CASUALTY ACTUARIAL SOCIETY FORUM

Spring 2007

Including a Reinsurance Call Paper

CASUALTY ACTUARIAL SOCIETY
ORGANIZED 1914

Printed for the Society by
United Book Press
Baltimore, Maryland

NOTICE

The Casualty Actuarial Society *Forum*
Spring 2007 Edition
Including a 2007 Reinsurance Call Paper

To CAS Members:

This is the Spring 2007 Edition of the Casualty Actuarial Society *Forum*. It contains a Reinsurance Call Paper, two reprinted 2007 Ratemaking Call Papers, and four additional papers. The *Forum* is a nonrefereed journal printed by the Casualty Actuarial Society.

The CAS is not responsible for statements or opinions expressed in the papers in this publication. See the Guide for Submission to CAS *Forum* (http://www.casact.org/about/index.cfm?fa=forum) for more information.

The CAS *Forum* is edited by the CAS Committee for the Casualty Actuarial Society *Forum*. Members of the committee invite all interested persons to submit papers on topics of interest to the actuarial community. Articles need not be written by a member of the CAS, but the paper's content must be relevant to the interests of the CAS membership. Members of the Committee for the Casualty Actuarial Society *Forum* request that the following procedures be followed when submitting an article for publication in the *Forum*:

1. Authors must submit papers in conformance with the research paper template established by the CAS. See the CAS Web Site at http://www.casact.org/research/index.cfm?fa=template for formatting instructions. Papers not conforming to the template will not be accepted.
2. Authors should submit a camera-ready original paper and two copies.
3. Authors should not number their pages.
4. All exhibits, tables, charts, and graphs should be in original format and camera-ready.
5. Authors should format graphs, tables, exhibits, and text in solid black and white. Avoid using color or gray-shaded graphs, tables, or exhibits as the printing process will produce muddy and moire patterns.
6. Authors should submit an electronic file of their paper using a popular word processing software (e.g., Microsoft Word and WordPerfect) for inclusion on the CAS Web Site.

The CAS *Forum* is printed periodically based on the number of call paper programs and articles submitted. The committee publishes two to four editions during each calendar year.

All comments or questions may be directed to the Committee for the Casualty Actuarial Society *Forum*.

Sincerely,

Glenn M. Walker, CAS *Forum* Chairperson

The 2007 CAS Reinsurance Call Paper
Presented at the
2007 CAS Seminar on Reinsurance
May 7–8, 2007
Sheraton Society Hill Hotel
Philadelphia, Pennsylvania

The Spring 2007 Edition of the CAS *Forum* is a cooperative effort between the Committee for the CAS *Forum* and the Committee on Reinsurance Research.

The CAS Committee on Reinsurance Research presents for discussion one paper prepared in response to their 2007 call for papers.

This *Forum* includes papers that will be discussed by the authors at the 2007 CAS Seminar on Reinsurance May 7–8, 2007, in Philadelphia, Pennsylvania.

2007 Reinsurance Call Paper

Reprinted 2007 Ratemaking Papers

Additional Papers

We're Skewed—The Bias in Small Samples from Skewed Distributions

By Kirk G. Fleming, FCAS, MAAA

Abstract People in insurance work all the time with financial processes that are best modeled with skewed distributions. Despite our constant exposure to skewed distributions, I believe when we study sample averages from these skewed distributions we think and work with them as if they were samples from normal symmetrical distributions. In this paper I want to discuss the idea that a sample average is biased lower than the actual mean of a skewed distribution – an amount that depends on the sample size and how skewed the distribution is. I will talk about the implications that this small sample bias has for credibility procedures. Why do people ignore outliers? I will offer up some possible reason for why we ignore outliers and why deals get done despite what the data indicates. I will talk about the winner's curse or why we lose even as we win. Finally, I will offer a small sample of skewed random thoughts on why these ideas explain everything from people engaging in risky behaviors to the property/casualty insurance cycle.

INTRODUCTION

People in insurance work all the time with financial processes that are best modeled with skewed distributions. Despite our constant exposure to skewed distributions, I believe when we study sample averages from these skewed distributions we think and work with them as if they were samples from normal symmetrical distributions.

In this paper, I will show through computer simulations that the expected value of a sample average from a skewed distribution varies between the mode of the distribution and the true mean of the underlying distribution. Where the expected value of the sample average falls between those two values will depend upon how skewed the distribution is and the sample size. For small samples, the expected value of the sample average will be near the mode of the distribution and for some skewed distributions, "small" samples can be unexpectedly big. The implication is that while we are searching for information on a population's mean by examining the averages of small data samples from skewed distributions, we will most likely be getting indications that could be significantly lower than the population mean. This is in contrast to the situation when we are sampling from a symmetrical distribution where the expected value of a sample average is equal to the mean of the distribution regardless of the sample size.

I would also like to talk about some of the implications of people not realizing or ignoring that the expected value of the sample average from a skewed distribution is biased lower than the mean of a positively skewed distribution. I will talk about small sample bias and credibility procedures. I will talk about why people tend to ignore outliers and why deals get done in spite of what the data indicates. I will offer an explanation on why we can't win for losing or why making money in insurance is no easy matter. Finally, I will offer up a small sample of skewed random thoughts on how these ideas help to explain everything from people engaging in risky pursuits to the property/casualty insurance cycle.

In the paper I talk a great deal about the mode and the mean because I think those are concepts that are common ground for all of us in insurance. I hope to reach a bigger audience of insurance professionals than just actuaries. To that end, I relegated all formulas to the appendices. However, I must share a word of caution to actuaries who want to discuss these ideas with others outside our field. I have tried it and I have seen strange reactions from professionals of all kinds. People have played dead so that I would just go away and leave them alone. Others have fought back violently. I have seen our outside audit partner a hardened insurance veteran who has "seen it all" practically break his leg as he tried to escape from my office when I even hinted at these ideas in answer to his question. You have been warned.

A PRACTICAL PROBLEM

Consider the following scenario – we have a customer who has written some business in a particular state and it turns out to be profitable business. The customer would like to expand into the state to write more of this good business. Our job is to produce forecasted financial statements for this customer so that they can present their business plan to management.

Because our customer does not have a great deal of existing business in the state, we use an industry average loss ratio – a ratio that happens to be higher than our customer's actual experience when we produce a first draft of the forecasted financials. Our customer objects to the higher loss ratio since he knows that his past business has been better than the industry result. In order to acknowledge his concerns we credibility weight his past experience with the industry average to give some credence to the actual experience. By using the credibility procedure, we are recognizing that our customer's experience might actually represent a profitable niche as opposed to being just be a random fluctuation from the industry average.

However there is another explanation for why the small sample average based on our customer's past experience is different from the long term industry average as opposed to it being a profitable niche or a random fluctuation. If we are sampling from a typical positively skewed distribution, the most likely value of that small sample average will be less than the true average of the distribution simply because it is a small sample from a skewed distribution. For very small samples from highly skewed distributions, the sample average will more likely be closer to the mode rather than to the mean.

When we do a single sample from any discrete distribution, the most likely value that we will see is the mode of the distribution. That's the definition of the mode – the observation that appears most often, or in other words, has the greatest probability of occurring. The mode is one of those statistics that we learn about when we first do statistics but then we never hear much about it again unless we are trying to avoid distortions associated with extreme values. That is an injustice to the mode; it actually deserves more attention.

For a symmetrical distribution with one mode like a bell curve, the mode is equal to the mean. But for a typical distribution that we might encounter in insurance that is skewed to the right and which has only one mode, the mode is less than the median which is less than the mean. (For an example of an atypical skewed distribution where the mode is greater than the mean, see Appendix A). When we do small samples from typical skewed distributions, the most likely value for the sample average will be somewhere between the mode and the mean of the distribution. How close to the mode or how close to the mean will depend on how skewed the distribution is and the sample size. Moreover, for some skewed distributions, "small" samples can be surprisingly big.

For some insurance examples, this relationship should be in the back of our minds. Take for example the annual sample from a highly skewed distribution like the annual hurricane losses in the city of Miami. For any particular year, the most likely loss we will observe is zero -- the mode of the distribution. Every so often there will be a hurricane loss that will bring the long-term average above the zero mark but most of the losses we see will be zero.

On the other hand, an industry average loss ratio is based on a sample size that we could consider for all practical purposes to be approaching infinity. If we are dealing with large samples, even from skewed distributions, we are confident that the most likely value for the sample average will be something close to the true average of the distribution. This is the law of large numbers. As the sample size increases, the probability approaches zero that the sample average differs from the mean of the distribution by any set amount as long as the samples are mutually independent and from a distribution with a finite mean and variance.

In between the two extreme cases – a sample size of one and a sample size approaching infinity – the most likely value for the average of the sample goes from the mode of the distribution up to the distribution average. How does the most likely value of the sample erage change with how skewed the distribution is and the sample size? Let us explore this question by examining results from a positively skewed distribution that is used in insurance modeling – the lognormal distribution.

RESULTS FROM A TYPICAL SKEWED DISTRIBUTION

The lognormal distribution has been used by actuaries to model losses since at least the early 1970's [1]. A lognormal has its hump to the left and a long tail to the right. Because of the shape of the curve, the lognormal implies that small losses are more likely than very big losses. How likely a small loss is as compared to a large loss depends on how skewed the particular lognormal distribution is. The chance of a large number of small losses increases with the skew of the distribution.

Rather than talk about the skew of a distribution, I am going to talk about the coefficient of variation (CV) of a distribution. When we are working with a lognormal, a higher CV is the same as a higher skew. The CV is defined as the standard deviation of the distribution divided by the mean of the distribution. Actuaries typically refer to the coefficient of variation (CV) of a distribution rather than how skewed a distribution so that they can compare the skews of two distributions with different means. Intuitively we should feel that for a family of skewed distributions, the higher the standard deviation, the higher the CV, and the more skewed the distribution. The lognormal is always positively skewed as shown in Appendix B.

Actuaries who use the lognormal for size of loss curves very often have rules of thumb for an appropriate CV depending upon the line of business. CV's of around 1 or 2 might represent low limits liability lines of business, CV's between 2 and 5 might represent mixed property and liability losses, and CV's on the order of 10 might be used for very volatile high limits excess lines of business.

Chart 1 shows three lognormal curves each with a mean of 1000 and with varying CV's. As the CV increases, the mode or highest point on the distribution is associated with lower and lower values than the mean. Other typically skewed distributions would have the same relationship between the mode and the mean – as the skew of the distribution increases, the mode gets lower and lower as compared to the mean.

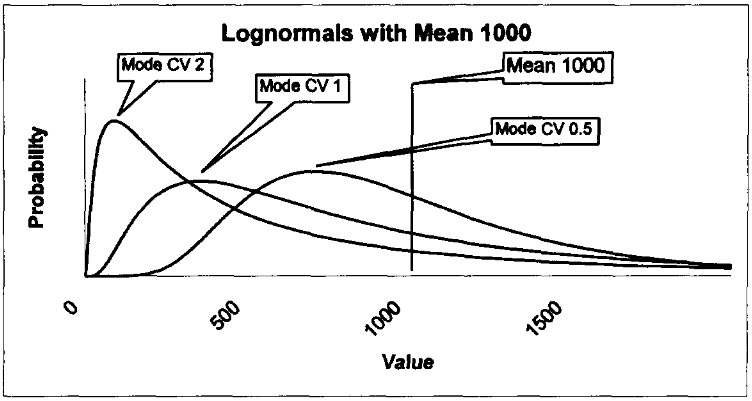


Chart 1

For the lognormal, the ratio of the mode to the mean can be written as a function of the CV. I have included that formula at the end of Appendix B for those who like formulas. Chart 2 shows the ratio of the mode to the mean for lognormal distributions with different coefficients of variation.

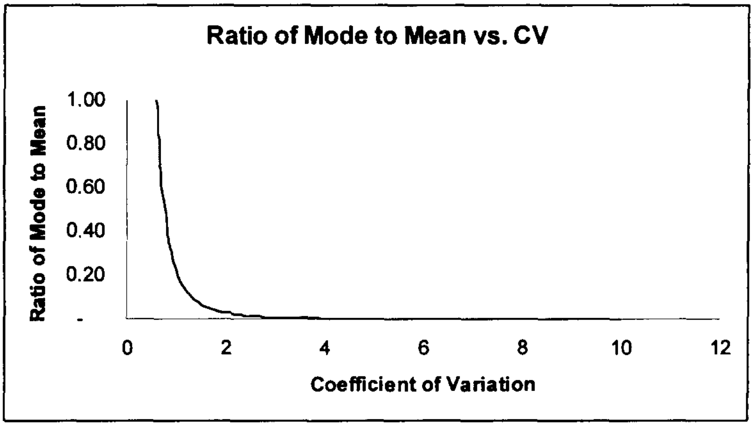


Chart 2

As the coefficient of variation increases, the ratio of the mode to the mean of a lognormal distribution drops off very quickly towards zero. What does that imply? The more skewed the distribution, the more likely a sampled mean will underestimate the true underlying mean. For a lognormal distribution with a CV over two, the most likely value for a sample of one is relatively close to zero no matter how big the mean of the distribution. For small samples, the expected value for the sample average will be close to zero.

To acknowledge that there are people who are uncomfortable with the idea of focusing on the mode rather than the mean, I offer some numbers that might help them get more comfortable. The mode is the point at the highest point on the probability density function. What I am going to show is the area under the probability density function for all points that have a value that is greater than the value associated with the mean. Chart 3 shows a lognormal distribution and we are interested in area "A" between the mean and the point to the left of the mode that has the same probably density function value as the mean.

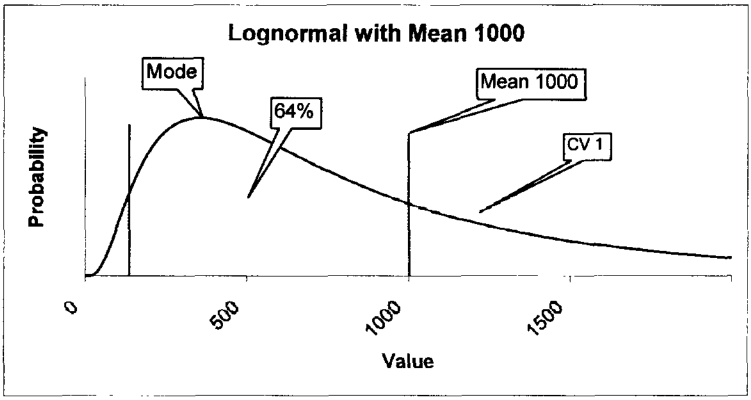


Chart 3

Table 1 shows for varying samples size averages from lognormals with different coefficients of variation the percentage of sample averages whose probability density function value is higher than the value at the mean. This is area "A" in chart 3.

Sample Size	Coefficient of Variation				
	0.5	1.0	2.0	5.0	10.0
1	47%	64%	74%	82%	86%
25	13	28	44	62	70
50	12	19	38	56	66
75	11	16	33	54	64
100	11	13	28	53	63
150	7	13	26	47	57
200	7	3	25	47	57
300	7	3	17	41	55
400	3	2	16	41	53
500	2	2	14	38	51

Table 1 Area "A" for different sample average sizes distributions and CV's

As an example, the numbers in this table says that if you are taking averages from a lognormal distribution with a coefficient of variation of 10.0, then there is better than a 50% chance that the sample average will be below the true average of the distribution if the sample size is 500 or less. I like to think that focusing on the mode makes it easy to capture a lot of this information and I hope to convince you of that with the following simulation exercises.

So what makes up a very small sample size? In order to answer this question, I simulated a single random value from a lognormal distribution with mean 1000 and varying CV's using an Excel add-in called @Risk by Palisades. The @Risk add-in has functions that will simulate random values from various statistical distributions and it has functions that will calculate statistics for the random results. Since I am sampling from a continuous distribution, it is unlikely that I would sample any single point more than once. So rather than find the most common single point, I set the program to keep track of the most common interval of width 5 as a proxy to finding the single point mode. As a check on this process and to see if @Risk actually does what it claims to do, I wanted to see if the mode for a sample size of one tracks with the formula mode of the distribution. The results in Table 2 show that the simulated results track closely to the formula mode of the distribution after 1,000,000 simulations.

CV	Simulated Mode	Formula Mode
0.5	717.91	715.54
1.0	361.67	353.55
2.0	79.75	89.44
5.0	7.86	7.54
10.0	2.51	0.99

Table 2

I then increased the sample size to 25 random values, 50 random values, 100, 200, 300, 400, 500 and 10,000 random values. I measured the midpoint of the most common interval for the average of those larger samples doing 500,000 simulations each time. (See Appendix C for additional details.) The results are shown in Chart 4 for lognormal distributions with mean of 1000 and CV's of 0.5, 1.0, 2.0, 5.0 and 10.0.

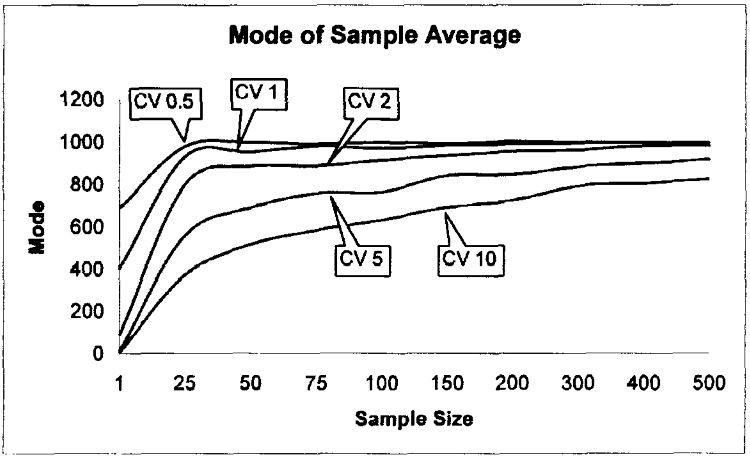


Chart 4

What does this Chart 4 tell us? For individual claim size distributions that have low CV's, the most likely value that we would see from a sample average very quickly approaches the mean of the distribution. We are getting the same result as if we were sampling from a symmetrical distribution where the mode is equal to the mean. However as the CV increases, it takes a very big sample size before the most likely value of the sample average approaches the mean of the individual claim distribution. For a distribution with a CV of 10, even at a sample size of 500 the most likely value we would see from the sample average is 85% of the distribution mean. Formal credibility formulas aside, I believe many actuaries would consider 500 homogeneous claims a fairly large database. Appendix D has charts, albeit more complicated charts, which show additional information about the entire distributions of the sample averages.

With a CV of 10 and a sample size of 10,000, the most likely value we would see is still only 96% of the mean of the distribution. William Blatcher, CFA, points out that a simulation size of 10,000 is a typical @Risk simulation size for actuaries working in reinsurance. Even at this large number of samples, there is still a downward bias of 4% from the actual average of the distribution.

Another thing to observe about these sample results is that the most likely values for the sample averages follow a pattern of rising quickly from the mode of the distribution and then hitting a fairly flat area that approaches the mean very slowly. In his book "Fooled by Randomness" [2], Nassim Taleb discusses how people are misled by skewed distributions.

He focuses on the rare extreme values in the tail of the distribution, which he calls the black swans that are usually missing from the sample results out of skewed distributions. People forget about these black swans or are unaware of them. However, for "small" samples out of skewed distributions, it is not just missing black swans that the observer can innocently miss that can cause problems. The small sample from the body of the distribution is actively misleading the observer because the mode is so much lower than the mean. It is almost as if the distribution is actively evil by feeding us misleading information from its body as opposed to passively withholding tail information from us.

What are some of the implications of this? When we are doing relatively small samples from skewed distributions, we should recognize that the most likely value of the sample average will be less than the mean of the distribution that we are trying to measure. We should adjust our sample results based on the CV of the distribution and the sample size to calculate the population mean of the sampled distribution. The correction factor should be the ratio of the population mean and the mode of the sample average. The mode of the sample average would vary by the sample size. It would equal the population mode for a sample size of one and would approach the population mean as the sample size approaches infinity. Table 3 shows the correction factors for a lognormal distribution for given sample sizes and coefficients of variation based on the simulation results. These values are just the ratio of the actual distribution mean and the mode of the sample means underlying Chart 4.

Sample Size	Coefficient of Variation				
	0.5	1	2	5	10
1	1.45	2.51	11.79	128.90	399.67
25	1.02	1.07	1.25	1.79	2.68
50	1.00	1.05	1.13	1.45	1.93
75	1.00	1.02	1.13	1.32	1.71
100	1.00	1.03	1.09	1.31	1.58
150	1.00	1.02	1.07	1.19	1.44
200	1.00	1.01	1.05	1.18	1.38
300	1.00	1.00	1.04	1.13	1.26
400	1.00	1.00	1.02	1.11	1.24
500	1.00	1.00	1.02	1.09	1.21
10000	1.00	1.00	1.00	1.02	1.05

Table 3: Correction factors

There is actually a precedent for a table of adjustment factors like this. The British

Department for Transport has recognized that planners for large public projects routinely underestimate the actual cost and time of the project [3]. To adjust for this tendency to be optimistic, project appraisers are required to make adjustments or 'uplifts" to the submitted costs, benefits and duration. The factors which depend on the type of engineering project under consideration could be up to 51% for building projects and up to 200% for IT projects. They are used to adjust the project costs to overcome this downward bias and increase them to more likely cost levels.

SMALL SAMPLE BIAS AND CREDIBILITY

When we are credibility weighting two results from skewed distributions, we should recognize that the small sample size average might be different from its population mean only because it is biased downwards. In his paper "An Examination of Credibility Concepts" [4], Stephen Philbrick presents an example of four people shooting at four different targets to help explain credibility. The diagrams in the paper show the historical results for the four shooters with their shots clustered around their four respective targets. The clustering is a simplifying assumption in order to focus on the main point of the paper. We are better able to guess who the shooter is if:

- We see more subsequent shots taken,
- The shooters are better shots or,
- The individual targets they are shooting at are moved further apart.

When the targets are widely separated and the shooters are good shots; we want to give high credibility to the hypothesis that A is the shooter when we see a subsequent shot fall near target A. This follows, in part, from the assumption that the shots are symmetrically distributed around the targets.

Now suppose a wind is blowing across the firing range affecting the results of shooter A. Most of the shots are blown away from target A and land near target B. Occasionally the wind will stop blowing and a shot will land near target A. Even more rarely, the wind will reverse direction and the shot will fall widely wide of the target on the other side. On average all the shots fall around target A. In this example, even if the means and standard deviations of the distribution of shooters has not changed from the symmetrical example, whatever standards we may have created for credibility when the shots were symmetrically clustered around the target have to be increased given that the distribution of shots is skewed. We need more shots, or the shooters have to compensate for the wind to improve their aim, or the targets have to be much further apart to achieve a given credibility standard when we are dealing with skewed distributions as opposed to symmetrical.

In this second example, if we had a small sample from this skewed distribution, we probably would have given little credibility to the idea that the shooter was A unless we really understood the process involved. If we assumed we were dealing with symmetrical distributions, we most likely would have concluded that the shooter was B since the small sample of shots would most likely have been grouped near the mode of the distribution – target B. It is important to understand what type of distribution we are working with and avoid convenient assumptions.

OUTLIERS AND THE ART OF THE DEAL

James MacGinnitie in his Address to New Fellows at the November 2006 Casualty Actuarial Society annual meeting stated that the world is not normal and warns against unexamined use of the bell curve or normal distribution as a model. In his book "The (Mis)Behaviour of Markets" [5], Benoit Mandelbrot of Chaos Theory fame discusses problems with assuming the financial markets behave according to the normal distribution. Why was the normal curve used in the first place? Mandelbrot states that at one time all of nature was assumed to behave according to the bell curve -- that is why it is called the normal curve. Independent observations from a normal curve are not an appropriate model for many financial market situations even though modelers have historically used normal curves. Actual observations come from more highly skewed correlated distributions. Nevertheless, many financial modelers say, "So what?" They argue that the normal curve is a convenient approximation and as long as this assumption does not cause any problems, then we should just ignore any theoretical refinements. MacGinnitie and Mandelbrot claim that assuming normal, independent observations does cause problems by underestimating the true risk associated with skewed processes. Mandelbrot demonstrates that Chaos Theory yields a better model of the financial market's behavior. He says that you cannot make money with this insight but he does assert that it allows you to understand better the risk that is involved which could help you avoid losing money.

If someone makes a statement it is good to check it if we are able. Can we underestimate the risk by using standard statistical techniques on small samples? Table 4 shows the most likely indicated CV from different sample sizes from a lognormal simulation with varying CV's.

Sample Size	Coefficient of Variation of Sampled Distribution				
	0.50	1.00	2.00	5.00	10.00
25	0.46	0.79	1.02	1.63	1.81
50	0.47	0.85	1.31	1.82	2.21
75	0.48	0.89	1.40	2.04	2.66
100	0.48	0.89	1.41	2.16	3.00
150	0.48	0.92	1.5	2.27	3.00
200	0.50	0.93	1.56	2.44	3.01
300	0.49	0.94	1.61	2.55	3.02
400	0.49	0.96	1.63	2.97	3.17
500	0.50	1.00	1.65	2.98	3.67
10,000	0.50	1.00	1.91	3.82	6.00

Table 4 Mode of the Sampled CV's

For small samples from skewed distributions, the most likely value for the CV underestimates the CV of the actual distribution since we are missing the values from the tail of the distribution. If we used these numbers as parameters for our models, then we would underestimate the risk of the situation we are modeling.

There is another reason to avoid assuming the normal distribution either consciously or unconsciously. Besides worrying about the tail of the distribution, we also have to worry about the body. In actuarial and financial work, we have to avoid assuming that "small" sample averages from skewed distributions will give unbiased indications of the true mean of the underlying distribution as they would if we were sampling from a normal distribution.

Actuaries sometimes go out of their way to create problems, for example, by creating smaller and smaller data samples as opposed to maintaining larger groups for large samples. I have seen reserve studies that will take a book of business, split it into 34 different rating groups, and then split each of those into three different currencies for over 100 different groups to study. By doing so many splits of data, we end up creating small sample averages where the results will be biased low. A few groups may have large claims but those are ignored as aberrations as opposed to being recognized as the extreme values from a skewed distribution. If only they were combined with the other sample values in larger groups, then there would be a better chance of yielding a more appropriate estimate of the true population mean.

Is there a psychological explanation for why people disregard outliers? One of the explanations for our tendency to disregard outliers has to do with our training. Students are taught that,

> *"An outlier is a point which your data set is better off without. If you can prove your point better by ignoring some small portion of your data, why not ignore it? It's probably just a blunder on the part of the person collecting data, or some special, irrelevant circumstance that we needn't investigate in detail."* [6]

I hope everyone is appropriately shocked with this advice and will acknowledge they have never followed it in the past nor will they ever follow it again. MacGinnitie and Mandelbrot strongly recommend not ignoring outliers. Taleb and others argue that properly accounting for outliers is how to win or lose the big money. Helping customers deal with outliers is what insurance is all about.

Ignoring outliers could be instinctual. As the herd moves on, the weak, the old and the sick fall behind becoming outliers to be picked off by the wolves. The clustering illusion is an identified psychological bias where people will pick out patterns even when none exist [7]. Because of this bias professionally designed standardized tests do not have long runs of a particular multiple choice answer. Students would feel such a pattern is unlikely and then feel pressured to answer incorrectly just to break the run.

Certainly actuaries make their living by finding and identifying patterns. Once a theory is formed about a particular pattern the confirmation bias in psychology is the tendency for people to search for or interpret information to confirm one's preconception [7]. Outliers don't fit the pattern and they don't support the basic idea that's being proposed, so ignore them. What is even worse, the more outrageous the outlier the more likely we are to throw it out of the sample. We can put all these biases all together to explain why people ignore outliers and call it the "Simon and Garfunkel Bias" – still the man hears what he wants to hear and disregards the rest [8].

Certainly, there may be business reasons for a person to leave an outlier unexamined when pricing a deal. There are no absolute rules. And for certain parties in the transaction, it is in their best interest to deemphasize the outliers. The negotiation skills and dedication of the brokers and market makers influence the final price. The best dealmakers that I have seen in action are those that continually work on the ego of the person they are trying to sell. The skilled insurance broker will set up a situation where the rejection of the proposed deal at the suggested price implies the insurance underwriter lacks cojones; they are not a real player. The broker will threaten that they have other underwriters at other companies – real business people – that are ready and willing to do the deal. Eventually, the ego driven underwriter will be dying to do the deal in order not to appear weak in front of the broker.

It doesn't matter about outliers. The broker now has the underwriter working for them in finding a way to do the deal rather than the underwriter working for their employer. This whole process is a beautiful thing to behold. You really have to admire a good broker at work.

This is where an objective actuary can be a valuable asset in these negotiations. Actuaries typically get their ego kicks from doing a thorough analysis and beating other people (either in competitive exams or in doing accurate forecasts). If another actuary has arrived at an estimate that is lower than your estimate but cannot give a satisfactory reason for why your answer is too high, then you will stubbornly stick to your result. This can be a valuable sanity check for the underwriter when evaluating a deal. Whether the deal finally gets done or not at a particular price will depend on many things. Ego is involved in a complex interaction with many different forces; forces that will vary from company to company. The actuary can be a big assistance in providing a quantitative estimate that takes into account all the available information.

Actuaries are also subject to ego problems and can be a liability to the process. Forecasts of indicated prices are the appropriate combination of all available information including outliers. As a deal is negotiated, very often new information is introduced that was not available when the first price forecast was produced, for example, a legitimate explanation of the outlier. That new information could cause the forecasted price to go up or it could cause the forecasted price to go down. It has been my experience that actuaries are more willing to allow their prices to go up rather than to go down based on new information. Part of this might be the natural reluctance to lower a price based on the suspicion that only good information is being shared and none of the bad information. Some of it might just be misplaced pride in that changing an answer somehow implies that the original forecast was wrong. Some of it might just be psychological.

There have been experiments done asking people to guess a particular number when they have no idea what the appropriate answer is. For some reason the first number that people hear sets the magnitude of the perceived correct answer whether or not it is anywhere close to the correct answer. This is known as the anchor effect [7]. All future answers will be judged against this initial answer. For example, what is the population of Brazil? Someone might throw out a guess that the population is 40 million people. It sounds like a reasonable number. From that point on people will be evaluating future answers to the question, including the correct one, based on this initial guess. And future guesses will tend to fluctuate in the neighborhood of this initial guess. (What is your guess for the population of

Brazil?*)

Are actuaries subject to the anchor effect? Are they more likely to be subject to it when they are the author of the original forecast or guess? Everyone else is affected by it so why not actuaries? If you have been around long enough, you have definitely seen this process in action. An early number sets the value for a deal, a transaction or an acquisition. Based on that early number a decision is made to do the deal or not. From that point on it doesn't matter what new information is brought forth and how the numbers change. A decision has been made and a course of action is in motion. The first numbers that are released are very important because those may be the last numbers anyone pays attention to.

The actuary has to bring a forecast to the table that reflects all the information available including outliers. A forecast is different than a prediction of the future. If data might have been withheld that can influence the answer or even if appropriate data are not available to do a proper forecast such as with a small sample from a skewed distribution, then that inadequacy of the data has to be built into the pricing of the deal. Exactly how the price is adjusted for the lack of the data is a judgment call. But that judgment call is made by those responsible for the deal. The actuary has to be upfront with the indicated forecast based on the information available and also explicit about any additional loadings in the price that are due to the quality of the data.

BLESSED ARE THE LOSERS

Economists are concerned with a problem called the winner's curse [9]. In this problem several bidders are competing for an item in an auction and the winner will be the highest bidder. This item is worth the same to all the bidders. The bidders only have incomplete information about the true value of the item and they have to make estimates about this value to prepare their bids. The average of all the bids is assumed to be around the true value of the item. If this is the case, the winner will tend to lose money since they will bid more than the item is worth – the winner's curse. Savvy bidders will avoid the winner's curse by bid shading or quoting a price below what they believe is the value of the item. The bidder who follows this strategy will lower their chance of winning a particular auction but increase their expected return over time. This is the ideal situation as described by economists.

For the reality of the insurance world we have to make some changes as the problem is more complicated. One change is the winner is the lowest bidder not the highest. Another

* The population of Brazil was reported to be 188 million people in 2006.

difference is that a particular account might be more valuable to one insurer than to another insurer for a variety of possible business reasons. An account might have value outside of its expected profit. For example, a company might have written premium goals and they will set the price to win the account as opposed to setting the price at the estimated value of the account. The winning bid does not necessarily lose money even if the bid is below the long term average cost of the account because the annual cost of an account is not fixed. An insured might have a good year and experience low losses for the year of the auction. Another difference is that the result of the traditional auction problem is known immediately but for an insurance contract, it could take years for the actual result to be known. Because there is this lagging feedback mechanism, inappropriate pricing might persist for a period of time as opposed to being corrected immediately. I would think these differences would tend to increase the monetary losses of the winner. A difference that would tend to ameliorate the losses of the winner is that all insurance companies are not all created equal. In real life, the insurance buyer bases their purchase decision on more than just price. They may go with a higher priced policy if they expect to get better service from a particular insurer.

Participants in the traditional auction problem are assumed to have incomplete advanced information about the value of the item being auctioned. This is certainly true for insurance. We also have the potential additional problem associated with small samples from skewed distributions. Gary Blumsohn points out that the more skewed the distribution, the more likely it is that bidders will be quoting prices based on downwardly biased sample averages and thus the winner's curse will be compounded.

Bidders in this case should build into their decision process how skewed the loss process is and how much actual loss information is available to price the account. The more biased the actual losses or the smaller the pool of available information on the particular account or similar accounts, the more we should be concerned about adding a charge to our bid to compensate for any potential biases in the available information. In some extreme situations, we might just want to quote a "go away" price to the risk or broker. That would be a situation where we are very uncomfortable with the risk involved and/or the deficiencies in the available pricing information. If we are concerned about pricing it too low, then we should just quote a price high enough so that it is unlikely to be accepted but high enough so that we still feel OK if for some reason we are the winning bidder.

If one is faced with the risk of an event that is likely to be a small sample from a highly skewed loss distribution, then there are a few things that can be done to improve the situation. The first is to increase the sample size and combine that risk with other risks to take advantage of the law of large numbers. If you are a single insured, you buy a policy from an insurance company who does the combining of risks for you. If you are an insurance company you can expand your writings until you have sufficient volume to produce stable results. As an insurance company, if it is impossible to combine the presented risk with a sufficiently large number of other independent risks, then the other alternative is to reduce the skew of the risk distribution. You can reduce the limits on the policy, restrict policy terms, or buy some form of appropriately priced reinsurance. Finally, the last thing that you can do is reduce the probability of the event to zero by not writing the risk at all.

A SMALL SAMPLE OF SKEWED RANDOM THOUGHTS

More than once, I have heard a story at a luncheon at a Casualty Actuarial Society meeting about either a start up company or a new branch of operation where the initial loss experience is good. The stories deal with heroic battles between an actuary and the naïve management team. The actuary wants to hold surplus and maintain high rate levels in anticipation of losses yet to come. Management wants to cut rates or pay out large dividends based upon the small but exceptional experience to date. In the stories that I have heard, either the actuary wins out or the company barely survives its first few years. Because those are the only endings that I have heard, I have to assume that there is a survivorship bias in these results – only the survivors are happy to share their stories.

The heroes of these stories recognized that skewed distributions give biased results not just due to small sample sizes but also because the mode is seen before the average result. Incremental claim reports follow skewed patterns. Once people in a company see incremental claim reports from a particular accident or policy year declining after the mode of the distribution has passed, they might think the worst is over and that claim reports will drop off as fast as they appeared. However, the tail that follows the mode could stretch out for years. Actuaries who have recognized this and have convinced their colleagues of claim reports yet to come have the right to boast [10]. Actuaries think accident year; everyone else thinks calendar year.

Speaking of start up companies, another lunchtime conversation has to do with the strategy of starting a reinsurance company devoted to catastrophes. The question is whether the company will be able to build capital by surviving its first year without sustaining a catastrophe. These companies are insuring events from highly skewed distributions. The most likely loss that they will see is zero. Chance is in their favor that they will survive the first year. This same type of thinking could explain why some investors are willing to rush in and refinance catastrophe reinsurance companies after they suffer a particularly bad season. The hope of stock price recoveries from big premium increases following the cat loss also has something to do with it.

In case the traditional age to age development method has not been beaten up enough in recent papers, what about the way we average development factors? In the past, I have used methods that take the average of the most recent x development factors excluding the high and the low value. I have also heard of methods that use x binomial factors as weights to apply to the most recent x development factors ranked from highest to lowest to get a weighted average estimate. It certainly sounds like these methods are making an implicit assumption that we are sampling from a symmetric distribution if not a normal one. However, if these unquestionably small sample averages of individual loss development factors are from a skewed distribution then these methods are throwing out or downplaying important information.

We become complacent about our safety or survival from repeated exposure to threatening situations that do not actually happen. Psychologists call this habituation [11]. When I worked in Jersey City, NJ, I would pass a good example of habituation in action every day when I went to work. Each morning I would drive past Nunez Restaurant on the corner of Montgomery Street and Jordan Avenue. The owner of the restaurant had put a couple of plastic owls on the ledge of his building to scare away the pigeons and save his customers untold embarrassing problems. However, the pigeons had become habituated to the owls. Initially the model of their natural predator would have scared the pigeons away but when nothing threatening ever happened, the pigeons learned to suppress their natural instinct to be afraid. So many pigeons have become habituated at Nunez Restaurant that on some mornings the corner could be used for the Jersey City/pigeon remake of Alfred Hitchcock's "The Birds". The pigeons have become conditioned to the mode of nothing happening and suppressed their fear of the extreme event of being attacked by the stationary plastic owls. (Some people might argue that my driving to work every day down Montgomery Street in Jersey City is a good enough example of habituation.)

People are not to be outdone by pigeons. Here is a possible explanation for riding a motorcycle without a helmet. People are probably encouraged to do any high risk activity because they get something out of it and, most likely, not suffer any consequences for at least the first few times. The loss distributions associated with any high risk activities are skewed – driving without a seat belt, cave diving, painting the outside of a house without securing the ladder. For a small number of trials, any individual is most likely to experience the mode of the distribution and not suffer any consequences. Based upon the lack of any immediate losses, the individual grows complacent, ignores any warnings, and continues with the activity. Yet the possibility that the individual will experience a loss increases over time as the sample size increases.

The difference between the mode and the mean in skewed industry loss distributions might be a contributing factor to the insurance cycle. The distribution of annual industry results appears skewed to me. If that is the case, the most common result for the industry during the cycle period will be something closer to the mode of the distribution. All other things being equal, competition will keep pricing levels below the mean of the distribution as people grow complacent and the sky doesn't seem to be falling as constantly predicted by actuaries. Every once and a while there will be a major industry loss event. The industry will feel the cash flow shock because it was pricing below the long-term average and it will overcorrect above the long-term average when it reacts. Mix in some skewed distributions associated with the asset side of the balance sheet and away we go.

Along these lines, Ted Kelly, CEO of Liberty Mutual was quoted in the November 27, 2006 [12] issue of the National Underwriter warning about pricing levels in the 2006 property market. Property insurance prices had increased dramatically in 2006 because of the losses associated with Hurricane Katrina in 2005 and presumably due to the early predictions by the hurricane forecasters of severe hurricanes for 2006 and beyond. He said, "The lack of catastrophes this year will create its own set of problems, including accusations that we cried wolf when we raised rates and are now price gouging." He joked that, "It's like saying someone who survives Russian roulette faced no risk just because the gun didn't go off, when we all know there is still a bullet in the chamber, and if you play the cat game long enough, it's going to go off." In my opinion, using the best estimate of the loss over the period in which the policy is exposed would be the correct way to fund for catastrophes. Currently, all the market forces seem to produce a collective behavior that is influenced by the results of small sample averages and then plays catch up after a major shock loss. If nothing else, funding at the best loss estimate for the exposure period would identify the costs that that market is facing. That being said, what is the loss distribution and what is the best cost estimate are among the difficult questions that all the participants in this market have to answer.

The only cure for complacency is a conscious effort to take measures guarding against extreme events. Insurance companies exist to help our customers guard against extreme unexpected financial consequences of life. As actuaries and managers of insurance companies, we have to make sure we are forecasting the true long-term results and acting appropriately to account for extreme events so that our companies will be there to pay the losses of our customers. We have to avoid complacency bred by constant exposure to the mode of distributions.

A quote attributed to former first lady Barbara Bush is, "Bias has to be taught." She was speaking about prejudices and that children learn prejudices from adults as opposed to being born with any biases. If we speak about the statistical bias of small sample averages, there are whole hosts of places where subtly we are being misled. No one is teaching us these things. We are forming theories based on small samples and then forgetting or not realizing that those theories might be wrong since they are based on small samples. Is that not a definition of prejudice – developing a theory and then forgetting that it is a theory and assuming that it is fact? It would have been better if Barbara Bush had said, "Bias has to be fought".

There is an old insurance joke that says an insurance company is a car being driven down the road by the blindfolded president of the company. The head of marketing is stepping on the gas, the underwriter is stepping on the brake and the actuary is looking out the rearview mirror yelling which way to turn. To paraphrase the warning label that appears on the passenger side rearview mirrors of US cars, those loss estimates that the actuary sees are larger than they appear.

In that joke, the actuary is the only person in the car who is looking at any section of the road. When working with small samples from skewed distributions, we should keep in mind that it might take many samples in order to get an average that provides a good estimate of the true average of the underlying distribution. We have to understand the loss process we are trying to model along with the limitations of our data samples, and make forecasts and recommendations accordingly.

Acknowledgements

I would like to thank William Blatcher and Larry Schober for their valuable feedback on earlier drafts of this paper. Also I would like to give many thanks to the reviewer of this paper Gary Blumsohn.

Appendix A

Counter Examples Where the Mode of a Distribution is Larger than the Mean of a Distribution

There is more than one definition of the skew of a distribution. The skew of a distribution is usually calculated as the third central moment of the distribution.

$$\frac{1}{n}\sum\left(\frac{(x_i-\bar{x})}{s}\right)^3$$

In this formula, n is the sample size, $\bar{x}$ is the sample mean and s is the sample standard deviation. A positively skewed distribution has a longer tail to the right.

In Excel, the formula for the skew of a distribution is the following:

$$\frac{n}{(n-1)(n-2)}\sum\left(\frac{(x_i-\bar{x})}{s}\right)^3$$

A distribution may have more than one mode but for this discussion I am going to assume that we are dealing with distributions with only one mode.

Using the Excel definition, it is easy to construct a discrete distribution that is skewed to the right and the mode is greater than the mean [13].

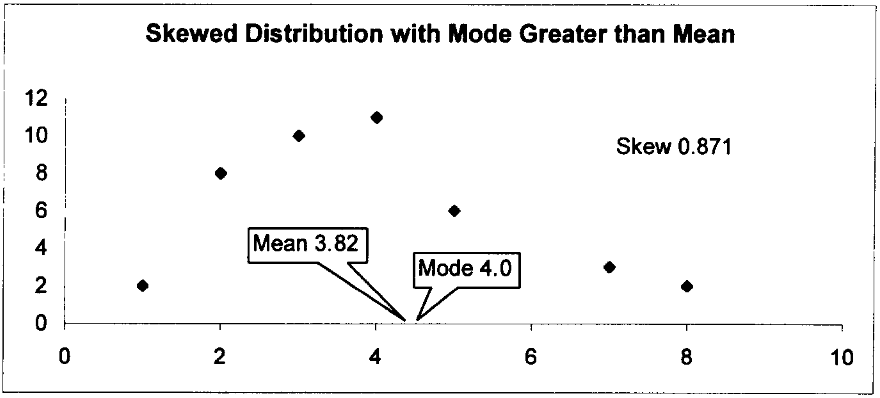


Chart 4

One way to remove the counter intuitive examples is to define them away. The Pearson mode skewness of a distribution is defined as:

$$\frac{\text{mean - mode}}{\text{standard deviation}}$$

Using this definition, a positively skewed distribution would always have the mean higher than the mode.

Appendix B
Some Formulas for the Lognormal Distribution

As a reference or reminder, the probability density function for a lognormal distribution with parameters $x < 0$, $-\infty < \mu < \infty$, and $\sigma > 0$ is the following [14]:

$$f(x,\mu,\sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln x-\mu}{\sigma}\right)^2}$$

The mean is equal to:

$$e^{\left(\mu+\sigma^2/2\right)}$$

The variance is equal to:

$$e^{\left(2\mu+\sigma^2\right)}\left(e^{\sigma^2}-1\right)$$

And the coefficient of variation (CV), the standard deviation of the distribution divided by the mean, is equal to:

$$\sqrt{\left(e^{\sigma^2}-1\right)}$$

Formulas that actuaries are probably not familiar with are the formulas for the median:

$$e^{\mu}$$

The formula for the mode:

$$e^{\left(\mu-\sigma^2\right)}$$

And the formula for the skew of the distribution is the following:

$$\sqrt{\left(e^{\sigma^2}-1\right)}\left(2+e^{\sigma^2}\right)$$

The lognormal is positively skewed for all values of σ.

The formula for the ratio of the mode to the mean as a function of the CV is the following:

$$\frac{\text{mode}}{\text{mean}}=\left(CV^2+1\right)^{-\frac{3}{2}}$$

Appendix C
Simulation of Results

For these simulations, I used @Risk Version 4.05. I did different sample averages for different sizes 1, 25, 50, 75, 100, 150, 200, 300, 400, 500 and 10,000. For each sample size I took a sample average of that many random values of the lognormal function and for each different sample size I used different random values. I did not generate 10,000 random values and then take the average of the first 25, the first 50, etcetera.

At each simulation, I selected as an output the RiskMod function to measure the mode of the simulated sample averages. Because we are dealing with a continuous distribution, I checked for the midpoint of the most common interval of width 5 as opposed to the most common single value. Excel/@Risk formulas for sample size 5 are below in Figure 1.

	A	B
1		
2	Mean	Standard Deviation
3		
4	1000	2000
5		
6	1	=RiskLognorm(A4,B4)
7	=+A6+1	=RiskLognorm(A4,B4)
8	=+A7+1	=RiskLognorm(A4,B4)
9	=+A8+1	=RiskLognorm(A4,B4)
10	=+A9+1	=RiskLognorm(A4,B4)
11		
12		Average 5
13	=RiskOutput() + TRUNC(AVERAGE(B6:B10)/5,0)*5+2.5	=+RiskOutput()+RiskMode(A13)
14		SD 5
15	=RiskOutput() + TRUNC(STDEV(B$6:B10)/5,0)*5+2.5	=RiskOutput() + RiskMode(A15)
16		CV 5
17	=RiskOutput() + TRUNC((+A15/A13)/0.01,0)*0.01+0.005	=RiskOutput() + RiskMode(A17)
18		

Figure 1

Appendix D
Contour Maps of Sample Average Distributions

In the paper, I showed charts on how the mode varies with the CV and how the mode varies in relation to the mean as the sample size varies. I wanted to get a feel for how the distribution of the lognormal sample average is shaped at different sample sizes. Even though the mode of the distribution is the most likely result, I was interested in whether it is really more likely than other values or just marginally more likely. Here are the results using standard output from @Risk and Excel graphing routines.

Charts 5 and 6 are contour maps of the distributions of sample averages of lognormal distributions with CV's of 1 and 5 as the sample size varies. These are two-dimensional representations of three-dimensional surfaces. If you are familiar with topographical maps, then think of these charts in the same way.

Contour Map of Sample Average Distributions CV = 1

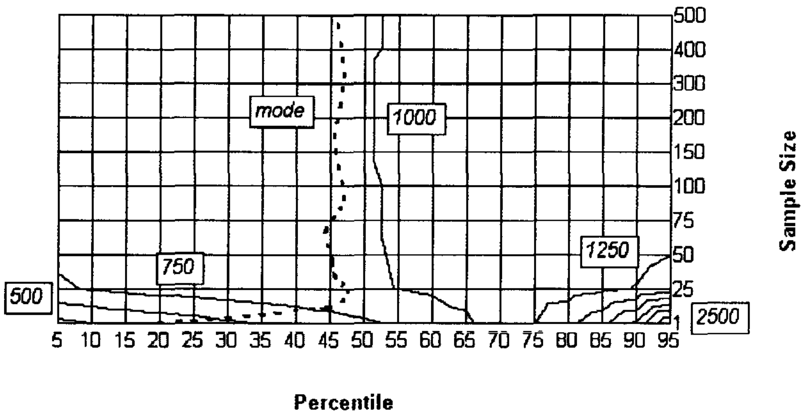


Chart 5

Reading across the gridline on Chart 5 for a sample size of 25 from left to right, the 5th percentile point is on a slope between the 500 and 750 contour lines. At the 10th percentile point, a wide gentle plateau begins going from 750 up to 1250 at about the 85th percentile point. The mode appears approximately the 47th percentile point and the mean is close by at the 54th percentile point. After we pass the 85th percentile, the distribution approaches a steeply rising area signified by the contour lines getting closer and closer. The 95th percentile point looks like it is just under a value of 1500. If we are sampling from a distribution with a

CV of 1 and the sample size if 25 or above, it is not going to be a problem that the mode of the average is lower than the mean. The values are close together straddling the median value at the 50th percentile line. It looks like we could get away with an assumption of a symmetrical distribution.

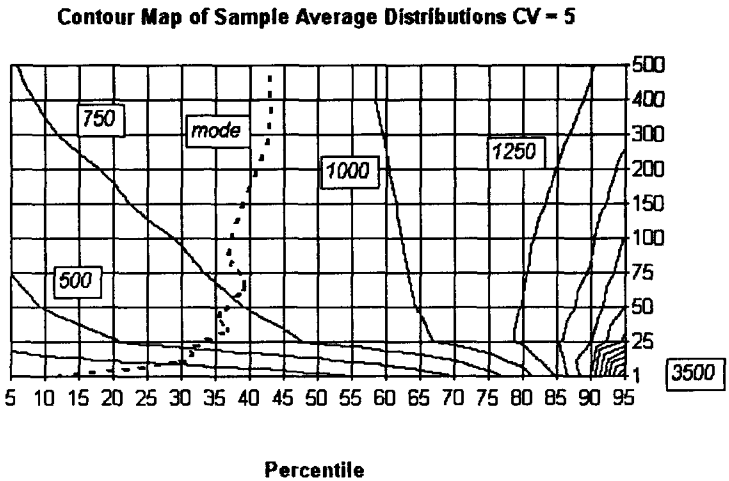


Chart 6

When the CV is 5 as in Chart 6, the map gets to be a little more interesting. If we look across the 25-sample size gridline again, the fifth percentile point is very close to the 250 contour line. Now rather than a gentle plateau stretching across the graph, we have a steadily increasing slope going across the chart up to the 1250 contour line at the 78th percentile. Here the surface starts to increase more steeply reaching just above the 2250 point at the 95th percentile. On this chart, the mode and the mean are widely separated for a sample size of 25. Approximately 67 percent of all the values of the distribution are below the mean. The mode looks like it is situated right in the middle of those values at the 33rd percentile. There are definitely higher values that are likely to occur out on the higher percentile part of the curve.

As the sample size increases, the situation is not as clear-cut that we can get away with a symmetrical distribution approximation as it was when the CV was lower. Here the 1000 contour line stays above the 60th percentile until the sample size reaches 250. Even at a sample size of 500, the contour line for 1000 is above the 57th percentile while the mode is at 43rd percentile.

These charts also show something interesting about company funding and size. Suppose we are collecting exactly the expected losses from each insured and we want to have enough surplus in the first year so that we have a 90% chance that surplus will not go negative just due to loss fluctuations. Using Chart 5, we can see for a small company producing only 25 claims it would need an additional 750 of surplus for each claim resulting in a premium to surplus ratio of approximately 1.33. A larger company producing 500 claims could get by with a premium to surplus ratio of approximately 4.0.

REFERENCES

[1] Hewitt, Charles C. Credibility for Severity Proceedings of the Casualty Actuarial Society Casualty Actuarial Society 1970 Vol. LVII pp.148-171
[2] Taleb, Nassim Nicholas. Fooled by Randomness: The Hidden Role of Chance in the Markets and in Life. New York: Texere LLC 2001
[3] Flyvbjerg, Bent, "Procedure for Dealing with Optimism Bias in Transport Planning." [Online] Available http://flyvbjerg.plan.aau.dk/0406DfT-UK%20OptBiasASPUBL.pdf 2007
[4] Philbrick, Stephen W. An Examination of Credibility Concepts. Proceedings of the Casualty Actuarial Society, 1981 Vol. LXVIII pp. 195 – 212
[5] Mandelbrot, Benoit, and Richard L. Hudson. The (Mis)Behavior of Markets: A Fractal View of Risk, Ruin and Reward. New York: Basic Books, 2004
[6] Mittledorf, Joshua. Response to, 'What is the definition of outlier?' [Online] Available http://www.mathforum.org/library/drmath/view/52801.html 2007
[7] Wikimedia Foundation. List of Cognitive Biases [Online] Available http://en.wikipedia.org/wiki/List_of_cognitive_biases 2007
[8] Simon, Paul. "The Boxer" 1968 [Online] Available http://www.lyricsfreak.com/s/simon+and+garfunkel/the+boxer_20124664.html 2007
[9] Wikimedia Foundation. Winner's Curse [Online] Available http://en.wikipedia.org/wiki/Winner's_curse 2007
[10] Weissner, Edward W. Estimation of the Distribution of Report Lags by Method of Maximum Likelihood. Proceedings of the Casualty Actuarial Society, 1978 Vol LXV pp. 1 – 9
[11] Wikimedia Foundation. Habituation [Online] Available http://en.wikipedia.org/wiki/Habituation 2007
[12] Friedman, Sam. "Top Dogs Barking." National Underwriter P&C, 27 November 2006, p. 5.
[13] von Hippel, Paul T., "Mean, Median, and Skew: Correcting a Textbook Rule." [Online] Available http://www.amstat.org/publications/jse/v13n2/vonhippel.html 2007
[14] Hogg, Robert V. and Stuart A. Klugman. Loss Distributions. New York: John Wiley & Sons 1984

An Exposure Based Approach to Automobile Warranty Ratemaking and Reserving

John Kerper, FSA, MAAA

Lee Bowron, ACAS, MAAA

ABSTRACT: Existing actuarial techniques for automobile warranty ratemaking and reserving rely heavily on emerging experience (loss development) for the pricing and unearned premium reserving of these products. Since terms for automobile warranties can extend up to 10 years, such data is typically not available or not credible to the degree that the actuary can take great reliance on it. In addition, changing coverage terms in the auto warranty products can often make past development even less meaningful. Exposure techniques that have been developed (Cheng, 1993) rely on overall averages for some critical assumptions instead of distributions or individual policy characteristics.

We propose a "miles-driven" approach in which claims are assumed to arise from auto warranties in proportion to the miles driven times a weight assigned to the overall mileage of the vehicle. The method we employ is much more complex than traditional methods, but relies on data that is typically available at warranty writers. Important data elements would include the mileage of the vehicle at the time of a claim and if the contract cancels. In addition, the underlying manufacturer's warranty is also critical.

In order to provide an accurate model of pricing, a distributional approach is utilized for each policy to model the different driving habits of the policyholders. For example, claim costs can be developed using 5 different driving habits for each policy.

Such a method is very useful for the pricing and premium reserving of new coverages or at start-up companies.

The method proposed utilizes "policy-event based loss estimation methodology" in which a predicted claim cost is derived from each warranty individually.

1. The Continuing Problem of Extended Warranty Coverages

Pricing issues continue to plague the extended warranty industry for vehicles, often known as "vehicle service contracts." Some of these issues are due to the structure of the industry which has historically had a low barrier to entry and a significant number of players with capital constraints. As such, the market can attract inexperienced players that are unaware of the complexities of this insurance product.

Warranties may be written as traditional insurance products, or may be in risk retention groups or captives. In some cases, warranties may not be classified as insurance for regulatory purposes. Regulation of warranty products varies widely and is constantly changing. Due to the fragmented nature of the industry and the variety of forms that warranties may take, it is difficult to compile industry level statistics.

The long warranty period gives rise to a long payout pattern that can mask optimistic pricing and reserving assumptions for several years. Terms for automobile warranties can range up to 10 years. For new car coverages, the effective coverage provided by the warranty over this time period is not uniform. For the first several years, relatively few claims are paid as manufacturer's warranty will cover most claims. As the manufacturer's warranties begin to expire, claims will begin to rise dramatically. Claims also should moderate at the end of the contract as many contract holders will "mile out" of their coverage – that is they will drive the allowed miles before the time has expired. In addition, the policyholder may sell or otherwise dispose of the vehicle without transferring the warranty to the new owner.

In general, this paper will use the term "warranty" which is common in the actuarial industry. However, the term "service contract" is increasingly being used in the industry. For the purposes of this paper, these terms are interchangeable.

2. The Structure of Automobile Extended Warranty Industry

Extended warranty or service contract underwriting is structurally different from other property/casualty products and an understanding of the structure and terminology may be helpful for the actuary who is unfamiliar with the business.

Although there are many different models, a common practice is that the extended warranty is sold at the dealership at the time of purchase of a new or used vehicle. Typically, the consumer may encounter several ancillary products which are sold at the time the vehicle is purchased. These would not only include extended warranties, but also pre-paid maintenance, GAP insurance (which covers the difference between the actual cash value and the loan balance at the time of an insurable event if the vehicle is a total loss), VIN etch, etc. These products are almost always financed with the vehicle. Once an extended warranty has been sold, the amount charged for the warranty will be divided into several components. These include:

- Retail markup (for the auto dealer)
- Agent's commission
- Administrator Fee
- Warranty Reserve

An administrator typically will perform all the processing and servicing of the warranty. An agent will represent the administrator to the dealer clients. The warranty reserve is remitted to an insurance company, which may or may not be owned by the administrator. For the actuary, there are two items of note:

1. The terminology of reserve is misleading because "reserve" in extended warranty typically refers to all funds used to pay claims, not just the outstanding portion, and is more analogous to written premium. For our purposes, we will use the term premium.
2. Since the vast majority of expenses are paid prior to the remittance of funds to the insurance company, the expected loss ratio is higher than other property/casualty products. Often, a book will be priced at an expected loss ratio of 95 to 100 percent.

Because these contracts are generally single premium and long term, there is a significant amount of investment income associated with extended warranties.

While this paper only concerns the calculation of expected loss costs for extended warranties, these techniques could also be used by administrators to recognize their fees in proportion to the expected claims from service contracts.

3. Warranty Exposure Bases

In general, exposure bases are measurements for insurers that tell of the relationship that exists between insurable objects and critical conditions where a claim can occur, that note the proportional size of hazard as measured by the losses (magnitude), and that are preferably practical and already in use. This means that exposure bases should have certain qualities, namely, accurate in measure of exposure to loss, easy to determine, and difficult to manipulate.[1]

The purpose of exposure bases is to determine the exposure to loss for an insurer based on the expected loss determined by a series of accepted calculations in order to use the simple and reliable data to develop correct premiums for the insurer and equitably distribute the premiums among the insureds.

For vehicle service contracts, exposure bases are somewhat unique in that the exposure base used to price and rate the coverage (Miles/Time) is not the exposure base that has been commonly used to evaluate the experience (Projected Claim Reporting Pattern).

Deriving an appropriate exposure base for vehicle warranty coverage is a fundamental question when analyzing this line. Fortunately, changing the exposure base in the analysis of the product does not imply changing the exposure base used to market the product.

- Time (Earned Warranty Year) is a poor choice. Warranty claims are not uniform during the policy period. For an extended warranty sold for a new car, the claims pattern will be especially non-uniform, with few claims arising during the initial period that is covered by the manufacturer's warranty. The majority of claims will occur after the manufacturer's warranty expires. In addition, there will be a drop in claims at the end of the warranty as many vehicles exceed the maximum mileage allowed under the warranty or are sold without the transfer of the warranty coverage.
- Indicated Claims Reporting Pattern – This is the most common exposure base used today. This is formulated by developing incremental pure premiums (Cheng, 1993) or simply developing losses by reporting period. This is typically done by loss triangulation. However, instead of aging the claims since the time of the accident, the age of claims are measured from the inception of the policy. This method is appropriate, however, only if:

    1. There is enough data to make these assumptions. While extended warranty achieves credibility at low volumes due to the high frequency/low severity

[1] See Bouska, 1989

nature of this coverage, there may be limited or no data at the latter points of the coverage being analyzed. If there is no data, common practice is to revert back to a benchmark pattern which may not be appropriate for the book being analyzed.

2. The data is homogenous in each cell. This assumption is difficult in that the underlying warranties analyzed may change over time. For example, if the average new car warranty on cars sold five years ago was 36 months but it has now increased to 48 months, the historical pure premium at 60 months will not be predictive of the projected pure premium. In addition, the mix of business may change (European makes typically have higher costs than Asian makes, for example). Another problem is that the coverage offered typically changes due to market conditions.

- Mileage Driven – This is the exposure base proposed in this paper. If mileage is hypothesized as an exposure base, then there is an assumption that claims are basically a function of the number of miles driven by the vehicle. This method is helpful for a number of reasons:

  1. Underlying warranty information is typically available at the individual contract level. Therefore, one could explicitly model the miles driven inside and outside the manufacturer's warranty.
  2. Historical claims information at the end of the contract is not necessary to make an estimate of future claims. Future claims can be modeled as a function of miles driven and the underlying cost per mile. While the claims cost per mile will increase with age, this assumption can also be modeled and tested.

3. A Different Approach

A better approach than loss development for estimating ultimate costs for either pricing or reserving is an exposure based modeling basis, where future losses are modeled for all contracts. This approach has been suggested for modeling other insurance liabilities, such as environmental and asbestos claims (Bouska, 1996). There are several advantages to modeling at the exposure level.

Unlike many insurance products, extended warranty is a high frequency/low severity coverage. It is common for most extended warranties to experience several claims during the life of the warranty. Because of the nature of extended warranty claims, loss data at specific evaluations is credible at relatively low levels, if credibility is defined by the number of claims reported.

The difficulty is estimating the exposure base. This paper proposes an exposure base consisting of the miles driven for the vehicle, so that each mile driven under the warranty is considered an exposure unit.

A miles based exposure base over the term of the contract is closely matched to the actual exposure of the vehicle, as claims can be considered a function of the miles driven during the contract.

One problem with using miles as an exposure base is that there will be some increase in claims per mile during the latter periods of the contract when the frequency of claims will rise due to the age and mileage of the vehicle. This problem can be alleviated by a trend factor, though for newer sets of contracts it will remain a source of uncertainty.

4. A Warranty Pure Premium For a New Vehicle Using a Mileage Function

T_{BASIC}	=	Maximum term of manufacturer basic (full) warranty in months
T_{PT}	=	Maximum term of manufacturer power train warranty in months
T_{START}	=	Age of Vehicle in months (since in service date) at start date of extended warranty
T_{BA_REM}	=	Remaining term of manufacturer basic (full) warranty in months at start date of extended warranty
	=	Max $(0, T_{BASIC} - T_{START})$
T_{PT_REM}	=	Remaining term of manufacturer power train warranty in months at start date of extended warranty
	=	Max $(0, T_{PT} - T_{START})$
T_{EXT}	=	Maximum term in Months of extended warranty at start date of extended warranty
M_{BASIC}	=	Maximum term of manufacturer basic (full) warranty in miles (actual odometer reading)
M_{PT}	=	Maximum term of manufacturer power train warranty in miles (actual odometer reading)
M_{START}	=	Actual odometer reading in miles at start date of extended warranty
M_{BA_REM}	=	Remaining miles of manufacturer basic (full) warranty at start date of extended warranty
	=	Max $(0, M_{BASIC} - M_{START})$
M_{PT_REM}	=	Remaining miles of manufacturer power train warranty at start date of extended warranty
	=	Max $(0, M_{PT} - M_{START})$
M_{EXT}	=	Maximum term of extended warranty in miles at start date of extended warranty

The following two formulas are based on the assumption that the miles driven for any particular vehicle is proportionate to time and that the number of miles driven per time period for each vehicle, A, is randomly distributed as a lognormal function.

A lognormal may be a reasonable approximation for the distribution of driving habits since it is positively skewed and one can model the "high mileage" drivers in the tail of the function.

$m(t)$	=	miles driven at time t in months
	=	At
$t(m)$	=	time in months at which m miles have been driven
	=	m/A
t_0	=	start of extended warranty
	=	0
t_1	=	time of true expiration of manufacturer basic (full) warranty, measured in months from start of extended warranty
	=	$\text{Min}\,(T_{BA\_REM}, t[M_{BA\_REM}])$
t_2	=	time of true expiration of manufacturer power train warranty, measured in months from start of extended warranty
	=	$\text{Min}\,(T_{PT\_REM}, t[M_{PT\_REM}])$
t_3	=	time of true expiration of extended warranty, measured in months from start of extended warranty
	=	$\text{Min}\,(T_{EXT}, t[M_{EXT}])$
$Cost_{BASIC}$	=	Extended Warranty cost per mile while manufacturer basic (full) warranty is in effect
$Cost_{PT}$	=	Extended Warranty cost per mile after manufacturer basic (full) warranty expires and while manufacturer power train warranty is in effect
$Cost_{EXT}$	=	Extended Warranty cost per mile after both manufacturer basic (full) and power train warranties have expired
$m(t)$	=	mileage driven during extended warranty
$k(t)$	=	trend of repair costs
$p(t)$	=	trend rate of probability of claims and size of the claims as the vehicle ages
Prem	=	Extended warranty pure premium (4.1)

$$= \int_0^{t_1} Cost_{BASIC} * m(t) * k(t) * p(t + T_{START}) * dt$$

$$+ \int_{t_1}^{t_2} Cost_{PT} * m(t) * k(t) * p(t + T_{START}) * dt$$

$$+ \int_{t_2}^{t_3} Cost_{EXT} * m(t) * k(t) * p(t + T_{START}) * dt$$

5. A Simple Example

In the example, we will use a new vehicle for an extended warranty. For a used vehicle, there is typically not an underlying warranty, so a similar analysis can be performed. A "Wrap Coverage" is often sold for vehicles with a long manufacturer's warranty and provides coverage in areas that the manufacturer's warranty excludes. This product can also be modeled using a similar technique.

Assume a contract is sold for a new vehicle for 6 years/72,000 miles for a vehicle with a 3 year/36,000 mile manufacturer's warranty. Assume that the inflation rate is 3% and claims will increase in proportion to the miles driven another 4%. In this example, the driver is assumed to drive 15,000 miles per year.

Warranty Example

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Year	Cumulative Miles	Miles in Manufacturer's Warranty	Cumulative Exposed Miles	Incremental Miles	Trend	Mileage Factor	Adjusted Exposed Miles	Percent Exposure
1	15,000	15,000	0	0	1.000	1.000	-	0%
2	30,000	30,000	0	0	1.030	1.040	-	0%
3	45,000	36,000	9,000	9,000	1.061	1.082	10,332	23%
4	60,000	36,000	24,000	15,000	1.093	1.125	18,444	41%
5	75,000	36,000	36,000	12,000	1.126	1.170	15,809	35%
6	90,000	36,000	36,000	0	1.160	1.217	-	0%
							44,585	100%

Assumptions:

15,000	Miles per Year
72,000	Contract Miles
36,000	Miles for the Manufacturer's Warranty
3.0%	Trend Rate for Repair Costs
4.0%	Mileage Trend

Column 1 represents the cumulative miles driven during the contract.

Column 2 is the cumulative miles covered by the manufacturer.

Column 3 is Column 1 – Column 2, subject to the limitations of the contract. In this example the warranty covers the 36,000 miles between the odometer readings of 36,000 and 72,000.

Column 4 is the incremental miles in Column 3 for each year

Column 5 is an estimate of the increase in repair costs.

Column 6 is an estimate of the rate of increase in claims due to the increased wear-and-tear on the vehicle.

Column 7 is Column 4 x Column 5 x Column 6. This is the adjusted miles.

Column 8 is the percentage of Column 7.

So in this example, we could assume that the earnings pattern should be 23% in Year 3, 41% in Year 4, and 35% in Year 5. Nothing would earn in Year 6 due to contract expiring due to miles. Years 1 and 2 would also earn nothing due to the manufacturer's warranty.

Issues with the Simplified Example

The example above is too simplified to utilize for a couple of reasons.

1. The assumption that no claims occur during the manufacturer's warranty is probably erroneous. Most contracts contain minimal coverage during the warranty period. This can be modeled by assuming the percentage of ultimate claims paid during the manufacturer's warranty.
2. Knowledge of the specific driving habits of a contract holder is unknown. In this example, we have assumed that the driver's mileage exceeds the maximum covered by the warranty in Year 5. That may be true for average driver on the book, but one could expect some earnings in the 6^{th} y ear for drivers who are driving fewer miles than the average for the book.

The next section will more closely examine estimating the average miles driven under Vehicle Service Contracts.

6. Estimating Miles Driven from the Contracts [m(t)]

A mileage function can be estimated from the average miles driven and therefore the percentage of the premium that ought to be earned in each period. One can examine all contracts that had a claim or cancellation (or both) and look at the average miles driven per month as of the last recorded event. This data will typically be available since coverage must be confirmed at the time of a claim and cancellations are typically "pro-rata" as to the greater of miles or time.

Instead of estimating a probability distribution for the mileage driven as shown above, it may be more practical to use a discrete approximation.

For our purposes, we will split the insured vehicles into five equal groups based on average miles driven per year at the time of the claim or cancellation with the arithmetic average calculated for each group. Then factors are calculated for each contract group assuming that claims are proportional to covered miles driven (miles under the contract but not under manufacturer warranty) and that the vehicle for each contract was driven at the respective average yearly rates. The final factor applied is the average of these five factors.

The factors thus derived for a new book of business may overstate earnings because the average miles generally decline as the warranty runs to expiration. This declining pattern is due to two factors - early claims are much more prevalent on cars with the most miles driven per month and as the higher mileage cars use up coverage, the average naturally declines.

Therefore, one can triangulate the data and project to ultimate the average miles driven per year.

For a new book of business, there may not be data available. In this case, the actuary may simply assume a distribution of miles or obtain driving mileage data from an external source.

For this example, the averages for the book have been estimated at the mileage rates below:

Estimated Mileage of Warranties Divided Into 5 Equal Groupings

	Base Average	Minimum Yearly	Maximum Yearly
Group 1	8,400	-	10,200
Group 2	12,000	10,201	13,200
Group 3	14,400	13,201	16,200
Group 4	18,000	16,201	20,400
Group 5	22,800	20,401	

7. A Better Example

Now we will redo the initial example with two changes. First, we will assume that 3% of claims occur during the manufacturer's warranty. Second, we will utilize the "5 bucket" approximation noted above and calculate the exposures for each scenario.

The results are shown below:

		(1)	(2)	(3)	(4)	(5)	(6)	(7)
		Adjusted Exposed Miles 8,400 per year	Adjusted Exposed Miles 12,000 per year	Adjusted Exposed Miles 14,400 per year	Adjusted Exposed Miles 18,000 per year	Adjusted Exposed Miles 22,800 per year	Exposure Average	Percent Exposure
Year								
	1	252	360	432	540	684	454	1%
	2	252	360	432	540	9,708	2,258	7%
	3	252	360	7,200	17,460	22,116	9,478	28%
	4	8,148	11,640	13,968	17,460	3,492	10,942	32%
	5	8,148	11,640	13,968	-	-	6,751	20%
	6	8,148	11,640	-	-	-	3,958	12%
Total		25,200	36,000	36,000	36,000	36,000	33,840	100.00%

8. Developing a Coverage Factor

The use of a "coverage factor" when calculating mileage can be a simplifying assumption. For example, one can calculate the mileage driven inside the manufacturer's warranty, inside the Power Train warranty, and outside the warranty. Claims can be aggregated by examining the mileage on the claim in relation to the underlying warranty.

Calculation of Coverage Factors
(Miles 000)

Warranty	Initial Covered Miles	Reported Losses	Cost per Mile	Coverage Adjustment	Adjusted Covered Miles
Manufacturers	174,831	349,662	0.002	0.071	12,413
Power Train	33,082	496,230	0.015	0.536	17,732

None	324,504	9,086,112	0.028	1.000	324,504

In this example, the cost per mile for each type of warranty is placed in ratio to the cost per mile for claims outside the manufacturer's warranty. Miles inside the warranty are then adjusted downward to reflect the substantially lower claims during this period. In this case the cost per mile during no manufacturer's warranty is 2.8 cents per mile (9,086,122/324,504,000).

9. Estimating the Trend [k(t), p(t)]

As noted above, there are two types of trend that impact the vehicle as the warranty ages:

The first type of trend [k(t)] is the general increase in repair costs. Information concerning repair costs can be estimated from industry repair information or by using the Consumer Price Index (CPI). While repair costs increase due to general inflation, it is important to realize that this trend has been tempered in the past by the increasing reliability of automobiles.

The second type of trend [p(t)] is the increase in costs due to the age of the vehicle. Theoretically, this would be offset by decreasing claims consciousness as the vehicle ages, i.e. a vehicle owner may be more accepting of minor issues as the car ages. In addition, the owner of the vehicle may not know the warranty is in effect. While the warranty can typically be transferred or cancelled for refund by a vehicle owner when the vehicle is sold, there may be some cases where this does not occur.

One could also estimate the two trends simultaneously, since the observed data will have trends due to both the inflationary [k(t)] and aging [p(t)] impact

Using this methodology, there is an assumption that all differences in loss costs between development periods are due to changing costs due to inflation and the aging of the vehicle. Therefore, one should be aware of any changes outside of these factors that would have a significant impact on the loss ratios. These would include:

- Changes in coverages. Administrators may change the coverages offered from time-to-time which can result in different expected loss costs.
- Changes in claims settlement practices. There appears to be significant leeway in how claims are settled. It is common that administrators place more resources in denying or reducing marginal claims when results are above the expected level.

Losses should now be segregated by the time since policy inception, and mileage calculated by the methodology above, also dividing the mileage into periods since policy inception and adjusting the mileage by the coverage levels above.

At this point, one can compare the cost per mile for various ages to calculate the underlying trend for both the aging of the vehicle and the underlying inflation rate. In the example below, used car experience will be used since it is easier to display and more credible at lower mileage levels.

Trend Estimation (Calculation of P(t), K(t))
(Miles 000)

Make	Term	Coverage	Covered Miles	*Miles During Policy Age Months* 0-12	12-24	24-36	36-48	Undriven Miles
European	36	Used	47,520	24,948	7,128	3,564		11,880
American	36	Used	69,863	32,696	12,575	5,030		19,562
Asian	36	Used	74,199	38,346	15,789	2,256		17,808
European	48	Used	38,475	17,006	5,233	2,616	1,308	12,312
American	48	Used	69,925	27,271	9,999	5,454	2,727	24,474
Asian	48	Used	54,667	16,531	6,298	2,624	787	28,427

Make	Term	Coverage	Overall Average Cost per Mile	Cost per Mile in Successive Time Periods 0-12	12-24	24-36	36-48
European	36	Used	0.0414	0.0403	0.0429	0.0458	
American	36	Used	0.0258	0.0250	0.0269	0.0285	
Asian	36	Used	0.0152	0.0149	0.0156	0.0175	
European	48	Used	0.0465	0.0446	0.0473	0.0519	0.0568
American	48	Used	0.0316	0.0304	0.0317	0.0347	0.0374
Asian	48	Used	0.0209	0.0202	0.0212	0.0234	0.0256

Change in Cost per Mile over Time

Make	Term	Coverage	12-24	24-36	36-48
European	36	Used	6.5%	6.8%	
American	36	Used	7.6%	5.9%	
Asian	36	Used	4.7%	12.2%	
European	48	Used	6.1%	9.7%	9.4%
American	48	Used	4.3%	9.5%	7.8%
Asian	48	Used	5.0%	10.4%	9.4%
	Weighted Avg*		5.7%	8.6%	8.5%

Selected Trend	**5.7%**	**8.6%**	**8.5%**

* Weighted by covered miles

Note in this example the trends for each year range from 5.7% to 8.6%. One must be careful to anticipate that the trend may increase in the outlying years. It might be advisable to simulate different trend levels, especially on the later years, to check the sensitivity of the loss estimate to the trend assumption.

The trend can either be modeled directly into the mileage function (by increasing the estimated miles in proportion to the selected trend) or by directly trending the results. The first method may be more practical when the selected trend varies significantly by product, term, or other variable.

10. Calculating the Future Claims Rate ($Cost_{BASIC}$, $Cost_{PT}$, $Cost_{EXT}$)

As noted above, future claims costs is a function of the expected mileage driven times the cost per mile. The historical cost per mile can be easily calculated by taking the reported claims divided by the historical estimated miles. For future claims, a claims rate should be calculated for each contract based on the characteristics for this contract. Important characteristics one should consider are:

- The type and term of the coverage
- The deductible of the coverage.
- The mileage of the vehicle when the contract was purchased. It is important to segregate contracts from "new" vehicles from "nearly new" vehicles (vehicles with perhaps 1,000 miles on them) because they are typically significant claims differences at this level.
- A general grouping of the vehicle type. Typical groupings are by vehicle national origin (American, European, and Asian) with a couple of sub groupings for each type to differentiate between high cost makes and low cost makes. Certain make groups exhibit different claims characteristics. For example, Asian makes tend to exhibit lower claims costs than North American makes, which in turn exhibit lower claims costs than European makes.
- Other differences that you can model with the available data. For example, some books may have different distribution sources. A common structure is a "Producer Owned Reinsurance Company" where the ultimate liability for covering the claim will be at the servicing dealer. Not surprisingly, these books can exhibit significantly lower clams costs than books with claims paid by a third-party.

In general the actuary should model all available variables and discard those with little relation to claims costs.

In modeling the claims costs, an iterative minimum bias approach is recommended since many variables with have significant correlations. Generalized linear modeling may also provide good results.

Once again, the high frequency/low severity nature of this line will tend to provide more credible relativities at lower loss levels than other property/casualty lines.

11. Cancellations

Future cancellations should also be considered when evaluating a book of business. In general, cancellations will result in a refund of premium equal to the lesser of the proportion of the miles remaining to total miles or time remaining to the total term of the warranty. No consideration of underlying manufacturer's warranty is usually given. For example, if the warranty holder with a 6 year/72,000 mile contract cancels after three years and 50,000 miles, the warranty holder will receive approximately 31% of the premium as a refund ((72000-50000)/72000). This is true even though the majority of the exposure of the warranty remains. In effect, the refund is stated as pro-rata to miles driven or time, but the impact is that of a short-rate cancellation. Therefore, it is generally advantageous for the underwriter of new vehicles for the warranty to be cancelled.

12. Case Reserves and IBNR

Case reserves may or may not be held by an administrator, and are generally not a significant liability compared to the unearned premium reserve. Amounts held for pure incurred but not reported claims are rare since most claims must be pre-approved by the administrator before work can commence. Since the date of loss is typically the date of approval from the administrator, this should eliminate unreported claims except for supplemental payments beyond the initial estimate to repair the vehicle.

If reported losses are used to analyze a book, it should not be necessary to include additional reserves in your estimate. If paid losses are used, the actuary can do a paid loss analysis for a development pattern and add this to observed cost per mile or extend the terms of the contracts by the average delay between claim report and claim payment date.

13. Building the Indicated Rates

Indicated rates should be trended by the inflationary measure [p(t)] from the average accident date on the book until the average accident date of the proposed rates. Assuming terms offered are similar, it is simpler to trend from the effective date of the contract until the effective date of the new rate change. The final indicated loss cost is defined by:

(Reported Losses + Future Claims) x Cost Trend/Number of Warranties

where Future Claims is the Adjusted Mileage (adjusted for trend and coverage factors x Future Claims Rate.

Depending on the situation, other expenses such as taxes, underwriting expense, profit and contingencies, administrator fees, dealer commission and retail markup must be considered. However, some of these items may be either a flat dollar amount or percentage.

14. Conclusion

The methodology proposed in this article is certainly more complex, but should estimate costs better than traditional methodology. Fortunately, the data required to do this type of analysis is typically available from a vehicle service contract database. The unique characteristics of the book (such as term, coverages, and underlying warranties) are explicitly modeled using such an approach.

Because of the high credibility of extended warranty losses, detailed analysis can be done with small and immature books. Indeed, this type of analysis is even more appropriate for such books since a traditional "triangle analysis" will not have enough data for a good estimate.

By explicitly modeling the exposures, the actuary is forced to consider the specific elements such as the trend rate which will have the most impact on the estimate.

15. References

1) Bouska, Amy S. "Exposure Bases Revisited," *Proceedings of the Casualty Actuarial Society*, **1989**, 1-23.

2) Cheng, Joseph S.; Bruce, Stephen J., "A Pricing Model for New Vehicle Extended Warranties," *Casualty Actuarial Society Forum* Year, **1993** Vol: Special Edition Page(s): 1-24

3) Hayne, Roger M. "Extended Service Contracts," *Casualty Actuarial Society*, **1994**, LXXXI, 243-302.

4) Noonan, Simon J. "The Use of Simulation in Addressing Auto Warranty Pricing and Reserving Issues," *Casualty Actuarial Society Forum*, **1993**, Special Ed. 25-52.

5) Weltmann, Jr., L. Nicholas; Mulhonen, David. "Extended Warranty Ratemaking," *Casualty Actuarial Society*, **2001**, Winter, 187-216.

6) Bouska, Amy S. "From Disability to Mega-Risks; Policy Based Loss Estimation," *Casualty Actuarial Society*, **1996**, Summer, 291-320.

Pricing the Hybrid

R. Stephen Pulis, ACAS, MAAA.

Abstract

The current literature describes pricing and reserving of medical malpractice insurance as written on either an occurrence or a claims-made basis. In current practice, many policies allow the reporting of incidents before a claim is submitted, to attach the claim to the current claims-made policy. This creates experience with characteristics of both types of experience. This paper addresses the blend of the two types of experience based on the acceleration of the attachment of claims from their true assertion date back into the claims-made period. The goal is to assign exposure in proportion to expected claims, and to determine the number of claims and the related reserves to expect to be assigned to the current claims-made policy and to the residual tail exposure, and to reflect the change in the final pricing of the policy.

Keywords. Medical malpractice, claims-made, pricing, reserving, Monte Carlo modeling

1. Introduction

Insurance contracts have been bound to provide coverage for events that occurred during the contract period since the inception of insurance. In the ninety years since its inception, the Casualty Actuarial Society has published papers outlining problems and methods to address these concerns in analyzing property/casualty experience for reserving and pricing.

The problems of estimating professional liability costs in the late 1970's led to the emphasis of providing insurance on a "claims-made" coverage basis. The claims-made coverage facilitated the analysis by concentrating on reserving and pricing the events that would be newly reported and deferred the more difficult effort to evaluate future reported claims. The claims-made policy continues to be used extensively for professional liability, and has been adopted for use on other difficult lines such as Directors and Officers Liability.

The occurrence policy attaches responsibility for the claim to the policy in effect when the event giving rise to the claim took place. While this definition seems precise, there has been substantial controversy and litigation over identifying a precise moment of occurrence, especially when a continuous event is taking place. It is not the purpose of this paper to investigate making this assignment, but to recognize that once this definition is accepted, the claim is attached to the occurrence policy in effect on the occurrence date even though it may be reported a substantial amount of time after the occurrence date. Once the claim is reported, a determination is made and the count of the claim and the costs for the claim are assigned back to the "occurrence" period. Tracking the history of the reporting and change in cost estimates provides historical development patterns. For simplicity, assume that these periods are 12-month continuous periods that will be called years.

The assignment of a claim and its associated costs back to the occurrence year means that there will be future changes to be anticipated in the number of claims reported, the costs of the new claims and any revisions in the estimate of the costs on claims previously reported. The estimate of the costs on future reported claims is the "pure" portion of what is normally referred to as IBNR (Incurred But Not Reported).

Under the claims-made policy, the assignment of a claim to the insuring policy is simplified. When the insurer receives notice of a claim, either directly or through its agent (either the insurance agent or the insured acting as a conduit to the insurer), the claim attaches to the policy in effect on that date. While there may be a short delay from the acceptance of the notice until the matter is recorded by the insurer, the "pure" IBNR is zero as all claims are known by the end of the policy term. There will not be any increases in claim counts except for the occasional clerical lag or mishandling. Any development of the case incurred losses will be from adjustments made on known claims, and the general "IBNR" fund for these changes is only for this more limited need. The claims-made insured will have some lingering exposure that will attach subsequent to the expiration of the current policy, and this is referred to as the "tail" of the experience.

Some claims-made policies provide for a claim to be attached to a current policy if the insured gives the insurer notice that an incident has occurred that may result in a claim being asserted in the future. The "assertion" of the claim is the official submission of a request for damages from the claimant to the insured/insurer. The traditional "report date" corresponds to the "assertion date" referred to in this paper. To distinguish from the pure IBNR claims, these reported but not asserted claims will be called "RBNA", and the remaining unknown claims will be the incurred but not known, "IBNK".

Marker and Mohl initially state as Principle #4[1], "Claims-made policies incur no liability for IBNR claims ..." and later state[2] that at the introduction of the claims-made policy, it was "assumed that, on average, claims would be reported sooner" and "that there would be

[1] Joseph Marker and James Mohl, "Rating Claims-Made Insurance Policies", CAS 1980 Discussion Paper Program, page 278.
[2] Joseph O. Marker and F. James Mohl, ibid, page 293.

some additional reporting of incidents that would never have come in under the occurrence policy". This acceleration was viewed as a one-time occurrence at the transition. Their on-going approach did not identify the RBNA component within the claims-made year, and treated its emergence in their backward-recursive development factors. However, an on-going acceleration of claim reporting may adversely affect the adequacy of the renewal premium.

There are pros and cons as to why an insured may give the insurer notice of a potential claim beyond simply providing the insurer additional time to prepare the insurer's defense of the potential claim. If the insured believes that this claim and the aggregate of the other expected claims for this period are within the limits currently purchased, then it is to the insured's benefit to submit an incident report to the insurer during the current policy period. This will maximize the benefit of the coverage already purchased and reduce the future liability under either another claims-made policy, or a "tail" coverage policy. If the insured is switching to self-insurance without purchasing a tail policy, then the reporting of incidences can only reduce potential self-insurance costs. A tort reform change may also simulate a change in the reporting and assertion pattern.

If the frequency of claims or magnitude of a particular claim would exceed current coverage, then there is a disincentive to report the incident until an actual assertion of a claim is received. There is also an incentive for the insured to purchase increased coverage in future policies when there is an increased likelihood of a need for such expanded coverage. When the renewal policy is for limits greater than the expiring limits, an endorsement could be attached that applies the expiring limits to claims reported subsequent to the occurrence year. If the underwriter is really concerned about this possibility, the new policy will not be permitted to have the limit changed.

2. Analysis of the Hybrid

It is the reporting of the incident prior to the claim's assertion that creates a hybrid between the claims-made and occurrence policy. The maximum number of potential claims will be known at the end of the policy, but the number of asserted claims will emerge over time and, therefore, have some characteristics similar to an occurrence policy. Not all potential claims that occur during the policy period will be recognized and reported as an incident within the policy period. The future asserted claims that were not reported as an incident in the first policy period (incurred but not known or IBNK) will attach to a future claims-made policy. The claim count on a hybrid policy will be a blend of:

- Claims that occurred during this period and are asserted during this period.
- Claims asserted during this period that were IBNK at the end of the prior period
- Claims that occurred during this period that are reported but not asserted (RBNA).
- Claims reported during this period that were IBNK at the end of the prior period, but that are not asserted yet (RBNA).

The reserve needed for this period consists of a provision for adjustments on case reserves on asserted claims (the first two types) plus a provision for RBNA claims as of the evaluation date (the last two types). The residual IBNK reserve is a separate issue to be handled as "tail" coverage or in a subsequent claims-made policy

For analysis purposes, some companies may set a subjective reserve and probability of assertion on individual incident reports if there is a substantial likelihood of a future claim with a payment. The hybrid therefore has reserves for development on known case reserves plus reserves on claims reported as incidents but not asserted (RBNA). The subjective reserves are part of the RBNA. They generally are not carried on the books as official reserves, but are used only in the reserve analysis for estimating case incurred development, claim frequency, and claim severity distributions.

In a perfect world, all risks would have experience available and be sufficiently large to be given full credence. If complete information were available, development triangles unique for this business could be calculated and applied. Lacking this, an estimate of the impact using a broad based distribution, and information and assumptions about the particular segment of business are used. This paper assumes complete information is not available and presents an approach to estimate the RBNA reserve. This approach is particularly useful when the pure IBNR is to be modeled using a Monte Carlo simulation.

The method requires knowledge of the claim reporting distribution between the occurrence date and the assertion date. If this distribution is defined in terms of the number of days between these dates, then an assumption should be made, such as "claims occur uniformly throughout the year", and the distribution converted into the portion reported by the end of 12 months, 24 months, etc. Edward Weissner's paper[3], "Estimation of the Distribution of Report Lags by the Method of Maximum Likelihood", describes a procedure for estimating the distribution when the final claim reporting is still unknown. For this paper's purpose, Exhibit 1 creates a claims-made reporting pattern using an estimated[4] probability distribution of the number of months between the occurrence and the assertion of a claim. Column (a) is the age, in months, from an occurrence that will produce a claim until the time of its assertion. Column (b) is the probability that the claim will be asserted in that month. Column (c) is the sum of the probabilities that the claim has been asserted by the end of the indicated month. Claims are assumed to occur uniformly throughout the year. An occurrence year would have an equal expected number of claims from each month but with varying ages of maturity. Column (d) calculates the 12-month rolling average of the monthly data by summing Column (c) for the 12 months ending at this age, and divides by 12. Note that if the occurrence year is a partial year (less than 12 months old), the rolling average needs to be adjusted for the period incurred.

Knowledge of the acceleration due to the incident reporting needs to be quantified when analyzing the hybrid. A development triangle of asserted claims by claims-made year can be compared to the distribution described in the previous paragraph. The claims-made distribution by report date defines how claims are assigned to current and future report years, and once assigned there is no development of claim counts. The measured development from the claims-made triangle is all emergence on claims reported as incidents by 12 months. At one extreme, if there is no incident reporting until a claim is asserted, the acceleration is 0% and the reporting distribution is a standard claims-made reporting pattern. At the other extreme, if incident reports are made on every situation inclusive of all claims ultimately asserted, then the acceleration is 100% and the resulting distribution is the same as the reporting pattern for an occurrence policy. Exhibit 2 is a table of the cumulative number of claims asserted for each evaluation of the hybrid year where the claim attaches.

---

[3] Edward W. Weissner, "Estimation of Distribution of Report Lags by the Method of Maximum Likelihood", PCAS LXV, page 1.

[4] The distributions used in this paper have been created to produce realistic results similar to observed data.

Acceleration could also be measured based on the additional change measured on developing losses. Quantifying the additional development resulting from late asserted claims over the case development on claims asserted within the first year, may be difficult to identify and measure. The probability of a severe claim having a higher likelihood of being reported early or late is debatable. Operating on the wrong part of the body or excessive anesthetics can be severe and immediately known damage. Missing a diagnosis, or leaving a foreign object in the body, may take years to recognize and cause irreparable harm or extended pain and suffering. Several large insurers now reflect different reporting patterns by specialty. It has been assumed in this paper that the severity of the claim is independent of the length of time for the claim to be asserted. Therefore, measuring the change in reporting patterns tracks with the associated costs. An adjustment for payment patterns is addresses later in the paper.

If the claim development shows that the number of claims reported at 12 months will ultimately increase by 48%, as in the example on Exhibit 2, then the quantity of 48% times the percentage of claims asserted at 12 months, divided by the percentage unreported at 12 months, gives a measure of the accelerated claim reporting. This calculation can be made at each successive12-month evaluation to determine the accelerated portion reported by that date. It is not obvious that an insured will be better at identifying and reporting an incident that will be asserted in the third year verses being asserted in the fifth year. It may be possible to report incidents occurring near the end of the policy period that are more likely to be asserted in the next 12 months. A uniform acceleration has been used in this paper, and is reflected in Exhibit 3.

A second possible measure of the acceleration can be estimated based on the frequency of the incident reports compared to a standard reporting frequency. If the underlying claim frequency is expected to be the same, then the ratio of incident frequency to asserted frequency is a measure of the acceleration. An adjustment may need to be applied to reflect a probability of less than 100% that all the incidents reported will result in an asserted claim. The initial incident reports should have a much higher probability of predicting an assertion. As the number of incident reports increase, the probability of identifying a future assertion should stay the same or decrease as marginal incidents are added. It is unlikely that an insured will be able to report all incidents that will result in an asserted claim without reporting an excessive number of incidents that will not result in an asserted claim. The example of neglecting to remove a foreign object from the body after an operation, will either be immediately known and treated, or remain unknown until such time that it is discovered and an immediate claim assertion is made. There is little expectation that an occurrence has taken place between those times that would warrant an incident report.

The hybrid year claim distribution, resulting from applying the acceleration, is separated into attachment years (hybrid year) on Exhibit 3. The change in the cumulative assertions (Exhibit 3, line (c)) is the amount of assertions during the calendar year as represented by each column. The probability that an RBNA will be asserted during the current calendar year is the ratio of asserted claims to the RBNA at the end of the prior year.

The cumulative development factor from Exhibit 2 provides a measure of the acceleration as a ratio of the projected the future assertions 0.09093 [=(0.18917)(0.481)] to the unasserted claims at the end of the first year 0.81083. The ratio indicates 11.214% of what would be claims in future claims-made years will now be attached to the current hybrid year. Assuming that the acceleration is uniform, the cumulative portion of occurrence claims attached is the sum of the claims asserted to date plus the acceleration ratio times the portion of claims not asserted as of the evaluation. The calendar year change in the cumulative total is the hybrid year's ultimate portion.

On Exhibit 4, the portion of the occurrence year accelerated and attached within the hybrid year is split into the amount asserted at each subsequent evaluation date, and the portion remaining as RBNA. These are expressed as proportions of the original occurrence-based incurred. Exhibit 4 assigns the ultimate hybrid year total [Exhibit 3, row (i)] to the initial subtotal for the hybrid year on Exhibit 4. The assertions during the calendar year [Exhibit 3 row (c)] correspond to the Total New Assertions at the bottom of Exhibit 4. The assertions during the calendar year are distributed between active hybrid years in proportion to the RBNA existing at the beginning of each the calendar year. Subtracting the asserted claims from the beginning RBNA produces the RBNA at the end of the current calendar year that will also be the RBNA at the beginning of the next calendar year. The probability that a RBNA will be asserted is the ratio of the assertions during the year to the RBNA at the beginning of the year.

Exhibits 4a and 4b provide the same information as Exhibit 4 but Exhibit 4a has 0% acceleration and, therefore, resembles a pure claims-made policy, and Exhibit 4b assumes 100% acceleration and, therefore, resembles an occurrence policy. As the acceleration increases, the tail diminishes as the exposure is shifted back into the prior years.

A multi-year analysis is modeled on Exhibit 5a and 5b. If the insureds are large self-insured hospitals or physician groups written on a claims-made policy. They want to know three things:

1. What is the reserve need at the end of the policy period?

2. What funding is needed for the next year?

3. What residual liability exists beyond next year?

The development on asserted claims can be measured using the standard actuarial techniques; however, care must be used not to include the pure IBNR emergence that is calculated separately. The cost of the unasserted and future claims is essentially a frequency time severity projection: multiplying frequency estimates times the underlying exposure, and multiplying the resulting expected number of asserted claims times an average claim cost amount.

A full-time equivalent exposure (FTE) is calculated as the sum of the product of the unit exposure and the rating relativities; such items as classification, territory, step factor[5], and fractional year exposed. These relativities recognize the variation in costs by medical specialty (classification), tendency for more or larger settlements depending on the location within the state (territory), number of years written under a claims-made policy (step factor), and portion of a year insured (fractional year). The historical claims are adjusted to a closed with payment basis, and developed to an ultimate occurrence basis for use in determining the underlying claim frequency. The historic claim frequency is used to project the ultimate frequency for each period under review. The product of the ultimate frequency and FTE produces the expected number of ultimate claims for each period.

On Exhibit 5a, the hybrid year proportions [Exhibit 3, row (i)] are multiplied times the calendar year exposures to distribute the exposures over the years in proportion to the expected claim assertions. The column can be summed to obtain the hybrid year total. A simplifying assumption could be made that either no exposure growth exists or that a fixed percentage of growth applies over all years. With these assumptions a modified distribution can be derived and applied to only the current calendar year exposure. This has not been done here. The proposed procedure has the benefits of: being sensitive to uneven growth that may arise from such things as general expansion of business or acquisitions; provides

[5] The step factor represents the cumulative percentage of an occurrence year that has been insured.

details of where the expected asserted claims were incurred; and facilitates applying trends and/or discounts related to the time lags. Exhibit 5a displays the allocation of the total exposures in proportion to the expected claim attachment distribution. Exhibit 5b multiplies the exposures times a frequency to project the expected ultimate claims for the occurrence year, and then uses the hybrid year proportions to distribute claims to the hybrid year.

The ultimate claims underlying the three desired quantities are found on Exhibit 5b. The ultimate claims for Hybrid Year 0 and prior are the asserted and RBNAs as of the experience evaluation date (claims in columns (r) thru (w)). The claims enclosed in the box produce the tail exposure at the Year0 year end evaluation. The claims under Year+1 (135 claims) will produce the loss experience to be funded for the next year, and the new tail subsequent to next year (160 claims) will be the losses produced by the claims in Columns (y) through (ad).

The separation of the asserted and the RBNA claims for Year 0 and prior is calculated on Exhibit 6. The ultimate claims on the upper portion of Exhibit 6 were calculated on Exhibit 5b. For each occurrence year, a line is shown with its contribution to the hybrid years in each column. The RBNA is the product of the ultimate occurrence year claims times the RBNA ratio for that assertion year and evaluation lag. The 12 RBNAs for Year0 is the product of 136 ultimate claims [column (b)] time 0.09093 on Exhibit 4 for Year0.

If a change in the acceleration has or is expected to take place, a probability of assertion can be calculated for each hybrid year and evaluation lag. The probability of assertion would be multiplied times the RBNA to determine the number of new asserted claims, and the remaining RBNA count. The probability of assertion may also be adjusted to reflect impacts of tort reform legislation. The cumulative emerged claims equals the ultimate minus the ending RBNAs. The hybrid year count is the total of the column.

The case incurred on known claims can be projected to ultimate using loss development factors if sufficient historical experience is available. However, including the open counts with the RBNA counts provides a mechanism to determine a range around the ultimate losses. Only the claims where a high likelihood that the case incurred is correct are treated as equivalent to a closed claim. The projected RBNA reserve is added to the "closed incurred" to determine the ultimate incurred.

The approached used to project the RBNA reserve is a Monte Carlo model similar to that described by Bickerstaff[6]. The loss dollars on closed claims and the subjective estimates for RBNAs with a high likelihood of payment, are trended to a common date, and fit with distribution curve(s), usually a single or compound log-normal curve(s) to project unlimited losses. A set of simulations (usually 1,000) are run to project first the number of claims based on a claim count distribution (a Poisson distribution is often used) with the expected number of claims as the mean. And second, for each random claim drawn by the Poisson, a random claim size is generated using the mean and variance of the severity-modeled lognormals. The lag between the time the incident is reported to the closing date can be accounted for by trending the (unlimited) severity mean used to generate the claim size.

A loss expense adjustment cost is also generated for each claim. On average, the loss expense increases as the size of the loss increases. Bickerstaff[7] demonstrated the development of a conditional Defense and Cost Containment (DCC) distribution. Its parameters and the generated loss size are used to generate a random DCC for the unlimited loss-size claim. After generating the DCC, the claim-size is limited to the policy provisions. If the policy terms include DCC within the coverage limit, then the combination of loss and DCC is limited and prorated.

The losses and DCC are summed for each sample, and the samples used to calculate the expected value, and the funding needed to meet the desired probability levels of confidence of adequate reserves. An additional loading is added for the reported incidents that are expected to produce loss adjustment expenses, but no indemnity payments.

One factor to consider for the hybrid is whether the paid development will be the same for claims reported and asserted in the first year, compared to claims asserted in future years. One large insurer has developed statistics that show the payout on claims asserted after the occurrence year is longer from occurrence than for claims asserted in the occurrence year, but when comparing the development from the year asserted, the payout is faster on the claims asserted after the year in which the event occurred giving rise to the claim. The speed up is faster during the first year after the assertion, and the differences diminish with age. This introduces a new dimension into determining the discounted value of the reserves.

---

[6] Dave Bickerstaff, "Hospital Self-Insurance Funding: A Monte Carlo Approach", CAS Forum, Spring 1989 Edition, page 89.

[7] Dave Bickerstaff, "Hospital Self-Insurance Funding: A Monte Carlo Approach", CAS Forum, Spring, 1989 Edition, page 105.

The statewide rate level change is based on comparing the indicated average premium to the current on-level average premium. Medical Malpractice policies generally carry high limits. It is a common practice to limit the analysis (premiums and losses) to a selected lower limit, such as 200/600 or 500/1000, to reduce the parameter variability. The fixed expenses are included in the premium as an Expense Constant added to the variable portion of the premium. The variable portion is the product of a "base rate" multiplied by relativity factors to adjust for territory, classification, time insured by claims-made coverage, and the other credit and debit adjustments. For the remainder of the paper I will use the more common term "claims-made" as inclusive of the "hybrid" coverage unless stated.

The current base rate is a know quantity. The average current relativity is calculated by sequentially applying the current relativity and measuring the average factor resulting from the application of a rating element. Exhibit 7 shows the determination of the average relativity as each rating element is added. The sequential calculation also facilitates measuring changes in relativities; however, none are taking place in this review. The product of the exposure, based on head-count, times the sum of the expense constant plus the base rate times the average factors (Exhibit 8) develops the premium at current rates.

The incurred losses and DCC expenses need to be increased for the Adjusting and Other expenses (AO, formerly known as unallocated loss adjustment expenses (ULAE)). Countrywide experience from the Annual Statement's Schedule P provides incurred Loss, DCC and AO experience. Ratios of the AO to loss plus DCC are calculated (see Exhibit 9) for the last 5 years. A loading is selected and applied to the state loss plus DCC to determine the ultimate incurred for all loss and loss adjustment expense.

The incurred loss and loss adjustment expense needs to be adjusted to the level expected under the new rates. A pure premium per base class equivalent exposure is calculated on Exhibit 10. Curves are fit by least squares to the average pure premiums for several lengths of time, and the best fit for each time span is shown. An annual trend amount is selected and used to project the historic loss and loss adjustment expense to the mid-point proposed under the new rates.

The expense loadings are separated between variable costs and fixed costs. The General Expenses and Other Acquisition are allocated on a per exposure insured basis to recognize that the costs to write and issue a policy do not materially vary with the location or classification of the risk. For this allocation the actual exposure are divided into the dollars of fixed expenses. The variable expenses are typically dependent on the state where the premium will be charged. The taxes, licenses and fees are dependent on the state laws. The brokerage and commissions are dependent on the contracts that will apply under the new rates. The adjustment for investment income recognizes the investment income on the available funds generated by the cash-flow and prevailing rates of return and taxes.

There are many papers on investment income calculations. This paper will not delve into a particular method, but it should be noted that with the shortened life of a claim under a claims-made policy, the investment income is significantly less than that realized under an occurrence policy. The hybrid policy will realize a return between the occurrence and pure claims-made amounts based on its payout pattern.

The premium from the expense constant will be subject to taxes, commissions, etc. The fixed expenses are loaded for these elements by dividing by the variable expense factor. The premium for fixed expenses is divided by the number of exposures that will be assessed the expense constant. One expense constant will be charged for every exposure, and will only be modified for a shortened policy term.

The statewide rate level indication uses premiums and losses limited to $500,000 per claim/$1,000,000 aggregate basic limit. These losses and loss adjustment expenses are trended to the average loss date under the proposed rates, and divided by the base class equivalent exposures to determine the indicated base pure premium at the future rate level. A base pure premium is selected, and a percentage, say 5%, is added for Death, Disability and Retirement[8]. The result is divided by the variable expense factor to determine the indicated base rate. The indicated average premium is the product of the base rate, the average proposed base class factor (which includes all factors other than the increased limit factor), and the average increased limit factor, and, as the final step, the expense constant is added. Dividing the indicated average premium by the current level average premium produces the indicated change.

This paper does not include revisions being made to the rate relativities, but the off-balance from each is used to adjust the base rate, and maintain the selected overall average premium.

[8] The Death, Disability and Retirement provision is a loading in an on-going business to provide for the average cost of tail coverage on individuals who have ceased to practice through death, disability or retirement.

3. Conclusion

A critical factor in evaluating medical malpractice insurance is to determine the period where claims will attach, and to align the losses and exposures. The claims-made policy provision allowing an insured to report an incident of a potential claim, and thereby attach that claim to a particular policy, creates experience that is a hybrid between a claims-made policy and an occurrence policy. The more aggressively the insured reports incidents in advance of the actual assertion of the claim, the greater the experience will resemble the experience expected under an occurrence policy. The procedure described in this paper facilitates measuring the shift and the calculation of the pure IBNR created for the claims-made policy by the acceleration of the attachment of the claims.

The shift in claims covered from a pure claims-made coverage, increases the pure premium needed, increases the step factors that apply, and increases the investment income. The amount of acceleration allowed determines the degree that the change moves from a pure claims-made basis to an occurrence basis.

4. References

Bickerstaff, Dave, "Hospital Self-Insurance Funding: A Monte Carlo Approach", CAS Forum. Spring 1989 Edition, 89-138.
Marker, Joseph, and James Mohl, "Rating Claims-Made Insurance Policies", CAS 1980 Discussion Paper Program, 265-304.
Weisnner, Edward W., "Estimation of Distribution of Report Lags by the Method of Maximum Likelihood", PCAS LXV, 1-9.

Abbreviations and notations

AO, all other loss adjustment expense
DCC, Defense & Cost Containment expense
FTE, full-time equivalent exposure
IBNK, incurred but not known
IBNR, incurred but not reported
RBNA, reported but not asserted
ULAE, unallocated loss adjustment expense

Biography of the Author

R. Stephen Pulis, ACAS, MAAA, is a consulting actuary at Actuarial Services and Programs in Houston, Texas. He has a Bachelor of Science Degree in Mathematics from Michigan State University. He is a past president of SWAF, and has participated on industry committees, and CAS research.

Exhibit 1

Development of Claim Reporting Distribution

(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)
	Single Claim - Month		Year-to-Date		Single Claim - Month		Year-to-Date
Month	Prob Rpt	Cum Rpt	Cum Rpt	Month	Prob Rpt	Cum Rpt	Cum Rpt
1	0.000	0.000	0.00000	37	0.006	0.939	0.81650
2	0.010	0.010	0.00083	38	0.006	0.945	0.74650
3	0.020	0.030	0.00333	39	0.006	0.951	0.67525
4	0.030	0.060	0.00833	40	0.005	0.956	0.60317
5	0.040	0.100	0.01667	41	0.005	0.961	0.53025
6	0.050	0.150	0.02917	42	0.005	0.966	0.45650
7	0.050	0.200	0.04583	43	0.004	0.970	0.38208
8	0.050	0.250	0.06667	44	0.004	0.974	0.30700
9	0.050	0.300	0.09167	45	0.004	0.978	0.23125
10	0.045	0.345	0.12042	46	0.003	0.981	0.15483
11	0.045	0.390	0.15292	47	0.003	0.984	0.07775
12	0.045	0.435	0.18917	48	0.003	0.987	0.96600
13	0.040	0.475	0.22875	49	0.002	0.989	0.97017
14	0.040	0.515	0.27083	50	0.002	0.991	0.97400
15	0.040	0.555	0.31458	51	0.002	0.993	0.97750
16	0.035	0.590	0.35875	52	0.001	0.994	0.98067
17	0.035	0.625	0.40250	53	0.001	0.995	0.98350
18	0.035	0.660	0.44500	54	0.001	0.996	0.98600
19	0.030	0.690	0.48583	55	0.001	0.997	0.98825
20	0.030	0.720	0.52500	56	0.001	0.998	0.99025
21	0.030	0.750	0.56250	57	0.001	0.999	0.99200
22	0.020	0.770	0.59792	58	0.001	1.000	0.99358
23	0.020	0.790	0.63125	59	0.000	1.000	0.99492
24	0.020	0.810	0.66250	60	0.000	1.000	0.99600
25	0.015	0.825	0.69167	61	0.000	1.000	0.99692
26	0.015	0.840	0.71875	62	0.000	1.000	0.99767
27	0.015	0.855	0.74375	63	0.000	1.000	0.99825
28	0.010	0.865	0.76667	64	0.000	1.000	0.99875
29	0.010	0.875	0.78750	65	0.000	1.000	0.99917
30	0.010	0.885	0.80625	66	0.000	1.000	0.99950
31	0.008	0.893	0.82317	67	0.000	1.000	0.99975
32	0.008	0.901	0.83825	68	0.000	1.000	0.99992
33	0.008	0.909	0.85150	69	0.000	1.000	1.00000
34	0.008	0.917	0.86375	70	0.000	1.000	1.00000
35	0.008	0.925	0.87500	71	0.000	1.000	1.00000
36	0.008	0.933	0.88525	72	0.000	1.000	1.00000

Exhibit 2

Calculation of Claim Development by Attachment Year

Attachment Year	Time to Reporting 12 mos	24 mos	36 mos	48 mos	60 mos	72 mos
1996	72	102	107	109	110	110
1997	74	100	104	108	108	108
1998	81	117	119	123	124	124
1999	85	120	126	127	127	
2000	82	107	110	113		
2001	94	131	137			
2002	86	118				
2003	90					
	Development Factor					
1996	1.417	1.049	1.019	1.009	1.000	
1997	1.351	1.040	1.038	1.000	1.000	
1998	1.444	1.017	1.034	1.008	1.000	
1999	1.412	1.050	1.008	1.000		
2000	1.305	1.028	1.027			
2001	1.394	1.046				
2002	1.372					
Average	1.385	1.038	1.025	1.004	1.000	
Cum.to Ulti.	1.481	1.069	1.030	1.004	1.000	

Exhibit 3

Calculation of Asserted Claim Emergence and Assignment to Hybrid Year

		Occurrence	Years Subsequent to Occurrence Year						
	y =	Year	+1	+2	+3	+4	+5	+6	Total
(a)	Cumulative Assertions	0.18917	0.66250	0.88525	0.96600	0.99600	1.00000	1.00000	
(b)	Remaining Unasserted at beginning of Year	1.00000	0.81083	0.33750	0.11475	0.03400	0.00400	0.00000	
(c)	Asserted During Year y	0.18917	0.47333	0.22275	0.08075	0.03000	0.00400	0.00000	1.00000
(d)	Probability IBNK Claim Asserted	0.18917	0.58376	0.66000	0.70370	0.88235	1.00000	1.00000	
(e)	Development Factor	1.481							
(f)	Attachments Moved to First Year	0.09093							
(g)	Acceleration	0.11214							
(h)	Incident Reporting Acceleration	0.28010	0.70035	0.89812	0.96981	0.99645	1.00000	1.00000	
(i)	Hybrid Year Assigned	0.28010	0.42025	0.19777	0.07169	0.02664	0.00355	0.00000	1.00000

Notes:
(a) from Exhibit 1.
(b) = 1.0 - (a) for prior year; ie 0.33750 = 1.0 - 0.66250
(c) = (a) - (a) for prior year; ie 0.47333 = 0.66250 - 0.18917
(d) = (c)/(b); ie 0.58376 = 0.47333 / 0.81083
(e) = from Exhibit 2.
(f) = (a) [(e)-1]; ie 0.09093 = 0.18917 [1.481 -1]
(g) = (f)/[1.0-(a)] ; ie 0.11214 = 0.09093 / [1.0 - 0.18917]
(h) = (a) +(g)[1.0-(a)]; ie 0.70035 = 0.66250 + 0.11214 [1- 0.66250]
(i) = (h) - (h) for prior year; ie 0.42025 = 0.70035 - 0.28010

Exhibit 4

Emergence of Assertions from One Occurrence Year

Hybrid Year		Years after Occurrence 0	+1	+2	+3	+4	+5	+6	Total
+0	New Assertions	0.18917	0.08420	0.00636	0.00036	0.00001	0.00000	0.00000	0.28010
	Remaining RBNA	0.09093	0.00673	0.00037	0.00001	0.00000	0.00000	0.00000	
	subtotal	0.28010	0.09093	0.00673	0.00037	0.00001	0.00000	0.00000	
+1	New Assertions		0.38913	0.02942	0.00162	0.00008	0.00000	0.00000	0.42025
	Remaining RBNA		0.03112	0.00170	0.00008	0.00000	0.00000	0.00000	
	subtotal		0.42025	0.03112	0.00170	0.00008	0.00000	0.00000	
+2	New Assertions			0.18697	0.01031	0.00048	0.00001	0.00000	0.19777
	Remaining RBNA			0.01080	0.00049	0.00001	0.00000	0.00000	
	subtotal			0.19777	0.01080	0.00049	0.00001	0.00000	
+3	New Assertions				0.06846	0.00318	0.00005	0.00000	0.07169
	Remaining RBNA				0.00323	0.00005	0.00000	0.00000	
	subtotal				0.07169	0.00323	0.00005	0.00000	
+4	New Assertions					0.02625	0.00039	0.00000	0.02664
	Remaining RBNA					0.00039	0.00000	0.00000	
	subtotal					0.02664	0.00039	0.00000	
+5	New Assertions						0.00355	0.00000	0.00355
	Remaining RBNA						0.00000	0.00000	
	subtotal						0.00355	0.00000	
+6	New Assertions							0.00000	0.00000
	Remaining RBNA							0.00000	
	subtotal							0.00000	
Total	New Assertions	0.18917	0.47333	0.22275	0.08075	0.03000	0.00400	0.00000	1.00000
	Remaining RBNA	0.09093	0.03785	0.01287	0.00381	0.00045	0.00000	0.00000	
	subtotal	0.28010	0.51118	0.23562	0.08456	0.03045	0.00400	0.00000	

Exhibit 4.a

Emergence of Assertions from One Occurrence Year
Assuming 0% Acceleration, ie Standard Claims-Made Year

Hybrid Year		Years after Occurrence 0	+1	+2	+3	+4	+5	+6	Total
+0	New Assertions	0.18917	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.18917
	Remaining RBNA	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	
	subtotal	0.18917	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	
+1	New Assertions		0.47333	0.00000	0.00000	0.00000	0.00000	0.00000	0.47333
	Remaining RBNA		0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	
	subtotal		0.47333	0.00000	0.00000	0.00000	0.00000	0.00000	
+2	New Assertions			0.22275	0.00000	0.00000	0.00000	0.00000	0.22275
	Remaining RBNA			0.00000	0.00000	0.00000	0.00000	0.00000	
	subtotal			0.22275	0.00000	0.00000	0.00000	0.00000	
+3	New Assertions				0.08075	0.00000	0.00000	0.00000	0.08075
	Remaining RBNA				0.00000	0.00000	0.00000	0.00000	
	subtotal				0.08075	0.00000	0.00000	0.00000	
+4	New Assertions					0.03000	0.00000	0.00000	0.03000
	Remaining RBNA					0.00000	0.00000	0.00000	
	subtotal					0.03000	0.00000	0.00000	
+5	New Assertions						0.00400	0.00000	0.00400
	Remaining RBNA						0.00000	0.00000	
	subtotal						0.00400	0.00000	
+6	New Assertions							0.00000	0.00000
	Remaining RBNA							0.00000	
	subtotal							0.00000	

Exhibit 4.b

Emergence of Assertions from One Occurrence Year
Assuming 100% Reporting, ie Occurrence Year

Hybrid Year		Years after Occurrence 0	+1	+2	+3	+4	+5	+6	Total
+0	New Assertions	0.18917	0.47333	0.22275	0.08075	0.03000	0.00400	0.00000	1.00000
	Remaining RBNA	0.81083	0.33750	0.11475	0.03400	0.00400	0.00000	0.00000	
	subtotal	1.00000	0.81083	0.33750	0.11475	0.03400	0.00400	0.00000	
+1	New Assertions		0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	Remaining RBNA		0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	
	subtotal		0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	
+2	New Assertions			0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	Remaining RBNA			0.00000	0.00000	0.00000	0.00000	0.00000	
	subtotal			0.00000	0.00000	0.00000	0.00000	0.00000	
+3	New Assertions				0.00000	0.00000	0.00000	0.00000	0.00000
	Remaining RBNA				0.00000	0.00000	0.00000	0.00000	
	subtotal				0.00000	0.00000	0.00000	0.00000	
+4	New Assertions					0.00000	0.00000	0.00000	0.00000
	Remaining RBNA					0.00000	0.00000	0.00000	
	subtotal					0.00000	0.00000	0.00000	
+5	New Assertions						0.00000	0.00000	0.00000
	Remaining RBNA						0.00000	0.00000	
	subtotal						0.00000	0.00000	
+6	New Assertions							0.00000	0.00000
	Remaining RBNA							0.00000	
	subtotal							0.00000	

Exhibit 5a

Exposure by Hybrid Year

(a) Occurrence Year	(b) Equivalent Exposures	(c) Hybrid Year > -5	(d) -4	(e) -3	(f) -2	(g) -1	(h) 0	(i) 1	(j) 2	(k) 3	(l) 4	(m) 5	(n) 6	(o) 7
Year -5	1,100	308	462	218	79	29	4	0						
Year -4	1,155		324	485	228	83	31	4	0					
Year -3	1,213			340	510	240	87	32	4	0				
Year -2	1,420				398	597	281	102	38	5	0			
Year -1	1,599					448	672	316	115	43	6	0		
Year 0	1,679						470	706	332	120	45	6	0	
Future Year	1,763							494	741	349	126	47	6	0
Total		308	786	1,043	1,215	1,397	1,545							
Reported In Year +1								1,654						
Reported After Year +1											1,983			

Exhibit 5b

Claims by Hybrid Year

(a) Occurrence Year	(b) Equivalent Exposures	(p) Expected Frequency	(q) Expected # Claims
Year -5	1,100	0.0810	89
Year -4	1,155	0.0810	94
Year -3	1,213	0.0810	98
Year -2	1,420	0.0810	115
Year -1	1,599	0.0810	130
Year 0	1,679	0.0810	136
Future Year	1,763	0.0810	143

(a)	(q)	(r)	(s)	(t)	(u)	(v)	(w)	(x)	(y)	(z)	(aa)	(ab)	(ac)	(ad)
Occurrence	Number of	Hybrid Year >												
Year	Claims	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7
Year -5	89	25	37	18	6	3	0	0						
Year -4	94		26	40	18	7	3	0	0					
Year -3	98			27	42	19	7	3	0	0				
Year -2	115				32	49	22	9	3	0	0			
Year -1	130					36	55	26	9	4	0	0		
Year 0	136						38	57	27	10	4	0	0	
Future Year	143							40	60	28	11	3	1	0
Total		25	63	85	98	114	125							
Reported In Year +1								135						
Reported After Year +1												160		

Exhibit 6

Claim Emergence By Hybrid Year as Evaluated at End of Year 0

Hybrid Year of Assignment

(a) Occurrence Year	(b) Number of Claims	(c) Year -5	(d) Year -4	(e) Year -3	(f) Year -2	(g) Year -1	(h) Year 0
		Ultimate Claims					
Year -5	89	25	37	18	6	3	0
Year -4	94		26	40	18	7	3
Year -3	98			27	42	19	7
Year -2	115				32	49	22
Year -1	130					36	55
Year 0	136						38
	Total	25	63	85	98	114	125

(a) Occurrence Year	(b) Number of Claims	(i) Year -5	(j) Year -4	(k) Year -3	(l) Year -2	(m) Year -1	(n) Year 0
		Cumulative Emergence of Assertions					
Year -5	89	25	37	18	6	3	0
Year -4	94		26	40	18	7	3
Year -3	98			27	42	19	7
Year -2	115				32	49	21
Year -1	130					35	51
Year 0	136						26
	Total	25	63	85	98	113	108

(a) Occurrence Year	(b) Number of Claims	(o) Year -5	(p) Year -4	(q) Year -3	(r) Year -2	(s) Year -1	(t) Year 0
		RBNA at End of Period					
Year -5	89	0	0	0	0	0	0
Year -4	94		0	0	0	0	0
Year -3	98			0	0	0	0
Year -2	115				0	0	1
Year -1	130					1	4
Year 0	136						12
	Total	0	0	0	0	1	17

Exhibit 7

Calculation of Average Rate Relativity

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
	Base	Territory			Classification			Step Factor			Schedule Rating		
Risk #	Rate	#	Rel.	Premium	Code	Rel.	Premium	Code	Rel.	Premium	Code	Rel.	Premium
1	6,422	1	1.00	6,422	1C	1.35	8,670	CMM	1.00	8,670	C1	0.95	8,237
2	6,422	1	1.00	6,422	2A	1.50	9,633	CM3	0.94	9,055	N	1.00	9,055
3	6,422	1	1.00	6,422	1B	1.00	6,422	CM2	0.88	5,651	D1	1.05	5,934
4	6,422	2	1.15	7,385	1A	0.90	6,647	CM0	0.30	1,994	N	1.00	1,994
5	6,422	3	1.25	8,028	1A	0.90	7,225	CMM	1.00	7,225	C1	0.95	6,864
6	6,422	3	1.25	8,028	2B	2.00	16,056	CMM	1.00	16,056	C3	0.85	13,648
7	6,422	4	1.50	9,633	8	6.00	57,798	CM3	0.94	54,330	N	1.00	54,330
8	6,422	4	1.50	9,633	3	2.25	21,674	CM2	0.88	19,073	N	1.00	19,073
9	6,422	2	1.15	7,385	1B	1.00	7,385	CMM	1.00	7,385	D2	1.10	8,124
10	6,422	3	1.25	8,028	1A	0.90	7,225	CMM	1.00	7,225	C2	0.90	6,503
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Total	64,220			84,000			163,800			151,515			148,333
Change Factor			(4)/(1) =	1.308		(7)/(4) =	1.950		(10)/(7) =	0.925		(13)/(10) =	0.979

Notes: (4) = (1)(3)
(7) = (4)(6)
(10) = (7)(9)
(13) = (10)(12)

Exhibit 8

Company X

State Y

Physicians & Surgeons

Average Rate Relativity

Source of Relativity	Factor
Territory Relativity	1.308
Classification Relativity	1.950
Claims Made Year (Step Factor)	0.925
Schedule Rating Credit/Debit Factor	0.979
New to Practice Credit Factor	0.988
Part Time Credit Factor	0.962
Risk Management Credit Factor	0.996
Claim Free Credit Factor	0.938
Combined Average Factor	2.051

Exhibit 9

Company X

Countrywide

Physicians and Surgeons

All Other Loss Adjustment Expense Factor

(1) Claims Made Year	(2) Total L&LAE Incurred	(3) All Other Loss Expense Paid	(4) All Other Loss Expense Unpaid	(5) AO LAE Factor
2000	73,825,290	2,499,355	232,181	0.038
2001	81,730,727	2,010,576	441,346	0.031
2002	105,054,866	2,045,418	1,421,392	0.034
2003	114,113,914	1,848,645	2,259,456	0.037
2004	137,487,266	1,495,861	3,178,706	0.035
Total	512,212,063	9,899,856	7,533,080	0.035

Notes: Countrywide Experience is from Schedule P - Part 1F.
(5) = [(3)+(4)]/[(2)-(3)-(4)]

Exhibit 10

Company X

State Y

Physicians and Surgeons

Development of Pure Premium Trend
500/1000 Limits

Report Year	(1) Base Class Equivalent Earned Exposures	(2) Selected Ultimate Loss & DCC	(3) Base Class Pure Premium Y=(2)/(1)	(4) X
1997	987	3,918,390	3,970	1
1998	1,004	4,151,540	4,135	2
1999	1,100	4,973,100	4,521	3
2000	1,155	5,509,350	4,770	4
2001	1,213	6,286,979	5,183	5
2002	1,420	8,828,140	6,217	6
2003	1,599	9,430,902	5,898	7
2004	1,679	11,269,448	6,712	8
2005				

# Points	Correlation Coefficient	Annual Trend
8	0.965	11.6%
6	0.920	11.1%
4	0.770	4.9%
	Selected =	9.2%

Exhibit 11

Company X

State Y

Physicians and Surgeons

Development of Expense Constant and Variable Expense Factor

Variable Expense Components:	
1. Brokerage and Commissions	10.0%
2. Taxes, Licenses and Fees	2.5%
3. Underwriting Profit Reflecting Investment Income	-1.7%
4. Total Variable Expenses excluding L&LAE	10.8%
5. Variable Expense Factor = 1.0 - Variable Expenses	0.892
Fixed Expense Component:	
6. Other Acquisition Expenses	237,074
7. General Expenses	355,611
8. Total Fixed Expenses = (6) + (7)	592,685
9. Base Class Equivalent Exposures [Exhibit 5a]	1,679
10. Average Base Class Factor [Exhibit 8]	2.051
11. Exposures = (10) / (9)	819
12. Fixed Expense per Exposure = (8) / (11)	724
13. Expense Constant = (12) / (5)	812

Exhibit 12

Company X

State Y

Physicians & Surgeons

Rate Level Indication
500/1000 Limits

Hybrid Year	(a) Base Class Equivalent Exposures	(b) Earned Premium at Current Rates	(c) Ultimate Incurred L&LAE	(d) Trended # Ultimate Incurred L&LAE	(e) = (d)/(a) Trended Pure Premium
1997	987	7,139,958	4,056,450	9,359,683	9,483
1998	1,004	7,262,936	4,297,814	9,081,134	9,045
1999	1,100	7,957,400	5,148,321	9,961,745	9,056
2000	1,155	8,355,270	5,703,465	10,106,155	8,750
2001	1,213	8,774,842	6,508,493	10,560,998	8,707
2002	1,420	10,272,280	9,139,189	13,580,305	9,564
2003	1,599	11,567,166	9,763,188	13,285,286	8,308
2004	1,679	12,145,886	11,666,513	14,537,766	8,659
Total	10,157	73,475,738	56,283,434	90,473,071	8,907
00-04	7,066	51,115,444	42,780,848	62,070,509	8,784

1. Selected Claims Made L&LAE Pure Premium	8,784
2. Death, Disability and Retirement (DDR) Load	1.05
3. Claims Made Pure Premium with DDR Load = (1)(2)	9,223
4. Variable Expense Factor [Exhibit 11]	0.892
5. Calculated Variable Base Rate = (3) / (4)	10,340
6. Average Proposed Base Class Factor [Exhibit 8]	2.051
7. Average Increased Limit Factor	1.123
8. Average Variable Premium = (5)(6)(7)	23,816
9. Expense Constant [Exhibit 11]	812
10. Average Indicated Premium = (8) + (9)	24,628
11. Current Average Premium = (b) / actual unit earned exposure	17,474
12. Indicated Change = (10) / (11) -1	40.9%

Note: # Trended to one year beyond 1/1/2006 Effective Date.

Incorporating Cancellations into Pricing and Reserving Extended Warranties

Richard Easton, FCAS, MAAA

Abstract: Accounting rules specify that extended warranty contracts with terms of thirteen months or longer use loss payment patterns to determine the unearned premium reserve. These payment patterns should incorporate cancellations. Ignoring cancellations overstates earned premium and understates the unearned premium reserve.

Disclaimer: The views expressed in this paper are solely the responsibility of the author and do not necessarily reflect the views of his employer, The Warranty Group.

Extended Warranties (EWs) are unusual property and casualty coverages due to the uncertainty about the estimate of unearned (and earned) premium. Generally, there is much less uncertainty about pending reserves and IBNR. The reverse is the case for the typical liability property and casualty line. Statutory Accounting Principle 65 requires that companies carry the highest of three estimates as the unearned premium reserve. Test 1 is the amount of refunds that would be paid if all the contracts canceled. Test 2 is the gross premium times the unpaid losses divided by the total losses. Test 3 is the unpaid losses with discounting allowed though at a less than market rate. Companies generally establish earnings patterns for their databases which calculate the unearned (and earned) premium for test 2. This paper asserts that the payment pattern should explicitly adjust for cancellations. Not adjusting for cancellations overestimates the earned premium by 2%-3% for a mature book of in-force business and by a substantially greater amount for a growing immature book.

EWs have been discussed in several Casualty Actuarial Society articles (see appendix). However, I have not been able to find any detailed consideration of how cancellations should be handled in terms of the earnings pattern. This issue pertains mainly to automobile and power sports EWs. Cancellations are not as significant on other EWs due to the difference in term and premium amount. For example, Electronics and Appliances generally have a lower EW premium and a shorter term than is the case for automobile. These two factors usually lead to less cancellations.

EW cancellation refunds are normally pro-rata. Thus, the refund for a six year contract with $1,000 premium after three years is $500. An additional cancellation fee is sometimes levied. Cancellation fees will be ignored in this paper. Generally, the manufacturer's warranty covers most if not all losses in the first three years for new vehicles. Once a contract is cancelled, any remaining premium is earned. No premium earns during the manufacturer's warranty unless the EW adds additional coverage. In this example, the canceled contract has $500 of earned premium against little or no exposure. This fact alone means that one should monitor cancellation rates closely since they greatly affect profitability. Most of these cancellations, except for the buyer's remorse ones just after the EW is purchased, arise from the existing vehicle being traded in for a new one. There is some ambiguity about cancellation rates. Thus, there is generally breakage in the latter part of the EW contract's term. Breakage is defined as the reduction in losses in the latter part of the contract period due to people forgetting that they have coverage or no longer owning the item. The cancellation rate could increase if fewer people forget that they have an EW when the covered car is sold or people owning vehicles for shorter periods of time. However, it is reasonable to assume that the rate of forgetfulness is relatively constant and an increase in cancellation rate implies a higher turnover rate for the covered car.

For used vehicles, the exposure is generally faster than pro-rata; thus the loss ratio on canceled contracts should be higher than that for contracts which run the full term and expire.

For the sake of simplicity, all of the examples in this paper will use term only. Most auto EWs have both a term and a mileage component. Thus, one could write a six year and 60,000 mile EW for a vehicle with a three year and 36,000 mile manufacturer's warranty. A few high mileage drivers will exceed the 36,000 limit in the first year with a much higher percentage exceeding it in the second and third year. Thus, they will mile out of the manufacturer's warranty before the three year term expires. These high mileage drivers will usually exceed the 60,000 limit prior to the expiration of the six year term limit.

Exhibit 1 shows a simple example of two year contracts. Note that EWs are not considered insurance in most states; thus, we will use the term contract not policy and effective year rather than policy year. 100 contracts are written on 1/1/2000 for $1 of premium per contract. Frequency is 10% per exposed year with severity uniform at $5. Thus, paid losses are 50% of the in-force premium per exposed year. The resulting payment pattern is 55.6% for the first year and 44.4% for the second year. Using this pattern mismatches premium and losses. Assuming no lag between accident date and payment date, which eliminates the need for lag IBNR, the $50 of losses in the first year divided by $55.6 of earned premium yields a 90% loss ratio. In the second year, $40 of losses divided by $34.4 of earned premium is a 116% loss ratio. The problem with Method 1 arises since the front loaded overall payment pattern stems from cancellations and not from the inherent risk being greater in the first half of the contract.

Method 2 measures the partial pure premiums in developing the payment pattern. Thus, there are $50 of losses in the first year against an in-force of $100 for a 50% in-force loss rate. Similarly, there are $40 of losses in the second year against an in-force of $80 for a 50% rate. The earned premium is $50 in the first year (0.5 x $100) and $40 in the second year (either 0.5 x $80 or 90 – 50). Method 3 projects the ultimate written premium net of cancellations. Thus, premium emergence patterns are used to estimate the ultimate written net of cancellations of $90. Using the standard payment pattern also yields earned of $50 in the first year and $40 in the second year. Method 2 is superior since the individual contracts are earned correctly and there is no need for an overall cancellation adjustment. This correct earning of contract data means that further splits, such as by class or SKU, will be correct. Alternately, one ignore all premium and losses from policies which have canceled.

Exhibit 2A shows a more realistic example for new vehicles with seven year contracts and three year manufacturer's warranties. In this example, 10% of the contracts cancel after the fourth year. Method 1, the unadjusted payment pattern, results in loss ratios of 92.5%, 90.3% 102.8% and 102.8% in years four to seven. Method 2 yields loss ratios of 100% except for year five. The lower year five loss ratio stems from all the cancellation profit being realized in the year in which the contracts cancel. Thus, the contracts earn 57.1% of the premium for covering 25% of the exposure. 6.4 = 20 x (.571 - 0.25). Method 3 gives a 104.5% loss ratio in year four and 94% in years five to seven. Once again, the partial pure premium after adjusting for cancellations, Method 2, yields the best result.

Exhibit 2B shows the effect of cancellations doubling. Underwriting profitability doubles as a result since the contracts which are not canceled have a 100% loss ratio. Note though that the 100% loss ratio probably reflects some breakage. Thus, individuals sell their car but forget to cancel their warranty contract will generally have even better experience than the cancellations since there is no return premium. Method 1 again sends out false profitability signs in years four and five. Method 2 shows break-even underwriting except for year five. Method 3 has an unprofitable year four and is profitable in years five to seven.

The long-term results from contracts in Exhibit 2A is shown in Exhibit 3A. Thus, it shows the effect on results of level writings with 10% cancellations in the beginning of the fifth year. The loss ratios in Method 1 are more profitable than the long-term average in years four to six and then equals the overall average of 96.6% after that. Method 2 is breakeven in year four, is better than average in years five and six, but higher than Method 1, and then is at the long-term average. Method 3 is unprofitable in year four and then declines gradually to the long-term average in years seven onwards. The UPR is consistently the highest in Method 3, reflecting the unprofitable results in the fourth year and less profitable results in years five and six. Exhibit 3B shows the effect of doubling cancellations.

Exhibits 4A and 4B show similar examples for used vehicles where losses, adjusted for cancellations, are faster than pro-rata. In these cases, the pro-rata cancellations increase the loss ratio from 100% to 100.7% and 101.5% in Exhibits 5A and 5B, respectively. Exhibit 5A shows that Method 1 gives a false underwriting profit in year one, whereas Method 3 shows far too unprofitable a loss ratio in the first year. Once again, Method 2 yields the best results. Exhibit 5B again shows the effect of doubling cancellation rates.

Exhibit 6 shows an example for a 60 month EW where most or all of the manufacturer's warranty has expired. Column 5 shows that with level written premium for at least five years, the earned premium without adjusting for cancels in column 2 is 2.3% higher than the adjusted earned in column 4. Column 6 shows an example where written premium is increasing by 4% per year. The larger premiums are given at the top since they represent more recent contract months (ages 1-12 are the first contract year, etc.). Adjusted for the premium increases, column 9 shows that the in-force earned premium without adjusting for cancels is 2.4% higher than the adjusted earned. For the most recent contract year, it is 10.6% higher; for the last two years, it is 7.5% higher, etc. The payment pattern in Exhibit 6 is given on an accident date basis rather than for payment date. Thus, pending reserves and IBNR are required to cover the liability for the payment lag. Earnings curves can also be done by payment date. They obviously will extend beyond the end of the contract period.

In conclusion, partial pure premiums, excluding contracts which canceled prior to the beginning of the period, are the best method for earning premium for EWs with significant cancellation rates. In general, I have found that the adjustment reduces earned premium on in-force contracts by about 2% as was shown in Exhibit 6. Thus, if no contracts have expired, the inception-to-date loss ratio using unadjusted payment patterns will be about 2% too low. Similarly, the carried UPR from Test 2 will also be too low.

ACKNOWLEDGMENTS

The author acknowledges the contributions of John Sopkowicz of GMAC and Steve Hayes, Rita Kwok, Mike Carzoli and Maureen Boyle of The Warranty Group to this paper.

BIOGRAPHY

Richard Easton is Vice President and Actuary at The Warranty Group (formerly AON Warranty Group). He has a degree in Mathematics from Brown University. He is a Fellow of the CAS and a Member of the American Academy of Actuaries.

Exhibit 1

2 year Warranties
All written on 1/1/2000
Pro-rata losses
Premium = $1
Contract Count = 100
Severity = $5
Frequency = 10% per exposed year
20% cancel on 1/1/2001 - $10 total return premium

Method 1 - overall payment pattern

	2000	2001	Total
Written Premium in-force	100	80	
Policies in-force	100	80	
Paid Losses	50.0	40.0	90.0
Payment Pattern	55.6%	44.4%	100.0%
Earned Premium from payment pattern	55.6	35.6	91.1
Adjusted Earned Premium (written - cancellation)	55.6	34.4	90.0
Refunds from Cancellations		10.0	10.0
Loss Ratio	90.0%	116.1%	100.0%

55.6 = 100 x 50/90.
34.4 =100 x 40/90 -10 or 90-55.6

Method 2 - use payment pattern excluding canceled policies

Partial Pure premium	50.0%	50.0%	100.0%
Resulting Earned Premium	50	40	90
Loss Ratio	100.0%	100.0%	100.0%

Method 3 - Project Ultimate Written Premium after cancellations

Projected Ultimate Premium	90		
Payment Pattern	55.6%	44.4%	100.0%
Resulting Earned Premium	50	40	90
Loss Ratio	100.0%	100.0%	100.0%

Exhibit 2A - New Vehicles

7 year Warranties
All written on 1/1/2000
Pro-rata losses
3 Year (36 month/36,000 miles) manufacturer's warranty - no losses during this period
Premium = $2
Contract Count = 100
Severity = $5
Frequency = 10% per exposed year
10% cancel on 1/1/2004

Method 1 - Unadjusted payment pattern

	2000	2001	2002	2003	2004	2005	2006	Total
Paid Losses	0.0	0.0	0.0	50.0	45.0	45.0	45.0	185.0
Payment Pattern	0.0%	0.0%	0.0%	27.0%	24.3%	24.3%	24.3%	100.0%
In Force Written Premium	200	200	200	200	180	180	180	
Policies in-force	100	100	100	100	90	90	90	
Earned Premium from payment pattern	0.0	0.0	0.0	54.1	43.8	43.8	43.8	185.4
Adjusted Earned Premium (written - cancellation)	0.0	0.0	0.0	54.1	49.8	43.8	43.8	191.4
Earned - Paid = profit from cancellations.	0.0	0.0	0.0	4.1	4.8	-1.2	-1.2	6.4
Refunds from Cancellations					8.6			8.6
Loss Ratio				92.5%	90.3%	102.8%	102.8%	96.6%

Method 2 - use payment pattern excluding canceled policies

	2000	2001	2002	2003	2004	2005	2006	Total
Partial Pure premium	0	0	0	25.0%	25.0%	25.0%	25.0%	100.0%
Resulting Earned Premium	0	0	0	50	45	45	45	185
Additional earned from cancellations					6.4			
Total Earned Premium	0	0	0	50	51.4	45	45	191.4
Earned - Paid = profit from cancellations.	0.0	0.0	0.0	0.0	6.4	0.0	0.0	6.4
Loss Ratio				100.0%	87.5%	100.0%	100.0%	96.6%

Method 3 - Project Ultimate Written Premium after cancellations

	2000	2001	2002	2003	2004	2005	2006	Total
Projected Ultimate Premium	191.4							
Payment Pattern	0.0%	0.0%	0.0%	25.0%	25.0%	25.0%	25.0%	
Total Earned Premium	0.0	0.0	0.0	47.9	47.9	47.9	47.9	191.4
Earned - Paid = profit from cancellations.	0.0	0.0	0.0	-2.1	2.9	2.9	2.9	6.4
Loss Ratio				104.5%	94.0%	94.0%	94.0%	96.6%

Exhibit 2B - New Vehicles

7 year Warranties
All written on 1/1/2000
Pro-rata losses
3 Year (36 month/36,000 miles) manufacturer's warranty - no losses during this period
Premium = $2
Contract Count = 100
Severity = $5
Frequency = 10% per exposed year
20% cancel on 1/1/2004

Method 1 - Unadjusted payment pattern

	2000	2001	2002	2003	2004	2005	2006	Total
Paid Losses	0.0	0.0	0.0	50.0	40.0	40.0	40.0	170.0
Payment Pattern	0.0%	0.0%	0.0%	29.4%	23.5%	23.5%	23.5%	100.0%
In Force Written Premium	200	200	200	200	160	160	160	
Policies in-force	100	100	100	100	80	80	80	
Earned Premium from payment pattern	0.0	0.0	0.0	58.8	37.6	37.6	37.6	171.8
Adjusted Earned Premium (written - cancellation)	0.0	0.0	0.0	58.8	48.7	37.6	37.6	182.9
Earned - Paid = profit from cancellations.	0.0	0.0	0.0	8.8	8.7	-2.4	-2.4	12.9
Refunds from Cancellations					17.1			17.1
Loss Ratio				85.0%	82.1%	106.3%	106.3%	93.0%

Method 2 - use payment pattern excluding canceled policies

	2000	2001	2002	2003	2004	2005	2006	Total
Partial Pure premium	0	0	0	25.0%	25.0%	25.0%	25.0%	100.0%
Resulting Earned Premium	0	0	0	50	40	40	40	170
Additional earned from cancellations					12.9			
Total Earned Premium	0	0	0	50	52.9	40	40	182.9
Earned - Paid = profit from cancellations.	0.0	0.0	0.0	0.0	12.9	0.0	0.0	12.9
Loss Ratio				100.0%	75.7%	100.0%	100.0%	93.0%

Method 3 - Project Ultimate Written Premium after cancellations

	2000	2001	2002	2003	2004	2005	2006	Total
Projected Ultimate Premium	182.9							
Payment Pattern	0.0%	0.0%	0.0%	25.0%	25.0%	25.0%	25.0%	
Total Earned Premium	0.0	0.0	0.0	45.7	45.7	45.7	45.7	182.9
Earned - Paid = profit from cancellations.	0.0	0.0	0.0	-4.3	5.7	5.7	5.7	12.9
Loss Ratio				109.4%	87.5%	87.5%	87.5%	93.0%

Exhibit 3A - New Vehicles

7 year Warranties
All written on 1/1/2000
Pro-rata losses
3 Year (36 month/36,000 miles) manufacturer's warranty - no losses during this period
Premium = $2
Contract Count = 100
Severity = $5
Frequency = 10% per exposed year
10% cancel on 1/1 of fifth year

Paid Losses

Effective Year	2000	2001	2002	2003	2004	2005	2006	2007	2008
2000	0.0	0.0	0.0	50.0	45.0	45.0	45.0		
2001		0.0	0.0	0.0	50.0	45.0	45.0	45.0	
2002			0.0	0.0	0.0	50.0	45.0	45.0	45.0
2003				0.0	0.0	0.0	50.0	45.0	45.0
2004					0.0	0.0	0.0	50.0	45.0
2005						0.0	0.0	0.0	50.0
2006							0.0	0.0	0.0
2007								0.0	0.0
2008									0.0
Total	0.0	0.0	0.0	50.0	95.0	140.0	185.0	185.0	185.0

Method 1 - use unadjusted payment pattern

Earned Premium

Effective Year	2000	2001	2002	2003	2004	2005	2006	2007	2008
2000	0.0	0.0	0.0	54.1	49.8	43.8	43.8		
2001		0.0	0.0	0.0	54.1	49.8	43.8	43.8	
2002			0.0	0.0	0.0	54.1	49.8	43.8	43.8
2003				0.0	0.0	0.0	54.1	49.8	43.8
2004					0.0	0.0	0.0	54.1	49.8
2005						0.0	0.0	0.0	54.1
2006							0.0	0.0	0.0
2007								0.0	0.0
2008									0.0
Total	0.0	0.0	0.0	54.1	103.9	147.6	191.4	191.4	191.4
Loss Ratio				92.5%	91.5%	94.8%	96.6%	96.6%	96.6%
Test 2 UPR	200.0	400.0	600.0	745.9	842.1	894.4	903.0	911.6	920.2

Method 2 - use payment pattern excluding canceled policies

Earned Premium

Effective Year	2000	2001	2002	2003	2004	2005	2006	2007	2008
2000	0.0	0.0	0.0	50.0	51.4	45.0	45.0		
2001		0.0	0.0	0.0	50.0	51.4	45.0	45.0	
2002			0.0	0.0	0.0	50.0	51.4	45.0	45.0
2003				0.0	0.0	0.0	50.0	51.4	45.0
2004					0.0	0.0	0.0	50.0	51.4
2005						0.0	0.0	0.0	50.0
2006							0.0	0.0	0.0
2007								0.0	0.0
2008									0.0
Total	0.0	0.0	0.0	50.0	101.4	146.4	191.4	191.4	191.4
Loss Ratio				100.0%	93.7%	95.6%	96.6%	96.6%	96.6%
Test 2 UPR	200.0	400.0	600.0	750.0	848.6	902.1	910.7	919.3	927.9

Method 3 - Project Ultimate Written Premium after cancellations

Earned Premium

Effective Year	2000	2001	2002	2003	2004	2005	2006	2007	2008
2000	0.0	0.0	0.0	47.9	47.9	47.9	47.9		
2001		0.0	0.0	0.0	47.9	47.9	47.9	47.9	
2002			0.0	0.0	0.0	47.9	47.9	47.9	47.9
2003				0.0	0.0	0.0	47.9	47.9	47.9
2004					0.0	0.0	0.0	47.9	47.9
2005						0.0	0.0	0.0	47.9
2006							0.0	0.0	0.0
2007								0.0	0.0
2008									0.0
Total	0.0	0.0	0.0	47.9	95.7	143.6	191.4	191.4	191.4
Loss Ratio				104.5%	99.3%	97.5%	96.6%	96.6%	96.6%
Test 2 UPR	200.0	400.0	600.0	752.1	856.4	912.9	921.4	930.0	938.6

Exhibit 3B - New Vehicles

7 year Warranties
All written on 1/1/2000
Pro-rata losses
3 Year (36 month/36,000 miles) manufacturer's warranty - no losses during this period
Premium = $2
Contract Count = 100
Severity = $5
Frequency = 10% per exposed year
20% cancel on 1/1 of fifth year

Paid Losses

Effective Year	2000	2001	2002	2003	2004	2005	2006	2007	2008
2000	0.0	0.0	0.0	50.0	40.0	40.0	40.0		
2001		0.0	0.0	0.0	50.0	40.0	40 0	40.0	
2002			0.0	0.0	0.0	50.0	40.0	40.0	40.0
2003				0.0	0.0	0.0	50.0	40.0	40.0
2004					0.0	0.0	0.0	50.0	40.0
2005						0.0	0.0	0.0	50.0
2006							0.0	0.0	0.0
2007								0.0	0.0
2008									0.0
Total	0.0	0.0	0.0	50.0	90.0	130.0	170.0	170.0	170.0

Method 1 - use unadjusted payment pattern

Earned Premium

Effective Year	2000	2001	2002	2003	2004	2005	2006	2007	2008
2000	0.0	0.0	0.0	58.8	48.7	37.6	37.6		
2001		0.0	0.0	0.0	58.8	48.7	37.6	37.6	
2002			0 0	0.0	0 0	58.8	48.7	37.6	37.6
2003				0.0	0.0	0.0	58.8	48.7	37.6
2004					0.0	0 0	0 0	58.8	48.7
2005						0 0	0.0	0.0	58.8
2006							0.0	0.0	0.0
2007								0.0	0.0
2008									0.0
Total	0.0	0.0	0.0	58.8	107.6	145.2	182.9	182.9	182.9
Loss Ratio				85 0%	83 7%	89.5%	93.0%	93.0%	93.0%
Test 2 UPR	200.0	400.0	600.0	741.2	833.6	888.4	905.5	922.7	939.8

Method 2 - use payment pattern excluding canceled policies

Earned Premium

Effective Year	2000	2001	2002	2003	2004	2005	2006	2007	2008
2000	0.0	0.0	0.0	50.0	52.9	40.0	40.0		
2001		0.0	0.0	0.0	50.0	52.9	40.0	40.0	
2002			0.0	0.0	0.0	50.0	52.9	40.0	40.0
2003				0.0	0.0	0.0	50.0	52.9	40.0
2004					0.0	0.0	0.0	50.0	52.9
2005						0.0	0.0	0.0	50.0
2006							0.0	0.0	0.0
2007								0 0	0 0
2008									0.0
Total	0 0	0.0	0 0	50.0	102.9	142.9	182.9	182 9	182.9
Loss Ratio				100.0%	87.5%	91.0%	93.0%	93.0%	93.0%
Test 2 UPR	200.0	400.0	600.0	750.0	847.1	904.3	921.4	938.6	955.7

Method 3 - Project Ultimate Written Premium after cancellations

Earned Premium

Effective Year	2000	2001	2002	2003	2004	2005	2006	2007	2008
2000	0.0	0.0	0.0	45.7	45.7	45.7	45.7		
2001		0.0	0.0	0.0	45.7	45.7	45.7	45.7	
2002			0.0	0.0	0.0	45.7	45.7	45.7	45.7
2003				0.0	0.0	0.0	45.7	45.7	45.7
2004					0.0	0.0	0.0	45.7	45.7
2005						0 0	0.0	0.0	45.7
2006							0.0	0.0	0.0
2007								0.0	0.0
2008									0.0
Total	0 0	0 0	0 0	45.7	91.4	137.1	182.9	182.9	182.9
Loss Ratio				109.4%	98.4%	94.8%	93.0%	93 0%	93.0%
Test 2 UPR	200.0	400.0	600.0	754.3	862.9	925.7	942.9	960.0	977.1

Exhibit 4A - Used Vehicles

3 year Warranties
All written on 1/1/2000
Losses emerge faster than pro-rata

Premium = $2
Contract Count = 100

10% cancel on 1/1/2001

Method 1 - use unadjusted payment pattern

	2000	2001	2002	Total
Paid Losses	80.0	54.0	54.0	188.0
Payment Pattern	42.6%	28.7%	28.7%	100.0%
In Force Written Premium	200	180	180	
Policies in-force	100	90	90	
Earned Premium from payment pattern	85.1	51.7	51.7	188.5
Adjusted Earned Premium (written - cancellation)	85.1	49.9	51.7	186.7
Difference (= extra profit from cancellations)	0.0	-1.8	0.0	-1.8
Refunds from Cancellations				0.0
Loss Ratio	94.0%	108.2%	104.4%	100.7%

Method 2 - use payment pattern excluding canceled policies

Partial Pure premium	40%	30%	30%	100.0%
Resulting Earned Premium	80.0	54.0	54.0	188.0
Additional earned from cancellations		-1.3		
Total Earned Premium	80.0	52.7	54.0	186.7
Loss Ratio	100.0%	102.5%	100.0%	100.7%

Method 3 - Project Ultimate Written Premium after cancellations

Projected Ultimate Premium	186.7			
Payment Pattern	40.0%	30.0%	30.0%	
Total Earned Premium	74.7	56.0	56.0	186.7
Loss Ratio	107.1%	96.4%	96.4%	100.7%

Exhibit 4B - Used Vehicles

3 year Warranties
All written on 1/1/2000
Losses emerge faster than pro-rata

Premium = $2
Contract Count = 100

20% cancel on 1/1/2001

Method 1 - use unadjusted payment pattern

	2000	2001	2002	Total
Paid Losses	80.0	48.0	48.0	176.0
Payment Pattern	45.5%	27.3%	27.3%	100.0%
In Force Written Premium	200	160	160	
Policies in-force	100	80	80	
Earned Premium from payment pattern	90.9	43.6	43.6	178.2
Adjusted Earned Premium (written - cancellation)	90.9	38.9	43.6	173.4
Difference (= extra profit from cancellations)	0.0	-4.8	0.0	-4.8
Refunds from Cancellations				
Loss Ratio	88.0%	123.5%	110.0%	101.5%

Method 2 - use payment pattern excluding canceled policies

	2000	2001	2002	Total
Partial Pure premium	40%	30%	30%	100.0%
Resulting Earned Premium	80.0	48.0	48.0	176.0
Additional earned from cancellations		-2.6		
Total Earned Premium	80.0	45.4	48.0	173.4
Loss Ratio	100.0%	105.7%	100.0%	101.5%

Method 3 - Project Ultimate Written Premium after cancellations

	2000	2001	2002	Total
Projected Ultimate Premium	173.4			
Payment Pattern	40.0%	30.0%	30.0%	
Total Earned Premium	69.4	52.0	52.0	173.4
Loss Ratio	115.3%	92.3%	92.3%	101.5%

Exhibit 5A - Used Vehicles

3 year Warranties
All written on 1/1 of policy year
Losses emerge faster than pro-rata

Premium = $2
Contract Count = 100

10% cancel on 1/1 of second year

Paid Losses

Effective Year	2000	2001	2002	2003	2004
2000	80.0	54.0	54.0		
2001		80.0	54.0	54.0	
2002			80.0	54.0	54.0
2003				80.0	54.0
2004					80.0
Total	80.0	134.0	188.0	188.0	188.0

Method 1 - use unadjusted payment pattern

Earned Premium

Effective Year	2000	2001	2002	2003	2004
2000	85.1	49.9	51.7		
2001		85.1	49.9	51.7	
2002			85.1	49.9	51.7
2003				85.1	49.9
2004					85.1
Total	85.1	135.0	186.7	186.7	186.7
Loss Ratio	94.0%	99.3%	100.7%	100.7%	100.7%
Test 2 UPR	114.9	179.9	193.2	206.5	219.8

Method 2 - use payment pattern excluding canceled policies

Earned Premium

Effective Year	2000	2001	2002	2003	2004
2000	80.0	52.7	54.0		
2001		80.0	52.7	54.0	
2002			80.0	52.7	54.0
2003				80.0	52.7
2004					80.0
Total	80.0	132.7	186.7	186.7	186.7
Loss Ratio	100.0%	101.0%	100.7%	100.7%	100.7%
Test 2 UPR	120.0	187.3	200.6	213.9	227.2

Method 3 - Project Ultimate Written Premium after cancellations

Earned Premium

Effective Year	2000	2001	2002	2003	2004
2000	74.7	56.0	56.0		
2001		74.7	56.0	56.0	
2002			74.7	56.0	56.0
2003				74.7	56.0
2004					74.7
Total	74.7	130.7	186.7	186.7	186.7
Loss Ratio	107.1%	102.5%	100.7%	100.7%	100.7%
Test 2 UPR	125.3	194.6	207.9	221.2	234.5

Exhibit 5B - Used Vehicles

3 year Warranties
All written on 1/1 of policy year
Losses emerge faster than pro-rata

Premium = $2
Contract Count = 100

20% cancel on 1/1 of second year

Effective Year	Paid Losses 2000	2001	2002	2003	2004
2000	80.0	48.0	48.0		
2001		80.0	48.0	48.0	
2002			80.0	48.0	48.0
2003				80.0	48.0
2004					80.0
Total	80.0	128.0	176.0	176.0	176.0

Method 1 - use unadjusted payment pattern

Effective Year	Earned Premium 2000	2001	2002	2003	2004
2000	90.9	38.9	43.6		
2001		90.9	38.9	43.6	
2002			90.9	38.9	43.6
2003				90.9	38.9
2004					90.9
Total	90.9	129.8	173.4	173.4	173.4
Loss Ratio	88.0%	98.6%	101.5%	101.5%	101.5%
Test 2 UPR	109.1	179.3	205.9	232.5	259.1

Method 2 - use payment pattern excluding canceled policies

Effective Year	Earned Premium 2000	2001	2002	2003	2004
2000	80.0	45.4	48.0		
2001		80.0	45.4	48.0	
2002			80.0	45.4	48.0
2003				80.0	45.4
2004					80.0
Total	80.0	125.4	173.4	173.4	173.4
Loss Ratio	100.0%	102.1%	101.5%	101.5%	101.5%
Test 2 UPR	120.0	194.6	221.2	247.8	274.4

Method 3 - Project Ultimate Written Premium after cancellations

Effective Year	Earned Premium 2000	2001	2002	2003	2004
2000	69.4	52.0	52.0		
2001		69.4	52.0	52.0	
2002			69.4	52.0	52.0
2003				69.4	52.0
2004					69.4
Total	69.4	121.4	173.4	173.4	173.4
Loss Ratio	115.3%	105.5%	101.5%	101.5%	101.5%
Test 2 UPR	130.6	209.3	235.9	262.5	289.1

Earned Premium Comparison **Exhibit 6**
Term: 60 months/ 60,000 miles

Age (Months)	With Cancels UPR (1)	With Cancels EPR (2)	Without Cancels UPR (3)	Without Cancels EPR (4)	Difference in Earned (5)	Written Premium (6)	Earned Premium With Cancels (7)	Earned Premium Without Cancels (8)	Difference (9)
1	1.0000	0.0000	1.0000	0.0000	-5.1%	1,213	0.0	0.0	0.0
2	0.9993	0.0007	0.9993	0.0007	1.1%	1,209	0.9	0.9	0.0
3	0.9956	0.0044	0.9961	0.0039	14.5%	1,205	5.3	4.7	0.7
4	0.9912	0.0088	0.9922	0.0078	13.6%	1,201	10.6	9.3	1.3
5	0.9847	0.0153	0.9865	0.0135	12.9%	1,197	18.3	16.2	2.1
6	0.9767	0.0233	0.9792	0.0208	12.2%	1,193	27.8	24.8	3.0
7	0.9673	0.0327	0.9706	0.0294	11.1%	1,189	38.9	35.0	3 9
8	0.9551	0.0449	0.9596	0.0404	11.2%	1,185	53.2	47.8	5.4
9	0.9401	0.0599	0.9460	0.0540	10.9%	1,181	70.8	63.8	7.0
10	0.9225	0.0775	0.9296	0.0704	10.2%	1,178	91.3	82.8	8.5
11	0.9032	0.0968	0.9121	0.0879	10.1%	1,174	113.6	103 2	10.4
12	0.8831	0.1169	0.8937	0.1063	10.0%	1,170	136.8	124.4	12 4
13	0.8615	0.1385	0.8734	0 1266	9.4%	1,166	161.5	147.6	13.9
14	0 8390	0.1610	0.8525	0.1475	9.2%	1,162	187.2	171 4	15.7
15	0.8156	0.1844	0.8301	0.1699	8.5%	1,158	213.6	196 8	16.7
16	0.7929	0.2071	0 8081	0.1919	8.0%	1,155	239.2	221.5	17.6
17	0 7698	0.2302	0.7862	0.2138	7 7%	1,151	265.0	246.0	18.9
18	0.7455	0.2545	0.7629	0.2371	7.4%	1,147	292.0	272.0	20.0
19	0.7219	0.2781	0.7405	0.2595	7.2%	1,143	318 0	296.7	21.3
20	0.6970	0.3030	0.7166	0.2834	6.9%	1,140	345.3	323.0	22.3
21	0.6709	0.3291	0.6912	0.3088	6.6%	1,136	373.9	350.8	23.1
22	0.6452	0.3548	0.6658	0.3342	6 2%	1,132	401.7	378.4	23 3
23	0.6196	0.3804	0.6407	0.3593	5.9%	1,129	429 4	405.5	23 8
24	0.5932	0.4068	0.6153	0.3847	5.8%	1,125	457 6	432.7	25 0
25	0.5653	0.4347	0.5878	0.4122	5.5%	1,121	487.4	462 2	25.2
26	0.5381	0.4619	0 5607	0.4393	5.1%	1,118	516.2	490 9	25 3
27	0.5106	0 4894	0.5327	0 4673	4.7%	1,114	545.1	520.5	24.7
28	0.4850	0 5150	0.5073	0.4927	4 5%	1,110	571 8	547.1	24 7
29	0 4591	0.5409	0 4820	0.5180	4.4%	1,107	598.6	573.3	25.3
30	0 4352	0.5648	0.4581	0.5419	4 2%	1,103	623.0	597 7	25.3
31	0.4108	0.5892	0.4330	0.5670	3 9%	1,099	647.8	623.4	24.3
32	0.3885	0 6115	0.4097	0.5903	3.6%	1,096	670.1	646.9	23.2
33	0.3659	0.6341	0.3866	0.6134	3.4%	1,092	692 6	670.0	22 6
34	0.3428	0.6572	0.3628	0.6372	3.1%	1,089	715.5	693.7	21.8
35	0 3202	0.6798	0.3393	0.6607	2.9%	1,085	737.7	717.0	20.7
36	0.2987	0.7013	0.3172	0.6828	2.7%	1,082	758.5	738.5	20.0
37	0.2784	0 7216	0.2959	0 7041	2 5%	1,078	777.9	759.1	18.8
38	0.2579	0.7421	0.2745	0.7255	2 3%	1,075	797.4	779.6	17.8
39	0 2408	0.7592	0.2565	0.7435	2.1%	1,071	813.1	796.3	16.8
40	0 2217	0.7783	0.2368	0.7632	2.0%	1,068	830 9	814.7	16 2
41	0.2050	0.7950	0.2193	0 7807	1 8%	1,064	846.0	830 7	15.3
42	0.1894	0.8106	0.2027	0.7973	1 7%	1,061	859 7	845.6	14 2
43	0.1736	0.8264	0.1861	0.8139	1 5%	1,057	873 6	860.4	13 2
44	0 1571	0 8429	0 1686	0.8314	1.4%	1,054	888 1	876 1	12 1
45	0.1439	0.8561	0.1546	0.8454	1 3%	1,050	899.1	887.9	11.2
46	0.1312	0.8688	0.1410	0.8590	1 1%	1,047	909.5	899.3	10.2
47	0.1178	0.8822	0.1266	0.8734	1 0%	1,043	920.5	911.3	9 2
48	0.1059	0.8941	0 1143	0.8857	0.9%	1,040	929.8	921.1	8.7
49	0.0949	0.9051	0.1024	0.8976	0.8%	1,037	938.3	930.5	7 8
50	0.0829	0.9171	0.0897	0.9103	0.8%	1,033	947.6	940.5	7.1
51	0.0710	0 9290	0.0770	0 9230	0.6%	1,030	956.7	950.6	6.2
52	0.0624	0.9376	0.0677	0.9323	0.6%	1,026	962.5	957.0	5.4
53	0.0545	0.9455	0.0592	0.9408	0 5%	1,023	967.4	962 6	4.8
54	0.0461	0.9539	0.0501	0.9499	0.4%	1,020	972.8	968.7	4.1
55	0.0387	0.9613	0.0421	0.9579	0.4%	1,016	977 1	973.7	3 5
56	0.0315	0.9685	0.0343	0.9657	0.3%	1,013	981.2	978.4	2.9
57	0 0261	0 9739	0.0284	0 9716	0.2%	1,010	983.5	981.1	2.4
58	0.0194	0.9806	0 0211	0 9789	0 2%	1,007	987.1	985.3	1.8
59	0.0144	0 9856	0.0157	0.9843	0 1%	1,003	988.8	987 5	1.3
60	0.0066	0 9934	0.0072	0.9928	0.1%	1,000	993.4	992 8	0 6
		31.8181		**31.1027**	2.3%		**33,818.0**	**33,027.2**	2.4%
						Year 1	567.4	512.8	10.6%
						Years 1-2	4,251 6	3,955 2	7.5%
						Years 1-3	11,815.9	11,236.5	5.2%
						Years 1-4	22,161.6	21,418 5	3.5%

Notes:
1. (2) = 1.0 - (1).
2. (4) = 1.0 - (3).
3. (5) = (2) / (4) - 1.0.
4. (6) is increasing by 4% per year.
5. (7) = (6) x (2).
6. (8) = (6) x (4).
7. (9) = (7) - (8).

INTERPRETATIONS OF SEMI-PARAMETRIC MIXTURE MODELS, UNBIASED ESTIMATORS OF ULTIMATE VALUE FOR INDIVIDUAL CLAIMS AND CONDITIONAL PROBABILITY APPLICATIONS TO CALCULATE BULK RESERVES

by Rajesh Sahasrabuddhe, FCAS, MAAA

Abstract

Semi-parametric mixture models have well documented technical advantages for modeling loss distributions. These technical advantages are documented in papers that focus on the estimation of the parameters of semi-parametric models.

This paper assumes that the parameters have already been determined and then provides an interpretation of the results of the parameter estimation. This interpretation is intended to make semi-parametric models intuitively appealing. If we accept this interpretation of the parameters, then we can use conditional probability concepts to calculate bulk reserves either deterministically or in a stochastic framework.

1. Introduction

A recent paper by Keatinge[1] discussed the virtues of semi-parametric mixture models vis-à-vis (fully) parametric models and non-parametric (empirical) models. The advantages discussed in Keatinge focus on the attractive compromise between smoothing and data responsiveness offered by semi-parametric models. Semi-parametric models have the following density function:

$$f(x) = w_1 \times f_1(x) + w_2 \times f_2(x) + \ldots + w_n \times f_n(x)$$

where:

i. $f(x)$ represents the probability density function for the mixture model,
ii. $f_i(x)$ represents the probability density function for the i-th component of the mixture, and
iii. w_i represents the mixing weight corresponding to i-th component of the mixture.

Furthermore, the mixing weights are subject to the constraints that:

i. $w_i > 0$
ii. $\sum w_i = 1$.

[1] Keatinge, Clive L., "*Modeling Losses with the Mixed Exponential Distribution*," Proceedings of the Casualty Actuarial Society 1999 Vol: LXXXVI Page(s): 654-698, Casualty Actuarial Society: Arlington, Virginia

The mixture is considered semi-parametric since each component of the mixture is a parametric model but the distribution of mixing weights is model free. Also, it should be noted that there is no restriction that the components of the mixture have the same model form (e.g. exponential, lognormal, Pareto) or, for that matter, any specific model form.

The remainder of this paper assumes that model forms and parameters have already been determined.

2. Interpreting Semi-parametric Models

While Keatinge's arguments are certainly persuasive, there may be a more important argument supporting the use of semi-parametric models: they are intuitively appealing.

Specifically, it is reasonable to assume that loss experience is comprised of observations from a discrete number of underlying loss processes. The table below provides some examples:

	Coverage			
	Auto Liability	**Workers Compensation**	**Medical Malpractice**	**Homeowners**
Loss Processes	Property Damage Only	Indemnity (Lost Time)	Nuisance	Theft
	Bodily Injury and Property Damage	Indemnity and Medical	Expense Only (Dismissals)	Partial Property Loss
		Death	Settlement without litigation	Total Loss of Property and Contents
			Settlements with litigation	

Under this assumption, it is then reasonable to interpret the mixing weights (w_i) as the percentage of total claims that are generated by each loss process. Logically, the named loss processes must therefore be exhaustive[2]. That is, all claims must fall into one of these categories and the sum of the probabilities associated with the loss processes must equal 1. It would then follow that the components of the mixture would be interpreted as describing the distribution of claims amounts resulting from each loss process.

Although in many cases the loss process is coded in the claims record[3], this is not always the case. The table above is meant to provide examples of types of multiple loss processes that might produce the observed claim distribution That is, we assume that the

[2]. Since we can define the loss process as narrowly or broadly as we desire, we are not concerned that a mixture model would be required to describe a single loss process nor are we concerned that multiple loss processes would be described by distributions that are not significantly different.

[3] For example, auto liability loss records will often indicate whether the loss is for bodily injury or property damage.

mixture model identifies all significantly different underlying loss processes[4]. In addition, it should be noted that there is no requirement that we identify the loss processes by name. In fact it may not be possible to identify each and every loss process.

3. Using Conditional Probability Concepts to Estimate Claim Level Bulk Reserves

The term "bulk reserve" is used to represent the reserve for development on known (or reported) losses and is often referred to as the "incurred but not enough reported" ("IBNER") reserve. As is common in analysis of claims-made coverages, the bulk reserve is often estimated in the aggregate for a body of claims. For occurrence coverages, the bulk reserve and the reserve for unreported claims are often estimated on a combined basis.

The mixture model represents the overall distribution of claim values *without any prior knowledge*. However for reported claims, we will have some knowledge about each claim. This "knowledge" will generally include amounts paid to date and case basis reserves.

For purposes of this discussion, it should be assumed that an unbiased estimator for the ultimate value of each claim is available. (We will return to this assumption in the next section.) This unbiased estimator will be denoted *U.*

We now focus on the likelihood of the various loss processes generating a claim of size *U.* What we are really concerned with is not the absolute probabilities but rather the relative probabilities. Recall the conditional probability relationship:

$$\Pr(A_i \mid B) = \frac{\Pr(A_i \cap B)}{\Pr(B)} \text{ where}$$

A_i represents the event that loss process i underlies the claim and
B represents event that the unbiased estimate of the claim is equal to U.

The relative probabilities i.e. $\Pr(A_1|B)$, $\Pr(A_2|B)$ … $\Pr(A_n|B)$ are proportional to:

$$\Pr(A_i \cap B) = \Pr(A_i \mid B) \times \Pr(B) = \Pr(B \mid A_i) * \Pr(A_i).$$

In this case we decide to use the second expression. Under the interpretation offered in Section 2:

$\Pr(A_i) = w_i$ and
$\Pr(B|A_i)$ is proportional to $f_i(\text{U})$.

So we can restate the mixture model with adjusted mixing weights defined as follows:

$$\tilde{w}_i = \Pr(A_i) \times \Pr(B \mid A_i) \propto w_i \times f_i(U). \tag{2.1}$$

[4] To the extent that those differences are represented in the data, of course.

where the ~ indicates that the parameter (or distribution) is applicable to an individual claim and has been adjusted to consider the unbiased estimator of the ultimate value of that claim.

Normalizing mixture weights, we obtain:

$$\widetilde{w}_i = w_i \times f_i(U) / \sum_i w_i \times f_i(U). \qquad (2.2)$$

The conditional density function is then equal to:

$$f(x \mid U) = \widetilde{f}(x) = \widetilde{w}_1 \times f_1(x) + \widetilde{w}_2 \times f_2(x) + \ldots + \widetilde{w}_n \times f_n(x). \qquad (2.3)$$

The table below provides an example of how mixing weights adjust for various *U* values for a mixture of 3 component lognormal models.

i	1	2	3	Total
mu_i	9	10	12	
$sigma_i$	1.5	1.75	1.5	
Initial Mixing Weight (w_i)	50%	20%	30%	100%
Mean	24,959	101,849	501,320	183,246
Standard Deviation	72,716	459,801	1,460,532	
Example #1				
Unbiased Estimator (U)	10,000			
$f_i(U)$	2.63E-05	2.06E-05	4.72E-06	
Adjusted Mixing Weight	70%	22%	8%	100%
Mean	24,959	101,849	501,320	77,943
Example #2				
Unbiased Estimator (U)	150,000			
$f_i(U)$	2.67E-07	8.33E-07	1.77E-06	
Adjusted Mixing Weight	16%	20%	64%	100%
Mean	24,959	101,849	501,320	344,701
Example #3				
Unbiased Estimator (U)	750,000			
$f_i(U)$	3.73E-09	3.98E-08	2.11E-07	
Adjusted Mixing Weight	3%	11%	87%	100%
Mean	24,959	101,849	501,320	445,681

In particular, readers should observe how the mixing weights shift given the unbiased estimator for the claim and the means and standard deviations of the components of the mixture.

Readers will also note that the mean value for each claim is not equal to the unbiased estimator. This is because the process is designed to produce individual distributions that, taken together, describe the distribution of a portfolio of claims. The process is not necessarily appropriate for any individual claim. (As discussed in the following section, determination of an unbiased estimator for a claim may also not be possible.) That is, as with most actuarial techniques, the predictive value of the results requires sufficient credibility of the claims portfolio being modeled. Finally, after reviewing the relative stability[5] of the distribution parameters and the stability of the unbiased estimator, it may be appropriate to balance the results of the two indications.

4. Calculation of Bulk Reserves

With this adjusted density function, we can calculate bulk reserves either deterministically or stochastically. The deterministic estimate is simply calculated by integrating and subtracting the current reported value (denoted R) of the claim as follows:

$$BULK = \frac{\int x \times \tilde{f}(x)dx}{\int \tilde{f}(x)dx} - R. \tag{3.1}$$

It should be noted that the limits of integration are not included in the formula above. A possible adjustment would be to truncate the distribution from below at the paid to date value of the claim or to truncate the distribution from above at the maximum probable loss. Without these adjustments, the denominator of the first term equals 1 and is not necessary.

In addition, the numerator can readily be modified to consider the effect of policy limits. This adjustment is left to the interested reader.

More powerfully however, we can use the adjusted mixing weights to simulate a range of ultimate values for each claim. This is done in a two step process. In the first step, we draw from a Discrete (x, p) distribution where the loss processes are the x values and the adjusted mixing weights are the p values. This step determines the loss model that describes the distribution of ultimate values of the claim. In the second step, we draw a loss value from the loss model from the first step. This amount represents the simulated ultimate value of the claim. Commercial simulation software can then be used to develop both mean estimates of the bulk reserve as well as bulk reserve estimates at various confidence levels.

5. The Unbiased Estimator

The discussion above assumes that an unbiased estimator of the ultimate value of each claim is available. As we know this is almost never the case. (If it were, there would be much less need for actuaries.) However, we should recognize a biased estimator is usually available and an adjustment factor can be applied to this estimator to remove the

[5] "Stability" here refers to the change in these items resulting from incremental (marginal) increases in the data underlying their estimation.

bias. That biased estimator is the reported (paid plus case reserve) value of the claim and the adjustment factor is related to the loss development factor.

The loss development factor would have to be adjusted to remove the distorting influences of closed claims and unreported claims. That is, a cumulative reported loss development factor at age (maturity) j could be written as follows[6]:

$$RLDF_j = \frac{\text{Paid on Closed}_j + \text{Ultimate on Open}_j + \text{Ultimate on Unreported}_j}{\text{Paid on Closed}_j + \text{Reported on Open}_j} \quad (4.1)$$

It should be noted that for this purpose, the loss development factors merely need to be for the same type of claim as the bulk reserve being estimated. That is, we are simply trying to develop the adjustment factor for known claims and liability for known claims exists regardless of whether coverage is written on a claims-made or occurrence basis. Therefore, we could use claims-made factors in this exercise to develop bulk reserves for occurrence basis coverage.

Rewriting the numerator of equation 4.1 as Ultimate Loss and taking reciprocals, we arrive at the following:

$$RLDF_j^{-1} = \frac{\text{Paid on Closed}_j + (\text{Paid on Open}_j - \text{Paid on Open}_j) + \text{Reported on Open}_j}{\text{Ultimate Loss}}$$

$$RLDF_j^{-1} = PLDF_j^{-1} - \frac{\text{Paid on Open}_j}{\text{Ultimate Loss}} + \frac{\text{Reported on Open Claims}_j}{\text{Ultimate Loss}} \quad (4.2)$$

where "$PLDF_j$" denotes the cumulative paid loss development factor at age j.

We can rewrite the second term of the right hand side of the equation using the following relationship:

$$\frac{\text{Paid on Open}_j}{\text{Ultimate Loss}} \times \frac{\text{Total Paid}_j}{\text{Total Paid}_j} = PLDF_j^{-1} \times \frac{\text{Paid on Open}_j}{\text{Total Paid}_j} \quad (4.3)$$

and we then rewrite equation 4.2 as:

$$RLDF_j^{-1} = PLDF_j^{-1} - PLDF_j^{-1} \times \frac{\text{Paid on Open}_j}{\text{Total Paid}_j} + \frac{\text{Reported on Open Claims}_j}{\text{Ultimate Loss}}. \quad (4.4)$$

Rearranging equation 4.4, we obtain:

$$RLDF_j^{-1} - PLDF_j^{-1} \times \left(1 - \frac{\text{Paid on Open}_j}{\text{Total Paid}_j}\right) = \frac{\text{Reported on Open Claims}_j}{\text{Ultimate Loss}}. \quad (4.5)$$

[6] Assumes no reserve is required for reopened claims.

Taking reciprocals, we have:

$$\left[RLDF_j^{-1} - PLDF_j^{-1} \times \left(1 - \frac{\text{Paid on Open}_j}{\text{Total Paid}_j}\right)\right]^{-1} = \frac{\text{Ultimate Loss}}{\text{Reported on Open Claims}_j}. \tag{4.6}$$

Focusing on the right side of this equation we have the following:

$$\begin{aligned} &\frac{\text{Ultimate Loss}}{\text{Reported on Open}_j} \\ &= \frac{\text{Paid on Closed}_j + \text{Ultimate on Open}_j + \text{Ultimate on Unreported}_j}{\text{Reported on Open}_j}. \end{aligned} \tag{4.7}$$

For convenience, we will refer to the three terms of the right hand side of this equation as ***F***, ***G***, and ***H***. We should recognize that the middle term (*G*) is the bias adjustment that we need to convert the reported value to an unbiased estimator of ultimate loss (*U*).

Substituting equation 4.7 into equation 4.6 and solving for G, we have:

$$G_j = \left(RLDF_j^{-1} - PLDF_j^{-1} \times \left(1 - \frac{\text{Paid on Open}_j}{\text{Total Paid}_j}\right)\right)^{-1} - F_j - H_j \tag{4.8}$$

$$\begin{aligned} G_j = &\left(RLDF_j^{-1} - PLDF_j^{-1} \times \left(1 - \frac{\text{Paid on Open}_j}{\text{Total Paid}_j}\right)\right)^{-1} - \\ &\frac{\text{Paid on Closed}_j}{\text{Reported on Open}_j} - \frac{\text{Ultimate on Unreported}_j}{\text{Reported on Open}_j} \end{aligned}. \tag{4.9}$$

The author recognizes that (1) $\frac{\text{Paid on Open}_j}{\text{Total Paid}_j}$, (2) $\frac{\text{Paid on Closed}_j}{\text{Reported on Open}_j}$ and (3) $\frac{\text{Ultimate on Unreported}_j}{\text{Reported on Open}_j}$ are statistics that are not "natural" and are not generally readily available.

However, it is the author's opinion that (1) and (2) should be straightforward to compile from a loss database since they are based strictly on reported values. Therefore it should not be inordinately more difficult to develop these statistics than it is to develop loss development factors.

The third statistic should also be straightforward to determine and in many cases it may not be necessary. That is, with respect to the numerator of this statistic, we can use a frequency / severity approach. We already have a severity model, *f(x)*, and unreported frequency should be readily estimated using a claim count development factors. The denominator is based only on reported data.

It should also be noted that for many lines of business substantially all claims are reported within two years so this term would be unnecessary after 24 months. For longer-tailed lines of business, loss development factors are generally available on a claims-made basis. If we are using these claims-made loss development factors, this term is, by definition, equal to 0 and is therefore not necessary.

Finally we should recognize that statistics (1) and (2) should be expected to approach zero as claims mature. Statistic (3) should be expected to become large as claims mature. In fact since (3) may become unstable at late maturities, the actuary may simply want to use the reported value of the claim (without adjustment) at late maturities. This is not altogether unwarranted since at late maturities the unknown facts associated with a claim will decrease and reported value of the claim is more likely to be an unbiased estimator of ultimate value.

Using $R_{j,k}$ to denote the reported value of the j-th claim at age k, and U_j to denote the unbiased estimator of the ultimate value of the j-th claim, we can now state the following:

$$U_j = R_{j,k} \times G_k . \tag{4.10}$$

Readers will note that G is based on aggregate statistics such as loss development factors. These statistics only consider the age of a claim and therefore ignore many other factors[7] that would influence development of a given claim. Determining an appropriate G for any single claim is extremely difficult. The framework described in this paper therefore provides an attractive compromise between the unbiased estimator and the *a priori* average claim size.

While this may seem like a significant effort, the reward is equally significant. Namely, the actuary now has insight into the average level of misstatement in case reserves. The actuary should recognize this as particularly important information in evaluating reserves.

6. Conclusion and Summary

Using the procedure above, we can transform a semi-parametric mixture model from its generic form of

$$f(x) = w_1 \times f_1(x) + w_2 \times f_2(x) + \ldots + w_n \times f_n(x)$$

to a form that may be used to describe the distribution of the ultimate value of a known claim:

$$f(x \mid U) = \tilde{f}(x) = \tilde{w}_1 \times f_1(x) + \tilde{w}_2 \times f_2(x) + \ldots + \tilde{w}_n \times f_n(x)$$

[7] Such as claim size.

where the mixing weights are adjusted based on an unbiased estimator of the ultimate value of the claim. This unbiased estimator can be calculated as a function of the reported value of the claim.

As actuarial analyses move from deterministic frameworks to stochastic frameworks, the distributions of ultimate values for known claims will gain in importance.

* * * * *

Acknowledgements

The author would like to acknowledge and thank the following individuals who reviewed this paper: David R. Lesieur, Paul Braithwaite and Don Doty

As always, any errors that remain are the responsibility of the author.

Resimulation

By James Shaheen

1. INTRODUCTION

One method of simulating random variables is to generate uniform 0 to 1 variables, then use them in the inverse of the cumulative distribution function of the random variable you want to simulate. For example, if you had U_1, a uniform 0 to 1 variable, and you wanted to use it to simulate S_1, the flip of a fair coin, you could use this function:

If $0<U_1<0.5$, then S_1=tails

If $0.5<U_1<1$, then S_1=heads

Historically it was thought that if you wanted to simulate two independent variables in this manner, you would have to generate two uniform variables. But in fact, you can simulate multiple independent discrete variables from a single uniform variable.

Example 1

If you wanted to simulate two flips of a fair coin (S_1 and S_2), you could use this function:

If $0<U_1<0.25$, then S_1=tails and S_2=tails

If $0.25<U_1<0.5$, then S_1=tails and S_2=heads

If $0.5<U_1<0.75$, then S_1=heads and S_2=tails

If $0.75<U_1<1$, then S_1=heads and S_2=heads

By simple inspection you can see that by this method S_1 and S_2 both have a 50% chance of being heads and a 50% chance of being tails, and that they are independent from each other.

2. ADDITIONAL VARIABLES

This method can work to simulate as many discrete variables as you want.

Example 2

If you wanted to simulate three flips of a fair coin, you could use this function:

If $0<U_1<0.125$, then S_1=tails, S_2=tails, and S_3=tails

If $0.125<U_1<0.25$, then S_1=tails, S_2=tails, and S_3=heads

If $0.25<U_1<0.375$, then S_1=tails, S_2=heads, and S_3=tails

If $0.375<U_1<0.5$, then S_1=tails, S_2=heads, and S_3=heads

If $0.5<U_1<0.625$, then S_1=heads, S_2=tails, and S_3=tails

If $0.625<U_1<0.75$, then S_1=heads, S_2=tails, and S_3=heads

If $0.75<U_1<0.875$, then S_1=heads, S_2=heads, and S_3=tails

If $0.875<U_1<1$, then S_1=heads, S_2=heads, and S_3=heads

So for example, if $U_1<0.43$, then S_1=tails, S_2=heads, and S_3=heads.

3. A SIMPLER FORMULA

The method from Sections 1 and 2 gets exponentially more complicated as you simulate more variables, so it can be a pain to use, especially in cases where you're simulating something more complicated than the flip of a coin. But it can be simplified if you use the first uniform 0 to 1 variable to simulate other uniform 0 to 1 variables, and use those other uniform variables to simulate subsequent flips of a coin, or subsequent variables of whatever distribution you want to simulate.

To do this, I introduce these definitions:

Definition 1

$\text{Min}(F^{-1}(F(U_n)))$=The smallest number which, when plugged into the function F, would produce the same result that U_n produced.

Definition 2

$\text{Max}(F^{-1}(F(U_n)))$=The largest number which, when plugged into the function F, would produce the same result that U_n produced.

Example 3

Suppose you were simulating a coin flip, using this function F:

If $0<U_n<0.5$, then $F(U_n)$=tails

If $0.5<U_n<1$, then $F(U_n)$=heads

If $U_n=0.43$, then $F(U_n)$=tails. Therefore $F^{-1}(F(U_n))= F^{-1}(\text{tails})$=anything from 0 to 0.5. Therefore

$\mathrm{Min}(F^{-1}(F(U_n)))=\mathrm{Min}(\text{anything from 0 to 0.5})=0.$

$\mathrm{Max}(F^{-1}(F(U_n)))=\mathrm{Max}(\text{anything from 0 to 0.5})=0.5.$

Example 4

Using the same function F from Example 3,

$\mathrm{Min}(F^{-1}(F(0.86)))=0.5.$

$\mathrm{Max}(F^{-1}(F(0.86)))=1.$

Using those new definitions, I introduce this recursive formula for simulating additional uniform 0 to 1 variables:

Theorem 1

$$U_{n+1}=(U_n-\mathrm{Min}(F^{-1}(F(U_n))))/(\mathrm{Max}(F^{-1}(F(U_n)))-\mathrm{Min}(F^{-1}(F(U_n))))$$

Example 5

Assume we have a uniform 0 to 1 variable, U_1, equal to 0.43, and from that we want to simulate three flips of a fair coin.

We'll use the simple simulation formula:

If $0<U_n<0.5$, then $S_n=F(U_n)=\text{tails}$

If $0.5<U_n<1$, then $S_n=F(U_n)=\text{heads}$

So if $U_1=0.43$, then $S_1=\text{tails}$

To get U_2, we use the formula

$U_{n+1}=(U_n-\mathrm{Min}(F^{-1}(F(U_n))))/(\mathrm{Max}(F^{-1}(F(U_n)))-\mathrm{Min}(F^{-1}(F(U_n))))$

$U_2=(U_1-\mathrm{Min}(F^{-1}(F(U_1))))/(\mathrm{Max}(F^{-1}(F(U_1)))-\mathrm{Min}(F^{-1}(F(U_1))))$

$U_2=(0.43-\mathrm{Min}(F^{-1}(F(0.43))))/(\mathrm{Max}(F^{-1}(F(0.43)))-\mathrm{Min}(F^{-1}(F(0.43))))$

$U_2=(0.43-0)/(0.5-0)=0.86$

To get S_2 we can again use our simple simulation formula:

If $0<U_n<0.5$, then $S_n=F(U_n)=\text{tails}$

If $0.5<U_n<1$, then $S_n=F(U_n)=\text{heads}$

So if $U_2=0.86$, then $S_2=\text{heads}$

To get U_3, we again use the recursive formula

$U_{n+1}=(U_n\text{-Min}(F^{-1}(F(U_n))))/(\text{Max}(F^{-1}(F(U_n)))\text{-Min}(F^{-1}(F(U_n))))$

$U_3=(U_2\text{-Min}(F^{-1}(F(U_2))))/(\text{Max}(F^{-1}(F(U_2)))\text{-Min}(F^{-1}(F(U_2))))$

$U_3=(0.86\text{-Min}(F^{-1}(F(0.86))))/(\text{Max}(F^{-1}(F(0.86)))\text{-Min}(F^{-1}(F(0.86))))$

$U_3=(0.86\text{-}0.5)/(1\text{-}0.5)=0.72$

And finally, we can get S_3 by using our simple simulation formula again:

If $0<U_n<0.5$, then $S_n=F(U_n)$=tails

If $0.5<U_n<1$, then $S_n=F(U_n)$=heads

So if $U_3=0.72$, then S_3=heads

Therefore if $U_1=0.43$, then S_1=tails, S_2=heads, and S_3=heads. And we were able to generate all three of those variables using just two simple formulas.

Note that neither U_2 nor S_2 are independent of U_1. But S_2 is independent of S_1, since S_2 has a 50% chance of being heads and a 50% chance of being tails regardless of whether S_1 is heads or tails.

Example 6

Assume we have a uniform 0 to 1 variable, U_1, equal to 0.29, and from that we want to simulate variables, S_1, S_2, and S_3, each of which have a 20% chance of being 0, a 50% chance of being 1, and a 30% chance of being 2.

If $0<U_n<0.2$, $S_n=F(U_n)=0$.

If $0.2<U_n<0.7$, $S_n=F(U_n)=1$.

If $0.7<U_n<1$, $S_n=F(U_n)=2$.

$U_{n+1}=(U_n\text{-Min}(F^{-1}(F(U_n))))/(\text{Max}(F^{-1}(F(U_n)))\text{-Min}(F^{-1}(F(U_n))))$

Therefore $S_1=1$.

$U_2=(0.29\text{-}0.2)/(0.7\text{-}0.2)=0.18$.

$S_2=0$.

$U_3=(0.18\text{-}0)/(0.2\text{-}0)=0.9$.

$S_3=2$.

Note that this theorem will not work with functions that have gaps in the definition of single outcomes. For example, it will work with this function:

If $0<U_1<0.5$, then S_1=tails

If $0.5<U_1<1$, then S_1=heads

But it would not work with this function, since it has a gap in the definition of S_1=tails:

If $0<U_1<0.25$, then S_1=tails

If $0.25<U_1<0.75$, then S_1=heads

If $0.75<U_1<1$, then S_1=tails

4. CONTINUOUS SIMULATED VARIABLES

In general, if you use a uniform 0 to 1 random variable to simulate a continuous variable, you will not be able to reuse it to simulate a second variable that is independent of the first simulated variable. The formula for U_{n+1} doesn't make sense if S_n is continuous.

If $F(U_n)$ is continuous, then $U_n=\text{Min}(F^{-1}(F(U_n)))=\text{Max}(F^{-1}(F(U_n)))$.

Therefore the formula for U_{n+1}:

$U_{n+1}=(U_n-\text{Min}(F^{-1}(F(U_n))))/(\text{Max}(F^{-1}(F(U_n)))-\text{Min}(F^{-1}(F(U_n))))$

Comes out to

$U_{n+1}=(U_n-U_n)/(U_n-U_n)=0/0$, which is undefined.

So in general, you can only reuse a uniform 0 to 1 random variable if it was first used to simulate a discrete variable.

5. PARTIALLY DISCRETE VARIABLES

If you're using a uniform 0 to 1 random variable to simulate a variable that is partially discrete and partially continuous, then you can reuse the uniform 0 to 1 random variable if and only *if it ends up simulating the part of the variable that is discrete.*

Example 7

Assume that we want to simulate three independent variables (S_1, S_2, and S_3) that each have a 50% chance of being uniformly distributed between 0 and 0.5 and a 50% chance of being 0.5. To do this we are given the uniform 0 to 1 variables 0.6 (which we'll call U_a), 0.3 (U_b), and 0.1 (U_c), but whenever possible we should reuse those uniform variables using resimulation, so that we use as few of them as possible. To simulate S_1, S_2, and S_3, we can use the formula:

If $0<U_n<0.5$, $S_n=F(U_n)=U_n$.

If $0.5<U_n<1$, $S_n=F(U_n)=0.5$.

We need a uniform 0 to 1 variable, U_1, to simulate S_1. For lack of anything else we can use, we'll use the U_a that we were given.

Therefore $U_1=0.6$, therefore $S_1=0.5$. That's the part of the variable that is discrete, therefore we can reuse U_1 to simulate U_2.

$U_{n+1}=(U_n\text{-Min}(F^{-1}(F(U_n))))/(\text{Max}(F^{-1}(F(U_n)))\text{-Min}(F^{-1}(F(U_n))))$

Therefore

$U_2=(U_1\text{-Min}(F^{-1}(F(U_1))))/(\text{Max}(F^{-1}(F(U_1)))\text{-Min}(F^{-1}(F(U_1))))$

$U_2=(0.6\text{-Min}(F^{-1}(F(0.6))))/(\text{Max}(F^{-1}(F(0.6)))\text{-Min}(F^{-1}(F(0.6))))$

$U_2=(0.6\text{-}0.5)/(1\text{-}0.5)=0.2$.

Therefore $S_2=0.2$. That's the part of the variable that is continuous, therefore we can't reuse U_2 to simulate U_3. If we tried, we'd get:

$U_{n+1}=(U_n\text{-Min}(F^{-1}(F(U_n))))/(\text{Max}(F^{-1}(F(U_n)))\text{-Min}(F^{-1}(F(U_n))))$

$U_3=(U_2\text{-Min}(F^{-1}(F(U_2))))/(\text{Max}(F^{-1}(F(U_2)))\text{-Min}(F^{-1}(F(U_2))))$

$U_3=(0.2\text{-Min}(F^{-1}(F(0.2))))/(\text{Max}(F^{-1}(F(0.2)))\text{-Min}(F^{-1}(F(0.2))))$

$U_3=(0.2\text{-}0.2)/(0.2\text{-}0.2)=0/0$, which is undefined.

So instead, for U_3 we'll have to use U_b, the second of the uniform 0 to 1 variables we were given.

$U_3=0.3$. Therefore $S_3=0.3$. And we don't have to use U_c.

Example 8

The James Insurance Company has losses at a Poisson rate of 3 per year. 40% of the losses are for \$100; 35% are for \$1,000; and 25% are for \$10,000. Simulate how much it had in losses in one year. To do this we are given the uniform 0 to 1 variables U_a to U_k: 0.57, 0.79, 0.63, 0.02, 0.33, 0.68, 0.18, 0.94, 0.12, 0.21, and 0.95, but whenever possible we should reuse those uniform variables using resimulation, so that we use as few of them as possible.

We simulate the number of losses first, then use resimulation to simulate the amount of each loss. If losses occur at a Poisson rate, then the times between losses are exponential. So we'll start by simulating them. The CDF for an exponential distribution with □ =1/3 is:

$1\text{-}e\text{^}(-3*S_n)$

We can simulate exponential variables if we set that equal to U_n, and solve for S_n, which gives us:

$S_n=F(U_n)=-1/3*\ln(1-U_n)$

We need a uniform 0 to 1 variable, U_1, to simulate S_1. For lack of anything else we can use, we'll use the U_a that we were given. Therefore $U_1=0.57$, therefore S_1, the time to the first loss is

$=-1/3*\ln(1-0.57)=0.2813$.

Now we need another uniform 0 to 1 variable, U_2, to simulate S_2. S_1 was continuous, so we can't reuse U_1 to get U_2. So for lack of anything else we can use, we'll use the U_b that we were given. Therefore $U_2=0.79$, therefore S_2, the time from the first loss to the second loss is

$=-1/3*\ln(1-0.79)=0.5202$.

The total time until the third loss is

$=0.2813+0.5202=0.8015$.

Similarly, for U_3 we'll use U_c. Therefore $U_3=0.63$, therefore S_3, the time from the second loss to the third loss is

$=-1/3*\ln(1-0.63)=0.3314$.

The total time until the third loss is

$=0.2813+0.5202+0.3314=1.1329$.

The second loss was the last one that occurred before the end of the first year, therefore there were two losses in the year. Now we need to simulate the amount of those two losses. To do that, we need another uniform 0 to 1 variable, U_4. And here's the trick. Since S_3 was continuous, one might imagine that we can't reuse U_3 to simulate U_4. But we can.

$1-S_1-S_2=1-0.2813-0.5202=0.1985$. Therefore if the time from the second loss to the third loss had been less than 0.1985, there would have been (at least) three losses in the year. But as long as the time from the second loss to the third loss was greater than 0.1985, there would only have been two losses in the year. And once we know that there were only two losses in the year, it doesn't matter to us when the third loss actually occurs. It could be in year 2 or it could be in year 100. It doesn't matter. And we can reflect that by rewriting $F(U_3)$ as a partially discrete function.

The CDF for an exponential distribution with □ =1/3 is:

$1\text{-}e\hat{\ }(-3^*S_n)=U_n$

Therefore the U_n that corresponds to a S_n of 0.1985 is:

$1\text{-}e\hat{\ }(-3^*0.1985)=0.449$.

So we can use this partially discrete function:

If $0<U_3<0.449$, then $S_3=F(U_3)=-1/3^*\ln(1-U_3)$.

If $0.449<U_3<1$, then S_3 is such that the third loss is after the end of the first year.

And since $U_3>0.449$, it falls into the discrete portion of our new function, and we can reuse U_3 to simulate U_4.

$U_{n+1}=(U_n\text{-Min}(F^{-1}(F(U_n))))/(\text{Max}(F^{-1}(F(U_n)))\text{-Min}(F^{-1}(F(U_n))))$

Therefore

$U_4=(U_3\text{-Min}(F^{-1}(F(U_3))))/(\text{Max}(F^{-1}(F(U_3)))\text{-Min}(F^{-1}(F(U_3))))$

$U_4=(0.63\text{-}0.449)/(1\text{-}0.449)=0.328$

And once we have a new uniform 0 to 1 random variable, we can simulate the size of the losses in the first year fairly easily.

If $0<U_n<0.4$, $S_n=100$.

If $0.4<U_n<0.75$, $S_n=1{,}000$.

If $0.75<U_n<1$, $S_n=10{,}000$.

$U_{n+1}=(U_n\text{-Min}(F^{-1}(F(U_n))))/(\text{Max}(F^{-1}(F(U_n)))\text{-Min}(F^{-1}(F(U_n))))$

$U_4=0.328$. Therefore the size of the first loss, $S_4=100$.

$U_5=(0.328\text{-}0)/(0.4\text{-}0)=0.82$. Therefore the size of the second loss, $S_5=10{,}000$.

Therefore the total losses are 100+10,000=10,100. All done using only the first three of the uniform random variables we were given.

Consistent Measurement of Property-Casualty Risk-Based Capital Adequacy

Michael G. Wacek, FCAS, MAAA

Abstract

This paper is a review and case study of Butsic's expected policyholder deficit (EPD) framework for measurement and maintenance of risk-based capital adequacy for property-casualty insurance companies, the promise of which is that long term solvency protection can be achieved by periodic assessment and adjustment of risk-based capital using a consistent and short time horizon, e.g., one year, for risks on both sides of the balance sheet. Using a common one-year EPD risk measure to assess all risks, the case study examines the exposure to capital exhaustion during the period 1999 through 2004 arising from 1) U.S. Commercial Auto Liability accident year 1999 underwriting and reserving and 2) investment in the stocks comprising the S&P 500. The case study results indicate that NAIC and rating agency risk-based capital requirements for Commercial Auto Liability are significantly higher than necessary to meet stated solvency objectives and much higher than those demanded for common stock investments. That disparity probably exists for other lines of business as well. The consistent measurement of all time-dependent risks described in the paper is relevant not only to risk-based capital applications but to enterprise risk assessment and management as well.

Keywords: risk-based capital, expected policyholder deficit, stochastic loss models, Commercial Auto Liability, enterprise risk management, transfer value

1. INTRODUCTION

The thesis of this paper is that U.S. regulatory and rating agency[1] risk-based capital factors used to allocate capital for at least some non-catastrophe underwriting and reserve risks are significantly higher than necessary to meet stated solvency objectives. These factors are too high both in absolute terms and relative to those that are applicable to insurance company assets such as common stocks. The reason for this disparity is that the risks related to insurance underwriting and reserving and those associated with investing in common stocks have been measured inconsistently. When the risks are measured consistently, less risk-based capital is required to support underwriting activity and loss reserves or more capital is required to support the holding of assets such as common stocks, or possibly both.

Our thesis is based on the application of Robert Butsic's approach to measurement of risk-based capital adequacy, which makes use of a clearly defined and consistent time horizon for assessing and managing underwriting, asset and other risks. In his Michelbacher-Prize-winning 1992 paper [5] on risk-based capital and solvency issues for the

[1] A.M. Best and Standard and Poor's.

property-casualty industry, Butsic showed that long term solvency protection can be achieved by periodically rebalancing risk-based capital to maintain a constant (and low) target exposure to capital exhaustion over a short prospective time horizon[2]. He used the *expected policyholder deficit* (EPD) with a time horizon of one year as the measure of exposure to capital exhaustion[3]. Butsic's framework incorporates a consistent time horizon for measuring and allocating capital for risks on both sides of the balance sheet. The risk-based capital requirements for asset risks as well as underwriting and reserve risks are all calibrated to the same target one-year EPD. The capital allocated to support a block of assets or underwriting risks is adjusted up or down at the end of each year to produce a prospective one-year EPD that matches the target value[4].

The idea that an insurer needs to allocate capital to minimize the risk of underwriting and reserve related capital exhaustion occurring only within the next year seems to run counter to the imperative that allocated capital be sufficient to deal with the risk of insolvency over the indefinite time horizon encompassing ultimate claim settlement. It apparently leaves the insolvency risk beyond the next year "unfunded." In fact, that is not the case. Butsic's breakthrough insight was that such longer term risk can be addressed effectively, one year at a time, as it comes into the one-year time horizon in future periods (in the same way that common stock risk has historically been handled). Because capital is recalibrated each year to the target EPD, any capital inadequacy short of total exhaustion that has emerged during a year can be corrected at the end of that year. In that way the small prospective exposure to capital exhaustion at the start of each successive one-year time horizon is maintained at the target level. If an insurer cannot recapitalize at the level consistent with the target EPD, in all circumstances except those in which the capital has been exhausted there still will be sufficient assets to facilitate liquidation of the risk portfolio. If the regulators do not immediately intervene, rating agencies can calculate the EPD associated with the reduced

---

[2] We refer, in particular, to his discussion on pages 327-335. A slightly amended version of Butsic's paper was later published in the *Journal of Risk and Insurance* under the same title [6]. In that version the discussion appears on pages 668-675.

[3] The expected policyholder deficit with a time horizon of one year is defined as the expected value of the amount by which available assets, including allocated capital, will be inadequate to satisfy all claims one year in the future. A policyholder deficit with respect to asset risk arises when a fluctuating asset value falls below the value of unpaid losses (which is assumed to be fixed). A policyholder deficit with respect to underwriting-related risks arises when the fluctuating transfer value of unpaid losses exceeds the value of the available assets (which is assumed to be fixed). The EPD expressed as a ratio to the expected unpaid losses as of the beginning of the year can be viewed as the expected value of the proportion of the outstanding policyholder claims that will be unrecoverable because of insurer insolvency. Butsic used a one-year time horizon to illustrate his framework. It could, of course, be more or less than one year.

[4] Note that this framework can easily be adapted to use a risk measure other than the EPD. The principle that the chosen risk measure be used consistently to assess all risks consistently over successive short time horizons is more important than the risk measure itself (provided the risk measure is a sound one).

capital level and adjust the insurer's financial strength rating to reflect the increased insolvency risk[5].

A major advantage of Butsic's approach is that its consistent measurement of all risks over a common time horizon allows us to compare and ultimately combine risks that have different natural time horizons. That enables us to make clear and meaningful statements about an insurer's exposure to capital exhaustion in the next year. In contrast, it is less clear what a statement about an insurer's exposure to capital exhaustion means when it reflects a mixture of time horizons. Should capital exposed over different time horizons be calibrated to different target EPDs, perhaps 1% for one-year horizons and 4% for four-year horizons, or should the target be fixed irrespective of time horizon? How do risks having different time horizons interact? For example, if we measure risks only at their ultimate time horizons, if we add a four-year horizon risk to a portfolio with a one-year horizon, how do we measure the one-year capital exhaustion risk of the new portfolio? Butsic's framework allows us to avoid these questions by focusing on a common time horizon from the start.

A significant obstacle to the full implementation of Butsic's approach has been that, while it is relatively easy to calculate for asset risks, it is more difficult to determine the one-year EPD for underwriting and reserve risks. Butsic explained the concept and illustrated the calculation, but he did not describe a model or method for doing the calculation in practice. The issue is that the value of the one-year EPD is a function of a time-dependent loss distribution for which actuaries historically have had no use. However, a recent paper by Wacek [12] on the path of the ultimate loss ratio estimate describes a framework that can be used to model that distribution. We will use the approach outlined in that paper together with actual industry loss experience to illustrate the application of Butsic's framework for measuring underwriting and loss reserve risk.

One of our aims is to revive interest in Butsic's approach to the assessment of risk-based capital requirements, and, in particular, the use of a clear and consistent time horizon for measuring all of the solvency risks faced by an insurance company. In this paper we present a review and illustration of the key concepts of his framework using insurance industry and stock market experience from the period 1999 through 2004. By using actual experience to parameterize stochastic stock price and loss ratio models, we show that Butsic's framework is not only of theoretical interest but can be practically applied in the real world. While we

---

[5] That is more or less what happens today (though both A.M. Best and Standard and Poor's base their capital factors for underwriting and reserve risks on an *ultimate time horizon* EPD methodology [1][11]). Best, for example, reports that an EPD ratio of greater than 1% indicates a BCAR score of less than 100 and a rating of less than B+, and makes clear that capital adequacy is a key element of its rating analysis. See [1], page 5.

focus on the analysis of non-catastrophe risks, Butsic's framework can also be applied to the analysis of the threat to solvency posed by property catastrophe loss events.

The late 1990s were challenging years for the U.S. insurance industry. For our case study of underwriting and reserve risk within the Butsic framework, we used industry data for Commercial Auto Liability, a line that experienced particularly poor accident year underwriting results in the late 1990s. We focused on accident year 1999, which had not only the highest estimated ultimate loss ratio of any accident year for that line in the period 1995 through 2004 but also one that proved difficult for the industry to estimate accurately. The ultimate loss ratio estimate of 78.3% booked as of December 1999 had to be increased repeatedly, reaching 92.1% as of December 2004[6]. While there is evidence that the industry could have made better estimates at early valuations using information available at the time, even those better estimates underestimated the ultimate loss ratio significantly[7]. The magnitude and unpredictability of the 1999 ultimate loss ratio makes that accident year a good choice for stress-testing the Butsic framework.

The period 1999 through 2004 was also a turbulent one for the U.S. stock market. The S&P 500 rose by more than 20% in two of the six years, declined for three consecutive years (including one year by more than 20%) and ended 2004 about 8% above its level at the beginning of 1999[8]. That volatility makes the S&P 500 in this period a good candidate for a case study of risk-based capital analysis of diversified common stock investments.

Our case study reveals that during this period, if the risk in both portfolios is measured consistently, the insolvency risk embedded in the industry Commercial Auto Liability underwriting and reserve portfolios was a small fraction of that inherent in the diversified common stock portfolio represented by the S&P 500. Our modeling of the increased risk associated with individual insurers (compared to the industry as a whole) also indicated much lower insolvency risk than investment in the S&P 500. Moreover, we found that the amount of risk-based capital required to achieve a target one-year EPD for underwriting and reserve risks consistent with the 1% target sometimes cited for common stocks [1][8] to be

---

[6] According to the industry 2004 Schedule P data reported in the 2005 edition of Best's Aggregates & Averages [4]. These loss ratios were calculated from "incurred net losses and cost containment expenses" reported in Part 2C and "net premiums earned" reported in Part 1C.

[7] See Wacek [13], which was a case study of the relative quality of clinical judgment and statistical prediction in Commercial Auto Liability loss reserving for accident years 1995 through 2001. The paper concluded that the statistical prediction methods performed far better than the clinical methods actually used to set the reserves, but noted that they also did not perform well. For example, the accident year 1999 loss ratio actually booked at twelve months underestimated the ultimate loss ratio by 13.8 loss ratio points, while the mean of the statistical estimates underestimated it by 6.9 loss ratio points.

[8] Including dividends, the annual S&P 500 total returns during the period 1999 through 2004 were as follows: +21.0%, -9.1%, -11.9%, -22.1%, +28.7% and +10.9%. Source: Berkshire Hathaway 2005 Annual Report [3].

much lower than NAIC and rating agency requirements as of December 2006, while we found the capital required for common stocks, at least during the period covered by our study, to be higher. Our study was too narrow in scope to support general conclusions, but the strikingly lower risk level we found for Commercial Auto Liability suggests that consistently calibrated risk-based capital factors would probably also be lower than those promulgated by the NAIC and the rating agencies for the underwriting and reserve risk associated with other insurance lines.

1.1 Organization of the Paper

The paper comprises three main sections including this introduction, plus four appendices containing more technical and detailed material. The heart of the paper is Section 2, where we first define the one-year policyholder deficit and its expected value with respect to 1) common stock asset risk and 2) underwriting and reserve risk, and then illustrate these definitions by applying them to the actual performance of the S&P 500 and the U.S. industry Commercial Auto Liability 1999 accident year between January 1999 and December 2004. In addition, we extend the industry analysis to model underwriting and reserve risks at the insurer level. Section 3 comprises a brief summary and our conclusions. Appendix A describes the source and use of the loss development data used in the paper. It also includes exhibits that summarize the calculation of statistical ultimate loss ratio estimates for accident year 1999 at successive annual valuations using unadjusted historical loss development patterns. Appendix B shows the derivation and illustration of a formula for Butsic's "transfer value of unpaid losses," a key element in the calculation of the one-year actual and expected policyholder deficits with respect to underwriting activity. Appendix C describes the stochastic modeling used to estimate the loss distributions underlying the calculation of the policyholder deficit with respect to underwriting and reserve risks. It discusses the sources of variation in future loss ratio estimates, describing in detail how this is manifested in the ultimate loss ratio estimates produced by the loss development methods used in the paper. It also discusses our modeling of the estimated ultimate loss ratio and the policyholder deficit distributions, explaining our application of Monte Carlo simulation and how we reflected parameter uncertainty in the modeling. Appendix D discusses the policyholder deficit and intermediate calculations pertaining to the U.S. industry Commercial Auto Liability 1999 accident year experience at annual valuations between December 1999 and December 2004.

2. THE POLICYHOLDER DEFICIT AND ITS EXPECTED VALUE

The distinctive element of Butsic's risk-based capital framework is its use of the EPD, calculated over a short time horizon, to calibrate risk-based capital consistently for both asset risks and underwriting and reserve risks. In order to determine the EPD for a specified time horizon, we first need to define the policyholder deficit for that time horizon and then to estimate the mean of its distribution.

For common stocks, modeling the distribution is relatively easy, because the idea that stock prices follow a time-dependent stochastic process, and one that can be modeled, is well-established. On the other hand, modeling the one-year EPD for underwriting and reserve risk is more difficult, because it requires a time-dependent perspective of ultimate loss ratio estimates, a perspective that is not required for most actuarial applications. For that reason we begin our discussion with common stocks.

2.1 Actual and Expected Policyholder Deficits - Stocks

If the required capital to asset ratio for common stocks is c, then an initial stock investment of A_0 made from assets matching expected unpaid losses L_0 requires a concurrent risk-based capital allocation of $C_0^R = c \cdot A_0$.

If the allocated capital C_0^R earns interest at the risk-free rate r and the value of the stock investment after one year is A_1, then the value of the capital at the end of the year is equal to the change in value of the stock investment plus the initial capital value with interest[9]:

$$C_1 = A_1 - A_0 + C_0^R(1+r) \tag{2.1}$$

If the value of the available assets at the end of the year, $A_1 + C_0^R(1+r)$, falls below the expected unpaid losses $L_0 = A_0$, then there is, by definition, a funding deficit with respect to the unpaid policyholder claims. Setting $S_1 = A_0 - C_0^R(1+r)$, we can express that policyholder deficit as:

$$PD_1 = S_1 - A_1 \tag{2.2}$$

To determine the *expected* policyholder deficit from the vantage point of investment inception, we need to model the prospective year-end policyholder deficit. We cannot use

[9] This formulation assumes no change in the estimate of the total claims value. It also assumes that any claims paid during the period are settled at the end of the year, allowing the stock investment to be held for the full year, and that any gain (or loss) $A_1 - A_0$ on the stock investment is transferred to (or from) the "capital account" at the end of the year, leaving assets in the "investment account" equal to the initial A_0 required to match the initial expected unpaid losses L_0.

A_1 and PD_1 for this purpose, because until a year after investment inception, they are unknown and uncertain. However, A_1 and PD_1 are prefigured by random variables a_1 and pd_1, defined as of investment inception, which represent the respective values, one year out, of the stock investment and the policyholder deficit. We can express the one-year expected policyholder deficit $E_0(pd_1)$ as of investment inception as the following function of a_1:

$$E_0(pd_1) = \int_0^{S_1} (S_1 - a_1) f(a_1) da_1 , \tag{2.3}$$

which is recognizable as the expected expiry value of a one-year European put option on the stock investment with a strike price of S_1.

$E_0(pd_1)$ is the expected value, as of investment inception (hence the E_0), of the policyholder deficit after one year of investment results. Because it is a measure of the capital exhaustion risk associated solely with prospective investment performance, the value of $E_0(pd_1)$ given by Formula (2.3) can be described as the one-year EPD with respect to common stock asset risk.

The classical stock price model assumes that price changes can be explained by geometric Brownian motion, which implies that future stock prices after any finite time interval are lognormally distributed. Accordingly, we will assume that a_1 is lognormal. This allows us to restate Formula (2.3) as:

$$E_0(pd_1) = E(a_1) \cdot (N(d_1) - 1) - S_1 \cdot (N(d_2) - 1) , \tag{2.4}$$

where $d_1 = \dfrac{\ln(E(a_1) / S_1) + 0.5\sigma^2}{\sigma}$ and $d_2 = d_1 - \sigma$. $N(d_1)$ and $N(d_2)$ are values of the standard normal cumulative distribution function evaluated at d_1 and d_2, respectively[10].

If we assume that the initial investment A_0 is funded by assets corresponding to the unpaid claims liability L_0 (i.e., $A_0 = L_0$), then $E_0(pd_1) > 0$ implies that policyholders can expect to recover less than 100% of the value of their unpaid claims. Butsic advocated that the risk-based capital factor *c* be chosen to target a selected EPD ratio that identifies this shortfall, namely, $\dfrac{E_0(pd_1)}{L_0}$.

[10] For some purposes it may be desirable to know the *present value* of the one-year EPD, which is given by the Black-Scholes formula for the value of a one-year European put option:

$PV(E_0(pd_1)) = A_0 \cdot (N(d_1) - 1) - S_1 e^{-r} \cdot (N(d_2) - 1)$, where $d_1 = \dfrac{\ln(A_0 / S_1) + r + 0.5\sigma^2}{\sigma}$ and $d_2 = d_1 - \sigma$.

Feldblum [8] reported that Butsic, as part of his work for the American Academy of Actuaries Property/Casualty Risk-Based Capital Task Force, had "calibrated the common stock charge using a 1% 'expected policyholder deficit'" and on that basis argued that a 15% risk-based capital charge was more appropriate than 30% (which was also under consideration)[11]. While we do not know what parameter assumptions Butsic used for his calibration, his conclusion seems about right for the time period in which he did his work. The long term standard deviation of U.S. stock market returns is about 20%[12], and the value of the CBOE VIX index, which measures the prospective annualized volatility (σ) of the S&P 500 index implied by the market prices of short term options on that index, hovered around 20% during the early 1990s[13]. If we assume $\sigma = 20\%$, together with a prospective expected annual stock return of 10%, risk-free rate $r = 5\%$ (both simple rates of return), $A_0 = 1$ and $c = 15\%$, then $S_1 = 1 - 0.15 \times 1.05 = 0.8425$ and Formula (2.4) produces an EPD ratio one year out of 0.81%:

$$\frac{E_0(pd_1)}{L_0} = 1.1 \cdot (0.9241 - 1) - 0.8425 \cdot (0.8916 - 1) = 0.81\%$$

Suppose the expected unpaid claim liability after one year is L_1 and, after transferring assets of $A_1 - A_0$ back to the investment account[14], the insurer makes a matching stock investment of A_1[15]. Because the capital $C_0^R = c \cdot A_0$ was intended to minimize the risk of capital exhaustion arising from the stock investment A_0 over the one-year time horizon just ended, the allocated risk-based capital needs to be adjusted to maintain the target EPD ratio with respect to the updated investment value A_1 for the year ahead. In particular, risk-based capital of $C_1^R = c \cdot A_1$ is required to hold the stock investment A_1. After the transfer of $A_1 - A_0$ back to the investment account, the capital account balance is $C_0^R(1+r)$, which means that a calibrating capital adjustment of $C_1^R - C_0^R(1+r)$ is necessary. If $C_1^R - C_0^R(1+r) > 0$, then the capital provider must contribute additional capital. $C_1^R - C_0^R(1+r) < 0$ implies that capital can be released to the capital provider.

If we reset $A_0 = A_1$ and $C_0^R = C_1^R$ at the beginning of each year, we can use Formulas (2.1) through (2.3) to determine C_1, PD_1 and $E_0(pd_1)$ for any one-year period.

---

[11] Best has also reported that its capital factor of 15% for common stocks "is consistent with A.M. Best's goal of calibrating the baseline capital factors to a 1% expected policyholder deficit." See [1], page 6.

[12] Dimson, Marsh and Staunton [7] put it at 20.2% for the period 1900 through 2000 (page 55).

[13] This can be seen in the chart for symbol ^VIX at the Yahoo! Finance website displayed for the maximum time range: (http://finance.yahoo.com/q/bc?s=%5EVIX&t=my).

[14] See footnote 9.

[15] It would be sheer coincidence, of course, for L_1 to match A_1. However, we want to illustrate the capital consequences of a buy-and-hold stock investment policy.

2.1.1 Case Study – S&P 500: 1999-2004

In this section we illustrate the calculation of one-year expected and actual policyholder deficits for each year from 1999 through 2004 with respect to $100 of assets invested on January 1, 1999 in the stocks comprising the S&P 500 index.

We begin by summarizing the performance of the S&P 500 during the period 1999 through 2004 against the backdrop of the estimated probability distributions from which it arose. We used the classical stock price model to estimate successive distributions of the value one year out of an investment in the stocks comprising the S&P 500 for each year from 1999 through 2004. For each of these distributions, we assumed a prospective expected annual return of 10% (9.53% continuously compounded) and annualized volatility (σ) equal to the closing value of the CBOE Volatility Index (VIX) on the last trading day before the beginning of each year[16].

Figure A is a plot of the actual performance during this period of a $100 investment made on January 1, 1999 against the backdrop of 95% confidence intervals from these six successive distributions of stock investment values one year out[17]. The connected square dots reflect the actual S&P 500 total return performance including dividends from the beginning of 1999 through the end of 2004. The triangles highlight the successive one-year confidence intervals. The vertical side of each triangle marks the confidence interval range. Because each confidence interval is a function of the state of knowledge as of the prior year valuation, in order to stress that temporal connection we connected the endpoints of each confidence interval to the investment value one year earlier.

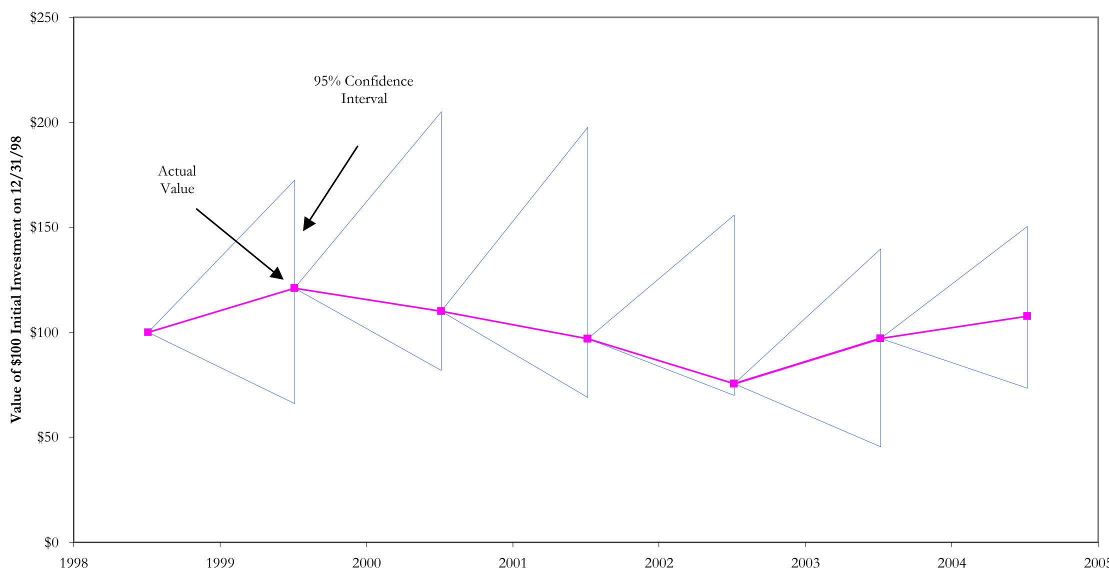
For example, during 1999 an investment in the S&P 500 returned 21.0% and the $100 initial investment grew to $121.00 at the end of December. Of course, on January 1, 1999, when the $100 investment was made, that result was far from certain. At that time the 95% confidence interval for the value of the investment on December 31, 1999 indicated a range of $66.16 to $172.31.

---

[16] These prospective σ estimates were: 24.42% (1999), 23.4% (2000), 26.85% (2001), 20.45% (2002), 28.62% (2003) and 18.31% (2004).

[17] See Table 1 for the actual investment values plotted here. According to the general formula for a 95% lognormal confidence interval, $E(x) \cdot \exp(\pm 1.96\sigma - 0.5\sigma^2)$, the endpoints of the confidence intervals are as follows: $66.16-$172.31 (1999), $81.87-$204.87 (2000), $68.95-$197.53 (2001), $69.91-$155.85 (2002), $45.48-$139.66 (2003) and $73.40-$150.46 (2004).

FIGURE A
Actual S&P 500 Performance: 1999-2004
Within 95% Confidence Intervals for One Year Horizon
Total Return Including Dividends



In 2000 the actual S&P 500 return was a loss of 9.1%, which reduced the value of the investment from $121.00 in January to $109.99 at the end of December. At the beginning of 2000 the 95% confidence interval for that ending value was a range of $81.87 to $204.87. The actual value of the investment at the end of each year during the 1999 through 2004 period fell within each year's 95% confidence interval. However, in 2002, when the total return on the S&P 500 was a loss of 22.1%, the ending investment value fell close to the bottom of the confidence interval.

Table 1 shows the expected and actual one-year policyholder deficit ratios for the period 1999 through 2004. The actual policyholder deficits were calculated using Formula (2.2), assuming capital ratio $c = 15\%$ [18] and risk-free rate $r = 5\%$, which imply a capital exhaustion threshold S_1 equal to 84.25% of the beginning market value each year. The EPDs were calculated from Formula (2.4) using the same assumptions together with an assumed prospective expected annual stock return of 10% and the previously described VIX-based σ estimates.

[18] We assumed the same 15% capital factor for common stocks used by the NAIC, A.M. Best and S&P in their capital models as of December 2006.

During most of this period the one-year EPD ratio, given $c = 15\%$, was significantly greater than 1%, averaging about 1.5% over the period. It reached 2.59% in 2003 when σ peaked at 28.6%. In 2002 the S&P 500 declined enough that an investment in it would have resulted in an actual policyholder deficit equal to 6.35% of the January 2002 investment value[19]. Given the high average EPD ratio during the period, it is not surprising that an actual policyholder deficit would have emerged in at least one year. A one-year EPD ratio of 1.5% corresponds roughly to an annualized 15% chance of a 10% deficit. Over six years the probability of a deficit in one or more years is over sixty percent[20]!

TABLE 1

Expected and Actual Policyholder Deficits 1999-2004
S&P 500 Investment (1)

Calendar Year	σ	Beginning Market Value A_0	Ending Market Value A_1	Capital Exhaustion Threshold S_1	Policyholder Deficit Expected $E_0(pd_1)$	Policyholder Deficit Actual PD_1
1999	24.4%	$100.00	$121.00	$84.25	1.63%	0.00%
2000	23.4%	121.00	109.99	101.94	1.42%	0.00%
2001	26.9%	109.99	96.90	92.67	2.17%	0.00%
2002	20.5%	96.90	75.49	81.64	0.89%	6.35%
2003	28.6%	75.49	97.15	63.60	2.59%	0.00%
2004	18.3%	97.15	107.74	81.85	0.57%	0.00%

(1) 15% capital, 5% risk-free rate, 10% expected stock return

Figure B shows how the risk-based capital allocated using a 15% factor at the beginning of each year was affected by the investment performance during the year. It also shows how, at the end of each year, the risk-based capital was rebalanced to match the prospective 15% requirement. In 2002 the capital was totally depleted and a policyholder deficit

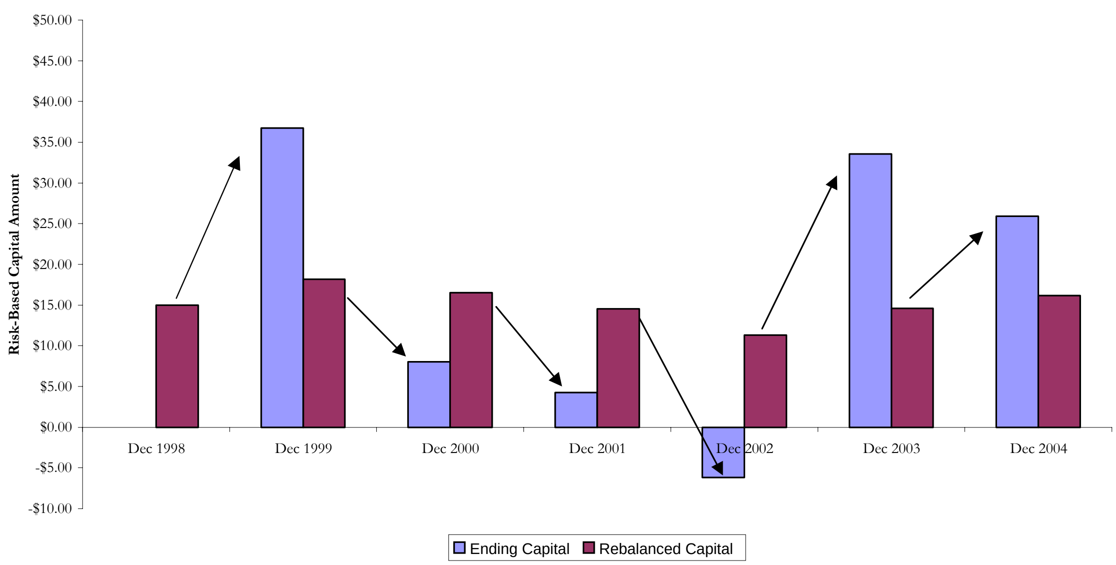
[19] $(\$81.64 - \$75.49)/\$96.90 = 6.35\%$.

[20] The binomial probability of no deficit in six years, given a 15% annual chance of deficit, is about 38%.

emerged. In order to continue trading forward into 2003, the capital provider would have needed both to fund that deficit and recapitalize to the 15% level.

This recap of historical policyholder deficit experience assumed a constant capital ratio of 15%. Given the widely varying EPD ratios indicated by the VIX estimates of σ, if the objective is to maintain a constant one-year EPD ratio, then it is necessary to adjust the capital ratio c to reflect expected S&P 500 volatility in the year ahead. If the c had been recalibrated at the beginning of each year to correspond to a prospective one-year EPD ratio of 1%, then the capital ratios would have been as follows: 19% (1999), 18% (2000), 22% (2001), 14% (2002), 24% (2003) and 11% (2004).

FIGURE B
Risk-Based Capital for S&P 500 Investment: 1999-2004
Investment of $100 on January 1, 1999
Required Capital = 15% of Investment Market Value



2.2 Actual and Expected Policyholder Deficits – Underwriting[21]

For our analysis of underwriting and reserve risks, we will focus initially on a single line of business for a single accident year and later discuss the implications of the more realistic scenario that involves reserves from multiple accident years. We will begin with

[21] To avoid a proliferation of variable names, we will reuse the capital and policyholder deficit related notation from Section 2.1. In particular, we will redefine C_0^R, C_1, C_1^R, $E_0(pd_1)$, PD_1, pd_1 and S_1 to reflect the underwriting related context of this section.

underwriting risk. The total available assets one year after accident year inception comprise the allocated risk-based capital and the underwriting assets derived from premiums[22], plus interest earned on these assets during the year[23]. If we assume that the underwriting assets $T(L_0)$ and allocated capital C_0^R earn interest at the risk-free rate *r*, then the value of the assets S_1 available at the end of the year to fund the claim obligation that was assumed at accident year inception is:

$$S_1 = (C_0^R + T(L_0)) \cdot (1 + \tfrac{3}{4} r)$$

In nominal terms, the end of year value of the estimated claim obligation L_0 that was assumed at inception is the sum of the estimated unpaid losses L_1 at year-end and the claims paid during the year P_1. The *transfer value* of that liability $T(L_1 + P_1)$ is the price at which $L_1 + P_1$ can be removed from the insurer's balance sheet one year after accident year inception.

In order to quantify $T(L_1 + P_1)$, let us look at the transfer values of the paid and unpaid loss components separately. $T(L_1)$ is the price a third party would charge to assume the liability for unpaid losses L_1 at one year of development, which Butsic defined as the present value of L_1 plus a risk charge to reflect the uncertainty in the unpaid losses[24]. $T(P_1)$ is the price claimants would demand to defer payment of their claims until year-end. Assuming that the claims comprising P_1 are paid, on average, halfway through the year, then their year-end transfer value is $T(P_1) = P_1 \cdot (1 + \tfrac{1}{2} r)$. The total year-end transfer value of $L_1 + P_1$ is $T(L_1 + P_1) = T(L_1) + T(P_1)$.

The capital position one year after accident year inception is equal to the value of the underwriting and capital assets less the transfer value of the loss liability[25]:

---

[22] These are premiums net of expenses only. Claims paid during the year are treated as part of the loss liability.

[23] We assume that half of the accident year earned premiums is written and collected before the beginning of the accident year and thus earns interest for the full year. We assume the other half of the earned premiums is collected, on average, halfway through the year and earns (simple) interest for six months. Capital is assumed to be allocated as premiums are collected and to earn interest accordingly.

[24] Conceptually, this is identical to the theoretical market price of a stock, which also reflects the present value of the future realizable cash flows of the company and an appropriate risk premium. In an efficient market, the theoretical and actual market prices should be the same. In his discussion of the value of insurance claims, Butsic used the terms "market value" and "transfer value" interchangeably. Because there is not an active market after policy inception for the buying and selling of loss reserves, we prefer the term "transfer value," which has a more theoretical connotation. We derive a formula for $T(L_n)$ for $n \geq 1$ in Appendix B, which reflects the recapture of the cost of the allocated risk-based capital. That approach has also been discussed in connection with the EU's Solvency II initiative. See the UK FSA's Solvency II discussion paper [10], page 25.

[25] This formulation assumes that any shortfall (or surplus) in the assets available to fund losses is transferred from (or to) the "capital account," leaving the correct amount in the "underwriting account" to fund losses exactly.

$$C_1 = S_1 - T(L_1 + P_1), \tag{2.5}$$

If $T(L_1 + P_1) > S_1$, then the ending capital $C_1 < 0$, which implies a policyholder deficit of:

$$PD_1 = T(L_1 + P_1) - S_1 \tag{2.6}$$

While at age twelve months $T(L_1 + P_1)$ and PD_1 take on specific values, at accident year inception their values are unknown and uncertain. Let t_1 and pd_1 be random variables, defined at accident year inception, that correspond to $T(L_1 + P_1)$ and PD_1, respectively. t_1 represents the transfer value, one year out, of the unpaid losses embedded in the premiums at accident year inception and pd_1 represents the policyholder deficit, one year out, viewed from the vantage point of accident year inception.

We can express the one-year expected policyholder deficit as of accident year inception as the following function of t_1:

$$E_0(pd_1) = \int_{S_1}^{\infty} (t_1 - S_1) f(t_1) dt_1, \tag{2.7}$$

which is recognizable as the expected expiry value of a one-year European call option, with a strike price of S_1, on the transfer value, one year out, of the estimated unpaid losses at inception[26].

$E_0(pd_1)$ is the expected value as of inception of the policyholder deficit after one year of development. It is calculated at accident year inception before any actual claims have been reported. Because it is a measure of the capital exhaustion risk associated solely with prospective underwriting activity, the value of $E_0(pd_1)$ given by Formula (2.7) can be described as the one-year EPD with respect to underwriting risk.

Butsic advocated calibration of risk-based capital ratios to produce consistent EPD ratios for all asset, underwriting and reserve risks. Suppose the required underwriting risk-based capital C_0^R at accident year inception is defined as a certain percentage c_0 of the premiums net of expenses $T(L_0)$, which implies $C_0^R = c_0 \cdot T(L_0)$. If the target one-year EPD ratio for common stocks is set at 1% of expected unpaid losses, then the underwriting risk-based capital factor c_0 should likewise be chosen to produce a value of $E_0(pd_1)$ equal to 1% of L_0. However, because L_0 is not observable, a practical alternative is to calibrate the EPD to 1% of the loss provision implied by premiums net of expenses $T(L_0)$.

[26] Note that while the classical theory of stock prices implies that a_1 (the stock price random variable) is lognormal, t_1 is lognormal only under very narrow circumstances, which means we cannot simply use the stock price model to value the underwriting risk EPD. We also cannot use the Black-Scholes call formula to determine the present value of the one-year EPD.

At twelve months after accident year inception, our interest turns from the adequacy of the loss provision in the premiums to the adequacy of the loss reserves. The loss reserve L_1 must be funded by assets equal to $T(L_1)$. In addition, capital of $C_1^R = c_1 \cdot L_1$ is required to support the loss reserves for the next twelve months, bringing the total required accident year underwriting and capital assets to $C_1^R + T(L_1)$. Because the ending capital C_1 will rarely match the prospective required capital C_1^R, a calibrating capital adjustment of $C_1^R - C_1$ is necessary. $C_1^R - C_1 > 0$ implies that the capital provider must contribute additional capital in that amount. $C_1^R - C_1 < 0$ implies that capital can be released to the capital provider in that amount.

From one year of accident year development and beyond, the successive one-year EPDs measure the capital exhaustion risk associated only with the prospective uncertainty in the loss reserve estimates. We *could* also refer to the risk arising from loss reserve uncertainty as underwriting risk, since it arises only as a result of past underwriting activity. However, because the risk-based capital convention is to refer to the risk arising from loss reserves as reserve risk, we follow that convention of separating the total risk in the accident year into its underwriting and reserve components.

2.3 Actual and Expected Policyholder Deficits – Loss Reserves

Following the capital rebalancing at the end of the first year of development, the combined risk-based capital and underwriting assets total $C_1^R + T(L_1)$. By the end of the second year the value of available assets S_2 (including interest earned for the full year) is:

$$S_2 = (C_1^R + T(L_1)) \cdot (1 + r)$$

During the second year of development, the loss reserve L_1 is reduced by paid claims P_2. At the end of the year $L_1 - P_2$ is replaced by a revised loss reserve L_2, which is based on the loss development observed during the year. $L_2 + P_2$ is the one-year hindsight estimate of L_1, with a transfer value of $T(L_2 + P_2) = T(L_2) + P_2 \cdot (1 + \frac{1}{2} r)$. The economic value of the allocated capital at the end of the second year of development is given by:

$$C_2 = S_2 - T(L_2 + P_2) \tag{2.8}$$

$T(L_2 + P_2) > S_2$ implies a policyholder deficit of:

$$PD_2 = T(L_2 + P_2) - S_2 \tag{2.9}$$

Let t_2 and pd_2 represent the random variables, defined at age one year, corresponding to $T(L_2 + P_2)$ and PD_2, respectively. We can then express the one-year EPD at age one year as the following function of t_2:

$$E_1(pd_2) = \int_{S_2}^{\infty} (t_2 - S_2) f(t_2) dt_2, \tag{2.10}$$

which is the same as Formula (2.7) except that the subscripts are different. The one-year EPD with respect to loss reserve risk at twelve months is expressible as the expected expiry value of a one-year European call option on the transfer value, one year out, of the unpaid losses as of twelve months.

The risk-based capital required to support the unpaid loss liability L_2 (which itself is funded by assets of $T(L_2)$) in the period from two to three years of development is $C_2^R = c_2 \cdot L_2$. Because the ending capital at two years is C_2 a calibrating capital adjustment of $C_2^R - C_2$ is required, resulting in an additional capital contribution or a capital release.

The same process is repeated at each successive valuation at one-year intervals until all accident year claims have been paid. The key loss-related formulas applicable to the valuation *n* years after accident year inception (for $n \geq 1$) are as follows:

$$C_n^R = c_n \cdot L_n \tag{2.11}$$

$$S_{n+1} = (C_n^R + T(L_n)) \cdot (1 + r) \tag{2.12}$$

$$C_{n+1} = S_{n+1} - T(L_{n+1} + P_{n+1}) \tag{2.13}$$

$$PD_{n+1} = T(L_{n+1} + P_{n+1}) - S_{n+1}, \text{ if } T(L_{n+1} + P_{n+1}) > S_{n+1} \tag{2.14}$$

$$E_n(pd_{n+1}) = \int_{S_{n+1}}^{\infty} (t_{n+1} - S_{n+1}) f(t_{n+1}) dt_{n+1} \tag{2.15}$$

In practice, an insurer's unpaid losses almost never pertain to a single accident year. This is important, because the objective of minimizing exposure to capital exhaustion due to reserve risk does not require minimizing that exposure with respect to each accident year individually, but rather for all accident years collectively. As we will see in our case study discussed in Section 2.4.1, this makes a big difference in the amount of required capital. The key formulas for working with unpaid losses arising from multiple accident years are given in Section C.7 of Appendix C.

2.4.1 Case Study – Commercial Auto Liability Accident Year 1999: 1999-2004

In this section we illustrate the calculation of one-year expected and actual policyholder deficits at the industry level for each year from 1999 through 2004 with respect to accident year 1999 claims arising from $100 of Commercial Auto Liability earned premiums. We also discuss the impact of loss reserves from multiple accident years as well as our modeling of the policyholder deficit calculations at the insurer level.

We begin by summarizing the performance of statistical ultimate loss ratio estimates during the period 1999 through 2004 against the backdrop of the estimated probability distributions from which they arose.

The statistical ultimate loss ratio estimates – so called because they reflected no actuarial or other clinical judgment – are simple averages of the unadjusted estimates produced by four traditional loss development methods: 1) paid chain ladder, 2) paid Bornhuetter-Ferguson, 3) case incurred chain ladder, and 4) case incurred Bornhuetter-Ferguson. We chose the four-method average because Wacek [13], in a comparison of the accuracy of various loss projection methods for accident years 1995 through 2001, found the four-method average to be the most accurate estimator of the ultimate loss ratio over that period for Commercial Auto Liability at twelve, twenty-four and thirty-six months of development.

TABLE 2

Accident Year 1999 Ultimate Loss Ratio Estimates
Commercial Auto Liability
As of December 31 each Year

Calendar Year	CL Paid	CL Case Incurred	B-F Paid	B-F Case Incurred	Mean of Methods
1999	90.3%	84.7%	83.7%	82.5%	85.3%
2000	91.7%	89.1%	90.8%	88.4%	90.0%
2001	92.7%	92.1%	92.4%	91.9%	92.3%
2002	93.5%	92.9%	93.4%	92.9%	93.2%
2003	93.1%	92.2%	93.1%	92.2%	92.6%
2004	91.6%	91.8%	91.7%	91.8%	91.7%

Table 2 shows the estimates of the ultimate loss ratio from the four methods and their mean at the end of each year from 1999 through 2004[27]. The mean of methods estimate as of the end of 1999 was 85.3%. Unanticipated loss development, i.e., development beyond that implied by the historical development patterns, during the next three years led to

[27] See Appendix Exhibit A-4 for the details of the calculation of each of these ultimate loss ratio estimates. For a full description of the four methods, see Appendix A, Section A.2.2, of [13].

increases in the ultimate loss ratio estimate in each of 2000, 2001 and 2002 to 90.0%, 92.3% and 93.2%, respectively[28]. Then the pattern of unfavorable development deviation reversed itself, and in 2003 and 2004 unexpectedly favorable development led to slight reductions in the ultimate loss ratio estimate to 92.6% and 91.7% at the end of 2003 and 2004, respectively.

Each of these actual ultimate loss ratio estimates can be viewed as one outcome from the distribution of potential loss ratio estimates arising from a stochastic loss development process. For example, from the vantage point of accident year inception, the ultimate loss ratio of 85.3% estimated as of the end of 1999 was one of many potential estimates. If the observed loss development during 1999 had been different, then the ultimate loss ratio estimate would also have been different. We estimated the distribution of these different ultimate loss ratio estimates using a Monte Carlo simulation process that stochastically modeled the loss development observed during 1999, combined it with what was already known at accident year inception, and then applied the four loss development methods described above to the simulated experience. We used the set of ultimate loss ratio estimates produced from 10,000 Monte Carlo trials as a discrete approximation of the distribution of the ultimate loss ratio estimate one year out (at the end of 1999) from the vantage point of accident year inception. We followed the same procedure to model the distribution of the ultimate loss ratio one year out at each successive annual valuation from accident year inception (which we have just described) through December 2003[29].

Figure C is a plot of the path of the 1999 accident year ultimate loss ratio estimate (the four-method average) for Commercial Auto Liability against the backdrop of 95% confidence intervals from the distributions of the estimated ultimate loss ratio one year out[30]. The connected square dots reflect the actual statistical ultimate loss ratio estimates for successive annual valuations ranging from the beginning of 1999 through the end of 2004. As in Figure A, the triangles highlight the successive one year confidence intervals, where the vertical side of each triangle marks the confidence interval range.

[28] Because the estimated ultimate loss ratios were purely statistical estimates calculated from the unadjusted indications of the four loss development methods, the main source of subsequent upward or downward revisions in the estimates was the deviation of observed loss development from that predicted by historical patterns. An additional source of minor deviations was the behavior of the five-year moving averages of historical development used to estimate prospective development. See footnote 32.

[29] See Appendix C for a detailed description of how these distributions were estimated.

[30] The endpoints of the confidence intervals are as follows: 76.1%-85.1% (1999), 83.6%-91.6% (2000), 87.9%-93.0% (2001), 90.4%-94.3% (2002), 91.5%-95.0% (2003) and 92.0%-93.4% (2004).

FIGURE C
Path of Accident Year 1999 Ultimate Loss Ratio Estimate: 1999-2004
Within 95% Confidence Intervals for One Year Horizon
Commercial Auto Liability

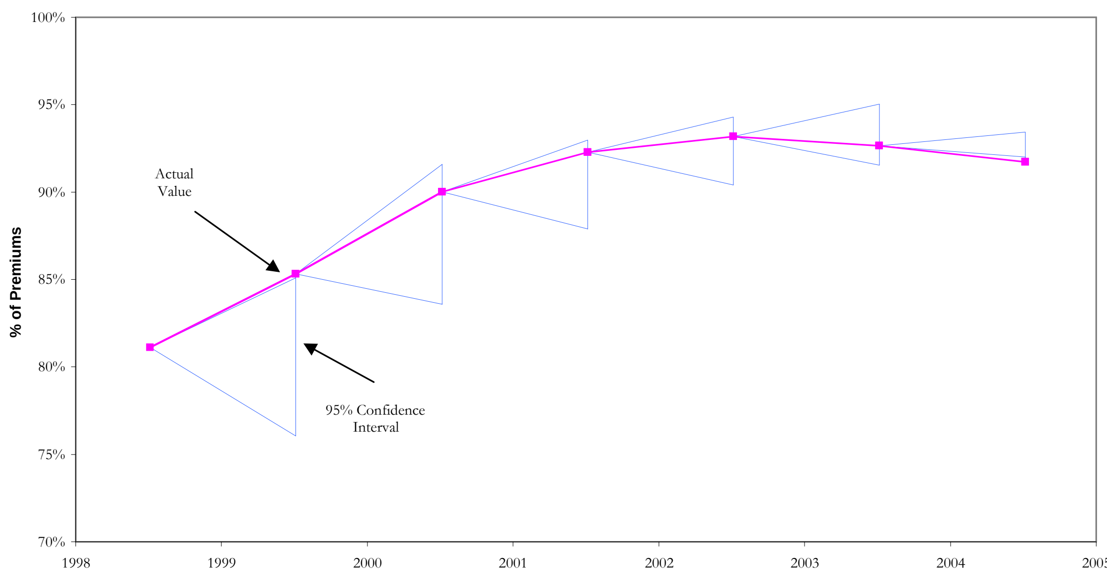


Figure C shows that at the beginning of the accident year on January 1, 1999, the initial ultimate loss ratio estimate was 81.1%[31]. At that time the 95% confidence interval for the ultimate loss ratio estimate one year out (i.e., at the December 31, 1999 valuation) was bounded by 76.1% on the low end and 85.1% on the upper end[32]. Based on the actual loss emergence during calendar year 1999 the four-method average ultimate loss ratio estimate as of December 31, 1999 was 85.3%. Unusual paid and case incurred loss development during calendar year 1999 led to an upward revision in the ultimate loss ratio estimate to just above the upper end of the 95% confidence interval.

At the December 31, 1999 valuation, the 95% confidence interval for the ultimate loss ratio estimate one year out (i.e., as of December 31, 2000) was a range of 83.6% to 91.6%. One year later, at the December 31, 2000 valuation, the ultimate loss ratio estimate was

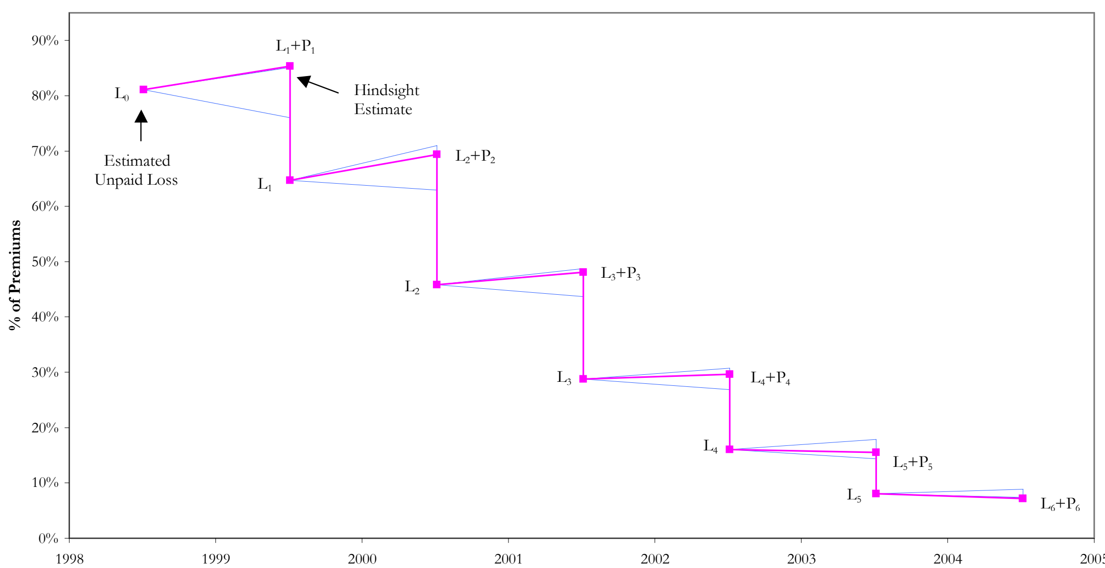
[31] This the mean of the paid and case incurred Bornhuetter-Ferguson initial expected loss ratios, 83.4% and 78.7%, respectively, which were based purely on 1998 and prior accident year experience. See [13] for more information about how that was done. This purely statistical estimate ignored other objective information, which, if available, might have improved this estimate.

[32] The mean of the ultimate loss ratio estimate one year out was 80.3%, which is different from the 81.1% estimate as of January 1, 1999 because it reflects a slight difference in the five data points comprising the development factor means. The estimate one year out drops the calendar year 1994 development observation from each development factor calculation and replaces it with the mean of the 1994-1998 observations.

revised from 85.3% to 90.0%, based on loss development observed during calendar year 2000.

The ultimate loss ratio estimates continued to increase at the December 31, 2001 and 2002 valuations, before declining slightly at the December 31, 2003 and 2004 valuations[33]. Note that the width of the confidence intervals became smaller in successive years, which reflects the declining proportion of unpaid claims (the only source of uncertainty) within the loss ratio.

FIGURE D
Accident Year 1999 Actual & Hindsight Unpaid Loss Ratio Estimates: 1999-2004
Within 95% Confidence Intervals for One Year Horizon
Commercial Auto Liability



Because the threat to solvency arises from the potential for adverse deviation inherent in the unpaid portion of the ultimate loss ratio estimate, let's look at the behavior of the accident year 1999 unpaid loss ratio over the same period. Figure D is a plot showing the succession of unpaid loss estimates (L_n) and their hindsight re-estimates one year later $(L_{n+1}+P_{n+1})$ against the backdrop of 95% confidence intervals for the latter. The connected square dots represent the actual and one-year hindsight estimates of the unpaid loss ratio at

[33] The development of both paid and case incurred losses during 2004 was extremely light compared to the historical pattern. Referring to the columns labeled "Age 5-6" in Appendix Exhibits A-1A and A-1B, we see that the paid age-to-age factor of 1.032 was the lowest by far of eleven factors and the case incurred age-to-age factor of 1.007 was tied for lowest of eleven.

successive annual valuation between accident year inception and 2004. The triangles show the 95% confidence intervals for the hindsight estimates one year out, and as such provide a visual representation of the potential exposure of the unpaid loss estimate to upward or downward revision in the next year.

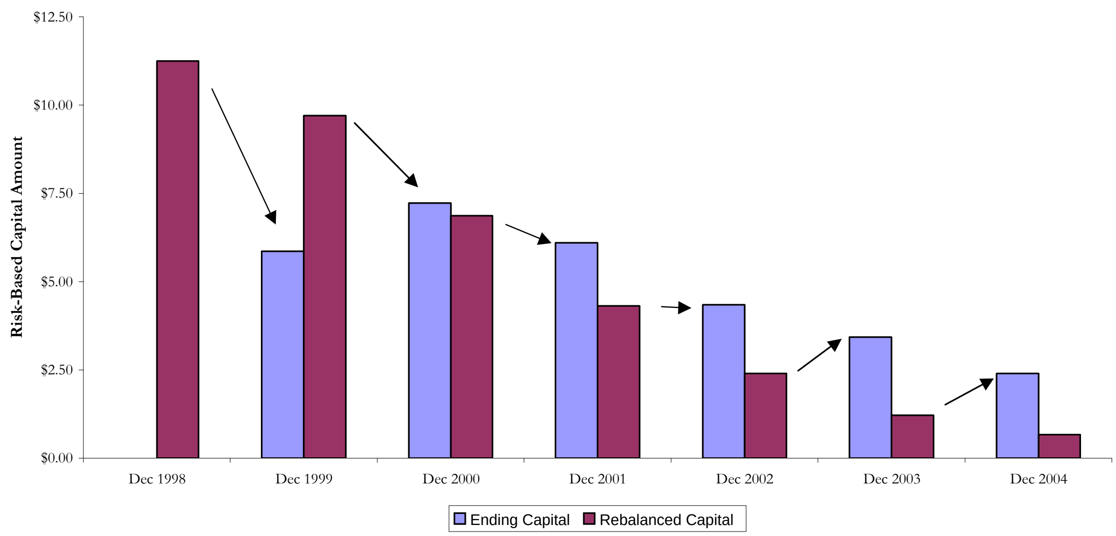
The initial unpaid loss ratio estimate at accident year inception was $L_0 = 81.1\%$. One year later the hindsight estimate of L_0 as of December 1999 was $L_1 + P_1 = 85.3\%$, representing the sum of the paid loss ratio P_1 of 20.6% and the unpaid loss ratio L_1 of 64.7%. The upward sloping line connecting L_0 and $L_1 + P_1$ indicates adverse loss development, which is quantified by the difference $L_1 + P_1 - L_0 = 4.2\%$ of premiums (5% of L_0).

The hindsight estimate of L_1 as of December 2000 was $L_2 + P_2 = 69.4\%$, the sum of the period paid loss ratio $P_2 = 23.6\%$ and the unpaid loss ratio $L_2 = 45.8\%$. The upward sloping line between L_1 and $L_2 + P_2$ indicates further adverse loss development of 4.7% of premiums (7% of L_1).

The next two years, 2001 and 2002 saw a continuation of the pattern of the hindsight estimates of unpaid losses exceeding the beginning of year estimates. The adverse development was 2.3% of premiums (5% of L_2) in 2001 and 0.8% of premiums (3% of L_3) in 2002. In 2003 and 2004 the unexpected loss development turned favorable.

The cumulative adverse loss development from accident year inception through December 2002 totaled 12% of premiums. That seems to support the argument for a large capital requirement for Commercial Auto Liability. However, the fact that this shortfall emerged over four years rather than a single year is extremely important. Because the required amount of risk-based capital is determined annually, any erosion of allocated capital caused by adverse loss development is replenished at the end of the year. If capital exhaustion can be avoided for each of the four years in succession, then clearly capital exhaustion is also avoided for the four years as a block. In that context the adverse loss development seen in Commercial Auto Liability between 1999 and 2002, which averaged about 5% of the unpaid loss estimate at the beginning of each year, was much more manageable than the volatility seen in the S&P 500, which lost 22% of its value in a single year (2002) and 38% over three years (2000-2002).

FIGURE E
Risk-Based Capital for Accident Year 1999 Unpaid Losses: 1999-2004
Commercial Auto Liability
1999 Premiums of $100
Required Capital = 15% of Unpaid Losses



This can be seen in Figure E, which shows the effect on capital of the accident year 1999 loss development observed between accident year inception and December 2004, assuming an expense ratio of 25%, risk free rate of 5% and a required capital ratio of 15%[34]. It shows, for example, that because the year-end 1999 ultimate loss ratio estimate of 85.3% exceeded the funding capacity of the premiums, a portion of the initial capital of $11.25 (15% of $75) had to be diverted to fund losses and capital ended the year at $5.65. That implied a loss to insurers, but capital was far from exhausted. At the end of 1999 capital had to be topped up to $9.70 (15% of unpaid losses of $64.69). During 2000 further adverse development resulted in a reduction in capital to $7.23. However, the required capital going forward (15% of unpaid losses of $45.80) was only $6.87, which meant that $0.36 of capital could be withdrawn. Subsequent increases in the ultimate loss ratio estimate in 2000 and 2001

[34] We chose the same 15% capital factor used for common stocks in order to facilitate the comparison of the relative riskiness of Commercial Auto Liability insurance and an investment in the S&P 500. As of December 2006, the NAIC and S&P both used capital factors for underwriting risk that equate to about 22% of premiums net of 25% expenses, and capital factors for reserve risk that equate to 16% and 10%, respectively, of undiscounted loss reserves. Note that, unlike the NAIC and Best, S&P does not use covariance or diversification adjustments, so its effective factors on a comparable basis are at least 50% higher than those given here. Best has not published its underwriting and reserve factors, but we have observed Best capital factors for Commercial Auto Liability greater than 15%.

resulted in further capital drawdowns. However, in none of the years was capital close to being exhausted[35].

Table 3 shows the expected and actual one-year policyholder deficit ratios for the period 1999 through 2004 in tabular form. The actual policyholder deficits were calculated using Formula (2.6) for the December 1999 valuation and Formula (2.9) for the 2000 through 2004 valuations. For further details of these calculations, see Appendix Exhibit D.

TABLE 3

Expected and Actual Policyholder Deficits 1999-2004

Industry Commercial Auto Liability - Accident Year 1999 (1)

Calendar Year	n (2)	Beginning Unpaid Loss Provision L_n (3)	Hindsight Unpaid Loss $L_{n+1}+P_{n+1}$	Transfer Value Hindsight Unpaid Loss $T(L_{n+1}+P_{n+1})$	Capital Exhaustion Threshold S_{n+1}	Policyholder Deficit Expected	Policyholder Deficit Actual
1999	0	\$75.00	\$85.32	\$83.84	\$89.48	0.01%	0.00%
2000	1	\$64.69	\$69.39	\$68.78	\$76.01	0.01%	0.00%
2001	2	\$45.80	\$48.07	\$47.89	\$54.05	0.00%	0.00%
2002	3	\$28.76	\$29.65	\$29.62	\$34.03	0.00%	0.00%
2003	4	\$16.01	\$15.48	\$15.48	\$18.95	0.04%	0.00%
2004	5	\$8.07	\$7.15	\$7.10	\$9.54	0.02%	0.00%

(1) 15% capital, 5% risk-free rate
(2) Lag from inception (in years) as of beginning of year
(3) $T(L_0)$ for 1999

[35] See Appendix Exhibit D for the details underlying the calculation of capital and policyholder deficits with respect to the underwriting and reserve risks associated with the Commercial Auto Liability accident year 1999 between 1999 and 2004.

The expected policyholder deficit $E_n(pd_{n+1})$ was calculated using Formula (2.7) for the 1999 valuation ($n=0$) and Formula (2.10) for the 2000 and subsequent valuations ($1 \le n \le 5$)[36]. A capital requirement of $15\% \cdot T(L_n)$ and risk-free rate $r = 5\%$ implied capital exhaustion thresholds $S_1 = 1.15 \cdot T(L_0) \cdot (1 + \frac{3}{4} \cdot 0.05)$ for $n=0$ and $S_{n+1} = 1.15 \cdot L_n \cdot 1.05$ for $1 \le n \le 5$. The policyholder deficit was determined by comparing the transfer value $T(L_{n+1} + P_{n+1})$ of the hindsight estimate $L_{n+1} + P_{n+1}$ to the capital exhaustion threshold S_{n+1}. We have expressed the policyholder deficits in Table 3 as ratios to L_n[37]. Throughout the 1999-2004 period the one-year EPD was barely greater than zero. The largest EPD value was 0.04% (4 basis points) in 2003 and its average was less than 1.5 basis points, just 1% of the EPD calculated for the investment in the S&P 500! Moreover, despite the persistent pattern of upward adjustment in the hindsight reserve estimates, the actual policyholder deficit remained zero throughout the 1999-2004 period.

These expected and actual policyholder deficit calculations assumed that accident year 1999 was the sole source of Commercial Auto Liability loss reserves. If, instead, we assume that there were also loss reserves from a number of other accident years, then the one-year EPD with respect to total reserve risk approaches zero. To illustrate this simply, let us pretend that the loss development statistics tabulated in Table 3 with respect to accident year 1999 over several calendar years instead pertained to loss development observed during calendar year 2000 with respect to accident years 1995 through 1999 as shown in Table 4. To create Table 4 we mapped the accident year 1999 loss reserves at each development age (shown in Table 3) to the accident year that would be the same age in calendar year 2000[38]. In effect, we assumed that the accident loss exposure was constant from 1995 through 1999 and the development patterns observed in that period were similar to those we saw for accident year 1999 as it developed.

At the beginning of 2000, the total unpaid loss provision with respect to these hypothetical accident years 1995-1999 was $163.33. At the end of 2000 the hindsight loss estimate for this block of reserves increased to $169.74 and the transfer value of that hindsight estimate was $168.27. The capital exhaustion threshold, which reflects the beginning of year total underwriting and risk-based capital assets plus interest, was $191.78.

[36] Strictly speaking we calculated the EPDs from discrete approximations of the underlying distributions achieved through Monte Carlo simulation, rather than by integrating the actual continuous density functions as implied by the references to Formulas (2.7) and (2.10). In particular, we approximated the application of Formulas (2.7) and (2.10) by using Formulas (2.6) and (2.9) for each Monte Carlo trial and then computing the mean policyholder deficit over all trials.

[37] $T(L_0)$ for $n=0$.

[38] This is a purely illustrative assumption for the purpose of showing the effect that holding multiple accident years' reserves has on the policyholder deficit calculations.

That implied an actual policyholder deficit of zero. We also found the one-year EPD to be negligible. We assumed the unpaid accident year losses were independent, but if they were anything less than totally correlated, the expected and actual policyholder deficits for the five accident years' unpaid losses would always be lower than for the accident years individually.

TABLE 4

Expected and Actual Policyholder Deficits

Illustration of Multiple Accident Years' Reserves Effect in 2000

Industry Commercial Auto Liability – Hypothetical Accident Years 1995-1999

Accident Year	Beginning Unpaid Loss Provision	Hindsight Unpaid Loss	Transfer Value Hindsight Unpaid Loss	Capital Exhaustion Threshold	Policyholder Deficit	
					Expected	Actual
1995	$8.07	$7.15	$7.10	$9.54	0.02%	0.00%
1996	$16.01	$15.48	$15.48	$18.95	0.04%	0.00%
1997	$28.76	$29.65	$29.62	$34.03	0.00%	0.00%
1998	$45.80	$48.07	$47.89	$54.05	0.00%	0.00%
1999	$64.69	$69.39	$68.78	$76.01	0.01%	0.00%
1995-99	$163.33	$169.74	$168.87	$191.78	0.00%+	0.00%

15% capital, 5% risk-free rate

Clearly, compared to the risk in the diversified common stock portfolio exemplified by the S&P 500, at the industry level the exposure to capital exhaustion posed by the accident year 1999 Commercial Auto Liability underwriting and reserve risks, given the same 15% capital ratio used with the S&P 500, was negligible. However, because solvency concerns are focused on the exposure that individual insurers have to capital exhaustion, and not on the exposure of the industry as a whole, we need to address the question of insolvency risk at the insurer level.

While we did not have access to the individual insurer data comprising the industry experience and thus could not model capital exhaustion exposure at the insurer level directly, we were able to model it indirectly by making use of the relationship between insurer "total risk" and "industry risk" reported by the American Academy of Actuaries Property/Casualty Risk-Based Capital Task Force [2]. The Task Force reported "total risk" coefficients of variation for ultimate time horizon underwriting and reserve risks of 23.3% and 17.2%, respectively, and corresponding "industry risk" coefficients of 12.2% and 6.4%, equating to ratios of 2.7 and 1.9[39]. To approximate the risk faced by individual insurers, we multiplied the standard deviations of the age-to-age development factor natural logarithms tabulated in Appendix Exhibits A-2A and A-2B by factor of 3 (rounding up from 2.7 and 1.9). We then repeated the same one-year EPD analysis that we had performed using the industry data[40].

This procedure produced a one-year EPD with respect to underwriting risk of 0.50% and a one-year EPD with respect to the reserve risk arising from the five accident years 1995-1999 of 0.03%, both of which are much lower than the average one-year EPD of 1.5% for the S&P 500 using the same risk-based capital factors.

Calibrated to a 1% target one-year EPD, the indicated Commercial Auto Liability capital factors for individual insurers would have been 5% for underwriting risk and 4% for reserve risk. These indicated factors are much lower than those promulgated by the NAIC and the rating agencies[41].

3. SUMMARY AND CONCLUSIONS

In this paper we have provided a detailed roadmap for the application of Butsic's framework for insurance company solvency protection and illustrated it with a case study using historical data. The results of the case study support Butsic's contention that insurance company solvency can be ensured by the periodic assessment and rebalancing of

---

[39] See [2], Exhibit 3, Sheets 1 and 2, pages 155-156.

[40] A factor of 3 increased the coefficient of variation of a lognormal random variable by slightly more than 3. See Appendix C for details of the loss development model used for the industry and company analyses.

[41] See footnote 34 for a recap of those factors. Another simple way to measure the relative variability of an investment in the S&P 500 and Commercial Auto Liability insurance is to compare the coefficients of variation of the random variables a_{n+1} and t_{n+1}. The c.v. of a_{n+1} ranged from 18% to 29%. In contrast the coefficients of variation for t_{n+1} (in particular, for t_1, which corresponds to underwriting risk, and $\sum t_i$, which corresponds to total reserve risk) were much lower at 9% for underwriting and 3% for total reserves. S&P states that its 15% capital factor for common stocks is equal to the standard deviation of S&P 500 annual returns since 1945 [11] (page 35). Ignoring the fact that our research indicated a higher standard deviation for the S&P 500, if S&P had been consistent in its approach, it would have set its capital factors at 9% for underwriting risk and 3% for reserve risk instead of at 22% and 10%.

capital to maintain a constant target EPD ratio over a short time horizon. A striking finding from the case study is that the amount of capital needed to support the problematical Commercial Auto Liability line in the worst accident year of the "soft market" of the late 1990s was significantly less than that required by the NAIC, Best and S&P. That Butsic's solvency framework with a capital ratio of 15% or less would work so well in the face of the severe deterioration in the accident year 1999 ultimate loss ratio estimate to 91.7% at the end of 2004 is a testament to its robustness. A second striking finding is that the riskiness of investment in a diversified common stock portfolio appears to be underappreciated by the NAIC and the rating agencies, both in absolute terms and relative to Commercial Auto Liability insurance. The risk-based capital they required to support such an investment was consistent with a one-year EPD ratio averaging 1.5% over the 1999 through 2004 period (fifty times the Commercial Auto Liability EPD ratio at the same capital level!) and was insufficient to prevent actual exhaustion of that allocated capital during 2002.

It is important that any solvency framework measure the risk of capital exhaustion *consistently* across lines of insurance and both sides of the balance sheet. While our case study is too limited in scope to permit sweeping conclusions, the results with respect to Commercial Auto Liability are startling enough to suggest that the capital requirements of other insurance lines should be studied as well. We have a strong suspicion that the capital requirements of the NAIC, Best and S&P with respect to other lines of insurance are also overstated, both in absolute terms and relative to their requirements for common stock investment[42].

We believe the bias in favor of common stock investment embedded in the current (December 2006) capital factors is unintentional and has resulted from the unconscious use of inconsistent methods of measuring risk. It appears that the risk associated with common stock investment has been measured using a time horizon of about one year, while underwriting and reserve risks have been measured over a much longer time horizon. Butsic argued the importance of using a consistent time horizon in the early 1990s at the time when the NAIC began implementing its risk-based capital framework. However, either because there was no practical way to incorporate his insights or because they were not properly understood, his ideas have languished, and for far too long[43].

[42] We have in mind the largest U.S. primary lines of business, which lend themselves well to Schedule P analysis. Unfortunately, because of data quality and heterogeneity issues, Schedule P does not shed much light on this question for International, Special Liability and the Nonproportional Reinsurance lines.

[43] Butsic was a member of the American Academy of Actuaries Property/Casualty Risk-Based Capital Task Force, but that did not prevent the Task Force from employing an EPD methodology for underwriting and

Butsic's concept of a short time horizon for risk measurement that is used consistently for all types of risk also has obvious application to enterprise risk management and other updated approaches to solvency risk management. While we have focused on the measurement and calibration of asset and underwriting-related risks separately, clearly the ultimate objective of solvency management is to minimize the likelihood and cost of insolvency from all of the risks, alone or in combination, inherent in an insurance enterprise. The EPD measured over a consistent short time horizon is a good measure of that enterprise-wide risk. We advocate calibrating the enterprise-wide capital requirement to a target EPD that measures all risks simultaneously over the same time horizon. We have illustrated the EPD measure using a time horizon of one year, but it is easy to see the potential merits of shorter time horizons, such as quarterly or even monthly. While there are obvious practical obstacles to implementing such a framework in the near term, conceptually we can imagine a solvency framework in which capital is recalibrated on a daily basis!

Meanwhile, it is important that the issue of the capital required by the existing risk-based capital models to support property-casualty insurance operations be taken up again and with some urgency. This is important, because in recent years rating agencies have shown an inclination to increase underwriting-related capital requirements by increasing capital factors directly and/or indirectly by increasing the capital adequacy ratios that correspond to their various ratings. While no responsible insurance professional can be opposed to a strong solvency regime, requiring more capital than is actually required to meet stated solvency objectives increases the cost of insurance and unnecessarily impedes the ability of insurers to compete with alternative methods of managing risk. Our aim in preparing this paper has been to stimulate thoughtful discussion of this important issue, which we hope will ultimately lead to actions by regulators and rating agencies to adapt their risk-based capital models to reflect more accurately the real risks embedded in insurance company underwriting and loss reserving activities.

---

reserve risks that assumed a time horizon that encompassed ultimate claim settlement [2]. Best and S&P both state that their risk-based capital models use that same ultimate time horizon EPD methodology [1] [11].

4. APPENDICES

APPENDIX A

Historical Loss Development and Accident Year 1999 Estimates

The main source of the loss development data used in this paper was the Best's Aggregates & Averages compilation of industry Schedule P information for Commercial Auto Liability that was tabulated in Wacek [13] as sets of paid and case incurred loss development factors (in that paper's Appendix Exhibits A-2A and A-5A, respectively)[44].

The upper half of Appendix Exhibit A-1A of this paper shows the 1) paid loss ratio through one year of development, and 2) age-to-age paid development factors (from age 1-to-2 through age 9-to-10), that were observed during calendar years 1994 through 2004 with respect to accident years 1999 and prior. The calendar year 1994 through 2003 data is from Best as tabulated by Wacek [13]. The calendar year 2004 information was derived directly from the industry Schedule P compilation contained within 2005 edition of Best's Aggregates & Averages [4]. The age 10-to-ultimate paid development factor implied by the relationship between the accident year 1995 reported ultimate and age 10 paid losses (1.009) is also tabulated here. The lower half of Appendix Exhibit A-1A displays the natural logarithms of the loss ratios and development factors shown in the upper half of the exhibit.

Appendix Exhibit A-1B is the case incurred loss analogue to Appendix Exhibit A-1A. The upper half of the exhibit displays the case incurred loss ratios through one year of development and age-to-age case incurred loss development factors. The source of that data is largely Appendix Exhibit A-5A of Wacek [13], supplemented by calendar year 2004 data from the 2005 edition of Best's Aggregates & Averages [4]. The age 10-to-ultimate case incurred development factor implied by the relationship between the accident year 1995 reported ultimate and age 10 case incurred losses (1.002) is also tabulated here. The lower half of the exhibit gives the corresponding natural logarithms.

Appendix Exhibits A-2A and A-2B display trailing five-year simple means and standard deviations of the loss ratio and development factor natural logarithms shown in the lower halves of Appendix Exhibits A-1A and A-1B, respectively. The means and standard deviations tabulated in Appendix Exhibit A-2A were used as estimates of the parameters μ and σ, respectively, of lognormal random variables representing 1) the paid loss ratio through one year of development, and 2) age-to-age paid loss development factors. For the

[44] Best's Aggregates & Averages, 1995-2005 editions. See [13] for full details.

insurer level analysis we multiplied these estimates σ by a factor of three. In order to preserve the same expected lognormal development factors as those found in the industry analysis, we adjusted the corresponding estimates of μ by adding the term $0.5\sigma^2 \cdot (1 - f^2)$, where $f = 3$.

The means and standard deviations tabulated in Appendix Exhibit A-2B were used to parameterize lognormal random variables representing 1) the case incurred loss ratio through one year of development, and 2) age-to-age case incurred loss development factors. For the insurer analysis, we made the same adjustment to the σ estimates that we described in the previous paragraph with respect to paid development factors.

Appendix Exhibits A-3A and A-3B display the expected values of the lognormal random variables parameterized using the means and standard deviations displayed in Appendix Exhibits A-2A and A-2B, respectively. To a very close degree of approximation, the implied mean age-to-age and age-to-ultimate development factors match those computed directly from the development factor data.

Appendix Exhibit A-4 summarizes the use of the historical loss development data to estimate accident year 1999 Commercial Auto Liability ultimate loss ratios using paid and case incurred chain ladder and Bornhuetter-Ferguson loss development methods at annual valuations from December 1999 through December 2004. We applied these four loss development methods as described in Appendix A of [13] using age-to-ultimate development factors from Appendix Exhibits A-3A and A-3B.

APPENDIX B

Estimating the Transfer Value of Unpaid Losses

According to Butsic, the transfer value of the unpaid loss liability should equal the present value of the expected future loss payments plus a risk charge for the potential for adverse deviation[45]. That definition was echoed in the UK FSA's February 2006 discussion paper on the EU's Solvency II initiative: "An unbiased valuation of insurance liabilities would reflect the best estimate plus a margin determined by the cost of capital required by the market to bear the risk of holding the liability[46]." In this appendix we derive a formula for this transfer value based on the capital required to support the unpaid loss liability and the required return on that allocated risk-based capital.

The transfer value $T(L_n)$ of unpaid losses L_n at development age $n \geq 1$ years is the sum:

[45] See [5], page 330, footnote 15.
[46] See [10], page 25.

$$T(L_n) = PV(L_n) + R'_n, \tag{B.1}$$

where $PV(L_n)$ is the present value sum of the future loss payments at the risk-free rate r and R'_n is the present value sum, at the same rate r, of the future risk charges associated with unpaid losses.

The calculation of the first term $PV(L_n)$ of $T(L_n)$ requires knowledge of the amounts and timing of the expected future loss payments $P_{n+1}, P_{n+2}, P_{n+3}, \ldots, P_{n+k}$, where k represents the number of future loss payments. If we assume that all loss payments are made at the midpoint of each payment year, then the value of $PV(L_n)$ is given by the formula:

$$PV(L_n) = (1 + \tfrac{1}{2}r) \cdot A_n, \tag{B.2}$$

where $A_n = P_{n+1} \cdot v + P_{n+2} \cdot v^2 + P_{n+3} \cdot v^3 + \ldots + P_{n+k} \cdot v^k$ and $v = \dfrac{1}{1+r}$. A_n is the present value sum of the loss payments under the assumption that they are made at the *end* of each year. $1 + \frac{1}{2}r$ is the adjustment factor required to reflect our assumption that loss payments are made at the *midpoint* of each year.

If the annual risk charge related to unpaid losses is expressed as a percentage return on the allocated risk-based capital $C_n^R = c_n \cdot L_n$, then the second term R'_n in Formula (B.1) can be expressed as:

$$R'_n = r'_n \cdot L_n \cdot v + r'_{n+1} \cdot L_{n+1} \cdot v^2 + r'_{n+2} \cdot L_{n+2} \cdot v^3 + \ldots + r'_{n+k-1} \cdot L_{n+k-1} \cdot v^k, \tag{B.3}$$

where $r'_n, r'_{n+1}, r'_{n+2}, \ldots, r'_{n+k-1}$ are the required annual returns expressed in terms of unpaid losses. To determine these required returns we assume that the capital provider demands an annualized after-tax return on equity of *roe* commensurate with the risk it is assuming for each year the capital is exposed. Given a tax rate of *tax*, the annual pre-tax return requirement on the allocated risk-based capital is $\dfrac{roe}{1-tax}$, of which r will be provided by interest earned on the capital itself. If the allocated capital is $c_n \cdot L_n$, then the required risk charge for each development period $n \geq 1$ is $c_n \cdot L_n \cdot (\dfrac{roe}{1-tax} - r)$. This risk charge can be expressed as an annual rate of return on L_n of:

$$r'_n = c_n \cdot (\frac{roe}{1-tax} - r) \tag{B.4}$$

If $c_1 = c_2 = c_3 = \cdots = c_n$ for all $n \geq 1$, i.e., the risk-based capital charges applicable to loss reserves are identical irrespective of the development age of the reserves, then we can drop the subscript from r'_n and restate Formula (B.3) as:

$$R'_n = r' \cdot (L_n \cdot v + L_{n+1} \cdot v^2 + L_{n+2} \cdot v^3 + \ldots + L_{n+k-1} \cdot v^k)$$

$$= r' \cdot (v \sum_{n+1}^{n+k} P_i + v^2 \sum_{n+2}^{n+k} P_i + v^3 \sum_{n+3}^{n+k} P_i + \ldots + v^k P_{n+k})$$

$$= r' \cdot (P_{n+1} \cdot v + P_{n+2} \cdot (v + v^2) + P_{n+3} \cdot (v + v^2 + v^3) + \ldots + P_{n+k} \cdot (v + v^2 + v^3 + \cdots + v^k))$$

$$= r' \cdot (P_{n+1} \cdot \frac{1-v}{r} + P_{n+2} \cdot \frac{1-v^2}{r} + P_{n+3} \cdot \frac{1-v^3}{r} + \ldots + P_{n+k} \cdot \frac{1-v^k}{r})$$

$$= \frac{r'}{r} \cdot (\sum_{n+1}^{n+k} P_i - (P_{n+1} \cdot v + P_{n+2} \cdot v^2 + P_{n+3} \cdot v^3 + \ldots + P_{n+k} \cdot v^k))$$

and finally as:

$$R'_n = \frac{r'}{r} \cdot (L_n - A_n) \qquad \text{(B.5)}$$

Then the transfer value $T(L_n) = PV(L_n) + R'_n$ can be expressed in terms of A_n as:

$$T(L_n) = (1 + \tfrac{1}{2} r) \cdot A_n + \frac{r'}{r} \cdot (L_n - A_n) \qquad \text{(B.6)}$$

or in terms of $PV(L_n)$ as:

$$T(L_n) = PV(L_n) + \frac{r'}{r} \cdot (L_n - PV(L_n) + \frac{r \cdot PV(L_n)}{2+r}) \qquad \text{(B.7)}$$

Appendix Exhibit B-1 illustrates the risk charge and transfer value calculations using the unpaid losses as of December 1999 associated with \$100 of premiums from accident year 1999. The expected payment pattern was derived from the simple average age-to-age (annual) paid development factors observed during the five calendar years 1995 through 1999[47]. The illustration assumes a capital factor of 15% of unpaid losses, a required after-tax return on allocated capital of 15%, tax rate of 35% and risk-free return of 5%.

The left side of the exhibit summarizes the transfer value calculation. The unpaid losses of \$64.69 at the end of 1999 (and beginning of 2000) had a present value of \$58.62. The present value sum of the future annual risk charges was \$4.06. The sum of these two components, \$62.69, represents the transfer value of the \$64.69 of loss reserves at the beginning of 2000. One year later at the end of 2000 (and beginning of 2001), the unpaid losses were expected to decline to \$43.59. On that basis and the expected future payment pattern, the present values of the unpaid losses and future risk charges, were \$39.93 and \$2.51, respectively, yielding a transfer value of \$42.44. Observe that a risk charge must be

[47] See the "1999" row in the "Trailing Five-Year Average Development to Ultimate" section of Appendix Exhibit A-3A.

added to the present value of the unpaid losses at each valuation date at which there remain unpaid losses, which in this illustration is out through the end of 2009.

The right side of the exhibit is a reconciliation of the transfer value calculations. It shows that invested cash equal to the transfer value of $62.69 at the beginning of 2000 would earn $2.61 during 2000. That principal and interest would be sufficient to pay expected claims of $21.10 plus a cash risk charge of $1.75 to the capital provider, leaving a balance of $42.44 at the end of the year. That ending balance matched the expected transfer value of unpaid losses at that time. The reconciliation shows that the transfer values calculated on the left side of the exhibit are such that all losses and risk charges can be paid as due (assuming the size and timing of loss payments are as expected.)

Appendix Exhibit B-2 summarizes the present value of the remaining loss reserves and the related risk charge (from Formula (B.5)), based on trailing five-year paid loss development experience, as of each calendar year-end from 1999 through 2004. The sum of these two present values is the transfer value of the remaining reserves (expressed as a percentage of remaining reserves).

APPENDIX C

C.1 Stochastic Modeling of Losses

The premise underlying the stochastic loss models used in this paper is that age-to-age loss development can be represented using a lognormal model. Our approach is closely related to the one described by Wacek [12], which was an elaboration of an idea first presented by Hayne [9]. We assumed that both paid and case incurred loss development patterns are lognormal.

Sources of Variation in Future Ultimate Loss Ratio Estimates

In general, a future estimate of the ultimate loss ratio with respect to a particular accident year depends mainly on the loss development that occurs between now and the time the future estimate is made[48]. That loss development affects the future ultimate loss estimate in two ways. The first and most direct effect arises from the loss development observed with respect to the subject accident year itself. More development generally implies a larger

[48] There can also be a minor effect that arises from the use of moving averages of historical development measures. For example, if prospective development in the tail is estimated using the five-year mean of historical development factors, then one year later when the tail is re-estimated, the earliest development factor will have dropped out of the calculation and a factor reflecting more recent development will have entered. The difference between the dropped factor and the added factor can have a small effect on the revised tail.

future ultimate loss ratio estimate than less development. The second effect arises from the loss development observed with respect to earlier accident years. That loss development affects the estimation of development in the tail of the subject accident year beyond the future valuation date. Again, more development generally implies a larger future ultimate loss ratio estimate than less development.

Our interest is in estimates of the ultimate loss ratio for accident year 1999 one year out from the vantage point of a succession of annual valuation dates from accident year inception ($n = 0$) through December 2003 ($n = 5$), where n refers to years of development.

From the vantage point of accident year 1999 inception ($n = 0$), the ultimate loss ratio estimate one year out will depend on loss development observed during calendar year 1999 with respect to: 1) accident year 1999 (from inception to age one year) and 2) 1998 and prior accident years.

In general, at each annual valuation through December 2003 corresponding to $0 \le n \le 5$ years of development, the ultimate loss ratio estimate one year out is a function of: 1) accident year 1999 development during calendar year $1998 + n + 1$ (from age n to $n + 1$), and 2) development on 1998 and prior accident years observed during calendar year $1998 + n + 1$.

To estimate the parameters of the random variables representing these loss development effects, we used industry Commercial Auto Liability loss development experience from Best's Aggregates & Averages, which is tabulated in Appendix Exhibits A-1A and A-1B mainly in the form of paid and case incurred age-to-age development factors (and their natural logarithms), respectively. See Appendix A for a full description of this data and its source.

C.2 Paid Chain Ladder

At n years of development the accident year 1999 paid chain ladder ultimate loss ratio estimate one year out is the product of the accident year 1999 cumulative paid loss ratio one year out (at age $n + 1$) and the paid age $n + 1$-to-ultimate tail factor one year out (at age $n + 1$). The first factor of this product reflects accident year 1999 development in the next year. The second factor reflects the effect of development of accident years 1998 and prior on the calculation of the tail factor one year out.

Modeling the First Source of Variation – Accident Year Development

To model accident year development over the course of the next year from the perspective of accident year 1999 inception ($n = 0$), we calculated the mean $\bar{y} = -1.678$ and standard deviation $s = 0.041$ of the natural logarithms of the paid loss ratios through one year of development observed over the five most recent calendar years (1994 through 1998)[49]. We used $\bar{y}$ and s as estimates of the parameters μ and σ of the lognormal random variable p_1 representing the paid loss ratio that will be observed one year out at the end of 1999. The *expected* value of the paid loss ratio at the end of 1999 $E(p_1)$ implied by these parameters was 18.7%. The *actual* paid loss ratio P_1 observed at the end of 1999 was 20.6%.

At the end of 1999 ($n = 1$) we went through a similar procedure. Let $\bar{y}$ and s represent the mean and standard deviation, respectively, of the natural logarithms of the age 1-to-2 development factors observed over the five most recent calendar years (1995 through 1999). We used $\ln P_1 + \bar{y} = \ln(20.6\%) + 0.743 = -0.836$ and $s = 0.024$ to estimate the parameters μ and σ of the lognormal random variable representing the cumulative paid loss ratio $P_1 + p_2$ that will be observed at the end of 2000[50]. These parameters implied an *expected* cumulative paid loss ratio as of the end of 2000 $E(P_1 + p_2)$ of 43.4%. The *actual* cumulative paid loss ratio $P_1 + P_2$ observed at the end of 2000 was 44.2%.

Generally, at $1 \le n \le 5$ years of development, to model the accident year 1999 paid loss ratio one year out at the end of $1999 + n$, we used $\ln(\sum_{i=1}^{n} P_i) + \bar{y}$ and s as estimates of the parameters μ and σ of the lognormal random variable $\sum_{i=1}^{n} P_i + p_{n+1}$, where P_i is the actual partial loss ratio paid during period *i*, and $\bar{y}$ and s are the mean and standard deviation, respectively, of the natural logarithms of the age n to $n+1$ development factors observed over the five most recent calendar years ($1999 + n - 5$ through $1999 + n - 1$). The *expected* cumulative paid loss ratio out year out, i.e., as of the end of $1999 + n$, is $E(\sum_{i=1}^{n} P_i + p_{n+1})$ and the *actual* cumulative paid loss ratio is $\sum_{i=1}^{n} P_i + P_{n+1}$.

---

49 Appendix Exhibit A-2A summarizes these calculations, which are based on data in Appendix Exhibit A-1A. For the insurer level analysis we used $3s$ in place of s and $\bar{y} + 0.5s^2 \cdot (1 - 3^2)$ to model the greater variability of an individual insurer's development factors, while preserving the original lognormal expected value development factors.

50 The random variable for the cumulative paid loss ratio can also be defined multiplicatively as $P_1 \cdot d_{1,1-2}$, where $d_{1,1-2}$ is the lognormal random variable at age $n = 1$ representing the age 1-to-2 development factor that will manifest itself over the next year, with parameters estimated by $\bar{y} = 0.743$ and $s = 0.024$. We prefer the additive formulation, because it preserves the annual components of the cumulative paid loss ratio.

We calculated these parameter estimates for $1 \le n \le 5$ and tabulated them, together with the expected and actual paid loss ratios one year out, in Appendix Exhibit C-1A in the column labeled "Paid L/R."

Modeling the Second Source of Variation – Tail Factor Revision

The revised tail factor one year out is the product of the mean age-to-age factors one year out. If five-year means are used, four of the five development factors to be used in the mean age-to-age factor calculations are already known. The fifth development factor is unknown, because it represents the development to be observed during the next year, but we can model it as a random variable. Because it involves four constants and a random variable, the mean age-to-age factor one year out is a random variable.

We modeled the revised tail factor one year out in three steps. First, we estimated the parameters of each age-to-age development factor random variable. These random variables modeled the age-to-age development to be observed during the next year. Next, we estimated the parameters of the *mean* age-to-age factors one year out. These mean age-to-age factor random variables combined the four known development factors and the random variable determined in step one. Finally, the mean age-to-age random variables were multiplied together to obtain the random variable for the revised tail factor out year out. Because the final step is difficult to carry out analytically, we used Monte Carlo simulation to model the revised tail factor random variables.

We illustrate the first two steps of this process for the age 1-to-2 development factor at accident year inception ($n = 0$). Referring to Appendix Exhibit A-1A, the age 1-to-2 development factors observed in calendar years 1994 through 1998 (with respect to accident years 1993 through 1997) and their natural logarithms were 2.265, 2.165, 2.115, 2.032, 2.079 and 0.817, 0.772, 0.749, 0.709, 0.732, respectively. That implied $\bar{y} = 0.756$ and $s = 0.041$, which we took as estimates of the μ and σ parameters of the random variable $d_{0,1-2}$ for the age 1-to-2 development factor to be observed in 1999 (with respect to accident year 1998). That is step one.

Because the historical development factors can be thought of as lognormal random variables with a σ parameter of zero, the random variable for the mean age 1-to-2 development factor one year out $\bar{d}_{1,1-2}$ has estimated parameters $\hat{y}$ (for μ) and $\hat{s}$ (for σ) of

$\frac{1}{5}(0.772+0.749+0.709+0.732+0.756)=0.744$ and $\frac{1}{5}\cdot 0.041=0.008$[51]. This random variable has an expected value of 2.104[52].

The same process was repeated for each age-to-age factor out to the factor for development from age nine to ten years. The age ten years to ultimate factor was treated as a constant. The product of all of the mean age-to-age factors one year out is the tail factor. The results from the perspective of accident year inception ($n=0$) are tabulated in Appendix Exhibit C-1A in the rows corresponding to Valuation Date "12/98." At the far right we also show the effect of multiplying the paid loss ratio one year out by the product of the revised age-to-age factors one year out to produce the estimated ultimate loss ratio estimate one year out. Here we see that at accident year inception the expected paid chain ladder ultimate loss ratio estimate one year out was 82.0%[53]. However, after observing the actual development during 1999, the paid chain ladder ultimate loss ratio estimate was revised to 90.3%.

In general, to model the tail factor one year out at $0 \le n \le 5$ years of development, we followed the same procedure that we described for $n=0$. In the first step we calculated the mean $\bar{y}$ and standard deviation s of the natural logarithms of the paid loss age-to-age development factors separately for each development period beyond age $n+1$ years (from age $n+1$-to-$n+2$ out through age 9-to-10) observed over the five calendar years $1999+n-5$ through $1999+n-1$. We took these as estimates of the parameters of the distributions of age-to-age factors that would be observed during $1998+n+1$. In the second step we combined the four age $n+1$-to-$n+2$ development factor observations from calendar years $1999+n-4$ through $1999+n-1$ with the random variable for $1999+n$ development whose parameters we estimated in step one. The results from the perspective of all annual valuation dates from December 1998 ($n=0$) through December 2003 ($n=5$) are tabulated in Appendix Exhibit C-1A. The combined effects of accident year development and tail factor revision are embodied in the actual and expected ultimate loss ratio estimates one year out shown at the far right.

---

51 For the insurer level analysis we did not adjust the four known data points (0.772, 0.749, 0.709 and 0.732) to offset the effect of multiplying s by a factor of three. This resulted in a slight upward bias in the distributions of the mean development factors making up the tail.

52 Note that this matches the simple average comprising the 1995 through 1998 development factors and the 1994 through 1998 development factor mean. If we were interested only in the development factor itself and not also its variability, it would be easier to work directly with the development factor data.

53 If the reader is puzzled about why this is different from the 83.5% shown in Appendix Exhibit A-3A as the implied paid chain ladder estimate at inception of the ultimate loss ratio, note that 82.0% is the estimate at inception of the paid chain ladder ultimate loss ratio *one year out*, which reflects the dropping of the 1994 development factors and addition of the estimate of 1999 development.

C.3 Case Incurred Chain Ladder

The modeling of the random variables for case incurred loss development was the same in every respect as that for paid loss development, except that case incurred loss data was used to estimate the parameters rather than paid loss data.

The parameter estimates for the case incurred random variables one year out, together with the expected and actual case incurred loss ratios one year out, are tabulated in Appendix Exhibit C-1B in the column labeled "Case Inc L/R" for annual valuation from accident year inception through December 2003. The parameter estimates for the case incurred age-to-age factor random variables one year out, together with the expected and actual case incurred age-to-age factors one year out, are tabulated in the body of the same Appendix Exhibit C-1B. The far right column shows the values one year out of the expected and actual ultimate loss ratio estimates.

C.4 Bornhuetter-Ferguson – Paid and Case Incurred

We applied the paid and case incurred Bornhuetter-Ferguson methods described in Appendix A, Section A.2.2, of Wacek [13] to loss development experience simulated using the random variables described in sections C.2 and C.3. At accident year inception ($n = 0$) we used the same initial expected loss ratios that were used in that paper: 83.4% for the paid method and 78.7% for the case incurred method. Separately for the paid and case incurred versions, we set the expected loss ratio for subsequent valuations equal to the chain ladder ultimate loss ratio estimate from the prior valuation, which was the convention used in [13].

C.5 Incorporation of Parameter Uncertainty

If we could have been certain about our lognormal parameter estimates, we would have simulated loss development experience using the lognormal random variables described in the foregoing sections of this appendix. Given a uniform random number R, the corresponding lognormal random number $LN^{-1}(\mu, \sigma, R)$ is:

$$LN^{-1}(\mu, \sigma, R) = \exp(\mu + N^{-1}(R) \cdot \sigma), \tag{C.1}$$

where μ and σ are the usual lognormal parameters and N^{-1} is the standard normal inverse distribution function.

However, we did not (and could not) know the true values of μ and σ. We had only parameter estimates $\hat{y}$ and $\hat{s}$. Because of that parameter uncertainty, we used a log t (rather than lognormal) random variable to simulate random values representing loss development experience:

$$LT^{-1}(\hat{y}, \hat{s}, R, k) = \exp(\hat{y} + T_{k-1}^{-1}(R) \cdot \hat{s}\sqrt{k + 1/k}) \qquad \text{(C.2)}$$

where $\hat{y}$ and $\hat{s}$ are the estimates of the lognormal parameters μ and σ, R is a uniform random number and T_{k-1}^{-1} is the inverse distribution function for the Student's *t* distribution with $k-1=4$ degrees of freedom (k representing the number of data points used to estimate the parameter). The $\sqrt{k+1/k}$ factor reflects the fact that both parameters are uncertain. In much statistical analysis involving the Student's *t* distribution it is assumed that μ is known and only σ is uncertain. We know here that both are uncertain. See Wacek [14] for a detailed discussion of parameter uncertainty in lognormal models.

C.6 Monte Carlo Simulation

Because the loss development random variables are hard to work with analytically (especially because we incorporated parameter uncertainty), we used Monte Carlo simulation to model chain ladder and Bornhuetter-Ferguson ultimate loss ratio estimates one year out as of each annual valuation date from inception through December 2003 using both paid and case incurred methods. For each of 10,000 Monte Carlo trials, we determined ultimate loss ratio estimates from each of the four loss development methods, and selected their unadjusted simple mean U_{n+1} as the best estimate of the ultimate loss ratio one year out. Appendix C-2A illustrates, for one Monte Carlo trial, the simulation of the paid chain ladder and Bornhuetter-Ferguson ultimate loss ratio estimates one year out from the vantage point of accident year inception ($n=0$). Appendix C-2B illustrates the simulation of the case incurred chain ladder and Bornhuetter-Ferguson ultimate loss ratio estimates one year out from the same vantage point[54]. We used the same uniform random numbers for the paid and case incurred simulations, reflecting our assumption that paid and case incurred loss development are not independent.

The ultimate loss ratio estimate has two stochastic elements corresponding to paid and unpaid losses, which we needed to separate in order to determine the transfer value of the ultimate loss ratio estimate one year out: $T(L_{n+1} + P_{n+1}) = T(L_{n+1}) + T(P_{n+1})$.

Therefore, in addition to U_{n+1}, for each trial we also tabulated the simulated values one year out of the period paid loss ratio P_{n+1} and the unpaid portion L_{n+1} of U_{n+1} (given by $L_{n+1} = U_{n+1} - \sum_{i=1}^{n+1} P_i$), as well as the transfer values $T(L_{n+1} + P_{n+1})$ and $T(L_{n+1})$. The transfer values were determined using the approach described in Appendix B.

[54] Note that Appendix Exhibits C-2A and C-2B use the same principles and format as Exhibit 11 in [12].

For each Monte Carlo trial we calculated the value of ending capital C_{n+1} and the ending policyholder deficit PD_{n+1} using the formulas in Section 2. The expected policyholder deficit was computed as the mean over 10,000 random trials:

$$E_n(pd_{n+1}) = \tfrac{1}{10{,}000}\sum_{i=1}^{10{,}000} PD_{n+1,i} \tag{C.3}$$

Formula (C.3) is a discrete approximation of Formula (2.15), which uses the continuous random variable t_{n+1} corresponding to $T(L_{n+1}+P_{n+1})$.

C.7 Reserves from Multiple Accident Years

If we let AY refer to the most recent of the accident years $AY-i$ ($i \geq 0$) with unpaid losses, the key loss-related formulas applicable to the valuation *n* years after AY's inception (for $n \geq 1$) of all accident years together are as follows:

$$_{AY-i}C^{R}_{n+i} = c_{n+i} \cdot {}_{AY-i}L_{n+i} \tag{C.4}$$

$$_{AY-i}S_{n+1+i} = ({}_{AY-i}C^{R}_{n+i} + {}_{AY-i}T(L_{n+i})) \cdot (1+r) \tag{C.5}$$

$$_{All}S_{n+1} = \sum_{i\geq 0}({}_{AY-i}C^{R}_{n+i} + {}_{AY-i}T(L_{n+i})) \cdot (1+r) \tag{C.6}$$

$$_{AY-i}C_{n+1+i} = {}_{AY-i}S_{n+1+i} - T({}_{AY-i}L_{n+1+i} + {}_{AY-i}P_{n+1+i}) \tag{C.7}$$

$$_{All}C_{n+1} = {}_{All}S_{n+1} - \sum_{i\geq 0} T({}_{AY-i}L_{n+1+i} + {}_{AY-i}P_{n+1+i}) \tag{C.8}$$

$$_{All}PD_{n+1} = \sum_{i\geq 0} T({}_{AY-i}L_{n+1+i} + {}_{AY-i}P_{n+1+i}) - {}_{All}S_{n+1}, \text{ if } {}_{All}C_{n+1} < 0 \tag{C.9}$$

$$E_n({}_{All}pd_{n+1}) = \int_{_{All}S_{n+1}}^{\infty} (t_{n+1} - {}_{All}S_{n+1}) f(t_{n+1}) dt_{n+1}, \tag{C.10}$$

where t_{n+1} is abbreviated notation for $_{All}t_{n+1} = \sum_{i\geq 0} {}_{AY-i}t_{n+1+i}$.

APPENDIX D

Accident Year 1999 Actual Policyholder Deficits: 1999-2004

This appendix gives details of the capital and policyholder deficit calculations arising from the *actual* Commercial Auto Liability accident year 1999 industry experience evaluated at successive annual intervals from December 1999 through December 2004. We used an analogous process to determine capital and policyholder deficits in the Monte Carlo analysis described in Appendix C. The centerpiece of our discussion is Appendix Exhibit D. Key results from that exhibit are summarized in Table 3 and Figure D in Section 2.

At the beginning of 1999, risk-based capital was established at 15% of premiums net of expenses and subsequently recalibrated to maintain funding equal to 15% of loss reserves at the end of each calendar year. A negative capital balance at the end of a year implied a policyholder deficit. If the "capital account" was under-funded at the end of any year, either because of a policyholder deficit or the recalibration requirement, the capital provider had to deposit additional cash. If this account was over-funded, the capital provider could withdraw the excess cash.

At accident year inception we assumed \$100 of premiums and an underwriting expense ratio of 25%, which implied initial underwriting assets $T(L_0)$ of \$75. The initial required capital was \$11.25 (15% of \$75). For purposes of calculating interest earned on the underwriting and capital assets, we assumed that half of the premium cash was available on January 1, 1999 in the form of an unearned premium portfolio and that the other half was available, on average, on July 1, 1999. Assuming the capital was allocated as the premiums were received and a risk free rate of 5%, interest of \$3.23 was earned during 1999. The December 1999 value of the underwriting and capital assets, including interest, was S_1=\$75.00+\$11.25+\$3.23=\$89.48.

Meanwhile, the hindsight re-estimate $L_1 + P_1$ as of December 1999 of the initial loss estimate L_0 was \$85.32, comprising paid losses P_1 of \$20.63 and an unpaid loss liability L_1 of \$64.69. The total transfer value of the hindsight losses $T(L_1 + P_1)$ was \$83.84, reflecting a paid loss transfer value $T(P_1)$ of \$21.15 and an unpaid loss transfer value $T(L_1)$ of \$62.69[55].

Now let's look at the "capital account". The ending capital as of December 1999 was \$5.65, which was the difference between the available assets S_1 and the transfer value of the hindsight losses $T(L_1 + P_1)$. Because the capital balance remained positive, the policyholder deficit was zero.

However, based on the loss reserve of \$64.69 as of December 1999, the prospective capital requirement was \$64.89×15%, or \$9.70, which meant that the capital account was under-funded by \$4.06. In order to meet the ongoing capital requirement, the capital provider had to contribute \$4.06 of additional capital at the end of 1999[56]. (This is summarized graphically in Figure D in Section 2 of the paper. The initial capital of \$11.25

[55] The paid loss transfer value assumes that claims were settled, on average, on July 1, 1999. The risk charge embedded in the unpaid loss transfer value is consistent with a 15% target after-tax return on the capital supporting the loss reserves, tax rate of 35%, risk-free interest rate of 5% and capital/reserve ratio of 15%. The loss payout pattern was derived from paid loss development experience through the end of calendar year 1999. See Appendix B for the theoretical basis and numerical illustration of the calculation.

[56] If the capital provider had failed to recapitalize, it would have been possible for the regulator to arrange an immediate transfer of the unpaid loss liability at the transfer value.

was reduced to $5.65 by the end of 1999, but was replenished to $9.70 to meet the prospective capital requirement based on year-end 1999 loss reserves.)

The same process was repeated for calendar year 2000. At the end of the year the sum of capital and underwriting assets ($9.70+$62.69) plus interest of $3.62 resulted in total available assets of $76.01, which was more than enough to cover the $68.78 transfer value of hindsight losses, despite an increase in the ultimate loss ratio estimate from 85.3% to 90.0%. Capital was reduced from $9.70 at the beginning of the year to $7.23 at the end, but it was far from exhausted, so again the actual policyholder deficit was zero.

At the beginning of 2001 capital again had to be reestablished at 15% of unpaid losses, or $6.87. Because the ending capital balance in December 2000 was $7.23, the capital provider could withdraw $0.36. 2001 saw further deterioration in the ultimate loss ratio estimate to 92.3%, and capital declined to $6.17 by year-end, but the policyholder deficit was zero.

We will leave it to the reader to review the details of the development of accident year 1999 from the end of 2001 through the end of 2004 as tabulated in Appendix Exhibit D, pointing out only that no policyholder deficit emerged at any valuation.

5. REFERENCES

[1] A.M. Best Company Methodology Paper, "Understanding BCAR", November 24, 2003, http://www.ambest.com/ratings/bcar.pdf

[2] American Academy of Actuaries Property/Casualty Risk-Based Capital Task Force, "Report on Reserve and Underwriting Risk Factors", *Casualty Actuarial Society Forum*, Volume: Summer, **1993**, 105-171, http://casact.org/pubs/forum/93sforum/93sf105.pdf

[3] Berkshire Hathaway, Inc., "2005 Annual Report", February 28, 2006, http://www.berkshirehathaway.com/2005arn/2005ar.pdf

[4] *Best's Aggregates & Averages (Property-Casualty) – United States,* 2005 Edition, Oldwick (NJ), A. M. Best Company, **2005**, 180, 199, 204, 209.

[5] Butsic, Robert P., "Solvency Measurement for Property-Liability Risk-Based Capital Applications", *Casualty Actuarial Society Discussion Paper Program*, May **1992** (Volume 1), 311-354, http://www.casact.org/pubs/dpp/dpp92/92dpp311.pdf

[6] Butsic, Robert P., "Solvency Measurement for Property-Liability Risk-Based Capital Applications", *The Journal of Risk and Insurance*, December **1994** (Volume 61, No. 4), 656-690

[7] Dimson, Elroy, Paul Marsh and Mike Staunton, "Long Run Global Capital Market Returns and Risk Premia", *London Business School Subject Area No. 035*, February **2002**, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=299335

[8] Feldblum, Sholom, "NAIC Property/Casualty Insurance Company Risk-Based Capital Requirements", *PCAS* LXXXIII, **1996**, 297-435, http://www.casact.org/pubs/proceed/proceed96/96297.pdf

[9] Hayne, Roger M., "An Estimate of Statistical Variation in Development Factor Models", *PCAS LXXII*, 1985, 25-43**,** http://www.casact.org/pubs/proceed/proceed85/85025.pdf

[10] UK Financial Services Authority, "Solvency II: a new framework for prudential regulation of insurance in the EU – A discussion paper", Her Majesty's Stationery Office, February **2006,** http://www.fsa.gov.uk/pubs/international/solvency2_discussion.pdf

[11] Standard and Poor's Ratings Services, *Insurance Ratings Criteria (Property/Casualty Edition)*, McGraw Hill, **1999**, 33-45.

[12] Wacek, Michael G., "The Path of the Ultimate Loss Ratio Estimate", *Casualty Actuarial Society Forum*, Volume: Winter, **2007**, 339-370, http://www.casact.org/pubs/forum/07wforum/07w345.pdf

[13] Wacek, Michael G., "A Test of Clinical Judgment vs. Statistical Prediction in Loss Reserving for Commercial Auto Liability", *Casualty Actuarial Society Forum*, Volume: Winter, **2007**, 371-404, http://www.casact.org/pubs/forum/07wforum/07w377.pdf

[14] Wacek, Michael G., "Parameter Uncertainty in Loss Ratio Distributions and its Implications", *Casualty Actuarial Society Forum*, Volume: Fall, **2005**, 165-202, http://www.casact.org/pubs/forum/05fforum/05f165.pdf

Abbreviations and notations

A_n, value of S&P 500 investment at time *n*

a_{n+1}, random variable, at time *n*, for value of S&P 500 investment one year out (*n*+1)

AY, accident year

C_n^R, required risk-based capital at time *n*

C_{n+1}, ending capital at time *n*+1

c, risk-based capital factor for common stocks

c_0, risk-based capital factor for underwriting

c_{n+1}, risk-based capital factor for loss reserves at time *n*+1

$d_{n,age-age+1}$, random variable for age-to-age+1 loss development factor

$\bar{d}_{n+1,age-age+1}$, random variable, at time *n*, for mean age-to-age+1 loss development factor one year out (*n*+1)

$E_n(pd_{n+1})$, expected value, at time n, of policyholder deficit one year out (*n*+1)

EPD, expected policyholder deficit

L_n, unpaid losses at time *n*

$L_{n+1}+P_{n+1}$, one-year hindsight estimate of L_n

LN^{-1}(μ,σ,prob), inverse lognormal distribution function

LT^{-1}($\hat{y}$,$\hat{s}$,prob,*k*), inverse log *t* distribution function based on k-point sample

N^{-1}(prob), inverse standard normal distribution function

n, lag (years) from inception at beginning of year

$n+1$, lag (years) from inception at end of year

P_{n+1}, paid losses between time *n* and *n*+1

PD_{n+1}, policyholder deficit at time *n*+1

p_{n+1}, random variable, as of time *n*, for paid losses between time *n* and *n*+1

pd_{n+1}, random variable, as of time *n*, for policyholder deficit one year out (*n*+1)

PV, present value operator

R, random number from unit uniform distribution

R_n', present value risk charge at time *n*

r, risk-free interest rate, per annum

r_n', risk charge, per annum, as a rate on L_n

roe, after-tax target return on equity capital

S_{n+1}, strike price, at time *n*, of insolvency option one year out (*n*+1)

s, five-year standard deviation of LDF logs

$\hat{s}$, estimate of σ used in log *t* simulations

$T(L_n)$, transfer value of unpaid losses at time *n*

$T(P_{n+1})$, transfer value of paid losses between time *n* and *n*+1

T_{k-1}^{-1}(prob), Student's *t* inverse distribution function with *k*-1 degrees of freedom

t_{n+1}, random variable, as of time *n*, transfer value of L_n one year out (*n*+1)

tax, corporate income tax rate

$\bar{y}$, five-year mean of LDF logs

$\hat{y}$, estimate of μ used in log *t* simulations

U_n, estimated ultimate loss at time *n*

v, one-year PV factor: $=1/(1+r)$

μ, first parameter of lognormal

σ, second parameter of lognormal

Biography of the Author

Michael Wacek is President of Odyssey America Reinsurance Corporation based in Stamford, CT. Over the course of more than 25 years in the industry, including nine years in the London Market, Mike has seen the business from the vantage point of a primary insurer, reinsurance broker and reinsurer. He has a BA from Macalester College (Math, Economics), is a Fellow of the Casualty Actuarial Society and a Member of the American Academy of Actuaries. He has authored a number of papers.

APPENDIX EXHIBIT A-1A

Commercial Auto Liability Accident Year Paid LDFs and their Natural Logarithms
By Calendar Year of Observed Development

Calendar Year	Age 1 Loss Ratio	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - 10	Age 10-Ult
1994	17.6%	2.265	1.456	1.196	1.101	1.050	1.028	1.016	1.008	1.004	
1995	18.2%	2.165	1.449	1.205	1.099	1.048	1.025	1.013	1.008	1.004	
1996	19.2%	2.115	1.422	1.202	1.104	1.047	1.024	1.011	1.005	1.004	
1997	19.2%	2.032	1.406	1.209	1.098	1.047	1.024	1.012	1.006	1.004	
1998	19.2%	2.079	1.422	1.197	1.096	1.046	1.020	1.012	1.007	1.003	
1999	20.6%	2.118	1.434	1.198	1.093	1.049	1.025	1.013	1.006	1.004	
2000		2.143	1.429	1.208	1.105	1.047	1.021	1.010	1.004	1.002	
2001			1.437	1.215	1.101	1.046	1.023	1.010	1.006	1.002	
2002				1.215	1.105	1.049	1.023	1.010	1.007	1.004	
2003					1.096	1.046	1.020	1.007	1.006	1.004	
2004						1.032	1.019	1.009	1.006	1.003	1.009

Natural Logarithms of Age 1 Loss Ratio and Age-to-Age Factors Shown Above

Calendar Year	Age 1 Loss Ratio	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - 10	Age 10-Ult
1994	-1.739	0.817	0.375	0.179	0.096	0.049	0.028	0.016	0.008	0.004	
1995	-1.703	0.772	0.371	0.187	0.094	0.047	0.025	0.013	0.008	0.004	
1996	-1.648	0.749	0.352	0.184	0.099	0.046	0.024	0.011	0.005	0.004	
1997	-1.652	0.709	0.341	0.189	0.093	0.046	0.024	0.012	0.006	0.004	
1998	-1.651	0.732	0.352	0.180	0.091	0.045	0.020	0.012	0.007	0.003	
1999	-1.578	0.750	0.360	0.181	0.089	0.048	0.025	0.013	0.006	0.004	
2000		0.762	0.357	0.189	0.100	0.046	0.020	0.009	0.004	0.002	
2001			0.362	0.195	0.097	0.045	0.023	0.010	0.006	0.002	
2002				0.194	0.099	0.048	0.023	0.010	0.007	0.004	
2003					0.092	0.045	0.020	0.007	0.006	0.004	
2004						0.032	0.019	0.009	0.006	0.003	0.009

APPENDIX EXHIBIT A-1B

Commercial Auto Liability Accident Year Case Incurred LDFs and their Natural Logarithms
By Calendar Year of Observed Development

Calendar Year	Age 1 Loss Ratio	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - 10	Age 10-Ult
1994	46.7%	1.363	1.123	1.048	1.024	1.010	1.006	1.004	1.002	1.002	
1995	46.8%	1.362	1.120	1.051	1.020	1.007	1.002	1.001	1.001	1.001	
1996	46.9%	1.337	1.121	1.050	1.024	1.009	1.002	1.002	1.000	1.001	
1997	48.0%	1.349	1.123	1.060	1.025	1.008	1.007	1.003	1.001	1.000	
1998	47.1%	1.336	1.137	1.065	1.024	1.010	1.003	1.002	1.001	1.000	
1999	50.3%	1.380	1.148	1.068	1.018	1.010	1.003	1.001	0.999	1.001	
2000		1.408	1.154	1.069	1.028	1.011	1.004	1.000	0.999	0.999	
2001			1.162	1.082	1.036	1.013	1.006	1.003	1.002	1.002	
2002				1.073	1.036	1.017	1.008	1.003	1.003	1.003	
2003					1.022	1.009	1.002	0.998	1.001	1.000	
2004						1.007	1.005	1.002	1.002	1.001	1.002
Natural Logarithms of Age 1 Loss Ratio and Age-to-Age Factors Shown Above											
1994	-0.761	0.310	0.116	0.047	0.023	0.010	0.006	0.004	0.002	0.002	
1995	-0.758	0.309	0.114	0.049	0.020	0.007	0.002	0.001	0.001	0.001	
1996	-0.758	0.291	0.114	0.049	0.024	0.009	0.002	0.002	0.000	0.001	
1997	-0.734	0.300	0.116	0.058	0.025	0.008	0.007	0.003	0.001	0.000	
1998	-0.754	0.289	0.128	0.063	0.023	0.010	0.003	0.002	0.001	0.000	
1999	-0.687	0.322	0.138	0.065	0.018	0.010	0.003	0.001	-0.001	0.001	
2000		0.342	0.143	0.067	0.028	0.010	0.004	0.000	-0.001	-0.001	
2001			0.150	0.079	0.035	0.013	0.006	0.003	0.002	0.002	
2002				0.070	0.035	0.017	0.008	0.003	0.003	0.003	
2003					0.022	0.009	0.002	-0.002	0.001	0.000	
2004						0.007	0.005	0.002	0.002	0.001	0.002

APPENDIX EXHIBIT A-2A

Commercial Auto Liability Accident Year Paid Loss Development
Mean and Standard Deviations of Natural Logarithms of LDFs

Trailing Five-Year Mean Age-to-Age Development Factor Natural Logarithms

Cal Year	Age 1 Loss Ratio	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - 10	Age 10-Ult *
1998	-1.678	0.756	0.358	0.184	0.095	0.047	0.024	0.013	0.007	0.004	0.009
1999		0.743	0.355	0.184	0.093	0.047	0.023	0.012	0.006	0.004	0.009
2000			0.352	0.185	0.094	0.046	0.023	0.011	0.006	0.003	0.009
2001				0.187	0.094	0.046	0.022	0.011	0.006	0.003	0.009
2002					0.095	0.046	0.022	0.011	0.006	0.003	0.009
2003						0.046	0.022	0.010	0.006	0.003	0.009
2004							0.021	0.009	0.006	0.003	0.009

Trailing Five-Year Standard Deviation of Age-to-Age Development Factor Natural Logarithms

Cal Year	Age 1 Loss Ratio	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - 10	Age 10-Ult *
1998	0.041	0.041	0.014	0.005	0.003	0.001	0.003	0.002	0.001	0.000	0.000
1999		0.024	0.011	0.004	0.004	0.001	0.002	0.001	0.001	0.000	0.000
2000			0.007	0.005	0.005	0.001	0.002	0.001	0.001	0.001	0.000
2001				0.006	0.004	0.001	0.002	0.001	0.001	0.001	0.000
2002					0.005	0.002	0.002	0.001	0.001	0.001	0.000
2003						0.002	0.002	0.002	0.001	0.001	0.000
2004							0.002	0.001	0.001	0.001	0.000

* Age 10 to Ultimate development implied in 2004 Annual Statement for accident year 1995

APPENDIX EXHIBIT A-2B

Accident Year Case Incurred Loss Development
Mean and Standard Deviations of Natural Logarithms of LDFs

Trailing Five-Year Mean Age-to-Age Development Factor Natural Logarithms

Cal Year	Age 1 Loss Ratio	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - 10	Age 10-Ult *
1998	-0.753	0.300	0.118	0.053	0.023	0.009	0.004	0.002	0.001	0.001	0.002
1999		0.302	0.122	0.057	0.022	0.009	0.003	0.002	0.001	0.001	0.002
2000			0.128	0.061	0.024	0.009	0.004	0.002	0.000	0.000	0.002
2001				0.067	0.026	0.010	0.004	0.002	0.001	0.000	0.002
2002					0.028	0.012	0.005	0.002	0.001	0.001	0.002
2003						0.012	0.004	0.001	0.001	0.001	0.002
2004							0.005	0.001	0.001	0.001	0.002

Trailing Five-Year Standard Deviation of Age-to-Age Development Factor Natural Logarithms

Cal Year	Age 1 Loss Ratio	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - 10	Age 10-Ult *
1998	0.011	0.010	0.006	0.007	0.002	0.001	0.002	0.001	0.001	0.001	0.000
1999		0.014	0.011	0.008	0.003	0.001	0.002	0.001	0.001	0.001	0.000
2000			0.013	0.007	0.004	0.001	0.002	0.001	0.001	0.001	0.000
2001				0.008	0.006	0.002	0.002	0.001	0.001	0.001	0.000
2002					0.008	0.003	0.002	0.001	0.002	0.001	0.000
2003						0.003	0.003	0.002	0.002	0.001	0.000
2004							0.002	0.002	0.001	0.001	0.000

* Age 10 to Ultimate development implied in 2004 Annual Statement for accident year 1995

APPENDIX EXHIBIT A-3A

Implied Lognormal Mean Accident Year Paid Loss Development Factors
Based on Mean and Standard Deviations of Natural Logarithms of LDFs
Commercial Auto Liability

Trailing Five-Year Average Age-to-Age Development

Cal Year	Age 1 Loss Ratio	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - 10	Age 10-Ult *
1998	18.7%	2.131	1.431	1.202	1.099	1.048	1.024	1.013	1.007	1.004	1.009
1999		2.102	1.427	1.202	1.098	1.048	1.024	1.012	1.006	1.004	1.009
2000			1.423	1.203	1.099	1.047	1.023	1.012	1.006	1.003	1.009
2001				1.205	1.099	1.047	1.023	1.011	1.006	1.003	1.009
2002					1.100	1.048	1.022	1.011	1.006	1.003	1.009
2003						1.047	1.022	1.010	1.006	1.003	1.009
2004							1.021	1.009	1.006	1.003	1.009

Trailing Five-Year Average Development to Ultimate

Cal Year	Age 1 Loss Ratio	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - 10	Age 10-Ult *
1998	83.5%	4.468	2.096	1.465	1.219	1.109	1.058	1.033	1.020	1.013	1.009
1999		4.378	2.083	1.460	1.215	1.107	1.056	1.032	1.019	1.013	1.009
2000			2.075	1.459	1.213	1.103	1.053	1.030	1.018	1.013	1.009
2001				1.460	1.211	1.102	1.053	1.030	1.018	1.012	1.009
2002					1.212	1.102	1.052	1.029	1.018	1.012	1.009
2003						1.101	1.051	1.028	1.018	1.012	1.009
2004							1.049	1.027	1.018	1.012	1.009

* Age 10 to Ultimate development implied in 2004 Annual Statement for accident year 1995

APPENDIX EXHIBIT A-3B

Implied Lognormal Mean Accident Year Case Incurred Loss Development Factors
Based on Mean and Standard Deviations of Natural Logarithms of LDFs
Commercial Auto Liability

Trailing Five-Year Average Age-to-Age Development

Cal Year	Age 1 Loss Ratio	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - 10	Age 10-Ult *
1998	47.1%	1.350	1.125	1.055	1.023	1.009	1.004	1.002	1.001	1.001	1.002
1999		1.353	1.130	1.059	1.022	1.009	1.003	1.002	1.001	1.001	1.002
2000			1.137	1.063	1.024	1.010	1.004	1.002	1.000	1.000	1.002
2001				1.069	1.026	1.010	1.004	1.002	1.001	1.000	1.002
2002					1.028	1.012	1.005	1.002	1.001	1.001	1.002
2003						1.012	1.004	1.001	1.001	1.001	1.002
2004							1.005	1.001	1.001	1.001	1.002

Trailing Five-Year Average Development to Ultimate

Cal Year	Age 1 Loss Ratio	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - 10	Age 10-Ult *
1998	78.7%	1.671	1.238	1.100	1.043	1.020	1.011	1.007	1.004	1.003	1.002
1999		1.684	1.244	1.101	1.040	1.018	1.009	1.005	1.004	1.003	1.002
2000			1.258	1.107	1.042	1.018	1.008	1.004	1.003	1.003	1.002
2001				1.119	1.046	1.020	1.009	1.005	1.003	1.003	1.002
2002					1.052	1.023	1.010	1.006	1.004	1.003	1.002
2003						1.021	1.009	1.005	1.004	1.003	1.002
2004							1.010	1.005	1.004	1.003	1.002

* Age 10 to Ultimate development implied in 2004 Annual Statement for accident year 1995

APPENDIX EXHIBIT A-4

Accident Year 1999 Ultimate Loss Ratio Estimates
Commercial Auto Liability

Chain Ladder Methods

Calendar Year	Dec Paid L/R	Dec Case L/R	Paid LDF	Case LDF	CL Paid Ult L/R	CL Case Ult L/R
1999	20.6%	50.3%	4.378	1.684	90.3%	84.7%
2000	44.2%	70.8%	2.075	1.258	91.7%	89.1%
2001	63.5%	82.3%	1.460	1.119	92.7%	92.1%
2002	77.2%	88.3%	1.212	1.052	93.5%	92.9%
2003	84.6%	90.2%	1.101	1.021	93.1%	92.2%
2004	87.3%	90.9%	1.049	1.010	91.6%	91.8%

Paid Bornhuetter-Ferguson Method

Calendar	Dec	BF Paid	Age to Ult LDF		BF Paid
Year	Paid L/R	ELR	Current	Prior	Ult L/R
1999	20.6%	83.4%	4.378	4.468	83.7%
2000	44.2%	90.3%	2.075	2.083	90.8%
2001	63.5%	91.7%	1.460	1.459	92.4%
2002	77.2%	92.7%	1.212	1.211	93.4%
2003	84.6%	93.5%	1.101	1.102	93.1%
2004	87.3%	93.1%	1.049	1.051	91.7%

Case Incurred Bornhuetter-Ferguson Method

Calendar	Dec	BF Case	Age to Ult LDF		BF Case
Year	Case L/R	ELR	Current	Prior	Ult L/R
1999	50.3%	78.7%	1.684	1.671	82.5%
2000	70.8%	84.7%	1.258	1.244	88.4%
2001	82.3%	89.1%	1.119	1.107	91.9%
2002	88.3%	92.1%	1.052	1.046	92.9%
2003	90.2%	92.9%	1.021	1.023	92.2%
2004	90.9%	92.2%	1.010	1.009	91.8%

Summary All Methods

Calendar Year	CL Paid Ult L/R	CL Case Ult L/R	BF Paid Ult L/R	BF Case Ult L/R	Mean Ult L/R
1999	90.3%	84.7%	83.7%	82.5%	85.3%
2000	91.7%	89.1%	90.8%	88.4%	90.0%
2001	92.7%	92.1%	92.4%	91.9%	92.3%
2002	93.5%	92.9%	93.4%	92.9%	93.2%
2003	93.1%	92.2%	93.1%	92.2%	92.6%
2004	91.6%	91.8%	91.7%	91.8%	91.7%

APPENDIX EXHIBIT B-1

Calculation of Expected Transfer Values of Accident Year 1999 Unpaid Losses
Loss Payment Patterns Based on Paid Development Through 1999
Per $100 of Commercial Auto Liability Premiums

	Calculation						Reconciliation				
Cal Year	Beginning Unpaid Losses	Paid Losses Expected in Period	PV Beginning Unpaid Losses	Risk Charge in Period	PV Total Risk Charge	Beginning Transfer Value of Unpaid Losses	Beginning Cash	Interest Earned	Paid Losses Expected in Period	Risk Charge Paid in Period	Ending Cash
2000	$64.69	$21.10	$58.62	$1.75	$4.06	$62.69	$62.69	$2.61	-$21.10	-$1.75	$42.44
2001	43.59	17.17	39.93	1.18	2.51	42.44	42.44	1.69	-17.17	-1.18	25.78
2002	26.42	11.60	24.33	0.72	1.46	25.78	25.78	1.00	-11.60	-0.72	14.47
2003	14.82	6.74	13.65	0.40	0.81	14.47	14.47	0.55	-6.74	-0.40	7.88
2004	8.08	3.61	7.42	0.22	0.45	7.88	7.88	0.30	-3.61	-0.22	4.35
2005	4.47	1.87	4.09	0.12	0.26	4.35	4.35	0.17	-1.87	-0.12	2.53
2006	2.59	0.99	2.38	0.07	0.15	2.53	2.53	0.10	-0.99	-0.07	1.56
2007	1.60	0.53	1.48	0.04	0.09	1.56	1.56	0.07	-0.53	-0.04	1.06
2008	1.07	0.32	1.01	0.03	0.05	1.06	1.06	0.05	-0.32	-0.03	0.76
2009	0.76	0.76	0.74	0.02	0.02	0.76	0.76	0.02	-0.76	-0.02	0.00

Interest Rate	5.0%	Eff Risk Chg Rate	2.71%
Capital Ratio	15.0%		
Target ROE	15.0%		
Tax Rate	35.0%		

APPENDIX EXHIBIT B-2

Projected Loss Payout Patterns for Use with Accident Year 1999 Commercial Auto Liability
Based on Trailing Five-Year Paid Development Experience

5-Year Period Ending	Remaining Reserves PV Factor	Remaining Reserves PV Risk Chg	Period Paid Development as % of Ultimate Losses: Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - 10	Age 10-Ult
1999	90.62%	6.28%	25.2%	20.5%	13.8%	8.0%	4.3%	2.2%	1.2%	0.6%	0.4%	0.9%
2000	91.66%	5.73%		20.4%	13.9%	8.2%	4.3%	2.2%	1.1%	0.6%	0.3%	0.9%
2001	92.25%	5.42%			14.1%	8.1%	4.3%	2.1%	1.1%	0.6%	0.3%	0.9%
2002	92.37%	5.36%				8.2%	4.3%	2.1%	1.1%	0.6%	0.3%	0.9%
2003	92.12%	5.49%					4.3%	2.1%	1.0%	0.6%	0.3%	0.9%
2004	91.44%	5.85%						2.0%	0.9%	0.6%	0.3%	0.9%

5-Year Period Ending	Cumulative Paid Development as % of Ultimate Losses: Age 1	Age 2	Age 3	Age 4	Age 5	Age 6	Age 7	Age 8	Age 9	Age 10
1999	22.8%	48.0%	68.5%	82.3%	90.4%	94.7%	96.9%	98.1%	98.7%	99.1%
2000		48.2%	68.6%	82.5%	90.6%	94.9%	97.1%	98.2%	98.8%	99.1%
2001			68.5%	82.6%	90.7%	95.0%	97.1%	98.2%	98.8%	99.1%
2002				82.5%	90.7%	95.0%	97.2%	98.2%	98.8%	99.1%
2003					90.8%	95.1%	97.3%	98.2%	98.8%	99.1%
2004						95.3%	97.3%	98.2%	98.8%	99.1%

Present values reflect 5% risk free rate
Capital allocation 15% of reserves, tax rate 35%, target return on equity 15%.

APPENDIX EXHIBIT C-1A

Commercial Auto Liability Accident Year Paid Loss Development
Applicable to Stochastic Modeling of Accident Year 1999 Losses One Year Out

Lognormal Parameters and Expected vs. Actual Values of Random Variables One Year Out

Val Date		Paid L/R	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - Ult	Est Ult L/R
12/98	ŷ	-1.678	0.744	0.355	0.185	0.094	0.046	0.023	0.012	0.007	0.013	
	ŝ	0.041	0.008	0.003	0.001	0.001	0.000	0.001	0.000	0.000	0.000	
	Expected	18.7%	2.104	1.426	1.203	1.099	1.047	1.023	1.012	1.007	1.013	82.0%
	Actual	20.6%	2.102	1.427	1.202	1.098	1.048	1.024	1.012	1.006	1.013	90.3%
12/99	ŷ	-0.836		0.352	0.183	0.093	0.047	0.023	0.012	0.006	0.013	
	ŝ	0.024		0.002	0.001	0.001	0.000	0.000	0.000	0.000	0.000	
	Expected	43.4%		1.422	1.201	1.098	1.048	1.023	1.012	1.006	1.013	89.9%
	Actual	44.2%		1.423	1.203	1.099	1.047	1.023	1.012	1.006	1.013	91.7%
12/00	ŷ	-0.464			0.185	0.094	0.046	0.022	0.012	0.006	0.012	
	ŝ	0.007			0.001	0.001	0.000	0.000	0.000	0.000	0.000	
	Expected	62.9%			1.203	1.098	1.048	1.023	1.012	1.006	1.012	91.7%
	Actual	63.5%			1.205	1.099	1.047	1.023	1.011	1.006	1.012	92.7%
12/01	ŷ	-0.267				0.094	0.046	0.022	0.011	0.006	0.012	
	ŝ	0.006				0.001	0.000	0.000	0.000	0.000	0.000	
	Expected	76.6%				1.099	1.047	1.022	1.011	1.006	1.012	92.7%
	Actual	77.2%				1.100	1.048	1.022	1.011	1.006	1.012	93.5%
12/02	ŷ	-0.164					0.047	0.023	0.011	0.006	0.012	
	ŝ	0.005					0.000	0.000	0.000	0.000	0.000	
	Expected	84.9%					1.048	1.023	1.011	1.006	1.012	93.6%
	Actual	84.6%					1.047	1.022	1.010	1.006	1.012	93.1%
12/03	ŷ	-0.121						0.022	0.010	0.006	0.012	
	ŝ	0.002						0.000	0.000	0.000	0.000	
	Expected	88.6%						1.022	1.010	1.006	1.012	93.1%
	Actual	87.3%						1.021	1.009	1.006	1.012	91.6%

APPENDIX EXHIBIT C-1B

Commercial Auto Liability Accident Year Case Incurred Loss Development
Applicable to Stochastic Modeling of Accident Year 1999 Losses One Year Out

Lognormal Parameters and Expected vs. Actual Values of Random Variables One Year Out

Val Date		Rptd L/R	Age 1 - 2	Age 2 - 3	Age 3 - 4	Age 4 - 5	Age 5 - 6	Age 6 - 7	Age 7 - 8	Age 8 - 9	Age 9 - Ult	Est Ult L/R
12/98	ŷ	-0.753	0.298	0.118	0.055	0.023	0.008	0.004	0.002	0.001	0.003	
	ŝ	0.011	0.002	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	
	Expected	47.1%	1.347	1.125	1.056	1.023	1.009	1.004	1.002	1.001	1.003	78.6%
	Actual	50.3%	1.353	1.130	1.059	1.022	1.009	1.003	1.002	1.001	1.003	84.7%
12/99	ŷ	-0.385		0.124	0.059	0.022	0.009	0.004	0.002	0.001	0.003	
	ŝ	0.014		0.002	0.002	0.001	0.000	0.000	0.000	0.000	0.000	
	Expected	68.1%		1.132	1.060	1.023	1.009	1.004	1.002	1.001	1.003	85.0%
	Actual	70.8%		1.137	1.062	1.024	1.010	1.004	1.002	1.000	1.003	89.1%
12/00	ŷ	-0.217			0.063	0.023	0.010	0.004	0.001	0.000	0.002	
	ŝ	0.013			0.001	0.001	0.000	0.000	0.000	0.000	0.000	
	Expected	80.5%			1.065	1.024	1.010	1.004	1.001	1.000	1.002	89.3%
	Actual	82.3%			1.069	1.026	1.010	1.004	1.002	1.001	1.003	92.1%
12/01	ŷ	-0.128				0.026	0.011	0.004	0.001	0.000	0.003	
	ŝ	0.008				0.001	0.000	0.000	0.000	0.000	0.000	
	Expected	88.0%				1.026	1.011	1.004	1.001	1.000	1.003	92.1%
	Actual	88.3%				1.028	1.012	1.005	1.002	1.001	1.003	92.9%
12/02	ŷ	-0.096					0.013	0.005	0.002	0.001	0.003	
	ŝ	0.008					0.001	0.000	0.000	0.000	0.000	
	Expected	90.8%					1.013	1.005	1.002	1.001	1.003	93.0%
	Actual	90.2%					1.012	1.004	1.001	1.001	1.003	92.2%
12/03	ŷ	-0.091						0.005	0.002	0.001	0.003	
	ŝ	0.003						0.001	0.000	0.000	0.000	
	Expected	91.3%						1.005	1.002	1.001	1.003	92.4%
	Actual	90.9%						1.005	1.001	1.001	1.003	91.8%

APPENDIX EXHIBIT C-2A

Monte Carlo Simulation of Estimated Ultimate Loss Ratio One Year Out - Paid Development Methods
Accident Year 1999 at Inception

Illustration of One Random Trial - Reflecting Parameter Uncertainty
Commercial Auto Liability

Devt Period	Expected LDF	Sample Size k	Degrees of Freedom k-1	Uniform Random Number R	$T_4^{-1}(R)$	$\hat{y}$	$\hat{s}$	$\sqrt{\frac{k+1}{k}}$	Random Paid LR: Accident Yr Devt *	Random LDF: Revised Tail *
9-Ult	1.013	5	4	0.034	-2.481	0.013	0.000	1.095		1.013
8-9	1.007	5	4	0.665	0.460	0.007	0.000	1.095		1.007
7-8	1.012	5	4	0.879	1.373	0.012	0.000	1.095		1.013
6-7	1.023	5	4	0.954	2.202	0.023	0.001	1.095		1.025
5-6	1.047	5	4	0.056	-2.032	0.046	0.000	1.095		1.047
4-5	1.099	5	4	0.110	-1.456	0.094	0.001	1.095		1.098
3-4	1.203	5	4	0.729	0.664	0.185	0.001	1.095		1.204
2-3	1.426	5	4	0.205	-0.918	0.355	0.003	1.095		1.422
1-2	2.104	5	4	0.025	-2.779	0.744	0.008	1.095		2.051
0-1	18.7%	5	4	0.333	-0.467	-1.678	0.041	1.095	18.3%	
	4.387								18.3%	4.270

* = $\exp(\hat{y} + T_{k-1}^{-1}(R) \cdot \hat{s}\sqrt{(k+1)/k})$

Revised **Paid Chain Ladder** Loss Ratio Estimate One Year Out
= 18.3% x 4.27 = **78.0%**

Revised **Paid Bornhuetter-Ferguson** Loss Ratio Estimate One Year Out
= 18.3% - 18.7% + 18.7% x 4.27 = **79.4%**

APPENDIX EXHIBIT C-2B

Monte Carlo Simulation of Estimated Ultimate Loss Ratio One Year Out - Case Incurred Development Methods
Accident Year 1999 at Inception

Illustration of One Random Trial - Reflecting Parameter Uncertainty
Commercial Auto Liability

Devt Period	Expected LDF	Sample Size k	Degrees of Freedom k-1	Uniform Random Number R	$T_4^{-1}(R)$	$\hat{y}$	$\hat{s}$	$\sqrt{\frac{k+1}{k}}$	Random Case LR: Accident Yr Devt *	Random LDF: Revised Tail *
9-Ult	1.003	5	4	0.034	-2.481	0.003	0.000	1.095		1.003
8-9	1.001	5	4	0.665	0.460	0.001	0.000	1.095		1.001
7-8	1.002	5	4	0.879	1.373	0.002	0.000	1.095		1.002
6-7	1.004	5	4	0.954	2.202	0.004	0.000	1.095		1.005
5-6	1.009	5	4	0.056	-2.032	0.008	0.000	1.095		1.008
4-5	1.023	5	4	0.110	-1.456	0.023	0.000	1.095		1.023
3-4	1.056	5	4	0.729	0.664	0.055	0.001	1.095		1.057
2-3	1.125	5	4	0.205	-0.918	0.118	0.001	1.095		1.124
1-2	1.347	5	4	0.025	-2.779	0.298	0.002	1.095		1.339
0-1	47.1%	5	4	0.333	-0.467	-0.753	0.011	1.095	46.8%	
	1.668								46.8%	1.658

$* \quad = \quad \exp(\hat{y} + T_{k-1}^{-1}(R) \cdot \hat{s}\sqrt{(k+1)/k})$

Revised Case Incurred **Chain Ladder** Loss Ratio Estimate One Year Out
= 46.8% x 1.658 = **77.6%**

Revised **Case Incurred Bornhuetter-Ferguson** Loss Ratio Estimate One Year Out
= 46.8% - 47.1% + 47.1% x 1.658 = **77.8%**

APPENDIX EXHIBIT D

Calculation of Actual Policyholder Deficits 1999-2004

Commercial Auto Liability – Accident Year 1999

Premiums of $100

25% Expenses / Required Capital 15% of Unpaid Losses / 5% Interest

Calendar Year	1999	2000	2001	2002	2003	2004
n [1]	0	1	2	3	4	5
$T(L_n)$	$75.00	$62.69	$44.61	$28.09	$15.65	$7.88
C_n^R	11.25	9.70	6.87	4.31	2.40	1.21
$r \cdot (C_n^R + T(L_n))$ [2]	3.23	3.62	2.57	1.62	0.90	0.45
S_n	$89.48	$76.01	$54.05	$34.02	$18.95	$9.54
U_{n+1} [3]	85.32	90.02	92.29	93.17	92.65	91.73
P_{n+1}	20.63	23.58	19.31	13.64	7.41	2.74
L_{n+1}	64.69	45.80	28.76	16.01	8.07	4.41
$L_{n+1} + P_{n+1}$	85.32	69.38	48.07	29.65	15.48	7.15
$T(P_{n+1})$	21.15	24.17	19.79	13.98	7.60	2.81
$T(L_{n+1})$	62.69	44.61	28.09	15.65	7.88	4.29
$T(L_{n+1} + P_{n+1})$	$83.84	$68.78	$47.89	$29.62	$15.48	$7.10
C_{n+1}	$5.86	$7.23	$6.17	$4.40	$3.47	$2.44
PD_{n+1}	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
C_{n+1}^R	9.70	6.87	4.31	2.40	1.21	0.66
$C_{n+1}^R - C_{n+1}$	4.06	(0.36)	(1.85)	(2.00	(2.26)	(1.78)

1 n is the lag in years from accident year inception at beginning of year

2 $\frac{3}{4} r \cdot (C_0^R + T(L_0))$ for n=0

3 U_{n+1} is the estimated ultimate loss amount at age n+1