The Casualty Actuarial Society *Forum*
Summer 2002 Edition
Including the Dynamic Financial Analysis Discussion Papers

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This is the Summer 2002 Edition of the Casualty Actuarial Society *Forum*. It contains four Dynamic Financial Analysis Discussion Papers, two committee reports, and two additional papers.

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All comments or questions may be directed to the Committee for the Casualty Actuarial Society *Forum*.

Sincerely,

Dennis L. Lange

Dennis L. Lange, CAS *Forum* Chairperson

The 2002 CAS Dynamic Financial Analysis Discussion Papers Presented at the 2002 Risk and Capital Management Seminar July 8-9, 2002 Toronto Marriott Eaton Centre Toronto, Ontario, Canada

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The CAS Committee on Dynamic Financial Analysis presents for discussion four papers prepared in response to its Call for 2002 Dynamic Financial Analysis Discussion Papers.

This Forum includes papers that will be discussed by the authors at the 2002 CAS Risk and Capital Management Seminar, July 8-9, 2002, in Toronto, Ontario, Canada.

Risks Considerations for the Allfinanz Organization

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Risk Considerations for the *Allfinanz* Organization

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Abstract

"*Allfinanz*" is the German expression used to describe an integrated financial services provider (Edwards [11]). Such *allfinanz* providers are becomingly increasingly common here in the U.S. and abroad. As firms redefine themselves through such integration, we must also redefine the way we evaluate such firms.

This paper will discuss many of the risks faced by an *allfinanz* organization and then look at the impact imposed by those risks. We will then review some important interrelationships between the various components of an integrated firm. We conclude by briefly discussing a question at the heart of the dynamic financial analysis of such a company: Which risk and performance measures are most important for such a firm?

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I. Introduction

Due to the growing level of integration within the industry, financial services corporations are increasingly comprised of units and subsidiaries in a variety of specific businesses. With respect to insurance, corporations often include multiple insurance subsidiaries for market segmentation, regulatory, and other purposes – and to a large degree always have. However, large property-liability (P-L) insurance corporations are now frequently more than just a collection of regional P-L insurers, or a property-liability and a life insurer, or even an insurer and a bank. A "full-service" financial services organization may include any or all of the following (and possibly more) units and subsidiaries:

- A property-liability (P-L) insurer
- A life insurer
- A reinsurance company
- A banking or asset management unit
- A unit devoted to helping clients arrange alternative risk transfer and integrated solutions
- A services unit, which might sell, on an unbundled consulting basis, a variety of financial- or insurance-related skills, including risk engineering, risk assessment and identification, claims handling, etc.

The reality is that the trend in the financial services industry seems to be toward this type of "full-service" approach. The challenge for everyone involved – corporate management, regulators, investors, etc. – is to analyze such organizations in an integrated and cohesive way. This involves recognizing and measuring the interdependencies and correlations between a diverse collection of economic, financial, and operational variables, and then identifying relative "success" measures to guide future changes in operational strategy.

In particular, this paper will look at the risks faced by a full-service financial enterprise. And for purposes of this paper, a firm of the "retail" or "main street" variety that may offer personal and small commercial lines property/liability coverages, life and health insurance, and personal and small business banking products and services, will be contemplated. After identifying many of the risks faced by such an organization, we will look at the impact of those risks to the firm followed by a discussion of some of the interrelationships of these risks within an integrated firm. Lastly, we will conclude this paper with a brief discussion of performance and risk measures.

II. Identification of Risks Faced by an Integrated Financial Services Organization

The integration of various components of an organization often means the integration of different cultures and languages. For example, the term "credit risk" conjures up one definition in the banking world and another to insurers. Thus, it is not only important to identify all of the risks facing the organization, but it is also important to define those risks such that everyone understands them the same way. The various risks faced by integrated financial services firms are identified (in alphabetical order) and robustly defined here as:

- ***Asset/Market risk*** – the risk to earnings arising from changes in the market price of assets held. Asset/Market risk is intended to include changes due to such things as overall market fluctuations, bond defaults and other factors, but for purposes of this paper is intended to exclude changes in asset price relating to changes in interest rates (which is defined as Interest Rate risk). However, these risks can be combined into one risk definition/category at the DFA professional's discretion.
- ***Credit risk*** – as defined here, refers to the risk of default or nonpayment by counterparties, but does not include the risk of such default/nonpayment by reinsurers or those with whom the firm has entered into hedge or other risk-sharing transactions (defined separately under "reinsurance/hedge risk"). The reason for this division of definitions is due to the varying levels of such risk between banks and insurers. As will be discussed later, the largest risk faced by retail banks is credit risk relating to loans, credit cards, and other such instruments. In the context of using DFA to evaluate credit risk, it seems prudent to separate this definition of credit risk from that associated with reinsurers, hedge and other risk-sharing partners.

- ***Economic risk*** – the risk to earnings arising out of changes in the economy.
- ***Foreign exchange (FX) risk*** – the risk associated with fluctuations in currency exchange rates. (Note: any counterparty risk associated with FX risk hedge transactions is intended to be included under the definition of Reinsurance/Hedge risk).
- ***Insurance risk*** – the risk that insurance operations will not perform as predicted. In a P-L context, this risk is comprised of the "reserving risk" and "premium risk" which the NAIC RBC calculation tries to capture. For life insurers, this risk is referred to as "business risk" in the NAIC RBC calculation and also tries to reflect the "underwriting" risk of a firm. This risk could also include any risk associated with the business cycle(s) of insurance lines written by the firm.
- ***Insurance affiliate investment and Off-Balance sheet risk*** – combined, this risk is the total encapsulated risk faced by affiliates, including any guarantees provided by, or contingent liabilities arising from, affiliates as well as any off-balance sheet risks.
- ***Interest rate risk*** – the risk to earnings (and to asset and liability values) arising from changes in interest rates.
- ***Legal risk*** – the risk associated with any instability in the legal process. Many would consider this to be part of Operational/Business risk, but as many P-L actuaries know, this risk is significant enough to be considered separately.
- ***Liquidity risk*** – the risk that a firm will be required to sell assets at an amount less than their market value in order to meet immediate liquidity needs.
- ***"Measurement" risk*** – the risk we incorrectly measure or that we measure the wrong thing as part of our strategic planning and/or DFA process. It is akin to the parameter risk associated with selecting and measuring strategies. If we use DFA as a tool to measure a firm's global performance, the "parameter" risk associated with this tool becomes increasingly important in the context of the integrated firm.

- ***Operational/Business risk*** – is defined here using the Basel Accord definition (Basel [6], p.2). It is "the risk of direct or indirect loss resulting from inadequate or failed internal processes, people or systems or from external events." This risk is often considered a "catch-all" bucket of risk and incorporates a broad range of risks ranging from business interruption to lawsuits to theft to natural disaster. In fact, some of these sub-divisions of operational risk are so significant that many are defined separately in this section.
- ***Political risk*** – the risk to earnings due to political instability.
- ***Prepayment risk*** – the risk of prepayment by mortgage or credit card holders. For example, a mortgage beneficiary (e.g. bank) holds an asset involving cash flows that are a function of underlying principal repayments. If unexpected changes in the mortgage prepayment rate occur, this will cause the beneficiary to receive cash flows either earlier or later than originally anticipated, and thus fail to earn the anticipated rate of return.
- ***Reinsurance/Hedge risk*** – the risk of default/nonpayment by reinsurers, hedge partners and/or by other risk-sharing partners. An example of "other" risk-sharing/hedging partners would be counterparties to swap transactions. Included in this definition of risk can be any "basis risk" arising from the use of derivative instruments (or this risk can be classified separately).
- ***Regulatory risk*** – the risk that changes in the regulatory environment will negatively impact a firm. Changes in the regulatory environment have been the condition precedent to the integration of financial services firms. Also, the recent happenings with Enron Corp. and the resulting threatened changes to accounting treatment of off-balance sheet risks/investments have increased awareness of regulatory risk. As such, the risk of changes in various regulations is an important consideration to the DFA practitioner.
- ***Reputation risk*** – the risk the firm's reputation will be sullied, perhaps causing an increase in one of the other risks listed in this section.

- ***"Shareholder" risk*** – the risk associated with fluctuations in the firm's market capitalization due to outside investors. It is, for example, the risk of a massive sell-off of your firm's stock over a short period of time. The importance of this risk relates to the firm's resulting cost of capital and ability to raise additional capital.
- ***Strategic risk*** – the risk that a given strategy or set of strategies will fail such that it impacts current or future earnings.

III. The Impact of Risks on the Organization

As can be seen from the previous section, there are a tremendous number of risks faced by *allfinanz* organizations. In this section, we attempt to add context to these risks by looking at their overall impact on the organization. In order to do this, we will first look at how some of these risks are treated by the various regulatory risk-based capital (RBC) requirements. This should provide a relative magnitude for these risks as well as provide insight into which risks the "experts" say are of most concern. After that, we will discuss those risks not specifically addressed in any RBC requirements but impact the organization nonetheless.

Risk-Based Capital

Property/casualty actuaries are well informed about the five risk components included in a P-L insurer's required National Association of Insurance Commissioners (NAIC) RBC calculation. Some important reminders with regard to these requirements are that "reserving risk" is generally the largest risk faced by P-L insurers. Asset risk associated with insurance affiliate investments and off-balance sheet risks is noteworthy as it is the only component outside of the co-variance adjustment (i.e. not subject to the square root) and therefore any increase in this risk will increase the overall RBC requirement by more than an equal increase in another risk charge. It is important to mention that much of the credit risk faced by P-L insurers involves the uncollectibility of reinsurance (and is thus included herein under "reinsurance/hedge risk"), whereupon 50% of the charge is its own charge while the remaining 50% is added in with the reserving risk charge under NAIC RBC.

The RBC requirements for life and health insurers are similar to those used in property-liability. An excellent comparison of these requirements versus P-L requirements can be found in the AAA's "Comparison of the NAIC Life, P&C and Health RBC Formula" (AAA [1]). One important note is that there are separate RBC requirements for life and health insurers, respectively. For life insurers, invested asset risk is typically the largest driver of the required RBC. Insurance risk or "underwriting" risk has the largest RBC charge for health insurers.

The invested asset risk for life insurers is split into two separate covariance items: 1) common stock (which has its own charge), and 2) all other invested assets including bonds, mortgage investments and other invested assets. Mortgage investments are more common, and therefore pose a greater risk, for life insurers than P-L insurers; and are especially important for health insurers that own hospitals, clinics or other real estate. For life insurers, the "other" invested asset charge also includes reinsurance credit risk. For health, the credit risk from reinsurance and capitations are combined in the invested asset charge inside the covariance formula.

While not explicitly incorporated in the RBC calculation, asset risk is also of concern to life insurers due to its effect on disintermediations. Disintermediations increase when returns on other assets go up as such returns are more attractive. Although disintermediation is an important consideration, it should be noted that variable life policies are reducing the risk of disintermediation (Browne, et al [7], p.10), and therefore the relative size of such a book of variable life business should be considered when modeling either the asset or disintermediation risk of a firm.

One important note is that "interest rate risk" is only reflected in the life RBC formula, and not for health or property-liability insurance. This is due in part to the magnitude of such risk faced by life insurers. Some reasons for this are a greater percentage of portfolios held as stocks and mortgage investments, a much higher asset/surplus ratio than P-L, as well as much longer "duration" for liabilities than P-L. Further, the interest rate risk charge is added to the invested asset risk charge of the NAIC RBC calculation before adjusting for covariance due to the higher covariance that exists between these two charges. The result is that these two charges strongly dictate the required RBC for life insurers.

Banking institutions are subject to different, yet just as strenuous, regulatory oversight and many countries mandate banks calculate their required RBC and submit it to a supervisory authority. The banking RBC formula used varies by country. However, the Basel Accord (established 1988, then revised in 1996) is considered to be foundation for these formulae. While the RBC formulae of the Basel Accord have often been criticized (refer to Matten [17] for further details), it is still regarded as the universal RBC standard for banking institutions.

Interestingly, the original Basel Accord RBC calculation only contemplated credit risk. Credit risk, especially with respect to loan and/or credit card holders, is the largest risk most banks face. Credit losses also tend to have a highly skewed distribution (James [15], p. 20). As such, the Basel Accord was concerned with defining the necessary minimum capital to be held to protect a bank from adverse credit experience.

Under the Basel approach, assets are assigned weights between 0% and 100% according to their riskiness. For example, most government-backed assets are given zero weight, most bank-backed assets are assigned a weight of 20%, property-backed assets such as mortgages are given 50% weighting, and most other assets are given a full 100% weight. Basel also specifies techniques and conversion factors for translating off-balance sheet exposures into their on-balance sheet equivalents so that the counterparty risks associated with such exposures are captured. The amount of eligible capital must then be calculated. Capital held is split into two tiers. Tier 1 capital consists of shareholders' equity plus disclosed reserves (including retained earnings) less any goodwill. Tier 2 capital is all other capital held. The total capital is the sum of these two tiers with the proviso that Tier 1 capital must be at least half of the total capital held.

The Basel Accord requires banks to maintain a ratio of eligible capital to risk-weighted assets of at least eight (8%) percent (Basel [4]).

In 1996, the Basel Accord was revised to include market risk as part of the RBC calculation. "Market risk" here is basically used to describe both "asset risk" and "interest rate risk" and includes several risk categories, such as interest rate, foreign exchange, equities and commodities. Basel allows banks to calculate the required RBC associated with market risk following either a set of Basel-prescribed guidelines, or banks may use their own internal models as long as those models meet certain criteria, including Basel-specified parameters such as holding periods and confidence intervals. By far, the most common models used for calculating the market risk faced by banks are Value-at-Risk (VaR) models (Matten [17] and Basel [4]).

Other Than Risk-Based Capital

Financial institutions of all kinds are confronted with myriad risks. Some of the most significant such risks are described and addressed by the various RBC formulae. However, RBC does not (nor does it intend to) capture all of the risks faced by insurers and banks and non-financial enterprises. Some other noteworthy risks for the DFA professional and their impacts are discussed below:

Perhaps the most important risk faced by each unit of our conglomerate but not addressed in any RBC calculation is operational risk. To a degree, one could argue that the "business risk" or the "reserving" and "premium" risk faced (and captured in RBC) by life/health and P-L insurers, respectively, are forms of operational risk as such risks are at the essence of their operations. In any event, according to the definition of operational risk above, insurers certainly face operational risk beyond underwriting risk.

The Basel Accord has clearly defined operational risk as one of the three most significant risks faced by banking institutions and Basel has reopened somewhat the issue of operational risk. In particular, Basel sought input on whether and how to determine the necessary RBC charge for such operational risk. While the Basel Committee on Banking Supervision has provided a consultative document on operational risk, the inclusion of operational risk in RBC was ultimately tabled, primarily due to the difficulty in quantifying this risk (Basel [6]).

Regardless, operational risk remains an important risk commonly faced by each of the core financial facets of the "integrated" financial services firm, including any non-financial unit(s). Thus far, the non-financial components of our organization have been scarcely mentioned. While operational risk is an important concern in managing insurance and other financial operations, it is often the most important concern for the risk manager of any non-financial firm. Various operational risks ranging from competition to supply chain management to product liability often represent the biggest risk faced by non-financial organizations.

One other important risk faced by the *allfinanz* organization is liquidity risk. Such risk is generally interrelated with asset, interest rate, credit and other risks. Generally speaking, liquidity risk should be higher for organizations that face greater variability in the timing of obligations becoming due (not to mention receiving amounts or supplies owed from others). This would suggest that such risk is higher for insurers (both life/health and P-L), less so for banks, and then even less so for non-financial companies. One important difference for non-financial companies, however, is that many non-financial companies often have a greater percentage of their cost of total revenues, as well as the volatility of those revenues, tied to external parties or other factors largely outside of the firm's control. For example, many service firms' largest cost of revenue relates to employee salaries, most of which is owed on a set schedule, whereas the income from job assignments may not be as predictable. So while the nominal liquidity risk may be greater for financial institutions, it can still be an important consideration for the non-financial entity.

A little-mentioned but noteworthy risk that should be considered in modeling any organization is shareholder risk. A big issue in evaluating this risk is the amount of equity capital held (and the "cost" of such capital) in comparison with debt capital. As a rule, banks tend to have a much larger portion of their capital base in the form of debt capital than insurers, which would suggest that banks might be less exposed to this risk. (It should be noted that the level of debt capital for non-financial firms can vary significantly). However, a rapid shareholder sell-off could result in a bank ending up in a not-so-favorable highly leveraged position, which could result in higher borrowing rates, a higher cost of capital, and perhaps invite acquisition of the firm. Regardless of the organization's function, a rapid sell-off can have an important effect on things such as credit rating, liquidity, employee morale and the firm's ability to execute its strategic plan, especially if that plan calls for raising additional capital in order to (e.g.) grow market share in a particular line of business.

Reinsurance/hedge risk as defined above is yet another important consideration for the DFA practitioner. It has been mentioned that such risk is captured for life, P-L and health in their NAIC RBC requirements. While banks do not use "reinsurance" per se, third party credit risk from risk-sharing partners is a significant issue. In particular, counterparty risk has become an important issue among banks with respect to derivatives, hedge instruments, etc. A default under these instruments will occur when a party to the contract owes a payment under the contract and the counterparty cannot obtain timely payment (Hentschel and Smith [14], p.11). Banks are increasing their use of derivatives and other instruments to hedge against various risks. To the extent this usage increases, so does any concern with respect to the creditworthiness of the counterparty to these transactions.

One last risk discussed here is economic risk. Like many categories of risk, economic risk is interrelated with other risks. Here, we are looking at how the firm's fortunes are affected by changes in the economy. For example, a receding economy may affect revenues and loss experience from homeowners and personal auto insurance. There is evidence that policy surrenders increase and purchases of life insurance products decrease if the economy weakens (Browne [7]). Loan and credit card defaults increase when the economy sours. Reciprocally, loan prepayments (associated with prepayment risk) go up when the economy is strong.

As can be seen, there are a number of important risks faced by the various aspects of financial services. When using DFA in decision-making, the key is to understand these risks and their impacts. The table below provides a summary comparison of the relative impact of risks (within each function) discussed above:

Table 1 – Risks and Overall Impact on Components of the Integrated Firm#

Risk	**P/C**	**Life/Health**	**Banking**	**Non-Financial**
Asset risk	Medium	Medium-High	High	Low
Interest rate risk	Medium	Medium-High	Medium	Low
Insurance risk##	Medium-High	Medium-High	Very Low	Very Low
Credit risk	Low	Low	High	Medium
Operational risk	Medium	Medium	Medium	High
Reins./Hedge risk	Low-Medium	Low	Medium	Very Low
Liquidity	Low-Medium	Low-Medium	Medium	Medium
Shareholder	Low	Low	Low-Medium	Low
Economic	Low-Medium	Low-Medium	Medium	Low-High

\# *It should be noted "Low" in the table above does not imply that there is no risk at all, nor does it imply that there is not any catastrophic risk. "Low" here simply means the overall expected value of risk is low.*

\## *Insurance risk refers to "business risk" in the context of the Life/Health NAIC RBC requirement and refers to the combination of reserving risk plus premium risk as defined under P&C RBC.*

IV. Interrelationships Among Risk and Corporate Variables

After identifying many of the risks faced by the various "components" of our integrated firm, the question becomes how these risks relate to each other now that we've put these components together. We can also ask what sorts of issues should be incorporated into the dynamic financial analysis of an integrated firm?

There is an ancient proverb – adopted by Chaos theory – about how the flap of a butterfly's wings in one part of the world can cause a hurricane in another part of the world. The point is that everything in life is related. So too is everything in the integrated financial services firm.

The primary motivator for integrating various financial units involves the strategic placement of the firm for success. That is, the trend toward integration seems to be driven by firms' desires to increase product offerings, leverage capabilities and fulfill other retail aspirations – whereas risk is seldom mentioned as a primary motivation. Risk, however, is a very important aspect to the firm, and identifying the effects of integration on the risks faced by the global firm will be equally as important. A good place to start is by looking at the specific risks discussed above and some of the ramifications of integrating all of the components of the firm with regard to these risks.

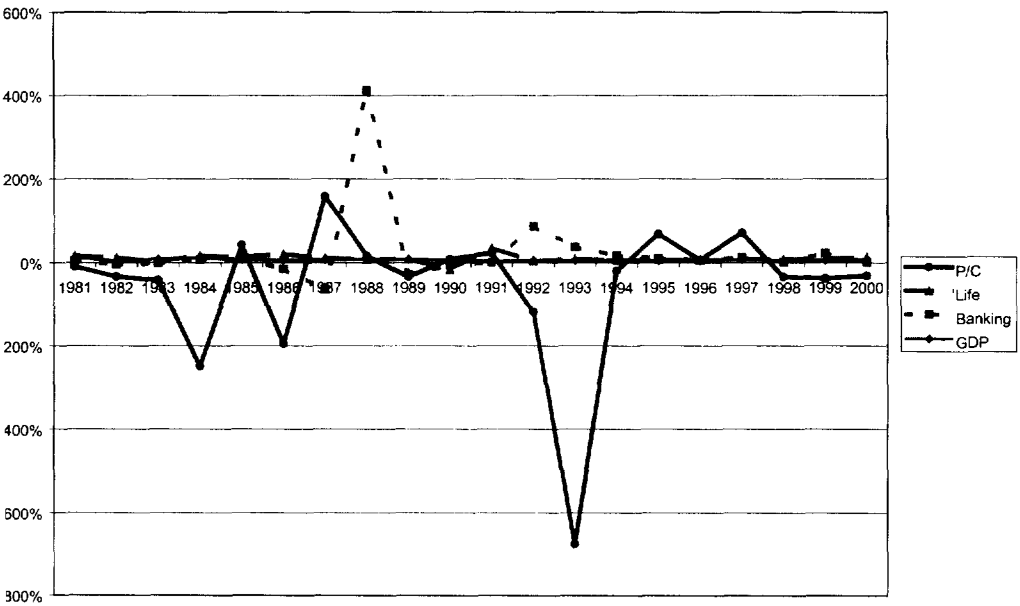
One obvious interrelationship question is to what degree will the risks and rewards of the integrated firm be leveraged versus diversified? This question goes to the very heart of the integration strategy. Presumably, integration means the potential for positive synergy as well as a way to diversify risk. But to what extent does such synergy or diversification take place?

On a macroeconomic basis, we could anticipate that different types of businesses might provide a natural diversification effect when combined. For example, due to differential reactions to economic and financial phenomena, different business segments might "complement" each other, perhaps even by having profit performance of opposite sign for a given set of economic conditions[•]. If this occurs for our integrated financial services firm, then by virtue of integrating multiple disciplines we are reducing the susceptibility of overall profits to annual volatility. In particular, this would imply that the integration of the firm has diversified our economic risk.

[•] A classic pedagogic example would involve the two businesses of daily umbrella sales and suntan lotion sales, which one might expect to have opposite signs regarding revenues for a given weather situation.

As a brief diversion, we can take an initial litmus test with regard to the level of diversification that might exist on a macroeconomic basis. The graph below compares the percentage change in income for the U.S. P-L insurance, life insurance, and banking industries for the past 20 years. The percentage change in GDP (U.S.) is also included as a very rough gauge of the diversification of economic risk for these financial segments.

% Change in Income



Sources: 1. 1998 Life Insurance Fact Book (published by the American Council of Life Insurance)
2. Life Insurers Fact Book 2001 (published by the American Council of Life Insurance)
3. Best's Aggregates and Averages 2001
4. FDIC (http://www2.fdic.gov/hsob/)
5. Bureau of Economic Analysis (http://www.bea.doc.gov)

The respective correlations for the changes shown above are as follows:

	P-L	Life	Banking	GDP
P-L	N/A	-0.06261	-0.02709	-0.01684
Life	-0.06261	N/A	-0.09741	-0.01610
Banking	-0.02709	-0.09741	N/A	0.092463
GDP	-0.01684	-0.01610	0.092463	N/A

The information used shows only a very faint, negative correlation between the change in income for the three respective financial disciplines; and shows no real relationship between economic changes and the income changes for these disciplines. Given the raw nature of this litmus test, it is not very clear to what extent any particular segment(s) will diversify each other, if at all. This is clearly an area where additional research would prove useful.

On a more microeconomic or internalized note, we can intuitively determine which risks are being magnified and which are being diversified as a result of integration. The magnification of risk can also be referred to as the aggregation of risk. This risk aggregation can come in many forms. It can come in the form of increased risk to the extent that "*Allfinanz* Corporation" has the same customers among its various functions. Perhaps due to adverse selection, customers who generate poor P&C loss experience will do the same for health insurance or for mortgage default. Certainly credit risk of customers could magnify to the extent a particular customer may not be able to pay amounts owed in a timely fashion (or pay at all).

The topic of customer aggregation could impact strategic planning beyond risk aggregation. Whether certain types of customers produce different financial results may give rise to the need for some sort of "class"-mapping technology. Different financial disciplines classify their customers in different ways, but most do use some sort of classification system. For example, a P-L personal lines insurer may classify auto risks by age, sex, and territory; a life insurer might classify whole or variable life insurance risks by age, sex, and whether that person smokes or not; and a retail bank would use credit scoring to classify credit card customers. The ability to "class"-map in order to connect the various "classes" of customers/insureds could be valuable in understanding the financial relevance of certain types of customers. Or, perhaps the firm could even create a universal "class" system for its customers and then develop rates/costs for various products for each of those classes. This approach would probably not be recommended for rate filings, but could be valuable in the strategic planning process and certainly could be an important consideration in modeling strategic outcomes.

Risk aggregation can also come in the form of leveraged reinsurance/hedge risk to the extent the firm uses the same reinsurers or other risk-sharing partners across functions. In addition, reputation risk would be magnified as the reputation of one unit may now affect other units. Imagine, for example, what the affect on Andersen Consulting (now Accenture™) would have been if it still had been a part of Arthur Andersen during the ongoing Enron debacle?

Along the same lines of reputation risk would be shareholder risk. To the extent that shareholder's equity is now co-mingled under the integrated firm, a sudden decrease in such equity would now affect multiple operations. If the total required capital/equity for the integrated firm is less than the sum of the needed capital/equity for each subdivision on a stand-alone basis, as many would expect, then shareholder risk could very well increase as a result of integration, especially due to any possible contagion that might occur if a rapid sell-off were to occur.

Converse to the risk aggregation mentioned above, there are clearly some risks that would be diversified as a result of integration. One risk that is likely to be diversified as a result of integration would be liquidity risk. The diversified operations should mean the firm is less susceptible to liquidity risk as it will now have more flexibility in how it meets any liquidity needs as they arise. Foreign exchange risks would also be diversified to the extent revenues are drawn from a greater number of countries.

Arguably, strategic risk would be diversified under the notion that the integrated firm should be more able to sustain one strategy gone bad. Also, if one were to treat each strategy as an independent random variable, then such strategy risk would be diversified as long as the distribution of individual probabilities for the strategies does not diverge as a result of integration. That is, if the distribution of success for the strategies stays the same, then our risk would diversify as we add each additional strategy. However, if the distribution of strategy "successes" did change, then it is possible strategic risk would be exacerbated by integration. It is unclear, however, whether the distribution of strategy successes would change solely due to integration in general.

The above paragraph regarding strategic risk ignores the human element of such risk. When evaluating strategic risk, one should consider the strategizers themselves. If integration results in the wrong people now creating strategy for the greater firm, strategic risk could very well increase, and vice versa. The same could be true to the extent that senior management chooses a strategy or set of strategies that could be construed as "putting all the firm's eggs into one basket." Many consider the human element to be un-model-able (not to mention a very sensitive topic in general), but it can still be an important consideration in any final analysis.

One risk that does not appear to have a clear answer whether integration will magnify or diversify the overall risk is operational risk. Using the definition above, will an *allfinanz* organization be more or less likely to suffer direct or indirect loss resulting from inadequate or failed internal processes, people or systems or from external events? On the one hand, one might argue that a larger, integrated firm would be more likely to withstand any loss arising from operational risk. On the other hand, such risk may be greater to the extent that the integrated firm now has to learn to cope and operate within its integrated structure. According to Frick and Torres, merger and acquisition destroys value for the acquiring company more than fifty (50%) percent of the time, while spin-offs and alliances produce similar results (Frick and Torres [13], p.1). However, Amslinger, et al. [2, p.4], suggest that some forms of restructuring, such as IPO's and spin-offs, on average create value. While "integrating" various disciplines may not be identical to merging, acquiring, or restructuring, a lot of the same dynamics and challenges are involved. The point is that integration is not guaranteed to succeed and therefore the risk of integration failure should be considered as part of the strategic planning analysis.

An interesting consideration that arises out of integration is the effect on interest rate risk. At first blush it would seem that one could predict the overall direction in risk based on the percentage allocation of various asset types. Generally speaking, the interest rate risk for a stand-alone life/health insurer will be higher than that of a P-L insurer, which will be higher than that of a bank using duration as our measure of interest rate sensitivity and assuming that each discipline will carry assets with durations similar to that discipline's duration of liabilities. As the duration of a combination of instruments is equal to the weighted average of the durations of the individual instruments (i.e., duration is additive) (Noris [19]), then the integrated firm's overall sensitivity to interest rate risk will be somewhere in-between the individual disciplines' sensitivity. This also means that regardless of whether a liability is insurance-related or banking-related, one can "immunize" the firm's surplus against interest rate risk for the integrated firm by matching the product of the asset portfolio value and the duration of the entire asset portfolio with the product of the total liability amounts and the overall duration of liabilities for the firm.

The use of derivatives can throw a wrinkle into the interest rate risk faced by our firm. Derivatives, for example, are often used to hedge against interest rate risk, asset risk, and foreign exchange risks. The obvious irony here is that many derivatives themselves are subject to potential losses due to changes in the price of the underlying assets or changes in interest rates. This risk is sometimes also called "pricing risk" (Hentschel and Smith [14], p.4). This means that this underlying "pricing risk" and the effectiveness of the hedge/derivative should be incorporated into our modeling methodology, perhaps through the use of some kind of subroutine. At a minimum, the incorporation of derivatives and other hedge instruments warrant consideration and can make for a complicated asset/liability model for our integrated firm.

Regulatory risk presents a topical risk for all *allfinanz* organizations. Changes in regulations have been a huge impetus in the creation of (not to mention legal ability to create) financially integrated firms. Because of this dependence on regulatory oversight, regulatory risk can impact our *allfinanz* organization if it rears its head. What if important current regulations were to change and what might the effect be? For example, what if there was a change (perhaps in light of the Enron ordeal, for instance) in how off-balance sheet risks were accounted for? What would the impact be to the balance sheet and income statement going forward? What if a change in regulations caused a necessary shift in current operations? Or what if such a change gave another competitor a new advantage in a certain market? These all are important questions at the heart of the regulatory risk faced by our firm.

One last source of interrelated risk not defined above is referred to here as "self-insurance risk." Self-insurance risk refers to the risk the integrated firm takes on when it self-insures against any exposures relating to their own operations, including those of subsidiaries. On the one hand, the integrated firm will likely be able to retain more risk due to its presumably diversified operations and more efficient capital base. On the other hand, what if our hypothetical services arm began to offer certain professional services that required professional liability insurance to be in place, and our firm self-insures much or all of this risk? Or what if a general increased "net" (of reinsurance) position is taken by the firm? The latter question is a favorite topic of DFA and any aggregation of this risk can be an issue, as can an overall shift in business practices to emphasize the offering of third-party services.

V. Brief Discussion of Measures

This paper has dealt with the risks associated with an integrated financial services firm. For purposes of strategic and operational modeling of such a firm – e.g., for purposes of doing dynamic financial analysis – identification of the specific risk characteristics of an integrated firm is a critical early step in the process. A great deal of additional work is required beyond that, however. In this concluding section, the selection of risk and performance measures is discussed briefly, and some commentary is provided. It is hoped that future research will provide additional consideration of these issues.

Which performance and risk measures should we use?

DFA has been defined as a systematic approach to financial modeling which projects financial results under a variety of possible scenarios, showing how outcomes might be affected by changing business, competitive, and economic conditions (CAS [9]). One of the goals of DFA is to provide management with a quantitative look at the risk-and-return tradeoffs inherent in emerging strategic opportunities, and to examine how these tradeoffs affect the entire organization. Many DFA models currently in use for P-L and Life insurers examine these tradeoffs by establishing both return (i.e. performance) and risk measures. Presumably, management can then evaluate their strategic opportunities by setting minimum and target thresholds and seeing how the firm holds up against these criteria when affected by changes in their business, competitive, and economic conditions.

There are a number of approaches to measuring and reflecting the interrelationship between risk and return for DFA or other application purposes. Some of these measures are uniquely used for specific financial services functions whereas others are used more universally among the gamut of financial services firms. Several examples of commonly used measures are:

- ***Probability of Ruin***
- ***Capital "adequacy" measures*** such as Risk-Based Capital, Best's Absolute Capital Adequacy Ratio, S&P's CAR calculation, or other similar internally developed measures.
- ***Risk-vs-return plot*** – many such plots are used to exhibit DFA results using such variables as expected profit, ROE, and ROA (e.g.). Also common are results encapsulated as probability density functions (p.d.f.'s) of a key financial measure such as the amount of surplus, profit, or ROE (e.g.). It should be noted that life insurers commonly focus on assets and/or the return on assets as a key performance measure (Browne [7]) as do many banks. Risk-vs-return and p.d.f. plots have the obvious benefit of being easy to understand and often provide excellent context in the risk versus reward tradeoff.

- ***Risk-adjusted Return on Capital (RAROC)*** – and other risk-adjusted measures try to incorporate the risk-vs-return tradeoff in a single measure. Like the efficient frontier (see below), the intent of such measures is to state returns on an apples-to-apples basis whereby the potential return is weighed against the risk of a particular asset, strategy, or whatever may apply. (For insight into how one large banking institution uses RAROC is assessing the performance of individual unit, see James [15]).
- ***Efficient frontier*** – the efficient frontier faced by a firm can be stated (e.g.) in terms of return on assets, usually from the investors' perspective (i.e. how will a firm's return to shareholders for a particular strategy compare to that of the returns offered by other investments given the same level of risk?). The efficient frontier can also be relayed (e.g.) in terms of economic value as well as other bases.

In addition to the above, I would like to provide further comment on two other important risk measures:

- ***Expected policyholder deficit (EPD)*** – the EPD concept works for not only P-L insurance, but can also be applied easily to life/health insurance exposures. But the contiguous term "policyholder" suggests that EPD may not be the best measure, or perhaps phrase, for banking or other non-insurance financial services functions. In terms of looking at capital adequacy of the integrated firm, however, the EPD concept of capturing both the probability and impact of insolvency makes sense regardless of the type of firm being analyzed. As such, perhaps the concept of Expected "Creditor" Deficit (ECD) as a more universal measure is appropriate? ECD would use the EPD concept, but be incorporated for all creditors. ECD could perhaps have a "tiered" result structure based on the security level of each creditor – for example, one tier for insurance policyholders, another for other secured creditors, one for general creditors, etc. While not called ECD, many financial ratings firms such as Moody's and Standard & Poor already perform a similar type of analysis for all types of firms, bond issues, etc. ECD would perhaps present such analyses' results in a slightly different manner.

- ***Value-at-Risk (VaR)*** – As mentioned previously, VaR is a risk measure commonly used by banking institutions, especially with regard to market risk. In this context, VaR is often used to evaluate the probability of a decline in the asset portfolio value (e.g.) by more than some percentage (e.g., five [5%] percent) over a stipulated period of time (often a very short period of time such as one day, but can be evaluated up to one year). Fallon [12] provides an excellent (albeit fairly technical) presentation of four VaR methodologies in use as well as offers his own VaR methodology to be used in banking risk management.

The use of VaR has now spread to life and P-L insurers alike. In fact, its use is already being scrutinized. Artzner et al. [3] (and Meyers [18] explains) show that VaR is not a "coherent" risk measure as it does not satisfy the subadditivity axiom. (The subadditivity axiom basically says that for all [bounded] random losses X and Y, the risk measure "amount" for X+Y combined {defined as $\rho(X+Y)$}, will be less than the sum of the individual risk measure "amounts" for X and Y, respectively {i.e. $\rho(X+Y) \leq \rho(X) + \rho(Y)$ }. Without question, Artzner et al offer important considerations when selecting risk measures. The authors also suggest the use of Tail Conditional Expectation (TCE) as an improved risk measure that satisfies all four of the coherency axioms/requirements (for more about TCE, see Artzner et al. [3] or Meyers [18]).

Before dismissing the use of VaR entirely, however, it is important to note that VaR does not fail to be subadditive every time – it is just not subadditive 100% of the time and really depends on the nature of the random variables X and Y (e.g. see [3] Remark 1, p. 14 as a limited example). Intuitively, many would expect that the overall risk of an *allfinanz* firm would be diversified as a result of integrating various functions. While it may be counter to this diversification expectation, the use of a "coherent" risk measure begs the question: what if such risk is not diversified? As discussed above, there are many possible dynamics within the integrated firm that would actually magnify the overall risk faced by such a firm. Again, while not immediately intuitive, if the overall risk of the firm is not diversified, then perhaps the use of non-"coherent" risk measures makes some sense.

One other interesting question the use of VaR for the integrated firm might raise is how should VaR be defined for such a firm? For example, if we verbally define VaR to be the most X we are willing to risk for a certain period of time (T), the question becomes what should X be for the integrated firm? In banking, the X is often the threshold for loss in asset value over the period (T) of one day. Should X be something different for the integrated firm? Perhaps market value or surplus should be used. Also, what should the threshold amount be and what time period (T) should be used? The reason that one day is often used for VaR purposes in banking is because it is commonly held that one day is the maximum amount of time it will take a bank to modify its portfolio holdings in order to stave off any further decrease in the asset portfolio. But for insurers, how long does it take to react to any precipitous fall in a particular balance sheet or other item? Given the length of policy periods, possible long-tail exposures, etc., the answer is unclear other than it could take a very long time. For this reason (as well as others), VaR may not make sense for the integrated firm – but its use certainly raises some interesting questions at a minimum.

Each of the above has its advantages and disadvantages. DFA in the context of the integrated firm can, however, result in these risk-return approaches having different advantages or disadvantages versus when used for one individual (or silo) financial services function (e.g., P-L insurance). Two important considerations in selecting a risk-return framework in an integrated-firm context are:

- ***Does the measure make sense for all aspects of the firm?*** For example, Carlton [8] states that non-financial firms should use a measure of risk that focuses on cashflow shortfall or cashflow-at-risk rather than variance in the market value of the firm's assets. Also, according to Browne, et al. [7], life insurers have historically focused more on returns than on risk.
- ***The purpose of the analysis.*** For example, a strong regulatory focus in an analysis might lead one to concentrate more on solvency-based measures such as the probability of ruin or the expected policyholder deficit.

Also, the performance measures used do not necessarily have to be in a risk-versus-reward tradeoff format. For many of the couplings above, one can simply strip the coupling of its risk metric and use the performance measure only. Or, the performance measures used could be in the form of "balanced scorecard" measures, such as the increase in customers, customer retention, market share, etc. The "balanced scorecard" measures have the benefit of being easy to understand, easy for the employee stakeholders to adopt, and in some cases are more in concert with a company's strategic objectives than the more financially-related measures originally discussed. That is, many chief executives verbalize a course for their firms based on objectives such as increasing the number of products per customer, rather than on achieving a certain position above the competitive efficient frontier.

The use of such measures does not mean they cannot be evaluated against risk measures. Such risk measures can certainly be added. For example, the performance measure of "doubling the number of products offered" can be used in conjunction with the risk measure of "a five-year standard deviation of calendar-year ROE of less than 0.10." One will certainly want to choose logical couplings; but DFA allows the freedom to choose the performance and risk measures that are most important to a particular firm.

VI. Conclusion

With the integration of the financial services organization comes new risks and a need to look at these risks in a more global context. This paper hopefully provides useful "food for thought" to the DFA professional and others regarding the risks faced by the integrated financial services firm, the impact of those risks, and the interrelationship of such risks within an *allfinanz* organization.

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Abstract

Since 1996, the Casualty Actuarial Society has issued a call for papers on one aspect or another of Dynamic Financial Analysis. In past years the calls have focused on the modeling of property/casualty companies. This year's call is the first to expand the focus of the call to DFA models that capture operations outside of the traditional property/casualty sphere. The process of developing and using models that incorporate more that P&C operations is, on one hand easier than developing a stand-alone P&C model, and on the other hand, more difficult. It is easier because it presumes the P&C model (and presumably the life or banking model to which the P&C model will be joined) has already been developed. As such, we can skip over the complex work of developing the stand-alone model or models and turn to the aspects that make the multi-operation model more complex. The greater complexities arise from the greater scope of the resulting model – there are more pieces to be thought about, more risk factors to be considered, more interrelationships to be quantified and programmed, and the end result is that much more complex with which to work.

This paper describes the process of creating a multi-operation model from three stand-alone models using the MoSes software package. The paper focuses on the process of bringing the three different pieces together into one combined model and discusses the nature of the risk factors and linkages between the pieces.

I. Introduction

The 2002 Casualty Actuarial Society call for papers on Dynamic Financial Analysis is focused on modeling financial institutions with more than just a property-casualty exposure. This paper describes a small subset of the operations of a banking institution with life and property-casualty interests in addition to the core banking operations. The company that provided the inspiration for this paper is a financial institution with a much larger focus than its property-casualty operations. In fact, one could easily say that the P&C operations are very minor in relation to the company's main banking operations. That does not mean that the company cannot benefit from an integrated modeling of their banking and insurance exposures, especially since the banking and insurance operations are subject to many of the same underlying drivers of profit or loss.

The process of creating and implementing an integrated banking / insurance model that captures all of the company's operations is still in its infancy. This paper describes the start of the process. As such, the emphasis of this paper is on the process of developing a model that integrates the various components rather than the results derived from the model. The paper describes the structure and the cross-model linkages of the consolidated model. It does not discuss modeled results or management conclusions drawn from the modeling, as these parts of the process are still going on at the time of writing. Even without these aspects, we feel there is value to be gained from merely developing the model and thinking about the nature of the interdependencies between the business units.

The modeling environment being used is one that allows models to be created as stand-alone constructs, and then linked together via the creation of a "parent" model. This capability allows the development of a simplified model at first that can be subsequently expanded to incorporate more aspects of the corporate entity.

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II. Model Overview

The subset of the banking entity being modeled consists of the following operations:

1. Collection of money from individuals through the sale of deferred annuity products.
2. Investment of that money through the financing of residential mortgages
3. Sharing in the private mortgage insurance (PMI) risk associated with some of the residential mortgages through a reinsurance arrangement with the private mortgage providers.

The model also has an aggregation component that pulls the results from the three operations into one holding company's financial statements. Lastly, there is a common economic scenario that applies to all modeled aspects of the company.

A. Deferred annuity product

In this model, the bank sells deferred annuities to the general public. The deferred annuity product being sold is a fairly standard life insurance product. In exchange for money paid to the bank (the deferred annuity premium), the bank credits the policyholder with a rate of interest that is typically greater than what would be available for an investment of a similar amount of money in a certificate of deposit or a savings account. The crediting rate is tied to a rolling average of U.S Treasury yields, with a contractually specified floor. This creates a linkage between the growth in value of the annuities and the economic environment in which the investment has been made. This also exposes the bank to two types of asset risk. The first of these is the potential for the bank to have invested the policyholder premiums in assets that return less than the promised crediting rate. Ordinarily, there is sufficient spread between the crediting rate and the yield on other assets in which the bank can invest policyholder premiums, but that might not always be the case. The second is the risk that policyholders will cancel their policies. In the event of policy cancellation, the bank must return the premiums to the policyholder, less any surrender charges. If the bank has invested the premiums in assets whose value has declined, the bank may not be able to sell the associated assets for as much as they owe to the policyholder. While the bank does have some protection against this risk by the surrender charge levied against early cancellation, there is always the potential that the surrender charge is not sufficiently large to offset the difference between amounts owed to policyholders and the value of the assets that must be sold to pay back the policyholders. In exchange for taking on these two forms of asset risk, the bank earns profits from any positive spreads between its investment of policyholder premiums and the crediting rate promised to policyholders.

B. Investment in residential mortgages

In this model, the bank takes the money it has collected from selling deferred annuities and invests it in residential mortgages. This is a vast oversimplification of what a bank would really do with its investable assets, but, for the purposes of this paper, let us assume this is what the bank does. In the model, the bank lends money to people looking to purchase residential properties. Some of the mortgages are provided to people who are able to put down twenty percent of the purchase price with their own funds. These mortgages do not require private mortgage insurance. Another portion of the mortgage portfolio goes to people who do not have sufficient funds to provide a twenty percent down payment. These mortgages require the mortgagor to purchase private mortgage insurance from a PMI provider. We will come back to PMI in a moment. For now we will focus on the mortgage process itself. In exchange for the bank loaning money to a mortgagor, the bank is promised a stream of monthly payments from the mortgagor that include some amount of interest payments. The interest rate is tied to the U.S. Treasury yield curve at the time of the mortgage origination.

What are the economics of PMI for a bank?

The bank requires mortgagors who make less than a twenty percent down payment to purchase PMI because of bank risk-based capital requirements. If a loan is made in which the loan amount is less than eighty percent of the underlying property value, the bank must set aside four percent of the loan amount to satisfy risk-based capital requirements. If the loan amount is greater than eighty percent of the underlying property value, the risk charges doubles to eight percent. However, if the mortgagor purchases private mortgage insurance, the bank's risk charge once again drops to four percent.

To simplify descriptions a bit, we will call a loan in which the loan amount is greater than eighty percent of the underlying property value a "higher-risk loan" and a loan in which the loan amount is less than eighty percent a "standard loan". Note that these are not descriptions with any meaning in the banking world – they are just being used in this paper.

If a bank is approached to make a higher-risk loan, the interest rate charged for that loan will be higher than what would be charged for an equal loan amount on a standard loan. The economic question facing the bank, then, is: "Is the potential extra profit I can make on a higher-risk loan worth the additional capital I will be forced to hold in support of that loan?" If the answer is No, then the bank's alternative is to require the mortgagor to purchase PMI. This allows the bank to treat the loan like a standard loan, since much of the risk associated with the higher-risk loan has been transferred to the PMI provider.

What are the risks the bank faces from making residential mortgage loans?
The bank is exposed to two types of risk from their residential mortgage investments. The first is a prepayment risk. If interest rates decline, mortgagors will be more likely to refinance their mortgages. This results in the return of the bank's loan to the bank much earlier than the bank had anticipated. The bank must now reinvest the repaid mortgage amount, most likely into investments yielding less than they were receiving on the now repaid mortgage. This can result in a narrowing of the spread between the crediting rate the bank has promised to its deferred annuity investors and what the bank can earn on the invested funds. This in turn leads to decreased profits or possibly even losses for the bank. The second risk is that of mortgage default. If the value of the property securing the mortgage falls below the level of the mortgage and the property owner defaults on the mortgage, the bank is left with an asset whose market value is less than the face value of the mortgage loan. This is exactly what happened in Texas during the savings and loan crisis of the early 1980's and in New England in the late 1980's.

C. Private mortgage insurance

It is to protect banks against the situation in which property values decline below the face amount of a mortgage that private mortgage insurance exists. The greater the "loan-to-value" ratio of the mortgage amount to the property value, the riskier the mortgage is. The standard rule is that if a property purchaser can provide twenty percent of the purchase price, no private mortgage insurance is required. If the property purchaser cannot put down twenty percent, he or she must purchase private mortgage insurance in addition to making the loan repayments. The private mortgage insurance protects the bank in the case of a mortgage default. If a property owner defaults on his or her mortgage and private mortgage insurance has been purchased, the PMI company will do one of two things: either pay the bank a contractually specified amount or pay the bank the face value of the mortgage. Either way, the bank recovers some or all of their potential loss from the mortgage defaults because of the PMI.

Example 1

An example of the way private mortgage insurance might work is as follows:

- Suppose a home purchaser buys a home for $100,000. To do so, the homebuyer takes out a $95,000 mortgage with an interest rate of 8%. The homebuyer is the mortgagor and the bank is the mortgagee.
- The bank requires the homebuyer purchase private mortgage insurance to protect the bank in the case of default. The PMI terms include the following items:
  - Coverage level – the maximum amount that will be paid by the private mortgage insurer to the bank in the event of a default. On this loan, the coverage level is equal to 25% of the original mortgage amount.
  - Premium rate – the annual amount, as a percentage of the loan's face value, which the mortgagor must pay to the PMI provider. On this loan, the premium rate is 0.67% of the original mortgage amount.
- The homeowner makes payments for two years before defaulting on the loan. The remaining principal equals $93,200. In the intervening two years, the housing market has deteriorated, leaving the house with a market value of $80,000.
- With no PMI, the bank would stand to lose $13,200, the difference between the outstanding principal and the market value at the time of the mortgagor's default.
- With PMI, the PMI provider has two alternatives – either pay the bank an amount equal to the coverage level or pay the bank the outstanding principal balance and take over the rights to the mortgage. In this case, the PMI provider would do the latter, because it is cheaper to pay the bank $93,200 in exchange for an asset worth $80,000 (i.e. a loss of $13,200) than to pay the bank 25% of 95,000, or $23,750.
- Because the bank receives an amount equal to the outstanding principal from the PMI provider, the bank suffers no loss. The PMI provider absorbs the entire loss.

Example 2

Suppose, instead, the market value declined more precipitously to $65,000. In this case:

- The PMI provider would pay the bank the contractually stipulated coverage level, or $23,750.
- The bank would retain the ownership of the defaulted property. Assuming the bank sold the property immediately, the bank would realize a net loss of $93,200 – 65,000 - 23,750, or $4,450.

To complicate matters, the bank has a reinsurance contract with the PMI provider. This allows the bank to share in the profits (or losses) arising from the sale of PMI. In this way the bank reacquires some of the mortgage default risk that was passed on to the PMI provider.

The reinsurance can either take the form of a quota share or an excess of loss arrangement. The underlying PMI coverage is organized according to "book years", or the year in which the mortgage getting the PMI coverage was originally written. One reinsurance contract covers claims arising from one book year.

For example, suppose the company loaned $1 billion for residential mortgages in calendar year 2000 and $1.1 billion in 2001. This is equivalent to two book years. There would be two separate reinsurance contracts in force, one for the 2000 book year and a second one for the 2001 book year. Any reinsurance payments made by the first reinsurance contract (the one for the 2000 book year) would be contingent upon claims arising from the $1 billion book of mortgage loans made in calendar year 2000. Any reinsurance payments made by the second reinsurance contract would be contingent upon claims arising from the $1.1 billion book of mortgage loans made in calendar year 2002.

Why would the bank have a reinsurance arrangement with the PMI provider?
What does the bank gain by reinsuring some of the risk that has been passed on to the PMI provider? Profits. Private mortgage insurance is generally a very profitable line of business. For example, consider the results reported by one of the largest writers of private mortgage insurance, Mortgage Guaranty Insurance Corporation (MGIC). According to MGIC's financial statements from 1997 through 2001, MGIC's combined ratio from 1993 through 2001 ranged from a low of 26.7% to a high of 66.0% and the company's return on equity was consistently in excess of twenty percent.[1] The bank wants a share of these profits. The bank, through its reinsurance contract with the PMI provider, is able to share in the PMI profits. Of course, by virtue of its re-acquiring some of the default risk that had been passed to the PMI provider, the bank's capital requirement increases. However, the additional capital the bank must hold is less than what it would have had to hold had it not required the mortgagor to purchase PMI. The end result, then, is the PMI provider profits and the bank shares in those profits and uses less capital in the process than if PMI were not used.

D. Aggregation component

This component aggregates balance sheet, income statement and cashflow items from the three operational areas. The aggregation creates financial statements at the holding company level.

E. Economic scenario

An economic scenario generator is used to produce values for various economic indices, including short and long-term interest rates, equity returns, and property inflation. All operational areas of the model access the same economic indices, insuring consistent responses to changes in economic conditions across the modeled environment.

[1] Information taken from MGIC's 1997-2001 annual statements, as posted on their web site http://www.mgic.com/.

III. Risk Factors

The company is exposed to a variety of risk factors, some of which cross multiple operational areas. It is by modeling this commonality of risk factors that the model will derive its ultimate value.

A. Deferred annuity risk factors

The bank is exposed to interest rate risk by selling deferred annuities. As noted in the section above that described the deferred annuity product, the bank pays the deferred annuity investor a contractually specified interest rate. If the bank cannot invest the policyholders' premiums in investments that provide a rate of return at least as high as the rate being paid to policyholders, the company will lose money on the annuity contracts. Additionally, if the deferred annuity policyholders cancel their policies, triggering a repayment requirement on the part of the bank, the bank must liquidate the investments that were purchased with the deferred annuitants' premiums. If the market value of those investments has declined below the repayment requirement, the bank must make up the difference out of surplus. Both of these are examples of an exposure to interest rate risk.

B. Residential mortgage risk factors

The bank is exposed to both interest rate risk and credit risk through its investment in residential mortgages. The interest rate risk arises from the potential for the mortgagors to prepay the mortgage if interest rates decline. This, in turn, puts the bank in the position of having to reinvest the repaid principal at a time when interest rates have declined. The result is that the bank will most likely have to invest in new assets with lower yields than the prepaid mortgages would have yielded, had they not prepaid. The credit risk reflects the potential for mortgagors to default on their loans at a time when the values of the properties securing the mortgages are less than the outstanding mortgage principal balance.

C. Private mortgage insurance risk factors

The PMI provider and the bank, through its reinsurance arrangement with the PMI provider, are both exposed to pricing and interest rate risk factors. The pricing risk arises from the potential that the (re)insurance is improperly priced vis-à-vis the underlying loss exposure arising from the insured mortgages. The underlying loss exposure is the same as the credit risk exposure faced by the bank when providing money for a mortgage that does not require PMI. This is the risk that, once a mortgagor has defaulted on his loan, the property value is worth less than the outstanding loan amount. The interest rate risk exposure is driven by the same factors as the residential mortgage interest rate risk – namely that in a period of declining interest rates, a higher percentage of mortgages will refinance their loans. When the underlying loans are refinanced, the PMI premiums that had been generated by those loans are cut off, leaving the PMI provider with lower than anticipated premium inflows. (The person refinancing the loan may need to purchase PMI on the refinanced loan, however there is no guarantee that either the refinancing or the reinsuring of the PMI provider will be done by this bank.)

IV. *Putting the pieces together*

A. Deferred Annuity → Residential Mortgages linkage

A bank must have an inflow of funds before it can loan funds out in the form of residential mortgages. There are many ways in which money can enter a bank. One is capital provided by investors in the bank. Another is through the sale of bonds or other debt instruments to outside parties. A third is by taking in funds through traditional avenues, such as customer deposits. Deferred annuities are very similar to bank certificate of deposits except that one is governed by insurance rules and the other by banking rules. We could have chosen to provide funds inflows into our bank through any of these avenues and in the future we expect we would increase the number of ways for money to flow into the bank. For the purposes of the current model, we have limited ourselves to just the sale of deferred annuities, as this is a new venture for the bank and they want to better understand the capital usage that would be involved in selling deferred annuities.

The primary link, then, between deferred annuities and residential mortgages is that the sale of deferred annuities provides the bank with funds that can be invested in residential mortgages.

A secondary link is through the application of a consistent set of economic conditions. Both deferred annuity products and residential mortgages are impacted by changes in underlying interest rates.

- A drop in interest rates will lead to a larger than anticipated number of mortgage refinancings. The bank will receive the outstanding principal from the refinancings and will need to reinvest the money in lower yielding investments. This will lower the spread between the crediting rate the bank has promised its deferred annuity policyholders and the yield the bank is receiving on the investment of the policyholder premiums, reducing or possibly eliminating the bank's profits from the deferred annuity sales.
- A rise in interest rates might lead the holders of the deferred annuities to cash in their policies so they can move their invested funds into higher yielding products. This will force the bank to sell some of its residential mortgages on the secondary market at a time when interest rates are higher than when the mortgages were originally written. As with bonds, the market value of a mortgage declines when interest rates rise. This means the bank is selling mortgages at a time when the market value of the mortgages is depressed relative to the book value of the mortgages. The bank is exposed to the potential for realized capital losses on the mortgages that must be sold to repay the deferred annuity policyholders. The bank is also exposed to a second potential source of loss in a time of rising interest rates, especially if the rising interest rates are accompanied by declining property values,

as happened in the mid-1980's in Texas (from 1986 to 1989, short term interest rates rose from 6.45% to 8.53% and house values declined by an average of 10%[2])

B. Residential Mortgages → Private Mortgage Insurance linkage

The linkage between the residential mortgages and the private mortgage insurance coverage is that the latter exists to protect the bank's investment in the former. The bank, through its reinsuring of the PMI provider, is electing to take back some of the exposure to loss that would otherwise be eliminated through the use of PMI. With this arrangement, the benefits of integrating the residential mortgage and PMI exposures into one model are quite clear. The actions that will lead to prepayments or losses on the residential mortgage portfolio will have a pass-through impact on the profits and losses of the PMI provider, and through the bank's reinsuring of the PMI provider, back to the bank.

When modeling the PMI business on a stand-alone basis, the following items were set up as inputs to model the gross PMI activity:

- Face value of mortgages being written
- Level of PMI coverage provided for each mortgage type
- PMI premium rate for each mortgage type
- Projected mortgage persistency (the percentage of mortgages on the books at time t that will prepay in time $t+1$)
- Expected ultimate loss amount, as a percentage of the face value of the mortgages being written
- Claims payout speed

When modeling the PMI business in conjunction with the residential mortgage business, many of these values are passed from the residential mortgage model to the PMI model. In the linked model, the face value of the mortgages being written is a function of the available cash flow, which is developed from both the amount of new deferred annuities being written and the profits derived across all three operating units. The level of PMI coverage provided for each mortgage type remains an input, but now it is captured as a characteristic of the mortgages being financed in the residential mortgage model. The same is true of the PMI premium rate for each mortgage type. The PMI mortgage persistency is now driven by persistency assumptions in the residential mortgage model. In turn, the residential mortgage persistency calculation is driven by the spread between the interest rate of each mortgage and the interest rate at time t. The modeler enters a table of prepayment rates that vary with the spread – the larger the spread, the higher the prepayment percentage.

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[2] Short-term interest rate data was taken from information on the Federal Reserve web site, http://www.federalreserve.gov/releases/. House values data was taken from the house price index compiled by the Office of Federal Housing Enterprise Oversight and provided on their web site, http://www.ofheo.gov/house/index.html.

The most interesting linkage is in the area of defaults and recoveries, which translate to claims against the private mortgage insurance. In the stand-alone PMI model, assumptions were made about the expected ultimate claims amount per $1,000 of mortgage face value and the timing with which the claims would be reported and paid. In the stand-alone residential mortgage application, each mortgage is given a credit rating (AAA, AA, etc.) and there is a user-entered annual default rate associated with each credit rating. The residential mortgage application applies the annual default rate to the outstanding loan balance at the start of each year to develop that year's default amount. Additionally, there is a recovery rate assumed for each credit rating that is used to calculate the percentage of defaulted principal that is recovered whenever there is a default.

In the merged model, both the default and recovery amounts are passed into the PMI model. At this point the recovery amounts are the recoveries that would be made in the absence of the PMI coverage. An additional calculation must occur – the calculation of the PMI claim, which will ultimately impact the recovery amount. The amount of the PMI claim will be the lesser of (a) and (b), where (a) and (b) are:

(a) Difference between default amount and recovery amount being passed from the residential mortgage application
(b) The percentage of the original mortgage portfolio remaining at time t * original principal amount of the mortgage portfolio * default rate at time t * PMI coverage percentage

Two examples will help to clarify the calculation of the PMI claim.

Example 3

Suppose at time 1, the bank loans $100,000,000 in residential mortgage loans. This can be thought of as 1,000 mortgages of $100,000 each. The next year, ten percent of the mortgages default. For simplicity sake, assume that none of the principal has been repaid yet. This equates to $10,000,000 in defaulted loans. Next, suppose that forty percent of the default amount is recoverable by the bank taking possession of the defaulted property and selling it on the open market. This produces a recovery amount of $4,000,000, or a net loss of $6,000,000.

Furthermore, suppose that PMI coverage exists on each of the mortgages with a thirty percent coverage rate. This exposes the PMI provider to a maximum loss of $30,000 per mortgage (30% of the 100,000 face amount of each mortgage). If we think about the defaults in terms of individual mortgages, there were 100 defaults with a net cost of $60,000 each. The PMI provider will pay the maximum claim amount of $30,000 on each of the defaulted mortgages for a total claim amount of $3,000,000.

In terms of (a) and (b), the calculations would be:

(a) $10,000,000 – 4,000,000 = $6,000,000
(b) Percentage of the original mortgage portfolio remaining = 100%, original principal amount of the mortgage portfolio = 100,000,000, default rate = 10%, PMI coverage percentage = 30%→100% * 100,000,000 * 10% * 30% = $3,000,000

Since (b) is less than (a), the PMI claim amount equals $3,000,000.

Example 4

Suppose we are now at time 20 and we are still looking at the block of mortgages written at time 1. By now, 85% of the original 1,000 mortgages have either prepaid or defaulted and the outstanding principal on the remaining 150 mortgages is $6,000,000, or $40,000 per mortgage. As before, ten percent of the remaining mortgages default and forty percent of default amount is recoverable on the open market. This equates to a $600,000 default amount and a $240,000 recovery amount, or a net loss of $360,000.

The PMI coverage still provides a maximum claim payment of $30,000 per mortgage. However, the PMI provider has the option of paying the mortgagee an amount equal to the outstanding principal balance instead of paying $30,000 per mortgage. If the PMI provider pays the mortgagee $40,000 per defaulted mortgage and recovers $16,000 per defaulted mortgage, the net loss will be $24,000 per mortgage instead of $30,000 per mortgage. Therefore, this is what the PMI provider will do, resulting in a claim amount of $24,000 * 15 = $360,000.

In terms of (a) and (b), the calculations would be:

(a) $600,000 – 240,000 = $360,000
(b) Percentage of the original mortgage portfolio remaining = 15%, original principal amount of the mortgage portfolio = 100,000,000, default rate = 10%, PMI coverage percentage = 30%➔15% * 100,000,000 * 10% * 30% = $450,000

Since (a) is less than (b), the PMI claim amount equals $360,000.

The amount of the PMI claim is passed back to the residential mortgage application and added to the amount that would have been recovered in the absence of the PMI claim. Referring back to the two examples above, the following table shows how the recovery amounts are modified in the residential mortgage model:

	Without PMI coverage	**With PMI coverage**	**Net loss to residential mortgage provider**
Example 3	$ 4,000,000	$ 7,000,000	$ 3,000,000
Example 4	$ 240,000	$ 600,000	$ 0

C. Private Mortgage Insurance ➔ Bank reinsurance of PMI provider linkage

The bank, through a reinsurance contract with the PMI provider, takes back some of the risk originally assumed by the PMI provider. One such contract might specify that the bank reinsures the PMI provider under a "10 x 5" excess of loss agreement. The 5 refers to five percent of the original risk exposure, so in the terms of Example 3, the bank would reinsure all claims payments between $1,500,000 and $4,500,000.[3] If the claims in Example 3 and Example 4 were the only claims paid by the PMI provider on this block of mortgages, the bank would have to repay $1,860,000 back to the PMI provider. The cash flow, balance sheet, and income statement values of the bank's reinsuring of the PMI provider all get incorporated into the bank's consolidated financial statements for accounting and tax purposes.

V. *Conclusion*

As will all papers of this nature, the conclusion begins with "There is much more still to do..." With models of this type, there is always more to do, whether it is refining assumptions, extending the model's scope, incorporating more cross-model linkages, etc. This model is certainly no different. The near term process involves the completion of the work described in this paper and drawing conclusions about capital needs and the volatility of results on an integrated basis and deciding if any changes ought to be made in the bank's strategy for these operations. Longer-term processes involve extending the scope of the model by adding more components that reflect different aspects of the bank's operations and seeing how the model reacts. Model building of this nature is a process that should not be rushed and that is the approach being taken here. It would be interesting to revisit this model in two or three years and see how it has evolved from what it is today.

[3] In Example 3, the PMI provider was exposed to a maximum loss of $30,000 per mortgage x 1,000 mortgages, or $30,000,000. Five percent of $30 million is $1.5 million and ten percent is $3 million. The excess of loss contract attaches at $1.5 million and covers the next $3 million of loss.

A Set of New Methods and Tools for Enterprise Risk Capital Management and Portfolio Optimization

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A SET OF NEW METHODS AND TOOLS FOR ENTERPRISE RISK CAPITAL MANAGEMENT AND PORTFOLIO OPTIMIZATION

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Abstract

The focus of this paper is on some new developments in the methodologies for enterprise risk management (ERM). The paper presents a set of new methods and tools, including (i) a universal risk measure for both assets and liabilities, (ii) a coherent method of determining the aggregate capital requirement for a firm, and (iii) a coherent method of allocating the cost of capital to individual business units. The discussed methods can be used for asset/loss portfolio optimization, and for quantifying the "value creation" of ERM. The paper also discusses some correlation models and methods for risk aggregation.

Introduction

The Casualty Actuarial Society (CAS) is presently promoting research in enterprise risk management and capital management. The current Call Paper Program focuses on analyzing, integrating, and optimizing the financial and insurance risks held by a financial institution or insurance company, so that capital may be efficiently deployed and consistently allocated, across the enterprise.

Recently, the CAS Advisory Committee on Enterprise Risk Management (ERM) recommended a conceptual "ERM framework," emphasizing that ERM should not solely be employed for defensive purposes, that is, to protect the firm's capital base against the "downside" of unexpected losses. ERM should also be employed for proactive purposes, that is, *to help manage the entire risk portfolio (including both assets and liabilities), and, ultimately, to enhance shareholder value.* It is believed that the pivotal role of ERM in "value creation" will become more evident in the near future.

The CAS conceptual "ERM framework" outlines a risk-management process that:

- Analyzes and quantifies risks, by obtaining and calibrating a probability distribution of outcomes for each major identified risk; then
- Integrates these major risks, by combining their outcome distributions, fully reflecting their correlations and portfolio effects; then
- Assesses and prioritizes these risks, by determining the contribution of each major identified risk to the firm's aggregate risk profile, and, in terms of their potential positive or negative impact to the firm's capital base; and then
- Optimizes the firm's aggregate risk profile, so that capital may be efficiently deployed and consistently allocated, across the global enterprise.

Outline and Focus of the Paper

Section 0. The growing pivotal role of ERM in the insurance industry
Section 1. A new universal risk measure for all assets and liabilities
Section 2. A coherent risk measure of required capital that captures overall loss distributions
Section 3. Allocating risk capital among the business units of the enterprise
Section 4. Aggregating correlated risks to produce an integrated risk profile of the firm
Section 5. Optimizing the "portfolio of the firm" to create new shareholder value

To meet the emerging ERM needs of the insurance industry, this paper presents a set of universal methods and tools that, taken together in a single framework, coherently analyzes, manages, integrates, prioritizes, and optimizes the capital requirements and risk-return trade-off of the firm.

In particular this paper presents:

- a universal risk measure for both assets and liabilities;
- a coherent method of determining the aggregate capital requirement for a firm;
- a coherent method of assessing the risk contribution, or the allocated cost of capital, of individual business units, so that RORAC (return on risk-adjusted capital) assessments can be made;
- aggregation methods for combining correlated risks;
- a proposed method for asset/loss portfolio optimization, and for quantifying the "value creation" of ERM.

Although I could have chosen to describe the detailed steps of some real-life ERM exercises, I decided to focus on a more urgent problem in that the industry lacks a sound, commonly agreed upon, methodology framework. To keep a reasonable scope for this paper, I will not detail an ERM exercise for a large financial institution. Instead I will present some new methodologies in risk measure, capital allocation, and portfolio optimization of the firm.

Please note that the methodologies discussed here are not exhaustive. Indeed, many unmentioned issues deserve separate discussions, to name a few: (1) cost of capital for long-tailed liabilities, (2) soft invisible correlation, (3) diversification versus area of expertise, and (4) macro- and micro- risk dynamics. With these caveats, this paper hopes to present innovations that can be formalized later into a set of ERM best practices, enabling insurance companies to prosper and grow in their risk taking.

Section 0. The Growing Pivotal Role Of ERM In The Insurance Industry

In this section I give a brief overview of important issues related to insurance risk and capital management, which serves as practical background for the technical discussions in later sections.

Insurance Risk and Capital

Unlike manufacturing, "insurance" is unique in that "capital" is not "spent" producing durable goods or building factories. Instead, capital is used as a cushion against the risk that insurance premiums combined with investment income are not sufficient to pay future policyholder claims. As a general principle, insurance companies with higher risks should carry higher levels of cushion capital. The very nature of insurance thus illustrates the universal link between *capital* and *risk*.

It is also because of this direct link, insurance risk managers often refer to "risk" and "capital" interchangeably. For example, when industry professionals refer to the allocation of insurance company "capital," they really mean the allocation of "risk contributions" from various business units. Insurance company capital is not legally divisible, so all of the capital available at any given time supports all insurance policies. Theoretically, a single policy with unlimited cover can claim the entire capital base of the whole insurance company.

Historically, regulatory cushion capital was determined by a simple rule-of-thumb, based on premium-to-surplus ratios, or reserve-to-surplus ratios. These simple rules-of-thumb did not reflect the true economic risk realities of insurance. The National Association of Insurance Commissioners has since tried to better link regulatory cushion capital with risk by developing a Risk Based Capital (RBC) system. So far, however, the RBC system has not been very effective in the property/casualty sector. This calls for advanced enterprise risk modeling that better captures the major risks of an insurance company.

Aggregate Capital Requirement

The capital requirements of an insurance company should measure the aggregate risk of the company risk portfolio, by incorporating asset risks, liability risks, event risks, and operational business risks. Enterprise risk modeling must properly incorporate *all of these disparate risks* in order to present an accurate profile of firm-wide risk.

Knowing the capital requirements of the firm is the first step to improved capital management. Excess capital, if any, can be transferred from treasury (risk-free) instruments, and re-deployed for more productive returns. A shortfall in capital can be rebalanced by infusions of fresh capital, purchases of reinsurance, or, by trimming risks from the company portfolio.

The Basel Accord[1] in the banking industry has inspired some insurance regulators to promote better practices in capital and risk management. For instance, Allan Brender, a Senior Director at the Office of the Superintendent of Financial Institutions Canada (OSFI), recently stated that the ultimate goal of insurance regulation is actually to help insurers better manage their capital and risk.

An Integrated View of Insurance Company Risks

There are at least two different views of the insurance business.

The Traditional Underwriting View: Insurance is mainly an underwriting operation, financing investments that earn low, but stable returns, just like a bank deposit. The emphasis is on managing liabilities. Many insurance company executives from the last century were from an underwriting background, and they guarded their companies against investing in unfamiliar risk vehicles that were outside their familiar turf.

A Financial Investment View: Insurance premiums are collected and held before claims are paid out. This creates a cash-flow float. This float provides opportunities for investing in a wide array of investment risk vehicles. In other words, the underwriting operation is essentially a "mutual fund," providing money for investment with higher returns. The emphasis is on managing assets.

The "underwriting" and the "investment" viewpoints reflect the flipsides of managing liabilities and assets in the insurance enterprise. It is better to take a more integrated view of underwriting and investment risks, where liabilities and assets are calibrated to maximize the company overall risk-return trade-off. Warren Buffett is an example of taking an integrated view of insurance operations. He has criticized some companies for aggressively accumulating investment funds using the underwritten cash-flow float, by sacrificing underwriting standards that subsequently resulted in unanticipated big losses. As another example of taking an integrated approach, some insurance companies were successful in operating high return hedge funds, but with sound risk-management in place

[1] See http://www.bis.org/bcbs/aboutbcbs.htm

to control the aggregate risk limit.

State of Affairs for the P/C Insurance Sector

In the years just before 2001, it was widely acknowledged that the property/casualty industry was over-capitalized. Company management tended to retain massive amounts of excess capital, to support company insurance and credit ratings, and to fortify reserves to withstand unexpected catastrophes. However, this excess capital was not utilized effectively. Instead of seeking better investment opportunities, insurance executives used the excess capital to subsidize price cutting in insurance premiums, so as to gain or defend market share over competitors. Actuarial indicated premium rates were useless in such a cutthroat competitive environment. Years of irresponsible pricing led to huge underwriting losses by many insurance companies.

The events of September 11 were a wake up call to the insurance industry, destroying a significant portion of the excess capital. Insurers suddenly found themselves in a dangerously weak capital position, and became much more responsible in taking on more risks. Insurance companies are now showing a higher appreciation for improved measurements of both liability and asset risks.

On the liability side, Renaissance Re is a catastrophe reinsurer that has achieved 20% annual returns on equity over the last decade. Jim Stanard, Chairman and CEO of Renaissance Re, is an early pioneer in enterprise risk modeling (see Lowe and Stanard, 1989). More generally, on the asset side, some insurance companies are taking on new kinds of market and credit investments for improved returns. These are welcome movements toward a holistic approach to actively manage *all* liability and asset risks within the insurance company.

Market Perspective versus Company Perspective

When it comes to measuring risks, the market and the company may have two different perspectives. In a market setting, transacted insurance prices are additive. To an individual company, however, the cost of taking on twice the amount of a specified risk exposure may be more than double, due to increases in portfolio concentration. For a company, different portfolio combinations can result in different aggregate risks.

Within the CAS, a group of prominent researchers are vigorously debating insurance capital allocation issues. Most of the differences in opinion can be attributed to the apparent incompatibility of market and company perspectives. I would argue that we look at insurance risk capital from both perspectives. Indeed, the interplay between these two perspectives lays the future foundation for responsive insurance risk capital management.

Traditionally, insurance transactions were driven by long-term relationships. Nowadays traded and underwritten risks are more and more becoming commodities. Consumers are becoming more conscious about shopping for the best price. In a competitive market, it is increasingly difficult for individual insurers to differentiate their offerings by pricing alone. However, these individual insurers have ample room to improve their enterprise risk capital management, and improve their shareholder return, by optimizing those liabilities and assets comprising the overall "portfolio of the firm."

To insurance company shareholders and executives, managing the asset return is becoming just as important as managing the liability risk. But more crucially, they realize that the risk/return trade-off for the integrated "portfolio of the firm" determines the day-to-day valuation of the insurance company.

The ERM Process

To succeed, the ERM process needs to be openly mandated, monitored, and managed by the executive suite. Insurance companies can contain people who are used to old ways of doing things and are skeptical to the ERM exercise. Unless these people are provided with imperative "marching orders from above," it can be difficult to get timely cooperation from the managers of individual business units.

The very first phase of ERM involves classifying major risk factors and business segments, so that data can be gathered and analyses performed in the most efficient and logical manner. Each business unit may have a particular way of obtaining, storing, and analyzing risk data. These peculiarities should be documented when the business unit data is gathered.

In the second phase, the ERM process compiles major risk factors, including:

- Market Risks, like fluctuations in equity portfolio valuation
- Credit Risks, like bond defaults and reinsurance receivables
- Interest Rate Risks, like shifts in the yield curve
- Foreign Exchange Risks, like changes in Euro/US Dollar currency exchange rates
- Catastrophe Events and Mass Tort Liabilities, like Hurricane losses, asbestos claims
- Loss Development Uncertainty, like the future unwinding of loss reserve estimates
- Business and Pricing Risks, like softening or hardening of California WC market

- Operational Risks, like captive agent compliance issues.

The industry is now moving to a standardized system for major risk factors, like those listed in the forthcoming IAA Solvency Working Party Report.

A common set of future economic scenarios, representative of stressed and unstressed conditions, should be then applied to these risk factors, across all business units, to capture correlations and concentrations of risk that may unduly impact the capital base simultaneously.

The third phase is a qualitative evaluation of the data by the managers of the individual business units. We should not be surprised if the first, raw compilations of data do not fully portray the true opportunities or risks of a given business unit. The ERM exercise succeeds only if it incorporates the practical knowledge and expertise of the business managers.

The biggest hurdle to ERM adoption by the insurance industry is the lack of a commonly accepted ERM methodology. One common mistake by many companies is spending too much effort on non-significant risks while ignoring the more important business risk dynamics. Another big hurdle is the lack of consistency of competing methods for capital risk analysis. Different methods applied to the same data can produce very different results. Venter (2002) gives a concise critique of certain quantitative approaches to capital allocation.

To be effective, the ERM methodology needs to do more than just consistently evaluate the relative levels of risk and return within the insurance enterprise. The ERM methodology must also help risk managers to take specific actions to enhance the bottom-line results of the enterprise.

Section 1. A New Universal Risk Measure For All Assets and Liabilities

In this section I propose a framework for measuring financial and insurance risks. A two-factor model of risk-adjustment for all moments in a distribution and for parameter uncertainty provides a "fair value" for a given risk vehicle. This risk vehicle can be an asset or a liability, traded or underwritten, whose outcomes have a normal or non-normal distribution.

Standard Deviation Method

Consider a risky asset with a one-period time horizon. Assume that the asset return R has a normal distribution. In a competitive market, the Capital Asset Pricing Model (CAPM) asserts that

$$E[R] = r + \lambda\, \sigma[R], \qquad \text{(eq-1.1)}$$

where r is the risk-free interest rate, and the parameter λ is the "market price of risk." In asset portfolio management, the parameter $\lambda=(E[R]-r)/\sigma[R]$ is called the Sharpe Ratio.

On the insurance side, the pricing of a liability usually starts with objective loss data, then calculates an expected loss (burning cost), and then loads for risk margin and expenses. The standard-deviation method (eq-1.1) has traditionally been used in risk-adjustment for losses:

Fair Premium = (Expected Loss) + λ (Standard Deviation of Losses),

where λ is a loading multiplier, analogous to the above "market price of risk" for assets.

Despite its popularity, the standard-deviation loading method fails to reflect the skew of a loss distribution. In fact, standard-deviation loading may unwittingly penalize upside skew and ignore downside skew. This drawback of standard-deviation loading has motivated actuarial researchers to develop various alternatives over the last decade.

Consider a loss variable X with a general exceedance curve $G(x)=\Pr\{X>x\}$. The following transform is a direct extension of the standard-deviation method of loading:

$$G^*(x) = \Phi\left(\Phi^{-1}(G(x)) + \lambda\right), \qquad \text{(eq-1.2)}$$

Here Φ represents the standard normal cumulative distribution function, where the parameter λ extends the concept of "market price of risk" or the Sharpe Ratio. Wang (2000, 2001) derived transform (eq-1.2) in the context of reinsurance pricing by layer, and showed that (eq-1.2) recovers CAPM for pricing underlying assets and replicates the results of the Black–Scholes formula for pricing options. The Wang Transform (eq-1.2) was inspired by an earlier work of Venter (1991).

Note that (eq-1.2) can be applied to risks with both positive and negative values. The mean value under the transformed distribution is

$$\mathrm{E}^*(X) = -\int_{-\infty}^{0}[1 - g(G(x))]dx + \int_{0}^{+\infty} g(G(x))dx.$$

For a given loss variable X with objective loss exceedance curve $G(x)$, the Wang Transform (eq-1.2) produces a "risk-adjusted" loss exceedance curve $G^*(x)$. The mean value $\mathrm{E}^*[X]$, under distribution $G^*(x)$, defines a risk-adjusted "fair value" of X at time T, which can be further discounted to time zero, using the risk-free interest rate.

One important property of the Wang Transform (eq-1.2) is that normal and lognormal distributions are preserved:

- If G has a normal(μ,σ^2) distribution, G^* is also a normal distribution with $\mu^* = \mu+\lambda\sigma$ and $\sigma^* = \sigma$.
- For a loss with a normal distribution, the Wang Transform (eq-1.2) recovers the traditional standard-deviation loading, with the parameter λ being the constant multiplier.
- If G has a lognormal(μ,σ^2) distribution such that $\ln(X) \sim$ normal(μ,σ^2), G^* is another lognormal distribution with $\mu^* = \mu+\lambda\sigma$ and $\sigma^* = \sigma$.

For any computer-generated distribution, the Wang Transform (eq-1.2) is fairly easy to compute numerically. Many software packages have both Φ and Φ^{-1} as built-in functions. In Microsoft Excel, $\Phi(y)$ can be evaluated by NORMSDIST(y) and $\Phi^{-1}(y)$ can be evaluated by NORMSINV(y).

Unified Treatment of Assets and Liabilities

A liability with loss variable X can be viewed as a negative asset with gain variable $Y = -X$, and vice versa. Mathematically, a liability with a "market price of risk" λ, can be treated as a negative asset whose market price of risk is $-\lambda$. That is, the "market price of risk" will have the same value but opposite signs, depending upon whether a risk vehicle is treated as an asset or liability. For an asset with gain variable X, the Wang Transform (eq-1.2) has an equivalent representation:

$$F^*(x) = \Phi\left[\Phi^{-1}(F(x)) + \lambda\right] \qquad \text{(eq-1.3)}$$

where $F(x) = 1-G(x)$ is the cumulative distribution function (cdf) of X.

The following operations are equivalent:

1. Apply transform (eq-1.2) with λ to the exceedance curve $G(x)$ of the loss variable X,
2. Apply transform (eq-1.2) with $-\lambda$ to the exceedance curve $G(y)$ of the gain variable $Y=-X$, and
3. Apply transform (eq-1.3) with λ to the cdf $F(y)=1-G(y)$ of the gain variable $Y=-X$.

These equivalences ensure that the same price is obtained for both sides of a risk transaction.

Stock prices are often modeled by lognormal distributions, which implies that stock returns are modeled by normal distributions. Equivalent results can be obtained by applying the Wang Transform (eq-1.3) either to the stock price distribution, or, alternatively, to the stock return distribution.

A Variation of the Wang Transform

For normal distributions, the Wang Transform (eq-1.3) represents a location-shift while preserving the volatility. As a variation of the (eq-1.3), we can simultaneously apply a location-shift and a volatility-multiplier:

$$F^*(x)=\Phi\left[b\cdot\Phi^{-1}(F(x))+\lambda\right]. \qquad \text{(eq-1.4)}$$

When $F(x)$ has a normal(μ, σ^2) distribution, (eq-1.4) represents an adjustment of the volatility by $\sigma^*=\sigma/b$, and a shift in the mean by $\mu^*=\mu+\lambda\sigma$. For most applications we would like to have $0<b<1$, so that $\sigma^*=\sigma/b$ is greater than σ (in other words, the volatility is inflated). In an unpublished result, Major and Venter (1999) first fitted model (eq-1.4) to a set of observed CAT-layer prices. Butsic (1999) applied both a location-shift and a volatility-multiplier to a lognormal CAT-loss distribution.

Adjustment for Parameter Uncertainty

So far we have assumed that probability distributions for risks under consideration are known without ambiguity. Unfortunately, this is seldom the case in real-life risk modeling. Parameter uncertainty is part of reality in risk modeling. Even with the best data and technologies available today, there are parameter uncertainties in the modeling of insurance losses (see Kreps, 1997; Major, 1999).

Consider the classic sampling theory in statistics. Assume that we have m independent observations from a given population with a normal(μ,σ^2) distribution. Note that μ and σ are not directly observable, we can at best estimate μ and σ by the sample mean $\tilde{\mu}$ and sample standard deviation $\tilde{\sigma}$. As a result, when we make probability assessments regarding a future outcome, we effectively need to use a student-t distribution with $k = m-2$ degrees-of-freedom.

The Student-t distribution with k degrees-of-freedom has a density

$$f(t;k) = \frac{1}{\sqrt{2\pi}} \cdot c_k \cdot \left[1+\frac{t^2}{k}\right]^{-(0.5k+1)}, \quad -\infty<t<\infty$$

where

$$c_k = \sqrt{\frac{2}{k}} \cdot \frac{\Gamma((k+1)/2)}{\Gamma(k/2)}$$

In terms of density at zero we have $f(0;k) = c_k \cdot \phi(0)$, where $\phi(0)$ is the standard normal density at x=0. Student-t has a lower density than standard normal at zero. As the degrees-of-freedom k increases, the factor c_k increases and approaches one:

k	3	4	5	6	7	8	9
c_k	0.921	0.940	0.952	0.959	0.965	0.969	0.973

The Student-t distribution can be generalized to having fractional degrees-of-freedom.

Following the statistical sampling theory that uses a Student-t distribution in place of a normal distribution, I suggest the following technique of adjusting for parameter uncertainty:

$$F^*(x) = Q\left(\Phi^{-1}(F(x))\right) \qquad \text{(eq-1.5)}$$

where Q has a Student-t distribution with degrees-of-freedom k.

Note that (eq-1.5) is an extension of the classic sampling theory, since there is no restriction imposed on the underlying distribution $F(x)$.

It may be argued that the adjustment (eq-1.5) represents a more objective view of the risk's probability distribution, instead of a form of profit loading. Empirical evidence suggests that market prices do often contain an adjustment for parameter uncertainty.

A Two-Factor Model

Let $G(x)$ be a best-estimate probability distribution, before adjustment for parameter uncertainty. The combination of parameter uncertainty adjustment in (eq-1.5) and pure risk adjustment using the Wang Transform in (eq-1.2) yields the following two-factor model:

$$G^*(y) = Q\left(\Phi^{-1}(G(y)) + \lambda\right) \qquad \text{(eq-1.6)}$$

where Q has a Student-t distribution with k degrees-of-freedom.

The two-factor model (eq-1.6) can also be written in terms of adjustments of local volatilities:

$$F^*(x) = Q\left[\Phi^{-1}(F(x)) + \lambda\right] = \Phi\left[b \cdot \Phi^{-1}(F(x)) + \lambda\right] \qquad \text{(eq-1.7)}$$

where the multiplier b depends on the value of $F(x)$, rather than being a constant.

As shown in Figure 1.1, the implied b-values in (eq-1.7) depend on the value of $F(x)$. In the middle range of a risk probability distribution, the implied b-values are closer to one, indicating a relatively smaller "volatility adjustment."

At the extreme tails of a risk probability distribution, the implied b-values deviate further below one, showing an increasing adjustment at the extreme tails. The extreme tails may represent many different pricing situations: deep out-of-the-money options, low-frequency but high-severity catastrophe losses, or, markets where risk vehicles are illiquid, benchmark data sparse, negotiations difficult, and the cost of keeping capital reserves is high.

If we choose Q as a Student-t without rescaling in the two-factor model (eq-1.7), the degrees-of-freedom will affect the simultaneous estimation of the Sharpe Ratio λ. To overcome this drawback, we can choose Q in (eq-1.7) being a rescaled Student-t distribution that matches the standard normal density at x=0. This rescaled Student-t distribution has a density function:

$$q(t;k) = \frac{1}{\sqrt{2\pi}} \cdot \left[1 + \frac{x^2}{k \cdot c_k^2}\right]^{-(0.5k+1)}, \quad -\infty < x < \infty$$

An advantage of using rescaled Student-t is to ensure a more robust estimate of the Sharpe Ratio λ. This can be useful to a fund manager comparing the Sharpe Ratio of risk vehicles from different asset classes.

Symmetry versus Asymmetry

Insurance risks are characterized by having skewed distributions. As Lane (2000) stated:

> "Any appraisal of the risks contained in insurance or reinsurance covers must take into account the fact that the statistical distribution of profit and loss outcomes may be severely skewed. Conventional risk measurement (i.e. the standard

deviation) deals with random outcomes that are symmetric in nature. Price volatility is usually viewed as symmetric. Event or outcome risk (a characteristic of insurance) is not. How is the asymmetry to be captured? What are the components of event risk and how they factor into price?"

Although the distributions Φ and Q are symmetric themselves, the one-factor Wang Transform (eq-1.3) and the two-factor model (eq-1.7) automatically reflect the skew in the input distribution $G(x)$. This ability to reflect the skew is an advantage over the standard deviation loading.

As an example, consider two bets X and Y with the following gain/loss probability distributions.

The bet X has a probability distribution of gain/loss:

x	−1	0	1	19
f(x)	0.29	0.6	0.1	0.01

The bet Y has a probability distribution of gain/loss:

y	−19	−1	0	1
f(y)	0.01	0.1	0.6	0.29

Both X and Y have the same mean=0 and variance=4. While X has an upside skew, Y has a downside skew.

1. Apply the Wang Transform (eq-1.3) with λ=0.4, to get fair values of $E^*[X]$= −0.33 and $E^*[Y]$= −0.52. Note that $E^*[X]$ −$E^*[Y]$= 0.19. As shown in the table below, for small values of lambda (say < 0.4), the one-factor Wang Transform (eq-1.3) differentiates slightly the upside skew from the downside skew. However, as the lambda value increases, this differentiating power increases "exponentially."

Lambda Value	**One-factor E*[X]**	**One-factor E*[Y]**	***Difference E*[X] - E*[Y]***
0.20	**-0.18**	**-0.23**	***0.05***
0.40	**-0.33**	**-0.52**	***0.19***
0.60	**-0.45**	**-0.90**	***0.45***
0.80	**-0.56**	**-1.39**	***0.83***
1.00	**-0.65**	**-2.01**	***1.36***

1.50	**-0.82**	**-4.27**	***3.44***
2.00	**-0.93**	**-7.47**	***6.55***
2.50	**-0.97**	**-11.14**	***10.16***

2. Apply a Student-t adjustment (eq-1.5) for parameter uncertainty with degrees-of-freedom k=6, to get fair values $E^*[X]$= 0.36 and $E^*[Y]$= −0.36. The Student-t adjustment (eq-1.6) clearly reflects the direction of the skew. We have $E^*[X] - E^*[Y]$= 0.72. As shown in the table below, the differentiating power decreases as the degrees-of-freedom increase.

Degrees of Freedom	Student-t E*[X]	Student-t E*[Y]	*Difference E*[X] - E*[Y]*
4	**0.56**	**-0.56**	***1.12***
5	**0.44**	**-0.44**	***0.88***
6	**0.36**	**-0.36**	***0.72***
7	**0.31**	**-0.31**	***0.62***
8	**0.27**	**-0.27**	***0.54***
9	**0.23**	**-0.23**	***0.46***
15	**0.14**	**-0.14**	***0.28***
20	**0.10**	**-0.10**	***0.20***

3. Apply the two-factor model (eq-1.7) with λ=0.4 and k=6, to get fair values of $E^*[X]$= −0.05 and $E^*[Y]$= −0.95. We have $E^*[X] - E^*[Y]$= 0.90, approximately equal to the combined differences, by separately using (eq-1.3) with λ=0.4, and using (eq-1.5) with 6 degrees-of-freedom.

Risk Premiums for Higher Moments

In classic CAPM where asset returns are assumed to follow multivariate normal distributions, the "market price of risk," $\lambda=(E[R]-r)/\sigma[R]$, represents the excess return per unit of volatility.

The classic CAPM has gone through important enhancements in modern finance and insurance research. In addition to risk premium associated with volatility, there is strong evidence of risk premium for higher moments (and for parameter uncertainty). This evidence has spurred extensions of classic CAPM, to include higher moments. In their recent paper, Kozik and Larson (2001) give a formal account of an n-moment CAPM. The authors offer insightful discussions on the risk premium for higher moments, pointing out that a three-moment CAPM significantly improves the fit of empirical financial data; however, there is little marginal gain by including higher moments beyond the third moment.

Obviously, the risk premium for higher moments has direct implications in pricing property catastrophe insurance, high excess-of-loss insurance layers, credit default risk, and deep-out-of-the-money options. From a risk management point of view, the cost of cushion capital increases with gearing and parameter uncertainty, as though they were extreme tail events.

The one-factor Wang Transform (eq-1.3), which can be viewed as an analog to the two-moment CAPM, does not produce sufficient risk adjustment at the extreme tails of the risk probability distribution.

The Student-t adjustment (eq-1.5) captures two opposing forces that often distort investors' rational behavior, namely *greed* and *fear*. Although investors may fear unexpected large losses, they desire unexpected large gains. As a result the tail probabilities are often inflated at both tails, with the magnitude of distortion increasing at the extremes. This distributional adjustment at both tails increases the kurtosis of the underlying distribution. The mean value of the transformed distribution under (eq-1.5) reflects the skew (asymmetry) of the underlying loss distribution.

The two-factor model (eq-1.7), however, as a combination of (eq-1.3) and (eq-1.5), provides risk premium adjustments not only for the second moment, but also for higher moments, and for parameter uncertainty.

The two-factor Wang Transform provides good fit to CAT-bond transaction data and corporate credit yield spreads (see Wang, 2002a). The parameter λ is directly linked to the Sharpe Ratio, a familiar concept to fund managers. With this universal pricing formula, investors can compare the risk/return trade-off of risk vehicles drawn from virtually any class of assets or liabilities.

Section 2. A Coherent Risk Measure Of Required Capital That Captures Overall Loss Distributions

In this section I propose a risk measure for measuring the capital requirements of the firm that goes beyond coherence. The popular VaR and the coherent Tail-VaR measures ignore information from a large part of the loss distribution. I propose a new coherent risk-measure that utilizes information from the entire loss distribution.

VaR as a Quantile Measure

Capital requirement risk-measures are used to decide the required levels of capital for a given risk portfolio, based on downside risk potential. A popular risk-measure for capital requirements in the banking industry is the Value-at-Risk (VaR), based on a percentile concept.

Consider a risk portfolio (e.g., investment portfolio, trading book, insurance portfolio) in a specified time-period (e.g., 10-day, one-year). Assume that the projected end-of-period aggregate loss (or shortfall) X has a probability distribution $F(x)$.

The Value-at-Risk is an amount of money such that the portfolio loss will be less than that amount with a specified probability α (e.g., α=99%):

$$\mathrm{VaR}(\alpha) = \mathrm{Min}\ \{x \mid F(x) \geq \alpha\}.$$

If the capital is set at VaR(α), the probability of ruin will be no greater than $1-\alpha$. For computer-generated discrete distributions, it is possible that $\Pr\{X > \mathrm{VaR}(\alpha)\} < 1-\alpha$.

VaR, as a risk-measure, is only concerned with the frequency of shortfall, but not the size of shortfall. For instance, doubling the largest loss may not impact the VaR at all. From the perspective of company executives, the quantile "VaR" at the enterprise level may be a meaningful risk-measure, as the primary concern is the occurrence of shortfall. However, as a risk measure for capital requirement, VaR has limitations since it ignores the size of shortfall and it may exhibit inconsistencies when used for comparing risk portfolios.

Tail-VaR as a Coherent Risk-Measure

From a regulatory perspective, Professors Artzner, Delbaen, Eber, and Heath (1999) advocated a set of consistency rules for a risk-measure.

1. Subadditivity: For all random losses X and Y, $\rho(X+Y) \leq \rho(X)+ \rho(Y)$.
2. Monotonicity: If $X \leq Y$ for each outcome, then $\rho(X) \leq \rho(Y)$.
3. Positive Homogeneity: For positive constant b, $\rho(bX) = b\rho(X)$.
4. Translation Invariance: For constant c, $\rho(X+c) = \rho(X) + c$.

They demonstrated that VaR does not satisfy these consistency rules. From a risk-management perspective, a consistent evaluation of the risks for business units and alternative strategies would require a coherent risk-measure other than VaR.

Artzner et al. (1999) proposed an alternative risk measure, called a "Conditional Tail Expectation" (CTE), also called the Tail-VaR. Letting α be a prescribed security level, Tail-VaR has the following expression (see Hardy, 2001):

$$\mathrm{CTE}(\alpha) = \mathrm{VaR}(\alpha) + \frac{\Pr\{X > \mathrm{VaR}(\alpha)\}}{1-\alpha} \cdot \mathrm{E}[X - \mathrm{VaR}(\alpha) \mid X > \mathrm{VaR}(\alpha)].$$

This lengthy expression is due to the complication that for computer-generated discrete distributions we may have $\Pr\{X > \mathrm{VaR}(\alpha)\} < 1 - \alpha$.

Tail-VaR reflects not only the frequency of shortfall, but also the expected value of shortfall. Tail-VaR is coherent, which makes it a superior risk-measure than VaR.

Recently there is a surge of interest in coherent risk-measures, evidenced in numerous discussions in academic journals and at professional conventions (see Yang and Siu, 2001; Meyers, 2001; among others). The Office of the Superintendent of Financial Institutions in Canada has put in regulation for the use of CTE(0.95) to determine the capital requirement for segregated fund risks.

The Tail-VaR, although being coherent, reflects only losses exceeding the quantile "VaR", and consequently lacks incentive for mitigating losses below the quantile "VaR". Moreover, Tail-VaR does not properly adjust for extreme low-frequency and high-severity losses, since it only accounts for the expected shortfall.

An Alternative Measure for Capital Requirement

Coherent risk-measure is by no means unique. The Wang Transform (eq-3) also satisfies the consistency rules of Artzner et al (1999). As an alternative to Tail-VaR, I propose a coherent risk-measure for capital requirements.

Definition 2.1 *For a loss (shortfall) variable X with distribution F, we define a new risk-measure for capital requirements as follows:*

1. *For a pre-selected security level* α, *let* $\lambda = \Phi^{-1}(\alpha)$.
2. *Apply the Wang Transform*: $F^*(x) = \Phi[\Phi^{-1}(F(x)) - \lambda]$.
3. *Set the capital requirement to be the expected value under* F^*:

$$\mathrm{WT}(\alpha) = \mathrm{E}^*[X].$$

For normal distributions, WT(α) is identical to VaR(α), the 100α-th percentile. For distributions other than normal, WT(α) may correspond to a percentile higher or lower than α, depending on the shape of the distribution.

When loss X has a log-normal distribution with $\ln(X) \sim \mathrm{Normal}(\mu,\sigma^2)$, the WT-measure has a simple formula:

$$\mathrm{WT}(\alpha)= \exp(\mu+\lambda\sigma+\sigma^2/2) \text{ with } \lambda = \Phi^{-1}(\alpha).$$

The WT(α) for the log-normal distribution corresponds to the percentile $\Phi(\lambda+\sigma/2)$, which is higher than α.

The following examples show that WT(α) improves differentiation at the extreme tails, and *provides the right incentives for risk management.*

Example 2.1. Consider two hypothetical portfolios with the following loss distributions.

Table 2.1. Loss Distributions for Portfolio A & Portfolio B

Portfolio A	
Loss x	**Prob f(x)**
$0	***0.600***
$1	***0.395***
$5	***0.005***

Portfolio B	
Loss x	**Prob f(x)**
$0	***0.600***
$1	***0.398***
$11	***0.002***

Table 2.2. Risk-Measures With α=0.99.

Portfolio	CTE(0.99)	WT(0.99)
A	$3.00	$2.59
B	$3.00	$3.89

At the security level α=0.99, given that a shortfall occurs, Portfolios A and B have the same expected shortfall. However, the maximal shortfall for Portfolio B ($11) is more than double that for portfolio A ($5). For most prudent individuals, Portfolio B constitutes a higher risk. Tail-VaR fails to recognize the differences between A and B. By contrast, WT(0.99) gives a higher capital requirement for Portfolio B ($3.89) than for Portfolio A ($2.59).

Example 2.2. Consider a risk portfolio with ten equally likely scenarios with loss amounts $1, $2, ..., $10, respectively. Assume that all loss-scenarios can be eliminated though active risk management, except that the worst-case $10 loss cannot be mitigated at all. Suppose a risk-manager is weighing the cost of active mitigation of risk against the benefit of capital relief. Tail-VaR would not encourage the active mitigation of risk, because there is no capital relief for removing losses below the worst-case loss. However, by removing all losses below $10, WT(0.99) drops from $9.71 to $8.52, showing a $1.19 capital relief. WT(0.95) drops from $9.12 to $6.42, showing a $2.70 capital relief.

RORAC Calculations

It is common practice for risk-managers to calculate the return on risk-adjusted capital (RORAC) for a given standalone portfolio. For such an exercise, our new risk-measure can be used in calculating the expression denominator, that is, for calculating the RAC, or risk-adjusted capital.

Comment on the Threshold

Regardless of the choice of risk measure, say, VaR(α), TailVaR(α), or WT(α), the value of the parameter α has significant implications to the financial performance of the enterprise. From the regulatory perspective, it may create market inefficiencies when selecting too low or too high a value of α. The optimal value for α may well depend on alternative investment opportunities in other industries. Kreps (1998) explores similar ideas in the context of reinsurance pricing. The optimal value of α is an important subject that deserves further research.

Section 3. Allocating Risk Capital Among the Business Units of the Enterprise

In this section I propose a framework for measuring risk and allocating capital among the business units of the company, based on exponential tilting.

Variance-Based Risk Measure

From a company's portfolio perspective, doubling a risk exposure may more than double the risk contribution to the aggregate "portfolio of the firm," due to increased risk concentration. Traditionally the aggregate risk concentration is better measured by "*variance*" rather than "*standard deviation.*"

Because "variance" is based on the second moment, it also suffers the drawback of "standard deviation" in failing to differentiate upside skew from downside skew. This drawback of the "variance" measure, however, can also be overcome by a probability transform:

$$f^*(x) = \frac{f(x)\exp(\lambda x)}{E[\exp(\lambda X)]}, \qquad \text{(eq-3.1)}$$

which is called the Esscher Transform (see Gerber and Shiu, 1994).

When X has a Normal(μ,σ^2) distribution, the Esscher Transform gives another normal distribution with $\mu^*=\mu+\lambda\sigma^2$, and $\sigma^*=\sigma$. Thus, for normally distributed risks, the Esscher premium recovers the variance-loading method:

$$H_{Esscher}[X;\lambda] = E[X] + \lambda \cdot Var[X].$$

In other words, the Esscher Transform extends variance-based risk-adjustment to risks with non-normal distributions. This is analogous to how the one-factor Wang Transform (eq-1.3) extends standard-deviation loading to risks with non-normal distributions.

States of the World

Let Ω represent a collection of possible *states* of the world. Each state of the world ω contains multivariate risk factors or events that could potentially happen in a specified time period. For instance, the collection of events that have had happened in 2001 can be viewed as a realized *state* of the world. In the U.S. insurance market, some major events happened in 2001 included the terror attacks of September 11, the collapse of Enron, increasing mold claims in Texas, and a lower domestic interest-rate environment.

Different business units, or lines of business, within an insurance company were impacted differently by these events.

Exponential Tilting

Consider a risk X and a reference portfolio with aggregate risk Z. Here the reference portfolio may be a company portfolio, industry portfolio, or the financial impact of a selected risk factor. We define an exponential tilting of X induced by Z:

$$X^*(\omega) = X(\omega)\frac{\exp(\lambda Z(\omega))}{E[\exp(\lambda Z(\omega))]},\text{ for every possible state } \omega \text{ in } \Omega.$$

We denote

$$H_\lambda[X,Z] = \frac{E[X\cdot\exp(\lambda Z)]}{E[\exp(\lambda Z)]}. \qquad \text{(eq-3.2)}$$

Remark: The theoretical foundation for "exponential tilting" is rooted in an equilibrium-pricing model of Buhlmann (1980, 1984). He considered an optimal risk exchange model where each participant aims to maximize his/her expected utility. Buhlmann showed that in the equilibrium the price for risk X has the same expression as (eq-3.2).

> **Example 3.1**: When X and Z have a bivariate normal distribution with correlation coefficient $\rho_{X,Z}$, the transformed variable X^* also has a normal distribution with
>
> $$\mu_X^* = \mu_X + \lambda\rho_{X,Z}\sigma_X\sigma_Z \text{ and } \sigma_X^* = \sigma_X.$$

Esscher Transform from Company Portfolio Perspective

For the aggregate risk of the reference portfolio, the exponential tilting gives

$$H_\lambda[Z,Z] = \frac{E[Z\cdot\exp(\lambda Z)]}{E[\exp(\lambda Z)]}.$$

This is exactly the Esscher premium, an extension of variance-based risk adjustment.

Systematic Risk for a Given Reference Portfolio

Let Z be the aggregate risk for a reference portfolio. Assume that risk X can be decomposed into two parts

$$X = X_{sys} + X_{non},$$

where

- X_{sys} (being co-monotone with Z) represents the systematic portion of X, and

- X_{non} (being uncorrelated with Z) represents the idiosyncrasy or non-systematic portion.
- By definition, X_{sys} and X_{non} are uncorrelated.

Note that the notion of "systematic risk" has a relative meaning, depending upon the reference portfolio Z.

It can be easily verified that

$$H_{\lambda}[X,Z] = E[X_{non}] + \frac{E[X_{sys} \cdot \exp(\lambda Z)]}{E[\exp(\lambda Z)]}. \qquad \text{(eq-3.3)}$$

In other words, exponential tilting induced by Z only adjusts for the non-diversifiable risk with respect to Z.

The following result can be found in Wang (2002b).

> **Theorem 3.1**: Let the reference portfolio be the market portfolio. Under the assumption that
>
> - Risk X_j is co-monotone with the aggregate risk Z,
> - The aggregate risk Z has a normal distribution with standard deviation $\sigma(Z)$,
>
> the exponential tilting (eq-3.2) is equivalent to the one-factor Wang Transform: $F^*(x) = \Phi\left[\Phi^{-1}(F(x)) + \lambda_0\right]$, where $\lambda_0 = \lambda\sigma(Z)$ represents the market price per unit of risk.

Conceptually, when we enlarge a company portfolio so that it approaches the market portfolio, we can reasonably expect that the risk measure based on the company portfolio perspective should converge to that of market perspective. We have seen that "exponential tilting" facilitates such a natural transition; it produces the Esscher Transform for the company portfolio perspective, and produces the Wang Transform for the market perspective.

We shall show that exponential tilting lends itself to a coherent allocation of risk contributions of various business units.

Intra-Company Allocation of the Cost of Capital

Consider a company with n individual business units. In making strategic evaluations, firms often need to allocate the total cost of capital to different business units, or among lines of business. For such an allocation exercise, the correlation structure between various business units, or among lines of business, becomes critically important.

For every possible state ω in Ω, let $X_1(\omega)$, $X_2(\omega)$, …, $X_n(\omega)$ represent the losses to n individual business units. The aggregate loss to the company is:

$$Z(\omega) = X_1(\omega) + X_2(\omega) + \ldots + X_n(\omega).$$

The correlations between $\{X_1(\omega), X_2(\omega), \ldots, X_n(\omega)\}$ are completely specified by their dependence of various states of the world. To describe such a correlation structure, we can use a representative sample of multivariate values based on historical data, and/or scenario-based simulations (see Section 4).

Assume that the aggregate capital requirement C_{aggr} has been given for the whole company. For instance, it can be based on a coherent risk measure such as WT(0.95), as in section 2.

We can solve out a number λ such that

$$C_{aggr} = H_\lambda[Z,Z] - E[Z]. \qquad \text{(eq-3.4)}$$

We propose the following allocation of cost of capital to individual business unit j (j=1, 2, …, n):

$$C_j = H_\lambda[X_j,Z] - E[X_j]. \qquad \text{(eq-3.5)}$$

Obviously this allocation method is additive: $C_{aggr} = \sum_{j=1}^{n} C_j$.

Theorem 3.2. Assume that $X_1(\omega), X_2(\omega), \ldots, X_n(\omega)$ have a multivariate normal distribution with a covariance matrix:

$$\begin{pmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \cdots & \rho_{1n}\sigma_1\sigma_n \\ \rho_{12}\sigma_1\sigma_2 & \sigma_2^2 & \cdots & \rho_{2n}\sigma_2\sigma_n \\ \vdots & \vdots & \vdots & \vdots \\ \rho_{1n}\sigma_1\sigma_n & \rho_{2n}\sigma_2\sigma_n & \cdots & \sigma_n^2 \end{pmatrix}.$$

The allocation method is equivalent to the covariance method with

$$C_{aggr} = \lambda\sigma_{aggr}^2 = \lambda\sum_{i,j=1}^{n}\rho_{ij}\sigma_i\sigma_j, \quad \text{and} \quad C_j = \lambda\sum_{i=1}^{n}\rho_{ij}\sigma_i\sigma_j, \text{ for } j=1, 2, \ldots, n.$$

Remark:

1. Under multivariate normal assumptions, the allocation method as outlined in (eq-3.4) & (eq-3.5) is exactly the same as the covariance method.

2. For other than multivariate normal distributions, the allocation method (eq-3.4) & (eq-3.5) is superior to the covariance method in that it better reflects tail correlation and skew/kurtosis in the individual risk distributions.

Market Implied Cost of Capital

From the market perspective, a formula-based benchmark price $E^*[X_j]$ for insurance risk X_j implies the following economic capital for X_j:

$$\pi(X_j) = (E^*[X_j] - E[X_j])/\mathrm{TEROE},$$

where TEROE is the *target excess return on equity*, over the risk-free rate r. Market benchmark pricing implies an aggregate capital requirement of $\pi(Z) = \pi(X_1)+\ldots+\pi(X_n)$.

Recall that C_{aggr} represents the aggregate capital requirement based on the company's own risk portfolio. It is useful to compare C_{aggr} with $\pi(Z)$, and C_j with $\pi(X_j)$, respectively. The relative sizes between C_{aggr} and $\pi(Z)$, C_j and $\pi(X_j)$, reveals the extent of company diversification, relative to an "average" industry portfolio.

Section 4. Aggregating Correlated Risks to Produce An Integrated Risk Profile of The Firm

The correlation structure between risks can significantly impact the aggregate portfolio risk, as well as the allocations of risk capital to business units. Here I discuss some useful correlation models.

Extreme Correlation

In many real-life situations, extreme correlation is often higher than what the linear correlation coefficient indicates. For instance, the terror attacks of September 11 resulted in big losses in many lines of business, including life insurance, property insurance, aviation insurance, and workers compensation. The collapse of Enron resulted in sudden increases in surety bond premium rates, which in turn forced the retailer K-Mart Stores to file for bankruptcy protection.

Normal Copula Is Sometimes Inadequate For Capital Allocation

One of the most popular correlation models is the normal copula, because (i) the correlation structure can be completely specified by a correlation matrix; and (ii) there are readily available simulation routines and software. Unfortunately, the normal copula does not give sufficient extreme correlations. Embrechts et al (1999) showed that a normal copula shows asymptotically zero-correlation at the extreme tails. An alternative, the student-t copula shows higher correlation at the tails. Mango and Sandor (2002) have cautioned against using the normal copula in capital allocation exercises. Venter (2001) analyzed various copulas using simulated catastrophe loss data.

The known drawbacks of the normal copula encourages the use of a statistical copula that properly incorporates a higher correlation at the extreme tails. This statistical copula can be empirically constructed from historical data, or modified with a set of stress tests embodied in scenarios.

Empirical Copula

There are numerous parametric copula models (see Frees and Valdz, 1998). Although a parametric copula can be fit to empirical multivariate data, the estimation of copula parameters often depends on the model choice of marginal distributions.

I propose using a type of empirical copula that is not affected by the choice of marginal distributions. When limited by historical multivariate data, we can supplement this empirical copula by scenario-based simulations.

Consider a sample of simultaneous observations $\{x_m, y_m\}$, $m=1, 2, \ldots, M$, of two variables X and Y.

Rank the values of x_m, $m=1, 2, \ldots, M$, in an ascending order. For each x_m we assign a sub-interval, $\mathrm{I}(x_m)$, situated within $[0,1]$.

The lower boundary of the interval $\mathrm{I}(x_m)$ is

$$\frac{\text{Number of observations strictly less than } x_m}{M}.$$

The upper boundary of the interval $\mathrm{I}(x_m)$ is

$$1-\frac{\text{Number of observations strictly greater than } x_m}{M}.$$

For each x_m, $m=1, 2, \ldots, M$, we define u_m as the mid-point of the interval $\mathrm{I}(x_m)$, Note that repetitive values of x_m will result in repetitive values of u_m.

We do the same for the values of y_m, $m=1, 2, \ldots, M$. Let v_m be the mid-point of the interval $\mathrm{I}(y_m)$.

We call the sample discrete distribution $\{u_m, v_m\}$, $m=1, 2, \ldots, M$, an empirical copula induced by the sample $\{x_m, y_m\}$, $m=1, 2, \ldots, M$.

A simple instance of this empirical copula is implied in multivariate traded prices for multiple stocks or stock indices. Multivariate insurance data, however, is much less abundant. We must rely heavily on scenario-based simulations to generate the appropriate multivariate data.

Use Empirical Copula in Modeling and Combining Correlated Risks

Consider two risks W_1 and W_2 with marginal distributions, F_1 and F_2, respectively. To simulate a bivariate sample of W_1 and W_2 with their correlation structure as specified by the empirical copula $\{u_m, v_m\}$, $m=1, 2, \ldots, M$, the following method can be employed:

$$\left\{ w_1(m) = F_1^{-1}(u_m),\ \ w_2(m) = F_2^{-1}(u_m) \right\},\quad m=1, 2, \ldots, M,$$

with each pair having a probability of $1/M$.

Using this simulation method, we can easily generate a sample of the aggregate risk: $W = W_1 + W_2$.

This aggregation method can be easily generalized to combining "n" risks.

Multivariate Normal Variance Mixture

Consider multivariate standard normal variables $(Z_1, Z_2, \ldots, Z_n)$ with correlation coefficients $\rho_{ij} = \text{corr}(Z_i, Z_j)$. Let B be a non-negative random variable. We define

$$(X_1, X_2, \ldots, X_n) = (BZ_1, BZ_2, \ldots, BZ_n).$$

Intuitively, this is a stochastic volatility model (that is, the variance itself is random). We say that $(X_1, X_2, \ldots, X_n)$ have a multivariate normal mixture distribution.

Normal variance mixtures preserve the linear correlation coefficients of the multivariate normal distribution:

$$\text{Corr}(X_i, X_j) = \text{Corr}(Z_i, Z_j).$$

Example 4.1. Let the multiplier B have a lognormal distribution with mean=1 and CV=γ.

Example 4.2. Let $B = \sqrt{\frac{k}{C}}$ where C has a Chi-square distribution with k degrees-of-freedom. Then $(X_1, X_2, \ldots, X_n)$ have a multivariate student-t distribution with k degrees-of-freedom. Frey et al (2001) compared the impacts on VaR calculations using student-t copula versus normal copula.

Section 5. Optimizing The "Portfolio Of The Firm" To Create New Shareholder Value

This section applies the two-factor Wang Transform to portfolio optimization, and to quantifying the value creation associated with portfolio strategies. This can be viewed as an extension of the Markowiz mean-variance framework.

The Sharpe Ratio and Mean-Variance Optimization

Let R be the return in the forthcoming time period on an asset portfolio. Let the benchmark portfolio be the risk-free rate r at the same time horizon. The Sharpe Ratio, as a measure of reward-to-variability ratio, is defined as

$$\lambda = \frac{E[R]-r}{\sigma[R]},$$

which also corresponds to the concept of the "market price of risk."

After its initial publication by economist William Sharpe in 1966, the Sharpe Ratio soon became a popular way for fund managers to calculate excess return for a given level of risk. As a simple rule of thumb, the higher the Sharpe ratio, the better the prospect of return, relative to a level of risk. Given some constraints of risk-tolerance set by the portfolio managers, an optimal portfolio can be constructed so as to maximize the prospective Sharpe Ratio for the aggregate portfolio.

In financial economics, the optimal asset portfolios lie on an efficient frontier, under the Markowiz mean-variance framework. For normally distributed risks, maximizing the Sharpe Ratio is consistent with the Efficient Frontier under the Markowiz framework.

The lambda parameter in the Wang Transform (eq-3) extends the Sharpe Ratio concept to assets/losses with non-normal distributions. With further incorporation of parameter uncertainty and higher moments, the two-factor Wang Transform enables an extension of the Markowiz mean-variance framework.

Portfolio Fair Economic Value

Consider a risk portfolio with a fixed time horizon, $[0, T]$. Suppose that we have projected a probability distribution, $F(x)$, of the aggregate profit/loss $X(T)$ at the end of the period for the whole portfolio.

With market benchmark parameters for the Sharpe Ratio λ and the Student-t degrees-of-freedom, we apply the two-factor Wang Transform

$$F^*(x) = Q\left[\Phi^{-1}(F(x)) + \lambda\right]. \qquad \text{(eq-5.1)}$$

We define fair economic value (EconVal) for the portfolio as the mean value under the transformed distribution, with further risk-free discounting for the fixed time horizon to present value as needed:

$$\text{EconVal} = \exp(-rT)\, E^*[X(T)]. \qquad \text{(eq-5.2)}$$

In real life most firms are operating under some sort of constraints (budgeting, capital requirement, and cash-flow liquidity). Insurance companies operate under stringent capital requirements imposed upon by regulators and rating agencies. A firm with excessive risk-taking may jeopardize its long-term viability, and incur substantial transaction costs when under financial stress.

Optimization Method #1

We can state our optimization problem as maximizing the expected value of the risk-adjusted returns *under certain operating constraints*. Maximizing expected profit *without constraint* would lead firms to engage in speculative investments with the highest risk-adjusted expected returns. One operating constraint is the probability of ruin within a given threshold. Another example of operating constraint is requiring the company to remain at a "AA" credit or insurance rating by the end of the given period.

The portfolio optimization process can then be formalized as maximizing the fair economic value in (eq-5.2) subject to some given operating constraints.

Based on the ERM analysis, a company may make strategic changes to its risk portfolio. The value creation can be simply quantified as

$$\text{ValCr} = \text{EconVal(Ending Portfolio)} - \text{EconVal (Initial Portfolio)}.$$

Optimization Method #2

Let X be the profit/loss distribution for the risk portfolio. Let the economic capital be determined by a pre-selected risk-measure, say, WT(α), for α=0.99, as in section 2. We can calculate the return on economic capital as:

$$R = X \,/\, \mathrm{WT}(\alpha). \qquad \text{(eq-5.3)}$$

We apply a two-factor Wang Transform in (eq-5.1) to the probability distribution of R, and calculate the Risk-Adjusted Return On Capital (*RAROC*) as its expected value.

Thus we can perform portfolio optimization by maximizing the *RAROC* under some operating constraints. By maximizing the RAROC in this way, on an enterprise-wide basis, ERM can lead to optimal decisions and shareholder value creation.

Conclusions

In this paper we have presented a set of methods and tools for measuring risks and allocating the cost of capital. These tools are inter-connecting parts of a common framework for enterprise risk capital management. A sound methodology framework lays the foundation for building a knowledge-based risk management system. To use these tools correctly in ERM practices, it is critical to first develop good risk metrics that captures the real risk dynamics of individual business units, and their inter-relations. Of course, many other issues remain to be addressed in future research. Stay tuned.

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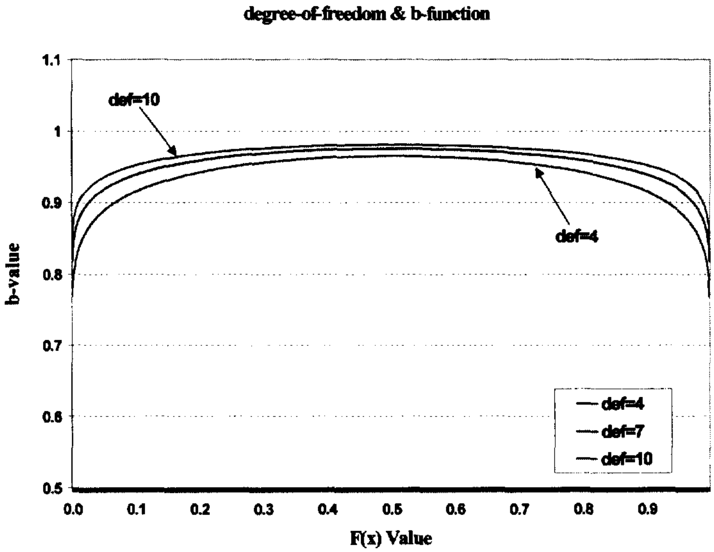
Figure 1.1 Implied *b*-values using Student-t distribution
(Here "def" refers to degrees-of-freedom)



Practical Application of the Risk-Adjusted Return on Capital Framework

Lisa S. Ward and David H. Lee

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ABSTRACT

This paper applies a risk-adjusted return on capital (RAROC) framework to the financial analysis of the risk and performance of an insurance company. A case study is presented for a diversified insurer with both property & casualty and life insurance business segments. The approach first quantifies the probability distributions of the different types of risk the institution faces: non-catastrophe liability risk, catastrophe risk, life risk, asset-liability mismatch (ALM) risk, credit risk, market risk, and operating risk. These risk type distributions are then aggregated to create an integrated risk distribution for the institution.

Economic Capital and RAROC are then calculated using this risk distribution in conjunction with income statement analysis to produce performance metrics and insights at both the line of business and total company level that support strategic as well as tactical decisions. Exhibits providing the case study numerical examples accompany the discussion of methodology throughout the paper.

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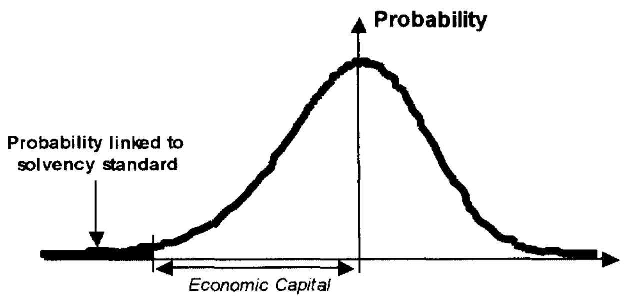
1. INTRODUCTION

1.1 Economic Capital and RAROC applied to the P&C industry

Insurers bear a responsibility both to shareholders and policyholders to maintain solvency throughout a variety of potential adverse events. Economic Capital, or the amount of capital required to support its risks to a given level of solvency, is an emerging standard in the insurance industry to help management fulfill this responsibility. The Economic Capital framework also lends itself to performance evaluation as the denominator of the Risk-Adjusted Return on Capital (RAROC) metric. With these tools, any financial institution can measure where its capital is invested, how much it is earning, how much capital it needs to hold to maintain a given debt rating, making risk-return tradeoff decisions as well as many other strategic decisions.

Economic Capital can be defined more precisely as the difference between the mean and the n^{th} percentile (i.e. the "solvency standard") of the value distribution for the entire company, where the value distribution represents the mark-to-market available capital, taking into account all risky assets and liabilities. The solvency standard, or probability of ruin, is typically linked to agency credit ratings, for example those from S&P or Moody's, e.g. an S&P rating of "AA" corresponding to an average default probability of 0.03%. As a result, an insurer that wishes to target a "AA" rating can quantify the capital to support its risks as the difference between the 0.03 percentile and the mean of its overall value distribution (see Figure 1-1).

Figure 1-1 -- Economic Capital in Relation to the Value Distribution[1]



While quantifying the overall risk of the company is important for strategic management, it is the allocation of overall economic capital back to the individual business units that enables the linking of tactical decisions with strategic goals, such as ROE targets. True insight into the economic performance of the organization comes only through linking risk and capital.

[1] Throughout this paper we represent all distributions as value distributions. This means that negative values represent an adverse outcome and positive values represent a favorable outcome.

1.2 Case Study Overview

The focus of this paper will be on the methodologies used to evaluate the risks an insurer faces. In order to facilitate the discussion, a case study insurer was created to provide a concrete example of the potential applications of the methodology. The case study company is a diversified insurer with both property & casualty and life insurance business segments. To keep things simple, the insurer has only five insurance business units in addition to an investments unit. We have selected business units in such a way as to illustrate the potential breadth of exposures an insurer may face. Table 1-1 illustrates the structure of the company and the risks to which each of the business units is exposed.

Table 1-1– Overview of Case Study Company

Segment	Business Unit	Non-Cat	Cat	Life	ALM	Credit	Market	Operating
P&C	Homeowners	X	X					X
P&C	General Liability	X						X
Specialty	Credit & Surety					X		X
Life	Term-Life			X				X
Life	Survival-Contingent Annuities			X	X			X
Investments	Investments				X	X	X	

2. RISK QUANTIFICATION

2.1 Non-Catastrophe Liability Risk

Non-catastrophe liability risk is a measure of the uncertainty in the amount and timing of insurance claims. The model used here incorporates process, parameter and systematic risks. The approach used is based on the volatility of loss-development factors, which are calculated using paid loss triangles.

The method involves back-casting ultimate loss estimates (ULE) based on a given paid loss triangle. First, the link ratio (or age-to-age factor) from one development year (DY) to the next is calculated. A cumulative development factor (CDF) for each DY is derived from the link ratio. By multiplying the CDF for each DY by the corresponding paid losses in the triangle, a triangle of ultimate loss estimates is generated.

The volatility of the ULEs and the change in ultimate estimates from one DY to the next within an accident year (AY) are used to calculate development factor volatility (a measure of process risk), and loss estimate uncertainty volatility (a contributor to parameter risk). In addition, systematic volatility is calculated as another indicator of parameter risk. Economic capital requirements are then calculated for the selected line after incorporating the diversification benefits resulting from AY and DY correlations. A lognormal loss distribution is assumed for each individual line of business. Finally, the individual loss distributions for each line are aggregated together while incorporating line-to-line diversification benefits using a line of business correlation matrix.

Table 2-1 – Paid Loss Triangles and Initial Loss Estimates (ILE) by Accident Year (AY)

Homeowners (HO)

		Cumulative Paid Loss by DY				
AY	ILE	1	2	3	4	5
1997	115,000,000	28,000,000	79,000,000	88,000,000	98,000,000	120,000,000
1998	110,000,000	31,000,000	73,000,000	89,000,000	92,000,000	
1999	107,000,000	18,000,000	72,000,000	103,000,000		
2000	100,000,000	22,000,000	101,000,000			
2001	93,000,000	23,000,000				

General Liability (GL)

		Cumulative Paid Loss by DY				
AY	ILE	1	2	3	4	5
1997	48,000,000	6,000,000	9,000,000	25,000,000	32,000,000	39,000,000
1998	70,000,000	4,000,000	23,000,000	35,000,000	45,000,000	
1999	72,000,000	7,000,000	15,000,000	30,000,000		
2000	63,000,000	3,000,000	4,000,000			
2001	55,000,000	10,000,000				

Table 2-2– Cumulative Development Factor (CDF) from One Development Year to the Final Development Year

DY	1 to 5	2 to 5	3 to 5	4 to 5	5 to 5
HO CDF	5.3938	1.6430	1.3144	1.2245	1.0000
GL CDF	7.6373	2.9950	1.5641	1.2188	1.0000

The back-casting of ultimate loss estimates uses a blended Chain-Ladder/Bornheutter-Ferguson approach:

$$ULE_{AY,DY} = \left(1 - \frac{1}{CDF_{DY}{}^{1-\gamma}}\right) ILE_{AY} + CDF_{DY}{}^{\gamma} PAID_{AY,DY} \quad (1)$$

where CDF_{DY} is the cumulative development factor from DY to final based on the link-ratio method, ILE_{AY} is the initial expected ultimate, i.e. premium times initial expected loss ratio, and γ is the degree of reliance on historical losses vs. initial expectations. The value of parameter γ is between zero and one, with zero resulting in the Bornheutter-Ferguson (BF) method, and one resulting in the pure chain-ladder method:

$$\gamma \to 1 \Rightarrow ULE_{AY,DY} = CDF_{DY} PAID_{AY,DY} \quad (2)$$

$$\gamma \to 0 \Rightarrow ULE_{AY,DY} = \left(1 - \frac{1}{CDF_{DY}}\right) ILE_{AY} + PAID_{AY,DY} \quad (3)$$

This blended approach is used to allow for flexibility in the relative importance of initial estimates versus observed results. The BF approach places greater weight on initial loss estimate (ILE) predictions. This solves the most significant problem with long-tailed triangles, namely that the initial development years exhibit dramatic percentage variations in paid losses magnified by CDF extrapolation. In this instance, we use $\gamma = 0.67$ for Homeowners and $\gamma = 0.33$ for General Liability, since GL is a much longer-tailed line. The BF approach requires the additional inputs of premium and loss ratio in order to derive the ILEs.

2.1.1 LDF Volatility (Process Risk)

The volatility of loss development is measured by taking a weighted standard deviation of observed results according to standard methods. Let $X_{i,j}$ denote the change in back-cast ultimate loss from development year *i*-1 to *i* for business from accident year *j*:

$$X_{i,j} = \frac{ULE_{i,j}}{ULE_{i-1,j}} \quad (4)$$

Let $w_{i,j}$ denote the relative weight of accident year *j* in development year *i*:

$$w_{i,j} = \frac{ULE_{i-1,j}}{\sum_k ULE_{i-1,k}} \tag{5}$$

Let Y be the random variable denoting change in ultimate loss from one year to the next, and let $\sigma_{LDF}^{(i)}$ represent the LDF volatility for development year i. $\sigma_{LDF}^{(i)}$ is computed from the basic definition of standard deviation:

$$\sigma_{LDF}^{(i)} = \sqrt{E\left[Y^2\right] - \left(E[Y]\right)^2} = \sqrt{\sum_j w_{i,j} X_{i,j}^2 - \left(\sum_j w_{i,j} X_{i,j}\right)^2} \tag{6}$$

Expanding w_j and $X_{i,j}$ we arrive at:

$$\sigma_{LDF}^{(i)} = \sqrt{\sum_j \frac{ULE_{i-1,j}}{\sum_k ULE_{i-1,k}} \left(\frac{ULE_{i,j}}{ULE_{i-1,j}}\right)^2 - \left(\sum_j \frac{ULE_{i-1,j}}{\sum_k ULE_{i-1,k}} \frac{ULE_{i,j}}{ULE_{i-1,j}}\right)^2} \tag{7}$$

Simplifying gives us:

$$\sigma_{LDF}^{(i)} = \sqrt{\left(\frac{1}{\sum_j ULE_{i-1,j}} \times \left(\sum_j \frac{ULE_{i,j}^2}{ULE_{i-1,j}}\right)\right) - \left(\sum_j \frac{1}{\sum_k ULE_{i-1,k}} ULE_{i,j}\right)^2} \tag{8}$$

And, finally:

$$\sigma_{LDF}^{(i)} = \sqrt{\left(\frac{1}{\sum_j ULE_{i-1,j}} \times \left(\sum_j \frac{ULE_{i,j}^2}{ULE_{i-1,j}}\right)\right) - \left(\frac{\sum_j ULE_{i,j}}{\sum_j ULE_{i-1,j}}\right)^2} \tag{9}$$

This method will produce n-1 $\sigma_{LDF}^{(i)}$ values, one for each column of the loss triangle that has more than one year of data. It is desirable to apply this method to n+1 different accident years, however: the n years embedded in the loss triangle, plus the current accident year, for which no losses have yet been recorded. To generate the last two values, $\sigma_{LDF}^{(n)}$ and $\sigma_{LDF}^{(n+1)}$, we compute a decay factor from the best-fit exponential curve through $\sigma_{LDF}^{(1)}, \cdots, \sigma_{LDF}^{(n-1)}$ using a weighted log-linear regression.

Let ω_i be the weight for development year i in the regression:

$$\omega_i = \frac{\sqrt{n-i+1}}{\sum_{j=1}^{n}\sqrt{j}} \tag{10}$$

The independent variable X in the regression corresponds to the development year index i, and the dependent variable is the natural log of the loss development factor volatility, $\ln\left(\sigma_{LDF}^{(1)}\right),\cdots,\ln\left(\sigma_{LDF}^{(n-1)}\right)$. The moments for the regression are:

$$E(X) = \sum_{i=1}^{n} i \cdot \omega_i \tag{11}$$

$$E(Y) = \sum_{i=1}^{n} \ln\left(\sigma_{LDF}^{(i)}\right) \cdot \omega_i \tag{12}$$

$$E\left(X^2\right) = \sum_{i=1}^{n} i^2 \cdot \omega_i \tag{13}$$

$$E\left(Y^2\right) = \sum_{i=1}^{n} \left(\ln\left(\sigma_{LDF}^{(i)}\right)\right)^2 \cdot \omega_i \tag{14}$$

$$E(XY) = \sum_{i=1}^{n} i \cdot \ln\left(\sigma_{LGD}^{(i)}\right) \cdot \omega_i \tag{15}$$

From the moments, we can calculate the slope and y-intercept of the log-linear regression line:

$$m = \frac{E(XY) - E(X) \cdot E(Y)}{E\left(X^2\right) - E(X)^2} \tag{16}$$

$$b = E(Y) - m \cdot E(X) \tag{17}$$

The decay factor d is defined as:

$$d = e^m \tag{18}$$

Finally, we use the decay factor to compute $\sigma_{LDF}^{(n)}$ and $\sigma_{LDF}^{(n+1)}$:

$$\sigma_{LDF}^{(n)} = d \cdot \sigma_{LDF}^{(n-1)} \tag{19}$$

$$\sigma_{LDF}^{(n+1)} = d^2 \cdot \sigma_{LDF}^{(n-1)} \tag{20}$$

Table 2-3 – Loss Development Factor (LDF) Volatility

DY	0 to 1	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6
HO LDF Vol	0.1176	0.2205	0.0973	0.0343	0.0230	0.0154
GL LDF Vol	0.0931	0.1634	0.0958	0.0012	0.0004	0.0001

2.1.2 ULE Volatility (Parameter Risk)

Given an estimate for the mean level of ultimate loss, σ_{LDF} represents the volatility of the actual loss outcome around the mean. However, there is additional uncertainty embedded in the estimated ultimate loss. "Ultimate Loss Estimate Volatility", or "Parameter Risk", represents the standard deviation of the mean loss estimate.

In general, given a random sample of a variable X, the standard deviation of its mean estimate $\overline{X}$ is:

$$s_{\overline{X}} = \sqrt{\frac{s_X^2}{n-1}} \tag{21}$$

where s_X^2 is the sample variance of X. In this case, the sample variance corresponds to LDF volatility, and the estimation error of $\overline{X}$ corresponds to ULE volatility:

$$s_X^2 = \sigma_{LDF}^2 \tag{22}$$

$$s_{\overline{X}}^2 = \sigma_{ULE}^2 \tag{23}$$

Let $\sigma_{ULE}^{(i)}$ represent the volatility of ultimate loss for development year i. $\sigma_{ULE}^{(i)}$ reduces to:

$$\sigma_{ULE}^{(i)} = \frac{\sigma_{LDF}^{(i)}}{\sqrt{n-1}} \tag{24}$$

Here, n is the number of observations (i.e. Accident Years) in the loss triangle at development year i. This risk is assumed to be independent of LDF volatility.

Table 2-4 – Ultimate Loss Estimate (ULE) Volatility

DY	0 to 1	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6
HO LDF Vol	0.0588	0.1273	0.0688	0.0343	0.0230	0.0154
GL LDF Vol	0.0466	0.0943	0.0678	0.0012	0.0004	0.0001

2.1.3 Systematic Risk

In addition to volatility that is observable in historical loss triangles, there is a risk that unforeseen and unprecedented systematic changes in legislation or market factors will have a negative impact on future results. This risk is intended to capture that which is unobserved in historical data; by definition it falls outside the realm of estimation from historical loss triangles and must be parameterized separately.

We begin by assuming that systematic risk is independent of process and parameter risk (LDF volatility and ULE volatility). That is, non-systematic factors provide no insight into the systematic risk faced by a given line of business. Also, we assume that systematic risk is, at the outset of development for a given accident year, proportional to ultimate loss. We further assume that, since it is proportional to ultimate loss, systematic risk can be proportionally be attributed to two sources: 1) the absolute level of ultimate loss; and 2) the unpaid portion of ultimate loss.

The formula for systematic risk is derived from these broad assumptions. Let q be the proportion of systematic risk attributable to unpaid ultimate loss (0 < q < 1). 1 - q is the proportion of systematic risk attributable to the absolute level of ultimate loss (the non-decaying portion). If σ_{Syst} is the total systematic risk, then the portion attributable to the level of ultimate loss is

$$\sigma_{Syst} \times (1-q) \tag{25}$$

This gives us one of the two components of systematic risk. The remaining component is built from the amount of unpaid ultimate loss. Let CDF_i be the cumulative development factor for development year i. By definition, $1/CDF_i$ is the percentage of total ultimate loss that has been paid at the end of development year i. Thus, $(1-1/CDF_i)$ is the percentage of ultimate loss that remains unpaid. With q as defined above - the proportion of systematic risk attributable to unpaid ultimate loss - the amount of systematic risk attributable to unpaid ultimate loss is:

$$\sigma_{Syst} \times q \times (1-1/CDF_i) \tag{26}$$

Combining equations 25 and 26, we arrive at the formula for allocating total systematic risk to development year i:

$$\sigma_{Syst}^{(i)} = \sigma_{Syst} \times \left(q \times \left(1 - \frac{1}{CDF_i} \right) + (1-q) \right) \tag{27}$$

where σ_{Syst} is the total systematic risk and q is the percentage of σ_{Syst} attributable to the unpaid portion of ultimate loss. CDF_i is the cumulative loss development factor at development year i. $\sigma_{Syst}^{(i)}$ is assumed to be perfectly correlated with $\sigma_{Syst}^{(j)}$, for any j, and uncorrelated with $\sigma_{LDF}^{(i)}$ and $\sigma_{ULE}^{(i)}$.

Assuming $\sigma_{Syst} = 0.05$ and $q = 0.9$:

Table 2-5 – Systematic Risk Volatility

DY	0 to 1	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6
HO Sys Vol	0.0500	0.0417	0.0226	0.0158	0.0133	0.0050
GL Sys Vol	0.0500	0.0441	0.0350	0.0212	0.0131	0.0050

2.1.4 Total Development Year Volatility

Let σ_i represent the total volatility for development year i. Assuming independence between the three components of total volatility, we compute σ_i in the standard fashion:

$$\sigma_i = \sqrt{\left(\sigma_{LDF}^{(i)}\right)^2 + \left(\sigma_{ULE}^{(i)}\right)^2 + \left(\sigma_{Syst}^{(i)}\right)^2} \tag{28}$$

Table 2-6 – Total Development Year Volatility

Homeowners

DY	0 to 1	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6
LDF Vol	0.1176	0.2205	0.0973	0.0343	0.0230	0.0154
ULE Vol	0.0588	0.1273	0.0688	0.0343	0.0230	0.0154
Systematic Vol	0.0500	0.0417	0.0226	0.0158	0.0133	0.0050
Overall Volatility	0.1407	0.2580	0.1213	0.0511	0.0351	0.0223

General Liability

DY	0 to 1	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6
LDF Vol	0.0931	0.1634	0.0958	0.0012	0.0004	0.0001
ULE Vol	0.0466	0.0943	0.0678	0.0012	0.0004	0.0001
Systematic Vol	0.0500	0.0441	0.0350	0.0212	0.0131	0.0050
Overall Volatility	0.1155	0.1938	0.1225	0.0213	0.0131	0.0050

2.1.5 Development Year Correlation

The total volatility σ for the line aggregates the σ_i from each year, taking into account correlation between development years. These correlations are derived from the total development year volatility and systematic volatility.

Let $X^{(i)}$ and $X^{(j)}$ be random variables denoting the loss distribution in development years i and j, respectively. Let ρ_{ij} denote the correlation between $X^{(i)}$ and $X^{(j)}$. By definition, ρ_{ij} is:

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} \tag{29}$$

σ_i and σ_j are known; they are the total volatilities for $X^{(i)}$ and $X^{(j)}$ respectively, as computed in equation 28. To calculate ρ_{ij}, we need to compute σ_{ij}, the covariance of $X^{(i)}$ and $X^{(j)}$. σ_{ij} is, by definition:

$$\sigma_{ij} = E\left[\left(X^{(i)} - \bar{X}^{(i)}\right)\left(X^{(j)} - \bar{X}^{(j)}\right)\right] \tag{30}$$

where $\bar{X}^{(i)}$ and $\bar{X}^{(j)}$ are the expected values of $X^{(i)}$ and $X^{(j)}$, respectively. Within an Economic Capital framework, we are primarily concerned with the distribution of change in value relative to expectations. Thus, we set the loss distributions to have mean 0. This leaves the following:

$$\sigma_{ij} = E\left[X^{(i)}X^{(j)}\right] \tag{31}$$

We assume that volatility is composed of 3 elements: LDF volatility (Process Risk), ULE volatility (Parameter Risk) and Systematic volatility. Thus, $X^{(i)}$ and $X^{(j)}$ can be thought of as the sum of three random variables:

$$X^{(i)} = X^{(i)}_{LDF} + X^{(i)}_{ULE} + X^{(i)}_{Syst} \tag{32}$$

$$X^{(j)} = X^{(j)}_{LDF} + X^{(j)}_{ULE} + X^{(j)}_{Syst} \tag{33}$$

Substituting, we have:

$$\sigma_{ij} = E\left[\left(X^{(i)}_{LDF} + X^{(i)}_{ULE} + X^{(i)}_{Syst}\right)\left(X^{(j)}_{LDF} + X^{(j)}_{ULE} + X^{(j)}_{Syst}\right)\right] \tag{34}$$

This expands to:

$$\sigma_{ij} = E\begin{bmatrix} X^{(i)}_{LDF}X^{(j)}_{LDF} + X^{(i)}_{ULE}X^{(j)}_{ULE} + X^{(i)}_{Syst}X^{(j)}_{Syst} + \\ X^{(i)}_{LDF}X^{(j)}_{ULE} + X^{(i)}_{LDF}X^{(j)}_{Syst} + \\ X^{(i)}_{ULE}X^{(j)}_{LDF} + X^{(i)}_{ULE}X^{(j)}_{Syst} + \\ X^{(i)}_{Syst}X^{(j)}_{LDF} + X^{(i)}_{Syst}X^{(j)}_{ULE} \end{bmatrix} \tag{35}$$

Because we have assumed independence between all non-systematic factors, all terms in equation 35 have expected value 0, with the exception of the systematic term:

$$\sigma_{ij} = E\left[X^{(i)}_{Syst}X^{(j)}_{Syst}\right] \tag{36}$$

The correlation between systematic factors is assumed to be 1, giving:

$$\sigma_{ij} = E\left[X^{(i)}_{Syst}X^{(j)}_{Syst}\right] = \sigma^{(i)}_{Syst}\sigma^{(j)}_{Syst} \tag{37}$$

Thus, returning to the original definition of ρ_{ij}, we have:

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} = \frac{\sigma_{Syst}^{(i)} \sigma_{Syst}^{(j)}}{\sigma_i \sigma_j} \tag{38}$$

Table 2-7 – Development Year Correlations

Homeowners

	0	1	2	3	4	5
0	1	0.0574	0.0662	0.1097	0.1341	0.0795
1	0.0574	1	0.0301	0.0499	0.0609	0.0361
2	0.0662	0.0301	1	0.0576	0.0703	0.0417
3	0.1097	0.0499	0.0576	1	0.1165	0.0691
4	0.1341	0.0609	0.0703	0.1165	1	0.0844
5	0.0795	0.0361	0.0417	0.0691	0.0844	1

General Liability

	0	1	2	3	4	5
0	1	0.0985	0.1236	0.4316	0.4325	0.4327
1	0.0985	1	0.0650	0.2269	0.2274	0.2275
2	0.1236	0.0650	1	0.2847	0.2853	0.2854
3	0.4316	0.2269	0.2847	1	0.9960	0.9965
4	0.4325	0.2274	0.2853	0.9960	1	0.9987
5	0.4327	0.2275	0.2854	0.9965	0.9987	1

2.1.6 Line of Business Loss Distribution

We compute the total volatility σ using the year-to-year correlation matrix:

$$\sigma^2 = \begin{pmatrix} \sigma_1 \cdot ULE_1 \\ \vdots \\ \sigma_{n+1} \cdot ULE_{n+1} \end{pmatrix}^T \begin{pmatrix} 1 & \rho_{1,2} & \cdots & \rho_{1,n+1} \\ \rho_{2,1} & 1 & & \vdots \\ \vdots & & \ddots & \rho_{n,n+1} \\ \rho_{n+1,1} & \cdots & \rho_{n+1,n} & 1 \end{pmatrix} \begin{pmatrix} \sigma_1 \cdot ULE_1 \\ \vdots \\ \sigma_{n+1} \cdot ULE_{n+1} \end{pmatrix} \tag{39}$$

We assume that losses within each line of business follow a lognormal distribution, with mean equal to the sum of the most recent ultimate loss estimates for all accident years ($\sum ULE_i$) and standard deviation equal to σ.

Table 2-8 – Line of Business Correlations

	HO	GL
HO	1	0.1
GL	0.1	1

Table 2-9 – Non-Cat Line of Business Change in Value Distribution

Probability	HO Value	GL Value
0.001%	-221,332,923	-87,407,075
0.010%	-206,669,526	-84,871,664
0.030%	-184,790,686	-69,960,747
0.050%	-170,505,552	-69,006,709
0.070%	-161,768,432	-65,435,595
:	:	:
:	:	:
99.930%	110,192,339	46,787,530
99.950%	113,192,983	48,752,368
99.970%	118,178,053	50,504,321
99.990%	126,805,831	55,164,647
99.999%	135,965,877	57,136,264

2.1.7 Total P&C Non-Catastrophe Loss Distribution

To compute the overall loss distribution, we convolve the individual loss distributions from each line of business (see section on aggregation). This requires an inter-line of business correlation matrix that is estimated using management judgement or from loss histories. (See upcoming paper from Weimin Dong and Jim Gant.)

Table 2-10 – Non-Cat Change in Value Distribution

Probability	Value
0.001%	-248,982,536
0.010%	-218,554,051
0.030%	-197,119,667
0.050%	-185,022,543
0.070%	-177,305,526
:	:
:	:
99.930%	127,100,263
99.950%	130,506,877
99.970%	135,547,274
99.990%	145,131,462
99.999%	161,932,821

2.2 Catastrophe Risk

Catastrophe Risk quantifies the potential financial loss due to severe natural catastrophes. To provide a complete view of the potential losses from such events, it is desirable to use a statistical model, such as RiskLink from Risk Management Solutions, for developing a complete loss distribution, rather than traditional metrics such as average annual loss or probable maximum loss. Typical software packages use a Monte Carlo simulation approach with stochastic loss events to generate a full range of possible losses.

2.2.1 AEP vs. OEP

It is important to draw a distinction between the two varieties of loss distributions associated with catastrophe risk models. One variety is the "occurrence exceedance probability" or OEP curve, and the other is the "aggregate exceedance probability" or AEP curve. An OEP curve is the cumulative loss distribution for any one occurrence in a given year. It shows the probability that losses from a single event will exceed a given amount. In contrast, an AEP curve is the combined cumulative loss distribution from all possible events in a given year. It shows the probability that total losses will exceed a given amount.

The method takes as input an AEP curve from one of the standard catastrophe modeling packages as the loss distribution for catastrophe risk. The AEP curve is converted it into a value distribution, which is then aggregated with the value distributions derived for other risk pillars.

In our case study company, the only line of business exposed to natural cat is Homeowners. The tables below illustrates this line's AEP curve and corresponding value distribution.

Table 2-11 – Cat AEP Curve

AEP	Loss
0.001%	341,143,958
0.010%	234,864,033
0.030%	183,122,205
0.050%	164,242,079
0.070%	149,441,501
:	:
:	:
30.000%	24,160,338
50.000%	14,989,184
70.000%	7,682,240
90.000%	823,453
99.999%	0

Table 2-12 – Cat Value Distribution

Probability	Value
0.001%	-341,143,958
0.010%	-234,864,033
0.030%	-183,122,205
0.050%	-164,242,079
0.070%	-149,441,501
:	:
:	:
99.930%	0
99.950%	0
99.970%	0
99.990%	0
99.999%	0

2.3 Mortality Risk

Mortality risk is defined as the volatility of contract value resulting from unexpected changes in mortality rates. This includes changes in current year mortality rates as well as expected future mortality rates. A subset of the contracts often exposed to mortality risk includes term life, whole life, and annuities.

Mortality risk is quantified through a Monte-Carlo simulation of portfolio value under various mortality scenarios. The resulting distribution of values constitutes the risk profile of the contracts. The simulation is achieved in the following stages:

- Identification of distinguishable sources of mortality risk
- Assignment of these risks to factors impacting mortality rates
- Re-evaluation of contract net present value under simulated factor scenarios

2.3.1 Sources of Mortality Risk

We may separate mortality risk into the following set of underlying risk drivers:

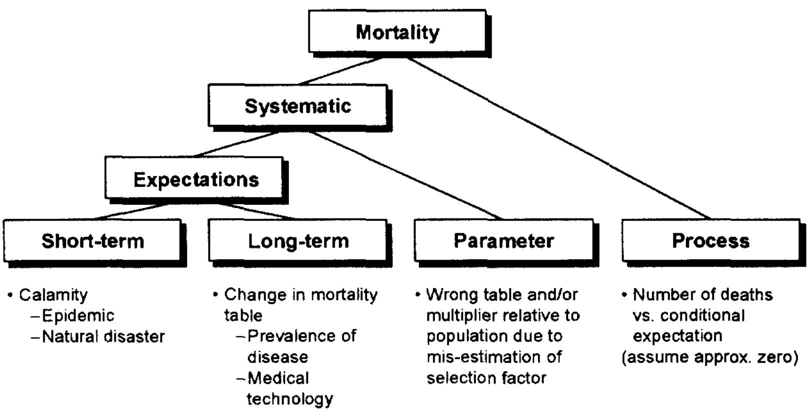
- Short-term systemic shocks
- Long-term changes in mortality expectations
- Parameter misestimation (Parameter Risk)
- Process volatility (Process Risk)

Short-term systemic shocks are the result of events that have a temporary impact on death rates across an insured population. For example, a particularly bad flu season will result in death rates increasing systematically across the entire life book for the coming year. They will not, however, necessarily change expectations of future death rates. Since economic capital is calculated on a one-year time horizon, only cash flows maturing within the coming year experience this risk.

Long-term changes in mortality expectations have an impact on multi-year products priced at the beginning of a term. Examples of long-term changes in mortality expectations include both positive impacts to mortality expectations (e.g. cure for cancer) and negative impacts (e.g. new diseases). Long-term systematic risk can impact expected future mortality rates (e.g. long-term impact of AIDS) and, to a lesser extent current year mortality (e.g. immediate impact of a new disease).

Parameter Risk results from a misestimation of the expected death rates of an insured population. Typically this is because the insured population differs from the population used to derive death rate estimates. A large portion of this risk derives from adverse selection of the insured population.

Figure 2-1 – Mortality Risk Taxonomy



Process risk derives from the difference between actual death rates and the true death rate mean adjusted for all the factors described above. For most books of policies, this is quickly diversified away; within our framework it is generally assumed to be zero risk[2].

[2] Consider a portfolio of N insured parties with an expected death rate of 20 basis points (0.20%). Each individual has a probability of dying within the next year of 0.20%, with standard deviation given by:

$$\sigma = \sqrt{p \times (1-p)} = \sqrt{0.2\% \times 99.8\%} = 4.47\%$$

Assuming each individual in the portfolio is independent, the total volatility due to Process Risk is:

2.3.2 Mortality Rate Factors

The above risks may be incorporated into a model via a set of risk factors. The factors are simulated random variables of a given distribution and correlation structure. When a factor is multiplied to an existing mortality rate, the resulting value represents a new hypothetical mortality rate. Our model uses a single "mortality occurrence" factor and a set of three "mortality expectation" factors to capture the above risks.

We can evaluate the impact of mortality risk on an institution via financial statements. When viewing from a one-year time horizon, unexpected changes in mortality can either create loss through higher benefits paid for the current year, or through an increase in reserves for future years. Volatility in benefits paid is captured in a mortality occurrence factor. This volatility is driven primarily by short-term systemic shocks but also captures parameter risk, long-term systematic risk and process risk.

An increase in reserves for losses in future years is captured with a set of three mortality expectation factors. These factors capture long-term systematic risk, parameter risk and process risk. The mortality expectation factors do not include the risk of short-term shocks since these shocks do not imply a change in mortality expectations for future years. Three factors are used to capture the varying degree of volatility and correlation between mortality changes within different age groups.

Table 2-13 – Risk Factors

Factor	Volatility	Age Min	Age Max
Occurrence Factor	0.05	n/a	n/a
Expectation Factor 1	0.10	0	40
Expectation Factor 2	0.09	41	60
Expectation Factor 3	0.05	61	120

$$\sigma_{Process} = \sqrt{\frac{\sigma^2}{N}}$$

If we assume that N is 1,000,000, then we find:

$$\sigma_{Process} = \sqrt{\frac{\sigma^2}{N}} \leq \frac{4.47\%}{1000} \leq 0.005\%$$

$\sigma_{Process}$ is small and decreasing as N grows.

Table 2-14 – Factor Correlations

	Expectation Factor 1	Expectation Factor 2	Expectation Factor 3	Occurrence Factor
Expectation Factor 1	1	0.4	0.2	0.0
Expectation Factor 2	0.4	1	0.4	0.0
Expectation Factor 3	0.2	0.4	1	0.0
Occurrence Factor	0.0	0.0	0.0	1

The variance of each of the mortality rate factors can be expressed as the sum of the variance due to systematic risk, parameter risk and process risk since we expect no correlation between these risk types.

$$\sigma^2 = \sigma_{systematic}{}^2 + \sigma_{parameter}{}^2 + \sigma_{process}{}^2$$

Systematic variance is attributable to volatility in industry-level mortality rates. In the case of the mortality expectation factors, systematic variance is determined by the annual volatility in expected mortality rates at the industry level. In the case of the mortality occurrence factor, the variance is the annual volatility of the difference between expected mortality and actual mortality. Parameter risk is specific to the institution and can be determined by comparing the historical systematic variance in industry level mortality rates with those at the institution level. Any difference is attributable to parameter risk. For a portfolio of a sizeable number of policyholders, process risk is negligible.

2.3.3 Mortality Sensitive Contract Value

Analogous to bond contracts, a mortality sensitive contract may be divided into a series of mortality sensitive cash flows. The nominal value of each cash flow is dependent on some set of mortality rates, either current or future. Changes in the nominal value of the cash flows result in a change in the mark-to-market value of the contract.

Let us define *surv(x,y,c)* as the percentage of policyholders of type *c* (here type can be gender, smoker status, country of residence, etc...), of age *x* at time zero expected to survive *y* years. We can see that $surv(x, y=0, c) = 1$ and that $\lim_{y \to \infty} surv(x, y, c) = 0$.

Let us also define *mort(z,y,c)* as the percentage of policyholders of type *c*, of age *z* at time *y-1* years that are expected to experience mortality by time *y*. We can then express the expected present value of the cash flows as:

$$F_m = \frac{a}{(1+r)^y} \times surv(x,y,c) = \frac{a}{(1+r)^y} \times \prod_{i=1}^{y} [1 - mort(x+i-1,i,c)] = \frac{a}{(1+r)^y} \times e^{-\sum_{i=1}^{y} mort(x+i-1,i,c)}$$

$$F_m = \frac{b}{(1+r)^y} \times surv(x, y-1, c) \times mort(x+y-1, y, c) =$$

$$= \frac{b}{(1+r)^y} \times mort(x+y-1, y, c) \times \prod_{i=1}^{y-1}[1 - mort(x+i-1, i, c)] = \frac{b}{(1+r)^y} \times mort(x+y-1, y, c) \times e^{-\sum_{i=1}^{y-1} mort(x+i-1, i, c)}$$

where a is the survival-contingent cash flow, b is the mortality-contingent cash flow and r is the discount rate.

Table 2-15 – SCA (Survival-Contingent) Cash Flows

	Initial Age →				
Yrs Forward	**38**	**48**	**58**	**68**	**78**
1	15,384,101	30,636,617	13,659,966	-2,681,184	-1,209,096
2	13,707,161	27,297,081	11,977,123	-2,758,095	-1,243,779
3	12,213,016	24,321,570	10,490,364	-2,802,538	-1,263,821
4	10,881,739	21,670,403	9,179,712	-2,815,391	-1,269,617
5	9,695,578	19,308,227	8,026,954	-2,798,224	-1,261,876
:	:	:	:	:	:
:	:	:	:	:	:
43	0	0	-54,030	-102,901	-46,404
44	0	0	-49,438	-94,154	-42,459
45	0	0	-45,235	-86,151	-38,850
46	0	0	-41,390	-78,828	-35,548
47	0	0	-37,872	-72,128	-32,526

Table 2-16 – Term Life (Death-Contingent) Cash Flows

	Initial Age →				
Yrs Forward	38	48	58	68	78
1	-5,021,260,205	-6,044,688,217	-1,222,039,843	-46,565,844	-20,999,141
2	-4,426,324,162	-5,328,492,931	-1,078,678,335	-43,771,893	-19,739,192
3	-3,943,833,850	-4,747,661,948	-962,108,661	-40,926,720	-18,456,145
4	-3,513,937,269	-4,230,144,295	-858,072,447	-38,061,850	-17,164,215
5	-3,130,901,453	-3,769,038,519	-765,218,074	-35,207,211	-15,876,899
:	:	:	:	:	:
:	:	:	:	:	:
43	0	0	-635,650	-1,210,598	-545,926
44	0	0	-581,620	-1,107,697	-499,523
45	0	0	-532,182	-1,013,543	-457,063
46	0	0	-486,947	-927,392	-418,213
47	0	0	-445,556	-848,563	-382,665

The above equations for contract value hold true for simulated scenarios with the modification of an appropriate factor multiplier applied to the mortality rate. For cash flows within one year, the mortality occurrence factor is used. For all other cash flows a mortality expectation factor is used dependant on the age of policyholder at the time of the cash flow. Table 2-16 displays the death-contingent cash flows, or the theoretical value of the portfolio if all insureds were to die at once.

Table 2-17 – Overall Change in SCA Cash Flows

	Initial Age →				
Yrs Forward	38	48	58	68	78
1	-4,746	-14,547	-14,677	7,393	7,288
2	-12,201	-52,593	-52,167	15,249	14,572
3	-19,511	-97,150	-99,343	25,951	24,527
4	-29,600	-129,867	-101,246	35,692	32,493
5	-36,932	-153,014	-100,468	44,227	38,478
:	:	:	:	:	:
:	:	:	:	:	:
43	0	0	479	178	16
44	0	0	379	139	12
45	0	0	299	108	10
46	0	0	236	84	7
47	0	0	186	66	6

Table 2-18 – Overall Change in Term Life Cash Flows

Yrs Forward	Initial Age → 38	48	58	68	78
1	-1,553,679	-2,882,886	-1,326,378	-131,830	-134,368
2	-2,581,646	-7,770,124	-3,574,737	-124,581	-119,355
3	-2,799,860	-9,878,489	-5,000,067	-158,700	-157,033
4	-3,957,926	-8,505,959	-1,360,273	-135,274	-118,190
5	-3,421,007	-7,325,814	-1,156,653	-114,575	-87,290
:	:	:	:	:	:
:	:	:	:	:	:
43	0	0	929	394	37
44	0	0	745	308	29
45	0	0	595	240	22
46	0	0	474	187	17
47	0	0	376	146	13

Table 2-19 – Life Value Distribution

Probability	Value
0.001%	-180,137,977
0.010%	-95,730,237
0.030%	-83,398,815
0.050%	-77,266,350
0.070%	-75,215,197
:	:
:	:
99.930%	56,811,041
99.950%	57,837,631
99.970%	60,174,618
99.990%	64,999,847
99.999%	74,015,472

2.4 Asset-Liability Mismatch Risk

Asset-liability mismatch (ALM) risk is the volatility in the value of the enterprise due to fluctuations in interest rates. Modeling ALM risk involves characterizing the portfolio of interest-rate sensitive positions on both the asset and liability side of the balance sheet, generating a set of change in rate scenarios, revaluing the enterprise under each scenario and finally generating a value distribution from the simulation results. This framework is analogous to typical Monte-

Carlo-based VaR models and is general enough to handle positions ranging from simple contractual cash flows to complex structured instruments.

2.4.1 Characterization of interest rate position

Interest-rate positions are classified into those that can be broken up into a series of deterministic cash flows, such as uncallable corporate bonds, and more complex instruments which are characterized using a tabulated rate versus value function. Cash flows positions are described by sets of cash flow amount, maturity pairs. Tabulated rate versus value data can be obtained from sources such as the Office for Thrift Supervision, or from an interest-rate sensitivity analysis in a spreadsheet or popular analytics packages.

Table 2-20 – Net Cash Flow

Maturity (Yrs)	Net Cash Flow
1	-500,000,000
2	-250,000,000
3	-150,000,000
4	-50,000,000
5	-25,000,000
7	200,000,000
10	1,100,000,000

Table 2-21 – SCA Rate vs. Value

Δ Rate	Δ Value
-3.0%	-85,000,000
-2.0%	-53,000,000
-1.0%	-21,000,000
0.0%	0
1.0%	6,000,000
2.0%	8,000,000
3.0%	9,000,000

2.4.2 Structure of the interest-rate simulation model

The interest-rate simulation seeks to generate scenarios corresponding to hypothetical changes in the yield curve. This is accomplished by characterizing a yield curve as a collection of rates which are themselves functions of the interest-rate factors. In this paper we have used a four-factor interest rate model with approximately N=50,000 simulations. The four-factors, the change in one-year rate, the change in the spread of the 10-year rate over the one-year rate and the change in the spread of the 30-year rate over the 10-year rate and the change in spread of the mortgage rate over the 10-year rate, are normally distributed and related via a Pearson correlation matrix.

Combined with the assumption of linearity of rate spreads between these three points, this suffices to determine the change in rate for all points along the yield curve. A Box-Muller approach is used to generate a set of correlated random draws for each of the N iterations.

Specifically, an $N \times m$ matrix of interest rate changes is calculated, where m represents all the relevant maturities. The change in rate for a particular maturity, Δr_m, is determined by the following formulas:

$$\Delta r_m = \begin{cases} \Delta rnd_1 + \left(\dfrac{m-1}{9}\right) \times \Delta rnd_2 & ,m \leq 10 \\ \Delta rnd_1 + \Delta rnd_2 + \left(\dfrac{m-10}{20}\right) \times \Delta rnd_3 & ,m > 10 \\ \Delta rnd_1 + \Delta rnd_2 + \Delta rnd_4 & ,m = mortgage \end{cases} \qquad (40)$$

where Δrnd_i is the randomly generated change in interest rate factor i.

Table 2-22 – Rate Curve Shift Simulations

	Δ Rate			
Rate (Yrs)	**Simulation 1**	**Simulation 2**	**....**	**Simulation *N***
1	0.41%	-0.26%	....	-0.29%
2	0.41%	-0.24%	....	-0.23%
3	0.40%	-0.22%	....	-0.18%
4	0.40%	-0.20%	....	-0.12%
5	0.39%	-0.18%	....	-0.07%
7	0.39%	-0.13%	....	0.04%
10	0.37%	-0.07%	....	0.20%
30	-0.22%	0.17%	....	-0.36%
Mortgage	0.11%	-0.26%	....	-0.38%

2.4.3 Valuing the Portfolio

Given a set of Δr_m's for a simulation, the change in value for a cash flow CF_m at maturity m is calculated as:

$$PV(CF_m) = CF_m e^{-(r+\Delta r_m)m} \qquad (41)$$

Table 2-23 – Change in Cash Flow Values for Simulations

	Δ Value			
Maturity (Yrs)	**Simulation 1**	**Simulation 2**	**....**	**Simulation *N***
1	1,989,396	-1,251,786	....	-1,380,780
2	1,877,409	-1,101,431	....	-1,070,782
3	1,592,850	-864,523	....	-705,514
4	664,214	-330,412	....	-207,173
5	387,654	-174,739	....	-68,776
7	-3,801,707	1,342,731	....	-385,740
10	-24,087,944	4,727,527	....	-13,066,148

For all instruments in the value-rate table, the change in value is found by looking up the specific Δr_m in the rate column, using linear interpolation.

Table 2-24 – Change in SCA Values for Simulations

Simulation	Δ Rate	Δ Value
1	0.41%	2,473,200
2	-0.26%	-5,428,500
:	:	:
:	:	:
N	-0.29%	-5,987,100

Finally, the change in value to all cash flows and instruments is calculated for each scenario and summed to yield the total change in value.

Table 2-25 – Overall Change in Value for Simulations

Simulation	Probability	Total Δ Value
1	1/N	-18,904,928
2	1/N	-3,081,133
:	:	:
:	:	:
N	1/N	-22,872,013

After each scenario is assigned a total change in value, the results across all simulations are sorted producing a cumulative probability distribution of change in value, with each scenario being equally probable with probability mass $1/N$. This distribution is then used for risk aggregation and capital allocation.

Table 2-26 – ALM Value Distribution

Probability	Value
0.001%	-247,325,565
0.010%	-206,414,145
0.030%	-185,755,073
0.050%	-179,171,451
0.070%	-175,236,860
:	:
:	:
99.930%	154,883,438
99.950%	159,731,919
99.970%	168,262,875
99.990%	181,303,688
99.999%	202,926,237

2.5 Credit Risk

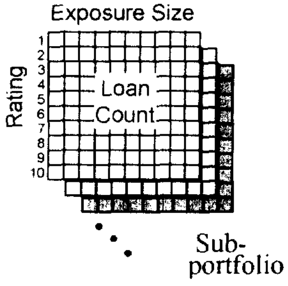
Credit risk is defined as the risk that a party to a contract, in most instances a borrower, defaults on an obligation, causing a loss of all or part of the replacement value of ongoing contracts. While default does not necessarily mean legal bankruptcy, it signals an inability or unwillingness of the party to fulfill its contractual obligations. Credit risk also includes the possibility that the obligor's credit quality weakens (i.e. the likelihood of default increases) causing a loss in value of obligations that are discounted for credit risk. For insurance companies, credit risk normally arises in a portfolio of bonds or loans, credit insurance, reinsurance recoverables, surety and financial derivatives.

The risk within a credit portfolio can be separated into three different types: systematic risk, non-systematic (or idiosyncratic) risk, and non-default economic loss risk. Systematic risk refers to the risk of default common to all counter-parties due to underlying economic factors that affect an industry, geography, etc. Idiosyncratic risk is specific to a particular company, for example fraud, and is statistically independent of sub-portfolio relationships. Non-default economic loss risk is the risk that the value of a credit changes over time even if the rating stays constant. For example, due to the credit cycle, a BBB credit may not be as credit worthy next year as a BBB credit is today, resulting in a loss of economic value. This type of risk captures the effect of credit movements over time on a systematic basis. The economic "mark-to-market" effect depends upon the maturity of the credit and the volatility of credit quality.

2.5.1 Characterization of credit exposures

Credit positions are bucketed into sub-portfolios constructed according to geographic, industry or other criterion. For each sub-portfolio, a credit matrix is constructed that groups credit obligations according to their credit quality (rating) and exposure size, as illustrated below:

Figure 2-2 – Illustration of Rating-Exposure Matrix



The credit risk for a single obligation depends upon the exposure at time of default, the probability of default (linked to the risk rating), the recovery rate in the event of default, the volatility of the recovery rate, the maturity of the obligation (for those obligations which are not systematically re-priced when credits weaken), and the correlation of the obligation to the rest of the sub-portfolio to which the position belongs. Correlations are specified between obligations within a sub-portfolio as well as between sub-portfolio types. For a portfolio of bonds or loans, correlations determine diversification benefits.

Table 2-27 – Mapping of S&P Ratings to Expected Default Frequency (EDF)

Rating	EDF
AAA	0.01%
AA	0.03%
A	0.07%
BBB	0.18%
BB	0.93%
B	4.46%

Table 2-28 – Corporate Bond Sub-portfolio Size Ranges

	Size Range 1	Size Range 2	Size Range 3	Size Range 4	Size Range 5
Minimum	0	6,000,000	11,000,000	20,000,000	25,000,000
Maximum	6,000,000	11,000,000	20,000,000	25,000,000	100,000,000
Average	2,500,000	9,000,000	14,000,000	21,000,000	26,000,000

Table 2-29 – Corporate Bond Sub-portfolio Rating-Exposure Matrix

	Bond Count				
Rating	**Size Range 1**	**Size Range 2**	**Size Range 3**	**Size Range 4**	**Size Range 5**
AAA	60	15	2	2	3
AA	10	15	5	5	0
A	10	10	5	5	0
BBB	0	5	2	1	0
BB	0	0	0	0	0
B	0	5	2	0	0

2.5.2 Expected loss

Credit loss can be described as the product of three terms:

Loss = Default · Exposure · Severity (Ong 94)

Loss is the amount that an institution is contractually owed but does not receive because of the borrower or borrower defaulting.

Default is the binomially distributed Bernoulli random variable that measures whether a borrower has defaulted or not, i.e., has fallen 3 months into arrears. It takes the values of either one in the case of default, or zero otherwise.

Exposure is the total amount of the institution's liability to a borrower.

Severity is the fraction of the exposure that is actually lost given a default of that borrower.

Table 2-30 – Corporate Bond Sub-portfolio Parameters

Parameter	Value
Recovery rate	50%
Recovery volatility	25%
Average maturity (yrs)	6
Intra-sub-portfolio correlation	0.5

Table 2-31 – Corporate Bond Sub-portfolio Exposure Summary

EDF	Size Range 1	Size Range 2	Size Range 3	Size Range 4	Size Range 5
0.0001	150,000,000	135,000,000	28,000,000	42,000,000	78,000,000
0.0003	25,000,000	135,000,000	70,000,000	105,000,000	0
0.0007	25,000,000	90,000,000	70,000,000	105,000,000	0
0.0018	0	45,000,000	28,000,000	21,000,000	0
0.0093	0	0	0	0	0
0.0446	0	45,000,000	28,000,000	0	0

Total Exposure ($) 1,225,000,000

The expected credit loss is the average annual loss rate over the course of a business cycle:

EL = E(Loss) = E(Default Frequency · Exposure · Severity) = E(Exp) · E(Sev) · E(DF) (Ong 94)

The expected loss for a portfolio is the sum of the ELs of the individual exposures.

2.5.3 Unexpected loss

Unexpected loss is the standard deviation of credit losses. There is typically little volatility in the size of the exposure amount (because the loan size is known upon origination), so $\sigma_{Exp}^2 = 0$. Exposure, default frequency and severity are treated as independent random variables. The standard deviation of default-based credit losses associated with an individual transaction is:

$$UL = \sigma_{loan} = \mu_{Exp}\sqrt{\sigma_{DF}^2 \cdot \mu_{Sev}^2 + \mu_{DF} \cdot \sigma_{Sev}^2} \text{ (Ong 113)}$$

The unexpected loss for a portfolio requires loss correlations between all pairs of borrowers. Let ρ_{ij} be the loss correlation between borrowers i and j, then

$$UL_{Portfolio} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} UL_i UL_j \rho_{ij}}$$ (Ong 133) where UL_i is the UL for loan i

In order to facilitate the calculation of the portfolio UL, the sub-portfolio UL can be divided into two components: a systematic piece and a non-systematic piece.

2.5.4 Systematic and idiosyncratic risk

The allocation of systematic risk and idiosyncratic risk is accomplished by splitting apart sources of variance in the $UL_{subportfolio}$ equation. Since a subportfolio is made up of a group of borrowers, the equation for $UL_{subportfolio}$ is analogous to the formula for $UL_{portfolio}$ (where the borrowers are those specific to the subportfolio). Then, we have:

$$UL_{Subportfolio}^{\;2} = \sum_{i=1}^{n}\sum_{j=1}^{n} UL_i UL_j \rho_{ij} = \sum_{i=1}^{n} UL_i^{\,2} + \sum_{i=1}^{n}\sum_{j\neq i}^{n} UL_i UL_j \rho_{ij}$$

where ρ_{ij} is the loss correlation between borrowers in the industry[3]. The first term represents borrower-specific risk (if borrower defaults were independent, this would be the total risk) and the second term represents additional risk owing to the correlation between borrowers within a subportfolio. As a result, the second term is purely systematic risk and the first term can be thought of as having systematic and idiosyncratic portions. It can be easily shown that UL_i (the UL for borrower i) can be split into a systematic portion, ULS_i, equal to:

$$ULS_i = \sqrt{\rho_i}\,UL_i$$

and an idiosyncratic, or non-systematic portion, ULN_i, equal to:

$$ULN_i = \left(\sqrt{1-\rho_i}\right)UL_i$$

where ρ_i is the loss correlation between 2 borrowers in the same industry (the industry for borrower i) each having probability of default equal to the probability of default for borrower i (i.e. the loss correlation between homogenous borrowers with the same credit rating). Therefore, the subportfolio non-systematic risk is calculated as:

$$ULN_{sp} = \sqrt{\sum_{i=1}^{n}\left(\sqrt{1-\rho_i}\right)UL_i^{\,2}}$$

and the systematic portion is calculated as:

$$ULS_{sp} = \sqrt{\sum_{i=1}^{n}\rho_i UL_i^{\,2} + \sum_{i=1}^{n}\sum_{j\neq i}^{n} UL_i UL_j \rho_{ij}}\,.$$

2.5.5 Non-default economic loss ("spread") risk

In addition to default risk over the 1-year time horizon, there is also the risk that longer-term loans (loans with maturities > 1 year) lose value resulting from changes in credit quality. More

[3]The loss correlation for loans in the same industry, $\rho_{i,j}$, is calculated using the Merton model of default. The calculation is a function of an industry asset correlation and default probabilities for borrowers i and j, as will be explained in more detail later in the document. Though the Merton model produces a default correlation, the assumption that loss correlation is approximatetly equal to default correlation is made since the majority of loss volatility is due to default volatility.

precisely, there is the risk of changes in the expectation of future losses that represents a risk to the value of the portfolio (analogous to a change in the "market value" of a loan). To model this risk, market parameters are used. Using yield spreads to risk-free securities, the non-default economic loss risk can be calculated analogously to interest rate risk for a bond.

The expected change in value is zero and the spread risk UL is assumed to be linear in maturity (making the effective maturity equal to the remaining maturity after 1 year, and weighted by principal payments). The volatility of spread is estimated using historical spreads on a universe of rated bonds. It is observed that the spread has a roughly constant coefficient of variation equal to 31%, making

$$Vol(r_{spread}) = r_{spread} \times \frac{\sigma_{spread}}{r_{spread}} = \sigma_{spread} \text{ with } r_{spread} = \%EL .$$

To be more exact, the spread loss variable for a loan is the product of a non-default indicator $\bar{D}$ and a spread loss variable P, $L_{sp} = \bar{D}P = (1-D)P$. We have that its mean is zero, so: $E[L_{sp}] = E[\bar{D}P] = u_{\bar{D}} E[P] = 0$, and $E[P] = 0$. Then, assuming independence of $\bar{D}$ and P, we can write $Var(L_{sp}) = \sigma_{\bar{D}} E[P]^2 + u_{\bar{D}} \sigma_P^{\,2} = (1-u_D)\sigma_P^{\,2}$. So, the unexpected loss owing to spread risk in a sub-portfolio is:

$$UL_{spread} = \sqrt{1-edf} \times EL \times \sigma_{spread} \times (T-1) \times Term\_Percent$$

where T is the Average Tenor (the quantity (T-1) is used for maturity to reflect the fact that spread risk is related to the remaining maturity after 1 year has passed, and the Term Percent is used to apply spread risk only to those loans with remaining maturities > 1 year). Since $\sqrt{1-edf}$ is approximately one, the term is dropped and the equation simplifies to $UL_{spread} = EL \times \sigma_{spread} \times (T-1) \times Term\_Percent$.

2.5.6 Portfolio unexpected loss

The equation for the portfolio UL as a function of systematic, non-systematic and spread risk components is:

$$UL_{TOTAL} = \sqrt{\left(\left(\left(\sqrt{\sum_i \sum_j UL_{Syst_i} \times UL_{Syst_j} \times \rho_{ij} - \sum_i (UL_{Syst_i})^2}\right) + \sum_i UL_{Spread_i}\right)^2 + \sum_i (UL_{Syst_i})^2 + \sum_i (UL_{NonSyst_i})^2\right)}$$

where UL_{Syst_i} is equal to the systematic portion of risk for industry i, $UL_{NonSyst_i}$ is the non-systematic and hence idiosyncratic portion of risk for industry i, UL_{Spread_i} is the spread risk for industry i, and $\rho_{i,j}$ is the correlation between industries i and j.

Table 2-32 – Sub-portfolio Correlations

	CorpBonds	GovtBonds	MBS	CreditIns	SuretyIns
CorpBonds	1	0.0	0.5	0.5	0.5
GovtBonds	0.0	1	0.0	0.0	0.0
MBS	0.5	0.0	1	0.5	0.5
CreditIns	0.5	0.0	0.5	1	0.5
SuretyIns	0.5	0.0	0.5	0.5	1

Since systematic risk is perfectly correlated within an industry, UL_{Syst_i} is computed as $\sum_{k \in i} UL_{Syst_k}$ where k is a subportfolio of type i. Since idiosyncratic risk is uncorrelated between all loans, $UL_{NonSyst_i}$ is computed as $\sqrt{\sum_{k \in i} UL_{NonSyst_k}^{\ 2}}$ where k is a subportfolio of type i. Since spread risk is perfectly correlated between all loans, UL_{Spread_i} is computed as $\sum_{k \in i} UL_{Spread_k}$ where k is a subportfolio of type i.

Non-systematic risk and systematic risk are by definition independent. Systematic risk and spread risk are assumed to be perfectly correlated and therefore additive. A correlation matrix between the subportfolios is required to capture the diversification effects of being exposed to different industries/geographies. Since only systematic volatility between subportfolio types is correlated, the total UL for the entire portfolio is a function of the independent nonsystematic volatilities (assuming they are independent of all other volatility), and correlated systematic volatilies and credit spread risk (assuming that they are correlated according to the correlation matrix and perfectly correlated to the credit spread risk).

Table 2-33 – Sub-portfolio Level Results

	CorpBonds	GovtBonds	MBS	CreditIns	SuretyIns
Loan count	162	180	153	4,572	8,535
Exposure	1,225,000,000	832,500,000	592,000,000	875,000,000	1,603,000,000
Expected Loss	1,885,900	33,450	11,840	787,500	7,453,950
Unexpected Loss - Systematic	3,793,515	17,775	53,296	1,938,542	13,219,292
Unexpected Loss - Idiosyncratic	3,473,965	532,212	237,163	807,546	1,667,388
Unexpected Loss - Spread	3,507,774	51,848	55,056	244,125	2,310,725
Total Unexpected Loss	8,085,620	536,746	260,742	2,327,265	15,619,270

Table 2-34 – Portfolio Level Results

	Total	As a % of Exposure
Expected Loss	10,172,640	0.1984%
Unexpected Loss - Systematic	19,022,419	0.3710%
Unexpected Loss - Idiosyncratic	3,979,980	0.0776%
Unexpected Loss - Spread	6,169,527	0.1203%
Total Unexpected Loss	21,074,259	0.4110%

2.5.7 Credit Loss Distribution

The final step of defining the Credit Loss Distribution is to assign a functional form to fit the characteristics of the distribution given the mean (EL) and standard deviation (UL). While there are several different ways to do this, the specific assumptions underlying our model lead to a natural choice. Because default is modelled as Bernoulli, the sum of a correlated portfolio of loans follows a Beta distribution. In mathematical terms, Beta is the continuous approximation to the distribution for a sum of Bernoulli random variables. While similar to the Gamma distribution, it is preferred because it does not allow firms to default repeatedly without curing. Between 0 and 1, the Beta distribution has a probability density function:

$$\beta(x,\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\cdot\Gamma(\beta)} \cdot x^{\alpha-1}(1-x)^{\beta-1} \quad \text{(Ong 165)}$$

where $\Gamma(z) = \int_0^\infty t^z e^{-t} dt$

The mean (EL) and standard deviation (UL) of the beta distribution, as a percent of exposure, can be solved through integration:

$$\text{Mean} = \text{EL\%} = \int_0^1 x \cdot \beta(x;\alpha,\beta)\, dx$$

$$= \frac{\alpha}{\alpha+\beta} \quad \text{(Ong 166)}$$

$$\text{Variance} = \text{UL\%}^2 = \int_0^1 x^2 \cdot \beta(x;\alpha,\beta)\, dx - \%EL^2$$

$$= \left[\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}\right]^2 \quad \text{(Ong 166)}$$

Rearranging for α and β

$$\alpha = (1 - EL\%) \left(\frac{EL\%}{UL\%}\right)^2 - EL\%$$

$$\beta = \frac{\alpha}{EL\%} - \alpha$$

The parameters $\alpha = 0.23$ and $\beta = 115.98$ can be used to generate the Credit Loss Distribution, which we translate into a value distribution.

Table 2-35 – Credit Risk Change in Value Distribution

Probability	Value
0.001%	-350,343,848
0.010%	-262,402,716
0.030%	-220,935,812
0.050%	-201,767,149
0.070%	-189,209,719
:	:
:	:
99.930%	10,183,496
99.950%	10,183,496
99.970%	10,183,496
99.990%	10,183,496
99.999%	10,183,496

2.6 Market Risk

Market risk is the risk associated with changes in the value of an investment portfolio or foreign exchange positions to market fluctuation. Market positions, henceforth called "sub-portfolios", are characterized by their current value and their β and tracking error relative to well-known market indices or individual securities. The potential for loss to tradable financial instruments resulting from unfavorable market movements is quantified by using a parametric model to calculate the Value at Risk (VaR) of the total investment portfolio. (Crouhy 198)

Table 2-36 – Tracking Indices

Tracking Index	Index Name	Volatility
SPX	S&P 500 Index	0.1986
BBREIT	Bloomberg REIT Index	0.1023
DH1	Direct Holding 1	0.2000
DH2	Direct Holding 2	0.2500

Table 2-37 – Investment Sub-portfolios

Subportfolio	Exposure	Tracking Index	β	Tracking Error
Equity Portfolio 1	100,000,000	SPX	1.05	3.2%
Equity Portfolio 2	50,000,000	SPX	1.10	5.0%
Direct Holding 1	50,000,000	DH1	1.00	0.0%
Direct Holding 2	50,000,000	DH2	1.00	0.0%
Real Estate	100,000,000	BBREIT	1.03	3.0%

2.6.1 Systematic and idiosyncratic risk

The volatility of each sub-portfolio's value is calculated in terms of a tracking index used as a benchmark. Once the amount of exposure in each index is determined, the systematic risk (due to the underlying movement of the index) and the idiosyncratic risk (due to the tracking error of the portfolio versus the index) are calculated.

Systematic risk is the volatility in the portfolio that arises from the fluctuations in the value of the underlying indices that the sub-portfolios are tracking. Systematic risk is calculated aggregating the β-weighted market values by index and calculating the total covariance:

$$\sigma_{Systematic} = \sqrt{\sum_{i,j} \beta_i \cdot MV_i \cdot \sigma_i^{Index} \cdot \rho_{i,j} \cdot \sigma_j^{Index} \cdot \beta_j \cdot MV_j}$$

Table 2-38 – Systematic Market Risk by Index

Index	β-weighted Exposure	Volatility	Systematic Risk
SPX	160,000,000	0.1986	31,776,000
BBREIT	103,000,000	0.1023	10,536,900
DH1	50,000,000	0.2000	10,000,000
DH2	50,000,000	0.2500	12,500,000
Total	363,000,000		41,971,271

Idiosyncratic risk is calculated by assuming independence across the idiosyncratic risks of each sub-portfolio:

$$\sigma_{Idiosyncratic} = \sqrt{\sum_i MV_i^2 \cdot TrackingError_i^2},$$

where MV_i is the market value of the position, and the index i represents sub-portfolios. Note that conventionally, tracking error is derived from the r^2 statistic (i.e. unexplained variance) obtained from a linear-regression of the sub-portfolio against its index.

Table 2-39 – Idiosyncratic Market Risk by Sub-portfolio

Sub-portfolio	Market Value	Tracking Error	Idiosyncratic Risk
Equity Portfolio 1	100,000,000	3.2%	3,200,000
Equity Portfolio 2	50,000,000	5.0%	2,500,000
Direct Holding 1	50,000,000	0.0%	0
Direct Holding 2	50,000,000	0.0%	0
Real Estate	100,000,000	3.0%	3,000,000
Total	350,000,000		5,048,762

Finally, we need to combine the idiosyncratic risk and the systematic risk, assuming independence between the two:

$$\sigma_{Total} = \left[\sigma_{Idiosyncratic}{}^{2} + \sigma_{Systematic}{}^{2}\right]^{1/2}$$

Table 2-40 – Portfolio Level Results

Total Systematic Risk	41,971,271
Total Idiosyncratic Risk	5,048,762
Total Risk	42,273,841

While it is accepted that the return on an individual security typically follows a log-normal distribution, there is some debate over whether a normal or lognormal distribution is appropriate for the value of a diversified portfolio. In this instance, we fit a normal distribution to the total volatility of $42,273,841.

Table 2-41 – Market Risk Value Distribution

Probability	Value
0.001%	-180,317,267
0.010%	-157,236,263
0.030%	-145,080,596
0.050%	-139,101,189
0.070%	-135,053,401
:	:
:	:
99.930%	135,053,401
99.950%	139,101,189
99.970%	145,080,596
99.990%	157,236,263
99.999%	180,317,267

Allocation of market risk capital to all of the activities that generate market risk is done using their contribution to total covariance.

2.7 Operating Risk

Operating Risk is used here to refer to the non-financial risks that arise in the course of running a business. Non-financial risks can be divided into two categories: event risks, which are one-off incidents that can cause large losses, and business risks, which are the risks associated with business decisions which relied on the wrong assumptions. Event Risk includes losses from systems failure, errors & omissions, fraud, uninsured damage to plant and equipment, and the impact these events have on customer behavior. Business Risk includes losses due to changes in the competitive environment or events that damage the franchise or operating economics of a business. Business Risk impacts the company through variation in volume, pricing, or costs.

An analog approach is used to quantify operating risk capital. The capital of analog non-financial companies is used as a proxy for their operational risk. "Pure-play" analog companies that have business processes subject to specific operating risks also faced by financial institutions were selected. Because these companies do not have significant financial risks, their economic capital supports only operating risk. These institutions' level of capital, along with their credit quality, yields an inferred estimate of the level of risk they face. Because these companies are more transparent, there is direct discipline from markets and rating agencies with respect to the amount of capital that they hold. We assume that these capital levels should be roughly equivalent to the levels of operating risk capital in similar business units of financial institutions.

2.7.1 Analogs

Table 2-42 describes each analog group and gives examples of companies in each analog.

Table 2-42 – Description of Operating Risk Analog Types

Analog Type	Description	Examples
Retail Services	• Fee-based services to consumers • High fixed costs due to many outlets • Low elasticity of demand	• Auto rental • Hair salons • Travel agencies
Business-to-Business Services	• Long-term relationships • No inventory • Low fixed costs	• Insurance brokers • Advertising agencies
Data Processing	• Process and track data, records, payments, etc. • Heavy investment in fixed cost systems, plant, and personnel	• ADP • EDS • Fiserv • First Data
Broker/ Dealer	• Transaction-based earnings from market-making and customer fees • Analogs selected have limited proprietary risk-taking activity	• PaineWebber • Legg Mason
Corporate Trust	• Performs similar roles as data processing companies but with added fiduciary responsibilities	• Northern Trust • U.S. Trust

2.7.2 Non-interest expense

The scale factor for calculating Operating Risk is an institution's non-interest expense (NIE). NIE has the benefit that it is the only common measure of size and scope between financial and non-financial companies. Therefore, for each analog group, we determined the Capital / NIE Multiplier adjusted for credit rating. We then apply these multipliers to financial institutions based on how their business units are divided between the five analogs' business lines.

Table 2-43 – Line of Business Contributions

Line of Business	NIE	Retail	B to B	D.P.	B/D	C.T.	Contribution to σ
Homeowners	30,000,000	0%	30%	70%	0%	0%	4,650,000
General Liability	15,000,000	0%	30%	70%	0%	0%	2,325,000
Credit & Surety	20,000,000	0%	30%	70%	0%	0%	3,100,000
Term Life	15,000,000	0%	40%	60%	0%	0%	2,250,000
SCA	7,500,000	0%	40%	60%	0%	0%	1,125,000
Total	87,500,000						13,450,000

Table 2-44 – Operating Risk Value Distribution

Probability	Value
0.001%	-57,370,402
0.010%	-50,026,865
0.030%	-46,159,374
0.050%	-44,256,944
0.070%	-42,969,084
:	:
:	:
99.930%	42,969,084
99.950%	44,256,944
99.970%	46,159,374
99.990%	50,026,865
99.999%	57,370,402

2.8 Risk Aggregation

In order to measure overall capital adequacy and derive accurate capital contributions, the "total" risk that an institution faces must be computed from the value distributions that describe its component risks. Since the underlying risk distributions for each risk type do not necessarily follow a particular distributional form (e.g. property catastrophe risk is frequently an empirical distribution), it is necessary to do a numerical integration or simulation in order to combine them. The method described here uses a numerical integration approach to "convolve" the underlying linearly correlated risk distributions.

The distribution aggregation method takes N distributions, specified as discrete cumulative density functions (i.e. a set of tables listing possible losses due to credit, market risk, etc. with the associated probabilities for exceeding that loss). To avoid simulation, the problem is parceled into a series of two-distribution convolutions, the result of each one subsequently convolved with the next input distribution, i.e. a "pair-wise roll-up" (e.g. aggregating market risk and credit risk, and then aggregating the resulting distribution with operating risk). This scheme generalizes to as many distributions as are desired with approximately linear cost in computational intensity, as opposed to multi-dimensional calculations or simulations that are exponential in computational cost.

Figure 2-3 – Tabular Discrete Density Functions in the Two-Distribution Case

$$A:\begin{bmatrix} x_1 & prob(x_1) \\ x_2 & prob(x_2) \\ \vdots & \vdots \\ x_{m-1} & prob(x_{m-1}) \\ x_m & prob(x_m) \end{bmatrix} \text{ and } B:\begin{bmatrix} y_1 & prob(y_1) \\ y_2 & prob(y_2) \\ \vdots & \vdots \\ y_{m-1} & prob(y_{m-1}) \\ y_m & prob(y_m) \end{bmatrix}$$

Convolution requires an assumption as to the form of the copula (the joint probability density function for the set of outcomes from multiple random variables, with each variable's outcome expressed in terms of it's marginal cumulative density function). The method assumes a multivariate normal copula[4].

The method for aggregating two distributions consists of first converting the input distributions to "Normal space" (using the cumulative density function) and using the bivariate normal density function to compute the probabilities for each possible combination of losses. This yields the desired resulting cumulative density function for the aggregate distribution (after sorting by loss and cumulating probability mass):

$$Z=\begin{bmatrix} z_{1,1} & prob(z_{1,1}) \\ z_{1,2} & prob(z_{1,2}) \\ \vdots & \vdots \\ z_{1,m-1} & prob(z_{1,m-1}) \\ z_{1,m} & prob(z_{1,m}) \\ z_{2,1} & prob(z_{2,1}) \\ z_{2,2} & prob(z_{2,2}) \\ \vdots & \vdots \\ z_{m,m-1} & prob(z_{m,m-1}) \\ z_{m,m} & prob(z_{m,m}) \end{bmatrix}$$

where $z_{i,j}=\dfrac{x_i+x_{i+1}}{2}+\dfrac{y_j+y_{j+1}}{2}$ and

[4] see Wang, "Aggregation of Correlated Risk Portfolios", Proceedings of the Casualty Actuarial Society, Volume LXXXV, Number 163, Page 887

$$prob\left(z_{i,j}\right)=\int_{F_{norm}^{-1}(prob(x_i))}^{F_{norm}^{-1}(prob(x_{i+1}))}\int_{F_{norm}^{-1}(prob(y_j))}^{F_{norm}^{-1}(prob(y_{j+1}))} f\left(x,y,\rho_{norm}\right)dx\,dy.$$

F_{norm}^{-1} is the inverse normal cumulative density function and $f(x,y,\rho_{norm})$ is the bivariate normal function defined by:

$$f\left(x,y,\rho_{norm}\right)=\frac{1}{2\pi\sqrt{1-\rho_{norm}^2}}\times e^{-\frac{1}{2}\left(\frac{x^2-2\rho_{norm}xy+y^2}{1-\rho_{norm}^2}\right)}$$

ρ_{norm} is calculated iteratively such that the equivalent correlation ρ_{equ}, defined as

$$\rho_{equ}=\frac{\left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f\left(x,y,\rho_{norm}\right)\times F_A^{-1}\left(F_{norm}(x)\right)\times F_B^{-1}\left(F_{norm}(y)\right)\right)-\mu_A\times\mu_B}{\sigma_A\times\sigma_B},$$

is approximately equal to ρ , the input correlation between distributions A and B, where $f(x,y,\rho_{norm})$ is the bivariate normal probability density function and F_{norm} is the standard normal cumulative density function.

Note that because the combined distributions are described by discrete cumulative density functions of m elements, and the algorithm evaluates each possible combination, the resulting convolution will be a tabulated cumulative density function containing approximately m^2 elements. To keep the subsequent calculation tractable, the result must be reduced in size by mapping to the given probability schedule; this is done with standard linear interpolation. Finally, this process is repeated, convolving the new distribution Z with the next input distribution and so on.

The total diversified economic capital is found by looking up the desired solvency standard on the aggregate distribution, then subtracting the mean of the distribution from the loss value.

Table 2-45 – Overall Risk Value Distribution

Probability	Value
0.001%	-580,173,921
0.010%	-522,774,051
0.030%	-464,762,707
0.050%	-430,156,396
0.070%	-411,064,239
:	:
:	:
99.930%	324,707,743
99.950%	334,632,692
99.970%	349,823,731
99.990%	378,717,198
99.999%	416,390,625

The mean of the above distribution is: -6,790,062.

Table 2-46 – Overall Risk Value Distribution

EDF	Required Economic Capital
0.01%	-515,983,989
0.03%	-457,972,646
0.07%	-404,274,177

2.9 Capital Allocation

The total diversified economic capital value must be attributed to the different risk types. Contributory capital for each risk type is calculated with a covariance and excess-skewness approach.

Let *EC* be the total economic capital at the desired solvency standard *S* and *NormEC* be the equivalent normal economic capital for the output distribution:

$$EC = F^{-1}(1-S) - \mu$$

$$NormEC = F_{Norm}^{-1}(1-S, \sigma)$$

$F^{-1}(1-S)$ is the inverse of the output distribution at the desired solvency standard, ${F_{NORM}}^{-1}(1-S,\sigma)$ is the inverse normal function, μ is the mean of the output distribution and σ is the standard deviation. *SkewEC*, the portion of economic capital that is due to shape (skewness), is defined as:

$$SkewEC = EC - NormEC$$

Let EC_i be the contributory economic capital of the *i*th input distribution. Let SAC_i be the stand-alone capital of the *i*th distribution. This is defined as:

$$SAC_i = F_i^{-1}(1-S) - \mu_i$$

where μ_i is the mean of the *i*th input distribution, and $F_i^{-1}(1-S)$ is the inverse of the *i*th distribution at the desired solvency standard. Let $NormSAC_i$ be the equivalent stand-alone normal economic capital for distribution *i*:

$$NormSAC_i = F_{norm}^{-1}(1-S, \sigma_i)$$

where σ_i is the standard deviation of distribution *i* and $F_{norm}^{-1}(1-S,\sigma_i)$ is the inverse normal function evaluated at the desired solvency standard *S*. Let $SkewSAC_i$ be the portion of stand-alone capital for distribution *i* that is due to shape (skewness), defined as:

$SkewSAC_i = SAC_i - NormSAC_i$

Contributory capital for distribution i is calculated as:

$$EC_i = NormEC \times \left(\frac{\sigma_i \sum_{j=1}^{n} \sigma_j \times \rho_{ij}}{\sum_{k=1}^{n} \sum_{j=1}^{n} \sigma_k \times \sigma_j \times \rho_{kj}} \right) + SkewEC \times \left(\frac{SkewSAC_i \times \sum_{j=1}^{n} SkewSAC_j \times \rho_{ij}}{\sum_{k=1}^{n} \sum_{j=1}^{n} SkewSAC_k \times SkewSAC_j \times \rho_{kj}} \right)$$

where n is the number of input distributions and ρ_{ij} is the Pearson correlation coefficient between distributions i and j.

Table 2-47 – Risk Pillar Correlations

	NonCat	Cat	Life	ALM	Credit	Market	Operating
NonCat	1	0.0	0.0	0.2	0.0	0.0	0.2
Cat	0.0	1	0.2	0.0	0.0	0.0	0.2
Life	0.0	0.2	1	0.0	0.0	0.0	0.2
ALM	0.2	0.0	0.0	1	0.3	0.2	0.2
Credit	0.0	0.0	0.0	0.3	1	0.2	0.2
Market	0.0	0.0	0.0	0.2	0.2	1	0.2
Operating	0.2	0.2	0.2	0.2	0.2	0.2	1

Table 2-48 – Capital Allocation to Risk Types

Risk Type	μ	σ	SAC_i	$NormSAC_i$	$SkewSAC_i$	EC_i	Allocation
Credit	0	21,082,000	-220,935,812	-72,351,814	-148,583,998	-84,949,758	19%
Market	0	42,279,492	-145,080,596	-145,099,991	19,395	-78,817,041	17%
NonCat	1,453	45,822,431	-197,121,120	-157,259,086	-39,862,034	-88,335,717	19%
ALM	-6,730,902	51,350,562	-179,024,171	-176,231,209	-2,792,962	-125,935,017	27%
Operating	0	13,451,798	-46,159,374	-46,165,544	6,171	-22,612,131	5%
Cat	0	19,853,644	-163,190,513	-68,136,192	-95,054,322	-38,943,801	9%
Life	0	20,423,027	-83,398,815	-70,090,270	-13,308,545	-18,379,180	4%
Total						-457,972,646	100%

3. RETURN QUANTIFICATION

Risk-Adjusted Return on Capital (RAROC) is the metric used to quantify the level of performance of line of business. For a given line of business, RAROC is defined as the following:

$$RAROC = \frac{UW + IC + CB}{EC}$$

where UW represents the calendar year underwriting result, IC is the investment credit, CB is the capital benefit and EC is the economic capital. RAROC can be computed either on a pre- or post-tax basis, with the components of the quotient adjusted accordingly. In all cases, economic capital in both instances must be measured on a contributory basis.

Frequently, economic capital is not equal to actual available capital. While RAROC is the return on equity, (ROE), that would result from holding an amount of capital equal to economic capital, under-capitalized companies have inflated ROE, while overcapitalized companies usually have a depressed ROE, except where the internal transfer rate on invested surplus is in excess of company-wide RAROC.

3.1 Calculation and Allocation of Investment Returns and Capital Benefit

Insurance lines of business generate reserves and surplus that earn an investment return. A portion or all of this return should be allocated back to the business that supplies the funds, as the reserves and surplus are on deposit with the investments unit. While there is a spectrum of opinion between allocating a risk-free rate of return or the entire investment return, the RAROC approach typically involves setting an internal cost of funds for the total amount supplied by the business.

The internal cost of funds rate should reflect a fair return for an investment that bears no credit, market or interest-rate risk. It should also reflect a premium for a guarantee of liquidity in the case of a sudden need to pay a large claim. Along with this, credit, market and interest-rate risk are managed by the investments unit, generating a need for economic capital. Investment returns in excess of the cost of funds are retained by the investment manager.

The total investment return is calculated as the sum of realized and unrealized gains, investment income, dividends, less expenses. The risk-adjusted income for the investments unit is the total investment return less the product of the cost-of-funds rate and the total invested assets. Because the investment credit and capital benefit reflect an internal transfer, the amount subtracted from the investment unit's return should be equal to the total added to the total investment credit and capital benefit allocated to the insurance lines of business.

Table 3-1 – Income Statement and Required Economic Capital by Line of Business

Lines	NEP	Other Revenue	Losses	Expenses	Reserves	Economic Capital
Homeowners	150,000,000		-97,500,000	-50,000,000	150,000,000	121,535,269
General Liability	90,000,000		-67,500,000	-30,000,000	250,000,000	17,470,615
Credit & Surety	115,000,000		-90,000,000	-38,000,000	175,000,000	62,470,270
Term-Life	100,000,000		-80,000,000	-30,000,000	200,000,000	21,603,260
SCA	45,000,000		-35,000,000	-15,000,000	200,000,000	32,210,652
Investments		100,000,000				202,682,580
Econ. Capital	500,000,000	100,000,000	-370,000,000	-163,000,000	975,000,000	457,972,646
Excess Capital						117,027,354
Total	500,000,000	100,000,000	-370,000,000	-163,000,000	975,000,000	575,000,000

3.2 Adjusting the Underwriting Result

The calendar year underwriting result can be adjusted to bring it closer to a true economic view of profitability. Specific adjustments are made to remove development in reserves for past accident years, allocate overhead expenses and reverse one-time special charges:

$$AdjUW = UW - \Delta Reserves + Overhead - One\text{-}Time\ Charges$$

Subtracting the change in reserves due to reassessment of prior accident years removes the "misdeeds of the past" to produce better forward-looking figures. Adding in corporate overhead ensures that the result uses "fully-loaded" expenses - it is not unheard of for a new business to launch a "profitable" product but manage to lose money every year. There are many theories of how to allocate corporate overhead, however we have found that the process can be contentious as it can affect P&L statements. Nevertheless, typical methods involve sizing the benefit received by each line of business from each cost center. Finally, true one-time charges are removed.

Table 3-2 – Return Adjustments and RAROC Calculations

Lines	UW Result	Investment Credit	Capital Benefit	Adjusted UW Result	RAROC	Tax Rate	Post-Tax Adjusted UW Result	Post-Tax RAROC
Homeowners	2,500,000	7,005,000	5,675,697	15,180,697	12%	35%	9,867,453	8%
General Liability	-7,500,000	11,675,000	815,878	4,990,878	29%	35%	3,244,071	19%
Credit & Surety	-13,000,000	8,172,500	2,917,362	-1,910,138	-3%	35%	-1,241,590	-2%
Term-Life	-10,000,000	9,340,000	1,008,872	348,872	2%	35%	226,767	1%
SCA	-5,000,000	9,340,000	1,504,237	5,844,237	18%	35%	3,798,754	12%
Investments	100,000,000	-72,385,000	9,465,276	37,080,276	18%	35%	24,102,180	12%
Econ Capital	67,000,000	-26,852,500	21,387,323	61,534,823	13%		39,997,635	9%
Excess Capital			5,465,177	5,465,177	5%	35%	3,552,365	3%
Total	67,000,000	-26,852,500	26,852,500	67,000,000	12%	35%	43,550,000	8%

4. EVALUATION OF RISK-ADJUSTED RETURN ON CAPITAL

4.1 Alternative Views

There are two different views of economic capital and RAROC in an insurance context. The first is "Calendar Year" RAROC, which is the approach taken in this paper. Calendar Year RAROC looks at the risk and return of a company's full balance sheet over the course of the next calendar year. The second view is "Accident Year" RAROC, which examines the lifetime risk and return of new business put on in the coming year.

4.1.1 Calendar Year RAROC

Calendar Year RAROC is the 'standard' approach to measuring risk and return, and was the method outlined in this paper.

4.1.2 Accident Year RAROC

Accident Year RAROC is an alternative to the Calendar Year approach. Rather than considering all business that has been written in the past - and therefore can't be changed - the Accident Year view focuses only on the risk and lifetime value embedded in new business.

In practice, the calculation of Accident Year RAROC for Non-Cat risk is very similar to the method outlined in this paper. It is equivalent to hypothesizing that the current accident year represents the firm's steady state; that is, all previous years are identical to the current in both volume and division between business units.

The computation of economic capital differs only in equation 39, where each ULE_i is just the initial loss estimate for the accident year in question.

The computation of risk-adjusted return is similar to the Calendar Year approach as well. The only difference is in the calculation of investment credit on reserves. Let R_i denote the expected reserve for the current year's contracts in development year i. The total reserve R for crediting investment returns is:

$$R = \sum_{i=1}^{\infty} R_i \tag{42}$$

The total reserve R is credited at the firm's cost of funds. This can be interpreted as crediting the specified accident year with all internal transfer income that will be accrued over the course of the contracts' lives.

The Accident Year vs. Calendar Year distinction is largely applicable only to P&C Non-Catastrophe risks. Other risk pillars, such as Credit, Market and Operating risks have no comparable notion of 'tenure'. Depending on the intended application, however, it may make sense in context to change the assumptions relating to these risks (e.g. amount of invested assets) to mirror the 'steady-state' view of the Accident Year Non-Catastrophe risk calculation.

4.1.3 Comparison

Both the Accident Year and Calendar Year approaches have important uses and interpretations. Accident Year RAROC is a measure of the lifetime value of new business. The true value of long-tailed insurance contracts is highly dependent on investment returns on reserves earned over the very long term; long-tailed lines can look extremely unprofitable during periods of growth as high loss ratios dominate relatively small levels of reserves (and therefore investment returns), even if the expected long-term profit is very high. Accident Year RAROC credits new business with this long-term income to clarify the tradeoff between underwriting profit and investment returns. Therefore, Accident Year RAROC is most useful for applications such as setting pricing targets and performing strategic planning.

In contrast, Calendar Year RAROC is a measure of realistic expected shareholder returns over a one-year period. It is the metric that is most closely comparable to budgeted financials. For this reason, it is the more useful measure for performance assessment and shareholder communication. Also, the Calendar Year methodology should be used for determining capital adequacy. It measures a company's true capital requirements in the short term. The Accident Year methodology captures the cumulative lifetime capital requirement of new business, which is not truly actionable in any reasonable manner.

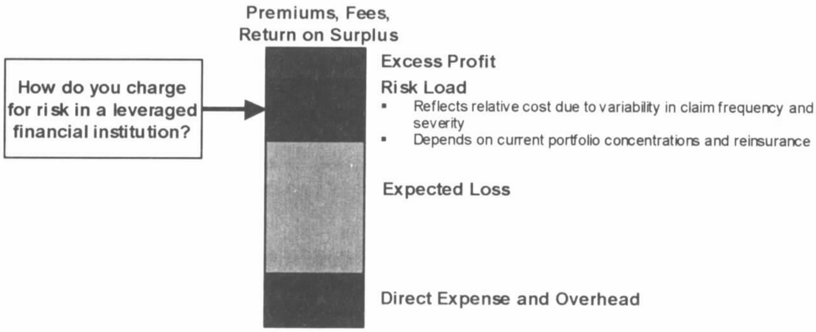
4.2 Other Applications

The RAROC framework lends itself to several applications including pricing, risk transfer evaluation and mergers & acquisitions analyses. In the pricing framework, economic capital and the hurdle ROE set the cost of risk that must be offset by the risk load. For evaluating the performance of reinsurance for risk transfer, RAROC is an effective risk-return metric that can be used to compare the efficiency of reinsurance across dissimilar lines of business. For M&A, RAROC enables a quick and straightforward calculation of the value of the potential target within the context of the acquirer's business portfolio. RAROC's versatility is a very compelling factor that is driving the adoption of the framework.

4.2.1 Risk-Based Pricing

The pricing cycle is an inevitable outcome of a pricing strategy that relies heavily upon observed market price-points rather than economic risk-based pricing. When capacity is plentiful, prices are reduced relative to the competition, trading current profitability for market share. Shareholder value destruction is a frequent result of this behavior, compounded by the rarity of explicit calculations of the economics of this trade. However, we can look to the banking industry for a way to escape this cycle. The answer is to know the economic break-even point by computing an appropriate risk load based on a hurdle RAROC that can be determined from market analysis and CAPM theory, and on the capital required to support the marginal risk of new business.

Figure 4-1 – The components of risk-based pricing



For example, by setting a 12% hurdle RAROC, prices that result in a lower return can be considered to destroy shareholder value, while prices that result in "Excess Profit" as shown above create shareholder value. Armed with this information, company management can assess the strategic value of market share initiatives relative to the near-term value destruction of ultra-competitive pricing. If uses for the excess capacity are found to be value destroying, management can and in many cases should decide to return that capital to shareholders for investment in other opportunities.

4.2.2 Reinsurance and M&A Evaluation

Evaluating reinsurance is never an easy task, but choosing between two programs in different business areas is a challenge that has proven elusive. Consider the case where the choice is between buying treaty reinsurance for a General Liability portfolio versus buying treaty reinsurance for a D&O Liability portfolio. The hypothetical company has a pre-treaty RAROC of 15% on $100 million in Economic Capital, with a hurdle rate of return of 15%.

Example Treaty	Risk-Adjusted Return	Economic Capital	RAROC	Intrinsic Value	Shareholder-Value Added
Gross	$ 15.0 MM	$ 100 MM	15.0%	$ 100.0 MM	$ 0.0 MM
10 x 10 on D&O	$ 14.0 MM	$ 90 MM	15.6%	$ 95.0 MM	$ 5.0 MM
15 x 5 on GL	$ 14.5 MM	$ 95 MM	15.3%	$ 97.5 MM	$ 2.5 MM

The D&O program results in higher RAROC and shareholder value creation despite the greater reduction in Risk-Adjusted Return. While this would technically "shrink" the business, it is more valuable than the alternatives

Evaluating Mergers & Acquisitions would involve a similar framework of computing the net reduction in total Economic Capital of the combined entity relative to the two standalone entities, and calculating the shareholder value creation for the acquirer.

5. CONCLUSION

5.1 Strategic Recommendations

The case study company is under-performing as its current RAROC of 9% is well below the hurdle, or target, return on capital of 15%. However, a glimmer of hope exists in the 19% RAROC posted by the General Liability business. Because General Liability consumes only 4% of total economic capital, there is room to grow the business without worrying about excess concentration risk. Conversely, Credit & Surety, which accounts for 14% of total economic capital, should be reigned in until its profitability can be addressed through a risk-based pricing initative, as described in section 4.2.1.

Additionally, we see in Table 3-1 that the company is overcapitalized by $117 MM, or about 25% ($117 MM/ $575 MM in total capital). This drags the actual ROE down from 9%, were it adequately capitalized, to 8% in its overcapitalized state. Note that only the additional investment return on the excess capital prevents the ROE from dropping even further. For this particular company, capital could be redeployed in the following ways:

- Redeploy capital from Credit & Surety to General Liability
- Return capital to shareholders via share buyback or increased dividends
- Expansion into new businesses that earn an adequate return

Table 5-1

Line of Business	Economic Capital (EC)	% of Total EC	Post-Tax RAROC
Homeowners	121,535,269	27%	8%
General Liability	17,470,615	4%	19%
Credit & Surety	62,470,270	14%	-2%
Term-Life	21,603,260	5%	1%
SCA	32,210,652	7%	12%
Investments	202,682,580	44%	12%
Total Economic	457,972,646	100%	9%

As these recommendations demonstrate, the Economic Capital and RAROC framework are designed around supporting specific decisions and strategic insights. The philosophy is to produce best results possible in a timely fashion, but with neither "perfect" accuracy nor excruciating detail. It is not intended to generate stochastic multi-year financial projections, set reserve requirements or model the particulars of a specific complex insurance policy. The adoption of RAROC as an industry standard in banking was predicated upon its ability to accommodate diverse risk types and businesses. RAROC's ease of use and cross-industry capabilities make it an emerging presence in the insurance industry.

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Dos and Don'ts in Dealing with the Media

CAS External Communications Committee

Dos and Don’ts in Dealing with the Media

Formal communication with external audiences is an important part of the Casualty Actuarial Society’s (CAS’s) communication and public relations activities. Public relations endeavors affect how our current customers, prospective customers, employers, and the public-at-large perceive casualty actuaries.

The following guidelines on media relations and public presentations were organized by the External Communications Committee (ECC) of the CAS in the interest of enhancing the external visibility and expertise of casualty actuaries and the image of the CAS. The guidelines are based on issues, goals, and considerations discussed by the Committee, and are consistent with the CAS Communication Plan and similar to guidelines used by other associations and Fortune 500 firms.

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When CAS members are quoted by the media, they help to improve the public perception of the casualty actuarial profession and promote the understanding of issues of concern to actuaries and our audiences. The ECC encourages CAS members to seek out this external visibility and endorses their full cooperation with the media.

However, because such activities must be undertaken with utmost sensitivity to the existing and potential relationships of CAS members, we should exercise caution when commenting on specific organizations by name. By avoiding comment on situations or events at a specific organization, we protect the confidentiality of our members and our specific audiences. The only exceptions are when a specific member of our audience has asked us to work with them on obtaining media coverage, when the organization in question has given us permission to comment, or when we are commenting on situations or events at organizations where we’re employed. In the last instance, we must take care to follow the media guidelines of our employers as well as those set forth in this CAS document.

The following key principles are intended to aid our members in responding effectively to the media:

Key Principles

- **Do Respond Quickly and Expertly.** If you are either not prepared or not able to provide an objective analysis for a reporter, don’t hesitate to suggest that someone else could assist the reporter. It’s also possible that what the reporter is asking for is not in your field of expertise. If you are unable to respond promptly to a media inquiry—or if you prefer not to because you are uncomfortable speaking with reporters—ask another CAS member to respond, or call either the American Academy of Actuaries (AAA), the CAS Office, or the Chairperson of the Media Relations Committee (see below), who will refer the reporter appropriately.

- **Do Respond to All Questions.** There is more to being a good source than being quoted regularly. A reporter will appreciate your candor if you don't know the answer to a question. If that happens, the appropriate response is "I can't answer that question for you." Then, if possible, explain why you can't, so there's no mystery. Not explaining can only lead to speculation. Don't reply by saying "No comment." Always explain why you can't comment, as, for example, "I'm sorry, but it is our policy not to comment on competitors." Otherwise you will sound suspicious, as if you are covering up. In these cases, if possible, offer to help put the reporter in touch with someone who can answer. If you are asked a question that is sensitive and you don't feel comfortable answering (for example, privileged business information is involved), it is still not recommended to use the "No comment." phrase. Politely say you are not able to answer the question, and then stop talking.
- **Do Clarify the Reporter's Question Before Replying.** Ask as many questions of the reporter as necessary to determine the scope and nature of the article. If you have been asked to supply illustrative data or research, ascertain ahead of time if your company or the CAS will be cited as the information source.
- **Don't Speak on Behalf of the CAS.** Make sure you make it clear that you are not speaking for the CAS, the actuarial profession, your company, or a political party. Convey that you are an interested citizen whose credentials as a professional offer a specific perspective on news events and that others may be interested in your viewpoint. When speaking or writing about an industry or legislative issue, you may wish to determine if the AAA or the CAS has formulated a position on the issue and if that position may have implications for your audience. It is necessary in all cases to distinguish between individual professional views and a position taken by your employer or the CAS. When the issue is controversial, such as pending legislation, even more care should be taken to guarantee that individual professional opinion is not mistaken for a policy statement of the CAS.
- **Do Prepare for Your Interview with Facts.** Don't ever be openly critical of the newspaper, magazine, news station, editor, publisher, or reporter, no matter how thoroughly you are provoked by a story. Assimilate their views and counter them with facts. If you build your case with understandable facts, it will be difficult for the press to ignore them. Gather facts and prepare a comprehensive but concise reply. Try to emphasize two or three key points in your remarks. When asked about a sensitive or complex issue, prepare a written statement or answers to anticipated questions to which you can refer during the interview. Use the same degree of care and attention when preparing presentation visuals as you would in making a presentation for your client and/or employer.

❑ **Do Be Accurate.** Even though reporters are always faced with urgent deadlines, you should never compromise the accuracy of your response. If you don't have the expertise or information to give an adequate reply, refer the reporter to someone who does, or ask the AAA, the CAS Office, or the Chairperson of the Media Relations Committee (see below) for guidance.

❑ **Do Use Examples.** Good reporters illustrate their stories with specific examples. By providing such illustrations, you improve the chances of being quoted. If you want to make a point based on a specific audience member's experience, you should either obtain that audience member's permission in advance or present it as a hypothetical example that masks the audience member's identity.

❑ **Don't Say Something if You Do Not Want it to Appear in Print.** If you want to give a reporter information, but don't want to be quoted as its source, provide the information only after you have agreed with the reporter that you are providing "background information," and as such the statement is "not for attribution." Never speak "off the record." Say nothing you don't want to read in tomorrow's edition. Don't be tempted to confide in a reporter off the record. Actually, most reporters try to keep such confidences, but the risk is unacceptable, as nothing you say to the media is ever truly "off the record." Bottom line: If you don't want something to appear in print or be included in a radio or television broadcast, don't say it.

❑ **Don't Expect to See an Advance Copy.** Don't badger reporters or editors to run your views. Don't call reporters unless you have a reason. Don't ever mention how much advertising your company runs in the paper or on the TV or radio station. You will lose credibility instantly. Most media rarely let outsiders review or edit their material. However, cooperate fully when asked by a "fact checker" to verify information about yourself or confirm a statement for attribution.

Some reporters will, in fact, allow sources to review quoted material for accuracy—especially when a subject is highly technical. But do not expect to be allowed to edit the article. If quoted material appears to be inaccurate, discuss it with the reporter. The reporter is likely to agree to change it. But do not expect to be allowed to reword quotes that both you and the reporter agree are accurate presentations of what you said. Only a very generous reporter will do this—and it is likely that the reporter will never call on you again for a future interview.

❑ **Don't Be Surprised by Misquotes or Unfortunate Contexts.** There are no guarantees in dealing with the media. You may not get the results you had hoped for. Putting yourself into the public eye is a risk and, at the very least, someone may misunderstand or disagree with you. However, if all your communications are honest and open and if you have no hidden or personal agendas, the media will consider you a valuable and reliable resource. If you are misquoted or the meaning of your statement is altered through its context in an article, discuss the situation with your CAS colleagues, the AAA, the CAS

Office, or the Chairperson of the Media Relations Committee (see below) to determine the appropriate response.

- **Do Be Aware of the Media at CAS Meetings and Seminars.** In addition to clients, prospects, and future employers, meeting and seminar audiences may include media representatives who may report your remarks and may want to interview you.

5 Common Interview Traps

When interviewed by a reporter, not all questions may be innocent. Reporters are trained in a variety of techniques to elicit the information they want. Beware the common traps itemized below and use the recommended counter technique.

- **Trap #1 - The oft-repeated question.** This is a common technique to get the source to say something he has already indicated he doesn't want to say. The interviewer will ask a question. The response will be, "I don't know" or "I can't say." The interviewer will then appear to go on to other topics but will come back to the unanswered question often.

Counter technique: There's only one: Keep answering the question the same way without getting irritated.

- **Trap #2 - The negatively phrased question.** This is something like, "Isn't it really the insurance industry's greed that keeps home insurance premiums so high?"

Counter technique: Turn your answer around and make it positive. Don't get suckered into responding in the negative. Say something like, "There are many uncontrollable factors that determine premiums." Then go into what they are.

- **Trap #3 - Multiple questions packaged as one.** Some reporters like to ask several questions at once without giving the interviewee time to respond to each one separately. They're hoping for an interesting slipup.

Counter technique: Don't try to answer all the questions as one. Ask the interviewer to rephrase them one at a time. You don't have to remember them all.

- **Trap #4 - Silence**. Often the reporter will ask a question, the interviewee will answer it and the reporter will just sit and stare. The reporter is hoping the source will add to the statements already made.

Counter technique: Say what you have to say and then be quiet—even if you're on TV or the radio. It isn't your job to keep the interview moving forward, it's the reporter's.

- **Trap #5 - Hostility.** The reporter uses loaded words and phrases designed to make you angry. For example, "Why do you sexists at ABC

Insurance charge women more than men? When will you stop ripping off half the insurance purchasers in America with this anti-woman policy?"

Counter technique: Rephrase the question using positive language. Don't answer the question the way it is. Say something like, "You want to know about our pricing policy. This is how we do it..."

Useful Techniques

The following are useful techniques to consider when being interviewed:

- **Change the subject by using conversation bridges.** Talk about what you want to talk about, as long as the information you want to provide is closely related to the questions being discussed. If the reporter asks a question that enables you to give a short reply, answer briefly and then "bridge" to another topic by saying something like, "I think your readers might also like to know..." or "Let me answer that another way..." or "That question reminds me of another point you might be interested in" and then elaborate on the topic of your choosing.
- **Buy thinking time.** If you need a few seconds to get an answer together, try these time-buying strategies. Repeat a part or the entire question. Use an introductory phrase like, "I'm really glad to have the opportunity to talk with you about that, because it's something I've thought about a great deal." It's a debater's technique, and it will help you to get your scattered thoughts together more than you might expect.
- **Count a few beats before answering.** If your answers are starting to sound "canned" or rehearsed, try the simple technique of waiting 3 seconds before answering. The slight pause before you answer will make your responses seem fresh, as though you've never uttered them before.
- **Taping the interview.** The reporter will probably want to tape the interview and will ask you if you mind. This is standard operating procedure for most reporters, and it helps ensure the accuracy of their news stories. Allow the taping.
- **Correct any misinformation.** If the reporter quotes the wrong statistic or fact to you and you have the right one, politely correct him or her. Do not let incorrect details go by unchallenged. If you know it's wrong but don't have the proper information at your disposal, offer to get it and call back.

The ECC encourages CAS members to enhance the profession's visibility in their communities by undertaking local media relations and civic activities. Appearing before external audiences and writing articles are an integral part of the CAS's public relations efforts and Communication Plan. Such public activities require the same care and attention as work being performed in your own company. Just as peer review is undertaken to help ensure the excellence of the advice and work product we provide to our employers, clients, and the public-at-large, that same process should be applied to speeches and articles.

It must be considered that reporters must gather and write about all kinds of information. To accomplish this goal in the very limited time they have, reporters must rely on sources—sources for everything. When it comes to breaking news, information that cannot be verified by at least two sources, independent of each other, generally will not be published or aired.

An exception to this "rule of two" occurs when an expert is quoted. The expert source needs no corroboration by another source. This is because the expert is offering a professional opinion, and it will be identified as such.

If you would like some guidance and peer-review before submitting an article for publication consideration by a national newspaper or journal, you may call the American Academy of Actuaries, the CAS Office, or the Chairperson of the Media Relations Committee.

If you have questions about these guidelines or need assistance with public relations activities, please contact Robert Wolf, Chairperson, Media Relations Committee (312-930-0648), Mike Boa, Manager, Communications and Research, CAS Office (703-276-3100), or Noel Card, Director of Communications, American Academy of Actuaries (202-223-8196).

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Interest Rate Risk: An Evaluation of Duration Matching as a Risk-Minimizing Strategy for Property/Casualty Insurers

CAS Valuation, Finance, and Investments Committee

INTEREST RATE RISK: AN EVALUATION OF DURATION MATCHING AS A RISK-MINIMIZING STRATEGY FOR PROPERTY/CASUALTY INSURERS

Casualty Actuarial Society

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INTEREST RATE RISK: AN EVALUATION OF DURATION MATCHING AS A RISK-MINIMIZING STRATEGY FOR PROPERTY/CASUALTY INSURERS

Executive Summary

In this analysis, the CAS Valuation, Finance and Investments Committee used Dynamic Financial Analysis to test the hypothesis that duration-matching of assets to a company's liability duration will yield an improved risk profile (e.g., reduced risk with essentially the same return) compared to longer or shorter investment strategies. Although the results varied by scenario tested, the overall conclusion was that duration matching does not appear to be the sole optimal strategy for most property casualty insurers. In many cases, a duration-matched strategy yields a result that belongs to a family of optimal strategies, from which the decisionmaker must choose based on desire for return and appetite for risk; i.e., the strategy lies on the efficient frontier, but is not inherently "better" than other strategies.

In several situations the matched strategy and longer strategies lay on the efficient frontier, with longer strategies yielding higher return at higher risk; but the shorter strategies were sub-optimal. This helps to explain the relatively long-duration investment posture assumed by the average property casualty insurer.

The Committee also noted that the results of the analysis were strongly influenced by the accounting convention used. Statutory results showed greatly reduced asset risk because of the amortized cost accounting method. As a result, in many cases longer investment strategies yielded higher return, and equal or lower risk, when viewed on a statutory basis.

Furthermore, neither statutory nor GAAP displayed the full measure of overall interest rate risk, because of the absence of discounting for the liabilities. Such effects tend to make the results highly specific to the accounting method under consideration. VFIC is considering undertaking a future research project in which "economic value" will be used as the accounting convention; in such a matching of asset and liability risk lies the final test of whether duration matching holds any benefit for property casualty insurers.

Introduction

The relationship between the portfolio of assets and the portfolio of liabilities of a property and casualty (P&C) insurance company is of interest to many audiences. People who influence this relationship include the insurer's management, board, actuaries and investment staff; as well as regulators and rating agencies. Other parties – such as policyholders, investors, investment bankers, and investment analysts - are also concerned with this relationship.

Insurers can choose one of many strategies for managing this relationship. Cash flow matching, or (more generally) duration matching of the asset and liability portfolios has been advocated by many as the preferred strategy. Duration measures the average time to maturity, using discounted cash flows as the weights, associated with a particular investment instrument or portfolio, usually a bond or group of bonds. It can be shown that duration so defined also approximately equals the units of change in the market value of a portfolio of assets and liabilities that would arise from a unit parallel shift in the market yield curve.[1] The unit of duration under this second definition is value per interest rate, but as interest rate is value per time, duration is also expressed as units of time.

Duration matching uses asset allocation to hedge the portfolio against parallel shifts in the yield curve; that is, interest rate (or reinvestment rate) risk. Specifically, if liabilities are discounted by current interest rates, then, if all else is equal, the value of the liabilities will decrease as interest rates increase. The bond market values also will decrease. Thus, surplus is potentially insulated. Managing the duration of the assets in this way immunizes the portfolio of assets and liabilities against this form of interest rate risk.

However, the adoption of a duration-matched strategy often leads to a reduction of the insurer's investment returns and thus of its potential increases in policyholder's surplus. These reductions reflect a combination of the relatively short duration of the liabilities of most P&C insurers, and the typically upward slope of the yield curve for available investments. Thus using duration matching to hedge against parallel shifts is not a costless strategy. Both income and long-term surplus growth can be adversely impacted.

Analysts who have looked at the actual behavior of P&C insurers find that most companies invest in portfolios of assets with durations longer than those of their liabilities. This suggests that most companies have either implicitly or explicitly concluded that the value of the duration hedge is not worth its cost. A contributing factor to this conclusion may be the fact that the effectiveness of the hedge varies depending on the accounting convention being used.

For instance, under GAAP accounting for P&C companies, liability values do not respond to changes in the yield curve. This prevents P&C companies from immunizing GAAP results through asset/liability management. The impact on GAAP equity of a change in interest rates would be the same as the impact on assets.

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[1] Panjer, Harry H. (ed.), <u>Financial Economics</u>, The Actuarial Foundation, Schaumburg, Illinois, 1998, p. 100. The two values differ by a factor, close to unity, which is described in the text.

Statutory accounting measures liabilities similarly, and in addition uses amortized value for bonds. Statutory surplus is thus automatically immunized against changes in interest rates. The only exception is if bonds have to be sold to pay losses. Then their value goes immediately from amortized to market. In this case the statutory surplus behaves somewhat like the GAAP surplus, in that some assets have been impacted by changes in interest rates while liabilities have not.

Life insurers have often used a matching strategy as a benchmark, but the typical life insurer's portfolio of liabilities is much longer in duration than that of a P&C insurer, which greatly reduces the cost of the duration hedge. These insurers also discount many of their liabilities, bringing them closer to an economic presentation than is the case for P&C insurers. Finally, because most life contracts provide a fixed amount of benefits, the only risk (other than mortality) is interest rate. As such, in the case of life contracts, duration matched assets are more likely to provide an optimal risk profile. Nevertheless, some regulators and other analysts schooled in the asset and liability matching strategies developed for life insurers have assumed that this strategy should apply to P&C insurers as well. Accordingly, regulators considering insurance investment laws and risk-based capital charges have at times developed proposals that would have the effect of penalizing P&C insurers that do not apply a matching strategy to their investment decisions.

In this paper, the Valuation, Finance and Investments Committee (VFIC) of the Casualty Actuarial Society (CAS) presents an analysis designed to shed light on the question of the appropriateness of an asset and liability matching strategy for P&C insurers under GAAP and statutory accounting conventions. VFIC is building upon work done by predecessor committees that have addressed this question. These previous studies were limited by the fact that their committees did not have access to the increasingly usable tools designed to support dynamic financial analyses. In particular, the former Financial Analysis Committee of the CAS did pioneering work on this subject that was presented in summary form but never published in detail, due to the difficulty of obtaining a validation of results using an independently developed model.[2]

Goals

For this analysis, we perform a simulation study of risk vs. return for a variety of investment strategies, and two company types (a monoline workers' compensation writer, and a monoline homeowners writer).

[2] Financial Analysis Committee, "A Study of the Effects of Asset/Liability Mismatch on P&C Insurers," Valuation Issues: Special Issues Seminar, Casualty Actuarial Society, 1989.

Under GAAP, we would expect the higher risk strategy (longer asset duration, higher interest rate risk) to generally have the higher return, with no strategy being clearly optimal. This risk-return relationship is what is seen in an "efficient frontier," wherein the risk bears a positive relationship with return and a higher return cannot be obtained without taking on more risk. In such cases the selection of strategy depends upon the entity's appetite for risk, because taking more risk will, on average, yield a greater return. For purposes of simplicity, we will assume that this traditional relationship, when observed in this analysis, describes the efficient frontier for the specific scenario under consideration.

The anticipated outcome for statutory results is that, unless assets have to be sold to pay losses, there is no risk from interest rate fluctuations. Since neither assets nor liabilities respond to interest rate shifts, the highest yielding investment portfolio would generally seem the optimal strategy, with equal or even *lower* risk than the lower-yielding portfolios. One reason that the risk could be lower, is that the higher investment income provides a cushion against loss experience.

In no case would there be any reason to expect that duration management would immunize the combined portfolio, or even lower its risk unless liabilities are discounted – this is expected to be the subject of a future VFIC paper related to the effect of interest rate risk on economic surplus.

The foregoing discussion primarily relates to traditional risk measures such as standard deviation. These describe risk purely in terms of variability. This is the case in this study for each of the two-sided risk measures. Other tests measure risk relative to some threshold; i.e., an adverse outcome larger than some value X. All of the down-side tests in this study are of this type. A key element of this type of test is that it is not independent of income, or return. A scenario with higher expected return has lower risk, all else being equal, because the higher return reduces the chance of the adverse outcome. In many respects this leads to a more meaningful and useful measure of risk.

On tests of this latter sort, such as Value at Risk (VaR) (an X% VaR calculation measures the Xth percentile of the projected annual change in capital), the risk measure itself is often inversely correlated with return. This powerfully contributes to turning many scenarios "upside down," causing their risk to decrease as their reward increases, in contradiction of traditional assumptions. We anticipate that this effect would often continue to exhibit itself if the scenarios were run on an "economic surplus" basis, as well.

This paper first discusses the design of the analysis, including:

- The standards used to compare the effect of various investment strategies on the modeled insurers, both with regard to risk and reward.

- The selection of representative types of insurers to model in order to develop conclusions of some generality with respect to P&C insurers.

- The description of the various investment strategies.

Next, this paper presents the findings for each selected modeled company. It then summarizes VFIC's conclusions, outlines some suggested directions for further research and then presents the limitations of this analysis.

The appendices to this paper present information regarding the following:

- Appendix A: A summary of the key assumptions used to parameterize each of the sample companies.

- Appendix B: The interest rate scenarios used in this analysis. (Note, we have assumed yields similar to historical averages, with randomness, but with no expected change in the mean interest rate over time.)

- Appendix C: Some sample graphs of the outcomes that resulted from this study.

We hope that this paper will generate discussion and further contributions on this very interesting subject.

Brief Description of Approach

To test the appropriateness of an asset and liability matching strategy for P&C insurers, we used random simulation to compare the selected measures of reward against various measures of risk for a range of investment strategies for eight sample insurance companies over a five year period.

Reward and Risk Measures

We chose two different bases for measuring reward to reflect the different accounting structures underlying statutory and GAAP financial statements. Although we could have selected others, we concluded that the results would be unlikely to change fundamentally. The return measures we chose are:

- Average annual statutory net income over the projected five-year period, as a percent of statutory policyholder surplus.

* Average annual GAAP net income over the projected five-year period (adjusted to remove the deferred taxes on the unrealized capital gains/losses), expressed as a percent of adjusted GAAP equity.

The important distinctions between the statutory and adjusted GAAP financial statements are displayed in Table 1.

Table 1

Value	Statutory Accounting	Adjusted GAAP/Total Return Accounting
Loss reserves	Undiscounted	Undiscounted
Unearned premium reserves	Pro rata *	Pro rata, with premium deficiency reserve
Bonds	Amortized cost	Market value
Deferred acquisition costs	Ignored	Recognized
Unrealized capital G/L	Ignored	Recognized
Deferred taxes	Ignored*	Recognized

* This analysis was done on the basis of pre-Codification Statutory Accounting Principles.

The risk measures were selected to consider both down-side risk and two-sided risk. Some companies or stakeholders may target consistency of results over time, whereas others will be more focused on threats to their ability to continue operations. Companies such as stock insurers, that are focused on consistency of results, might be interested in two-sided risk. We therefore selected as a measure of two-sided risk the standard deviation of income (on both a statutory and adjusted GAAP basis), as a percentage of surplus or equity.

We consider two-sided risk measures elegant, but less likely to be relevant to a company's actual performance and long-term success. Therefore, we focused more on income-sensitive measures of down-side risk. The following risk measures were selected to evaluate this risk, on a GAAP and statutory basis separately.

* Probability of a drop in surplus of more than 25% in any one year.

* Probability of ruin (where surplus or equity drops below zero, causing insolvency).

* The 5-year 5% surplus (or equity) VaR.

* The 5th projected year 5% surplus (or equity) VaR.

The VaR measures reflect the expected loss at the 5% level, as a percent of surplus. For example, a 5% surplus VaR of 15% implies that, 5% of the time, the company results will entail a financial loss of 15% or more of the company's surplus. The 5-year measure calculates this factor for all five projected calendar years; the 5^{th}-year measure calculates it only for the final year of the projection period.

We also tested a newer risk measure called "Tail Value at Risk", or Tail VaR. We found that the results would not lead us toward significantly different conclusions than did the measures shown above. Furthermore, Tail VaR shares with standard deviation the weakness of being heavily affected by extreme outliers. For these reasons we did not include the Tail VaR measure among the results displayed in this report.

Sample Companies

The sample companies were selected to represent a relatively wide range of P&C insurers. The companies can be generally characterized (in approximate order of worsening results) as follows (specific assumptions are shown on Appendix A):

(1) Growing premiums at 5% per annum with a "typical" loss ratio.

(2) Declining premiums at 5% per annum with a "typical" loss ratio.

(3) Declining premiums at 5% per annum with a worse-than-"typical" loss ratio.

(4) Growing premiums at 5% per annum with a worse-than-"typical" loss ratio.

Each of these four sets of characteristics were applied to a hypothetical monoline workers' compensation writer, and a monoline homeowners writer, separately. This generated eight company scenarios.

Our intent is to capture a range of company conditions typical of the insurance industry. For example, the workers' compensation writer is intended to be representative of companies which, either due to their size or mix of business, have relatively stable cash flows and long-duration liabilities from year to year. We expect that this type of company will not have many cash calls and therefore will not be subject to significant interest rate risk when being viewed from a statutory accounting perspective.

In contrast, the homeowners writer is intended to be representative of companies which have erratic cash flows and short-duration liabilities. The average cash flows for these companies are expected to be positive, but for some years, such as those in which large catastrophes occur, or in scenarios where loss ratios are unusually high, cash flows can be negative and assets might be liquidated. This type of company will, at times, feel the effects of changes in the market value of bonds even on a statutory basis.

The various premium and loss ratio characteristics further stress test the results of these companies to evaluate our conclusions under a wider range of scenarios.

Investment Strategies

To focus the analysis on the issue of the duration of the invested assets and to keep the analyses relatively simple, we assumed that the insurers invested only in US government bonds and cash. For each of the eight companies, we tested strategies with the approximate durations shown in Table 2.

Table 2

Investment Strategy	Line of Business	Duration of Invested Assets and Cash	Duration of Claims Liabilities
Short	Workers' Compensation	1.0	3.8
	Homeowners	1.0	2.2
Matched	Workers' Compensation	3.8	3.8
	Homeowners	2.2	2.2
Long	Workers' Compensation	7.5	3.8
	Homeowners	7.5	2.2

In the above strategies, 1% of assets is held in cash and the rest in government bonds, so that the combined duration of bonds and cash is equal to the amounts shown above.

The insurer is assumed to hold bonds to maturity, unless available cash is insufficient to meet obligations, in which case bonds are sold in proportion to the mix owned on the balance sheet date. At that time, the mix of assets between cash and bonds is re-balanced. If bonds are purchased, they are purchased to maintain the target average duration of liabilities (which is assumed to be approximately constant throughout the projection period).

The durations are calculated relative to liabilities and assets carried as of each financial evaluation date. There is no consideration in this analysis of the duration of future cash flows (relating, for example, to losses to be incurred and premiums to be written in the future), as we considered this to somewhat depart from the classical concept of asset-liability duration matching as it is commonly understood.[3]

Economic Projections

We used randomly varying macroeconomic variables (such as short and long-term interest rates) and random variations in payment patterns to simulate a wide variety of future outcomes. Parameters for the variables we used for interest rate are shown on Appendix B. We projected these future outcomes five years from the statement date of the company being modeled.

We assumed no upward or downward movement on average for future interest rates (although there was substantial variation in the actual interest rates from year to year and iteration to iteration). We used the same assumption for the other modeled economic variables. The model used is mean-reverting, meaning that any deviation from the average in a modeled year results in an increased probability that the following year's observation will be closer to the mean rather than farther from it.

To check the sensitivity of the model to these assumptions, we tested models with increasing and decreasing average interest rates over time. We found that the results for such environments, which generally pertained for relatively short intervals, yielded quite different optimal investment strategies for that interval than our base model. However, these results did not have a major effect upon the companies' long-term investment strategies for either statutory or GAAP accounting. This is because the changing interest rate environment will tend to level off at some point in time.

Note further that, because the level interest rate scenarios we used include a wide variety of random economic deviates, a spread of alternative interest rate environments is already reflected in this analysis. Specifics regarding the interest rate model we did use are found in Appendix B.

Findings of Each Modeled Company

Tables 3 through 10 show the analytical results for each of the eight hypothetical insurers under the three investment strategies.

[3] Panjer, p. 100 ff. We used the Macaulay duration in our analysis, which in general terms is the weighted average time to maturity, with present-valued cash flows used as weights. See the text for a precise definition, and a discussion of some other measures of duration.

Workers' Compensation Insurer – Growing premiums at 5% per annum with a "typical" loss ratio (80%)

Table 3

Strategy	**Short**	**Matched**	**Long**
Reward Measures			
Avg. Stat. Net Income	6.3%	8.8%	9.6%
Avg. Adj. GAAP Net Income	7.2%	10.7%	12.2%
Down-Side Risk Measures			
P{ΔStat. Surplus<-25%}	0.0%	0.0%	0.0%
P{Stat. Surplus<0}	0.0%	0.0%	0.0%
5-Year 5% VaR (Stat.)	2.0%	-6.4%	-7.7%
5^{th} Year 5% VaR (Stat.)	5.1%	-5.4%	-6.9%
P{ΔGAAP Equity<-25%}	0.0%	0.0%	1.2%
P{GAAP Equity<0}	0.0%	0.0%	0.0%
5-Year 5% VaR (GAAP)	1.1%	4.0%	15.7%
5^{th} Year 5% VaR (GAAP)	4.3%	6.0%	16.9%
Two-Sided Risk Measures			
Standard Deviation of Net Income (Stat.)	4.7%	1.5%	1.1%
Standard Deviation of Net Income (Adjusted GAAP)	4.9%	9.9%	18.7%

Notes: 1. Net income is net of taxes.
2. Stat. is statutory.
3. Δ is "percentage change in."
4. VaR is value at risk. A negative value represents a gain.
5. All values as % of starting surplus/equity for the modeled iteration/year.
6. GAAP has been adjusted to reflect the deferred taxes on unrealized capital gains/losses.

For this company, both measures of net income are maximized when the long strategy is selected. This finding reflects the impact of the usually positively sloping yield curve. That is, the insurer benefits from the additional yield gained by investing in long maturities. Because the company, as modeled, rarely experiences negative cash flows, its income measures appear to be only modestly affected by losses from the sale of bonds.

Under statutory accounting, all of the measures of risk are minimized when the long strategy is selected. The longer duration investment strategies benefit from the amortized accounting convention and the lack of bond liquidations. As a result of these characteristics, there is little risk to surplus arising from the effect of changing interest rates on assets. Statutory risk measures are optimized under the long strategy because, under statutory accounting, the investors in long bonds lock in an interest rate and thus shield themselves from interest rate fluctuations for a longer period of time. An example of the risk-return curve resulting from a two-sided risk measure is found on Appendix C, Sheet 1. See Sheet 2 for a sample down-side measure.

However, under GAAP accounting, all measures of risk are maximized when the long strategy is selected, and therefore the increased reward is coupled with increased risk. Sheets 3 and 4 of Appendix C provide examples of these risk-return analyses.

In no case, however, does duration matching appear objectively to be a superior approach. It is either inferior to longer investments (in the case of statutory accounting), or is arguably an equally viable pick to the longer term investments due to the classic risk-return tradeoff (in the case of GAAP), where greater risk yields greater return.

Workers' Compensation Insurer – Declining premiums at 5% per annum with a "typical" loss ratio (80%)

Table 4

Strategy	**Short**	**Matched**	**Long**
Reward Measures			
Avg. Stat. Net Income	8.9%	10.9%	11.7%
Avg. Adj. GAAP Net Income	8.8%	11.8%	13.3%
Down-Side Risk Measures			
P{ΔStat. Surplus<-25%}	0.0%	0.0%	0.0%
P{Stat. Surplus<0}	0.0%	0.0%	0.0%
5-Year 5% VaR (Stat.)	-2.0%	-8.9%	-9.9%
5^{th} Year 5% VaR (Stat.)	0.0%	-8.1%	-9.1%
P{ΔGAAP Equity<-25%}	0.0%	0.0%	0.4%
P{GAAP Equity<0}	0.0%	0.0%	0.0%
5-Year 5% VaR (GAAP)	-1.4%	1.9%	12.6%
5^{th} Year 5% VaR (GAAP)	0.7%	2.6%	11.7%
Two-Sided Risk Measures			
Standard Deviation of Net Income (Stat.)	4.0%	1.2%	1.0%
Standard Deviation of Net Income (Adjusted GAAP)	4.3%	9.1%	17.2%

Notes: 1. Net income is net of taxes.
2. Stat. is statutory.
3. Δ is "percentage change in."
4. VaR is value at risk. A negative value represents a gain.
5. All values as % of starting surplus/equity for the modeled iteration/year.
6. GAAP has been adjusted to reflect the deferred taxes on unrealized capital gains/losses.

The results in Table 4 are similar to the results seen in Table 3, that is, under statutory accounting, risk decreases with duration and in contrast, risk increases with duration under GAAP accounting. For example, because of the relatively low loss ratio and the long tail of the business, the declining premium does not frequently cause liquidation of bonds to be necessary under this scenario (even though, with payment pattern fluctuations, such a cash call could occasionally occur).

Workers' Compensation Insurer – Declining premiums at 5% per annum with a worse-than-"typical" loss ratio (110%)

Table 5

Strategy	**Short**	**Matched**	**Long**
Reward Measures			
Avg. Stat. Net Income	-78.1%	-22.3%	-17.4%
Avg. Adj. GAAP Net Income	-21.9%	-11.4%	-8.2%
Down-Side Risk Measures			
P{ΔStat. Surplus<-25%}	57.3%	22.4%	7.8%
P{Stat. Surplus<0}	37.2%	0.1%	0.0%
5-Year 5% VaR (Stat.)	128.8%	38.9%	27.8%
5th Year 5% VaR (Stat.)	648.8%	58.6%	37.4%
P{ΔGAAP Equity<-25%}	22.3%	13.1%	23.9%
P{GAAP Equity<0}	5.1%	0.0%	1.0%
5-Year 5% VaR (GAAP)	50.4%	30.8%	46.6%
5th Year 5% VaR (GAAP)	101.1%	39.4%	60.6%
Two-Sided Risk Measures			
Standard Deviation of Net Income (Stat.)	1038.4%	9.0%	6.1%
Standard Deviation of Net Income (Adjusted GAAP)	19.8%	13.6%	29.4%

Notes: 1. Net income is net of taxes.
2. Stat. is statutory.
3. Δ is "percentage change in."
4. VaR is value at risk. A negative value represents a gain.
5. All values as % of starting surplus/equity for the modeled iteration/year.
6. GAAP has been adjusted to reflect the deferred taxes on unrealized capital gains/losses.

This is a particularly adverse scenario, in which the company is subject to negative underwriting results which prevent the new premium from adequately shielding the company from its looming cash flow problems. Thus, its risk profile differs somewhat from the previous examples studied.

Under the statutory environment, the relationship between investment strategies in Table 5 is similar to the relationships evident in Tables 3 and 4. Specifically, the risk to this insurer decreases, and the net loss is reduced, the longer the asset duration is relative to the duration of the claim liabilities.

For the GAAP risk measures in this scenario, the poor overall results drive the risk upward for the short (low investment income) approach relative to the matched (higher investment income) strategy – that is, investing short yields higher risk and lower reward in this case. As a result, the matched strategy is superior to the shorter options in this case, because it yields higher return with lower risk. The choice between the matched and longer strategies, however, is a tradeoff conditioned upon the insurer's appetite for risk.

Workers' Compensation Insurer – Growing premiums at 5% per annum with a worse-than-"typical" loss ratio (110%)

Table 6

Strategy	**Short**	**Matched**	**Long**
Reward Measures			
Avg. Stat. Net Income	-261.6%	-506.7%	-180.7%
Avg. Adj. GAAP Net Income	-123.2%	-41.2%	-36.9%
Down-Side Risk Measures			
P{ΔStat. Surplus<-25%}	99.7%	77.3%	78.5%
P{Stat. Surplus<0}	99.0%	99.1%	97.3%
5-Year 5% VaR (Stat.)	582.2%	439.0%	595.8%
5th Year 5% VaR (Stat.)	6452.0%	5716.7%	2616.1%
P{ΔGAAP Equity<-25%}	79.7%	54.9%	49.9%
P{GAAP Equity<0}	71.3%	35.0%	36.2%
5-Year 5% VaR (GAAP)	246.2%	119.3%	127.0%
5th Year 5% VaR (GAAP)	1281.9%	263.5%	327.3%
Two-Sided Risk Measures			
Standard Deviation of Net Income (Stat.)	2520.6%	12663.3%	1306.2%
Standard Deviation of Net Income (Adjusted GAAP)	1639.2%	83.5%	273.2%

Notes: 1. Net income is net of taxes.
2. Stat. is statutory.
3. Δ is "percentage change in."
4. VaR is value at risk. A negative value represents a gain.
5. All values as % of starting surplus/equity for the modeled iteration/year.
6. GAAP has been adjusted to reflect the deferred taxes on unrealized capital gains/losses.

This is the most adverse scenario, because the company is writing progressively greater amounts of unprofitable business.

Although this is a highly adverse scenario under both the statutory and the GAAP environments, the long strategy still minimizes the loss (i.e., maximizes the income measure).

Because of the high frequency of insolvency, the risk measures are substantially distorted by missing values (observations which are removed from the analysis because the starting surplus is negative), and by outlier values caused by very small starting surplus (and correspondingly very large risk percentages). As a result, two of the statutory risk measures are minimized by a duration-matched strategy, while two of them are maximized. It is therefore difficult to draw any strong conclusions from statutory results in this scenario.

The GAAP risk pattern continues to be broadly similar to that observed under the previous scenario.

Homeowners Insurer – Growing premiums at 5% per annum with a "typical" loss ratio (72.5%)

Table 7

Strategy	**Short**	**Matched**	**Long**
Reward Measures			
Avg. Stat. Net Income	4.8%	5.2%	8.3%
Avg. Adj. GAAP Net Income	4.6%	5.1%	6.4%
Down-Side Risk Measures			
P{ΔStat. Surplus<-25%}	5.8%	5.8%	5.8%
P{Stat. Surplus<0}	2.7%	2.8%	1.8%
5-Year 5% VaR (Stat.)	46.8%	46.2%	42.3%
5th Year 5% VaR (Stat.)	38.9%	40.5%	39.0%
P{ΔGAAP Equity<-25%}	5.7%	5.7%	4.8%
P{GAAP Equity<0}	0.5%	0.6%	0.4%
5-Year 5% VaR (GAAP)	28.1%	27.4%	24.1%
5th Year 5% VaR (GAAP)	42.6%	44.2%	35.0%
Two-Sided Risk Measures			
Standard Deviation of Net Income (Stat.)	17.9%	17.6%	145.8%
Standard Deviation of Net Income (Adjusted GAAP)	13.1%	12.9%	14.3%

Notes: 1. Net income is net of taxes.
2. Stat. is statutory.
3. Δ is "percentage change in."
4. VaR is value at risk. A negative value represents a gain.
5. All values as % of starting surplus/equity for the modeled iteration/year.
6. GAAP has been adjusted to reflect the deferred taxes on unrealized capital gains/losses.

For homeowners the matched strategy uses a duration of 2.2 years, in contrast to the 3.8 years for workers' compensation, and so it is closer to the short strategy (the duration of which is one year).

As with the workers' compensation companies, the homeowners companies (Tables 7 though 10) also produce measures of net income that are maximized when the long strategy is selected.

Under statutory accounting, generally speaking, return increases and risk decreases as the duration of assets increases. However, at least three of the five statutory risk measures indicate that short has no greater risk than the matched strategy. This pattern might be attributable to the fact that the shorter asset duration creates smaller asset value fluctuations in a line where liquidation of assets is unavoidable due to catastrophes; this effect works to offset the slightly lower investment income of the short strategy. For the long strategy, on the other hand, the additional stability arising from the locked-in investments causes the risk to be reduced relative to a matched strategy.

For the statutory standard deviation risk measure, note that the seeming riskiness of the long strategy arises from a single extreme outlier in the model. Without that observation (which is one of a thousand in this run), the risk for the long strategy is lower than for the matched. This is an indicator of how a measure like standard deviation, which is more heavily affected by observations that are further from the mean, can be greatly influenced by a very few or even a single outlier. The VaR measure is more robust in this regard. However, Tail VaR suffers from the same limitations as standard deviation, though to a lesser degree.

Under GAAP accounting, the results are generally similar to statutory results, for the down-side tests. That is, the longer duration strategy has the highest return and lowest risk relative to the other strategies, because the extra income of the longer strategy provides risk reduction. This is a significant result, because it represents a situation in which the income sensitivity of the selected down-side risk measures has inverted the risk-return relationship when compared to a pure variability measure. The two-sided GAAP standard deviation of net income, is highest for the long strategy, as asset value fluctuation is greatest there. However, we believe the down-side results are more meaningful, because they put the variability into the context of the average income being generated by the scenario.

Note that the result is different than for the corresponding workers' compensation scenario in this regard. The reason is that in homeowners, with its catastrophe exposure, the higher yield of the investments from the long scenario is able to shield the company from adverse underwriting results, which are relatively large in comparison to the extra investment risk that the longer scenario entails.

Homeowners Insurer – Declining premiums at 5% per annum with a "typical" loss ratio (72.5%)

Table 8

Strategy	**Short**	**Matched**	**Long**
Reward Measures			
Avg. Stat. Net Income	6.1%	6.2%	7.6%
Avg. Adj. GAAP Net Income	5.0%	5.5%	6.7%
Down-Side Risk Measures			
P{ΔStat. Surplus<-25%}	5.4%	5.3%	4.7%
P{Stat. Surplus<0}	0.6%	0.6%	0.7%
5-Year 5% VaR (Stat.)	27.5%	26.5%	22.4%
5th Year 5% VaR (Stat.)	18.5%	19.4%	19.2%
P{ΔGAAP Equity<-25%}	2.6%	2.5%	2.3%
P{GAAP Equity<0}	0.0%	0.0%	0.0%
5-Year 5% VaR (GAAP)	17.2%	16.2%	14.7%
5th Year 5% VaR (GAAP)	20.3%	21.6%	14.2%
Two-Sided Risk Measures			
Standard Deviation of Net Income (Stat.)	35.7%	19.9%	59.4%
Standard Deviation of Net Income (Adjusted GAAP)	9.2%	9.2%	11.1%

Notes: 1. Net income is net of taxes.
2. Stat. is statutory.
3. Δ is "percentage change in."
4. VaR is value at risk. A negative value represents a gain.
5. All values as % of starting surplus/equity for the modeled iteration/year.
6. GAAP has been adjusted to reflect the deferred taxes on unrealized capital gains/losses.

Most of the statutory and GAAP risk measures indicate the common theme of higher return and lower risk as asset durations increase. This is explained as previously discussed. In two cases, the matched scenario actually has the *greatest* risk.

However, the GAAP two-sided risk measure indicates the opposite result; that is, return and risk have a positive relationship. The two-sided risk measures both the up-side and the down-side fluctuations in annual income, with no mitigation of risk when return increases. Therefore, it continues to display the classical risk-reward relationship.

The statutory two-sided risk measure shows the duration-matched scenario to have the minimum risk. However, as in Table 7, this is due to a single outlier driving the value for the long scenario upward. Without this outlier, the long scenario would be the least risky of the three investment strategies.

Homeowners Insurer – Declining premiums at 5% per annum with a worse-than-"typical" loss ratio (87.5%)

Table 9

Strategy	**Short**	**Matched**	**Long**
Reward Measures			
Avg. Stat. Net Income	0.9%	1.4%	2.5%
Avg. Adj. GAAP Net Income	1.1%	1.6%	2.9%
Down-Side Risk Measures			
P{ΔStat. Surplus<-25%}	5.8%	5.8%	5.8%
P{Stat. Surplus<0}	1.9%	1.9%	1.6%
5-Year 5% VaR (Stat.)	40.2%	38.8%	34.4%
5^{th} Year 5% VaR (Stat.)	30.0%	31.6%	30.9%
P{ΔGAAP Equity<-25%}	4.6%	4.3%	4.1%
P{GAAP Equity<0}	0.0%	0.0%	0.0%
5-Year 5% VaR (GAAP)	23.4%	22.6%	20.5%
5^{th} Year 5% VaR (GAAP)	28.5%	30.2%	22.8%
Two-Sided Risk Measures			
Standard Deviation of Net Income (Stat.)	15.9%	15.7%	15.1%
Standard Deviation of Net Income (Adjusted GAAP)	10.1%	10.1%	11.8%

Notes: 1. Net income is net of taxes.
2. Stat. is statutory.
3. Δ is "percentage change in."
4. VaR is value at risk. A negative value represents a gain.
5. All values as % of starting surplus/equity for the modeled iteration/year.
6. GAAP has been adjusted to reflect the deferred taxes on unrealized capital gains/losses.

Under both statutory and GAAP accounting, we see from the above table that, in general, the risk and the return have an inverse relationship as the duration increases.

However, we again see that the GAAP two-sided risk measure indicates a positive relationship between risk and return as the asset duration increases. We would expect this relationship since the adjusted GAAP risk measure considers the additional volatility due to the inclusion of unrealized capital gains and/or losses, which are substantially higher for the long-duration strategy.

Homeowners Insurer – Growing premiums at 5% per annum with a worse-than-"typical" loss ratio (87.5%)

Table 10

Strategy	**Short**	**Matched**	**Long**
Reward Measures			
Avg. Stat. Net Income	-1.4%	-0.8%	0.5%
Avg. Adj. GAAP Net Income	-0.9%	-0.6%	0.9%
Down-Side Risk Measures			
P{ΔStat. Surplus<-25%}	5.8%	5.8%	5.8%
P{Stat. Surplus<0}	2.8%	2.8%	2.8%
5-Year 5% VaR (Stat.)	65.0%	64.5%	59.7%
5th Year 5% VaR (Stat.)	58.1%	60.4%	58.0%
P{ΔGAAP Equity<-25%}	6.2%	6.1%	6.2%
P{GAAP Equity<0}	1.8%	1.7%	1.6%
5-Year 5% VaR (GAAP)	37.0%	36.4%	33.2%
5th Year 5% VaR (GAAP)	62.2%	64.5%	52.6%
Two-Sided Risk Measures			
Standard Deviation of Net Income (Stat.)	23.2%	22.5%	21.1%
Standard Deviation of Net Income (Adjusted GAAP)	26.1%	16.6%	17.7%

Notes: 1. Net income is net of taxes.
2. Stat. is statutory.
3. Δ is "percentage change in."
4. VaR is value at risk. A negative value represents a gain.
5. All values as % of starting surplus/equity for the modeled iteration/year.
6. GAAP has been adjusted to reflect the deferred taxes on unrealized capital gains/losses.

The results on this table appear generally consistent with Table 9. The two-sided GAAP risk measure, however, shows a result more like Table 7.

Conclusions

Under statutory accounting, the majority of the eight modeled companies have an inverse risk-return relationship (including the two-sided risk measures) as the asset duration increases. Variability in income arises primarily from changes in bond yields; since these occur more slowly with a long-investment strategy in an amortized cost environment, longer strategies yield lower risk.

When the risk and return have an inverse relationship, purely statutory decision-making will result in a preference for longer investments, since additional income can be had for less risk. But when underwriting losses force liquidation of assets, a direct relationship can take hold between risk and reward, with the trade-off that that entails among equally viable alternative strategies. We observed this last pattern infrequently in this analysis.

Under GAAP accounting, for workers' compensation writers, the majority of our results indicate a positive correlation between risk and return (including the two-sided risk measures). This is generally what we would expect under GAAP since the higher the duration, the higher the fluctuations in the market value of assets.

However, under GAAP accounting for the homeowners writer, most of our modeled results indicate a generally inverse relationship between return and down-side risk. This we consider to be due to the higher underwriting risk (due to catastrophe exposure). The increased income due to longer investment offsets more of this risk, but the increase in investment variability is modest in comparison. Since our selected down-side risk measures are income sensitive, this results in a reduction in risk under longer strategies.

Regardless of the accounting convention, line of business, or the company's underwriting experience, the surplus (or equity) at the end of the projected period (i.e., year 5) had, on average, a positive relationship with the length of the asset duration. That is, long duration strategies performed better than matched duration strategies, on average. Using traditional risk-return analysis, then, a matched portfolio is not inherently superior to a longer one. Although it may be less risky, it is also less profitable.

Note that some of the strength of our conclusions arises from our reliance on income-sensitive down-side risk measures such as VaR, which have the characteristic of being favorably influenced by increased return. This widens the range of scenarios under which increased return will yield lower risk, because for these measures an increase in return can actually *cause* a decrease in risk.

These findings are also consistent with the conclusions reached in the following CAS work on asset/liability matching:

- The 1989 Financial Analysis Committee article[4] identified the risk-return tradeoff (matching is less risky, and also less rewarding, but not necessarily better or worse than longer investments). Special cases of

[4] Financial Analysis Committee, "A Study of the Effects of Asset/Liability Mismatch on P&C Insurers."

expected value outcomes were examined under a limited set of scenarios such as rising and declining interest rates.

- In 1992, preliminary research[5] indicated that income-adjusted down-side risk measures might decrease as insurers invest in assets with longer duration. This result obtained under a variety of surplus assumptions, including market (i.e., economic) surplus, and is similar to those we observed under some of our scenarios.

Further Research

Our modeled results do not consider economic surplus, which would include discounting of the claim liabilities. Further research is needed to assess the impact on duration analysis of using economic assumptions across both assets and liabilities. Such an analysis is the subject of an anticipated followup to this paper.

Limitations on Analysis

A major source of uncertainty surrounding these findings is the appropriateness of the models used to derive the findings. Two components of model risk are (1) errors in the model and (2) appropriateness of the model as an approximation of the situation being modeled. We have addressed the first of these components by performing these analyses using two independent dynamic financial analysis models: the proprietary model developed and used by Milliman USA; and a proprietary model developed by Guy Carpenter. The results presented herein were derived from the Milliman model. The findings of the Carpenter model, which were used for verification and validation of the results shown, were generally consistent with those presented in this paper.

Many simplifications and approximations remain. Therefore, care must be taken to consider the scope of this analysis when seeking to draw conclusions from its findings.

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[5] Grannan, Patrick J., Transcript of presentation at Asset/Liability Matching Session, at the 1992 Casualty Actuarial Society's Valuation Issues Seminar (unpublished). Copies available from Casualty Actuarial Society upon request.

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INTEREST RATE RISK: DURATION ANALYSIS

Line: Workers' Compensation

	Historical Base Period	Projected Period				
	0	1	2	3	4	5
Selected Loss and ALAE Ratio by Accident Year:						
Typical:	80.0%	80.0%	80.0%	80.0%	80.0%	80.0%
Adverse:	80.0%	110.0%	110.0%	110.0%	110.0%	110.0%
Underwriting Expense Ratio by Acc. Year:	30.0%	30.0%	30.0%	30.0%	30.0%	30.0%
Unearned Premium:	4,762					
Tax Payment Pattern:	Used the most recent IRS WC pattern by accident year.					
Written Premium: (+5%)	9,524	10,000	10,500	11,025	11,576	12,155
Written Premium: (-5%)	9,524	9,048	8,595	8,165	7,757	7,369
Balance Sheet						
Cash	337					
Bonds	33,375					
Income Taxes Payable:	50					
Surplus	7,714					
Income Statement (Statutory values = GAAP values)						
Earned Premium	9,297					
Investment Income	1,500					
Incurred Losses	7,438					
Underwriting Expenses	2,857					
Income Taxes Incurred	200					
Cash Flow Statement						
Premium Collected	9,524					
Interest Dividends Received	1,500					
Losses Paid	6,558					
Underwriting expenses paid	2,857					
Income taxes paid	200					
Tax Discount Rate by Accident Year		0.063	0.063	0.063	0.063	0.063

Note: Dollars are in thousands.

INTEREST RATE RISK: DURATION ANALYSIS

Line: Workers' Compensation

Historical Period	Selected Loss & ALAE Reserves at End of Historical Period	Age	Expected Incremental Payment Pattern (*)
-19	$0	0	24.0%
-18	31	1	28.0%
-17	65	2	13.0%
-16	102	3	7.0%
-15	143	4	4.0%
-14	188	5	3.0%
-13	236	6	2.0%
-12	290	7	2.0%
-11	391	8	2.0%
-10	502	9	2.0%
-9	623	10	2.0%
-8	755	11	2.0%
-7	899	12	2.0%
-6	1,055	13	1.0%
-5	1,224	14	1.0%
-4	1,469	15	1.0%
-3	1,799	16	1.0%
-2	2,361	17	1.0%
-1	3,400	18	1.0%
0	5,653	19	1.0%
Total	$21,186	Total	100.0%

Note
(*) Variability was added to the payment pattern at each incremental payment date based on lognormal draws. The variability of the draws was set such that the coefficient of variation of the claims liabilities duration as of the statement date is approximately 25%.

Appendix A
Exhibit 2
Sheet 1

INTEREST RATE RISK: DURATION ANALYSIS

Line: Homeowners

	Historical Base Period	Projected Period				
	0	1	2	3	4	5
Selected Loss and ALAE Ratio by Accident Year: (excluding CAT loss ratio)						
Typical:	62.5%	62.5%	62.5%	62.5%	62.5%	62.5%
Adverse:	62.5%	77.5%	77.5%	77.5%	77.5%	77.5%
Underwriting Expense Ratio by Acc Year:	30.0%	30.0%	30.0%	30.0%	30.0%	30.0%
CAT Expected Loss Ratio	10.0%	10.0%				
Unearned Premium:	4,762					
Tax Payment Pattern:	Used the most recent IRS Homeowners pattern by accident year.					
Written Premium: (+5%)	9,524	10,000	10,500	11,025	11,576	12,155
Written Premium: (-5%)	9,524	9,048	8,595	8,165	7,757	7,369
Balance Sheet						
Cash	337					
Bonds	33,375					
Income Taxes Payable:	50					
Surplus	21,741					
Income Statement (Statutory values = GAAP values)						
Earned Premium	9,297					
Investment Income	1,500					
Incurred Losses	6,740					
Underwriting Expenses	2,857					
Income Taxes Incurred	200					
Cash Flow Statement						
Premium Collected	9,524					
Interest Dividends Received	1,500					
Losses Paid	6,400					
Underwriting expenses paid	2,857					
Income taxes paid	200					
Tax Discount Rate by Accident Year		0.063	0.063	0.063	0.063	0.063

Note: Dollars are in thousands

Appendix A
Exhibit 2
Sheet 2

INTEREST RATE RISK: DURATION ANALYSIS

Line: Homeowners

Historical Period	Selected Loss & ALAE Reserves at end of Historical Period	Age	Expected Incremental Payment Pattern (*)
-19	$0	0	56.0%
-18	0	1	21.9%
-17	0	2	6.1%
-16	0	3	7.3%
-15	0	4	0.8%
-14	0	5	2.2%
-13	0	6	2.1%
-12	15	7	0.9%
-11	31	8	0.6%
-10	49	9	0.4%
-9	69	10	0.4%
-8	91	11	0.4%
-7	126	12	0.4%
-6	176	13	0.4%
-5	297	14	0.0%
-4	434	15	0.0%
-3	505	16	0.0%
-2	978	17	0.0%
-1	1,419	18	0.0%
0	2,969	19	0.0%
Total	$7,159		100.0%

Note

(*) Variability was added to the payment pattern at each incremental payment date based on lognormal draws. The variability of the draws was set such that the coefficient of variation of the claims liabilities duration as of the statement date is approximately 25%.

Appendix A
Exhibit 2
Sheet 3

INTEREST RATE RISK: DURATION ANALYSIS

Line: Homeowners

Annual Probability of a CAT event	CAT Severity Under +5% Premium Growth Scenario — Projected Year					CAT Severity under -5% Premium Decline Scenario — Projected Year				
	1	2	3	4	5	1	2	3	4	5
6.25%	$15,619.20	$16,400.16	$17,220.17	$18,081.18	$18,985.24	$14,857.30	$14,114.44	$13,408.71	$12,738.28	$12,101.36

Appendix B

INTEREST RATE RISK: DURATION ANALYSIS

Approximate Interest Rate Scenario
Used In Model

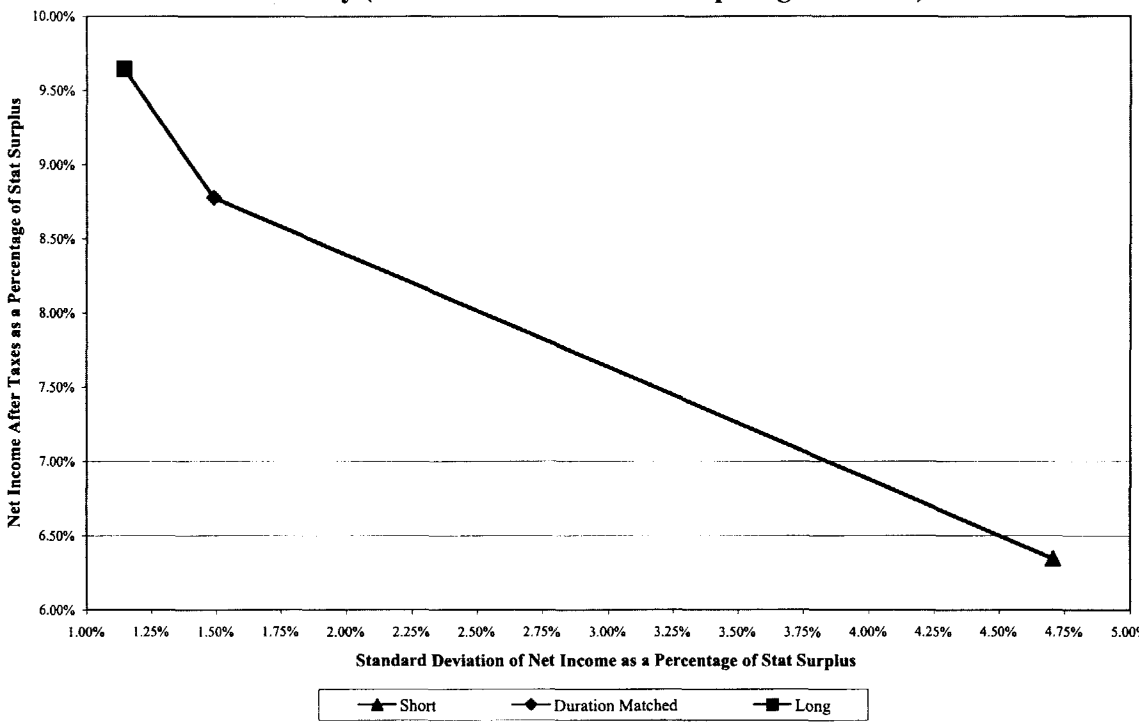
	Projected Year				
Year	1	2	3	4	5
Short Term Rate					
Mean	5.5%	5.5%	5.5%	5.5%	5.5%
Std. Dev.	1.2%	1.6%	1.8%	1.9%	2.0%
Min.	2.0%	1.0%	1.0%	1.1%	1.0%
Max.	9.5%	11.9%	11.7%	12.2%	12.1%
CV	21.1%	29.5%	32.3%	34.6%	36.0%
Long Term Rate					
Mean	7.0%	7.0%	7.0%	7.0%	7.0%
Std. Dev.	0.9%	1.2%	1.4%	1.4%	1.5%
Min.	4.2%	3.1%	2.8%	2.9%	2.8%
Max.	9.7%	11.2%	11.6%	12.0%	11.8%
CV	12.1%	17.3%	19.4%	20.6%	21.2%

Rates other than short and long term reflect
selected yield curve.

WORKERS COMPENSATION COMPANY

Normal Loss Ratio (80%), Increasing Premium (5% per year)

Statutory (excludes bond unrealized capital gains/losses)



Note: Lines between plotted points do not represent tested scenarios, but only approximate the shape of the actual curve.

WORKERS COMPENSATION COMPANY

Normal Loss Ratio (80%), Increasing Premium (5% per year)

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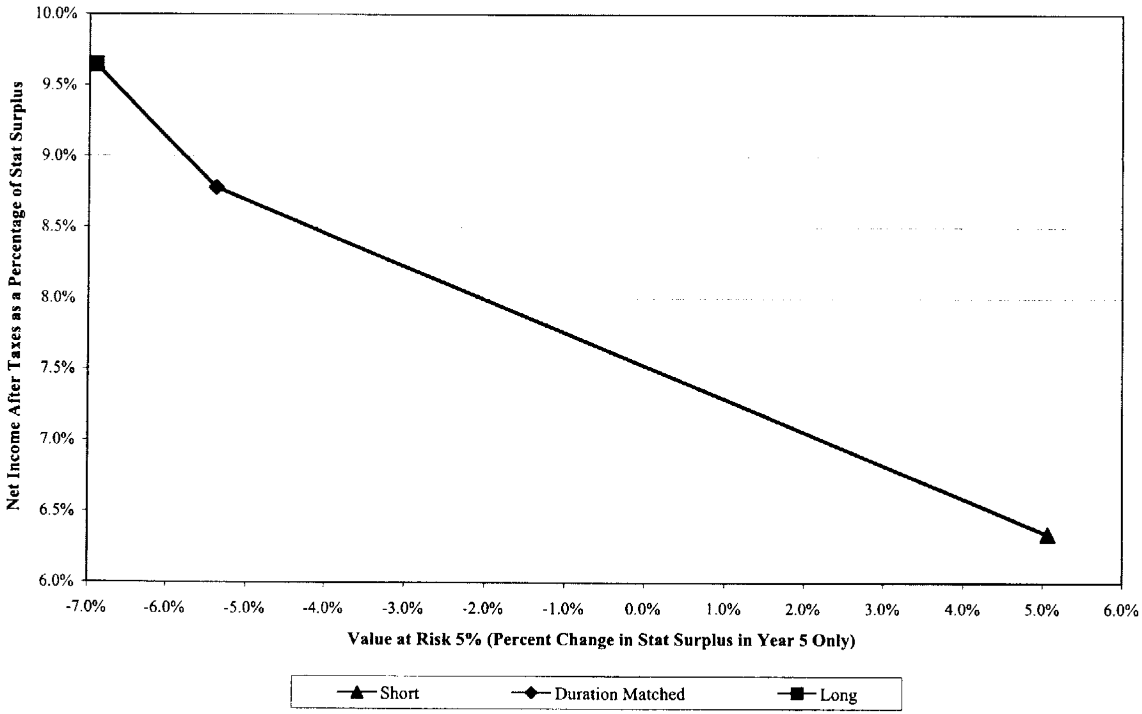
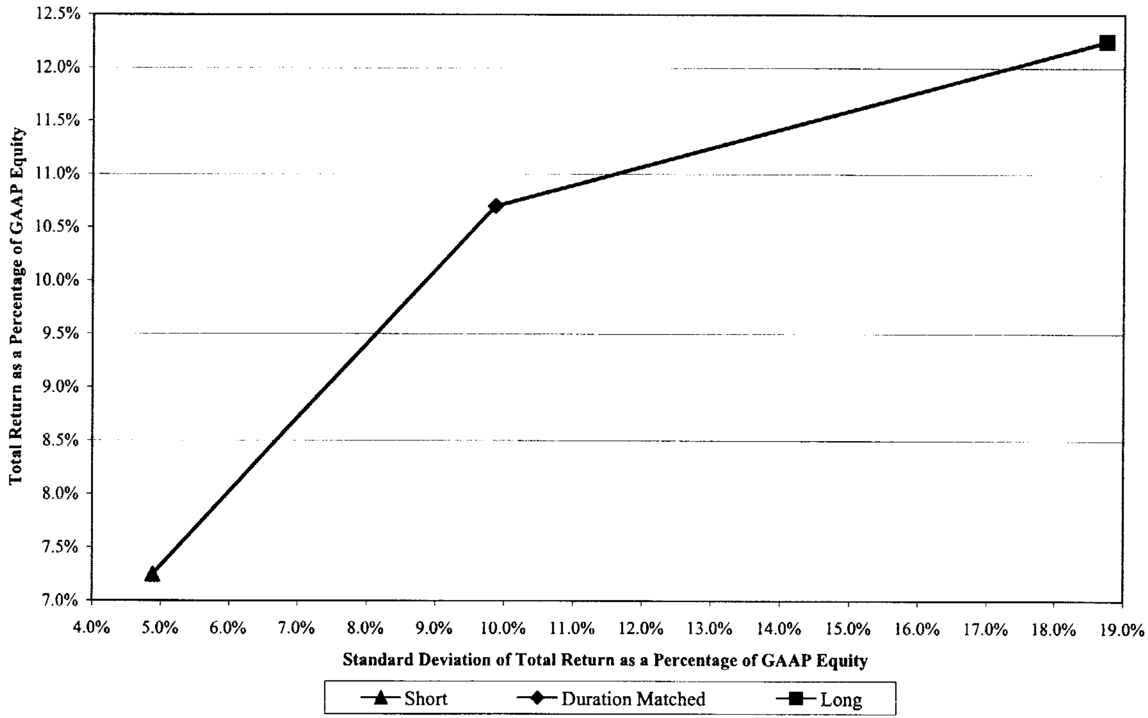


Note: Lines between plotted points do not represent tested scenarios, but only approximate the shape of the actual curve.

WORKERS COMPENSATION COMPANY

Normal Loss Ratio (80%), Increasing Premium (5% per year)

Adjusted GAAP (includes tax-adjusted bond unrealized capital gains/losses)



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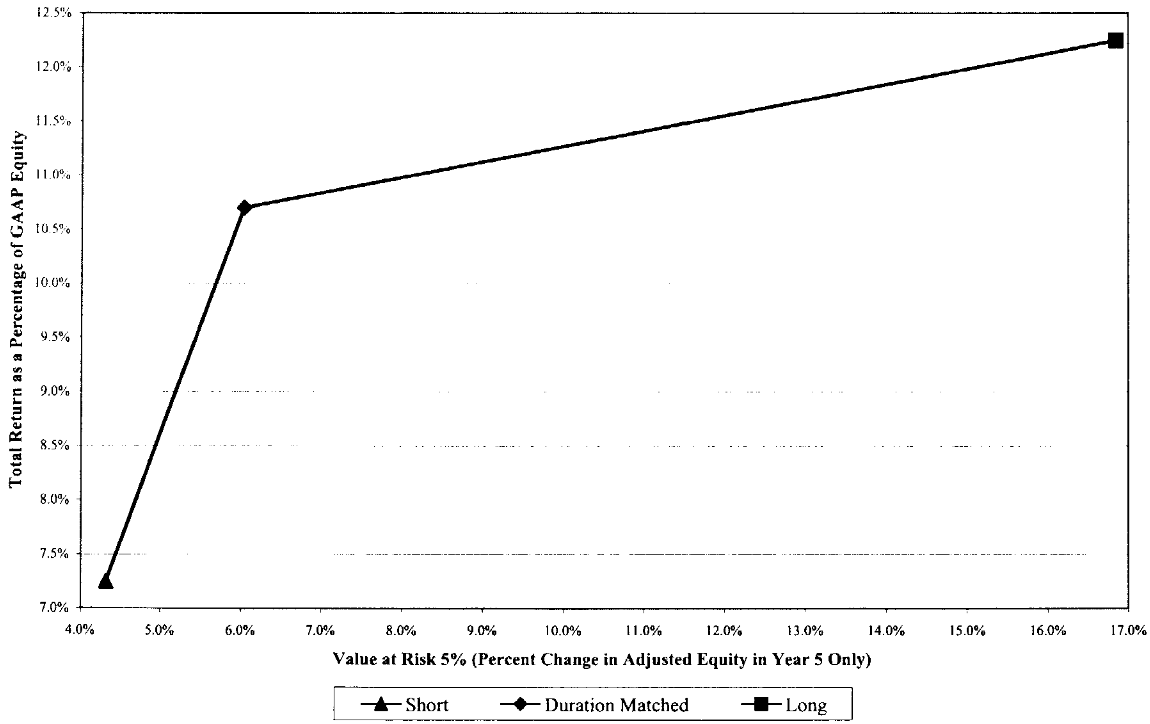
WORKERS COMPENSATION COMPANY

Normal Loss Ratio (80%), Increasing Premium (5% per year)

Adjusted GAAP (includes tax-adjusted bond unrealized capital gains/losses)



Note: Lines between plotted points do not represent tested scenarios, but only approximate the shape of the actual curve.

Fitting Beta Densities to Loss Data

Daniel R. Corro

Fitting Beta Densities to Loss Data

Dan Corro
National Council on Compensation Insurance, Inc.
March, 2002

Abstract: *This short note details how to match the mean and variance of any loss distribution on a finite interval to a Beta density, scaled to that interval.*

Most loss distributions that actuaries use (c.f. [1], Appendix) are naturally defined on $(0, \infty)$. In this note we consider instead loss distributions defined on a finite interval *(0,L)* of positive width *L*>0. We require throughout that all loss distributions considered have a positive mean and finite mean and variance. So let $\mu > 0$ denote the mean and σ^2 the variance of any such distribution. Also, let $\gamma = \dfrac{\sigma}{\mu}$ be the coefficient of variation and X be the associated random variable of such a loss distribution. Then, since $X < L$, we have the following inequality that will come in handy later:

$$\mu(1+\gamma^2) = \frac{\mu^2+\sigma^2}{\mu} = \frac{E(X^2)}{E(X)} = \frac{\sum x^2 p(x)}{\sum xp(x)} < \frac{\sum Lxp(x)}{\sum xp(x)} = L.$$

The Beta density on (0,1) is among the most useful of this class of loss densities. Recall that the Beta distribution is a two-parameter, α, β, distribution that is usually defined in terms of its probability density function [PDF]:

$$f(\alpha,\beta;x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\mathrm{B}(\alpha,\beta)} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} \quad x \in (0,1), \alpha > 0, \beta > 0$$

where B and Γ denote the usual Beta and Gamma functions (c.f. [1], p. 48). For this Beta density, the mean and variance are:

$$\mu = \frac{\alpha}{\alpha+\beta} \quad \text{and} \quad \sigma^2 = \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}.$$

Indeed, the reduction formula $\Gamma(x) = (x-1)\Gamma(x-1)$ leads directly to:

$$E(X^n) = \frac{\mathrm{B}(\alpha+n,\beta)}{\mathrm{B}(\alpha,\beta)} = \prod_{i=0}^{n-1} \frac{\alpha+i}{\alpha+\beta+i}.$$

Thus, the moments of the Beta density are easy to compute from the parameters.

It is also easy to verify that two ordered pairs of parameters α, β and α', β' have the same mean if and only if the points (α, β) and (α', β') lie on the same line through the origin, i.e., if and only if $\alpha' = \rho\alpha$ and $\beta' = \rho\beta$ for some fixed proportionality constant $\rho > 0$. It is then apparent that the pair α, β is uniquely determined by the mean and variance.

With these preliminaries out of the way, suppose we have bounded loss data with loss amounts in $(0,L)$ that we want to model or otherwise approximate using a continuous PDF on $(0,L)$. Assume we have determined the following statistics for the data:

$$\text{mean} = m > 0 \qquad \text{and} \qquad \text{variance} = s^2 > 0.$$

Let $c = \dfrac{s}{m}$ and consider what the data looks like scaled into the interval (0,1). Evidently the scaled data has mean and standard deviation:

$$\hat{m} = \frac{m}{L} > 0 \qquad \hat{s} = c\hat{m} = \frac{s}{L}.$$

In particular, the above inequality implies [$\Rightarrow$] that:

$$\hat{m}(1+c^2) < 1 \Rightarrow 1 - \hat{m} - c^2\hat{m} > 0.$$

So we may define:

$$\alpha = \frac{1-\hat{m}-c^2\hat{m}}{c^2} > 0 \quad \text{and} \quad \beta = \left(\frac{1-\hat{m}}{\hat{m}}\right)\alpha > 0$$

as the parameters of a Beta density. Now observe that this Beta density has mean

$$\frac{\alpha}{\alpha+\beta} = \frac{\alpha}{\alpha + \left(\frac{1-\hat{m}}{\hat{m}}\right)\alpha} = \frac{1}{1+\left(\frac{1-\hat{m}}{\hat{m}}\right)} = \frac{\hat{m}}{\hat{m}+1-\hat{m}} = \hat{m}.$$

Observe next that:

$$\frac{\alpha}{\alpha+\beta}=\hat{m}\Rightarrow\frac{\alpha+\beta}{\alpha}=\frac{1}{\hat{m}}\Rightarrow\alpha+\beta=\frac{\alpha}{\hat{m}}.$$

And then we find, as no surprise, that the variance is:

$$\frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}=\frac{\alpha^2\left(\frac{1-\hat{m}}{\hat{m}}\right)}{\left(\frac{\alpha}{\hat{m}}+1\right)\left(\frac{\alpha}{\hat{m}}\right)^2}=\hat{m}^2\left(\frac{1-\hat{m}}{\alpha+\hat{m}}\right)$$

$$=\hat{m}^2\left(\frac{1-\hat{m}}{\frac{1-\hat{m}-c^2\hat{m}}{c^2}+\hat{m}}\right)=\hat{m}^2\left(\frac{1-\hat{m}}{\frac{1-\hat{m}}{c^2}}\right)=(c\hat{m})^2=\hat{s}^2.$$

It follows that these parameters define a Beta density whose mean and variance equal those of the rescaled loss data.

In terms of the original scale, the approximating Beta continuous PDF is:

$$g(\alpha,\beta;z)=\frac{z^{\alpha-1}(L-z)^{\beta-1}}{\mathrm{B}(\alpha,\beta)L^{\alpha+\beta+1}}\quad z\in(0,L).$$

It is worth emphasizing that this construction is quite sensitive to the choice of L. In general, unless some other applicable loss limitation prevails, it is usually best to select L at or near the maximum observed loss.

By a continuous density on the finite interval $(0,L)$, we mean a density that can be specified via a PDF, $f(x)$, that is defined and continuous on $(0,L)$. We may summarize what we have shown in two simple results:

Proposition 1: *The following condition is both necessary and sufficient for a pair μ,σ^2 of positive real numbers to be the mean and variance of a continuous density on* $(0,1)$:

$$\mu\left(1+\left(\frac{\sigma}{\mu}\right)^2\right)<1.$$

Proof: Necessity follows from the inequality shown earlier and sufficiency from our discussion of the Beta density, which also establishes:

Proposition 2: *Within the class of all continuous densities on* (0,1) *with a given mean* = μ *and variance* = σ^2 *, there is exactly one Beta density that is uniquely determined by the parameters:*

$$\alpha = \frac{1-\mu-\gamma^2\mu}{\gamma^2} > 0$$

$$\beta = \left(\frac{1-\mu}{\mu}\right)\alpha > 0$$

where $\gamma = \dfrac{\sigma}{\mu}$.

The so-called "central moments" μ, σ^2 may not be the most convenient for this purpose. Let $\mu = \mu_1 = E(X)$ and $\mu_2 = E(X^2)$ be the first and second moments of a continuous density on (0,1), then:

$$X \le X^2 \Rightarrow \mu_1 = E(X) \le E(X^2) = \mu_2$$

$$E((X-E(X))^2) \ge 0 \Rightarrow E(X^2) - E(X)^2 \ge 0 \Rightarrow \mu_2 = E(X^2) \ge E(X)^2 = \mu_1^2 .$$

And, equivalent to the above, we obtain the corresponding Beta density parameters:

$$\alpha = \mu_1\left(\frac{\mu_1-\mu_2}{\mu_2-\mu_1^2}\right) > 0 \quad \text{and} \quad \beta = \frac{(\mu_1-\mu_2)(1-\mu_1)}{\mu_2-\mu_1^2} > 0.$$

References:

[1] *Hogg, Robert V., Klugman, Stuart A.,* Loss Distributions, *Wiley Series in Probability and Statistics, John Wiley & Sons, Inc., 1984.*

Does the NAIC Risk-Based Capital Suffice? And Are Property & Casualty Insurance Company Asset Allocations Rational?

Chris K. Madsen, ASA, CFA, MAAA

Does the NAIC Risk-Based Capital Suffice?

And

Are Property & Casualty Insurance Company Asset Allocations Rational?

May, 2002

This paper examines the asset allocation for a typical property & casualty insurer, and the effect of asset allocation changes on the NAIC Risk-Based Capital (RBC) requirements. The effect on performance measures such as Return on Equity (ROE), growth in capital and surplus, and the ratio of capital and surplus to RBC are studied in parallel to determine if RBC properly rewards good risk decision-making. This paper further examines the extent to which the RBC requirements favor asset or insurance risk and whether or not this is a desirable quality of RBC.

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Introduction

Traditionally, property & casualty insurers have invested very conservatively. Generally, these companies favor treasuries and investment grade corporate bonds. During the last decade a few exceptions have emerged, but for the most part property & casualty insurers have opted to accept little or no asset risk. Is this a desirable and a rational decision?

DFA Model

The analysis in this paper was performed using DFA Capital Management Inc.'s Dynamic Financial Analysis (DFA) software. This software is an enterprise-wide model built specifically for insurance companies. The model calculates transaction level detail on both sides of the balance sheet, and produces all the major accounting and tax schedules and forms at each node of the simulation. In addition, the model calculates the regulatory requirements (NAIC Risk-Based Capital (RBC) and Insurance Regulatory Information System (IRIS) ratios) at each node[1].

Simulating at this level of detail is necessary to address the questions that this paper poses, namely:

- Does RBC suffice?
- Are property & casualty insurance company asset allocations rational?

Does RBC Suffice?

The NAIC Risk-Based Capital (RBC) measure consists of six components, referred to as R0 through R5. R0 is based on off balance sheet investments and investments in insurance company affiliates. R1 is based on the company's fixed income portfolio and R2 is based on the company's equity portfolio. R3 is a charge based on credit risk, which can arise from either side of the balance sheet. R4 is a charge based on the

[1] A DFA simulation simulates thousands of paths (sometimes called iterations) across time. A node is any one point in time along any one path.

loss and the loss adjustment expense reserves. R5 is a charge based on premium written. Overall, RBC is determined as shown in Equation 1.

Equation 1

$$RBC = R0 + \sqrt{R1^2 + R2^2 + R3^2 + R4^2 + R5^2}$$

For all of these components, R0 through R5, percentage charges are tallied based upon certain criteria. For example, R2, the equity component, is simply 15% of the market value of common stock, if the company holds only common stock (as opposed to preferred stock, for example).

If the company's capital and surplus falls below the RBC amount, the company has to submit a plan of corrective action to the regulators. If the company's capital and surplus falls below half of the RBC amount, then the regulators will intervene. It should be clear that it is essential for an insurance company to monitor its RBC amount since it represents a minimum threshold to regulators. In fact, most insurers will maintain a healthy margin between their capital and surplus and their RBC amount.

The charges for some of the main asset components of a property & casualty insurer are shown in Table 1. Each percentage is applied to the market value of the category. For fixed income assets, classes 1 and 2 are considered investment grade. Classes 3 and higher are considered high yield.

Table 1

Security	Rating	RBC Category	Charge
Fixed Income	US Treasury	US Treasury	0.0%
Fixed Income	AAA, AA, A	Class 1	0.3%
Fixed Income	BBB	Class 2	1.0%
Fixed Income	BB	Class 3	2.0%
Fixed Income	B	Class 4	4.5%
Fixed Income	CCC, CC, C	Class 5	10.0%
Fixed Income	CI, D (Default)	Class 6	30.0%
Common Stock		Common Stock	15.0%
Real Estate		Real Estate	10.0%

Table 1 shows the NAIC risk-based capital charges for each asset class as a percentage of market value.

In reviewing these charges, we can see that investing in US Treasury bonds carries no capital charge. For fixed income securities, the charge relates to default risk as opposed to price fluctuation. Since there is little or no default risk associated with US Treasury debt, there is no charge for holding it. This is the case despite the fact that prices can fluctuate quite significantly for longer maturity bonds. Investment grade corporate bonds carry with them a small capital charge of 0.3% to 1.0%, whereas equities carry a charge that is fifteen times as large (before the independence assumption adjustment) as the charge for the lowest rated investment grade corporate bonds. It would appear that the deck is stacked against equities here.

For a little background, take a look at the following over-simplified and generalized example: a property & casualty insurance company is completely invested in US treasuries and has no reinsurance or other credit risk. Thus, the company has only R4 (reserves) and R5 (premium written) charges. Also say that the insurer has a ratio of R4 to assets of 10% and a ratio of R5 to assets of 4%. Then we can write the following:

Equation 2

$$\frac{RBC}{Assets} = \frac{R0}{Assets} + \sqrt{\left(\frac{R1}{Assets}\right)^2 + \left(\frac{R2}{Assets}\right)^2 + \left(\frac{R3}{Assets}\right)^2 + \left(\frac{R4}{Assets}\right)^2 + \left(\frac{R5}{Assets}\right)^2}$$

$$\Updownarrow$$

$$\frac{RBC}{Assets} = 0.00 + \sqrt{(0.00 \cdot 1.00)^2 + (0.15 \cdot 0.00)^2 + (0.00)^2 + (0.10)^2 + (0.04)^2}$$

$$\Updownarrow$$

$$\frac{Assets}{RBC} = 9.28$$

Let us say that we shift some funds that were in treasuries (with no RBC charge) to equities (with 15% RBC charge). Specifically, say we shift 10% of assets from US Treasury bonds to common stocks. The amount of total assets will not change, but RBC will.

Equation 3

$$\frac{RBC_{post}}{Assets} = \frac{R0}{Assets} + \sqrt{\left(\frac{R1_{post}}{Assets}\right)^2 + \left(\frac{R2_{post}}{Assets}\right)^2 + \left(\frac{R3}{Assets}\right)^2 + \left(\frac{R4}{Assets}\right)^2 + \left(\frac{R5}{Assets}\right)^2}$$

$$\Updownarrow$$

$$\frac{RBC_{post}}{Assets} = 0.00 + \sqrt{\left(\frac{0.00 \cdot FixedIncome}{Assets}\right)^2 + \left(\frac{0.15 \cdot Equities}{Assets}\right)^2 + (0.00)^2 + (0.10)^2 + (0.04)^2}$$

$$\Updownarrow$$

$$\frac{RBC_{post}}{Assets} = \sqrt{(0.00 \cdot 0.90)^2 + (0.15 \cdot 0.10)^2 + (0.10)^2 + (0.04)^2}$$

$$\Updownarrow$$

$$\frac{Assets}{RBC_{post}} = 9.20$$

We see that the asset-to-RBC ratio has changed from 9.28 to 9.20. In terms of the capital and surplus to RBC ratio, if the company previously had capital and surplus equal to one third of assets, then the company has just reduced its ratio from 3.095 to 3.065 – a reduction of roughly 1% due to an increase in the allocation to equities. While this difference may seem trivial at first, it is not. In order to return the ratio to its prior level, equities (assuming no equities prior to the change) need to return approximately 10%[2].

But it is actually worse than that. To truly bring the ratio back to its prior level one year in the future, equities will need to outperform treasuries by 10%. If treasuries return 5% for the next year, equities will need to return 15% to be equivalent to the all-treasuries scenario. This is greater than the long-term equity performance, which depending on time horizon has been about 8-10% per annum. In short, it seems virtually impossible to justify an equity allocation in terms of RBC.

Another implication of RBC is that the lower the R4 and R5 to asset ratios are, the higher the hurdle rate for equities in the example above. While we

[2] Equities make up 10% of assets. A 10% return from equities is a 1% return on assets.

will not pursue this particular aspect any further here, it is suggested as an area for further research. It suggests that the higher the capital-to-asset ratio of an insurer is, the higher the assumed equity return needs to be in order to justify it in terms of RBC. On the other end of the scale, this also suggests there is an incentive for low capital-to-asset ratio companies to increase their equity exposure. This is the exact opposite of the behavior that regulators should encourage.

It is interesting to note that A.M. Best in its Best's Capital Adequacy Ratio (BCAR) calculation charges relatively more for underwriting risk than the NAIC model does[3]. This is another way of leveling the field between the asset and underwriting risk.

There are other measures of risk that better balance performance such as the distribution of return on equity (ROE), growth in capital and surplus, down-side risk of capital and surplus and so on. We will examine some of these, along with RBC, in a stochastic environment in the paragraphs that follow.

Reviewing the Simulations

Three simulations were run. All simulations were based on a typical property & casualty insurer. The insurer had ten product lines and twelve treaties covering losses from those products. Measured in terms of expected net losses, 70% of the product lines covered automobile losses with a slightly greater exposure to liability as opposed to physical damage. The greater part of the remaining 30% (of expected net losses) was from commercial property. At the start of the simulation, the insurer had about $200 million in total assets and about $70 million in capital and surplus. Twenty quarters were simulated.

Every simulation started with the same asset allocation. Only the investment strategy was changed for each of the three simulations. Transaction costs were incorporated and the shift in asset allocation was gradual over time just as it would be in reality. The different strategies are summarized below:

[3] Mosher, M., "Understanding BCAR", A.M. Best, August, 2001.

- Base Case: This investment strategy matches duration and convexity of the fixed income portfolio to the company's liabilities. This means that the company is investing mostly in short-term (five years or less, average duration is less than two years) fixed income securities. Treasury bonds must make up at least 25% of the total bond portfolio and common stock must make up roughly 20% of the entire portfolio. Corporate bonds cannot exceed 80% of the bond portfolio. All bonds must be investment grade.

- Alternative 1: Common stocks must make up 20-30% of portfolio. Corporate bonds must make up 30-50% of the entire portfolio. All bonds must be investment grade. Treasury bonds cannot make up more than 30% of the entire portfolio. The duration of the company's fixed income securities must be maintained around five years.

- Alternative 2: Mid-term Treasury bonds must make up 70% of the entire portfolio. 10% of the portfolio must be in common stocks and 20% must be in corporate bonds.

In the table below, the starting portfolio and the average[4] portfolio after five years are summarized.

Table 2

	Asset Allocations		
	Base Case	**Alternative 1**	**Alternative 2**
At Beginning of Simulation	10% Equity 45% Corporate 32% Treasury 13% Municipal	10% Equity 45% Corporate 32% Treasury 13% Municipal	10% Equity 45% Corporate 32% Treasury 13% Municipal
After Five Years (20 Quarters)	20% Equity 20% Corporate 47% Treasury 13% Municipal	30% Equity 30% Corporate 30% Treasury 10% Municipal	10% Equity 20% Corporate 70% Treasury 0% Municipal

[4] Since the investment strategy is modeled, each path can have a different asset allocation depending on the company's circumstances on that particular path.

Table 2 shows the beginning asset allocation and the expected asset allocation after five years. Note, that the beginning portfolio is the same for all three scenarios. The only difference is the strategy applied over the five years. All transaction costs and tax consequences were considered in applying these strategies.

Transaction costs and all tax consequences were considered in adjusting the portfolio over time. In Tables 3 and 4 we look at the expected value of the capital and surplus to RBC ratio to get a sense of the expected impact of the change in investment strategy. Later, we will look at the entire distribution of the ratio.

Table 3

Expected Ratio of Capital and Surplus to RBC			
	Base Case	**Alternative 1**	**Alternative 2**
After 4 Quarters	3.31	3.15	3.57
After 20 Quarters	3.75	3.59	3.67

Table 3 shows the expected value of the ratio (averaged over all simulated paths) of capital and surplus to RBC at different points in time for the three different asset strategies. After 4 quarters, Alternative 2 has the highest ratio due to the immediate drop in RBC charges. After 20 quarters, the Base Case has the highest expected ratio.

Not surprisingly, the company gets penalized in the first year for holding equities. The ratio of capital and surplus to RBC drops to 3.15 from 3.31. But over the next four years, the company is able to grow its surplus relative to RBC and the relative difference between the Base Case and Alternative 1 decreases.

Alternative 2 looks good in the first year relative to RBC, but over the next four years, the portfolio barely grows relative to RBC and the company pays the price for being too conservatively invested.

Table 4

Expected Percentage Change in Capital and Surplus to RBC Ratio			
	Base Case	**Alternative 1**	**Alternative 2**
20 Quarters Divided by 4 Quarters	13%	14%	3%

Table 4 shows the expected change in the ratio of capital and surplus to RBC. Alternative 1 shows the greatest improvement in this ratio over 5 years, though the Base Case shows roughly the same improvement.

In short, it appears that the Base Case is the best strategy of the three when viewed in the context of the capital and surplus to RBC ratio. Alternative 1 does appear to offer a decent alternative, but even over five years, it does not quite match the Base Case, while short-term there is definitely a price to pay.

But is capital and surplus to RBC really a measure that property & casualty insurers should care about? Of course, but only because it is imposed by regulators. In and of itself it is not that meaningful and may even encourage sub-optimal decision making by property & casualty decision-makers.

Since the ratio of capital and surplus to RBC is reviewed once a year, it is implied that the time frame inherent in RBC is one year. Below, we review the actual level of capital and surplus after one year.

Figure 1: One Year Horizon, Cumulative Distribution Function of Statutory Capital and Surplus

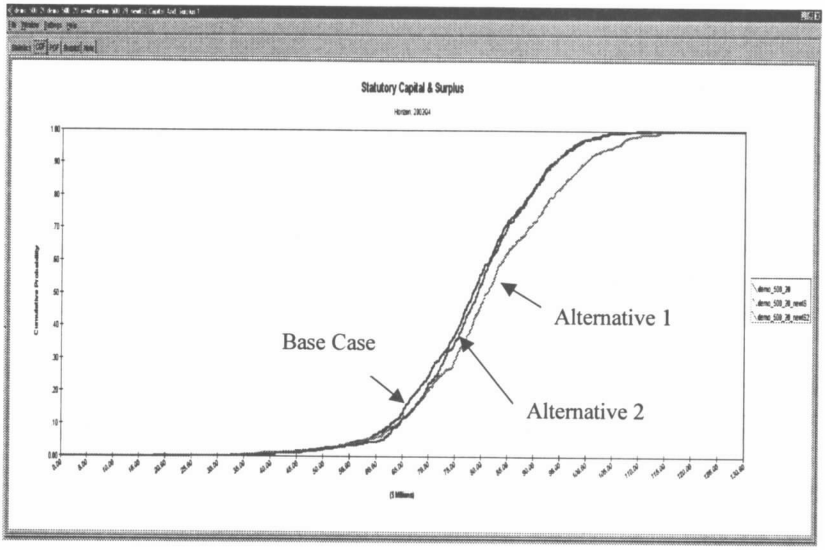

Figure 1 shows the cumulative distribution function of capital and surplus after one year for each of the three strategies: "demo_500_20" is the Base Case (blue), "demo_500_20_newIS" is Alternative 1 (green), and "demo_500_20_newIS2" is Alternative 2 (red). Alternative 1 is almost completely to the right of the Base Case and Alternative 2 suggesting that in terms of surplus growth and even down-side risk, Alternative 1 is the superior strategy.

The start of the simulation is 1/1/2002, so the above chart (Figure 1) is at the end of the first year. It should be easy to see that Alternative 1 (labeled demo_500_20_newIS) almost completely dominates the Base Case. *At virtually all levels of probability, Alternative 1 produces a higher capital and surplus position after one year. Clearly, this should be a desirable outcome. Yet, as we saw earlier, RBC penalizes the move from US Treasuries to investment grade corporate bonds and equities enough such that the ratio of capital and surplus to RBC drops.*

Below (Figure 2) is an enlarged image of the down-side tail. Again, it is clear that Alternative 1 (green) almost completely dominates the Base Case (blue). Alternative 2 (red), being very conservative, has less down-side to capital and surplus after one year as illustrated by its tail region (see Figure 2).

Figure 2: One Year Horizon, Down-side Tail of Cumulative Distribution Function of Statutory Capital and Surplus

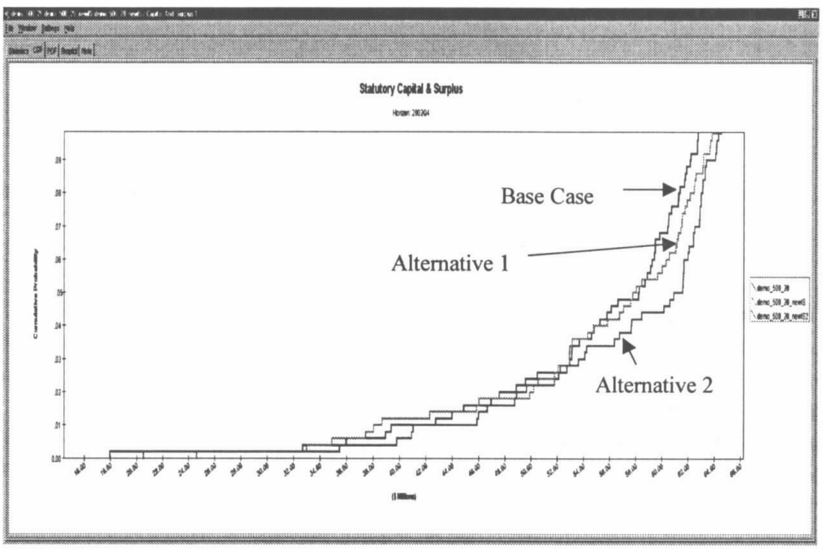

Figure 2 is an enlarged image of the tail of Figure 1. Here we see clearly that in terms of down-side risk, Alternative 2 (red) is slightly superior. Mostly, however, the three strategies have similar tails.

What does the picture look like after five years? Here (Figure 3), Alternative 1 is the clear-cut best choice. The worst scenario is the one which is mostly invested in treasuries (Alternative 2).

Figure 3: Five Year Horizon, Cumulative Distribution Function of Statutory Capital and Surplus

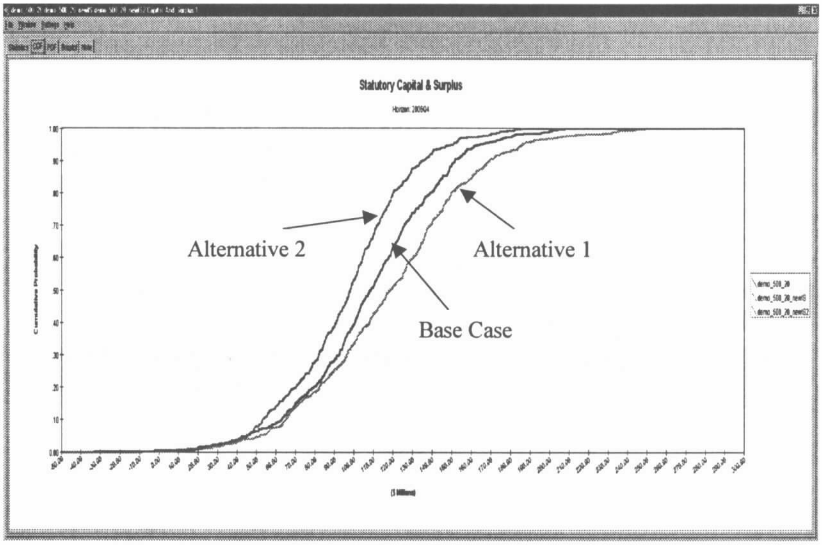

Figure 3 is similar to Figure 1 except that the time horizon is five years rather than one. At this time frame, Alternative 1 completely dominates the other strategies for all levels of probability, suggesting that Alternative 1 is the preferable strategy.

In terms of absolute dollars, capital and surplus is expected to be $10 million higher under Alternative 1 relative to the Base Case after five years (see Table 5). Even Alternative 1's worst case, as represented by the lowest observation, is more desirable than the Base Case. Yet, RBC penalizes this strategy due to its greater concentration of corporate bonds and equities. Certainly, equities and corporate bonds are more risky than treasuries, but it seems RBC charges unfairly for this risk.

Table 5

Capital and Surplus at 5 Year Horizon

	Base Case	Alternative 1	Alternative 2
Average	108,154,635	118,268,283	96,333,953
St. Dev.	36,276,521	42,473,574	32,129,526
Minimum	-33,111,610	-9,906,571	-8,962,279
Maximum	212,220,244	250,642,713	188,112,618
1st Percentile	19,689,593	22,372,340	22,057,790
99th Percentile	199,905,756	234,605,286	174,620,491

Table 5 shows the various levels of capital and surplus associated with Figure 3. As can be seen, even the worst case (minimum) outcome is $23 million better under Alternative 1 when compared to the Base Case. It is worth noting that the volatility is higher under Alternative 1, but it is up-side volatility, which is attractive.

If we look at return on equity (ROE) to obtain some insight into the return for shareholders, we see a picture that is similar to what we just saw. Over a five year time horizon, Alternative 1 dominates (see Table 7). Even over a one year time frame (Table 6), Alternative 1 looks the most attractive, though the first percentile (-13.9%) is slightly less than the Base Case (-10.1%).

Table 6

Economic ROE at 1 Year Horizon

	Base Case	Alternative 1	Alternative 2
Average	9.5%	12.9%	10.4%
St. Dev.	7.8%	10.5%	6.6%
Minimum	-26.5%	-26.0%	-23.0%
Maximum	33.6%	42.5%	26.6%
1st Percentile	-10.1%	-13.9%	-6.4%
99th Percentile	26.1%	36.2%	23.4%

Table 6 shows economic return on equity after one year. The result here is consistent with Table 5 in that Alternative 1 looks the most attractive.

Table 7

Economic ROE at 5 Year Horizon

	Base Case	Alternative 1	Alternative 2
Average	9.2%	10.9%	7.1%
St. Dev.	6.2%	6.6%	5.3%
Minimum	-32.5%	-17.9%	-21.9%
Maximum	23.2%	27.0%	19.2%
1st Percentile	-10.9%	-6.9%	-7.0%
99th Percentile	20.6%	24.0%	17.1%

Table 7 shows economic return on equity after five years. The result here is also consistent with Table 5 in that Alternative 1 looks the most attractive.

It seems that focusing on the capital and surplus to RBC ratio can lead to sub-optimal company performance. Below, we take a closer look at RBC under the Base Case and Alternative 1, the two strategies that appear most attractive.

Table 8

RBC at the End of First Simulated Quarter[5]

	Base Case	Alternative 1
R0: RBC not subject to co-variance	0	0
R1: Fixed Income RBC	985,063	579,558
R2: Equity RBC	8,633,360	12,925,410
R3: Credit & Reinsurance RBC	946,806	937,272
R4: Loss and LAE Reserve RBC	14,672,659	14,672,659
R5: Premium Written RBC	6,401,369	6,401,369
RBC	18,250,214	20,618,702

Table 8 breaks down RBC for the Base Case and Alternative 1. As can be seen, increasing the equity exposure from 20% to 30% is very costly in terms of RBC. The fixed income charge actually drops in the first quarter (longer duration does not impact RBC).

A few observations from this table can be highlighted:

[5] Each entry in this table is the expected value across all paths of the simulations. This means that the RBC listed at the bottom cannot be calculated directly from the table entries as the distribution of RBC and its components is skewed.

- Changing the asset allocation strategy from the Base Case to Alternative 1 causes RBC to increase by almost $2.4 million or over 13% over the first year.
- In the Base Case strategy at the end of the first year, the charge for asset risk (R1, R2 and some of R3) is roughly half that of the charge for insurance risk (R4, R5 and some of R3).

Even though the capital and surplus in Alternative 1 almost completely dominates (i.e., higher for each level of probability meaning the entire distribution has shifted right) the Base Case over both the one- and five-year horizons, the increase that this change brings is not enough to offset the increase in RBC. Thus, the ratio of capital and surplus to RBC deteriorates suggesting that monitoring this ratio beyond what is absolutely essential is counter-productive.

In Figures 4 and 5 below, we break down the Statutory Income Statement into underwriting income/(loss) on the vertical axis and investment income/(loss) on the horizontal axis. Figure 4 shows underwriting income plotted against investment income after one year - the implied timeframe of RBC. A best-fit regression line has also been added. As can be seen, there does not appear to be any relationship between the two, which suggests that the independence assumption that RBC makes among the various components of RBC is valid. Though this is a model result, the model is based on a parameterization of real life tying underwriting cash flows to the appropriate economic measures, and while not definitive proof, it does seem to support the RBC independence assumption.

Figure 4: One Year Horizon, Statutory Underwriting Income Versus Statutory Investment Income

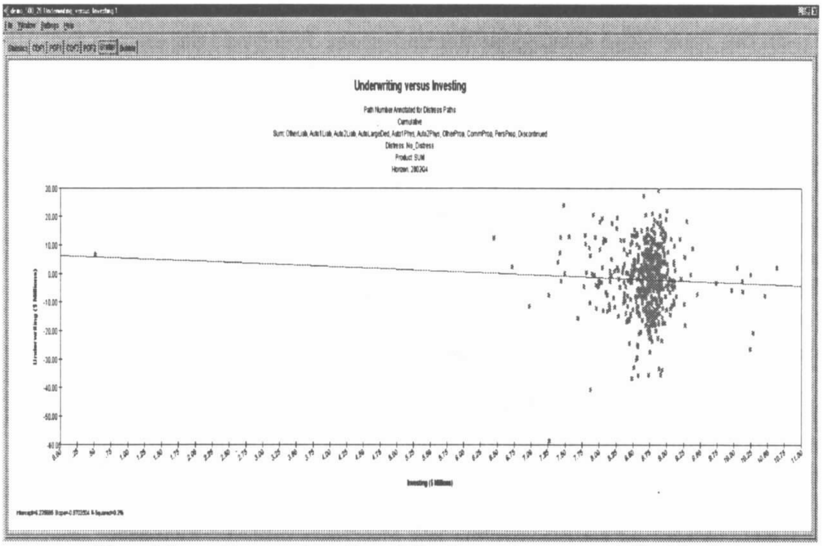

Figure 4 shows statutory underwriting gain/(loss) versus investment gain/(loss) for the Base Case strategy. By simple inspection, the volatility of the underwriting gain/(loss) is much greater than the volatility of the investment gain/(loss). Yet, in Table 8, we saw that RBC ranks investment risk as half of underwriting risk.

However, if we look at the ranges in Figure 4, we see that – with one exception – investment income is in the range of $6 million to $11 million while underwriting income is in the range of $-40 million to $30 million. Thus, the range of investment results is $5 million, whereas the range of insurance results is $70 million. That is a relative risk of 1 to 14. Yet, the capital charge for investments is half of that of insurance or 1 to 2.

Figure 5: Five Year Horizon, Statutory Underwriting Income Versus Statutory Investment Income

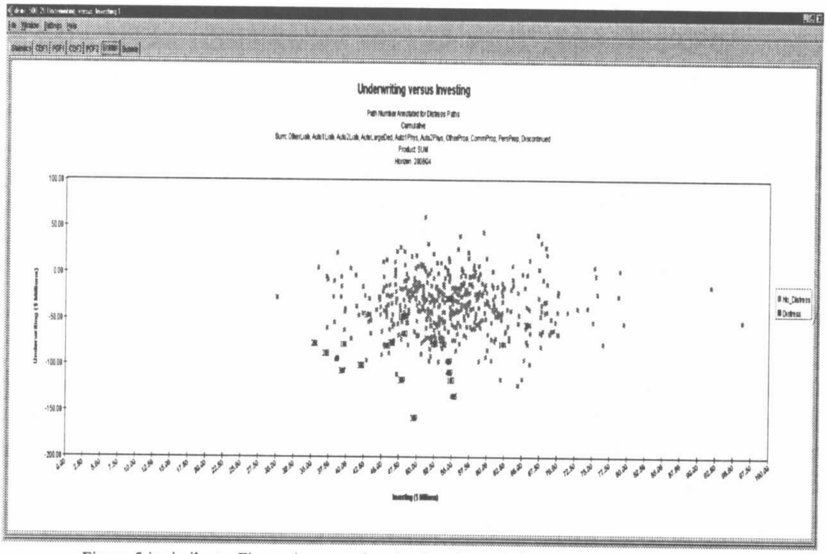

Figure 5 is similar to Figure 4 except that the time frame is five years. In this case, points where capital and surplus fell below RBC have been colored red and path numbers have been annotated. Notice that in general, paths where capital and surplus fall below RBC are located in the lower left quartile of the scatter plot (low investment income and low underwriting gain).

Over five years, the story is similar. The cumulative ranges are now roughly $35 million to $80 million for investments and $-150 million to $50 million for insurance. In this case, the ranges are $45 million versus $200 million, or 1 to 4.4. Relatively speaking, the asset risk has increased, but this is not the timeframe that RBC is concerned with. Even if it were, it is difficult to see how the RBC charge is reasonable. It would seem that an RBC asset charge of about 50% or less of what is currently indicated would be more reasonable.

The "Real World"

Most property & casualty insurers are very conservatively invested. The author has often wondered why that is. Why do companies that are so willing to take enormous risks on the liability side of the balance sheet shy away from asset risk? The answer often is that they choose to do one thing and do it to the best of their ability is too convenient in a market place with many players. The fact is that it appears that regulators, through the RBC standard, provide a disincentive for property & casualty insurers to take asset risk. Since asset risk and traditional insurance risk are mostly unrelated, regulators are in fact providing a disincentive for companies to diversify risk and maximize shareholder value.

Thus, it appears based on the analysis offered in these pages, that the choice of most property & casualty insurers to invest conservatively is in fact a rational choice if they are focused on satisfying regulators.

Conclusion

In a regulated insurance world, the obstacles that insurers have to navigate through are complex. Not only need insurers be concerned with regulators, they also need to concern themselves with shareholders. As we have shown, these issues are often at odds with one another. In fact, an insurer specifically focused on satisfying traditional regulator measures, such as the ratio of capital and surplus to the NAIC Risk-Based Capital, may be sacrificing shareholder value and even the overall long-term health of the company.

As technology has improved and as the banking and insurance lines have become blurred, it would seem to make sense for regulators to adopt new standards for charging for asset risk that would encourage maximization of shareholder value thereby aligning shareholder and regulator objectives more closely.

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