4

Estimating ULAE Liabilities: Rediscovering and Expanding Kittel's Approach

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ABSTRACT

Existing actuarial literature provides guidance on the use of dollar and count-based methods for the estimation of ULAE liabilities. Traditional dollar-based methods are based on widely available, and usually audited, company financial data, while count-based methods rely on relatively detailed information regarding the number and cost of various claim-handling activities and events. In the case of fast reporting, slow paying lines of business, traditional dollar-based methods may not produce the best estimate of ULAE liabilities, since the familiar "50/50" assumption does not apply. On the other hand, the application of count-based methods is sometimes impractical. For example, the detailed claim count and activity cost data used in the structural methods can be quite difficult to compile and verify – especially to "outside" actuaries. This article describes a generalization to a familiar ULAE liability estimation approach, which attempts to duplicate some of the benefits of the structural methods, while relying exclusively on aggregate loss data.

INTRODUCTION[1]

The need for a refinement to the traditional ULAE reserving methods surfaced while we were evaluating the liabilities of a mid-size, single-state workers compensation insurer. The long duration of claim payments, as well as rapid expansion since the company had started operations a few years earlier, made the traditional paid-to-paid ratio approach inappropriate. Also, conversations with management, together with our knowledge of the specific characteristics of workers compensation claims handling, made it clear that the usual 50/50 assumption (half of the ULAE is incurred in opening claims, the other half in closing claims) did not apply. Count-based approaches to estimating ULAE liabilities, although perhaps conceptually appropriate for accurately modeling the dynamics of the organization in question, were not practical due to the unavailability of accurate claim count, or refined transaction and expense information.

In the subsequent sections, we present a brief survey of several established approaches to ULAE liability estimation, directing readers interested in the details of each method to relevant literature. We then present a description of a generalized dollar-based methodology, and its relationship to some of the traditional approaches. Our generalized dollar-based method is anticipated in a little-noted formula in an article by Kittel [4], and thus we refer to the method as "generalized Kittel" method or "generalized" method or formula, for short. We show how to expand this generalized formula to allow its application as a count-based method, and we also outline several conceptual refinements that would incorporate various reserving refinements

1 We are indebted to Jon Michelson for building a spreadsheet model to assess the relative accuracy of the generalized method.

suggested by other authors, as well as a simplified application of the method. Finally, we discuss practical complications such as errors in the estimations of parameters, and suggest areas for further research and improvement.

We have included illustrations of some of the concepts presented in this article. The attached exhibits display an actual application of the generalized method in comparison to the traditional method and Kittel's method. The Appendix sets forth the detailed derivation of the generalized formula to estimate an ULAE-to-loss ratio.

Throughout this article, we use the term "losses" to refer to "losses and allocated loss adjustment expenses"[2].

BRIEF SURVEY OF TRADITIONAL METHODS OF ESTIMATING ULAE LIABILITIES

We begin our discussion by briefly surveying the actuarial literature regarding methodologies for the estimation of ULAE liabilities. In our particular case, several of the methods surveyed were inappropriate, either due to the lack of detailed historical information, or to the specific characteristics of the company in question. Ideally, the actuary would have access to sufficient

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[2] Throughout this paper, we refer to ULAE as the traditional categorization of general overhead expenses associated with the claims-handling process, and particularly including the costs of investigating, handling, paying and resolving claims. Several issues associated with the 1998 change in loss adjustment expense categories are discussed in the Practical Difficulties section of this paper.

data to employ both dollar-based and count-based methods, and make a choice of methodology based on which approach is likely to produce the best estimate.

Dollar-based versus count-based methods – We first describe several dollar-based methods (Classical, Kittel Refined, and Mango-Allen Smoothing), followed by a description of count-based methods. These two broad classes of methods differ significantly in the amount of data and calculations required, and are based on fundamentally different assumptions. In the case of the dollar-based methods, a fundamental assumption is that ULAE expenditures track with loss dollars. Most importantly, this assumption means that the general timing of ULAE expenditures (or of specified portions of ULAE expenditures) follows the timing of the reporting or payment of loss dollars. In addition, this assumption implies that a $1,000 claim requires 10 times as much ULAE resources as a claim with losses of $100. By contrast, count-based methods incorporate fundamental assumption that the same kind of transaction costs the same amount of ULAE, regardless of claim size. However, because these count-based methods typically include some parameter to reflect the cost of ongoing management and maintenance of claims, they also imply that a claim that stays open longer will cost proportionately more than a quick-closing claim, at least with respect to some component of ULAE.

In practice, these seemingly divergent assumptions may not affect the results of the methods quite as severely as it might seem at first glance. Since the methods are being used for an entire population of claims, they need to be correct only for the "average" claim being reported, handled, paid, or closed during a time period – not for each individual claim. In other cases, the gulf can be bridged by stratifying the claims data and types of transactions and making assumptions about the relative ULAE resources required in the various sub-populations. In every case, it is a useful exercise for the actuary to reflect upon the assumptions underlying a selected method, and the implications of those assumptions regarding the underlying ULAE process and resources, as well as the implications for the results of the reserving method.

We also describe several triangle-based ULAE projection methodologies towards the end of this section.

Classical Paid-to-Paid Ratio Method – By reviewing the ratios of calendar year paid ULAE to calendar year paid losses, the actuary estimates an ULAE-to-loss ratio. To reflect the assumption that half of ULAE is incurred when new claims are set up, and the remaining half is spent closing them[3], this ratio is applied to the incurred but not reported (IBNR) loss reserves, plus half of case reserves. This method has several implicit assumptions, including (a) the specific company's ULAE-to-loss relationships have achieved a steady state (so that the ratio of paid ULAE to paid losses provides a reasonable approximation of the relationship of ultimate ULAE to ultimate losses); (b) that the relative volume and cost of future claims-management activity on

[3] In descriptions of the classical method, the concepts of "closing" a claim, and "paying" a claim seem to be used interchangeably, implying that the descriptions were written in the context of claim types for which the only payment occurs at closing. Given the use of paid loss dollars and case reserve dollars to apply the classical method, it would be more accurate to describe the classical assumption as "half the ULAE is spent with the *payment* of claims."

not-yet-reported claims and reported-but-not-yet-closed claims, respectively, will be proportional to the dollars of IBNR reserves and case reserves.[4]

As described in Kittel's article, the Classical Paid-to-Paid Ratio Method can lead to inaccurate results whenever the volume of losses is growing – since the paid-to-paid ratios will be overstated due to the mismatch between ULAE and losses paid. As mentioned above, the company in question had been expanding rapidly since its incorporation. That led to the material overstatement of ULAE reserves by purely mechanical application of this methodology. Also, we believed that the 50/50 assumption did not describe this company's application of resources to the various stages in the life cycle of its claims[5].

Kittel's Refinement to the Classical Method – A refinement to the Classical Method, detailed in Kittel's paper, explicitly recognizes the fact that ULAE is incurred as claims are reported, even if no loss payments are made. That is, ULAE payments for a specific calendar year would not be expected to track loss payments perfectly, because actual ULAE is related to both the reporting and the payment of losses. In contrast, the Classical Method, by assuming a steady state, makes the implicit simplifying assumption that paid losses are approximately equal to reported losses, and thus that the two quantities can be used interchangeably. To derive the indicated ULAE ratio

[4] Another imprecision with the usual description and frequent application of the classical method is the equating of "IBNR" reserves with reserves on not-yet-reported claims. In practice, IBNR reserve dollars typically include not only provision for not-yet-reported claims (IBNYR), but also provision for development on known cases (IBNER, or Incurred But Not Enough Reported). A more correct application of the classical method is to apply the full ULAE-to-loss ratio to the IBNYR reserve, and to apply half of that ratio to the sum of case reserves and IBNER.

[5] Kittel notes that inflation also can create distortions in the classical method.

under Kittel's refined method, the actuary reviews several years' ratios of calendar year paid ULAE to the average between paid and reported losses for that year. Conceptually, Kittel's use of the ratio of ULAE to the average of paid losses and reported losses derives directly from the assumption that half of a claim's ULAE is expended when a loss is reported, half when it is paid. As in the classical method, the actuary's selected ULAE-to-loss ratio is applied fully to the IBNR reserve and half the ratio is applied to the case reserve dollars to obtain the estimate of unpaid ULAE. Although the Kittel refinement addresses the distortion in the Classical Method associated with a growing company, it maintains the traditional "50/50" assumption regarding ULAE expenditures. Therefore, it does not allow for the particular allocation of ULAE cost between opening, maintaining and closing claims exhibited by the company in question.

While Kittel's paper typically is associated with the refined formula described in the preceding paragraph, the paper also shows a brief outline of a potential generalization. In this generalization, the cost of ULAE is described as the sum of incurred losses multiplied by an "opening factor", paid losses multiplied by a "closing factor", and mean loss reserves, multiplied by an "open factor."

With the "opening factor" set at 50%, the "closing factor" set at 50%, and the "open factor" set at 0%, this formula simplifies to the familiar Kittel formula. As specifically presented by Kittel, this generalization suffers from the same incorrect equating of paid losses with closed claims, and of incurred losses with the ultimate cost of reported claims. Nonetheless, as described in the following pages, it provides the core of the approach elaborated in our paper.

Mango-Allen Smoothing Adjustment – Mango and Allen [5] provide a general discussion of Kittel's refinement to the classical method. They specifically suggest a possible variation on the application of the formula when the actuary is working with a line of business where the actual historical calendar period paid losses are volatile, perhaps due to the random timing associated with the reporting or settling of large claims. In this case, Mango and Allen suggest replacing the actual calendar period losses with "expected" losses for those historical calendar periods, which can be estimated by applying a selected reporting and payment pattern to a set of accident year estimated ultimate losses. We expect that this type of adjustment would most likely be necessary in a line of business with a relatively small number of claims of widely varying sizes.

Early count-based methods – Skurnick [7] summarizes an early (1967) proposal for a count-based method by R.E.Brian in the Insurance Accounting and Statistical Association Proceedings. Brian suggested breaking the ULAE process into five kinds of transactions: transactions associated with setting up new claims, maintaining outstanding claims, making a single payment, closing a claim, and reopening a claim. To estimate the future ULAE effort required for a set of claims, the actuary projects the future numbers of each type of transaction. Brian estimated that each of these transactions would bear a similar cost, and suggested estimating the cost per transaction using ratios of historical ULAE expenditures to the number of claim transactions occurring during the same calendar periods. Conceptually, this approach is based on the assumption that all kinds of claim transactions require similar ULAE resources and expenditures. However, this weakness could be remedied by refining the formula to allow for different cost levels for the different types of transactions. The need to forecast the numbers of future transactions is a considerable practical difficulty in the application of this approach. For our particular client situation, reliable claim count and claim transaction data were not available. Thus, we were unable to consider this and other count-based methods.

Wendy Johnson Method – This count-based method, presented in Johnson's 1989 paper [3], follows a line similar to Brian's. Johnson's specific example suggests using the reporting and maintenance as the key transactions. Johnson, like Brian, then projects the future number of newly reported claims, as well as the number of claims that will be in a pending status each year – and thus will have required maintenance during the year. Also like Brian, Johnson estimates the cost of each transaction by comparing historical aggregate ULAE expenditures to the number of transactions occurring in the same time historical period.

Johnson introduces a clever innovation by allowing for an explicit differential in the amount of ULAE resource or cost required for different types of claim transactions. Johnson's specific example assumes, based on qualitative input, that the process of opening a claim costs $\$x$, and the process of maintaining existing claims costs additional $\$x$.

Alternative weights, as well as additional transaction types, could be introduced directly into Johnson's formula (for example, our model assumes that the cost of closing a claim is in addition to the cost of maintenance.) The benefit of Johnson's innovation is that it requires only that the actuary estimate the *relative* amount of resources required for each transaction type, and does not require that the actuary perform detailed time-and-motion studies to calculate the *actual* cash cost of each transaction type.

The mechanics of the Johnson method involve estimating the ULAE cost per claim activity by calculating weighted claim counts based on historical data[6] and comparing those weighted claim counts to the total ULAE costs in the same historical period. The estimate of unpaid ULAE is obtained by projecting the number of, and the ULAE cost associated with, weighted claim counts at each subsequent year-end, related only to claims occurring prior to the reserve evaluation date.

Rahardjo and Mango-Allen: costs varying over time – Whereas Johnson introduces the concept that opening a claim requires a different quantity of resources, Rahardjo [6] and Mango-Allen [5] focus on the situation in which the annual (or quarterly) cost of maintaining and managing a claim varies over the life of the open claim. Mango and Allen, for example, introduce the concept that claims (liability claims, from the context of the paper) which are still open after a long period of time are likely to be complex claims requiring more claim adjuster time, and from a more senior (and probably more highly paid) adjuster. Their paper also introduces a specific inflation adjustment. The final reserve indication is likely to be quite sensitive to the magnitude of the parameters used, as the reader of the Mango-Allen paper will realize after working with the illustrative parameters presented. In addition, the estimates will be affected by parameters not explicitly considered in the articles, such as Mango and Allen's implicit assumption that equal amounts of ULAE resources are required to open, close, and handle one average claim for a year.

Spalla: quantifying transaction costs – Spalla describes that manual time-and-motion studies no longer are necessary to determine the cost of various claim-related activities and transactions.

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[6] For example, by adding newly opened and open claim counts at each evaluation, after multiplying the counts by the relative ULAE effort.

Rather, because so many of these activities are computer-supported, modern claim department management systems are equipped to track the amount of time spent on individual claim activities, by level of employee. By combining the individual claim management activities into somewhat more macroscopic transactions, it is feasible to calculate the average cost of each type of claim transaction. These average costs, loaded for overhead and other costs not captured by the computerized tracking systems, can be applied within analytical frameworks as described by Rahardjo and Mango-Allen. Another benefit of working with the underlying cost data that Spalla describes is that it allows for more detailed analysis of the claim activity costs. Using the detailed information, the actuary can determine which types of claim, which types of claim transactions and which stages of the claim life cycle have relatively similar (or relatively different) costs. The insight gained allows the actuary to treat those transactions with different costs (e.g., opening a workers compensation medical only claim versus opening a lost-time claim) separately for ULAE reserving purposes. Spalla describes her method of loading unmeasured costs on top of the costs specifically measured. We suggest that the actuary using Spalla's method consider an equally important additional step as a "reality check": if the selected costs per transaction were applied to the numbers of transactions that were undertaken last year, would the result match that period's actual total ULAE expenditures?

While Spalla describes determining the actual cost of various transactions, the process she describes could be effectively used to quantify the *relative* amount of cost per transaction, as compared to the cost of other kinds of claim transactions. This relativity is less subject to annual change, versus the dollar cost per transaction or per activity. With such relativities in hand, the general approaches described in Rahardjo and Mango-Allen could be used, but now with some quantitative basis for the magnitudes of the parameters.

Triangle Projection Methods – In the paid loss development method, losses paid to date for a particular accident year are used as the basis for estimating unpaid losses. Similar approaches can be applied to the paid ULAE that has been reported to date for a particular accident year. The projection of paid ULAE to ultimate ULAE can use selected parameters based on historical observations regarding the amount of paid ULAE reported before or after a comparable age of maturity. The literature describes three methods for quantifying the parameters to project paid-to-date ULAE to ultimate.

In much the same kind of methodology as used with paid losses in the paid loss development method, an accident year by evaluation year triangle of paid ULAE can be used to calculate development factors from evaluation point to ultimate. Note, however, that the construction of paid ULAE triangles relies on the manner in which ULAE payments are allocated to accident year – since "actual" ULAE by accident year is not observable, at least not for all categories of ULAE expenditures. This allocation of ULAE payments is typically based on the pattern of claim payments, which *can* be observed. The accident year triangles of ULAE may be distorted if either the method of allocating calendar year ULAE to accident years changes over time, or if the loss payment patterns change.

The triangle projection method also raises a philosophical question in cases where a company is using a simplistic ULAE allocation method (e.g., the historical 50/50 rule) that does not mirror the actual distribution of expenses. In this case, the triangle projection method may produce a good estimate of the ULAE dollars that will be reported on future Schedule Ps for the current and prior accident years. However, this estimate may not accurately estimate the actual ULAE expenditures that will be required to handle and settle claims for the same period. Which one of the two is the actuary really trying to accomplish? We believe that the answer should be that the actuary is estimating a reserve for the actual future expenditures, but one might also argue that the objective is to predict the future values on Schedule P. This question may arise less frequently now that carriers are allowed to allocate ULAE payments to accident years based on their own analysis.

Slifka [8] and Kittel [4] each describe methodologies that project ultimate or unpaid ULAE based on historical ULAE payments. Slifka suggests using a time-and-motion study to estimate the claim department's allocation of resources between current accident year claims and prior accident year claims. This relationship between the "cost" of current year's claim management activities and prior years' claim management activities can be used then to estimate the future payment activity. Let us assume for example that such a study suggests that 60% of the current accident year's ULAE remains unpaid, 15% of the prior accident year's ULAE remains unpaid, and 5% of the second prior accident year's ULAE remains unpaid as of December 31. Then the total unpaid ULAE at this evaluation date is estimated as 80% (60% + 15% + 5%) of a typical calendar year's ULAE payment. Although this approach presumes a steady state, it can be refined it to reflect volume growth as well as the effects of inflation.

A third approach we have used in some real-life applications is to construct paid ULAE triangles, not by using the actual historical allocations of ULAE to accident year, but by restating those allocations using current time-and-motion studies, and/or relationships to loss payment patterns. For example, let us assume that these studies suggest that half of ULAE is paid at the time a claim is reported, and half is paid in proportion to claim payments. Then historical calendar year ULAE can be assigned to accident year-calendar year cohorts: half according to the distribution of reported claims across current accident year, prior accident year, second prior accident year, and so on; and half according to the distribution of paid losses – as indicated by an appropriate accident year loss payment pattern (e.g., 10% to the current accident year, 15% to the prior, and so on). Once the ULAE triangle is constructed, traditional triangle projections can be applied.

THE GENERALIZED APPROACH

During the course of our client assignment, we set out to define a procedure to estimate ULAE liabilities which would recognize this company's rapid growth, and be consistent with our understanding of the patterns of the company's ULAE expenditures over the life of a claim. The objectives were (a) to reproduce the key concepts behind the Johnson method, while using commonly available – and usually reliable – aggregate payment and reserve data; and (b) to develop an extension to the Kittel refinement which could allow for alternatives to the traditionally-assumed "half and half" pattern of ULAE expenditures over the life cycle of the claim. The generalized method described in this section accomplishes both of these objectives.

Indeed, the reader will recognize the roots of the generalized method in both Johnson's and Kittel's methods. As done by Wendy Johnson, our approach employs the concept of "weighted" claims, by which claims "use up" different amounts of ULAE at each different stage of their life cycle, from opening to closing. Therefore, newly opened, open and newly closed claims should be given different weights when determining the "loss basis" to which ULAE payments during a past or future calendar period would be related. However, because we believed that handling costlier claims warrants and requires relatively more resources than handling smaller claims, we sought to use claim dollars rather than claim counts.

Following this thought process, we defined our *loss basis* for a particular time period as a weighted average of the ultimate cost of claims reported during the period, the ultimate cost of claims closed during the period, and losses paid during the period. Note that by "ultimate cost of claims reported" we mean the reported amounts as well as any future development on known claims. Analogously, we define "ultimate cost of claims closed" as the final cost of claims that are currently closed, that is, we include in that amount any future payments made after the closing of the claim[7].

Kittel in fact introduced a weighted average of this sort, but Kittel's average includes only incurred losses and paid losses, and Kittel's weights are fixed at 50/50. By comparison, the generalized method introduces a third loss measure that allows distinguishing the cost of

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[7] As discussed in a subsequent section, this approach assumes that there is no additional cost associated with reopening or "reclosing" a reopened claim. The formulas do provide, however, for the cost of maintaining reopened claims.

maintenance from the cost of closing (an important distinction for workers compensation), and allows the flexibility of selecting weights appropriate to the company and segment of business.

In the following paragraphs, we present the explicit definition of the loss basis, and how it is used to calculate the projected ultimate ULAE, as well as the estimated ULAE liability.

Let U_1, U_2 and U_3 be such that $U_1 + U_2 + U_3 = 100\%$, and U_1, U_2 and U_3 are defined as follows:

- U_1 is the percentage of ultimate ULAE spent opening claims,
- U_2 is the percentage of ultimate ULAE spent maintaining claims, and
- U_3 is the percentage of ultimate ULAE spent closing claims.

In the course of a loss and loss adjustment expense reserve review, it would be appropriate to determine reasonable ranges for U_1, U_2 and U_3, and test the sensitivity of the final result (unpaid ULAE) to variations within those ranges. Several considerations for the selection of U_1, U_2 and U_3 are discussed in a subsequent section.

Also, for a particular time period T, let us define:

- R as the ultimate cost of claims that have been reported during the period T,
- C as the ultimate cost of claims that have been closed during the period T, and
- P as the losses paid during the period T,

following the definition of "ultimate cost" of reported and closed claims stated in a preceding paragraph.

Conceptually, the time period T could represent activity occurring between t_1 and t_2 related to a particular accident year, or activity occurring between t_1 and t_2 related to all accident years, where t_1 and t_2 are selected points in time.

The use of the aggregate claim dollar values R, C, and P as driving values in the generalized method reveals an assumption that the expenditure of ULAE resources is proportional to the dollars of losses being handled. This assumption is in contrast to Wendy Johnson's assumption that ULAE costs are independent of the claim size and nature. More specifically, the generalized method is based on the following assumptions:

- ULAE amounts spent opening claims are proportional to the ultimate cost of claims being reported,
- ULAE amounts spent maintaining claims are proportional to payments made, and
- ULAE amounts spent closing claims are proportional to the ultimate cost of claims being closed.

The appropriateness and sensitivity of these assumptions warrant further analysis, both as a matter of general research, and for a particular application of either method. We concluded that the dollar proportionality was an assumption that would produce reasonable indications for our particular application.

From the preceding definitions and assumptions, the total amount spent on ULAE during a time period T would be described by the relationship:

$$M = (R \times U_1 \times W) + (P \times U_2 \times W) + (C \times U_3 \times W),$$

where W represent the ratio of ultimate ULAE to ultimate losses (L), and M represents the total ULAE expenditures during time period T.

We can now define our loss basis B for the time period T as:

$$B = (U_1 \times R) + (U_2 \times P) + (U_3 \times C).$$

By simple algebra:

$M = B \times W$, and $W = M / B$.

Each component of the loss basis B can be understood conceptually as the value of the claims underlying the ULAE payments. Thus,

- $U_1 \times R$ represents the loss basis for ULAE spent setting up new claims,
- $U_2 \times P$ represents the basis for ULAE spent maintaining open claims, and
- $U_3 \times C$ represents the basis for ULAE spent closing existing claims.

A more detailed algebraic and intuitive description of the derivation of the loss basis B and the ULAE ratio W can be found in the Appendix.

In practice, companies typically observe, measure and report M, the ULAE payments during a period, such as a calendar year[8]. Once U_1, U_2 and U_3 are estimated or selected, the loss basis B can be calculated from loss amounts R, P, and C (defined in a preceding paragraph) that can typically be determined from data and calculations underlying an actuarial loss reserve analysis. In particular, M and B can be calculated for historical calendar periods. By computing the ratio $W = M / B$, where both M and B are expressed on a calendar-year basis, we obtain ratios of ULAE to loss by calendar year. We can then select an overall ratio of ULAE to loss, which we will name W^*, to be used in estimating future ULAE expenditures.

Based on the concepts and notation defined above, we could estimate the ultimate ULAE (U) for a group of accident years as the product between W^* and L, where L represents the independently estimated ultimate losses for the same group of accident years, and W^* represents the selected ultimate ULAE to loss ratio. That is, $U = W^* \times L$.

This representation of ultimate ULAE suggests different ways to estimate a reserve for unpaid ULAE for a group of accident years. As discussed below, these approaches will produce different results, and may be appropriate for use under different circumstances.

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[8] It must be noted that the 1998 revisions to the statutory classification of loss adjustment expenses create a practical difficulty for the application of this and other ULAE reserving methods, as discussed in a subsequent section of this article.

A possible method (not the one we prefer, as described below) is to estimate unpaid ULAE from the estimate of ultimate ULAE ($V = W^* \times L$), reduced by the amount of ULAE already paid (M). That is, we can compute:

Unpaid ULAE = $(W^* \times L) - M$.

In many situations, this method presents both practical and conceptual difficulties. From a practical perspective, it may be difficult to quantify the historical paid ULAE that corresponds only to the accident year losses represented by L. And, conceptually, this approach has some similarities to, and shares the potential distortions of, an expected loss ratio approach to unpaid losses, in which unpaid losses are estimated as the product of a pre-set expected loss ratio and premium (expected ultimate), reduced for actual paid amounts. As a period matures, the reserve estimate can become increasingly distorted if actual paid losses do not approach the pre-set expected ultimate.

Another method, which we prefer, is analogous to the Bornhuetter-Ferguson loss reserving method, in that an *a priori* provision of unpaid ULAE is computed. Using the notation introduced in this article, we would calculate:

Unpaid ULAE $= W^* \times \{L - B\}$

To understand the derivation of this estimate, let

- $R(t)$ be the ultimate cost of claims known as of time t,
- $C(t)$ be the ultimate cost of claims closed as of time t,
- $P(t)$ be the total amount paid as of time t.

If L, $R(t)$, $C(t)$ and $P(t)$ relate to a specific group of accident years, then we could express ULAE liabilities on these accident years at time t as:

$$\text{Unpaid ULAE} = W^* \times \{U_1 \times [L - R(t)] + U_2 \times [L - P(t)] + U_3 \times [L - C(t)]\}.$$

Each component of this formula represents a provision for the expenses associated with:

- opening claims not yet reported,
- making payments on currently active claims and on those claims that will be reported in the future, and
- closing "unclosed" claims, i.e., closing claims open at time t and closing those claims that will be reported/opened in the future.

Rearranging the terms in the equation above, we obtain:

$$\text{Unpaid ULAE} = W^* \times \{L \times (U_1 + U_2 + U_3) - [U_1 \times R(t) + U_2 \times P(t) + U_3 \times C(t)]\}$$
$$= W^* \times \{L - [(U_1 \times R(t)) + (U_2 \times P(t)) + (U_3 \times C(t))]\}$$
$$= W^* \times \{L - B\},$$

As noted above, this methodology implies that the amount of ULAE paid to date and the ULAE liability are not directly related, except to the extent that these payments influence the selection of the ratio W^*. The reader may recall that a similar assumption is the basis behind the popular Bornhuetter-Ferguson reserving approach (a thorough discussion of the Bornhuetter Ferguson loss reserving approach, as well as of the expected loss ratio method, can be found in the 1972 *Proceedings* article by Bornhuetter and Ferguson [1].)

A third possible approach implied by the definition of B would be analogous to the loss development reserving method. ULAE liabilities could be estimated as:

$$\text{Unpaid ULAE} = M \times \left(\frac{L}{B} - 1 \right).$$

Such approach, which warrants further investigation, would imply that ULAE liabilities are proportional to paid amounts reported to date. Aside from the practical difficulty of establishing the ULAE amounts paid that correspond to accidents occurring during a particular period, this methodology, similarly to the paid loss development approach, may be overly responsive to random fluctuations in ULAE emergence.

Readers interested in the comparison of the three corresponding loss reserving methodologies are directed to a study note by Brosius [2].

The foregoing discussion presents the generalized approach based on relating historical ULAE payments and estimated ULAE reserves to loss amounts. This approach and notation can easily be adapted to using claim counts or transaction counts. For example, if the analyst believes that ULAE is best described as being related to the number of claims reported, the number of claims open at any point during the period, and the number of claims closing during the period, then our "loss basis" – using lower case notation to differentiate from the standard generalized method – is:

$$b = (v_1 \times r) + (v_2 \times o) + (v_3 \times c)$$

where r represent reported claims, o are open claim counts, and c, closed claims. We find it most convenient in this formula to describe v_1, v_2 and v_3 as being estimates of the relative cost of handling the reporting of a claim, managing an open claim for one year (or portion thereof), and closing a claim, respectively. As in Johnson's paper, discussed earlier, it is not necessary to predetermine the actual cost of these various activities, just their relative magnitudes – Johnson, for example, assumes $v_1 = 2$, $v_2 = 1$ and $v_3 = 0$. We now select w^*, representing the cost of an activity having $v = 1$, and estimated from historical data as $w = M / b$, where M still represents ULAE payments.

After the analyst selects a value of w^* for use in projecting future costs (perhaps a series of w^*_i's, reflecting explicit future inflation adjustments), the ULAE reserve can be estimated as

$$\text{Unpaid ULAE} = \sum_i w_i^* \times \left[(v_1 \times r_i) + (v_2 \times o_i) + (v_3 \times c_i) \right],$$

where:

- r_i represents the number of claims to be reported in each calendar year i,
- o_i is the number of open claims at the end of calendar year i,
- c_i represents the claims to be closed during calendar year i, and
- i represents the series of future calendar year-ends until all claims are closed.

In each case, only claims occurring on or before the date of evaluation of ULAE liabilities should be considered. The reader will note that, according to the formula above, a claim that stays open for several years is counted multiple times in the summation. This is consistent with the assumption that ULAE is incurred each year such a claim stays open.

It is relatively straightforward to see that this formulation of ULAE based on claim counts is equivalent to that presented by Wendy Johnson. It could be adapted to recognize the Rahardjo and Mango-Allen concepts of costs varying over time by stratifying the claims activities more finely than just reporting, open, and closing, i.e., by having more than three categories of claims activities.

U_1, U_2 AND U_3

No doubt the reader will have identified by now that there is no convenient handbook providing the values of U_1, U_2 and U_3 for a particular category of business. Certainly, we expect that the values could vary significantly from carrier to carrier, and between coverages. For example, a litigation-intense liability book of business might have a strong concentration of activity close to the time of claim settlement and payment, versus a large front-end cost for workers compensation.

We have found it feasible to develop a range of values for U_1, U_2 and U_3 for a particular company and line of business by interviewing claims personnel. The resulting ranges can be used to test the consistency of the resulting ULAE ratios, as well as to assess the sensitivity of the ULAE ratios to different choices of U_1, U_2 and U_3 within the range suggested by the interview process. Time and motion studies as described by Spalla could be used to develop an empirical basis for the parameters needed.

An interesting research project would be to develop a series of benchmark values of U_1, U_2 and U_3 (and v_1, v_2 and v_3) by line of business, market segment or carrier characteristics.

A SIMPLIFICATION

We realize that in many cases, the estimation of R and C, that is, the ultimate cost of reported and closed claims, may not be a trivial exercise.

As defined in the beginning of this article, the ultimate cost of claims reported as of a certain date represents the total payments that will ultimately be made in connection with all claims known to the carrier as of that date. Another way of thinking about these costs is as the ultimate for the accident period ending on that date, reduced for the pure IBNR amounts, which represent the ultimate cost of not-yet-reported claims. Analogously, the ultimate cost of closed claims as of a certain evaluation point represents the final cost of claims that are closed as of the evaluation date, including any subsequent payments.

Although an actuary familiar with the reserving and claims handling practices of the specific company would normally be able to produce accurate estimates of R and C, the necessary detailed information may not be available, or the additional effort may not be justified. To address situations like these, we explored a simplification of the generalized methodology that does not require the estimation of R or C.

First, we used the estimated ultimate losses for the *accident* year as a proxy for the ultimate cost of claims reported in the *calendar* year. This calendar year amount can be expressed exactly as the sum of the corresponding *accident* year ultimate and the pure IBNR at year-end, reduced by the amount of pure IBNR at the beginning of the year. The actuary can evaluate the error inherent in using the suggested approximation after considering the difference in exposures between accident years as well as the characteristics of the coverage being analyzed, and then make judgmental adjustments as necessary. For example, given the minimal delay in the reporting of workers compensation claims, we can often assume the pure IBNR component of the ultimate is not likely to vary much from one year to the next. Therefore, the accident year ultimate would be a reasonable approximation of the true value of the parameter R.

Secondly, if no particular additional effort is required to close an existing claim (as is the case in the example presented in a subsequent section), we can assume that U_3 equals zero. This assumption may be inappropriate for some lines of business. For example, a significant portion of the cost of handling an employment practices liability claim will be incurred in connection with its settlement.

If the particular coverage allows making the assumption that U_3 equals zero, then $U_1 + U_2$ should equal 1.0, and we can approximate the loss basis B for each calendar year as

$$\hat{B} = (U_1 \times A) + (U_2 \times P),$$

where A represents the ultimate losses for the corresponding accident year, and compute the observed W values as $M / \hat{B}$ for each year. After reviewing those observed ULAE ratios, the actuary will select an appropriate ratio W^* for use in estimating ULAE liabilities.

An overall estimate of pure IBNR as of the evaluation date can be obtained (perhaps by analyzing claim reporting patterns and ultimate severities), which could then be deducted from L to obtain an estimation of the ultimate cost of claims reported to date, which we denote R. Unpaid ULAE can be then calculated according to the formulas presented above, as

$$\text{Unpaid ULAE} = W^* \times \{L - [(U_1 \times R) + (U_2 \times P)]\},$$

which can also be expressed as:

$$\text{Unpaid ULAE} = W^* \times [U_1 \times (L - R) + U_2 \times (L - P)]$$

AN EXAMPLE

To illustrate our approach, we included an example of the application of the traditional, Kittel and generalized methodologies in the evaluation of ULAE liabilities for a workers compensation book of business.

This sample insurance company began operations in 1997, and over the course of its 6 years of operations, paid ULAE has averaged 18% of paid losses (as seen in Exhibit B). Following a review of the paid-to-paid ratios by year, the traditional method might lead the analyst to select 16% as the ratio of ULAE to loss for use in establishing a ULAE reserve. This would be appropriate if ULAE payments were proportional to paid losses for a particular calendar year. We have found, however, that for workers compensation, ULAE expenditures are concentrated more heavily towards the front end of the claim than are the loss payments. Consider a hypothetical extreme, in which all ULAE is incurred at the moment the claim occurs, with the amount of the ULAE being proportional to the size of the claim. In this hypothetical case, the appropriate relationship to examine would be the ratio of ULAE to ultimate losses for an accident period[9]. Furthermore, the growth experienced by this company will cause the indicated ULAE liability using the traditional methodology to be overstated.

Interviews with management of this company's claims department, and examination of the flows of work and allocation of resources in the claims department suggested that approximately 60%

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[9] The reader will recognize elements of the suggested simplification of the generalized method in the discussion of this extreme case.

to 70% of the work for a claim is concentrated at the time the claim is reported, and 30% to 40% of the work is spread over the remaining life of the claim. For this company, no particular extra degree of effort is associated with closing the claims. Since ULAE expenditures are heavier at the beginning of a claim's life cycle, it should come as no surprise to the reader that the standard Kittel method (shown in Exhibit C) indications of unpaid ULAE are overstated.

Applying the generalized method, and setting U_1 equal to a value in the range 60% to 70%, U_2 in the range 40% to 30%, and U_3 equal to zero, the observed ULAE to loss ratios range between 8% and 11% for the various years, as can be seen in Exhibits D and E. The selected ratio in this illustration is 10%. While this selection was based on the company's total history, rather than the individual accident periods, we note that, for individual periods, the ULAE ratios implied by this method behave much more regularly than if the traditional paid-to-paid ratios are used. This behavior provides some support for the reasonableness of the selected values of U_1, U_2 and U_3. The reader will note the significant difference between this ratio and the ratios indicated by the traditional and Kittel methods.

Given the selected W^* ratio of 10%, Exhibits D and E display the application of the three alternative ULAE reserving formulas derived from the generalized method ("expected loss", "Bornhuetter-Ferguson" and "development" methods) which we describe in a preceding section.

The simplified version of the generalized method is shown in Exhibit F. In this case, we chose to present a likely range for estimated pure IBNR to be used in computing the ULAE reserve.

PRACTICAL DIFFICULTIES

Inconsistencies in the reporting of claim adjustment expenses would create obvious difficulties to the application of ULAE reserve estimation methodologies. With the 1998 change in statutory rules requiring the classification and reporting of "Other Adjusting Expenses", some insurers may no longer capture traditional ULAE, or a consistent history of such expense payments may not be available. The methods described in this paper could be applied to traditional ULAE, to the new "Other Adjusting" expenses, to the individual component activities and expenses that comprise these broader categories of loss adjustment expense, or to historical loss adjustment expenses reclassified to approximate the current Defense and Cost Containment and Other Expense definitions. These methods could also be applied to the whole, or components of, ALAE or its statutory replacement, "Defense and Cost Containment" expenses, although likely using different weighting parameters.

Furthermore, as noted in a preceding section, estimation of R and C, that is, the ultimate cost of reported and closed claims, may not be trivial . The simplification shown in this article is only one of the many approaches that an actuary could take to sidestep that difficulty.

As noted earlier in this paper, the generalized methodology is consistent with the assumption that the claims adjusting activities associated with reopening and "reclosing" a claim have no cost. An alternative approach, which we have not used in practice, is to assume that the ultimate cost of closed claims C equals the sum of total amounts paid on closed claims as of the evaluation date (noted here as $\hat{C}$). An approximated loss basis $\hat{B}$ can be expressed as:

$$\hat{B} = (U_1 \times R) + (U_2 \times P) + (U_3 \times \hat{C})$$

Under this approach, the cost of "reclosing" a claim is assumed to be equal to the cost of closing a claim of the same size. However, this alternative approach would still fail to capture the cost of reopening claims.

In cases where reopenings of claims are more than negligible, and the ULAE cost of such reopenings (and subsequent "reclosings") is not immaterial, the actuary could obtain a separate provision for the cost of future claims handling activities relating to claims that are closed as of the evaluation of ULAE liabilities. This provision could perhaps be based on a study of the frequency of reopenings and average cost in ULAE of handling the reopened claims.

As noted by Kittel, loss inflation can cause material distortions in the projection of future ULAE payments. We have not attempted to measure the relative accuracy of the generalized method (as compared to other dollar-based methods) in an inflationary environment. Two other issues that warrant further investigation are: the effect of reopened claims on the accuracy of the estimates of unpaid ULAE, and how to modify the approach to properly reflect the change over time in the quantity or cost of resources dedicated to the handling of a claim, as that claim ages.

As mentioned before, the actuary may introduce a measure of estimation error for parameters U_1, U_2 and U_3, and obtain an associated range of reasonable ULAE liability estimates.

As with any reserving methodology, the practicing actuary should carefully examine the explicit and implicit assumptions of the generalized method, as well as the potential effect of external issues when estimating ULAE liabilities, and customize the approach accordingly.

KITTEL'S REFINED APPROACH AS A SPECIAL CASE OF THE GENERALIZED APPROACH

We can quite easily prove that the approach described in this article is simply a generalization of the familiar Kittel refined method described in a preceding section of this article. Indeed, each of the assumptions in Kittel's refined approach can be translated to assumptions about the parameters of the generalized approach.

For example, Kittel's refined method implicitly assumes no future case reserve development or reopened claims. In other words, the estimated IBNR reserves amount to "pure IBNR" only. The Kittel approach also assumes implicitly that all payments associated with a claim occur at closing. Therefore, according to Kittel's implicit assumptions, and using the notation described in the preceding section, R equals reported losses and $P = C$ equal paid losses. Furthermore, by selecting $U_1 = U_3 = 50\%$, and $U_2 = 0\%$, the two approaches are algebraically equivalent.

Table 1 shows the equivalence between the refined and generalized methods, given the assumptions for the refined approach.

Table 1 – Equivalence of Kittel's refined method and generalized approach

Kittel's assumptions and calculations	Adapted to generalized approach notation
• There are no partial payments or reopened claims	• $P = C =$ paid losses
• 50% of ULAE is spent opening claims, and 50% is spent closing claims	• $U_1 = 50\%$, $U_2 = 0\%$, $U_3 = 50\%$
• W = paid ULAE / [50% x (paid loss + reported loss)]	• $W = M / B =$ $= M / (R \times U_1 + P \times U_2 + C \times U_3) =$ $= M / [50\% \times (R + C)] =$ = paid ULAE / [50% x (paid loss + reported loss)]
• Unpaid ULAE = $= W^* \times [\text{IBNR} + 50\% \times \text{case reserves}]$	• Unpaid ULAE $= W^* \times (L - B) =$ $= W^* \times [L - 50\% \times (R + C)] =$ $= W^* \times [L - R + 50\% \times (R - C)] =$ $= W^* \times [\text{IBNR} + 50\% \times \text{case reserves}]$

Where W^* is the selected ULAE-to-loss ratio, based on observed W's.

XYZ INSURANCE COMPANY
REVIEW OF ULAE RESERVES AS OF 12/31/2002
($000's)

EXHIBIT A.1 -- INPUT PARAMETERS

Calendar Year	Cal. Year Paid ULAE	Cal. Year Paid Loss & ALAE	Cal. Year Reported Loss & ALAE	Est. Ultimate Loss & ALAE on Claims Reported in Cal. Year
(1)	(2)	(3)	(4)	(5)
1997	$1,978	$4,590	$19,534	$27,200
1998	4,820	14,600	57,125	76,700
1999	8,558	38,390	85,521	106,900
2000	12,039	58,297	128,672	154,300
2001	13,143	86,074	145,070	163,100
2002	15,286	105,466	163,626	176,400
Total	$55,824	$307,417	$599,547	$704,600

Notes:

(2), (3), (4) As shown in XYZ's 2002 and prior years' Annual Statement.
(5) Estimated in year-end 2002 actuarial analysis.

XYZ INSURANCE COMPANY
REVIEW OF ULAE RESERVES AS OF 12/31/2002
($000's)

EXHIBIT A.2 -- INPUT PARAMETERS

Accident Year	Ultimate Loss & ALAE	IBNR Loss & ALAE at 12/31/2002	Reported Loss & ALAE at 12/31/2002
(1)	(2)	(3)	(4)
1997	$28,600	$257	$28,343
1998	79,200	1,742	77,458
1999	108,400	5,095	103,305
2000	156,700	16,140	140,560
2001	163,400	34,477	128,923
2002	177,100	56,141	120,959
Total	$713,400	$113,853	$599,547

Notes:

(2), (3) Estimated in year-end 2002 actuarial analysis.
(4) As shown in XYZ's 2002 Annual Statement.

XYZ INSURANCE COMPANY
REVIEW OF ULAE RESERVES AS OF 12/31/2002
($000's)

EXHIBIT B -- APPLICATION OF TRADITIONAL METHOD

Calendar Year	Cal. Year Paid ULAE	Cal. Year Paid Loss & ALAE	Paid-to-Paid ULAE Ratio
(1)	(2)	(3)	(4)
1997	$1,978	$4,590	0.431
1998	4,820	14,600	0.330
1999	8,558	38,390	0.223
2000	12,039	58,297	0.207
2001	13,143	86,074	0.153
2002	15,286	105,466	0.145
Total	$55,824	$307,417	0.182

(5) Selected ULAE ratio	0.160
(6) Case reserve	$292,130
(7) IBNR	$113,853
(8) Indicated ULAE Reserve	$41,587

Notes:

(2), (3), (6) As shown in XYZ's 2002 and prior years' Annual Statement.
(4) Equals (2) / (3).
(5) Judgmentally selected.
(7) Estimated in year-end 2002 actuarial analysis.
(8) Equals (5) x [(7) + 50% x (6)].

XYZ INSURANCE COMPANY
REVIEW OF ULAE RESERVES AS OF 12/31/2002
($000's)

EXHIBIT C -- APPLICATION OF KITTEL METHOD

Calendar Year	Cal. Year Paid ULAE	Cal. Year Paid Loss & ALAE	Cal. Year Reported Loss & ALAE	ULAE Ratio
(1)	(2)	(3)	(4)	(5)
1997	$1,978	$4,590	$19,534	0.164
1998	4,820	14,600	57,125	0.134
1999	8,558	38,390	85,521	0.138
2000	12,039	58,297	128,672	0.129
2001	13,143	86,074	145,070	0.114
2002	15,286	105,466	163,626	0.114
Total	$55,824	$307,417	$599,547	0.123

(6) Selected ULAE ratio	0.115
(7) Case reserve	$292,130
(8) IBNR	$113,853
(9) Indicated ULAE Reserve	$29,891

Notes:

(2), (3), (4), (7) As shown in XYZ's 2002 and prior years' Annual Statement.
(5) Equals (2) / { 50% x [(3) + (4)] }.
(6) Judgmentally selected.
(8) Estimated in year-end 2002 actuarial analysis.
(9) Equals (6) x [(8) + 50% x (7)].

XYZ INSURANCE COMPANY
REVIEW OF ULAE RESERVES AS OF 12/31/2002
($000's)

EXHIBIT D -- APPLICATION OF GENERALIZED METHOD
USING 60/40 ASSUMPTION

Calendar Year	Cal. Year Paid ULAE	Est. Ultimate Loss & ALAE on Claims Reported in Cal. Year	Cal. Year Paid Loss & ALAE	Loss Basis	ULAE Ratio
(1)	(2)	(3)	(4)	(5)	(6)
1997	$1,978	$27,200	$4,590	$18,156	0.109
1998	4,820	76,700	14,600	51,860	0.093
1999	8,558	106,900	38,390	79,496	0.108
2000	12,039	154,300	58,297	115,899	0.104
2001	13,143	163,100	86,074	132,290	0.099
2002	15,286	176,400	105,466	148,026	0.103
Total	$55,824	$704,600	$307,417	$545,727	0.102

(7) Selected ULAE ratio	0.100
(8) Ultimate loss and LAE	$713,400
(9) Indicated ULAE Reserve	
(a) Using "expected loss" method	$15,516
(b) Using "Bornhuetter - Ferguson" method	$16,767
(c) Using "development" method	$17,152

Notes:

(2), (4), (8) As shown in XYZ's 2002 and prior years' Annual Statement.
(3) Estimated in year-end 2002 actuarial analysis.
(5) Equals 60% x (3) + 40% x (4).
(6) Equals (2) / (5).
(7) Judgmentally selected.
(8) Estimated in year-end 2002 actuarial analysis.
(9a) Equals (7) x (8) - [Total (2)].
(9b) Equals (7) x { (8) - [Total (5)] }.
(9c) Equals { (8) / [Total (5)] - 1.0 } x [Total (2)].

XYZ INSURANCE COMPANY
REVIEW OF ULAE RESERVES AS OF 12/31/2002
($000's)

EXHIBIT E -- APPLICATION OF GENERALIZED METHOD
USING 70/30 ASSUMPTION

Calendar Year	Cal. Year Paid ULAE	Est. Ultimate Loss & ALAE on Claims Reported in Cal. Year	Cal. Year Paid Loss & ALAE	Loss Basis	ULAE Ratio
(1)	(2)	(3)	(4)	(5)	(6)
1997	$1,978	$27,200	$4,590	$20,417	0.097
1998	4,820	76,700	14,600	58,070	0.083
1999	8,558	106,900	38,390	86,347	0.099
2000	12,039	154,300	58,297	125,499	0.096
2001	13,143	163,100	86,074	139,992	0.094
2002	15,286	176,400	105,466	155,120	0.099
Total	$55,824	$704,600	$307,417	$585,445	0.095

(7) Selected ULAE ratio	0.100
(8) Ultimate loss and LAE	$713,400
(9) Indicated ULAE Reserve	
(a) Using "expected loss" method	$15,516
(b) Using "Bornhuetter - Ferguson" method	$12,795
(c) Using "development" method	$12,201

Notes:

(2), (4), (8) As shown in XYZ's 2002 and prior years' Annual Statement.
(3) Estimated in year-end 2002 actuarial analysis.
(5) Equals 70% x (3) + 30% x (4).
(6) Equals (2) / (5).
(7) Judgmentally selected.
(8) Estimated in year-end 2002 actuarial analysis.
(9a) Equals (7) x (8) - [Total (2)].
(9b) Equals (7) x { (8) - [Total (5)] }.
(9c) Equals { (8) / [Total (5)] - 1.0 } x [Total (2)].

XYZ INSURANCE COMPANY
REVIEW OF ULAE RESERVES AS OF 12/31/2002
($000's)

EXHIBIT F -- APPLICATION OF SIMPLIFIED GENERALIZED METHOD
USING 60/40 ASSUMPTION

Calendar Year	Cal. Year Paid ULAE	Acc. Year Ultimate Loss & ALAE	Cal. Year Paid Loss & ALAE	Loss Basis	ULAE Ratio
(1)	(2)	(3)	(4)	(5)	(6)
1997	$1,978	$28,600	$4,590	$18,996	0.104
1998	4,820	79,200	14,600	53,360	0.090
1999	8,558	108,400	38,390	80,396	0.106
2000	12,039	156,700	58,297	117,339	0.103
2001	13,143	163,400	86,074	132,470	0.099
2002	15,286	177,100	105,466	148,446	0.103
Total	$55,824	$713,400	$307,417	$551,007	0.101

(7) Selected ULAE ratio	0.100
(8) Ultimate loss and LAE	$713,400
(9) Estimated pure IBNR based on	
(a) Pure IBNR amounts to 4% of latest accident year ultimate	$7,084
(b) Pure IBNR amounts to 6% of latest accident year ultimate	$10,626
(10) Indicated ULAE Reserve	
(a) If pure IBNR amounts to 4% of latest accident year ultimate	$16,664
(b) If pure IBNR amounts to 6% of latest accident year ultimate	$16,877

Notes:

(2), (4), (8) As shown in XYZ's 2002 and prior years' Annual Statement.
(3) Estimated in year-end 2002 actuarial analysis.
(5) Equals 60% x (3) + 40% x (4).
(6) Equals (2) / (5).
(7) Judgmentally selected.
(8) Estimated in year-end 2002 actuarial analysis.
(9a), (9b) Based on claims reporting pattern and severity analysis in year-end 2002 actuarial analysis.
(10a) Equals (7) x { [(8) - (9a)] x 60% + 40% x [(8) - [Total (4)]]}.
(10b) Equals (7) x { [(8) - (9b)] x 60% + 40% x [(8) - [Total (4)]]}.

REFERENCES

[1] Bornhuetter, R.L.; and Ferguson, R.E., "The Actuary and IBNR," *Proceedings of the Casualty Actuarial Society* LIX, 1972, pp. 181-195.

[2] Brosius, E., "Loss Development Using Credibility," *CAS Study Note*, March 1993.

[3] Johnson, Wendy A., "Determination of Outstanding Liabilities for Unallocated Loss Adjustment Expenses," *Proceedings of the Casualty Actuarial Society* LXXVI, 1989, pp. 111-125.

[4] Kittel, John, "Unallocated Loss Adjustment Expense Reserves in an Inflationary Economic Environment," Inflation Implications for Property-Casualty Insurance, Casualty Actuarial Society Discussion Paper Program, 1981, pp. 311-331.

[5] Mango, Donald F.; Allen, Craig A., "Two Alternative Methods for Calculating the Unallocated Loss Adjustment Expense Reserve," Casualty Actuarial Society *Forum*, Fall 1999, pp. 225-250.

[6] Rahardjo, Kay Kellogg, "A Methodology for Pricing and Reserving for Claim Expenses in Workers Compensation," Casualty Actuarial Society *Forum*, Summer 1996, pp. 151-184.

[7] Skurnick, David, "A Survey of Loss Reserving Methods," *Proceedings of the Casualty Actuarial Society* LX, 1973, pp.16-58

[8] Slifka, R. S., "Testing of Loss Adjustment (Allocated) Expense Reserve," *Proceedings of the Insurance Accounting and Statistical Association* LX,1968, p.291.

[9] Spalla , Joanne S., "Using Claim Department Work Measurement Systems to Determine Claim Adjustment Expense Reserves," *Proceedings of the Casualty Actuarial Society* LXXXVIII, 2001, pp. 64-115.

APPENDIX: DERIVATION OF THE FORMULAS FOR LOSS BASIS *B* AND ULAE RATIO *W*

Over the life of a cohort of claims corresponding to a particular accident year, let us assume that *L* dollars will be spent on losses. That is, the ultimate losses for claims occurred during that accident year amount to *L* dollars. Using the notation described in this paper, the insurer will spend *W* times *L* dollars on ULAE during the life of these claims, as follows:

- $U_1 \times W \times L$ dollars are spent on the initial opening and set up of claims,
- $U_2 \times W \times L$ dollars are spent on the ongoing maintenance and payment of claims, and
- $U_3 \times W \times L$ dollars are spent closing claims.

If *t* is some point in time after the start of the accident year *y*, the amount of ULAE spent cumulatively through time *t*, or *M(y,t)* is the sum of:

- $U(1,y,t) = U_1 \times W \times R(y,t)$, where *R*(*y*, *t*)is the ultimate cost of accident year *y* claims reported by time *t*,
- $U(2,y,t) = U_2 \times W \times P(y,t)$, where *P*(*y*, *t*) are the loss dollars paid by time *t*, in connection with accident year *y* claims, and
- $U(3,y,t) = U_3 \times W \times C(y,t)$, where *C*(*y*, *t*) is the ultimate cost of accident year *y* claims closed on or before *t*.

The reader will note that each component of total ULAE payments to date is assumed to be proportional to the ultimate cost of claims reported or closed, or payments made, respectively.

Naturally, as t grows, $R(y, t)$, $P(y, t)$ and $C(y, t)$ all approach the ultimate losses for year y, or $L(y)$, and $U(1,y,t) + U(2,y,t) + U(3,y,t)$ approaches the ultimate ULAE for y.

By summing across all past accident years, we can express the total ULAE paid on or before time t, in connection with claims occurring on or before time t, or $M(t)$, as the sum of:

- $\sum_{y \le t} U(1, y, t) = U_1 \times W \times \sum_{y \le t} R(y, t) = U_1 \times W \times R(t)$;
- $\sum_{y \le t} U(2, y, t) = U_2 \times W \times \sum_{y \le t} P(y, t) = U_2 \times W \times P(t)$; and
- $\sum_{y \le t} U(3, y, t) = U_3 \times W \times \sum_{y \le t} C(y, t) = U_3 \times W \times C(t)$.

That is, $M(t) = W \times [U_1 \times R(t) + U_2 \times P(t) + U_3 \times C(t)]$, or, ULAE paid to date is proportional to a weighted average of the ultimate cost of reported and closed claims, and loss payments to date.

Finally, if s and t are January 1st and December 31st of a specific year, the amount of ULAE paid during that year can be described as the difference between $M(t)$ and $M(s)$. We can now derive a formula for W as:

$$W = \frac{[M(t) - M(s)]}{U_1 \times [R(t) - R(s)] + U_2 \times [P(t) - P(s)] + U_3 \times [C(t) - C(s)]}.$$

That is, the actuary can obtain indications of the ratio of ULAE to losses by observing the ratio of ULAE paid during a certain period to the weighted average of the ultimate cost of claims reported, ultimate cost of claims closed and loss amounts paid in that same period.

The reader will recognize the formula for $W = M/B$, where $M = M(t) - M(s)$, $R = R(t) - R(s)$, $P = P(t) - P(s)$, and $C = C(t) - C(s)$.

Typically, the actuary would calculate observed W's for several calendar years, and select the appropriate W^* to be applied for reserving purposes, based on his or her knowledge of any special company circumstances.

The reader will note that this procedure is not restricted to accident years, and could have been as easily applied to accident quarters, or inception-to-date losses, for example.