Practical Application of the Risk-Adjusted Return on Capital Framework

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ABSTRACT

This paper applies a risk-adjusted return on capital (RAROC) framework to the financial analysis of the risk and performance of an insurance company. A case study is presented for a diversified insurer with both property & casualty and life insurance business segments. The approach first quantifies the probability distributions of the different types of risk the institution faces: non-catastrophe liability risk, catastrophe risk, life risk, asset-liability mismatch (ALM) risk, credit risk, market risk, and operating risk. These risk type distributions are then aggregated to create an integrated risk distribution for the institution.

Economic Capital and RAROC are then calculated using this risk distribution in conjunction with income statement analysis to produce performance metrics and insights at both the line of business and total company level that support strategic as well as tactical decisions. Exhibits providing the case study numerical examples accompany the discussion of methodology throughout the paper.

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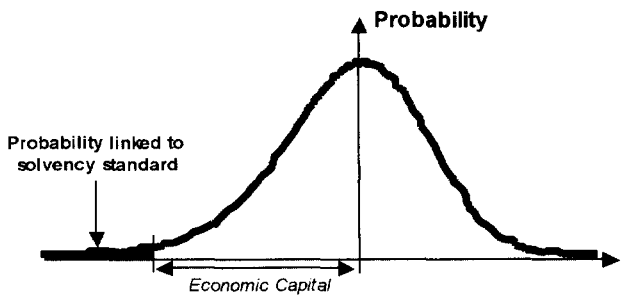
1. INTRODUCTION

1.1 Economic Capital and RAROC applied to the P&C industry

Insurers bear a responsibility both to shareholders and policyholders to maintain solvency throughout a variety of potential adverse events. Economic Capital, or the amount of capital required to support its risks to a given level of solvency, is an emerging standard in the insurance industry to help management fulfill this responsibility. The Economic Capital framework also lends itself to performance evaluation as the denominator of the Risk-Adjusted Return on Capital (RAROC) metric. With these tools, any financial institution can measure where its capital is invested, how much it is earning, how much capital it needs to hold to maintain a given debt rating, making risk-return tradeoff decisions as well as many other strategic decisions.

Economic Capital can be defined more precisely as the difference between the mean and the n^{th} percentile (i.e. the "solvency standard") of the value distribution for the entire company, where the value distribution represents the mark-to-market available capital, taking into account all risky assets and liabilities. The solvency standard, or probability of ruin, is typically linked to agency credit ratings, for example those from S&P or Moody's, e.g. an S&P rating of "AA" corresponding to an average default probability of 0.03%. As a result, an insurer that wishes to target a "AA" rating can quantify the capital to support its risks as the difference between the 0.03 percentile and the mean of its overall value distribution (see Figure 1-1).

Figure 1-1 -- Economic Capital in Relation to the Value Distribution[1]



While quantifying the overall risk of the company is important for strategic management, it is the allocation of overall economic capital back to the individual business units that enables the linking of tactical decisions with strategic goals, such as ROE targets. True insight into the economic performance of the organization comes only through linking risk and capital.

[1] Throughout this paper we represent all distributions as value distributions. This means that negative values represent an adverse outcome and positive values represent a favorable outcome.

1.2 Case Study Overview

The focus of this paper will be on the methodologies used to evaluate the risks an insurer faces. In order to facilitate the discussion, a case study insurer was created to provide a concrete example of the potential applications of the methodology. The case study company is a diversified insurer with both property & casualty and life insurance business segments. To keep things simple, the insurer has only five insurance business units in addition to an investments unit. We have selected business units in such a way as to illustrate the potential breadth of exposures an insurer may face. Table 1-1 illustrates the structure of the company and the risks to which each of the business units is exposed.

Table 1-1– Overview of Case Study Company

Segment	Business Unit	Non-Cat	Cat	Life	ALM	Credit	Market	Operating
P&C	Homeowners	X	X					X
P&C	General Liability	X						X
Specialty	Credit & Surety					X		X
Life	Term-Life			X				X
Life	Survival-Contingent Annuities			X	X			X
Investments	Investments				X	X	X	

2. RISK QUANTIFICATION

2.1 Non-Catastrophe Liability Risk

Non-catastrophe liability risk is a measure of the uncertainty in the amount and timing of insurance claims. The model used here incorporates process, parameter and systematic risks. The approach used is based on the volatility of loss-development factors, which are calculated using paid loss triangles.

The method involves back-casting ultimate loss estimates (ULE) based on a given paid loss triangle. First, the link ratio (or age-to-age factor) from one development year (DY) to the next is calculated. A cumulative development factor (CDF) for each DY is derived from the link ratio. By multiplying the CDF for each DY by the corresponding paid losses in the triangle, a triangle of ultimate loss estimates is generated.

The volatility of the ULEs and the change in ultimate estimates from one DY to the next within an accident year (AY) are used to calculate development factor volatility (a measure of process risk), and loss estimate uncertainty volatility (a contributor to parameter risk). In addition, systematic volatility is calculated as another indicator of parameter risk. Economic capital requirements are then calculated for the selected line after incorporating the diversification benefits resulting from AY and DY correlations. A lognormal loss distribution is assumed for each individual line of business. Finally, the individual loss distributions for each line are aggregated together while incorporating line-to-line diversification benefits using a line of business correlation matrix.

Table 2-1 – Paid Loss Triangles and Initial Loss Estimates (ILE) by Accident Year (AY)

Homeowners (HO)

		Cumulative Paid Loss by DY				
AY	ILE	1	2	3	4	5
1997	115,000,000	28,000,000	79,000,000	88,000,000	98,000,000	120,000,000
1998	110,000,000	31,000,000	73,000,000	89,000,000	92,000,000	
1999	107,000,000	18,000,000	72,000,000	103,000,000		
2000	100,000,000	22,000,000	101,000,000			
2001	93,000,000	23,000,000				

General Liability (GL)

		Cumulative Paid Loss by DY				
AY	ILE	1	2	3	4	5
1997	48,000,000	6,000,000	9,000,000	25,000,000	32,000,000	39,000,000
1998	70,000,000	4,000,000	23,000,000	35,000,000	45,000,000	
1999	72,000,000	7,000,000	15,000,000	30,000,000		
2000	63,000,000	3,000,000	4,000,000			
2001	55,000,000	10,000,000				

Table 2-2– Cumulative Development Factor (CDF) from One Development Year to the Final Development Year

DY	1 to 5	2 to 5	3 to 5	4 to 5	5 to 5
HO CDF	5.3938	1.6430	1.3144	1.2245	1.0000
GL CDF	7.6373	2.9950	1.5641	1.2188	1.0000

The back-casting of ultimate loss estimates uses a blended Chain-Ladder/Bornheutter-Ferguson approach:

$$ULE_{AY,DY} = \left(1 - \frac{1}{CDF_{DY}^{\;1-\gamma}}\right) ILE_{AY} + CDF_{DY}^{\;\gamma} PAID_{AY,DY} \tag{1}$$

where CDF_{DY} is the cumulative development factor from DY to final based on the link-ratio method, ILE_{AY} is the initial expected ultimate, i.e. premium times initial expected loss ratio, and γ is the degree of reliance on historical losses vs. initial expectations. The value of parameter γ is between zero and one, with zero resulting in the Bornheutter-Ferguson (BF) method, and one resulting in the pure chain-ladder method:

$$\gamma \to 1 \Rightarrow ULE_{AY,DY} = CDF_{DY} PAID_{AY,DY} \tag{2}$$

$$\gamma \to 0 \Rightarrow ULE_{AY,DY} = \left(1 - \frac{1}{CDF_{DY}}\right) ILE_{AY} + PAID_{AY,DY} \tag{3}$$

This blended approach is used to allow for flexibility in the relative importance of initial estimates versus observed results. The BF approach places greater weight on initial loss estimate (ILE) predictions. This solves the most significant problem with long-tailed triangles, namely that the initial development years exhibit dramatic percentage variations in paid losses magnified by CDF extrapolation. In this instance, we use γ = 0.67 for Homeowners and γ = 0.33 for General Liability, since GL is a much longer-tailed line. The BF approach requires the additional inputs of premium and loss ratio in order to derive the ILEs.

2.1.1 LDF Volatility (Process Risk)

The volatility of loss development is measured by taking a weighted standard deviation of observed results according to standard methods. Let $X_{i,j}$ denote the change in back-cast ultimate loss from development year *i*-1 to *i* for business from accident year *j*:

$$X_{i,j} = \frac{ULE_{i,j}}{ULE_{i-1,j}} \tag{4}$$

Let $w_{i,j}$ denote the relative weight of accident year *j* in development year *i*:

$$w_{i,j} = \frac{ULE_{i-1,j}}{\sum_k ULE_{i-1,k}} \tag{5}$$

Let Y be the random variable denoting change in ultimate loss from one year to the next, and let $\sigma_{LDF}^{(i)}$ represent the LDF volatility for development year i. $\sigma_{LDF}^{(i)}$ is computed from the basic definition of standard deviation:

$$\sigma_{LDF}^{(i)} = \sqrt{E\left[Y^2\right] - \left(E[Y]\right)^2} = \sqrt{\sum_j w_{i,j} X_{i,j}^2 - \left(\sum_j w_{i,j} X_{i,j}\right)^2} \tag{6}$$

Expanding w_j and $X_{i,j}$ we arrive at:

$$\sigma_{LDF}^{(i)} = \sqrt{\sum_j \frac{ULE_{i-1,j}}{\sum_k ULE_{i-1,k}} \left(\frac{ULE_{i,j}}{ULE_{i-1,j}}\right)^2 - \left(\sum_j \frac{ULE_{i-1,j}}{\sum_k ULE_{i-1,k}} \frac{ULE_{i,j}}{ULE_{i-1,j}}\right)^2} \tag{7}$$

Simplifying gives us:

$$\sigma_{LDF}^{(i)} = \sqrt{\left(\frac{1}{\sum_j ULE_{i-1,j}} \times \left(\sum_j \frac{ULE_{i,j}^2}{ULE_{i-1,j}}\right)\right) - \left(\sum_j \frac{1}{\sum_k ULE_{i-1,k}} ULE_{i,j}\right)^2} \tag{8}$$

And, finally:

$$\sigma_{LDF}^{(i)} = \sqrt{\left(\frac{1}{\sum_j ULE_{i-1,j}} \times \left(\sum_j \frac{ULE_{i,j}^2}{ULE_{i-1,j}}\right)\right) - \left(\frac{\sum_j ULE_{i,j}}{\sum_j ULE_{i-1,j}}\right)^2} \tag{9}$$

This method will produce n-1 $\sigma_{LDF}^{(i)}$ values, one for each column of the loss triangle that has more than one year of data. It is desirable to apply this method to n+1 different accident years, however: the n years embedded in the loss triangle, plus the current accident year, for which no losses have yet been recorded. To generate the last two values, $\sigma_{LDF}^{(n)}$ and $\sigma_{LDF}^{(n+1)}$, we compute a decay factor from the best-fit exponential curve through $\sigma_{LDF}^{(1)}, \cdots, \sigma_{LDF}^{(n-1)}$ using a weighted log-linear regression.

Let ω_i be the weight for development year i in the regression:

$$\omega_i = \frac{\sqrt{n-i+1}}{\sum_{j=1}^{n} \sqrt{j}} \tag{10}$$

The independent variable X in the regression corresponds to the development year index i, and the dependent variable is the natural log of the loss development factor volatility, $\ln\left(\sigma_{LDF}^{(1)}\right), \cdots, \ln\left(\sigma_{LDF}^{(n-1)}\right)$. The moments for the regression are:

$$E(X) = \sum_{i=1}^{n} i \cdot \omega_i \tag{11}$$

$$E(Y) = \sum_{i=1}^{n} \ln\left(\sigma_{LDF}^{(i)}\right) \cdot \omega_i \tag{12}$$

$$E\left(X^2\right) = \sum_{i=1}^{n} i^2 \cdot \omega_i \tag{13}$$

$$E\left(Y^2\right) = \sum_{i=1}^{n} \left(\ln\left(\sigma_{LDF}^{(i)}\right)\right)^2 \cdot \omega_i \tag{14}$$

$$E(XY) = \sum_{i=1}^{n} i \cdot \ln\left(\sigma_{LGD}^{(i)}\right) \cdot \omega_i \tag{15}$$

From the moments, we can calculate the slope and y-intercept of the log-linear regression line:

$$m = \frac{E(XY) - E(X) \cdot E(Y)}{E\left(X^2\right) - E(X)^2} \tag{16}$$

$$b = E(Y) - m \cdot E(X) \tag{17}$$

The decay factor d is defined as:

$$d = e^m \tag{18}$$

Finally, we use the decay factor to compute $\sigma_{LDF}^{(n)}$ and $\sigma_{LDF}^{(n+1)}$:

$$\sigma_{LDF}^{(n)} = d \cdot \sigma_{LDF}^{(n-1)} \tag{19}$$

$$\sigma_{LDF}^{(n+1)} = d^2 \cdot \sigma_{LDF}^{(n-1)} \tag{20}$$

Table 2-3 – Loss Development Factor (LDF) Volatility

DY	0 to 1	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6
HO LDF Vol	0.1176	0.2205	0.0973	0.0343	0.0230	0.0154
GL LDF Vol	0.0931	0.1634	0.0958	0.0012	0.0004	0.0001

2.1.2 ULE Volatility (Parameter Risk)

Given an estimate for the mean level of ultimate loss, σ_{LDF} represents the volatility of the actual loss outcome around the mean. However, there is additional uncertainty embedded in the estimated ultimate loss. "Ultimate Loss Estimate Volatility", or "Parameter Risk", represents the standard deviation of the mean loss estimate.

In general, given a random sample of a variable X, the standard deviation of its mean estimate $\overline{X}$ is:

$$s_{\overline{X}} = \sqrt{\frac{s_X^2}{n-1}} \tag{21}$$

where s_X^2 is the sample variance of X. In this case, the sample variance corresponds to LDF volatility, and the estimation error of $\overline{X}$ corresponds to ULE volatility:

$$s_X^2 = \sigma_{LDF}^2 \tag{22}$$

$$s_{\overline{X}}^2 = \sigma_{ULE}^2 \tag{23}$$

Let $\sigma_{ULE}^{(i)}$ represent the volatility of ultimate loss for development year i. $\sigma_{ULE}^{(i)}$ reduces to:

$$\sigma_{ULE}^{(i)} = \frac{\sigma_{LDF}^{(i)}}{\sqrt{n-1}} \tag{24}$$

Here, n is the number of observations (i.e. Accident Years) in the loss triangle at development year i. This risk is assumed to be independent of LDF volatility.

Table 2-4 – Ultimate Loss Estimate (ULE) Volatility

DY	0 to 1	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6
HO LDF Vol	0.0588	0.1273	0.0688	0.0343	0.0230	0.0154
GL LDF Vol	0.0466	0.0943	0.0678	0.0012	0.0004	0.0001

2.1.3 Systematic Risk

In addition to volatility that is observable in historical loss triangles, there is a risk that unforeseen and unprecedented systematic changes in legislation or market factors will have a negative impact on future results. This risk is intended to capture that which is unobserved in historical data; by definition it falls outside the realm of estimation from historical loss triangles and must be parameterized separately.

We begin by assuming that systematic risk is independent of process and parameter risk (LDF volatility and ULE volatility). That is, non-systematic factors provide no insight into the systematic risk faced by a given line of business. Also, we assume that systematic risk is, at the outset of development for a given accident year, proportional to ultimate loss. We further assume that, since it is proportional to ultimate loss, systematic risk can be proportionally be attributed to two sources: 1) the absolute level of ultimate loss; and 2) the unpaid portion of ultimate loss.

The formula for systematic risk is derived from these broad assumptions. Let q be the proportion of systematic risk attributable to unpaid ultimate loss ($0 < q < 1$). $1 - q$ is the proportion of systematic risk attributable to the absolute level of ultimate loss (the non-decaying portion). If σ_{Syst} is the total systematic risk, then the portion attributable to the level of ultimate loss is

$$\sigma_{Syst} \times (1-q) \tag{25}$$

This gives us one of the two components of systematic risk. The remaining component is built from the amount of unpaid ultimate loss. Let CDF_i be the cumulative development factor for development year i. By definition, $1/CDF_i$ is the percentage of total ultimate loss that has been paid at the end of development year i. Thus, $(1-1/CDF_i)$ is the percentage of ultimate loss that remains unpaid. With q as defined above - the proportion of systematic risk attributable to unpaid ultimate loss - the amount of systematic risk attributable to unpaid ultimate loss is:

$$\sigma_{Syst} \times q \times (1-1/CDF_i) \tag{26}$$

Combining equations 25 and 26, we arrive at the formula for allocating total systematic risk to development year i:

$$\sigma_{Syst}^{(i)} = \sigma_{Syst} \times \left(q \times \left(1 - \frac{1}{CDF_i} \right) + (1-q) \right) \tag{27}$$

where σ_{Syst} is the total systematic risk and q is the percentage of σ_{Syst} attributable to the unpaid portion of ultimate loss. CDF_i is the cumulative loss development factor at development year i. $\sigma_{Syst}^{(i)}$ is assumed to be perfectly correlated with $\sigma_{Syst}^{(j)}$, for any j, and uncorrelated with $\sigma_{LDF}^{(i)}$ and $\sigma_{ULE}^{(i)}$.

Assuming $\sigma_{Syst} = 0.05$ and $q = 0.9$:

Table 2-5 – Systematic Risk Volatility

DY	0 to 1	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6
HO Sys Vol	0.0500	0.0417	0.0226	0.0158	0.0133	0.0050
GL Sys Vol	0.0500	0.0441	0.0350	0.0212	0.0131	0.0050

2.1.4 Total Development Year Volatility

Let σ_i represent the total volatility for development year i. Assuming independence between the three components of total volatility, we compute σ_i in the standard fashion:

$$\sigma_i = \sqrt{\left(\sigma_{LDF}^{(i)}\right)^2 + \left(\sigma_{ULE}^{(i)}\right)^2 + \left(\sigma_{Syst}^{(i)}\right)^2} \tag{28}$$

Table 2-6 – Total Development Year Volatility

Homeowners

DY	0 to 1	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6
LDF Vol	0.1176	0.2205	0.0973	0.0343	0.0230	0.0154
ULE Vol	0.0588	0.1273	0.0688	0.0343	0.0230	0.0154
Systematic Vol	0.0500	0.0417	0.0226	0.0158	0.0133	0.0050
Overall Volatility	0.1407	0.2580	0.1213	0.0511	0.0351	0.0223

General Liability

DY	0 to 1	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6
LDF Vol	0.0931	0.1634	0.0958	0.0012	0.0004	0.0001
ULE Vol	0.0466	0.0943	0.0678	0.0012	0.0004	0.0001
Systematic Vol	0.0500	0.0441	0.0350	0.0212	0.0131	0.0050
Overall Volatility	0.1155	0.1938	0.1225	0.0213	0.0131	0.0050

2.1.5 Development Year Correlation

The total volatility σ for the line aggregates the σ_i from each year, taking into account correlation between development years. These correlations are derived from the total development year volatility and systematic volatility.

Let $X^{(i)}$ and $X^{(j)}$ be random variables denoting the loss distribution in development years *i* and *j*, respectively. Let ρ_{ij} denote the correlation between $X^{(i)}$ and $X^{(j)}$. By definition, ρ_{ij} is:

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} \tag{29}$$

σ_i and σ_j are known; they are the total volatilities for $X^{(i)}$ and $X^{(j)}$ respectively, as computed in equation 28. To calculate ρ_{ij}, we need to compute σ_{ij}, the covariance of $X^{(i)}$ and $X^{(j)}$. σ_{ij} is, by definition:

$$\sigma_{ij} = E\left[\left(X^{(i)} - \overline{X}^{(i)}\right)\left(X^{(j)} - \overline{X}^{(j)}\right)\right] \tag{30}$$

where $\overline{X}^{(i)}$ and $\overline{X}^{(j)}$ are the expected values of $X^{(i)}$ and $X^{(j)}$, respectively. Within an Economic Capital framework, we are primarily concerned with the distribution of change in value relative to expectations. Thus, we set the loss distributions to have mean 0. This leaves the following:

$$\sigma_{ij} = E\left[X^{(i)}X^{(j)}\right] \tag{31}$$

We assume that volatility is composed of 3 elements: LDF volatility (Process Risk), ULE volatility (Parameter Risk) and Systematic volatility. Thus, $X^{(i)}$ and $X^{(j)}$ can be thought of as the sum of three random variables:

$$X^{(i)} = X^{(i)}_{LDF} + X^{(i)}_{ULE} + X^{(i)}_{Syst} \tag{32}$$

$$X^{(j)} = X^{(j)}_{LDF} + X^{(j)}_{ULE} + X^{(j)}_{Syst} \tag{33}$$

Substituting, we have:

$$\sigma_{ij} = E\left[\left(X^{(i)}_{LDF} + X^{(i)}_{ULE} + X^{(i)}_{Syst}\right)\left(X^{(j)}_{LDF} + X^{(j)}_{ULE} + X^{(j)}_{Syst}\right)\right] \tag{34}$$

This expands to:

$$\sigma_{ij} = E\begin{bmatrix} X^{(i)}_{LDF}X^{(j)}_{LDF} + X^{(i)}_{ULE}X^{(j)}_{ULE} + X^{(i)}_{Syst}X^{(j)}_{Syst} + \\ X^{(i)}_{LDF}X^{(j)}_{ULE} + X^{(i)}_{LDF}X^{(j)}_{Syst} + \\ X^{(i)}_{ULE}X^{(j)}_{LDF} + X^{(i)}_{ULE}X^{(j)}_{Syst} + \\ X^{(i)}_{Syst}X^{(j)}_{LDF} + X^{(i)}_{Syst}X^{(j)}_{ULE} \end{bmatrix} \tag{35}$$

Because we have assumed independence between all non-systematic factors, all terms in equation 35 have expected value 0, with the exception of the systematic term:

$$\sigma_{ij} = E\left[X^{(i)}_{Syst}X^{(j)}_{Syst}\right] \tag{36}$$

The correlation between systematic factors is assumed to be 1, giving:

$$\sigma_{ij} = E\left[X^{(i)}_{Syst}X^{(j)}_{Syst}\right] = \sigma^{(i)}_{Syst}\sigma^{(j)}_{Syst} \tag{37}$$

Thus, returning to the original definition of ρ_{ij}, we have:

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} = \frac{\sigma_{Syst}^{(i)} \sigma_{Syst}^{(j)}}{\sigma_i \sigma_j} \tag{38}$$

Table 2-7 – Development Year Correlations

Homeowners

	0	1	2	3	4	5
0	1	0.0574	0.0662	0.1097	0.1341	0.0795
1	0.0574	1	0.0301	0.0499	0.0609	0.0361
2	0.0662	0.0301	1	0.0576	0.0703	0.0417
3	0.1097	0.0499	0.0576	1	0.1165	0.0691
4	0.1341	0.0609	0.0703	0.1165	1	0.0844
5	0.0795	0.0361	0.0417	0.0691	0.0844	1

General Liability

	0	1	2	3	4	5
0	1	0.0985	0.1236	0.4316	0.4325	0.4327
1	0.0985	1	0.0650	0.2269	0.2274	0.2275
2	0.1236	0.0650	1	0.2847	0.2853	0.2854
3	0.4316	0.2269	0.2847	1	0.9960	0.9965
4	0.4325	0.2274	0.2853	0.9960	1	0.9987
5	0.4327	0.2275	0.2854	0.9965	0.9987	1

2.1.6 Line of Business Loss Distribution

We compute the total volatility σ using the year-to-year correlation matrix:

$$\sigma^2 = \begin{pmatrix} \sigma_1 \cdot ULE_1 \\ \vdots \\ \sigma_{n+1} \cdot ULE_{n+1} \end{pmatrix}^T \begin{pmatrix} 1 & \rho_{1,2} & \cdots & \rho_{1,n+1} \\ \rho_{2,1} & 1 & & \vdots \\ \vdots & & \ddots & \rho_{n,n+1} \\ \rho_{n+1,1} & \cdots & \rho_{n+1,n} & 1 \end{pmatrix} \begin{pmatrix} \sigma_1 \cdot ULE_1 \\ \vdots \\ \sigma_{n+1} \cdot ULE_{n+1} \end{pmatrix} \tag{39}$$

We assume that losses within each line of business follow a lognormal distribution, with mean equal to the sum of the most recent ultimate loss estimates for all accident years ($\sum ULE_i$) and standard deviation equal to σ.

Table 2-8 – Line of Business Correlations

	HO	GL
HO	1	0.1
GL	0.1	1

Table 2-9 – Non-Cat Line of Business Change in Value Distribution

Probability	HO Value	GL Value
0.001%	-221,332,923	-87,407,075
0.010%	-206,669,526	-84,871,664
0.030%	-184,790,686	-69,960,747
0.050%	-170,505,552	-69,006,709
0.070%	-161,768,432	-65,435,595
:	:	:
:	:	:
99.930%	110,192,339	46,787,530
99.950%	113,192,983	48,752,368
99.970%	118,178,053	50,504,321
99.990%	126,805,831	55,164,647
99.999%	135,965,877	57,136,264

2.1.7 Total P&C Non-Catastrophe Loss Distribution

To compute the overall loss distribution, we convolve the individual loss distributions from each line of business (see section on aggregation). This requires an inter-line of business correlation matrix that is estimated using management judgement or from loss histories. (See upcoming paper from Weimin Dong and Jim Gant.)

Table 2-10 – Non-Cat Change in Value Distribution

Probability	Value
0.001%	-248,982,536
0.010%	-218,554,051
0.030%	-197,119,667
0.050%	-185,022,543
0.070%	-177,305,526
:	:
:	:
99.930%	127,100,263
99.950%	130,506,877
99.970%	135,547,274
99.990%	145,131,462
99.999%	161,932,821

2.2 Catastrophe Risk

Catastrophe Risk quantifies the potential financial loss due to severe natural catastrophes. To provide a complete view of the potential losses from such events, it is desirable to use a statistical model, such as RiskLink from Risk Management Solutions, for developing a complete loss distribution, rather than traditional metrics such as average annual loss or probable maximum loss. Typical software packages use a Monte Carlo simulation approach with stochastic loss events to generate a full range of possible losses.

2.2.1 AEP vs. OEP

It is important to draw a distinction between the two varieties of loss distributions associated with catastrophe risk models. One variety is the "occurrence exceedance probability" or OEP curve, and the other is the "aggregate exceedance probability" or AEP curve. An OEP curve is the cumulative loss distribution for any one occurrence in a given year. It shows the probability that losses from a single event will exceed a given amount. In contrast, an AEP curve is the combined cumulative loss distribution from all possible events in a given year. It shows the probability that total losses will exceed a given amount.

The method takes as input an AEP curve from one of the standard catastrophe modeling packages as the loss distribution for catastrophe risk. The AEP curve is converted it into a value distribution, which is then aggregated with the value distributions derived for other risk pillars.

In our case study company, the only line of business exposed to natural cat is Homeowners. The tables below illustrates this line's AEP curve and corresponding value distribution.

Table 2-11 – Cat AEP Curve

AEP	Loss
0.001%	341,143,958
0.010%	234,864,033
0.030%	183,122,205
0.050%	164,242,079
0.070%	149,441,501
:	:
:	:
30.000%	24,160,338
50.000%	14,989,184
70.000%	7,682,240
90.000%	823,453
99.999%	0

Table 2-12 – Cat Value Distribution

Probability	Value
0.001%	-341,143,958
0.010%	-234,864,033
0.030%	-183,122,205
0.050%	-164,242,079
0.070%	-149,441,501
:	:
:	:
99.930%	0
99.950%	0
99.970%	0
99.990%	0
99.999%	0

2.3 Mortality Risk

Mortality risk is defined as the volatility of contract value resulting from unexpected changes in mortality rates. This includes changes in current year mortality rates as well as expected future mortality rates. A subset of the contracts often exposed to mortality risk includes term life, whole life, and annuities.

Mortality risk is quantified through a Monte-Carlo simulation of portfolio value under various mortality scenarios. The resulting distribution of values constitutes the risk profile of the contracts. The simulation is achieved in the following stages:

- Identification of distinguishable sources of mortality risk
- Assignment of these risks to factors impacting mortality rates
- Re-evaluation of contract net present value under simulated factor scenarios

2.3.1 Sources of Mortality Risk

We may separate mortality risk into the following set of underlying risk drivers:

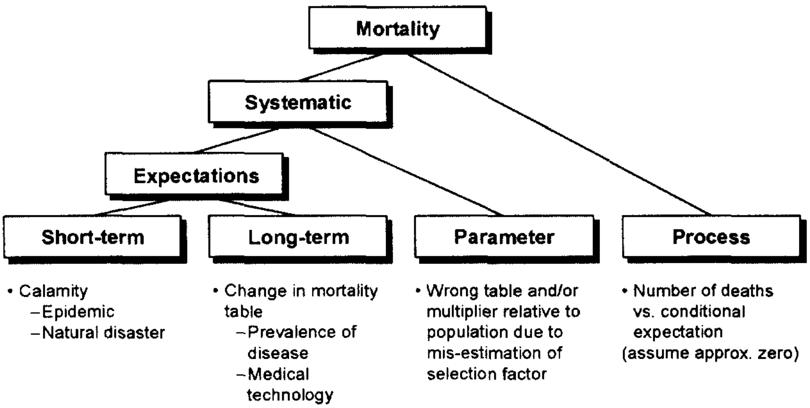
- Short-term systemic shocks
- Long-term changes in mortality expectations
- Parameter misestimation (Parameter Risk)
- Process volatility (Process Risk)

Short-term systemic shocks are the result of events that have a temporary impact on death rates across an insured population. For example, a particularly bad flu season will result in death rates increasing systematically across the entire life book for the coming year. They will not, however, necessarily change expectations of future death rates. Since economic capital is calculated on a one-year time horizon, only cash flows maturing within the coming year experience this risk.

Long-term changes in mortality expectations have an impact on multi-year products priced at the beginning of a term. Examples of long-term changes in mortality expectations include both positive impacts to mortality expectations (e.g. cure for cancer) and negative impacts (e.g. new diseases). Long-term systematic risk can impact expected future mortality rates (e.g. long-term impact of AIDS) and, to a lesser extent current year mortality (e.g. immediate impact of a new disease).

Parameter Risk results from a misestimation of the expected death rates of an insured population. Typically this is because the insured population differs from the population used to derive death rate estimates. A large portion of this risk derives from adverse selection of the insured population.

Figure 2-1 – Mortality Risk Taxonomy



Process risk derives from the difference between actual death rates and the true death rate mean adjusted for all the factors described above. For most books of policies, this is quickly diversified away; within our framework it is generally assumed to be zero risk[2].

---

[2] Consider a portfolio of *N* insured parties with an expected death rate of 20 basis points (0.20%). Each individual has a probability of dying within the next year of 0.20%, with standard deviation given by:

$$\sigma = \sqrt{p \times (1-p)} = \sqrt{0.2\% \times 99.8\%} = 4.47\%$$

Assuming each individual in the portfolio is independent, the total volatility due to Process Risk is:

2.3.2 Mortality Rate Factors

The above risks may be incorporated into a model via a set of risk factors. The factors are simulated random variables of a given distribution and correlation structure. When a factor is multiplied to an existing mortality rate, the resulting value represents a new hypothetical mortality rate. Our model uses a single "mortality occurrence" factor and a set of three "mortality expectation" factors to capture the above risks.

We can evaluate the impact of mortality risk on an institution via financial statements. When viewing from a one-year time horizon, unexpected changes in mortality can either create loss through higher benefits paid for the current year, or through an increase in reserves for future years. Volatility in benefits paid is captured in a mortality occurrence factor. This volatility is driven primarily by short-term systemic shocks but also captures parameter risk, long-term systematic risk and process risk.

An increase in reserves for losses in future years is captured with a set of three mortality expectation factors. These factors capture long-term systematic risk, parameter risk and process risk. The mortality expectation factors do not include the risk of short-term shocks since these shocks do not imply a change in mortality expectations for future years. Three factors are used to capture the varying degree of volatility and correlation between mortality changes within different age groups.

Table 2-13 – Risk Factors

Factor	Volatility	Age Min	Age Max
Occurrence Factor	0.05	n/a	n/a
Expectation Factor 1	0.10	0	40
Expectation Factor 2	0.09	41	60
Expectation Factor 3	0.05	61	120

$$\sigma_{Process} = \sqrt{\frac{\sigma^2}{N}}$$

If we assume that N is 1,000,000, then we find:

$$\sigma_{Process} = \sqrt{\frac{\sigma^2}{N}} \leq \frac{4.47\%}{1000} \leq 0.005\%$$

$\sigma_{Process}$ is small and decreasing as N grows.

Table 2-14 – Factor Correlations

	Expectation Factor 1	Expectation Factor 2	Expectation Factor 3	Occurrence Factor
Expectation Factor 1	1	0.4	0.2	0.0
Expectation Factor 2	0.4	1	0.4	0.0
Expectation Factor 3	0.2	0.4	1	0.0
Occurrence Factor	0.0	0.0	0.0	1

The variance of each of the mortality rate factors can be expressed as the sum of the variance due to systematic risk, parameter risk and process risk since we expect no correlation between these risk types.

$$\sigma^2 = \sigma_{systematic}{}^2 + \sigma_{parameter}{}^2 + \sigma_{process}{}^2$$

Systematic variance is attributable to volatility in industry-level mortality rates. In the case of the mortality expectation factors, systematic variance is determined by the annual volatility in expected mortality rates at the industry level. In the case of the mortality occurrence factor, the variance is the annual volatility of the difference between expected mortality and actual mortality. Parameter risk is specific to the institution and can be determined by comparing the historical systematic variance in industry level mortality rates with those at the institution level. Any difference is attributable to parameter risk. For a portfolio of a sizeable number of policyholders, process risk is negligible.

2.3.3 Mortality Sensitive Contract Value

Analogous to bond contracts, a mortality sensitive contract may be divided into a series of mortality sensitive cash flows. The nominal value of each cash flow is dependent on some set of mortality rates, either current or future. Changes in the nominal value of the cash flows result in a change in the mark-to-market value of the contract.

Let us define *surv(x,y,c)* as the percentage of policyholders of type *c* (here type can be gender, smoker status, country of residence, etc...), of age *x* at time zero expected to survive *y* years. We can see that $surv(x, y = 0, c) = 1$ and that $\lim_{y \to \infty} surv(x, y, c) = 0$.

Let us also define *mort(z,y,c)* as the percentage of policyholders of type *c*, of age *z* at time *y-1* years that are expected to experience mortality by time *y*. We can then express the expected present value of the cash flows as:

$$F_m = \frac{a}{(1+r)^y} \times surv(x,y,c) = \frac{a}{(1+r)^y} \times \prod_{i=1}^{y} [1 - mort(x+i-1,i,c)] = \frac{a}{(1+r)^y} \times e^{-\sum_{i=1}^{y} mort(x+i-1,i,c)}$$

$$F_m = \frac{b}{(1+r)^y} \times surv(x, y-1, c) \times mort(x+y-1, y, c) =$$

$$= \frac{b}{(1+r)^y} \times mort(x+y-1, y, c) \times \prod_{i=1}^{y-1} [1 - mort(x+i-1, i, c)] = \frac{b}{(1+r)^y} \times mort(x+y-1, y, c) \times e^{-\sum_{i=1}^{y-1} mort(x+i-1, i, c)}$$

where a is the survival-contingent cash flow,b is the mortality-contingent cash flow and r is the discount rate.

Table 2-15 – SCA (Survival-Contingent) Cash Flows

Yrs Forward	Initial Age → **38**	**48**	**58**	**68**	**78**
1	15,384,101	30,636,617	13,659,966	-2,681,184	-1,209,096
2	13,707,161	27,297,081	11,977,123	-2,758,095	-1,243,779
3	12,213,016	24,321,570	10,490,364	-2,802,538	-1,263,821
4	10,881,739	21,670,403	9,179,712	-2,815,391	-1,269,617
5	9,695,578	19,308,227	8,026,954	-2,798,224	-1,261,876
:	:	:	:	:	:
:	:	:	:	:	:
43	0	0	-54,030	-102,901	-46,404
44	0	0	-49,438	-94,154	-42,459
45	0	0	-45,235	-86,151	-38,850
46	0	0	-41,390	-78,828	-35,548
47	0	0	-37,872	-72,128	-32,526

Table 2-16 – Term Life (Death-Contingent) Cash Flows

	Initial Age →				
Yrs Forward	**38**	**48**	**58**	**68**	**78**
1	-5,021,260,205	-6,044,688,217	-1,222,039,843	-46,565,844	-20,999,141
2	-4,426,324,162	-5,328,492,931	-1,078,678,335	-43,771,893	-19,739,192
3	-3,943,833,850	-4,747,661,948	-962,108,661	-40,926,720	-18,456,145
4	-3,513,937,269	-4,230,144,295	-858,072,447	-38,061,850	-17,164,215
5	-3,130,901,453	-3,769,038,519	-765,218,074	-35,207,211	-15,876,899
:	:	:	:	:	:
:	:	:	:	:	:
43	0	0	-635,650	-1,210,598	-545,926
44	0	0	-581,620	-1,107,697	-499,523
45	0	0	-532,182	-1,013,543	-457,063
46	0	0	-486,947	-927,392	-418,213
47	0	0	-445,556	-848,563	-382,665

The above equations for contract value hold true for simulated scenarios with the modification of an appropriate factor multiplier applied to the mortality rate. For cash flows within one year, the mortality occurrence factor is used. For all other cash flows a mortality expectation factor is used dependant on the age of policyholder at the time of the cash flow. Table 2-16 displays the death-contingent cash flows, or the theoretical value of the portfolio if all insureds were to die at once.

Table 2-17 – Overall Change in SCA Cash Flows

	Initial Age →				
Yrs Forward	**38**	**48**	**58**	**68**	**78**
1	-4,746	-14,547	-14,677	7,393	7,288
2	-12,201	-52,593	-52,167	15,249	14,572
3	-19,511	-97,150	-99,343	25,951	24,527
4	-29,600	-129,867	-101,246	35,692	32,493
5	-36,932	-153,014	-100,468	44,227	38,478
:	:	:	:	:	:
:	:	:	:	:	:
43	0	0	479	178	16
44	0	0	379	139	12
45	0	0	299	108	10
46	0	0	236	84	7
47	0	0	186	66	6

Table 2-18 – Overall Change in Term Life Cash Flows

Yrs Forward	Initial Age → 38	48	58	68	78
1	-1,553,679	-2,882,886	-1,326,378	-131,830	-134,368
2	-2,581,646	-7,770,124	-3,574,737	-124,581	-119,355
3	-2,799,860	-9,878,489	-5,000,067	-158,700	-157,033
4	-3,957,926	-8,505,959	-1,360,273	-135,274	-118,190
5	-3,421,007	-7,325,814	-1,156,653	-114,575	-87,290
:	:	:	:	:	:
:	:	:	:	:	:
43	0	0	929	394	37
44	0	0	745	308	29
45	0	0	595	240	22
46	0	0	474	187	17
47	0	0	376	146	13

Table 2-19 – Life Value Distribution

Probability	Value
0.001%	-180,137,977
0.010%	-95,730,237
0.030%	-83,398,815
0.050%	-77,266,350
0.070%	-75,215,197
:	:
:	:
99.930%	56,811,041
99.950%	57,837,631
99.970%	60,174,618
99.990%	64,999,847
99.999%	74,015,472

2.4 Asset-Liability Mismatch Risk

Asset-liability mismatch (ALM) risk is the volatility in the value of the enterprise due to fluctuations in interest rates. Modeling ALM risk involves characterizing the portfolio of interest-rate sensitive positions on both the asset and liability side of the balance sheet, generating a set of change in rate scenarios, revaluing the enterprise under each scenario and finally generating a value distribution from the simulation results. This framework is analogous to typical Monte-

Carlo-based VaR models and is general enough to handle positions ranging from simple contractual cash flows to complex structured instruments.

2.4.1 Characterization of interest rate position

Interest-rate positions are classified into those that can be broken up into a series of deterministic cash flows, such as uncallable corporate bonds, and more complex instruments which are characterized using a tabulated rate versus value function. Cash flows positions are described by sets of cash flow amount, maturity pairs. Tabulated rate versus value data can be obtained from sources such as the Office for Thrift Supervision, or from an interest-rate sensitivity analysis in a spreadsheet or popular analytics packages.

Table 2-20 – Net Cash Flow

Maturity (Yrs)	Net Cash Flow
1	-500,000,000
2	-250,000,000
3	-150,000,000
4	-50,000,000
5	-25,000,000
7	200,000,000
10	1,100,000,000

Table 2-21 – SCA Rate vs. Value

Δ Rate	Δ Value
-3.0%	-85,000,000
-2.0%	-53,000,000
-1.0%	-21,000,000
0.0%	0
1.0%	6,000,000
2.0%	8,000,000
3.0%	9,000,000

2.4.2 Structure of the interest-rate simulation model

The interest-rate simulation seeks to generate scenarios corresponding to hypothetical changes in the yield curve. This is accomplished by characterizing a yield curve as a collection of rates which are themselves functions of the interest-rate factors. In this paper we have used a four-factor interest rate model with approximately N=50,000 simulations. The four-factors, the change in one-year rate, the change in the spread of the 10-year rate over the one-year rate and the change in the spread of the 30-year rate over the 10-year rate and the change in spread of the mortgage rate over the 10-year rate, are normally distributed and related via a Pearson correlation matrix.

Combined with the assumption of linearity of rate spreads between these three points, this suffices to determine the change in rate for all points along the yield curve. A Box-Muller approach is used to generate a set of correlated random draws for each of the N iterations.

Specifically, an $N \times m$ matrix of interest rate changes is calculated, where m represents all the relevant maturities. The change in rate for a particular maturity, Δr_m, is determined by the following formulas:

$$\Delta r_m = \begin{cases} \Delta rnd_1 + \left(\dfrac{m-1}{9}\right) \times \Delta rnd_2 & , m \le 10 \\ \Delta rnd_1 + \Delta rnd_2 + \left(\dfrac{m-10}{20}\right) \times \Delta rnd_3 & , m > 10 \\ \Delta rnd_1 + \Delta rnd_2 + \Delta rnd_4 & , m = mortgage \end{cases} \tag{40}$$

where Δrnd_i is the randomly generated change in interest rate factor i.

Table 2-22 – Rate Curve Shift Simulations

	Δ Rate			
Rate (Yrs)	**Simulation 1**	**Simulation 2**	**....**	**Simulation N**
1	0.41%	-0.26%	....	-0.29%
2	0.41%	-0.24%	....	-0.23%
3	0.40%	-0.22%	....	-0.18%
4	0.40%	-0.20%	....	-0.12%
5	0.39%	-0.18%	....	-0.07%
7	0.39%	-0.13%	....	0.04%
10	0.37%	-0.07%	....	0.20%
30	-0.22%	0.17%	....	-0.36%
Mortgage	0.11%	-0.26%	....	-0.38%

2.4.3 Valuing the Portfolio

Given a set of Δr_m's for a simulation, the change in value for a cash flow CF_m at maturity m is calculated as:

$$PV(CF_m) = CF_m e^{-(r+\Delta r_m)m} \tag{41}$$

Table 2-23 – Change in Cash Flow Values for Simulations

	Δ Value			
Maturity (Yrs)	**Simulation 1**	**Simulation 2**	**....**	**Simulation N**
1	1,989,396	-1,251,786	....	-1,380,780
2	1,877,409	-1,101,431	....	-1,070,782
3	1,592,850	-864,523	....	-705,514
4	664,214	-330,412	....	-207,173
5	387,654	-174,739	....	-68,776
7	-3,801,707	1,342,731	....	-385,740
10	-24,087,944	4,727,527	....	-13,066,148

For all instruments in the value-rate table, the change in value is found by looking up the specific Δr_m in the rate column, using linear interpolation.

Table 2-24 – Change in SCA Values for Simulations

Simulation	Δ Rate	Δ Value
1	0.41%	2,473,200
2	-0.26%	-5,428,500
:	:	:
:	:	:
N	-0.29%	-5,987,100

Finally, the change in value to all cash flows and instruments is calculated for each scenario and summed to yield the total change in value.

Table 2-25 – Overall Change in Value for Simulations

Simulation	Probability	Total Δ Value
1	1/N	-18,904,928
2	1/N	-3,081,133
:	:	:
:	:	:
N	1/N	-22,872,013

After each scenario is assigned a total change in value, the results across all simulations are sorted producing a cumulative probability distribution of change in value, with each scenario being equally probable with probability mass $1/N$. This distribution is then used for risk aggregation and capital allocation.

Table 2-26 – ALM Value Distribution

Probability	Value
0.001%	-247,325,565
0.010%	-206,414,145
0.030%	-185,755,073
0.050%	-179,171,451
0.070%	-175,236,860
:	:
:	:
99.930%	154,883,438
99.950%	159,731,919
99.970%	168,262,875
99.990%	181,303,688
99.999%	202,926,237

2.5 Credit Risk

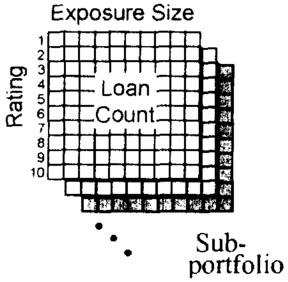
Credit risk is defined as the risk that a party to a contract, in most instances a borrower, defaults on an obligation, causing a loss of all or part of the replacement value of ongoing contracts. While default does not necessarily mean legal bankruptcy, it signals an inability or unwillingness of the party to fulfill its contractual obligations. Credit risk also includes the possibility that the obligor's credit quality weakens (i.e. the likelihood of default increases) causing a loss in value of obligations that are discounted for credit risk. For insurance companies, credit risk normally arises in a portfolio of bonds or loans, credit insurance, reinsurance recoverables, surety and financial derivatives.

The risk within a credit portfolio can be separated into three different types: systematic risk, non-systematic (or idiosyncratic) risk, and non-default economic loss risk. Systematic risk refers to the risk of default common to all counter-parties due to underlying economic factors that affect an industry, geography, etc. Idiosyncratic risk is specific to a particular company, for example fraud, and is statistically independent of sub-portfolio relationships. Non-default economic loss risk is the risk that the value of a credit changes over time even if the rating stays constant. For example, due to the credit cycle, a BBB credit may not be as credit worthy next year as a BBB credit is today, resulting in a loss of economic value. This type of risk captures the effect of credit movements over time on a systematic basis. The economic "mark-to-market" effect depends upon the maturity of the credit and the volatility of credit quality.

2.5.1 Characterization of credit exposures

Credit positions are bucketed into sub-portfolios constructed according to geographic, industry or other criterion. For each sub-portfolio, a credit matrix is constructed that groups credit obligations according to their credit quality (rating) and exposure size, as illustrated below:

Figure 2-2 – Illustration of Rating-Exposure Matrix



The credit risk for a single obligation depends upon the exposure at time of default, the probability of default (linked to the risk rating), the recovery rate in the event of default, the volatility of the recovery rate, the maturity of the obligation (for those obligations which are not systematically re-priced when credits weaken), and the correlation of the obligation to the rest of the sub-portfolio to which the position belongs. Correlations are specified between obligations within a sub-portfolio as well as between sub-portfolio types. For a portfolio of bonds or loans, correlations determine diversification benefits.

Table 2-27 – Mapping of S&P Ratings to Expected Default Frequency (EDF)

Rating	EDF
AAA	0.01%
AA	0.03%
A	0.07%
BBB	0.18%
BB	0.93%
B	4.46%

Table 2-28 – Corporate Bond Sub-portfolio Size Ranges

	Size Range 1	Size Range 2	Size Range 3	Size Range 4	Size Range 5
Minimum	0	6,000,000	11,000,000	20,000,000	25,000,000
Maximum	6,000,000	11,000,000	20,000,000	25,000,000	100,000,000
Average	2,500,000	9,000,000	14,000,000	21,000,000	26,000,000

Table 2-29 – Corporate Bond Sub-portfolio Rating-Exposure Matrix

	Bond Count				
Rating	Size Range 1	Size Range 2	Size Range 3	Size Range 4	Size Range 5
AAA	60	15	2	2	3
AA	10	15	5	5	0
A	10	10	5	5	0
BBB	0	5	2	1	0
BB	0	0	0	0	0
B	0	5	2	0	0

2.5.2 Expected loss

Credit loss can be described as the product of three terms:

Loss = Default · Exposure · Severity (Ong 94)

Loss is the amount that an institution is contractually owed but does not receive because of the borrower or borrower defaulting.

Default is the binomially distributed Bernoulli random variable that measures whether a borrower has defaulted or not, i.e., has fallen 3 months into arrears. It takes the values of either one in the case of default, or zero otherwise.

Exposure is the total amount of the institution's liability to a borrower.

Severity is the fraction of the exposure that is actually lost given a default of that borrower.

Table 2-30 – Corporate Bond Sub-portfolio Parameters

Parameter	Value
Recovery rate	50%
Recovery volatility	25%
Average maturity (yrs)	6
Intra-sub-portfolio correlation	0.5

Table 2-31 – Corporate Bond Sub-portfolio Exposure Summary

EDF	Size Range 1	Size Range 2	Size Range 3	Size Range 4	Size Range 5
0.0001	150,000,000	135,000,000	28,000,000	42,000,000	78,000,000
0.0003	25,000,000	135,000,000	70,000,000	105,000,000	0
0.0007	25,000,000	90,000,000	70,000,000	105,000,000	0
0.0018	0	45,000,000	28,000,000	21,000,000	0
0.0093	0	0	0	0	0
0.0446	0	45,000,000	28,000,000	0	0

Total Exposure ($) 1,225,000,000

The expected credit loss is the average annual loss rate over the course of a business cycle:

EL = E(Loss) = E(Default Frequency · Exposure · Severity) = E(Exp) · E(Sev) · E(DF) (Ong 94)

The expected loss for a portfolio is the sum of the ELs of the individual exposures.

2.5.3 Unexpected loss

Unexpected loss is the standard deviation of credit losses. There is typically little volatility in the size of the exposure amount (because the loan size is known upon origination), so $\sigma_{Exp}^2 = 0$. Exposure, default frequency and severity are treated as independent random variables. The standard deviation of default-based credit losses associated with an individual transaction is:

$$UL = \sigma_{loan} = \mu_{Exp}\sqrt{\sigma_{DF}^2 \cdot \mu_{Sev}^2 + \mu_{DF} \cdot \sigma_{Sev}^2} \text{ (Ong 113)}$$

The unexpected loss for a portfolio requires loss correlations between all pairs of borrowers. Let ρ_{ij} be the loss correlation between borrowers i and j, then

$$UL_{Portfolio} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} UL_i UL_j \rho_{ij}}$$ (Ong 133) where UL_i is the UL for loan i

In order to facilitate the calculation of the portfolio UL, the sub-portfolio UL can be divided into two components: a systematic piece and a non-systematic piece.

2.5.4 Systematic and idiosyncratic risk

The allocation of systematic risk and idiosyncratic risk is accomplished by splitting apart sources of variance in the $UL_{subportfolio}$ equation. Since a subportfolio is made up of a group of borrowers, the equation for $UL_{subportfolio}$ is analogous to the formula for $UL_{portfolio}$ (where the borrowers are those specific to the subportfolio). Then, we have:

$$UL_{Subportfolio}^{\ 2} = \sum_{i=1}^{n}\sum_{j=1}^{n} UL_i UL_j \rho_{ij} = \sum_{i=1}^{n} UL_i^{\ 2} + \sum_{i=1}^{n}\sum_{j\neq i}^{n} UL_i UL_j \rho_{ij}$$

where ρ_{ij} is the loss correlation between borrowers in the industry[3]. The first term represents borrower-specific risk (if borrower defaults were independent, this would be the total risk) and the second term represents additional risk owing to the correlation between borrowers within a subportfolio. As a result, the second term is purely systematic risk and the first term can be thought of as having systematic and idiosyncratic portions. It can be easily shown that UL_i (the UL for borrower i) can be split into a systematic portion, ULS_i, equal to:

$$ULS_i = \sqrt{\rho_i} UL_i$$

and an idiosyncratic, or non-systematic portion, ULN_i, equal to:

$$ULN_i = \left(\sqrt{1-\rho_i}\right) UL_i$$

where ρ_i is the loss correlation between 2 borrowers in the same industry (the industry for borrower i) each having probability of default equal to the probability of default for borrower i (i.e. the loss correlation between homogenous borrowers with the same credit rating). Therefore, the subportfolio non-systematic risk is calculated as:

$$ULN_{sp} = \sqrt{\sum_{i=1}^{n} \left(\sqrt{1-\rho_i}\right) UL_i^{\ 2}}$$

and the systematic portion is calculated as:

$$ULS_{sp} = \sqrt{\sum_{i=1}^{n} \rho_i UL_i^{\ 2} + \sum_{i=1}^{n}\sum_{j\neq i}^{n} UL_i UL_j \rho_{ij}}\ .$$

2.5.5 Non-default economic loss ("spread") risk

In addition to default risk over the 1-year time horizon, there is also the risk that longer-term loans (loans with maturities > 1 year) lose value resulting from changes in credit quality. More

[3]The loss correlation for loans in the same industry, $\rho_{i,j}$, is calculated using the Merton model of default. The calculation is a function of an industry asset correlation and default probabilities for borrowers i and j, as will be explained in more detail later in the document. Though the Merton model produces a default correlation, the assumption that loss correlation is approximately equal to default correlation is made since the majority of loss volatility is due to default volatility.

precisely, there is the risk of changes in the expectation of future losses that represents a risk to the value of the portfolio (analogous to a change in the "market value" of a loan). To model this risk, market parameters are used. Using yield spreads to risk-free securities, the non-default economic loss risk can be calculated analogously to interest rate risk for a bond.

The expected change in value is zero and the spread risk UL is assumed to be linear in maturity (making the effective maturity equal to the remaining maturity after 1 year, and weighted by principal payments). The volatility of spread is estimated using historical spreads on a universe of rated bonds. It is observed that the spread has a roughly constant coefficient of variation equal to 31%, making

$$Vol(r_{spread}) = r_{spread} \times \frac{\sigma_{spread}}{r_{spread}} = \sigma_{spread} \text{ with } r_{spread} = \%EL .$$

To be more exact, the spread loss variable for a loan is the product of a non-default indicator $\bar{D}$ and a spread loss variable P, $L_{sp} = \bar{D}P = (1-D)P$. We have that its mean is zero, so: $E[L_{sp}] = E[\bar{D}P] = u_{\bar{D}} E[P] = 0$, and $E[P] = 0$. Then, assuming independence of $\bar{D}$ and P, we can write $Var(L_{sp}) = \sigma_{\bar{D}} E[P]^2 + u_{\bar{D}} \sigma_P^2 = (1-u_D)\sigma_P^2$. So, the unexpected loss owing to spread risk in a sub-portfolio is:

$$UL_{spread} = \sqrt{1-edf} \times EL \times \sigma_{spread} \times (T-1) \times Term\_Percent$$

where T is the Average Tenor (the quantity (T-1) is used for maturity to reflect the fact that spread risk is related to the remaining maturity after 1 year has passed, and the Term Percent is used to apply spread risk only to those loans with remaining maturities > 1 year). Since $\sqrt{1-edf}$ is approximately one, the term is dropped and the equation simplifies to $UL_{spread} = EL \times \sigma_{spread} \times (T-1) \times Term\_Percent$.

2.5.6 Portfolio unexpected loss

The equation for the portfolio UL as a function of systematic, non-systematic and spread risk components is:

$$UL_{TOTAL} = \sqrt{\left(\left(\left(\sqrt{\sum_i \sum_j UL_{Syst_i} \times UL_{Syst_j} \times \rho_{ij} - \sum_i (UL_{Syst_i})^2}\right) + \sum_i UL_{Spread_i}\right)^2 + \sum_i (UL_{Syst_i})^2 + \sum_i (UL_{NonSyst_i})^2\right)}$$

where UL_{Syst_i} is equal to the systematic portion of risk for industry i, $UL_{NonSyst_i}$ is the non-systematic and hence idiosyncratic portion of risk for industry i, UL_{Spread_i} is the spread risk for industry i, and $\rho_{i,j}$ is the correlation between industries i and j.

Table 2-32 – Sub-portfolio Correlations

	CorpBonds	GovtBonds	MBS	CreditIns	SuretyIns
CorpBonds	1	0.0	0.5	0.5	0.5
GovtBonds	0.0	1	0.0	0.0	0.0
MBS	0.5	0.0	1	0.5	0.5
CreditIns	0.5	0.0	0.5	1	0.5
SuretyIns	0.5	0.0	0.5	0.5	1

Since systematic risk is perfectly correlated within an industry, UL_{Syst_i} is computed as $\sum_{k \in i} UL_{Syst_k}$ where k is a subportfolio of type i. Since idiosyncratic risk is uncorrelated between all loans, $UL_{NonSyst_i}$ is computed as $\sqrt{\sum_{k \in i} UL_{NonSyst_k}^{2}}$ where k is a subportfolio of type i. Since spread risk is perfectly correlated between all loans, UL_{Spread_i} is computed as $\sum_{k \in i} UL_{Spread_k}$ where k is a subportfolio of type i.

Non-systematic risk and systematic risk are by definition independent. Systematic risk and spread risk are assumed to be perfectly correlated and therefore additive. A correlation matrix between the subportfolios is required to capture the diversification effects of being exposed to different industries/geographies. Since only systematic volatility between subportfolio types is correlated, the total UL for the entire portfolio is a function of the independent nonsystematic volatilities (assuming they are independent of all other volatility), and correlated systematic volatilies and credit spread risk (assuming that they are correlated according to the correlation matrix and perfectly correlated to the credit spread risk).

Table 2-33 – Sub-portfolio Level Results

	CorpBonds	GovtBonds	MBS	CreditIns	SuretyIns
Loan count	162	180	153	4,572	8,535
Exposure	1,225,000,000	832,500,000	592,000,000	875,000,000	1,603,000,000
Expected Loss	1,885,900	33,450	11,840	787,500	7,453,950
Unexpected Loss - Systematic	3,793,515	17,775	53,296	1,938,542	13,219,292
Unexpected Loss - Idiosyncratic	3,473,965	532,212	237,163	807,546	1,667,388
Unexpected Loss - Spread	3,507,774	51,848	55,056	244,125	2,310,725
Total Unexpected Loss	8,085,620	536,746	260,742	2,327,265	15,619,270

Table 2-34 – Portfolio Level Results

	Total	As a % of Exposure
Expected Loss	10,172,640	0.1984%
Unexpected Loss - Systematic	19,022,419	0.3710%
Unexpected Loss - Idiosyncratic	3,979,980	0.0776%
Unexpected Loss - Spread	6,169,527	0.1203%
Total Unexpected Loss	21,074,259	0.4110%

2.5.7 Credit Loss Distribution

The final step of defining the Credit Loss Distribution is to assign a functional form to fit the characteristics of the distribution given the mean (EL) and standard deviation (UL). While there are several different ways to do this, the specific assumptions underlying our model lead to a natural choice. Because default is modelled as Bernoulli, the sum of a correlated portfolio of loans follows a Beta distribution. In mathematical terms, Beta is the continuous approximation to the distribution for a sum of Bernoulli random variables. While similar to the Gamma distribution, it is preferred because it does not allow firms to default repeatedly without curing. Between 0 and 1, the Beta distribution has a probability density function:

$$\beta(x,\alpha,\beta)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\cdot\Gamma(\beta)}\cdot x^{\alpha-1}(1-x)^{\beta-1}\ \text{(Ong 165)}$$

where $\Gamma(z)=\int_0^\infty t^z e^{-t}dt$

The mean (EL) and standard deviation (UL) of the beta distribution, as a percent of exposure, can be solved through integration:

$$\text{Mean} = \text{EL\%} \quad = \quad \int_0^1 x \cdot \beta(x;\alpha,\beta)\,dx$$

$$= \quad \frac{\alpha}{\alpha+\beta}\ \text{(Ong 166)}$$

$$\text{Variance} = \text{UL\%}^2 \quad = \quad \int_0^1 x^2 \cdot \beta(x;\alpha,\beta)\,dx - \%EL^2$$

$$= \quad \left[\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}\right]^2\ \text{(Ong 166)}$$

Rearranging for α and β

$$\alpha = (1 - EL\%) \left(\frac{EL\%}{UL\%}\right)^2 - EL\%$$

$$\beta = \frac{\alpha}{EL\%} - \alpha$$

The parameters $\alpha = 0.23$ and $\beta = 115.98$ can be used to generate the Credit Loss Distribution, which we translate into a value distribution.

Table 2-35 – Credit Risk Change in Value Distribution

Probability	Value
0.001%	-350,343,848
0.010%	-262,402,716
0.030%	-220,935,812
0.050%	-201,767,149
0.070%	-189,209,719
:	:
:	:
99.930%	10,183,496
99.950%	10,183,496
99.970%	10,183,496
99.990%	10,183,496
99.999%	10,183,496

2.6 Market Risk

Market risk is the risk associated with changes in the value of an investment portfolio or foreign exchange positions to market fluctuation. Market positions, henceforth called "sub-portfolios", are characterized by their current value and their β and tracking error relative to well-known market indices or individual securities. The potential for loss to tradable financial instruments resulting from unfavorable market movements is quantified by using a parametric model to calculate the Value at Risk (VaR) of the total investment portfolio. (Crouhy 198)

Table 2-36 – Tracking Indices

Tracking Index	Index Name	Volatility
SPX	S&P 500 Index	0.1986
BBREIT	Bloomberg REIT Index	0.1023
DH1	Direct Holding 1	0.2000
DH2	Direct Holding 2	0.2500

Table 2-37 – Investment Sub-portfolios

Subportfolio	Exposure	Tracking Index	β	Tracking Error
Equity Portfolio 1	100,000,000	SPX	1.05	3.2%
Equity Portfolio 2	50,000,000	SPX	1.10	5.0%
Direct Holding 1	50,000,000	DH1	1.00	0.0%
Direct Holding 2	50,000,000	DH2	1.00	0.0%
Real Estate	100,000,000	BBREIT	1.03	3.0%

2.6.1 Systematic and idiosyncratic risk

The volatility of each sub-portfolio's value is calculated in terms of a tracking index used as a benchmark. Once the amount of exposure in each index is determined, the systematic risk (due to the underlying movement of the index) and the idiosyncratic risk (due to the tracking error of the portfolio versus the index) are calculated.

Systematic risk is the volatility in the portfolio that arises from the fluctuations in the value of the underlying indices that the sub-portfolios are tracking. Systematic risk is calculated aggregating the β-weighted market values by index and calculating the total covariance:

$$\sigma_{Systematic} = \sqrt{\sum_{i,j} \beta_i \cdot MV_i \cdot \sigma_i^{Index} \cdot \rho_{i,j} \cdot \sigma_j^{Index} \cdot \beta_j \cdot MV_j}$$

Table 2-38 – Systematic Market Risk by Index

Index	β-weighted Exposure	Volatility	Systematic Risk
SPX	160,000,000	0.1986	31,776,000
BBREIT	103,000,000	0.1023	10,536,900
DH1	50,000,000	0.2000	10,000,000
DH2	50,000,000	0.2500	12,500,000
Total	363,000,000		41,971,271

Idiosyncratic risk is calculated by assuming independence across the idiosyncratic risks of each sub-portfolio:

$$\sigma_{Idiosyncratic} = \sqrt{\sum_i MV_i^2 \cdot TrackingError_i^2} ,$$

where MV_i is the market value of the position, and the index i represents sub-portfolios. Note that conventionally, tracking error is derived from the r^2 statistic (i.e. unexplained variance) obtained from a linear-regression of the sub-portfolio against its index.

Table 2-39 – Idiosyncratic Market Risk by Sub-portfolio

Sub-portfolio	Market Value	Tracking Error	Idiosyncratic Risk
Equity Portfolio 1	100,000,000	3.2%	3,200,000
Equity Portfolio 2	50,000,000	5.0%	2,500,000
Direct Holding 1	50,000,000	0.0%	0
Direct Holding 2	50,000,000	0.0%	0
Real Estate	100,000,000	3.0%	3,000,000
Total	350,000,000		5,048,762

Finally, we need to combine the idiosyncratic risk and the systematic risk, assuming independence between the two:

$$\sigma_{Total} = [\sigma_{Idiosyncratic}{}^2 + \sigma_{Systematic}{}^2]^{1/2}$$

Table 2-40 – Portfolio Level Results

Total Systematic Risk	41,971,271
Total Idiosyncratic Risk	5,048,762
Total Risk	42,273,841

While it is accepted that the return on an individual security typically follows a log-normal distribution, there is some debate over whether a normal or lognormal distribution is appropriate for the value of a diversified portfolio. In this instance, we fit a normal distribution to the total volatility of $42,273,841.

Table 2-41 – Market Risk Value Distribution

Probability	Value
0.001%	-180,317,267
0.010%	-157,236,263
0.030%	-145,080,596
0.050%	-139,101,189
0.070%	-135,053,401
:	:
:	:
99.930%	135,053,401
99.950%	139,101,189
99.970%	145,080,596
99.990%	157,236,263
99.999%	180,317,267

Allocation of market risk capital to all of the activities that generate market risk is done using their contribution to total covariance.

2.7 Operating Risk

Operating Risk is used here to refer to the non-financial risks that arise in the course of running a business. Non-financial risks can be divided into two categories: event risks, which are one-off incidents that can cause large losses, and business risks, which are the risks associated with business decisions which relied on the wrong assumptions. Event Risk includes losses from systems failure, errors & omissions, fraud, uninsured damage to plant and equipment, and the impact these events have on customer behavior. Business Risk includes losses due to changes in the competitive environment or events that damage the franchise or operating economics of a business. Business Risk impacts the company through variation in volume, pricing, or costs.

An analog approach is used to quantify operating risk capital. The capital of analog non-financial companies is used as a proxy for their operational risk. "Pure-play" analog companies that have business processes subject to specific operating risks also faced by financial institutions were selected. Because these companies do not have significant financial risks, their economic capital supports only operating risk. These institutions' level of capital, along with their credit quality, yields an inferred estimate of the level of risk they face. Because these companies are more transparent, there is direct discipline from markets and rating agencies with respect to the amount of capital that they hold. We assume that these capital levels should be roughly equivalent to the levels of operating risk capital in similar business units of financial institutions.

2.7.1 Analogs

Table 2-42 describes each analog group and gives examples of companies in each analog.

Table 2-42 – Description of Operating Risk Analog Types

Analog Type	Description	Examples
Retail Services	• Fee-based services to consumers • High fixed costs due to many outlets • Low elasticity of demand	• Auto rental • Hair salons • Travel agencies
Business-to-Business Services	• Long-term relationships • No inventory • Low fixed costs	• Insurance brokers • Advertising agencies
Data Processing	• Process and track data, records, payments, etc. • Heavy investment in fixed cost systems, plant, and personnel	• ADP • EDS • Fiserv • First Data
Broker/ Dealer	• Transaction-based earnings from market-making and customer fees • Analogs selected have limited proprietary risk-taking activity	• PaineWebber • Legg Mason
Corporate Trust	• Performs similar roles as data processing companies but with added fiduciary responsibilities	• Northern Trust • U.S. Trust

2.7.2 Non-interest expense

The scale factor for calculating Operating Risk is an institution's non-interest expense (NIE). NIE has the benefit that it is the only common measure of size and scope between financial and non-financial companies. Therefore, for each analog group, we determined the Capital / NIE Multiplier adjusted for credit rating. We then apply these multipliers to financial institutions based on how their business units are divided between the five analogs' business lines.

Table 2-43 – Line of Business Contributions

Line of Business	NIE	Retail	B to B	D.P.	B/D	C.T.	Contribution to σ
Homeowners	30,000,000	0%	30%	70%	0%	0%	4,650,000
General Liability	15,000,000	0%	30%	70%	0%	0%	2,325,000
Credit & Surety	20,000,000	0%	30%	70%	0%	0%	3,100,000
Term Life	15,000,000	0%	40%	60%	0%	0%	2,250,000
SCA	7,500,000	0%	40%	60%	0%	0%	1,125,000
Total	87,500,000						13,450,000

Table 2-44 – Operating Risk Value Distribution

Probability	Value
0.001%	-57,370,402
0.010%	-50,026,865
0.030%	-46,159,374
0.050%	-44,256,944
0.070%	-42,969,084
:	:
:	:
99.930%	42,969,084
99.950%	44,256,944
99.970%	46,159,374
99.990%	50,026,865
99.999%	57,370,402

2.8 Risk Aggregation

In order to measure overall capital adequacy and derive accurate capital contributions, the "total" risk that an institution faces must be computed from the value distributions that describe its component risks. Since the underlying risk distributions for each risk type do not necessarily follow a particular distributional form (e.g. property catastrophe risk is frequently an empirical distribution), it is necessary to do a numerical integration or simulation in order to combine them. The method described here uses a numerical integration approach to "convolve" the underlying linearly correlated risk distributions.

The distribution aggregation method takes N distributions, specified as discrete cumulative density functions (i.e. a set of tables listing possible losses due to credit, market risk, etc. with the associated probabilities for exceeding that loss). To avoid simulation, the problem is parceled into a series of two-distribution convolutions, the result of each one subsequently convolved with the next input distribution, i.e. a "pair-wise roll-up" (e.g. aggregating market risk and credit risk, and then aggregating the resulting distribution with operating risk). This scheme generalizes to as many distributions as are desired with approximately linear cost in computational intensity, as opposed to multi-dimensional calculations or simulations that are exponential in computational cost.

Figure 2-3 – Tabular Discrete Density Functions in the Two-Distribution Case

$$A:\begin{bmatrix} x_1 & prob(x_1) \\ x_2 & prob(x_2) \\ \vdots & \vdots \\ x_{m-1} & prob(x_{m-1}) \\ x_m & prob(x_m) \end{bmatrix} \text{ and } B:\begin{bmatrix} y_1 & prob(y_1) \\ y_2 & prob(y_2) \\ \vdots & \vdots \\ y_{m-1} & prob(y_{m-1}) \\ y_m & prob(y_m) \end{bmatrix}$$

Convolution requires an assumption as to the form of the copula (the joint probability density function for the set of outcomes from multiple random variables, with each variable's outcome expressed in terms of it's marginal cumulative density function). The method assumes a multivariate normal copula[4].

The method for aggregating two distributions consists of first converting the input distributions to "Normal space" (using the cumulative density function) and using the bivariate normal density function to compute the probabilities for each possible combination of losses. This yields the desired resulting cumulative density function for the aggregate distribution (after sorting by loss and cumulating probability mass):

$$Z=\begin{bmatrix} z_{1,1} & prob(z_{1,1}) \\ z_{1,2} & prob(z_{1,2}) \\ \vdots & \vdots \\ z_{1,m-1} & prob(z_{1,m-1}) \\ z_{1,m} & prob(z_{1,m}) \\ z_{2,1} & prob(z_{2,1}) \\ z_{2,2} & prob(z_{2,2}) \\ \vdots & \vdots \\ z_{m,m-1} & prob(z_{m,m-1}) \\ z_{m,m} & prob(z_{m,m}) \end{bmatrix}$$

where $z_{i,j} = \dfrac{x_i + x_{i+1}}{2} + \dfrac{y_j + y_{j+1}}{2}$ and

[4] see Wang, "Aggregation of Correlated Risk Portfolios", Proceedings of the Casualty Actuarial Society, Volume LXXXV, Number 163, Page 887

$$prob\left(z_{i,j}\right)=\int_{F_{norm}^{-1}(prob(x_i))}^{F_{norm}^{-1}(prob(x_{i+1}))}\int_{F_{norm}^{-1}(prob(y_j))}^{F_{norm}^{-1}(prob(y_{j+1}))} f\left(x,y,\rho_{norm}\right)dx\,dy.$$

F_{norm}^{-1} is the inverse normal cumulative density function and $f(x,y,\rho_{norm})$ is the bivariate normal function defined by:

$$f\left(x,y,\rho_{norm}\right)=\frac{1}{2\pi\sqrt{1-\rho_{norm}^2}}\times e^{-\frac{1}{2}\left(\frac{x^2-2\rho_{norm}xy+y^2}{1-\rho_{norm}^2}\right)}$$

ρ_{norm} is calculated iteratively such that the equivalent correlation ρ_{equ}, defined as

$$\rho_{equ}=\frac{\left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f\left(x,y,\rho_{norm}\right)\times F_A^{-1}\left(F_{norm}(x)\right)\times F_B^{-1}\left(F_{norm}(y)\right)\right)-\mu_A\times\mu_B}{\sigma_A\times\sigma_B},$$

is approximately equal to ρ , the input correlation between distributions A and B, where $f(x,y,\rho_{norm})$ is the bivariate normal probability density function and F_{norm} is the standard normal cumulative density function.

Note that because the combined distributions are described by discrete cumulative density functions of m elements, and the algorithm evaluates each possible combination, the resulting convolution will be a tabulated cumulative density function containing approximately m^2 elements. To keep the subsequent calculation tractable, the result must be reduced in size by mapping to the given probability schedule; this is done with standard linear interpolation. Finally, this process is repeated, convolving the new distribution Z with the next input distribution and so on.

The total diversified economic capital is found by looking up the desired solvency standard on the aggregate distribution, then subtracting the mean of the distribution from the loss value.

Table 2-45 – Overall Risk Value Distribution

Probability	Value
0.001%	-580,173,921
0.010%	-522,774,051
0.030%	-464,762,707
0.050%	-430,156,396
0.070%	-411,064,239
:	:
:	:
99.930%	324,707,743
99.950%	334,632,692
99.970%	349,823,731
99.990%	378,717,198
99.999%	416,390,625

The mean of the above distribution is: -6,790,062.

Table 2-46 – Overall Risk Value Distribution

EDF	Required Economic Capital
0.01%	-515,983,989
0.03%	-457,972,646
0.07%	-404,274,177

2.9 Capital Allocation

The total diversified economic capital value must be attributed to the different risk types. Contributory capital for each risk type is calculated with a covariance and excess-skewness approach.

Let EC be the total economic capital at the desired solvency standard S and $NormEC$ be the equivalent normal economic capital for the output distribution:

$$EC = F^{-1}(1-S) - \mu$$

$$NormEC = F_{Norm}^{-1}(1-S,\sigma)$$

$F^{-1}(1-S)$ is the inverse of the output distribution at the desired solvency standard, $F_{NORM}{}^{-1}(1-S,\sigma)$ is the inverse normal function, μ is the mean of the output distribution and σ is the standard deviation. *SkewEC*, the portion of economic capital that is due to shape (skewness), is defined as:

$$SkewEC = EC - NormEC$$

Let EC_i be the contributory economic capital of the *i*th input distribution. Let SAC_i be the stand-alone capital of the *i*th distribution. This is defined as:

$$SAC_i = F_i^{-1}(1-S) - \mu_i$$

where μ_i is the mean of the *i*th input distribution, and $F_i^{-1}(1-S)$ is the inverse of the *i*th distribution at the desired solvency standard. Let $NormSAC_i$ be the equivalent stand-alone normal economic capital for distribution *i*:

$$NormSAC_i = F_{norm}^{-1}(1-S,\sigma_i)$$

where σ_i is the standard deviation of distribution *i* and $F_{norm}^{-1}(1-S,\sigma_i)$ is the inverse normal function evaluated at the desired solvency standard S. Let $SkewSAC_i$ be the portion of stand-alone capital for distribution *i* that is due to shape (skewness), defined as:

$$SkewSAC_i = SAC_i - NormSAC_i$$

Contributory capital for distribution i is calculated as:

$$EC_i = NormEC \times \left(\frac{\sigma_i \sum_{j=1}^{n} \sigma_j \times \rho_{ij}}{\sum_{k=1}^{n} \sum_{j=1}^{n} \sigma_k \times \sigma_j \times \rho_{kj}} \right) + SkewEC \times \left(\frac{SkewSAC_i \times \sum_{j=1}^{n} SkewSAC_j \times \rho_{ij}}{\sum_{k=1}^{n} \sum_{j=1}^{n} SkewSAC_k \times SkewSAC_j \times \rho_{kj}} \right)$$

where n is the number of input distributions and ρ_{ij} is the Pearson correlation coefficient between distributions i and j.

Table 2-47 – Risk Pillar Correlations

	NonCat	Cat	Life	ALM	Credit	Market	Operating
NonCat	1	0.0	0.0	0.2	0.0	0.0	0.2
Cat	0.0	1	0.2	0.0	0.0	0.0	0.2
Life	0.0	0.2	1	0.0	0.0	0.0	0.2
ALM	0.2	0.0	0.0	1	0.3	0.2	0.2
Credit	0.0	0.0	0.0	0.3	1	0.2	0.2
Market	0.0	0.0	0.0	0.2	0.2	1	0.2
Operating	0.2	0.2	0.2	0.2	0.2	0.2	1

Table 2-48 – Capital Allocation to Risk Types

Risk Type	μ	σ	SAC_i	$NormSAC_i$	$SkewSAC_i$	EC_i	Allocation
Credit	0	21,082,000	-220,935,812	-72,351,814	-148,583,998	-84,949,758	19%
Market	0	42,279,492	-145,080,596	-145,099,991	19,395	-78,817,041	17%
NonCat	1,453	45,822,431	-197,121,120	-157,259,086	-39,862,034	-88,335,717	19%
ALM	-6,730,902	51,350,562	-179,024,171	-176,231,209	-2,792,962	-125,935,017	27%
Operating	0	13,451,798	-46,159,374	-46,165,544	6,171	-22,612,131	5%
Cat	0	19,853,644	-163,190,513	-68,136,192	-95,054,322	-38,943,801	9%
Life	0	20,423,027	-83,398,815	-70,090,270	-13,308,545	-18,379,180	4%
Total						-457,972,646	100%

3. RETURN QUANTIFICATION

Risk-Adjusted Return on Capital (RAROC) is the metric used to quantify the level of performance of line of business. For a given line of business, RAROC is defined as the following:

$$RAROC = \frac{UW + IC + CB}{EC}$$

where *UW* represents the calendar year underwriting result, *IC* is the investment credit, *CB* is the capital benefit and *EC* is the economic capital. RAROC can be computed either on a pre- or post-tax basis, with the components of the quotient adjusted accordingly. In all cases, economic capital in both instances must be measured on a contributory basis.

Frequently, economic capital is not equal to actual available capital. While RAROC is the return on equity, (ROE), that would result from holding an amount of capital equal to economic capital, under-capitalized companies have inflated ROE, while overcapitalized companies usually have a depressed ROE, except where the internal transfer rate on invested surplus is in excess of company-wide RAROC.

3.1 Calculation and Allocation of Investment Returns and Capital Benefit

Insurance lines of business generate reserves and surplus that earn an investment return. A portion or all of this return should be allocated back to the business that supplies the funds, as the reserves and surplus are on deposit with the investments unit. While there is a spectrum of opinion between allocating a risk-free rate of return or the entire investment return, the RAROC approach typically involves setting an internal cost of funds for the total amount supplied by the business.

The internal cost of funds rate should reflect a fair return for an investment that bears no credit, market or interest-rate risk. It should also reflect a premium for a guarantee of liquidity in the case of a sudden need to pay a large claim. Along with this, credit, market and interest-rate risk are managed by the investments unit, generating a need for economic capital. Investment returns in excess of the cost of funds are retained by the investment manager.

The total investment return is calculated as the sum of realized and unrealized gains, investment income, dividends, less expenses. The risk-adjusted income for the investments unit is the total investment return less the product of the cost-of-funds rate and the total invested assets. Because the investment credit and capital benefit reflect an internal transfer, the amount subtracted from the investment unit's return should be equal to the total added to the total investment credit and capital benefit allocated to the insurance lines of business.

Table 3-1 – Income Statement and Required Economic Capital by Line of Business

Lines	NEP	Other Revenue	Losses	Expenses	Reserves	Economic Capital
Homeowners	150,000,000		-97,500,000	-50,000,000	150,000,000	121,535,269
General Liability	90,000,000		-67,500,000	-30,000,000	250,000,000	17,470,615
Credit & Surety	115,000,000		-90,000,000	-38,000,000	175,000,000	62,470,270
Term-Life	100,000,000		-80,000,000	-30,000,000	200,000,000	21,603,260
SCA	45,000,000		-35,000,000	-15,000,000	200,000,000	32,210,652
Investments		100,000,000				202,682,580
Econ. Capital	500,000,000	100,000,000	-370,000,000	-163,000,000	975,000,000	457,972,646
Excess Capital						117,027,354
Total	500,000,000	100,000,000	-370,000,000	-163,000,000	975,000,000	575,000,000

3.2 Adjusting the Underwriting Result

The calendar year underwriting result can be adjusted to bring it closer to a true economic view of profitability. Specific adjustments are made to remove development in reserves for past accident years, allocate overhead expenses and reverse one-time special charges:

$$AdjUW = UW - \Delta Reserves + Overhead - One\text{-}Time\ Charges$$

Subtracting the change in reserves due to reassessment of prior accident years removes the "misdeeds of the past" to produce better forward-looking figures. Adding in corporate overhead ensures that the result uses "fully-loaded" expenses - it is not unheard of for a new business to launch a "profitable" product but manage to lose money every year. There are many theories of how to allocate corporate overhead, however we have found that the process can be contentious as it can affect P&L statements. Nevertheless, typical methods involve sizing the benefit received by each line of business from each cost center. Finally, true one-time charges are removed.

Table 3-2 – Return Adjustments and RAROC Calculations

Lines	UW Result	Investment Credit	Capital Benefit	Adjusted UW Result	RAROC	Tax Rate	Post-Tax Adjusted UW Result	Post-Tax RAROC
Homeowners	2,500,000	7,005,000	5,675,697	15,180,697	12%	35%	9,867,453	8%
General Liability	-7,500,000	11,675,000	815,878	4,990,878	29%	35%	3,244,071	19%
Credit & Surety	-13,000,000	8,172,500	2,917,362	-1,910,138	-3%	35%	-1,241,590	-2%
Term-Life	-10,000,000	9,340,000	1,008,872	348,872	2%	35%	226,767	1%
SCA	-5,000,000	9,340,000	1,504,237	5,844,237	18%	35%	3,798,754	12%
Investments	100,000,000	-72,385,000	9,465,276	37,080,276	18%	35%	24,102,180	12%
Econ Capital	67,000,000	-26,852,500	21,387,323	61,534,823	13%		39,997,635	9%
Excess Capital			5,465,177	5,465,177	5%	35%	3,552,365	3%
Total	67,000,000	-26,852,500	26,852,500	67,000,000	12%	35%	43,550,000	8%

4. EVALUATION OF RISK-ADJUSTED RETURN ON CAPITAL

4.1 Alternative Views

There are two different views of economic capital and RAROC in an insurance context. The first is "Calendar Year" RAROC, which is the approach taken in this paper. Calendar Year RAROC looks at the risk and return of a company's full balance sheet over the course of the next calendar year. The second view is "Accident Year" RAROC, which examines the lifetime risk and return of new business put on in the coming year.

4.1.1 Calendar Year RAROC

Calendar Year RAROC is the 'standard' approach to measuring risk and return, and was the method outlined in this paper.

4.1.2 Accident Year RAROC

Accident Year RAROC is an alternative to the Calendar Year approach. Rather than considering all business that has been written in the past - and therefore can't be changed - the Accident Year view focuses only on the risk and lifetime value embedded in new business.

In practice, the calculation of Accident Year RAROC for Non-Cat risk is very similar to the method outlined in this paper. It is equivalent to hypothesizing that the current accident year represents the firm's steady state; that is, all previous years are identical to the current in both volume and division between business units.

The computation of economic capital differs only in equation 39, where each ULE_i is just the initial loss estimate for the accident year in question.

The computation of risk-adjusted return is similar to the Calendar Year approach as well. The only difference is in the calculation of investment credit on reserves. Let R_i denote the expected reserve for the current year's contracts in development year i. The total reserve R for crediting investment returns is:

$$R = \sum_{i=1}^{\infty} R_i \qquad (42)$$

The total reserve R is credited at the firm's cost of funds. This can be interpreted as crediting the specified accident year with all internal transfer income that will be accrued over the course of the contracts' lives.

The Accident Year vs. Calendar Year distinction is largely applicable only to P&C Non-Catastrophe risks. Other risk pillars, such as Credit, Market and Operating risks have no comparable notion of 'tenure'. Depending on the intended application, however, it may make sense in context to change the assumptions relating to these risks (e.g. amount of invested assets) to mirror the 'steady-state' view of the Accident Year Non-Catastrophe risk calculation.

4.1.3 Comparison

Both the Accident Year and Calendar Year approaches have important uses and interpretations. Accident Year RAROC is a measure of the lifetime value of new business. The true value of long-tailed insurance contracts is highly dependent on investment returns on reserves earned over the very long term; long-tailed lines can look extremely unprofitable during periods of growth as high loss ratios dominate relatively small levels of reserves (and therefore investment returns), even if the expected long-term profit is very high. Accident Year RAROC credits new business with this long-term income to clarify the tradeoff between underwriting profit and investment returns. Therefore, Accident Year RAROC is most useful for applications such as setting pricing targets and performing strategic planning.

In contrast, Calendar Year RAROC is a measure of realistic expected shareholder returns over a one-year period. It is the metric that is most closely comparable to budgeted financials. For this reason, it is the more useful measure for performance assessment and shareholder communication. Also, the Calendar Year methodology should be used for determining capital adequacy. It measures a company's true capital requirements in the short term. The Accident Year methodology captures the cumulative lifetime capital requirement of new business, which is not truly actionable in any reasonable manner.

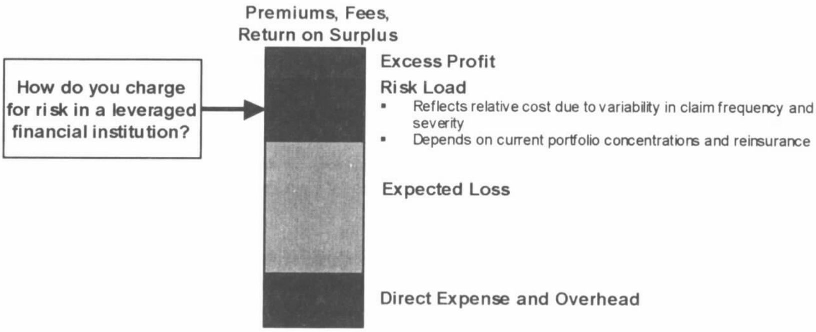
4.2 Other Applications

The RAROC framework lends itself to several applications including pricing, risk transfer evaluation and mergers & acquisitions analyses. In the pricing framework, economic capital and the hurdle ROE set the cost of risk that must be offset by the risk load. For evaluating the performance of reinsurance for risk transfer, RAROC is an effective risk-return metric that can be used to compare the efficiency of reinsurance across dissimilar lines of business. For M&A, RAROC enables a quick and straightforward calculation of the value of the potential target within the context of the acquirer's business portfolio. RAROC's versatility is a very compelling factor that is driving the adoption of the framework.

4.2.1 Risk-Based Pricing

The pricing cycle is an inevitable outcome of a pricing strategy that relies heavily upon observed market price-points rather than economic risk-based pricing. When capacity is plentiful, prices are reduced relative to the competition, trading current profitability for market share. Shareholder value destruction is a frequent result of this behavior, compounded by the rarity of explicit calculations of the economics of this trade. However, we can look to the banking industry for a way to escape this cycle. The answer is to know the economic break-even point by computing an appropriate risk load based on a hurdle RAROC that can be determined from market analysis and CAPM theory, and on the capital required to support the marginal risk of new business.

Figure 4-1 – The components of risk-based pricing



For example, by setting a 12% hurdle RAROC, prices that result in a lower return can be considered to destroy shareholder value, while prices that result in "Excess Profit" as shown above create shareholder value. Armed with this information, company management can assess the strategic value of market share initiatives relative to the near-term value destruction of ultra-competitive pricing. If uses for the excess capacity are found to be value destroying, management can and in many cases should decide to return that capital to shareholders for investment in other opportunities.

4.2.2 Reinsurance and M&A Evaluation

Evaluating reinsurance is never an easy task, but choosing between two programs in different business areas is a challenge that has proven elusive. Consider the case where the choice is between buying treaty reinsurance for a General Liability portfolio versus buying treaty reinsurance for a D&O Liability portfolio. The hypothetical company has a pre-treaty RAROC of 15% on $100 million in Economic Capital, with a hurdle rate of return of 15%.

Example Treaty	Risk-Adjusted Return	Economic Capital	RAROC	Intrinsic Value	Shareholder-Value Added
Gross	$ 15.0 MM	$ 100 MM	15.0%	$ 100.0 MM	$ 0.0 MM
10 x 10 on D&O	$ 14.0 MM	$ 90 MM	15.6%	$ 95.0 MM	$ 5.0 MM
15 x 5 on GL	$ 14.5 MM	$ 95 MM	15.3%	$ 97.5 MM	$ 2.5 MM

The D&O program results in higher RAROC and shareholder value creation despite the greater reduction in Risk-Adjusted Return. While this would technically "shrink" the business, it is more valuable than the alternatives

Evaluating Mergers & Acquisitions would involve a similar framework of computing the net reduction in total Economic Capital of the combined entity relative to the two standalone entities, and calculating the shareholder value creation for the acquirer.

5. CONCLUSION

5.1 Strategic Recommendations

The case study company is under-performing as its current RAROC of 9% is well below the hurdle, or target, return on capital of 15%. However, a glimmer of hope exists in the 19% RAROC posted by the General Liability business. Because General Liability consumes only 4% of total economic capital, there is room to grow the business without worrying about excess concentration risk. Conversely, Credit & Surety, which accounts for 14% of total economic capital, should be reigned in until its profitability can be addressed through a risk-based pricing initative, as described in section 4.2.1.

Additionally, we see in Table 3-1 that the company is overcapitalized by $117 MM, or about 25% ($117 MM/ $575 MM in total capital). This drags the actual ROE down from 9%, were it adequately capitalized, to 8% in its overcapitalized state. Note that only the additional investment return on the excess capital prevents the ROE from dropping even further. For this particular company, capital could be redeployed in the following ways:

- Redeploy capital from Credit & Surety to General Liability
- Return capital to shareholders via share buyback or increased dividends
- Expansion into new businesses that earn an adequate return

Table 5-1

Line of Business	Economic Capital (EC)	% of Total EC	Post-Tax RAROC
Homeowners	121,535,269	27%	8%
General Liability	17,470,615	4%	19%
Credit & Surety	62,470,270	14%	-2%
Term-Life	21,603,260	5%	1%
SCA	32,210,652	7%	12%
Investments	202,682,580	44%	12%
Total Economic	457,972,646	100%	9%

As these recommendations demonstrate, the Economic Capital and RAROC framework are designed around supporting specific decisions and strategic insights. The philosophy is to produce best results possible in a timely fashion, but with neither "perfect" accuracy nor excruciating detail. It is not intended to generate stochastic multi-year financial projections, set reserve requirements or model the particulars of a specific complex insurance policy. The adoption of RAROC as an industry standard in banking was predicated upon its ability to accommodate diverse risk types and businesses. RAROC's ease of use and cross-industry capabilities make it an emerging presence in the insurance industry.

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