A Structural Simulation Model for Measuring General Insurance Risk

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Abstract

Motivation. As use of economic capital models expands, the need for a robust approach to the measurement of reserving and pricing risk becomes increasingly important. The paper describes a stochastic simulation model developed by the authors that has some attractive advantages over other published approaches to risk measurement. A particular issue is the need to measure reserving and pricing risk over a one-year time horizon; the model does both one-year and run-off risk measurement.

Method. The Structural Model separates overall insurance risk into systematic and non-systematic risk elements, using an Economic Scenario Generator to simulate the former and the Practical method to simulate the latter. In addition, it combines stochastic chain ladder projections of past years with a stochastic ARMA loss ratio model for recent and future years, facilitating the measurement of reserving and pricing risk in an integrated way.

Results. The Structural Model offers several benefits, described in the paper and illustrated using an empirical dataset. Illustrative validation results are also presented. These include some useful ideas about validation that may be applied to other approaches, as well.

Conclusions. The Structural Model is a practical approach to measuring reserve and pricing risk in an integrated way, over either a one-year or run-off risk horizon. Its ability to separate systematic economic risks from general claim misestimation risk is particularly relevant, given the concerns about a resurgence of inflation. It can be successfully validated using historical data on past reserve and pricing errors.

Availability. No software is being made available with the paper.

Keywords

Economic capital, stochastic reserving, financial modeling, inflation risk, economic scenario generator, ARMA model

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1. INTRODUCTION

The measurement of General Insurance[1] risk is becoming increasingly important, as insurers work to create stronger linkages between portfolio risk, capital utilization and value creation. When combined with a security standard, a General Insurance risk profile can be used to calculate the economic capital required to support a given class of business. It is also a necessary input to models used in the measurement of the Fair Value of claim liabilities and other market-consistent liability measurement schemes. The same risk profile can also be used to assess the adequacy of returns in relationship to the risk, and to set risk-based technical price margins.

This paper describes a method for measuring General Insurance risk, using a structural stochastic simulation approach. It builds upon work by many previous authors, particularly Butsic [1]; Kelly [7]; and Hodes, Feldblum, and Blumsohn [4].

We describe the method as *structural* because it seeks to separate the overall risk into (a) systematic[2] elements that are driven by key socio-economic factors such as inflation and unemployment, and (b) non-systematic elements that are more truly random in nature (although there may be a dependency structure to the randomness). Application of the method requires that the user specify the structure of the risk, in contrast to other simulation approaches which typically assume that risk can be represented by a series of independent random variables within a stationary stochastic process. The method seems particularly apropos given the heightened concerns about a resurgence of inflation.

After this introductory section, the next section provides a formal definition of General Insurance risk and articulates the desired characteristics of the model that we sought to develop. We believe the structural model performs well against these desired characteristics.

Section 3 provides a description of the structural simulation model, using a "peel the onion" approach; we start with a basic overview, and then gradually drill down to provide successive detail on the parameterization and operation of the model. The structural model requires specification of many parameters, giving the user a great deal of flexibility. The price one pays for this flexibility is

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[1] In the U.S., General Insurance is usually referred to as "Property & Casualty Insurance." In other parts of the world it is referred to as "Non-Life Insurance." Because our model was developed and tested using data from a multinational insurer, we have chosen to adopt their more global terminology.

[2] In this context *systematic risk* is a general term reflecting socio-economic risks that affect all claims; it should not be taken as limited to *equity-systematic* risk as defined in the Capital Asset Pricing Model.

the need for care in setting the parameters, and the need for validation via comparison to historical experience. We conclude this section with a discussion of alternative approaches to validation.

Finally, in Section 4 we provide some test results from the model using publicly available data from a large personal lines insurer. We compare results from the structural model to comparable results for other methods; and also show some validation results, comparing model results to historical reserve errors and pricing volatility.

2. MEASURING GENERAL INSURANCE RISK

Before describing the model itself, we begin with a definition and discussion of the problem we are trying to solve.

2.1 The Nature of General Insurance Risk

The emergence of General Insurance claim payments is subject to a number of underlying stochastic processes relating to the incidence of insured events, the recognition and reporting of claims, the adjusting of the claim, estimates of the value of the claim by the adjuster, and the claim settlement process. These underlying stochastic processes reflect both randomness and systematic variation due to changing socio-economic conditions that may influence the numbers and costs of all claims. As a result, forecasts of future General Insurance claim payments are uncertain.

> *Definition: General Insurance risk is the risk of adverse development in the cost*[3] *of unpaid claims (net of any offsetting premium or tax benefits) from their current actuarial central estimate.*

Often this risk is represented by a conveniently tractable probability distribution, such as a Lognormal or Pareto. In other instances it is represented by an empirical distribution, for example produced as the output of a hurricane or earthquake simulation model.

[3] We will define "cost" more fully below.

For an insurer who has been in business for many years, the total insurance risk associated with a particular class of business will include the risk associated with all unpaid claims on past business, and also the risk associated with potential claims on future business about to be written. Actuarial central estimates for both types of claim liabilities will be based on the actuary's interpretation of historical experience. While central estimates of the ultimate claims for older accident[4] years will be relatively certain (because many of the claims are paid, and case reserves are relatively reliable), central estimates for more recent accident years will reflect increasing levels of uncertainty (as successively fewer claims will be reported and paid). The actuary's estimation problem is depicted schematically in Chart 2.1, with the central estimates for each accident year represented by a bar and the increasing level of uncertainty represented by the dotted-line "funnel of doubt."

A key point in this schematic is that the "reserving risk" and "pricing risk" components of insurance risk are interrelated. The central estimates on past business are used to construct the central estimate claim costs for pricing of future business. If the central estimates of the former are too low, the central estimates of the latter are likely to be too low as well. The extent of dependency will depend on the characteristics of the class of business. For many classes, we would expect the two risk components to be highly correlated.

Note that under our definition not all variation in underwriting results, or all types of reserve inadequacies, would be classified as insurance risk. For example if a company knowingly followed market prices down in a softening market, any economic losses sustained would be expected, not a variance from expected. Similarly, if management chose to hold claim reserves below the central actuarial estimate, then any resulting adverse development would be expected, not a variance from expected. These types of risks are more appropriately classified as operational risks, rather than insurance risks, as they relate to failures in underwriting and financial controls.

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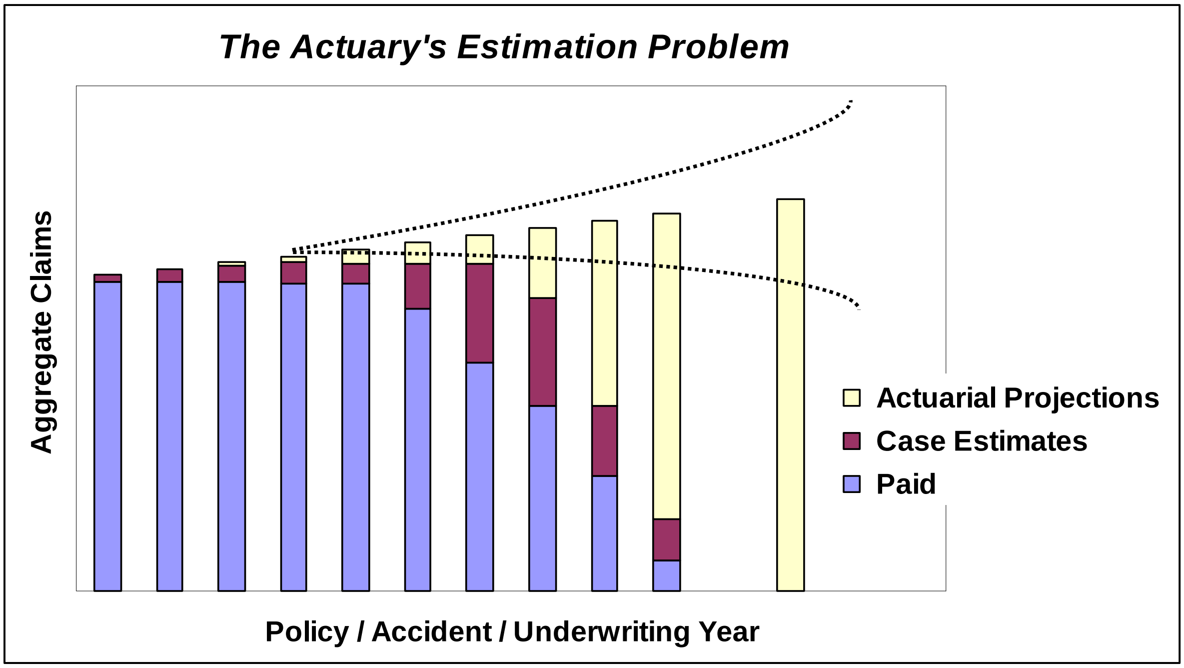
[4] For ease of exposition, the term accident year is used throughout this paper. The concepts apply equally in a policy year, underwriting year, or other cohort-based configuration of claim data.

Chart 2.1: Actuarial Estimation of Claim Liabilities Creates Insurance Risk



While our definition of General Insurance risk and the chart above provide a simple and intuitive depiction of the uncertainty, our framework needs to be refined to account for varying risk horizons. An analogy may be helpful to understand the risk horizon issue. When we discuss market or credit risk in the context of assets, we have to specify the holding period over which we want to measure the risk. For example, the default probability of an investment grade corporate bond is relatively low over a one-year holding period, but can be substantially higher over a longer period. It is highest when the bond is held to maturity.

In an analogous way, insurance risk depends on the period over which it is measured. This period is referred to generically as the risk horizon. The two horizons of greatest interest are the run-off risk horizon (i.e., to "maturity" when all claims are settled) and the one-year risk horizon. Fundamentally, these two risk horizons ask the actuary two different, but related questions:

- In the context of a run-off risk horizon: ***What is the potential adverse variation in the ultimate cost[5] of claim liabilities from the current actuarial central estimate?***
- In the context of a one-year risk horizon: ***What is the potential adverse change in the actuarial central estimate of ultimate claim costs that could occur, with the benefit of one additional year of actual claim emergence, other relevant information, and changes in circumstances that affect the valuation?***

In the context of the run-off risk horizon, the cost of the claim liabilities is measured as the present value of the claim payments and the associated claim handling expenses, discounted using a risk-free yield curve. However, in the context of a one-year risk horizon, the cost of the claim liabilities is measured as the theoretical price at which they could be transferred to a willing third party. In addition to the expected present value of the future claim payments, the price would include a margin for the cost of the economic capital required to support the risk of the liabilities. Returning to our bond analogy, the credit risk associated with a bond over its lifetime is simply the risk of non-payment of any of its coupons or principal; however, the credit risk associated with a bond over a single year will depend on credit-driven movements in the price of the bond, including changes in the probabilities of non-payment of the remaining coupons and principal and the change in the market's price of the remaining credit risk. Concepts and issues relating to alternative risk horizons are discussed more fully in a forthcoming paper by Jing, Lowe and Morin [6].

The one-year risk horizon introduces a new complexity to the problem, because it requires us to go beyond estimating the degree to which our estimates are uncertain. To measure risk over a one-year horizon we must also model how the uncertainty resolves over time — and in particular how much uncertainty resolves during the upcoming year.

[5] Technically, the risk relates to adverse movements in claim costs, net of any offsetting premium or tax benefits. Premium benefits would relate to the additional premiums due on loss sensitive insurance products. Tax benefits would relate the additional tax deductions associated with the adverse development; these would be limited by recoverability considerations. Consideration of these adjustments is outside of the scope of our paper.

Mathematically, we can express the two questions above as follows. Let:

$$C_{a,d}, \quad a = 1,...,n, \qquad d = 1,...,m, \tag{2.1}$$

represent the actual incremental paid claims on the a^{th} accident year in the d^{th} development period. Since they are the result of stochastic processes relating to claim occurrence, reporting and settlement, it is customary to treat all the $C_{a,d}$ as random variables.

We define time $t = 0$ to be the starting point of the first accident year. At time $t = n$ we will then typically have a triangular array of actual values of $C_{a,d}$, for all $C_{a,d}$ where $a + d \le n + 1$, as illustrated below for the case where $n = m = 4$.

$$\begin{bmatrix} C_{1,1} & C_{1,2} & C_{1,3} & C_{1,4} \\ C_{2,1} & C_{2,2} & C_{2,3} & . \\ C_{3,1} & C_{3,2} & . & . \\ C_{4,1} & . & . & . \\ . & . & . & . \end{bmatrix}$$

Note that the array depicted above includes an extra, empty row at the bottom. This row represents the incepting accident year, on which no claims will have occurred or been paid at time $t = n$. Inclusion of this row in the array allows us to consider pricing risk as well as reserving risk, in an integrated way.[6]

These actual $C_{a,d}$ are a sample realization from the underlying stochastic processes; they are an incomplete realization, which will be completed when the balance of the array is filled in. The *reserving* process involves developing a central estimate for all of the missing future $C_{a,d}$, except for the latest row. These latter values are developed via the *pricing* process. The reserving process typically involves a projection across the rows, applying some variant of the chain ladder method to the observed values for each accident year. Since there are not yet any observed values for the incepting accident year, the pricing process typically involves some form of projection down the columns. Insurance risk stems fundamentally from the stochastic nature of the claim payments,

[6] For simplicity we will assume that all policies are annual and they all incept on the first day of the year, such that the accident year and the policy year are identical. In actual application, one may need to consider how to treat the unexpired risk at the end of each year.

which manifests itself as (a) the inability to observe more than a single random sample of the past claim payments, and (b) the randomness of the future claim payments.

The present value of the actual outstanding claim liabilities for an accident year at time t are given by[7]:

$$L_a^{(t)} = \sum_{d=t+2-a}^{m} (v_{d+a-t-1}) \times (C_{a,d}) \tag{2.1a}$$

where $v_{d+a-t-1}$ is the discount factor applicable to payments made $d+a-t-1$ periods after t[8]. For example, $L_{n+1}^{(n)}$ are the actual outstanding claim liabilities for the incepting accident year at time $t=n$, before any claim payments have been made.

The aggregate present value of the total outstanding claim liabilities are given by:

$$L^{(t)} = \sum_{a=1}^{n+1} L_a^{(t)} \tag{2.1b}$$

Let $\hat{C}_{a,d}^{(t)}$ represent the actuarial central estimate of the expected incremental paid claims on accident year a in development period d, based on information at time t. Then the present value of the estimated outstanding claim liabilities for an accident year at time t_1, based on information at time t_2, are given by:

$$\hat{L}_a^{(t_1,t_2)} = \sum_{d=t_1+2-a}^{m} (v_{d+a-t_1-1}) \times (\hat{C}_{a,d}^{(t_2)}) \tag{2.2a}$$

The aggregate actuarial central estimate of the outstanding claim liabilities are given by:

$$\hat{L}^{(t_1,t_2)} = \sum_{a=1}^{n+1} \hat{L}_a^{(t_1,t_2)} \tag{2.2b}$$

The normal case in equations (2.2a) and (2.2b) would be for $t_1 = t_2 = n$. (When $t_1 = t_2$ we will simplify the notation to a single superscript parameter.) The introduction of the additional time variable t_2 allows us to generalize the liability estimates to reflect varying degrees of hindsight, in which the $\hat{C}_{a,d}^{(n)}$ estimated payments are successively replaced by $C_{a,d}$ actual payments, and the remaining $\hat{C}_{a,d}^{(n)}$ estimates are replaced by revised estimates $\hat{C}_{a,d}^{(n+1)}$, $\hat{C}_{a,d}^{(n+2)}$, etc.

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[7] The use of parentheses around the parameter *t* is to distinguish it from an exponent in the formulae.
[8] The nominal value of the liabilities is, of course, found by setting the present value factors to one.

As was noted at the outset of this section, under the run-off risk horizon, risk is measured from the potential variation of the ultimate cost of outstanding claim liabilities from the current actuarial central estimate. From the definitions above, we can see that General Insurance run-off risk at time $t = n$ can be measured using the probability distribution of the error quantity:

$$e_r = \hat{L}^{(n)} - L^{(n)} \tag{2.3a}$$

$$e_r = \sum_{a=1}^{n+1} \sum_{d=t_1+2-a}^{m} (v_{d+a-t_1-1}) \times (\hat{C}_{a,d}^{(t_2)} - C_{a,d}) \tag{2.3b}$$

In contrast, under the one-year risk horizon, risk is measured from the potential changes in the market-consistent value of the outstanding claim liabilities over a single year. Since the one-year risk horizon postulates the cost of the potential transfer of the outstanding liabilities to a third party at the end of the year, it is necessary to focus on valuations rather than just on net present values. For simplicity we define a market-consistent valuation by the addition of a risk margin to the calculated present value. This approach has been suggested by the CRO Forum in their recommendations for Solvency II implementation, based on cost-of-capital considerations.

$$\hat{V}^{(t_1,t_2)} = \sum_{a=1}^{n+1} \sum_{d=t_1+2-a}^{m} (v_{d+a-t_1-1}) \times [(\hat{C}_{a,d}^{(t_2)})] + M^{(t_1,t_2)} \tag{2.4}$$

Employing the definition above, General Insurance risk can be measured at time $t = n$ using the probability distribution of the error quantity:

$$e_1 = \hat{V}^{(n.n)} - V^{(n,n+1)} \tag{2.5a}$$

$$e_1 = \hat{V}^{(n,n)} - v_1 \left(\sum_{a=1}^{n+1} C_{a,t_1+2-a} + \hat{V}^{(n+1,n+1)}\right) \tag{2.5b}$$

Where the C_{a,t_1+2-a} are the diagonal of actual claim payments in the next calendar year and v_1 is the discount factor for 1 year at the risk-free rate.

In the one-year risk horizon, risk measurement is concerned both with the movement in the actuarial central estimate over the course of one year, and also with any movement in the risk margin during the same period. At its current state of development, the structural simulation model focuses solely on the movement of the actuarial central estimate. The authors hope to incorporate movements in the risk margin in a subsequent version.

2.2 Desired Characteristics of the Risk Model

While many papers have been written proposing stochastic models for measuring reserving risk, the majority of those papers do not address how the models can be extended to include pricing risk. We sought to develop a model that would measure the totality of insurance risk, incorporating both reserving and pricing components, and explicitly recognize that the two components are interrelated.

General Insurance risk can be measured in two ways.

- *Hindsight Testing*, in which one measures historical claim estimation errors by comparing actuarial central estimates made in the past to the actual subsequent claim emergence. (For example, see Jing, Lebens, and Lowe [5]).
- *Stochastic Modeling*, in which one postulates a model of the stochastic process and measures the risk by applying the model with parameters derived from historical claim experience. Stochastic models can be analytic (for example, regression-based as in Murphy [10]), or simulation-based (for example, Bootstrap, as outlined by England and Verrall [3]).

The principal advantage of hindsight testing is that it is non-parametric; in other words, it does not rely on the assumption of any specific underlying model. Risk measures based on hindsight testing therefore reflect the total risk present in the underlying stochastic processes (even if we can't specify them). Its principal disadvantage is that it requires an extensive history of past estimates and associated run-off claim data. In contrast, stochastic modeling requires less historical information; the data requirements are usually the same as those necessary for estimating claim liabilities, typically the current paid or incurred loss development triangle. The principal disadvantage of stochastic methods is that they are model dependent, requiring estimation of parameter and model error. If one is to use the stochastic modeling approach, it therefore seems important to perform validation testing of the approach. Unfortunately, some of the validation testing of published stochastic models indicates that they perform relatively poorly in many real-world situations[9]. We sought to develop a stochastic model that performed well in validation tests.

As was mentioned earlier, the stochastic processes underlying General Insurance claim emergence include systematic as well as non-systematic (i.e., random) elements. Both general price

[9] See, for example, the ROC/GIRO Working Party Paper [12], presenting test results suggesting that the Mack method may understate risk.

inflation and sector-specific cost inflation (for example, traumatic medical care or auto damage repair) affect the ultimate cost of claims. Levels of economic activity, unemployment rates, capital market and credit market conditions all may also affect claim frequency and severity. Historical claim development triangles will reflect the changing state of these systematic variables. In contrast, many of the published stochastic methods assume that the historical loss development is a realization of a stationary random process. We sought to develop a stochastic model that accommodated both systematic and non-systematic components of risk. Such an approach has the advantage of offering much more explanatory value, for example by allowing the user to isolate the impact of inflation risk. This seems particularly relevant in the current economic environment.

To the extent that claims from different classes of business are affected by the same socio-economic variables, the model will provide insight into the correlation of insurance risk across classes of business. Also, since these socio-economic variables may influence asset behavior (at least in the longer term), they provide a linkage between risks on the asset and liability sides of the balance sheet.

Another shortcoming of many of the published stochastic models is that they rely on a particular projection method (typically the chain ladder), usually applied to paid claim development data to avoid the problems of downward development present in reported claim development data. While such models are reasonably well-behaved when applied to classes of business with stable short-tail claim development, several problems arise when they are applied to classes with more volatile long-tail development. First, they often "blow up" when applied to these datasets, indicating extremely high uncertainty. Second, since they usually assume that the development factors from one maturity to the next are independent, they often mistake volatility in the timing of payments for volatility in the ultimate amounts of the payment. More fundamentally, they are really measuring the uncertainty associated with a paid chain ladder projection; and for volatile long-tail lines most actuaries would agree that a paid projection is not an appropriate estimation method, due to the high leverage in the compound development factors. For these classes, actuaries would be more likely to utilize the reported chain ladder method, or a Bornhuetter-Ferguson approach, or even an expected loss ratio approach as the basis of their estimates of the ultimate claims — especially for the more recent accident years. We sought to develop a stochastic model that more faithfully reflected the actual approach to estimation that would take place for these classes.

Finally, we sought to develop a stochastic method that would measure risk over a one-year risk horizon in addition to a run-off risk horizon. The need to look at risk over a one-year horizon is becoming increasingly important in the context of economic capital. Unfortunately, virtually all of the published stochastic models have focused entirely on the run-off risk horizon question. To our knowledge, only Wacek [13] and Wuthrich, Merz and Lysenko [14] have addressed the issue of modeling the one-year risk horizon.

In summary, we sought to develop a stochastic model that met the following needs.

- It would measure reserving and pricing risk in an integrated way, capturing the relationship between the two.
- It would perform well in validation tests.
- It would incorporate both systematic and non-systematic sources of risk.
- It would facilitate the measurement of correlation between classes of business.
- It would reflect actuarial estimation methods actually employed for each class of business.
- It would be capable of measuring General Insurance risk in the context of either a one-year or a run-off risk horizon.

3. THE STRUCTURAL SIMULATION MODEL

In this section, we describe the mechanics of the structural simulation model. We describe both the general concept and the current implementation. The latter reflects where we have traveled thus far, while the former reflects where we ultimately want to go with the development and implementation of the model.

3.1 Overview of the Structural Model

In the structural simulation, claim emergence and development is assumed to be decomposable into the following two components:

- An underlying stationary claim emergence pattern that is the culmination of underlying stochastic claim processes subject to random noise.
- One or more socio-economic factors which distort the stationary emergence pattern, "stretching" or "shrinking" it in some way. Each of these socio-economic factors may itself entail a stochastic process subject to random noise.

Mathematically, we can describe this generically as:

$$C_{a,d} = f(\overline{C}_d, Z) + \varepsilon_{a,d} \tag{2.6}$$

Where $\overline{C}_d$ is the expected payment generated from the underlying stationary emergence, Z is the set of socio-economic factors that modify the payment, and $\varepsilon_{a,d}$ is the residual random error from the stochastic processes. Note that in this general form Z can include past, as well as current values of any socio-economic variables. For example, if calendar year inflation affects development year claim payments, then Z would need to include a vector of calendar year inflation rates for the entire development period.

Since the socio-economic factors affect all claims, they represent the systematic aspect of insurance risk, while the error term $\varepsilon_{a,d}$ represents the non-systematic risk. In the balance of the paper, we will sometimes refer to the socio-economic variables as systematic risk variables.

Typically, the socio-economic factors will be represented by a system of stochastic equations that describe the behavior of each variable over time and how each variable interacts with other variables. Because the equations describe a dynamic system with structure, they are sometimes referred to as a structural model. Hence, we describe our overall model as a structural simulation.

To apply the model, one must determine the relevant Z for the class of business, and the functional form of f. Methods for doing this are beyond the scope of this paper. Taylor [11], Zehnwirth [15], and Christofides [2] have suggested methodologies that may represent workable approaches. In addition, some of the approaches we have taken reflect as-yet unpublished research by the authors' firm.

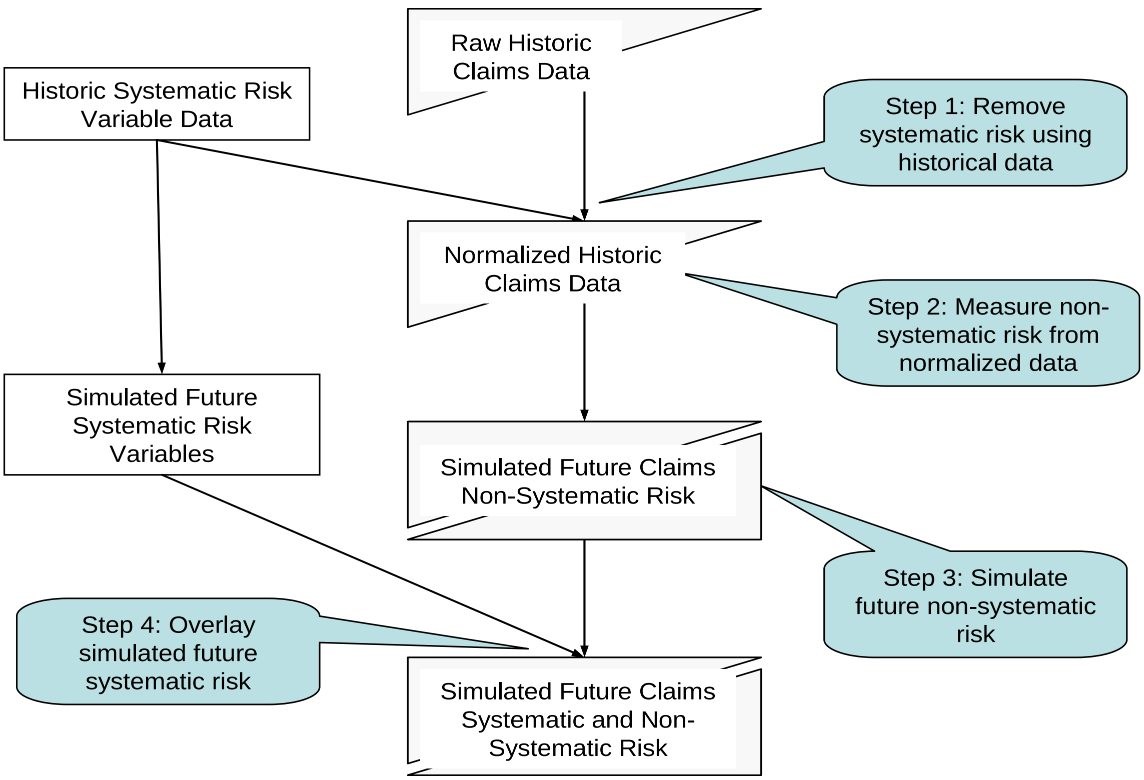
As a matter of practicality, rather than removing the effects of Z completely from the claim development, it is often simpler to substitute constant "steady-state" expected values of the systematic risk variables Z for the actual varying historical values. Then $\overline{C}_d$ would reflect the stationary emergence under the chosen steady-state values of Z, and the "distortions" would reflect variation of Z from those chosen constant values. The chosen steady-state values can either be normative long-term historical means or current expectations of long-term future means.

Chart 3.1 presents a high-level schematic of the key steps in applying the structural simulation model. As can be seen in Chart 3.1, the model proceeds in four distinct steps. The first two steps entail parameterization of the model; the last two entail running it.

- The first step is to remove systematic risk elements from the historical data, by substituting the effect of constant normative systematic variables for the varying effect of the historical values of those variables.

- Once the systematic risk elements have been normalized out of the historical data, the triangle should reflect only non-systematic noise emanating from a stationary stochastic process. Stationarity can be tested at this point to assure that it is present within a reasonable tolerance. The second step is to measure the amount of non-systematic noise in the normalized historical data and select non-systematic risk parameters for the simulation.

Chart 3.1: Overview of Structural Simulation Procedure



- The third step is to simulate future non-systematic risk for purposes of applying to future development, using the parameters developed in the second step. For each trial, we simulate future development factors and other projection calculations based on the observed historical volatility of the normalized stationary data set.
- The fourth and final step is to overlay future systematic risk. To do this, we use an Economic Scenario Generator ("ESG"), to generate plausible paths of future values for each systematic risk variable. For each trial, we adjust the simulated future development for the effect of the difference between the normative values of systematic variables and the simulated values for that specific trial.

The output from the model are simulated distributions of e_r and e_1, the run-off and one-year claim estimation errors, respectively. Estimation errors can be calculated on a nominal basis, i.e., by setting the discount factors, $(v_{d+a-t-1})$ equal to one. Alternatively, they can be calculated on a present value basis, typically using discount factors based on the risk-free yield curve.

Each of the steps outlined above will be discussed in greater detail in subsequent sections, but before jumping directly to the full detail, we present a simplified example to help the reader understand the basics of the process. The same four steps outlined above are presented in the example. Note that the example ignores a number of technical parameterization issues, which are covered in the subsequent sections.

3.2 A Simplified Example

In Step 1, we begin the model parameterization effort by removing the systematic risk from the historical development triangle. For the purposes of our simple example, we will assume that the only systematic risk is monetary inflation, as measured by the U.S. Consumer Price Index (CPI). Further, we will assume that inflation fully accrues up to the time of the claim payment. Removing the systematic inflation risk requires that we de-cumulate the cumulative claim payment triangle into an incremental triangle, adjust the incremental claim payments to reflect stationary inflation conditions, and re-accumulate the adjusted incremental claim payment triangle to produce a normalized cumulative claim payment triangle.

In our example in Chart 3.2, the actual historical incremental claim payment amounts are 'inflated' by the actual historical CPI index to 2007 levels, and then 'deflated' using an inflation index reflecting constant 2.5% inflation. The net adjustment factors are shown in the upper left table of Chart 3.2. For example, the actual claim payments for the first evaluation period for accident year 2005 are $755. Since these payments occurred in calendar year 2005, they would be inflated to 2007 levels by dividing them by .964; then they would be deflated to reflect constant inflation by multiplying them by .952. The net of these factors is .9874, producing the adjusted paid claims of $745.

In this case, the best choice for the constant inflation rate would be the expected future inflation rate, so that the central value of the resulting claim projections would explicitly reflect current inflation expectations.

Chart 3.2: Step 1 — Removing Systematic Risk

Factors to adjust payments from actual to expected inflation

Calendar Year	Actual Inflation Index	Expected Inflation Index	Net Adjust. Factor
2004	0.948	0.929	0.9797
2005	0.964	0.952	0.9874
2006	0.981	0.976	0.9941
2007	1.000	1.000	1.0000

Actual cumulative paid claim triangle

Accident Year	Earned Premium	Evaluation Date 12	24	36	48
2004	2,243	650	1,049	1,257	1,406
2005	2,340	755	1,218	1,477	
2006	3,288	881	1,483		
2007	2,937	950			
2008	3,000				

Normalized cumulative paid claim triangle

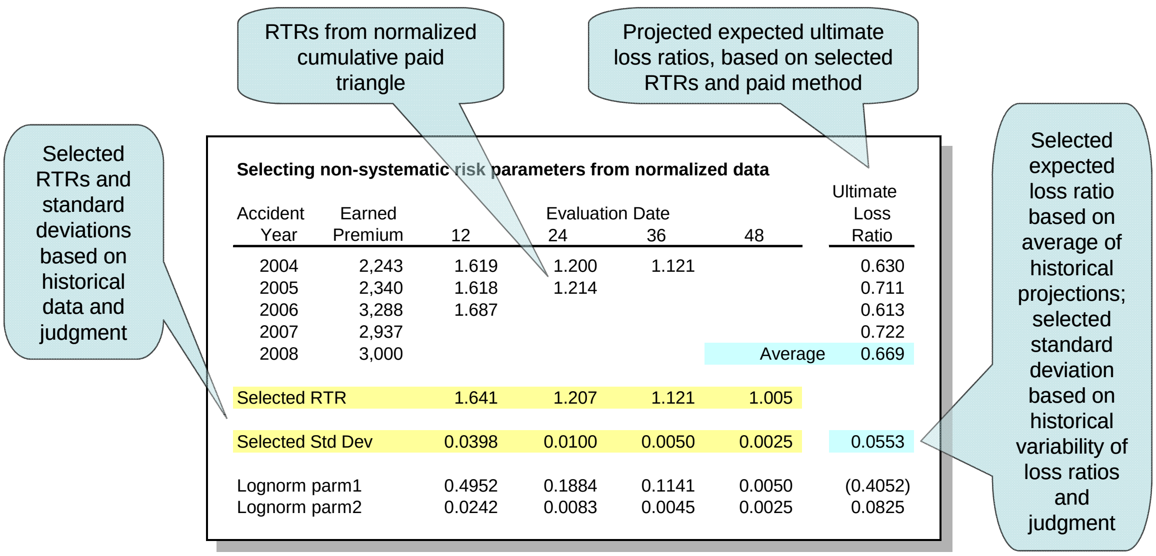
Accident Year	Earned Premium	Evaluation Date 12	24	36	48
2004	2,243	637	1,031	1,237	1,387
2005	2,340	745	1,206	1,465	
2006	3,288	876	1,478		
2007	2,937	950			
2008	3,000				

The historic claim data is normalized by removing historic inflation disturbances from the incremental payments in each calendar year (i.e., by diagonal) and substituting constant inflation expectations

The remaining noise in the development pattern implied by the normalized cumulative loss triangle is attributable solely to non-systematic factors. The normalized triangle can be tested to confirm that the remaining noise is reasonably random, indicating that the normalized triangle reflects a stationary development process.

In Step 2, we develop the parameters necessary to simulate future non-systematic risk. Chart 3.3 below displays the parameterization process. For this simple example, we will assume that (a) the future paid claims on all accident years follows a stochastic paid chain ladder process; and (b) the ultimate losses for the current accident year follow a lognormal process around a temporally stationary expected loss ratio. Ignoring sampling error, the parameters for the stochastic chain ladder process are based on the historical normalized report-to-report development factors (RTRs). Since the observed normalized RTRs are random draws from the underlying lognormal distribution, we can calculate the lognormal parameters from the means and standard deviations of the observed RTRs at each development maturity. The standard deviations for the last two development periods are judgmentally selected, as is the mean for the last development period.

Chart 3.3: Step 2 — Selecting Non-Systematic Risk Parameters



Selecting non-systematic risk parameters from normalized data

Accident Year	Earned Premium	Evaluation Date 12	24	36	48	Ultimate Loss Ratio
2004	2,243	1.619	1.200	1.121		0.630
2005	2,340	1.618	1.214			0.711
2006	3,288	1.687				0.613
2007	2,937					0.722
2008	3,000				Average	0.669
Selected RTR		1.641	1.207	1.121	1.005	
Selected Std Dev		0.0398	0.0100	0.0050	0.0025	0.0553
Lognorm parm1		0.4952	0.1884	0.1141	0.0050	(0.4052)
Lognorm parm2		0.0242	0.0083	0.0045	0.0025	0.0825

The selected RTRs can be interpreted as the expected development at the expected inflation rate. The standard deviations can be interpreted as the variability around the selected RTRs that is due to non-systematic risks associated with the underlying stochastic claim development process. For example, from 12 to 24 months we expect cumulative paid claims to develop upward by 64.1%; the actual development in any year will vary lognormally from that expected value, with a standard deviation of 3.98%.

We also project the expected ultimate loss ratios for the four past years, and measure the mean and the standard deviation of the loss ratios across the four years. This gives us the parameters for the stochastic lognormal accident year loss ratio process, applicable to the 2008 accident year. In our example, the average loss ratio over the past four years is .669. Again, ignoring sampling error, we will assume that .669 is the expected loss ratio for all accident years, and that the standard deviation of the past four years around that average is the expected standard deviation for the 2008 accident year loss ratio due to non-systematic risk.

Based on our selected parameters, the expected unpaid claims on the prior accident year are $1,898. The expected losses on the current accident year are $2,007.

Having parameterized the model, we are now ready to begin the simulation process. In Step 3 we simulate the future emergence of claims using the paid chain ladder model, and the expected loss ratio model for the current year. This process is displayed in Chart 3.4.

We start with the current levels of paid losses (from the actual triangle, the first diagonal in the lower table). We then simulate the future emergence of the past accident years via random draws of the lognormal RTRs to complete the triangle and obtain ultimate losses. For example, the simulated RTR of 1.122 for the 2005 accident year from 36 to 48 months is a random draw from a lognormal distribution with a mean of 1.121 and a standard deviation of .0050 (the selected parameters in Chart 3.3). When applied to actual paid losses of $1,477, we obtain simulated cumulative claim payments at 48 months of $1,657. The same process simulates the development from 48 to ultimate, producing simulated ultimate losses for the 2005 accident year of $1,671.

To simulate losses for the pricing accident year, we take the average of the simulated past year loss ratios and perform a random draw from a lognormal distribution with that average as our mean. In the simulation trial shown, the average loss ratio is .673 (versus an original expectation of .669) and the simulated loss ratio is .744.

As a final step, the simulated 2008 losses are "spread back" to create simulated claim payments using the simulated RTRs for the current accident year.

With this step, each trial is an alternative realization of the ultimate claim liabilities for past and current accident years, reflecting only non-systematic risk, and with constant normative systematic levels. Note that the ultimate claims for the current accident year are dependent on what happens with the development of the past accident years. When simulated RTRs are high, ultimate claims for past years will be high, and the average historical loss ratio (used as the expected loss ratio for the current accident year) will be high. Thus, reserve and pricing risk are inter-linked in the model. In our simulated trial, the actual unpaid claim liabilities on past accident years are $1,944 (2.4% higher than expected), and the expected claim liabilities on the current accident year are $2,231 (11.1%

higher than expected). The latter difference reflects the combined effect of both a higher expected value, and a higher than expected random draw around the higher expected value.

Chart 3.4: Step 3 — Simulating Non-Systematic Risk

Simulated RTRs based on assumed lognormal distribution and selected parameters

Simulating future RTRs and ELRs

Accident Year	Earned Premium	Evaluation Date 12	24	36	48	Ultimate Loss Ratio
2004	2,243				1.006	0.631
2005	2,340			1.122	1.008	0.714
2006	3,288		1.208	1.119	1.005	0.613
2007	2,937	1.672	1.211	1.122	1.001	0.735
2008	3,000	1.668	1.223	1.120	1.003	
2008					Average	0.673
2008					Simulated	0.744

Expected ultimate loss ratio for latest accident year is average of prior years; simulated ultimate loss ratio is based on assumed lognormal distribution with selected CV

Squaring the triangle using simulated RTRs and ELRs

Accident Year	Earned Premium	Evaluation Date 12	24	36	48	Ultimate Losses
2004	2,243				1,406	1,415
2005	2,340			1,477	1,657	1,671
2006	3,288		1,483	1,791	2,003	2,014
2007	2,937	950	1,588	1,922	2,157	2,160
2008	3,000	973	1,623	1,985	2,224	2,231

Dark shaded: actual cumulative paid claims to-date
Light shaded: simulated values based on the paid chain ladder method and simulated RTRs

A key point of the model is that it is a mixture of a stochastic chain ladder and a stochastic loss ratio model. In the simple example, the former is applied to prior accident years and the latter is applied to the future accident year. As will be seen subsequently, the mixing of the two models can be generalized to include use of one or the other, or a mixture of the two, for any accident year.

To complete our example, all that is left to do is reintroduce future systematic variability into our model. This is step 4 in the process, displayed in Chart 3.5.

To reintroduce systematic risk, the simulated triangle from Step 3 is decumulated into incremental paid claims, the incremental paid claims are adjusted to reflect simulated inflation, and the triangle is re-accumulated. The adjustment factors are calculated in a manner analogous to Step 1. Future deviations from the long term expected inflation rate are calculated by comparing the constant inflation index based on our 2.5% inflation expectation to a simulated inflation index using an ESG. The net adjustment factors are applied to the simulated incremental payments; after re-accumulating these incremental amounts, we have the simulated ultimate claims reflecting both systematic and non-systematic risk.

To produce the final output, the simulated future claim payments are discounted for time value of money using a risk-free yield curve derived from swap rates. This gives us a distribution of the present value claim liabilities for past accident years, as well as the current accident year. Subtracting the expected values gives us the estimation error distribution.

The above example illustrates the structural approach to simulating ultimate losses, capturing both reserving and pricing risk in an integrated model. The example reflects a number of simplifications over the actual approach that we have implemented, including:

- The use of a simple calendar year inflation model for systematic risk;
- Exclusive reliance on the paid development projection method, without consideration of the information value of case-basis claim reserves or claim counts;
- Use of the historical average loss ratio for the current year, without consideration of changes in price levels or the underwriting cycle; and
- No consideration of parameter risk in the use of the lognormal model for the RTRs and the current year loss ratio.

In subsequent sections we will address these simplifications.

Chart 3.5: Step 4 — Overlaying Systematic Risk

Factors to adjust payments from expected to simulated inflation

Calendar Year	Expected Inflation Index	Simulated Inflation Index	Net Adjust. Factor
2007	1.000	1.000	1.0000
2008	1.025	1.030	1.0049
2009	1.051	1.063	1.0117
2010	1.077	1.095	1.0167
2011	1.104	1.133	1.0266
2012	1.131	1.171	1.0346

Simulated cumulative payments -- without systematic risk

Accident Year	Evaluation Date 12	24	36	48	60
2004	650	1,049	1,257	1,406	1,415
2005	755	1,218	1,477	1,657	1,671
2006	881	1,483	1,791	2,003	2,014
2007	950	1,588	1,922	2,157	2,160
2008	973	1,623	1,985	2,224	2,231

The simulated constant-inflation claim data is disturbed by re-introducing simulated inflation from the ESG; net inflation adjustment factors are applied to the incremental payments by calendar year diagonal

Simulated cumulative payments -- with systematic risk

Accident Year	Evaluation Date 12	24	36	48	60
2004	650	1,049	1,257	1,406	1,415
2005	755	1,218	1,477	1,658	1,672
2006	881	1,483	1,792	2,007	2,019
2007	950	1,591	1,929	2,168	2,171
2008	978	1,635	2,003	2,248	2,256

Finally, the example illustrates how the general structural framework can be applied to the run-off risk horizon to measure reserve and pricing risk. The implementation of the one-year risk horizon is significantly different from that of the run-off horizon, but the general structural concept of simulating systematic and non-systematic risk separately remains.

3.3 Removing Historical Systematic Risk

The starting point in building a structural simulation model is a body of historical claim experience, consisting of historical premiums and claim development triangles for a class of business. These could either be the available actual experience of a company, or they could be the experience of peer competitors, drawn from published Schedule P information. In our experience, the latter is often useful as a supplement to the company's own data. Our firm maintains a Schedule P database extending back in time more than twenty-five years for this purpose.

Typically, we make use of both a paid claim and a reported claim triangle. These triangles reflect the actual socio-economic conditions that existed in the historical period. Once the relevant social-economic variables are identified and their functional relationship to the claim development is specified, the historical triangles can be restated from their historical-conditions basis to a normative constant-conditions basis.

In the current implementation of the model, the only economic variable utilized is inflation. We use the inflation model based on that first proposed by Butsic [1]. The core of the Butsic approach is that the impact of inflation on claims paid in a particular period is a blend of the accident date and the payment date (i.e., calendar date). Mathematically, this can be expressed as:

$$Q_{a,d} = (I_a / I_0)^{\alpha} \times (I_{a+d} / I_d)^{(1-\alpha)} \tag{3.1}$$

Where: $Q_{a,d}$ is the blended inflation index for payments in accident year *a* and development period *d*;

I_t is the actual inflation index value at time *t*; and

α is the blending factor that determines the degree to which the accident year or payment year inflation predominates.

The extent of blending will vary with the characteristics of the line, depending on the degree to which claim costs are related to price levels at the time of the accident versus price levels at the time of payment. For example, since Workers Compensation wage loss benefits are keyed to the average weekly wage of the worker at the time of the injury, it is reasonable to assume that the accident date model predominates for indemnity claim payments. Conversely, since Workers Compensation medical benefits are paid at costs prevailing at the time of payment, it is reasonable to assume that the payment date model predominates for medical claim payments. If one is modeling Workers Compensation claim development on a combined basis, it would be reasonable to assume an intermediate value of the α blending factor.

The estimation of the α parameter from empirical data is difficult; our implementation therefore makes a judgmental selection of α. Since our goal is to measure volatility around the mean, and not predict the mean itself, the choice of α need not be based on "best fit" considerations. Sensitivity tests can demonstrate the sensitivity of the overall volatility to the selection of alternative values for α.

As noted above, the current implementation of the removal of systematic risk involves the substitution of a "steady-state" value of Z. Such an approach results in incremental inflation adjustments that take the form of:

$$A_{a,d} = [(I_a / I_0)/(R_a / R_0)]^{\alpha} \times [(I_{a+d} / I_d)/(R_{a+d} / R_d)]^{(1-\alpha)} \qquad (3.2)$$

Where: $A_{a,d}$ is the excess inflation over the long term expected inflation rate for accident year *i* and development period *d*;

R is the inflation index constructed from expected inflation; and

all other variables retain the same definition as above.

The adjustment factor above removes only the net deviation from the expected inflation rate.

The inflation index is chosen so that it relates specifically to the underlying drivers of the claim settlement costs for the class of insurance being modeled. For example, in Workers Compensation, the inflation index could be a mixture of medical cost inflation and wage inflation. In Auto Liability it could be a mixture of auto repair costs, legal costs, and general inflation. The inflation adjustment is applied to the paid development triangle to obtain an adjusted paid triangle. Typically, the actual case reserve triangle is then added to the adjusted paid triangle to produce an adjusted reported triangle; however, it would be possible to adjust the case reserves for inflation as well.

In addition to monetary inflation, if other systematic variables have been introduced, these would also be factored out of the claims experience in a similar manner.

3.4 Parameterizing Non-systematic Risk

The objective of removing systematic risk from the triangle is to reduce the remaining variability in the triangle to random noise of the type that would be generated from a stationary stochastic claim development process. Once the systematic risk has been removed, the adjusted triangle can be tested to see whether or not it reasonably meets the stationarity criterion, by inspection of the standardized residuals along the accident period, development period, and calendar period dimensions.

In some instances, the test results are not very good, indicating that there is still an unexplained factor influencing one or more of the three dimensions. At this stage one has three choices: (1) go back, and identify and add an additional systematic variable that explains the remaining pattern in the residuals; (2) go forward, but allow for correlation in the RTRs along the relevant dimensions; (3) go forward, accepting that the test results are "good enough". From our experience, the second option is sometimes a reasonable compromise, with explicit correlation in the factors along a calendar year diagonal. This introduces acceleration or deceleration in the calendar year payments or claim reporting.

In the simple example presented earlier, we illustrated how the non-systematic portion of the structural model combined a stochastic chain ladder model with a stochastic loss ratio model. The parameterization of each of those two components in the structural simulation model is described below.

3.4.1 The Stochastic Chain Ladder Method

Once suitably stationary adjusted paid and reported claim development triangles have been produced, the structural simulation model calculates RTRs at each maturity. These are assumed to be a random sample drawn from a stationary distribution.

At each maturity, one must choose a form of the distribution (e.g., lognormal) for the RTRs, and choose parameters for the selected distribution by treating the available observations as a sample. Typically, one would use the sample weighted mean across all observations as the mean RTR, as this is consistent with the assumption that the adjusted triangle reflects a stationary process. However, this is not required by the model; the user may select any suitable set of mean RTRs. To the extent that there are outliers or externalities that would suggest an alternative selection, the model allows the user to override any of the weighted average RTRs with a selected value.

The sample variances of the RTRs understate the overall risk, because they do not contemplate parameter risk. In order to properly consider total risk, we increase the sample variances of the RTRs to account for the parameter risk component of total risk. To do this, we used a by-product of the unpaid claims variability approach developed by Mack [8] and Murphy [10]. Murphy's equations, which were developed in a different but fundamentally equivalent form to those in the Mack paper, provide a quantification of process and parameter risk related to each RTR which are used in his final determination of overall unpaid claims risk. In our model, we calculate Murphy's parameter risk component only and combine it with the process risk determined above to estimate the total risk associated with each RTR.

The parameterization of the RTRs allows us to simulate future paid development or future reported development "across the row" for each historical accident year in the triangle. This gives us simulated estimates of ultimate claims using either a paid chain ladder or a reported chain ladder method. Based on experience, our preference is to use reported claim development data as the basis for the model.

3.4.2 The Stochastic Loss Ratio Method

The structural simulation model also uses a stochastic loss ratio method to project "down the column" to obtain ultimate claims. This is used for the current accident year, where there are no reported losses; and it can also be used for recent accident years in volatile classes such as excess liability, where the paid and reported chain ladder projections are highly volatile.

Rather than using the average loss ratio, the model uses a stochastic auto-regressive moving average (ARMA) model to project future loss ratios. The ARMA method is a simple time series method used to project future values based on past values.

The functional form of the ARMA model is:

$$LR_a = [LR_{a-1} + MRF \times (LR_\mu - LR_{a-1}) + MF \times (LR_{a-1} - LR_{a-2})] \times \varepsilon \qquad (3.3)$$

Where: LR_a is the simulated loss ratio for accident year *a*;

LR_μ is the selected normative long term average loss ratio;

MRF is the selected mean reversion factor;

MF is the momentum factor; and

ε is the percentage error factor, assumed to be an independent lognormal random variable with a mean of one.

In its traditional form, the ARMA model has an additive error term. In our model, we adjusted the error to be a multiplicative lognormal error factor to ensure that LR_a is never negative. While we have chosen to use a lognormal model for loss ratios, it would be relatively easy to use a different distributional form, for example to achieve a "fatter tail" in the simulated loss ratios.

Since the lognormal error term has a mean of one, the expected value of LR_a is given by equation 3.3 without the ε term.

The second term of equation 3.3 causes the loss ratios generated by the ARMA model to be mean reverting over time; the higher the value of the *MRF* parameter, the greater the mean reversion tendency. The third term causes the loss ratio to move in the same direction as it did between the prior two accident years; the higher the *MF* parameter, the greater the momentum tendency. In combination, the two terms can be used to introduce apparent "cyclicality" to the time series, a behavior that mimics that of actual loss ratio time series.

The *MRF* and *MF* parameters can be estimated in any number of ways. Our implementation makes judgmental selection for the *MRF* and *MF*. Since our goal is to measure volatility around the mean, and not predict the mean itself, the choices of *MRF* and *MF* need not be based on best-fit parameterization.

3.4.3 Mixing the Two Projection Methods

The chain ladder projections rely solely on the reported or paid claims within the given accident year to project its ultimate claims, ignoring all knowledge of the experience in adjacent accident years. Conversely, since the ARMA loss ratio projections rely solely on the prior two accident year loss ratios, the loss ratio method ignores all knowledge of the paid or reported claims in the given accident year to project its ultimate claims. This is obviously necessary for future years, where there are no paid or reported claims. It is equally useful for past accident years where the actual cumulative paid or reported claims have no credibility. Jing, Lebens and Lowe [5] outline a method for empirically measuring the skill of an actuarial projection method; when measured skill is equal to or less than zero, the projection is more volatile than the overall volatility around the expected loss ratio, and the expected loss ratio approach is a more accurate estimator. Skill can be measured by maturity, providing a basis for deciding when to use the chain ladder method and when to use the expected loss ratio method.

Between these two extremes, the structural simulation model allows for a blending of the two methods. For each accident year, the user selects the method that will determine the selected ultimate losses. For each year, the user may chose to set the ultimate claims equal to any of the following: (a) the results of the chain ladder method (either paid or reported); (b) the expected loss ratio method multiplied by the premium; or (c) the Bornhuetter-Ferguson result (either paid or reported), which is viewed as a weighted average of the two prior methods with weighting based on the expected reporting or payment pattern.

The simulation proceeds iteratively, starting with the oldest accident year, determining the selected ultimate losses for that year, and then using the result as an input to the next year. (The selection for the oldest two years must be determined by the chain ladder method.)

For example, for a model involving ten prior plus the current accident year, the selections might be:

{CL, CL, CL, CL, CL, CL, CL, BF, BF, ELR, ELR}

The intent is to imitate the actual reserving and pricing process, with the selected methods reflecting that which might actually be used on the specific class of business in the particular circumstances.

It is important to understand that the ARMA method uses the *selected* ultimate loss ratio from the two prior years to simulate the loss ratio for the next year. The error term in the ARMA model is estimated based on the variance of the errors between the chain ladder projected loss ratio and the ARMA projected loss ratio over all accident years that use the chain ladder method as the selected ultimate projection method. We do not use accident years where the selected ultimate is based on the Bornhuetter-Ferguson or loss ratio methods, as this would introduce a downward bias in the estimated variance since both of these latter methods rely on the ARMA projections. Thus, the error term can be interpreted as the error associated with the ARMA model's ability to accurately project our best estimate of ultimate losses.

In summary, non-systematic risk is introduced into the simulation in three ways:

1) In the chain ladder method through the simulated RTRs, where the selected distribution and parameters reflect non-systematic variability inherent in the claim reporting or payment process.

2) In the ARMA loss ratio method through the simulated loss ratios, where the selected variance reflects non-systematic variability inherent in the ultimate loss ratio over time.

3) In the interaction between the two methods, as the simulated ultimate claims from the chain ladder method enter into the ARMA method, affecting the expected loss ratio for the next year.

Once the ultimate claims are simulated for an accident year, the simulated paid RTRs are used to construct the claim payments that produce that ultimate value. The RTRs are scaled up or down to match the ultimate value.

3.5 Measuring and Simulating Systematic Risk

Systematic risk is incorporated via an economic scenario generator (ESG). The relevant socio-economic variables are simulated using the ESG. A detailed description of ESGs is beyond the scope of this paper. The reader is referred to Mulvey and Thorlacius [9] for a description of Towers Watson's Global CAP:Link ESG, which is used in the current structural model. Global CAP:Link uses a system of stochastic differential equations to generate plausible future paths of a variety of economic variables, including equity market returns, treasury yield curves, credit spreads above treasuries for several key debt instruments, GDP growth rates, unemployment rates, and several key inflation rates. It generates these indices for multiple economies, and also generates dynamic foreign exchange rates between the economies.

In addition to the standard economic variables, Global CAP:Link has the facility to generate customized indices that are related to the standard indices. This facility is used, for example, to generate medical inflation rates that are linked to the CPI and GDP growth rates.

Each scenario from the ESG represents a plausible future path of the economy. The parameters of the system of equations are selected to reproduce a stylized set of facts developed from the historical data. Examples would be the frequency of inverted yield curves, the volatility of equity returns over various holding periods, and the degree of correlation between interest rates and inflation rates over selected time horizons. Economic principles such as purchasing power parity also affect the form and parameterization of the equations.

Any of the generated indices can be used to induce systematic risk into the claim development simulation, to the extent that they are believed to be drivers of claim frequency or claim settlement costs. For example, the frequency of Workers Compensation claims might be related to levels of unemployment, or D&O claim experience might be linked to the behavior of the stock market. Many liability lines are subject to *social inflation* in addition to monetary inflation, which can also be modeled as a systematic risk variable.

The economic scenario indices can be generated on a monthly, quarterly, or annual basis.

The Global CAP:Link scenarios are *real-world* scenarios, suitable for modeling the risk of changes in future socio-economic conditions over time. They are not *risk-neutral* scenarios and therefore are not suitable for valuation of assets or liabilities with prices that are sensitive to the market price of risk.

In the current model, for each trial the simulated inflation index for each class of business is used to adjust the paid losses to account for the difference between expected cumulative inflation and actual cumulative inflation. As was indicated earlier, the process uses the Butsic formula to mix accident date and payment date inflation.

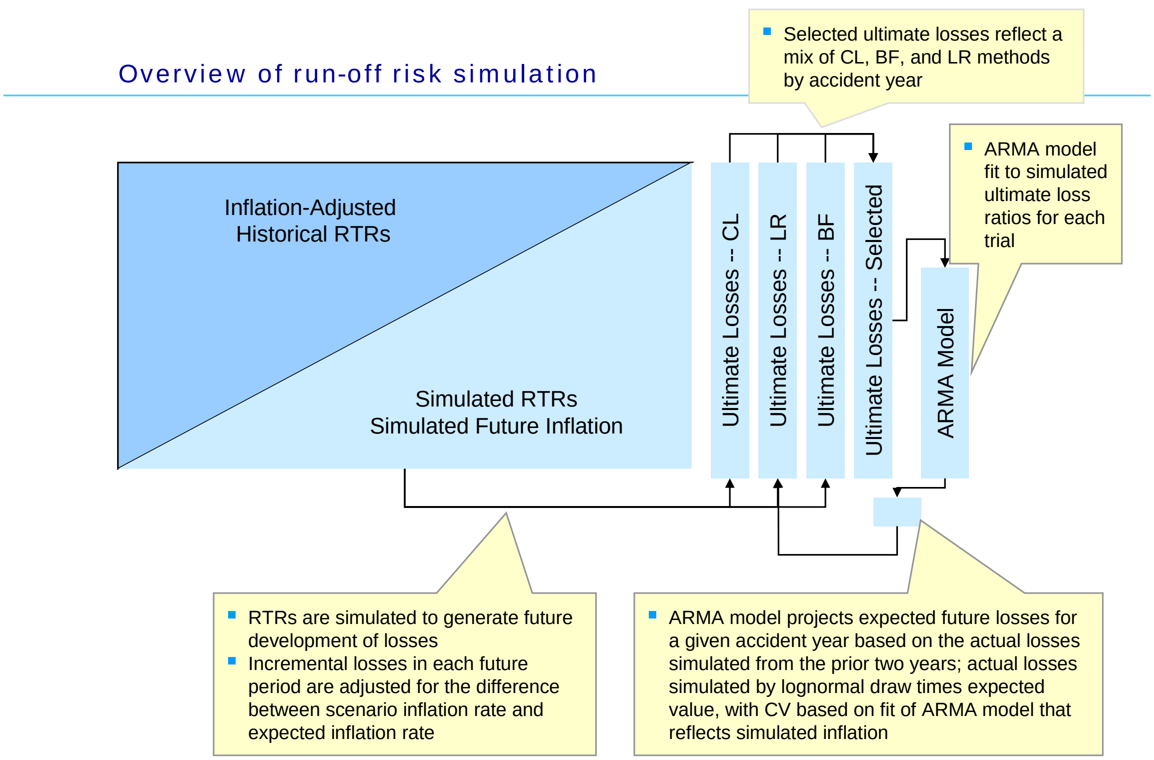
3.6 The Run-off Risk Horizon Model

Chart 3.6 portrays the general operation of the model for measuring risk using a run-off horizon.

Recall that the run-off model aims to answer the question "What is the potential variation in the ultimate cost of claim liabilities from the current actuarial central estimate?" The run-off model measures the variability of the actual ultimate claim liabilities around the current best estimate by simulating ultimate claims using the three stochastic methods: chain ladder (either paid or reported), ARMA loss ratio and Bornhuetter-Ferguson. The run-off model simulates the complete claim development process by generating all of the future missing values for the entire paid claim $C_{a,d}$ array. Systematic risk is overlaid on the simulated paid claim array to obtain an array that incorporates both systematic and non-systematic risk.

The simulated future claim payments from each trial are discounted for time value of money using the risk-free yield curve, and compared to the actuarial central estimate, to produce a distribution of the error term $e_r = \hat{L}^{(n)} - L^{(n)}$. This distribution is a representation of run-off insurance risk.

Chart 3.6: Schematic of the Run-Off Horizon Model

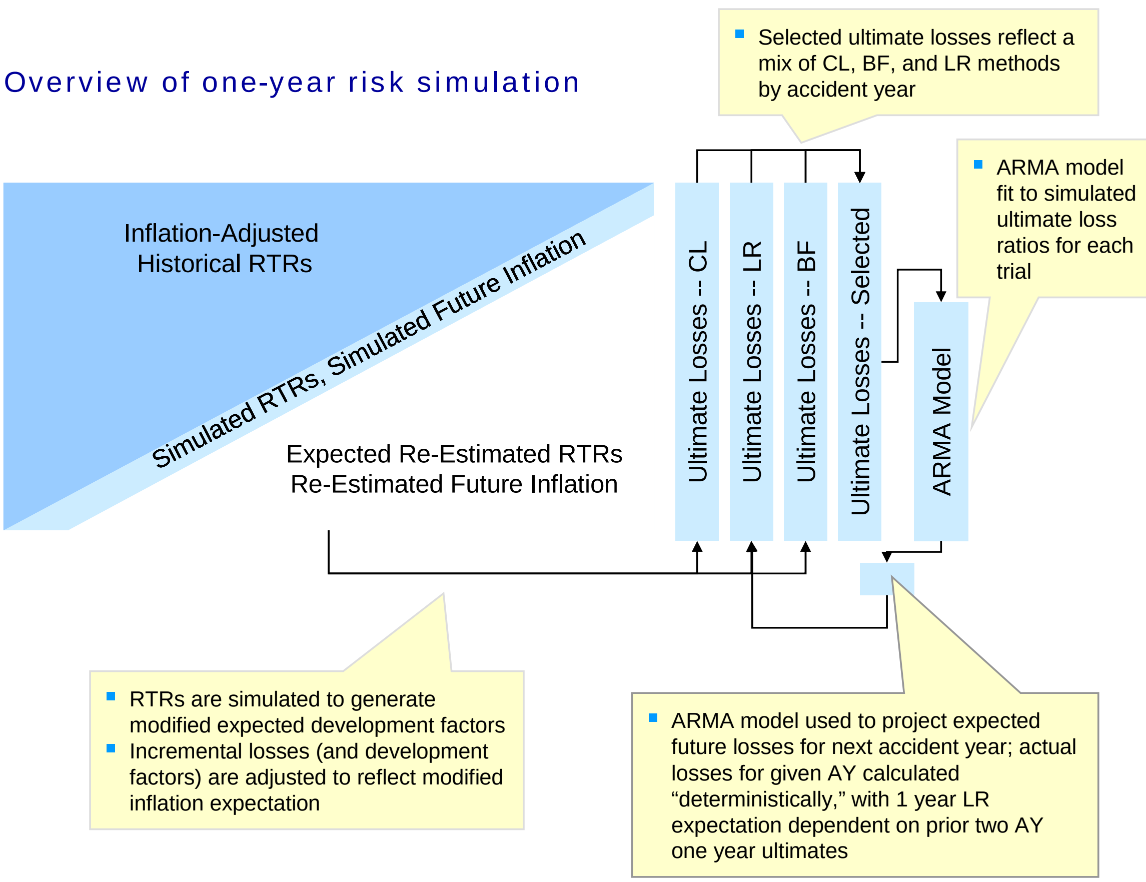


3.7 The One-year Risk Horizon Model

Chart 3.7 portrays the general operation of the model when measuring risk over a one-year horizon.

Recall that the one-year model aims to answer the question "What is the potential change in the actuarial central estimate of ultimate claim costs that could occur as a result of one additional year of actual claim emergence, other information, and changes in circumstances that affect the valuation?" The one-year model measures the variability of the one-year changes in actuarial central estimates by simulating how an additional "diagonal" of calendar year information would change the central estimate from its current value.

Chart 3.7: Schematic of the One-Year Horizon Model



The one-year chain ladder simulates RTRs and future inflation rates using the same model, parameters and inflation scenarios as in the run-off. The key difference is that only one calendar year (systematic and non-systematic risk) is simulated. That is, one diagonal of RTRs (non-systematic risk) and one calendar year of inflation (systematic risk) are simulated and combined to create a simulated realization of the next diagonal of the claim array. This simulated calendar year is then used to re-estimate the *expected* RTRs and the expected inflation rate. In general, this is done by examining two items:

1) ***The impact of the new simulated diagonal on the expected RTRs.*** The model selects new expected RTRs with the benefit of the new observed values along the simulated diagonal. The selection process for the new expected RTRs is a credibility weighted average of the observed simulated RTR, adjusted for the impact of the revised expected long term inflation, and the prior expected RTR. The credibility is a user input into the model.

2) ***The impact of the simulated inflation rate on the expected future inflation rate.*** The model uses an exponential smoothing technique to blend together the prior expectation with the new information, to obtain a revised expectation. The exponential smoothing equation is of the general form:

$$E' = E \times \beta + O \times (1 - \beta)$$

The β term determines the degree to which the new information alters the expectation.

E' is the revised expected long term inflation rate, E is the prior long term expected inflation rate and O is the simulated observed inflation rate

Finally, the development over the one-year simulation accretes into the re-estimated expectation of the remaining claim liabilities. That is, the simulated ultimate loss is a deterministic projection using the revised expected RTRs, applied to the simulated realization of the new diagonal. In addition, the entire triangle is adjusted to reflect the new level of expected inflation, so that the projection reflects the new inflation expectation.

This procedure is intended to exactly imitate the process that would be followed to derive the actuarial central estimate of ultimate losses one year hence, assuming we were using the inflation-adjusted chain ladder method. A comparison of the original estimate using this method to the simulated estimates using this method with one year of additional information is a measure of the one-year risk associated with the method.

The one-year ARMA loss ratio method follows a similar procedure. The new one-year chain ladder results are used to generate future expected loss ratios. As before, the ARMA model uses the selected loss ratio for the two prior years to project a given accident year's loss ratio. However, in this case the inputs for the ARMA model for a given accident year are the two prior years' *one-year* selected estimated ultimate losses. Since the one-year horizon is trying to measure the effect of one year's worth of information on the *expected* value, we are only concerned with the expected ARMA value and thus do not need to simulate an error factor (the expected error factor is 1.00). Again, this process can be considered a stepwise method since the one-year selected ultimate loss ratios for the two prior accident years are simulated and selected prior to projecting the given accident year loss ratio.

Given that all the inputs for the one-year ARMA model reflect the impact of one year's information, the one-year loss ratio method allows us to measure the impact of this extra year's worth of information on the given year's estimated ultimate losses.

The BF method is simply the credibility-weighted average of the chain ladder method and the loss ratio method, where the credibility and its complement are determined by the re-estimated expected percentage paid/reported pattern from the chain ladder method.

3.8 Correlation Between Classes of Business

This structural model's framework presents an intuitive way to measure the correlation between classes of business. Correlation between classes is introduced through the model in two ways:

1) *Through shared systematic risk*. Since each class of business being simulated will use the same scenario generated from the economic scenario generator, co-dependency in the results will be induced via the common systemic risk variables. For example, if two classes of business are both influenced by medical cost inflation, then in each trial of the simulation the two classes will be influenced by an identical medical inflation rate. Even if one class is affected by medical inflation and another is affected by wage inflation, a degree of co-dependency will be introduced to the extent that medical and wage inflation are correlated in the ESG.

2) *Through shared non-systematic risk*. Additional co-dependency can also be introduced by drawing all simulated values from a bivariate distribution. The dependency structure of the distribution is constructed by examining the empirical dependence structure of the historic RTRs, transformed to the cumulative probability space. Once this empirical dependence structure is determined, correlated bivariate values can be simulated by simulating a

correlated pair of values in the cumulative probability space and transforming that pair into their original bivariate space.

3.9 Final Calibration and Validation

In addition to the specification of the structure of systematic risk, the structural simulation requires the specification of many parameters, including:

— Means, variances (including process and parameter elements), and distributional form of the paid and reported claim RTRs;

— The Butsic mixing parameter for payment versus accident date inflation;

— ARMA mean reversion and momentum parameters;

— ARMA loss ratio model variance (including process and parameter elements), and distributional form;

— Choice of claim projection method (CL, BF, ELR) by accident year;

— The beta parameter that determines how inflation expectations adapt to new information; and

— All of the parameters within the ESG.

As a final calibration step, it is advisable to compare the results from the model to hindsight tests of historical claim estimation errors. For the run-off model, several comparisons can be made fairly readily. All of these comparisons should be on a present value basis, consistent with the risk measurements produced by the model.

- First, one can look back at historical reserve estimation errors, comparing actual emergence to original estimates. These can be done by accident year organized by maturity, or in the overall. The hindsight empirical reserve errors should look like a sample drawn from the modeled reserve risk distribution.
- Second, one can look at the volatility of historical ultimate loss ratios. These are cross-sectional rather than estimation errors, but they are nevertheless indicative of the volatility of loss ratios.

This volatility should look like a sample statistic drawn from the modeled pricing risk distribution.

- Alternatively, one can compare the initial estimate of loss ratios on the new accident year (for example, as is often used in reserving at the end of the first quarter) to the ultimate loss ratios. While this is technically a better comparison, few companies have the data available to make this comparison.

For the one-year model, several comparisons can be made, again on a present value basis, consistent with the model.

- One can look at historical one-year reserve development, either by accident year/maturity or in the overall. This data is generally available, and can be viewed as a sample from the model's one-year reserve risk distribution.
- One can look at the development of the estimated ultimate loss ratio on the current accident year from the beginning of the calendar year to the end of the year (i.e., at twelve months maturity). The latter values are typically available from year-end reserve analyses; the former values are sometimes available from planning or reserving work earlier in the year.

If the actual empirical reserve and pricing risk data do not compare favorably to the model, then the model's parameters should be reconsidered and adjusted where appropriate. Consistency tests across classes of business are also useful in this regard.

4. TESTING AND EMPIRICAL RESULTS

This section presents some illustrative empirical test results of the structural model. Unless noted to the contrary, the datasets used were the Personal Auto Liability and Commercial Auto Liability claim development data for a large U.S. insurer, as published in Schedule P of their statutory annual statements. This data is net of reinsurance (although little-to-no reinsurance is purchased by this insurer) and combines claim and claim defense costs together.

4.1 Comparison to Other Methods

Table 4.1 compares the results from the structural model to the results obtained from three other published stochastic reserving methods. Due to the limitations of those methods, the comparison relates only to reserve risk using a run-off risk horizon. In applying each method, the mean RTRs were selected as the volume weighted average of all years, and no tail factors or tail variability was used. Parameter risk was introduced into each method, except for the Practical Method, where only process risk is measured. To the extent possible, we strove to create an "apple-to-apples" comparison across the methods. In the case of the structural model, the results reflect a relatively modest level of expected inflation of 3.5%, with an alpha value of 0.5 (meaning that one-half of the inflation affects the development dimension). In each case, the methods were applied to both reported and paid claim development data.[10]

While the models generally produce a complete distribution of the possible outcomes, we have chosen to display the results in terms of the 99th percentile expressed as a ratio to the mean. Our choice of this statistic is merely one of convenience; it is an intuitive measure of the risk, relevant to economic capital issues.

Table 4.1: Indicated Reserve Risk from Structural Model versus Other Methods

	Ratio of 99th Percentile to Mean -- Run-Off Risk Horizon			
	Personal Auto Liability		Commercial Auto Liability	
	Reported	Paid	Reported	Paid
Structural Model	1.120	1.143	1.159	1.136
Bootstrap Method	1.085	1.087	1.166	1.151
Mack / Murphy Method	1.095	1.119	1.152	1.128
Practical Method	1.086	1.075	1.134	1.103

The results in Table 4.1 reflect the typical variations in indicated risk from the application of different methods to different datasets. The variations reflect differences in how the models measure the noise in the data. Note than in Commercial Auto Liability, across all methods, the indicated risk of adverse development is lower when the method is applied to paid claim development data. However, the reverse is the case in Personal Auto Liability, where the risk indications from the paid data are higher in three of the four methods. This shows that sometimes

[10] Note that the results are designed to facilitate comparisons, and that in actual practice the measured level of risk would be slightly higher than the values shown, due to the inclusion of tail factor variability.

the noise in the paid data is relatively higher, while at other times the noise in the reported data is higher; since all methods are keying off the same underlying historical development data, it is not surprising that all methods pick up the same relative relationship.

In general, from our work with these models we believe that the results obtained from application of the methods to the reported claim development data are a better indicator of reserve risk.

Focusing on the reported claim development results, we would observe that the structural model produces distinctly higher indicated reserve risk than the other three methods for the Personal Auto Liability class of business. Since other research has suggested that the Mack method may understate reserve risk and the Practical method excludes consideration of parameter risk, this is not an entirely surprising result. The indicated reserve risk for Commercial Auto Liability is more consistent across the different methods, with only the Practical Method lagging.

The structural model results are intuitively reasonable. They indicate that, at the 1-in-100 probability level, Personal Auto Liability reserves will develop adversely by as much as 12.0%; and that Commercial Auto Liability reserves will develop adversely by as much as 15.9%.

4.2 Integration of Pricing and Reserving Risk

Table 4.2 displays the indicated pricing and reserving risk when the structural model was applied to the reported claim development data for the two classes of business. Also shown are the indicated correlation between reserving and pricing risk generated by the model, and the indicated level of overall insurance risk (i.e., combined reserving and pricing risk).

Here, we show results on both a run-off and one-year risk horizon basis.

Table 4.2: Indicated Reserving, Pricing, and Insurance Risk from Structural Model

Ratio of 99th Percentile to Mean
Structural Model Applied to Reported Claim Development Data

	Personal Auto Liability		Commercial Auto Liability	
	Run-Off	**One-Year**	**Run-Off**	**One-Year**
Reserve Risk	1.120	1.093	1.159	1.129
Pricing Risk	1.195	1.118	1.289	1.125
Indicated Correlation	68%	87%	48%	74%
Insurance Risk	1.142	1.102	1.182	1.120

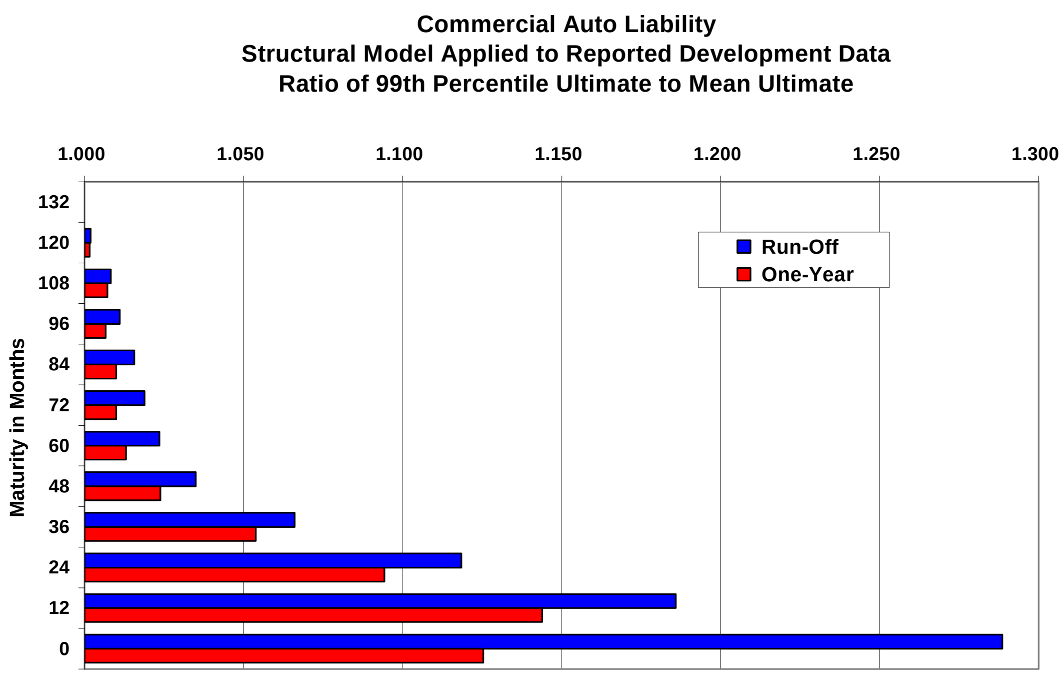
Not surprisingly, the indicated risk on a one-year horizon basis is distinctly lower than on a run-off horizon basis. This merely reflects the fact that the former focuses only on movements in estimates in a single year, rather than over the entire lifetime of the claim liabilities where annual movements can compound.

While indicated pricing risk is higher than reserve risk under the run-off risk horizon, the situation is less pronounced in the case of the one-year risk horizon. In fact, in the case of commercial auto liability, the one year pricing risk is less than the reserve risk. While somewhat counter-intuitive, this result also stems from the way that risk emerges. For longer-tailed liability lines, we sometimes learn more about the ultimate values of claims as they develop from 12 to 24 months than we do from the development from 0 to 12 months.

Finally, one can observe that the model produces relatively high correlation between reserve and pricing risk. This reflects the common pattern of inadequate reserves leading to inadequate prices.

The quoted results for reserve risk in Tables 4.1 and 4.2 are for all maturities combined. One can see the model results more clearly by examining results by individual maturity, as in Chart 4.3 below.

Chart 4.3: Standard Model Output – Risk by Maturity



Note that in Chart 4.3, risk is expressed as the ratio of the 1-in-100 ultimate claim amount (i.e., including paid-to-date) to the expected ultimate claim amount, rather than the ratio of the 1-in-100 liability to the expected liability. This is done to make the results more intuitive, showing how the uncertainty of the ultimate claims for an accident year resolves as the year matures. Chart 4.3 illustrates several important relationships, described below:

- The indicated risk ratios are much higher at early maturities, reflecting the "funnel of doubt" relationship referred to earlier in the paper.
- The indicated one-year and run-off risks are significantly different, with the one-year risk being smaller in every year. The run-off model is designed to capture the variability of all reasonably predictable future outcomes in all future calendar years. In contrast, the one-year model

contemplates the impact of outcomes for only one future calendar year. Consequently, one would expect that the one-year risks would be smaller than their run-off counterparts.

- In the chart, maturity zero is the prospective year. We refer to the indicated risk associated with this year as the pricing risk. Under the run-off horizon, the indicated pricing risk is much higher than all of the other years, as would be expected. However, under the one-year horizon the opposite is true; there is a slight dip in the pricing risk. Initially this may seem counter-intuitive, but recall that we are trying to answer the question "What is the potential change in my estimate of ultimate losses that could occur as a result of one additional year of actual claim emergence?" The dip in the one-year pricing risk can be explained by the fact that in many classes there may be very little expected emergence in the first year. With very little information, our estimate of ultimate losses will not change drastically one year hence. (This reversal is not necessarily the case, for example in short-tailed lines where much of the emergence occurs in the first year.)

4.3 Sensitivity to Inflation Parameters

We performed sensitivity tests on several of the key inflation parameters of the structural model, using the Commercial Auto Liability reported claim development data. For comparability with the results presented earlier, we focused on run-off reserve risk. Results are displayed in Table 4.4.

In the base case, the expected future inflation rate is at a constant level of 3.5%; and the Butsic alpha factor is 0.5, meaning that half of the calendar period inflation affects the development dimension. The base case incorporates systematic risk in the form of varying inflation rates about the expected value, as well as non-systematic risk due to variations in the future development factors. In the base case, the 1-in-100 adverse development is 15.9% above the best estimate.

In the first set of sensitivity tests, we turned one or the other of the two sources of risk off, so that we could observe the relative contribution of each risk. One can see that when the inflation volatility is turned off, the 1-in-100 adverse development drops slightly, to 15.6%. Conversely, when the development volatility is turned off, one can see that inflation alone contributes reserve risk of 4.7%. One can also see that the two risk sources are not additive, as the sum of the two stand-alone risks is substantially greater than the combined result. As expected, the introduction of inflation as a systematic risk adds to the risk that is otherwise modeled by the stochastic reserving method. In this case, the contribution is small, reflecting the relatively fast-paying nature of the claims and the low level of expected inflation.

Table 4.4: Selected Sensitivity Test Results

Structural Model Applied to Reported Claim Development Data

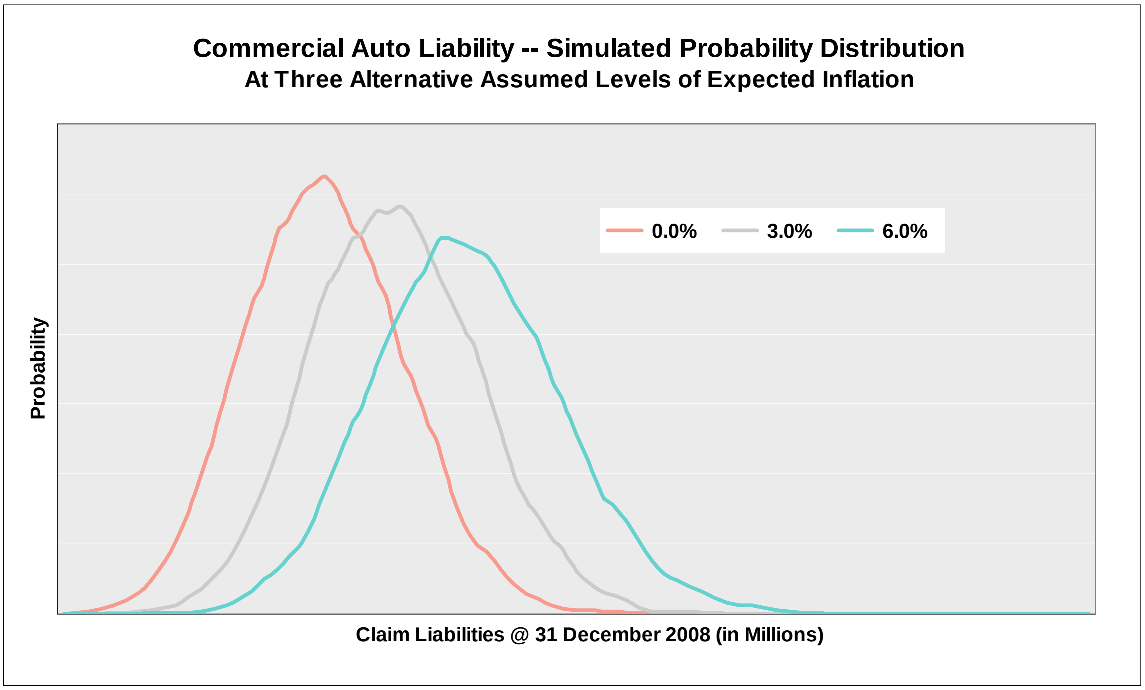
	Ratio of 99th Percentile to Mean Commercial Auto Liability Run-Off Reserve Risk
Base Case	1.159
No inflation volatility, just development volatility	1.156
No development volatility, just inflation volatility	1.047
Bustic alpha factor = 1, inflation does not affect development	1.160
Butsic alpha factor = 0, inflation fully affects development	1.173
Expected inflation lower, at constant 0.0%	1.137
Expected inflation higher, at constant 6.0%	1.174

We also tested the effect of varying the alpha factor from zero to one, around the base case value of 0.5. When the alpha value is decreased to a value of 0 (implying that inflation fully affects development), the risk increases to 17.3%. This is intuitive because the variability of inflation is allowed to fully affect development. In contrast, when the alpha factor is increased to a value of one (implying that inflation does not affect development), rather than decreasing as expected we obtain what initially appears to be an anomalous result; the 1-in-100 adverse development risk rises marginally from 15.9% to 16.0%. Because this is marginal increase, it could simply be attributed to noise. However, it turns out that these results are a consequence of the underlying data, and not just noise. When we set the alpha factor to one, we are implicitly assuming that inflation has no effect on the development dimension of the triangle. Hence the triangle is unadjusted for historical inflation. However, it turns out that some of the variation is actually explained by inflation. As a consequence, the unadjusted triangle exhibits more historical volatility than the adjusted triangle in the base case, and this higher level of volatility manifests itself in the marginal increase of the indicated level of risk.

Finally, we tested the sensitivity of the results to different expected inflation rates. Not unexpectedly, as the assumed inflation rate rises, the risk also rises. If expected inflation rises from 3.5% to 6%, then the 1-in-100 adverse reserve development rises from 15.9% to 17.4%. This result is qualitatively consistent with the experience of the late 1970s and early 1980s, where high inflation destabilized claim trends, resulting in significant adverse reserve development. This is an important result, as — consistent with history — the model implies that reserving risk is higher when inflation rates are higher.

The impact of varying the expected inflation rate can be seen graphically in Chart 4.5, where we show the claim reserve probability distributions generated by the model for Commercial Auto Liability at the three alternative assumptions regarding the expected levels of future inflation. Here, one sees the gradual flattening of the distribution as the expected inflation rate rises. In addition, the expected value of the claim liabilities also rises with the assumed inflation rate. This chart demonstrates a principal advantage of the structural simulation: the ability to delve into inflation as a potential driver of insurance risk.

Chart 4.5: Impact of the Assumed Inflation Rate on the Claim Liability Distribution



4.4 Validation Using Actual Historical Experience

As discussed in the previous section, validation against historical experience is a critical step in the calibration of all stochastic reserving methods. This is particularly true for the structural model, as it has many parameters that must be specified. While these parameters offer the user the advantage of a great deal of flexibility, they necessitate validation to assure that the model is producing results that are realistic.

Usually the available historical data on past reserving and pricing errors is limited, making it an insufficient resource for measuring reserve and pricing risk directly. (Simply stated, it is hard to estimate the 1-in-100 reserve error when the historical reserving database only has observations from about a dozen prior years.) The better approach is often to employ a stochastic model, and validate it by testing whether the historical data could reasonably be a sample drawn from the distributions produced by the stochastic model.

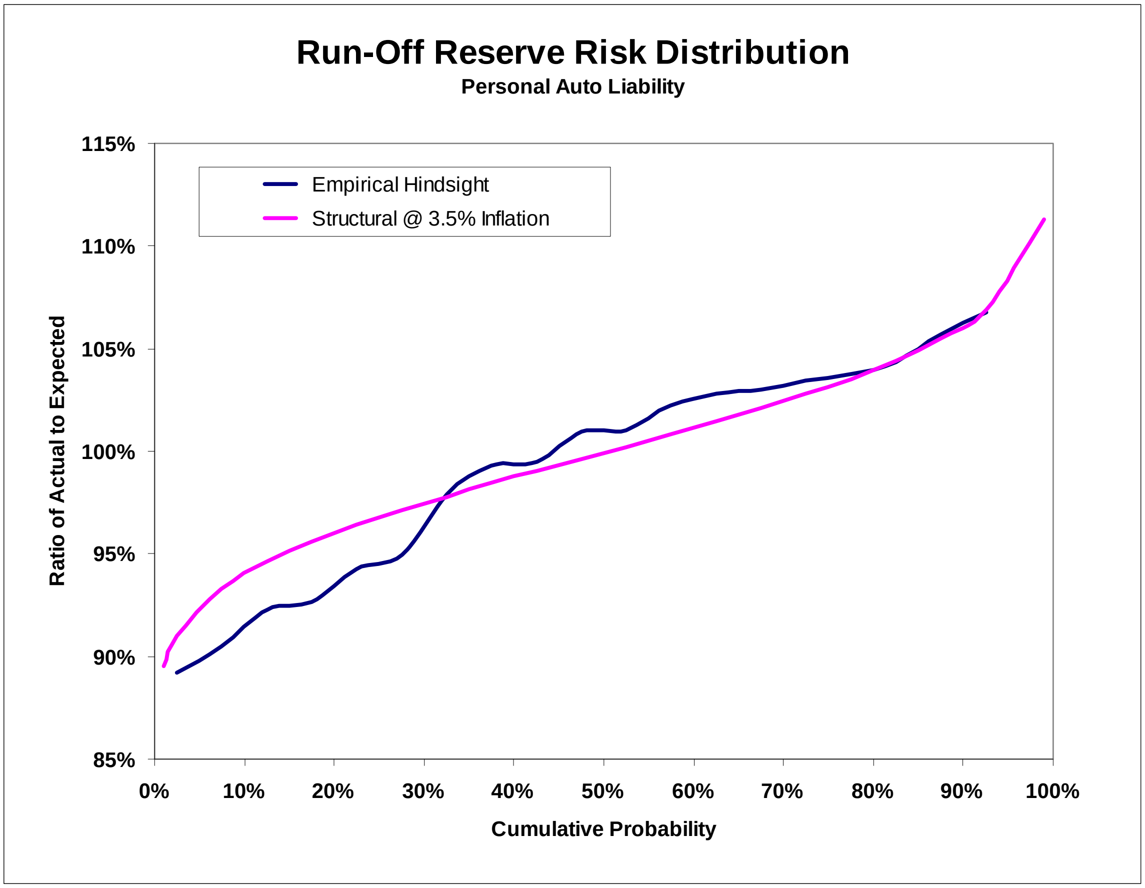
To illustrate some of the validation approaches, we performed hindsight tests on the Personal Auto Liability dataset. We developed best estimates of the claim liabilities for nineteen prior year-ends (i.e., from year-end 1989 to 2007) using only the information that would have been available at the time, and compared those estimates to the actual run-off experience through year-end 2008 to determine historical estimation errors. In addition to estimates of claim liabilities for past accident years, we also estimated claim liabilities for two prospective accident years based on premiums, price changes, and assumptions regarding claim cost trends for use in validating the pricing risk aspect of the structural model.

Over the nineteen year historical period, the overall reserve estimation errors for Personal Auto Liability ranged from a 10.8% redundancy to a 6.8% inadequacy. The highest redundancies occurred during the period from 1994 to 1998, and reflect the effects of disinflation that occurred around that time. While the estimation errors are slightly biased in magnitude towards redundancy, the number of years of redundancy is roughly equal to the number of years of inadequacy.

Chart 4.6 compares the cumulative probability distribution of reserve errors (expressed as a ratio of actual outcome to expected value) implied by the historical data to the cumulative probability distribution generated by the structural model for Personal Auto Liability. While the correspondence is not perfect, the model result compares favorably to the empirical evidence. This is not always the case; in unpublished tests of other models on other classes of business, the authors have observed a lack of correspondence between the model and the historical data that was sufficiently obvious to clearly reject the model. Even with the structural model, adjustments to one or more of the parameters are sometimes required to achieve good validation results.

The standard deviation of the historical errors is slightly higher than that produced by the structural model (5.4% versus 4.7%), driven largely by the higher historical redundancies. Note also that our comparisons are based on structural model output under an assumed expected inflation rate of 3.5%, which is consistent with the actual inflation rate in the latter part of the historical period. Use of a higher expected inflation rate would move the two standard deviations closer together.

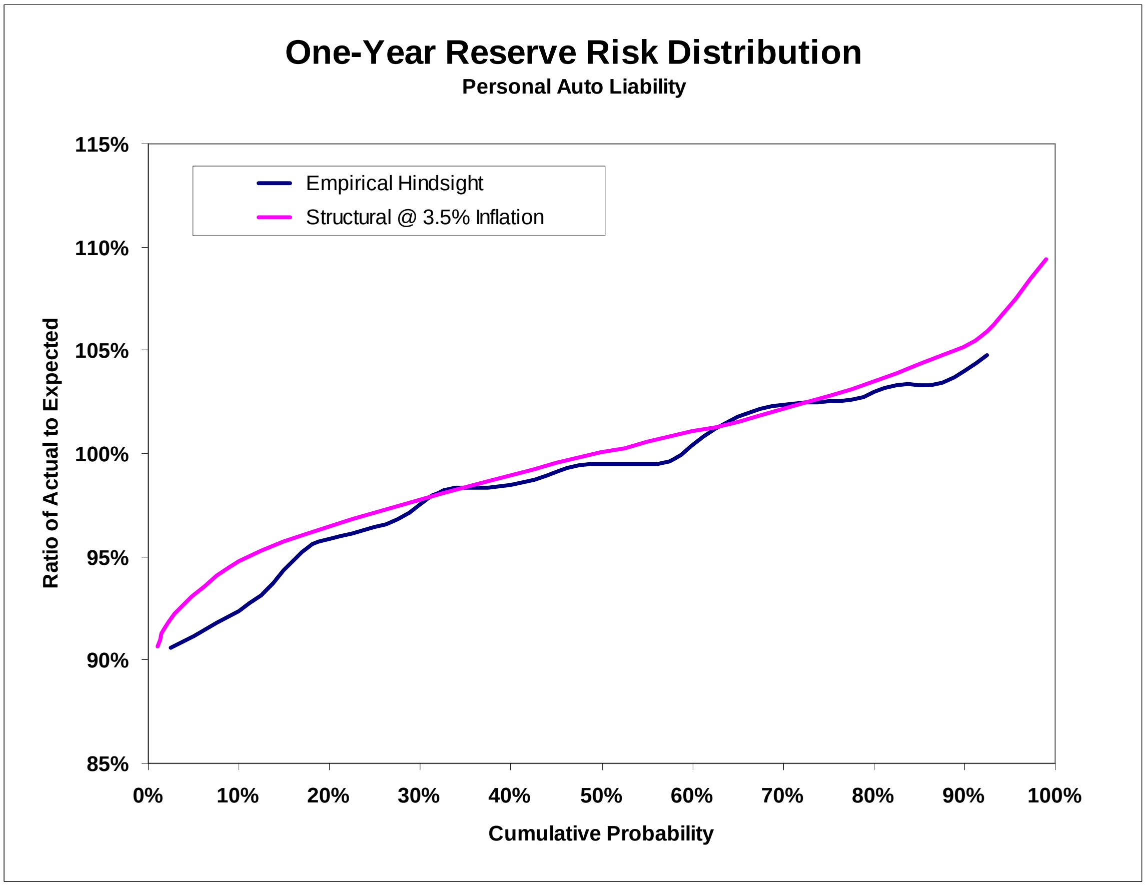
Chart 4.6: Validation of Overall Run-Off Reserve Risk in the Structural Model



The same historical datasets can be used to validate the one-year insurance risk model. Instead of comparing the ultimate run-off estimation errors to the model results, one compares the one-year historical development to the corresponding one-year structural model results. This comparison is displayed in Chart 4.7.

From the same analysis of Personal Auto Liability, we observed that, over the nineteen year historical period, the one-year movements in the best estimate of liabilities for past accident years ranged from a favorable development of 9.4% to an adverse development of 4.7%. As with the run-off results, high favorable development occurred in the calendar years from 1994 to 1998, reflecting disinflation. Once again, the model result compares favorably to this empirical evidence.

Chart 4.7: Validation of Overall One-Year Reserve Risk in the Structural Model



The standard deviation of the one-year historical movements in estimates is slightly higher than that produced by the structural model (4.2% versus 3.9%).

Rather than simply validating the overall result for reserve risk, one can validate the results by maturity. This is done in Chart 4.8, where we compare the ratio of the estimated ultimate claims to the actual ultimate claims, for each accident year at each maturity. The hindsight empirical data consists of the movement of the estimated ultimate claims for each of 25 past accident years, from 12 months prior to inception through to 120 months maturity.[11] The model output consists of the simulated 98% confidence interval for the same statistic at each maturity.

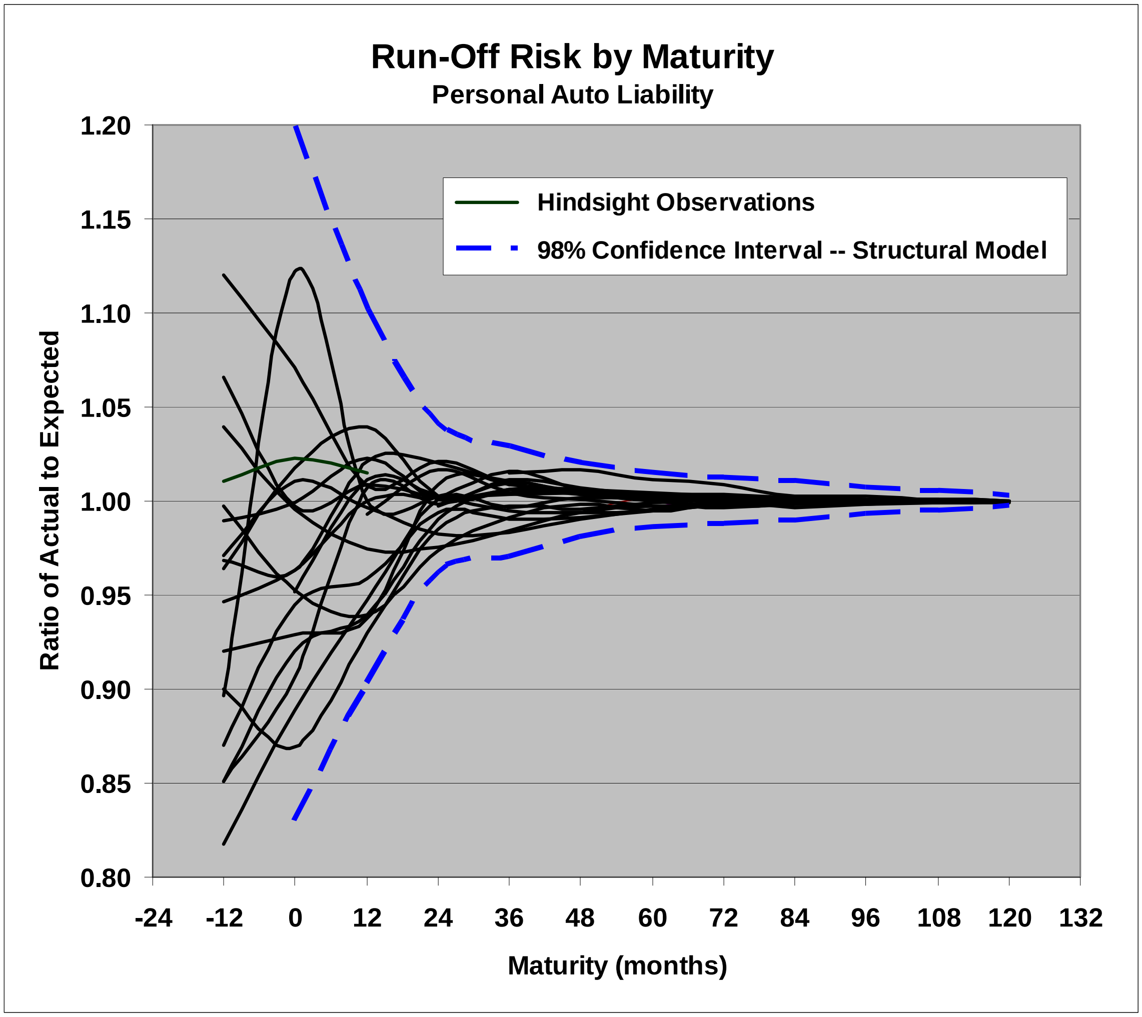
Collectively, the historical data paints an empirical "funnel of doubt", as described earlier in Section 2, except that the funnel here is an inverted mirror image of the funnel displayed in Chart 2.1.

The structural model results in Chart 4.8 compare reasonably well with the empirical hindsight data, with most empirical values falling within the simulated 98% confidence interval. The general shape of the structural model funnel is consistent with that of the hindsight data, with initial pricing risk dissipating relatively rapidly as actual claim experience becomes available. The results shown in Chart 4.8 are those of a "second iteration" of the model. Comparison of the initial results to the empirical data suggested that the model was understating reserve risk slightly at maturities in the 48 to 84 months range (the funnel was too narrow, and quite close to the empirical results. This was addressed by selecting slightly larger CVs for the development factors at these maturities. The final selected CVs are not inconsistent with the loss development data; the adjustments made can be attributed to sampling error.

Note that in Chart 4.8, the hindsight data includes estimates of two future accident years, whereas the structural model reflects only one prospective accident year. Some have suggested that, given the natural lag between available experience data and implementation of price changes, it may be more realistic to measure pricing risk by looking at estimation errors beyond the immediate prospective accident year. If this is the case, it would be easy to extend the structural model to include additional future years.

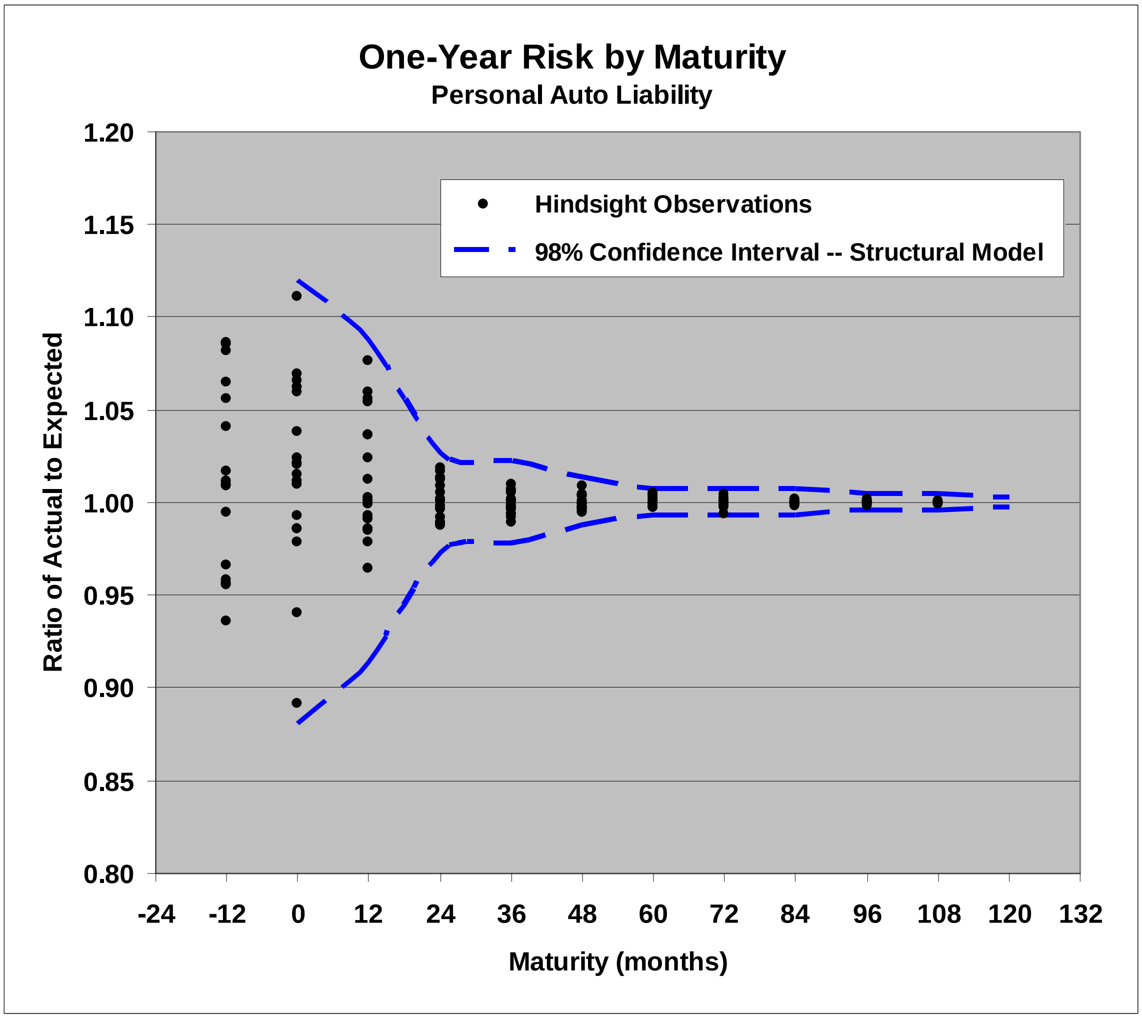
[11] Due to the limitations of the dataset, not all values are available for all accident years. For example the 1983 accident year is only available at valuations from 84 months onward. Similarly, the 2006 accident year is only available at valuations up to 24 months.

Chart 4.8: Validation of Run-Off Risk in the Structural Model by Maturity



A similar analysis to that depicted in Chart 4.8 can be produced to validate the one-year model, rather than the run-off model. Instead of comparing the estimated ultimate claims at one maturity to the true ultimate claims, one instead compares the estimated ultimate claims at one maturity to the estimate at the next maturity. One-year validation results are shown in Chart 4.9. These results are also those of the "second iteration" of the model, to be consistent.

***Chart 4.9:* Validation of One-Year Risk in the Structural Model by Maturity**



Other validation tests can also be devised to test various aspects of the model, for example focusing on run-off and one-year pricing risk. Tests can also be performed that examine the correlation between estimation errors between classes of business. In the interests of time and space these will not be presented here.

4.5 Sensitivity to Choice of Reserving Methods

A key set of parameters to the structural model is the choice of reserving method (i.e., chain ladder, Bornhuetter-Ferguson, or expected loss ratio) by accident year. The baseline approach is to use the chain ladder method for all historical accident years, introducing the other two methods only when the chain ladder method introduces more volatility than is actually present.

For both Personal and Commercial Auto Liability, we used the chain ladder method throughout the historical accident years, as this is what was indicated by the validation test results. To see the importance of this parameter to the final results, we performed some sensitivity tests on the Commercial Auto Liability dataset. These sensitivity test results are displayed in Table 4.10.

Table 4.10: Testing the Choice of Reserving Methods in the Structural Model

Structural Model Applied to Reported Claim Development Data

	Ratio of 99th Percentile to Mean Commercial Auto Liability Run-Off Reserve Risk
Actual claim development data	
Chain ladder for all years	1.159
B-F for latest accident year; CL for all prior	1.162
ELR for latest accident year; CL for all prior	1.215
Modified claim development -- highly volatile 12-to-24 months	
Chain ladder for all years	5.238
B-F for latest accident year; CL for all prior	2.977
ELR for latest accident year; CL for all prior	1.218

In the first set of sensitivity tests, we simply re-ran the structural model using either the BF or the ELR methods for the latest historical accident year (i.e., the year valued at 12 months maturity). Results are shown in the upper portion of Table 4.10. As one can see, the indicated reserve risk is slightly higher than the baseline case when the Bornhuetter-Ferguson method is employed and substantially higher than the baseline case when the expected loss ratio method is employed.

The reader may find these results somewhat counter-intuitive, as generally the purpose of introducing the Bornhuetter-Ferguson and/or expected loss ratio methods into any actuarial projection is to reduce the volatility of the chain ladder projection. Here, because the initial expected loss ratio is a random variable drawn from the ARMA model, the use of the latter methods actually raises the volatility. These results indicate that the chain ladder method does an adequate job at projecting ultimate losses, and the use of the latter two methods is injecting spurious volatility into the model.

Of course, there are situations where the chain ladder method is not the appropriate method to use. When the historic RTRs are highly volatile, the chain ladder method may have "negative skill" in that the projections it produces have greater volatility than that of the actual loss ratios around the expected. In such cases the chain ladder method is not an appropriate method for reserving, and its use in the structural model will overstate the reserve risk. This point is illustrated by considering the results in the lower half of Table 4.10. Here we have artificially modified the historical claim development data by dividing the historical reported claims at 12 months maturity by a lognormal error term with a mean of 1 and standard deviation of 1, making the development factor from 12 to 24 months highly volatile. (The RTRs in the remaining development periods are unchanged.) In this case, the use of the chain ladder method for the latest historical accident year in the structural model generates simulated losses for the latest year that are extremely volatile, causing reserve risk to be overstated. Even the introduction of the Bornhuetter-Ferguson method is insufficient to dampen the volatility of the projected ultimate claims for the latest historical accident year. Therefore, the correct choice is to use the expected loss ratio method for the latest accident year.

As discussed earlier, a simple validation test to determine whether the chain ladder method is appropriate is to compare the volatility of the historical chain ladder method projections at a given maturity to the volatility of historic ultimate loss ratios. Generally, the chain ladder method would be appropriate when the former is smaller than the latter.

4.6 Correlation Between Classes of Business

As noted earlier, the structural model can be used with multiple classes of business to generate an aggregate distribution across the classes. In order to compare the results of the aggregate distributions, we constructed several normal copula models using Bootstrap simulation results using differing correlation coefficient assumptions. Results are displayed in Table 4.11.[12]

Table 4.11: Structural Model Correlation Results versus a Normal Copula Structure

	Ratio of Indicated Percentile to Mean				
	Structural Method	Normal Copula - Bootstrap			
Confidence Intervals					
Mean	1.000	1.000	1.000	1.000	1.000
50.0th Percentile	1.000	0.999	0.999	0.999	1.000
75.0th Percentile	1.028	1.028	1.028	1.029	1.035
90.0th Percentile	1.055	1.054	1.055	1.057	1.070
95.0th Percentile	1.071	1.071	1.072	1.074	1.092
99.0th Percentile	1.102	1.103	1.104	1.108	1.131
99.5th Percentile	1.116	1.114	1.116	1.120	1.147
Correlation	17.6%	5.0%	10.0%	20.0%	100.0%

The structural model's risk margins are similar to those produced by the bootstrap normal copula model under the various correlation assumptions. Slight differences would be expected because the two methodologies are fundamentally different. However, the fact that the results are generally similar suggests that they are both producing reasonable conclusions.

A key strength of the structural model is that it does not require the user to specify the correlation between the classes nor the form of the bivariate distribution; the correlation is simply a by-product of the simulation. In contrast, an explicit correlation assumption and the assumption that the dependence structure follows a normal copula must be made when using the bootstrap models.

The structural model introduces correlation between classes through shared systematic risk and an empirically defined dependence in the non-systematic risk. The shared systematic risk component is of particular interest because it allows the user to reflect the impact of changing socio-economic factors on the aggregate book of business. Table 4.12 compares the structural correlation results under various inflation scenarios.

[12] To facilitate the comparison in this section, we adjusted the volume of the Commercial Auto data by a factor of 50 in order to prevent the Personal Automobile loss distribution from dominating the aggregate distribution. The 50-times adjustment resulted in the sizes of the two classes being approximately equal.

Table 4.12: Structural Model Correlation —Results Under Various Inflation Scenarios

Scenario	Correlation
0.0% mean future inflation	16.3%
3.5% mean future inflation	17.6%
6.0% mean future inflation	18.6%

As the expected future inflation increases, the correlation also increases. Consequently, the benefits of diversification decrease in highly inflationary environments. Unlike the normal copula bootstrap model, the structural model is able to capture this phenomenon.

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Abbreviations and Notations

α, Butsic mixing parameter, determines the degree to which inflation affects the development dimension
ARMA, Auto-Regressive Moving Average
CL, Chain Ladder projection method
CV, Coefficient of Variation
BF, Bornhuetter-Ferguson projection method
ELR, Expected Loss Ratio projection method
ESG, Economic Scenario Generator
RTR, Report-to-Report development factor

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