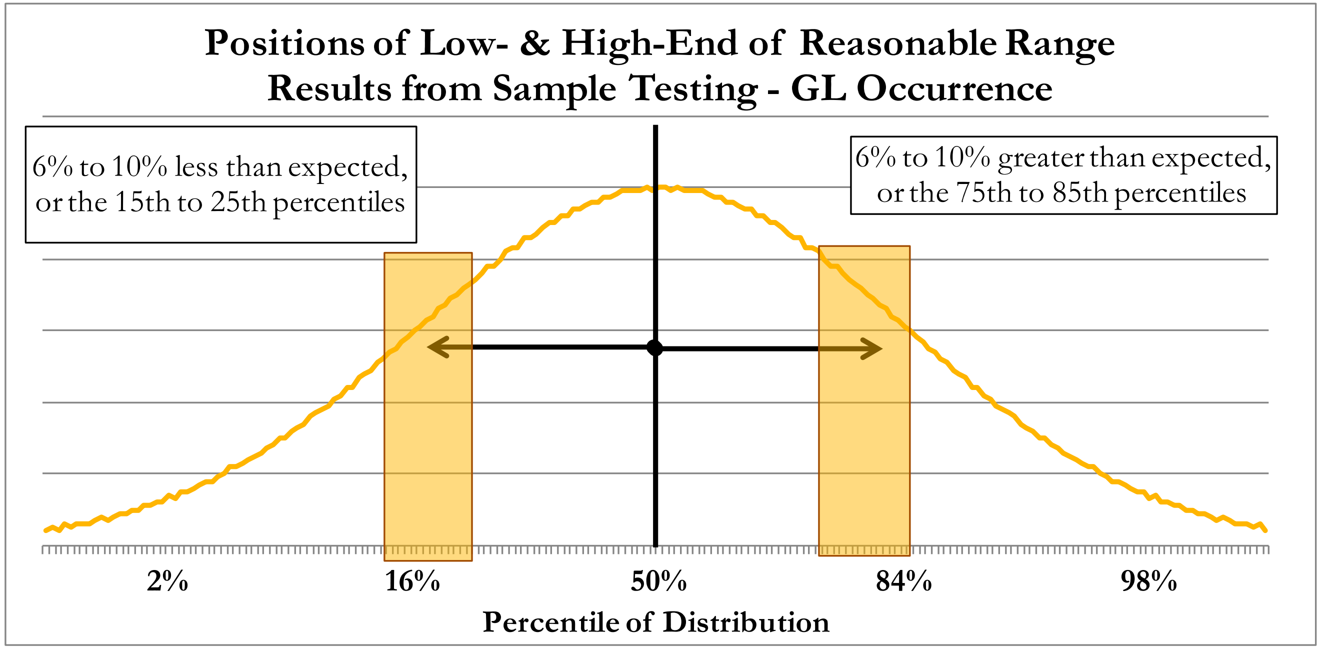


Table 10 summarizes our observations from our testing for the three lines.

Table 10

	High-End Reasonable Range as % Reserves	Percentiles of Distribution aligning with High-End of Reasonable Range	# Std Dev's from Mean to High-End of Reasonable Range	Estimated Standard Deviation of Distribution
Personal Auto Liability	3% to 6%	75th to 85th	0.7 to 1.0	3% to 7%
Homeowners	8% to 12%	70th to 80th	0.6 to 0.9	12% to 16%
GL Occurrence	6% to 10%	75th to 85th	0.7 to 1.0	6% to 12%

The breadth of the range (difference from the reasonable range high-end to the mean/baseline estimate of unpaid claims) expressed in relation to the unpaid claim estimate for Homeowners tended to be larger than for GL-Occurrence and Personal Auto Liability. We believe this is a consequence of Homeowners having a greater proportion of the ultimate loss estimates being paid for a given accident year maturity than the other lines, and thus the measure of uncertainty/range was relatively greater in proportion to the unpaid claim estimate. We also observe that actual reserves for these lines recorded by insurance companies may have subsequently developed within these indicated reasonable ranges or beyond the end-points (low or high) of the ranges. Tracking the actual development of recorded amounts may be another way to consider an evaluation of ranges of reasonable estimates, but that would incorporate an element of hindsight testing, while we are considering the reasonability of estimates based on information available at a point in time.

6. Considerations of Ranges on an Aggregate Basis[2]

While an evaluation of a range of reasonable estimates for an individual business segment has an inherent degree of difficulty, the challenge is elevated for an evaluation of a reasonable range or a distribution of outcomes on an aggregated basis. The higher degree of difficulty is, in part, due to the need to consider and reflect potential correlations among reserve segments. Nevertheless, since actuarial opinions are primarily given for a company, for which there are generally multiple analysis segments, or for a corporate group, comprised of multiple

[2]In this section we explore considerations for an aggregate range of reasonable estimates only. The CAS literature contains a variety of papers describing approaches for evaluating aggregate distributions of possible outcomes. Our primary focus throughout this Call paper has been on ranges of reasonable estimates.

companies, the actuary must consider how to approach the analysis of ranges on an aggregated basis. We believe that there are merits in a "bottom-up" approach and a "top-down" approach as discussed below.

6.1 Bottom-Up Approach

Under a bottom-up approach, the actuary would first evaluate ranges for individual reserve segments, and then form an aggregate range. An analysis of the ranges for individual lines, coverages, or other attributes by which the data are organized or the business is managed can provide management with insights on the relative certainty of estimates of ultimate losses and the associated unpaid claims. The fundamental issue in the aggregation is the consideration of potential correlations among the various reserve segments. If all segments are deemed to be independent of each other, than a simple "square root of the sum of the squares" may be practical and sufficient in evaluating an aggregate range. If all segments are deemed to be fully (and positively) correlated with each other, then the sum of the high and low ends of the individual ranges would be indicative of an aggregate range.

Situations in-between these two extremes can be tricky. The practitioner can make judgments for the correlations, or may perform advanced calculations in an attempt to quantify correlations in development among the different pairs of segments. As a simplified alternative, the actuary could assume 100% and 0% correlations to calculate the two aggregate indications, and form an aggregate view on correlation in order to construct a weighted-average of the two aggregate indications.

In practice, we often observe actuaries simply summing up the "low" ends and, similarly, the "high" ends, to development a range of reserves in the aggregate.

6.2 Top-Down Approach

An alternative to a bottom-up approach to evaluate a range of reasonable estimates at an aggregate level would be to evaluate the potential variation in central estimates by applying sensitivity testing or the Mack technique to aggregated data. We do not generally advocate an analysis of aggregated data for evaluating a point estimate, but consider it potentially useful to perform sensitivity testing or stochastic analysis in order to assess an aggregate range of reasonable estimates. We observe that the mix of the underlying coverages should be relatively stable over the experience period for such an analysis of aggregate data; to the extent that there are substantial shifts of the mix of business (for instance, relative proportion of long and short tail business), we would caution against this approach. When the underlying data is satisfactory for this purpose, the top-down approach has a relative advantage of implicitly handling correlation among the underlying business segments.

The illustration presented in section 5.1 above yielded an estimated range of reasonable estimates of the unpaid claims that extended from 7% less than to 7% greater than the point-estimate of $2,250; this segment will be referenced as Line 1. We performed a similar analysis for Line 2, for which the estimated range of reasonable estimates of unpaid claims extended from 9% less than to 9% greater than the point-estimate of $1,000. We also performed a similar analysis on the combined data for the two lines, for which the range extended from 6% less than to 6% greater than the point estimate of $3,168[3]. The illustrative results are summarized in Table 11.

Table 11

	Unpaid Claims Estimate (UCE)	High-end of Reasonable Range minus UCE	High-end of Reasonable Range as % Reserves
Line 1	2,250	168	7%
Line 2	1,000	91	9%
Combined	3,168	191	6%

If the two lines were 100% correlated, then the difference from the central estimate to the high-end of the reasonable range for the combined data would be the sum of the two lines' differences, or $259. If the two lines were deemed independent of each other, the difference from the combined central estimate to the high-end could be reasonably approximated as the square root of the sum of the squares of the lines' metrics, or $191. As the difference to the high-end of a reasonable range based on the combined data was evaluated at $191 greater than the point estimate for the combined data, we infer that the two lines have an approximate 0% correlation.

From our testing on the Personal Auto Liability and Homeowners data for five companies in our sample, we observed implied correlations between the reserve ranges for the two lines ranging from (0.3) to +0.8. The implied correlations were highly sensitive to alternate judgments around the reasonable range on the combined data; thus, we do not believe the reader should take away any particular "rule of thumb" on correlations.

7. Conclusion

We wrote this Call paper with the goal being to describe a variety of practical approaches that we have observed for assessing variability of unpaid claim estimates and to present illustrations of the application of chosen methods for evaluating and comparing ranges of

3 We acknowledge that the sum of the point-estimates for the two lines is $3,250, which is slightly greater than the point-estimate based on the combined data. LDF's for the analysis of aggregate data were calibrated based on the parameters for the two lines; the small difference arose from small differences in the ELR's and the weights applied to the various projections to form the point-estimate. We do not consider the differences significant in the context of our discussion of the framework of the analysis.

reasonable estimates and distributions of possible outcomes. We believe the framework described herein is practical and can be reasonably explained to the variety of stakeholders who seek insights and opinions from actuaries on point-estimates and the associated uncertainty.

In the course of preparing this paper, we discovered an apparent relationship that the illustrative ranges of reasonable estimates for the three lines reviewed tended to align with portions of the distribution of outcomes that extend up to one standard deviation above and below the mean. While the estimated standard errors for each segment reflected the inherent nature of the line and the company's claims development experience, the ranges of reasonable estimates tended to be subject to similar degrees of variability. This should be an area of further and more robust research.

Just as there is uncertainty and judgment inherent in the process for determining a central estimate of unpaid claim liabilities, these attributes are inherent in evaluating a range of reasonable estimates. While the accuracy of a point estimate will ultimately be known when all subject claims are settled and paid, expressions of a range of reasonable estimates are much more tenuous and cannot be tested with hindsight; therefore, such expressions primarily serve as indications of the effects of plausible differences in assumptions. We believe that the days of expressions of reasonable ranges based solely on judgment or rules of thumb are over, as stakeholders seek a more-reasoned response to questions regarding the basis of a stated range.

Acknowledgment

The authors acknowledge the contributions of Barbara Jackie and Sun Sun in preparing the analyses of the data summarized in this paper.

8. References

[1] Bornhuetter, Ronald L and Ferguson, Ronald E, "The Actuary and IBNR," *PCAS* **1972**, Vol. LIX, 181-195.

[2] Mack, Thomas. "Measuring the Variability of Chain Ladder Reserve Estimates," *Casualty Actuarial Society Forum, Spring 1994,* 101-182.

Abbreviations and notations

ESE, estimated standard error

LDF, loss development factor

SEC, Securities & Exchange Commission

GAAP, generally accepted accounting principles

SAP, statutory accounting principles

Biographies of the Authors

Christopher Walker is a principal in the Actuarial & Insurance Management Solutions practice of PricewaterhouseCoopers LLP. His primary technical responsibilities include loss reserve, ratemaking, and regulatory assistance for dozens of insurance carriers and self-insurers. Particular experience is in personal lines, primary/excess workers' compensation, warranty, and insurance company valuation. He is a Fellow of the CAS and a Member of the American Academy of Actuaries.

Mark Littmann is a principal in the Actuarial & Insurance Management Solutions practice of PricewaterhouseCoopers LLP. He leads actuarial teams supporting external financial audits of insurance organizations and corporations with self-insured exposures and providing consulting services regarding reserving valuations and processes and other analytical applications. He has a degree in Mathematics and Economics from Valparaiso University. He is a Fellow of the CAS and a Member of the American Academy of Actuaries. He is a member of the program planning committee for the Casualty Loss Reserve Seminar.

Effects of Loss Reserve Margins on Calendar Year Results - Balcarek Expanded

Robert J. Walling III, FCAS, MAAA and Erich A. Brandt, FCAS, MAAA

Motivation. Reserve uncertainty is a significant risk to many insurance companies, captive insurers and self-insurance programs. Understanding and quantifying this risk is essential to insurance related enterprise risk management efforts. Using publicly available data, this paper examines reserve uncertainty for a majority of the U.S. property-casualty insurance industry on both an industry and by-company basis.
Method. The authors apply analyses similar to those used by Rafal Balcarek in his 1966 Proceedings of the Casualty Actuarial Society article entitled, "Effect of Loss Reserve Margins in Calendar Year Results." The authors are able to greatly expand the amount of data reviewed and the methods of analysis greatly due to changes in publicly available data and computing power in the intervening years. Data organized in this manner may provide opportunities for understanding industry and company reserving behaviors and loss development risk potentials.
Results/Conclusions.

- For personal lines, industry loss development from initial reserve estimates has generally been favorable.
- The three main commercial lines, CMP, CAL, and WC all show significant cyclicality between years of material adverse development and material favorable development.
- Medical professional liability shows even stronger cyclical swings between a high of 26.0% adverse development on the 2001 calendar year loss ratio and a 31.9% favorable impact on calendar year 2010.
- Each of the lines reviewed have calendar year reserve adjustments that are positively correlated to the others. Particularly strong correlations were seen between:
    - Homeowners (HMP/FMP) and personal auto liability (PPAL).
    - Personal auto and commercial auto liability (CAL).
    - The three predominant commercial lines, CMP, CAL and WC.
    - Medical professional liability and the other three commercial lines.
- At a company level, the commercial lines, especially WC and MM have greater potential for significant calendar year loss ratios changes due to development from prior years.
- The commercial lines show cyclical behaviors in unexpected loss reserve development both at the industry composite and insurance company/group level.
- Calendar year loss ratios do not appear to be more stable than accident year results, but do appear to delay the recognition of underwriting losses and profits, particularly for commercial lines.
- For Personal lines, adverse development for the industry as a whole is realized by 36 months of maturity.
- For CMP and CAL, adverse development for the industry as a whole is generally under 1% at 72 months of maturity and beyond
- WC and MM both experience the widest fluctuations in AY loss reserves in more mature observations.

Keywords. Loss Reserving, Risk Margins, Variability, Schedule P.

1. BACKGROUND AND SCOPE

1.1 Research Context

The 1966 Proceedings of the Casualty Actuarial Society published one of the most interesting and enduring articles in the actuarial literature, Rafal Balcarek's, "Effect of Loss Reserve Margins in Calendar Year Results." The article has remained relevant in the actuarial literature for decades for at least a couple of reasons: the straightforward, clean approach to the analysis and the importance of the underlying business behavior it measures. This succinct, 16 page article with its handwritten tables, took a measured approach to one of the most basic risk factors property-casualty insurance companies and actuaries faced then and face now – loss reserve variability.

The Balcarek paper recognized the inherent variability in loss reserves and their impact on calendar year (CY) results. It also recognized the importance of calendar year results in insurance company decision making. The "nagging question" of the validity of business decisions based on calendar year data "if the calendar year results on which they are based contain major distortions" is as relevant today as in 1966.

The approach used by Balcarek was to compare estimated ultimate losses for a given company and accident year (AY) as of twelve months of development to the comparable results as of 60 months. For example, one could look at accident year 1959 data evaluated at December 31, 1959 and again evaluated as of December 31, 1963. The change in this estimate represents an over or understatement of calendar year 1959 results that impacts subsequent calendar years. The original paper sought to answer several questions:

- How materially can reserve changes impact calendar year results?
- Do companies' reserve changes move together?
- Do reserve changes by line move in sympathy with one another or do they offset?
- Do companies manipulate reserve margins to stabilize results? Does this work?

1.2 Objectives

This paper is our attempt to update and expand the data used in the original analysis, expand the number of coverages and companies reviewed, and update and extend its findings, while honoring the approach of the original work.

Our analysis will strive to answer questions similar, if somewhat more expansive than the scope of Balcarek's original paper. At both a company and industry level, we will try to answer the following questions:

- What is the impact of initial reserve development on calendar year results on the company and/ or industry level?
- Do lines of insurance move in sympathy at the company and/or industry level?
- Can you compare company CY or AY[1] results?
- At what level do companies/lines exhibit a risk of material adverse deviation (RMAD)?
- At what maturity do accident year loss reserves no longer have an RMAD?

[1] Accident year and calendar year are assumed to have their typical definitions, consistent with their use in the NAIC annual statement and the actuarial literature.

1.3 Outline

The remainder of the paper proceeds as follows. Section 2 will discuss the data and technical approach used in our analysis. Section 3 presents our discussion and analysis of each of the issues being considered. Section 4 provides a brief conclusion.

2. DATA AND TECHNICAL APPROACH

In order to update the analysis and increase its robustness, we have made a number of changes in approach from the original analysis. These changes relate to the lines of business considered, the number of years in the analysis, the maturity of the accident year loss evaluations examined, and the number of companies reviewed.

The original Balcarek paper looked at the auto bodily injury, general bodily injury and workers compensation lines of business. We have expanded the analysis to include the first six lines of business, or parts, of the current Schedule P:

- Part A - Homeowners/Farmowners (HMP/FMP)
- Part B - Private Passenger Auto Liability/Medical (PPAL)
- Part C - Commercial Auto/Truck Liability/Medical (CAL)
- Part D – Workers' Compensation (WC)
- Part E - Commercial Multiple Peril (CMP)
- Part F - Medical Malpractice – Section 1 (Occurrence) and Section 2 (Claims-made) (MM)

This approach allows a consideration of the two main lines of business for personal lines insurance, the three main lines of business for "main street" commercial lines, and a key specialty line (medical professional liability) that has demonstrated significant reserve variability and several severe market disruptions since the Balcarek paper was published.

As previously mentioned, Balcarek looked at estimated ultimate losses as of twelve months and sixty months of development. His rationale was "it is suggested that the five year period is sufficiently long to account for the bulk of reserve developments." He looked at accident years 1953-1960 from these two perspectives.

We have made two changes in our approach due to the substantial increases in the ability to compile Schedule P data and other annual statement data and the substantial lengthening of the "tail" on loss development. First, we are going to look at more years. Accident years 1991 through 2010 are readily available to us from data provided by AM Best Company. Second, we will capture data for each of the first ten valuations of held net ultimate losses.

For our accident year held ultimate loss data we will use Schedule P – Part 2 that includes held ultimate loss and defense and cost containment expenses (DCCE), net of reinsurance. For our calendar year data, we will use data from Part 3 of the Insurance Expense Exhibit (IEE) which is also net and includes loss and DCCE. We recognize that during the experience period under review, the NAIC changed from allocated loss adjustment expenses to DCCE. We also recognize that each company adjusted their financial reporting data to reflect this change in their own way. We have not attempted to adjust the insurance companies' actual annual statement in any way to address this issue. For the remainder of this paper, the term loss will include DCCE.

Balcarek examined ten insurance companies in his original analysis. We have expanded the companies reviewed substantially. We desired to increase the number of companies reviewed so that the current market share of the companies would be greater than 60%. We deemed this volume of data sufficient to be representative of the overall U.S. property–casualty industry for these lines. It required between sixteen (16) and twenty-six (26) insurance companies or groups to achieve this critical mass for each line. Our analysis includes the following insurance groups:

- Accident Fund
- ACE
- Allstate
- AIG
- Amerisure
- APCapital
- Auto-Owners
- Berkshire Hathaway
- Canal
- Chubb
- Cincinnati
- CNA
- The Doctors Company
- Erie
- Farmers
- FPIC
- Great American
- Hartford
- ISMIE
- Liberty Mutual
- MAG Mutual
- Medical Mutual (MD)
- Medical Mutual (NC)
- Medical I.C. (AZ)
- Nationwide
- New Jersey Manufacturers (NJM)
- Norcal
- Old Republic
- ProAssurance
- Progressive
- ProMutual
- QBE
- Safeco
- SAIF
- SCIF
- State Farm
- State Volunteer Mutual
- Travelers
- USAA
- WR Berkley
- Zenith
- Zurich

Data for these companies was included in the analysis for all reviewed lines in which they had at least a 0.5% marketshare. Company names have been masked in the analysis and each company or group has been assigned a unique number that applies to that organization throughout the analysis.

During the experience period under review, many of these companies have undergone significant changes due to mergers, acquisitions, divestitures, etc. We have made a daunting number of adjustments to restate the historical data to the current composition of the organization. The authors would like to thank A.M. Best Company and a Pinnacle team led by Greg Fears for their efforts to scrub the data until we were confident in its appropriateness for this analysis.

3. DISCUSSION AND ANALYSIS

This section is organized along the lines of the key questions we are addressing.

How material is the impact of initial reserve development on calendar year results?

In order to evaluate this question, we calculated the calendar year change in prior ultimate loss and DCCE estimates divided by current year calendar year net earned premium. The results for our industry composites are shown in Exhibit 1. This metric should measure the impact of unexpected prior year development on current calendar year loss ratios[2]. Because ten prior accident years of data are needed to compute this metric, only calendar years 2000-2010 are shown. The results are quite interesting.

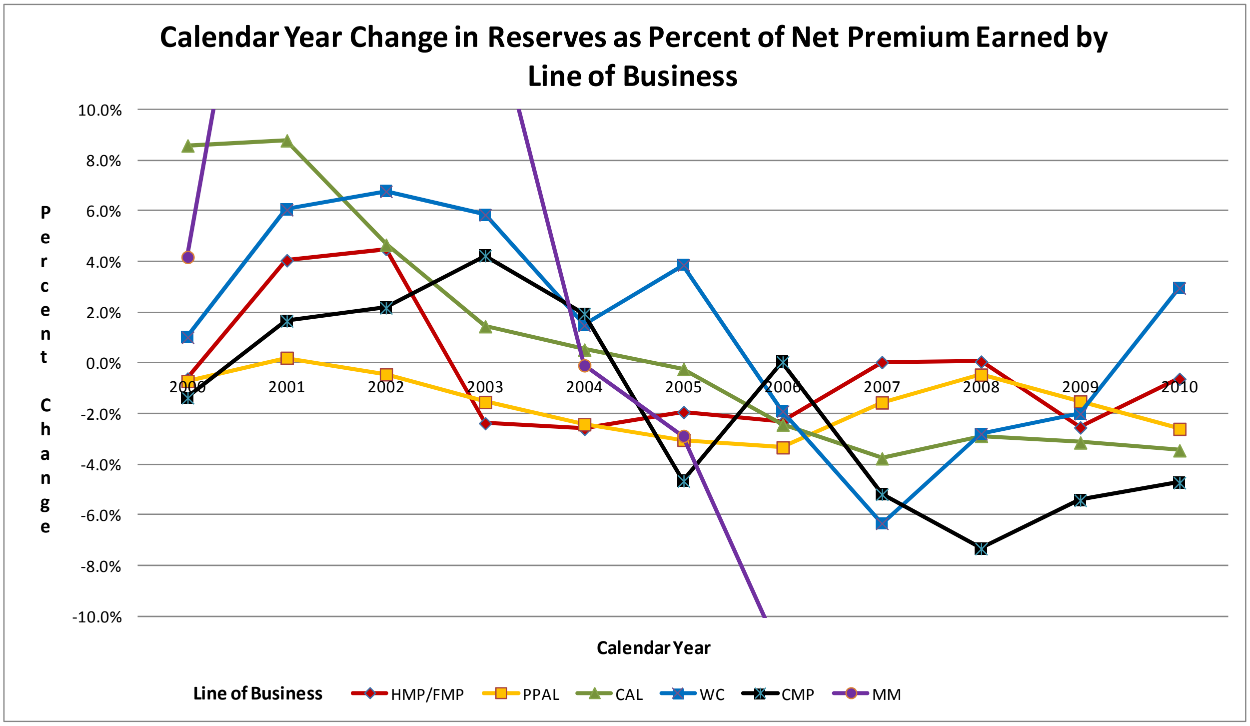
For PPAL, every year but one shows favorable loss development. This would appear to reflect the relatively predictable nature of PPAL claims and the conservative loss reserving philosophy of several of the leading insurers in this line. For HMP/FMP and CMP, there are four and five years respectively (out of eleven) that show adverse development. However, the magnitude of these reserve increases is never more than 4.5% of current year earned premiums. The three main commercial lines, CMP, CAL, and WC all show significant cyclicality between years of material adverse development (e.g. 8.8% for CAL in 2001) and material favorable development (e.g. -7.3% for CMP in 2008). WC in particular had adverse development up through CY 2005 and positive development in the four subsequent years. Medical professional liability shows even stronger cyclical swings between a high of 26.0% adverse development on the 2001 calendar year loss ratio and a 31.9% favorable impact on calendar year 2010. Using the ratio of CY reserve development as a percentage of carried reserves, the adverse deviation seen in years 2000 and 2001 for CAL, WC and

[2] This approach does not consider development from accident periods more than ten years prior to the current year. This simplifying assumption makes working with the AM Best Schedule P data easier and usually does not have a material impact on the lines reviewed. Some other lines, such as products liability, periodically see material development more than ten years after an accident year has expired.

MM could easily meet the threshold for material adverse deviation as used for a commercial lines insurer. Clearly this type of unexpected development on prior year reserves would materially impact calendar year net income.

Do lines of business tend to have reserve development that move in sympathy with one another? If so, which ones are most highly correlated?

The second table in Exhibit 1 computes the correlations between the calendar year impact of reserve development as a percentage of current calendar year earned premium shown in the exhibit. All of the lines have calendar year reserve adjustments that are positively correlated. This correlation can be seen in the following graph.



Graph 1 - Calendar Year Change in Reserves as Percent of Net Premium Earned by Line of Business

In addition, several of the lines are highly correlated to one another. The first noteworthy example is that homeowners (HMP/FMP) and personal auto liability (PPAL) are highly correlated with a correlation coefficient of 0.7291. This would suggest that at an industry level, when HMP/FMP has favorable or adverse development, PPAL is likely to move similarly as well, and vice versa. In addition, reserve development from initial estimates for personal auto and commercial auto liability (CAL) are highly correlated (0.6023) which seems intuitive. The three predominant commercial lines, CMP, CAL and WC, all show significant positive reserve development correlations as well. CMP, however, shows very little correlation with the two personal lines observed. Finally, medical professional liability shows strong positive correlations to the other three commercial lines.

How material is the impact of prior year reserve development on calendar year results for an individual company?

Exhibit 2 examines the same metrics used in Exhibit 1 to examine unexpected reserve development by calendar year at the company level for leading insurers. In addition, several statistics have been added examining the range, average, and standard deviation of these reserve development metrics both across multiple years for a given company and across all companies in a given calendar year.

For HMP/FMP and PPAL, the generally favorable development seen in the industry composite is also seen for most individual companies. A few companies have one or two years with adverse development of more than 5.0% of current year premiums, but overall the potential for adverse development in these lines appears pretty modest. Almost all of the HMP/FMP companies had their worst year in the exposure period in 2001 or 2002, while PPAL does not show a systematic pattern. In addition, the standard deviations between the companies in a given year are generally very low. The 2010 observation of 51.4% development for company 33 relates to a group who is taking a substantial reduction to their PPAL writings while realizing the effect of several past accident years that weren't adequately reserved.

As would be expected, the three main commercial lines, CMP, CAL and WC, show much more divergence in reserve development results between companies. The standard deviation between companies in a given year is larger than in personal lines and most companies have adverse development of more than 10% of premium in at least one year and some companies have a year with an impact on current calendar year loss ratios in excess of 40% (e.g. 2000 CAL for company 20, 2009 WC for company 15 and 2010 CMP for company 17).

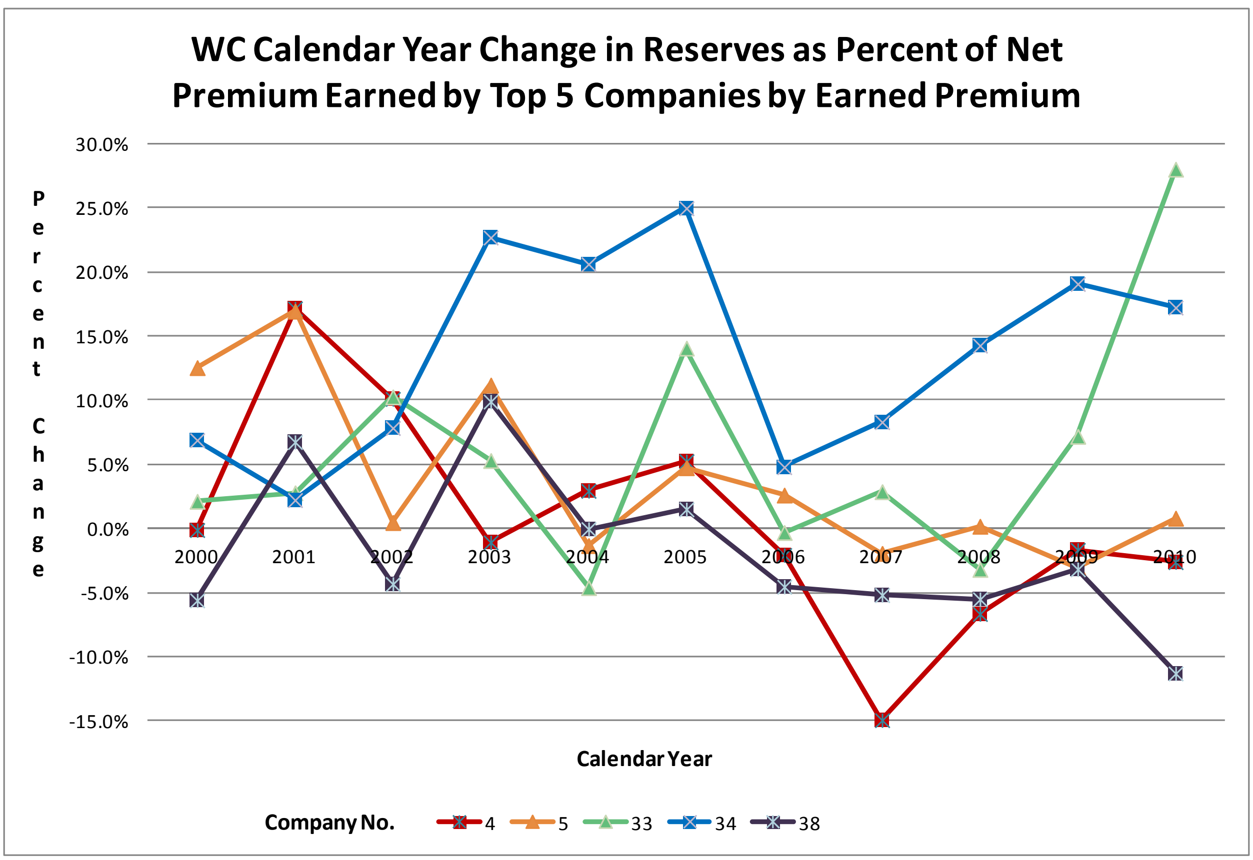
MM shows dramatically more variability than any other line reviewed. In some cases a company experienced adverse development of more than 300% of current year earned premium. In other cases a company saw loss ratio reductions of as much as 78.4% of current year earned premium.

Do companies in a given line of business tend to have reserve development that move in sympathy with one another? If so, which ones are most highly correlated?

A natural extension of the finding of strong positive correlations between lines at an industry composite level is "do companies in a given line also exhibit positively correlated movements?" The same metrics used in Exhibit 1 are provided at a company level for each line in Exhibit 2. The correlation statistics in Exhibit 2 compute correlations in unexpected reserve development between companies in a given line.

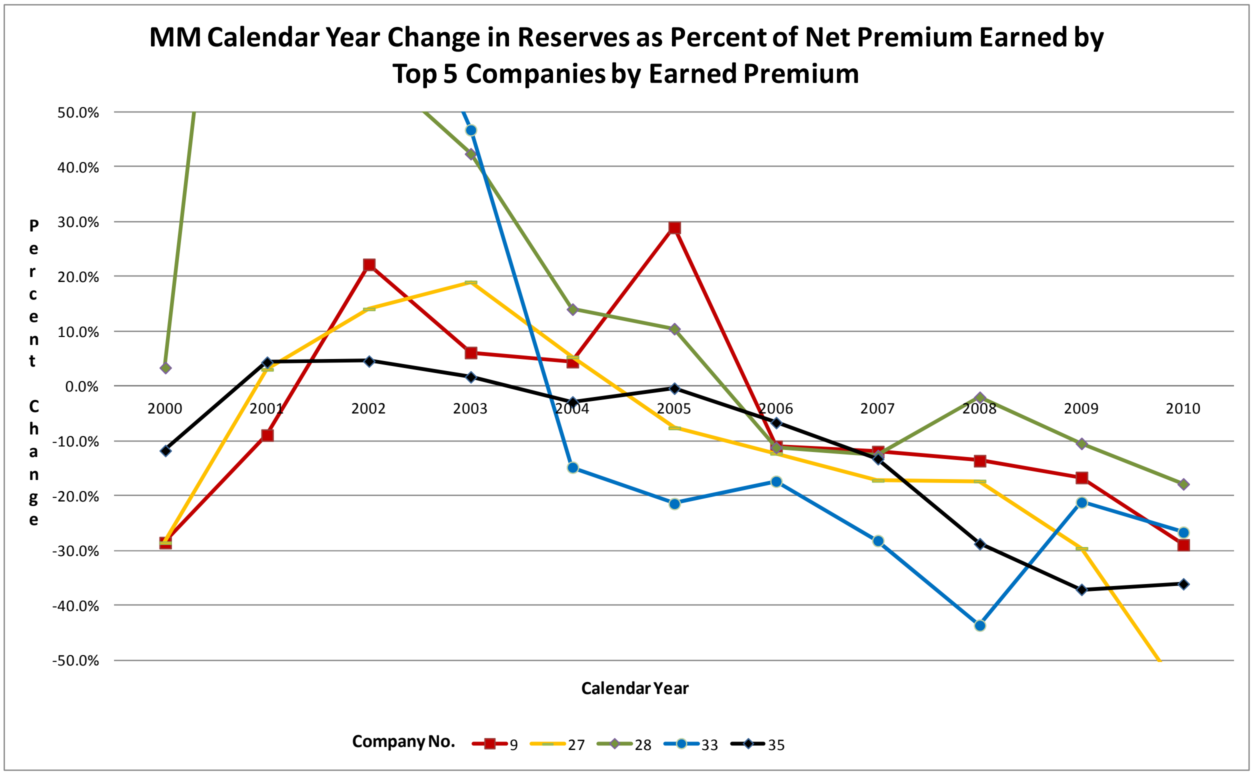
Interestingly, for HMP/FMP and PPAL, while some pairings of companies show reserve developments that are highly correlated, overall average correlations are only mildly positive at 0.3107 and 0.0127, respectively.

The divergence in reserve development results between companies for the three main commercial lines, results in small positive average correlations between companies in these lines, less than 0.20 for each. This positive correlation does seem to exhibit itself with a number of companies showing the same cyclical changes in reserves that was seen at the industry composite level. For example, consider the pattern seen in the following graph for the five largest WC insurers.



Graph 2 - WC Calendar Year Change in Reserves as Percent of Net Premium Earned by Top 5 Companies by Earned Premium

Interestingly, MM shows the most significant correlation in company reserve development on calendar year results with an average correlation of 0.4724. One could posit that factors such as the impact of countrywide changes in MM claims trends, the role of an industry association such as the Physician Insurers Association of America (PIAA), and the impact of a high concentration of these carriers with certain external service providers may contribute to this higher correlation of insurance company behaviors for this line relative to the others reviewed. Graph 3 shows the results for the five leading carriers.



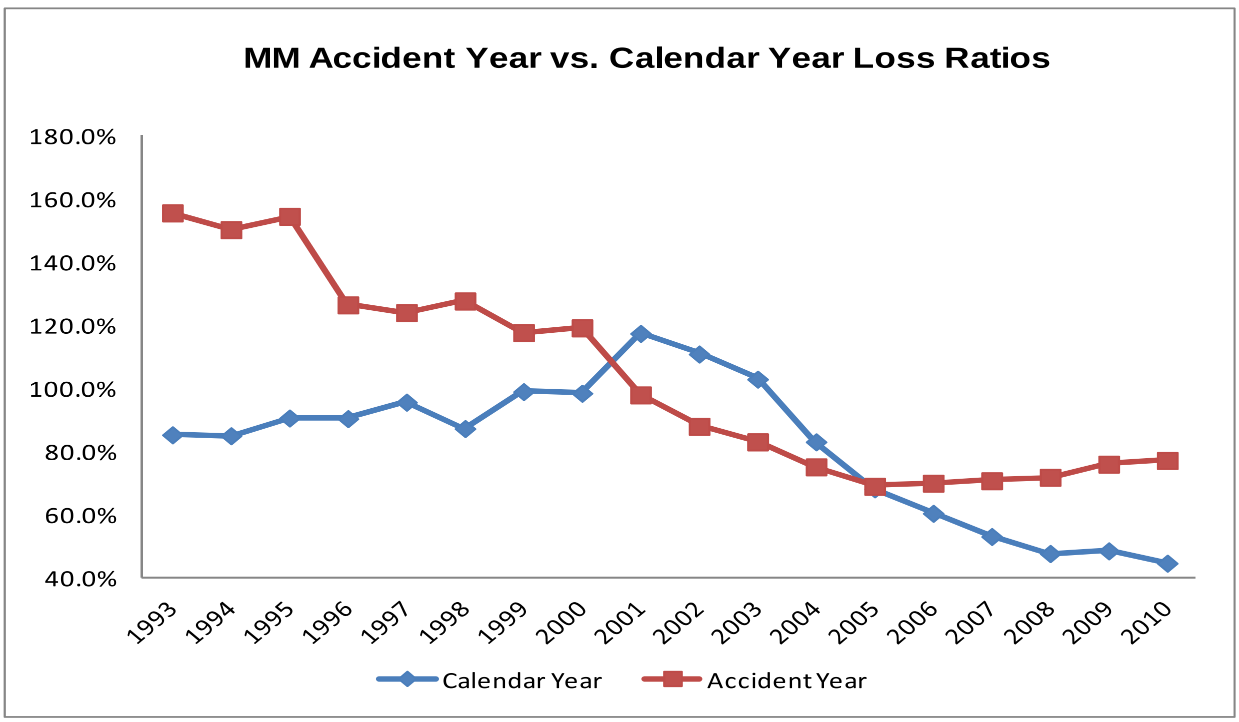
Graph 3 - MM Calendar Year Change in Reserves as Percent of Net Premium Earned by Top 5 Companies by Earned Premium

For a given company or group, do lines of business tend to have reserve development that move in sympathy with one another?

Exhibit 3 reorganizes the same calendar year loss ratio impacts of prior year reserve changes and presents them by company for all reviewed lines. The focus of these exhibits was insurance companies and groups with material market shares in most or all of the reviewed lines. The results are quite interesting. Several of the companies, for example numbers 4, 20, 21 33 and 38 show very little consistent correlation or pattern between lines as it related to reserve development from prior years. Within those companies however, are some strong examples of individual lines moving in synch with each other like the correlation of .9073 for WC and PPAL for company 33. There are a couple of notable exceptions. Group/companies 5 and 19 show consistently high correlations between lines that seem to suggest a tendency for these company management teams to adjust loss reserve levels at a corporate level resulting in all lines moving together. There are some pairs of lines that appear to move together in other companies/groups, but nothing as pronounced as the phenomenon seen in these two companies.

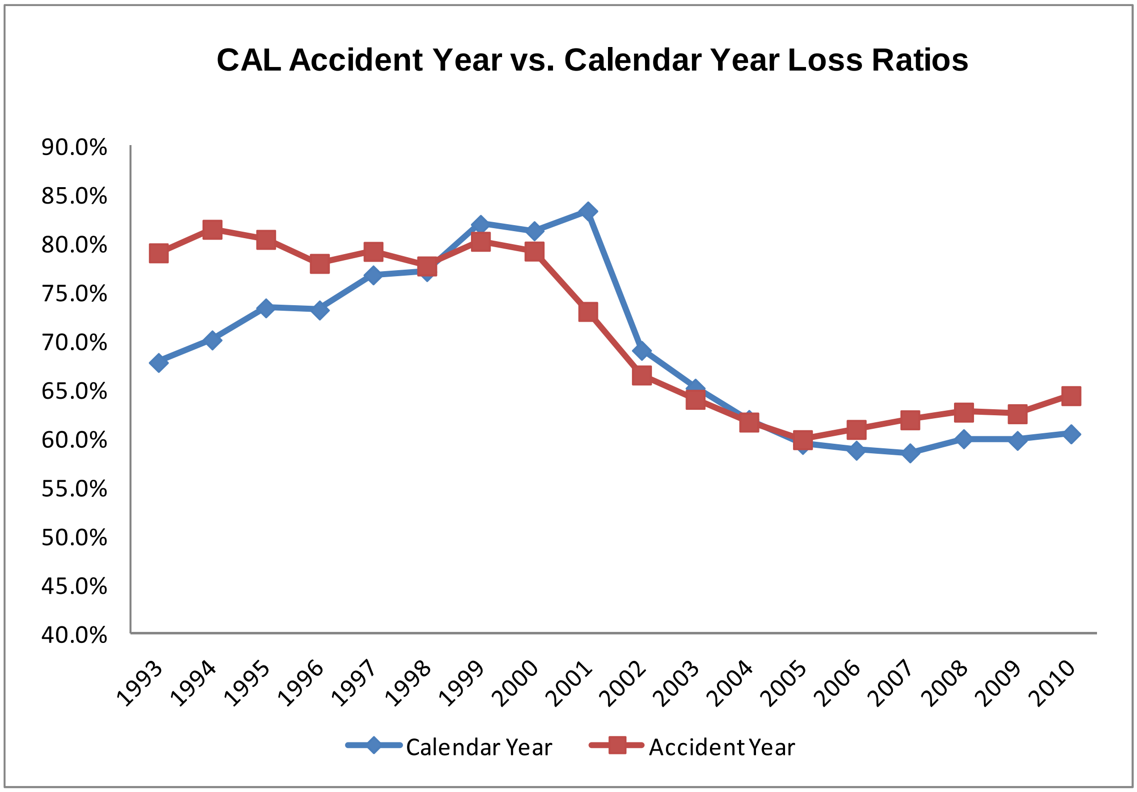
Do calendar year adjustments of loss reserves increase the stability of loss ratios? How do they affect the timing of when results are realized on financial statements? (Exhibits 4 and 5)

To begin to evaluate these questions, we examine Exhibit 4. Page 1 contains initial held ultimate loss ratios for our industry composite, while page 2 contains calendar year results for the same years. In addition, several summary statistics are provided by line. If companies were adjusting calendar year reserve levels to stabilize results, we would expect less variability (and therefore smaller standard deviations) in calendar year loss ratios than accident year loss ratios. With the sole exception of medical professional liability, this simply does not seem to be the case. For the MM line, calendar year adjustments appear to limit the highest of the highs and the lowest of the lows. This can also be seen in the following graph (Graph 4) comparing the MM calendar year and accident year results.

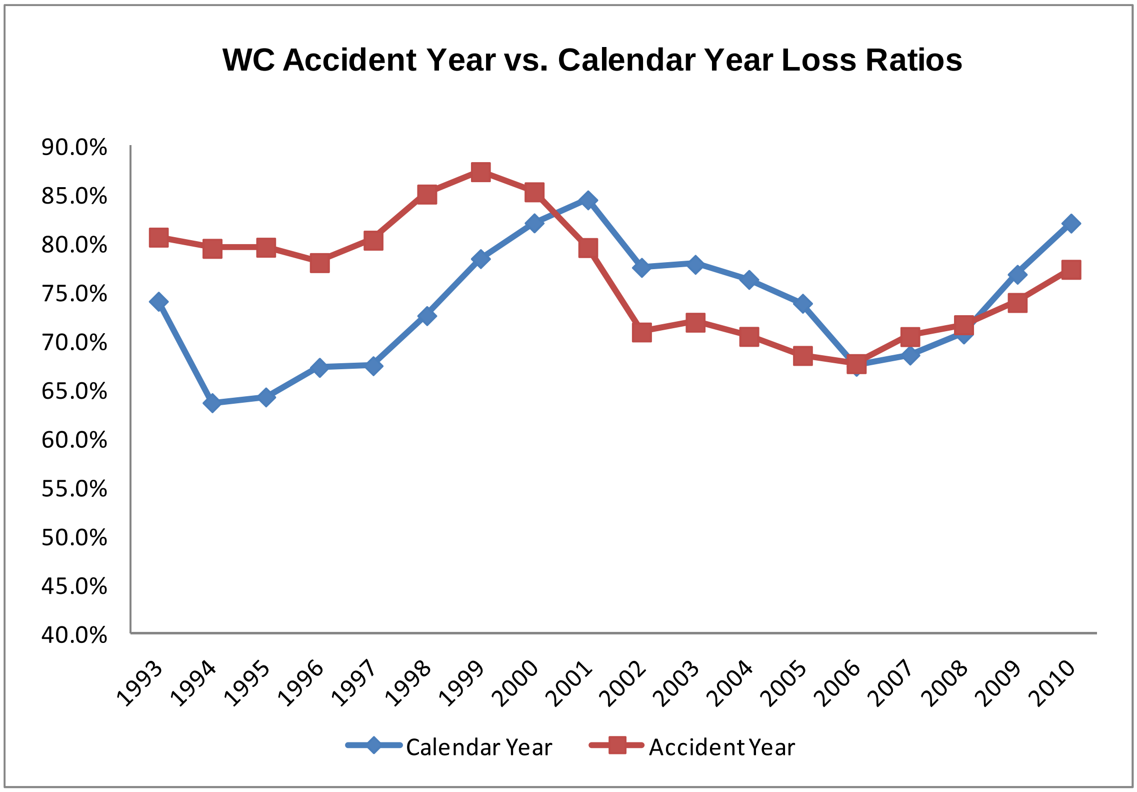


Graph 4 - Medical Professional Liability Accident Year and Calendar Year Loss Ratios

This graph also suggests that insurance company management reserving decisions may delay the recognition of underwriting profits and losses for the MM line. This concept can be reinforced by a couple of the commercial lines that have enough "tail" to result in timing shifts. The following graphs for both commercial auto liability and workers compensation show pronounced lags in the loss ratio cycles between the accident year and calendar year results suggesting that calendar year results delay the recognition of underwriting results compared to initial accident year estimates.



Graph 5 - Commercial Auto Liability Accident Year and Calendar Year Loss Ratios



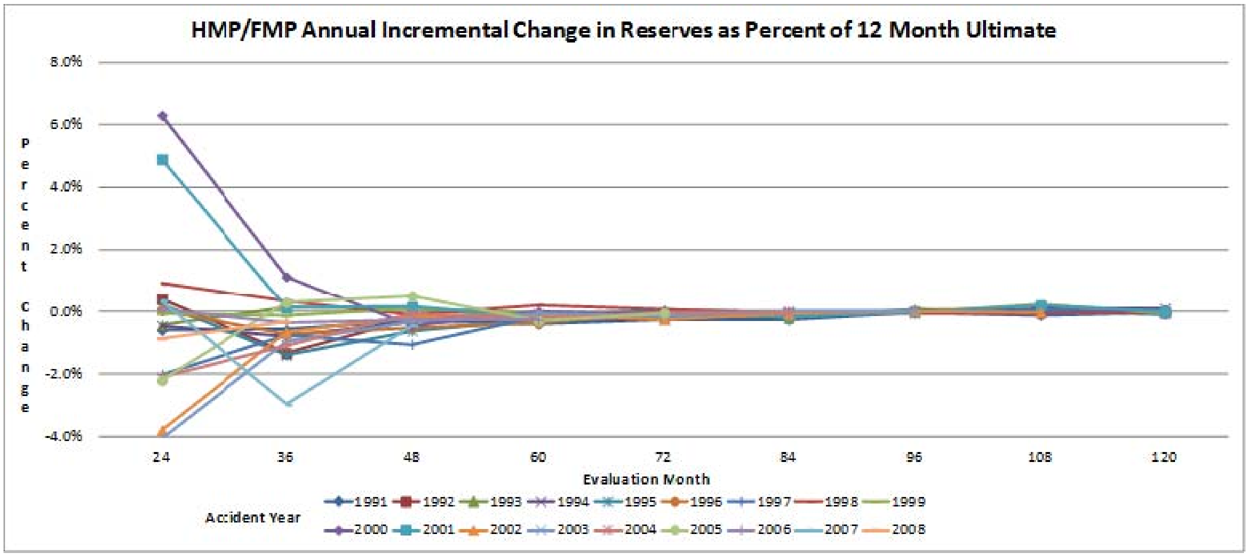
Graph 6 - Workers Compensation Accident Year and Calendar Year Loss Ratios

Exhibit 5 provides comparable data to Exhibit 4 for individual companies/groups. It can be noted that both AY and CY loss ratios for the major Homeowners writers appear to be positively correlated with each other, CAL shows a similar phenomenon. Workers' Compensation is interesting because the correlation of AY loss ratios between the major writers appear to be positively correlated while the CY correlations are split between negative and positive. This could lead one to conclude that while market forces effect WC results with a degree of consistency across accident years, there are internal factors such as case reserve setting practices that influence CY results for WC.

How material a risk of material adverse deviation (RMAD) do the accident year loss reserves for a line of business exhibit at different maturities? At what maturity do accident year loss reserves for a given line no longer have an RMAD?

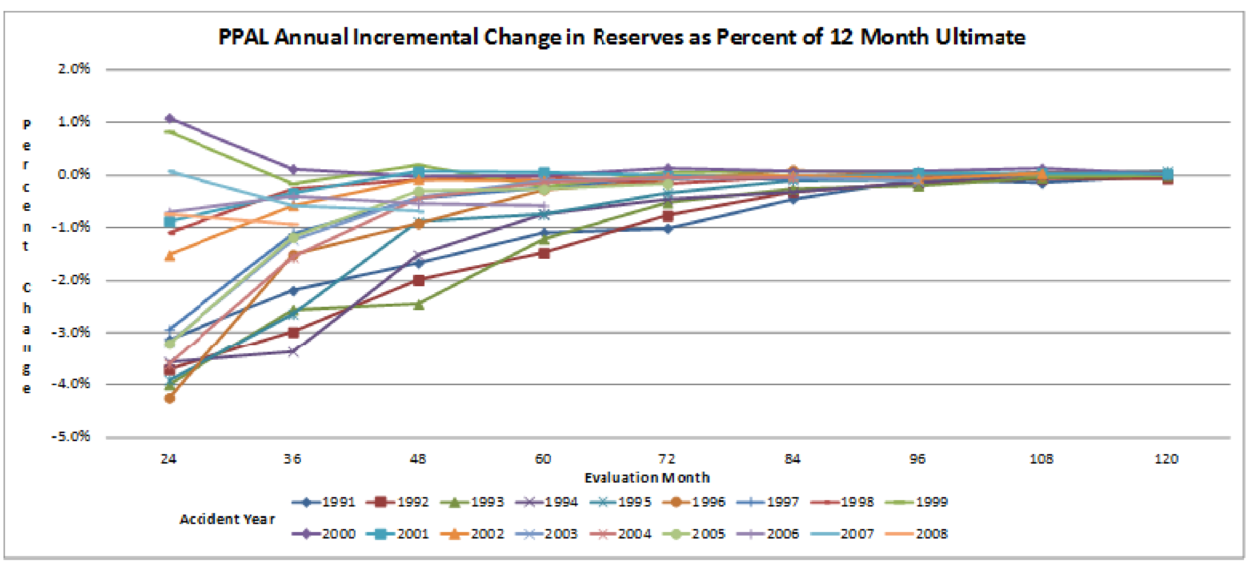
The first perspective we will examine on how material adverse loss reserve development can be in a given accident year is Exhibit 6 which shows held ultimate loss development by annual valuations for our industry composite divided by the initial ultimate loss estimate.

For HMP/FMP, only one year, 2000 shows adverse development of more than 1% of initial ultimate losses after 24 months of maturity. Graph 7 shows visually how fast loss reserves stabilize for this line.



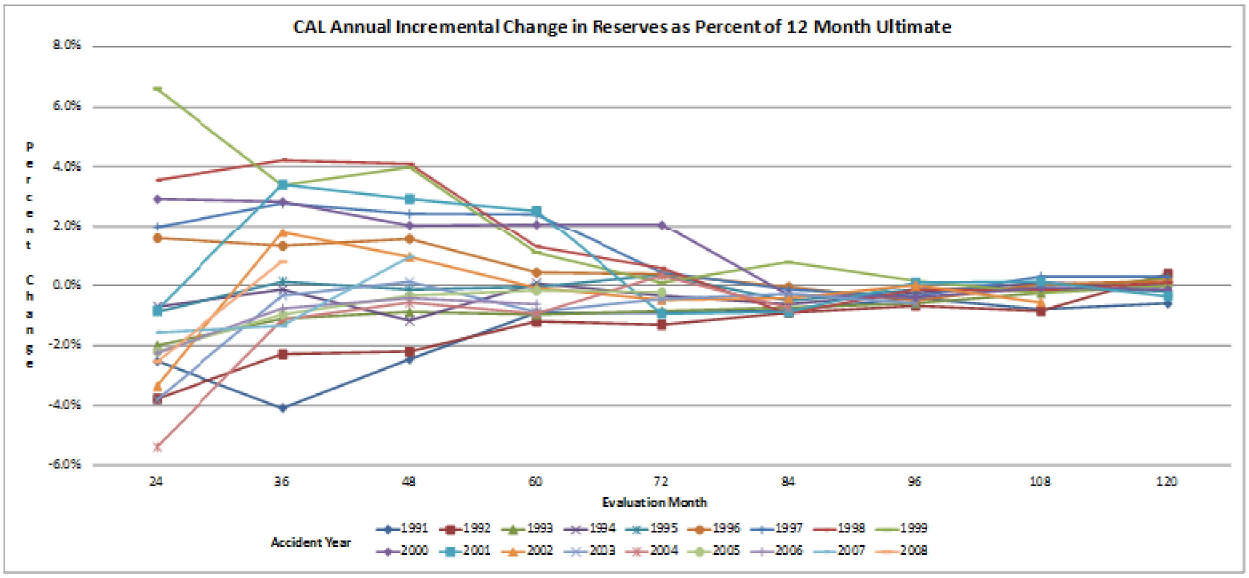
Graph 7 - Homeowners Accident Year Development by Maturity

The result for PPAL is even more dramatic as only two years, 2000 and 2001 show more than 1% adverse development on initial ultimate loss estimates after only 12 months. In fact, the graph below implies a tendency for PPAL reserves to be inherently redundant on an industry-wide basis across early maturities. Things get more interesting in commercial lines. Again, the following graph shows both the conservatism of reserve levels for this line and the lack of potential for adverse development at the industry composite level.

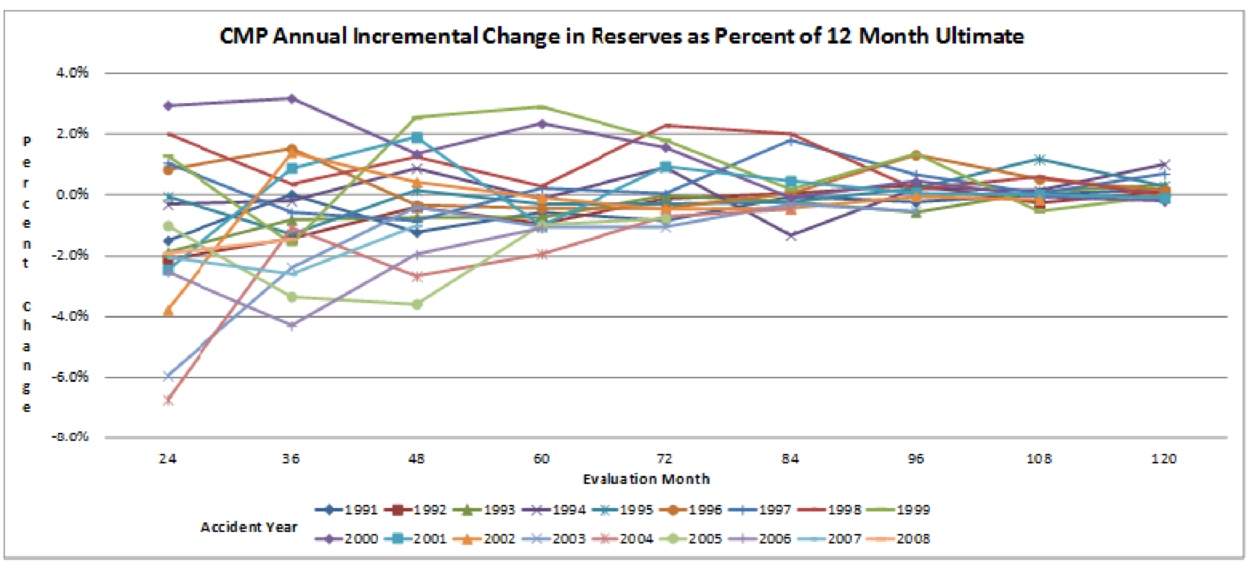


Graph 8 – Private Passenger Auto Liability Accident Year Development by Maturity

In CAL, the potential for development of more than 2% of the initial ultimate loss estimate exists certainly until 60 months and actually was seen once between 60 and 72 months. In addition, during the calendar year diagonals 2005-2009 there appears to be a systematic improvement in ultimate loss estimates across a number of accident years. This could be seen as evidence of an asymptote in the soft curve of the underwriting cycle. Some of this same phenomenon can also be seen in the CMP line. The CMP line also shows development of more than 2% of initial ultimate losses as late as 84 months of development. Graphs 9 and 10 show these results.

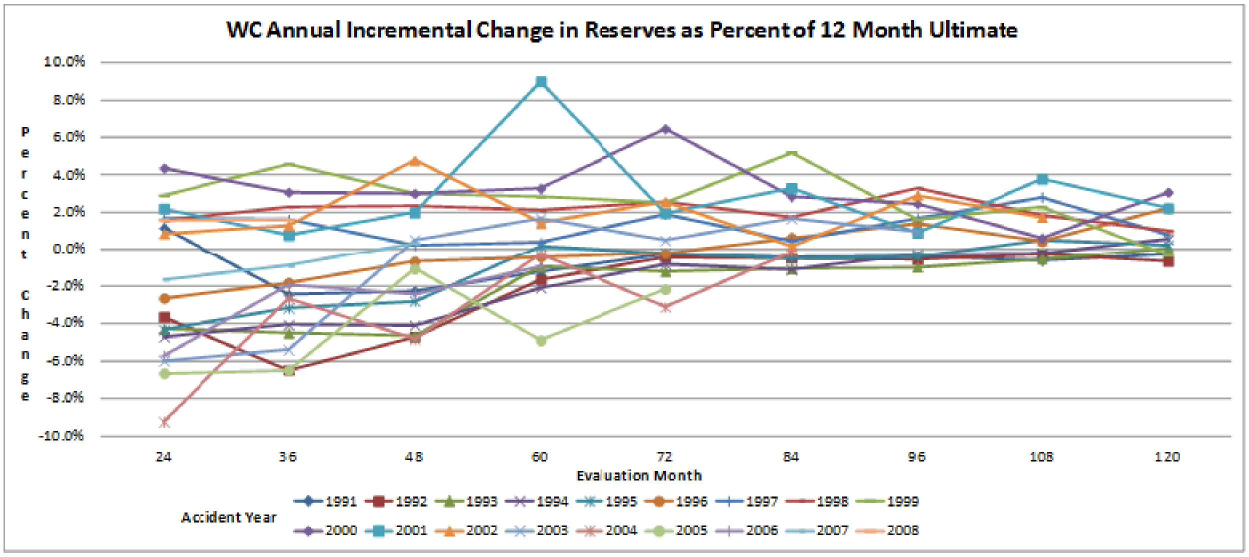


Graph 9 – Commercial Auto Liability Accident Year Development by Maturity



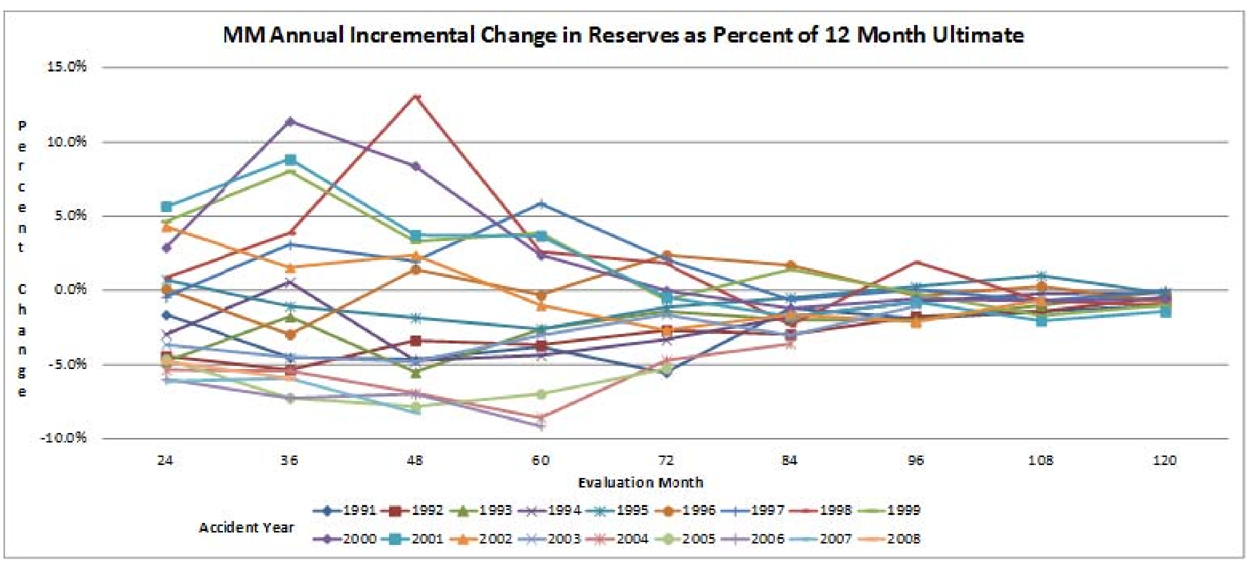
Graph 10 – Commercial Multiple Peril Accident Year Development by Maturity

The WC line shows even longer potential for significant adverse development. Numerous accident years show adverse development of more than 2% of initial ultimate loss estimates at the last valuation (between 108 and 120 months). The standard deviations of the amount of development are also greater in WC. Graph 10 shows these results.



Graph 11 – Workers Compensation Accident Year Development by Maturity

Finally, MM shows the largest potential for adverse development in the first 60 months of maturity with the 1997 year having development of 5.8% of initial held ultimates during calendar 2001. After 60 months, the potential for adverse development settles down to standard deviations comparable to WC. After 96 months of maturity, WC still exhibits the greatest amount of volatility of all lines, MM appears to have little if any adverse development at these intervals.



Graph 12 – Medical Professional Liability Accident Year Development by Maturity

At the company level, how material a risk of material adverse deviation (RMAD) do the accident year loss reserves for a line of business exhibit at different maturities? At what maturity do accident year loss reserves for a given line no longer have an RMAD?

Exhibits 7-9 show company/group level development between 12 and 24 months, 12 and 36 months, and 12 and 120 months respectively. In Exhibit 9, accident years 2002 through 2009 are shown even though they have not reached the full 120 month maturity.

The HMP/FMP and PPAL results at the company level mirror the industry composite results. Most companies show the potential for adverse development of 5-10% of initial held ultimate loss and DCCE between 12 and 36 months, but rarely show significant development after 36 months. Several large inadequacies from 12 to 120 months can be observed in the 2000 and 2001 accident years. Since portions of these cumulative inadequacies developed by over 10% between 12 and 24 months as well as 24 to 36 months, we can conclude the personal lines insurance as a group can have an industry RMAD across a single accident year.

For both CAL and CMP, many companies show at least one year that manifested adverse development of more than 10% of initial ultimates after 36 month.

For workers compensation, the development between 12 and 36 months and the development between 12 and 120 months appear to be highly correlated. Years that show favorable development in held ultimate losses at 36 months often show additional favorable development at 120 months. Similarly, years with adverse development at 36 months often show additional adverse development by 120 months.

Maybe the most interesting line for these exhibits is MM. We have already presented evidence of correlation between companies in their held reserve development, significant levels of favorable and adverse development, and cyclicality in the development of initial reserves. It should be no surprise then that Exhibit 9 MM shows all of these trends in the company data. Most years between 1991 and 2001 have at least one company with adverse development of more than 100% of initial held ultimates. Further, the company development for accident years 1991-2001 often shows the same cyclicality seen in the averages and the industry composite.

4. CONCLUSION

A review of publicly available insurance company annual statement data leads to several findings regarding industry and company loss reserve development. These findings include:

- For personal lines, industry loss development from initial reserve estimates has generally been favorable.
- The three main commercial lines, CMP, CAL, and WC all show significant cyclicality between years of material adverse development and material favorable development.
- Medical professional liability shows even stronger cyclical swings between a high of 26.0% adverse development on the 2001 calendar year loss ratio and a 31.9% favorable impact on calendar year 2010.
- Each of the lines reviewed have calendar year reserve adjustments that are positively correlated to the others. Particularly strong correlations were seen between:
    - Homeowners (HMP/FMP) and personal auto liability (PPAL).
    - Personal auto and commercial auto liability (CAL).
    - The three predominant commercial lines, CMP, CAL and WC.
    - Medical professional liability and the other three commercial lines.
- At a company level, the commercial lines, especially WC and MM have greater potential for significant calendar year loss ratios changes due to development from prior years.
- The commercial lines show cyclical behaviors in unexpected loss reserve development both at the industry composite and insurance company/group level.
- Calendar year loss ratios do not appear to be more stable than accident year results, but do appear to delay the recognition of underwriting losses and profits, particularly for commercial lines.
- For Personal lines, adverse development for the industry as a whole is realized by 36 months of maturity.
- For CMP and CAL, adverse development for the industry as a whole is generally under 1% at 72 months of maturity and beyond
- WC and MM both experience the widest fluctuations in AY loss reserves in more mature observations.
- Data organized in this manner may provide opportunities for understanding industry and company reserving behaviors and loss development risk potentials.

INDEX OF EXHIBITS

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Exhibit 2	Impact of Prior Year Development on Calendar Year Loss Ratios by Company
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Impact of Prior Year Development on Calendar Year Loss Ratios **Exhibit 1**
Industry Composite

Calendar Year Change in Reserves as a Percentage of Net Premiums Earned

	Line of Business					
Calendar Year	HMP/ FMP	PPAL	CAL	WC	CMP	MM
2000	-0.6%	-0.7%	8.6%	1.0%	-1.4%	4.2%
2001	4.0%	0.2%	8.8%	6.0%	1.7%	26.0%
2002	4.5%	-0.5%	4.7%	6.8%	2.2%	17.6%
2003	-2.4%	-1.5%	1.4%	5.8%	4.2%	15.2%
2004	-2.6%	-2.4%	0.5%	1.5%	1.9%	-0.1%
2005	-1.9%	-3.0%	-0.2%	3.9%	-4.7%	-2.9%
2006	-2.3%	-3.3%	-2.4%	-1.9%	0.0%	-11.5%
2007	0.0%	-1.6%	-3.8%	-6.3%	-5.2%	-19.6%
2008	0.1%	-0.5%	-2.9%	-2.8%	-7.3%	-24.8%
2009	-2.5%	-1.5%	-3.1%	-2.0%	-5.4%	-27.8%
2010	-0.6%	-2.6%	-3.4%	3.0%	-4.7%	-31.9%

Correlation

	Line of Business					
Line of Business	HMP/ FMP	PPAL	CAL	WC	CMP	MM
HMP/FMP		0.7291	0.5560	0.3972	0.2009	0.4925
PPAL			0.6023	0.1978	0.1225	0.4270
CAL				0.6232	0.5930	0.8461
WC					0.6547	0.7334
CMP						0.8332
MM						

Impact of Prior Year Development on Calendar Year Loss Ratios
By Company
Homeowners/Farmowners

Exhibit 2
HMP/FMP

Calendar Year Change in Reserves as a Percentage of Net Premiums Earned

Calendar Year	Group / Company 1	2	3	4	5	6	15	18	19	20	21	25	33	38	40	High	Low	Average	Range	Std Dev
2000	-2.4%	-0.2%	-3.8%	1.2%	0.9%	0.2%	-2.6%	0.3%	2.1%	-2.2%	-4.6%	4.6%	-7.2%	-0.8%	-1.9%	4.6%	-7.2%	-1.1%	11.8%	0.0291
2001	5.5%	6.4%	4.2%	-1.2%	4.0%	4.7%	4.1%	2.7%	1.1%	2.2%	1.2%	3.7%	-10.5%	0.3%	5.7%	6.4%	-10.5%	2.3%	16.8%	0.0412
2002	7.5%	9.7%	11.9%	2.7%	3.6%	2.4%	-2.1%	4.7%	3.4%	-1.6%	0.4%	-1.6%	-4.4%	-0.4%	3.9%	11.9%	-4.4%	2.7%	16.2%	0.0455
2003	-0.2%	2.9%	-3.6%	-2.2%	-0.8%	-3.0%	-4.8%	-2.9%	-4.3%	-4.2%	-2.6%	-1.0%	-0.8%	-10.0%	-1.4%	2.9%	-10.0%	-2.6%	12.9%	0.0284
2004	-3.0%	-2.3%	-1.1%	1.8%	-2.6%	-1.4%	-3.0%	-2.7%	-3.7%	-3.6%	-3.3%	-3.5%	1.1%	-10.8%	-4.3%	1.8%	-10.8%	-2.8%	12.7%	0.0281
2005	-3.1%	-3.3%	-2.8%	1.1%	-4.2%	-2.3%	1.4%	0.1%	-1.7%	-2.7%	-5.4%	2.6%	-2.1%	0.8%	-5.1%	2.6%	-5.4%	-1.8%	8.0%	0.0244
2006	-5.6%	-3.7%	-0.4%	-0.7%	0.0%	-0.5%	-6.5%	-2.2%	-3.6%	0.2%	-2.2%	-1.4%	-7.9%	-8.6%	-3.1%	0.2%	-8.6%	-3.1%	8.7%	0.0288
2007	0.1%	-3.7%	-2.5%	-0.3%	-1.9%	2.6%	0.4%	-2.5%	-2.4%	-4.4%	-3.0%	2.6%	-16.5%	-0.7%	2.4%	2.6%	-16.5%	-2.0%	19.1%	0.0460
2008	1.2%	-3.4%	-1.8%	3.6%	-0.9%	1.9%	-1.2%	-1.0%	-3.0%	-4.3%	-0.1%	0.1%	1.0%	-3.8%	-3.6%	3.6%	-4.3%	-1.0%	7.9%	0.0234
2009	-3.6%	-4.7%	-2.9%	3.5%	-4.4%	-1.9%	-4.1%	-4.2%	0.5%	-2.2%	1.5%	-1.0%	-1.2%	-4.3%	-2.0%	3.5%	-4.7%	-2.1%	8.2%	0.0241
2010	-1.6%	-0.3%	-2.6%	6.2%	1.0%	-0.1%	-1.0%	-2.8%	-1.3%	-3.1%	-1.4%	-0.6%	-4.7%	0.5%	-0.1%	6.2%	-4.7%	-0.8%	10.9%	0.0246
High	7.5%	9.7%	11.9%	6.2%	4.0%	4.7%	4.1%	4.7%	3.4%	2.2%	1.5%	4.6%	1.1%	0.8%	5.7%					
Low	-5.6%	-4.7%	-3.8%	-2.2%	-4.4%	-3.0%	-6.5%	-4.2%	-4.3%	-4.4%	-5.4%	-3.5%	-16.5%	-10.8%	-5.1%					
Average	-0.5%	-0.2%	-0.5%	1.4%	-0.5%	0.2%	-1.8%	-0.9%	-1.2%	-2.4%	-1.8%	0.4%	-4.8%	-3.4%	-0.9%					
Range	13.1%	14.5%	15.7%	8.4%	8.4%	7.7%	10.5%	8.9%	7.7%	6.6%	6.9%	8.1%	17.6%	11.6%	10.7%					
Std Dev	0.0396	0.0471	0.0467	0.0247	0.0278	0.0239	0.0297	0.0269	0.0258	0.0203	0.0230	0.0257	0.0537	0.0442	0.0350					

Correlation

Group / Company	Group / Company 1	2	3	4	5	6	15	18	19	20	21	25	33	38	40
1		0.8472	0.7913	(0.0490)	0.7156	0.7200	0.5108	0.7812	0.5274	0.2387	0.5129	0.1490	(0.1855)	0.4167	0.7953
2			0.7962	(0.1635)	0.8274	0.4434	0.2540	0.7792	0.5630	0.4080	0.3507	0.0436	(0.0977)	0.2413	0.7269
3				0.0136	0.6813	0.5638	0.1935	0.8158	0.5786	0.4783	0.4965	(0.1674)	(0.1157)	0.2423	0.6409
4					(0.0837)	(0.0288)	(0.0010)	(0.1055)	0.2475	(0.2761)	0.2550	(0.2764)	0.3724	0.3245	(0.1664)
5						0.6864	0.2383	0.7085	0.5015	0.5492	0.3673	0.1834	(0.3434)	0.3049	0.7379
6							0.6047	0.6108	0.4370	0.4216	0.4580	0.4071	(0.5979)	0.5246	0.7655
15								0.4738	0.2986	0.2295	0.0707	0.6307	(0.3610)	0.7213	0.4741
18									0.6824	0.4957	0.1732	0.3346	(0.1841)	0.5178	0.5533
19										0.4512	0.3456	0.3777	(0.2186)	0.6614	0.5571
20											0.3769	0.2498	(0.3310)	0.2026	0.4520
21												(0.2511)	0.0446	0.0843	0.5248
25													(0.5756)	0.6813	0.2905
33														(0.4412)	(0.6321)
38															0.4743
40															

Impact of Prior Year Development on Calendar Year Loss Ratios **Exhibit 2**
By Company **PPAL**
Private Passenger Auto Liability

Calendar Year Change in Reserves as a Percentage of Net Premiums Earned

Calendar	Group / Company																					
Year	1	2	3	4	5	6	8	9	15	18	19	20	21	25	33	38	40	High	Low	Average	Range	Std Dev
2000	-2.8%	2.2%	-3.2%	-1.4%	-1.6%	-0.1%	3.1%	-4.7%	-13.0%	9.4%	2.8%	-1.0%	-21.0%	2.4%	0.2%	-3.5%	37.5%	37.5%	-21.0%	0.3%	58.5%	0.1170
2001	-0.1%	-2.6%	-2.4%	-0.3%	0.2%	2.8%	-2.0%	-4.7%	-16.5%	2.5%	9.1%	-2.1%	2.9%	-1.1%	7.0%	0.3%	4.1%	9.1%	-16.5%	-0.2%	25.6%	0.0552
2002	1.0%	-1.3%	-1.1%	1.4%	3.1%	-0.8%	-0.2%	-4.8%	-11.5%	4.0%	13.5%	-4.1%	7.3%	-7.5%	10.0%	-3.7%	0.8%	13.5%	-11.5%	0.4%	24.9%	0.0616
2003	-1.4%	2.6%	8.1%	-3.5%	1.4%	-5.5%	-0.8%	-2.8%	-10.6%	-1.2%	-2.2%	1.7%	6.1%	-0.2%	6.4%	-4.8%	2.0%	8.1%	-10.6%	-0.3%	18.6%	0.0469
2004	-7.0%	1.8%	8.4%	-1.5%	-1.5%	-3.9%	-1.2%	-3.3%	-11.4%	-0.4%	-2.3%	-0.6%	5.2%	-0.6%	1.1%	-9.9%	-1.5%	8.4%	-11.4%	-1.7%	19.9%	0.0482
2005	-6.0%	9.4%	1.5%	-6.1%	-3.4%	-2.7%	-3.2%	-7.2%	-3.3%	2.0%	-4.1%	-1.1%	3.6%	-0.6%	20.4%	-16.5%	-5.4%	20.4%	-16.5%	-1.3%	36.8%	0.0781
2006	-5.4%	-8.5%	-0.9%	-2.1%	-3.7%	-1.9%	-2.6%	-6.3%	-12.5%	2.0%	-10.8%	0.6%	-2.8%	-5.0%	2.3%	-7.4%	-2.4%	2.3%	-12.5%	-4.0%	14.8%	0.0419
2007	-2.2%	-11.4%	-0.7%	-0.1%	-5.7%	1.2%	0.8%	-5.2%	-4.4%	-1.5%	-16.1%	3.4%	-1.2%	-1.8%	1.8%	-4.3%	-8.6%	3.4%	-16.1%	-3.3%	19.5%	0.0504
2008	0.6%	-11.5%	-1.2%	-5.8%	-4.4%	2.5%	0.7%	-2.7%	-1.2%	-6.7%	-5.9%	-7.4%	-0.9%	1.9%	-0.7%	-0.8%	-9.5%	2.5%	-11.5%	-3.1%	14.1%	0.0410
2009	1.3%	-14.0%	-5.0%	-3.7%	-4.5%	0.3%	-1.4%	-2.6%	-1.2%	-10.6%	-0.1%	-4.7%	5.7%	-0.2%	-3.3%	0.5%	3.2%	5.7%	-14.0%	-2.4%	19.8%	0.0474
2010	0.2%	-11.6%	-5.4%	-6.9%	-2.8%	-1.7%	-3.1%	-5.2%	-5.8%	-6.6%	-4.8%	-4.5%	7.3%	-4.0%	51.4%	-0.2%	-1.1%	51.4%	-11.6%	-0.3%	63.0%	0.1389
High	1.3%	9.4%	8.4%	1.4%	3.1%	2.8%	3.1%	-2.6%	-1.2%	9.4%	13.5%	3.4%	7.3%	2.4%	51.4%	0.5%	37.5%					
Low	-7.0%	-14.0%	-5.4%	-6.9%	-5.7%	-5.5%	-3.2%	-7.2%	-16.5%	-10.6%	-16.1%	-7.4%	-21.0%	-7.5%	-3.3%	-16.5%	-9.5%					
Average	-2.0%	-4.1%	-0.2%	-2.7%	-2.1%	-0.9%	-0.9%	-4.5%	-8.3%	-0.6%	-1.9%	-1.8%	1.1%	-1.5%	8.8%	-4.6%	1.7%					
Range	8.2%	23.4%	13.9%	8.3%	8.8%	8.4%	6.3%	4.7%	15.4%	20.0%	29.5%	10.8%	28.3%	9.9%	54.7%	16.9%	47.0%					
Std Dev	0.0297	0.0770	0.0461	0.0272	0.0272	0.0261	0.0192	0.0151	0.0529	0.0566	0.0831	0.0317	0.0815	0.0294	0.1556	0.0511	0.1268					

Correlation

Group /	Group / Company																
Company	1	2	3	4	5	6	8	9	15	18	19	20	21	25	33	38	40
1		(0.5414)	(0.5579)	(0.0065)	0.1819	0.5163	0.1901	0.4614	0.2156	(0.4181)	0.3904	(0.5757)	0.2017	(0.0845)	0.1048	0.8553	0.0100
2			0.5791	0.1024	0.5262	(0.5215)	(0.0161)	(0.3076)	(0.4164)	0.6842	0.3619	0.3293	(0.1136)	0.1435	(0.0057)	(0.7044)	0.3048
3				0.1098	0.3247	(0.6952)	(0.0430)	0.2062	(0.2284)	0.1713	(0.1001)	0.4870	0.2379	0.1334	(0.2578)	(0.5621)	(0.2104)
4					0.4304	0.1841	0.4052	(0.0060)	(0.6286)	0.5524	0.3660	0.3955	(0.1775)	(0.3258)	(0.5149)	0.1074	0.2444
5						(0.3209)	0.0231	0.1124	(0.6721)	0.4702	0.8022	(0.0095)	0.2101	(0.3435)	0.0709	0.0491	0.2725
6							0.3202	0.0486	0.1418	(0.0987)	0.0990	(0.4076)	(0.2768)	0.1584	(0.2258)	0.5728	0.0205
8								0.3615	(0.0593)	0.3337	0.0745	0.0581	(0.7028)	0.4330	(0.5384)	0.2815	0.5549
9									0.1555	(0.4949)	0.1631	(0.3312)	0.1466	0.4144	(0.4051)	0.5509	0.0405
15										(0.7279)	(0.4798)	(0.3534)	0.2290	0.2418	0.1115	(0.0093)	(0.4604)
18											0.3438	0.4286	(0.5683)	(0.0830)	(0.1636)	(0.3790)	0.5740
19												(0.4102)	0.1011	(0.1682)	(0.0107)	0.2379	0.4121
20													(0.1826)	(0.0356)	(0.1939)	(0.4394)	0.0568
21														(0.4660)	0.3531	(0.0226)	(0.7469)
25															(0.3729)	0.0209	0.3349
33																(0.0093)	(0.1547)
38																	0.1711
40																	

Impact of Prior Year Development on Calendar Year Loss Ratios **Exhibit 2**
By Company **CAL**
Commercial Auto Liability

Calendar Year Change in Reserves as a Percentage of Net Premiums Earned

Calendar	Group / Company																												
Year	1	2	3	4	5	6	7	8	9	14	15	17	19	20	21	22	23	25	28	31	33	34	38	40	High	Low	Average	Range	Std Dev
2000	1.6%	34.9%	5.3%	0.1%	18.0%	7.0%	-1.3%	-0.9%	4.5%	-13.4%	-3.1%	12.6%	19.5%	45.6%	-2.0%	16.1%	3.6%	5.7%	5.0%	11.4%	23.1%	2.4%	6.7%	3.1%	45.6%	-13.4%	8.6%	59.0%	0.1277
2001	2.2%	-1.3%	17.0%	0.4%	20.2%	12.9%	-11.8%	-2.6%	0.6%	0.7%	12.1%	-2.6%	13.9%	8.0%	12.5%	3.5%	-2.2%	5.4%	40.7%	36.7%	-5.8%	6.8%	7.8%	6.0%	40.7%	-11.8%	7.5%	52.5%	0.1223
2002	2.4%	-6.1%	-4.9%	4.3%	1.6%	8.3%	-7.9%	3.7%	7.6%	3.3%	-8.4%	-3.4%	20.0%	0.1%	18.4%	1.4%	7.2%	0.9%	-3.8%	15.5%	-7.3%	14.5%	11.0%	6.3%	20.0%	-8.4%	3.5%	28.4%	0.0823
2003	5.6%	-3.8%	-11.7%	4.1%	0.2%	3.6%	-12.6%	0.2%	-6.2%	-2.1%	-20.5%	-8.2%	1.4%	-10.6%	13.8%	3.9%	4.5%	-3.0%	0.8%	5.9%	7.8%	7.2%	5.0%	-6.4%	13.8%	-20.5%	-0.9%	34.4%	0.0793
2004	2.2%	0.4%	8.6%	4.9%	-4.2%	5.2%	-18.5%	-2.6%	-9.6%	-0.3%	13.8%	-0.2%	2.9%	-11.7%	7.0%	7.0%	5.1%	-3.9%	3.5%	11.9%	7.9%	9.2%	-1.7%	3.6%	13.8%	-18.5%	1.7%	32.2%	0.0755
2005	0.6%	-1.9%	3.3%	3.2%	-3.0%	12.6%	-9.0%	-4.0%	-9.9%	1.3%	5.2%	-9.2%	-4.3%	-4.8%	5.9%	10.7%	-5.6%	-2.6%	16.7%	-7.8%	2.7%	15.7%	-8.8%	-0.6%	16.7%	-9.9%	0.3%	26.6%	0.0780
2006	-8.4%	-3.3%	-1.6%	1.9%	-7.3%	15.2%	-7.9%	0.1%	-16.3%	3.7%	-8.3%	4.0%	-11.1%	-4.9%	6.8%	-0.1%	-12.9%	4.9%	-1.1%	5.8%	-1.4%	11.6%	-11.2%	-5.0%	15.2%	-16.3%	-2.0%	31.4%	0.0782
2007	-7.9%	-7.1%	1.9%	-3.9%	-5.3%	13.3%	-7.7%	6.0%	-7.7%	8.4%	-12.8%	-5.1%	-22.8%	-6.0%	6.4%	-2.1%	-7.6%	1.2%	-10.8%	-5.2%	-6.5%	3.5%	-15.0%	1.2%	13.3%	-22.8%	-3.8%	36.2%	0.0805
2008	-4.9%	-12.4%	-11.2%	-3.3%	-3.1%	19.9%	-10.1%	1.9%	-12.4%	10.2%	19.1%	4.0%	-12.7%	-2.2%	0.1%	-0.9%	-10.4%	-0.3%	3.9%	-12.2%	-2.8%	1.0%	-10.4%	-3.8%	19.9%	-12.7%	-2.2%	32.6%	0.0912
2009	-10.6%	-9.6%	2.3%	-6.0%	-7.4%	14.5%	-12.3%	-4.1%	-16.1%	29.7%	32.9%	-13.9%	-16.8%	-5.8%	4.8%	4.4%	-9.3%	-4.4%	1.6%	7.9%	23.0%	-11.8%	-15.2%	-2.3%	32.9%	-16.8%	-1.0%	49.8%	0.1400
2010	0.7%	-9.4%	-5.1%	-11.0%	-4.7%	10.3%	-16.3%	-7.6%	-13.8%	16.7%	7.2%	-2.9%	-7.2%	-9.6%	0.2%	-9.3%	-7.8%	-10.2%	-17.6%	-16.2%	17.0%	1.0%	-1.1%	5.3%	17.0%	-17.6%	-3.8%	34.7%	0.0969
High	5.6%	34.9%	17.0%	4.9%	20.2%	19.9%	-1.3%	6.0%	7.6%	29.7%	32.9%	12.6%	20.0%	45.6%	18.4%	16.1%	7.2%	5.7%	40.7%	36.7%	23.1%	15.7%	11.0%	6.3%					
Low	-10.6%	-12.4%	-11.7%	-11.0%	-7.4%	3.6%	-18.5%	-7.6%	-16.3%	-13.4%	-20.5%	-13.9%	-22.8%	-11.7%	-2.0%	-9.3%	-12.9%	-10.2%	-17.6%	-16.2%	-7.3%	-11.8%	-15.2%	-6.4%					
Average	-1.5%	-1.8%	0.4%	-0.5%	0.5%	11.2%	-10.5%	-0.9%	-7.2%	5.3%	3.4%	-2.3%	-1.6%	-0.2%	6.7%	3.2%	-3.2%	-0.6%	3.5%	4.9%	5.2%	5.5%	-3.0%	0.7%					
Range	16.1%	47.3%	28.7%	15.9%	27.7%	16.3%	17.2%	13.6%	23.8%	43.1%	53.5%	26.5%	42.8%	57.3%	20.3%	25.5%	20.0%	15.9%	58.3%	53.0%	30.3%	27.5%	26.2%	12.7%					
Std Dev	0.0543	0.1281	0.0851	0.0504	0.0963	0.0483	0.0463	0.0386	0.0814	0.1113	0.1572	0.0729	0.1455	0.1615	0.0621	0.0676	0.0715	0.0489	0.1514	0.1492	0.1148	0.0770	0.0960	0.0456					

Correlation

Group /	Group / Company																							
Company	1	2	3	4	5	6	7	8	9	14	15	17	19	20	21	22	23	25	28	31	33	34	38	40
1		0.3265	0.0474	0.4590	0.5138	(0.7398)	(0.1648)	(0.1927)	0.5992	(0.6378)	(0.3400)	0.1253	0.7673	0.1763	0.3475	0.2749	0.7974	(0.0841)	0.2630	0.2831	0.0029	0.5033	0.8597	0.3851
2			0.3660	0.2678	0.6648	(0.4318)	0.5867	(0.0384)	0.5316	(0.7162)	(0.2166)	0.6454	0.5905	0.8978	(0.3066)	0.7547	0.4429	0.4910	0.1887	0.3103	0.4318	0.0694	0.4247	0.2231
3				0.1471	0.5196	(0.0614)	(0.0066)	(0.2480)	0.2402	(0.2010)	0.3028	0.0555	0.2905	0.3230	0.0265	0.4187	0.1205	0.3672	0.6447	0.6438	0.0139	0.0403	0.1226	0.5394
4					0.2126	(0.4431)	0.1550	0.3266	0.4377	(0.6904)	(0.4054)	0.1103	0.5207	0.0673	0.5931	0.5825	0.6117	0.4417	0.3985	0.4889	(0.3762)	0.7322	0.3987	(0.0999)
5						(0.2424)	0.3961	(0.0193)	0.7562	(0.6083)	(0.0850)	0.4404	0.7661	0.7762	0.0803	0.4792	0.4596	0.5942	0.6504	0.6707	0.0204	0.0906	0.7085	0.4668
6							0.1691	0.1105	(0.4572)	0.5071	0.4571	(0.0031)	(0.5726)	(0.1208)	(0.3413)	(0.3397)	(0.8535)	0.1679	0.1155	(0.2879)	(0.3095)	(0.2764)	(0.6543)	(0.2564)
7								0.4905	0.4660	(0.4266)	(0.3762)	0.4591	0.2283	0.7406	(0.1338)	0.4446	(0.0022)	0.7232	0.0655	0.0762	(0.0692)	0.1136	0.0624	(0.0765)
8									0.3645	(0.2501)	(0.5230)	0.1850	(0.0427)	0.0642	0.3461	(0.0406)	0.1382	0.5366	(0.1631)	0.0681	(0.6300)	0.2300	(0.0405)	(0.1543)
9										(0.6306)	(0.3851)	0.3082	0.8550	0.6114	0.4307	0.4063	0.7664	0.5017	0.2954	0.5744	(0.2003)	0.3574	0.8168	0.5733
14											0.5869	(0.6110)	(0.6709)	(0.5561)	(0.1339)	(0.5865)	(0.6037)	(0.5687)	(0.3209)	(0.3415)	0.1735	(0.6424)	(0.5955)	(0.1389)
15												(0.1855)	(0.2097)	(0.0764)	(0.3900)	0.0221	(0.3449)	(0.2931)	0.2689	0.0110	0.3371	(0.5752)	(0.3330)	0.1114
17													0.3840	0.6724	(0.4257)	0.2219	0.1120	0.5419	(0.0041)	0.0705	0.0191	0.1625	0.2809	0.1558
19														0.5783	0.3632	0.4883	0.8009	0.3668	0.4082	0.6436	0.0160	0.4146	0.9362	0.5465
20															(0.3325)	0.6187	0.2784	0.6143	0.2675	0.3317	0.3137	(0.0840)	0.4112	0.2870
21																(0.0566)	0.4884	0.1616	0.2429	0.5590	(0.5576)	0.4960	0.4456	0.1084
22																	0.4623	0.4190	0.4820	0.3918	0.3018	0.1554	0.2350	(0.0222)
23																		0.0827	0.1238	0.4874	0.0478	0.3637	0.8123	0.4087
25																			0.4893	0.5780	(0.4000)	0.2723	0.2004	0.0075
28																				0.6730	(0.2410)	0.1992	0.2776	0.1294
31																					(0.1526)	0.1375	0.5617	0.3451
33																						(0.6055)	(0.0192)	(0.0428)
34																							0.3753	0.1431
38																								0.5302
40																								

Impact of Prior Year Development on Calendar Year Loss Ratios
By Company
Workers Compensation

Exhibit 2
WC

Calendar Year Change in Reserves as a Percentage of Net Premiums Earned

Calendar Year	Group / Company 2	3	4	5	6	7	9	10	11	15	16	17	19	20	21	22	23	25	28	31	33	34	38	39	40	High	Low	Average	Range	Std Dev
2000	-6.5%	0.2%	-0.1%	12.5%	-8.5%	-12.6%	13.7%	1.6%	-28.8%	-21.6%	3.6%	-7.3%	0.3%	-14.9%	-26.8%	-28.0%	10.4%	-1.2%	9.8%	-85.0%	2.1%	6.9%	-5.6%	-3.6%	65.0%	65.0%	-85.0%	-5.0%	150.0%	0.2488
2001	-0.4%	17.8%	17.2%	16.9%	-0.1%	-0.6%	15.6%	1.0%	-59.1%	-2.2%	5.5%	3.4%	10.2%	-0.9%	-6.3%	12.6%	6.2%	26.9%	-28.3%	6.0%	2.8%	2.2%	6.7%	-8.4%	14.3%	26.9%	-59.1%	2.4%	86.0%	0.1681
2002	1.5%	22.1%	10.1%	0.5%	2.5%	-0.6%	10.6%	6.7%	-9.3%	-5.0%	12.0%	1.4%	6.6%	0.7%	-8.1%	2.4%	7.2%	7.9%	15.4%	4.0%	10.3%	7.8%	-4.3%	0.1%	3.5%	22.1%	-9.3%	4.2%	31.4%	0.0721
2003	8.2%	29.0%	-1.1%	11.2%	7.0%	6.9%	10.4%	2.8%	-27.7%	-12.0%	1.2%	-4.6%	10.3%	4.0%	4.1%	7.0%	2.8%	8.8%	18.0%	8.3%	5.3%	22.7%	9.9%	-1.8%	-1.1%	29.0%	-27.7%	5.2%	56.7%	0.1084
2004	5.5%	7.2%	2.9%	-1.3%	13.3%	2.5%	-15.3%	-0.4%	-20.2%	-23.6%	2.4%	-10.9%	0.3%	3.9%	2.3%	13.4%	5.7%	0.0%	14.3%	-1.7%	-4.6%	20.6%	0.0%	0.1%	1.2%	20.6%	-23.6%	0.7%	44.3%	0.1015
2005	1.5%	2.1%	5.2%	4.7%	4.5%	0.5%	2.3%	-4.6%	-20.1%	-9.6%	-3.4%	-9.0%	-8.8%	14.1%	-17.4%	-0.9%	-3.7%	0.7%	28.7%	-4.4%	14.1%	25.0%	1.5%	-2.3%	-1.0%	28.7%	-20.1%	0.8%	48.8%	0.1107
2006	-5.7%	-0.8%	-2.1%	2.6%	5.8%	-1.1%	-8.4%	-17.5%	-3.0%	-9.2%	-3.2%	-12.8%	-15.6%	2.1%	-14.8%	-6.2%	-7.4%	-14.4%	10.3%	-3.7%	-0.3%	4.8%	-4.5%	6.5%	-1.9%	10.3%	-17.5%	-4.0%	27.8%	0.0735
2007	-7.3%	-12.2%	-14.9%	-1.9%	2.0%	-2.8%	-12.9%	-15.5%	-14.2%	-8.5%	0.7%	-16.3%	2.4%	0.8%	-24.4%	-7.0%	-10.9%	-10.8%	7.9%	-169.8%	2.9%	8.3%	-5.2%	1.2%	6.1%	8.3%	-169.8%	-12.1%	178.1%	0.3394
2008	-3.9%	-4.2%	-6.7%	0.2%	8.8%	-4.5%	-23.7%	-13.0%	-15.7%	-0.8%	-3.8%	-11.4%	-16.7%	3.0%	13.7%	-3.9%	-31.2%	-3.3%	19.2%	-6.9%	-3.2%	14.3%	-5.5%	2.6%	-11.8%	19.2%	-31.2%	-4.3%	50.4%	0.1137
2009	-3.9%	1.4%	-1.7%	-3.1%	-2.8%	-6.4%	2.0%	-1.9%	-3.5%	43.5%	-58.3%	-18.1%	-50.3%	15.1%	24.8%	-2.3%	-10.9%	-5.0%	-2.6%	-7.8%	7.2%	19.1%	-3.2%	7.1%	1.4%	43.5%	-58.3%	-2.4%	101.8%	0.2004
2010	-4.7%	4.4%	-2.6%	0.8%	7.6%	12.9%	-23.3%	5.8%	-9.0%	-10.7%	-2.9%	-20.7%	-39.1%	-8.7%	10.8%	-5.5%	-7.3%	-10.8%	8.6%	-1.2%	28.0%	17.2%	-11.3%	9.4%	3.5%	28.0%	-39.1%	-1.9%	67.1%	0.1399
High	8.2%	29.0%	17.2%	16.9%	13.3%	12.9%	15.6%	6.7%	-3.0%	43.5%	12.0%	3.4%	10.3%	15.1%	24.8%	13.4%	10.4%	26.9%	28.7%	8.3%	28.0%	25.0%	9.9%	9.4%	65.0%					
Low	-7.3%	-12.2%	-14.9%	-3.1%	-8.5%	-12.6%	-23.7%	-17.5%	-59.1%	-23.6%	-58.3%	-20.7%	-50.3%	-14.9%	-26.8%	-28.0%	-31.2%	-14.4%	-28.3%	-169.8%	-4.6%	2.2%	-11.3%	-8.4%	-11.8%					
Average	-1.4%	6.1%	0.6%	3.9%	3.6%	-0.5%	-2.6%	-3.2%	-19.1%	-5.4%	-4.2%	-9.7%	-9.1%	1.8%	-3.8%	-1.7%	-3.6%	-0.1%	9.2%	-23.8%	5.9%	13.5%	-2.0%	1.0%	7.2%					
Range	15.5%	41.2%	32.1%	20.0%	21.8%	25.5%	39.3%	24.2%	56.1%	67.1%	70.3%	24.1%	60.6%	30.0%	51.6%	41.4%	41.6%	41.3%	56.9%	178.2%	32.6%	22.8%	21.2%	17.8%	76.8%					
Std Dev	0.0510	0.1219	0.0844	0.0668	0.0599	0.0669	0.1470	0.0850	0.1584	0.1769	0.1856	0.0758	0.2001	0.0858	0.1649	0.1133	0.1196	0.1159	0.1471	0.5485	0.0918	0.0786	0.0608	0.0520	0.2017					

Correlation

Group / Company	Group / Company 2	3	4	5	6	7	9	10	11	15	16	17	19	20	21	22	23	25	28	31	33	34	38	39	40
2		0.7791	0.4628	0.2112	0.4778	0.4455	0.2938	0.4774	(0.2883)	(0.2187)	0.2264	0.4909	0.4415	0.3479	0.2191	0.7257	0.4173	0.5467	0.2187	0.5568	(0.0751)	0.5397	0.7418	(0.3862)	(0.3128)
3			0.6698	0.4869	0.1078	0.4067	0.5866	0.7062	(0.4001)	(0.0932)	0.2677	0.7003	0.4229	0.0088	0.1960	0.5506	0.5806	0.7216	(0.1354)	0.6109	0.1321	0.1085	0.6478	(0.3914)	(0.0693)
4				0.5394	(0.1177)	0.0751	0.6224	0.5855	(0.5744)	(0.0345)	0.2272	0.7658	0.3185	0.0331	(0.0130)	0.4630	0.6130	0.8043	(0.4034)	0.6209	0.0810	(0.1618)	0.4693	(0.5548)	0.1208
5					(0.3899)	(0.0840)	0.6898	0.3193	(0.8513)	(0.3168)	0.3915	0.6687	0.5400	(0.3829)	(0.3427)	(0.0110)	0.5289	0.6988	(0.4053)	0.1313	(0.0794)	(0.2992)	0.5975	(0.7492)	0.5270
6						0.6851	(0.6506)	(0.1449)	0.2141	(0.3119)	0.2521	(0.1766)	0.0188	0.2589	0.3469	0.6078	(0.3002)	(0.1571)	0.3973	0.3948	(0.0347)	0.4475	0.0741	0.2780	(0.7533)
7							(0.3349)	0.3211	0.0572	(0.2343)	0.2292	(0.1393)	(0.0723)	0.0272	0.3213	0.5329	0.0328	0.0111	0.1195	0.4144	0.5776	0.4064	0.1134	0.2793	(0.5325)
9								0.4467	(0.5216)	0.0821	0.0621	0.7366	0.4619	(0.0107)	(0.3237)	0.0079	0.7043	0.6804	(0.3199)	0.0965	(0.0663)	(0.2314)	0.5882	(0.6710)	0.4944
10									(0.2857)	(0.0554)	0.0969	0.3665	0.0752	(0.2526)	0.2328	0.1908	0.6429	0.5134	(0.1373)	0.4087	0.4942	0.2174	0.1801	(0.2205)	0.2947
11										0.2731	(0.3915)	(0.6749)	(0.6027)	0.2447	0.2288	(0.2914)	(0.4394)	(0.8427)	0.5613	(0.0580)	0.2140	0.2598	(0.6511)	0.8680	(0.3571)
15											(0.8889)	(0.2499)	(0.6471)	0.5739	0.6044	0.0753	(0.3906)	(0.0311)	(0.3401)	0.1564	0.0943	0.0981	(0.0454)	0.3599	(0.2894)
16												0.5314	0.7870	(0.5504)	(0.5998)	0.0688	0.3664	0.2850	0.1731	(0.0913)	(0.0715)	(0.3270)	0.1166	(0.4938)	0.1756
17													0.7711	(0.0984)	(0.2980)	0.3440	0.5668	0.8696	(0.2460)	0.2869	(0.2669)	(0.3482)	0.6242	(0.8210)	0.2014
19														(0.2443)	(0.6185)	0.2328	0.5284	0.5724	0.0023	(0.1820)	(0.4235)	(0.2994)	0.5611	(0.8186)	0.2347
20															0.3697	0.5007	(0.3170)	0.0144	0.1709	0.2996	(0.1025)	0.5351	0.3679	0.1142	(0.6782)
21																0.4014	(0.4247)	0.0045	(0.1090)	0.5758	0.1513	0.4594	(0.0078)	0.4803	(0.5275)
22																	0.1081	0.5323	(0.2211)	0.5358	(0.1012)	0.2619	0.6068	(0.2072)	(0.6549)
23																		0.4986	(0.2131)	0.0709	0.0602	(0.1550)	0.3812	(0.5368)	0.5608
25																			(0.4930)	0.3612	(0.1294)	(0.1554)	0.7344	(0.8110)	0.1231
28																				(0.0029)	0.0890	0.5714	(0.1970)	0.2563	(0.2165)
31																					0.1690	0.3151	0.3108	0.0716	(0.4081)
33																						0.2791	(0.3097)	0.3609	(0.0615)
34																							0.2020	0.2200	(0.3974)
38																								(0.6815)	(0.1158)
39																									(0.4009)
40																									

Impact of Prior Year Development on Calendar Year Loss Ratios **Exhibit 2**
By Company **CMP**
Commercial Multiple Peril

Calendar Year Change in Reserves as a Percentage of Net Premiums Earned

Calendar	Group / Company																									
Year	1	2	3	4	5	6	7	9	17	19	20	21	22	23	25	28	31	33	34	38	40	High	Low	Average	Range	Std Dev
2000	-6.2%	-2.9%	-1.5%	-7.3%	2.0%	-1.0%	-28.6%	45.4%	-2.6%	6.8%	-4.9%	6.3%	-0.3%	17.7%	1.2%	1.5%	13.2%	11.4%	4.0%	-6.7%	-32.2%	45.4%	-32.2%	0.7%	77.6%	0.1551
2001	4.0%	-3.6%	-16.3%	-7.1%	6.9%	0.9%	-9.1%	-12.4%	-11.9%	5.6%	0.7%	6.2%	2.5%	-2.0%	1.8%	28.8%	1.6%	-0.6%	8.0%	-3.0%	-5.7%	28.8%	-16.3%	-0.2%	45.1%	0.0946
2002	5.0%	2.0%	9.3%	5.5%	-0.8%	-8.1%	-0.2%	1.2%	8.5%	5.2%	1.1%	-7.6%	6.9%	10.8%	-4.5%	3.3%	-32.4%	-17.5%	24.1%	1.9%	-0.6%	24.1%	-32.4%	0.6%	56.5%	0.1118
2003	1.6%	-1.0%	17.9%	-0.3%	-3.1%	-2.7%	11.6%	-6.6%	-0.5%	9.9%	1.4%	13.1%	1.0%	18.0%	1.3%	30.7%	81.8%	10.2%	4.4%	-1.6%	2.6%	81.8%	-6.6%	9.0%	88.4%	0.1894
2004	8.2%	-10.0%	6.0%	-3.4%	-0.3%	-0.9%	4.0%	16.0%	-3.5%	-12.9%	-0.3%	12.7%	2.0%	8.8%	0.6%	-8.4%	11.3%	-1.6%	82.5%	-7.1%	3.3%	82.5%	-12.9%	5.1%	95.4%	0.1927
2005	1.2%	-12.0%	3.1%	5.4%	-4.6%	0.6%	-5.5%	-25.9%	-4.6%	-4.6%	-6.5%	-7.2%	2.3%	-4.0%	-1.3%	5.7%	-83.1%	-12.7%	-31.2%	-10.6%	1.3%	5.7%	-83.1%	-9.2%	88.8%	0.1930
2006	1.5%	-6.2%	1.3%	11.3%	2.3%	18.4%	1.0%	-3.8%	-0.1%	-1.3%	3.9%	-12.4%	2.6%	-5.1%	-0.5%	-1.3%	7.8%	-6.5%	-6.2%	-10.7%	3.7%	18.4%	-12.4%	0.0%	30.8%	0.0708
2007	-0.2%	-9.3%	-2.0%	-4.8%	-9.7%	4.8%	1.7%	-11.0%	-3.4%	-4.0%	-4.8%	-6.3%	-1.5%	-4.2%	-1.5%	-8.3%	6.8%	8.5%	-15.9%	-10.0%	1.1%	8.5%	-15.9%	-3.5%	24.4%	0.0611
2008	5.6%	-8.0%	-5.7%	9.6%	-10.2%	8.9%	5.8%	-6.8%	-2.6%	-6.4%	-10.6%	-6.4%	0.9%	-5.3%	2.5%	-29.5%	-17.4%	-18.0%	-14.7%	-21.2%	-10.1%	9.6%	-29.5%	-6.7%	39.1%	0.1014
2009	0.5%	-15.6%	-1.0%	-2.3%	-9.1%	-1.3%	4.8%	-11.7%	23.9%	9.8%	-7.5%	-20.9%	-7.5%	-3.5%	-0.6%	-11.4%	10.3%	16.1%	-8.9%	-9.6%	4.8%	23.9%	-20.9%	-1.9%	44.8%	0.1080
2010	-0.8%	-6.3%	-1.7%	-2.3%	-1.7%	-4.6%	-1.6%	-9.7%	41.6%	-13.8%	-7.5%	-22.7%	-11.2%	2.8%	-9.2%	-17.6%	-11.7%	3.0%	-11.5%	0.5%	6.3%	41.6%	-22.7%	-3.8%	64.3%	0.1266
High	8.2%	2.0%	17.9%	11.3%	6.9%	18.4%	11.6%	45.4%	41.6%	9.9%	3.9%	13.1%	6.9%	18.0%	2.5%	30.7%	81.8%	16.1%	82.5%	1.9%	6.3%					
Low	-6.2%	-15.6%	-16.3%	-7.3%	-10.2%	-8.1%	-28.6%	-25.9%	-11.9%	-13.8%	-10.6%	-22.7%	-11.2%	-5.3%	-9.2%	-29.5%	-83.1%	-18.0%	-31.2%	-21.2%	-32.2%					
Average	1.8%	-6.6%	0.9%	0.4%	-2.6%	1.4%	-1.5%	-2.3%	4.1%	-0.5%	-3.2%	-4.1%	-0.2%	3.1%	-0.9%	-0.6%	-1.1%	-0.7%	3.1%	-7.1%	-2.3%					
Range	14.4%	17.6%	34.2%	18.7%	17.0%	26.5%	40.2%	71.2%	53.5%	23.7%	14.6%	35.8%	18.1%	23.3%	11.7%	60.3%	164.9%	34.1%	113.7%	23.1%	38.5%					
Std Dev	0.0384	0.0509	0.0868	0.0654	0.0545	0.0720	0.1058	0.1880	0.1548	0.0856	0.0472	0.1233	0.0505	0.0915	0.0337	0.1809	0.3949	0.1185	0.3011	0.0652	0.1100					

Correlation

Group /	Group / Company																				
Company	1	2	3	4	5	6	7	9	17	19	20	21	22	23	25	28	31	33	34	38	40
1		(0.0224)	0.0795	0.3479	(0.0196)	0.0342	0.6206	(0.3179)	(0.2383)	(0.3068)	0.2471	0.2152	0.4421	(0.2018)	0.1709	(0.0522)	(0.1112)	(0.5808)	0.5407	(0.1099)	0.4759
2			0.2488	0.0173	0.5467	(0.2515)	(0.2033)	0.3422	(0.1642)	0.3150	0.5477	0.3937	0.4700	0.6290	(0.0787)	0.4903	0.2761	(0.2064)	0.1944	0.5739	(0.3110)
3				0.2488	(0.1633)	(0.2876)	0.4141	0.0894	0.0898	0.1270	0.3296	0.2231	0.2060	0.5920	(0.1261)	0.1958	0.3042	0.0271	0.2553	0.3081	0.2856
4					(0.2250)	0.5119	0.4113	(0.3475)	(0.0417)	(0.1600)	0.0558	(0.3360)	0.3544	(0.3558)	(0.0151)	(0.2835)	(0.3486)	(0.7356)	(0.2566)	(0.4152)	0.2663
5						(0.0533)	(0.4959)	0.3536	(0.2163)	0.1306	0.6767	0.3864	0.3242	0.3400	0.0150	0.5816	0.0993	(0.0393)	0.3828	0.5450	(0.2271)
6							0.1118	(0.1245)	(0.3601)	(0.1672)	0.1040	(0.1261)	0.1524	(0.5797)	0.3927	(0.2272)	0.0171	(0.1944)	(0.2774)	(0.6911)	0.0074
7								(0.6001)	0.1698	(0.1586)	0.0830	(0.1401)	(0.0454)	(0.2449)	(0.0452)	(0.1256)	0.2463	(0.1359)	0.0832	(0.1181)	0.7991
9									(0.1437)	0.1205	0.1320	0.4426	0.1260	0.6737	0.2094	(0.0530)	0.3115	0.2516	0.4956	0.1048	(0.7364)
17										(0.1995)	(0.3578)	(0.7296)	(0.7953)	(0.0199)	(0.8007)	(0.4307)	(0.0331)	0.2437	(0.1690)	0.3429	0.3698
19											0.2938	0.1933	0.2264	0.3202	0.3929	0.5975	0.3873	0.3611	(0.1645)	0.2139	(0.2935)
20												0.4441	0.5794	0.3305	0.1289	0.6676	0.3772	(0.0381)	0.4712	0.4945	0.1894
21													0.5719	0.5910	0.6406	0.5763	0.4379	0.0785	0.5610	0.1107	(0.3710)
22														0.1752	0.4772	0.4239	(0.1326)	(0.5830)	0.3169	(0.0441)	(0.1957)
23															0.0190	0.4043	0.4924	0.2814	0.4778	0.5767	(0.3810)
25																0.2805	0.3156	0.0540	0.1348	(0.5406)	(0.4188)
28																	0.3409	0.1985	0.0881	0.5403	(0.0260)
31																		0.6494	0.2811	0.2121	(0.0269)
33																			(0.0141)	0.2349	(0.0791)
34																				0.3033	0.0375
38																					0.1830
40																					

Impact of Prior Year Development on Calendar Year Loss Ratios **Exhibit 2**
By Company **MM**
Medical Professional Liability

Calendar Year Change in Reserves as a Percentage of Net Premiums Earned

Calendar Year	Group / Company																						
	9	12	13	20	22	24	26	27	28	29	30	31	32	33	34	35	36	37	High	Low	Average	Range	Std Dev
2000	-28.6%	-14.8%	-3.0%	-39.7%	309.6%	-38.7%	-1.7%	-28.6%	3.4%	17.5%	4.8%	102.6%	-41.6%	181.4%	159.9%	-11.8%	-7.7%	-0.5%	309.6%	-41.6%	31.3%	351.2%	0.9533
2001	-9.0%	-21.6%	63.0%	-0.2%	53.6%	-29.9%	-3.4%	3.0%	165.0%	-0.8%	7.5%	108.2%	-10.1%	379.3%	242.2%	4.4%	-20.3%	-28.5%	379.3%	-29.9%	50.1%	409.2%	1.1005
2002	22.2%	-18.0%	-59.4%	1.1%	9.5%	-18.9%	-4.9%	14.1%	59.2%	2.4%	2.0%	53.4%	-9.2%	89.3%	66.4%	4.6%	15.7%	26.9%	89.3%	-59.4%	14.2%	148.8%	0.3530
2003	6.1%	-21.3%	11.4%	-18.0%	-1.3%	-9.2%	-1.0%	19.0%	42.4%	1.6%	0.4%	-0.8%	4.8%	46.8%	111.7%	1.7%	11.9%	4.3%	111.7%	-21.3%	11.7%	133.0%	0.3027
2004	4.5%	-2.0%	6.0%	-60.9%	-3.3%	3.5%	2.0%	5.3%	14.0%	-4.6%	1.0%	30.3%	0.1%	-14.8%	-24.1%	-2.9%	-4.3%	13.6%	30.3%	-60.9%	-2.0%	91.2%	0.1862
2005	28.9%	-1.7%	5.6%	-66.5%	-35.6%	-16.2%	-4.5%	-7.6%	10.5%	-8.7%	-0.7%	3.6%	-5.9%	-21.4%	-27.1%	-0.4%	-4.9%	0.5%	28.9%	-66.5%	-8.5%	95.5%	0.2040
2006	-11.0%	-19.6%	4.5%	-51.4%	-2.1%	-27.7%	-4.4%	-12.3%	-11.2%	-14.0%	-4.0%	-3.0%	-25.9%	-17.4%	1.1%	-6.7%	-4.7%	-6.7%	4.5%	-51.4%	-12.0%	55.9%	0.1315
2007	-12.0%	-33.2%	-9.2%	-66.4%	-16.4%	-42.6%	-17.3%	-17.2%	-12.7%	-24.3%	-14.9%	20.0%	-22.9%	-28.2%	-72.3%	-13.3%	-25.3%	-16.5%	20.0%	-72.3%	-23.6%	92.2%	0.2086
2008	-13.6%	-55.2%	-4.5%	-44.8%	-8.0%	-35.7%	-28.8%	-17.3%	-2.0%	-43.2%	-7.8%	-52.3%	-45.6%	-43.6%	-23.9%	-28.8%	-40.2%	-12.0%	-2.0%	-55.2%	-28.2%	53.2%	0.1755
2009	-16.7%	-23.3%	-11.5%	-31.5%	-8.4%	-45.6%	-24.0%	-29.6%	-10.5%	-78.4%	-10.7%	-32.2%	-44.6%	-21.2%	-28.6%	-37.1%	-32.7%	-17.2%	-8.4%	-78.4%	-28.0%	70.0%	0.1689
2010	-29.0%	-45.8%	-32.1%	-38.8%	-5.1%	-43.8%	-35.9%	-56.6%	-17.9%	-33.2%	-15.0%	-5.7%	-35.4%	-26.7%	-28.1%	-36.1%	-26.7%	-15.3%	-5.1%	-56.6%	-29.3%	51.5%	0.1362
High	28.9%	-1.7%	63.0%	1.1%	309.6%	3.5%	2.0%	19.0%	165.0%	17.5%	7.5%	108.2%	4.8%	379.3%	242.2%	4.6%	15.7%	26.9%					
Low	-29.0%	-55.2%	-59.4%	-66.5%	-35.6%	-45.6%	-35.9%	-56.6%	-17.9%	-78.4%	-15.0%	-52.3%	-45.6%	-43.6%	-72.3%	-37.1%	-40.2%	-28.5%					
Average	-5.3%	-23.3%	-2.7%	-37.9%	26.6%	-27.7%	-11.3%	-11.6%	21.8%	-16.9%	-3.4%	20.4%	-21.5%	47.6%	34.3%	-11.5%	-12.7%	-4.7%					
Range	57.9%	53.6%	122.4%	67.7%	345.3%	49.1%	37.9%	75.6%	182.8%	95.9%	22.5%	160.5%	50.4%	422.9%	314.4%	41.7%	55.8%	55.4%					
Std Dev	0.1883	0.1634	0.2985	0.2404	0.9633	0.1587	0.1298	0.2194	0.5321	0.2679	0.0773	0.5066	0.1847	1.2917	0.9842	0.1570	0.1788	0.1577					

Correlation

Group / Company	Group / Company																	
	9	12	13	20	22	24	26	27	28	29	30	31	32	33	34	35	36	37
9		0.5711	(0.0800)	0.0603	(0.4417)	0.7248	0.5232	0.7398	0.2621	0.3213	0.3942	(0.0093)	0.7582	(0.0817)	(0.0549)	0.6785	0.6374	0.6260
12			0.2066	(0.0864)	0.1570	0.6418	0.8397	0.4948	0.2056	0.5007	0.6274	0.4420	0.6111	0.1988	0.2095	0.6629	0.6590	0.4906
13				0.0246	0.0993	0.1531	0.3683	0.2509	0.5599	0.1896	0.4345	0.3107	0.2484	0.5502	0.5039	0.3213	(0.1589)	(0.5376)
20					0.1474	(0.0051)	0.1213	0.3464	0.7216	0.1752	0.4484	0.3946	0.1482	0.6632	0.7207	0.2481	0.2823	0.0739
22						(0.2248)	0.2890	(0.1863)	0.0783	0.4756	0.4565	0.6619	(0.3180)	0.5252	0.5881	0.0644	0.1002	0.0319
24							0.7144	0.7742	0.2533	0.5318	0.5697	0.1073	0.8509	(0.0157)	0.0974	0.7093	0.7085	0.6830
26								0.7553	0.4368	0.7771	0.8365	0.5969	0.7000	0.4289	0.5039	0.8977	0.7757	0.5032
27									0.5603	0.5075	0.6533	0.2470	0.8008	0.2778	0.3596	0.8460	0.6805	0.5216
28										0.4209	0.6904	0.6386	0.4801	0.8720	0.8029	0.6067	0.2381	(0.1016)
29											0.7480	0.7353	0.5835	0.5089	0.5918	0.8208	0.7352	0.4786
30												0.7026	0.5045	0.7149	0.7911	0.7771	0.6053	0.3781
31													0.2763	0.8526	0.7708	0.5848	0.3705	0.0902
32														0.1830	0.2238	0.8344	0.7310	0.4746
33															0.9392	0.4715	0.1572	(0.2053)
34																0.5041	0.3247	(0.0580)
35																	0.7869	0.4762
36																		0.7956
37																		

Impact of Prior Year Development on Calendar Year Loss Ratios **Exhibit 3**
By Group/Company and Line **Page 1**
Group/Company No. 4

Calendar Year Change in Reserves as a Percentage of Net Premiums Earned

	Line of Business					
Calendar Year	HMP/ FMP	PPAL	CAL	WC	CMP	MM
2000	1.2%	-1.4%	0.1%	-0.1%	-7.3%	
2001	-1.2%	-0.3%	0.4%	17.2%	-7.1%	
2002	2.7%	1.4%	4.3%	10.1%	5.5%	
2003	-2.2%	-3.5%	4.1%	-1.1%	-0.3%	
2004	1.8%	-1.5%	4.9%	2.9%	-3.4%	
2005	1.1%	-6.1%	3.2%	5.2%	5.4%	
2006	-0.7%	-2.1%	1.9%	-2.1%	11.3%	
2007	-0.3%	-0.1%	-3.9%	-14.9%	-4.8%	
2008	3.6%	-5.8%	-3.3%	-6.7%	9.6%	
2009	3.5%	-3.7%	-6.0%	-1.7%	-2.3%	
2010	6.2%	-6.9%	-11.0%	-2.6%	-2.3%	

Correlation

	Line of Business					
Line of Business	HMP/ FMP	PPAL	CAL	WC	CMP	MM
HMP/FMP		(0.4678)	(0.6357)	(0.1381)	0.0829	
PPAL			0.4682	0.2705	(0.2791)	
CAL				0.4574	0.2086	
WC					(0.0990)	
CMP						
MM						

Impact of Prior Year Development on Calendar Year Loss Ratios **Exhibit 3**
By Group/Company and Line **Page 2**
Group/Company No. 5

Calendar Year Change in Reserves as a Percentage of Net Premiums Earned

Calendar Year	Line of Business: HMP/ FMP	PPAL	CAL	WC	CMP	MM
2000	0.9%	-1.6%	18.0%	12.5%	2.0%	
2001	4.0%	0.2%	20.2%	16.9%	6.9%	
2002	3.6%	3.1%	1.6%	0.5%	-0.8%	
2003	-0.8%	1.4%	0.2%	11.2%	-3.1%	
2004	-2.6%	-1.5%	-4.2%	-1.3%	-0.3%	
2005	-4.2%	-3.4%	-3.0%	4.7%	-4.6%	
2006	0.0%	-3.7%	-7.3%	2.6%	2.3%	
2007	-1.9%	-5.7%	-5.3%	-1.9%	-9.7%	
2008	-0.9%	-4.4%	-3.1%	0.2%	-10.2%	
2009	-4.4%	-4.5%	-7.4%	-3.1%	-9.1%	
2010	1.0%	-2.8%	-4.7%	0.8%	-1.7%	

Correlation

Line of Business	Line of Business: HMP/ FMP	PPAL	CAL	WC	CMP	MM
HMP/FMP		0.6490	0.6343	0.5088	0.6685	
PPAL			0.4762	0.4862	0.5801	
CAL				0.8592	0.6510	
WC					0.6738	
CMP						
MM						

Impact of Prior Year Development on Calendar Year Loss Ratios **Exhibit 3**
By Group/Company and Line **Page 3**
Group/Company No. 19

Calendar Year Change in Reserves as a Percentage of Net Premiums Earned

	Line of Business					
Calendar Year	HMP/ FMP	PPAL	CAL	WC	CMP	MM
2000	2.1%	2.8%	19.5%	0.3%	6.8%	
2001	1.1%	9.1%	13.9%	10.2%	5.6%	
2002	3.4%	13.5%	20.0%	6.6%	5.2%	
2003	-4.3%	-2.2%	1.4%	10.3%	9.9%	
2004	-3.7%	-2.3%	2.9%	0.3%	-12.9%	
2005	-1.7%	-4.1%	-4.3%	-8.8%	-4.6%	
2006	-3.6%	-10.8%	-11.1%	-15.6%	-1.3%	
2007	-2.4%	-16.1%	-22.8%	2.4%	-4.0%	
2008	-3.0%	-5.9%	-12.7%	-16.7%	-6.4%	
2009	0.5%	-0.1%	-16.8%	-50.3%	9.8%	
2010	-1.3%	-4.8%	-7.2%	-39.1%	-13.8%	

Correlation

	Line of Business					
Line of Business	HMP/ FMP	PPAL	CAL	WC	CMP	MM
HMP/FMP		0.7594	0.6053	0.0205	0.4576	
PPAL			0.8463	0.2582	0.4741	
CAL				0.5628	0.3401	
WC					0.1994	
CMP						
MM						

Impact of Prior Year Development on Calendar Year Loss Ratios **Exhibit 3**
By Group/Company and Line **Page 4**
Group/Company No. 20

Calendar Year Change in Reserves as a Percentage of Net Premiums Earned

Calendar Year	Line of Business HMP/ FMP	PPAL	CAL	WC	CMP	MM
2000	-2.2%	-1.0%	45.6%	-14.9%	-4.9%	-39.7%
2001	2.2%	-2.1%	8.0%	-0.9%	0.7%	-0.2%
2002	-1.6%	-4.1%	0.1%	0.7%	1.1%	1.1%
2003	-4.2%	1.7%	-10.6%	4.0%	1.4%	-18.0%
2004	-3.6%	-0.6%	-11.7%	3.9%	-0.3%	-60.9%
2005	-2.7%	-1.1%	-4.8%	14.1%	-6.5%	-66.5%
2006	0.2%	0.6%	-4.9%	2.1%	3.9%	-51.4%
2007	-4.4%	3.4%	-6.0%	0.8%	-4.8%	-66.4%
2008	-4.3%	-7.4%	-2.2%	3.0%	-10.6%	-44.8%
2009	-2.2%	-4.7%	-5.8%	15.1%	-7.5%	-31.5%
2010	-3.1%	-4.5%	-9.6%	-8.7%	-7.5%	-38.8%

Correlation

Line of Business	Line of Business HMP/ FMP	PPAL	CAL	WC	CMP	MM
HMP/FMP		(0.0493)	0.2710	(0.0864)	0.5036	0.4915
PPAL			(0.0232)	(0.0432)	0.5303	(0.3130)
CAL				(0.6234)	(0.0701)	0.1662
WC					(0.0696)	(0.1763)
CMP						0.3667
MM						

Impact of Prior Year Development on Calendar Year Loss Ratios **Exhibit 3**
By Group/Company and Line **Page 5**
Group/Company No. 21

Calendar Year Change in Reserves as a Percentage of Net Premiums Earned

Calendar Year	HMP/ FMP	PPAL	CAL	WC	CMP	MM
2000	-4.6%	-21.0%	-2.0%	-26.8%	6.3%	
2001	1.2%	2.9%	12.5%	-6.3%	6.2%	
2002	0.4%	7.3%	18.4%	-8.1%	-7.6%	
2003	-2.6%	6.1%	13.8%	4.1%	13.1%	
2004	-3.3%	5.2%	7.0%	2.3%	12.7%	
2005	-5.4%	3.6%	5.9%	-17.4%	-7.2%	
2006	-2.2%	-2.8%	6.8%	-14.8%	-12.4%	
2007	-3.0%	-1.2%	6.4%	-24.4%	-6.3%	
2008	-0.1%	-0.9%	0.1%	13.7%	-6.4%	
2009	1.5%	5.7%	4.8%	24.8%	-20.9%	
2010	-1.4%	7.3%	0.2%	10.8%	-22.7%	

Correlation

Line of Business	HMP/ FMP	PPAL	CAL	WC	CMP	MM
HMP/FMP		0.4297	0.3055	0.6317	(0.3360)	
PPAL			0.5418	0.5849	(0.2356)	
CAL				(0.0539)	0.2955	
WC					(0.3066)	
CMP						
MM						

Impact of Prior Year Development on Calendar Year Loss Ratios **Exhibit 3**
By Group/Company and Line **Page 6**
Group/Company No. 28

Calendar Year Change in Reserves as a Percentage of Net Premiums Earned

	Line of Business					
Calendar Year	HMP/ FMP	PPAL	CAL	WC	CMP	MM
2000			5.0%	9.8%	1.5%	3.4%
2001			40.7%	-28.3%	28.8%	165.0%
2002			-3.8%	15.4%	3.3%	59.2%
2003			0.8%	18.0%	30.7%	42.4%
2004			3.5%	14.3%	-8.4%	14.0%
2005			16.7%	28.7%	5.7%	10.5%
2006			-1.1%	10.3%	-1.3%	-11.2%
2007			-10.8%	7.9%	-8.3%	-12.7%
2008			3.9%	19.2%	-29.5%	-2.0%
2009			1.6%	-2.6%	-11.4%	-10.5%
2010			-17.6%	8.6%	-17.6%	-17.9%

Correlation

	Line of Business					
Line of Business	HMP/ FMP	PPAL	CAL	WC	CMP	MM
HMP/FMP						
PPAL						
CAL				(0.5165)	0.5656	0.7879
WC					(0.3238)	(0.6449)
CMP						0.7230
MM						

Impact of Prior Year Development on Calendar Year Loss Ratios **Exhibit 3**
By Group/Company and Line **Page 7**
Group/Company No. 33

Calendar Year Change in Reserves as a Percentage of Net Premiums Earned

	Line of Business					
Calendar Year	HMP/ FMP	PPAL	CAL	WC	CMP	MM
2000	-7.2%	0.2%	23.1%	2.1%	11.4%	181.4%
2001	-10.5%	7.0%	-5.8%	2.8%	-0.6%	379.3%
2002	-4.4%	10.0%	-7.3%	10.3%	-17.5%	89.3%
2003	-0.8%	6.4%	7.8%	5.3%	10.2%	46.8%
2004	1.1%	1.1%	7.9%	-4.6%	-1.6%	-14.8%
2005	-2.1%	20.4%	2.7%	14.1%	-12.7%	-21.4%
2006	-7.9%	2.3%	-1.4%	-0.3%	-6.5%	-17.4%
2007	-16.5%	1.8%	-6.5%	2.9%	8.5%	-28.2%
2008	1.0%	-0.7%	-2.8%	-3.2%	-18.0%	-43.6%
2009	-1.2%	-3.3%	23.0%	7.2%	16.1%	-21.2%
2010	-4.7%	51.4%	17.0%	28.0%	3.0%	-26.7%

Correlation

	Line of Business					
Line of Business	HMP/ FMP	PPAL	CAL	WC	CMP	MM
HMP/FMP		0.0179	0.3398	(0.0110)	(0.2510)	(0.3516)
PPAL			0.1591	0.9073	(0.1052)	(0.1263)
CAL				0.2813	0.6555	(0.1233)
WC					0.0487	(0.1293)
CMP						0.1078
MM						

Impact of Prior Year Development on Calendar Year Loss Ratios **Exhibit 3**
By Group/Company and Line **Page 8**
Group/Company No. 34

Calendar Year Change in Reserves as a Percentage of Net Premiums Earned

	Line of Business					
Calendar Year	HMP/ FMP	PPAL	CAL	WC	CMP	MM
2000			2.4%	6.9%	4.0%	159.9%
2001			6.8%	2.2%	8.0%	242.2%
2002			14.5%	7.8%	24.1%	66.4%
2003			7.2%	22.7%	4.4%	111.7%
2004			9.2%	20.6%	82.5%	-24.1%
2005			15.7%	25.0%	-31.2%	-27.1%
2006			11.6%	4.8%	-6.2%	1.1%
2007			3.5%	8.3%	-15.9%	-72.3%
2008			1.0%	14.3%	-14.7%	-23.9%
2009			-11.8%	19.1%	-8.9%	-28.6%
2010			1.0%	17.2%	-11.5%	-28.1%

Correlation

	Line of Business					
Line of Business	HMP/ FMP	PPAL	CAL	WC	CMP	MM
HMP/FMP						
PPAL						
CAL				(0.0753)	0.2023	0.1426
WC					0.0198	(0.4588)
CMP						0.1491
MM						

Impact of Prior Year Development on Calendar Year Loss Ratios **Exhibit 3**
By Group/Company and Line **Page 9**
Group/Company No. 38

Calendar Year Change in Reserves as a Percentage of Net Premiums Earned

Calendar Year	HMP/ FMP	PPAL	CAL	WC	CMP	MM
	Line of Business					
2000	-0.8%	-3.5%	6.7%	-5.6%	-6.7%	
2001	0.3%	0.3%	7.8%	6.7%	-3.0%	
2002	-0.4%	-3.7%	11.0%	-4.3%	1.9%	
2003	-10.0%	-4.8%	5.0%	9.9%	-1.6%	
2004	-10.8%	-9.9%	-1.7%	0.0%	-7.1%	
2005	0.8%	-16.5%	-8.8%	1.5%	-10.6%	
2006	-8.6%	-7.4%	-11.2%	-4.5%	-10.7%	
2007	-0.7%	-4.3%	-15.0%	-5.2%	-10.0%	
2008	-3.8%	-0.8%	-10.4%	-5.5%	-21.2%	
2009	-4.3%	0.5%	-15.2%	-3.2%	-9.6%	
2010	0.5%	-0.2%	-1.1%	-11.3%	0.5%	

Correlation

Line of Business	HMP/ FMP	PPAL	CAL	WC	CMP	MM
	Line of Business					
HMP/FMP		0.1831	0.1272	(0.3409)	0.1447	
PPAL			0.1488	(0.2328)	0.1371	
CAL				0.3179	0.7212	
WC					0.1691	
CMP						
MM						

Initial Net Held Ultimate Loss & LAE Ratios **Exhibit 4**
Industry Composite **Page 1**
Accident Year Basis

12 Month Held / Premium Earned

	Line of Business					
Accident Year	HMP/ FMP	PPAL	CAL	WC	CMP	MM
1993	77.7%	86.5%	79.1%	80.7%	75.2%	155.7%
1994	82.2%	86.0%	81.5%	79.5%	81.5%	150.4%
1995	77.4%	81.5%	80.5%	79.6%	73.4%	154.6%
1996	82.7%	76.4%	78.0%	78.1%	78.0%	126.6%
1997	61.4%	72.4%	79.2%	80.4%	71.2%	124.1%
1998	68.2%	71.2%	77.7%	85.1%	78.4%	127.8%
1999	68.4%	75.2%	80.3%	87.4%	76.4%	117.8%
2000	72.6%	79.5%	79.2%	85.3%	72.4%	119.3%
2001	80.4%	79.1%	73.1%	79.6%	75.8%	98.1%
2002	68.3%	76.2%	66.6%	71.0%	59.7%	88.1%
2003	62.2%	70.2%	64.1%	72.0%	56.7%	83.2%
2004	59.6%	67.0%	61.7%	70.5%	60.2%	75.3%
2005	64.1%	66.8%	59.9%	68.5%	62.3%	69.1%
2006	51.0%	65.3%	61.0%	67.7%	52.2%	70.1%
2007	56.0%	68.7%	62.0%	70.5%	53.5%	70.9%
2008	78.4%	69.7%	62.7%	71.7%	68.6%	72.0%
2009	68.8%	73.2%	62.6%	74.0%	59.1%	76.1%
2010	68.4%	73.8%	64.4%	77.4%	63.8%	77.3%
High	82.7%	86.5%	81.5%	87.4%	81.5%	155.7%
Low	51.0%	65.3%	59.9%	67.7%	52.2%	69.1%
Average	69.3%	74.4%	70.7%	76.6%	67.7%	103.1%
Range	31.8%	21.2%	21.6%	19.7%	29.3%	86.6%
Std Dev	0.0924	0.0628	0.0849	0.0608	0.0933	0.3144

Correlation

	Line of Business					
Line of Business	HMP/ FMP	PPAL	CAL	WC	CMP	MM
HMP/FMP		0.7732	0.5872	0.4919	0.7921	0.6001
PPAL			0.7633	0.6133	0.7019	0.8186
CAL				0.8826	0.8801	0.9448
WC					0.8224	0.7487
CMP						0.8205
MM						

Calendar Year Net Loss & LAE **Exhibit 4**
Industry Composite **Page 2**
Calendar Year Basis

Calendar Year Incurred & DCC / Premium Earned

Calendar Year	Line of Business HMP/ FMP	PPAL	CAL	WC	CMP	MM
1993	74.9%	77.5%	67.8%	74.1%	72.9%	85.4%
1994	77.5%	75.4%	70.1%	63.7%	81.1%	85.1%
1995	74.7%	71.4%	73.4%	64.3%	70.9%	90.7%
1996	82.1%	67.0%	73.2%	67.3%	77.9%	90.6%
1997	59.7%	64.3%	76.8%	67.5%	71.0%	95.8%
1998	66.3%	66.4%	77.1%	72.6%	79.4%	87.4%
1999	68.0%	72.0%	82.0%	78.5%	75.0%	99.1%
2000	71.9%	78.7%	81.3%	82.1%	69.9%	98.6%
2001	84.5%	79.1%	83.4%	84.5%	77.4%	117.5%
2002	72.9%	76.2%	69.1%	77.6%	63.7%	111.0%
2003	59.8%	68.6%	65.2%	77.9%	63.7%	103.0%
2004	57.1%	65.1%	62.0%	76.3%	64.1%	83.2%
2005	62.2%	64.5%	59.4%	73.9%	58.2%	68.3%
2006	48.8%	62.0%	58.8%	67.4%	54.2%	60.6%
2007	56.1%	67.3%	58.6%	68.6%	49.2%	53.2%
2008	78.6%	69.4%	60.0%	70.8%	61.4%	47.8%
2009	66.7%	71.9%	59.8%	76.9%	53.9%	48.7%
2010	68.3%	71.3%	60.5%	82.1%	59.7%	44.8%
High	84.5%	79.1%	83.4%	84.5%	81.1%	117.5%
Low	48.8%	62.0%	58.6%	63.7%	49.2%	44.8%
Average	68.3%	70.5%	68.8%	73.7%	66.9%	81.7%
Range	35.7%	17.1%	24.8%	20.8%	31.9%	72.8%
Std Dev	0.0973	0.0527	0.0874	0.0629	0.0958	0.2255

Correlation

Line of Business	Line of Business HMP/ FMP	PPAL	CAL	WC	CMP	MM
HMP/FMP		0.6962	0.4818	0.1163	0.6256	0.3282
PPAL			0.4626	0.4665	0.3817	0.4023
CAL				0.2039	0.8132	0.7921
WC					(0.0572)	0.2121
CMP						0.6819
MM						

Initial Net Held Ultimate Loss & LAE Ratios **Exhibit 5**
Homeowners/Farmowners **HMP/FMP**
Accident Year Basis **Page 1**

Accident Year Loss Ratios by Company

Accident	Group / Company														
Year	4	5	19	20	21	28	33	34	38	High	Low	Average	Range	Std Dev
1993	70.2%	101.9%	68.8%	72.0%	75.6%		19.0%		67.6%	101.9%	19.0%	67.9%	82.8%	0.2457
1994	76.1%	107.0%	116.6%	75.3%	76.7%		14.0%		79.8%	116.6%	14.0%	77.9%	102.6%	0.3277
1995	71.9%	96.7%	66.9%	84.1%	83.4%		19.0%		64.9%	96.7%	19.0%	69.6%	77.7%	0.2493
1996	93.6%	91.1%	109.6%	99.5%	83.8%		19.9%		81.7%	109.6%	19.9%	82.7%	89.7%	0.2927
1997	61.7%	63.5%	58.6%	74.4%	78.9%		18.2%		55.3%	78.9%	18.2%	58.6%	60.7%	0.1975
1998	64.7%	72.8%	62.5%	94.2%	97.9%		24.9%		67.8%	97.9%	24.9%	69.3%	73.0%	0.2411
1999	66.4%	70.4%	69.0%	70.5%	83.1%		24.7%		66.3%	83.1%	24.7%	64.3%	58.4%	0.1839
2000	66.3%	78.7%	66.3%	90.9%	93.6%		25.9%		68.4%	93.6%	25.9%	70.0%	67.7%	0.2253
2001	72.0%	83.0%	66.4%	92.9%	105.1%		43.6%		67.5%	105.1%	43.6%	75.8%	61.5%	0.2006
2002	61.1%	63.7%	75.8%	88.8%	79.1%		57.9%		53.1%	88.8%	53.1%	68.5%	35.7%	0.1293
2003	60.7%	59.4%	73.0%	85.8%	71.5%		52.7%		53.7%	85.8%	52.7%	65.2%	33.2%	0.1204
2004	68.7%	54.0%	48.2%	87.8%	81.2%		51.9%		55.8%	87.8%	48.2%	63.9%	39.7%	0.1555
2005	49.1%	57.5%	41.3%	69.7%	62.8%		126.4%		47.8%	126.4%	41.3%	65.0%	85.1%	0.2876
2006	54.1%	50.2%	58.7%	71.1%	66.6%		41.8%		41.8%	71.1%	41.8%	54.9%	29.3%	0.1141
2007	55.9%	51.0%	53.3%	56.5%	76.7%		57.9%		41.7%	76.7%	41.7%	56.1%	35.1%	0.1058
2008	71.2%	75.8%	67.2%	90.1%	104.3%		110.9%		59.1%	110.9%	59.1%	82.7%	51.8%	0.1955
2009	71.2%	65.6%	68.8%	97.6%	91.6%		53.4%		57.0%	97.6%	53.4%	72.2%	44.2%	0.1665
2010	74.4%	61.5%	87.4%	84.3%	94.9%		57.3%		61.9%	94.9%	57.3%	74.5%	37.7%	0.1475
High	93.6%	107.0%	116.6%	99.5%	105.1%	0.0%	126.4%	0.0%	81.7%					
Low	49.1%	50.2%	41.3%	56.5%	62.8%	0.0%	14.0%	0.0%	41.7%					
Average	67.2%	72.4%	69.9%	82.5%	83.7%		45.5%		60.6%					
Range	44.4%	56.8%	75.3%	42.9%	42.3%	0.0%	112.4%	0.0%	40.0%					
Std Dev	0.0986	0.1745	0.1882	0.1167	0.1203		0.3113		0.1117					

Correlation

Group /	Group / Company								
Company	4	5	19	20	21	28	33	34	38
4		0.6544	0.7544	0.5882	0.4840		(0.4244)		0.8330
5			0.6159	0.2223	0.2321		(0.4901)		0.8431
19				0.3066	0.1438		(0.4344)		0.7332
20					0.6340		(0.0755)		0.4779
21							(0.0427)		0.4225
28									
33									(0.5446)
34									
38									

Calendar Year Net Loss & LAE
Homeowners/Farmowners
Calendar Year Basis

Exhibit 5
HMP/FMP
Page 2

Calendar Year Loss Ratios by Company

Accident Year	Group / Company 4	5	19	20	21	28	33	34	38	High	Low	Average	Range	Std Dev
1993	73.5%	76.1%	68.5%	68.4%	76.4%		20.5%		68.7%	76.4%	20.5%	64.6%	55.9%	0.1975
1994	79.9%	81.0%	115.3%	78.5%	78.6%		17.2%		74.7%	115.3%	17.2%	75.0%	98.1%	0.2901
1995	66.9%	74.8%	66.9%	81.7%	81.2%		73.6%		64.5%	81.7%	64.5%	72.8%	17.2%	0.0697
1996	97.9%	90.6%	108.0%	97.9%	84.2%		68.3%		83.5%	108.0%	68.3%	90.0%	39.7%	0.1289
1997	63.5%	65.0%	60.2%	74.5%	80.3%		48.2%		60.3%	80.3%	48.2%	64.6%	32.1%	0.1043
1998	62.6%	70.5%	61.6%	95.0%	96.2%		65.9%		67.0%	96.2%	61.6%	74.1%	34.6%	0.1497
1999	64.9%	69.5%	67.6%	70.6%	84.8%		55.8%		62.3%	84.8%	55.8%	67.9%	29.0%	0.0896
2000	62.7%	74.5%	65.3%	85.7%	88.5%		43.6%		66.4%	88.5%	43.6%	69.5%	44.9%	0.1523
2001	67.1%	83.6%	65.2%	100.7%	106.7%		42.4%		70.1%	106.7%	42.4%	76.5%	64.3%	0.2224
2002	64.8%	71.6%	85.8%	90.4%	84.8%		57.9%		63.7%	90.4%	57.9%	74.2%	32.5%	0.1278
2003	60.7%	60.9%	73.2%	84.9%	73.1%		56.1%		54.5%	84.9%	54.5%	66.2%	30.4%	0.1113
2004	70.2%	56.0%	46.6%	88.0%	85.4%		54.9%		44.4%	88.0%	44.4%	63.6%	43.6%	0.1781
2005	48.9%	52.9%	42.9%	67.2%	59.6%		132.0%		62.6%	132.0%	42.9%	66.6%	89.1%	0.3002
2006	50.3%	51.7%	57.4%	75.7%	65.4%		52.0%		34.4%	75.7%	34.4%	55.3%	41.3%	0.1297
2007	54.7%	52.6%	53.5%	53.1%	73.5%		60.3%		43.4%	73.5%	43.4%	55.9%	30.0%	0.0921
2008	70.6%	74.7%	64.1%	89.0%	102.3%		100.6%		56.1%	102.3%	56.1%	79.6%	46.2%	0.1800
2009	72.6%	60.6%	71.3%	97.2%	93.8%		70.2%		51.3%	97.2%	51.3%	73.8%	45.9%	0.1657
2010	81.3%	62.5%	86.1%	81.1%	93.5%		52.3%		62.4%	93.5%	52.3%	74.2%	41.3%	0.1512
High	97.9%	90.6%	115.3%	100.7%	106.7%	0.0%	132.0%	0.0%	83.5%					
Low	48.9%	51.7%	42.9%	53.1%	59.6%	0.0%	17.2%	0.0%	34.4%					
Average	67.4%	68.3%	70.0%	82.2%	83.8%		59.5%		60.6%					
Range	48.9%	38.9%	72.3%	47.6%	47.0%	0.0%	114.9%	0.0%	49.1%					
Std Dev	0.1156	0.1127	0.1877	0.1250	0.1210		0.2604		0.1183					

Correlation

Group / Company	Group / Company 4	5	19	20	21	28	33	34	38
4		0.6871	0.7682	0.4648	0.4347		(0.2982)		0.6177
5			0.6560	0.5026	0.5062		(0.3143)		0.8546
19				0.3197	0.1693		(0.4217)		0.6266
20					0.6933		(0.0011)		0.3333
21							(0.1256)		0.2821
28									
33									(0.1432)
34									
38									

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Accident Year Loss Ratios by Company

Accident	Group / Company													
Year	4	5	19	20	21	28	33	34	38	High	Low	Average	Range	Std Dev
1993	80.6%	86.8%	75.7%	85.7%	75.0%		20.3%		68.0%	86.8%	20.3%	70.3%	66.5%	0.2298
1994	80.4%	92.7%	78.5%	86.4%	74.4%		17.7%		72.4%	92.7%	17.7%	71.8%	75.0%	0.2484
1995	74.2%	86.0%	76.8%	85.5%	76.5%		12.0%		68.9%	86.0%	12.0%	68.5%	74.1%	0.2568
1996	69.2%	72.6%	78.8%	89.3%	72.7%		7.2%		71.5%	89.3%	7.2%	65.9%	82.1%	0.2676
1997	64.0%	73.5%	74.0%	88.0%	72.6%		15.0%		70.4%	88.0%	15.0%	65.4%	72.9%	0.2333
1998	63.1%	74.4%	70.2%	83.2%	73.1%		14.4%		72.9%	83.2%	14.4%	64.5%	68.8%	0.2287
1999	65.8%	79.6%	74.5%	78.6%	75.1%		30.1%		76.6%	79.6%	30.1%	68.6%	49.5%	0.1758
2000	70.9%	84.1%	79.5%	81.5%	81.2%		33.3%		82.5%	84.1%	33.3%	73.3%	50.8%	0.1816
2001	80.7%	81.3%	79.9%	77.4%	77.4%		100.4%		79.1%	100.4%	77.4%	82.3%	23.0%	0.0813
2002	79.5%	79.3%	79.5%	79.8%	83.0%		97.0%		75.3%	97.0%	75.3%	81.9%	21.7%	0.0703
2003	70.1%	72.1%	73.0%	73.4%	82.4%		89.9%		68.6%	89.9%	68.6%	75.6%	21.3%	0.0769
2004	63.7%	65.5%	67.2%	69.2%	72.4%		73.0%		62.5%	73.0%	62.5%	67.6%	10.5%	0.0410
2005	63.2%	63.2%	61.7%	64.3%	73.0%		83.3%		60.1%	83.3%	60.1%	67.0%	23.3%	0.0831
2006	64.3%	63.3%	60.5%	64.3%	75.7%		70.7%		60.0%	75.7%	60.0%	65.5%	15.7%	0.0570
2007	70.0%	65.6%	61.7%	69.8%	84.5%		81.1%		63.5%	84.5%	61.7%	70.9%	22.8%	0.0873
2008	70.1%	64.9%	61.9%	66.4%	86.9%		77.8%		62.2%	86.9%	61.9%	70.0%	25.0%	0.0923
2009	72.9%	67.1%	66.0%	69.9%	85.2%		94.7%		64.2%	94.7%	64.2%	74.3%	30.5%	0.1140
2010	74.7%	67.9%	67.9%	70.3%	79.2%		52.4%		62.1%	79.2%	52.4%	67.8%	26.8%	0.0869
High	80.7%	92.7%	79.9%	89.3%	86.9%	0.0%	100.4%	0.0%	82.5%					
Low	63.1%	63.2%	60.5%	64.3%	72.4%	0.0%	7.2%	0.0%	60.0%					
Average	71.0%	74.5%	71.5%	76.8%	77.8%		53.9%		68.9%					
Range	17.6%	29.5%	19.4%	25.0%	14.5%	0.0%	93.3%	0.0%	22.6%					
Std Dev	0.0631	0.0912	0.0700	0.0851	0.0487		0.3452		0.0673					

Correlation

Group /	Group / Company								
Company	4	5	19	20	21	28	33	34	38
4		0.6396	0.5457	0.3206	0.3143		0.0870		0.3357
5			0.8514	0.7846	(0.1984)		(0.5161)		0.7327
19				0.8404	(0.2193)		(0.4188)		0.8587
20					(0.4031)		(0.7651)		0.6774
21							0.5881		(0.0716)
28									
33									(0.2982)
34									
38									

Calendar Year Net Loss & LAE
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Calendar Year Loss Ratios by Company

Accident	Group / Company													
Year	4	5	19	20	21	28	33	34	38	High	Low	Average	Range	Std Dev
1993	78.7%	81.0%	72.0%	64.9%	88.6%		3.2%		73.0%	88.6%	3.2%	65.9%	85.4%	0.2865
1994	79.8%	74.8%	77.5%	83.5%	79.2%		3.0%		66.7%	83.5%	3.0%	66.3%	80.4%	0.2842
1995	81.3%	70.7%	79.2%	79.6%	72.4%		74.5%		70.8%	81.3%	70.7%	75.5%	10.6%	0.0448
1996	76.2%	67.4%	83.4%	91.2%	69.0%		71.0%		62.5%	91.2%	62.5%	74.4%	28.7%	0.0997
1997	74.6%	69.8%	79.7%	85.4%	69.5%		59.8%		63.5%	85.4%	59.8%	71.7%	25.6%	0.0892
1998	68.7%	66.0%	67.9%	85.8%	58.6%		63.0%		67.9%	85.8%	58.6%	68.3%	27.2%	0.0852
1999	73.6%	73.6%	68.4%	73.2%	74.9%		78.9%		65.9%	78.9%	65.9%	72.6%	13.0%	0.0429
2000	75.1%	78.3%	74.6%	83.3%	61.0%		76.4%		82.3%	83.3%	61.0%	75.9%	22.3%	0.0739
2001	78.6%	79.4%	85.3%	79.1%	78.3%		69.5%		85.5%	85.5%	69.5%	79.4%	16.0%	0.0535
2002	82.7%	85.4%	97.6%	76.7%	84.5%		92.7%		81.9%	97.6%	76.7%	85.9%	20.9%	0.0702
2003	72.6%	75.3%	80.6%	76.3%	80.3%		90.6%		78.4%	90.6%	72.6%	79.2%	18.0%	0.0579
2004	71.1%	70.7%	76.0%	67.5%	75.1%		77.3%		67.5%	77.3%	67.5%	72.2%	9.8%	0.0401
2005	61.4%	66.6%	75.2%	63.0%	75.5%		108.5%		57.7%	108.5%	57.7%	72.5%	50.8%	0.1723
2006	65.8%	61.6%	61.7%	64.3%	74.3%		73.4%		56.1%	74.3%	56.1%	65.3%	18.1%	0.0656
2007	74.9%	63.4%	51.3%	70.4%	83.8%		78.5%		57.2%	83.8%	51.3%	68.5%	32.5%	0.1174
2008	66.3%	62.6%	54.4%	62.4%	82.6%		73.4%		61.7%	82.6%	54.4%	66.2%	28.2%	0.0921
2009	72.2%	63.0%	71.8%	66.1%	87.6%		90.3%		63.9%	90.3%	63.0%	73.6%	27.2%	0.1111
2010	67.9%	64.8%	62.2%	65.8%	86.1%		101.1%		61.8%	101.1%	61.8%	72.8%	39.3%	0.1500
High	82.7%	85.4%	97.6%	91.2%	88.6%	0.0%	108.5%	0.0%	85.5%					
Low	61.4%	61.6%	51.3%	62.4%	58.6%	0.0%	3.0%	0.0%	56.1%					
Average	73.4%	70.8%	73.3%	74.4%	76.7%		71.4%		68.0%					
Range	21.4%	23.9%	46.2%	28.8%	30.0%	0.0%	105.5%	0.0%	29.4%					
Std Dev	0.0581	0.0706	0.1124	0.0915	0.0852		0.2783		0.0895					

Correlation

Group /	Group / Company								
Company	4	5	19	20	21	28	33	34	38
4		0.7058	0.5734	0.5313	0.1075		(0.4490)		0.6104
5			0.7183	0.3160	0.0559		(0.3123)		0.8600
19				0.5151	(0.1233)		(0.0179)		0.6542
20					(0.6445)		(0.2357)		0.3754
21							(0.0321)		(0.0793)
28									
33									(0.0995)
34									
38									

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Accident Year Loss Ratios by Company

Accident Year	Group / Company 4	5	19	20	21	28	33	34	38	High	Low	Average	Range	Std Dev
1993	62.7%	68.7%	77.1%	76.6%	80.0%	75.7%	91.7%	78.7%	68.4%	91.7%	62.7%	75.5%	29.0%	0.0834
1994	64.6%	85.3%	86.5%	88.3%	86.2%	79.6%	90.8%	78.8%	70.3%	90.8%	64.6%	81.2%	26.3%	0.0877
1995	65.1%	88.1%	80.5%	80.7%	79.3%	82.0%	98.5%	78.3%	77.6%	98.5%	65.1%	81.1%	33.4%	0.0890
1996	71.3%	86.7%	82.6%	81.5%	84.0%	80.4%	109.9%	85.7%	75.9%	109.9%	71.3%	84.2%	38.6%	0.1078
1997	68.5%	96.0%	89.4%	80.0%	74.3%	92.2%	93.0%	95.3%	86.2%	96.0%	68.5%	86.1%	27.4%	0.0977
1998	84.8%	90.7%	75.6%	96.8%	76.3%	97.2%	76.6%	107.6%	90.4%	107.6%	75.6%	88.4%	31.9%	0.1111
1999	86.4%	104.9%	74.9%	94.5%	86.6%	94.2%	131.8%	93.5%	93.7%	131.8%	74.9%	95.6%	56.8%	0.1581
2000	79.6%	92.8%	85.1%	88.8%	89.0%	80.8%	124.8%	83.9%	84.6%	124.8%	79.6%	89.9%	45.2%	0.1371
2001	70.8%	71.1%	88.1%	70.4%	80.7%	103.7%	102.8%	75.9%	78.4%	103.7%	70.4%	82.4%	33.3%	0.1309
2002	58.2%	61.9%	73.3%	60.5%	68.5%	69.1%	91.7%	75.8%	61.4%	91.7%	58.2%	68.9%	33.5%	0.1047
2003	53.2%	54.6%	75.6%	59.7%	64.2%	59.1%	98.1%	67.6%	55.0%	98.1%	53.2%	65.2%	44.9%	0.1426
2004	47.5%	54.0%	65.0%	57.7%	61.8%	58.0%	59.7%	58.5%	53.9%	65.0%	47.5%	57.4%	17.6%	0.0508
2005	53.8%	55.2%	58.3%	57.1%	59.9%	60.7%	59.0%	66.2%	49.3%	66.2%	49.3%	57.7%	16.9%	0.0476
2006	48.1%	59.8%	58.0%	60.6%	63.5%	58.9%	63.5%	67.0%	50.6%	67.0%	48.1%	58.9%	18.9%	0.0611
2007	56.5%	60.2%	54.1%	59.8%	68.4%	62.0%	75.4%	66.8%	56.3%	75.4%	54.1%	62.2%	21.3%	0.0686
2008	51.4%	60.1%	58.4%	61.1%	61.9%	71.0%	70.9%	65.1%	57.3%	71.0%	51.4%	61.9%	19.6%	0.0633
2009	56.1%	56.8%	58.4%	56.3%	62.1%	67.6%	64.4%	64.2%	54.8%	67.6%	54.8%	60.1%	12.8%	0.0458
2010	55.8%	60.1%	62.5%	61.1%	62.1%	68.3%	64.8%	70.9%	59.1%	70.9%	55.8%	62.7%	15.1%	0.0466
High	86.4%	104.9%	89.4%	96.8%	89.0%	103.7%	131.8%	107.6%	93.7%					
Low	47.5%	54.0%	54.1%	56.3%	59.9%	58.0%	59.0%	58.5%	49.3%					
Average	63.0%	72.6%	72.4%	71.8%	72.7%	75.6%	87.1%	76.7%	68.0%					
Range	38.9%	50.8%	35.4%	40.5%	29.1%	45.7%	72.8%	49.0%	44.5%					
Std Dev	0.1190	0.1698	0.1189	0.1414	0.1018	0.1421	0.2189	0.1275	0.1457					

Correlation

Group / Company	Group / Company 4	5	19	20	21	28	33	34	38
4		0.8914	0.6450	0.9081	0.8268	0.8574	0.7449	0.8934	0.9506
5			0.7124	0.9426	0.8515	0.7962	0.7422	0.8704	0.9463
19				0.7044	0.7998	0.7252	0.7373	0.6392	0.7550
20					0.8797	0.7784	0.6857	0.8849	0.9194
21						0.7311	0.8484	0.6824	0.8333
28							0.6044	0.8073	0.9106
33								0.5619	0.7506
34									0.9075
38									

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Calendar Year Loss Ratios by Company

Accident Year	Group / Company 4	5	19	20	21	28	33	34	38	High	Low	Average	Range	Std Dev
1993	64.4%	73.8%	64.0%	55.6%	86.6%	88.8%	27.4%	68.7%	60.9%	88.8%	27.4%	65.6%	61.4%	0.1818
1994	57.4%	68.7%	88.2%	88.3%	94.7%	84.6%	39.1%	72.6%	68.2%	94.7%	39.1%	73.5%	55.6%	0.1769
1995	69.7%	81.9%	76.1%	80.4%	86.4%	75.8%	69.9%	73.5%	68.5%	86.4%	68.5%	75.8%	18.0%	0.0615
1996	78.4%	63.6%	79.4%	82.0%	83.1%	81.0%	154.5%	76.3%	74.7%	154.5%	63.6%	85.9%	90.8%	0.2636
1997	62.2%	74.6%	78.8%	72.9%	75.2%	74.9%	154.9%	90.6%	75.9%	154.9%	62.2%	84.4%	92.7%	0.2741
1998	60.1%	89.0%	62.5%	75.3%	64.8%	66.5%	72.8%	96.2%	85.1%	96.2%	60.1%	74.7%	36.0%	0.1276
1999	70.8%	89.9%	66.7%	88.8%	76.7%	90.3%	134.7%	80.4%	72.2%	134.7%	66.7%	85.6%	68.0%	0.2037
2000	72.1%	100.3%	87.2%	131.3%	73.9%	29.0%	164.9%	70.4%	77.4%	164.9%	29.0%	89.6%	135.9%	0.3916
2001	60.7%	98.9%	89.9%	84.2%	75.4%	155.0%	101.5%	66.5%	81.4%	155.0%	60.7%	90.4%	94.2%	0.2775
2002	61.1%	71.5%	98.1%	67.8%	77.6%	66.3%	64.2%	85.8%	76.8%	98.1%	61.1%	74.4%	37.0%	0.1176
2003	66.3%	63.1%	83.0%	52.8%	68.2%	68.4%	94.7%	69.2%	68.6%	94.7%	52.8%	70.5%	42.0%	0.1196
2004	64.5%	57.5%	78.4%	48.3%	58.8%	63.2%	91.6%	66.2%	59.6%	91.6%	48.3%	65.3%	43.3%	0.1271
2005	57.0%	55.4%	71.2%	57.9%	63.2%	77.6%	60.1%	80.0%	50.5%	80.0%	50.5%	63.7%	29.5%	0.1031
2006	54.9%	53.6%	60.0%	57.9%	66.3%	64.0%	67.5%	78.9%	45.8%	78.9%	45.8%	61.0%	33.1%	0.0958
2007	53.0%	60.0%	49.1%	57.9%	68.9%	54.2%	59.5%	70.6%	45.4%	70.6%	45.4%	57.6%	25.2%	0.0838
2008	51.1%	62.0%	51.4%	64.7%	55.8%	75.8%	58.8%	71.3%	50.5%	75.8%	50.5%	60.1%	25.3%	0.0912
2009	51.6%	54.8%	47.0%	56.0%	66.1%	76.3%	85.3%	56.2%	43.8%	85.3%	43.8%	59.7%	41.5%	0.1369
2010	45.0%	54.9%	55.6%	51.6%	62.3%	52.8%	82.1%	67.2%	57.8%	82.1%	45.0%	58.8%	37.1%	0.1079
High	78.4%	100.3%	98.1%	131.3%	94.7%	155.0%	164.9%	96.2%	85.1%					
Low	45.0%	53.6%	47.0%	48.3%	55.8%	29.0%	27.4%	56.2%	43.8%					
Average	61.1%	70.8%	71.5%	70.8%	72.5%	74.7%	88.0%	74.5%	64.6%					
Range	33.4%	46.7%	51.1%	83.0%	38.9%	126.0%	137.5%	40.0%	41.2%					
Std Dev	0.0849	0.1539	0.1522	0.2016	0.1054	0.2479	0.4009	0.0962	0.1325					

Correlation

Group / Company	Group / Company 4	5	19	20	21	28	33	34	38
4		0.5153	0.5704	0.5406	0.5018	0.0653	0.5689	0.1957	0.6110
5			0.4499	0.7830	0.3870	0.2949	0.3933	0.2632	0.7845
19				0.4627	0.4992	0.2132	0.2926	0.2202	0.7129
20					0.4455	(0.0290)	0.5509	0.1669	0.6001
21						0.2818	0.0006	0.0765	0.4341
28							(0.0936)	(0.1037)	0.2423
33								0.1364	0.4672
34									0.4883
38									

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Accident Year Loss Ratios by Company

Accident Year	4	5	19	20	21	28	33	34	38	High	Low	Average	Range	Std Dev
	Group / Company													
1993	55.0%	61.9%	64.3%	67.6%	59.3%	66.4%	92.5%	69.7%	64.7%	92.5%	55.0%	66.8%	37.4%	0.1059
1994	61.5%	66.7%	53.3%	53.2%	60.0%	55.4%	84.7%	69.4%	68.3%	84.7%	53.2%	63.6%	31.5%	0.1006
1995	61.1%	71.8%	48.8%	54.3%	51.2%	60.4%	85.0%	64.6%	71.6%	85.0%	48.8%	63.2%	36.2%	0.1153
1996	62.4%	79.4%	48.2%	57.5%	54.7%	60.9%	82.9%	67.5%	74.7%	82.9%	48.2%	65.4%	34.6%	0.1168
1997	74.6%	95.2%	59.8%	65.2%	59.1%	85.6%	89.6%	75.6%	81.1%	95.2%	59.1%	76.2%	36.1%	0.1294
1998	82.4%	108.4%	68.5%	72.3%	55.2%	104.4%	90.9%	79.8%	93.7%	108.4%	55.2%	84.0%	53.2%	0.1725
1999	89.8%	119.4%	108.2%	84.4%	64.9%	97.2%	114.7%	92.7%	104.5%	119.4%	64.9%	97.3%	54.6%	0.1682
2000	85.3%	102.6%	94.6%	93.9%	66.6%	91.1%	103.8%	126.2%	115.6%	126.2%	66.6%	97.7%	59.6%	0.1725
2001	83.7%	74.7%	95.1%	86.5%	77.4%	124.4%	106.1%	98.1%	103.2%	124.4%	74.7%	94.4%	49.7%	0.1575
2002	62.3%	67.9%	77.9%	77.5%	78.6%	82.8%	81.7%	84.2%	88.1%	88.1%	62.3%	77.9%	25.8%	0.0811
2003	56.4%	66.7%	77.8%	75.3%	73.9%	75.9%	74.7%	76.9%	75.0%	77.8%	56.4%	72.5%	21.4%	0.0684
2004	55.1%	66.2%	62.6%	77.2%	67.7%	50.7%	71.0%	64.4%	64.1%	77.2%	50.7%	64.3%	26.5%	0.0790
2005	54.5%	60.0%	48.9%	73.1%	70.1%	52.9%	70.5%	56.7%	62.9%	73.1%	48.9%	61.1%	24.3%	0.0865
2006	54.9%	64.7%	56.6%	74.0%	68.5%	55.8%	70.8%	60.8%	65.4%	74.0%	54.9%	63.5%	19.1%	0.0691
2007	59.3%	73.1%	53.7%	76.6%	71.0%	65.3%	74.0%	75.7%	69.5%	76.6%	53.7%	68.7%	23.0%	0.0784
2008	60.8%	77.7%	60.9%	82.7%	83.5%	69.8%	84.0%	89.5%	76.6%	89.5%	60.8%	76.2%	28.7%	0.1030
2009	61.4%	83.7%	69.0%	85.2%	80.3%	76.4%	79.6%	79.2%	74.4%	85.2%	61.4%	76.6%	23.8%	0.0749
2010	64.3%	87.0%	84.7%	91.0%	80.4%	74.7%	81.9%	88.4%	77.9%	91.0%	64.3%	81.2%	26.7%	0.0816
High	89.8%	119.4%	108.2%	93.9%	83.5%	124.4%	114.7%	126.2%	115.6%					
Low	54.5%	60.0%	48.2%	53.2%	51.2%	50.7%	70.5%	56.7%	62.9%					
Average	65.8%	79.3%	68.5%	74.9%	67.9%	75.0%	85.5%	78.9%	79.5%					
Range	35.3%	59.4%	60.0%	40.7%	32.4%	73.7%	44.2%	69.6%	52.7%					
Std Dev	0.1176	0.1708	0.1778	0.1183	0.0985	0.1980	0.1256	0.1645	0.1546					

Correlation

Group / Company	4	5	19	20	21	28	33	34	38
	Group / Company								
4		0.8581	0.7140	0.3718	(0.1338)	0.8517	0.8795	0.7256	0.9262
5			0.5793	0.3562	(0.1652)	0.6201	0.6891	0.5968	0.7464
19				0.7347	0.3613	0.7517	0.7220	0.7867	0.8141
20					0.7378	0.4819	0.2685	0.7008	0.5311
21						0.1526	(0.1691)	0.3294	0.0695
28							0.7577	0.6945	0.8547
33								0.6924	0.8289
34									0.8813
38									

Calendar Year Net Loss & LAE
Workers Compensation
Calendar Year Basis

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Calendar Year Loss Ratios by Company

Accident Year	Group / Company 4	5	19	20	21	28	33	34	38	High	Low	Average	Range	Std Dev
1993	73.3%	85.8%	77.7%	83.2%	69.9%	75.7%	15.8%	80.3%	98.5%	98.5%	15.8%	73.4%	82.7%	0.2315
1994	66.3%	70.6%	56.8%	76.5%	66.9%	65.7%	21.2%	66.5%	77.9%	77.9%	21.2%	63.2%	56.7%	0.1692
1995	70.4%	62.6%	58.6%	60.9%	46.4%	54.1%	82.0%	53.7%	68.1%	82.0%	46.4%	61.9%	35.6%	0.1057
1996	67.4%	63.3%	37.9%	50.7%	53.8%	49.3%	65.0%	73.9%	92.3%	92.3%	37.9%	61.5%	54.5%	0.1596
1997	66.7%	73.6%	46.4%	67.4%	43.5%	36.6%	76.3%	63.3%	69.4%	76.3%	36.6%	60.4%	39.7%	0.1439
1998	65.3%	79.0%	42.1%	64.1%	59.3%	74.0%	52.9%	71.5%	76.3%	79.0%	42.1%	64.9%	36.9%	0.1200
1999	68.3%	83.9%	75.9%	66.0%	27.9%	76.4%	82.2%	53.4%	72.3%	83.9%	27.9%	67.4%	56.0%	0.1741
2000	63.6%	86.0%	80.1%	68.7%	62.1%	84.9%	87.9%	71.2%	76.3%	87.9%	62.1%	75.7%	25.8%	0.0974
2001	86.1%	96.6%	86.8%	75.4%	65.0%	63.3%	82.9%	72.3%	87.2%	96.6%	63.3%	79.5%	33.3%	0.1118
2002	67.7%	70.7%	79.8%	73.4%	69.4%	75.7%	74.1%	73.2%	75.4%	79.8%	67.7%	73.3%	12.0%	0.0363
2003	63.6%	87.3%	94.0%	74.7%	64.8%	109.5%	77.1%	74.8%	87.0%	109.5%	63.6%	81.4%	45.9%	0.1464
2004	68.5%	77.2%	85.8%	73.4%	81.4%	88.7%	87.6%	82.8%	88.8%	88.8%	68.5%	81.6%	20.3%	0.0720
2005	70.6%	81.7%	66.4%	84.0%	64.8%	106.0%	81.6%	95.1%	75.4%	106.0%	64.8%	80.6%	41.1%	0.1346
2006	64.2%	78.1%	59.9%	77.5%	72.5%	83.9%	71.9%	77.9%	80.1%	83.9%	59.9%	74.0%	24.0%	0.0776
2007	60.1%	81.7%	70.6%	78.7%	59.7%	92.3%	72.7%	85.2%	74.8%	92.3%	59.7%	75.1%	32.7%	0.1086
2008	52.5%	74.3%	63.8%	82.6%	85.2%	128.6%	73.4%	95.1%	82.2%	128.6%	52.5%	82.0%	76.1%	0.2147
2009	58.2%	79.1%	14.1%	113.3%	97.7%	85.9%	95.3%	91.8%	72.1%	113.3%	14.1%	78.6%	99.2%	0.2892
2010	62.5%	86.9%	48.1%	86.9%	89.5%	86.2%	115.1%	107.3%	70.3%	115.1%	48.1%	83.6%	66.9%	0.2092
High	86.1%	96.6%	94.0%	113.3%	97.7%	128.6%	115.1%	107.3%	98.5%					
Low	52.5%	62.6%	14.1%	50.7%	27.9%	36.6%	15.8%	53.4%	68.1%					
Average	66.4%	78.8%	63.6%	75.4%	65.5%	79.8%	73.0%	77.2%	79.1%					
Range	33.6%	34.0%	79.9%	62.7%	69.8%	92.0%	99.3%	53.9%	30.4%					
Std Dev	0.0693	0.0870	0.2042	0.1313	0.1685	0.2212	0.2364	0.1419	0.0853					

Correlation

Group / Company	Group / Company 4	5	19	20	21	28	33	34	38
4		0.2850	0.3865	(0.3053)	(0.3552)	(0.5044)	(0.1651)	(0.3890)	0.3080
5			0.4289	0.3754	0.1727	0.3485	0.1964	0.2906	0.1806
19				(0.2507)	(0.2058)	0.2957	(0.0728)	(0.1874)	0.3769
20					0.7484	0.4844	0.1795	0.6582	(0.1211)
21						0.5163	0.1783	0.8154	0.1735
28							0.1935	0.6246	0.1152
33								0.3021	(0.4784)
34									0.1045
38									

Initial Net Held Ultimate Loss & LAE Ratios **Exhibit 5**
Commercial Multiple Peril **CMP**
Accident Year Basis **Page 1**

Accident Year Loss Ratios by Company

Accident Year	Group / Company 4	5	19	20	21	28	33	34	38	High	Low	Average	Range	Std Dev
1993	67.5%	68.3%	63.9%	67.7%	69.2%	76.1%	44.5%	78.5%	65.4%	78.5%	44.5%	66.8%	34.0%	0.0963
1994	65.2%	70.4%	90.3%	73.6%	80.4%	83.1%	43.8%	93.3%	70.8%	93.3%	43.8%	74.5%	49.4%	0.1488
1995	59.9%	68.0%	63.1%	67.8%	76.7%	76.4%	59.4%	90.8%	62.9%	90.8%	59.4%	69.4%	31.4%	0.1023
1996	67.9%	85.8%	85.6%	80.1%	80.0%	85.6%	67.8%	99.2%	70.4%	99.2%	67.8%	80.3%	31.4%	0.1033
1997	63.3%	77.6%	68.8%	68.3%	83.1%	78.0%	55.1%	102.8%	65.2%	102.8%	55.1%	73.6%	47.7%	0.1392
1998	73.1%	79.5%	66.1%	77.9%	96.9%	102.4%	87.1%	110.1%	80.0%	110.1%	66.1%	85.9%	44.0%	0.1448
1999	74.5%	82.6%	74.4%	70.5%	94.3%	90.9%	73.7%	104.0%	79.2%	104.0%	70.5%	82.7%	33.6%	0.1140
2000	68.6%	81.1%	68.2%	79.0%	89.5%	79.5%	66.1%	108.7%	74.6%	108.7%	66.1%	79.5%	42.6%	0.1326
2001	69.6%	69.9%	79.3%	68.8%	80.6%	79.2%	71.7%	86.3%	73.0%	86.3%	68.8%	75.4%	17.4%	0.0612
2002	56.1%	56.3%	65.7%	59.9%	59.7%	62.2%	53.6%	89.6%	47.0%	89.6%	47.0%	61.1%	42.6%	0.1193
2003	50.1%	50.8%	72.7%	58.2%	50.7%	54.1%	45.0%	65.8%	42.8%	72.7%	42.8%	54.5%	29.8%	0.0965
2004	51.8%	48.7%	53.9%	53.2%	66.1%	53.0%	54.3%	68.3%	41.5%	68.3%	41.5%	54.5%	26.7%	0.0820
2005	48.1%	49.4%	47.4%	54.7%	48.9%	60.7%	50.4%	53.5%	56.1%	60.7%	47.4%	52.1%	13.2%	0.0442
2006	49.3%	48.7%	54.4%	50.1%	54.2%	48.2%	47.2%	51.3%	38.7%	54.4%	38.7%	49.1%	15.7%	0.0465
2007	55.3%	53.0%	54.7%	49.8%	55.2%	55.3%	40.0%	48.5%	45.4%	55.3%	40.0%	50.8%	15.4%	0.0539
2008	68.9%	75.1%	62.1%	67.0%	80.7%	65.8%	42.6%	49.5%	56.1%	80.7%	42.6%	63.1%	38.1%	0.1206
2009	56.5%	67.6%	56.9%	59.1%	75.9%	65.2%	47.2%	32.6%	52.0%	75.9%	32.6%	57.0%	43.3%	0.1255
2010	61.7%	70.6%	68.6%	74.0%	78.6%	65.9%	60.4%	46.0%	57.8%	78.6%	46.0%	64.8%	32.6%	0.0972
High	74.5%	85.8%	90.3%	80.1%	96.9%	102.4%	87.1%	110.1%	80.0%					
Low	48.1%	48.7%	47.4%	49.8%	48.9%	48.2%	40.0%	32.6%	38.7%					
Average	61.5%	66.9%	66.5%	65.5%	73.4%	71.2%	56.1%	76.6%	59.9%					
Range	26.4%	37.2%	42.9%	30.3%	48.0%	54.2%	47.1%	77.5%	41.3%					
Std Dev	0.0846	0.1265	0.1124	0.0975	0.1467	0.1462	0.1290	0.2498	0.1332					

Correlation

Group / Company	Group / Company 4	5	19	20	21	28	33	34	38
4		0.9086	0.5966	0.8374	0.8974	0.8751	0.6068	0.6207	0.8841
5			0.6063	0.9067	0.9130	0.8600	0.5889	0.5962	0.8617
19				0.7121	0.5064	0.5982	0.3281	0.5874	0.5931
20					0.8341	0.8542	0.6309	0.6655	0.8636
21						0.8580	0.6884	0.5989	0.8419
28							0.7375	0.7701	0.9595
33								0.6548	0.7041
34									0.7303
38									

Calendar Year Net Loss & LAE **Exhibit 5**
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Calendar Year Loss Ratios by Company

Accident Year	4	5	19	20	21	28	33	34	38	High	Low	Average	Range	Std Dev
	Group / Company													
1993	60.7%	90.0%	66.7%	66.3%	55.6%	72.4%	4.2%	78.8%	76.1%	90.0%	4.2%	63.4%	85.8%	0.2444
1994	72.5%	75.8%	74.7%	79.2%	64.1%	82.5%	202.5%	85.6%	69.4%	202.5%	64.1%	89.6%	138.4%	0.4285
1995	65.3%	69.6%	66.8%	76.8%	73.5%	72.7%	-18.7%	92.6%	67.2%	92.6%	-18.7%	62.9%	111.2%	0.3166
1996	74.1%	78.1%	82.1%	86.9%	77.7%	96.4%	102.8%	75.9%	77.4%	102.8%	74.1%	83.5%	28.6%	0.0997
1997	59.4%	75.7%	63.6%	56.1%	76.4%	90.6%	67.1%	69.3%	62.2%	90.6%	56.1%	68.9%	34.5%	0.1063
1998	70.1%	71.6%	66.1%	71.5%	76.0%	84.3%	140.6%	107.0%	82.1%	140.6%	66.1%	85.5%	74.4%	0.2398
1999	67.1%	76.4%	64.9%	67.4%	82.7%	91.1%	93.4%	54.8%	67.9%	93.4%	54.8%	74.0%	38.6%	0.1288
2000	52.3%	75.2%	70.5%	70.5%	98.3%	67.3%	104.1%	79.1%	62.5%	104.1%	52.3%	75.5%	51.8%	0.1649
2001	58.4%	89.9%	73.7%	71.3%	95.2%	109.1%	76.1%	76.2%	71.5%	109.1%	58.4%	80.2%	50.7%	0.1524
2002	51.2%	68.4%	71.1%	64.0%	59.9%	77.0%	36.5%	98.1%	59.8%	98.1%	36.5%	65.1%	61.6%	0.1712
2003	80.0%	50.4%	88.8%	59.9%	73.6%	93.7%	51.1%	74.6%	54.7%	93.7%	50.4%	69.6%	43.3%	0.1629
2004	53.0%	56.6%	46.4%	56.9%	88.9%	59.4%	67.6%	180.9%	49.6%	180.9%	46.4%	73.2%	134.5%	0.4226
2005	51.4%	51.9%	44.6%	49.9%	50.2%	70.5%	47.4%	63.5%	67.7%	70.5%	44.6%	55.2%	25.9%	0.0943
2006	64.0%	58.1%	57.0%	59.2%	50.9%	63.4%	44.4%	66.9%	42.0%	66.9%	42.0%	56.2%	24.9%	0.0874
2007	53.2%	47.8%	54.3%	51.1%	59.1%	57.5%	50.0%	43.3%	37.7%	59.1%	37.7%	50.4%	21.4%	0.0678
2008	79.9%	67.8%	59.3%	62.1%	76.3%	29.2%	18.5%	40.8%	37.5%	79.9%	18.5%	52.4%	61.4%	0.2166
2009	52.6%	62.2%	75.0%	57.2%	53.2%	57.1%	68.5%	30.3%	41.4%	75.0%	30.3%	55.3%	44.7%	0.1345
2010	60.6%	69.9%	55.4%	66.7%	54.2%	44.2%	66.9%	54.0%	57.7%	69.9%	44.2%	58.9%	25.8%	0.0812
High	80.0%	90.0%	88.8%	86.9%	98.3%	109.1%	202.5%	180.9%	82.1%					
Low	51.2%	47.8%	44.6%	49.9%	50.2%	29.2%	-18.7%	30.3%	37.5%					
Average	62.5%	68.6%	65.6%	65.2%	70.3%	73.2%	67.9%	76.2%	60.3%					
Range	28.8%	42.2%	44.2%	37.0%	48.1%	79.9%	221.2%	150.6%	44.6%					
Std Dev	0.0969	0.1227	0.1154	0.0983	0.1516	0.1986	0.5041	0.3294	0.1387					

Correlation

Group / Company	4	5	19	20	21	28	33	34	38
	Group / Company								
4		0.1177	0.4605	0.4602	0.1428	0.1368	0.1747	(0.1281)	0.1383
5			0.3538	0.6559	0.3675	0.3996	0.1884	0.0117	0.6460
19				0.5647	0.2418	0.5608	0.2528	(0.1591)	0.3009
20					0.3648	0.3944	0.3936	0.1469	0.6325
21						0.3742	0.2363	0.3949	0.2546
28							0.3398	0.1874	0.6817
33								0.1275	0.3438
34									0.2789
38									

Initial Net Held Ultimate Loss & LAE Ratios
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Accident Year Loss Ratios by Company

Accident	Group / Company													
Year	4	5	19	20	21	28	33	34	38	High	Low	Average	Range	Std Dev
1993				61.4%		87.7%	44.2%	16.4%		87.7%	16.4%	52.4%	71.3%	0.2995
1994				97.6%		107.2%	44.0%	28.0%		107.2%	28.0%	69.2%	79.3%	0.3911
1995				140.3%		119.1%	171.2%	43.5%		171.2%	43.5%	118.5%	127.7%	0.5439
1996				104.3%		122.2%	189.7%	211.3%		211.3%	104.3%	156.9%	107.0%	0.5169
1997				138.5%		131.0%	190.7%	303.4%		303.4%	131.0%	190.9%	172.5%	0.7958
1998				155.2%		147.0%	212.8%	395.4%		395.4%	147.0%	227.6%	248.4%	1.1562
1999				176.8%		141.7%	174.9%	252.9%		252.9%	141.7%	186.6%	111.2%	0.4706
2000				161.8%		110.6%	154.6%	207.2%		207.2%	110.6%	158.5%	96.7%	0.3957
2001				98.1%		138.3%	183.8%	141.7%		183.8%	98.1%	140.5%	85.7%	0.3503
2002				89.7%		98.3%	62.3%	105.8%		105.8%	62.3%	89.0%	43.5%	0.1899
2003				63.9%		55.2%	39.3%	24.3%		63.9%	24.3%	45.7%	39.5%	0.1749
2004				47.0%		48.8%	43.4%	38.1%		48.8%	38.1%	44.3%	10.7%	0.0474
2005				33.8%		50.2%	38.2%	44.7%		50.2%	33.8%	41.7%	16.4%	0.0722
2006				46.2%		45.6%	46.7%	43.1%		46.7%	43.1%	45.4%	3.5%	0.0156
2007				44.4%		59.4%	47.9%	48.7%		59.4%	44.4%	50.1%	15.0%	0.0645
2008				67.9%		57.0%	55.0%	61.6%		67.9%	55.0%	60.4%	12.9%	0.0573
2009				62.9%		58.8%	67.2%	70.3%		70.3%	58.8%	64.8%	11.4%	0.0501
2010				100.6%		60.3%	79.3%	70.0%		100.6%	60.3%	77.6%	40.3%	0.1723
High	0.0%	0.0%	0.0%	176.8%	0.0%	147.0%	212.8%	395.4%	0.0%					
Low	0.0%	0.0%	0.0%	33.8%	0.0%	45.6%	38.2%	16.4%	0.0%					
Average				93.9%		91.0%	102.5%	117.0%						
Range	0.0%	0.0%	0.0%	143.0%	0.0%	101.4%	174.6%	379.0%	0.0%					
Std Dev				0.4454		0.3673	0.6732	1.1103						

Correlation

Group /	Group / Company								
Company	4	5	19	20	21	28	33	34	38
4									
5									
19									
20						0.8496	0.8356	0.7583	
21									
28							0.8789	0.7570	
33								0.8405	
34									
38									

Calendar Year Net Loss & LAE **Exhibit 5**
Medical Professional Liability **MM**
Calendar Year Basis **Page 2**

Calendar Year Loss Ratios by Company

Accident Year	Group / Company 4	5	19	20	21	28	33	34	38	High	Low	Average	Range	Std Dev
1993				119.9%		62.3%	50.8%	73.6%		119.9%	50.8%	76.6%	69.1%	0.3031
1994				70.0%		108.9%	16.8%	80.9%		108.9%	16.8%	69.2%	92.0%	0.3853
1995				163.2%		94.1%	84.4%	87.8%		163.2%	84.4%	107.4%	78.8%	0.3741
1996				100.3%		114.3%	57.4%	92.9%		114.3%	57.4%	91.2%	56.9%	0.2425
1997				70.5%		107.9%	103.3%	132.2%		132.2%	70.5%	103.5%	61.7%	0.2537
1998				108.4%		86.3%	41.2%	165.9%		165.9%	41.2%	100.4%	124.7%	0.5182
1999				169.2%		108.9%	153.1%	246.4%		246.4%	108.9%	169.4%	137.5%	0.5730
2000				151.5%		65.7%	201.5%	261.0%		261.0%	65.7%	169.9%	195.3%	0.8265
2001				130.8%		255.3%	195.0%	345.4%		345.4%	130.8%	231.6%	214.6%	0.9132
2002				138.2%		129.9%	149.0%	158.6%		158.6%	129.9%	143.9%	28.7%	0.1251
2003				107.8%		110.2%	135.2%	173.4%		173.4%	107.8%	131.6%	65.6%	0.3047
2004				44.2%		77.8%	92.7%	27.1%		92.7%	27.1%	60.5%	65.6%	0.3009
2005				11.6%		70.1%	36.3%	29.3%		70.1%	11.6%	36.8%	58.5%	0.2450
2006				30.1%		50.5%	51.6%	59.9%		59.9%	30.1%	48.0%	29.7%	0.1263
2007				21.1%		44.7%	38.7%	-21.4%		44.7%	-21.4%	20.8%	66.0%	0.2983
2008				27.1%		60.3%	21.7%	53.2%		60.3%	21.7%	40.6%	38.7%	0.1906
2009				48.3%		51.4%	47.7%	49.2%		51.4%	47.7%	49.2%	3.8%	0.0164
2010				61.2%		42.7%	51.5%	40.9%		61.2%	40.9%	49.1%	20.3%	0.0933
High	0.0%	0.0%	0.0%	169.2%	0.0%	255.3%	201.5%	345.4%	0.0%					
Low	0.0%	0.0%	0.0%	11.6%	0.0%	42.7%	16.8%	-21.4%	0.0%					
Average				87.4%		91.2%	84.9%	114.3%						
Range	0.0%	0.0%	0.0%	157.6%	0.0%	212.7%	184.7%	366.8%	0.0%					
Std Dev				0.5134		0.4917	0.5840	0.9527						

Correlation

Group / Company	Group / Company 4	5	19	20	21	28	33	34	38
4									
5									
19									
20						0.4770	0.6872	0.7447	
21									
28							0.5920	0.7405	
33								0.8456	
34									
38									

Calendar Year Change in Accident Year Ultimate Losses Over Time **Exhibit 6**
Industry Composite **HMP/FMP**
Homeowners/Farmowners

Calendar Year Change in Ultimate Losses as a Percent of 12 Month Ultimate

	Months of Maturity								
Accident	12	24	36	48	60	72	84	96	108
Year	24	36	48	60	72	84	96	108	120
1991	-0.6%	-0.6%	-0.3%	-0.4%	-0.2%	-0.3%	0.0%	-0.1%	0.0%
1992	0.4%	-1.3%	-0.2%	-0.2%	-0.1%	-0.1%	0.0%	0.0%	0.0%
1993	-0.4%	0.1%	0.1%	-0.2%	0.0%	-0.2%	0.1%	0.0%	0.0%
1994	-0.5%	-0.8%	-0.3%	-0.2%	-0.3%	0.0%	0.0%	0.1%	0.1%
1995	0.3%	-1.4%	-0.6%	-0.2%	-0.1%	0.0%	0.0%	0.0%	0.0%
1996	0.1%	-0.7%	-0.5%	-0.4%	0.0%	0.0%	0.0%	0.0%	0.0%
1997	-2.0%	-0.8%	-1.1%	-0.2%	0.1%	0.0%	0.0%	0.0%	-0.1%
1998	0.9%	0.3%	-0.1%	0.2%	0.1%	0.0%	-0.1%	-0.1%	0.0%
1999	-0.1%	-0.1%	0.1%	-0.1%	-0.1%	0.0%	0.0%	0.2%	-0.1%
2000	6.3%	1.1%	-0.5%	0.0%	-0.1%	-0.1%	0.0%	0.0%	0.1%
2001	4.9%	0.1%	0.2%	-0.2%	-0.2%	-0.1%	0.0%	0.2%	0.0%
2002	-3.8%	-0.7%	-0.2%	-0.1%	-0.2%	-0.1%	0.0%	0.0%	
2003	-4.1%	-1.0%	-0.3%	-0.1%	-0.1%	0.0%	0.0%		
2004	-2.1%	-1.1%	0.0%	-0.2%	0.0%	0.0%			
2005	-2.2%	0.3%	0.5%	-0.3%	-0.1%				
2006	0.1%	-0.4%	-0.3%	-0.3%					
2007	0.3%	-3.0%	-0.5%						
2008	-0.9%	-0.3%							
2009	0.0%								
2010									
High	6.3%	1.1%	0.5%	0.2%	0.1%	0.0%	0.1%	0.2%	0.1%
Low	-4.1%	-3.0%	-1.1%	-0.4%	-0.3%	-0.3%	-0.1%	-0.1%	-0.1%
Average	-0.2%	-0.6%	-0.2%	-0.2%	-0.1%	-0.1%	0.0%	0.0%	0.0%
Range	10.4%	4.1%	1.6%	0.6%	0.3%	0.3%	0.1%	0.4%	0.2%
Std Dev	0.0248	0.0089	0.0036	0.0015	0.0011	0.0009	0.0003	0.0010	0.0006

Calendar Year Change in Accident Year Ultimate Losses Over Time **Exhibit 6**
Industry Composite **PPAL**
Private Passenger Auto Liability

Calendar Year Change in Ultimate Losses as a Percent of 12 Month Ultimate

	Months of Maturity								
Accident Year	12 24	24 36	36 48	48 60	60 72	72 84	84 96	96 108	108 120
1991	-3.1%	-2.2%	-1.7%	-1.1%	-1.0%	-0.5%	-0.1%	-0.2%	0.0%
1992	-3.7%	-3.0%	-2.0%	-1.5%	-0.8%	-0.3%	-0.1%	-0.1%	-0.1%
1993	-4.0%	-2.6%	-2.4%	-1.2%	-0.5%	-0.3%	-0.2%	-0.1%	0.0%
1994	-3.6%	-3.4%	-1.5%	-0.7%	-0.5%	-0.3%	-0.1%	0.0%	0.0%
1995	-3.9%	-2.6%	-0.9%	-0.7%	-0.3%	-0.1%	-0.1%	0.0%	0.1%
1996	-4.3%	-1.5%	-0.9%	-0.3%	-0.1%	0.1%	0.0%	0.0%	0.0%
1997	-2.9%	-1.1%	-0.4%	-0.2%	0.0%	-0.1%	0.0%	0.0%	0.0%
1998	-1.1%	-0.3%	-0.1%	0.0%	-0.2%	0.0%	-0.1%	0.0%	0.1%
1999	0.8%	-0.2%	0.2%	-0.2%	0.1%	0.1%	0.1%	0.0%	0.0%
2000	1.1%	0.1%	0.0%	0.0%	0.1%	0.1%	0.1%	0.1%	0.0%
2001	-0.9%	-0.3%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%
2002	-1.5%	-0.6%	-0.1%	-0.1%	0.0%	0.0%	-0.1%	0.0%	
2003	-3.2%	-1.2%	-0.4%	-0.1%	-0.1%	0.0%	-0.1%		
2004	-3.6%	-1.6%	-0.4%	-0.1%	-0.1%	0.0%			
2005	-3.2%	-1.2%	-0.3%	-0.3%	-0.2%				
2006	-0.7%	-0.4%	-0.5%	-0.6%					
2007	0.1%	-0.6%	-0.7%						
2008	-0.7%	-0.9%							
2009	-1.4%								
2010									
High	1.1%	0.1%	0.2%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%
Low	-4.3%	-3.4%	-2.4%	-1.5%	-1.0%	-0.5%	-0.2%	-0.2%	-0.1%
Average	-2.1%	-1.3%	-0.7%	-0.4%	-0.2%	-0.1%	-0.1%	0.0%	0.0%
Range	5.3%	3.5%	2.6%	1.5%	1.1%	0.5%	0.3%	0.3%	0.1%
Std Dev	0.0172	0.0105	0.0077	0.0047	0.0032	0.0017	0.0008	0.0007	0.0004

Calendar Year Change in Accident Year Ultimate Losses Over Time **Exhibit 6**
Industry Composite **CAL**
Commercial Auto Liability

Calendar Year Change in Ultimate Losses as a Percent of 12 Month Ultimate

	Months of Maturity								
Accident	12	24	36	48	60	72	84	96	108
Year	24	36	48	60	72	84	96	108	120
1991	-2.5%	-4.1%	-2.5%	-0.9%	-0.9%	-0.8%	-0.4%	-0.8%	-0.6%
1992	-3.8%	-2.3%	-2.2%	-1.2%	-1.3%	-0.9%	-0.7%	-0.9%	0.4%
1993	-2.0%	-1.1%	-0.9%	-1.0%	-0.9%	-0.7%	-0.6%	-0.3%	0.2%
1994	-0.7%	-0.2%	-1.2%	0.1%	-0.3%	-0.6%	-0.3%	0.1%	-0.1%
1995	-0.9%	0.1%	-0.2%	0.0%	0.3%	-0.5%	-0.1%	-0.1%	-0.2%
1996	1.6%	1.3%	1.5%	0.4%	0.4%	-0.1%	-0.5%	0.0%	0.1%
1997	2.0%	2.8%	2.4%	2.4%	0.4%	-0.2%	-0.4%	0.3%	0.3%
1998	3.6%	4.2%	4.1%	1.3%	0.6%	-1.0%	-0.2%	-0.2%	0.1%
1999	6.6%	3.4%	4.0%	1.1%	0.1%	0.8%	0.2%	-0.2%	0.0%
2000	2.9%	2.8%	2.0%	2.1%	2.1%	-0.3%	-0.4%	-0.1%	-0.1%
2001	-0.8%	3.4%	2.9%	2.5%	-0.9%	-0.9%	0.1%	0.1%	-0.4%
2002	-3.4%	1.8%	1.0%	-0.1%	-0.5%	-0.4%	0.0%	-0.6%	
2003	-3.8%	-0.3%	0.1%	-0.9%	-0.5%	-0.3%	-0.6%		
2004	-5.4%	-1.2%	-0.6%	-0.9%	0.3%	-0.8%			
2005	-2.2%	-1.0%	-0.4%	-0.2%	-0.2%				
2006	-2.3%	-0.8%	-0.4%	-0.6%					
2007	-1.6%	-1.4%	1.0%						
2008	-2.6%	0.8%							
2009	-3.9%								
2010									
High	6.6%	4.2%	4.1%	2.5%	2.1%	0.8%	0.2%	0.3%	0.4%
Low	-5.4%	-4.1%	-2.5%	-1.2%	-1.3%	-1.0%	-0.7%	-0.9%	-0.6%
Average	-1.0%	0.5%	0.6%	0.3%	-0.1%	-0.5%	-0.3%	-0.2%	0.0%
Range	12.0%	8.3%	6.6%	3.7%	3.4%	1.7%	0.8%	1.2%	1.0%
Std Dev	0.0305	0.0225	0.0198	0.0126	0.0084	0.0046	0.0027	0.0036	0.0029

Calendar Year Change in Accident Year Ultimate Losses Over Time **Exhibit 6**
Industry Composite **WC**
Workers Compensation

Calendar Year Change in Ultimate Losses as a Percent of 12 Month Ultimate

Accident	Months of Maturity								
	12	24	36	48	60	72	84	96	108
Year	24	36	48	60	72	84	96	108	120
1991	1.2%	-2.5%	-2.3%	-1.2%	-0.3%	-0.4%	-0.4%	-0.6%	-0.3%
1992	-3.6%	-6.5%	-4.7%	-1.6%	-0.4%	-0.5%	-0.6%	-0.3%	-0.6%
1993	-4.3%	-4.5%	-4.7%	-1.0%	-1.2%	-1.0%	-1.0%	-0.5%	0.0%
1994	-4.7%	-4.1%	-4.1%	-2.1%	-0.8%	-1.1%	-0.2%	-0.2%	0.5%
1995	-4.3%	-3.2%	-2.8%	0.2%	-0.2%	-0.5%	-0.3%	0.5%	0.2%
1996	-2.6%	-1.8%	-0.7%	-0.4%	-0.2%	0.6%	1.4%	0.4%	2.2%
1997	1.7%	1.6%	0.2%	0.4%	1.9%	0.5%	1.7%	2.8%	0.8%
1998	1.6%	2.3%	2.3%	2.1%	2.6%	1.7%	3.3%	1.8%	1.0%
1999	2.9%	4.6%	3.0%	2.9%	2.5%	5.2%	1.6%	2.3%	-0.3%
2000	4.4%	3.1%	3.0%	3.3%	6.5%	2.9%	2.5%	0.6%	3.1%
2001	2.2%	0.8%	2.0%	9.0%	2.0%	3.3%	0.9%	3.8%	2.2%
2002	0.9%	1.3%	4.8%	1.4%	2.6%	0.2%	2.9%	1.7%	
2003	-6.0%	-5.4%	0.5%	1.7%	0.5%	1.7%	0.9%		
2004	-9.2%	-2.7%	-4.9%	-0.3%	-3.1%	-0.2%			
2005	-6.6%	-6.5%	-1.1%	-4.9%	-2.2%				
2006	-5.7%	-2.0%	-2.4%	-0.9%					
2007	-1.6%	-0.9%	0.3%						
2008	1.6%	1.6%							
2009	1.2%								
2010									
High	4.4%	4.6%	4.8%	9.0%	6.5%	5.2%	3.3%	3.8%	3.1%
Low	-9.2%	-6.5%	-4.9%	-4.9%	-3.1%	-1.1%	-1.0%	-0.6%	-0.6%
Average	-1.6%	-1.4%	-0.7%	0.5%	0.7%	0.9%	1.0%	1.0%	0.8%
Range	13.6%	11.1%	9.6%	13.9%	9.6%	6.3%	4.3%	4.4%	3.7%
Std Dev	0.0389	0.0335	0.0304	0.0305	0.0238	0.0186	0.0141	0.0143	0.0122

Calendar Year Change in Accident Year Ultimate Losses Over Time **Exhibit 6**
Industry Composite **CMP**
Commercial Multiple Peril

Calendar Year Change in Ultimate Losses as a Percent of 12 Month Ultimate

	Months of Maturity								
Accident Year	12 24	24 36	36 48	48 60	60 72	72 84	84 96	96 108	108 120
1991	-1.5%	0.0%	-1.2%	-0.6%	-0.8%	-0.1%	-0.2%	0.0%	0.2%
1992	-2.1%	-1.4%	-0.4%	-0.9%	-0.1%	0.0%	0.3%	-0.3%	0.1%
1993	-1.9%	-0.8%	-0.7%	-0.7%	0.0%	-0.2%	-0.6%	0.1%	0.3%
1994	-0.3%	-0.2%	0.9%	-0.1%	0.9%	-1.3%	0.2%	0.1%	1.0%
1995	-0.1%	-1.3%	0.1%	-0.3%	-0.3%	-0.2%	0.2%	1.2%	0.3%
1996	0.8%	1.5%	-0.3%	-0.4%	-0.4%	0.1%	1.3%	0.5%	0.1%
1997	1.0%	-0.6%	-0.8%	0.2%	0.0%	1.8%	0.7%	0.1%	0.7%
1998	2.0%	0.3%	1.3%	0.3%	2.3%	2.0%	0.2%	0.6%	0.0%
1999	1.3%	-1.6%	2.5%	2.9%	1.8%	0.2%	1.3%	-0.5%	0.0%
2000	2.9%	3.2%	1.3%	2.3%	1.6%	-0.1%	0.4%	-0.1%	-0.2%
2001	-2.5%	0.9%	1.9%	-1.0%	0.9%	0.5%	0.1%	0.0%	-0.1%
2002	-3.8%	1.4%	0.4%	-0.1%	-0.5%	-0.5%	0.0%	-0.1%	
2003	-5.9%	-2.4%	-0.4%	-1.1%	-1.0%	-0.3%	-0.5%		
2004	-6.7%	-1.1%	-2.7%	-1.9%	-0.7%	-0.5%			
2005	-1.0%	-3.3%	-3.6%	-1.0%	-0.8%				
2006	-2.5%	-4.3%	-2.0%	-1.1%					
2007	-2.1%	-2.6%	-1.0%						
2008	-1.9%	-1.4%							
2009	-1.9%								
2010									
High	2.9%	3.2%	2.5%	2.9%	2.3%	2.0%	1.3%	1.2%	1.0%
Low	-6.7%	-4.3%	-3.6%	-1.9%	-1.0%	-1.3%	-0.6%	-0.5%	-0.2%
Average	-1.4%	-0.8%	-0.3%	-0.2%	0.2%	0.1%	0.3%	0.1%	0.2%
Range	9.7%	7.5%	6.1%	4.8%	3.3%	3.3%	1.9%	1.7%	1.2%
Std Dev	0.0247	0.0183	0.0160	0.0124	0.0105	0.0086	0.0059	0.0044	0.0036

Calendar Year Change in Accident Year Ultimate Losses Over Time **Exhibit 6**
Industry Composite **MM**
Medical Professional Liability

Calendar Year Change in Ultimate Losses as a Percent of 12 Month Ultimate

Accident Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
	Months of Maturity								
1991	-1.6%	-4.5%	-4.7%	-3.8%	-5.5%	-1.2%	-1.9%	-1.4%	-1.0%
1992	-4.4%	-5.3%	-3.4%	-3.7%	-2.7%	-3.0%	-1.8%	-1.4%	-0.4%
1993	-4.8%	-1.8%	-5.5%	-2.6%	-1.4%	-2.0%	-2.0%	-1.0%	-0.1%
1994	-3.0%	0.5%	-4.7%	-4.4%	-3.3%	-1.8%	-0.8%	-0.2%	-0.2%
1995	0.7%	-1.1%	-1.8%	-2.6%	-1.2%	-0.5%	0.2%	1.0%	-0.2%
1996	0.1%	-3.0%	1.4%	-0.3%	2.4%	1.7%	-0.4%	0.3%	-0.9%
1997	-0.5%	3.1%	1.9%	5.8%	2.1%	-0.6%	0.1%	-0.7%	0.0%
1998	0.8%	3.9%	13.1%	2.6%	1.8%	-2.4%	1.9%	-0.7%	-1.0%
1999	4.7%	8.0%	3.3%	3.9%	-0.7%	1.4%	-0.2%	-1.7%	-1.0%
2000	2.9%	11.4%	8.4%	2.4%	0.0%	-1.2%	-0.6%	-0.7%	-0.6%
2001	5.7%	8.8%	3.7%	3.7%	-0.4%	-1.7%	-0.8%	-2.1%	-1.4%
2002	4.3%	1.5%	2.4%	-1.0%	-2.7%	-1.6%	-2.1%	-0.7%	
2003	-3.6%	-4.5%	-4.8%	-3.0%	-1.6%	-3.0%	-1.0%		
2004	-5.4%	-5.4%	-6.9%	-8.6%	-4.7%	-3.6%			
2005	-4.6%	-7.2%	-7.8%	-7.0%	-5.3%				
2006	-6.0%	-7.3%	-7.0%	-9.1%					
2007	-6.1%	-5.9%	-8.2%						
2008	-4.8%	-6.0%							
2009	-5.1%								
2010									
High	5.7%	11.4%	13.1%	5.8%	2.4%	1.7%	1.9%	1.0%	0.0%
Low	-6.1%	-7.3%	-8.2%	-9.1%	-5.5%	-3.6%	-2.1%	-2.1%	-1.4%
Average	-1.6%	-0.8%	-1.2%	-1.7%	-1.5%	-1.4%	-0.7%	-0.8%	-0.6%
Range	11.8%	18.6%	21.3%	15.0%	7.9%	5.3%	4.0%	3.1%	1.4%
Std Dev	0.0388	0.0581	0.0607	0.0450	0.0254	0.0153	0.0112	0.0086	0.0047

Calendar Year Change in Accident Year Ultimate Losses between 12 and 24 Months **Exhibit 7**
By Group/Company **HMP/FMP**
Homeowners/Farmowners

12 to 24 Months of Maturity Change in Ultimate Losses as a Percent of 12 Month Ultimate

Accident	Group/Company																			
Year	1	2	3	4	5	6	15	18	19	20	21	25	33	38	40	High	Low	Average	Range	Std Dev
1991	1.3%	0.4%	-1.3%	-0.6%	-0.3%	-0.7%	-3.5%	1.7%	-1.5%	-5.3%	-2.6%	-3.9%	-0.1%	-3.1%	-4.0%	1.7%	-5.3%	-1.6%	7.1%	0.0210
1992	-2.3%	0.8%	-1.2%	1.6%	-1.4%	3.8%	-19.7%	-1.2%	-2.2%	-0.9%	-5.0%	-1.6%	-2.4%	-1.0%	-5.2%	3.8%	-19.7%	-2.5%	23.5%	0.0525
1993	0.6%	-2.4%	-3.6%	-0.2%	-1.3%	2.4%	-2.0%	-4.4%	-2.3%	-0.3%	-4.8%	-2.9%	-7.5%	-6.3%	-2.6%	2.4%	-7.5%	-2.5%	9.9%	0.0261
1994	0.9%	1.4%	-1.7%	-5.6%	-1.9%	0.8%	0.5%	-4.5%	1.3%	-4.1%	-6.3%	-0.1%	-6.9%	-0.9%	-0.8%	1.4%	-6.9%	-1.9%	8.3%	0.0290
1995	1.6%	2.9%	-0.3%	1.3%	-1.5%	1.5%	-3.3%	-7.6%	0.8%	0.1%	-3.7%	3.3%	-19.5%	-1.5%	-2.7%	3.3%	-19.5%	-1.9%	22.8%	0.0564
1996	1.6%	-1.4%	-2.1%	-0.2%	1.5%	0.9%	-3.6%	-8.0%	1.6%	2.8%	-3.3%	0.1%	-27.6%	0.4%	0.5%	2.8%	-27.6%	-2.4%	30.5%	0.0748
1997	-3.7%	-3.1%	1.3%	-0.2%	-1.6%	-0.6%	-11.3%	-8.3%	-1.4%	2.2%	-3.6%	-4.7%	-28.2%	-2.8%	-5.2%	2.2%	-28.2%	-4.7%	30.4%	0.0736
1998	-0.8%	4.2%	3.3%	5.0%	1.1%	2.3%	-8.2%	-0.3%	1.0%	-0.7%	-0.9%	-3.6%	-28.7%	-1.0%	-2.7%	5.0%	-28.7%	-2.0%	33.7%	0.0809
1999	-2.5%	1.0%	-0.7%	1.3%	1.1%	0.1%	-8.3%	-0.3%	3.0%	-2.9%	-4.7%	4.1%	-24.7%	2.5%	0.1%	4.1%	-24.7%	-2.1%	28.8%	0.0702
2000	7.3%	13.4%	6.6%	3.3%	5.8%	5.7%	9.2%	3.7%	4.4%	4.5%	2.4%	11.3%	-11.2%	3.1%	10.1%	13.4%	-11.2%	5.3%	24.6%	0.0560
2001	7.9%	15.1%	10.4%	2.6%	3.2%	2.7%	-0.6%	6.9%	3.9%	0.0%	-0.5%	0.4%	-15.9%	2.1%	1.4%	15.1%	-15.9%	2.6%	31.0%	0.0677
2002	-0.5%	5.4%	-8.8%	-5.7%	-3.0%	-3.1%	-3.0%	-5.3%	-5.5%	-2.0%	-5.1%	-1.6%	-6.1%	-14.1%	-3.2%	5.4%	-14.1%	-4.1%	19.5%	0.0423
2003	-1.5%	-4.2%	-11.3%	-3.7%	-1.2%	-1.4%	-4.4%	-4.3%	-4.8%	-1.5%	-3.9%	-5.9%	1.0%	-15.3%	-4.9%	1.0%	-15.3%	-4.5%	16.3%	0.0409
2004	-3.7%	-3.3%	-4.2%	1.7%	-5.0%	-2.5%	0.5%	1.9%	-2.2%	-2.3%	-6.2%	3.8%	-2.4%	1.4%	-7.2%	3.8%	-7.2%	-2.0%	11.0%	0.0321
2005	-4.5%	-3.3%	-0.1%	-1.9%	0.6%	1.2%	-5.2%	-1.5%	-5.4%	0.2%	-2.2%	-0.6%	-9.9%	-13.0%	-3.5%	1.2%	-13.0%	-3.3%	14.3%	0.0396
2006	2.0%	-4.7%	-2.9%	1.4%	-1.8%	2.6%	-2.1%	-2.2%	-1.9%	-3.9%	-0.5%	-1.0%	-26.7%	-1.5%	2.4%	2.6%	-26.7%	-2.7%	29.3%	0.0699
2007	0.6%	-2.6%	-2.1%	2.6%	-0.8%	2.6%	-5.7%	-1.4%	-4.0%	-2.6%	0.7%	0.1%	-0.3%	-2.0%	-3.0%	2.6%	-5.7%	-1.2%	8.4%	0.0233
2008	-2.1%	-0.5%	-3.3%	4.8%	0.1%	-1.1%	-2.5%	-2.0%	1.0%	-2.4%	2.2%	1.0%	13.4%	-0.1%	-2.0%	13.4%	-3.3%	0.4%	16.7%	0.0418
2009	0.2%	-0.4%	-4.2%	3.4%	1.2%	1.4%	-0.6%	-3.4%	0.6%	-1.8%	-0.7%	-0.9%	-8.8%	2.6%	0.1%	3.4%	-8.8%	-0.8%	12.2%	0.0299
2010																0.0%	0.0%		0.0%	
High	7.9%	15.1%	10.4%	5.0%	5.8%	5.7%	9.2%	6.9%	4.4%	4.5%	2.4%	11.3%	13.4%	3.1%	10.1%					
Low	-4.5%	-4.7%	-11.3%	-5.7%	-5.0%	-3.1%	-19.7%	-8.3%	-5.5%	-5.3%	-6.3%	-5.9%	-28.7%	-15.3%	-7.2%					
Average	0.1%	1.0%	-1.4%	0.6%	-0.3%	1.0%	-3.9%	-2.1%	-0.7%	-1.1%	-2.6%	-0.1%	-11.2%	-2.7%	-1.7%					
Range	12.4%	19.8%	21.7%	10.8%	10.8%	8.8%	28.9%	15.2%	9.9%	9.8%	8.8%	17.2%	42.1%	18.4%	17.3%					
Std Dev	0.0329	0.0544	0.0481	0.0308	0.0236	0.0221	0.0572	0.0398	0.0300	0.0245	0.0267	0.0388	0.1201	0.0560	0.0379					

Calendar Year Change in Accident Year Ultimate Losses between 12 and 24 Months **Exhibit 7**
By Group/Company **PPAL**
Private Passenger Auto Liability

12 to 24 Months of Maturity Change in Ultimate Losses as a Percent of 12 Month Ultimate

Accident	Group/Company																					
Year	1	2	3	4	5	6	8	9	15	18	19	20	21	25	33	38	40	High	Low	Average	Range	Std Dev
1991	-3.1%	-7.5%	-3.7%	-7.1%	0.7%	-2.4%	-6.4%	-4.7%	-1.9%	-3.2%	-3.6%	-1.7%	-0.7%	-5.3%	-3.6%	-5.9%	0.9%	0.9%	-7.5%	-3.5%	8.4%	0.0250
1992	-2.1%	-6.1%	-5.7%	-4.6%	-3.8%	-2.9%	-8.6%	-4.0%	-12.1%	-3.4%	-6.6%	-9.8%	-7.1%	-6.1%	-6.6%	-2.2%	-1.4%	-1.4%	-12.1%	-5.5%	10.7%	0.0289
1993	-2.7%	1.2%	-5.2%	-0.6%	-5.7%	-2.9%	-4.8%	-6.4%	-5.7%	-8.0%	-4.7%	0.6%	-12.3%	-6.0%	1.1%	-2.0%	-2.8%	1.2%	-12.3%	-3.9%	13.5%	0.0351
1994	-1.3%	-3.3%	-4.1%	3.0%	-3.4%	-3.6%	-0.1%	-5.1%	-6.2%	-11.5%	-6.8%	-2.3%	-10.1%	-4.3%	0.0%	-2.1%	0.9%	3.0%	-11.5%	-3.5%	14.5%	0.0374
1995	-1.5%	-4.7%	-5.3%	-0.2%	-4.9%	-5.1%	-3.3%	-4.1%	-5.3%	-1.9%	-4.1%	-1.9%	-10.4%	-1.2%	-10.2%	-3.4%	0.8%	0.8%	-10.4%	-3.9%	11.2%	0.0302
1996	-2.4%	-2.3%	-1.8%	0.4%	-2.1%	-6.7%	-4.2%	-4.5%	-2.7%	-15.0%	0.5%	-0.9%	-7.5%	-0.1%	-0.3%	-1.5%	0.3%	0.5%	-15.0%	-3.0%	15.6%	0.0389
1997	-2.7%	3.0%	-2.7%	-0.5%	-4.5%	-2.8%	-5.2%	-4.0%	-6.6%	1.3%	-3.5%	-2.1%	-9.8%	-0.7%	-18.2%	-1.3%	-2.6%	3.0%	-18.2%	-3.7%	21.2%	0.0475
1998	0.5%	3.4%	1.2%	0.2%	-1.8%	-3.2%	1.0%	-2.7%	-2.1%	2.4%	-0.9%	-4.6%	-2.0%	-3.0%	4.7%	-1.3%	4.8%	4.8%	-4.6%	-0.2%	9.4%	0.0281
1999	1.6%	7.8%	0.6%	-0.7%	-0.4%	-0.1%	4.9%	-2.3%	-0.1%	8.4%	2.7%	-0.9%	-10.1%	-0.7%	1.9%	1.9%	8.5%	8.5%	-10.1%	1.4%	18.6%	0.0450
2000	1.8%	-1.6%	-0.9%	0.8%	2.4%	1.6%	-3.2%	-4.2%	-4.7%	2.6%	4.6%	-1.8%	0.3%	4.7%	16.1%	2.4%	9.5%	16.1%	-4.7%	1.8%	20.8%	0.0513
2001	0.7%	-2.3%	-2.7%	0.8%	0.5%	-0.8%	-1.3%	-5.6%	-8.8%	4.1%	6.0%	-2.5%	3.3%	-4.3%	4.7%	-0.3%	-1.6%	6.0%	-8.8%	-0.6%	14.8%	0.0382
2002	-1.6%	3.2%	2.5%	-6.3%	-0.6%	-4.1%	0.2%	-3.1%	-4.0%	0.3%	0.3%	2.5%	9.2%	0.2%	7.3%	0.0%	3.4%	9.2%	-6.3%	0.5%	15.6%	0.0399
2003	-5.4%	0.2%	0.8%	-4.8%	-1.6%	-4.3%	-0.7%	-5.5%	-7.2%	-0.2%	-2.9%	3.4%	11.0%	-1.6%	-4.2%	-6.4%	0.6%	11.0%	-7.2%	-1.7%	18.2%	0.0440
2004	-5.2%	3.5%	-0.4%	-7.5%	-2.6%	-3.3%	-2.9%	-6.7%	-2.5%	0.1%	-3.3%	3.0%	6.7%	-2.2%	-4.8%	-11.7%	-5.3%	6.7%	-11.7%	-2.7%	18.4%	0.0440
2005	-4.0%	-4.1%	-1.5%	-2.7%	-3.0%	-2.3%	-1.8%	-6.6%	-6.9%	0.1%	-6.8%	3.7%	1.3%	-3.5%	-9.2%	-6.0%	-0.5%	3.7%	-9.2%	-3.2%	12.8%	0.0330
2006	-0.1%	-2.5%	-0.8%	0.5%	-0.4%	1.0%	1.3%	-5.2%	-4.0%	0.2%	-9.1%	6.4%	0.3%	-0.8%	-1.6%	-4.0%	-6.7%	6.4%	-9.1%	-1.5%	15.6%	0.0357
2007	1.0%	-4.4%	-1.4%	-3.3%	-1.3%	1.4%	0.9%	-1.8%	-0.4%	-0.9%	-5.3%	7.1%	0.7%	1.7%	-1.4%	1.9%	-5.7%	7.1%	-5.7%	-0.7%	12.8%	0.0310
2008	0.8%	-6.7%	-4.0%	-0.5%	-0.6%	0.3%	-0.5%	-2.7%	-4.6%	-5.3%	0.7%	-2.6%	7.5%	0.7%	-0.5%	1.5%	8.4%	8.4%	-6.7%	-0.5%	15.2%	0.0398
2009	1.2%	-6.3%	-0.9%	-3.9%	-0.1%	0.2%	-3.0%	-5.0%	-3.5%	-4.9%	-2.7%	-1.8%	3.5%	-3.1%	2.0%	1.1%	4.9%	4.9%	-6.3%	-1.3%	11.1%	0.0316
2010																		0.0%	0.0%		0.0%	
High	1.8%	7.8%	2.5%	3.0%	2.4%	1.6%	4.9%	-1.8%	-0.1%	8.4%	6.0%	7.1%	11.0%	4.7%	16.1%	2.4%	9.5%					
Low	-5.4%	-7.5%	-5.7%	-7.5%	-5.7%	-6.7%	-8.6%	-6.7%	-12.1%	-15.0%	-9.1%	-9.8%	-12.3%	-6.1%	-18.2%	-11.7%	-6.7%					
Average	-1.3%	-1.6%	-1.9%	-1.9%	-1.7%	-2.1%	-2.0%	-4.4%	-4.7%	-1.8%	-2.4%	-0.3%	-1.4%	-1.9%	-1.2%	-2.1%	0.9%					
Range	7.2%	15.3%	8.3%	10.5%	8.1%	8.3%	13.5%	4.9%	12.0%	23.4%	15.1%	16.9%	23.3%	10.8%	34.3%	14.1%	16.2%					
Std Dev	0.0226	0.0425	0.0235	0.0301	0.0211	0.0234	0.0315	0.0144	0.0294	0.0547	0.0404	0.0398	0.0733	0.0278	0.0731	0.0358	0.0475					

Calendar Year Change in Accident Year Ultimate Losses between 12 and 24 Months **Exhibit 7**
By Group/Company **CAL**
Commercial Auto Liability

12 to 24 Months of Maturity Change in Ultimate Losses as a Percent of 12 Month Ultimate

Accident	Group/Company																												
Year	1	2	3	4	5	6	7	8	9	14	15	17	19	20	21	22	23	25	28	31	33	34	38	40	High	Low	Average	Range	Std Dev
1991	-4.3%	-2.7%	-0.1%	-4.1%	0.3%	-0.6%	-5.8%	7.6%	-8.1%	-3.1%	29.2%	-4.4%	22.3%	0.0%	7.3%	4.0%	2.9%	-7.3%	-2.2%	-8.9%	1.4%	-0.9%	-7.2%	-0.9%	29.2%	-8.9%	0.6%	38.1%	0.0895
1992	2.1%	2.7%	-12.0%	-5.7%	-3.1%	0.7%	-12.5%	-9.5%	-7.0%	-1.6%	11.7%	15.6%	3.8%	2.0%	7.0%	5.0%	13.7%	-4.2%	-2.1%	-11.4%	-7.8%	3.1%	-6.5%	5.6%	15.6%	-12.5%	-0.4%	28.1%	0.0793
1993	2.8%	-0.2%	-13.0%	-3.1%	-4.1%	0.1%	-6.8%	-19.7%	-9.4%	-6.1%	14.0%	-6.7%	-5.4%	12.4%	20.0%	3.3%	-4.0%	0.3%	-2.6%	-9.6%	-0.2%	4.8%	-1.3%	1.3%	20.0%	-19.7%	-1.4%	39.7%	0.0858
1994	-1.9%	-3.8%	-7.8%	1.5%	0.3%	-0.3%	-7.5%	-11.4%	10.9%	-6.9%	2.1%	-5.4%	-4.3%	5.3%	3.9%	0.8%	-5.5%	0.6%	-5.9%	-0.5%	3.4%	5.7%	-0.3%	11.0%	11.0%	-11.4%	-0.7%	22.4%	0.0572
1995	-3.9%	-0.9%	-23.3%	0.2%	-4.4%	-2.8%	-9.0%	-3.3%	-37.3%	-6.2%	13.2%	-12.7%	-1.7%	2.0%	-3.7%	-2.2%	-1.3%	-3.9%	1.7%	-7.5%	10.9%	4.5%	6.1%	6.7%	13.2%	-37.3%	-3.3%	50.4%	0.1043
1996	1.4%	11.8%	-5.1%	-1.6%	-1.5%	-4.4%	-6.0%	-7.3%	-13.1%	-3.2%	31.5%	9.5%	2.1%	3.3%	1.0%	-1.8%	8.2%	-2.1%	6.3%	4.6%	17.6%	5.3%	-0.2%	7.6%	31.5%	-13.1%	2.7%	44.6%	0.0914
1997	3.5%	17.7%	-4.6%	-3.6%	10.7%	3.2%	1.6%	-7.3%	-0.6%	0.2%	13.1%	-0.3%	-5.3%	7.2%	-3.5%	0.0%	7.7%	3.9%	-4.4%	9.3%	-12.2%	6.8%	3.8%	-1.2%	17.7%	-12.2%	1.9%	29.9%	0.0694
1998	0.0%	16.1%	19.0%	3.2%	2.0%	1.0%	1.8%	-0.3%	2.7%	-4.1%	11.1%	4.2%	1.9%	8.9%	4.6%	1.0%	-0.2%	7.3%	11.8%	10.5%	2.9%	4.5%	-2.3%	5.1%	19.0%	-4.1%	4.7%	23.1%	0.0569
1999	3.6%	12.5%	9.7%	0.1%	11.5%	6.9%	0.9%	-3.8%	2.3%	-8.5%	3.5%	21.8%	6.4%	21.5%	2.7%	12.1%	2.7%	3.7%	8.6%	13.2%	6.7%	-0.7%	8.0%	7.1%	21.8%	-8.5%	6.4%	30.2%	0.0705
2000	0.4%	-1.8%	-2.4%	-0.8%	4.0%	10.3%	-8.0%	-2.0%	4.9%	3.4%	11.1%	-2.2%	16.0%	12.5%	8.7%	-2.7%	-0.6%	5.6%	15.9%	21.0%	-12.0%	-3.2%	4.0%	7.9%	21.0%	-12.0%	3.8%	33.0%	0.0801
2001	-2.8%	-4.4%	-7.6%	-1.6%	-13.1%	4.3%	-4.0%	8.3%	14.4%	3.9%	4.0%	-0.6%	10.2%	-0.4%	17.3%	-0.4%	6.1%	-2.4%	14.2%	141.3%	-26.8%	5.8%	-0.5%	1.3%	141.3%	-26.8%	6.9%	168.1%	0.3008
2002	6.4%	-10.9%	-12.6%	-2.2%	-10.2%	1.1%	-10.6%	-0.1%	-9.6%	-3.7%	-11.9%	-1.2%	0.2%	-3.2%	9.2%	1.8%	0.8%	-4.5%	-5.7%	-1.7%	4.3%	0.8%	-3.2%	-12.4%	9.2%	-12.6%	-3.3%	21.8%	0.0613
2003	1.7%	0.9%	-1.5%	-2.7%	-8.1%	5.0%	-12.3%	-3.7%	-16.5%	-0.1%	12.2%	-2.4%	-0.7%	-2.0%	14.0%	-1.1%	0.7%	-5.8%	-2.1%	-20.4%	-0.6%	-9.6%	-2.8%	-9.9%	14.0%	-20.4%	-2.8%	34.4%	0.0769
2004	0.1%	-7.3%	0.8%	-3.2%	-6.4%	11.3%	-5.2%	-4.5%	-2.7%	1.4%	5.6%	-7.4%	1.3%	-3.2%	8.7%	7.2%	-7.2%	-1.0%	0.7%	-21.3%	-26.3%	1.8%	-12.1%	-2.5%	11.3%	-26.3%	-3.0%	37.5%	0.0848
2005	2.4%	-2.3%	-4.5%	2.0%	-4.6%	14.6%	-3.1%	0.9%	-9.6%	0.5%	-10.5%	2.7%	-7.1%	-5.4%	2.7%	-2.8%	-10.3%	2.9%	0.6%	13.3%	-8.1%	0.7%	-7.5%	1.9%	14.6%	-10.5%	-1.3%	25.1%	0.0647
2006	-1.5%	5.5%	1.0%	0.2%	-4.6%	9.4%	-5.2%	5.7%	-7.8%	-6.0%	-8.9%	-3.7%	-11.2%	-5.1%	7.7%	-0.4%	-9.3%	-0.9%	-5.5%	-0.4%	-8.0%	-1.4%	-4.6%	3.5%	9.4%	-11.2%	-2.1%	20.6%	0.0555
2007	-2.7%	-5.5%	-8.4%	1.5%	-4.5%	15.2%	-4.2%	3.9%	-6.1%	-6.1%	5.9%	0.5%	-11.2%	-1.9%	5.3%	-2.8%	-8.2%	-0.2%	0.5%	-5.7%	-3.9%	1.5%	-3.4%	2.4%	15.2%	-11.2%	-1.6%	26.4%	0.0568
2008	-5.3%	-1.2%	9.5%	-2.5%	-8.9%	6.8%	-1.9%	-1.8%	-6.5%	7.9%	35.7%	-12.7%	-5.3%	-4.4%	10.5%	3.7%	-5.4%	-2.5%	5.0%	21.6%	3.2%	-8.5%	-13.1%	1.2%	35.7%	-13.1%	1.0%	48.8%	0.1088
2009	2.7%	0.5%	-4.1%	-3.5%	-7.6%	3.0%	-8.1%	-7.3%	-9.2%	8.8%	12.0%	0.2%	-5.6%	-9.0%	1.4%	-5.1%	-2.7%	-4.7%	-3.2%	0.0%	4.2%	-3.8%	-6.5%	-1.2%	12.0%	-9.2%	-2.0%	21.2%	0.0545
2010																									0.0%	0.0%		0.0%	
High	6.4%	17.7%	19.0%	3.2%	11.5%	15.2%	1.8%	8.3%	14.4%	8.8%	35.7%	21.8%	22.3%	21.5%	20.0%	12.1%	13.7%	7.3%	15.9%	141.3%	17.6%	6.8%	8.0%	11.0%					
Low	-5.3%	-10.9%	-23.3%	-5.7%	-13.1%	-4.4%	-12.5%	-19.7%	-37.3%	-8.5%	-11.9%	-12.7%	-11.2%	-9.0%	-3.7%	-5.1%	-10.3%	-7.3%	-5.9%	-21.3%	-26.8%	-9.6%	-13.1%	-12.4%					
Average	0.2%	1.4%	-3.5%	-1.4%	-2.8%	4.5%	-5.6%	-2.9%	-5.7%	-1.5%	9.7%	-0.3%	0.3%	2.1%	6.6%	1.0%	-0.6%	-0.8%	1.7%	7.8%	-2.7%	0.9%	-2.6%	1.8%					
Range	11.7%	28.6%	42.4%	8.9%	24.6%	19.6%	14.3%	27.9%	51.7%	17.3%	47.6%	34.5%	33.5%	30.5%	23.7%	17.2%	24.0%	14.6%	21.7%	162.6%	44.4%	16.5%	21.1%	23.4%					
Std Dev	0.0315	0.0790	0.0946	0.0238	0.0647	0.0565	0.0422	0.0686	0.1105	0.0496	0.1284	0.0858	0.0869	0.0762	0.0620	0.0413	0.0648	0.0403	0.0680	0.3461	0.1132	0.0470	0.0563	0.0587					

Calendar Year Change in Accident Year Ultimate Losses between 12 and 24 Months **Exhibit 7**
By Group/Company **WC**
Workers Compensation

12 to 24 Months of Maturity Change in Ultimate Losses as a Percent of 12 Month Ultimate

Accident	Group/Company																													
Year	2	3	4	5	6	7	9	10	11	15	16	17	19	20	21	22	23	25	28	31	33	34	38	39	40	High	Low	Average	Range	Std Dev
1991	4.6%	5.7%	0.7%	3.8%	32.9%	-9.7%	-0.2%	1.9%	-13.6%	1.2%	6.4%	15.8%	26.1%	22.0%	-21.2%	1.7%	13.8%	2.8%	-0.5%	-4.9%	0.7%	3.1%	-1.7%	0.7%	0.2%	32.9%	-21.2%	3.7%	54.0%	0.1160
1992	3.8%	-13.0%	-2.4%	-0.3%	26.3%	-5.3%	2.5%	-1.8%	-9.6%	2.9%	-11.8%	-5.7%	15.4%	3.1%	-13.1%	1.3%	-0.7%	-7.7%	-1.0%	-10.2%	-6.3%	7.2%	-6.7%	-0.9%	-7.0%	26.3%	-13.1%	-1.6%	39.4%	0.0889
1993	1.0%	-6.4%	-6.8%	-2.6%	3.2%	-4.7%	1.9%	-5.9%	0.5%	0.4%	-13.5%	-11.6%	0.3%	-4.4%	-14.6%	-9.0%	-2.5%	-0.5%	-0.6%	-8.5%	-0.3%	-4.5%	-10.0%	2.9%	-8.6%	3.2%	-14.6%	-4.2%	17.9%	0.0515
1994	0.5%	-11.8%	3.6%	-5.8%	-0.2%	-1.2%	-1.3%	4.3%	-1.5%	-4.9%	-2.5%	-13.2%	10.4%	-12.2%	-19.2%	-1.3%	-4.3%	-9.0%	-4.2%	0.9%	-4.7%	0.1%	-11.9%	0.4%	-9.3%	10.4%	-19.2%	-3.9%	29.6%	0.0654
1995	1.2%	-9.3%	-3.7%	-2.7%	1.5%	-9.4%	-1.7%	2.7%	-13.8%	-8.9%	6.3%	-4.3%	-9.4%	-16.5%	-22.3%	-2.5%	1.1%	-5.5%	-1.8%	4.5%	-8.3%	1.2%	-11.5%	-3.7%	-8.2%	6.3%	-22.3%	-5.0%	28.6%	0.0683
1996	4.7%	4.1%	0.1%	-6.7%	-5.5%	-1.1%	5.8%	3.8%	-10.5%	-7.4%	3.4%	0.3%	-19.9%	-3.2%	-25.6%	-2.1%	11.0%	-9.3%	-3.9%	-4.6%	-5.7%	-1.4%	5.3%	-1.3%	-3.0%	11.0%	-25.6%	-2.9%	36.6%	0.0793
1997	5.6%	5.0%	-0.4%	0.8%	4.7%	-3.2%	0.7%	0.0%	-9.6%	-9.5%	5.2%	3.9%	0.2%	-2.4%	-21.2%	-0.6%	3.5%	9.3%	7.8%	1.0%	-2.6%	3.3%	4.8%	-1.4%	-2.8%	9.3%	-21.2%	0.1%	30.5%	0.0640
1998	7.6%	-9.8%	-1.6%	4.9%	-12.9%	4.3%	9.0%	-0.4%	-1.4%	8.7%	4.7%	-1.7%	2.1%	2.0%	-38.6%	2.7%	1.9%	-2.4%	4.1%	5.1%	-8.5%	8.1%	2.1%	-0.3%	-0.8%	9.0%	-38.6%	-0.4%	47.5%	0.0966
1999	2.4%	-2.2%	0.9%	5.7%	-1.5%	4.3%	-6.4%	5.2%	-9.3%	-9.2%	-0.4%	-4.4%	4.5%	-2.1%	-27.3%	3.7%	20.7%	11.2%	4.7%	12.5%	-4.3%	12.6%	8.1%	0.3%	14.4%	20.7%	-27.3%	1.8%	48.0%	0.0960
2000	5.4%	4.2%	9.7%	4.4%	1.8%	5.1%	2.7%	-2.8%	-6.8%	8.2%	9.2%	3.8%	6.3%	4.2%	-10.6%	10.3%	-3.0%	19.1%	1.1%	-1.1%	-3.1%	7.1%	8.4%	-0.7%	21.3%	21.3%	-10.6%	4.2%	31.9%	0.0713
2001	-3.6%	1.9%	5.5%	-3.2%	7.4%	3.2%	5.5%	4.6%	-2.9%	-4.9%	11.4%	2.9%	5.6%	6.2%	-7.3%	-2.0%	-6.9%	6.9%	10.6%	-7.1%	-6.2%	-1.7%	-1.6%	0.5%	0.4%	11.4%	-7.3%	1.0%	18.7%	0.0558
2002	11.7%	16.1%	-0.9%	-3.5%	1.6%	1.2%	1.0%	4.2%	-6.9%	-1.6%	1.1%	-4.4%	12.4%	2.2%	0.4%	12.7%	0.9%	3.0%	1.7%	13.2%	2.9%	6.5%	-5.6%	-1.4%	-1.5%	16.1%	-6.9%	2.7%	22.9%	0.0616
2003	-1.8%	-1.1%	-0.9%	-15.5%	2.5%	2.5%	-14.8%	-6.8%	-0.9%	-8.7%	-2.4%	-6.8%	-5.9%	-1.2%	0.2%	-0.6%	-10.7%	-5.7%	0.8%	-11.3%	-8.7%	-6.5%	-10.6%	-2.5%	-3.4%	2.5%	-15.5%	-4.8%	18.0%	0.0511
2004	-7.3%	-6.9%	-2.6%	-5.5%	7.0%	1.3%	5.4%	-15.1%	-3.4%	-8.1%	-13.9%	-6.4%	-14.8%	6.4%	-32.0%	-13.2%	-12.4%	-2.2%	-2.2%	-16.1%	-12.3%	-4.4%	-8.0%	-0.6%	0.1%	7.0%	-32.0%	-6.7%	39.0%	0.0855
2005	-8.4%	-0.5%	-2.3%	-11.7%	1.1%	-3.0%	-20.0%	-23.1%	-1.4%	-4.2%	-4.9%	-6.8%	-18.1%	1.5%	-27.1%	-8.3%	-5.4%	-7.8%	4.5%	-11.1%	-3.4%	-11.0%	-7.8%	-3.0%	-1.8%	4.5%	-27.1%	-7.3%	31.5%	0.0781
2006	-4.8%	-8.5%	-7.0%	-8.1%	6.0%	-0.2%	-10.0%	-10.2%	-8.5%	-3.9%	-0.1%	-10.4%	2.6%	-2.8%	-35.0%	-11.5%	-11.2%	-7.2%	0.9%	-7.5%	-3.6%	-11.7%	-9.0%	-0.5%	6.6%	6.6%	-35.0%	-6.2%	41.6%	0.0806
2007	-0.8%	3.2%	-3.6%	-1.4%	9.8%	-1.6%	-9.0%	11.1%	-5.6%	-2.1%	-0.2%	-4.8%	-8.2%	-1.6%	-22.5%	-3.8%	-3.4%	2.2%	2.2%	-9.2%	-0.4%	4.5%	-2.2%	4.3%	-3.1%	11.1%	-22.5%	-1.8%	33.5%	0.0662
2008	0.1%	2.7%	-1.1%	-1.1%	-2.0%	1.9%	6.8%	0.0%	3.7%	1.3%	8.0%	4.6%	-14.3%	2.4%	11.8%	-0.3%	13.3%	2.2%	2.5%	-5.1%	1.9%	5.7%	1.2%	7.9%	-1.3%	13.3%	-14.3%	2.1%	27.6%	0.0547
2009	-1.4%	14.4%	0.1%	0.8%	-0.4%	0.7%	-4.7%	3.4%	2.1%	-2.9%	-0.2%	-0.1%	-12.6%	-9.2%	13.2%	-1.7%	2.0%	-8.1%	1.3%	-8.3%	5.0%	7.1%	-1.4%	6.2%	3.1%	14.4%	-12.6%	0.3%	27.0%	0.0626
2010																										0.0%	0.0%		0.0%	
High	11.7%	16.1%	9.7%	5.7%	32.9%	5.1%	9.0%	11.1%	3.7%	8.7%	11.4%	15.8%	26.1%	22.0%	13.2%	12.7%	20.7%	19.1%	10.6%	13.2%	5.0%	12.6%	8.4%	7.9%	21.3%					
Low	-8.4%	-13.0%	-7.0%	-15.5%	-12.9%	-9.7%	-20.0%	-23.1%	-13.8%	-9.5%	-13.9%	-13.2%	-19.9%	-16.5%	-38.6%	-13.2%	-12.4%	-9.3%	-4.2%	-16.1%	-12.3%	-11.7%	-11.9%	-3.7%	-9.3%					
Average	1.1%	-0.6%	-0.7%	-2.5%	4.4%	-0.8%	-1.4%	-1.3%	-5.2%	-2.8%	0.3%	-2.6%	-0.9%	-0.3%	-16.4%	-1.3%	0.4%	-0.5%	1.5%	-3.6%	-3.6%	1.3%	-3.0%	0.4%	-0.2%					
Range	20.1%	29.1%	16.6%	21.3%	45.8%	14.7%	28.9%	34.1%	17.5%	18.2%	25.4%	29.1%	46.0%	38.4%	51.8%	25.9%	33.1%	28.4%	14.9%	29.3%	17.2%	24.3%	20.3%	11.6%	30.6%					
Std Dev	0.0508	0.0826	0.0390	0.0561	0.1026	0.0429	0.0771	0.0806	0.0515	0.0553	0.0735	0.0686	0.1239	0.0803	0.1470	0.0645	0.0898	0.0790	0.0374	0.0800	0.0444	0.0668	0.0665	0.0301	0.0759					

Calendar Year Change in Accident Year Ultimate Losses between 12 and 24 Month **Exhibit 7**
By Group/Company **CMP**
Commercial Multiple Peril

12 to 24 Months of Maturity Change in Ultimate Losses as a Percent of 12 Month Ultimat

Accident	Group/Company																								
Year	1	2	3	4	5	6	7	9	17	19	20	21	22	23	25	28	31	33	34	38	40	High	Low	Average	Range	Std Dev
1991	-1.6%	-3.9%	-2.3%	-4.0%	-0.4%	-3.0%	4.6%	13.1%	-15.3%	2.3%	9.9%	-16.7%	-0.7%	-10.6%	0.9%	-3.5%	4.0%	2.8%	0.0%	-1.2%	15.9%	15.9%	-16.7%	-0.5%	32.6%	0.0792
1992	-1.8%	0.1%	-7.7%	-11.1%	0.5%	-3.1%	1.3%	-5.7%	17.2%	4.9%	1.2%	-11.6%	-1.1%	-13.5%	3.5%	-1.7%	0.1%	20.7%	-0.3%	-2.7%	-4.1%	20.7%	-13.5%	-0.7%	34.2%	0.0814
1993	1.1%	-0.3%	-5.3%	-7.7%	-3.4%	-2.9%	-5.8%	-19.5%	0.9%	-11.9%	7.8%	-3.6%	5.0%	-3.8%	0.9%	1.6%	-4.5%	10.9%	-2.8%	-1.6%	-1.1%	10.9%	-19.5%	-2.2%	30.4%	0.0645
1994	0.5%	2.7%	2.9%	0.2%	-1.7%	-2.9%	-0.6%	-20.1%	-1.5%	4.0%	-3.6%	3.4%	-0.4%	6.9%	6.5%	-0.9%	-3.5%	-15.1%	-2.9%	0.4%	-2.8%	6.9%	-20.1%	-1.4%	27.0%	0.0627
1995	4.8%	2.8%	-4.9%	-5.2%	-2.0%	-2.2%	-2.2%	-54.9%	2.7%	-5.2%	-3.0%	7.7%	-1.3%	7.2%	7.3%	1.3%	-4.7%	21.8%	-1.0%	3.2%	-7.0%	21.8%	-54.9%	-1.7%	76.8%	0.1383
1996	5.6%	0.4%	-2.4%	-6.1%	3.0%	-1.6%	10.7%	-0.9%	-4.0%	-3.7%	-7.1%	-3.3%	0.2%	1.1%	4.9%	5.0%	11.4%	21.0%	-3.9%	0.4%	-4.9%	21.0%	-7.1%	1.2%	28.1%	0.0678
1997	-0.7%	-0.1%	17.0%	0.3%	0.3%	1.3%	-4.8%	0.9%	-4.1%	4.7%	-4.5%	-5.3%	-5.9%	-2.3%	-2.1%	-2.1%	-3.7%	43.5%	8.4%	1.6%	-6.6%	43.5%	-6.6%	1.7%	50.1%	0.1099
1998	2.5%	7.7%	-3.1%	1.1%	3.5%	3.0%	-1.3%	24.2%	-8.0%	-4.4%	-3.9%	-3.4%	-2.6%	3.3%	-2.4%	9.2%	17.3%	18.7%	2.0%	-3.4%	-5.3%	24.2%	-8.0%	2.6%	32.2%	0.0852
1999	-4.3%	0.8%	16.6%	0.6%	4.1%	-0.3%	-11.1%	33.9%	2.3%	5.9%	-3.8%	2.5%	-3.6%	-0.6%	0.1%	2.0%	0.2%	-3.8%	1.4%	-1.5%	3.3%	33.9%	-11.1%	2.1%	45.0%	0.0894
2000	-0.6%	5.4%	2.3%	3.0%	5.4%	0.4%	-4.5%	-22.2%	-3.7%	9.2%	3.7%	2.0%	2.8%	7.2%	10.3%	7.7%	15.8%	-10.4%	-1.0%	-0.8%	-1.9%	15.8%	-22.2%	1.4%	38.1%	0.0790
2001	5.1%	-1.2%	5.2%	1.4%	-9.3%	-0.5%	-0.2%	6.9%	-1.6%	5.5%	3.7%	-10.8%	-1.5%	1.3%	-1.9%	5.4%	-15.0%	-42.9%	-6.9%	-2.7%	-8.8%	6.9%	-42.9%	-3.3%	49.8%	0.1086
2002	-1.8%	0.3%	-4.3%	-3.3%	-7.1%	0.2%	6.0%	-14.6%	-1.3%	7.4%	1.3%	-13.7%	2.9%	-17.7%	-0.9%	-8.5%	11.9%	-2.4%	-1.2%	-4.7%	-12.0%	11.9%	-17.7%	-3.0%	29.6%	0.0740
2003	5.4%	-9.6%	-10.4%	-6.5%	-9.9%	-0.1%	-8.3%	-0.6%	-10.7%	-6.7%	1.9%	-1.8%	-4.7%	-17.6%	-4.7%	-8.3%	-32.3%	-10.3%	0.4%	-6.5%	-7.6%	5.4%	-32.3%	-7.1%	37.7%	0.0782
2004	-0.9%	-9.4%	-4.0%	-10.8%	-5.8%	6.8%	-7.7%	25.5%	-10.2%	-7.0%	-1.1%	-6.5%	-0.1%	-15.0%	-0.5%	-7.0%	-14.2%	-21.7%	-19.5%	-8.6%	-3.8%	25.5%	-21.7%	-5.8%	47.1%	0.0979
2005	-1.5%	-0.8%	-3.9%	7.3%	-2.1%	39.1%	1.2%	-27.1%	2.5%	1.6%	5.1%	-9.2%	0.1%	-14.1%	3.9%	0.3%	-28.3%	-7.5%	-20.7%	-11.1%	4.8%	39.1%	-28.3%	-2.9%	67.4%	0.1402
2006	8.8%	0.0%	-0.2%	-2.8%	-1.7%	4.1%	-2.1%	-33.2%	-11.3%	-4.8%	-1.2%	-7.2%	-3.6%	-7.9%	-2.1%	-2.3%	-10.5%	-0.8%	-11.2%	-4.4%	3.9%	8.8%	-33.2%	-4.3%	42.0%	0.0831
2007	-3.3%	-3.0%	-2.9%	2.7%	-2.0%	8.7%	2.5%	-1.4%	-9.0%	-6.4%	-6.0%	-4.0%	-1.3%	-8.5%	2.5%	-7.9%	-19.7%	-8.0%	-5.4%	-4.0%	1.3%	8.7%	-19.7%	-3.6%	28.4%	0.0579
2008	0.0%	-6.1%	-0.6%	3.8%	-2.9%	1.0%	10.5%	-14.2%	-3.4%	0.8%	-5.6%	-8.0%	-4.9%	-7.3%	1.8%	-3.0%	10.0%	6.7%	-9.4%	-3.6%	1.9%	10.5%	-14.2%	-1.5%	24.7%	0.0621
2009	2.5%	-1.9%	-4.3%	1.2%	0.2%	0.6%	0.1%	5.7%	8.8%	-7.4%	-8.5%	-1.2%	-6.1%	-4.2%	-7.0%	-3.3%	-1.4%	-4.6%	-21.4%	1.8%	0.5%	8.8%	-21.4%	-2.4%	30.2%	0.0613
2010																						0.0%	0.0%		0.0%	
High	8.8%	7.7%	17.0%	7.3%	5.4%	39.1%	10.7%	33.9%	17.2%	9.2%	9.9%	7.7%	5.0%	7.2%	10.3%	9.2%	17.3%	43.5%	8.4%	3.2%	15.9%					
Low	-4.3%	-9.6%	-10.4%	-11.1%	-9.9%	-3.1%	-11.1%	-54.9%	-15.3%	-11.9%	-8.5%	-16.7%	-6.1%	-17.7%	-7.0%	-8.5%	-32.3%	-42.9%	-21.4%	-11.1%	-12.0%					
Average	1.0%	-0.8%	-0.6%	-1.9%	-1.6%	2.6%	-0.6%	-5.5%	-2.6%	-0.6%	-0.7%	-4.8%	-1.4%	-5.1%	1.1%	-0.8%	-3.5%	1.0%	-5.0%	-2.6%	-1.8%					
Range	13.1%	17.2%	27.4%	18.4%	15.3%	42.2%	21.8%	88.8%	32.5%	21.1%	18.4%	24.4%	11.1%	24.9%	17.3%	17.7%	49.6%	86.4%	29.8%	14.4%	28.0%					
Std Dev	0.0351	0.0432	0.0713	0.0507	0.0423	0.0942	0.0588	0.2192	0.0765	0.0616	0.0513	0.0620	0.0299	0.0831	0.0429	0.0523	0.1403	0.1923	0.0813	0.0358	0.0630					

Calendar Year Change in Accident Year Ultimate Losses between 12 and 24 Months **Exhibit 7**
By Group/Company **MM**
Medical Professional Liability

12 to 24 Months of Maturity Change in Ultimate Losses as a Percent of 12 Month Ultimate

Accident	Group/Company																					
Year	9	12	13	20	22	24	26	27	28	29	30	31	32	33	34	35	36	37	High	Low	Average	Range	Std Dev
1991	0.3%	15.7%	-3.9%	85.1%	0.0%	-0.7%	-12.0%	-1.8%	1.3%	-5.6%	18.5%	84.0%	-1.1%	-11.2%		-3.0%	-0.2%	-12.0%	85.1%	-12.0%	9.0%	97.1%	0.2955
1992	-1.4%	14.8%	-1.0%	-25.1%	13.4%	0.7%	-19.4%	-1.0%	-13.9%	-5.0%	-12.9%	-10.9%	-5.0%	5.4%		-4.0%	1.0%	-5.8%	14.8%	-25.1%	-4.1%	39.9%	0.1039
1993	1.1%	-11.9%	1.4%	33.6%	-0.8%	5.9%	-14.9%	-6.1%	0.7%	-18.4%	-4.4%	62.6%	-17.4%	-10.9%	0.6%	-3.4%	-1.5%	2.1%	62.6%	-18.4%	1.0%	81.0%	0.1920
1994	1.6%	-1.4%	1.7%	9.9%	0.0%	0.4%	-5.6%	-7.7%	-3.7%	-0.3%	2.4%	33.7%	-7.4%	-7.8%	23.7%	-3.0%	-0.8%	-2.6%	33.7%	-7.8%	1.8%	41.4%	0.1079
1995	0.5%	-3.4%	2.4%	-10.3%	18.6%	2.0%	-6.4%	-4.0%	14.3%	-5.9%	6.4%	41.6%	-4.9%	9.7%	16.6%	-4.9%	3.4%	2.3%	41.6%	-10.3%	4.3%	51.9%	0.1245
1996	-3.4%	-5.9%	3.3%	-21.3%	-7.4%	1.1%	-7.9%	-2.4%	18.6%	-9.5%	4.9%	6.8%	-7.1%	12.4%	30.8%	-6.1%	0.4%	-13.3%	30.8%	-21.3%	-0.3%	52.1%	0.1214
1997	0.5%	-1.2%	0.0%	-5.4%	9.4%	3.5%	-2.8%	-1.6%	18.3%	-11.9%	0.2%	27.7%	-7.2%	-15.6%	5.6%	-3.2%	3.6%	2.3%	27.7%	-15.6%	1.2%	43.2%	0.1003
1998	1.7%	0.0%	0.0%	2.3%	75.7%	0.6%	-5.2%	-2.3%	10.5%	-3.4%	-0.7%	72.9%	-8.7%	5.9%	32.3%	-4.1%	3.7%	-3.0%	75.7%	-8.7%	9.9%	84.4%	0.2501
1999	-6.4%	-3.7%	5.0%	8.3%	122.5%	6.8%	-2.8%	-2.2%	26.0%	0.0%	6.3%	55.8%	-0.1%	19.0%	41.4%	2.3%	0.2%	8.7%	122.5%	-6.4%	16.0%	128.9%	0.3127
2000	-6.4%	1.4%	-6.1%	11.3%	-6.5%	1.1%	-6.4%	5.0%	4.9%	3.5%	7.6%	-57.4%	-4.6%	41.3%	19.4%	5.4%	0.4%	2.2%	41.3%	-57.4%	0.9%	98.7%	0.1851
2001	6.6%	-2.6%	11.3%	9.0%	-6.1%	3.5%	-2.5%	6.7%	1.1%	5.7%	3.4%	-14.0%	-0.9%	3.6%	65.0%	-1.3%	19.5%	6.0%	65.0%	-14.0%	6.3%	79.0%	0.1633
2002	-3.6%	-13.4%	5.3%	2.6%	-5.2%	15.5%	-1.2%	8.2%	25.2%	4.7%	-7.7%	-25.7%	0.0%	14.6%	11.4%	2.3%	8.6%	1.9%	25.2%	-25.7%	2.4%	50.9%	0.1153
2003	0.8%	-6.9%	8.8%	-13.7%	-3.4%	8.9%	4.7%	-4.2%	1.6%	3.3%	-14.7%	-9.3%	-7.4%	-18.4%	-22.0%	0.2%	10.4%	-4.7%	10.4%	-22.0%	-3.7%	32.5%	0.0948
2004	-0.1%	-4.4%	2.1%	-30.1%	-29.0%	-4.6%	-5.1%	-5.0%	-8.7%	2.5%	-24.0%	-12.3%	-8.2%	-11.2%	-2.7%	-1.5%	-6.3%	-3.9%	2.5%	-30.1%	-8.5%	32.5%	0.0974
2005	-4.6%	-4.1%	2.6%	-15.7%	-6.2%	-6.4%	-15.5%	-6.6%	-0.6%	-2.1%	-2.1%	-5.9%	-6.0%	-7.6%	-11.4%	-1.8%	-3.6%	-6.7%	2.6%	-15.7%	-5.8%	18.3%	0.0472
2006	-6.7%	-1.9%	4.7%	-27.9%	-33.0%	-11.0%	-8.1%	-5.2%	-0.1%	-4.5%	-5.3%	3.0%	-3.5%	-13.6%	-17.7%	0.3%	-10.4%	-7.7%	4.7%	-33.0%	-8.2%	37.7%	0.0988
2007	-6.1%	-15.3%	-5.2%	-26.6%	-3.5%	-2.1%	-15.0%	-4.3%	1.1%	-5.2%	-0.7%	-6.1%	-10.1%	-15.6%	-1.8%	0.6%	-7.6%	-3.6%	1.1%	-26.6%	-7.1%	27.7%	0.0710
2008	-5.0%	5.2%	-8.6%	-1.3%	-0.1%	0.6%	5.7%	-0.2%	2.2%	-14.3%	0.0%	-7.8%	-17.4%	-8.2%	-3.7%	1.7%	-12.5%	-2.9%	5.7%	-17.4%	-3.7%	23.1%	0.0658
2009	-4.9%	0.2%	-13.1%	-24.2%	-3.4%	-9.9%	-15.0%	-5.9%	-0.3%	-5.3%	-0.3%	0.7%	-9.4%	-4.0%	-14.2%	0.5%	-5.3%	-5.1%	0.7%	-24.2%	-6.6%	25.0%	0.0668
2010																			0.0%	0.0%		0.0%	
High	6.6%	15.7%	11.3%	85.1%	122.5%	15.5%	5.7%	8.2%	26.0%	5.7%	18.5%	84.0%	0.0%	41.3%	65.0%	5.4%	19.5%	8.7%					
Low	-6.7%	-15.3%	-13.1%	-30.1%	-33.0%	-11.0%	-19.4%	-7.7%	-13.9%	-18.4%	-24.0%	-57.4%	-17.4%	-18.4%	-22.0%	-6.1%	-12.5%	-13.3%					
Average	-1.9%	-2.0%	0.5%	-2.1%	7.1%	0.8%	-7.1%	-2.1%	5.2%	-3.8%	-1.2%	12.6%	-6.7%	-0.6%	10.2%	-1.2%	0.2%	-2.4%					
Range	13.4%	31.0%	24.5%	115.2%	155.5%	26.5%	25.1%	15.8%	39.8%	24.1%	42.4%	141.4%	17.4%	59.7%	87.0%	11.4%	32.0%	22.0%					
Std Dev	0.0369	0.0792	0.0593	0.2719	0.3541	0.0626	0.0683	0.0442	0.1083	0.0655	0.0932	0.3705	0.0491	0.1524	0.2330	0.0294	0.0748	0.0568					

Calendar Year Change in Accident Year Ultimate Losses between 12 and 36 Months **Exhibit 8**
By Group/Company **HMP/FMP**
Homeowners/Farmowners

12 to 36 Months of Maturity Change in Ultimate Losses as a Percent of 12 Month Ultimate

Accident	Group/Company																			
Year	1	2	3	4	5	6	15	18	19	20	21	25	33	38	40	High	Low	Average	Range	Std Dev
1991	0.5%	-3.7%	-1.3%	1.1%	-0.6%	-0.8%	-2.3%	-1.3%	-1.0%	-8.7%	-2.9%	-4.5%	-2.7%	-5.1%	-5.7%	1.1%	-8.7%	-2.6%	9.8%	0.0260
1992	-6.7%	0.4%	-1.1%	1.9%	-2.9%	3.9%	-18.5%	-5.2%	-0.2%	-1.9%	-6.6%	-3.2%	-2.9%	-2.0%	-6.1%	3.9%	-18.5%	-3.4%	22.4%	0.0520
1993	-0.7%	-5.6%	-0.9%	-0.2%	-1.7%	2.9%	1.2%	-5.8%	-1.9%	-0.4%	-5.8%	-2.6%	-12.0%	-6.6%	-3.1%	2.9%	-12.0%	-2.9%	14.9%	0.0375
1994	-0.3%	-1.4%	-3.0%	-4.5%	-2.2%	0.1%	0.3%	-7.0%	1.4%	-4.4%	-8.5%	0.0%	-10.4%	-1.2%	-1.5%	1.4%	-10.4%	-2.8%	11.7%	0.0348
1995	0.7%	0.9%	-7.5%	1.8%	-1.8%	1.0%	0.2%	-12.4%	0.4%	0.0%	-4.2%	3.3%	-25.0%	-0.8%	-3.6%	3.3%	-25.0%	-3.1%	28.3%	0.0726
1996	-1.1%	-2.1%	-3.4%	0.3%	1.5%	0.8%	-1.5%	-10.4%	1.9%	1.6%	-4.8%	1.0%	-42.4%	-1.3%	1.6%	1.9%	-42.4%	-3.9%	44.3%	0.1113
1997	-7.1%	-4.7%	0.7%	-2.6%	-0.8%	-0.6%	-13.9%	-8.5%	-1.5%	1.7%	-4.5%	-3.2%	-42.1%	-7.3%	-3.8%	1.7%	-42.1%	-6.6%	43.8%	0.1062
1998	-1.5%	3.7%	3.5%	7.5%	1.6%	2.8%	-4.9%	0.4%	0.9%	-0.3%	-1.5%	-1.8%	-38.0%	-2.0%	-1.9%	7.5%	-38.0%	-2.1%	45.5%	0.1039
1999	-1.8%	0.7%	-2.0%	-2.0%	2.0%	0.5%	-6.9%	0.2%	2.6%	-2.1%	-4.8%	0.9%	-32.9%	2.4%	1.7%	2.6%	-32.9%	-2.8%	35.4%	0.0875
2000	9.4%	13.4%	8.2%	4.1%	6.6%	5.8%	5.7%	4.9%	5.5%	3.3%	3.3%	9.1%	-12.9%	4.0%	9.5%	13.4%	-12.9%	5.3%	26.3%	0.0578
2001	7.1%	14.4%	11.9%	3.2%	4.6%	1.9%	-4.2%	6.0%	3.7%	-2.9%	1.3%	0.4%	-12.6%	-0.6%	2.9%	14.4%	-12.6%	2.5%	27.0%	0.0648
2002	-2.1%	4.5%	-6.4%	-2.7%	-5.9%	-4.3%	-3.4%	-6.7%	-7.3%	-2.3%	-5.1%	-1.9%	-6.9%	-19.7%	-4.4%	4.5%	-19.7%	-5.0%	24.2%	0.0504
2003	-3.6%	-7.1%	-13.8%	-4.7%	-3.2%	-1.9%	-3.3%	-5.9%	-5.6%	-2.3%	-4.4%	-6.4%	-4.0%	-17.2%	-6.5%	-1.9%	-17.2%	-6.0%	15.3%	0.0421
2004	-5.5%	-4.5%	-4.5%	0.0%	-7.4%	-3.3%	-2.9%	-0.2%	-3.8%	-1.7%	-6.6%	1.8%	-0.6%	1.4%	-7.4%	1.8%	-7.4%	-3.0%	9.2%	0.0305
2005	-5.6%	-5.5%	-0.7%	-2.1%	0.2%	3.4%	-3.4%	-3.1%	-7.9%	0.3%	-2.7%	1.1%	-9.9%	-14.3%	-3.5%	3.4%	-14.3%	-3.6%	17.7%	0.0457
2006	1.2%	-7.1%	-2.5%	3.2%	-2.5%	2.4%	0.8%	-2.7%	-3.7%	-5.7%	-0.2%	-0.5%	-36.2%	-3.2%	1.1%	3.2%	-36.2%	-3.7%	39.4%	0.0946
2007	-2.9%	-8.9%	-2.9%	0.8%	-6.1%	0.8%	-6.4%	-6.8%	-4.0%	-1.5%	0.8%	-3.4%	-26.4%	-6.4%	-3.3%	0.8%	-26.4%	-5.1%	27.3%	0.0661
2008	-2.4%	-0.4%	-3.4%	6.2%	0.4%	-1.7%	-1.2%	-2.9%	0.2%	-3.6%	1.8%	1.6%	9.9%	-1.4%	-2.8%	9.9%	-3.6%	0.0%	13.5%	0.0374
2009																0.0%	0.0%		0.0%	
2010																0.0%	0.0%		0.0%	
High	9.4%	14.4%	11.9%	7.5%	6.6%	5.8%	5.7%	6.0%	5.5%	3.3%	3.3%	9.1%	9.9%	4.0%	9.5%					
Low	-7.1%	-8.9%	-13.8%	-4.7%	-7.4%	-4.3%	-18.5%	-12.4%	-7.9%	-8.7%	-8.5%	-6.4%	-42.4%	-19.7%	-7.4%					
Average	-1.2%	-0.7%	-1.6%	0.6%	-1.0%	0.8%	-3.6%	-3.8%	-1.1%	-1.7%	-3.1%	-0.5%	-17.1%	-4.5%	-2.0%					
Range	16.6%	23.3%	25.7%	12.2%	14.0%	10.1%	24.3%	18.4%	13.3%	12.0%	11.8%	15.5%	52.3%	23.7%	17.0%					
Std Dev	0.0427	0.0650	0.0562	0.0342	0.0359	0.0261	0.0554	0.0487	0.0368	0.0284	0.0332	0.0347	0.1593	0.0657	0.0413					

Calendar Year Change in Accident Year Ultimate Losses between 12 and 36 Months **Exhibit 8**
By Group/Company **PPAL**
Private Passenger Auto Liability

12 to 36 Months of Maturity Change in Ultimate Losses as a Percent of 12 Month Ultimate

Accident	Group/Company																					
Year	1	2	3	4	5	6	8	9	15	18	19	20	21	25	33	38	40	High	Low	Average	Range	Std Dev
1991	-5.4%	-8.3%	-6.1%	-10.1%	-1.9%	-4.5%	-10.5%	-6.0%	-6.6%	-4.5%	-4.9%	-10.0%	-3.7%	-9.5%	-8.6%	-8.7%	0.0%	0.0%	-10.5%	-6.4%	10.5%	0.0301
1992	-4.5%	-9.8%	-10.2%	-5.0%	-9.4%	-5.1%	-14.4%	-6.0%	-14.1%	-8.4%	-5.9%	-13.0%	-15.2%	-10.3%	-8.4%	-7.0%	-3.0%	-3.0%	-15.2%	-8.8%	12.2%	0.0375
1993	-6.7%	-1.7%	-7.0%	-0.1%	-8.9%	-5.9%	-6.8%	-7.8%	-0.6%	-12.9%	-5.0%	-1.8%	-21.2%	-8.8%	-5.0%	-3.2%	-3.0%	-0.1%	-21.2%	-6.3%	21.1%	0.0511
1994	-6.4%	-6.1%	-6.5%	3.2%	-6.7%	-7.3%	-1.7%	-6.5%	-4.7%	-25.7%	-5.5%	-1.7%	-18.1%	-4.8%	-1.9%	-4.2%	1.8%	3.2%	-25.7%	-6.0%	28.9%	0.0682
1995	-4.1%	-6.2%	-4.3%	0.8%	-7.8%	-9.7%	-5.6%	-5.3%	-5.2%	-10.4%	-5.6%	-1.8%	-16.8%	-1.8%	-14.7%	-8.0%	1.3%	1.3%	-16.8%	-6.2%	18.0%	0.0483
1996	-4.9%	-0.3%	-2.5%	-1.0%	-4.9%	-7.9%	-6.9%	-7.2%	3.8%	-21.8%	-1.2%	-2.3%	-14.8%	0.7%	-14.5%	-3.2%	-4.3%	3.8%	-21.8%	-5.5%	25.6%	0.0643
1997	-4.0%	3.9%	-3.1%	-1.4%	-5.2%	-4.6%	-5.3%	-7.8%	2.2%	1.7%	-3.9%	-1.8%	-13.9%	-1.4%	-22.8%	-3.7%	-4.5%	3.9%	-22.8%	-4.4%	26.7%	0.0621
1998	-0.1%	2.6%	0.2%	0.7%	-1.3%	-3.8%	1.0%	-3.4%	-6.6%	4.6%	1.0%	-4.0%	-7.9%	-1.7%	2.2%	-0.9%	6.0%	6.0%	-7.9%	-0.7%	13.8%	0.0367
1999	1.0%	8.9%	0.0%	-1.2%	-0.9%	1.1%	4.5%	-2.5%	-4.2%	7.0%	6.5%	3.8%	-8.4%	-3.3%	-4.5%	1.9%	8.2%	8.9%	-8.4%	1.1%	17.3%	0.0491
2000	2.0%	-3.4%	-0.5%	2.9%	4.2%	1.3%	-2.5%	-4.2%	-8.4%	2.7%	9.6%	0.3%	4.6%	1.0%	18.2%	-0.2%	12.4%	18.2%	-8.4%	2.4%	26.6%	0.0637
2001	0.3%	-0.9%	2.4%	1.3%	2.6%	-2.8%	-1.7%	-6.3%	-13.0%	1.8%	5.2%	0.3%	7.1%	-5.2%	7.4%	-5.7%	-6.1%	7.4%	-13.0%	-0.8%	20.4%	0.0534
2002	-3.8%	3.3%	8.4%	-4.9%	-1.8%	-5.4%	-0.4%	-3.2%	-11.5%	-1.5%	-0.9%	3.1%	9.7%	0.2%	4.9%	-5.6%	0.6%	9.7%	-11.5%	-0.5%	21.2%	0.0532
2003	-7.7%	2.0%	0.7%	-8.9%	-3.1%	-5.6%	-2.5%	-8.7%	-10.5%	1.8%	-5.6%	3.6%	11.2%	-1.8%	6.1%	-14.7%	-0.3%	11.2%	-14.7%	-2.6%	26.0%	0.0659
2004	-8.4%	1.7%	-1.5%	-9.5%	-6.1%	-3.9%	-4.4%	-8.9%	-11.3%	1.0%	-6.7%	5.4%	6.9%	-4.0%	-6.7%	-17.5%	-8.9%	6.9%	-17.5%	-4.9%	24.4%	0.0618
2005	-6.9%	-8.0%	-2.2%	-3.1%	-7.5%	-1.8%	-1.2%	-9.1%	-14.3%	-1.0%	-14.2%	6.5%	1.8%	-4.7%	-9.1%	-11.3%	-3.4%	6.5%	-14.3%	-5.3%	20.8%	0.0556
2006	-0.3%	-7.5%	-1.6%	-2.0%	-2.3%	1.7%	2.2%	-5.9%	-7.2%	-4.0%	-12.5%	3.1%	-0.9%	-0.3%	-1.6%	-6.1%	-10.0%	3.1%	-12.5%	-3.2%	15.5%	0.0436
2007	1.2%	-8.5%	-2.4%	-4.9%	-4.7%	1.3%	0.9%	-2.0%	-3.4%	-4.9%	-5.8%	3.9%	0.1%	1.7%	-1.8%	3.2%	-6.2%	3.9%	-8.5%	-1.9%	12.4%	0.0362
2008	1.9%	-11.7%	-6.6%	-2.8%	-3.5%	-0.6%	-0.8%	-4.0%	-6.7%	-7.6%	-1.1%	-5.1%	5.9%	-1.0%	4.7%	-0.2%	6.2%	6.2%	-11.7%	-1.9%	17.9%	0.0489
2009																		0.0%	0.0%		0.0%	
2010																		0.0%	0.0%		0.0%	
High	2.0%	8.9%	8.4%	3.2%	4.2%	1.7%	4.5%	-2.0%	3.8%	7.0%	9.6%	6.5%	11.2%	1.7%	18.2%	3.2%	12.4%					
Low	-8.4%	-11.7%	-10.2%	-10.1%	-9.4%	-9.7%	-14.4%	-9.1%	-14.3%	-25.7%	-14.2%	-13.0%	-21.2%	-10.3%	-22.8%	-17.5%	-10.0%					
Average	-3.2%	-2.8%	-2.4%	-2.6%	-3.8%	-3.5%	-3.1%	-5.8%	-6.8%	-4.6%	-3.1%	-0.6%	-4.1%	-3.1%	-3.1%	-5.3%	-0.7%					
Range	10.4%	20.6%	18.6%	13.3%	13.6%	11.5%	18.9%	7.1%	18.1%	32.7%	23.9%	19.5%	32.4%	12.0%	41.0%	20.8%	22.4%					
Std Dev	0.0355	0.0576	0.0420	0.0401	0.0372	0.0342	0.0463	0.0221	0.0523	0.0879	0.0603	0.0517	0.1064	0.0361	0.0960	0.0546	0.0600					

Calendar Year Change in Accident Year Ultimate Losses between 12 and 36 Months **Exhibit 8**
By Group/Company **CAL**
Commercial Auto Liability

12 to 36 Months of Maturity Change in Ultimate Losses as a Percent of 12 Month Ultimate

Accident	Group/Company																												
Year	1	2	3	4	5	6	7	8	9	14	15	17	19	20	21	22	23	25	28	31	33	34	38	40	High	Low	Average	Range	Std Dev
1991	-7.3%	-6.7%	-10.5%	-4.6%	-8.3%	-0.3%	-15.8%	1.2%	-14.8%	-4.5%	21.8%	-9.6%	22.8%	-7.3%	16.6%	2.6%	11.3%	-13.0%	-4.3%	-16.9%	-2.2%	-2.4%	-12.5%	2.2%	22.8%	-16.9%	-2.6%	39.7%	0.1107
1992	4.7%	2.4%	-23.4%	-7.5%	-10.5%	0.1%	-19.9%	-23.6%	-13.7%	-4.6%	7.4%	5.1%	1.7%	6.3%	10.4%	3.4%	16.6%	-3.1%	-6.6%	-8.5%	-8.5%	2.4%	-8.1%	4.4%	16.6%	-23.6%	-3.0%	40.2%	0.1040
1993	2.7%	-1.0%	-17.4%	-5.1%	-6.9%	-0.1%	-14.5%	-13.1%	-1.5%	-10.4%	11.2%	-9.7%	-3.3%	11.5%	18.4%	4.9%	-5.0%	-2.6%	-1.5%	-9.3%	-5.2%	3.7%	-2.6%	7.9%	18.4%	-17.4%	-2.0%	35.8%	0.0868
1994	0.4%	-0.5%	-18.6%	0.2%	-1.8%	-2.9%	-13.1%	-13.1%	-14.3%	-10.2%	3.9%	-6.5%	-3.5%	7.8%	4.5%	-0.4%	-4.7%	0.3%	-1.0%	-8.2%	19.2%	6.5%	-1.8%	10.6%	19.2%	-18.6%	-2.0%	37.9%	0.0856
1995	-5.7%	0.9%	-19.7%	-3.1%	-4.6%	-7.2%	-13.4%	-5.9%	-70.1%	-11.4%	23.8%	-17.0%	2.4%	9.0%	-1.8%	-6.1%	-3.9%	-5.2%	6.4%	-8.3%	16.3%	4.8%	4.5%	4.2%	23.8%	-70.1%	-4.6%	93.9%	0.1708
1996	-0.2%	15.6%	-6.8%	-6.8%	5.3%	-2.8%	-8.3%	-12.2%	-8.9%	-8.1%	28.8%	3.9%	-0.2%	2.0%	2.6%	1.9%	4.2%	-1.4%	5.8%	13.3%	21.1%	6.2%	0.5%	4.8%	28.8%	-12.2%	2.5%	40.9%	0.0974
1997	6.6%	15.7%	-6.1%	-1.7%	15.5%	3.6%	6.2%	-8.4%	4.3%	-3.9%	25.7%	0.4%	-2.0%	13.9%	2.7%	8.8%	8.2%	11.1%	4.3%	6.3%	-7.3%	6.9%	-0.2%	4.4%	25.7%	-8.4%	4.8%	34.1%	0.0804
1998	3.5%	32.3%	20.0%	5.9%	7.4%	6.2%	0.7%	-4.9%	7.3%	-11.1%	8.2%	5.3%	11.3%	30.0%	5.7%	5.9%	-0.8%	10.8%	10.8%	10.3%	12.0%	8.2%	2.3%	5.1%	32.3%	-11.1%	8.0%	43.4%	0.0937
1999	5.1%	13.1%	6.0%	2.0%	16.7%	11.7%	-1.8%	-3.0%	-3.4%	-10.6%	10.9%	15.7%	10.1%	25.4%	13.1%	15.4%	6.9%	5.8%	9.2%	17.5%	5.0%	6.5%	11.8%	10.9%	25.4%	-10.6%	8.3%	36.1%	0.0793
2000	4.3%	-2.3%	-4.2%	-0.8%	6.4%	14.6%	-12.8%	0.7%	9.9%	5.0%	5.7%	-5.7%	22.6%	10.7%	19.5%	2.5%	6.4%	7.2%	3.9%	22.0%	-4.1%	5.1%	11.3%	12.7%	22.6%	-12.8%	5.9%	35.4%	0.0875
2001	0.3%	5.6%	-13.4%	-0.6%	-10.9%	3.6%	-7.5%	8.3%	13.9%	5.5%	-18.2%	1.3%	9.7%	-1.1%	27.6%	1.0%	14.6%	-0.5%	11.2%	35.1%	-34.5%	12.9%	9.0%	4.9%	35.1%	-34.5%	3.3%	69.6%	0.1423
2002	8.5%	-1.4%	-12.7%	-5.2%	-13.1%	3.7%	-20.1%	-1.3%	-12.3%	-4.3%	-10.7%	-8.1%	2.1%	-8.3%	17.0%	10.4%	11.5%	-3.2%	-1.1%	21.4%	20.4%	0.9%	-1.8%	-6.9%	21.4%	-20.1%	-0.6%	41.5%	0.1091
2003	-0.2%	3.0%	-11.6%	-7.4%	-11.0%	9.9%	-18.8%	-5.1%	-24.7%	1.6%	16.1%	-5.4%	-4.1%	-4.5%	17.7%	6.2%	-3.4%	-9.8%	-4.7%	-28.9%	3.3%	3.8%	-6.0%	-10.0%	17.7%	-28.9%	-3.9%	46.6%	0.1112
2004	-6.2%	-10.8%	-12.4%	-1.1%	-11.5%	16.7%	-9.3%	-4.6%	-6.0%	2.1%	-0.3%	-3.8%	-6.4%	-3.4%	18.4%	9.9%	-16.5%	3.8%	3.0%	-6.9%	-35.6%	2.8%	-19.6%	-13.0%	18.4%	-35.6%	-4.6%	54.0%	0.1149
2005	-7.0%	-7.2%	-5.6%	5.7%	-7.6%	25.9%	-5.6%	3.7%	-10.1%	10.7%	-20.3%	-0.9%	-19.3%	-8.5%	5.6%	-2.5%	-13.0%	3.9%	0.7%	11.7%	-11.8%	-0.1%	-13.6%	1.3%	25.9%	-20.3%	-2.7%	46.2%	0.1049
2006	-4.6%	2.3%	-16.4%	-3.7%	-4.4%	21.1%	-8.8%	6.5%	-8.2%	-1.0%	-14.4%	-3.0%	-16.4%	-3.2%	8.1%	0.4%	-13.4%	-0.2%	-0.9%	-1.2%	-12.1%	-0.9%	-10.5%	2.0%	21.1%	-16.4%	-3.4%	37.5%	0.0854
2007	-4.8%	-16.1%	-14.4%	3.3%	-7.6%	22.2%	-10.8%	1.7%	-6.4%	5.0%	14.3%	-4.2%	-25.7%	-4.5%	5.5%	-0.6%	-12.9%	-1.7%	1.2%	-2.0%	6.0%	-3.5%	-12.4%	-0.1%	22.2%	-25.7%	-2.9%	47.9%	0.1011
2008	-5.8%	-8.3%	2.6%	-6.2%	-9.3%	9.6%	-7.0%	-3.0%	-8.2%	14.4%	25.5%	-12.6%	-9.4%	-9.2%	9.3%	-0.2%	-7.8%	-7.0%	4.5%	18.8%	13.0%	-9.2%	-6.3%	3.6%	25.5%	-12.6%	-0.3%	38.1%	0.1046
2009																									0.0%	0.0%		0.0%	
2010																									0.0%	0.0%		0.0%	
High	8.5%	32.3%	20.0%	5.9%	16.7%	25.9%	6.2%	8.3%	13.9%	14.4%	28.8%	15.7%	22.8%	30.0%	27.6%	15.4%	16.6%	11.1%	11.2%	35.1%	21.1%	12.9%	11.8%	12.7%					
Low	-7.3%	-16.1%	-23.4%	-7.5%	-13.1%	-7.2%	-20.1%	-23.6%	-70.1%	-11.4%	-20.3%	-17.0%	-25.7%	-9.2%	-1.8%	-6.1%	-16.5%	-13.0%	-6.6%	-28.9%	-35.6%	-9.2%	-19.6%	-13.0%					
Average	-0.3%	2.0%	-9.1%	-2.0%	-3.1%	7.5%	-10.0%	-4.2%	-9.3%	-2.0%	7.7%	-3.0%	-0.4%	3.7%	11.1%	3.5%	-0.1%	-0.3%	2.3%	3.7%	-0.3%	3.0%	-3.1%	2.7%					
Range	15.8%	48.4%	43.4%	13.4%	29.8%	33.1%	26.3%	31.9%	84.0%	25.8%	49.1%	32.7%	48.5%	39.2%	29.4%	21.5%	33.1%	24.1%	17.8%	64.0%	56.8%	22.1%	31.4%	25.7%					
Std Dev	0.0515	0.1147	0.1049	0.0424	0.0934	0.0956	0.0706	0.0788	0.1799	0.0790	0.1543	0.0766	0.1293	0.1156	0.0773	0.0524	0.1042	0.0653	0.0522	0.1630	0.1680	0.0504	0.0890	0.0682					

Calendar Year Change in Accident Year Ultimate Losses between 12 and 36 Months
By Group/Company
Workers Compensation

Exhibit 8
WC

12 to 36 Months of Maturity Change in Ultimate Losses as a Percent of 12 Month Ultimate

Accident	Group/Company																													
Year	2	3	4	5	6	7	9	10	11	15	16	17	19	20	21	22	23	25	28	31	33	34	38	39	40	High	Low	Average	Range	Std Dev
1991	10.1%	-2.6%	6.4%	0.2%	25.6%	-8.6%	-0.3%	1.3%	-29.7%	-2.1%	6.2%	9.7%	17.3%	14.8%	-27.2%	-1.1%	8.6%	-0.4%	-5.6%	-6.2%	0.5%	4.2%	-7.3%	2.5%	0.6%	25.6%	-29.7%	0.7%	55.3%	0.1185
1992	1.9%	-19.1%	-2.2%	-10.3%	13.4%	-3.7%	3.9%	-1.8%	-23.2%	5.1%	-20.8%	-10.8%	6.2%	2.4%	-21.0%	-5.9%	-5.8%	-15.9%	-8.4%	-18.1%	-10.4%	-3.7%	-15.2%	-3.7%	-13.2%	13.4%	-23.2%	-7.2%	36.6%	0.0968
1993	-0.4%	-10.8%	-8.1%	-10.3%	-2.4%	-10.0%	0.1%	-11.1%	-2.9%	-2.6%	-22.2%	-9.4%	-7.0%	-7.0%	-29.5%	-12.9%	-3.4%	-6.6%	-9.3%	14.9%	-1.8%	-10.4%	-17.3%	2.3%	-12.6%	14.9%	-29.5%	-7.6%	44.4%	0.0850
1994	-7.1%	-24.9%	3.4%	-9.7%	-2.7%	-15.9%	-3.3%	4.9%	-16.0%	-14.5%	-4.0%	-11.1%	-10.5%	-24.3%	-28.9%	-4.0%	-9.0%	-15.8%	-9.9%	-0.8%	-6.3%	-3.3%	-17.9%	-4.2%	-15.4%	4.9%	-28.9%	-10.1%	33.8%	0.0857
1995	-3.4%	-17.0%	-5.5%	-6.0%	-2.2%	-11.3%	9.9%	6.3%	-27.7%	-11.4%	6.2%	-12.0%	-27.5%	-23.3%	-35.0%	-5.7%	2.2%	-13.9%	-10.8%	4.6%	-14.7%	0.5%	-13.1%	-4.8%	-8.6%	9.9%	-35.0%	-9.0%	44.9%	0.1134
1996	1.6%	4.2%	-1.1%	-7.4%	-8.3%	-3.3%	10.6%	3.6%	-23.6%	-7.9%	3.8%	-3.8%	-26.3%	-5.3%	-42.9%	-5.3%	9.8%	-7.7%	-3.9%	-0.4%	-11.7%	0.3%	-3.5%	-4.3%	-9.6%	10.6%	-42.9%	-5.7%	53.5%	0.1144
1997	1.7%	1.5%	-1.2%	4.2%	6.3%	-3.8%	11.0%	2.8%	-19.0%	-4.4%	8.3%	5.3%	-2.2%	-7.6%	-29.8%	2.1%	7.9%	5.8%	6.5%	10.0%	-0.5%	6.1%	5.5%	-1.6%	1.9%	11.0%	-29.8%	0.7%	40.8%	0.0899
1998	6.9%	12.9%	-0.9%	8.1%	-18.7%	5.3%	13.6%	0.8%	-11.0%	-0.6%	7.6%	-1.0%	3.6%	-3.0%	-64.8%	7.4%	0.3%	-0.8%	10.6%	5.1%	-8.7%	10.3%	3.7%	-0.6%	0.0%	13.6%	-64.8%	-0.6%	78.4%	0.1531
1999	4.6%	5.8%	10.1%	12.0%	0.0%	4.9%	2.0%	8.6%	-24.2%	-8.3%	2.6%	-4.1%	5.1%	-1.0%	-27.9%	20.2%	20.3%	8.1%	4.9%	11.7%	6.2%	13.7%	10.6%	-0.9%	20.5%	20.5%	-27.9%	4.2%	48.4%	0.1164
2000	8.9%	11.5%	12.2%	5.7%	0.9%	4.4%	9.8%	1.0%	-12.9%	6.6%	14.6%	3.6%	10.9%	4.7%	-10.9%	14.2%	0.3%	17.2%	6.5%	-2.0%	-3.2%	19.8%	8.8%	1.2%	18.4%	19.8%	-12.9%	6.1%	32.7%	0.0827
2001	0.6%	8.7%	4.9%	-3.4%	10.2%	13.8%	13.7%	2.7%	-11.4%	-8.5%	11.4%	5.1%	8.9%	7.4%	4.2%	5.6%	-8.4%	10.3%	-28.7%	3.5%	0.3%	6.9%	0.1%	-0.1%	-1.5%	13.8%	-28.7%	2.3%	42.5%	0.0930
2002	11.7%	13.6%	0.1%	-5.2%	4.8%	2.4%	1.1%	6.8%	-9.4%	-8.0%	3.0%	-7.1%	9.1%	2.6%	-6.2%	25.0%	-0.6%	0.1%	-2.0%	6.5%	6.7%	11.4%	0.1%	-3.2%	1.7%	25.0%	-9.4%	2.6%	34.3%	0.0778
2003	-1.7%	-4.9%	-3.9%	-24.3%	1.8%	1.7%	-16.2%	-14.8%	-5.1%	-16.4%	-17.6%	-10.9%	-7.5%	5.3%	4.0%	-3.8%	-17.6%	-6.2%	-10.6%	-7.7%	-8.9%	0.2%	-9.2%	-7.7%	-6.3%	5.3%	-24.3%	-7.5%	29.6%	0.0742
2004	-14.2%	-11.0%	-3.7%	-6.1%	6.9%	0.0%	-5.0%	-27.7%	-2.0%	-12.9%	-19.6%	-13.9%	-24.2%	8.1%	-40.6%	-21.1%	-18.7%	-9.2%	-8.0%	-15.9%	-16.1%	-3.6%	-17.6%	2.8%	-4.6%	8.1%	-40.6%	-11.1%	48.7%	0.1114
2005	-14.6%	-8.4%	-13.2%	-18.2%	4.6%	-5.3%	-37.6%	-39.7%	-6.1%	-3.8%	-14.3%	-15.5%	-34.7%	2.0%	-35.7%	-15.1%	-12.8%	-14.7%	-8.0%	-84.3%	-1.1%	-21.6%	-13.5%	-3.3%	-6.5%	4.6%	-84.3%	-16.9%	88.9%	0.1847
2006	-11.0%	-23.2%	-10.2%	-7.6%	7.1%	-0.8%	-17.4%	-6.7%	-10.7%	-3.3%	0.1%	-14.7%	-8.0%	-2.8%	-35.4%	-17.5%	-25.3%	-12.4%	-7.6%	-7.6%	-6.6%	-10.9%	-17.3%	-1.5%	2.1%	7.1%	-35.4%	-10.0%	42.6%	0.0934
2007	-3.5%	5.0%	-6.9%	-4.7%	8.8%	-1.6%	-7.6%	10.0%	-8.3%	3.3%	-0.3%	-12.7%	-17.2%	0.3%	-19.1%	-6.1%	-7.5%	0.5%	3.7%	-13.1%	-1.1%	11.4%	-3.1%	5.6%	-2.0%	11.4%	-19.1%	-2.6%	30.5%	0.0797
2008	-3.9%	5.7%	0.6%	-0.1%	-1.8%	2.0%	1.7%	3.1%	0.1%	-0.5%	4.7%	-2.8%	-25.8%	4.2%	17.8%	-6.4%	9.1%	0.9%	4.9%	-5.7%	9.2%	10.4%	1.0%	10.9%	0.2%	17.8%	-25.8%	1.6%	43.6%	0.0797
2009																										0.0%	0.0%		0.0%	
2010																										0.0%	0.0%		0.0%	
High	11.7%	13.6%	12.2%	12.0%	25.6%	13.8%	13.7%	10.0%	0.1%	6.6%	14.6%	9.7%	17.3%	14.8%	17.8%	25.0%	20.3%	17.2%	10.6%	14.9%	9.2%	19.8%	10.6%	10.9%	20.5%					
Low	-14.6%	-24.9%	-13.2%	-24.3%	-18.7%	-15.9%	-37.6%	-39.7%	-29.7%	-16.4%	-22.2%	-15.5%	-34.7%	-24.3%	-64.8%	-21.1%	-25.3%	-15.9%	-28.7%	-84.3%	-16.1%	-21.6%	-17.9%	-7.7%	-15.4%					
Average	-0.6%	-3.0%	-1.1%	-4.6%	3.0%	-1.6%	-0.6%	-2.8%	-13.5%	-5.0%	-1.7%	-5.9%	-7.2%	-1.2%	-23.8%	-1.7%	-2.8%	-3.4%	-4.2%	-5.9%	-3.8%	2.3%	-5.9%	-0.6%	-1.9%					
Range	26.3%	38.6%	25.4%	36.3%	44.3%	29.8%	51.2%	49.7%	29.8%	23.0%	36.8%	25.3%	52.0%	39.2%	82.6%	46.1%	45.7%	33.2%	39.3%	99.2%	25.3%	41.4%	28.6%	18.5%	35.9%					
Std Dev	0.0773	0.1253	0.0671	0.0892	0.0933	0.0708	0.1303	0.1311	0.0916	0.0653	0.1185	0.0778	0.1572	0.0995	0.1978	0.1242	0.1165	0.0968	0.0934	0.2169	0.0721	0.1026	0.0969	0.0436	0.0953					

Calendar Year Change in Accident Year Ultimate Losses between 12 and 36 Month
By Group/Company
Commercial Multiple Peril

Exhibit 8
CMP

12 to 36 Months of Maturity Change in Ultimate Losses as a Percent of 12 Month Ultimat

Accident	Group/Company																								
Year	1	2	3	4	5	6	7	9	17	19	20	21	22	23	25	28	31	33	34	38	40	High	Low	Average	Range	Std Dev
1991	-7.2%	-7.1%	5.7%	-6.6%	-2.9%	-2.9%	9.9%	9.2%	-14.3%	-3.0%	5.3%	-22.3%	-5.0%	-13.7%	5.8%	-4.1%	5.6%	5.2%	3.2%	-1.4%	13.2%	13.2%	-22.3%	-1.3%	35.5%	0.0891
1992	-4.7%	-1.2%	-10.4%	-16.8%	-0.1%	-3.8%	0.9%	0.4%	15.8%	-3.5%	-0.4%	-16.2%	-3.0%	-14.3%	5.3%	-3.4%	-2.7%	4.5%	-1.8%	-2.8%	-1.6%	15.8%	-16.8%	-2.9%	32.6%	0.0733
1993	1.1%	3.9%	-5.6%	-9.9%	-5.4%	-6.7%	-6.1%	-30.2%	15.3%	-12.7%	2.7%	-1.0%	2.4%	-9.4%	-2.3%	3.1%	-8.5%	-17.4%	-0.1%	-1.3%	2.3%	15.3%	-30.2%	-4.1%	45.5%	0.0926
1994	-6.2%	1.6%	-3.6%	-2.9%	-5.4%	-4.7%	6.1%	-35.2%	-4.1%	3.2%	-5.1%	13.0%	-2.1%	8.6%	7.3%	8.1%	-3.7%	-8.1%	-6.8%	0.1%	-5.4%	13.0%	-35.2%	-2.2%	48.1%	0.0965
1995	1.7%	5.2%	-7.9%	-6.1%	-0.1%	-6.4%	-3.2%	-122.3%	1.6%	-6.0%	-12.5%	12.0%	-1.4%	8.4%	9.5%	0.6%	-7.4%	23.4%	-9.5%	1.9%	-8.8%	23.4%	-122.3%	-6.1%	145.7%	0.2797
1996	2.6%	0.0%	-3.4%	-6.4%	2.6%	-2.6%	11.7%	9.8%	-6.8%	-7.5%	-10.5%	-1.8%	2.7%	2.1%	6.5%	9.1%	10.0%	52.5%	4.0%	2.9%	-8.8%	52.5%	-10.5%	3.3%	63.0%	0.1302
1997	-1.2%	-1.9%	15.0%	0.6%	2.7%	2.9%	-7.1%	3.6%	-8.8%	6.4%	-5.0%	-3.7%	4.4%	-1.6%	-1.4%	-3.6%	-7.2%	33.2%	8.8%	-3.4%	-11.3%	33.2%	-11.3%	1.0%	44.5%	0.0962
1998	1.4%	5.8%	9.8%	-0.5%	4.4%	1.1%	-6.7%	8.8%	-5.7%	-5.8%	-3.6%	1.3%	1.9%	2.0%	-1.9%	8.3%	20.3%	19.3%	-1.1%	-3.2%	-7.9%	20.3%	-7.9%	2.3%	28.3%	0.0770
1999	-1.2%	-2.9%	9.4%	-4.5%	8.2%	-2.6%	-13.1%	33.6%	3.0%	7.2%	-3.9%	2.7%	-3.0%	1.5%	-3.5%	-1.5%	-7.6%	2.7%	-6.6%	-1.5%	0.4%	33.6%	-13.1%	0.8%	46.7%	0.0922
2000	3.5%	9.1%	9.8%	3.6%	8.2%	-4.3%	-3.1%	-19.5%	0.3%	13.4%	5.1%	1.0%	14.6%	7.8%	8.6%	9.2%	-23.7%	-1.1%	14.4%	5.4%	3.4%	14.6%	-23.7%	3.1%	38.4%	0.0982
2001	5.6%	-1.3%	11.2%	-0.8%	-14.4%	-3.5%	8.0%	-3.8%	-7.9%	14.5%	3.0%	-3.0%	-2.0%	7.5%	0.1%	3.2%	45.4%	-44.9%	-14.7%	-0.1%	2.3%	45.4%	-44.9%	0.2%	90.3%	0.1600
2002	1.6%	-6.4%	0.5%	-0.7%	-6.2%	0.3%	5.1%	-16.5%	0.3%	0.1%	0.9%	-0.9%	9.8%	-25.5%	2.4%	-11.0%	25.2%	-2.6%	11.6%	-8.4%	-7.0%	25.2%	-25.5%	-1.3%	50.7%	0.1029
2003	7.2%	-16.3%	-14.1%	-9.5%	-11.2%	-1.1%	-13.0%	-16.0%	-13.3%	-5.5%	-3.4%	-1.1%	-0.7%	-28.2%	-5.6%	-10.6%	-65.8%	-13.3%	-2.7%	-12.0%	-9.5%	7.2%	-65.8%	-11.7%	73.1%	0.1449
2004	1.6%	-16.7%	-14.4%	-2.9%	-3.8%	5.4%	-9.4%	12.4%	-15.2%	-10.7%	-0.8%	-5.4%	1.0%	-16.7%	-2.1%	-10.8%	-38.3%	-28.2%	-33.1%	-11.5%	-0.7%	12.4%	-38.3%	-9.5%	50.8%	0.1254
2005	-6.9%	-6.8%	-6.9%	5.6%	-8.1%	26.7%	-1.2%	-34.7%	-1.6%	-1.0%	1.6%	-5.7%	-1.5%	-12.4%	4.3%	-1.9%	-17.2%	-8.0%	-45.6%	-17.1%	2.1%	26.7%	-45.6%	-6.5%	72.2%	0.1452
2006	6.2%	-1.7%	-5.8%	1.7%	-9.7%	6.7%	-2.3%	-68.3%	-20.0%	-7.3%	-6.9%	-9.7%	-3.3%	-14.1%	-2.2%	-15.5%	-20.8%	-10.4%	-20.5%	-14.8%	-1.8%	6.7%	-68.3%	-10.5%	75.0%	0.1550
2007	-7.3%	-9.2%	-5.7%	3.2%	-7.9%	8.1%	-1.2%	-4.5%	-3.0%	-1.1%	-6.4%	-11.6%	-5.4%	-11.3%	0.2%	-9.8%	-14.8%	-4.5%	-13.4%	-8.7%	0.0%	8.1%	-14.8%	-5.4%	22.9%	0.0566
2008	-3.7%	-9.6%	-2.8%	4.8%	-3.8%	-0.9%	11.5%	-21.0%	0.6%	-6.1%	-8.4%	-9.9%	-10.2%	-4.0%	-0.2%	-6.7%	3.6%	17.1%	-16.6%	-3.5%	4.7%	17.1%	-21.0%	-3.1%	38.1%	0.0868
2009																						0.0%	0.0%		0.0%	
2010																						0.0%	0.0%		0.0%	
High	7.2%	9.1%	15.0%	5.6%	8.2%	26.7%	11.7%	33.6%	15.8%	14.5%	5.3%	13.0%	14.6%	8.6%	9.5%	9.2%	45.4%	52.5%	14.4%	5.4%	13.2%					
Low	-7.3%	-16.7%	-14.4%	-16.8%	-14.4%	-6.7%	-13.1%	-122.3%	-20.0%	-12.7%	-12.5%	-22.3%	-10.2%	-28.2%	-5.6%	-15.5%	-65.8%	-44.9%	-45.6%	-17.1%	-11.3%					
Average	-0.3%	-3.1%	-1.1%	-2.7%	-2.9%	0.6%	-0.7%	-16.3%	-3.6%	-1.4%	-2.7%	-3.5%	0.0%	-6.3%	1.7%	-2.1%	-6.0%	1.1%	-7.3%	-4.4%	-1.9%					
Range	14.5%	25.8%	29.4%	22.4%	22.7%	33.4%	24.8%	155.9%	35.8%	27.2%	17.8%	35.2%	24.9%	36.8%	15.1%	24.7%	111.3%	97.4%	59.9%	22.5%	24.5%					
Std Dev	0.0473	0.0715	0.0906	0.0587	0.0644	0.0778	0.0794	0.3524	0.0948	0.0768	0.0521	0.0869	0.0568	0.1153	0.0457	0.0776	0.2448	0.2257	0.1525	0.0629	0.0631					

Calendar Year Change in Accident Year Ultimate Losses between 12 and 36 Months **Exhibit 8**
By Group/Company **MM**
Medical Professional Liability

12 to 36 Months of Maturity Change in Ultimate Losses as a Percent of 12 Month Ultimate

Accident	Group/Company																						
Year	9	12	13	20	22	24	26	27	28	29	30	31	32	33	34	35	36	37	High	Low	Average	Range	Std Dev
1991	2.8%	7.2%	-12.4%	54.2%	-5.5%	0.2%	-17.9%	-7.1%	-10.9%	-11.8%	15.9%	58.9%	-4.3%	-17.5%		-8.7%	-8.2%	-12.0%	58.9%	-17.9%	1.3%	76.9%	0.2252
1992	-5.2%	-5.8%	-8.4%	-39.5%	6.7%	0.0%	-21.3%	-6.2%	-10.7%	-27.1%	-16.7%	-27.1%	-20.5%	0.9%		-8.5%	3.8%	-4.2%	6.7%	-39.5%	-11.2%	46.2%	0.1249
1993	-0.9%	-13.0%	-3.8%	-3.3%	-14.8%	4.8%	-23.2%	-11.4%	0.3%	-24.9%	-10.3%	70.2%	-19.7%	0.1%	-8.6%	-9.5%	-3.1%	3.4%	70.2%	-24.9%	-3.8%	95.1%	0.2041
1994	-0.4%	2.7%	4.2%	-20.6%	11.2%	0.7%	-17.2%	-13.0%	18.0%	-7.2%	0.4%	36.9%	-9.8%	-21.6%	5.5%	-5.5%	3.8%	2.6%	36.9%	-21.6%	-0.5%	58.5%	0.1417
1995	2.0%	-5.4%	1.6%	-31.0%	28.7%	2.2%	-5.8%	-10.1%	22.6%	-15.0%	8.4%	60.0%	-5.2%	14.6%	16.6%	-12.9%	2.1%	-0.8%	60.0%	-31.0%	4.0%	91.0%	0.1994
1996	-2.2%	-6.8%	5.2%	-38.6%	6.0%	0.4%	-19.1%	-7.7%	20.7%	-20.3%	-0.3%	21.1%	-12.9%	-6.1%	32.1%	-7.3%	-1.9%	-22.9%	32.1%	-38.6%	-3.4%	70.7%	0.1701
1997	-1.0%	-3.3%	0.1%	-11.5%	49.0%	-5.7%	-11.5%	-4.2%	29.1%	-12.5%	-3.4%	36.1%	-12.5%	2.8%	63.6%	-6.3%	7.0%	-8.1%	63.6%	-12.5%	6.0%	76.1%	0.2270
1998	-8.0%	-0.5%	9.7%	-6.8%	164.2%	-2.3%	-7.9%	-1.2%	12.7%	-5.2%	3.6%	55.9%	-15.7%	48.2%	61.7%	-5.6%	4.0%	-2.4%	164.2%	-15.7%	16.9%	179.9%	0.4329
1999	-5.5%	-7.3%	3.5%	9.4%	66.4%	7.4%	1.0%	4.9%	46.2%	-1.3%	6.4%	116.0%	0.6%	76.2%	77.1%	3.5%	-0.5%	1.7%	116.0%	-7.3%	22.5%	123.4%	0.3673
2000	10.3%	-0.4%	-8.2%	11.9%	-0.9%	3.5%	-4.4%	18.0%	50.5%	5.2%	7.5%	-125.4%	-3.9%	66.7%	78.6%	5.6%	7.6%	5.6%	78.6%	-125.4%	7.1%	204.0%	0.4122
2001	4.8%	-14.3%	20.6%	-0.6%	-11.4%	10.0%	-6.8%	17.8%	48.8%	4.3%	4.6%	-20.9%	11.4%	22.5%	70.2%	3.4%	20.1%	7.8%	70.2%	-20.9%	10.7%	91.1%	0.2181
2002	-3.9%	-7.3%	5.7%	-19.4%	-10.0%	13.2%	-1.7%	9.6%	38.3%	3.7%	-13.9%	5.6%	-0.8%	20.2%	13.0%	0.1%	1.2%	9.3%	38.3%	-19.4%	3.5%	57.7%	0.1327
2003	1.7%	-15.4%	15.7%	-44.5%	-36.3%	1.5%	3.1%	-7.6%	6.4%	-1.2%	-22.6%	-28.1%	-11.5%	-44.8%	-37.3%	-0.2%	-0.3%	-5.4%	15.7%	-44.8%	-12.6%	60.5%	0.1862
2004	-12.3%	-11.2%	1.6%	-54.9%	-51.9%	-13.8%	-16.1%	-10.8%	-8.8%	-3.8%	-43.9%	-22.6%	-18.0%	-17.4%	-1.0%	-4.2%	-6.4%	-7.1%	1.6%	-54.9%	-16.8%	56.5%	0.1669
2005	-12.4%	-14.3%	-4.5%	-51.2%	-22.4%	-24.5%	-29.6%	-14.7%	-1.5%	-6.7%	-15.4%	-16.9%	-11.7%	-12.5%	-27.2%	-7.5%	-12.5%	-18.1%	-1.5%	-51.2%	-16.9%	49.7%	0.1141
2006	-12.8%	-16.8%	1.7%	-60.6%	-48.2%	-21.0%	-25.2%	-9.7%	-2.0%	-11.5%	-23.2%	12.5%	-14.5%	-35.5%	-19.2%	-6.1%	-16.8%	-11.7%	12.5%	-60.6%	-17.8%	73.1%	0.1720
2007	-12.1%	-38.9%	-12.2%	-47.3%	-11.7%	-7.8%	-33.6%	-5.9%	4.7%	-21.6%	-5.9%	-13.7%	-20.4%	-27.2%	-9.7%	-1.7%	-13.8%	-9.8%	4.7%	-47.3%	-16.0%	52.1%	0.1337
2008	-10.3%	-13.1%	-17.8%	-5.1%	-5.1%	-7.9%	-18.8%	-4.7%	1.7%	-26.2%	-8.3%	-4.1%	-33.1%	-18.9%	-17.4%	1.8%	-15.2%	-7.2%	1.8%	-33.1%	-11.7%	34.9%	0.0926
2009																			0.0%	0.0%		0.0%	
2010																			0.0%	0.0%		0.0%	
High	10.3%	7.2%	20.6%	54.2%	164.2%	13.2%	3.1%	18.0%	50.5%	5.2%	15.9%	116.0%	11.4%	76.2%	78.6%	5.6%	20.1%	9.3%					
Low	-12.8%	-38.9%	-17.8%	-60.6%	-51.9%	-24.5%	-33.6%	-14.7%	-10.9%	-27.1%	-43.9%	-125.4%	-33.1%	-44.8%	-37.3%	-12.9%	-16.8%	-22.9%					
Average	-3.6%	-9.1%	0.1%	-20.0%	6.3%	-2.2%	-14.2%	-3.6%	14.8%	-10.2%	-6.5%	11.9%	-11.2%	2.8%	18.6%	-3.9%	-1.6%	-4.4%					
Range	23.1%	46.1%	38.5%	114.8%	216.2%	37.7%	36.8%	32.8%	61.4%	32.2%	59.8%	241.4%	44.6%	121.0%	115.9%	18.5%	36.9%	32.2%					
Std Dev	0.0672	0.1000	0.0987	0.2924	0.4945	0.0995	0.1056	0.0982	0.2065	0.1058	0.1455	0.5315	0.1002	0.3362	0.3985	0.0521	0.0939	0.0876					

Calendar Year Change in Accident Year Ultimate Losses between 12 and 120 Months
By Group/Company
Homeowners/Farmowners

Exhibit 9
HMP/FMP

12 to 120 Months of Maturity Change in Ultimate Losses as a Percent of 12 Month Ultimate

Accident	Group/Company																			
Year	1	2	3	4	5	6	15	18	19	20	21	25	33	38	40	High	Low	Average	Range	Std Dev
1991	-0.9%	-4.9%	-2.9%	0.8%	-2.4%	-1.9%	-4.0%	-4.7%	-3.8%	-10.9%	-6.0%	-4.7%	-5.0%	-6.0%	-8.4%	0.8%	-10.9%	-4.4%	11.6%	0.0287
1992	-7.1%	-1.0%	-1.6%	2.9%	-4.3%	4.0%	-17.0%	-6.8%	-3.9%	-4.7%	-8.4%	-4.4%	-3.8%	-2.7%	-9.1%	4.0%	-17.0%	-4.5%	21.1%	0.0506
1993	-0.1%	-7.0%	2.2%	1.1%	-2.6%	2.2%	-1.7%	-7.2%	-3.8%	-3.7%	-7.8%	-2.3%	-13.1%	-7.0%	-5.1%	2.2%	-13.1%	-3.7%	15.3%	0.0425
1994	-2.1%	-2.3%	-1.4%	-5.2%	-3.3%	-0.2%	-5.9%	-6.9%	-0.2%	-7.1%	-9.8%	0.9%	-10.7%	-3.1%	-5.0%	0.9%	-10.7%	-4.2%	11.6%	0.0347
1995	-1.1%	0.0%	-10.4%	1.2%	-2.6%	0.9%	-1.8%	-10.9%	0.2%	-2.5%	-5.7%	4.9%	-20.7%	-3.0%	-5.4%	4.9%	-20.7%	-3.8%	25.6%	0.0630
1996	-2.0%	-2.2%	-7.1%	-0.5%	1.5%	0.3%	-5.9%	-8.8%	1.4%	0.2%	-6.2%	0.9%	-31.8%	-5.1%	1.8%	1.8%	-31.8%	-4.2%	33.6%	0.0838
1997	-7.3%	-7.3%	-5.1%	-2.1%	-1.1%	-1.2%	-9.8%	-7.2%	-1.9%	1.1%	-6.1%	-3.3%	-29.7%	-11.3%	-3.2%	1.1%	-29.7%	-6.4%	30.8%	0.0733
1998	-1.2%	3.4%	5.5%	6.4%	1.8%	3.1%	-6.4%	0.6%	0.3%	-2.6%	-2.5%	-4.0%	-26.6%	-4.6%	-0.3%	6.4%	-26.6%	-1.8%	32.9%	0.0777
1999	-1.4%	0.3%	-1.8%	-0.4%	2.6%	0.9%	-4.6%	0.4%	2.2%	-2.2%	-4.9%	-0.3%	-23.3%	0.2%	2.3%	2.6%	-23.3%	-2.0%	25.8%	0.0630
2000	9.3%	13.9%	8.9%	7.8%	6.8%	4.7%	3.3%	4.3%	5.0%	3.0%	3.0%	10.7%	-8.1%	1.8%	8.9%	13.9%	-8.1%	5.6%	21.9%	0.0507
2001	6.7%	17.9%	14.8%	6.2%	4.3%	1.2%	-2.1%	6.3%	3.3%	-5.8%	-0.4%	3.4%	5.2%	-3.3%	1.7%	17.9%	-5.8%	4.0%	23.8%	0.0627
2002	-2.3%	2.9%	-6.7%	-1.3%	-6.4%	-5.1%	-2.9%	-6.7%	-7.9%	-3.6%	-6.4%	-0.9%	5.1%	-17.4%	-5.3%	5.1%	-17.4%	-4.3%	22.5%	0.0517
2003	-3.8%	-9.9%	-12.5%	-3.6%	-3.7%	-2.3%	-2.8%	-5.8%	-5.9%	-3.7%	-5.6%	-6.0%	-3.2%	-16.6%	-5.9%	-2.3%	-16.6%	-6.1%	14.3%	0.0400
2004	-5.0%	-6.8%	-4.7%	0.8%	-7.8%	-3.6%	-3.6%	-0.5%	-4.2%	-4.1%	-8.4%	2.7%	-1.9%	0.9%	-5.6%	2.7%	-8.4%	-3.5%	11.1%	0.0327
2005	-4.2%	-6.3%	-1.3%	3.1%	1.0%	4.8%	-7.4%	-2.9%	-7.5%	-0.3%	-3.3%	1.5%	-5.4%	-22.8%	-3.6%	4.8%	-22.8%	-3.6%	27.6%	0.0649
2006	-0.4%	-8.0%	-2.3%	6.1%	-2.8%	2.0%	-2.4%	-3.5%	-3.6%	-5.9%	-1.5%	-1.0%	-30.1%	-2.4%	0.7%	6.1%	-30.1%	-3.7%	36.3%	0.0802
2007	-4.6%	-9.5%	-2.8%	1.8%	-6.4%	0.4%	-7.5%	-7.0%	-4.7%	-1.7%	0.5%	-3.6%	-24.2%	-6.0%	-3.5%	1.8%	-24.2%	-5.3%	26.0%	0.0614
2008	-2.3%	-0.4%	-3.3%	6.5%	0.4%	-1.7%	-1.1%	-2.8%	0.2%	-3.6%	1.9%	1.6%	11.2%	-1.4%	-2.8%	11.2%	-3.6%	0.2%	14.8%	0.0401
2009	0.2%	-0.4%	-4.2%	3.4%	1.2%	1.4%	-0.6%	-3.4%	0.6%	-1.8%	-0.7%	-0.9%	-8.8%	2.6%	0.1%	3.4%	-8.8%	-0.8%	12.2%	0.0299
2010																0.0%	0.0%		0.0%	

For 2001 & Prior only

	1	2	3	4	5	6	15	18	19	20	21	25	33	38	40
High	9.3%	17.9%	14.8%	7.8%	6.8%	4.7%	3.3%	6.3%	5.0%	3.0%	3.0%	10.7%	5.2%	1.8%	8.9%
Low	-7.3%	-7.3%	-10.4%	-5.2%	-4.3%	-1.9%	-17.0%	-10.9%	-3.9%	-10.9%	-9.8%	-4.7%	-31.8%	-11.3%	-9.1%
Average	-0.7%	1.0%	0.1%	1.6%	0.1%	1.3%	-5.1%	-3.7%	-0.1%	-3.2%	-5.0%	0.2%	-15.2%	-4.0%	-2.0%
Range	16.7%	25.2%	25.2%	13.0%	11.1%	6.6%	20.3%	17.3%	8.8%	13.9%	12.7%	15.4%	37.0%	13.1%	18.0%
Std Dev	0.0496	0.0808	0.0730	0.0391	0.0357	0.0208	0.0520	0.0570	0.0300	0.0393	0.0371	0.0474	0.1197	0.0352	0.0539

Note: 2002 & subsequent is evaluated through the end of 2010

Calendar Year Change in Accident Year Ultimate Losses between 12 and 120 Months **Exhibit 9**
By Group/Company **PPAL**
Private Passenger Auto Liability

12 to 120 Months of Maturity Change in Ultimate Losses as a Percent of 12 Month Ultimate

Accident	Group/Company																					
Year	1	2	3	4	5	6	8	9	15	18	19	20	21	25	33	38	40	High	Low	Average	Range	Std Dev
1991	-10.3%	-10.1%	-10.1%	-10.8%	-6.2%	-7.7%	-15.0%	-8.6%	-15.6%	-13.9%	-5.5%	-17.8%	-11.1%	-11.7%	-10.0%	-16.6%	-4.9%	-4.9%	-17.8%	-10.9%	13.0%	0.0383
1992	-11.5%	-15.9%	-12.9%	-5.8%	-14.4%	-8.7%	-15.5%	-9.6%	-24.4%	-18.4%	-5.1%	-18.5%	-23.0%	-10.7%	-10.3%	-14.8%	-9.0%	-5.1%	-24.4%	-13.4%	19.3%	0.0548
1993	-14.2%	-3.5%	-8.3%	-0.7%	-14.8%	-9.2%	-8.1%	-11.0%	-16.9%	-21.6%	-3.7%	-7.4%	-28.4%	-8.5%	-9.8%	-11.5%	-5.9%	-0.7%	-28.4%	-10.8%	27.6%	0.0687
1994	-13.1%	-2.4%	-7.7%	3.1%	-11.2%	-9.1%	-3.9%	-11.0%	-16.8%	-23.9%	-7.1%	-6.2%	-26.6%	-3.9%	-8.8%	-7.5%	-0.5%	3.1%	-26.6%	-9.2%	29.7%	0.0771
1995	-8.6%	-10.5%	-5.3%	-1.7%	-11.3%	-9.8%	-6.8%	-9.5%	-15.0%	-8.7%	-4.0%	-6.1%	-22.9%	-1.2%	-22.3%	-12.7%	-2.1%	-1.2%	-22.9%	-9.3%	21.7%	0.0633
1996	-7.5%	1.1%	-4.0%	-3.9%	-7.6%	-7.1%	-6.8%	-9.1%	-8.4%	-15.1%	0.5%	-7.1%	-19.9%	0.3%	-17.7%	-8.0%	-5.7%	1.1%	-19.9%	-7.4%	21.0%	0.0585
1997	-5.1%	-0.2%	-5.5%	-3.6%	-6.1%	-4.1%	-5.1%	-11.9%	-11.0%	5.7%	-0.8%	-3.9%	-14.5%	-3.2%	-8.7%	-6.2%	-6.2%	5.7%	-14.5%	-5.3%	20.2%	0.0469
1998	-1.3%	-0.5%	0.2%	1.4%	-1.2%	-3.3%	1.1%	-4.9%	-11.2%	5.8%	4.9%	-8.3%	-8.8%	-4.8%	13.6%	-2.6%	3.7%	13.6%	-11.2%	-1.0%	24.8%	0.0604
1999	0.5%	10.4%	1.3%	-0.3%	1.0%	0.7%	4.2%	-3.4%	-10.3%	10.2%	10.8%	-0.9%	-10.0%	-4.8%	7.9%	0.3%	10.3%	10.8%	-10.3%	1.6%	21.1%	0.0669
2000	0.5%	-3.3%	3.4%	6.6%	5.6%	0.5%	-3.9%	-4.1%	-11.4%	3.7%	9.4%	-4.0%	2.8%	3.9%	41.5%	-1.3%	18.0%	41.5%	-11.4%	4.0%	52.8%	0.1166
2001	-1.1%	1.9%	6.9%	2.7%	2.8%	-2.7%	-3.3%	-7.2%	-14.1%	2.2%	3.8%	-5.8%	5.2%	-4.8%	23.0%	-7.1%	-7.3%	23.0%	-14.1%	-0.3%	37.1%	0.0817
2002	-4.7%	4.1%	11.3%	-2.2%	-3.4%	-5.0%	-2.1%	-5.1%	-7.2%	-0.9%	-3.8%	-1.5%	9.4%	-1.0%	18.0%	-10.0%	-0.9%	18.0%	-10.0%	-0.3%	28.0%	0.0717
2003	-8.7%	-1.9%	0.7%	-8.8%	-3.2%	-5.0%	-3.8%	-9.9%	-10.4%	1.1%	-10.3%	-1.6%	10.3%	-4.0%	8.9%	-15.0%	-2.3%	10.3%	-15.0%	-3.8%	25.4%	0.0670
2004	-8.8%	-4.6%	-2.0%	-12.2%	-8.7%	-2.9%	-5.3%	-9.7%	-7.2%	-1.2%	-10.1%	1.2%	8.9%	-5.3%	-4.9%	-15.8%	-10.4%	8.9%	-15.8%	-5.8%	24.6%	0.0572
2005	-6.3%	-13.1%	-3.5%	-6.6%	-9.5%	-1.4%	-2.7%	-9.4%	-11.2%	-5.7%	-14.5%	0.3%	3.0%	-5.1%	-5.5%	-11.8%	-5.3%	3.0%	-14.5%	-6.4%	17.5%	0.0479
2006	-1.3%	-11.5%	-3.5%	-5.1%	-4.4%	0.9%	0.1%	-6.3%	-4.8%	-5.8%	-12.0%	0.9%	-1.1%	-0.7%	-0.5%	-5.1%	-10.7%	0.9%	-12.0%	-4.2%	12.9%	0.0419
2007	0.8%	-12.0%	-3.3%	-6.2%	-5.1%	0.5%	-0.1%	-2.7%	-2.2%	-7.2%	-6.6%	4.5%	0.0%	0.8%	5.5%	3.2%	-7.7%	5.5%	-12.0%	-2.2%	17.5%	0.0477
2008	1.9%	-10.9%	-6.3%	-2.8%	-3.4%	-0.6%	-0.8%	-3.9%	-6.4%	-7.2%	-1.1%	-5.0%	6.3%	-1.0%	4.7%	-0.3%	6.8%	6.8%	-10.9%	-1.8%	17.6%	0.0482
2009	1.2%	-6.3%	-0.9%	-3.9%	-0.1%	0.2%	-3.0%	-5.0%	-3.5%	-4.9%	-2.7%	-1.8%	3.5%	-3.1%	2.0%	1.1%	4.9%	4.9%	-6.3%	-1.3%	11.1%	0.0316
2010																		0.0%	0.0%		0.0%	

For 2001 & Prior only

	1	2	3	4	5	6	8	9	15	18	19	20	21	25	33	38	40
High	0.5%	10.4%	6.9%	6.6%	5.6%	0.7%	4.2%	-3.4%	-8.4%	10.2%	10.8%	-0.9%	5.2%	3.9%	41.5%	0.3%	18.0%
Low	-14.2%	-15.9%	-12.9%	-10.8%	-14.8%	-9.8%	-15.5%	-11.9%	-24.4%	-23.9%	-7.1%	-18.5%	-28.4%	-11.7%	-22.3%	-16.6%	-9.0%
Average	-6.5%	-3.0%	-3.8%	-1.2%	-5.8%	-5.5%	-5.7%	-8.2%	-14.1%	-6.7%	0.3%	-7.8%	-14.3%	-4.5%	-0.1%	-8.0%	-0.9%
Range	14.8%	26.3%	19.8%	17.4%	20.4%	10.5%	19.7%	8.4%	16.1%	34.1%	17.8%	17.5%	33.6%	15.6%	63.7%	16.9%	27.0%
Std Dev	0.0553	0.0714	0.0609	0.0479	0.0701	0.0391	0.0590	0.0291	0.0444	0.1250	0.0616	0.0549	0.1123	0.0462	0.1939	0.0548	0.0839

Note: 2002 & subsequent is evaluated through the end of 2010

Calendar Year Change in Accident Year Ultimate Losses between 12 and 120 Months **Exhibit 9**
By Group/Company **CAL**
Commercial Auto Liability

12 to 120 Months of Maturity Change in Ultimate Losses as a Percent of 12 Month Ultimate

Accident	Group/Company																												
Year	1	2	3	4	5	6	7	8	9	14	15	17	19	20	21	22	23	25	28	31	33	34	38	40	High	Low	Average	Range	Std Dev
1991	-11.2%	-11.2%	-22.7%	-8.1%	-15.6%	-0.6%	-26.5%	-6.4%	-23.5%	-34.9%	16.3%	-8.7%	28.5%	-5.3%	7.4%	1.2%	-4.8%	-15.5%	-17.7%	-19.2%	-0.7%	-14.1%	-17.7%	-4.6%	28.5%	-34.9%	-9.0%	63.5%	0.1378
1992	-3.7%	-0.1%	-31.0%	-11.0%	-15.2%	-0.5%	-29.5%	-25.3%	-18.6%	-24.2%	-6.6%	-0.1%	0.9%	3.3%	6.1%	-3.9%	4.4%	-5.7%	-17.8%	-13.0%	-12.1%	-6.6%	-14.7%	1.6%	6.1%	-31.0%	-9.3%	37.0%	0.1094
1993	-0.9%	-3.1%	-16.4%	-10.3%	-11.8%	-1.6%	-18.8%	-17.7%	-22.6%	-14.6%	7.7%	-9.0%	0.0%	12.5%	17.1%	-7.2%	-6.9%	-7.3%	-11.3%	-13.9%	0.6%	5.6%	-7.9%	4.3%	17.1%	-22.6%	-5.6%	39.6%	0.1010
1994	-4.1%	-5.4%	-16.8%	-9.0%	-1.6%	-2.0%	-17.1%	-14.4%	-19.4%	-13.5%	5.0%	-10.6%	3.1%	7.2%	-4.6%	-3.5%	-6.8%	1.0%	-4.8%	-6.8%	31.5%	4.4%	-7.5%	3.5%	31.5%	-19.4%	-3.8%	50.9%	0.1053
1995	-9.4%	3.1%	-25.3%	-7.5%	2.1%	-5.7%	-14.2%	-5.3%	-41.6%	-25.5%	19.4%	-15.5%	6.5%	2.3%	-9.1%	3.5%	-7.1%	-6.5%	3.5%	-14.8%	23.5%	4.7%	3.0%	-0.6%	23.5%	-41.6%	-4.9%	65.1%	0.1405
1996	-0.3%	27.1%	-12.0%	-9.9%	12.8%	0.0%	-9.1%	-10.7%	-8.8%	-10.3%	35.9%	11.7%	6.4%	2.9%	-4.4%	3.0%	11.0%	1.8%	10.5%	19.7%	33.8%	9.2%	-0.9%	-0.4%	35.9%	-12.0%	4.9%	47.8%	0.1365
1997	7.1%	17.0%	48.7%	0.3%	25.9%	5.5%	4.8%	-12.2%	0.8%	-7.2%	25.7%	-10.2%	22.6%	26.4%	-4.1%	16.2%	7.8%	14.8%	10.6%	14.2%	15.0%	9.0%	4.1%	0.9%	48.7%	-12.2%	10.2%	60.9%	0.1372
1998	0.3%	29.6%	23.6%	16.3%	16.0%	14.0%	-2.5%	-4.8%	12.9%	-11.9%	4.6%	6.9%	16.4%	22.8%	4.7%	7.6%	-9.3%	11.0%	19.9%	23.6%	23.3%	19.0%	8.2%	4.0%	29.6%	-11.9%	10.7%	41.5%	0.1110
1999	3.6%	12.5%	10.6%	10.5%	26.5%	18.1%	-5.0%	-4.1%	4.2%	-5.9%	1.3%	9.8%	18.1%	15.6%	13.4%	23.1%	10.4%	11.1%	14.8%	28.3%	19.6%	21.7%	20.1%	18.1%	28.3%	-5.9%	12.3%	34.2%	0.0950
2000	5.3%	-10.3%	3.7%	10.5%	12.5%	24.2%	-16.5%	0.3%	24.5%	2.0%	2.5%	-10.1%	28.0%	3.6%	19.8%	6.2%	3.7%	-2.6%	12.0%	51.8%	16.7%	16.3%	18.3%	25.6%	51.8%	-16.5%	10.3%	68.3%	0.1479
2001	-0.7%	7.1%	-0.7%	11.0%	-7.1%	8.6%	-14.2%	7.4%	4.6%	11.2%	-15.3%	5.2%	15.3%	-6.1%	27.4%	15.2%	23.7%	-1.5%	20.7%	76.5%	-6.4%	28.8%	5.3%	8.8%	76.5%	-15.3%	9.4%	91.8%	0.1857
2002	7.8%	-3.4%	1.9%	0.2%	-11.2%	14.7%	-22.7%	-3.6%	-22.2%	-5.7%	-1.5%	-6.8%	-3.8%	-10.9%	16.3%	20.7%	-1.1%	-2.6%	-2.5%	4.9%	38.8%	7.3%	-9.4%	-7.8%	38.8%	-22.7%	-0.1%	61.6%	0.1334
2003	-4.1%	3.3%	-3.0%	-14.5%	-14.8%	23.4%	-19.5%	-5.9%	-31.5%	8.0%	25.9%	-5.0%	-8.1%	-5.7%	17.1%	2.8%	-12.1%	-5.6%	-9.6%	-31.4%	11.1%	8.9%	-14.2%	-10.3%	25.9%	-31.5%	-4.0%	57.4%	0.1470
2004	-4.5%	-23.3%	-9.2%	-15.7%	-9.9%	27.4%	-12.6%	-5.1%	-12.9%	9.6%	6.4%	-6.4%	-13.1%	-4.7%	20.3%	10.7%	-22.1%	5.6%	-1.2%	-24.1%	-23.2%	2.8%	-22.4%	-14.2%	27.4%	-24.1%	-5.9%	51.4%	0.1399
2005	-11.1%	-13.4%	-12.3%	1.5%	-6.8%	45.4%	-12.0%	1.9%	-13.8%	20.4%	-1.3%	1.4%	-21.8%	-9.0%	4.3%	-3.1%	-19.3%	2.4%	4.3%	14.4%	-6.6%	2.5%	-16.3%	-3.6%	45.4%	-21.8%	-2.2%	67.2%	0.1433
2006	-9.7%	-2.7%	-15.0%	-9.8%	-2.7%	32.7%	-11.8%	4.6%	-8.5%	0.2%	5.4%	-5.3%	-18.6%	-3.7%	7.1%	0.2%	-18.4%	-3.9%	-5.6%	-0.9%	-7.4%	-0.3%	-11.7%	2.4%	32.7%	-18.6%	-3.5%	51.4%	0.1042
2007	-4.7%	-15.9%	-9.3%	2.4%	-7.6%	29.3%	-13.9%	1.0%	-7.7%	5.1%	-1.6%	-9.4%	-23.3%	-6.6%	9.7%	-2.4%	-14.5%	-4.0%	-1.3%	-10.1%	15.1%	-2.1%	-6.1%	2.6%	29.3%	-23.3%	-3.1%	52.7%	0.1086
2008	-5.5%	-8.2%	2.9%	-6.1%	-8.5%	10.3%	-6.8%	-3.0%	-7.7%	15.6%	34.6%	-11.0%	-8.9%	-8.8%	10.3%	-0.2%	-7.4%	-6.8%	4.7%	22.9%	13.4%	-8.4%	-5.5%	3.6%	34.6%	-11.0%	0.6%	45.6%	0.1177
2009	2.7%	0.5%	-4.1%	-3.5%	-7.6%	3.0%	-8.1%	-7.3%	-9.2%	8.8%	12.0%	0.2%	-5.6%	-9.0%	1.4%	-5.1%	-2.7%	-4.7%	-3.2%	0.0%	4.2%	-3.8%	-6.5%	-1.2%	12.0%	-9.2%	-2.0%	21.2%	0.0545
2010																									0.0%	0.0%		0.0%	

For 2001 & Prior only

	1	2	3	4	5	6	7	8	9	14	15	17	19	20	21	22	23	25	28	31	33	34	38	40
High	7.1%	29.6%	48.7%	16.3%	26.5%	24.2%	4.8%	7.4%	24.5%	11.2%	35.9%	11.7%	28.5%	26.4%	27.4%	23.1%	23.7%	14.8%	20.7%	76.5%	33.8%	28.8%	20.1%	25.6%
Low	-11.2%	-11.2%	-31.0%	-11.0%	-15.6%	-5.7%	-29.5%	-25.3%	-41.6%	-34.9%	-15.3%	-15.5%	0.0%	-6.1%	-9.1%	-7.2%	-9.3%	-15.5%	-17.8%	-19.2%	-12.1%	-14.1%	-17.7%	-4.6%
Average	-1.3%	6.0%	-3.5%	-0.7%	4.0%	5.5%	-13.5%	-8.5%	-8.0%	-12.3%	8.8%	-2.8%	13.3%	7.8%	6.7%	5.6%	2.4%	0.1%	3.7%	13.3%	13.2%	8.9%	0.9%	5.5%
Range	18.2%	40.8%	79.6%	27.3%	42.2%	29.9%	34.3%	32.7%	66.1%	46.2%	51.2%	27.2%	28.5%	32.5%	36.5%	30.4%	33.0%	30.2%	38.5%	95.7%	45.9%	42.9%	37.8%	30.2%
Std Dev	0.0564	0.1410	0.2392	0.1063	0.1561	0.0961	0.1011	0.0894	0.1923	0.1291	0.1461	0.0967	0.1049	0.1054	0.1173	0.0941	0.1039	0.0918	0.1434	0.3087	0.1549	0.1238	0.1226	0.0890

Note: 2002 & subsequent is evaluated through the end of 2010

Calendar Year Change in Accident Year Ultimate Losses between 12 and 120 Months
By Group/Company
Workers Compensation

Exhibit 9
WC

12 to 120 Months of Maturity Change in Ultimate Losses as a Percent of 12 Month Ultimate

Accident	Group/Company																													
Year	2	3	4	5	6	7	9	10	11	15	16	17	19	20	21	22	23	25	28	31	33	34	38	39	40	High	Low	Average	Range	Std Dev
1991	2.6%	-5.5%	4.0%	-5.7%	32.2%	-11.6%	14.8%	0.3%	-49.0%	-0.9%	2.9%	-2.1%	14.8%	12.3%	-28.5%	-16.1%	6.2%	-12.1%	-15.0%	-8.3%	-1.5%	-0.8%	-11.6%	-6.4%	-9.0%	32.2%	-49.0%	-3.8%	81.1%	0.1543
1992	-6.2%	-20.5%	-10.3%	-21.6%	12.4%	-0.8%	56.3%	-3.4%	-44.6%	0.6%	-22.2%	-16.9%	1.8%	-10.4%	-25.8%	-32.9%	-12.4%	-18.0%	-21.6%	-32.1%	-17.1%	-9.5%	-22.6%	-14.7%	-17.2%	56.3%	-44.6%	-12.4%	100.9%	0.1886
1993	-21.6%	-20.3%	-18.2%	-22.1%	-1.9%	-19.3%	15.5%	-9.1%	-22.5%	-5.3%	-22.7%	-20.0%	-14.8%	-19.0%	-32.4%	-47.5%	-16.2%	-15.2%	-19.9%	-14.6%	-4.6%	-12.0%	-24.4%	-13.9%	-17.3%	15.5%	-47.5%	-16.8%	63.0%	0.1129
1994	-26.4%	-17.3%	1.8%	-14.6%	-1.2%	-23.1%	18.6%	8.2%	-37.1%	-15.4%	-3.6%	-18.2%	-20.8%	-29.5%	-28.8%	-31.1%	-11.8%	-21.8%	-25.6%	-20.4%	-12.5%	-6.7%	-27.8%	-23.3%	-21.9%	18.6%	-37.1%	-16.4%	55.7%	0.1318
1995	-23.4%	-12.2%	-6.7%	-5.5%	-0.7%	-29.7%	5.9%	12.2%	-41.9%	-23.0%	10.7%	-17.4%	-25.6%	-28.5%	-23.2%	-33.2%	7.0%	-14.2%	-12.5%	12.9%	-11.8%	-4.7%	-22.0%	-28.8%	-20.8%	12.9%	-41.9%	-13.5%	54.8%	0.1525
1996	-7.3%	-23.4%	0.5%	-0.3%	-6.4%	-2.9%	17.8%	8.2%	-35.4%	-13.1%	21.9%	-3.9%	-22.3%	-18.3%	-32.1%	-24.6%	15.4%	-8.6%	-4.9%	9.6%	6.6%	-1.9%	-6.2%	-28.7%	-11.4%	21.9%	-35.4%	-6.9%	57.2%	0.1541
1997	2.0%	9.1%	7.4%	18.0%	6.8%	-7.1%	22.5%	11.8%	-29.4%	-15.3%	28.0%	0.1%	2.5%	-8.1%	-25.9%	-4.0%	17.4%	6.6%	15.8%	3.7%	14.5%	12.6%	11.1%	-13.0%	6.3%	28.0%	-29.4%	3.8%	57.4%	0.1416
1998	18.9%	50.2%	15.0%	26.7%	-17.8%	6.9%	52.4%	10.7%	-24.9%	-5.7%	27.4%	-9.0%	1.6%	-2.4%	-43.0%	26.1%	14.2%	0.6%	43.4%	6.8%	10.3%	20.9%	12.7%	-1.0%	10.1%	52.4%	-43.0%	10.1%	95.5%	0.2187
1999	16.7%	30.6%	29.8%	27.0%	16.3%	5.0%	39.6%	15.0%	-32.6%	-9.5%	24.8%	-5.6%	60.3%	2.1%	-15.3%	29.7%	38.9%	13.9%	40.4%	22.9%	20.0%	54.0%	26.2%	2.4%	32.6%	60.3%	-32.6%	19.4%	92.9%	0.2138
2000	31.5%	30.5%	39.3%	23.8%	-0.2%	9.5%	14.0%	3.7%	-26.2%	9.2%	55.5%	1.8%	20.7%	14.2%	-9.3%	52.6%	7.6%	25.9%	28.6%	-2.0%	22.4%	88.9%	32.4%	26.7%	23.2%	88.9%	-26.2%	21.0%	115.1%	0.2322
2001	39.6%	19.2%	4.1%	9.3%	29.4%	24.9%	11.3%	2.2%	-18.6%	-8.7%	51.8%	-0.8%	12.9%	15.6%	15.5%	37.0%	5.1%	10.9%	32.8%	11.3%	57.3%	51.0%	5.8%	5.3%	-0.4%	57.3%	-18.6%	16.9%	75.9%	0.1924
2002	16.1%	17.7%	0.1%	4.6%	13.0%	7.0%	-7.3%	2.5%	-17.5%	-9.1%	31.9%	-18.7%	3.2%	10.1%	6.5%	45.7%	-0.9%	-4.0%	12.7%	-40.9%	34.2%	32.9%	5.7%	12.6%	4.8%	45.7%	-40.9%	6.5%	86.5%	0.1839
2003	-13.8%	-7.6%	-13.3%	-4.7%	9.8%	0.7%	-21.0%	-26.1%	-8.2%	-15.0%	-6.1%	-18.9%	-11.7%	8.4%	23.0%	4.3%	-24.0%	-14.8%	-7.1%	-61.0%	-2.1%	20.2%	-5.2%	-6.1%	-2.7%	23.0%	-61.0%	-8.1%	84.0%	0.1648
2004	-23.4%	-25.3%	-16.6%	-8.1%	4.5%	-6.0%	-17.1%	-39.6%	-6.7%	-6.2%	-37.0%	-23.3%	-27.8%	13.0%	-17.5%	-22.6%	-37.8%	-12.0%	-11.2%	-66.6%	-10.8%	2.5%	-23.8%	4.4%	-4.2%	13.0%	-66.6%	-16.8%	79.6%	0.1712
2005	-20.6%	-28.7%	-14.1%	-18.2%	4.2%	-8.3%	-37.9%	-33.7%	-10.6%	2.9%	-45.1%	-22.7%	-35.3%	4.5%	-13.2%	-17.5%	-37.1%	-20.1%	-14.2%	-74.2%	-0.3%	-14.9%	-20.3%	-4.1%	-8.6%	4.5%	-74.2%	-19.5%	78.7%	0.1772
2006	-14.4%	-23.4%	-12.8%	-8.7%	7.0%	-3.1%	-21.6%	-7.5%	-13.7%	7.5%	-18.1%	-24.0%	-23.8%	-0.1%	-22.0%	-19.1%	-28.9%	-14.5%	-10.7%	-11.4%	-5.5%	-6.7%	-19.3%	0.7%	1.3%	7.5%	-28.9%	-11.7%	36.4%	0.1011
2007	-6.4%	6.4%	-9.8%	-4.2%	11.5%	-0.3%	-9.0%	13.5%	-10.9%	1.3%	-0.3%	-20.4%	-26.3%	0.2%	-13.5%	-5.1%	-9.0%	0.0%	4.3%	-7.9%	1.8%	14.6%	-5.6%	11.8%	-2.6%	14.6%	-26.3%	-2.6%	40.9%	0.1001
2008	-3.9%	5.8%	0.6%	-0.1%	-1.8%	2.0%	1.8%	3.1%	0.1%	-0.5%	5.1%	-2.9%	-22.1%	4.3%	19.9%	-6.3%	10.3%	0.9%	5.0%	-5.4%	9.3%	11.0%	1.1%	11.7%	0.2%	19.9%	-22.1%	2.0%	42.0%	0.0779
2009	-1.4%	14.4%	0.1%	0.8%	-0.4%	0.7%	-4.7%	3.4%	2.1%	-2.9%	-0.2%	-0.1%	-12.6%	-9.2%	13.2%	-1.7%	2.0%	-8.1%	1.3%	-8.3%	5.0%	7.1%	-1.4%	6.2%	3.1%	14.4%	-12.6%	0.3%	27.0%	0.0626
2010																										0.0%	0.0%		0.0%	
For 2001 & Prior only																														
High	39.6%	50.2%	39.3%	27.0%	32.2%	24.9%	56.3%	15.0%	-18.6%	9.2%	55.5%	1.8%	60.3%	15.6%	15.5%	52.6%	38.9%	25.9%	43.4%	22.9%	57.3%	88.9%	32.4%	26.7%	32.6%					
Low	-26.4%	-23.4%	-18.2%	-22.1%	-17.8%	-29.7%	5.9%	-9.1%	-49.0%	-23.0%	-22.7%	-20.0%	-25.6%	-29.5%	-43.0%	-47.5%	-16.2%	-21.8%	-25.6%	-32.1%	-17.1%	-12.0%	-27.8%	-28.8%	-21.9%					
Average	2.4%	3.7%	6.1%	3.2%	6.3%	-4.4%	24.4%	5.5%	-32.9%	-7.9%	15.9%	-8.3%	2.8%	-6.5%	-22.6%	-4.0%	6.5%	-2.9%	5.6%	-0.9%	7.6%	17.4%	-2.4%	-8.7%	-2.4%					
Range	66.0%	73.6%	57.5%	49.1%	50.0%	54.7%	50.4%	24.1%	30.3%	32.2%	78.3%	21.8%	85.9%	45.2%	58.5%	100.1%	55.1%	47.7%	69.0%	55.1%	74.4%	100.9%	60.2%	55.5%	54.4%					
Std Dev	0.2225	0.2558	0.1681	0.1881	0.1515	0.1600	0.1706	0.0742	0.0967	0.0891	0.2602	0.0827	0.2486	0.1640	0.1542	0.3437	0.1587	0.1544	0.2689	0.1642	0.2126	0.3314	0.2127	0.1653	0.1844					

Note: 2002 & subsequent is evaluated through the end of 2010

Calendar Year Change in Accident Year Ultimate Losses between 12 and 120 Month
By Group/Company
Commercial Multiple Peril

Exhibit 9
CMP

12 to 120 Months of Maturity Change in Ultimate Losses as a Percent of 12 Month Ultimat

Accident	Group/Company																									
Year	1	2	3	4	5	6	7	9	17	19	20	21	22	23	25	28	31	33	34	38	40	High	Low	Average	Range	Std Dev
1991	-11.0%	-21.9%	0.5%	-8.8%	-7.9%	-6.5%	7.6%	1.2%	-14.2%	-12.1%	-3.2%	-24.0%	-7.3%	-11.2%	-3.2%	9.3%	0.3%	19.6%	-3.5%	-10.2%	9.0%	19.6%	-24.0%	-4.6%	43.6%	0.1045
1992	-7.3%	-15.4%	-17.7%	-14.0%	-5.4%	-2.9%	-7.6%	-4.2%	13.5%	-8.3%	-11.2%	-16.6%	-5.8%	-8.2%	7.5%	9.3%	-10.2%	2.3%	-2.1%	-7.1%	-3.8%	13.5%	-17.7%	-5.5%	31.2%	0.0823
1993	-4.6%	-2.1%	-12.5%	-5.5%	-7.4%	-9.9%	-11.7%	-29.8%	-2.2%	-11.5%	-10.0%	0.1%	0.4%	-8.9%	5.3%	11.5%	-13.4%	-7.5%	1.9%	-11.2%	-10.9%	11.5%	-29.8%	-6.7%	41.3%	0.0841
1994	-7.0%	0.7%	-5.4%	-8.5%	-4.7%	-2.6%	2.1%	-31.2%	-18.2%	-1.9%	-12.7%	16.8%	-2.5%	17.9%	15.6%	17.3%	0.0%	1.7%	8.3%	-3.0%	-9.2%	17.9%	-31.2%	-1.3%	49.1%	0.1212
1995	-0.1%	6.7%	-6.0%	-14.2%	-0.6%	-4.6%	-3.2%	-53.7%	-13.5%	-8.8%	-17.4%	12.5%	10.3%	17.2%	12.0%	14.5%	-7.2%	53.4%	10.5%	-8.9%	-15.4%	53.4%	-53.7%	-0.8%	107.1%	0.1999
1996	-13.6%	1.7%	-2.7%	-8.8%	4.7%	-4.4%	5.4%	2.8%	-15.8%	-4.8%	-15.1%	19.6%	13.0%	17.9%	8.9%	20.6%	1.6%	70.1%	26.7%	-6.0%	-5.6%	70.1%	-15.8%	5.5%	85.9%	0.1908
1997	-1.4%	-4.8%	17.2%	-1.5%	5.2%	4.1%	-10.8%	-1.2%	-18.3%	0.6%	-8.1%	9.6%	6.7%	11.4%	-5.1%	1.4%	-13.5%	61.7%	47.0%	-5.9%	-11.3%	61.7%	-18.3%	3.9%	80.0%	0.1900
1998	5.8%	6.0%	18.7%	5.3%	9.6%	-2.7%	-6.1%	10.3%	5.4%	-3.8%	-6.7%	13.1%	1.7%	14.5%	-7.0%	27.9%	24.3%	34.5%	38.5%	-1.2%	-3.8%	38.5%	-7.0%	8.8%	45.5%	0.1351
1999	4.2%	-7.1%	31.7%	4.5%	14.8%	-7.7%	-5.5%	49.7%	13.6%	12.8%	-6.0%	13.4%	1.8%	22.2%	-4.6%	16.1%	8.7%	16.4%	41.2%	2.6%	12.2%	49.7%	-7.7%	11.2%	57.4%	0.1546
2000	12.6%	6.5%	23.4%	16.0%	11.5%	-7.9%	12.5%	11.9%	11.7%	8.8%	4.9%	-0.2%	20.3%	24.4%	10.4%	15.3%	-16.5%	18.5%	46.8%	6.2%	12.2%	46.8%	-16.5%	11.9%	63.4%	0.1250
2001	13.2%	-15.4%	17.6%	1.7%	-13.4%	-4.1%	15.6%	-3.1%	1.7%	17.0%	-1.6%	-15.1%	3.6%	18.0%	0.1%	2.3%	106.6%	-18.9%	33.7%	-5.5%	6.0%	106.6%	-18.9%	7.6%	125.5%	0.2621
2002	6.0%	-11.2%	3.9%	18.8%	-9.7%	-2.1%	5.7%	-23.5%	35.0%	0.8%	-4.0%	-15.2%	21.1%	-25.9%	-0.5%	-14.0%	-2.3%	8.7%	24.9%	-17.6%	-3.3%	35.0%	-25.9%	-0.2%	60.9%	0.1584
2003	8.8%	-26.8%	-12.4%	-5.3%	-7.9%	-1.4%	-4.2%	-31.5%	4.1%	-7.8%	0.2%	-15.2%	7.5%	-28.3%	-8.1%	-19.3%	-33.4%	-8.7%	-5.6%	-21.1%	-10.7%	8.8%	-33.4%	-10.8%	42.2%	0.1220
2004	5.7%	-26.9%	-12.7%	-3.5%	-15.7%	6.6%	0.7%	1.8%	2.7%	-8.0%	-5.4%	-11.9%	-0.9%	-18.4%	-3.6%	-21.9%	-39.3%	-18.4%	-29.7%	-25.9%	1.0%	6.6%	-39.3%	-10.7%	45.9%	0.1295
2005	-3.9%	-14.1%	-11.0%	-1.6%	-13.8%	40.1%	-12.1%	-32.4%	11.4%	-3.5%	-5.0%	-14.3%	-0.3%	-16.6%	5.9%	-7.9%	-24.2%	-13.1%	-42.9%	-26.4%	3.4%	40.1%	-42.9%	-8.7%	83.0%	0.1703
2006	11.1%	-10.3%	-2.4%	-6.0%	-11.8%	4.1%	-4.1%	-31.9%	-6.6%	-6.1%	-7.7%	-14.3%	-7.8%	-16.2%	-4.1%	-19.0%	-28.6%	-5.5%	-21.7%	-19.9%	1.6%	11.1%	-31.9%	-9.9%	43.0%	0.1044
2007	-8.1%	-12.6%	-3.7%	1.3%	-7.1%	8.8%	0.1%	-6.1%	0.1%	-6.1%	-9.1%	-15.4%	-9.1%	-4.4%	-0.8%	-13.3%	-11.5%	-2.4%	-18.5%	-5.8%	0.6%	8.8%	-18.5%	-5.9%	27.3%	0.0646
2008	-3.7%	-9.0%	-2.8%	5.0%	-3.7%	-0.9%	12.6%	-18.0%	0.6%	-6.1%	-8.0%	-9.1%	-9.7%	-3.7%	-0.2%	-6.5%	3.9%	18.2%	-15.0%	-3.4%	4.8%	18.2%	-18.0%	-2.6%	36.2%	0.0844
2009	2.5%	-1.9%	-4.3%	1.2%	0.2%	0.6%	0.1%	5.7%	8.8%	-7.4%	-8.5%	-1.2%	-6.1%	-4.2%	-7.0%	-3.3%	-1.4%	-4.6%	-21.4%	1.8%	0.5%	8.8%	-21.4%	-2.4%	30.2%	0.0613
2010																						0.0%	0.0%		0.0%	
For 2001 & Prior only																										
High	13.2%	6.7%	31.7%	16.0%	14.8%	4.1%	15.6%	49.7%	13.6%	17.0%	4.9%	19.6%	20.3%	24.4%	15.6%	27.9%	106.6%	70.1%	47.0%	6.2%	12.2%					
Low	-13.6%	-21.9%	-17.7%	-14.2%	-13.4%	-9.9%	-11.7%	-53.7%	-18.3%	-12.1%	-17.4%	-24.0%	-7.3%	-11.2%	-7.0%	1.4%	-16.5%	-18.9%	-3.5%	-11.2%	-15.4%					
Average	-0.8%	-4.1%	5.9%	-3.1%	0.6%	-4.5%	-0.1%	-4.3%	-3.3%	-1.1%	-7.9%	2.6%	3.8%	10.5%	3.6%	13.2%	7.3%	22.9%	22.6%	-4.6%	-1.9%					
Range	26.8%	28.5%	49.5%	30.2%	28.1%	14.1%	27.3%	103.4%	31.9%	29.1%	22.2%	43.6%	27.7%	35.6%	22.6%	26.5%	123.1%	89.0%	50.5%	17.3%	27.6%					
Std Dev	0.0900	0.0986	0.1634	0.0928	0.0915	0.0372	0.0938	0.2702	0.1312	0.0991	0.0640	0.1506	0.0827	0.1324	0.0787	0.0770	0.3496	0.2900	0.1999	0.0535	0.1002					

Note: 2002 & subsequent is evaluated through the end of 2010

Calendar Year Change in Accident Year Ultimate Losses between 12 and 120 Months **Exhibit 9**
By Group/Company **MM**
Medical Professional Liability

12 to 120 Months of Maturity Change in Ultimate Losses as a Percent of 12 Month Ultimate

Accident Year	Group/Company 9	12	13	20	22	24	26	27	28	29	30	31	32	33	34	35	36	37	High	Low	Average	Range	Std Dev
1991	-9.0%	-32.7%	-36.0%	-10.4%	-90.6%	-42.5%	-7.5%	-29.3%	-22.2%	-41.5%	-8.5%	91.3%	-35.4%	-38.3%		-29.6%	-24.2%	-5.7%	91.3%	-90.6%	-21.9%	181.9%	0.3551
1992	-29.4%	-25.1%	-31.8%	-66.2%	-79.7%	-30.9%	-20.2%	-25.5%	-6.0%	-42.0%	-37.6%	1.3%	-35.8%	-21.2%		-29.6%	-30.0%	-17.3%	1.3%	-79.7%	-31.0%	81.0%	0.1926
1993	-15.9%	-26.9%	-19.2%	-55.5%	-42.5%	-29.5%	-18.7%	-30.5%	-10.4%	-28.8%	-38.5%	158.9%	-24.7%	-37.7%	62.3%	-24.3%	-21.8%	1.4%	158.9%	-55.5%	-11.3%	214.4%	0.4896
1994	-9.7%	-14.8%	-25.1%	-52.6%	20.5%	-33.0%	-10.7%	-28.4%	9.6%	-20.6%	-24.4%	70.7%	-26.7%	-26.1%	39.0%	-25.5%	-7.9%	-23.4%	70.7%	-52.6%	-10.5%	123.3%	0.2914
1995	-7.3%	0.8%	-1.5%	-41.7%	30.3%	-30.5%	3.0%	-27.4%	33.5%	-26.4%	-4.4%	109.7%	-24.3%	81.6%	69.9%	-26.2%	-24.9%	10.8%	109.7%	-41.7%	7.0%	151.4%	0.4267
1996	-5.1%	-7.6%	-15.0%	-53.4%	-30.7%	-27.9%	-4.9%	-13.6%	27.0%	-23.9%	-21.8%	48.2%	-25.0%	118.1%	153.7%	-15.7%	-9.0%	-10.0%	153.7%	-53.4%	4.6%	207.0%	0.5281
1997	-2.3%	-26.0%	4.7%	-31.5%	54.4%	-45.4%	-9.9%	-14.0%	48.1%	-15.8%	3.4%	110.0%	-23.8%	136.6%	169.9%	-9.6%	-11.9%	-10.3%	169.9%	-45.4%	18.1%	215.2%	0.6145
1998	1.8%	-14.2%	17.7%	-21.5%	341.2%	-26.4%	-24.8%	-0.3%	48.0%	-11.6%	15.7%	210.0%	-21.5%	168.8%	214.9%	0.2%	12.1%	10.6%	341.2%	-26.4%	51.1%	367.6%	1.0677
1999	19.4%	-6.7%	-8.7%	-10.0%	167.6%	-13.3%	-9.9%	3.2%	56.0%	-16.4%	15.4%	299.8%	-10.4%	191.6%	165.0%	4.9%	5.8%	18.3%	299.8%	-16.4%	48.4%	316.3%	0.9228
2000	44.3%	-11.1%	-12.5%	-19.1%	-7.3%	-7.3%	-0.1%	29.0%	48.8%	5.9%	26.1%	67.0%	0.6%	181.8%	105.7%	-0.6%	9.9%	6.6%	181.8%	-19.1%	26.0%	200.9%	0.5050
2001	41.5%	-3.9%	3.2%	-30.5%	-0.3%	-12.0%	6.6%	11.4%	46.0%	9.6%	32.9%	206.9%	8.7%	28.6%	36.5%	2.5%	4.4%	16.6%	206.9%	-30.5%	22.7%	237.4%	0.4991
2002	25.1%	-26.4%	2.0%	-35.1%	8.2%	-12.1%	-1.2%	-0.3%	27.5%	-26.9%	25.0%	85.5%	-15.7%	-15.5%	18.1%	-1.0%	-8.0%	4.7%	85.5%	-35.1%	3.0%	120.5%	0.2757
2003	-6.7%	-19.2%	12.7%	-49.7%	-37.2%	-24.7%	6.2%	-21.2%	-10.2%	-28.9%	-4.3%	-13.0%	-30.3%	-56.8%	-60.6%	-18.9%	-17.8%	2.9%	12.7%	-60.6%	-21.0%	73.3%	0.2061
2004	-32.4%	-33.5%	3.0%	-55.2%	-47.7%	-42.7%	-34.0%	-36.9%	-20.8%	-35.2%	-44.0%	-53.7%	-33.8%	-39.4%	-24.9%	-39.6%	-35.3%	-21.3%	3.0%	-55.2%	-34.9%	58.2%	0.1339
2005	-31.5%	-36.3%	-2.5%	-57.9%	-15.1%	-45.5%	-32.5%	-41.9%	-14.6%	-35.1%	-33.5%	-46.8%	-24.6%	-34.0%	-27.7%	-36.4%	-27.2%	-17.3%	-2.5%	-57.9%	-31.1%	55.4%	0.1326
2006	-32.5%	-40.1%	-4.2%	-43.7%	-34.7%	-35.9%	-39.8%	-35.7%	-22.0%	-33.9%	-35.3%	-4.4%	-24.4%	-29.9%	-30.5%	-28.1%	-28.5%	-18.4%	-4.2%	-43.7%	-29.0%	39.5%	0.1103
2007	-16.6%	-46.0%	-23.1%	-49.5%	-11.4%	-22.6%	-39.7%	-24.4%	3.1%	-24.8%	-17.8%	-15.7%	-25.6%	-27.0%	-17.1%	-9.2%	-22.3%	-15.4%	3.1%	-49.5%	-22.5%	52.5%	0.1272
2008	-9.8%	-13.8%	-16.3%	-5.1%	-5.1%	-7.9%	-19.8%	-4.7%	1.7%	-22.5%	-8.3%	-3.8%	-27.4%	-17.4%	-16.7%	1.8%	-13.3%	-7.0%	1.8%	-27.4%	-10.9%	29.2%	0.0809
2009	-4.9%	0.2%	-13.1%	-24.2%	-3.4%	-9.9%	-15.0%	-5.9%	-0.3%	-5.3%	-0.3%	0.7%	-9.4%	-4.0%	-14.2%	0.5%	-5.3%	-5.1%	0.7%	-24.2%	-6.6%	25.0%	0.0668
2010																			0.0%	0.0%		0.0%	
For 2001 & Prior only																							
High	44.3%	0.8%	17.7%	-10.0%	341.2%	-7.3%	6.6%	29.0%	56.0%	9.6%	32.9%	299.8%	8.7%	191.6%	214.9%	4.9%	12.1%	18.3%					
Low	-29.4%	-32.7%	-36.0%	-66.2%	-90.6%	-45.4%	-24.8%	-30.5%	-22.2%	-42.0%	-38.5%	1.3%	-35.8%	-38.3%	36.5%	-29.6%	-30.0%	-23.4%					
Average	2.6%	-15.3%	-11.3%	-35.7%	33.0%	-27.1%	-8.8%	-11.4%	25.3%	-19.2%	-3.8%	124.9%	-19.9%	71.3%	113.0%	-13.9%	-8.9%	-0.2%					
Range	73.7%	33.5%	53.7%	56.2%	431.8%	38.1%	31.4%	59.4%	78.2%	51.6%	71.4%	298.5%	44.6%	230.0%	178.4%	34.5%	42.1%	41.7%					
Std Dev	0.2317	0.1091	0.1646	0.1943	1.2418	0.1203	0.0979	0.1981	0.2790	0.1650	0.2482	0.8646	0.1399	0.9301	0.6499	0.1380	0.1517	0.1404					

Note: 2002 & subsequent is evaluated through the end of 2010

Exhibit 10

Company	HO	PPAL	CAL	WC	CMP	MM
Allstate Insurance Group	x	x	x		x	
Chubb Group of Insurance Companies	x	x	x	x	x	
Farmers Insurance Group	x	x	x	x	x	
Hartford Insurance Group	x	x	x	x	x	
Liberty Mutual Insurance Companies	x	x	x	x	x	
State Farm Group	x	x	x	x	x	
Old Republic General Insurance Group			x	x	x	
Progressive Insurance Group		x	x			
Berkshire Hathaway Insurance Group		x	x	x	x	x
Zenith National Insurance Group				x		
SAIF Corporation				x		
Mutual Insurance Company of Arizona						x
State Volunteer Mutual Insurance Company						x
Canal Group			x			
NJM Insurance Group	x	x	x	x		
State Compensation Insurance Fund of CA				x		
Amerisure Companies			x	x	x	
USAA Group	x	x				
Erie Insurance Group	x	x	x	x	x	
Cincinnati Insurance Companies	x	x	x	x	x	x
Auto-Owners Insurance Group	x	x	x	x	x	
W. R. Berkley Group			x	x	x	x
Great American P & C Insurance Grp			x	x	x	
Medical Mutual Group (MD)						x
Nationwide Group	x	x	x	x	x	
Medical Mutual Group (NC)						x
Doctors Company Insurance Group						x
CNA Insurance Companies			x	x	x	x
ProMutual Group						x
FPIC Insurance Group						x
ACE INA Group			x	x	x	x
NORCAL Group						x
American International Group Inc	x	x	x	x	x	x
Zurich Financial Services NA Group			x	x	x	x
ProAssurance Group						x
MAG Mutual Group						x
ISMIE Mutual Group						x
Travelers Insurance Companies	x	x	x	x	x	
Accident Fund Group				x		
QBE Americas Group	x	x	x	x	x	