The Report of the Research Working Party on Correlations and Dependencies Among All Risk Sources

Part 4

Serial Correlations of Interest and Inflation Rates

Hans E. Waszink AAG, MSc.

Abstract

This chapter discusses an approach to model the value of an outstanding, discounted liability under the impact of uncertain interest and inflation rates. Interest and inflation rates are modeled separately as time series to take into account autocorrelation. Subsequently, the dependence between interest and inflation is modeled using copulas. The goodness of fit of some copulas can be evaluated on the basis of historic data using a quantile plot. This is done for the Gumbel, Clayton and Independent copulas. The Gumbel copula, which gives the best fit, is then compared with the Normal copula to show that the two copulas are very similar with the parameters chosen. The distribution of the required reserve is shown under four different copula assumptions: comonotonicity, which represent the best case, countermonotonicity which represents the worst case, and the Gumbel and Normal copulas which represent more realistic scenarios. The choice of copula has considerable impact on the higher percentiles of the required reserve, and the adopted approach is effective in selecting a suitable copula.

1. INTRODUCTION

In this chapter the following are investigated:

1. Correlations between the same variable, i.e. interest or inflation, at different points in time (autocorrelation).
2. Correlations between interest and inflation over an extended period of time.
3. Impact of these correlations on the present value of a discounted and inflated liability.

The effect of both types of correlations is demonstrated in a case study investigating the effect of interest and inflation rates fluctuations on outstanding claims liabilities. Interest and inflation rates are modeled as time series. Time series models are commonly used for variables of which observations are available sequentially in time, and consecutive observations are dependent. Both these properties typically apply to interest as well as inflation rates.

A simple example of a time series is an autoregressive process of order 1 (AR(1)) which is given below:

$$X(t) = a + bX(t\text{-}1) + \varepsilon(t),\quad t = 1,\dots,T$$

with

$X(.)$: array of stochastic variables, $t= 0,1,\dots,T$, $X(0)$ a given constant.
$\varepsilon(t)$: random error within period $(t\text{-}1,t)$, with $N(0,\sigma)$ distribution.
a,b: model parameters.

It can be shown that this structure defines a correlation structure between all $X(t)$, with correlations depending on b and σ and the elapsed time between observations. More complex time series models are often required to adequately capture specific characteristics such as cyclicality or heteroskedasticity.

2. OUTSTANDING LIABILITY UNDER UNCERTAIN INTEREST AND INFLATION RATES

We consider the value of an outstanding claims reserve as the present value of inflated and discounted future claim payments. Interest and inflation rates are modeled as random variables. As a starting point, we use uninflated projections of future claim payments in each future payment period. These can be derived from triangular reserving methods which include an explicit inflationary effect.

Define:

C(t): Uninflated, fixed and given cashflow projection at time *t*.
Inf(t): Inflation rate in period *(t,t+1)*, $t = 0,1,2,\ldots$
Int(t): Interest rate in period *(t, t+1)*, $t = 0,1,2,\ldots$

Ac(t): Actual cashflow at time *t*.

Ac(t) is equal to:

$$Ac(t) = C(t) \times \prod_{\tau=0}^{t-1} [1 + Inf(\tau)], \quad t = 1,2,3,\ldots$$

For simplicity it is assumed that *Ac(t)* is the product of the cashflow projection *C(t)*, which is fixed and given, and future inflation rates only. Therefore the only uncertain factor in actual future cashflows is future inflation which can represent general inflation, superimposed inflation or a line-specific inflation. In this study we have used medical inflation, a line-specific inflation impacting on health insurance related liabilities.

The inflation rates represent a component of systematic risk in the cash flow projection, i.e. they affect all individual claims simultaneously and to the same extent. To relax the assumption that inflation is the only uncertain factor affecting future cashflows, additional components of unsystematic risk can be added without any difficulty, however these are excluded here.

Df(t): Discount factor in period *(t,t+1)*, $t=0,1,2,\ldots$:

$$Df(t) = \frac{1}{1 + Int(t)}$$

RR(t): Required reserve at time *t* , $t=0,1,2,\ldots$:

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$$RR(t) = \sum_{s>t} [Ac(s) \times \prod_{\tau=0}^{s-t-1} Df(t+\tau)]$$

The required reserve is the total of all actual future cashflows discounted at actual future interest rates. Obviously *RR(t)* is not known in advance as it is a function of *C(t)*, *Inf(t)* and *Int(t)* with future interest and inflation rates unknown.

The distribution of the *RR(t)* is a function of the marginal distributions of the interest and inflation rates after time *t* and the dependencies between interest rates in different periods, the dependence between inflation rates in different periods, and the dependence between inflation and interest rates in the same period and in different periods.

3. MODELLING THE DISTRIBUTION OF INTEREST AND INFLATION RATES AND THEIR MUTUAL DEPENDENCE

3.1 Interest rates

A discrete version of the CIR[1]-model for a single interest rate is used. A single interest rate is used for simplicity, although the CIR-model allows for the generation of the entire yield curve with full dependence between different maturities. Different yield curve structures can be generated using various other interest rate models of a similar time series structure.

The discrete CIR-model is a time-series model of the following form:

$$Int(t) = max\{0, a[b - Int(t-1)] + \sqrt{Int(t-1)}\varepsilon_{int}(t)\}$$

with

Int(t):	the interest rate in the period $(t,t+1)$
a:	the average speed of reversion to the long term mean interest rate;
b:	the long term mean interest rate.
$\varepsilon_{int}(t)$:	random deviation in period $(t,t+1)$. The $\varepsilon_{int}(t)$ are mutually independent with marginal distributions $N(0,\sigma^2)$.

The model has several desirable properties such as:

- Interest rates are mean reverting;
- Interest rates are non-negative.
- Interest rates are heteroskedastic, i.e. variance increases with mean.
- Interest rates at adjacent points in time are correlated.
- Confidence intervals widen for interest rates projections further into the future.

For the parameterization of the time series, we have used 3 year interest rates on US government securities which are shown in appendix I. The estimated parameters are shown in appendix II, simulated autocorrelations of interest rates are shown in appendix III.

[1] Cox Ingersoll Ross, see Kaufmann (2001)

3.2 Inflation rates

For inflation rates a second order autoregressive process (AR(2)) is used:

$$Inf(t) = c_0 + c_1\, Inf(t\text{-}1) + c_2\, Inf(t\text{-}2) + \varepsilon_{inf}(t)$$

with

c_0, c_1, c_2: model parameters.
$\varepsilon_{inf}(t)$: random deviations in period $(t,t+1)$.

The $\varepsilon_{inf}(t)$ are mutually independent with identical marginal distributions $N(0,\sigma^2)$.

Some properties of the AR(2) model are:

- If $c_2 < 0$, inflation rates may exhibit cyclicality.
- Observations at adjacent points in time are correlated.
- Confidence intervals widen for projections further into the future.

For the parameterization of the time series, we have used US medical care index figures provided by the Bureau of Labor Statistics, which are shown in appendix I. The estimated parameters are shown in appendix II, simulated autocorrelations of inflation rates are shown in appendix III.

The (analytically determined) autocorrelations between the inflation rate in time period 1 and all other periods, derived from the time series parameterization, are shown below:

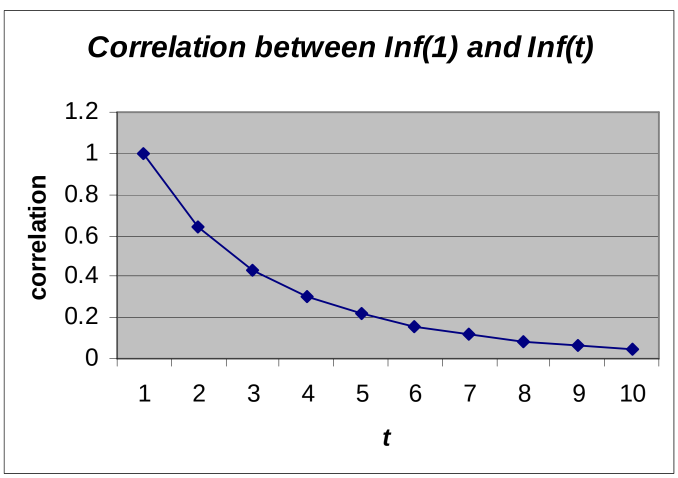

Figure 1: Modeled autocorrelations of inflation rates

As the parameter c_2 is very close to, and not significantly different from 0, there is no cyclical pattern in the correlation structure and the process is virtually identical to an AR(1) process.

3.3 Dependence between inflation and interest rates

The dependence relation between interest and inflation rates in the same period is determined by, on the one hand, the structure of the time series model for both interest and inflation rates, and on the other hand by the dependence between the random errors $\varepsilon_{int}(t)$ and $\varepsilon_{inf}(t)$ in the same period.

Both time series as well as the dependence between them are parameterized on the basis of actual historic data shown in appendix I. It can be expected that there is a dependency between $\varepsilon_{int}(t)$ and $\varepsilon_{inf}(t)$ as changes in both inflation and interest rates are driven by the same or related macro-economic factors. Various types of dependence relationships, i.e. copulas, can be used to model the dependency between $\varepsilon_{int}(t)$ and $\varepsilon_{inf}(t)$. We assume that the dependence relation is the same for all values of *t*, hence does not change over time.

Given that both error terms are assumed to follow a Normal distribution, the simplest form of dependency is the linear correlation which corresponds to the Normal copula. However the Normal copula does not always capture dependencies in the tail of the distributions appropriately[2] hence the Gumbel and Clayton copulas are also investigated

As $\varepsilon_{int}(t)$ and $\varepsilon_{int}(t')$ are independent if $t \neq t'$, so are $\varepsilon_{int}(t)$ and $\varepsilon_{inf}(t')$. Thus the choice of the time series models for interest and inflation rates together with the copula representing the dependence between $\varepsilon_{int}(t)$ and $\varepsilon_{inf}(t)$ fully define the joint distribution of interest and inflation rates. As *RR(t)* is fully determined by the deterministic uninflated cashflows *C(t)* in combination with interest and inflation rates during the projection period, the distribution of all *RR(t)* is fully defined by the joint distribution of inflation and interest rates and *C(t)* . The distribution of *RR(t)* is derived by means of simulation.

For the uninflated cashflow projection *C(t)* we set *C(t) = 1* for *t= 1,2,…,10* and 0 otherwise. For the choice of the copula defining the dependence between $\varepsilon_{int}(t)$ and $\varepsilon_{inf}(t)$, several alternative scenarios are investigated:

1. $\varepsilon_{int}(t)$ and $\varepsilon_{inf}(t)$ are comonotonic, i.e. the dependence between the two is maximum. As both $\varepsilon_{int}(t)$ and $\varepsilon_{inf}(t)$ are Normal random variables, the linear correlation between them is 100%. This is the best case scenario for the insurer with respect to the dependence between the two error terms. The underlying assumption is that random deviations of interest rates are fully correlated with random deviations of inflation rates, hence unexpected increases in inflation are always accompanied by unexpected increases in interest rates. As increases in inflation rates lead to increases in *RR(.)* whereas increases in interest rates lead to a decreases of *RR(.)*, the comonotonic assumption implies that there always is a compensating effect of the two random errors on the liability for the insurer. Therefore this scenario represents a best case for the insurer with respect to the occurrence of extremely high values of *RR(t)*.

[2] See Embrechts (2001)

2. In the second scenario, the dependence between $\varepsilon_{int}(t)$ and $\varepsilon_{inf}(t)$ is assumed to be 'countermonotonic'[3], meaning unexpected increases in inflation rates are always accompanied by unexpected decreases in interest rates. Contrary to the first alternative, this scenario represents the worst case with respect to the occurrence of extremely high values of *RR(.)*, as the effects of unexpected inflation in any particular period are aggravated by lower interest earnings in the same period.

3. In the third scenario, the dependence between $\varepsilon_{int}(t)$ and $\varepsilon_{inf}(t)$ is parameterized on the basis of historic observations. Historic observations of the error terms are obtained by substituting observed historic interest/inflation rates in the time series equations for *Int(t)* and *Inf(t)*. Sufficient credible historic data needs to be available to justify a choice and parameterization of a copula in this way.

   The copula chosen here is the Gumbel copula, with parameter $\alpha = 1.4$. Appendix IV shows the fit of the Gumbel and Clayton copulas, on the basis of which the Gumbel copula is the preferred choice. Appendix V shows correlations between inflation and interest rates under the Gumbel copula.

4. In the fourth alternative, the dependence between $\varepsilon_{int}(t)$ and $\varepsilon_{inf}(t)$ is modeled as a multivariate Normal distribution, with the dependence between the two random variables fully characterized by their linear correlation coefficient.

The simulated results of each of the four methods are shown in figure 2 below[4], with BC (Best Case) , WC (Worst Case), Gumbel and Normal depicting *RR(0)* in alternatives 1-4 respectively. As the graphs of alternative 3 and 4 seem to overlap completely, the right tail is shown in more detail in figure 3.

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[3] Characterization of comonotonicity and countermonotonicity can be found in Denuit (2003)

[4] Results were generated using IGLOO software.

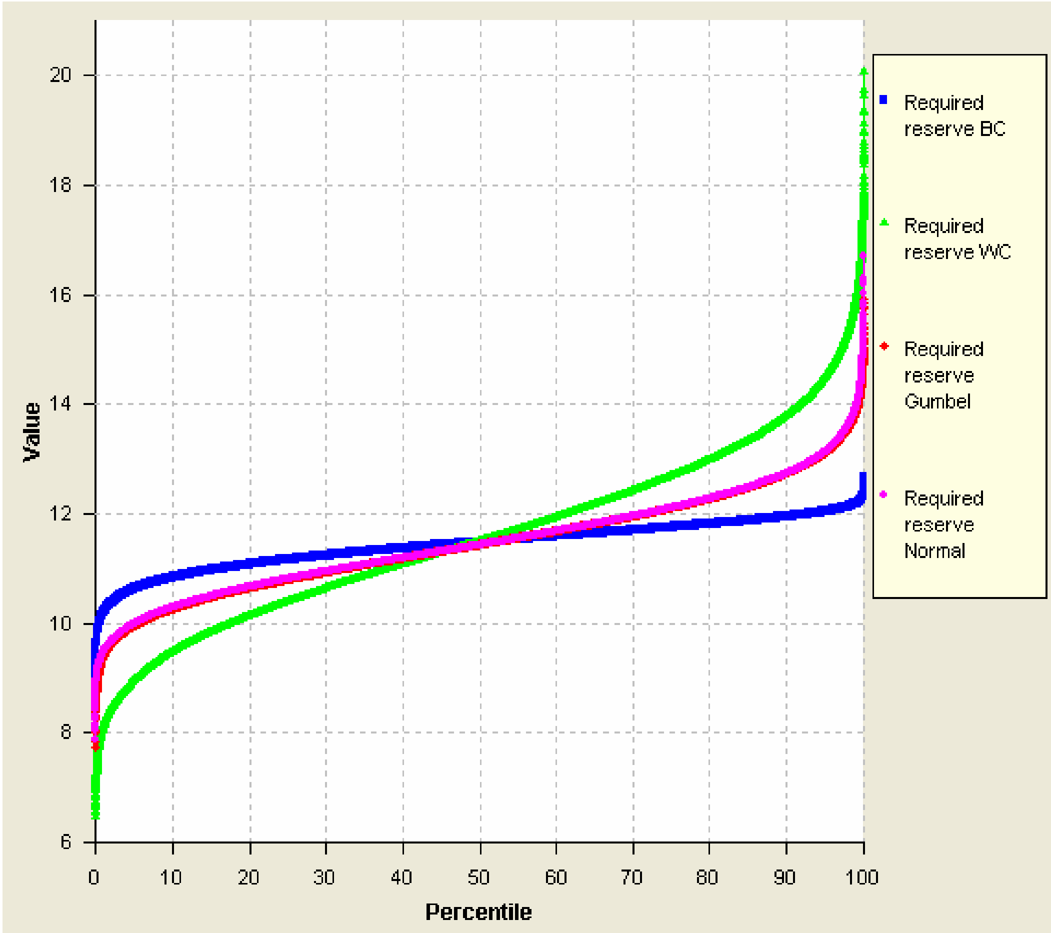

Figure 2: simulated distributions of RR(0) scenario 1-4

In alternative 4, a linear correlation between $\varepsilon_{int}(t)$ and $\varepsilon_{inf}(t)$ of 0.44 is applied. This is the historically observed correlation between the residuals. In alternative 3, the Gumbel copula is parameterized using the algorithm described in Valdez (1998). The Gumbel copula in this case gives rise to the same linear correlation of 0.44 as the Normal copula.

The right tail of the distributions resulting from the Normal and Gumbel copulas are shown below.

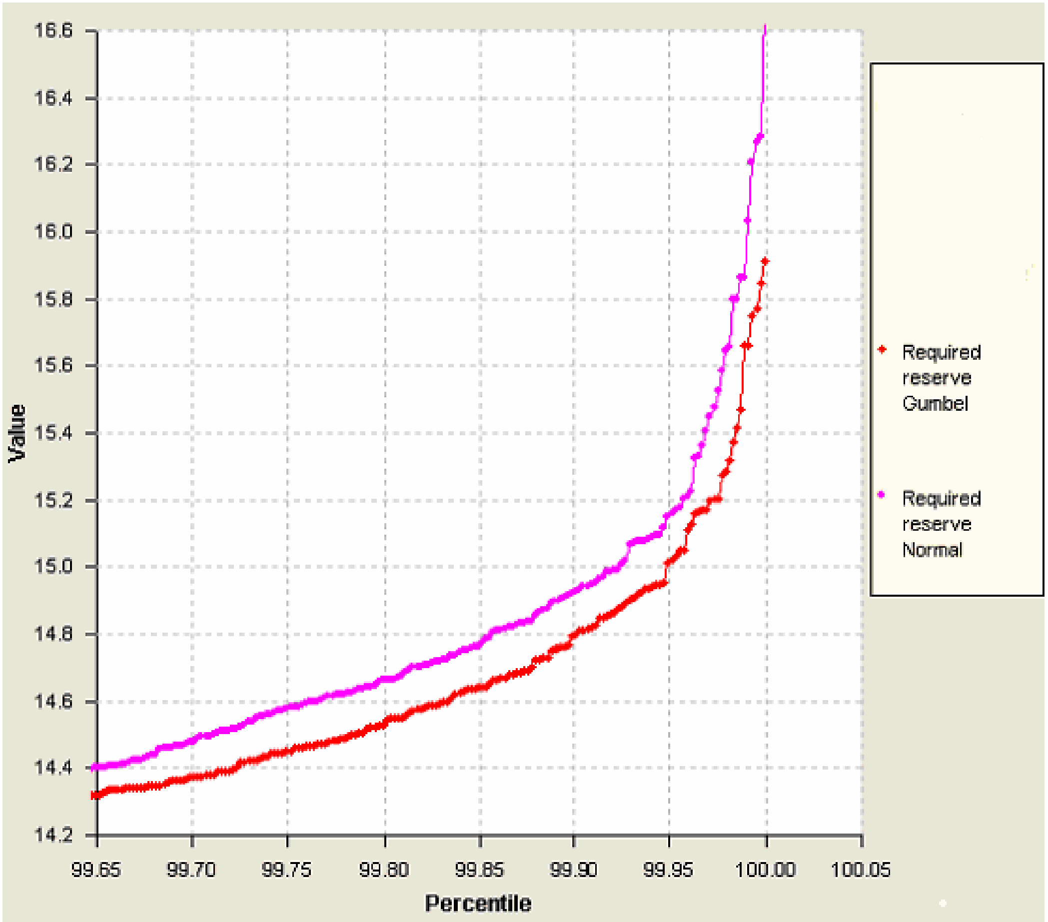

Figure 3: right tail of simulated distributions of RR(0) scenario 3,4

A marginal difference between the two reserves can be observed. The fact that the difference between the distributions under the two copulas is so small suggests that the two copulas generate very similar dependence structures. This is confirmed by the simulated rank scatter plots of the two copulas shown in figure 4 below.

A rank scatter plot shows simulated pairs of uniform random variables under a given dependence structure between the two variables. When realizations are spread evenly across the square, this indicates a low degree of dependence. A high degree of dependence is indicated by concentrations of points in certain parts of the square. For example tail correlation leads to a higher concentration of realizations in the corners of the square.

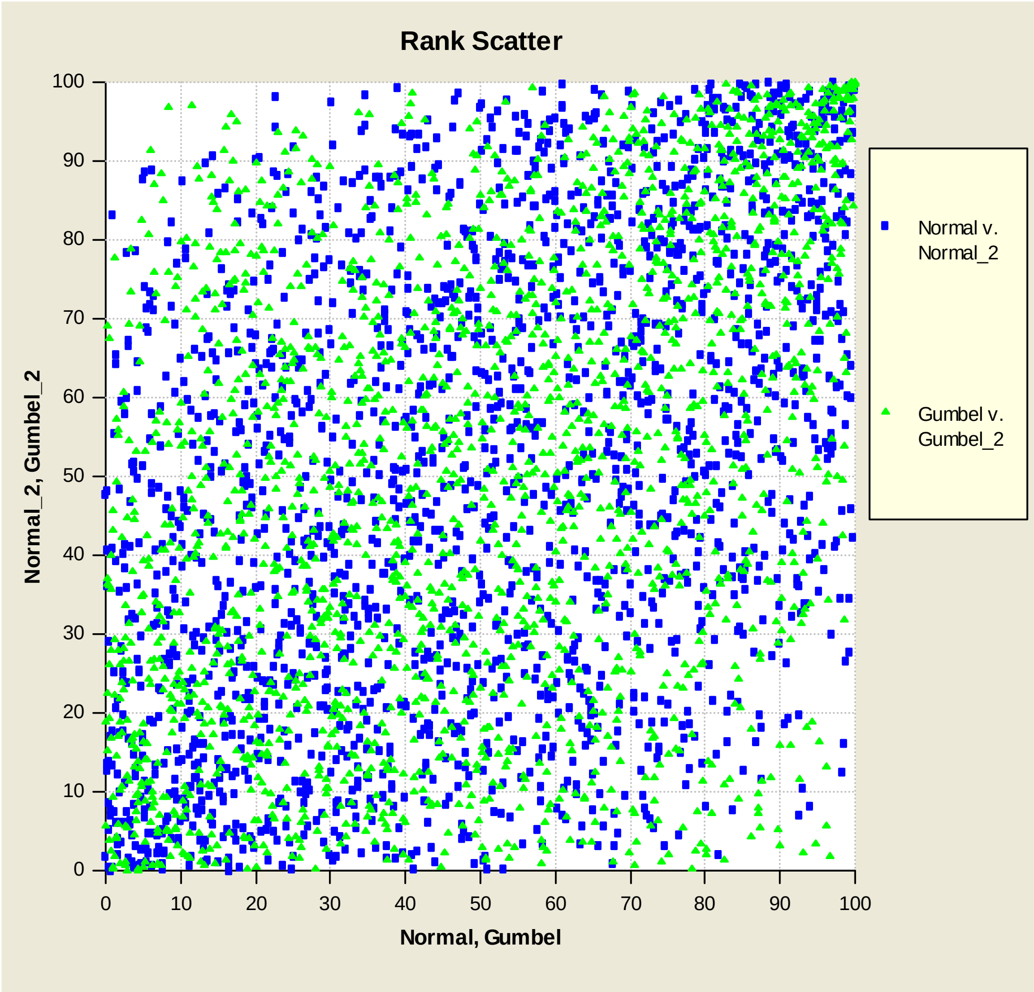

figure 4: rank scatter plot of simulated Normal and Gumbel copulas.

The two scatter plots shown in figure 4 show very similar patterns, both with a slightly lower density of points towards the upper left hand and lower right hand corner, and higher towards the other two corners. This indicates the dependence structures simulated by the two copulas are very similar.

4. RESULTS AND DISCUSSION

Dependence between interest and inflation rates has a considerable impact on the distribution of the required reserve. The parameterization of the copulas in alternatives 3 and 4 require a sufficiently large history of reliable data, and one needs to assume that the dependence structure does not change over time. The approach in alternative 2 however provides an upper bound with regard to the dependence between the random errors of the two time series. Hence alternative 2 may be preferable if a prudent approach is sought and historic data are not considered sufficiently reliable.

The difference between the Normal and the Gumbel copula and the impact on the distribution of the required reserve is minimal. The Gumbel copula gives a better fit to the data than the Clayton copula. A fit of the Normal copula can not be shown in the same way as it does not belong to the family of so-called 'Archimedean' copulas, see Valdez (1998).

Parameterization of an interest rate model based on historically observed rates may lead to results which are inconsistent with current market rates. Also, the use of a one-factor model can be regarded as too simplistic. However additional prudence can be built in by reducing the long term mean parameter *b* for example on the basis of projections by an economic forecasting bureau.

The long term average interest rate parameter *b* of 6.7% appears high in the current environment, and leads to a continuous upward trend in the projected future interest rate. Reducing *b* to 3% leads to an increase of the liability by about 6% across the distribution. Alternatively the CIR model can be parameterized on the basis of the current yield curve but this would not allow for the measurement of the correlation with inflation rates. Such measurement requires the availability of simultaneous observations of interest and inflation over an extended historic period.

Supplementary Material

Two spreadsheets are attached. One contains the parameterization of the time series, the other the parameterization of the copulas and the quantile plot.

Appendix I

1. Medical Inflation rates:

Area: *U.S. city*
Item: *Medical care*
Source:http://data.bls.gov/servlet/SurveyOutputServlet?data_tool=latest_numbers&series_id=CUUR0000SAM&output_view=pct_1mth

2. Interest rates:

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Rate of interest in money and capital markets
Federal Reserve System
Long-term or capital market
Government securities
Federal
Constant maturity
Three-year
Not seasonally adjusted
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Twelve months ending December

Source: http://www.federalreserve.gov/releases/h15/data.htm#fn12

Year	Medical Inflation rate (%)	Interest rate (%)
1962	2.02	3.47
1963	2.42	3.67
1964	2.02	4.03
1965	2.83	4.22
1966	6.69	5.23
1967	6.38	5.03
1968	6.27	5.68
1969	6.05	7.02
1970	7.44	7.29
1971	4.80	5.66
1972	3.45	5.72
1973	5.52	6.96
1974	12.56	7.84
1975	9.70	7.5
1976	10.14	6.77
1977	8.73	6.68
1978	8.73	8.29
1979	10.25	9.70
1980	10.03	11.51
1981	12.45	14.46
1982	11.02	12.93
1983	6.38	10.45
1984	6.48	11.92
1985	6.48	9.64
1986	7.87	7.06

1987	5.75	7.68
1988	6.91	8.26
1989	8.73	8.55
1990	9.70	8.26
1991	7.97	6.82
1992	6.48	5.30
1993	5.22	4.44
1994	4.80	6.27
1995	3.97	6.25
1996	3.04	5.99
1997	3.04	6.10
1998	3.45	5.14
1999	3.76	5.49
2000	4.18	6.22
2001	4.70	4.09
2002	4.90	3.10
2003	3.66	2.10
2004	4.28	2.78

Appendix II Parameterization of the time series

Interest rates

Parameters of the CIR model are:

a	0.085823
b	6.684528
σ	0.450159

Parameters are estimated by minimizing the sum of squared residuals on the basis of the data in appendix I.

Inflation rates

parameter	Estimate	Standard error
c_0	1.3470	
c_1	0.8441	0.13
c_2	- 0.0806	0.13
σ'	1.7151	

N.B. as c_2 is very small and not significantly different from 0, an AR(1) process (with $c_2 = 0$) will produce very similar results.

Parameter estimates are derived as[5]:

$$c_1 = \frac{r_1(1-r_2)}{1-r_1^2}$$

$$c_2 = \frac{r_2 - r_1^2}{1-r_1^2}$$

with r_1 and r_2 estimates of the first and second order autocorrelation:

$$r_k = \frac{\sum_{t=1}^{n-k}(x_t-\bar{x})(x_{t+k}-\bar{x})}{\sum_{t=1}^{n}(x_t-\bar{x})^2}$$, $k = 1,2$ and n the number of observations x.

c_0 is estimated such that the mean inflation rate is stationary and equal to the historical average:

$$c_0 = \frac{1}{n}\sum_{t=1}^{n} Inf(t)(1-c_1-c_2).$$

[5] See Box (1994)

Appendix III Simulated autocorrelations of interest and inflation rates

	Interest[1]	Interest[2]	Interest[3]	Interest[4]	Interest[5]	Interest[6]	Interest[7]	Interest[8]	Interest[9]	Interest[10]
Interest [1]	1.000	0.660	0.509	0.416	0.352	0.306	0.267	0.230	0.206	0.186
Interest [2]	0.660	1.000	0.766	0.625	0.529	0.457	0.399	0.347	0.310	0.277
Interest [3]	0.509	0.766	1.000	0.815	0.688	0.593	0.519	0.453	0.400	0.357
Interest [4]	0.416	0.625	0.815	1.000	0.842	0.724	0.633	0.556	0.490	0.435
Interest [5]	0.352	0.529	0.688	0.842	1.000	0.857	0.749	0.657	0.579	0.514
Interest [6]	0.306	0.457	0.593	0.724	0.857	1.000	0.868	0.760	0.670	0.594
Interest [7]	0.267	0.399	0.519	0.633	0.749	0.868	1.000	0.876	0.773	0.685
Interest [8]	0.230	0.347	0.453	0.556	0.657	0.760	0.876	1.000	0.881	0.780
Interest [9]	0.206	0.310	0.400	0.490	0.579	0.670	0.773	0.881	1.000	0.884
Interest [10]	0.186	0.277	0.357	0.435	0.514	0.594	0.685	0.780	0.884	1.000

	Inflation[1]	Inflation[2]	Inflation[3]	Inflation[4]	Inflation[5]	Inflation[6]	Inflation[7]	Inflation[8]	Inflation[9]	Inflation[10]
Inflation [1]	1.000	0.629	0.420	0.289	0.206	0.151	0.108	0.071	0.055	0.043
Inflation [2]	0.629	1.000	0.709	0.493	0.351	0.254	0.185	0.128	0.098	0.072
Inflation [3]	0.420	0.709	1.000	0.739	0.528	0.383	0.280	0.197	0.144	0.105
Inflation [4]	0.289	0.493	0.739	1.000	0.754	0.547	0.398	0.290	0.209	0.151
Inflation [5]	0.206	0.351	0.528	0.754	1.000	0.760	0.555	0.402	0.291	0.211
Inflation [6]	0.151	0.254	0.383	0.547	0.760	1.000	0.760	0.550	0.401	0.291
Inflation [7]	0.108	0.185	0.280	0.398	0.555	0.760	1.000	0.765	0.561	0.410
Inflation [8]	0.071	0.128	0.197	0.290	0.402	0.550	0.765	1.000	0.767	0.560
Inflation [9]	0.055	0.098	0.144	0.209	0.291	0.401	0.561	0.767	1.000	0.767
Inflation [10]	0.043	0.072	0.105	0.151	0.211	0.291	0.410	0.560	0.767	1.000

Appendix IV Quantile plot for Independent, Gumbel and Clayton copulas

A quantile plot (also know as Q-Q plot) can be used to inspect the goodness of fit of Archimedean copulas, and is derived as follows.

Archimedean copulas are of the form:

> $C_{\varphi}(u,v) = \varphi^{-1}(\varphi(u) + \varphi(v))$ with $0 < u,v \leq 1$ and φ a convex decreasing function with domain (0,1].

For two random variables X and Y with dependence defined by the Archimedean copula C_{φ}, it can be shown that the random Variable $Z = C_{\varphi}(F_X(X), F_Y(Y))$ has the following distribution function:

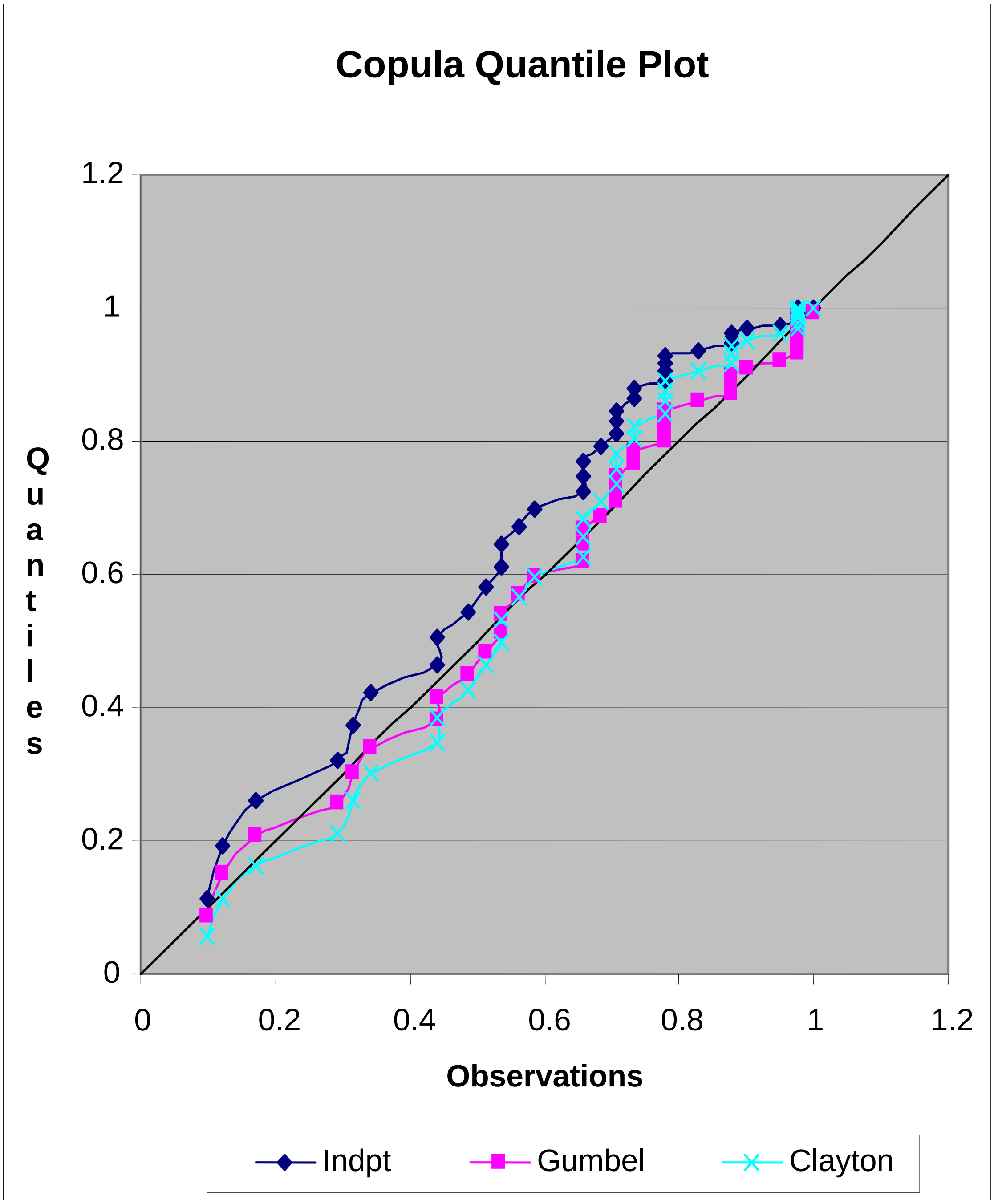
> $F_Z(z) = z - \varphi(z)/\varphi'(z)$.

This implies that, assuming the dependence between X and Y is described by a given Archimedean copula C_{φ}, the variable Z should follow the distribution function given above. Hence comparing n ordered (pseudo)-observations of Z with the percentiles of the distribution function of Z in a Q-Q plot allows for inspection of the goodness of fit of the assumed distribution of Z hence of the copula function C_{φ}. The observations of Z are derived from the observations of X and Y and the relation $Z = C_{\varphi}(F_X(X), F_Y(Y))$. The process of constructing the quantile plot and the underlying theory can be found in Valdez (1998).

The interpretation of the Q-Q plot is no different than the Q-Q plot for any other single random variable. The closer observations are to the corresponding percentiles of the theoretical distribution, the better the fit of the distribution. Hence a Q-Q plot showing a pattern close to the straight line through the origin and (1,1) indicates a good fit of the distribution.

The copulas used are:

Gumbel:	$C(u,v) = \exp\{-[(-\ln u)^{\alpha} + (-\ln v)^{\alpha}]^{1/\alpha}\}$,	$\varphi(u) = (-\ln u)^{\alpha}$
Clayton:	$C(u,v) = (u^{-\alpha} + v^{-\alpha} - 1)^{-1/\alpha}$,	$\varphi(u) = u^{-\alpha} - 1$
Independent:	$C(u,v) = uv$,	$\varphi(u) = -\ln u$.



Parameterization:

Gumbel (α)	1.41716
Clayton (α)	0.62773

Appendix V Linear correlations of interest and inflation rates

Simulated linear correlations between interest and inflation rates in alternative 3 under the Gumbel copula are as follows:

	Interest[1]	Interest[2]	Interest[3]	Interest[4]	Interest[5]	Interest[6]	Interest[7]	Interest[8]	Interest[9]	Interest[10]
Inflation [1]	0.436	0.289	0.214	0.176	0.148	0.124	0.105	0.094	0.083	0.070
Inflation [2]	0.277	0.430	0.322	0.259	0.219	0.187	0.159	0.140	0.126	0.111
Inflation [3]	0.188	0.314	0.431	0.346	0.290	0.248	0.214	0.187	0.169	0.151
Inflation [4]	0.131	0.220	0.324	0.423	0.356	0.305	0.263	0.229	0.204	0.183
Inflation [5]	0.094	0.158	0.236	0.324	0.420	0.360	0.312	0.274	0.242	0.220
Inflation [6]	0.067	0.114	0.169	0.234	0.320	0.417	0.362	0.320	0.284	0.254
Inflation [7]	0.052	0.086	0.127	0.173	0.238	0.323	0.414	0.366	0.323	0.288
Inflation [8]	0.035	0.059	0.089	0.121	0.169	0.236	0.319	0.409	0.360	0.319
Inflation [9]	0.027	0.045	0.068	0.090	0.127	0.175	0.237	0.316	0.403	0.358
Inflation [10]	0.022	0.037	0.052	0.069	0.096	0.131	0.177	0.235	0.313	0.398

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Biography of Hans Waszink

Hans Waszink works for Lloyds TSB General Insurance in the United Kingdom, where he is responsible for capital adequacy, investment strategy and reinsurance. He holds a Master's degree in Mathematics from the University of Groningen in the Netherlands, and a Master's degree in Actuarial Science from the University of Amsterdam, the Netherlands. He is a full member of the Actuarial Society of The Netherlands. Prior to joining Lloyds TSB in 2003, Hans worked for ING Group headquarters in Amsterdam as a general insurance actuary, and as an actuarial consultant for William M. Mercer in Amsterdam.

Hans currently serves on the IAA (International Actuarial Association) Solvency Subcommittee and the ABI (Association of British Insurers) Non-life Capital Working Party. Hans was co-author of the IAA publication 'A Global Framework for Insurer Solvency Assessment' which was published in 2004. He is currently pursuing an MBA at London Business School.