The Casualty Actuarial Society *Forum*

Fall 2004 Edition

Including the 2004 Reserves Call Papers and Reports from the 2003 Membership Survey Task Force and Research Working Party on Executive-Level Decision-Making Using Dynamic Risk Modeling

To CAS Members:

This is the Fall 2004 Edition of the Casualty Actuarial Society *Forum*. It contains eight Reserves Call Papers and five additional papers. Also included is the report of the 2003 CAS Membership Survey Task Force, which summarizes the results of the CAS five-year survey. In addition, we are pleased to include for the first time a report from the CAS Working Party on Executive-Level Decision-Making Using DRM. This working party is one of several new working parties created under the CAS Research and Development division.

The *Forum* is a nonrefereed journal printed by the Casualty Actuarial Society. The CAS is not responsible for statements or opinions expressed in the papers in this publication. See the Guide for Submission to CAS *Forum* (www.casact.org/aboutcas/forum.htm) for more information.

The CAS *Forum* is edited by the CAS Committee for the Casualty Actuarial Society *Forum*. Members of the committee invite all interested persons to submit papers on topics of interest to the actuarial community. Articles need not be written by a member of the CAS, but the paper's content must be relevant to the interests of the CAS membership. Members of the Committee for the Casualty Actuarial Society *Forum* request that the following procedures be followed when submitting an article for publication in the *Forum*:

1. Authors should submit a camera-ready original paper and two copies.
2. Authors should not number their pages.
3. All exhibits, tables, charts, and graphs should be in original format and camera-ready.
4. Authors should avoid using gray-shaded graphs, tables, or exhibits. Text and exhibits should be in solid black and white.
5. Authors should submit an electronic file of their paper using a popular word processing software (e.g., Microsoft Word and WordPerfect) for inclusion on the CAS Web Site.

The CAS *Forum* is printed periodically based on the number of call paper programs and articles submitted. The committee publishes two to four editions during each calendar year.

All comments or questions may be directed to the Committee for the Casualty Actuarial Society *Forum*.

Sincerely,

Glenn M. Walker, CAS *Forum* Chairperson

The 2004 CAS Reserves Call Papers
Presented at the
2004 Casualty Loss Reserve Seminar
September 13-14, 2004
The Mirage
Las Vegas, NV

The Fall 2004 Edition of the CAS *Forum* is a cooperative effort between the Committee for the CAS *Forum* and the CAS Committee on Reserves.

The CAS Committee on Reserves present for discussion eight papers prepared in response to their 2004 call for papers. This *Forum* includes papers that will be discussed by the authors at the 2004 Casualty Loss Reserves Seminar, September 13-14, 2004 in Las Vegas, NV.

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Reserves Call Papers

Working Party Report

Task Force Report

Additional Papers

Reconciliation and The Actuarial Opinion

Wendy Germani, FCAS, MAAA, and
Holmes Gwynn, ACAS, MAAA

Reconciliation And The Actuarial Opinion

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2004 Reserves Call Paper Program

Abstract

The opining actuary is required by ASOP 36 and the NAIC Property/Casualty Annual Statement Instructions to reconcile the data used in his or her analysis of the loss and loss adjustment reserves with Schedule P Part 1 in the Annual Statement.

This paper reviews the importance of a reconciliation, what data to include in a reconciliation, a description of the reconciliation process (including illustrative examples), and discussions of applicable Actuarial Standards of Practice.

While the emphasis here is on Schedule P, it is no less true for GAAP or for ratemaking exercises.

The authors gratefully acknowledge the help of Nicole Elliott, who has participated in the editing of this document. In addition we want to thank those regulatory actuaries who recognize the problems associated with reconciliations and encouraged us to produce this document. We hope it will bring some illumination to an area that we believe is critical to the credibility of the stated actuarial opinion.

CAVEAT: This paper is intended only as an aid and does not supercede the actuary's professional judgment, any Actuarial Standards of Practice or NAIC instructions.

The Importance of a Detailed Reconciliation

The actuarial opinion has become an increasingly important supplement to the Annual Statement since becoming a requirement in 1990. Even though the insurer is not committed to booking the reserves developed by the actuary, management has been under a growing pressure to book actuarially sound reserves. That pressure will likely continue to grow in years to come. As it does, actuarial integrity will be questioned and challenges will surface, be it from management, regulators, or actuarial peers.

Opinions will vary, but what should not vary is the underlying data used to determine the ultimate losses and the relationship that data has to the financial results of the company. A portion of the opining actuary's work product, which does not appear to be fully understood, is the reconciliation requirement.

The opiner states in the formal opinion that:

> "In forming my opinion on the loss and loss adjustment expense reserves, I relied upon data prepared by the responsible officers or employees of the company or group to which it belongs."[1]

[1] NAIC Annual Statement Instructions for Property/Casualty, 2003, p38

With these words, the actuary places a major caveat on the opinion being rendered. In this statement of reliance, the actuary is saying the opinion is only as good as the information given by company management. However, when things go wrong at a company, in one way or another, management has been responsible. So this statement, while perhaps making the opining actuary feel better, may detract from the credibility of the opinion itself.

Adding credibility back into the opinion, the opining actuary continues:

> "I evaluated that data for reasonableness and consistency. I also reconciled that data to Schedule P, Part 1 of the company's current Annual Statement."[2]

This language is very important to the integrity of the opinion. This paper is written to suggest ways to help the opining actuary take the necessary steps to demonstrate that the data used to form the opinion, relates to the data presented in Schedule P of the Annual Statement. This in turn relates to the financial pages of the Statement which reflect the statutory financial well-being of the company.

In general, the actuarial workpapers are very good at presenting the analysis performed by the actuary, but are less effective when attempting to substantiate that the data is reliable. When a reconciliation is included, it is frequently done on a total case reserves basis. This is often not enough to give the reviewer a

[2] IBID

comfort level that all losses have been considered and/or that the data is in the appropriate cells.

In one major case, an actuary used these words, but failed to reconcile the data. As a result, incorrect data was used to find the company's reserves so inadequate as to make it insolvent, placing the company in receivership. At a subsequent trial, it was found that the actuary used incorrect data. The company was not insolvent, and a jury made an $11 million malpractice damage award against the actuaries.[3] This example demonstrates why the actuary needs to do more than just rely on the data given.

More frequently the example works the other way. If one were to review the insolvencies in recent history, more often than not, inadequate reserves would be involved. The reserve inadequacy is usually not due to actuarial incompetence, but to data quality issues. Therefore, the actuary when reviewing the data for reasonableness and consistency should consider every possible aspect of the data. The cases in this paper not only relate the reconciliation to the bottom line, but also to the detail.

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[3] Dailey, Joseph and Selznick, Loren, "Navigating The Litigation Minefield: A Guide To Actuarial Malpractice Claims", Mealey's Litigation Report; Insurance Insolvency, Vol. 14 #5

When considering the detail necessary, the actuary should be guided by ASOP #9:

> "Documentation should be sufficient for another actuary practicing in the same field to evaluate the work. The documentation should describe clearly the sources of data, the material assumptions, and methods."[4]

The greater the detail in the reconciliation, the more credibility can be placed on the words "reasonable" and "consistent" used by the opining actuary.

Relationship to Accounting

As opining actuaries complete their analysis, they generally turn their results over to accountants to complete the financial reporting. The accountants may then allocate the IBNR reserves to the various lines of business and/or companies based on a predefined allocation process. The greatest difficulty usually occurs when the two disciplines within the company have an undefined dual responsibility which is not mutually understood. Communication between the two disciplines is vital to the reconciliation process.

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[4] Actuarial Standards Board of the American Academy of Actuaries, "Actuarial Standard of Practice No. 9, Documentation and Disclosure in Property and Casualty Insurance Ratemaking, Loss Reserving, and Valuations, Paragraph 5.2

Two links should be established with regard to Schedule P. The first link is between Schedule P and the reserves on Page 3 (Liabilities.) This tie is rarely violated and is well understood by the accountant and the opining actuary.

The second link is the tie between the actuarial reserves and the Schedule P reserves. The reconciliation provides that link. In addition to increasing the credibility of the opinion, a good reconciliation provides the reviewing actuary with a better understanding how the actuarial workpapers relate to Schedule P.

As the actuary states reliance on other officers of the company for data quality, the actuary should assume or share responsibility for how the actuarial work product is reflected or relates to the financial statements of the company. The financial statements, of course, contain the numbers on which the actuary states the opinion. The opining actuary, while not responsible for the audit of Schedule P, needs to be sure the work product is represented correctly in Schedule P, and/or that Schedule P correctly reflects the opinion. The actuary does not have to personally do the reconciliation, but is responsible for the work product and the level of detail in the reconciliation.

As an additional check, in 2004 the NAIC has added an instruction to the auditor to subject the data used by the appointed actuary to testing procedures.

> "The auditor is required to determine what historical data and methods have been used by management in developing the loss reserve estimate and whether the auditor will rely on the same data or different statistical data in evaluating the reasonableness of the loss reserve estimate... Through inquiry of the Appointed Actuary, the auditor should obtain an understanding of the data identified by the Appointed Actuary as significant." [5]

These instructions point to a need for the auditor to better understand the actuarial database on which the reserve estimate is based. While the construction of a good reconciliation may be an arduous task, the benefits are worthwhile. If the actuary and the accountant have an understanding up front that the reconciliation is an important part of his work product, then they can make the construction of the reconciliation a joint project. In this instance the auditors will also derive an extra benefit in their review of the company.

The reconciliation should be thorough enough to demonstrate the actuary has considered all loss information from the actuarial database, as well as loss information not included in the actuarial analysis, but is reflected in Schedule P.

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[5] 2004 NIAC Annual Statement Instructions Property & Casualty, 11/2003 Nonsubstantive Revisions, Page 48: Annual Audited Financial Reports- Item 9.

Data To Reconcile

The P&C Practice Note[6] gives a good synopsis of what is required. Data elements from Schedule P, Part 1 (by line) to be reconciled to the actuarial database are:

"A. each of the following types of data, if relied upon significantly in forming the actuarial opinion...

- paid losses;
- incurred (case basis) losses;
- paid defense and cost containment expenses;
- incurred (case basis) defense and cost containment expense;
- paid adjusting and other expenses, and
- earned premiums.

B. the reconciliation consisted of comparing the changes from the prior year-end values (e.g., current calendar year paid losses and changes in case basis loss reserves), in detail by line of business and year in which losses were incurred to the extent that such detail was relied upon significantly and is provided in Schedule P...."

This language suggests that incremental reconciliations may be acceptable.

[6] Property and Casualty Practice Note, December 2003, Statements of Actuarial Opinion on P & C Loss Reserves as of December 31, 2003, Appendix 1

Notice IBNR is not included in this list. The company is not obligated to book the opining actuary's indicated IBNR. An exhibit of the IBNR as computed in the actuarial workpapers versus the booked IBNR by line and accident year is recommended. It would assist the reviewer in the analysis of the opining actuary's workpapers.

Organization of Data

Schedule P has very precise definitions regarding its data elements. The actuary seeks data that fits the characteristics of the risk. The actuarial data and Schedule P data frequently differ in groupings or amounts. The actuary uses different groupings because of homogeneity issues, such as claims handling and underwriting characteristics, and unique coverage applications such as excess and deductible coverages. Examples are:

- The company may write a commercial risk(s) on a program basis since this is the way internal underwriting is structured. The data can be grouped with similar programs or even stand on its own, or data can be assembled by other than an accident year basis.
- Workers compensation laws vary by state, to the point one may want to combine only certain states for actuarial analysis.
- Workers comp data may allow analysis by medical verses indemnity, or deductible options.

- The multi-perils lines may cause problems where property and liability coverages are combined under one Schedule P line of business (LOB.) In such instances, the experience may be split into separate components for actuarial review.
- Policy or report year data is often used because of the better fit for the coverage under evaluation. If policy year or report year data is used the actuary should know how these losses are sorted back into accident year components.
- With personal lines, data may be grouped by state to reflect differing tort laws, different coverages such as uninsured motorist or PIP, or different hazard conditions, (e.g., exposure to mold.)
- A common situation occurs when the actuary performs detailed analysis based on nine month data. This gives the actuary a head start toward producing year-end ultimate losses. The idea is to use nine month triangles for the analysis, then make a projection of year-end ultimates, including a projection of fourth quarter paid losses and reserves for the current year.

These are all legitimate reasons to perform reserve analysis on other than a Schedule P basis. Regardless of how the actuarial database is constructed, at the end of the process:

- the actuary should have a good working knowledge of how his data is brought into Schedule P,

- what additional data may be in Schedule P that was not part of the actuarial database or analysis, and
- what assumptions have been made to get the data into accident year and line of business format.

The opining actuary should demonstrate, in exhibit form, how Schedule P reflects the actuarial database, including explanations for any differences. Without the reconciliation, the reviewer, can not establish that the actuary's work product is related to the company's financial statement.

How to Reconcile

A reconciliation should be made of any data relied on significantly in evaluating the reserves of a company. According to the CAS "Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expense Reserves":

> "Whatever data are used in analysis of reserves, they must reconcile to the insurer's financial records."

If the actuary opines on gross as well as net reserves, the reconciliation should be done on each basis. The reconciliation should also be done by accident year, usually for the current ten years.

For paid data it is best to reconcile cumulative paid loss, defense and cost containment expenses (DCCE) and/or adjusting and other expenses (A&OE). Reconciliations on a cumulative paid basis may be difficult to do if the company has data from assigned risk pools or other external sources. If a reconciliation is done every year, as should be the case, it is acceptable to have a reconciliation done for the incremental paid amounts. The actuary needs to state in the Actuarial Report that a reconciliation has been done this way for x years and to make the reconciliations available for x years (most likely in an appendix).

If the reserve analysis is done before paid salvage and subrogation with a separate analysis of salvage and subrogation, the reconciliation should also include the salvage and subrogation data.

If earned premium is used to apply the Bornhuetter-Ferguson method or other methods which rely on earned premium, the earned premium also needs to be reconciled. If the Schedule P Part 1 earned premium reconciliation was made the previous year, only the current accident year earned premium needs to be reconciled. If there has been a change in the premium data due to errors in the data or reflections of the effects of pooling , then earned premiums for all accident years need to be reconciled.

Claim counts are frequently used for a variety of purposes including: (1) a check for reasonableness of severities; (2) methods such as Fisher-Lange and

Berquist-Sherman; or (3) evaluating asbestos reserves. If the actuary has placed significant reliance on the claim counts (reported and/or outstanding), then they should be reconciled to the data in Schedule P Part 5.

Schedule P Part 1 includes data after tabular discounting, while Part 2 is before tabular discounting. Analysis done using Part 2 should include the adjustments made for tabular discounting to reconcile the data to Part 1.

Given some of the circumstances discussed above, several types of reconciliations are presented at the end of the paper. The data may be divided into several subgroups for a particular line of business, such as medical and indemnity for Workers Comp (Case 1); or several affiliated companies may pool their data and the actuary looks at the combined data for all companies (Case 2); or the actuary may group data across several lines of business, such as bodily injury data for Other Liability Occurrence and Products Occurrence (Case 3).

Normally in a reserve analysis, there are groups of data or adjustments which the actuary does not evaluate. This includes data from assigned risk pools, residual markets, or adjustments for reinsurance treaties which affect several lines of business at once. This data should be shown in the reconciliation in separate columns. (Cases 1 and 2)

If the data and/or Schedule P data needs to be aggregated; this should be minimized to the extent possible, i.e. maintain as much detail as possible. For example, the data used for analysis may be subdivided by the line of business in order to be reconciled back to the Schedule P line of business. (Case 3)

Loss adjustment expense reserves are a particular problem observed in many workpapers. Most often, the opining actuary has continued to collect data according to the pre-1998 definitions of allocated (ALAE) and unallocated loss adjustment expense (ULAE). However, the actuary should try to have the underlying data in accordance with the new definition. This may not be possible due to the way the company collects and/or records the expenses. The reconciliation should reflect this fact and indicate what has been done to allocate the data to the expense components as currently defined. The data should still be reconciled by accident year to the appropriate columns in Schedule P, showing the amounts reclassified as DCCE or A&OE. (Case 4)

Another problem occurs when A&OE data are grouped by payment year. If the analysis is done this way, the reconciliation should include the incremental paid amounts for all accident years by line of business, or an explanation of how the payment year data has been allocated to accident year.

If the analysis is done on a policy year or report year basis, the actuary should try to get the data refined to an accident year/policy year basis; e.g. accident year 2000 policy year 1999, 2000. The data by policy year can then be reconciled to the actuary's data and the total for each accident year can be reconciled to Schedule P. If this is not possible, then only total case reserves and incremental paids can be reconciled to Schedule P by line of business. An explanation should then be included of how Schedule P was constructed. (Case 5)

Often the actuary's analysis is based on some date prior to the calendar year end. At year end, the actuary normally compares expected year-end values (as calculated in the analysis at 9 months) with actual year-end results. The reconciliation would then be based on a comparison of the actual year-end results as shown in the workpapers with the Schedule P data. (Case 6)

When IBNR becomes part of the reconciliation process then diligence is necessary to be sure it is allocated to the appropriate accident years and lines of business. (Case 7)

Examples

The following cases are examples of how reconciliations might be done. Different formats may certainly be used but the key points are to show how the actuarial data relates to the Annual Statement line of business and accident year detail and to document/ discuss any differences. The exhibits underlying each of the cases are included in the Appendix.

Case 1

XYZ Insurance Company writes Workers Compensation coverage. The opining actuary has chosen to evaluate Medical and Indemnity losses separately due to their different development patterns. The company also has experience from its share of the NCCI Workers Comp Pool and it books the reserves as reported by the pool. Exhibit 1 illustrates a reconciliation of the net case loss reserves. It shows Schedule P data in the first column, followed by the case reserves for medical and indemnity used by the actuary (columns 2 and 3.) Columns 2 and 3 should reference the exhibits used in the Actuarial Report as shown in the footnotes in Exhibit 1. Column 5 shows NCCI Pool data as provided by the Company. Column 7 shows the unreconciled differences which on an accident year basis show larger differences than for the total for all years combined.

Case 2

Exhibits 2 and 3 show a reconciliation for paid loss and DCCE on an incremental paid basis and on a cumulative paid basis. ABC and XYZ Insurance Companies are affiliated companies and their homeowners policies are covered by a single Stop Loss Treaty. The actuary combines the data for the two companies for his/her analysis excluding the effects of the Stop Loss Treaty.

Exhibit 2 shows the reconciliation on an incremental paid basis. Using cumulative paid amounts from the current year and prior year Schedule P Part 1, the incremental paid amount is calculated in Column 5. Columns 6, 7, and 8 are from the actuarial triangles of paid data. Column 9 shows the paid amounts from the Stop Loss Treaty. To adjust the actuarial data, the Stop Loss Treaty data is subtracted as shown in Column 10.

A comparison of the difference in adjusted actuarial data and Schedule P data for all accident years combined shows a difference of less than 1%, while individual accident years have a difference of as high as 28%. It is probable that some stop loss information was put in the wrong accident year, but this difference needs to be explored and either corrected or explained by the actuary in the Actuarial Report.

Exhibit 3 shows the reconciliation on a cumulative basis. Accident year 1998 now has a large difference, which did not show up in the incremental comparison. Again, there are large differences for individual accident years, but overall the difference for all years combined is less than 1%. If the actuarial data is wrong, a large difference for an individual accident year could influence the choice of development factors. Note the paid development factor for accident year 1998, based on Schedule P data in Exhibit 2, is 1.19 = ((236+92)/(207+69)), while the paid development factor base on the actuarial data is 1.34 = (369/276).

Case 3

In Case 3, the experience for Other Liability and Products Liability is combined for analysis and then divided into Bodily Injury and Property Damage. A reconciliation of the underlying data to the actuarial data should be made (see Exhibit 4) as well as a separate reconciliation to Schedule P data (see Exhibit 5). There are some differences in the reconciliation of the underlying data to Schedule P data. This may lead to incorrect allocations of IBNR back to the individual Schedule P lines of business.

Case 4

In this analysis of loss adjustment expenses, the company has provided the actuary data based on the old definitions of ALAE and ULAE. The actuary's analysis is done on this basis and the actuary gives a point estimate for ultimate values for ALAE and ULAE. The Company then allocates IBNR to the appropriate categories of DCCE and A&OE. The reconciliation should include the amounts transferred from ALAE to A&OE and the amount transferred back from ULAE to DCCE. This information should be available from the company. Exhibit 6 illustrates an example of this type of reconciliation for Workers Compensation net paid amounts.

Case 5

In this case of Workers Compensation for Casualty Insurance Company, the actuary prefers to use policy year data (see Exhibit 7.) The actuarial data is first reconciled to the policy year information. For example policy year 2002, (Exhibit 7 Column 5), total case reserves of 36,000 are used as the latest evaluation point to compare with the data used for the actuarial analysis. Then the total amounts in column 7 are reconciled to Schedule P. This example shows unreconciled data points by policy year and by accident year with the overall unreconciled difference less than 1%. The actuary needs to determine whether or not the differences by individual year are material.

Case 6

In this case, the actuary does the analysis of Commercial Auto experience based on third quarter data. The actuary makes projections of expected year-end results for paid loss and DCCE and case reserves, and compares them to the year-end data for reasonableness. Exhibit 8 has a reconciliation for the paid losses and DCCE. Column 3 reflects the expected paid losses based on the third quarter analysis. Column 4 contains the actual paid losses from the actuarial data as supplied by the company at year end. The differences between the actual actuarial data and Schedule P for accident year 2003 (Column 5) probably should be investigated and discussed.

CASE 7

As mentioned earlier, it is sometimes helpful to include a reconciliation of IBNR. Exhibit 9 shows such a reconciliation. Case reserves and IBNR reserves are both reconciled. The case reserves are reasonably close, but the IBNR has moved around to different accident years. The current accident year is carried at a much lower ultimate value. The opining actuary might want to investigate this further.

What To Do If Data Does Not Reconcile

Frequently, differences between the actuarial data and schedule P data occur as illustrated in the examples. When does this become a problem? If the differences are due to voluntary or involuntary pools such as workers compensation pools which are reviewed by another actuary, the opining actuary can accept the review and include appropriate documentation in the reconciliation.

Other times, the discrepancy is due to inaccurate or incomplete data. ASOP No. 23 – "Data Quality" states:

> "The actuary may be aware that the data are incomplete, inaccurate, or not as appropriate as desired. In such cases, the actuary should consider whether the use of such imperfect data may produce material biases in the results of

the study, or whether the data are so inadequate that the data cannot be used to satisfy the purpose of the study."[7]

If this is the case and the material difference cannot be reconciled, the opining actuary should so state in the Opinion, with more detail in the Actuarial Report.

Conclusion

In order to achieve the maximum level of credibility for the Actuarial Opinion, the reconciliation of the actuarial database to Schedule P is of utmost importance. While bottom line reconciliations are important, the actuary should also make sure there is a documented relationship between Schedule P and the data underlying the opinion.

Schedule P is usually constructed by non-actuaries and it is possible that Schedule P integrity can be compromised even if bottom line results are not. The actuary needs to take co-responsibility in the development of the Schedule. In doing so , the reconciliations discussed are easier and more meaningful.

Several examples of detailed reconciliations have been presented here. Although guidance on reconciliations is limited, the COPFLR Practice Note contains further useful information in Appendix 1.

[7] Actuarial Standards Board of the American Academy of Actuaries, "Actuarial Standard of Practice No. 23, Data Quality", Section 5.2

When the opining actuary makes the statement "I have reconciled the data…," the opiner can be satisfied that a detailed reconciliation will provide a high level of confidence that the actuary's reserve analysis is correctly recognized in Schedule P.

This paper is written not only to encourage better reconciliations from actuaries, but to help any preparer of Schedule P better understand the importance of including the actuary in the process.

References

[1.] Actuarial Standards Board of the American Academy of Actuaries, "Actuarial Standard of Practice, No. 9, "Documentation and Disclosure in Property and Casualty Insurance Ratemaking, Loss Reserving, and Valuations"

[2.] Actuarial Standards Board of the American Academy of Actuaries, "Actuarial Standard of Practice No. 23, Data Quality"

[3.] Actuarial Standards Board of the American Academy of Actuaries, "Actuarial Standard of Practice, No. 36, Statements of Actuarial Opinion Regarding Property/Casualty Loss and Loss Adjustment Expense Reserves."

[4.] CAS, "Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expense Reserves." (Adopted by the Board of Directors of the CAS, May 1988)

[5.] CAS Valuation, Finance, and Investments Committee, "Materiality and ASOP No. 36: Considerations for the Practicing Actuary."

[6.] Committee on Property and Liability Financial Reporting, (COPLFR), American Academy of Actuaries, "Property and Casualty Practice Note, Statements of Actuarial Opinion on P&C loss Reserves as of December 31, 2002."

[7.] National Association of Insurance Commissioners, "Annual Statement Instructions, Property and Casualty, 2003"

[8.] National Association of Insurance Commissioners, "Official 2003 NAIC Annual Statement Blanks, Property and Casualty."

[9.] National Association of Insurance Commissioners, Accounting Practices and Procedures Manual, 2003, Preamble

[10.] Quigley, Robert C., "Underlying Data the Actuary's Achilles' Heel," Mealey's Litigation Report: Insurance Insolvency, 2001

Appendix

Seven Case Examples

Of Reconciliations

Exhibit 1

Case 1
Reconciliation to Schedule P
Workers Compensation
For XYZ Insurance Company

Reconciliation of Net Case Loss Reserves
Evaluation as of 12/31/2003

	Schedule P	Actuarial Data						
	Case Loss Reserves	Case Reserves Medical	Case Reserves Indemnity	Total Case Reserves	Work Comp Pool Case Reserves	Adjusted Actuarial Case Reserves	Unreconciled Difference $	Unreconciled Difference as % of Carried Case Loss Reserves
Accident Year	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Prior	42,663	8,168	33,751	41,919	600	42,519	144	0.3%
1994	13,270	2,553	10,547	13,100	42	13,142	128	1.0%
1995	14,744	2,725	11,719	14,444	300	14,744	-	0.0%
1996	21,378	5,245	13,636	18,881	2,598	21,479	(101)	-0.5%
1997	20,622	6,233	10,389	16,622	4,000	20,622	-	0.0%
1998	20,528	7,212	10,303	17,515	3,000	20,515	13	0.1%
1999	13,358	4,838	7,928	12,766	580	13,346	12	0.1%
2000	20,497	8,233	10,703	18,936	1,500	20,436	61	0.3%
2001	29,235	12,775	13,646	26,421	3,164	29,585	(350)	-1.2%
2002	49,525	22,354	22,354	44,708	4,500	49,208	317	0.6%
2003	72,774	36,612	25,628	62,240	10,500	72,740	34	0.0%
Total	318,593	116,948	170,604	287,551	30,784	318,335	258	0.1%

Notes:

(1) = Col 13 - 14 from Schedule P Part 1D
(2) See Actuarial Data (eg. EX 3, Col 2)
(3) See Actuarial Data (eg. Ex. 4 Col 2)
(4) = (2) + (3)

(5) From Company Data
(6) = (4) + (5)
(7) = (1) - (6)
(8) = (7) / (1)

Exhibit 2

Case 2
Reconciliation to Schedule P
Homeowners
For ABC Insurance Company and XYZ Insurance Company on a Combined Basis

Reconciliation of Net Incremental Paid Losses and DCCE
('000's omitted)

	Schedule P Data					Actuarial Data						
	ABC Insurance Cum. Paid Loss & DCCE as of 12/31/2002	ABC Insurance Cum. Paid Loss & DCCE as of 12/31/2003	XYZ Insurance Cum. Paid Loss & DCCE as of 12/31/2002	XYZ Insurance Cum. Paid Loss & DCCE as of 12/31/2003	Incremental Paid Loss & DCCE in 2003	Cum. Paid Loss and DCCE as of 12/31/2002	Cum. Paid Loss and DCCE as of 12/31/2003	Incremental Paid Loss & DCCE in 2003	Excess Stop Loss Treaty	Total Adjusted Actuarial Data	Difference $	%
Year	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1994	14	14	5	5	-	19	19	-	-	-	-	
1995	180	190	60	63	13	240	253	13	-	13	-	0.00%
1996	233	291	78	97	77	310	388	78	-	78	(1)	-1.30%
1997	209	246	153	132	16	462	529	67	50	17	(1)	-6.25%
1998	207	236	69	92	52	276	369	93	40	53	(1)	-1.92%
1999	126	180	52	70	72	207	280	73	-	73	(1)	-1.39%
2000	183	262	61	87	105	244	349	105	-	105	-	0.00%
2001	271	327	90	113	79	362	452	90	-	90	(11)	-13.92%
2002	596	643	199	210	58	795	837	42	-	42	16	27.59%
2003	-	608	-	207	815	-	827	827	12	815	-	0.00%
Total	2,019	2,997	767	1,076	1,287	2,915	4,303	1,388	102	1,286	1	0.08%

Notes:

(1), (2), (3), (4) = Col 4 - 5 + 6 - 7 from Schedule P Part 1A
(5) = (1) - (2) + (3) - (4)
(6) See Actuarial Data (eg. Ex. 1 Col 4)
(7) See Actuarial Data (eg.Ex. 1 Col. 3)
(8) = (7) - (6)

(9) From Company
(10) = (8) - (9)
(11) = (5) - (10)
(12) = (11) / (5)

Exhibit 3

Case 2

Reconciliation to Schedule P

Homeowners

For ABC Insurance Company and XYZ Insurance Company on a Combined Basis

Reconciliation of Net Cummulative Paid Losses and DCCE

('000's omitted)

	Schedule P Data			Actuarial Data				
	ABC Insurance Cum. Paid Loss & DCCE as of 12/31/2003	XYZ Insurance Cum. Paid Loss & DCCE as of 12/31/2003	Cummulative Paid Loss & DCCE	Cum. Paid Loss and DCCE as of 12/31/2003	Paid Excess Stop Loss Treaty	Adjusted Actuarial Paid	Difference $	Difference %
Year	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1994	14	5	19	19	-	19	-	0.00%
1995	190	63	253	253	-	253	-	0.00%
1996	291	97	388	388	-	388	-	0.00%
1997	246	132	378	529	150	379	(1)	-0.26%
1998	236	92	328	369	40	329	(1)	-0.30%
1999	180	70	250	280	-	280	(30)	-12.00%
2000	262	87	349	349	-	349	-	0.00%
2001	327	113	440	452	-	452	(12)	-2.73%
2002	643	210	853	837	-	837	16	1.88%
2003	608	207	815	827	12	815	-	0.00%
Total	2,997	1,076	4,073	4,303	202	4,101	(28)	-0.69%

Notes:

(1), (2) = Col 4 - 5 + 6 - 7 from Schedule P Part 1A

(3) = (1) + (2)

(4) See Actuarial Data (eg. Ex. 1 Col 4)

(5) From Company

(6) = (4) - (5)

(7) = (5) - (6)

(8) = (7) / (3)

Exhibit 4

Case 3
Reconciliation to Schedule P
Other Liability & Products Liability
For XYZ Insurance Company

Reconciliation of Net Case Loss Reserves Underlying Data and Actuarial Data

Evaluation as of 12/31/2003

	Underlying Data			Actuarial		Underlying Data			Actuarial	
	Case Reserves BI Other Liability	Case Reserves BI Products Liability	Total BI Case Reserves	BI Case Reserves	Difference Underlying Data - Actuarial Data	Case Reserves PD Other Liability	Case Reserves PD Products Liability	PD Case Reserves	PD Case Reseves	Difference Underlying Data - Actuarial Data
Year	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1994	71	594	665	665	-	-	-	-	-	-
1995	725	3,719	4,444	4,444	-	-	-	-	-	-
1996	5,245	13,636	18,881	18,881	-	100	200	300	300	-
1997	6,233	10,389	16,622	16,622	-	550	350	900	900	-
1998	7,212	10,303	17,515	17,515	-	1,050	2,000	3,050	3,050	-
1999	4,838	7,928	12,766	12,766	-	5,200	1,500	6,700	6,700	-
2000	8,233	10,703	18,936	18,937	(1)	2,500	1,500	4,000	4,000	-
2001	12,775	13,646	26,421	26,421	-	8,700	6,500	15,200	15,200	-
2002	22,354	22,354	44,708	44,708	-	15,000	8,000	23,000	23,000	-
2003	36,612	25,628	62,240	62,239	1	23,000	21,000	44,000	44,000	-
Total	104,297	118,898	223,198	223,198	-	56,094	41,043	97,150	97,150	-

Notes

(1), (2), (6), (7) From Underlying Data

(3) = (1) + (2)

(4) See Actuarial Data (eg. Ex 3, Col 2)

(5) = (3) - (4)

(8) = (6) +(7)

(9) See Actuarial Data (eg Ex 6 Col 2)

(10) = (8) - (9)

Exhibit 5

Case 3
Reconciliation to Schedule P
Other Liability & Products Liability
For XYZ Insurance Company

Reconciliation of Net Case Loss Reserves Underlying Data and Actuarial Data

Evaluation as of 12/31/2003

Year	**Schedule P** Other Liability Case Reserves (1)	**Actuarial** Case Reserves (2)	Unreconciled Difference (3)	Difference as % of Reserves (4)	**Schedule P** Products Liability Case Reserves (5)	**Actuarial** Case Reserves (6)	Unreconciled Difference (7)	Difference as % of Reserves (8)
1994	71	71	-		594	594	-	
1995	725	725	-	0.0%	3,719	3,719	-	0.0%
1996	5,095	5,345	(250)	-4.9%	13,836	13,836	-	0.0%
1997	6,783	6,783	-	0.0%	10,989	10,739	250	2.3%
1998	8,262	8,262	-	0.0%	12,303	12,303	-	0.0%
1999	10,163	10,038	125	1.2%	9,303	9,428	(125)	-1.3%
2000	10,733	10,733	-	0.0%	12,203	12,203	-	0.0%
2001	21,475	21,475	-	0.0%	20,146	20,146	-	0.0%
2002	37,354	37,354	-	0.0%	30,354	30,354	-	0.0%
2003	59,312	59,612	(300)	-0.5%	47,078	46,628	450	1.0%
Total	159,973	160,398	(425)	-0.3%	160,525	159,950	575	0.4%

Notes:

(1) From Schedule P Part 1H1 (Col 13 - 14)
(2) = Ex 4 col 1 + Ex 4 Col 6
(3) = (1) - (2)
(4) = (3) / (1)

(5) From Schedule P Part 1 R1 (Col 13 - Col 14)
(6) = Ex 4 Col 2 + Ex 4 Col 7
(7) = (5) - (6)
(8) = (7) / (5)

Exhibit 6

Case 4
Reconciliation to Schedule P
Workers Compensation
For XYZ Insurance Company

Reconciliation of Net Paid DCCE and A & OE

Evaluation as of 12/31/2003

	Schedule P		Actuarial Data				Adjusted Actuarial Data			
	Paid DCCE	Paid A & OE	Paid ALAE per analysis	Paid ULAE per analysis	Transferred from ALAE to A & OE	Transferred from ULAE To DCCE	Paid DCCE	Paid A & OE	Unreconciled Difference DCCE	Unreconciled Difference A & OE
Year	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1994	1,451	5,563	1,185	5,829	290	556	1,451	5,563	-	-
1995	5,529	18,277	4,807	18,999	1,106	1,828	5,529	18,277	-	-
1996	11,619	34,350	11,148	34,921	2,424	3,395	12,119	33,950	(500)	400
1997	11,267	37,447	9,776	38,938	2,253	3,745	11,267	37,447	-	-
1998	8,877	31,053	7,547	32,383	1,775	3,105	8,877	31,053	-	-
1999	6,025	20,712	4,819	21,418	1,145	2,051	5,725	20,512	300	200
2000	8,705	14,070	8,989	14,286	1,741	1,457	8,705	14,570	-	(500)
2001	10,761	15,627	11,351	15,038	2,152	1,563	10,761	15,627	-	-
2002	6,474	15,357	6,233	15,598	1,295	1,536	6,474	15,357	-	-
2003	2,514	11,109	1,906	11,717	503	1,111	2,514	11,109	-	-
Total	73,222	203,565	67,760	209,127	14,684	20,347	73,422	203,465	(200)	100

Notes:

(1) = Col 6-7 from Schedule P Part 1 D

(2) = Col 8 - 9 From Schedule P Part 1 D

(3) See Actuarial Data (eg. Ex. 1 Col 4)

(4) See Actuarial Data (eg. Ex. 2 Col. 4)

(5), (6) From Company Data (or from accounting)

(7) = (3) - (5) + (6)

(8) = (4) + (5) - (6)

(9) = (1) - (7)

(10) = (2) - (8)

Exhibit 7

Case 5
Data is Analyzed on a Report Year Basis
Reconciliation to Schedule P
Workers Compensation
For Casualty Insurance Company

Reconciliation of Case Loss Reserves

Evaluation as of 12/31/2003

Data from Accounting

Accident Year	Policy Year 1998 (1)	Policy Year 1999 (2)	Policy Year 2000 (3)	Policy Year 2001 (4)	Policy Year 2002 (5)	Policy Year 2003 (6)	Total (7)
1998	10,400	9,050	8,560	7,500	6,500	5,000	47,010
1999		4,388	7,928	9,500	8,400	6,000	36,216
2000			8,233	10,703	12,543	10,000	41,479
2001				12,775	13,646	43,000	69,421
2002					22,354	54,200	76,554
2003						36,612	36,612
Total	10,400	13,438	24,721	40,478	63,443	154,812	307,292

Reconciliation with Actuarial Data

Policy Year	Data From Accounting (8)	Actuarial Data (9)	Difference (10)	Difference as % of Reserves (11)
1998	10,400	9,988	412	4.0%
1999	13,438	13,400	38	0.3%
2000	24,721	24,021	700	2.8%
2001	40,478	41,502	(1,024)	-2.5%
2002	63,443	63,444	(1)	0.0%
2003	154,812	154,800	12	0.0%
Total	307,292	307,155	137	0.0%

Reconciliation with Schedule P Data

Accident Year	Schedule P Case Reserves (12)	Case Reserves From Underlying Data (13)	Unreconciled Difference (14)	Difference as % of Reserves (15)
1998	47,422	47,010	412	0.9%
1999	36,216	36,216	-	0.0%
2000	40,455	41,479	(1,024)	-2.5%
2001	69,421	69,421	-	0.0%
2002	77,254	76,554	700	0.9%
2003	36,612	36,612	-	0.0%
Total	307,380	307,292	88	0.0%

Notes:

(1), (2), (3), (4), (5) , (6) From Company
(7) = Sum of (1) thru (6)
(8) Column Totals from (1) - (6)
(9) From Actuarial Analysis (eg Ex. 2, Col 3)
(10) = (8) - (9)
(11) = (10) / (8)
(12) Schedule P Part 1D Col 13 - 14
(13) Col (7)
(14) = (12) - (13)
(15) = (14) / (12)

Case 6 **Exhibit 8**

Reconciliation to Schedule P
Commercial Auto Liability
For Auto Insurance Company

Reconciliation of Net Cummulative Paid Loss and DCCE
Evaluation as of 12/31/2003

	Schedule P	Actuarial	Actuarial	Actuarial		
	Paid Loss & DCCE as of 12/31/2003	Paid Loss & DCCE at 9/30/2003	Paid Loss & DCCE 4th Qt 2003	Paid Loss & DCCE as of 12/31/2003	Difference Schedule P - Actuarial Data	Difference as %
Year	(1)	(2)	(3)	(4)	(5)	(6)
1994	10,700	10,700	-	10,700	-	-
1995	9,010	9,005	5	9,010	-	-
1996	8,920	8,913	7	8,920	-	-
1997	10,248	10,217	31	10,248	-	-
1998	21,425	21,345	80	21,425	-	-
1999	29,200	29,148	352	29,500	(300)	(0)
2000	44,900	43,106	1,619	44,725	175	0
2001	41,500	39,054	2,696	41,750	(250)	(0)
2002	32,500	29,993	2,758	32,751	(251)	(0)
2003	13,988	12,219	2,431	14,650	(662)	(0)
Total	222,390	213,698	9,976	223,675	(1,285)	(0)

Notes:

(1) = Col 4 - 5 +6 -7 from Schedule P Part 1C
(2) See Actuarial Data (eg. Ex 3, Col 2)
(3) See Actuarial Data (eg. Ex. 4 Col 2)
(4) = (2) + (3)

(5) = (1) - (4)
(6) = (5) / (4)

Exhibit 9

Case 7
Reconciliation to Schedule P
Commercial Auto Liability
For Auto Insurance Company

Reconciliation of Net Reserves for Losses

Evaluation as of 12/31/2003

	Schedule P			Actuarial			Difference Carried - Indicated			
	Net Case Reserves	Net Carried IBNR	Total Carried Reserves	Net Case Reserves	Net Indicated IBNR	Total Indicated Reserves	Net Case Reserves	Net IBNR	Net Reserves	Difference as %
Year	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1994	1,736	260	1,996	1,736	271	2,007	-	(10)	(10)	-0.6%
1995	2,906	523	3,429	2,906	513	3,419	-	10	10	0.4%
1996	4,243	849	5,092	4,243	840	5,083	-	8	8	0.2%
1997	3,372	1,012	4,384	3,372	996	4,368	-	15	15	0.5%
1998	6,612	2,645	9,257	6,612	2,650	9,262	-	(5)	(5)	-0.1%
1999	43,050	24,108	67,158	43,050	24,180	67,230	-	(72)	(72)	-0.2%
2000	107,459	64,475	171,934	107,459	64,153	171,612	-	322	322	0.3%
2001	224,853	157,397	382,250	224,555	158,184	382,739	298	(787)	(489)	-0.2%
2002	414,462	331,570	746,032	414,526	324,938	739,464	(64)	6,631	6,567	1.6%
2003	560,724	1,121,448	1,682,172	560,851	1,143,877	1,704,728	(127)	(22,429)	(22,556)	-4.0%
Total	1,369,416	1,704,285	3,073,701	1,369,306	1,720,598	3,089,907	100	(16,324)	(16,209)	-1.2%

Notes:

(1) = Col 13 - 14 from Schedule P Part 1C
(2) = Col 15 - 16 From Part 1C
(3) = (1) + (2)
(4) See Actuarial Data (eg. Ex 1 Col 2)
(5) See Actuarial Data (eg Ex 1 Col1)
(6) = (4) + (5)
(7) = (1) - (4)
(8) = (2) - (5)
(9) = (3) - (6)
(10) = (9) / (3)

Atypical Circumstances in Statements of Actuarial Opinion on P&C Loss Reserves: Professional Considerations and Sample Wordings

Thomas L. Ghezzi, FCAS, MAAA, and
David S. Powell, ACAS, MAAA

Atypical Circumstances in Statements of Actuarial Opinion on P&C Loss Reserves: Professional Considerations and Sample Wordings

Thomas L. Ghezzi and David S. Powell

While the majority of statements of actuarial opinion regarding loss reserves are relatively straightforward, occasionally the actuary is confronted with atypical situations. These situations may include financially distressed or insolvent companies, ceded reinsurance collectibility issues, significant influence of a small group of claims, items requiring the disclosure of a significant risk of material adverse deviation, exceptional values on IRIS tests, and other unusual circumstances. The paper discusses the professional considerations and judgments applicable to such situations, and provides sample wordings.

1. Introduction

> *"When I use a word," Humpty Dumpty said in a rather scornful tone, "it means just what I choose it to mean - neither more nor less." -- Lewis Carroll -- Through The Looking Glass*

Statements of actuarial opinion regarding loss reserves are an important manifestation of actuarial work. They present actuarial conclusions regarding what is often the most significant balance sheet item of property/casualty insurers in a concise manner while also conveying to the reader information needed to understand those conclusions.

The majority of loss reserve opinions are relatively straightforward; reserves are reasonable and there are no unusual features. The content of such opinions generally reflects boilerplate language as well as disclosure of specific details related to the company. For statutory opinions in the United States, such language is contained in the Annual Statement Instructions promulgated by the National Association of Insurance Commissioners (NAIC) and the *Property and Casualty Practice Note, Statements of Actuarial Opinion on P&C Reserves* (the Practice Note), published annually by the American Academy of Actuaries.

The discussion and analysis contained here is directed primarily towards statements of opinion required to be attached to the financial statements issued by property/casualty insurers in the United States (referred to here as US statutory opinions), and reflect the statutory requirements for year-end 2003. However, in many cases, the underlying professional considerations are applicable to other forms of opinion as well.

The sample wordings presented here are what we believe to be reasonable interpretations of the application of actuarial standards and practices to the special situations included. There are certainly other wordings and approaches that would also be reasonable. The actual approach to any specific situation is the responsibility of the actuary issuing the statement of opinion.

In addition, we have not tried to replicate the detailed guidance included in the Practice Note. While the Practice Note provides guidance in all segments of the US statutory opinion, we focus on the key considerations and wordings for unusual situations that occasionally arise.

2. Relevant Actuarial Standards of Practice and Principles

There are several regulations, standards of practice, and actuarial principles affecting the issuance of statements of actuarial opinion. Most relevant are the following:

- Property and Casualty Annual Statement Instructions issued by the National Association of Insurance Commissioners (NAIC), especially those sections related to the US statutory statement of actuarial opinion.

- Actuarial Standard of Practice No. 36 (ASOP No. 36) – *Statements of Actuarial Opinion Regarding Property/Casualty Loss and Loss Adjustment Expense Reserves*, adopted by the Actuarial Standards Board (ASB), effective for all statements of actuarial opinions on loss reserves evaluated on or after October 15, 2000.

- ASOP No. 9 – *Documentation and Disclosure in Property and Casualty Insurance Ratemaking, Loss Reserving and Valuations*. ASOP No. 9 became effective July 14, 1989 for documentation and disclosure of loss reserving analyses. ASOP No. 9 includes the *Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expense Reserves.*

- ASOP No, 20 – *Discounting of Property and Casualty Loss and Loss Adjustment Expense Reserves*, adopted by the ASB, April 1992.

- *Code of Professional Conduct.*

While all of these (and other) ASOPs and statements of principles and actuarial precepts are important and relevant to the preparation of statements of actuarial opinion, ASOP No. 36 provides the most guidance. ASOP No. 36 provides professional guidance regarding the loss reserve analysis, uncertainty, discussion of the range of reasonable estimates, and other issues. It includes at a somewhat summarized level all of the issues covered in greater detail by the other relevant ASOPs.

A requirement of ASOP No. 36 that affects all statements of actuarial opinion to some degree is the need for the actuary to disclose if there are circumstances that would create a significant risk of material adverse deviation in the reserves. This is an important feature for atypical opinions.

3. Key Considerations

There are several key considerations that we believe guide the evaluation of issues falling within the scope of the statement of opinion. These include the following.

- **Purpose of the statement of opinion** – We believe the primary intent of U. S. statutory opinions is to assist regulators in monitoring the solvency of property/casualty insurers. It focuses primarily on the loss and loss adjustment expense reserves contained in the insurer's statutory financial statements, and it is intended to inform the reader – usually the relevant insurance regulatory authorities – regarding the reasonableness of the insurer's carried loss reserves, and the risk factors affecting the reserves of the individual carrier.

- **Knowledgeable User** - It is reasonable to assume that the user of the statement of opinion has a high degree of knowledge regarding the relevant underlying concepts, as well as some knowledge of the insurer.

- **No Guarantee** - It is important to recognize that the opinion is not intended to provide a guarantee that the loss reserves will prove to be adequate and not redundant, nor that the insurer is or will remain financially solvent.

- **Materiality** - We believe items covered by the statement of opinion (e.g., reinsurance collectibility issues, risk factors) are material if they would be reasonably expected to have an effect on the readers conclusions related to the purpose of the opinion. Consequently, given the purpose of the US statutory opinion of solvency monitoring noted above, we believe that an item is material if its eventual disposition is likely to have a significant effect on the insurer's solvency. For comparison, a statement of opinion intended for use in GAAP financial statements may also have purposes related to the income statement. Materiality for such an opinion may be related to net income rather than or in addition to solvency.

- **Focus on Loss Reserves** - The opinion is focused on the reasonableness of loss reserves, and related risk factors. It is not an opinion on the financial condition of the company. It is possible for a financially troubled company to have reasonable reserves.

These considerations imply that the actuary is required to perform at least some level of financial analysis in rendering the opinion. It may not be possible to be concerned only with the loss reserves without regard to other balance sheet and possibly income statement items. For all opinions, and particularly for atypical circumstances, it is important to understand the level of financial analysis required.

We believe a reasonable benchmark for the level of financial analysis is for the actuary to be aware of the income statement and balance sheet, but not necessarily of items that are only contained on the underlying schedules. Under this guide, the actuary should recognize a probable liquidity issue for a company whose assets are all in real estate (which is shown on the balance sheet), but not for a company whose assets are all in long term bonds (which requires a review of Schedule D).

4. Reasonable Reserves

This paper uses the definition of the term "reasonable reserves" as contained in ASOP No. 36; reserves that fall within "... a range of estimates that could be produced by appropriate actuarial methods or alternative sets of assumptions that the actuary judges to be reasonable."

This definition of reasonable relates solely to the actuarial estimate of reserves. In certain of the situations discussed below, one may believe that "adequate," "proper," or "prudent" reserves should exceed a reasonable amount as defined. However, "adequate" would allow a redundant reserve, and has an implication of a guarantee. The terms "proper" and "prudent" seem to go beyond the actuarial calculations to include that the reserve is somehow appropriate to the circumstances of the company. Neither ASOP No. 36 nor the annual statement instructions use these words; they use reasonable.

By using the definition of ASOP No. 36, we are implicitly relegating the additional provision over a reasonable reserve that the other terms imply to another balance sheet item such as a reinsurance bad debt reserve or capital or surplus. When such unusual features exist that make the actuary believe that greater conservatism is required, and the additional amounts are material to the balance sheet or required surplus, disclosures are required. This can be summarized as what is perhaps the key point of this paper:

Reserves can be reasonable, even if the viability of the company is uncertain or even doubtful; this situation, however, requires certain disclosures.

5. Atypical Situations

Many statements of actuarial opinion are prepared for companies with relatively strong reinsurance protection, relatively strong surplus positions, and few or no unusual risk factors. In such cases, the actuarial opinion generally contains the required scope section, required disclosures and the opinion section.

In many cases, however, the statement of opinion applies to less secure situations. This section provides analysis and discussion of several such scenarios, and provides possible wordings to handle them.

5.1. Financially Weak or Insolvent Company

The situation where the insurance company is financially weak or even insolvent poses an unusual challenge to the actuary providing the statement of actuarial opinion. In such cases, it is possible that the carried loss reserves are reasonable, or even conservative, yet the company's surplus is low or negative. Given the use of the statement of actuarial opinion as an important tool in the monitoring of solvency, this situation requires special wording and discussion in the statement of actuarial opinion.

Financially weak is a difficult concept to define in a precise quantifiable manner. One manifestation would be surplus below the Risk Based Capital ("RBC") Authorized Control Level (ACL). For this paper we define insolvency as negative surplus. There are several different situations, including the following:

- Surplus is negative or below RBC action level based on the carried reserve.
- Surplus is positive or above RBC action level based on carried reserves, but there are points in the range of reasonable estimates that cause surplus to cross the boundary.

In addition to disclosing the insolvency or whether reasonably expected reserve fluctuations might cause the company's surplus to fall below certain levels, the actuary may consider whether the company's situation could affect the reserve indications. Since typical reserve formulations implicitly assume an ongoing company, it is relevant to consider whether any change in the operation of the company due to regulatory control, rehabilitation or liquidation may cause changes to the key parameters implicitly underlying the loss reserve analysis. Claim frequency, severity, payment patterns, case reserving, and other key factors will likely be different under such scenarios. However, unless the specifics of the change are known, the effects may not be measurable.

As noted above, in each of these situations reserves may be reasonable in that they are the result of appropriate methods and assumptions applied to the available information. The sample wordings that follow assume that the actuary judges the reserves to be reasonable. However, the statement of opinion requires more that an assertion that reserves are reasonable. These suggested wordings are examples of ways that the actuary can inform the reader of the consequences of the reserve and its inherent uncertainty.

In the situation where the company is insolvent based on carried reserves, we believe that the fact of the insolvency must be disclosed. The reader's understanding of the company's financial condition is enhanced by the actuary's statement that the reserves are the result of reasonable methods and assumptions, and the actuary may also wish to disclose whether the reserve analysis reflected the possible effects caused by the insolvency. However, opining that the reserves make a reasonable provision is likely to be insufficient discussion when there aren't enough assets to go around. In such cases, whether the reserves of an insolvent company meet the requirements of insurance law seems more a legal (or perhaps a philosophical) question. For these reasons we would suggest that the actuary consider including a discussion of the company's position as a risk factor, and we would modify the Opinion section as follows.

> ***Example 5.1.1*** *– The Company's carried reserves are within a reasonable range, however recorded surplus is below zero. The financial condition of the Company creates an additional risk factor. My analysis of reserves implicitly assumes the Company is viable. If it is not viable (e.g., due to developments such as regulatory actions, inability to meet claim payments, etc.), reserves may be affected in ways that cannot be quantified at this time. Therefore I believe that there are significant risks and uncertainties that could result in material adverse deviation in the loss and loss adjustment expense reserves.*

The Opinion section may contain wording such as the following.

> *Given that the Company's surplus is below zero, I believe that the reserves may be affected in ways that cannot be quantified at this time. Therefore I cannot express an opinion on the carried loss and loss adjustment expense reserves.*
>
> *Were it not for the financial condition of the Company, the amounts carried in the Scope paragraph on account of the items identified:*
>
> *a) would meet the requirements of the insurance laws of [state];*

b) would be consistent with amounts computed in accordance with accepted loss reserving standards and principles; and

c) would make a reasonable provision for all unpaid loss and loss expense obligations of the Company under the terms of its contracts and agreements.

Note that this language informs the reader of both the financial condition of the company and the specifics of the actuary's view on the reserves.

The situation where points within the actuary's range of reasonable reserve estimates would cause surplus to be negative clearly requires disclosure of a significant risk of material adverse deviation per ASOP No.36. It may also affect the Opinion section. Possible wording in this situation is as follows.

> ***Example 5.1.2*** *- The Company's carried reserves are within a reasonable range, however other points within the range would cause surplus to be below zero. Therefore I believe that there are significant risks and uncertainties that could result in material adverse deviation in the loss and loss adjustment expense reserves, possibly by amounts exceeding surplus.*
>
> *The financial condition of the Company creates an additional risk factor. My analysis of reserves implicitly assumes the Company is viable. If it is not viable (e.g., due to developments such as regulatory actions, inability to meet claim payments, etc.), reserves may be affected in ways that cannot be quantified at this time.*

The actuary may also change the Opinion section as follows.

> *Because of the uncertainties noted above I cannot express an opinion on the carried loss and loss adjustment expense reserves.*
>
> *Were it not for the financial condition of the Company, the amounts carried in the Scope paragraph on account of the items identified:*
>
> *a) would meet the requirements of the insurance laws of [state];*
>
> *b) would be consistent with amounts computed in accordance with accepted loss reserving standards and principles; and*
>
> *c) would make a reasonable provision for all unpaid loss and loss expense obligations of the Company under the terms of its contracts and agreements.*

Similar risk factor language would be appropriate when the reserve movement would cause surplus to remain positive, but cross a Risk Based Capital threshold.

> ***Example 5.1.3*** *- The Company's carried reserves are within a reasonable range, however other points within the range would cause surplus to be below the Risk Based Capital Authorized Control Level. Therefore I believe that there are significant risks and uncertainties that could result in*

material adverse deviation in the loss and loss adjustment expense reserves.

The financial condition of the Company creates an additional risk factor. My analysis of reserves implicitly assumes the Company is viable. If it is not viable (e.g., due to developments such as regulatory actions, inability to meet claim payments, etc.), reserves may be affected in ways that cannot be quantified at this time.

Strictly speaking, ASOP No.36 requires that the actuary disclose the amount of adverse deviation considered to be material. In the cases of weak or insolvent insurers, we believe that the materiality threshold is implicit in the above.

The language dealing with insolvency and the changes in the Opinion section are possibly not needed when the only issue is RBC triggers.

5.2. Reserve leverage

When reserves are large in relation to surplus, reasonably expected variations in actual results may have a material impact on surplus. In such cases, disclosures may be appropriate even if financial viability is not threatened. The following wording may be considered.

Example 5.2.1 *- The Company's reserves are large in relation to surplus. As a result, reasonably expected fluctuations of actual versus expected results may be material to surplus. Consequently, I believe that there are significant risks and uncertainties that could result in material adverse deviation in the loss and loss adjustment expense reserves. In consideration of the use of this opinion for purposes of solvency monitoring, I consider X% of surplus to be material for this Company.*

When appropriate, the phrase *possibly by amounts exceeding surplus, or by amounts that would cause surplus to fall below an RBC trigger* may be added and language similar to Section 5.1 should be considered.

5.3. Reinsurance Collectibility Concerns

The NAIC Instructions require that the statement of actuarial opinion include comment on topics affecting loss reserves, including reinsurance collectibility. In most cases, the comment on reinsurance collectibility cites the portion of the ceded loss reserves that is with reinsurers rated highly or secured by other means. Occasionally, however, the actuary determines that a material amount of the ceded reserves is with troubled reinsurers, or are uncertain to be collected. In such cases, the statement of actuarial opinion should include additional discussion.

The two important considerations in dealing with ceded reinsurance collection concerns are the definition of reasonable and the fact that the opinion is rendered in the context of insurance law and accounting requirements. Reasonable reserves are defined as the result of appropriate methods and assumptions. Law and accounting rules govern balance sheet credit for ceded reinsurance. We believe that the fact that material amounts are ceded to a financially weak reinsurer does not make the net reserve not reasonable. To do so would require the actuary to consider unreasonable a reserve that law and regulation otherwise allow. This does not seem to be a reasonable place in which to put the actuary. The resolution of this possible conflict is disclosure.

In the case of a material amount of cessions to troubled reinsurer(s), the following collectibility wording may be considered.

> ***Example 5.3.1*** *- My opinion on the loss and loss adjustment expense reserves net of ceded reinsurance assumes that all ceded reinsurance is valid and collectible. Approximately X% of the Company's ceded loss and loss adjustment expense reserves are with companies having secure ratings by a reputable insurance rating agency.* ***The majority of the remaining cessions are to a financially-troubled reinsurer. Based on discussions with Company management, I have assumed that cessions to this reinsurer will be collectible.*** *Other cessions are not material. In addition, the Company has represented to me that it knows of no material uncollectible reinsurance cessions. I have not anticipated any contingent liabilities that could arise if the reinsurers do not meet their obligations to the Company as reflected in the data and other information provided to me.*

In this case where cessions to troubled reinsurer(s) are material, the actuary may consider adding this issue to the discussion of risk factors, as required by the Annual Statement Instructions. If the cessions to troubled reinsurer(s) are a high percentage of surplus, it is possible that the situation represents a material risk of significant adverse deviation as per ASOP No. 36. In that case, the following wording may be appropriate.

> ***Example 5.3.2*** *- As noted above, the Company cedes an amount of loss and loss adjustment expense reserves that is material to (or exceeds) its surplus to a troubled reinsurer. While the probability of failure to collect the full amount of the ceded reserves from this reinsurer is unknown, it is more than remote. Therefore I believe that there are significant risks and uncertainties that could result in material adverse deviation in the loss and loss adjustment expense reserves. In consideration of the use of this opinion for purposes of solvency monitoring, I consider X% of surplus to be material for this Company.*

Another scenario that may require additional comment by the opining actuary is where the company's interpretation of its ceded reinsurance coverage is in dispute with its reinsurers' interpretations. Disputed reinsurance amounts are disclosed in the Notes to Financial Statements in the US Statutory Annual Statement. If the dispute related to any particular issue is considered to be material, a comment such as the following may be appropriate.

> ***Example 5.3.3*** *– The Company has interpreted certain of its ceded reinsurance contracts in a manner that is currently disputed by the Company's reinsurers. If the Company does not prevail in its interpretation, net reserves can increase by approximately $X million.*

As noted above in Example 5.3.2, the actuary may also include this issue in the discussion of risk factors, and possibly include it as a material risk of significant adverse deviation, if appropriate.

Another situation is where net reserves are reasonable, but gross reserves are not. A separate opinion is required on each. When gross reserves are inadequate, the reader's understanding of the financial condition of the company is enhanced by further disclosure related to the potential collectibility of the shortfall. Possible wording for the Opinion section is a follows.

> ***Example 5.3.4*** *- In my opinion, the amounts carried in the Scope paragraph for the sum of items A, B, E and F:*
>
> *a) meet the requirements of the insurance laws of [state];*
>
> *b) are consistent with amounts computed in accordance with accepted loss reserving standards and principles; and*
>
> *c) make a reasonable provision for all unpaid loss and loss expense obligations of the Company under the terms of its contracts and agreements.*
>
> *Further, in my opinion, the amounts carried in the Scope paragraph for the sum of items C, D and F are inadequate. The amounts recorded are $X below my range of reasonable estimates. Because this amount is ceded to reinsurers with [A] or better ratings from a reputable insurance rating agency it is unlikely that this deficiency will have an effect on surplus.*

Of course, if the reinsurers' ratings are less secure, this wording regarding the gross reserves would need to be adjusted, as appropriate.

In addition to the above wording in the Opinion section, this situation may require further disclosures in the reinsurance collectibility section, since Annual Statement values imply ceded amounts below the actuary's estimate. Possible wording is as follows.

> *The ceded reserve amount reflected in the Annual Statement is below my range of reasonable estimates. Consequently, the cessions to one or more individual reinsurers are below my estimate. I assume that the additional amounts will be collectible.*

5.4. Exceptional Values on IRIS Tests

The NAIC Instructions require that statement of actuarial opinion include an explanation of any exceptional values produced for the reserve-related IRIS tests. These tests are:

- One Year Reserve Development to Surplus,
- Two Year Reserve Development to Surplus, and
- Estimated Current Reserve Deficiency to Policyholders Surplus.

It is important to remember that the one year and two year reserve development tests are based on a comparison of historical carried reserves (i.e., from the prior and next prior annual statements) to the current reserves. The implicit assumption underlying these tests is that the amount (percentage) of loss reserve development experienced over the last one and two years may be predictive of the amount of development that the current reserves will experience. If the test results exceed the prescribed tolerances (currently +/- 20%), an exceptional value is produced.

Assuming that the company's current carried reserves are considered to be reasonable, the explanation of the exceptional value(s) needs to focus on why the adverse development experienced by prior reserve levels is not predictive of the developments to be experienced on the current reserves. Possible explanatory paragraphs for several possible causes of an exceptional value on the one-year reserve development IRIS test are as follows. Note that similar language would apply to an adverse result on the two year development test.

Assuming that the exceptional value was related generally to adverse developments on prior years, the following language might be considered.

> ***Example 5.4.1a*** *- I have reviewed the calculations of IRIS Test numbers 10, 11 and 12. The Company shows an exceptional value for IRIS Test 10, One Year Reserve Development to Surplus. The exceptional value on Test 10 is due to significant adverse development during [the most recent calendar year] on prior years' reserves. The associated parameters in my analysis of the loss and loss adjustment expense reserves have been modified accordingly. Therefore, I do not believe that this test indicates a deficiency in the current reserves.*

If the cause of the adverse development can be attributed to a particular type of claim or situation, the following approach could be considered.

> ***Example 5.4.1b*** *- I have reviewed the calculations of IRIS Test numbers 10, 11 and 12. The Company shows an exceptional value for IRIS Test 10, One Year Reserve Development to Surplus. The exceptional value on Test 10 is due to significant adverse development during the most recent calendar year related to reserves for [asbestos, pollution, construction defect, reinsurance assumed] losses. The associated parameters in my analysis of the loss and loss adjustment expense reserves have been modified accordingly. Therefore, I do not believe that this test indicates a deficiency in the current reserves.*

Another possible scenario for an exceptional value on the development test would be changes in inter-company reinsurance arrangements whereby the Company's share of total pooled reserves for older years increases. Possible wording for this scenario is as follows.

> ***Example 5.4.1c*** *- I have reviewed the calculations of IRIS Test numbers 10, 11 and 12. The Company shows an exceptional value for IRIS Test 10, One Year Reserve Development to Surplus. The exceptional value on Test 10 is due to significant adverse development during the most recent calendar year resulting from changes to the Company's inter-company*

pooling (or reinsurance) arrangements. The changes caused a significant increase in prior year loss reserves that are the responsibility of the Company. Therefore, I do not believe that this test indicates a deficiency in the current reserves.

Similar language can be used for other common reasons for exceptional values on the reserve run-off tests such as the development of a single large claim and ceded reinsurance commutation. Exceptional values can also be produced when reserves are large in relation to surplus such that small adverse movements in reserve produce ratios exceeding the tolerance.

The explanation of an exceptional value for the Estimated Current Reserve Deficiency to Policyholders Surplus test is more involved. That test calculates the ratio of loss reserves to earned premium for each of the two prior annual statement evaluations. These ratios reflect the current carried reserves associated with the prior evaluation dates. The average of these two ratios is then applied to the current statement's earned premium to derive the implied needed current loss reserves. The difference between this implied needed reserve and the carried reserve is compared to current policyholders' surplus. If the result exceeds the prescribed tolerance (+/- 25%), an exceptional value is produced.

It is tempting to explain an exceptional value on this test by citing adverse development in the last two years. However, that explanation is usually not appropriate, since the test is based on the assumption that the current indication of the prior two year-end reserves, as a percentage of the prior years' premium, is indicative of what the current reserves to premium relationship should be. Consequently, the explanation needs to focus on reasons why the reserve to premium relationship from prior years is not an appropriate indication of what the current ratios should be. Reasons why the appropriate reserve to premium ratio might change over time include the following:

- Significant rate level activity in the recent year;
- Change in mix of business to significantly shorter or longer tailed lines of business;
- Change in inter-company pooling or reinsurance arrangements;
- Other.

Assuming that the result is due to significant rate activity, the following wording may be appropriate.

***Example 5.4.2a** – I have reviewed the calculations of IRIS Test numbers 10, 11 and 12. The Company produces an exceptional value for IRIS Test 12, Estimated Current Reserve Deficiency to Policyholders Surplus. This exceptional value is caused by the effect of the Company's premium growth in [the most recent calendar year] related to significant rate activity, and the long-tail nature of its business. Therefore, I do not believe that this test indicates a deficiency in the current reserves.*

Another reason why an exceptional value on this test may not be an accurate indication of the reasonableness of current reserves is an abrupt change in mix of business. If the company switched between long-tail and short-tail lines of business, the implied reserve to premium relationship would change, perhaps by enough to create the exceptional value. In this case, the following language could be included in the statement of actuarial opinion.

> ***Example 5.4.2b*** *– I have reviewed the calculations of IRIS Test numbers 10, 11 and 12. The Company produces an exceptional value for IRIS Test 12, Estimated Current Reserve Deficiency to Policyholders Surplus. The exceptional value for Test 12 is due to significant growth [in recent years] in shorter-tailed lines of business. Therefore, I do not believe that this test indicates a deficiency in the current reserves.*

If the exceptional value resulted from a significant change in the terms of inter-company pooling or reinsurance arrangements affecting the share of older years' reserves that are reflected in the company's reserves, the following wording might be appropriate.

> ***Example 5.4.2c*** *– I have reviewed the calculations of IRIS Test numbers 10, 11 and 12. The Company produces an exceptional value for IRIS Test 12, Estimated Current Reserve Deficiency to Policyholders Surplus. This exceptional value is caused by the effect of changes in the most recent year to the Company's inter-company pooling (or reinsurance) arrangements. The changes caused a significant (increase or decrease) in prior year loss reserves that are the responsibility of the Company. Therefore, I do not believe that this test indicates a deficiency in the current reserves.*

5.5. Unquantifiable Situation

It is possible at the time loss reserves are established, and the statement of actuarial opinion is prepared, that some material uncertainty exists due to lack of information or developing legal proceedings. It is possible that in such a case the ultimate loss will be either negligible or highly significant. In this instance, it might be appropriate in the statement of actuarial opinion to provide an explanation of the underlying situation. An example of a hypothetical scenario and possible opinion wording is as follows.

> ***Example 5.5.1*** *– Allegations of fraud and negligent care have recently been made against a specific insured. The Company is exposed to loss from existing coverage and guaranteed tail coverage. Because the underlying facts are still being developed, the ultimate liability under these coverages cannot be estimated at this time. However, because of the possibility that multiple policy limits will be exposed to loss, the ultimate liability could be material to the Company's surplus. Therefore, I believe that this situation represents a significant risk and uncertainty that could result in material adverse deviation. In consideration of the use of this opinion for purposes of solvency monitoring, I consider X% of surplus to be material for this Company.*

5.6. Change In Operations, Data Availability

Sometimes an insurer's operation changes so materially that there is no loss history or other basis on which to estimate loss reserves. The types of changes include changes to policy terms, changes in the lines of business or exposures written, and changes to operational units such as claims and underwriting. In these cases, the added uncertainty associated with these changes may require exceptional opinion wording, such as the following.

> ***Example 5.6.1*** *– The Company wrote liability coverages for commercial risks on both a direct and an assumed basis. Inherent in these coverages are risk factors that expose the Company's loss and loss adjustment expense reserves to variability. Besides the usual risk factors associated with these coverages, I have identified additional risk factors as the lack of detailed statistical information for some of the Company's segments of business, and recent changes in the claim handling and case reserving practices of the Company. The potential impact of these risk factors is unknown at this time. The absence of other risk factors from this listing does not imply that additional factors will not be identified in the future as having been a significant influence on the Company's reserves.*

5.7 Non-Tabular Discounting

Occasionally, insurers carry loss reserves reflecting non-tabular discounting. Both the NAIC Instructions and ASOP No. 36 require that the actuary comment in the statement of actuarial opinion on such discounting. Since the insurance laws of all states do not allow discounting, it is important that special permission to reflect non-tabular discounting be obtained by the insurer from the relevant insurance commissioner. The authority to use non-tabular discount is normally disclosed in the Notes to Financial Statements in the Statutory Annual Statement. The statement of actuarial opinions should also include reference to the reasons why such discounting is allowed.

The actuary's comment should also be guided by ASOP No. 20. Specifically, the comment should disclose the basis of the discounting (e.g., derivation of the payment pattern and interest rate assumptions). ASOP No. 20 also requires that the actuary disclose clearly if the interest rate is not included in the opinion. Possible wording incorporating these considerations is as follows.

> ***Example 5.7.1*** *- With the permission of the (state) Department of Insurance, the Company reflects in the details of write-ins section of the Liabilities, Surplus and Other Funds page a contra-liability for the discount related to its net loss and loss adjustment expense reserves based on an actuarially determined payment pattern and a Y% interest rate. I am not expressing an opinion on this rate. The amount of discount is $X.*

This disclosure should be tailored to the specific way in which the Company reflects the discount. The above assumes that a contra-liability is established. Other scenarios are possible. Also, if the actuary is opining on the interest rate, the penultimate sentence in the above paragraph should be omitted.

5.8. Significant Claim

There are situations where a single claim can be significant. Examples include a company in runoff for a long time, a company just beginning operations and company writing very low frequency/very high severity coverages. In each situation the emergence of a single claim can represent a large portion or even exceed the carried reserves. Actuarial techniques cannot predict a single claim.

For example, a company in runoff may have only $500,000 of reserve, but may have issued policies with $1 million limit. The $500,000 reserve is reasonable as it is based on appropriate methods and assumptions. However one unexpected policy limits claim would cause the reserve to be significantly inadequate. Disclosure of this potential should be considered in particular as it is reasonable to assume that the situation may affect the reader's view of surplus. In this situation, the following wording may be inserted.

> ***Example 5.8.1*** *– The Company wrote liability coverages with policy limits that are large in relation to reserves. Consequently, it is possible that a single claim could occur that would represent a high percentage of reserves. This situation represents a risk factor that exposes the Company's loss and loss adjustment expense reserves to variability.*

5.9. Uncertainty

In practice actuaries often include a comment toward the end of the Opinion section on the inherent uncertainty in loss reserves. Typical wording is as follows.

> ***Example 5.9*** *- In evaluating whether the reserves make a reasonable provision for unpaid losses and loss expenses, it is necessary to project future loss and loss adjustment expense payments. Actual future losses and loss adjustment expenses will not develop exactly as projected and may, in fact, vary significantly from the projections.*

In atypical situations this language could be modified to include statements such as the following:

> *The uncertainty inherent in any estimate of loss reserves is increased because ... [cite reasons for the added uncertainty].*

6. Conclusion

Application of basic principles can guide the actuary in providing actuarial opinions on the loss reserves of property and casualty insurance companies. It is important to evaluate each situation in the context of the intended use of the opinion, and to provide ample disclosure for the user of the opinion to understand the implication of any unusual situations.

Estimating and Incorporating Correlation in Reserve Variability

Roger M. Hayne, FCAS, MAAA

ESTIMATING AND INCORPORATING CORRELATION IN RESERVE VARIABILITY

By

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Abstract

An actuarial analysis of a book of reserves usually focuses on ultimate loss estimates by exposure period (accident year, report year, etc.) and by business segment. If an actuary is interested in the distribution of total reserves for all segments and exposure periods combined, then the actuary must find a way to combine the distribution estimates from the various parts analyzed. In this paper we present a method that can be used to estimate the correlation of reserve estimates among the various components analyzed, allowing the actuary to combine the resulting distributions in a meaningful way.

Biography

Roger Hayne is a Fellow of the Casualty Actuarial Society, a Member of the American Academy of Actuaries and a Consulting Actuary in the Pasadena, California office of Milliman USA. He holds a Ph.D. in mathematics from the University of California and joined Milliman in 1977. Roger has been involved in reserve estimation for a wide range of property and liability coverages with emphasis on exposures with longer tails and in situations where full data may not be readily available. The winner of the 1995 Dorweiler Prize he long has had interest in reserve variability and has authored several papers on the topic that have appeared in both the *Proceedings* and in the *Forum*.

ESTIMATING AND INCORPORATING CORRELATION IN RESERVE VARIABILITY

1. Introduction

The traditional collective risk model that is sometimes used in estimating reserve variability usually depends on the assumption that the various claim count and severity variables are all independent of one another. Though the assumption makes estimating the resulting aggregate distributions rather tractable, it often may not be realistic in practice.

Heckman and Meyers[1] recognized this in their 1985 paper and incorporated dependency among the various distributions by recognizing the uncertainty that affects the estimates of the parameters of the underlying distributions.

The Casualty Actuarial Society (CAS) recognized the importance of this issue and commissioned research into estimating aggregate distributions when the underlying random variables were correlated. This resulted in Shaun Wang's 1998 paper[2].

Meyers et al.[3] have taken up this issue again recently and considered correlation of risks within the framework of the Heckman and Meyers model. There they build on Shaun Wang's work to use the parameter uncertainty variables as a means of incorporating correlation among variables in the collective risk model. We will take a similar tack here, first measuring correlation, and then using its effects on the variance of the aggregate distribution to select the "mixing" parameter for the Heckman and Meyers version of the collective risk model.

In reserve problems we are faced with potential correlation of distributions for various accident years since inflation, court decisions, and other factors could induce correlation in reserves among various accident years. In addition, such factors could also cause correlation among lines of insurance.

In this paper we consider a hindsight method, similar to the bootstrap approach, to measure correlation among various distributions going into the collective risk model. Such correlations could be between accident years in a single line of business or more broadly across accident years and lines of business. With this measure we modify the "mixing parameter" in the Heckman and Meyers algorithm to incorporate the effects of correlation in the estimate of the aggregate distribution of reserves.

This paper builds on and refines the approach discussed in "Measurement of Reserve Uncertainty"[4] and explores an approach that provides insight into the correlation of reserve estimates, both across accident years and across lines of business. We will assume that the reader has access to that paper for detailed background. We will, however, briefly cover the major items relevant to this discussion.

2. A Brief Digression

One of the primary conclusions in "Measurement of Reserve Uncertainty" is that current stochastic methods of reserve estimation lack much of the robustness of the traditional reserving approach. That traditional approach recognizes its limitations by incorporating several different forecasting methods to assist the actuary in estimating reserves.

That paper used the Heckman and Meyers modification of the collective risk model to estimate the aggregate distribution of reserves. There reserves for an accident year were first looked on as the aggregate of an unknown number N of independent open and IBNR claims all drawn from the same distribution, with N and each of those variables all independent. This is simply the formulation of the classic collective risk model.

Heckman and Meyers introduced additional random variables χ_i and β that are independent from the claim count random variables N_i and claim size random variables X_i with

$$\text{(2.1)} \qquad \begin{aligned} &\mathrm{E}(\chi_i) = 1, \ \mathrm{Var}(\chi_i) = c_i \\ &\mathrm{E}(1/\beta) = 1, \ \mathrm{Var}(1/\beta) = b. \end{aligned}$$

They then used Fourier transforms to calculate the aggregate distribution resulting from the following algorithm repeated several times:

1. Randomly select values for each χ_i ,

2. Randomly select the number of claims N_i from a Poisson distribution with expected value $\chi_i\lambda_i$,

3. Randomly select N_i claims from the i[th] claim size distribution,

4. Sum all claims from all distributions,

5. Randomly select a value for β, and divide the sum of claims by β.

In this case λ_i is the expected number of claims for the i[th] accident year. The key in this algorithm is that the variable β affects all claims. Heckman and Meyers called the parameters c_i the "contagion parameters" and the parameter b the "mixing parameter." The contagion parameters affect the distributions of reserves for each accident year separately, while the mixing parameter affects the distribution of all years combined.

The approach set forth in "Measurement of Reserve Uncertainty" suggests considering the range of accident year results from various traditional methods to estimate the c_i parameters and then to consider variation in severities to estimate the b parameter. Here we maintain the first concept but follow Meyers, Klinker, and LaLonde and incorporate correlation among accident years (or across lines of business) to assist in the estimate of the b parameter. For this analysis, we will use the same data set as that used in "Measurement of Reserve Uncertainty."

3. A Hindsight Measure of Correlation

We will use hindsight results from traditional actuarial methods in an attempt to measure correlation among accident years. We begin with the traditional approach that calls upon a variety of different methods to assist the actuary in deriving ultimate loss and reserve estimates. Exhibit 1 shows the data underlying the examples in this paper as well as in "Measurement of Reserve Uncertainty." In Exhibit 2, we applied a range of methods to arrive at our final estimates of ultimate losses. The range of estimates from the various methods may provide some information regarding the uncertainty inherent in the reserve estimates. In fact in that earlier paper, the parameter c for a particular accident year is selected using the variance in reserve estimates implied by the various methods along with the following relationship derived in Heckman and Meyers[5]

(3.1) $$\text{Var}(R) = \lambda\left(\mu^2 + \sigma^2\right) + c\lambda^2\mu^2 .$$

Here μ and σ represent the mean and standard deviation of the claim size distribution for a single accident year and λ the expected number of claims for that year.

These forecast methods and final selection can also be used to assess the behavior of the individual forecast methods. For example, the development factor (link ratio, chain ladder) method assumes that there is a variable d_j such that an estimate of the ultimate losses for an accident year at age j is given by:

(3.2) $$U_i = d_j C_{ij} .$$

Here C_{ij} is the amount paid at age j for accident year i. Thus given our set of ultimate loss selections from Exhibit 2 that take into account the information provided by all the forecast methods the factors in (3.2) give us a hindsight view of what development factors would have resulted in the final selections. These hindsight historical factors are then:

(3.3) $$d_{ij} = \frac{U_i}{C_{ij}}.$$

We can then get a set of "what if" alternate ultimate loss estimates for a particular accident year, implied by our final selections and the historical development data by simply applying these hindsight factors at the appropriate age to the amount paid to date for that accident year. For example, paid amounts to date for accident year 1991 times the 12-to-ultimate factor implied by the 1978 forecasts and the 1978 losses at 12 months gives an "alternate" forecast for the 1991 losses. These alternative "what if" reserve estimates for the i^{th} accident year based on the development of the k^{th} accident year would then be given as:

(3.4) $$R_i^k = C_{iN-i}\left(d_{kN-i} - 1\right), k = 1, 2, \ldots, i.$$

Here N represents the number of years of experience in the triangles which, for ease in notation, are assumed to have as many years of development as accident years. Using the sample data and the forecasts from Exhibit 2, Exhibit 3 shows the resulting hindsight factors and resulting alternate reserve loss estimates for the development factor method.

We can take the same type of approach for the incremental severity method projections. These are similar to the forecast methods presented by Berquist and Sherman[6] where the ultimate loss forecast is the sum of the amount paid to date and the incremental average payment in each future development period times an exposure base, very often estimated ultimate claims or exposures earned. In this case, rather than taking the age-to-ultimate development factor implied by the loss emergence for a previous accident year, we take the future average incremental average implied by that year, adjust for expected trend, and multiply by the expected exposure units. The following formula shows this, assuming an annual trend of τ and exposure units for year j of e_j.

(3.5) $$R_i^k = e_i\left(\frac{U_k - C_{kN-i}}{e_k}\right)(1+\tau)^{i-k},\ k = 1, 2, \ldots, i.$$

One can obtain similar formulations for other forecasting methods brought to bear on a particular reserve review.

In this manner, we can construct triangles of hindsight alternative reserve estimates for each of the methods used in the analysis. At this time, one could review these triangles to assess the correlation between the various forecast methods. However, since we have taken the final selections on Exhibit 2 as weighted averages of the forecasts of the various methods, we focus on the corresponding weighted averages of the alternate estimates for each accident year in order to measure correlation among accident years. We note that this same approach can be used to measure correlation across both accident years and lines of business. Exhibit 4 shows the hindsight alternate reserve estimates taken as the weighted averages of the alternates for each accident year, using the selected weights shown in Exhibit 2.

Just as the bootstrap method uses historical data to estimate distributions, one can look at a row of the triangle in Exhibit 4 as one potential realization of future losses for subsequent accident years. With this view we can calculate the correlation coefficient between the alternates in pairs of accident years. For example, to estimate the correlation coefficient between accident years 1976 and 1991, we would calculate the correlation coefficient between the first three amounts in the 1991 column and those in the 1976 column. The top portion of Exhibit 5 shows the results of these calculations.

We note at this point that there is no restriction to applying this approach only to measuring correlations among accident years for a single line of business. The same approach could be just as easily used to measure correlation both across accident years and lines of business. In that case, the rows and columns of the correlation and covariance matrices would represent combinations of lines of business and accident years. For example, one entry may be Accident Year #1 for Line #1 and another may be Accident Year #5 for Line #3, and so forth.

From here we would suggest reference to Shaun Wang's paper.[7] This paper, the product of a CAS research project, does an excellent job at discussing issues that arise when dealing with correlated distributions and presents several approaches that can be used in modeling and estimating such distributions. In all cases, however, there is the need to assume some sort of structure on the correlation of the distributions. One approach would be to assume some joint distribution as a model, such as a joint lognormal. Wang gives an algorithm to model such a distribution if the covariance matrix of the corresponding joint normal distribution is positive definite. It happens that the matrix associated with our correlation structure is not positive definite and Wang's algorithm breaks down.

We will follow the approach in Meyers, Klinker, and LaLonde and use the Heckman and Meyers algorithm with the covariance structure guiding our choice of mixing parameter *b*. Given the covariance matrix in Exhibit 5, it is a simple matter to obtain the standard deviation of total reserves using the standard formula

$$(3.6)\qquad \begin{aligned}\operatorname{Var}\left(\sum_{i=1}^{n} X_i\right) &= \sum_{i=1}^{n}\operatorname{Var}(X_i)+\sum_{i=1}^{n}\sum_{j=1}^{i-1}2\operatorname{Cov}(X_i,X_j)\\ &= \sum_{i=1}^{n}\operatorname{Cov}(X_i,X_i)+\sum_{i=1}^{n}\sum_{\substack{j=1\\ i\neq j}}^{n}\operatorname{Cov}(X_i,X_j)\\ &= \sum_{i=1}^{n}\sum_{j=1}^{n}\operatorname{Cov}(X_i,X_j).\end{aligned}$$

The calculations on the bottom of Exhibit 5 show that correlation of reserves among accident years does add substantially to the standard deviation of reserves.

We can then use this information to estimate the mixing parameter b in the Heckman and Meyers algorithm. To this end we note that the variance for the total distribution is given by:

$$(3.7)\qquad \begin{aligned}\operatorname{Var}\left(\sum_{i=1}^{n} R_i\right) &= \mathrm{E}_\beta\left(\operatorname{Var}\left(\sum_{i=1}^{n} R_i|\beta\right)\right)+\operatorname{Var}_\beta\left(\mathrm{E}\left(\sum_{i=1}^{n} R_i|\beta\right)\right)\\ &= \mathrm{E}_\beta\left(\left(\sum_{i=1}^{n}\lambda_i\left(\mu_i^2+\sigma_i^2\right)+c_i\lambda_i^2\mu_i^2\right)\Big/\beta^2\right)+\operatorname{Var}_\beta\left(\left(\sum_{i=1}^{n}\lambda_i\mu_i\right)\Big/\beta\right)\\ &= \left(\sum_{i=1}^{n}\lambda_i\left(\mu_i^2+\sigma_i^2\right)+c_i\lambda_i^2\mu_i^2\right)\mathrm{E}_\beta\left(1/\beta^2\right)+\left(\sum_{i=1}^{n}\lambda_i\mu_i\right)^2\operatorname{Var}_\beta\left(1/\beta\right)\\ &= \left(\sum_{i=1}^{n}\lambda_i\left(\mu_i^2+\sigma_i^2\right)+c_i\lambda_i^2\mu_i^2\right)(1+b)+\left(\sum_{i=1}^{n}\lambda_i\mu_i\right)^2 b\\ &= \sum_{i=1}^{n}\operatorname{Var}(R_i)+b\left(\sum_{i=1}^{n}\operatorname{Var}(R_i)+\left(\sum_{i=1}^{n}\mathrm{E}(R_i)\right)^2\right).\end{aligned}$$

Solving for b we have

$$(3.8)\qquad b=\frac{\operatorname{Var}\left(\sum_{i=1}^{n} R_i\right)-\sum_{i=1}^{n}\operatorname{Var}(R_i)}{\sum_{i=1}^{n}\operatorname{Var}(R_i)+\left(\sum_{i=1}^{n}\mathrm{E}(R_i)\right)^2}$$

Using the results from Exhibits 2 and 5 we derive a value of the mixing parameter *b* of 0.025748. We can now use this *b* value with the other parameter estimates derived in "Measurement of Reserve Uncertainty" to derive an estimate of the distribution of total reserves in this example. Using the program CRIMCALC written by Glenn Meyers to implement the algorithm set out in his paper with Phil Heckman, we derived the estimates shown graphically in Exhibit 6.

As can be seen in that exhibit, parameter uncertainty is by far the most significant contributor to overall reserve uncertainty in this case. The correlation among accident years also has a marked contribution to overall uncertainty, as evidenced by the difference between the "With Parameter Uncertainty" and the "Independent Accident Year" distributions. The only difference between these two is that the "Independent Accident Year" distribution assumed that the mixing parameter *b* was 0 instead of the estimated 0.025748.

4. Conclusion

We believe this approach, though somewhat ad-hoc in nature, can provide very useful information with regards to the correlation structure of reserve estimates, both across years and across lines of business. Doubtlessly, there remains much more to be done.

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[1] Heckman, P.E., and Meyers, G.G., "The Calculation of Aggregate Loss Distriubtions from Claim Severity and Claim Count Distributions," *Proceedings of the Casualty Actuarial Society*, LXX, 1983, pp. 22-61, addendum in LXXI, 1984, pp. 49-66.

[2] Wang, S. "Aggregation of Correlated Risk Portfolios: Models and Algorithms," *Proceedings of the Casualty Actuarial Society*, LXXXV, 1998, pp. 848-939.

[3] Meyers, G.G., Klinker, F.L., and LaLonde, D.A., "The Aggregation and Correlation of Reinsurance Exposure," *Casualty Actuarial Society Forum*, Spring 2003, pp. 69-152.

[4] Hayne, R.M., "Measurement of Reserve Variability," *Casualty Actuarial Society Forum,*" Fall 2003, pp. 141-172.

[5] Heckman, P.E., and Meyers, G.G., op.cit.

[6] Berquist, J.R., and Sherman, R.E., "Loss Reserve Adequacy Testing: A Comprehensive, Systematic Approach," *Proceedings of the Casualty Actuarial Society*, LXXIV, 1977, pp. 123-184.

[7] Ibid.

EXAMPLE PRIVATE PASSENGER AUTO BODILY INJURY LIABILITY DATA

Cumulative Paid Losses

ccident	Months of Development																	
Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216
1974	$267	$1,975	$4,587	$7,375	$10,661	$15,232	$17,888	$18,541	$18,937	$19,130	$19,189	$19,209	$19,234	$19,234	$19,246	$19,246	$19,246	$19,246
1975	310	2,809	5,686	9,386	14,884	20,654	22,017	22,529	22,772	22,821	23,042	23,060	23,127	23,127	23,127	23,127	23,159	
1976	370	2,744	7,281	13,287	19,773	23,888	25,174	25,819	26,049	26,180	26,268	26,364	26,371	26,379	26,397	26,397		
1977	577	3,877	9,612	16,962	23,764	26,712	28,393	29,656	29,839	29,944	29,997	29,999	29,999	30,049	30,049			
1978	509	4,518	12,067	21,218	27,194	29,617	30,854	31,240	31,598	31,889	32,002	31,947	31,965	31,986				
1979	630	5,763	16,372	24,105	29,091	32,531	33,878	34,185	34,290	34,420	34,479	34,498	34,524					
1980	1,078	8,066	17,518	26,091	31,807	33,883	34,820	35,482	35,607	35,937	35,957	35,962						
1981	1,646	9,378	18,034	26,652	31,253	33,376	34,287	34,985	35,122	35,161	35,172							
1982	1,754	11,256	20,624	27,857	31,360	33,331	34,061	34,227	34,317	34,378								
1983	1,997	10,628	21,015	29,014	33,788	36,329	37,446	37,571	37,681									
1984	2,164	11,538	21,549	29,167	34,440	36,528	36,950	37,099										
1985	1,922	10,939	21,357	28,488	32,982	35,330	36,059											
1986	1,962	13,053	27,869	38,560	44,461	45,988												
1987	2,329	18,086	38,099	51,953	58,029													
1988	3,343	24,806	52,054	66,203														
1989	3,847	34,171	59,232															
1990	6,090	33,392																
1991	5,451																	

Claims Closed with Payment

ccident	Months of Development																	
Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216
1974	268	607	858	1,090	1,333	1,743	2,000	2,076	2,113	2,129	2,137	2,141	2,143	2,143	2,145	2,145	2,145	2,145
1975	294	691	913	1,195	1,620	2,076	2,234	2,293	2,320	2,331	2,339	2,341	2,343	2,343	2,343	2,343	2,344	
1976	283	642	961	1,407	1,994	2,375	2,504	2,549	2,580	2,590	2,596	2,600	2,602	2,603	2,603	2,603		
1977	274	707	1,176	1,688	2,295	2,545	2,689	2,777	2,809	2,817	2,824	2,825	2,825	2,826	2,826			
1978	269	658	1,228	1,819	2,217	2,475	2,613	2,671	2,691	2,706	2,710	2,711	2,714	2,717				
1979	249	771	1,581	2,101	2,528	2,816	2,930	2,961	2,973	2,979	2,986	2,988	2,992					
1980	305	1,107	1,713	2,316	2,748	2,942	3,025	3,049	3,063	3,077	3,079	3,080						
1981	343	1,042	1,608	2,260	2,596	2,734	2,801	2,835	2,854	2,859	2,860							
1982	350	1,242	1,922	2,407	2,661	2,834	2,887	2,902	2,911	2,915								
1983	428	1,257	1,841	2,345	2,683	2,853	2,908	2,920	2,925									
1984	291	1,004	1,577	2,054	2,406	2,583	2,622	2,636										
1985	303	1,001	1,575	2,080	2,444	2,586	2,617											
1986	318	1,055	1,906	2,524	2,874	2,958												
1987	343	1,438	2,384	3,172	3,559													
1988	391	1,671	3,082	3,771														
1989	433	1,941	3,241															
1990	533	1,923																
1991	339																	

EXAMPLE PRIVATE PASSENGER AUTO BODILY INJURY LIABILITY DATA

Cumulative Reported Claims

ccident	Months of Development																	
Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216
1974	1,912	2,854	3,350	3,945	4,057	4,104	4,149	4,155	4,164	4,167	4,169	4,169	4,169	4,170	4,170	4,170	4,170	4,170
1975	2,219	3,302	3,915	4,462	4,618	4,673	4,696	4,704	4,708	4,711	4,712	4,716	4,716	4,716	4,716	4,716	4,717	
1976	2,347	3,702	4,278	4,768	4,915	4,983	5,003	5,007	5,012	5,012	5,013	5,014	5,015	5,015	5,015	5,015		
1977	2,983	4,346	5,055	5,696	5,818	5,861	5,884	5,892	5,896	5,897	5,900	5,900	5,900	5,900	5,900			
1978	2,538	3,906	4,633	5,123	5,242	5,275	5,286	5,292	5,298	5,302	5,304	5,304	5,306	5,306				
1979	3,548	5,190	5,779	6,206	6,313	6,329	6,339	6,343	6,347	6,347	6,348	6,348	6,348					
1980	4,583	6,106	6,656	7,032	7,128	7,139	7,147	7,150	7,151	7,153	7,154	7,154						
1981	4,430	5,967	6,510	6,775	6,854	6,873	6,883	6,889	6,892	6,894	6,895							
1982	4,408	5,849	6,264	6,526	6,571	6,589	6,594	6,596	6,600	6,602								
1983	4,861	6,437	6,869	7,134	7,196	7,205	7,211	7,212	7,214									
1984	4,229	5,645	6,053	6,419	6,506	6,523	6,529	6,531										
1985	3,727	4,830	5,321	5,717	5,777	5,798	5,802											
1986	3,561	5,045	5,656	6,040	6,096	6,111												
1987	4,259	6,049	6,767	7,206	7,282													
1988	4,424	6,700	7,548	8,105														
1989	5,005	7,407	8,287															
1990	4,889	7,314																
1991	4,044																	

Outstanding Claims

ccident	Months of Development																	
Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216
1974	1,381	1,336	1,462	1,660	1,406	772	406	191	98	57	23	13	3	4	0	0	0	0
1975	1,289	1,727	1,730	1,913	1,310	649	358	167	73	30	9	6	4	2	2	1	1	
1976	1,605	1,977	1,947	1,709	1,006	540	268	166	79	48	32	18	14	10	10	7		
1977	2,101	2,159	2,050	1,735	988	582	332	139	66	38	27	21	21	8	3			
1978	1,955	1,943	1,817	1,384	830	460	193	93	56	31	15	9	7	2				
1979	2,259	2,025	1,548	1,273	752	340	150	68	36	24	18	13	4					
1980	2,815	1,991	1,558	1,107	540	228	88	55	28	14	8	6						
1981	2,408	1,973	1,605	954	480	228	115	52	27	15	11							
1982	2,388	1,835	1,280	819	354	163	67	44	21	10								
1983	2,641	1,765	1,082	663	335	134	62	34	18									
1984	2,417	1,654	896	677	284	90	42	15										
1985	1,924	1,202	941	610	268	98	55											
1986	1,810	1,591	956	648	202	94												
1987	2,273	1,792	1,059	626	242													
1988	2,403	1,966	1,166	693														
1989	2,471	2,009	1,142															
1990	2,642	2,007																
1991	2,366																	

EXAMPLE PRIVATE PASSENGER AUTO BODILY INJURY LIABILITY DATA

Outstanding Losses

ccident Year	Months of Development 12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216
1974	$5,275	$8,867	$12,476	$11,919	$8,966	$5,367	$3,281	$1,524	$667	$348	$123	$82	$18	$40	$0	$0	$0	$0
1975	6,617	11,306	13,773	14,386	10,593	4,234	2,110	1,051	436	353	93	101	10	5	5	3	3	
1976	7,658	11,064	13,655	13,352	7,592	4,064	1,895	1,003	683	384	216	102	93	57	50	33		
1977	8,735	14,318	14,897	12,978	7,741	4,355	2,132	910	498	323	176	99	101	32	14			
1978	8,722	15,070	15,257	11,189	5,959	3,473	1,531	942	547	286	177	61	67	7				
1979	9,349	16,470	14,320	10,574	6,561	2,864	1,328	784	424	212	146	113	38					
1980	11,145	16,351	14,636	11,273	5,159	2,588	1,290	573	405	134	81	54						
1981	10,933	15,012	14,728	9,067	5,107	2,456	1,400	584	269	120	93							
1982	13,323	16,218	12,676	6,290	3,355	1,407	613	398	192	111								
1983	13,899	16,958	12,414	7,700	4,112	1,637	576	426	331									
1984	14,272	15,806	10,156	8,005	3,604	791	379	159										
1985	13,901	15,384	12,539	7,911	3,809	1,404	827											
1986	15,952	22,799	16,016	8,964	2,929	1,321												
1987	22,772	24,146	18,397	8,376	3,373													
1988	25,216	26,947	17,950	8,610														
1989	24,981	30,574	19,621															
1990	30,389	34,128																
1991	28,194																	

ccident Year	Earned Exposures
1974	11,000
1975	11,000
1976	11,000
1977	12,000
1978	12,000
1979	12,000
1980	12,000
1981	12,000
1982	11,000
1983	11,000
1984	11,000
1985	11,000
1986	12,000
1987	13,000
1988	14,000
1989	14,000
1990	14,000
1991	13,000

EXAMPLE ULTIMATE LOSS FORECASTS

Accident Year	Reserve Estimates by Ultimate Forecast Method: Incurred Development	Paid: Development	Paid: Severity	Paid: Pure Premium	Paid: Hindsight	Adjusted Incurred	Paid Adjusted for Claim Closing Changes: Development	Paid Adjusted: Severity	Paid Adjusted: Pure Premium	Paid Adjusted: Hindsight	Weighted Average	Weighted Standard Deviation
1974	$0	$0	$0	$0		$0	$0	$0	$0		$0	0
1975	3	0	0	0		3	0	0	0		0	0
1976	33	0	0	0		33	21	0	0		11	14
1977	5	0	0	0		8	24	0	0		5	8
1978	-15	10	9	10		7	26	0	0		6	11
1979	-10	35	34	33		-35	28	0	0		11	24
1980	-7	54	55	50		-29	61	33	31		31	30
1981	-37	49	73	75		-20	77	47	49		39	41
1982	-41	107	136	131		-58	100	79	75		66	70
1983	114	275	297	297		-68	200	176	172		156	126
1984	-161	416	394	446		-135	352	318	351		181	258
1985	403	761	713	812		130	692	702	779		567	249
1986	744	2,143	1,760	1,909	$1,687	394	1,936	1,842	1,950	$675	1,357	637
1987	2,335	6,847	5,583	5,128	5,128	2,348	6,000	5,790	5,220	2,301	4,260	1,620
1988	8,371	19,768	16,246	13,451	14,428	10,391	17,352	16,433	13,399	8,001	12,866	3,525
1989	25,787	44,631	36,887	29,232	32,199	26,048	39,241	36,431	28,512	19,174	30,212	6,426
1990	60,211	83,760	73,987	61,846	62,974	55,734	79,667	70,246	57,192	43,286	62,516	10,197
1991	83,093	130,907	95,283	95,185	78,616	79,573	154,268	87,625	84,688	72,157	90,014	19,165

Selected Weights

1974	1	1	1	1		1	1	1	1	
1975	0	1	1	1		0	1	1	1	
1976	1	1	1	1		1	1	1	1	
1977	1	1	1	1		1	1	1	1	
1978	1	1	1	1		1	1	1	1	
1979	1	1	1	1		1	1	1	1	
1980	1	1	1	1		1	1	1	1	
1981	1	1	1	1		1	1	1	1	
1982	1	1	1	1		1	1	1	1	
1983	3	1	2	2		3	1	2	2	
1984	3	1	2	2		3	1	2	2	
1985	3	1	2	2		3	1	2	2	
1986	3	1	2	2	2	3	1	2	2	2
1987	3	1	2	2	2	3	1	2	2	2
1988	3	1	2	2	2	3	1	2	2	2
1989	3	1	2	2	2	3	1	2	2	2
1990	3	1	2	2	2	3	1	2	2	2
1991	3	1	2	2	2	3	1	2	2	2

Exhibit 3

HINDSIGHT ALTERNATE PAID LOSS DEVELOPMENT ESTIMATES

Accident Year	Age-to-Ultimate Factors Implied by Ultimate Selections at Age 12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216
1974	72.0824	9.7448	4.1958	2.6096	1.8053	1.2635	1.0759	1.0380	1.0163	1.0061	1.0030	1.0019	1.0006	1.0006	1.0000	1.0000	1.0000	1.0000
1975	74.7065	8.2446	4.0730	2.4674	1.5560	1.1213	1.0519	1.0280	1.0170	1.0148	1.0051	1.0043	1.0014	1.0014	1.0014	1.0014	1.0000	
1976	71.3726	9.6239	3.6270	1.9875	1.3356	1.1055	1.0490	1.0228	1.0138	1.0087	1.0053	1.0017	1.0014	1.0011	1.0004	1.0004		
1977	52.0860	7.7518	3.1267	1.7718	1.2647	1.1251	1.0585	1.0134	1.0072	1.0037	1.0019	1.0018	1.0018	1.0002	1.0002			
1978	62.8524	7.0810	2.6512	1.5078	1.1764	1.0802	1.0369	1.0241	1.0125	1.0032	0.9997	1.0014	1.0008	1.0002				
1979	54.8169	5.9925	2.1094	1.4327	1.1871	1.0616	1.0194	1.0102	1.0071	1.0033	1.0016	1.0011	1.0003					
1980	33.3887	4.4623	2.0546	1.3795	1.1316	1.0623	1.0337	1.0144	1.0108	1.0016	1.0010	1.0009						
1981	21.3919	3.7547	1.9525	1.3211	1.1266	1.0550	1.0270	1.0065	1.0025	1.0014	1.0011							
1982	19.6375	3.0601	1.6701	1.2365	1.0983	1.0334	1.0112	1.0063	1.0037	1.0019								
1983	18.9470	3.5601	1.8005	1.3041	1.1198	1.0415	1.0104	1.0071	1.0041									
1984	17.2274	3.2311	1.7300	1.2782	1.0825	1.0206	1.0089	1.0049										
1985	19.0559	3.3482	1.7149	1.2856	1.1105	1.0367	1.0157											
1986	24.1310	3.6271	1.6988	1.2278	1.0649	1.0295												
1987	26.7449	3.4440	1.6349	1.1989	1.0734													
1988	23.6521	3.1875	1.5190	1.1943														
1989	23.2504	2.6176	1.5101															
1990	15.7485	2.8722																
1991	17.5133																	

Accident Year Paid to Date

1991	1990	1989	1988	1987	1986	1985	1984	1983	1982	1981	1980	1979	1978	1977	1976	1975	1974
5,451	33,392	59,232	66,203	58,029	45,988	36,059	37,099	37,681	34,378	35,172	35,962	34,524	31,986	30,049	26,397	23,159	19,246

Based on Accident Year	Hindsight Alternate Reserve Estimates for Accident Year 1991	1990	1989	1988	1987	1986	1985	1984	1983	1982	1981	1980	1979	1978	1977	1976	1975	1974
1974	387,470	292,007	189,292	106,562	46,729	12,119	2,737	1,411	615	208	104	69	22	20	0	0	0	0
1975	401,774	241,911	182,019	97,146	32,262	5,578	1,870	1,037	640	509	179	154	48	44	42	37	0	
1976	383,601	287,968	155,600	65,375	19,472	4,851	1,767	846	519	299	187	60	48	35	12	11		
1977	278,470	225,455	125,967	51,097	15,359	5,753	2,109	497	271	126	66	65	63	5	5			
1978	337,157	203,056	97,803	33,616	10,238	3,688	1,330	893	470	111	-11	51	29	6				
1979	293,356	166,709	65,710	28,644	10,859	2,832	699	379	269	114	57	38	11					
1980	176,551	115,613	62,468	25,125	7,637	2,864	1,215	534	408	54	35	31						
1981	111,156	91,983	56,418	21,261	7,349	2,529	972	240	96	49	39							
1982	101,593	68,790	39,691	15,655	5,707	1,536	406	235	140	66								
1983	97,829	85,488	47,414	20,132	6,954	1,909	377	263	156									
1984	88,456	74,500	43,240	18,415	4,785	947	322	181										
1985	98,423	78,410	42,346	18,911	6,410	1,686	567											
1986	126,087	87,725	41,394	15,083	3,764	1,357												
1987	140,335	81,611	37,608	13,171	4,260													
1988	123,477	73,045	30,740	12,866														
1989	121,287	54,013	30,212															
1990	80,394	62,516																
1991	90,014																	

Exhibit 4

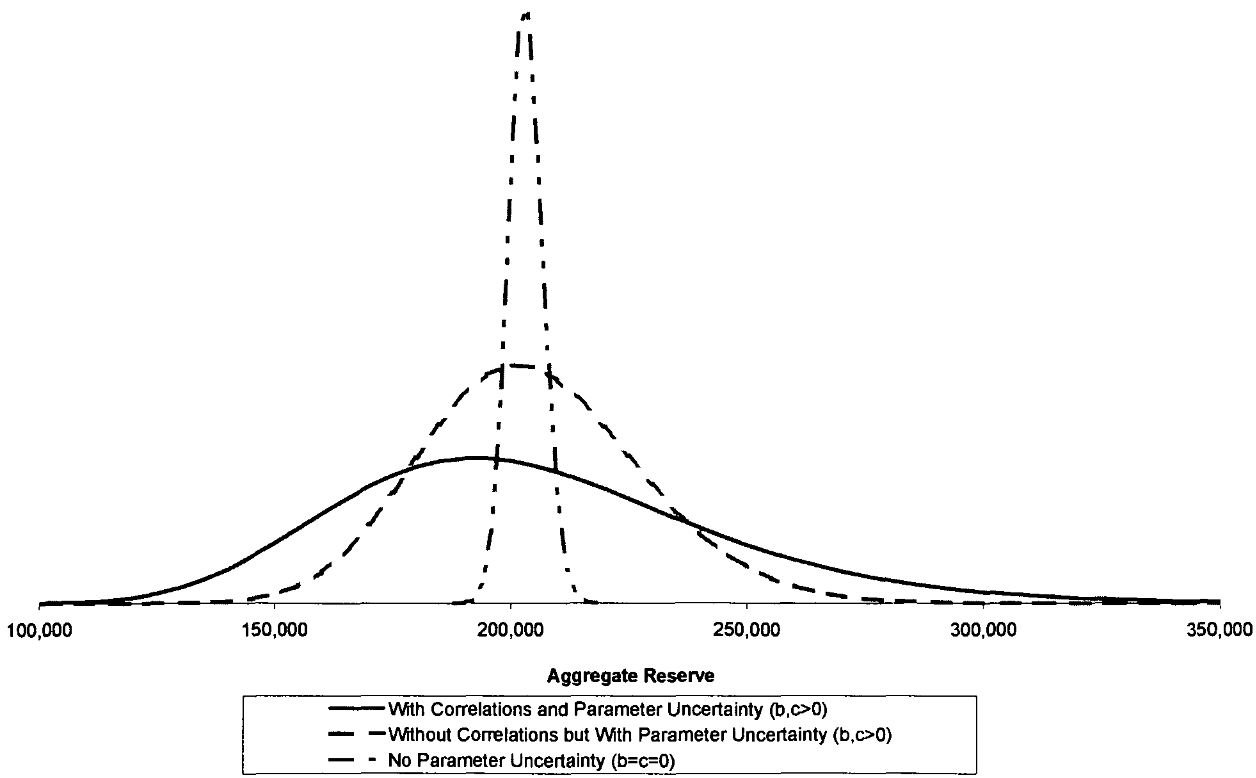
COMPOSITE HINDSIGHT ALTERNATE RESERVE ESTIMATES

Based on Accident Year	Estimates for Accident Year																	
	1991	1990	1989	1988	1987	1986	1985	1984	1983	1982	1981	1980	1979	1978	1977	1976	1975	1974
1974	119,864	77,831	41,971	22,816	9,430	2,449	60	101	-79	15	50	25	17	6	6	11	0	0
1975	123,373	73,746	43,816	22,471	7,441	1,134	373	314	295	324	170	125	47	41	41	42	0	
1976	115,554	76,766	41,440	19,596	6,356	1,399	677	234	122	147	102	31	11	15	3	11		
1977	97,719	70,593	38,461	17,741	5,412	1,803	736	62	44	55	45	41	10	-1	5			
1978	108,995	70,843	36,227	14,207	4,003	1,282	801	447	199	43	-16	35	13	6				
1979	98,437	65,969	31,990	13,968	4,681	1,253	418	156	151	67	36	15	11					
1980	82,177	62,553	31,790	13,422	4,286	1,776	1,074	374	319	63	47	31						
1981	74,869	56,710	28,954	12,966	4,293	1,433	648	76	79	46	39							
1982	68,912	51,023	26,148	11,271	3,776	1,151	494	141	152	66								
1983	72,409	59,085	31,041	14,104	4,914	1,625	569	203	156									
1984	68,211	53,945	29,544	12,040	3,297	991	492	181										
1985	69,543	55,614	26,719	12,349	4,611	1,490	566											
1986	80,939	58,132	29,839	12,341	3,928	1,274												
1987	82,822	62,227	29,517	12,076	4,009													
1988	85,755	62,235	29,363	12,238														
1989	89,367	56,699	28,871															
1990	82,522	60,382																
1991	88,967																	

ESTIMATES OF CORRELATION AND COVARIANCE AMONG RESERVE PROJECTIONS

Accident Year	Accident Year 1991	1990	1989	1988	1987	1986	1985	1984	1983	1982	1981	1980	1979	1978	1977	1976
	Indicated Correlation Coefficients															
1991	1.0000	0.9432	0.9215	0.8734	0.7696	0.2875	-0.2808	0.2565	-0.1211	0.4603	0.4737	0.4767	0.6662	0.7310	0.5456	0.8307
1990	0.9432	1.0000	0.9422	0.8714	0.7791	0.4501	-0.1859	0.2023	-0.2232	0.2893	0.3644	0.2248	0.2232	0.1849	-0.1977	-0.9699
1989	0.9215	0.9422	1.0000	0.9538	0.8146	0.3558	-0.2467	0.1785	-0.1498	0.5005	0.6072	0.5800	0.6042	0.7182	0.7191	0.9754
1988	0.8734	0.8714	0.9538	1.0000	0.9422	0.4732	-0.4501	-0.0227	-0.2841	0.4906	0.6739	0.5157	0.5869	0.5167	0.5370	0.4059
1987	0.7696	0.7791	0.8146	0.9422	1.0000	0.6381	-0.6052	-0.1570	-0.4458	0.2991	0.5301	0.2915	0.4139	0.2778	0.1618	-0.1753
1986	0.2875	0.4501	0.3558	0.4732	0.6381	1.0000	-0.1700	-0.2617	-0.6127	-0.4809	-0.2305	-0.4074	-0.2994	-0.5792	-0.5933	-0.6619
1985	-0.2808	-0.1859	-0.2467	-0.4501	-0.6052	-0.1700	1.0000	0.4781	0.5570	-0.1380	-0.3085	-0.1644	-0.3493	-0.2700	-0.2493	0.0171
1984	0.2565	0.2023	0.1785	-0.0227	-0.1570	-0.2617	0.4781	1.0000	0.7592	0.3085	-0.0020	0.2528	0.3177	0.3986	0.7332	0.7910
1983	-0.1211	-0.2232	-0.1498	-0.2841	-0.4458	-0.6127	0.5570	0.7592	1.0000	0.5344	0.2792	0.4589	0.5704	0.7256	0.8042	0.8476
1982	0.4603	0.2893	0.5005	0.4906	0.2991	-0.4809	-0.1380	0.3085	0.5344	1.0000	0.9063	0.9074	0.8738	0.9610	0.8801	0.9079
1981	0.4737	0.3644	0.6072	0.6739	0.5301	-0.2305	-0.3085	-0.0020	0.2792	0.9063	1.0000	0.7843	0.7879	0.8632	0.8623	0.9055
1980	0.4767	0.2248	0.5800	0.5157	0.2915	-0.4074	-0.1644	0.2528	0.4589	0.9074	0.7843	1.0000	0.9565	0.9047	0.9829	0.9987
1979	0.6662	0.2232	0.6042	0.5869	0.4139	-0.2994	-0.3493	0.3177	0.5704	0.8738	0.7879	0.9565	1.0000	0.9300	0.9937	0.9891
1978	0.7310	0.1849	0.7182	0.5167	0.2778	-0.5792	-0.2700	0.3986	0.7256	0.9610	0.8632	0.9047	0.9300	1.0000	0.9140	0.9705
1977	0.5456	-0.1977	0.7191	0.5370	0.1618	-0.5933	-0.2493	0.7332	0.8042	0.8801	0.8623	0.9829	0.9937	0.9140	1.0000	0.9957
1976	0.8307	-0.9699	0.9754	0.4059	-0.1753	-0.6619	0.0171	0.7910	0.8476	0.9079	0.9055	0.9987	0.9891	0.9705	0.9957	1.0000
	Estimated Standard Deviation by Year															
	19,165	10,197	6,426	3,525	1,620	637	249	258	.126	70	41	30	24	11	8	14
Total Standard Deviation Assuming Independence Among Accident Years																22,983
	Estimated Covariance															
1991	367,314,112	184,342,232	113,490,346	59,002,642	23,900,201	3,511,144	-1,342,580	1,266,255	-291,818	616,198	371,817	278,107	308,697	153,055	82,154	230,220
1990	184,342,232	103,982,827	61,738,485	31,324,197	12,873,454	2,924,467	-472,885	531,256	-286,074	206,038	152,187	69,773	55,026	20,601	-15,844	-143,013
1989	113,490,346	61,738,485	41,296,208	21,605,944	8,483,346	1,456,783	-395,555	295,520	-120,958	224,675	159,802	113,464	93,867	50,421	36,310	90,642
1988	59,002,642	31,324,197	21,605,944	12,425,915	5,381,949	1,062,726	-395,866	-20,606	-125,857	120,794	97,281	55,345	50,020	19,900	14,874	20,688
1987	23,900,201	12,873,454	8,483,346	5,381,949	2,625,950	658,817	-244,652	-65,514	-90,788	33,859	35,177	14,383	16,215	4,918	2,061	-4,108
1986	3,511,144	2,924,467	1,456,783	1,062,726	658,817	405,924	-27,026	-42,945	-49,062	-21,403	-6,014	-7,902	-4,612	-4,032	-2,970	-6,098
1985	-1,342,580	-472,885	-395,555	-395,866	-244,652	-27,026	62,242	30,721	17,464	-2,405	-3,152	-1,248	-2,107	-736	-489	62
1984	1,266,255	531,256	295,520	-20,606	-65,514	-42,945	30,721	66,341	24,576	5,551	-21	1,982	1,979	1,122	1,484	2,946
1983	-291,818	-286,074	-120,958	-125,857	-90,788	-49,062	17,464	24,576	15,797	4,692	1,437	1,756	1,733	996	794	1,541
1982	616,198	206,038	224,675	120,794	33,859	-21,403	-2,405	5,551	4,692	4,880	2,593	1,930	1,476	733	483	917
1981	371,817	152,187	159,802	97,281	35,177	-6,014	-3,152	-21	1,437	2,593	1,677	978	780	386	277	536
1980	278,107	69,773	113,464	55,345	14,383	-7,902	-1,248	1,982	1,756	1,930	978	927	704	301	235	440
1979	308,697	55,026	93,867	50,020	16,215	-4,612	-2,107	1,979	1,733	1,476	780	704	584	246	189	346
1978	153,055	20,601	50,421	19,900	4,918	-4,032	-736	1,122	996	733	386	301	246	119	78	153
1977	82,154	-15,844	36,310	14,874	2,061	-2,970	-489	1,484	794	483	277	235	189	78	62	113
1976	230,220	-143,013	90,642	20,688	-4,108	-6,098	62	2,946	1,541	917	536	440	346	153	113	209
Indicated Total Standard Deviation																39,942
Implied b value:																

ESTIMATED DISTRIBUTION OF RESERVES



An Introduction to Reserving and Financial Reporting Issues for Non-Traditional Reinsurance

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An Introduction to Reserving and Financial Reporting Issues for Non-Traditional Reinsurance

Abstract

Non-traditional reinsurance contracts, and finite risk reinsurance contracts in particular, are structured differently from traditional reinsurance. The incorporation of special features that make each contract unique tends to preclude standard portfolio loss reserving. This paper introduces the basic features related to common types of finite risk reinsurance contracts that provide prospective (e.g., aggregate stop-loss) or retroactive (e.g., adverse development cover) coverage. This paper will also discuss some of the considerations related to financial reporting issues for non-traditional reinsurance. The appendix will provide basic examples of prospective and retroactive deals to illustrate the balance sheet and income statement impacts for both the buyer and seller of finite risk reinsurance.

I. INTRODUCTION

Non-traditional reinsurance is characterized by the transfer of risk through customized arrangements that are produced for the specific needs of a cedant. For finite risk arrangements, the risk transferred from the ceding entity will be limited and correspond to a limited upside for the reinsurer. Though finite risk reinsurance is a subset of non-traditional reinsurance, the terms "non-traditional" and "finite" are used interchangeably throughout this paper.

When finite risk reinsurance first emerged, it provided an alternative to traditional reinsurance for both reinsurers and cedants. Ceding companies found a less expensive mechanism to smooth earnings and to address other issues such as adverse loss development and diminished underwriting capacity. Reinsurers, on the other hand, began to incorporate overall aggregate limits of liability and were better able to protect themselves against adverse selection and catastrophic losses. As cedants participated to a greater degree in their own ultimate loss exposure, finite reinsurance began to align the interests of the ceding company with the reinsurer. This, in turn, led to increased flexibility in the structure of reinsurance arrangements and enabled cedants to address needs that were not satisfactorily met by traditional reinsurance. Common uses of finite risk reinsurance were:

- Deferral of taxes
- Discounting of loss reserves
- Earnings stabilization
- Risk management related to mergers and acquisitions

- Surplus protection via all of the above

These uses continue to drive the demand for finite risk reinsurance (although the current interest rate environment has reduced the impact of the time value of money). In recent years, however, the significant increases in the cost of traditional reinsurance have contributed to the demand for finite risk arrangements. Additionally, for emerging issues like terrorism or mass torts such as asbestos and toxic mold, finite risk reinsurance may be the most appropriate approach, from both the cedant and reinsurer perspective, to provide adequate protection.

II. Types of Contracts and Common Structural Features

A. Types of Non-Traditional Reinsurance Arrangements – Retroactive

The most common retroactive arrangements are loss portfolio transfers (LPT's) and adverse development covers (ADC's). For both types of deals, the reinsurer provides protection from the loss reserve deterioration for claims that have already been incurred. The reinsurer assumes a portion of the ceding entity's reserve uncertainty in return for a fixed premium.

Loss portfolio transfers. With respect to LPT deals, the ceding entity is able to reduce future loss payment uncertainty by transferring a "portfolio" of reserves off of its balance sheet to the reinsurer. The premium paid to transfer the reserve uncertainty is based on the present value of the liabilities, plus an additional amount to reflect the risk to the reinsurer of further development of the transferred liabilities. LPT's protect the ceding entity from the deterioration of past written business and are often used in mergers or acquisitions in order to wall off future exposure to loss from discontinued operations.

Adverse development covers. ADC deals are also intended to protect the ceding entity against unexpected development of past liabilities. In these cases, however, the ceding entity retains the underlying portfolio of loss reserves. As a result, ADC deals do not reduce reserve leverage to the same extent as with LPT's. For these deals, the premium is based on the reinsurer's evaluation of both the potential for adverse development and the expected timing of additional loss payments. ADC deals typically provide a specific dollar amount of coverage for potential development in excess of the ceding entity's carried reserves at the selected accounting date.

In general, LPT deals tend to apply to smaller segments of business (e.g., a single line of business that the cedant has exited) than ADC deals, which commonly address larger groupings (e.g., all casualty lines of business combined).

B. Types of Non-Traditional Reinsurance Arrangements – Prospective

The most common prospective finite reinsurance arrangements are aggregate stop-loss covers, finite quota share treaties, and spread loss covers.

Aggregate stop-loss covers. The typical use of aggregate stop-loss covers is to stabilize earnings of the ceding entity. For this type of deal, the reinsurer typically provides a loss ratio corridor of protection above the ceding entity's planned future loss ratio in return for a fixed premium. Aggregate stop-loss reinsurance contracts often cover multiple (typically three to five) years together; this further reduces the volatility of the ceding entity's earnings. (See Illustration 1 in the Appendix for a sample of this type of deal.)

Finite quota share treaties. In a traditional quota share agreement, the reinsurer assumes a fixed percentage of the ceding entity's premium and corresponding losses and returns a ceding commission to the cedant. Finite quota share agreements are generally similar to and provide the same benefits as traditional quota share reinsurance. Like traditional quota share agreements, the primary benefit of finite quota share protection to the cedant is surplus relief, which in turn provides an increase in underwriting capacity. The main difference between finite quota share and traditional quota share is the aggregate limit of liability. For a finite quota share agreement, this is typically reflected via features such as a loss ratio cap for the reinsurer or a loss corridor, which defines a layer of loss for which the reinsurer does not pay the cedant. Also, the net cost of finite quota share reinsurance is typically less than traditional quota share because profits tend to be returned to the cedant. (See Illustration 2 in the Appendix for a sample of this type of deal.)

Spread loss covers. Spread loss covers are similar to multi-year aggregate stop-loss deals; their focus is also to stabilize future years' earnings. With spread loss covers, the reinsurer commits to pay a defined level of loss across a number of future years. Like aggregate stop-loss covers, spread loss coverage can reduce the impact on earnings of specific covered events (e.g., catastrophes) or claim experience that is worse than expected.

C. Common Features of Finite Risk Reinsurance Deals

Although each finite risk reinsurance deal is tailored to the ceding entity's specific needs, finite risk contracts tend to have a number of common structural features. The most significant feature is the contractual limitation on the ultimate amount of losses to be paid under the arrangement. By definition, this is found in all finite reinsurance deals, but aggregate limits are increasingly common in traditional reinsurance arrangements as well.

Other features that are frequently incorporated into finite risk reinsurance deals include the following:

- Recognition of the time value of money
- Cedant participation in upside (profit sharing) and downside (additional premiums)
- Sub-limits of liability
- Multiple years
- Cancellation and commutation provisions

Time value of money. The time value of money is most commonly recognized in finite reinsurance by the use of an "experience account" that is initially funded by the premium paid by the ceding entity. For both retroactive and prospective deals, the experience account is typically established as the initial premium paid by the ceding entity, less the reinsurer's explicit provision for profit (the "margin") and brokerage fees. Loss payments under the contract are paid from the experience account and, while the experience account balance (EAB) is positive, it accrues interest at a negotiated interest rate. When the experience account is held by the ceding entity ("funds withheld" basis), the interest credit tends to be higher than when the experience account is held by the reinsurer ("funds held/transferred"). In a funds transferred scenario, the credit is usually based on the risk-free interest rate. In a funds withheld scenario, the credit is higher because the reinsurance premium is essentially loaned back to the ceding entity. The higher interest rate for funds withheld scenarios also accounts for the credit risk to which the reinsurer is exposed; the reinsurer is still obligated to the cedant if the experience account is inadequate.

Cedant participation. In finite risk reinsurance, it is common for the ceding entity to share both the potential upside and downside of the contract. When experience is favorable, most contracts allow for any positive experience account balance to be refunded to the ceding entity. The reinsurer, in fact, typically has a limited and small upside that is contractually defined as its margin. Due to the limited upside to the reinsurer, finite reinsurance contracts may be "overfunded" in order to minimize the downside to the reinsurer. This tends to be acceptable to cedants because of the profit sharing arrangement, which makes it likely that the reinsurer will return any initial overfunding to the cedant.

Most prospective reinsurance arrangements also have provisions that ensure the ceding entity participates in the downside. For stop-loss and spread loss covers, this is commonly reflected in additional premiums to be paid depending on the cedant's loss experience. When these additional premiums (AP's) are contractually defined, they may be referred to as "hard AP's." On the other hand, "relationship" agreements by which a ceding entity promises to renew or extend a current contract in order to make a reinsurer whole for adverse experience represent "soft AP" arrangements. Soft AP arrangements continue to exist, but they are increasingly rare in the current reinsurance environment. For finite quota share contracts, the ceding entity typically participates in the upside and downside by way of a sliding scale ceding commission, which is increased for favorable experience and decreased for poor experience.

Sub-limits of liability. Another means for reinsurers to reduce its downside is to incorporate sub-limits of liability. For retroactive deals, sub-limits are typically used to reduce the reinsurer's exposure to losses that are unusually difficult to estimate. For prospective deals, sub-limits are used to limit the exposure to shock losses.

Reduced life span of contracts. In most cases, profit sharing occurs at commutation of the reinsurance contract. This is typically initiated by the ceding entity although when the commutation may occur is contractually defined. Unlike most traditional reinsurance agreements, finite risk reinsurance is expected to commute soon after the cedant has achieved the intended benefit. From the reinsurer's perspective, early commutation can be appealing because it accelerates the recognition of the margin. Assuming any related experience account balance is projected to be positive, finite risk deals tend to commute shortly after the contractual window opens. Although the life span tends to be longer when interest rates are lower (and thus the experience account grows more slowly), the average life span of finite risk deals is shorter than that of traditional reinsurance arrangements.

III. RESERVING ISSUES

The basic characteristics of finite risk reinsurance (limited risk transfer, investment income credit, profit sharing between cedant and reinsurer, commutation clauses) are unique for each contract. In addition to the non-homogeneous nature of finite risk reinsurance contracts, the underlying exposure typically varies for each contract. As each finite risk reinsurance arrangement is tailored to meet the specific needs of the cedant, it is practically impossible to apply standard actuarial loss reserving methods to a group of finite contracts. As a result, ultimate loss estimation by the reinsurer is done on a deal-by-deal basis.

Included below is a list of basic issues to consider when estimating the reinsurer's liabilities for a particular finite risk deal.

A. Understanding the Structure – Start with the Pricing Analysis

A key initial step to projecting the reinsurer's ultimate liabilities associated with a particular finite risk deal is to understand its structural features. Following is a list of some preliminary questions to address for this step:

- What is the purpose of the deal? Does the cedant have surplus constraints, rating agency concerns, etc?

- Is the contract retroactive or prospective?
- What lines of business are covered?
- What type of coverage is provided? For other than quota share arrangements, what layers of coverage does the reinsurer provide? For quota share arrangements, what is the assumed percentage and are there any loss corridors for the cedant?
- What annual limits, sub-limits and aggregate limits of the reinsurer's liability exist?
- Does the contract qualify for reinsurance treatment or is deposit accounting required?
- Is loss reserve discounting used?
- Is there an experience account? If so, what is the initial funding and how is the interest credit determined?
- Is there a provision for additional premiums from the cedant?
- What is the reinsurer's margin?
- Is there a commutation provision? If so, which party (reinsurer or cedant) can commute and under what circumstances?

This is not intended to be an exhaustive list and these are generally not unique to finite risk deals. It is, however, particularly important to address some of these items in order to appropriately reflect the issues specific to each individual deal. When estimating losses at the individual deal level, the relative importance of these issues is magnified. From this list, it is clear that many of the key issues should be addressed in the pricing analysis from the initial underwriting process.

B. Considering the Experience Account

In many cases, finite risk deals that fund contractual loss payments via an experience account will include a provision that allows the cedant, and sometimes also the reinsurer, to commute the contract. At the commutation date, a significant portion of the experience account balance is typically returned to the ceding entity and the reinsurer is released from future obligations to the cedant. The experience account refund is sometimes known as the "profit commission" and is frequently equal to 100% of the experience account balance. As a result, many reinsurers tend to hold reserves (including unearned premium) based on a 100% combined ratio, less its brokerage costs and margin. This approach makes sense when the experience account balance is projected to be positive. A question arises, however, of how to address situations in which the experience account is projected to be negative.

As discussed earlier, an experience account is typically equal to premium payments by the cedant, less the reinsurer's margin and contractual loss payments, plus investment income accrued via an interest credit on the balance. If, however, the loss payments for the deal are requested earlier than expected or the interest rate environment deteriorates and the interest credit is lower than expected, it is possible that the experience account may be exhausted. If additional premiums are not available to replenish the experience account in these cases, the reinsurer may not realize its full margin and could be exposed to an economic loss for the contract.

C. Monitoring Actual and Expected Loss Emergence

One of the central issues related to reserve estimation is how the actual loss emergence compares to the expected emergence. This is a common issue for reserving, but it has extra significance for non-traditional reinsurance due to the impact of the time value of money.

For the reinsurer, problems can emerge with changes in either the timing or magnitude of reported losses from the ceding entity. Generally, a slowdown or decrease in loss reporting is favorable to the reinsurer. If, however, actual losses exceed the expectations, there are different issues to consider.

First, a temporary speed-up in loss reporting by the ceding entity will reduce the reinsurer's benefits from the time value of money. As the experience account is utilized to pay losses to the cedant earlier than anticipated, the interest credit will not grow as expected. As a result, the experience account could be exhausted before the reinsurer's obligations have been settled. Thus, even if the initial ultimate loss estimate were accurate on an undiscounted basis, acceleration in claim payments could lead to an economic loss for the reinsurer.

If actual losses during a reporting period are consistently greater than expected, a second problem may emerge: the initial loss projection could be understated. Clearly, this can also exhaust the experience account earlier than anticipated. A related and more subtle issue is whether the cedant begins to under-report losses to the reinsurer. As most finite deals include provisions for additional premiums from the cedant, the cedant will have incentive to delay the triggering of any AP payment to the reinsurer. A delay in the transfer of AP's form the cedant will increase the likelihood of an economic loss to the reinsurer.

(See Illustrations 3A and 3B for examples of the potential impact of a reporting speed-up and slowdown.)

Unlike deals with favorable claim experience, which the cedant is expected to commute, deals that generate net losses to the reinsurer will require a more rigorous analysis for the purpose of estimating the reinsurer's ultimate liabilities.

D. Projecting the Interest Credit

The time value of money is most frequently reflected in finite risk reinsurance via an experience account, which accumulates interest until losses are paid from the account. In most cases, the interest credit for the experience account is based on a risk-free interest rate. The credit typically reflects a modest spread above the risk-free rate – the ceding entity and reinsurer will negotiate the spread, which tends to vary depending on whether the experience account is a funds withheld or funds transferred arrangement. For the purpose of projecting the future experience account balance, it is necessary to estimate the future values of the risk-free interest rate. A common and simple approach is to utilize the term structure of interest rates based on the spot rates of U.S. Treasury securities.

E. Testing the Sensitivity of Loss Projections

For any reinsurance deal, it is important to test the sensitivity of the subject losses to variations in the assumptions that underlie the reinsurer's loss projections. The key concerns to the reinsurer are the level of subject losses and the corresponding timing of the payout of those losses. Understanding the potential variability of the losses is critical in order for the reinsurer to determine a range of reasonable loss estimates as well as the best estimate within that range.

For retroactive reinsurance, sensitivity testing is often more simplistic, though no less important, than for prospective reinsurance. For LPT and ADC deals, the subject losses have already been incurred so potential adverse (or favorable) development of the subject losses is the initial focus. Varying the tail of the loss development patterns that underlie the initial loss projections is a simple and reasonable approach to testing the sensitivity of the nominal loss amounts.

For prospective reinsurance, a common approach to sensitivity testing is stochastic simulation of future loss levels. Given that the subject losses have not been incurred for prospective reinsurance, this usually involves modeling the claim frequency and severity components of loss. A notable benefit of simulations is that the user can identify confidence level percentiles for the expected reinsured losses.

From the reinsurer's perspective, the timing of the loss payout can be as important as the actual amount to be paid. Thus, for both prospective and retroactive reinsurance deals, it is also important to review alternative payout patterns together with the various loss projections. By combining alternative payout patterns with various expectations of the nominal loss amounts, the reinsurer can produce a range of estimates of the economic value of the coverage provided.

F. Considering Bulk Reserves

Due to the large size of most individual finite reinsurance deals and the intensive underwriting process involved, these books of business tend to be comprised of a small number of contracts. As each deal has unique features and is reserved individually, the law of large numbers with respect to loss reserving does not typically apply to finite reinsurance. Thus, it is worthwhile to consider the appropriateness of bulk or "non-specific" reserves for the overall book of finite reinsurance.

At issue is whether the total carried reserve for all contracts reflects an adequate provision for the potential of adverse scenarios. A key consideration in debating this topic is how the reinsurer defines its "best estimate" of loss for individual contracts. While many approaches are possible, three approaches are readily available based on the reinsurer's simulation of future loss outcomes.

First, there is the most likely outcome (i.e., the mode) of the loss distribution. The mode might be appealing because it is the single outcome with the greatest probability of occurring. The problem with this, however, is that using the mode completely ignores all other possible outcomes. Consider an example in which 90% of the possible outcomes for a contract produce loss estimates of $0 and 10% produce $1,000,000 – is it reasonable to carry $0 for this contract? Suppose each contract in the book has a similar loss distribution – would $0 be an appropriate reserve to carry for the entire book? The positively skewed nature of most aggregate loss distributions implies that the mode could be grossly inadequate in some cases. As the contracts' loss distributions are increasingly skewed to the right, there is a greater need for a bulk reserve when the mode underlies the loss reserve best estimate.

To address the basic problem with the mode, an alternative is the expected value of the loss distribution (i.e., the mean). The mean is a weighted average of all projected outcomes and reflects the expected probability that each could occur. The mean value for each contract, therefore, explicitly reflects a provision for all expected scenarios.

A different approach would be to book loss estimates that correspond to a specific confidence level for each contract. The likely expectation underlying this approach is that the selected percentile produces a conservative estimate (otherwise the mean or mode would likely be selected).

In practice, bulk reserves for finite reinsurance are not often used. As noted earlier, many finite risk reinsurance contracts are booked to 100% combined ratios, which will tend to produce conservative estimates in aggregate. A secondary argument against bulk reserves is that booked reserves are generally undiscounted, so the amount of potential discount is an implicit buffer.

G. Establishing Claim Liabilities When Deposit Accounting is Required

As discussed in Section IV – FINANCIAL STATEMENT REPORTING ISSUES, one of the key issues related to finite risk reinsurance is whether a contract qualifies for reinsurance accounting or deposit accounting. This is strictly a financial reporting issue, however, and does not affect the loss estimation process. The preceding discussion applies equally regardless of whether reinsurance or deposit accounting is used. One difference to note is that, unlike under reinsurance accounting, the deposits and liabilities recorded by the ceding and assuming entities are typically based on the discounted values of the expected subject losses.

IV. FINANCIAL STATEMENT REPORTING ISSUES

A. Reinsurance versus Deposit Accounting

Regardless of the reporting purpose (i.e., GAAP versus statutory), the key issue to address when accounting for finite risk reinsurance contracts is whether reinsurance accounting is permitted or deposit accounting is required. U.S. GAAP financial statements rely on Statement of Financial Accounting Standards (SFAS) No. 113 while statutory accounting depends on Statement of Statutory Accounting Principles (SSAP) No. 62 for guidance in determining when reinsurance treatment is permissible.

With both forms of accounting, reinsurance treatment requires that both underwriting and timing risk be transferred to the reinsurer. The language used to define the conditions of insurance risk transfer is essentially identical; in fact, the U.S. statutory guidance is copied almost verbatim from SFAS 113. Following are the conditions as defined by SFAS 113:

> "a. The reinsurer assumes significant risk under the reinsured portions of the underlying insurance contracts.
>
> "b. It is *reasonably possible* that the reinsurer may realize a *significant loss* from the transaction." (Emphasis added.)

If either of these conditions is not met, deposit accounting is required. For the purpose of evaluating insurance risk transfer, SFAS 113 and SSAP 62 state "an outcome is reasonably possible if its probability is more than remote." In reviewing the potential significance of loss, the accounting statements establish that it is necessary to evaluate the net present value of the cash flows (premiums, commissions, losses, and loss adjustment expenses) from reasonably possible outcomes of the transaction.

It has been frequently observed that the language in SFAS 113 does not specify how to quantify the amount of risk transfer. While some rules of thumb exist, there is a great deal of uncertainty related to the terms *reasonably possible* and *significant loss*. The most commonly cited target is the "10/10 rule," which implies sufficient risk is transferred if the reinsurer has a 10% probability of sustaining a 10% loss. This discussion, however, is not intended to address how to determine whether sufficient risk is transferred. The interested reader is referred to guidance from the CAS Committee on Valuations, Finance, and Investments ("Considerations in Risk Transfer Testing"). Note, however, that the accounting statements are clear about contractual features that delay the timing of payments from the reinsurer to the cedant. As SSAP 62 states, "any feature that can delay timely reimbursement violates the conditions for reinsurance accounting" and thus requires deposit accounting.

B. Reinsurance Accounting – Prospective versus Retroactive

Contracts that qualify for reinsurance accounting are treated differently depending on whether a contract provides prospective or retroactive coverage. Generally, prospective reinsurance covers incurred losses assumed from future events while retroactive reinsurance covers liabilities from past insurable events. It is possible that some contracts contain both prospective and retroactive provisions. When this occurs, the provisions should be accounted for separately unless this is not feasible, in which case the full contract should be treated as retroactive reinsurance.

Under U.S. statutory accounting, there are some exceptions to the rule for retroactive reinsurance. The following should instead receive prospective reinsurance treatment:

- Structured settlement annuities for individual claims;
- Novations – these are primarily agreements by which the liabilities of the cedant are completely extinguished;
- Termination of or reduced participation in reinsurance treaties; and

- Intercompany agreements that do not produce a gain in surplus as a direct result of the arrangement.

From the ceding entity's perspective, retroactive reinsurance is most commonly used to increase policyholder surplus. This occurs via implicit loss reserve discounting that underlies the pricing of retroactive reinsurance. For example, the ceding entity may be required to book an undiscounted reserve of $100 million related to claims for past events it covered. If the discounted value of these liabilities at the reinsurance contract effective date were $80 million, the reinsurer and cedant might agree to a premium of $88 million. The intent of this deal would be to create an additional $12 million of surplus for the ceding entity as it pays $88 million up front to the reinsurer to assume the future payment obligations with an estimated nominal value of $100 million. For the reinsurer, the $8 million difference between the reinsurance premium and the discounted reserve estimate reflects a provision for both profit and the risk of adverse development of the assumed book of business.

As explicit loss reserve discounting is allowed only in very limited circumstances, accounting treatment of retroactive reinsurance is somewhat different from prospective reinsurance. As the accounting guidance states, this is due to potential abuses related to surplus creation by cedants and the corresponding distortion of underwriting results.

For the ceding entity, it must reflect loss and loss adjustment expense reserves gross of retroactive reinsurance on the balance sheet and all other schedules and exhibits of the financial statements. The amount of retroactive reinsurance must be shown as a contra-liability on the balance sheet and be reported as a write-in item specifically identified as Retroactive Reinsurance Ceded. In addition, any surplus created by the retroactive reinsurance transaction must be restricted as a special surplus fund. This fund is not released into unassigned surplus until the reinsurance recoveries exceed the consideration paid for the retroactive reinsurance agreement. (See Illustration 4 for an example of the treatment from the ceding entity's perspective.)

For the reinsurer, it must exclude the assumed retroactive reinsurance from loss and loss adjustment expense reserves on the balance sheet and all other schedules and exhibits of the financial statements. The amount of retroactive reinsurance must be shown as a contra-liability on the balance sheet and be reported as a write-in item specifically identified as Retroactive Reinsurance Assumed.

While the balance sheet effects of retroactive reinsurance are similar between GAAP and statutory accounting, one notable difference between the two is reflected on the income statement. Unlike GAAP, statutory accounting allows the immediate recognition of the retroactive reinsurance gain (for the ceding entity) or loss (for the assuming entity) on the income statement. This must be recorded as a write-in item, reflected in Other Income, and specifically identified as Retroactive Reinsurance Gain or Loss. Under GAAP, the immediate recognition of gains or losses from retroactive reinsurance is permissible only if the ceding entity no longer has any obligation to its policyholder.

C. Deposit Accounting

When a finite risk arrangement requires deposit accounting, there is no initial impact on the loss and loss adjustment expense reserve entries on the balance sheet of either party. There is also no initial impact on their income statements.

At the onset, the ceding entity records a deposit (i.e., asset) equal to the net consideration paid to the assuming entity. The assuming entity records a corresponding liability on its balance sheet. Note that this liability is not part of the loss reserve; instead, it is a separate item on the balance sheet and can be viewed as a "loss-equivalent" reserve. As noted in Section III – RESERVING ISSUES, the amount of the deposit or liability is based on the discounted value of the ceded obligation.

After the initial financial reporting date, the balance sheet and income statement reflect adjustments that address: (a) actual payments between ceding and assuming entity; (b) unwinding of the underlying discount; and (c) revisions to the expected amount and timing of future "loss" payments. Item (a) is reflected as a direct adjustment to the deposit or liability held. Items (b) and (c), however, affect both the income statement and the balance sheet.

As long as the timing and amount of the actual cash flows are as expected, item (b) is the only adjustment to the income statement. This is calculated as the product of the effective yield and the remaining deposit. For the ceding entity, item (b) is a credit to interest income and an increase to the deposit asset. For the assuming entity, item (b) is reflected as an interest expense and an increase to the liability.

If, however, the timing or amount of an annual cash flow differs from expected, the effective yield will be recalculated to reflect the revised expected future timing and amounts. The intent is to ensure that the deposit declines to zero at the same time as when the loss payments are completed. Also, the difference between actual and expected cash flow during the reporting period will also be reflected as interest income/expense and a corresponding increase/decrease to the deposit or liability. (See Illustration 5 for an example of the accounting from the reinsurer's perspective.)

Note that the accounting guidance does not require that both cedant and reinsurer account for a reinsurance contract the same way. While it is unusual, there can be instances when one party utilizes reinsurance accounting while the counterparty uses deposit accounting.

V. CONCLUSIONS

For most finite deals, which are addressed on a deal-by-deal basis, reserving is based on the initial pricing analysis with the monitoring of critical deal-specific variables. When subject claim experience or the interest rate environment is favorable, cedants are expected to commute contracts in order to gain the profits embedded in the experience account balance. In these scenarios, reinsurers frequently base reserves on a 100% combined ratio for the contract. If, however, loss emergence is faster or greater than expected or if interest rates are lower than projected, the cedant's experience account balance might not be sufficient to cover the reinsurer's liabilities. As a result, the reinsurer could suffer a net loss, so it is very important to monitor both loss emergence and the projected interest credit. This will enable the reinsurer to assess the adequacy of the experience account, to determine whether reserves in excess of the experience account balance are necessary, and to determine whether additional premiums will be required. Clearly, these considerations combined, together with the accounting issues that apply to all reinsurance contracts, present some different challenges from traditional reinsurance. Hopefully, this paper will provide the reader with a foundation from which to address the main reserving and financial reporting issues related to this family of insurance products, which continue to emerge.

VI. Bibliography

1. CAS Committee on Valuations, Finance, and Investments, "Accounting Rule Guidance Statement of Financial Accounting Standards No. 113 – Considerations in Risk Transfer Testing," Casualty Actuarial Society *Forum*, Fall 2002, pp. 305-338.
2. Lane, Morton, *Alternative Risk Strategies*, Risk Books, 2002.
3. Feldblum, Sholom, "Selected Notes to the Fire and Casualty Annual Statement", Fourth Edition, Casualty Actuarial Society Study Note, 1999.
4. Fell, Bruce, "Finite Reinsurance," Casualty Loss Reserve Seminar, September 2003.
5. Financial Accounting Standards Board, "Statement of Financial Accounting Standards No. 113, Accounting and Reporting for Reinsurance of Short-Duration and Long-Duration Contracts," Financial Accounting Standards Board, December 1992.
6. International Risk Management Institute, "The Risk Report," January 1998.
7. National Association of Insurance Commissioners, *Accounting Practices and Procedures Manual*, 2003 Edition.

VII. Appendix

Illustration 1 – Sample Aggregate Stop-Loss Deal

Illustration 2 – Sample Finite Quota Share Deal

Illustration 3 – Monitoring Loss Emergence

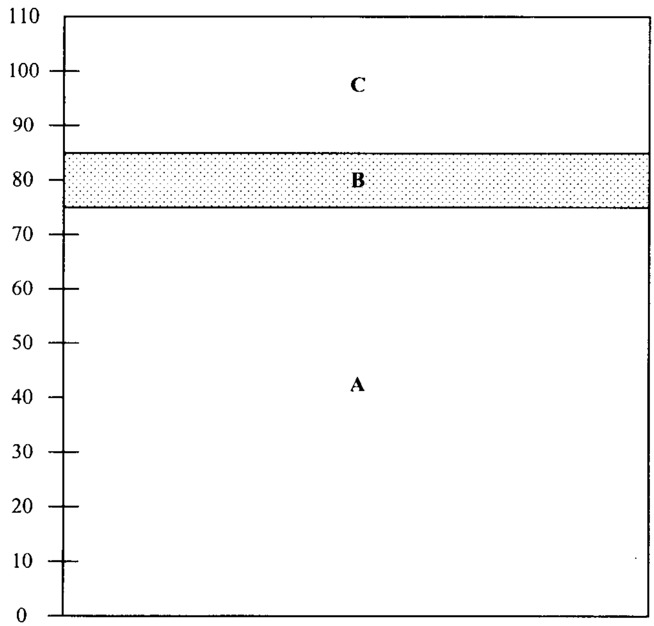
Illustration 4 – Retroactive Reinsurance

Illustration 5 – Deposit Accounting

Background:

Coverage Period:	1/1/03-12/31/05
Subject premium:	$50,000,000
Stop-loss attachment point:	75%
Limit:	10%
Coverage:	10% XS 75% or $5M XS $37.5M
Interest credit:	4.25%

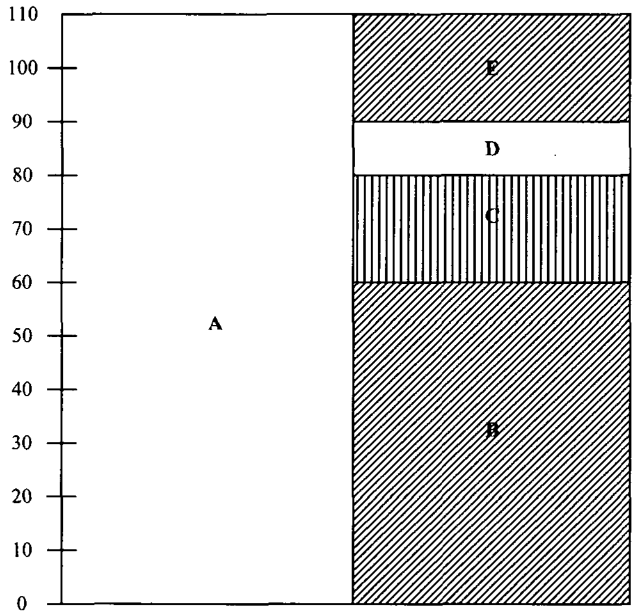
Commutation provision: cedant will receive 100% of experience account balance if commutation occurs after the end of the exposure period (i.e., 1/1/06 or later).



A = retained by ABC below the stop-loss attachment point loss ratio of 75%
B = assumed by XYZ; loss ratio layer from 75% to 85%
C = retained by ABC above the stop-loss ratio limit of 85%

Background: ABC would like to reduce its premium leverage in order to expand its volume. ABC enters into a quota share with XYZ in the following scenario:

Quota share percentage:	50%
Expected loss ratio:	65%
Aggregate limit loss ratio:	110%
Expected expense ratio:	30%
Cedingcommission:	
minimum at 60% loss ratio	39%
minimum at 80% loss ratio	19%



A = 50% share retained by ABC
B = 50% assumed by XYZ for loss ratio < 60%, ceding commission = 39%
C = 50% share assumed by XYZ with sliding scale ceding commission
D = loss ratio corridor from 80% to 90%, retained by ABC
E = 50% share assumed by XYZ for loss ratio from 90% to 110% (aggregate limit)

Monitoring Loss Emergence - Reporting Speed-Up Illustration 3A

Expected Loss Payout and Experience Account Balance

Nominal Ult. 100,000,000

Calendar Year	Payout Pattern %	Payout Pattern $	Interest Credit at 5%	Losses Paid	Exp. Acct. Balance
0					85,635,238
1	15.0%	15,000,000	4,281,762	(15,000,000)	74,917,000
2	25.0%	25,000,000	3,745,850	(25,000,000)	53,662,850
3	25.0%	25,000,000	2,683,143	(25,000,000)	31,345,993
4	10.0%	10,000,000	1,567,300	(10,000,000)	22,913,292
5	10.0%	10,000,000	1,145,665	(10,000,000)	14,058,957
6	10.0%	10,000,000	702,948	(10,000,000)	4,761,905
7	5.0%	5,000,000	238,095	(5,000,000)	0
Total	100.0%	100,000,000			

Present Value of Expected Loss 85,635,238

Actual Loss Payout and Experience Account Balance

Calendar Year	Payout Pattern %	Payout Pattern $	Interest Credit at 5%	Losses Paid	Exp. Acct. Balance
0					85,635,238
1	25.0%	25,000,000	4,281,762	(25,000,000)	64,917,000
2	30.0%	30,000,000	3,245,850	(30,000,000)	38,162,850
3	30.0%	30,000,000	1,908,143	(30,000,000)	10,070,993
4	10.0%	10,000,000	503,550	(10,000,000)	574,542
5	5.0%	5,000,000	28,727	(5,000,000)	(4,396,731)
6	0.0%	0	NA	0	NA
7	0.0%	0	NA	0	NA
Total	100.0%	100,000,000			

Comments: When the payout pattern is accelerated, the experience account is exhausted before all claims are settled. The reinsurer is still obligated to pay the remaining $4.4 million and thus incurs a net loss for this deal.

Expected Loss Payout and Experience Account Balance

Nominal Ult. 100,000,000

Calendar Year	Payout Pattern %	Payout Pattern $	Interest Credit at 5%	Losses Paid	Exp. Acct. Balance
0					85,635,238
1	15.0%	15,000,000	4,281,762	(15,000,000)	74,917,000
2	25.0%	25,000,000	3,745,850	(25,000,000)	53,662,850
3	25.0%	25,000,000	2,683,143	(25,000,000)	31,345,993
4	10.0%	10,000,000	1,567,300	(10,000,000)	22,913,292
5	10.0%	10,000,000	1,145,665	(10,000,000)	14,058,957
6	10.0%	10,000,000	702,948	(10,000,000)	4,761,905
7	5.0%	5,000,000	238,095	(5,000,000)	0
Total	100.0%	100,000,000			

Present Value of Expected Loss 85,635,238

Actual Loss Payout and Experience Account Balance

Calendar Year	Payout Pattern %	Payout Pattern $	Interest Credit at 5%	Losses Paid	Exp. Acct. Balance
0					85,635,238
1	15.0%	15,000,000	4,281,762	(15,000,000)	74,917,000
2	20.0%	20,000,000	3,745,850	(20,000,000)	58,662,850
3	20.0%	20,000,000	2,933,143	(20,000,000)	41,595,993
4	15.0%	15,000,000	2,079,800	(15,000,000)	28,675,792
5	10.0%	10,000,000	1,433,790	(10,000,000)	20,109,582
6	10.0%	10,000,000	1,005,479	(10,000,000)	11,115,061
7	10.0%	10,000,000	555,753	(10,000,000)	1,670,814
Total	100.0%	100,000,000			

Comments: When the payout pattern is slower than expected, there is a positive experience account balance when all claims are settled. This is profit that will typically be returned to the cedant. In many cases, the cedant will commute the contract in order to recognize this gain prior to the final claim settlement.

Background: ABC Insurance Company (ABC) would like to get surplus relief via a loss portfolio transfer to XYZ Reinsurance Company (XYZ) effective 12/31/03. ABC chose to transfer the reserves for its book of accountants professional liability, which it has been running off since exiting that market. At 12/31/03, the undiscounted unpaid losses for this book were $100.0 million.

Prior to effecting the LPT, total assets are $1.25 billion, total loss reserves are $1.0 billion. Assume no balance sheet activity other than the LPT and its runoff.

Expected payout pattern:

Calendar Year	2004	2005	2006	2007	2008	2009	2010
% Paid	15%	25%	25%	10%	10%	10%	5%

PV at 5% (millions) $85.6

Reinsurance premium: $90.0

Ceding Entity Accounting:

12/30/03 (Prior to LPT)

Assets		Liabilities, Surplus, and Other Funds	
Cash	$1,250.0	Unpaid loss	$1,000.0
		Policyholder surplus	250.0

12/31/03 (Subsequent to LPT)

Assets		Liabilities, Surplus, and Other Funds	
Cash	$1,160.0	Unpaid loss	$1,000.0
		Retro reinsurance ceded	(100.0)
		Total liabilities	$900.0
		Special surplus from retro re	$10.0
		Unassigned surplus	250.0
		Policyholder surplus	$260.0

Comments:

(1) Cash decreases by the amount of the LPT premium ($90.0 million) while liabilities decrease by the amount of the transferred reserve ($100.0 million).

(2) The cedant cannot gain from the surplus relief until the losses paid/reimbursed exceed the consideration paid to the reinsurer. As a result, the surplus gain ($10.0 million) is restricted and recorded as "special surplus from retroactive reinsurance."

...

12/31/08

Assets		Liabilities, Surplus, and Other Funds	
Cash	$1,160.0	Unpaid loss	$915.0
		Retro reinsurance ceded	(15.0)
		Totalliabilities	$900.0
		Special surplus from retro re	$10.0
		Unassigned surplus	250.0
		Policyholdersurplus	$260.0

12/31/09

Assets		Liabilities, Surplus, and Other Funds	
Cash	$1,160.0	Unpaid loss	$905.0
		Retro reinsurance ceded	(5.0)
		Totalliabilities	$900.0
		Special surplus from retro re	$5.0
		Unassigned surplus	255.0
		Policyholdersurplus	$260.0

12/31/10

Assets		Liabilities, Surplus, and Other Funds	
Cash	$1,160.0	Unpaid loss	$900.0
		Retro reinsurance ceded	0.0
		Totalliabilities	$900.0
		Special surplus from retro re	$0.0
		Unassigned surplus	260.0
		Policyholdersurplus	$260.0

Comments:

(1) As of 12/31/08, $85.0 million of the $100.0 million transferred has been paid. This does not exceed the LPT premium, so the $10.0 million of surplus relief is still restricted.

(2) As of 12/31/09, $95.0 million of the $100.0 million transferred has been paid. The $5.0 million of transferred loss still to be paid is restricted surplus; the remaining $5.0 million of the $10.0 million of surplus relief is earned as unassigned surplus.

(3) As of 12/31/10, all transferred liabilities have been paid and the full $10.0 million of relief has been earned.

Deposit Accounting Illustration 5

Background: XYZ Re provides excess-of-loss coverage to ABC Primary Insurance Company. XYZ will not begin to reimburse ABC until 2 years from the effective date of the contract.

Expected Loss: 50,000,000 (initial)
51,000,000 (revised at end of year 1)

Expected Payout:

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7
0%	0%	25%	30%	30%	10%	5%

Premium: 40,399,180 (present value at 5% of initial expected loss)

XYZ Accounting:

	Interest Expense	Cash Payment	Deposit Liability
Initial liability			40,399,180
Y1 interest at 5%	2,019,959		42,419,139
EOY1		0	42,419,139
Upward Revaluation	1,000,000		43,419,139
Y2 interest at 3.81%	1,614,448		45,033,587
EOY2		0	45,033,587
Y3 interest at 3.81%	1,713,952		46,747,539
EOY3		(12,750,000)	33,997,539
Y4 interest at 3.81%	1,779,184		35,776,723
EOY4		(15,300,000)	20,476,723
Y5 interest at 3.81%	1,361,641		21,838,364
EOY5		(15,300,000)	6,538,364
Y6 interest at 3.81%	831,155		7,369,520
EOY6		(5,100,000)	2,269,520
Y7 interest at 3.81%	280,480		2,550,000
EOY7		(2,550,000)	0

Comments:

(1) Due to the 2-year delay before payments by XYZ, this deal does not transfer timing risk and therefore requires deposit accounting.
(2) XYZ initially records a liability equal to the consideration paid by ABC.
(3) Each year, unwinding of discount is reflected as interest expense on the income statement and as an increase to the deposit liability on the balance sheet.
(4) At the end of Year 1, the estimated subject losses are revised upward by $1,000,000. This is reflected as interest expense to XYZ and also as an increase to the deposit liability. In addition, the effective yield is revised from 5% to 3.81% to reflect the expect timing and amount of future payments. The effective yield is calculated so that the liability declir to $0 at the same time as the final loss payment is made.

Reserving in a Changing Environment: Responding to the Impact of Layoffs, Plant Closures and Downsizing in Reserving for Workers Compensation Liabilities

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Casualty Actuarial Society

Committee on Reserves — 2004 Call Paper Program

Reserving in a Changing Environment: Responding to the Impact of Layoffs, Plant Closures and Downsizing in Reserving for Workers Compensation Liabilities

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ABSTRACT

In some instances, the impacts of layoffs and plant closures on workers compensation costs have resulted in a doubling of the pure premiums whereas, in other instances there were no appreciable effects on workers compensation pure premiums. This paper discusses some of the issues surrounding estimating workers compensation losses during periods of layoffs and plant closures. We have also developed a simplistic and practical approach for incorporating the estimated impacts into traditional reserving methodologies.

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INTRODUCTION

As companies change their workforce due to economic conditions, companies may experience changes in its workers compensation costs per employee. As actuaries, we sometimes find it difficult to interpret trends and changes in benefit levels (and resulting utilization changes) on loss development and pure premiums. Combining these normal challenges with company-specific issues, such as staff reductions, can lead to additional challenges. The staff reductions may include such actions as plant closures, layoffs, and geographical relocation of production capacity. Other staff actions such as strikes may have similar impacts.

In researching the potential impacts of staff downsizing, we started by looking at prior "downsizing" impacts on workers compensation costs for the company we were analyzing. The diagnostics used will be discussed latter. Then, we did a literary search on articles correlating to staffing actions and workers compensation costs. The following paragraphs refer to some of the associated costings.

In 1996, Cigna Group, in association with the American Management Association[1] conducted a survey of approximately 300 large and midsized employers that underwent organizational staff changes between 1990 and 1995. The survey results showed that staff reductions may have reduced the payroll, but increased the workers compensation costs (as a function of payroll). The survey showed that staff reductions contributed to a rise in claims for occupational and non-occupational disabilities, particularly stress-related claims. The survey concluded that claims not only rose among employees that lost their jobs, but also among the surviving employees. The results of the survey showed that 33% of the entities going through staff reductions saw an increase in occupational disability claims, whereas 24% of the entities saw an increase in non-occupational claims.

Studies of Recessions and Workers Compensation Costs

Some of the literature we reviewed included studies of impact of recessions on workers compensation costs. Rather than focusing on individual company impacts, these studies reflected the impacts on a state's entire workers compensation system. We believe that the impacts noted in these studies would be significantly magnified for a specific company undergoing staff actions.

A study by the Workers Compensation Research Institute (WCRI) in 1994[2] - conducted on the cost drivers of the New Jersey workers compensation system during the 1989 - 1991 recession - concluded that the recession was estimated to have reduced costs by 3.8% and to have increased costs by 5.6% through other effects. The reductions were due to reduced employment and the changed mix of employment. The researchers noted that increased wages drove up costs by 1.7%, and increased costs of medical services drove up costs by 1.9%. In addition to the effects on unemployment, the recession had the following impact on workers compensation costs in the state:

- increased average duration of temporary total disability cases, contributing 1.8% to the total costs; average duration in the construction and manufacturing industry rose from 8.8 weeks to 23.0 weeks

- increased medical costs, contributing 1.6% to the total costs due to increased utilization of medical services; this increase was in addition to the increase resulting from the price of medical services

- higher permanent partial disability ratings, contributing 0.9% to the total costs; the research found evidence that the higher ratings in the industries most affected by the recession were not related to the severity of injuries but rather more sympathetic adjudicators

- increased claims for occupational disease or cumulative injury, contributing 0.7% to the total costs; according to the study, the onset of recession substantially increased occupational disease and cumulative injury cases; the researchers believed that the cases are ones that would not have been filed otherwise; unlike most other such cases, many of these did not name a specific problem

- other indemnity benefits, contributing 0.6% to the total costs.

A similar WCRI study[3] on the Massachusetts workers compensation system concluded that the recovery from the 1991 recession led to a reduction in costs of 4.0% per year in the Massachusetts workers compensation system. The reductions were a result of reduced indemnity benefits. The largest cost savings came from industries where employment was steady or grew steadily.

Another WCRI study[4] conducted on workers compensation costs in the states of Florida, Georgia, Illinois, Massachusetts, Michigan and Pennsylvania from 1984 - 1988 concluded that the recessions have an impact on claims severity because of the increased use of the workers compensation system, longer duration of claims, and more frequent and larger lump sum settlements.

A similar study conducted by WCRI[5], examined the effects of recessions on medical costs, and it concluded that medical costs grow fast during recessions. Researchers concluded that the increase is likely due to the increase in utilization of medical benefits and a change in the mix of claims. According to the study the increase in medical costs may be to establish and maintain entitlement to workers compensation benefits or may be due to the shift in costs from employer-provided medical insurance to the workers compensation system (as medical insurance might be eliminated).

PURPOSE

This paper discusses issues that should be considered when reserving for workers compensation liabilities of large entities undergoing staff reductions. Additionally, we present diagnostic techniques to detect the impact of the changed conditions and a practical approach to incorporating these changes into the reserving model.

During transition periods such as staff reductions, entities may experience abrupt changes in claim frequency and severity, and in the rate at which workers compensation claims/losses are reported and settled. These abrupt changes in claim frequency and severity may be caused by the population of laid-off employees, as well as ongoing employees. As a result, the use of historical data patterns and traditional actuarial reserving methods without modifications may result in erroneous estimates. The underlying assumptions for the traditional reserving methodologies only allow for random variations in parameters such as type of exposure, mix of claims and so on. Any non random variation of these parameters will result in the traditional reserving methodologies yielding results that are systematically distorted.

This paper develops an analytical approach that may help the actuary cope with the challenges of the changing environment, such as those experienced during a staff reduction. We use diagnostics such as emerging frequency and severity at different evaluation points by accident year to discern shifts in data patterns. The mix of claims by type of claim should also be investigated. The results of the diagnostic analysis are used to develop an approach that allows the reserving actuary to adjust estimates of indicated liabilities based on historical data for the estimated impact of changes as a result of staff reductions.

The concepts presented in this paper pertain to reserving for large employers. However, some of the ideas presented and issues discussed are equally pertinent to insurance company reserving for workers compensation in a recessionary environment. For smaller employers it may be difficult to separate the impacts of staff reductions from random variations that typically occur in the data.

APPROACH

This paper is organized in two parts:

Section I will discuss the potential considerations and impacts of staff reductions on workers compensation losses.

Section II will discuss the modifications that the actuary can incorporate in the reserving model to account for the impacts or changes as a result of staff reductions.

SECTION I: DISCUSSION OF THE POTENTIAL IMPACTS OF STAFF ACTIONS ON WORKERS COMPENSATION LOSSES

The following is a broad overview of the type of contributing factors and changes the entity might experience after undergoing a staff action.

Contributing Factors

The impacts of staff actions can vary significantly from company to company or even within a company. The impacts of staff reductions on workers compensation costs can vary from 0% to 100%. The impact can be influenced by a number of factors including:

- level of severance benefits
- "downsizing" announcement tactics
- employee loyalty (from downsized employees and ongoing employees)
- psychology of ongoing employees
- union relations
- economic environment
- local unemployment rates
- skill level of downsized staff and their ability to learn new skills
- socioeconomic issues that can vary by geographic areas

Below is a discussion of the effects of the contributing factors and other factors during staff reductions on workers compensation cost components.

Frequency of Claims

Some sources[6] estimate that as many as 40% to 50% of the laid-off employees may file a workers compensation claim. General Electric[6], during a gradual shutdown of a Southern California plant that employed 250 workers, received 70 workers compensation claim filings from just 125 workers who were laid off in the initial phase of the plant closure in the first six months alone.

Claim frequency from ongoing employees can also be affected. The primary incentives for the increased claim filings by the laid-off employees are as follows:

- Workers compensation benefits (which are nontaxable) can partially substitute loss of income.

- The differential penalty between full pay and workers compensation indemnity benefits is absent as the worker is laid off.

- Laying off employees who have open workers compensation claims is much more difficult.

- Workers compensation benefits are usually larger, and paid over a longer period of time than unemployment benefits[9].

- Additional surgeries/treatment may be scheduled to improve positioning for next job (e.g., surgery to correct carpal tunnel syndrome).

- Usually plant closures are accompanied by deterioration of relations between the management and employees, which further leads to an increase in claims.

- When workers fear they might lose their jobs[7], they:
  - exhibit a lower level of knowledge about appropriate safety behaviors
  - demonstrate less motivation to comply with organizational safety policies.

In some instances, claim frequency can decrease as "downsized" employees have less work to do or payroll is continued temporarily due to severance package or due to change in the nature of the work that is performed after the staff reduction.

A WCRI study on New Jersey[2] suggested that during the 1989 -1991 recession, workers compensation claim frequency declined because of reduced employment. Some individual company data that we have reviewed show a similar picture. In a staff reduction setting, while the frequency of claims in the laid-off population rises, there is an offsetting decline in the claim frequency of the surviving population in the year of the staff reduction, which may lead to an overall decline in frequency.

Severity of Claims

Severity of claims increases significantly during a plant closure. The reasons for this increase in severity may be the following:

- Workers who are getting laid off may try to shift medical costs for chronic injuries or ailments from the employer-sponsored group health care plans to the first-dollar workers compensation system.

- Workers getting laid off may have an incentive to hire attorneys to get larger settlements in the court system than mandated by the workers compensation laws[6].

- The distribution by type of claim may shift (short term versus long term versus medical only) as a higher proportion of claims are for longer-duration injuries such as psychological, stress, lower-back injury claims and cumulative injury claims.

- Absence of return to work and rehabilitation programs may prolong the duration of injuries.

- The distribution of surviving employees may influence costs. Some hypothesize that younger, less experienced workers tend to be injured more often but less severely than older, more experienced workers (who usually survive layoffs), who are injured less often but more severely.

- The loss of loyalty to an employer may result in a higher incidence of fraud and other moral hazard issues.

- Staff reductions may lead to increases in workers compensation benefits by increasing the time it takes for a worker to find a job.

- Severity may be higher due to type of injury; some chronic injuries may have been concealed for an extended period of time, only to be revealed upon layoff[9].

- Laid-off employees objective is to achieve a workers compensation benefit that exceeds the expected unemployment benefit[9].

A plethora of the studies cited above note that one of the primary drivers of workers compensation costs during recessionary periods is increased claims severity due to increased duration, increased medical utilization, claims mix shift due to increased claim filings for occupational disease and cumulative injuries and more frequent and larger lump sum settlements.

Allocated Loss Adjustment Expenses (ALAE)

Increase in ALAE severities during a plant closure can be associated with the increased litigation rate of claims and a mix shift towards a higher proportion of indemnity claims. Increased litigation rate is one of the primary factors driving the increased duration of claims in addition to the change in the mix of claims in a staff reduction environment.

In a staff reduction employees getting laid off are more apt to get an attorney involved to ensure a higher settlement of their workers compensation claim. The increased litigation rate and duration of claims may result in a different ratio of ALAE to loss (on both a paid and reported basis during the life of the claim).

Settlement and Reporting Rate of Claims

Claims settlement rate in a plant closure layoff scenario could change for the following reasons:

1. The entity undergoing the staff reduction may decide to close claims faster by offering lump sum settlements to claimants. This strategy could be adopted to get rid of the liability associated with the plant closure quickly and also limit the impact of attorney involvement from the claimant side.

2. One of the inadvertent results of a staff reduction scenario is that claims adjuster loads may increase. This could be the result of either faster reporting of claims or higher volume of claims in a staff reduction scenario. This in turn usually results in a change (slowdown) in the claims settlement rate as more claims are reported.

3. Another factor that may be affecting claims closure rate may be the change in the mix of claims. The shift in the mix of claims is usually toward the higher duration claims. For example, a claim that before a plant closure would have been filed as a medical-only claim may in a staff-reduction scenario be filed as an indemnity claim.

4. The rate at which claims are reported during the year of the plant closure and prior may change in a staff-reduction scenario. It is common to experience a wave of reporting activity soon after staff reductions are announced or unemployment benefits expire. Another suggested trigger for claims filings is the expiration of the supplemental disability benefits[6].

We would also like to note that an entity undergoing staff reductions can have extensive exposure to employment practices liability claims such as age-based or gender based employment discrimination during the layoff process. Such claims are usually filed as class action suits and have large attorney involvement. These suits could represent a huge exposure that an actuary should consider while reserving for an entity undergoing plant closure or downsizing. However, the impact of employment practices liability losses is beyond the scope of this paper.

Another consideration that the actuary reserving for an entity undergoing staff reductions should be aware of is the issue of re-opening of closed claims for older accident years. In some of the data we reviewed, we found several instances of a substantial number of claim re-openings for older accident years. It was difficult to ascertain whether this effect was a result of improper closing of claims or whether this was purely due to staff reductions.

The factors noted above, combined with the fact that claims and loss emergence has a random component to them, makes it extremely difficult to accurately measure the contribution of each of the above components in the actual experience. For example, at the end of the year if the reported number of claims and/ or claim severity is higher than the historical average, it is difficult to ascertain whether it is purely due to the staff reduction, the general deterioration in the entity's experience or just random worse experience.

SECTION II: SUGGESTED METHODOLOGIES THAT THE ACTUARY CAN INCORPORATE IN TRADITIONAL RESERVING METHODOLOGIES TO REFLECT THE IMPACTS OF STAFF REDUCTIONS

We discussed the impact of staff reductions on the workers compensation cost components in Section I. As a result of these changes the overall propensity to loss in terms of claims frequency and claims severity changes going forward for the entity undergoing staff reductions. As a result during this transition period the entity will have a propensity to loss that is different from that of its historical propensity to loss. For example, more injury claims may be reported during these transition periods as employee awareness to safety in the workplace declines during stressful periods of staff reduction and employees being laid off try to substitute employment income with workers compensation benefits. Similarly the frequency of claims during this period might be significantly higher or lower than what the historical data might suggest.

Essentially, the entity undergoing staff reductions has two different exposures to loss. One component contributing to the exposure is the surviving population of employees, which may exhibit loss characteristics closer to the entity's historical propensity to loss. The other component is the population of laid-off employees that shows a much higher propensity to loss. If possible, the actuary may want to separate certain facilities into those that are fully affected ("closed"), partially affected, and not-affected.

Our approach to working around the distortions in the latest diagonal and the change in propensity to loss for the recent accident years affected by staff reduction has some components similar to those outlined in the paper "Loss Reserving Without Loss Development Patterns – Beyond Berquist-Sherman" by Thomas L. Ghezzi and Berquist-Sherman. However, complete application of the approaches outlined in these papers is not possible as we still need to account for the changed exposure/propensity to loss in the most recent accident years as a result of the staff reduction. We considered selecting loss and claim development patterns by excluding the latest few diagonals to avoid the distortions due to staff reductions, but this approach ignores the shift in the exposure and the changed rate at which losses are being reported or paid for the entity undergoing staff reductions.

The approach we adopt in this paper will be to make adjustments to the fundamental components of the loss process, the claim frequency and severity. Essentially, we develop an adjusted estimate of the pure premium. The adjustments to the claims frequency will be made by segregating the exposure of the entity into those employees who are laid off and the surviving employees. The adjustments to severity are carried out by calculating on level claim severities by type of injury (claim). Using this approach we forego the use of loss and claim development history of the entity and thus avoid the systemic distortions present in the history during this transition phase.

If the actuary is also faced with a situation in which the staff reduction has affected the rate at which claims are being closed or changes in case reserving philosophy and/ or the rate at which losses are being paid out, as is often the case in such situations, then by adopting an approach which does away with the use of loss and claim development will also mitigate the problem.

We also developed a B-F based approach. To estimate workers compensation ultimate losses for the most recent years, actuaries usually rely on Bayesian methodologies such as the Bornhuetter-Ferguson (B-F) method as the loss development methods are extremely leveraged and unstable for a slow developing line of business such as workers compensation.

One of the inputs to the B-F method is the initial expectation of ultimate loss. Indeed for a long tailed line such as workers compensation, the method produces ultimate loss estimates for the relatively new accident years that are quite sensitive to the initial expectation of loss. In a regular environment the initial expectation of loss for the most recent years is estimated as a function of the historical loss experience of the entity per unit of exposure (pure premium) and the estimated exposure for the recent accident years. The assumption behind this technique is that the type and extent of hazard or propensity to workers compensation losses for the most recent years is similar to that of the entity's historical exposure. However, for an entity undergoing staff reductions this symmetry is destroyed. In such a situation, one of the issues facing the actuary is how to arrive at a meaningful estimate of initial expectation of ultimate loss as an input to the B-F method. We used the estimates of ultimate loss — arrived at by making adjustments to the frequency and severity of loss— as our initial expectation of loss in the B-F method. The other input required for the B-F method is the loss emergence patterns. We develop modified loss emergence patterns for this purpose. These modified loss emergence patterns are also used to develop estimates of ultimate loss.

We also considered modifying the loss development factors or to speed up or lag the historical loss and claim development patterns based on the observed effects of the staff reduction scenario. However, the drawback of this approach is that it is very difficult to come up with appropriate speed-up or lags to modify the loss and claim development patterns.

A related adjustment may also be needed for the accident year of staff reduction. After the plant closure the average accident date for the plant closure year will be earlier than the usual middle of the accident year. To factor this earlier average accident date we may need to speed up the loss development patterns.

We will consider the following hypothetical example to discuss the adjustments proposed above and to measure the results of these adjustments:

Example:

- An entity, XYZ, announces staff reductions on January 1, and the staff reduction will be completed by the end of the year.
- The entity's management has decided not to make any changes to its case reserving strategy.
- The number of employees has remained constant over the last two years at 20,000.
- Out of a total of 20,000 employees, the entity is downsizing 6,000 employees.
- The entity's on level annual claims frequency per employee is 5.7% in the accident year of staff reductions.
- There are no benefit level changes in the most recent five accident years.
- Claim frequency is calculated as number of claims per employee.

Loss Information

Exhibits 8 through 17 show the historical loss experience of the hypothetical entity under consideration, in the form of triangles. Losses and claims are aggregated by accident year. We created loss and claim information for 15 accident years at 15 annual evaluations. The data for the first year are based on hypothetical ultimate claims and succeeding years is derived by assuming 1% per year trend in ultimate claim counts and a 4% per year trend in ultimate severity (i.e., total ultimate loss trend of 5% per year, assuming constant exposure level). We adopted this approach as we wanted to focus on just the impacts of the staff reductions and did not want to deal with the noise involved in the history of losses. Accident years are numbered 1 through 15.

We assume that the staff reduction is announced on January 1 of calendar-year 15 and completed by the end of year 15. As shown by the loss and claim count development triangles, the older years are not affected by the staff reductions. However, accident years 15 through 13 show development factors which are different from historical averages.

We assume that the impact of the staff reductions on the most recent three accident years claims is as follows:

Accident Year	Frequency Impact	Severity Impact
15	Yes	Yes
14	Yes	Yes
13	No	Yes
12	No	No

These assumptions makes sense as usually most of the workers compensation claims for a given accident year are reported by the end of 24 months. In the example considered in this paper, historically approximately 89% of the claims are reported by the end of 24 months for any given accident year. Claim severity however, is still emerging for the recent accident years and, as a result, will be affected for more accident years as claim durations increase on new and already open claims and changes in the mix of claims take place during the transition phase.

As we show later in this section, a diagnostic technique to discern the accident years that are affected by staff reductions is to chart reported claims severity (average reported loss per reported claim) at different evaluations and compare the results with those of older accident years. This chart is shown later in this section. In our example we assume that claims severity for the most recent three accident years is impacted.

The exhibits in this paper discuss estimates for accident years 15 through 13, the accident years that are assumed will be affected by the staff reduction.

The entity's historical claim distribution by type of claim is as follows:

Entity XYZ Historical Distribution of Workers Compensation Claims by Type of Claim	
Claim Type	% Of Total
Permanent	8.0%
Temporary	30.0%
Medical Only	62.0%

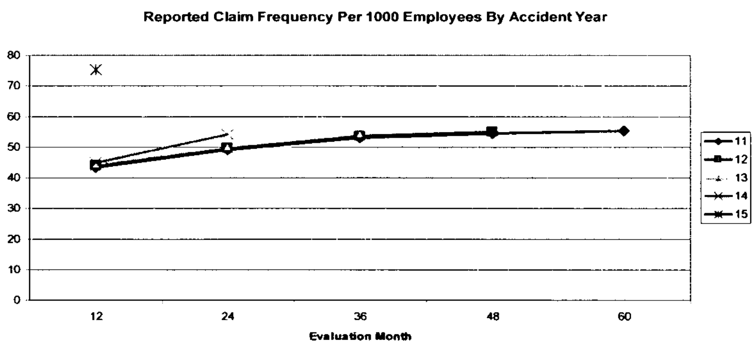
Discussion of Adjustments

In this paper we will approach the adjustment process by making adjustments to the individual loss components of the loss process — the frequency and severity of claims.

Claim Frequency

The adjustment to claim frequency is done by constructing an exposure-based model. The adjustment to frequency is shown in Exhibit 7 of the appendix. The process begins by comparing the reported claims frequency for the accident year of staff reduction and the accident year prior with the emerged claims data at the 12- and 24-month evaluation points, respectively.

The following diagnostic chart shows the explosion in emerged claims frequency for accident years 14 through 15 in the latest calendar year (calendar year of staff reduction).



We segregate the population of employees into those who have survived the staff reduction and those who are laid off. As discussed above, for an entity undergoing staff reduction, there are two different types of exposures. The surviving workers may have a different claims frequency compared with those workers who are being laid off. It is probable that the claim frequency for the surviving employees will be similar to the historical frequency of the entity (this may not hold true if the surviving population has a different exposure mix, e.g., a company shuts down the entire manufacturing facility but keeps the office staff). The claims frequency for the laid-off employees may be much higher. Additionally, the claims emergence patterns for the surviving and laid-off employees may be very different. Furthermore, the claims emergence pattern for the laid off employees will be much faster than that of the surviving employees.

In a scenario in which an entire plant is shut down, we can go a step further and segregate the overall historical exposure into two separate components: 1) the exposure of the plant that was shut down 2) ongoing facilities. We can then select claim frequency for the ongoing operations according to historical averages and select estimates of claim frequency for the plant that was shut down using the approach described below.

Initial estimates of the claim frequency of the laid-off workers can be arrived at by talking to the claims management personnel, the entity's management and the Risk Manager. Additionally, the list of contributing factors mentioned in Section I (p. 4) of this paper should be considered when arriving at the a priori estimate of frequency of claims as a result of the laid-off employees. Other exposure-based methods could also be used. Some estimates in literature put the estimate of claim frequency at roughly 40% to 50% of the laid-off employees.

Once the initial estimates of claim frequency for the laid-off population are selected, we can then use a Bayesian approach to update these frequencies after the end of the year when reserves are being estimated, and when the staff reduction has already taken place and actual claims information is available.

We assumed that the a priori estimate of the claim frequency of the surviving population is the same as the historical claim frequency of the entity (5.7%). We assumed the a priori estimate of claim frequency for the laid-off population of employees to be 10.0% for our study.

The next issue that we have to deal with is the reporting pattern of claims from the laid-off employees. It is to be expected that the reporting pattern for these claims will be much faster than those for the surviving employees. Our review of the literature on this subject and data on reporting patterns for plant closures indicates that most of the claims after the staff reduction are filed within the first year of the staff reduction. The reporting of claims may coincide with the ceasing of the unemployment benefits and social security disability benefits. Input from claims management personnel should also be considered when arriving at the estimate. For our analysis we assumed the following:

— 95% of the total claims filed by the laid-off employees will be reported by the end of the year of the staff reduction; the balance 5% will be reported in the following year.

— 90% of the total claims resulting from downsizing will be due to occurrences in the current accident year; the remaining 10% will be due to occurrences in the accident year prior to the staff reduction year.

The above pattern of the claims filed by the laid-off employees was selected on the basis of a review of accident year by report year claims reporting patterns of the downsized employees at the end of the staff reduction year and the diagnostic reported claim count chart below. Consideration was also given to the fact that claims filed by laid-off employees (after the layoff) may be denied by employers using the legal doctrine of post-termination defense. Under the post-termination defense the employer can argue that the employee is filing a workers compensation claim because they are downsized and that the injury may not be work related. Only in instances where the laid off employee has a medical history of injury prior to layoff, can the laid-off employee successfully file a workers compensation claim. As a result, the longer the time elapsed between layoff and the filing of the claim, the more difficult it becomes for the laid off employees claim to be accepted by the employer. Additionally, some of the literature that we reviewed suggested the following two[6] likely triggers for workers compensation claims in a staff reduction scenario:

a) expiration of unemployment benefits by the end of six months and

b) expiration of supplemental disability income benefits by the end of twelve months.

Based on all of the above considerations we assumed most of the workers compensation claims by the laid-off employees will be reported by the end of the staff reduction calendar year.

We note that the reporting pattern of claims due to a staff reduction will vary depending on the particular situation at hand. The actuary should consult the employer's risk manager and claims personnel before arriving at the claims occurrence and the reporting pattern assumptions for the analysis.

We assume that the surviving population of employees will have a claims reporting pattern similar to the self-insured entity's historical reporting pattern for the purpose of this analysis.

Armed with the above information above we can compare the actual claim frequency at year-end of the plant closure for both the surviving and the laid-off populations with their respective a priori estimates and calculate estimates posterior to the observation using a Bayesian approach. The calculations are shown in Exhibit 7. We used a B-F approach to come up with our estimates of ultimate claims.

Claim Severity

Since the underlying mix of claims by type of claim has changed we cannot develop new estimates of ultimate severity by just completing the claim severity triangle of all claim types combined.

Adjustments to the claims severity can be made by calculating historical severities by the usual type of disability classifications used in workers compensation analysis (i.e., temporary partial disability, temporary total disability, permanent partial disability permanent total disability, and medical only). If the claims data are not available in sufficient detail then the actuary can request data broken down in much lesser refinement such as short-term and long-term claims and medical-only claims or temporary, permanent disability and medical-only claims. Estimates of ultimate claim severity by claim type can be arrived at by reviewing a historical sample of closed claim severities by claim type. This information is usually available in the claims database of the entity. The selected severities by claim type can then be brought to current levels using trend and benefit-level factors.

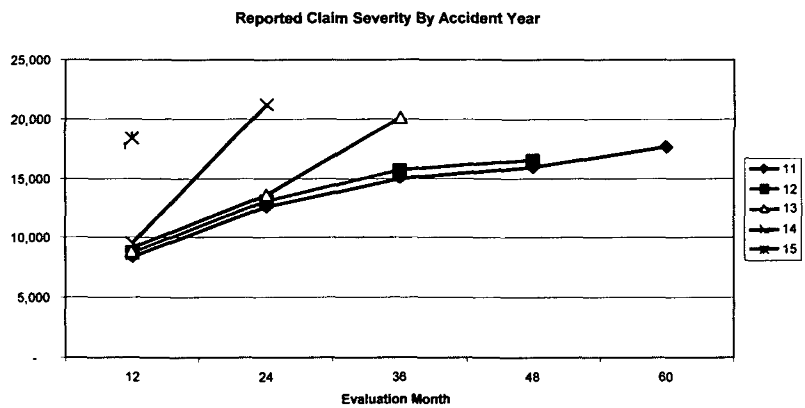
We note that during the course of development some temporary claim injuries usually convert from temporary disability to permanent partial disability. When reviewing the claims mix an actuary should be cognizant of this fact and make appropriate adjustments to the claims mix, to arrive at the overall severity as described above.

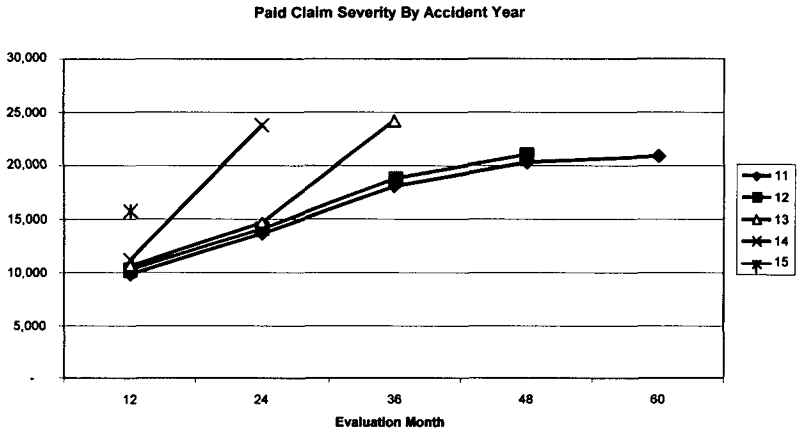
Another way to work around the problem of claim loss data being unavailable by type of claim is to segregate claims loss data by size of loss and then calculating average historical claim severities for the different buckets/intervals of size of loss. We can use this approach to create estimates of ultimate claim severity by type of claim. This approach is similar to one of the approaches outlined in Loss Reserve Adequacy Testing: A Comprehensive Systematic Approach: Berquist, James R.; Sherman, Richard E.

The adjustment to severity is shown in Exhibit 6 of the appendix. The process of adjustment begins by comparing the average reported severity for the accident year of staff reduction and the most recent two accident years prior to the plant closure year at the different evaluations with similar historical data as shown in the chart below. The following diagnostic chart shows the explosion in claim severity for accident years 13 through 15 in the calendar year of staff reduction. To test whether the explosion in reported severity is not due to case reserve strengthening the actuary should also review paid claim severity (paid loss to paid claims) shown in the following chart, and/ or paid loss to reported loss ratios at different evaluation points. As shown both the paid and reported severity have exploded in the calendar year of staff reduction for the most recent three accident years that are affected.

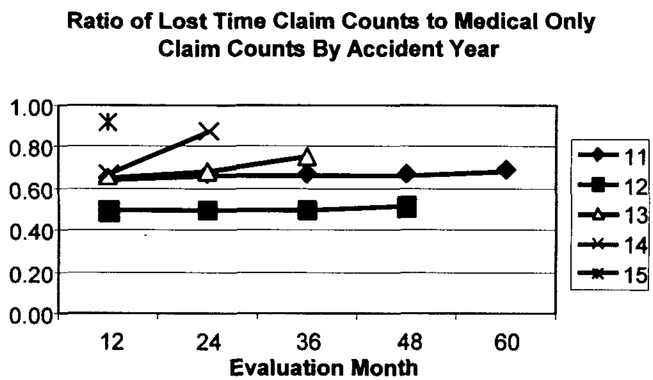
The next step is to review the mix of claims by type of claim for each accident year. Using the current reported mix of claims by accident year and the historical on level ultimate severities we calculate the posterior estimate of overall severity.

We wish to note that this calculation will yield imprecise results if there are strong calendar-year effects influencing the severities. This has often been the case for medical severity in states such as California.





The following chart below shows the change in the ratio of lost time claim counts to medical-only claims counts for a typical entity that has undergone staff reduction in calendar-year 15.



We note that if historical loss development information segregated by claim type is available, then we can develop the ultimate loss by each claim type and the above calculations are not necessary. However, frequently self-insured entities do not track loss development information by claim type.

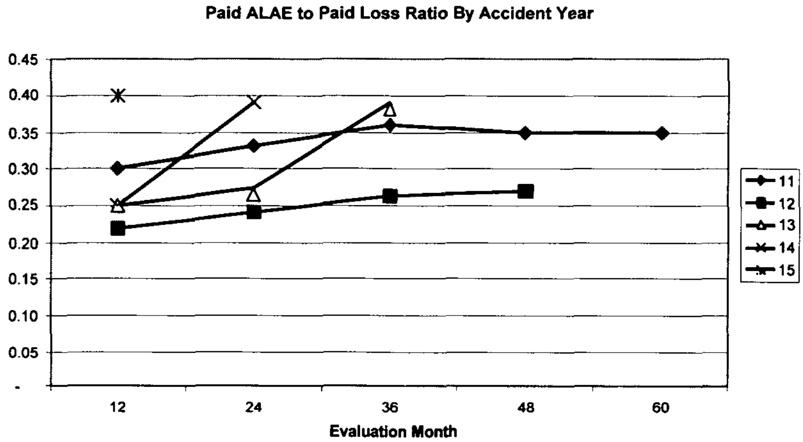
Allocated Loss Adjustment Expenses

As discussed earlier, one of the impacts of a staff reduction is an increase in ALAE severities and the paid ALAE to paid loss ratio for the accident years affected by staff reduction. As a result the B-F methodology on paid ALAE to loss ratio to calculate ultimate ALAE for the most recent accident years may underestimate the ultimate ALAE, if we use the historical paid ALAE to paid loss ratio as the initial estimate in the B-F methodology.

The diagnostic chart below shows the explosion of paid ALAE to paid loss ratio for accident years 15 and 14.

To make adjustments to the calculations of ultimate ALAE, we start by reviewing the litigation rate (number of reported cases in litigation to total number of reported cases) of the claims reported to date for the most recent five accident years at similar evaluation points. Exhibit 5 shows a log-linear model[10] which predicts ALAE to loss ratio based on the independent variables such as litigation rate and the ratio of indemnity to medical only claims. We used historical data on litigation rate and indemnity claims to med only claims ratio on accident years 13 and prior to develop this log-linear model to project paid ALAE to paid loss ratio for accident years 14 and 15. We used a simplistic log-linear model for this paper. However, the reader should endeavor to build a more robust model to estimate ALAE costs.

Exhibit 4 shows the calculation of ultimate ALAE given the B-F methodology applied to the projected ALAE to loss ratio both on an adjusted and unadjusted basis.



Ultimate Loss & ALAE Calculations and Calculation of the Impact of Staff Reduction

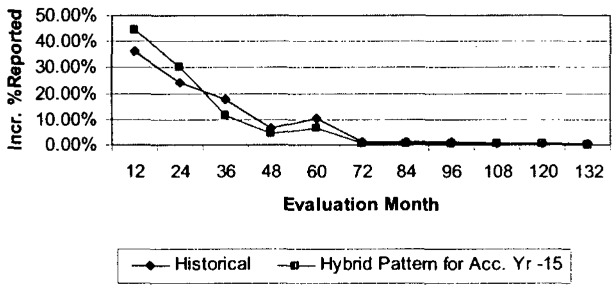
The adjusted claim frequency and severity, along with estimates of ALAE to loss ratio calculated above, can then be used to calculate an estimate of the ultimate loss and ALAE for accident years 13 through 15. This approach foregoes the use of historical loss development factors, thus avoiding the distortions in the loss development history due to the changed circumstances. A similar approach was used to calculate estimates of ultimate loss for accident years 13 through 15 based on unadjusted estimates of claim frequency and claim severity. The difference between the two estimates of ultimate loss for each accident year gave us the impact of staff reduction by accident year. The calculations for the staff reduction impact by accident year are shown in Exhibit 2.

We show a reporting pattern for the additional losses as a result of staff reduction in Exhibit 2. We assumed that these additional losses will be reported in the calendar year of staff reduction and the subsequent calendar year, much faster than the historical loss reporting pattern. We then overlaid the reported development of the additional staff reduction impact over the expected reported amounts (assuming there was no staff reduction) to derive a hybrid reporting pattern. Exhibit 2 shows the derivation of a hybrid loss reporting pattern for accident years 13 through 15. The following chart shows the relationship between the incremental historical loss reporting pattern and the incremental hybrid reporting pattern. As shown the hybrid reporting pattern is faster at the initial evaluations but slower at the later evaluations compared with the historical reporting pattern.

We used the hybrid reporting pattern developed above to calculate estimates of ultimate loss using the B-F method. We refer to these estimates as the adjusted B-F method estimates. The initial expected loss for the adjusted B-F method was based on the adjusted frequency severity method ultimate loss estimates developed above. We also developed estimates of ultimate loss using the B-F method but using an initial expected loss developed based on historical loss data without any adjustments (unadjusted B-F method). The loss development pattern used to develop the unadjusted B-F estimates is the historical loss development pattern of the entity.

We note that in this study that we have relied on reported loss development patterns to come up with estimates of our ultimate loss. As a result we did not endeavor to develop a hybrid paid loss development pattern for the purpose of this study. However, similar principles can be applied to arrive at a hybrid paid loss pattern. For example we can assume that for each of the accident years affected, the additional staff-reduction impact will be paid out in a manner similar to the entity's historical payment pattern for a new accident year. This additional paid amount can then be overlaid on the expected paid amount, assuming no staff reduction impact, to arrive at a hybrid payment pattern.

Incremental Reporting Patterns



The estimates based on the B-F method are developed in Exhibit 3. The hybrid incremental loss reporting pattern shown above is faster than the historical loss reporting pattern for the first 24 months and slower than the historical loss reporting pattern subsequently.

We also calculated estimates of ultimate losses based on the traditional loss development technique using the hybrid loss reporting pattern and the unadjusted loss development pattern. These estimates are also shown in Exhibit 3.

The ultimate ALAE estimates are developed in Exhibit 4. We developed estimates for ultimate ALAE using the paid ALAE development method. Additionally, a B-F approach is used on the adjusted and unadjusted paid ALAE to paid loss ratio.

DISCUSSION OF RESULTS

Exhibit 1 shows estimates of ultimate loss and ALAE produced by the various methods. As shown the estimates produced by the adjusted and unadjusted methods are markedly different for the affected accident years. The adjusted frequency-severity and the adjusted B-F approach produce estimates of ultimate loss for the most recent three accident years that are lower than the unadjusted loss development approach but higher than the unadjusted B-F method estimates. The unadjusted B-F method is slow in responding to the changing conditions, whereas the unadjusted reported loss development method is over responsive to the changing conditions.

The estimates of ultimate ALAE based on adjusted paid ALAE to paid loss ratio method and the adjusted B-F method estimates (based on adjusted paid ALAE to paid loss ratio) are higher compared to estimates of ALAE based on the unadjusted paid ALAE development and the B-F method applied to unadjusted paid ALAE to paid loss ratio. The estimates of ultimate ALAE based on the adjusted methods are almost similar.

We note that the results of the various methods could be higher or lower depending on the impact of staff reduction.

CONCLUSION

In this paper we have discussed various issues related to reserving for a self-insured entity which has recently undergone staff reductions. We discussed why the traditional loss reserving techniques may not produce accurate estimates of ultimate loss and ALAE and reserves. During such a transition phase reserve estimates can be calculated by employing an alternate frequency severity type approach, as appropriate changes can be made to this approach to account for the changing circumstances. We showed how the results of the adjusted frequency severity approach can be incorporated into the B-F approach. We also developed an exposure-based approach to calculate ultimate ALAE.

The advantage of the frequency severity approach adopted in this paper is that it avoids the distortions that may exist in the loss development history for the most recent accident years as a result of the staff reductions in addition to providing additional information about loss drivers. So even if losses and claims are being reported or settled faster or slower than what the historical development data would suggest, our projections are not affected by these distortions. This approach allows explicit consideration of factors such as the shift in mix of claims and propensity to loss. Considering this approach also provides the actuary with a range of estimates of ultimate loss.

One drawback of the approach used in this paper is that it does not help us completely delineate the effect of staff reduction from the other trends affecting the loss process. Despite incorporating the frequency severity approach and building hybrid loss development patterns into the B-F methods, we still have some distortion in the adjusted B-F estimates in that the B-F methods still rely on the historical reporting or paid loss pattern to some extent to come up with estimates of ultimate loss. A related shortcoming of the approach adopted in this paper is that our estimates of ultimate losses and reserves are contingent on the accuracy of the assumption of how the additional impact of staff reductions both in terms of claims and losses, will emerge.

An improvement to this methodology would be to perform sensitivity testing to ascertain the impact of changes in various assumptions that are built into the model. This can be accomplished by building and testing different scenarios according to different assumptions of the staff reduction impact on losses. This will help the actuary devise a range of estimates for ultimate losses and reserves and provide the actuary with a better idea of uncertainty associated with the reserves resulting from staff reduction impact.

References:

1. Plant Closings, Layoffs and Workers Compensation, Doug McCoy, McCoy Consulting

2. Gardner, Victor, Telles, and Moss, Cost Drivers in New Jersey, WCRI 1994. Minnesota Department of Labor & Industry Research Reporter

3. Gardener, Telles, and Moss, The 1991 Reforms in Massachusetts: An Assessment of Impact, WCRI, May 1996. Minnesota Department of Labor & Industry Research Reporter

4. Cost Drivers in Six States, WCRI, December 1992, Gardner, Victor, Telles, and Moss. Minnesota Department of Labor & Industry Research Reporter

5. How Choice of Provider and Recessions Affect Medical Costs in Workers Compensation WCRI, June 1990, Victor and Fleischman. Minnesota Department of Labor & Industry Research Reporter

6. Avoiding Workers Comp Claims Crush Following Layoffs and Plant Closings, Joanne Wojcik

7. Journal of Occupational Health Psychology called "The Effects of Job Insecurity on Employee Safety Outcomes"

8. California Workers Compensation Medical Payments, Litigation and Claim Duration — A Post-Reform Report Card, CWCI May 2002, Laura B. Gardner, Alex Swedlow

9. Workers Compensation and Economic Cycles: A Longitudnal Approach, Robert P. Hartwig, Ronald C. Retterath, Tanya E. Restrepo & William J. Kahley

10. Evaluating Reserves in a Changing Claims Environment: Aaron Halpert, Scott Weinstein, Christopher Gonwa

11. Loss Reserving Without Loss Development Patterns — Beyond Berquist - Sherman: Thomas L. Ghezzi

12. Loss Reserve Adequacy Testing: A Comprehensive Systematic Approach: Berquist, James R., Sherman, Richard E.

13. Public Policy Monograph, The American Academy of Actuaries: The Worker's Compensation System: An Analysis of Past, Present and Potential Future Crises

14. Preparing for Downsizing Costs: Letting employees go can lead to rising workers compensation claims: are you prepared?: Business Insurance

Exhibit 1

Ultimate Loss ('000)					
Accident Year	Adjusted Frequency-Severity Method	Unadjusted Reported Loss Development Method	Adjusted Reported Loss Development Method	Adjusted Reported Loss B-F Method	Unadjusted Reported Loss B-F Method
13	26,479	27,980	28,957	28,350	26,810
14	33,058	38,041	38,127	36,117	32,402
15	38,961	53,794	43,457	40,970	35,373
Total	98,499	119,816	110,541	105,437	94,586

Ultimate ALAE ('000)				
Accident Year	Adjusted Paid ALAE to Loss Ratio Method	Paid ALAE Development Method	Adjusted B-F Method	Unadjusted B-F Method
13	10,040	10,609	10,347	8,533
14	19,726	15,735	18,665	9,625
15	37,499	25,104	36,126	9,703
Total	67,266	51,448	65,138	27,862

Exhibit 2

Estimation of Impact of Staff Reduction

Accident Year	Freq.-Sev. Adjusted Ultimate Loss	Freq.-Sev. Unadjusted Ultimate Loss	Staff Reduction Impact	% Impact
13	26,479	22,631	3,849	17.0%
14	33,058	23,771	9,287	39.1%
15	38,961	24,970	13,992	56.0%
Total	98,499	71,372	27,127	38.0%

Development of Unadjusted Frequency-Severity Ultimate Loss

Accident Year	Selected Historical Ultimate Claim Severity	Trend Factor	Projected On Level Ultimate Claim Severity	Selected Historical Claim Frequency	Trend Factor	No. of Employees	Unadjusted Ultimate Claims	Freq.-Sev. Unadjusted Ultimate Loss
11	18,569	1.000	18,569	5.5%	1.000	20,000	1,105	20,511
12	18,569	1.040	19,311	5.5%	1.010	20,000	1,116	21,545
13	18,569	1.082	20,084	5.5%	1.020	20,000	1,127	22,631
14	18,569	1.125	20,887	5.5%	1.030	20,000	1,138	23,771
15	18,569	1.170	21,723	5.5%	1.041	20,000	1,149	24,970

Development of Hybrid Reported Loss Development Pattern

Accident Year 15												Total
Development Pattern	12	24	36	48	60	72	84	96	108	120	132	
Regular loss reporing	36.09%	60.49%	78.13%	84.90%	95.41%	96.70%	97.73%	98.71%	99.20%	99.70%	100.00%	
Incremental loss reporting	36.09%	24.39%	17.64%	6.77%	10.51%	1.29%	1.04%	0.98%	0.49%	0.50%	0.30%	100.0%
Staff Red. impact loss reporting	60.00%	40.00%										100.0%
Loss Payout												
Regular	9,012	6,091	4,405	1,692	2,624	321	259	244	123	124	75	
Staff Reduction Impact	8,395	5,597										
Cumulative Loss Payout	17,407	11,688	4,405	1,692	2,624	321	259	244	123	124	75	38,961
Hybrid Reporting Pattern	44.7%	74.7%	86.0%	90.3%	97.1%	97.9%	98.5%	99.2%	99.5%	99.8%	100.0%	
Incr. Hybrid Reporting Pattern	44.7%	30.0%	11.3%	4.3%	6.7%	0.8%	0.7%	0.6%	0.3%	0.3%	0.2%	100.0%

Accident Year 14												Total
Development Pattern	12	24	36	48	60	72	84	96	108	120	132	
Regular loss reporing	36.09%	60.49%	78.13%	84.90%	95.41%	96.70%	97.73%	98.71%	99.20%	99.70%	100.00%	
Incremental loss reporting	36.09%	24.39%	17.64%	6.77%	10.51%	1.29%	1.04%	0.98%	0.49%	0.50%	0.30%	100.0%
Staff Red. impact loss reporting	0.00%	60.00%	40.00%									100.0%
Loss Payout												
Regular	8,580	5,799	4,194	1,610	2,498	305	247	232	117	118	71	
Staff Reduction Impact		5,572	3,715									
Cumulative Loss Payout	8,580	11,371	7,908	1,610	2,498	305	247	232	117	118	71	33,058
Hybrid Reporting Pattern	26.0%	60.3%	84.3%	89.1%	96.7%	97.6%	98.4%	99.1%	99.4%	99.8%	100.0%	
Incr. Hybrid Reporting Pattern	26.0%	34.4%	23.9%	4.9%	7.6%	0.9%	0.7%	0.7%	0.4%	0.4%	0.2%	100.0%

Accident Year 13												Total
Development Pattern	12	24	36	48	60	72	84	96	108	120	132	
Regular loss reporing	36.09%	60.49%	78.13%	84.90%	95.41%	96.70%	97.73%	98.71%	99.20%	99.70%	100.00%	
Incremental loss reporting	36.09%	24.39%	17.64%	6.77%	10.51%	1.29%	1.04%	0.98%	0.49%	0.50%	0.30%	100.0%
Staff Red. impact loss reporting	0.00%	0.00%	60.00%	40.00%								100.0%
Loss Payout												
Regular	8,168	5,520	3,992	1,533	2,378	291	235	221	112	112	68	
Staff Reduction Impact			2,309	1,539								
Cumulative Loss Payout	8,168	5,520	6,302	3,073	2,378	291	235	221	112	112	68	26,479
Hybrid Reporting Pattern	30.8%	51.7%	75.5%	87.1%	96.1%	97.2%	98.1%	98.9%	99.3%	99.7%	100.0%	
Incr. Hybrid Reporting Pattern	30.8%	20.8%	23.8%	11.6%	9.0%	1.1%	0.9%	0.8%	0.4%	0.4%	0.3%	100.0%

Development of Ultimate Loss Estimates ('000)

Adjusted Frequency-Severity Method

Accident Year	Adjusted Ultimate Claim Count	Adjusted Ultimate Claim Severity	Ultimate Loss
13	1,127	23,499	26,479
14	1,215	27,198	33,058
15	1,251	31,155	38,961
Total			98,499

Loss Development Method

Accident Year	Actual Reported	Unadjusted % Reported	Adjusted % Reported	Unadj. Est. Reported Ult. Loss	Adjusted Reported Ult. Loss
13	21,861	78.1%	75.5%	27,980	28,957
14	23,010	60.5%	60.3%	38,041	38,127
15	19,416	36.1%	44.7%	53,794	43,457
Total	64,286			119,816	110,541

Adjusted BF Method

Accident Year	Initial Ultimate Loss	Adjusted % Reported	Expected Reported	Actual Reported	Estimated Reported Ult. Loss
13	26,479	75.5%	19,990	21,861	28,350
14	33,058	60.3%	19,950	23,010	36,117
15	38,961	44.7%	17,407	19,416	40,970
Total	98,499				105,437

Unadjusted BF Method

Freq. Exp. Model	Incurred Ultimate Loss	Paid Ultimate Loss	Loss Trend to Acc. Yr 15 Level	On Level Ultimate Loss
7	16,849	16,849	1.482	24,970
8	17,698	17,698	1.411	24,970
9	18,590	18,590	1.343	24,970
10	19,527	19,527	1.279	24,970
11	20,511	20,511	1.217	24,970
12	21,545	21,545	1.159	24,970
	Selected Initial Ultimate Loss			24,970

		Initial Ult. Loss
13	1.103	22,631
14	1.050	23,771
15	1.000	24,970

Accident Year	Initial Ultimate Loss	Unadjusted % Reported	Expected Reported	Actual Reported	Estimated Reported Ult. Loss
13	22,631	78.1%	17,681	21,861	26,810
14	23,771	60.5%	14,379	23,010	32,402
15	24,970	36.1%	9,012	19,416	35,373
Total	71,372				94,586

Exhibit 4

Development of Ultimate ALAE Estimates ('000)

Ultimate ALAE Development (Adjusted Paid ALAE to Loss Ratio)

Accident Year	Projected ALAE to Ultimate Loss Ratio	Ultimate Loss	Ultimate ALAE
13	0.38	26,479	10,040
14	0.60	33,058	19,726
15	0.96	38,961	37,499
Total		98,499	67,266

Ultimate ALAE Development (B-F Adjusted)

Accident Year	Projected ALAE to Ultimate Loss Ratio	Initial Ultimate Loss	Initial Ultimate ALAE	% Paid ALAE	Expected Paid ALAE	Actual Paid ALAE	Estimated Paid Ult. ALAE
13	0.38	26,479	10,040	53.9%	5,407	5,714	10,347
14	0.60	33,058	19,726	26.6%	5,247	4,186	18,665
15	0.96	38,961	37,499	11.1%	4,154	2,781	36,126
Total		98,499	67,266				65,138

Ultimate ALAE Development (B-F Unadjusted)

Accident Year	Projected ALAE to Ultimate Loss Ratio	Initial Ultimate Loss	Initial Ultimate ALAE	% Paid ALAE	Expected Paid ALAE	Actual Paid ALAE	Estimated Paid Ult. ALAE
13	0.27	22,631	6,110	53.9%	3,291	5,714	8,533
14	0.31	23,771	7,411	26.6%	1,971	4,186	9,625
15	0.31	24,970	7,785	11.1%	862	2,781	9,703
Total		71,372	21,306				27,862

Paid ALAE Development Method

Accident Year	Actual Paid ALAE	% Paid ALAE	Estimated Paid Ult. ALAE
13	5,714	53.9%	10,609
14	4,186	26.6%	15,735
15	2,781	11.1%	25,104
Total	12,680		51,448

Exposure Based ALAE Model

Accident Year	Reported Claims	Litigation Rate	Indemnity to Medical Ratio	Litigated Claims	Ultimate Paid ALAE	Paid Ultimate Loss	Ultimate Paid ALAE To Ultimate Loss	Projected Paid ALAE To Paid Loss
7	1,062	23.0%	61.3%	244	4,549	16,849	0.27	0.27
8	1,072	26.0%	61.3%	279	5,309	17,698	0.30	0.30
9	1,083	27.0%	61.3%	292	5,949	18,590	0.32	0.32
10	1,094	22.0%	61.3%	241	5,077	19,527	0.26	0.25
11	1,105	29.0%	61.3%	320	7,179	20,511	0.35	0.34
12	1,102	24.9%	61.3%	274	5,817	21,545	0.27	0.29
13	1,085	24.7%	78.6%	268	10,609	27,980	0.38	0.38
14	1,085	30.0%	92.3%	326				***0.60***
15	1,053	35.0%	108.3%	369				***0.96***
Overall	9,740	26.8%		2,613	44,490	142,701	0.31	
Overall		25.2%					0.31	
Average (14 to 15)		32.5%						

Regression Statistics		Coefficent Indem. to Med Rattio	Coeffient Litigation Rate
	Slope	5.15	74.25
	Constant	0.04	
	R-Squared	0.96	

Exhibit 6

Exposure Based Loss Severity Model

Assumption:
Change in Severity being caused by change in the duration of claims.
The change in duration of claims is being caused by the mix shift in claims

Accident Year	Selected Historical Ultimate Claim Severity	Claim Severity Trend Factor	Projected On Level Ultimate Claim Severity	Actual Reported Claims	A Priori Temporary % of Total	A Priori Permanent % of Total	A Priori Medical Only % of Total	Severity Temporary	Severity Permanent	Severity Medical Only
11	18,569	1.000	18,569	1,105	30%	8%	62%	36,267	88,355	1,000
12	18,569	1.040	19,311	1,102	30%	8%	62%	37,718	91,889	1,040
13	18,569	1.082	20,084	1,085	30%	8%	62%	39,226	95,565	1,082
14	18,569	1.125	20,887	1,085	30%	8%	62%	40,795	99,387	1,125
15	18,569	1.170	21,723	1,053	30%	8%	62%	42,427	103,363	1,170

Staff Reduction Accident Year-15

Claim Type	A Priori Distribution	Actual Reported Distribution	Actual Reported Claims	Posterior Distribution	A Priori On Level Ultimate Severity	Posterior Ultimate Severity
Permanent	8.0%	14.0%	147	14.0%	103,363	103,363
Temporary	30.0%	38.0%	400	38.0%	42,427	42,427
Medical Only	62.0%	48.0%	505	48.0%	1,170	1,170
	100.0%	100.0%	1,053	100.0%	21,723	31,155

Accident Year - 14

Claim Type	A Priori Distribution	Actual Reported Distribution	Actual Reported Claims	Posterior Distribution	A Priori On Level Ultimate Severity	Posterior Ultimate Severity
Permanent	8.0%	12.0%	130	12.0%	99,387	99,387
Temporary	30.0%	36.0%	391	36.0%	40,795	40,795
Medical Only	62.0%	52.0%	564	52.0%	1,125	1,125
	100.0%	100.0%	1,085	100.0%	20,887	27,198

Accident Year-13

Claim Type	A Priori Distribution	Actual Reported Distribution	Actual Reported Claims	Posterior Distribution	A Priori On Level Ultimate Severity	Posterior Ultimate Severity
Permanent	8.0%	10.0%	109	10.0%	95,565	95,565
Temporary	30.0%	34.0%	369	34.0%	39,226	39,226
Medical Only	62.0%	56.0%	608	56.0%	1,082	1,082
	100.0%	100.0%	1,085	100.0%	20,084	23,499

Accident Year-12

Claim Type	A Priori Distribution	Actual Reported Distribution	Actual Reported Claims	Posterior Distribution	A Priori On Level Ultimate Severity	Posterior Ultimate Severity
Permanent	8.0%	8.0%	88	8.0%	91,889	91,889
Temporary	30.0%	30.0%	331	30.0%	37,718	37,718
Medical Only	62.0%	62.0%	683	62.0%	1,040	1,040
	100.0%	100.0%	1,102	100.0%	19,311	19,311

Exposure Based Frequency Model

Scenario 1 - No Speed up & No Change in Reserve Adequacy

Assumptions:

Accident Year of Staff Reduction	15
Announcement of Staff Reduction	1/1/15
Years Affected by the Staff Reduction	15, 14, 13
Historical On Level & Trended Claim Frequency	5.7%

Claims as a result of the downsizing will be reported in the first two years after the plant closure year

A Priori Frequency of the Laid-Off employees 10%

Claims Reporting Pattern for the Laid Off Employees

Accident Year	% of Total Claims due to staff red. alloc. to Accident Year	A Priori Frequency Staff Reduction Year	A Priori Frequency Subsequent Report Year	Total
		Plant Closure Report Year	Subsequent Report Year	
	Emergence Pattern	95%	5%	
14	10%	1.0%	0.1%	1.0%
15	90%	8.6%	0.5%	9.0%
	Total	9.5%	0.5%	

Calcultion of the Ultimate Claims-Exposure Based Bayesian Approach

Employee Type	Number of Employees	A Priori Ultimate Claim Frequency	A Priori Est. Ultimate Claims	Expected % Reported	Expected A Priori Reported Frequency	Expected Reported Claims	Actual Claim Count	Actual Reported Claim Frequency	Posterior Ultimate Claim Frequency	Posterior Ultimate Claim Count
Staff Reduction Accident Year -15										
Surviving	14,000	5.7%	805	78.80%	4.5%	634	663	4.7%	6.0%	834
Laid-Off	6,000	9.0%	540	95.00%	8.6%	513	390	6.5%	7.0%	417
Overall	20,000	6.7%	1,345	83.66%	5.7%	1,147	1,053	5.3%	6.3%	1,251
Accident Year Immediately Prior to Staff reduction Year -14										
Surviving*	14,000	5.7%	797	88.80%	5.1%	707	975	7.0%	7.6%	1,064
Laid-Off	6,000	6.7%	401	89.73%	6.0%	360	110	1.8%	2.5%	151
Overall	20,000	6.0%	1,198	89.08%	5.3%	1,068	1,085	5.4%	6.1%	1,215

Exhibit 8

Estimated Loss Development Pattern
Reported Claims Severity ($000)

Accident Year	Evaluation Age in Months														
	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
1	5,746	8,545	10,177	10,780	11,969	12,130	12,260	12,383	12,445	12,507	12,544	12,544	12,544	12,544	12,544
2	5,976	8,886	10,584	11,211	12,447	12,615	12,750	12,878	12,942	13,007	13,046	13,046	13,046	13,046	
3	6,215	9,242	11,008	11,659	12,945	13,120	13,260	13,393	13,460	13,527	13,568	13,568	13,568		
4	6,463	9,612	11,448	12,126	13,463	13,644	13,791	13,929	13,998	14,068	14,111	14,111			
5	6,722	9,996	11,906	12,611	14,002	14,190	14,342	14,486	14,558	14,631	14,675				
6	6,991	10,396	12,382	13,115	14,562	14,758	14,916	15,065	15,141	15,216					
7	7,270	10,812	12,877	13,640	15,144	15,348	15,513	15,668	15,746						
8	7,561	11,244	13,392	14,185	15,750	15,962	16,133	16,295							
9	7,863	11,694	13,928	14,753	16,380	16,600	16,779								
10	8,178	12,162	14,485	15,343	17,035	17,264									
11	8,505	12,648	15,065	15,957	17,716										
12	8,845	13,154	15,667	16,595											
13	9,199	13,680	20,146												
14	9,567	21,207													
15	18,439														

Accident Year	Age Interval in Months														
	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132	132-144	144-156	156-168	168-180	180-
1	1.487	1.191	1.059	1.110	1.013	1.011	1.010	1.005	1.005	1.003	1.000	1.000	1.000	1.000	
2	1.487	1.191	1.059	1.110	1.013	1.011	1.010	1.005	1.005	1.003	1.000	1.000	1.000		
3	1.487	1.191	1.059	1.110	1.013	1.011	1.010	1.005	1.005	1.003	1.000	1.000			
4	1.487	1.191	1.059	1.110	1.013	1.011	1.010	1.005	1.005	1.003	1.000				
5	1.487	1.191	1.059	1.110	1.013	1.011	1.010	1.005	1.005	1.003					
6	1.487	1.191	1.059	1.110	1.013	1.011	1.010	1.005	1.005						
7	1.487	1.191	1.059	1.110	1.013	1.011	1.010	1.005							
8	1.487	1.191	1.059	1.110	1.013	1.011	1.010								
9	1.487	1.191	1.059	1.110	1.013	1.011									
10	1.487	1.191	1.059	1.110	1.013										
11	1.487	1.191	1.059	1.110											
12	1.487	1.191	1.059												
13	1.487	1.473													
14	2.217														
15															
Selected Factors	1.487	1.191	1.059	1.110	1.013	1.011	1.010	1.005	1.005	1.003	1.000	1.000	1.000	1.000	1.000
Factors to Ultimat	2.183	1.468	1.233	1.164	1.048	1.034	1.023	1.013	1.008	1.003	1.000	1.000	1.000	1.000	1.000
Est. Ultimate Clai	40,256	31,134	24,831	19,311	18,569	17,854	17,168	16,507	15,872	15,262	14,675	14,111	13,568	13,046	12,544

Exhibit 9

Estimated Loss Development Pattern
Average Paid Claim-Cumulative

Paid Losses ('000)

Accident Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
	Evaluation Age in Months														
1	1,621	3,539	6,568	9,060	9,740	10,421	11,368	12,209	12,392	12,454	12,492	12,517	12,542	12,544	12,544
2	1,703	3,717	6,899	9,517	10,231	10,947	11,941	12,825	13,017	13,082	13,121	13,147	13,174	13,176	
3	1,789	3,905	7,247	9,996	10,746	11,498	12,543	13,471	13,673	13,741	13,783	13,810	13,838		
4	1,879	4,102	7,612	10,500	11,288	12,078	13,175	14,150	14,362	14,434	14,477	14,506			
5	1,973	4,308	7,996	11,030	11,857	12,687	13,839	14,863	15,086	15,161	15,207				
6	2,073	4,525	8,399	11,585	12,454	13,326	14,536	15,612	15,846	15,925					
7	2,177	4,753	8,822	12,169	13,082	13,998	15,269	16,399	16,645						
8	2,287	4,993	9,267	12,783	13,741	14,703	16,039	17,225							
9	2,402	5,245	9,734	13,427	14,434	15,444	16,847								
10	2,523	5,509	10,224	14,104	15,161	16,223									
11	2,651	5,787	10,740	14,814	15,926										
12	2,784	6,078	11,281	15,561											
13	2,924	6,385	14,651												
14	3,072	10,732													
15	6,951														

No. of Paid Claims

Accident Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
	Evaluation Age in Months														
1	243	385	538	664	691	717	736	750	750	750	750	750	750	750	750
2	245	389	543	671	698	724	743	758	758	758	758	758	758	758	
3	248	393	549	677	705	731	751	765	765	765	765	765	765		
4	250	397	554	684	712	739	758	773	773	773	773	773			
5	253	401	560	691	719	746	766	780	780	780	780				
6	255	405	565	698	726	754	774	788	788	788					
7	258	409	571	705	734	761	781	796	796						
8	261	413	577	712	741	769	789	804							
9	263	417	583	719	748	776	797								
10	266	421	588	726	756	784									
11	268	425	594	733	763										
12	271	430	600	741											
13	274	434	606												
14	277	450													
15	440														

Average Paid Claim

Accident Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
	Evaluation Age in Months														
1	6,671	9,192	12,209	13,645	14,095	14,535	15,446	16,279	16,523	16,606	16,656	16,689	16,722	16,726	16,726
2	6,938	9,560	12,697	14,191	14,659	15,116	16,063	16,930	17,184	17,270	17,322	17,356	17,391	17,395	
3	7,215	9,942	13,205	14,758	15,245	15,721	16,706	17,607	17,871	17,961	18,015	18,051	18,087		
4	7,504	10,340	13,733	15,349	15,855	16,350	17,374	18,312	18,586	18,679	18,735	18,773			
5	7,804	10,754	14,282	15,963	16,489	17,004	18,069	19,044	19,330	19,426	19,485				
6	8,116	11,184	14,854	16,601	17,149	17,684	18,792	19,806	20,103	20,203					
7	8,441	11,631	15,448	17,265	17,835	18,391	19,544	20,598	20,907						
8	8,778	12,096	16,066	17,956	18,548	19,127	20,325	21,422							
9	9,129	12,580	16,708	18,674	19,290	19,892	21,138								
10	9,495	13,083	17,377	19,421	20,062	20,688									
11	9,874	13,607	18,072	20,198	20,864										
12	10,269	14,151	18,795	21,006											
13	10,680	14,717	24,167												
14	11,107	23,849													
15	15,799														

Annual % Change

Accident Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
	Evaluation Age in Months														
1	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
2	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	
3	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%		
4	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%			
5	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%				
6	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%					
7	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%						
8	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%							
9	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%								
10	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%									
11	4.0%	4.0%	4.0%	4.0%	4.0%										
12	4.0%	4.0%	4.0%	4.0%											
13	4.0%	4.0%	28.6%												
14	4.0%	62.1%													
15	42.2%														

Estimated Loss Development Pattern
Average Outstanding

Outstanding Losses ('000)

Accident Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
1	2,907	4,049	3,232	1,590	2,229	1,708	892	173	52	52	53	28	3	-	-
2	3,053	4,253	3,395	1,670	2,341	1,794	937	182	55	55	55	29	3	-	
3	3,207	4,467	3,566	1,755	2,459	1,885	984	191	58	58	58	30	3		
4	3,369	4,692	3,746	1,843	2,583	1,980	1,034	201	60	61	61	32			
5	3,538	4,929	3,935	1,936	2,713	2,080	1,086	211	64	64	64				
6	3,717	5,177	4,133	2,033	2,850	2,184	1,141	222	67	67					
7	3,904	5,438	4,342	2,136	2,994	2,295	1,198	233	70						
8	4,101	5,712	4,560	2,244	3,145	2,410	1,258	245							
9	4,307	6,000	4,790	2,357	3,303	2,532	1,322								
10	4,525	6,302	5,032	2,475	3,470	2,659									
11	4,753	6,620	5,285	2,600	3,644										
12	4,992	6,954	5,552	2,731											
13	5,244	7,304	7,210												
14	5,508	12,277													
15	12,465														

No. of Open Claims

Accident Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
1	420	290	200	94	59	40	20	15	10	7	5	3	1		
2	424	293	202	95	60	40	20	15	10	7	5	3	1		
3	428	296	204	96	60	41	20	15	10	7	5	3	1		
4	433	299	206	97	61	41	21	15	10	7	5	3			
5	437	302	208	98	61	42	21	16	10	7	5				
6	441	305	210	99	62	42	21	16	11	7					
7	446	308	212	100	63	42	21	16	11						
8	450	311	214	101	63	43	21	16							
9	455	314	217	102	64	43	22								
10	459	317	219	103	65	44									
11	464	320	221	104	65										
12	469	324	223	105											
13	473	327	225												
14	478	416													
15	608														

Average Outstanding

Accident Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
1	6,920	13,961	16,162	16,917	37,778	42,708	44,600	11,560	5,217	7,491	10,519	9,203	2,577		
2	7,197	14,519	16,808	17,594	39,289	44,416	46,384	12,022	5,426	7,790	10,939	9,571	2,680		
3	7,485	15,100	17,481	18,297	40,860	46,193	48,239	12,503	5,643	8,102	11,377	9,954	2,787		
4	7,785	15,704	18,180	19,029	42,495	48,040	50,169	13,003	5,869	8,426	11,832	10,352			
5	8,096	16,332	18,907	19,791	44,195	49,962	52,176	13,524	6,104	8,763	12,305				
6	8,420	16,985	19,664	20,582	45,962	51,960	54,263	14,065	6,348	9,114					
7	8,757	17,665	20,450	21,405	47,801	54,039	56,433	14,627	6,602						
8	9,107	18,371	21,268	22,262	49,713	56,200	58,691	15,212							
9	9,471	19,106	22,119	23,152	51,701	58,448	61,038								
10	9,850	19,870	23,004	24,078	53,769	60,786									
11	10,244	20,665	23,924	25,041	55,920										
12	10,654	21,492	24,881	26,043											
13	11,080	22,352	31,992												
14	11,523	29,513													
15	20,501														

Annual % Change

Accident Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
1	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
2	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	N/A	
3	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%		
4	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%			
5	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%				
6	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%					
7	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%						
8	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%							
9	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%								
10	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%									
11	4.0%	4.0%	4.0%	4.0%	4.0%										
12	4.0%	4.0%	4.0%	4.0%											
13	4.0%	4.0%	28.6%												
14	4.0%	32.0%													
15	77.9%														

Estimated Claims Development Pattern
Paid Claims(i.e., Claims Closed With Payment)

Accident Year	Evaluation Age in Months														
	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
1	243	385	538	664	691	717	736	750	750	750	750	750	750	750	750
2	245	389	543	671	698	724	743	758	758	758	758	758	758	758	
3	248	393	549	677	705	731	751	765	765	765	765	765	765		
4	250	397	554	684	712	739	758	773	773	773	773	773			
5	253	401	560	691	719	746	766	780	780	780	780				
6	255	405	565	698	726	754	774	788	788	788					
7	258	409	571	705	734	761	781	796	796						
8	261	413	577	712	741	769	789	804							
9	263	417	583	719	748	776	797								
10	266	421	588	726	756	784									
11	268	425	594	733	763										
12	271	430	600	741											
13	274	434	606												
14	277	450													
15	440														

Accident Year	Age Interval in Months														
	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132	132-144	144-156	156-168	168-180	180-
1	1.584	1.397	1.234	1.041	1.038	1.026	1.019	1.000	1.000	1.000	1.000	1.000	1.000	1.000	
2	1.584	1.397	1.234	1.041	1.038	1.026	1.019	1.000	1.000	1.000	1.000	1.000	1.000		
3	1.584	1.397	1.234	1.041	1.038	1.026	1.019	1.000	1.000	1.000	1.000	1.000			
4	1.584	1.397	1.234	1.041	1.038	1.026	1.019	1.000	1.000	1.000	1.000				
5	1.584	1.397	1.234	1.041	1.038	1.026	1.019	1.000	1.000	1.000					
6	1.584	1.397	1.234	1.041	1.038	1.026	1.019	1.000	1.000						
7	1.584	1.397	1.234	1.041	1.038	1.026	1.019	1.000							
8	1.584	1.397	1.234	1.041	1.038	1.026	1.019								
9	1.584	1.397	1.234	1.041	1.038	1.026									
10	1.584	1.397	1.234	1.041	1.038										
11	1.584	1.397	1.234	1.041											
12	1.584	1.397	1.234												
13	1.584	1.397													
14	1.627														
15															

Estimated Claims Development Pattern
Closed Claims

Accident Year	Evaluation Age in Months: 12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
1	368	598	763	894	941	960	980	985	990	993	995	997	999	1,000	1,000
2	372	604	771	903	950	970	990	995	1,000	1,003	1,005	1,007	1,009	1,010	
3	375	610	778	912	960	979	1,000	1,005	1,010	1,013	1,015	1,017	1,019		
4	379	616	786	921	970	989	1,010	1,015	1,020	1,023	1,025	1,027			
5	383	622	794	930	979	999	1,020	1,025	1,030	1,033	1,035				
6	387	629	802	940	989	1,009	1,030	1,035	1,040	1,044					
7	391	635	810	949	999	1,019	1,040	1,046	1,051						
8	395	641	818	958	1,009	1,029	1,051	1,056							
9	398	648	826	968	1,019	1,040	1,061								
10	402	654	834	978	1,029	1,050									
11	407	661	843	988	1,039										
12	411	667	851	997											
13	415	674	860												
14	419	669													
15	445														

Accident Year	Age Interval in Months: 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132	132-144	144-156	156-168	168-180	180-
1	1.625	1.276	1.172	1.053	1.020	1.021	1.005	1.005	1.003	1.002	1.002	1.002	1.001	1.000	
2	1.625	1.276	1.172	1.053	1.020	1.021	1.005	1.005	1.003	1.002	1.002	1.002	1.001		
3	1.625	1.276	1.172	1.053	1.020	1.021	1.005	1.005	1.003	1.002	1.002	1.002			
4	1.625	1.276	1.172	1.053	1.020	1.021	1.005	1.005	1.003	1.002	1.002				
5	1.625	1.276	1.172	1.053	1.020	1.021	1.005	1.005	1.003	1.002					
6	1.625	1.276	1.172	1.053	1.020	1.021	1.005	1.005	1.003						
7	1.625	1.276	1.172	1.053	1.020	1.021	1.005	1.005							
8	1.625	1.276	1.172	1.053	1.020	1.021	1.005								
9	1.625	1.276	1.172	1.053	1.020	1.021									
10	1.625	1.276	1.172	1.053	1.020										
11	1.625	1.276	1.172	1.053											
12	1.625	1.276	1.172												
13	1.625	1.276													
14	1.597														
15															

Exhibit 13

Estimated Claims Development Pattern
Reported Claims

Accident Year	Evaluation Age in Months 12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
1	788	888	963	988	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000
2	796	897	973	998	1,010	1,010	1,010	1,010	1,010	1,010	1,010	1,010	1,010	1,010	
3	804	906	982	1,008	1,020	1,020	1,020	1,020	1,020	1,020	1,020	1,020	1,020		
4	812	915	992	1,018	1,030	1,030	1,030	1,030	1,030	1,030	1,030	1,030			
5	820	924	1,002	1,028	1,041	1,041	1,041	1,041	1,041	1,041	1,041				
6	828	933	1,012	1,038	1,051	1,051	1,051	1,051	1,051	1,051					
7	836	943	1,022	1,049	1,062	1,062	1,062	1,062	1,062						
8	845	952	1,032	1,059	1,072	1,072	1,072	1,072							
9	853	962	1,043	1,070	1,083	1,083	1,083								
10	862	971	1,053	1,081	1,094	1,094									
11	870	981	1,064	1,091	1,105										
12	879	991	1,074	1,102											
13	888	1,001	1,085												
14	897	1,085													
15	1,053														

Accident Year	Age Interval in Months 12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132	132-144	144-156	156-168	168-180	180-
1	1.127	1.084	1.026	1.012	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	
2	1.127	1.084	1.026	1.012	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000		
3	1.127	1.084	1.026	1.012	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000			
4	1.127	1.084	1.026	1.012	1.000	1.000	1.000	1.000	1.000	1.000	1.000				
5	1.127	1.084	1.026	1.012	1.000	1.000	1.000	1.000	1.000	1.000					
6	1.127	1.084	1.026	1.012	1.000	1.000	1.000	1.000	1.000						
7	1.127	1.084	1.026	1.012	1.000	1.000	1.000	1.000							
8	1.127	1.084	1.026	1.012	1.000	1.000	1.000								
9	1.127	1.084	1.026	1.012	1.000	1.000									
10	1.127	1.084	1.026	1.012	1.000										
11	1.127	1.084	1.026	1.012											
12	1.127	1.084	1.026												
13	1.127	1.084													
14	1.210														
15															

Selected Factors	1.127	1.084	1.026	1.012	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Factors to Ultimat	1.269	1.126	1.038	1.012	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Est. Ultimate Clai	1,336	1,222	1,127	1,116	1,105	1,094	1,083	1,072	1,062	1,051	1,041	1,030	1,020	1,010	1,000

% Reported @	
12 Months	78.8%
24 Months	88.8%
36 Months	96.3%
48 Months	98.8%

Exhibit 14

Estimated Loss Development Pattern
Paid ALAE to Paid Loss Ratio

	Evaluation Age in Months														
Accident Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
1	0.20	0.22	0.24	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
2	0.20	0.22	0.24	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	
3	0.20	0.22	0.24	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25		
4	0.20	0.22	0.24	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25			
5	0.20	0.22	0.24	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35				
6	0.20	0.22	0.24	0.25	0.25	0.25	0.25	0.25	0.25	0.25					
7	0.22	0.24	0.26	0.27	0.27	0.27	0.27	0.27	0.27						
8	0.25	0.28	0.30	0.30	0.30	0.30	0.30	0.30							
9	0.27	0.30	0.32	0.32	0.32	0.32	0.32								
10	0.21	0.23	0.25	0.26	0.26	0.26									
11	0.30	0.33	0.36	0.35	0.35										
12	0.22	0.24	0.26	0.27											
13	0.25	0.28	0.39												
14	0.25	0.39													
15	0.40														

Exhibit 15

Estimated Loss Development Pattern
Paid ALAE ($000)

	Evaluation Age in Months														
Accident Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
1	324	779	1,576	2,265	2,435	2,605	2,842	3,052	3,098	3,114	3,123	3,129	3,135	3,136	3,136
2	341	818	1,656	2,379	2,558	2,737	2,985	3,206	3,254	3,270	3,280	3,287	3,293	3,294	
3	358	859	1,739	2,499	2,687	2,875	3,136	3,368	3,418	3,435	3,446	3,453	3,459		
4	376	902	1,827	2,625	2,822	3,019	3,294	3,537	3,591	3,608	3,619	3,627			
5	395	948	1,919	3,860	4,150	4,440	4,844	5,202	5,280	5,306	5,322				
6	415	996	2,016	2,896	3,114	3,332	3,634	3,903	3,962	3,981					
7	479	1,150	2,329	3,286	3,532	3,779	4,123	4,428	4,494						
8	572	1,373	2,780	3,835	4,122	4,411	4,812	5,168							
9	649	1,558	3,154	4,297	4,619	4,942	5,391								
10	530	1,273	2,577	3,667	3,942	4,218									
11	795	1,910	3,866	5,185	5,574										
12	613	1,471	2,978	4,201											
13	731	1,756	5,714												
14	768	4,186													
15	2,781														

	Age Interval in Months														
Accident Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132	132-144	144-156	156-168	168-180	180-
1	2.402	2.025	1.437	1.075	1.070	1.091	1.074	1.015	1.005	1.003	1.002	1.002	1.000	1.000	
2	2.402	2.025	1.437	1.075	1.070	1.091	1.074	1.015	1.005	1.003	1.002	1.002	1.000		
3	2.402	2.025	1.437	1.075	1.070	1.091	1.074	1.015	1.005	1.003	1.002	1.002			
4	2.402	2.025	1.437	1.075	1.070	1.091	1.074	1.015	1.005	1.003	1.002				
5	2.402	2.025	2.012	1.075	1.070	1.091	1.074	1.015	1.005	1.003					
6	2.402	2.025	1.437	1.075	1.070	1.091	1.074	1.015	1.005						
7	2.402	2.025	1.411	1.075	1.070	1.091	1.074	1.015							
8	2.402	2.025	1.379	1.075	1.070	1.091	1.074								
9	2.402	2.025	1.362	1.075	1.070	1.091									
10	2.402	2.025	1.423	1.075	1.070										
11	2.402	2.025	1.341	1.075											
12	2.402	2.025	1.411												
13	2.402	3.254													
14	5.450														
15															
Selected Factors	2.402	2.025	1.341	1.075	1.070	1.091	1.074	1.015	1.005	1.003	1.002	1.002	1.000	1.000	1.000
Factors to Ultimat	9.028	3.759	1.857	1.385	1.288	1.204	1.103	1.027	1.012	1.007	1.004	1.002	1.000	1.000	1.000
Est. Ultimate ALA	25,104	15,735	10,609	5,817	7,179	5,077	5,949	5,309	4,549	4,010	5,345	3,635	3,460	3,294	3,136

% Paid ALAE @		Acc. Yr.	Ultimate ALAE
12 Months	11.1%	7	4,549
24 Months	26.6%	8	5,309
36 Months	53.9%	9	5,949
48 Months	72.2%	10	5,077
		11	7,179
		12	5,817

Exhibit 16

Estimated Loss Development Pattern
Paid Losses ($000)

Accident Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
	Evaluation Age in Months														
1	1,621	3,539	6,568	9,060	9,740	10,421	11,368	12,209	12,392	12,454	12,492	12,517	12,542	12,544	12,544
2	1,703	3,717	6,899	9,517	10,231	10,947	11,941	12,825	13,017	13,082	13,121	13,147	13,174	13,176	
3	1,789	3,905	7,247	9,996	10,746	11,498	12,543	13,471	13,673	13,741	13,783	13,810	13,838		
4	1,879	4,102	7,612	10,500	11,288	12,078	13,175	14,150	14,362	14,434	14,477	14,506			
5	1,973	4,308	7,996	11,030	11,857	12,687	13,839	14,863	15,086	15,161	15,207				
6	2,073	4,525	8,399	11,585	12,454	13,326	14,536	15,612	15,846	15,925					
7	2,177	4,753	8,822	12,169	13,082	13,998	15,269	16,399	16,645						
8	2,287	4,993	9,267	12,783	13,741	14,703	16,039	17,225							
9	2,402	5,245	9,734	13,427	14,434	15,444	16,847								
10	2,523	5,509	10,224	14,104	15,161	16,223									
11	2,651	5,787	10,740	14,814	15,926										
12	2,784	6,078	11,281	15,561											
13	2,924	6,385	14,651												
14	3,072	10,732													
15	6,951														

Accident Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132	132-144	144-156	156-168	168-180	180-
	Age Interval in Months														
1	2.183	1.856	1.379	1.075	1.070	1.091	1.074	1.015	1.005	1.003	1.002	1.002	1.000	1.000	
2	2.183	1.856	1.379	1.075	1.070	1.091	1.074	1.015	1.005	1.003	1.002	1.002	1.000		
3	2.183	1.856	1.379	1.075	1.070	1.091	1.074	1.015	1.005	1.003	1.002	1.002			
4	2.183	1.856	1.379	1.075	1.070	1.091	1.074	1.015	1.005	1.003	1.002				
5	2.183	1.856	1.379	1.075	1.070	1.091	1.074	1.015	1.005	1.003					
6	2.183	1.856	1.379	1.075	1.070	1.091	1.074	1.015	1.005						
7	2.183	1.856	1.379	1.075	1.070	1.091	1.074	1.015							
8	2.183	1.856	1.379	1.075	1.070	1.091	1.074								
9	2.183	1.856	1.379	1.075	1.070	1.091									
10	2.183	1.856	1.379	1.075	1.070										
11	2.183	1.856	1.379	1.075											
12	2.183	1.856	1.379												
13	2.183	2.295													
14	3.494														
15															
Selected Factors	2.183	1.856	1.379	1.075	1.070	1.091	1.074	1.015	1.005	1.003	1.002	1.002	1.000	1.000	1.000
Factors to Ultimat	7.739	3.545	1.910	1.385	1.288	1.204	1.103	1.027	1.012	1.007	1.004	1.002	1.000	1.000	1.000
Est. Ultimate Loss	53,794	38,041	27,980	21,545	20,511	19,527	18,590	17,698	16,849	16,041	15,271	14,538	13,841	13,176	12,544

% Paid @		Acc. Yr.	Ultimate Paid Loss
12 Months	12.9%	7	16,849
24 Months	28.2%	8	17,698
36 Months	52.4%	9	18,590
48 Months	72.2%	10	19,527
		11	20,511
		12	21,545

Exhibit 17

Estimated Loss Development Pattern
Incurred Losses ($000)

Accident Year	Evaluation Age in Months														
	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
1	4,528	7,588	9,801	10,650	11,969	12,130	12,260	12,383	12,445	12,507	12,544	12,544	12,544	12,544	12,544
2	4,756	7,970	10,295	11,187	12,572	12,741	12,878	13,007	13,072	13,137	13,176	13,176	13,176	13,176	
3	4,995	8,372	10,813	11,751	13,205	13,383	13,527	13,662	13,731	13,799	13,841	13,841	13,841		
4	5,247	8,794	11,358	12,343	13,871	14,058	14,209	14,351	14,423	14,495	14,538	14,538			
5	5,512	9,237	11,931	12,965	14,570	14,766	14,925	15,074	15,149	15,225	15,271				
6	5,790	9,702	12,532	13,619	15,304	15,511	15,677	15,834	15,913	15,993					
7	6,081	10,191	13,164	14,305	16,076	16,292	16,467	16,632	16,715						
8	6,388	10,705	13,827	15,026	16,886	17,113	17,297	17,470							
9	6,710	11,245	14,524	15,784	17,737	17,976	18,169								
10	7,048	11,811	15,256	16,579	18,631	18,882									
11	7,403	12,407	16,025	17,415	19,570										
12	7,776	13,032	16,833	18,292											
13	8,168	13,689	21,861												
14	8,580	23,010													
15	19,416														

Accident Year	Age Interval in Months														
	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132	132-144	144-156	156-168	168-180	180-
1	1.676	1.292	1.087	1.124	1.013	1.011	1.010	1.005	1.005	1.003	1.000	1.000	1.000	1.000	
2	1.676	1.292	1.087	1.124	1.013	1.011	1.010	1.005	1.005	1.003	1.000	1.000	1.000		
3	1.676	1.292	1.087	1.124	1.013	1.011	1.010	1.005	1.005	1.003	1.000	1.000			
4	1.676	1.292	1.087	1.124	1.013	1.011	1.010	1.005	1.005	1.003	1.000				
5	1.676	1.292	1.087	1.124	1.013	1.011	1.010	1.005	1.005	1.003					
6	1.676	1.292	1.087	1.124	1.013	1.011	1.010	1.005	1.005						
7	1.676	1.292	1.087	1.124	1.013	1.011	1.010	1.005							
8	1.676	1.292	1.087	1.124	1.013	1.011	1.010								
9	1.676	1.292	1.087	1.124	1.013	1.011									
10	1.676	1.292	1.087	1.124	1.013										
11	1.676	1.292	1.087	1.124											
12	1.676	1.292	1.087												
13	1.676	1.597													
14	2.682														
15															
Selected Factors	1.676	1.292	1.087	1.124	1.013	1.011	1.010	1.005	1.005	1.003	1.000	1.000	1.000	1.000	1.000
Factors to Ultimat	2.771	1.653	1.280	1.178	1.048	1.034	1.023	1.013	1.008	1.003	1.000	1.000	1.000	1.000	1.000
Proj. Ultimate Lo:	53,794	38,041	27,980	21,545	20,511	19,527	18,590	17,698	16,849	16,041	15,271	14,538	13,841	13,176	12,544

% Reported @		Acc. Yr.	Ultimate Incurred Loss
12 Months	36.1%	7	16,849
24 Months	60.5%	8	17,698
36 Months	78.1%	9	18,590
48 Months	84.9%	10	19,527
		11	20,511
		12	21,545

International Accounting Standards Applied to Property and Casualty Insurance—Overview of Reserving Issues

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Abstract

This paper provides an overview of the newly developing International Accounting Standards (IAS or IFRS) for Insurance, with emphasis on issues impacting property and casualty insurers and the reserving work that actuaries do to support that. Those standards will emerge in two phases, with the more challenging actuarial issues deferred to Phase II. This paper focuses on the Phase II actuarial issues but also provides a brief overview of Phase I issues. The paper is intended to serve two purposes – providing background information for those actuaries not yet familiar with IAS developments, and encouraging discussion and research on new challenges that casualty actuaries will face in determining reserves on the new bases required to implement these standards.

1. INTRODUCTION

The International Accounting Standards Board (IASB) is spearheading a global effort to transform financial reporting that has significant implications for insurance companies worldwide. The IASB's objective is to develop a single set of global accounting standards that provide useful, understandable and comparable information in financial statements, thereby helping participants in the world's capital markets make sound economic decisions. The direction of these standards is toward fair value or "fair value like" measurement of financial assets and liabilities. Some expect the new IAS standard for insurance to eventually be carried into U.S. GAAP.

The European Commission (EC) has mandated that by 2005 all companies with shares trading on stock markets within the European Union (EU) must report using IASB standards. This requirement may also be extended to some other financial institutions operating in the EU, even if not listed.

There is currently no International Financial Reporting Standard (IFRS) covering insurance. Thus accounting for insurance has been a top priority project for the IASB. While considerable work had been done on developing a new IAS standard which would value insurance contracts at Fair Value, by 2002 it became clear that the task could not be completed in time for the EU 2005 deadline. Consequently, the IASB decided to split the insurance project into two Phases.

Phase I is intended for implementation in 2005, along with all the other IFRS's applicable to insurer operations. This will allow EU insurers to produce full IAS statements for 2005 (and comparative statements for 2004). The general intent of the IASB was to make as few changes as possible to existing accounting for insurance contracts, since there would be major changes needed again shortly thereafter when Phase II is introduced.

Phase II will introduce Fair Value accounting for insurance contracts. While the timing of Phase II is not yet set, the intent of the IASB seems to be to have it effective for 2007 or 2008.

Much of the work on developing a Fair Value standard was summarized in the **Draft Statement of Principles for Insurance Contracts** (DSOP), released in 2001. This DSOP is likely to form the basis for the Phase II International Financial Reporting Standard (IFRS) on insurance contracts, moving to Fair Value. It reflects discussions since 1997 by the Insurance Accounting Steering Committee of the IASC (predecessor of the IASB), with substantial input from the International Actuarial Association and many others. The DSOP applies to all forms of insurance (life, property/casualty, and health) and is a set of principles upon which an IFRS can be built.

While most of the recent work of the IASB on insurance has focused on Phase I, some "tentative" conclusions have been reached on the direction of Phase II although the project has been more or less dormant since January 2003. The IASB has agreed that the project should be restarted in May 2004 with the aim of completing an Exposure draft by June 2005. On restarting the project the Board will return to a study of the major issues and use the assistance of experts from national standard setters and selected industry participants.

An overview of the implications of Phase I for P&C insurers is given in the next section. For most property and casualty insurers, Phase I will not present major issues. That is followed by a discussion of another important standard for insurers, IAS 39, on financial instruments, which will govern accounting for insurer assets. This description is based on IAS 39 as of early 2004. There may be subsequent amendments. Thereafter, we discuss in more detail some key technical and business implications of Phase II, based largely on the DSOP. While the timing of Phase II is still not set, it is important to remember that these principles will impact insurers soon. Phase II will likely raise many new issues for casualty actuaries, with many new concepts introduced into the reserving process. This paper focuses on those, with most of the discussion focused on these four broad areas:

Insurance Contract Definition
Estimating the Timing and Amount of Cash Flows
Adjustments for Risk and Uncertainty
Discounting

This paper is intended both to educate those not familiar with IAS, and to stimulate discussion and research among casualty actuaries on how to handle these new issues.

2. PHASE I - 2005 REQUIREMENTS

The Staff of the IASB was previously working on a Fair Value based approach to accounting for insurance, but concluded that, due to the lack of time and to fierce opposition to a fair value standard, that would not be ready for 2005. Consequently, the IASB has introduced a "two-phased" approach and released a Phase I Exposure Draft (ED 5) for insurance contract reporting. Comments on ED 5 were due 31 October 2003. At its November, December and January meetings, the IASB reviewed the comment letters and made some decisions. The final Phase I Standard, IFRS 4, *Insurance Contracts*, was released on 31 March 2004, reflecting those decisions.

Product Classification

The first step for valuing a contract is to determine whether it is classified as insurance and valued under the insurance contracts guidance. Contracts issued by insurers that do not meet the definition of insurance will be classified as investment contracts and valued under IAS 39. For contracts to meet the definition of insurance, they must include significant insurance risk— namely, a plausible event that adversely affects the policyholder or beneficiary. The definition includes most property/casualty insurance contracts. The definition is intended to be very broad. In addition, "unbundling" is required in certain circumstances. Many insurance contracts can be viewed as a "bundle" of insurance and a non-insurance financial instrument. This is most obvious in the case of many life insurance contracts, but a similar view could be taken of some retrospectively rated property and casualty insurance contracts. "Unbundling" means splitting these two components of the bundle, and accounting for them separately. In that case, the non-insurance deposit component would be accounted for using deposit accounting and IAS 39 valuation rules.

Unbundling of contracts is required to recognize deposit components or features of insurance contracts that are "hidden" on the balance sheet, where those can be separately measured. It would apply to many large retrospectively rated commercial lines or reinsurance contracts, where portions of the premium or loss payments could be viewed as deposits, if the existing accounting did not appropriately recognize them.

Phase I Insurance Contract Accounting

IFRS 4, *Insurance Contracts,* provides the guidance for accounting for insurance contracts during Phase I. The general intent is to allow companies to continue to use existing accounting policies, while at the same time introducing some key modifications. The Board considers the modifications necessary to ensure that existing accounting methods more closely conform to the principles of the IAS framework. Consequently, the IASB expects these modifications to continue to be in effect in Phase II. The main modifications impacting property and casualty insurers that must be made to existing accounting policies include:

- **Catastrophe provisions** for future claims beyond the term of the existing contracts are not allowed
- **Claims equalization provisions** to cover random fluctuations in claims costs are not allowed
- **Recognition of future losses,** measured by analysis of future cash flows, is mandated
- **Liabilities must be shown gross of reinsurance,** with the reinsurance asset shown separately

Some accounting systems, for example, US GAAP, already comply with most of the modifications or concepts described above. Significant changes are necessary for others, including the removal of catastrophe and claims equalization reserves as often found in French, German, Spanish and UK GAAP reporting.

The rules for derecognizing financial assets and liabilities also apply to insurance contracts. Therefore, insurance liabilities and assets can be removed from the balance sheet only if fully extinguished, discharged, cancelled, or expired. This means that reinsurance will not enable a company to derecognize a direct liability; rather gross presentation of liabilities and the recognition of reinsurance assets is required. Revenues, benefits, and expenses must be presented gross of reinsurance, with reinsurance amounts affecting the accounts shown in the profit or loss.

Existing Accounting Policies Allowed to Continue

Some existing accounting policies that are likely to be disallowed in Phase II may continue in Phase I if they are already in place. However, if an entity currently does not apply these policies, it cannot adopt them, even though other entities may be permitted to use them. The accounting policies that are allowed to continue in Phase I include:

- **Undiscounted measurement basis** for claims reserves — there is no requirement for discounting now, although it is very likely to be required in Phase II
- **Excessive prudence or deliberate overstatement** of insurance liabilities that may be a result of applying local regulatory requirements
- **Reflecting future investment margins** in the measurement of insurance liabilities — for instance, assuming a realistic portfolio investment return as a discount rate
- **Investment management fees** recognized at amounts above fair value
- **Recognition of deferred acquisition costs (DAC)**
- **Non-uniform accounting policies** for insurance subsidiaries

Changing Accounting Policies

A company may change their existing accounting methods under Phase I, if the change is more relevant, prudent, without bias, and more faithfully represents the economic substance of the insurance contracts. In practice, it is unlikely that companies will change their existing accounting very much for Phase I. There may be some exceptions, such as where the regulatory reserves are already moving towards a fair value-type standard.

Reinsurance

A reinsurance contract that contains significant insurance risk (i.e. is not merely a financial contract) is classified as an insurance contract and falls within the scope of IFRS 4. As originally proposed in ED 5, cedants could not recognize a gain at the inception of a reinsurance treaty. The break-even position was to be achieved by deferring and amortizing the difference between:

- The net amounts paid by the cedant, adjusted for any amount that represents a reimbursement for expensed acquisition costs; and
- The carried amount of the related portion of the cedant's liability.

While this was proposed in ED 5, at the November IASB meeting, the Board softened the position to just requiring disclosure of the gains at inception, although the details of how to do this have yet to be finalized.

Recognition of Future Losses

Impairment testing and loss recognition are similar concepts. The aim of such tests is to assess whether a liability valuation is inadequate or an asset value is overstated.

Loss recognition testing applies to insurance contracts under ED 5 and requires (i) the application of the loss recognition test under existing accounting policies, or (ii) where a test does not exist under existing accounting, IAS 37 must be applied, as discussed below.

Loss Recognition on Insurance Contracts

For insurance contracts, loss recognition or liability adequacy tests may follow existing accounting policies where such a test exists that meets specified criteria. The test examines whether the liability held on the valuation date is sufficient to cover the expected future loss payments. The loss recognition test should cover insurance liabilities, including loss reserves and unearned premium reserves, as well as related deferred acquisition costs (if any) and related intangible assets recognized in a business combination or portfolio transfer. The traditional actuarial approaches to determining loss reserves clearly meet this requirement. For unearned premium reserves, it would be necessary to examine whether the unearned premium reserve less the DAC asset is sufficient to cover the future loss payments on future losses under the contract.

Where such formal loss recognition tests do not exist under existing accounting policies, the adequacy requirements of IAS 37 must be applied. Inconsistencies may result between companies that are able to use an existing accounting loss recognition test and those required to apply IAS 37.

Application of IAS 37

The adequacy requirements under IAS 37 are applied to insurance contracts where no formal loss recognition test currently exists. The minimum liability under IAS 37 is essentially a fair value type provision. The fair value is calculated as the present value of future projected loss and expense cash flows.

The cash flows should include margins for uncertainty, so the minimum liability may be greater than the liability when measured by the discounted present value of future cash flows using realistic assumptions. It is unclear whether a market discount rate is limited to a risk-free discount rate adjusted for credit standing (as in the current Phase II proposals, discussed later in this paper). For property/casualty companies, undiscounted best estimate loss reserves (without any implicit allowance for the effects of discounting) would likely exceed a fair value type IAS 37 minimum requirement, although testing should be done to confirm that this is the case. Adding provisions for uncertainty and projected future administration expenses and reflecting rate inadequacies on unearned premiums act to increase the reserve required, and in some cases may more than offset the lack of discounting.

If current estimates of future cash flows indicate the existence of a loss, the insurer should increase the carrying amount of the liability in question to the amount that would be required under IAS 37. Any loss recognition resulting from this test and subsequent changes in the best estimate liability is reflected in earnings for the period. The amount of the loss provision can decrease, but the liability cannot be less than the value under the initial accounting basis.

IAS 37 is written in the context of a single contract or single event. For investment or insurance contracts, the testing will likely be made for a group of contracts. The results may depend on the level of aggregation, so companies will need to develop a policy for the aggregation, and apply it consistently from period to period.

ED 5 Expanded Disclosure Requirements

ED 5 contained three high-level disclosure principles that are likely to significantly increase the current level of qualitative and quantitative financial statement disclosures:

- **Principle 1: Explanation of reported amounts** — "an insurer shall disclose information that identifies and explains the amounts in its balance sheet and income statement that arise from insurance contracts" (paragraph 26)
- **Principle 2: Amount, timing and uncertainty of cash flows** — "an insurer shall disclose information that enables users to understand the estimated amount, timing and uncertainty of future cash flows from insurance contracts" (paragraph 28)
- **Principle 3: Fair value of insurance liabilities and insurance assets** — "an insurer shall disclose the fair value of its insurance liabilities and insurance assets" (paragraph 30)

This third principle was very controversial, as it seemed the IASB was requiring disclosure of something that it found impossible to define. The IASB agreed to remove this requirement at its November 2003 meeting. But IFRS 4 did retain the first two Principles, and implementing them will prove to be a major challenge for many insurers.

Practical Implications of Disclosure Requirements

The implementation of the detailed requirements of Principles 1 and 2 is likely to lead to a significant increase in the length and complexity of insurance contract disclosures. These additional disclosures are also likely to be of significant interest to analysts and, as such, their presentation will require careful consideration. The disclosures likely to generate the greatest added effort or the greatest interest include:

- Risk management objectives and the policies established to mitigate insurance risk
- Terms and conditions of insurance contracts that are likely to have a material impact on the amount, timing and certainty of future cash flows
- Information on credit risk that is likely to be particularly important for reinsurance contracts
- Insurance risk, including sensitivity analysis, and information about concentration of insurance risk

- Details of actual claims compared to previous estimates (e.g., claims development for a general insurer for periods where incurred claims are still outstanding)

3. ACCOUNTING FOR ASSETS - IAS 39

While the focus of this paper is accounting for insurance, an understanding of the accounting for insurer's invested assets is important to understand the likely earnings impact of the new requirements valuing insurance reserves. The main standard that applies for the accounting of insurers' invested assets is IAS 39, *Financial Instruments: Recognition and Measurement.* Many aspects of IAS 39 are common to all financial institutions and other entities, including measurement principles for invested assets and macro-hedging, recognition and derecognition guidance, and disclosure requirements.

Measurement of Invested Assets

IAS 39 requires many, but not all financial assets to be carried at fair value in the financial statements, and allows an amortized cost approach for most financial liabilities. Similar to US GAAP FAS 115, financial assets (except for originated loans) are classified as held-to-maturity (HTM), trading, or available-for-sale (AFS). Trading and AFS financial assets are valued at fair value, while HTM assets are valued at amortized cost. For trading assets, unrealized gains and losses are recorded in the income statement. Unrealized gains and losses for AFS assets are recorded directly in equity except for impairment losses that are taken into income. Loans and receivables originated by the entity are measured at amortized cost.

Although the basis of classification of financial assets is similar to US GAAP, the IAS 39 Exposure Draft permits entities the option of designating any financial instrument (including originated loans) as trading at inception. Further, ED5 permits insurers who change their accounting policies for insurance liabilities the option to reclassify some or all financial assets into the trading category.

In all likelihood, most insurers will classify most financial assets as AFS during Phase I. For most insurers, Phase I liability values will not vary with changes in market interest rates, so insurers will want asset values to be similarly unaffected. This is the approach commonly taken now by U.S. GAAP reporters.

In the longer-term, companies will need to consider the ability and impact of redesignating assets to be consistent with the ultimate measurement basis under Phase II for insurance contracts. As discussed below, insurance liability values will be based on the yield curve on the valuation date. That is consistent with the fair value used for assets classified as trading. When a fair value standard is implemented for insurance contracts, companies will likely want to classify assets as trading to achieve consistency in the measurement between assets and liabilities. Assuming assets and liabilities are reasonably matched, this will reduce the volatility of earnings.

4. PHASE II INSURANCE STANDARD - OVERVIEW

This section contains a brief overview of the major changes to financial reporting that Phase II is likely to introduce and some key business implications relating to these changes. These are each described in further detail in later sections.

The requirement for fair value or "fair value like" accounting represents a significant departure from current accounting practice, based on the deferral and matching approach. Implementation of the new reporting framework will be a major challenge, surpassed in difficulty only by the challenge of explaining reported earnings after the new principles are implemented.

Insurance Contract Definition

As with Phase I, Phase II will apply to insurance contracts not insurance companies. In addition, the definition of an insurance contract requires the presence of "significant insurance risk." The Definition provided would include some contracts that are not considered to be insurance policies under most current accounting standard, while excluding some that are.

Single Measurement Approach

A single measurement approach applies to valuing all insurance contracts, whether they are long or short term, life, annuity, health or property/casualty. This approach contrasts with some current practice, for example, under US GAAP, where different products are accounted for under different standards.

Valuing Options and Guarantees

Options and guarantees contained in contracts should be explicitly valued and reserved for. For example, this includes minimum interest rate guarantees, guaranteed annuity rates, and guaranteed death benefits on variable or unit-linked products. For most property and casualty insurance contracts, the value of options and guarantees is probably not material. But in some cases, it will be necessary to value these.

In such cases, option pricing and stochastic valuation techniques, which use random scenarios to project outcomes, would be required when such techniques are likely to have a material impact on the result. This would require many companies to significantly enhance their existing financial measurement and modeling systems.

Estimating Cash Flows and Adjustment for Risk and Uncertainty

The liability valuation begins with projections of expected cash flows under the contract. The present value of those cash flows, discounted at the risk-free rate plus a spread to reflect the insurer's credit risk, is the liability value prior to any adjustment for risk and uncertainty. Both fair value and entity-specific value should always both contain a market-based adjustment for risk. The risk adjustment is preferably made through adjusting the cash flows, or adjusting the discount rate, or both, without double counting.

The risk adjustments are referred to as "market value margins" and should be set to be consistent with market-risk preferences. The market-based adjustment for risk and uncertainty effectively act as a market mechanism for pricing uncertainty. However, there is no guidance in the DSOP on how this should be done.

Financial Statement Disclosures

The disclosure requirements in the DSOP are voluminous and burdensome. Companies will need to disclose expected earnings based on prior period valuation assumptions together with the effects on earnings of new business written, release of margins, deviations due to differences between actual and expected experience by source, and changes in assumptions. In addition, new business impact may need to be split between contracts sold to existing customers and contracts sold to new customers.

Business Implications

The business implications of the Phase II reporting framework are far reaching. Some of the most significant are described below:

- **Increased financial volatility:** reported financial results will be more volatile, making it more difficult to understand results and explain them to management, investors and other stakeholders.

- **Tighter matching of assets and liabilities:** as a means to reduce earnings volatility, assets and liabilities will tend to be more tightly matched and assets backing surplus may be less risky. In the process, policyholders and investors may lose some potential upside gain.

- **Fewer constraints on portfolio management:** there may be fewer constraints on managing asset portfolios on a total-return basis if insurers classify assets backing insurance liabilities as Trading, to keep their valuation consistent with the liabilities measured at Fair Value.

- **Investment portfolio and credit quality:** the credit quality of the fixed-income portfolio will become more transparent as changing credit spreads may materially impact reported income.

- **Forecasting challenges:** it will become nearly impossible to forecast results accurately, since results will depend on future economic conditions. In this new environment, stakeholders may require multiple forecasts based on differing future economic assumptions. As a result, companies will need to develop techniques to quickly estimate the impacts of changing economic scenarios. By the time reported financials are published, they may already be "out of date".

The DSOP is the primary source of explanation of, and rationale for, the Phase II standard. The IASB reached various "tentative conclusions" in its discussions that appear in the Basis for Conclusions section of IFRS 4, which in some cases differ from the DSOP. Those are reflected here as well.

The DSOP contains 14 chapters, each of which addresses specific principles that apply to insurance accounting. The principles are numbered within each chapter; for example, Principle 4.2

is the second principle of chapter four. We refer to the main principles we discuss throughout the report by these numbers so that you can readily refer back to the relevant section of the DSOP.

In the next sections, we consider the key financial reporting principles of the DSOP that have the greatest potential impact on property and casualty insurers.

5. INSURANCE CONTRACT DEFINITION

Insurance Contracts Not Companies

The DSOP applies to insurance contracts not insurance companies. Therefore, the same rules will apply regardless of the type of company that issues the contract. For example, a bank that issues insurance contracts must apply the same rules as an insurance company that issues insurance contracts. The rules apply to assets and liabilities that arise from insurance contracts, so-called "insurance assets and insurance liabilities". An example of an insurance liability is a liability for future benefits under the contract. An example of an insurance asset is reinsurance recoverable. A bond held by an insurance company is not an "insurance asset".

Definition of Insurance Contracts

For Phase I, a very broad definition of insurance contracts was adopted. In part, this was done to avoid forcing insurers to define fair value for contracts not qualifying as insurance (and therefore to be valued under IAS 39), where the valuation principles were not yet clear, and would likely be addressed in Phase II. At this time, it is not known if the definition of insurance will be narrowed in Phase II.

Under Phase I a contract qualifies as an insurance contract if the insurer accepts "significant insurance risk". If only "financial risk" is present, the contract will be classified as a financial liability (investment contract) and will be accounted for under IAS 39.

Certain criteria have to be met in order for a risk to be an insurance risk under the Phase I definition:

- The risk must arise from a specified uncertain future event that adversely affects the policyholder. For example, death adversely affects a life insurance policyholder and living too long adversely affects an immediate annuity policyholder.
- Changes in a specified interest rate, security price, commodity price, foreign exchange rate and similar items are specifically excluded as being "financial risks."
- It must be plausible that the uncertain future event will cause a significant adverse change in the present value of the insurer's cash flows under the contract. This condition is met even if the insured event is extremely unlikely.

This definition would include most property and casualty insurance contracts.

The DSOP covers many types of contracts not issued by insurers. Examples are automobile club repair services, warranty contracts issued by non-insurance companies, (but not warranties provided by the manufacturers) and contracts of some health care organizations such as CCRC's and HMO's.

The DSOP also covers some contracts that are not financial instruments. For example, contracts that provide payments in kind or services rather than cash payments in the event of an insured event are covered. This includes performance bonds, and some types of health insurance arrangements.

The DSOP would also exclude many types of contracts that have been issued by insurers. There is a requirement of risk shifting similar to that in the US GAAP under FAS 113. Weather derivatives and some catastrophe bonds would be excluded, if the payment amount is not linked to the actual losses by the insured.

In general, unbundling of the investment and insurance elements of an insurance contract would not be permitted under the DSOP.

6. ESTIMATING THE TIMING AND AMOUNT OF CASH FLOWS

The Starting Point - Expected Value

The starting point for measuring insurance assets and insurance liabilities is the expected value of future pre-tax, pre-reinsurance cash flows associated with the closed book of insurance contracts in force on the valuation date. These expected cash flows are then adjusted for risk and uncertainty (discussed in the next section), and the result is then discounted to get the present value (discussed in the following section).

These projected cash flows would, of course, include loss payments and loss adjustment expenses, and insurance premiums. In addition, other company expenses for marketing and administrative would be included, which is not current practice in most property and casualty insurance accounting systems. Overhead expenses that can be allocated to the policies on a "reasonable and consistent" basis would be included in the projections as well. There would seem to be wide room for judgment in doing that.

Salvage and subrogation rights are to be recognized as assets when they meet certain criteria – e.g. the insurer controls those rights and can measure them reliably. Prior to that time, the potential future salvage and subrogation rights should be provided for in the estimated cash flows used to calculate the liability.

Under the DSOP, there is no unearned premium reserve or deferred acquisition cost asset. Instead, there is a provision for future payments on in force contracts in addition to the payments provided for in the loss reserve, the estimated future expenses and payments on claims related to future coverage periods under contracts in the closed book. This provision is sometimes referred to as the "unexpired risk reserve" but it is different from what a similar term (provision for unexpired risk) refers to under U.K. GAAP rules.

The unexpired risk reserve as of the date of issue does not have to be equal to the premium less acquisition costs. The insurer may recognize a gain or loss at issue. However, the Board believes that in the absence of market evidence to the contrary the estimated Fair Value (FV) of an insurance liability shall not be less, but may be more, than the entity would charge to accept new contracts with identical contractual terms and remaining maturity from new policyholders.

Therefore, an insurer should not recognize a net gain on inception of an insurance contract unless such market evidence is available.

The expected present value of cash flows is not necessarily the same as the present value of expected cash flows. If there are significant options or guarantees provided under the contract, these two may be very different values. In that case, it may be necessary to use stochastic models or option pricing approaches to determine the liability. For most property and casualty products that will not be an issue, but for some such as longer-term savings oriented products, that may be necessary.

In some cases property and casualty policies generate unusual types of cash flows - e.g., residual market assessments or obligations to insure poor risks, guarantee fund assessments. These obligations should also be reflected in the liabilities, and approaches to doing that must be developed.

Setting Appropriate Assumptions

Assumptions may be classified as economic assumptions (such as interest rates and equity prices) and non-economic assumptions (such as expenses and mortality). Economic assumptions have to be set to be consistent with current market prices and data. Non-economic assumptions are set consistent with the market's expectations of experience that will result on that block of business are used for fair value.

In practice, there may not be market-based assumptions that are observable or available. In such cases a company's own estimates can serve as a proxy for market estimates, unless there is specific evidence that this is not appropriate. Some have suggested that reinsurance rates might be used as a source of market-based mortality assumptions. However, reinsurers differ greatly in their assessments of risk and often have different assessments than direct writing companies. On the other hand, data on industry expense levels may be more readily available than other types of information.

The assumptions should reflect "all future events, including changes in legislation and future technology changes, that may affect future cash flows." In contrast, under US GAAP, only legislation that has already been enacted, or for which enactment is imminent and certain, would normally be reflected.

Inflation should be reflected in the cash flows, in a way that is consistent with the interest rates used for discounting. That linkage does not exist in most loss reserve methods now in use.

The assumptions should reflect constructive obligations to make payments, as well as the explicit contractual obligations.

Assumptions should be reviewed and reset at each valuation date, at the then current best estimates. For economic assumptions, this will be necessary to maintain consistency with current market values of assets. That may require a change in the inflation assumptions underlying projected losses as well.

The Closed Book

The closed book concept poses several interesting questions for property and casualty insurers, both with respect to when a liability is recognized, and to what extent cash flows in future contract renewal periods are reflected in the liability.

The liability should be recognized at the time that an insurance contract is established. The event that creates insurance assets and liabilities is becoming a party to the insurance contract. That will generally not be the same as the starting date of the coverage. For some types of business, it will often be in a different year. That would be the case, for example, for January 1 reinsurance renewals agreed the previous year.

Becoming a party to an insurance contract is an event that gives the insurer and the policyholder control over their contractual rights and creates contractual obligations that gives them little, if any, discretion to avoid the net cash flows resulting from their contractual obligations. In some cases, it may not be clear when a contract is established. Is it necessary for the actual policy to be signed, or is a signed application or reinsurance slip sufficient? What about a verbal assurance that coverage will be provided, or even draft agreed contracts (e.g., World Trade Center)? Or a signed application binding coverage, but giving the insurer a specified period to underwrite and reject? Is the contract created when the application is signed, or when the insurer's right to reject expires? Policy renewals where the insurer must give advance notice (e.g., 30 days) in order to non-renew raise similar issues. Whatever the answers to these questions may be, most insurers do not now have business processes or information systems that would enable them to implement those answers.

Under the DSOP, an insurer may recognize a loss at issue, although gains should be recognized on issue only if there is clear market evidence to justify them. So determining when the contract comes into existence will be more important than is the case now.

Renewals

The closed book includes cash flows in future renewal coverage periods in determining the liability only to the extent that:

(a) the policyholder has non-cancelable continuation or renewal rights constraining the insurer's ability to reprice; and

(b) those rights lapse if the policyholder ceases to pay premiums.

Considerable effect will be needed to determine exactly what that means, and how it should apply to the multitude of different regulatory and contractual approaches to renewal that exist in the market. Note that this definition has been significantly changed by the IASB discussions from what was proposed in the DSOP. This is indicative of the difficulty of the issue.

Unexpired Risk Reserve

As noted above, the Unearned Premium Reserve and Deferred Acquisition Cost items used now in deferred and matching accounting approaches will disappear. They will be replaced by a new Unexpired Risk Reserve (URR). It would be the present value of loss and expense payments to be provided for by premiums covering the period from the valuation date to expiry on all contracts in force on the valuation date, whether those premiums have been paid or not. If they have not yet been paid, there would be an offsetting receivable for premium due.

Calculating this URR would resemble a simplified ratemaking exercise, with the loss reserve analysis as a base. For example, the last few accident year selected ultimate loss ratios and claim payment patterns, from the loss reserve analysis, would be the starting point. A trend factor would project those to the average exposure date of the unearned premium. Rate level adjustment factors would reflect the impact of rate changes. The "averages" of these projected loss payment streams (e.g., average of last 3, 3-2-1 weights, etc.) would give the expected loss ratio on current rate level. In some cases, it may be desirable to credibility weight recent actual amounts with payments based on an a priori expected loss ratio. And in some cases judgmental adjustments may be appropriate - e.g., to reflect an actual or expected change in the law that will impact losses.

Everything in the previous paragraph is standard ratemaking technique, and readers of this paper will likely know many variations of the approach used to fit various circumstances.

The URR calculation will also require a number of new elements, not now common in ratemaking.

1. The level of aggregation of the business will be different - most likely more similar to that commonly used for loss reserve analysis. Perhaps ratemaking and reserving processes will become more integrated.

2. Additional elements of cash flows will need to be projected - e.g., future maintenance and acquisition expenses for contracts in force on the valuation date.

3. Market Value Margins (MVM's) to reflect risk and uncertainty will need to be added to the expected cash flows. Under some approaches to MVM's (e.g., setting gain at issue to zero), the MVM's will be reset at each valuation date for the new business issued since the prior valuation date. In practice, many companies may leave those MVM's unchanged for the life of those contracts, so the selection of MVM's for the URR will also determine the levels of MVM's for loss reserves. MVM issues are discussed in more detail in a later section.

4. The projected payments must be discounted using the risk-free yield curve on the valuation date, plus a spread for the insurer's own credit risk on that date.

5. The process described so far is a deterministic, best-estimate approach. But where the policy contains significant options or guarantees, those must be reflected as well. Stochastic methods may be required to do that, but this should not be a material issue for most property and casualty insurance contracts.

6. Renewal provisions must be considered for contracts where the closed book includes future renewal periods. These will be many cases where property and casualty insurance falls in a gray area in this respect.

7. This has to all be done on a gross (of reinsurance) basis, and then again for the amounts reinsured. But for the reinsured business, the assumptions may change - e.g., credit rating spreads may change, MVM's will change sign, and perhaps amount, maintenance expenses may not be included.

Projected Expenses

As noted in point 2 above this calculation will require projections of future expenses on contracts in force - both maintenance expenses and acquisition expenses. That is a new area for most P&C insurers. It will raise a number of challenging issues. How should overhead expenses be reflected? What should be done to project expense trends? What about anticipated changes in expenses levels - e.g., planned cost level reductions. To the extent those are reflected by changes in the URR, they impact earnings at the time they are planned, not at the time they are carried out.

7. ADJUSTMENTS FOR RISK AND UNCERTAINTY

Definition

The fair value of a liability consists of the expected value of the cash flows discounted for the time value of money, and a risk adjustment. This risk adjustment to liabilities will be referred to as the Market Value Margin (MVM). "Own credit risk", i.e., the risk that the insurer will default, will not be considered here.

The MVM is not directly observable for a P&C insurer's policy liabilities because they are not actively traded. Consequently, the MVM needs to be estimated. In this section, the MVM will consider three types of risk:

1) process risk - random statistical fluctuations will cause the value of the liability to be different than expected. Process risk may often be regarded as diversifiable risk.
2) parameter risk - misestimation of parameters used in the modeling process
3) model risk - the wrong model was used to estimate the liability

In the current DSOP, Principle 5.4 states, "The entity-specific value or fair value of an insurance liability or insurance asset should always reflect both diversifiable and non-diversifiable risk." This implies that the insurer should estimate process risk, parameter risk, and model risk. However, Section 5.10 states that while it is "conceptually preferable" to reflect parameter risk and model risk, "it is appropriate to exclude such adjustments unless there is persuasive evidence that enables an insurer to [quantify] them by reference to observable market data." Consequently, it may be at the insurer's discretion whether it wants to estimate model and parameter risk.

Here are four examples of practical approaches to estimating the MVM's. This is clearly an area where more research by CAS members would be useful.

1) Canadian Provision for Adverse Deviation (for non-diversifiable risk only)
2) Initial Expected Profit Margin (for both non-diversifiable and diversifiable risk)
3) Poisson Frequency / Lognormal Severity Simulation (for diversifiable risk only)
4) Mack's Approach using historical loss triangles (for both non-diversifiable and diversifiable risk)

1) Canadian Provision for Adverse Deviation

The Canadian Institute of Actuaries (CIA) introduced a standard of practice covering provisions for adverse deviations (PFAD's) for P&C insurers effective January 1, 1994. Before this, the general direction from the CIA advised,

> *"For several reasons, it is not possible to determine expected experience with complete confidence. The member should, therefore, define a margin for adverse deviation in each assumption to add a provision to the liabilities. This provision should be appropriate for income statement purposes and appropriate to the company circumstances... For each assumption, the margin is for the misestimation of its mean and for possible deterioration of this mean. Statistical fluctuation, catastrophic or similar major unexpected events should not be covered by the margin.*

The 1994 CIA standard of practice described three major valuation variables: claims development, reinsurance recovery, and discount rate, which the PFAD's should cover. The standard described low margin and high margin situations for each variable and asked the actuary to determine where within that continuum a particular insurer fell. The standard set a range for the PFAD for each variable, namely,

- claims development — from 2.5% to 15%,
- reinsurance recovery — from 0% to 15%, and

- discount rate from 50 to 200 basis points.

In practice Canadian actuaries first estimate policy liabilities on the traditional ultimate undiscounted basis and then determine payment patterns. A discount rate is selected, generally based on the expected future book returns of the insurer's invested assets including, if necessary, assumptions about the yield on reinvestments. The expected book yield is used for the discount rate so that the policy liabilities and corresponding assets are on a comparable basis. This is a difference from the DSOP approach, under which the discount rate would not reflect the assets held by the insurer.

The claim liabilities are discounted once at the discount rate and a second time at a rate equal to the discount rate less the basis points required in the circumstances. The difference between the two estimates is the PFAD for interest rate.

The PFAD for claim development is typically a percentage of the discounted gross unpaid claim liabilities.

The PFAD for reinsurance recovery is typically a percentage of the discounted ceded unpaid claim liabilities.

2) Initial Expected Profit Margin

Under the DSOP and the conclusions of subsequent IASB discussion, the discounted value of expected future cash outflows (claims and operating expenses) from a policy, or group of policies, is in most circumstances less than the discounted value of expected future cash inflows (premiums and policy service fees). This difference would be the present value of expected profit. When MVM's are added to the policy liabilities, they effectively defer the recognition of profit until the passage of time replaces fact for estimates and the MVM's are removed. There is no guarantee that any theoretical MVM will exactly offset the expected profit margin at issue. Nevertheless, if markets are efficient, the DSOP suggests there should be no gain at issue. Consequently if a profit is indicated at issue, any theoretical MVM should be scaled so that the result is simply breakeven, unless there is clear market evidence supporting a gain at issue.

There are cases when a gain at issue is permitted. For example if one group of policies is sold a price X and shortly afterwards the market price for other policyholders with identical risk characteristics is reduced, this may suggest that there is a legitimate gain at issue for the first group.

Conversely, if prices are later increased, it may imply there should be a loss at issue for the earlier policies.

3) Poisson Frequency / Lognormal Severity Simulation

Loss reserves are calculated based on a Monte Carlo simulation. Parameters for this simulation are based on future claims with payment and pending severity estimates from the insurer.

Future claims with payment are assumed to be Poisson distributed. The lambda parameter for the Poisson future claims with payment distribution is determined by projecting ultimate claims with payment and subtracting closed claims with payments.

Pending severities are assumed to be lognormally distributed. The expected value of the pending severity equals ultimate losses less paid losses divided by future claim counts. Loss data should be gross of reinsurance. The coefficient of variation for the pending severity distribution can be derived from the increased limit factors of the insurer or appropriate industry standards such as ISO.

An example of the calculations for this method can be found in Appendix A, Exhibit A.

The advantages of this method are:

- The data needed to calculate this method are readily available (most of this data can be found in the US Schedule P by line of business)
- The simulations could be run on Microsoft Excel or other readily available software
- The method is already in use by some insurance entities to estimate process risk.
- Parameters used in simulation are fairly easy to disclose and results can be replicated by outsiders.

Disadvantages of this method are:

- Loss data may not have claim data that is Poisson distributed and it may not have severity data that is log-normally distributed.

- The method only measures process risk. This can be a potential advantage if the insurer cannot accurately measure non-diversifiable risk or the insurer already has a method that calculates only non-diversifiable risk (e.g., the Canadian Provision for Adverse Deviation, the CAPM method, etc.)
- All claims with payment may not have the same coefficient of variation parameters.
- The method is dependent on the insurer having adequate reserves. If the insurer's reserves are inadequate, the MVM from this method will be inadequate.

4). Mack's Approach Using Historical Loss Triangles

In this approach, we use historical loss triangles of the insurer to calculate the MVM. Full documentation of this approach can be found in Thomas Mack's article, "Measuring the Variability of Chain Ladder Estimates". This approach relies on the chain-ladder technique to develop expected ultimate losses. It then uses the actual data's variation around the insurer's expected losses to estimate the variance of the insurer's losses.

This method can be applied to paid losses, case incurred losses, and ultimate losses. The paid loss triangle is independent of claim adjusters, actuaries, and upper management's opinions on reserves but is vulnerable to changes in payout patterns. The case incurred loss triangle is independent of actuaries' and upper management's opinions on reserves, but is vulnerable to both changes in payout patterns and claim adjusters' case reserving practices. The ultimate loss triangle is dependent on the opinions of actuaries and upper management as well as changes in payout patterns and reserve level.

Expected loss reserves from these three versions of Mack's approach can provide widely disparate results. However the ratio of the standard deviation of reserves to the expected value of reserves (or, as shown in the example in Appendix A, the ratio of the 75th percentile of reserves to expected reserves) can provide more consistent results. The ratio of standard deviation to expected reserves should be smaller for the incurred loss and ultimate methods if the judgments of claims adjusters, actuaries, and upper management provide some insight into the true estimate of ultimate losses (and if the implicit assumptions of the chain-ladder method are true for the insurer's data; see the list of disadvantages below).

Advantages of this method:

- The data needed to calculate this method are readily available (most of this data can be found in the US Schedule P by line of business).
- This method calculates both process risk and parameter risk. The ultimate loss method also measures the historical method risk for the company.
- This method does not make any assumptions about the underlying distribution of the insurer's losses.
- This method can be readily calculated on a spreadsheet, although a number of formulas are needed to determine the standard error of ultimate lossess.
- An insurer that historically under-reserves will have a larger MVM than one that accurately estimates its reserves if the ultimate loss version of this approach is used.

Disadvantages of this method:

- This method assumes that future losses will develop in the same way that losses have developed historically. Dramatic changes in current payout patterns, case incurred reporting patterns, and ultimate loss reporting patterns can render this method unusable.
- Mack shows that the chain-ladder relies on a number of implicit assumptions, most notably that accident year data are independent of one another. Mack provides a number of tests that can be used to test whether these implicit assumptions are true for an individual insurer's data.
- For long tailed reserves, a number of years of experience are needed to estimate the variance in reserves.
- This method can provide strange results for lines of business with sparse data.
- This approach is not commonly used for valuing process risk. Further research is needed to determine the viability of the suggested approach.

8. DISCOUNTING

Discount Rates

Discounting estimates of future cash flows is a significant step in estimating the fair value of an insurance liability. Whilst the process of discounting does not pose the same level of technical difficulties as estimating market value margins it still requires a degree of care and it will be a new process for many property and casualty insurers.

The starting point for the discount rate, before any adjustment for risk and uncertainty, is the pre-tax market yield on risk-free assets at the balance sheet date.

Some observers have suggested that yields on high-grade corporate bonds, properly adjusted for expected default costs, are a better measure of risk-free rates than are yields on government securities. However, this approach is not permitted under the DSOP unless "there is no active market in government securities".

The liability value should reflect the company's own credit, and this would probably be accomplished by adjusting the discount rate. This was a very controversial issue during much of the discussion leading up to the DSOP, with many objecting strongly to a system in which an insurer's deteriorating financial condition automatically leads to a reduction in the value assigned to its liability. Many actuaries objected strongly. Others assert that this is merely reflecting reality, and refer to situations in which companies have been able to buy back their own debt at prices reflecting reduced credit rates. Whatever the views one has, it seems now that the decision has been made to reflect the insurer's own credit rating in the discount rate.

However, there is no guidance on what the proper adjustment to the risk-free rate should be. The credit spread that corresponds to the company's debt rating is not necessarily the right answer because insurance contracts have a different priority in liquidation. The spread corresponding to the company's claims paying ability rating may be more appropriate.

Many IAS reporters will have assets and liabilities in foreign currencies, often in currencies where meaningful risk-free yield curves and credit spreads are difficult or impossible to determine.

Discounting Reflecting Option and Guarantees

In theory, the DSOP calls for a discounting approach that properly values options and guarantees. A stochastic approach as follows would accomplish that.

- Each scenario of cash flows for an insurance liability would be discounted and then the present values added together weighted by the probability of each scenario. The cash flows should include the appropriate market value margins if it has been decided to incorporate them by altering cash flows rather than adjusting the discount rate.

- The discount rate should be the risk-free rate consistent with the timing and currency of the cash flows, adjusted for the insurer's own credit risk. (If market value margins are not included within the cash flows then the discount rate needs to be reduced appropriately).

- The present value of foreign cash flows would be converted into the measurement currency using the spot rate at the reporting date.

In practice, the expected cash flows can be discounted in most cases without significant loss of accuracy for most P&C insurance contracts.

An example of how to do this discounting is provided in Appendix B.

Choosing the Discount Rate

For most developed countries the interest rate paid on Government securities can be reasonably used as the benchmark for the risk-free rate. This is because the risk of default is usually regarded as negligible and also in those countries such securities have a lower credit risk than other securities.

This will not be appropriate for some developing countries where such a benchmark rate may not be appropriate as the risk of default is not minimal. One possibility is to use the rate implied by highly rated corporate bonds if such bonds carry a lower default risk than Government securities.

However quite often in such jurisdictions high quality corporate bonds are also not available. One way around this may be to try to convert the yield available on the highest quality securities available into a risk-free rate. This can be done by adding the value of the expected default level of such securities onto the market price of the security to estimate a risk-free rate. The expected default value can be estimated using write-off factors from credit rating agencies for a particular credit rating of the security. If it is relatively straightforward to estimate, the risk premium for bearing the risk of volatile defaults should also be added onto the market price of the security when estimating the risk-free rate of return.

Similarly, in many foreign currencies it will be more difficult to judge the credit spread required to reflect the insurer's own credit risk than is the case in developed markets.

9. OTHER ISSUES

Performance-linked Contracts

The DSOP defines performance-linked contracts as an insurance contract under which the payments to policyholders depend partly on one or more of:

- Performance of the contract itself, a specified pool of contracts or a specified type of contract
- Realized and/or unrealized investment returns on a specific pool of assets held by the insurer
- The net profit or loss of the company, fund or other entity that issues the performance-linked insurance contract

Traditional participating (with profits) and variable (unit linked) life insurance and annuity contracts are the most obvious examples of performance-linked contracts.

Property and casualty insurers also have performance-linked plans e.g. retrospective rating, experience based dividend plans. They also have plans that to some may appear to be performance – linked but probably do not fit here – e.g., prospective experience rating, bonus/malus systems. Many property and casualty insurance retrospectively rated contracts may be taken out of this category by the Unbundling approach proposed for retrospectively rated contracts in Phase I.

Reinsurance Ceded

The Phase II approach for reinsurance largely carries forward the principles introduced for direct insurance in Phase I. The same general approach should be used to value reinsurance ceded as used for direct insurance. Again, "one size fits all" and there is no difference in the treatment of reinsurance and direct insurance.

In addition, the effect of reinsurance ceded should be carved out and presented separately as below:

- Reinsurance amounts recoverable are shown as assets on the balance sheet. They may not be set up as negative reserves to offset against direct liability.

- Reinsurance premiums are shown as expenses and reinsurance claims are shown as income. They may not be netted from direct premiums and claims.

Contracts that do not transfer a significant amount of "insurance risk" will not qualify for reinsurance accounting.

The accounting approach for reinsurers will be the same as for insurance companies. However, there is no requirement for "mirror reserving" between reinsurers and ceding companies. In fact, since the insurer's and reinsurer's credit ratings will likely differ, and since MVM's should increase the liability and decrease the asset (i.e., MVM's are additions to expected insurance cash flows in valuing liabilities, and reductions from expected cash flow in valuing insurance assets), the direct and ceded values for the same business may be quite different. Where large portions of the business are reinsured on a quota share basis, a common practice in many P&C markets, this may tend to produce a loss at inception of the coverage.

Savings-oriented and other long-term policies

In many countries, property and casualty insurers issue long-term policies, and in many cases there is an explicit savings function involved in the policy. These policies raise many of the same issues that apply to life insurance contracts. They are beyond the scope of this paper, and will not be discussed here. These contracts will require property and casualty insurers to develop financial modeling tools and skills that they do not have now, perhaps including stochastic modeling tools.

Deferred and fund methods of accounting

These approaches, historically used at Lloyd's, will not be permitted by the DSOP. The Lloyd's market will drop this in 2005 anyway under UK accounting rules.

10. CONCLUSION

The new requirements for International Accounting Standards for insurers will present challenges to reserving actuaries for property and casualty insurance companies over the next few years, especially when Phase II is introduced. Already, the profession is busy in both a research and an advocacy role, trying to influence the IASB and its staff, to help them develop new standards that are practical and meaningful for property and casualty insurers, and that will provide useful information to the investing public and other users of IAS financial reports. As the new standards are finalized, the profession will need to develop practical approaches to doing the required reserve analyses. This paper focused on some of the issues involved in that. There are clearly many open issues that will need to be resolved as this moves forward. And just developing the new reserve methods alone is not enough – it will be necessary for a large number of reserving actuaries to be educated about them, and to develop the tools and skills necessary to apply them as part of the regular support to the financial reporting process.

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The authors wish to thank many people who have contributed, directly or indirectly, to this paper. All of the authors are employed at Ernst & Young. Some of the thinking here, and in some cases the actual text used, was developed by committees that we participated on. We have also participated in professional and industry committees addressing IAS for insurance, and the discussions there have deepened our understanding of the issues. While we acknowledge the contributions of many, the ideas presented here are solely those of the authors, and do not represent the views of their employers or any others organization.

Guide to Abbreviations Used

DSOP – Draft Statement of Principles, a document setting forth some basic principles for Phase II

ED 5 – Exposure Draft 5, a document exposing the proposed Phase I accounting standard

IAS – International Accounting Standards

IASB – International Accounting Standards Board

IAS 37 – International Accounting Standard dealing with accounting for contingencies

IAS 39 – International Accounting Standard dealing with accounting for financial instruments

IFRS – International Financial Reporting Standards

IFRS 4 – International Financial Reporting Standard dealing with accounting for insurance, Phase I

MVM – Market Value Margin, a provision for risk and uncertainty

PFAD – Provision for Adverse Deviations, a Canadian term for MVM's

URR – Unearned Revenue Reserve, a new reserve providing for future costs on contracts in force

EXAMPLES OF MVM APPROACHES

Summary of Exhibits

Poisson/Lognormal Simulation

Exhibit A, Sheet 1
Summarizes Market Value Margin Calculation for Process Risk

Exhibit A, Sheet 2
Shows Results of Poisson/Lognormal Simulation

Exhibit A, Sheet 3
Shows Calculation of Parameters for Simulation

Mack's Approach

Exhibit 1, Sheet 1
Summarizes Market Value Margin Calculation for Process, Parameter & Model Risk

Exhibit 2, Sheet 1
Calculates Reserves at the 75th percentile for paid loss & ALAE triangle

Exhibit 2, Sheet 2
Calculates standard deviation of total reserves for paid loss & ALAE triangles

Exhibit 2, Sheet 3
Calculates standard deviation of reserves for each year for paid loss & ALAE triangles

Exhibit 2, Sheet 4
Calculates parameters used in standard deviation calculations for paid loss & ALAE triangles

Exhibit 3, Sheet 1
Calculates Reserves at the 75th percentile for incurred loss & ALAE triangle

Exhibit 3, Sheet 2
Calculates standard deviation of total reserves for incurred loss & ALAE triangles

Exhibit 3, Sheet 3
Calculates standard deviation of reserves for each year for incurred loss & ALAE triangles

Exhibit 3, Sheet 4
Calculates parameters used in standard deviation calculations for incurred loss & ALAE triangles

Exhibit 4, Sheet 1
Calculates Reserves at the 75th percentile for ultimate loss & ALAE triangle

Exhibit 4, Sheet 2
Calculates standard deviation of total reserves for ultimate loss & ALAE triangles

Exhibit 4, Sheet 3
Calculates standard deviation of reserves for each year for ultimate loss & ALAE triangles

Exhibit 4, Sheet 4
Calculates parameters used in standard deviation calculations for ultimate loss & ALAE triangles

Exhibit A
Sheet 1

Insurer X

Reserve Analysis As of December 31, 1997
Summary of Simulation Results
Poisson/Lognormal Simulation

Loss & ALAE @ 75th percentile	Expected Loss & ALAE	MVM Process Risk Load
265,234	263,210	0.8%

Notes:

(1) From Exhibit 1, Sheet 2
(2) From Exhibit 1, Sheet 2
(3) = (1) / (2)

Insurer X
Reserve Analysis As of December 31, 1997
Summary of Simulation Results
Modeling Future Closed Claims (IBNR and Open)
Poisson/Lognormal Simulation

Exhibit A
Sheet 2

	(1)	(2)
Percentile Levels	Loss & ALAE Reserve ($000s)	Risk Margin
Expected	263,210	0
Low	257,409	(5,801)
10%	260,686	(2,524)
20%	261,841	(1,369)
30%	262,447	(763)
40%	263,234	24
50%	263,742	532
55%	263,889	679
60%	264,043	833
65%	264,285	1,075
70%	264,984	1,774
75%	**265,234**	**2,024**
80%	265,501	2,291
85%	265,904	2,694
90%	267,030	3,820
95%	267,702	4,492
High	268,256	5,046

(1) Monte Carlo Simulation with underlying loss assumptions:

Claim count distribution is approximated by a Poisson Distribution with mean 98,25[illegible]
Claim severity distribution is approximated by a Lognormal Distribution with mean = $2,679 and coefficient of variation = 3.0

(2) = (1) - mean ultimate loss

Exhibit A
Sheet 3

Insurer X
Data as of December 31, 1997
Gross of Reinsurance
Determination of Frequency and Severity Parameters for Poisson / Lognormal Simulation
Poisson/Lognormal Simulation

1) Gross Ultimate Loss & ALAE Reserves ($000s)	1,418,282
2) Gross Paid Loss & ALAE Reserves ($000s)	1,155,072
3) Gross Loss & ALAE Reserves ($000s)	263,210
4) Ultimate Counts Closed With Payments	486,079
5) Counts Closed With Payments to Date	387,820
6) Future Closed With Payments to Date (λ parameter for Poisson dist.	98,259
7) Pending Severity	2,679
8) Coefficient of Variation	3.00
9) Pending Severity - σ lognormal parameter	1.51743
10) Pending Severity - μ lognormal parameter	6.74181

Notes:
(1) Amount Booked by Insurer X
(2) Amount Booked by Insurer X
(3) = (2) - (1)
(4) We projected ultimate counts with payments ourselves; documentation available upon request
(5) Provided by Insurer X
(6) = (4) - (5)
(7) = (3) / (6)
(8) Determined by Analyzing Increased Limit Factors used to price Insurer X's policies
(9) = square root of $(\ln[(8)^2+1])$)
(10) = $\ln[(7)] - (9)^2/2$

Insurer X
Commercial Auto Liability
Data as of December 31, 1997
Selection of Market Value Margin
Based on ratio of 75th percentile to expected reserves
Mack's Approach

(1) Paid Method	9.4%
(2) Incurred Method	8.1%
(3) Ultimate Method	6.9%
(4) Selected MVM	8.1%

Notes:

(1) From Exhibit 2, Page 1
(2) From Exhibit 3, Page 1
(3) From Exhibit 4, Page 1
(4) Judgmentally Selected Based on (1), (2), & (3)

Insurer X
Commercial Auto Liability
Data as of December 31, 1997
Mack's Approach

	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*	*(11)*
Accident Year	Paid Losses To Date	Loss Development Factor	Ultimate Losses	Total Reserves	Standard Error of Reserves	Ratio of Standard Error to Expected Reserves	σ_i^2	μ_i	Reserves @ 75th percentile	% Larger Than Expected Reserves	Ultimates @ 75th percentile
1988	145,282	1.000	145,282	0	0						145,282
1989	179,147	1.004	179,798	651	514	79.0%	0.485	6.236	713	9.5%	179,860
1990	151,891	1.017	154,448	2,557	930	36.4%	0.124	7.784	2,845	11.2%	154,736
1991	111,829	1.038	116,051	4,222	1,560	36.9%	0.128	8.284	4,700	11.3%	116,529
1992	108,757	1.075	116,921	8,164	2,527	30.9%	0.091	8.962	9,014	10.4%	117,771
1993	135,502	1.130	153,157	17,655	4,939	28.0%	0.075	9.741	19,389	9.8%	154,891
1994	108,001	1.261	136,233	28,232	8,128	28.8%	0.080	10.208	31,053	10.0%	139,054
1995	101,862	1.519	154,774	52,912	13,187	24.9%	0.060	10.846	57,742	9.1%	159,604
1996	75,558	2.226	168,189	92,631	18,246	19.7%	0.038	11.417	99,780	7.7%	175,338
1997	35,251	4.889	172,325	137,074	41,621	30.4%	0.088	11.784	151,193	10.3%	186,444
TOTAL	1,153,080		1,497,179	344,099	51,906	15.1%	0.022	12.737	376,503	9.4%	1,384,227

(1) From Exhibit 2, Page 3
(2) From Exhibit 2, Page 3
(3) = (1) * (2)
(4) = (3) - (1)
(5) Annual Standard Error from Exhibit 2, Page 3
Total Standard Error from Exhibit 2, Page 2
(6) = (5) / (4)

(7) = ln[(1 + (6)^2)]
(8) = ln[(4)] - (7)/2
(9) total reserves = (4)*exp(.675*sqrt[(7)]-(7)/2)
annual reserves = (4)*exp(0.479*sqrt[(7)]-(7)/2)
0.479 is the factor needed so that the sum of annual reserves = total reserves
(10) = (9) / (4)
(11) = (9) + (1)

Insurer X
Commercial Auto Liability
Data as of December 31, 1997
Mack's Approach

Calculation of Variance for Total Reserves

	k = 1	2	3	4	5	6	7	8	9
(1) α_{k-1}^2	7,042.27	708.25	520.02	280.76	81.74	28.04	12.83	2.91	0.66
(2) All-Yr Wtd Ave Incremental LDF	2.196	1.465	1.205	1.116	1.051	1.036	1.021	1.013	1.004
(3) Sum of Paid Losses as of Time k for 1988 to 1997-k	271,013	519,625	659,378	686,266	630,368	553,992	462,081	319,698	144,756

Accident Year	*(4)* Ultimate Losses	*(5)* Annual Standard Error	*(6)* Annual Variance	*(7)* Total Reserves Variance Component$_{k-1}$	*(8)* Total Reserves Variance Component$_{k-2}$	*(9)* Total Reserves Variance Component$_{k-3}$	*(10)* Total Reserves Variance Component$_{k-4}$	*(11)* Total Reserves Variance Component$_{k-5}$	*(12)* Total Reserves Variance Component$_{k-6}$	*(13)* Total Reserves Variance Component$_{k-7}$	*(14)* Total Reserves Variance Component$_{k-8}$	*(15)* Total Reserves Variance Component$_{k-9}$	*(16)* Total Reserves Total Variance	*(17)* Total Reserves Standard Deviation
1988	145,282	0	0											
1989	179,798	514	264,196									1,905,158	2,169,354	
1990	154,448	930	865,445								2,785,148	1,420,899	5,071,492	
1991	116,051	1,560	2,432,696							5,576,991	1,854,091	945,901	10,809,679	
1992	116,921	2,527	6,383,609						8,653,164	4,890,136	1,625,744	829,405	22,382,058	
1993	153,157	4,939	24,397,987					22,691,396	9,122,517	5,155,380	1,713,925	874,393	63,955,599	
1994	136,233	8,128	66,065,615				44,328,865	15,829,826	6,363,992	3,596,463	1,195,657	609,988	137,990,406	
1995	154,774	13,187	173,890,645			57,291,335	34,624,256	12,364,313	4,970,767	2,809,115	933,900	476,448	287,360,779	
1996	168,189	18,246	332,905,880		36,813,885	31,506,664	19,041,182	6,799,601	2,733,612	1,544,838	513,587	262,017	432,121,266	
1997	172,325	41,621	1,732,343,026	0	0	0	0	0	0	0	0	0	1,732,343,026	
	1,497,179												2,694,203,659	51,906

(1) From Exhibit 2, Page 4
(2) From Exhibit 2, Page 4
(3) From Exhibit 2, Page 4
(4) From Exhibit 2, Page 3
(5) From Exhibit 2, Page 3
(6) From Exhibit 2, Page 3

(7) No Component for k = 1
(8) = 2 * [(4)]*[sum of (4) for all acc. Yrs. After Current Acc. Yr]*[(1) / (2)^2 * [1/(3)] for k = 2
(9) = 2 * [(4)]*[sum of (4) for all acc. Yrs. After Current Acc. Yr]*[(1) / (2)^2 * [1/(3)] for k = 3
(10) = 2 * [(4)]*[sum of (4) for all acc. Yrs. After Current Acc. Yr]*[(1) / (2)^2 * [1/(3)] for k = 4
(11) = 2 * [(4)]*[sum of (4) for all acc. Yrs. After Current Acc. Yr]*[(1) / (2)^2 * [1/(3)] for k = 5
(12) = 2 * [(4)]*[sum of (4) for all acc. Yrs. After Current Acc. Yr]*[(1) / (2)^2 * [1/(3)] for k = 6

(13) = 2 * [(4)]*[sum of (4) for all acc. Yrs. After Current Acc. Yr]*[(1) / (2)^2 * [1/(3)] for k = 7
(14) = 2 * [(4)]*[sum of (4) for all acc. Yrs. After Current Acc. Yr]*[(1) / (2)^2 * [1/(3)] for k = 8
(15) = 2 * [(4)]*[sum of (4) for all acc. Yrs. After Current Acc. Yr]*[(1) / (2)^2 * [1/(3)] for k = 9
(16) = sum of (6) to (15)
(17) = square root of (the sum of (16) for all accident years)

Insurer X
Commercial Auto Liability
Data as of December 31, 1997
Mack's Approach

Calculation of Standard Deviation by Accident Year

	k = 1	2	3	4	5	6	7	8	9
(1) α_k^2	7,042.27	708.25	520.02	280.76	81.74	28.04	12.83	2.91	0.66
(2) All-Yr Wtd Ave Incremental LDF_k	2.196	1.465	1.205	1.116	1.051	1.036	1.021	1.013	1.004
(3) Sum of Paid Losses as of Time k for 1988 to 1997-k	271,013	519,625	659,378	686,266	630,368	553,992	462,081	319,698	144,756

Accident Year	*(4)* Paid Losses	*(5)* LDF	*(6)* Ultimate Losses	*(7)* Annual Reserves Variance Component$_{k=1}$	*(8)* Annual Reserves Variance Component$_{k=2}$	*(9)* Annual Reserves Variance Component$_{k=3}$	*(10)* Annual Reserves Variance Component$_{k=4}$	*(11)* Annual Reserves Variance Component$_{k=5}$	*(12)* Annual Reserves Variance Component$_{k=6}$	*(13)* Annual Reserves Variance Component$_{k=7}$	*(14)* Annual Reserves Variance Component$_{k=8}$	*(15)* Annual Reserves Variance Component$_{k=9}$	*(16)* Annual Reserves Total Variance	*(17)* Annual Reserves Standard Deviation
1988	145,282	1.000	145,282											
1989	179,147	1.004	179,798									264,196	264,196	514
1990	151,891	1.017	154,448								656,196	209,250	865,445	930
1991	111,829	1.038	116,051							1,842,027	453,582	137,087	2,432,696	1,560
1992	108,757	1.075	116,921						3,928,608	1,858,544	457,882	138,575	6,383,609	2,527
1993	135,502	1.130	153,157					15,552,117	5,407,864	2,582,441	648,959	206,607	24,397,987	4,939
1994	108,001	1.261	136,233				44,835,154	13,563,110	4,701,550	2,235,628	556,819	173,354	66,065,615	8,128
1995	101,862	1.519	154,774			97,304,025	51,879,868	15,745,669	5,476,761	2,616,375	658,028	209,919	173,890,645	13,187
1996	75,558	2.226	168,189		141,514,302	106,964,242	57,117,727	17,375,097	6,057,864	2,903,282	735,053	238,313	332,905,880	18,246
1997	35,251	4.889	172,325	1,390,029,992	145,447,245	109,982,235	58,756,563	17,886,014	6,240,458	2,993,678	759,446	247,395	1,732,343,026	41,621
	1,153,080		1,497,179											

(1) From Exhibit 2, Page 4
(2) From Exhibit 2, Page 4
(3) From Exhibit 2, Page 4
(4) From Exhibit 2, Page 4
(5) From Exhibit 2, Page 4
(6) = (4) * (5)

(7) = [(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)] for k = 1
(8) = [(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)] for k = 2
(9) = [(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)] for k = 3
(10) = [(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)] for k = 4
(11) = [(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)] for k = 5
(12) = [(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)] for k = 6

(13) = [(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)] for k = 7
(14) = [(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)] for k = 8
(15) = [(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)] for k = 9
(16) = sum of (7) to (15)
(17) = square root of (16)

Insurer X
Commercial Auto Liability
Data as of December 31, 1997
Mack's Approach

Paid Loss Data

	Evaluation Period k									
	1	2	3	4	5	6	7	8	9	10
1988	27,683	72,332	102,742	122,927	133,314	138,612	141,647	143,350	144,756	145,282
1989	36,190	84,658	120,453	144,360	164,614	168,391	173,056	176,348	179,147	**179,798**
1990	21,457	65,935	97,648	122,587	131,843	140,522	147,378	151,891	**153,889**	**154,448**
1991	17,892	50,911	84,750	88,043	97,681	106,467	111,829	**114,130**	**115,631**	**116,051**
1992	24,154	46,187	66,032	84,314	102,916	108,757	**112,667**	**114,985**	**116,498**	**116,921**
1993	24,007	62,224	100,473	124,035	135,502	**142,463**	**147,585**	**150,621**	**152,602**	**153,157**
1994	30,797	57,273	87,280	108,001	**120,529**	**126,720**	**131,276**	**133,977**	**135,739**	**136,233**
1995	46,368	80,105	101,862	**122,700**	**136,933**	**143,967**	**149,143**	**152,212**	**154,214**	**154,774**
1996	42,465	75,558	**110,691**	**133,335**	**148,801**	**156,445**	**162,070**	**165,405**	**167,580**	**168,189**
1997	35,251	**77,416**	**113,413**	**136,614**	**152,461**	**160,292**	**166,055**	**169,472**	**171,701**	**172,325**

Note: Numbers in bold font are projections based on the All Yr Wtd Ave Incremental LDF;
numbers in regular font are actual historical data

Historical Paid Incremental Loss Development Factors (LDFs)

	Evaluation Period k								
	1:2	2:3	3:4	4:5	5:6	6:7	7:8	8:9	9:10
1988	2.613	1.420	1.196	1.084	1.040	1.022	1.012	1.010	1.004
1989	2.339	1.423	1.198	1.140	1.023	1.028	1.019	1.016	
1990	3.073	1.481	1.255	1.076	1.066	1.049	1.031		
1991	2.845	1.665	1.039	1.109	1.090	1.050			
1992	1.912	1.430	1.277	1.221	1.057				
1993	2.592	1.615	1.235	1.092					
1994	1.860	1.524	1.237						
1995	1.728	1.272							
1996	1.779								

*Squared Residuals of Historical Loss Development Factors (All Year Weighted Average Incremental LDFs used as expected LDFs)**

	Evaluation Period k								
	1	2	3	4	5	6	7	8	9
1988	4,807	144	7	122	18	27	10	2	0
1989	741	150	4	85	133	11	0	1	
1990	16,494	17	252	201	28	23	15		
1991	7,544		2,327	4	145	22			
1992	1,948	58	345	923	3				
1993	3,760	1,395	90	69					
1994	3,486	199	94						
1995	10,180	2,995							
1996	7,379								
	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9
All Year Squared-Sum Incremental LDF**	2.096	1.451	1.204	1.113	1.048	1.035	1.021	1.013	1.004
All Year Wtd Ave Incremental LDF	2.196	1.465	1.205	1.116	1.051	1.036	1.021	1.013	1.004
All Year Ave Incremental LDF	2.305	1.479	1.205	1.120	1.055	1.037	1.021	1.013	1.004
LDF to Ult									
All Year Squared-Sum Cumulative LDF**	4.587	2.189	1.509	1.253	1.126	1.074	1.038	1.017	1.004
All Year Wtd Ave Cumulative LDF	4.889	2.226	1.519	1.261	1.130	1.075	1.038	1.017	1.004
All Year Ave Cumulative LDF	5.225	2.267	1.533	1.272	1.135	1.076	1.037	1.017	1.004
k	1	2	3	4	5	6	7	8	9
α_{k^2}***	7,042.27	708.25	520.02	280.76	81.74	28.04	12.83	2.91	0.66

Notes:
* Squared Residuals = Paid Losses_k*(Paid Losses_{k+1}/Paid Losses_k - All Year Wtd Ave Incremental LDF_k)^2
** Squared-Sum Incremental LDF = Σ(Paid Losses_k*Paid Losses_{k+1})/Σ(Paid Losses_k^2)
*** α_{k^2} = 1/(9-k)*(Sum of Squared Residuals for all years for k)

Insurer X
Commercial Auto Liability
Data as of December 31, 1997
Mack's Approach

Accident Year	*(1)* Paid Losses To Date	*(2)* Incurred Losses To Date	*(3)* Loss Development Factor	*(4)* Ultimate Losses	*(5)* Total Reserves	*(6)* Standard Error of Reserves	*(7)* Ratio of Standard Error to Expected Reserves	*(8)* σ_i^2	*(9)* μ_i	*(10)* Reserves @ 75th percentile	*(11)* % Larger Than Expected Reserves	*(12)* Ultimates @ 75th percentile
1988	145,282	146,228	1.000	146,228	946	0						145,282
1989	179,147	181,604	1.001	181,747	2,600	14	0.6%	0.000	7.863	2,607	0.3%	181,754
1990	151,891	158,529	1.011	160,236	8,345	185	2.2%	0.000	9.029	8,429	1.0%	160,320
1991	111,829	116,892	1.029	120,289	8,460	2,460	29.1%	0.081	9.002	9,272	9.6%	121,101
1992	108,757	114,776	1.045	119,984	11,227	3,071	27.4%	0.072	9.290	12,268	9.3%	121,025
1993	135,502	147,836	1.081	159,792	24,290	5,774	23.8%	0.055	10.070	26,350	8.5%	161,852
1994	108,001	126,101	1.153	145,365	37,364	6,972	18.7%	0.034	10.511	40,025	7.1%	148,026
1995	101,862	122,446	1.208	147,975	46,113	11,986	26.0%	0.065	10.706	50,257	9.0%	152,119
1996	75,558	115,141	1.340	154,345	78,787	18,992	24.1%	0.056	11.246	85,532	8.6%	161,090
1997	35,251	76,019	1.934	147,032	111,781	29,573	26.5%	0.068	11.590	121,938	9.1%	157,189
TOTAL	1,153,080	1,305,572		1,482,992	329,912	42,278	12.8%	0.016	12.698	356,677	8.1%	1,364,476

(1) From Exhibit 2, Page 3
(2) From Exhibit 3, Page 3
(3) From Exhibit 3, Page 3
(4) = (2) * (3)
(5) = (3) - (1)
(6) Annual Standard Error from Exhibit 3, Page 3
Total Standard Error from Exhibit 3, Page 2
(7) = (6) / (5)
(8) = ln[(1 + (7)²)]
(9) = ln[(5)] - (8)/2
(10) total reserves = (5)*exp(.675*sqrt[(8)]-(8)/2);
annual reserves = (5)*exp(0.464*sqrt[(8)]-(8)/2)
0.464 is the factor needed so that the sum of annual reserves = total reserves
(11) = (10) / (5)
(12) = (10) + (1)

Insurer X
Commercial Auto Liability
Data as of December 31, 1997
Mack's Approach

Calculation of Variance for Total Reserves

								k						
				1	2	3	4	5	6	7	8	9		
			(1) α_{k-1}	3,515.46	1,104.26	505.54	109.83	101.59	23.33	40.51	0.14	0.00		
			(2) All-Yr Wtd Ave Incremental LDF	1.443	1.109	1.048	1.067	1.034	1.016	1.018	1.010	1.001		
			(3) Sum of Incurred Losses as of Time k for 1988 to 1997-k	690,723	881,480	855,314	770,560	673,975	582,089	474,425	324,482	146,113		
Accident Year	*(4)* Ultimate Losses	*(5)* Annual Standard Error	*(6)* Annual Variance	*(7)* Total Reserves Variance $Component_{k=1}$	*(8)* Total Reserves Variance $Component_{k=2}$	*(9)* Total Reserves Variance $Component_{k=3}$	*(10)* Total Reserves Variance $Component_{k=4}$	*(11)* Total Reserves Variance $Component_{k=5}$	*(12)* Total Reserves Variance $Component_{k=6}$	*(13)* Total Reserves Variance $Component_{k=7}$	*(14)* Total Reserves Variance $Component_{k=8}$	*(15)* Total Reserves Variance $Component_{k=9}$	*(16)* Total Reserves Total Variance	*(17)* Total Reserves Standard Deviation
1988	146,228	0	0											
1989	181,747	14	208									1,465	1,673	
1990	160,236	185	34,166								138,534	1,112	173,812	
1991	120,289	2,460	6,053,468							17,331,581	91,422	734	23,477,205	
1992	119,984	3,071	9,430,197						7,032,817	14,915,705	78,678	632	31,458,029	
1993	159,792	5,774	33,344,779					26,796,676	7,382,596	15,657,542	82,591	663	83,264,848	
1994	145,365	6,972	48,612,461				16,371,131	18,418,795	5,074,455	10,762,270	56,770	456	99,296,338	
1995	147,975	11,986	143,669,579			47,967,977	11,177,149	12,575,162	3,464,510	7,347,781	38,759	311	226,241,227	
1996	154,345	18,992	360,710,613		46,212,067	24,409,441	5,687,710	6,399,116	1,762,983	3,739,062	19,723	158	448,940,875	
1997	147,032	29,573	874,589,708	0	0	0	0	0	0	0	0	0	874,589,708	
	1,482,992												1,787,443,715	42,278

(1) From Exhibit 3, Page 4
(2) From Exhibit 3, Page 4
(3) From Exhibit 3, Page 4
(4) From Exhibit 3, Page 3
(5) From Exhibit 3, Page 3
(6) From Exhibit 3, Page 3

(7) No Component for k = 1
(8) = 2 * [(4)]*[sum of (4) for all acc. Yrs. After Current Acc. Yr]*[(1) / (2)) * [1/(3)] for k = 2
(9) = 2 * [(4)]*[sum of (4) for all acc. Yrs. After Current Acc. Yr]*[(1) / (2)) * [1/(3)] for k = 3
(10) = 2 * [(4)]*[sum of (4) for all acc. Yrs. After Current Acc. Yr]*[(1) / (2)) * [1/(3)] for k = 4
(11) = 2 * [(4)]*[sum of (4) for all acc. Yrs. After Current Acc. Yr]*[(1) / (2)) * [1/(3)] for k = 5
(12) = 2 * [(4)]*[sum of (4) for all acc. Yrs. After Current Acc. Yr]*[(1) / (2)) * [1/(3)] for k = 6

(13) = 2 * [(4)]*[sum of (4) for all acc. Yrs. After Current Acc. Yr]*[(1) / (2)) * [1/(3)] for k = 7
(14) = 2 * [(4)]*[sum of (4) for all acc. Yrs. After Current Acc. Yr]*[(1) / (2)) * [1/(3)] for k = 8
(15) = 2 * [(4)]*[sum of (4) for all acc. Yrs. After Current Acc. Yr]*[(1) / (2)) * [1/(3)] for k = 9
(16) = sum of (6) to (15)
(17) = square root of (the sum of (16) for all accident years)

Insurer X
Commercial Auto Liability
Data as of December 31, 1997
Mack's Approach

Calculation of Standard Deviation by Accident Year

	k: 1	2	3	4	5	6	7	8	9
(1) α_{k-1}	3,515.46	1,104.26	505.54	109.83	101.59	23.33	40.51	0.14	0.00
(2) All-Yr Wtd Ave Incremental LDF_k	1.443	1.109	1.048	1.067	1.034	1.016	1.018	1.010	1.001
(3) Sum of Incurred Losses as of Time k for 1988 to 1997-k	690,723	881,480	855,314	770,560	673,975	582,089	474,425	324,482	146,113

Accident Year	*(4)* Incurred Losses	*(5)* LDF	*(6)* Ultimate Losses	*(7)* Annual Reserves Variance $\text{Component}_{k=1}$	*(8)* Annual Reserves Variance $\text{Component}_{k=2}$	*(9)* Annual Reserves Variance $\text{Component}_{k=3}$	*(10)* Annual Reserves Variance $\text{Component}_{k=4}$	*(11)* Annual Reserves Variance $\text{Component}_{k=5}$	*(12)* Annual Reserves Variance $\text{Component}_{k=6}$	*(13)* Annual Reserves Variance $\text{Component}_{k=7}$	*(14)* Annual Reserves Variance $\text{Component}_{k=8}$	*(15)* Annual Reserves Variance $\text{Component}_{k=9}$	*(16)* Annual Reserves Total Variance	*(17)* Annual Reserves Standard Deviation
1988	146,228	1.000	146,228											
1989	181,604	1.001	181,747									208	208	14
1990	158,529	1.011	160,236								33,994	171	34,166	185
1991	116,892	1.029	120,289							6,029,925	23,431	112	6,053,468	2,460
1992	114,776	1.045	119,984						3,395,111	6,011,618	23,356	111	9,430,197	3,071
1993	147,836	1.081	159,792					20,011,881	4,768,641	8,530,216	33,869	171	33,344,779	5,774
1994	126,101	1.153	145,365				18,829,163	17,909,358	4,256,626	7,587,266	29,900	148	48,612,461	6,972
1995	122,446	1.208	147,975			94,034,416	19,215,649	18,285,382	4,348,058	7,755,317	30,605	152	143,669,579	11,986
1996	115,141	1.340	154,345		209,945,788	98,611,399	20,166,100	19,211,196	4,573,435	8,170,184	32,350	162	360,710,613	18,992
1997	76,019	1.934	147,032	533,061,003	198,903,065	93,360,585	19,075,816	18,149,304	4,314,963	7,694,471	30,349	150	874,589,708	29,573
	1,305,572		1,482,992											

(1) From Exhibit 3, Page 4
(2) From Exhibit 3, Page 4
(3) From Exhibit 3, Page 4
(4) From Exhibit 3, Page 4
(5) From Exhibit 3, Page 4
(6) = (4) * (5)

(7) = $[(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)]$ for k = 1
(8) = $[(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)]$ for k = 2
(9) = $[(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)]$ for k = 3
(10) = $[(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)]$ for k = 4
(11) = $[(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)]$ for k = 5
(12) = $[(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)]$ for k = 6

(13) = $[(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)]$ for k = 7
(14) = $[(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)]$ for k = 8
(15) = $[(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)]$ for k = 9
(16) = sum of (7) to (15)
(17) = square root of (16)

Insurer X
Commercial Auto Liability
Data as of December 31, 1997
Mack's Approach

Incurred Loss Data

	Evaluation Period k									
	1	2	3	4	5	6	7	8	9	10
1988	66,042	109,982	131,364	130,763	134,169	140,803	143,732	144,777	146,113	146,228
1989	99,902	150,119	149,956	154,698	169,729	177,697	177,812	179,705	181,604	**181,747**
1990	83,606	108,107	120,211	140,978	150,807	150,170	152,881	158,529	**160,109**	**160,236**
1991	56,825	91,368	102,206	101,617	106,564	113,419	116,892	**119,007**	**120,194**	**120,289**
1992	86,924	89,280	102,918	102,216	112,706	114,776	**116,596**	**118,706**	**119,889**	**119,984**
1993	94,293	130,363	131,972	140,288	147,836	**152,857**	**155,280**	**158,090**	**159,667**	**159,792**
1994	50,428	89,009	116,687	126,101	**134,488**	**139,056**	**141,260**	**143,817**	**145,251**	**145,365**
1995	74,850	113,252	122,446	**128,365**	**136,903**	**141,553**	**143,797**	**146,399**	**147,859**	**147,975**
1996	77,853	115,141	**127,717**	**133,891**	**142,797**	**147,646**	**149,987**	**152,701**	**154,224**	**154,345**
1997	76,019	**109,685**	**121,666**	**127,547**	**136,030**	**140,650**	**142,880**	**145,466**	**146,916**	**147,032**

Note: Numbers in bold font are projections based on the All Yr Wtd Ave Incremental LDF;
numbers in regular font are actual historical data

Historical Incurred Incremental Loss Development Factors (LDFs)

	Evaluation Period k								
	1:2	2:3	3:4	4:5	5:6	6:7	7:8	8:9	9:10
1988	1.665	1.194	0.995	1.026	1.049	1.021	1.007	1.009	1.001
1989	1.503	0.999	1.032	1.097	1.047	1.001	1.011	1.011	
1990	1.293	1.112	1.173	1.070	0.996	1.018	1.037		
1991	1.608	1.119	0.994	1.049	1.064	1.031			
1992	1.027	1.153	0.993	1.103	1.018				
1993	1.383	1.012	1.063	1.054					
1994	1.765	1.311	1.081						
1995	1.513	1.081							
1996	1.479								

*Squared Residuals of Historical Loss Development Factors (All Year Weighted Average Incremental LDFs used as expected LDFs)**

	Evaluation Period k								
	1	2	3	4	5	6	7	8	9
1988	3,269	798	368	214	32	3	17	0	0
1989	357	1,827	42	145	29	41	10	0	
1990	1,876	1	1,861	1	220	1	54		
1991	1,547		299	32	98	25			
1992	15,026	169	313	133	27				
1993	343	1,224	28	23					
1994	5,235	3,622	122						
1995	369	89							
1996	101								

	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9
All Year Squared-Sum Incremental LDF**	1.421	1.095	1.049	1.067	1.033	1.014	1.018	1.010	1.001
All Year Wtd Ave Incremental LDF	1.443	1.109	1.048	1.067	1.034	1.016	1.018	1.010	1.001
All Year Ave Incremental LDF	1.471	1.123	1.047	1.066	1.035	1.018	1.018	1.010	1.001
LDF to Ult									
All Year Squared-Sum Cumulative LDF**	1.878	1.321	1.207	1.150	1.078	1.044	1.029	1.011	1.001
All Year Wtd Ave Cumulative LDF	1.934	1.340	1.208	1.153	1.081	1.045	1.029	1.011	1.001
All Year Ave Cumulative LDF	1.998	1.359	1.210	1.156	1.084	1.047	1.029	1.011	1.001
k	1	2	3	4	5	6	7	8	9
α_{k^2}***	3,515.46	1,104.26	505.54	109.83	101.59	23.33	40.51	0.14	0.00

Notes:
* Squared Residuals = Incurred Losses_k*(Incurred Losses_{k+1}/Incurred Losses_k - All Year Wtd Ave Incremental LDF_k)^2
** Squared-Sum Incremental LDF = Σ(Incurred Losses_k*Incurred Losses_{k+1})/Σ(Incurred Losses_k^2)
*** α_{k^2} = 1/(9-k)*(Sum of Squared Residuals for all years for k)

Insurer X
Commercial Auto Liability
Data as of December 31, 1997
Mack's Approach

Accident Year	(1) Paid Losses To Date	(2) Actual Ultimate Losses To Date	(3) Loss Development Factor	(4) Projected Ultimate Losses	(5) Total Reserves	(6) Standard Error of Reserves	(7) Ratio of Standard Error to Expected Reserves	(8) σ_i^2	(9) μ_i	(10) Reserves @ 75^{th} percentile	(11) % Larger Than Expected Reserves	(12) Ultimates @ 75th percentile
1988	145,282	146,782	1.000	146,782	1,500	0						145,282
1989	179,147	182,732	0.995	181,796	2,649	1,680	63.4%	0.338	7.713	2,930	10.6%	182,077
1990	151,891	161,836	1.001	162,079	10,188	2,045	20.1%	0.040	9.209	10,954	7.5%	162,845
1991	111,829	118,718	1.006	119,474	7,645	1,936	25.3%	0.062	8.911	8,320	8.8%	120,149
1992	108,757	117,746	1.017	119,725	10,968	3,809	34.7%	0.114	9.246	12,119	10.5%	120,876
1993	135,502	151,101	1.044	157,715	22,213	8,191	36.9%	0.127	9.945	24,600	10.7%	160,102
1994	108,001	133,297	1.073	142,996	34,995	8,294	23.7%	0.055	10.436	37,956	8.5%	145,957
1995	101,862	133,436	1.074	143,273	41,411	10,567	25.5%	0.063	10.600	45,089	8.9%	146,951
1996	75,558	139,987	1.100	154,054	78,496	14,598	18.6%	0.034	11.254	84,073	7.1%	159,631
1997	35,251	128,763	1.131	145,670	110,419	17,616	16.0%	0.025	11.599	117,372	6.3%	152,623
TOTAL	1,153,080	1,414,398		1,473,562	320,482	34,655	10.8%	0.012	12.672	342,679	6.9%	1,351,212

(1) From Exhibit 2, Page 3
(2) From Exhibit 4, Page 3
(3) From Exhibit 4, Page 3
(4) = (2) * (3)
(5) = (3) - (1)
(6) Annual Standard Error from Exhibit 4, Page 3
Total Standard Error from Exhibit 4, Page 2
(7) = (6) / (5)

(8) = $\ln[(1 + (7)^2)]$
(9) = $\ln[(5)] - (8)/2$
(10) total reserves = (5)*exp(.675*sqrt[(8)]-(8)/2);
annual reserves = (5)*exp(0.464*sqrt[(8)]-(8)/2)
0.464 is the factor needed so that the sum of annual reserves = total reserves
(11) = (10) / (5)
(12) = (10) + (1)

Insurer X
Commercial Auto Liability
Data as of December 31, 1997
Mack's Approach

Calculation of Variance for Total Reserves

	k = 1	2	3	4	5	6	7	8	9
(1) α_{k-1}	637.94	501.83	244.29	53.94	246.00	75.36	6.90	7.60	6.90
(2) All-Yr Wtd Ave Incremental LDF	1.028	1.025	1.001	1.028	1.027	1.010	1.005	1.007	0.995
(3) Sum of Incurred Losses as of Time k for 1988 to 1997-k	1,173,771	1,066,649	959,810	827,370	699,248	600,046	487,553	328,086	147,538

Accident Year	*(4)* Projected Ultimate Losses	*(5)* Annual Standard Error	*(6)* Annual Variance	*(7)* Total Reserves Variance Component$_{k=1}$	*(8)* Total Reserves Variance Component$_{k=2}$	*(9)* Total Reserves Variance Component$_{k=3}$	*(10)* Total Reserves Variance Component$_{k=4}$	*(11)* Total Reserves Variance Component$_{k=5}$	*(12)* Total Reserves Variance Component$_{k=6}$	*(13)* Total Reserves Variance Component$_{k=7}$	*(14)* Total Reserves Variance Component$_{k=8}$	*(15)* Total Reserves Variance Component$_{k=9}$	*(16)* Total Reserves Total Variance	*(17)* Total Reserves Standard Deviation
1988	146,782	0	0											
1989	181,796	1,680	2,822,572									19,671,457	22,494,029	
1990	162,079	2,045	4,182,976								7,281,963	15,055,350	26,520,288	
1991	119,474	1,936	3,748,266							2,891,768	4,715,322	9,748,858	21,104,213	
1992	119,725	3,809	14,508,625						21,907,985	2,496,026	4,070,025	8,414,717	51,397,379	
1993	157,715	8,191	67,088,292					61,710,650	22,739,508	2,590,764	4,224,504	8,734,099	167,087,817	
1994	142,996	8,294	68,784,378				7,819,952	42,297,941	15,586,197	1,775,771	2,895,575	5,986,558	145,146,373	
1995	143,273	10,567	111,669,657			21,820,187	5,301,097	28,673,510	10,565,786	1,203,784	1,962,892	4,058,251	185,255,162	
1996	154,054	14,598	213,113,650		20,100,881	11,402,962	2,770,288	14,984,425	5,521,550	629,083	1,025,783	2,120,792	271,669,414	
1997	145,670	17,616	310,317,132	0	0	0	0	0	0	0	0	0	310,317,132	
	1,473,562												1,200,991,807	34,655

(1) From Exhibit 4, Page 4
(2) From Exhibit 4, Page 4
(3) From Exhibit 4, Page 4
(4) From Exhibit 4, Page 3
(5) From Exhibit 4, Page 3
(6) From Exhibit 4, Page 3

(7) No Component for k = 1
(8) = 2 * [(4)]*[sum of (4) for all acc. Yrs After Current Acc. Yr]*[(1) / (2)^2 * [1/(3)] for k = 2
(9) = 2 * [(4)]*[sum of (4) for all acc. Yrs. After Current Acc. Yr]*[(1) / (2)^2 * [1/(3)] for k = 3
(10) = 2 * [(4)]*[sum of (4) for all acc. Yrs. After Current Acc. Yr]*[(1) / (2)^2 * [1/(3)] for k = 4
(11) = 2 * [(4)]*[sum of (4) for all acc. Yrs. After Current Acc. Yr]*[(1) / (2)^2 * [1/(3)] for k = 5
(12) = 2 * [(4)]*[sum of (4) for all acc. Yrs. After Current Acc. Yr]*[(1) / (2)^2 * [1/(3)] for k = 6

(13) = 2 * [(4)]*[sum of (4) for all acc. Yrs. After Current Acc. Yr]*[(1) / (2)^2 * [1/(3)] for k = 7
(14) = 2 * [(4)]*[sum of (4) for all acc. Yrs. After Current Acc. Yr]*[(1) / (2)^2 * [1/(3)] for k = 8
(15) = 2 * [(4)]*[sum of (4) for all acc. Yrs. After Current Acc. Yr]*[(1) / (2)^2 * [1/(3)] for k = 9
(16) = sum of (6) to (15)
(17) = square root of (the sum of (16) for all accident years)

Insurer X
Commercial Auto Liability
Data as of December 31, 1997
Mack's Approach

Calculation of Standard Deviation by Accident Year

	k								
	1	2	3	4	5	6	7	8	9
(1) α_{k+1}	637.94	501.83	244.29	53.94	246.00	75.36	6.90	7.60	6.90
(2) All-Yr Wtd Ave Incremental LDF_k	1.028	1.025	1.001	1.028	1.027	1.010	1.005	1.007	0.995
(3) Sum of Incurred Losses as of Time k for 1988 to 1997-k	1,173,771	1,066,649	959,810	827,370	699,248	600,046	487,553	328,086	147,538

Accident Year	*(4)* Actual Ultimate Losses	*(5)* LDF	*(6)* Projected Ultimate Losses	*(7)* Annual Reserves Variance Component$_{k+1}$	*(8)* Annual Reserves Variance Component$_{k+2}$	*(9)* Annual Reserves Variance Component$_{k+3}$	*(10)* Annual Reserves Variance Component$_{k+4}$	*(11)* Annual Reserves Variance Component$_{k+5}$	*(12)* Annual Reserves Variance Component$_{k+6}$	*(13)* Annual Reserves Variance Component$_{k+7}$	*(14)* Annual Reserves Variance Component$_{k+8}$	*(15)* Annual Reserves Variance Component$_{k+9}$	*(16)* Annual Reserves Total Variance	*(17)* Annual Reserves Standard Deviation
1988	146,782	1.000	146,782											
1989	182,732	0.995	181,796									2,822,572	2,822,572	1,680
1990	161,836	1.001	162,079								1,817,539	2,365,437	4,182,976	2,045
1991	118,718	1.006	119,474							1,021,709	1,223,435	1,503,122	3,748,266	1,936
1992	117,746	1.017	119,725						10,749,952	1,024,278	1,226,693	1,507,702	14,508,625	3,809
1993	151,101	1.044	157,715					46,734,771	14,898,142	1,433,273	1,752,875	2,269,230	67,088,292	8,191
1994	133,297	1.073	142,996				9,095,961	41,670,434	13,248,801	1,270,009	1,541,179	1,957,993	68,784,378	8,294
1995	133,436	1.074	143,273			42,728,157	9,116,025	41,764,379	13,279,340	1,273,024	1,545,071	1,963,660	111,669,657	10,567
1996	139,987	1.100	154,054		91,617,280	46,365,619	9,904,564	45,461,854	14,483,003	1,392,105	1,699,305	2,189,919	213,113,650	14,598
1997	128,763	1.131	145,670	110,395,482	86,084,081	43,531,901	9,290,129	42,579,864	13,544,522	1,299,222	1,578,908	2,013,023	310,317,132	17,616
	1,414,398		1,473,562											

(1) From Exhibit 4, Page 4
(2) From Exhibit 4, Page 4
(3) From Exhibit 4, Page 4
(4) From Exhibit 4, Page 4
(5) From Exhibit 4, Page 4
(6) = (4) * (5)

(7) = $[(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)]$ for k = 1
(8) = $[(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)]$ for k = 2
(9) = $[(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)]$ for k = 3
(10) = $[(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)]$ for k = 4
(11) = $[(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)]$ for k = 5
(12) = $[(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)]$ for k = 6

(13) = $[(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)]$ for k = 7
(14) = $[(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)]$ for k = 8
(15) = $[(6)^2]*[(1)/(2)^2]*[1/(4)+1/(3)]$ for k = 9
(16) = sum of (7) to (15)
(17) = square root of (16)

** Squared-Sum Incremental LDF = $\Sigma(\text{Ultimate Losses}_k * \text{Ultimate Losses}_{k+1}) / \Sigma(\text{Ultimate Losses}_k^2)$

Insurer X
Commercial Auto Liability
Data as of December 31, 1997
Mack's Approach

Actual Ultimate Loss Data

	Evaluation Period k									
	1	2	3	4	5	6	7	8	9	10
1988	130,227	132,540	133,667	132,518	134,515	141,630	146,095	145,783	147,538	146,782
1989	151,110	158,917	169,141	165,735	175,061	182,584	180,334	182,303	182,732	**181,796**
1990	149,231	148,077	152,365	156,660	162,867	156,766	161,124	161,836	**162,913**	**162,079**
1991	116,227	119,345	113,790	111,500	111,686	119,066	118,718	119,295	**120,089**	**119,474**
1992	132,417	121,936	121,802	113,047	115,119	117,746	**118,968**	**119,546**	**120,341**	**119,725**
1993	152,845	152,205	143,114	147,910	151,101	**155,108**	**156,717**	**157,479**	**158,527**	**157,715**
1994	108,590	109,329	125,931	133,297	**136,999**	**140,632**	**142,091**	**142,782**	**143,732**	**142,996**
1995	114,326	124,300	133,436	**133,555**	**137,264**	**140,905**	**142,366**	**143,058**	**144,011**	**143,273**
1996	118,798	139,987	**143,478**	**143,606**	**147,594**	**151,508**	**153,080**	**153,824**	**154,848**	**154,054**
1997	128,763	**132,368**	**135,669**	**135,790**	**139,561**	**143,263**	**144,749**	**145,452**	**146,420**	**145,670**

Note: Numbers in bold font are projections based on the All Yr Wtd Ave Incremental LDF;
numbers in regular font are actual historical data

Historical Incurred Incremental Loss Development Factors (LDFs)

	Evaluation Period k								
	1:2	2:3	3:4	4:5	5:6	6:7	7:8	8:9	9:10
1988	1.018	1.009	0.991	1.015	1.053	1.032	0.998	1.012	0.995
1989	1.052	1.064	0.980	1.056	1.043	0.988	1.011	1.002	
1990	0.992	1.029	1.028	1.040	0.963	1.028	1.004		
1991	1.027	0.953	0.980	1.002	1.066	0.997			
1992	0.921	0.999	0.928	1.018	1.023				
1993	0.996	0.940	1.034	1.022					
1994	1.007	1.152	1.058						
1995	1.087	1.073							
1996	1.178								

*Squared Residuals of Historical Loss Development Factors (All Year Weighted Average Incremental LDFs used as expected LDFs)**

	Evaluation Period k								
	1	2	3	4	5	6	7	8	9
1988	14	36	12	21	94	63	7	4	0
1989	85	247	75	135	47	94	7	3	
1990	191	2	114	22	667	48	0		
1991	0		50	76	175	21			
1992	1,520	83	645	10	2				
1993	158	1,091	152	6					
1994	49	1,761	418						
1995	401	293							
1996	2,686								

	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9
All Year Squared-Sum Incremental LDF**	1.025	1.023	1.002	1.030	1.024	1.009	1.005	1.006	0.995
All Year Wtd Ave Incremental LDF	1.028	1.025	1.001	1.028	1.027	1.010	1.005	1.007	0.995
All Year Ave Incremental LDF	1.031	1.027	1.000	1.025	1.029	1.011	1.004	1.007	0.995
LDF to Ult									
All Year Squared-Sum Cumulative LDF**	1.125	1.097	1.073	1.071	1.040	1.016	1.006	1.001	0.995
All Year Wtd Ave Cumulative LDF	1.131	1.100	1.074	1.073	1.044	1.017	1.006	1.001	0.995
All Year Ave Cumulative LDF	1.138	1.104	1.074	1.074	1.048	1.018	1.006	1.002	0.995
k	1	2	3	4	5	6	7	8	9
α_{k^2}***	637.94	501.83	244.29	53.94	246.00	75.36	6.90	7.60	6.90
minimum LDF from historical LDF triangle	0.921	0.940	0.928	1.002	0.963	0.988	0.998	1.002	0.995
maximum LDF from historical LDF triangle	1.178	1.152	1.058	1.056	1.066	1.032	1.011	1.012	0.995
minimum age-ult LDF	0.761	0.827	0.879	0.948	0.946	0.983	0.995	0.997	0.995
maximum age-ult LDF	1.699	1.441	1.251	1.182	1.119	1.050	1.018	1.007	0.995

Notes:

* Squared Residuals = Ultimate Losses$_k$*(Ultimate Losses$_{k+1}$/Ultimate Losses$_k$ - All Year Wtd Ave Incremental LDF$_k$)^2

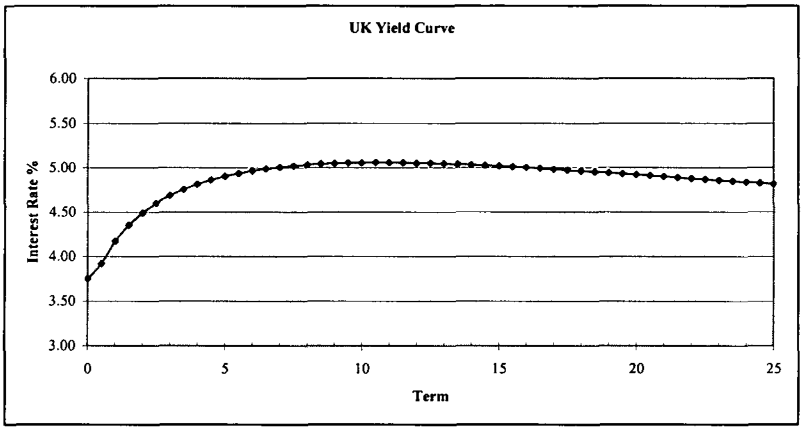
** Squared-Sum Incremental LDF = Σ(Ultimate Losses$_k$*Ultimate Losses$_{k+1}$)/Σ(Ultimate Losses$_k^2$)

*** α_{k^2} = 1/(9-k)*(Sum of Squared Residuals for all years for k)

DISCOUNTING EXAMPLE

Example of Discounting

Consider the following example of the liabilities of a US insurer writing general liability business in the UK. The UK yield curve is as illustrated below and in common with other developed economies is publicly available, for example from the Central Bank. From this the relevant discount rates needed for each term can be ascertained.



The actual calculations are relatively straightforward as illustrated below. The cash flows have been calculated by standard actuarial techniques and the market value margins would be calculated as described in the section of the paper "Adjustments for Risk and Uncertainty". We need to make the assumption that the cash flows occur on average in the middle of each year and make our chosen discount rate for these cash flows in the middle of the year as well. Each cash flow can then be discounted and the resulting total added up to come up with an expected present value of the cash flow. This is then converted to US$ as the prevailing spot rate as this is the measurement currency of the US insurer.

Year	Expected Cashflows (£000's)	Market Value Margins (£000's)	Total (£000's)	Discount Rate to Middle of Year	Discounted Cashflows (£000's)	Discounted Cashflows ($000's) (1£=$1.66)
1	319	32	351	3.92%	344	
2	3,877	388	4,265	4.35%	4,001	
3	10,548	1,055	11,603	4.60%	10,369	
4	21,688	2,169	23,856	4.60%	20,382	
5	49,935	4,993	54,928	4.76%	44,563	
6	42,895	4,290	47,185	4.86%	36,340	
7	40,612	4,061	44,673	4.94%	32,661	
8	42,481	4,248	46,729	4.99%	32,440	
9	47,848	4,785	52,633	5.02%	34,708	
10	12,207	1,221	13,428	5.04%	8,415	
11	7,324	732	8,056	5.05%	4,801	
12	3,653	365	4,018	5.06%	2,278	
13	1,536	154	1,690	5.06%	912	
14	732	73	805	5.04%	415	
15	265	27	292	5.03%	143	
	285,921	28,592	314,513		232,773	140,224

Estimating the Workers' Compensation Tail

Richard E. Sherman, FCAS, MAAA, and
Gordon F. Diss, ACAS, MAAA

ESTIMATING THE WORKERS' COMPENSATION TAIL

RICHARD E. SHERMAN AND GORDON F. DISS

Abstract

The workers' compensation tail largely consists of the medical component of permanent disability claims (MPD). Yet the nature of MPD payments is not widely understood and is counter to that presumed in common actuarial models.

This paper presents an analysis of medical payments based on 160,000 permanently disabled claimants—for accident years 1926-2002. It introduces a method for utilizing incremental payment data prior to the standard triangle to extend development factors beyond the end of the triangle.

A close-fitting model is presented that explicitly reflects the opposing effects of 1) medical cost escalation on average incremental payments, and 2) the force of mortality in closing claims. It clearly demonstrates that:

- *Paid development factors will tend to increase over many successive, "mature" years of development.*
- *Paid development factors and tails will trend upward over time—if past declines in mortality rates continue in the future.*

The paper also demonstrates that case reserves based on inflating payments until the expected year of death are significantly less than the expected value of such reserves. A method is introduced for realistically simulating the high expected value and variability of MPD reserves. It is based on a Markov chain model of annual payments on individual claims.

1. INTRODUCTION AND SUMMARY

Historically, the ability of workers' compensation (WC) insurers to reasonably estimate tail factors has been hampered by a dearth of available development experience at maturities beyond 10 to 20 years. Substantive advances in WC tail estimation are dependent on the availability of a substantial database extending to 50 or more years of development.

This paper presents the results of a thorough analysis of the extensive paid loss development database of the SAIF Corporation, Oregon's state fund. That database extends out to 65 years of development—separately for medical and indemnity, separately ***by injury type***.

Some of the key findings from this analysis include:

- Medical tail factors calculated empirically are significantly greater than those derived from extrapolation techniques.

- There is an effective, systematic way (the Mueller Incremental Tail Method) to utilize incremental payment data prior to the standard triangle to extend paid development factors beyond the end of the triangle.

- Medical cost escalation and the force of mortality are the key drivers of tail factors.

- In the early stages of the tail, medical cost escalation overpowers the force of mortality, leading to increases in incremental paid losses.

- Mortality rates combined with medical inflation fit the empirical data very well out to 40 years of development, but then tend to understate losses for the next 15 years of development. This understatement appears to be due to the added costs of caring for the elderly—who make up a rapidly increasing percentage of surviving claimants.

- Declining mortality rates have a substantial effect on medical tail factors. Mortality improvement will also cause individual paid loss development factors to trend upward slowly for any given year of development.

- The expected value of an MPD case reserve is much greater than cumulative inflated payments through the expected year of death. This is similar to the situation that occurs when reinsurance contracts are commuted—where usage of the life expectancy of the claimant produces an estimate well below the weighted average of outcomes based on a mortality table[1].

- The variability of total MPD reserves can be gauged realistically by a Markov chain simulation model that separately estimates payments for each future year of development by claimant.

- The potential for common actuarial methods to understate the MPD reserve, and consequently, the entire WC reserve, is significant. This is also true regarding common methods for estimating the degree of variability in the WC reserve as well as its expected value.

This paper is divided into ten sections:

1. Introduction and Summary
2. MPD Tails Indicated by SAIF's Loss Experience
3. Incorporating the Static Mortality Model into the Incremental Paid to Prior Open Method
4. Mortality Improvement
5. The Trended Mortality Model
6. A Comparison of Indicated Tail Factors
7. Sensitivity Considerations
8. Estimating the Expected Value of MPD Reserves
9. Estimating the Variability of the MPD Reserve with a Markov Chain Simulation
10. Concluding Remarks

The paper also includes four appendices:

A. The Mueller Incremental Tail (MIT) Method
B. Historical PLDFs for All Other WC
C. Incorporating the Static Mortality Model into the Incremental Paid to Prior Open Method
D. Incorporating the Trended Mortality Model into the Incremental Paid to Prior Open Method

WC tail estimation would not be a problem if gross WC losses in the tail adhered to behavior anticipated by common actuarial methods. That, however, is not the case for ***medical losses for permanently disabled claimants*** (MPD). This is a serious concern because MPD loss reserves make up the bulk of total WC loss reserves for all but the most recent accident years.

The deviations from ordinary development patterns for MPD losses are persistent and substantial—because of the compounding effects of anticipated rates of future medical cost escalation on services provided to claimants until death.

A severely injured worker in his or her early twenties could receive medical benefits for up to 90 years in the future. Loss development data is usually available only for significantly shorter development periods. Schedule P data contains loss development for ten individual accident years. Financial Statistical calls provide loss development for twenty accident years. Consequently actuaries frequently have to estimate loss development factors with little or no experience in the tail beyond 20 years.

SAIF's data base for MPD tracks the paid loss development of over 160,000 PD claims for accident years 1926-2002. It thus provides a credible source for actuarial analysis.

Table 1.1 displays SAIF's historical paid loss development factors (PLDFs) for unlimited MPD up through the 15^{th} year of development.

Table 1.1
Historical Age to Age Paid Loss Development Factors
Medical Losses of Permanently Disabled Claimants
(By Development Year)

	Development Year													
AY	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
4th Prior	6.645	1.516	1.169	1.061	1.042	1.024	1.012	1.028	1.016	1.011	1.009	1.011	1.008	1.010
3rd Prior	6.402	1.489	1.115	1.066	1.039	1.027	1.017	1.013	1.010	1.013	1.010	1.010	1.011	1.012
2nd Prior	5.745	1.539	1.153	1.058	1.036	1.017	1.022	1.015	1.011	1.010	1.014	1.013	1.011	1.010
1st Prior	6.867	1.509	1.113	1.064	1.045	1.035	1.012	1.027	1.020	1.011	1.013	1.016	1.010	1.009
Latest	7.460	1.572	1.149	1.111	1.041	1.031	1.030	1.018	1.020	1.016	1.012	1.014	1.018	1.011
Average	6.624	1.525	1.140	1.072	1.041	1.027	1.019	1.020	**1.015**	**1.013**	**1.012**	**1.013**	**1.012**	**1.010**

In Table 1.1, as well as throughout this paper, a PLDF for a given development year (DY) is denoted by the maturity at the end of that year. For example, the factors in the column headed by "2" are for development from 1 to 2 years of age—since this is the second year of development.

Ordinarily, it would be expected that PLDFs for subsequent development years would slowly decline below the last factor (1.010) as a continuation of the pattern of slowly decreasing factors exhibited, for example, during development years 10 through 15.

Table 1.2 displays SAIF's actual MPD PLDFs, calculated as the averages of the latest five factors. These historical factors generally increase during the 16^{th} through the 26^{th} development years.

Table 1.2
A Comparison of Historical MPD PLDFs with
Projections Based on Development Years 10 through 15

	Development Year										
	16	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**
Historical	1.011	1.013	1.011	1.011	1.012	1.012	1.014	1.012	1.015	1.015	1.016
Projections Based on Development Years 10-15											
Linear Decay	1.009	1.008	1.007	1.006	1.005	1.004	1.003	1.002	1.001	1.000	1.000
Exp. Decay	1.009	1.009	1.008	1.007	1.007	1.006	1.006	1.005	1.005	1.005	1.004
Inverse Power	1.010	1.010	1.009	1.009	1.008	1.008	1.007	1.007	1.007	1.006	1.006

Projections of these PLDFs based on three common actuarial methods[2] applied to the historical PLDFs for development years 10 through 15 are also shown in Table 1.2.

Since each of these methods assumes that the pattern of declining factors for these development years will continue in the future, the projected PLDFs fall increasingly below the actual historical factors. This pattern of divergence continues during development years 27 through 37, as shown in Table 1.3.

Table 1.3
A Comparison of Historical MPD PLDFs with Projections Based on Development Years 10 through 15

	Development Year										
	27	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**
Historical	1.020	1.023	1.027	1.026	1.022	1.018	1.015	1.017	1.018	1.029	1.033
Projections Based on Development Years 10-15											
Linear Decay	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Exp. Decay	1.004	1.004	1.003	1.003	1.003	1.003	1.003	1.002	1.002	1.002	1.002
Inverse Power	1.006	1.005	1.005	1.005	1.005	1.004	1.004	1.004	1.004	1.004	1.003

It is evident that these three common actuarial methods of projecting subsequent PLDFs will produce projections of MPD ultimate losses that are severely inadequate. Table 1.4 provides a direct comparison of the PLDF from 15 to 37 years indicated by SAIF's actual experience with those based on PLDFs extrapolated by common methods.

Table 1.4
A Comparison of SAIF's Historical Factor from 15 to 37 Years with Those Based on Extrapolated Development Factors (Based on a Fit to Historical PLDFs for DYs 10-15)

Extrapolation Method	Paid Development Factor from 15 to 37 Years
Linear Decay	1.046
Exponential Decay	1.108
Inverse Power Curve	1.145
SAIF's Historical Factor	1.471

It is natural to ask whether the phenomenon of increasing PLDFs from the 16th through the 37th development years might be due to some unusual cause peculiar to Oregon WC in general or the Oregon State Fund in particular. Medical PLDFs compiled by the California Workers Compensation Insurance Rating Bureau (WCIRB) [3] provide evidence that the phenomenon is widespread. While the PLDFs displayed in Table 1.5 for years of development 2.5 (18 to 30 months) through 15.5 (174 to 186 months) consistently decline in a manner similar to the SAIF experience, the PLDFs shown in Table 1.6 for subsequent ages reflect definite upward movement.

Table 1.5
WCIRB Historical Medical Paid Loss Development Factors for Development Years 2.5 Through 15.5

	Development Year													
	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5	15.5
Selected	1.740	1.296	1.152	1.104	1.069	1.058	1.030	1.022	1.015	1.012	1.009	1.007	1.006	1.005

Source for Tables 1.5 and 1.6: WCIRB Bulletin No. 2003-24, p. 9.

Table 1.6
WCIRB Historical Medical Paid Loss Development Factors for Development Years 16.5 Through 28.5

	Development Year												
	16.5	17.5	18.5	19.5	20.5	21.5	22.5	23.5	24.5	25.5	26.5	27.5	28.5
Selected	1.006	1.005	1.005	1.005	1.008	1.006	1.007	1.007	1.007	1.009	1.010	1.008	1.009

The WCIRB factors are for all medical losses—including those for the large number of quickly settling claims. Consequently, the California factors would naturally be lower than SAIF's, which only include medical losses for the permanently disabled.

Additional confirmation of this phenomenon appears in the medical paid loss history of the Washington State Fund (for all types of claims)—shown in Tables 1.7 and 1.8.

Table 1.7
Washington State Fund Historical Medical Paid Loss Development Factors for DYs 8 Through 21

	Development Year													
	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Avg. Last 5	1.027	1.023	1.020	1.017	1.016	1.015	1.013	1.013	1.012	1.010	1.010	1.009	1.009	1.008

Table 1.8
Washington State Fund Historical Medical Paid Loss Development Factors for DYs 22 Through 35

	Development Year													
	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Avg. Last 5	1.009	1.009	1.009	1.009	1.008	1.009	1.009	1.011	1.009	1.010	1.013	1.013	1.015	1.010

When confronted with SAIF's historical pattern of PLDFs through 37 years of development, or the WCIRB's rising PLDFs through 28.5 years, or Washington's PLDFs through 35 years, what should one select for subsequent factors? Obviously, the standard pattern of declining factors does not apply to the 16th through 37th development years, but, absent pertinent data for later development years, how should one proceed to extrapolate these factors? Clearly, later PLDFs cannot continue to increase indefinitely. At some point these factors should start decreasing, if only because all claimants will eventually die. But how can one determine when, and by how much? A common approach has been to estimate such future development based on the ratio of incurred to paid at the most mature year of development. We will see that this approach may not always result in unbiased estimates of ultimate losses. This is particularly true when individual case reserves are established by multiplying *current* annual medical costs times the life expectancy of the claimant.

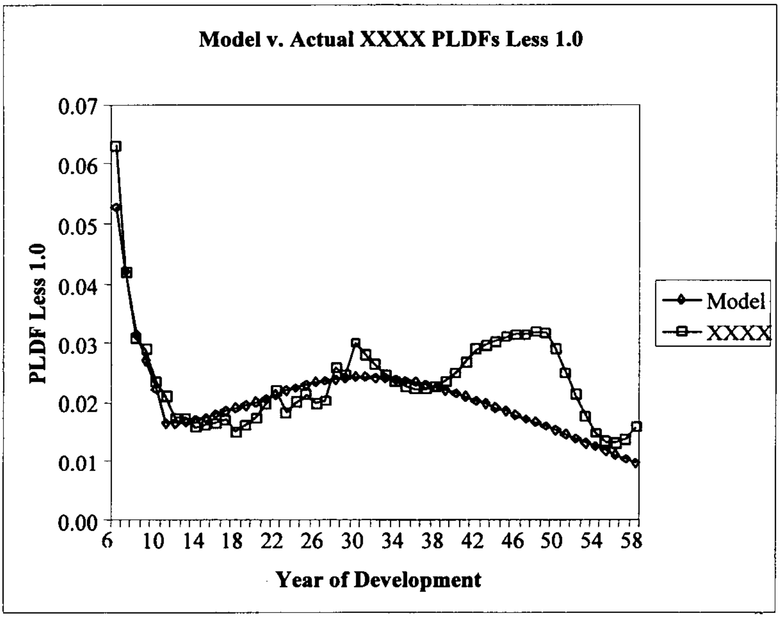
In addressing the problem of extrapolating paid development when the most mature PLDFs are increasing, some insurers or self-insureds may have data for longer periods of time than the latest 20 years. However, because of system changes or acquisitions, cumulative loss development data for old accident years is frequently lacking. In these cases incremental calendar year data for old accident years may be available because payments are still being made on the old open claims. Section 2 (and Appendix A) presents the Mueller Incremental Tail Method for making full use of the incremental data to calculate empirical tail factors. We have used this method to derive empirically based PLDFs out to 57 years of development based on SAIF's actual MPD loss experience.

This paper presents a reserving model that largely explains the seemingly anomalous behavior of increasing PLDFs at "mature" DYs. The model explicitly accounts for the separate effects of inflation and mortality on paid MPD during all years of development. This is done by directly incorporating *recent* mortality rates into an incremental paid per prior open loss reserving method. It will be referred to as the ***static mortality model*** and will be presented in Section 3.

In Figure 1.1, the PLDFs indicated by the static mortality model are compared with SAIF's empirical PLDFs. The static mortality model PLDFs are shown in the last column of Tables 3.2 and 3.5. The empirical PLDFs for the first 29 DYs are the averages of the latest 15 historical factors. For DYs 30-58, the PLDFs appear in Tables A.1, A.2 and A.3, where the Mueller Incremental Tail Method is applied.

As Figure 1.1 shows, SAIF's actual development experience for DYs 40-54 is consistently worse than the model predicts. While it may be speculated that this might simply be due to some unusual series of influences on SAIF's MPD payment experience, this same pattern is also evident in the medical PLDFs for the Washington State Fund.

Figure 1.1



We also applied the Bbbbb Method to derive a comparable analysis out to 60 years of development based on the incremental payment experience of the Zzzz State Fund for medical losses—under the assumption that all medical payments after 20 years of development should be attributable to PD claimants. The phenomenon of worse than expected development for DYs 40-54 is even more pronounced in the Zzzz experience. (This is detailed in Tables A.7 and A.8 at the end of Appendix A.)

We believe that the bulge in adverse paid development evident for DYs 40-54 is attributable to the rapidly increasing percentage of surviving claimants who are elderly. Not uncommonly, elderly PD claimants simply require more extensive and expensive medical care than younger claimants. And as PD claimants age, so do their spouses. Often a spouse reaches an age where they can no longer provide as much care as previously, and the insurer then pays for the increased cost of hiring outside assistants. Table 1.9 indicates the percentage of surviving claimants who will be 80 or older at the beginning of various years of development. It also shows the percentage of surviving claimants expected to die within the succeeding five years. It has also been observed that incremental severities tend to undergo an increase during the last years before a claimant's death that exceeds normal rates of medical cost escalation.

Table 1.9
Two Indicators of an Increasing Proportion of the Elderly Among Surviving Claimants

DY	% 80+ Years Old	% Who Will Die Within Five Years
0	0.0%	4.4%
10	0.9%	9.4%
20	10.9%	18.3%
30	36.5%	30.1%
40	51.2%	39.0%
50	64.7%	47.2%
60	100.0%	60.3%

Table 1.9 indicates that for DYs 40 and higher, over half of the surviving claimants will be 80 or more years old. Clearly, this fact could have been anticipated on an a priori basis. After all, if the average claimant were age 40 when injured, it should be expected that 40 years after the injury year, the average surviving claimant would be about 80 years old. However, the above table underscores a reality that casualty actuaries may not have heretofore given much consideration. The behavior of loss development for DYs 40+ may well differ noticeably from what would be expected on the basis of earlier DYs—because of the increasing infirmities of surviving claimants and their spouses. The percentages in Table 1.9 are based on 2000 mortality tables published by the Social Security Administration (SSA), assuming 75% of the claimants are male, and a census of SAIF's permanent total disability claimants by age-at-injury.

Paid loss development factors (PLDFs) for MPD are not monotonically decreasing. Because of this seemingly anomalous behavior, estimates of the MPD tail by common actuarial methods could be seriously understated. This potentially surprising behavior is due to the fact that medical inflation rates are expected to be greater than the rate of closure of PD claims due to death during these years of development. For the most mature years of development, the increasing force of mortality overtakes the effects of medical inflation and causes a slow reduction in incremental payments. That rate of reduction is surprisingly small.

Earlier a comparison was made in Table 1.4 of SAIF's actual PLDFs for DYs 16-37 with estimates of the same based on three different common extrapolation techniques, as applied to PLDFs through DY 15. Table 1.10 provides an analogous comparison of the tail factors at 15 years produced by those extrapolation techniques compare with that based on SAIF's historical experience.

Table 1.10
A Comparison of SAIF's Historical Factor
with Extrapolated Tail Factors At 15 Years
(Based on a Fit to Historical PLDFs for DYs 10-15)

Extrapolation Method	Indicated Tail Factor At 15 Years	Extrapolated Reserve as a %-age of the Reserve Indicated by SAIF's History
Linear Decay	1.046	3.5%
Exponential Decay	1.128	9.8%
Inverse Power Curve	1.234	17.9%
SAIF's Historical Factors	2.309	100.0%

As Table 1.10 shows, the extrapolated MPD loss reserves at 15 years of maturity are only a small fraction of the MPD reserve indicated by SAIF's development history.

As high as the paid tail factor at 15 years is (2.309), it is understated because it implicitly assumes that past mortality rates will continue indefinitely into the future. As noted in Section 4, mortality rates have been declining steadily for at least the past four decades, and the SSA reasonably expects such declines to continue throughout the next century.

A second reserving model is presented that explicitly accounts for the compounding effects of ***downward trends in future mortality rates*** and persistently high rates of future medical inflation. It will be referred to as the ***trended mortality model***, and will be described in Section 5.

The indications of the trended mortality model for MPD are significant and troubling.

- Paid tail factors at the end of any selected year of development are not constant. They should be expected to increase slowly but steadily over successive accident years (AYs).

- Incremental PLDFs for any selected year of development are not constant. They will also trend upward slowly but inexorably for successive AYs.

- The above effects on MPD will cause corresponding upward trends in paid tails and incremental PLDFs ***for all WC losses in the aggregate. This finding is contrary to the common practice of assuming no trend in paid tails or PLDFs.***

We will see in Section 5 that the common practice of using constant tail factors and constant PLDFs for each development year is not properly founded. While this practice may be defended using intuitive reasoning, that reasoning is flawed. Unless the effects of downward trends in mortality rates are incorporated into a WC reserve analysis, the resulting reserve estimates will be low when numerous AYs are involved. Fortunately, this effect may not be material for net experience at retentions (such as $250,000) that are low enough to virtually eliminate paid development at later maturities.

We believe that the most appropriate approach to estimating gross WC loss reserves is to separately evaluate MPD loss reserves by one (or more) of the methods presented in this paper. Lacking separate MPD loss experience, the static mortality and trended mortality models, and the Mueller Incremental Tail method can be applied satisfactorily to total medical loss experience for DYs 20 and higher—since virtually all medical payments are MPD payments at such maturities.

2. MPD TAILS INDICATED BY SAIF'S LOSS EXPERIENCE

A careful analysis of SAIF's historical development of paid gross MPD losses through 65 years of development indicates a large paid tail factor of 1.581 from 37 to 65 years. This indication is higher than the 1.480 tail factor from the static mortality model presented in Section 3, as one would expect given the bulge in SAIF's empirical factors shown in Figure 1.1 for DYs 40-54. The trended mortality model supports a higher paid development factor (1.600) from 37 to 65 years. If the ratio of the tail factors for the trended vs. static mortality models were applied to the empirical tail factor of 1.581, a tail factor of 1.695 would be indicated when the effect of future mortality trends is reflected.

To get a sense of how such large paid tail factors are possible at 37 years, it is useful to review patterns in SAIF's actual incremental MPD paid at mature stages of development. Table 2.1 presents incremental paid MPD for different five year development periods for each of several five year groupings of AYs.

For example, consider the incremental payments for AYs 1951-55. During development years 35 through 39 $746 thousand was paid on open MPD. Surprisingly, $830 thousand was paid out in the five years after that and $943 thousand in the five years after that.

What is remarkable about the track record of actual five year incremental paid MPD at mature stages of development shown in Table 2.1 is that in 7 out of 15 cases (highlighted by shading and bold type), payments in the subsequent five development years are greater than in the preceding five years.

Table 2.1
Five Year Incremental Paid MPD ($000's)
for Five Year Groupings of AYs—SAIF Corporation

					Years of Development				
AYs	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64
1931-35								10	8
1936-40							62	2	**5**
1941-45						259	**289**	204	
1946-50					327	**797**	302		
1951-55				746	**830**	**943**			
1956-60			1,570	1,506	**2,227**				
1961-65			2,933	1,908					
1966-70	6,818	6,583	5,121						
1971-75	10,227	**12,158**							

It was the fact that SAIF's actuaries were observing many instances of rising incremental MPD payments over successive development years that led to the recognition of the need for a more in depth review and analysis of MPD paid losses beyond 20 years of development.

Appendix A includes a compilation of SAIF's historical incremental MPD payments for AYs 1926-1965 during development years 29 through 60. This history reveals a high proportion of incidents where incremental payments increase from one development year to the next. This phenomenon is also apparent in the incremental medical payments of the Washington State Fund, as displayed in Table 2.2.

Table 2.2
Five Year Incremental Paid Medical Losses ($000's)
For Five Year Groupings of AYs—Washington State Fund

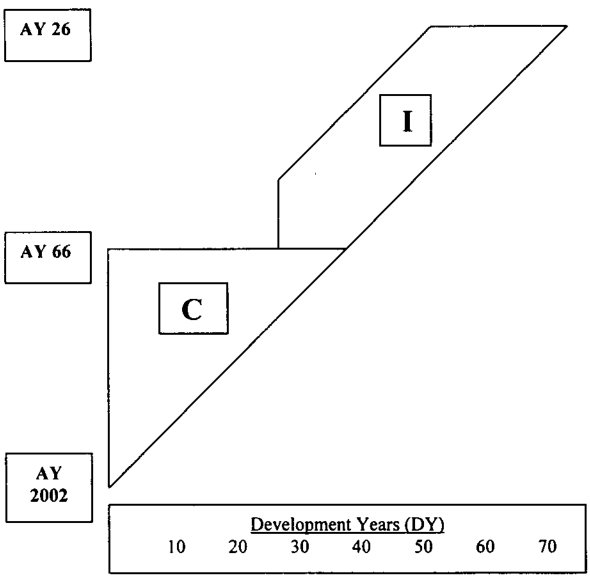
				Years of Development				
AYs	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59
1940-44			149	**294**	271	243	101	**131**
1945-49		183	**258**	**384**	273	194	**291**	
1950-54	840	750	**989**	**1,115**	**1,757**	1,400		
1955-59	1,055	**2,244**	2,153	1,879	**2,170**			
1960-64	1,589	**1,765**	1,581	1,186				
1965-69	5,380	4,741	**5,318**					
1970-74	7,014	**8,383**						

As in Table 2.1, instances where total incremental payments for five calendar years exceeded those for the preceding five calendar years, are shown in bold type in shaded cells. This occurs 13 out of 25 possible times. It is also interesting to scan across each row and to note that incremental payments for the least mature block of five years of development are often lower than during a number of the subsequent blocks of five years of development.

The Mueller Incremental Tail Method

Figure 2.1 provides a graphic summary of the portions of the incremental MPD payments experience of the SAIF Corporation that are available. A complete triangle of MPD payments exists for AYs 1966-2002. This region is the triangle labeled "C" to designate that cumulative paid losses are available for all of these AYs. In addition, since calendar year 1985, incremental MPD payments have been captured for AYs 1926-1965 for DYs 29 and higher. This region is the diagonally shaped area labeled "I" to designate that only incremental payments are available.

Figure 2.1
Configuration of SAIF's MPD Paid Loss Data



Since paid MPD for AYs 1926-1965 has only been available for calendar years since 1985, it was necessary to construct an actuarial method of estimating the tail factor based on decay ratios of incremental payments. This method is called the Mueller Incremental Tail (MIT) method, in recognition of the work done by Conrad Mueller, ACAS, in developing this method. We will use SAIF Corporation experience as an example. This section of the paper describes the method and provides key results.

Detailed calculations are included in Appendix A.

The MIT method was used to calculate empirical 37 to 65 tail factors using the incremental data on old accident years. We describe the method in three stages:

1. Incremental age-to-age factors
2. Anchored decay factors
3. Tail factors

1. **Incremental age-to-age factors.** The first step is to calculate incremental age to age factors. With the SAIF data, we are able to calculate incremental paid at age (n+1) to incremental paid at age (n) for n ranging from 29 to 57 years, using twenty-year weighted averages. A factor for 57 to 65 was calculated using the sum of the payments in years 57 to 65 relative to payments made in year 57.

   Because of the sparseness of claims of this age, the empirical development factors needed to be smoothed before they could be used. The smoothing was done using five year centered moving averages.

2. **Anchored decay factors.** After calculating incremental age to age factors, we then anchor them to a base year. We illustrate this using development year 37 as our anchor year. The anchored decay factors represent incremental payments made in year n relative to payments made in the anchor year. Table 2.3 shows the anchored decay factors for payments made in accident years of age 40, 45, 50 and 55 relative to payments made in an accident years of age 37 (our anchor year).

Table 2.3
Indicated Decay Factors Relative to Anchor Year 37 Incremental Payments

Year of Development	Decay Factors
55	.962
50	1.880
45	1.724
40	1.211
Anchor Year 37	1.000

For example – payments made in DY 50 are 88.0% greater than the payments made in DY 37. The main reason that incremental payments rise over time is because the force of medical cost escalation exceeds the force of mortality—until most of the claimants are fairly advanced in age, when the force of mortality becomes greater.

By summing the decay factors from 38 to 65, we get the payments made in age 38 to 65 relative to the payments made in year 37. The sums of the decay factors are similar to tail factors, but instead of being relative to cumulative payments they are relative to the incremental payments made in year 37.

The process can be repeated using a different anchor year. In addition to anchor year 37, the calculations were also performed using anchor years 36, 35, 34 and 33. In each case, the payments from 38 to 65 were compared to the payments made in the selected anchor year. Table 2.4 shows the cumulative decay factors for each of these anchor years:

Table 2.4
Cumulative Decay Factors Relative to
Incremental Payments During Different Anchor Years

Anchor Year	Cumulative Decay Factors
37	30.071
36	30.115
35	29.508
34	28.280
33	26.961

The cumulative decay factors can be interpreted as follows: Payments made in ages 38 to 65 are 30.071 times the payments made in age 37. Similarly, payments made in ages 38 to 65 are 30.115 times the payments made in age 36 etc.

3. **Tail Factors.** To convert these cumulative decay factors into tail factors, we make use of the selected cumulative loss development factors from the customary cumulative paid loss development triangle.

We illustrate the 37 to 65 tail factor calculation using the anchor year 37 cumulative decay factor and the 37:36 cumulative loss development factor (PLDF). SAIF's 37:36 PLDF was 1.03311.

The algebra is shown below:

The 37 to 36 PLDF of 1.03311 = $\frac{\text{Sum(1 to 37)}}{\text{Sum(1 to 36)}}$

Therefore .03311 = $\frac{\text{Sum(1 to 37)}}{\text{Sum(1 to 36)}} - \frac{\text{Sum(1 to 36)}}{\text{Sum(1 to 36)}} = \frac{\text{Paid in 37}}{\text{Sum(1 to 36)}}$

Consequently, $\frac{.03311}{1.03311} = \frac{\text{Paid in 37}}{\text{Sum(1 to 36)}}$ divided by $\frac{\text{Sum(1 to 37)}}{\text{Sum(1 to 36)}}$

$$= \frac{\text{Paid in 37}}{\text{Sum(1 to 37)}}$$

The cumulative decay factor of 30.071 = $\frac{\text{Sum(38 to 65)}}{\text{Paid in 37}}$

Therefore the product of the 30.071 and .03311 / 1.0331 = $\frac{\text{Sum(38 to 65)}}{\text{Paid in 37}} * \frac{\text{Paid in 37}}{\text{Sum(1 to 37)}}$

$$= \frac{\text{Sum(38 to 65)}}{\text{Sum(1 to 37)}}$$

The 37 to 65 tail factor is $\frac{\text{Sum(1 to 65)}}{\text{Sum(1 to 37)}} = 1 + \frac{\text{Sum(38 to 65)}}{\text{Sum(1 to 37)}} = 1 + 30.071 * \frac{.03311}{1.03311}$

$$= 1.964$$

The general formula for the tail factor is:

$$\text{Tail factor}_n = f_n D_{n+1}/[1 + f_n],$$

where f_n is the paid loss development factor, less one, for the nth year of development, and D_{n+1} is the cumulative decay factor for payments made during years (n+1) to ultimate relative to payments made in anchor year n.

This method is sensitive to the 37:36 cumulative PLDF. For this reason the analysis can be repeated using the 36, 35, 34 or 33 anchor years. Table 2.5 shows the 37 to 65 tail factor calculated using each of these anchor years:

Table 2.5
37 to 65 MPD Tail Factors Based on Different Anchor Years

Anchor Year	37 to 65 MPD Tail Factors
37	1.964
36	1.808
35	1.496
34	1.439
33	1.369
Selected	1.581 *

* Average excluding the high and low.

The empirically calculated 37 to 65 MPD medical tail factors range from a low of 1.369 to a high of 1.964. The value is sensitive to relatively small changes either in incremental age-to-age factors in the tail or in the cumulative age to age factors at the end of the cumulative triangle.

Another approach for reducing the high level of volatility of the tail factors shown in Table 2.5 is presented in Table A.6 of Appendix A. Each of the average PLDFs for ages 30 through 36 are adjusted to what they would be for age 37—using the appropriate products of incremental decay factors from AYs 1965 and prior. A weighted average of all of these adjusted PLDFs (1.022) is then used to replace the actual PLDF for DY 37 (1.0331). The final selected tail factor from age 37 to 65 is then 1.0 plus the product of the cumulative decay factor of 30.071 and .022/1.022 (1.647).

SAIF's Indicated Paid Tail Factors

When the indications from SAIF's incremental paid estimation of the tail from 37 years to 65 years are combined with those of a standard paid loss development approach up to 37 years of maturity, the MPD tails shown in the left column of Table 2.6 at different maturities were derived. The Total WC tail factors in Table 2.6 assume an ultimate mix of MPD and Other WC of 50% for each.

Table 2.6
SAIF's Indicated Paid Tail Factors

Maturity (Years)	MPD	Other WC	Total WC
10	2.469	1.263	1.671
15	2.328	1.234	1.613
25	2.054	1.129	1.457
35	1.680	1.052	1.294

In addition to MPD tail factors, Table 2.6 also displays indicated paid tail factors for all other types of WC losses, and for WC in total. Most of the Other WC tail factors are reflective of paid development for indemnity losses of permanently disabled claimants. A small portion is also due to paid development on fatal cases. The above table puts the impact of MPD paid tails in perspective relative to the indicated paid tail for all WC losses (i.e., for all injury types and for medical and indemnity combined).

Appendix B provides a comparison of SAIF's historical PLDFs for MPD, All Other WC and Total WC by DY. MPD is the primary reason why PLDFs for Total WC decline much more slowly than generally expected.

To gain an appreciation for the relative contribution of MPD versus All Other WC to the total loss reserves for a given AY at each of the above years of maturities, Table 2.7 provides a comparison of what the reserve would be, assuming that total ultimate losses for that AY were $100 million.

The MPD reserve makes up an increasing percentage of the total WC loss reserve at later maturities.

It should be borne in mind that Tables 2.6 and 2.7 provide indications specific to SAIF's loss experience in the state of Oregon, and not that of WC insurers in general.

Table 2.7
Indicated Loss Reserve at Different Maturities
(Dollars in millions)

Maturity (Years)	MPD Reserve	Other WC Reserve	MPD Reserve as a %-age of Total WC Reserve
10	$29.8	$10.4	74%
15	$28.5	$9.5	75%
25	$25.7	$5.7	82%
35	$20.2	$2.5	89%

Table 2.8 provides a comparison of indicated tails at different maturities for California WC experience, as projected by the WCIRB.

Table 2.8
WCIRB's Indicated California Paid Tail Factors

Maturity (Years)	Medical Tail	Indemnity Tail	Total WC Loss Tail
10	1.276	1.064	1.168
15	1.217	1.041	1.129
25	1.143	1.025	1.086

Source: WCIRB Bulletin No. 2003-24, pp. 8-9.

Although the California tails are consistently smaller than SAIF's, it is again true that the medical tails are decidedly greater than the indemnity tails. Table 2.9 provides a comparison of the size of the medical and indemnity loss reserves at different maturities, again assuming an AY with $100 million of ultimate losses.

Table 2.9
WCIRB Indicated Loss Reserve by Loss Type at Different Maturities
(Dollars in millions)

Maturity (Years)	Medical Loss Reserve	Indemnity Loss Reserve	Medical Reserve as a Percentage of Total Reserve
10	$11.7	$2.7	81%
15	9.6	1.8	84%
25	6.8	1.1	86%

In California, medical loss reserves make up an increasing percentage of the total WC loss reserve at later maturities.

3. INCORPORATING THE STATIC MORTALITY MODEL INTO THE INCREMENTAL PAID TO PRIOR OPEN METHOD

This section presents the incremental paid to prior open method of reserve estimation. The basics of this method bear much resemblance to the structural methods developed by Fisher/Lange [4] and Adler/Kline [5]. In essence, incremental payments for every development year are estimated by taking the product of the number of open claims at the end of the prior development year and an estimated claim severity.

While this method is of limited value for early DYs, its merit relative to other reserving methods is substantial in estimating reserves for future MPD payments for more mature DYs. For such mature DYs, future incremental payments are essentially a function of how many claims are still open and the average size of incremental payments per open claim. In contrast, future incremental MPD payments have almost no causal linkage to payments for rapidly settled claims during early DYs.

Table 3.1 provides a specific example of how this method is applied. The specific steps to be taken in applying the incremental paid per prior open claim method are:

1) Incremental paid losses (A) and open counts (B) are compiled by AY and DY.

2) Historical averages of incremental paid per prior open (C) are computed as A) divided by B).

3) Each historical average is trended to the expected severity level for the first CY (2003) after the evaluation date (12/31/2002) and a representative average is selected for each DY (last row of D). A trend factor of 9% per year was assumed in this example.

4) Ratios of open counts at successive year-ends are computed (E).

5) The selected ratios from (E) by DY are used to project the number of open claims for each future DY of each AY, thereby completing (B).

6) Future values of incremental paid per prior open (C) are projected on the basis of the representative averages in the last row of (D).

7) Projections of incremental paid losses for future DYs for each AY (A) are determined as the product of the projected open counts from the lower right portion of (B) and the projected values of incremental paid per prior open from (C).

The descriptions in the lower right portion of sections A), B) and C) of Table 3.1 also detail how the estimates in that portion are derived.

Table 3.2 presents a sample application of this method in estimating incremental payments for accident year 2002—assuming 5,000 ultimate PD claims and a series of additional assumptions derived from SAIF's historical loss experience (as described in Appendix C).

Table 3.1
Sample Application of the Incremental Paid per Prior Open Method

A) Incremental Paid Losses ($000's)

AY	12	24	36	48	60	72
1997	2,822.8	15,936.1	9,182.3	4,281.6	2,063.8	1,411.4
1998	2,638.0	14,249.9	9,096.4	2,935.8	3,214.7	
1999	3,331.3	15,805.8	9,734.9	4,308.9		
2000	3,170.4	18,602.1	12,462.0			
2001	3,143.1	20,305.9		*Product of Projected B)*		
2002	4,263.1			*and Projected C).*		

B) Open Counts

AY	12	24	36	48	60	72
1997	362	1112	793	490	375	324
1998	338	888	628	431	352	
1999	343	840	664	492		
2000	268	867	731			
2001	276	897		*Use Ratios from (D) to*		
2002	333			*Project Future Open Counts*		

C) Incremental Paid per Prior Open

AY	24	36	48	60	72
1997	44,022	8,257	5,399	4,212	3,764
1998	42,159	10,244	4,675	7,459	
1999	46,081	11,589	6,489		
2000	69,411	14,374			
2001	73,572	*Selected Average at CY 2003*			
2002		*Level (E) Adjusted for 9% Inflation*			

D) Incremental Paid per Prior Open Trended to CY 2003 at 9%/Yr.

AY	24	36	48	60	72
1997	67,734	11,656	6,992	5,004	4,102
1998	59,511	13,266	5,554	8,130	
1999	59,676	13,769	7,073		
2000	82,467	15,667			
2001	80,194				
Avg. Latest 3	74,112	14,234	6,540	6,567	4,102

E) Ratio of Open Counts at Successive Year-Ends

AY	24	36	48	60	72
1997	3.072	0.713	0.618	0.765	0.864
1998	2.627	0.707	0.686	0.817	
1999	2.449	0.790	0.741		
2000	3.235	0.843			
2001	3.250				
Avg. Latest 3	2.978	0.780	0.682	0.791	0.864

Table 3.2
Estimation of Incremental Payments by Static Mortality Model
MPD Losses for Accident Year 2002

Development Year	# Prior Open	Paid/ Prior Open ($000's)	Incremental Paid Loss ($000,000's)	Cumulative Paid Loss ($000,000's)	PLDF	Paid Factor to Ultimate
1	460 *	13.5	6.2	6.2		44.579
2	460	78.4	36.1	42.3	6.8187	6.538
3	1531	16.6	25.4	67.7	1.6014	4.082
4	1366	8.4	11.5	79.2	1.1692	3.492
5	949	7.9	7.5	86.7	1.0948	3.189
6	677	6.8	4.6	91.2	1.0530	3.029
7	554	6.9	3.8	95.1	1.0420	2.907
8	396	7.5	3.0	98.1	1.0314	2.818
9	323	8.2	2.7	100.7	1.0271	2.744
10	249	9.0	2.2	103.0	1.0222	2.684
11	209	8.0	1.7	104.6	1.0163	2.641
12	197	8.8	1.7	106.4	1.0165	2.598
13	187	9.5	1.8	108.1	1.0167	2.556
14	178	10.4	1.8	110.0	1.0171	2.513
15	170	11.3	1.9	111.9	1.0175	2.469
16	163	12.4	2.0	113.9	1.0180	2.426
17	156	13.5	2.1	116.0	1.0185	2.382
18	150	14.7	2.2	118.2	1.0190	2.337
19	144	16.0	2.3	120.6	1.0195	2.293
20	139	17.5	2.4	123.0	1.0201	2.248
21	133	19.0	2.5	125.5	1.0205	2.202
22	128	20.7	2.7	128.2	1.0212	2.157
23	124	22.6	2.8	130.9	1.0218	2.111
24	119	24.6	2.9	133.9	1.0223	2.065
25	114	26.9	3.1	136.9	1.0228	2.018
26	109	29.3	3.2	140.1	1.0232	1.973
27	104	31.9	3.3	143.4	1.0236	1.927
28	98	34.8	3.4	146.8	1.0239	1.882
29	93	37.9	3.5	150.4	1.0241	1.838
30	88	41.3	3.6	154.0	1.0242	1.795
31	83	45.0	3.7	157.7	1.0242	1.752
32	78	49.1	3.8	161.5	1.0242	1.711
33	73	53.5	3.9	165.4	1.0240	1.671
34	68	58.3	3.9	169.4	1.0238	1.632
35	63	63.6	4.0	173.4	1.0236	1.594
36	58	69.3	4.0	177.4	1.0232	1.558
37	54	75.5	4.1	181.4	1.0229	1.523
38	49	82.3	4.1	185.5	1.0224	1.490
39	45	89.7	4.1	189.6	1.0220	1.458
40	42	97.8	4.1	193.7	1.0215	1.427
41	38	106.6	4.1	197.7	1.0209	1.398
42	35	116.2	4.0	201.7	1.0204	1.370
43	31	126.7	4.0	205.7	1.0198	1.343
44	29	138.1	3.9	209.7	1.0192	1.318
45	26	150.5	3.9	213.6	1.0185	1.294

* For the first development year only, the number of claims open at the end of the year is shown.
** The PLDFs in this table closely fit SAIF's ten year historical average factors.

Two striking phenomena are exhibited in Table 3.2. First, incremental payments consistently increase for every development year from the 11th through the 39rd, a counter-intuitive pattern. Second, the PLDFs consistently increase for every development year from the 11th through the 31st.

To understand why incremental payments, as well as PLDFs tend to increase during many "mature" years of development, it is helpful to examine how the two key components of the incremental paid to prior open method change over successive development years.

This section illustrates how a static mortality model has been incorporated into the incremental paid to prior open method. It describes the main framework of the method, while Appendix C covers the derivation of various assumptions that involve a complex analysis.

As is evident from Column (4) in Table 3.3, it was assumed that incremental payments per prior open claim would increase by 9% per year for every DY beyond the 7th, except for the 11th DY. This was based on an analysis of SAIF's historical incremental severities for these DYs (see Section C.2 of Appendix C). The fact that SAIF's historical PLDFs for DYs 40-54 are noticeably higher than those predicted by this model is evidence that the rate of medical cost escalation for these DYs was perceptibly higher than 9%.

The basis for our selection of 9% as the long-term rate of medical inflation is presented in Section C.3 of Appendix C. This assumed rate of increase also includes a provision for increases in utilization such as have occurred historically.

In Table 3.3 incremental payments continue to increase until age 39 because the impact of claims inflation [Column (4)] is greater than the force of mortality in closing existing claims [Column (2)]. Incremental payments first clearly decline when the product of the inflation factor (1.09) and the ratio of claims remaining open (0.915) is less than one—for development year 40.

Table 3.3
Estimation of Incremental Payments by Static Mortality Model

Development Year (DY)	(1) # Open at End of Prior DY	(2) % Decline in Prior Open Counts	(3) Increm. Pd/ Prior Open ($000's)	(4) % Severity Change
1	0.0		13.478	
2	460.0		78.425	481.9%
3	1531.0		16.607	-78.8%
4	1366.0	10.8%	8.388	-49.5%
5	949.0	30.5%	7.903	-5.8%
6	677.0	28.7%	6.781	-14.2%
7	554.0	18.2%	6.924	2.1%
8	396.0	28.5%	7.547	9.0%
9	323.0	18.4%	8.226	9.0%
10	249.0	22.9%	8.967	9.0%
11	209.0	16.1%	8.036	-10.4%
12	196.9	5.8%	8.759	9.0%
13	186.5	5.3%	9.548	9.0%
14	177.5	4.8%	10.407	9.0%
15	169.7	4.4%	11.343	9.0%
16	162.9	4.0%	12.364	9.0%
17	156.1	4.2%	13.477	9.0%
18	150.2	3.8%	14.690	9.0%
19	144.0	4.1%	16.012	9.0%
20	138.5	3.8%	17.453	9.0%
21	132.8	4.2%	19.024	9.0%
22	128.2	3.4%	20.736	9.0%
23	123.6	3.6%	22.603	9.0%
24	118.7	3.9%	24.637	9.0%
25	113.8	4.2%	26.854	9.0%
26	108.8	4.4%	29.271	9.0%
27	103.6	4.7%	31.905	9.0%
28	98.4	5.0%	34.777	9.0%
29	93.2	5.3%	37.907	9.0%
30	88.0	5.6%	41.318	9.0%
31	82.8	5.9%	45.037	9.0%
32	77.6	6.2%	49.090	9.0%
33	72.5	6.5%	53.509	9.0%
34	67.6	6.8%	58.324	9.0%
35	62.8	7.1%	63.574	9.0%
36	58.2	7.4%	69.295	9.0%
37	53.7	7.7%	75.532	9.0%
38	49.5	7.9%	82.330	9.0%
39	45.4	8.2%	89.739	9.0%
40	41.6	8.4%	97.816	9.0%
41	38.0	8.6%	106.619	9.0%
42	34.6	8.9%	116.215	9.0%
43	31.5	9.1%	126.674	9.0%
44	28.5	9.3%	138.075	9.0%
45	25.8	9.6%	150.502	9.0%

The percentage declines in prior open counts reflect the composite effects of three factors affecting the number of open claims: 1) increases due to newly reported claims; 2) decreases due to the death of a few claimants; and 3) net decreases due to other reasons (including increases due to reopened claims). After 20 years of development newly reported claims become negligible, as do net claim closures. Thus, after 20 years of development, virtually all claim closures are attributable to the death of claimants. Consequently, changes in the number of open claims at the end of each development year beyond 20 years can be predicted entirely on the basis of mortality rates. And changes in the number of open claims can be estimated beyond 15 years via mortality rates and inclusion of the small number of newly reported claims and net closures for other reasons. This is subject to fine-tuning due to the possibility that the mortality rates of disabled claimants might be higher than those of the general populace, although recent improvements in medical technology have reduced the influence of medical impairment on mortality rates.

Table 3.4 presents an accounting of how each of the above factors affect the number of open MPD claims during the development of a typical accident year. Derivation of these assumptions is disclosed in Appendix C.

SAIF's historical database only includes the total number of closed claims. So the number of claimant deaths was estimated based on mortality tables and any additional claim closures are presumed to be for other reasons. The breakdown was derived by estimating the number of claim closures due to death from the 2000 SSA mortality tables.

The SSA tables were not modified by a disabled lives scale factor because key values predicted by the model either: 1) closely fit SAIF's actual experience; or 2) underestimated actual development (e.g., DYs 40-54). Furthermore, prior actuarial inquiries into this question have been mixed regarding whether such a factor is justified. This is discussed in two papers in the Winter 1991 Edition of the CAS Forum ("*Injured Worker Mortality*" Gillam, William R. [6] and "*Review of Report of Committee on Mortality for Disabled Lives*" Venter, Gary G., Schill, Barbara, and Barnett, Jack [7]). It is quite possible that permanently disabled workers receive better medical care, on average, than do non-disabled people, helping to close a gap in mortality rates that would otherwise exist.

Table 3.4
Factors Affecting the Number of Open MPD Claims for a Single Accident Year

Develop-ment Year (DY)	(1) # Open at End of Prior DY [(5) of Prior DY End]	(2) Newly Reported Claims	(3) Estimated # of Claimant Deaths	(4) Estimated Claims Closed for Other Reasons	(5) # Open at End of Current DY [(1)+(2)-(3)-(4)]
1		926	3.5	462.5	460.0
2	460.0	2790	15.0	1704.0	1531.0
3	1531.0	866	17.3	1013.7	1366.0
4	1366.0	215	14.1	617.9	949.0
5	949.0	91	10.3	352.7	677.0
6	677.0	47	7.9	162.1	554.0
7	554.0	19	6.9	170.1	396.0
8	396.0	11	5.3	78.7	323.0
9	323.0	8	4.7	77.3	249.0
10	249.0	5	3.9	41.1	209.0
11	209.0	4	3.5	12.5	196.9
12	196.9	3	3.6	9.8	186.5
13	186.5	3	3.6	8.4	177.5
14	177.5	3	3.7	7.1	169.7
15	169.7	3	3.8	5.9	162.9
16	162.9	2	3.9	4.9	156.1
17	156.1	2	4.0	3.9	150.2
18	150.2	1	4.2	3.0	144.0
19	144.0	1	4.3	2.2	138.5
20	138.5	0	4.4	1.4	132.8
21	132.8	0	4.5	0.0	128.2
22	128.2	0	4.7	0.0	123.6
23	123.6	0	4.8	0.0	118.7
24	118.7	0	4.9	0.0	113.8
25	113.8	0	5.1	0.0	108.8

This method produces projected PLDFs out to 85 years of development. Such development is possible because a worker could be injured at age 16 and live to be over 100. Table 3.5 displays the basis for projections of incremental payments and PLDFs beyond 45 years of development. It is a continuation of Table 3.2.

Table 3.5
Estimation of Incremental Payments by Static Mortality Model
For Development Years 45 and Greater

Development Year	# Prior Open	Paid/ Pr Open ($000's)	Incremental Paid Loss ($000,000's)	Cumulative Paid Loss ($000,000's)	PLDF	Paid Factor to Ultimate
45	25.8	150.5	3.9	213.6	1.0185	1.2940
46	23.3	164.0	3.8	217.4	1.0179	1.2715
47	21.0	178.8	3.7	221.1	1.0172	1.2499
48	18.8	194.9	3.7	224.8	1.0166	1.2295
49	16.8	212.4	3.6	228.4	1.0159	1.2103
50	15.0	231.6	3.5	231.8	1.0152	1.1921
51	13.4	252.4	3.4	235.2	1.0146	1.1750
52	11.9	275.1	3.3	238.5	1.0139	1.1589
53	10.5	299.9	3.1	241.6	1.0132	1.1438
54	9.3	326.9	3.0	244.7	1.0125	1.1296
55	8.1	356.3	2.9	247.6	1.0118	1.1164
56	7.1	388.4	2.8	250.3	1.0112	1.1041
57	6.2	423.3	2.6	253.0	1.0105	1.0926
58	5.4	461.4	2.5	255.4	1.0098	1.0820
59	4.6	502.9	2.3	257.8	1.0091	1.0722
60	4.0	548.2	2.2	260.0	1.0085	1.0632
61	3.4	597.5	2.0	262.0	1.0078	1.0549
62	2.9	651.3	1.9	263.9	1.0072	1.0474
63	2.4	709.9	1.7	265.6	1.0065	1.0406
64	2.0	773.8	1.6	267.2	1.0059	1.0345
65	1.7	843.5	1.4	268.6	1.0053	1.0291
66	1.4	919.4	1.3	269.8	1.0047	1.0242
67	1.1	1,002.1	1.1	271.0	1.0042	1.0200
68	0.90	1,092.3	1.0	272.0	1.0036	1.0163
69	0.71	1,190.6	0.85	272.8	1.0031	1.0131
70	0.56	1,297.8	0.73	273.5	1.0027	1.0104
71	0.43	1,414.6	0.61	274.1	1.0022	1.0082
72	0.33	1,541.9	0.51	274.7	1.0019	1.0063
73	0.25	1,680.7	0.41	275.1	1.0015	1.0048
74	0.18	1,831.9	0.33	275.4	1.0012	1.0036
75	0.13	1,996.8	0.26	275.7	1.0009	1.0026
76	0.093	2,176.5	0.20	275.9	1.0007	1.0019
77	0.065	2,372.4	0.15	276.0	1.0006	1.0013
78	0.044	2,585.9	0.11	276.1	1.0004	1.0009
79	0.029	2,818.7	0.08	276.2	1.0003	1.0006
80	0.019	3,072.3	0.06	276.3	1.0002	1.0004
81	0.012	3,348.8	0.04	276.3	1.0001	1.0003
82	0.008	3,650.2	0.03	276.3	1.0001	1.0002
83	0.005	3,978.8	0.02	276.4	1.0001	1.0001
84	0.003	4,336.9	0.01	276.4	1.0000	1.0001
85	0.002	4,727.2	0.01	276.4	1.0000	1.0000

Incremental payments in Tables 3.2 and 3.5:

- Are at a local minimum of \$1.68 million during the 11^{th} year of development.
- Increase to a local maximum of \$4.08 million during the 39^{th} year of development.
- Do not decrease down below the local minimum of \$1.68 million during the 11^{th} year of development until the 64^{th} year of development.

This is an example of the extreme degree to which the behavior of MPD payments departs from those of other WC loss costs (or those for other casualty coverages).

The paid factors to ultimate in the last column of Table 3.2 and 3.5 above are exceptionally sensitive to future rates of claim inflation. Table 3.6 provides a comparison of the indicated tail factors with and without inflation at various representative ages of development.

Table 3.6
Indicated Paid Factors to Ultimate

End of Year of Development	With 9% Inflation	Without Inflation	Ratio of 9% Inflation Reserve to Zero Inflation Reserve
10	2.684	1.152	11.1
15	2.469	1.110	13.4
25	2.019	1.054	18.9
35	1.594	1.022	27.0
50	1.192	1.003	64.0

An example will put the implications of Table 3.6 into practical terms. Suppose a claims adjuster reviews all PD claims open at the end of 25 years of development. For each PD claim, he estimates the medical portion by multiplying current medical payments times an annuity factor that is the life expectancy of the claimant at their current age. The ratio of 18.9 in the right column of Table 3.6 is saying is that future medical payments will be 18.9 times the case reserve derived by this method. One might think the error would decrease the more mature the accident year became, but in actuality the percentage error dramatically increases at high maturities. In addition, the mortality table used by the claims adjuster may be out of date.

Just as we have modeled the expected PLDF patterns for MPD losses, analogous ILDF patterns can be estimated if we define total case reserves as the product of the latest year's incremental payments times the average annuity factor for all living PD claimants. This is presented in Table 3.7.

Table 3.7
Expected ILDFs if Case Reserves are Based on Zero Inflation Annuity Factors

DY	# Prior Open	Upward Sum of # Prior Open	Zero Inflation Annuity Factor	Incremental Pd/ Pr Open	Zero Inflation Case Reserve	Cum Paid	Zero Inflation Case Incurred	ILDF	Incurred Tail
5	949	6,912.6	6.28	7.9	47.1	86.7	133.8	0.9756	2.066
6	677	5,963.6	7.81	6.8	35.8	91.2	127.1	0.9500	2.175
7	554	5,286.6	8.54	6.9	32.8	95.1	127.9	1.0059	2.162
8	396	4,732.6	10.95	7.5	32.7	98.1	130.8	1.0231	2.113
9	323	4,336.6	12.43	8.2	33.0	100.7	133.7	1.0225	2.066
10	249	4,013.6	15.12	9.0	33.8	103.0	136.7	1.0222	2.022
11	209	3,764.6	17.01	8.0	28.6	104.6	133.2	0.9744	2.075
12	196.9	3,555.6	17.05	8.8	29.4	106.4	135.8	1.0193	2.035
13	186.5	3,358.7	17.01	9.5	30.3	108.1	138.4	1.0195	1.996
14	177.5	3,172.1	16.87	10.4	31.2	110.0	141.2	1.0197	1.958
15	169.7	2,994.6	16.65	11.3	32.0	111.9	144.0	1.0199	1.920
16	162.9	2,824.9	16.34	12.4	32.9	113.9	146.8	1.0200	1.882
17	156.1	2,662.0	16.05	13.5	33.8	116.0	149.8	1.0202	1.845
18	150.2	2,505.9	15.69	14.7	34.6	118.2	152.9	1.0203	1.808
19	144.0	2,355.8	15.36	16.0	35.4	120.6	156.0	1.0204	1.772
20	138.5	2,211.8	14.96	17.5	36.2	123.0	159.2	1.0204	1.737
21	132.8	2,073.2	14.62	19.0	36.9	125.5	162.4	1.0205	1.702
22	128.2	1,940.5	14.13	20.7	37.6	128.2	165.7	1.0205	1.668
23	123.6	1,812.2	13.67	22.6	38.2	130.9	169.1	1.0204	1.634
24	118.7	1,688.7	13.22	24.6	38.7	133.9	172.6	1.0203	1.602
25	113.8	1,569.9	12.80	26.9	39.1	136.9	176.0	1.0202	1.570
26	108.8	1,456.1	12.39	29.3	39.4	140.1	179.6	1.0200	1.539
27	103.6	1,347.4	12.00	31.9	39.7	143.4	183.1	1.0198	1.509
28	98.4	1,243.8	11.64	34.8	39.8	146.8	186.7	1.0195	1.481
29	93.2	1,145.4	11.29	37.9	39.9	150.4	190.3	1.0192	1.453
30	88.0	1,052.2	10.96	41.3	39.8	154.0	193.8	1.0189	1.426
31	82.8	964.2	10.65	45.0	39.7	157.7	197.4	1.0185	1.400
32	77.6	881.5	10.36	49.1	39.5	161.5	201.0	1.0181	1.375
33	72.5	803.9	10.08	53.5	39.1	165.4	204.6	1.0177	1.351
34	67.6	731.3	9.82	58.3	38.7	169.4	208.1	1.0172	1.328
35	62.8	663.7	9.57	63.6	38.2	173.4	211.6	1.0167	1.306
36	58.2	600.9	9.33	69.3	37.6	177.4	215.0	1.0163	1.286
37	53.7	542.8	9.11	75.5	36.9	181.4	218.4	1.0157	1.266
38	49.5	489.0	8.89	82.3	36.2	185.5	221.7	1.0152	1.247
39	45.4	439.6	8.68	89.7	35.4	189.6	225.0	1.0147	1.229
40	41.6	394.2	8.48	97.8	34.5	193.7	228.1	1.0142	1.211
41	38.0	352.6	8.28	106.6	33.5	197.7	231.3	1.0136	1.195
42	34.6	314.6	8.08	116.2	32.5	201.7	234.3	1.0131	1.180
43	31.5	279.9	7.89	126.7	31.5	205.7	237.2	1.0125	1.165
44	28.5	248.5	7.71	138.1	30.4	209.7	240.0	1.0119	1.151
45	25.8	219.9	7.52	150.5	29.2	213.6	242.8	1.0114	1.138
46	23.3	194.1	7.33	164.0	28.0	217.4	245.4	1.0108	1.126
47	21.0	170.8	7.15	178.8	26.8	221.1	247.9	1.0103	1.115
48	18.8	149.9	6.97	194.9	25.5	224.8	250.3	1.0097	1.104
49	16.8	131.0	6.78	212.4	24.3	228.4	252.6	1.0092	1.094
50	15.0	114.2	6.60	231.6	23.0	231.8	254.8	1.0086	1.085

A review of Table 3.7 reveals the following:

- Although there are ILDFs less than 1.0 for the 5th, 6th and 11th development years, subsequent factors become noticeably greater than 1.0—even up through the 50th year of development, and beyond.

- Incurred loss development factors are expected to increase during each development year from the 12th through the 21st years.

- The rate of decrease in ILDFs after the 21st development year is surprisingly small, resulting in very large incurred tails for nearly all ages.

This example raises concerns about the practice of estimating the paid tail by taking the ratio of incurred (perhaps with some modest upward adjustment) to paid at the most mature development year.

4. MORTALITY IMPROVEMENT

Life expectancies have been increasing steadily and noticeably for at least the past several decades, and are expected to continue to increase throughout the next century, if not beyond. Consider these trends in life expectancies that have occurred over past decades, and those projected by the Social Security Administration (SSA).

Table 4.1
Life Expectancies at Different Ages—Male
Based on Social Security Administration Mortality Tables

Current Age	1960	1980	2000	*2020*	*2040*	*2060*	*2080*
20	49.7	51.7	54.7	*56.8*	*58.7*	*60.3*	*61.8*
40	31.3	33.5	36.2	*38.1*	*39.8*	*41.4*	*42.7*
60	15.9	17.3	19.3	*20.8*	*22.2*	*23.4*	*24.6*
80	6.0	6.8	7.2	*7.8*	*8.6*	*9.4*	*10.1*

Table 4.1 presents male life expectancies since a high percentage of permanently disabled claimants are male. Table 4.2 displays the percentage increases in life expectancy corresponding to the estimates in Table 4.1.

Typically, PD claimants receive a percentage of replacement wages until their retirement age and coverage of their medical expenses related to their work injuries are paid until they die. Since medical expenses are expected to continue rising at high rates of inflation, coverage of such expenses significantly compounds the effects of expected increases in life expectancies.

Table 4.2
Percentage Increase in Male Life Expectancies
Based on Social Security Administration Mortality Tables

Current Age	1980 1960	2000 1980	*2020 2000*	*2040 2020*	*2060 2040*	*2080 2060*
20	4.2%	5.8%	*3.7%*	*3.3%*	*2.8%*	*2.5%*
40	7.0%	8.2%	*5.2%*	*4.5%*	*3.8%*	*3.3%*
60	9.1%	11.7%	*7.6%*	*6.6%*	*5.6%*	*4.9%*
80	11.9%	6.5%	*8.7%*	*10.0%*	*8.6%*	*7.6%*

Consequently, the difference between MPD reserves calculated using constant recent mortality rates and those calculated with trended mortality rates is substantial. The latter calculations are unusually complex. They can best be measured and understood with the aid of a heuristic model.

While the effects of declining mortality rates are almost undetectable over the short run, their magnitude over future decades is quite substantial on gross MPD reserves. However, the extent of these effects is negligible on net MPD when retentions are relatively low. The effect is also fairly small for indemnity loss reserves for permanently disabled claimants.

5. THE TRENDED MORTALITY MODEL

This method is similar to the static mortality model adaptation of the incremental paid to prior open method described in Section 3 and Appendix C. The key thing that differs is that the change in the number of open claims for every future development year of every AY is determined by applying mortality tables forecasted by the SSA for the appropriate future development year. The rest of the method is essentially unchanged. A sample of these differences is provided in Tables 5.1a and 5.1b for several different DYs for AY 2002.

As is evident from Tables 5.1a and 5.1b, small improvements in the annual survival rate of remaining claimants result in major differences in the number of claims still open at higher development years. Given that the greatest differences occur during development years in the distant future, when the effects of medical inflation have had an opportunity to compound over decades, the total reserve indicated by the trended mortality method is decidedly greater than that indicated by the static mortality method.

Table 5.1a
Comparison of Mortality Rates and Claims Open at Different Development Years for Accident Year 2002

	Mortality Table Assumed		Group Survival Rate		Claims Open at Prior Year End		% Greater Open Claims
DY	Static	Trended	Static	Trended	Static	Trended	
30	2000	2031	0.941	0.946	88.0	91.5	4.0%
31	2000	2032	0.938	0.943	82.8	86.6	4.6%
32	2000	2033	0.935	0.941	77.6	81.6	5.2%
33	2000	2034	0.932	0.938	72.5	76.8	5.9%
34	2000	2035	0.929	0.935	67.6	72.0	6.6%
35	2000	2036	0.926	0.933	62.8	67.4	7.3%
36	2000	2037	0.923	0.931	58.2	62.9	8.1%
37	2000	2038	0.921	0.928	53.7	58.5	8.9%
38	2000	2039	0.918	0.926	49.5	54.3	9.8%
39	2000	2040	0.916	0.924	45.4	50.3	10.7%
40	2000	2041	0.914	0.922	41.6	46.5	11.7%
41	2000	2042	0.911	0.920	38.0	42.8	12.7%
42	2000	2043	0.909	0.918	34.6	39.4	13.8%
43	2000	2044	0.907	0.916	31.5	36.2	14.9%
44	2000	2045	0.904	0.914	28.5	33.1	16.1%
45	2000	2046	0.902	0.912	25.8	30.3	17.3%
46	2000	2047	0.900	0.910	23.3	27.6	18.5%
47	2000	2048	0.898	0.908	21.0	25.1	19.9%
48	2000	2049	0.895	0.906	18.8	22.8	21.2%
49	2000	2050	0.893	0.904	16.8	20.7	22.7%
50	2000	2051	0.890	0.902	15.0	18.7	24.2%
51	2000	2052	0.887	0.899	13.4	16.8	25.9%
52	2000	2053	0.885	0.897	11.9	15.1	27.6%
53	2000	2054	0.882	0.895	10.5	13.6	29.4%
54	2000	2055	0.878	0.892	9.3	12.2	31.3%
55	2000	2056	0.875	0.889	8.1	10.8	33.3%
56	2000	2057	0.871	0.886	7.12	9.6	35.5%
57	2000	2058	0.867	0.883	6.20	8.5	37.9%
58	2000	2059	0.863	0.880	5.38	7.6	40.4%
59	2000	2060	0.859	0.876	4.64	6.65	43.1%
60	2000	2061	0.853	0.872	3.99	5.82	46.1%

Table 5.1b
Comparison of Mortality Rates and Claims Open at Different Development Years for Accident Year 2002

	Mortality Table Assumed		Group Survival Rate		Claims Open at Prior Year End		% Greater Open
DY	Static	Trended	Static	Trended	Static	Trended	Claims
60	2000	2061	0.853	0.872	3.99	5.82	46.1%
61	2000	2062	0.848	0.868	3.40	5.08	49.3%
62	2000	2063	0.842	0.863	2.89	4.41	52.8%
63	2000	2064	0.835	0.858	2.43	3.81	56.7%
64	2000	2065	0.828	0.852	2.03	3.26	60.8%
65	2000	2066	0.821	0.846	1.68	2.78	65.4%
66	2000	2067	0.812	0.840	1.38	2.35	70.6%
67	2000	2068	0.803	0.833	1.12	1.98	76.3%
68	2000	2069	0.794	0.826	0.90	1.65	82.8%
69	2000	2070	0.783	0.819	0.715	1.36	90.3%
70	2000	2071	0.772	0.811	0.560	1.11	98.9%
71	2000	2072	0.761	0.803	0.432	0.90	108.8%
72	2000	2073	0.749	0.794	0.329	0.72	120.3%
73	2000	2074	0.736	0.785	0.246	0.576	133.7%
74	2000	2075	0.723	0.776	0.181	0.452	149.4%
75	2000	2076	0.709	0.767	0.131	0.351	167.9%
76	2000	2077	0.695	0.758	0.093	0.269	189.7%
77	2000	2078	0.681	0.748	0.065	0.204	215.7%
78	2000	2079	0.667	0.739	0.044	0.153	246.9%
79	2000	2080	0.652	0.729	0.029	0.113	284.4%
80	2000	2081	0.637	0.719	0.019	0.082	329.8%
81	2000	2082	0.621	0.756	0.012	0.059	385.3%
82	2000	2083	0.604	0.748	0.008	0.045	491.2%
83	2000	2084	0.586	0.738	0.005	0.033	632.3%
84	2000	2085	0.566	0.728	0.003	0.025	823.2%
85	2000	2086	0.545	0.716	0.002	0.018	1086.8%

To fully present the projections of the trended mortality model would require the display of arrays consisting of 37 rows and about 90 columns—the rows representing accident years and the columns years of development. Since this would be unwieldy, ***summary arrays in which data for every fifth accident year is shown at the end of every fifth development year*** will be presented. An example is presented below in Table 5.2.

Table 5.2 shows the calendar year mortality table that should be used in determining the probability of continuation of a claim for each AY-DY combination. If a current table (e.g., 2000) is used, differences between the static and trended mortality rates will increase the farther the year of the appropriate mortality table is from CY 2000.

Table 5.2
Sample Layout of Summarized Results

Calendar Year of Payments—
For Every Fifth Accident Year at Every Fifth Development Year

	Development Year															
AY	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**	**55**	**60**	**65**	**70**	**75**	**80**
1970	1974	1979	1984	1989	1994	1999	2004	2009	2014	2019	2024	2029	2034	2039	2044	2049
1975	1979	1984	1989	1994	1999	2004	2009	2014	2019	2024	2029	2034	2039	2044	2049	2054
1980	1984	1989	1994	1999	2004	2009	2014	2019	2024	2029	2034	2039	2044	2049	2054	2059
1985	1989	1994	1999	2004	2009	2014	2019	2024	2029	2034	2039	2044	2049	2054	2059	2064
1990	1994	1999	2004	2009	2014	2019	2024	2029	2034	2039	2044	2049	2054	2059	2064	2069
1995	1999	2004	2009	2014	2019	2024	2029	2034	2039	2044	2049	2054	2059	2064	2069	2074
2000	2004	2009	2014	2019	2024	2029	2034	2039	2044	2049	2054	2059	2064	2069	2074	2079

What effects will the above trends in mortality have on MPD loss reserves? It is not hard to foresee the general effects. PD claimants for more recent accident years are expected to live longer than their counterparts from old accident years. This is a direct consequence of declining mortality rates. As a result, a higher percentage of PD claimants will still be alive at any given age of development. Therefore, the percentage of claims closed will decline at any given age and thus simple paid loss development projections will need to be adjusted upward to reflect these declines in claims disposal ratios. Hence, tail factors that reflect the effects of declining mortality rates must increase over successive accident years *for every possible development age*.

While the general effects of anticipated future mortality trends are easy to grasp, the best way to quantify these effects is to construct a heuristic model designed to isolate the effects of mortality trends on PLDFs and paid tails. The trended mortality model we have constructed is such that:

- The only thing that changes over time are mortality rates—as historically compiled and as officially forecasted by the Social Security Administration.

- Medical inflation is a constant 9% per year, both historically and prospectively. Support for this assumption is provided in Section C.3 of Appendix C.

- The number of ultimate reported claims for every accident year, from 1966 through 2002, is held at a constant level of 5,000 per year.

- Claim reporting and closure patterns for SAIF's PD claimants over the past ten calendar years served as the basis for these key assumptions—in order to make the model as realistic as possible.

By designing a model where claimant mortality rates are the only thing that changes from AY to AY, the effects of mortality trends can clearly be seen. Details of the model are presented in Appendices C and D.

Projections of the number of open claims were derived from the heuristic model for each accident year from 1966 through 2002 at the end of every development year from the first to the eightieth. As noted above, each accident year was assumed to have 5,000 ultimate reported claims. Claim closure patterns, for reasons other than death of the claimant, were held constant for all accident years. The only thing that varied from accident year to accident year in the model was the number of claims closed due to death. In this way the effects of mortality trends on the number of open claims at the end of each development year for each accident year can be isolated.

What is evident from the summarized results presented in Table 5.3 is that the expected number of open claims at any given year of development will slowly increase as one moves from the oldest accident years to the most recent—by scanning down any chosen column in this table.

Table 5.3
Number of Open Claims for Representative
Accident Years at Five Year Intervals of Development

	End of Development Year															
AY	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**	**55**	**60**	**65**	**70**	**75**	**80**
1970	653	196	149	119	95	71	50	33	21	12	6.9	3.5	1.5	0.5	0.1	0.02
1975	655	197	150	120	97	73	52	34	22	13	7.2	3.7	1.6	0.6	0.1	0.03
1980	659	200	153	123	100	76	54	36	23	14	7.7	3.9	1.7	0.6	0.2	0.03
1985	662	202	156	126	103	79	56	38	24	14	8.1	4.2	1.9	0.7	0.2	0.04
1990	665	204	158	128	105	81	58	39	25	15	8.5	4.4	2.0	0.7	0.2	0.04
1995	668	206	160	130	108	83	60	41	26	16	9.0	4.7	2.1	0.8	0.2	0.05
2000	670	207	161	132	110	86	62	42	27	17	9.5	5.0	2.3	0.9	0.3	0.06

For example, at the end of 35 years of development, the number of open claims is expected to increase from 50 for accident year 1970 to 62 for accident year 2000. This is an increase of 24% in the number of open claims. And at the end of 60 years of development, the number of open claims is expected to increase from 3.5 to 5.0, an increase of 42.9%. The percentage rate of increase in the number of open claims for each given column increases as one moves from the earlier development years on the left to the later development years on the right. This is due to the compounding effect of expected declines in future mortality rates. Table 5.4 displays the total percentage increase for each development year column.

Table 5.4
Percentage Increases in the Number of Open Claims at the End of Representative Development Years—From Accident Year 1970 to Accident Year 2000

End of Development Year															
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
2.6%	5.6%	8.3%	11.6%	15.6%	19.8%	23.9%	27.4%	30.3%	33.5%	37.5%	43.7%	54.3%	73.2%	106.8%	164.5%

Since the number of open claims at any given development year will be increasing steadily over successive accident years, the total proportion of ultimate losses paid through that development year will decline slightly over time. Because of this we would naturally expect that the appropriate tail factors at any given development year will also increase steadily over time. The projected results are displayed in Table 5.5 below.

Table 5.5
Indicated Tail Factors

	End of Development Year															
AY	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
1970	3.037	2.570	2.375	2.177	1.973	1.773	1.592	1.438	1.311	1.210	1.132	1.075	1.037	1.015	1.004	1.001
1975	3.108	2.628	2.428	2.223	2.012	1.805	1.617	1.456	1.325	1.220	1.139	1.080	1.040	1.016	1.005	1.001
1980	3.197	2.701	2.492	2.279	2.058	1.842	1.645	1.477	1.340	1.231	1.146	1.085	1.043	1.018	1.006	1.001
1985	3.286	2.774	2.558	2.336	2.106	1.879	1.674	1.499	1.356	1.242	1.154	1.090	1.046	1.020	1.007	1.002
1990	3.376	2.848	2.624	2.393	2.154	1.918	1.704	1.521	1.372	1.253	1.162	1.095	1.049	1.021	1.007	1.002
1995	3.466	2.921	2.690	2.451	2.203	1.957	1.733	1.543	1.388	1.265	1.170	1.101	1.053	1.023	1.008	1.002
2000	3.549	2.990	2.752	2.505	2.248	1.993	1.761	1.563	1.402	1.275	1.177	1.105	1.054	1.023	1.008	1.002

Table 5.6 displays the percentage understatement in AY 2000 loss reserves at different development ages—if such reserves were based on AY 1970 tail factors. It clearly indicates that the usage of constant tail factors will result in material inadequacies in the indicated loss reserves.

Table 5.6
Indicated Percentage Understatement in AY 2000 Loss Reserves (If Based on AY 1970 Tail Factors)

End of Development Year															
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
25%	27%	27%	28%	28%	28%	28%	29%	29%	31%	34%	39%	47%	59%	85%	102%

The heuristic model also indicates that incremental PLDFs at any given maturity will trend upward over time. In Table 5.7 below, five year paid loss development factors, each of which are the cumulative products of five successive paid loss development factors, inch upward over time within any given development column.

Table 5.7
Trends in Five Year Paid Loss Development Factors

	Development Years															
AY	10/5	15/10	20/15	25/20	30/25	35/30	40/35	45/40	50/45	55/50	60/55	65/60	70/65	75/70	80/75	85/80
1970	1.182	1.082	1.091	1.103	1.113	1.114	1.107	1.097	1.084	1.069	1.053	1.037	1.022	1.010	1.004	1.001
1975	1.183	1.083	1.092	1.105	1.115	1.116	1.110	1.099	1.086	1.071	1.055	1.039	1.023	1.011	1.004	1.001
1980	1.184	1.084	1.094	1.107	1.118	1.119	1.114	1.103	1.089	1.073	1.057	1.040	1.024	1.012	1.004	1.001
1985	1.185	1.084	1.095	1.109	1.120	1.123	1.117	1.106	1.092	1.076	1.059	1.042	1.026	1.013	1.005	1.002
1990	1.186	1.085	1.096	1.111	1.123	1.126	1.120	1.109	1.094	1.078	1.061	1.044	1.027	1.014	1.005	1.002
1995	1.186	1.086	1.097	1.113	1.126	1.129	1.123	1.112	1.097	1.081	1.063	1.046	1.029	1.015	1.006	1.002
2000	1.187	1.087	1.098	1.114	1.128	1.132	1.126	1.115	1.100	1.083	1.065	1.048	1.030	1.015	1.006	1.002

Table 5.7 rebuts the conjecture that the paid loss development factors for earlier (as well as middle) development years will hold constant over successive accident years. However, it is also evident that the rate of increase in these paid development factors is small. It is small enough that it would not be detectable to an experienced actuary reviewing historical PLDFs. This becomes even more evident if we look at different sections of the typical triangle of paid loss development factors that are generated by the heuristic model. In Table 5.8 the individual PLDFs generated by the model are displayed for AYs 1990-2002 for the earliest development years.

Table 5.8
PLDFs Factors Indicated by the Heuristic Model During Early Years of Development

	Years of Development										
AY	2	3	4	5	6	7	8	9	10	11	12
1990	6.81875	1.59471	1.16775	1.09383	1.05240	1.04154	1.03101	1.02670	1.02182	1.01604	1.01618
1991	6.81875	1.59488	1.16781	1.09387	1.05243	1.04157	1.03104	1.02673	1.02185	1.01607	1.01621
1992	6.81875	1.59505	1.16786	1.09392	1.05246	1.04160	1.03107	1.02676	1.02187	1.01609	1.01623
1993	6.81875	1.59522	1.16792	1.09396	1.05250	1.04163	1.03110	1.02679	1.02190	1.01611	1.01625
1994	6.81875	1.59539	1.16797	1.09400	1.05253	1.04166	1.03113	1.02681	1.02192	1.01613	1.01628
1995	6.81875	1.59557	1.16803	1.09405	1.05256	1.04169	1.03115	1.02684	1.02195	1.01615	1.01630
1996	6.81875	1.59571	1.16807	1.09408	1.05259	1.04172	1.03118	1.02686	1.02197	1.01617	1.01632
1997	6.81875	1.59586	1.16812	1.09412	1.05261	1.04174	1.03120	1.02688	1.02199	1.01618	1.01634
1998	6.81875	1.59601	1.16816	1.09415	1.05263	1.04176	1.03122	1.02691	1.02201	1.01620	1.01636
1999	6.81875	1.59616	1.16821	1.09419	1.05266	1.04179	1.03124	1.02693	1.02203	1.01622	1.01638
2000	6.81875	1.59631	1.16825	1.09422	1.05268	1.04181	1.03126	1.02695	1.02205	1.01623	1.01639
2001	6.81875	1.59647	1.16830	1.09426	1.05271	1.04184	1.03129	1.02697	1.02208	1.01625	1.01642
2002	6.81875	1.59662	1.16835	1.09430	1.05273	1.04186	1.03131	1.02699	1.02210	1.01627	1.01644

The constant PLDFs in the column for DY 2 merely reflect a simplifying assumption in the model.

In Table 5.9 individual PLDFs generated by the model are displayed for accident years 1966-1977 for the most mature historical development years. Projected PLDFs for the short term future are also shown below the diagonal.

Table 5.9
PLDFs Indicated by the
Heuristic Model During Later Years of Development

	Year of Development										
AY	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**
1966	1.02103	1.02124	1.02139	1.02147	1.02149	1.02146	1.02136	1.02121	1.02101	1.02077	1.02049
1967	1.02112	1.02134	1.02149	1.02157	1.02160	1.02156	1.02147	1.02132	1.02113	1.02088	1.02060
1968	1.02121	1.02143	1.02159	1.02168	1.02170	1.02167	1.02158	1.02143	1.02124	1.02100	1.02072
1969	1.02130	1.02153	1.02168	1.02178	1.02181	1.02178	1.02169	1.02154	1.02135	1.02111	1.02083
1970	1.02140	1.02163	1.02179	1.02189	1.02192	1.02189	1.02180	**1.02166**	1.02147	1.02123	1.02095
1971	1.02148	1.02171	1.02187	1.02198	1.02201	1.02199	1.02190	1.02176	1.02157	1.02133	1.02106
1972	1.02155	1.02179	1.02196	1.02207	1.02211	1.02209	1.02200	1.02187	1.02168	1.02144	1.02116
1973	1.02163	1.02187	1.02205	1.02216	1.02220	1.02218	1.02211	1.02197	1.02178	1.02155	1.02127
1974	1.02170	1.02195	1.02213	1.02225	1.02230	1.02228	1.02221	1.02208	1.02189	1.02165	1.02138
1975	1.02178	1.02203	1.02222	1.02234	1.02239	1.02238	1.02231	**1.02218**	1.02200	1.02176	1.02148
1976	1.02188	1.02214	1.02233	1.02245	1.02250	1.02250	1.02243	1.02230	1.02211	1.02188	1.02160
1977	1.02199	1.02225	1.02244	1.02256	1.02262	1.02261	1.02254	1.02241	1.02223	1.02200	1.02172

Table 5.10 provides an example of the kinds of errors in estimating future incremental payments that can occur when it is assumed that PLDFs for each year of development hold constant. First, a PLDF of 1.02138 is selected as the average of the latest four historical factors during the 34th year of development (shaded in gray in Table 5.9). By comparing this selection with the true underlying trended PLDF, the percentage error in incremental payments for that development year is shown for every fifth AY. These errors assume, however, that other similar errors did not occur during preceding development years.

Table 5.10
Errors in PLDFs During 34th Year of Development
Due to Selecting a Constant Historical Average PLDF

Accident Year	Selected PLDF	True Underlying PLDF	% Error in Incremental Payments
1970	1.02138	**1.02166**	-1.3%
1975	1.02138	**1.02218**	-3.6%
1980	1.02138	1.02276	-6.1%
1985	1.02138	1.02336	-8.5%
1990	1.02138	1.02395	-10.7%
1995	1.02138	1.02452	-12.8%
2000	1.02138	1.02507	-14.7%

Though all of the errors above are small, these errors compound significantly in the calculation of tail factors, which are the product of numerous individual PLDFs.

Even though it is true that past declines in mortality rates are implicitly embedded in historical PLDFs, the above example clearly illustrates that it would be incorrect to assume that the selection of historical factors as estimates of future PLDFs would implicitly incorporate the effects of future declines in mortality rates. What would be more appropriate would be to select representative PLDFs for each development year based on recent historical factors and then to trend these upward in a manner parallel to the PLDFs indicated by a realistic model.

6. A COMPARISON OF INDICATED TAIL FACTORS

Table 6.1 provides a comparison of the MPD tails indicated by SAIF's own loss experience with those indicated by the static and trended mortality methods. This table repeats the MPD tails indicated by SAIF's experience in Table 2.6.

Table 6.1
A Comparison of Indicated MPD Tail Factors

Maturity (Years)	Based on SAIF's Experience	Based on Static Mortality Model	Based on Trended Mortality Model
10	2.469	2.684	3.025
15	2.328	2.469	2.783
25	2.054	2.019	2.271
35	1.680	1.594	1.776

As noted earlier, the indications of the static mortality model reasonably fit those from SAIF's historical loss experience—except that the model somewhat understates development for DYs 40-54.

The relative contribution of MPD versus All Other WC to the total loss reserves for a given AY is much greater if the trended mortality model is assumed. Those percentages at various maturities are shown in the last column of Table 6.2.

Table 6.2
Indicated Loss Reserve at Different Maturities
(Dollars in millions)

Maturity (Years)	MPD Reserve	Other WC Reserve	MPD Reserve as a %-age of Total WC Reserve
10	$41.3	$10.4	80%
15	39.6	9.5	81%
25	34.6	5.7	86%
35	27.0	2.5	92%

The above table is analogous to Table 2.8, which shows results based on SAIF's historical loss experience. In deriving these estimates, total AY ultimate losses of $100 million were assumed, together with a 50-50 split between MPD and Other WC. However, the $50 million figure for ultimate MPD was changed to the product of cumulative paid MPD at 10 years of development times the 10 to ultimate tail factor from the trended mortality model. That increased ultimate MPD to $61.75 million.

Table 6.3 provides a side-by-side comparison of the percentages of the total WC loss reserve attributable to MPD—as estimated using historical PLDFs and PLDFs indicated by the trended mortality model.

Table 6.3
Comparison of MPD Loss Reserve as a Percentage of the
Total WC Loss Reserve (Based on Different PLDF Assumptions)

Maturity (Years)	Indicated by Historical PLDFs	Indicated by Trended Mortality PLDFs	Percentage Increase in MPD Reserve Due to Using Trended Mortality Rates
10	$29.6	$41.3	+39.7%
15	28.3	39.6	+39.6
25	25.5	34.6	+35.8
35	20.0	27.0	+34.9

Clearly, the trended mortality model indicates MPD loss reserves that are significantly larger than straight historical experience would indicate.

7. SENSITIVITY CONSIDERATIONS

The most significant factor affecting the indications in this paper is the applicable retention. This paper presents indications on an unlimited basis. Tail factors and PLDFs at more mature years of development should be expected to be significantly less at relatively low retentions. This is evident on an a priori basis.

Consider a hypothetical PD claim injured on December 15, 2003 at age 35.9

years, with a life expectancy of 40 years. His medical costs are $5,000 during 2004, and future medical inflation is 9% per year. Indemnity losses are a flat $25,000 per year, beginning in 2004. Table 7.1 indicates what cumulative loss payments would total at the end of each of the first 41 years of development.

For this hypothetical PD claimant, net paid losses would top out by the end of the ninth year of development at a retention of $ $250,000; after 16 years with a $500,000 retention; after 26 years at a $1 million retention; and after 37 years at a $2 million retention.

While this dampening effect of retentions can obviously serve to greatly mitigate the magnitude of the applicable tail factors for different insurers and self-insureds, that effect can rapidly dissipate when retentions rise significantly from year to year. It is quite common for insurers as well as self-insureds to significantly increase retentions when faced with costs for excess coverage that have risen substantially as the market has hardened. The effect of recognizing the upward impact greater retentions will have on assumed tails can be sizeable.

Other factors that can have a material impact on MPD tail factors are:

- The assumed future rate of medical cost escalation.
- The observed tendency of medical losses to step up noticeably as a claimant nears death.
- The possibility that actual mortality rates of PD claimants might be higher (or lower) than those for the general populace.
- Variations in the gender mix and age-at-injury mix of PD claimants.

An entire paper could be devoted to quantifying the effects of changes in any or all of the above factors would have on indicated tail factors. Of the above factors, the first is the most significant. While some believe that the long term future rate of medical cost escalation will be less than the historical rate of 9%, others believe a constant 9% assumption is reasonable. Arguably, the differential between medical inflation and general inflation may lessen over future decades. However, long term general inflation may move upward as a result of shortages in critical commodities (such as petroleum) and their ubiquitous derivative products (e.g., plastics and synthetics).

We note that SAIF's actual age-at-injury distribution is weighted heavily toward the middle-age groups. If a much younger distribution were assumed, this would dramatically increase the survival probabilities during each year of development and the resulting tails would be considerably greater than those presented in this paper. The age-at-injury distribution can vary significantly depending on statutory provisions for qualification for a permanent disability award and the nature of the risks insured or self-insured.

Table 7.1
Cumulative Loss Payments for Hypothetical PD Claimant

(A) Age of Claim-Ant	(B) DY	(C) Incre-mental Medical Payments	(D) Cumu-lative Medical Payments	(E) Cumu-lative Indemnity Payments	(F) Cumu-lative Loss Payments	Effects of Retention on Development
35	1	0.0	0.0	0.0	0.0	
36	2	5.0	5.0	25.0	30.0	
37	3	5.5	10.5	50.0	60.5	
38	4	5.9	16.4	75.0	91.4	
39	5	6.5	22.9	100.0	122.9	
40	6	7.1	29.9	125.0	154.9	
41	7	7.7	37.6	150.0	187.6	
42	8	8.4	46.0	175.0	221.0	
43	9	9.1	55.1	200.0	255.1	Development Stops if Ret'n is $250K
44	10	10.0	65.1	225.0	290.1	
45	11	10.9	76.0	250.0	326.0	
46	12	11.8	87.8	275.0	362.8	
47	13	12.9	100.7	300.0	400.7	
48	14	14.1	114.8	325.0	439.8	
49	15	15.3	130.1	350.0	480.1	
50	16	16.7	146.8	375.0	521.8	Development Stops if Ret'n is $500K
51	17	18.2	165.0	400.0	565.0	
52	18	19.9	184.9	425.0	609.9	
53	19	21.6	206.5	450.0	656.5	
54	20	23.6	230.1	475.0	705.1	
55	21	25.7	255.8	500.0	755.8	
56	22	28.0	283.8	525.0	808.8	
57	23	30.5	314.4	550.0	864.4	
58	24	33.3	347.7	575.0	922.7	
59	25	36.3	383.9	600.0	983.9	
60	26	39.6	423.5	625.0	1,048.5	Development Stops if Ret'n is $1,000K
61	27	43.1	466.6	650.0	1,116.6	
62	28	47.0	513.6	675.0	1,188.6	
63	29	51.2	564.8	700.0	1,264.8	
64	30	55.8	620.7	725.0	1,345.7	
65	31	60.9	681.5	750.0	1,431.5	
66	32	66.3	747.9	775.0	1,522.9	
67	33	72.3	820.2	800.0	1,620.2	
68	34	78.8	899.0	825.0	1,724.0	
69	35	85.9	984.9	850.0	1,834.9	
70	36	93.6	1,078.6	875.0	1,953.6	
71	37	102.1	1,180.6	900.0	2,080.6	Development Stops if Ret'n is $2,000K
72	38	111.3	1,291.9	925.0	2,216.9	
73	39	121.3	1,413.1	950.0	2,363.1	
74	40	132.2	1,545.3	975.0	2,520.3	
75	41	144.1	1,689.4	1,000.0	2,689.4	

In the static mortality model, we started with the assumption of a beginning gender mix of 75% male and 25% female. Because of the higher mortality rates of males at all ages, by the 50th year of development, the percentage of surviving claimants that are male is expected to drop to 64.5%. By the 72nd year of development, a 50-50 gender split is expected.

8. ESTIMATING THE EXPECTED VALUE OF MPD RESERVES

In Table 7.1 cumulative loss payments for a hypothetical PD claimant are displayed. This might be a profile of paid losses for a male claimant injured on December 15, the reserve evaluation date. At age 35.9, the claimant is expected to live another 40 years. Two different methods of estimating the ***medical*** ***case reserve*** for this claimant at the end of the first year of development are common. They are:

1. **First Method:** *Zero Inflation Case Reserve Based on Projected Payments Through Expected Year of Death*. Estimated annual medical expenses of $5,000 per year (during the first full year of development) are multiplied by the life expectancy of 40 years to obtain a case reserve of $200,000.
2. **Second Method:** *9% Inflation Case Reserve Based on Projected Payments Through Expected Year of Death*. Escalating medical expenses are cumulated up through age 75, yielding a total incurred of $1,689,000.

Two additional methods may also be applied. Each of these produces much higher, and more accurate, estimates of the expected value of the case reserve:

3. **Third Method:** *Expected Total Payout over Scenarios of All Possible Years of Death*. This method, described below, yields an expected reserve of $2,879,000.
4. **Fourth Method:** *Expected Value of Trials from a Markov Chain Simulation.* This method, described in Section 9, yields an expected reserve of $2,854,000.

In applying the third method, cumulative payments are calculated through each possible future year of death. Each of these estimates represents the scenario of the claimant's death during a specific (n-th) year of development. The probability of occurrence of the n-th scenario is the product of the probability the claimant will live through all prior years of development, and then die during the n-th year of development. The expected value of the case reserve is then the weighted average of all of these estimates of final cumulative payments, weighted by their associated probability of occurrence. In this example, the expected value of total incurred is $2,879,000, which is 70.5% higher than the second estimate. This kind of estimate is not customarily calculated for PD claimants. Yet it is in keeping with the standard definition of the expected value of total incurred.

The total case reserve based on this third approach is dramatically higher than that derived from the second approach because the cumulative paid amounts associated with death at ages beyond the claimant's expected year of death are given more weight—due to the compounding effects of medical cost escalation.

In Tables 8.1a and 8.1b the medical case reserve for the hypothetical PD claimant is calculated for the second and third methods. For the second method, Projected Payments Through Expected Year of Death, the cumulative payments from Column (F) at the end of the expected year of death (at age 75), yields the estimate of $1,689,000.

For the third method, each row is treated as a different scenario, with its probability of occurrence shown in Column (C). These probabilities are the weights applied to the estimates of cumulative medical payments in Column (F) to obtain the components of the expected total payout in Column (G) that are cumulated in Column (H). Hence, the expected value of the case reserve is the bottom number in Column (H) in Table 8.1b ($2,879,000).

The distribution of deaths by age of death (Column (C)) would be the same as the distribution of the different scenarios for the *indemnity* case reserve, since incremental indemnity payments are not subject to inflation. Figure 8.1 illustrates the shift in the distribution of the different scenarios for the *medical* case reserve (Column (I) decumulated, or Column (G)/(Total of Column (G))—due to the effects of compounding medical cost escalation in giving more dollar weight to scenarios where the claimant lives beyond his expected year of death.

The impact of medical cost escalation shifts the median age of death (and of total indemnity payments) from 77 to 87 (for total medical payments), or ten years. This can be seen by comparing the age corresponding to a cumulative probability of 50% in Column (D) to the age when Column (I) reaches 50%. To further appreciate the significance of this shift, consider the following observations drawn from Table 8.1b:

- While 83% of such claimants die before they reach the age of 87, medical payments to claimants who live beyond 86 years of age account for over half of total expected future medical payments.
- While 90% of such claimants die before they reach the age of 90, medical payments to claimants who live beyond 89 years of age account for over 30% of total expected medical losses.

The ratio of the estimated case reserve based on the second method to that from the first method varies dramatically with the age of the claimant at the reserve date. It is also dependent on gender. This is also true, though to a lesser degree, for the ratio of the third method case reserve to the second method reserve. These ratios are displayed in Table 8.2.

Table 8.1a

Calculation of Case Reserve by Second and Third Methods

Male Claimant, Age 35.9 at Reserve Date, 9% Future Inflation Assumed

Age	(A) l(x)	(B) d(x)	(C) Probability of Dying at Age x (B)/(A)@36	(D) Cumulative Probability of Dying at Age x	(E) Incre-mental Medical Paid	(F) Cumu-lative Medical Paid	(G) Expected Total Payout (C) x (G)	(H) Cumulative Expected Total Payout	(I) % of Expected Total Payout
36	96023	198	0.00206	0.206%	5	5	0.01	0.01	0.000%
37	95825	209	0.00218	0.424%	5	10	0.02	0.03	0.001%
38	95616	225	0.00234	0.658%	6	16	0.04	0.07	0.003%
39	95391	241	0.00251	0.909%	6	23	0.06	0.13	0.005%
40	95150	260	0.00271	1.180%	7	30	0.08	0.21	0.007%
41	94890	279	0.00291	1.470%	8	38	0.11	0.32	0.011%
42	94611	300	0.00312	1.783%	8	46	0.14	0.46	0.016%
43	94311	321	0.00334	2.117%	9	55	0.18	0.65	0.023%
44	93990	343	0.00357	2.474%	10	65	0.23	0.9	0.031%
45	93647	368	0.00383	2.858%	11	76	0.29	1.2	0.041%
46	93279	395	0.00411	3.269%	12	88	0.36	1.5	0.054%
47	92884	422	0.00439	3.708%	13	101	0.44	2.0	0.069%
48	92462	448	0.00467	4.175%	14	115	0.54	2.5	0.088%
49	92014	477	0.00497	4.672%	15	130	0.65	3.2	0.111%
50	91537	508	0.00529	5.201%	17	147	0.78	3.9	0.138%
51	91029	544	0.00567	5.767%	18	165	0.9	4.9	0.171%
52	90485	583	0.00607	6.375%	20	185	1.1	6.0	0.210%
53	89902	629	0.00655	7.030%	22	207	1.4	7.3	0.257%
54	89273	679	0.00707	7.737%	24	230	1.6	9	0.314%
55	88594	735	0.00765	8.502%	26	256	2.0	11	0.383%
56	87859	797	0.00830	9.332%	28	284	2.4	13	0.466%
57	87062	865	0.00901	10.233%	31	314	2.8	16	0.565%
58	86197	936	0.00975	11.208%	33	348	3.4	20	0.684%
59	85261	1014	0.01056	12.264%	36	384	4.1	24	0.826%
60	84247	1096	0.01141	13.405%	40	424	4.8	28	0.995%
61	83151	1184	0.01233	14.638%	43	467	5.8	34	1.197%
62	81967	1287	0.01340	15.978%	47	514	6.9	41	1.438%
63	80680	1405	0.01463	17.442%	51	565	8.3	49	1.728%
64	79275	1532	0.01595	19.037%	56	621	10	59	2.075%
65	77743	1669	0.01738	20.775%	61	682	12	71	2.490%
66	76074	1803	0.01878	22.653%	66	748	14	85	2.982%
67	74271	1923	0.02003	24.656%	72	820	16	102	3.558%
68	72348	2023	0.02107	26.762%	79	899	19	120	4.222%
69	70325	2109	0.02196	28.959%	86	985	22	142	4.980%
70	68216	2203	0.02294	31.253%	94	1,079	25	167	5.847%
71	66013	2305	0.02400	33.653%	102	1,181	28	195	6.840%
72	63708	2407	0.02507	36.160%	111	1,292	32	228	7.903%

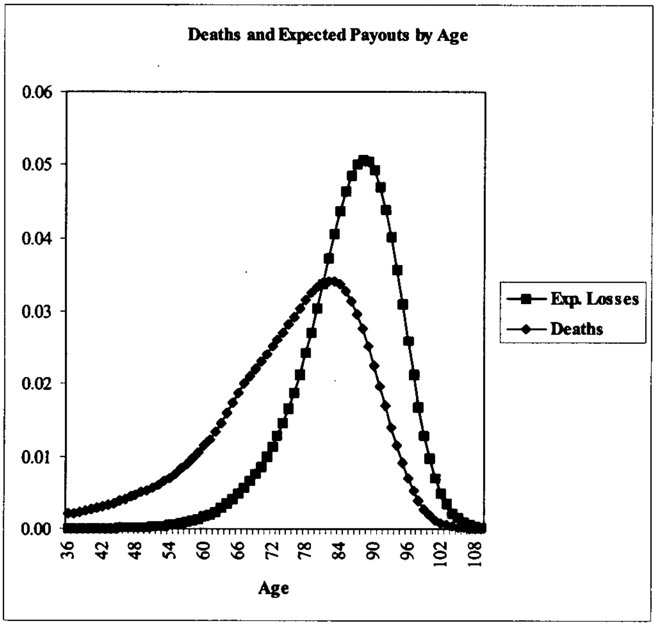
Table 8.1b

Calculation of Case Reserve by Second and Third Methods

Male Claimant, Age 35.9 at Reserve Date, 9% Future Inflation Assumed

Age	(A) l(x)	(B) d(x)	(C) Probability of Dying at Age x (B)/(A)@36	(D) Cumulative Probability of Dying at Age x	(E) Incre-mental Medical Paid	(F) Cumu-lative Medical Paid	(G) Expected Total Payout (C) x (G)	(H) Cumulative Expected Total Payout	(I) % of Expected Total Payout
73	61301	2504	0.02608	38.768%	121	1,413	37	264	9.182%
74	58797	2603	0.02711	41.479%	132	1,545	42	306	10.637%
75	56194	2704	0.02816	44.295%	144	1,689	48	354	12.289%
76	53490	2808	0.02924	47.219%	157	1,846	54	408	14.164%
77	50682	2915	0.03036	50.255%	171	2,018	61	469	16.292%
78	47767	3021	0.03146	53.401%	187	2,204	69	538	18.700%
79	44746	3119	0.03248	56.649%	203	2,408	78	617	21.416%
80	41627	3199	0.03331	59.980%	222	2,629	88	704	24.458%
81	38428	3253	0.03388	63.368%	242	2,871	97	802	27.836%
82	35175	3281	0.03417	66.785%	263	3,134	107	909	31.555%
83	31894	3276	0.03412	70.197%	287	3,421	117	1,025	35.609%
84	28618	3232	0.03366	73.563%	313	3,734	126	1,151	39.974%
85	25386	3147	0.03277	76.840%	341	4,075	134	1,285	44.612%
86	22239	3020	0.03145	79.985%	372	4,447	140	1,424	49.470%
87	19219	2852	0.02970	82.955%	405	4,852	144	1,569	54.475%
88	16367	2649	0.02759	85.714%	442	5,294	146	1,715	59.547%
89	13718	2414	0.02514	88.228%	481	5,776	145	1,860	64.589%
90	11304	2159	0.02248	90.476%	525	6,300	142	2,002	69.509%
91	9145	1890	0.01968	92.445%	572	6,873	135	2,137	74.207%
92	7255	1619	0.01686	94.131%	624	7,496	126	2,263	78.596%
93	5636	1355	0.01411	95.542%	680	8,176	115	2,379	82.603%
94	4281	1106	0.01152	96.694%	741	8,916	103	2,481	86.169%
95	3175	878	0.00914	97.608%	807	9,724	89	2,570	89.257%
96	2297	676	0.00704	98.312%	880	10,604	75	2,645	91.850%
97	1621	506	0.00527	98.839%	959	11,563	61	2,706	93.966%
98	1115	367	0.00382	99.221%	1,046	12,609	48	2,754	95.639%
99	748	258	0.00269	99.490%	1,140	13,749	37	2,791	96.922%
100	490	178	0.00185	99.675%	1,242	14,991	28	2,819	97.888%
101	312	119	0.00124	99.799%	1,354	16,346	20	2,839	98.591%
102	193	77	0.00080	99.879%	1,476	17,822	14	2,853	99.087%
103	116	49	0.00051	99.930%	1,609	19,431	10	2,863	99.432%
104	67	29	0.00030	99.960%	1,754	21,185	6	2,870	99.654%
105	38	18	0.00019	99.979%	1,912	23,096	4	2,874	99.804%
106	20	10	0.00010	99.990%	2,084	25,180	3	2,876	99.895%
107	10	5	0.00005	99.995%	2,271	27,451	1	2,878	99.945%
108	5	4	0.00004	99.999%	2,476	29,927	1	2,879	99.988%
109	1	1	0.00001	100.000%	2,698	32,625	0	2,879	100.00%

Figure 8.1



There are a number of reasons to believe that the reserve estimates produced by the static mortality model presented in Section 3 are analogous to estimates produced by the second method. If that is true, then it would be necessary to multiply reserve estimates based on the static mortality model by some weighted average of the ratios in Column (E) of Table 8.2 to arrive at an estimated reserve at the expected level. Whether that ratio is 1.25 or 1.40 or 1.55, it represents a substantial add-on to a reserve estimate that is likely higher than what would be obtained using more traditional methods.

Why are reserve estimates based on the static mortality model similar to those produced by the second method? A fundamental assumption of the model is that all claimants die according to a schedule dictated by current mortality tables. When an expected value of the reserve is calculated, it is based on a weighted average of a full range of scenarios, including those where many claimants die earlier than plan and others die later. Total future payments for those claimants that die later will be given more dollar weight. Hence, the expected value of the reserve will be correspondingly greater than that projected by the static mortality model.

Table 8.2
Comparison of Different Types of MPD Reserve Estimates
Assuming SSA 2000 Male & Female Mortality Tables and 9% Medical Cost Escalation

	(A)	(B)	(C)	(D)	(E)
	Reserve ($000's) at Eval. Date				
Age at Reserve Date	First Method (Zero Inflation Case Reserve)	Second Method (9% Inflation Case Reserve)	Third Method (Total Expected Future Payout)	Ratio of Second Method Reserve to First Method Reserve	Ratio of Third Method Reserve to Second Method Reserve
			Male Claimants		
20	273.7	7,333.9	11,318.1	26.795	1.543
30	227.3	2,989.5	4,816.3	13.155	1.611
40	181.2	1,321.0	2,042.3	7.290	1.546
50	137.3	590.0	864.0	4.298	1.464
60	96.7	265.3	362.9	2.744	1.368
70	62.9	123.5	153.2	1.965	1.240
80	36.0	57.1	63.4	1.587	1.110
			Female Claimants		
20	301.0	10,796.0	16,724.2	35.867	1.549
30	252.4	4,641.7	7,069.1	18.390	1.523
40	204.7	2,005.7	2,983.6	9.800	1.488
50	158.4	873.8	1,254.5	5.516	1.436
60	115.1	384.3	524.0	3.341	1.363
70	77.0	165.0	217.3	2.144	1.317
80	45.2	76.3	87.2	1.690	1.142

9. ESTIMATING THE VARIABILITY OF THE MPD RESERVE WITH A MARKOV CHAIN SIMULATION

The size of loss distribution for the medical component of a single PD claim is far more skewed to the right than can be modeled by distributions commonly used by casualty actuaries. This distribution can be described by the ultimate costs in Column (F) of Tables 8.a and 8.b, with the associated confidence levels taken from Column (D). In attempting to find a distribution that produced a reasonable fit, it was necessary to first transform the ultimate cost amounts by taking the natural log of the natural log of the natural log and then taking the n-th root—before a common distribution could be found. Taking the fifth root of the triple natural log appears to produce a distribution of ultimate costs that conforms well with an extreme value distribution. The fact that such intense transformations were needed suggests that a totally different approach than fitting commonly used distributions should be used.

As is indicated from Table 8.2, the ratio of the Expected Value of the individual case reserve to the Projected Payments Through Expected Year of Death estimate varies dramatically according to the gender and current age of each claimant. This suggests that the variability of the total MPD reserve can best be modeled by simulating the variability of the future payout for each claim separately. Table 9.1 provides a sample framework for this type of simulation. The example insurer has ten open PD claims.

Table 9.1
Layout for Simulation of Variability of Total MPD Reserve at Year-End 2003

Claim Number	Gender	Current Age	Projected Annual Medical Costs ($000's) During: 2004	2005	2006	...	2038	2039	2040	...	2068	2069	2070	Total Future Medical Payments
1	F	75	3.2	3.5	3.8		0	0	0		0	0	0	10.5
2	F	47	5.6	6.1	6.7		105	114	125		0	0	0	2,354.8
3	M	22	1.9	2.1	2.3		35.6	38.8	42.2		472	515	0	6,211.4
4	M	46	0.7	0.8	0.8		13.1	14.3	15.6		0	0	0	312.1
5	M	55	12.7	13.8	15.1		0	0	0		0	0	0	181.4
6	F	82	6.3	6.9	7.5		0	0	0		0	0	0	55.2
7	M	66	8.1	8.8	9.6		0	0	0		0	0	0	99.7
8	M	34	1.2	1.3	1.4		22.5	24.5	26.7		0	0	0	443.8
9	F	57	4.4	4.8	5.2		82.4	0	0		0	0	0	949.1
10	M	71	3.6	3.9	4.3		67.4	73.5	80.1		0	0	0	1,468.3
													Total:	**12,086.3**

An individual row in Table 9.1 is devoted to each open claim. Census data on the gender and current age of each living PD claimants appears in two columns on the left side of the table. Consider claim number 1 in the top row. Actual medical payments in 2003 were $3,000. A random number between 0 and 1 is generated. If that number is between 0 and q_{75}, the claimant dies during 2004. If the random number is greater than q_{75}, the claimant lives throughout 2004.

In effect, in Table 9.1, projected annual medical costs for each future year are estimated via a Markov chain simulation model. The state space consists of two outcomes from each trial: 1) the claimant does not die during a given future DY; or 2) the claimant dies during that DY. The transition probabilities in this model are simply the $(1 - q_x)$ and q_x values from a mortality table. The outcome of any trial depends at most upon the outcome of the immediately preceding trial and not upon any other previous outcome. Death is an "absorbing" state, since one cannot transition out of it.

An assumed rate of medical cost escalation of 9% per year is applied to the prior year's payments if the claimant lives throughout the year. Otherwise, if the claimant dies during the year, projected medical payments for the year are still shown, after which medical losses drop to zero for every future year of development. While projected medical payments may arguably be only for half a year, assuming the average claimant dies in the middle of the final year of development, in reality medical costs are often higher during the year of death. So assuming a full year's worth of medical payments is a reasonable assumption.

For each trial, total projected future payments from the cell at the bottom right are recorded and confidence levels for the reserve can be derived from a ranking of all of the simulated total reserve estimates. If this is done for a single claim, the resulting probability distribution closely conforms to that described in the first paragraph of this section.

Simulating the variability of the MPD reserve for unreported claims is naturally more complicated. First, the total number of IBNR claims should be represented by a Poisson (or similar) distribution. Then census data of the age at injury of recent claimants can be used to randomly generate these ages for unreported claimants. Then additional rows can be added to Table 9.1 to further simulate future payments for each unreported claimant. The degree of variability of the MPD reserve for unreported claimants is exceptionally high—because some of those claimants may have been quite young when injured, and the total expected future payment for workers injured at a young age is dramatically higher than for those injured at an older age. An appreciation for this can be gained by reviewing either Columns (B) or (C) of Table 8.2. For example, the total expected future payout for a female who is 20 at the reserve date is $16.7 million, while it is only $3.0 million if she is 40.

10. CONCLUDING REMARKS

In this paper we have seen that common actuarial methods will tend to underestimate the true MPD loss reserve. This is also the case for typical methods of estimating MPD reserves at higher confidence levels—based on commonly used size-of-loss distributions. The need to develop and apply new methods that directly reflect the characteristics of MPD payments is substantial.

REFERENCES

[1] Blumsohn, Gary, "Levels of Determinism in Workers' Compensation Reinsurance Commutations", *Proceedings of the Casualty Actuarial Society,* 1999 Vol. LXXXVI, Part I, pp. 1 – 79.

[2] Sherman, Richard E., "Extrapolating, Smoothing and Interpolating Development Factors", *Proceedings of the Casualty Actuarial Society,* 1984 Vol: LXXI pp. 122-155

[3] Workers' Compensation Insurance Rating Bureau of California Bulletin No. 2003-24, p. 1-38.

[4] Fisher, W.H.; and Lange, J.T., "Loss Reserve Testing: A Report Year Approach," *PCAS* LX, 1973, pp. 189-207

[5] Adler, M.; and Kline, C.D. Jr., "Evaluating Bodily Injury Liabilities Using a Claims Closure Model," *Evaluating Insurance Company Liabilities*, Casualty Actuarial Society *Discussion Paper Program*, 1988, pp. 1-66

[6] Gillam, William R., "Injured Worker Mortality", *Casualty Actuarial Society Forum,* Winter 1991, pp.89-114.

[7] Venter, Gary G., Schill, Barbara, and Barnett, Jack, "Review of Report of Committee on Mortality for Disabled Lives", *Casualty Actuarial Society Forum,* Winter 1991, pp.115-137.

APPENDIX A
THE MUELLER INCREMENTAL TAIL (MIT) METHOD

The MIT method calculates tail factors based of cumulative paid loss development triangles augmented by incremental calendar year payments from older accident years.

The method was described in Section 2 of the paper as consisting of three stages:

1 Incremental age-to-age factors
2 Anchored decay factors
3 Tail factors

This appendix provides more specifics regarding the first two stages.

1. **Incremental age-to-age factors.** The first step is to calculate incremental age to age factors. With the SAIF data, we can calculate incremental paid at age (n+1) to incremental paid at age (n) for n ranging from 29 to 57 years, using twenty-year weighted averages.

Tables A.1 through A.3 display incremental MPD payments for DYs 29 through 40, 40 through 50, and 50 through 60 respectively.

Because the underlying data for any individual accident year is volatile, the age to age factors were smoothed using centered moving averages. The empirical age to age decay factors and smoothed factors are shown in Table A.4.

The empirical factors are calculated directly from the raw data. The centered average is a simple five year average based on the empirical factor averaged with the two factors above and the two below. When it was not possible to calculate a five year average, shorter term centered averages were used.

The weighted average is similar but uses corresponding paid losses as weights. The geometric mean provides another level of smoothing. It is also a five year centered average, but it is the fifth root of the product of the five weighted average factors.

2. **Anchored decay factors.** After selecting the geometric mean incremental age to age factors, they are then anchored to a base year. Table A.5 shows the anchored decay factors using five different anchor years. The anchored decay factors represent incremental payments made in year n relative to payments made in the anchor year.

Table A.1
Derivation of Incremental Age-to-Age Decay Factors for DYs 30 to 40

	Incremental Payments ($000's) During Development Year X:											
AY	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
1943												4
1944											1	3
1945										16	14	20
1946									1	11	19	5
1947								0	0	3	1	5
1948							7	3	9	16	6	17
1949						49	27	52	29	12	15	48
1950					16	28	20	26	8	30	16	22
1951				17	16	5	7	6	16	6	2	11
1952			4	3	16	11	9	26	32	16	62	9
1953		32	21	14	17	16	28	11	11	9	31	17
1954	54	43	48	59	80	52	109	44	65	81	59	63
1955	25	16	20	14	13	26	33	8	41	22	12	14
1956	45	66	35	68	44	48	24	68	17	40	36	13
1957	53	57	51	35	21	20	39	60	38	36	79	51
1958	26	30	29	33	23	24	30	14	9	75	7	4
1959	110	138	75	81	81	195	122	161	127	148	116	84
1960	47	89	56	71	107	94	69	46	30	26	89	203
1961	97	97	146	118	140	105	101	109	91	121	81	95
1962	96	80	60	46	55	114	57	23	85	108	64	118
1963	82	239	84	81	82	101	85	47	48	36	56	46
1964	465	177	210	178	28	65	106	34	36	55	168	
1965	143	123	107	191	150	153	53	75	69	93		
A) Sum x 1st:		1155	943	994	874	1056	919	812	762	942	933	847
B) Sum Prior x Last:		1241	1187	947	1011	890	1105	926	812	763	865	766
C) Indicated Decay Ratios:		0.931	0.794	1.050	0.864	1.187	0.832	0.877	0.938	1.235	1.079	1.106
D) Selected Decay Ratios:		0.930	0.933	0.937	0.937	0.953	0.958	0.980	1.001	1.048	1.063	1.088
PLDF - 1.0:	0.025	0.030	0.028	0.026	0.025	0.023	0.023	0.022	0.022	0.023	0.022	0.021
PLDF:	1.025	1.030	1.028	1.026	1.025	1.023	1.023	1.022	1.022	1.023	1.022	1.021
Model:	1.024	1.024	1.024	1.024	1.024	1.024	1.024	1.023	1.023	1.022	1.022	1.021

Notes:

1) The selected decay ratios were derived in Table A.4. See last column.
2) The PLDFs for DYs 29-37 were derived in Table A.6. See Row I).
3) The (PLDF - 1.0)'s for ages 38 through 40 were computed as the product of the previous (PLDF - 1.0) and the current decay ratio, divided by the prior PLDF.

Table A.2
Derivation of Incremental Age-to-Age Decay Factors for DYs 40 to 50

	Incremental Payments ($000's) During Development Year X:										
AY	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
1933											2
1934										2	0
1935									1	4	14
1936								15	0	3	5
1937							0	4	0	1	0
1938						0	0	1	1	0	2
1939					0	3	0	1	0	0	0
1940				1	2	1	3	0	0	4	0
1941			4	1	1	0	0	0	0	0	1
1942		0	7	8	3	1	16	2	0	2	0
1943	4	1	11	4	3	2	4	1	6	7	8
1944	3	3	6	2	1	3	1	1	0	0	0
1945	20	24	17	14	6	15	(1)	50	73	75	63
1946	5	5	(5)	4	9	4	2	30	31	29	31
1947	5	(2)	0	4	0	32	0	0	0	3	5
1948	17	7	2	1	1	12	0	6	7	14	3
1949	48	42	17	9	39	7	20	41	83	225	116
1950	22	18	43	24	11	165	71	9	2	4	1
1951	11	32	12	13	6	4	23	26	19	10	18
1952	9	48	7	7	170	44	12	1	1	22	1
1953	17	10	7	23	13	18	37	15	43	70	68
1954	63	83	49	67	70	142	67	62	96	101	
1955	14	21	26	28	26	21	67	15	13		
1956	13	9	15	21	35	17	8	33			
1957	51	66	367	116	51	28	94				
1958	4	10	19	32	41	21					
1959	84	93	88	83	87						
1960	203	133	181	230							
1961	95	74	158								
1962	118	105									
1963	46										
Sum x 1st:		782	1027	691	575	540	424	298	375	574	336
Sum Prior x Last:		806	677	873	462	488	519	330	280	363	475
Indicated Decay Ratios:		0.970	1.517	0.792	1.245	1.107	0.817	0.903	1.339	1.581	0.707
Selected Decay Ratios:		1.098	1.101	1.056	1.054	1.058	1.044	1.031	1.047	1.023	0.946
PLDF - 1.0:	0.025	0.027	0.029	0.029	0.030	0.031	0.031	0.031	0.032	0.032	0.029
PLDF:	1.025	1.027	1.029	1.029	1.030	1.031	1.031	1.031	1.032	1.032	1.029
Model:	1.021	1.021	1.020	1.020	1.019	1.019	1.018	1.017	1.017	1.016	1.015

Notes:
1) The selected decay ratios were derived in Table A.4. See last column.
2) The (PLDF - 1.0)'s for ages 40 through 50 were computed as the product of the previous (PLDF - 1.0) and the current decay ratio, divided by the prior PLDF.

Table A.3
Derivation of Incremental Age-to-Age Decay Factors for DYs 50 to 60

	Incremental Payments During Development Year X:										
AY	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
1926								0	3	0	0
1927							0	0	2	0	0
1928						0	0	0	0	0	15
1929					9	5	1	4	0	0	0
1930				0	0	0	0	0	0	0	0
1931			0	0	0	0	0	0	0	0	0
1932		0	0	0	0	0	0	0	0	0	0
1933	2	0	0	0	0	0	0	0	0	0	0
1934	0	0	0	0	0	0	0	0	0	0	0
1935	14	4	0	1	0	0	0	9	0	0	1
1936	5	2	7	1	0	0	0	0	0	0	4
1937	0	0	15	13	0	0	0	0	0	0	0
1938	2	10	0	3	4	0	0	1	0	0	0
1939	0	0	0	0	0	0	0	0	0	0	0
1940	0	0	0	0	0	0	0	0	0	0	0
1941	1	1	0	1	5	4	10	37	9	0	0
1942	0	0	1	1	0	0	0	0	0	0	0
1943	8	2	7	8	3	1	0	0	10	0	0
1944	0	0	0	0	0	1	3	1	4	2	
1945	63	63	48	43	35	34	71	11	6		
1946	31	32	7	14	23	6	1	2			
1947	5	0	0	4	0	1	0				
1948	3	0	4	0	35	5					
1949	116	5	8	1	0						
1950	1	3		2							
1951	18	44	32								
1952	1	1									
1953	68										
Sum x 1st:		167	129	92	105	57	86	65	34	2	20
Sum Prior x Last:		270	166	97	90	114	52	86	63	28	0
Indicated Decay Ratios:		0.619	0.777	0.948	1.167	0.500	1.654	0.756	0.540	0.071	
Selected Decay Ratios:		0.888	0.868	0.850	0.851	0.919	1.002	1.067	1.151		
PLDF - 1.0:	0.029	0.025	0.021	0.018	0.015	0.013	0.013	0.014	0.016		
PLDF:	1.029	1.025	1.021	1.018	1.015	1.013	1.013	1.014	1.016		
Model:	1.015	1.015	1.014	1.013	1.013	1.012	1.011	1.010	1.010		

Notes: 1) The selected decay ratios were derived in Table A.4. See last column.
2) The (PLDF - 1.0)'s for ages 50 through 58 were computed as the product of the previous (PLDF - 1.0) and the current decay ratio, divided by the prior PLDF.

Table A.4
Calculation of Age to Age Decay Factors

Age to Age	Empirical	Centered Average	Weighted Average	Geometric Mean
58+	1.151	1.151	1.151	1.151
57/56	0.744	1.186	1.108	1.067
56/55	1.661	1.046	0.952	1.002
55/54	0.502	1.001	0.918	0.919
54/53	1.171	1.011	0.907	0.851
53/52	0.928	0.801	0.745	0.850
52/51	0.792	0.843	0.756	0.868
51/50	0.610	0.924	0.946	0.888
50/49	0.712	1.008	1.019	0.946
49/48	1.579	1.028	1.016	1.023
48/47	1.345	1.070	1.022	1.047
47/46	0.892	1.149	1.117	1.031
46/45	0.824	1.081	1.063	1.044
45/44	1.107	0.971	0.946	1.058
44/43	1.237	1.096	1.080	1.054
43/42	0.793	1.125	1.093	1.056
42/41	1.516	1.125	1.094	1.101
41/40	0.970	1.093	1.074	1.098
40/39	1.108	1.182	1.169	1.088
39/38	1.079	1.066	1.064	1.063
38/37	1.235	1.047	1.040	1.048
37/36	0.939	0.992	0.977	1.001
36/35	0.877	1.014	0.999	0.980
35/34	0.832	0.940	0.932	0.958
34/33	1.186	0.962	0.954	0.953
33/32	0.864	0.945	0.931	0.937
32/31	1.049	0.965	0.952	0.937
31/30	0.795	0.925	0.916	0.933
30/29	0.930	0.930	0.930	0.930

For example – payments made in year of development 50 are 88.0% greater than the payments made in year 37 etc. The main reason that payments rise over time is because the force of medical cost escalation exceeds the force of mortality—until most of the claimants are fairly advanced in age, when the force of mortality becomes stronger than the force of medical cost escalation.

By summing the decay factors from 38 to 65, we get the payments made in age 38 to 65 relative to the payments made in the selected anchor year. The sums of the decay factors are similar to tail factors, but instead of being relative to cumulative payments they are relative to the incremental payments made in given anchor year.

The cumulative decay factors can be interpreted as follows: Payments made in ages 38 to 65 are 30.071 times the payments made in age 37. Similarly, payments made in ages 38 to 65 are 26.961 times the payments made in age 33 etc.

Table A.5
Anchored Decay Factors

Year of			Anchor Year		
Development	**37**	**36**	**35**	**34**	**33**
> 57	1.184	1.186	1.162	1.113	1.062
57	1.028	1.030	1.009	0.967	0.922
56	0.964	0.966	0.946	0.907	0.864
55	0.962	0.964	0.944	0.905	0.863
54	1.047	1.049	1.028	0.985	0.939
53	1.231	1.233	1.208	1.158	1.104
52	1.448	1.450	1.421	1.362	1.298
51	1.669	1.671	1.637	1.569	1.496
50	1.880	1.882	1.844	1.768	1.685
49	1.987	1.990	1.950	1.869	1.782
48	1.943	1.946	1.907	1.827	1.742
47	1.856	1.859	1.821	1.746	1.664
46	1.800	1.803	1.766	1.693	1.614
45	1.724	1.727	1.692	1.622	1.546
44	1.630	1.633	1.600	1.533	1.462
43	1.547	1.550	1.518	1.455	1.387
42	1.466	1.468	1.438	1.378	1.314
41	1.331	1.332	1.306	1.251	1.193
40	1.211	1.213	1.189	1.139	1.086
39	1.114	1.116	1.093	1.048	0.999
38	1.048	1.049	1.028	0.985	0.939
37	1.000	1.001	0.981	0.940	0.897
36		1.000	0.980	0.939	0.895
35			1.000	0.958	0.914
34				1.000	0.953
33					1.000
Totals (38 to ultimate)	30.071	30.115	29.508	28.280	26.961
Relative to anchor year	37	36	35	34	33

Because this approach produces volatile indicated tail factors, Table A.6 presents an approach for stabilizing those indications (see Table 2.6). Each of the average PLDFs for ages 30 through 36 are adjusted to what they would be for age 37—using the appropriate products of incremental decay factors from AYs 1965 and prior. A weighted average of all of these adjusted PLDFs is then used to replace the actual PLDF for DY 37. In this way, the PLDF for DY 37 is changed from being entirely determined by only one historical PLDF for one AY, to being an indication based on all 36 PLDFs for DYs 30 through 37. This results in a reduction of the PLDF for anchor year 37 from 1.0331 to 1.022. The final selected tail factor from age 37 to 65 is then the product of 0.022/1.022 and the cumulative decay factor of 30.071 and .022/1.022 (= 1.634).

Table A.6
Using the Mueller Incremental Tail Method to Produce a More Stable Estimate of the PLDF for Anchor Year 37

AY	29	30	31	32	33	34	35	36	37
1966	1.015	1.025	1.020	1.017	1.021	1.017	1.026	1.027	1.033
1967	1.019	1.030	1.026	1.026	1.023	1.025	1.025	1.030	
1968	1.013	1.009	1.006	1.004	1.003	1.003	1.004		
1969	1.018	1.017	1.019	1.021	1.013	1.023			
1970	1.017	1.016	1.030	1.013	1.017				
1971	1.014	1.040	1.040	1.026					
1972	1.036	1.021	1.015						
1973	1.042	1.037							
1974	1.025								
A) Average	1.022	1.024	1.023	1.018	1.015	1.017	1.018	1.029	1.033
B) Avg. - 1.0	0.022	0.024	0.023	0.018	0.015	0.017	0.018	0.029	0.033
C) Decay Ratios		0.930	0.933	0.937	0.937	0.953	0.958	0.980	1.001
D) Adjustment Factor to Age 37		0.734	0.787	0.840	0.897	0.940	0.981	1.001	1.000
E) B) Adjusted to Age 37		0.018	0.018	0.015	0.014	0.016	0.018	0.029	0.033
F) Weights for E)		5%	5%	10%	10%	15%	15%	20%	20%
G) Weighted Avg. of E)									0.022
H) Revised B)		0.030	0.028	0.026	0.025	0.023	0.023	0.022	0.022
I) Revised PLDFs		1.030	1.028	1.026	1.025	1.023	1.023	1.022	**1.022**

Notes:
C) From Table A.4, Last Column.
D) Product of all decay ratios to the right of given age.
E) B) x D).
H) G) / D).

Once the best estimate of the PLDF for the anchor year (DY 37) is selected, then all of the subsequent PLDFs can be easily generated using the iterative formula:

$$f_{n+1} = f_n \, d_{n+1}/[1 + f_n],$$

where f_n is the paid loss development factor, less one, for the nth year of development, and d_{n+1} is the decay ratio between incremental paid during year (n+1) and year (n).

Figure 1.1 provides a comparison between SAIF's indicated PLDFs and those based on the static mortality model presented in Section 3 and Appendix C of this paper. The indicated PLDFs for the more mature DYs in Figure 1.1 were derived using the above iterative formula and the revised PLDF of 1.022 for anchor year 37 (as derived in Table A.6.

Table A.7 documents the calculation of decay factors from the medical paid loss experience of the Washington State Fund in the same manner as Table A.4. The Washington experience was for all medical losses, but it should be virtually the same as the Oregon experience for DYs 20 and higher—on the assumption that all medical payments made for those more mature years are associated with PD claimants.

The Washington decay factors are quite similar to SAIF's, as shown in the comparison provided in Table A.8.

While SAIF's average annual decay factors tend to be slightly higher than those for the Washington State Fund for DYs 21-45, the WA factors are higher for DYs 46-55. This would suggest that a comparison graph such as that provided in Figure 1.1 for the Washington State Fund would show an even more pronounced bulge for the most mature DYs than is the case for SAIF.

Table A.7
Age to Age Decay Factors for the Washington State Fund

					Averages			
	Total ($000's)	Total ($000's)	Age to	Age to age	# of years	Simple	Weighted	Geometric
Age	(ex last)	(ex first)	Age to	Factors		Average	Average	Mean
60	14	14	60/59	0.502	5	0.567	0.675	0.615
59	28	28	59/58	0.881	5	0.796	1.086	0.925
58	31	35	58/57	0.816	5	1.107	1.277	1.083
57	42	138	57/56	1.648	5	1.160	1.219	1.215
56	84	92	56/55	1.687	5	1.285	1.306	1.213
55	55	57	55/54	0.767	5	1.311	1.199	1.131
54	75	86	54/53	1.506	5	1.192	1.076	1.095
53	57	184	53/52	0.945	5	1.001	0.902	1.051
52	195	440	52/51	1.054	5	1.131	1.038	1.024
51	417	417	51/50	0.732	5	1.078	1.061	1.019
50	570	594	50/49	1.416	5	1.083	1.052	1.063
49	419	439	49/48	1.243	5	1.082	1.051	1.055
48	353	451	48/47	0.968	5	1.130	1.117	1.032
47	465	480	47/46	1.050	5	1.016	0.997	1.013
46	458	471	46/45	0.972	5	0.956	0.950	0.998
45	484	493	45/44	0.849	5	0.961	0.958	0.961
44	581	859	44/43	0.944	5	0.968	0.976	0.952
43	911	939	43/42	0.989	5	0.928	0.924	0.962
42	949	920	42/41	1.087	5	0.958	0.950	0.979
41	847	874	41/40	0.771	5	1.013	1.005	0.991
40	1,133	1,164	40/39	0.998	5	1.056	1.042	1.019
39	1,166	1,240	39/38	1.220	5	1.055	1.042	1.028
38	1,017	1,039	38/37	1.205	5	1.072	1.061	1.020
37	862	948	37/36	1.081	5	1.031	0.991	0.989
36	876	954	36/35	0.855	5	0.992	0.966	0.965
35	1,116	1,394	35/34	0.794	5	0.914	0.894	0.941
34	1,756	1,969	34/33	1.027	5	0.912	0.921	0.940
33	1,917	2,036	33/32	0.813	5	0.934	0.938	0.947
32	2,504	2,580	32/31	1.071	5	0.985	0.985	0.975
31	2,409	2,613	31/30	0.965	5	0.994	1.000	0.997
30	2,709	3,092	30/29	1.050	5	1.036	1.037	1.019
29	2,944	3,676	29/28	1.072	5	1.028	1.030	1.030
28	3,430	3,787	28/27	1.024	5	1.046	1.046	1.032
27	3,696	4,519	27/26	1.027	5	1.039	1.037	1.021
26	4,400	4,877	26/25	1.058	5	1.017	1.014	1.008
25	4,611	5,347	25/24	1.013	5	0.993	0.981	0.986
24	5,281	5,965	24/23	0.965	5	0.975	0.964	0.963
23	6,178	6,876	23/22	0.901	5	0.946	0.939	0.943
22	7,634	8,313	22/21	0.940	3	0.918	0.918	0.923
21	8,840	9,518	21/20	0.912	1	0.912	0.912	0.912
20	10,433	11,398						

Table A.8
Comparison of Average Annual Decay Factors for the Oregon and Washington State Funds

	Average Annual Decay Factor		WA Avg. Factor as a %-age
DYs	SAIF	WA	of SAIF's
21-25	1.059	0.945	-10.7%
26-30	1.098	1.022	-6.9%
31-35	0.969	0.960	-0.9%
36-40	1.044	1.004	-3.9%
41-45	1.073	0.969	-9.7%
46-50	1.018	1.032	1.4%
51-55	0.875	1.063	21.5%
56-60	1.072	0.981	-8.5%

APPENDIX B
HISTORICAL PLDFs FOR ALL OTHER WC

This section presents SAIF's historical PLDFs for MPD losses as well as WC losses other than MPD. The averages of the latest five PLDFs are shown for each development year in Table B.1. These factors are counterparts to the MPD PLDFs shown in Tables 1.1, 1.2 and 1.3.

Table B.1
A Comparison of Historical Age to Age Paid Loss Development Factors (By Year of Development)

	Years of Development													
	2	3	4	5	6	7	8	9	10	11	12	13	14	15
MPD	6.624	1.525	1.140	1.072	1.041	1.027	1.019	1.020	1.015	1.013	1.012	1.013	1.012	1.010
Other WC	1.843	1.131	1.043	1.023	1.018	1.013	1.009	1.006	1.005	1.004	1.004	1.004	1.005	1.006
Total WC	2.168	1.213	1.069	1.036	1.025	1.017	1.012	1.010	1.008	1.007	1.006	1.007	1.007	1.007

	Year of Development										
	16	17	18	19	20	21	22	23	24	25	26
MPD	1.011	1.013	1.011	1.011	1.012	1.012	1.014	1.012	1.015	1.015	1.016
Other WC	1.006	1.008	1.010	1.009	1.009	1.009	1.008	1.009	1.010	1.010	1.010
Total WC	1.008	1.010	1.010	1.010	1.010	1.010	1.010	1.010	1.012	1.011	1.012

	Year of Development										
	27	28	29	30	31	32	33	34	35	36	37
MPD	1.020	1.023	1.027	1.026	1.022	1.018	1.015	1.017	1.018	1.029	1.033
Other WC	1.009	1.008	1.008	1.007	1.007	1.006	1.006	1.005	1.004	1.006	1.006
Total WC	1.012	1.012	1.013	1.013	1.011	1.010	1.009	1.009	1.009	1.013	1.014

The 37 to 65 tail factor indicated for other WC is 1.039. In Oregon, escalation of indemnity benefits is paid out of a second injury fund. The above Other WC development factors do not include the escalation of indemnity benefits. The Other than MPD tail factor of 1.039 can be compared to the MPD tail factor of 1.581. It is medical losses that contribute significantly to the tail factor and it is the medical cost escalation component of the medical tail factor that that contributes significantly to the medical tail factor. Without medical cost escalation, the medical factor drops from 1.581 to 1.030 when put on a current cost basis.

The above PLDFs serve as the basis for the tail factors presented in Tables 2.7 and 2.8.

APPENDIX C
INCORPORATING THE STATIC MORTALITY MODEL INTO THE INCREMENTAL PAID TO PRIOR OPEN METHOD

SECTION C.1 OVERVIEW

Given the complexity of this method, Table C.1 provides a roadmap to the key steps involved in the application of the method and the location of tables and/or narrative describing those steps. The method was originally introduced in Section 3 by presenting Step 11)—since this is easily understood.

Table C.1
Guide to Location of Description and/or Display of Key Steps of Method

Step	Appendix C	Section 3 of Main Text
1) Select representative historical claim reporting pattern	Section C.5	
2) Select representative historical claim closing pattern	Section C.5	
3) Derive historical open count pattern by subtracting 2) from 1)	Section C.5	
4) Derive projections of number of claims closed due to death	Section C.2	Table 3.4
5) Derive assumptions regarding %-age of claims closed for other reasons	Section C.5	Table 3.4
6) Synchronize open count estimates of historical experience and mortality model	Section C.5	Table 3.4
7) Select appropriate medical inflation assumption	Section C.4	
8) Trend historical incremental paid to prior open averages to current level	Section C.3	
9) Select representative paid severities	Section C.3	
10) Trend paid severities to year of payment	Section C.3	Tables 3.2 and 3.5
11) Estimate incremental payments as the product of trended paid severities and projections of the number of prior open claims		Tables 3.2 and 3.5

As is evident from Table C.1, this method has been presented in reverse order to how it is applied.

This appendix consists of four sections: 1) Derivation of Number of Open Claims at the End of Each Development Year; 2) Selection of Representative Values of Incremental Paid per Prior Open; 3) Basis for Selection of Future Medical Inflation Assumption of 9%; and 4) Derivation of Assumed Claim Reporting and Closure Patterns.

SECTION C.2 DERIVATION OF NUMBER OF OPEN CLAIMS AT THE END OF EACH DEVELOPMENT YEAR

The first part of this appendix describes the derivation of the estimated number of PD claimant deaths shown in Column (3) of Table 3.4. Such estimates also directly become the number by which total open claims declines for each development year after the twentieth year. After that year, it is assumed that no new claims will be reported and that the number of claim closures for reasons other than death will be cancelled out by the number of reopened claims for each development year.

The survival probabilities for each development year were derived from a claimant mortality model and these were compared with the actual probabilities of a claim remaining open throughout each given development year. For each development year under 10, the probability of a claim remaining open during a given development year was substantially less than the survival probability—since most (or many) claims will close for reasons other than death of the claimant. However, these two sets of probabilities converge for increasing development years until they are virtually identical—for development years 20 and higher.

Mortality rates were used to derive a claims closure pattern (due to death) by development year in the following way. A two-dimensional array was created, with the age-at-injury down the leftmost column and the development years as column headings.

Table C.2.1 presents a small portion of the array, including only ages-at-injury from 40 through 49 shown at the beginning of the first five development years, and at the beginning of the 10th and 20th development years.

Appendix D provides a more detailed description of the array structure. The arrays described in these two appendices differ only in the applicable mortality tables. For the static method, the 2000 mortality table is assumed for all future years. In the trended method (Appendix D), projections of future mortality tables are used.

Table C.2.1 is a segment of the male lives array. We assumed that the initial PD claimant population consisted of 750 males and 250 females. A corresponding array was used for the female claimants.

Table C.2.1
Number of Living Male Claimants for Accident Year 2002
At Successive Year-Ends Assuming a 2000 Mortality Table

Age-at-	Beginning of Development Year						
Injury	1	2	3	4	5	10	20
40	12.99	12.96	12.92	12.88	12.83	12.56	11.50
41	14.71	14.66	14.62	14.57	14.51	14.19	12.89
42	16.09	16.04	15.99	15.93	15.87	15.48	13.94
43	16.03	15.97	15.91	15.85	15.78	15.38	13.71
44	17.48	17.41	17.34	17.27	17.19	16.72	14.74
45	18.86	18.79	18.71	18.62	18.53	17.98	15.66
46	20.12	20.03	19.94	19.84	19.74	19.10	16.41
47	21.43	21.34	21.23	21.12	21.01	20.27	17.14
48	22.69	22.58	22.46	22.34	22.20	21.36	17.75
49	23.02	22.90	22.77	22.63	22.49	21.56	17.59
40-49	**183.41**	**182.68**	**181.89**	**181.06**	**180.16**	**174.61**	**154.38**

The first column to the right of the age-at-injury values is a portion of the distribution of 750 male PD claimants by age—based on individual permanent total disability (PTD) claimant data from SAIF for accident years 1975-1990. By doing so, we assumed that the age-at-injury distribution for PD claims would be the same as for PTD claims. The actual census data was smoothed among different age-at-injury categories to derive the numbers in Column "1".

Consider the row for the age-at-injury of 40. Suppose that 12.99 of the 1000 total claimants were injured at age 40. The probability of living from age 40 to age 41 from the male 2000 SSA mortality table is used to calculate the expected number of male claimants still alive one year after the accident, and so forth for each subsequent age and year of development out to development year 90. In this way each age-at-injury row is filled out in the array. For each development year column, the expected total number of surviving claimants is simply the sum of the expected number of surviving claimants for each age-at-injury ranging from 40 through 49.

The same calculations were performed for all possible ages-at-injury and all development years from 1 through 90. The resulting estimates of the number of surviving male claimants is summarized in Table C.2.2 for different age-at-injury groupings at different selected years of development. The totals derived in Table C.2.1 immediately above are displayed below in bold type in shaded boxes.

Table C.2.2
Number of Surviving Male Claimants at the Beginning of Various Development Years for Accident Year 2002

Age-at-	Number of Surviving Male Claimants at the Beginning of Development Year											
Injury	1	5	10	15	20	25	30	40	50	60	70	80
16-29	30.7	30.5	30.2	29.9	29.4	28.8	27.9	24.6	18.0	8.7	1.5	0.0
30-39	78.9	78.2	77.0	75.4	73.0	69.5	64.3	47.3	22.8	4.0	0.1	0.0
40-49	183.4	180.2	174.6	166.5	154.4	137.0	114.3	56.2	10.2	0.3	0.0	0.0
50-59	321.3	309.0	286.9	255.0	213.4	162.7	106.1	19.7	0.6	0.0	0.0	0.0
60+	135.7	124.2	105.6	83.2	58.0	33.0	13.8	0.6	0.0	0.0	0.0	0.0
TOTAL	750.0	722.1	674.4	609.9	528.1	431.0	326.4	148.3	51.7	13.0	1.6	0.0
Survival Probability		96.3%	93.4%	90.4%	86.6%	81.6%	75.7%	45.4%	34.8%	25.1%	12.3%	2.6%

The expected number of surviving claimants at the beginning of development year 5 is 722.1 and at development age 10 is 674.4. Hence the probability of survival during the fifth through ninth development years for all male claimants is 93.4%. It is evident from a review of the bottom row of Table C.2.2 that the survival probabilities steadily decline as the claimant population ages.

Table C.2.3 displays the survival probabilities for each age-at-injury grouping during each grouping of development years.

Table C.2.3
Indicated Male Claimant Survival Probabilities

Range of	Beginning of Development Year										
Ages-at-Injury	5	10	15	20	25	30	40	50	60	70	80
16-29	99.4%	99.2%	98.9%	98.5%	97.8%	96.9%	88.1%	73.4%	48.0%	17.0%	2.8%
30-39	99.1%	98.5%	97.8%	96.9%	95.2%	92.5%	73.5%	48.2%	17.5%	3.0%	0.2%
40-49	98.2%	96.9%	95.4%	92.7%	88.7%	83.5%	49.1%	18.2%	3.1%	0.2%	0.0%
50-59	96.2%	92.8%	88.9%	83.7%	76.3%	65.2%	18.6%	3.1%	0.2%	0.0%	
60+	91.6%	85.0%	78.8%	69.7%	56.9%	41.8%	4.1%	0.2%	0.0%		

Given that survival probabilities vary significantly for different ages-at-injury groups, it is clear that the group survival probabilities will be highly sensitive to the distribution of claimants by age-at-injury. The greater the proportion of younger claimants, the bigger the MPD tail.

SECTION C.3 SELECTION OF REPRESENTATIVE VALUES OF INCREMENTAL PAID PER PRIOR OPEN

Historical incremental paid per prior open claim averages were trended to CY 2003 cost level using an assumed annual medical inflation rate of 9% per year. The resultant on-leveled averages are displayed in Tables C.3.1 and C.3.2.

Table C.3.1
Incremental Paid/Prior Open Averages Trended to 2003 Cost Level

AY	1	2	3	4	5	6	7	8	9	10	11	12	13
1979													9.50
1980												6.90	6.86
1981											8.22	3.80	3.00
1982										6.34	6.49	2.45	3.08
1983									10.42	4.92	3.11	3.28	3.05
1984								6.93	5.43	3.86	3.77	2.95	2.40
1985							6.51	4.64	4.72	2.70	2.65	3.66	2.21
1986						10.22	6.21	5.36	2.76	4.48	3.52	3.14	3.12
1987					8.50	6.05	3.67	2.30	2.69	1.91	2.60	2.38	3.10
1988				11.58	6.11	5.88	2.83	2.18	2.62	3.33	3.03	3.13	4.47
1989			20.00	9.42	7.03	4.79	3.02	2.90	3.26	3.67	3.46	4.05	5.43
1990		84.33	18.60	8.04	5.93	3.68	2.89	3.65	4.90	2.31	2.74	4.60	4.74
1991	13.41	85.99	16.21	5.03	5.69	3.25	2.05	1.84	2.44	2.47	2.89	3.09	
1992	13.20	70.53	14.73	5.69	3.16	3.59	3.34	3.18	3.65	6.05	4.69		
1993	12.95	76.82	13.62	6.66	4.01	4.70	3.96	4.12	7.13	5.78			
1994	11.46	68.64	12.33	3.58	4.26	4.11	2.39	2.31	4.10				
1995	12.52	66.15	10.24	7.55	4.77	4.25	5.63	5.49					
1996	12.17	67.86	13.15	5.24	4.82	5.54	4.52						
1997	14.25	73.83	12.71	7.62	5.45	4.47							
1998	13.09	64.87	14.46	6.05	8.86								
1999	14.94	65.05	15.01	7.71									
2000	16.70	89.89	17.08										
2001	14.75	87.41											
2002	15.21												
Average	13.59	75.11	14.84	7.01	5.72	5.05	3.92	3.74	4.51	3.99	3.93	3.62	4.25
X Hi/Lo	13.48	74.66	14.79	6.90	5.66	4.71	3.85	3.61	4.13	3.96	3.64	3.41	3.92
Avg. Last 3	15.46	80.78	15.52	7.13	6.38	4.76	4.18	3.97	4.96	4.77	3.44	3.91	4.88
Wtd. Avg.	14.69	78.42	15.24	7.06	6.10	4.80	4.06	3.85	4.70	4.45	3.58	3.75	4.56
Selected	14.69	78.42	15.24	7.06	6.10	4.80	4.50	4.50	4.50	4.50	3.70	3.70	3.70

Table C.3.2
Incremental Paid/Prior Open Averages Trended to 2003 Cost Level

AY	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1966										3.68	4.79	4.24	2.75	4.31
1967									7.66	6.49	7.71	11.75	4.00	6.04
1968								11.84	13.27	9.82	2.97	2.69	2.94	2.03
1969							5.31	7.53	5.22	5.70	3.46	5.69	3.00	2.98
1970						4.34	3.23	3.87	3.51	2.55	2.90	2.95	2.66	2.45
1971					6.17	4.87	3.35	2.60	2.57	2.10	2.16	2.63	2.12	5.88
1972				4.45	3.22	3.05	2.66	2.93	1.55	1.30	1.89	2.30	6.07	3.50
1973			7.38	5.53	4.26	3.99	4.16	4.55	4.56	5.19	6.55	7.27	7.90	6.90
1974		5.35	4.28	4.47	2.95	4.18	2.30	2.43	4.00	0.99	2.80	3.84	4.33	
1975	5.69	4.28	4.52	4.65	6.11	3.91	3.85	4.18	3.44	3.96	5.33	4.56		
1976	5.28	3.46	3.77	2.91	2.89	3.41	2.73	2.48	2.11	2.44	1.77			
1977	3.56	2.80	2.73	3.48	3.03	2.77	3.02	3.36	2.86	3.97				
1978	3.22	3.81	2.85	3.33	3.05	4.55	4.39	3.79	3.74					
1979	5.90	4.58	4.14	4.89	3.97	4.73	4.17	4.10						
1980	3.47	3.49	2.98	2.49	3.53	3.15	2.22							
1981	3.25	3.37	3.43	3.28	2.79	4.54								
1982	3.00	2.86	3.15	4.05	3.55									
1983	3.69	3.93	4.98	3.27										
1984	4.19	3.28	2.87											
1985	5.89	4.77												
1986	4.69													
Average	4.32	3.83	3.92	3.90	3.79	3.96	3.45	4.47	4.54	4.02	3.85	4.79	3.97	4.26
X Hi/Lo	4.29	3.78	3.70	3.88	3.66	3.99	3.39	3.94	3.97	3.74	3.65	4.23	3.68	4.19
Avg. Last 3	4.93	3.99	3.67	3.53	3.29	4.14	3.59	3.75	2.90	3.46	3.30	5.22	6.10	5.42
Wtd. Avg.	4.68	3.92	3.72	3.68	3.46	4.07	3.52	3.93	3.44	3.63	3.48	4.94	5.19	4.94
Selected	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70	3.70

SECTION C.4 BASIS FOR SELECTION OF FUTURE MEDICAL INFLATION ASSUMPTION OF 9%

Future medical inflation rate forecasts are based on an analysis of medical severity since 1966. Future medical severity is expected to grow on average at the same rate observed over this 37 year period. Internal studies have shown that the best predictor of long term medical cost escalation is the long term historical average itself. Short term medical escalation rates are more accurately predicted using shorter term historical averages.

SAIF has two ways of estimating future medical escalation rates:

1. The average severity growth of medical-only claims.

2. The average severity growth of medical on indemnity claims.

The medical-only method is straight-forward because Medical-Only claims develop very quickly and the ultimate average medical only cost by accident year can be accurately calculated.

Table C.4.1 shows the medical only severity by accident year, the growth rates of this index and the mean, median and geometric mean of the series.

Table C.4.1
Medical Only Severity Escalation Rates

Accident Year	Medical Severity	Escalation Rate
1966	$28	
1967	32	13.9%
1968	34	4.4
1969	37	11.1
1970	37	0.0
1971	42	12.2
1972	46	9.5
1973	48	5.8
1974	56	15.7
1975	63	12.2
1976	73	16.3
1977	79	7.9
1978	88	12.1
1979	99	11.8
1980	114	15.5
1981	144	26.5
1982	162	12.2
1983	188	16.2
1984	205	8.9
1985	230	12.1
1986	247	7.4
1987	258	4.6
1988	264	2.4
1989	268	1.4
1990	279	4.4
1991	298	6.8
1992	335	12.3
1993	361	7.7
1994	372	3.0
1995	383	3.1
1996	403	5.1
1997	434	7.7
1998	454	4.5
1999	501	10.4
2000	540	7.7
2001	604	12.0
2002	658	9.0
Mean		**9.3%**
Median		**9.0%**
Geometric mean		**9.2%**

Although the severities of medical only claims give an indication of medical escalation trends, the types of services provided to medical only claimants are not the same as the services provided to permanently disabled claimants. SAIF has developed a technique to compute medical escalation indices for the MPD claims. In this paper, we describe the method.

Given a triangle of incremental MPD payments and claim counts, it is possible to compute triangles of MPD severities. For any particular calendar year (one of the diagonals of the triangle) one can calculate the growth in severity for each age of development. The escalation rate for the calendar year is the weighted average of the severities for each age of development, where the weights are the calendar year paid losses by accident age.

Using SAIF's data on MPD claims, the mean, median and geometric means of the MPD index is shown in comparison to the Medical Only index in Table C.4.2.

Table C.4.2
Medical Escalation Rates
(Average from 1966 to 2002)

	Medical Only claims	MPD claims
Mean	9.3%	9.7%
Median	9.0	9.5
Geometric mean	9.2	9.8

Medical escalation rates on MPD claims have been even higher than the Medical Only inflation rates and have averaged in the 9 to 10% range over a period of 37 years.

SECTION C.5 DERIVATION OF ASSUMED CLAIM REPORTING AND CLOSURE PATTERNS

Exhibits C.5.1 and C.5.2 disclose the SAIF specific assumptions that form the basis for the PLDF static and trended mortality model estimates. These assumptions are held constant for all accident years in the model:

- 5,000 ultimately reported PD claims.
- A claim reporting pattern based on recent historical experience.
- Percentages of cumulative reported claims still open at the end of each DY—based on recent historical experience.
- Estimates of PD claims closed by death—based on SSA mortality tables.
- Estimates of PD claims closed for reasons other than death—calculated as total claim closures less expected deaths.

From the above, the percentage of claims available for closure that closed for reasons other than death was derived from AY 2002 for the static mortality model. These percentages were also assumed for the trended mortality model. Consequently, the only thing difference between the two models is the expected number of claimant deaths during each DY.

Table C.5.1

Derivation of Key Assumptions of the Static and Trended Models
Accident Year 2002 MPD Losses

	Development Year						
	1	**2**	**3**	**4**	**5**	**6**	**7**
1) % of Claims Reported	18.52%	74.33%	91.64%	95.93%	97.77%	98.69%	99.08%
2) Selected Reported Counts	926	3,716	4,582	4,797	4,888	4,935	4,954
3) % of Reported Still Open	49.65%	41.20%	29.82%	19.78%	13.85%	11.23%	8.00%
4) Selected Open Counts	460	1,531	1,366	949	677	554	396
5) Group Survival Probability	0.993	0.992	0.991	0.990	0.990	0.989	0.988
6) Number Closed by Death	3.46	15.05	17.30	14.09	10.33	7.89	6.88
7) Total Closed Counts	466	2185	3216	3848	4211	4381	4558
8) Closed for Other Causes	462.54	1703.95	1013.70	617.91	352.67	162.11	170.12
9) Newly Reported Counts	926	2,790	866	215	91	47	19
10) Open + Newly Reported	926	3,250	2,397	1,581	1,040	724	573
11) Indicated % Closed (Other)	99.26%	52.43%	42.29%	39.08%	33.91%	22.39%	29.69%
12) Selected % Closed (Other)	99.26%	52.43%	42.29%	39.08%	33.91%	22.39%	29.69%

NOTES:
1) Based on average reported count development factors for the latest 10 CYs.
2) 5,000 x 1). The constant ultimate claim count of 5,000 was assumed for all years.
3) Based on the average percentage open for the most recent CYs.
4) 2) x 3), for DYs 1-10; [Prior 4) + 9) – 6 – 8)], for later DYs.
5) See Section C.1 of Appendix C.
6) [4) + 0.5 x 9)] x (1 - 5)).
7) 2) - 4).
8) [Change in 7)] - 6).
9) Change in 2).
10) 4) + 9).
11) 8) / 10).
12) Selected on the basis of 11). Actual %-ages were selected for the 1st 10 DYs.

Table C.5.2

Derivation of Key Assumptions of the Static and Trended Models
Accident Year 2002 MPD Losses

	Development Year						
	8	**9**	**10**	**11**	**12**	**13**	**14**
1) % of Claims Reported	99.30%	99.46%	99.56%	99.64%	99.69%	99.76%	99.81%
2) Selected Reported Counts	4,965	4,973	4,978	4,982	4,985	4,988	4,991
3) % of Reported Still Open	6.50%	5.00%	4.20%	3.95%	3.74%	3.56%	3.40%
4) Selected Open Counts	323	249	209	197	187	178	170
5) Group Survival Probability	0.987	0.986	0.985	0.983	0.982	0.981	0.979
6) Number Closed by Death	5.31	4.68	3.89	3.52	3.56	3.63	3.72
7) Total Closed Counts	4642	4724	4769	4785	4798	4810	4821
8) Closed for Other Causes	78.69	77.32	41.11	12.54	9.85	8.39	7.10
9) Newly Reported Counts	11	8	5	4	3	3	3
10) Open + Newly Reported	407	331	254	213	200	190	181
11) Indicated % Closed (Other)	19.33%	23.36%	16.19%	5.89%	4.92%	4.43%	3.93%
12) Selected % Closed (Other)	19.33%	23.36%	16.19%	6.00%	5.00%	4.50%	4.00%

APPENDIX D
INCORPORATING THE TRENDED MORTALITY MODEL INTO THE INCREMENTAL PAID TO PRIOR OPEN METHOD

Table C.1 displays each of the steps taken in incorporating the static mortality model into the incremental paid to prior open method. The trended mortality method is the same as the static mortality method, except for step 4), where projections of the number of claims closed due to death are derived. In the trended method, mortality tables forecasted by the SSA for the appropriate future development year are used instead of some fixed historical mortality table. The differences between these tables grows exponentially for development years that are decades into the future. A sample of these differences is disclosed in Tables 5.1a and 5.1b of Section 5. These differences are compounded by medical costs that have risen dramatically due to expected high future rates of medical inflation.

The focus of this appendix is to disclose the specific manner by which a series of 90 different mortality tables were derived and applied to the expected number of surviving claimants by age-at-injury for every future development year. The final result is a slowly evolving and elongating series of claims closure patterns for each AY out to 90 years of development.

Standard mortality tables for each decade since 1970 and projected tables for each decade through 2080 were obtained from the SSA web site (http://www.ssa.gov/OACT/NOTES/as116/as116_Tbl_6_2020.html#wp1085674).

The separate male and female tables were combined into one using an assumed 75%/25% male/female mix, the proportion indicated from SAIF's PD claimant census data. The resulting weighted mortality rates were then compiled into an array of expected mortality rates for each age at each future calendar year.

Six models of the number of PD claimants who would still be alive at the end of each future development year were derived—separately for accident years 1975, 1980, 1985, 1990, 1995 and 2000. Each of these models consists of a separate two-dimensional array, such as presented in Tables C.2 and C.3 of Appendix C.

The first step in deriving these arrays was to compile mortality rates from the SSA tables. Table D.1 displays a sampling of these q(x), or probability of death, values.

Each of the one-year q(x) values were converted into survival rates by taking their complement, yielding the ratios in Table D.2.

The entire array of resulting one-year l(x)'s was then shifted so that the rows of the original array became the diagonals of a new array – i.e., each successive column was shifted up one row. After the shift, the l(x)'s were arranged as shown in Table D.3.

Table D.1
Sample Q(x) Values

	Calendar Year						
Age	**1970**	**1980**	**1990**	**2000**	**2020**	**2040**	**2060**
20	.00175	.00156	.00130	.00110	.00091	.00078	.00066
35	.00239	.00187	.00217	.00172	.00154	.00130	.00110
50	.00861	.00685	.00556	.00496	.00397	.00330	.00278
65	.02961	.02524	.02206	.01938	.01615	.01371	.01182
80	.09386	.08308	.07604	.07028	.05929	.04976	.04261

Table D.2
Sample One Year L(x) Values

	Calendar Year						
Age	**1970**	**1980**	**1990**	**2000**	**2020**	**2040**	**2060**
20	.99825	.99844	.99870	.99890	.99909	.99922	.99934
35	.99761	.99813	.99783	.99828	.99846	.99870	.99890
50	.99139	.99315	.99444	.99504	.99603	.9967	.99722
65	.97039	.97476	.97794	.98062	.98385	.98629	.98818
80	.90614	.91692	.92396	.92972	.94071	.95024	.95739

Table D.3
Shifted L(x) Array: Age

	Year of Development						
Age at Injury	**1**	**2**	**3**	**4**	**5**	**6**	**7**
20	21	22	23	24	25	26	27
21	22	23	24	25	26	27	28
22	23	24	25	26	27	28	29
23	24	25	26	27	28	29	30
24	25	26	27	28	29	30	31

Each row thus has a structure similar to an accident year reporting format, as displayed below.

This shift facilitated multiplication of the survival ratios times the preceding number of surviving claimants for each age-at-injury row, working successively from left to right within each age-at-injury row.

Table D.5 provides a side-by-side comparison of parallel calculations of the expected number of surviving claimants at the end of each calendar year—for the static and trended mortality methods. The example presented is for claimants who were 50 years old when they were injured (during AY 2002).

Table D.4
Calendar Year of Payments and Applicable Mortality Table—
For Each Accident Year and Development Year

	Year of Development								
AY	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1996	1996	1997	1998	1999	2000	2001	2002	2003	2004
1997	1997	1998	1999	2000	2001	2002	2003	2004	2005
1998	1998	1999	2000	2001	2002	2003	2004	2005	2006
1999	1999	2000	2001	2002	2003	2004	2005	2006	2007
2000	2000	2001	2002	2003	2004	2005	2006	2007	2008
2001	2001	2002	2003	2004	2005	2006	2007	2008	2009
2002	2002	2003	2004	2005	2006	2007	2008	2009	2010

Table D.5
Comparison of the Estimation of the Number of
Living Claimants with Age-at-Injury of 50 for
Accident Year 2002 At Successive Year-Ends—
Under the Static and Trended Mortality Methods

STATIC MORTALITY METHOD					
	Calendar Year				
	2031	**2032**	**2033**	**2034**	**2035**
Number of Surviving Claimants	100.00	93.63	87.05	80.30	73.42
CY of Mortality Table	**2000**	**2000**	**2000**	**2000**	**2000**
Survival Probability	.93633	.92972	.92242	.91439	.90562
TRENDED MORTALITY METHOD					
	Calendar Year				
	2031	**2032**	**2033**	**2034**	**2035**
Number of Surviving Claimants	100.00	95.12	90.05	84.79	79.30
CY of Mortality Table	**2031**	**2032**	**2033**	**2034**	**2035**
Survival Probability	.95121	.94671	.94152	.93526	.92769

In Table D.5 we started with the same number of surviving claimants at the beginning of CY 2031 (100.00). Nevertheless, at the beginning of CY 2035, we would be expecting 73.42 such claimants to still be alive using a 2000 mortality table while 79.30 claimants would be alive using a series of mortality tables corresponding to CYs 2031 through 2034. In this example, we would be expecting 8% more claimants to still be alive at the beginning of CY 2035 assuming the trended mortality method (versus the static method).

Although there is little difference in the survival probabilities shown in Table D.5, these differences become fairly significant during future decades. This can be seen by comparing these rates in the fourth and fifth columns of Tables 5.1a and 5.1b.

Obtaining Predictive Distributions for Reserves Which Incorporate Expert Opinion

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Obtaining Predictive Distributions for Reserves Which Incorporate Expert Opinion

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Abstract
This paper shows how expert opinion can be inserted into a stochastic framework for claims reserving. The reserving methods used are the chain-ladder technique and Bornhuetter-Ferguson, and the stochastic framework follows England and Verrall (2002). Although stochastic models have been studied, there are 2 main obstacles to their more frequent use in practice: ease of implementation and adaptability to user needs. This paper attempts to address these obstacles by utilising Bayesian methods, and describing in some detail the implementation, using freely available software and programs supplied in the Appendix.

Keywords
Bayesian Statistics, Bornhuetter-Ferguson, Chain-ladder, Claims Reserving, Risk

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1. Introduction

There has been a lot of attention given to stochastic reserving methods in the actuarial literature over recent years, of which useful summaries can be found in England and Verrall (2002) and Taylor (2000). The reader is strongly recommended to read England and Verrall (2002), which contains more details on the basic models, before reading this paper. There have been many useful things which have resulted from the recent papers on stochastic claims reserving: it is now possible to use a variety of methods to obtain reserve estimates, prediction intervals, predictive distributions and so on. It is possible to use these for assessing the reserving risk, for modelling a portfolio, line of business or a whole company in a dynamic financial analysis, etc. In short, the research published in recent years has been very successful in enhancing the understanding of claims reserving methods. This has been done by establishing stochastic approaches to models that are commonly used for claims reserving, for example, the chain-ladder technique, the Hoerl curve, and other parametric and non-parametric models. In addition to this, the stochastic approaches have added further models to the range of possible approaches. To take just one example, England and Verrall (2000) showed how a non-parametric approach can be used to define a complete spectrum of models, with the chain-ladder technique at one end and the Hoerl curve at the other end.

In practical terms, it appears that the stochastic approaches that have found most popularity are those which are the simplest to implement. To pick out two examples, both Mack's model (Mack, 1993) and the bootstrap (England and Verrall, 1999 and England, 2000) are straightforward to implement in a spreadsheet. In contrast, using the full statistical model requires the use of statistical software, with some careful programming. It is not surprising, therefore, that a practitioner requiring prediction intervals as well as reserve estimates, or simply wanting to investigate the use of a stochastic approach, should choose the methods which are simplest to implement.

One aspect of reserving which has not, so far, received a great deal of attention in the literature is the question of intervention in the process by the actuary. In other words, the stochastic models have largely concentrated on providing a framework for the basic, standard methods. When these are used in practise, it is common to apply some expert knowledge or opinion to adjust the results before they are used. Examples of situations when intervention may be desirable is when there has been a change in the payment pattern, due to a change in company policy, or where legislatures have enacted benefit limitations that restrict the potential for loss development and require an adjustment to historical development factors. While it is possible to intervene in some models, the tendency is for this intervention to disrupt the assumptions made in the stochastic framework. For example, it is possible to change one or more of the residuals before applying a bootstrapping procedure, if the observed residuals appear to be out of line with what might be expected. But if this is done, the validity of the stochastic assumptions may be compromised. To take another example, consider the chain-ladder technique. This method involves the estimation of development factors, but is often the case that these are adjusted before being applied to obtain reserve estimates. If this is done, the estimates from the stochastic model are being abandoned, and it is not clear what effect this might have on the prediction errors. For example, it is possible to calculate estimation errors for any parameter estimated in a stochastic model, but what estimation error should be used for a parameter that is simply inserted? The only way to address this properly is to use the Bayesian approach, and this provides an important motivation for the ideas discussed in this paper.

A second area where expert knowledge is applied is when the Bornhuetter-Ferguson technique is used (Bornhuetter and Ferguson, 1972). This method uses the development factors from the chain-ladder technique, but does not apply these to the latest cumulative claims to estimate the outstanding claims. Instead, an estimate is first procured separately, using background knowledge about the claims. This is then used with the development factors to obtain reserve estimates. Although not originally formulated using a Bayesian philosophy, the Bornhuetter-Ferguson technique is quite clearly suited to this approach because of the basic idea of what it is trying to do: incorporate expert opinion. Thus, we have a second important motivation for considering the use of Bayesian reserving methods. These are two very important examples of reserving approaches commonly used, which are best modelled using Bayesian methods. Among previous papers to discuss Bayesian claims reserving, we would mention de Alba (2002) and Ntzoufras and Dellaportas (2002).

One important property of Bayesian methods can be seen which makes them suitable for using when a stochastic reserving model is used: they allow us to incorporate expert knowledge in a natural way, overcoming any difficulties about the effect on the assumptions made. In this paper, we consider the use of Bayesian models for claims reserving in order to incorporate expert opinion into the prediction of reserves. We concentrate on two areas as mentioned above: the Bornhuetter-Ferguson technique and the insertion of prior knowledge about individual development factors in the chain-ladder technique. The possibility of including expert knowledge is an important property of Bayesian models, but there is another equally important point: the ease with which they can be implemented. This is due to the modern developments in Bayesian methodology based on so-called "Markov chain Monte Carlo" methods. It is difficult to emphasize enough the effect these methods have had on Bayesian statistics, but the books by Congdon (Congdon, 2001 and 2003) give some idea of the scope of the applications for which they have been used. The crucial aspect as far as this paper is concerned is that they are based on simulation, and therefore have some similarities with bootstrapping methods that, as was mentioned above, have gained in popularity for claims reserving. It is also important that there is software available, which is relatively easy to use, which allows us to implement the Bayesian models for claims reserving. While it is straightforward to define a Bayesian model, it is not always so easy to find the required posterior distributions for the parameters, and predictive distributions for future observations. However, this has been made much easier in recent years by the development of MCMC methods, and by the software package winBUGS (Spiegelhalter et al, 1996). This software package is freely available from *http://www.mrc-bsu.cam.ac.uk/bugs*, and the programs for carrying out the Bayesian analysis for the models described in this paper are contained in the Appendix.

The paper is set out as follows. In section 2, we describe the notation and basic methods used in this paper, and in section 3 we summarize the stochastic models used in the context of the chain-ladder technique. Sections 4 and 5 describe the Bayesian models for incorporating prior information into the reserving process. In section 6 we describe in some detail how to implement the Bayesian models so that the reader can investigate the use of these models himself/herself, using the programs given in the Appendix. In section 7 we state some conclusions.

2. Notation and basic methods

To begin with, we define the notation used in this paper, and in doing so briefly summarize the chain-ladder technique and the Bornhuetter-Ferguson method. Although the methods can also be applied to other shapes of data, in order that the notation should not get too complicated we make the assumption that the data is in the shape of a triangle. Thus, without loss of generality, we assume that the data consist of a triangle of incremental claims:

$$\begin{array}{llll} C_{11} & C_{12} & \cdots & C_{1n} \\ C_{21} & \cdots & C_{2,n-1} & \\ \vdots & & & \\ C_{n1} & & & \end{array}$$

This can be also written as $\{C_{ij} : j = 1,\ldots,n-i+1; i = 1,\ldots,n\}$, where n is the number of accident years. C_{ij} is used to denote incremental claims, and D_{ij} is used to denote the cumulative claims, defined by:

$$D_{ij} = \sum_{k=1}^{j} C_{ik} .$$

One of the methods considered in this paper is the chain-ladder technique, and the development factors $\{\lambda_j : j = 2,\ldots,n\}$. The usual estimates of the development factors from the standard chain-ladder technique are

$$\hat{\lambda}_j = \frac{\sum_{i=1}^{n-j+1} D_{ij}}{\sum_{i=1}^{n-j+1} D_{i,j-1}} .$$

Note that we only consider forecasting claims up to the latest development year (n) so far observed, and no tail factors are applied. It would be possible to extend this to allow a tail factor, using the same methods, but no specific modelling is carried out in this paper of the shape of the run-off beyond the latest development year. Thus, we refer to cumulative claims up to development year n, $D_{in} = \sum_{k=1}^{n} C_{ik}$, as "ultimate claims". For the chain-ladder technique, the estimate of outstanding claims is $D_{i,n-i+1}\left(\hat{\lambda}_{n-i+2}\hat{\lambda}_{n-i+3}\cdots\hat{\lambda}_n - 1\right)$.

The first case we consider is when these development factor estimates are not used for all rows. In other words, we consider the more general case where there is a separate development factor in each row, $\lambda_{i,j}$. The standard chain-ladder model sets $\lambda_{i,j} = \lambda_j$, for $i = 1,2,\ldots,n-i+1; j = 1,2,\ldots n$, but we consider allowing the more general case where development factors can change from row to row. Section 4 describes the Bayesian approach to this, allowing expert knowledge to be used to set prior distributions for these parameters. In this way, we will be able to intervene in the estimation of the development factors, or else simply leave them for the standard chain-ladder model to estimate.

In section 5 we consider the Bornhuetter-Ferguson method. This method uses the development factors from the chain-ladder technique, but it incorporates knowledge about the "level" of each row by replacing the chain-ladder estimate of outstanding claims, $D_{i,n-i+1}\left(\hat{\lambda}_{n-i+2}\hat{\lambda}_{n-i+3}\cdots\hat{\lambda}_n - 1\right)$ by $M_i \frac{1}{\hat{\lambda}_{n-i+2}\hat{\lambda}_{n-i+3}\cdots\hat{\lambda}_n}\left(\hat{\lambda}_{n-i+2}\hat{\lambda}_{n-i+3}\cdots\hat{\lambda}_n - 1\right)$.
Here, M_i denotes an value for the ultimate claims for accident year *i* which is obtained using expert knowledge about the claims, for example taken from the premium calculation. Thus, $M_i \frac{1}{\hat{\lambda}_{n-i+2}\hat{\lambda}_{n-i+3}\cdots\hat{\lambda}_n}$ replaces the latest cumulative claims for accident year *i*, to which the usual chain-ladder parameters are applied to obtain the estimate of outstanding claims. From this, it can be seen that the difference between the Bornhuetter-Ferguson method and the chain-ladder technique is that the Bornhuetter-Ferguson technique uses an external estimate of the "level" of each row in the triangle, while the chain-ladder technique uses the data in that row itself. The Bornhuetter-Ferguson method can be formulated using a Bayesian approach, with the information about the external estimates for each row being used to form the prior distributions, as in section 5.

This section has defined the notation used in the paper, and outlined the basic reserving methods which will be considered using stochastic approaches. In order to do this, a brief introduction to the stochastic models is needed, and this is given in section 3.

3. Stochastic Models for the Chain-ladder Technique

This section gives a brief summary of stochastic models that are related to the chain-ladder technique. A much fuller account may be found in England and Verrall (2002), and in the references in that paper and its discussion. We consider the chain-ladder technique, and note that it is possible to apply Bayesian methods in a similar way to other models.

There are a number of different approaches that can be taken to the chain-ladder technique, with various positivity constraints, all of which give the same reserve estimates as the chain-ladder technique. The connections between the chain-ladder technique and various stochastic models have been explored in a number of previous papers. For example, Mack (1993) takes a non-parametric approach and specifies only the first 2 moments for the cumulative claims. In Mack's model the mean and variance of $D_{ij} \mid D_{i,j-1}, \lambda, \sigma^2$ are $\lambda_j D_{i,j-1}$ and $\sigma_j^2 D_{i,j-1}$, respectively. Estimates of all the parameters are derived, and the properties of the model are examined. As was stated in the introduction, one of the advantages of this approach is that the parameter estimates and prediction errors can be obtained just using a spreadsheet, without having recourse to a statistical package or any complex programming. The consequence of not specifying a distribution for the data is that there is no predictive distribution. Also, there are separate parameters in the variance that must also be estimated, separately from the estimation of the development factors.

As a separate stream of research, models in the form of generalized linear models have also been considered. Renshaw and Verrall (1998) used an approach based on generalized linear models (McCullagh and Nelder, 1989) and examined the over-dispersed Poisson model for incremental claims:

$C_{ij} \mid c, \alpha, \beta, \varphi \sim$ independent over-dispersed Poisson, with mean, m_{ij}, where $\log(m_{ij}) = c + \alpha_i + \beta_j$ and $\alpha_1 = \beta_1 = 0$.

The term "over-dispersed" requires some explanation. It is used here in connection with the Poisson distribution, and it means that if $X \sim$ Poisson (μ), then $Y = \varphi X$ follows the over-dispersed Poisson distribution with $E(Y) = \varphi\mu$ and $V(Y) = \varphi^2 E(X) = \varphi^2\mu$. φ is usually greater than 1 – hence the term "over-dispersed" – but this is not a necessity. It can also be used for other distributions, and we make use of it for the negative binomial distribution. As with the Poisson distribution, the over-dispersed negative binomial distribution is defined such that if $X \sim$ negative binomial then $Y = \varphi X$ follows the over-dispersed negative binomial distribution. Furthermore, a quasi-likelihood approach is taken so that the claims data are not restricted to the positive integers.

It can be seen that this formulation has some similarities with the model of Kremer (1982), but it has a number of advantages. It does not necessarily break down if there are negative incremental claims values, it gives the same reserve estimates as the chain-ladder technique, and it has been found to be more stable than the log-normal model of Kremer. For these reasons, we concentrate on it in this paper. There are a number of ways of writing this model, which are useful in different context (note that the reserve estimates are unaffected by the way the model is written). Another way of writing the over-dispersed Poisson model for the chain-ladder technique is as follows:

$C_{ij} \mid x, y, \varphi \sim$ independent over-dispersed Poisson, with mean $x_i y_j$, and $\sum_{k=1}^{n} y_k = 1$.

Here $x = \{x_1, x_2, \ldots, x_n\}$ and $y = \{y_1, y_2, \ldots, y_n\}$ are parameter vectors relating to the rows (accident years) and columns (development years), respectively, of the run-off triangle. The parameter $x_i = E[D_{in}]$, and so represents expected ultimate cumulative claims (up to the latest development year so far observed, n) for the ith accident year. The column parameters, y_j, can be interpreted as the proportions of ultimate claims which emerge in each development year.

Although the over-dispersed Poisson models give the same reserve estimates as the chain-ladder technique (as long as the row and column sums of incremental claims are positive), the connection with the chain-ladder technique is not immediately apparent from this formulation of the model. For this reason, the negative binomial model was developed by Verrall (2000), building on the over-dispersed Poisson model. Verrall (2000) showed that the same predictive distribution can be obtained from a negative binomial model (also with the inclusion of an over-dispersion parameter). In this recursive approach, the incremental claims have an over-dispersed negative binomial distribution, with mean and variance

$$(\lambda_j - 1)D_{i,j-1} \quad \text{and} \quad \phi\lambda_j(\lambda_j - 1)D_{i,j-1}, \quad \text{respectively.}$$

Again, the reserve estimates are the same as the chain-ladder technique, and the same positivity constraints apply as for the over-dispersed Poisson model. It is clear from this that the column sums must be positive since a negative sum would result in a development factor less than 1 ($\lambda_j < 1$), causing the variance to be negative. It is important to note that exactly the same predictive distribution can be obtained from either the Poisson or negative binomial models. Verrall (2000) also argued that the model could be specified either for incremental or cumulative claims, with no difference in the results. The negative binomial model has the advantage that the form of the mean is exactly the same as that which naturally arises from the chain-ladder technique. In fact, by adding the previous cumulative claims, an equivalent model for $D_{ij} \mid D_{i,j-1}, \lambda, \phi$ has an over-dispersed negative binomial distribution, with mean and variance

$$\lambda_j D_{i,j-1} \quad \text{and} \quad \phi\lambda_j(\lambda_j - 1)D_{i,j-1}, \quad \text{respectively.}$$

Here the connection with the chain-ladder technique is immediately apparent because of the format of the mean.

A further model, which is not considered further in this paper, is closely connected with Mack's model, and deals with the problem of negative incremental claims. This model replaces the negative binomial by a Normal distribution, whose mean is unchanged, but whose variance is altered to accommodate the case when $\lambda_j < 1$. Preserving as much of $\lambda_j(\lambda_j - 1)D_{i,j-1}$ as possible, the variance is still proportional to $D_{i,j-1}$, with the constant of proportionality depending on j, but a Normal approximation is used for the distribution of incremental claims. Thus, $C_{ij} \mid D_{i,j-1}, \lambda, \phi$ is approximately Normally distributed, with mean and variance

$$D_{i,j-1}(\lambda_j - 1) \text{ and } \phi_j D_{i,j-1}, \text{ respectively,}$$

or $D_{ij} \mid D_{i,j-1}, \lambda, \phi$ is approximately Normally distributed, with mean and variance

$$\lambda_j D_{i,j-1} \text{ and } \phi_j D_{i,j-1}, \text{ respectively.}$$

As for Mack's model, there is now another set of parameters in the variance that needs to be estimated.

For each of these models, the mean square error of prediction can be obtained, allowing the construction of prediction intervals, for example. Claims reserving is a predictive process: given the data, we try to predict future claims. These models apply to all the data, both observed and future observations. The estimation is based on the observed data, and we require predictive distributions for the future observation. We use the expected value of the distribution of future claims as the prediction. When considering variability, attention is focused on the root mean squared error of prediction (RMSEP), also known as the prediction error. To explain what this is, we consider, for simplicity, a random variable, y, and a predicted value $\hat{y}$. The mean squared error of prediction (MSEP) is the expected square difference between the actual outcome and the predicted value, $E\left[(y-\hat{y})^2\right]$, and can be written as follows:

$$E\left[(y-\hat{y})^2\right] = E\left[((y-E[y])-(\hat{y}-E[y]))^2\right].$$

In order to obtain an estimate of this, it is necessary to plug-in $\hat{y}$ instead of y in the final expectation. Then the MSEP can be expanded as follows:

$$E\left[(y-\hat{y})^2\right] \approx E\left[(y-E[y])^2\right] - 2E[(y-E[y])(\hat{y}-E[\hat{y}])] + E\left[(\hat{y}-E[\hat{y}])^2\right],$$

Assuming future observations are independent of past observations, the middle term is zero, and

$$E\left[(y-\hat{y})^2\right] \approx E\left[(y-E[y])^2\right] + E\left[(\hat{y}-E[\hat{y}])^2\right].$$

In words, this is

prediction variance = process variance + estimation variance.

It is important to understand the difference between the prediction error and the standard error. Strictly, the standard error is the square root of the estimation variance. The prediction error is concerned with the variability of a forecast, taking account of uncertainty in parameter estimation and also of the inherent variability in the data being forecast. Further details of this can be found in England and Verrall (2002).

Using non-Bayesian methods, these two components – the process variance and the estimation variance – are estimated separately, and section 7 of England and Verrall (2002) goes into a lot of detail about this. The direct calculation of these quantities can be a tricky process, and this is one of the reasons for the popularity of the bootstrap. The bootstrap uses a fairly simple simulation approach to obtain simulated estimates of the prediction variance in a spreadsheet. Fortunately, the same advantages apply to the Bayesian methods: the full predictive distribution can be found using simulation methods, and the RMSEP can be obtained directly by calculating its standard deviation. In addition, it is preferable to have the full predictive distribution, rather than just the first 2 moments, which is another advantage of Bayesian methods.

The purpose of this paper is to show how expert opinion, from sources other than the specific data set under consideration, can be incorporated into the predictive distributions of the reserves. We use the approach of generalized linear models outlined in this section, concentrating on the over-dispersed Poisson and negative binomial models. We begin with considering how it is possible to intervene in the development factors for the chain-ladder technique in section 4, and then consider the Bornhuetter-Ferguson method in section 5.

4. Incorporating expert opinion about the development factors

In this section, a Bayesian model is specified which allows the practitioner to intervene in the estimation of the development factors for the chain-ladder technique. There are a number of ways in which this could be used, and we describe some possibilities in this section. It is expected that a practitioner would be able to extend these to cover situations which, although not specifically covered here, would also be useful. The cases considered here are the intervention in a development factor in a particular row particular, and the choice of how many years of data to use in the estimation. The reasons for intervening in these ways could be that there is information that the settlement pattern has changed, making it inappropriate to use the same development factor for each row.

For the first case, what may happen in practice is that a development factor in a particular row is simply changed. Thus, although the same development parameters (and hence run-off pattern) is usually applied for all accident years, if there is some exogenous information that indicates that this is not appropriate, the practitioner may decide to apply a different development factor (or set of factors) in some, or all, rows.

In the second case, it is common to look at, say, 5-year volume weighted averages in calculating the development factors, rather than using all the available data in the triangle. The Bayesian methods make this particularly easy to do, and are flexible enough to allow many possibilities.

We use the negative binomial model described in section 3, with different development factors in each row. This is the model for the data, and we then specify prior distributions for the development factors. In this way, we can choose prior distributions that reproduce the chain-ladder results, or we can intervene and use prior distributions based on external knowledge. The model for incremental claims, $C_{ij} \mid D_{i,j-1}, \lambda, \phi$, is an over-dispersed negative binomial distribution, with mean and variance

$\left(\lambda_{i,j}-1\right)D_{i,j-1}$ and $\varphi\lambda_{i,j}\left(\lambda_{i,j}-1\right)D_{i,j-1}$, respectively.

We next need to define prior distributions for the development factors, $\lambda_{i,j}$. It is possible to set some of these equal to each other (within each column) in order to revert to the standard chain-ladder model. This is done by setting

$$\lambda_{i,j}=\lambda_j \qquad \text{for} \qquad i=1,2,\ldots,n-i+1;\ j=1,2,\ldots n$$

and defining vague prior distributions for λ_j $\left(j=1,2,\ldots n\right)$. This was the approach taken in section 8.4 of England and Verrall (2002) and is very similar to that taken by de Alba (2002). This can provide a very straightforward method to obtain prediction errors and predictive distributions for the chain-ladder technique.

However, we really want to move away from the basic chain-ladder technique, and construct Bayesian prior distributions that encompass the expert opinion about the development parameters. Suppose, for example, that we have a 10×10 triangle. We consider the 2 possibilities for incorporating expert knowledge described above.

To illustrate the first case, suppose that there is information that implies that the second development factor (from column 2 to column 3) should be given the value 2, for rows 8, 9,and 10, and that there is no indication that the other parameters should be treated differently from the standard chain-ladder technique. An appropriate way to treat this would be to specify

$$\begin{aligned}
&\lambda_{i,j}=\lambda_j && \text{for} && i=1,2,\ldots,n-i+1;\ j=1,3,4,\ldots,n\\
&\lambda_{i,2}=\lambda_2 && \text{for} && i=1,2,\ldots,7\\
&\lambda_{8,2}=\lambda_{9,2}=\lambda_{10,2}
\end{aligned}$$

The means and variances of the prior distributions of the parameters are chosen to reflect the expert opinion:

$\lambda_{8,2}$ has a prior distribution with mean 2 and variance *W*, where *W* is set to reflect the strength of the prior information

λ_j have prior distributions with a large variances.

For the second case, we divide the data into two parts using the prior distributions. To do this, we set

$$\begin{aligned}
&\lambda_{i,j}=\lambda_j \text{ for } i=n-i-3,n-i-2,n-i-1,n-i,n-i+1\\
&\lambda_{i,j}=\lambda_j^{*} \text{ for } i=1,2,\ldots,n-i-4
\end{aligned}$$

and give both λ_j and λ_j^* prior distributions with large variances so that they are estimated from the data. Adjustments to the specification are made in the later development years, where there are less than 5 rows. For these columns there is just one development parameter, λ_j.

The specific form of the prior distribution (gamma, log-normal, etc) is usually chosen so that the numerical procedures in winBUGS work as well as possible.

These models are used as illustrations of the possibilities for incorporating expert knowledge about the development pattern, but it is (of course) possible to specify many other prior distributions. In Appendix 1, the winBUGS code is supplied, which can be cut and pasted directly in order to examine these methods. Section 6 contains a number of examples including the ones described in this section.

5. A Bayesian model for the Bornhuetter-Ferguson method.

In this section, we show how the Bornhuetter-Ferguson method can be considered in a Bayesian context, using the approach of Verrall (2004). For further background on the Bornhuetter-Ferguson method, see Mack (2000).

In section 3, the over-dispersed Poisson model was defined as follows.

$C_{ij} \mid x, y, \varphi \sim$ independent over-dispersed Poisson, with mean $x_i y_j$, and $\sum_{k=1}^{n} y_k = 1$,

and in the Bayesian context, we also require prior distributions for the parameters. The Bornhuetter-Ferguson method assumes that there is expert opinion about the level of each row, and we therefore concentrate first on the specification of prior distributions for these. The most convenient form to use is gamma distributions:

$$x_i \mid \alpha_i, \beta_i \sim \text{ independent } \Gamma(\alpha_i, \beta_i).$$

There is a wide range of possible choices for the parameters of these prior distributions, α_i and β_i. It is easiest to consider the mean and variance of the gamma distribution, $\frac{\alpha_i}{\beta_i}$ and $\frac{\alpha_i}{\beta_i^2}$, respectively. These can be written as M_i and $\frac{M_i}{\beta_i}$, from which it can be seen that, for a given choice of M_i, the variance can be altered by changing the value of β_i. To consider a simple example, suppose it has been decided that $M_i = 1000$. The table below shows how the value of β_i affects the variance of the prior distribution, while M_i is kept constant.

α_i	β_i	M_i	$\frac{M_i}{\beta_i}$
10000	10	1000	100
1000	1	1000	1000
100	0.1	1000	10000

Clearly, choosing a larger value of β_i implies we are more sure about the value of M_i, and choosing a smaller value means we are less sure.

We now consider the effect of using these prior distributions on the model for the data. Recall that, for the chain-ladder technique, the mean of the distribution of incremental claims may be written as $(\lambda_j - 1)D_{i,j-1}$. It can shown that the equivalent mean for the Bayesian model is

$$Z_{ij}(\lambda_j - 1)D_{i,j-1} + (1 - Z_{ij})(\lambda_j - 1)M_i \frac{1}{\lambda_j \lambda_{j+1} \cdots \lambda_n}$$

where $Z_{ij} = \dfrac{\sum_{k=1}^{j-1} y_k}{\beta_i \varphi + \sum_{k=1}^{j-1} y_k}$.

This can be seen to be in the form of a credibility formula, and is a trade-off between the chain-ladder ($(\lambda_j - 1)D_{i,j-1}$) and the Bornhuetter-Ferguson ($(\lambda_j - 1)M_i \frac{1}{\lambda_j \lambda_{j+1} \cdots \lambda_n}$). The credibility factor, Z_{ij}, governs the trade-off between the prior mean and the data. We can influence the balance of this trade-off through the choice of β_i. In line with the discussion above, the larger the value of β_i the closer we get to the Bornhuetter-Ferguson method, and the smaller the value of β_i, the closer we get to the chain-ladder technique. In this way, we can use different specifications of the prior distributions for the row parameters in order to use the chain-ladder technique, the Bornhuetter-Ferguson method, or for a complete spectrum of methods between these two extremes. If we choose to use prior distributions with large variances, we do not influence the parameter estimates and the result will be the same as (or extremely close to) the chain-ladder technique. If we use very small variances, we are saying that we are very sure what the parameter values should be and the results will be the same as (or very close to) the Bornhuetter-Ferguson method. Thus, we can use these methods within a stochastic framework, and we can also consider using the whole range of models that lie between these two.

We have yet to consider the estimation of the column parameters, other than to point out at the Bornhuetter-Ferguson method, being deterministic, simply plugs in the chain-ladder parameter estimates. We now consider this issue in more detail, and define a Bayesian approach to the Bornhuetter-Ferguson method. One option is to simply use plug-in estimates, obtained, for example, from the straightforward chain-ladder technique. This is the approach used in the deterministic application of the Bornhuetter-Ferguson method, but it is not suitable here since we would prefer a stochastic approach. A better option is to define improper prior distributions for the column parameters, and estimate the column parameters *first*, before applying prior distributions for the row parameters and estimating these. This second option allows us to take into account the fact that the column parameters have been estimated when calculating the prediction errors, predictive distribution, etc. It is not required to include any information about the column parameters, and hence we use improper gamma distributions for the column parameters, and derive the posterior distributions of these using a standard Bayesian prior-posterior analysis. The result of this is a distribution which looks similar to the negative binomial model for the chain-ladder technique, but which is recursive in i instead of j:

$C_{ij} \mid C_{1,j}, C_{2,j}, \ldots, C_{i-1,j}, x, \varphi \sim$ over-dispersed negative binomial, with mean $(\gamma_i - 1)\sum_{m=1}^{i-1} C_{mj}$.

Comparing this to the mean of the chain-ladder model, $(\lambda_j - 1) D_{i,j-1} = (\lambda_j - 1)\sum_{m=1}^{j-1} C_{i,m}$, it can be seen that they are identical in form, with the recursion either being across the rows, or down the columns.

In the context of the Bornhuetter-Ferguson method, we now have the stochastic version of this model. The Bornhuetter-Ferguson method inserts values for the expected ultimate claims in each row, x_i, in the form of the values, M_i. In the Bayesian context, prior distributions will be defined for the parameters x_i, as discussed above. However, the model has been reparameterised, with a new set of parameters, γ_i. Hence, it is necessary to define the relationship between the new parameters, γ_i, and the original parameters, x_i. This is given in the equation below, which can be used to find values of γ_i from the values of x_i given in the prior distributions.

$$\gamma_i = \frac{x_i\left(1 - \frac{1}{\prod_{k=n-i+2}^{n} \lambda_k}\right)}{C_{1n}} \frac{1}{\prod_{k=1}^{i-1} \gamma_k} + 1$$

The Bornhuetter-Ferguson technique can be reproduced by using strong prior information for the row parameters, x, and the chain-ladder technique can be reproduced by using improper priors for the row parameters. In other words, the Bornhuetter-Ferguson technique assumes that we are completely sure about the values of the row parameters, and their prior distributions have very small variances, while the chain-ladder technique assumes there is no information and has very large variances.

This has now defined a stochastic version of the Bornhuetter-Ferguson technique. Since the column parameters (the development factors) are dealt with first, using improper prior distributions, their estimates will be those implied by the chain-ladder technique. Prior information can be defined in terms of distributions for the parameters x_i, which can then be converted into values for the parameters γ_i, and this is implemented in section 6.

6. Implementation

This section explains how the Bayesian models can be implemented, using the software package winBUGS (Spiegelhalter et al, 1996) which is available from http://www.mrc-bsu.cam.ac.uk/bugs. The programs used in these illustrations are contained in the Appendix.

The data set we consider in this section is taken from Taylor and Ashe (1983), and has also been used in a number of previous papers on stochastic reserving. The incremental claims data is given in table 1, together with the chain-ladder results for comparison purposes.

Table 1. Data from Taylor and Ashe (1983) with the chain-ladder estimates

357,848	766,940	610,542	482,940	527,326	574,398	146,342	139,950	227,229	67,948
352,118	884,021	933,894	1,183,289	445,745	320,996	527,804	266,172	425,046	
290,507	1,001,799	926,219	1,016,654	750,816	146,923	495,992	280,405		
310,608	1,108,250	776,189	1,562,400	272,482	352,053	206,286			
443,160	693,190	991,983	769,488	504,851	470,639				
396,132	937,085	847,498	805,037	705,960					
440,832	847,631	1,131,398	1,063,269						
359,480	1,061,648	1,443,370							
376,686	986,608								
344,014									

Chain-ladder development factors:

3.4906	1.7473	1.4574	1.1739	1.1038	1.0863	1.0539	1.0766	1.0177

Chain-ladder reserve estimates:

2	94,634
3	469,511
4	709,638
5	984,889
6	1,419,459
7	2,177,641
8	3,920,301
9	4,278,972
10	4,625,811
Overall	18,680,856

Before looking at the uses of the Bayesian models, we should discuss the nuisance parameter ϕ. In a full Bayesian analysis, we should also give this a prior distribution and estimate it along with the other parameters. However, for ease of implementation we instead use a plug-in estimate, in line with the approach taken in classical methods (in England and Verrall, 2002, for example). The value used is that obtained from the straightforward application of the over-dispersed Poisson model, estimating the row and column parameters using maximum likelihood estimation (it is possible to use S-Plus or excel for this).

6.1 Using the Software

Before considering the results from the programs in any detail, we first describe how to set up the software and run one of the programs from scratch. An excellent reference in the context of actuarial modelling is Skollnik (2001). Table 1 shows the standard chain-ladder results, and in this section we will implement the model described in section 5, but use the assumptions of the chain-ladder technique, rather than the Bornhuetter-Ferguson method. This means that we will use large variances for the prior distributions for the ultimate claims in each row, implying that there is no prior knowledge about them, and hence the results we obtain should be close to the chain-ladder results. Thus, we will first reproduce the results which can also be obtained using non-Bayesian methods (see England and Verrall, 2002, for more details of the non-Bayesian methods). After going through this example in detail, the remainder of this section will show how the Bayesian models incorporating prior knowledge described in sections 4 and 5 can be implemented, and illustrate the effect that the choice of prior distributions can have.

The steps necessary for implementing the chain-ladder technique in winBUGS are lised below.

1. Go to the web site, download the latest version of the software and install it on a pc.
2. Go back to the web site and register, and you will be sent a copy of the key to unlock the software. Follow the instructions in the email for unlocking the software.
3. Once you have a fully functioning version of winBUGS on a pc, you can run the programs in the Appendix. Open winBUGS and click on "File" in the top toolbar, and then "New" in the pop-down list. This will open a new window.
4. Copy the program in (i) of the Appendix, including the word "model" at the top and all the data at the bottom, right down to where it the next subsection begins at (ii). The last line is 0,0,0,0,0,0,0,0,0)) Paste all of this into the new window in winBUGS.
5. In winBUGS select "Model" in the toolbar at the top and "Specification" in the pop-down list. This opens a new window called "Specification Tool".
6. Highlight the word "model" at the top of the program, and then click "check model" in the Specification Tool window. If all is well, it will say "model is syntactically correct" in the bottom left corner.
7. Now move down in the window containing the program until you get to #DATA. Highlight the word "list" immediately below that, and click "load

data" in the Specification Tool window. It should say "data loaded" in the bottom left corner.

8. Click "compile" in the Specification Tool window. After a few seconds, it should say "model complied" in the bottom left corner.
9. Now move down in the window containing the program until you get to #INITIAL VALUES. Highlight the word "list" immediately below that, and click "load inits" in the Specification Tool window. It should say "model is initialised" in the bottom left corner.
10. Select "Model" in the toolbar at the top and "Update" in the pop-down list. This opens a new window called "Update Tool". The number of iterations in the simulation process can be changed in this window, by changing the figure next to "updates". Just at the moment, 1000 is sufficient, so just click on "update". This runs 1000 simulations without storing the results. This may take a few minutes: don't be concerned if nothing appears to be happening! When it is complete, a message appears in the bottom left corner saying how long the updates took (for my laptop it was 221 seconds).
11. Select "Inference" in the toolbar at the top and "Samples" in the pop-down list. This opens a new window called "Sample Monitor Tool". We want to look at the row totals and overall total, which have been defined as a vector R and Total in the program. In the Sample Monitor Tool window, click in the box to the right of the word "node", and type R. Then click on "set". Repeat for Total, noting that *it is case sensitive*.
12. Return to the Update Tool Window and click on Update to perform 1000 simulations. This should be quicker (6 seconds for my laptop). This time the values of R and Total will be stored.
13. Return to the Sample Monitor Tool window, type * in the box to the right of the word "node", and click "stats". This will give a new window with something like the results below. This completes the steps necessary for fitting the Bayesian model.

node	mean	sd	MC error	2.5%	median	97.5%	start	sample
R[2]	92750.0	110600.0	2963.0	779.2	56320.0	412800.0	1001	1000
R[3]	473900.0	223100.0	6424.0	1.52E+5	4.4E+5	1.011E+6	1001	1000
R[4]	7.05E+5	2.58E+5	9085.0	307600.0	674500.0	1.288E+6	1001	1000
R[5]	985800.0	304600.0	8127.0	467600.0	960600.0	1.667E+6	1001	1000
R[6]	1.417E+6	378300.0	13430.0	768500.0	1.399E+6	2.217E+6	1001	1000
R[7]	2.174E+6	5.19E+5	16850.0	1.271E+6	2.132E+6	3.233E+6	1001	1000
R[8]	3.925E+6	776900.0	28100.0	2.585E+6	3.885E+6	5.555E+6	1001	1000
R[9]	4.284E+6	1.066E+6	36840.0	2.464E+6	4.207E+6	6.731E+6	1001	1000
R[10]	4.641E+6	2.002E+6	61630.0	1.73E+6	4.407E+6	9.345E+6	1001	1000
Total	1.87E+7	3.056E+6	101600.0	1.314E+7	1.861E+7	2.554E+7	1001	1000

The columns headed mean and sd give the predicted reserves and prediction errors, and these values can be compared with the chain-ladder results above. Since this is a simulation process, the results will depend on the prior distributions, the initial values and the number of iterations carried out. The prior distributions in the program had reasonably large variances so the results should be close to the chain-ladder results. More simulations should be used in steps 10 and 12 (we use 10,000 in the illustrations below), and the prior variances could be increased. Using this number of simulations gives the results shown in Table 2.

Table 2. Chain-ladder results. The Prediction Error is equal to the Bayesian Standard Deviation

	Chain-ladder Reserve	Bayesian Mean	Bayesian Standard Deviation	Prediction Error (%)
Year 2	94,634	94,440	111,100	118%
Year 3	469,511	471,400	219,400	47%
Year 4	709,638	716,300	263,600	37%
Year 5	984,889	991,600	308,100	31%
Year 6	1,419,459	1,424,000	374,700	26%
Year 7	2,177,641	2,186,000	497,200	23%
Year 8	3,920,301	3,935,000	791,000	20%
Year 9	4,278,972	4,315,000	1,068,000	25%
Year 10	4,625,811	4,671,000	2,013,000	43%
Overall	18,680,856	18,800,000	2,975,000	16%

However, the results certainly confirm that we can reproduce the chain-ladder results, and produce the prediction errors. It is also possible to obtain other information about the model from winBUGS. For example, it is possible to produce full predictive distributions, using "density" in the Sample Monitor Tool window.

We have now described one implementation of a Bayesian model using winBUGS. In the rest of this section, we consider the Bayesian models described in sections 4 and 5, in order to consider how expert opinion can be incorporated into the predictive distribution of reserves. In each case, the programs are available in the Appendix, and the results can be reproduced using steps 3 to 13, above. It should be noted that this is a simulation-based program, so that the results obtained may not match exactly the results given below. However, there should be no significant differences, with the differences that there are being due to simulation error.

6.2 Intervention in the chain-ladder technique

We now consider using a prior distribution to intervene in some of the parameters of the chain-ladder model, instead of using prior distributions with large variances which just reproduce the chain-ladder estimates. The implementation is set up in section (ii) of the Appendix, and the program can be cut and pasted into winBUGS and run following steps 3 onwards, above.

We consider 2 cases, as discussed in section 4. For the first case, we assume that there is information that implies that the second development factor (from column 2 to column 3) should be given the value 1.5, for rows 7, 8, 9,and 10, and that there is no indication that the other parameters should be treated differently from the standard chain-ladder technique. In order to implement this, the parameter for the second development factor for rows 7-10 is given a prior distribution with mean 1.5. We then look at two different choices for the prior variance for this parameter. Using a large variance means that the parameter is estimated separately from the other rows, but using the data without letting the prior mean influence it too greatly. We then use a standard deviation of 0.1 for the prior distribution, so that the prior mean has a greater influence.

We consider first the estimate of the second development factor. The chain ladder estimate is 1.7473 and the individual development factors for the triangle are shown in table 3. The rows for the second development factor that are modelled separately are shown in italics. The estimate using the Bayesian models is 1.68 for rows 1-6. When a large variance is used for the prior distribution of the development factor for rows 7-10, the estimate using the Bayesian model is 1.971. With the smaller variance for this prior distribution, the estimate is 1.673, and has been drawn down towards the prior mean of 1.5. This clearly shows how the prior distributions can be used to influence the parameter estimates.

Table 3. Individual development factors

3.143	1.543	1.278	1.238	1.209	1.044	1.040	1.063	1.018
3.511	1.755	1.545	1.133	1.084	1.128	1.057	1.086	
4.448	1.717	1.458	1.232	1.037	1.120	1.061		
4.568	1.547	1.712	1.073	1.087	1.047			
2.564	1.873	1.362	1.174	1.138				
3.366	1.636	1.369	1.236					
2.923	*1.878*	1.439						
3.953	*2.016*							
3.619								

The effect on the reserve estimates is shown in table 4, which compares the reserves and prediction errors for the two cases outlined above with the results for the chain-ladder model (which could be produced using the program in 6.1 on this set of data). The chain-ladder figures are slightly different from those given in table 2 because this is a simulation method.

Table 4. Reserves and prediction errors for the chain-ladder and Bayesian models

	Chain-ladder		Large variance		Small variance	
	Reserve	Prediction Error (%)	Reserve	Prediction Error (%)	Reserve	Prediction Error (%)
Year 2	97,910	115%	95,920	116%	95,380	117%
Year 3	471,200	46%	475,700	46%	470,500	47%
Year 4	711,100	38%	721,700	37%	714,400	37%
Year 5	989,200	31%	996,800	31%	994,700	31%
Year 6	1,424,000	27%	1,429,000	26%	1,428,000	27%
Year 7	2,187,000	23%	2,196,000	23%	2,185,000	23%
Year 8	3,930,000	20%	3,937,000	20%	3,932,000	20%
Year 9	4,307,000	24%	4,998,000	27%	4,044,000	25%
Year 10	4,674,000	43%	5,337,000	44%	4,496,000	43%
Overall	18,790,000	16%	20,190,000	17%	18,360,000	16%

It is interesting to note that, in this case, the intervention has not had a marked effect on the prediction errors (in percentage terms). However, the prediction errors themselves have changed considerably, and this indicates that it is important to think of the prediction error as a percentage of the prediction. Other prior distributions could have a greater effect on the percentage prediction error.

The second case we consider is when we use only the most recent data for the estimation of each development factor. For the last 3 development factors, all the data is used because there is no more than 3 years for each. For the other development factors, only the 3 most recent years are used. The estimates of the development factors are shown in table 5. The estimates of the first development factor are not affected by the change in the model (the small differences could be due to simulation error or the changes elsewhere). For the other development factors, the estimates can be seen to be affected by the model assumptions.

Table 5. Development factors using 3 most recent years data separately

	3.143	1.543	1.278	1.238	1.209	1.044	1.040	1.063	1.018
	3.511	1.755	1.545	1.133	1.084	1.128	1.057	1.086	
	4.448	1.717	1.458	1.232	1.037	1.120	1.061		
	4.568	1.547	1.712	1.073	1.087	1.047			
	2.564	1.873	1.362	1.174	1.138				
	3.366	1.636	1.369	1.236					
	2.923	1.878	1.439						
	3.953	2.016							
	3.619								
Earlier rows	3.575	1.688	1.513	1.197	1.139	1.045			
Recent rows	3.579	1.852	1.393	1.155	1.085	1.099	1.054	1.076	1.018
All rows	3.527	1.751	1.46	1.175	1.104	1.087	1.054	1.076	1.018

The effect of using only the latest 3 years in the estimation of the development factors in the forecasting of outstanding claims can be seen in table 6.

Table 6 Reserve estimates using 3 most recent years data

	Chain-ladder		Bayesian Model	
	Reserve	Prediction Error (%)	Reserve	Prediction Error (%)
Year 2	97,910	115%	94,860	115%
Year 3	471,200	46%	469,300	46%
Year 4	711,100	38%	712,900	37%
Year 5	989,200	31%	1,042,000	30%
Year 6	1,424,000	27%	1,393,000	27%
Year 7	2,187,000	23%	2,058,000	24%
Year 8	3,930,000	20%	3,468,000	22%
Year 9	4,307,000	24%	4,230,000	27%
Year 10	4,674,000	43%	4,711,000	47%
Overall	18,790,000	16%	18,180,000	18%

In this case, the effect on the reserves is not particularly great. The prediction errors have increased for most years, although the effect is not great on these either. The importance of the Bayesian method is to actually be able to assess the effect of using different sets of data on the uncertainty of the outcome.

6.3 The Bornhuetter-Ferguson method

In this section, we consider intervention on the level of each row, using the Bornhuetter-Ferguson method. We consider two examples. The first uses small variances for the prior distributions of the row parameters, thus reproducing the Bornhuetter-Ferguson method. The second example uses less strong prior information, and produces results that lie between the Bornhuetter-Ferguson method and the chain-ladder technique. We use the negative binomial model for the data that was described in section 5, and the winBUGS code for this is given in Appendix 2 (i). Section 6.1 used this method with large variances for the prior, thereby reproducing the chain-ladder technique.

First we consider the Bornhuetter-Ferguson method, exactly as it usually applied. For this, we begin by use prior distributions for the row parameters which all have standard deviation 1000 (which is small compared with the means), and whose means are:

x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
5,500,000	5,500,000	5,500,000	5,500,000	5,500,000	6,000,000	6,000,000	6,000,000	6,000,000

The Bornhuetter-Ferguson estimates of outstanding claims, and the results from the Bayesian model are shown in table 7.

Table 7. Negative binomial model: Bayesian model with precise priors for all rows: mean and prediction error of reserves.

	Bayesian Mean Reserve	Bayesian Prediction Error	Bayesian Prediction Error %	Bornhuetter-Ferguson Reserve
Year 2	95,680	111,100	116%	95,788
Year 3	482,500	211,900	44%	480,088
Year 4	736,400	250,100	34%	736,708
Year 5	1,118,000	296,500	27%	1,114,999
Year 6	1,533,000	339,700	22%	1,527,444
Year 7	2,305,000	410,300	18%	2,308,139
Year 8	3,474,000	497,500	14%	3,466,839
Year 9	4,547,000	555,000	12%	4,550,270
Year 10	5,587,000	610,900	11%	5,584,677
Overall	19,880,000	1,854,000	9%	19,864,951

In this case, it can be seen that the results are very close to those of the Bornhuetter-Ferguson technique. Thus, if it is desired to use the Bornhuetter-Ferguson method within this stochastic framework, this is the approach that should be used. The added information which is available are the prediction errors. Further, it is possible to generate predictive distributions rather than just the mean and prediction error.

The Bornhuetter-Ferguson technique assumes that there is strong prior information about the row parameters, so that the standard deviations of the prior distributions used in this example are small. The other end of the spectrum is constituted by the chain-ladder technique, when large standard deviations are used for the prior distributions. Between these two extremes is a whole range of possible models, which can be specified by using different standard deviations. We now illustrate the results when less strongly informative prior distributions are used for the row parameters. We use the same prior means as above, but this time use a standard deviation of 1,000,000. We are incorporating prior belief about the ultimate claims for each year, but allowing for uncertainty in this information. The associated reserve results are shown in Table 8. Notice that the reserves are between the chain-ladder and Bornhuetter-Ferguson results. Notice also that the precision of the prior has influenced the prediction errors, but to a lesser extent. This provides an extra level of flexibility, to choose a range of models in a continuous spectrum between the chain-ladder technique and Bornhuetter-Ferguson.

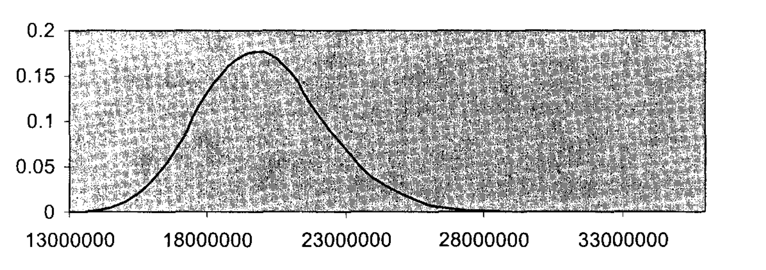
Table 8. Negative binomial model: Bayesian model with informative priors: mean and prediction error of reserves.

	Bayesian Mean Reserve	Bayesian Prediction Error	Bayesian Prediction Error %	Bornhuetter-Ferguson Reserve	Chain-Ladder Reserve
Year 2	94,660	111,500	118%	95,788	94,634
Year 3	470,400	218,800	47%	480,088	469,511
Year 4	717,100	265,900	37%	736,708	709,638
Year 5	994,900	308,900	31%	1,114,999	984,889
Year 6	1,431,000	376,800	26%	1,527,444	1,419,459
Year 7	2,198,000	488,900	22%	2,308,139	2,177,641
Year 8	3,839,000	727,200	19%	3,466,839	3,920,301
Year 9	4,417,000	865,500	20%	4,550,270	4,278,972
Year 10	5,390,000	1,080,000	20%	5,584,677	4,625,811
Overall	19,550,000	2,252,000	12%	19,864,951	18,680,856

7. Conclusions

This paper has shown how expert opinion, separate from the reserving data, can be incorporated into the prediction intervals for a stochastic model. The advantages of a stochastic approach are that statistics associated with the predictive distribution are also available, rather than just a point estimate. In fact, it is possible to produce the full predictive distribution, rather than just the first two moments. As was emphasized by England and Verrall (2002), the full predictive distribution contains a lot more information than just its mean and standard deviation, and it is a great advantage to be able to look at this distribution. As an illustration of this, figure 1 shows the predictive distribution of outstanding claims for the final example considered above, in section 6.3, table 5.

Figure 1. Distribution of reserve for Bornhuetter-Ferguson estimation



A further possibility for including expert knowledge within a stochastic framework applies when the Bornhuetter-Ferguson technique is used. This is an adaptation of the method used in sections 5 and 6.3, whereby the reserve is specified rather than the ultimate claims, u_i. The reserve value can be used to infer a value for u_i, from which the stochastic version of the Bornhetter-Ferguson method can be applied.

We have concentrated on two important situations, which we believe are the most common situations when expert opinion is used. However, the same approach could also be taken in other situations and for other modelling methods, such as the Hoerl curve (for example). This would allow us to add tail factors to the models considered in this paper. This paper has been more concerned with the general approach rather than specific reserving methods. However, we do acknowledge that methods based on the chain-ladder set-up are very commonly used and we hope therefore that, by using this framework, we enable actuaries to appreciate the suggestions made in this paper, and experiment with the programs supplied.

References

Bornhuetter, R.L. and Ferguson, R.E. (1972) *The Actuary and IBNR* Proc. Cas. Act. Soc., LIX, pp 181-195

Congdon, P. (2001) *Bayesian Statistical Modelling*. Wiley.

Congdon, P. (2003) *Applied Bayesian Modelling*. Wiley.

De Alba, E. (2002) *Bayesian Estimation of Outstanding Claims Reserves* NAAJ, Vol. 6, No 4, pp 1-20.

England, P.D. (2001). Addendum to "Analytic and bootstrap estimates of prediction errors in claims reserving". *Insurance: Mathematics and Economics*

England, P.D. and Verrall, R.J. (1999). Analytic and bootstrap estimates of prediction errors in claims reserving. *Insurance: Mathematics and Economics* **25**, 281-293.

England, P.D. and Verrall, R.J. (2000) *A Flexible Framework for Stochastic Claims Reserving*. Casualty Actuarial Society, Vol. 88.

England, P.D. and Verrall, R.J. (2002) *Stochastic Claims Reserving in General Insurance (with discussion)*. British Actuarial Journal, 8, pp 443-544

Ntzoufras, I. and Dellaportas, P. (2002) *Bayesian Modelling of Outstanding Liabilities Incorporating Claim Count Uncertainty* NAAJ, Vol 6, No 1, pp 113-128

Mack, T. (1993). Distribution-free calculation of the standard error of chain-ladder reserve estimates. *ASTIN Bulletin*, **23**, 213-225.

Mack, T. (2000) *Credible Claims Reserves: The Benktander Method.* ASTIN Bulletin, Vol. 30, pp 333-347

Renshaw, A.E. and Verrall, R.J. (1998) *A stochastic model underlying the chain ladder technique*. British Actuarial Journal, 4, IV, pp 903-923

Scollnik, D.P.M. (2001). *Actuarial Modeling with MCMC and BUGS*. North American Actuarial Journal, **5** (2), 96-124.

Spiegelhalter, D.J., Thomas, A., Best, N.G. and Gilks, W.R. (1996) *BUGS 0.5: Bayesian Inference using Gibbs Sampling*, MRC Biostatistics Unit, Cambridge, UK.

Taylor, G.C. (2000) *Loss Reserving: An Actuarial Perspective*, Kluwer.

Taylor, G.C. and Ashe, F.R. (1983) *Second moments of estimates of outstanding claims*. Journal of Econometrics 23, pp37-61.

Verrall, R.J. (2000) *An Investigation into Stochastic Claims Reserving Models and the Chain-Ladder Technique*. Insurance: Mathematics and Economics, Volume 26, pp 91-99.

Verrall, R.J. (2000) *A Bayesian Generalised Linear Model for the Bornhuetter-Ferguson Method of Claims Reserving* NAAJ (to appear)

Appendix

The code for winBUGS is shown below for the models used in section 6. This can be cut and pasted directly into winBUGS. Anything to the right of "#" is ignored, so the code can be changed by adding and removing this at the start of a line.

(i) This section contains the code for the Bornhuetter-Ferguson method in section 5, which was used for the illustrations in sections 6.1 and 6.3.

```
model
{
# Model for Data
	for( i in 1 : 45 ) {
	Z[i] <- Y[i]/1000
	pC[i]<-D[i]/1000
# Zeros trick
	zeros[i]<- 0
	zeros[i] ~ dpois(phi[i])
	phi[i]<- (-pC[i]*log(1/(1+g[row[i]]))-Z[i]*log(g[row[i]]/(1+g[row[i]])))/scale
		}
# Cumulate down the columns:
	DD[3]<-DD[1]+Y[46]
	for( i in 1 : 2 ) {DD[4+i]<-DD[4+i-3]+Y[49+i-3]}
	for( i in 1 : 3 ) {DD[7+i]<-DD[7+i-4]+Y[52+i-4]}
	for( i in 1 : 4 ) {DD[11+i]<-DD[11+i-5]+Y[56+i-5]}
	for( i in 1 : 5 ) {DD[16+i]<-DD[16+i-6]+Y[61+i-6]}
	for( i in 1 : 6 ) {DD[22+i]<-DD[22+i-7]+Y[67+i-7]}
	for( i in 1 : 7 ) {DD[29+i]<-DD[29+i-8]+Y[74+i-8]}
	for( i in 1 : 8 ) {DD[37+i]<-DD[37+i-9]+Y[82+i-9]}

# Needed for the denominator in definition of gammas
	E[3]<-E[1]*gamma[1]
	for( i in 1 : 2 ) {E[4+i]<-E[4+i-3]*gamma[2]}
	for( i in 1 : 3 ) {E[7+i]<-E[7+i-4]*gamma[3]}
	for( i in 1 : 4 ) {E[11+i]<-E[11+i-5]*gamma[4]}
	for( i in 1 : 5 ) {E[16+i]<-E[16+i-6]*gamma[5]}
	for( i in 1 : 6 ) {E[22+i]<-E[22+i-7]*gamma[6]}
	for( i in 1 : 7 ) {E[29+i]<-E[29+i-8]*gamma[7]}
	for( i in 1 : 8 ) {E[37+i]<-E[37+i-9]*gamma[8]}

	EC[1]<-E[1]/1000
	EC[2]<-sum(E[2:3])/1000
	EC[3]<-sum(E[4:6])/1000
	EC[4]<-sum(E[7:10])/1000
	EC[5]<-sum(E[11:15])/1000
	EC[6]<-sum(E[16:21])/1000
	EC[7]<-sum(E[22:28])/1000
	EC[8]<-sum(E[29:36])/1000
	EC[9]<-sum(E[37:45])/1000

# Model for future observations
```

```
	for( i in 46 : 90 ) {
		a1[i]<- max(0.01,a[row[i]]*DD[i-45]/(1000*scale))
		b1[i]<- 1/(gamma[row[i]]*1000*scale)
		Z[i]~dgamma(a1[i],b1[i])
		Y[i]<-Z[i]
		fit[i]<-Y[i]
				}
scale<-52.8615
#Convert row parameters to gamma using (5.6)
	for (k in 1:9) {
		gamma[k]<-1+g[k]
		g[k]<-u[k]/EC[k]
		a[k]<-g[k]/gamma[k]
				}
# Prior distributions for row parameters.
	for (k in 1:9) {
			u[k]~dgamma(au[k],bu[k])
			au[k]<-bu[k]*(ultm[k+1]*(1-1/f[k]))
			bu[k]<-(ultm[k+1]*(1-1/f[k]))/pow(ultsd[k+1],2)
					}
# The prior distribution can be changed by changing the data input values for the
# vectors ultm and ultsd

# Row totals and overall reserve
		R[1] <- 0
		R[2] <- fit[46]
		R[3] <- sum(fit[47:48])
		R[4] <- sum(fit[49:51])
		R[5] <- sum(fit[52:55])
		R[6] <- sum(fit[56:60])
		R[7] <- sum(fit[61:66])
		R[8] <- sum(fit[67:73])
		R[9] <- sum(fit[74:81])
		R[10] <- sum(fit[82:90])
		Total <- sum(R[2:10])
		}

# DATA
list(
row=c(1,1,1,1,1,1,1,1,1,
2,2,2,2,2,2,2,2,
3,3,3,3,3,3,3,4,4,
4,4,4,4,5,5,5,5,5,
6,6,6,6,7,7,7,8,
8,9,1,2,2,3,3,3,4,4,4,
4,5,5,5,5,5,6,6,6,6,6,6,
7,7,7,7,7,7,7,8,8,8,8,8,
8,8,8,9,9,9,9,9,9,9,9,
9),
Y=c(352118,884021,933894,1183289,445745,320996,527804,266172,425046,
```

```
290507,1001799,926219,1016654,750816,146923,495992,280405,
310608,1108250,776189,1562400,272482,352053,206286,
443160,693190,991983,769488,504851,470639,
396132,937085,847498,805037,705960,
440832,847631,1131398,1063269,
359480,1061648,1443370,
376686,986608,
344014,
NA,
NA,NA,
NA,NA,NA,
NA,NA,NA,NA,
NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,NA,NA,NA),

D=c(357848,766940,610542,482940,527326,574398,146342,139950,227229,
709966,1650961,1544436,1666229,973071,895394,674146,406122,
1000473,2652760,2470655,2682883,1723887,1042317,1170138,
1311081,3761010,3246844,4245283,1996369,1394370,
1754241,4454200,4238827,5014771,2501220,
2150373,5391285,5086325,5819808,
2591205,6238916,6217723,
2950685,7300564,
3327371,
NA,
NA,NA,
NA,NA,NA,
NA,NA,NA,NA,
NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,NA,NA,NA),

DD=c(67948,
652275,NA,
686527,NA,NA,
1376424,NA,NA,NA,
1865009,NA,NA,NA,NA,
3207180,NA,NA,NA,NA,NA,
6883077,NA,NA,NA,NA,NA,NA,
7661093,NA,NA,NA,NA,NA,NA,NA,
8287172,NA,NA,NA,NA,NA,NA,NA,NA),

E=c(67948,
652275,NA,
686527,NA,NA,
```

```
1376424,NA,NA,NA,
1865009,NA,NA,NA,NA,
3207180,NA,NA,NA,NA,NA,
6883077,NA,NA,NA,NA,NA,NA,
7661093,NA,NA,NA,NA,NA,NA,NA,
8287172,NA,NA,NA,NA,NA,NA,NA,NA),

f=c(1.017724725, 1.095636823, 1.154663551, 1.254275641, 1.384498969,
1.625196481, 2.368582213, 4.138701016, 14.44657687),
ultm=c(NA,5500, 5500, 5500, 5500, 5500, 6000, 6000, 6000, 6000),
ultsd=c(NA,10000,10000,10000,10000,10000,10000,10000,10000,10000))
```

These values for the ultsd will give the chain-ladder results. To obtain the Bornhuetter-Ferguson results, replace the last line with the following line:
ultsd=c(NA,1,1,1,1,1,1,1,1,1))
The other illustration in section 6.3 uses:
ultsd=c(NA,1000,1000,1000,1000,1000,1000,1000,1000,1000))

```
#INITIAL VALUES
list(u = c(5500, 5500, 5500, 5500, 5500, 6000, 6000, 6000, 6000),
Z=c(NA,NA,NA,NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,
NA,NA,NA,NA,
NA,NA,NA,
NA,NA,
NA,
0,
0,0,
0,0,0,
0,0,0,0,
0,0,0,0,0,
0,0,0,0,0,0,
0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0))
```

(i) Code for the model in section 4, which was used for the illustrations in section 6.2.

```
model
{
#Model for data:
        for( i in 1 : 45 ) {
                Z[i] <- Y[i]/(scale*1000)
                pC[i]<-D[i]/(scale*1000)
                C[i]<-Z[i]+pC[i]
         zeros[i]<- 0
                zeros[i] ~ dpois(phi[i])
```

```
            phi[i]<-(loggam(Z[i]+1)+loggam(pC[i])-loggam(C[i])-
pC[i]*log(p1[row[i],col[i]])-Z[i]*log(1-p1[row[i],col[i]]))
            }

                                          DD[3]<-DD[2]+Y[47]
      for( i in 1 : 2 ) {DD[4+i]<-DD[4+i-1]+Y[49+i-1]}
      for( i in 1 : 3 ) {DD[7+i]<-DD[7+i-1]+Y[52+i-1]}
      for( i in 1 : 4 ) {DD[11+i]<-DD[11+i-1]+Y[56+i-1]}
      for( i in 1 : 5 ) {DD[16+i]<-DD[16+i-1]+Y[61+i-1]}
      for( i in 1 : 6 ) {DD[22+i]<-DD[22+i-1]+Y[67+i-1]}
      for( i in 1 : 7 ) {DD[29+i]<-DD[29+i-1]+Y[74+i-1]}
      for( i in 1 : 8 ) {DD[37+i]<-DD[37+i-1]+Y[82+i-1]}

#Model for future observations
      for( i in 46 : 90 ) {
            a1[i]<- max(0.01,(1-p1[row[i],col[i]])*DD[i-45]/(1000*scale))
                  b1[i]<- p1[row[i],col[i]]/(1000*scale)
                  Z[i]~dgamma(a1[i],b1[i])
                  Y[i]<-Z[i]
                              }
      scale <- 52.8615

# Set up the parameters of the negative binomial model.
      for (k in 1:9) {
                  p[k]<-1/lambda[k]
                  lambda[k]<-exp(g[k])+1
                  g[k]~dnorm(0.5,1.0E-6)
            }
# Choose one of the folllowing (1,2 or 3) and delete the "#" at the start of each line
before running.

# 1. Vague Priors: Chain-ladder model
#     for (j in 1:9) {
#     for (i in 1:10) {p1[i,j]<-p[j]}
#                 }

# 2. Intervention in second development factor.
#     for (i in 1:10) {p1[i,1]<-p[1]}
#     for (i in 1:6) {p1[i,2]<-p[2]}
#     p1[7,2]<-p82
#     p1[8,2]<-p82
#     p1[9,2]<-p82
#     p1[10,2]<-p82
#     for (j in 3:9) {
#     for (i in 1:10) {p1[i,j]<-p[j]}
#                 }
#     lambda82<-g82+1
#     p82<-1/lambda82
# Use one of the following 2 lines:
#     g82~dgamma(0.005,0.01) #This is a prior with a large variance
```

```
#       g82~dgamma(25,50) #This is a prior with a small variance

#3. Using latest 3 years for estimation of development factors.
#       for (j in 1:6) {
#       for (i in 1:(7-j)) {p1[i,j]<-op[j]}
#       for (i in (8-j):10) {p1[i,j]<-p[j]}
#       }
#       for (j in 7:9) {
#       for (i in 1:10) {p1[i,j]<-p[j]}
#               }
#       for (k in 1:6) {
#                       op[k]<-1/olambda[k]
#                       olambda[k]<-exp(og[k])+1
#                       og[k]~dnorm(0.5,1.0E-6)
#                               }

# Row totals and overall reserve
                R[1] <- 0
                R[2] <- Y[46]
                R[3] <- sum(Y[47:48])
                R[4] <- sum(Y[49:51])
                R[5] <- sum(Y[52:55])
                R[6] <- sum(Y[56:60])
                R[7] <- sum(Y[61:66])
                R[8] <- sum(Y[67:73])
                R[9] <- sum(Y[74:81])
                R[10] <- sum(Y[82:90])
                Total <- sum(R[2:10])

        }


# DATA
list(
row=c(1,1,1,1,1,1,1,1,1,
2,2,2,2,2,2,2,2,
3,3,3,3,3,3,3,4,4,
4,4,4,4,5,5,5,5,5,
6,6,6,6,7,7,7,8,
8,9,2,3,3,4,4,
4,5,5,5,5,6,6,6,6,6,
7,7,7,7,7,7,8,8,8,8,
8,8,8,9,9,9,9,9,9,9,
9,10,10,10,10,10,10,10,10,10),
col=c(1,2,3,4,5,6,7,8,9,
1,2,3,4,5,6,7,8,
1,2,3,4,5,6,7,1,2,3,
4,5,6,1,2,3,4,5,1,
2,3,4,1,2,3,1,
```

```
2,1,9,8,9,7,8,9,
6,7,8,9,5,6,7,8,9,4,
5,6,7,8,9,3,4,5,6,7,
8,9,2,3,4,5,6,7,8,9,
1,2,3,4,5,6,7,8,9),
Y=c(
766940,610542,482940,527326,574398,146342,139950,227229,67948,
884021,933894,1183289,445745,320996,527804,266172,425046,
1001799,926219,1016654,750816,146923,495992,280405,
1108250,776189,1562400,272482,352053,206286,
693190,991983,769488,504851,470639,
937085,847498,805037,705960,
847631,1131398,1063269,
1061648,1443370,
986608,
NA,
NA,NA,
NA,NA,NA,
NA,NA,NA,NA,
NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,NA,NA,NA),
D=c(
357848,1124788,1735330,2218270,2745596,3319994,3466336,3606286,3833515,
352118,1236139,2170033,3353322,3799067,4120063,4647867,4914039,
290507,1292306,2218525,3235179,3985995,4132918,4628910,
310608,1418858,2195047,3757447,4029929,4381982,
443160,1136350,2128333,2897821,3402672,
396132,1333217,2180715,2985752,
440832,1288463,2419861,
359480,1421128,
376686,
NA,
NA,NA,
NA,NA,NA,
NA,NA,NA,NA,
NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,NA,NA,NA),
DD=c(5339085,
4909315,NA,
4588268,NA,NA,
3873311,NA,NA,NA,
3691712,NA,NA,NA,NA,
3483130,NA,NA,NA,NA,NA,
2864498,NA,NA,NA,NA,NA,NA,
```

```
1363294,NA,NA,NA,NA,NA,NA,NA,
344014,NA,NA,NA,NA,NA,NA,NA,NA))
```

#INITIAL VALUES
This is what is used for 1.

For 2, replace the first line by
list(g=c(0,0,0,0,0,0,0,0,0), g82=0.5,

For 3, replace the first line by
list(g=c(0,0,0,0,0,0,0,0,0), og=c(0,0,0,0,0,0),

```
list(g=c(0,0,0,0,0,0,0,0,0),
Z=c(NA,NA,NA,NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,
NA,NA,NA,NA,
NA,NA,NA,
NA,NA,
NA,
0,
0,0,
0,0,0,
0,0,0,0,
0,0,0,0,0,
0,0,0,0,0,0,
0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0))
```

Presenting DRM Results to Decision Makers: A Summary Report

CAS Working Party on Executive-Level Decision-Making Using DRM

Presenting DRM Results to Decision Makers: A Summary Report

CAS Working Party on Executive Level Decision Making Using DRM

Prepared by the Working Party members:
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Abstract

Motivation. Creating an effective Dynamic Risk Modeling (DRM) presentation to management is a crucial part of a DRM project. Unless the management team can see results in a form that helps them make decisions, there is no incentive for them to support the use of DRM.

History. The Casualty Actuarial Society (CAS) has recognized the importance of Dynamic Risk Modeling (DRM) for many years and has actively supported research in DRM issues through its committee structure and calls for research papers. In 2003, the Dynamic Financial Analysis Committee (DFAC) of the CAS changed its name to the Dynamic Risk Modeling Committee (DRMC) to recognize, in part, the broader family of risk modeling implied by the name Dynamic Risk Modeling. Accordingly, DRM and DFA could be used interchangeably in many instances, although the DRMC considers DFA to be a specific subset of DRM modeling. Prior to 2004, the DFAC issued calls for research papers under the DFA heading.

Method. The Working Party reviewed slides from past DRM and DFA presentations to find examples of effective slides. The presentations were also reviewed to understand how to sequence the slides to walk an audience through the parts of the study relevant to the decision making process.

Results. A PowerPoint template containing slides developed from the review of past DRM and DFA presentations was produced along with examples of how to use the slides to assemble a presentation and a guideline on giving DRM presentations.

Conclusions. An effective DRM presentation focuses on the financial measures that matter to the management team, which implies that one should establish those financial measures early in the life of a DRM project. Graphs provide the best approach to conveying the likely range of potential results and how those results can change over time. A number of slides in the PowerPoint template contain graphs that can be adapted to a particular presentation.

Availability. The PowerPoint template as well as the DRM presentation examples in PowerPoint can be downloaded from the CAS Web Site, www.casact.org. This summary report, the PowerPoint template, examples of presentations using the template, and the presentation guideline are on the CAS Web Site with hyperlinks between them at appropriate places.

Keywords. Dynamic Risk Modeling, DRM, Dynamic Financial Analysis, DFA, graphs, presentations.

1. INTRODUCTION

The Working Party (WP) was formed to give practicing actuaries help in developing effective Dynamic Risk Modeling (DRM) presentations for senior management. DRM model usage in practice has been more limited than was anticipated in the early 1990s when the CAS began promoting DFA. One potential reason for the limited acceptance of DRM models in practice may be the lack of effective presentation of such models' results.

The WP reviewed existing DRM and DFA presentations to identify techniques or slides that are effective in communicating to management the results of a DRM study. From that survey and from the ensuing discussions and targeted research, we produced the following items to help practicing actuaries in their presentations:

- this report from the Working Party;
- a PowerPoint template that can be used as a source for final slides;
- a paper describing how the slides in the PowerPoint template help solve some of the unique presentation problems for DRM studies;
- three sample DRM PowerPoint presentations based on the template, discussing reinsurance, investment, and mix of business options; and
- a collection of guidelines for the assembly and presentation of DRM concepts and results.

The report from the WP is a summary document. The other items listed above are hyperlinked[1] attachments to the report that expand upon selected parts of the project. Each item is available to be downloaded from the CAS Web Site. The remainder of this report gives a summary of our findings and a description of the other items listed above.

2. CATEGORIES OF EFFECTIVE DRM SLIDES

The sequence of slides for an effective DRM presentation can be broken down into three categories: Orientation, Presentation of Results, and Conclusion. The content of the slides is dependent on the specific study presented, but the sequence of slides is common across effective presentations.

2.1 Orientation

The goal in the orientation section is to prepare the audience for the presentation of financial results. The items to be presented in this section include:

- Overall goals of the study
- Options to be evaluated
- Financial measures used to evaluate the options
- Modeling assumptions
- Overview of modeling process

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[1] Clicking on a hyperlink in the Working Party's papers will open the referred document. (In Word XP, hold down CTRL when clicking.) If the hyperlink's properties include a location in the referred document, the document will open to the specified location. The hyperlinks in the Working Party's papers use "relative references" – i.e., all materials must be downloaded into the same directory to enable the hyperlinks to function properly.

2.1.1 Overall Goals of the Study

One of the initial slides in an effective DRM presentation states the goal(s) or business purpose(s) of the study. This should briefly summarize the problem being solved with the study, make clear why the presentation is being held and set the stage for the rest of the presentation.

2.1.2 Options to be Evaluated

Another of the initial slides in an effective DRM presentation lists the options to be evaluated as potential solutions to the stated problem. The management team has alternative courses of action among which to choose. These courses of action are the options to be evaluated, and the presenter will provide information that will affect the management team's decision. A slide that lists the options will set up the labeling convention used on the subsequent slides of financial results. The focus of the DRM study will determine the style in which the options are presented. The options may be stated as a series of investment strategies, reinsurance structures, or business growth plans, for example. For some presentations, the overall goal(s) and options being evaluated can be effectively combined.

2.1.3 Financial Measures used to Evaluate the Options

A slide that states the financial measures used to evaluate the options is a second background item for the later slides on the financial results. Such a slide gives an opportunity to state the definitions of the financial measures used in the presentation and to affirm that the results will be stated in terms that the management team can use to select the best course of action. Focusing on the pre-selected set of financial measures also aids the presenter, as it limits the number of items in later slides. Different management teams select different financial measures as the key items to evaluate in making decisions; therefore, the slides in this section are dependent on the management team's preferences. The determination of those financial measures is a process that should be completed at the start of a DRM project and is the subject of another Working Party, "The CAS Working Party on Elicitation and Elucidation of Risk Preferences."

2.1.4 Modeling Assumptions

The slide describing high-level modeling assumptions allows the presenter to describe the relative breadth and depth of the DRM study in various areas of modeling. One may state the areas the study focused on while building the model as well as areas where simplifying assumptions were used to keep the scope of the study within reasonable bounds. The list of modeling assumptions should contain only those items that the presenter can reasonably anticipate would carry significance with the management team. When modeling alternative investment options, comments on the interest rate model are appropriate. If modeling reinsurance program options, one can probably leave out comments on the interest rate generator. The modeling assumptions should be stated in non-mathematical terms. Instead of giving a formula used to drive a particular part of the model, state the behavior the formula models. Sometimes, a DRM study's results are heavily dependent on items external to the company, such as the path short term interest rates will follow. Stating the assumptions on those key external drivers is useful.

In summary, it is important to identify the "key drivers" of the model for the audience, while the inclusion of assumptions not on the "key drivers" list will depend on the project and your knowledge of the intended audience.

2.1.5 Overview of Modeling Process

Giving an overview of the modeling process is an opportunity to make the audience more familiar with the process and increase their confidence in the results to be presented by making the model less mysterious. A high level flow chart is the best route to accomplish that goal. A flow chart can illustrate that the model links different parts of company operations together within its analysis, without losing the audience in the complexity of the DRM modeling process.

2.2 Presentation of Results

There are three questions the presenter should address in this section of the presentation, the answers to which should be related to the overall goal(s) or business purpose(s) of the project:

- What is the likely range of financial results for each option?
- How do the financial results vary over time?
- What is the risk vs. return trade-off between the options?

Graphs offer the best means to answer these questions. A large number of data points can be summarized on a well-designed graph.

The successful communication of DRM results often requires fine attention to detail in formatting the graphs and use of consistent labeling and color schemes. Formatting mistakes can distract the audience by causing them to lose focus on the information the graph is intended to convey. For example, a graph that is commonly used to display the risk and return measures of each option is the "efficient frontier" type of graph with risk plotted on the X axis and return on the Y axis. Switching the axes would create a graph with the same information, yet the presenter will likely have to take additional time to explain the graph's meaning. Retaining the convention that risk is measured on the X axis and return is measured on the Y axis saves time during the presentation and keeps the audience focused on the results.

Even with the template provided by the Working Party, selecting the best graph to display the results for a given study and adjusting the formatting of the graph can be time consuming. The project timeline for a DRM study should allow time for those activities as well as for a dry run of the presentation to improve its flow and to catch formatting errors that can detract from a presentation.

2.3 Conclusion

In general, any presentation needs a slide that draws conclusions from the presented material. The need for a conclusion is particularly acute in a DRM presentation. After the actuary has presented the results of a dynamic risk model, the management team is left with the task of making a decision using results from a process that is probably outside the scope of their experience. It's reasonable to assume that the management team has some familiarity with accounting concepts, but it's unlikely they will have practical experience using simulation models or the probability density functions and interest rate models that are part of the driving force within a DRM model.

The speaker should do the following at the conclusion of the presentation:

- Restate the goal(s) or business purpose(s) of the study.
- Summarize the results of the study in terms of the financial measures selected.
- Offer an opinion on the best course of action given the financial measures selected.

Referring back to the slide that stated the goal of the study is useful in summarizing the presentation and reaching a conclusion. A slide with a table summarizing the results for the selected financial measure results by option is useful. While the responsibility for the decision lies with the management team, offering an opinion on how to interpret the results may help them process the information given during the presentation.

Drawing a conclusion on the course of action to be taken involves comparing results between the options. Keeping the number of comparisons to be made to a reasonable level is the reason the number of options is limited in defining the goal for the study.

3. WORKING PARTY PRODUCT

This section describes the end products from the project. Our goal is to provide some practical help to an actuary faced with developing and presenting the results of a DRM study.

3.1 PowerPoint Template

The goal of providing practical help led us to create Microsoft PowerPoint slides with embedded Excel charts, since we assume those are tools that are commonly available to practicing actuaries. The use of an embedded Excel chart allows both the slide and the chart to remain fully editable by their parent applications subsequent to the placement of the chart in the slide. The template is available to the public and can be downloaded from the CAS Web Site. The template offers a variety of graphs that will suit the needs of a particular DRM study. The graphs were developed by extracting and enhancing the best graphs or slides from the review of past DRM presentations. The Working Party has sought to maximize the graphs' efficiency in presenting DRM concepts and to illustrate the capabilities of commonly available software.

3.2 Design of Slides in PowerPoint Template

One member of the Working Party, Aleksey Popelyukhin, wrote a paper, "Presenting DRM Results: Helping Executives Make Sense of DRM." Designing graphs for DRM presentations is the focus of his paper. The paper describes how graphs such as those in this Working Party's template may be built and the various purposes they serve.

3.3 Guidelines for the Assembly and Presentation of DRM Concepts and Results

The PowerPoint slides provide some building blocks that can be used to assemble a DRM presentation. The outline is meant to provide a checklist that the presenter can refer to while assembling the presentation.

3.4 Sample DRM Presentations

This Working Party created sample presentations to illustrate the use of the PowerPoint template and to make our general observations on effective DRM presentations more concrete. The presentations are based on the results of DRM analyses that were also created by the Working Party, but they should only be viewed as a means to demonstrate use of the slides from the template and the type of comments that could be offered to orient the audience when viewing the results. The slides are available with speaker notes in PDF format. They may also be downloaded as PowerPoint files.

Three sample presentations were created:

- A reinsurance study generated with proprietary software.
- An investment study generated with the public access DRM model.[2]
- A mix of business study generated with the public access DRM model.

The speaker notes were included to describe why a given slide was included and the intended benefit to the audience. The sample presentations complement both the general, conceptual findings on what makes an effective DRM presentation and the PowerPoint slides in the template.

[2] The public access DRM model can be freely downloaded at www.pinnacleactuaries.com/pages/products/dynamo.asp

3.5 Caveats

The Working Party created these materials with the express intent that they be freely downloaded and used. By downloading these materials, the user recognizes that they are intended to be guidelines to assist the user and that they can be readily modified or otherwise changed. Furthermore, the user accepts all responsibility for the final slides used in their presentation and recognizes that the CAS is not responsible for any user content.

3.6 Future Additions to Power Point Template

The Working Party anticipates that as the template is used, actuaries will have suggestions for additional graphs or slides to be added to the PowerPoint template. Anyone wishing to make a contribution to the PowerPoint template should forward that suggestion to the chairperson of the Dynamic Risk Modeling Committee (DRMC) for review. A submission to change the template should include an explanation of the purpose of the slide and should follow the convention of using Excel objects with PowerPoint to develop the slides. If the DRMC decides the slide should be added to the template, the committee will modify the PowerPoint template and ask the Casualty Actuarial Society staff to post the revised template on the Web Site along with an acknowledgment to the contributor.

4. CONCLUSIONS

This report is intended to provide an actuary with the tools to assemble a presentation that will make a DRM study useful to management. In order to meet that goal, actuaries should keep the following ideas in mind while preparing their presentations.

The focus of a DRM presentation to management should be on the financial measures with which management members are familiar and which they accept as criteria for evaluating success in their company.

The presenter should avoid including detailed, technical information on the slides. In general, however, the presenter should be able to answer detailed, probing questions related to the functioning of the model or to the modeling assumptions.

To use a common analogy, our assumption is that your audience really only wants to know what time it is, not the details of how the watch was built. The audience needs some information about your results, though. Continuing with the time analogy, they need to know if you are giving them the time on Eastern Daylight Savings time or Pacific Coast standard time. They would like some assurance that you have recently checked your watch against some other reliable source. In the context of a DRM presentation, the audience needs to be sure your answer really matches the question they have on their minds. They want some assurance that a well-defined process was used to produce the results you are presenting, and that it captures the behavior of key items in a manner that can be reviewed for reasonability.

Supplementary Material

Index of hyperlinks to related Working Party documents
PowerPoint Template
Design of Slides in PowerPoint Template
Sample Reinsurance Study
Sample Investment Study
Sample Mix of Business Study
Guidelines for the Assembly and Presentation of DRM Concepts and Results

SELECTED REFERENCES

[1] Bohra, Raju. DFA and Reinsurance – Using DFA to Align Reinsurance with Corporate Strategy. Risk and Capital Management Seminar, 2003.

[2] DFA Capital Management Inc. – Web Site

[3] Isaac, Daniel. Analyzing Reinsurance with DFA – Practical Examples. Risk and Capital Management Seminar, 2003.

[4] Liebowitz, Neal. Make the Connection – Presenting DFA to Management. Seminar on Dynamic Financial Analysis, 2000.

[5] Newman, Henry E. Asset and Liability Dynamics. 1999 CAS Special Interest Seminar on Dynamic Financial Analysis.

[6] Schwartz, Nathan. Using DFA to Optimize the Value of Reinsurance. 2001 CAS

Special Interest Seminar on Dynamic Financial Analysis.
[7] Stricker, Markus. Using Dynamic Financial Analysis to Structure Reinsurance. 2001 CAS Special Interest Seminar on Dynamic Financial Analysis.
[8] Venter, Gary. Measuring Value in Reinsurance. CAS Seminar on Reinsurance, 2001.

Abbreviations and notations

DFA – Dynamic Financial Analysis	WP – Working Party
DRM – Dynamic Risk Modeling	

Biographies of Working Party Members

Nathan J. Babcock is an Assistant Vice President within the Insurance Advisory group of Conning Asset Management, where he is responsible for developing analytical software for Conning's asset-liability and integrated risk management advisory services to insurance companies. Prior to joining Conning, Mr. Babcock was a Senior ALM Analyst within Swiss Re Investors' asset-liability management unit. He has been involved in the Property/Casualty insurance field since 1990. He is a graduate of the University of Maryland with a BS in Mathematics, and is an Associate of the CAS. Mr. Babcock participates on the CAS Dynamic Risk Modeling Committee, and is a co-chairperson of the CAS Working Party on Executive Level Decision Making Using DRM.

Raju Bohra currently serves as a Vice President and Client Reinsurance Modeling Analyst within the Client Analysis & Advisory team of American Re-Insurance and provides dynamic financial analysis (DFA), reinsurance modeling, and financial planning advisory services to AmRe's direct clients as well as business prospects with related issues. Mr. Bohra functions as AmRe's central expert on the development, implementation, and use of DFA and reinsurance models. Mr. Bohra received a BA in Economics from the Johns Hopkins University in Baltimore, MD. Also, he has attained his FCAS designation from the Casualty Actuarial Society. He has published papers concerning the practical use of DFA modeling and communicating DFA results to non-technical audiences. He also holds an ARe designation from the AICPCU.

Patrick J. Crowe is currently Vice President, Market Research and Actuary for Kentucky Farm Bureau Insurance Companies. Patrick is a Fellow of the Casualty Actuarial Society, a Member of the American Academy of Actuaries, and an Associate of Risk Management. During his career he has been a member of the CAS examination, continuing education, Call Paper Review and Dynamic Risk Modeling committees. He has served as Chairperson of the 1995 DFA seminar and the American Academy of Actuaries Automobile Insurance Subcommittee. He has also been a speaker at CAS conventions, ratemaking and loss reserve seminars. Patrick graduated from Northern Illinois University with a B.S. degree in Mathematics and Physics.

Michael R. Larsen works in the Personal Lines Actuarial Department at The Hartford where he functions as an internal consultant adapting modeling techniques like those in the 2004 Call Paper program – Adapting Generalized Linear Models to Personal Lines ratemaking problems. Mike is a Member of the American Academy of Actuaries and became a Fellow of the Casualty Actuarial Society in 1982. He has served on the Examination Committees, the Dynamic Risk Modeling Committee and has spoken at Loss Reserve Seminars. He is is a co-chairperson of the CAS Working Party on Executive Level Decision Making Using DRM. Mike has a Masters in Actuarial Science from The Graduate

School of Business at The University of Michigan.

Aleksey Popelyukhin is a Senior Vice-President of Technology with the Sam Sebe LLC and a Vice-President of Information Systems with the 2 Wings Risk Services in Stamford, Connecticut. He holds a Ph.D. in Mathematics and Mathematical Physics from Moscow University (1989). Aleksey is presently developing an integrated pricing/reserving/DFA computer system for reinsurance and also an action/adventure computer game tentatively called "Actuarial Judgment." Dr. Popelyukhin is an active member of several scientific societies and an author of almost 20 scientific publications. His article "The Big Picture: Actuarial Process from the Data Management point of view" (1996) won the prize for the Data Management Discussion Call Paper Program in 1997.

Nathan Schwartz is a Senior Vice President of Benfield, Inc., and is currently the head of the financial modeling department. His primary function is to illustrate and optimize the benefits of traditional and non-traditional reinsurance structures. He does this using dynamic financial analysis and Benfield's proprietary software product ReMetrica(tm). In addition, Nathan provides client consultation on non-reinsurance strategic analysis and is involved with modeling capability enhancements. Before joining the company in 1999, he spent three years with The St. Paul Cos., in the medical malpractice division and reserving divisions. Nathan received his bachelor's degree in mathematics and computer science from Carleton College and a master's degree in mathematics from Northwestern University. Nathan obtained his Fellowship in the Casualty Actuarial Society in 1999 and is a Member of the American Academy of Actuaries.

Scott Sobel works at the NCCI in Workers Compensation ratemaking and overall industry analysis. Scott obtained his Fellowship in the Casualty Actuarial Society in 2003, is a Member of the American Academy of Actuaries, and has about thirteen years of actuarial experience. He holds a BS in Statistics from the University of Florida.

Robert J. Walling III is a Principal and Consultant with Pinnacle Actuarial Resources, Inc., in the Bloomington, Illinois, office. He holds a Bachelor of Science degree in secondary mathematics education from Miami University. He has worked in the insurance industry since 1989. Mr. Walling is a Fellow of the Casualty Actuarial Society and a member of the American Academy of Actuaries. He has served the CAS as Chairman of the Ratemaking Seminar Committee, Chairman of the Risk and Capital Management Seminar Committee, Chairman of the New Fellows Committee, and faculty member of the Limited Attendance Seminar on Dynamic Financial Analysis (DFA). Mr. Walling is a frequent speaker at industry meetings on topics related to commercial lines pricing, DFA, and generalized linear modeling (GLM). Prior to joining Pinnacle, Mr. Walling was employed for five years by Anthem Casualty Insurance Group where he was responsible for the pricing and product monitoring functions of the Commercial Lines and Managed Care Workers Compensation Divisions. He also has work experience at Providence Washington Insurance Company and Great American Insurance Group. His experience includes, personal and commercial lines ratemaking and product development, funding and reserving studies for self-insureds, personal and commercial lines reserving, rate filings and regulatory compliance, dynamic financial analysis of insurance companies, and legislative review and costing. His published articles include "Using the Public Access DFA Model: A Case Study,"CAS Forum, Summer, 1998, "A Dynamic Approach to Modeling Free Tail Coverage," CAS Forum, Fall, 1999, "Customizing the Public Access Model Using Publicly Available Data," CAS Forum, Summer, 1999, "Are You Ready to Unlock the Power Hidden in Your BOP Application," Pinnacle Actuarial Resources Monograph Program, July 2003, and "The Case of the Medical Malpractice Crisis: A Classic Who Dunnit," CAS Forum, Summer, 2004.

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Craig Allen	Larry Johnson
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Thomas Conway	Timothy Pratt
Patrick J. Crowe	James E. Rech
Karl Goring	Chester Szezepanski
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Report of the 2003 CAS Membership Survey Task Force

CAS Membership Survey Task Force

REPORT OF THE

2003 CAS MEMBERSHIP SURVEY

TASK FORCE

CAS Membership Survey Task Force

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TABLE OF CONTENTS

EXECUTIVE SUMMARY

The 2003 CAS Membership Survey provided a good deal of positive news for the Casualty Actuarial Society (CAS). First of all, the commitment of CAS members was evidenced by the high response rate to the survey and the high number of members who took the time to respond to write-in questions. 1,934 members completed the survey, for a response rate of 52%.

The 2003 CAS Membership Survey asked respondents to rate their overall satisfaction with the CAS. Most notably, over 80% of the respondents indicated that they were very satisfied or satisfied. In addition to asking about overall satisfaction, the survey asked about satisfaction with five specific aspects of the CAS. CAS Staff garnered the highest satisfaction ratings, followed by Communications and Publications, and Meetings and Professional Education. Ratings for the leadership and committee chairs were somewhat lower, although still very high. Satisfaction levels were consistent across all demographic groups and sub-groups. Consistent with the CAS Core Values, education and the CAS "community" were noted as the strongest parts of the Society. There seemed to be a positive correlation between the level of involvement in the CAS and satisfaction.

Almost 40% of the respondents indicated they serve the actuarial profession in some way. Almost 75% of the respondents indicated that time is the major obstacle preventing them from increasing their participation in CAS committees and task forces. Retirees represent a potential future source of volunteers. However, the limited numbers of retirees that responded to the survey indicated that lack of interest was a reason for not increasing their participation in the CAS. Finding a way to engage this growing segment of our membership may become a critical issue in the future, given the large number of respondents that indicated that they expect to retire in the coming decades and the CAS goal of having at least half of its members volunteer. Respondents also indicated that the CAS could make better use of academics in promoting the profession, research, literature, education, examinations, and continuing education.

Another positive observation from the survey is the fact that the CAS leadership has already begun to address some of the issues that are important to the membership. The respondents reacted favorably to the changes that were made for the 2002 election process. Furthermore, the CAS Board has formed two task forces to address the voting and other rights of Associates, which was an area that generated a lot of comments from respondents. The CAS has also formed a Task Force on Publications, which submitted several questions to the 2003 survey. The responses to the survey indicated that the CAS should strive to maintain some form of refereed journal and should concentrate on improving the overall organization of CAS papers and the quality of non-refereed papers. Respondents also reacted favorably to the concept of "Working Parties" as a research vehicle. Recent initiatives to hire a CAS Librarian and develop a research taxonomy should enable members to take better advantage of CAS research. In the International arena, the initiatives that the CAS has launched in the past two years are consistent with what respondents felt the CAS should be doing.

Some survey responses indicated areas where the CAS should increase communications. For example, although all CAS Fellows are members of the IAA by virtue of the CAS paying

Fellows' IAA dues, only 3% of the respondents reported being members of the IAA, while 68% of the respondents were Fellows.

Another potential area for increased communications involves educating members on what research is currently available. While a large number of the respondents felt that CAS research was useful and valuable, less than 20% felt that they were aware of the research that was performed or sponsored by the CAS. In a related area, a number of respondents suggested improvements to the search engine on the CAS Web Site. Other respondents suggested improvements that were already part of the Web Site, which indicated the need for ongoing education of members about the features of the Web Site.

The Membership Survey Task Force noted a couple of trends that may raise potential concerns to the CAS as it strives to achieve its Centennial Goal. The Task Force was surprised by an increase in the proportion of respondents involved in the traditional actuarial activities of ratemaking and reserving. At the same time, the Task Force was concerned about a decline in involvement in executive management, strategic and financial planning, marketing, and underwriting. There was also a marked decline in the membership in international actuarial associations, particularly ASTIN and AFIR. The Task Force was unable to ascertain if sample bias between surveys accounted for these differences. Because these trends may have negative implications on the achievement of the CAS Centennial Goal, the Task Force recommends that the CAS investigate these trends further.

The 2003 Membership Survey Task Force offers the following recommendations based on the results of the survey. The recommendations are listed in the order that they appear in the report.

1. The CAS may wish to further explore the apparent trend away from executive management and non-traditional activities and its potential implications for the CAS Centennial Goal.

2. The CAS should consider improving communication of IAA membership to CAS Fellows, given that the CAS pays Fellows' IAA dues. The CAS should examine the reasons behind the decreasing trend in the number of CAS members who are also members of the AAA.

3. The CAS may want to consider requiring all members to take the Course on Professionalism. The CAS may also want to determine if there are other forms of professionalism/ethics education that may be more appropriate for members that have not attended the current course.

4. The CAS needs to better publicize the availability and organization of its research.

5. The CAS may want to consider expanding current continuing education requirements to apply to all members performing actuarial work.

6. The Committee on General Business Skills should consider offering sessions on Strategic Thinking and/or Negotiation Skills at future CAS meetings. The CAS should consider including the cost of the sessions in the registration fee in order to increase participation in these sessions.

7. The individual Regional Affiliates, in conjunction with the Regional Affiliates Committee, may want to survey candidates and CAS members in their geographic region for further input on the value of Regional Affiliate meetings.

8. The CAS should continue to explore ways to make use of the unique talents found in the academic community to improve its education and examination process.

9. The CAS should stay the course it has already embarked on to make the CAS Syllabus and research accessible and useful to actuaries practicing outside of the United States.

10. The CAS should regularly educate members on the current capabilities of the CAS Web Site.

11. The CAS should evaluate the feasibility of improving the Web Site search engine. The CAS should increase its promotion of web casts of the Spring and Annual Meetings to encourage more members to take advantage of them.

12. The CAS should strive to maintain some form of refereed journal and should concentrate on improving the overall organization of CAS papers and the quality of non-refereed papers.

The 2003 Membership Survey Task Force would like to thank the CAS leadership for their input into the survey and the CAS members that took the time to respond to the survey. The Task Force hopes that the members and leadership of the CAS find the information in 2003 Membership Survey Report useful. The Task Force notes that this report does not attempt to provide comprehensive results, which would have numbered hundreds of pages. There is a great deal of additional detail contained in the survey results and the Task Force would welcome the opportunity to work with CAS Committee and Task Force members to explore the findings in more detail.

Finally, we would like to express our sincerest appreciation to Todd Rogers, Mike Boa and the CAS office staff for their extensive help throughout the entire survey process, from the selection of the vendor and administration of the online survey through the editing of the final report. Having had the pleasure of working with these very dedicated professionals, it was easy to see why the CAS Staff earned the extremely high satisfaction ratings and praise from the survey respondents.

INTRODUCTION

Every five years, the CAS conducts a major survey of its members. The results of these membership surveys provide the CAS leadership with valuable input that helps to shape the short and long-term direction of the Society. A Membership Survey Task Force (MSTF) was formed in 2002 to coordinate the 2003 Membership Survey. The MSTF was chaired by Joanne S. Spalla and included Roger M. Hayne, Douglas W. Oliver, Stephen W. Philbrick, Alessandrea C. Quane, and James B. Rowland. CAS office liaisons Todd P. Rogers and J. Michael Boa provided staff support to the Task Force. Association Research Inc. (ARI) was hired to administer the 2003 Membership Survey and advise the Task Force.

To develop questions for the 2003 Survey, the MSTF requested input from the CAS Board, Executive Council, and all Committee Chairs. The MSTF also elected to include a number of questions from prior surveys to enable it to observe trends in CAS members' demographics and attitudes.

In order to maximize the number of questions in the survey without making its length excessive, the Task Force elected to implement a recommendation by ARI to issue two different survey forms. ARI advised that, given the size of the CAS membership and historical response rates, the sample size for each survey form would be adequate. Accordingly, two versions of the 2003 Membership Survey were prepared with 55 questions each. 34 of the questions, including the 13 demographic items, were included in both versions of the survey. The remaining questions were different.

The 2003 Membership Survey was conducted online for the first time during the month of July 2003. Paper copies were provided only to members with no e-mail address on file, or upon request. Only 32 members submitted paper surveys.

1,934 members completed the survey, for a response rate of over 52%. For comparison purposes, the response rates were 32% in 1998, 41% in 1993, and 62% in 1988. The demographic profiles of respondents to the two survey forms were virtually identical and were representative of the entire CAS membership.

The survey was peer-reviewed by members of the CAS Membership Advisory Panel Committee.

1. DEMOGRAPHICS

1.1 Gender

	2003	1998
Male	72%	78%
Female	26%	22%
No Response	2%	N/A

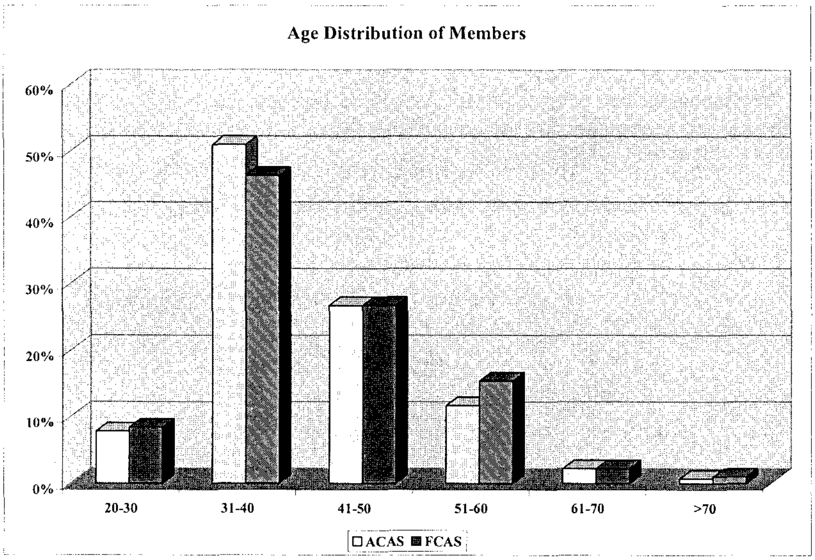
1.2 Designation

	2003	1998
Fellows	68%	64%
Associates	31%	36%
Affiliates	1%	N/A

The average Associate who responded has been an ACAS for 8.3 years. The average Fellow who responded has been an FCAS for 9.4 years.

1.3 Age

The average age of the responding Associates is 40.1 years, while the average age of the responding Fellows is 40.9 years.



1.4 Business Affiliation

	2003	1998	1993
Insurance Company	55%	57%	58%
Reinsurance Company	14%	13%	9%
Consulting Actuary	16%	18%	21%
Service Organization	2%	4%	6%
Regulatory Organization	3%	2%	N/A
Broker	2%	2%	N/A
Retired	3%	2%	N/A
Academic	0.3%	0%	N/A
Other	5%	2%	6%

The majority of insurance company actuaries are 31-35 years old while the majority of Reinsurance, Consulting, Service and Regulatory actuaries are 36-40 years old.

1.5 Geographic Area of Primary Business Responsibility

	2003	1998	1993
United States	83%	84%	80%
Canada	10%	10%	14%
Worldwide	7%	7%	6%
Europe	7%	5%	7%
Bermuda	5%	N/A	N/A
Asia	4%	5%	4%
Central & South America	3%	2%	2%
Australia & New Zealand	1%	Incl. in Asia	Incl. in Asia
Africa	0.4%	Incl. in Asia	Incl. in Asia

Note that respondents were able to indicate multiple areas of primary business responsibility.

Respondents were asked whether they would be likely to accept a job opportunity outside of their primary place of work. About one-fourth (24%) of the respondents would be likely to accept a relocation lasting at least one year if they were presented with a job opportunity within the next five years. Interestingly, there did not appear to be any significant difference in the willingness to relocate between genders, actuarial designations, or ages.

1.6 Primary Place of Work (United States)

Of the 86% of the respondents who indicated that their primary place of work was located in the United States, 10 states comprise approximately 72% of the respondents:

State	2003	1998
Illinois	13%	12%
Connecticut	11%	9%
New York	10%	12%
New Jersey	9%	6%
California	8%	10%
Pennsylvania	7%	7%
Wisconsin	4%	3%
Texas	4%	4%
Ohio	3%	4%
Minnesota	3%	3%

Region	2003
Northeast	44%
South	15%
North Central	30%
West	12%

1.7 Other Actuarial Organizations

There was an across the board decrease in membership in other professional organizations, particularly ASTIN and AFIR. The decline in CAS membership in these two international organizations is potentially a concern, given the international aspects of the CAS Centennial Goal.

	2003	1998
American Academy of Actuaries	80%	82%
Canadian Institute of Actuaries	7%	8%
IAA	3%	4%
ASTIN	5%	13%
AFIR	2%	8%
Society of Actuaries	4%	5%

It is interesting to note that although all CAS Fellows are members of the IAA by virtue of the CAS paying Fellows' IAA dues, only 3% of the respondents reported being members of the IAA, while 68% of the respondents were Fellows.

The New Fellows Committee examined membership in the AAA by years since designation and found that both new Fellows and new Associates (<10 years since designation) are far less likely to be AAA members than their more senior peers. The New Fellows Committee has prepared a separate report on this trend for the CAS and AAA leadership's review.

Recommendation: ***The CAS should consider improving communication of IAA membership to CAS Fellows, given that the CAS pays Fellows' IAA dues. The CAS should examine the reasons behind the decreasing trend in the number of CAS members who are also members of the AAA.***

1.8 Professional Designations

The 2003 survey tracked (for the first time) other professional designations held by respondents. Approximately 10% of the respondents held designations by other professional organizations such as Chartered Property & Casualty Underwriter (CPCU) (4%), Associate in Reinsurance (ARe) (2%), Associate in Risk Management (ARM) (2%) and Chartered Financial Analyst (CFA) (1%).

1.9 Education

While a large majority of the respondents had a four year BA or BS degree (74%), advanced degrees held by respondents included MA/MS (18%), MBA (4%), and PhD (3%).

1.10 Areas of Practice

The figures shown below represent the percentage of time the respondents spent over the past two years on each category:

	2003	**1998**	**1993**	**1987**
Ratemaking	29%	23%	24%	21%
Reserving	21%	19%	23%	20%
Subtotal Ratemaking and Reserving	**50%**	**42%**	**47%**	**41%**
Management of an Actuarial Unit	11%	13%	12%	12%
Executive Management	5%	7%	9%	
Planning – Strategic & Financial	5%	7%	4%	7%
Risk & Capital Management (e.g. DFA)	3%	3%		
Marketing/Underwriting	5%	7%	4%	4%
Data Management	3%	6%	4%	
Programming – Software Development	3%	4%	3%	
Teaching – Research	2%	4%	3%	
Investments	1%	1%		6%
Valuation	1%	1%	2%	
Other	11%	6%	13%	26%

The most frequent write-ins for the "other" category included reinsurance, pricing and retirement. One surprising observation was the fact that the proportion of time spent in the traditional actuarial activities of ratemaking and reserving has actually increased from 41% in 1987 to 50% in 2003. At the same time, involvement in executive management, strategic and financial planning and marketing and underwriting has declined. This movement puzzled the Task Force. While these activities were likely to be performed by more seasoned actuaries, there was no change in the distribution of responses by age or tenure from the previous survey that would explain this trend. The Task Force was unable to determine if other forms of sample bias between surveys accounted for the differences. The Task Force was concerned that this trend may have negative implications on the CAS Centennial Goal.

Recommendation: ***The CAS may wish to further explore the apparent trend away from executive management and non-traditional activities and its potential implications for the CAS Centennial Goal.***

Other observations drawn from these responses include:

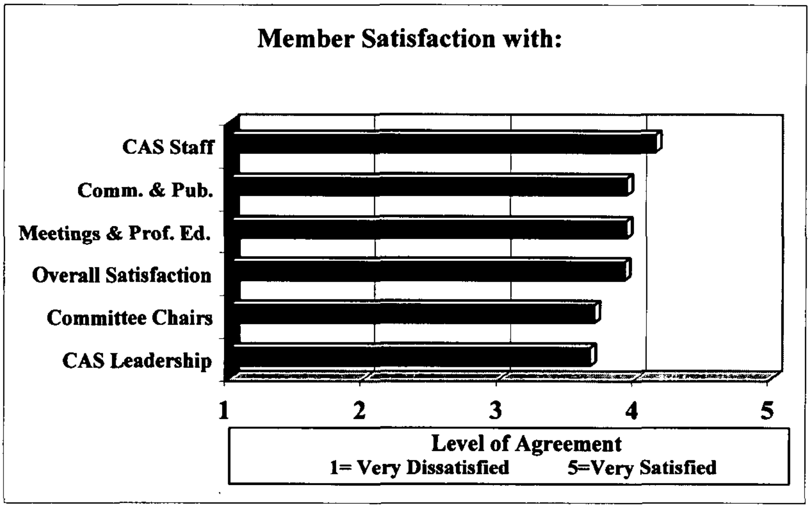
- Nearly two thirds of the respondents have, at one point in their career, been the Manager of an Actuarial Unit.
- Almost 90% have been involved with Ratemaking at some point, but only 80% have spent time in Reserving.
- More than 20% of the respondents have taught or done research at some point in their careers.

1.11 CAS Service

Forty percent of the respondents are active in the actuarial profession: 30% serve as a CAS committee member, 5% are in a CAS leadership role (Board or Executive Council member or CAS committee chair) and 9% play a role in another actuarial organization.

2. MEMBER SATISFACTION

The 2003 Membership Survey, for the first time, asked members to rate their level of satisfaction with five aspects of the CAS, as well as their overall satisfaction with the CAS.



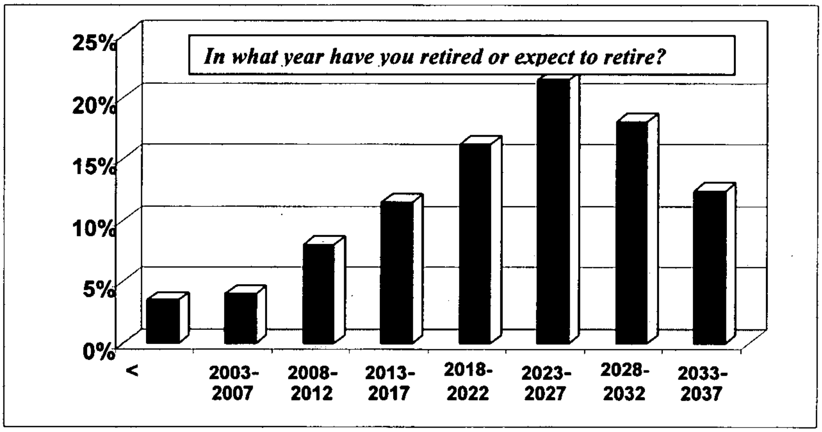
Overall, satisfaction rates are tremendous, with over 80% of the respondents satisfied or very satisfied. CAS Staff garnered the highest satisfaction ratings, followed by Communications and Publications, and Meetings and Professional Education. Ratings for the leadership and committee chairs were somewhat lower, although still very high.

No demographic groups showed particularly high levels of dissatisfaction. There was a positive correlation between the level of involvement in the CAS and satisfaction.

Members were asked to write-in their opinions about the strongest and weakest parts of the CAS. Almost 700 members took the time to write in their thoughts about CAS strengths and over 600 wrote in about CAS weaknesses. They cited the members themselves and the volunteer culture as the strongest parts of the CAS. As expected, exams and admissions generated a large number of responses—as both a strength and weakness of the CAS. Negative comments about exams outweighed positive by more than two to one. About 40 comments about the weaknesses cited issues with the Board and CAS leadership. In addition, there were about a dozen comments that mentioned arrogance and elitism as the biggest weaknesses of the CAS.

3. RETIREMENT

Shown below is a histogram of the expected retirement year of those who completed the survey:



The individual year with the largest percentage of expected retirees is 2030, with 10% of the respondents expecting to retire.

4. VOLUNTEERISM

4.1 CAS Committee and Task Force Involvement

Members were asked if there was anything preventing them from increasing their participation on CAS committees and task forces. Over 73% of the respondents indicated that a lack of time is the major reason. Nearly 23% of respondents mentioned lack of interest as a major issue. Only 5% indicated that nothing is preventing them from increasing their participation levels. There were no significant deviations when the responses were examined by demographic group.

Retirees were a rather small sample of respondents (N=60); however, their input may become more important in future years if the CAS wishes to rely on this group's volunteer participation. Fifty-two percent of retirees indicated a lack of interest, while 28% and 13% responded that cost and time respectively were issues.

4.2 Impact of Travel Costs on Volunteer Activities

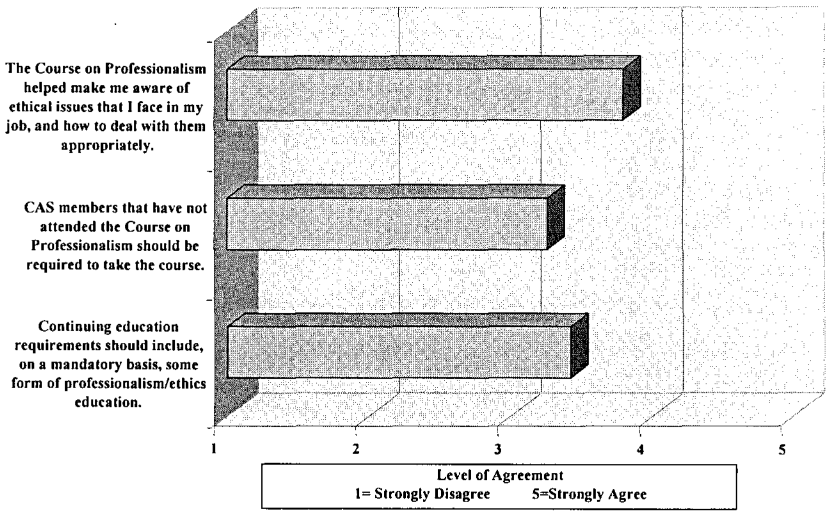
Subsidization of travel costs does not appear to be a major issue in increasing volunteer efforts. Only 27% of the respondents indicated that subsidization of travel costs would help them increase their volunteer efforts. Respondent groups where subsidization would appear to have a greater impact on volunteer efforts include regulators (47%) and west coast actuaries (40%). Although very few respondents were from the academic community, 50% of those respondents indicated that subsidized costs would help their volunteer efforts. The response for retirees was similar to respondents that are currently employed.

5. PROFESSIONALISM

Only 20% of respondents indicated that they sign prescribed statements of actuarial opinion in the course of their practice. However, approximately 35% of respondents who practice outside of the United States indicate that they sign these statements, with 45% of those who practice in Bermuda topping the list. Nearly 72% of respondents indicated they meet the general qualification standards for prescribed statements of actuarial opinion. However, this figure drops to 50% when asked if they meet the specific qualification standards for NAIC Statements of Opinion. Regarding the specific standards, nearly 13% of respondents indicated that they do not know if they meet the standards. For both the general and specific qualification standards, those who practice outside the United States tend to meet the qualification standards more frequently. Only 14% of respondents indicated that they serve as the appointed actuary for one or more U.S. domiciled P&C insurance companies.

Over two-thirds of the respondents have attended the CAS Course on Professionalism. However, only 20% of Fellows with greater than 10 years of tenure have attended the course, compared to 90% of those with tenure of less than 10 years. This may be a concern because 34% of the respondents that indicated that they sign prescribed statements of actuarial opinion achieved their designation more than ten years ago.

The following graph reflects the respondents' level of agreement with three statements about the Course on Professionalism:



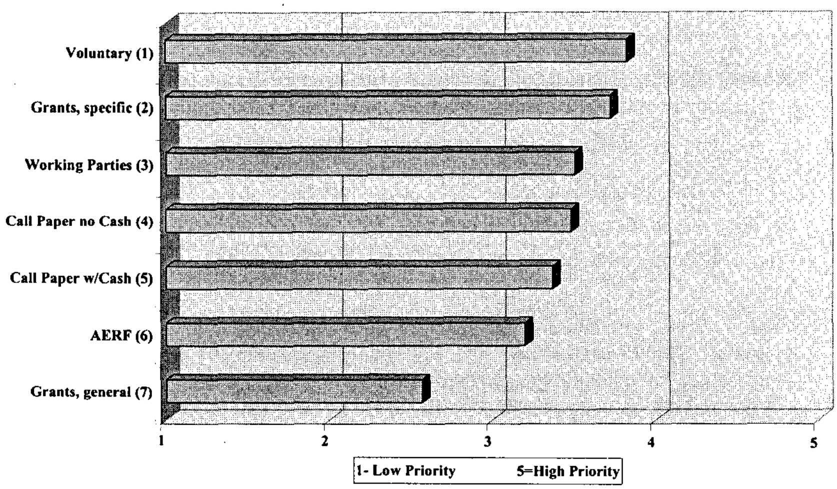
Recommendations: ***There was not a clear consensus on whether all CAS members should attend the Course on Professionalism. The CAS may want to discuss the value of making this a requirement. The CAS may also want to determine if there are other forms of professionalism/ethics education that may be more appropriate for members that have not attended the current course.***

6. RESEARCH

6.1 Prioritization of research "channels"

CAS members were asked to prioritize seven specific "channels" for conducting research. Respondents were provided the opportunity to write in alternatives.

Research Channels



1. Voluntary research and submission of papers
2. Funded research grants for specific topics
3. Working parties (papers written by a group of researchers)
4. Call paper programs without cash awards
5. Call paper programs with cash awards
6. Funded research through Actuarial Education Research Fund (AERF)
7. Funded research grants allowing proposers to choose subject

These results reflect the strong volunteer culture of the membership, with voluntary research at the top of the list of the types of research on which the CAS should focus. There is strong support for the new "channel," working parties. Call paper programs continue to get support, but the existence of a cash award appears to be unimportant. Funded research gets strong support, but this support is much stronger when the CAS has complete control (specific topics), drops when there is moderate control (through AERF), and fairly low when the researcher gets to choose the topic. The relatively low priority to the "other" category may be interpreted as a determination that the channels listed are largely sufficient.

The write-in responses for this question can be grouped into four categories:

1. Alternative channels for research, such as joint studies with a larger academic community, funded research through The Actuarial Foundation, and requiring new fellows to submit original research.
2. General areas for study, such as better syllabus material/papers, and research in applied actuarial science.
3. Specific research topics, such as derivation of loss and LAE reserve ranges, high profile P&C industry topics (e.g., medical malpractice), realistic loss trend analyses, salary studies, and studies on the use of credit in rating.
4. Alternative format for the delivery of results, such as software and/or spreadsheets.

6.2 Techniques

Members were asked to describe the techniques they are using for the majority of their work by assigning them to one of three categories.

Technique	Percentage
Basic, traditional	32%
Some advanced, some traditional	62%
Cutting edge, advanced	6%

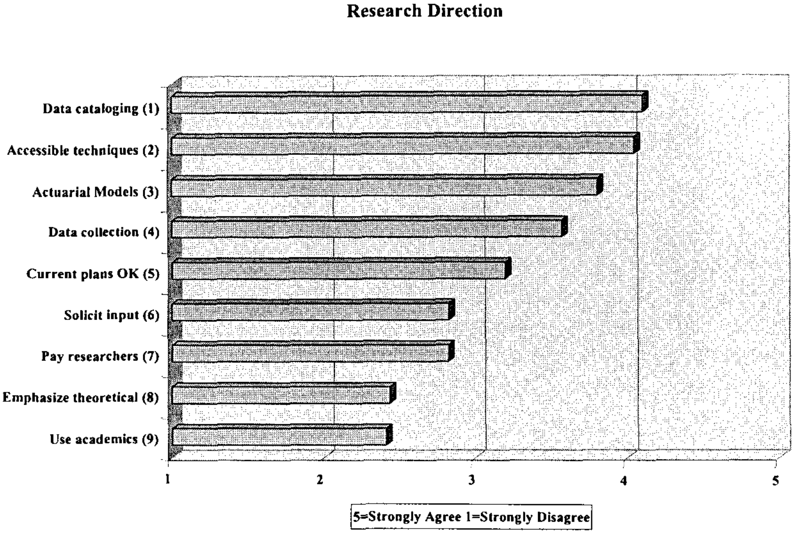
Roughly similar distributions prevailed for most demographic groups, with one notable exception. Respondents whose Area of Primary Responsibility was **other** than U.S. or Canada identified double digit percentages for cutting edge or advanced techniques, which is twice as high as the U.S. and Canada. (These results must be interpreted with some caution, as the number of respondents is not large for some categories.)

The percentages by geographic area of primary responsibility were:

Area of Primary Responsibility	Number of respondents	Percentage assigned to Cutting Edge, Advanced
Worldwide	75	13%
Africa/Asia	26	12%
Australia/New Zealand	12	17%
Bermuda	36	11%
Canada	81	5%
Latin America	16	19%
Europe	45	13%
US	786	5%

6.3 Research Direction

One question had nine parts, identifying a number of activities that could be undertaken with respect to research, and asking respondents to indicate their strength of agreement with undertaking the activity.



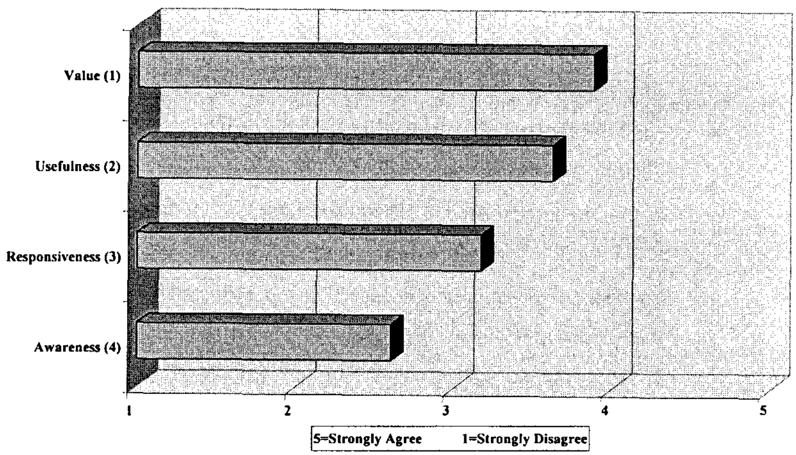
The labels on the bars are short descriptions of the actual options, as follows:

1. The CAS should identify and catalog sources of data that could be useful to actuaries.
2. The CAS should sponsor research to make advanced techniques more accessible to, and more widely used by, the CAS membership.
3. The CAS should conduct research that involves the development of actuarial models.
4. The CAS should conduct research studies that involve the collection, combination and analysis of data.
5. The CAS is acting aggressively enough to provide research and education to its members on the subject of enterprise risk management.
6. I would like the opportunity to provide input about areas in need of CAS research.
7. The CAS should pay researchers to conduct projects and rely less on volunteers and prize/awards for research papers.
8. CAS research should be primarily theoretical. Individual practitioners and companies should develop their own practical applications.
9. The CAS should primarily use academics for paid research projects.

6.4 Research Reactions

One set of questions asked for reactions to the usefulness and awareness of CAS research.

Research Reactions



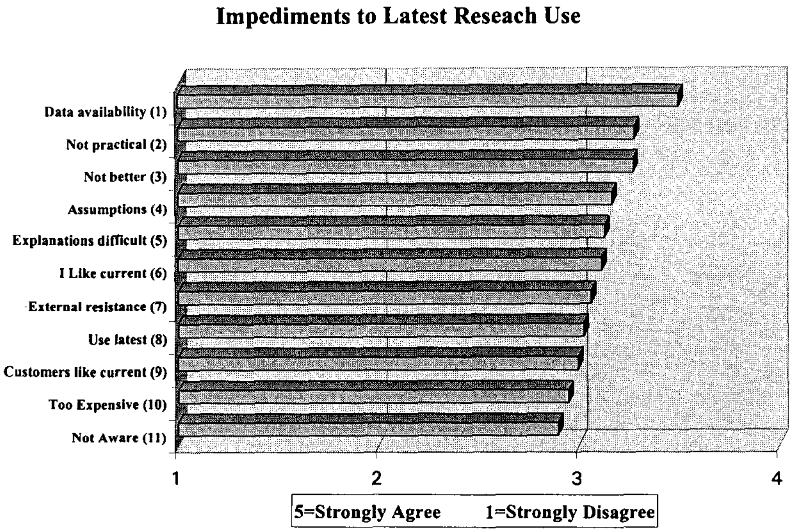
1. I view CAS research as a valuable resource when I have specific problems to address.
2. I have used research in my work that was completed by or sponsored by the CAS.
3. CAS sponsored research is generally responsive to my needs as a practicing actuary.
4. I am well aware of most of the research done by and sponsored by the CAS.

When research exists, respondents use it and find it valuable. The breadth of research is more of a question. Given the responses to the first two questions, a reasonable interpretation of the less positive responses to the third question is "What I can find is fine, but I can't always find what I want." The relatively low response to the final question suggests we need better communication of the work that has been done.

Recommendation: ***The CAS needs to better publicize the availability and organization of its research. Recent efforts to develop a research taxonomy should support this objective.***

6.5 Research Impediments

A set of questions listed eleven potential impediments to using CAS research, and asked for reactions.



1. The required data is usually not available
2. The techniques are not practical enough to use in practice
3. I'm not sure they produce better results
4. Too many assumptions need to be made
5. They are too difficult to explain to non-technical audiences
6. I like my current methods
7. Auditor, regulators, etc. may not accept these approaches
8. I do use the latest techniques
9. My management, or my clients, like the way it's done now
10. They are too expensive to use in practice
11. I am not aware of recent research in my area of practice

Most of these values should be cause for moderate concern. In only one case did a majority of respondents agree that a potential problem was a "real" issue. However, these are substantial values. A fair number of respondents were in the neutral category, so a relatively small minority report that these are NOT areas of concern. The relatively high response to data issues mirrors the "lukewarm" response to data collection as a possible CAS research topic (section 6.3) and may support targeted opportunities in that area.

6.6 Access to Research

Respondents were asked to identify places they would like to access research, with an option to write-in alternatives. Not surprisingly, the CAS Web Site led the list of options. Only two groups gave this option low marks, retired members, and, not surprisingly, members who never accessed the Web Site. Traditional paper formats, call papers, *Proceedings*, and the *Forum* came next, with comparable weight to face-to-face options such as meetings and seminars. Online biographies came in lower, along with the Actuarial Review. A quarterly research newsletter received the fewest votes of the listed options, but this should be tempered by the fact that it does not yet exist, so this vote may reflect lack of familiarity. Only a fraction of one per cent wrote in alternatives, including options such as: interactive web site, searchable CD-ROM, and e-mail notification.

7. PROFESSIONAL EDUCATION

7.1 Continuing Education

Respondents indicated they spent an average of 42 hours of organized continuing education activities and 110 hours via other continuing education activities during the three years prior to the survey, which is in excess of the Qualification Standards for Prescribed Statements of Actuarial Opinion requirement of 24 hours every two years.

Members were asked for their opinions about continuing education requirements and how changes should be applied:

What form of continuing education requirement should exist for CAS members?	None	5%
	Required only for those signing actuarial opinions	4%
	Required only for those signing public statements	13%
	Required only for all actuaries doing actuarial work	51%
	Required only for those currently employed	25%
	Required for all CAS members, even if retired	2%
If changes are made to continuing education requirements, to whom should they apply?	Only to new members	6%
	All currently practicing members	79%
	All currently listed members	15%

Over 75% of respondents believe that continuing education requirements should be required for either all members doing actuarial work or for all actuaries currently employed. Nearly 80% believe that if changes are made to the continuing education requirements, they should apply to all currently practicing actuaries.

The majority of respondents believe that the current ASB continuing education requirement of 24 hours every two years is appropriate:

Assuming that the CAS has a continuing education requirement, what should be the extent of the requirement?	More than current ASB standard	9%
	Less than current ASB standard	7%
	Same as the current ASB standard	58%
	It should vary based on category (new, practicing, or listed member)	11%
	There should be no requirement	4%
	No opinion	11%

7.2 General Business Skills

The most popular venue to receive education on general business skills appears to be in the actuary's own company or via a suggested reading list:

Where would you prefer to receive education on General Business Skills? (check all that apply)	In my own company	23%
	Suggested reading list	23%
	CAS meetings	18%
	CAS seminars	17%
	Desktop application learning tools	17%
	Regional Affiliate meetings	12%
	Limited Attendance seminars	11%
	Not interested	4%
	Other	1%

Interest levels vary widely based on the type of general business skills education offered. Strategic thinking and negotiation skills appear to hold the greatest interest level.

Would you be interested in attending a workshop on the following topics if offered at future CAS meetings?			
	No	Yes, if included in meeting registration fee	Yes, even if it requires additional fee
Strategic Thinking	26%	56%	18%
Negotiation	34%	49%	17%
Project Management	36%	49%	15%
Marketing/Networking	41%	46%	13%
Working with Others	60%	35%	5%
Writing Skills	66%	27%	7%
Survey Writing Skills	79%	19%	2%
Other	67%	15%	18%

Recommendation: ***The Committee on General Business Skills should consider offering sessions on Strategic Thinking and/or Negotiation Skills at future CAS meetings. The CAS should consider including the cost of the sessions in the registration fee in order to increase participation in these sessions.***

8. REGIONAL AFFILIATES

Two questions regarding Regional Affiliates may prove to be informative for the leadership of the various organizations, as well as the Regional Affiliates Committee of the CAS.

If you are not an active member/participant of a CAS Regional Affiliate, why not?	Time and travel costs	36%
	Low relevance of subject matter	19%
	I prefer meetings with more activities or in more interesting locations	13%
	My company doesn't encourage or sponsor my attendance	10%
	Other	9%
	RA meetings perceived as "student's CAS meeting" with limited value to members	6%
	Networking opportunity limited	4%
	I don't get timely notification of meetings	3%

Considering the difference in time and travel costs, how do Regional Affiliate (RA) meetings compare to other CAS meeting/seminar opportunities?	RA meetings have significantly less value	22%
	RA meetings have somewhat less value	39%
	Both provide about the same value	25%
	RA meetings have somewhat more value	11%
	RA meetings have significantly more value	3%

61% of respondents indicated that Regional Affiliate meetings provide less value when compared to other CAS meetings and seminars. Caution must be used in interpreting this finding, since CAS candidates, who make up a large portion of the audience at Regional Affiliate Meeting, were not part of this survey audience,

<u>Recommendation</u>: ***The individual Regional Affiliates, in conjunction with the Regional Affiliates Committee, may want to survey candidates and CAS members in their geographic region for further input on the value of Regional Affiliate meetings.***

9. INTERNATIONAL

9.1 Need for Recognition from Another Actuarial Organization

While only 5% of the respondents have ever had a need for recognition from another actuarial organization, over 20% of the CAS respondents living outside of the United States and Canada have had a need in the past.

9.2 International Travel

Over half the respondents never travel internationally for work purposes. The vast majority of respondents who travel internationally for work are located outside of the United States and Canada. 93% of our members living abroad travel internationally at least once a year and 63% of them take more than four international business trips per year.

9.3 CAS Support for Actuarial Profession in Developing Countries

There is an overwhelming consensus to provide support through literature, education and sharing of techniques to aid in the development of the actuarial profession outside the United States. The majority of respondents, however, are not in favor of using CAS funds to directly support this development.

The table below shows the percentage of respondents who rated each area as important or very important and the percentage that rated the area as not important or not important at all.

The CAS has launched initiatives to address the issues below and currently provides material via the Web Site, has regional teams within the International Issues Committee which act as a liaison with local organizations, sends representatives to meetings, and is active within the IAA. The CAS appears to be doing what the membership in general believes is necessary.

Recognizing that financial and human resources are required, in which areas should the CAS be actively working to support the development of the actuarial profession in countries where the profession is in the development stages?

	Agree	Disagree
Provide crucial casualty actuarial literature through the CAS Web Site and links to other web sites	70%	3%
Send CAS leaders to participate in key meetings	62%	7%
Send CAS members to speak at general insurance/actuarial seminars	61%	8%
Make exam sites available to interested candidates for CAS exams, wherever they are located in world	54%	12%
Send study materials to universities	53%	12%
Send CAS members to teach specific subjects, including exam-oriented subjects, at local seminars	50%	13%

Work with local regulators, policymakers, and actuarial bodies to gain official recognition of the CAS credential in various jurisdictions	50%	9%
Establish ambassadors or liaisons to cooperate with other international actuarial societies on matters involving casualty areas outside of North America	47%	11%
Create an international referral service whereby foreign actuaries could ask specific questions and be referred to CAS volunteers for comment on North American approaches to similar issues	45%	12%
Assist local organizations in developing the casualty content for their own exams	44%	15%
Organize CAS seminars	43%	15%
Proactively develop CAS regional affiliates in other countries or regions	42%	15%
Actively participate in the International Actuarial Association (IAA)	42%	10%
Organize a program for CAS members in the U.S. to donate their personal libraries of CAS publications to university or similar libraries	36%	18%
Encourage (including monetary subsidies) local practitioners and academics to become Affiliates of the CAS and/or to take the CAS exams to Fellowship	29%	26%
Subsidize the registration and travel cost for actuaries and academics from these countries to speak at CAS meetings and seminars (i.e., in North America)	29%	28%
Use CAS funds to help finance the efforts of organizations such as the International Actuarial Association to support the development of the actuarial profession in these countries	29%	24%
Offer discounted CAS dues	18%	41%
Subsidize the registration and travel cost for actuaries and academics from these countries to attend CAS meetings and seminars	11%	48%

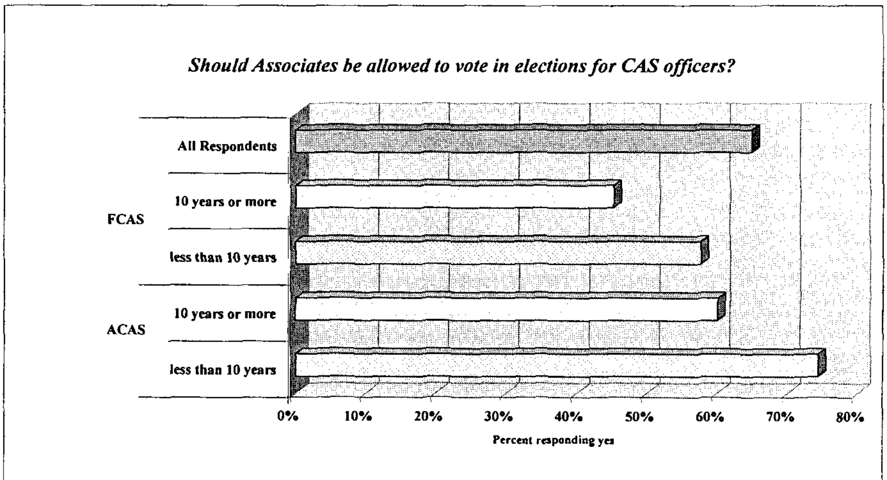
10. GOVERNANCE/ELECTIONS

10.1 Election Process

65% of the respondents voted in the 2002 CAS election. Of those that did not vote, the main reason given was insufficient knowledge of the candidates. "Meet the Candidates" material was introduced on the CAS Web Site for the 2002 election. 65% of the respondents read the material and 56% found it helpful. Less than 7% of the respondents were not aware that the material was published on the Web Site. Over 50% of respondents believe the changes made to the election process in 2002 will improve the governance of the CAS. Only 7% believe the changes will have no impact.

10.2 Voting Rights for Associates

The majority of the respondents (65%) agree that Associates should have voting rights within the CAS. Associates were more likely to favor these voting rights than Fellows (69% versus 52%). In addition, both Fellows and Associates that achieved their designations less than ten years ago were more likely to favor granting voting rights to Associates.



While there is general consensus about granting voting rights to Associates, there is disagreement over when those rights should begin. Approximately half believe they should begin upon achievement of the ACAS designation, while the other half think a waiting period after achieving the designation is appropriate. It is interesting to note that in the 1993 survey only 34% of respondents thought that voting rights should be extended to ACAS members. Based on the written comments received, there is a consensus among those that believe that ACAS should have the right to vote, that this right should not be extended to exam-related issues or volunteering on exam committees. The CAS Board has commissioned two Task Forces to address the rights of Associates. The Task Forces have been provided with the feedback from this survey.

11. ADMISSIONS

Five of the survey questions dealt with admissions, education, and syllabus issues. Not surprisingly, the CAS does not speak with one voice, but there are some apparent trends.

11.1 Alternate Means for Meeting Educational Requirements

Respondents were asked to indicate how actuaries practicing in casualty insurance outside the United States should be able to satisfy educational requirements for CAS membership. Despite the significant majority of Fellows voting in favor of mutual recognition, more than half the respondents indicated that actuaries should have to satisfy the current requirements for U.S. candidates (pass seven exams) before being admitted as CAS members. Of the remainder, about 75% indicated that being credentialed in the actuary's home country and one or two CAS exams was sufficient. Only approximately 7% indicated that mutual recognition should be automatic and about 4% said "Not at All."

Affiliates were more inclined to allow relaxed requirements than Fellows, with Associates even less inclined than Fellows. Also, those practicing outside the United States seem to be more willing to accept less rigorous requirements than those practicing inside the United States

11.2 Supply of Candidates

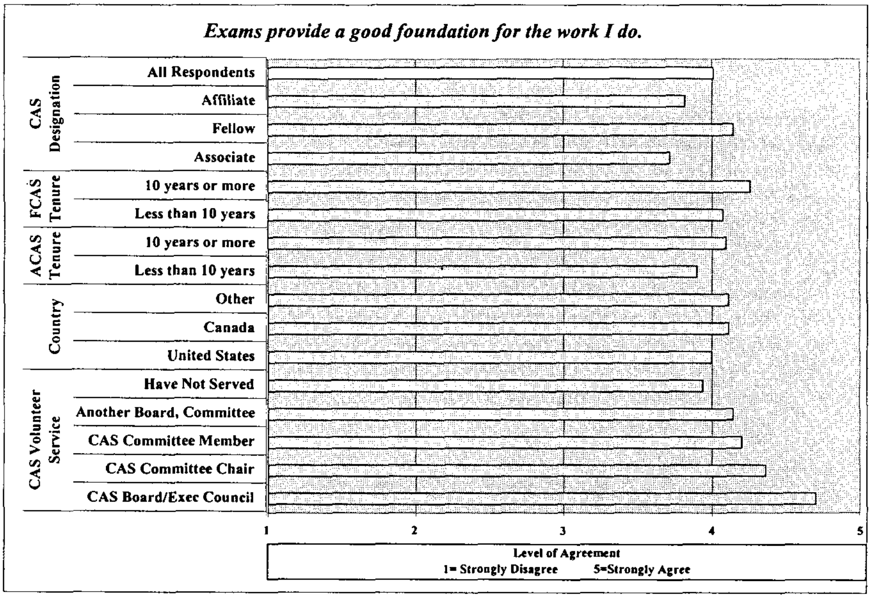
Respondents did not believe that there was an oversupply of casualty actuaries, with only 6% indicating that there are too many coming into the profession. The vast majority (65%) reported that there was a sufficient supply. Only 29% said there were not enough, including 4% saying there are far too few.

11.3 Exams and Career Preparation

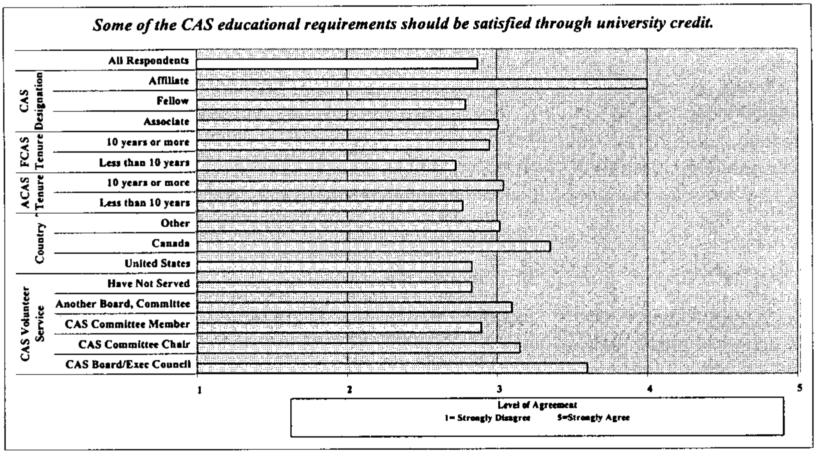
The survey included a question asking the respondent to indicate the degree of agreement with three statements about exams and career preparation on a five-point scale.

More than 80% of the respondents either agreed or strongly agreed that the exams are a good foundation for the work they do. More respondents indicated that the exams were an impediment (46%) than felt that they were not (38%).

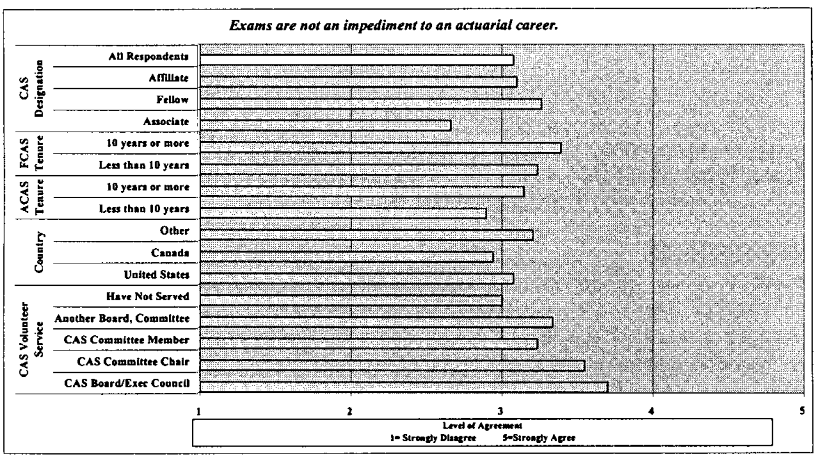
Digging deeper into the demographics sheds some light on these responses. When asked if the exams provide a good foundation for actuarial work, the Fellows were much more likely to agree (88% either Agree or Strongly Agree) than Associates and Affiliates (69% and 73%, respectively).



Members were then asked whether some of the CAS educational requirements should be satisfied through college credit. College credit was strongly favored by Affiliates, but not nearly as much by Fellows or Associates. Those with more than 10 years tenure seemed more in favor than newer members. Those practicing outside the United States, where a college degree is a more common path to qualification, favored college credit more than those practicing in the United States. Moreover, Board members and committee chairs were more likely to agree with the statement than those not in CAS leadership positions.



Associates and Affiliates were less likely than Fellows to agree with the statement that exams are not an impediment to an actuarial career. As might be expected, tenure is positively correlated. Respondents outside the United States seem to be more likely to agree with this statement than those in the United States although the difference is small. There was also a marked difference between the responses of the CAS leadership.



11.4 Value of CAS Designation

Only about 25% of the respondents practice outside of the United States. Of these, 58% feel the CAS designations have equal or greater value than designations in other countries and 33% thought they had some value, leaving only 9% with little value or uncertain. In short, respondents perceive the status as having great value outside of the U.S. The perceived value generally increases with designation with 50% of the Associates practicing outside of the United States saying the designation has great value, 62% of the Fellows, and only 17% of the Affiliates.

11.5 Participation of the Academic Community

The respondents indicate that we can make better use of the academic community, virtually across the board. In all areas excluding continuing education more than half the respondents favored greater academic involvement. One write-in comment said, we "want their help, but must keep it practical."

	Yes	No	No Opinion
Promoting the profession	69.3%	14.0%	16.7%
Research	63.3%	15.2%	21.5%
Literature (esp. examination readings)	62.1%	17.5%	20.5%
Training / examination preparation sessions	55.5%	23.2%	21.4%
Examination structure and design	52.1%	30.8%	17.1%
Continuing education	46.2%	32.3%	21.4%

Answers were fairly consistent across nearly all demographic strata, with the exception of the desire for more help from academics on the exam structure. Here Associates favored academic participation a bit more than Fellows.

Recommendations:

- ***The CAS should continue to explore ways to make use of the unique talents found in the academic community to improve its education and examination process.***
- ***The CAS should stay the course it has already embarked on to make the CAS Syllabus and research accessible and useful to actuaries practicing outside of the United States.***

12. ADMINISTRATION

12.1 Electronic Services

CAS members are well connected electronically. At work, virtually all respondents have some form of Internet access with 93% having broadband. 52% have broadband access at home. Moreover, only 5% of the respondents have no Internet access at home. As Internet access has been growing for CAS members, access to the CAS Web Site has been increasing. 20% of the respondents report that they access the Web Site more than once a week, compared to only 12% in 1998. Similarly, 71% of respondents now access the Web Site at least once a month, compared to 52% in 1998. Only 1% of respondents have never accessed the Web Site, compared to 24% in 1998.

Over 100 members responded to the write-in opportunity to suggest changes they would like to see in the CAS Web Site. Forty of these responses indicated satisfaction with the site and recommended no changes; many highly praised the site and CAS Web Site staff. These comments were further supported by the high quality rating that the CAS Web Site received in the publication questions on the survey (section 13 below). The most frequent recommendation, suggested by 19 respondents, was improving the search engine capability. Other suggestions included adding more research material and improving user-friendliness. Interestingly, a number of the suggested improvements are already part of the Web Site, which may indicate that there is a need for more member education on the capabilities of the Web Site.

The CAS currently sends out e-mails in text format only. More than half of the respondents expressed no preference about the format; the members that expressed a preference were equally split between text and HTML.

The CAS piloted webcasts of the business sessions of the CAS meetings in 2002. Five percent of the respondents have seen live webcasts and another 8% of respondents have viewed webcasts afterward. Of the remaining respondents, 16% did not know that the web cast was available and 8% did not have the proper technology to view it. 64% of the members chose not to view the webcast. Despite the small number of respondents that actually viewed a webcast, 65% of respondents said that they would view a future webcast if it were free. Only 10% of respondents would pay to see a webcast. 25% of the respondents did not think they would view a webcast in the future. Only 31% would substitute viewing the webcast for attending a CAS meeting in person.

Recommendations:

- ***Regularly educate members on the current capabilities of the CAS Web Site.***
- ***Evaluate the feasibility of improving the Web Site search engine. The CAS should increase its promotion of web casts of the Spring and Annual Meetings to encourage more members to take advantage of them.***

12.2 Dues and Meeting Fees

87% of the respondents do not pay dues out of their own pockets. This percentage is down only slightly from the prior survey. Eleven percent of the respondents pay for all of their dues personally and 7% pay for all of their meeting fees. The remainder pays for a portion of these fees personally. Actuaries employed by reinsurance companies and service companies have the highest proportion of fully reimbursed fees (98% and 99%, respectively), followed by actuaries employed by insurance companies (92%). Consulting actuaries and actuaries working for regulatory organizations have much lower reimbursement rates at 77% and 63%, respectively. Of course, retirees and full-time parents had the lowest reimbursement rates. The reimbursement patterns for meeting fees were very similar to dues.

Members were asked if they would pay for dues and meeting fees out of their own pockets. 77% of respondents indicated that they would be willing to pay for dues themselves. Respondents over 45 years of age, respondents with tenure of more than ten years, and CAS leaders were much more likely than other groups to be willing to pay for dues out of their own pockets. When it came to paying for meeting fees out of their own pockets, only 36% of respondents expressed a willingness to pay the fees themselves.

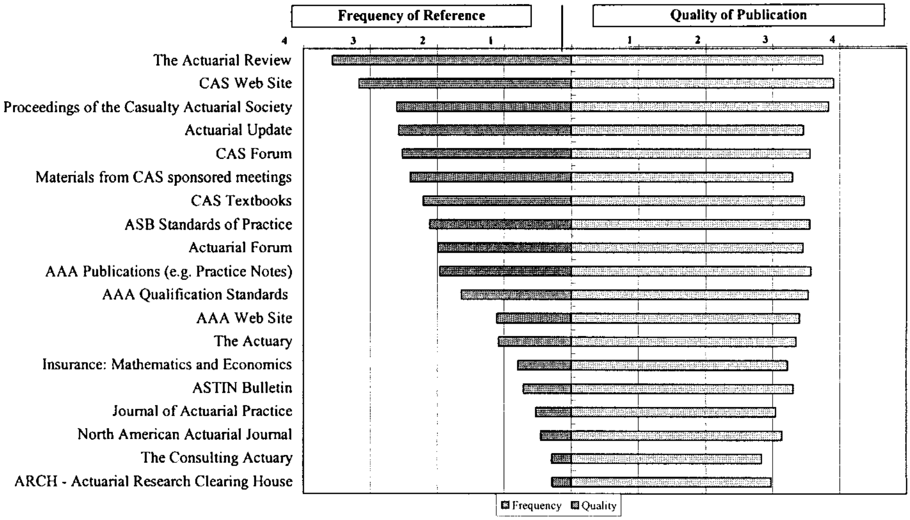
12.3 Reimbursement for Volunteer Activities

Members were asked whether they paid for all, some or none of their volunteer activities. 11% percent paid for all of their volunteer activities and another 13% paid for a portion out of pocket; these percentages were about twice as high as the proportion of members paying for dues out of pocket. The pattern of relative reimbursement by employer was similar to dues and meeting fees. Only one third of the total respondents indicated that they would pay for volunteer activities out of their own pocket. However, two thirds of CAS Board and Executive Council members and half of the CAS committee chairs would pay for their volunteer activities.

13. PUBLICATIONS

The CAS launched a Task Force on Publications, which was charged with examining the entire CAS publication structure. As part of their research, the Task Force on Publications submitted several questions for the 2003 Membership Survey and five were included in the final survey.

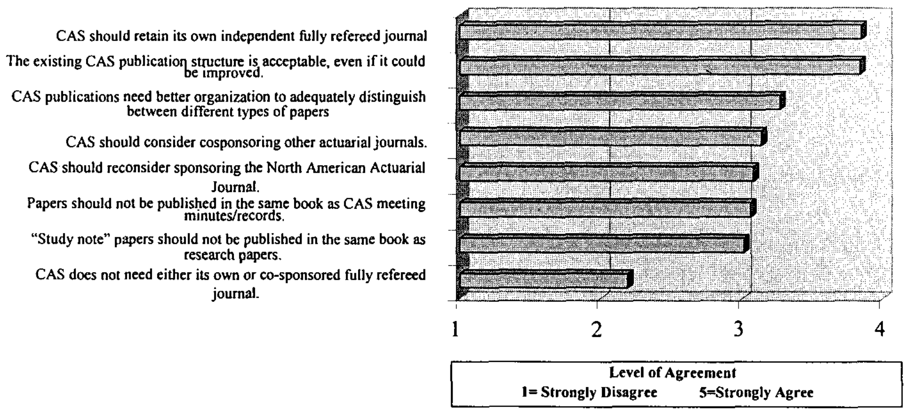
CAS members were asked about how frequently they read eighteen different actuarial publications and were asked to rate the quality of the publications. The frequency was evaluated on a five-point scale plus a choice of "Never Read," and quality was rated on a five-point scale. Frequency and quality measures were calculated by taking a weighted average of the responses, which were converted to show five as the highest rating. The results are summarized in the following chart.



Respondents most frequently read documents published by the CAS. Publications of the American Academy of Actuaries followed the CAS publications in popularity. Members were given the opportunity to write in the names of other publications that they read. The AAA Contingencies magazine was the most popular write-in item, cited by 23 respondents. Not surprisingly, respondents rated the most-read publications as the highest in quality.

Members were asked to rank six general publication sources according to the relative importance they place on them in their own actuarial research and continuing education efforts. Once again, CAS publications topped the list by a wide margin with fully 80% of the respondents rating them as important or very important. The next most important group was other economic, scientific or mathematical publications, which were rated as important or very important by 23% of the respondents. The remaining categories were rated as unimportant by between 40% and 50% of the respondents. Even though they rated CAS publications as the most important, respondents that have primary business responsibility outside of the United States placed somewhat more importance on the SOA and IAA publications than their U.S. counterparts.

Members were also asked to rate their level of agreement with eight statements about CAS publications. The results are summarized in the graph below:



Almost 75% of the respondents indicated "the existing CAS publication structure is acceptable, even if it could be improved." Less than 5% disagreed with this statement and the remainder was neutral. More than 60% of the respondents felt that the CAS should retain its own independent fully refereed journal. Approximately the same proportion disagreed with the statement that CAS does not need either its own or co-sponsored fully refereed journal. 37% felt that "CAS publications need better organization to adequately distinguish between different types of papers."

Respondents were less opinionated about whether papers should be published in the same book as CAS meeting minutes/records and whether "study note" papers should be published in the same book as research papers. When asked about cosponsoring other actuarial journals and reconsidering sponsorship of the North American Actuarial Journal, almost two-thirds of the respondents had no opinion. Those that expressed an opinion were more inclined to be in favor of co-sponsorship.

Members were given the opportunity to respond with written comments regarding how they view the structure and organization of existing CAS publications. Respondents had a clear diversity of opinion on this topic with some respondents recommending no change and others calling for a complete overhaul. The most common criticisms of the publications were the confusing organization structure and the poor editorial review of Forum and Call Papers. Several respondents felt that the CAS needs to maintain an independent set of publications. As one respondent wrote, "a strong independent set of publications is an important element of the identity of the CAS — and partly what keeps the CAS focus very sharp and not diluted by any other priorities."

Respondents were asked to describe their interest in writing papers for CAS publications. Only 6% of those responding indicated that they have written papers in the past and a similar proportion indicated that they would be interested in submitting papers for CAS publications in the future. Only 3% said they would prefer to publish papers in the *Proceedings* because it is fully refereed and 3% said they would prefer to avoid the burden of review by the Committee on Review of Papers and submit papers only to Call Paper programs or the *Forum* directly. 15% indicated that they have less interest in writing papers than in other CAS activities and the same proportion said they had no interest in writing papers for future CAS publication. 10% said they were unsure whether they were qualified to write papers sufficient for CAS publication.

It is interesting to note that a much larger proportion (over one fourth) of the CAS leadership (Board and Executive Council members and Committee Chairs) have written papers for CAS publications. At the same time, the leadership was more likely to express their opinion about the CAS publication structure and was more in favor of making changes to it.

Recommendation: ***The Task Force on Publications has already received the feedback from the Membership Survey and is planning to follow up by conducting focus groups to gather further input from the Member Advisory Panel. Based on the feedback from the Membership Survey, CAS should strive to maintain some form of refereed journal and should concentrate on improving the overall organization of CAS papers and the quality of non-refereed papers.***

CONCLUSION

This report has summarized the key findings from the responses to the 2003 CAS Membership Survey. There is a great deal of additional detail contained in the survey results and cross-tabulations by demographic group that various CAS Committee and Task Force members may find relevant and interesting. The 2003 Membership Survey Task Force would welcome the opportunity to work with Committee and Task Force members to explore these findings in more detail.

Casualty Actuarial Society
2003 MEMBERSHIP SURVEY QUESTIONNAIRE

Every five years, the CAS conducts a survey of its membership to determine the needs of the actuarial profession and how those needs can be better met. We appreciate the time and effort you are spending in completing the 2003 survey. All responses to the survey, and the identity of respondents, will be kept in strictest confidence. A full report on the results of the survey will be published in Fall 2003.

We encourage you to complete the survey online by going to the following web site:
www.ari-surveys.com/run/CASMemberA

However, you may also fill it out and fax it back to CAS at *(703) 276-3108*, or send it by mail to:

CASUALTY ACTUARIAL SOCIETY
1100 N. Glebe Road
Suite 600
Arlington, VA 22201

Please complete this survey by ***July 31, 2003.*** *Thank you for your participation.*

Demographics

1. Are you (check only one):
   ❑ Associate ❑ Fellow ❑ Affiliate

2. What were the year(s) you attained your CAS designation(s) or affiliate membership?

   ACAS _________ FCAS _________ Affiliate _________

3. I am:
   ❑ Male ❑ Female

4. Age Range (check one):
   ❑ <20
   ❑ 20 to 25
   ❑ 26 to 30
   ❑ 31 to 35
   ❑ 36 to 40
   ❑ 41 to 45
   ❑ 46 to 50
   ❑ 51 to 55
   ❑ 56 to 60
   ❑ 61 to 65
   ❑ 66 to 70
   ❑ 71 to 75
   ❑ >75

5. What is your business affiliation? (check one)

a. ❑ Insurance company
b. ❑ Consulting actuary
c. ❑ Service company
d. ❑ Regulatory organization
e. ❑ Retired
f. ❑ Reinsurance company
g. ❑ Insurance broker
h. ❑ University or college
i. ❑ Full-time parent
j. ❑ Other

6. Where is your primary place of work?
State/Province: ________________________________
Country: ___________________________________

7. Geographic area of your primary business responsibility: (check all that apply)

a. ❑ Worldwide
b. ❑ Africa
Asia (c-g)
c. ❑ Central (e.g., India, Pakistan)
d. ❑ Southeast (e.g. Singapore, Hong Kong)
e. ❑ China
f. ❑ Japan
g. ❑ Other parts of Asia
h. ❑ Australia / New Zealand
i. ❑ Bermuda
j. ❑ Canada
k. ❑ Central America
Europe (l-m)
l. ❑ Eastern Europe
m. ❑ Western Europe
n. ❑ Middle East
o. ❑ South America
p. ❑ United States

8. If you were presented with a job opportunity (i.e., a relocation lasting at least 1 year) outside of your primary place of work (as specified in question # 6) within the next 5 years, what is the likelihood that you would accept it?

Very Likely	Somewhat Likely	Undecided	Somewhat Unlikely	Very Unlikely
a ❑	b ❑	c ❑	d ❑	e ❑

9. I am a member of the following actuarial organizations: (check all that apply)

a ❑ American Academy of Actuaries
b ❑ American Society of Pension Actuaries
c ❑ Conference of Consulting Actuaries
d ❑ Canadian Institute of Actuaries
e ❑ Faculty of Actuaries
f ❑ Institute of Actuaries
g ❑ Institute of Actuaries of Australia
h ❑ International Actuarial Association
i ❑ International Actuarial Association – ASTIN
j ❑ International Actuarial Association – AFIR
k ❑ International Association of Consulting Actuaries
l ❑ Society of Actuaries
m ❑ Other ________________

10. Highest level of academic education completed:

a ❑ HS/GED
b ❑ AA/AS (two-year degree)
c ❑ BA/BS
d ❑ MA/MS
e ❑ MBA
f ❑ JD
g ❑ PhD
h ❑ MD/DDS/Other Medical
i ❑ Other (specify) ____________________

11. Non-actuarial professional designations (check all that apply):

a ❑ ARe
b ❑ ARM
c ❑ AIMR
d ❑ CFA
e ❑ CPA
f ❑ CPCU
g ❑ Other (specify) _________

12. A. Please indicate what percentage of your time over the past two years you have spent in each of the following areas (total should be 100%). B. Please also indicate which of the following roles you've played in your career by checking the box to the right.

Function	(A) Time In/During Past 2 Years Percentage	(B) Over Your Career
A Data Management / Systems Administrator	________	❑
B Risk & Capital Management (e.g., DFA)	________	❑
C Management Advisor	________	❑
D Management of Actuarial Unit	________	❑
E Executive Management	________	❑
F Expert Witness	________	❑
G Investments / Financial Decision Maker	________	❑
H Marketing / Underwriting	________	❑
I Planning - Strategic and Financial	________	❑
J Programming / Software Development	________	❑
K Ratemaking	________	❑
L Reserving	________	❑
M Regulator	________	❑
N Teaching / Researching	________	❑
O Valuation	________	❑
P Reinsurance Pricing	________	❑
Q Other (please write in) ______________________	________	❑
Total	100%	

13. In the last three years, have you served: (check all that apply)

a ❑ On the CAS Board or Executive Council?
b ❑ As Chair of a CAS Committee?
c ❑ As a member of a CAS Committee?
d ❑ On another actuarial organization's Board, Executive Council or Committee?
e ❑ None of the above

Administration - Electronic Services and Finance

14. How do you access to the Internet?

	Dial-up	Broadband	Do Not Have Access
Home	a ❑	b ❑	c ❑
Work	a ❑	b ❑	c ❑

15. How often do you access the CAS Web Site?

a ❑ Daily
b ❑ More than once per week
c ❑ Once per week
d ❑ Once per month
e ❑ Less than once per month
f ❑ Never accessed it

16. What changes would you like to see on the CAS Web Site?

17. What portion of the following do you pay for personally?

	All	Some	None
A. Dues	❑	❑	❑
B. Meeting fees	❑	❑	❑
C. Volunteer activities	❑	❑	❑

If you were asked to pay for the following yourself, would you pay for?

	Yes	No
A. Dues	❑	❑
B. Meeting fees	❑	❑
C. Volunteer activities	❑	❑

Research and Development

18. On which types of research should the CAS focus?

	High Priority				Low Priority	No Opinion
Call paper programs with cash awards	1	2	3	4	5	❑
Call paper programs without cash awards	1	2	3	4	5	❑
Funded research grants for specific topics	1	2	3	4	5	❑
Funded research grants allowing proposers to choose subject	1	2	3	4	5	❑
Voluntary research and submission of papers	1	2	3	4	5	❑
Funded research through Actuarial Education Research Fund (AERF)	1	2	3	4	5	❑
Experience studies	1	2	3	4	5	❑
Working parties (papers written by a group of researchers)	1	2	3	4	5	❑
Other (please write in) ____________________	1	2	3	4	5	❑

19. What best describes the techniques you are using today for the majority of your work?

a ❑ Basic, traditional
b ❑ Some advanced, some traditional
c ❑ Cutting edge, advanced

20. Please indicate your level of agreement with the statements below.

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
A. The CAS should sponsor research to make advanced techniques more accessible to, and more widely used by, the CAS membership.	1	2	3	4	5
B. The CAS should conduct research that involves the development of actuarial models.	1	2	3	4	5
C. The CAS should conduct research studies that involve the collection, combination and analysis of data.	1	2	3	4	5
D. The CAS should pay researchers to conduct projects and rely less on volunteers and prize/awards for research papers.	1	2	3	4	5
E. The CAS should primarily use academics for paid research projects.	1	2	3	4	5
F. CAS research should be primarily theoretical. Individual practitioners and companies should develop their own practical applications.	1	2	3	4	5
G. I would like the opportunity to provide input about areas in need of CAS research.	1	2	3	4	5
H. The CAS should identify and catalog sources of data that could be useful to actuaries.	1	2	3	4	5
I. The CAS is acting aggressively enough to provide research and education to its members on the subject of enterprise risk management.	1	2	3	4	5

International

21. Have you ever had the need for recognition from an actuarial society other than one in which you were already a member?

❑ Yes ❑ No

If yes, what was the reason recognition was necessary?____________________
In what country(ies)?____________________

22. How often do you travel internationally?

	More than 4 time per year	1-4 times per year	Less than once a year	Never
For business	❑	❑	❑	❑
For pleasure	❑	❑	❑	❑

23. Recognizing that financial and human resources are required, in which areas should the CAS be actively working to support the development of the actuarial profession in countries where the profession is in the development stages? Rate each of the following using a scale from 1–5 with 1 being very important and 5 being not important at all. If you have no opinion, please indicate so by selecting #6.

	Very Important				Not Important At All	No Opinion
A. Send CAS members to speak at general insurance/actuarial seminars.	1	2	3	4	5	6
B. Send CAS members to teach specific subjects, including exam-oriented subjects, at local seminars.	1	2	3	4	5	6
C. Send CAS leaders to participate in key meetings.	1	2	3	4	5	6
D. Organize CAS seminars.	1	2	3	4	5	6
E. Make exam sites available to interested candidates for CAS exams, wherever they are located in the world.	1	2	3	4	5	6
F. Send study materials to universities.	1	2	3	4	5	6
G. Assist local organizations in developing the casualty content for their own exams.	1	2	3	4	5	6
H. Organize a program for CAS members in the US to donate their personal libraries of CAS publications to university or similar libraries.	1	2	3	4	5	6
I. Encourage (including monetary subsidies) local practitioners and academics to become Affiliates of the CAS and/or to take the CAS exams to Fellowship.	1	2	3	4	5	6
J. Proactively develop CAS regional affiliates in other countries or regions.	1	2	3	4	5	6
K. Other ______________________	1	2	3	4	5	6

Volunteerism

24. Is there anything that is preventing you from increasing your participation on CAS committees/task forces (Check all that apply)?

a ❑ No limitation
b ❑ Cost
c ❑ Time
d ❑ Lack of interest at this time
e ❑ Not supported by my employer
f ❑ Other (Please describe) ______________________________

25. Would you volunteer more if your travel costs were subsidized?

❑ Yes ❑ No

Governance – Elections

Questions 26-29 are to be answered by Fellows only

26. Did you vote in the last CAS election?

a ❑ Yes, I cast votes for all offices.
b ❑ Yes, I cast votes for some, but not all of the offices.
c ❑ No
d ❑ I can't remember.

27. If the answer to the above question was either b or c, what was the reason for not voting for all offices? (check all that apply)

a ❑ I did not agree with the positions of the candidates.
b ❑ I did not have sufficient knowledge of the candidates.
c ❑ Other ______________________________

28. Did you read the "Meet the Candidates" material on the CAS Web Site for the last election?

a ❑ Yes, I found the material helpful in making my choices.
b ❑ Yes, but the material was not helpful.
c ❑ No
d ❑ I was unaware that this material was on the CAS Web Site.

29. The CAS made several changes to the election process in 2002, including the process for nominating candidates. Do you feel that these changes will improve the governance of the Casualty Actuarial Society?

a ❑ The changes will significantly improve the governance of the CAS
b ❑ The changes will somewhat improve the governance of the CAS
c ❑ The changes will have no impact on the governance of the CAS
d ❑ Uncertain
e ❑ I was not aware of any changes.

30. Should Associates be allowed to vote in elections for CAS officers?

a ❑ No
b ❑ Yes, immediately upon achieving ACAS.
c ❑ Yes, after a period of 1-3 years
d ❑ Yes, after a period of 4-5 years
e ❑ Yes, after a period of 6-9 years
f ❑ Yes, after a period of 10 or more years

Please share any comments you may have relative to ACAS voting rights.

The Actuarial Profession

31. During your actuarial career, How many...

Distinct jobs have you held? _______
Employers have you worked for (including self)? _______

32. Important emerging areas of actuarial practice include the following. Please indicate and rank the three that you believe are creating the greatest new demand for actuaries (1 being the highest demand, 2 being the second highest demand, and 3 being the third highest demand):

	Emerging Areas of Practice
a	International insurance
b	Finance
c	Catastrophe modeling and securitization
d	Risk management and self insurance
e	Managed care
f	Capital allocation and corporate structure
g	Other #1 (please write in) ____________________
h	Other #2 (please write in) ____________________
i	Other #3 (please write in) ____________________

A) From the list above, write in the letter of the area creating the highest demand for actuaries

____

B) Using the same list, write in the letter of the area creating the second highest demand for actuaries:

____

C) Using the same list, write in the letter of the area creating the third highest demand for actuaries:

____

33. The CAS is always looking for new areas where we can expand actuarial practice. Please list any suggestions for areas to expand practice.

If you do not practice in the United States, please skip questions 34-37.

34. In the course of your practice, do you sign prescribed statements of actuarial opinion?
   - ❑ Yes
   - ❑ No

35. Do you meet the **general** qualifications standards for prescribed statements of actuarial opinion? (a statement of actuarial opinion issued for purposes of compliance with law or regulation or compliance with Actuarial Standards of Practices as promulgated by the Actuarial Standards Board or an Accounting Standards Board)
   - ❑ Yes
   - ❑ No
   - ❑ Don't Know

36. Do you function as the appointed actuary for one or more US-domiciled property & casualty insurance companies?
   - ❑ Yes
   - ❑ No

37. Do you meet the **specific** qualification standard for statements of opinion, NAIC Property & Casualty Annual Statement?
   - ❑ Yes
   - ❑ No
   - ❑ Don't Know

Professionalism Issues

38. Have you attended the CAS Course on Professionalism? ☐ Yes ☐ No
Please indicate your level of agreement with the statements below.

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree	Not Applicable
A. The Course on Professionalism helped make me aware of ethical issues that I face in my job, and how to deal with them appropriately.	1	2	3	4	5	6
B. CAS members that have not attended the Course on Professionalism should be required to take the course.	1	2	3	4	5	6
C. Continuing education requirements should include, on a mandatory basis, some form of professionalism/ethics education.	1	2	3	4	5	6

Professional Education

39. How many hours of continuing education have you completed in the last three years?
_______ Organized Activities (e.g., attendance at meetings or seminars)
_______ Other Activities (e.g., reading research articles)

40. What form of continuing education requirement should exist for CAS members? (check only one)
a ☐ None
b ☐ Required only for those signing actuarial opinions.
c ☐ Required only for those signing public statements (e.g., actuarial opinions, rate filing certifications) as a credentialed actuary.
d ☐ Required for all actuaries doing actuarial work.
e ☐ Required for all actuaries currently employed, even if currently in a non-actuarial profession.
f ☐ Required for all CAS members, even if retired.

41. If changes are made to continuing education requirements, to whom should they apply? (check only one)

a ☐ Only to new members
b ☐ All currently practicing members
c ☐ All currently listed members

42. Assuming that the CAS has a continuing education requirement, what should be the extent of the requirement? (check only one)

A ❑ More than the current Actuarial Standards Board (ASB) standard for actuarial opinion signers (24 hours for each 2 year period). Enter the recommended # of hours ______

B ❑ Less than the current ASB standard (24 hours for each 2 year period). Enter the recommended # of hours ______

C ❑ The same as the current ASB standard (24 hours for each 2 year period)

D ❑ It should vary based on the categories from question 41 above.

E ❑ There should be no requirement

F ❑ No opinion

Retirement Issues

43. In what year did you retire or do you expect to retire?

____

44. Have you participated in the following CAS activities since retiring or do you plan to participate in the following CAS activities upon retirement? (check all that apply)

a ❑ Committees

b ❑ Meetings/Seminars

c ❑ Other (please write in) ______________________________

d ❑ Don't know

Overall Member Satisfaction

45. As a CAS member, how satisfied are you with the following?

	Very Satisfied				Very Dissatisfied
a) CAS leadership (elected officers)	1	2	3	4	5
b) Committee chairs	1	2	3	4	5
c) CAS staff	1	2	3	4	5
d) Communications/Publications	1	2	3	4	5
e) Meetings/Professional education	1	2	3	4	5
f) Overall satisfaction with CAS	1	2	3	4	5

46. What is the strongest part of the CAS?

____

What is the weakest?

____

47. How can the CAS add more value for its members? What else can the CAS do?

____

Regional Affiliates

48. Indicate the Regional Affiliate(s) and Special Interest Section(s) in which you are active: (check all that apply)

a ☐ Casualty Actuaries of the Bay Area (CABA)
b ☐ Casualty Actuaries of Bermuda (CABER)
c ☐ Casualty Actuaries of Europe (CAE)
d ☐ Casualty Actuaries of the Far East (CAFE)
e ☐ Casualty Actuaries of Desert States (CADS)
f ☐ Casualty Actuaries of Greater New York (CAGNY)
g ☐ Casualty Actuaries of the Mid-Atlantic Region (CAMAR)
h ☐ Casualty Actuaries of New England (CANE)
i ☐ Casualty Actuaries of the Northwest (CANW)
j ☐ Casualty Actuaries of the Southeast (CASE)
k ☐ Central States Actuarial Forum (CSAF)
l ☐ Midwestern Actuarial Forum (MAF)
m ☐ Ontario Conference of Casualty Actuaries (OCCA)
n ☐ Southern California Casualty Actuarial Club (SCCAC)
o ☐ Southwest Actuarial Forum (SWAF)
p ☐ Casualty Actuaries in Regulation (AIR)
q ☐ Casualty Actuaries in Reinsurance (CARe)
r ☐ Not currently active in any regional affiliate or special interest section.

49. If you are NOT an active member/participant of a CAS Regional Affiliate, why not? (check all that apply)

a ☐ I am an active participant.
b ☐ Low relevance of subject matter.
c ☐ Networking opportunity limited due to small number/practice area of attendees.
d ☐ Perceive Regional Affiliate meetings as a "*student's CAS meeting*" with limited value for members.
e ☐ Prefer to go to meetings with more activities and more interesting locations.
f ☐ Don't get timely information regarding the dates and locations of the meetings.
g ☐ Company doesn't encourage or sponsor my attendance.
h ☐ Time and Travel Costs
i ☐ Other (specify)____

50. Considering the difference in time and travel costs, how do Regional Affiliate meetings compare to the other CAS meeting/seminar opportunities? (check one):

a ❑ Regional Affiliate meetings provide significantly less value
b ❑ Regional Affiliate meetings provide somewhat less value
c ❑ Both provide about the same value
d ❑ Regional Affiliate meetings provide somewhat more value
e ❑ Regional Affiliate meetings provide significantly more value

51. Special Interest Sections (currently Casualty Actuaries in Reinsurance (CARe) and Actuaries in Regulation (AIR)) serve the needs of actuaries in particular practice areas. Are there other practice areas in which you feel the CAS should consider forming a Section?

a ❑ Yes, please specify ______________________________
b ❑ No
c ❑ No Opinion

52. If a new special interest section were formed in an area of interest to you, and it had regular meetings requiring travel and time commitment, would you: (check only one)

a ❑ Volunteer to be on the organizing committee for the section and perhaps serve on its Board
b ❑ Definitely go
c ❑ Consider going often
d ❑ Maybe go once in a while
e ❑ Read about it but probably not attend

Admissions - Education - Examinations and Syllabus

53. Please indicate your level of agreement with the statements below.

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
A. Passing CAS exams provides a good foundation for the work that I do.	1	2	3	4	5
B. Some of the CAS educational requirements should be able to be satisfied through college or university credit.	1	2	3	4	5
C. Exams are not an impediment to an actuarial career.	1	2	3	4	5

54. If you practice outside of the United States, what do you perceive as the value of CAS examinations or admission: (check only one)

a ❑ I do not practice outside of the United States.
b ❑ CAS status has no value.
c ❑ CAS status has little value.
d ❑ Uncertain.
e ❑ CAS status has some value.
f ❑ CAS status has a great deal of value (almost equal to or greater than the country's own credentialing).

55. Should the CAS seek greater participation with the academic community with respect to:

	Yes	No	No Opinion
A Research	❑	❑	❑
B Literature (esp. examination readings)	❑	❑	❑
C Training / examination preparation sessions	❑	❑	❑
D Examination structure and design	❑	❑	❑
E Continuing education	❑	❑	❑
F Promoting the profession	❑	❑	❑
G Other ______________________	❑	❑	❑

Casualty Actuarial Society
2003 MEMBERSHIP SURVEY QUESTIONNAIRE

Every five years, the CAS conducts a survey of its membership to determine the needs of the actuarial profession and how those needs can be better met. We appreciate the time and effort you are spending in completing the 2003 survey. All responses to the survey, and the identity of respondents, will be kept in strictest confidence. A full report on the results of the survey will be published in Fall 2003.

We encourage you to complete the survey online by going to the following web site:
www.ari-surveys.com/run/CASMemberB

However, you may also fill it out and fax it back to CAS at *(703) 276-3108*, or send it by mail to:

CASUALTY ACTUARIAL SOCIETY
1100 N. Glebe Road
Suite 600
Arlington, VA 22201

Please complete this survey by ***July 31, 2003.*** *Thank you for your participation.*

Demographics

1. Are you (check only one):
   ❑ Associate ❑ Fellow ❑ Affiliate

2. What were the year(s) you attained your CAS designation(s) or affiliate membership?

   ACAS ________ FCAS ________ Affiliate ________

3. I am:
   ❑ Male ❑ Female

4. Age Range (check one):
   ❑ <20
   ❑ 20 to 25
   ❑ 26 to 30
   ❑ 31 to 35
   ❑ 36 to 40
   ❑ 41 to 45
   ❑ 46 to 50
   ❑ 51 to 55
   ❑ 56 to 60
   ❑ 61 to 65
   ❑ 66 to 70
   ❑ 71 to 75
   ❑ >75

5. What is your business affiliation? (check one)

a. ❑ Insurance company
b. ❑ Consulting actuary
c. ❑ Service company
d. ❑ Regulatory organization
e. ❑ Retired
f. ❑ Reinsurance company
g. ❑ Insurance broker
h. ❑ University or college
i. ❑ Full-time parent
j. ❑ Other

6. Where is your primary place of work?
State/Province: ______________________________
Country: ________________________________

7. Geographic area of your primary business responsibility: (check all that apply)

a. ❑ Worldwide
b. ❑ Africa
Asia (c-g)
c. ❑ Central (e.g., India, Pakistan)
d. ❑ Southeast (e.g. Singapore, Hong Kong)
e. ❑ China
f. ❑ Japan
g. ❑ Other parts of Asia
h. ❑ Australia / New Zealand
i. ❑ Bermuda
j. ❑ Canada
k. ❑ Central America
Europe (l-m)
l. ❑ Eastern Europe
m. ❑ Western Europe
n. ❑ Middle East
o. ❑ South America
p. ❑ United States

8. If you were presented with a job opportunity (i.e., a relocation lasting at least 1 year) outside of your primary place of work (as specified in question # 6) within the next 5 years, what is the likelihood that you would accept it?

Very Likely	Somewhat Likely	Undecided	Somewhat Unlikely	Very Unlikely
a ❑	b ❑	c ❑	d ❑	e ❑

9. I am a member of the following actuarial organizations: (check all that apply)

a ❑ American Academy of Actuaries
b ❑ American Society of Pension Actuaries
c ❑ Conference of Consulting Actuaries
d ❑ Canadian Institute of Actuaries
e ❑ Faculty of Actuaries
f ❑ Institute of Actuaries
g ❑ Institute of Actuaries of Australia
h ❑ International Actuarial Association
i ❑ International Actuarial Association – ASTIN
j ❑ International Actuarial Association – AFIR
k ❑ International Association of Consulting Actuaries
l ❑ Society of Actuaries
m ❑ Other ________________

10. Highest level of academic education completed:

a ❑ HS/GED
b ❑ AA/AS (two-year degree)
c ❑ BA/BS
d ❑ MA/MS
e ❑ MBA
f ❑ JD
g ❑ PhD
h ❑ MD/DDS/Other Medical
i ❑ Other (specify) ____________________

11. Non-actuarial professional designations (check all that apply):

a ❑ ARe
b ❑ ARM
c ❑ AIMR
d ❑ CFA
e ❑ CPA
f ❑ CPCU
g ❑ Other (specify) __________

12. A. Please indicate what percentage of your time over the past two years you have spent in each of the following areas (total should be 100%). B. Please also indicate which of the following roles you've played in your career by checking the box to the right.

Function	(A) Time In/During Past 2 Years Percentage	(B) Over Your Career
A Data Management / Systems Administrator	______	❑
B Risk & Capital Management (e.g., DFA)	______	❑
C Management Advisor	______	❑
D Management of Actuarial Unit	______	❑
E Executive Management	______	❑
F Expert Witness	______	❑
G Investments / Financial Decision Maker	______	❑
H Marketing / Underwriting	______	❑
I Planning - Strategic and Financial	______	❑
J Programming / Software Development	______	❑
K Ratemaking	______	❑
L Reserving	______	❑
M Regulator	______	❑
N Teaching / Researching	______	❑
O Valuation	______	❑
P Reinsurance Pricing	______	❑
Q Other (please write in) ____________________	______	❑
Total	100%	

13. In the last three years, have you served: (check all that apply)

a ❑ On the CAS Board or Executive Council?
b ❑ As Chair of a CAS Committee?
c ❑ As a member of a CAS Committee?
d ❑ On another actuarial organization's Board, Executive Council or Committee?
e ❑ None of the above.

Administration - Electronic Services and Finance

14. How do you access to the Internet?

	Dial-up	Broadband	Do Not Have Access
Home	a ☐	b ☐	c ☐
Work	a ☐	b ☐	c ☐

15. How often do you access the CAS Web Site?

a ☐ Daily
b ☐ More than once per week
c ☐ Once per week
d ☐ Once per month
e ☐ Less than once per month
f ☐ Never accessed it

16. Within the past 12 months, the CAS has offered a live and archived webcast of a portion of the CAS Annual and Spring Meetings.

a. Did you view either of the webcasts live or after? (check only one)
1 ☐ Yes, I viewed it live.
2 ☐ Yes, I viewed it afterward.
3 ☐ No, I chose not to view it.
4 ☐ No, I did not have the proper technology.
5 ☐ No, I did not know it was available.

b. Would you view future webcasts of CAS meetings and seminars? (check only one)
1 ☐ Yes, only if it were free.
2 ☐ Yes, even if there were a cost involved.
3 ☐ No.

c. Would you substitute viewing webcasts of meeting/seminar sessions for in-person attendance at meetings/seminars?
☐ Yes. ☐ No.

17. Currently, the CAS sends out membership e-mails in text format only. In what format would you prefer to receive e-mails form the CAS? (check only one)

a ☐ Text only.
b ☐ HTML (Note: HTML formatted e-mails are more attractive and easier to read, but may be more difficult to receive on some e-mail systems.)
c ☐ No preference.

18. What changes would you like to see on the CAS Web Site?

19. What portion of the following do you pay for personally?

	All	Some	None
A. Dues	❑	❑	❑
B. Meeting fees	❑	❑	❑
C. Volunteer activities	❑	❑	❑

If you were asked to pay for the following yourself, would you?

	Yes	No
A. Dues	❑	❑
B. Meeting fees	❑	❑
C. Volunteer activities	❑	❑

Administration - Publications

20. Please indicate the frequency that you read or reference the following actuarial materials using a scale of 1-5 with 1 being very frequently and 5 being not at all.
For those that you read or reference, rate the quality of each on a scale of 1-5, with 1 being the highest rating.

Periodical	Frequency	Never Read	Quality
a. AAA Publications (e.g., Practice Notes)	1 2 3 4 5	❑	1 2 3 4 5
b. AAA Qualification Standards	1 2 3 4 5	❑	1 2 3 4 5
c. AAA Web Site	1 2 3 4 5	❑	1 2 3 4 5
d. Actuarial Forum	1 2 3 4 5	❑	1 2 3 4 5
e. ARCH - Actuarial Research Clearing House	1 2 3 4 5	❑	1 2 3 4 5
f. The Actuarial Review	1 2 3 4 5	❑	1 2 3 4 5
g. Actuarial Update	1 2 3 4 5	❑	1 2 3 4 5
h. ASB Standards of Practice	1 2 3 4 5	❑	1 2 3 4 5
i. ASTIN Bulletin	1 2 3 4 5	❑	1 2 3 4 5
j. CAS Forum	1 2 3 4 5	❑	1 2 3 4 5
k. CAS Textbooks	1 2 3 4 5	❑	1 2 3 4 5
l. CAS Web Site	1 2 3 4 5	❑	1 2 3 4 5
m. Insurance: Mathematics and Economics	1 2 3 4 5	❑	1 2 3 4 5
n. Journal of Actuarial Practice	1 2 3 4 5	❑	1 2 3 4 5
o. Materials from CAS sponsored meetings	1 2 3 4 5	❑	1 2 3 4 5
p. North American Actuarial Journal	1 2 3 4 5	❑	1 2 3 4 5
q. Proceedings of the Casualty Actuarial Society	1 2 3 4 5	❑	1 2 3 4 5
r. The Actuary	1 2 3 4 5	❑	1 2 3 4 5
s. The Consulting Actuary	1 2 3 4 5	❑	1 2 3 4 5
t. Other #1 (please write in) ________________	1 2 3 4 5	❑	1 2 3 4 5
u. Other #2 (please write in) ________________	1 2 3 4 5	❑	1 2 3 4 5
v. Other #3 (please write in) ________________	1 2 3 4 5	❑	1 2 3 4 5

21. In which format would you prefer to receive the following:

Publication	Hard Copy Only	Web Copy Only	Both
a The Actuarial Review	❑	❑	❑
b Forum	❑	❑	❑
c Discussion Papers	❑	❑	❑
d Continuing Education Catalog / Calendars	❑	❑	❑
e Proceedings	❑	❑	❑
f Meeting Notices	❑	❑	❑
g Syllabus	❑	❑	❑
h Membership Directory section of Yearbook	❑	❑	❑
i Other sections (than Membership Directory) of Yearbook	❑	❑	❑
j Other ____________________	❑	❑	❑

22. In your own actuarial research/continuing education efforts, what relative importance do you place on the following general publication sources? Please rank the following in order of importance to you:

	Very Important	Important	Neutral	Somewhat Important	Not Important
A. Casualty Actuarial Society Publications: *Proceedings, Forum, etc.*	1	2	3	4	5
B. Society of Actuaries Publications: *North American Actuarial Journal, etc.*	1	2	3	4	5
C. *Journal of Actuarial Practice*	1	2	3	4	5
D. International Actuarial Association Publications: *ASTIN Bulletin, etc.*	1	2	3	4	5
E. Insurance: Mathematics and Economics	1	2	3	4	5
F. Other economic, scientific or mathematical publications (*Journal of Finance, Journal of Risk and Insurance, etc.)*	1	2	3	4	5
G. Other ____________________ ____________________ ____________________	1	2	3	4	5

23. How do you view the structure and organization of existing CAS publications?

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
A. The existing CAS publication structure is acceptable, even if it could be improved.	1	2	3	4	5
B. CAS publications need better organization to adequately distinguish between different types of papers (i.e., educational/study notes, pure research, practical applications, short notes, long exhaustive ' thesis', etc.).	1	2	3	4	5
C. Papers should not be published in the same book as CAS meeting minutes/records.	1	2	3	4	5
D. "Study note" papers should not be published in the same book as research papers.	1	2	3	4	5
E. CAS should reconsider sponsoring the *North American Actuarial Journal.*	1	2	3	4	5
F. CAS should consider cosponsoring other actuarial journals.	1	2	3	4	5
G. CAS should retain its own independent fully refereed (each paper is subject to thorough peer reviews) journal.	1	2	3	4	5
H. CAS does not need either its own or co-sponsored fully refereed journal.	1	2	3	4	5

Comments: ____

24. Please describe your interest in writing and submitting papers for CAS publication: (Check all that apply).

a ☐ I have written papers for CAS publications in the past.
b ☐ I am interested and plan to submit papers for CAS publication in the future.
c ☐ I (would) prefer to publish papers in the *Proceedings* because it is fully refereed.
d ☐ I (would) prefer to avoid the burden of review by the Committee on Review of Papers and submit papers only to call paper programs or the *Forum* directly.
e ☐ I have less interest in writing papers than in other CAS activities.
f ☐ I'm unsure whether I'm qualified to write papers sufficient for CAS publication.
g ☐ I have no interest in writing papers for future CAS publication.

Comments: ____

Research and Development

25. Please indicate your level of agreement with the statements below:

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
A. I have used research in my work that was completed by or sponsored by the CAS.	1	2	3	4	5
B. I view CAS research as a valuable resource when I have specific problems to address.	1	2	3	4	5
C. I seek out CAS research information only in response to specific job assignments.	1	2	3	4	5
D. I am well aware of most of the research done by and sponsored by the CAS.	1	2	3	4	5
E. CAS sponsored research is generally responsive to my needs as a practicing actuary.	1	2	3	4	5

26. To what extent do the following prevent you from employing the results of recent CAS research in your work:

	Strongly Agree				Strongly Disagree
a. Too many assumptions need to be made	1	2	3	4	5
b. They are too difficult to explain to non-technical audiences	1	2	3	4	5
c. The required data is usually not available	1	2	3	4	5
d. They are too expensive to use in practice	1	2	3	4	5
e. I am not aware of recent research in my area of practice	1	2	3	4	5
f. The techniques are not practical enough to use in practice	1	2	3	4	5
g. I'm not sure they produce better results	1	2	3	4	5
h. Auditor, regulators, etc. may not accept these approaches	1	2	3	4	5
i. My management, or my clients, like the way it's done now	1	2	3	4	5
j. I like my current methods	1	2	3	4	5
k. I do use the latest techniques	1	2	3	4	5

27. How would you like to access the results or products of CAS research?
(Check all that apply):

a. ❑ CAS Web Site
b. ❑ CAS *Forum* and call paper publications
c. ❑ CAS Meetings and Seminars (including Regional Affiliates)
d. ❑ Online bibliographies
e. ❑ CAS *Proceedings*
f. ❑ Regular section of the *Actuarial Review*
g. ❑ Online searchable database of abstracts, with links to full texts of papers.
h. ❑ Quarterly research newsletter
i. ❑ Other (please describe) ______________________________

International

28. Have you ever had the need for recognition from an actuarial society other than one in which you were already a member?

❑ Yes ❑ No

If yes, what was the reason recognition was necessary? ______________________________
In what country(ies)? ______________________________

29. How often do you travel internationally?

	More than 4 time per year	1-4 times per year	Less than once a year	Never
For business	❑	❑	❑	❑
For pleasure	❑	❑	❑	❑

30. Recognizing that financial and human resources are required, in which areas should the CAS be actively working to support the development of the actuarial profession in countries where the profession is in the development stages? Rate each of the following using a scale from 1–5 with 1 being very important and 5 being not important at all. If you have no opinion, please indicate so by selecting #6.

	Very Important				Not Important At All	No Opinion
A. Subsidize the registration and travel cost for actuaries and academics from these countries to attend CAS meetings and seminars (i.e., in North America).	1	2	3	4	5	6
B. Subsidize the registration and travel cost for actuaries and academics from these countries to speak at CAS meetings and seminars (i.e., in North America).	1	2	3	4	5	6
C. Offer discounted CAS dues.	1	2	3	4	5	6
D. Work with local regulators, policymakers, and actuarial bodies to gain official recognition of the CAS credential in various jurisdictions.	1	2	3	4	5	6
E. Use CAS funds to help finance the efforts of organizations such as the International Actuarial Association to support the development of the actuarial profession in these countries.	1	2	3	4	5	6
F. Provide crucial casualty actuarial literature through the CAS Web Site and links to other Web sites.	1	2	3	4	5	6
G. Create an international referral service whereby foreign actuaries could ask specific questions and be referred to CAS volunteers for comment on North American approaches to similar issues.	1	2	3	4	5	6
H. Establish ambassadors or liaisons to cooperate with other international actuarial societies on matters involving casualty areas outside of North America.	1	2	3	4	5	6
I. Actively participate in the International Actuarial Association (IAA).	1	2	3	4	5	6
J. Other__________	1	2	3	4	5	6

Volunteerism

31. Is there anything that is preventing you from increasing your participation on CAS committees/task forces (Check all that apply)?
    - a ❑ No limitation
    - b ❑ Cost
    - c ❑ Time
    - d ❑ Lack of interest at this time
    - e ❑ Not supported by my employer
    - f ❑ Other (Please describe) ______________________________

32. Would you volunteer more if your travel costs were subsidized?

    ❑ Yes ❑ No

Governance – Elections

Questions 31-34 are to be answered by Fellows only

33. Did you vote in the last CAS election?
    - a ❑ Yes, I cast votes for all offices.
    - b ❑ Yes, I cast votes for some, but not all of the offices.
    - c ❑ No
    - d ❑ I can't remember

34. If the answer to the above question was either b or c, what was the reason for not voting for all offices? (check all that apply)
    - a ❑ I did not agree with the positions of the candidates.
    - b ❑ I did not have sufficient knowledge of the candidates.
    - c ❑ Other ______________________________

35. Did you read the "Meet the Candidates" material on the CAS Web Site for the last election?
    - a ❑ Yes, I found the material helpful in making my choices.
    - b ❑ Yes, but the material was not helpful.
    - c ❑ No
    - d ❑ I was unaware that this material was on the CAS Web Site.

36. The CAS made several changes to the election process in 2002, including the process for nominating candidates. Do you feel that these changes will improve the governance of the Casualty Actuarial Society?
    - a ❑ The changes will significantly improve the governance of the CAS
    - b ❑ The changes will somewhat improve the governance of the CAS
    - c ❑ The changes will have no impact on the governance of the CAS
    - d ❑ Uncertain
    - e ❑ I was not aware of any changes.

37. Should Associates be allowed to vote in elections for CAS officers?

a ☐ No
b ☐ Yes, immediately upon achieving ACAS.
c ☐ Yes, after a period of 1-3 years
d ☐ Yes, after a period of 4-5 years
e ☐ Yes, after a period of 6-9 years
f ☐ Yes, after a period of 10 or more years

Please share any comments you may have relative to ACAS voting rights.

The Actuarial Profession

38. During your actuarial career, How many...

Distinct jobs have you held? _______
Employers have you worked for (including self)? _______

39. Which of the following do you consider to be important to the long-term job security (or demand) for property-casualty actuaries? Please check all that apply:

a. ☐ Expansion of P&C actuarial experience outside of the insurance industry
b. ☐ Knowledge of global issues
c. ☐ Better communication and business skills
d. ☐ Application of actuarial skills to other types of risk such as operational and strategic risk.
e. ☐ All of the above.

40. Employment opportunities for CAS members are (choose only one)

a. ☐ Increasing faster than CAS membership
b. ☐ Increasing at about the same rate as CAS membership
c. ☐ Increasing more slowly than CAS membership
d. ☐ Don't know

41. The CAS is always looking for new areas where we can expand actuarial practice. Please list any suggestions for areas to expand practice.

If you do not practice in the United States, please skip questions 40-43.

42. In the course of your practice, do you sign prescribed statements of actuarial opinion?

- ❑ Yes
- ❑ No

43. Do you meet the **general** qualifications standards for prescribed statements of actuarial opinion? (a statement of actuarial opinion issued for purposes of compliance with law or regulation or compliance with Actuarial Standards of Practices as promulgated by the Actuarial Standards Board or an Accounting Standards Board.)

- ❑ Yes
- ❑ No
- ❑ Don't Know

44. Do you function as the appointed actuary for one or more US-domiciled property & casualty insurance companies?

- ❑ Yes
- ❑ No

45. Do you meet the **specific** qualification standard for statements of opinion, NAIC Property & Casualty Annual Statement?

- ❑ Yes
- ❑ No
- ❑ Don't know

Retirement Issues

46. In what year did you retire or do you expect to retire? ________________

47. Have you participated in the following CAS activities since retiring or do you plan to participate in the following CAS activities upon retirement? (check all that apply)

a ❑ Committees
b ❑ Meetings/Seminars
c ❑ Other (please write in) ____________________________________
d ❑ Don't know

Overall Member Satisfaction

48. How satisfied are you with the following as a CAS member?

	Very Satisfied				Very Dissatisfied
a) CAS leadership (elected officers)	1	2	3	4	5
b) Committee chairs	1	2	3	4	5
c) CAS staff	1	2	3	4	5
d) Communications/Publications	1	2	3	4	5
e) Meetings/Professional education	1	2	3	4	5
f) Overall satisfaction with CAS	1	2	3	4	5

49. What is the strongest part of the CAS?

____

What is the weakest?

____

50. How can the CAS add more value for its members? What else can the CAS do?

____

Admissions - Education - Examinations and Syllabus

51. Actuaries practicing in casualty (general, non-life) insurance outside of the United States should be able to satisfy educational requirements for CAS membership by: (choose one)

a ☐ Not at all.
b ☐ Satisfying current requirements for US candidates (first seven examinations).
c ☐ Being credentialed in the actuary's home country and passing one or two CAS specific examinations.
d ☐ Being credentialed in the actuary's home country.
e ☐ Automatically.

52. How would you assess the current supply of qualified candidates entering the actuarial profession? (check only one)

a ☐ Far too few
b ☐ Not enough
c ☐ A sufficient supply exists
d ☐ Too many

Professional Education

53. How many hours of continuing education have you completed in the last three years?

_______ Organized Activities (e.g., attendance at meetings or seminars)
_______ Other Activities (e.g., reading research articles)

54. Where would you prefer to receive education on General Business Skills? (check all that apply)

a ☐ CAS Meetings
b ☐ CAS Seminars
c ☐ Regional Affiliate Meetings
d ☐ Limited Attendance Seminars
e ☐ In my own Company
f ☐ Suggested reading list (books, articles, etc.)
g ☐ Desktop application learning tools (Web-based or CD-ROM based)
h ☐ Not interested in education in General Business Skills
i ☐ Other (Please Specify) ______________________________

55. What General Business Skills topics would you be interested in attending if offered at future CAS meetings? (check all that apply)

	Not interested	Yes, if included in meeting registration fee	Yes, even if it requires an additional fee
a. Writing Skills	❑	❑	❑
b. Negotiation Skills	❑	❑	❑
c. Project Management Skills	❑	❑	❑
d. Strategic Thinking	❑	❑	❑
e. Marketing/Networking	❑	❑	❑
f. Survey Writing	❑	❑	❑
g. "Working with Others"	❑	❑	❑
h. Other (Please Specify) ______________________	❑	❑	❑

Exposure Dependent Modeling of Percent of Ultimate Loss Development Curves

Ira Robbin, Ph.D.

EXPOSURE DEPENDENT MODELING

OF

PERCENT OF ULTIMATE LOSS DEVELOPMENT CURVES

by

IRA ROBBIN, PhD

Abstract

This paper presents a loss development model in which exposure period dependence is fundamental to the structure of the model. The basic idea is that an exposure period, such as an accident year or policy year, gives rise to a particular distribution of accident date lags, where the accident date lag is the time elapsed from the start of the exposure period till the accident date. The paper shows how to derive the density of the accident date lag from a familiar parallelogram diagram. A fairly general theory of development is then presented and simplified under certain conditions to arrive at a total development random variable whose cumulative distribution is related to the usual percent of ultimate development curve. After presenting the theory, the paper turns to practical applications. Simulation is used to generate consistent patterns for different exposure periods. A convenient accident period development formula is derived and then used to fit and convert factors. The average date of loss approximation is generalized. To summarize, this paper will demonstrate that modeling loss development with exposure dependent percent of ultimate curves is a theoretically sound procedure with many practical uses.

1. INTRODUCTION

A key step in the usual procedure for modeling a loss development pattern is to fit formulas to empirical age-to-age or age-to-ultimate factors. Having a fitted formula is useful because it provides an easy way to smooth the bumps found in most series of empirical factors. Also, if the fit is to age-to-ultimate factors, the formula usually provides a convenient way to interpolate the factors.

While the fitting is convenient and practical, it can hardly be said to have a substantive conceptual foundation. A formula is chosen because it is easy to compute and because it nicely fits the age-to-age factors. It is not derived from more basic assumptions in the sense that nothing is specifically built in to reflect that it is being fitted to data that represent ratios of loss for a particular exposure period as of given evaluation ages.

While a formula serves perfectly well for smoothing, it may not suffice, in and of itself, to handle other applications such as tail factor extrapolation, early age extrapolation or conversion of the factors from one exposure basis to another. Tail factor extrapolation is needed to get age-to-ultimate factors after a fit is obtained to age-to-age factors. Yet, an age-to-age factor formula may not immediately lead to the extrapolation. To obtain the desired age-to-ultimate factors the actuary may have to derive the product of an infinite series, make cut-off assumptions, or use a computerized numerical algorithm.

In early age extrapolation, the actuary is seeking factors at an evaluation age younger than the earliest evaluation age associated with the fitted factors. For example, the actuary may have accident year age-to-ultimate factors for evaluations at 12, 24, 36... months, yet may need to have factors at 6, 18, 30, ...months. The problem is that the back extrapolation of a formula fit may or may not yield plausible results at earlier ages (i.e. the factor at 6 months). Some additional techniques may be needed to get reasonable factors at these ages.

Finally with regard to conversion, the actuary may have fitted accident year factors, but may want to have policy year factors. Yet a good fit to accident year factors may not directly lead to a good fit to the corresponding policy year factors. Actuaries have usually dealt with this conversion problem by using an average date of loss adjustment. Under this adjustment, the development factor for one type of exposure period at a given evaluation age is estimated by the development factor for the original type of exposure period at an adjusted evaluation age. The adjustment is equal to the difference in the average dates of loss for the different exposure periods. While this adjustment works well at mature ages after all exposures are earned, it goes awry at immature evaluation ages.

The conclusion is that fitting with general formulas is a useful and flexible approach that must often be supplemented for extrapolation and conversion applications. The supplemental procedures may not be too difficult to implement. So, in the end, from a practical perspective, not too much should be made of the need to introduce them. However, it would be more convenient to have a model of loss development that would automatically handle extrapolation and conversion. Such a model would not start with a formula for age-to-age factors, but would instead be based on percent of ultimate or age-to-ultimate curves having an explicit dependence on the underlying exposure period.

Models such as this have been previously proposed. Yet they have not been widely adopted. Why? We speculate the reluctance stems from two essential areas of concern. First, there may be questions about the theoretical underpinnings of such models. Second, there may be doubts about whether the proposed models are practical.

In order to address these concerns, we will present a general, yet accessible, conceptual foundation for exposure dependent percent of ultimate models. We will start by relating an exposure period, such as an accident year or policy year, to an associated distribution of accident date lags. The accident date lag for a claim is defined as the length of time from the start of the exposure period to the accident date. We will show that the familiar parallelogram or rectangle diagram representation of an exposure period can be readily converted into a graph of the density of this accident date lag random variable. The cumulative distribution of the accident date lag may be identified with the percent of premium earned to date assuming the earning of premium corresponds exactly to the exposure to loss. We will argue that under certain conditions the percent of ultimate loss development curve may be expressed as the cumulative distribution of the sum of the accident date lag random variable plus another random variable that summarizes the claims process. The claims process in this context includes the delay between the accident date and report date, as well as the changes in the valuation of a claim and the time lags between these valuation changes. Perhaps the key insight underlying this construction is that exposure dependence can be isolated in the accident lag distribution.

We will then turn to applications. We will use the model to simulate patterns for different exposure periods, derive a convenient accident period development formula, fit and convert patterns, extend the average date of loss approximation, and approximate a converted pattern as the weighted sum of shifted versions of the original pattern. In the end we will hope to have shown that exposure dependent percent of ultimate models are not only pleasing to the theorist, but also useful to the practical actuary.

2. EXPOSURE MODELING

We start by establishing the key concept that an exposure period is defined by a distribution of accident date lags, where an accident date lag is the length of time from the start of an exposure period until an accident occurs.

To state this mathematically, define:

- W = Exposure Random Variable = Accident Date Lag

  = accident date - date of start of exposure period (2.1)

We identify the cumulative distribution of W with the percentage of exposure earned to date and sometimes write:

$$F_W(w) = ETD_W(w) \quad (2.2)$$

The assumption here is that the earning of premium corresponds exactly with exposure to accidents so that the percent of premium earned as of a given date equals the expected percent of accidents that have occurred by that date.

It is easy to define the accident date lag distributions for the most commonly encountered exposure periods. For an accident year under the usual uniformity assumptions, the exposure random variable is a uniform random variable.

$$f_{AY}(w) = \begin{cases} 1 & \text{for } 0 < w < 1 \\ 0 & \text{otherwise} \end{cases} \tag{2.3a}$$

$$F_{AY}(w) = \begin{cases} w & \text{for } 0 < w < 1 \\ 1 & \text{for } w \geq 1 \end{cases} \tag{2.3b}$$

The policy year exposure random variable has density that increases linearly for one year and then decreases linearly for the second year.

$$f_{PY}(w) = \begin{cases} w & \text{for } 0 < w < 1 \\ 2 - w & \text{for } 1 \leq w < 2 \\ 0 & \text{otherwise} \end{cases} \tag{2.4a}$$

$$F_{PY}(w) = \begin{cases} \frac{w^2}{2} & \text{for } 0 < w < 1 \\ \left(\frac{1}{2} + \frac{1-(2-w)^2}{2}\right) & \text{for } 1 \leq w < 2 \\ 1 & \text{for } w \geq 2 \end{cases} \tag{2.4b}$$

Though it may appear initially a bit different, this view of an exposure period as being synonymous with a distribution of accident date lags is equivalent to the standard actuarial approach involving rectangles and parallelograms. It is generally straightforward to convert these geometric objects into the density of the exposure random variable defined here. The idea is to collapse the parallelogram down towards the "x-axis" and then normalize so that the area under the curve is unity.

For example, consider how the policy year parallelogram in Figures 1 can be collapsed to yield the policy year density shown in Figure 2.

Figure 1

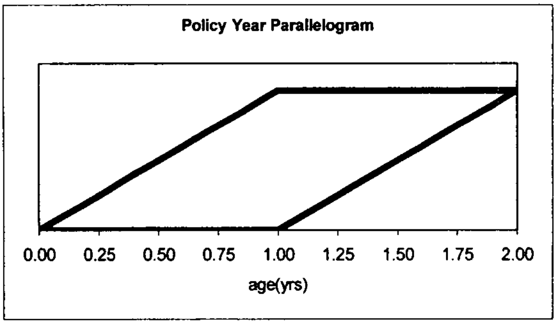
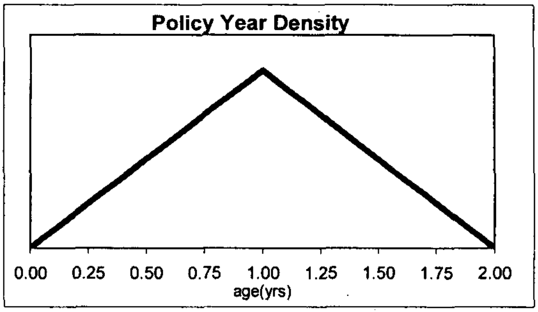


Figure 2



Similarly the policy quarter parallelogram in Figures 3 is readily converted to the policy quarter density shown in Figure 4. The policy quarter density is typical of policy periods: the density starts with an exposure growth triangle, then reaches an exposure plateau, and finally ends with an exposure decay triangle

Figure 3

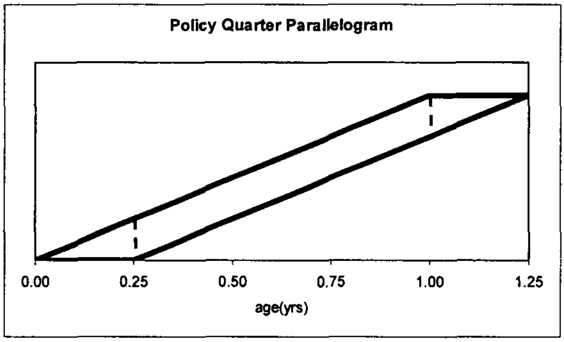
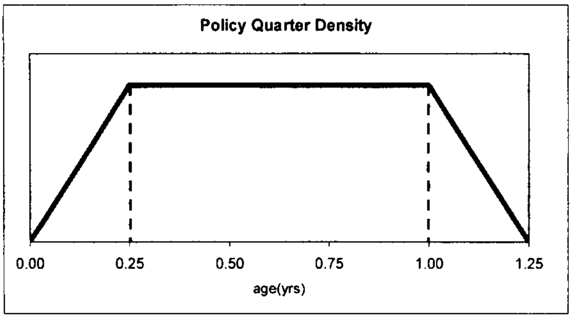


Figure 4



To summarize, the accident date lag for an exposure period is a random variable that captures differences between different types of exposure periods. The density of this random variable may be easily constructed from the parallelogram diagrams with which actuaries are familiar. To put it in other words, we start our exposure dependent development model by characterizing different exposure periods by their Earned to Date functions.

3. MODELING THE CLAIMS PROCESS

Next we model the development of a claim after the original accident has occurred. We model this development with a series of paired random variables, where each pair in the series describes a step in the claims development process. Each pair consists of:

- a time lag random variable that measures the time since the previous step and,
- an amount change random variable that equals the change in the value of the claim at that step.

After the accident has occurred, the first step in the claim process is that the claim is reported. The length of time between the accident date and report date is called the report lag. If we are interested in development of case incurred losses, the amount change variables will measure changes in the case incurred loss. If we are looking at paid development, the amount changes will equal payments made a various points in time as defined by the lags.

To describe this in general mathematical terms, we define:

- M = Number of steps (3.1)
- $\Delta V(i)$ = Process lag at the i^{th} step (3.2a)

  = the time between $(i-1)^{st}$ step and the i^{th} step

  (where the 1^{st} step is the report lag)
- $V(i)$ = Total lag since the claim occurred = $\Delta V(1)+\Delta V(2)\ldots+\Delta V(i)$ (3.2b)
- $\Delta A(i)$ = Change in the amount of a claim at the i^{th} step (3.3a)
- $A(i)$ = Claim amount after the i^{th} step = $\Delta A(1)+\Delta A(2)\ldots+\Delta A(i)$ (3.3b)

Diagrams can be helpful in understanding the definitions of these variables. Figure 5 depicts the lag variables in a claim count development model

Figure 5

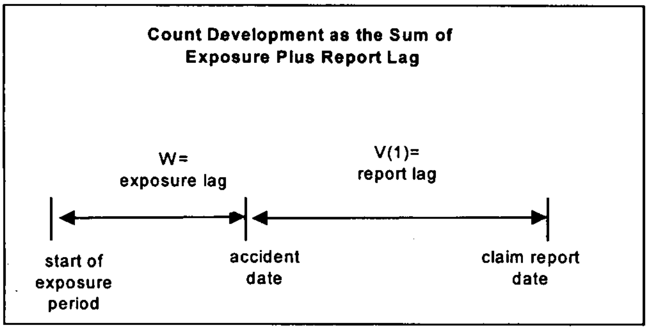
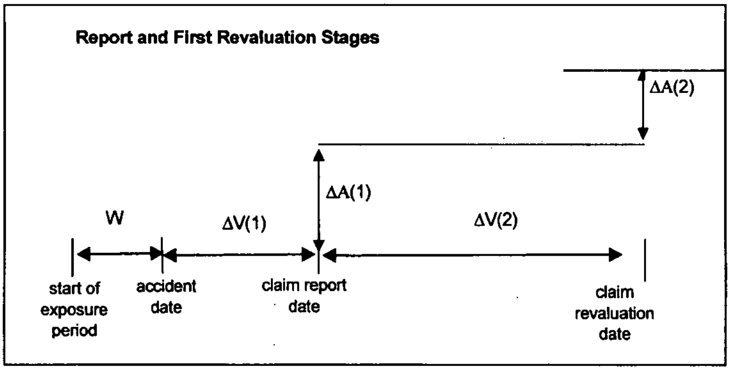


Figure 6 shows the lag and amount change variables for the claim reporting and first revaluation stages of a claim.

Figure 6



We now use the time lags to define a function, B(t), which is the claim amount expressed as a function of the time, t, that has elapsed since the accident.

$$B(t) = \begin{cases} 0 & \text{if } t < V(1) \\ A(i) & \text{if } \; V(i) \le t < V(i+1) \text{ for } i = 1,2,\ldots M-1 \\ A(M) & \text{if } \; V(M) \le t \end{cases} \qquad (3.4)$$

Now we define P(t) as the ratio of the expected value of B(t) over the expected ultimate value of B.

$$P(t) = \frac{E[B(t)]}{E[B(\infty)]} \qquad (3.5)$$

While the diagrams can be drawn for as many transitions as necessary, it is clear that the final evaluation of E[B(t)] could become fairly messy. One would need assumptions on the distribution of the number of revaluations a claim will undergo. One would also need assumptions about the distributions of the lags and the amount changes. Further, in general, the number of steps, the length of the lags, and the amount of the changes might not be independent of one another. Rather than try to evaluate all full model in detail, we will first attempt to simplify it.

As preparation for simplifying the model, we first note that in the general case some of the amount change variables could well be negative or even have a negative expectation. We have allowed this because we want a model that could handle negative development such as can arise from downward reserve revaluations, closing of claims without payment, salvage and subrogation, and other factors.

However, if we now restrict the model and assume that all of the amount change variables must be non-negative, it will follow that B(t) is an increasing function of t and that E[B(t)] is increasing as well. We can therefore conclude that P(t) is an increasing function between zero and unity that tends to unity as time approaches infinity. Thus P(t) is the cumulative distribution of some random variable. We call this random variable the Process Lag and denote it as S. Sometimes we may write $F_S(t)$ in place of P(t). Observe that S effectively summarizes the amount change and step lag random variables that describe the development of claims after their accident dates. It is the existence of this single Process Lag that allows us to simplify the model.

Before going further with our simplified model, we first observe that under these definitions the Report Lag (from accident date to report date) is included in the Process Lag. We also observe that the Process Lag distribution defined here is equivalent to the percent of ultimate loss development pattern for loss on an exposure of infinitesimal duration as given in Robbin and Homer [4] and similar functions defined in Brosius [1], Philbrick [3], and Wiser [6].

4. EXPOSURE DEPENDENT DEVELOPMENT

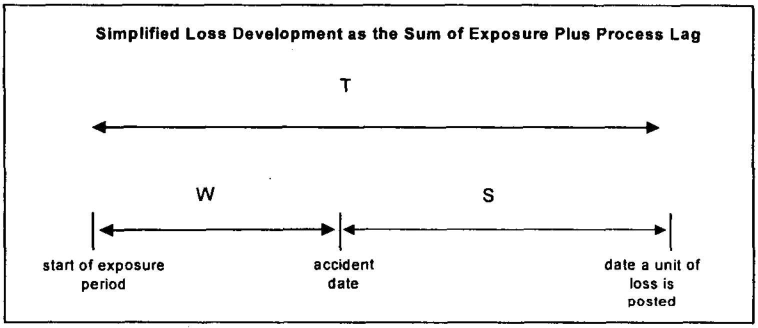
We now add the Accident Date Lag to the Process Lag to obtain the Total Lag for exposure period loss development.

Define:

- T_W = Total Lag = W+S (4.1)

We may view T as the difference between the start of the exposure period and the date a unit of loss is posted on the books. The term, "unit of loss", is here meant to be a general term that could apply to claim counts reported, loss dollars incurred, loss dollars paid or other quantities that actuaries display in triangles. The random variables are shown in the diagram in Figure 7.

Figure 7



In principle, the claims reporting and settlement process should not depend on how the claims are grouped into exposure period buckets. We formalize this by assuming that W and S are independent. As necessary, we index the total lag distribution, T, by W to indicate its dependence on the exposures.

We next make the critical observation that the cumulative distribution of T_W is the same as the percent of ultimate curve for losses arising from the exposures specified by W. Let $PCT_W(t)$ denote the expected percent of ultimate for losses arising from exposures given by accident date lag W as of time, t, since the start of the exposure period. Our observation is mathematically expressed by the equation:

$$PCT_W(t) = F_{W+S}(t) = F_{T_W}(t) \qquad (4.3)$$

For example, in a claim reporting model, let N(t) be the number of claims reported as of time, t, and let N(ult) be the ultimate number of claims. The report date measured from the start of the exposure period can be regarded as a sample of the random variable, T_W. It follows that N(t) will be binomially distributed with parameters, N(ult) and $F_{T_W}(t)$. Thus E[N(t)] = N(ult) $F_{T_W}(t)$ and it follows that $PCT_W(t) = F_{T_W}(t)$. For example, if the percent of ultimate curve is at 60% as of a particular evaluation age, then if we look at the total report lags for a sufficiently large set of claims, we will find that 60% of these lags are less than or equal to the given evaluation age.

In general, the loss development factor from age t to ultimate is given as the inverse of the percent of ultimate. We can thus relate standard age-to-ultimate factors to the inverse of the cumulative distribution of the Total Lag:

$$AULDF_W(t) = \frac{1}{PCT_W(t)} = \frac{1}{F_{T_W}(t)} \tag{4.4}$$

Assuming W and S are independent, it is known that the cumulative distribution of their sum is given as a convolution integral. Thus we can write:

$$F_{T_W}(t) = \int_0^t dw\, f_W(w) \cdot F_S(t-w) = \int_0^t dw\, f_W(w) \cdot P(t-w) \tag{4.5}$$

This is equivalent to percent of ultimate loss development formulas seen in the literature (Robbin and Homer [4], Brosius[1], and Philbrick [3]). What we have done here is base the formula on well-defined random variables. The derivation is based on the assumption the underlying amount change random variables were all non-negative. Later, we will relax this assumption, but for now we see that it is critical, for it allows us to summarize all the changes a claim undergoes with a single process random variable

Next, we will use our Exposure Lag plus summarized Process Lag model to directly simulate loss development patterns.

5. SIMULATION

A big advantage in having a development model based on process and exposure random variables is that we may simulate these variables and thereby generate loss development patterns. Given any non-negative random variable as a model for S and a particular exposure period with accident lag random variable, W, we can use simulation models to quickly generate a few thousand samples of S and W. With these, we can compute the cumulative distribution of T=S+W at various evaluation ages. By retaining our original set of simulated process lags and using a different exposure random variable, we can see how the development pattern changes in response to a change in the underlying exposures.

Exhibit 1 provides a small sample demonstration of the procedure. The accident year and policy year patterns shown in the exhibit were generated from the sample of 20 random trials listed in Sheet 2 of Exhibit 1. The Process was assumed to follow a Pareto distribution with shape parameter equal to 2.0. Given the extremely small sample size, it is no surprise these simulated patterns differ significantly from the true patterns displayed in the exhibit. The small sample size was used so the reader could follow the computation of the percent of ultimate from the simulated values. Much larger samples would be required in any real application. The formulas for the true patterns are shown in Appendix A. A more realistic sample size of 2,000 was used to generate the simulated patterns displayed in Exhibit 2. These fit the true formula-generated patterns quite nicely.

When applying this simulation technique to actual problems, the required sample size ought to be large enough to guarantee that the simulated percent of ultimate values or incremental percentages are highly likely to fall within a desired tolerance. A binomial test can be applied using the normal approximation to the binomial in order to estimate this requisite sample size. Simulations run with that sample size will still typically yield age-to-age patterns with small statistical fluctuations. To get a smoother curve requires a larger sample size.

In practice, if we have a model for the Process Lag that generates simulated factors that closely match given accident year factors, we can reuse the simulated values of the process variable to generate the factors for another exposure period. To do this we simply add each previously simulated process lag to a simulated accident lag for the other exposure period. Since the simulated Total Lag for the accident year already fits the accident year pattern, the simulated Total Lag for the other exposure period should also be reasonably close to its true value.

Simulation provides a powerful all-purpose tool for solving problems using the exposure dependent model. It may be especially useful when trying to estimate development patters for an irregular exposure period. For example, we could use simulation to estimate development patterns on a risks attaching reinsurance contract covering a mix of 3 month and 12 month term policies where the contract was cut-off so that it only covers accidents occurring during the first 12 months. We will next derive a formula for accident period development and use it as the basis for other application techniques.

6. A SIMPLE ACCIDENT PERIOD DEVELOPMENT FORMULA

Though the convolution integral formula 4.5 may initially look forbidding, it reduces to a quite tractable formula when applied to accident period exposures. For a uniform accident period of duration, D, the cumulative distribution and density of the accident lag variable, W, are given as:

$$F_{A(D)}(w) = \frac{w}{D} \text{ for } 0 < w < D \tag{6.1a}$$

$$f_{A(D)}(w) = \frac{1}{D} \text{ for } 0 < w < D \tag{6.1b}$$

Here for clarity we have written A(D) instead of W when subscripting the cumulative distribution and density. The cumulative distribution for the loss development pattern generated from a uniform accident period is thus given as:

$$F_{T_{A(D)}}(t) = \int_0^{\min(t,D)} dw \frac{1}{D} \cdot F_S(t-w) = \int_0^{\min(t,D)} dw \frac{1}{D} \cdot (1 - G_S(t-w)) \tag{6.2}$$

where G denotes the tail probability.

We simplify this percent of ultimate formula using the fact that the integral of the tail probability is the limited expected value:

If t<D:

$$F_{T_{A(D)}}(t) = \frac{t}{D} - \int_0^t dw \frac{1}{D} \cdot G_S(t-w) = \frac{t}{D} - \frac{1}{D}\int_0^t du G_S(u) = \frac{t}{D} - \frac{E[S;t]}{D} \qquad (6.3a)$$

If t>D:

$$F_{T_{A(D)}}(t) = \frac{D}{D} - \int_0^D dw \frac{1}{D} \cdot G_S(t-w) = 1 - \frac{1}{D}\int_{t-D}^t du G_S(u)$$

$$= \quad 1 - \frac{E[S;t] - E[S;t-D]}{D} \qquad (6.3b)$$

In these formulas, E[S;s] is the limited expected value of S at s. Limited expected value formulas for many distributions are given in various books on loss distributions and statistics [2]. With 6.3, we can then use any one of these to generate consistent accident period curves for accident periods of different duration.

7. CURVE FITTING AND CONVERSION

The accident period development formula can be readily applied to fitting accident year-by-year data. After fitting some data, we will then use the formula to generate the associated accident quarter-by-quarter development pattern.

We will fit age-to-age factors using three different parametric distributions: the Pareto, the Gamma and a two-parameter form of the Burr. The limited expected value functions are as follows:

Pareto: $$E[S;s] = \mu \cdot \left(1 - \left(\frac{\mu(\alpha-1)}{\mu(\alpha-1)+s}\right)^{\alpha-1}\right) \qquad (7.1)$$

Gamma: $$E[S;s] = \mu \cdot \Gamma(s \mid \alpha+1, \frac{\mu}{\alpha}) + s \cdot \left(1 - \Gamma(s \mid \alpha, \frac{\mu}{\alpha})\right) \qquad (7.2)$$

Two Parameter Burr: $$E[S;s] = s \cdot \left(1 + \left(\frac{s}{\mu}\right)^{\alpha}\right)^{-1/\alpha} \qquad (7.3)$$

We have parameterized all of these so they have two parameters: μ, the mean, and α, the shape. It is the experience of the author that numerical fitting routines often work better if the mean is isolated as a single parameter. The reader can find sources (see Hogg and Klugman, [2]) for all of these except for the two-parameter form of the Burr. To illustrate how accident year percent of ultimate values would be derived for this modified Burr distribution, let $\alpha = 1$ and $\mu = 2$. We compute E[S;1] = 2/3, E[S;2] = 1, and E[S;3] = 3*(1+3/2) $^{-1}$ = 6/5. Using formula 7.3, this yields percent of ultimate values of 33.10%, 66.7%, and 80.0% at the end of the first three years respectively.

Next, we use these limited expected value formulas to derive age-to-age factors and fit them to one set of age-to-age factor data shown in the Sherman's paper [5]. The results are shown in Exhibit 3. The fits were obtained so as to minimize the sum of square errors in the running back-products of the age-to-age factors. Other fitting criteria could be used, but this one is easy to program. Also, it naturally assigns more weight to the shape of the tail of the available data and seems more forgiving if there happens to be a strange factor or two in the data. Sherman's fit with a power curve is shown for comparison. Reviewing our results, we see the Burr fit is good, the Pareto fit is fair, and the Gamma fit is not good. Perhaps the Gamma would fare better with a different fitting criterion, or perhaps this curve form just does not fit the data. In any event, the Burr fit is arguably as good as that obtained by Sherman using the power curve. However, the conclusion from the example is <u>not</u> that the Two-parameter Burr fits better than the power curve or that the exposure dependent percent of ultimate model does a better job of fitting the factors. It merely demonstrates that the exposure dependent model is practical and can produce good fits. In real applications, it would be advisable to look at more than three curves and to try different fitting criteria.

While the exposure dependent model has no advantage over pure curve formulas in fitting a given set of development factors, some advantages come to light after the fit is obtained. Suppose we have just fitted accident year-by-year age-to-age factors. With our model, we automatically get the resulting age-to-ultimate factors. With a power curve or other age-to-age formula, one may have to posit an arbitrary cut-off age. This difficulty arises because the product of the infinite series of formula generated age-to-age factors may be infinite or at least difficult to compute. The root of the problem stems from viewing the age-to-age factors as a series of numbers, instead of deriving them from a percent of ultimate curve, as was done in our model. Second, with our model, interpolation is easy. One can simply compute limited expected values at requisite intermediate ages and use them to compute the percent of ultimate curve at the desired evaluation ages. With the pure curve fitting approaches, interpolation may entail rebalancing and refitting procedures [5]. Another advantage of our model is that we can quickly generate the associated accident quarter-by-quarter factors. These are shown in Exhibit 4 for the Burr fit in our example. The pure curve fitting methods run into difficulty with this problem [5], whereas our model handles it with ease precisely because dependence on the exposure period is built in from the start.

8. AVERAGE MATURITY OF LOSS APPROXIMATION

Next we generalize and extend the usual average date of loss approximation so that it handles immature evaluation ages. We call the generalization the Average Maturity of Loss Approximation. Under the average date of loss approximation, loss development for one exposure period as of a given evaluation age is estimated by the development for another exposure period at an adjusted age. The adjustment is equal to the difference between the average dates of loss for the two exposure periods.

To express this mathematically, let W be an exposure random variable and define $\mu_W = E[W]$ as its average date of loss. Given another exposure random variable, W*, we define the average date of loss approximation of W* using W via:

$$PCT^*(t^*) \approx PCT(t^* + \mu_W - \mu_{W^*}) \tag{8.1}$$

Here PCT denotes the percent of ultimate loss.

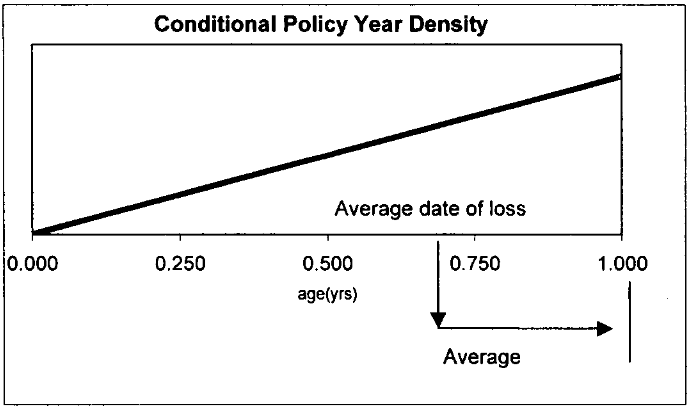
For example, if W represents uniform accident year exposure and W* is the exposure variable for a policy year, then $\mu_W = 6$ months, $\mu_{W^*} = 12$ months and we approximate the policy year using the accident year factor at the age six months earlier. For evaluation ages greater than two years, the approximation has some error but is not unreasonable. It becomes fairly accurate at ages above three years. However, for ages less than two years, the logic of the fixed six-month shift breaks down and for ages below six months the shift fails to yield an answer at all.

Following Robbin and Homer [4], we extend the approximation so that it works at immature ages by first defining the conditional average date of loss, $\mu_W(t)$ =E[W| W< t]. We next the define the average maturity of loss, $m_W(t)$, via:

$$m_W(t) = t - \mu_W(t). \tag{8.2}$$

A loss that occurred at the average date of loss has developed, as of time t, for a period equal to the average maturity. For example, an accident year as of 8 months has a conditional average date of loss equal to 4 months and an average maturity of loss also equal to 4 months. Using 2.2 we can show a policy year as of 12 months has an average date of loss equal to 8 months and an average maturity equal to 4 months. This can be seen geometrically by observing that the policy year density forms an upward sloping triangle over the first 12 months. The average for a triangle occurs 2/3 of the way along its base. The picture is shown in Figure 8.

Figure 8



In general, we approximate:

$$PCT^*(t^*) \approx PCT(t)\frac{ETD^*(t^*)}{ETD(t)} \qquad (8.3)$$

where $m_W(t) = m_{W^*}(t^*)$.

In words, we first find the date, t, at which W has the same average maturity as W* does at t*. We call "t" the evaluation age of equivalent maturity. The percent of ultimate loss curve for W at the evaluation age of equivalent maturity is then used to approximate the percent of ultimate for W* at t*, where the denominators in the formula adjust for differences in the exposures earned to date. Applying 8.3, we would for example approximate the policy year as of 12 months using the using the accident year as 8 months and the multiplying by ½ and dividing by 8/12. So if the accident year percent of ultimate as of 8 months was 40%, the policy year percent of ultimate as of 12 months would be estimated as 40%*(3/4) = 30%. The corresponding age-to-ultimate factors would be 2.5 and 3.3.

To show why this approximation works, we first follow Robbin and Homer [4] and expand the percent of ultimate convolution formula, 4.5, using the Taylor series expansion of the process distribution. For notational brevity, we will drop subscripts at times during the derivations; for instance writing $\mu(t)$ in place of $\mu_W(t)$. We expand up to second order as follows:

$$\begin{aligned} &P(t-w) = P(t-\mu(t)+\mu(t)-w) = P(m(t)+\mu(t)-w) = \\ &P(m(t)) + (-1)\cdot(\mu(t)-w)\cdot P'(m(t)) + \tfrac{1}{2}(\mu(t)-w)^2 \cdot P''(\delta) \end{aligned} \tag{8.4}$$

where $0<\delta<m(t)$

Note that $\mu(t)$ has been defined so that the integral of the first order term times the exposure density vanishes over the interval from 0 to t. If we now only use the expansion up to first order and plug 8.4 into 4.5, we obtain the approximation:

$$PCT(t) = F_{T_w}(t) = ETD_W(t)\cdot P(m(t)) \tag{8.5}$$

The approximation says that the percent of ultimate loss pattern as of time t for exposures given by W is equal to the percent earned to date times the cumulative distribution of the process distribution as of the conditional average maturity. We see that P(m(t)) approximates the percent of ultimate for the exposures earned to date. If we now write the approximation 8.5 for W* and have t such that $m_W(t) = m_{W^*}(t^*)$, it is then a small rearrangement of terms to arrive at our average maturity of loss approximation as shown in 8.3.

In Exhibit 5 an average maturity of loss approximation for policy year development is computed based on accident year factors. The first sheet of the exhibit shows the derivation of the conditional policy year average date of loss and average maturity of loss at quarterly evaluations. To simplify the calculations, the derivation is done using the exposure growth and decay triangles for the policy year density. The first sheet also shows the accident year evaluation age of equivalent maturity. Then in the second sheet the accident year percent of ultimate and age-to-ultimate factors at the original evaluation ages are shown. This is for information and comparison purposes only. The subsequent derivation of the average maturity approximation makes no use of them. As shown in the second sheet of Exhibit 5, accident year factors are posted for the ages of equivalent maturity. These are then multiplied by the appropriate Earned to Date ratios to obtain the Average Maturity Approximation. Finally, the approximation is compared against the true policy year factors. The accident year factors and the true policy year factors were generated used a Pareto Process with shape equal to 2.0. In actual applications, one should not develop policy year losses evaluated at ages below one year as the data is too immature and the corresponding factors are so large that results are too unstable to be reliable. Note that after two years the approximation reduces to a six-month shift as per the usual Average Date of Loss Approximation.

To summarize, there are two key aspects of the Average Maturity Approximation. First, it adjusts evaluation dates so losses for the two exposure periods have the same conditional average maturity. Second, it adjusts for differences in exposures earned to date. This second adjustment is critical when dealing with immature exposures. Because exposure dependence is built into our model, this earned to date exposure adjustment falls out naturally from the basic equations.

Next we turn to another approximation techniques in which a desired pattern is estimated using a weighted average of the shifted accident period patterns.

9. MULTI-SHIFTED ACCIDENT PERIOD APPROXIMATION

The idea here is that if we can approximate an exposure period random variable as the weighted average of shifted accident period distributions, then we could approximate its development pattern as a weighted average of shifted accident period patterns. Since we have a convenient formula that allows us to evaluate an accident period pattern at arbitrary ages, we will then arrive at a practical way to approximate the development pattern for the original exposure period. After explaining the technique in mathematical terms, we will use it to approximate a policy year pattern as a weighted sum of shifted accident quarter patterns.

Let $A(D_i, c_i)$ be the exposure random variable for an accident period of duration, D_i, which begins at time c_i. Given a process random variable, S, we can write the resulting percent of ultimate, T, as:

$$F_{T(A(D,c))}(t) = \begin{cases} \dfrac{\max(0,(t-c)) - E[S;t-c]}{D} & \text{if } t-c \le D \\ 1 - \dfrac{E[S;t-c] - E[S;t-D-c]}{D} & \text{if } t-c > D \end{cases} \tag{9.1}$$

Now take a finite sequence, $(A(D_1,c_1), A(D_2, c_2), \ldots, A(D_m, c_m))$ of such shifted uniform random variables, and corresponding weights, $(p_1, p_2, \ldots, p_m)$ that sum to unity. Define the mixed multi-shifted exposure random variable, W as follows:

$$F_W(w) = \sum_{i=1}^{m} p_i \cdot \min(1, \max(0, w - c_i)) / D_i \tag{9.2}$$

Given a process random variable, S, the percent of ultimate, T, based on the mixed exposures, W, can be written as:

$$F_{T|\vec{A}(\vec{D},\vec{c}),\vec{p}}(t) = \sum_{i=1}^{m} p_i \cdot F_{T(A(D_i,c_i))}(t) \tag{9.3}$$

While these formulas may look terribly complicated, they are very easy to apply in practice. When the durations are all the same and the shifts follow a simple pattern, one can typically generate the pattern for the common duration and then "copy and paste" to apply (9.3). Generating the basic pattern involves taking limited expected values; so that step is not too difficult either.

The conclusion is that if we can approximate a given exposure random variable as a weighted average of shifted accident period variables, then we can approximate the loss development pattern for the given exposures. In Exhibit 6 we approximate a policy year as the weighted average of five shifted accident year patterns. The weights are: (1/8,1/4, 1/4, 1/4,1/8) and the shifts are: (0,1,2,3,4,5) quarters. While the fit against the true pattern is not exact, it is nonetheless fairly good and we could refine it further by using thirteen accident months with monthly shifts. Note the multi-shift approximation does not inherently fall apart at early ages.

10. MIXING AND NEGATIVE DEVELOPMENT

So far we have used a single Process to describe the underlying multi-step development of claims. While we have proved such a single summary process exists when all the amount changes are non-negative, in practice it may still be useful to regard the single process as a mix of two or more processes. For example, if we know there are two types of claims in our data, one type that develops quickly and the other type more slowly, it may be best to try a model with two processes. Exhibit 7 shows the accident year pattern resulting from a mix of two Gammas, one short-tailed and the other long-tailed.

Also, in all we have done so far, it has been assumed that incremental development must always be non-negative. We now extend the model to handle negative development. For clarity, we will consider a model for the development of the number of non-zero claims. Negative development occurs when a claim is closed without payment. We count the number of non-zero claims as the difference between the total number of claims reported less the number closed without payment.

Let N be the ultimate total number reported, CNP the number closed without payment, and define M as the ultimate number of non-zero claims. Thus M=N-

CNP. For each of the CNP claims, we define a closing lag, U, as the difference between when the claim was reported and when it was closed without payment. Given values of the exposure lag, W, the reporting process lag, S, and the closed without pay lag, U, a claim will be counted as a non-zero claim as of time t if t is between W+S and W+S+U. .

The percent of ultimate for the number of non-zero claims is given as:

$$PCT_M(t) = E\left[\frac{M(t)}{M}\right] = E\left[\frac{N(t) - CNP(t)}{N - CNP}\right] \qquad (10.1)$$

If N and CNP are assumed fixed for the moment, it follows that N(t)-CNP(t) is the sum of two binomially distributed random variables with parameters :

- $(N\text{-}CNP,\ F_{W+S}(t))$ (10.2a)
- $(CNP,\ F_{W+S}(t) - F_{W+S+U}(t))$ (10.2b)

Thus

$$E[M(t)] = (N - CNP)\, F_{W+S}(t)) + CNP(F_{W+S}(t) - F_{W+S+U}(t))$$

$$= N\, F_{W+S}(t) - CNP\, F_{W+S+U}(t) \qquad (10.3)$$

Let r denote the expected ratio at ultimate of the number of claims closed without over the total number of claims ever reported. Then for any reasonably large number of claims we can approximate the percent of ultimate curve as follows:

$$PCT_M(t) = E\left[\frac{M(t)}{M}\right] = E\left[\frac{N(t) - CNP(t)}{N - CNP}\right] \approx \frac{F_{W+S}(t) - rF_{W+S+U}(t)}{1-r} \qquad (10.4)$$

In Exhibit 8 we use patterns based on a Gamma base process with a Gamma decrementing process.

Finally, in Exhibit 9 we generate patterns from a mix of two processes, one of which undergoes negative development. The resulting shape of the development curve is fairly complex with age-to-age factors above unity, then below unity, then back above unity till they taper off in the tail. Yet loss data sometimes exhibits this type of behavior. This could happen when reserves on some claims are taken down as quick settlements are made, but the remaining claims slowly develop upwards over many years.

11. CONCLUSION

It is useful to end with a brief review of what we have done. First we have established a conceptual foundation by identifying exposure with a distribution of accident date lags and then viewing total lag as the sum of exposure and process lag random variables. The single process lag was obtained as a simplified summary of a more general multi-step model of non-negative amount changes and step lags. We were able to connect our model with standard actuarial descriptions of loss development by proving the percent of ultimate development curve is synonymous with the cumulative distribution of the total lag random variable. Having separated exposure from process, we were able to vary the exposure to obtain exposure dependent development curves. Just having a random variable model of loss development was shown to be useful, because it allowed us to simulate loss development patterns. A key result was the derivation of an accident period loss development formula in terms of limited expected values. Because the formula is readily programmable for a large number of distributions, we were able to use it in fitting accident year factors, generating accident quarter factors, and computing multi-shift approximations. Adding in mixed processes and negative development allowed us to structure a model that can reflect our knowledge of the claims process and capture more complex patterns of development.

Hopefully, the reader now has a solid understanding of the conceptual foundation of exposure dependent modeling of loss development patterns and has seen that it may be put to good practical use. Future research along these lines will likely yield new insights and techniques.

REFERENCES

1. Brosius, Eric, Foundations of Casualty Actuarial Science, Second Edition, 1990, Appendix A, p594-598.
2. Hogg, Robert V. and Klugman, Stuart A., Loss Distributions, John Wiley & Sons, Inc., 1984, Appendix, p217-232.
3. Philbrick, Steven, "Reserve Review of a Reinsurance Company", CAS Discussion Paper Program, 1986, May, p147-162.
4. Robbin, Ira and Homer, David, "Analysis of Loss Development Patterns Using Infinitely Decomposable Percent of Ultimate Curves", CAS Discussion Paper Program, 1988, May, p501-538.
5. Sherman, Richard E., "Extrapolating, Smoothing and Interpolating Development Factors", PCAS, 1984, p122-155
6. Wiser, Ronald, Foundations of Casualty Actuarial Science, Second Edition, 1990, Chapter 4, p154-157.

APPENDIX

Accident Year and Policy Year Percent of Ultimate Formulas for a Pareto Process with Shape Equal to 2.0

Let S be a Pareto distribution with scale parameter, λ, and shape parameter equal to 2.0. Then

$$E[S;s]=\frac{\lambda}{2-1}\left(1-\left(\frac{\lambda}{\lambda+s}\right)^{2-1}\right)=\lambda\frac{s}{\lambda+s} \qquad \text{(A.1)}$$

It follows that:

For t<D

$$F_{T_{A(D)}}(t)=\frac{t-E[S;t]}{D}=\frac{1}{D}\left(t-\frac{\lambda t}{\lambda+t}\right)=\frac{t}{D}\left(\frac{(\lambda+t)-\lambda}{\lambda+t}\right)=\frac{1}{D}\left(\frac{t^2}{\lambda+t}\right) \qquad \text{(A.2a)}$$

For t >D

$$\begin{aligned} F_{T_{A(D)}}(t) &= 1-\frac{E[s;t]-E[S;t-D]}{D} \\ &= 1-\frac{1}{D}\left(\frac{\lambda t}{\lambda+t}-\frac{\lambda(t-D)}{\lambda+t-D}\right)=1-\left(\frac{\lambda^2}{(\lambda+t)(\lambda+t-D)}\right) \end{aligned} \qquad \text{(A.2b)}$$

For an accident year, D=1, and we get:

For t<1,

$$F_{T_{AY}}(t)=\left(\frac{t^2}{\lambda+t}\right) \tag{A.3a}$$

For t >1,

$$F_{T_{AY}}(t)=1-\left(\frac{\lambda^2}{(\lambda+t)(\lambda+t-1)}\right) \tag{A.3b}$$

For a policy year, we first consider the exposures from the first calendar year. We derive:

For t<1,

$$F_{T_{PY1}}(t)=\int_0^t dw\, w\cdot\left(1-\left(\frac{\lambda}{\lambda+t-w}\right)^2\right)=\frac{t^2}{2}-\int_0^t du(t-u)\cdot\left(\frac{\lambda}{\lambda+u}\right)^2 \tag{A.4}$$

After several integrations by parts and various standard manipulations, this reduces to:

$$F_{T_{PY1}}(t)=\frac{t^2}{2}+\lambda^2\ln(1+t/\lambda)-\lambda t \tag{A.5}$$

Again restricting our attention for the moment to only those exposures earned in the first year of the policy year but now looking at evaluations exceeding unity, we derive the percent of ultimate:

For t>1,

$$F_{T_{PY1}}(t)=\int_0^1 dw\, w\cdot\left(1-\left(\frac{\lambda}{\lambda+t-w}\right)^2\right)=\frac{1}{2}-\int_{t-1}^t du(t-u)\cdot\left(\frac{\lambda}{\lambda+u}\right)^2 \tag{A.6}$$

After various standard manipulations, this simplifies to:

For t>1,

$$F_{T_{PY1}}(t) = \frac{1}{2} + \lambda^2 \ln\left(\frac{\lambda + t}{\lambda + t - 1}\right) - \frac{\lambda^2}{\lambda + t - 1} \tag{A.7}$$

Now we turn our attention to policy exposures earned in the second calendar year. We first consider evaluation dates in the second year and derive:

For 1<t<2

$$\begin{aligned} F_{T_{PY2}}(t) &= \int_1^t dw\,(2 - w) \cdot \left(1 - \left(\frac{\lambda}{\lambda + t - w}\right)^2\right) \\ &= \frac{1 - (2-t)^2}{2} - \int_0^{t-1} du(2 + u - t) \cdot \left(\frac{\lambda}{\lambda + u}\right)^2 \end{aligned} \tag{A.8}$$

This reduces to:

For 1<t<2,

$$\begin{aligned} F_{T_{PY2}}(t) &= \int_1^t dw\,(2 - w) \cdot \left(1 - \left(\frac{\lambda}{\lambda + t - w}\right)^2\right) \\ &= \frac{1 - (2-t)^2}{2} - \lambda^2 \ln\left(\frac{\lambda + t - 1}{\lambda}\right) - \frac{(\lambda + t - 2)\lambda(t-1)}{\lambda + t - 1} \end{aligned} \tag{A.9}$$

Again considering only policy year exposures earned in the second year, but now looking at evaluation dates beyond two years, we derive:

For t>2,

$$\begin{aligned} F_{T_{PY2}}(t) &= \int_1^2 dw\,(2 - w) \cdot \left(1 - \left(\frac{\lambda}{\lambda + t - w}\right)^2\right) \\ &= \frac{1}{2} - \int_{t-2}^{t-1} du(2 + u - t) \cdot \left(\frac{\lambda}{\lambda + u}\right)^2 \end{aligned} \tag{A.10}$$

After some rather tedious but straightforward manipulations, this simplifies to:

For t>2,

$$F_{T_{PY2}}(t) = \frac{1}{2} - \frac{\lambda^2}{\lambda + t - 1} - \lambda^2 \ln\left(\frac{\lambda + t - 1}{\lambda + t - 2}\right) \quad \text{(A.11)}$$

Now we finally have all the pieces to evaluate the policy year percent of ultimate.

For example, at t=3, we would add together A.7 plus A.11 to get:

$$\begin{aligned} F_{T_{PY1}}(3) + F_{T_{PY2}}(3) &= \frac{1}{2} + \lambda^2 \ln\left(\frac{\lambda + 3}{\lambda + 2}\right) - \frac{\lambda^2}{\lambda + 2} + \frac{1}{2} - \frac{\lambda^2}{\lambda + 2} - \lambda^2 \ln\left(\frac{\lambda + 2}{\lambda + 1}\right) \\ &= 1 - \lambda^2 \ln\left(\frac{(\lambda + 2)^2}{(\lambda + 2)^2 - 1}\right) \end{aligned} \quad \text{(A.12)}$$

With the scale equal to 1.5, we obtain:

$$F_{T_{PY}}(3) = 1 - 2.25 \ln\left(\frac{(3.5)^2}{(3.5)^2 - 1}\right) = .80839 \quad \text{(A.13)}$$

Exhibit 1
Sheet 1

Sample Simulation of Exposure Dependent Development
Development Patterns Generated from Random Trails
20 = Number of Random Trials

	Exposure: W Accident Year				*Exposure: W* Policy Year			
Evaluation Age	***Formula Pct of Ultimate***	***Formula AU LDF***	***Simulated Pct of Ultimate***	***Simulated AU LDF***	***Formula Pct of Ultimate***	***Formula AU LDF***	***Simulated Pct of Ultimate***	***Simulated AU LDF***
0.250	3.57%	28.000	0.00%	#DIV/0!	0.31%	323.726	0.00%	#DIV/0!
0.500	12.50%	8.000	5.00%	20.000	2.23%	44.874	0.00%	#DIV/0!
0.750	25.00%	4.000	15.00%	6.667	6.85%	14.589	5.00%	20.000
1.000	40.00%	2.500	25.00%	4.000	14.94%	6.695	5.00%	20.000
1.250	53.25%	1.878	55.00%	1.818	26.39%	3.790	15.00%	6.667
1.500	62.50%	1.600	65.00%	1.538	39.00%	2.564	25.00%	4.000
1.750	69.23%	1.444	65.00%	1.538	50.88%	1.965	45.00%	2.222
2.000	74.29%	1.346	70.00%	1.429	60.77%	1.646	65.00%	1.538
2.250	78.18%	1.279	70.00%	1.429	68.09%	1.469	65.00%	1.538
2.500	81.25%	1.231	75.00%	1.333	73.50%	1.361	70.00%	1.429
2.750	83.71%	1.195	75.00%	1.333	77.62%	1.288	70.00%	1.429
3.000	85.71%	1.167	80.00%	1.250	80.84%	1.237	70.00%	1.429
3.250	87.37%	1.145	95.00%	1.053	83.40%	1.199	75.00%	1.333
3.500	88.75%	1.127	95.00%	1.053	85.48%	1.170	85.00%	1.176
3.750	89.92%	1.112	95.00%	1.053	87.19%	1.147	95.00%	1.053
4.000	90.91%	1.100	95.00%	1.053	88.61%	1.129	95.00%	1.053
4.250	91.76%	1.090	100.00%	1.000	89.80%	1.114	95.00%	1.053
4.500	92.50%	1.081	100.00%	1.000	90.82%	1.101	95.00%	1.053
4.750	93.14%	1.074	100.00%	1.000	91.68%	1.091	100.00%	1.000
5.000	93.71%	1.067	100.00%	1.000	92.44%	1.082	100.00%	1.000

Exhibit 1
Sheet 2

Sample Simulation of Exposure Dependent Development- Trial Listing
20 Random trials

	Process: S Pareto		*Exposure Generator*	*Exposure: W* Accident Year		*Exposure: W* Policy Year	
	Mean **1.5000** Shape **2.0000**						
Trial	***Process Random Number***	***Simulated S***	***Exposure Random Number***	***Simulated W***	***Total Lag T=S+W***	***Simulated W***	***Total Lag T=S+W***
1	0.9203	3.8131	0.4196	0.4196	4.2326	0.9160	4.7291
2	0.7438	1.4634	0.0127	0.0127	1.4761	0.1594	1.6228
3	0.8923	3.0703	0.0396	0.0396	3.1099	0.2813	3.3516
4	0.3144	0.3116	0.8249	0.8249	1.1366	1.4083	1.7199
5	0.3636	0.3803	0.7865	0.7865	1.1668	1.3465	1.7268
6	0.3508	0.3617	0.0799	0.0799	0.4416	0.3997	0.7614
7	0.3905	0.4213	0.8603	0.8603	1.2816	1.4714	1.8927
8	0.8827	2.8795	0.1994	0.1994	3.0790	0.6316	3.5111
9	0.1185	0.0976	0.9674	0.9674	1.0650	1.7445	1.8422
10	0.8309	2.1480	0.7681	0.7681	2.9162	1.3190	3.4671
11	0.7886	1.7623	0.1612	0.1612	1.9235	0.5679	2.3301
12	0.8543	2.4298	0.7724	0.7724	3.2022	1.3254	3.7551
13	0.3334	0.3372	0.2267	0.2267	0.5639	0.6733	1.0105
14	0.3494	0.3597	0.4011	0.4011	0.7608	0.8957	1.2554
15	0.7848	1.7332	0.7657	0.7657	2.4989	1.3154	3.0486
16	0.4073	0.4484	0.8111	0.8111	1.2594	1.3853	1.8337
17	0.2008	0.1779	0.8887	0.8887	1.0666	1.5283	1.7062
18	0.3040	0.2980	0.5561	0.5561	0.8541	1.0578	1.3557
19	0.5301	0.6882	0.6978	0.6978	1.3860	1.2226	1.9108
20	0.1742	0.1506	0.7604	0.7604	0.9110	1.3077	1.4584
Average	0.5267	1.1666	0.5500	0.5500	1.7166	1.0479	2.2145

Exhibit 2

Simulation of Exposure Dependent Development

Development Patterns Generated from Random Trails

2000 = Number of Random Trials

	Exposure: W Accident Year				*Exposure: W* Policy Year			
Evaluation Age	***Formula Pct of Ultimate***	***Formula AU LDF***	***Simulated Pct of Ultimate***	***Simulated AU LDF***	***Formula Pct of Ultimate***	***Formula AU LDF***	***Simulated Pct of Ultimate***	***Simulated AU LDF***
0.250	3.57%	28.000	3.95%	25.316	0.31%	323.726	0.40%	250.000
0.500	12.50%	8.000	12.50%	8.000	2.23%	44.874	2.60%	38.462
0.750	25.00%	4.000	24.70%	4.049	6.85%	14.589	7.60%	13.158
1.000	40.00%	2.500	40.35%	2.478	14.94%	6.695	15.15%	6.601
1.250	53.25%	1.878	52.55%	1.903	26.39%	3.790	25.95%	3.854
1.500	62.50%	1.600	62.30%	1.605	39.00%	2.564	39.70%	2.519
1.750	69.23%	1.444	69.35%	1.442	50.88%	1.965	50.65%	1.974
2.000	74.29%	1.346	74.15%	1.349	60.77%	1.646	60.20%	1.661
2.250	78.18%	1.279	78.05%	1.281	68.09%	1.469	67.60%	1.479
2.500	81.25%	1.231	81.15%	1.232	73.50%	1.361	73.40%	1.362
2.750	83.71%	1.195	83.85%	1.193	77.62%	1.288	77.80%	1.285
3.000	85.71%	1.167	85.45%	1.170	80.84%	1.237	80.90%	1.236
3.250	87.37%	1.145	87.05%	1.149	83.40%	1.199	83.40%	1.199
3.500	88.75%	1.127	88.60%	1.129	85.48%	1.170	84.85%	1.179
3.750	89.92%	1.112	90.00%	1.111	87.19%	1.147	87.15%	1.147
4.000	90.91%	1.100	90.75%	1.102	88.61%	1.129	88.40%	1.131
4.250	91.76%	1.090	91.60%	1.092	89.80%	1.114	89.75%	1.114
4.500	92.50%	1.081	92.45%	1.082	90.82%	1.101	90.75%	1.102
4.750	93.14%	1.074	93.15%	1.074	91.68%	1.091	91.50%	1.093
5.000	93.71%	1.067	93.80%	1.066	92.44%	1.082	92.40%	1.082

Exhibit 3
Sheet 1

Accident Year - AA LDF Fitting Summary

		Fitted AA LDF			
Age (year)	*Given AA LDF*	*Gamma*	*Pareto*	*Burr*	*Sherman Power Curve*
1	**1.920**	1.884	1.960	1.924	1.889
2	**1.228**	1.238	1.205	1.216	1.224
3	**1.098**	1.115	1.094	1.101	1.100
4	**1.051**	1.064	1.054	1.059	1.056
5	**1.036**	1.039	1.036	1.038	1.036
6	**1.025**	1.024	1.025	1.026	1.025
7	**1.019**	1.016	1.019	1.019	1.018
8	**1.014**	1.010	1.015	1.014	1.014
9	**1.011**	1.007	1.012	1.011	1.011
10	**1.009**	1.004	1.009	1.009	1.009
11	**1.008**	1.003	1.008	1.007	1.008

Exhibit 3
Sheet 2

Accident Year - AA LDF fitting

Process Distribution	Gamma
Mean	1.7731
Shape	0.6416
Scale	2.7636

Fitting Criteria	
Minimize Square Error	
Error	Difference in AALDF Back Product
Square Error	**0.0030**

Fitting

Age (year)	*Given AA LDF*	*Fitted LEV*	*Fitted % of Ult*	*Fitted AU LDF*	*Fitted AA LDF*	*Error in AA LDF*	*Back Product*	*Fitted Back Product*	*Error*
1	**1.9200**	0.6757	32.43%	3.0831	1.8843	-0.0357	3.0698	3.0638	-0.0060
2	**1.2280**	1.0645	61.12%	1.6362	1.2379	0.0099	1.5988	1.6259	0.0271
3	**1.0980**	1.3079	75.66%	1.3218	1.1145	0.0165	1.3020	1.3135	0.0115
4	**1.0510**	1.4647	84.32%	1.1859	1.0643	0.0133	1.1858	1.1785	-0.0073
5	**1.0360**	1.5672	89.74%	1.1143	1.0388	0.0028	1.1282	1.1073	-0.0209
6	**1.0250**	1.6350	93.22%	1.0727	1.0243	-0.0007	1.0890	1.0660	-0.0231
7	**1.0190**	1.6801	95.49%	1.0472	1.0156	-0.0034	1.0625	1.0407	-0.0218
8	**1.0140**	1.7103	96.98%	1.0311	1.0102	-0.0038	1.0427	1.0247	-0.0180
9	**1.0110**	1.7306	97.97%	1.0207	1.0067	-0.0043	1.0283	1.0143	-0.0140
10	**1.0090**	1.7442	98.63%	1.0139	1.0045	-0.0045	1.0171	1.0075	-0.0096
11	**1.0080**	1.7535	99.08%	1.0093	1.0030	-0.0050	1.0080	1.0030	-0.0050
12		1.7598	99.37%						

Exhibit 3
Sheet 3

Accident Year - AA LDF Fitting

Process Distribution	Pareto
Mean	64.8752
Shape	1.0164

Fitting Criteria	
Minimize Square Error	
Error	Difference in AALDF Back Product
Square Error	**0.00094**

Fitting

Age (year)	*Given AA LDF*	*Fitted LEV*	*Fitted % of Ult*	*Fitted AU LDF*	*Fitted AA LDF*	*Error in AA LDF*	*Back Product*	*Fitted Back Product*	*Error*
1	**1.9200**	0.7015	29.85%	3.3506	1.9604	0.0404	3.0698	3.0775	0.0078
2	**1.2280**	1.1165	58.51%	1.7091	1.2049	-0.0231	1.5988	1.5698	-0.0290
3	**1.0980**	1.4115	70.50%	1.4185	1.0939	-0.0041	1.3020	1.3029	0.0009
4	**1.0510**	1.6404	77.11%	1.2968	1.0544	0.0034	1.1858	1.1911	0.0054
5	**1.0360**	1.8273	81.31%	1.2299	1.0357	-0.0003	1.1282	1.1296	0.0014
6	**1.0250**	1.9852	84.21%	1.1875	1.0252	0.0002	1.0890	1.0907	0.0017
7	**1.0190**	2.1218	86.34%	1.1583	1.0188	-0.0002	1.0625	1.0638	0.0014
8	**1.0140**	2.2422	87.96%	1.1369	1.0146	0.0006	1.0427	1.0442	0.0016
9	**1.0110**	2.3498	89.24%	1.1206	1.0116	0.0006	1.0283	1.0292	0.0010
10	**1.0090**	2.4470	90.28%	1.1077	1.0095	0.0005	1.0171	1.0174	0.0003
11	**1.0080**	2.5357	91.13%	1.0973	1.0079	-0.0001	1.0080	1.0079	-0.0001
12		2.6172	91.85%						

Exhibit 3
Sheet 4

Accident Year - AA LDF fitting

Process Distribution	**Burr**
Mean	3.2549
Shape	0.8505

Fitting Criteria	
Minimize Square Error	
Error	Difference in AALDF Back Product
Square Error	**0.00082**

Fitting

Age (year)	*Given AA LDF*	*Fitted LEV*	*Fitted % of Ult*	*Fitted AU LDF*	*Fitted AA LDF*	*Error in AA LDF*	*Back Product*	*Fitted Back Product*	*Error*
1	**1.9200**	0.6927	30.73%	3.2540	1.9240	0.0040	3.0698	3.0878	0.0180
2	**1.2280**	1.1014	59.13%	1.6913	1.2164	-0.0116	1.5988	1.6049	0.0061
3	**1.0980**	1.3822	71.92%	1.3904	1.1015	0.0035	1.3020	1.3193	0.0174
4	**1.0510**	1.5899	79.22%	1.2622	1.0587	0.0077	1.1858	1.1978	0.0120
5	**1.0360**	1.7512	83.87%	1.1923	1.0379	0.0019	1.1282	1.1314	0.0031
6	**1.0250**	1.8806	87.05%	1.1487	1.0263	0.0013	1.0890	1.0900	0.0010
7	**1.0190**	1.9872	89.34%	1.1193	1.0191	0.0001	1.0625	1.0622	-0.0003
8	**1.0140**	2.0768	91.05%	1.0984	1.0144	0.0004	1.0427	1.0423	-0.0004
9	**1.0110**	2.1532	92.36%	1.0828	1.0112	0.0002	1.0283	1.0275	-0.0008
10	**1.0090**	2.2194	93.39%	1.0708	1.0089	-0.0001	1.0171	1.0161	-0.0010
11	**1.0080**	2.2772	94.22%	1.0614	1.0072	-0.0008	1.0080	1.0072	-0.0008
12		2.3283	94.89%						

Accident Quarter by Quarter - LDF Generation

Process Distribution	Burr
Mean	3.2549
Shape	0.8505

AQ by Q LDF Generated by LEV Formula

Age (year)	*LEV*	*% of Ult*	*AU LDF*	*AA LDF*
0.250	0.2205	11.80%	8.4716	2.3132
0.500	0.4022	27.30%	3.6624	1.3881
0.750	0.5575	37.90%	2.6384	1.2114
1.000	0.6927	45.92%	2.1779	1.1378
1.250	0.8121	52.24%	1.9141	1.0984
1.500	0.9186	57.38%	1.7426	1.0743
1.750	1.0145	61.65%	1.6222	1.0582
2.000	1.1014	65.23%	1.5329	1.0469
2.250	1.1807	68.29%	1.4642	1.0386
2.500	1.2533	70.93%	1.4098	1.0324
2.750	1.3203	73.23%	1.3656	1.0275
3.000	1.3822	75.24%	1.3291	1.0236
3.250	1.4396	77.02%	1.2984	1.0205
3.500	1.4931	78.60%	1.2723	1.0179
3.750	1.5431	80.01%	1.2499	1.0158
4.000	1.5899	81.27%	1.2304	1.0140
4.250	1.6339	82.41%	1.2134	1.0125
4.500	1.6753	83.45%	1.1983	1.0113
4.750	1.7143	84.39%	1.1850	1.0102
5.000	1.7512	85.25%	1.1731	1.0092
5.250	1.7861	86.03%	1.1624	1.0084
5.500	1.8192	86.75%	1.1527	1.0076
5.750	1.8507	87.41%	1.1440	1.0070
6.000	1.8806	88.02%	1.1361	

Exhibit 5
Sheet 1

Average Maturity of Loss Approximation of Policy Year Development Based on Accident Year

Derivation of PY Average Maturity and AY Age of Equivalent Maturity

*Evaluation Age t**	*Exposure Growth Triangle Prob*	*Exposure Growth Triangle Avg Date*	*Exposure Decay Triangle Prob*	*Exposure Decay Triangle Avg Date*	*PY ETD ETD*(t*)*	*PY Average Date of Loss*	*PY Average Maturity*	*AY Age of Equivalent Maturity t*	*AY ETD ETD(t)*
0.250	3.125%	0.167	0.000%	1.000	3.125%	0.167	0.083	0.167	16.667%
0.500	12.500%	0.333	0.000%	1.000	12.500%	0.333	0.167	0.333	33.333%
0.750	28.125%	0.500	0.000%	1.000	28.125%	0.500	0.250	0.500	50.000%
1.000	50.000%	0.667	0.000%	1.000	50.000%	0.667	0.333	0.667	66.667%
1.250	50.000%	0.667	21.875%	1.119	71.875%	0.804	0.446	0.891	89.130%
1.500	50.000%	0.667	37.500%	1.222	87.500%	0.905	0.595	1.095	100.000%
1.750	50.000%	0.667	46.875%	1.300	96.875%	0.973	0.777	1.277	100.000%
2.000	50.000%	0.667	50.000%	1.333	100.000%	1.000	1.000	1.500	100.000%
2.250	50.000%	0.667	50.000%	1.333	100.000%	1.000	1.250	1.750	100.000%
2.500	50.000%	0.667	50.000%	1.333	100.000%	1.000	1.500	2.000	100.000%
2.750	50.000%	0.667	50.000%	1.333	100.000%	1.000	1.750	2.250	100.000%
3.000	50.000%	0.667	50.000%	1.333	100.000%	1.000	2.000	2.500	100.000%
3.250	50.000%	0.667	50.000%	1.333	100.000%	1.000	2.250	2.750	100.000%
3.500	50.000%	0.667	50.000%	1.333	100.000%	1.000	2.500	3.000	100.000%
3.750	50.000%	0.667	50.000%	1.333	100.000%	1.000	2.750	3.250	100.000%
4.000	50.000%	0.667	50.000%	1.333	100.000%	1.000	3.000	3.500	100.000%
4.250	50.000%	0.667	50.000%	1.333	100.000%	1.000	3.250	3.750	100.000%
4.500	50.000%	0.667	50.000%	1.333	100.000%	1.000	3.500	4.000	100.000%
4.750	50.000%	0.667	50.000%	1.333	100.000%	1.000	3.750	4.250	100.000%
5.000	50.000%	0.667	50.000%	1.333	100.000%	1.000	4.000	4.500	100.000%

Exhibit 5
Sheet 2

Average Maturity of Loss Approximation of Policy Year Development Based on Accident Year

Derivation of PY AU LDF Approximation

Evaluation Age t*	PY ETD ETD*(t*)	AY PCT of ULT PCT(t*)	AY AU LDF(t*)	AY Age of Equivalent Maturity t	AY ETD ETD(t)	AY PCT of ULT PCT(t)	AY AU LDF(t)	PY Avg Maturity Approx PCT of ULT	PY Avg Maturity Approx AU LDF	PY True AU LDF	Error
0.250	3.125%	3.57%	28.000	0.167	16.667%	1.67%	60.000	0.31%	320.000	323.726	-3.726
0.500	12.500%	12.50%	8.000	0.333	33.333%	6.06%	16.500	2.27%	44.000	44.874	-0.874
0.750	28.125%	25.00%	4.000	0.500	50.000%	12.50%	8.000	7.03%	14.222	14.589	-0.366
1.000	50.000%	40.00%	2.500	0.667	66.667%	20.51%	4.875	15.38%	6.500	6.695	-0.195
1.250	71.875%	53.25%	1.878	0.891	89.130%	33.22%	3.010	26.79%	3.733	3.790	-0.057
1.500	87.500%	62.50%	1.600	1.095	100.000%	45.65%	2.190	39.95%	2.503	2.564	-0.061
1.750	96.875%	69.23%	1.444	1.277	100.000%	54.40%	1.838	52.70%	1.898	1.965	-0.068
2.000	100.000%	74.29%	1.346	1.500	100.000%	62.50%	1.600	62.50%	1.600	1.646	-0.046
2.250	100.000%	78.18%	1.279	1.750	100.000%	69.23%	1.444	69.23%	1.444	1.469	-0.024
2.500	100.000%	81.25%	1.231	2.000	100.000%	74.29%	1.346	74.29%	1.346	1.361	-0.014
2.750	100.000%	83.71%	1.195	2.250	100.000%	78.18%	1.279	78.18%	1.279	1.288	-0.009
3.000	100.000%	85.71%	1.167	2.500	100.000%	81.25%	1.231	81.25%	1.231	1.237	-0.006
3.250	100.000%	87.37%	1.145	2.750	100.000%	83.71%	1.195	83.71%	1.195	1.199	-0.004
3.500	100.000%	88.75%	1.127	3.000	100.000%	85.71%	1.167	85.71%	1.167	1.170	-0.003
3.750	100.000%	89.92%	1.112	3.250	100.000%	87.37%	1.145	87.37%	1.145	1.147	-0.002
4.000	100.000%	90.91%	1.100	3.500	100.000%	88.75%	1.127	88.75%	1.127	1.129	-0.002
4.250	100.000%	91.76%	1.090	3.750	100.000%	89.92%	1.112	89.92%	1.112	1.114	-0.001
4.500	100.000%	92.50%	1.081	4.000	100.000%	90.91%	1.100	90.91%	1.100	1.101	-0.001
4.750	100.000%	93.14%	1.074	4.250	100.000%	91.76%	1.090	91.76%	1.090	1.091	-0.001
5.000	100.000%	93.71%	1.067	4.500	100.000%	92.50%	1.081	92.50%	1.081	1.082	-0.001

Multi-shifted Approximation of Policy Year
Using 5 Shifted Accident Years

	Exposure: W Accident Year		*Exposure: W* Policy Year		*Policy Year* *Multi-shifted Approximation*						
						Shift	0.000	0.250	0.500	0.750	1.000
						Weight	0.125	0.250	0.250	0.250	0.125
Evaluation Age	***Formula Pct of Ultimate***	***Formula AU LDF***	***Formula Pct of Ultimate***	***Formula AU LDF***	***Pct of Ult***	***AU LDF***	***Pct of Ultimate***	***Pct of Ultimate***	***Pct of Ultimate***	***Pct of Ultimate***	***Pct of Ultimate***
0.250	3.57%	28.000	0.31%	323.726	0.45%	224.000	3.57%				
0.500	12.50%	8.000	2.23%	44.874	2.46%	40.727	12.50%	3.57%			
0.750	25.00%	4.000	6.85%	14.589	7.14%	14.000	25.00%	12.50%	3.57%		
1.000	40.00%	2.500	14.94%	6.695	15.27%	6.550	40.00%	25.00%	12.50%	3.57%	
1.250	53.25%	1.878	26.39%	3.790	26.48%	3.777	53.25%	40.00%	25.00%	12.50%	3.57%
1.500	62.50%	1.600	39.00%	2.564	38.94%	2.568	62.50%	53.25%	40.00%	25.00%	12.50%
1.750	69.23%	1.444	50.88%	1.965	50.72%	1.972	69.23%	62.50%	53.25%	40.00%	25.00%
2.000	74.29%	1.346	60.77%	1.646	60.53%	1.652	74.29%	69.23%	62.50%	53.25%	40.00%
2.250	78.18%	1.279	68.09%	1.469	67.93%	1.472	78.18%	74.29%	69.23%	62.50%	53.25%
2.500	81.25%	1.231	73.50%	1.361	73.39%	1.363	81.25%	78.18%	74.29%	69.23%	62.50%
2.750	83.71%	1.195	77.62%	1.288	77.55%	1.290	83.71%	81.25%	78.18%	74.29%	69.23%
3.000	85.71%	1.167	80.84%	1.237	80.79%	1.238	85.71%	83.71%	81.25%	78.18%	74.29%
3.250	87.37%	1.145	83.40%	1.199	83.36%	1.200	87.37%	85.71%	83.71%	81.25%	78.18%
3.500	88.75%	1.127	85.48%	1.170	85.45%	1.170	88.75%	87.37%	85.71%	83.71%	81.25%
3.750	89.92%	1.112	87.19%	1.147	87.16%	1.147	89.92%	88.75%	87.37%	85.71%	83.71%
4.000	90.91%	1.100	88.61%	1.129	88.59%	1.129	90.91%	89.92%	88.75%	87.37%	85.71%
4.250	91.76%	1.090	89.80%	1.114	89.79%	1.114	91.76%	90.91%	89.92%	88.75%	87.37%
4.500	92.50%	1.081	90.82%	1.101	90.80%	1.101	92.50%	91.76%	90.91%	89.92%	88.75%
4.750	93.14%	1.074	91.68%	1.091	91.68%	1.091	93.14%	92.50%	91.76%	90.91%	89.92%
5.000	93.71%	1.067	92.44%	1.082	92.43%	1.082	93.71%	93.14%	92.50%	91.76%	90.91%

Exhibit 7

Accident Year - LDF Generation
Mixed Processes

Process1	
Distribution	Gamma
Mean	1.0000
Shape	1.0000
Scale	1.0000

Process2	
Distribution	Gamma
Mean	8.0000
Shape	2.0000
Scale	4.0000
Weight	10.00%

Derivation of Development Pattern for Total Mixed Process

Age (year)	*Process1 LEV*	*Process1 % of Ult*	*Process2 LEV*	*Process2 % of Ult*	*Total Process % of Ult*	*Total Process AU LDF*	*Total Process AA LDF*
1	0.6321	36.79%	0.9908	0.92%	33.20%	3.0119	2.0973
2	0.8647	76.75%	1.9347	5.61%	69.63%	1.4361	1.2007
3	0.9502	91.45%	2.8040	13.07%	83.61%	1.1961	1.0687
4	0.9817	96.85%	3.5854	21.85%	89.35%	1.1192	1.0303
5	0.9933	98.84%	4.2754	31.00%	92.06%	1.0863	1.0168
6	0.9975	99.57%	4.8762	39.93%	93.61%	1.0683	1.0115
7	0.9991	99.84%	5.3934	48.28%	94.69%	1.0561	1.0090
8	0.9997	99.94%	5.8346	55.88%	95.54%	1.0467	1.0074
9	0.9999	99.98%	6.2082	62.64%	96.25%	1.0390	1.0063
10	1.0000	99.99%	6.5225	68.57%	96.85%	1.0325	1.0053
11	1.0000	100.00%	6.7854	73.71%	97.37%	1.0270	1.0045
12	1.0000	100.00%	7.0043	78.11%	97.81%	1.0224	1.0038
13	1.0000	100.00%	7.1857	81.85%	98.18%	1.0185	1.0032
14	1.0000	100.00%	7.3357	85.01%	98.50%	1.0152	1.0027
15	1.0000	100.00%	7.4591	87.66%	98.77%	1.0125	

Accident Year - LDF Generation
Negative Development

Process Distribution	Gamma
Mean	1.0000
Shape	1.0000
Scale	1.0000

Decrement Lag Distribution	Gamma
Mean	2.0000
Shape	2.0000
Scale	1.0000
Weight	30.00%

Process +Decrement Lag	Gamma
Mean	3.0000
Shape	3.0000
Scale	1.0000

Derivation of Development Pattern

Age (year)	*Process LEV*	*Process % of Ult*	*Process +Decrement LEV*	*Process + Decrement % of Ult*	*% of Ult*	*AU LDF*	*AA LDF*
1	0.6321	36.79%	0.9767	2.33%	51.55%	1.9397	1.9648
2	0.8647	76.75%	1.7820	19.47%	101.29%	0.9872	1.0975
3	0.9502	91.45%	2.3279	45.41%	111.17%	0.8995	0.9840
4	0.9817	96.85%	2.6520	67.59%	109.40%	0.9141	0.9680
5	0.9933	98.84%	2.8282	82.38%	105.90%	0.9443	0.9750
6	0.9975	99.57%	2.9182	91.00%	103.25%	0.9685	0.9846
7	0.9991	99.84%	2.9622	95.60%	101.66%	0.9837	0.9916
8	0.9997	99.94%	2.9829	97.93%	100.81%	0.9920	0.9957
9	0.9999	99.98%	2.9924	99.05%	100.38%	0.9962	0.9980
10	1.0000	99.99%	2.9967	99.57%	100.17%	0.9983	0.9990
11	1.0000	100.00%	2.9986	99.81%	100.08%	0.9992	0.9996
12	1.0000	100.00%	2.9994	99.92%	100.03%	0.9997	0.9998
13	1.0000	100.00%	2.9997	99.96%	100.01%	0.9999	0.9999
14	1.0000	100.00%	2.9999	99.99%	100.01%	0.9999	1.0000
15	1.0000	100.00%	3.0000	99.99%	100.00%	1.0000	

Exhibit 9

Accident Year - LDF Generation
Mixed Processes- One with Negative Development

Process1 Distribution	Gamma
Mean	1.0000
Shape	1.0000
Scale	1.0000

Decrement Lag Distribution	Gamma
Mean	2.0000
Shape	2.0000
Scale	1.0000
Weight	**30.00%**

Process1 +Decrement Lag	Gamma
Mean	3.0000
Shape	3.0000
Scale	1.0000

Process2 Distribution	Gamma
Mean	8.0000
Shape	2.0000
Scale	4.0000
Weight	**20.00%**

Derivation of Development Pattern

Age (year)	*Process1 LEV*	*Process1 % of Ult*	*Process1 +Decrement LEV*	*Process1 + Decrement % of Ult*	*Process1 After Decrement % of Ult*	*Process2 LEV*	*Process2 % of Ult*	*Total % of Ult*	*Total AU LDF*	*Total AA LDF*
1	0.6321	36.79%	0.9767	2.33%	51.55%	0.9908	0.92%	41.43%	2.4139	1.9831
2	0.8647	76.75%	1.7820	19.47%	101.29%	1.9347	5.61%	82.16%	1.2172	1.1144
3	0.9502	91.45%	2.3279	45.41%	111.17%	2.8040	13.07%	91.55%	1.0923	1.0036
4	0.9817	96.85%	2.6520	67.59%	109.40%	3.5854	21.85%	91.89%	1.0883	0.9895
5	0.9933	98.84%	2.8282	82.38%	105.90%	4.2754	31.00%	90.92%	1.0999	0.9963
6	0.9975	99.57%	2.9182	91.00%	103.25%	4.8762	39.93%	90.58%	1.1039	1.0044
7	0.9991	99.84%	2.9622	95.60%	101.66%	5.3934	48.28%	90.98%	1.0991	1.0092
8	0.9997	99.94%	2.9829	97.93%	100.81%	5.8346	55.88%	91.82%	1.0891	1.0110
9	0.9999	99.98%	2.9924	99.05%	100.38%	6.2082	62.64%	92.83%	1.0772	1.0110
10	1.0000	99.99%	2.9967	99.57%	100.17%	6.5225	68.57%	93.85%	1.0655	1.0101
11	1.0000	100.00%	2.9986	99.81%	100.08%	6.7854	73.71%	94.80%	1.0548	1.0089
12	1.0000	100.00%	2.9994	99.92%	100.03%	7.0043	78.11%	95.65%	1.0455	1.0077
13	1.0000	100.00%	2.9997	99.96%	100.01%	7.1857	81.85%	96.38%	1.0375	1.0065
14	1.0000	100.00%	2.9999	99.99%	100.01%	7.3357	85.01%	97.01%	1.0309	1.0054
15	1.0000	100.00%	3.0000	99.99%	100.00%	7.4591	87.66%	97.53%	1.0253	

Pricing for Systematic Risk

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Pricing for Systematic Risk

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Abstract

In recent years, financial methods have emerged as the dominant approach for establishing insurance profit loadings. Financial theory suggests that prices should reflect systematic risk only, with no reward for diversifiable risk. This principle is applied to the pricing of insurance exposures actively traded in a secondary market. The resulting Systematic Risk Pricing Model differs from the Capital Asset Pricing Model in that it determines the price rather than the rate of return for each exposure. In order to reconcile the two pricing models, the amount of capital invested in a security in the Capital Asset Pricing Model is reinterpreted as the price for the exposure. Under the Systematic Risk Pricing Model, the price for the exposure is determined without regard for the insurer's cost of capital. In this method, an exposure's rate of return represents the profit margin, that is, the expected profit for an exposure in relation to its price. Due to the inconsistency of the CAPM with this result, the interpretation of CAPM rate of return as the market capitalization rate used to discount future income to present value is abandoned. An in-depth examination of the CAPM identifies a number of conceptual errors with the model, the most serious of these being that the CAPM substitutes the variability of the price of the exposure over time for the true risk of the exposure. A mathematical derivation of the CAPM from the Systematic Risk Pricing Model is presented to identify the faulty assumptions underlying the model.

Pricing for Systematic Risk

Introduction

One of the ongoing challenges in insurance risk pricing is to determine an appropriate profit margin to include in an insurer's rates. Several distinct approaches to this problem have been developed. In recent years, the actuarial literature has focused primarily on the use of financial analysis methods to evaluate the insurer's rate of return on capital. In comparison, the economic literature emphasizes the use of risk pricing methods such as the expected utility theory model to determine the certainty equivalent price for a transfer of risk. A discussion of these and other methods can be found in Feldblum (1990).

At first glance, the financial and economic approaches to insurance risk pricing appear to be irreconcilable. Financial methods such as the discounted cash flow models described in Bingham (1993) and Feldblum (1992) stress the role of capital, risk-adjusted rates of return, and returns on alternative investment opportunities. In comparison, the expected utility theory (EUT) model as described in Borch (1990) gives little or no consideration to the insurer's capital or to the exposure's rate of return. While financial methods take into account the timing of future cash flows in the analysis and operate under the assumption that time and risk are essentially inseparable, the EUT model avoids any reliance on the time value of money by assuming that the indemnities are paid immediately after the premium is collected. The two methods also differ in their treatment of risk diversification. D'Arcy and Dyer (1997) note that the Capital Asset Pricing Model (CAPM) rewards an investor only for systematic risk, that portion of the risk that cannot be eliminated by diversification over the market. In comparison, the expected utility theory approach evaluates the price for each risk transfer in isolation from all other transactions. In essence, the EUT model determines the price for each exposure based on its own risk and gives no consideration to the effect of risk diversification on price.

Rather than attempt to address all of the differences between the financial and economic approaches to insurance risk pricing, this paper will focus exclusively on the issue of risk diversification and its effect on price. Diversification is a strategy for reducing the risk of an insurer or investor. For insurance, risk diversification relies on the law of large numbers. By insuring a large number of independent identically distributed exposures, an insurer is able to reduce the variance of the average insured damages so that its results become more stable and predictable. The effect of risk diversification in securities markets is similar except that the returns on securities tend to be correlated with one another, thereby limiting an investor's ability to reduce risk. Markowitz (1991) provides a discussion of optimizing portfolio selection in security markets based on the objective of minimizing the investor's variance while simultaneously achieving a selected expected return.

In addition to reducing risk, diversification may also have an effect on price. Based on the principle that risk determines return, the reduction in risk that an insurer can achieve through diversification should be expected to lead to a reduction in the price it requires for a transfer of risk. For securities markets, an explicit measurement of the impact of risk diversification on price is provided by the CAPM pricing formula, which determines the expected return for a security based on its systematic risk and the expected return for the market as a whole. However, since insurance exposures are not actively traded in securities markets, the CAPM approach is not directly applicable to insurance pricing. Despite this, a variety of insurance pricing methods that rely directly or indirectly on the CAPM have been developed. One example is the use of the CAPM for determining underwriting betas, as discussed in Feldblum (1990). Feldblum discounts this particular approach since it "quantifies the risk faced by the investor in insurance stocks, not the risk of the insurer." As an alternative, he proposes an insurance pricing formula (p. 187) analogous to the CAPM in which the market return is replaced by the rate of return on a fully diversified insurance portfolio. One aspect of this formula is that it quantifies the effect of risk diversification on the price for an insurance exposure. On the other hand, Feldblum (p. 189) observes that his proposed approach leaves unanswered the question of whether the insurer should be rewarded solely for its insurance risk or also for unrelated risks such as asset valuation fluctuations. A review of the actuarial literature shows that this issue is also relevant to other insurance risk pricing methods, particularly those based on a total rate of return approach.

Since the CAPM, either directly or indirectly, has been the basis for any number of insurance pricing models, it seems appropriate to consider whether the interpretations and conclusions drawn from the CAPM are valid. In order to provide a fresh perspective to this issue, this paper will address the fundamental principle underlying the CAPM, the concept that risk diversification has an effect on price. The following section considers the effect of risk diversification on the insurer's price for the portfolio as a whole. Pricing for the individual exposures within a portfolio is considered in the remainder of this paper.

Risk Diversification and the Price for the Portfolio

In order to examine the effect of risk diversification on price, consider an insurer that provides coverage to a large number of identically distributed exposures from a single market segment. The notation $X_1, \ldots, X_n$ will be used to represent the insurer's damages from a set of n exposures selected from the market segment. The insurer is assumed to have a systematic, consistent, and non-judgmental procedure for determining a unique price $P(X)$ for each exposure X. This price is required to depend entirely on the risk of each individual exposure, without consideration of the effect of risk diversification on price. For instance, the insurer might evaluate its price by means of an expected utility theory model. Since the insurer needs to be rewarded for risk, the price for each exposure is required to be no less than the expected damages and no greater than the maximum damages so that $E(X) <= P(X) <= max(X)$. This inequality indicates that the insurer expects to earn a profit but that it has the potential to lose money on each transaction.

From the insurer's perspective, the worst possible portfolio is one in which the exposures are perfectly correlated with one another. In this situation, the insurer obtains no benefit from risk diversification since the variance of the average damages per exposure is identical to the variance of any individual exposure, i.e., $V(\sum X_i/n) = V(X_1)$. Since each exposure is priced for its own risk, the insurer's premium for the portfolio is $\sum P(X_i)$ or $nP(X_1)$. In comparison, if the insurer's portfolio consists of n independent exposures, the variance of the average damages per exposure is significantly reduced since $V(\sum X_i/n)$ is now equal to $V(X_1)/n$. Due to the greater risk of the first portfolio, the insurer should be willing to insure the second portfolio at a lower price. This effect of risk diversification on the price for the portfolio would also reduce the insurer's price for the individual exposures within the portfolio. As a result, the final price charged for each exposure would depend not only on the risk of the exposure but also on the amount of risk diversification the insurer achieves.

Secondary Market Pricing

The discussion of risk diversification in the previous section raises two issues. The first issue, determining the effect of risk diversification on the price for the portfolio, is beyond the scope of this paper. The second issue, how to determine the price for the individual exposures given the price for the portfolio, is considered below.

The basis for this analysis will be the assumption that all insurance exposures are actively traded in a secondary market (i.e., an insurance exchange) that functions as an intermediary between insurers and the capital markets. Even though this is not a realistic model of insurance markets, the benefit of this approach is that it provides a basis for analyzing the relationship between insurance prices and the capital markets. The secondary market is considered to consist of a large number of buyers and sellers with no market participant being large enough to have an influence on price. The market is required to clear, with each exposure selected by some market participant. The secondary market premium for each exposure is determined by competition within the market subject to certain restrictions to be discussed. Due to the effect of risk diversification on price, the premium for the secondary market as a whole may be less than the sum of the prices that would be charged if each exposure were priced for its own risk. Also, since insurers require a return for accepting risk, the premium for the secondary market as a whole needs to exceed the total expected damages. In order to eliminate opportunities for insurers to earn risk-free returns, the price established in the secondary market can be assumed to determine the price charged in the primary market. At the outset of this analysis, the time value of money will be disregarded by requiring that all premiums and indemnities be paid instantaneously. Transaction and insurer overhead expenses will be disregarded.

Let the exposures $X_1, \ldots, X_n$ represent the entire collection of exposures transferred into the secondary market. Each X_i is a random variable whose outcome x_i represents the actual damages incurred. To improve marketability, each exposure X_i is permitted to be divided into smaller units aX_i, where $0 < a <= 1$. The aggregate exposure for the entire market will be designated as $W = \Sigma X_i$, with P_W being the premium for the secondary market as a whole. P_W may differ from $P(W)$, the insurer's price for W when considered as a single exposure. The premium P_W is required to be no greater than the total of the individual risk premiums $\Sigma P(X_i)$ in recognition of the effect of risk diversification. Also, since insurers need to be rewarded for accepting risk, the premium P_W is required to be not less than $E(W)$ so that $E(W) <= P_W <= \Sigma P(X_i)$. The secondary market premium for an individual exposure X_i will be designated as $P(X_i\,;\,W)$. This premium is assumed to depend exclusively on X_i and the portfolio W. Subjective elements are not permitted to influence the price.

Since the exposures are actively traded, the secondary market prices needs to be consistent with the ability of market participants to take advantage of opportunities for risk-free returns. For example, consider two exposures X_1 and X_2 with premiums of $P(X_1\,;\,W)$ and $P(X_2\,;\,W)$, respectively. If market prices are such that the premium $P(X_1 + X_2\,;\,W)$ for the combined exposure $X_1 + X_2$ differs from the sum $P(X_1\,;\,W) + P(X_2\,;\,W)$ of the premiums for the individual exposures, a market participant may be able to acquire and restructure the exposures in order to earn a risk-free return. Arbitrage opportunities such as this can be eliminated by requiring that prices in the secondary market be additive:

$$(1) \qquad P(X_1 + X_2 ; W) = P(X_1 ; W) + P(X_2 ; W)$$

For the market as a whole, the sum of the premiums for the individual exposures in the portfolio must be equal to the premium for the market portfolio W. Since $\Sigma X_i = W$:

$$(2) \qquad P_W = \Sigma P(X_i ; W) = P(W ; W)$$

The risk margin for each exposure will be defined as the amount of premium in excess of the expected damages. Using the notation M_i to represent the risk margin for X_i, M_i is defined as $P(X_i ; W) - E(X_i)$. Similarly, the risk margin for W is defined as $M_W = P_W - E(W)$. Given these definitions, equation (2) can be restated as:

$$(3) \qquad M_W = \Sigma M_i$$

Based on the price additivity rule in (1), the price for a pro-rata portion of an exposure should be the pro-rata price. This can be demonstrated for any positive integer m since the price additivity rule requires that $P(X_i ; W) = mP(X_i/m ; W)$. Similarly, for any rational number k/m, where k is a positive integer such that $k <= m$, $P((k/m)X_i ; W) = (k/m)P(X_i ; W)$. While this suggests that $P(aX_i ; W) = aP(X_i ; W)$ for all a in the range $0 <= a <= 1$, this paper will adopt a more limited pricing rule that applies only to W. For all multipliers a with $0 <= a <= 1$, it will be assumed that:

$$(4) \qquad P(aW ; W) = aP(W ; W) = aP_W$$

Since $aW + (-a)W = 0$, equation (4) can be extended to all constants a in the range from -1 to 1, and subsequently to any positive or negative value.

In addition, the market price for a certain outcome c is required to be equal to that outcome:

$$(5) \qquad P(c ; W) = c$$

These pricing rules provide the basis for the development of the Systematic Risk Pricing Model.

The Systematic Risk Pricing Model

In order to determine market prices, the first requirement is to consider how each exposure contributes to the risk of the portfolio W. Each exposure X_i can be decomposed into two components $\beta_i W$ and U_i, where:

$$(6) \qquad X_i = \beta_i W + U_i$$

The value for β_i can be selected to ensure that $\beta_i W$ and U_i are uncorrelated with one another:

(7) $\beta_i = Cov(X_i, W) / V(W)$

Mathematically, $\beta_i W$ is the projection of the vector X_i on the vector W, where the covariance function is used as the inner product operator. Since $\Sigma\beta_i = 1$, equation (6) implies that $\Sigma U_i = 0$. This result indicates that the uncertainty arising from the uncorrelated components U_i for each exposure is completely eliminated by diversification over the portfolio. Since this implies that $\Sigma E(U_i) = 0$ and $\Sigma P(U_i\,;\,W) = 0$, this implies the diversifiable risk adds nothing to the risk margin for the portfolio:

(8) $\Sigma [P(U_i\,;\,W) - E(U_i)] = 0$

In essence, the U_i represent a zero sum game. Equation (8) indicates that any surcharge for diversifiable risk included in the premium for exposure X_i would be offset by a credit on the premium for another exposure. Since exposures having positive risk margins on their diversifiable risk components will be more attractive than those having negative risk margins, competition among market participants should be expected to eliminate both the positive and negative risk margins for diversifiable risk. More generally, it will be assumed that the market does not reward diversifiable risk. Consequently, for any exposure U included in but uncorrelated with the portfolio W:

(9) $P(U\,;\,W) = E(U)$

This result can now be used to determine the market price for exposure X_i. Based on equations (6) and (9), the price for X_i is:

(10) $P(X_i\,;\,W) = E(X_i) + \beta_i M_W$

The Systematic Risk Pricing Model in equation (10) determines the price for X_i entirely in terms of its contribution $\beta_i W$ to the systematic (i.e., non-diversifiable) risk of the portfolio, while the diversifiable component of risk U_i makes no contribution to the risk margin for X_i.

This result can also be stated in terms of the standard deviation of the exposure and correlation between the exposure and the market portfolio. As a first step, express β_i as $\rho_i\sigma_i / \sigma_W$, where the correlation coefficient ρ_i is defined as $Cov(X_i, W) / (\sigma_i\ \sigma_W)$. Next, define λ as M_W / σ_W, the risk margin of the portfolio per unit of standard deviation. Substituting these into equation (10) yields an alternate form of the Systematic Risk Pricing Model:

(11) $P(X_i\,;\,W) = E(X_i) + \lambda\rho_i\sigma_i$

Based on this result, the premium for an exposure can be less than the expected damages, that is, $P(X_i\,;\,W) < E(X_i)$, whenever the correlation coefficient with the portfolio is negative. Even though this is contrary to expectations, it arises because the Systematic Risk Pricing Model determines price based on the systematic risk rather than on the total risk of the exposure.

Equation (10) can also be expressed in terms of the relationship between the risk margins for X_i and the portfolio W:

(12) $$M_i = \beta_i M_W$$

Using the results developed above, it can now be shown that the pro-rata pricing rule in (4) applies to every exposure and not simply to the portfolio W. Let X_i be an exposure in W and define Y as aX_i for some a. Equation (10) states that the price for Y is $P(Y\,;\,W) = E(Y) + \beta_Y M_W$. Substituting $E(Y) = aE(X)$ and $\beta_Y = Cov(Y, W) / V(W) = a\beta_i$ into this formula demonstrates that $P(aX_i\,;\,W) = aP(X_i\,;\,W)$.

At this point, it may be worthwhile to briefly review these results. The Systematic Risk Pricing Model in (10) has several elements in common with the Capital Asset Pricing Model. Both models are developed for markets in which exposures are actively traded, and both reward only systematic risk. The CAPM develops its results for an identifiable set of exposures in a real secondary market, while the Systematic Risk Pricing Model presumes the existence of an imaginary insurance exchange. The precise nature of this exchange has not been defined – it may include exposures from only a single market segment or from all market segments combined.

One important difference between the two models is that the Systematic Risk Pricing Model determines the price for an exposure while the CAPM determines the expected rate of return on capital for an investment. As indicated by equations (3) and (12), the Systematic Risk Pricing Model is simply a method for allocating the risk margin for the portfolio to the individual exposures with the portfolio. This is not the only method that can be used to accomplish this result. For example, the standard actuarial expense loading formula allocates a portfolio's risk margin to individual exposures through the use of a loss cost multiplier applied to the expected damages. Other allocation bases, such as standard deviation or variance, could also be used for this purpose.

One of the most interesting aspects of the Systematic Risk Pricing Model is that the insurer's prices are determined without the need to allocate the insurer's capital to market segments. Rather than being a flaw in the model, this demonstrates that insurance exposures can be priced for systematic risk without reference to the capital markets. According to equation (10), the risk margin for an insurance exposure depends on the relationship between the exposure and the portfolio W and on the risk margin M_W for the portfolio, and not on the insurer's cost of capital. The portfolio W may represent a single market segment for one insurer or it may be the complete book of business across the entire insurance industry. The model also indicates that the price for an exposure should be based exclusively on its systematic risk and not on unrelated risks, such as asset valuation fluctuations, to which the insurer is exposed.

As a final observation, it should be noted that the Systematic Risk Pricing Model places only very minimal restrictions on the market premium P_W. The only requirement is that P_W falls within the range from $E(W)$ to $\Sigma P(X_i)$. If P_W is at the lower end of the range, the price $P(X_i\,;\,W)$ for each exposure would be equal to its expected value $E(X_i)$. If P_W is at the upper end of the range, then the secondary market simply redistributes the individual risk premiums $P(X_i)$ among the exposures. In this situation, some premiums could increase while others would decrease in relation to the individual risk prices $P(X_i)$ offered in the primary market. However, whenever the secondary market price $P(X_i\,;\,W)$ for an exposure exceeds its individual risk price $P(X_i)$, the policyholder's willingness to participate in the secondary market may be affected.

The Variance Pricing Formula for Independent Exposures

For independent exposures, the Systematic Risk Pricing Model implies that the insurer's risk margin for each exposure should be proportional to the variance of each exposure. To demonstrate this result, evaluate β_i under the assumption that the exposures are independent:

$$(13) \qquad \beta_i = Cov(X_i, W) \,/\, V(W) = V(X_i) \,/\, V(W) = \sigma_i^2 \,/\, \sigma_W^2$$

Next, define k as $M_W \,/\, \sigma_W^2$, the portfolio's risk margin per unit of variance. This can be substituted into (10) to obtain the variance pricing formula:

$$(14) \qquad P(X_i\,;\,W) = E(X_i) + k\sigma_i^2$$

Miccolis (1977) provides an application of the variance pricing formula to the pricing of liability increased limits factors. The difference between this result and the approach described by Miccolis is that the Systematic Risk Pricing Model determines the value for k based on the risk margin M_W for the portfolio, whereas Miccolis determines the value for k based on a judgmentally selected risk margin for the basic limits policy.

As an application of this result, suppose that an insurance advisory organization (i.e., ISO) provides both $E(X_i)$ and σ_i^2 at each policy limit. Given this information, the insurer can select an arbitrary value for the parameter k in order to determine $P(X_i\,;\,W)$ at each policy limit. Based on these results, the insurer's increased limits factor at a selected policy limit can be defined as the value of $P(X_i\,;\,W)$ at the higher limit divided by the corresponding value at the basic limit. This approach enables an insurer to revise its increased limits factors without the need to develop revised estimates of $E(X_i)$ and σ_i^2.

The Insurance Analogue to the Capital Asset Pricing Model

The next topic to be considered is the relationship of the Systematic Risk Pricing Model to the Capital Asset Pricing Model. According to Brealey and Myers (1996), the CAPM states that the expected rate of return for a security is determined by the security's beta:

$$(15) \qquad E(r_s) - r_f = \beta_s\,(E(r_M) - r_f)$$

where:

$$(16) \qquad \beta_s = Cov(r_s, r_M) \,/\, V(r_M)$$

and r_s and r_M represent the rate of return on the security and the market, respectively. The value r_s is generally described as the risk adjusted rate of return or the cost of capital.

In order make a comparison between the two models, the Systematic Risk Pricing Model needs to be restated in terms of its implied rate of return. For any particular outcome x_i of X_i, define R_i as the observed return $P(X_i\,;\,W) - x_i$ for that outcome. Since $E(R_i) = M_i$, the expected return is simply the risk margin in the premium. Similarly, let R_W represent the observed return $P_W - w$ for the market segment given the outcome w so that the expected return is $E(R_W) = M_W$. With these definitions, equation (12) can be restated as:

$$(17) \qquad E(R_i) = \beta_i\, E(R_W)$$

Next, express the expected returns R_i and R_W as rates of return in relation to the price for the exposure. Define $r_i = R_i / P(X_i\,;\,W)$ and $r_W = R_W / P(W)$ and substitute in the equation above:

$$(18) \qquad E(r_i)P(X_i\,;\,W) = \beta_i\, E(r_W)P(W)$$

To be consistent with the CAPM, define β'_i as:

$$(19) \qquad \beta'_i = Cov(r_i\,,\,r_W) \,/\, V(r_W)$$

Since β'_i is equal to $\beta_i P(W) / P(X_i\,;\,W)$, equation (18) can be restated as:

$$(20) \qquad E(r_i) = \beta'_i\, E(r_W)$$

This result represents the insurance analogue to the Capital Asset Pricing Model. Since the damages for each exposure are paid at time 0, the adjustment for the time value of money in equation (15) is unnecessary. However, in order to analyze the two models on a consistent basis, assume that the premium and the damages for each exposure X_i in market segment W are paid at time 1 rather than at time 0. Let $P(X_i\,;\,W)$ be the market price for X_i at time 1 as determined by equation (10). Since the premium is a constant, it can be discounted to present value at the risk-free rate. Denote the discounted premium for X_i as $PV_i = vP(X_i\,;\,W)$. Similarly, let $PV_W = vP_W$ be the discounted premium for the portfolio. Next, reverse the sign on the damages so that cash outflows are treated as negative values. This adjustment is needed for consistency with the CAPM treatment of investment gains as positive values. Finally, define the rate of return for an exposure as the return earned at the end of the period divided by the price for the exposure at the start of the period. On this basis, the rate of return on X_i is defined as $r_i = (x_i - PV_i)/PV_i$ while the rate of return r_W on the portfolio is $(w - PV_W)/PV_W$. Given these definitions, and with β'_i defined as in (19), the Systematic Risk Pricing Model can be used to show that:

$$(21) \qquad E(r_i) - r_f = \beta'_i\,(E(r_W) - r_f)$$

This structure of this result is identical to that of the Capital Asset Pricing Model formula in equation (15). The primary difference between this formula and the CAPM is that the rate of return in (21) is defined in relation to the price for the exposure rather than in relation to the amount of capital invested. This result demonstrates that the exposure's expected rate of return has no relationship to its cost of capital. Instead, the rate of return in (21) is simply the insurer's profit margin, that is, the expected profit divided by the premium for the exposure. It should also be noted that equation (21) was obtained by discounting the premium rather than the uncertain damages for the exposure. For this reason, the rate of return in equation (21) does not represent the risk-adjusted rate at which uncertain future cash flows for an insurance exposure can be discounted to present value.

To complete this analysis, the relationship between the insurance pricing formula in (21) and the CAPM formula in (15) needs to be addressed. Since the rationale used to develop the Systematic Risk Pricing Model can also be applied to security pricing, the two models should be consistent with one another. The primary difference between the models can be immediately reconciled by recognizing that the price for a security in a secondary market is equivalent to the amount of capital invested. In other words, the return on capital for a security is also its return on price. One issue this raises is that the conclusion from the previous paragraph can also be applied to security pricing. This result shows that the interpretation of the CAPM rate of return as the risk-adjusted rate at which uncertain future cash flows can be discounted to present value is inconsistent with the Systematic Risk Pricing Model.

Discounting Future Cash Flows to Present Value

In the previous section, the different interpretations of the Capital Asset Pricing Model and the Systematic Risk Pricing Model were reconciled by recognizing that the amount of capital invested in a security is equivalent to its price. A second difference that needs to be addressed is the relationship between time and risk in the two models. For the Capital Asset Pricing Model, time is treated as an essential element of risk. Specifically, the CAPM rate of return represents the rate at which uncertain future cash flows are discounted to present value. In comparison, the Systematic Risk Pricing Model considers time and risk to be independent of one another. The difference between the two models is illustrated by equation (20), in which the Systematic Risk Pricing Model has been used to determine the rate of return for an exposure whose outcomes are paid at time 0. Since the exposure has no time element, the CAPM cannot be used to determine its rate of return.

One issue that arises from the independence of time and risk in the Systematic Risk Pricing Model is that the model provides no information on how to discount uncertain future cash flows to present value. In order to investigate this issue, recall that equation (10) determines the relationship between the price for each exposure and the portfolio risk margin $M_W = P_W - E(W)$. Provided that the portfolio price P_W meets certain reasonability conditions, it may be possible to determine how uncertain future damages should be discounted to present value. Let W_0 and W_1 be two portfolios having identical damage distributions except that the damages for W_0 are paid at time 0 while the damages for W_1 are paid at time 1. The first assumption is that the price for the portfolio is independent of time. If P_0 and P_1 represent the insurer's pricing functions at times 0 and 1 respectively, this requires that the price $P_0(W_0)$ at time 0 be identical to the price $P_1(W_1)$ at time 1. Second, for positive values k close to 1, the price for a portfolio kW is assumed to be k times the price for the original portfolio, $P(kW) = kP(W)$. By substituting v for k, where v is the discount factor corresponding to the risk-free rate r_f, this implies that $vP_0(W_0) = P_1(vW_1)$.

To apply these assumptions to the pricing of an individual exposure within the portfolio, let X_1 be an exposure with damages paid at time 1 and let X_0 be the identical set of damages paid at time 0. Since the premium at time 1 for exposure X_1 is $P_1(X_1 ; W_1)$, the premium payable at time 0 is $vP_1(X_1 ; W_1)$. This premium can be compared to the premium based on discounting the uncertain damages to present value at the risk-free rate. The discounted payments for X_1 at time 0 are represented by vX_0 while the discounted payments for W_1 are vW_0. In accordance with equation (10), the premium at time 0 for the discounted damages is:

$$(22) \qquad P_0(vX_0 ; vW_0) = E(vX_0) + \beta_0 (P_0(vW_0) - E(vW_0))$$

By applying the portfolio pricing assumptions, this can be expressed as:

$$(23) \qquad P_0(vX_0 ; vW_0) = v[E(X_0) + \beta_0 (P_0(W_0) - E(W_0))]$$

The right hand side of this formula is equal to $vP_0(X_0 ; W_0)$. Since the values for β at time 0 and time 1 are identical, equation (10) ensures that $P_0(X_0 ; W_0)$ is identical to $P_1(X_1 ; W_1)$. Based on this result, the premium $vP_1(X_1 ; W_1)$ for the exposure X_1 at time 0 is equivalent to the premium $P_0(vX_0 ; vW_0)$ obtained by discounting the uncertain future damages X_1 and W_1 to time 0 at the risk-free rate.

Given the two reasonability assumptions regarding the portfolio price, the preceding analysis has shown that the price for the uncertain future cash flows can be obtained by discounting each outcome to present value at the risk-free rate. This result reaffirms that the rate of return shown in equation (21) does not represent the rate at which uncertain future cash flows should be discounted to present value. While this conclusion has been developed from the context of Systematic Risk Pricing Model, it also applies to the CAPM. Accordingly, the assumption that the CAPM rate of return can be used to discount uncertain future cash flows to present value needs to be abandoned.

The Capital Asset Pricing Model

In equation (21), the insurance analogue to the Capital Asset Pricing Model, r_i has been defined as $(x_i - PV_i)/PV_i$. This expresses the rate of return for an exposure X in terms of the uncertain future outcome x_i and the price PV_i at time 0 for the exposure. The systematic risk of the exposure, represented by $\beta_i W$, is based on the correlation between the outcomes for the exposure and the outcomes for the secondary market W. In comparison, the CAPM defines the rate of return for a security in terms of its price at two points in time. More specifically, the CAPM rate of return is defined as $r_X = (P_{X1} - P_{X0})/P_{X0}$, where P_{X0} and P_{X1} represent the price for a security X at times 0 and 1 respectively. The systematic risk of the security is based on the correlation between the rate of return r_X for the security and the rate of return $r_W = (P_{W1} - P_{W0})/P_{W0}$ for the market W as a whole. At first glance, the two methods for pricing for systematic risk appear to be reasonably consistent with one another. However, a more careful examination of the both methods leads to the identification of a number of significant conceptual problems with the CAPM. These problems are severe enough to undermine the validity of the CAPM as risk pricing model.

The most basic problem with the CAPM is that the expected rate of return for a security is determined without any consideration being given to the performance of the business that underlies the security. Since a security represents ownership of the income generated by a business, a relationship between the performance of the business and the price for its security is essential. For insurance exposures, the Systematic Risk Pricing Model determines the price for an exposure based on the uncertainty of its insured damages rather than on the variability of its price in the secondary market. Similarly, the risk for a security should be based on the uncertain future cash flows (e.g., stockholder dividends) for the business underlying the security rather than on the variability of the price for the security in the secondary market. In essence, the CAPM mistakenly substitutes the price variability of an exposure in the secondary market for the uncertainty of the future cash flows from the business. Price variability over time is not the proper measure of the risk of an exposure.

The CAPM suffers from a second problem that arises out of its relationship between present and future prices. In the CAPM approach, the price P_{X0} for a security at time 0 adjusts to ensure that the security achieves its cost of capital. Since the cost of capital expresses the relationship between the current and future price, this implies that the current price P_{X0} for a security is a function of the distribution of its future price P_{X1}. However, if similar reasoning is applied to any specific future price P_{X1}, the price P_{X1} for the security at time 1 depends on the distribution of its price P_{X2} even further in the future. This establishes an iterative and indeterminate process for determining the price for a security. The pricing procedure fails because price is treated as both an input and an output of the analysis. The Systematic Risk Pricing Model avoids this problem by evaluating price in terms of the true risk of an exposure rather than in terms of the variability of the price of the exposure over time.

An additional problem with the Capital Asset Pricing Model is that the CAPM approach is based on a fundamental misinterpretation of a security as a risk exposure. For an insurance policy, the risk of an insured exposure is essentially static throughout the policy term. That is, the insurer expects the risk characteristics of the exposure throughout the policy period to be consistent with the insurer's expectations at the time the policy is issued. If the risk characteristics for a policy change mid-term, the insurer often has a contractual right to cancel coverage. In comparison, a security represents ownership of the future income of a business. Unlike an insurance policy, a business is dynamic in the sense that the company management can take actions that affect its future income. Actions that might influence the risk profile of a business include product pricing changes and decisions to enter or exit individual market segments. Since a business has a measure of control over its risk profile, a business cannot be interpreted as a static risk exposure. More properly, a business represents a risk exposure X_0 at one point in time and a different risk exposure X_1 at another point in time. Since the risk profile of a business can change over time, there is no reason to think that the CAPM β value for its security will remain stationary over time. However, without a stable value for β, the CAPM formula is not useful for determining the expected rate of return for a security.

Another problem with the CAPM becomes apparent based on a direct comparison between the Systematic Risk Pricing Model and the CAPM. In order to determine the CAPM β for a security, the correlation between the rate of return for the security and the rate of return for the market as a whole needs to be evaluated. However, the Systematic Risk Pricing Model in equation (10) considers the price $P_X = P(X\,;\,W)$ for the exposure X and P_W for the market W to be constants rather than random variables. If the price for a security is not a random variable, neither is the security's rate of return in the secondary market. Accordingly, the correlation between the two rates of return, and consequently the CAPM β for the security, is not a meaningful concept.

Since the Capital Asset Pricing Model is based on portfolio selection theory, the observation that the CAPM is invalid also raises a challenge to portfolio selection theory. Portfolio selection theory describes how to construct a portfolio with the least risk for a selected expected rate of return. In order to apply this procedure, the investor needs to know the mean and variance of the rate of return for each security in the secondary market. The problem with this statement is that the types of information available to the investor are more likely to be related to the performance of the business underlying the security than they are to the rate of return for the security in the secondary market. For instance, an investor may have information on prospective economic conditions that can be anticipated to affect the income for a business in the current year. Whether or not this information has already been incorporated into the market price for the security may not be evident. If the information is consistent with the prior assumptions that were used to set the current market price, it may have no effect on the future rate of return for the security. On the other hand, the information may alter the perceived risk profile of the business so that the initial risk exposure X_0 is transformed into a new risk exposure X_1. In this situation, the information may have an effect on the security's future rate of return.

The problems discussed above can also be demonstrated mathematically in the derivation of the CAPM from the Systematic Risk Pricing Model. Consider a security X that has an unknown selling price of P_{X1} versus an original purchase price of P_{X0}, and let the rate of return for X be defined as $r_X = (P_{X1} - P_{X0})/P_{X0}$. Since the future prospects for the business underlying the security can change over time, the investor can be considered to own a share of the original exposure X_0 at time 0 and a different exposure X_1 at time 1. The price for X at each point in time is determined in a secondary market W, or more accurately, a secondary market W_0 at time 0 and a different secondary market W_1 at time 1. The price at each point in time, $P_{X0} = P_0(X_0 ; W_0)$ and $P_{X1} = P_1(X_1 ; W_1)$, can be developed from the Systematic Risk Pricing Model for times $t = 0$ and $t = 1$ such that $P_t(X_t ; W_t) - E(X_t) = \beta_t(P_t(W_t) - E(W_t))$ for each value of t. In this formulation, X_t and W_t are random variables that represent the underlying primary exposures. However, in order to develop the CAPM, the random variables X_t and W_t need to be treated as constants while the corresponding prices $P_t(X_t ; W_t)$ and $P_t(W_t)$ are treated as variables. Following this reasoning, define $y(t) = P_t(X_t ; W_t)$ and $z(t) = P_t(W_t)$. Since the risk of the exposure and the risk of the secondary market both change over time, the value of β can also change over time. For this analysis, make the assumption that β_t can be considered to be constant over brief periods of time so that $d\beta_t/dt = 0$. From this, it follows immediately that $dy/dt = \beta_t\, dz/dt$. This indicates that the instantaneous rate of change of price $y(t)$ for the exposure X in the secondary market is proportional to the instantaneous rate of change of $z(t)$, the price for the market as a whole, where the proportionality constant β_t is evaluated in terms of the underlying exposures X_t and W_t, consistent with the Systematic Risk Pricing Model.

A different perspective on this result can be obtained by treating the prices $P_t(X_t ; W_t)$ and $P_t(W_t)$ for $t = 0$ and 1 as random variables rather than as real valued functions of time. For this analysis, let Y_t be defined as $P_t(X_t ; W_t)$ and Z_t as $P_t(W_t)$. An immediate problem with this analysis is that equation (10) requires Y_t and Z_t to be perfectly correlated with one another. To circumvent this issue, introduce independent error terms ε_t with an expected value of 0 into the Systematic Risk Pricing Model so that the price for each exposure at the two points in time can be expressed as:

(24) $$Y_0 - E(X_0) = \beta_0(Z_0 - E(W_0)) + \varepsilon_0$$

and:

(25) $$Y_1 - E(X_1) = \beta_1(Z_1 - E(W_1)) + \varepsilon_1$$

Under the assumption that the expected cash flows for W_0 and X_0 both increase at the risk-free rate so that $E(X_1) = (1 + r_f)E(X_0)$ and $E(W_1) = (1 + r_f)E(W_0)$, and assuming that the proportionality term is constant over time so that $\beta_0 = \beta_1, = \beta$, it follows immediately that:

(26) $$(Y_1 - (1 + r_f)Y_0) = \beta(Z_1 - (1 + r_f)Z_0) + \varepsilon_1 - \varepsilon_0$$

where $\beta = cov(X_1, W_1)/V(W_1)$. Using equation (25), β can be expressed in terms of the price random variables Y_1 and Z_1, so that $\beta = cov(Y_1,Z_1)/V(Z_1)$. After applying the expectation operator (and removing unnecessary parentheses), equation (26) can be restated as:

(27) $$EY_1 - (1 + r_f)EY_0 = \beta(EZ_1 - (1 + r_f)EZ_0)$$

Next, recall that the CAPM formula defines beta as $\beta' = cov(r_Y,r_Z)/V(r_Z)$ where $r_Y = (Y_1 - Y_0)/Y_0$ and $r_Z = (Z_1 - Z_0)/Z_0$. If Y_0 and Z_0 can be considered to be constants, then β' can be evaluated as $\beta Z_0/Y_0$. Substituting this into equation (27) gives $(EY_1 - Y_0)/Y_0 - r_f = \beta'((EZ_1 - Z_0)/Z_0 - r_f)$, which can be expressed more succinctly as:

(28) $$E(r_Y) - r_f = \beta'(E(r_Z) - r_f) \quad \text{where } \beta' = cov(r_Y,r_Z)/V(r_Z)$$

This completes the proof of the CAPM formula. It should be noted that the proof relies on a number of questionable assumptions. The most questionable of these is that the variability of the price for the security over time, rather than the uncertainty of the underlying exposure itself, is the proper measure of the risk of an exposure. In addition, the proof requires β to be constant over time so that $\beta_0 = \beta_1$. It also assumes that $E(X_1) = (1 + r_f)E(X_0)$, and $E(W_1) = (1 + r_f)E(W_0)$. While these conditions might be reasonable approximations over brief periods of time, they are not likely to be valid over longer periods. For example, β (and hence β') will immediately change each time a new exposure enters the secondary market. If β is not constant, the CAPM is not a useful method for security market pricing. The CAPM proof also requires that the prices Y_0 and Z_0 at time 0 to be known values. If the current prices are known, this negates the value of the CAPM as a means for evaluating the current price for a security based on its future cash flows.

Conclusion

Financial theory suggests that prices should reflect systematic risk only, with no compensation given to the diversifiable risk of each exposure. By applying this principle to the pricing of insurance exposures actively traded in a secondary market, the Systematic Risk Pricing Model in equation (10) has been derived. This model determines the price for an exposure based on its contribution to the risk of the portfolio in which it resides. Given the price for the portfolio, the price for each exposure within the portfolio can be determined without reference to the insurer's or the exposure's cost of capital. The Systematic Risk Pricing Model can be interpreted as being simply a method for allocating the risk margin for the portfolio to the individual exposures within the portfolio. In the special case where the exposures are independent, the risk margin for each exposure is proportional to its variance. Other methods, such as the standard actuarial loss cost multiplier approach, can also be used for this purpose.

The Systematic Risk Pricing Model differs from the Capital Asset Pricing Model in that it determines the price rather than the rate of return for each exposure. A careful examination of the CAPM leads to the identification of a number of significant conceptual problems within the model, the most serious of which is that the model substitutes the variability of the price of an exposure over time for the true risk of the exposure. It also relies on the questionable assumption that the value of the CAPM β is stable over time. Due to these problems, the CAPM cannot be considered to be a valid risk pricing model.

The principle that exposures should be priced solely on the basis of their systematic risk, as described in this paper, is also open to interpretation. Based on its construction, the Systematic Risk Pricing Model is relevant for exposures that are actively traded in a secondary market. Since this is not a realistic assumption for insurance markets, the Systematic Risk Pricing Model may not be the most realistic method for determining prices for insurance exposures. Other methods for allocating the risk margin of the portfolio to the individual exposures within the portfolio, such as the use of loss cost multipliers, may be more suitable.

A related issue with regard to pricing for systematic risk is the size of the portfolio over which the insurer's systematic risk is evaluated. Due to the insurer's ability to reduce its risk by insuring a large number of independent exposures, the insurer's price should decrease in response to its success in diversifying risk within each market segment. What may not be as evident is that the insurer's ability to diversify its risk across market segments need not have an effect on the insurer's price. This issue is addressed in the companion piece to this paper, "The Cost of Conditional Risk Financing." As described in that paper, the risk pricing function for a well-diversified insurer that retains the benefits of risk diversification across market segments can be completely determined provided that the insurer operates under a capital preservation objective. After the insurer uses its risk pricing function to determine its premium for each market segment, the Systematic Risk Pricing Model or another model can be applied to determine the premium for the individual exposures within each market segment. In combination, the two pricing models are capable of completely determining the price an insurer should charge for each exposure.

Author's note: This paper is based on material presented at the 11th AFIR Colloquium.

References

[1] Brealey, R. A., and Myers, S.C., *Principles of Corporate Finance, Fifth Edition.* New York: The McGraw-Hill Companies, Inc. 1996

[2] Bingham, R.E., "Rate of Return – Policyholder, Company, and Shareholder Perspectives." Proceedings of the Casualty Actuarial Society, Volume LXXX, Nos. 152 and 153, 1993.
http://www.casact.org/pubs/proceed/proceed93/93110.pdf

[3] Borch, K.H. *Economics of Insurance.* Aase, K.K. and Sandmo, A. (Ed.), New York: North-Holland, 1990.

[4] D'Arcy, S.P., and Dyer, M.A., "Ratemaking: A Financial Economics Approach." Proceedings of the Casualty Actuarial Society, Volume LXXXIV, pp. 301-390, 1997.
http://www.casact.org/pubs/proceed/proceed97/97301.pdf

[5] Feldblum, S., "Risk Loads for Insurers." Proceedings of the Casualty Actuarial Society, Volume LXXVII, pp. 160-195, 1990.
http://www.casact.org/pubs/proceed/proceed90/90160.pdf

[6] Feldblum, S., "Pricing Insurance Policies: The Internal Rate of Return Model." Casualty Actuarial Society, Study Notes, Part 9, May 1992.
http://www.casact.org/library/studynotes/feldblum9.pdf

[7] Grimmett, G.R. and Stirzaker, D.R., *Probability and Random Processes, Second Edition.* Oxford, England: Oxford University Press, 1992.

[8] Mahler, H., "Investment Income in Ratemaking in Massachusetts (Seminar on Profitability, 4/91)."
http://casact.org/pubs/forum/91fforum/91ff197.pdf

[9] Markowitz, H.M. Portfolio Selection: Efficient Diversification of Investments (Second Edition). Malden, Massachusetts: Blackwell Publishers, 1991.

[10] Miccolis, R.S. "On the Theory of Increased Limits and Excess of Loss Pricing." *Proceedings of the Casualty Actuarial Society*, Volume LXIV, pp. 60-73, 1977.
http://www.casact.org/pubs/proceed/proceed77/77027.pdf

[11] Schnapp, F.F., "The Diversification Property." Presented at the 11th AFIR Colloquium, Toronto, Canada, September 2001.

[12] Schnapp, F.F., "The Effect of Risk Diversification on Price." Presented at the 11th AFIR Colloquium, Toronto, Canada, September 2001.

[13] Schnapp, F.F., "The Cost of Conditional Risk Financing." CAS Forum, pp. 121-151, Winter 2004.
http://www.casact.org/pubs/forum/04wforum/04wf121.pdf

[14] Starmer, C. "Developments in Non-Expected Utility Theory: The Hunt for a Description Theory of Choice under Risk." *Journal of Economic Literature*, XXXVIII, June 2000, 332-382

The Seventh Game—
An Example of Pricing Arbitrage

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The Seventh Game—An Example of Pricing Arbitrage

Tom Struppeck, FCAS, MAAA

Two friends of mine happen to each have a pair of tickets to the seventh game of the World Series between the Chicago Cubs and the Boston Red Sox. For those who don't know, the World Series is the annual championship of Major League Baseball, and it (currently) is a best of seven series[1]. What that means is that the first team to win four games wins the Series and play stops. In baseball, there can be no ties[2], so if the seventh game is played it will be the final game, but the Series could end before the seventh game.

I happen to know that if there is a Game 7, a pair of tickets will be worth \$1000. If there is no Game 7, a pair of tickets can be returned for a refund of \$300. How much are the tickets worth before Game 1 is played?

What is the value of something? For our purposes, we will assume that the value of a ticket is the amount that a willing buyer and willing seller would agree upon as a price. This is the "fair market price." In the problem posed, my friends would be the sellers. They know that if there is a Game 7, they could sell their pair of tickets for \$1000, but would they want to? I don't know, perhaps they would not be willing sellers. To get around this problem, let's assume that we know that we could either sell <u>or buy</u> the tickets immediately before Game 7 (if there is one) for \$1000.

One of my friends, Steve, works in an office where there are fans of both teams. Before any game he can, if he chooses to, place a bet on that game on either team for any amount of money at even odds.

In Steve's office, the tickets must sell for exactly \$518.75 for the pair[3]. Let's see why.

Suppose that five games have been played and that the Cubs have won three and the Red Sox have won two (the case when the Red Sox lead, is similar). Now if the Cubs win again, Steve's tickets are worth \$300 (because the series is over) while if the Red Sox tie the series, Game 7 will be played and his tickets will be worth \$1000. Steve is \$700 better off if the Red Sox win. Such a big swing in value makes Steve nervous.

Steve has an idea. He approaches one of the Red Sox fans in his office and asks, "Who's going to win Game 6?" The Red Sox supporter answers, "Why the Red Sox, of course!" Steve says, "Here's \$350, if the Red Sox win you keep it, if they don't you pay me \$700." The Red Sox supporter answers, "Done!"

This looks like a good deal to Steve, if the Cubs win he gets the \$300 refund and he collects \$700 from his colleague, \$1000 total. If the Red Sox win, he has a pair of tickets

[1] The 1903 Series was best-of-nine and ended in eight. The Series was also best-of-nine from 1919-21.
[2] Well, okay, maybe there can be in the All-Star Game.
[3] Since this amount has an odd number of cents, a transaction for one ticket at the fair price isn't possible.

worth $1000 total. Steve can put himself in this happy position for a cost of $350. If he does this, Steve won't care what happens in Game 6. He pays $350 and his tickets and bet together are worth $1000, so each pair of tickets must be worth $650 at the start of Game 6.

We are making progress. We now know the value of the tickets at the start of Game 6. What about at the start of Game 5? Well, if the series is tied, 2 games each, we know that no matter what happens, it will be 3 games to 2 next and that our tickets are worth $650 in that case, so they must be worth $650 at the start of Game 5 (if it's 2-2). What if it's three games to one? Again, there are two possible outcomes, either the series will end and Steve's tickets will be worth $300 or the team that has won one game will win and we will have a 3-2 situation, which is the case we just valued (the tickets are worth $650).

Just as before, Steve can place an even-money bet to make himself ambivalent as to which team wins. This time he will bet $175 on the team that is ahead. If they win, he gets $350 and has a pair of tickets worth $300 for a total of $650; if they lose, he has tickets worth $650. The cost of this insurance is $175, so the value of the tickets before Game 5 is played (if the series is 3-1) is $475.

The reader should continue this process to see that the price before Game 1 for the pair of tickets is $518.75.

Steve's bookies think that he is odd. He had no interest in betting on Game 1, but suddenly wanted to bet on Game 2 for a very specific amount on the team that was ahead. As it happened, he lost that bet. The series was now tied and again, he had no interest in betting on Game 3, but when Game 4 came along, he was suddenly interested in betting even more than before (and on the other team as it happened, since they were now ahead in the series). Again, he lost. What upsets his bookies the most is that he doesn't care whether he wins or loses these bets!

I said that I have two friends with pairs of tickets, the other is Glenn. Glenn works in a different office from Steve and, as it happens, his coworkers also like to bet. Glenn, too, can buy or sell a pair of tickets for $1000 if there is a Game 7 and can get the $300 refund if Game 7 is not held. Glenn's coworkers have reviewed the history of the Series and believe that there is a 50/50 chance that Game 7 will be held. In Glenn's office, before Game 1 is held a pair of Game 7 tickets must sell for $650. This is because Glenn, like Steve before, could bet $350 against there being a Game 7. If there is a Game 7, he has tickets worth $1000 and if there is no Game 7, he has tickets with a refund value of $300 and he wins his side bet and collects $700 --- again $1000. The cost of this insurance is $350, so the tickets must be worth $650.

I have lunch with Steve and Glenn (before Game 1) and they each tell me about their tickets and how much they are worth. In Steve's office, the tickets are worth $518.75; in Glenn's office they are worth $650. In order to pay for lunch, I will buy tickets from Steve for $518.75 a pair and sell them to Glenn at $650 a pair. This will pay for lunch (and after rescaling, much more). We've all heard the expression that there are no free lunches, but here we have one; we have found an inter-market arbitrage.

The people in Steve's office offer him even money bets on each game. We do not know the actual probabilities for the Cubs winning a given game --- the bookies in Steve's office may not either. It doesn't matter. So long as Steve knows that he can place his bets at those odds before each game, he can guarantee a payoff of $1000 if there is a Game 7 and $300 if there is no Game 7 for an initial cost of exactly $518.75.

Some questions to consider:

1) Suppose the bookies in Steve's office offer two to one odds on the visiting team and one to two odds on the home team. How does this change the price of his tickets? The home teams are scheduled: Red Sox, Red Sox, Cubs, Cubs, Cubs, Red Sox. Does the order matter?
2) In the original problem, as I keep buying from Steve and selling to Glenn what eventually happens? Why?
3) Suppose that in the original problem the bookies in Glenn's office change their odds and decide to offer bets at two to one against there being a game 7, now what should Steve, Glenn, and I do to get our free lunches?

Estimation of Trends Using Ordinary Differential Equations: An Application to Occupational Injuries

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Estimation of Trends using Ordinary Differential Equations: An Application to Occupational Injuries

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Abstract

In this paper, we present and discuss the dynamic time varying version of trend estimation. These models often underlie the analytical functions that are used in practice by actuaries and economists. We also show how one of the most frequently used softwares (the SAS systems) by practitioners and researchers can be used to fit the dynamics to data. An alternate formulation of the Ordinary Least Squares (OLS) is also given. Using this technique, we analyze occupational injury and illness data from 12 countries. The results for most countries have shown an average decline.

I. Introduction

In this paper, we estimate trends in occupational injuries and illnesses for 12 countries using the dynamics as represented by the ordinary differential equations (ODEs)[1]. We show that the most commonly employed models in practice are analytical solutions of the basic differential equations. Differential equations are used in many applications in real life such as engineering. For the cases of the linear and exponential trend models, we demonstrate that these models yield the same results.

To provide further insight into the modeling and estimation of trends, we present and discuss in some detail the relationship between the continuous time dynamics of the time series variable (injuries) and their analytical solutions. In other words, we want to highlight the link between the continuous time dynamics and their solutions, which are often used in regression analysis. Dynamic estimation, i.e., fitting models that are represented by equations that describe the time evolution of the economic/actuarial variables, is also performed (Ussif, Sandal and Steinshamn, 2002a, b). In areas such as oceanography and meteorology such dynamic parameter estimation technique is called data assimilation (Evensen, Dee and Schroeter, 1998, Matear, 1995). We use the SAS dynamic estimation capability (see the SAS Institute's online documentation) which is not provided by most software packages (see also Ussif et al., 2002 a, b). The goal is to point out that such a capability exists and can be used to perform more advanced dynamic systems estimation. This can be used to fit nonlinear dynamic systems that arise from the relaxation of the linearity assumptions often made in economics. Pesaran and Potter

[1] Please note the differences in the number of observations available for each country. This should however note be a problem in this application.

(1992) in their introductory notes on nonlinear dynamics and econometrics argue that the rich dynamics in nonlinear models in economics be explored.

The structure of this paper is as follows. In the next section, we discuss the dynamic representation of the models. This is followed by the estimation of the trend coefficients using SAS software and a discussion of the results. A technical note on the adjoint method is also presented. We then summarize and conclude the paper.

Dynamic representation of the trend models

In this section, we show how the purely linear and exponential functions of time that are being used for trend estimation can be derived as solutions of their corresponding continuous time dynamic equations, i.e., equations used to describe how systems change or evolve over time. This is important because understanding the relationships can be very useful to economists and researchers. It is often the case that reality necessitates the relaxation of the linearity assumptions in economics giving rise to nonlinear dynamic systems. Analytical solutions of these systems are in general unattainable for some relatively more complicated dynamics and the only method of estimation may be the dynamic approach. The dynamic capability is quite rare in most software packages in part because conventional economic analysis has in the past focused on simpler models.

Linear trend function: In the dynamic continuous time formulation, it is assumed that the absolute change with respect to time of the series is equal to a constant. That is, the average growth is constant during the period. Hence, the dynamics are given by

$$\frac{dy}{dt}=\beta,\ y(0)=\alpha \qquad (1)$$

where α is the initial value of the series. This is equivalent to assuming that $\frac{d^2y}{dt^2}=0$, where $\frac{dy(0)}{dt}=\beta$, $y(0)=\alpha$ are the initial conditions. It is quite easy to see that this equation has the solution

$$y_t=\alpha+\beta t \qquad (2)$$

which is the linear trend function in time. Thus, we can view the estimation of the parameters in (1) as fitting the solution (2) to a discrete data set. Note that $y(t)$ and y_t are used interchangeably in this case.

Exponential trend function: The dynamics in this case can be described by

$$\frac{dy}{dt}=\beta y,\ y(0)=y_0=e^{\alpha} \qquad (3)$$

that is, the percent growth rate is equal to a constant or that the absolute change is proportional to the current value of the series. We denote by y_0 the initial condition for the problem. It can be seen by inspection that this equation has the solution

$$y_t=\exp(\alpha+\beta t). \qquad (4)$$

This is the familiar exponential trend function of time used in estimating trends and growth rates. Its advantage is that the estimated coefficient is the average growth rate. The linear and exponential functions of time are often used in economics, business, and finance to forecast trends. Recent advances and progress in most statistical software packages allow us to fit nonlinear regression equations using nonlinear least squares techniques without having to use, for example, logarithmic transformation.

The SAS software has the additional capability of fitting dynamic systems to data without requiring that analytical or closed form solutions be available. This is important and very useful because for some relatively complicated dynamics, closed form solutions are often not attainable. Hence, dynamic estimation becomes the only option available. In dynamic estimation, the parameters of interest are estimated by fitting the dynamic equations rather than their solutions.

To illustrate the use of the dynamic estimation capability of SAS, we fit (see sample program) the dynamics represented by equation (3) and the results are compared with the results of the purely exponential function in Table 1. It can be observed that the results are consistent with those obtained using the log transformed function (shown in the log-linear column of Table 1). The agreement is quite impressive both qualitatively and quantitatively. The dynamic estimation was performed using the procedure "proc model" in SAS. Included is a sample SAS program for interested readers. In static dynamic option, the initial data point is used as the initial condition of the differential equation, while in the dynamic option; the initial condition(s) is estimated as an additional parameter. The nice thing about this procedure is that the dynamics are written as they are seen in the model equations. It is very important to understand the difference between the static and dynamic options when fitting dynamic models to data. For further details, readers are referred to the SAS manual (www.sasonline.com).

Technical note

It is the goal in this note to describe an alternative technique for estimating dynamic systems. The method in this section is much used in areas such as meteorology, oceanography, etc. (Evensen et al., 1998; Thacker, 1989). It has recently been applied to resource economics (see Ussif et al., 2002a-b). The approach is a data assimilation technique called the Adjoint Method (AM). In the AM, a loss or penalty function measuring the distance between the model solution and the observations is minimized. This is formulated in the following sections.

The Adjoint Method

The formulation of the adjoint method is as follows. We minimize the penalty function $J = \sum_{t=0}^{T-1} (y_t - y_t^{obs})^2$ subject to the dynamics in (3). Thus the statement of the problem is

$$\min_{\alpha,\beta} \sum_{t=0}^{T-1} (y_t - y_t^{obs})^2 \qquad (5)$$

subject to

$$\frac{dy}{dt} = \beta\, y, y(0) = y_0 = e^{\alpha}$$

where y_t, y_t^{obs} are the model solution and the observed value respectively and y_0 is the initial condition. The model solution is the numerical approximation often obtained by finite difference methods (Ussif et al., 2002b, Gerald and Wheatley, 1992). Note that, in determining the best fit to the data we estimate the initial condition as a parameter. It is important to note that the static option in SAS uses the first data point as the initial condition and thus does not estimate it. The problem in this example is trivial because the dynamics are linear. It becomes more complicated if the dynamics are nonlinear and coupled, that is, a simultaneous system of differential equations that are linked together through the variables (see Ussif et al., 2002b).

The constrained optimization problem can be solved by using the calculus of variations or optimal control theory. By constructing the continuous form of the Lagrangian L using μ and λ as the Lagrange multipliers, we have,

$$L[y, y_0, \beta] = J + \mu(y(0) - y_0) + \int_0^T \lambda(\frac{dy}{dt} - \beta\, y)dt \ . \qquad \text{(A6)}$$

The constrained problem (5) is now transformed into the unconstrained optimization problem of finding the extreme values of $L[y, y_0, \beta]$ in (6).

Using the calculus of variations, we can derive the adjoint equation. Detailed derivation of the adjoint equation is not given in this paper since the goal here is to present and formulate the problem. However, interested readers are referred to (Ussif et al., 2002 a-b or Evensen et al., 1998). The first order conditions are

$$\frac{\partial L}{\partial \mu} = 0, \qquad \frac{\partial L}{\partial \lambda} = 0 \qquad \text{(7)}$$

$$\frac{\partial L}{\partial y}=0\,, \qquad (8)$$

Note that differentiating L with respect to μ and λ give back the initial condition and the model dynamics respectively, while differentiating with respect to y results in the so-called adjoint equation.

To make the analysis easier and for consistency with the continuous model dynamics, assume we have data continuously, i.e., the time period of observation or reporting is small (e.g. on the scale of a day or even more frequently), and also, rewriting $J=\frac{J}{2}$, then the Adjoint equation is

$$\frac{d\lambda}{dt}=-\beta\lambda+(y-y^{obs}),\quad \lambda(T)=0 \qquad (9)$$

$$\frac{\partial J}{\partial \beta}=-\int_{t=0}^{T}\lambda\, y\, dt \qquad (10)$$

$$\frac{\partial J}{\partial y_0}=-(\mu+\lambda(0)\beta)=-(1+\beta)\lambda(0),\ \mu=\lambda(0)\,. \qquad (11)$$

The gradients of the penalty function (10-11) are obtained by differentiating the Lagrangian with respect to the independent parameters (y_0,β) and are used together with an optimization routine (e.g. the Newton-Raphson method), to find the minimum of the penalty function. It can be shown that, one can estimate either α directly or its exponent

$y_0 = e^{\alpha}$ which is by definition the initial condition. We have chosen the latter for convenience and also for practical purposes since we may be able to guess the starting value from available data.

Implementation of the Algorithm

Implementation of the adjoint technique is quite straightforward. The algorithm is outlined below

- Choose the first guess for the disposable or free parameters i.e. the parameters that can be tuned in order to minimize the penalty function
- Integrate the forward model (3) over the time horizon[2]
- Calculate the penalty function
- Integrate the adjoint equation (9) and calculate the gradients (10-11)[3]
- Use an iterative procedure[4] to find the minimum of the penalty function

For this simple dynamic problem convergence of the iterative procedure to the absolute minimum may be possible. However, for more complex problems, i.e., highly nonlinear dynamics with many parameters to estimate, multiple extrema may exist and convergence to the absolute minimum can be difficult. Note that the problem reduces to solving a two point boundary value problem (Equations 3 and 9) and then calculating the gradients This makes it possible to calculate the gradients of several parameters simultaneously and more accurately compared to when using the finite difference methods (Huiskes, 1998).

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[2] This is usually done numerically since the models are much more complicated than the example used in this paper.
[3] See how the adjoint variable enters the gradient relationships in equations A10-11.
[4] This requires setting an appropriate convergence criterion for the minimization. Please see Ussif 2002b for example.

The Error-Covariance Matrix

While point estimates are often useful, their utility is greatly enhanced if their error bounds are also provided. Statistical tests can be performed and confidence intervals can also be constructed. When the errors in the observations are assumed to be normally distributed, the uncertainty in the optimal parameters is obtained by analyzing the Hessian matrix. The Hessian matrix is the second derivative of the penalty function with respect to the parameters. By differentiating J two times with respect to each of the parameters the Hessian matrix (H) is obtained as

$$H = \begin{bmatrix} \dfrac{\partial^2 J}{\partial^2 \alpha} & \dfrac{\partial^2 J}{\partial \alpha \partial \beta} \\ \dfrac{\partial^2 J}{\partial \alpha \partial \beta} & \dfrac{\partial^2 J}{\partial^2 \beta} \end{bmatrix}. \qquad (12)$$

The Hessian matrix is symmetric and positive definite and is often called the Fisher information matrix in the econometrics literature. Inverting the Hessian matrix gives the approximate Variance-Covariance matrix (Greene, 1997; Matear, 1995). Hence, the diagonal elements of the Variance-Covariance matrix are the variances, which can be used to construct confidence intervals for the parameters.

The Adjoint Method is an efficient method for the minimization of the penalty function. It provides an efficient and reliable way of calculating the gradient(s) of the penalty function which allows for the simultaneous estimation of a large number of parameters (Huiskes, 1998; Matear, 1995). So called derivative free methods such as the simplex algorithm (Nelder and Mead, 1965) can also be used to optimize the penalty function. Other methods of minimizing the penalty function are simulated annealing (Greene, 1997; Matear, 1995) and the Markov Chain Monte Carlo technique (Harmon and Challenor, 1997). Kruger (1992) stated that, the adjoint method is about 100 times faster than simulated annealing. In general, the adjoint method is faster than the other methods.

Conclusions

This paper has demonstrated the utility of trend estimation using dynamic representation. The SAS systems have been used to fit two prototypes to data on occupational injuries for 12 countries. The conclusion is that such techniques are equally applicable. However, once the analysis becomes more complicated, this approach can be of tremendous help.

Reference

Evensen, G., Dee, D.P. & Schroeter, J., (1998). Parameter Estimation in Dynamical Models. NATO ASI, Ocean Modeling and Parameterizations edited by E. P.Chassignet and J. Verron, (Eds).

Gerald, C.F. & Wheatley, P.O., (1992). Applied Numerical Analysis. Addison-Wesley Publishing Company Inc.

Greene W.H., 1997. Econometric Analysis, Third Edition, Prentice-Hall, Inc., New Jersey.

Harmon, R. and Challenor, P. 1997. Markov Chain Monte Carlo Method for Estimation and Assimilation into Models. Ecological Modeling.101:41-59.

Huiskes, M.J.1998. Virtual Population Analysis with the Adjoint Method. Fisheries Stock Assessment Models, p.639-657. Alaska Sea Grant College Program, AK-Sg-98-01.

Kruger, J. 1992. Simulated Annealing: A Tool for Data Assimilation into Almost Steady Model State. Journal of Physical Oceanography 23:679-88.

Matear, R.J. 1995. Parameter Optimization and Analysis of Ecosystem Models Using Simulated Annealing: a Case Study at Station P. Journal of Marine Research. 53:571-607.

Nelder, J. and Mead, U. 1965. A Simplex Method for Function Minimization. The Computer Journal.

Pesaran, H.M. and Potter, S.M. 1992. Nonlinear Dynamics and Econometrics: An Introduction, Journal of Applied Econometrics. 7:s1-s7.

Thacker, W.C., 1989. On the Role of the Hessian Matrix in Fitting Models to Measurements. Journal of Geophysical Research. 94:6177-96.

Ussif A. M. An International Analysis of Workplace Injuries, Forthcoming Monthly Labor Review, 2003a.

Ussif A. M. Econometric Analysis of Insurance Frequency Trends: Which Model Should we Choose? Casualty Actuarial Forum, Summer 2003b.

Ussif, A.M., Leif, S.K. and Steinshamn, S.I. 2002a. Estimation of Biological and Economic Parameters of a Bioeconomic Fisheries Model Using Dynamical Data Assimilation. Journal of Bioeconomics. 4:39-48.

Ussif, A.M., L.K. Sandal and Steinshamn, S.I. 2002b. On the Dynamics of Commercial Fishing and Parameter Identification. Marine Resource Economics. 17:35-46.

```
* SAMPLE SAS PROGRAM ******************************************;
DATA REGDATA;
INPUT YEAR Y;
DATALINES;
   1     3.21
   2     4.01
   3     3.89
;

*******************************************************************
* Dynamic Estimation Program: Model: dy/dt = β y, y(0) = y0        *
*******************************************************************;

PROC MODEL DATA=REGDATA;
      PARMS A B; /* MODEL PARAMETERS*/
      DERT.Y = B *Y; /* DEFINING THE EQUATION*/
      FIT Y INITIAL=(Y=A) / TIME=YEAR DYNAMIC; /* OPTIONS*/
RUN;
*********************************************************************;
```

Table 1. Estimated Coefficients of the Dynamic Model and Log Linear Function.

	Sample Period	Log-linear	Dynamic
Country		**Value**	**Value**
Australia	1992-1999	-0.0583(0.0001)	-0.0585(0.0001)
Canada	1970-1999	-0.0158(0.0001)	-0.0145(0.0001)
Denmark	1979-1999	0.0176(0.0022)	0.0146(0.0009)
Finland	1976-1997	-0.0404(0.0001)	-0.0346(0.0001)
France	1975-1999	-0.0214(0.0001)	-0.0226(0.0001)
Japan	1990-1999	-0.0443(0.0001)	-0.0440(0.0001)
Mexico	1991-1999	-0.0058(0.1045)	-0.0078(0.0225)**
Norway	1975-1999	0.0281(0.0002)	0.0361(0.0001)
Sweden	1976-1999	-0.0633(0.0001)	-0.0522(0.0001)
Switzerland	1984-1998	-0.0293(0.0003)	-0.0325(0.0001)
United Kingdom	1987-1999	-0.0674(0.0001)	-0.0908(0.1048)
United States	1978-1999	0.0098 (0.0019)	0.0093(0.0026)

Table A1: Annual average trend estimates (p-values in parentheses) for 12 countries. ** the estimate for Mexico is suspect because, there are only 4 observations with missing values in between them. Results not corrected for serial correlation.

Commercial Lines Price Monitoring

Trent R. Vaughn, FCAS, MAAA

COMMERCIAL LINES PRICE MONITORING

TRENT R. VAUGHN

Abstract

This paper examines price monitoring techniques for the commercial lines of property/ liability insurance. Section 1 discusses the rationale for commercial lines price monitoring. Next, Sections 2 and 3 cover the two major categories of price monitoring reports: renewal rate change reports and overall rate level change reports. Section 4 considers the subtle relationship between manual rate changes and experience rating factors. Section 5 includes a short note on the concept of insurance-to-value and increases in exposure units. Finally, Section 6 concludes with some brief comments on the importance and role of price monitoring in the property/ liability industry.

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1. INTRODUCTION

Price monitoring techniques can be utilized for both major categories of property/liability insurance: personal lines and commercial lines. In general, however, price monitoring is a much more important and necessary tool for the commercial lines of business. This greater importance stems from the dichotomy between the personal and commercial rating mechanisms.

Specifically, personal lines rating plans contain numerous rating variables, but provide very little judgmental flexibility to the agent or underwriter. These rating plans often contain a tiered rate structure; for example, a personal automobile insurer may provide both a standard and a preferred program. Most state insurance departments, however, require well-defined and objective underwriting guidelines that specify which program applies to a given insured. After specifying the applicable program, the rate manual then determines a unique and fixed premium for each potential insured on the basis of that insured's individual rating characteristics.

In contrast, commercial lines rating plans generally provide fewer rating variables and more judgmental flexibility. Several elements of commercial rating plans allow the agent or underwriter to judgmentally modify the premium for each individual insured. If properly utilized, these rating mechanisms allow the underwriter to properly match the insured's premium to the corresponding loss exposure. For instance, the following rating techniques are widely utilized in commercial insurance:

1. *Experience rating* utilizes the insured's own historical loss experience to calculate an experience modification factor. This factor is then applied to the manual rate. In theory, risks with better-than-average loss experience will obtain a lower rate. Section 4 of this paper will provide a numeric example of a typical experience rating calculation.

2. *Schedule rating* allows the underwriter to judgmentally adjust the manual rate on the basis of the individual insured's characteristics. In theory, the schedule rating modification only reflects characteristics of the risk that are not already reflected in the risk's historical loss experience. For instance, an insured may have recently implemented a comprehensive loss control program that was not in effect during the experience period.

The maximum schedule rating modification varies by state, but it can often be as great as plus or minus 40%. In general, the schedule rating rules allow for a great deal of subjective judgment on the part of the underwriter or agent. For instance, the underwriter may choose to apply an adjustment factor of plus or minus 10% to reflect the quality of the insured's management team. Appendix A provides an example of a hypothetical schedule rating plan.

3. *Multi-company tiering* establishes a different rate level for two or more distinct legal entities within the same insurance group. Some (but not all) state insurance departments require the filing of underwriting guidelines that describe the rationale for assigning business to companies with different rate levels. Even so, there is generally a certain degree of judgment or subjectivity allowed in these filed underwriting guidelines.

4. *(a)-rating* allows the underwriter to judgmentally select the rate for certain classes. (a)-rating is generally only permitted by regulators for certain commercial classes, such as classes with widely heterogeneous members, or classes with insufficient data to determine a manual rate. (a)-rating is most common in commercial general liability insurance, due to the heterogeneous nature of the general liability exposure in many classes. (a)-rating may also be utilized for smaller, miscellaneous classes and coverages in other commercial lines.

5. *Composite rating* plans facilitate the rating of large risks. These plans calculate the rate per some simplified, or proxy, exposure base. The final premium charged to the

insured then depends on the actual level of this proxy exposure base during the policy period. Composite rating plans simplify the rating and the premium audit process for large commercial accounts. Section 2 will provide more information about composite rating, including the similarity between the composite rating process and the techniques involved in renewal rate change reports.

6. In *retrospective rating*, the final premium depends on the insured's own loss experience during the policy period. Generally, the contract stipulates certain maximum and minimum premium amounts. Policies written under retrospective rating plans are sometimes called *loss-sensitive* contracts, since the insured's final premium depends on the actual losses incurred during the contract period. In contrast, contracts that are not retrospectively rated are called *guaranteed cost* policies, since the insured's final premium does not depend on the actual loss experience during the contract period. Due to the unique nature of retrospective rating, the pricing levels and underwriting results on these contracts are generally evaluated separately from the guaranteed cost policies. Thus, retrospectively-rated contracts are outside the scope of this article.

Unfortunately, these distinctive rating mechanisms also make it very difficult to determine the actual changes in the insurance company's overall commercial price level over a certain period of time. In fact, a manual rate change history alone often provides a misleading picture of the actual changes in overall price level, since the agent or underwriter can utilize the techniques described above to offset or reduce the impact of the manual change.

As an example, assume that the actuary of ABC Insurance Company has determined the need for a substantial rate level increase for commercial auto business in the state of Maine. ABC subsequently obtains approval for a 10% increase in commercial auto manual rates. Prior to the rate increase, ABC's underwriters provided, on average, a 10% schedule credit to this business. After the rate increase, competition forces the underwriters to offer a 15% average schedule credit to maintain the business. Thus, the net rate level increase is not the +10% change filed by the actuary, but only +3.9% [1.10 x (0.85/0.90) -1 = 0.039].

Thus, in order to obtain an accurate picture of the overall price level in commercial lines, we need to look at more than just manual rate changes; we also need to quantify the impact of these discretionary rating tools on the insured's revenue. In response to this challenge, insurance companies and managing general agencies have developed various price monitoring tools for the commercial lines. In general, there are two broad categories of price monitoring reports: renewal rate change reports and overall rate level change reports. Each of these categories will be described in more detail in the following two sections.

2. RENEWAL RATE CHANGE REPORTS

The first step in producing a renewal rate change report is to track the change in the average final rate per unit of exposure on each renewal policy during a given period of time. The second, and final, step is to determine the premium-weighted average of these changes across all renewal policies in the given time period. In order to more fully describe this process, we need to carefully define several terms.

For a given unit of exposure, the *final rate* is defined as the manual rate after all discretionary rating adjustments. As an example, let's assume that the rating manual for commercial auto liability provided a manual rate of $1,000 for a commercial vehicle with certain rating characteristics (for example, territory, gross vehicle weight, radius of use, etc.). In addition, the underwriter will apply a schedule credit of 10% and an experience debit of 5%. In this case, the final rate is given $945 (that is, $1,000 x 0.90 x 1.05 = $945).

Likewise, for policies with multiple exposures, we can distinguish between the *final premium* and the manual premium. For example, assume that the commercial auto policy in the example above also provides coverage for a second commercial vehicle with a manual rate of $2,000. The final premium is then equal to ($1,000 + $2,000) x 0.90 x 1.05 = $2,835, whereas the manual premium is equal to the full $3,000.[1]

The *average final rate per unit of exposure* is then equal to the final premium divided by the total number of exposures on the policy. In our simple example, the average final rate per unit of exposure is equal to the final premium of $2,835 divided by two the vehicles, or $1,417.50 per vehicle.[2] This average final rate would then be compared to the comparable rate on the expiring policy to determine the change in the average final rate on the renewal policy. Finally, these changes would be averaged across all renewal policies (using final premium per policy as the weight for the average) to produce a measure of the overall price change for policies renewing during the given period of time.

[1] This example assumes that both vehicles on the policy receive the same schedule and experience rating mod.

[2] In more precise terms, the exposure base in this example is "vehicle-years", or number of vehicles insured for a one-year period, assuming annual policies.

Varying Exposure Bases

In practice, the calculation of the average final rate per unit of exposure is often complicated by two factors. First, the exposure base on a given commercial policy may vary by class code. For instance, in commercial general liability insurance, the exposure base varies considerably by class code, and may even vary between premises/operations and products/completed operations for a given class code. Second, the underwriter may charge an additional policy premium for a unique endorsement, and this additional premium may not have an associated exposure base. For example, in commercial auto insurance, for an additional premium amount the underwriter may be willing to eliminate the fellow-employee exclusion.

For these reasons, it is often necessary to determine a "proxy exposure base" for each commercial line of business. The average final rate per unit of exposure is then determined in relation to this proxy exposure base. In order to illustrate the procedure, the following table provides a simple example from commercial general liability insurance.

Class Code	Exposure Base	Manual Rate	# of Exposures	Manual Premium
XX455	Area (in square feet)	$0.20	2,000	$400
XX567	# of watertowers	$500	1	$500
XX454	Gross Receipts (in 000's)	$0.10	$4,000	$400

In this example, there are three class codes on the policy, and the manual premium for the policy is $1,300. If we assume that there is no experience rating, schedule rating, or other discretionary rating modifications, then the final premium is also equal to $1,300. In addition, let's assume that we have selected gross receipts (in thousands) as the proxy exposure base for general liability. In this case, the average final rate per unit of exposure is

$1,300 / $4,000 = $0.325. This average final rate would then be compared to the comparable average final rate on the expiring policy to determine the overall rate change at renewal.

This procedure is, in fact, very similar to the procedure used in composite rating, one of the commercial lines rating tools listed in Section 1. In composite rating, which is generally utilized to facilitate the rating of large commercial accounts, the underwriter first determines the average rate per unit of some proxy exposure base. At the expiration of the policy term, the actual value of the proxy exposure base is then utilized to determine the final policy premium. In this manner, the premium audit process is greatly simplified by allowing the premium auditor to focus only on one primary exposure base.

Since composite rating begins by determining the average final rate per unit of exposure on the policy, this rating approach is well-suited to renewal rate change reports. Essentially, for composite rated policies, we have simply eliminated a step from the process of creating the renewal rate change report.

Of course, in order to determine a renewal rate change report for each line of business, the actuary must determine the necessary proxy exposure bases. Fortunately, there are obvious candidates for most major commercial lines. The following table provides a suggested list of proxy exposure bases by line of business.

Line of Business	Suggested Proxy Exposure Base
Commercial Auto	Vehicle-Years
General Liability	Gross Receipts
Workers Compensation	Payroll
Property	Insured Value

Advantages and Disadvantages of Renewal Rate Change Reports

Since the renewal rate change report begins by analyzing the rate by class on each individual commercial policy, all of the major discretionary rating components are monitored. That is, by directly analyzing the final rate on each policy, we necessarily include the impact of experience rating, schedule rating, (a)-rating, company shift, and composite rating.

On the other hand, there are several drawbacks associated with renewal rate change reports. For instance, the renewal rate change report does not monitor the price level changes associated with new-business policies. Potentially, pricing could remain strong on renewals, while underwriters are forced to aggressively cut rates to write new business; this troubling situation would not be detected by renewal rate change reports.

Moreover, extensive programming changes are generally required to implement renewal rate change reports. Specifically, the renewal report must analyze rates by class code at the renewal effective date for each commercial policy, which requires very detailed premium coding and thousands of records. Also, there is generally no easy way to handle changes in coverage or classification during the policy term. For instance, if a vehicle is added or deleted on a commercial auto policy mid-term, it may be very difficult to determine the impact of this change on the average final rate per vehicle, and then incorporate this information into the renewal report.

Lastly, and most importantly, renewal rate change reports often provide misleading indications of rate changes, due to changes in the underlying mix of business on each policy.

In other words, the change in the final rate per unit of exposure on a renewal policy may be distorted by changes in the exposure mix by class. As a simple example, consider a commercial auto liability policy that provides coverage for one vehicle, a heavy truck. At the time of renewal, the insured has replaced this heavy truck with an extra-heavy truck. Assume that there have been no changes in manual rates or any of the discretionary rating tools, but that the insurance company's rate manual requires a higher rate for extra-heavy trucks than for heavy trucks. In this case, the renewal rate change report will imply that there has been a rate increase on this renewal policy; in reality, however, the higher underlying rate merely reflects the greater loss exposure on the new vehicle.

3. OVERALL RATE LEVEL CHANGE REPORTS

Instead of drilling down to the final rate for each class code at the individual policy level, an overall rate level change report separately tackles each of the manual and discretionary pricing components. These separate pricing components are then combined multiplicatively to determine the overall rate level change for a given period of time. This procedure includes both new and renewal business. In general, there are three categories, or sources, of rate level changes that are considered in the report: (1) manual rate changes, (2) discretionary rating mods, and (3) company shift. Each of these sources will be discussed below, using a hypothetical example to illustrate the ideas.

Manual Rate Changes

The manual rate change reflects any changes to the manual rates during the period, including changes in the underlying loss costs, loss cost multipliers, and package mod factors. For most insurance companies, a manual rate change history is readily available, since it is a key component in the pricing indications procedure. For example, the table below provides an illustrative manual rate change history for a given line of business, for the period from 1998 through 2003:

Effective Date	Rate Change
1-1-98	+10.0%
7-15-98	-6.4%
8-1-99	+5.6%
5-1-00	+4.3%
12-15-01	+6.5%
7-1-02	+5.5%
11-1-03	+1.0%

If the rate changes result from a change in the rating bureau's underlying loss costs, then the impact should be calculated on the basis of the individual company's premium distribution by class, territory, etc. This impact is generally completed as part of the rate change procedure, and then recorded in the manual rate change history database.

In the overall rate level change report, the full impact of each rate level change is reflected in its effective year. For instance, in the example above, the impact of the 11-1-03 rate change on the 2003 overall rate level is +1.0%.

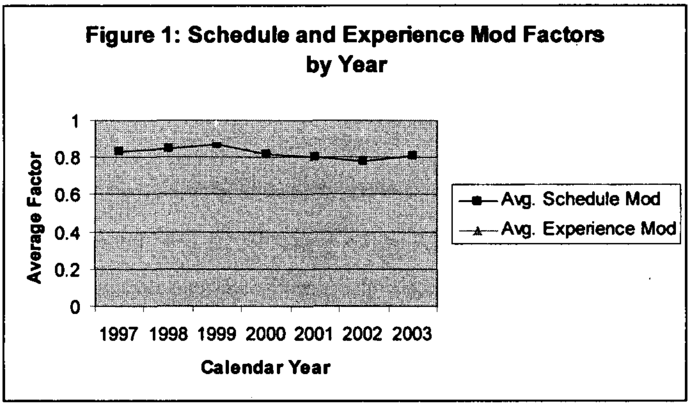
For workers compensation insurance, a portion of the manual rate change may be intended to offset a corresponding benefit level change. In the final price monitoring report, the actuary may choose to show the rate change *net* of benefit level changes, or the two impacts may be displayed separately.

Discretionary Rating Mods

The second category of rate level change involves the change during the period in the average level of discretionary rating modification factors (or "mods"). Discretionary rating mods may include both schedule and/or experience mods, depending on the line of business. The average modification factor for a given period is the premium-weighted average factor across all policies (new and renewal) with an effective date during that period. As an example, assume that we have determined the following average schedule and experience modification factors for the same period and line of insurance as our manual rate change example:

Calendar Year	Avg. Schedule Mod	Average Experience Mod
1997	0.83	0.92
1998	0.85	0.92
1999	0.87	0.91
2000	0.82	0.93
2001	0.80	0.92
2002	0.78	0.94
2003	0.81	0.90

Figure 1 provides a graphical display of the average schedule mod and experience mod by calendar year.



For each calendar year, we then determine the change in the average modification factor from the previous year. For instance, since the average schedule mod in our example increased from 0.78 in 2002 to 0.81 in 2003, the impact on the 2003 overall rate level is +3.8% (that is 0.81 / 0.78 -1 = 0.038).

In theory, experience rating may or may not be properly regarded as a *discretionary* rating mod. In workers compensation insurance, for example, experience rating is generally required and strictly enforced on all eligible accounts. Consequently, a change in the average level of the experience rating mod for workers compensation may simply reflect a change in the quality and exposure of the book of business – as opposed to a change in the overall rate level. For other lines, the application of experience rating may be more lax, due to the difficulty of obtaining the necessary data for all eligible insureds. For some lines at certain companies, experience rating may only be applied if it is requested by the agent – often for the purpose of generating an additional credit. In this case, experience rating is more properly considered as a discretionary rating mod.

Moreover, there is a subtle connection between manual rate changes and experience mod factors; specifically, the presence of an experience rating plan may cause a tempering, or "watering down", of the filed manual rate change. This relationship between manual rate changes and experience mod factors will be discussed further in Section 5.

Lastly, some overall rate level change reports may also monitor changes in the average *premium discount* factor for each given period. Premium discount factors provide a rate credit for certain large policies; the amount of the credit typically increases as the size of the account increases. Premium discount factors are common in many workers compensation rate manuals.

In theory, premium discount factors reflect the economies of scale involved in writing and servicing large accounts; that is, the expense load (as a percentage of premium) often decreases as the size of the account increases, due to the presence of certain "fixed expenses" per policy. For this reason, the premium discount factor is generally not regarded as a *discretionary* rating mod. For instance, an increase in the average premium discount factor from 10% in one period to 15% in the following period may simply reflect a change in the mix of business by size of account – as opposed to a true decrease in price level.

Even so, changes in the average premium discount factor are worth noting, and may be included for informational purposes in the final report. Such changes, however, should not contribute to the measure of the overall rate level change for the period, with one caveat: if there are any changes to the premium discount factors themselves, or to the structure of the premium discount table, then these changes should be quantified and included in the manual rate change history for the line.

Company Shift

Company shift measures the rate impact produced by moving business between companies with different rate levels. In order to quantify the impact of company shift, we need to examine the change in the premium distribution between companies. As an illustration of the procedure, assume the following data for our hypothetical example:

Rating Company/Tier	Deviation	2002 WP Distribution	2003 WP Distribution
High	+40%	25%	50%
Medium	+20%	50%	25%
Low	0%	25%	25%

The "Deviation" column displays the rating relationship between tiers; in this example, the rates in the "High" company are 40% higher than the rates in the "Low" company. In the loss cost environment, this implies that the Loss Cost Multiplier (LCM) for the "High" company is 40% greater than the LCM for the "Low" company. On the basis of this data, the average deviation for 2002 is +20%, whereas the average deviation for 2003 has increased to +25%. Thus, the impact of company shift on the 2003 pricing level is +4.2% (that is, 1.25/1.20 -1 = 0.042).

Lastly, note that the deviation in this chart should apply to the deviation at the *beginning* of the 2003 year. Any change in deviations or LCM's that occurred during 2003 is reflected in the manual rate change for 2003. In this sense, the company shift item of the report is intended only to reflect pricing changes due to movement between companies. In the business world, this phenomenon is sometimes referred to as "up-tiering" or "down-tiering".

Continuing with our example, assume that the impact of company shift by calendar year is as shown in the following table.

Calendar Year	Company Shift
1998	-3.1%
1999	+2.3%
2000	+1.5%
2001	-0.5%
2002	+2.2%
2003	+4.2%

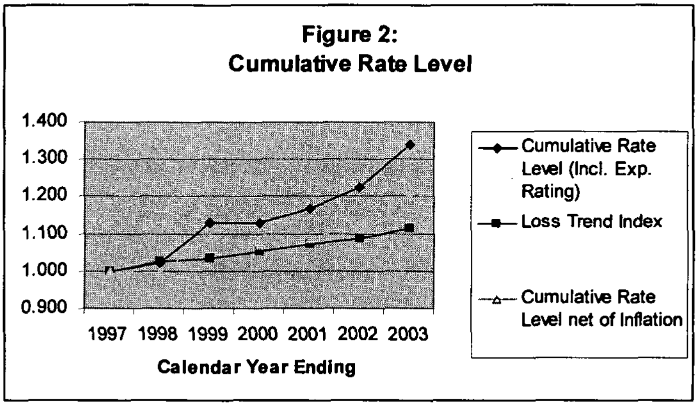
Total Pricing Change

The overall rate change for a given year is then defined as the product of the manual rate change, the discretionary mod change, and the company shift change. For example, if there was no manual rate change during a given year, but discretionary mods were down 10% and company shift was up 5%, then the overall rate change for the year would be -5.5% (1.00 x 0.90 x 1.05 = 0.945). Due to the theoretical considerations involved with experience rating, the overall rate change might be shown both including and excluding the change in experience rating mods.

For our example, the following chart summarizes the overall rate change by year, both by individual component and in total:

Calendar Yr.	Manual Rate Change	Company Shift	Schedule Mod Change	Exper. Mod Change	Total Change Incl. Exper. Rating	Total Change Excl. Exper. Rating
1998	+3.0%	-3.1%	+2.4%	0.0%	+2.2%	+2.2%
1999	+5.6%	+2.3%	+2.4%	-1.1%	+9.4%	+10.6%
2000	+4.3%	+1.5%	-5.7%	+2.2%	+2.0%	-0.2%
2001	+6.5%	-0.5%	-2.4%	-1.1%	+2.3%	+3.4%
2002	+5.5%	+2.2%	-2.5%	+2.2%	+7.4%	+5.1%
2003	+1.0%	+4.2%	+3.8%	-4.3%	+4.6%	+9.3%

In addition to the changes from year to year, we can also display the total accumulated rate level over the entire period. In our example, we set the rate level index as of 12/31/97 equal to 1.00. The rate level index (including experience rating) as of the end of calendar year 1998 (that is, 12/31/98) is then given by 1.022 (that is, 1.00 x (1 + 2.2%) = 1.022). As a final step, the actuary may incorporate some measure of the corresponding loss trend index over the same period; the total accumulated rate level can then be shown both gross and net of claim inflation. Figure 2 provides a graphical viewpoint of the total accumulated rate level over the period from 12/31/97 to 12/31/03.



Advantages and Disadvantages of Overall Rate Level Change Reports

The overall rate level change report offers the following key advantages over the renewal rate change report: (1) overall rate change reports include the pricing impact on both new and renewal policies; (2) overall rate change reports are not as impacted by distortions in the mix of business within a given policy; and (3) the report may be easier to program and implement than a renewal rate change report. On the other hand, because it does not focus on the individual rate by class for each policy, the overall rate level change may ignore the impact of certain discretionary pricing tools, such as (a)-rating or any judgmental over-rides in final rates.

4. RELATIONSHIP BETWEEN EXPERIENCE MOD FACTORS AND MANUAL RATE CHANGES

In the previous discussion, both the renewal rate change report and the overall rate level change report included experience rating changes as part of the overall price change. As noted in the previous section, however, a change in the average level of the experience modification factor may simply reflect a change in the quality or composition of the book of business, as opposed to a true pricing change. On the other hand, the experience rating formula *itself* may serve to mitigate or "water down" the impact of any *manual* rate change; for this reason, it is important to monitor experience rating changes along with manual rate changes. In this section, we will use a simple example to demonstrate the relationship between experience rating and manual rate changes.

In general, experience rating plans compare the *actual loss ratio* (or "ALR") on a given policy or account to an *expected loss ratio* (or "ELR"). For the denominator in the actual loss ratio (the so-called *subject premium*), the plan typically will utilize the manual premium (that is, the premium prior to any discretionary or experience modifications) for the upcoming policy period, with a *detrend* factor to adjust for premium and loss trend for each year of the experience period. As an example, let's assume that we are determining the experience mod factor for a policy with a 7/1/04 effective date. Assume the manual premium for the upcoming policy period is $10,000. The following table uses this manual premium, along with some hypothetical detrend factors, to determine the subject premium for the applicable experience period.

Policy Period	Manual Premium	Detrend	SubjectPremium
7/1/02-6/30/03	10,000	0.82	8,200
7/1/01-6/30/02	10,000	0.74	7,400
7/1/00-6/30/01	10,000	0.67	6,700
Total			22,300

For the numerator in the actual loss ratio (the so-called *subject losses*) the plan typically multiplies the case-incurred losses for each year of the experience period by an appropriate loss development factor. In our example, the following table demonstrates this calculation:

Policy Period	Case-Inc Loss	LDF	Subject Losses
7/1/02-6/30/03	4,000	1.20	4,800
7/1/01-6/30/02	4,500	1.10	4,950
7/1/00-6/30/01	5,500	1.05	5,775
Total			15,525

Thus, the actual loss ratio is the quotient of the subject losses and the subject premium, or 69.6% in our example (that is, 15,525 / 22,300 = 0.696). Let's assume that the expected loss ratio for the plan is 65%. The experience mod factor is generally given by the following formula:

Experience Mod Factor = 1.0 + (ALR / ELR – 1) x Credibility Factor

The *credibility factor* is defined in the plan, and generally is a function of the total subject premium for the policy. Let's assume that our plan indicates a credibility factor of 0.70 for a policy with a subject premium of $22,300. In this case, the experience mod factor equals 1.050 (that is, 1.0 + (0.696/0.650 -1) x 0.7 = 1.050). Assuming that there are no schedule credits, the final premium for the policy will be $10,500 (that is, the manual premium of $10,000 times the experience mod of 1.050).

However, let's now add a wrinkle to the story. Assume that the actuary for this company has implemented a 10% across-the-board manual rate change for this book of business – on the basis of some recent indications – and that this change is implemented prior to the effective date of our hypothetical policy. The manual rate for our policy increases to $11,000 – but, this new, larger manual premium will also impact the experience rating calculation. Specifically, the subject premium in the experience rating formula increases by 10%, as demonstrated in the following table:

Policy Period	Manual Premium	Detrend	Subject Premium
7/1/02-6/30/03	11,000	0.82	9,020
7/1/01-6/30/02	11,000	0.74	8,140
7/1/00-6/30/01	11,000	0.67	7,370
Total			24,530

As a result, the actual loss ratio decreases to 63.3%, reflecting the new, higher manual premium on the policy. As a result of the higher subject premium, the credibility factor may also increase. For simplicity, however, let's assume that the credibility factor stays at 0.70. The new experience mod factor is then 0.982, and the final premium on the policy is $10,802. Thus, while the manual premium on this policy increased by 10%, the final premium – after the application of experience rating – only increased by +2.9%. This is an example of the mitigating impact of experience rating on manual rate changes.

For this reason alone, it may be necessary to include experience rating changes in the price monitoring report. Even so, there are other potential methods for dealing with the issue. For instance, the actuary can adjust the manual rate change history to reflect the mitigating impact of experience rating. Alternatively, at the time of the manual rate change, the actuary may choose to adjust the expected loss ratio in the experience rating plan in order to offset the "watering down" phenomenon, and ensure that the desired manual rate change is realized. The details of such an adjustment depend on the premium distribution of the book of business, the credibility table in the plan, and several other factors. The exact calculation is beyond the scope of this article.

5. INSURANCE-TO-VALUE AND INCREASES IN EXPOSURE UNITS

For a given policy, an increase in the exposure units on the policy often indicates a true increase in that policy's exposure to loss; an example would be an increase in the number of vehicles covered on a commercial auto liability policy. For this reason, the renewal rate change reports consider the change in the average final rate per unit of exposure, as opposed to simply considering the change in the final premium on the policy. Likewise, the overall rate level change reports do not consider changes in exposure units as one of the sources of rate change during the period.

There are, however, certain cases where an increase in exposure units may reflect – at least partially – an increase rate adequacy. In particular, this may be true for inflation-sensitive exposure bases, such as gross receipts, payroll, or insured value. For these types of exposure bases, the claims inflation rate may be at least partially offset by the inflation rate on the exposure base.

The overall goal of any price monitoring report is to measure the overall rate change during a given period. This result can then be compared to the corresponding loss trend, in order to determine the net change in rate during the period. If the line of business utilizes an inflation-sensitive exposure base, then the annual rate changes should be compared to a loss trend that is *net* of the exposure trend.

6. CONCLUSION

One of the primary responsibilities of an actuary is to ensure that the premiums collected by the insurance company are adequate to pay for future loss costs and expenses. In order to fulfill this responsibility, the actuary must monitor the impact of *all* rating variables on the insurance premium, including the discretionary rating tools that are inherent in commercial rating plans.

Moreover, commercial lines price monitoring tools, if effectively designed and produced, will alert company management to changes in the level of underwriting discipline. As a result, the proper usage of price monitoring tools may result in a mitigation of the underwriting cycle in the property/liability industry. Thus, actuaries should champion the cause of price monitoring, by producing timely reports and effectively communicating the results to senior management.

APPENDIX A -- HYPOTHETICAL SCHEDULE RATING PLAN FOR COMMERCIAL AUTOMOBILE

A schedule rating modification may also be applied to the otherwise chargeable premium in accordance with the following table, subject to a maximum credit or debit of 40%. The schedule rating modification is intended to reflect such characteristics of the risk as are not reflected in its experience.

Schedule Rating Modifications

		Range of Modification		
Risk Characteristic	Description	Credit		Debit
Management	Cooperation with insurance company, interest in insurance program, quality of relationship with employees.	10%	to	10%
Employees	Selection, training, supervision, experience and basis of remuneration.	10%	to	10%
Equipment	Type, condition, servicing, risk's own repair facilities, establishment and maintenance of safety equipment.	10%	to	10%
Safety Organization	Periodic meetings, distribution of safety literature, award and penalty system, review of accidents with drivers, quality of accident reports.	10%	to	10%