# The Estimation of Loss Development Tail Factors: A Summary Report

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**ABSTRACT**

**Motivation.** Tail factors are used by actuaries to estimate the additional development that will occur after the eldest maturity in a given loss development triangle, or after the eldest credible link ratio. Over the years, many valuable contributions have been made to the CAS literature that describes various methods for calculating tail factors. The CAS Tail Factor Working Party prepared this paper on the methods currently used by actuaries to estimate loss development 'tail' or 'completion' factors. Standard terminology for discussing aspects of link ratios and tail development is communicated within the paper. Descriptions of the advantages and disadvantages of each method are included as well general indications of what types of entities (companies, rating bureaus, or consulting firms) typically use each method.

**Method**. An extensive survey of existing CAS literature was performed, along with surveys of methods currently in use by various rating bureaus, insurers, and consulting organizations. The methods identified by the Working Party are grouped into six basic categories: (1) "Bondy Methods"; (2) algebraic methods that focus on relationships between paid and incurred loss; (3) methods based on use of benchmark data; (4) curve-fitting methods; (5) methods based on remaining open counts; (6) methods based on peculiarities of the remaining open claims; and (7) the remaining unclassified methods.

**Results**. Comparisons of the results of several key tail factor methodologies to the actual post-ten year development for a number of long-tail lines using multiple realistic data sets are included, along with the advantages and vulnerabilities of each method.

**Availability**. A copy of the Working Party's paper and companion Excel template can be found on the CAS website at http://www.casact.org/pubs/forum/13fforum/.

**Keywords**. Tail Factors; Completion Factors; Link Ratios; Age-to-Age Factors; Development Factors; Loss Reserving; Curve Fitting; Bondy Method; Benckmark; Loss Development.

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# 1. INTRODUCTION

## 1.1 Importance of Loss Development Tail Factors

The loss development tail factors (sometimes referred to as completion factors) are an important part of any reserve analysis. They have a highly leveraged impact since they form a portion of the loss development applied to each of the accident years being analyzed. However, the discussions of tail factor estimation methods used, when they are contained in the CAS literature at all, are generally just as adjuncts to the main topics of papers. Further, some methods are used in practice that are not described in the CAS literature at all. Therefore, the CAS Committee on Reserving sponsored a Tail Factor Working Party to undertake an exhaustive survey of the tail factor estimation methods in use and describe and comment on each method.

## 1.2 Research Context

As stated above, tail factors have a highly leveraged impact on loss development since they form a portion of the loss development of all accident years analyzed. Further, tail loss development reflects development occurring after the last development period in the reserving data triangle and is therefore somewhat more difficult to estimate than the various link ratios developed from the data triangle. For both those reasons, the Tail Factor Working Party believes it is helpful to provide information concerning tail factor estimation methods to practitioners.

## 1.3 Objective

This paper is designed to be as exhaustive a listing of methods used to estimate tail loss development as is reasonably possible at the time of its writing. The Tail Factor Working Party hopes this will expose the various approaches to a wider audience, and help actuaries choose the best method for each reserving circumstance from a larger toolkit. Further, this paper lists at least some of the advantages and disadvantages of each method, which could help the practitioner decide which method to use in a given circumstance.

## 1.4 Disclaimer

While this paper is the product of a CAS Working Party, its findings do not represent the official view of the Casualty Actuarial Society. Moreover, while we believe the approaches we describe are very good examples of how to estimate tail development in reserving, ratemaking and selecting the best method for a given circumstance, we do not claim they are the only acceptable ones or that we have ultimately addressed all of the issues that must be considered in selecting a tail factor or tail factor methodology.

### 1.5 Section References to Methods

The classes of methods presented are discussed in the next sections. Within each class of method, an introduction to the class of method, a summary of the methods, any particular findings, and conclusions are presented.

### 1.6 Alternate Grouping of Methods Included in the Paper

While organizing this paper, working party members noted that the groupings of methods were not inherently absolute and that the methods could be grouped in alternate ways. The commentary and listing in Appendix A represents an alternate but still logical view of how the various methods relate to each other.

### 1.7 Notation

This paper describes many tail factor methods identified in the actuarial literature and elsewhere. For the sake of uniform notation, where appropriate we have adopted (and expanded) the notation used by the CAS Working Party on Quantifying Variability in Reserve Estimates. In the paper produced by that Working Party, some models visualize loss statistics as a two-dimensional triangle array. In the notation, the row dimension is the period[1] by which the loss information is subtotaled, most commonly an accident period.[2] For each accident period $w$, development age $d$ the $(w, d)$ element of the array is the total of the loss information as of development age $d$.[3]

For this discussion, we assume that the loss information available is an upper left triangular subset of the two-dimensional array for rows $w = 1, 2, \ldots, n$. For each row $w$, the information is available for development ages 1 through $n - w + 1$. If we think of period $n$ as the latest accounting period for which loss information is available, the triangle represents the loss information as of accounting dates 1 through $n$. The diagonal for which $w + d$ equals a constant $k$ represents the loss information for each accident period $w$ as of accounting

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1 Most commonly the periods are annual (years), but as most methods can accommodate periods other than annual we will use the more generic term "period" to represent year, half-year, quarter, month, etc. unless noted otherwise.

2 Other exposure period types, such as policy period and report period, also utilize tail factor methods. For ease of description, we will use the generic term "accident" period to mean all types of exposure periods, unless otherwise noted.

3 Depending on the context, the $(w, d)$ cell can represent the cumulative loss statistic as of development age $d$ or the incremental amount occurring during the $d^{th}$ development period.

period $k$.[4]

In general, the two-dimensional array will also extend to columns $d = 1, 2, \ldots, n$. For purposes of calculating tail factors, we are interested in understanding the development beyond the observed data for periods $d = n+1, n+2, \ldots, u$, where $u$ is the ultimate time period for which any claim activity occurs – i.e., $u$ is the period in which all claims are final and paid in full.

The paper uses the following notation for certain important loss statistics:

| | |
|---|---|
| $c(w,d)$: | cumulative paid or incurred loss from accident period $w$ as of development ages $d$. ($w$ and $d$ may be thought of as representing "when" and "delay," respectively.) In the context of this and other notation, $c_{Paid}(w,d)$ denotes cumulative paid loss and $c_{Inc}(w,d)$ denotes cumulative case incurred loss. |
| $q(w,d)$: | incremental paid or incurred loss on accident period $w$ during the development age from $d-1$ to $d$. Also denoted as $q_{Paid}(w,d)$ or $q_{Inc}(w,d)$. |
| $s(w,d)$: | case reserves at end of development age $d$ for accident period $w$. |
| $c(w,u) = U(w)$: | total loss from accident period $w$ when at the end of ultimate development. |
| $R(w)$: | future development after age $d = n - w + 1$ for accident period $w$, i.e., = $U(w) - c(w, n-w+1)$. |
| $S(d)$: | estimated ratio of unpaid costs to case reserves at the end of the triangle data $d$. |
| $S$: | estimated ratio of unpaid costs to case reserves as of the end of the triangle data. |
| $f(d) = 1 + v(d)$: | factor applied to $c(w,d)$ to estimate $c(w,d+1)$ or more generally any factor relating to age $d$. This is commonly referred to as a link ratio. $v(d)$ is referred to as the 'development portion' of the link ratio, which is used to estimate $q(w,d+1)$. The other portion, the number one, is referred to |

4 For a more complete explanation of this two-dimensional view of the loss information see the *Foundations of Casualty Actuarial Science* [5], Chapter 5, particularly pages 210-226.

as the 'unity portion' of the link ratio.

$\hat{f}(d) = 1 + \hat{v}(d)$: an estimate of the link ratio for development age to development age $d+1$.

$F(d) = 1 + V(d)$: ultimate development factor relating to development age $d$. The factor applied to $c(w,d)$ to estimate $c(w,u)$ or more generally any cumulative development factor relating to development age $d$. The capital indicates that the factor produces the ultimate loss level. As with link ratios, $V(d)$ denotes the 'development portion' of the loss development factor, the number one is the 'unity portion' of the loss development factor. $G(d)$ is used interchangeably with $F(d)$ and by convention, $G$ may also be used to denote the ultimate loss development factor needed for period $w$ when written as $G(w)$.

$T = T(n)$: tail factor at end of triangle data.

$\hat{T}$: estimate of the tail factor.

$h(w+d)$: factor relating to the diagonal $k$ along which $w+d$ is constant.

$e(w,d)$: a mean zero random fluctuation that occurs at the $w$, $d$ cell.

$r(k)$: annual rate of loss cost inflation, in this case related to payment period, although in cases where $r$ is either constant or estimated as a constant, $r$ is the cumulative impact over $k$ years $(1+r)^k$.

$\hat{r}$: an estimate of the rate of annual loss cost inflation.

$m$: development or delay time in months.

$D(m)$: rate of loss cost inflation per month, when $D$ is constant over $m$, the impact over $m$ months is $(1+D)^m$.

$\hat{D}$: an estimate of the rate of monthly loss cost inflation.

$l$: lag until payouts start. Used in McClenahan and Sherman methods.

$B(d) = 1 + b(d)$: notation for a benchmark link ratio and the 'development portion' of the benchmark. Note that $B_T = 1 + b_T$ represents the benchmark tail factor.

$i$: a specific accident month, similar to $w$.

$p_i$: the month-to-month decay rate of the pre-inflation loss payouts for a given accident month, also used as a constant over all months, $p$.

$q_i = 1 - p_i$: the complement of $p$, also used as a constant over all months, $q$.

$A(i)$: constant of proportionality reflecting total expected pre-inflation losses in a given accident month $i$.

$H(w)$: a constant of proportionality used in curve-fitting. Often, for global curve-fitting across an entire triangle, simply used as $H$.

$a$ and $b$: constant terms representing the multiplier and exponent of an inverse power curve, respectively.

$RE$: the reinsurance retention applying to a given triangle. $RE(w)$ refers to the retention of a specific period $w$.

$E(x)$: the expectation of the random variable $x$.

$Var(x)$: the variance of the random variable $x$.

$U(w)$: ultimate loss amount in accident period $w$ = c(*w,u*).

Also, for some methods, additional or slightly different notation is used.

## 2. BONDY-TYPE METHODS

### 2.1 Introduction and Description of Bondy-Type Methods

This class of methods is discussed first due to its simplicity. Martin Bondy suggested this method of just repeating the last observed link ratio for use as the tail factor. Note, that at the time Bondy developed his method in the 1960s, most lines of insurance were believed to be "short-tailed" in nature compared to assumptions assumed for many casualty lines of insurance today. Bondy's Original Method (see section 2.2) may seriously understate the needed tail factor for "long-tail" lines or for any case where substantial development occurs in the tail. Several alternate versions of the Bondy approach have been developed in an attempt to mitigate the original method's shortcomings.

The formulas for the Bondy-Type methods are described in the sub-sections below. Starting with the original method, we move through modifications that lead to a fully generalized method.

### 2.2 Bondy's Original Method

Bondy's Original Method used the link ratio $f(n-1)$ at the last observed development age, $n$, to develop losses to ultimate; that is

$$F(n) = f(n-1). \tag{2.1}$$

The assumption for age-to-age development factors in the tail is that

$$f(d) = \sqrt{f(d-1)}\,. \tag{2.2}$$

### 2.3 Modified Bondy Method

In these revisions of Bondy's Original Method, some recognition is given to more extended development patterns; the first approach is multiplicative, the second additive.

The first approach consists of simply squaring the last link ratio, rather than just repeating it:

$$F(n) = f(n-1)^2\,. \tag{2.3}$$

The second approach, utilized by some practitioners, is to merely double the development portion of the last link ratio:

$$F(n) = 1 + [2 \times v(n-1)]\,. \tag{2.4}$$

### 2.4 Generalized Bondy Method

Subsequently, Weller [16] suggested a generalization by setting $f(n) = f(n-1)^B$, where $B$ is a number between 0 and 1. We call $B$ the Bondy exponent. It follows that

$$F(n) = f(n-1)^B f(n-1)^{B^2} \cdots = f(n-1)^{B/(1-B)}\,. \tag{2.5}$$

Thus, if $B = \frac{1}{2}$, we recover the original Bondy method.

Let $f(d)$ be the development ratio chosen for age $d-1$ to age $d$. In his paper, Weller used the average of the latest three observed development ratios for $f(d)$. (Fewer or more observations could be utilized.) Set $l_d = \log f(d)$, $\hat{B}$ the estimated Bondy parameter, $\hat{f}(i)$ the estimated development ratio for the earliest development period used to estimate the parameters, and $\hat{l}_i = \log \hat{f}(i)$. The parameters, $\hat{f}(i)$ and $\hat{B}$, are chosen to minimize

$$\sum_{d=i}^{n} \left( l_d - \hat{l}_i \hat{B}^{d-1} \right)^2 . \tag{2.6}$$

The parameters, $\hat{f}(i)$ and $\hat{B}$, can be calculated easily using a readily available spreadsheet optimization function such as the "Solver" function in Microsoft® Excel.

### 2.5 Fully Generalized Bondy Method

Gile [6] devised a further generalization by letting the estimated development ratios vary by accident period, while using the same estimated Bondy parameter for each accident period. Two parameters, as well as the development ratios, are chosen for each accident period by minimizing the sum of squared differences using more than one development period for each accident period.

### 2.6 Examples

See Appendix B, Section B.2.

### 2.7 Advantages and Disadvantages of the Bondy Methods

The method is easily implemented using standard spreadsheet functions. It only uses the data in cumulative paid or incurred loss triangles. Finally, loss development is described in terms of only one factor, the Bondy exponent.

The fully generalized Bondy method is not always useful for incurred loss data because it may produce Bondy exponents not in the range from 0 to 1. For this same reason, the method fails to give meaningful answers when the pattern of development factors is increasing. Since the Bondy method describes loss development in terms of only one parameter, the method may also fail if the development pattern is complicated in some other way.

### 2.8 Users

The Bondy-type methods (including the specific forms discussed above) are widely accepted and used in current practice.

### 2.9 Summary

Bondy methods give a simple solution to the problem of determining tail factors. They are easy to explain and to implement. However, they describe loss development in terms of only one parameter so that complicated development patterns may not be accurately projected.

## 3. ALGEBRAIC METHODS

### 3.1 Introduction to Algebraic Methods

Algebraic methods are methods that focus on the relationships between the paid and incurred loss triangles. They are based on relatively simple calculations in the sense that complex mathematical formulae and curve fitting, etc. is not required. Additionally, ancillary information beyond readily available paid and incurred data is not required for any of these methods.

### 3.2 Equalizing Paid and Incurred Development Ultimate Losses

This method is one of the oldest tail factor methods used and also has perhaps the broadest usage of all the methods. It was designed to provide an easy methodology for determining a paid loss tail factor when the incurred loss tail factor is available.

#### 3.2.1 Description[5]

This method is most useful when incurred loss development essentially stops after a certain stage (i.e., the link ratios are near to unity or are equal to unity). Then, due to the absence of continuing development, the current case incurred (e.g., case incurred as of end of most recent accounting period, sometimes called reported) losses are a good predictor of the ultimate losses for the older or oldest years without the need for additional tail factor development. A tail factor suitable for paid loss development can then be computed as the ratio of the case incurred for the oldest accident period in the triangle divided by the paid losses to date for the same accident period. This results in a paid to ultimate development factor estimate which when multiplied by the cumulative paid equals the ultimate (which are also the current) incurred losses for that oldest accident year.

This method relies on one axiomatic (meaning plainly true rather than an assumption as such) assumption and two true assumptions. The axiomatic assumption is that the paid loss and incurred loss development estimates are estimating the same quantity, therefore the ultimate loss estimates they produce should be equal. The second assumption (the first true assumption) is that the incurred loss estimate of the ultimate losses for the oldest accident period is accurate. The last assumption is that the other periods will show the same development in the tail as the oldest period. An appropriate way to test this assumption is to estimate the paid loss tail based on several accident periods.

This method may also be generalized to the case where the current case incurred is still showing development near the tail. In this situation, the implied paid loss tail factor is

$$\frac{\text{ultimate incurred loss development estimate for the oldest accident period}}{\text{paid losses to date for the oldest accident period}}, \text{ or}$$

$$\frac{c_{Inc}(1,u)}{c_{Paid}(1,n)}. \tag{3.1}$$

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5 Section 3.2.1 is reproduced from [1] with permission. Minor edits have been made for consistency with the rest of this Report.

In this instance, the incurred loss development estimate for the oldest accident period is usually the current case incurred losses for the oldest period multiplied by an incurred loss tail factor developed using other methods.

### 3.2.2 Example

We are given the following selected incurred loss development factors:

| | |
|---|---|
| 12-24 months | 2.000 |
| 24-36 | 1.500 |
| 36-48 | 1.250 |
| 48-60 | 1.125 |
| 60-72 | 1.063 |
| 72-84 | 1.031 |
| 84-96 | 1.016 |
| 96-108 | 1.008 |
| 108-120 | 1.004 |

Incurred losses for the oldest year in the triangle as of 120 months is $50,000,000 and the corresponding paid loss is $40,000,000. The incurred estimated ultimate using the 1.004 tail factor is $50,200,000. The paid loss tail factor to equalize the paid estimated ultimate to the incurred estimated ultimate would be $50,200,000 divided by $40,000,000 or 1.255.

### 3.2.3 Advantages and Disadvantages

This method has a substantial advantage in that it is based solely on the information in the triangle itself. One of its weaknesses is that a reliable estimate of the ultimate loss for the oldest year is needed before it can be used. In addition, if the ultimate incurred loss development of the oldest accident year is estimated using a tail factor estimate, then this method also relies on the incurred loss tail factor. Lastly, there is an assumption that the ratio of the case incurred loss to the paid loss will be the same for less mature years once they reach the level of maturity used initially to calculate the paid tail. This assumption can be tested by looking at the stability of the paid to incurred ratio.

### 3.2.4 Users

This method is such a basic part of most loss development analyses that it is probably under-reported on surveys. For example, most users will attempt to at least compare the estimated ultimate paid and estimated ultimate incurred loss for the oldest years.

### 3.2.5 Summary

This method is both simple and widely used. However, a major limitation is that unless development of the oldest accident period is complete at least one tail factor (incurred or paid) must be calculated by other means before this approach can be used.

## 3.3 Sherman-Boor Method

This method was developed by Sherman in [13], and later by Joseph Boor in the course of analyzing very long tail workers compensation data during the 1987-1989 periods. Although it was originally published some time ago as an adjunct to other tail factor methods, it has only recently received much attention. Thus, a comparatively small percentage of practicing actuaries are aware of it. It was developed largely to provide an alternative to the use of fitted curves and their heavy reliance on theoretical assumptions.

### 3.3.1 Description

This method relies solely on the triangles themselves and does not require a pre-existing ultimate loss estimate, involve curve-fitting assumptions, or require external data. For data triangles with high statistical reliability as predictors, this can represent a powerful and reliable predictor of tail development.

This method involves simply determining the ratio of case reserves to paid loss for the oldest period in the triangle, then adjusting the case reserves by an estimate of the ratio of the unpaid loss to carried case reserves. In essence, the case reserves of the oldest accident period are 'grossed up' to estimate the true unpaid loss using a factor. The estimate of the (true unpaid loss)/(case reserves) factor is based on how many dollars of payments are required to 'eliminate' a dollar of case reserve.

The mathematical formula requires computing a triangle containing incremental rather than cumulative paid losses. The formula for incremental paid losses for accident period $w$, from development age $d-1$ to $d$ is:

$$q_{Paid}(w,d) = c_{Paid}(w,d) - c_{Paid}(w,d-1). \qquad (3.2)$$

The next step begins with a triangle of case reserves. The incremental case reserve disposed of in a development period is calculated as the beginning case reserve of that period minus the ending case reserve of that period. The formula for case reserves disposed of is essentially a decrement-type process (process of reduction rather than process of increase), so it is stated in negative terms as:

$$-q_{Case}(w,d) = s(w,d) - s(w,d-1). \qquad (3.3)$$

Alternately, it may be stated positively as:

$$q_{Case}(w,d) = s(w,d-1) - s(w,d)\,, \tag{3.3.1}$$

where $s(w,d)$ represents case reserves at the end of development age $d$ for accident period $w$. Next the ratios of incremental paid to reserve disposed for each element in the triangles is computed. Noting that the case decrement at the first column (which may be either $d=0$ or $d=1$ in context) is essentially undefined, we get a triangle relating the costs of disposing of case reserves to the amount of case reserves that are disposed of

$$\text{Relative Disposal Costs}\,(w,d) = q_{paid}(w,d)\,/\,q_{Case}(w,d) \tag{3.4}$$

Reviewing the above matrix (triangle) of relative disposal costs, a final adjustment ratio for ending case reserves, $S$ is selected.[6] The final step involves multiplying that selected $S$ ratio times the ratio of the remaining case reserves of the oldest accident period (which provides an estimate of remaining payments) and dividing by the cumulative paid loss of the oldest accident period. The result is an estimate of the development portion of the paid loss tail factor. The tail factor formula is:

$$\hat{T}_{Paid} = 1.0 + S \times \frac{s(1,n)}{c_{inc}(1,n)} \tag{3.5}$$

For the incurred tail factor, it must be recognized that the unity (1.0) portion of the case is already accrued in the incurred loss. So, the incurred tail factor formula is:

$$\hat{T}_{Inc} = 1.0 + (S-1) \times \frac{s(1,n)}{c_{inc}(1,n)} \tag{3.6}$$

### 3.3.2 Example

See Appendix B, Section B.3.1.

#### 3.3.2.1 Considerations

It is important to consider the primary activity within each development stage.

When using multiple periods to estimate a tail factor, it is relatively important that the periods reflect the same general type of claims department activity as that which takes place in the tail. For example, in the early 12 to 24 month stage of workers compensation, the primary development activity is the initial reporting of claims and the settlement and closure of small claims. The primary factors influencing development are how quickly the claims are reported and entered into the system, and the average reserves (assuming the claims department

[6] However, it is important to focus the review on the period in the triangle where the same 'type' of activity is occurring, as will be discussed later.

initially just sets a 'formula reserve', or a fixed reserve amount for each claim of a given type such as medical or lost time) used when claims are first reported.

In the 24 to 36-48 month period, claims department activity is focused on ascertaining the true value of long-term claims and settling claims. After 48-60 months most of the activity centers on long-term claims. So, the 12-24 link ratio has relatively little relevance for the tail, as the driver behind the link ratio is reporting and the size of initial formula reserves rather than the handling of long-term cases. Similarly, if the last credible link ratio in the triangle is the 24 to 36 or 36 to 48 link ratio, that triangle may be a poor predictor of the required tail factor.

Another consideration that could improve this method is using multiple years to estimate the tail factor. This method assumes that the current ratio of case incurred loss to paid loss that exists in the oldest year will apply to the other years when they reach that same level of maturity. For a large, high dollar volume triangle with relatively low underlying policy limits that may be a reasonable assumption, but for many reserving applications the 120-month ratio of case incurred to paid loss may depend on whether a few large, complex claims remain open or not. Therefore, it may be wise to supplement the tail factor derived from the oldest available accident period with that implied by the following accident period or even the second following accident period. This method is particularly useful when the later development portion of the triangle has some credibility, but the individual link ratio estimates from the development triangle are not fully credible.

The process is fairly straightforward: compute the tail factor for each succeeding accident period by the method above, and divide each such tail factor by remaining link ratios in the triangle.

An example using the data in Appendix B may help clarify matters. The 2000 accident period at development age 108 has $7,934 of paid loss and $584 of case reserves. Assume that the best estimate of the 108-120 paid loss link ratio is (using 2000 accident period data) 1.024. Assuming $S$ is 3.073, then the 108-month paid loss tail would be 1.0+ (3.073*584/7,934) = 1.226. Then, dividing out the 108-120 link ratio of 1.024 would give a 108-month paid tail factor of 1.226/1.024=1.197. By comparison, the analysis in the Appendix using 2000 instead of 2001 gives a 120-month tail factor estimate of 1.149. Both indicate tail factors in the 1.15-1.20 range and averaging the estimates would be reasonable. The use of averaging greatly limits the impact of any unusually low or high case reserves that may be present in the oldest year in the triangle.

Note also, that the improvement above involved computing an alternate tail factor using the accident period with one year less development age than the oldest accident period. A similar analysis could also be performed on the next oldest year, except that two paid development link ratios plus the tail factor are needed to estimate the paid loss tail factor.

### 3.3.3 Advantages and Disadvantages

The significant strengths of this method are that it requires only the data already in the triangles. The weakness is that it can be distorted if the adequacy of the ending case reserve has changed significantly over time.

### 3.3.4 Users

At present this method has not been published and as such is not widely known or used.

### 3.3.5 Summary

This method can be a reasonable approach in predicting tail factors without reliance on extensive assumptions, but it needs to be focused on data mature enough so that the overwhelming majority of claims have been reported.

## 3.4 NCCI Method

This section describes the methodology used by the National Council on Compensation Insurance (NCCI) to derive an indicated 19th-to-ultimate tail factor for use in aggregate ratemaking specifically for workers compensation. NCCI applies this method in most states where it provides ratemaking services.

### 3.4.1 Introduction

NCCI uses the Accident Year Call for Experience (Call 5) submitted by its affiliates for the calculation of the accident year incurred 19$^{th}$-to-ultimate tail factor used for ratemaking. The loss data collected on Call 5 includes cumulative paid losses, case loss reserves, bulk reserves, and IBNR for the most recent 20 accident years individually, and in total for years prior to the 20$^{th}$ accident year. [7] Throughout the examples in this section, the notation $c(w,d)$ will be used to denote cumulative incurred losses including paid, case, bulk and IBNR reserves for accident year $w$ and development period $d$. Similarly, $q(w,d)$ will be used to denote incremental incurred (paid plus change in case, bulk and IBNR) losses for accident year $w$ during the period from $d-1$ to $d$.

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7 Beginning with data valued as of December 31, 2007, NCCI began the process of expanding Call 5 by adding an additional accident year each reporting year until 30 accident years are reported individually, with years prior to the 30th accident year reported in total. However, as of the time of this writing, NCCI continues to calculate a 19th-to-ultimate tail factor as described in this section.

### 3.4.2 Calculation of the Accident Year Incurred 19th-to-Ultimate Tail Factor

An estimate of all future incurred development beyond 19$^{th}$ report for a given accident year is estimated as the sum of i) reported incurred development from 19$^{th}$ to 20$^{th}$ report on the given accident year and ii) adjusted reported incurred development during the same calendar year for all prior accident years.[8] The incurred development on prior accident years is adjusted by a "growth factor" to reflect the difference in overall loss levels between those years and the given accident year.

The incurred 19$^{th}$-to-ultimate tail factor for a given accident year is then obtained by adding unity to the ratio of a) estimated future incurred development beyond 19$^{th}$ report to b) incurred losses at 19$^{th}$ report for the given accident year:

$$\text{AY incurred } 19^{th}\text{-to-ultimate tail factor} = 1 + \frac{\text{Estimated AY incurred development beyond } 19^{th}}{\text{AY incurred losses at } 19^{th}}$$

Where:

$$\text{Estimated AY incurred development beyond } 19^{th} = \text{Incurred development on given AY from } 19^{th} \text{ to } 20^{th} + \frac{\text{Nominal CY incurred development on all prior AYs}}{\text{Growth factor}}$$

$$= \textbf{(a)} + \frac{\textbf{(b)}}{\textbf{(c)}}.$$

OR:

$$F(19) = 1 + \frac{R(w)}{c(w,19)}. \tag{3.7}$$

Where:

$$R(w) = q(w,20) + \frac{\sum_{d=21}^{n} q(n-d+1,d)}{g}. \tag{3.8}$$

This is best illustrated by an example. Displayed below is a historical incurred loss triangle through 2010 evaluated at 12/31/2010. Note that values to the right of the jagged line (for

[8] The development on all prior accident years during a calendar year, i.e., calendar year development, is a reasonable approximation of the future development on the given accident year assuming development patterns and exposure levels are constant.

development periods beyond 20th) are not available individually, but are shown for the purpose of this example.

Cumulative Incurred Loss Triangle $c(w,d)$

| | 1 | … | 19 | 20 | 21 | 22 | 23 | … |
|---|---|---|---|---|---|---|---|---|
| 1986 | $c(1986,1)$ | … | $c(1986,19)$ | $c(1986,20)$ | $c(1986,21)$ | $c(1986,22)$ | $c(1986,23)$ | … |
| 1987 | $c(1987,1)$ | … | $c(1987,19)$ | $c(1987,20)$ | $c(1987,21)$ | $c(1987,22)$ | $c(1987,23)$ | … |
| 1988 | $c(1988,1)$ | … | $c(1988,19)$ | $c(1988,20)$ | $c(1988,21)$ | $c(1988,22)$ | $c(1988,23)$ | |
| 1989 | $c(1989,1)$ | … | $c(1989,19)$ | $c(1989,20)$ | $c(1989,21)$ | $c(1989,22)$ | | |
| 1990 | $c(1990,1)$ | … | $c(1990,19)$ | $c(1990,20)$ | $c(1990,21)$ | | | |
| 1991 | $c(1991,1)$ | … | $c(1991,19)$ | $c(1991,20)$ | | | | |
| 1992 | $c(1992,1)$ | … | $c(1992,19)$ | | | | | |
| ⋮ | ⋮ | | | | | | | |
| 2010 | $c(2010,1)$ | | | | | | | |

The values below are shown for illustrative purposes and are not intended to reflect realistic incurred loss development patterns.

Cumulative Incurred Loss Triangle $c(w,d)$

| | 1 | … | 19 | 20 | 21 | 22 | 23 | … |
|---|---|---|---|---|---|---|---|---|
| ⋮ | | … | | | | | | |
| 1986 | 6,000 | … | 30,000 | 30,600 | 30,906 | 31,061 | 31,154 | … |
| 1987 | 8,000 | … | 40,000 | 40,800 | 41,208 | 41,414 | 41,538 | … |
| 1988 | 10,000 | … | 50,000 | 51,000 | 51,510 | 51,768 | 51,923 | |
| 1989 | 12,000 | … | 60,000 | 61,200 | 61,812 | 62,121 | | |
| 1990 | 14,000 | … | 70,000 | 71,400 | 72,114 | | | |
| 1991 | 16,000 | … | 80,000 | 81,600 | | | | |
| 1992 | 18,000 | … | 90,000 | | | | | |
| ⋮ | ⋮ | | | | | | | |
| 2010 | 50,000 | | | | | | | |

Note that in this example, accident year 1991 is the most recent accident year for which data is available at 20$^{th}$ report. The 19$^{th}$-to-ultimate tail factor for this accident year is calculated below. Since the underlying data is evaluated as of 12/31/2010, the formula uses incurred loss development on all prior accident years that occurred during calendar year 2010. The components of formula (3.8) are calculated as follows:

**(a)** Incurred development on given AY from 19$^{th}$ to 20$^{th}$ report

$$= c(1991,20) - c(1991,19)$$
$$= 81{,}600 - 80{,}000$$
$$= 1{,}600.$$

**(b)** Incurred development on all prior AYs

$$= \sum_{d=21}^{n} q\left(2010 - d + 1, d\right)$$
$$= q\left(1990,21\right) + q\left(1989,22\right) + q\left(1988,23\right) + \cdots$$
$$= \left[c(1990,21) - c(1990,20)\right] + \left[c(1989,22) - c(1989,21)\right] + \left[c(1988,23) - c(1988,22)\right] + \cdot$$
$$= \left(72{,}114 - 71{,}400\right) + \left(62{,}121 - 61{,}812\right) + \left(51{,}923 - 51{,}768\right) + \cdots$$
$$= 714 + 309 + 155 + \cdots$$
$$= 3{,}000 \quad \text{(datapoints not shown).}$$

**(c)** Growth factor, $g$ (the rationale for the selection of the elements used to calculate $g$ is discussed below.)

$$= \frac{\left(\frac{1}{5}\right)\times\left[c(1986,19)+c(1987,19)+c(1988,19)+c(1989,19)+c(1990,19)\right]}{c(1991,19)}$$

$$= \frac{\left(\frac{1}{5}\right)\times(30{,}000+40{,}000+50{,}000+60{,}000+70{,}000)}{80{,}000}$$

$$= 0.625.$$

Substituting,

| | | | | | |
|---|---|---|---|---|---|
| (3.8) | Estimated AY incurred development beyond 19th | = | (a) | + | (b) / (c) |
| | | = | 1,600 | + | 3,000 / 0.625 |
| | | = | 6,400. | | |

| | | | |
|---|---|---|---|
| (3.7) | AY incurred 19th-to-ultimate tail factor | = 1 + | Estimated AY incurred development beyond 19th / AY incurred losses at 19th |
| | | = 1 + | 6,400 / 80,000 |
| | | = 1.08. | |

### 3.4.3 Derivation of the Formula

Assuming that all claims are closed and all losses paid out at $n^{th}$ report, the actual incurred development on accident year 1991 from 19th report to ultimate is:

$$F(1991,19) = \frac{c(1991,\, n)}{c(1991,\, 19)}$$

$$= \frac{c(1991,19)+q(1991,20)+\sum_{d=21}^{n} q(1991,d)}{c(1991,19)}$$

$$= 1+\frac{q(1991,20)}{c(1991,19)}+\frac{q(1991,21)+q(1991,22)+\ldots}{c(1991,19)}$$

$$= 1+\frac{q(1991,20)}{c(1991,19)}+\frac{\left[q(1990,21)\times\frac{q(1991,21)}{q(1990,21)}\right]+\left[q(1989,22)\times\frac{q(1991,22)}{q(1989,22)}\right]+\ldots}{c(1991,19)}$$

$$= 1+\frac{q(1991,20)}{c(1991,19)}+\frac{\sum_{d=21}^{n}\left[q(2010-d+1,d)\times\frac{q(1991,d)}{q(2010-d+1,d)}\right]}{c(1991,19)}$$

$$=1+\frac{q(1991,20)}{c(1991,19)}+\frac{\sum_{d=21}^{n} q(2010-d+1,d)}{c(1991,19)}\times h, \tag{3.9}$$

where
$$h=\frac{\sum_{d=21}^{n}\left[q(2010-d+1,d)\times\frac{q(1991,d)}{q(2010-d+1,d)}\right]}{\sum_{d=21}^{n} q(2010-d+1,d)}, \tag{3.10}$$

which can be described as a weighted average of the terms

$$\frac{q(1991,d)}{q(2010-d+1,d)},$$

using as weights

$$\frac{q(2010-d+1,d)}{\sum_{d=21}^{n} q(2010-d+1,d)}.$$

Each of the terms in this series is a ratio of incremental incurred losses for accident year 1991 relative to an earlier accident year. However, in each term the numerator is unknown (because this development has yet to occur), and the denominator is not available (because only 20 development years of data are reported individually). Therefore, NCCI approximates these terms by measuring accident year 1991 incurred losses against each of the earlier accident years at an earlier, known report level. Because the incremental incurred losses for one report can vary widely, cumulative losses are compared in each of the terms, as follows:

$$\frac{c(1991,19)}{c(1990,19)},\frac{c(1991,19)}{c(1989,19)},\frac{c(1991,19)}{c(1988,19)},\ldots$$

Substituting into formula (3.10):

$$h\approx\frac{\sum_{d=21}^{n}\left[q(2010-d+1,d)\times\frac{c(1991,19)}{c(2010-d+1,19)}\right]}{\sum_{d=21}^{n} q(2010-d+1,d)}. \tag{3.11}$$

For a given term (which measures accident year 1991 against a given accident year) in the weighted average described by formula (3.11), the weight applied to that term is the given accident year's proportion of calendar year incurred development on all accident years prior to 1991. Since the calendar year incurred development on accident years prior to 1991 is only available in total and not by accident year, NCCI approximates the weighted average on the right-hand side of equation (3.11) with a simple average of a subset of the first $k$ terms. With this approximation, equation (3.11) simplifies to:

$$h \approx \frac{c(1991,19)}{\frac{1}{k} \times \sum_{d=21}^{21+(k-1)} c(2010-d+1,19)}.$$

Currently, NCCI uses a simple average of the first five terms ( $k=5$ ) to approximate this "growth factor." This selection is discussed in further detail below. With $k=5$, we have:

$$h \approx \frac{1}{g} = \frac{c(1991,19)}{\frac{1}{5} \times \sum_{d=21}^{25} c(2010-d+1,19)}. \tag{3.12}$$

Substituting into formula (3.9):

$$\begin{aligned} F(1991,19) &\approx 1 + \frac{q(1991,20)}{c(1991,19)} + \frac{\sum_{d=21}^{n} q(2010-d+1,d)}{c(1991,19)} \times \frac{1}{g} \\ &\approx 1 + \frac{q(1991,20) + \left[\frac{1}{g} \times \sum_{d=21}^{n} q(2010-d+1,d)\right]}{c(1991,19)}. \end{aligned} \tag{3.13}$$

Formula (3.13) is the form used by NCCI. [9]

#### 3.4.3.1 Growth Factor

The tail factor method used by NCCI has evolved since its initial implementation. While the derivation of the formula above accurately describes the rationale underlying the current approach, the method originated from a simpler form that initially did not incorporate the growth factor adjustment. Using the current formula (3.13), removing the growth adjustment would be equivalent to setting $g=1$. In an environment of increasing exposure (loss volume), failure to incorporate a growth adjustment would result in an understated tail factor. Conversely, the tail factor would be overstated if exposure is decreasing and $g=1$.

Since it is not possible to calculate the growth adjustments shown in formulas (3.10) or (3.11) with the data collected on financial calls, NCCI approximates the growth adjustment using formula (3.12). This approximation compares the cumulative incurred losses at 19th report for the most recent accident year to the average cumulative incurred losses at 19th report for the five prior accident years.[10] The five-year average was selected (as opposed to

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9 The discussion above illustrates the calculation of the incurred 19th-to-ultimate tail factor for a single accident year using the most recent data. In NCCI filings, the final tail factor is selected based on a review of at least the most recent five accident year tail factors.

10 For tail factors using data valued prior to December 31, 2008, NCCI used 8th report losses in the calculation

shorter- or longer-term averages) judgmentally with consideration given to the following items:

1. **Incurred loss development pattern beyond 19th report –** Workers compensation is a long-tailed line of insurance in which the ultimate cost of claims incurred during a given accident year may not be known for several decades. When using a simple average of a fixed number of accident years for the growth factor adjustment, a longer tail would suggest using more years in the average. Conversely, a shorter tail would suggest using fewer years in the average.
2. **Exposure growth rates –** Exposure (loss volume) can increase or decrease over time due to a number of factors (e.g., inflation, benefit changes). Given constant incurred loss development beyond 19th report for all accident years, a higher rate of exposure growth would suggest using a fewer number of years in the average for the growth factor adjustment.
3. **Impact of the growth factor adjustment –** In some states, incurred loss development beyond 19th report may be minimal (especially for indemnity benefits, which are typically limited in duration by statute). In these cases, the growth factor has little to no impact on the calculated tail factor, making the number of years used in calculating the growth factor an immaterial selection.
4. **Data constraints –** The number of years used in the average for the growth adjustment is limited on the upper end by data constraints. Specifically, the oldest accident year for which data was reported individually at 19th report is 1979.

#### 3.4.3.2 Conversion Ratios

In determining 1st-to-19th loss development factors, NCCI organizes loss data in a variety of ways (policy year or accident year, on a paid or paid + case basis). Therefore, a "conversion ratio" is required to convert the accident year incurred 19th-to-ultimate tail factor to the corresponding 1st-to-19th loss development basis. For instance, in a state where link ratios from 1st-to-19th report are based on accident year paid + case losses, a paid + case-to-incurred

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of the growth factor. When the growth adjustment was introduced to the formula in the late 1980s, data reported to NCCI included only eight individual accident years. Over time, the financial calls were expanded to include 20 individual accident years of data—adding one additional accident year at each subsequent reporting date. Growth factors could not be calculated using data at a 19th report until there were six valuations of data that each included 20 individual accident years of loss experience.

conversion ratio at $19^{th}$ report is divided into the accident year incurred $19^{th}$-to-ultimate tail factor to calculate an accident year paid + case 19th-to-ultimate tail factor.

For 1st-to-19th development on a policy year basis, the 18th-to-19th policy year link ratio is first raised to the two-thirds power to approximate accident year experience at 19th report.[11]

The various conversions are illustrated in the following table:

| $1^{st}$-to-$19^{th}$ Loss Development Basis | $18^{th}$-to-$19^{th}$ Link Ratio | | Tail Factor Conversion Formula |
|---|---|---|---|
| AY Paid+Case: | AY Paid+Case $18^{th}$-to-$19^{th}$ Link Ratio | x | Incurred $19^{th}$-to-Ult Tail / AY Paid+Case-to-Inc Conv Ratio @ $19^{th}$ |
| AY Paid: | AY Paid $18^{th}$-to-$19^{th}$ Link Ratio | x | Incurred $19^{th}$-to-Ult Tail / AY Paid-to-Inc Conv Ratio @ $19^{th}$ |
| PY Paid+Case: | (PY Paid+Case $18^{th}$-to-$19^{th}$ Link Ratio)$^{2/3}$ | x | Incurred $19^{th}$-to-Ult Tail / AY Paid+Case-to-Inc Conv Ratio @ $19^{th}$ |
| PY Paid: | (PY Paid $18^{th}$-to-$19^{th}$ Link Ratio)$^{2/3}$ | x | Incurred $19^{th}$-to-Ult Tail / AY Paid-to-Inc Conv Ratio @ $19^{th}$ |

As part of ongoing efforts to improve its ratemaking methodologies, NCCI continues to research alternative methods to address tail development. As of the time of this writing, NCCI is currently considering the following potential changes to the method described above:

1. Elimination of bulk and IBNR reserves from the calculation[12]
2. Change to the number of years used in the growth factor
3. Algebraic revision to the growth factor formula

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11 The justification of the two-thirds power adjustment to bring the maturity level of the policy year experience more in line with the maturity level of the accident year experience is beyond the scope of this paper.

12 Calculating the tail factor using paid + case losses would eliminate the need for the paid + case-to-incurred conversion ratios. In addition, without the need for IBNR data (only reported on an accident year basis), a policy year 19th-to-ultimate tail factor could be calculated directly, eliminating the need for the "two-thirds power" adjustment.

#### 3.4.3.3 Adjustment for Capped Methodology

In 2004, NCCI enhanced its aggregate ratemaking methodology to mitigate the possible distortions that catastrophic events and extremely large individual claims can create in state premium level indications.[13] NCCI uses this large loss ratemaking procedure in most of the states where it provides ratemaking services. Essentially, the methodology derives ultimate losses using reported losses capped at a given dollar threshold per claim and later adds a provision for expected losses in excess of that threshold.

In order to develop capped losses to ultimate, loss development factors on a capped basis are needed. From 1st to 19th report, NCCI caps individual claims prior to calculating loss development factors. However, individual claim detail for large claims is only reported for claims with accident dates on or after January 1, 1984. Therefore, to calculate the capped 19th-to-ultimate tail factor, NCCI derives a factor to adjust the selected uncapped paid + case tail factor to a capped basis.

In general terms, the tail adjustment factor is the ratio of capped (for a given threshold) to uncapped paid + case loss development beyond 19th report on a countrywide basis. NCCI uses excess ratios and excess loss development factors to calculate the adjustment factor by threshold and then applies the factor as follows: [14]

$$\text{Capped 19th-to-ultimate paid + case tail factor} = 1 + \left[ \text{Tail adjustment factor} \times \left( \text{Uncapped 19th-to-ultimate paid + case tail factor} - 1 \right) \right]$$

### 3.4.4 Advantages and Disadvantages

One strong advantage of this method is that it uses the total for all prior accident years (the 'prior' row) available in the financial call data submitted to NCCI. Further, although this calculation may appear relatively complex, the core approach of the method (looking at one year's runoff of all prior years during the current calendar year) is actually fairly simple. A disadvantage is that the growth factor used by NCCI is an approximation, and the number of years of data used in the calculation is selected judgmentally. Also of note, this method requires that a sufficient history of accident years and volume of loss activity exists in the

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13 In this paper, discussion of NCCI's large loss methodology is restricted to that portion affecting the tail factor calculation. For a more thorough treatment of the procedure used by NCCI, see "Catastrophes and Workers Compensation Ratemaking," by Tom Daley, CAS *Forum*, Winter 2007.

14 If the selected uncapped 19th-to-ultimate paid + case tail factor is less than 1.0, the tail adjustment factor is set equal to 1 so that the capped tail factor equals the uncapped tail factor.

'prior' row.

#### 3.4.5 Users

This method is used most by its developers, NCCI, but it is sometimes used by consulting firms as well.

#### 3.4.6 Summary

At its core, this method was designed by a rating bureau for their specific situation. However, it has evolved from a fairly simple and understandable concept. Therefore, as long as there is an adequate volume in the runoff from prior years and an appropriate and reliable growth correction can be made, it can be a very useful method.

### 3.5 Summary of Algebraic Methods

The algebraic methods key off basic and very reasonable assumptions about the relationship of development in the tail to quantities which are relatively simple to compute from basic reserving data. As such, they are very useful reserving tools.

## 4. BENCHMARK-BASED METHODS

### 4.1 Introduction to Benchmark-Based Methods

If a suitable benchmark can be found, the use of benchmark data from a larger pool of losses, typically those that contain development detail at greater maturity than the data being developed, can supplement the data being developed. This can feature advantages due to a higher credibility of the link ratios near the tail, or may have more years of development than a start-up type program.

### 4.2 Directly Using Tail Factors from Benchmark Data

#### 4.2.1 Description

Many actuaries review benchmark data when selecting a tail factor. Benchmark data can be used in place of or as a supplement to more company-specific data when selecting the tail factor. In some cases, the benchmark is comprised of industry data triangles and the tail must be derived; in other cases the tail factor and development pattern have been selected by the organization producing the benchmark data. At its simplest, the benchmark method involves copying the benchmark age to ultimate development factor at the maturity desired for the tail factor. If the tail factor needed is a different age than available, it will be necessary to interpolate (assuming the age is in between two ages available in the benchmark) or extrapolate (if the age needed is outside the range of ages available in the benchmark data).

For extrapolation, it may be possible to use one of the other methods described in this paper. If the source does not directly compute a tail factor, it will be necessary to derive a tail factor.

### 4.2.2 Data Sources

Perhaps the most common benchmark data triangles are those that can be developed from Best's Aggregates and Averages for each of the Schedule P lines. This source presents summarized development triangles on an industry basis out to 120 months. Triangles are available for Paid, IBNR and Total Incurred (paid loss + case reserves + IBNR) to 120 months for the last 10 accident years. An incurred loss triangle excluding IBNR can be derived by subtracting the IBNR triangle from the Total Incurred triangle. Aggregates and Averages do not generate a tail factor or development pattern directly; a tail factor must be calculated. This can be done using one of the other methods described in this paper (on what should be a very credible set of data) or a tail factor can be inferred based on the IBNR booked by the industry. For example, if one needed a paid tail factor from 96 months to ultimate for a particular period, you could compute the ratio of the ultimate losses of the accident period at 96 months to paid loss at 96 months to determine the tail factor. Alternatively, you could use the ratio of ultimate loss for all accident periods older than 96 months to the sum of paid loss at 96 months for those same accident periods.

The two larger rating bureaus, the National Council on Compensation Insurance (NCCI) and Insurance Services Office (ISO), as well as the Reinsurance Association of America (RAA), all publish benchmark loss development data. Benchmarks are also available from the state workers compensation rating bureaus. The rating bureaus will generally select a development pattern and tail factor based on the statistical data reported to them by insurance companies and other writers in the case of workers compensation coverage.

Another source of benchmark data is the annual statements of individual insurance companies. This data is basically in the same form as Aggregates and Averages. The annual statements can be found at each state's insurance department. Tail factors can be derived as described above, but this method is more heavily dependent upon the adequacy of the reserve estimates for a single company, and would be less credible. On the other hand, this data would more specifically capture the reserving practices of the company used. Also, the annual statement of a company known to be writing business on risks similar to those of the company under review may be of particular interest.

### 4.2.3 Usage

This method is very commonly used by consulting actuaries and actuaries at smaller companies where data either are inadequate or do not exist.

#### 4.2.4 Advantages and Disadvantages

One key advantage of using tail factors from benchmark data is that benchmark data is easily available through common industry sources. In addition, benchmark tail factors are typically based on a high volume of data, which can help reduce process variance that is often inherent in smaller data sets.

The primary disadvantage of this method is that the benchmark tail development may not be representative of the book of business being analyzed. Considerations such as differences in the way claims are adjusted or reserved, differences in the types or mix of types of claims (medical vs. indemnity), differences in the potential for long-developing high-value claims, differences in the initial reporting pattern of claims (claims-made vs. occurrence, whether or not there is an innately long discovery period, etc.), and differences in the adjudication process of litigated claims can all cause differences in development patterns. It is important to consider those factors along with the statistical reliability of the benchmark triangle when selecting the most appropriate benchmark tail factor.

#### 4.2.5 Summary

This is the most basic and most common of the benchmark-based methods. It is dependent on the benchmark data being a 'good match' to the data in question. However, for low-credibility data, where it is most often used, any mismatch in data must be measured against the unreliability of the data in the triangle being analyzed.

### 4.3 Use of Benchmark Tail Factors Adjusted to Match Pre-Tail Link Ratios

#### 4.3.1 Description

One way to address differences between the benchmark development pattern and the development pattern of a given book of business is to try to adjust the benchmark data to take into account differences in the subject book of business. One common practice is to compare the age-to-age link ratios from the subject data to the benchmark age-to-age link ratios prior to the tail development stage. The relativities from those stages are used to estimate an adjustment multiplier for the benchmark tail factor. Of note, generally just the development portions of the link ratios ($v(d)$ of $1+v(d)$) are compared.

#### 4.3.2 An Example

An example will help to illustrate how the process works. Consider the following two patterns where we simply compute the ratio of the development portion of our triangle-based link ratios to the development portion of the matching benchmark link ratios:

| (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|
| | Selected Link Ratio | | Benchmark Link Ratio | | Selected to Benchmark Ratio = |
| Maturity | Estimated by Triangle $f(d)=1+v(d)$ | Development portion $v(d)$ | Ratio | Development portion $v(d)$ | (3)/ (5) |
| 12 | 2.000 | 1.000 | 2.000 | 1.000 | 100% |
| 24 | 1.450 | .450 | 1.350 | .350 | 129% |
| 36 | 1.200 | .200 | 1.150 | .150 | 133% |
| 48 | 1.150 | .150 | 1.100 | .100 | 150% |
| 60 | 1.100 | .100 | 1.050 | .050 | 200% |
| 72 | 1.080 | .080 | 1.030 | .030 | 267% |
| 84 | 1.050 | .050 | 1.025 | .025 | 200% |
| 96 | 1.035 | .035 | 1.020 | .020 | 175% |
| 108 | 1.010 | .010 | 1.010 | .010 | 100% |
| Tail | | | 1.050 | | |
| Chosen Ratio | | 200% | | | |

$T(n)$ = 1+.050*200% =1 + .100 = 1.100

In the example above, 200% is chosen as the ratio of subject development portions of the age-to-age factors to the benchmark based on the 60- through 108-month relativities.

The underlying assumption of this adjustment is the underlying processes in our subject data that are causing the (in this case) higher development than seen in the benchmark data will continue throughout the life of the claim. This may or may not be the case. From a practical standpoint, it is generally not possible to examine all aspects of claims handling to the degree necessary to make this determination. The example above is representative of a reasonable adjustment one might make based on the data, but it is a qualitative adjustment, not a statistically based adjustment.

#### 4.3.3 Usage of This Method

This method is very commonly used by consulting actuaries and actuaries at smaller companies where data either are inadequate or do not exist.

#### 4.3.4 Advantages and Disadvantages

The main advantages of this method are (1) it is easy to apply and (2) it presents a very broad representation of the potential outcomes of the subject data. Industry-wide benchmark data represents an industry-wide view of the possible outcomes of the claims adjustment process. Even a complete set of data for a smaller company may not adequately represent the potential for very long-term claims. This broad perspective can also be one of the major weaknesses of this method. The benchmark may be too broad, as it is often difficult to find a perfect match in terms of all the factors (claims handling, case reserving, potential for large claims, etc.) that affect loss development.

Another issue with benchmarks is the availability of data beyond 120 months. Most available benchmark data does not extend beyond 108 or 120 months. Deriving a tail factor for an age beyond this time frame would require some form of extrapolation.

#### 4.3.5 Summary

This method is a relatively simple way to improve the tail predictions generated by benchmark data. It is used a little less commonly than the 'straight benchmark,' though, there are many different ways to adjust benchmark data. Presumably, adjustment can improve the effectiveness of benchmark data significantly.

### 4.4 Benchmark Average Ultimate Severity Method

#### 4.4.1 Description

This method relies on a benchmark average severity and the reported average severity near the tail to derive a tail factor. It requires two key assumptions. Specifically, one must first assume that the average ultimate severity of the oldest accident period being analyzed is equal to or similar to some benchmark ultimate severity. Second, one must assume that the number of reported claims is equal to the total ultimate number of claims (or, equivalently, one must be able to derive a highly reliable estimate of the total ultimate number of claims generated by the oldest period). The method then involves the simple act of using the ratio of the benchmark average ultimate severity to the reported severity as the tail factor. If, for an accident period, the estimate of the ultimate number of claims is higher than the oldest period's number of reported claims, then the ratio of benchmark ultimate severity to the reported average severity must be multiplied by the reported claims count tail factor to derive the tail factor.

In mathematical terms, the first case may be stated as:

$$T(n) = Average\,Severity(u) \,/\, \frac{c_{inc}(1,n)}{c_{reported\,count}(1,n)}\,. \tag{4.1}$$

The second case may be stated as:

$$T'(n) = T(n) \times T_{reported\,count}(n)\,, \tag{4.2}$$

where we recall that $n$ represents the last development age of a given accident period as well as the most recent accounting period for which data is available. $T_{reported\ count}(n)$ is the development factor to ultimate for reported claim counts for the oldest maturity in the triangle $n$.

When using this method it is absolutely imperative that the benchmark severity be appropriate for the eldest period. Note that when the triangle data has low or medium credibility, the true average severity may be strongly affected by the vicissitudes of fortune with respect to large, late settling claims. If more than the average number of large claims are present in the data, this test may improperly suggest negative development in the tail. If the oldest period contains fewer large claims than average (which is more common), then this method will suggest more development than actually occurs. On the other hand, in the rare cases where a large number of large claims emerge and balance out the average number of large claims, the development will be relatively greater than this method implies. Of note, if only a limited amount of development is available in the triangle (say four to five periods in long tail lines), and the larger claims all occur after the oldest maturity $n$, then (as long as the benchmark is appropriate) this test may have higher relative reliability. That is because this test can comfortably assume an average number of large losses without the data being distorted by variance in the number of large losses already reported. Further, note that the class of business must be such that a reliable benchmark that matches the type of data in the triangle is available. For example, if one has a large volume of private passenger auto data all from standard classes, for which benchmark data is readily available, this method may prove to be useful.

**4.4.3 Example**

Consider a triangle going out 10 periods (120 months) containing private passenger auto data. Suppose that all the claims are clearly reported by 120 months but some remain unsettled. Specifically, suppose that the total incurred loss for the oldest period is

$c_{inc}(1,10)$ = $120 million.

Further, suppose the corresponding reported counts are

$c_{reported\ count}(1,10)$ = 6,000.

So, the reported severity at 120 months is $20,000. If the benchmark average ultimate severity is $20,200, then the implied tail factor would be

$T(10)$ = 20,200/20,000 = 1.01.

**4.4.3.1 A Second Example**

The other utility mentioned for this approach involves long-tail data that requires a tail for a medium term triangle. Say, for example, that a workers compensation triangle is available, but it only has five 12-month periods of data and hence stops at 60 months. Suppose you know that the average severity benchmark data, for the hazard group mix contained in your data, at ultimate is $50,000 per claim, counting both initial claims closed with any type of payment and reopened claims closed with any type of payment separately in the denominator. Further, suppose that this larger benchmark workers compensation data says that the reported claims count tail factor at 60 months is 1.02.

Then all you need from your data are the reported counts and incurred losses for the oldest period. (Again, all reported count figures only include those with payment, and count reopened claims as claims in themselves in this example). Suppose they are:

$c_{inc}(1,5)$ = $4 million, and

$c_{reported\ count}(1,5)$ = 100 claims.

Then the current reported severity of the oldest period would be $4 million/100 = $40,000 per claim;

and the implied tail factor would be

1.02 ×$50,000/40,000 = 1.02 ×1.25 = 1.275.

#### 4.4.4 Advantages and Disadvantages

As mentioned above this method is only suitable when a reliable benchmark average severity is available and when the presence or absence of a few large losses are not factors in the eldest period's data. Due to the relative rarity of those situations, this method is not widely used.

#### 4.4.5 Users

This method does not currently have widespread usage. A few actuaries in consulting and primary company actuaries have been observed to use this method.

#### 4.4.6 Summary

This method involves applying an average severity from benchmark data to correct the severity shown in the case incurred data. Because of the difficulty in finding reliable benchmark severities, its utility and use in practice is somewhat limited.

### 4.5 Use of Industry-Booked Tail Factors

This method is also referred to as the "industry-booked" method, as it relies on the adequacy of booked industry IBNR in older accident years to determine the tail factor. While it can be argued that this factor should represent the "best" estimate of the industry actuaries of the additional reserve need, history has shown that this figure has often been inadequate. This would suggest that tail factors based on this method would be understated.

#### 4.5.1 Description

The general practice while using this method is to simply look at the (direct or net) IBNR booked by the industry (per Best's Aggregates and Averages, or other sources) for the oldest year in schedule P, then divide that by the (direct or net) case incurred loss for that year. The result forms the industry booked incurred tail. Similarly, dividing the (direct or net) case reserves + IBNR for the oldest year by the (direct or net) paid loss for that year yields the industry-booked paid loss tail.

#### 4.5.2 Example

Assume that the industry Schedule P for the year 2007 shows the following values for accident year 1998 (the oldest year in that Schedule P):

| | |
|---|---|
| A. Direct Paid Loss | 5,000,000 |
| B. Direct Loss Case Reserves | 2,500,000 |
| C. Direct IBNR | 2,500,000 |

Then, we first compute some intermediate values (the total incurred and total reserve):

| | |
|---|---|
| D. Total Case Incurred Loss (= A+B) | 7,500,000 |
| E. Total Reserves (Case + IBNR) (= B+C) | 5,000,000 |

We then can compute the development portions of the tail factors as described above, and the tail factors themselves.

| | |
|---|---|
| F. Development Portion of Incurred Tail Factor (= C/D) | 0.33 |
| G. Incurred Loss Tail Factor (= 1.0 + F) | 1.33 |
| H. Development Portion of Paid Tail Factor (= E/A) | 1.00 |
| I. Paid Loss Tail Factor (= 1.0 + H) | 2.00 |

#### 4.5.3 Usage of this Method

In spite of the potential problems with industry reserve inadequacy, this method is in broad usage in consulting firms and by actuaries at small- to medium-sized insurance companies. Generally, larger companies tend to have better alternatives. There is a smaller group of large firms that prefer to benchmark relative to their peers that may use this approach.

### 4.5.4 Advantages and Disadvantages of this Method

The pros and cons of this method revolve around two main points: first, the data is easy to obtain and the method itself is easy to perform; but, second, for many lines it may be unrealistic to expect industry booked IBNR to be adequate. Another important concern would be whether or not the industry would be a suitable benchmark for the book of business being analyzed.

### 4.5.5 Summary

It must be noted that this method is based on what may be an incorrect assumption (that industry IBNR is adequate). Nonetheless, many actuaries use this method. That is perhaps a tribute to its simplicity.

## 4.6 Benchmark Tail Factors Adjusted for Company-Specific Case Reserving

The use of benchmark data, as discussed earlier, is often necessary due to lack of credibility in triangles with low data volume near the tail. However, this can be problematic when the entity handling claims for the subject book of business uses different case reserving standards than the industry at large. In such cases, it is common to include a correction to the benchmark tail factors to reflect the specific case reserve adequacy of the subject book of business.

### 4.6.1 Description

This method is very similar to the use of benchmark tail factors, excepting that a secondary factor is included that adjusts the case reserves near the tail to industry level. Most commonly, the adjustment will be generated by a claims audit. That will involve sending a highly experienced claims person, preferably one specializing in claims audits, to the claims handling office for a formal audit. Typically, such a claims auditor will review a sample of the claim files and, based on what is in the file and his or her claims expertise, estimate what case reserve should be carried on the file at industry standard case reserve levels. Such efforts may be focused on the tail by sampling solely from the most mature years in the triangle. That is because the case adequacy may be different near the tail than it is an early and intermediate maturity. Using the results of the audit, one can compute a case reserve adjustment factor to industry reserving levels as the ratio of the total case reserves suggested by the claims auditor divided by the carried case reserves on the claims in the sample.

As the final step in producing the corresponding tail factor, one need only multiply the benchmark tail factor by a factor to adjust the business' total case incurred losses to industry levels. To compute that factor to adjust the business' total case incurred to industry levels we sum up the ratio of the eldest years' cumulative paid losses to its case incurred losses plus the claims auditor's case reserve adjustment factor times the eldest years' ratio of case reserves to case incurred losses. So, in total we have the following equation.

$$\hat{T} = T_{Benchmark} \times \{(c_{Paid} / c_{Incurred}) + [Adj\ Factor \times (s / c_{Incurred})]\} \quad (4.3)$$

**4.6.2 Example**

Suppose the benchmark tail factor is 1.2. Further, suppose the cumulative paid loss for the eldest year is 85% of the case incurred, so the case reserves ('*s*' above) are 15% of the case incurred at the tail. Then suppose a claims audit says that in order to bring case reserves in line with industry reserve adequacy, the case reserves should be twice what they are. Then, the adjusted benchmark tail is:

1.2 × (.85 + 2 × .15) = 1.2 × 1.15 = 1.38.

**4.6.3 Advantages and Disadvantages**

This method offers a significant opportunity to improve the accuracy of benchmark tail factors. However, claims audits can impose a significant cost and, more importantly, require the use of highly trained claims auditors. These resources are not available to every actuary. Further, the auditors need not only to be highly trained but also have to be extremely objective, or else the results will be misleading. Perhaps another approach would be to ask an objective auditor for 'industry best practices,' which might be different from the 'industry average.' In order to recognize that difference, claims adjusted using industry best practices could be developed using a benchmark that is more mature than the data. Certainly the more experience an actuary has in working with an auditor and watching the tails develop, the more trust he or she can place in this method. Also the fact that history has shown benchmark IBNR data is often inadequate must be considered.

**4.6.4 Users**

This method is used primarily by large commercial and reinsurance carriers that must reserve data from a multitude of different claims handlers. Some actuarial firms that work with data from many different claims entities use it as well.

**4.6.5 Summary**

In summary, this method can be a useful adjunct to the use of benchmark tail factors, but does require an extensive set of resources. Further, it requires a great deal of vetting of not just the case reserves but the claims auditor as well.

### 4.7 Summary of Benchmark-Based Methods

Benchmark data can serve a useful function, especially in the small-to-medium credibility situations. One must be careful, though, to make sure that either the benchmark data is a good match for the book of business being analyzed, or that appropriate adjustments are applied either to the benchmark data or the book of business being analyzed.

## CURVE-FITTING METHODS

### 5.1 Introduction to Curve-Fitting Methods

One strategy for developing tail factors is to posit some relationship between the link ratios at various development ages (or, some similar quantity such as incremental paid by development age), and use that relationship as an assumption to fit a curve to the link ratios. Projected link ratios in the development ages covered by the tail factor can then be generated. All those projected link ratios can then be multiplied together to provide an estimate of the tail factor. The methods below represent only those methods where curve-fitting is the primary source of the tail factor. There are several methods (e.g., Mueller's method, which is discussed later) that involve curve-fitting but are not solely curve-fitting type methods.

The topic of modeling loss development for various purposes such as projecting ultimate losses or estimating variability in development factors has been discussed in various actuarial articles and papers such as McClenahan [9], Finger [4] and Hayne [8]. A common characteristic of probability distributions selected for modeling is that they indicate that incremental losses emerge or are paid out at a monotonically decreasing rate (decay function). The exponential distribution is one of many probability distributions used in practice for modeling a decay process.

### 5.2 Exponential Decay Method

### 5.2.1 Description

The method utilizes link ratios, $f(d_i)$, as opposed to cumulative or incremental paid loss. Define the function $v(d_i)$, the development portion of the link ratio, as follows: $f(d_i) = 1 + v(d_i)$. In contrast to the McClenahan method (see section 5.3) and Skurnick method (see section 5.4), this method assumes that the $v(d_i)$'s decay at a constant rate, $r$, i.e., $v(d_{i+1}) = v(d_i) \times r$.

The process consists of first fitting an exponential curve to $v(d_i)$'s. This can be accomplished by using a regression to the natural logarithms (natural log) of $v(d_i)$'s. Next, the decay constant $r$ can be estimated as the inverse natural log of the slope of the fitted curve. The remaining development, from a given development age $d$, can be estimated as:

$$T(d) = \prod_{m=1}^{\infty} = (1 + v(d) \times r^m)\,. \tag{5.1}$$

For small $v(d)$, remaining development can be approximated by:

$$T(d) \approx 1 + v(d) \times \sum_{m=1}^{\infty} r^m = 1 + v(0) \times r^{(d)} / (1 - r)\,. \tag{5.2}$$

### 5.2.2 Example

Appendix B, Section 5.2, shows a contrived example of fitting the following link ratios:

| Age in Months | Period | Link Ratio |
|---|---|---|
| 12 | 1 | 1.5 |
| 24 | 2 | 1.25 |
| 36 | 3 | 1.125 |
| 48 | 4 | 1.0625 |
| 60 | 5 | 1.03125 |
| 72 | 6 | 1.015625 |
| 84 | 7 | 1.007813 |

The outputs from the curve fit and actual and approximated tail calculations are shown below:

| From curve fit to column (5) in the Appendix | |
|---|---|
| $\ln(r)$ = -0.6931 | $r$ = 0.5000 |
| $\ln[v(0)]$ = 0.0000 | $v(0)$ = 1.0000 |

| | | |
|---|---|---|
| Product of Age 8 to Age 22 Link Ratios | 1.007830 | $= T(8)$ |
| Approximation formula | 1.007813 | $= 1 + v(0) \times r^8 / (1 - r)$ |

#### 5.2.2.1 Another Example (Appendix B, Section 4.1)

The above example was contrived for purposes of demonstrating the method. A more realistic data pattern helps highlight certain issues that can arise when using this method.

In the Appendix example the "error in fit" (actual minus fitted) suggests a possible poor fit of the curve to the data. In the next section, a method of addressing the issue of less than optimal fit is presented.

#### 5.2.2.2 Adjustment to Exact Fitting

In this enhancement, the development portion of the derived tail factor is adjusted by the "actual to fitted ratio" from the last stage. For example, suppose that the development portion of the 108-month tail factor is 0.03 and the ratio of the actual-to-fitted link ratio is 1.7. The adjusted tail factor is now 1 + (0.03) x (1.7) = 1.051. This 'adjustment' increases the tail factor, but this resulting value is considerably different from the tail factor produced by the method. Without further knowledge of the underlying data, such as what is the line of business, what are the claims department's reserving/payment practices, etc., there remains uncertainty as to which result is the better estimate or whether either estimate is appropriate.

#### 5.2.2.3 Fitting Curve to Mature Periods Only

Since the focus of the curve-fitting is in estimating the development in the more mature ages, one possible enhancement to the methodology is to only fit the curve to the latter development periods.

### 5.2.3 Advantages and Disadvantages of Exponential Decay Method

This method is fairly straightforward to construct, intuitive in nature and there exists a closed-form approximation, which can be applied in most situations. The assumptions underlying the method are: (1) loss development from period to period decays in a constantly decreasing pattern, (2) the exponential decay rate is constant throughout the entire loss development pattern.

Exponential decay can produce relatively fast development compared to the development resulting from other distributional models. In certain circumstances (for instance high excess lines or long tail liability lines) other models might produce a more appropriate development result. In addition, in the case where paid losses do not continue to decay at a constant rate such as workers compensation indemnity, an alternative approach might be more appropriate. This method is not generally applicable to incurred losses for such reasons as (1) changing reserve patterns and (2) negative development, which would refute the decay constant assumption and can produce erroneous results from the fitting, if any at all.

### 5.2.4 Users

This method is used to a varying extent by consulting actuaries and actuaries at smaller companies where data either are inadequate or do not exist, or when development experience to date for a newly underwritten line of business does not reflect patterns from alternative sources such as industry aggregated data. Since a key assumption of this technique is a constant decay rate this might generally run contrary to other assumptions underlying the development patterns assumed in a reinsurance application.

### 5.2.5 Summary of Exponential Decay Method

The exponential decay method is based on a few assumptions concerning the rate of decay in incremental loss paid. From these assumptions, a curve can be fit to the development portion of age-to-age factors, which are calculated from observed paid loss data, and a resulting tail factor can be developed from the slope of the fitted curve. This method will produce suboptimal results for lines of business for which the decay rate "stalls out" or varies by development period, but adjustments such as fitting the curve to the most mature development periods will sometimes improve results.

## 5.3 McClenahan's Method

This method is derived from Charles McClenahan's loss model [9], which assumes incremental paid losses decay at a constant monthly rate after an initial few months lag in which no claims are paid.

### 5.3.1 Description

Let the monthly decay rate, $p$, be defined as the ratio of {accident month $m$ incremental losses paid during month $d$ to $d+1$} to {accident month $m$ incremental losses paid during month $d-1$ to $d$},

$$p = q^{*}_{Paid}(m, d+1) / q^{*}_{Paid}(m, d) \tag{5.3}$$

for all accident months $m$ and accident maturities (in months) $d \geq a$, where $a$ is the average lag time (in months) until a claim begins to be paid. Since total loss from accident month $m$ can be expressed as the sum of all monthly payments made on these claims over time, we have

$$U^{*}(m) = \sum_{d=a}^{\infty} q^{*}_{Paid}(m, d). \tag{5.4}$$

Since we assume a constant monthly decay rate, for some constant $A$, the incremental losses paid in month $a$ can be expressed as $q_{Paid}(m,a) = A \times (1-p)$. Using the theorem $\sum_{n=0}^{\infty} p^n = 1/(1-p)$, it can be shown the constant $A$ is in fact the ultimate or total loss incurred in accident month $m$.

$$U^{*}(m) = \sum_{d=0}^{\infty} A \times (1-p) \times p^d = A. \tag{5.5}$$

Under this assumption, additional payments are theoretically determined once the parameters $p$ and $a$ are estimated. The monthly decay rate is constant, so the annual decay rate, $r$, for annual periods after the initial lag period in which no claims are paid is also a constant, $r = p^{12}$. Given an average annual decay rate, the monthly decay rate $p$ can be estimated as the 12th root of the average annual decay rate. McClenahan suggests estimation of $a$ can be derived from the average report lag (average date of report – average date of occurrence). In any event, the final selection of the parameter $a$ should consider the overall fit of the decay curve to the selected link ratios.

For any accident period at $d$ months development, the tail factor is just unity divided by the percentage of total losses paid at $d$ months, or

$$\mathrm{T}(d/12) = 1 \,/\, (1 - \text{percentage unpaid at } d \text{ months}). \tag{5.6}$$

In his paper, McClenahan presents several closed-form formulas for various loss statistics.[15]

Assuming $U^*(m)$ is constant for all $m$, and letting $q = 1 - p$, we can derive the following expressions for the total loss for year $w$, and future development of accident year $w$ respectively,

$$U(w) = \sum_{m=0}^{11} U^*(m) = (12 \times A \times q)/(1-p)\,, \tag{5.7}$$

$$R(w, m/12) = \left\{U(w) \times q \times p^{m-a-10} \times (1-p^{12})\right\}/(1-p)^2\,. \tag{5.8}$$

Substituting (5.7) and (5.8) into equation (5.6) produces the closed-form expression for a tail factor at $m$ months in terms of $a$, $m$, and $p$

$$T(m/12) = \left\{12 \times (1-p)\right\} / \left\{12 \times (1-p) - p^{m-a-10} \times (1-p^{12})\right\} \tag{5.9}$$

$R$ is used here with the same meaning as in McClenahan's work, rather than as defined in Section 1.7.

### 5.3.2 Example (An additional example is in Appendix B, Section 4.2)

Reviewing an example may help the reader follow the application of the model discussed above. Even though this method is presented as applicable to incremental paid loss, with actual loss data, it would be highly unlikely that paid incremental losses for different accident periods will be the same, therefore we begin with the selection of age-to-age factors from an eight year (96-month) triangle:

**Selected Age-to-age Factors**

15 For the purpose of this exercise, the variables McClenahan incorporates in his model for trend in severity, frequency, etc. can be collapsed into the decay rate and total loss for the accident period, hence there can be certain simplifications utilized in applying McClenahan's formulas for $U(w)$ and $R(w, m/12)$.

| 12-24 | 24-36 | 36-48 | 48-60 | 60-72 | 72-84 | 84-96 |
|---|---|---|---|---|---|---|
| 5.7720 | 1.5290 | 1.1870 | 1.0851 | 1.0424 | 1.0220 | 1.0116 |

Next we convert these to cumulative paid loss amounts by selecting a base amount for the first development period paid loss, for simplicity sake we use $100 in our example. To determine incremental paid losses by period we subtract successive cumulative loss amounts, and then we have the following table:

DEVELOPMENT DATA

| (1) Development Age | (2) Selected Age-to-age Factors | (3) Age in Months | (4) Cumulative Paid | (5) Incremental Paid |
|---|---|---|---|---|
| | | 12 | 100.00 | 100.00 |
| 12-24 | 5.7720 | 24 | 577.20 | 477.20 |
| 24-36 | 1.5290 | 36 | 882.54 | 305.34 |
| 36-48 | 1.1870 | 48 | 1,047.57 | 165.03 |
| 48-60 | 1.0851 | 60 | 1,136.72 | 89.15 |
| 60-72 | 1.0424 | 72 | 1,184.92 | 48.20 |
| 72-84 | 1.0220 | 84 | 1,210.99 | 26.07 |
| 84-96 | 1.0116 | 96 | 1,225.04 | 14.05 |

Taking successive ratios of incremental paid amounts for the accident periods produces estimates of the annual decay constant $r$. This example was contrived to produce an estimate of $r$, but in practice any of a variety of curve-fitting techniques using the incremental paid loss regressed on age can be employed to develop an estimate of $r$ from Column 5.

In order to avoid distortions in the "true" annual decay rate caused by the payment lag, for our example we will next fit the curve to incremental accident period losses starting with the third annual development period.

By definition $p = r^{1/12}$, and for the sake of the example, we will assume a lag constant of $a = 7$ months (see above discussion on estimating $a$). Once the value of $r$ is calculated, with the value of $p$ estimated, we can develop an estimate of $T(8)$ using equation (5.7) above.

| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
|---|---|---|---|---|---|---|
| | Selected | | | | | Fitted |
| | Age-to-age | Age in | Cumulative | Incremental | | Incremental |
| Age | Factors | Months | Paid | Paid | Age-to-age | Paid |
| | | 12 | 100.00 | 100.00 | | |
| 12-24 | 5.7720 | 24 | 577.20 | 477.20 | 4.7720 | |
| 24-36 | 1.5290 | 36 | 882.54 | 305.34 | 0.6399 | 306.02 |
| 36-48 | 1.1870 | 48 | 1047.57 | 165.03 | 0.5405 | 165.35 |
| 48-60 | 1.0851 | 60 | 1136.72 | 89.15 | 0.5402 | 89.35 |
| 60-72 | 1.0424 | 72 | 1184.92 | 48.20 | 0.5407 | 48.28 |
| 72-84 | 1.0220 | 84 | 1210.99 | 26.07 | 0.5409 | 26.09 |
| 84-96 | 1.0116 | 96 | 1225.04 | 14.05 | 0.5389 | 14.09 |

$r$ =0.5403 From Curve Fit to column (5)
$p$ = 0.9500
$a$ = 7
$m$ = 96

$$T(8) = 1.0135 = \{12 \times q\} / \{12 \times q - p^{m-a-10} \times (1 - p^{12})\}$$

**5.3.2.1 Exact Fitting to the Oldest Period**

Curve fitting commonly has the problem of producing parameters that result in a less than desired fit in the tail of the curve, relative to actual results observed for these older periods. This can be due to a variety of factors relating to the assumptions underlying the structure or parameters of the fitted curve or random fluctuations within the actual data in earlier development periods. By comparing actual incremental paid loss to fitted results at the latest stage of development, we can usually improve the quality of the tail prediction.

In the above example, assume the actual link ratio for the development stage 84-96 was 1.0175 producing an incremental paid amount significantly greater than the overall annual decay rate $r$ which is still expected to be 0.5403. In this case, the actual decay rate is less in the older development periods, hence the incremental paid loss in these latter development stages maybe expected to be higher than is implied by the model.

One approach would be to adjust the development portion of the initial estimate of the tail factor $\{T(m/12)-1\}$ by the ratio of the actual to the fitted incremental paid loss,

$$T(m/12_A) = 1 + (q_{Paid}^{Actual} / q_{Paid}^{Fitted}) \times \{T(m/12)-1\}. \quad (5.10)$$

Applying this adjustment to the example above, we have the following table:

| (1) Age | (2) Selected Age-to-age Factors | (3) Age in Months | (4) Cumulative Paid | (5) Incremental Paid | (6) Incremental Age-to-age | (7) Fitted Incremental Paid |
|---|---|---|---|---|---|---|
| | | 12 | 100.00 | 100.00 | | |
| 12-24 | 5.7720 | 24 | 577.20 | 477.20 | 4.7720 | |
| 24-36 | 1.5290 | 36 | 882.54 | 305.34 | 0.6399 | 306.00 |
| 36-48 | 1.1870 | 48 | 1047.57 | 165.03 | 0.5405 | 165.33 |
| 48-60 | 1.0851 | 60 | 1136.72 | 89.15 | 0.5402 | 89.33 |
| 60-72 | 1.0424 | 72 | 1184.92 | 48.20 | 0.5407 | 48.27 |
| 72-84 | 1.0220 | 84 | 1210.99 | 26.07 | 0.5409 | 26.08 |
| 84-96 | 1.0175 | 96 | 1232.18 | 21.19 | 0.8128 | 14.09 |

$T(8) = 1.0135$ From initial Example

$Act / Fit = 150\% = 21.19 / 14.09$

$T_A(8) = 1.0203 = 1 + (Act / Fit) \times [T(8)-1]$

#### 5.3.2.2 Using Multiple Periods to Estimate the Tail

This enhancement is similar to exact fitting to the oldest period adjustment, but provides an alternative in situations when the "tail" of the triangle is believed to possess some credibility, but individual link ratios are less than fully credible.

For example, assume as above, the actual (selected) link ratio for the 84-96 development period is 1.0175. In addition, assume the actual link ratio for the 72-84 period is 1.0440 instead of 1.0220. As in the prior example, the ratio of actual to fitted incremental paid loss for the 72-84 development period is now significantly different than 1.000. It should be noted the change in the link ratio for the 72-84 development period also has an effect on the paid incremental loss for the 84-96 development period, hence changes the adjustment ratio (actual to fitted) for this development period as well. These two adjustment ratios can be credibility-weighted to reflect the predictive accuracy of each factor. For the purpose of the example, each factor is assigned 50% weight. This results in an estimate of the tail factor as outlined in the following table:

| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
|---|---|---|---|---|---|---|
| | Selected | | | | | Fitted |
| | Age-to-age | Age in | Cumulative | Incremental | | Incremental |
| Age | Factors | Months | Paid | Paid | Age-to-age | Paid |
| | | 12 | 100.00 | 100.00 | | |
| 12-24 | 5.7720 | 24 | 577.20 | 477.20 | 4.7720 | |
| 24-36 | 1.5290 | 36 | 882.54 | 305.34 | 0.6399 | 306.00 |
| 36-48 | 1.1870 | 48 | 1047.57 | 165.03 | 0.5405 | 165.33 |
| 48-60 | 1.0851 | 60 | 1136.72 | 89.15 | 0.5402 | 89.33 |
| 60-72 | 1.0424 | 72 | 1184.92 | 48.20 | 0.5407 | 48.27 |
| 72-84 | 1.0440 | 84 | 1237.06 | 52.14 | 1.0817 | 26.08 |
| 84-96 | 1.0175 | 96 | 1258.71 | 21.65 | 0.4152 | 14.09 |

$T(8) = 1.0135$ From initial Example

$$(Act/Fit)_2 = 200\% = 52.14/26.08$$

$$(Act/Fit)_1 = 154\% = 21.65/14.09$$

$$(Act/Fit)_{Avg} = 177\%$$

$$T_{A2}(8) = 1.0239 = 1 + (Act/Fit)_{Avg} \times [T(8) - 1]$$

### 5.3.3 Advantages and Disadvantages

This method is relatively easy to apply and produces a closed-form solution. The assumptions underlying the method are: (1) for a given accident period, losses decay at a constant decreasing pattern after an initial payment lag; (2) the reduction in paid incremental losses is proportional to the most current payout; and (3) the exponential decay rate is constant throughout the entire payout pattern (all accident periods, all development periods). If little is known about the "true" development pattern for the data, these assumptions appear to be minimal and reasonable, but care should be taken to assure that these assumptions do apply to the situation in which the method is being applied.

This method is subject to many of the same disadvantages as the exponential decay method such as (1) not being applicable to incurred loss or lines with potential for negative development between evaluation periods, (2) exponential decay at an indicated rate developed from the observed data that can produce a relatively faster development than other models for certain long tail liability lines, and (3) a suboptimal fit would be obtained for lines with variable decay rates across evaluation periods such as workers compensation or if the decay rate varies by accident period.

#### 5.3.4 Users

This method is a variation of the exponential decay method utilizing incremental paid loss in place of the development portion of the link ratio. Usage of this method or similar variants of the exponential decay method (for example see Skurnick's method in section 5.4) are used to varying extents by consulting actuaries and actuaries at smaller companies where data either is inadequate or does not exist or when development experience to date for a newly underwritten line of business does not reflect patterns from alternative sources (i.e., industry aggregated data). Usage by reinsurance actuaries is assumed to be infrequent due to the constant rate of decay assumption.

#### 5.3.5 Summary

Based on most of the assumptions underlying McClenahan's loss model along with the rate of decay estimated from the incremental paid experience data, a closed form equation for the tail factor can be developed. The results of the method can be adjusted in cases where the fit using all periods is less than optimal (different decay rate at later maturities) or credibility in the older development periods is less than fully credible.

### 5.4 Skurnick's Method

This method is derived from the loss model developed by David Skurnick [15] in his discussion of Charles McClenahan's loss model [9].

#### 5.4.1 Description

This method is based on the same underlying loss model as McClenahan's method discussed in section 5.3 with a few simplifying assumptions. First, the model is developed on annual incremental payments and an annual decay rate. Second, no average delay constant is assumed (i.e., no delay between accident occurrence and accident payment). Third, we assume the annual rate of decay can vary by accident period (this assumption is not necessarily a simplifying one).

More formally stated, the annual decay rate, $r_w$, is defined as ratio of {accident year $w$ incremental losses paid during development period $d$ to $d+1$} to {accident year $w$ incremental losses paid during development period $d-1$ to $d$}, i.e., $q_{Paid}(w,d+1)/q_{Paid}(w,d)$. Since total loss from accident period $w$ can be expressed as the sum of all annual payments made on these claims over time we have,

$$U(w)=\sum_{d=0}^{\infty}q_{Paid}(w,d). \tag{5.11}$$

Given a constant rate of decay, the incremental losses paid in period 0 can be expressed in terms of some constant $A$ and the decay rate, $q_{Paid}(w,0)=A\times(1-r)$. Using the theorem

$$\sum_{d=0}^{\infty}r_w^d=1/(1-r_w), \tag{5.12}$$

we can show the constant $A$ is the total loss for accident period $w$,

$$U(w)=\sum_{d=0}^{\infty}q_{Paid}(w,d)=\sum_{d=0}^{\infty}A\times(1-r_w)\times r_w^d=A. \tag{5.13}$$

For any accident year $w$ at $D$ period's development, by definition, the tail factor times the sum of the incremental loss paid to date will produce an estimate of the ultimate loss for accident year $w$. In equation format this can be expressed as:

$$T(D)*\sum_{d=0}^{D}U(w)\times(1-r_w)\times r_w^d=U(w). \tag{5.14}$$

Based on the following theorem for finite summations:

$$\sum_{d=0}^{D}ar^i=\left[ar^{D+1}-a\right]/(r-1)\ \text{ if } r\neq 1 \tag{5.15}$$

we can develop a closed form solution for the tail factor as:

$$T(D)=1/(1-r_w^{D+1}). \tag{5.16}$$

### 5.4.2 Example

Assume the following incremental loss payouts for an accident period:

| Age in Months | Period | Incremental Paid |
|---|---|---|
| 12 | 0 | 4,000 |
| 24 | 1 | 2,000 |
| 36 | 2 | 1,000 |
| 48 | 3 | 500 |
| 60 | 4 | 250 |
| 72 | 5 | 125 |
| 84 | 6 | 62.5 |
| 96 | 7 | 31.25 |

Fitting a line to the natural logarithms of the incremental paid losses in each development period, we can develop the estimates for $\ln(r_w)$ and $\ln\left[U(w)\times(1-r_w)\right]$ by using the identity derived above:

$$q_{Paid}(w,0)=U(w)\times(1-r)\times r_w^d, \quad \text{hence}$$

$$\ln\left[q_{Paid}(w,0)\right]=\ln\left[U(w)\times(1-r)\right]+d\times\ln\left[r_w\right].$$

| (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|
| Age in Months | Period | Incremental Paid | Log of (3) | Fitted Incremental Loss | Fit Error |
| 12 | 0 | 4000 | 8.294050 | 4000 | 0 |
| 24 | 1 | 2000 | 7.600902 | 2000 | 0 |
| 36 | 2 | 1000 | 6.907755 | 1000 | 0 |
| 48 | 3 | 500 | 6.214608 | 500 | 0 |
| 60 | 4 | 250 | 5.521461 | 250 | 0 |
| 72 | 5 | 125 | 4.828314 | 125 | 0 |
| 84 | 6 | 62.5 | 4.135167 | 62.5 | 0 |
| 96 | 7 | 31.25 | 3.442019 | 31.25 | 0 |

$\ln(r)=$ -0.6931 $\qquad r=$ 0.5000

From Curve Fit to column (4)

$\ln[U(w)\times(1-r)]=$ 8.294 $\qquad U(w)\times(1-r)=$ 4000

$T_w(6)=$ 1.0079

$T_w(7)=$ 1.0039

Taking the exponential of the estimate of the natural log of $r$ produces estimates of the annual decay constant, from which we can estimate the tail factor at given development stages for this accident period. This example was contrived to produce an estimate of $r$ with no error term (column (6) = column (3) minus column (5)). The next example demonstrates the effect of an increase in incremental loss in an early development period, followed by a return to a constant decay pattern.

| (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|
| Age in Months | Period | Incremental Paid | Log of (3) | Fitted Incremental Loss | Fit Error |
| 12 | 0 | 1000 | 6.907755 | 2,245 | -1,245 |
| 24 | 1 | 2000 | 7.600902 | 1,260 | 740 |
| 36 | 2 | 1000 | 6.907755 | 707 | 293 |
| 48 | 3 | 500 | 6.214608 | 397 | 103 |
| 60 | 4 | 250 | 5.521461 | 223 | 27 |
| 72 | 5 | 125 | 4.828314 | 125 | 0 |
| 84 | 6 | 62.5 | 4.135167 | 70 | -8 |
| 96 | 7 | 31.25 | 3.442019 | 39 | -8 |

$\ln(r) =$ -0.5776 $r =$ 0.5612

From Curve Fit to column (4)

$\ln[U(w)\times(1-r)] =$ 7.7164 $U(w)\times(1-r) =$ 2245

$T_w(6) =$ 1.0178 $=1/[1-r^{D+1}]$

$T_w(7) =$ 1.0099

The resulting curve does not fit the incremental losses as well in the earlier development periods. The tail factor produced by the estimated decay constant, $r$, is much larger than in the previous example, though the observed decay rate in the incremental losses in the later development periods are the same for both examples.

### 5.4.2.1 Limit Curve Fitting to the More Mature Development Periods

Increases in incremental paid losses from period to period, especially in early stages of development, are a common phenomenon. As demonstrated in the second example above, this can lead to less than an optimal curve fit, and possible distortions in the estimated tail factor. Putting more emphasis on the behavior of losses in the latter stages of development, at a point where a strictly monotonic decease in incremental paid losses is observed, is one approach that can provide a more optimal fit. An example of fitting a curve to actual incremental paid loss, only in the latter stages of development, is shown below:

| (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|
| Age in Months | Period | Incremental Paid | Log of (3) | Fitted Incremental Loss | Fit Error |
| 60 | 4 | 250 | 5.521461 | 250.00 | 0 |
| 72 | 5 | 125 | 4.828314 | 125.00 | 0 |
| 84 | 6 | 62.5 | 4.135167 | 62.50 | 0 |
| 96 | 7 | 31.25 | 3.442019 | 31.25 | 0 |

$\ln(r) =$ -0.6931 $\qquad r =$ 0.5000

From Curve Fit to column (4)

$\ln[U(w)\times(1-r)] =$ 8.294 $\qquad U(w)\times(1-r) =$ 4000

$T_w(6) =$ 1.0079 $\qquad =1/[1-r^{D+1}]$

$T_w(7) =$ 1.0039

In this example, the tail factors produced for the 84- and 96-month development periods are the same as those produced in the original, contrived example.

**5.4.2.2 Excluding the Latest Development Periods to Estimate the Tail**

This enhancement can be used when the last development period incremental data is believed to be less than credible. The procedure is to (1) fit the curve to all periods but the last development period incremental paid loss, (2) compute the corresponding tail factor for the next to last stage of development, and (3) divide this result by the last observed link ratio.

**5.4.2.3 Adjustment to Exact Fitting**

In this enhancement, the development portion of the derived tail factor is adjusted by the "actual-to-fitted ratio" from the last stage. Using the second example above, the development portion of the 96-month tail factor is 0.0099 and the actual to fitted ratio is 31.25/39 = 0.795. The adjusted tail factor is now 1 + (.0099) * (0.795) = 1.0079. Given the observed data utilized in these examples, this 'correction' appears to move the factor in the right direction. This is an example of a situation in which the type of curve fitted to the data is not appropriate, based on the pattern of the data.

**5.4.3 Advantages and Disadvantages**

This method is simpler in construction than the McClenahan model, and produces a closed-form solution. The assumptions underlying the method are: (1) for a given accident period losses decay at a constant decreasing pattern; (2) the reduction in paid incremental losses is proportional to the most current payout; (3) the exponential decay rate, though constant over evaluation periods for a given accident period, may be a different rate for other accident periods; and (4) there is no lag between accident occurrence and accident payment.

Some of the draw backs to this method include: (1) Exponential decay assumes a monotonically decreasing function, therefore this method does not accommodate increases in incremental paid losses from one period to the next (hump-shaped patterns) very well; (2) The method breaks down when, in a given accident period, there are periods of no payments or negative payments; (3) This method is not applicable to incurred losses since they are often subject to negative development or changes in reserving practices (refutes constant decay rate); (4) For less mature accident periods with few valuations, the regression line fit could be less than optimal; and (5) This method is subject to most of the potential pitfalls of the McClenahan method such as the fitted exponential decay rate might be faster than is appropriate for the line of business, or the decay rate might vary by development period for lines such as workers compensation.

#### 5.4.4 Users

This method is a variation of the exponential decay method utilizing incremental paid loss in place of the development portion of the link ratio. Usage of this method or similar variants of the exponential decay method (for example see McClenahan's method in section 5.3) are used to varying extents by consulting actuaries and actuaries at smaller companies where data either is inadequate, does not exist or when development experience to date for a newly underwritten line of business does not reflect patterns from alternative sources (i.e., industry-aggregated data). Usage by reinsurance actuaries is assumed to be infrequent due to the constant rate of decay assumption.

#### 5.4.5 Summary

This method is similar in many respects to the McClenahan method (see section 5.3). Differences of this method from the McClenahan method include that (1) simplifications that reduce the calculations required in the closed-form solution, (2) the ability to vary the decay rate by accident periods, and (3) there is no need for payment lag in the calculation.

### 5.5 Sherman's Method

This method, first articulated by Richard Sherman [14], relies on fitting "inverse power" curves to link ratios.

#### 5.5.1 Description

##### 5.5.1.1 Sherman's Original Method

In this method, we fit "inverse power" curves of the form $1+ad^b$ ($d$ representing development age) to the link ratios. The identity below enables us to base the fitted curve on a simple regression.

$$\ln(\text{f(d)} - 1) = \ln(\text{v(d)}) \approx \ln(1 + ad^b - 1) = \ln(ad^b) = \ln(a) + b[\ln(d)] \quad (5.17)$$

Unfortunately, there does not appear to exist a simple closed-form approximation to the tail this curve generates. The tail factor must then be estimated by multiplying together successive link ratios after the tail begins, until the impact of additional link ratios is negligible.

**5.5.1.2 Sherman's Revised Method**

In his study of the inverse power curve, Sherman [14] noted that the fit could sometimes be improved by adding a lag parameter to the curve. He used the formula

$$\text{f(d)} = 1 + \text{v(d)} \approx 1 + a(d - c)^b \quad (5.18)$$

In this case, the mechanics of fitting the curve are somewhat more complex.

**5.5.2 Example (See Appendix B, Section B.4.3 for additional example)**

The following illustrative data will be used in the appendix to illustrate Sherman's Methods.

First determine the development portion, $v(d)$, of each link ratio. The natural logarithms of $v(d)$ and the age $d$ then represent the dependent and independent variables in our regression, respectively.

| (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|
| Development Age $d$ | Link Ratio $f(d) = 1 + v(d)$ | Development Portion $v(d)$ | 'X' $\ln[d]$ | 'Y' $\ln[v(d)]$ |
| 1 | 2.034 | 1.034 | 0.000 | 0.034 |
| 2 | 1.560 | 0.560 | 0.693 | (0.580) |
| 3 | 1.321 | 0.321 | 1.099 | (1.137) |
| 4 | 1.184 | 0.184 | 1.386 | (1.692) |
| 5 | 1.106 | 0.106 | 1.609 | (2.240) |
| 6 | 1.074 | 0.074 | 1.792 | (2.601) |
| 7 | 1.047 | 0.047 | 1.946 | (3.065) |
| 8 | 1.032 | 0.032 | 2.079 | (3.438) |
| 9 | 1.024 | 0.024 | 2.197 | (3.731) |

The fitted parameters of the dependent and independent variables of the fitted curve then are:

| Fitted-Curve Parameters | |
|---|---|
| Slope = $b$ | (2.386) |
| Intercept | 4.806 |
| $a = e^{Intercept}$ | 1.137 |

The tail factor ($T$) is then estimated as the product of link ratios for development ages 10 through $d$, where $d$ is sufficiently large that the fitted age-to-age is close 1.00.

Several possible alternatives to Sherman's method exist. For example, in determining the appropriate curve, we could rely on link ratios of only the first 5 or 10 development ages or we could rely on the link ratios of only "mature" development ages. In addition, as discussed above, Sherman's revised formula introduces a lag parameter to the curve.

#### 5.5.3 Advantages and Disadvantages

As with all curve-fitting methods, Sherman's method of fitting inverse-power curves to link ratios has advantages and disadvantages. The primary advantage is its relative simplicity and flexibility in evaluating multiple variations, once established in spreadsheet form. The primary disadvantage, on the other hand, is that it makes specific mathematical assumptions about the link ratio pattern when there is no compelling reason for the link ratios to follow any pattern whatsoever.

Sherman's revised formula has an added level of complexity. The modeler must evaluate whether the resulting degree of accuracy warrants the added level of complexity and work.

#### 5.5.4 Users

This method enjoys fairly broad acceptance both with consulting firms and within insurance companies. It is not used quite as often as some of the other methods (e.g., industry booked tail), but is perhaps the most common medium complexity method in use.

#### 5.5.5 Summary

Per Sherman's analysis in the paper describing this method, this method does appear to fit link ratio data better than the various exponential approaches (exponential decay of development, McClenahan's method, and Skurnick's method). The calculations, though they are readily doable by most actuaries, involve a little more mathematics than most audiences are prepared for. Nevertheless, this generates a very useful estimator of the tail factor.

### 5.6 Pipia's Method

This method determines the tail factor that best fits selected age-to-age factors by fitting a Weibul curve to the historical age-to-age data. The best fitting curve is determined by minimizing the squared ratio of the difference between the fitted age-to-age factors derived from the curve and the historical age-to-age factors. The curve represents the age to ultimate factor. The indicated age-to-age factor from the curve is found by dividing the value of the curve at time $d$ by the value at time $d+1$.

#### 5.6.1 Description

Age-to-age factors are selected from historical data or from an industry source; age to ultimate factors are calculated from this data. A tail factor is selected that minimizes the squared differences between selected age-to-age factors and the age-to-age factors implied by the curve representing the age to ultimate factors. For workers compensation, the Weibul distribution, $1-e^{-\lambda(d+c)^t}$ , has been found to provide a good fit to age-to-ultimate factors. The age to ultimate factor at time $d$ equals $1/1-e^{-\lambda(d+c)^t}$ where $c$ is a shift parameter.

### 5.6.2 Pipia's Example (See Appendix B, Section B.4.4)

### 5.6.3 Advantages and Disadvantages

This method is relatively easy to apply and produces a tail factor consistent with the underlying historical observations. It is also easily adaptable to alternative selections of the distribution to be used for other lines of business. A good starting point may be the underlying loss distribution for the line of business since development is often related to the claim size distribution. This method, although it does not produce development factors less than 1.000, does not fail when actual factors below unity are in the historical data being fitted. Another advantage is that the historical data need not be complete or have consistent evaluation dates for each accident year. It provides a means to calculate development factors for a risk that only has scattered loss reports at different and inconsistent evaluation dates. This model can also be used to calculate development factors at intermediate points as well as points prior to or after the historical data. This last item is useful when one is using some benchmark data such as the NCCI Annual Statistical Bulletin, which provides incremental development factors at annual evaluations through 96 months.

This method is subject to many of the same disadvantages as all loss development methods such as changes in case reserving, payout pattern, statutory changes that affect loss development and the appropriateness of the selected distribution for a line of business.

### 5.6.4 Users

It is understood that the developer has used this method to provide another estimate of the tail factor in conjunction with other methods, and that he has also used it when using benchmark data such as Schedule P data from the annual statement. However, due to its limited distribution to date, the specific Weibull-curve method is only used by a few actuaries.

### 5.6.5 Summary

The method fits expected incremental development factors to the actual historical factors to generate an age to ultimate curve. The curve provides the age to ultimate for the average age of an accident year. The average age input can be outside the historical data range as well as at an intermediate point within the historical data period. It provides an alternative estimate of development factors as well as a tail factor. This should be used as one of several alternative methods in making a tail factor selection.

### 5.7 England-Verrall Method

For an excellent introduction to this method, see Section 8, "Discussion And Conclusions," of the research paper itself. Sections of the paper are quoted in their entirety below, although not in the same order in which they appear in the research paper. For consistency, the notation in the following subsections differs slightly from the notation of Section 1.7. The notation of this section follows that of mathematical probability while the notation of Section 1.7 is that of loss development in actuarial science. Table 5.7.2.1 below retains the notation of Section 1.7.

### 5.7.1 Description

Currently, given a triangle of data, a simple reserving exercise might proceed by fitting a chain ladder model (usually a 3, 4, or 5 period volume-weighted average chain ladder) and looking at the resultant development factors. It would then be common to smooth the factors and consider the necessity of a tail factor for projecting beyond the range of data observed. A number of methods, including judgment might be used to smooth the factors with the aim of smoothing out random variations, particularly in the later stages of development, while leaving the systematic trend intact. A tail factor might be chosen, by a variety of methods.

To construct a flexible framework for stochastic claims reserving, within which several of the models can be regarded as special cases, for incremental paid claims $c(w,d)$ define

$$E\left[c(w,d)\right] = m_{w,d}\,, \tag{5.19}$$

$$Var\left[c(w,d)\right] = \phi m_{w,d}^{\rho} \tag{5.20}$$

and

$$\ln(m_{w,d}) = \eta_{w,d} = \mu_{w,d} + \delta k + c + s\theta_w(w) + s\theta_d(d) + s\theta_d(\ln(d))\,. \tag{5.21}$$

Equations (5.19), (5.20), and (5.21), which correspond to Equations (3.3), (3.4), and (3.5) on page 16 of the original research paper, specify a generalized additive model with power variance function and constant scale parameter. The power $\rho$ dictates the choice of error distribution, with normal, Poisson, gamma and inverse Gaussian specified by $\rho$ = 0, 1, 2, and

3, respectively. The predictor is linked to the expected value of the response through the logarithmic link function. The offsets $\mu_{w,d}$ and inflation term $\delta k$ are optional (where $k = w + d$), and may be suggested by a particular context. The function $s(w)$ represents a smooth of accident period $w$, obtained using a smoothing spline with smoothing parameter $\theta_w$. Similarly, the functions $s(d)$ and $s(\ln(d))$ represent smoothing splines specifying the shape of the runoff pattern, with smoothing parameter $\theta_d$ chosen (for simplicity) to be the same for both functions. In practice, it may not be necessary to include smoothers in both $d$ and $\ln(d)$. It should be noted that both accident period $w$ and development age $d$ are considered as continuous covariates.

When $\theta_w$ is zero, there is no smoothing and the model is forced to pass through each value of $w$, which treats accident period $w$ as though it is a factor. The same is true of $\theta_d$; when $\theta_d$ is zero, the model is forced to pass through each value of $d$, and development time is treated as though it is a factor. When $\theta_d$ tends to infinity, the part of the model relating to development time is linear in $d$ and $\ln(d)$, giving the Hoerl curve. It is also necessary to choose the power function $\rho$ to complete the model specification.

Having chosen the model specification, the model can be fitted using maximum quasi likelihood to obtain parameter estimates (and their approximate standard errors). At this point the authors make use of standard statistical software packages which have the facility to fit generalized additive models. Currently the choice is limited, although greater choice is likely in the future as the popularity of generalized additive models increases. The authors used S-PLUS for the example.

Having fitted the model, reserve estimates are obtained by summing the appropriate predicted values in the southeast region of the claims rectangle. All that remains is the estimation of variability in the reserve estimates.

One of the principal advantages of stochastic reserving models is the availability of estimates of precision. Commonly used in prediction problems is the standard error of prediction, also known as the prediction error, or root mean square error of prediction. For claim payments in development period $d$ for accident period $w$ (yet to be observed), the mean square error of prediction is given by

$$E\left[\left\{c(w,d)-\hat{c}(w,d)\right\}^2\right] \approx Var\left[c(w,d)\right]+Var\left[\hat{c}(w,d)\right]. \tag{5.22}$$

Note that the mean square error of prediction can be considered as the sum of two components: variability in the data (process variance) and variability due to estimation (estimation variance).

For the general model defined above, the process variance is given by Equation (5.20). For the estimation variance, note that

$$\hat{c}(w,d) = \hat{m}_{w,d} = e^{\hat{\eta}_{w,d}} . \tag{5.23}$$

$$E\left[\left\{c(w,d) - \hat{c}(w,d)\right\}^2\right] \approx \phi \hat{m}_{w,d}^p + \hat{m}_{w,d}^p Var\left[\hat{\eta}_{w,d}\right]. \tag{5.24}$$

The final component of Equation (5.24), the variance of the (linear) predictor, is usually available directly from statistical software packages, enabling the mean square error to be calculated without difficulty. The standard error of prediction is the square root of the mean square error of prediction.

The mean square error of prediction of the origin period reserve, the total reserve estimate, and the mean square error of prediction of the total reserve are found in the original research paper as Equations (4.3) and (4.4).

Although Equations 4.3 and 4.4 of the original research paper look fairly complex, they are relatively easy to calculate by summing the appropriate elements. The only components not readily available from statistical software packages are the covariance terms. Provided the design matrix and variance-covariance matrix of the parameter estimates can be extracted from the statistical software package used, a full matrix of the covariance terms can be calculated without difficulty for any specification of the predictor $\eta$. Indeed, the variances of the (linear) predictors are simply the diagonal of such a matrix. Although natural in stochastic claims reserving, it is unusual to focus on the shape of the decay of incremental claims using traditional actuarial methods, in which it is common to focus on the relative increase in cumulative claims through development factors, the traditional "parameters" in a standard chain ladder exercise. After fitting a stochastic claims reserving model, it is straightforward to obtain equivalent development factors by applying the standard chain ladder model to the fitted values of the stochastic model. If the model is fully parametric, it may be possible to obtain a relationship between the model parameters and the chain ladder development factors.

Incremental paid losses from an aggregation of classes of business are shown in Table 6.1 on page 23 of the paper, and are used to illustrate the methodology. The incremental claims fall fairly rapidly, but are not completely run-off by the end of the tenth development period, implying the necessity for a tail factor greater than 1.0 when using the traditional chain ladder model.

### 5.7.2 Example (See Appendix B, Section B.4.5)

### 5.7.3 Advantages and Disadvantages

Advantages of this procedure are that it is extremely flexible, and it forces the actuary to look at the data. Disadvantages are that it is time-consuming and statistically inefficient.

The main strength of the method presented in this paper is that both the smoothing and extrapolating can be performed at the same time in the same model. The actuary simply has to choose one parameter for smoothing across the whole range of development time, choose an error distribution, and choose how far to extrapolate (an additional parameter is necessary if smoothing over accident years). Further advantages are that it is also possible to obtain measures of precision of the reserve estimates, and investigate where the data deviate from the fitted model by viewing residual plots. Choosing smoothing parameters at the extremes is a useful additional feature since at one extreme the model may be considered over-parameterized, and at the other the structure may be too rigid.

Incremental data are used for the method put forward in this paper: This is both an advantage and a disadvantage. It is advantageous since the method can be used when the data history is incomplete. If incremental data were recorded by accident year only after a certain date, accident years prior to that date will have incomplete runoff information, and a section of the claims triangle in the northwest corner will be missing (this is a reasonably common occurrence). This presents difficulties using standard deterministic techniques that rely on cumulative data, but is not a problem for stochastic techniques, which treat the unobserved data as "missing" and estimate the data as part of the fitting procedure. The disadvantage is that negative incremental values sometimes occur in data based on paid losses, and frequently occur in data based on incurred losses where case estimates are often set on a conservative basis and overestimated. The method proposed is robust to a small number of negative incremental claims (as in the example), but will always produce positive fitted values (due to the use of the logarithmic link function) and hence will always produce development factors greater than one. For this reason, the techniques are often not suitable for use with incurred data, which often include a series of negative incremental losses in the later stages of development requiring development factors less than one.

#### 5.7.4 Users

As a newly developed method, there were no known users identified in our survey at this time.

#### 5.7.5 Summary

Stochastic models have been constructed with the aim of producing exactly the same reserve estimates as the traditional deterministic chain ladder model. Advantages are that measures of precision are readily available, and the assumptions underlying the chain ladder model are clarified. More importantly, the models provide a bridge between traditional methods and stochastic methods, which is useful for the practitioner who is familiar with traditional methods and needs a starting point for exploring stochastic methods.

The aim of the England-Verrall paper is to present a flexible framework for stochastic claims reserving which allows the practitioner to choose whether to use the basic chain ladder model, or to apply some smoothing, or to use a parametric curve for the runoff. Several of the models proposed to date fit within this framework, and further extensions are possible that have not yet been tried.

### 5.8 Summary of Curve-Fitting Methods

Several curve-fitting methods were presented, three that involve some sort of exponential decay process, and one that involves alternate assumptions about the decay of the development portion of the link ratios. It must be recognized that, by their very nature, the exponential decay methods will all tend to produce similar answers. So, the addition of the Sherman method is a welcome improvement. However, it must be recognized that all curve-fitting methods make some very significant assumptions as to how development factors will decay. In using curve-fitting methods, it is a good idea to compare the results of several different curve-fitting techniques, considering the potential for bias in (1) the choice of the function, (2) actual points used in the fit and, (3) estimation of parameters. So, the user is cautioned to not just use them blindly.

## 6. METHODS BASED ON REMAINING OPEN COUNTS

### 6.1 Introduction to Open-Count Based Methods

There is a class of methods that involve first estimating an average cost per open count for each calendar period and multiplying by the projected number of claims remaining open in that period. Summing together the results for all calendar periods in the tail gives the unpaid loss at the tail period. Dividing that by the paid loss up to the tail produces a paid loss tail factor. As it happens, the two methods presented use mortality to project the claims remaining open, although other approaches are possible.

### 6.2. Static Mortality Method

The static mortality method is also known as the incremental paid to prior open method. It separately treats changes in workers compensation incremental severities (due to annual rates of medical cost escalation) and the slow decline in the number of open claims (due to mortality). It is an adaptation of the (classic) structural methods of Fisher/Lange and Adler/Kline.

#### 6.2.1 Description

Incremental payments for every development year are estimated by taking the product of the number of open claims at the end of the prior development year and an estimated claim severity. For mature development years, future incremental payments are essentially a function of how many claims are still open and the average size of incremental payments per open claim. Changes in the number of open claims can be estimated beyond years in the triangle via mortality rates and inclusion of the small number of newly reported claims and net closures for other reasons. Analogous incurred loss development patterns can be estimated if one defines total case reserves as the product of the latest year's incremental payments times the average annuity factor for all living permanent disability (PD) claimants.

#### 6.2.2 Example

Section 3 of the Sherman-Diss paper includes a detailed example of this method.

While the static mortality method is of limited value for early development periods, its merit relative to other reserving methods is substantial in estimating reserves for future MPD payments (the medical component of permanent disability claims) for more mature development periods. For such mature development periods, future incremental payments are essentially a function of how many claims are still open and the average size of incremental payments per open claim. In contrast, future incremental MPD payments have almost no causal linkage to payments for rapidly settled claims during early development periods.

The specific steps to be taken in applying the incremental paid per prior open claim method are:

(1) Incremental paid losses and open counts are compiled by accident year and development period.

(2) Historical averages of incremental paid per prior open are compiled in triangle format starting at 24 months, computed using the above incremental paid and open count data.

(3) Each historical average is trended to the expected severity level for the first calendar year after the evaluation date.

(4) Development factors of open counts at successive period-ends are computed.

(5) The selected ratios from (4) by development period are used to project the number of open claims for each future development period of each accident year, thereby completing the triangle of open counts.

(6) Future values of incremental paid per prior open are selected for each development period based on the trended data in (3) above.

(7) Projections of incremental paid losses for future development periods for each accident year are determined as the product of the projected open counts from the completed triangle and the projected values of incremental paid per prior open selected in (6).

The percentage declines in prior open counts reflect the composite effects of three factors affecting the number of open claims: (1) increases due to newly reported claims, (2) decreases due to the death of a few claimants, and (3) net decreases due to other reasons (including increases due to reopened claims). After 20 periods of development, newly reported claims and net claim closures (1 and 3 above) become negligible. Thus, after 20 periods of development, virtually all claim closures are attributable to the death of claimants. Consequently, changes in the number of open claims at the end of each development period beyond 20 periods can be predicted almost entirely on the basis of mortality rates. And changes in the number of open claims can be estimated beyond 15 periods via mortality rates and inclusion of the small number of newly reported claims and net closures for other reasons. This is subject to fine-tuning due to the possibility that the mortality rates of disabled claimants might be higher than those of the general populace, although recent improvements in medical technology have reduced the influence of medical impairment on mortality rates.

If the historical database includes only the total number of closed claims, the number of claimant deaths may be estimated based on mortality tables and any additional claim closures are presumed to be for other reasons. In the Sherman-Diss model of Section 8.5, the breakdown is derived by estimating the number of claim closures due to death from the 2000 Social Security Administration (SSA) mortality tables.

Just as the authors have modeled the expected paid loss development factor (PLDF) patterns for MPD losses, analogous incurred loss development factor (ILDF) patterns can be estimated by defining total case reserves as the product of the latest period's incremental payments times the average annuity factor for all living PD claimants.

### 6.2.3 Advantages and Disadvantages

While this method is of limited value for early development years, its merit relative to other reserving methods is substantial in estimating reserves for future MPD (medical permanent disability) payments for more mature development periods. The method is subject to fine-tuning due to the possibility that the mortality rates of disabled claimants might be higher than those of the general populace. In other words, a substandard mortality table may be required. It is important to note that the applicability of the method is not only dependent on the open claim count retention level, but also (1) the presence (or absence) of PD claimants with ongoing medical costs, and (2) the specific provisions of state workers compensation laws. However, the Sherman-Diss paper focuses primarily on MPD claims, which generally do not vary significantly between states.

#### 6.2.4 Users

The method is utilized by the SAIF Corporation and the Oregon State Fund.

#### 6.2.5 Summary

After 20 years of development, virtually all workers compensation claim closures are attributable to the death of claimants. Consequently, changes in the number of open claims at the end of each development year beyond 20 years can be predicted almost entirely on the basis of mortality rates. Medical cost escalation rates and the force of mortality are the key drivers of MPD tail factors. The paid loss development method is not designed to treat these two influences separately. This method (incremental paid per prior open) provides for the separate, explicit treatment of the effects of these two drivers. The above method can be applied satisfactorily to workers compensation total medical loss experience for development years 20 and higher.

### 6.3 Trended Mortality Method

The trended mortality method is an adaptation of the (classic) structural methods of Fisher/Lange and Adler/Kline. The model explicitly accounts for the compounding effects of downward trends in future mortality rates and persistently high rates of future medical cost escalation.

#### 6.3.1 Description

The method is similar to the static mortality method of Section 6.2. The key difference is that the change in the number of open claims for every future development period of every accident year is determined by applying mortality tables forecasted by the SSA for the appropriate future development year. The rest of the method is essentially unchanged. The use of *forecasted* mortality is the distinctive feature of the trended mortality method.

#### 6.3.2 Example

An example of the method is given in Section 6.2, the static mortality method. A few comments should be made, which refer specifically to the trended mortality method.

Small improvements in the annual survival rate of remaining claimants result in major differences in the number of claims still open at higher development periods. Given that the greatest differences occur during development periods in the distant future, when the effects of medical inflation have had an opportunity to compound over decades, the total reserve indicated by the trended mortality method is decidedly greater than that indicated by the static mortality method.

Paid loss development factors for earlier (as well as middle) development periods will not hold constant over successive accident periods. However, it is also evident that the rate of increase over the short to middle term in these paid development factors on account of mortality is small. It is small enough that it would not be detectable to an experienced actuary reviewing historical PLDFs (paid loss development factors).

Even though it is true that past declines in mortality rates are implicitly embedded in historical PLDFs, it would be incorrect to assume that the selection of historical factors as estimates of future PLDFs would implicitly incorporate the effects of future declines in mortality rates. With respect to mortality, the past experience of the data under review may very well not be a good indication of future mortality. What would be more appropriate would be to select representative PLDFs for each development period based on recent historical factors and then to trend these upward in a manner parallel to the PLDFs indicated by a realistic model such as mortality tables forecasted by SSA.

#### 6.3.3 Advantages and Disadvantages

Advantages and disadvantages are similar to those for the static mortality method of Section 6.2.

#### 6.3.4 Users

The method is utilized by the SAIF Corporation and the Oregon State Fund.

#### 6.3.5 Summary

The Trended Mortality Method is similar to the Static Mortality Method described above but additionally, incorporates the compounding effects of the drivers. The above method can be applied satisfactorily to workers compensation total medical loss experience for development years 20 and higher.

### 6.4 Summary of Future Remaining Open Claims Methods

Two methods were presented, both of which rely on mortality to estimate open claim counts. These methods are correct to point out that workers' compensation link ratios can actually increase at certain durations due to medical inflation and the slow rate of withdrawals/deaths from the system.

## 7. METHODS BASED ON PECULIARITIES OF THE REMAINING OPEN CLAIMS

### 7.1 Introduction

Although tail factors are generally intended to cover 'average' development beyond the data triangle, the actual development of the oldest year may be heavily driven by whether some particularly difficult claims are left in the oldest year. So, while these methods do not generally result in a tail factor applicable to the less mature years (that may or may not have a similar open claim portfolio when they become the oldest year in the triangle), it can be very useful for analyzing the oldest year and other years near the top of the triangle.

### 7.2 The Maximum Possible Loss Method

#### 7.2.1 Description

This method is a variant of the unclosed count method. However, it does not create a tail factor per se but establishes a maximum tail for the older years. The core idea of this method is that, given that the maximum net liability of an insurer is some net retention $R$, the liability for all the open claims should not be more than the sum of $R$ minus paid to date across all the open claims. For simplicity we assume the coverage period of the pertinent reinsurance agreement coincides to an accident period. To use this method, given that an accident year is sufficiently mature that no IBNR claims are reasonably possible, the remaining amounts to reach the retention ($R$ - paid-to-date) are summed across all remaining open claims in the accident year to produce the liability of open claims.

The result is an upper bound on tail development for that specific year. So, if application of the tail factor to a given year suggests more development than is 'possible' per the remaining amounts to reach the retention in the accident year, the ultimate unpaid loss for that accident year might be capped at the amounts remaining to reach the retention.

In the (fairly unusual) event that there are enough claims left open for this to be a statistically valid predictor of the development of the more recent years, it could be used in estimating the tail factor for all the accident years. But, one would have to be certain that this finding was statistically consistent with the initial tail factor analysis. For example, if the initial tail factor came from a curve fitting, it might be statistically reasonable that the curve fitting was simply using the wrong curve. However, if the initial tail factor came from a 'paid overdisposed' method that also used the actual data in the triangle itself, the tail findings would suggest the data is internally inconsistent. In that case, greater care must be taken to understand which method is most accurate for the tail factor to be applied to the more recent years.

### 7.2.2 Example

Consider the following list of claims remaining open for the oldest year in a triangle (assumed to be 1991)

Claims Remaining Open in Oldest Year (1991)

| Claim Number | Retention | Paid at Year-End |
|---|---|---|
| 1 | 300,000 | 150,000 |
| 2 | 300,000 | 200,000 |
| 3 | 300,000 | 250,000 |
| 4 | 300,000 | 275,000 |
| Total | | 875,000 |

Note that retention is the same for all claims as it is presumed that one reinsurance program was in place throughout all of accident year 1991. Then, we compute the total amount unpaid up to the retention, on each individual claim.

| Claim Number | Retention | Retention-Paid at Year-End |
|---|---|---|
| 1 | 300,000 | 150,000 |
| 2 | 300,000 | 100,000 |
| 3 | 300,000 | 50,000 |
| 4 | 300,000 | 25,000 |
| Total | | 325,000 |

In the event that no closed claims reopen, the total of the remainders to hit the retention is the maximum possible unpaid loss. Continuing in that vein, we divide the total possible maximum loss by the paid-to-date on all 1991 claims, and get a corresponding maximum possible tail factor.

Paid-to-Date (All Claims) for Oldest Year (1991)
2,000,000
Cap on Development Portion (Total Max Unpaid/Paid All Claims)
0.16
Maximum Possible Tail Factor for 1991 (1+Cap)
1.16

A similar process can be used to compute maximum IBNR, using case-incurred loss instead of paid losses.

### 7.2.3 Advantages and Disadvantages

This method improves on the average unpaid loss method by dint of the fact that the amount to reach the retention need not be estimated. Rather, it is fact. However, it only produces an upper bound, not an actual best estimate.

Like the average unpaid loss method, there are often statistical reliability issues when making inferences about the tail factors of the more recent years. But, one cannot readily dispute the results as an upper bound for the older years on which the method is applied, at least as long as one is certain the prospect of additional IBNR claims is immaterial. So, like the average unpaid loss method, one must be very careful to make sure the proper assumptions hold when using it. But, unlike the average unpaid loss method, it has far more certainty surrounding the loss sizes.

#### 7.2.4 Users

This method is used by some consulting firms and some insurance companies.

#### 7.2.5 Summary

As stated, this method may be a powerful tool for setting an upper bound on development on the oldest year or years. Yet, it does not generalize well to the more recent years. So, it does not lend itself to a tail factor that can be applied to all the years.

### 7.3 Judgment Estimate Method

#### 7.3.1 Description

A method to derive the tail for the oldest claims is to examine the particular fact pattern of each reported outstanding claim and rely upon claims evaluation expertise to estimate the remaining settlement value for each claim. The sum of the estimated outstanding reported remaining settlement values by accident period is added to the cumulative payments by accident period to derive estimated ultimate settlement values by accident period. The estimated ultimate settlement value divided by the reported (or cumulative paid) losses to date by accident period results in the incurred (or paid) tail factors implied by this method. As this method is essentially a claims audit for the oldest claims, the method should probably not be strictly classified as an actuarial method.

The method is intended to be applied only to the oldest periods where there is no reasonable expectation that additional claims will be reported. Of course, the resulting estimate of the tail will only be as useful as the quality of the claims expertise used to evaluate remaining claim settlement values.

#### 7.3.1.2 Example

Consider the following cumulative paid loss triangle:

Cumulative Paid Loss Triangle $c_{Paid}(w,d)$

| | 12 | 24 | 36 | 48 | 60 | 72 |
|---|---|---|---|---|---|---|
| 1991 | 1,000 | 2,000 | 2,500 | 2,800 | 2,950 | 3,100 |
| 1992 | 1,100 | 2,400 | 3,000 | 3,500 | 3,900 | |
| 1993 | 1,300 | 2,500 | 3,000 | 3,400 | | |
| 1994 | 1,200 | 2,300 | 3,100 | | | |
| 1995 | 1,400 | 2,800 | | | | |
| 1996 | 1,490 | | | | | |

By 72 months of development, it is believed that all claims have been reported for the oldest accident year—1991. There are six (6) claims outstanding for accident year 1991 as of 72 months of development. A professional claims examiner is engaged to evaluate the fact pattern of each of the six claims in order to derive an estimate of the remaining settlement value for each outstanding claim.. The claims examiner estimates are as follows: claim #1- 100; claim #2- 300 (the policy limit); claim #3- 0 (i.e., expected to be closed without payment); claim #4- 300; claim #5- 250; and claim #6- 250. The actuary reviews the claims examiner estimates for possible additional adjustments. Although claim #2 is expected to settle at the policy limit, the actuary believes there will be some loss adjustment expense to settle the case and, as such, adds 50 to the estimate for this claim. Similarly, the actuary adds 50 to the claim #3 estimate to reflect future allocated loss adjustment expenses. Claim #6 is expected to be settled in several years and the actuary believes the claims examiner has not fully reflected severity inflation through time of settlement. The actuary adds 50 to this claim in order to account for additional severity inflation beyond which has been reflected by the claims examiner. After actuarial adjustment, the individual claim estimates are as follows: claim #1- 100; claim #2- 350; claim #3- 50 claim #4- 300; claim #5- 250; and claim #6- 300. These claim estimates total 1,350. Accordingly, the 72-ultimate payment tail development factor is derived as

$$(3{,}100+1{,}350)/3{,}100=1.435$$

The actuary further notes that the payment tail factor is only based upon an evaluation of six (6) claims and, as such, may not have full credibility.

**7.3.2 Advantages and Disadvantages**

Strengths of this method are:

(1) The tail estimate is based upon the particulars of actual reported outstanding claims without reliance on theoretical models.

(2) The tail estimates may be improved by better claims settlement evaluation expertise.

(3) The method is readily understood by nontechnical users of the resulting actuarial

work product.

(4) The method may provide insight into the plausible upper and lower bounds for the tail by period. A lower bound may be derived by assuming all reported remaining outstanding claims are closed without payment. An upper bound may be obtained by assuming all reported remaining outstanding claims are settled at the retention or policy limits. However, the use of these upper and lower bounds has its limitations, as discussed below.

Weaknesses of this method are:

(1) The method is only applicable: where there is access to individual claim information; when individual claim evaluation expertise is available; and for periods where there is no reasonable expectation that additional claims will be reported.

(2) The results of the method are highly subject to the expertise and judgment of the examiner/auditor performing the claim evaluation. There is typically no fitting or testing of historical experience and no statistical support for the assumptions used in the claim evaluation.

(3) Claims that are subject to worsening of claimant condition, such as long-term workers compensation (or short-term benefits that are escalated to long-term), or liability claims where adverse facts may have yet to emerge, are difficult or impossible to quantify. A claims examiner/auditor estimate may have a tendency to underestimate the liability for such claims as the emergence of adverse facts might be difficult for a claims examiner/auditor to justify for any particular claim. Additional actuarial adjustments would be required to the extent that the examiner/auditor has omitted consideration of the potential for future adverse facts.

(4) Claims examiners/auditors may have a tendency to perform their evaluation on the basis of the estimated current value to dispose of the claim. Claims estimated on this basis tend to be underestimated since severity inflation through the time of final settlement is not considered. Additional actuarial adjustments would be required to the extent that the claims examiner/auditor has omitted consideration of severity inflation through final settlement.

(5) Even where there is reasonable expectation that all claims have been reported, there may be risk that additional claims may emerge due to unexpected new claims; reopened claims (e.g., for workers compensation); changes or broadening in interpretation of coverage; changes in classification of claims by period; or other unforeseen circumstances. Additional actuarial adjustments would be required to the extent that the examiner/auditor has omitted these considerations.

(6) Even where there is reasonable expectation that all claims have been reported, there is risk that the remaining settlement value of outstanding claims may be effectively less than

zero because of changes in classification of claims by period; salvage and subrogation recoveries; other recoveries on prior claims; or other unforeseen circumstances.

(7) Even where there is reasonable expectation that all claims have been reported, there is risk that the remaining settlement value of outstanding claims may be greater than the sum of the remaining policy limit amounts for each claim by period as a result of the emergence of additional claims; ALAE costs (if included in the reserve provision); changes or broadening in interpretation of coverage; bad faith claims; punitive damage awards; or other unforeseen circumstances.

#### 7.3.3 Users

One of the key hurdles to overcome in using this approach is the need for experienced claims auditors. So, this method tends to be used the most often by those with access to claims auditors, which includes, insurance companies that work with multiple third-party administrators, consulting firms, and, occasionally, state insurance solvency regulators.

#### 7.3.4 Summary

This method has the advantage of reflecting only the claims left open, even if the judgment estimate may sometimes be biased. It can certainly be used, though, in conjunction with tail factors developed from industry benchmark data. It is perhaps better thought of as a method for developing older years, than as a method for developing greener years that may have a different open claims pattern near the tail. It has its disadvantages in terms of the limits of what a claims auditor can reasonably ascertain. But, it is also fairly easy to explain to lay people.

### 7.4 Summary of Methods Based on Peculiarities of the Remaining Open Claims

These methods can produce significant improvements in estimates of the total costs of the oldest years, especially when only a few claims remain open in those years. But, the user is cautioned to avoid assuming that similar tail factors will be accurate for the less mature years.

## 8. OTHER METHODS

### 8.1 Introduction

There are several other methods discussed below that do not fall into any of the previous classes.

### 8.2 Restate Historical Experience Method

#### 8.2.1 Description

When the historical reported losses are inconsistent (e.g., there has been a substantive change in the claim counting or case reserving philosophy) and/or incompatible with industry benchmark experience (e.g., the most recent case reserves are substantially lower than comparable industry case reserves), it may be useful to attempt to restate the historical experience using concepts from the judgment estimate method .

One possibility is to restate the entire reported loss history using claims evaluation expertise to estimate the case reserves of the outstanding reported losses as of each stage of development. After restatement of historical reported losses in this manner, the tail factor may be estimated using many of the methods described in this summary report. Indeed, once the historical reported losses have been restated on a consistent basis, all development factors may be estimated using traditional actuarial methods. This method shares several of the strengths and weaknesses of the judgment estimate method. However, this method has several serious additional weaknesses: (1) it is ordinarily extremely difficult to reconstruct the contemporaneous claim file information as of each previous historical development period; (2) in order to properly implement this method, the claims auditor must ignore claim developments that are known or knowable subsequent to each development period; and (3) in order to properly implement this method, the claims auditor must evaluate each previous open claim as if the evaluation were performed at a prior historical date corresponding to the development period.

Generally, a more practical approach is to use claims evaluation expertise to estimate the current value of all open claims only as discussed in the judgment estimate method and apply comparable industry tail development factors. If the current open claims are estimated at industry standard levels and the industry development factors are truly comparable, then this method is applicable for all periods rather than only the oldest periods where there is no reasonable expectation that additional claims will be reported.

### 8.2.2 Example

Consider the following cumulative paid loss triangle:

Cumulative Paid Loss Triangle $c_{Paid}(w,d)$

| | 12 | 24 | 36 | 48 | 60 | 72 |
|---|---|---|---|---|---|---|
| 1991 | 1,000 | 2,000 | 2,500 | 2,800 | 2,950 | 3,100 |
| 1992 | 1,100 | 2,400 | 3,000 | 3,500 | 3,900 | |
| 1993 | 1,300 | 2,500 | 3,000 | 3,400 | | |
| 1994 | 1,200 | 2,300 | 3,100 | | | |
| 1995 | 1,400 | 2,800 | | | | |
| 1996 | 1,490 | | | | | |

Remaining claims open as of 72 months for accident year 1991 and remaining claims open as of 60 months for accident year 1992 are evaluated at industry standard levels. A claims examiner estimates that the industry standard value of the six (6) accident year 1991 outstanding claims as 1,000 and the eleven (11) accident year 1992 outstanding claims as 1,400. An appropriate source of compatible industry-incurred development factors indicates that the 72-ultimate comparable industry incurred development factor is 1.100 and the 60-ultimate comparable industry incurred development factor is 1.150. Accordingly, the indicated accident year 1991 72–ultimate payment tail development factor is:

$$[(3{,}100 + 1{,}000)/(3{,}100)] \times 1.100 = 1.455.$$

Similarly, the indicated accident year 1992 60-ultimate payment tail development factor is

$$[(3{,}900 + 1{,}400)/(3{,}900)] \times 1.150 = 1.563.$$

A similar procedure could be adopted for each accident year.

The actuary considers whether the industry is truly reserving up to the levels of the claims examiner industry standard. If the actuary believes that the industry is not reserving up to the level of the claims examiner industry standard, then the actuary would increase the indicated tail development factors to reflect additional expected development.

### 8.2.2 Advantages and Disadvantages

Strengths of this method are:

(1) Estimates of ultimate losses may be improved by better claims settlement evaluation expertise at the industry standard.

(2) The method of adjustment is more readily understood by non-technical users of the resulting actuarial work product than highly theoretical models.

(3) The method relies upon industry development factors which are often compiled and

may be readily available.

Weaknesses of this method are:

(1) The method is only applicable where there is access to individual claim information and when individual claim evaluation expertise is available.

(2) The results of the method are highly subject to the expertise and judgment of the examiner/auditor performing the claim evaluation. Evaluation of claims at the industry standard is subjective. There is often no fitting or testing of historical experience and no statistical support for the assumptions used in the claim evaluation.

(3) Appropriate industry development factors may not be readily available. Selection of appropriate industry development factors is not always clear in consideration of policy limits, mix of business, reinsurance, deductibles, etc. There is often considerable judgment required to select appropriate industry development factors. It may be appropriate to use a weighted average of several industry development factors in order to improve the comparability of the development factors with the restated historical experience. The appropriate weighting scheme of industry development factors itself may also be subject to a high degree of judgment.

(4) Industry standard may be a higher value than the industry actuarial reserves. An adjustment (i.e., increase) to industry development factors may be required to reflect that the industry may actually reserve at values lower than industry standard levels.

#### 8.2.3 Users

As with the judgment estimate method, one of the key hurdles to overcome in using this approach is the need for experienced claims auditors. So, this method tends to be used the most often by those with access to claims auditors, which includes insurance companies that work with multiple third-party administrators, consulting firms, and, occasionally, state insurance solvency regulators.

#### 8.2.4 Summary

This subsection is a brief summary of the method and its utility. This method has the advantage of reflecting only the claims left open, even if the judgment estimate may sometimes be biased. Successful application of the method requires that the claims auditor accurately tracks the industry standard and that the industry development factors selected are appropriate for the line of business under consideration. Its disadvantages are the limits of the claims auditor's ability to ascertain industry standard and the uncertainty in the appropriate industry development factor to apply to develop the auditor's recast incurred loss to ultimate.

As with most methods, the uncertainty is greatest for the least mature years. On the other hand, the method is relatively easy to explain to non-technical users.

### 8.3 Mueller Incremental Tail Method

Named in recognition of the work done by Conrad Mueller, ACAS, the Mueller Incremental Tail (MIT) method was developed by Mueller internally at the SAIF Corporation.

#### 8.3.1 Description

The MIT method is used in the Sherman-Diss model to calculate empirical 37 to 65 tail factors using paid incremental data on old accident years. See Section 8.5 of this paper for a synopsis of the Sherman-Diss paper. The method involves three stages:

1. Incremental age-to-age factors

2. Anchored decay factors

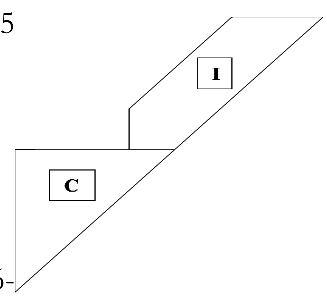
3. Tail factors

#### 8.3.2 Example

In the following example, table and figure numbers shown in parentheses refer to the original research paper by Sherman and Diss. Figure 8.3.2.1 provides a graphic summary of the portions of the incremental medical component of permanent disability claims (MPD) payments experience of the SAIF Corporation that are available. A complete triangle of MPD payments exists for AYs 1966-2002. This region is the triangle labeled "C" to designate that cumulative paid losses are available for all of these AYs. In addition, since calendar period 1985, incremental MPD payments have been captured for AYs 1926-1965 for development years 29 and higher. This region is the diagonally shaped area labeled "I" to designate that only incremental payments are available.

Figure 8.3.2.1 (Figure 2.1) Configuration of SAIF's MPD Paid Loss Data

AY 1926-1965




AY 1966-

Since paid MPD for AYs 1926-1965 has only been available for calendar periods since 1985, it was necessary to construct an actuarial method of estimating the tail factor based on decay ratios of incremental payments. This method is called the MIT method.

The MIT method was used to calculate empirical 37 to ultimate tail factors using the incremental data on old accident periods. The empirical data ended at 65 years of development, which, for purposes of this section, will be considered to be ultimate. The method is described in three stages mentioned earlier:

(1) Incremental age-to-age decay ratios
(2) Anchored decay factors
(3) Tail factors

$$f(d)=c_{\text{Paid}}(w,d)/c_{\text{Paid}}(w,d\text{-}1)=[c_{\text{Paid}}(w,d\text{-}1)+q_{\text{Paid}}(w,d)]/c_{\text{Paid}}(w,d\text{-}1)=1+q_{\text{Paid}}(w,d)/c_{\text{Paid}}(w,d\text{-}1)$$

Then, $f(d)-1=q_{Paid}(w,d)/c_{Paid}(w,d-1)$, which is equal to $v(d)$.

1. **Incremental age-to-age decay ratios.** The first step is to calculate incremental age to age decay ratios:

$q_{Paid}(w,d+1)/q_{Paid}(w,d)$, $q_{Paid}(w,d+2)/q_{Paid}(w,d+1)$, $q_{Paid}(w,d+3)/q_{Paid}(w,d+2)$, etc.

With the SAIF data, Sherman and Diss were are able to calculate ratios of incremental paid at age $d+1$ to incremental paid at age $d$, for $d$ ranging from 29 to 65 years, using 20-year-weighted averages. Because of the sparseness of claims of this age, the empirical development ratios needed to be smoothed before they could be used. The smoothing was done using five-year centered moving averages.

2. **Anchored decay factors.** After calculating incremental age-to-age decay ratios, the factors are anchored to a base year and thereafter termed anchored age-to-age factors. In the illustration that follows, development year $d$ is the anchor year.

$d_d=q_{Paid}(w,d)/q_{Paid}(w,d)=1$, $d_{d+1}=q_{Paid}(w,d+1)/q_{Paid}(w,d)$, $d_{d+2}=q_{Paid}(w,d+2)/q_{Paid}(w,d)$, … all relative to $q_{Paid}(w,d)$.

In general

$$q_{Paid}(w,d+r)/q_{Paid}(w,d)=q_{Paid}(w,d+1)/q_{Paid}(w,d)\times q_{Paid}(w,d+2)/q_{Paid}(w,d+1)\times\ldots\times q_{Paid}(w,d+r)/q_{Paid}(w,d+r-1).$$

The anchored decay factors are cumulative products of the age-to-age decay ratios and represent payments made in year $d+r$ relative to payments made in the anchor year $d$.

Table 8.3.2.2 shows the anchored decay factors for payments made in accident years of age

40, 45, 50, and 55 relative to payments made in an accident years of age 37 (our anchor year).

Table 8.3.2.2 (Table 2.3)

Indicated Decay Factors Relative to Anchor Year 37 Incremental Payments

| Year of Development | Decay Factors |
|---|---|
| 55 | .962 |
| 50 | 1.880 |
| 45 | 1.724 |
| 40 | 1.211 |
| Anchor Year 37 | 1.000 |

For example, payments made in development year 50 are, on average, almost double (88.0% greater) the payments made in development year 37.

Payments made in ages 38 to 65 relative to payments made in year 37 are obtained by summing the anchored decay factors from 38 to ultimate. The authors refer to these as anchored cumulative decay factors, $D_d$ s, where

$D_{d+1} = q_{Paid}(w,d+1)/q_{Paid}(w,d) + q_{Paid}(w,d+2)/q_{Paid}(w,d) + ... = \sum d_i$ for $i = d+1$ to 65.

The sums of the decay factors are similar to tail factors, but instead of being relative to cumulative payments they are relative to the incremental payments made in the anchor year.

The process can be repeated using a different anchor year. In addition to anchor year 37, the calculations were also performed using anchor years 36, 35, 34 and 33. In each case, the payments from 38 to ultimate were compared to the payments made in the selected anchor year. Table 8.3.2.3 shows the cumulative decay factors for each of these anchor years:

Table 8.3.2.3 (Table 2.4)

Cumulative Decay Factors Relative to Incremental Payments During Different Anchor Years

| Anchor Year | Cumulative Decay Factors |
|---|---|
| 37 | 30.071 |
| 36 | 30.115 |
| 35 | 29.508 |
| 34 | 28.280 |
| 33 | 26.961 |

The cumulative decay factors can be interpreted as follows: Payments made from ages 38 to ultimate are 30.071 times the payments made in age 37. Similarly, payments made in ages 38 to ultimate are 30.115 times the payments made in age 36, etc.

3. **Tail Factors.** To convert these cumulative decay factors into tail factors, the authors make use of the selected cumulative loss development factors from the customary cumulative paid loss development triangle.

The Tail Factor from $d$ to ultimate $=\left\{c_{Paid}(w,d)+\left[\sum_{d=1}^{65}q_{Paid}(w,d)\right]\right\}/c_{Paid}(w,d)$,

$$=1+\left[\sum_{d=1}^{65}q_{Paid}(w,d)\right]/c_{Paid}(w,d)$$

$$=1+q_{Paid}(w,d+1)/c_{Paid}(w,d)+q_{Paid}(w,d+2)/c_{Paid}(w,d)+\ldots$$

$$=1+\left[q_{Paid}(w,d)/c_{Paid}(w,d)\right]\times\left[q_{Paid}(w,d+1)/q_{Paid}(w,d)+q_{Paid}(w,d+2)/q_{Paid}(w,d)+\ldots\right]$$

.

But $q_{Paid}(w,d)/c_{Paid}(w,d)=\left[q_{Paid}(w,d)/c_{Paid}(w,d-1)\right]/\left[c_{Paid}(w,d)/c_{Paid}(w,d-1)\right]=$ $(f(d)-1)/f(d)$.

So the tail factor is $1+[(f(d)-1)/f(d)]\times D_{d+1}$ where $f(d)$ is the paid loss development factor for the *d*th year of development, and $D_{d+1}$ is the cumulative decay factor for payments made during years ($d+1$) to ultimate relative to payments made in anchor year $d$.

In a similar way, an age-to-age loss development factor (less 1.0) extending beyond the cumulative triangle is

$$\left[d(d+1)-1\right]=[(f(d)-1)]\times d_{n+1}/f(d),$$

where $d_{n+1}$ is the decay factor for payments made in year ($n+1$) relative to payments made in anchor year $n$.

This method is sensitive to $f_n$, the 37:36 paid loss development factor less 1. For this reason the analysis can be repeated using the 36, 35, 34 or 33 anchor years. Table 8.3.2.4 shows the 37 to 65 tail factor calculated using each of these anchor years.

Table 8.3.2.4 (Table 2.5)

37 to Ultimate MPD Tail Factors Based on Different Anchor Years

| AnchorYear | 37 to Ultimate MPD Tail Factors |
|---|---|
| 37 | 1.964 |
| 36 | 1.808 |
| 35 | 1.496 |
| 34 | 1.439 |
| 33 | 1.369 |

| | |
|---|---|
| Selected | 1.581* |

* Average excluding the high and low.

The empirically calculated 37 to ultimate MPD tail factors range from a low of 1.369 to a high of 1.964. The value is sensitive to relatively small changes either in incremental age-to-age factors in the tail or in the cumulative age-to-age factors at the end of the cumulative triangle.

**8.3.3 Advantages and Disadvantages**

The Mueller Incremental Tail method can be applied satisfactorily to workers compensation total medical loss experience for development years 20 and higher since virtually all medical payments are MPD payments at such maturities. A disadvantage is that it may be sensitive to the anchor year. However, the process may be repeated with various anchor years to reduce the high volatility of the tail data. This method may not be predictive if the payment patterns are changing over time but this is a disadvantage of any tail factor methodology.

**8.3.4 Users**

The method is utilized by the SAIF Corporation, Oregon's State Fund.

**8.3.5 Summary**

Workers compensation tail data is often difficult to obtain and may be of dubious quality. The Mueller method is based on decay ratios of incremental paid data and may be used to anchor a tail factor at 20 to 35 years of maturity.

**8.4 Corro's Method**

Daniel R. Corro published this method in his 2003 research paper titled "Annuity Densities with Application to Tail Development."

**8.4.1 Description**

The paper considers the task of modeling "pension" claims whose durations may vary, but whose payment pattern is uniform and flat. The aggregate payout pattern is derived from the duration density and can be applied to calculating tail development factors.

For consistency, the notation in the following subsections differs slightly from the notation of Section 1.7. The tail factor notation is the same as in Corro's original research paper.

The following assumptions are made. All payments on all claims are of the same amount. Payments are made periodically at a common uniform time interval immediately following a common time of loss, $t=0$, to claim closure. For every claim of duration $x$, the model assumes a continuous and constant payment rate of $1 until the claim closes. For pension claims, as described here, the entire payment schedule of a claim is completely determined by the claim duration. With the assumption that for any time $t$, $0<t<b$, all claims with duration $t$ have the same predetermined and differentiable payment pattern.

Let $S(t)$ denote a survival function on the time interval $(0,b)$. Regard $S(t)$ as a distribution of closure times and let $F(t)=1-S(t)$ be the corresponding cumulative distribution function [CDF]. In effect, all claims are assumed to close on or before time *b*.

We are interested in a related CDF, denoted by $\tilde{F}(t)$ to emphasize its relation with $F(t)$, which models the paid loss development as a function of time. More precisely, $\tilde{F}(t)$ is the proportion of total loss paid by time $t$, i.e., the proportion paid out during $(0,t)$ (without any discount adjustment). $\tilde{F}(t)$ is the reciprocal of the paid to ultimate loss development factor and $\tilde{F}(t)$ is referred to as the paid loss development divisor [PLDD].

Consider the case when aggregate paid losses are followed over a series of $N$ time units with $N<b$. The usual paid loss development patterns built from these $N$ evaluations will not account for the "tail paid loss development" beyond the final evaluation at time $t=N$. With this notation, observe that this tail development factor is just $\lambda=\tilde{F}(N)^{-1}$.

It is reasonable to assume that workers compensation payments beyond some valuation, say after 10 periods, will be primarily made on pension-like claims. A model suited to such pension claims may be helpful in projecting the full payout pattern beyond 10 periods. Suppose you have a collection of PLDDs that covers the portion of the loss "portfolio" that is expected to develop beyond 10 periods. That is, for each type of claim you have a PLDD that is appropriate, at least over the time frame beyond 10 periods. The paper illustrates how to translate the mix of claims in the loss portfolio into a mixed distribution of those PLDDs. That mixed distribution then provides an estimated tail factor.

In the workers compensation work that motivated this paper, the author seeks to find a 19th to ultimate paid loss development factor. Consider a weighted sum (mixture) of PLDDs of the form

$$w\tilde{F}_{\alpha}(b_1;t)+(1-w)\tilde{F}_{\beta}(b_2;t) \text{ for } 0<w<1. \tag{8.1}$$

The assumption here is that all claims close after $Max(b_1,b_2)$ periods; one part of the loss portfolio closes by time $t=b_1$ and the complement by $t=b_2$.

Empirical loss development factor data is used to fit a non-linear model in which the mixing weight variable $w$ is a parameter. When these simple functions are used with $b_1$, $b_2$ as selected constants, it is straightforward to set up the calculation so as to assure a closed form solution for the value of $w$ that gives the best fit to the data.

### 8.4.2 Example

See the example in the Excel spreadsheet that accompanyies this paper.

### 8.4.3 Advantages and Disadvantages

Advantages of the method include that it is a nonsubjective, nonlinear "fit" of the tail data, which has a closed-form solution. The subjectivity of curve fitting is removed, at least to some extent, since the same mathematical assumptions are made for any tail data to which the method is applied. Tail factors calculated empirically are often significantly greater than those derived from extrapolation techniques. The greater weight given to tail data in this method reduces the likelihood of underestimation of reserves. The added complexity of the nonlinear fit involves no added work on the part of the user. The sum of squared difference minimization is easily calculated and is a well-known procedure. Another advantage is that the procedure addresses the nature of workers compensation tail data, comprised largely of permanent disability claims.

A disadvantage of the method is that the mathematical notation may not be readily understood.

### 8.4.4 Users

As a newly developed method, there may be few users of the method at this time.

### 8.4.5 Summary

This paper considers the task of modeling "pension" claims whose durations may vary, but whose payment pattern is uniform and flat. The authors derive the aggregate payout pattern from the duration density, provide examples to show how this idea can be applied to calculating tail development factors and discuss the process.

## 8.5 Sherman-Diss Method

The workers compensation tail largely consists of the medical component of permanent disability claims (MPD). Yet the nature of MPD payments is not widely understood and is counter to that presumed in common actuarial models. In the Sherman-Diss paper, it is shown that common actuarial methods tend to underestimate the true MPD loss reserve. This is a serious concern because MPD loss reserves make up the bulk of total workers compensation loss reserves for all but the most recent accident periods. The authors state that the need to develop and apply new methods that directly reflect the characteristics of MPD payments is substantial.

### 8.5.1 Description

The Sherman-Diss paper presents an analysis of medical payments based on 160,000 permanently disabled claimants for accident periods 1926-2002, and a method utilizing incremental payment data prior to the standard triangle to extend development factors beyond the end of the triangle.

Presented is an analysis of the extensive paid loss development database of the SAIF Corporation, Oregon's state fund, extending out to 77 periods of development, separately for medical and indemnity, and separately by injury type. Medical paid loss development factors compiled by the California Workers Compensation Insurance Rating Bureau (WCIRB) and the medical paid loss history of the Washington Department of Labor and Industries (WA LNI) are presented as additional support.

Ordinarily, it would be expected that paid loss development factors for subsequent development periods would slowly decline below the last factor as a continuation of the pattern of slowly decreasing factors exhibited, for example, during development periods 10 through 15. Since common actuarial methods assume that the pattern of declining factors for these development periods will continue in the future, the projected paid loss development factors fall increasingly below the actual historical factors. This pattern of divergence continues during development periods 27 through 37, as shown in Table 8.5.1.1. Table and figure numbers shown in parentheses throughout this section refer to the original research paper.

Table 8.5.1.1 (Table 1.3) A Comparison of Historical MPD Paid Loss Development Factors with Projections Based on Development Periods 10 through 15

| | **Development Period** | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 37 | 38 |
| Historical | 1.020 | 1.023 | 1.027 | 1.026 | 1.022 | 1.018 | 1.015 | 1.017 | 1.018 | 1.029 | 1.033 |
| **Projections Based on Development Periods 10 – 15** | | | | | | | | | | | |
| Linear Decay | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| Exp. Decay | 1.004 | 1.004 | 1.003 | 1.003 | 1.003 | 1.003 | 1.003 | 1.002 | 1.002 | 1.002 | 1.002 |
| Inverse Power | 1.006 | 1.005 | 1.005 | 1.005 | 1.005 | 1.004 | 1.004 | 1.004 | 1.004 | 1.004 | 1.003 |

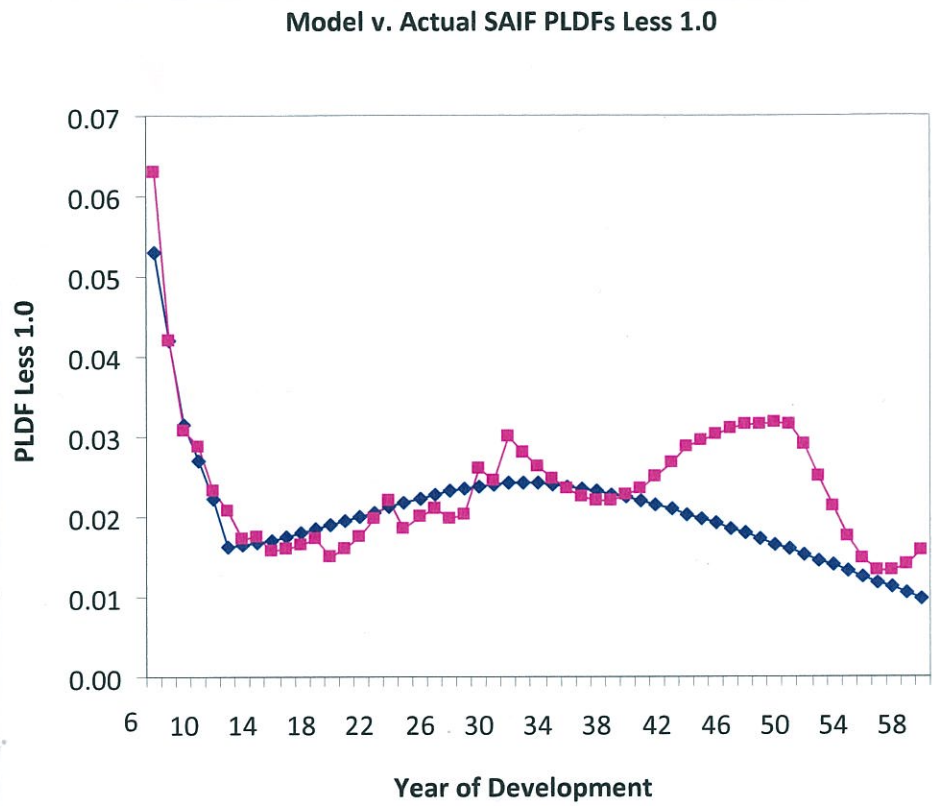
Paid loss development factors for MPD are not monotonically decreasing. Because of this seemingly anomalous behavior, estimates of the MPD tail by common actuarial methods could be seriously understated. This potentially surprising behavior is due to the fact that medical inflation rates are expected to be greater than the rate of closure of permanent disability claims due to death during these periods of development. For the most mature periods of development, the increasing force of mortality overtakes the effects of medical inflation and causes a slow reduction in incremental payments. That rate of reduction is surprisingly small.

This paper presents a reserving model that largely explains the seemingly anomalous behavior of increasing paid loss development factors at "mature" development years. The Sherman-Diss model explicitly accounts for the separate effects of inflation and mortality on paid MPD during all periods of development. This is done by directly incorporating recent mortality rates into an incremental paid per prior open loss reserving method. It will be referred to as the static mortality model.

A second reserving model is presented that explicitly accounts for the compounding effects of downward trends in future mortality rates and persistently high rates of future medical inflation. It will be referred to as the trended mortality model.

In Figure 8.5.1.1, the paid loss development factors indicated by the static mortality model are compared with SAIF's empirical paid loss development factors.

**Figure 8.5.1.1 (Figure 1.1)**




### 8.5.2 Organization of the Sherman-Diss Paper

This paper is divided into ten sections:

1. Summary and Introduction
2. Using Prior Incremental Paid Data to Extend the PLDF Triangle
3. Incorporating the Static Mortality Model into the Incremental Paid to Prior Open Method
4. Mortality Improvement
5. The Trended Mortality Model
6. A Comparison of Indicated Tail Factors
7. Sensitivity Considerations
8. Estimating the Expected Value of MPD Reserves
9. Estimating the Variability of the MPD Reserve with a Markov Chain Simulation
10. Concluding Remarks

The paper also includes five appendices:

A. The Mueller Incremental Tail (MIT) Method
B. Historical PLDFs for All Other workers compensation

C. Incorporating the Static Mortality Model into the Incremental Paid to Prior Open Method

D. Incorporating the Trended Mortality Model into the Incremental Paid to Prior Open Method

E. Quantifying the Elder Care Cost Bulge

### 8.5.3 Example

The authors of the research paper believe that the most appropriate approach to estimating gross workers compensation loss reserves is to separately evaluate MPD loss reserves by one (or more) of the methods presented in their paper. Lacking separate MPD loss experience, the Static Mortality and Trended Mortality models, and the Mueller Incremental Tail method can be applied satisfactorily to total medical loss experience for DYs 20 and higher since virtually all medical payments are MPD payments at such maturities. Examples of the above three methods are given in Section 6.2, Section 6.3, and Section 8.3, of this paper, respectively.

#### 8.5.3.1 A Comparison of Indicated Tail Factors

Table 8.5.2.1 provides a comparison of the MPD tails indicated by SAIF's own loss experience with those indicated by the static and trended mortality methods.

**Table 8.5.2.1 (Table 6.1) A Comparison of Indicated MPD Tail Factors**

| Maturity (Years) | Based on SAIF's Experience | Based on Static Mortality Model | Based on Trended Mortality Model |
|---|---|---|---|
| 10 | 2.469 | 2.684 | 3.025 |
| 15 | 2.328 | 2.469 | 2.783 |
| 25 | 2.054 | 2.019 | 2.271 |
| 35 | 1.680 | 1.594 | 1.776 |

#### 8.5.3.2 Estimating the Expected Value of MPD Reserves

Consider a hypothetical permanent disability male claimant injured at age 35.9, and expected to live another 40 periods. Two different methods of estimating the medical case reserve for this claimant at the end of the first period of development are common. They are:

1. **First Method:** *Zero Inflation Case Reserve Based on Projected Payments Through Expected Period of Death.* Estimated annual medical expenses of $5,000 per period (during the first full period of development) are multiplied by the life expectancy of 40 periods to obtain a case reserve of $200,000.

2. **Second Method:** *9% Inflation Case Reserve Based on Projected Payments Through Expected Period of Death.* Escalating medical expenses are cumulated up through age 75, yielding a total incurred of $1,689,000. (Other rates of inflation may be considered appropriate.)

Two additional methods may also be applied. Each of these produces much higher, and more accurate, estimates of the expected value of the case reserve:

3. **Third Method:** *Expected Total Payout Weighted by Probability of Occurrence Over Scenarios of All Possible Periods of Death*. This method yields an expected reserve of $2,879,000.

4. **Fourth Method:** *Expected Value of Trials from a Markov Chain Simulation*. This method yields an expected reserve of $2,854,000.

#### 8.5.3.3 Estimating the Variability of the MPD Reserve with a Markov Chain Simulation

The size of loss distribution for the medical component of a single permanent disability claim is far more skewed to the right than can be modeled by distributions commonly used by actuaries. In attempting to find a distribution to produce a reasonable fit, the authors found it necessary to first transform the ultimate cost amounts by taking the natural log of the natural log of the natural log and then taking the $n$th root—before a common distribution could be found. Taking the fifth root of the triple natural log appears to produce a distribution of ultimate costs that conforms well with an extreme value distribution. The fact that such intense transformations were needed suggests that a totally different approach than fitting commonly used distributions should be used.

Simulating the variability of the MPD reserve for unreported claims is naturally more complicated. First, the total number of IBNR claims should be represented by a Poisson (or similar) distribution. Then census data of the age at injury of recent claimants can be used to randomly generate these ages for unreported claimants. Then, future payments for each unreported claimant can be simulated. The degree of variability of the MPD reserve for unreported claimants is exceptionally high—because some of those claimants may have been quite young when injured, and the total expected future payment for workers injured at a young age is dramatically higher than for those injured at an older age. Estimates also vary dramatically according to the gender and age of each claimant at the time of the analysis. This suggests that the variability of the total MPD reserve can best be modeled by simulating the variability of the future payout for each claim separately.

### 8.5.4 Advantages and Disadvantages

The methods presented in the Sherman-Diss paper were tested against actual historical data and provide a reasonable estimate of future loss development extending out to 85 years of development. Such development is possible; a worker could be injured at age 16 and live to be over 100. No other method in the actuarial literature has been successful in doing so. One disadvantage is that total medical loss experience for development years 20 and higher is needed to successfully implement the methods. Such data may be difficult for a user to obtain. Another disadvantage is that medical and mortality rates may be difficult to obtain or estimate. A sub-standard mortality table may be necessary.

#### 8.5.5 Users

The method is currently utilized by the SAIF Corporation, Oregon's State Fund.

#### 8.5.6 Summary

The Sherman-Diss paper presents an analysis of medical payments based on 160,000 permanently disabled claimants—for accident years 1926-2002, and a method utilizing incremental payment data prior to the standard triangle to extend development factors beyond the end of the triangle, up to 85 years of development.

## 9. COMPARISON OF SELECTED RESULTS

### 9.1 Discussion

The working party obtained data from a number of different sources with the goal of applying the methods presented in order to (1) provide a comparison of results, and (2) enhance the discussion of each the method's value and validity under various circumstances. To the extent possible, we used a common data set to illustrate the various methods and also used this data in the companion Excel workbook (which illustrates, where possible, many of the examples shown in the appendix). One exception to this approach involves methods previously detailed in CAS papers. In these cases, we generally used the data as originally presented in the paper.

In general, the methods discussed may require different types of data – such as different historical periods, differing granularity of data (i.e., separate medical versus indemnity losses), incremental versus cumulative, absence of incurred data and completeness of data, as examples. As a result, it was not always meaningful to use the same data set for each method.

Even using the same data set, different methods produce a range of results. In addition to differences caused by the dynamics of the methods themselves, individual judgments and selections may also contribute to differences in results. For example, methods that require an assumption of link ratios or ratios of incurred loss to paid loss for each evaluation point may require actuarial judgment of the most appropriate "average" to differ between methods. To the extent possible, we have held actuarial judgment and assumptions consistent among the various methods for testing and comparison purposes. The actuary should be aware that differences in indicated tail factors can vary both as a result of the method used as well as due to the underlying assumptions which rely on actuarial judgment or selection.

The table below shows the indicated ten-period to ultimate (120 months) paid loss development tail for the methods using the common 10 year loss history shown in the appendix.

| Method | Indicated Paid Tail |
|---|---|
| Generalized Bondy Method | 1.025 |
| Fully Generalized Bondy Method | 1.043 |
| Sherman-Boor | 1.096 |
| Exponential Fit | |
| Using all Points | 1.032 |
| Using last 6 Points | 1.044 |
| McClenahan's Method | 1.055 |
| McClenahan's Adjusted Method | 1.040 |
| Sherman's Method | 1.137 |
| Sherman's Method with Lag Adjustment | 1.135 |
| Pipia's Method (Weibull Fit) Using all Factors | |
| Using all historical factors | 1.098 |
| Using selected development factors | 1.049 |

As is evidenced by the range of indicated tail factors, it is important for the actuary to understand the underlying exposure being evaluated and to use judgment in determining the most appropriate method(s) for each situation. The coverage being evaluated, the layer (i.e., excess versus primary), and claims handling practices are examples of items that should be considered in selecting the appropriate methodology for calculating a tail factor. As discussed in the sections above, each method has its own specific advantages and disadvantages and therefore, some advice was provided on whether each specific method is optimum in the reserving context of specific situations; this is intended to be helpful when selecting a method to estimate tail loss development (i.e., a method to compute a tail factor).

### 9.2 Future Research

The Working Party believes that this is an area of future research using simulated data wherein the ultimate values of the simulated data are also known. Thus testing of the various methods would provide a clearer sense of which methods work best based on the different types of data aberrations built into the simulations. One key point is to create as many varying simulations as possible to properly test all methods.

## 10. CONCLUSIONS

The Tail Factor Working Party undertook an exhaustive study of all the methods for computing tail factors that are believed to be available to actuaries. While it is possible that some methods in use were not identified by the working party, this document is believed to present the vast majority of the available methods. As the document shows, each method has its own specific advantages and disadvantages.

It should also be noted that many methods were identified that had only a handful of current users. Therefore this document can serve an important function by introducing these new approaches to a broader actuarial audience.

Again, as this document is primarily a survey paper, listing and describing all or most of the methods in existence, it is difficult to draw conclusions on tail factor methods in general. The most appropriate approach for a given analysis is likely to depend on the circumstances of the analysis. As stated above, it is certainly reasonable to conclude that there are more methods available to actuaries than are in general use. Hopefully this document will act to expand the repertoire of tail factor methods in the resources of the typical actuary.

## Acknowledgments

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## Supplementary Material

A companion Excel file with a sample of each of the methods in this paper is on the CAS website at http://www.casact.org/pubs/forum/13fforumpt/.

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# APPENDIX A – Alternative Organization of the Methods

Many methods in this report could conceivably be placed in different categories than the ones the working party assigned them in this document. We have listed some alternate groupings below (along with each method's reference section within this document).

## A.1 Bondy-Type & Decay Methods

- Bondy methods (2.1.1-2.1.4)
- Exponential decay method (5.2)
- McClenahan's loss model (5.3)
- Skurnick method (5.4)
- Mueller Incremental tail method (8.3)

In reviewing the relationships between these distinct, but related methods, the Working Party has the following comments about the underlying decay concept and how it weaves through the methods. Of note, other methods use similar decay concepts, but may not show an explicit year-to-year decay.

Bondy methods decay the last estimated development factor over time. This is in accordance with a half-life type function where the rate of decay is assumed to be constant over time. The most common form of this method assumes a decay rate of 0.5 (i.e., each successive factor is the square root of the previous), which generates a result where the tail factor is equal to the last estimated development factor.

The physical interpretation is that the claims are being settled at a rate proportional to the current outstanding claims. This is probably not an accurate model of how claims departments work in practice but does have the benefit of generating a smooth function.

Other variations of this method include decaying with a constant number between 0 and 1. Certain lines of business are expected to exhibit thicker tails.

The exponential decay method is a way of obtaining an appropriate factor using curve-fitting techniques. It is described more fully in section 5.1.1.

The McClenahan and Skurnick methods are variations on this basic concept. With the Skurnick method the decay rate is allowed to vary by accident period.

Similar drawbacks apply, to a varying extent, to all these methods. These include:

- They are generally not applicable to lines with negative development between evaluation periods without additional adjustment – i.e., they will generally fail on incurred data.

- Exponential decay assumes a monotonically decreasing function, therefore these methods do not accommodate increases in incremental losses from one period to the next ("hump" shaped patterns).
- Exponential decay at an indicated rate developed from the observed data can produce a relatively faster development than other models for certain long tail liability lines.
- A sub-optimal fit will be obtained for lines with variable decay rates across evaluation periods such as workers compensation.

The Mueller Incremental Decay Method and the Generalized Bondy Method tackle some of the first couple of points above by considering a variety of decay factors based on differing anchor periods and estimating tail factors. It is to be noted however that this method is relatively sensitive to the choice of anchor periods and small changes in the incremental age-to-age factors.

**A.2 Algebraic Methods that Focus on Relationships between Paid and Incurred**

- Equalizing Paid and Incurred Development (3.2)
- Sherman-Boor Method (3.3)
- NCCI Method (3.4)
- Static Mortality Method (6.2)
- Trended Mortality Method (6.3)
- Judgment Estimate Method (7.3)

This batch of methods considers the information available from the case handlers estimates of outstanding claim reserves in estimating a tail factor for the paid claims data.

Assumptions:

- The case reserves for the final year are a true reflection of the reserves required.
- Settlement and reporting patterns are unchanged over time and claims department reserving is similar over time.
- No future pure IBNR claims will materialize for the benchmark year.

The static and trended mortality methods examine the incremental paid per prior open to estimate the paid tail going forward. The number of open claims in any period is determined using mortality tables. These two methods have been applied in practice only to the medical component of permanent disability claims. Other algebraic methods may use alternate projection techniques to estimate the number of open claims.

By their nature, algebraic methods that focus on the paid to incurred loss amounts cannot be used to estimate an incurred tail. However, once a payment stream is calculated by means of the static or trended mortality methods, expected values of case reserves may be estimated for the same payment stream. The Sherman-Diss model of Section 8.5 describes the procedure.

### A.3 Methods Based on Benchmark Data

- Benchmark Data Based Methods (4.2-4.6)
- Restate historical experience (8.2)

Benchmark methods are used when the data/experience of the book is not robust. This could be due to a number of reasons including data scarcity, change in the mix of business over time or where the historical development has been distorted by changes in settlement/reporting or claims estimation practices.

In addition these methods are often used as fall-back to test the reasonability of other approaches.

The major disadvantage of this approach is that appropriate industry development factors are not always available. In addition the performance of the book may be faster/slower than the industry average; for this reason it is often instructive to compare the actual historical development to that indicated by the benchmark data and adjust as required.

### A.4 Stochastic and Curve-Fitting Methods

- Exponential decay method (5.2)
- McClenahan's loss model (5.3)
- Skurnick's method (5.4)
- Sherman's method (5.5)
- Pipia's method (5.6)
- England and Verrall (5.7)

This selection of methods aims to fit curves to the data and extrapolate an appropriate tail factor. The process is similar and involves four stages:

(a) specification of the functional form (this is normally defined by the method)

(b) optimizing function and assessment of goodness of fit

(c) estimation of parameters using curve-fitting techniques

(d) reading off the curve to develop an implied tail factor.

Most of the curves that tend to be used are exponential/logs based and are generally monotonically decreasing. As such they do not allow for "humps" or negative developments in the data. Specific features like these, or even structural breaks in the development, are smoothed out as part of the fitting process; these curves do not capture these phenomena even if they are a consequence of a true underlying process rather than just as a result of random data volatility.

The Sherman-Diss Method of Section 8.5 allows for breaks in structural development. In fact, the static and trended mortality methods of the Sherman-Diss model bear much resemblance to the classic structural methods developed by Fisher/Lange and Adler/Kline.

The England-Verrall Method allows for humps and negative development by the stochastic nature of the method although the development may also be judgmentally smoothed. Stochastic methods are an enhancement of traditional methods in this respect. The England-Verrall Method simulates paid claim amounts by stochastic means. Traditional chain ladder reserving techniques may then be applied to the triangle of simulated claims payments.

The curve-fitting methods do have the advantage that they tend to consider the entire loss development, rather than focusing on the northwest corner of the triangle, where arguably there is the most volatility.

### A.5 Methods Based on Future Remaining Open Counts

- Static mortality method: incremental paid per open count (6.2)
- Trended mortality method (6.3)

The static and trended mortality methods examine the incremental paid per prior open to estimate the paid tail going forward. The number of open claims in any period is determined using mortality tables. These two methods have been applied in practice only to the medical component of permanent disability claims. Other algebraic methods may use alternate projection techniques to estimate the number of open claims. Once a payment stream is calculated by means of the static or trended mortality methods, expected values of case reserves may be estimated for the same payment stream. The Sherman-Diss Method of Section 8.5 describes the procedure.

### A.6 Methods Based on the Peculiarities of the Remaining Open Claims

- Maximum Possible Loss Method (7.2)
- Judgment Estimate Method (7.3)

While these methods do not generally result in a tail factor for the less mature years (that may or may not have a similar open claim portfolio when they become the oldest year in the triangle), they can be very useful for analyzing the oldest year and other years near the top of the triangle.

### A.7 Other Methods

- Restate Historical Experience Method (8.2)
- Mueller Incremental Tail Method (8.3)
- Corro's Method (8.4)
- Sherman-Diss Method (8.5)

Corro's technique can be used to estimate tail factors for claims, which are duration dependent but whose payment period is flat and uniform (e.g., credit insurance claims). A "mixing weight parameter" is calculated to allocate probabilities to two specified durations.

# APPENDIX B – Examples

## B.1 Introduction

This appendix will show additional details and illustrations of specific methods discussed in the main body of the paper. To the extent possible, the examples shown in this appendix reference a single data set, which is shown below. This data is also included in the accompanying Excel file.

### B.1.1 Paid Loss

| Cumulative Paid Loss Data | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Accident Year | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 |
| 2000 | 1,202 | 2,685 | 4,132 | 5,323 | 6,059 | 6,406 | 6,812 | 7,208 | 7,440 | 7,618 |
| 2001 | 1,297 | 2,712 | 4,232 | 5,314 | 6,062 | 6,786 | 7,375 | 7,687 | 7,934 | |
| 2002 | 1,342 | 2,566 | 4,058 | 5,388 | 6,480 | 7,141 | 7,801 | 8,109 | | |
| 2003 | 1,293 | 2,716 | 4,228 | 5,587 | 6,661 | 7,626 | 8,040 | | | |
| 2004 | 1,387 | 2,555 | 4,017 | 5,460 | 6,743 | 7,479 | | | | |
| 2005 | 1,487 | 2,738 | 4,125 | 5,683 | 6,793 | | | | | |
| 2006 | 1,499 | 2,920 | 4,781 | 6,285 | | | | | | |
| 2007 | 1,587 | 3,287 | 5,006 | | | | | | | |
| 2008 | 1,221 | 2,775 | | | | | | | | |
| 2009 | 1,321 | | | | | | | | | |
| | | | | | | | | | | |
| Paid Loss Development Triangle | | | | | | | | | | |
| Accident Year | 12-24 | 24-36 | 36-48 | 48-60 | 60-72 | 72-84 | 84-96 | 96-108 | 108-120 | |
| 2000 | 2.234 | 1.539 | 1.288 | 1.138 | 1.057 | 1.063 | 1.058 | 1.032 | 1.024 | |
| 2001 | 2.092 | 1.560 | 1.256 | 1.141 | 1.119 | 1.087 | 1.042 | 1.032 | | |
| 2002 | 1.911 | 1.582 | 1.328 | 1.203 | 1.102 | 1.092 | 1.039 | | | |
| 2003 | 2.100 | 1.557 | 1.321 | 1.192 | 1.145 | 1.054 | | | | |
| 2004 | 1.842 | 1.572 | 1.359 | 1.235 | 1.109 | | | | | |
| 2005 | 1.841 | 1.507 | 1.378 | 1.195 | | | | | | |
| 2006 | 1.948 | 1.637 | 1.314 | | | | | | | |
| 2007 | 2.071 | 1.523 | | | | | | | | |
| 2008 | 2.272 | | | | | | | | | |
| | | | | | | | | | | |
| Straight Average | 2.034 | 1.560 | 1.321 | 1.184 | 1.106 | 1.074 | 1.047 | 1.032 | 1.024 | |
| Volume Weighted Average | 2.026 | 1.559 | 1.320 | 1.185 | 1.107 | 1.074 | 1.046 | 1.032 | 1.024 | |
| 5 Year Volume Weighted | 1.988 | 1.559 | 1.339 | 1.193 | 1.107 | 1.074 | 1.046 | 1.032 | 1.024 | |
| 3 Year Volume Weighted | 2.085 | 1.555 | 1.349 | 1.207 | 1.119 | 1.077 | 1.046 | 1.032 | 1.024 | |
| | | | | | | | | | | |
| Selected LDF | 2.034 | 1.560 | 1.321 | 1.184 | 1.106 | 1.074 | 1.047 | 1.032 | 1.024 | |

## B.1.2 Incurred Loss

| Cumulative Incurred Loss Data | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Accident Year | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 |
| 2000 | 2,539 | 4,479 | 5,650 | 6,639 | 7,224 | 7,224 | 7,464 | 7,778 | 7,892 | 7,987 |
| 2001 | 2,672 | 4,667 | 6,049 | 6,988 | 7,355 | 7,819 | 8,171 | 8,296 | 8,518 | |
| 2002 | 2,808 | 4,676 | 6,207 | 7,064 | 7,601 | 7,984 | 8,390 | 8,628 | | |
| 2003 | 3,073 | 5,099 | 6,292 | 7,237 | 7,749 | 8,386 | 8,604 | | | |
| 2004 | 3,070 | 4,527 | 5,915 | 6,986 | 7,780 | 8,197 | | | | |
| 2005 | 2,932 | 4,750 | 6,041 | 7,144 | 7,771 | | | | | |
| 2006 | 3,095 | 5,104 | 6,770 | 7,821 | | | | | | |
| 2007 | 3,228 | 5,526 | 7,204 | | | | | | | |
| 2008 | 2,877 | 5,122 | | | | | | | | |
| 2009 | 2,890 | | | | | | | | | |
| | | | | | | | | | | |
| Incurred Loss Development Triangle | | | | | | | | | | |
| Accident Year | 12-24 | 24-36 | 36-48 | 48-60 | 60-72 | 72-84 | 84-96 | 96-108 | 108-120 | |
| 2000 | 1.764 | 1.261 | 1.175 | 1.088 | 1.000 | 1.033 | 1.042 | 1.015 | 1.012 | |
| 2001 | 1.747 | 1.296 | 1.155 | 1.053 | 1.063 | 1.045 | 1.015 | 1.027 | | |
| 2002 | 1.665 | 1.327 | 1.138 | 1.076 | 1.050 | 1.051 | 1.028 | | | |
| 2003 | 1.659 | 1.234 | 1.150 | 1.071 | 1.082 | 1.026 | | | | |
| 2004 | 1.474 | 1.307 | 1.181 | 1.114 | 1.054 | | | | | |
| 2005 | 1.620 | 1.272 | 1.183 | 1.088 | | | | | | |
| 2006 | 1.649 | 1.326 | 1.155 | | | | | | | |
| 2007 | 1.712 | 1.304 | | | | | | | | |
| 2008 | 1.781 | | | | | | | | | |
| | | | | | | | | | | |
| Straight Average | 1.675 | 1.291 | 1.162 | 1.081 | 1.050 | 1.039 | 1.029 | 1.021 | 1.012 | |
| Volume Weighted Average | 1.671 | 1.291 | 1.162 | 1.081 | 1.050 | 1.039 | 1.028 | 1.021 | 1.012 | |
| 5 Year Volume Weighted | 1.646 | 1.289 | 1.161 | 1.080 | 1.050 | 1.074 | 1.046 | 1.032 | 1.024 | |
| 3 Year Volume Weighted | 1.712 | 1.301 | 1.172 | 1.090 | 1.062 | 1.040 | 1.028 | 1.032 | 1.024 | |
| | | | | | | | | | | |
| Selected | 1.675 | 1.291 | 1.162 | 1.081 | 1.050 | 1.039 | 1.029 | 1.021 | 1.012 | |

## B.1.3 Case Reserves

| Case Reserve | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Accident Year | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 |
| 2000 | 1,337 | 1,795 | 1,518 | 1,316 | 1,164 | 817 | 652 | 570 | 452 | 369 |
| 2001 | 1,376 | 1,955 | 1,816 | 1,674 | 1,293 | 1,033 | 796 | 609 | 584 | |
| 2002 | 1,466 | 2,111 | 2,149 | 1,677 | 1,121 | 843 | 589 | 520 | | |
| 2003 | 1,780 | 2,384 | 2,063 | 1,650 | 1,087 | 760 | 565 | | | |
| 2004 | 1,684 | 1,972 | 1,899 | 1,526 | 1,036 | 718 | | | | |
| 2005 | 1,445 | 2,012 | 1,916 | 1,460 | 978 | | | | | |
| 2006 | 1,596 | 2,184 | 1,988 | 1,537 | | | | | | |
| 2007 | 1,640 | 2,239 | 2,199 | | | | | | | |
| 2008 | 1,656 | 2,348 | | | | | | | | |
| 2009 | 1,569 | | | | | | | | | |

## B.2 Bondy-Type Methods

Using the paid loss data shown above and specifically the selected paid loss development pattern, we can estimate the tail factor based on the various Bondy methods. A summary of those results and the cumulated development pattern is shown below.

| Selected Paid Loss Development Factor | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | Tail | |
| Selected | 2.034 | 1.560 | 1.321 | 1.184 | 1.106 | 1.074 | 1.047 | 1.032 | 1.024 | | |
| Bondy Estimated Age-Ultimate | | | | | | | | | | | |
| Original | 6.680 | 3.283 | 2.105 | 1.594 | 1.346 | 1.217 | 1.133 | 1.082 | 1.049 | **1.024** | **= Prior A-A** |
| Modified 1 | 6.840 | 3.362 | 2.156 | 1.632 | 1.379 | 1.246 | 1.160 | 1.108 | 1.074 | **1.049** | **= (Prior A-A)^2** |
| Modified 2 | 6.837 | 3.360 | 2.154 | 1.631 | 1.378 | 1.245 | 1.159 | 1.108 | 1.073 | **1.048** | **= (Prior A-A) * 2** |
| Generalized | 6.632 | 3.260 | 2.092 | 1.586 | 1.334 | 1.197 | 1.119 | 1.073 | 1.045 | **1.028** | |
| Fully Generalized | 8.119 | 3.574 | 2.264 | 1.695 | 1.413 | 1.243 | 1.153 | 1.102 | 1.067 | **1.043** | |

The calculation of the generalized Bondy method is shown in the table below. Column 4 in this table uses formula 2.6 from the main body of the report and the sum of column 4 is minimized using the Excel "solver" function.

| Generalized Bondy: Parameters and Development Factors | | | | | | | |
|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | |
| Period | LDF | Ln (2) | Formula 2.6 | Fitted A-A | Fitted A-U | Index i | |
| 12-24 | 2.034 | 0.710 | 0.000000 | 2.034 | 6.632 | 1 | |
| 24-36 | 1.560 | 0.444 | 0.000001 | 1.558 | 3.260 | 2 | |
| 36-48 | 1.321 | 0.278 | 0.000001 | 1.319 | 2.092 | 3 | |
| 48-60 | 1.184 | 0.169 | 0.000017 | 1.189 | 1.586 | 4 | |
| 60-72 | 1.106 | 0.101 | 0.000048 | 1.114 | 1.334 | 5 | |
| 72-84 | 1.074 | 0.072 | 0.000016 | 1.070 | 1.197 | 6 | |
| 84-96 | 1.047 | 0.046 | 0.000012 | 1.043 | 1.119 | 7 | |
| 96-108 | 1.032 | 0.032 | 0.000028 | 1.027 | 1.073 | 8 | |
| 108-120 | 1.024 | 0.024 | 0.000052 | 1.017 | 1.045 | 9 | |
| | | | | | **1.028** | ***= [Last AA ^ (B / 1 - B)]*** | |
| | | Total | 0.000175 | *Note: Must use "Solver" to minimize least squares [Sum of (4)]* | | | |
| | | | | | | | |
| Bondy Parameter | | 0.625 | *= B* | | | | |
| Estimated Ratio for 12-24 | | 2.034 | *= d^* | | | | |

The fully generalized Bondy method allows the estimated development ratios (*d*) to vary by accident period, while using the same Bondy parameter (B). In example shown below, the formula 2.6 is calculated for each of the last three accident periods at every maturity and the sum of the entire triangle is minimized using Excel.

| Fully Generalized Bondy:Parameters, Development Factors, and Squared Error | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **=[Ln(Actual AA) - Ln(d)*B^(Index - 1)]^2** | | | | | | | | | |
| Accident Year | Parameter Estimate (d) | 12-24 | 24-36 | 36-48 | 48-60 | 60-72 | 72-84 | 84-96 | 96-108 | 108-120 | |
| 2000 | 2.090 | | | | | | | 0.000 | 0.000 | 0.000 | |
| 2001 | 1.969 | | | | | | 0.000 | 0.000 | 0.000 | | |
| 2002 | 1.835 | | | | | 0.000 | 0.000 | 0.000 | | | |
| 2003 | 1.932 | | | | 0.000 | 0.000 | 0.000 | | | | |
| 2004 | 2.070 | | | 0.000 | 0.000 | 0.001 | | | | | |
| 2005 | 1.954 | | 0.001 | 0.002 | 0.000 | | | | | | |
| 2006 | 1.994 | 0.001 | 0.002 | 0.000 | | | | | | | |
| 2007 | 2.023 | 0.001 | 0.001 | | | | | | | | |
| 2008 | 2.272 | 0.000 | | | | | | | | | |
| | | | | | | | | | | | |
| Bondy: | 0.648 | *= B* | | | | | | | | | |
| | | | | | | | | | | | |
| | Index | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | |
| | | | | | | | | | | | |
| Minimum Least Squares | | 0.009 | *Note: Must use "Solver" to minimize least squares [Sum of triangle]* | | | | | | | | |
| | | | | | | | | | | | |
| | Period | 12-24 | 24-36 | 36-48 | 48-60 | 60-72 | 72-84 | 84-96 | 96-108 | 108-120 | Tail |
| | Fitted LDF | 2.272 | 1.579 | 1.336 | 1.200 | 1.137 | 1.078 | 1.046 | 1.033 | 1.023 | |
| | Factor to Ultimate | 8.119 | 3.574 | 2.264 | 1.695 | 1.413 | 1.243 | 1.153 | 1.102 | 1.067 | 1.043 |

## B.3 Algebraic Methods

### B.3.1 Sherman-Boor Method

This method requires two triangles, one of paid loss and one of case reserves. Using the triangle shown in the introduction and the formulas from the main body of the paper, we can then calculate triangles of the incremental paid loss and incremental disposed case reserves. Specifically, the incremental paid loss triangle is computed as: given a cell in the cumulative paid loss triangle, then we subtract the previous cell in the same row of the cumulative paid loss triangle. Subtracting the current cell from the previous cell in the case reserve triangle to obtain the triangle of case reserves disposed of. The incremental triangles are shown below:

| Accident | Incremental Paid Loss (Formula 3.2) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Year | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 |
| 2000 | 1,202 | 1,483 | 1,448 | 1,191 | 736 | 347 | 406 | 396 | 232 | 178 |
| 2001 | 1,297 | 1,416 | 1,520 | 1,082 | 748 | 723 | 589 | 312 | 247 | |
| 2002 | 1,342 | 1,223 | 1,493 | 1,329 | 1,093 | 661 | 659 | 308 | | |
| 2003 | 1,293 | 1,422 | 1,513 | 1,359 | 1,074 | 965 | 413 | | | |
| 2004 | 1,387 | 1,168 | 1,462 | 1,443 | 1,283 | 736 | | | | |
| 2005 | 1,487 | 1,251 | 1,387 | 1,559 | 1,109 | | | | | |
| 2006 | 1,499 | 1,421 | 1,861 | 1,503 | | | | | | |
| 2007 | 1,587 | 1,700 | 1,719 | | | | | | | |
| 2008 | 1,221 | 1,553 | | | | | | | | |
| 2009 | 1,321 | | | | | | | | | |
| | | | | | | | | | | |
| Accident | Incremental Case Reserves Disposed Of (Formula 3.3) | | | | | | | | | |
| Year | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 |
| 2000 | | (457) | 277 | 202 | 151 | 347 | 166 | 82 | 118 | 83 |
| 2001 | | (579) | 139 | 142 | 381 | 260 | 237 | 187 | 25 | |
| 2002 | | (644) | (38) | 472 | 556 | 278 | 254 | 70 | | |
| 2003 | | (604) | 320 | 414 | 562 | 327 | 196 | | | |
| 2004 | | (289) | 74 | 373 | 489 | 318 | | | | |
| 2005 | | (567) | 96 | 456 | 482 | | | | | |
| 2006 | | (588) | 195 | 452 | | | | | | |
| 2007 | | (598) | 40 | | | | | | | |
| 2008 | | (692) | | | | | | | | |
| 2009 | | | | | | | | | | |

Then divide the incremental paid loss by the case reserves eliminated. These ratios will be used to calculate estimators of '$S$'.

| Accident | Relative Disposal Costs (Formula 3.4 = Ipaid(w,d) / Icase(w,d)) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Year | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 |
| 2000 | | (3.241) | 5.232 | 5.885 | 4.862 | 1.000 | 2.446 | 4.846 | 1.965 | 2.151 |
| 2001 | | (2.444) | 10.950 | 7.612 | 1.964 | 2.781 | 2.486 | 1.670 | 9.972 | |
| 2002 | | (1.898) | (39.244) | 2.815 | 1.965 | 2.378 | 2.601 | 4.427 | | |
| 2003 | | (2.355) | 4.725 | 3.285 | 1.910 | 2.949 | 2.113 | | | |
| 2004 | | (4.047) | 19.857 | 3.871 | 2.622 | 2.314 | | | | |
| 2005 | | (2.206) | 14.469 | 3.419 | 2.302 | | | | | |
| 2006 | | (2.416) | 9.522 | 3.326 | | | | | | |
| 2007 | | (2.841) | 42.912 | | | | | | | |
| 2008 | | (2.244) | | | | | | | | |
| | | | | | | | | | | |
| | | | | Adjustment Factor | | 3.073 | *='S', which is selected here as* | | | |
| | | | | | | | *as average of last 5 columns* | | | |
| | | Oldest Period, Current Case Reserve | | | | 369 | | | | |
| | | | Older Period, Cumulative Paid | | | 7,618 | | | | |
| | | | | Paid Tail Factor | | 1.149 | *Formula 3.5* | | | |
| | | | | Incurred Tail Factor | | 1.096 | *Formula 3.6* | | | |

Because the early development involves not just elimination of case reserves through payments, but also substantial emergence of IBNR claims, the early maturities could be potentially distorted. Looking at the various ratios at the 'mature' development stage it would appear that they average around 3.0, so we will use that as our adjustment factor 'S' for the case reserves.

Utilizing $369 of case left on the 2000 accident period at 120 months development, and the cumulative paid on 2000 accident period of $7,618, the development portion of the paid loss tail factor would be ($369/$7,618)×3.079 = .149 So, the paid loss tail factor would be 1.149.

For the incurred loss tail factor, first note that only the 'development portion' of the $S$ =3.073, or $S-1$=2.073, need be applied (the remaining case is already contained in the incurred). Second, a ratio of the case reserves to incurred loss is needed (which is $c(1,n)/c_{Incurred}(1,n)=c(2000,120)/c_{Incurred}(2000,120)=$ $369/$7,987 = .046. Multiplying the two numbers creates an estimate of the development portion of the tail at 2.073 ×.046 = 0.096. So, the incurred loss tail factor estimate would be 1.096.

## B.4 Curve-Fitting Methods

## B.4.1 Exponential Method

The main body of the report illustrates an exponential fit using data provided by Joe Boor. Below, the exponential fit is applied to the same data used in other sections of this appendix to illustrate the fit two different ways. Specifically, the table below develops the fit using all of the selected development factors (result in column 6) and a fit using only the 6 most mature periods (with result in column 8).

| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|
| | | | | | Curve Fit Results | | | |
| Development Period | | Selected | v(d) | ln [v(d)] | Using All Periods | | Using Last 6 Periods | |
| | | LDF | = (2) - 1 | = ln(3) | Fitted A-A | Fit Error | Fitted A-A | Fit Error |
| 1 | 12-24 | 2.034 | 1.034 | 0.034 | 1.855 | 0.180 | | |
| 2 | 24-36 | 1.560 | 0.560 | -0.580 | 1.532 | 0.027 | | |
| 3 | 36-48 | 1.321 | 0.321 | -1.137 | 1.332 | -0.011 | | |
| 4 | 48-60 | 1.184 | 0.184 | -1.692 | 1.207 | -0.023 | 1.169 | 0.015 |
| 5 | 60-72 | 1.106 | 0.106 | -2.240 | 1.129 | -0.022 | 1.113 | -0.006 |
| 6 | 72-84 | 1.074 | 0.074 | -2.601 | 1.080 | -0.006 | 1.075 | -0.001 |
| 7 | 84-96 | 1.047 | 0.047 | -3.065 | 1.050 | -0.003 | 1.050 | -0.003 |
| 8 | 96-108 | 1.032 | 0.032 | -3.438 | 1.031 | 0.001 | 1.033 | -0.001 |
| 9 | 108-120 | 1.024 | 0.024 | -3.731 | 1.019 | 0.005 | 1.022 | 0.002 |
| 10 | | | | | 1.012 | | 1.015 | |
| 11 | | | | | 1.008 | | 1.010 | |
| 12 | | | | | 1.005 | | 1.007 | |
| 13 | | | | | 1.003 | | 1.004 | |
| 14 | | | | | 1.002 | | 1.003 | |
| 15 | | | | | 1.001 | | 1.002 | |
| 16 | | | | | 1.001 | | 1.001 | |
| 17 | | | | | 1.000 | | 1.001 | |
| 18 | | | | | 1.000 | | 1.001 | |
| 19 | | | | | 1.000 | | 1.000 | |
| 20 | | | | | 1.000 | | 1.000 | |

Fitting a line to the natural logarithms of the development portion of the link ratios (column 6), we estimate the slope and intercept of the fitted line. The inverse natural logarithm of the slope parameter becomes the decay constant, $r$. The complete fitted parameters are shown below. Note that for this data set and truncating the age-to-age factors through period 20, the tail factor based on the approximate formula and the cumulative of the age-to-age factors is very similar.

| Curve Fit Parameters | Decay Rate | Coefficient | Tail Factor At Period 10 Truncated | Approximate |
|---|---|---|---|---|
| Using All Points | 0.623 | 1.372 | 1.032 | 1.032 |
| Using Last 6 Points | 0.666 | 0.863 | 1.044 | 1.044 |

Decay = e^[slope of the linear fit of (1) and (5)]
Coefficient = intercept of linear fit of (1) and (5)
Truncated Tail = Product of remaining fitted A-A
Approximate = 1 + Coefficient x Decay ^ [Period / (1-Decay)]
From Formula 5.2

### B.4.2 McClenahan's Method

Here we have replicated the McClenahan method discussed in the body of the report using the same data shown here in the Appendix. Specifically, we are using selected paid loss development factors and again converting these to cumulative paid loss amounts by selecting a base amount for the first development period paid loss, for simplicity sake we use $100. To determine incremental paid losses by period we subtract successive cumulative loss amounts, and then we have the following:

| (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|
| | | | | | |
| Development Period | | Age | Selected A-A Factor | Cumulative Paid | Incremental Paid |
| | | | | | |
| 1 | | 12 | | 100 | 100 |
| 2 | 12-24 | 24 | 2.034 | 203 | 103 |
| 3 | 24-36 | 36 | 1.560 | 317 | 114 |
| 4 | 36-48 | 48 | 1.321 | 419 | 102 |
| 5 | 48-60 | 60 | 1.184 | 496 | 77 |
| 6 | 60-72 | 72 | 1.106 | 549 | 53 |
| 7 | 72-84 | 84 | 1.074 | 590 | 41 |
| 8 | 84-96 | 96 | 1.047 | 617 | 28 |
| 9 | 96-108 | 108 | 1.032 | 637 | 20 |
| 10 | 108-120 | 120 | 1.024 | 652 | 15 |

We can continue this table by taking successive ratios of incremental paid amounts for the accident periods to produces estimates of the annual decay constant $r$. In practice any of a variety of curve-fitting techniques using the incremental paid loss regressed on age can be employed to develop an estimate of $r$ from Column 8, in this example we have used a linear fit of the natural log of the $r$'s.

| (7) | (8) | (9) | (10) |
|---|---|---|---|
| | | Curve Fit Results | |
| Incremental | Log Incremental | Fitted | Actual / Fitted |
| A-A | Ln (6) | Incremental | Ratio |
| | | | |
| | | 377 | 0.265 |
| 1.034 | 4.639 | 273 | 0.379 |
| 1.101 | 4.735 | 197 | 0.577 |
| 0.893 | 4.622 | 143 | 0.713 |
| 0.758 | 4.346 | 103 | 0.747 |
| 0.685 | 3.967 | 75 | 0.707 |
| 0.771 | 3.707 | 54 | 0.753 |
| 0.675 | 3.314 | 39 | 0.703 |
| 0.721 | 2.988 | 28 | 0.701 |
| 0.770 | 2.726 | 20 | 0.745 |
| | | | |
| | | Average | 0.724 |
| | | | |
| Curve Fit on Boxed Points | | | |
| Monthy Decay | 0.973 | | |
| Annual Decay | 0.724 | | |

The decay rates shown are the result of a linear fit of the boxed values in column 8. The monthly decay uses monthly maturities in column 3 and the annual decay uses in period number in column 1. Note that .973^12 = .724 (when accounting for decimals rounded off), however, McLenahan's formula uses the monthly decay rate to calculate the tail (where $p$ is the monthly decay rate and $= r(1/12)$ ).

For the sake of the example, we will assume a lag constant of $a = 6$. Once the value of $p$ is calculated, we can develop an estimate of the tail at 120 months or $T(10)$ using equation 5.7. We can also estimate an adjusted tail using the actual to fitted ratio from column 10.

| | Months | |
|---|---|---|
| Initial Lag in Report | 6 | |
| Tail at 120 | 1.055 | From Formula 5.7 |
| Adjusted Tail at 120 | 1.040 | Calculated Tail, Adjusted for Actual / Fitted Ratio |

Formula 5.7: $T(m/12) = \{12 \times (1 - p)\} / \{12 \times (1 - p) - p^{m-a-10} \times (1 - p^{12})\}$

### B.4.3 Sherman's Method

Given the selected paid loss ink ratios, we first determine the development portion, $v(d)$, of each link ratio. The natural logarithms of the age $d$ and $v(d)$ then represent the dependent and independent variables in our regression, respectively.

| Curve Fit Using An Inverse Power Function | | | | | |
|---|---|---|---|---|---|
| Development Period | | Selected LDF | Development Portion | Log of Development Age | Log of Development Portion |
| 1 | 12-24 | 2.034 | 1.034 | 0.000 | 0.034 |
| 2 | 24-36 | 1.560 | 0.560 | 0.693 | (0.580) |
| 3 | 36-48 | 1.321 | 0.321 | 1.099 | (1.137) |
| 4 | 48-60 | 1.184 | 0.184 | 1.386 | (1.692) |
| 5 | 60-72 | 1.106 | 0.106 | 1.609 | (2.240) |
| 6 | 72-84 | 1.074 | 0.074 | 1.792 | (2.601) |
| 7 | 84-96 | 1.047 | 0.047 | 1.946 | (3.065) |
| 8 | 96-108 | 1.032 | 0.032 | 2.079 | (3.438) |
| 9 | 108-120 | 1.024 | 0.024 | 2.197 | (3.731) |
| 10 | | | | | |
| 11 | | | | | |
| 12 | | **I. Curve Fit With No Lag Parameter** | | | |
| 13 | | | | | |
| 14 | | Exponent = slope | (2.386) | | |
| 15 | | Coefficient = e ^ Intercept | 4.806 | | |
| 16 | | Tail Factor | 1.137 | | |
| 17 | | | | | |
| 18 | | | | | |
| 19 | | **II. Curve Fit With Optimal Lag Parameter** | | | |
| 20 | | | | | |
| 21 | | Lag Parameter | (0.076) | | |
| 22 | | Minimal Squared Error | 0.000 | | |
| 23 | | Tail Factor | 1.135 | | |
| 24 | | | | | |
| 25 | Note: Must use "Solver" to minimize squared error each time new ratios are selected | | | | |

The fitted parameters of the curve are based on a linear regression of the boxed factors. The tail factor is the determined by cumulating the estimated age-to-age factors for each future period, where the factor $f(d) = 1 +$ coefficient * age slope.

Several possible alternatives to the above example exist. For example, we might have chosen a to rely on link ratios of only the first 5 or 8 development ages, we could rely on the link ratios of only "mature" development ages, etc.

To estimate the optimum lag, you can use a bisection process, specifically following the process above using different potential lags; finding the lowest value of the squared error across a group of values; and progressively narrowing the range. Alternatively, you can also use the 'solver' Excel function.

### B.4.4 Pipia's Method

The following example is based the cumulative paid loss; the method can also be applied to incurred losses. In addition, other choices for the dimensions of the triangle can easily be substituted.

The parameter being minimized is the square of the ratio of the difference between the actual and fitted incremental development to the expected incremental development. As shown in the triangle below, the difference is taken for each of the age-to-age factors and the total difference for the triangle is minimized using Excel.

| Squared Difference | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Accident Year | 12-24 | 24-36 | 36-48 | 48-60 | 60-72 | 72-84 | 84-96 | 96-108 | 108-120 |
| 2000 | 0.100 | 0.001 | 0.004 | 0.031 | 0.240 | 0.037 | 0.001 | 0.048 | 0.047 |
| 2001 | 0.156 | 0.006 | 0.003 | 0.026 | 0.004 | 0.011 | 0.062 | 0.049 | |
| 2002 | 0.245 | 0.014 | 0.043 | 0.044 | 0.008 | 0.032 | 0.090 | | |
| 2003 | 0.153 | 0.005 | 0.034 | 0.021 | 0.085 | 0.095 | | | |
| 2004 | 0.285 | 0.010 | 0.105 | 0.161 | 0.001 | | | | |
| 2005 | 0.286 | 0.001 | 0.154 | 0.027 | | | | | |
| 2006 | 0.226 | 0.050 | 0.025 | | | | | | |
| 2007 | 0.166 | 0.000 | | | | | | | |
| 2008 | 0.088 | | | | | | | | |

Total Squared Difference 3.2776

Note: Must use "Solver" to minimize least squares each time new ratios are selected

| Curve Parameters | |
|---|---|
| $\lambda$ | -0.231 |
| $c$ | 0.000 |
| $t$ | 1.044 |
| Implied Tail | 1.098 |

| Curve Fit Using All Historical Development Factors | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Age | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 |
| Average Age of Claim | 0.5 | 1.5 | 2.5 | 3.5 | 4.5 | 5.5 | 6.5 | 7.5 | 8.5 | 9.5 |
| Fitted Age to Age | 2.806 | 1.521 | 1.271 | 1.168 | 1.112 | 1.078 | 1.056 | 1.041 | 1.031 | |
| Fitted Age to Ultimate | 9.454 | 3.369 | 2.215 | 1.742 | 1.492 | 1.342 | 1.244 | 1.178 | 1.131 | 1.098 |

The above estimated tail is only one way to minimize the squared difference. The estimated tail shown below was determined after minimizing the difference between the fitted and selected factors for only the 24 to 108 age-to-age factors, rather than the entire triangle.

| Curve Fit Using Selected Factors Only | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Selected** | | **1.560** | **1.321** | **1.184** | **1.106** | **1.074** | **1.047** | **1.032** | **1.024** | |
| Fitted Age to Age | 3.219 | 1.597 | 1.298 | 1.176 | 1.112 | 1.073 | 1.049 | 1.033 | 1.023 | |
| Fitted Age to Ultimate | 10.889 | 3.383 | 2.119 | 1.632 | 1.388 | 1.249 | 1.163 | 1.109 | 1.073 | 1.049 |

| Squared Difference | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 12-24 | 24-36 | 36-48 | 48-60 | 60-72 | 72-84 | 84-96 | 96-108 | 108-120 |
| | 0.0039 | 0.0058 | 0.0022 | 0.0021 | 0.0001 | 0.0030 | 0.0015 | 0.0027 |

Total Squared Difference 0.0212

| Curve Parameters | |
|---|---|
| $\lambda$ | -0.218 |
| $c$ | 0.000 |
| $t$ | 1.175 |
| Implied Tail | 1.049 |

### B.4.5 England-Verrall

Sections 6 and 7 of the England-Verrall paper present examples to illustrate the methodology. A comparison of predictor structures is included in Section 6. The Tables shown below include original table numbers in parentheses. Three models are fitted utilizing an over-dispersed Poisson model ( $\rho = 1$ in Equation (5.20)) with a logarithmic link function. For all three models:

$$E\left[c(w,d)\right] = m_{w,d} \, , \tag{5.19}$$

$$Var\left[c(w,d)\right] = \phi m_{w,d}^{\rho} \tag{5.20}$$

$$\ln(m_{w,d}) = \eta_{w,d} \tag{5.21}$$

The models differ only in the choice of the predictor, $\theta_w$ and $\theta_d$ .

Model 1: $\eta_{w,d} = c + \alpha_w + \beta_d$ (5.21.1)

Model 2: $\eta_{w,d} = u_d + c + \alpha_w + \beta \ln(d) + \gamma_d$ (5.21.2)

Model 3: $\eta_{w,d} = u_d + c + \alpha_w + s\theta_d \ln(d)$ (5.21.3)

Models 1 and 2, shown in Table 5.7.2.1, can be fitted in any statistical software package that fits generalized linear models. Model 3 can only be fitted in statistical software packages that fit generalized additive models. Equivalent development factors are shown in Table5.7.2.2, together with the actual development factors obtained by applying the standard chain ladder model to the data in Table5.7.2.1. The reserve estimates implied by Models 1, 2 and 3 are shown in Table 5.7.2.4, together with their prediction errors (as a percentage of the reserves).

TABLE 5.7.2.1 (TABLE 6.1)

Incremental Paid Losses Formed by Aggregating Across Different Classes

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 45,630 | 23,350 | 2,924 | 1,798 | 2,007 | 1,204 | 1,298 | 563 | 777 | 621 |
| 2 | 53,025 | 26,466 | 2,829 | 1,748 | 732 | 1,424 | 399 | 537 | 340 | |
| 3 | 67,318 | 42,333 | 1,854 | 3,178 | 3,045 | 3,281 | 2,909 | 2,613 | | |
| 4 | 93,489 | 37,473 | 7,431 | 6,648 | 4,207 | 5,762 | 1,890 | | | |
| 5 | 80,517 | 33,061 | 6,863 | 4,328 | 4,003 | 2,350 | | | | |
| 6 | 68,690 | 33,931 | 5,645 | 6,178 | 3,479 | | | | | |
| 7 | 63,091 | 32,198 | 8,938 | 6,879 | | | | | | |
| 8 | 64,430 | 32,491 | 8,414 | | | | | | | |
| 9 | 68,548 | 35,366 | | | | | | | | |
| 10 | 76,013 | | | | | | | | | |

TABLE 5.7.2.2 (TABLE 6.2)

Equivalent Development Factors: Overdispersed-Poisson Model

| Delay Year | Standard Chain Ladder | Model 1 Stochastic Chain Ladder | Model 2 Hoerl Curve | Model 3 GAM (dof = 5) |
|---|---|---|---|---|
| 2 | 1.4906 | 1.4906 | 1.4496 | 1.489 1 |
| 3 | 1.0516 | 1.0516 | 1.0796 | 1.0537 |
| 4 | 1.0419 | 1.0419 | 1.0372 | 1.0395 |
| 5 | 1.0268 | 1.0268 | 1.0238 | 1.0292 |
| 6 | 1.0254 | 1.0254 | 1.0180 | 1.0224 |
| 7 | 1.0149 | 1.0149 | 1.0150 | 1.0163 |
| 8 | 1.0130 | 1.0130 | 1.0135 | 1.0120 |
| 9 | 1.0067 | 1.0067 | 1.0127 | 1.0091 |
| 10 | 1.0078 | 1.0078 | 1.0124 | 1.0071 |
| 11 | | | 1.0125 | 1.0057 |
| 12 | | | 1.0129 | 1.0047 |
| 13 | | | 1.0135 | 1.0039 |
| 14 | | | 1.0144 | 1.0033 |
| 15 | | | 1.0156 | 1.0029 |
| 16 | | | 1.0171 | 1.0025 |

A comparison of error structures is included in Section 7 of the original paper. The same three model predictors are used, but with a Gamma error structure ($\rho = 2$) giving:

$$E\left[c(w,d)\right] = m_{w,d}\,, \quad (5.19)$$

$$Var\left[c(w,d)\right] = \phi m_{w,d}^2 \quad (5.20)$$

$$\ln(m_{w,d}) = \eta_{w,d} \quad (5.21)$$

and

Model 4: $\eta_{w,d} = c + \alpha_w + \beta_d$ (5.21.4)

Model 5: $\eta_{w,d} = u_d + c + \alpha_w + \beta \ln(d) + \gamma_d$ (5.21.5)

Model 6: $\eta_{w,d} = u_d + c + \alpha_w + s\theta_d \ln(d)$ (5.21.6)

Equivalent development factors are shown in Table 5.7.2.3

TABLE5.7.2.3 (TABLE7.1)

Equivalent Development Factors: Gamma Model

| Delay Period | Standard Chain Ladder | Model 4 Stochastic Chain Ladder | Model 5 Hoerl Curve | Model 6 GAM (dof = 5) |
|---|---|---|---|---|
| 2 | 1.4906 | 1.4969 | 1.4515 | 1.4771 |
| 3 | 1.0516 | 1.0470 | 1.0799 | 1.0512 |
| 4 | 1.0419 | 1.0381 | 1.0372 | 1.0357 |
| 5 | 1.0268 | 1.0259 | 1.0237 | 1.0280 |
| 6 | 1.0254 | 1.0251 | 1.0178 | 1.0221 |
| 7 | 1.0149 | 1.0154 | 1.0148 | 1.0165 |
| 8 | 1.0130 | 1.0131 | 1.0131 | 1.0125 |
| 9 | 1.0067 | 1.0084 | 1.0123 | 1.0098 |
| 10 | 1.0078 | 1.0086 | 1.0119 | 1.0079 |
| 11 | | | 1.0119 | 1.0066 |
| 12 | | | 1.0122 | 1.0055 |
| 13 | | | 1.0127 | 1.0048 |
| 14 | | | 1.0135 | 1.0041 |
| 15 | | | 1.0145 | 1.0036 |
| 16 | | | 1.0157 | 1.0032 |

Reserve estimates and prediction errors are shown in Table 5.7.2.5

TABLE 5.7.2.4 (TABLE 6.3)

Reserve Estimates and Prediction Errors: Overdispersed-Poisson Model

| | Reserve Estimates | | | Prediction Error | | |
|---|---|---|---|---|---|---|
| Accident Period | Model 1 Stochastic Chain Ladder | Model 2 Hoerl Curve | Model 3 GAM (dof = 5) | Model 1 Stochastic Chain Ladder | Model 2 Hoerl Curve | Model 3 GAM (dof = 5) |
| 1 | 0 | 0 | 0 | — | — | — |
| 2 | 683 | 1,085 | 622 | 159% | 95% | 110% |
| 3 | 1,792 | 3,101 | 1,998 | 100% | 61% | 62% |
| 4 | 4,363 | 6,129 | 4,470 | 63% | 46% | 43% |
| 5 | 5,657 | 7,173 | 5,940 | 50% | 43% | 38% |
| 6 | 8,209 | 8,689 | 8,106 | 40% | 39% | 33% |
| 7 | 10,914 | 11,031 | 11,106 | 34% | 34% | 29% |
| 8 | 15,199 | 14,765 | 15,112 | 28% | 30% | 25% |
| 9 | 21,135 | 24,002 | 21,293 | 24% | 23% | 22% |
| 10 | 60,335 | 59,625 | 60,377 | 17% | 17% | 16% |
| Total | 128,286 | 135,600 | 129,024 | 15% | 15% | 12% |

TABLE 5.7.2.5 (TABLE 7.2)

Reserve Estimates And Prediction Errors: Gamma Model

| | Reserve Estimates | | | Prediction Error | | |
|---|---|---|---|---|---|---|
| Accident Period | Model 4 Stochastic Chain Ladder | Model 5 Hoerl Curve | Model 6 GAM (dof = 5) | Model 4 Stochastic Chain Ladder | Model 5 Hoerl Curve | Model 6 GAM (dof = 5) |
| 1 | 0 | 0 | 0 | — | — | — |
| 2 | 488 | 675 | 450 | 62% | 46% | 43% |
| 3 | 2,086 | 3,296 | 2,205 | 43% | 36% | 33% |
| 4 | 5,240 | 6,818 | 5,300 | 36% | 32% | 29% |
| 5 | 6,169 | 7,061 | 6,313 | 32% | 30% | 28% |
| 6 | 9,750 | 9,305 | 9,427 | 31% | 29% | 28% |
| 7 | 15,080 | 13,029 | 15,097 | 31% | 29% | 29% |
| 8 | 18,498 | 15,069 | 17,671 | 32% | 30% | 31% |
| 9 | 20,470 | 24,400 | 20,896 | 36% | 35% | 35% |
| 10 | 60,043 | 59,576 | 58,519 | 52% | 48% | 48% |
| Total | 137,824 | 139,229 | 135,878 | 25% | 23% | 24% |