Structured Tools to Help Organize One's Thinking When Performing or Reviewing a Reserve Analysis

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Abstract

Actuaries regularly update the results of prior analyses that leverage more current information. Actuaries will often apply similar methodologies and thought processes from the prior analysis to the current one. In doing so, actuaries are employing techniques that help them to evaluate the reasonability of prior assumptions as compared to the most recent data emergence and using judgment to update assumptions ranging from selection of loss development factors to initial expected loss rates to point estimates or ranges of ultimate losses. However, actuarial literature to date provides little guidance on the questions that one can ask during each of these steps and the calculations that can be done to help bring a repeatable rigor to the analysis being done.

This paper will identify three distinct series of exercises that can be performed to help bring just such a repeatable rigor to the analysis. Along the way, the exercises will help the actuary frame answers to the following questions:

1. How did losses emerge between the prior review and the current review in relation to what was expected to emerge?
2. Are the selected loss development factors (LDFs) generally in line with the patterns in the underlying data triangles?
3. What is driving the change in ultimate loss estimates from the prior to the current analysis? Is it data (i.e., loss emergence), change of assumptions (i.e., loss development factors or initial expected loss rates), or change in judgment (i.e., the manner in which a point ultimate is chosen relative to a paid or incurred ultimate loss projection for a given accident period)?

By giving actuaries a structured and repeatable methodology to apply in search of answers to these questions, we are providing actuaries with a framework that will bring them a structure to their analyses and help them to identify areas in their analyses that might benefit from further investigation and study.

Keywords: reserving, suitability testing, data diagnostics

1. INTRODUCTION

The approach described in this paper was developed over several years of working as actuarial consultants and training actuarial students in how to perform an actuarial analysis in a way that considers the work that was done before. It was very easy to hand an actuarial student a client project that they had not worked on before and ask that they update the study, only to have the student do so in a very mechanical way that did not engender the student asking insightful questions about where and why things might have changed from the prior study to the current one.

To remedy this gap, we developed a series of three structured processes that were intended to stimulate critical thinking about the data and the current analysis in a way that would lead the student to identify potential data issues, pattern changes, or other things that would benefit from deeper investigation before concluding the current analysis.

The three structured processes are:

1. Review of Actual versus Expected loss emergence
2. Review of selected loss development factors relative to the factors indicated by the data without an overlay of actuarial judgment
3. A calculation of the source of change between prior and current ultimate loss selections, broken down into three subsets: data, assumptions, and judgment

2. REVIEW OF ACTUAL VERSUS EXPECTED LOSS EMERGENCE

We assume that actuaries are doing an "actual versus expected" study as part of updating an actuarial study. We also assume, unless specifically stated, that there have been no changes to claims handling or case reserving practices. Our methodology employs two actual versus expected calculations and asks questions about the results. Our thought process going into this structured process is to enable us to comment on the following questions:

1. How have the assumptions and conclusions reached in the prior reserve analyses held up when compared to the most recent claims emergence?
2. Are there any significant differences between the actual versus expected results for incurred versus paid claims emergence?
3. Are there any significant differences between the actual versus expected results for direct versus indirect expected claims projections?
4. If the current claims activity is in line with the prior projection, we might reasonably expect current assumptions and ultimate losses to be close to prior assumptions and ultimate losses. Are they?

The two calculation methods we employ in this structured process are "direct" emergence and "indirect" or "percent of reserves" emergence.

2.1 Direct Emergence Method

The formula for calculating expected cumulative incurred losses[1] at time t for Accident Year X using the Direct Emergence Method between time t-1 and time t is as follows:

$$\begin{pmatrix}\text{Cumulative Incurred Losses}\\ \text{@ time } t\text{ - }1 \text{ for Accident Year X}\end{pmatrix} \times \begin{pmatrix}\dfrac{\begin{matrix}\text{Cumulative Development Factor}\\ \text{@ time } t\text{ - }1 \text{ for Accident Year X}\end{matrix}}{\begin{matrix}\text{Cumulative Development Factor}\\ \text{@ time } t \text{ for Accident Year X}\end{matrix}}\end{pmatrix} \tag{1.1}$$

The following table, Table 1, provides an example of the calculation of Direct Emergence Expected Cumulative Losses. The example assumes time t-1 was 12/31/2011 and time t is 12/31/2012.

[1] The methodology for calculating expected emergence of cumulative paid losses is identical to what is shown in the cumulative incurred loss expected emergence formula, except that cumulative paid losses and paid loss development factors are used in place of cumulative incurred losses and incurred loss development factors.

Table 1: Example of Direct Emergence Expected Cumulative Loss Calculation

Accident Year	Current Age	Prior Age	Cumulative Incurred Losses at 12/31/2011	Cumulative Development Factor (CDF) from Prior Actuarial Study	CDF Interpolated to Current Claim Age[2]	Expected Cumulative Incurred Losses at 12/31/2012
			(1)	(2)	(3)	(4) = (1) * (2) / (3)
2004	108	96	621	1.025	1.012	629
2005	96	84	1,468	1.046	1.025	1,498
2006	84	72	1,283	1.072	1.046	1,315
2007	72	60	1,064	1.104	1.072	1,096
2008	60	48	1,510	1.181	1.104	1,615
2009	48	36	857	1.264	1.181	917
2010	36	24	847	1.706	1.264	1,143
2011	24	12	108	22.182	1.706	1,404
TOTAL			**7,758**			**9,618**

2.2 Indirect (Percent of Reserves) Emergence Method

The formula for calculating expected cumulative incurred losses at time *t* for Accident Year X

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[2] The CDF for the oldest loss year cannot be interpolated from the CDFs calculated in the prior study. Instead, the CDF must be extrapolated from the decay pattern in the CDFs in the prior study. The methodology used to derive the 1.012 value was to (a) calculate the rate of change in the three oldest CDFs in Column (2); (b) fit an exponential curve to the resulting rates of change using Excel's "Growth" function; (c) extrapolate the fitted exponential curve one time period into the future; and (d) apply the extrapolated value to the 1.025 value from column (2). The mathematics of this process were as follows:

Accident Year	Current Age	Prior Age	CDF from Prior Actuarial Study	CDF from Prior Actuarial Study - 1	Rate of Change in Column (2) Values	Fitted Rate of Change using Current Age as "X" Value	Extrapolated CDF
			(1)	(2)	(3)	(4)	(5)
2004	108	96	1.025	0.025		0.497	(0.025 * 0.497) + 1 = 1.012
2005	96	84	1.046	0.046	0.025 / 0.046 = 0.551	0.557	
2006	84	72	1.072	0.072	0.046 / 0.072 = 0.636	0.623	
2007	72	60	1.104	0.104	0.072 / 0.104 = 0.691	0.698	

using the Indirect Emergence Method between time *t*-1 and time *t* is as follows:

$$\left\{ \begin{pmatrix} \text{Selected IBNR @} \\ \text{time } t\text{ - }1 \text{ for} \\ \text{Accident Year X} \end{pmatrix} \times \left[\frac{\begin{pmatrix} \text{Percent Incurred} \\ \text{@ time } t \text{ for} \\ \text{Accident} \\ \text{Year X} \end{pmatrix} - \begin{pmatrix} \text{Percent Incurred} \\ \text{@ time } t\text{ - }1 \text{ for} \\ \text{Accident} \\ \text{Year X} \end{pmatrix}}{\begin{matrix} 1 - \text{Percent Incurred @} \\ \text{time } t\text{ - }1 \\ \text{for Accident Year X} \end{matrix}} \right] \right\} + \begin{pmatrix} \text{Cumulative} \\ \text{Incurred} \\ \text{Losses} \\ \text{@ time } t\text{ - }1 \\ \text{for Accident} \\ \text{Year X} \end{pmatrix} \quad (1.2)$$

Note: A CDF is converted into a percent incurred factor by taking the reciprocal of the CDF, i.e., the percent incurred at a given loss year age equals (1 / CDF).

Note 2: When applying this formula to paid losses, instead of using IBNR as shown in formula (1.2), use unpaid losses.

The following table provides an example of the calculation of Indirect Emergence Expected Cumulative Losses.

Table 2: Example of Indirect Emergence Expected Cumulative Loss Calculation

Accident Year	Current Age	Prior Age	Cumulative Incurred Losses at 12/31/11	Selected IBNR at 12/31/11	Percent Incurred at Prior Age	Percent Incurred at Current Age	Expected Cumulative Incurred Losses at 12/31/2012
			(1)	(2)	(3) *	(4) **	(5) ***
2004	108	96	621	0	97.6%	98.8%	621
2005	96	84	1,468	50	95.6%	97.6%	1,490
2006	84	72	1,283	67	93.3%	95.6%	1,306
2007	72	60	1,064	86	90.6%	93.3%	1,089
2008	60	48	1,510	240	84.7%	90.6%	1,602
2009	48	36	857	443	79.1%	84.7%	975
2010	36	24	847	703	58.6%	79.1%	1,195
2011	24	12	108	1,417	4.5%	58.6%	911
TOTAL			**7,758**	**3,006**			**9,190**

* Values in column (3) equal 1 / value in Table 1, Column (2).
** Values in column (4) equal 1 / value in Table 1, Column (3).
*** Values in column (5) equal $\left\{(2)\times\left[{(4)-(3)}\big/{1-(3)}\right]\right\}+(1)$

2.3 Comparing Direct and Indirect Expected Results Using a Simplified Example

If ultimate losses are selected to be exactly equal to the direct development ultimate loss value, there will be no difference in actual versus expected results under either method. Differences only arise when selected ultimate losses are different than the direct development ultimate loss value.

This can be demonstrated with the following simplified data and example:

Table 3: A Priori Expected Loss Emergence Pattern

Development Age	0 – 1	1 – 2	2 – 3	Total
Incremental Loss Emergence	1,000	500	250	
Cumulative Loss Emergence	1,000	1,500	1,750	1,750

Table 4: Selected Loss Development Pattern Based on A Priori Expected Loss Emergence Pattern

Development Age	0 – 1	1 – 2	2 – 3
Incremental LDF	n/a	1.500	1.167
Cumulative LDF	n/a	1.750	1.167
Percent Incurred	57.1%	85.7%	100.0%

Table 5: Direct versus Indirect Expected Loss Emergence @ Time 2 – assumes that incurred losses at Time 1 = 1,000 and selected ultimate losses = 1,750

Development Age	Selected Ultimate Loss	Actual Cumulative Incurred Losses @ Time 1	Expected Cumulative Incurred Losses @ Time 2
Direct Expected Loss Emergence	1,750	1,000	1000 * (1.750 / 1.167) = 1,500
Indirect Expected Loss Emergence	1,750	1,000	$750 * \frac{0.857 - 0.571}{1 - 0.571} + 1000 = 1{,}500$

Table 5 demonstrates that when selected ultimate losses exactly equal the direct development projection of ultimate losses, the direct and indirect expected loss emergence calculations produce equivalent results. Table 6 shows what happens when selected ultimate losses are not exactly equal to the direct development projection of ultimate losses. For Table 6, we change the example as follows:

Incurred losses at Time 1 = 1,400

Direct Development Ultimate Loss Projection = 1,400 * 1.750 = 2,450

Selected Ultimate Losses = 2,000

Table 6: Direct versus Indirect Expected Loss Emergence @ Time 2 – assumes that incurred losses at Time 1 = 1,400 and selected ultimate losses = 2,000

Development Age	Selected Ultimate Loss	Actual Cumulative Incurred Losses @ Time 1	Expected Cumulative Incurred Losses @ Time 2
Direct Expected Loss Emergence	2,000	1,400	1400 * (1.750 / 1.167) = 2,100
Indirect Expected Loss Emergence	2,000	1,400	$600 * \frac{(0.857 - 0.571)}{1 - 0.571} + 1400 = 1{,}800$

As can be seen in Table 6, when selected ultimate losses do not equal the direct development ultimate loss projection, the direct and indirect expected loss calculation produce different expected loss amounts in the projected time period.

2.4 Interpreting Actual versus Expected Results from Simplified Example

Continuing with the simplified example from Table 6, suppose the actual incurred loss amount at Time 2 was 2,000. As shown in Table 7 below, our direct development actual versus expected result shows actual losses are $100 below expected and our indirect development actual versus expected result shows actual losses to be $200 above expected.

Table 7: Actual versus Direct and Indirect Expected Loss Emergence @ Time 2 – assumes that incurred losses at Time 1 = 1,400 and selected ultimate losses = 2,000

Development Age	Expected Cumulative Incurred Losses @ Time 2	Actual Cumulative Incurred Losses @ Time 2	Actual minus Expected Losses
Direct Expected Loss Emergence	1400 * (1.750 / 1.167) = 2,100	2,000	(100)
Indirect Expected Loss Emergence	$600 * \frac{0.857 - 0.571}{1 - 0.571} + 1400 = 1{,}800$	2,000	200

Focusing first on the direct development result, we can interpret the result to mean that actual losses have not emerged as quickly as expected. This might reasonably lead us to conclude that any subsequent development will also be lower than what we would have expected from the selected loss development pattern underlying the direct development calculation. This would lead us to consider a new selected ultimate loss that is something less than the direct development ultimate loss projection based on actual loss emergence through Time 2 (= $2,000 * 1.167 = $2,333). Alternatively, in this example, we might argue that we saw higher than expected loss emergence during Time 1 and lower than expected loss emergence during Time 2 and going forward, we will return to a loss emergence pattern that is more consistent with the historical expectations for Time 3 and beyond than what we have seen for Times 1 and 2. This counter-argument would be a reason to select $2,333 as our new ultimate loss indication.

Turning next to the indirect development result, we can interpret the result to mean that actual losses have emerged more quickly than our selected ultimate loss pick would have led us to expect. This might reasonably suggest that our selected ultimate loss pick was too low and, given what we now know, should be increased. When taking this information in conjunction with the observation that the direct development expectation for Time 2 was higher than the actual loss emergence in Time 2, we might consider selecting a new ultimate loss estimate that is higher than the $2,000 that we chose in the prior actuarial analysis but is not as high as is indicated by the current direct development ultimate loss projection ($2,000 * 1.167 = $2,333) because we think the remaining loss emergence will follow the Time 2 pattern where actual losses emerge lower than the direct development expectation.

We can summarize our analysis methodology from this section as providing actuaries with tools to critically consider how well the ultimate loss picks from prior years' reviews are holding up when compared to actual loss emergence in the most recent time period and give guidance for the direction and magnitude by which we might want to adjust ultimate loss selections.

2.5 Actual vs. Expected Results for Original Example

We can now return to the original example from Tables 1 and 2 and compare actual loss emergence to the direct and indirect expected emergence.

Table 8: Actual vs. Expected Loss Emergence for Original Example

	Expected Loss	Actual Loss	Actual - Expected
	(1)	(2)	(2) – (1)
Direct Method	9,618	9,458	(160)
Indirect Method	9,190	9,458	268

We can see in Table 8 that actual losses have emerged $160 below expectations on a direct basis but $268 above expectation on an indirect basis. The lower than expected emergence on a direct basis implies that the selected loss development factors may be too high, as losses projected to emerge in the period were higher than losses that actually emerged. However, the higher than expected emergence on an indirect basis implies that the selected ultimates might be low. The direct method is independent of the prior selected ultimate losses and uses only the cumulative incurred losses and selected loss development pattern while the indirect method uses the cumulative incurred losses, selected loss development pattern, and the prior selected ultimate losses. Understanding this, the actuary might want to consider decreasing loss development factors but increasing initial expected losses or selecting ultimate losses based on higher methods.

2.6 Considerations When Assessing the Direct and Indirect Expected Loss Emergence Results

As was noted in Section 1.3, when ultimate losses are selected to be exactly equal to the direct development ultimate loss value, there will be no difference in actual versus expected results under either method. Differences only arise when selected ultimate losses are different than the direct development ultimate loss value. With this understanding of the driver of differences in Direct versus Indirect results, we can better evaluate the meaning of the results being produced by the two methods.

1. The Indirect method incorporates a judgmental element that the direct method does not, namely the selected ultimate loss value from the prior analysis. The Indirect Actual versus Expected result provides us with a quantitative way of assessing the consistency of the selected ultimate loss value from the prior analysis with the most recent actual loss emergence.

2. The Direct method provides us with a quantitative way of assessing the extent to which the most recent actual loss emergence is or is not consistent with the emergence pattern we believe should exist (as quantified through the emergence pattern implicit in our LDF pattern). If we know that Accident Year X losses through time t-1 were lower than (higher than) what we were expecting to see at time t-1, but we do not see the actual emergence during time t coming in higher than (lower than) the Direct method expectation, we may need to dig deeper to understand why Accident Year X's losses are coming in below (above) our a priori ultimate loss expectation. For example, is claim frequency in Accident Year X different from other accident years? Or is claim severity distorting results, as might occur if there are fewer than (more than) the expected number of large losses reported to date?

To summarize, when the Direct and Indirect methods produce results that either differ in magnitude or, as in the examples shown previously, direction, the actuary has an opportunity to think about his or her a priori ultimate loss expectations as compared to the actual data reported to date. If the actual data is deviating from the a priori expectations, is this because something structural is changing in the data, such as a change in claim frequency? Or is it because the data is inherently volatile and the differences are due to random events that do not require the actuary to change his or her long term expectations?

For example, when we see large divergences between actual and expected results when the two methods are applied to what had been blocks of business with historically stable emergence patterns, we have good reason to call into question the reliability of the actual results. In this situation, we might want to ask ourselves questions along the lines of:

- Might there be something wrong with the data we are seeing?
- Has there been a change in claims handling practices that we were not aware of that would lead to an acceleration or deceleration in claim reporting?
- Has there been a change in the way case reserves are being set?

The thought process is more complex when we are looking at blocks of business that are more volatile because we have more randomness in the data with which to contend. This does not mean we should not ask the same questions as we would ask when looking at a more stable block of business. It just means that we may need to accept more volatility in the actual versus expected results. It also means we may need to dig deeper to understand if there really is a structural change occurring as opposed to just random noise in the data. Digging deeper may mean we need to look at:

- Claim counts instead of total losses;
- Data stratifications by claim size to assess if the differences are coming from changes in the mix of large versus small losses;
- Capped versus excess losses[3]
- Historical levels of volatility in less versus more mature accident periods to evaluate if the magnitude of the differences being observed have been seen before or not.
- Adjusting the data to remove calendar year inflationary trends, if the trend rates have fluctuated significantly over the time period being used to derive the expected loss development patterns

Neither method is inherently "better" than the other. We believe maximum value is achieved when they are used in conjunction so that differences between the two methods can be identified, analyzed and understood. Additionally, we have no hard and fast rule for when ultimate loss selections should be adjusted in response to actual versus expected emergence differences. Our objective with these methods is not to provide a formulaic way to get to the "right" answer, but rather to describe tools that we have found effective at helping us identify the right questions to be asked during our analysis.

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[3] When performing an actual versus expected analysis using capped losses or excess losses, an additional layer of quantitative rigor needs to be incorporated into the application of formulas 1.1 and 1.2. The expected losses being calculated need to align with the capping or claim attachment points being applied to the actual data. For example, if there are large losses in the actual data at time t-1 and the application of the Direct Development method loss development factor would cause one or more claims to exceed the selected loss cap, this excess amount needs to be removed from the Direct Development expectation.

3. REVIEW OF SELECTED LOSS DEVELOPMENT FACTORS VERSUS "LETTING THE DATA SPEAK"

Once loss development factors have been selected, a reviewer must assess the overall reasonability of the LDF selections. We split this exercise into two subsets:

1. Age to age factors for which there is historical data
2. Tail factors, where there are no (or no reliable) observable data points

3.1 Review of Age to Age Factors

The review of age to age factors is done by performing a series of sensitivity tests on the underlying data while keeping the selected tail factor (and possibly the oldest age to age factors for which there is limited data) unchanged. The objective of these sensitivity tests is to assess the extent to which the selected LDFs are in line with the patterns in the data. However, having the selected LDFs in line with the patterns in the data does not necessarily mean the selected LDFs are reasonable. There could be numerous reasons that the selected LDFs should not be in line with the patterns in the data. For example, the data might contain large loss distortions that should be ignored or smoothed when selecting LDFs. Another example is changes in business mix in the historical data that is driving a change in the loss emergence pattern. In this case, the history might not be reasonably reflective of the current book of business.

These questions serve to highlight that the true importance of this test is not if the selected LDFs align or do not align with the historical patterns; rather it is so the reviewer can think about what the selected LDFs ought to look like as compared to the historical patterns and assess if the selections are consistent with his/her expectations.

The sensitivity testing compares the selected LDFs to the LDFs that would be indicated by different calculated averages. The calculated averages to use for sensitivity comparison should include different time periods (e.g., 3 year average, 5 year average) and different weighting schemes (e.g., 5 ex hi/lo, highest or 2nd highest (lowest) of the last five, weighted versus straight averages). This range of weighting schemes will include some combinations that can reasonably be expected to be biased high (such as the highest of the last five observations) and others that we expect will be biased low (such as the lowest of the last five or the 5 ex hi/lo average[4]). By including weighting schemes that will tend to be biased in one direction or the other, the selected LDFs can be compared to a sufficiently wide range of LDF alternatives to facilitate a comparison of the selected LDFs to the unadjusted patterns present in the data alone.

Additionally, we can tie our Section 1 analysis into this analysis. Where the most recent diagonal of claim emergence differs from what was expected to emerge, this difference will be reflected the most recent diagonal of LDFs. By including this most recent diagonal in the various LDF averages being calculated for comparison against the selected LDFs, we are implicitly factoring into this section's analysis our Direct Development actual versus expected results so that we can further assess how we might want to adjust our new LDF picks in response to the actual versus expected results.

For this example, we will use the following table of incurred losses:

Table 9: Incurred Loss Data Triangle (Dollars in Thousands)

Accident	Development Age								
Year	12	24	36	48	60	72	84	96	108
2004	49	402	504	570	569	624	652	621	621
2005	37	1,297	1,529	1,448	1,384	1,423	1,468	1,452	
2006	122	777	988	1,086	1,300	1,283	1,232		
2007	137	804	935	888	1,064	1,131			
2008	57	751	1,407	1,510	1,759				
2009	56	830	857	850					
2010	38	847	1,122						
2011	108	1,291							
2012	114								

[4] For discussion of the downward bias in the 5 ex hi/lo average, see "Downward Bias of Using High-Low Averages for Loss Development Factors" by Cheng-Sheng Peter Wu, Casualty Actuarial Society Summer 1997 Forum, Volume 1, pages 197-240 and 1999 Proceedings of the Casualty Actuarial Society, Volume LXXXVI, pages 699 – 735.

The corresponding loss development factors from this data triangle are as follows:

Table 10: Incurred Loss Development Factors and Loss Development Factors Averages[5]

	Development Period								
Accident Year	**12 - 24**	**24 - 36**	**36 - 48**	**48 - 60**	**60 - 72**	**72 - 84**	**84 - 96**	**96 - 108**	**108 - Ult**
2004	8.204	1.254	1.131	0.998	1.097	1.045	0.952	1.000	
2005	35.054	1.179	0.947	0.956	1.028	1.032	0.989		
2006	6.369	1.272	1.099	1.197	0.987	0.960			
2007	5.869	1.163	0.950	1.198	1.063				
2008	13.175	1.874	1.073	1.165					
2009	14.821	1.033	0.992						
2010	22.289	1.325							
2011	11.954								
2012									
3 point average	16.355	1.410	1.005	1.187	1.026	1.012	0.971	1.000	
5 point average	13.622	1.333	1.012	1.103	1.044	1.012	0.971	1.000	
7 point average	15.647	1.300	1.032	1.103	1.044	1.012	0.971	1.000	
3 point wtd avg	14.693	1.395	1.015	1.183	1.024	1.007	0.978	1.000	
5 point wtd avg	11.422	1.324	1.012	1.104	1.033	1.007	0.978	1.000	
7 point wtd avg	11.886	1.286	1.021	1.104	1.033	1.007	0.978	1.000	
5 point ex hi/lo	13.317	1.253	1.005	1.120	1.044	1.012			
Largest LDF	35.296	1.873	1.132	1.198	1.096	1.045	0.989	1.000	
2nd largest LDF	22.131	1.324	1.099	1.197	1.063	1.032	0.953		
2nd smallest LDF	6.363	1.162	0.950	0.997	1.028	1.032	0.989		
Smallest LDF	5.876	1.032	0.947	0.955	0.987	0.960	0.953	1.000	
Selected LDF	13.000	1.400	1.070	1.070	1.030	1.020	1.015	1.007	1.005

For the remainder of this example, we replace the calculated averages for ages 84 and beyond with the selected LDF for ages 84 and beyond. Doing this provides stability to the different averages where the data is very sparse.

[5] When fewer data points are available than are needed to calculate a particular average or weighted average loss development factor, the averaging formula is adjusted to use the number of data points that are available. For example, the age 27-39 "7 point average" value is an average of the six available age 27-39 LDFs and the age 39-51 "7 point average" value is an average of the five available age 39-51 LDFs.

Table 11: Loss Development Factors Averages Being Used for Sensitivity Testing

	Development Period								
	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108	108 - Ult
3 point average	16.355	1.410	1.005	1.187	1.026	1.012	1.015	1.007	1.005
5 point average	13.622	1.333	1.012	1.103	1.044	1.012	1.015	1.007	1.005
7 point average	15.647	1.300	1.032	1.103	1.044	1.012	1.015	1.007	1.005
3 point wtd avg	14.693	1.395	1.015	1.183	1.024	1.007	1.015	1.007	1.005
5 point wtd avg	11.422	1.324	1.012	1.104	1.033	1.007	1.015	1.007	1.005
7 point wtd avg	11.886	1.286	1.021	1.104	1.033	1.007	1.015	1.007	1.005
5 point ex hi/lo	13.317	1.253	1.005	1.120	1.044	1.012	1.015	1.007	1.005
Largest LDF	35.296	1.873	1.132	1.198	1.096	1.045	1.015	1.007	1.005
2nd largest LDF	22.131	1.324	1.099	1.197	1.063	1.032	1.015	1.007	1.005
2nd smallest LDF	6.363	1.162	0.950	0.997	1.028	1.032	1.015	1.007	1.005
Smallest LDF	5.876	1.032	0.947	0.955	0.987	0.960	1.015	1.007	1.005
Selected LDF	13.000	1.400	1.070	1.070	1.030	1.020	1.015	1.007	1.005

Table 12: Cumulative Loss Development Factors Averages Being Used for Sensitivity Testing

	12 - Ult	24 - Ult	36 - Ult	48 - Ult	60 - Ult	72 - Ult	84 - Ult	96 - Ult	108 - Ult
3 point average	29.344	1.794	1.272	1.266	1.067	1.040	1.027	1.012	1.005
5 point average	21.997	1.615	1.211	1.197	1.085	1.040	1.027	1.012	1.005
7 point average	25.118	1.605	1.235	1.197	1.085	1.040	1.027	1.012	1.005
3 point wtd avg	26.062	1.774	1.272	1.253	1.059	1.034	1.027	1.012	1.005
5 point wtd avg	18.054	1.581	1.194	1.180	1.068	1.034	1.027	1.012	1.005
7 point wtd avg	18.424	1.550	1.205	1.180	1.068	1.034	1.027	1.012	1.005
5 point ex hi/lo	20.383	1.531	1.222	1.216	1.085	1.040	1.027	1.012	1.005
Largest LDF	105.476	2.988	1.595	1.409	1.176	1.073	1.027	1.012	1.005
2nd largest LDF	43.437	1.963	1.482	1.349	1.127	1.060	1.027	1.012	1.005
2nd smallest LDF	7.632	1.199	1.032	1.086	1.090	1.060	1.027	1.012	1.005
Smallest LDF	5.338	0.908	0.880	0.930	0.973	0.986	1.027	1.012	1.005
Selected LDF	22.487	1.730	1.236	1.155	1.079	1.048	1.027	1.012	1.005

To calculate ultimate losses using the different CDF averages, we take the cumulative incurred losses for each accident year at time t and multiply by the CDFs in Table 12.

Table 13: Projected Ultimate Losses Using Different CDF Averages from Table 12

Accident Year	2012	2011	2010	2009	2008	2007	2006	2005	2004
Incurred Loss	114	1,291	1,122	850	1,759	1,131	1,232	1,452	621
3 point average	3,345	2,316	1,428	1,076	1,877	1,176	1,266	1,469	624
5 point average	2,508	2,085	1,359	1,017	1,909	1,176	1,266	1,469	624
7 point average	2,863	2,072	1,386	1,017	1,909	1,176	1,266	1,469	624
3 point wtd avg	2,971	2,290	1,427	1,065	1,862	1,169	1,266	1,469	624
5 point wtd avg	2,058	2,041	1,339	1,003	1,879	1,169	1,266	1,469	624
7 point wtd avg	2,100	2,001	1,352	1,003	1,879	1,169	1,266	1,469	624
5 point ex hi/lo	2,324	1,976	1,371	1,033	1,909	1,176	1,266	1,469	624
Largest LDF	12,024	3,858	1,790	1,198	2,069	1,214	1,266	1,469	624
2nd largest LDF	4,952	2,534	1,663	1,147	1,982	1,199	1,266	1,469	624
2nd smallest LDF	870	1,548	1,158	924	1,917	1,199	1,266	1,469	624
Smallest LDF	609	1,173	988	790	1,712	1,115	1,266	1,469	624
Selected LDF	2,564	2,233	1,386	982	1,898	1,185	1,266	1,469	624

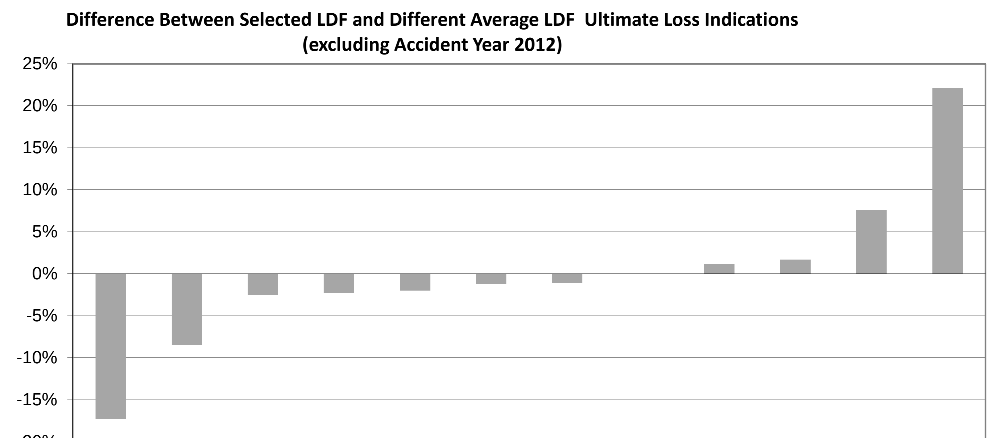
A judgmental decision is required relating to the inclusion or exclusion of the least mature accident years in the LDF comparison. Because the loss development factors being applied to the least mature accident years can contain a high degree of volatility from average to average, the analysis might benefit from excluding one or more years from consideration. In Table 14, we exclude the 2012 year for just this reason.

Table 14: Comparison of Ultimate Loss Indications Between Selected and Different Average LDF Calculations

	Total Ultimate Losses	Dollar Variance with Selected Total	Percentage Variance with Selected Total	Total Ultimate Losses ex. AY 2012	Dollar Variance with Selected Total ex. AY 2012	Percentage Variance with Selected Total ex. AY 2012
Incurred Loss	9,572			9,458		
3 point average	14,577	970	7%	11,232	188	2%
5 point average	13,413	-194	-1%	10,905	-138	-1%
7 point average	13,783	176	1%	10,919	-124	-1%
3 point weighted average	14,143	536	4%	11,172	129	1%
5 point weighted average	12,849	-758	-6%	10,791	-253	-2%
7 point weighted average	12,864	-743	-5%	10,764	-279	-3%
5 point ex hi/lo	13,147	-460	-3%	10,824	-220	-2%
Largest LDF	25,513	11,906	87%	13,489	2,445	22%
2nd largest LDF	16,836	3,229	24%	11,884	840	8%
2nd smallest LDF	10,975	-2,632	-19%	10,105	-938	-8%
Smallest LDF	9,745	-3,862	-28%	9,137	-1,906	-17%
Selected LDF	13,607			11,043		

We can also look at the results of Table 14 graphically, as follows:

Table 15: Comparison of Ultimate Loss Indications Between Selected and Different Average LDF Calculations



	Smallest LDF	2nd smallest LDF	7 point weighted average	5 point weighted average	5 point ex hi/lo	5 point average	7 point average	Selected LDF	3 point weighted average	3 point average	2nd largest LDF	Largest LDF
Variance to Selected	-17%	-8%	-3%	-2%	-2%	-1%	-1%	0%	1%	2%	8%	22%

From the results in Tables 14 and 15, we observe that if we consider just the 3, 5 and 7 year averages for accident years excluding 2012, the ultimate indications are all within 2 to 3% of the ultimate indications using the selected LDFs. While the consistency of the different averages should give us comfort that the selected LDFs are a reasonable representation of the historical data pattern, we might also observe that the shorter the average, the higher the indicated ultimates. This observation should cause us to (a) examine the historical data more closely for indications LDFs are increasing and (b) assess if our selected LDFs might be aligned more closely to the 3 year averages than being somewhere between the 3 and 5 year averages.

3.2 Review of Tail Factors

As the tail factor selection impacts the ultimate loss indication for every accident period not yet at ultimate, the value being selected can have a considerable impact on the overall reserve indication. We do not ignore the importance of this actuarial assumption; however it is one that has been written about in several other papers. Rather than reiterate what was discussed in those other papers, we refer the reader to a 2006 paper by Joseph A. Boor ("Estimating Tail Development Factors: What to Do When the Triangle Runs Out", by Joseph A. Boor, Casualty Actuarial Society Winter 2006 Forum, pages 345-390) for further guidance in selecting appropriate tail factors and assessing the reasonability of the tail factors being selected.

4. "SOURCE OF CHANGE" CALCULATION

Often the first thing we will look at when reviewing an analysis is the change in ultimate losses. Our methodology breaks down the drivers of this change into three categories: data, assumptions, and judgment. This enables us to comment on the following questions:

1. What is the impact on ultimate loss estimates of data emerging in a different pattern than expected?
2. What impact will changing an assumption have on the ultimate loss estimates?
3. Do any changes in assumptions make sense in relation to what is happening in the data?
4. Are ultimates selected in a consistent manner relative to the method results? And if not, is this inconsistency reasonable and explainable?

In order to measure the sources of change, we must first calculate three Bornhuetter-Ferguson (BF)[6] method values[7].

A. BF method with prior data and prior assumptions: this is the BF method from the prior analysis that uses data as of time t-1 and assumptions underlying the analysis as of time t-1

B. BF method with current data and prior assumptions: this is an interim value that uses updated data as of time t but assumptions underlying the analysis as of time t-1. Note that LDFs must be interpolated to the proper ages as of time t.

C. BF method with current data and current assumptions: this is the BF method from the current analysis that uses updated data as of time t and updated assumptions underlying the analysis as of time t.

These method results will be referred to as Method A, Method B, and Method C throughout the remainder of this section.

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[6] "The Actuary and IBNR" by Ronald L, Bornhuetter and Ronald E. Ferguson, 1972 Proceedings of the Casualty Actuarial Society, Volume LIX, pages 181-195.

[7] If exposures are not available, the same process can be followed using the Loss Development Method as a base. However, our experience in using this methodology is that it works best with paid and incurred BF method results because the BF methods tend to stabilize potential swings in the indicated ultimate losses as compared with direct development methods.

The following table, Table 16, provides an example of the calculation of Methods A, B, and C. The example assumes time *t*-1 was 12/31/2011 and time *t* is 12/31/2012.

Table 16: Example of BF Method Recalculation

Accident Year	Prior Incurred Loss	Prior Initial Expected Loss	Prior Percent Incurred at time *t*-1	Method A
	(1)	(2)	(3)	(4) *
2004	621	682	97.6%	638
2005	1,468	1,470	95.6%	1,533
2006	1,283	1,405	93.3%	1,377
2007	1,064	1,045	90.6%	1,162
2008	1,510	1,600	84.7%	1,755
2009	857	1,574	79.1%	1,186
2010	847	1,539	58.6%	1,484
2011	108	1,539	4.5%	1,578
TOTAL	**7,758**	**10,854**		**10,713**

Accident Year	Current Incurred Loss	Prior Percent Incurred at time *t*	Method B
	(5)	(6)	(7) **
2004	621	98.8%	629
2005	1,452	97.6%	1,488
2006	1,232	95.6%	1,294
2007	1,131	93.3%	1,201
2008	1,759	90.6%	1,910
2009	850	84.7%	1,091
2010	1,122	79.1%	1,443
2011	1,291	58.6%	1,928
TOTAL	**9,458**		**10,984**

Accident Year	Current Initial Expected Loss	Current Percent Incurred at time *t*	Method C
	(8)	(9)	(10) ***
2004	621	99.5%	624
2005	1,475	98.8%	1,470
2006	1,350	97.4%	1,268
2007	1,150	95.4%	1,183
2008	1,750	92.7%	1,887
2009	1,300	86.6%	1,024
2010	1,442	80.9%	1,397
2011	1,875	57.8%	2,082
TOTAL	**10,963**		**10,935**

* Values in column (4) equal (2) * [100% - (3)] + (1)
** Values in column (7) equal (2) * [100% - (6)] + (5)
** Values in column (10) equal (8) * [100% - (9)] + (5)

4.1 Change Due to Data

The first source of change considered is the change driven by the actual data. Unless losses have emerged exactly as expected, updating the loss experience in the analysis will change the resulting method values. We can quantify the difference driven by the data by recalculating the BF test using current data but keeping the assumptions the same as the prior analysis (interpolated to the current ages) and comparing this to the BF test in the prior analysis. This is Method B minus Method A.

Continuing the example from above, Table 17 shows the change due to data

Table 17: Example of Ultimate Loss Change Due to Data

Method B	Method A	Data Difference
(1)	(2)	(1) – (2)
10,984	10,713	272

The results should be similar to the indirect actual vs. expected results. However, rather than just telling us how much actual loss emergence differed from expected within the period, the change due to data extrapolates that difference to tell us how much the change in data impacts the ultimate loss estimates.

An increase in method results due to the change in data implies that either the assumptions underlying the prior analysis projected too little development in the period or that the ultimate losses from the prior analysis should be increased or some combination of the two.

4.2 Change Due to Changes in Assumptions

The next source of change considered is the change due to changing assumptions, such as loss development factors or initial expected losses. The additional insight that comes from having an additional year of data may lead us to change our assumptions. We can isolate this change by comparing the BF tests calculated with the same data where the only difference is changing the prior assumptions to the current assumptions. This is Method C minus Method B.

Continuing the example from above, Table 18 shows the change due to changes in assumptions. [8]

Table 18: Example of Ultimate Loss Change Due to Change in Assumptions

Method C	Method B	Assumptions Difference
(1)	(2)	(1) – (2)
10,935	10,984	-49

The results show us that the actuary has lowered assumptions from the prior analysis to the current analysis.

4.3 Change Due to Judgment

The remaining change in ultimate loss is attributable to the often elusive concept of "actuarial

[8] For methods with multiple assumptions, we can break out the change into assumptions into each individual assumption change, if desired. To accomplish this, calculate successive method values changing one assumption at a time and calculating the difference between each successive step. As an example, we look at the BF method and isolate the change in age to age factors, tail factor, and initial expected loss. Calculate the following:
Method B1: BF method using current data and all prior assumptions (interpolated to the current age)
Method B2: BF method using current data, current age to age factors, prior tail factor (interpolated to the current age) and prior initial expected loss
Method B3: BF method using current data, current age to age factors, current tail factor, and prior initial expected loss
Method C: BF method using current data and all current assumptions

It then follows that:
Method B2 – Method B1 = change due to change in age to age factors
Method B3 – Method B2 = change due to change in tail factor
Method C – Method B3 = change due to change in initial expected loss

judgment". However, we can actually calculate this judgment component from the selected ultimate losses and the calculated method values.

If we define judgment as the amount that the selected ultimate loss differs from the method values, we can then calculate the change in judgment in successive actuarial analyses. The key is that the base method used for comparison (whether a single method or some combination of methods) must be the same base method used in the change in assumptions analysis and the same method must be used as a base for both the prior and current analysis. In this example, our base method is the incurred BF method.

Table 19 calculates the judgment built into ultimate loss selections in both the current and prior analyses in our example.

Table 19: Calculation of Judgment

Accident Year	Prior BF Method (Method A)	Prior Selected Ultimate Loss	Judgment in Prior Analysis	Current BF Method (Method C)	Current Selected Ultimate Loss	Judgment in Current Analysis
	(1)	(2)	(3) = (2) – (1)	(4)	(5)	(6) = (5) – (4)
2004	638	621	-17	624	621	-3
2005	1,533	1,475	-58	1,470	1,425	-45
2006	1,377	1,350	-27	1,268	1,250	-18
2007	1,162	1,150	-12	1,183	1,168	-15
2008	1,755	1,750	-5	1,887	1,788	-99
2009	1,186	1,300	114	1,024	1,038	14
2010	1,484	1,550	66	1,397	1,450	53
2011	1,578	1,525	-53	2,082	1,900	-182
TOTAL	**10,713**	**10,721**	**8**	**10,935**	**10,640**	**-295**

The change due to judgment is thus Column (6) minus Column (3). This can also be written out as Change due to Judgment = [Current Selected Ultimate Loss – Current BF Method] – [Prior Selected Ultimate Loss – Prior BF Method].

Table 20: Example of Ultimate Loss Change Due to Change in Judgment

Judgment in Current Analysis	Judgment in Prior Analysis	Judgment Difference
(1)	(2)	(1) – (2)
-295	8	-304

We can also demonstrate that this is simply the remaining difference in selected ultimate losses after defining the difference due to data and the difference due to assumptions.

Table 21: Remaining Difference in Ultimate Loss Change

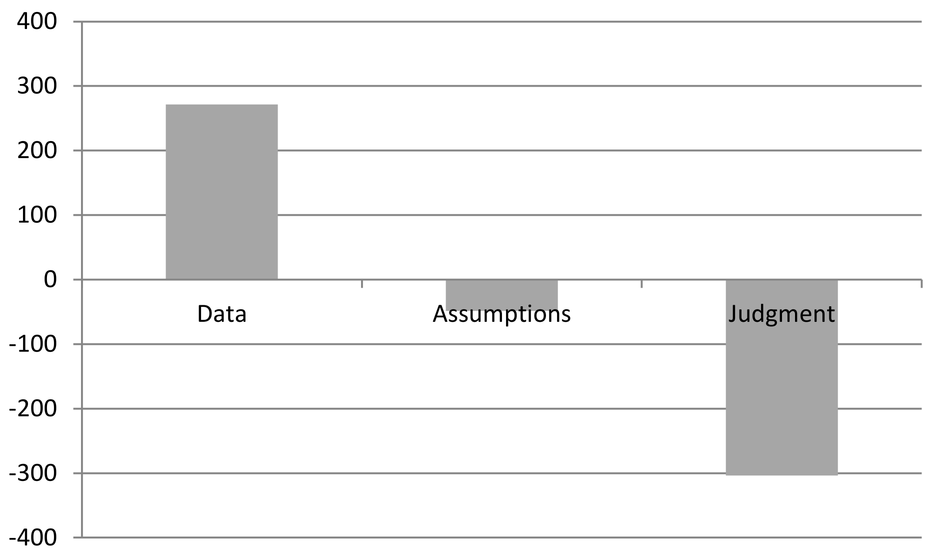
Prior Selected Ultimate Loss	Current Selected Ultimate Loss	Change in Selected Ultimate Loss	Data Difference	Assumption Difference	Remaining Difference (Judgment Difference)
(1)	(2)	(3) = (2) – (1)	(4)	(5)	(6) = (3) – (4) – (5)
10,721	10,640	-81	272	-49	-304

4.4 Interpreting Source of Change Results

Examining the sources of change allows us to ask and answer many questions about the analysis. We now know how much of the change in ultimate losses is due to the actual loss data emerging differently than expected as opposed to changes that the actuary is making in either assumptions or judgment.

We have also often found it beneficial to present this information graphically, such as the following.

Table 22: Sources of Change



In our example we see that the impact of data is an increase of $272 which is offset by a decrease in assumptions of $49 and a decrease in judgment of $304. At this point, we may ask why the actuary is lowering assumptions and judgment when the data is indicating an increase. There may be valid reasons for this, such as if the increase in data is driven by a single large loss or adverse loss emergence in a single year.

If the results do not make sense at first glance, it is often helpful to break the changes down into smaller steps. One can break out the assumptions into individual changes as discussed in footnote 5, or look at the change for each component for each individual accident year. Often there is a single year that skews overall results and if we look at results excluding that year, the picture becomes clearer. Continuing our example from above, we look at the changes for 2011 alone and all years excluding 2011.

Table 23: Sources of Change for Accident Year 2011

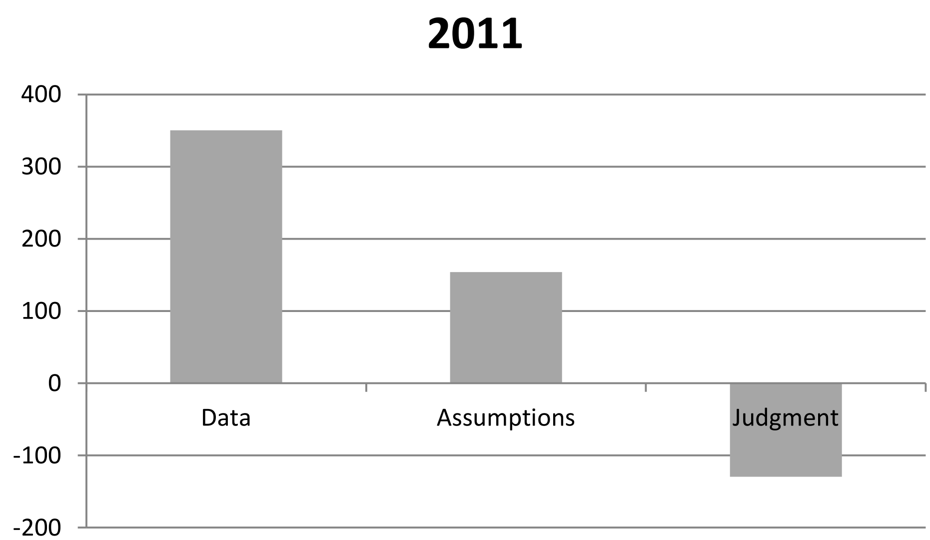
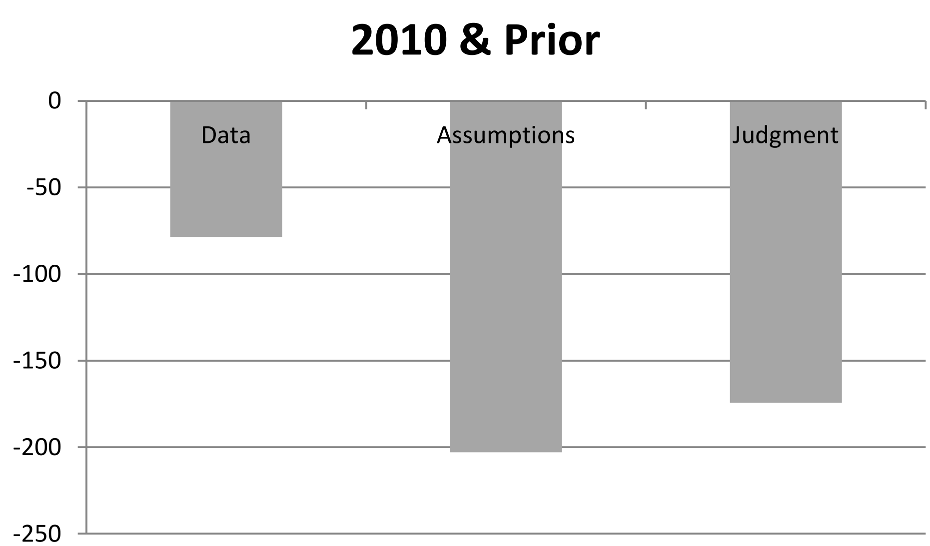


Table 24: Sources of Change for Accident Years 2010 and Prior



We can see that in our example, the increase in data is driven primarily by the 2011 year. When Accident Year 2011 is excluded, the data implies a decrease of $79. Given that data is the source of a decrease, it now makes sense that the actuary might lower assumptions so that if the current loss emergence patterns continue the data emergence in the next period might line up with lower expectations.

4.5 Common Questions

The following questions are common questions that may come up in the course of the analysis.

Do I worry if the change due to data is inconsistent with the actual vs. expected results?

In our application of the Source of Change methodology, we will often average the paid and incurred loss results at each step of the Source of Change process. This can lead to situations in which the average change due to data is greater (less) than zero, while one of the actual versus expected results is less (greater) than zero. In this case, the perceived inconsistency is not really an inconsistency at all, but rather a distortion that comes from averaging the paid and incurred data in the Source of Change but not averaging the paid and incurred data in the Actual versus Expected calculations.

Another way in which a potential inconsistency might arise is if the prior analysis Initial Expected Loss (IEL) is very different than the prior analysis direct development ultimate loss indication or the prior analysis selected ultimate loss. The Source of Change calculation uses the prior analysis IEL value to calculate the effect of data changes, whereas the Actual versus Expected calculation are based on either actual losses without regard to the prior analysis IEL or the prior analysis selected ultimate loss. A sizable difference between the values entering each of these calculations can result in one calculation showing actual loss emergence to be greater (less) than expected while another shows actual loss emergence to be less (greater) than expected.

Either of these apparent inconsistencies can be explained by an examination of the data and the calculations being done, thereby eliminating the perception of an inconsistency between the Source of Change and Actual versus Expected results.

Do I worry if I see different directional changes in my LDF picks and my IELR?

This is the type of result that should lead to some follow-up questions about the conclusions being drawn. We can imagine an example of when such an outcome might be reasonable as follows: we see that actual versus expected experience is showing different results for older versus immature accident periods. We have seen just such results when older accident years do not wind down as quickly as we had previously expected, resulting in higher LDFs for these older development ages. At the same time, we see immature accident periods showing accelerated claim closure rates that can be attributed to greater emphasis being placed on resolving claims early. In these situations we do not necessarily believe the higher LDFs that we have selected for the older accident periods are going to be needed for the immature accident periods. However, we only have one set of LDFs for the entire loss triangle, so the CDFs being calculated for the immature accident periods are now overstated because they include the accumulation of the older period LDFs. In order to counteract this LDF overstatement, we might lower our IELR pick from what was previously selected.

Do I worry if I see a large judgment impact?

There are various reasons why the judgment change may be significant.

- If ultimate losses are not selected based on the method(s) used for the baseline in the source of change, the judgment change could be large even though selection methodology is consistent from one analysis to the next.
- Consider the case where the baseline method is the average of the paid and incurred BF methods, and suppose there is a large loss in the prior analysis where very little has been paid. The incurred method will give higher results than the paid method, and the actuary would likely select closer to the incurred method since the paid method is skewed low by virtue of not including the large loss. "Judgment" in the prior analysis would appear to be a large positive number as the incurred method is above the average of the two methods. In the current analysis, a portion of the large loss is paid, bringing the paid method in line with the incurred method. Since the method results are now similar, the selected ultimate loss will now be close to the average and the "judgment" in the current analysis will be minimal. This would manifest in the source of change as a large decrease in judgment. However, there is not really a change in judgment, but is rather driven by the fact that the prior paid BF method was skewed by the large loss.
- The actuary may have a valid reason for changing the methods relied upon when selecting ultimate losses (e.g., change in case reserving practices leads the actuary to rely more on paid methods, or discovery that exposure estimates are not reliable leading the actuary to rely more on non-exposure based estimates).

5. CONCLUSION

This methodology is not designed to provide answers, but rather provide a structured framework through which to examine a reserve analysis. The results of each step in the analysis lead the actuary to ask questions that lead to a better understanding of the results of the actuarial analysis. We have used this methodology with great success as a way of teaching less experienced actuarial practitioners the types of critical thinking questions that should be asked when doing an analysis. We have also used multiple years' worth of Source of Change results to evaluate the trends in our analysis over time. For example, because the Source of Change methodology provides a consistent structural format for dissecting movement in ultimate losses into component parts, we are able to understand if we are lowering LDFs in one analysis, only to raise them in the subsequent analysis, or if we are steadily increasing (decreasing) them from analysis to analysis. Lastly, we have found the visual depiction of the Source of Change results shown in Tables 22, 23, and 24 to be very effective when communicating results to a non-actuarial audience.

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Abbreviations and notations

BF: Bornhuetter Ferguson
LDF: loss development factor
CDF: cumulative loss development factor

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