Casualty Actuarial Society E-Forum, Winter 2010

The Winter 2010 CAS *E-Forum*

The Winter 2010 Edition of the CAS *E-Forum* is a cooperative effort between the Committee for the CAS *E-Forum* and various other CAS committees.

The CAS Committee on Dynamic Risk Modeling presents for discussion one paper prepared in response to their 2010 Call for Papers on the topic of "Solving Problems Using a Dynamic Risk Modeling Process." This *E-Forum* also includes one additional paper.

This call paper that will be discussed by the authors at the 2010 Enterprise Risk Management Symposium scheduled for April 12-14, 2010, in Chicago, IL.

2010 Winter *E-Forum*

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E-Forum Committee

For information on submitting a paper to the *E-Forum*, visit http://www.casact.org/pubs/forum/.

Holistic Approach to Setting Risk Limits
ERM for the Masses

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Abstract

Enterprise risk management and its holistic approach appear to have attained permanency as a best-in-class approach to risk management. Yet, we continue to see insurers utilizing risk limits that have been set in isolation and remain untested from an enterprise-wide perspective. Explanations range from "our conservative approach to setting individual risk limits renders the holistic approach unnecessary" to "we simply don't have the resources to tackle this problem."

This paper uses a hypothetical, medium-sized, multi-line, mutual insurer and the Public Access DFA Dynamo 4 Model (Dynamo 4) to holistically evaluate a company's current risk limits. Historically, the company's risk limits were set in isolation with an eye towards capital preservation. The risk limits reviewed include those pertaining to growth rates, retentions within the company's reinsurance program, and investment policy statement limits. We also test some of the underlying risk assumptions used by Dynamo 4.

We utilized the Dynamo 4 model to test and suggest improvements to the current risk limits from an enterprise-wide capital preservation perspective. We concluded that certain risk limits that were set in isolation and originally appeared to mitigate risk were actually unnecessarily increasing the risks of violating the company's long-term solvency goals.

Keywords. Risk Limits, Setting Risk Limits, ERM, Dynamo 4

INTRODUCTION

Insurance companies are in the business of assuming risk. Central to successfully executing on this business model is an understanding of how much risk is desirable (both in terms of overall risk exposure and individual sources of risk) and how much risk is being assumed. With the current focus on enterprise risk management (ERM), many companies have expended significant time and resources in defining their risk policy in terms of risk appetite and risk tolerance, and the risk acceptance requirements that are intended to keep individual business units and the company overall in compliance with the risk policy.

> ***Risk Policy:*** *a governance document that describes how the organization views risk, the role risk assumption or risk avoidance plays in the management and oversight of company operations, and the processes the company has established to monitor and, when necessary, to intervene to keep the organization's operations aligned with the level of risk the company has established as being acceptable.*
>
> ***Risk Appetite:*** *one or more statements that describe the levels of risk that company management deems to be acceptable in the pursuit of overall financial and solvency goals.*
>
> ***Risk Tolerance:*** *one or more statements that establish boundaries on how much variation away from expected financial return the entity is willing to accept.*
>
> ***Risk Acceptance Requirements:*** *detailed tactical statements that provide guidance to the organization's staff about the procedures they are expected to follow on a day-to-day basis to support the organization's risk appetite and risk policy.*

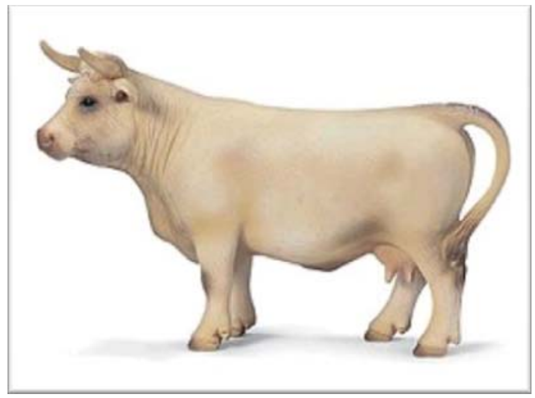
Ideally, a company's risk policy will take a holistic view of the entity's operations. The risk acceptance requirements being established for one area of the company appropriately interact with those being established for another area so as to realize the company's short- and long-term financial and risk objectives. Working on the assumption that the risk tolerances are appropriately aligned with the risk appetite, management can monitor the risk tolerances and gain comfort that its risk appetite is not being exceeded and the company is operating in a way that is in accordance with the risk policy.

Our paper describes a company's use of quantitative analysis tools to (a) evaluate its existing risk acceptance requirements vis-à-vis its risk policy and risk appetite, and (b) identify modifications to the risk acceptance requirements that would better satisfy the risk policy, through either reductions in downside risk while maintaining the overall rate of return or increases in the company's overall rate of return at similar levels of risk.

INTRODUCTION TO COMPANY

Introduction

The insurance company (DynaMOO Insurance Group) that we are using to illustrate the aims of this paper is fictitious. Any resemblance to an actual insurance company is purely coincidental. While the situations and options discussed in this paper can be applied to actual insurance companies, the individual results and conclusions will be different for each insurer. As such, no results presented in this paper should be blindly relied on for managing a real insurance company.



History

DynaMOO Insurance Group (DIG) was originally founded as a cooperative aimed at protecting local farmers from weather related losses in 1935. The original structure was a mutual where the policyholders owned the business. During the early 1950s, DIG expanded its underwriting to include workers compensation and homeowners lines of business, and effectively ceased to write weather-related lines of business in the late 1960s. However, it has retained its mutual ownership structure.

Business Model

DIG's business model is aimed at providing quality insurance protection to its policyholders. DIG prides itself on supplying superior claims handling services and consistently ranks extremely high in satisfaction surveys that target claims satisfaction.

As such, DIG's business model is built around its claims management services. It strongly prefers lines of business and groups of policies that are relatively high frequency so that it can "play to its strength."

Lines of Business

DIG currently operates two (2) lines of business:

- Workers Compensation – DIG writes approximately $8.5M of workers compensation business in selected states.
- Homeowners – DIG writes approximately $2.5M of homeowners business in selected states.

Insurance Constraints

DIG's initial risk acceptance policies were established several years ago through a combination of judgmental and quantitative estimations of the amount of risk inherent in different aspects of the company's operations.

The company history and its business model result in the following constraints on the insurance business:

- The geographical spread is limited to select states so that its claims handling services are not spread "too thin."
- The growth in business is restricted so that DIG can maintain the highest quality of claims management services to its policyholders (it takes approximately six months for a new claims officer to complete internal DIG claims training and move to handling claims).
- The claims handling teams are very closely tied to the two lines of business given the different nature of workers compensation and homeowners claims, thus resulting in little or no cross functional support.
- Annual policy growth is restricted to 3.5% per year. This is a rule of thumb that was developed as a risk acceptance requirement to keep the company in compliance with its risk tolerance.

Investment Constraints

The company history and its business model result in the following investment-related constraints:

- The current investment portfolio is structured for short-term capital preservation and liquidity. Investments include $10 million in cash, $15 million in bonds maturing in one to five years, and $15 million in bonds maturing in less than one year.
- Historically, investment in equities has not been allowed. This position was established as a risk-acceptance requirement to keep the company in compliance with its risk tolerance.

Company/Solvency Goals

Historically, DIG did not have an enterprise-wide statement of risk tolerance. DIG management has observed the evolution of more sophisticated analytic tools that can provide them with greater insight into the overall effectiveness of their risk assessment policies.

Risk-based capital is the vehicle used by insurance regulators to monitor company solvency. The aim of this paper is to highlight how the risk-based capital concept can be used as a holistic risk measurement and not to get overly involved in the intricacy of the calculations. As such, we (the authors) have used a simplified approach to define the required solvency level (RSL). We have defined RSL as 30% of loss reserves plus unearned premium reserves. If surplus falls below the RSL, DIG's regulators will force the company to take action.

Management does not want to approach a surplus level where they are in danger of having regulatory action taken. As such, they are managing DIG to a solvency margin in excess of the RSL. DIG management has set such a desired minimum at 175% of the RSL. We define this level as the management solvency margin level (MSML). DIG tracks its actual solvency margin, which is defined as surplus divided by RSL, to ensure they are not approaching the MSML.

DIG management has historically placed the protection of its policyholders through unquestioned solvency among its primary goals. Intuitively, this was believed to be achieved through building long-term value in conjunction with conservative risk management practices. DIG has decided to introduce a long-term statement of risk tolerance. DIG has determined that modeled surplus should exceed the RSL 99.9% of the time measured over five years. In addition, modeled surplus should exceed the MSML 90% of the time measured over five years.

CURRENT STATE RESULTS

Introduction to How We Are Looking at Dynamo Results

In the past, DIG management focused on one year underwriting results and its goal was to achieve short-term surplus preservation. The company did not possess the necessary tools for a rigorous enterprise-wide view or a rigorous multi-year view. DIG has decided to utilize a simple but realistic financial model called "Dynamo" to model the company's results over a five-year time horizon. The Dynamo model has enabled the company to create a more sophisticated risk management view that supports a more holistic approach to enterprise risk management. In keeping with DIG's past goal of surplus preservation, the primary focus is on modeled surplus over each of the next five years and comparing this modeled surplus to the MSML and RSL. Surprisingly, when modeling the current set of risk limits, we found that they led to an unsatisfactory number of MSML violations in years 2012 and 2013, the fourth and fifth modeled years. See Table 1 and Chart 1 for these results.

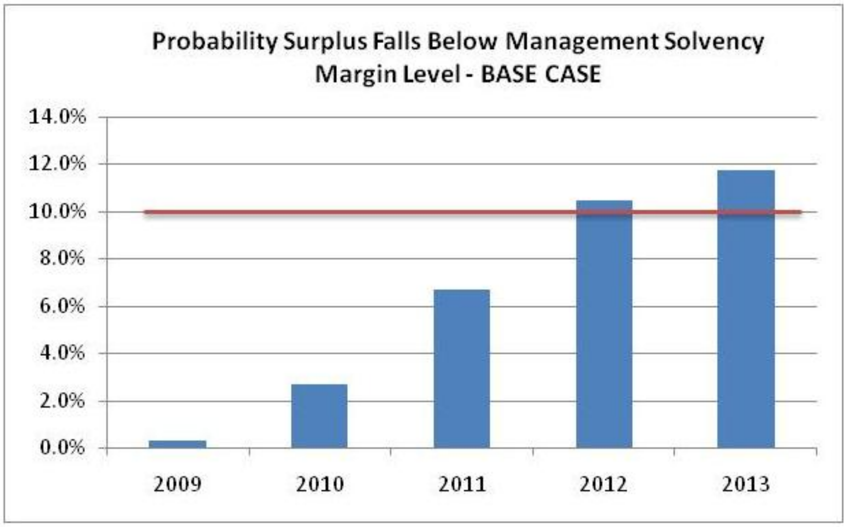
What Dynamo Tells Us About the Current State of the Company

We defined our base case as including DIG's current investment portfolio and current excess of loss (XOL) protection. In order to stress test DIG's risk limits, we assumed the currently allowed maximum policy growth rate of 3.5%. The base case results are shown in Table 1 and Chart 1.

Table 1: Projected Financial Metrics Under Current Business Plan (000 omitted)

Base case	2009	2010	2011	2012	2013
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus falls below management solvency margin level	0.3%	2.7%	6.7%	10.4%	11.7%
Expected value of surplus	18,526	19,738	21,210	22,913	25,047
Expected value of solvency margin	233%	224%	220%	217%	217%

Chart 1



DIG Management Response to Initial Findings

While DIG management was pleased by the average growth of surplus over time, they were surprised by the amount of risk embedded in their current selection of risk limits when viewed from an enterprise-wide perspective. After seeing these results, DIG management has decided they would like to keep the probability of surplus falling below the MSML at or below 10% over both a one-year and a five-year time horizon. Further, management was concerned with the downward trend in the expected value of the solvency margin, even though the probability of surplus falling below the RSL stayed below DIG's 0.1% tolerance over the five-year period.

After seeing these results, DIG management has chosen to undertake a review of the effects of changing various risk limits. The desired outcome is to identify a holistic risk management framework that will continue the long-term growth of surplus but with a reduction in risk over the five-year period.

This risk limit review, and the associated risk/reward tradeoffs, is first explored on a stand-alone risk basis. Later in the evaluation process, combinations of changes in the risk limits from the base case are examined.

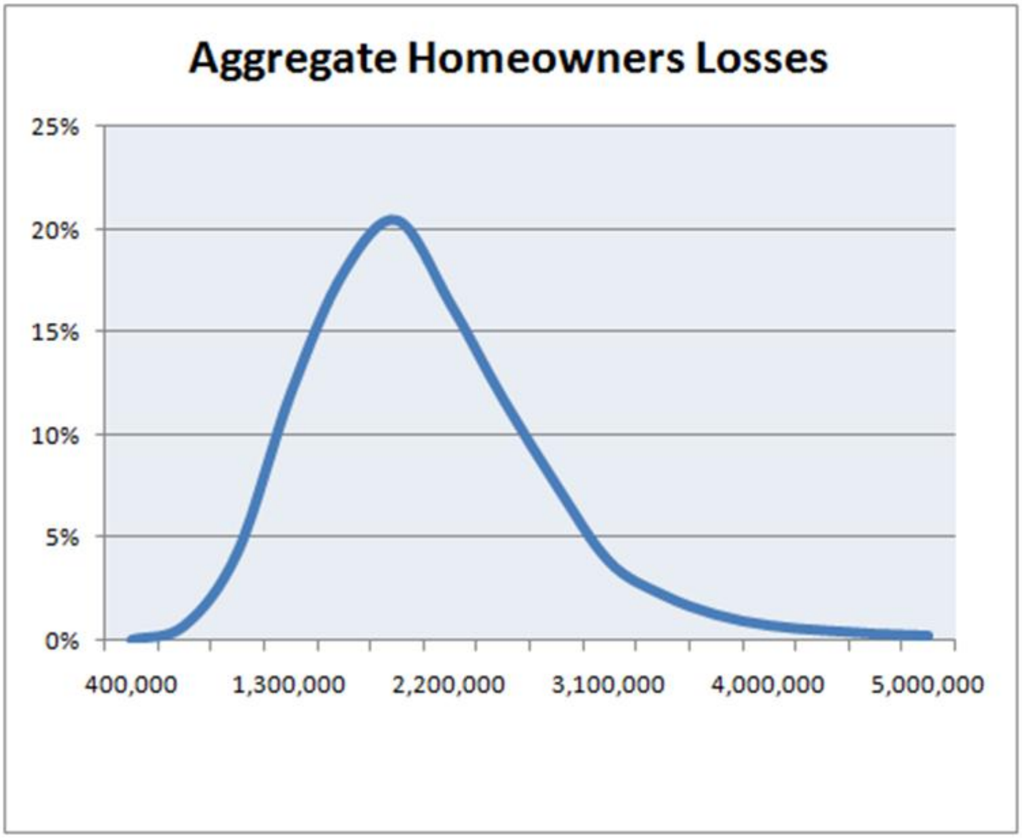
In addition to measuring risk by the frequency of scenarios violating MSML and RSL levels, DIG management has added a "cost of failure" measurement to supplement the two "probability of failure" measurements. Specifically, DIG management has defined the company's cost of failure measurement to be the expected value of deficit relative to MSML. Quantitatively, this is defined as the expected value of the surplus deficit relative to MSML taken over all scenarios where modeled surplus is below MSML. DIG management sometimes requires this further measure of risk, in order to better understand the possible severity within the tail of adverse scenarios.

IMPLICATIONS OF BUSINESS UNIT/COMPANY CONSTRAINTS

Business Unit Reinsurance

The homeowners business unit (BU) was concerned about the number and size of large claims that their operation generates. They retained an external consultant to perform some simulation work in an attempt to analyze the range of large claims that they are likely to experience and to review their current XOL reinsurance. The consultant provided the graph shown in Chart 2 to illustrate the likely range of aggregate losses that the homeowners BU might see in 2013, assuming the company adopted growth rate.

Chart 2: Distribution of Aggregate Gross Homeowners Losses



With an average expected gross premium income of approximately $3.5M in 2013 and gross claims projected to exceed this value in approximately 3.8% of the time, the homeowners BU has decided to purchase XOL reinsurance.

The details of the reinsurance purchased (in Dynamo terminology) are as follows:

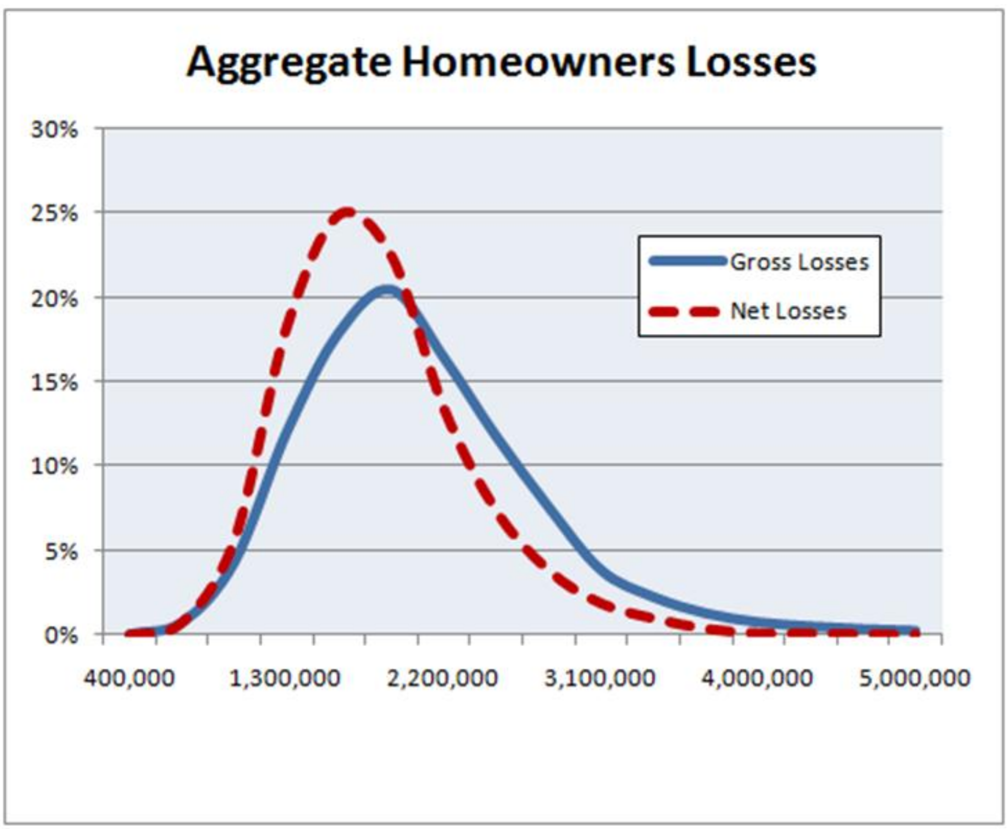
Retention: $100,000
Limit: $5,000,000
Cost of XOL: 17%
Ceding Commission: 25%

The consultant described DIG's large loss profile in the following statistical terms:

> ***Large losses occur approximately 1.5 times per year and are modeled in Dynamo via a Poisson distribution with $\lambda = 1.5$***
>
> ***Large claim severities are approximately lognormally distributed with an average loss amount of \$300,000. Large loss severities are modeled in Dynamo via a lognormal distribution with a μ value of 12.5 and a σ value of 0.500.***

The same consultant produced the histogram shown in Chart 3 illustrating the likely impact of this XOL treaty in 2013.

Chart 3: Distribution of Aggregate Homeowners Losses Gross and Net of Excess of Loss Coverage



The homeowners BU was certainly encouraged by the estimated results of purchasing this reinsurance contract and felt that the premium (and the associated profits) ceded to the reinsurance company would be more than justified by the significant reduction in the net claims.

From an "expected value" point of view, the following summarizes the impact of the retaining the annual average XOL treaty, as measured over the years 2010-2013.

Expected Gross Premium: $3.17M
Expected Gross Claims: $1.92M
Expected Ceded Premium: $0.54M
Expected Ceded Claims: $0.31M
Expected Ceding Commission: $0.14M (25% of $0.54M)
Expected Reduction in Underwriting Gain from retaining the XOL: $0.10M

The DIG Risk Management Committee (a Group level function) is a little concerned about the expected annual $0.10M of profits that DIG is ceding to the reinsurance company. They are currently in discussions with the homeowners BU about replacing this external reinsurance contract with an internal treaty to provide the protection that the homeowners BU is looking for while retaining the expected profits within the DIG. The general thinking behind replacing external reinsurance with internal reinsurance is that while the small BU requires protection from the volatility of its claims (and pays for that protection), the Group does not require protection from the volatility of the BU's claims (because it is larger and can weather any claim fluctuation) and, hence, the expected foregone profits are a true cost to the Group for no material benefit.

Policyholder Growth Constraint

As stated above, DIG is a mutual and, as such, its avenues for raising capital (if required) are more constrained than those of a stock or privately owned company. Effectively, it has limited sources of capital:

- Capital it generates itself from the existing and new business
- Capital it raises from its policyholders
- Surplus notes sold to third parties
- Demutualization

The first source of capital has limited volume capacity and can be difficult to organize due to market price competition. The second source of capital can generate larger, quicker quantities but involves considerable more policyholder interaction and is rarely encountered in practice. The third source gives a company access to outside investors but the notes themselves may require higher than market rates to attract investors as insurance departments typically must approve the payment of any principal or interest amounts to the note holders. The final source is a method that is seen in practice but its execution contains significant political risk and involves significant investment of time and effort into policyholder communication.

As such, DIG has significant capital constraints that potentially impede its ability to grow. Effectively, its capital requirements (driven by solvency pressures) can easily exceed its capital generation ability.

DIG has decided to view this as a strategic advantage instead of a disadvantage and has deliberately constrained its growth rate. Further, it has built its claims handling approach and thus its policy acquisition business model around this constraint. As stated above, DIG prides itself on its superior claims handling practices. It has targeted business that generates claim volume so that it can "play to its strength." Given the training the DIG claim administrators receive, it takes a significant amount of time for a newly hired claims manager to progress through training and reach the front line of actively managing claims. In this situation, DIG cannot "ramp up" its front-line claims managers and cannot afford a significant increase in the volume of claims.

The current business plan has the company growing at 3.5% per annum (policy count)[1] over the next five years. Table 2 shows the results from the DIG's stochastic model based on this growth rate.

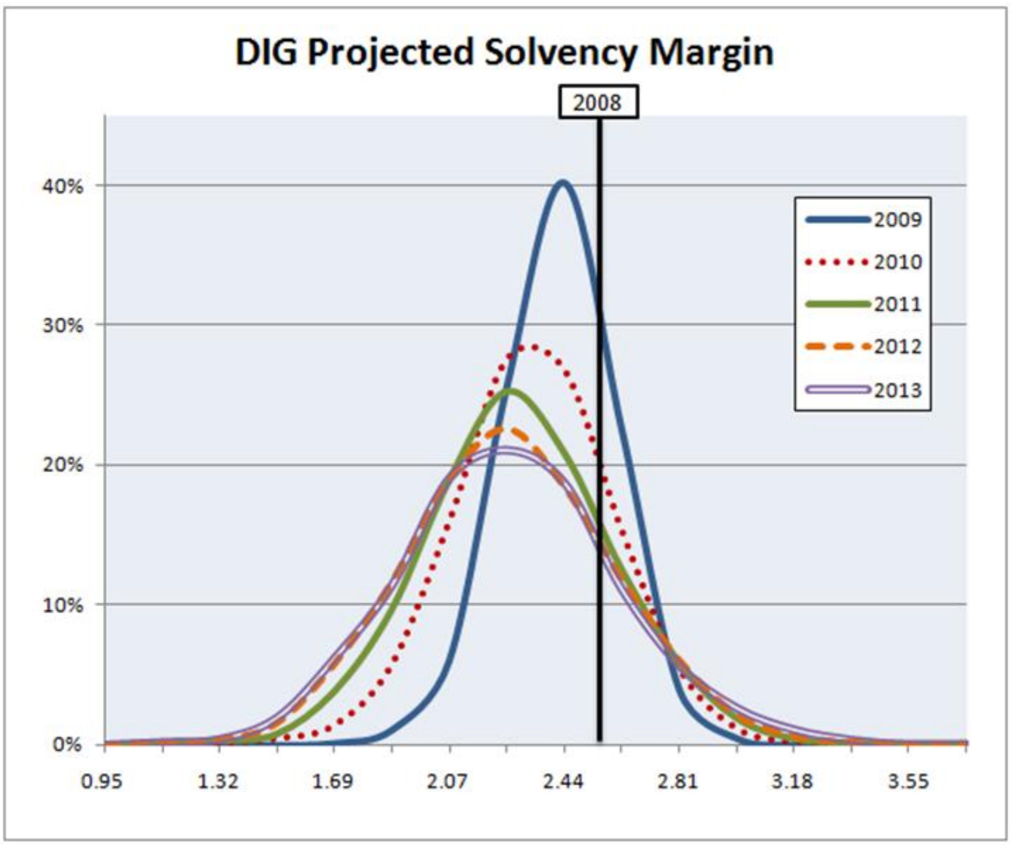
Table 2: Projected Financial Metrics Under Current Business Plan (000 omitted)

Base case	2009	2010	2011	2012	2013
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus falls below management solvency margin level	0.3%	2.7%	6.7%	10.4%	11.7%
Expected value of surplus	18,526	19,738	21,210	22,913	25,047
Expected value of solvency margin	233%	224%	220%	217%	217%

[1] Note that this is not a premium growth rate. The premium growth rate includes the policy growth rate and a premium per policy growth rate. The latter growth rate is closely linked to the inflationary pressure on claims.

We have also examined the solvency margin range that the more holistic model is forecasting. The starting solvency margin is a little under 2.5 and the results of the model using the base assumptions shows that, based on DIG's current plans, this solvency margin is expected to decline to 2.17 by the end of 2013. The forecast solvency margin range is illustrated below in Chart 4.

Chart 4: Range of Projected Solvency Margins by Year Under Current Business Plan



IMPACT OF RELAXING CONSTRAINTS

New Reinsurance—Description/Impact

As mentioned above, the risk management people from Group are interested in bringing the homeowners BU's XOL treating within the group. In effect, Group would provide the protection that the homeowners BU is looking for while retaining the expected profits within the group.

Prior to rushing into setting up such an arrangement, they have decided to review the consultant's work within the context of DIG's enterprise-wide risk tolerance statement. Holistic risk management requires consistent measures of risk across the organization.

Table 3: Impact of XOL Reinsurance on Key Financial Risk Metrics (000 omitted)

Base case	**2009**	**2010**	**2011**	**2012**	**2013**
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus falls below management solvency margin level	0.3%	2.7%	6.7%	10.4%	11.7%
Expected value of surplus	18,526	19,738	21,210	22,913	25,047
Expected value of solvency margin	233%	224%	220%	217%	217%
Internal Reinsurance	**2009**	**2010**	**2011**	**2012**	**2013**
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus falls below management solvency margin level	0.6%	4.3%	9.6%	13.0%	13.9%
Expected value of surplus	18,336	19,584	21,109	22,888	25,121
Expected value of solvency margin	227%	219%	214%	212%	213%

As expected, the expected surplus position at the end of 2013 under the "internal XOL" approach is greater than the "external XOL." This supports the Group's expectations of retaining the expected ceded profits within the DIG. However, there was surprise that the probability of surplus falling below the solvency margin increased by 19% (from 11.7% to 13.9% probability) and, consequently, it was decided that such an increase did not support the "internal XOL" proposition.

Additional Growth—Description/Impact

DIG made a comparison of the current allowed annual policy growth rate of 3.5% with a zero policy growth scenario and a 5% policy growth scenario. We note that these different growth scenarios assume that new business can be written at an acceptable loss ratio. More specifically, we did not assume that greater growth rates implied a higher expected loss ratio or greater volatility. Actual growth rates will not reach the growth rate limitation unless suitably profitable business is available to be written. Our modeled scenarios assume that new business can always be written with equal expected loss ratios for each growth rate.

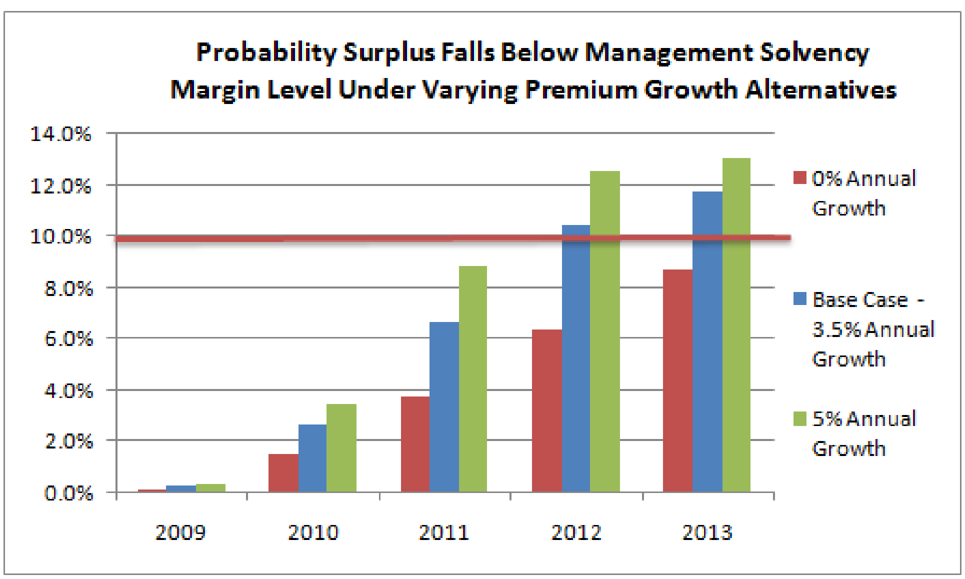
The results are shown in Table 4 as follows.

Table 4: Projected Financial Metrics Under 3.5% Annual Growth versus 0% and 5% Annual Growth Alternatives (000 omitted)

Base case	2009	2010	2011	2012	2013
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus falls below management solvency margin level	0.3%	2.7%	6.7%	10.4%	11.7%
Expected value of surplus	18,526	19,738	21,210	22,913	25,047
Expected value of solvency margin	233%	224%	220%	217%	217%
No growth	**2009**	**2010**	**2011**	**2012**	**2013**
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus falls below management solvency margin level	0.2%	1.5%	3.8%	6.4%	8.7%
Expected value of surplus	18,590	19,664	20,716	21,665	22,662
Expected value of solvency margin	238%	234%	231%	228%	226%
5% Growth	**2009**	**2010**	**2011**	**2012**	**2013**
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus falls below management solvency margin level	0.4%	3.5%	8.9%	12.5%	13.0%
Expected value of surplus	18,500	19,775	21,448	23,532	26,276
Expected value of solvency margin	231%	220%	215%	212%	214%

Although a no growth scenario brings the five-year probability of violating the MSML below 10%, the reduction to long-term surplus growth is substantial. In particular, the currently allowed 3.5% policy growth rate would grow expected surplus by 41% over a five-year period relative to year-end 2008's surplus of $17,739. Implementing a zero policy growth scenario would reduce the growth of expected surplus to 28%.

Chart 5: Surplus Implications of Alternative Premium Growth Scenarios



The five-year risk of allowing a 5% policy growth rate was deemed too high as DIG's risk tolerance is to keep the probability of violating MSML at below 10% over the next five years. In addition, under the 5% policy growth assumption, the downward trend in expected solvency margin is exacerbated, due to the total of reserves and unearned premium growing faster than surplus despite the fact that the additional business is anticipated to be profitable. It is interesting to note, however, that the risk of violating MSML under a 5% policy growth rate begins to level off from 2012 to 2013, and the expected value of solvency margin actually increases from 212% to 214%. Because we assume DIG's underwriting is profitable on average, the effect of writing more business will build expected surplus and eventually counter the additional risks of rapid growth.

We can see a similar outcome for the 3.5% base case policy growth scenario, i.e., the increased risk of violating MSML starts to mitigate between 2012 and 2013 and the expected value of solvency margin is no longer decreasing.

Equity Investments—Description/Impact

DIG has never allowed equities within their investment policy statement. The addition of equities has never been thought of as being consistent with the firm's conservative approach. DIG's current asset manager has been told that the investments belong to their policyholders, and investment in risky assets is therefore inappropriate.

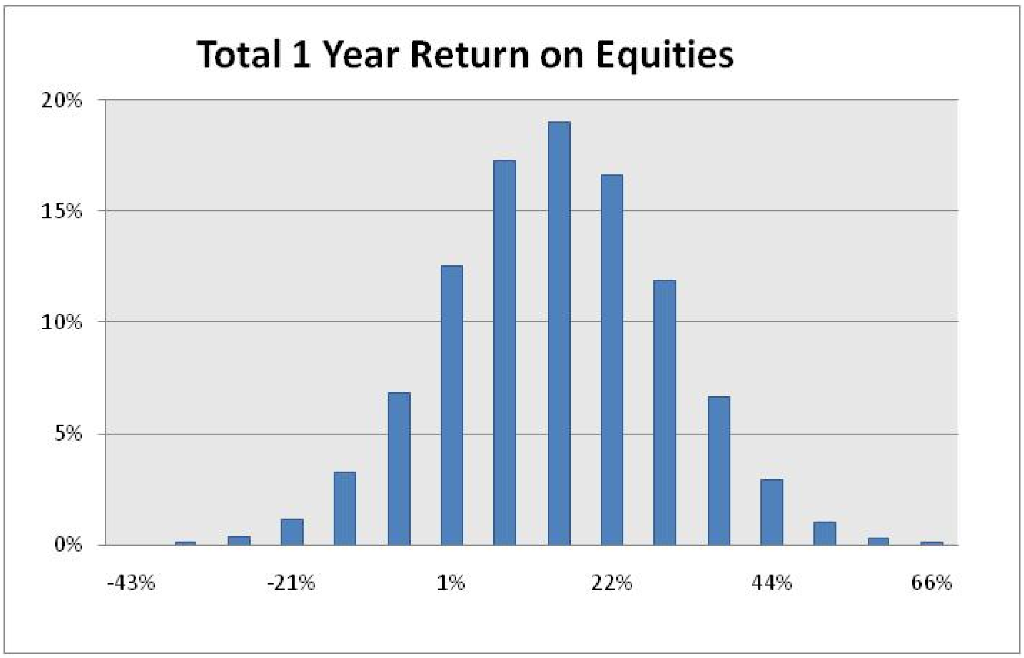
In the spirit of re-evaluating all current risk limits, DIG modeled the case of a 10% investment in equities. The positive effects of diversification in the portfolio produced results that surprised DIG management. These results are shown in Table 5. A modest asset allocation to equities both increased expected surplus and reduced the risk of violating MSML over the five-year horizon.

Table 5: Projected Financial Metrics Under 0% versus 10% Equity Investment Alternatives (000 omitted)

Base case	**2009**	**2010**	**2011**	**2012**	**2013**
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus falls below management solvency margin level	0.3%	2.7%	6.7%	10.4%	11.7%
Expected value of surplus	18,526	19,738	21,210	22,913	25,047
Expected value of solvency margin	233%	224%	220%	217%	217%
10% Equities	**2009**	**2010**	**2011**	**2012**	**2013**
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus falls below management solvency margin level	0.2%	2.0%	3.4%	4.4%	4.7%
Expected value of surplus	18,948	20,641	22,652	24,950	27,790
Expected value of solvency margin	238%	235%	234%	236%	241%

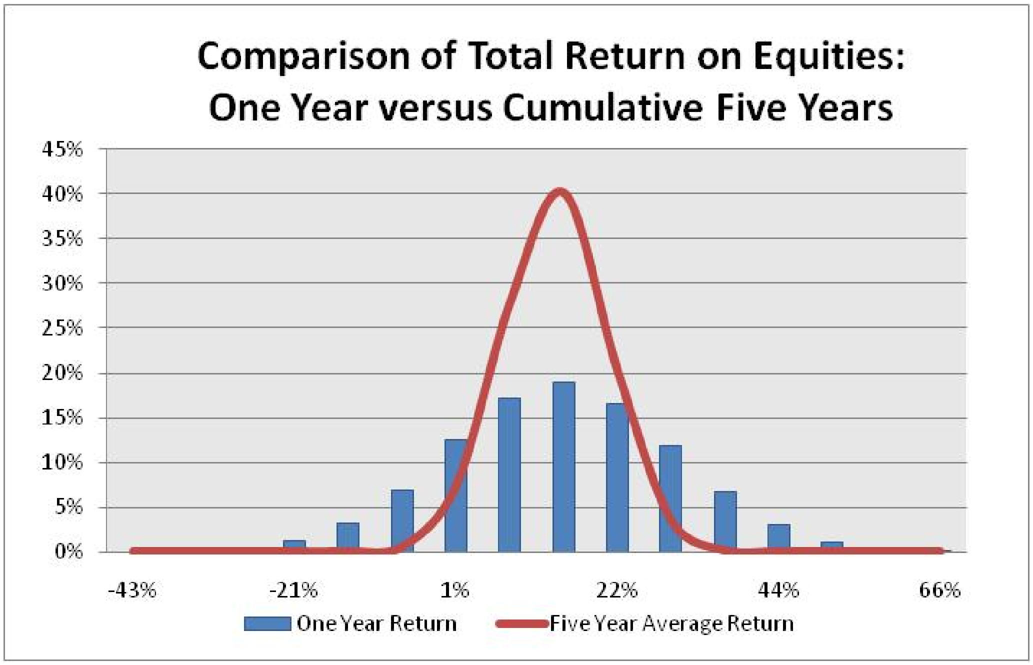
DIG management is appropriately skeptical of any unexpected results that are generated by the Dynamo model. In particular, the expected risk premium and assumed volatility for equities within the Dynamo model were called into question. The initial equities scenario assumed an expected market return for stocks at 8.5% above the risk-free rate. The histogram in Chart 6 presents the modeled distribution of returns over a one-year time horizon. We are presenting total returns that combine investment income and capital gains.

Chart 6: Modeled Distribution of Total Returns on Equities Over a One-Year Time Horizon



We note the considerable variability in returns in both the positive and negative tails and that most of the returns are between -20% and +40%. It is instructive to compare the one-year returns with the variability of average cumulative returns over a five-year horizon. For Chart 7, we computed the geometric averages to obtain an annualized return over the five-year period. The five-year annualized return lies predominantly between -5% and +25%. The reduction in variability over the five-year horizon is dramatic. From DIG's perspective, equities appear less risky over a multi-year horizon than over a one-year horizon.

Chart 7: One Year Versus Five Year Annualized Total Return on Equities



Given cash returns of 1.6% per annum to 2.9% per annum and relatively low expected bond yields over the next five years (see Chart 9 with future expected bond yields in subsequent section), it is not surprising that the scenario including equities is generating higher returns.

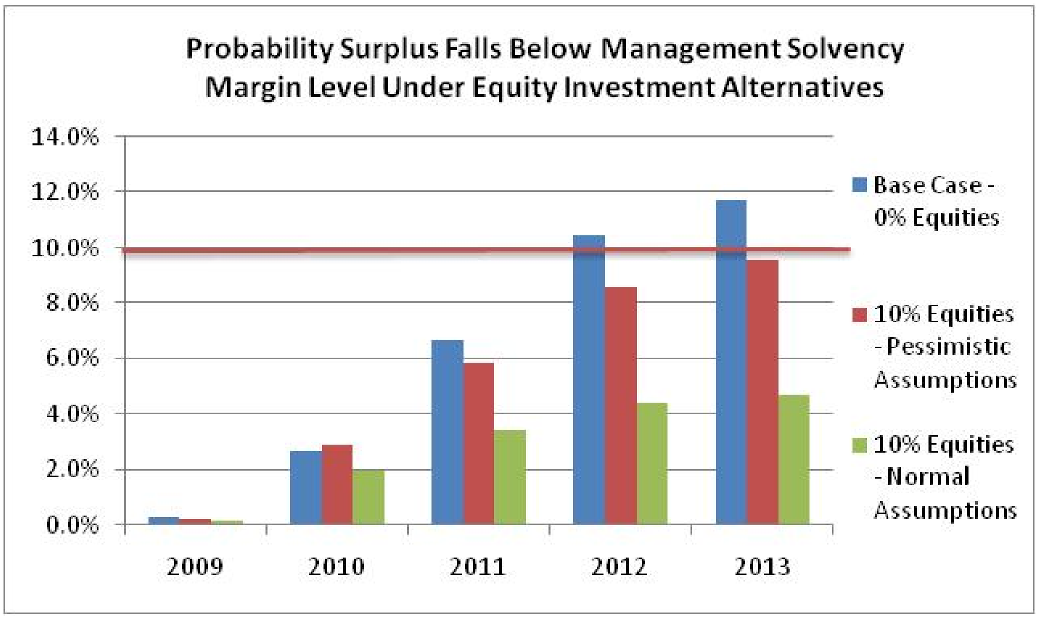
After discussion with DIG management, an additional model run was performed with more pessimistic assumptions for future equity performance. Here the expected market return for stocks was modeled at 1.5% above the risk-free rate. In addition, the volatility of equity returns was increased from 15% to 30%. The results, as shown in Table 6, were as follows:

Table 6: Projected Financial Metrics Under 10% Equity Investment in Pessimistic Market Conditions (000 omitted)

10% Equities with pessimistic market conditions	2009	2010	2011	2012	2013
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus falls below management solvency margin level	0.2%	2.9%	5.9%	8.6%	9.6%
Expected value of surplus	18,668	20,042	21,688	23,576	25,953
Expected value of solvency margin	235%	228%	224%	223%	225%

Even with the pessimistic assumptions, the alternative hypothesis of investing in equities continues to outperform the base case on both a risk and return basis as measured by surplus, as can be seen in Chart 8.

Chart 8: Surplus Implications of Alternative Equity Investment Scenarios



It is interesting to note that the probability of violating MSML in 2010 is slightly greater when equities are included (with pessimistic market assumptions). As noted in the histograms in Chart 6 and Chart 7, the model is indicating that equities are relatively more risky over the short term than over the long term according to this measure of risk. As with the premium growth assumption, we attribute this to an expected increase in company surplus arising from the higher long-term expectations of equity asset returns over the existing bond returns that serves to mitigate downside risk in later years.

Bond Duration Lengthening—Description/Impact

DIG has historically maintained a short-duration, highly liquid portfolio. As a potential alternative, it was decided to consider a longer-duration fixed-income portfolio, while maintaining the same amount of cash historically carried.

Table 7: Distribution of Bond Maturities in Base Case versus Alternative Case

	Base Case	Longer Duration
Cash	25%	25%
Bonds (< 1 year)	38%	13%
Bonds (1-5 years)	38%	13%
Bonds (6-10 years	0%	25%
Bonds (10-20 years)	0%	25%
Total	100%	100%

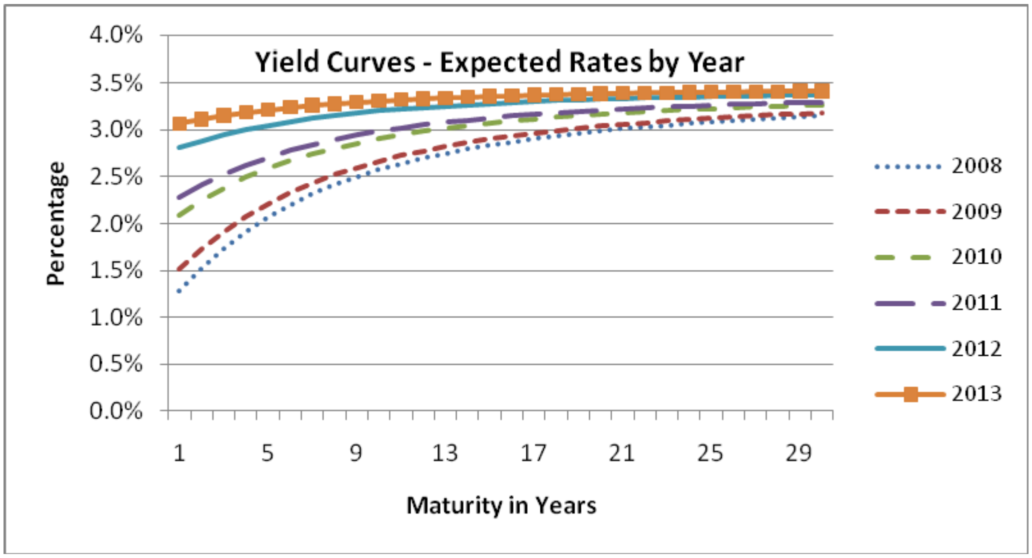
In the past, DIG has considered a short-duration portfolio to be safer because of its reduced sensitivity to movement in interest rates. A comparison of strategies, however, seems to indicate otherwise. As shown in Table 8, a longer-duration fixed-income portfolio appears to both increase expected surplus and decrease the risk of violating the MSML.

Table 8: Projected Financial Metrics Under Current versus Longer-Duration Bond Investment Alternatives (000 omitted)

Base case	**2009**	**2010**	**2011**	**2012**	**2013**
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus falls below management solvency margin level	0.3%	2.7%	6.7%	10.4%	11.7%
Expected value of surplus	18,526	19,738	21,210	22,913	25,047
Expected value of solvency margin	233%	224%	220%	217%	217%
Long duration	**2009**	**2010**	**2011**	**2012**	**2013**
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus falls below management solvency margin level	0.2%	2.0%	4.6%	6.8%	7.4%
Expected value of surplus	18,740	20,094	21,770	23,711	26,101
Expected value of solvency margin	236%	228%	225%	224%	226%

In order to better understand the apparent advantages of the longer portfolio, let us review the current yield curve and the expectation of future modeled yield curves.

Chart 9: Dynamo Projected Yield Curves



The current yield curve is below the model's long-term expectation of interest rates. Because the model simulates interest rates utilizing a process that reverts to the long-term mean, it is to be expected that, on average, interest rates will increase over the next five years, thereby decreasing the market value of bonds in the portfolio. From an enterprise-wide operational perspective, this creates a potential problem if bonds must be sold when their market values are depressed due to higher interest rates. We note this anticipated reversion to higher interest rates is greater at the shorter durations. Hence, in addition to the higher yield of a longer duration portfolio, the Dynamo model anticipates less expected interest rate movement in the longer-duration portfolio.

Combined Equities & Bond Duration Lengthening—Description/Impact

Based upon the initial analysis utilizing the Dynamo model, DIG is considering two significant changes to their current investment strategy. These changes include a modest investment in equities and an increase in the average duration of the fixed-income portfolio. At this point, we have only tested each of these changes in isolation. Dynamo provides a tool for better understanding the combined effects of making both of these changes. The scenario comparisons in Table 9 indicate that the combined effects of both changes are even more favorable than either change in isolation.

Table 9: Projected Financial Metrics Under Current versus Alternative Investment Option (000 omitted)

Base case	2009	2010	2011	2012	2013
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus falls below management solvency margin level	0.3%	2.7%	6.7%	10.4%	11.7%
Expected value of surplus	18,526	19,738	21,210	22,913	25,047
Expected value of solvency margin	233%	224%	220%	217%	217%

Long duration	2009	2010	2011	2012	2013
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus falls below management solvency margin level	0.2%	2.0%	4.6%	6.8%	7.4%
Expected value of surplus	18,740	20,094	21,770	23,711	26,101
Expected value of solvency margin	236%	228%	225%	224%	226%

10% Equities	2009	2010	2011	2012	2013
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus falls below management solvency margin level	0.2%	2.0%	3.4%	4.4%	4.7%
Expected value of surplus	18,948	20,641	22,652	24,950	27,790
Expected value of solvency margin	238%	235%	234%	236%	241%

Long Duration + 10% Equities	2009	2010	2011	2012	2013
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus falls below management solvency margin level	0.1%	1.4%	2.1%	2.9%	2.5%
Expected value of surplus	19,162	20,992	23,197	25,719	28,794
Expected value of solvency margin	241%	239%	240%	243%	249%

We see improvements in both risk and reward from executing the alternative investment strategy, i.e., with the longer duration and equity strategy, there is both a greater long-term build up of expected surplus and a reduction in the frequency of violating the MSML. To further test these proposed changes, the Dynamo model was run with the pessimistic market scenario for equities. The results are as follows.

Table 10: Projected Alternative Investment Scenario in Pessimistic Market Conditions (000 omitted)

Long Duration + 10% Equities with pessimistic market conditions	2009	2010	2011	2012	2013
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus falls below management solvency margin level	0.1%	2.1%	4.1%	5.6%	6.1%
Expected value of surplus	18,882	20,396	22,249	24,378	27,012
Expected value of solvency margin	237%	232%	230%	231%	234%

Again, there is improvement in both risk and reward as defined by expected value of surplus and frequency of violating the MSML. In addition, under both combined long duration and equity scenarios, the expected value of the solvency margin is increasing in the long term.

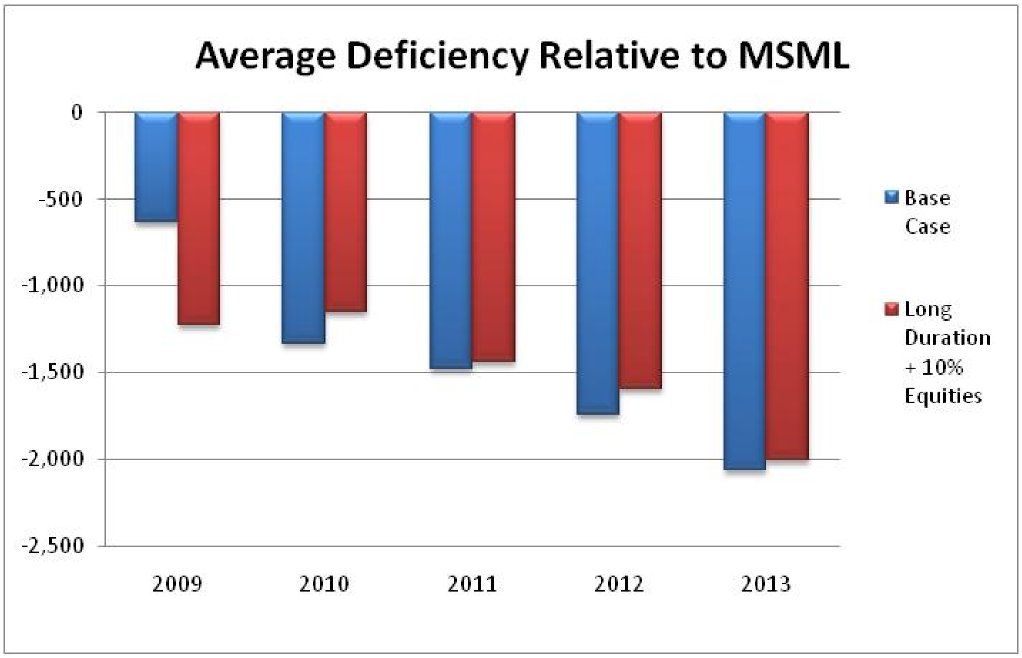
Finally, it was noted by DIG management that the severity of losses within the tail should also be considered. If surplus does fall below the MSML, how bad might the situation be? Note that throughout our model runs, we have been considering the probability of violating the RSL. Because violation of the RSL is indicative of a more serious situation, it is helpful in understanding the severity within the tail. DIG management has indicated that they would also like to consider a more comprehensive measure of risk in the tail. Utilizing the Dynamo model, a calculation of the average deficiency relative to MSML was calculated. This risk measure is defined as the expected value of the difference between surplus and MSML over all simulations where surplus falls below MSML.

Table 11: Projected Average Deficiency/Surplus Under Current versus Alternative Investment Option (000 omitted)

Base case	**2009**	**2010**	**2011**	**2012**	**2013**
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Expected value of surplus	18,526	19,738	21,210	22,913	25,047
Average Deficiency amount relative to management solvency level	(635)	(1,335)	(1,478)	(1,740)	(2,067)
Long duration	**2009**	**2010**	**2011**	**2012**	**2013**
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Expected value of surplus	18,740	20,094	21,770	23,711	26,101
Average Deficiency amount relative to management solvency level	(788)	(1,243)	(1,338)	(1,576)	(1,862)
10% Equities	**2009**	**2010**	**2011**	**2012**	**2013**
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Expected value of surplus	18,948	20,641	22,652	24,950	27,790
Average Deficiency amount relative to management solvency level	(1,179)	(1,236)	(1,433)	(1,769)	(1,905)
Long Duration + 10% Equities	**2009**	**2010**	**2011**	**2012**	**2013**
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Expected value of surplus	19,162	20,992	23,197	25,719	28,794
Average Deficiency amount relative to management solvency level	(1,229)	(1,150)	(1,440)	(1,594)	(2,008)

It is interesting to note that while there is a substantial difference in the average deficiency between our long duration plus 10% equities scenario and the base case in the short term according to this measure of risk, the results are relatively close to the base case in the long term. An interpretation here might be that while the probability of violating MSML is decreased by the proposed investment strategy, the severity of such a violation will be similar on average in the long run.

Chart 10: Comparison of Average Deficiency / MSML Under Base Case versus Alternative Investment



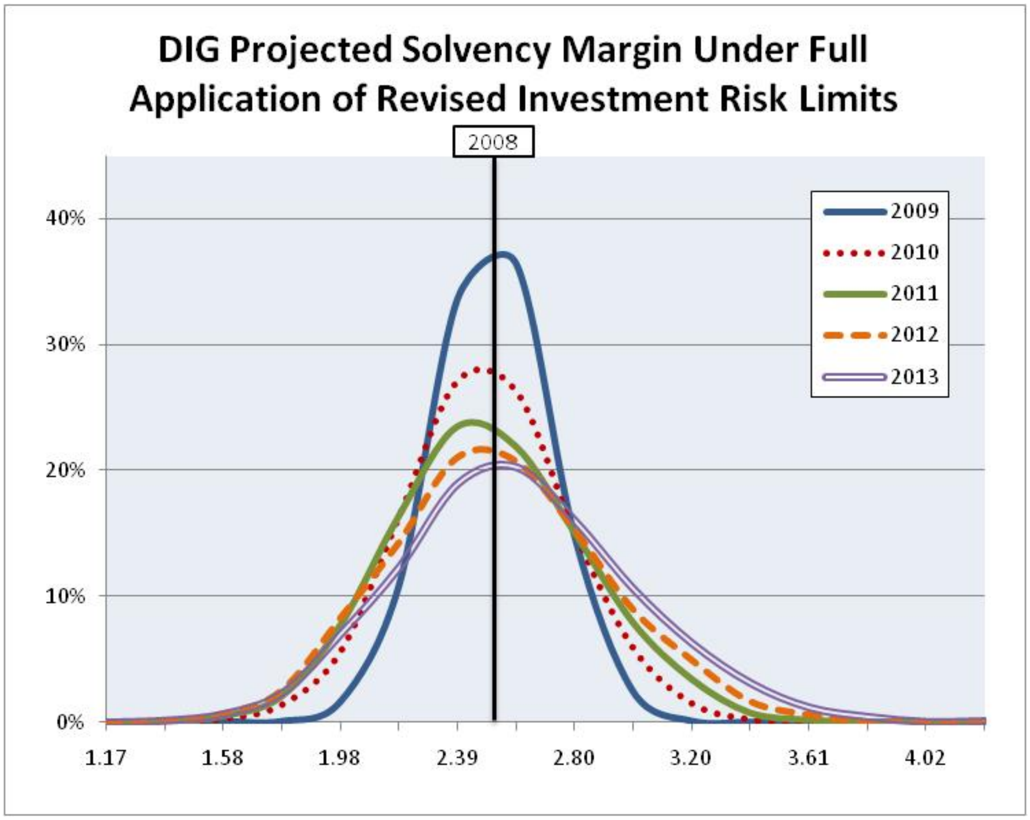
DIG management was happy with this result. The Dynamo model implies that the new investment strategy significantly reduces the probability of strained surplus. When surplus strain does occur, DIG will be no worse off on average than when surplus strain occurred under the base case. However, the severity in the tail may be greater over a one-year horizon under the new investment strategy.

After seeing these results, DIG management is favorably inclined to adjust the company's risk limits as follows:

- Premium Growth: retain at 3.5% per annum
- External Homeowners Excess of Loss Coverage: continue to purchase
- Equity investments: increase to 10% of portfolio
- Bond duration: lengthen by changing the mix of bonds as described in Table 7

The expected range of surplus relative to DIG's solvency margin can be seen in Chart 11.

Chart 11: Range of Projected Solvency Margins by Year Based on Revised Risk Limits



Other Combinations with New Investment Strategy—Description/Impact

Earlier in this paper, we evaluated possible changes to the allowed policy growth rate and reinsurance program. These changes were evaluated in isolation from other possible changes. The conclusion from this initial evaluation was to maintain the current growth cap and reinsurance program. Our Dynamo model allows us to again evaluate these two possible changes in conjunction with the new investment strategy. Such an evaluation is consistent with the holistic approach espoused throughout this paper. The model results are as follows.

Table 12: Blending Growth, Reinsurance and Investment Alternatives (000 omitted)

Long Duration + 10% Equities	**2009**	**2010**	**2011**	**2012**	**2013**
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus falls below management solvency margin level	0.1%	1.4%	2.1%	2.9%	2.5%
Expected value of surplus	19,162	20,992	23,197	25,719	28,794
Expected value of solvency margin	241%	239%	240%	243%	249%
Average deficiency amount relative to management solvency level	(1,229)	(1,150)	(1,440)	(1,594)	(2,008)
Long Duration + 10% Equities +5% growth	**2009**	**2010**	**2011**	**2012**	**2013**
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus falls below management solvency margin level	0.1%	1.8%	3.1%	3.6%	3.1%
Expected value of surplus	19,135	21,029	23,436	26,343	30,034
Expected value of solvency margin	249%	238%	234%	235%	237%
Average deficiency amount relative to management solvency level	(1,236)	(1,203)	(1,386)	(1,731)	(1,984)
Long Duration + 10% Equities +5% growth+ Internal Reinsurance	**2009**	**2010**	**2011**	**2012**	**2013**
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus falls below management solvency margin level	0.3%	3.1%	4.5%	4.8%	3.9%
Expected value of surplus	18,950	20,888	23,365	26,377	30,206
Expected value of solvency margin	233%	229%	229%	233%	240%
Average deficiency amount relative to management solvency level	(886)	(1,249)	(1,493)	(1,814)	(2,058)
Long Duration + 10% Equities + Internal Reinsurance	**2009**	**2010**	**2011**	**2012**	**2013**
Surplus falls below required solvency level	0.0%	0.0%	0.0%	0.0%	0.0%
Surplus falls below management solvency margin level	0.3%	2.3%	3.4%	3.8%	3.3%
Expected value of surplus	18,976	20,843	23,102	25,704	28,880
Expected value of solvency margin	235%	233%	234%	238%	244%
Average deficiency amount relative to management solvency level	(879)	(1,271)	(1,469)	(1,795)	(2,084)

Comparing the new investment strategy (long duration plus equities) to the new investment strategy with elimination of external reinsurance (long duration plus equities plus internal reinsurance), we note that elimination of external reinsurance provides a small boost to expected surplus along with a significant increase in the likelihood of violating MSML. In particular, note the probability of violating MSML in year 2013 increases from 2.5% to 3.3%; i.e., violation of MSML would occur over 30% more often. Based on this, DIG remains convinced that external reinsurance should be maintained.

As part of sharing these additional results with DIG management, we observed, the combination of the new investment strategy with 5% growth now appears to present a more favorable risk reward tradeoff than previously thought. The increased risk of violating MSML with 5% growth is similar to the scenario with internal reinsurance, 3.1% versus 3.3% of the time. Unlike the scenario with internal reinsurance, there is a meaningful increase of 4% to expected surplus in the scenario with 5% growth, from $28,880,000 to $30,034,000.

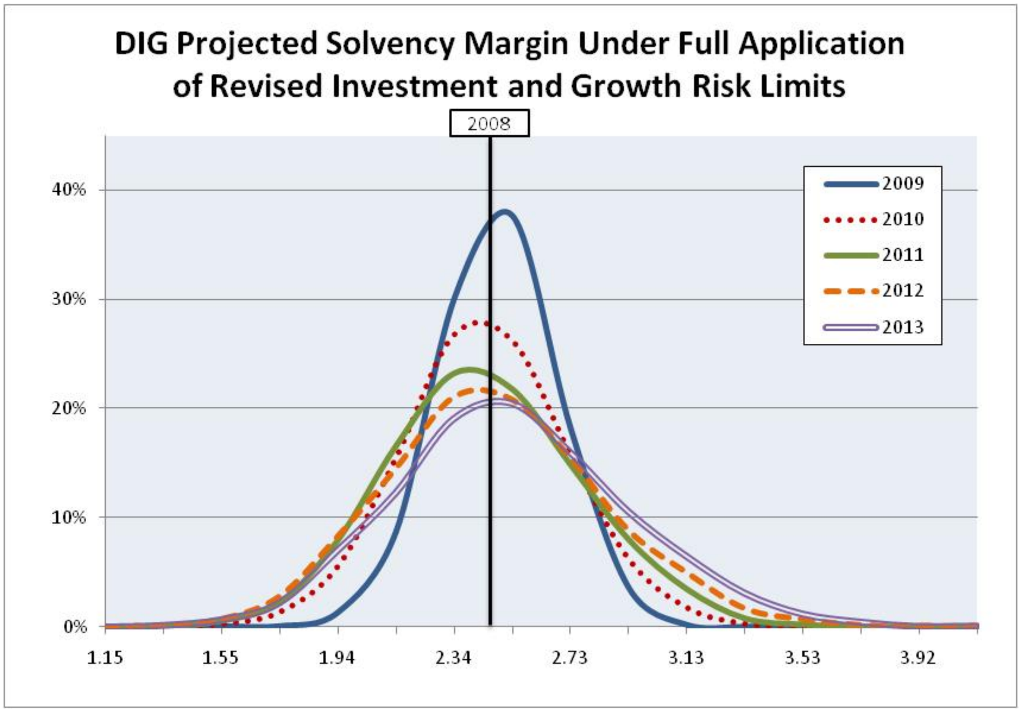
We recalled that the 5% growth cap was previously rejected because DIG was attempting to reduce the violation of MSML to below 10% over a five-year period. Note that the 5% growth option, in conjunction with the new investment strategy, does not violate the MSML below 10% condition. By considering the changes together, DIG is able to reap the expected rewards of faster growth when market conditions are favorable, while maintaining an acceptable enterprise-wide risk profile for the company. As a result of this additional analysis, DIG management is now favorably inclined to revise the new business growth limit from 3.5% per year to 5% per year. Such a revision has operational implications outside of the purely financial metrics being reviewed in this paper. As mentioned earlier in the paper, DIG's capacity to effectively handle claims is a consideration in the policy growth constraint. Before relaxing the growth constraint to allow 5% annual growth, DIG will need to increase its claims handling capacity. DIG will also need to re-evaluate enforcement of underwriting standards to ensure that the possibility of faster growth does not lead to deterioration in the underwriting book.

To summarize, DIG management is now favorably inclined to adjust the company's risk limits as follows:

- Premium Growth: increase to 5.0% per annum
- External Homeowners Excess of Loss Coverage: continue to purchase
- Equity investments: increase to 10% of portfolio
- Bond duration: lengthen by changing the mix of bonds as described in Table 7

The expected range of surplus relative to DIG's solvency margin can be seen in Chart 12.

Chart 12: Range of Projected Solvency Margins by Year Based on Revised Risk Limits



For sake of completeness, we note that the new investment strategy with 5% growth and elimination of external reinsurance does not represent a favorable risk reward tradeoff when compared to the new investment strategy with 5% growth and maintaining the external reinsurance program, i.e., the external reinsurance program is still desirable in conjunction with the new investment and higher growth strategy.

Other Combinations?

At this point, the mathematically minded reader may point out that DIG could have initially constructed a grid of all possible combinations of changes.

Table 13: Options being Considered by DIG

	Number of Options Considered
Growth	3
Reinsurance	2
Equities	2
Bond Duration	2

For our highly simplified example, this creates 24 (=3x2x2x2) different possible combinations while a more realistic setting would likely lead to a much higher number of possible combinations. Note also that our considered options are special cases of variables better defined as continuous. The number of possible combinations can quickly become impractical and lead to a "black box" approach and if one blindly attempts a strictly mathematical approach, the problem very quickly becomes intractable.

In reality, our chosen approach is more an attempt to build management confidence in the holistic approach and its benefits. By initially considering individual changes, DIG management was better able to understand underlying factors in the risk/reward tradeoff for each proposed change. Only later in the process did DIG management begin to selectively consider combinations of changes.

CONCLUSION

When the analysis described here was initially proposed to DIG management, management viewed it as a harmless but not very interesting exercise that they expected to merely validate DIG's already existing business constraints. However, as the analysis proceeded and management was advised of areas in which current constraints were impeding the achievement of management's financial objectives without adequately satisfying management's risk tolerances, DIG management became substantially more invested in the analysis. As a result of the findings of this study, DIG management has expressed an interest in developing a full-time enterprise risk management team with a charter to continue analyzing and monitoring company operations using state of the art modeling tools and techniques.

Additionally, DIG management has authorized changes in the company's investment strategy as well as signing off on the continuing purchase of the homeowners XOL reinsurance cover. DIG is also in the process of increasing their growth cap on number of policies written.

From an overall enterprise risk management standpoint, this paper highlights the pitfalls of establishing risk tolerances in an isolated or "siloed" manner and demonstrates the benefits of having a periodic review of all aspects of a company's risk policy. From a purely analytical standpoint, the use of a dynamic financial analysis (DFA) tool such as Dynamo 4 brings technical rigor to the overall analysis process and facilitates the evaluation of interactions between different parts of the organization in ways that are not possible using traditional methods. While the situation described in this paper is simplistic, the following general conclusions are transferrable to a real-life insurer:

- A company's risk policy should not be viewed as a static construct—it requires periodic review and consideration to maintain its relevance.
- Construction of isolated or "siloed" risk acceptance requirements, no matter how well intentioned, is often not an optimal way to support a company's risk policy.
- Conventional wisdom and "gut instincts" are no substitute for rigorous analysis.
- It is important to include a multi-year view of risk as certain risks aggregate over time.
- A DFA model is an essential tool in analyzing the extent to which a company's risk acceptance requirements are satisfying the company's risk policy.

APPENDIX A: A POTENTIAL INCONSISTENCY?

When presenting the results of the base case versus the internal reinsurance options to DIG management, a query was raised by one executive about why the expected profitability of the internal reinsurance scenario was not greater in relation to the base case scenario. The executive's logic was that if the average increase in underwriting gain from moving to the internal reinsurance alternative was $0.16M per year over five years, the average pre-tax benefit to DIG at the end of five years should be $0.8M and the average post-tax benefit should be $0.5M. However, the results derived from Dynamo indicate the internal reinsurance scenario's average surplus at the end of 2013 to be $0.073M higher than the base case's average surplus at the end of 2013.

To address this potential inconsistency, we firstly reviewed the large claim assumptions and results. As described by DIG's external consultant, DIG expects to experience, on average, 1.5 large losses per year with an average cost per large loss of $300,000. When the Dynamo simulation results were reviewed, we observed an average claim frequency of 1.499 and an average claim severity of $303,994 over twenty-five thousand simulations.

Next, we reviewed the financial statement results from the model. After reviewing the modeled results, we concluded that these results do appear to present a potential inconsistency with the a priori expectations based on the model inputs and simulated model results. We reviewed the mechanics of the Dynamo model but could find no specific reason that might be driving the difference between expectation and modeled results. We believe the differences might be attributable to sheer number of moving parts within a model such as Dynamo that can lead to results that do not fully align with the a priori expectations, items such as taxes, the timing with which premiums are collected and losses are paid and the resulting impact on investment income, and random fluctuations in general.

Additionally, we observed that the financial statement exhibits in the Dynamo model do not contain accurate reconciliations between the balance sheet, income statement and cash flow statement. Reconciliation exhibits (such as those on the Statement of Income and Cash Flow Statement pages of in insurance company statutory filings) are crucial to validating that a model such as Dynamo has internal consistency in its calculations. An evaluation of the Dynamo model provided as the starting point for this paper shows that the model's initial setup produces internally inconsistent results, such as the year over year change in policyholder surplus on the Statutory Liabilities exhibit not matching the change in policyholder surplus on the Income Statement exhibit.

Our take-away from this is that a model's results should always be subjected to rigorous testing, including both a peer review of the technical model construction and a series of reasonability tests to identify potential inconsistencies in the modeled results.

APPENDIX B: DYNAMO MODEL USAGE

Dynamo is a spreadsheet based tool that simulates five years of GAAP and statutory financial statements, including balance sheet, income statement, and IRIS ratios, for a property casualty insurance company. The model covers risks such as reserve development, pricing, natural catastrophes, and investments. It also models economic variables like inflation, interest rates, and underwriting cycles.

When testing different strategies, DIG focused on the interactions between the surplus and reserve development, investment, catastrophe risk, and reinsurance. Thus, the company used some parts of the model more extensively than others.

Heavily Utilized Items

Premiums

Written premiums are modeled by separately projecting the number of exposures and rate per exposure. In its current state, DIG has limited exposure growth to 3.5% per year. The rate level depends on implied rate changes, inflation levels, and the adequacy of current rate levels. DIG has assumed a steady mix of new and renewal business over the five-year projection period.

Earned premiums are modeled by selecting an earning pattern to apply to written premiums. DIG writes 12-month policies uniformly throughout the year, and thus earns 50% of the premium in the year it is written and 50% of the premium in the following year.

Expenses

Expenses can be entered as fixed or variable expenses as a percent of written or earned premium. Fixed expenses must be estimated for each of the years in the projection period. Variable expenses will be modeled as a percent of the appropriate premium. Unallocated loss adjustment expenses (ULAE) are modeled as a percentage of paid losses.

Losses

Losses are modeled using a normal distribution to model frequency on a per exposure basis and severity on a per loss basis. Parameters for the normal distributions are selected after analyzing historical data including:

- Historical paid loss triangles and net paid loss at current valuation
- Estimation of net and gross ultimate loss and ALAE
- Estimation of net and gross earned premium
- Estimation of ultimate claim counts
- Historical written exposures

Large losses and catastrophes are modeled separately and are discussed in subsequent sections of this paper.

Catastrophe generator

The Dynamo model has an integrated catastrophe loss generator. It uses Poisson and lognormal distributions to simulate catastrophes by state, for each year. The user can also enter correlations between geographically proximate states.

The company used this model to generate catastrophe losses. The number of losses that impact the company is a function of its market share in each state. DIG's homeowners line of business covers 0.1% of the exposure in Massachusetts and 0.05% of the exposure in Florida. Historically, the DIG workers compensation line of business did not experience catastrophic losses. To simplify the analysis, we have assumed that there is no catastrophic exposure for this line.

Reinsurance

The Dynamo model has the ability to model quota share and excess of loss reinsurance at the line of business level, and stop loss and catastrophe reinsurance at the company level. DIG has not purchased any quota share or stop loss reinsurance. DIG has purchased a $10M xs $1M catastrophe policy in the past, and projects to continue with this coverage through the modeled period. DIG's excess of loss reinsurance is discussed in the section on Modifications.

Asset input

Dynamo allows for investment in many varied asset classes. In its current state, DIG has invested only in cash and Treasury bonds of short duration. In its scenario planning, DIG allows for investment in equities. In Dynamo, we have decided to model these equities as common stock. Investment profits must be allocated back to an investment class for reinvestment. DIG has reinvested its assets to keep a constant mix between cash and Treasury bonds through the projection period. When allowing for investment in equities, DIG has tested both expected market conditions as well as poor market conditions. Dynamo allows for the user to project the expected risk premium and standard deviation of the market returns.

Simplifications in the Analysis

Required solvency level

We have defined required solvency level (RSL) as 30% of loss reserves plus unearned premium reserves. A more appropriate definition of a risk based capital measure would likely include an adjustment for investment risk and other factors such as rapid growth. Note that the addition of such factors into the RSL calculation would generally impact the evaluation of the different risk limits considered in this paper, i.e., the conclusions may differ.

Exposure to workers compensation catastrophe losses

We have not attempted to incorporate a catastrophe element for workers compensation. While a real-world analysis might include such a component, we believe including it in the scenarios contemplated for this paper would not add to the overall value of the holistic ERM exercise being performed here.

Exposure units

DIG has exposure only in Massachusetts and Florida.

Investment inputs

Being a conservative mutual company, DIG has a relatively simple investment portfolio that contains only non-callable U.S. government bonds and cash. Thus, DIG used only a small part of the Dynamo's ability to model different types of assets.

Inflation generator

The modification of inflation was not in DIG's priorities. The Dynamo model has the ability to implement frequency and severity and unexpected inflation. For homeowners lines, the company assumed inflation of 2% in the payment pattern, 0% frequency inflation, and less than 2.5% severity inflation. For workers compensation lines, the company assumed inflation of 3% in the payment pattern, 0% frequency inflation, and less than 5% severity inflation

Items with Limited Use

Dynamo allows the user to enter different loss assumptions for different classes of business (new, first renewal, and mature renewal). DIG has not selected to model any distinction between their classes of business. Since they have a constant mix of new and renewal business, overall average frequency and severity are appropriate.

Dynamo also has the ability to model various underwriting market conditions. DIG has assumed that the market will not go through a major change during the projection period, and their growth and pricing assumptions will remain unchanged.

Modifications

DIG has modified Dynamo to include an individual large loss generator for the homeowners line of business. A number of large losses are generated and a severity is modeled for each large loss. DIG's excess of loss reinsurance is then applied to limit each individual loss. Large losses are modeled using Poisson frequency and Lognormal severity. DIG has purchased an excess of loss policy with a $100,000 per occurrence retention and $5M policy limit.

Limits of the Dynamo Model

While using the Dynamo model to project the company's financial statements, some areas for improvement of the model were identified.

Distribution of dividends

As DIG is a mutual company, it will not pay stockholder dividends. Dynamo does not have the built-in ability to pay policyholder dividends. The current practice is that the company would pay policyholder dividends in the event the surplus reached a certain level, but this is not explicitly modeled in the scenarios testing.

Allocation of the asset portfolio

Another limit of Dynamo is related to the allocation of assets. In the current version, it is possible to allocate the excess of cash at the end of the year to any type of assets. And this is the only way to change the existing asset portfolio. The user can not implement strategies where, for example, at the end of the second simulated year, all bonds are sold and the money is invested in stocks. Even if this is an extreme example that will rarely happen in reality, the model prevents even strategies of type "percentage allocation" where the company wants to have a constant percentage of each assets in each year. (e.g., 10% in cash, 10% in equities, and 80% in bonds).

Insolvency scenarios

Dynamo continues to model future years, even if the company goes insolvent. In the case of insolvency a real company will cease operations. This will have an impact on the analysis of the mean of simulated values.

Balance sheet-income statement-cash flow statement reconciliations

The financial statement exhibits in the Dynamo model do not contain reconciliations between the balance sheet, income statement, and cash flow statement. Users of Dynamo would be more comfortable about the accounting consistency of Dynamo if there were reconciliation exhibits such as those on the statement of income and cash flow statement pages of an insurance company's statutory filings.

Direct Analysis of Pre-Adjusted Loss Cost, Frequency or Severity in Tweedie Models

Sheng G. Shi, Ph.D.

Abstract

Response data (loss cost, claim frequency or claim severity) are often pre-adjusted with known factors and directly analyzed with generalized linear models (GLM). This paper shows that the exposure weights should also be adjusted if the Tweedie distribution with log link is used in such direct analysis. An advantage of the direct analysis over GLM offsetting is that the structure of the original dataset may be simplified significantly after removing the known factors. Direct analysis is a convenient tool for directly modeling loss ratio and for removing known territory factors from the dataset. Implementation in EMBLEM and SAS is discussed, and a computationally efficient SAS macro is provided for Tweedie models.

Keywords. Predictive Modeling; GLM offset; Ratemaking; Tweedie Model; EMBLEM; SAS.

1. INTRODUCTION

In insurance ratemaking, response data are often pre-adjusted with known factors before predictive modeling. However, the effect of adjustment on exposure weights is usually either ignored or not linked to the response distribution. This is particularly the case when the response variable is loss cost, which is assumed to follow a Tweedie distribution of power p ($1<p<2$).

Application of the GLM offset feature in property-casualty predictive modeling has been discussed recently by Yan et. al.[7]. They translated the analysis on loss ratio into an analysis on loss cost with premium offset. In this paper, we will show how loss ratio, viewed as loss cost pre-adjusted with premium rates, can be analyzed directly. An advantage of the direct analysis is that the structure of original dataset may be simplified significantly for subsequent analysis. We first show, in general, how the exposure weights should be modified in Tweedie models (including the special case of Poisson and Gamma) with pre-adjusted loss cost, claim frequency or claim severity as the response.

2. CONNECTION BETWEEN OFFSETS AND PRE-ADJUSTMENT

In this section, we give two propositions that connect GLM offsets with pre-adjustment. Proposition 1 builds a simple linkage between the offsets and pre-adjustment. Proposition 2 establishes a foundation for data simplification.

2.1 Assumptions and Notations

Suppose that there are two rating factors *U* and *V*, where *U* has *m* categories and *V* has *n* categories. Denote u_i as the relativity of the i^{th} category ($i = 1,2,...,m$) of *U* and v_j as the relativity of the j^{th} category ($j = 1,2,...,n$) of *V*. Let Y_{ij} be a random variable for the ratio of interest in the rating cell with the i^{th} level of *U* and the j^{th} level of *V* such that $Y_{ij} = X_{ij} / w_{ij}$. When the ratio of interest is loss cost, $X_{ij} = L_{ij}$ as loss amount and $w_{ij} = e_{ij}$ as earned exposure. When the ratio of interest is claim frequency, $X_{ij} = c_{ij}$ as claim count and $w_{ij} = e_{ij}$. When the ratio of interest is claim severity, $X_{ij} = L_{ij}$ and $w_{ij} = c_{ij}$. Assume that the Y_{ij}'s are mutually independent and Y_{ij} follows a Tweedie distribution with power parameter *p* such that

$$E(Y_{ij}) = u_i v_j, \tag{2.1}$$

$$Var(Y_{ij}) = \phi(u_i v_j)^p / w_{ij} \tag{2.2}$$

where ϕ is a constant dispersion parameter [3]. To include the dispersed Poisson and Gamma as special cases of the Tweedie distribution, we focus on the range of power parameter, $1 \le p \le 2$.

As in a typical analysis, we assume that the power parameter *p* and the constant dispersion parameter ϕ are known or have been pre-determined. We will use log link in all the models.

2.2 Propositions

2.2.1 Simple link between GLM offsets and pre-adjustment

With the Tweedie model, an offset problem can be translated into a pre-adjustment problem and vice versa as shown in the proposition below. This interchangeability also allows us to have a model with both pre-adjustment and offsets.

Proposition 1

Under the assumptions and notations above, if u_i's are known, then fitting the following Tweedie model (in Eq. (2.3)) of power *p* with weights w_{ij} and $\log(u_i)$ as an offset,

$$\log E(Y_{ij}) = \log(v_j) + \log(u_i), \tag{2.3}$$

where $i = 1,2,...,m$ and $j = 1,2,...,n$, is equivalent to fitting the Tweedie model of power *p* below (in Eq. (2.4)) with pre-adjusted response variable $Z_{ij} = Y_{ij} / u_i$ and weights $w_{ij} u_i^{2-p}$,

$$\log E(Z_{ij}) = \log(v_j), \tag{2.4}$$

where $i = 1,2,...,m$ and $j = 1,2,...,n$.

In other words, Z_{ij} can be viewed to follow Tweedie distribution of the same power *p* and dispersion parameter ϕ as Y_{ij}, but with different weights.

Proof

Note that the Tweedie distribution belongs to the exponential dispersion family, which is closed under a scale transformation (cf. [3] Formula 6 on p. 72). Thus, Z_{ij} follows a Tweedie distribution with power parameter *p*. Based on Eq. (2.1) and Eq. (2.2) above,

$$E(Z_{ij}) = E(Y_{ij})/u_i = u_i v_j / u_i = v_j \tag{2.5}$$

and

$$Var(Z_{ij}) = Var(Y_{ij})/u_i^{\,2} = \phi(u_i v_j)^p /(w_{ij} u_i^{\,2}) = \phi v_j^{\,p} /(w_{ij} u_i^{\,2-p}). \tag{2.6}$$

To show that two models are equivalent, let l_{ij} be the log-likelihood function for $Y_{ij} = y_{ij}$. Then, according to the property of the exponential dispersion family, we have

$$\frac{\partial l_{ij}}{\partial \mu_{ij}} = \frac{y_{ij} - \mu_{ij}}{\phi V(\mu_{ij})/w_{ij}} \tag{2.7}$$

where the mean $\mu_{ij} = E(Y_{ij}) = u_i v_j$ and the variance function $V(\mu_{ij}) = \mu_{ij}^{\,p}$. To obtain the maximum likelihood estimate $\hat{v}_j$ of v_j, we set for $j = 1,2,\ldots,n$,

$$\sum_i \frac{\partial l_{ij}}{\partial v_j} = \sum_i \frac{\partial l_{ij}}{\partial \mu_{ij}} \frac{\partial \mu_{ij}}{\partial v_j} = \sum_i \frac{y_{ij} - u_i v_j}{\phi(u_i v_j)^p / w_{ij}} u_i = 0 \tag{2.8}$$

which leads to the estimate for the model specified in Eq. (2.3),

$$\hat{v}_j = \frac{\sum_i w_{ij} u_i^{\,2-p} y_{ij} / u_i}{\sum_i w_{ij} u_i^{\,2-p}}. \tag{2.9}$$

Now, let l_{ij}^* be the log-likelihood function for $Z_{ij} = z_{ij}$. Then,

$$\frac{\partial l_{ij}^*}{\partial v_j} = \frac{z_{ij} - v_j}{\phi V(v_j)/(w_{ij} u_i^{\,2-p})} \tag{2.10}$$

where the mean $v_j = E(Z_{ij})$ and the variance function $V(v_j) = v_j^{\,p}$. To obtain the maximum likelihood estimate $\hat{v}_j^*$ of v_j, we set for $j = 1,2,\ldots,n$,

$$\sum_i \frac{\partial l_{ij}^*}{\partial v_j} = \sum_i \frac{z_{ij} - v_j}{\phi v_j^{\,p} /(w_{ij} u_i^{\,2-p})} = 0 \tag{2.11}$$

which leads to the estimate for the model specified in Eq. (2.4),

$$\hat{v}_j^* = \frac{\sum_i w_{ij} u_i^{2-p} z_{ij}}{\sum_i w_{ij} u_i^{2-p}} = \hat{v}_j \,. \tag{2.12}$$

It is easy to verify that $\sum_i \frac{\partial^2 l_{ij}(\hat{v}_j)}{\partial v_j^2} < 0$ and $\frac{\partial^2 l_{ij}^*(\hat{v}_j^*)}{\partial v_j^2} < 0$ for the maxima. Q.E.D.

Note that the right side of Eq. (2.4) is not related to the index *i*. Thus, it may be simplified by collapsing over the rating factor *U* as discussed in Section 2.2.2.

Example 1

Loss ratio can be viewed as loss cost L_{ij} / e_{ij} pre-adjusted with the premium rates u_i in a rating plan:

Loss Ratio = Losses/Earned Premiums

= Losses/(Exposures*Rates) = (Losses/Exposures)/Rates

= (Loss Cost)/Rates.

Assume that the loss cost L_{ij} / e_{ij} follows Tweedie of power *p*. Then, the loss ratio $L_{ij} /(e_{ij} u_i)$ can be analyzed with the Tweedie model of power *p*, but the model weights need to be adjusted to Exposures*Rates^(2-*p*) = $e_{ij} u_i^{2-p}$.

2.2.2 Pre-adjustment for data simplification

Aggregating data reduces the number of records in a dataset and simplifies the data structure. This can be especially beneficial when aggregating across high-dimensional variables, such as territory. From a modeling perspective, this is achieved by collapsing on the GLM offset variable, but subsequent analyses will then need to be done with pre-adjusted data as shown in the proposition below.

Proposition 2

Under the assumptions and notations above, if u_i's are known, then fitting the following Tweedie model (in Eq. (2.13)) of power *p* with weights w_{ij} and $\log(u_i)$ as an offset

$$\log E(Y_{ij}) = \log(v_j) + \log(u_i), \tag{2.13}$$

where $i = 1,2,...,m$ and $j = 1,2,...,n$, is equivalent to fitting the simplified Tweedie model of power *p* below (in Eq. (2.14)) with weights $\sum_i w_{ij} u_i^{2-p}$,

$$\log E(Z_j) = \log(v_j);\ j = 1,2,...,n, \tag{2.14}$$

where

$$Z_j = \frac{\sum_i w_{ij} u_i^{2-p} (Y_{ij} / u_i)}{\sum_i w_{ij} u_i^{2-p}};\ j = 1,2,...,n. \tag{2.15}$$

In other words, Z_j can be viewed to follow the Tweedie distribution of the same power *p* and dispersion parameter ϕ as Y_{ij}, but with different weights (cf. [4]).

Proof

Note that the Tweedie distribution belongs to the exponential dispersion family, which is closed under a scale transformation and follows the convolution formula (cf. [3] Formula 10 on p. 74). Write $Z_{ij} = Y_{ij} / u_i$. We know from Proposition 1 that Z_{ij} follows the Tweedie distribution of the power *p* with mean v_j, dispersion parameter ϕ and prior weights $w_{ij} u_i^{2-p}$. Therefore, for $j = 1,2,...,n$,

$$Z_j = \frac{\sum_i w_{ij} u_i^{2-p} Z_{ij}}{\sum_i w_{ij} u_i^{2-p}} \tag{2.16}$$

is still Tweedie distributed with the power parameter *p* and

$$E(Z_j) = \frac{\sum_i w_{ij} u_i^{2-p} E(Y_{ij} / u_i)}{\sum_i w_{ij} u_i^{2-p}} = \frac{\sum_i w_{ij} u_i^{2-p} (u_i v_j / u_i)}{\sum_i w_{ij} u_i^{2-p}} = v_j, \tag{2.17}$$

$$Var(Z_j) = \frac{\sum_i (w_{ij} u_i^{2-p})^2 * Var(Y_{ij} / u_i)}{(\sum_i w_{ij} u_i^{2-p})^2}$$

$$= \frac{\sum_i (w_{ij} u_i^{2-p})^2 * (\phi u_i^{p} v_j^{p} / w_{ij} u_i^{2})}{(\sum_i w_{ij} u_i^{2-p})^2} \tag{2.18}$$

$$= \phi v_j^{p} * \frac{\sum_i (w_{ij} u_i^{2-p})^2 / (w_{ij} u_i^{2-p})}{(\sum_i w_{ij} u_i^{2-p})^2}$$

$$= \phi v_j^{p} / \sum_i w_{ij} u_i^{2-p}.$$

To show the two models are equivalent for estimating v_j, we note from the proof of Proposition 1 that the maximum likelihood estimate for the model specified by Eq. (2.13) is given in Eq. (2.9). From Eq. (2.17), it is rather trivial that the maximum likelihood estimate for the model specified by Eq. (2.14) is the same as that in Eq. (2.9), because only a single Z_j is involved for estimating v_j. Q.E.D.

Example 2

In a loss ratio analysis, a dataset with numerous premium rate levels may be simplified by collapsing over the premium variable. Note that a unique premium rate level is defined by a unique combination of all rating variables in a rating plan. The data size can be reduced drastically in many cases by collapsing over the premium variable. Before collapsing, loss ratios $L_{ij} / (e_{ij} u_i)$ are recorded for each exposure, where u_i's are premium rates. We are interested in fitting a Tweedie model of power *p* with other covariates that are combined into v_j. After collapsing, we can equivalently model "weighted loss ratios" $(\sum_i L_{ij} u_i^{1-p}) / (\sum_i e_{ij} u_i^{2-p})$ with adjusted exposure weights $\sum_i e_{ij} u_i^{2-p}$. Note that the weighted loss ratios are not of the form $(\sum_i L_{ij}) / (\sum_i e_{ij} u_i)$.

Example 3

In a loss cost analysis, a dataset with numerous territories may be simplified by collapsing over the territory variable. Both loss cost and exposure weights need to be adjusted by known territory relativities for Tweedie models.

3. IMPLEMENTATION

Suppose that the EMBLEM (cf. [1] and [2]) data source is in a summarized table such that each record has an observed level (indexed by *i*) of a rating factor (for example, territory) to be collapsed, an observed level (indexed by *j*) of a populated combination of other rating factors, along with the number of claims (c_{ij}), the loss amount (L_{ij}) and the exposure (e_{ij}) at the level (*i, j*). Assume that the original predictive models are as in Table 1 and log link is used for all models. With the log link, u_i is specified as an offset in accordance with the EMBLEM logic. The dispersion parameter is either specified or estimated wherever appropriate.

With pre-adjustment, the response variable and the weight variable before collapsing are given in Table 2 in accordance with Proposition 1. After collapsing, the response variable and the weight variable in Table 3 are ready for simplified analysis in accordance with Proposition 2.

Table 1. Description of original predictive models

Model	Distribution	Response Variable	Weight Variable	Offset
Frequency	Poisson	Claim frequency, c_{ij}/e_{ij}	Number of exposures, e_{ij}	u_i
Severity	Gamma	Claim severity, L_{ij}/c_{ij}	Number of claims, c_{ij}	u_i
Loss cost	Tweedie(*p*)	Loss cost, L_{ij}/e_{ij}	Number of exposures, e_{ij}	u_i

Table 2. Description of pre-adjustment models before collapsing

Model	Distribution	Response Variable	Weight Variable
Frequency	Poisson	Adjusted claim frequency, $c_{ij}/(e_{ij}u_i)$	Number of adjusted exposures, $e_{ij}u_i$
Severity	Gamma	Adjusted claim severity, $L_{ij}/(c_{ij}u_i)$	Number of claims, c_{ij}
Loss cost	Tweedie(*p*)	Adjusted loss cost, $L_{ij}/(e_{ij}u_i)$	Number of adjusted exposures, $e_{ij}u_i^{2-p}$

Table 3. Description of pre-adjustment models after collapsing

Model	Distribution	Response Variable	Weight Variable
Frequency	Poisson	Weighted sum of adjusted claim frequency, $(\sum_i c_{ij})/(\sum_i e_{ij}u_i)$	Total number of adjusted exposures, $\sum_i e_{ij}u_i$
Severity	Gamma	Weighted sum of adjusted claim severity, $(\sum_i L_{ij}/u_i)/(\sum_i c_{ij})$	Total number of claims, $\sum_i c_{ij}$
Loss cost	Tweedie(p)	Weighted sum of adjusted loss amount $(\sum_i L_{ij}u_i^{1-p})/(\sum_i e_{ij}u_i^{2-p})$	Total number of adjusted exposures, $\sum_i e_{ij}u_i^{2-p}$

Implementation in SAS can be done similarly. With known Tweedie power and dispersion parameters, the GENMOD procedure can be adopted with user defined distribution.[5]

4. REMARKS

Throughout this paper, we assumed that both the Tweedie power and dispersion parameters are known. In practice, the power parameter p is often taken from prior modeling experience, while the dispersion parameter is estimated using the Pearson, Deviance or the likelihood approach [3]. Compared to the likelihood approach, an estimated dispersion parameter using either the Pearson or Deviance can be significantly different for p in the mid-range of the interval (1, 2). In the SAS environment, PROC NLMIXED may be used for simultaneous estimation of all Tweedie parameters using the code written by Flynn [7], but convergence may become a problem with a large dataset and numerous class variables. As an alternative, the code in Appendix A may be applied.

We assumed that the dispersion parameter ϕ is a constant. However, it is often more appropriate to allow ϕ to vary with different rating cells such that $\phi = \phi_{ij}$, especially in a loss cost model [6]. In such a case, if we insist on fitting a model with fixed ϕ, then a different set of weights may be necessary for an accurate solution. On the other hand, if ϕ is allowed to vary, we may put any adjustment on weights into ϕ, leaving the original weights untouched.

The choice of weights in Eq. (2.4) and Eq. (2.14) affects both the accuracy of the model estimates and the validity of hypothesis tests even if an estimate of v_j is unbiased.

Both propositions 1 and 2 can be generalized to the case with more than two rating factors. Note that multiple adjustments can be combined and sequenced with index *i* and other covariates may be combined and sequenced with index *j*.

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Appendix A

The SAS macro provided in this appendix may be used experimentally for Tweedie models. The macro is based on the orthogonal property between the mean parameter μ and the power/dispersion parameter (p,ϕ) [3], which allows their separate optimizations. It iterates until convergence between the μ-step with PROC GENMOD and the (p,ϕ)-step with PROC NLMIXED. The GENMOD procedure is easy to converge and has a handy CLASS statement, which is suitable for Tweedie models with known (p,ϕ) and high dimension of μ. This approach reduces the burden on PROC NLMIXED so that it is used only to estimate (p,ϕ) with μ assumed known.

```
****************************************************************
*                 MACRO FOR TWEEDIE MODEL                      *
*                                                              *
*       Author: Sheng G. Shi                                   *
*       Paramters:                                             *
*         dn -- dataset name                                   *
*         vformat -- list of formats                           *
*         vclass -- class variables                            *
*         wght -- weight variable                              *
*         resp -- response variable (must be non-negative)     *
*         pred -- predictors                                   *
*         clmcnt -- claim counts                               *
*         offset -- offset variable                            *
*      Warning:                                                *
*        Check output for convergence of GENMOD and NLMIXED;   *
*        Check log for results;                                *
*        Title3 will be over-written.                          *
****************************************************************;
%macro tweedie(dn=,vformat=,vclass=,wght=,resp=,pred=,clmcnt=,offset=);
/* Initialization */
title3;
data Est_save_;
  format p_ phi_ sigma_ p_lower p_upper phi_lower phi_upper
         sigma_lower sigma_upper 15.4 p_change sigma_change 15.4;
  p_ = 1.5;
  p_lower = .;
```

```
  p_upper = .;
  phi_ = 1;
  phi_lower = .;
  phi_upper = .;
  sigma_ = 1;
  sigma_lower = .;
  sigma_upper = .;
  p_change = .;
  sigma_change = .;
  call symput('p',trim(left(put(p_,15.4))));
  call symput('phi',trim(left(put(phi_,15.4))));
  call symput('sigma',trim(left(put(sigma_,15.4))));
  call symput('p_lower',trim(left(put(p_,15.4))));
  call symput('p_upper',trim(left(put(p_,15.4))));
  call symput('phi_lower',trim(left(put(phi_,15.4))));
  call symput('phi_upper',trim(left(put(phi_,15.4))));
  call symput('sigma_lower',trim(left(put(sigma_,15.4))));
  call symput('sigma_upper',trim(left(put(sigma_,15.4))));
run;
/* Maximum likelihood estimation */
%let converge = 0;
%let i=1;
%do %until ((&converge eq 1) or (&i gt 10));
  title3 "Optimization Step &i";
  %optimize(&dn,&vformat,&vclass,&wght,&resp,&pred,&clmcnt,&offset,0);
  %let i = %eval(&i+1);
    data Est_save_(drop=p_old sigma_old);
        set Est_save_ end=last;
        p_old = p_;
        sigma_old = sigma_;
        retain p_old sigma_old;
        output;
        if last then do;
          p_ = &p;
          p_lower = &p_lower;
          p_upper = &p_upper;
          phi_ = &phi;
          phi_lower = &phi_lower;
          phi_upper = &phi_upper;
          sigma_ = &sigma;
          sigma_lower = &sigma_lower;
          sigma_upper = &sigma_upper;
          p_change = abs(p_-p_old);
          sigma_change = abs(sigma_-sigma_old);
          p_old = p_;
          sigma_old = sigma_;
          if (p_change le 1e-5) and (p_change ne .)
                and (sigma_change le 1e-5) and (sigma_change ne .) then
              call symput('converge','1');
          output;
        end;
    run;
%end;
```

```
/* Results */
%if (&converge eq 1) %then %do;
  title3 'Tweedie Model with Converged Parameter Estimates';
  %optimize(&dn,&vformat,&vclass,&wght,&resp,&pred,&clmcnt,&offset,1);
  %put Converged;
  %put Power parameter = &p with 95% C.I. (&p_lower, &p_upper);
  %put Dispersion parameter = &phi with 95% C.I. (&phi_lower, &phi_upper);
  %put SAS scale parameter = &sigma with 95% C.I. (&sigma_lower,
&sigma_upper);
%end; %else %do;
  %put Not converged after 10 iterations: ;
  %put Power parameter = &p;
  %put Dispersion parameter = &phi;
  %put SAS scale parameter = &sigma;
  %put at the end of 10th iteration.;
%end;
title3;
%mend tweedie;
%macro optimize(dn,vformat,vclass,wght,resp,pred,clmcnt,offset,flag);
proc genmod data=&dn;
  format &vformat;
  class  &vclass
       /param=glm;
  p_ = &p;
  mu_ = _MEAN_;
  y_ = _RESP_;
  v_ = mu_**p_;
  if y_ gt 0 then
    d_ = 2*(y_*(y_**(1-p_)-mu_**(1-p_))/(1-p_)-(y_**(2-p_)-mu_**(2-p_))/(2-
p_));
  else
    d_ = 2*(mu_**(2-p_))/(2-p_);
  variance var = v_;
  deviance dev = d_;
  weight &wght;
  model &resp = &pred
      /link=log noscale scale=&sigma
            %if %length(&offset) eq 0 %then ;
            %else offset=&offset;;
  output out=Out_mu_ pred=yhat_;
run;
%if &flag ne 1 %then %do;
ods trace on;
ods output ParameterEstimates=Est_;
proc nlmixed data=Out_mu_;
  format p_ 15.4 phi_ 15.4;
  parms  p_=&p phi_=&phi;
  bounds 1<p_<2, phi_>0;
  n_ = &clmcnt;
  w_ = &wght;
  y_ = &resp;
  mu_ = yhat_;
  t_ = y_*mu_**(1-p_)/(1-p_)-mu_**(2-p_)/(2-p_);
```

```
  a_ = (2-p_)/(p_-1);
  if (n_ eq 0) then
    rll_ = (w_/phi_)*t_;
  else
    rll_ = n_*((a_+1)*log(w_/phi_)+a_*log(y_)-a_*log(p_-1)-log(2-p_))
        -lgamma(n_+1)-lgamma(n_*a_)-log(y_)+(w_/phi_)*t_;
  /* log likelihood of (p_,phi_) with mu_ known */
  model y_ ~ general(rll_);
run;
ods trace off;
data _null_;
  set Est_;
  if Parameter eq 'p_' then do;
    call symput('p',trim(left(put(Estimate,15.4))));
    call symput('p_lower',trim(left(put(Lower,15.4))));
    call symput('p_upper',trim(left(put(Upper,15.4))));
  end; else if Parameter eq 'phi_' then do;
    call symput('phi',trim(left(put(Estimate,15.4))));
    call symput('phi_lower',trim(left(put(Lower,15.4))));
    call symput('phi_upper',trim(left(put(Upper,15.4))));
    call symput('sigma',trim(left(put(sqrt(Estimate),15.4))));
    call symput('sigma_lower',trim(left(put(sqrt(Lower),15.4))));
    call symput('sigma_upper',trim(left(put(sqrt(Upper),15.4))));
  end;
run;
%end;
%mend optimize;
***************************************************************;
```

Here is an example that calls the **%*tweedie*** macro:

```
%tweedie(dn=CarData,
         vformat=ModelYr 4.
                 SYM $symfmt.,
         vclass=ModelYr SYM,
         wght=EExp,
         resp=LossCost,
         pred=ModelYr SYM,
         clmcnt=ClaimCnt,
         offset=LogEP
);
```

5. REFERENCES

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Biography of the Author

Sheng G. Shi has fifteen years of experience in analyzing insurance data. He has a Ph.D. in statistics from the University of Texas at Austin. He is currently with Safeco Insurance, a member of Liberty Mutual Group.