Classification Ratemaking Using Decision Trees

Nasser Hadidi, Ph.D., FCAS, MAAA

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By

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Introduction

Manual rating of specific risks begin with a base rate, which is then modified by appropriate relativity factors depending on characteristics of each risk. Classical methods of deriving indicated relativities, are described by McClenahan (1996) and Finger (1996). A number of different modeling procedures are described in Brown's (1988) "minimum bias" paper and Venter's (1990) review of Brown's paper. These methods generally rely on the "multiplicative" or "additive" assumptions, which may not be reasonable for all types of risk. In this paper an alternative method of calculating indicated relativities is described, and demonstrated using a commercial Business Owners' Product (BOP) data set. Accident year 1997 to 2000 data is used to describe the method. The results are then applied to claims with accident year 2001. The derived 2001 relativities are compared with observed relativities, thereby demonstrating the extent of suitability of this method. It should be stressed that the intent here is entirely demonstration of a procedure. For actual practical implementation, modification would be required.

First a few words about the terminology and the data set. Relativities are based on grouping of risks with similar risk characteristics. This is essentially a classification problem. The purpose of any classification procedure is partitioning of objects - in our case risks - into demonstrably more homogeneous groups. For the BOP data we seek groups of risks with significantly differing claim frequencies, severities, pure premiums or loss ratios. Typically partitioning is based on a number of risk factors, which for BOP might be Coverage, Risk State, ISO territory, ISO coverage code, Property versus Liability, etc.

Distinctions must be made between these rating factors, which are used to group risks together and variables such as frequency, severity, pure premium or loss ratio, which must be estimated partially based on these risk factors. The former are independent or predictor variables while the latter are dependent variables. Both of these variables may be assumed to be either categorical or vary continuously in a given interval. The statistics used as the basis of classification depends on whether the dependent and/or the independent variables are categorical or interval scale. For risk classification the independent variables are typically categorical. For example the BOP data includes losses for three different coverages, in 51 different risk states, and with 178 different ISO territory codes, 6 different ISO construction codes, etc. It is customary to say that the classification variable – risk state - has 51 different levels. Similarly, there are six levels of ISO construction code, etc.

The BOP data set under consideration includes 27,854 claims with accident years 1997 through 2000, and 9011 claims with accident year 2001. These are broken down by:

	Number of Levels
Risk State	51
Company	4
ISO protection code	1014
ISO territory code	178
ISO construction code	6
Property/Liability indicator	2
Building/Content/Other indicator	3
Coverage code	3
ISO subline	2
ISO coverage code	4,

as well as a few other variables.

The data includes paid and incurred loss and expense (combined), basic limit (300k) loss and expense, and excess as well as cat losses.

The customary and classical method of splitting data for the purposes of deriving relativities is to consider all possible combinations of levels of permissible classification factors. For auto coverage, for example, in some states a 2x2 table of gender by rural/urban may be considered. Or in this case for any risk state, company and territory we may have a typical 2x3 table of property versus liability by building versus content versus other. The biggest drawback of such splits is that a specific rating factor may have a large impact on one level of another rating factor, but no impact at all on other levels. Or if it does, the nature of the impact may be quite different. For example ISO construction code may influence losses in one way in one ISO territory and in no way or a completely different way in another territory. The methodology described here is intended to avoid this drawback without the necessity of introducing numerous "interaction" parameters.

Rather than splitting the data in all possible ways, the data can be sequentially and selectively split in mutually exclusive homogeneous groups. Splits would be made only if clearly indicated by data to be meaningful. Meaningful in the sense that, risks are indeed more homogeneous in whatever it is that we are measuring- typically frequency or severity. The term decision "tree" is derived from this sequential approach. That is, start with the whole data - appropriately split the data into branches by one "best" rating factor to begin with. Then concentrate on one branch by itself, disregarding the rest of the branches, and find the next "best" rating factor for it, thereby creating new branches for this branch. Then concentrate on the next branch and find the next "best" rating factor for it, which may be – and in fact often is - a different rating factor. New branches are then created for this second branch and so on. Details of how exactly this can be done in practice is given below, but first it should be stated that this method is based on principles underpinning a large number of classification procedures – referred to as classification or decision tree procedures – which are amply reported in the statistical data mining literature.

These procedures appear with a variety of acronyms including:

AID Automatic Interaction Detection

THAID	Theta AID
CHAID	Chi-Squared Automatic Interaction Detection
Exhaustive CHAID	Modified CHAID
C&RT / CART	Classification And Regression Trees
QUEST	Quick, Unbiased, Efficient Statistical Tree
FACT	Fast Algorithm for Classification Trees
FIRM	Formal Inference-based Recursive Modeling
C4.5	A set of computer programs that construct classification models
ID3	The predecessor of C4.5
GUIDE	Generalized Unbiased Interaction Detection and Estimation
CRUISE	Classification Rule with Unbiased Interaction Selection and Estimation
Etc.	

AID, THAID, CHAID and Exhaustive CHAID

AID was first described by Morgan and Sonquist in1963 as a sequential procedure for analysis of survey data. It is intended to avoid the problem of interaction between variables used for classification. In the case of classifying risks, the problem of interaction translates into the possibility that type of coverage, for example, may have a different impact on rates for one territory as opposed to another territory. They propose *bisecting* the data sequentially, one factor at a time, based on maximizing the between levels sum of squared deviation. This is somewhat similar to the ordinary analysis of variance procedure, though in their 1963 paper they propose stopping the splits simply when the reduction in error sum of squares is less than a specified value. THAID, which was proposed by Messenger and Mandell in 1972, similarly *bisects* data, but based on a different statistic. This statistic, which they call THeta, is related to the proportional reduction in misclassification errors. CHAID was described by Kass in (1980). As the acronym indicates the predominant statistic used for splits in this procedure is the Chi-square statistic. Whereas for AID the dependent variable is interval scaled, here the dependent variable is nominal. As an example one would look at the overall proportion of policies that resulted in none, one, or 2 or more claims; that is three categories. And then split policies in groups so that the proportion of policies in each category would differ significantly between groups. Independent variables also being categorical, typical two way contingency tables are constructed and Chi-square statistics are calculated which form the basis of the splits. This procedure is demonstrated by Gallager Monroe, and Fish (2001) for private passenger automobile experience. Exhaustive CHAID (spss.com) is based on Biggs, de Ville, and Suen. (1991). It is a refinement and expansion of the method given by Kass. The significance level of the utilized Chi-square statistic is appropriately adjusted for the number of independent variables.

CART, C&RT, QUEST, FACT, GUIDE and CRUISE

CART, (cart.com) was introduced by Breiman, Friedman, Olshen, and Stone in (1984). It is similar to AID in that to achieve the final classification a series of binary splits are made. But it is far different from AID or CHAID in the splitting mechanism. Here at each step of the classification a series of queries is made regarding the value of each of the independent variables. For categorical independent variables such queries take the form of whether or not each case belongs to a given subset of levels of each independent variable. All possible subsets of all independent variables are considered. For interval scale independent variables the percentage of cases with values less than all observed values of this variable are considered. A misclassification cost is then calculated and the split is based on minimizing that misclassification cost. C&RT is another vendor's (spss.com) version of CART. QUEST is proposed by of Loh and Shih (1997). It is intended to reduce the bias in favor of splits that are based on independent variables for which more branching is possible. Categorical independent variables with more levels and interval scale independent variables with more distinct values are more likely to be selected first in classification tree procedure. This bias is a frequently recurring criticism of these classification procedures and several efforts in minimizing the bias are reported in the literature. FACT is also proposed by Loh and Vanichsetakul, (1988). It is described as an algorithm combining CART and Linear Discriminant Analysis (LDA). Discriminant analysis is the classical method of predicting group membership based on predictive characteristics. Depending on the number of groups one or more discriminant functions are estimated from the data. These functions are linear combinations of independent variables, and are in turn used to predict group membership. The values of these functions are, ideally, substantially different for each group. This would be the case if predictors have sufficient discriminating information. FACT differs from CART in that it uses a different misclassification cost based on these discriminant functions. GUIDE is proposed by Loh (2002). It is also intended to eliminate the variable selection bias. As mentioned for QUEST this bias refers to the fact that categorical independent variables with more levels as well as interval scale variables with more distinct observed values are more likely to be selected first in the tree structure. The bias is eliminated by an adjustment to the Chi-square p-value. CRUISE is the described by Kim and Loh (2001). It borrows ideas from FACT, QUEST, GUIDE, and CART, and is claimed to be faster and further reduce the variable selection bias.

FIRM , C4.5 and ID3

FIRM is a collection of codes presented by Hawkins (1990) for implementation of CHAID. Two versions, CATFIRM and CONFIRM, are given respectively for categorical and interval scale dependent variable. Details of the procedure are given in Hawkins and Kass (1982). Here essentially the interval scale variables are converted to categorical variables by clustering adjacent values in one category. C4.5 and its predecessor ID3 are presented by Quinlan (1993). They are a collection of computer programs that construct classification trees. The construction method is based on what they refer to as "divide and conquer algorithm" which uses the "gain" criterion. They refer to the data as "training set" and for any split of the training set the gain is defined in terms of the information or entropy obtained thereby.

Procedure Description

Almost all of the above procedures are packaged, some more elegantly than others, and are available commercially. But none can be used without modifications with actuarial data since they are not specifically designed as such. Many of the splits automatically tested in these procedures are meaningless for actuarial data. CART would routinely test if a BOP policy belongs to *all subsets* of the rating variable ISO construction code. Clearly only subsets including only one element are meaningful for actuarial data. None address the credibility issue. Furthermore routine 'black box' style use of these packages usually mask the statistics used as splitting criteria. It is therefore not clear whether the assumptions required, especially with regard to the distribution of such statistics are indeed valid for the data at hand. Therefore, with actuarial data the common underlying principles of these procedures should be grasped, modified appropriately and implemented directly.

These underlying principles are the sequential consideration of the rating factors, splitting the data based on an appropriate metric and at each split combining the levels of each rating factor as long as they are not significantly different. How this can be done in practice would now be demonstrated using the described BOP data. For this illustration the natural log of basic limit losses is considered the dependent variable, and the following factors are independent variables:

Coverage code, Risk State, Company, ISO protection code, ISO territory code, ISO construction code, Property/Liability indicator, Building/Content/Other indicator, ISO Subline, ISO coverage code

The selected metric is the F statistics (or equivalently its p-value) given by the ratio of between and within mean squares as described below. The choice of this statistic is justified by the fact that basic limit losses here very closely follow the lognormal distribution. It is essential to check this lognormal assumption which results in the F distribution for the mean square ratios when we use log of losses.

Splits will not be made if the p-value of the F statistic is more than 0.01 or the resulting splits will have less than 200 claims.

As stated earlier risk state, ISO protection code, and ISO territory code have, 51, 1014, and 178 levels respectively. Most of these levels have very few claims. Risk state '54' (Alaska) and '99' (miscellaneous) have 3 and 1 claims respectively. Therefore before any analysis, for each factor the number of levels is reduced by appropriate level combinations and/or introduction of an 'all other' category. The exact number of levels to reduce to is not crucial at this stage of the analysis since groups will be recombined objectively with the tree structure in the next steps. Simply inspecting the mean severity by level of each factor *along with the standard errors* of each mean (confidence interval) provides an adequate means of combining levels.

In this fashion the rating factor risk state was combined into nine distinct groups. Similarly the levels of ISO protection code, and ISO territory were regrouped into 9 and 8 levels respectively. Exhibit 3 is a description of these recodes along with the number of claims in each group.

A standard multivariate split of this data would result in at most

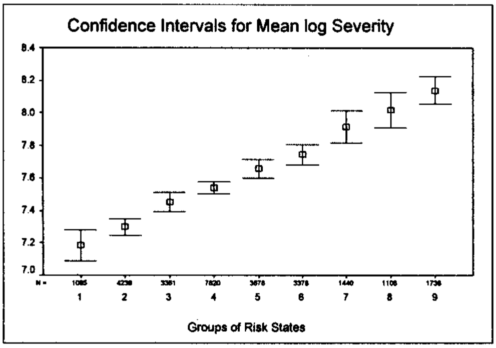
(9 Risk State)x(4 Company)x (9 ISO protection code)x(8 ISO territory code)x (6 ISO construction code)x(2 Property/Liability indicator)x(3 Building/Content/Other indicator)x(3 Coverage code)x(2 ISO subline)x(4 ISO coverage code)=2,239,488 cells,

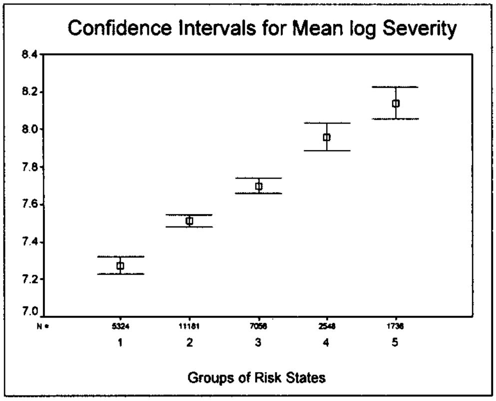
The whole idea of this method is which of these 2,239,488 cells are indeed materially different from the rest, and must be evaluated individually. The tree structure is intended to isolate these significantly different cells from the total. As shown below for the BOP data only 21 tiers or nodes , need be considered separately. Obviously each risk would belong to one and only one of these 21 tiers. Here is how the procedure works.

For the entire data, the so called node 0, the mean and standard deviation of log severity based on 27,845 claims is 7.5944 and 1.7692 respectively. At this stage the "best" predictor of this log severity is risk state. The best predictor means the predictor producing the highest F ratio, which here is the selected metric of choice for splitting, or equivalently the lowest p-value for the F statistic. This factor would divide risks into five groups:

Risk States	log Severity	Number of claims
9	8.1405	1736
7,8	7.9598	2548
5,6	7.6994	7056
3,4	7.5127	11181
1,2	7.2742	5324
All	7.5944	27845

The following graphs demonstrate the reasoning behind these combinations.





From the first graph it is observed that there is no clear reason not to combine 1 and 2, 3 and 4, 5 and 6, 7,8 and 9. But once 7 and 8 are combined, 9 would be significantly different from that combination. From the second graph it is clear that the five new groups have significantly different means. Graphical descriptions aside, the F statistic is the ratio of between mean square

$$[1736(8.1405-7.5944)^2 + 2548(7.9598-7.5944)^2 + 7056(7.6994-7.5944)^2 + 11181(7.5127-7.5944)^2 + 5324(7.2742-7.5944)^2]/4 = 389.0511$$

and within (error) mean square

$(27844 \times 1.7692^2 - 4 \times 389.0511)/27840 = 3.0746$

This ratio equals 126.5, which is of course highly significant leading to the necessity of the above split.

Let us now concentrate on states 3 and 4, disregarding other states for now. For this branch the next best predictor is property/liability indicator, which again based on the F ratio of 69.0 divides risk into 2 branches:

	log Severity	Number of claims
Property	7.2632	2533
Liability	7.5858	8648
All	7.5127	11181

Next, consider liability claims in states 3 and 4. These have to be broken down by ISO territory. The resulting F statistic is 28.5 based on 3 distinct groups of territories:

Territory	log Severity	Number of claims
2,6,7	7.6988	4357
1,3,4,5	7.5001	3961
8	7.1206	330
All	7.5858	8648

Next, consider liability claims in states 3 and 4 and ISO territories 1,3,4,5. These have to be broken down by ISO constuction code. The resulting F statistic is 16.35.

Construction	log Severity	Number of claims
3,4,5,6	7.6257	1632
1,2	7.4121	2329
All	7.5001	3961

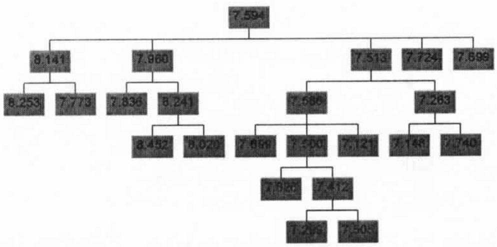
Next, consider liability claims in states 3 and 4, ISO territories 1,3,4,5 and ISO construction code 1,2. These have to be broken down by building/content indicator. The resulting F statistic is 9.61.

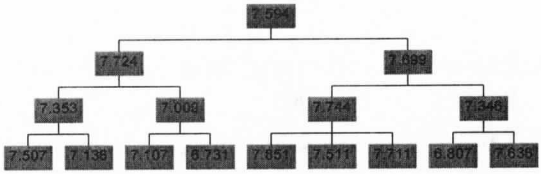
	log Severity	Number of claims
Building	7.2992	1050
Cotent	7.5049	1279
All	7.4121	2329

With the constraint of a p-value less than 0.01 and at least 200 claims, no further splits based on any other independent variable is implied by this procedure.

In this manner a number of distinct tiers, or the so called terminal nodes can be identified.

It is customary to depict these tiers with a tree structure as follows:





There are corresponding structures containing tier standard deviations and sample sizes needed for credibility adjustments, which are given in Exhibits 1 and 2.

The point is that while the overall mean is 7.594, 21 distinct tiers have in this manner been identified with means ranging from as low as 6.731 and as high as 8.452.

The profiles of risks in each tier are listed below:

Tier	Profile	log Severity	Number of Claims
1	Risk State 1 Liability	6.7313	317
2	Risk State 5,6 ISO Territories 4,6,8 ISO Subline 2	6.8069	279
3	Risk State 2 Liability	7.1072	898
4	Risk State 3,4 ISO Territories 8 Property	7.1206	330
5	Risk State 1,2 ISO Territories 2,5,6,8 Property	7.1355	1708
6	Risk States 3,4 Liability Building, Other	7.1477	2039

7	Risk State 3,4 ISO Territories 1,3,4,5 Property ISO Construction 1,2 Building, Other	7.2992	1050
8	Risk State 3,4 ISO Territories 1,3,4,5 Property ISO Construction Code 1,2 Content	7.5049	1279
9	Risk State 1,2 ISO Territories 1,3,4,7 Property	7.5071	2401
10	Risk State 5,6 ISO Territory Code 1,2,3,5,7 ISO Construction Code 1	7.5107	989
11	Risk State 3,4 ISO Territory Code 2,6,7 Property	7.6988	4357
12	Risk States 3,4 ISO Territories 1,3,4,5 Property ISO Construction Code 3,4,5,6	7.6257	1632
13	Risk State 5,6 ISO Territories 4,6,8 ISO Subline 1	7.6359	518

14	Risk State 5,6 ISO Territories 1,2,3,5,7 ISO Construction Code 2,3,5	7.7114	2359
15	Risk State 3,4 Liability Content	7.7398	494
16	Risk State 9 Building	7.7726	407
17	Risk State 5,6 ISO Territories 1,2,3,5,7 ISO Construction Code 4,6	7.8505	2911
18	Risk State 7,8 Coverage 1,3	7.8360	1769
19	Risk State 7 Coverage 2	8.0208	381
20	Risk State 9 Content, Other	8.2532	1329
21	Risk State 8 Coverage 2	8.4521	398
All		7.5944	27,845

Based on this procedure, certain independent variables do not impact claim costs at most levels of other independent variables. Coverage for example is only relevant for risk state groups 7 and 8.

Credibility Adjustments

Consider tier 20 with mean and standard deviations equal to 8.2532 and 1.8208 respectively. The estimated mean here is

$$e^{8.2532+0.5x1.8208^{\wedge}2} = 20{,}148$$

which once compared with the overall mean of

$$e^{7.5944+0.5x1.7692^{\wedge}2} = 9{,}504$$

results in relativity of 2.120. This figure is based on 1329 claims. So it is not fully credible.

The standard of full credibility utilized here is to be within one percent of the estimated mean with a probability of 0.99. As stated before limited losses being very closely distributed as a lognormal random variable, for this class the full credibility standard would be at least

$$(2.575x1.8208/0.01x8.2532)^2 = 3{,}227$$

claims. Using the square root rule, the partial credibility of 2.120 is thus

$$(1329/3227)^{0.5} = 0.642 .$$

The complement of credibility is assigned here to the non-terminal node immediately preceding this tier, the so called parent node. If there are not sufficient claims in this parent node to attain full credibility one can first adjust this relativity with its own parent node before using it as a complement. In this case the parent of tier 20 has log severity mean and standard deviation of 8.1405 and 1.8078 respectively, giving the severity mean of

$$e^{8.1405+0.5\times1.8078^2} = 17{,}581$$

and relativity of 1.850. But the number of claims here is only 1736 not reaching its own full credibility standard of

$$(2.575\times1.8078/0.01\times8.1405)^2 = 3{,}270.$$

The 1.850 estimate therefore has partial credibility of 0.729. The node immediately preceding this node has a relativity of 1, which has full credibility. Therefore the complement of credibility for tier 20 is attached to

$$0.729\times1.850 + 0.271\times1 = 1.619$$

Hence the credibility adjusted estimate of relativity for tier 20 is

$$0.642\times2.120 + 0.358\times1.619 = 1.941.$$

The necessary calculations for all 21 tiers are given in Exhibits 4-9.

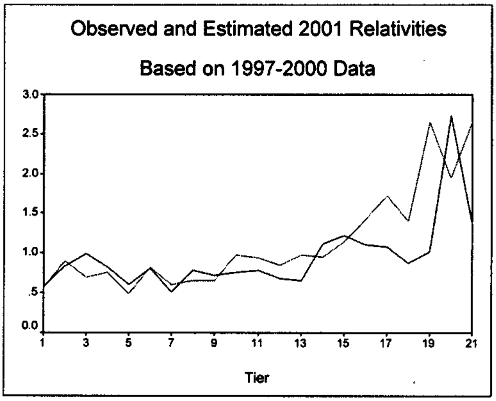
Cross Validation

As stated earlier the BOP data set includes 9011 claims with accident year 2001 which were not used for this classification scheme. These were deliberately left out for cross validation of the procedure. The accident year 2001 claims are grouped into 21 tiers based on the above scheme. For example coverage 2 claims in risk state 8 form tier 21, etc. Tier 1 includes 126 claims with the observed mean of $7,053. This value compared with the overall observed mean of $12,253 gives an observed relativity of **0.575**.
How does this compare with the estimated relativity based on 1997-2000 data?
The unadjusted relativity in that tier computed as explained above and listed in Exhibit 4, is 0.392 with a partial credibility of $(317/4368)^{0.5} = 0.269$. The complement of this credibility is assigned to the adjusted relativity of the parent node, which is

0.509x0.664 + 0.491x0.604 = 0.634. Thus the credibility adjusted estimated relativity of tier 1 is 0.269x0.392 + 0.731x0.634 = **0.569.**

This sort of calculation and comparison of course has to be done for all tiers. Details are given in Exhibits 4-9 in the appendix. The resulting credibility adjusted relativities and the actual observed 2001 relativities are listed below. The extent of association between these values can be observed from the chart.

Accident Year 2001 Observed Relativities	Credibility Adjusted Relativities
0.575	0.569
0.833	0.904
0.994	0.700
0.820	0.760
0.602	0.492
0.813	0.820
0.510	0.602
0.791	0.652
0.717	0.658
0.758	0.975
0.781	0.920
0.677	0.853
0.655	0.982
1.116	0.951
1.217	1.148
1.107	1.431
1.086	1.729
0.870	1.405
1.021	2.651
2.725	1.941
1.384	2.655



Summary and Conclusions

Classical methods of deriving rate relativities, are based on either a univariate or multivariate analysis of the data. The former requires the additive or multiplicative assumption and the latter may require estimation of numerous interaction parameters. An alternative method based on classification tree procedures is described in this paper. It is shown how risks with homogeneous loss severities can be grouped, based on appropriate combinations of levels of rating factors. Using accident year 1997-2000 data for a particular product, relativities are computed for 2001 accident year claims. With appropriate adjustment for credibility, these relativities are then compared with actual observed relativities demonstrating the suitability of this method. Because of the particular description of risk tiers, results obtained by these procedures might be somewhat difficult to implement. However, as an additional underwriting guideline, especially when deviating from manual rates, these procedures can be quite useful.

Acknowledgements

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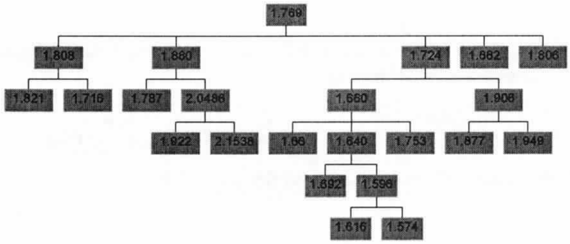
Exhibit 1

Node log Severity Standard Deviations



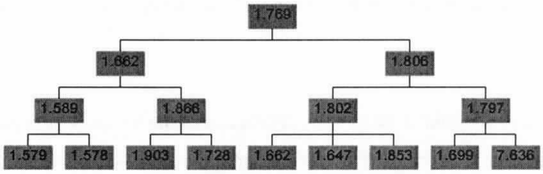


Exhibit 2
Node Number of Claims

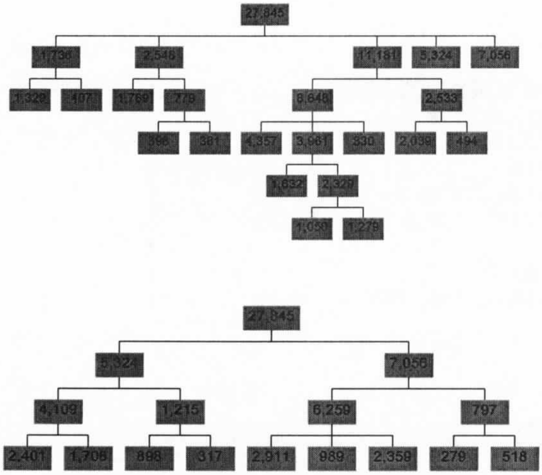


Exhibit 3
Recoded Levels of Risk Factors

Risk State

5,45,32,49,36,3,26,18	1
10,37,35,22,28	2
47,2,25,30,43,15	3
29,17	4
6,16,21,8,38,42	5
12,19,41,27,46,23,24,1,39,44,33,54	6
31	7
9,20,4,7	8
34,14,11,13,48,99,40	9

Company

BD	1
BE	2
BG	3
Others	4

ISO Protection Code

1,10	1
2	2
3	3
4	4
5	5
6	6
7,11	7
8,9	8
Missing	9

ISO Territory Code

1	1
2	2
3	3
4	4
5	5
6	6
Missing	8

Others	7

ISO Construction Code

1	1
2	2
3	3
4	4
Missing	6
Others	5

ISO Coverage Code

21	1
22	2
Missing	3
Others	4

Coverage

81	1
84	2
Others	3

ISO Subline

915	1
Missing	2

Exhibit 4

Unadjusted Observed Relativities by Tier

Tier	(a1) Underlying Normal Mean	(b1) Number of Claims	(c1) Underlying Normal Std. Deviation	(d1) Mean Severity	(e1) Unadjusted Relativity
1	6.7313	317	1.7276	3,728	0.392
2	6.8069	279	1.8531	5,034	0.530
3	7.1072	898	1.9032	7,467	0.786
4	7.1206	330	1.7528	5,749	0.605
5	7.1355	1708	1.5776	4,359	0.459
6	7.1477	2039	1.8772	7,403	0.779
7	7.2992	1050	1.6163	5,461	0.575
8	7.5049	1279	1.5741	6,272	0.660
9	7.5071	2401	1.5785	6,329	0.666
10	7.5107	989	1.6621	7,273	0.765
11	7.6988	4357	1.6600	8,748	0.920
12	7.6257	1632	1.6927	8,590	0.904
13	7.6359	518	1.6985	8,764	0.922
14	7.7114	2359	1.6474	8,676	0.913
15	7.7398	494	1.9485	15,339	1.614
16	7.7726	407	1.7156	10,345	1.088
17	7.8505	2911	1.9540	17,319	1.822
18	7.8360	1769	1.7872	12,494	1.315
19	8.0208	381	2.1538	30,953	3.257
20	8.2532	1329	1.8208	20,148	2.120
21	8.4521	398	1.9216	29,684	3.123
Total	7.5944	27845	1.7692	9,504	1.000

$(d_1) = \text{Exp}[(a_1)+0.5\text{x}(c_1)^2]$

$(e_1) = (d_1)/[\text{Total Entry of }(d_1)]$

Exhibit 5
Required Number of Claims for Full Credibility by Tier

	(a_2)	(b_2)
Tier	Number of Claims for full Credibility	Partial Credibility
1	4,368	0.269
2	4,914	0.238
3	4,755	0.435
4	4,018	0.287
5	3,241	0.726
6	4,573	0.668
7	3,251	0.568
8	2,917	0.662
9	2,932	0.905
10	3,247	0.552
11	3,083	1.000
12	3,267	0.707
13	3,281	0.397
14	3,026	0.883
15	4,202	0.343
16	3,230	0.355
17	4,108	0.842
18	3,449	0.716
19	4,781	0.282
20	3,227	0.642
21	3,427	0.341

$(a_2)=[2.575x(c_1)/(.01)x(a_1)]^2$
$(b_2)=[(b_1)/(a_2)]^{0.5}$

Exhibit 6
Parent Node Partial Credibility

Tier	Parent Node Normal Mean	Std. Deviation	Number of Claims	Parent Node Mean Severity	(a3) Unadjusted Relativity	(b3) Number of claims for Full Credibility	Partial Credibility
1	7.0092	1.8656	1,215	6,307	0.664	4,697	0.509
2	7.3457	1.7971	797	7,789	0.819	3,969	0.448
3	7.0092	1.8656	1,215	6,307	0.664	4,697	0.509
4	7.5858	1.6597	8,648	7,810	0.822	3,174	1.000
5	7.3526	1.5885	4,109	5,510	0.580	3,095	1.000
6	7.2632	1.9055	2,533	8,766	0.922	4,564	0.745
7	7.4121	1.5962	2,329	5,920	0.623	3,075	0.870
8	7.4121	1.5962	2,329	5,920	0.623	3,075	0.870
9	7.3526	1.5885	4,109	5,510	0.580	3,095	1.000
10	7.7444	1.8023	6,259	11,714	1.233	3,591	1.000
11	7.5858	1.6597	8,648	7,810	0.822	3,174	1.000
12	7.5001	1.6398	3,961	6,937	0.730	3,170	1.000
13	7.3457	1.7971	797	7,789	0.820	3,969	0.448
14	7.7477	1.8023	6,259	11,753	1.237	3,588	1.000
15	7.2632	1.9055	2,533	8,766	0.922	4,564	0.745
16	8.1405	1.8078	1,736	17,581	1.850	3,270	0.729
17	7.7444	1.8023	6,259	11,714	1.233	3,591	1.000
18	7.9598	1.8798	2,548	16,758	1.763	3,698	0.830
19	8.2409	2.0486	779	30,924	3.254	4,098	0.436
20	8.1405	1.8078	1,736	17,581	1.850	3,270	0.729
21	8.2409	2.0486	779	30,924	3.254	4,098	0.436

(a_3) and (b_3) are calculated as in Exhibit 4

Exhibit 7
Parent of Parent Node Relativity

(a4)

Tier	Parent of Parent Node Mean	Std. Deviation	Number of Claims	Mean Severity	Relativity*
1	7.2742	1.6619	5,324	5,740	0.604
2	7.6994	1.8060	7,056	11,274	1.186
3	7.2742	1.6619	5,324	5,740	0.604
4					
5					
6	7.5127	1.7237	11,181	8,089	0.851
7	7.5001	1.6398	3,961	6,937	0.730
8	7.5001	1.6398	3,961	6,937	0.730
9					
10					
11					
12					
13	7.6994	1.8060	7,056	11,274	1.186
14	7.6994	1.8060	7,056	11,274	1.186
15	7.5127	1.7237	11,181	8,089	0.851
16	7.5944	1.7692	27,845	9,504	1.000
17	7.5127	1.7237	11,181	8,089	0.851
18	7.5944	1.7692	27,845	9,504	1.000
19	7.9598	1.8798	2,548	16,758	1.763
20	7.5944	1.7692	27,845	9,504	1.000
21	7.9598	1.8798	2,548	16,758	1.763

*) Same as Exhibit 3 and 4

Exhibit 8
Observed Claims and Relativities for Accident Year 2001

Tier	Observed Mean Severity	Number of Claims	Observed Relativity
1	7,053	126	0.575
2	10,213	69	0.833
3	12,179	339	0.994
4	10,052	75	0.820
5	7,380	344	0.602
6	9,968	344	0.813
7	6,250	216	0.510
8	9,689	229	0.791
9	8,788	775	0.717
10	9,287	300	0.758
11	9,578	1443	0.781
12	8,297	456	0.677
13	8,033	157	0.655
14	13,679	691	1.116
15	14,913	489	1.217
16	13,574	131	1.107
17	13,314	1072	1.086
18	10,661	810	0.870
19	12,518	234	1.021
20	33,395	579	2.725
21	16,964	132	1.384
All	12,253	9011	1.000

Exhibit 9
Comparison of 2001 Observed and
1997-2000 Credibility Adjusted
Relativities

Accident Year 2001 Observed Relativities	(a_5) Credibility Adjusted Relativities
0.575	0.569
0.833	0.904
0.994	0.700
0.820	0.760
0.602	0.492
0.813	0.820
0.510	0.602
0.791	0.652
0.717	0.658
0.758	0.975
0.781	0.920
0.677	0.853
0.655	0.982
1.116	0.951
1.217	1.148
1.107	1.431
1.086	1.729
0.870	1.405
1.021	2.651
2.725	1.941
1.384	2.655

$(a_5) = (e_1)(b_2)+[1-(b_2)][(a_3)(b_3) + \{1-(b_3)\}(a_4)]$