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E-Forum, Fall 2011

The CAS *E-Forum,* Fall 2011

The Fall 2011 Edition of the CAS *E-Forum* is a cooperative effort between the Committee for the CAS *E-Forum* and various other CAS committees. This *E-Forum* includes one paper.

CAS *E-Forum*, Fall 2011

Table of Contents

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Bayesian Underwriting of Sinkhole Exposure Revisited

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Abstract: Sinkholes have emerged as a significant cost for homeowners and dwelling insurance in Florida. The conditions contributing to the formation of a sinkhole are generally similar over a local area. Thus the observation of known sinkholes and their proximity to a particular area provides a Bayesian predictor of the probability of a future insured claim. Measuring the relative increase in the number of sinkholes and the closeness of these sinkholes to the potential risk gives an indicator of the increased risk.

1. INTRODUCTION

Sinkholes are a unique type of earth movement. They are generally confined to a smaller area than is affected by an earthquake or volcano, but their presence is an indicator of potential future sinkholes within their proximity. The reason for this predictability is the nature of the local land's makeup, the local aquifer, local climatology, and local land use.

The typical land formation in sinkhole areas consists of a deeper layer of limestone or dolomite covered by a layer of clay, or sedimentary soil. The limestone/dolomite was formed from deposits of the remains of shells and micro-skeletons of minute sea creatures that flowed to the sea bed. The organisms decayed and were covered by other sediment. The overlying layers of sediment subjected the organisms to high pressure, and formed calcified rock. Thus the layer of limestone/dolomite is spread over an adjoining area and will be at about the same depth below the surface. Large earth movements may raise or bend these layers as mountains are formed, and erosion can start thinning the surface layers.

Aquifers are the underground waters that flow along layers of the subterranean. This flow is generally over a wide area. Underground springs will have a more intense flow along a narrower but directional area. The local climate also contributes to the water percolating through the earth.

The dissolution of the limestone and dolomite is accentuated by acidic water. A weak carbonic acid is created as rain or surface water reacts with carbon dioxide in the air or in the soil. These chemical reactions are shown in "Sinkhole Type, Development, and Distribution in Florida" by U.S. Geological Survey (1985) as:

$$CO_2 + H_2O \rightarrow H_2CO_3$$
(carbon dioxide) (water) (carbonic acid)

$$H_2CO_3 + CaCO_3 \rightarrow Ca^{++} + 2(HCO_3)^{-}$$
(carbonic acid) (limestone) (calcium ions) (bicarbonate ions)

$$2H_2CO_3 + (CaMg(CO_3)_2 \rightarrow Ca^{++} + Mg^{++} + 4(HCO_3)^{-}$$
(carbonic acid) (dolomite) (calcium ions) (magnesium ions) (bicarbonate ions)

Sinkholes are the result of water dissolving the underlying limestone or dolomite. When the dissolved material drains off it leaves a depression on the surface or a cavity under the surface. Depressions on the surface can be visually detected. Cavities under the surface are generally not known until a cataclysmic collapse occurs. If the water table is above the cavity, the water may support the soil above it. If the water table is lowered, the surface soil may fall into the cavity creating a sinkhole. The water table can be lowered by reduced water at the source of the aquifer, reduced local rain fall such as during a drought, and increased extraction of water from the aquifer for business use or to support the population living in the area.

Sinkholes occur throughout the world. The largest identified sinkholes receive significant publicity. Carnegie University identifies a sinkhole named, "Zacation", as the deepest sinkhole with its depth unknown, but in excess of 282 meters. In Kansas, there is a sinkhole named "Big Basin", that is a mile across and 100 feet deep. In 2007 British divers identified the interconnection of underground caves in Mexico's Yucatan peninsula. The cave system is 95 miles long and connects to numerous sinkholes and sinkhole lakes.

Sinkholes occur in many states. A U.S. Geological Survey of eastern U.S. states, found sinkholes in 19 states. The survey identified Alabama, Florida, Kentucky, Missouri, Pennsylvania, Tennessee and Texas as the states most likely to have sinkholes, and Florida has more sinkholes than any other state.

The topography conducive to sinkholes is called "karst" topography. The Insurance Study of Sinkholes (April, 2005) by the Florida Geological Survey (FGS) reported the following average costs:

- Cost of a sinkhole in 2003 is $9,944.
- Cost for land damage not covered by homeowners policy $2,632 (1996) increased to $12,070 (2001).
- Average property damage claim $40,218 (1996) increased to $62,628 (2001).

2. ANALYSIS OF THE PROXIMITY OF SINKHOLES

This study does not investigate the costs resulting from sinkholes, but is concerned with the emergence of sinkholes based on the observation of the number and proximity of sinkholes to an insured risk. It is expected and assumed that the topography, aquifer, climatology, and water usage will be identical over a small area. This study is looking at the probabilities of another sinkhole within a limited radius (less than 1 mile).

Florida requires insurers to offer sinkhole coverage. Florida has a "Subsidence Incidence Report Form" for submitting the descriptive details to the Office of the Florida Geological Survey (FGS), Florida Department of Environmental Protection. The FGS website includes a disclaimer regarding the use of the subsidence data. The disclaimer is included in the appendix. The Florida subsidence data, after excluding incidence identified as not sinkholes, through July, 2011 is analyzed in this study (3,199 reports). The 2010 Sinkhole Data Call Report identifies 24,671 closed or open sinkhole claims for the 2006 through 2010 period, but it does not provide the precise location and dimensions of the sinkholes needed for this analysis. The majority of this analysis is based on the FGS information.

Florida consists of 67 counties totaling 53,598 square miles. The FGS experience has 3,199 sinkholes that have at least 1 sinkhole in 49 counties that comprise 41,012 square miles. The 18 counties without a recorded sinkhole represent 23% of the area of Florida. There is some correlation between the number of sinkholes reported and the size of the county (0.17), between the county's population and the number of reported (0.25), and a stronger correlation between the number of sinkholes reported and the county's population density (0.38). This last correlation may reflect both the awareness of sinkholes, and the impact of local water usage, such as extracting water from the aquifer and irrigation of the land.

County	Population	Area	Number Sinkholes	Density Pop/Area	Density SH/Area
Alachua	247,336	874	54	283.0	0.06178
Baker	27,115	585	0	46.4	0.00000
Bay	168,852	764	1	221.0	0.00131
Bradford	28,520	293	0	97.3	0.00000
Brevard	543,376	1,018	0	533.8	0.00000
Broward	1,748,066	1,209	4	1,445.9	0.00331
Calhoun	14,625	567	0	25.8	0.00000
Charlotte	159,978	694	1	230.5	0.00144
Citrus	141,236	584	353	241.8	0.60445
Clay	190,865	601	3	317.6	0.00499
Collier	321,520	2,026	2	158.7	0.00099
Columbia	67,531	797	31	84.7	0.03890
DeSoto	34,862	637	0	54.7	0.00000
Dixie	16,422	704	12	23.3	0.01705
Duval	864,263	774	8	1,116.6	0.01034
Escambia	297,619	664	0	448.2	0.00000
Flagler	95,696	485	0	197.3	0.00000
Franklin	11,549	534	0	21.6	0.00000
Gadsden	46,389	516	2	89.9	0.00388
Gilchrist	16,939	349	46	48.5	0.13181
Glades	12,884	774	0	16.6	0.00000
Gulf	15,863	565	0	28.1	0.00000
Hamilton	14,799	515	13	28.7	0.02524
Hardee	27,731	637	22	43.5	0.03454
Hendry	39,140	1,153	1	33.9	0.00087
Hernando	172,778	478	232	361.5	0.48536
Highlands	98,786	1,028	11	96.1	0.01070
Hillsborough	1,229,226	1,051	511	1,169.6	0.48620
Holmes	19,927	482	3	41.3	0.00622
Indian River	138,028	503	6	274.4	0.01193
Jackson	49,746	916	19	54.3	0.02074
Jefferson	14,761	598	3	24.7	0.00502
Lafayette	8,870	543	6	16.3	0.01105
Lake	297,052	953	115	311.7	0.12067
Lee	618,754	804	3	769.6	0.00373
Leon	275,487	667	115	413.0	0.17241
Levy	40,801	1,118	68	36.5	0.06082
Liberty	8,365	836	1	10.0	0.00120
Madison	19,224	692	5	27.8	0.00723
Manatee	322,833	741	5	435.7	0.00675
Marion	331,298	1,579	337	209.8	0.21343
Martin	146,318	556	1	263.2	0.00180
Miami-Dade	2,496,435	1,945	1	1,283.5	0.00051
Monroe	73,090	997	1	73.3	0.00100

County	Population	Area	Number Sinkholes	Density Pop/Area	Density SH/Area
Nassau	73,314	652	2	112.4	0.00307
Okaloosa	180,822	936	2	193.2	0.00214
Okeechobee	39,996	774	0	51.7	0.00000
Orange	1,145,956	908	194	1,262.1	0.21366
Osceola	268,685	1,322	11	203.2	0.00832
Palm Beach	1,320,134	2,034	5	649.0	0.00246
Pasco	464,697	745	254	623.8	0.34094
Pinellas	916,542	280	72	3,273.4	0.25714
Polk	602,095	1,875	267	321.1	0.14240
Putnam	74,364	722	2	103.0	0.00277
Santa Rosa	151,372	1,016	0	149.0	0.00000
Sarasota	379,448	572	6	663.4	0.01049
Seminole	422,718	308	130	1,372.5	0.42208
St. Johns	190,039	609	4	312.1	0.00657
St. Lucie	277,789	572	0	485.6	0.00000
Sumter	93,420	546	24	171.1	0.04396
Suwannee	41,551	688	63	60.4	0.09157
Taylor	22,570	1,042	20	21.7	0.01919
Union	15,535	240	0	64.7	0.00000
Volusia	494,593	1,106	87	447.2	0.07866
Wakulla	30,776	607	54	50.7	0.08896
Walton	55,043	1,058	3	52.0	0.00284
Washington	24,896	580	3	42.9	0.00517
Florida Total	**18,801,310**	**53,998**	**3,199**	**348.2**	**0.05924**

The 2010 Sinkhole Data Call Report shows 24,671 sinkholes reported with no paid claims in only 10 counties: Baker, Hardee, Hendry, Holmes, Indian River, Glades, Nassau, Okaloosa, Santa Rosa, and Walton.

The depth of a sinkhole affects the cost to rectify the land's structure and repair any structural damage. Even a minor land's shift can result in significant costs. As the depth of a sinkhole increases, the costs escalate rapidly. Two thirds of the recorded sinkholes are less than 10 feet deep, but the average depth is 10.5 feet, and the median depth is 6 feet.

The likelihood of a sinkhole causing damage depends on the surface size of the sinkhole. The FGS data has measurements or estimates on the surface size of 2,492 sinkholes. The shape is described as either circular or elongated. The circular area is calculated using its diameter, and the elongated as a rectangular with its length and width given.

Sinkhole Depth in Feet		
From	To	Percentage
under	9.99	67.25%
10	19.99	18.87%
20	29.99	6.06%
30	39.99	3.29%
40	49.99	1.67%
50	59.99	0.98%
60	69.99	0.64%
70	79.99	0.34%
80	89.99	0.17%
90	99.99	0.26%
100	149.99	0.30%
150	199.99	0.09%
200	249.99	0.04%
250	299.99	0.04%
300	& over	0.00%
	Sample Size	2,342
	Mean Depth	10.5
	Median Depth	6.0
	Std. Deviation	250

Area in Square Feet		
From	To	Percentage
under	1,000	89.00%
1,000	1,999	4.29%
2,000	2,999	1.85%
3,000	3,999	0.88%
4,000	4,999	0.64%
5,000	5,999	0.40%
6,000	6,999	0.16%
7,000	7,999	0.44%
8,000	8,999	0.08%
9,000	9,999	0.20%
10,000	19,999	0.88%
20,000	29,999	0.24%
30,000	39,999	0.44%
40,000	49,999	0.24%
50,000	89,999	0.12%
90,000	& over	0.12%
	Sample Size	2,492
	Mean Area	998
	Median Area	50
	Std. Deviation	5,424

The FGS coordinates for each sinkhole are used to calculate the distance to every other sinkhole. The study is only concerned with neighboring sinkholes within one mile. Initially the study assumed an adjustment for the curvature of the earth was insignificant, and the points were assumed to be on a flat surface. The distance between points was calculated as:

A=latitude radians = degrees latitude/(2*π)

B=longitude radians = degrees longitude/(2*π)

$$\text{Distance}_{0,1} = 3{,}963 * \text{square root } \{ +[\sin(A_0) - \sin(A_1)]^2 + [\sin(B_0) - \sin(B_1)]^2 + [\cos(A_0) - \cos(A_1)]^2 \}$$

While the conversion to Cartesian coordinates is correct for the latitudes, it is not correct for the longitude measurements as it is measured on a smaller circle of radius 3,963 * cos(B). Using the above formula produced calculated distances greater than the actual surface distance, thus understating the concentration of sinkholes.

Using the FGS coordinates for each sinkhole, the correct formulas for calculating the surface distance to every other sinkhole are the following:

A=latitude radians = degrees latitude/(2*π)

B=longitude radians = degrees longitude/(2*π)

$$\Delta\lambda = B_0 - B_1$$

$$C = \frac{\{[\cos(A_0) * \sin(\Delta\lambda)]^2 + [\cos(A_0) * \sin(A_1) - \sin(A_0) * \cos(A_1) * \cos(\Delta\lambda)]^2\}^{0.5}}{\sin(A_0) * \sin(A_1) + \cos(A_0) * \cos(A_1) * \cos(\Delta\lambda)}$$

$$\text{Distance}_{0,1} = 3{,}963 * \arctan(C)$$

The expected number of sinkholes is estimated as the average actual number of sinkholes within the selected radial distance(r) given an observed number of sinkholes (n) or E[n/r].

Set the base as the statewide average number of sinkholes within a one-mile radius when no sinkholes are observed. Using the 2010 Sinkhole Data Call Report of 24,671 reported sinkholes:

E[0/1] = 24,671 / [53,998 / π] = 1.4359 per a circular mile radius.

The FGS experience produced the following results based on the observed number of sinkholes within the selected distance:

Average Number of Sinkholes within Selected Radius When Observe N Sinkholes

N	1/8 mile	2/8 mile	3/8 mile	4/8 mile	5/8 mile	6/8 mile	7/8 mile	1 mile
Base 0								1.4359
1	3.3313	4.1613	4.8358	5.3282	5.5497	5.8967	6.2893	6.6267
2	4.9361	5.6024	6.2236	6.6562	6.8091	7.0849	7.4310	7.6827
3	6.2972	6.8657	7.3131	7.6777	7.5954	7.8808	8.2601	8.4972
4	7.5938	7.8547	8.3046	8.7662	8.5154	8.6989	9.1088	9.3987
5	8.7586	8.7273	9.3424	9.6767	9.3411	9.4648	9.7752	10.0742
6	10.0463	9.8180	10.3217	10.5827	10.1359	10.2924	10.5116	10.6897
7	11.5316	10.5900	11.3047	11.4911	10.8966	10.9075	11.1409	11.3369
8	12.3433	11.3784	11.7876	12.2287	11.7843	11.6011	11.7504	12.1035
9	12.9322	12.1494	12.5085	12.9012	12.4584	12.2510	12.3168	12.6432
>=10	14.0989	12.8923	13.2007	13.6928	13.2203	12.9175	12.7288	13.1014

The probability of being "hit" by a sinkhole is estimated as the number of sinkholes(n) expected within the selected radial distance(r), times to size of an average sinkhole (1,000 square feet) divided by the circular area of the radial distance selected ($\pi * r^2$); or

$P[d/r] = E[n/r] * 1000 / (\pi * r^2)$.

Probability of Being "Hit" by a Sinkhole When Observe N Sinkholes within Selected Radius

N	1/8 mile	2/8 mile	3/8 mile	4/8 mile	5/8 mile	6/8 mile	7/8 mile	1 mile
Base 0								0.000016
1	0.002434	0.000760	0.000393	0.000243	0.000162	0.000120	0.000094	0.000076
2	0.003607	0.001023	0.000505	0.000304	0.000199	0.000144	0.000111	0.000088
3	0.004602	0.001254	0.000594	0.000351	0.000222	0.000160	0.000123	0.000097
4	0.005549	0.001435	0.000674	0.000400	0.000249	0.000177	0.000136	0.000107
5	0.006400	0.001594	0.000759	0.000442	0.000273	0.000192	0.000146	0.000115
6	0.007341	0.001794	0.000838	0.000483	0.000296	0.000209	0.000157	0.000122
7	0.008427	0.001935	0.000918	0.000525	0.000319	0.000221	0.000166	0.000129
8	0.009020	0.002079	0.000957	0.000558	0.000344	0.000235	0.000175	0.000138
9	0.009450	0.002220	0.001016	0.000589	0.000364	0.000249	0.000184	0.000144
>=10	0.010303	0.002355	0.001072	0.000625	0.000386	0.000262	0.000190	0.000150

The value of knowing the number of sinkholes within a selected distance is the relative change in probability of being "hit" by a sinkhole.

Relative Probability of Being "Hit" by a Sinkhole Compared to Statewide Average on Not Observing

N	1/8 mile	2/8 mile	3/8 mile	4/8 mile	5/8 mile	6/8 mile	7/8 mile	1 mile
Base 0								1.000
1	148.481	46.368	23.949	14.843	9.894	7.301	5.721	4.615
2	220.011	62.427	30.822	18.542	12.140	8.772	6.759	5.350
3	280.675	76.504	36.217	21.388	13.541	9.757	7.514	5.918
4	338.464	87.523	41.127	24.420	15.182	10.770	8.286	6.545
5	390.384	97.247	46.267	26.956	16.654	11.718	8.892	7.016
6	447.777	109.400	51.117	29.480	18.071	12.743	9.562	7.445
7	513.981	118.003	55.985	32.011	19.427	13.504	10.134	7.895
8	550.157	126.787	58.377	34.066	21.010	14.363	10.688	8.429
9	576.406	135.379	61.947	35.939	22.212	15.168	11.204	8.805
>=10	628.407	143.657	65.375	38.144	23.570	15.993	11.578	9.124

Generally, the observation of an additional sinkhole increases the probability of damage from a sinkhole, but at a decreasing rate. The relative additional information obtained by the Bayesian observation of additional sinkholes within a particular area is:

Relative Value of Observing an Addition Sinkhole within a Selected Radius

N	1/8 mile	2/8 mile	3/8 mile	4/8 mile	5/8 mile	6/8 mile	7/8 mile	1 mile
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.482	1.346	1.287	1.249	1.227	1.201	1.182	1.159
3	1.276	1.226	1.175	1.153	1.115	1.112	1.112	1.106
4	1.206	1.144	1.136	1.142	1.121	1.104	1.103	1.106
5	1.153	1.111	1.125	1.104	1.097	1.088	1.073	1.072
6	1.147	1.125	1.105	1.094	1.085	1.087	1.075	1.061
7	1.148	1.079	1.095	1.086	1.075	1.060	1.060	1.061
8	1.070	1.074	1.043	1.064	1.081	1.064	1.055	1.068
9	1.048	1.068	1.061	1.055	1.057	1.056	1.048	1.045
>=10	1.090	1.061	1.055	1.061	1.061	1.054	1.033	1.036

There is a monotonic increase in the probability of damage from a sinkhole as the same number of sinkholes is observed in a smaller radial distance. The relative increase in probability as the radial distance is decreased is:

Relative Increase in Probability of "Hit" for Radial Distance Compared to Base of 1-Mile

N	1/8 mile	2/8 mile	3/8 mile	4/8 mile	5/8 mile	6/8 mile	7/8 mile	1 mile
1	3.202	1.936	1.613	1.500	1.355	1.276	1.240	1.000
2	3.524	2.025	1.662	1.527	1.384	1.298	1.263	1.000
3	3.669	2.112	1.693	1.579	1.388	1.299	1.270	1.000
4	3.867	2.128	1.684	1.609	1.410	1.300	1.266	1.000
5	4.014	2.102	1.716	1.619	1.421	1.318	1.267	1.000
6	4.093	2.140	1.734	1.631	1.418	1.333	1.284	1.000
7	4.356	2.108	1.749	1.648	1.439	1.333	1.284	1.000
8	4.339	2.172	1.714	1.621	1.463	1.344	1.268	1.000
9	4.258	2.185	1.724	1.618	1.464	1.354	1.272	1.000
>=10	4.374	2.197	1.714	1.618	1.474	1.381	1.269	1.000

3. CONCLUSION

The FGS experience demonstrates the value in using a Bayesian analysis of sinkhole experience. This experience does not include the costs to repair or indemnify insured losses, but if the average insured cost is known, the Bayesian evaluation will provide information for underwriters to determine the risk associated with a particular exposure. The risk increases as the number of identified sinkholes increases and is more pronounced as the proximity of these sinkholes becomes closer. Access to the detailed information on all of the sinkholes reported under the "Subsidence Incidence Report Form" will improve the Bayesian analysis and relative importance of additional information.

4. APPENDIX

FGS Disclaimer

Geographic Information Systems (GIS) data and maps produced by the Florida Geological Survey (FGS), an office of the Florida Department of Environmental Protection (FDEP), are provided solely as a general reference for state geologic features, are not warranted for any other use or purpose, and are not intended to replace site-specific or use-specific investigations Use of FGS data and maps by an end-user for any purpose other than general reference shall free the FDEP and the FGS from any and all liability for outcomes resulting from unintended or inappropriate applications of FGS spatial data. Examples of such applications include but are not limited to the mapping of point data with respect to cadastral boundaries, the use of geologic data for engineering purposes, and the interpolation of results of broad-scale GIS modeled surfaces to local areas that may or may not be deemed geologically analogous.

As with all spatial data, the attributes and exact coordinates of features depicted in FGS maps, coverages and datasets are subject to inherent errors due to limitations in the resources and technology available to record data. Please keep in mind that older maps, projects, and data were developed using the best tools and knowledge available at the time, and that spatial data standards and the tools used to develop these data have improved significantly over the last several years. Existing maps and projects are sometimes updated and enhanced with new data of higher precision, and thus are subject to change. The FGS makes every effort to provide the most up-to-date and accurate geographic information possible, but cannot be held liable for the use of outdated data regardless of whether more current data have been obtained, analyzed, or made available by the FGS or FDEP.

The use of information from the FGS site by a third party does not indicate that the FGS recommends or endorses the third party user or their services.

5. REFERENCES

[1.] "Karst and the Hydrogeology of the Florida Aquifer, FL, USA" by Laura Torres Coral and Ann Whealan, January 2005.

[2.] "Final Report: Insurance Study of Sinkholes", submitted to State of Florida, April 2005.